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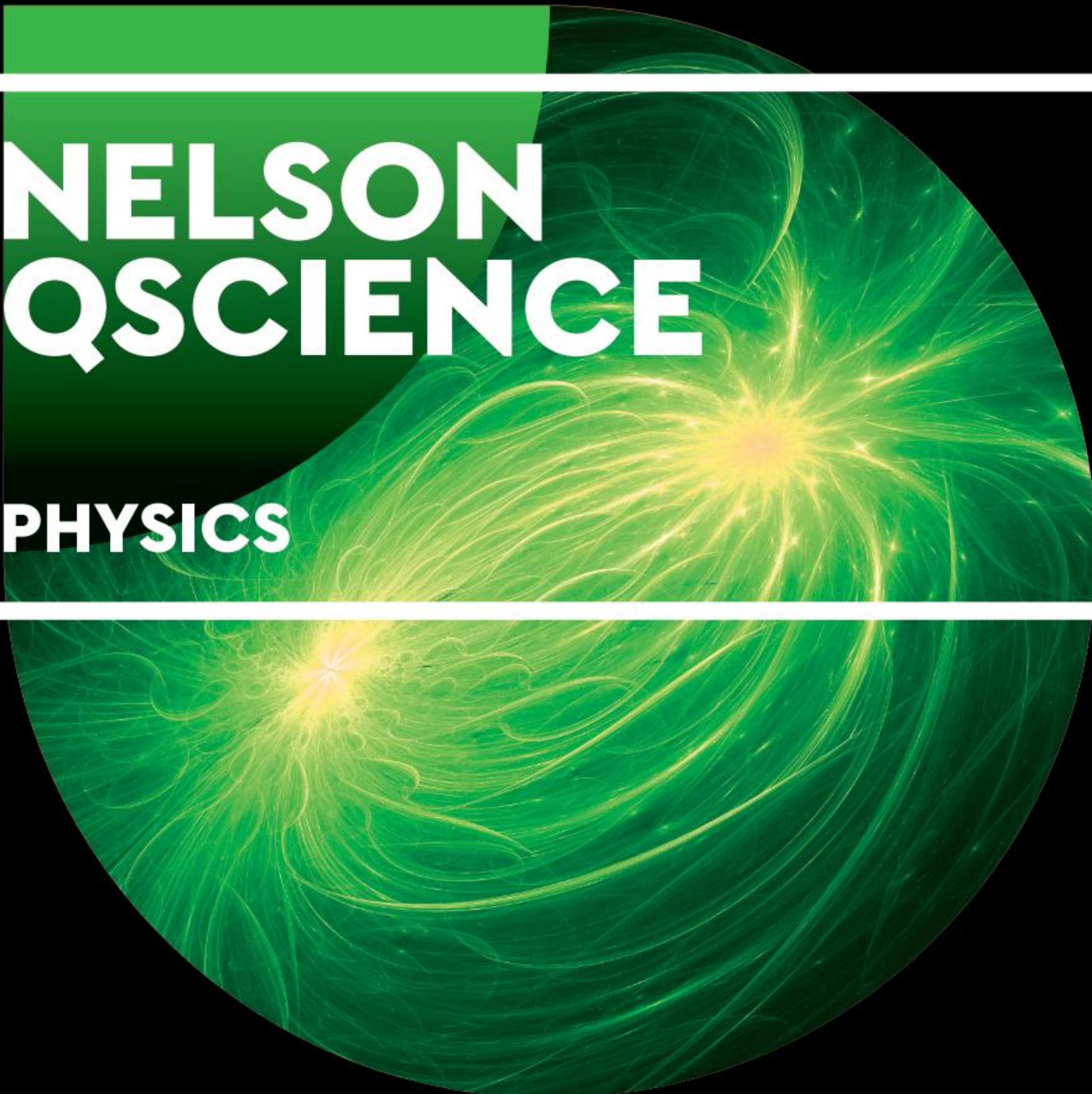

UNITS 3 4

Scott Adamson
Oliver Alini
Neil Champion
Tara Kuhn

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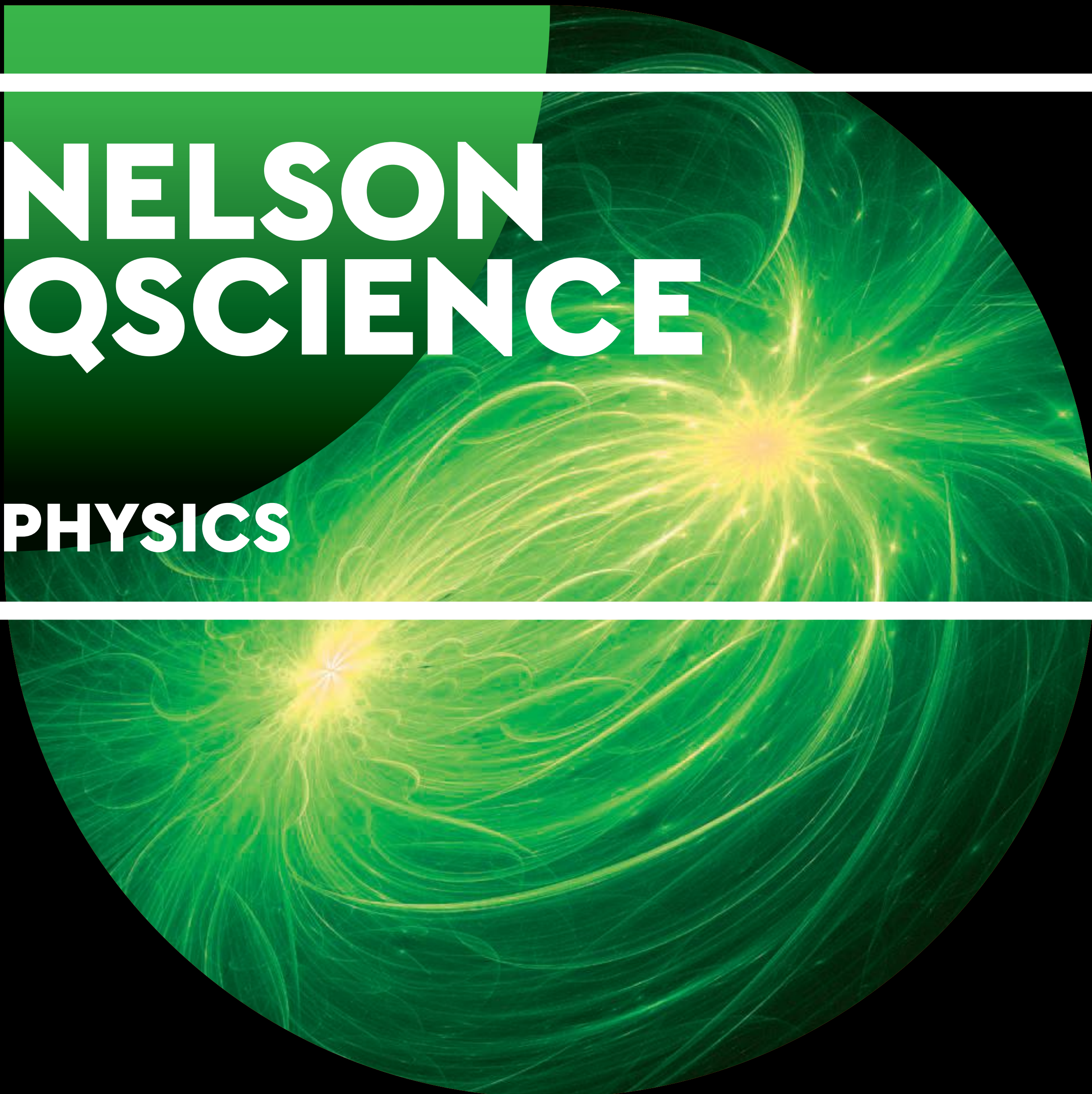

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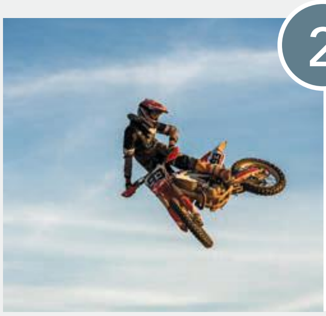

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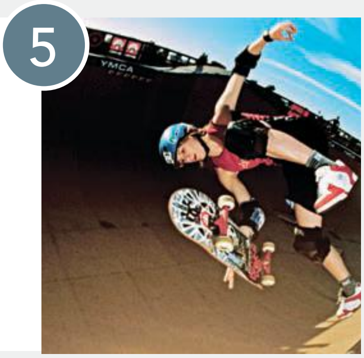

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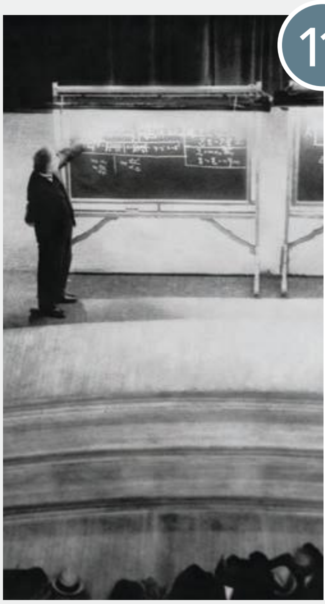

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PREFACE

Nelson QScience Physics Units 3 & 4 has been written to meet the requirements of the QCAA Senior Secondary Science Syllabus – Physics. Each page has been carefully considered to provide students with all of the information they need to meet the content and skills requirements of the new syllabus.

With the introduction of the QCE external examination, *Nelson QScience Physics* includes features such as practice exams at the end of each section, a Units 3 & 4 practice examination, chapter quizzes (available on NelsonNet) and ExamView (available on NelsonNet).

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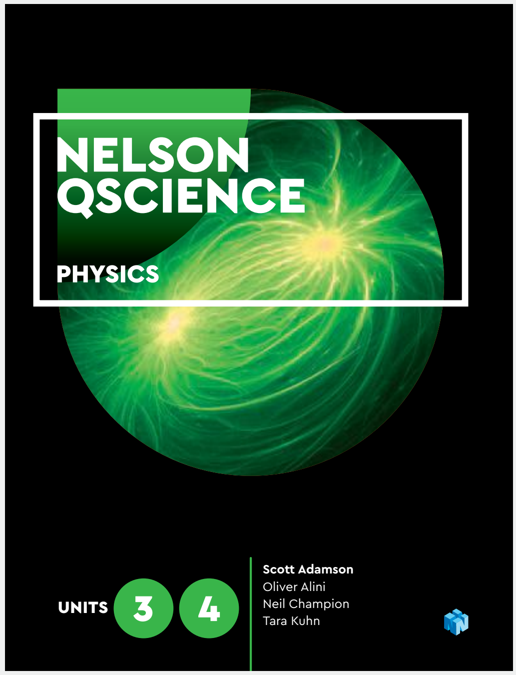

AUTHORS AND REVIEWER TEAM

Nelson QScience Physics Units 3 & 4 has been adapted from the following titles: *Nelson Physics Units 1 & 2 for the Australian Curriculum* and *Nelson Physics Units 3 & 4 for the Australian Curriculum* by Neil Champion, Robert Farr, Megan Mundy, Geoff Cody and Kate Wilson.

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An experienced maths author, science reviewer, HOD Science and member of the QCAA Physics State Panel and QCAA Science LARG, Scott brings a wealth of knowledge in teaching physics, as well as teaching senior science, to the author team. Scott's dedication to teaching physics has enabled him to lead the *QScience* team in the development of this text.

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Oliver is a passionate senior physics teacher who is dedicated to using the content of Physics to develop the critical thinking and problem-solving ability of his students. In addition to his experience in teaching Physics, Oliver has worked in a variety of fields including as a Director of Assessment Innovation, Associate Dean of Data Analysis and as an analyst in the Soft-Condensed Matter Laboratory at the University of Melbourne.

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Tara Kuhn

An experienced science teacher, Tara brings her enthusiasm for physics and maths to the writing team.

Review team

The following people have contributed to the review of the *QScience* series: David Austin, Laura Bagnall, Professor Alan E.W. Knight, Rebecca Delaney and Catherine Munro.

SYLLABUS REFERENCE GRID

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UNIT THREE » GRAVITY AND ELECTROMAGNETISM	
TOPIC 1: GRAVITY AND MOTION	
Gravity and motion	Chapter 1
Projectile motion	Chapter 2
Inclined planes	Chapter 3
Circular motion	Chapter 4
Gravitational force and field	Chapter 5
Orbital motion	Chapter 6
TOPIC 2: ELECTROMAGNETISM	
Electrostatics	Chapter 7
Magnetic fields	Chapter 8
Electromagnetic induction	Chapter 9
Electromagnetic radiation	Chapter 10
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Special relativity	Chapter 11
TOPIC 2: QUANTUM THEORY	
Quantum theory	Chapter 12
TOPIC 3: THE STANDARD MODEL	
The Standard Model	Chapter 13
Particle interactions	Chapter 14

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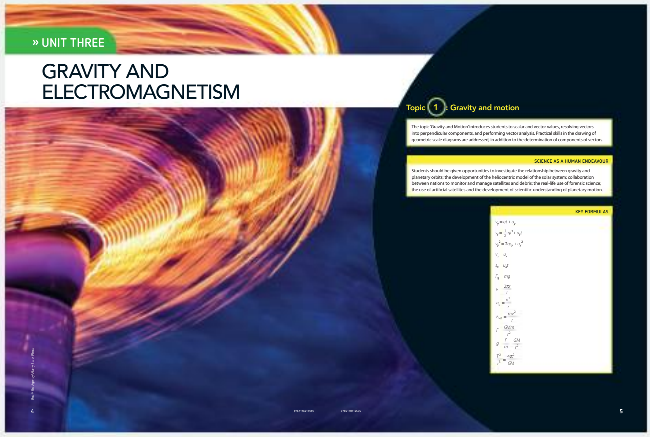
ABOUT THIS BOOK

At the beginning of Unit and Topic

- Unit introductions are an overview of the key content in the unit.
- Topic introductions are an overview of the key content in the topic.



At the beginning of each chapter

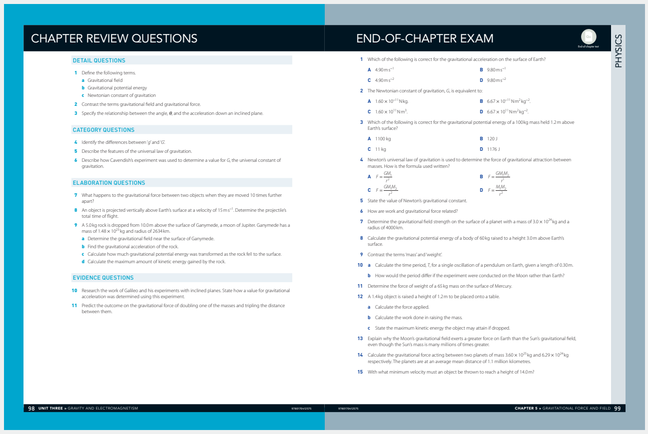
- A short chapter summary introduces students to the key content and skills covered.
- Stimulus questions are relevant to the syllabus.

In each chapter

- **Key formulas** are highlighted throughout the text.
- **Key glossary terms** are highlighted in the margin.
- **Science as a Human Endeavour** provides opportunities for students to connect to the importance of science as a human endeavour and develop scientific research skills.
- **Inquiring further** provides opportunities for students to further investigate scientific concepts and develop scientific research skills.
- **Section reviews** are written in the style of Bloom's revised taxonomy.
- **Practical experiments** contain guided instructions on the materials, procedure, collection and analysis of results, and discussion.

At the end of each chapter

- **Chapter review questions** written in the style of Marzano and Simms (2014) questioning sequences.
- **End-of-chapter examinations** help students develop skills in decoding and answering exam-style questions.



At the end of the book

- **Practice exam** questions provide an extended practice of the content and skills learnt across the text.
- **Glossary** provides explanations of all of the new terms introduced in the text.
- **Answers** short answers for student reference.

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» UNIT THREE

GRAVITY AND ELECTROMAGNETISM

- Topic 1: Gravity and motion
- Topic 2: Electromagnetism

Gravity and electromagnetism provide a basis for students' understanding of motion and its causes through the application of Newton's laws of motion and the gravitational field model. Field theories are applied to the natural phenomena of gravity and electromagnetism and students investigate the production of electromagnetic waves. The analysis of motion of objects on inclined planes, projectiles and orbital satellites is explored, including technologies such as artificial satellites, navigation devices, large-scale electrical power generation and distribution, motors and generators. Students' skills in graphical representation of data and the identification of relationships between variables, including in two or three dimensions, are supported as students develop skills in conducting investigations, interpreting results and evaluating the validity of primary and secondary data.

UNIT OBJECTIVES

By the end of this unit, students should:

1. describe and explain gravity and motion, and electromagnetism
2. apply understanding of gravity and motion, and electromagnetism
3. analyse evidence about gravity and motion, and electromagnetism
4. interpret evidence about gravity and motion, and electromagnetism
5. investigate phenomena associated with gravity and motion, and electromagnetism
6. communicate understandings, findings and conclusions about gravity and motion, and electromagnetism.

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» UNIT THREE

GRAVITY AND ELECTROMAGNETISM

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Topic 1: Gravity and motion

The topic 'Gravity and Motion' introduces students to scalar and vector values, resolving vectors into perpendicular components, and performing vector analysis. Practical skills in the drawing of geometric scale diagrams are addressed, in addition to the determination of components of vectors.

SCIENCE AS A HUMAN ENDEAVOUR

Students should be given opportunities to investigate the relationship between gravity and planetary orbits; the development of the heliocentric model of the solar system; collaboration between nations to monitor and manage satellites and debris; the real-life use of forensic science; the use of artificial satellites and the development of scientific understanding of planetary motion.

KEY FORMULAS

$$v_y = gt + u_y$$

$$s_y = \frac{1}{2}gt^2 + u_y t$$

$$v_y^2 = 2gs_y + u_y^2$$

$$v_x = u_x$$

$$s_x = u_x t$$

$$F_g = mg$$

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r}$$

$$F_{net} = \frac{mv^2}{r}$$

$$F = \frac{GMm}{r^2}$$

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

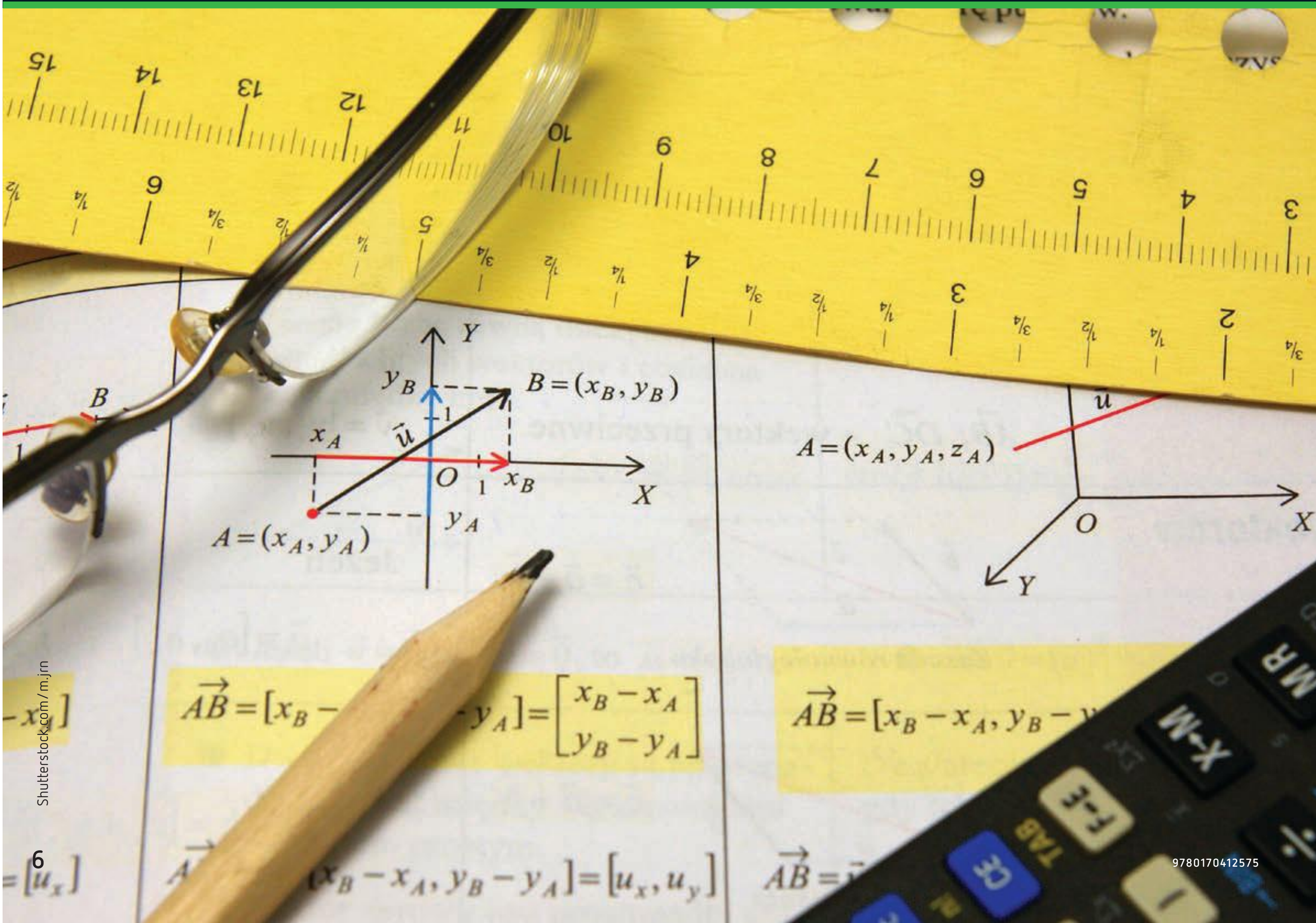
1 GRAVITY AND MOTION

Introduction

A quantity can be specified completely by a single scale (scalar) or by two or more scales (vector). In symbol form, vectors are distinguished from scalars by placing an arrow on top of the letter symbol: $\vec{A}$, $\vec{B}$, $\vec{C}$ and so on.

Stimulus question

How is it possible to add two quantities together when they have different magnitudes and point in different directions?



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1.1 Vector and directional analysis

In the description of motion (kinematics), displacement, velocity and acceleration are all vectors comprising scalar magnitude and direction. Similarly, in the explanation of motion (dynamics), force and momentum are vectors. Magnitude is given in standard SI units, such as metres, kilograms, seconds or newtons. Direction is given with respect to an agreed axis system: compass points (quadrant or true/azimuth bearings), Cartesian axes (positive x-axis), one of the vectors involved, or the plane of a sloping surface.

SYLLABUS LINK

Chapter 12 of *Nelson QScience Physics Units 1 & 2*, introduces vectors in relation to one- and two-dimensional displacement and one-dimensional velocity and acceleration along a number line. Other symbols can also be used to distinguish between vectors and scalars.

Chapter 12 also discusses quadrant and true (azimuth) bearings.

Vector geometric addition in two dimensions: scale drawing

All vectors can be added and subtracted geometrically. Solutions can be found by careful scale drawing using Cartesian graph paper, protractor, ruler and a fine-point pencil.

Tip-to-tail method

Using the tip-to-tail method, the two force vectors form two sides of a triangle.

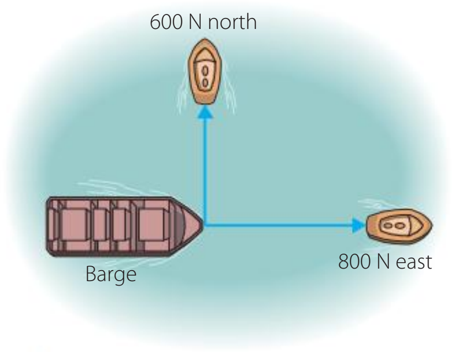

FIGURE 1.1.1 An example of the tip-to-tail method. The vector sum of two vectors, $\vec{C} = \vec{A} + \vec{B}$. In the tip-to-tail method, vectors add head tip to tail.

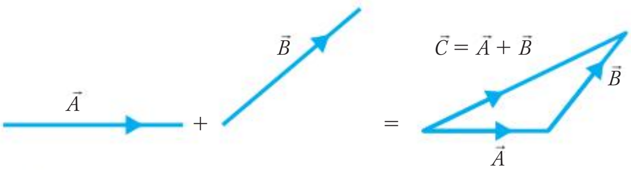

FIGURE 1.1.2 The vector sum of two vectors, $\vec{C} = \vec{A} + \vec{B}$. In the tip-to-tail method, vectors add head (tip) to tail.

Parallelogram method

Vector arrows are just representations of the reality and can be moved to positions appropriate for the solution. In the **parallelogram method**, both vectors are aligned so that their tails are at the same position. A parallelogram is constructed using the vectors as adjacent sides. The resultant is the diagonal that starts at the tails of the vectors being added.

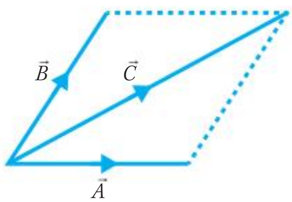

FIGURE 1.1.3 Adding vectors by the parallelogram method. The resultant starts on the tails of the vectors being added.

parallelogram method
vector addition in which the tail of each vector is connected at the same position; the vectors are used as adjacent sides of a parallelogram and the resultant is the diagonal that starts at the tails of the vectors being added

Vector subtraction

As in ordinary subtraction, vector subtraction is the addition of the negative. Thus:

$$\vec{C} = \vec{A} - \vec{B}$$

$$\Rightarrow \vec{C} = \vec{A} + ({}^{-}\vec{B})$$

1.1.1 Vector addition

Subtraction of vectors: addition of the negative

$\vec{C} = \vec{A} - \vec{B}$

$\Rightarrow \vec{C} = \vec{A} + (^{-}\vec{B})$

KEY FORMULA

component two or more vectors into which a vector can be resolved

resolute component of a vector

rectangular components components that are at right-angles to each other; perpendicular components

Components of vectors

Any vector can be constructed as the sum of any two other vectors. Each of these two vectors is a **component** of the original vector. When a vector is resolved into two components, each component is called a **resolute**.

It is very useful to resolve vectors into **rectangular components** that are perpendicular to each other. This enables the use of the geometry and trigonometry of right-angle triangles.

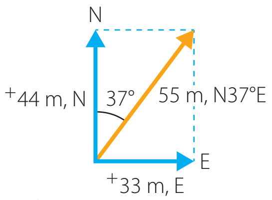

FIGURE 1.1.4 Resolutes of the vector 55 m, N37°E: north component = $^{+}$44 m; east component = $^{+}$33 m.

Figure 1.1.4 shows the rectangular resolutes of the displacement vector 55 m, N37°E. The resolutes are taken in the north and east directions respectively.

On a Cartesian grid, resolutes are taken with respect to the *x*- and *y*-axis respectively. The angle is taken with respect to the positive direction of the *x*-axis.

Vectors may have nothing to do with compass points or graphs. For example, resolutes for projectile motion are usually defined relative to horizontal and vertical directions. For motion on an inclined plane, the resolutes are usually taken parallel and perpendicular to the plane.

Adding vectors by components

Resolutes can be added algebraically in the *x*- and *y*-directions respectively. This enables the magnitude of the resultant to be calculated by Pythagoras' theorem, and the angle by the tangent ratio from trigonometry.

Consider vector $\vec{A}$, which is oriented at an angle θ to the positive *x*-axis. The rectangular components are A_x and A_y.

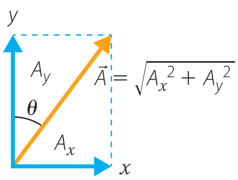

FIGURE 1.1.5 Vector $\vec{A}$ has rectangular components, A_x and A_y. The angle is taken relative to the positive direction of the *x*-axis

KEY FORMULA

Rectangular components of vector $\vec{A}$:

x-component: $A_x = A\cos\theta$

y-component: $A_y = A\sin\theta$

Angle:

$$\tan\theta = \frac{y\text{-component}}{x\text{-component}}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{y\text{-component}}{x\text{-component}}\right)$$

Length is found by Pythagoras' theorem:

$$A = \sqrt{A_x^2 + A_y^2}$$

Where:

A = magnitude of vector, $\vec{A}$

A_x = magnitude of the *x*-component

A_y = magnitude of the *y*-component

θ = angle relative to the positive direction of the *x*-axis

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KEY FORMULA

Figure 1.1.6, across, shows the addition of two vectors, $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} + \vec{B}$$

Using the convention described above, the components of the resultant, $\vec{R}$ are:

x-component: $R_x = A_x + B_x$

y-component: $R_y = A_y + B_y$

Angle:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

The length, R, of the resultant vector, $\vec{R}$, is found by Pythagoras' theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Rightarrow R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

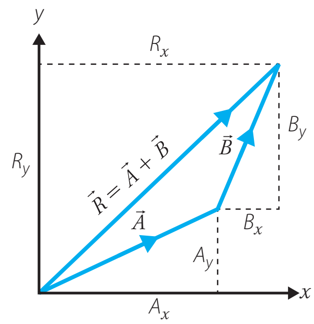

FIGURE 1.1.6 $\vec{R} = \vec{A} + \vec{B}$. Components in the *x*- and *y*- directions add algebraically.

Multiplication of a vector by a scalar

The magnitude of a vector is a scalar, which can be multiplied by a scalar factor. This **scalar multiplier** makes the vector longer or shorter by the scale factor without changing its direction. However, if a scalar multiplier is negative, the magnitude is changed and the direction is reversed.

scalar multiplier
positive or negative number that can change the magnitude and/or the direction of a vector

KEY FORMULA

A vector may be multiplied by a scalar quantity.

- Multiplication by a positive scalar changes the magnitude but not the direction of the vector.
- Multiplication by a negative scalar changes the magnitude *and* reverses the direction of the vector.

Division of a vector by a scalar is the same as multiplication of the vector by the inverse of the divisor.

1.1.2 Scalar multiplication of vectors

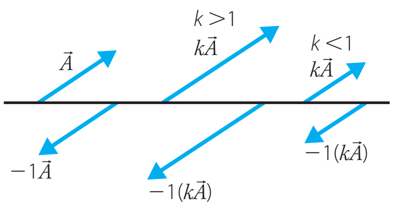

FIGURE 1.1.7 Multiplication of a vector by the scalar, *k*. (a) If $0 < k < 1$, magnitude is reduced and the direction is *not* altered. (b) If $k > 1$, magnitude is increased and the direction is *not* altered. (c) If $k < 0$, magnitude is affected *and* the direction is reversed.

SECTION REVIEW

1.1

REMEMBERING

1 Identify the two methods of geometric vector addition.

2 Explain what must be done in order to add vectors geometrically.

3 Describe what happens to a vector when it is multiplied by a scalar, k. Consider multipliers:

 a $k > 1$

 b $0 < k < 1$

 c $k < 0$.

4 a Write the components of vector $\vec{A}$, in terms of the angle relative to the x-axis.

 b Recall how to find the angle in terms of components.

UNDERSTANDING

5 Compare the two types of geometric vector addition methods.

6 Explain how vector subtraction can be treated as vector addition.

7 Explain why a consistent scale is needed when constructing answers to vector equations using a ruler and protractor.

8 Describe the conditions that must be met before Pythagoras' theorem and trigonometric ratios can be used to solve vector additions and subtractions.

APPLYING

9 For vectors $\vec{A}, \vec{B}, \vec{C}$, write vector *addition* equations to show:

 a $\vec{C}$ as the resultant when $\vec{A}$ and $\vec{B}$ are added

 b $\vec{C}$ as the resultant when $\vec{A}$ is subtracted from $\vec{B}$

 c $\vec{A}$ as the difference between $\vec{B}$ and $\vec{C}$

 d $\vec{C}$ as the resultant when $2\vec{A}$ and $3\vec{B}$ are added.

ANALYSING

10 $\vec{P}, \vec{Q}$ *and* $\vec{R}$ are all vectors on a Cartesian plane. Write equations to show the components in x- and y-directions when:

 a $\vec{R} = \vec{P} + \vec{Q}$

 b $\vec{R} = \vec{P} - \vec{Q}$

 c $\vec{R} = 2\vec{P} - 3\vec{Q}$.

1.2 Solving problems: vectors

Vectors involving magnitude and direction can be added and subtracted geometrically. In order to do this accurately, select Cartesian graph paper or a clean sheet of paper. Lay the paper on a flat surface. Use a ruler, protractor and a fine-point pencil to draw the vectors.

Tip-to-tail method

Follow the steps below for geometrically adding vectors using the tip-to-tail method.

1 Decide on an appropriate scale for the drawing – state the scale in the form, p units represents q units or (p units: q units).

2 Draw one of the vectors to scale, showing tail and head clearly. Label as $\vec{A}$.

3 Draw the second vector to scale, so that its tail is on the head of the first vector, and its head is pointing in the correct direction. Label as $\vec{B}$.

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4 The arrow drawn from the tail of the first vector to the head of the second vector is the resultant (sum) of the two vectors: $\vec{R} = \vec{A} + \vec{B}$.

5 Measure the length of the resultant and use the scale to convert it to the correct value.

6 Measure the angle carefully with the protractor – angles may be related to the compass points (quadrant or azimuth bearings), the x-axis, or relative to one of the vectors in the problem.

WORKED EXAMPLE 1.2.1

A ball-bearing slides around a horizontal circular track at a constant speed of $6.0\,\mathrm{m\,s^{-1}}$. At one point it is travelling east. One-quarter turn later it is pointing south.
Use the tip-to-tail method to find the change of velocity for the ball-bearing.

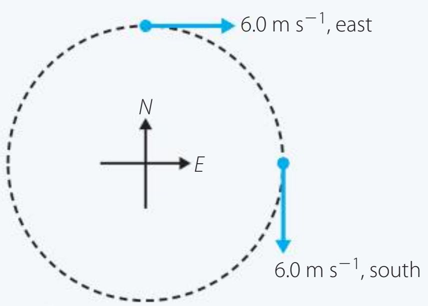

FIGURE 1.2.1

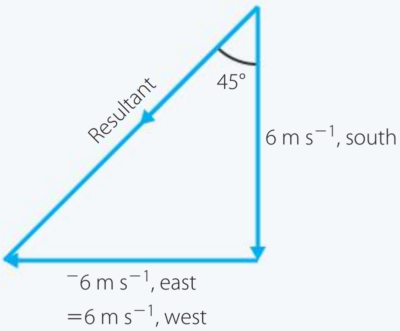

FIGURE 1.2.2

ANSWER

Change of velocity is found by subtracting $6.0\,\mathrm{m\,s^{-1}}$ east from $6.0\,\mathrm{m\,s^{-1}}$ south. This is done by adding the negative of the initial velocity.

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$\Delta\vec{v} = \vec{v}_f + (-\vec{v}_i)$

Scale: $1.0\,\mathrm{cm} = 1.0\,\mathrm{m\,s^{-1}}$

The resultant, $\Delta\vec{v}$, is measured to be 8.4 cm. This converts to $8.4\,\mathrm{m\,s^{-1}}$.

The angle is measured as S45°W.

Thus, $\Delta\vec{v} = 8.40\,\mathrm{m\,s^{-1}}$, S45°W.

Parallelogram method

Follow the steps below for adding vectors geometrically by the parallelogram method.

1 Decide on an appropriate scale for the drawing – state the scale in the form, p units represents q units or (p units: q units).

2 Draw one of the vectors to scale, showing tail and head clearly. Label as $\vec{A}$.

3 Draw the second vector to scale, so that its tail is on the tail of the first vector, and its head is pointing in the correct direction. Label as $\vec{B}$.

4 Construct a parallelogram, using the two vectors as adjacent sides.

5 The arrow drawn along the diagonal that starts from the tails of the two vectors is the resultant (sum) of the two vectors: $\vec{R} = \vec{A} + \vec{B}$.

6 Measure the length of the resultant and use the scale to convert it to the correct value.

7 Measure the angle carefully with the protractor – angles may be related to the compass points (quadrant or azimuth bearings), the x-axis, or relative to one of the vectors in the problem.

WORKED EXAMPLE 1.2.2

An oil rig is being towed into place by two tug boats, T_1 and T_2. T_1 pulls on the rig with a force of 4.0×10^5 N, 135° true. The true bearing of T_2 is 240° and it pulls with a force of 6.0×10^5 N. Determine the resultant of these two forces.

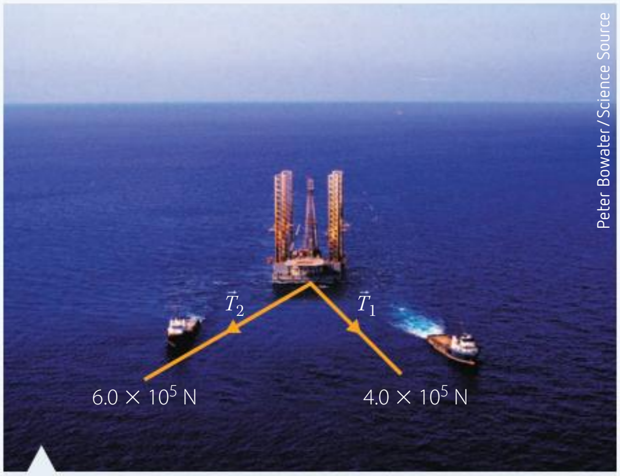

FIGURE 1.2.3 An oil rig is towed by two tug boats.

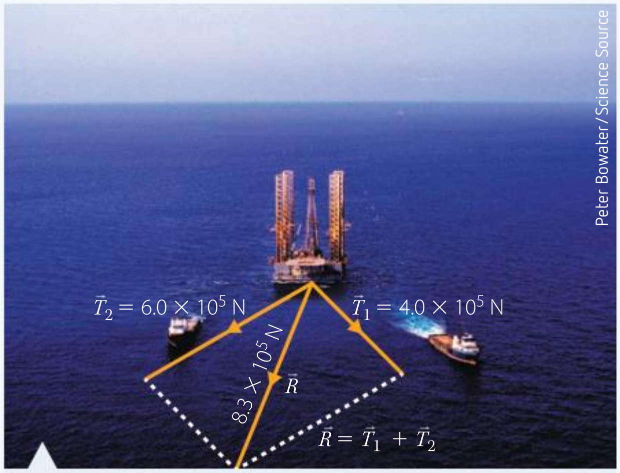

FIGURE 1.2.4 Addition of forces by parallelogram method

ANSWER

Scale: 1.0 cm represents 1.0×10^5 N

1 The resultant, $\vec{R}$, is measured to be 12.6 cm. This converts to 6.3×10^5 N.

2 The angle is measured as 207° true.

3 Thus, $\vec{R} = 6.3 \times 10^5$ N, 207° true.

Rectangular components

Follow this general procedure for adding (or subtracting) two or more vectors using components.

1 Sketch a diagram that clearly shows the vector addition.

2 Choose rectangular x- and y-axes. Sometimes this is obvious, as in the case of using the compass points. Sometimes it is better to select the x-axis to be along one of the vectors so that one of the vectors has a single component.

3 Resolve each vector into rectangular resolutes (x- and y-components, compass points, etc.). Be sure to identify positive and negative values for the resolutes.

4 Calculate the magnitude of each component.

KEY FORMULA

For vector $\vec{A}$, which is oriented at an angle, θ, to the defined positive direction (x-axis), the magnitude of the components are:

along the x-axis: $A_x = A\cos\theta$

along the y-axis: $A_y = A\sin\theta$

5 Find the magnitude of the components of the resultant vector.

KEY FORMULA

The x-component, R_x, of the resultant, $\vec{R}$, is the sum of all the x-components of the individual vectors being added. Similarly, the y-component, R_y, is the sum of all the y-components of the individual vectors being added.

6 Find the magnitude of the resultant.

7 Find the angle.

KEY FORMULA

The magnitude of the resultant is, by Pythagoras' theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

KEY FORMULA

The angle with respect to the axis is found by the trigonometric ratio for tan:

$$\tan\theta = \frac{R_y}{R_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

8 Check that the angle is given in the terms required by the question.

WORKED EXAMPLE 1.2.3

Solve the tug boat–oil rig problem using components.

ANSWER

1 a x-components:

The resultant is likely to be more towards the west ⇒ take west as positive x-direction:

$$R_x = T_{1x} + T_{2x}$$

$$\Rightarrow R_x = {}^{-}4.0\times10^5\text{ N}\times\sin45^\circ + 6.0\times10^5\text{ N}\times\cos30^\circ$$

$$\Rightarrow R_x = 2.368\times10^5\text{ N}$$

b y-components:

The resultant is likely to be more towards the south => take south as positive y-direction:

$$R_y = T_{1y} + T_{2y}$$

$$\Rightarrow R_y = 4.0\times10^5\text{ N}\times\cos45^\circ + 6.0\times10^5\text{ N}\times\cos60^\circ$$

$$\Rightarrow R_y = 5.828\times10^5\text{ N}$$

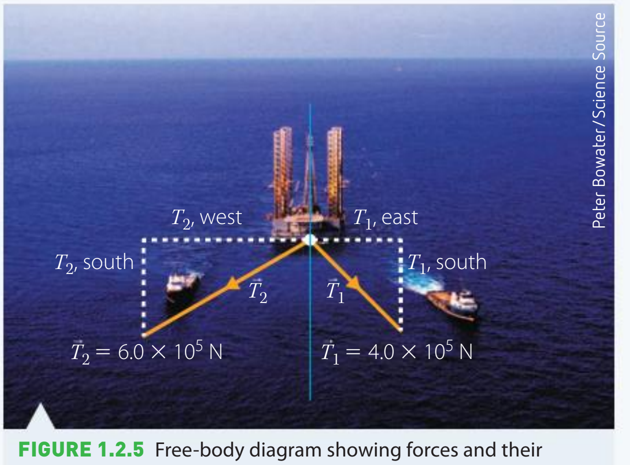

FIGURE 1.2.5 Free-body diagram showing forces and their components for the oil rig being towed by two tug boats (see Figure 1.2.3).

2 Magnitude of resultant:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Rightarrow R = \sqrt{(2.368 \times 10^5)^2 + (5.828 \times 10^5)^2}$$

$$\Rightarrow R = 8.3 \times 10^5 \text{ N}$$

3 Angle:

Let θ be the angle opposite the y-component:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{5.828 \times 10^5 \text{ N}}{2.368 \times 10^5 \text{ N}}\right)$$

$$\Rightarrow \theta = 68°$$

4 Quadrant or azimuth bearing:

Let α be the angle with respect to south.

$$\alpha = 90° - 68°$$

$$\Rightarrow \alpha = 22° \text{ (S22°W)}$$

5 True bearing:

$$\text{True bearing} = 180° + 22°$$

$$\Rightarrow \text{True bearing} = 202°$$

WORKED EXAMPLE 1.2.4

A mass slides without friction down a slope that is inclined at 30° to the horizontal. Find the component of the gravitational acceleration, $g = 9.8 \text{ m s}^{-2}$, parallel to the surface.

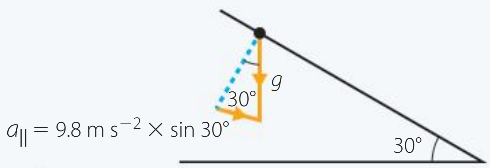

FIGURE 1.2.6 The component of the acceleration due to gravity down the slope.

ANSWER

The acceleration due to gravity is $g = 9.8 \text{ m s}^{-2}$ vertically down.

Parallel to the slope, the component of the gravitational acceleration, $a_{||}$, is:

$$a_{||} = g \sin\theta$$

$$\Rightarrow a_{||} = 9.8 \text{ m s}^{-2} \times \sin 30°$$

$$\Rightarrow a_{||} = 4.9 \text{ m s}^{-2}$$

SECTION REVIEW 1.2

REMEMBERING

1 Recall the steps for solving vector problems by the tip-to-tail method.

2 Recall the steps for solving vector problems by the parallelogram method.

3 Recall the steps for solving vector problems by the method of components.

UNDERSTANDING

4 Explain how, when two vectors are added on the Cartesian plane, the x- and y-components can be used in the addition.

APPLYING

5 Use the tip-to-tail method to add the following vectors on a Cartesian plane:

a $\vec{A} = 50\,\text{m}, \theta = 30°; \vec{B} = 80\,\text{m}, \theta = 75°$

b $\vec{A} = 10\,\text{m}, \theta = 30°; \vec{B} = 30\,\text{m}, \theta = 45°$.

6 Use the parallelogram method to add the following vectors on a Cartesian plane:

a $\vec{A} = 3 \times 10^3\,\text{N}, \theta = 30°; \vec{B} = 4 \times 10^3\,\text{N}, \theta = 60°$

b $\vec{A} = 320\,\text{N}, \theta = 40°; \vec{B} = 250\,\text{N}, \theta = 80°$.

ANALYSING

7 Use the components method to subtract $\vec{B}$ from $2\vec{A}$ on a Cartesian plane, given the following values:

a $\vec{A} = 15\,\text{m}\,\text{s}^{-1}, \theta = 30°; \vec{B} = 25\,\text{m}\,\text{s}^{-1}, \theta = 60°$

b $\vec{A} = 35\,\text{N}, \theta = 120°; \vec{B} = 25\,\text{N}, \theta = 45°$.

8 Sketch the following vector sums, $\vec{P} + \vec{Q}$, then find the resultant, $\vec{R}$, using the component method.

a $\vec{P} = 20\,\text{N}$, 120° true; $\vec{Q} = 50\,\text{N}$, 300° true

b $\vec{P} = 300\,\text{N}$, N30°W; $\vec{Q} = 450\,\text{N}$, S60°W

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Identify the scalars used when:
- **a** describing motion
- **b** explaining how motion is caused.

2 In what ways can vectors be used in the study of movement and its causes?

3 Describe how components of vectors can be useful.

CATEGORY QUESTIONS

4 Consider the following scenario.

A student who is provided with good quality materials (graph paper, ruler, pencil and protractor), but no calculator, is asked to perform the following vector subtraction:

$\vec{R} = \vec{C} - 2\vec{D}$

where $\vec{C}$ = 50 m, 20° and $\vec{D}$ = 30 m, 50°.

- **a** Identify and justify an appropriate solution method involving the materials and equipment available.
- **b** Use the method identified in part **a** to find $\vec{R}$.

The student is now provided with a scientific calculator.

- **c** Identify and justify an appropriate solution method involving the calculator.
- **d** Use the method identified in part **c** to find $\vec{R}$.
- **e** Which of the methods selected in parts **a** and **c** respectively do you think is the more accurate? Justify your answer.

ELABORATION QUESTIONS

5 A yacht travels at a velocity of 20 m s^{-1}, N50°E before changing its speed to 10 km h^{-1} in the direction 140° true. Find the average acceleration of the yacht if the change takes 25 s.

6 A drone enthusiast practises for the Drone Nationals on a circular course of radius 10.0 m. In one practice flight, the drone travels at a constant speed of 4.2 m s^{-1}.
- **a** Calculate how long it takes for the drone to complete one circuit of the course.
- **b** Construct the change of velocity vector for a one-eighth segment of the circuit.
- **c** Give a value for the magnitude of the average acceleration for this one-eighth of the circuit.

EVIDENCE QUESTIONS

7 Use the race course outline for a yachting competition, such as a division of the Airlie Beach Race Week regatta or the America's Cup, to specify each of the following vectors:
- **a** each point on the race course where a change of displacement occurs, relative to the start
- **b** change of displacement for each point relative to the preceding point on the race course.

8 From a search engine's 'list of amusement rides', choose one that demonstrates non-uniform motion. Estimate several changes of displacement and velocity for selected sections of the ride.

END-OF-CHAPTER EXAM

End-of-chapter test

1 Vectors and scalars differ because:

A a vector must have only two scales; a scalar must have only one scale.

B a vector must have at least two scales; a scalar must have only one scale.

C a scalar must have only two scales; a vector must have only one scale.

D a scalar must have at least two scales; a vector must have only one scale.

2 Which of the following statements is true?

A True bearings are taken with respect to the positive x-axis.

B Quadrant bearings are taken with respect to the positive y-axis.

C True bearings can take any value between 0° and 360°.

D Quadrant bearings can take any value between 0° and 360°.

3 A vector has a magnitude M and true bearing of θ, where $0° < \theta < 90°$. This means that its component in the direction of east is:

A $M\sin\theta$.

B $M\cos\theta$.

C $M\tan\theta$.

D M, E$(90° - \theta)$N.

4 A vector has a magnitude M and true bearing of θ, where $180° < \theta < 270°$. This means that its component in the direction of east is:

A $^{-}M\sin(270° - \theta)$.

B $M\cos(\theta)$.

C $^{-}M\cos(270° - \theta)$.

D M, S$(270° - \theta)$W.

5 On a Cartesian plane, a vector, $\vec{R}$, has an x-component of p and a y-component of q. The magnitude, R, and direction, θ, of $\vec{R}$ are, respectively:

A $\sqrt{p^2+q^2}$; $\tan\left(\frac{p}{q}\right)$

B $\sqrt{p^2+q^2}$; $\tan\left(\frac{q}{p}\right)$

C $\sqrt{p^2+q^2}$; $\tan^{-1}\left(\frac{p}{q}\right)$

D $\sqrt{p^2+q^2}$; $\tan^{-1}\left(\frac{q}{p}\right)$

6 The x- and y-components of a vector are also known as ______________ resolutes.

7 On a Cartesian plane, the angle of a vector is measured with respect to the ______________ axis.

8 On a Cartesian plane, explain how the x- and y-components of the resultant are calculated.

9 What is the effect on vector, $\vec{A}$, when it is multiplied by $^{-}4$?

10 Use a geometrical method on a Cartesian grid to find $\vec{R} = 2\vec{A} + \vec{B}$ where $\vec{A} = 10\,\text{m}$, $\theta = 45°$, and $\vec{B} = 20\,\text{m}$, $\theta = 120°$.

11 An inclined plane is raised 30° above the horizontal. Find the components of the acceleration due to gravity:

a parallel to the plane

b perpendicular to the plane.

12 Use the method of components to find $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ where $\vec{v}_1 = 16.7\,\text{m}\,\text{s}^{-1}$, 32° true, and $\vec{v}_2 = 18.6\,\text{m}\,\text{s}^{-1}$, 312° true.

13 Explain why, in general, the average velocity of an object has the same direction, but not the same magnitude, as the direction associated with a displacement interval. Support your answer with an example.

14 An object is swung on a horizontal circle of radius 20 m. At one moment it is at P, a point due north of the centre, and travelling at $10\,\text{m}\,\text{s}^{-1}$. Two seconds later (2.0 s), the object passes Q, a point that is 45° around towards the east, at a speed of $15\,\text{m}\,\text{s}^{-1}$.

a Use quadrant bearings to specify the displacement of the object at points P and Q respectively.

b Find the average velocity of the object between P and Q.

c Find the average acceleration of the object.

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2 PROJECTILE MOTION

Introduction

A projectile is an object that goes up and down vertically at the same time as it moves horizontally. Projectile motion is, therefore, motion in two dimensions. Near the surface of Earth, the gravitational field acts vertically throughout the motion. For current purposes, air resistance will be regarded as negligible. Thus, the vertical motion is only affected by the gravitational field. But, as there is no force component in the horizontal direction, the horizontal motion is not affected by a force or component of force.

Stimulus question

How far and how high does an object travel when it is launched at an angle to the horizontal?

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2.1 Horizontal and vertical vector components

2.1.1 Describing projectiles with numbers: (horizontal and vertical velocity)

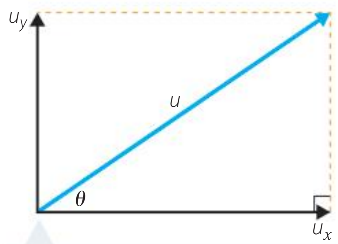

FIGURE 2.1.1 A projectile is launched at speed, u, and angle, θ, to the horizontal. The horizontal and vertical components of the launch velocity, u_x and u_y respectively are shown.

A projectile is launched at speed, u and angle, θ, relative to the horizontal.

Horizontal component of the launch velocity, u_x

The horizontal component of the launch velocity is:

$$u_x = u\cos\theta$$

Vertical component of the launch velocity, u_y

Typically the direction upwards from the launch position is defined as positive. Then, the vertical component of the launch velocity is:

$$u_y = u\sin\theta$$

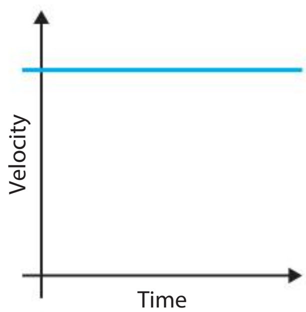

FIGURE 2.1.2 The horizontal component of motion of a projectile remains the same, neglecting air resistance.

KEY FORMULA

Components of the launch velocity

Horizontal component: $u_x = u\cos\theta$

Vertical component: $u_y = u\sin\theta$

Where:

u = launch speed (typically $m\,s^{-1}$)

u_x = horizontal component of the launch velocity

u_y = vertical component of the launch velocity

θ = angle of the launch velocity relative to the horizontal

WORKED EXAMPLE 2.1.1

A projectile is fired at an angle of 30° to the horizontal with a speed of $120\,m\,s^{-1}$. For the initial velocity, find:

1 the horizontal component

2 the vertical component.

ANSWER

1 $u_x = u\cos\theta$
$\Rightarrow u_x = 120\,m\,s^{-1} \times \cos 30°$
$\Rightarrow u_x = 104\,m\,s^{-1}$

2 $u_y = u\sin\theta$
$\Rightarrow u_y = 120\,m\,s^{-1} \times \sin 30°$
$\Rightarrow u_y = 60\,m\,s^{-1}$

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The vertical motion is affected by Earth's gravitational field, which applies a force downwards on the object. As we define the vertical (upwards) direction as positive, the constant acceleration due to Earth's gravitational field near Earth is negative:

$$g = {}^{-}9.8\,\mathrm{m\,s^{-2}}$$

The speed–time graph of the vertical component of motion of a projectile is shown in Figure 2.1.1.

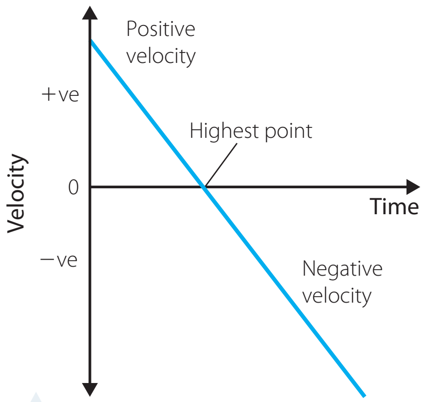

FIGURE 2.1.3 When drag is considered negligible, the vertical component of motion of a projectile proceeds at constant negative acceleration: $g = {}^{-}9.8\,\mathrm{m\,s^{-2}}$.

Combining horizontal and vertical components of projectile motion

For a projectile, the complete set of motion variables can be specified by treating the horizontal component of the motion separately from the vertical component of the motion. Algebraically, the *suvat* equations can be rewritten using the symbols defined above.

Horizontal component of motion

There is no horizontal component of force to affect the horizontal motion. The horizontal component of acceleration $a_x = 0$. Consequently, the horizontal component of the motion remains constant and $u_x = u_y$. Therefore, the horizontal distance, s_x, travelled in a time interval t is:

$$s_x = u_x t = v_x t$$

The velocity–time graph of the horizontal component of motion of a projectile is shown in Figure 2.1.3.

KEY FORMULA

Horizontal component of motion at zero acceleration:

$s_x = u_x t = v_x t$

Where:

s_x = horizontal distance travelled

u_x = horizontal component of the launch velocity

v_x = horizontal velocity

t = time

Vertical component of motion at constant acceleration, *g*

The vertical component of motion is affected by the constant gravitational field, g, which produces a constant acceleration of $g = {}^{-}9.8\,\mathrm{m\,s^{-2}}$ on every mass. Thus, the *suvat* equations can be used to analyse the vertical component of the motion. The initial and final vertical components of speed are, respectively, u_y and v_y. In the vertical direction, the object travels a distance interval of $s_y = y$ over a time interval t. The *suvat* equations can be written for projectile motion as follows:

KEY FORMULA

$v_y = gt + u_y$

$s_y = \frac{1}{2}gt^2 + u_y t$

$v_y^2 = 2gs_y + u_y^2$

Where:

u_y = vertical component of the launch velocity

v_y = vertical component of the velocity some time after launch

$s_y = y$ = vertical height interval relative to the launch position

g = acceleration due to gravity

t = time interval

SECTION REVIEW 2.1

REMEMBERING

1 Consider a projectile launched at speed, u, and angle relative to the horizontal, θ.

a Draw a vector diagram to show the launch velocity and its horizontal and vertical components.

b Write the equation for each of the components.

2 Write the kinematic equations for projectile motion. Define all variables.

UNDERSTANDING

3 'Near the surface of Earth, the gravitational field is taken to be constant'. Explain this approximation.

4 Explain why the horizontal and vertical components of motion of projectiles are independent of each other.

APPLYING

5 Complete the following table.

LAUNCH SPEED ($m\,s^{-1}$)	ANGLE TO THE HORIZONTAL (°)	HORIZONTAL COMPONENT ($m\,s^{-1}$)	VERTICAL COMPONENT ($m\,s^{-1}$)
20	30		
15.6	45		
2.41	60		

6 A ball is thrown upwards with a speed of $12\,m\,s^{-1}$ at an angle of 70° to the horizontal.

a Calculate the velocity of the ball when it reaches its highest point.

b Determine the acceleration of the ball at the top of its flight.

c Find the time when the ball is at a height of 4.0 m above its launch position.

7 A rocket leaves the launch pad with a speed of $300\,m\,s^{-1}$ at an angle of elevation of 35°. Calculate the horizontal distance travelled, in kilometres, when it returns to the same height as the launch site.

ANALYSING

8 Write down the *suvat* equations for vertical motion and compare them to the equations for the vertical component of projectile motion (see Key Formula, page 21) to show their similarities and differences.

9 A projectile is launched at an angle of 60° above the horizontal. It rises to a maximum height of 25 m. Find the launch speed.

REFLECTING

10 Qualitatively compare projectile motion with and without the effect of air resistance.

2.2 The trajectory of projectiles 'near Earth'

Projectiles may be launched with initial velocities that are vertical (up or down), horizontal or at some angle to the horizontal. 'Near Earth' the magnitude of the acceleration due to gravity is $9.8\,m\,s^{-2}$ directed vertically downwards.

Falling object compared to a horizontally projected object

Two balls are released simultaneously from the same position above the ground. Ball A, initially at rest, falls vertically down. Ball B is projected horizontally at initial speed u_x (Figure 2.2.1).

Both objects fall at the same rate, but the path of ball B is 'stretched' horizontally by the horizontal component of the velocity. After a time interval t, the velocity of each ball can be found by analysing the horizontal and vertical motions independently of each other, then combining the results.

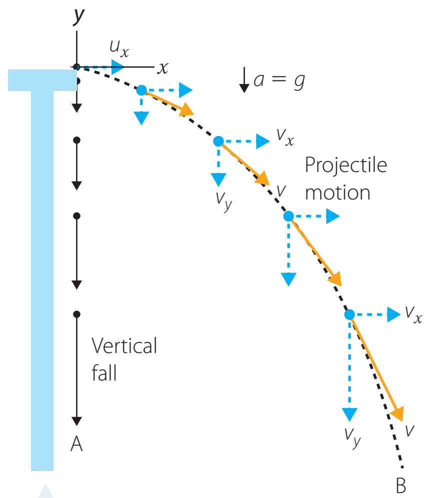

FIGURE 2.2.1 Two balls are released simultaneously from the same height.

2.2.1 Characteristics of a projectile's trajectory

Horizontal motion of ball B

The horizontal component of the velocity after a time interval t is the same as the original horizontal component of the launch velocity:

$$v_x = u_x \text{ for all time intervals}$$

Vertical motion of ball A

The vertical component of the launch velocity is changed by an amount equal to the area under the acceleration–time or g–t graph.

$$v_x - u_x = gt$$

$$v_y - u_y = gt$$

$$\Rightarrow v_y = gt \quad (u_y = 0)$$

This is the same vertical component of the speed as ball B.

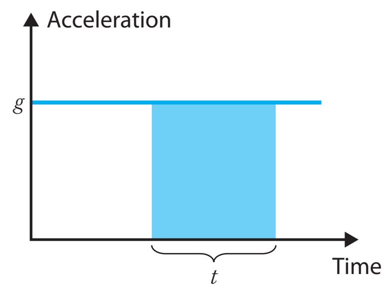

FIGURE 2.2.2 The area under an acceleration–time graph, such as the g–t graph shown, is the change in velocity.

Combining vertical and horizontal components for ball A

The velocity, $\vec{v}$, can now be specified by the use of Pythagoras' theorem (magnitude) and some trigonometry (direction):

$$|\vec{v}| = \sqrt{u_x^2 + v_y^2}$$

$$\Rightarrow |\vec{v}| = \sqrt{u_x^2 + (gt)^2} \quad (u_y = 0)$$

$$\tan\theta = \frac{v_y}{v_x}$$

Projectile launched at an angle to the horizontal

For the motion of a projectile launched at speed, u, and angle, θ, a general analysis, similar to that for horizontal projection ($u_y = 0$), can be undertaken. The velocity is tangential to the path of the projectile.

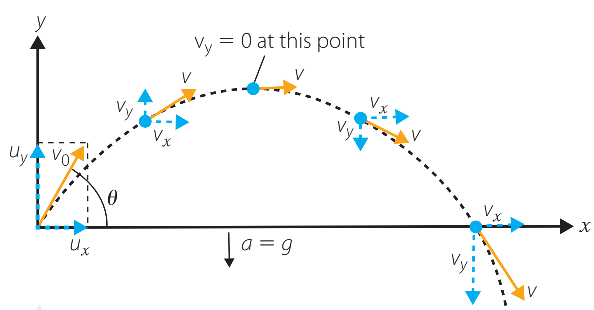

FIGURE 2.2.3 Path of a projectile fired with initial speed, u, and angle, θ. At each point, the horizontal component of the velocity is the same. The vertical component changes according to the time for which the gravitational field acts.

Horizontal motion of projectile

The horizontal component of the velocity after a time interval, t, is the same as the original horizontal component of the launch velocity:

$$v_x = u_x \text{ for all time intervals}$$

Vertical motion of projectile

The vertical component of the launch velocity is changed by an amount equal to the area under the acceleration–time or g–t graph (Figure 2.2.2, page 23).

$$v_y - u_y = gt$$

$$\Rightarrow v_y = gt + u_y \ (u_y > 0)$$

Combining vertical and horizontal components for a projectile

The velocity, $\vec{v}$, at any time can now be specified by the use of Pythagoras' theorem (magnitude) and some trigonometry (direction).

KEY FORMULA

$$|\vec{v}| = \sqrt{u_x^2 + v_y^2}$$

where $v_y = gt + u_y$

$$\tan\theta = \frac{v_y}{u_x}$$

Where:

$|\vec{v}|$ = magnitude of the final velocity ($m\,s^{-1}$)

u_x = horizontal component of launch velocity ($m\,s^{-1}$)

u_y = vertical component of launch velocity ($m\,s^{-1}$)

v_y = vertical component of final velocity ($m\,s^{-1}$)

θ = angle of the launch velocity relative to the horizontal (°)

g = the acceleration due to gravity ($m\,s^{-2}$)

t = time interval (s)

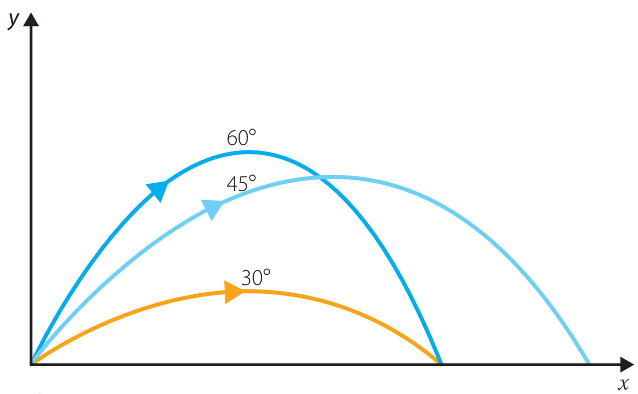

FIGURE 2.2.4 The range of a projectile over level ground depends on the angle of projection. There are two solutions for the same range, except where they coincide at 45°.

Range of a projectile over level ground

A projectile that is launched over level ground at 60° stays above ground for longer than one launched at 30°. However, they both travel the same horizontal distance. In general, for a particular launch velocity, there are two solutions to the horizontal range of the projectile. At a projection angle of 45° the two solutions coincide and the range is a maximum. This can be deduced from the equations for projectile motion.

Vertical component of motion

The time of flight can be calculated from the vertical component of the motion. The position, y, above the ground is zero when the projectile is launched and when it lands.

$$y = 0$$

$$\Rightarrow \frac{1}{2}gt^2 + u_y t = 0$$

$$\Rightarrow t\left(\frac{1}{2}gt + u_y\right) = 0$$

$\Rightarrow t = 0$ (initial condition); or

$$\Rightarrow \frac{1}{2}gt + u_y = 0$$

$$\Rightarrow t = \frac{2u_y}{g} \text{ (Positive because } g < 0)$$

Horizontal component of motion

The range, R, can be calculated by finding values for s_x from the constant horizontal component of the launch velocity:

$$s_x = u_x t = R$$

$$\Rightarrow R = u_x\left(\frac{2u_y}{g}\right) \text{(from vertical analysis)}$$

$$\Rightarrow R = \frac{2u_x u_y}{g}$$

but $u_x = u\cos\theta$ and $u_y = u\sin\theta$

$$\Rightarrow R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$\Rightarrow R = \frac{u^2 \times 2\sin\theta\cos\theta}{g} \text{ (trig identity: } \sin 2\theta = 2\sin\theta\cos\theta)$$

$$\Rightarrow R = \frac{u^2 \sin 2\theta}{g},\ 0° < \theta < 90°$$

KEY FORMULA

$$\Rightarrow R = \frac{u^2 \sin 2\theta}{g},\ 0° < \theta < 90°$$

Where:

R = range over level ground from launch site to landing position (m)

u = initial launch speed ($m\,s^{-1}$)

θ = launch angle (°)

g = acceleration due to gravity ($m\,s^{-2}$)

The range is a maximum when:

$$\sin 2\theta = 1$$

$$\Rightarrow 2\theta = 90°$$

$$\Rightarrow \theta = 45°$$

Notice that the sine function is positive in the first and second quadrants. This means that, if $2\theta = \alpha$ is a solution, then $2\theta = 180° - \alpha$ is a solution. Both solutions for θ lie in the range $0° < \theta < 90°$.

A word of caution

This analysis only works for projectiles that travel over level ground or between two positions at the same vertical distance above the horizontal. If the projectile lands above or below the launch position, the analysis will not provide correct answers.

SECTION REVIEW 2.2

REMEMBERING

1 Consider the equation $|\vec{v}| = \sqrt{u_x^2 + v_y^2}$.

a Describe the situation for which it is applicable.

b Define all variables.

2 A projectile launched from ground level travels over a horizontal plane before landing. Write an equation for the range, making sure to define all variables.

UNDERSTANDING

3 Consider the equation $|\vec{v}| = \sqrt{u_x^2 + v_y^2}$.

a Explain how the time interval is included.

b Explain how the launch angle is involved.

4 Explain why a projectile launched horizontally lands with the same vertical component of velocity as a projectile dropped from the same height.

5 Describe the conditions under which the range equation can be applied. Justify your answer.

APPLYING

6 Calculate the velocity of a projectile 5.0 s after it is launched at a speed of $50\,m\,s^{-1}$ and an angle of 50° to the horizontal.

7 For a projectile that leaves and lands at the same horizontal height above ground, complete the following table.

LAUNCH SPEED ($m\,s^{-1}$)	SMALLEST LAUNCH ANGLE TO THE HORIZONTAL (°)	LARGEST LAUNCH ANGLE TO THE HORIZONTAL (°)	RANGE (m)
20	30		
30			91
	39		48

ANALYSING

8 A paintball is fired horizontally at a bag up a tree that is 12.0 m away. The paintball and bag are both initially 10.0 m above the ground. The bag is let drop at the same time as the shot is fired.

a Explain why the bag is splattered with paint.

b Find the minimum launch speed of the paintball shot.

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2.3 Solving problems: projectile motion

When solving problems involving projectile motion follow the steps below.

1. Read the question carefully.
2. Sketch the real situation described.
3. On the sketch:
   a. show the path from launch to landing
   b. draw the initial velocity vector
   c. show the horizontal and vertical components of the initial velocity vector
   d. add any data provided in the question, including quantitative values for the vertical and horizontal components of the initial velocity
   e. show the direction and magnitude of the gravitational acceleration: g.
4. Separate the analysis into horizontal and vertical components and select appropriate formulae:
   a. horizontal motion

      $u_x = u\cos\theta$

      $s_x = u_x t = v_x t$

   b. vertical motion.

      $u_y = u\sin\theta$

      $v_y = gt + u_y$

      $s_y = \frac{1}{2}gt^2 + u_y t$

      $v_y^2 = 2gs_y + u_y^2$

      $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

      $\tan\theta = \frac{v_y}{v_x}$

5. For the special case of projectiles that are launched from and return to the same horizontal height:

   $$\Rightarrow R = \frac{u^2 \sin 2\theta}{g},\ 0° < \theta < 90°.$$

   Pay attention to the two possible solutions.
6. Transpose formulas for the required unknown variable or substitute values directly into the equation.
7. Solve the equations.
8. Check to ensure the answers are those required.

WORKED EXAMPLE 2.3.1

QUESTION

1 For a projectile fired at an angle of 45° above the horizontal with a speed of $86\,\text{m s}^{-1}$, find:

a the vertical component of the initial velocity

b the maximum height reached by the projectile

c the time taken to reach the maximum height

d the acceleration at maximum height

e the velocity at maximum height.

ANSWER

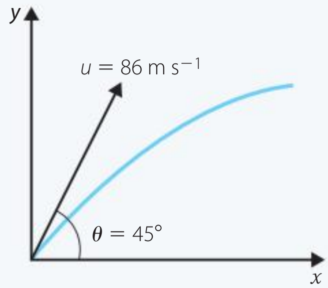

FIGURE 2.3.1 Sketch of projectile motion

1 a $u_y = u\sin\theta$

$\Rightarrow u_y = 86\,\text{m s}^{-1} \times \sin 45°$

$\Rightarrow u_y = 61\,\text{m s}^{-1}$

b $v_y^2 = u_y^2 + 2gs_y$

$$\Rightarrow s_y = \frac{v_y^2 - u_y^2}{2g}$$

$$\Rightarrow s_y = \frac{0^2 - (61\,\text{m s}^{-1})^2}{2 \times {}^{-}9.8\,\text{m s}^{-2}}$$

$\Rightarrow s_y = 190\,\text{m}$

c $v_y = u_y + gt$

$$\Rightarrow t = \frac{v_y - u_y}{g}$$

$$\Rightarrow t = \frac{0 - 61\,\text{m s}^{-1}}{{}^{-}9.8\,\text{m s}^{-2}}$$

$\Rightarrow t = 6.2\,\text{s}$

d $a = g = {}^{-}9.8\,\text{m s}^{-2}$

e $u_x = u\cos\theta$

$\Rightarrow u_x = 86\,\text{m s}^{-1} \times \cos 45°$

$\Rightarrow u_x = 61\,\text{m s}^{-1}$

QUESTION

2 Find the distance covered over level ground by a projectile that is launched from ground level with a speed of $35\,\text{m s}^{-1}$ and an angle 55° above the horizontal.

ANSWER

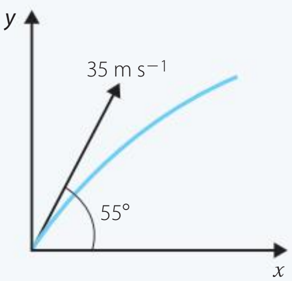

FIGURE 2.3.2 Sketch of projectile motion

2 $$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow R = \frac{(35\,\text{m s}^{-1})^2 \times \sin(2 \times 55°)}{9.8\,\text{m s}^{-2}}$$

$$\Rightarrow R = \frac{(35\,\text{m s}^{-1})^2 \times \sin(180° - 110°)}{9.8\,\text{m s}^{-2}}$$

$$\Rightarrow R = \frac{(35\,\text{m s}^{-1})^2 \times \sin 70°}{9.8\,\text{m s}^{-2}}$$

$\Rightarrow R = 117\,\text{m}$

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QUESTION

3 A projectile is launched from a 96 m high cliff at a speed of $32\ \mathrm{m\,s^{-1}}$. The angle of launch is upwards at 10° to the horizontal. Find:

a the maximum height attained above the cliff

b the time taken to reach the maximum height

c the time taken to land

d the distance from the base of the cliff to the landing position.

ANSWER

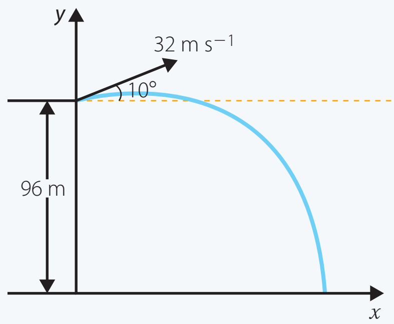

FIGURE 2.3.3 Sketch of projectile motion

3 a $v_y{}^2 = 2gs_y + u_y{}^2$

$$\Rightarrow s_y = \frac{v_y{}^2 - u_y{}^2}{2g}$$

$$\Rightarrow s_y = \frac{(0\ \mathrm{m\,s^{-1}})^2 - (5.557\ \mathrm{m\,s^{-1}})^2}{2 \times {}^{-}9.8\ \mathrm{m\,s^{-2}}}$$

$$\Rightarrow s_y = 1.6\ \mathrm{m}$$

b $v_y = gt + u_y$

$$\Rightarrow t = \frac{v_y - u_y}{g}$$

$$\Rightarrow t = \frac{0\ \mathrm{m\,s^{-1}} - 5.557\ \mathrm{m\,s^{-1}}}{{}^{-}9.8\ \mathrm{m\,s^{-2}}}$$

$$\Rightarrow t = 0.57\ \mathrm{s}$$

c Vertically from top of flight:

$y = (96\ \mathrm{m} + 1.6)\ \mathrm{m} = 97.6\ \mathrm{m}$, $u_y = 0\ \mathrm{m\,s^{-1}}$, $v_y = ?$, $g = {}^{+}9.8\ \mathrm{m\,s^{-1}}$, $t = ?$

$$y = \frac{1}{2}gt^2 + u_y t$$

$$\Rightarrow t = \sqrt{\frac{2y}{g}},\ u_y = 0$$

$$\Rightarrow t = \sqrt{\frac{2 \times 97.6\ \mathrm{m}}{{}^{+}9.8\ \mathrm{m\,s^{-2}}}}$$

$$\Rightarrow t = 4.5\ \mathrm{s}$$

Time of flight, T:

$T = t_{\text{to top}} + t_{\text{from top to ground}}$

$\Rightarrow T = 0.56\ \mathrm{s} + 4.46\ \mathrm{s}$

$\Rightarrow T = 5.0\ \mathrm{s}$

d Horizontally:

$u_y = u\cos\theta = 32\ \mathrm{m\,s^{-1}} \times \cos 10° = 31.5\ \mathrm{m\,s^{-1}}$

$s_x = u_x t$

$\Rightarrow s_x = 31.5\ \mathrm{m\,s^{-1}} \times 5.0\ \mathrm{s}$

$\Rightarrow s_x = 158\ \mathrm{m}$

SECTION REVIEW 2.3

REMEMBERING

1 List the steps to use when solving projectile motion problems.

2 Sketch the scenario for projectile motion for which the landing position is horizontally opposite the launch position.

UNDERSTANDING

3 State the magnitude of the speed and acceleration at the top of a projectile's flight, for a projectile that was launched:

  a vertically

  b at an angle to the horizontal.

4 Identify the conditions under which it is possible to use a positive value for g.

5 Identify the conditions under which the following formulas can be applied.

  a $s_x = u_x t = v_x t$

  b $t = \dfrac{2u_y}{g}$

  c $|\vec{v}| = \sqrt{u_x^2 + (gt)^2}$

APPLYING

6 For a projectile fired with a speed of $40\,\text{m}\,\text{s}^{-1}$ at an angle of 30° above the horizontal find:

  a the maximum height above its launch height

  b the time taken to reach maximum height

  c the acceleration at the top of its flight

  d the velocity of the projectile after 1.0 s.

7 A river flows between two cliffs of equal height. A rocket is fired across the river from the edge of one cliff and lands on the edge of the other. If the rocket is launched with a speed of $50\,\text{m}\,\text{s}^{-1}$ at an angle 75° above the horizontal, find the distance across the river.

ANALYSING

8 Near Earth, the speed of a projectile at any position after launch can be given in terms of three variables: the initial speed, u, launch angle, θ, and time of flight, t. Demonstrate this proposition, starting from the equation $|\vec{v}| = \sqrt{u_x^2 + v_y^2}$.

9 Find the range of a projectile that is launched with a speed of $70\,\text{m}\,\text{s}^{-1}$ at an angle of 30° above the horizontal and which lands 30 m below its launch position. Is this range the same as for a launch angle of 60°?

2.4 Mandatory practical

EXPERIMENT 2.4.1

Projectile motion

Projectiles with the same launch speed over level ground travel different horizontal distances depending on the angle of launch. When air resistance is negligible and the acceleration due to gravity near the surface of Earth is constant, the range, R, is related to the magnitude of the launch velocity, u, and the angle of launch, θ, by the equation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

AIM

For a projectile that travels over level ground:

1 to demonstrate the relationship between launch angle and distance travelled

2 to compare theoretical with measured values for:

  a range

  b launch speed.

MATERIALS

- flat, horizontal surface
- curved track, such as a toy car track, mounted on a solid rigid base so that the end of the track is parallel to the base
- small ball, such as a ball bearing or glass marble
- wedges of different angles: 15°, 30°, 45°, 60° and 75°
- ruler
- carbon paper and A3 plain paper or sand tray for recording the landing position

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO REMAIN SAFE?
Small balls may roll across the floor and create a tripping hazard.	Keep balls in a secure container when not in use. Assign one group member to collect projected balls and return them to the container immediately.

RISK ASSESSMENT

PROCEDURE

1 Arrange the curved track on a table so that the ball can be projected at an angle to the horizontal.

2 Use the wedges to raise and lower the base of the track to different angles of elevation.

3 Arrange the recording system so that the landing position is horizontally opposite the launch position.

4 Release the ball from a position on the track, that is the same height above the launch position for each launch angle.

5 Record the horizontal distance, R, travelled by the ball.

6 For each angle, θ, record the horizontal distance, R, at least three times. »

RESULTS

- Ensure each data point is recorded in appropriately constructed data tables as it is produced.
- Estimate the uncertainty in each of the variables.
- Produce a summary data table of values measured for R and θ, including the uncertainty in each value.

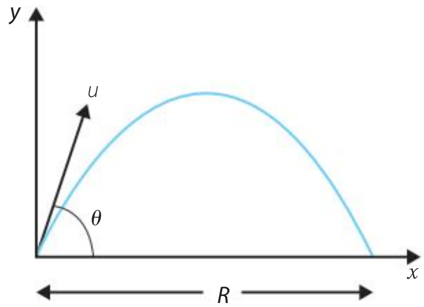

FIGURE 2.4.1 A ball is launched at an angle θ from a constant height, h, above the launch position. The landing position is at the same height above the ground as the launch position.

ANALYSIS OF RESULTS

1. Plot the data from the summary table on a correctly constructed graph of R vs θ, including uncertainty bars.
2. Draw the line of best fit.
3. Use the line of best fit to plot R vs $\sin 2\theta$.
4. Use the graph to find the experimentally determined launch speed.

DISCUSSION

1. Explain why the ball must always be released from the same vertical height above the launch position.
2. Explain why the landing position must always be the same height above the table as the launch position.
3. Explain the reason for taking three range measures for each angle.
4. Use conservation of energy to deduce the launch speed; hence, deduce the theoretical range for each of the launch angles.
5. Compare the theoretical values of R to the measured values of R, taking into account measurement uncertainties.
6. Explain how the experiment could be improved to collect more precise data.

CONCLUSION

1. Describe the experiment in two or three short sentences.
2. State any relationship that can be justified between measured variables.
3. Compare the measured values of R and u with the results predicted from theory.
4. Identify limitations in the experiment in two or three short sentences.

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTION

1 Two projectiles are launched from a cliff top. One projectile is launched upwards at an angle above the horizontal. The other is launched downwards at an angle below the horizontal. Compare the use of the acceleration due to gravity in each case when solving problems.

CATEGORY QUESTIONS

2 A projectile is launched from, and returns to, the ground. The launch velocity has horizontal and vertical components v_x and v_y respectively. Sketch the velocity–time graph for:

a the horizontal component of the motion

b the vertical component of the motion.

Use the gradient and areas under the graphs to derive the kinematic equations for projectile motion.

ELABORATION QUESTIONS

3 Show that the path of any projectile is a parabola, if air resistance is negligible and the value of g does not vary appreciably over the range of the trajectory.

4 An object is launched at speed u from different angles, θ, from the horizontal. It lands at a horizontal distance, x, which is the same distance, H, below the launch position in each case. Find the relationship between u, x, and θ.

EVIDENCE QUESTIONS

5 Relief supplies of mass 100 kg are to be dropped into a small clearing in a remote village by light aircraft. The pilot plans to release the supplies from a height of 200 m when the aircraft is 400 m from the drop zone and travelling horizontally at 250 km h^{-1}. Find the conditions under which the supplies will land on the drop zone. Discuss whether the pilot's plan is sensible or not.

6 The hammer throw is a field event at an athletics carnival. Use realistic values of launch speed, u, and height of release, h, to show how the distance travelled by the hammer depends on the launch angle, θ, and the height above the ground from which it is launched. Discuss the effect on the range of changes to the launch angle in the interval, $40° < \theta < 50°$.

END-OF-CHAPTER EXAM

End-of-chapter test

1 The vertical and horizontal components of a launch velocity, u, and angle of launch above the horizontal, α, are, respectively:

A $u \sin \alpha$; $u \tan \alpha$.

B $u \tan \alpha$; $u \cos \alpha$.

C $u \sin \alpha$; $u \cos \alpha$.

D $u \cos \alpha$; $u \sin \alpha$.

2 The maximum height reached by a rocket, which is launched at 30° to the horizontal at 320 m s^{-1} is closest to:

A 1300 m.

B 1.3×10^4 m.

C 0.13 km.

D 1.3 km.

3 A ball takes 1.5 s to travel from the thrower to the catcher, who is 45 m away. The ball is caught at its maximum height, which is 5.0 m above the launch height. The speed at which the ball was thrown was approximately:

A 30 m s^{-1}.

B 31 m s^{-1}.

C 33 m s^{-1}.

D 39 m s^{-1}.

4 Find the launch speed of a projectile that travels a horizontal distance of 200 m in 5.0 seconds after being launched at an angle below the horizontal of 45°.

A 56 m s^{-1}

B 40 m s^{-1}

C 28 m s^{-1}

D 20 m s^{-1}

5 For an object that lands below its launch point, the acceleration due to gravity is taken to be negative when the positive launch velocity is:

A above the horizontal and the vertical displacement on landing is positive.

B above the horizontal and the vertical displacement on landing is negative.

C below the horizontal and the vertical displacement on landing is negative.

D below the horizontal and the vertical displacement on landing is positive.

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6 A projectile is launched horizontally from a cliff at a speed of $25\,m\,s^{-1}$. It lands 75 m below the cliff. Calculate:

a the time taken for the projectile to land

b the speed of the projectile on landing

c the angle at which the projectile lands.

7 A rock is thrown downwards from the bank of a 235 m wide river at an angle of 15° to the vertical. It takes 1.76 s to land on the other side. Find the minimum launch speed.

8 A soccer ball is kicked off the ground at speed $30.0\,m\,s^{-1}$ and angle 42.0°. It lands on the ground some distance away. Calculate:

a the time of flight of the ball until it lands

b the maximum height above the ground attained by the soccer ball

c the velocity of the ball at the top of its flight

d the acceleration of the ball at the top of its flight.

9 State the conditions under which the horizontal and vertical components of projectile motion can be treated independently of each other.

10 A hammer is dropped out of a window 24.7 m above the ground at the same time as a ball is launched directly towards it from the ground and 42.8 m out from the wall. Find:

a the minimum speed of launch for which ball and hammer collide

b the maximum time elapsed before the ball and hammer collide.

11 Provide a connected narrative for scenario **A** *or* **B**. The narrative must include diagrams, calculations and quantitative data.

A An object is launched with a velocity of $30.0\,m\,s^{-1}$ at 30° to the horizontal. Compare the range of the object over level ground with the range for landing positions that are 8.0 m above and 10.0 m below the launch position. Discuss the range for these three landing positions if the launch angle is changed to 60°.

OR

B A shot put is launched from 2.00 m above the ground at an angle of 45.0°. It travels 18.00 m horizontally before landing. Discuss the possibility that, with the same launch speed, the shot put could have been thrown further. Support your answer quantitatively.

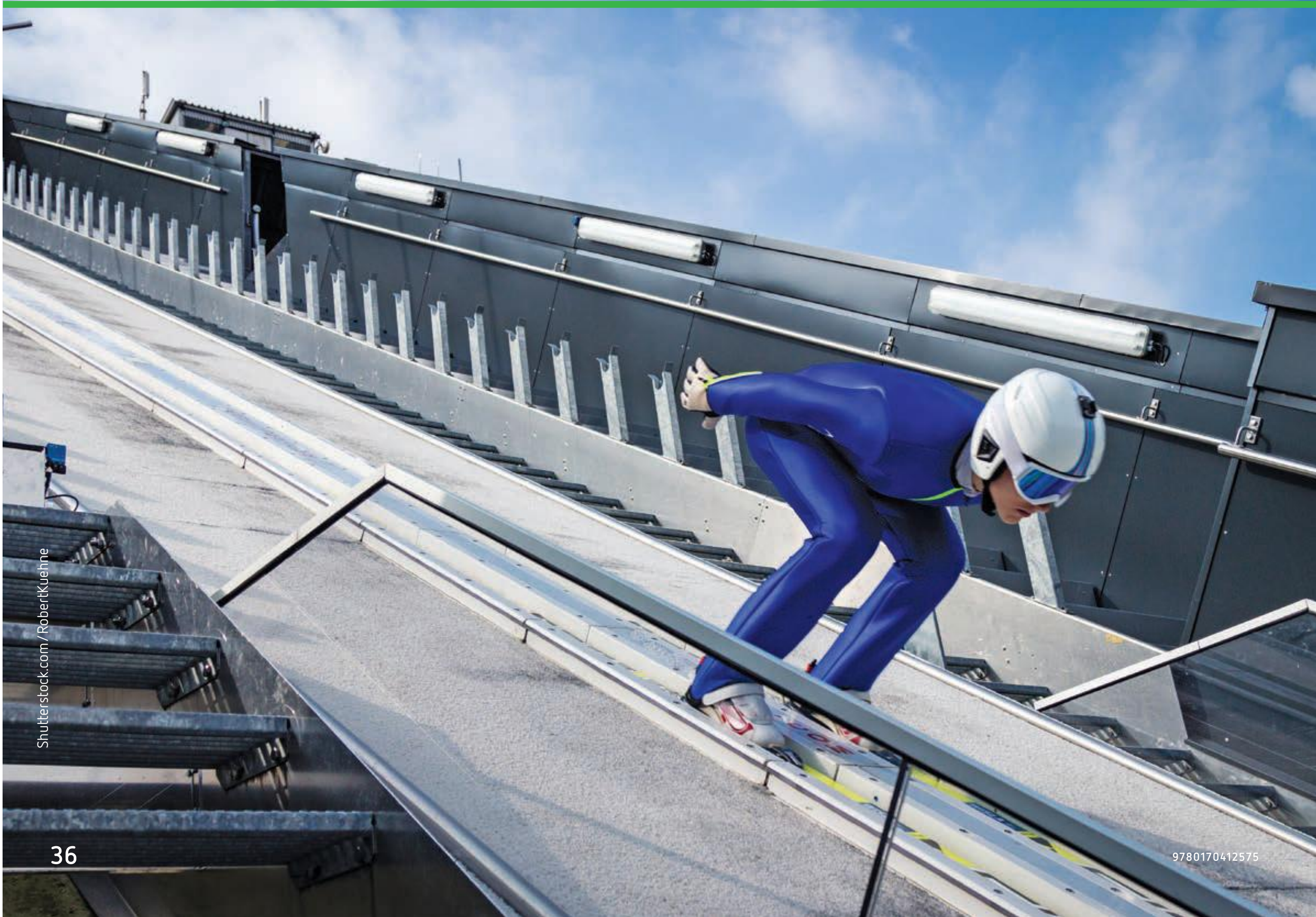
3 INCLINED PLANES

Introduction

An inclined plane is one of the simplest of the simple machines. Inclined planes are used as wedges and levers, and can be seen in winding roads and screw threads. Movement along a slope involves gravitational force and the forces applied by the surface – friction (parallel to the slope) and the normal force (perpendicular to the slope).

Stimulus question

How can the movement of objects up and down a slope be explained?



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3.1 Inclined planes

A box will stay right where it is on a horizontal floor unless it is forced to move. That is the consequence of Newton's first law. If the floor is raised up at an angle, the box will eventually start to slide down the slope. It accelerates down the slope when the net force is greater than zero. That is a consequence of Newton's second law. The box accelerates when the effect of the gravitational force on the box along the surface overcomes the frictional force applied by the surface.

The inclined plane

An inclined plane is a surface that has been tilted at an angle to the horizontal. This angle is called the **angle of inclination**. The angle of inclination always lies between zero and a right angle:

$$0° < \theta < 90°$$

angle of inclination
angle, θ, relative to the horizontal; $0° < \theta < 90°$

KEY FORMULA

Angle of inclination, θ

$0° < \theta < 90°$

Forces on an inclined plane

A mass on an inclined plane is subject to forces parallel to the plane and perpendicular to the plane. Some forces are gravitational in origin, others are electrostatic in origin. The combined effect of forces can be considered in terms of vector sums. These can be analysed as forces applied parallel to the surface and forces applied perpendicular to the surface.

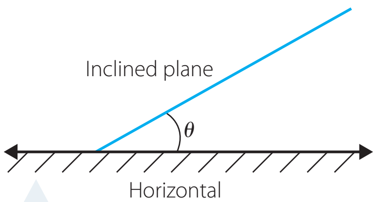

FIGURE 3.1.1 The angle of inclination, θ, is an angle relative to the horizontal.

Horizontal surface

An object that is not accelerating on a horizontal surface is subject to two forces:

- the normal force applied by the surface on the object perpendicular to the surface

$$\vec{F}_{\perp} \text{ (by surface on object) or } \vec{N}$$

- the friction force applied by the surface on the object parallel to the surface.

$$\vec{F}_{\parallel} \text{ (by surface on object) or } \vec{f}$$

3.1.1 Inclined planes
3.1.2 What are inclines?
3.1.3 Forces and motion basics

KEY FORMULA

Normal force:

$\vec{N} = \vec{F}_{\perp}$ (by surface on object)

Where:

$\vec{N}$ = normal force applied by a surface on an object, perpendicular to the surface (N)

$\vec{F}_{\perp}$ (by surface on object) = force applied by a surface on an object, perpendicular to the surface (N)

Friction force:

$\vec{f} = \vec{F}_{\parallel}$ (by surface on object)

Where:

$\vec{f}$ = friction force applied by a surface on an object, parallel to the surface (N)

$\vec{F}_{\parallel}$ (by surface on object) = force applied by a surface on an object, parallel to the surface (N)

When there is no acceleration, the net force on an object is zero. For a stationary box, there is no friction. In the vertical direction, the net force on the box is zero. It comprises the upwards normal force, $\vec{N}$ and the downwards weight force, $\vec{w}$. Applying Newton's second law:

$$\sum F = 0$$

$$\Rightarrow N - w = 0$$

$$\Rightarrow N = w$$

$$\Rightarrow N = mg$$

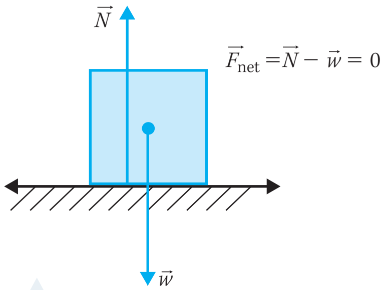

FIGURE 3.1.2 Horizontal plane: the net force acting on a stationary box is zero. In the vertical direction, $\sum \vec{F} = \vec{N} - \vec{w} = 0$.

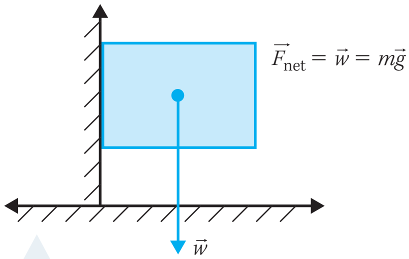

FIGURE 3.1.3 Vertical plane: In the vertical direction, the net force on the box is the weight force: $\sum \vec{F} = \vec{w} = m\vec{g}$.

Forces on a vertical surface

When the surface is tilted so that it is vertical, the box is no longer in contact with the surface. There is no friction. The normal force is zero. The box falls due to the gravitational force (weight). The net force is the weight.

Sliding on a frictionless inclined plane

If the surface is tilted at an angle somewhere between horizontal and vertical, and in the absence of friction, the normal force and the weight force combine to form a net force down the slope. The box slides down the slope with increasing speed (acceleration).

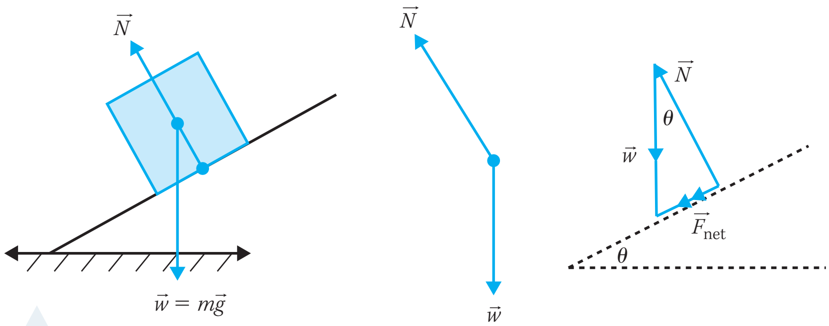

FIGURE 3.1.4 Inclined plane: for a frictionless surface, the net force is the vector sum of the normal force and the weight force.

Applying trigonometry to the vector sum gives values for $\sum \vec{F}$ and $\vec{N}$ in terms of the weight. The net force is found as follows:

$$\frac{\sum \vec{F}}{\vec{w}} = \sin\theta$$

$$\Rightarrow \sum F = mg\sin\theta$$

Thus, the net force down the slope is the rectangular component of the weight force acting down the slope. It is this component that causes the object to accelerate.

Similarly, the normal force is found as follows:

$$\frac{\vec{N}}{\vec{w}} = \cos\theta$$

$$\Rightarrow N = mg\cos\theta$$

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The normal force is the rectangular component of the weight force perpendicular to the slope. This component pulls the box into the surface at right angles to the surface, enabling the surfaces more or less to stick together. Gravitational force is responsible for the object being pulled into the surface. Electrostatic force is responsible for the surfaces sticking together. The net force perpendicular to the surface is zero, since the object does not leave or fall through the surface. Applying Newton's second law:

$$\Sigma F_{\perp} = 0$$
$$\Rightarrow N - mg\cos\theta = 0$$
$$\Rightarrow N = mg\cos\theta$$

KEY FORMULA

Parallel to surface:

$\Sigma F_{\parallel} = mg\sin\theta = ma$

$\Rightarrow a = g\sin\theta$

Where:

$\Sigma F_{\parallel}$ = net force parallel to the surface (N)

m = mass (kg)

g = gravitational force = 9.8 m s^{-2}

θ = angle of inclination (°)

a = acceleration along the slope m s^{-2}

Sliding on an inclined plane with friction

Friction is a force that occurs when one surface affects the movement of another surface. At the microscopic level, the base of a box and an inclined plane both have bumps and hollows. When the two surfaces are pushed together, strong attractive forces stick the molecules of both surfaces to each other.

This **static friction** must be overcome before the box can move. Since static friction depends on the force applied by the box on the surface, static friction depends on the perpendicular component of the weight force. Static friction rises to a maximum value, at which point the box begins to accelerate.

Once the box begins to move, the friction force between the surfaces reduces. This **kinetic or sliding friction** can be considered to be a constant force that opposes the motion of the box. Kinetic friction is also dependent on the gravitational force component perpendicular to the surface because this component of the weight force pulls the surfaces together so that they more readily stick.

KEY FORMULA

Perpendicular to surface

$\Sigma F_{\perp} = N - mg\cos\theta = 0$

Where:

$\Sigma F_{\perp}$ = net force perpendicular to the surface

N = normal force (N)

m = mass (kg)

g = gravitational force = 9.8 m s^{-2}

θ = angle of inclination (°)

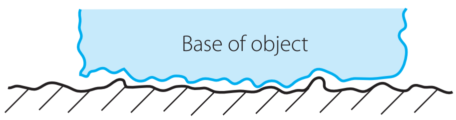

FIGURE 3.1.5 Microscopic and sub-microscopic surface irregularities contribute to the sticking together of surfaces, hence friction.

static friction force that impedes motion up to the point where motion begins

kinetic or sliding friction force that impedes motion once motion has begun

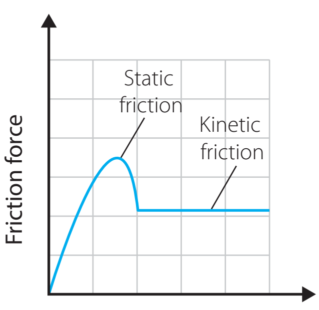

FIGURE 3.1.6 As force is applied to a box, the static friction rises linearly to a maximum. Once moving, the kinetic friction applied to the box is constant.

Figure 3.1.6 shows that static friction rises to a maximum as force is applied to the box. At this point, the kinetic friction becomes a constant, but lesser value, than the maximum static friction.

On surfaces involving friction, the kinetic friction opposes the motion caused by the component of the weight parallel to the slope. Newton's second law is used to find the net force:

$$\Sigma F_{\parallel} = ma$$
$$\Rightarrow mg\sin\theta - f = ma$$

KEY FORMULA

When friction is involved

Parallel to the slope:

$\Sigma F_{\parallel} = ma$

$\Rightarrow mg\sin\theta - f = ma$

Where:

$\Sigma F_{\parallel}$ = net force object parallel to the slope (N)

m = mass (kg)

g = gravitational force = 9.8 m s^{-2}

θ = angle of inclination (°)

f = kinetic friction (N)

a = acceleration of the mass m s^{-2}

KEY FORMULA

Perpendicular to the slope:

$\Sigma F_{\perp} = 0$

$\Rightarrow N - mg\cos\theta = 0$

Where:

$\Sigma F_{\perp}$ = net force object perpendicular to the slope (N)

N = normal force (N)

m = mass (kg)

g = gravitational force = 9.8 m s^{-2}

θ = angle of slope, $0° < \theta < 90°$ (°)

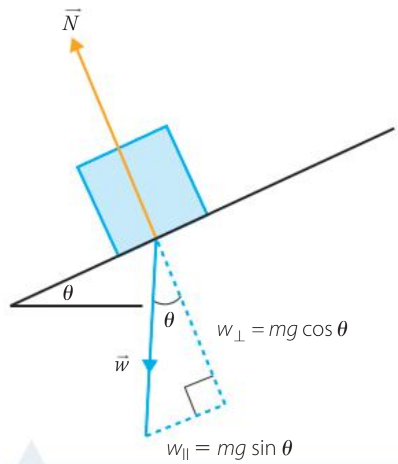

FIGURE 3.1.7 Perpendicular and parallel components of forces acting on a box on an incline

Sliding and rolling friction

rolling friction force that acts to oppose motion and that rotates an object as it rolls along a slope

Kinetic friction acts against the motion of an object sliding down a slope. The friction involved in rolling an object down a slope is typically less than sliding friction. This **rolling friction** causes an object to roll, even as the object moves down a slope under the effect of the weight component. Rolling friction contributes to the linear acceleration of a rolling object down the slope being less than that of a sliding object. The distribution of the mass throughout the object also contributes to the acceleration along the slope. These two reasons are why the linear acceleration down the slope of a rolling object is not equal to the linear acceleration due to the component of the weight acting parallel to the surface.

PRACTICAL ACTIVITY 3.3.1

Measuring friction

INTRODUCTION

An object will start to slide when the angle of inclination of the plane on which the object is sitting reaches a particular value. The angle can be used to find the static friction between the object and the plane. When sliding, the object will experience a force of sliding or kinetic friction that is less than the static friction.

AIM

To measure static and sliding friction between an object, for example a smartphone, and an inclined plane

MATERIALS

- flat surface made from a solid material such as wood, melamine or plastic
- a range of rectangular blocks ranging in height from 1 cm to 10 cm
- smartphone with protective cover and accelerometer app
- ruler
- weighing scale

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PROCEDURE

1. Weigh the smartphone and record its mass.
2. Place the smartphone, cover down, horizontally on the plane and activate the accelerometer app.
3. Record the distance from the end of the plane to the front end of the smartphone.
4. Use the blocks to lift the end of the plane until the smartphone just starts to move.
5. Record the height of the front end of the smartphone above the horizontal at the angle for which the smartphone just starts to move.
6. Increase the angle by approximately 10° (do this 5 times).
7. Let the smartphone slide down the slope.
8. For each angle tested, collect data for the acceleration at a point halfway down the slope.
9. Repeat the procedure (steps **2**–**8**) three times.

RESULTS

1. Calculate the following quantities and their uncertainties:
   - a angle of inclination
   - b component of weight: (**i**) parallel to the surface and (**ii**) perpendicular to the surface
   - c normal force
   - d static friction
   - e sliding friction.
2. Draw the following graphs:
   - a friction vs angle of inclination
   - b friction vs applied force parallel to the slope
   - c friction vs applied force perpendicular to the slope.

DISCUSSION

1. On the graphs, identify the sections for which:
   - a no motion occurred
   - b motion occurred
   - c there was the maximum static friction
   - d there was sliding friction.
2. Explain why the procedure was repeated three times (step **9**).
3. Specify any experimentally justifiable relationships between friction and other variables.
4. Show one representative set of calculations for all derived quantities.
5. Identify limitations in the experimental design and procedures.
6. Discuss how the experiment could be improved to produce more precise results.

CONCLUSION

1. Summarise the purpose of the experiment.
2. Use a diagram to describe the method.
3. State precise values of:
   - a static friction
   - b sliding friction.
4. Summarise the limitations and improvements that were identified in the Discussion section above.

SECTION REVIEW 3.1

REMEMBERING

1 Define the following terms of forces applied by the surface.
   a Friction force
   b Normal force

2 Write vector equations for the net force on a mass that is sliding on a frictionless surface when the surface is:
   a horizontal
   b inclined at an angle to the horizontal
   c vertical.

3 For an inclined plane where friction is involved, write equations for net force:
   a parallel to the surface
   b perpendicular to the surface.

UNDERSTANDING

4 Explain the origin of friction between surfaces.

5 Explain why friction depends on the mass of an object.

6 Compare static friction and kinetic friction.

7 Draw a vector diagram to show the forces applied to an object that is accelerating down:
   a a frictionless surface
   b a surface where friction is involved.

APPLYING

8 A maximum static friction force of 15 N is applied to a 3.0 kg mass on an inclined plane.
   a Find the net force on the mass perpendicular to the surface.
   b Find the net force on the mass parallel to the surface.
   c Determine the angle of inclination of the slope.

9 A 120 kg toboggan and rider slides from rest for 120 m down a frictionless icy surface at an angle of 25° to the horizontal.
   a Calculate the normal force applied by the surface on the toboggan and rider.
   b Calculate the acceleration of the toboggan.
   c Determine the increase in speed of the rider by the end of the slope.

ANALYSING

10 A mass of 20 kg is placed on a rough horizontal surface. The surface is then rotated to an angle of inclination of 30°. At this angle, the object is just about to move.
   a Explain why this angle can be used to measure static friction, not kinetic friction.
   b Calculate the static friction.

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3.2 Solving problems: inclined planes

When solving problems involving motion on an inclined plane, follow the steps below.

1. Read the question carefully.
2. Visualise or sketch the real situation described.
3. Draw a free-body diagram.
   a. Identify each force acting on the object in question.
   b. Write each force in symbol form: normal, weight and, if appropriate, friction.
   c. Add any data provided in the question.
   d. Define the direction of the net force (or acceleration).
4. Consider whether to draw a vector diagram.
   a. For frictionless motion: $\vec{N} + \vec{w} = m\vec{a}$, noting that $\vec{a}$ must be parallel to the surface (the vector signs reduce to a directed number line).
   b. For motion involving friction: $\vec{N} + \vec{w} - \vec{f} = m\vec{a}$, noting that $\vec{f}$ and $\vec{a}$ must be parallel to the surface (the vector signs reduce to a directed number line).
5. Separate the analysis into motion that is:
   a. parallel to the surface. *Use Newton's second law to set up equations along the surface:*
      i. without friction: $\Sigma F_{||} = mg\sin\theta = ma$; $a = g\sin\theta$
      ii. with friction: $\Sigma F_{||} = mg\sin\theta - f = ma$
   b. perpendicular to the surface. *Use Newton's second law to set up equations at right angles to the surface:* $\Sigma F_{\perp} = N - mg\cos\theta = 0$.
6. Transpose formulas for the required unknown variable or substitute values directly into the equation.
7. Consider whether any of the *suvat* kinematic formulas should be used to complete your answer to the question.
8. Solve the equations.
9. Check to ensure the answers are those required.

WORKED EXAMPLE 3.2.1

A 1000 kg car is sliding down a frictionless plane that is inclined at an angle of 15° to the horizontal.

1. On the diagram use vectors to show the forces acting on the car.
2. Sketch a vector sum diagram.
3. Calculate values for:
   a. the force acting parallel to the surface
   b. the force that stops the car being pushed off the surface
   c. the acceleration of the car.

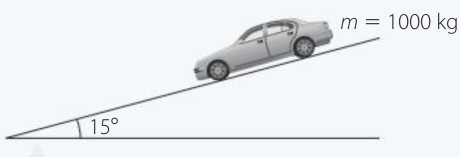

FIGURE 3.2.1

ANSWER

1

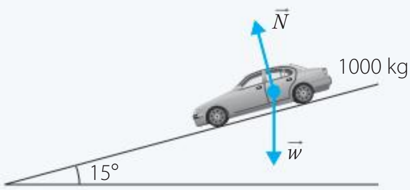

FIGURE 3.2.2

2

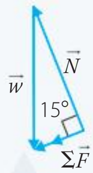

FIGURE 3.2.3

3 **a** Parallel to the surface:

$\Sigma F_{||} = mg\sin\theta$

$\Rightarrow F_{||} = 1000\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 15°$

$\Rightarrow F_{||} = 2.5 \times 10^{3}\,\text{N}$

b Perpendicular to the surface:

$F_{\perp} = mg\cos\theta$

$\Rightarrow F_{\perp} = 1000\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \cos 15°$

$\Rightarrow F_{\perp} = 9.5 \times 10^{3}\,\text{N}$

c $a = g\sin\theta$

$\Rightarrow a = 9.8\,\text{m s}^{-2} \times \sin 15°$

$\Rightarrow a = 2.5\,\text{m s}^{-2}$

WORKED EXAMPLE 3.2.2

A 50 kg box slides down a ramp that has an angle of inclination of 60°. There is a 15 N friction force between the ramp and the box.

1. Find the net force on the box.
2. Calculate the acceleration of the box.
3. Determine the normal force on the box.

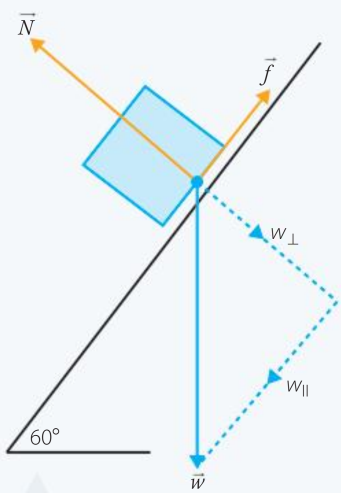

FIGURE 3.2.4

ANSWER

1 Parallel to the slope:

$\Sigma F_{||} = ma$

$\Rightarrow mg\sin\theta - f = ma$

Perpendicular to the slope:

$\Sigma F_{\perp} = ma$

$\Rightarrow N - mg\cos\theta = 0$

Therefore, the net force on the box is the net force parallel to the slope:

$\Sigma F_{||} = mg\sin\theta - f$

$\Rightarrow \Sigma F_{||} = 50\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 60° - 15\,\text{N}$

$\Rightarrow \Sigma F_{||} = 424.3\,\text{N} - 15\,\text{N}$

$\Rightarrow \Sigma F_{||} = 4.1 \times 10^{2}\,\text{N}$

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2 $a = \frac{\Sigma F}{m}$

$\Rightarrow a = \frac{409.34\ \text{N}}{50\ \text{kg}}$

$\Rightarrow a = 8.2\ \text{m s}^{-2}$

3 Perpendicular to the slope

$N - mg\cos\theta = 0$

$\Rightarrow N = mg\cos\theta$

$\Rightarrow N = 50\ \text{kg} \times 9.8\ \text{m s}^{-2} \times \cos 60°$

$\Rightarrow N = 245\ \text{N}$

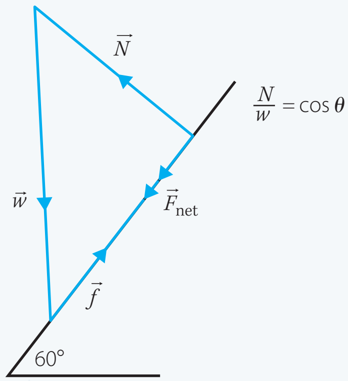

FIGURE 3.2.5

SECTION REVIEW 3.2

REMEMBERING

1 List the steps used when solving problems involving an inclined plane.

2 For a mass sliding down a frictionless inclined plane, draw and label with force arrows the following diagrams:

a a reality diagram b a free-body diagram c a vector diagram.

3 Write net equations in terms of forces acting on an object:

a perpendicular to the surface

b parallel to a frictionless surface

c parallel to a surface involving friction.

UNDERSTANDING

4 Explain why the net force on an object that is sliding on an inclined plane is equal to the net force parallel to the plane only.

5 Use Newton's laws to explain how a mass can slide on an inclined plane at constant speed.

APPLYING

6 A frictionless plane is inclined at an angle of 20° to the horizontal. A 100 kg box slides down the plane. Calculate values for:

a the force acting parallel to the surface

b the normal force

c the acceleration of the car.

7 A 400 kg motorcycle is sliding down a 6° plane at $3.0\ \text{m s}^{-1}$. Find the friction force applied to the motorcycle.

ANALYSING

8 A 2.0 kg box on a plane inclined at 25° is just about to start to slide.

a Draw and label with force arrows the following diagrams:

i a reality diagram ii a free-body diagram iii a vector diagram.

b Find the maximum static friction force acting on the box by the surface.

9 A 60 kg box slides on a frictionless plane inclined at 15°. Find the magnitude and direction of the external force other than the weight that must be applied to the box to achieve an acceleration of $3.2\ \text{m s}^{-2}$:

a down the slope

b up the slope.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Compare free-body and force diagrams for a mass sliding on an incline where friction is, or is not, involved.

2 Explain why static friction and kinetic friction are not the same.

CATEGORY QUESTION

3 'The normal reaction and the component of the weight force perpendicular to the surface are an action–reaction pair.' Is this statement true? Discuss.

ELABORATION QUESTIONS

4 A 150 kg mass on a frictionless plane is 3.2 m from the lower end. The mass is raised to a height of 145 cm. At this point, the mass is about to slide. At a height of 1.50 m, the mass accelerates at 1.4 m s^{-2}. Draw the graph of friction vs net force applied to the mass.

5 A mass is free to slide on a plane that is rotated from horizontal to vertical.

a Construct graphs to demonstrate the relationship between the angle of inclination and the forces applied to the mass:

i perpendicular to the surface

ii parallel to the surface.

b Explain how these graphs would differ if friction were involved.

EVIDENCE QUESTION

6 Speed-skating events are undertaken indoors in front of an audience. As a session proceeds, the temperature of the air in the arena increases. Compare the forces applied by skaters who compete early in a session compared to those who skate later in the same session.

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END-OF-CHAPTER EXAM

End-of-chapter test

PHYSICS

1 The angle of inclination of a plane is the angle:

A to the horizontal.

B to the vertical.

C perpendicular to the plane.

D parallel to the plane.

2 On an inclined plane, the normal force is:

A the reaction to the component of the weight parallel to the plane.

B the reaction to the weight.

C the force applied on a mass by the surface perpendicular to the plane.

D the force applied on a mass by the surface parallel to the plane.

3 The acceleration of a 10 kg mass sliding on a plane inclined at 30° is:

A $9.8\,m\,s^{-2}$ **B** $8.5\,m\,s^{-2}$ **C** $4.9\,m\,s^{-2}$ **D** $2.8\,m\,s^{-2}$

4 A 5.0 kg mass is situated 1.8 m from the lower end of an inclined plane. It is about to slide when it is 45 cm above the horizontal. The magnitude and type of force keeping the mass in place is:

A 13 N; static friction.

B 13 N; kinetic friction.

C 49 N; static friction.

D 49 N; kinetic friction.

5 A rope is tied to a 65 kg box on a 30° slope. The friction between the box and the slope is 65 N. The tension in the rope is:

A 384 N. **B** 319 N. **C** 254 N. **D** 65 N.

6 Friction on an inclined plane may point up or down the slope. Explain.

7 Compare static and kinetic friction.

8 A 750 kg car is about to slide down a frictionless plane that is inclined at an angle of 10° to the horizontal.

a On the diagram, use vectors to show the force or forces applied on the surface by the car. Label each force.

b Sketch the vector sum of the forces that act on the car.

c Calculate the magnitude of the force that stops the car being pushed off the surface.

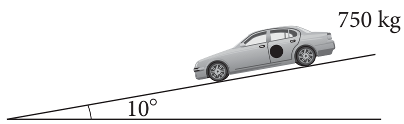

FIGURE 3.3.1

9 A 90 kg snowboarder accelerates uniformly from rest at $2.70\,m\,s^{-2}$ down a 60° slope.

a Calculate the total resistance force on the snowboarder.

b Calculate the speed of the snowboarder after 100 m of travel down the slope.

10 A 31 kg child on a 4.0 kg luge accelerates uniformly from rest to a maximum speed of $20\,m\,s^{-1}$ in 5.0 s down a 30° slope. The child maintains this speed for a further 5.0 s.

a Calculate the normal force on the child and luge system.

b Find the net force on the child 3.0 s after the start.

c Determine the maximum kinetic frictional force on the child and luge system.

4 CIRCULAR MOTION

Introduction

An ice skater glides effortlessly in a straight line at constant speed. That is a consequence of Newton's first law: a moving object travels at constant speed in a straight line unless forced to change speed, direction, or speed and direction simultaneously. An ice skater travelling at constant speed can change direction without a change of speed. In order to change direction, a force must be applied to the ice skater. If the force is applied continuously, the skater can change direction continuously. With the right amount of continuous force, the skater can complete a circle.

Stimulus questions

Is it possible to travel at a constant speed yet be accelerating?

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4.1 Uniform circular motion

Ice skaters, cars, cyclists and runners can all move at constant speed while changing direction. Each motion can be modelled in terms of a point particle for which the mass is concentrated at the centre of mass.

4.1.1 Uniform circular motion: interactive

Period and frequency

Uniform circular motion of a point particle is described in terms of the radius of the circle, r, and the time taken for the particle to complete one revolution, T, called the **period**.

For repetitive circular motion, such as a ball being swung around on the end of a string, the time taken for one revolution, T, is related to the number of times per second that the object describes a circle in a unit of time. The number of times per second is called the **frequency**, f. It is measured in units of hertz, Hz or s^{-1}. For example, if a ball takes 0.1 second to go once around a circle ($T = 0.1$ s), the number of times per second it goes around is 10 ($f = 10$ Hz). Thus:

$$f = \frac{1}{T}$$

period
time taken for an object undergoing circular motion to complete one revolution

frequency
number of times a circular motion is completed in a time period

KEY FORMULA

$$f = \frac{1}{T}$$

Where:

f = frequency (Hz; s^{-1})

T = period (s)

Average speed and period

The speed of a particle, v, is related to the two fundamental variables of distance and time. For uniform circular motion, the instantaneous speed at any point, v_{inst}, is the same as the average speed, v_{av}, between any two points or measured over one complete revolution. The distance covered in one revolution of a circle is the circumference. Thus, from the definition of average speed:

$$v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\Rightarrow v_{av} = \frac{\text{circumference}}{\text{period}}$$

$$\Rightarrow v_{av} = \frac{2\pi r}{T}$$

$$v = \frac{2\pi r}{T}$$

$$\text{But } f = \frac{1}{T}$$

$$\Rightarrow v = 2\pi r f$$

Since v_{inst} and v_{av} are the same for uniform circular motion, the subscripts will be dropped and the symbol, v, will be used.

KEY FORMULA

$$v = \frac{2\pi r}{T} = 2\pi r f$$

Where:

r = radius (m)

T = period (s)

f = frequency (Hz)

v = average and instantaneous speed ($m\,s^{-1}$)

KEY CONCEPT

Circular functions and radians

Circular functions are projections of circular motion onto x- and y-axes.

The role of the factors $\frac{2\pi}{T}$ or $2\pi f$ in circular functions relate directly to the way in which angles change. Angular measurement can be done in radians as well as degrees.

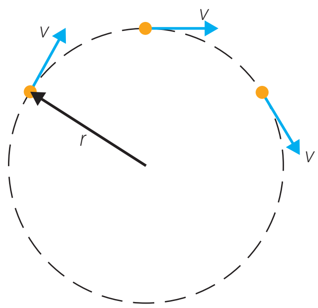

FIGURE 4.1.1 In circular motion, the velocity vector changes direction continuously. It travels the circumference of the circle in one time period.

SECTION REVIEW 4.1

REMEMBERING

1 Define the following terms:

a frequency

b period.

2 Write down the formula that links:

a frequency to period

b speed to radius and period

c speed to radius and frequency.

UNDERSTANDING

3 Explain why average and instantaneous speeds are always equal for uniform motion.

APPLYING

4 Find the frequency for an object that has a period of:

a 50 s

b 0.34 s

c 2.8×10^{-3} s.

5 Find the speed of an object moving at uniform circular speed for the following conditions:

a radius = 1.2 m; period = 0.3 s

b radius = 0.37 m; period = 2.9×10^{-2} s

c 1.5×10^{8} km (Earth's orbital radius), period = 365.25 days.

ANALYSING

6 From the point of view of an observer on Earth, the planet Jupiter appears to be moving away in a straight line along its orbit around the Sun. A point, J, on Jupiter's equator is rotating towards the observer. The observer on Earth wants to find the speed of the point, J.

a Identify the data the observer needs to collect.

b Describe the calculations the observer must undertake.

4.2 Solving problems: circular motion

Period and frequency

For questions requiring the conversion of period to frequency or frequency to period follow the steps below.

- Read the question carefully.
- Identify the quantity provided.
- Identify the appropriate equation:

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

- Substitute values, including units.
- Solve the equation.
- Check to ensure the question has been answered in the correct units.

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Average speed, radius, period and frequency

For questions requiring the calculation of speed, radius, period or frequency follow the steps below.

- Read the question carefully.
- Identify the quantities provided.
- Write the appropriate equation:

$$v = \frac{2\pi r}{T} \text{ or } v = 2\pi r f$$

- If necessary, transpose the equation to make the required variable the subject.
- Substitute values, including units.
- Solve the equation.
- Check to ensure the question has been answered in the correct units.

SECTION REVIEW 4.2

REMEMBERING

1 Write down the sequence of steps for converting period to frequency and frequency to period.

2 Write down the sequence of steps for calculating speed, radius, period or frequency.

UNDERSTANDING

3 **a** Explain why period and frequency are the inverse of each other.

b Show the steps that lead from $v = \frac{2\pi r}{T}$ to $v = 2\pi rf$.

4 Transpose $v = \frac{2\pi r}{T}$ and $v = 2\pi rf$ to make equations with the following subjects:

a r

b T

c f.

APPLYING

5 Find the period of an object that has the following frequency (provide answers in ms):

a 12 Hz

b 4.9×10^3 Hz

c 2.5×10^{14} Hz.

6 Find the period of an object moving at uniform circular speed for the following conditions:

a radius = 35 m; speed = 4.2×10^7 m s^{-1}

b radius = 5.6×10^4 km (Earth's radius); speed = 4.1×10^3 m s^{-1}.

7 Find the radius of orbit for an object moving at uniform circular speed for the following conditions:

a speed = 3.0×10^5 m s^{-1}; frequency = 3.0×10^5 Hz

b speed = 4.5 m s^{-1}; period = 28 days (the Moon's rotational period).

ANALYSING

8 The factor $\frac{2\pi}{T}$ is the angular speed. Explain.

9 A vinyl record rotates at 33.3 revolutions per minute (33.3 rpm). Point A is halfway from the centre to point B, which is on the edge of the record. Calculate the ratio of the speed of point A to the speed of point B. Show your working.

REFLECTING

10 Create a set of mnemonics to help transpose the equations in this section.

4.3.1 What is centripetal acceleration?

4.3.2 Visual understanding of centripetal acceleration formula

4.3 Centripetal acceleration and force

Uniform circular motion means that the speed is constant. But the speed is constantly changing direction. Consequently, the velocity changes. This velocity change occurs over a time interval. Thus, circular motion is accelerated motion. According to Newton's second law, this acceleration is caused by a net force and affected by the mass that is being forced to change its state of motion.

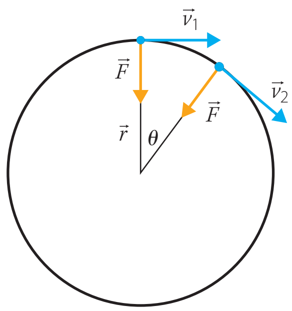

FIGURE 4.3.1 Derivation of centripetal acceleration: two equal magnitude velocity vectors are shown separated by a time interval, which is represented by the angle between the equal magnitude radius vectors.

Centripetal acceleration

An object circling around a point must be pushed continuously inwards. The acceleration is therefore directed inwards. In the case of uniform circular motion, the acceleration is always directed towards the exact centre of the motion. Figure 4.3.1 is used to justify this assertion.

The change in velocity is:

$$\Delta\vec{v} = \vec{v}_2 + {}^{-}\vec{v}_1 = \vec{v}_2 - \vec{v}_1$$

This is shown geometrically as a vector subtraction in Figure 4.3.2.

The average acceleration, $\vec{a}_{av}$ is the change in velocity over the time interval:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\Rightarrow \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

The direction of the average acceleration vector is the same as the direction of the change in velocity vector.

For uniform circular motion, the change in velocity is smooth. The instantaneous change of velocity vector for a time interval occurs approximately halfway through the time interval. The smaller the time interval, the closer this approximation comes to the actual value. In the limit, when the time interval approaches zero, the change in the velocity vector points directly towards the centre of the circle. The average and instantaneous acceleration vectors become the same. Acceleration always points to the centre of the circle. This pointing to the centre is called **centripetal**; the acceleration is a centre-seeking, centripetal acceleration.

centripetal centre-seeking; directed towards the centre

The relationship between the speed and the radius can now be deduced. In Figure 4.3.4, the length of the circumference interval, $v\Delta t$, is approximated by a straight line because the time interval is very

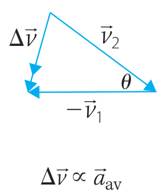

FIGURE 4.3.2 Change of velocity is a geometrical, vector difference

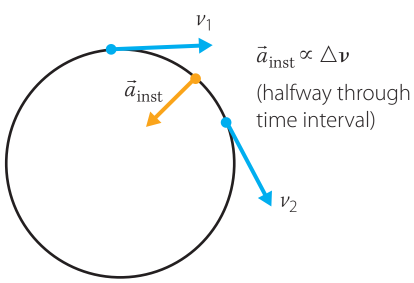

FIGURE 4.3.3 For smooth changes, the instantaneous acceleration occurs approximately halfway through the time interval. For very small time intervals, the approximation becomes $\vec{a}_{av} = \vec{a}_{inst}$. The acceleration is centripetal (centre-seeking).

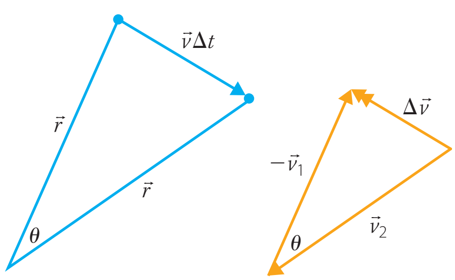

FIGURE 4.3.4 The displacement and velocity change vector diagrams are similar for small time intervals and uniform circular motion. In similar triangles, ratios of corresponding sides are equal.

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small. The vector subtraction in Figure 4.3.4 is similar to this triangle because both triangles are isosceles triangles with the same angle between the equal sides.

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\Rightarrow \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$\Rightarrow a = \frac{v^2}{r}$$

KEY FORMULA

$$a = \frac{v^2}{r}$$

Where:

a = acceleration ($m\,s^{-2}$)

v = speed ($m\,s^{-1}$)

r = radius (m)

This equation links a, v and r. By substituting $v = \frac{2\pi r}{T}$ into this equation, a related equation can be deduced to connect a, r and T:

$$a = \frac{v^2}{r}$$

$$\Rightarrow a = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\left(\frac{2\pi}{T}\right)^2 \times r^2}{r}$$

$$\Rightarrow a = \left(\frac{2\pi}{T}\right)^2 \times r$$

$$\Rightarrow a = \frac{4\pi^2 r}{T^2}$$

KEY FORMULA

$$a = \frac{4\pi^2 r}{T^2}$$

Where:

a = acceleration ($m\,s^{-2}$)

r = radius (m)

T = period (s)

Net force causes circular motion

An object moving in uniform circular motion undergoes centripetal acceleration. This means that the net force applied to the object is also directed towards the centre. Another name for this net force is **centripetal force**. Centripetal force is not a new kind of force like the normal force, friction, tension, gravitational force, electrostatic force or magnetic force. These forces are real forces in their own right. Centripetal force is the name given to the sum of the real forces that act on a particle to cause it to undergo circular motion.

centripetal force in uniform circular motion, the sum of real forces that point towards the centre of the circle

Newton's second law applies to circular motion as follows. The centre-seeking (centripetal) acceleration, a, on a mass, m, is caused by a net force, $\Sigma\vec{F}$:

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

$$\Rightarrow \Sigma\vec{F} = m\vec{a}$$

Substituting the expressions for acceleration derived in Section 4.1, equations can be derived for the net force:

When m, v and r are provided: $\Sigma\vec{F} = m\frac{v^2}{r}$

When m, r and T are provided: $\Sigma\vec{F} = m\frac{4\pi^2 r}{T^2}$

KEY FORMULA

$$\Sigma\vec{F} = m\frac{v^2}{r} \text{ and } \Sigma\vec{F} = m\frac{4\pi^2 r}{T^2}$$

Where:

$\Sigma\vec{F}$ = net force directed towards the centre of the circle (N)

m = mass (kg)

v = speed ($m\,s^{-1}$)

r = radius (m)

T = period (s)

Types of force that contribute to net force

Real forces are responsible for the net force that causes circular motion. Real forces include electrostatic forces (normal, friction, electric, tension), gravitational force (weight) and magnetic forces.

The normal force is always at right angles to a surface. For example, it is the normal force that causes a person to go around inside a funpark rotor.

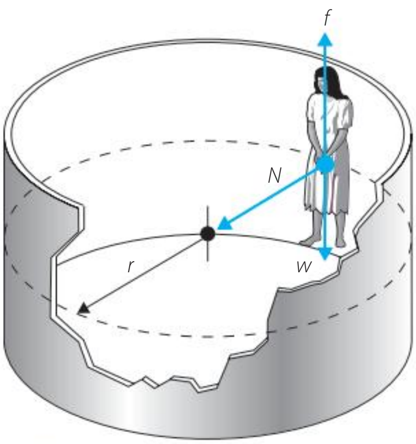

FIGURE 4.3.5 The normal force, N, applied by the rotor wall causes a person to move in a circular path.

FIGURE 4.3.6 The outwards directed action force by the car on the surface affects the surface. The inwards reaction force applied by the surface friction on the wheels of the car causes the car to move in a circular path.

Friction by the ground on an object, such as an athlete's foot or the tyres of bicycles or the wheels of cars, causes the object to turn corners. In terms of Newton's third law, the action force applied by the object on the surface, which is directed away from the centre is equal to the opposite reaction force by the surface on the object. It is this reaction force by the surface on the object that causes the object to change direction.

Tension force by a string makes it possible to whirl an object around in a circle.

Gravitational force is responsible for the motion of the Moon and other satellites around Earth, and the planets around the Sun.

Electrostatic force causes moving charged particles to circulate around a central charge, such as the electron in a hydrogen atom, which goes around the proton.

Magnetic forces act at right angles to the motion of charged particle streams to cause them to travel in circles. This occurs in a mass spectroscope where ions are given kinetic energy, then injected into a magnetic field. The different circular paths depend on the charge of and mass of the ions.

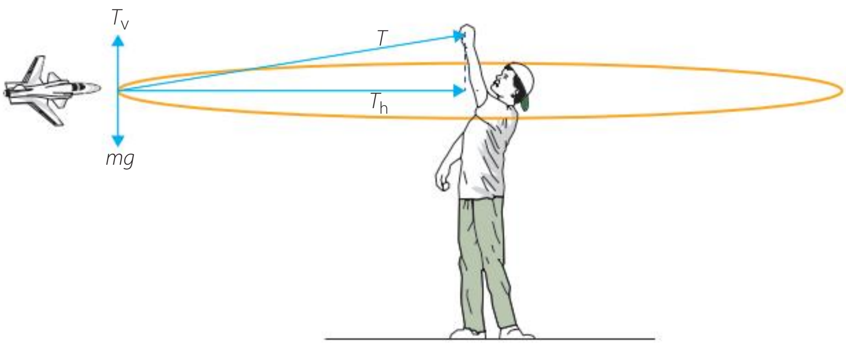

FIGURE 4.3.7 The horizontal component, T_h, of the force applied by the string in tension, T, on the toy plane causes it to move in a circular path.

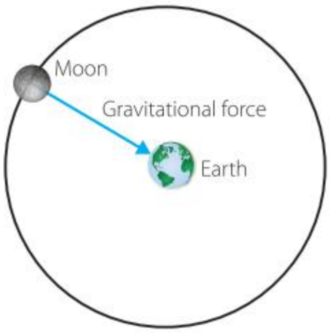

FIGURE 4.3.8 The force applied by the mass of Earth on the mass of the Moon causes the Moon to move in a circular orbit.

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One or more of these forces may be applied to a particle in order to move it in a circle. For example, the normal force and the weight force may act on a cyclist to enable uniform circular movement on the banked track of a velodrome. These two forces combine to produce the net force on the cyclist. This sum of forces is centre-directed, hence centripetal. This centripetal force is the sum of different forces, but is not itself a new kind of force.

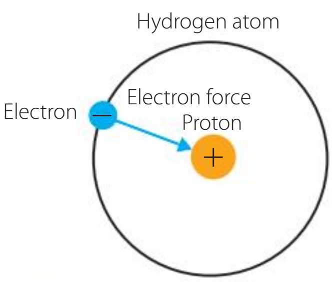

FIGURE 4.3.9 The electrostatic force applied by the central charge (proton) on the moving electron causes the electron to move in a circular orbit.

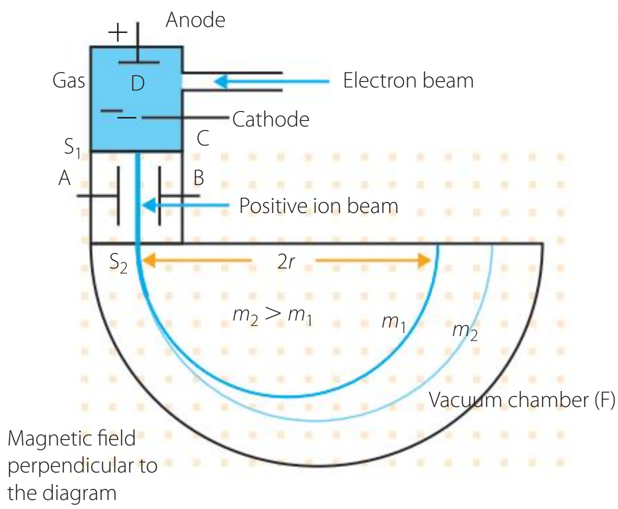

FIGURE 4.3.10 In a mass spectrometer, the force applied by the magnetic field on the flow of charged ions causes the ions to move in circular orbits that depend on their mass. The smaller the mass the smaller the radius.

PRACTICAL ACTIVITY 4.3.1

Net force and circular motion

INTRODUCTION

A mass can be whirled around in a horizontal circle on the end of a string. For a particular radius, the force applied to the mass is related to the frequency:

$$F(\text{by string on mass}) = \frac{2\pi r}{T} = 2\pi r f$$

$$\Rightarrow f = \frac{F(\text{by string on mass})}{2\pi r}$$

In this practical activity, the radius is kept at a constant length, the force is changed and the frequency measured.

MATERIALS

- two-hole rubber stopper
- 2 m of strong inextensible string (or fishing line)
- 15–20 cm long glass or plastic tube with polished ends
- 30 metal washers of equal mass
- paperclip
- crocodile clip
- metre ruler
- stopwatch

»

RISK ANALYSIS

Construct a table similar to the one below. Identify specific risks to a person's safety and ways to manage these risks.

Do not proceed to undertake any trial run or any actual measurements until the risk analysis has been approved.

RISK ASSESSMENT

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

PROCEDURE

1 Tie the rubber stopper securely to one end of the string.
2 Pass the string through the tube.
3 Measure 40 cm of string from the top of the tube to the stopper.
4 Attach the crocodile clip to the string just below the bottom of the tube.
5 Open the paperclip and secure it to the other end of the string.
6 Use the paper clip as a hook and secure 30 washers to it.
7 Hold the tube vertically and whirl the rubber stopper around in a horizontal circle.

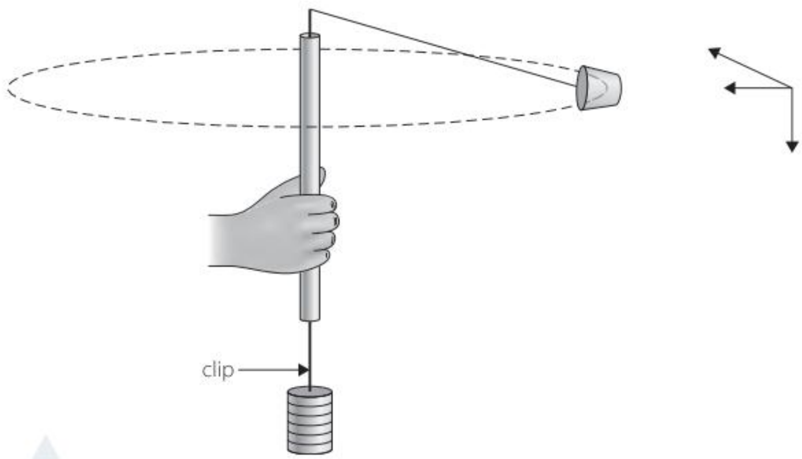

FIGURE 4.3.11 Rubber stopper whirled in a horizontal circle. Note that the crocodile clip must always remain a fixed distance below the bottom end of the tube.

8 Measure and record the time taken for 20 revolutions of the stopper three times.
9 Repeat the procedure with different numbers of washers, e.g. 25, 20, 18, 15, 10.

DATA ANALYSIS

1 Plot a graph of frequency versus number of washers, including uncertainty bars.
2 Describe mathematically any relationship that is justifiable between frequency and force applied.

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DISCUSSION

1 Explain why it is important not to allow the crocodile clip to touch the tube.

2 Draw a free-body diagram to show:

   a the forces applied to the rubber stopper

   b the net force on the rubber stopper.

3 Explain why using the measured length of the string, ℓ, which is greater than the radius of the conical circle, *r*, on which the stopper actually moves, has no effect on the result.

CONCLUSION

Summarise the activity, its purpose and the relationship identified.

Uniform circular motion on a banked track

A car that travels horizontally around a banked roadway at constant speed is acted upon by two forces: the normal force and the weight force. Friction is considered to be negligible in this analysis.

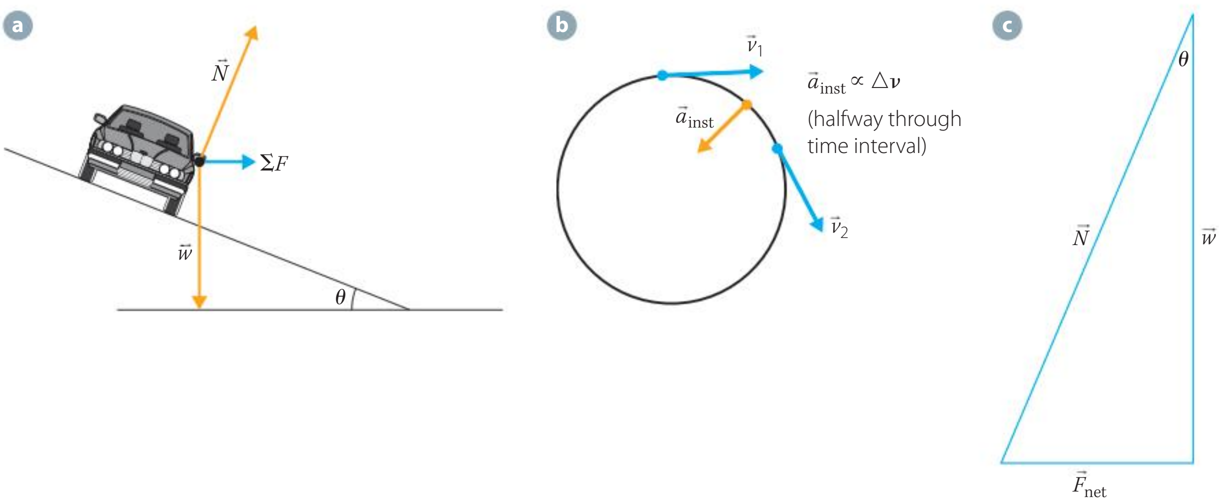

FIGURE 4.3.12 (**a**) Forces on a car cornering on a frictionless banked road. The net force is the horizontal component of the normal force. (**b**) Free-body diagram and (**c**) the vector sum of the forces.

The normal force and the weight force are vectors. The vector sum is equal to the net force, which is horizontally directed towards the centre of the curve of the road. The net force can be found in terms of the weight force and the angle of banking of the road:

$$\frac{F_{net}}{w} = \tan\theta$$

$$\Rightarrow F_{net} = w\tan\theta$$

$$\Rightarrow F_{net} = mg\tan\theta$$

$$\Rightarrow mg\tan\theta = \frac{mv^2}{r}$$

$$\Rightarrow g\tan\theta = \frac{v^2}{r}$$

KEY FORMULA

$$F_{net} = mg\tan\theta$$

$$mg\tan\theta = \frac{mv^2}{r}$$

$$T + w = \frac{mv^2}{r}$$

Where:

F_{net} = vector sum of the normal force and the weight force (N)

m = mass (kg)

v = speed ($m\,s^{-1}$)

r = radius (m)

θ = angle of banking (°)

Friction by the road is never really negligible. The friction force acts parallel to the slope. If the friction force acts up the slope, it tends to prevent the car from sliding down the slope. This occurs when the car is travelling too slowly such that the net force, without friction, would point below the horizontal. If the friction force acts down the slope, it tends to prevent the car from sliding up the slope. This occurs when the car is travelling too fast such that the net force, without friction, would act above the horizontal.

Vertical circular motion

Motion in a vertical circle can be quite complicated to analyse; however, forces at the upper and lower positions are relatively straightforward. An object on the end of a string, such as a toy aeroplane, is subject to two forces: tension, T, and weight, w. These combine to form the net (centripetal) force on the object:

- At the top: $T + w = \frac{mv^2}{r}$
- At the bottom: $T - w = \frac{mv^2}{r}$

If the object is travelling at constant speed, the tension at the bottom must be greater than at the top.

A car travelling at speed v is subject to the normal force, N, by the road and the weight force, w, on the car. These combine to form the net (centripetal) force on the car.

- Travelling over a rise:

$$w - N = \frac{mv^2}{r}$$

$$\Rightarrow N = \frac{mv^2}{r} - mg$$

This means that it is possible for the normal force applied by the road on the car to reduce to zero:

$$a = \frac{v^2}{r} - g = 0$$

$$\Rightarrow v = \sqrt{gr}$$

The faster the car travels over a rise, the less control the driver can exert on the car, because it is the normal force that governs the ability of the driver to exert a force on the road in order to change direction. When the normal force is reduced to zero, the speed is $v = \sqrt{gr}$ and the car becomes airborne, therefore uncontrollable.

- Travelling through a dip:

$$N - w = \frac{mv^2}{r}$$

$$\Rightarrow N = \frac{mv^2}{r} + mg$$

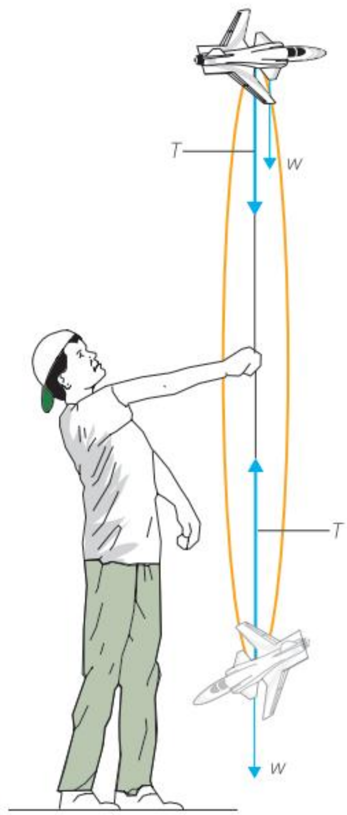

FIGURE 4.3.13 An object being whirled in a vertical circle

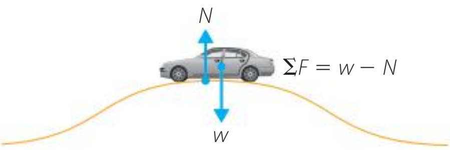

FIGURE 4.3.14 The net force for a car travelling over a rise is downwards.

This means that the faster the car travels through a dip, the greater the normal force applied on the car by the surface. Steering may become more difficult because the larger normal force means that more effort is needed from the driver to change direction.

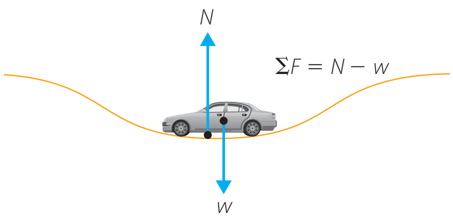

FIGURE 4.3.15 The net force for a car travelling through a dip is upwards.

SECTION REVIEW 4.3

REMEMBERING

1 Define the following terms:

- **a** centripetal
- **b** uniform circular motion.

2 For circular motion, write down key formulas for the following, and describe the conditions under which they can be applied:

- **a** centripetal acceleration
- **b** net force.

3 Name five different forces that are able to cause objects to travel in circular paths.

4 Draw a free-body diagram to show the forces applied on a car that is travelling on a banked track in a horizontal circle. From this free-body diagram, sketch a vector diagram to show the net force on the car. Ignore friction.

UNDERSTANDING

5 'Centripetal force is not a new kind of force.' Explain this.

6 Explain why there is a maximum speed at which it is possible to steer a car while travelling over a rise in the road.

APPLYING

7 Complete the table below.

m (kg)	v (m s^{-1})	r (m)	a (m s^{-2})	ΣF (N)
1.0	2.0	0.55		
400	20	150		
1.5×10^3	28	50		

ANALYSING

8 A 400 kg motorcycle travels around a corner of radius 80 m at 25 m s^{-1}. Calculate the net force applied by the motorcycle on the road.

9 In circular motion, net force equations can be written as $\Sigma F = m\omega^2 r$.

- **a** In terms of ω and r write the equation for:
  - **i** acceleration
  - **ii** speed.
- **b** Explain the physical significance of the variable ω.

REFLECTING

10 Draw a mind map to show the relationship between net force and centripetal force. Add to the mind map by adding types of force and relevant formulas.

4.4 Solving problems: centripetal force and acceleration

When solving problems involving acceleration, speed, radius and period for circular motion, follow the steps below.

1 Read the question carefully.
2 Visualise or sketch the real situation described.
3 Identify all variables provided.
   a Connect v, r and T with the equation:
   $$v = \frac{2\pi r}{T}$$
   b Connect a, v and r with the equation:
   $$a = \frac{v^2}{r}$$
   c Connect a, r and T with the equation:
   $$a = \frac{4\pi^2 r}{T^2}$$
4 Transpose the equation to make the required variable the subject.
5 Solve the equations.
6 Check to ensure the answers are those required.

WORKED EXAMPLE 4.4.1

An object accelerates at $25\,\text{m}\,\text{s}^{-2}$ when swung in a circle of radius 80 cm. Calculate:

1 the speed of the object
2 the period of rotation.

ANSWER

1 $$a = \frac{v^2}{r}$$
$$\Rightarrow v = \sqrt{ar}$$
$$\Rightarrow v = \sqrt{25\,\text{m}\,\text{s}^{-2} \times 0.80\,\text{m}}$$
$$\Rightarrow v = 4.5\,\text{m}\,\text{s}^{-1}\ (4.47\,\text{m}\,\text{s}^{-1})$$

2 $$v = \frac{2\pi r}{T}$$
$$\Rightarrow T = \frac{2 \times \pi r}{v}$$
$$\Rightarrow T = \frac{2 \times \pi \times 0.80\,\text{m}}{4.47\,\text{m}\,\text{s}^{-1}}$$
$$\Rightarrow T = 1.1\,s$$

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When solving problems involving forces and circular motion, follow the steps below.

1 Read the question carefully.

2 Visualise or sketch the real situation described.

3 Draw a free-body diagram.

   a Identify each real force acting on the object in question. (Do not make the mistake of adding centripetal force as a separate force – it is the vector sum of all the real forces acting on a single object.)

   b Write each force in the form: *F*(by A on B) or use the symbols provided in the question.

   c Identify the direction of the centripetal acceleration or net force.

   d Add any data provided in the question.

4 Consider the application of Newton's laws to the question asked.

   a Newton's second law: the equations relate to $\Sigma F = ma$.

   b Newton's third law: the inwards force can be the reaction to an outward push.

5 Set up any equations, using the symbols from the free-body diagram.

6 Transpose the equation to make the desired variable the subject.

7 Solve the equations.

8 Check to ensure the answers are those required.

WORKED EXAMPLE 4.4.2

A 250 g aeroglider on the end of a string is swung in a horizontal circle with a radius of 1.2 m. The aeroglider makes a revolution every 2.0 s.

1 Calculate the horizontal component of the tension force applied by the string to the aeroglider.

2 Calculate the acceleration of the aeroglider.

3 Find the force applied by the aeroglider to the string.

ANSWER

Sketch the situation:

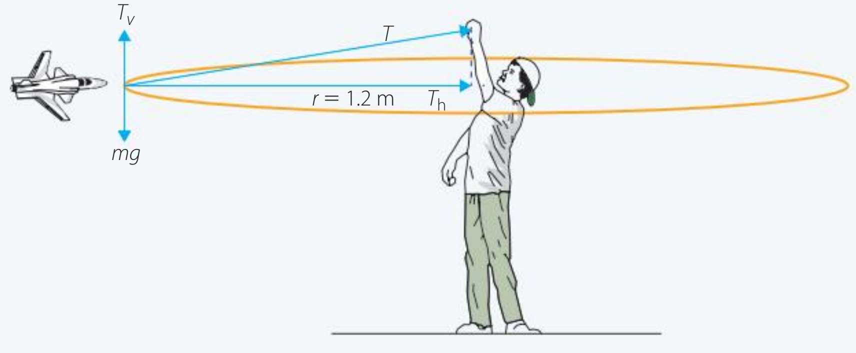

FIGURE 4.4.1

1 Identify the forces applied to the aeroglider.
The horizontal component of the tension keeps the aeroglider in the circle (the vertical component of the tension balances the weight, but neither affects motion at right angles to their direction of action).

Set up the appropriate equation and solve it .

$$\Sigma F = T_h$$

$$\Rightarrow T_h = m\frac{4\pi^2 r}{T^2}$$

$$\Rightarrow T_h = 0.250\ \text{kg} \times \frac{4 \times \pi^2 \times 1.2\ \text{m}}{(2.0\ \text{s})^2}$$

$$\Rightarrow T_h = 3.0\ \text{N}$$

2 Consider the application of Newton's second law.
Set up the appropriate equation and solve it.

$$a = \frac{\Sigma F}{m}$$

$$\Rightarrow a = \frac{3.0\ \text{N}}{0.250\ \text{kg}}$$

$$\Rightarrow a = 1.20\ \text{m s}^{-2}$$

3 Consider the application of Newton's third law.
The tension applied by the string in the aeroglider is equal and opposite to the force applied by the aeroglider on the string:

$$|F(\text{by aeroglider on string})| = |T|$$

$$\Rightarrow F(\text{by aeroglider on string}) = \sqrt{T_h^2 + T_v^2}$$

$$T_h = 3.0\ \text{N and } T_v - mg = 0$$

$$\Rightarrow T_v = 0.250\ \text{kg} \times 9.8\ \text{m s}^{-2}$$

$$\Rightarrow T_v = 2.45\ \text{N}$$

$$\Rightarrow F(\text{by aeroglider on string}) = \sqrt{(3.0\ \text{N})^2 + (2.45\ \text{N})^2}$$

$$\Rightarrow F(\text{by aeroglider on string}) = 3.9\ \text{N}$$

WORKED EXAMPLE 4.4.3

A car of mass 1500 kg travels horizontally at 20 m s^{-1} around a bend that is banked at 10° to the horizontal. Friction along the slope is negligible.

1 Calculate the normal force acting on the car.

2 Calculate the net force acting on the car.

3 Determine the radius of curvature of the road.

ANSWER

Sketch the situation.

Draw the free-body diagram.

Draw the vector diagram.

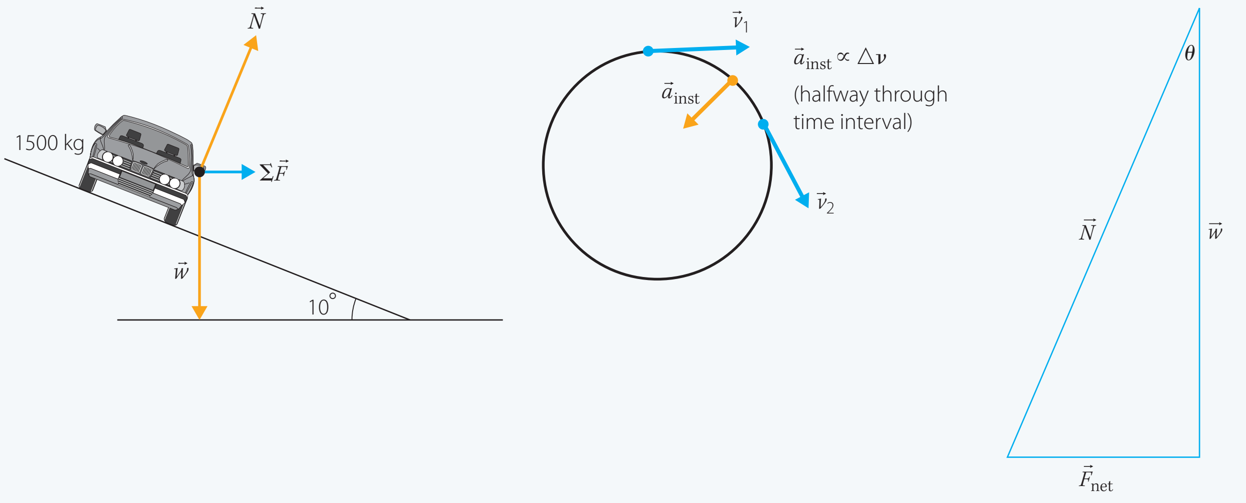

FIGURE 4.4.2

1 From the vector diagram:

$$\frac{w}{N} = \cos\theta$$

$$\Rightarrow N = \frac{w}{\cos\theta}$$

$$\Rightarrow N = \frac{1500\text{ kg} \times 9.8\text{ m s}^{-2}}{\cos 10°}$$

$$\Rightarrow N = 1.49 \times 10^3\text{ N}$$

2

$$\frac{\Sigma F}{w} = \tan\theta$$

$$\Rightarrow \Sigma F = w\tan\theta$$

$$\Rightarrow \Sigma F = 1500\text{ kg} \times 9.8\text{ m s}^{-2} \times \tan 10°$$

$$\Rightarrow \Sigma F = 2.59 \times 10^3\text{ N, centre-directed}$$

3 Select the appropriate formula:

$$a = \frac{v^2}{r}$$

$$\Rightarrow r = \frac{v^2}{a}$$

$$\Rightarrow r = \frac{(20\text{ m s}^{-1})^2}{9.8\text{ m s}^{-2} \times \tan 10°}$$

$$\Rightarrow r = 5.23\text{ m}$$

WORKED EXAMPLE 4.4.4

A 0.20 kg aeroglider is whirled in a vertical circle on the end of a string of length 0.60 m at a constant speed of 3.0 m s^{-1}. Calculate the tension in the string:

1 at the top of the circle

2 at the bottom of the circle.

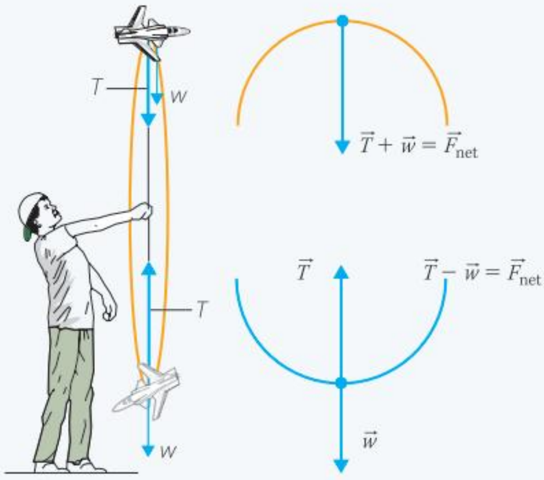

FIGURE 4.4.3

ANSWER

Sketch the situation.

Draw the free-body diagram.

1 $\Sigma F = T + w = m\dfrac{v^2}{r}$

$\Rightarrow T + w = m\dfrac{v^2}{r}$

$\Rightarrow T = m\dfrac{v^2}{r} - mg$

$\Rightarrow T = 0.20\text{ kg} \times \dfrac{(3.0\text{ m s}^{-1})^2}{0.60\text{ m}} - 0.20\text{ kg} \times 9.8\text{ m s}^{-2}$

$\Rightarrow T = 1.0\text{ N}$

2 $\Sigma F = T - w = m\dfrac{v^2}{r}$

$\Rightarrow T - w = m\dfrac{v^2}{r}$

$\Rightarrow T = m\dfrac{v^2}{r} + mg$

$\Rightarrow T = 0.20\text{ kg} \times \dfrac{(3.0\text{ m s}^{-1})^2}{0.60\text{ m}} + 0.20\text{ kg} \times 9.8\text{ m s}^{-2}$

$\Rightarrow T = 5.0\text{ N}$

SECTION REVIEW 4.4

REMEMBERING

1 Write down the steps for solving questions involving:

a centripetal acceleration

b net (centripetal) force.

UNDERSTANDING

2 Transpose the following equations to make v and T the subject respectively.

a $a = \dfrac{v^2}{r}$

b $\Sigma F = m\dfrac{4\pi^2 r}{T^2}$.

APPLYING

3 A 1.2 t car travels around a horizontal curve of radius 120 m at 100 km h^{-1}.

a Calculate the acceleration of the car.

b Determine the force applied by the car on the road.

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4 A velodrome is banked at 42°. A 75 kg cyclist travels horizontally at 18 m s^{-1} around the curved part of the track. Friction is negligible.

a Calculate the net force acting on the cyclist.

b Calculate the radius of curvature of the cyclist's path.

5 The bottom of a rollercoaster ride has a track with a radius of curvature of 28 m. The ride passes this point with a speed of 12 m s^{-1}. Find the normal force on a 50 kg person at the lowest point.

6 A car goes over a crest in the road that has a radius of curvature of 34 m. Determine the speed at which the car will lose contact with the road.

7 A car travels through a dip in a road that has a radius of curvature of 45 m at 35 m s^{-1}. Calculate the force applied by the seat of the car on the 82 kg driver.

ANALYSING

8 The acceleration of an object travelling with uniform circular motion in a horizontal circle of radius r and period T is given by the expression $a = 25r^2$.

a Find an expression for the period of this motion.

b Calculate the frequency, f, of rotation for a 10 kg mass circulating on a 4.0 m radius.

9 Explain why it is dangerous to travel at high speed over a rise in the road.

10 Explain why the velocity vector must rotate at the same rate as the radius vector.

11 Derive the acceleration formula:

$$a = \frac{2\pi v}{T}.$$

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Use Newton's first law to explain how it is possible to travel at constant speed while changing direction.

2 Use Newton's third law to explain how a person can run around a corner.

3 On a bend in the road, the road surface eventually starts to creep up over the kerb. Why does this happen?

CATEGORY QUESTIONS

4 Explain why centripetal force is not a real force such as gravitational, electrostatic or magnetic force.

5 Describe the conditions for which different uniform circular motion acceleration equations are used.

6 Compare the net force equations for horizontal circular motion, horizontal circular motion on banked tracks and vertical circular motion.

ELABORATION QUESTIONS

7 A 150 kg satellite orbits the Earth at an altitude of 20 200 km from the surface. It takes 12 hours to complete one orbit. The radius of Earth is 6.37×10^6 m.

- **a** Find the orbital speed of the satellite.
- **b** Compare the force applied on the satellite at 20 200 km to the force applied at the surface of Earth.

8 A marble is able to go around a vertical circular track in one of two ways. It can go around so that at its highest point it is either under or over the track. In both cases it travels above the track at the bottom. In each case determine the range of speeds at the bottom of the loop such that the marble can stay on the track and complete the loop by going:

- **a** under the top of the loop
- **b** over the top of the loop

EVIDENCE QUESTIONS

9 A baseball player runs from first to third base along a circular path at constant speed. If the baseballer runs too fast he will break a leg. Explain using realistic secondary data.

10 A cyclist at a velodrome is able to maintain a horizontal circular path at uniform speed only because a frictional force is applied to stop movement up the slope. Explain this, with reference to a set of reasonable data.

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END-OF-CHAPTER EXAM

End-of-chapter test

1 Which of the following types of force could be responsible for an object moving on a circular path?

A Tension force, magnetic force, centripetal force and gravitational force

B Centripetal force, electrostatic force, gravitational force and net force

C Net force, magnetic force, gravitational force and electrostatic force

D Tension, gravitational force, electrostatic force and magnetic force

2 A 25 g object takes 13 s to revolve 15 times around a fixed point. Calculate the speed of the object when it is 45 cm from the fixed point.

A $3.3\,m\,s^{-1}$

B $33\,cm\,s^{-1}$

C $0.82\,m\,s^{-1}$

D $8.2 \times 10^{-2}\,cm\,s^{-1}$

3 An object is rotating at 1200 Hz at a speed of $2.5 \times 10^{3}\,m\,s^{-1}$. What is its acceleration?

A $1.88 \times 10^{7}\,m\,s^{-2}$

B $1.88 \times 10^{6}\,cm\,s^{-2}$

C $1.88 \times 10^{5}\,m\,s^{-2}$

D $1.88 \times 10^{3}\,m\,s^{-2}$

4 A rollercoaster car travels to the right through its lowest point at $10\,m\,s^{-1}$. At this point the track has a radius of curvature of 25 m. What is the magnitude and direction of the force applied by the seat on a 50 kg person sitting in the car?

A 290 N, right

B 290 N, vertically up

C 690 N, vertically up

D 690 N, vertically down

5 A car travelling at $28\,km\,h^{-1}$ encounters a speed hump. At this speed the car just leaves the road. What was the radius of curvature of the speed hump?

A 80 m

B 16.6 m

C 6.2 m

D 2.9 m

6 Explain the difference between centripetal acceleration and centripetal force.

7 When whirling a rubber stopper around her head, Emily experiences an outwards force on her hand. Explain using one of Newton's laws.

8 Calculate the acceleration of a point on the edge of a 30 cm disc that is rotating at 33.3 revolutions per minute.

9 A 65 kg motorcyclist rides a 350 kg motorcycle at a constant speed of 100 km h^{-1} around a horizontal bend of radius 85 m.

a Calculate the force applied on the motorcyclist.

b Determine the force applied by the motorcycle and rider on the road.

10 An object of mass 2.5 kg is whirled on the end of a 75 cm long string in a vertical circle at constant speed. The string will break if the force exceeds 200 N. Find how fast the object can be whirled before the string breaks.

11 The diagram below shows a 1.3 t car travelling in a horizontal circle of radius 265 m on a road that is banked at 12°.

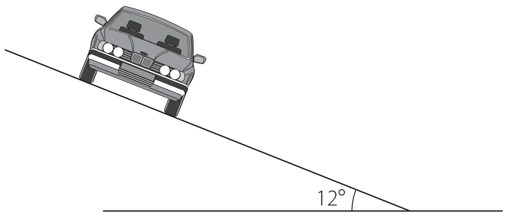

FIGURE 4.5.1

a On the diagram, draw force arrows to show all the forces acting on the car.

b Calculate:

i the acceleration of the car

ii the speed of the car.

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5 GRAVITATIONAL FORCE AND FIELD

Introduction

Newton's universal law of gravitation was developed from the prior observations and work of Brahe, Kepler, Copernicus and others. In this chapter, the nature of gravitational fields is explored, including the historical models of gravity and the development of the universal law of gravitation. The inverse squared nature of the gravitational force is demonstrated and the magnitude of the force is calculated for a range of scenarios.

Stimulus questions

How does our force of weight vary between the surface of the Moon, Earth and other planets of the solar system?

How does our force of weight vary as our altitude above Earth varies?

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5.1 The history of gravity

gravitas
Aristotelian idea about the 'heaviness' of objects made of earth that allowed them to fall in straight lines towards Earth

empirical methods
the central tenet of the scientific method whereby hypotheses are tested using observation and experimentation

5.1.1 The history of gravity

5.1.2 How to think about gravity

5.1.3 Misconceptions about gravity

inverse square law
describes a relationship in which the dependent variable is proportional to the square of the inverse of the independent variable

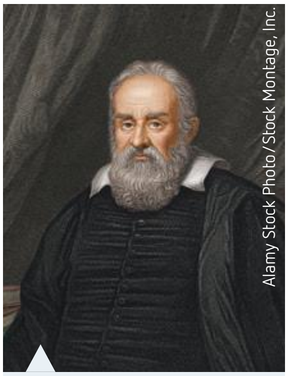


FIGURE 5.1.1 Galileo Galilei used experimental inquiries to investigate scientific phenomena.

FIGURE 5.1.2 Sir Isaac Newton, the 'father' of the universal law of gravitation

FIGURE 5.1.3 Monument to Tycho Brahe and Johannes Kepler, Prague, Czech Republic

Why do things fall to the ground? To us, this seems obvious: gravity pulls things down towards Earth. For Aristotle (384–322 BCE) and his contemporaries, the answer was also obvious: things fell to the ground because they were made of earth, and earth naturally moved towards Earth. They had a kind of heaviness or '**gravitas**' that enabled them to fall straight down.

The emergence of **empirical methods** to test Aristotle's ideas led to the development of kinematics (the relationship between measurements of distance and time) as a way of understanding motion. It was not until the 15th and 16th centuries that actual experiments on projectiles and the motion of falling objects were carried out. The most significant of these experiments were undertaken by Galileo (1564–1642), who showed that falling objects accelerated uniformly towards Earth. This was famously conducted using an inclined plane that decreased the acceleration due to gravity to just a fraction of the vertical acceleration.

From the mid-17th century, Newton (1643–1727) united the work in gravitational acceleration of those scientists who preceded him, including Copernicus and Kepler. By 1687 Newton had developed a description of gravitational force to involve a relationship between force, mass and distance that obeyed an **inverse square law**.

This law was 'universal' because it incorporated all motion and could be applied to Earth as well as across the universe. Newton's understanding of gravity was built on the impressive measurements of Tycho Brahe (1546–1601) and the mathematical interpretation of these data by Johannes Kepler (1571–1630). Earlier work by Nicolaus Copernicus (1473–1543), itself indebted to the accuracy of Muslim astronomers such as Muhammad al-Battani (c. 868–929), and Galileo on the motion of the planets around the Sun, as well as data from the Royal Observatory at Greenwich (1675 onwards) contributed to Newton's confidence in the universality of his gravitational theory.

Newton's law of universal gravitation remained undisputed until Albert Einstein (1879–1955) made significant modifications in his 1915 paper on general relativity. These changes, and the expansion in high-quality Earth and space-based astronomical observations during the past hundred years have enhanced our knowledge of the universe immensely. Current theories of dark matter and dark energy are significantly based on our developing understanding of gravity.

KEY FORMULA

$$F = G\frac{Mm}{r^2}$$

Where:

F = gravitational force (N)

G = Newtonian constant of gravitation ($6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$)

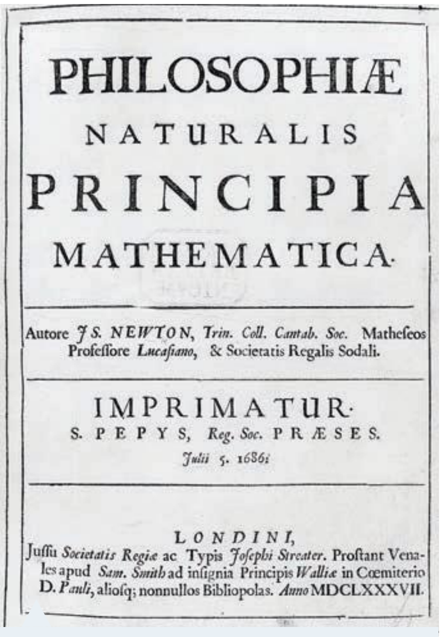
M = mass of object 1 (kg)

m = mass of object 2 (kg)

r = radius or distance between the objects (m)

Edmund Halley (1656–1742), Robert Hooke (1635–1703) and Newton all recognised that the elliptical orbits of planets, first described by Kepler, could be explained by a force that depended on the object's distance from the Sun. Newton wrote to Halley: 'It is now established that this force is *gravitas*, and therefore we shall call it gravitas from now on.' Newton used the Aristotelian idea of 'heaviness' to describe what we now refer to in English as gravity.

In 1687, Newton published *Philosophiae Naturalis Principia Mathematica* in which he described the law of universal gravitation.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtant Venales apud *Sam. Smith* ad inſignia Principis *Walliæ* in Cœmiterio D. *Pauli*, aliosq; nonnullos Bibliopolas. *Anno* MDCLXXXVII.

FIGURE 5.1.4 *Philosophiae Naturalis Principia Mathematica*, 1687, by Isaac Newton

INQUIRING FURTHER

The famous quote 'Standing on the shoulders of giants' has been attributed to Bernard of Chartres. Explain how this saying may be applied to the development of scientists' concepts of gravitation.

SECTION REVIEW 5.1

REMEMBERING

1 State a contribution to the understanding of gravitational acceleration by these scientists:

- **a** Aristotle
- **b** Einstein
- **c** Galileo
- **d** Newton.

2 What was Aristotle's term to describe the falling of heavy objects towards Earth?

3 State two examples of inverse square relationships.

UNDERSTANDING

4 Describe the key difference between the scientific methods applied by Aristotle and Galileo.

5 Describe what Halley, Hooke and Newton all noted about the paths of planets in the solar system.

APPLYING

6 Explain why the term 'universal' is applicable to Newton's law of gravitation.

ANALYSING

7 Predict the net difference (increases, decreases, remains the same) for the value of the gravitational force, F, when:

- **a** the mass, M, increases
- **b** the radius, r, increases.

REFLECTING

8 Draw a historical timeline for the development of our understanding of gravity, including the work of Aristotle, Copernicus, Galileo, Brahe, Kepler and Newton.

5.2 Gravitational potential energy

potential energy
energy stored in a system due to the interaction of components in the system via forces; energy stored in a field. Potential energy gives a system the ability to do work

work
energy transferred due to the action of a force: $W = Fs$

Energy may be classified in two distinct types: kinetic energy and potential energy. Energy is transferred when a force acts over a distance. When a force, F, acts on an object and moves it through some displacement, s, in the direction of the force, work is done.

KEY FORMULA

$W = Fs$

Where:

W = work (joules, J)

F = force (newton, N)

s = displacement (metres, m)

WORKED EXAMPLE 5.2.1

Determine the work done when a force of 100 N is applied against gravity to raise an object 1.50 m.

ANSWER

$W = Fs$

$W = 100\,\text{N} \times 1.50\,\text{m}$

$W = 150\,\text{J}$

5.2.1 What is gravitational potential energy?

5.2.2 Gravitational potential energy

Both force and displacement are vectors; however, work is done only when the force and the component of the displacement are parallel to each other.

When work is done on an object, its kinetic energy changes. When the component of the displacement is in the same direction as the force, the work done causes the kinetic energy of the object to increase. At the same time, the potential energy of the system must decrease so that energy is conserved. Conversely, when the component of the displacement is in the opposite direction to the force, kinetic energy decreases and the potential energy increases. Thus, the change in kinetic energy is the opposite of the change in potential energy:

$$\Delta E_k = -\Delta E_p$$

Remember that potential energy belongs to a system of interacting objects. It is not meaningful to refer to the potential energy of a single object.

Gravitational potential energy is due to the interaction of objects via their gravitational fields.

It is the gravitational field that mediates or exerts the force on one object due to the mass of another. Hence, we say that the gravitational field does work when an object falls in Earth's gravitational field. In this case, the work is done by the field on the object because the object moves in the direction of the field. The kinetic energy increases and the potential energy decreases.

Consider a pencil allowed to fall in Earth's gravitational field, as in Figure 5.2.1a. If we define our system as the pencil and Earth, the gravitational field of Earth does work on the pencil, increasing its kinetic energy. The potential energy of the Earth–pencil system has decreased. If we then pick up the pencil and lift it through some height, we must apply a force in the direction opposite to the gravitational field (Figure 5.2.1a). We do work equal to Fs on the pencil, where F is the force we apply and s is the distance through which the pencil is moved.

gravitational potential energy
the potential energy associated with the interaction of objects via the gravitational force; the potential energy is stored in the gravitational field

gravitational field
the field that mediates the gravitational force between all objects with mass; the field surrounding all objects with mass: $g = \frac{GM}{r^2}$

field
the means by which action-at-a-distance forces are exerted

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If the pencil begins and ends at rest, there is no change in the pencil's kinetic energy, yet we have done work, so energy must have been transferred. The field was also doing work at the same time. In this case the work done by the field was negative. The total work done *on the pencil* is zero. The total work done *on the Earth–pencil system* is the amount of work that we have done, and is equal to the change in potential energy of the Earth–pencil system. We say that this work was done *on* the field. So the energy transferred by the application of the force appears as an increase in the potential energy of the system.

When you hold the pencil stationary, both you and the gravitational field are exerting a force on it. However the displacement is zero, so no work is done by either force. This is also the case when the displacement is perpendicular to the field; that is, it is in the horizontal direction. In this case there is no component of displacement in the direction of the field, so the work done by or on the field is zero (Figure 5.2.1b).

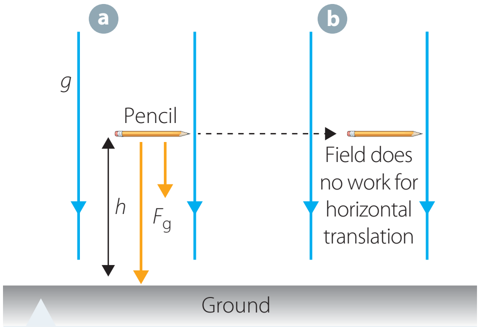

FIGURE 5.2.1 (**a**) When we lift the pencil at constant speed, we do work against the gravitational field: $W = Fs = F_{applied}h$. Work is done on the field and the kinetic energy is unchanged, the potential energy of the pencil–Earth system is increased. When the pencil falls through a distance of h, the field does work $= F_g h$ on the pencil, increasing its kinetic energy. (**b**) When we move the pencil horizontally, no work is done on or by the field and there is no change in potential energy.

The gravitational potential energy belongs to the *system*, which is both the object creating the field *and* the object experiencing a force due to the field. But, you might wonder, where is the energy stored? A pencil does not contain gravitational potential energy. Gravitational potential energy is *not* stored in the object, rather, it is stored in the field.

The field is able to do work because it exerts a force. The energy is distributed throughout all space where the field exists. The energy in a given volume (the energy density) depends on the field strength. In field theory, we model the action-at-a-distance forces, including gravity, as being mediated by a field. The field applies a force to objects in the field and because the field is able to do work, we say that the potential energy of the system is stored in the field.

Choosing a zero of gravitational potential energy

To be able to say how much potential energy a system has, we need to be able to define a zero energy position or configuration for the system. Note that we are again talking about *systems*, not isolated objects.

Both kinetic and potential energy cannot be defined for an isolated object. An object only has potential energy because a force is exerted on it, and the force must have some agent. Kinetic energy must be measured against some reference frame.

The potential energy of an object is always dependent on other objects, which generate the field. Even kinetic energy is not truly the property of a single object because it is due to motion, which is always relative to other objects in a frame of reference.

Choosing a zero for the potential energy of the Earth–pencil system may seem obvious. If we take the zero as being when the pencil is on the ground, then the potential energy of the system when the pencil is at any height, h, above the ground is simply $mg\Delta h = mg(h - 0) = mgh$.

The work done must be equal and opposite to the work done by the gravitational field if the pencil is to begin and end at rest.

KEY FORMULA

$W = \Delta E_p = F_g s$

$\Delta E_p = mg\Delta h$

Where:

W = work (J)

ΔE_p = change in potential energy (J)

F_g = force due to gravity (N)

s = displacement (m)

g = acceleration due to gravity (9.80 m s^{-2})

Δh = change in height (m)

WORKED EXAMPLE 5.2.2

Determine the work done in increasing the potential energy of a 50.0 kg object by lifting it 1.20 m in Earth's gravitational field. Let $g = 9.80\ \text{m s}^{-2}$.

ANSWER

$W = Fs$

$W = mgh$

$W = 50.0\ \text{kg} \times 9.80\ \text{m s}^{-2} \times 1.20\ \text{m}$

$W = 588\ \text{J}$

This is a useful working definition when considering forces and motion close to Earth's surface. However, you must be careful to define exactly what you mean by the surface or ground level, as this may vary according to the situation. It also means that the potential energy of any Earth–object system becomes negative whenever the object falls below the defined zero level. There is nothing wrong with a negative potential energy: the negative sign simply means that the potential energy at the end is less than the potential energy at the beginning. As we only ever measure changes in potential energy these changes can be positive or negative.

This 'ground level' definition for zero potential energy is not applicable to other planets or the behaviour of objects that move a long way above the surface of Earth. The simplest way to define a meaningful zero value that is not based on any single particular object or position is to take the zero as being when all objects in a system are infinitely separated. Consider a system of massive objects very far apart from each other. When all the objects in the system are infinitely separated, the forces acting on them are zero and we define the potential energy of this configuration as zero. If the objects are not moving there is no kinetic energy, so the total energy of the system is zero.

KEY FORMULA

The law of conservation of energy means that no change occurs to the total energy in a system:

$\Delta E_{\text{total}} = 0$

$\Delta E_{\text{k}} + \Delta E_{\text{p}} = 0$

$\Delta E_{\text{k}} = -\Delta E_{\text{p}}$

Where:

ΔE = the change in energy (J)

KEY FORMULA

$$\Delta E_{\text{k}} = \frac{1}{2}mv^2$$

Where:

m = mass (kg)

v = velocity (m s^{-1})

The gravitational force is always attractive. Any change from this zero configuration lowers the potential energy of the system to a negative value. The gravitational field due to each object does work on the other objects, bringing them closer together. The work done by the field decreases the potential energy of the system, as it attracts the objects closer together. This means that the kinetic energy must increase in accordance with the conservation of energy. As the objects accelerate closer together, their kinetic energy increases.

When an object moves in the direction of the gravitational field, the gravitational field does positive work – the kinetic energy of the object increases while the potential energy of the system decreases.

When an object moves against the gravitational field in an isolated system (where no other force is exerted on it), the gravitational field does negative work and the kinetic energy of the object decreases, while the potential energy of the system increases.

In an open system, work can be done on the objects in the system by an external agent, for example, lifting a pencil in the Earth–pencil system. In this case, if an object is moved against the field, the potential energy of the system again increases. These energy changes in a gravitational field are summarised in Table 5.2.1.

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TABLE 5.2.1 Summary of energy changes in a gravitational field

SYSTEM	OBJECT MOVES	WORK IS DONE	POTENTIAL ENERGY	KINETIC ENERGY
Closed	with the field	by the field	decreases	increases
Closed	against the field	on the field	increases	decreases
Open	with the field	by external agent	decreases	increases
Open	against the field	by external agent	increases	either

Note that when work is done by an external agent to move an object in a field, the field may still do work. The work done by the field is positive if the object moves with the field, and negative if it moves against the field.

WORKED EXAMPLE 5.2.3

A 10.0 kg school bag is lifted onto a shelf 2.00 m above the ground.

1 How much work was done on the bag?
2 By how much did the gravitational potential energy of the bag change?
3 If the bag fell off the shelf, with what kinetic energy would the bag land on the ground?

ANSWER

1 $W = Fs$
$W = mg \times s$
$W = 10.0\,\text{kg} \times 9.80\,\text{m s}^{-2} \times 2.00\,\text{m}$
$W = 196\,\text{J}$

2 $E_p = 196\,\text{J}$

3 $E_k = 196\,\text{J}$

SECTION REVIEW 5.2

REMEMBERING

1 Write the conservation of energy law in terms of energy changes.
2 As an object falls in a gravitational field, potential energy is reduced. Where does this energy go?
3 Define:
   a gravitational potential energy
   b kinetic energy
   c potential energy.

UNDERSTANDING

4 The gravitational potential energy in the field does not change as an object near Earth moves horizontally. Explain this.
5 Explain why a falling object is having work done on it by the gravitational field.

APPLYING

6 How much work is done in raising a 200 kg mass through a vertical height of 30 m?
7 The gravitational potential energy associated with a stationary object is 2.352×10^3 J.
   a What is its kinetic energy? Explain.
   b Determine the height of the object above the ground (zero) if it has a mass of 60 kg.

8 a How much work is done by the gravitational field when a 60 kg diver falls through a vertical height of 3.0 m?

b Using the conservation of energy law, determine the velocity of the diver as they enter the water.

9 A 400 kg rocket is launched from ground level. When it is at an altitude of 100 m its vertical velocity is 50 m s^{-1}.

a What is the kinetic energy of the rocket when it is at 100 m altitude?

b How much work was done on the rocket to change its gravitational potential energy?

c How much work in total was done on the rocket?

10 What is the minimum work that must be done to move a 1500 kg car up a 300 m high hill?

ANALYSING

11 Explain why the gravitational potential energy of an asteroid is small compared to Earth's gravitational potential energy.

12 400 J of work is done on a stationary 5.0 kg mass to raise it from a position 100 m above the ground.

a What is its new height above the ground?

b The mass is dropped from its new height. What is its velocity as it passes its original position?

c What is its velocity when it strikes the ground, assuming all other forces are negligible?

5.3 Gravitational fields

Each fundamental force (gravitational, electromagnetic, strong and weak nuclear) can be described as acting via a field. These fundamental forces are all action-at-a-distance forces. They allow us to explain how one object is able to exert a force on a second object without being in contact with it.

The gravitational field model allows us to explain how objects can exert forces without being in contact. It also allows us to:

- predict the acceleration of an object in any gravitational field
- calculate the mass of an object from the observed force it exerts on another object
- calculate the mass of distant objects, such as planets, by observing their orbits about the Sun.

Measuring the acceleration of objects dropped on the surface of the Moon tells us about the mass of the Moon. The radius of the Moon can be measured from astronomical observations, and then combining the size and mass information tells us that the density of the Moon is very similar to that of Earth's crust. This information was important in the development of modern theories of the formation of the Moon. These theories state that the Moon was actually formed when a massive object collided with Earth, breaking off some material that reformed in orbit, becoming our natural satellite, the Moon.

negligible
any value or variation in a value that is too small to be taken into account

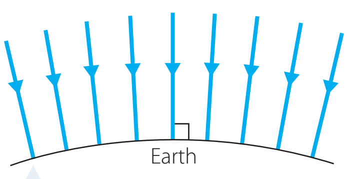

FIGURE 5.3.1 The gravitational field near Earth's surface is perpendicular to the surface and directed towards the centre of Earth.

Gravity 'near Earth'

At Earth's surface, we are subject to the force of attraction applied by the combined mass of Earth. Unless otherwise constrained, all objects fall from a height to the surface with an acceleration of 9.80 m s^{-2}. This is the effect of Earth's gravitational field on the masses. 'Near Earth' is an approximation that we are able to apply, as the field lines in a local area are very nearly parallel to each other, striking the surface at right angles (Figure 5.3.1). Up to several kilometres – well above the tallest buildings – the field strength varies by very little. We say that there is **negligible** variation in field strength over any local region.

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Force of weight on a planet

All matter has mass; however, the weight associated with each mass varies. In fact, the force of weight on an object differs according to the gravitational field in which it lies. On the surface of Earth, our force weight is the product of our mass and the acceleration due to gravity at this radius ($g = 9.80\,m\,s^{-2}$): force weight, $F_w = mg$. On another astronomical body, such as the Moon ($g = 1.63\,m\,s^{-2}$) or Jupiter ($g = 23.1\,m\,s^{-2}$) our weights would be considerably different. Table 5.3.1 lists the acceleration due to gravity on the Moon and various planets.

KEY FORMULA

Force weight is found using Newton's second law, $F = ma$.

$F_w = mg$

Where:

F = force weight (N)

m = mass (kg)

g = acceleration due to gravity for the astronomical body, typically Earth, where $g = 9.80\,m\,s^{-2}$

TABLE 5.3.1 Acceleration due to gravity on the Moon and various planets

OBJECT	MASS (kg)	RADIUS (m)	ACCELERATION DUE TO GRAVITY ($m\,s^{-2}$)
Sun	1.98×10^{30}	6.95×10^{8}	–
Moon	7.34×10^{22}	1.74×10^{6}	1.63
Mercury	3.28×10^{23}	2.57×10^{6}	3.70
Venus	4.83×10^{24}	6.31×10^{6}	8.89
Earth	5.97×10^{24}	6.37×10^{6}	9.80
Mars	6.37×10^{23}	3.43×10^{6}	3.69
Jupiter	1.90×10^{27}	7.18×10^{7}	23.10
Saturn	5.67×10^{26}	6.03×10^{7}	8.98
Uranus	8.80×10^{25}	2.67×10^{7}	8.71
Neptune	1.03×10^{26}	2.48×10^{7}	11.00

WORKED EXAMPLE 5.3.1

Calculate the force weight of a 60 kg mass on the surface of Mercury. Refer to Table 5.3.1 for the acceleration due to gravity on the surface of the planet.

ANSWER

According to Newton's second law, $F_w = mg$, where $g = 3.70\,m\,s^{-2}$:

$F_w = mg$

$F_w = 60\,kg \times 3.70\,m\,s^{-2}$

$F_w = 222\,N$

WORKED EXAMPLE 5.3.2

Determine the force weight of a 75 kg astronaut on the surface of the Moon. Refer to Table 5.3.1 for the acceleration due to gravity on the surface of the Moon.

ANSWER

According to Newton's second law, $F_w = mg$, where $g = 1.63\,\text{m s}^{-2}$:

$F_w = mg$

$F_w = 75\,\text{kg} \times 1.63\,\text{m s}^{-2}$

$F_w = 122.25\,\text{N}$

The gravitational force of attraction has an infinite range – every object in the universe is attracted to every other object due to their masses. The gravitational force reduces quickly with distance, however, due to its inverse square relationship. We personally experience a noticeable gravitational force due to the significant combined mass of Earth and our mass. We do not, however, experience a similar force of attraction from smaller objects, such as the person sitting next to us. This is simply because the force is negligible, due to our smaller masses.

SECTION REVIEW 5.3

REMEMBERING

1 Describe the difference between force weight and mass.

2 List the planets of our solar system in order of their gravitational pull on their surface, from least to greatest. (Refer to Table 5.3.1.)

UNDERSTANDING

3 Explain why the weight of an object may vary but its mass remains constant.

APPLYING

4 Calculate the force weight of a 1200 kg lunar lander on the surface of Earth and on the surface of the Moon. (Refer to Table 5.3.1.)

5 Calculate the difference in the force weight of an 80 kg astronaut on the surface of Mars and Venus. (Refer to Table 5.3.1.)

6 The Mars rover has a mass of 533 kg. Determine the force weight of the Mars rover on the Martian surface. (Refer to Table 5.3.1.)

7 The Cassini spacecraft had a mass of 2523 kg. Determine its weight on the surface of Earth and on the surface of Neptune. (Refer to Table 5.3.1.)

8 Titan (one of the moons of Saturn) has a mass of 1.35×10^{23} kg, a radius of 2.58×10^{6} m and an acceleration due to gravity of $1.35\,\text{m s}^{-2}$. Determine the weight force of an 860 kg probe on the surface of Titan.

ANALYSING

9 The gravitational field strength of a planet varies with distance above the surface. Explain why we may use the value $g = 9.80\,\text{m s}^{-2}$ as a 'near Earth's surface' value for nearly all calculations.

10 Why is the gravitational field of a planet considered as 'acting at a distance' even though it may be applying forces to objects that are in contact?

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5.4 Gravitational field strength

We can imagine that every mass has a gravitational field, g, surrounding it. This field reaches to infinity; however, as distance increases, its strength decreases non-uniformly, in an inverse square relationship. The gravitational field of a mass, M, exerts a force on another mass, m, as shown in Figure 5.4.1. Although the masses are not in contact, the force acts at a distance; we say that the force is mediated by the field.

Although the gravitational field due to Earth's mass is non-uniform and radial in its nature, in everyday life on the surface of Earth the field experienced is very nearly uniformly vertical.

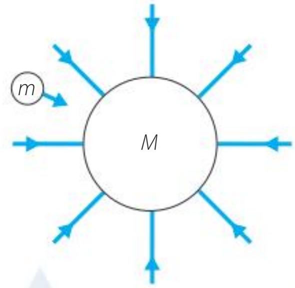

FIGURE 5.4.1 The gravitational field surrounding a mass mediates the gravitational force, accelerating the objects towards each other.

KEY FORMULA

$$g = \frac{GM}{r^2}$$

Where:

g = gravitational field ($N\,kg^{-1}$ or $m\,s^{-2}$)

G = universal gravitational constant ($6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$)

M = mass of the planet (kg)

r = radius of the planetary body (m)

WORKED EXAMPLE 5.4.1

Calculate the magnitude of the gravitational field at the surface of Earth. Use:

- mass of Earth, M, is 5.97×10^{24} kg
- radius of Earth, r, is 6.37×10^{6} m.

ANSWER

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67 \times 10^{-11}\,N\,m^2\,kg^{-2} \times 5.97 \times 10^{24}\,kg}{(6.37 \times 10^{6}\,m)^2}$$

$$g = 9.81\,m\,s^{-2}$$

WORKED EXAMPLE 5.4.2

Calculate the gravitational field strength on the surface of Mars. Use:

- mass of Mars, M, is 6.37×10^{23} kg
- radius of Mars, r, is 3.43×10^{6} m.

ANSWER

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67 \times 10^{-11}\,N\,m^2\,kg^{-2} \times 6.37 \times 10^{23}\,kg}{(3.43 \times 10^{6}\,m)^2}$$

$$g = 3.61\,m\,s^{-2}$$

WORKED EXAMPLE 5.4.3

Determine the gravitational field strength on the surface of the Moon.
Use:

- $\text{mass}_{\text{Moon}} = 7.34 \times 10^{22}\,\text{kg}$
- $\text{radius}_{\text{Moon}} = 1.74 \times 10^{6}\,\text{m}$.

ANSWER

$$g = \frac{GM}{r^2}$$

$$g = \frac{6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2} \times 7.34 \times 10^{22}\,\text{kg}}{(1.74 \times 10^6\,\text{m})^2}$$

$$g = 1.62\,\text{m s}^{-2}$$

KEY FORMULA

It follows from the definition of gravitational field, $g = \frac{F}{m}$, that the field of M at distance r is given by:

$$g = G\frac{M}{r^2}$$

Note that the field associated with M is independent of the mass m in the field. The gravitational field due to M exists whether we put another mass near it or not.

INQUIRING FURTHER

- Gravitational force acts at a distance.
- Gravitational force is modelled as acting at the centre of an object's mass (its centre of gravity).
- Gravitational field is calculated as the force per unit mass, N kg^{-1}, directed towards the centre of mass of an object.
- Gravitational force and gravitational field are vector quantities (having both magnitude and direction) acting in the same direction.

Create a short interactive or multimodal presentation that demonstrates your understanding of this concept.

When measuring the local value of the gravitational field, g, due to the mass, M, we use a small test mass, m. The force on the small test mass, m, due to the gravitational field of M causes m to accelerate towards it in accordance with Newton's second law of motion, $F = ma$. This acceleration is the gravitational field strength, g.

KEY FORMULA

$$g = \frac{F(\text{by } M \text{ acting on } m)}{m}$$

The gravitational field has the units of N kg^{-1}, which is equivalent to the units for acceleration, m s^{-2}.

$1.0\,\text{N kg}^{-1} = 1.0\,\text{m s}^{-2}$

Thus, if we can measure the acceleration of a mass, m, placed near the larger mass, M, we can find the value of the gravitational field. This means that all objects, independent of their mass, fall at the same rate of acceleration. At the surface of Earth this acceleration is approximately equal to $9.80\,\text{m s}^{-2}$, although this value does vary, depending on the height above sea level, among other factors.

Gravitational field near Earth's surface

At any point near Earth's surface, an object experiences the effect of its mass as an acceleration due to gravity and ultimately as a force weight. Newton was the first to realise that this same effect was the force that held the Moon in orbit around Earth and the planets in their orbits about the Sun. Newton's consideration of how Earth's gravitational field emanates through space led him to the invention of integration in the field of mathematics.

Due to Earth's nearly spherical shape, an object anywhere near Earth's surface is about the same distance from its centre of mass. In theory, this field will apply a force to every object in the universe. However, the great distances within our own solar system and neighbouring galaxies means that, in reality, Earth's gravitational field only has an influence on objects within a few hundred million kilometres in any real sense.

The approximation that the gravitational field is constant is reasonable when close to the surface of Earth (or any other massive body or planet); however, the field decreases in accordance with the radius, following an inverse square relationship, $\frac{1}{r^2}$. For an object at the height of the International Space Station (about 400 km above Earth), the gravitational field is approximately 90% of that at Earth's surface. For satellites that may be several thousand kilometres above Earth's surface, the near-Earth approximation cannot be used because g is substantially less than $9.80\,\mathrm{m\,s^{-2}}$.

The gravitational field strength around any mass is determined by the distance from the centre of mass and the mass itself. The force applied to any mass within the gravitational field is, in turn, determined by the strength of the gravitational field. Near Earth's surface, a 1.0 kg mass has a force of 9.80 N applied to it by Earth's gravitational field.

According to Newton's third law of equal and opposite reactions, Earth's mass and the mass of another object on its surface will exert a gravitational force of equal magnitude on each other. These forces act in opposing directions, along a line joining their centres of mass.

KEY FORMULA

The gravitational field strength is:

$$g = \frac{F(\text{by mass of Earth on mass } m)}{m}\ \mathrm{N\,kg^{-1}}$$

KEY FORMULA

For any mass, m, the magnitude of the force applied to it by Earth's gravitational field, that is, its weight, is given by:

$$F_w = mg$$

Where:

F_w = force of weight (N)

m = mass (kg)

g = gravitational field strength ($\mathrm{N\,kg^{-1}}$ or $\mathrm{m\,s^{-2}}$)

The value of g near Earth's surface is $9.80\,\mathrm{N\,kg^{-1}}$.

Newton's second law, $a = \frac{F_{net}}{m}$, can be applied to the gravitational force acting on any mass, m, near Earth's surface. Using $F_{net} = mg$, we get:

$$a = \frac{F_{net}}{m}$$

$$a = \frac{mg}{m}$$

$$a = g$$

Therefore, the acceleration of a mass free to move near Earth's surface is $9.80\,\mathrm{m\,s^{-2}}$. As g is the acceleration due to gravity, the direction of the acceleration must be towards the centre of mass of Earth.

This gives us a very simple way of measuring the gravitational field at any point in space. We simply need to ensure that no forces other than gravity are acting on a test mass, and then observe its acceleration.

KEY FORMULA

The gravitational field at any point is equal to the acceleration of a mass due to the gravitational force at that point:

$$g = \frac{F_{\text{gravitational}}}{m}$$

Close to Earth's surface, $g = 9.80\,\mathrm{N\,kg^{-1}} = 9.80\,\mathrm{m\,s^{-2}}$.

EXPERIMENT 5.4.1

Earth's gravitational field strength

The period, T, of a pendulum is dependent on two variables: length of pendulum, ℓ, and the gravitational field strength, g, in which it swings. The relationship between the variables T, ℓ and g is given by $T = 2\pi\sqrt{\frac{\ell}{g}}$.

AIM

To measure the strength of Earth's gravitational field, g, near the surface using a pendulum.

KEY FORMULA

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Where:

T = period (s)

ℓ = length (m)

g = acceleration due to gravity ($\mathrm{m\,s^{-2}}$)

MATERIALS

- retort stand
- boss head and clamp
- length of string (approximately 1.0 m length)
- mass bob
- stopwatch or timing device
- ruler
- data logging apparatus (optional)

PROCEDURE

1. Set up the apparatus as shown in Figure 5.4.2.
2. Measure the effective length of the pendulum from the top of the string to the centre of mass of the bob. Begin with a length of approximately 1.00 m.
3. Pull the bob back until it makes an angle of approximately 5° to the vertical.
4. Record the time taken for 10 complete oscillations of the pendulum, using the stopwatch (or data logging device).
5. Change the length of the string to approximately 0.8 m and repeat steps 3 and 4.
6. Repeat again with string lengths of approximately 0.6 m, 0.4 m and 0.2 m.
7. Calculate the period of the pendulum for each length.

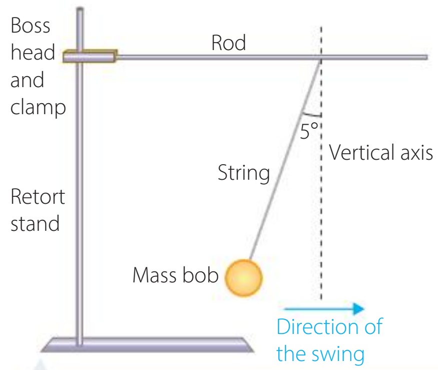

FIGURE 5.4.2 Apparatus to determine the acceleration due to gravity

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RESULTS

Record the data in a table similar to that below.

TABLE 5.4.1 Experimental pendulum data

LENGTH OF PENDULUM (ℓ) (m)	TIME FOR 10 OSCILLATIONS (s)	PERIOD OF PENDULUM (T) (s)

ANALYSIS OF RESULTS

Two alternative methods may be used to determine the value of g.

METHOD 1: ALGEBRAIC – USING EQUATIONS TO MODEL THE PENDULUM

1 Each period and length measured may be substituted into the equation for period, to determine values for g:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = (2\pi)^2\frac{\ell}{g}$$

$$g = \frac{4\pi^2\ell}{T^2}$$

2 The multiple calculated values for g may then be used to determine an average.

METHOD 2: GRAPHICAL – USING GRAPHICAL REPRESENTATIONS TO MODEL THE PENDULUM

1 Plot the relationship between ℓ and T. You will note that it is non-linear.

2 Considering the equation, it is seen that $\ell \propto T^2$; hence, plotting a graph of ℓ versus T^2 will exhibit a linear relationship. Be sure to plot ℓ (m) on the x axis, as it is the independent variable, and T^2 (s^2), the dependent variable, on the y axis. A line of best fit should be drawn through the data points.
Using the equation for T:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = (2\pi)^2\frac{\ell}{g}$$

$$T^2 = \frac{4\pi^2}{g}\ell$$

it can be seen that the value of the gradient of the graph of T^2 vs ℓ is $\frac{4\pi^2}{g}$.

3 Using any two points on the line of best fit, determine the gradient of the graph and hence calculate the value of g.
This value of g may also be determined by finding the gradient of the relationship on your graphing calculator.

DISCUSSION

1 What are the sources of uncertainty in this experiment?
2 Suggest ways in which these uncertainties could be minimised.
3 Explain why the time for 10 oscillations was measured, and then divided by 10, to determine the period, T.
4 If the length of the pendulum was consistently overestimated, how might this affect the value of g obtained in each method of analysis?
5 Give your best estimate of g to the correct number of significant figures.

EXPERIMENT 5.4.2

Acceleration due to gravity using an inclined plane

AIM

To measure the acceleration due to gravity of an object moving down an inclined plane.

MATERIALS

- steel ball or trolley cart
- inclined plane (ramp and block or desk placed on an incline)
- tape measure or ruler
- stopwatch (or data logger and electronic timing gate)

PROCEDURE

1 Set up the inclined plane and measure the length of ramp, ℓ (m), and the height of ramp, h (m).
2 Measure the length of the incline for the ball or trolley cart to roll down.
3 Use a stopwatch or electronic timing apparatus to record the time taken for the ball or trolley cart to roll down the ramp when released from rest.
4 Repeat step 3 to gather a minimum of three trials for this angle of incline.
5 Set up another incline of different height and angle. Record the measurements of length and height and repeat step 3.
6 Repeat step 5 for a third height and angle of incline.

RESULTS

Record your results in a table similar to that below.

TABLE 5.4.2 Experimental data for an inclined plane

INCLINE	LENGTH OF INCLINE (m)	HEIGHT OF INCLINE (m)	ANGLE OF INCLINE (°)	TIME FOR BALL / TROLLEY CART TO ROLL DOWN INCLINE (s)			
				TRIAL 1	TRIAL 2	TRIAL 3	AVERAGE
1							
2							
3							

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ANALYSIS OF RESULTS

1 Calculate the angle of incline, using the length of the incline as the hypotenuse and the height of the incline as the opposite side to the angle, θ.
2 Find the average time interval taken for the ball or trolley cart to roll down the incline. Include the uncertainty in your result.
3 Use the appropriate equation to find the speed of the ball or trolley cart at the bottom of the slope. Estimate the uncertainty in this value.
4 Use the appropriate equation to find the average acceleration of the ball down the slope. Estimate the uncertainty in this value.

DISCUSSION

1 Calculate the expected value of the acceleration and the final speed of the ball or trolley cart (ignoring its rotational kinetic energy). To do this, take $a = g\sin\theta$, where θ is the angle between the horizontal and the incline.
2 Compare the expected velocity and acceleration with the measured values.
3 How can these differences (if any) be explained by rotational kinetic energy?
4 When we calculate a value of g from an experiment such as this, we are modelling a rolling ball or trolley cart as an object sliding without friction. Comment on how appropriate this model is in this situation.

SCIENCE AS A HUMAN ENDEAVOUR

PRIMORDIAL GRAVITATIONAL WAVES

Einstein's theory of general relativity describes gravity as an artefact of the warping of the fabric of space–time due to the presence of mass. This can be visualised by thinking of space–time as the surface of a trampoline where a mass, such as our Sun, is represented by a bowling ball placed on the trampoline. The stretching of the trampoline means that a tennis ball would roll around the bowling ball in a similar way to a planet orbiting around a star in space. When massive events occur, such as two black holes colliding, the ripples through the curvature of space–time are thought to radiate outwards at the speed of light.

One of the last predictions based on Einstein's 1916 general theory of relativity, such ripples in space–time have now been detected by the LIGO and Virgo collaborative experiments, following other experiments conducted at Australia's Parkes radio telescope and the Bicep2 experiment at the South Pole. The 2017 Nobel Prize in Physics was awarded to three of the leading scientists behind the 1000-strong international project that first detected these tiny ripples in the fabric of space-time.

Vertical motion under gravity

An object that is free to fall near Earth's surface will experience a gravitational force vertically, regardless of its direction of motion. This gravitational force results in the object accelerating vertically downwards with a value equivalent to g.

As the value of g at Earth's surface is approximately $9.80\,\text{m s}^{-2}$ downwards, we get:

$$a = \frac{v_y - u_y}{t} \text{ (the definition of acceleration)}$$

$$\Rightarrow v_y = u_y + at$$

so:

$$v_y = u_y + gt$$

Notice that the direction is indicated by the sign for each unit; that is, if upwards is taken as being the positive direction, then the value of g, as it is acting downwards, must be written as $-9.80\,\text{m s}^{-2}$. Worked examples 5.4.4 and 5.4.5 demonstrate how the downwards direction may be treated as being positive or as negative. Whichever way this is defined at the outset of solving a problem, you need to ensure that it is consistently applied. Quantities with a downwards direction, such as acceleration and displacement, are typically assigned negative values. It is possible to assign the reverse – assigning negative values to quantities with an upwards direction. If this is done, the answer will have the same physical meaning and value.

WORKED EXAMPLE 5.4.4

A parcel is dropped from a hot air balloon that is sitting momentarily at a height of 200 m above the ground.

1 With what speed does the parcel hit the ground?

2 How long does the parcel take to fall?

ANSWER

1 a Identify known and required variables:
$u = 0\,\text{m s}^{-1}$, $a = 9.80\,\text{m s}^{-2}$, $s = 200\,\text{m}$, $v = ?$
Let the downwards direction be taken as positive.

b Select the equation and solve:
$v_y^2 = u_y^2 + 2as$
$v_y^2 = 0^2 + 2 \times 9.80\,\text{m s}^{-2} \times 200\,\text{m}$
$v_y^2 = 3920$
$v_y = 62.6\,\text{m s}^{-1}$ vertically down

2 a Identify known and required variables:
$u = 0\,\text{m s}^{-1}$, $a = 9.80\,\text{m s}^{-2}$, $s = 200\,\text{m}$, $t = ?$
Let the downwards direction be taken as positive.

b Select the equation and solve:
$s = ut + \frac{1}{2}at^2$
$200 = 0 \times t + \frac{1}{2} \times 9.80 \times t^2$
$\frac{200}{4.90} = t^2$
$t = \sqrt{40.82}$
$t = 6.39$ seconds
It takes 6.39 s for the parcel to fall.

WORKED EXAMPLE 5.4.5

A ball is thrown vertically upwards at $20.0\,\text{m s}^{-1}$. Determine the maximum height reached.

ANSWER

1 Identify known and required variables:
$u = 20.0\,\text{m s}^{-1}$, $a = -9.80\,\text{m s}^{-2}$, $v = 0\,\text{m s}^{-1}$, $s = ?$
Let the downwards direction be taken as negative.
At the maximum height, the vertical velocity is $0\,\text{m s}^{-1}$.

2 Select the equation and solve:
$v_y^2 = u_y^2 + 2as$
$0^2 = 20.0^2 + 2 \times -9.80\,\text{m s}^{-2} \times s$
$0 = 400 + -19.6 \times s$
$s = \frac{400}{19.6}$
$s = 20.4\,\text{m}$
The maximum height reached is 20.4 m.

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WORKED EXAMPLE 5.4.6

A ball is thrown vertically upwards at 20.0 m s^{-1}. Determine total time of flight.

ANSWER

1 Identify known and required variables:
$u = 20.0\,\text{m s}^{-1}$, $a = -9.80\,\text{m s}^{-2}$, $v = 0\,\text{m s}^{-1}$, $t = ?$
Let the downwards direction be taken as negative.
At the maximum height, the vertical velocity is 0 m s^{-1}.

2 Select the equation and solve:
$v_y = u_y + at$
$0 = 20.0 + -9.80 \times t$
$t = \frac{-20.0}{-9.80}$
$t = 2.04\,\text{s}$
The total time of flight is $2 \times 2.04\,\text{s} = 4.08\,\text{s}$. This is because the path is symmetrical and it takes the same time to travel up to the maximum height as it does to fall back down (ignoring air resistance).

SECTION REVIEW 5.4

REMEMBERING

1 What are the units of g?

2 Recall the kinematics formulas that relate the variables s, u, v, a and t.

UNDERSTANDING

3 Identify the purpose of using the notation v_y rather than simply v for vertical motion.

4 Explain why $a_y = g$ for free-falling objects near Earth's surface.

5 When finding the maximum height reached by a tennis ball hit vertically upwards, the value of v may be assigned as zero. Why is this possible?

APPLYING

6 A ball is dropped from a very tall building. How long does it take the ball to reach a velocity of 60.0 m s^{-1}?

7 A ball is thrown from a window with an initial downwards velocity of 4.40 m s^{-1}. It hits the ground after 1.40 s. Determine the height of the window above the ground.

8 With what minimum velocity must a student throw an object vertically upwards so that it reaches a point 5.5 m above its starting height?

9 Titan (one of the moons of Saturn) has a mass of 1.35×10^{23} kg and a radius of 2.58×10^{6} m. Determine the acceleration due to gravity on its surface.

ANALYSING

10 An object is thrown vertically upwards with an initial velocity of 30.0 m s^{-1}.

- **a** What is the maximum height reached by the object?
- **b** How long will the object take to fall back to its original position?

11 Calculate the gravitational acceleration, g, at a range of altitudes above Earth's surface and construct a table of values to demonstrate the relationship. Use 6.37×10^{6} m for the radius of Earth, 5.97×10^{24} kg for the mass of Earth and altitudes of 0.0 m (surface), 100 km, 1000 km and 10 000 km.

REFLECTING

12 Neptune's acceleration due to gravity (11.0 m s^{-2}) is greater than that of Saturn (8.98 m s^{-2}) even though Saturn is nearly five times as massive. Explain how this may be the case.

13 Given that the acceleration of objects near Earth's surface is directed vertically downwards, explain why a bullet shot horizontally from a height, h, hits the ground at the same time as one dropped from the same height.

5.5 Newton's law of universal gravitation and gravitational force

centre of mass
the average position of the mass in an object or group of objects. It is the point at which the gravitational force can be modelled as acting when the object is in a gravitational field

Newton determined that the gravitational force that keeps us on the ground and the planets in orbit about the Sun is a product of the masses of the objects and varies with an inverse square relationship with the distance between the objects. That is, the force, F, is dependent on both masses M and m, as well as the inverse of the square of the distance, r, between them. The distance, r, is measured between the centre of mass of each object. This relationship is termed Newton's law of universal gravitation.

KEY FORMULA

$$F = G\frac{Mm}{r^2}$$

Where:

F = gravitational force (N)

G = Newtonian constant of gravitation ($6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$)

M = mass of object 1 (kg)

m = mass of object 2 (kg)

r = radius or distance between the objects (m)

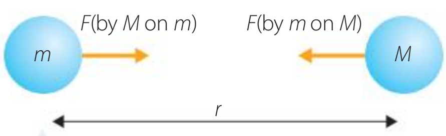

FIGURE 5.5.1 Gravitational force acts between bodies with mass with an equal and opposite force, in accordance with Newton's law of universal gravitation.

Although we experience the force of gravity as an everyday phenomenon, it is actually relatively weak by comparison to the other fundamental forces (Table 5.5.1).

TABLE 5.5.1 Comparison of the four fundamental forces

TYPE OF FUNDAMENTAL FORCE	RELATIVE MAGNITUDE
Electromagnetic	$\times 10^{36}$
Gravitational	$\times 10^{0}$
Strong nuclear	$\times 10^{38}$
Weak nuclear	$\times 10^{29}$

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EXPERIMENT 5.5.1

The inverse square law

Light intensity, like the force of gravity, is subject to the inverse square law: that is, intensity $\propto \frac{1}{d^2}$; doubling the distance from the light source leads to one quarter of the intensity of the source.

KEY FORMULA

INVERSE SQUARE NATURE OF FORCE OF GRAVITY	INVERSE SQUARE NATURE OF LIGHT INTENSITY
$F \propto \frac{1}{r^2}$ Where: F = gravitational force (N) r = radius or distance between the objects (m)	$E \propto \frac{1}{r^2}$ Where: E = light intensity (lux) r = distance from the point source (m)

AIM

To explore and to model the inverse square law.

MATERIALS

- light source (torch, lamp, LED)
- light meter
- metre ruler or tape measure

PROCEDURE

1. Turn on the light meter to an appropriate setting for measuring light intensity.
2. Turn on a light source.
3. Measure the light intensity at a minimum of five distances from the source using the light meter. Record the results for light intensity and the distance from the source in an appropriate table.
4. Calculate the values for the final column and complete the table.

RESULTS AND ANALYSIS

TABLE 5.5.2 Experimental light intensity and distance data

LIGHT INTENSITY (LUX, OR SIMILAR)	DISTANCE (m)	$\frac{1}{\text{Distance}^2}\left(\frac{1}{\text{m}^2}\right)$

1. Draw a scatter graph of light intensity (lux) on the *y* axis versus distance (m) on the *x* axis. Describe the relationship between the variables.
2. Draw a scatter graph of light intensity (lux) on the *y* axis versus $\frac{1}{\text{distance}^2}$ (m^{-2}) on the *x* axis. Add the line of best fit, and include the R^2 correlation coefficient.

»

» DISCUSSION

1 Consider the precision of the data from your experiment with reference to the R^2 correlation coefficient. How could this precision be improved?

2 Complete the following predictive sentences, based on the inverse square law.

   a If an observer moves to twice the distance from a light source, the light intensity would …

   b If an observer moves three times closer to a light source, the light intensity would …

Newton's third law and universal gravitation

5.5.1 Newton's law of universal gravitation

Recall that the gravitational force acting on a mass, *m*, at a distance, *r*, from the centre of another mass, *M*, is given by Newton's law of universal gravitation.

$$F = G\frac{Mm}{r^2}$$

5.5.2 The Cavendish experiment

The Cavendish experiment was the first experiment to measure the strength of the gravitational constant *G*.

We have seen that the field due to mass *M* acts on a mass *m* with a force *F*:

$$F = G\frac{Mm}{r^2}$$

But, what about the force applied by *m* on *M*? This force has the same magnitude:

$$F = G\frac{Mm}{r^2}$$

This is in accordance with Newton's third law of motion: for every action force there is an equal and opposite reaction force. That is, Earth is attracted towards you with the same magnitude of force, but opposite direction, as you are attracted towards Earth.

KEY FORMULA

The gravitational acceleration of *m* is the field strength of *M* at distance *r*: $g_M = G\frac{M}{r^2}$

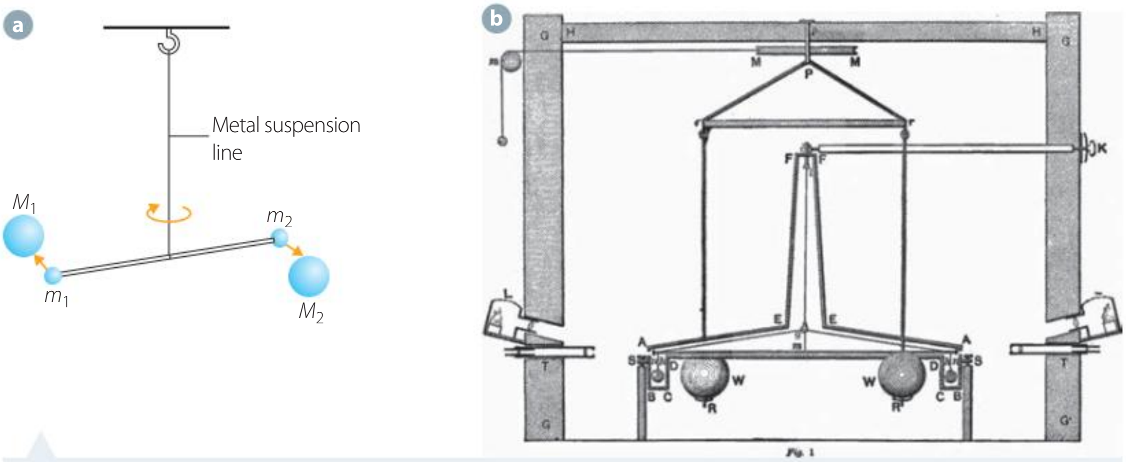
The gravitational acceleration of *M* is the field strength of *m* at distance *r*: $g_m = G\frac{m}{r^2}$

Thus, *m* and *M* accelerate at different rates, even though the magnitude of the force applied to each is the same. For example, the gravitational field of Earth acts on a 0.1 kg (100 g) apple with a force of 0.98 N. The apple consequently accelerates at $9.80\,m\,s^{-2}$. The apple acts on Earth with the same force, 0.98 N, but due to Earth's mass of approximately 5.98×10^{24} kg, Earth accelerates at approximately $1.63 \times 10^{-25}\,m\,s^{-2}$. (Earth will not accelerate appreciably!)

The universal gravitational constant, *G*

In 1798, 71 years after Newton's death, Henry Cavendish (1731–1810) measured the value of the constant of proportionality, *G*, in Newton's law of universal gravitation. He placed massive lead balls near two much smaller balls at the end of a long rod, as in Figure 5.5.2. The forces applied to each of the smaller balls caused a rotation of the rod. This rotation was opposed by the torsion (twisting) in the metal suspension line. The amount by which the suspension line rotated was used in the calculations to determine *G*.

Cavendish's experimental method was extraordinarily accurate in determining a value for G and this value of $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ remains applicable for all calculations of gravitational force.



from Cavendish, H. (1798), 'Experiments to determine the Density of the Earth' in McKenzie, A.S. ed. *Scientific Memoirs Vol.9: The Laws of Gravitation*, American Book Co. 1900, p.62

FIGURE 5.5.2 Cavendish's experimental apparatus. He measured the value of G to within about 1% of the currently accepted value of $G = 6.673\,84 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$.

SCIENCE AS A HUMAN ENDEAVOUR

TESTING THE GRAVITATIONAL INVERSE SQUARE LAW

Henry Cavendish was the first person to measure the universal gravitational constant, G. He used a system of very large masses to attract other smaller masses. By a cunning arrangement involving a torsion balance, he was able to deduce a value of G in the 18th century that was within 1% of the current experimental value (Figure 5.5.2b). Cavendish understood the importance of extraneous variables on the results and took action to ensure these effects were minimised; consequently, his accuracy was not bettered for more than 100 years! Extremely precise equipment can now be used to measure the universal gravitational constant. The current value of the universal gravitational constant is $G = 6.673\,846\,80 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$.

In order to achieve this result, Cavendish calibrated a torsion balance using known forces to twist the wire. He performed experiments to develop an empirical relationship between the angle of twist and the reaction force applied by the wire in response to the force applied. This was shown as force per unit of twist angle. A torsion balance makes use of torque, the rotational equivalent of translational force. A force, F, applied at a perpendicular distance, $r_\perp$, from a point of suspension or axis of rotation causes a rotation or torque, τ: $\tau = r_\perp F$. For the torsion pendulum, the torques from both sides cause a twist. When the torsion wire is twisted enough, its restoring force per twist angle is just equal to the combined torques of the two masses. The forces measured in this way are of the order of 10^{-10} N.

WORKED EXAMPLE 5.5.1

Determine the gravitational force of attraction between Earth and the Sun.

- Mass of Earth: 5.97×10^{24} kg
- Mass of the Sun: 1.98×10^{30} kg
- Radius of Earth's orbit around the Sun: 1.49×10^{11} m

ANSWER

$$F = \frac{GM_1M_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11}\,\mathrm{N\,kg^{-2}\,m^2} \times 5.97 \times 10^{24}\,\mathrm{kg} \times 1.98 \times 10^{30}\,\mathrm{kg}}{(1.49 \times 10^{11})^2\,\mathrm{m^2}}$$

$F = 3.55 \times 10^{22}$ N

The gravitational force between various masses can be determined in a similar way. Table 5.5.3 contains a range of planetary data, including acceleration due to gravity values, for performing further calculations.

TABLE 5.5.3 Planetary data

OBJECT	MASS (kg)	RADIUS (m)	MEAN ORBITAL RADIUS (m)	MEAN ORBITAL RADIUS (AU)	PERIOD OF REVOLUTION (s)	ACCELERATION DUE TO GRAVITY ($m\,s^{-2}$)
Sun	1.98×10^{30}	6.95×10^{8}	–	–	–	–
Moon	7.34×10^{22}	1.74×10^{6}	3.84×10^{8}	–	2.36×10^{6}	1.63
Mercury	3.28×10^{23}	2.57×10^{6}	5.79×10^{10}	0.387	7.60×10^{6}	3.70
Venus	4.83×10^{24}	6.31×10^{6}	1.08×10^{11}	0.723	1.94×10^{7}	8.89
Earth	5.97×10^{24}	6.37×10^{6}	1.49×10^{11}	1.000	3.16×10^{7}	9.80
Mars	6.37×10^{23}	3.43×10^{6}	2.28×10^{11}	1.520	5.94×10^{7}	3.69
Jupiter	1.90×10^{27}	7.18×10^{7}	7.78×10^{11}	5.200	3.74×10^{8}	23.10
Saturn	5.67×10^{26}	6.03×10^{7}	1.43×10^{12}	9.540	9.30×10^{8}	8.98
Uranus	8.80×10^{25}	2.67×10^{7}	2.87×10^{12}	19.19	2.66×10^{9}	8.71
Neptune	1.03×10^{26}	2.48×10^{7}	4.50×10^{12}	30.07	5.20×10^{9}	11.00

WORKED EXAMPLE 5.5.2

Earth and the Moon form a binary system – they are bound by their equal and opposite forces of gravitational attraction, orbiting around their common centre of mass. Determine the gravitational force of attraction between Earth and the Moon.
Use:

- $\text{mass}_{\text{Earth}} = 5.97 \times 10^{24}\,\text{kg}$
- $\text{mass}_{\text{Moon}} = 7.34 \times 10^{22}\,\text{kg}$
- radius of the Moon's orbit around Earth $= 3.84 \times 10^{8}$ m.

ANSWER

$$F = \frac{GM_1M_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg} \times 7.34 \times 10^{22}\,\text{kg}}{(3.84 \times 10^{8})^2\,\text{m}^2}$$

$$F = 1.98 \times 10^{20}\,\text{N}$$

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EXPERIMENT 5.5.2

The variation in gravitational force between objects due to mass and distance

AIM

To calculate the variation in gravitational force acting between two objects with varying distances and masses

MATERIALS

- Newton's universal law of gravitation: $F = \frac{GM_1M_2}{r^2}$
- the universal gravitational constant, as determined through Cavendish's experiment, $G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$

PROCEDURE

Use Newton's universal law of gravitation and the gravitational constant to calculate the force of gravitational attraction acting between each pair of objects in Table 5.5.4.

RESULTS

Perform each calculation and record the values in Table 5.5.4.

TABLE 5.5.4 Calculations of gravitational force

MASS OF OBJECT 1 (kg)	MASS OF OBJECT 2 (kg)	SEPARATION DISTANCE (m)	GRAVITATIONAL FORCE (N)
Football player 100	Earth 5.97×10^{24}	6.37×10^6 (on surface)	
Ballerina 40	Earth 5.97×10^{24}	6.37×10^6 (on surface)	
Physics student 70	Earth 5.97×10^{24}	6.60×10^6 (low-height orbit)	
Physics student 70	Physics student 70	1	
Physics student 70	Physics student 70	0.2	

ANALYSIS OF RESULTS

Compare the calculated values.

FURTHER EXPLORATION

Use a physics simulation to perform experiments to explore the effect of changing mass and changing distance on the force of gravitational attraction.

5.5.3 Gravity force lab

DISCUSSION

1 State the effect that an increase in mass has on the gravitational force.
2 State the effect that an increase in distance has on the gravitational force.
3 Explain which of these values has the greatest impact on the gravitational force.

The vector nature of gravitational force

The force exerted on an object within a gravitational field is applied in a direction towards the centre of mass of the object. Gravitational fields exist even for the smallest of masses. Therefore, we can say that a gravitational force is acting between you and the person nearest you. However, because this force is so small, it goes unnoticed. The very large mass of Earth results in a gravitational force on any object near it that cannot be ignored. The position of the centre of mass of Earth means that the gravitational force is exerted vertically downwards. The variations in the direction of this force caused by local influences such as mountains or dense bodies of rock beneath the surface are very small.

Vector addition of forces

Newton's second law, $\vec{a} = \frac{\vec{F}_{net}}{m}$, allows for the fact that any number of forces may be acting on a mass, m, at any time. The symbol $\vec{F}_{net}$ signifies the resultant force (or net force) – the sum of all the forces acting.

To find the sum of the forces acting on an object, the magnitudes, or sizes, of the forces cannot simply be added. The vector nature of force means that the directions of the individual forces must be taken into account. When adding two force vectors acting on an object at the same time, the resultant force may be found either geometrically or by drawing a scale diagram.

WORKED EXAMPLE 5.5.3

An 8000 kg spacecraft positioned 100 000 km from Earth and travelling directly to the Moon experiences a gravitational force from both Earth and the Moon, but in different directions. The net force acting on the spacecraft is the vector sum of the two gravitational forces.

Determine the net force acting on the spacecraft:

- Earth–Moon distance = 3.84×10^8 m
- $m_{Earth} = 5.97 \times 10^{24}$ kg
- $m_{Moon} = 7.34 \times 10^{22}$ kg

1 Determine the gravitational force and direction between Earth and the spacecraft.

2 Determine the gravitational force and direction between the Moon and the spacecraft.

3 Determine the net force acting on the spacecraft.

ANSWER

1 $F_{Earth} = \frac{GM_1M_2}{r^2}$

$$F_{Earth} = \frac{6.67 \times 10^{-11}\ \text{N kg}^{-2}\ \text{m}^2 \times 5.97 \times 10^{24}\ \text{kg} \times 8000\ \text{kg}}{(1.00 \times 10^8)^2\ \text{m}^2}$$

$= 318.6$ N acting towards Earth

2 $F_{Moon} = \frac{GM_1M_2}{r^2}$

$$F_{Moon} = \frac{6.67 \times 10^{-11}\ \text{N kg}^{-2}\ \text{m}^2 \times 7.34 \times 10^{22}\ \text{kg} \times 8000\ \text{kg}}{(2.84 \times 10^8)^2\ \text{m}^2}$$

$= 0.490$ N acting towards the Moon

3 The net force acting on the spacecraft is the vector sum of the F_{Earth} and F_{Moon}:

$F_{Earth} = 318.6$ N towards Earth, and $F_{Moon} = 0.490$ N towards the Moon

$F_{Earth} = +318.6\ \text{N} + (-0.490\ \text{N})$ towards Earth

$F_{net} = +318.1$ N towards Earth

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INQUIRING FURTHER

Lagrangian points are points in space at which the gravitational fields of Earth, the Moon and Sun combine to produce a net centripetal force, inducing a specific orbit. One such point is positioned approximately 1.5 million km towards the Sun from Earth and is known as Lagrange 1 (L1). A spacecraft can be positioned here so that it orbits the Sun with the same period as Earth. The solar observatory SOHO is positioned at L1, as are a few other observatories. SOHO monitors the solar wind and can give us a few hours warning of the approach of dangerously high levels of energetic particles before they hit Earth's magnetic field.

Write a short report on how SOHO monitors the solar wind, and Lagrangian points.

Gravitational equilibrium

Further to our calculation of net force, we can explore points of gravitational equilibrium between massive objects such as Earth and our natural satellite, the Moon.

WORKED EXAMPLE 5.5.4

Use Newton's law of universal gravitation to determine the point of gravitational equilibrium between Earth and the Moon.

- Earth–Moon distance = 3.80×10^8 m
- $m_{\text{Earth}} = 5.97 \times 10^{24}$ kg
- $m_{\text{Moon}} = 7.34 \times 10^{22}$ kg

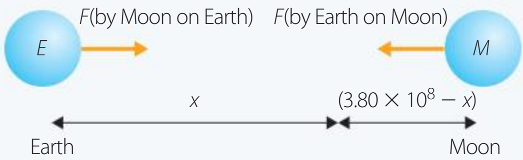

FIGURE 5.5.3 Diagram of the Earth–Moon system. There is a point of gravitational equilibrium that exists between Earth and the Moon where the forces are equal in magnitude and opposite in their direction.

ANSWER

1 Let F_g of Earth acting on the Moon = F_g of the Moon acting on Earth:

$$F_{g\text{ Earth}} = F_{g\text{ Moon}}$$

Distance from Earth to the point of gravitational equilibrium = x metres; therefore, the distance from the Moon to the point of gravitational equilibrium = $(3.80 \times 10^8 - x)$ metres:

$$\frac{Gm_{\text{Earth}}m_{\text{spacecraft}}}{x^2} = \frac{Gm_{\text{Moon}}m_{\text{spacecraft}}}{(3.80 \times 10^8 - x)^2}$$

2 Cancel the mass of the craft and the gravitational constant (they appear on both sides of the equation):

$$\frac{m_{\text{Earth}}}{x^2} = \frac{m_{\text{Moon}}}{(3.80 \times 10^8 - x)^2}$$

3 Substitute known values:

$$\frac{5.97 \times 10^{24}}{x^2} = \frac{7.34 \times 10^{22}}{(3.80 \times 10^8 - x)^2}$$

4 Cross multiply the denominators:

$$5.97 \times 10^{24}(3.80 \times 10^8 - x)^2 = 7.34 \times 10^{22}x^2$$

5 Simplify the equation into the form of $ax^2 + bx + c = 0$:

$$5.97 \times 10^{24}(1.444 \times 10^{17} - 7.60 \times 10^8 x + x^2) = 7.34 \times 10^{22}x^2$$

$$8.62068 \times 10^{41} - 4.5372 \times 10^{33}x + 5.97 \times 10^{24}x^2 = 7.34 \times 10^{22}x^2$$

$$8.62068 \times 10^{41} - 4.5372 \times 10^{33}x + 5.8966 \times 10^{24}x^2 = 0$$

6 Solve the equation for both solutions (use a graphics calculator equation function; degree 2 polynomial):

$x = 3.42 \times 10^8$ or $x = 4.27 \times 10^8$ m

7 State the valid solution, including the direction from the source:

Reject $x = 4.27 \times 10^8$ m as this point of gravitational equilibrium is on the far side of the Moon (not between the two bodies). Therefore, the gravitational point of equilibrium is positioned 3.42×10^8 m from Earth.

WORKED EXAMPLE 5.5.5

Using Newton's law of universal gravitation, determine the point of equilibrium between Jupiter and its moon Io. The distance between Jupiter and Io, the mass of Io and other values for Jupiter's Galilean moons are listed in Table 5.5.5.

TABLE 5.5.5 Table of data for Jupiter and its four Galilean moons

MOON	MEAN DISTANCE FROM JUPITER (m)	MASS (kg)	ORBITAL PERIOD AROUND JUPITER (EARTH DAYS)	ORBITAL PERIOD AROUND JUPITER (EARTH SECONDS)	MEAN DIAMETER (km)
Jupiter	–	1.90×10^{27}	–	–	69 911
Io	4.22×10^{8}	8.93×10^{22}	1.769	5.58×10^{7}	1822
Europa	6.71×10^{8}	4.80×10^{22}	3.551	1.12×10^{8}	1561
Ganymede	1.070×10^{9}	1.48×10^{23}	7.155	2.26×10^{8}	2634
Callisto	1.883×10^{9}	1.08×10^{23}	16.689	5.27×10^{8}	2410

ANSWER

1 Let the F_g of Jupiter acting on Io = F_g of Io acting on Jupiter:

$F_{g\,Jupiter} = F_{g\,Io}$

Distance from Jupiter to the point of gravitational equilibrium = x metres; therefore, the distance from the moon Io to the point of gravitational equilibrium = $(4.22 \times 10^8 - x)$ metres:

$$\frac{Gm_{Jupiter}m_{spacecraft}}{x^2} = \frac{Gm_{Io}m_{spacecraft}}{(4.22\times10^8 - x)^2}$$

2 Cancel the mass of the craft and the Gravitational constant (they appear on both sides of the equation):

$$\frac{m_{Jupiter}}{x^2} = \frac{m_{Io}}{(4.22\times10^8 - x)^2}$$

3 Substitute known values:

$m_{Jupiter} = 1.90 \times 10^{27}$ kg

$m_{Io} = 8.93 \times 10^{22}$ kg

$$\frac{1.90\times10^{27}}{x^2} = \frac{8.93\times10^{22}}{(4.22\times10^8 - x)^2}$$

4 Cross multiply the denominators:

$$1.90\times10^{27}(4.22\times10^8 - x)^2 = 8.93\times10^{22}x^2$$

5 Simplify the equation into the form of $ax^2 + bx + c = 0$:

$$1.90\times10^{27}(1.781\times10^{17} - 8.44\times10^8 x + x^2) = 8.93\times10^{22}x^2$$

$$3.384\times10^{44} - 1.604\times10^{36}x + 1.90\times10^{27}x^2 = 8.93\times10^{22}x^2$$

$$3.384\times10^{44} - 1.604\times10^{36}x + 1.899\times10^{27}x^2 = 0$$

6 Solve the equation for both solutions (use a graphics calculator equation function; degree 2 polynomial):

$x = 4.35 \times 10^8$ or $x = 4.09 \times 10^8$

7 State the valid solution, including the direction from the source:

Reject $x = 4.35 \times 10^8$ m as this point of gravitational equilibrium is on the far side of Io (not between the two bodies). Therefore, the gravitational point of equilibrium is positioned 4.09×10^8 m from Jupiter

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Other forces at a distance

Gravitational force is not the only force that acts at a distance. The other such forces are the electrostatic forces between charged particles, the magnetic force that is easily observed using magnets and compasses, and the strong and weak nuclear forces that act over very small distances within the nuclei of atoms. Without these two forces atoms would not be stable, but they are not detectable over distances usually encountered in everyday life.

When forces act at a distance, the force is a result of the objects interacting with the surrounding field. It takes a finite time for the interaction to be transmitted from one object to the other; the effect is not instantaneous, as is sometimes thought. Field theory does not explain why interactions are not instantaneous. This limitation of field theory was one factor that led to the development of a different model of forces – the exchange-particle model.

SECTION REVIEW 5.5

REMEMBERING

1 Contrast gravitational field strength with gravitational force.

2 Contrast the formulas used for gravitational field and gravitational force.

3 Name the two variables that determine the acceleration due to gravity at the surface of a planet.

UNDERSTANDING

4 'Gravitational field does not depend on the mass, *m*, in the field.' Explain this statement.

5 Explain how Newton's third law applies to the force of gravitational attraction between any two objects.

6 The force of gravitational attraction and light intensity both exhibit the inverse square law. Provide details to explain this.

APPLYING

7 Two masses, *m* and *M*, have a gravitational force of attraction, *F*, when they are a distance, *r*, apart. What is the relative magnitude of the force when this distance is increased to 4*r*?

8 Using Newton's law of universal gravitation, determine the point of equilibrium between Earth and Venus. The distance between Earth and Venus is 4.10×10^{10} m. Refer to Table 5.5.3, page 92, for other planetary data values.

9 An astronaut on the Moon drops a ball from a height of 1.50 m. It takes 1.36 s for the ball to fall to the surface of the Moon. Determine the acceleration due to gravity on the surface of the Moon.

ANALYSING

10 Are you satisfied that masses really do attract other masses by fields that stretch out to infinity? Justify your answer.

11 At what distance from Earth's centre is the net gravitational field of Earth and the Moon zero? Take the distance between Earth and the Moon to be 3.84×10^5 km. ($m_{Earth} = 5.97 \times 10^{24}$ kg, $m_{Moon} = 7.34 \times 10^{22}$ kg)

REFLECTING

12 Draw a concept map to show the relationships between mass, weight, gravitational field, gravitational force, measurement of mass and weight, normal force and weightlessness.

13 Reflect on the effect that a changing mass or distance has on gravitational force. Which variable has the greater impact?

14 Define the term 'centre of mass' and explain why it is necessary to consider it when calculating gravitational attraction between masses.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Gravitational field
   b Gravitational potential energy
   c Newtonian constant of gravitation

2 Contrast the terms gravitational field and gravitational force.

3 Specify the relationship between the angle, θ, and the acceleration down an inclined plane.

CATEGORY QUESTIONS

4 Identify the differences between '*g*' and '*G*'.

5 Describe the features of the universal law of gravitation.

6 Describe how Cavendish's experiment was used to determine a value for *G*, the universal constant of gravitation.

ELABORATION QUESTIONS

7 What happens to the gravitational force between two objects when they are moved 10 times further apart?

8 An object is projected vertically above Earth's surface at a velocity of $15\,m\,s^{-1}$. Determine the projectile's total time of flight.

9 A 5.0 kg rock is dropped from 10.0 m above the surface of Ganymede, a moon of Jupiter. Ganymede has a mass of 1.48×10^{23} kg and radius of 2634 km.
   a Determine the gravitational field near the surface of Ganymede.
   b Find the gravitational acceleration of the rock.
   c Calculate how much gravitational potential energy was transformed as the rock fell to the surface.
   d Calculate the maximum amount of kinetic energy gained by the rock.

EVIDENCE QUESTIONS

10 Research the work of Galileo and his experiments with inclined planes. State how a value for gravitational acceleration was determined using this experiment.

11 Predict the outcome on the gravitational force of doubling one of the masses and tripling the distance between them.

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END-OF-CHAPTER EXAM

End-of-chapter test

PHYSICS

1 Which of the following is correct for the gravitational acceleration on the surface of Earth?

A $4.90\,m\,s^{-1}$ **B** $9.80\,m\,s^{-1}$

C $4.90\,m\,s^{-2}$ **D** $9.80\,m\,s^{-2}$

2 The Newtonian constant of gravitation, G, is equivalent to:

A $1.60 \times 10^{-11}\,N\,kg$. **B** $6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$.

C $1.60 \times 10^{11}\,N\,m^3$. **D** $6.67 \times 10^{11}\,N\,m^2\,kg^{-2}$.

3 Which of the following is correct for the gravitational potential energy of a 100 kg mass held 1.2 m above Earth's surface?

A 1100 kg **B** 120 J

C 11 kg **D** 1176 J

4 Newton's universal law of gravitation is used to determine the force of gravitational attraction between masses. How is the formula written?

A $F = \frac{GM_1}{r^2}$ **B** $F = \frac{GM_1M_2}{r^1}$

C $F = \frac{GM_1M_2}{r^2}$ **D** $F = \frac{M_1M_2}{r^2}$

5 Determine the units of Newton's gravitational constant.

6 How are work and gravitational potential related?

7 Determine the gravitational field strength on the surface of a planet with a mass of 3.0×10^{24} kg and a radius of 4000 km.

8 Calculate the gravitational potential energy of a body of 60 kg raised to a height 3.0 m above Earth's surface.

9 Contrast the terms 'mass' and 'weight'.

10 **a** Calculate the time period, T, for a single oscillation of a pendulum on Earth, given a length of 0.30 m.

b How would the period differ if the experiment were conducted on the Moon rather than Earth?

11 Determine the force of weight of a 65 kg mass on the surface of Mercury. Refer to Table 5.5.3, page 92.

12 A 1.4 kg object is raised a height of 1.2 m to be placed onto a table.

a Calculate the force applied.

b Calculate the work done in raising the mass.

c State the maximum kinetic energy the object may attain if dropped.

13 Explain why the Moon's gravitational field exerts a greater force on Earth than the Sun's gravitational field, even though the Sun's mass is many millions of times greater.

14 Calculate the gravitational force acting between two planets of mass 3.60×10^{25} kg and 6.29×10^{24} kg respectively. The planets are at an average mean distance of 1.1 million kilometres.

15 With what minimum velocity must an object be thrown to reach a height of 14.0 m?

6 ORBITAL MOTION

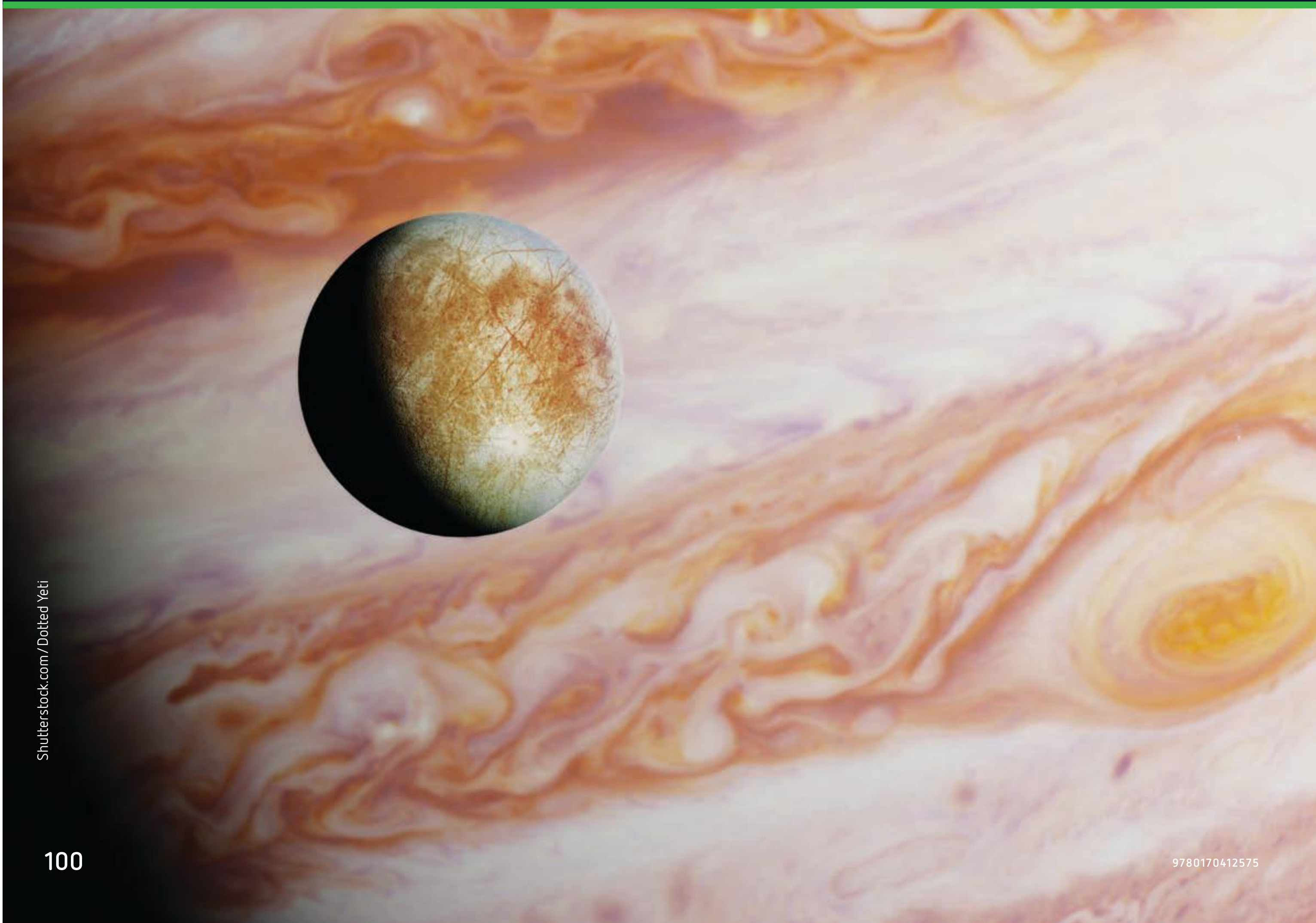
Introduction

Orbital motion has dictated the revolutions of planets, comets and natural satellites for aeons. Humans have recently taken advantage of the relationships that guide this motion to place artificial satellites into orbit. In this chapter, the nature of orbits is explored, including Kepler's three laws of planetary motion as well as the relationship between centripetal force and gravitational force that allows satellites to orbit Earth.

Stimulus questions

The phrase 'standing on the shoulders of giants' has been applied to many situations in history and science. How might it apply to the work of Aristotle, Ptolemy, Copernicus and Galileo?

What biological effects do astronauts experience when living in a zero-gravity environment, such as the International Space Station?



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6.1 Early models of planetary motion

Models of planetary motion have been proposed, and revised, for centuries. The ancient Greek philosopher Plato had the heavenly bodies fixed in **concentric spheres** (Figure 6.1.1). The stars were fixed in one sphere while the Moon and Sun had different spheres to account for their different movements among the stars. The planets were observed to wander across the sky (the word 'planet' is Greek for 'wanderer'). This example of a model designed to explain natural observations is one of many developed over the centuries. As the precision of measurements, number of points of data or the thinking changed, the models were improved. Claudius Ptolemy's (100–168) later models retained the circles but added **epicycles** (circles on circles) (Figure 6.1.2). These were required to describe the retrograde motion observed in the motion of the planets in the sky. Better observations required better models. However, the models quickly became complex, with no explanation of how the epicycles were maintained.

6.1.1 Classical astronomy

6.1.2 The Ptolemaic model

concentric spheres
spheres that share a common centre

epicycles
smaller circles whose centre is on the radius of larger circles; used by Ptolemy to describe the motion of planets

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FIGURE 6.1.1 The ancient Greek philosopher Plato had the heavenly bodies fixed in concentric spheres circling Earth.

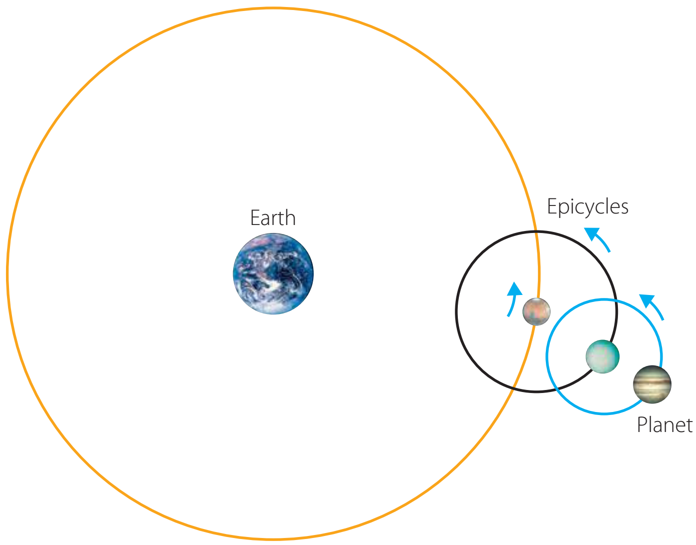

FIGURE 6.1.2 Epicycles, or circles on circles, were added to Plato's model of the universe to better describe the motion of planets.

geocentric model
a superseded model of the solar system with the Sun, Moon and planets revolving about Earth at its centre

heliocentric model
a current model of the solar system with the Sun (Helios) at its centre and all planets revolving about it. Closely associated with the work of Nicolaus Copernicus and Galileo Galilei

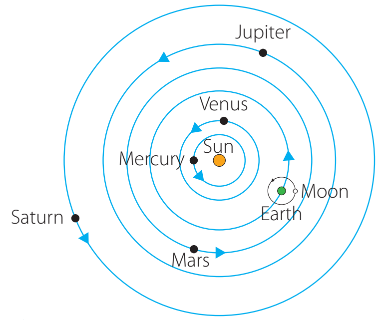

FIGURE 6.1.3 The Sun-centred heliocentric model of the universe, as determined by Nicolaus Copernicus.

The scientific revolution transformed the key scientific ideas of the Aristotelian tradition. Aristotle's cosmological understandings positioned Earth in the centre of the known universe – the **geocentric model**. Ptolemy's subsequent model of planetary motion was also geocentric, predicting the positions of the Sun, Moon, planets and stars, but not always accurately.

Nicolaus Copernicus (1473–1543), Galileo Galilei (1564–1642), Johannes Kepler (1571–1630) and Sir Isaac Newton (1643–1727) each determined an alternative explanation of the motions of heavenly bodies, placing the Sun at the centre of the cosmos in what was termed a **heliocentric model**.

SCIENCE AS A HUMAN ENDEAVOUR

REFINEMENT OF MODELS AND THEORIES

Models and theories are contested and refined or replaced when new evidence challenges them, or when a new model or theory has greater explanatory power. This has certainly been the case for our model of the solar system and the universe, highlighted by the difficulties experienced by scientists, such as Galileo, who supported a heliocentric model of the solar system.

Investigate the role of Galileo Galilei in supporting the heliocentric model of the universe, against the prevailing geocentric view of the time.

SECTION REVIEW 6.1

REMEMBERING

1 Name one contribution to the understanding of our universe by each of these scientists:
- **a** Copernicus
- **b** Galileo
- **c** Kepler
- **d** Newton.

2 Define 'concentric'.

UNDERSTANDING

3 Explain the term 'epicycle'. Use a diagram to assist your explanation.

APPLYING

4 Contrast the geocentric model with the heliocentric model of the solar system.

ANALYSING

5 How has the scientific method enabled our understanding of the universe to change?

REFLECTING

6 Explain the process of how scientific knowledge develops and changes. Refer to the various models of the solar system to assist in your response.

6.2 Kepler's laws of planetary motion

6.2.1 Kepler's laws

Johannes Kepler inherited the volumes of Tycho Brahe's (1546–1601) meticulous observations of the motions of the planets, the Moon and the stars. Built up over many years, Brahe's measurements and recordings, all made before the invention of the telescope, enabled the mathematically minded Kepler to propose a new model for the motion of the planets.

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Kepler's first law: the law of ellipses

It had always been assumed that the planets orbited Earth, and, in later models, the Sun, in perfectly circular orbits. This was, in part, due to the belief that the heavens were perfect and that circles were considered to be a perfect shape. Moving in anything but a circle had not been proposed previously. Kepler found that, if the planets were considered as moving in elliptical orbits, then their observed positions in the sky could be predicted almost perfectly. Natural satellites, such as moons, and planets typically revolve about their planets or stars in elliptical, at times, near circular orbits.

ellipse
a regular, curved shape that is a conic section (formed by cutting a cone obliquely); the path of satellites in orbit around larger bodies

KEY LAW

Kepler's first law: the law of ellipses
All planets move in elliptical orbits with the Sun at one focus.

An **ellipse** is a curved shape, such as that shown in Figure 6.2.1. Its major axis is the longest line between two points on the edge drawn through the geometric centre. The minor axis of an ellipse is the shortest line joining two points on the edge drawn through the geometric centre. An ellipse has two foci. A circle is a special case of an ellipse in which the two axes are equal in length and the two foci are located at the same position. The orbits of many planets, moons and satellites are very close to circular, while the orbits of comets are highly elliptical. How pronounced an ellipse, is indicated mathematically by its eccentricity, a value between 0 and 1. The elliptical eccentricity of Earth's path around the Sun is 0.167 while that of Halley's comet is 0.967.

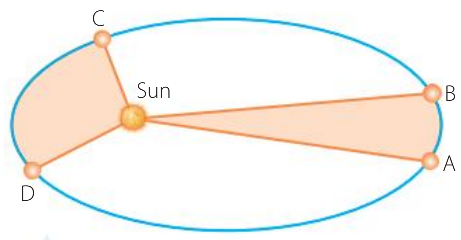


FIGURE 6.2.1 Kepler's first law, the law of ellipses, illustrates that the path of the planets about the Sun is elliptical in shape, the Sun being at one focus. Segments AB and CD are swept out in equal time intervals (Kepler's second law).

Kepler's second law: the law of equal areas

Kepler noticed that the speeds of the planets changed during their orbits. Nearer to the Sun their speeds increased; further away their speeds decreased. He was able to conclude, through application of the conservation of angular momentum, that the areas covered in equal time intervals were the same.

KEY LAW

Kepler's second law: the law of equal areas
A line that connects a planet to the Sun sweeps out equal areas in equal time periods.

Kepler's third law: the law of periods

By doing further work on Tycho Brahe's data and using his own observations, Kepler showed that there is a relationship between the average radius of orbit of the planets and their period of revolution around the Sun, such that the period of revolution squared was proportional to the **mean orbital radius** of the orbit cubed; that is, $T^2 \propto R^3$.

mean orbital radius
the average radius of orbit of one massive object about another, e.g. Earth revolving about the Sun

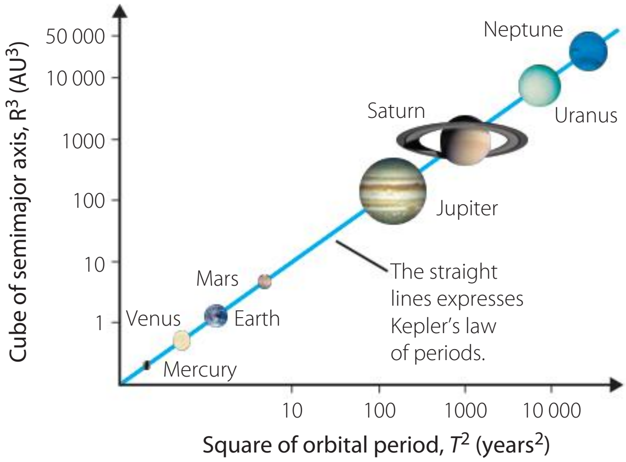
$$\frac{T^2}{R^3} = k \text{ (}k\text{ is a constant)}$$

KEY LAW

Kepler's third law: the law of periods
The square of the period of a planet's orbit is proportional to the cube of its mean orbital distance.

$$T^2 \propto R^3$$

FIGURE 6.2.2 Kepler's third law, the law of periods, is constant for all orbiting bodies within a given system. This is illustrated for the planets of our solar system.



WORKED EXAMPLE 6.2.1

Kepler's third law, the law of periods, $\frac{T^2}{R^3}$ = constant, may be used to graphically illustrate the relationship that applies for the planets of our solar system. This may also be used to determine whether other bodies belong to this system, such as Halley's comet.

Manipulate and graph the selection of data for our solar system to confirm that these planets may be classified as belonging within our solar system.

TABLE 6.2.1 Solar system planetary data

PLANETARY BODY	ORBITAL RADIUS ($\times 10^9$ m)	ORBITAL RADIUS^3 (m^3)	ORBITAL PERIOD (s)	ORBITAL PERIOD^2 (s^2)	$\frac{R^3}{T^2}$
Mercury	57.9		1.00×10^7		
Earth	149.6		3.16×10^7		
Mars	227.9		6.74×10^7		

ANSWER

The $\frac{T^2}{R^3}$ constant for the planetary system is consistent for the planets Mercury, Earth and Mars (see completed Table 6.2.2). The $\frac{T^2}{R^3}$ value is also the inverse of the gradient of the R^3 versus T^2 graph, confirming that the planets all belong to the same system. Note that the units do not necessarily need to be in SI units; that is, the orbital radius may be given in metres, kilometres or astronomical units and the orbital period may be given in seconds, days or years.

TABLE 6.2.2 Solar system planetary data (completed)

PLANETARY BODY	ORBITAL RADIUS ($\times 10^9$ m)	ORBITAL RADIUS^3 (m^3)	ORBITAL PERIOD (s)	ORBITAL PERIOD^2 (s^2)	$\frac{T^2}{R^3}$
Mercury	57.9	1.94×10^{32}	1.00×10^7	1.00×10^{14}	5.15×10^{-19}
Earth	149.6	3.35×10^{33}	3.16×10^7	9.99×10^{14}	2.99×10^{-19}
Mars	227.9	1.18×10^{34}	6.74×10^7	4.54×10^{15}	3.83×10^{-19}

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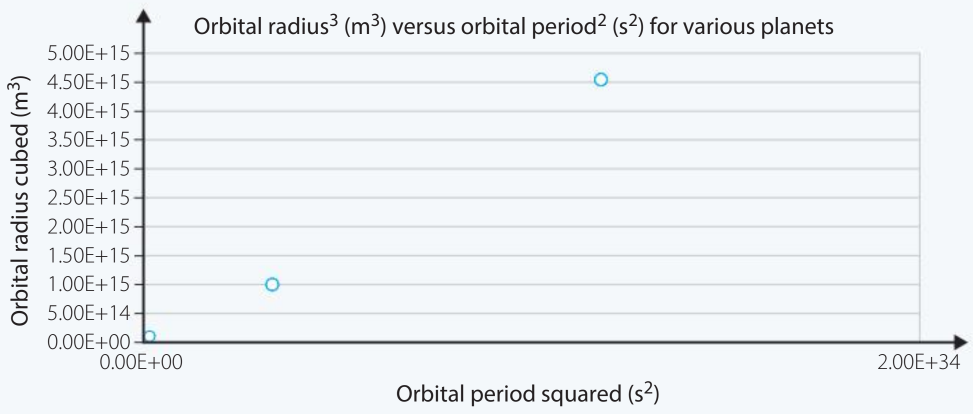

FIGURE 6.2.3 Graph of orbital radius cubed (m^3) vs orbital period squared (s^2)

WORKED EXAMPLE 6.2.2

Kepler's third law applies for all bodies within a given planetary system. Use Kepler's law of periods, $\frac{T^2}{R^3}$ = constant, to determine whether planet X is part of the same planetary system as planets A, B and C orbiting a central star.

TABLE 6.2.3 Planetary system data

	PLANET A	PLANET B	PLANET C	PLANET X
Orbital radius (m)	2.30×10^9	7.50×10^9	3.75×10^9	1.04×10^9
Mass (kg)	1.49×10^{24}	1.43×10^{24}	7.10×10^{23}	8.82×10^5
Orbital period (s)	2.09×10^5	1.25×10^6	4.35×10^5	8.82×10^5

ANSWER

The $\frac{T^2}{R^3}$ value for planet A $= \frac{(2.09 \times 10^5)^2}{(2.30 \times 10^9)^3} = 3.60 \times 10^{-18}$

The $\frac{T^2}{R^3}$ value for planet B $= \frac{(1.25 \times 10^6)^2}{(7.50 \times 10^9)^3} = 3.70 \times 10^{-18}$

The $\frac{T^2}{R^3}$ value for planet C $= \frac{(4.35 \times 10^5)^2}{(3.75 \times 10^9)^3} = 3.60 \times 10^{-18}$

The $\frac{T^2}{R^3}$ value for planet X $= \frac{(8.82 \times 10^5)^2}{(1.04 \times 10^9)^3} = 6.90 \times 10^{-16}$

The $\frac{T^2}{R^3}$ constant for the planetary system is consistent for planets A, B and C; however, the value for planet X is considerably different; therefore, it does not form part of this planetary system.

Kepler arrived at his three laws empirically, basing the laws on an analysis of the data that Tycho Brahe had provided as well as his own observations. Kepler's laws had excellent predictive power, although they were not based on any underlying models or theoretical basis; they did not give any explanation of the observed behaviour of the planets. Newton's model for gravity and the principle of conservation of momentum provided the theoretical framework needed to explain *why* planets and other orbiting bodies moved as described by Kepler's laws.

SECTION REVIEW 6.2

REMEMBERING

1 State Kepler's first, second and third laws of planetary motion.

2 Draw a diagram to illustrate Kepler's first law.

UNDERSTANDING

3 Describe Kepler's third law.

4 State the physical phenomena used to explain the motion of planetary bodies in Kepler's second law.

APPLYING

5 Data values for several natural satellites of the Jovian system are provided. Use the data for the moon Io to determine the ratio of $\frac{T^2}{R^3}$ and hence determine the orbital radius of Jupiter's moons Europa, Ganymede and Callisto.

TABLE 6.2.4 Jovian system data

MOON	ORBITAL PERIOD, *T* (DAYS)	ORBITAL RADIUS, *R* (m)
Io	1.78	4.22×10^8
Europa	3.56	
Ganymede	7.16	
Callisto	16.70	

6 A recently discovered exoplanet, planet X, is found to travel within a nearby galaxy near a known star, star Y. It has been suggested that planet X orbits star Y; however, this is yet to be confirmed. Use Kepler's law and the data provided for planets A and B, which are known to be part of this system, to confirm whether planet X should be classified as part of this system.

TABLE 6.2.5 Table of Exoplanet system data

	PLANET A	PLANET B	EXOPLANET X (NOT YET CONFIRMED)
Radius ($\times 10^3$ km)	21.1	6.50	42.2
Mass (kg)	1.96×10^{24}	1.84×10^{24}	2.65×10^{24}
Orbital period (Earth seconds)	4.15×10^6	2.45×10^7	4.42×10^8
Orbital radius (m)	4.52×10^{10}	1.44×10^{11}	10.2×10^{11}
Rotational period (days)	8.33	1.92	0.83

REFLECTING

7 Define 'mean orbital distance' and explain why this is required when applying Kepler's laws.

8 Describe the difference in motion of a comet compared to a planet, with reference to the eccentricity of its elliptical path.

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6.3 Newton's law of universal gravitation and Kepler's third law

In the case of the periodic circular motion of a planet around the Sun or a moon about a planet, the centripetal force that accelerates the orbiting body about the primary focal point is due to the gravitational force. Kepler's third law can be deduced from Newton's law of universal gravitation and the equation for uniform circular motion. For simplicity, we will assume that the planet follows a circular orbit, although a more complex geometrical analysis of elliptical orbits provides the same result.

KEY FORMULA

$$F_c = \frac{mv^2}{r}$$

Where:

F_c = centripetal force (N)

m = mass of planetary body (kg)

v = velocity ($m\,s^{-1}$)

r = radius (m)

$$F_g = \frac{GMm}{r^2}$$

Where:

F_g = gravitational force (N)

$G = 6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$

M = mass of body 1 (that being orbited) (kg)

m = mass of body 2 (orbiting body) (kg)

r = radius (m)

WORKED EXAMPLE 6.3.1

Determine the force required to keep a 1200 kg satellite in orbit around Earth at an altitude of 300 km. Use:

- $M_{Earth} = 5.97 \times 10^{24}$ kg
- $r_{Earth} = 6.37 \times 10^6$ m.

ANSWER

The centripetal force is equal to the gravitational force.

$$F_g = \frac{GMm}{r^2}$$

$$F_g = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1.2 \times 10^3}{(6.37 \times 10^6)^2}$$

$$F_g = \frac{4.48 \times 10^{17}}{4.06 \times 10^{13}}$$

$$F_g = 1.10 \times 10^4\,N$$

KEY FORMULA

$$F\text{(due to Sun's gravitational field)} = G\frac{Mm}{r^2}$$

$$F\text{(due to circular motion)} = \frac{mv^2}{r}$$

$$F\text{(due to circular motion)} = m\frac{(2\pi r)^2}{T^2 r}$$

$$F\text{(due to circular motion)} = m\frac{4\pi^2 r}{T^2}$$

Let the gravitational force = centripetal force

$$G\frac{Mm}{r^2} = m\frac{4\pi^2 r}{T^2}$$

$$\text{Then: } \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = k \text{ (constant)}$$

The equivalence between the centripetal force accelerating the orbiting body and the gravitational force can be further developed to determine Kepler's third law, the law of periods. Consider a planet (mass, m) orbiting the Sun (mass, M) with a mean orbital distance of r. The only force applied to the planet is the force mediated by the gravitational field of the Sun, providing the centripetal acceleration and hence uniform circular motion.

Figure 6.2.2 (page 104) shows this constant relationship for the planets in our solar system. Note that, in this example, the radius is given in astronomical units (AU), which is Earth's mean orbital radius (approximately 1.50×10^{11} m). The orbital period is given in years. It should also be noted that the equation remains valid regardless of the units used, as long as they are consistent. The equation is also valid if inverted.

Astronomical distances

astronomical unit (AU)
a unit of measure equivalent to Earth's mean orbital radius about the Sun (1.50×10^{11} m)

megaparsec (Mpc)
the distance subtended by an angle of one arcsecond $\times 1 \times 10^6$ (3.09×10^{22} m)

light-year (ly)
a measure of the distance that light would travel in one year (9.47×10^{15} m)

Although the international standard (SI) unit of length is the metre, m, the typical distances between planets, stars and galaxies are enormous, hence it is often more convenient for astronomers to use a range of longer distance units to perform calculations. Typically, such units include the astronomical unit, the megaparsec and the light-year. The **astronomical unit (AU)**, is a unit of measure equivalent to Earth's mean orbital radius about the Sun; the **megaparsec (Mpc)**, is the distance subtended by an angle of one arcsecond $\times 1 \times 10^6$, and the **light-year (ly)**, which, despite its name, is actually a measure of the distance that light would travel in one year.

KEY FORMULA

Astronomical unit, 1.0 AU = 1.50×10^8 km = 1.50×10^{11} m

Megaparsec, 1.0 Mpc = 3.09×10^{19} km = 3.09×10^{22} m

Light-year, 1.0 ly = 9.47×10^{12} km = 9.47×10^{15} m

The speed of light in a vacuum, $c = 3.00 \times 10^8$ m s^{-1}

WORKED EXAMPLE 6.3.2

The Andromeda galaxy, otherwise known as Messier 31, is the nearest large galaxy to our Milky Way. It is a spiral galaxy found approximately 780 000 parsecs, or 0.78 megaparsecs, from Earth. State the distance to the Andromeda galaxy in:

1 kilometres
2 light years.

ANSWER

1 $0.78 \text{ Mpc} \times 3.09 \times 10^{19}$ km per Mpc
$= 2.41 \times 10^{19}$ km

2 $$\frac{2.41 \times 10^{19} \text{ km}}{9.47 \times 10^{12} \text{ km ly}^{-1}}$$
$= 2.54 \times 10^6$ ly

The analysis does not apply just to the planetary motion of our solar system, but to all systems where satellites (moons, planets) orbit larger masses; for example, the 67 known moons orbiting Jupiter. The value of the constant $\frac{T^2}{R^3}$ is unique to each system, varying with each different central mass as the constant $\frac{4\pi^2}{GM}$ depends on the mass, M, around which the bodies are circulating. Hence, the constant, k, is not universal, but a constant that is unique to a given system.

WORKED EXAMPLE 6.3.3

Calculate the period T for a satellite of Earth with an orbital radius of 42 000 km. Use Earth's mass as:

- 5.97×10^{24} kg
- $G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$

ANSWER

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$T^2 = \frac{4\pi^2}{GM} \times R^3$$

$$T = \sqrt{\frac{4\pi^2}{GM} \times R^3}$$

$$T = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg}} \times (42\,000 \times 10^3)^3\,\text{m}^3}$$

$$T = \sqrt{7\,345\,264\,562}$$

Therefore, $T = 8.6 \times 10^4$ s (or 23.8 hours)

WORKED EXAMPLE 6.3.4

A small planet, planet A, is observed to orbit a star every 30 days. A second planet, planet B, orbits the same star at a distance that is nine times the orbital radius of planet A.
What is the period of planet B?

ANSWER

Here, the mass M of the star is unknown. The term $\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$ is the same for both planets, as they are in the same system and orbit the same star, so we can use:

$$\frac{T^2}{R^3} = \text{constant}$$

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$$

$$T_2^2 = \frac{T_1^2}{R_1^3} \times R_2^3 \quad \text{where } R_2 = 9 \times R_1 \text{ and } T_1 = 30 \text{ days}$$

$$T_2 = \sqrt{\frac{(30\text{ days})^2 \times (9R_1)^3}{R_1^3}}$$

$$T_2 = \sqrt{656\,100}$$

$T_2 = 810$ days

SECTION REVIEW 6.3

REMEMBERING

1 State the formula used to derive:

- **a** centripetal force
- **b** gravitational force.

2 Define 'astronomical unit'.

3 State the speed of light in a vacuum.

UNDERSTANDING

4 Contrast the units of length measurement the megaparsec and the light-year.

5 Convert 12.8 Mpc into light-years.

6 Explain how Kepler's third law can be used to determine whether an unknown comet or planet should be classified as belonging to a given solar system.

APPLYING

7 Calculate the orbital period, T, for an artificial satellite orbiting Earth at an altitude of 300 km. Use Earth's mass as 5.97×10^{24} kg and Earth's radius as 6.37×10^{6} m.

8 The mean orbital radius of Saturn is 1.43×10^{12} m. Determine the orbital period of Saturn, given that Kepler's third law, the law of periods, for our solar system has an average value of $\frac{T^2}{r^3}$ of approximately $3.41 \times 10^{18}\,\mathrm{s^2\,m^{-3}}$.

9 The Whirlpool galaxy, otherwise known as Messier 51a, is approximately 23 million light-years from the Milky Way.

State the distance to the Whirlpool galaxy in:

- **a** kilometres
- **b** megaparsecs
- **c** parsecs.

ANALYSING

10 A newly identified exoplanet, planet P, has been observed to orbit its nearby star with a period of 18 days. A second exoplanet, planet Q, orbits the same star at double the orbital radius of planet P. Determine the orbital period of planet Q.

11 A natural satellite (moon) orbiting a planet of mass 7.5×10^{25} kg has an orbital radius of 29 000 km. Using $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, determine the period of revolution for this satellite.

12 Calculate the orbit of an Earth-orbiting satellite that is found to have a period of 19 hours.

REFLECTING

13 Explain why the astronomical unit, AU, is used for measuring astronomical distances rather than the SI unit of the metre.

14 State what occurs to the period of a satellite if its radius of orbit is decreased.

15 Could the $\frac{T^2}{r^3}$ value for Earth orbiting the Sun be applied for satellites of another planet? Justify your response.

satellite
a natural (e.g. moon) or synthetic (e.g. GPS or communications satellite) body that orbits a significantly larger mass

weight
the gravitational force that acts on an object, $F_w = mg = \frac{GMm}{r^2}$

6.4 Satellite motion

The motion of a **satellite** can be modelled as uniform circular motion. Most satellites have circular or very nearly circular orbits around Earth. They are in a constant state of free-fall; the only force acting on them is gravity or their **weight**. The gravitational force by Earth's mass on a satellite is directed towards Earth's centre, which is also the centre of the satellite's circular orbit. Therefore, the net force acting on the satellite is perpendicular to the velocity of the satellite.

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Orbital velocity

A satellite in **orbit** is still very much within Earth's gravitational field. It is falling to Earth with an acceleration equal to the gravitational acceleration, g, at that distance from Earth. An orbiting spacecraft or satellite is given a horizontal velocity, its **orbit velocity**, such that, as it falls, it is also moving horizontally with such a speed that its path is circular. If no force other than that due to the gravitational field acts on the satellite, the satellite will continue in its orbit forever.

The gravitational force acting on satellites provides the centripetal force to keep them travelling in their circular orbit (Figure 6.4.1). The gravitational force $F_g = \frac{GMm}{r^2}$ can be found from the key formula:

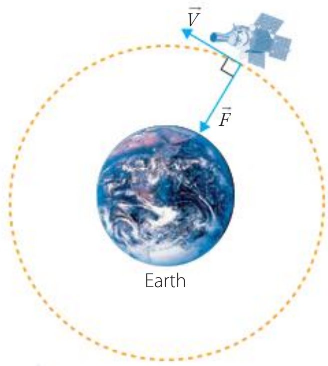

FIGURE 6.4.1 The net force on a satellite is perpendicular to its velocity and directed towards the centre of its circular orbit.

KEY FORMULA

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

Where:

v = orbital velocity (m s^{-1})

This provides the centripetal force, $F_c = \frac{mv^2}{r}$ to determine the orbital velocity, $v = \sqrt{\frac{GM}{r}}$.

Travelling at the precise orbital velocity enables a satellite to fall towards Earth at the same rate as Earth curves away from it; hence, the satellite is able to continually accelerate towards Earth, yet remain at the same orbit altitude. It is in this situation that passengers and their craft undergo **free-fall** and experience **apparent weightlessness**.

6.4.1 Catalogue of satellite orbits

6.4.2 The science: orbital mechanics

orbit
a regularly repeated elliptical path of one object about another massive object, such as a planet about a sun

orbit velocity
the precise velocity required for an object to continue to orbit a mass at a given altitude

free-fall
falling with the acceleration g, the local gravitational field strength

apparent weightlessness
the experience of having no normal force exerted on you; this occurs during free-fall

WORKED EXAMPLE 6.4.1

Determine the velocity of a satellite orbiting Earth at an altitude of 630 km. Use:

- the mass of Earth as 5.97×10^{24} kg
- radius of Earth as 6.37×10^6 m.

ANSWER

Note that the radius of orbit is the sum of Earth's radius and the altitude above the surface.

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg}}{6.37 \times 10^6 + 630 \times 10^3\,\text{m}}}$$

$$v = \sqrt{56\,885\,571}$$

$v = 7542$ m s^{-1} or $v = 7.54$ km s^{-1}

6.4.3 Gravity, weightlessness, and apparent weightlessness

WORKED EXAMPLE 6.4.2

Determine the orbital period of a satellite orbiting Earth at an altitude of 23 000 km. Use:

- the mass of Earth as 5.97×10^{24} kg
- the radius of Earth as 6.37×10^{6} m.

ANSWER

1 Determine the orbital velocity:

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11}\ \text{N kg}^{-2}\ \text{m}^2 \times 5.97 \times 10^{24}\ \text{kg}}{6.37 \times 10^6 + 23\,000 \times 10^3\ \text{m}}}$$

$$v = \sqrt{13\,558\,018}$$

$v = 3682\ \text{m s}^{-1}$ or $v = 3.68\ \text{km s}^{-1}$

2 Determine the period using the formulae for velocity and the circumference of a circle:

$$v = \frac{s}{t}$$

and $s = 2\pi r$

$$v = \frac{2\pi r}{t}$$

where $v = 3682\ \text{m s}^{-1}$

$$3682\ \text{m s}^{-1} = \frac{2 \times \pi \times (6.37 \times 10^6 + 23\,000 \times 10^3)}{t}$$

$t = 50118$ s, or $t = 13.92$ hours

SCIENCE AS A HUMAN ENDEAVOUR

SCIENTIFIC LITERACY: NASA'S GRAIL MISSION TO MAP THE MOON'S GRAVITATIONAL FIELD

In September 2011, NASA's Gravity Recovery and Interior Laboratory (GRAIL) mission placed two spacecraft into the same orbit around the Moon. As they flew over areas of greater and lesser gravity, caused both by visible features such as mountains and craters and by masses hidden beneath the lunar surface, the two spacecraft moved slightly towards and away from each other. An instrument aboard each spacecraft measured the changes in their relative velocity very precisely. Scientists translated this information into a high-resolution map of the Moon's gravitational field.

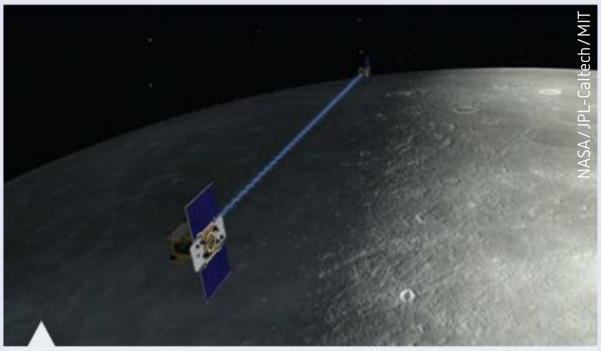

FIGURE 6.4.2 An artist's impression of the twin spacecraft that comprise GRAIL.

This gravity-measuring technique is essentially the same as that of the Gravity Recovery and Climate Experiment (GRACE), which has been mapping Earth's gravity since 2002. GRAIL's engineering objectives were to map lunar gravity and to use that information to increase understanding of the Moon's interior and thermal history. Spacecraft have been observed to change orbit unexpectedly as a result of unobserved regions of mass concentrations or mascons. Accurate gravitational maps of the Moon will enable future Moon missions to be safer and more precise. Getting the two spacecraft where they needed to be, when they needed to be there, was extremely challenging. It required a set of manoeuvres never before carried out in solar system exploration missions. The two GRAIL spacecraft were near-twins, each about the size of a washing machine, with minor differences resulting from the need for one specific spacecraft (GRAIL A) to follow the other (GRAIL B) as they circled the Moon at a height of 55 km. They each described the same polar orbit lasting 113 minutes.

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The on-board Lunar Gravity Ranging Systems measured changes in the distance between the two spacecraft down to a few microns – about the diameter of a red blood cell. That's not bad, considering they were flying 150 km apart. Each spacecraft carried a set of cameras for MoonKAM (Moon Knowledge Acquired by Middle school students). This was the first time a NASA planetary mission had carried instruments expressly for an education and public outreach project. Among other things, these cameras recorded the final crash site at the end of the mission. The two GRAIL spacecraft communicated with each other and the Earth to record any changes in their distance. One of the antennas on each craft was mounted on the sunny side of the spacecraft and another on the dark side. The sunny-side antennas pointed to Earth during the full moon and the dark-side antennas pointed to Earth during new moons. This system avoided the need to rotate the antennas mechanically during the mission. Any changes would have altered the spacecraft's **centre of mass** and disturbed the measurements. The mission lasted for about a year, terminating at the crash site Sally Ride in December 2012.

Source: NASA: GRAIL Mission overview, July 12, 2011 (http://www.nasa.gov/mission_pages/grail/overview/index.html#)

QUESTIONS

1 What do the acronyms GRAIL, GRACE and MoonKAM stand for?

2 What was the purpose of the GRAIL mission?

3 Why were two spacecraft used for this mission?

4 Describe how measuring the distance between the two craft enabled the local value of g to be calculated.

5 The Moon has a mean radius of 1.74×10^6 km.

  a Calculate the orbital speed of the GRAIL twins at their altitude of 55 km above the surface.

  b Calculate the orbital acceleration of the GRAIL twins.

6 Explain why NASA thought that it would be useful to have a map of the Moon's gravitational field?

centre of mass
the average position of the mass in an object or group of objects. It is the point at which the gravitational force can be modelled as acting when the object is in a gravitational field

Satellite orbits

A **geostationary satellite** remains above one place on Earth. It must travel directly above a point on the equator. **Geosynchronous satellites** travel above any great circle. A great circle is any circle on Earth whose radius extends from Earth's centre. Both geostationary and geosynchronous orbits have approximately 24 hour periods (23 h 56 min 4 s or 86 164 s).

Satellites in **low Earth orbit (LEO)** are high enough to be modestly affected by atmospheric friction but low enough to be relatively easy to service from Earth. This region extends from about 250 km to 1000 km above Earth's surface. The International Space Station (ISS) has been in low Earth orbit since November 1998. Its altitude averages 370 km. Even at this altitude the friction of the very low density atmosphere found there is sufficient to slow the ISS down; hence, its rocket motors must be fired every few weeks to boost it back into a higher orbit.

As a satellite in LEO slows down, its orbital radius decreases. At lower altitude there is greater friction and the process, without booster rocket intervention, would result in the spacecraft eventually crashing back to Earth or burning up in the atmosphere during a fiery re-entry.

FIGURE 6.4.3 The International Space Station is in low Earth orbit (LEO).

geostationary satellite
a satellite positioned directly above a point on the equator; they have periods of approximately 24 hours

geosynchronous satellite
a satellite that completes one orbit of Earth in the same time as Earth completes one revolution; geosynchronous orbits have a period of approximately 24 hours.

low Earth orbit (LEO)
a satellite orbit within the range of approximately 250 km to 1000 km above Earth's surface

EXPERIMENT 6.4.1

The relationship between radius and mass for an object in orbit

AIM

To simulate and examine the relationship between the radius of orbit and the mass of an orbiting object on its orbit velocity.

MATERIALS

Simulation involving the use of the formula for orbit velocity:

$$v = \sqrt{\frac{GM}{r}}$$

PROCEDURE 1

1. Consider the role of radius in the orbit velocity of a satellite. Let a satellite of mass 2000 kg be placed into orbit at varying altitudes above Earth, as below.

TABLE 6.4.1 Satellite orbit altitudes

ORBIT ALTITUDE (km)	ORBIT VELOCITY ($km\,s^{-1}$)
1000 (low Earth orbit, LEO)	
20 000 (medium Earth orbit, MEO)	
42 164 (geosynchronous Earth orbit, GSO)	

Note: Be sure to add the radius of Earth, $r_E = 6.37 \times 10^6$ m, to the altitude. Use $M_E = 5.97 \times 10^{24}$ kg.

2. Using the orbital velocity formula, determine the velocity for this satellite at each altitude.
3. Compare the velocities for each altitude and draw a conclusion.

PROCEDURE 2

1. Consider the role of mass in the orbit velocity of a satellite. Let satellites of varying masses be placed into orbit at the altitude of 3000 km, as below.

TABLE 6.4.2 Satellite orbit masses

SATELLITE MASS (kg)	ORBIT VELOCITY ($km\,s^{-1}$)
7 (pico-satellite)	
3500 (communication satellite)	
419 455 (International Space Station, ISS)	

Note: Be sure to add the radius of Earth, $r_E = 6.37 \times 10^6$ m, to the altitude. Use $M_E = 5.97 \times 10^{24}$ kg.

2. Using the orbital velocity formula, determine the velocity for each satellite of different mass at this altitude.
3. Compare the velocities for each mass. Consider the role of mass in the formula and draw a conclusion.

DISCUSSION

1. Explain the effect of altitude on orbital velocity.
2. Explain the effect of varying mass on orbital velocity.

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SCIENCE AS A HUMAN ENDEAVOUR

SCIENTIFIC KNOWLEDGE CAN BE USED TO EVALUATE ENVIRONMENTAL IMPACTS AND TO ASSIST SUSTAINABLE PRACTICES

An important, yet less known application of satellite technology is that of the continual monitoring of Earth's environment. Earth observation is the gathering of information about the physical, chemical and biological systems of Earth and includes remote sensing by satellites. Data is used to monitor the state of the natural environment, to inform models, to track biodiversity and wildlife, and to observe the impact of human activity. Understanding these relationships assists in supporting governmental policy and decision making to ensure a more sustainable future.

Investigate the role of global positioning system (GPS) satellites as well as remote sensing in providing accurate time, imagery, greenhouse gas and chemical concentration data and how this data is used to monitor deforestation, desertification and geomorphologic surveying.

Escape velocity

In order to escape the effect of Earth's gravitational field, extra energy must be expended. A rocket fired into the atmosphere may reach a specific height and then fall back to Earth. Given enough energy, it may reach a critical velocity, allowing it to orbit Earth. Given further energy still, the rocket may exceed the gravitational attraction of Earth and escape it entirely. **Escape velocity** is the minimum velocity needed for an object to escape the gravitational field of a planet or other large mass so that it no longer experiences a gravitational force due to that large mass. For Earth, the escape velocity has been determined to be $11.2\,\text{km}\,\text{s}^{-1}$. The escape velocity may be derived by considering the initial and final kinetic energies of the object, where this change in kinetic energy is equal to the change in the potential energy of the system.

escape velocity the minimum velocity required for an object to escape the gravitational field of a planet or other large mass

KEY FORMULA

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$

Where:

$G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$

M = mass of the planetary body (kg)

r = radius of planet (m)

WORKED EXAMPLE 6.4.3

Determine the escape velocity required for a rocket to escape Earth's gravitational attraction.
Use:

- $G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$
- Earth's mass as $5.97 \times 10^{24}\,\text{kg}$
- Earth's radius as $6.37 \times 10^{6}\,\text{m}$.

ANSWER

Use the formula for escape velocity:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$

$$v = \sqrt{\frac{2 \times 6.67 \times 10^{-11}\,\text{Nm}^2\,\text{kg}^{-2} \times 5.97 \times 10^{24}\,\text{kg}}{6.37 \times 10^{6}\,\text{m}}}$$

$$v = \sqrt{125\,023\,234}$$

$v = 11181\,\text{m}\,\text{s}^{-1}$ or $v = 11.2\,\text{km}\,\text{s}^{-1}$

SCIENCE AS A HUMAN ENDEAVOUR

INTERNATIONAL COLLABORATION TO DETECT GRAVITATIONAL WAVES

Scientific understanding is often gained through international collaboration on projects. The Laser Interferometer Gravitational-Wave Observatory (LIGO) is an example of such a large-scale experiment and observatory. The LIGO structure includes two gigantic instruments, in Washington and Louisiana, that are primarily used to search for gravity waves.

Recent discoveries have marked a triumph for the team of more than 1000 physicists involved in this project.

Investigate what LIGO has recently discovered through its detection of gravitational waves, as well as the international collaboration required to sustain such a project.

SECTION REVIEW 6.4

REMEMBERING

1 State the formula for determining orbit velocity.

2 State the formula for determining escape velocity.

UNDERSTANDING

3 Describe the change in orbit velocity as the radius of orbit increases.

4 Explain why astronauts orbiting Earth experience weightlessness.

APPLYING

5 Determine the velocity of a satellite orbiting Earth at an altitude of 2300 km. Use M_E as 5.97×10^{24} kg and the r_E as 6.37×10^6 m.

6 Calculate the gravitational force acting on a satellite of orbital radius 650 km and mass 4000 kg. Use M_E as 5.97×10^{24} kg and the r_E as 6.37×10^6 m.

7 Determine the orbital period, in hours, of a satellite of mass 950 kg and altitude of 12 000 km. Use M_E as 5.97×10^{24} kg and the r_E as 6.37×10^6 m.

ANALYSING

8 Determine the escape velocity required for a rocket to escape Mars' gravitational attraction. Use $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$, Mars' mass as 6.37×10^{23} kg and Mars' radius as 3.43×10^6 m.

9 Determine the gravitational field strength for a satellite orbiting Earth at an altitude of 36 000 km. Use Earth's mass as 5.97×10^{24} kg and Earth's radius as 6.37×10^6 m.

10 Earth's gravitational field strength is found to be $4.90\ \text{m s}^{-2}$ at the height of a satellite in orbit. Determine how far away from Earth's centre the satellite must be.

REFLECTING

11 State one advantage and one disadvantage of placing satellites in a low Earth orbit (LEO).

12 Explain why the escape velocity from a planet will always be greater than the velocity required to orbit the planet.

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define Kepler's three laws of orbital motion. State the formula for the constant used for Kepler's third law.

2 Contrast the formulas for determining orbit velocity and escape velocity. Explain why the escape velocity is necessarily larger than the orbit velocity for a given space craft and planet.

3 Describe three different astronomical distances, their units and their equivalent distances in metres.

CATEGORY QUESTIONS

4 Describe the type of force that is required to keep a satellite in orbit.

5 Describe how an increase in the orbit radius (altitude) of a satellite changes the orbit velocity.

6 Explain why it can be advantageous to place a satellite in a low Earth orbit (LEO).

ELABORATION QUESTIONS

7 Explain how the acceleration of a satellite with an orbital radius of 9000 km differs from that of a satellite with an orbital radius of 12 000 km.

8 A satellite is orbiting the Sun with a radius of 1.25×10^{11} m. Determine its orbital speed, given the mass of the Sun is 2.0×10^{30} kg.

9 Calculate the gravitational force acting on a satellite of orbital radius 12 500 km and mass 8850 kg. Use M_E as 5.97×10^{24} kg and the r_E as 6.37×10^6 m.

10 Determine the orbital period, in hours, of a satellite of mass 4950 kg and altitude of 9000 km. Use M_E as 5.97×10^{24} kg and the r_E as 6.37×10^6 m.

11 Determine the escape velocity required for a rocket to escape Mercury's gravitational attraction. Use G = 6.67×10^{-11} N m^2 kg^{-2}, Mercury's mass as 3.28×10^{23} kg and Mercury's radius as 2.57×10^6 m.

EVIDENCE QUESTIONS

12 Use the data in Table 6.5.2 to sketch a graph to confirm Kepler's third law. Use the relationship to determine the missing value for Saturn's orbital period.

TABLE 6.5.2 Mean orbital radius and orbital period of selected planets

PLANET	MEAN ORBITAL RADIUS (m)	ORBITAL PERIOD (s)
Earth	1.50×10^{11}	3.15×10^7
Jupiter	7.78×10^{11}	3.74×10^8
Saturn	1.43×10^{12}	
Uranus	2.98×10^{12}	2.65×10^9

13 Determine the gravitational field strength for a satellite orbiting Earth at an altitude of 9500 km. Use Earth's mass as 5.97×10^{24} kg and Earth's radius as 6.37×10^6 m.

END-OF-CHAPTER EXAM

End-of-chapter test

1 What would be required to happen to the orbit velocity of a satellite if it were moved to an orbit of higher altitude?

A It would be slowed down.

B It would be sped up.

C Its velocity would remain constant.

D It would need to exceed the escape velocity of the planet.

2 What are epicycles?

A A necessary aspect of the heliocentric model of the solar system.

B A way of explaining the phases of the Moon.

C 'Circles around circles' to describe the observed retrograde motion of the planets.

D The period of time for a planet to orbit its main star.

3 Who proposed the heliocentric model of the solar system?

A Nicolaus Copernicus

B Galileo Galilei

C Johannes Kepler

D Tycho Brahe

4 The mechanism that provides the centripetal acceleration that satellites in orbit experience towards Earth is:

A force tension.

B mass.

C force friction.

D force gravity.

5 Find the altitude of a satellite in orbit around Earth when its orbital speed is $5.0\,\text{km}\,\text{s}^{-1}$.

Use $r_E = 6.37 \times 10^6$ m and $M_E = 5.97 \times 10^{24}$ kg.

6 Titan, a moon of Saturn, has an orbital radius of 1.22×10^6 km. It takes Titan 15 days and 22 hours to revolve around Saturn. Use this data to determine the mass of Saturn.

7 Determine the force of gravitational attraction between Earth and the Moon. Use $M_E = 5.97 \times 10^{24}$ kg, $M_M = 7.35 \times 10^{22}$ kg and the mean Earth–Moon distance is 3.84×10^8 m.

8 The average $\frac{T^2}{r^3}$ value for our solar system is 2.99×10^{-19} ($\text{s}^2\,\text{m}^{-3}$). The mean orbital radius of Mars is 2.28×10^{11} m. Use this value and Kepler's third law to determine the orbital period of Mars.

9 **a** State Kepler's three laws.

b Why are Kepler's laws referred to as empirical laws?

10 Describe the difference between the heliocentric and geocentric models of the solar system.

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11 Distinguish between orbital velocity, orbital acceleration and escape velocity.

12 Use the data in Table 6.5.1 to plot a graph of mean orbital radius cubed versus orbital period squared to confirm Kepler's third law. Calculate the average $\frac{T^2}{R^3}$ value.

TABLE 6.5.1 Mean orbital radius and orbital period of select planets

PLANET	MEAN ORBITAL RADIUS (m)	ORBITAL PERIOD (s)
Earth	1.50×10^{11}	3.15×10^{7}
Jupiter	7.78×10^{11}	3.74×10^{8}
Saturn	1.43×10^{12}	9.29×10^{8}
Uranus	2.98×10^{12}	2.65×10^{9}

13 Explain why the acceleration of a satellite with an orbital radius of 42 000 km is less than that for a satellite with 7000 km orbital radius.

14 A satellite is orbiting the Sun with an orbit of 1.5×10^{10} m. What is its orbital speed, given the mass of the Sun is 2.0×10^{30} kg?

15 The law of parsimony (Occam's razor) states that the simpler explanation is to be preferred. How does this law apply to the explanations of planetary motion provided by Ptolemy, Kepler and Newton?

» UNIT THREE

GRAVITY AND ELECTROMAGNETISM

Getty Images/SasinParaksa

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Topic 2: Electromagnetism

The topic 'Electromagnetism' introduces students to concepts in electrostatics, including Coulomb's law, electric fields and electric field strength, and concepts in magnetism, including magnetic fields and the forces on charged particles moving within magnetic fields. Electromagnetic induction is explored through Faraday's law of induction and Lenz's law, applying these concepts to the production and transmission of alternating current. Electromagnetic radiation is also explored.

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7 ELECTROSTATICS

Introduction

There are four fundamental forces. The strong force, the weak force, the gravitational force and the electromagnetic force. The electromagnetic force is a combination of both the electrostatic and magnetic forces. All objects with charge emanate an electric field. As charged particles come into close proximity, their behaviour can be described by determining the type of charge on the particle, its electric field, and how far it is from another particle of charge.

Stimulus question

How do stationary charged particles in electric fields behave differently from those moving in an electric circuit?

Getty Images/Robert Downie/EyeEm

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7.1 Coulomb's law

If a particle has a surplus or deficit of negatively charged electrons, the particle is considered to be negatively or positively charged respectively. If two charged particles are separated by some distance r, they exert an equal and opposite force on each other. This force can be classified as attractive or repulsive, depending on the nature of the charge on each particle.

7.1.1 Coulomb's law

7.1.2 Coulomb's law video

Force exerted on charged particles

If two positively charged particles are brought into close proximity, they experience a **repulsive force**; they will repel and move away from each other. If two negatively charged particles are brought into close proximity, they will also repel. Conversely, if a positively charged particle is brought into close enough proximity to a negatively charged particle, they will experience an **attractive force** and move towards each other. This can be summarised as the **first law of electrostatics**.

repulsive force when two particles of like charge move away from each other

attractive force when two particles of unlike charge move towards each other

first law of electrostatics like charges repel and unlike charges attract

The force that any two particles exert on each other is equal (Figure 7.1.1), and the magnitude of the force can be determined with **Coulomb's law**.

The movement of charges due to a repulsive force is described simply by Newton's third law. The forces F(by q on Q) and F(by Q on q):

- are equal in magnitude
- are opposite in direction
- have the same fundamental nature
- each act on a different object.

7.1.3 Attractive and repulsive force

KEY FORMULA

Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

Where:

F = electrostatic force q exerts on Q (N)

q = charge (C) of one point charge

Q = charge (C) of the other point charge

r = separation distance of q and Q (m)

ε_0 = permittivity of free space

$\frac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^9\,\text{N}\,\text{m}^2\,\text{C}^{-2}$; also known as Coulomb's constant, k

Coulomb's law (simplified)

$$F = \frac{kqQ}{r^2}$$

Coulomb's law the second law of electrostatics; states that the force exerted between two point charges is directly proportional to the product of their electric charges, inversely proportional to the square of the distance between them and inversely proportional to the permittivity of the surrounding medium

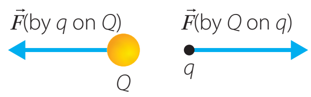

FIGURE 7.1.1 An example of two like-charged particles. They exert the exact same repulsive force on each other. This causes the particle Q to move to the left (from the force exerted by charge q), and causes the charge q to move to the right (from the force exerted by charge Q).

Coulomb's law states that the force that one charged particle exerts on a second charged particle can be found if the magnitudes of both charges are known, and the distance between them can be measured. Negatively charged particles are substituted into Coulomb's law with a negative sign. If a force is calculated to be negative, it means the force between the two particles is attractive. If the force is calculated to be positive, it means the force between the two particles is repulsive. Coulomb's law is also known as the **second law of electrostatics**.

second law of electrostatics Coulomb's law

WORKED EXAMPLE 7.1.1

Calculate the force exerted by a proton on an electron at a distance of 8×10^{-11} m. This is the orbital radius of an electron in hydrogen.

ANSWER

1 Recall that the charge on an electron is -1.6×10^{-19} C and the charge on a proton is 1.6×10^{-19} C.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

$$F = 9.0 \times 10^9 \times \frac{-1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(8 \times 10^{-11})^2}$$

$$F = -3.6 \times 10^{-8}\,\text{N}$$

2 The negative sign indicates a force of attraction between the proton and the electron.

INQUIRING FURTHER

A nucleus contains protons and neutrons, leaving it with a net positive charge. Negatively charged electrons buzz around the nucleus in atoms. How is it that if there is an attractive force between the electrons and protons in an atom, the electrons do not spiral inwards towards the nucleus?

PRACTICAL ACTIVITY 7.1.1

Electrostatic force

AIM

To observe how charged objects are able to move electrically charged stationary objects due to the electrostatic force.

RISK ASSESSMENT

RISK INVOLVED	MINIMISATION STRATEGY
Depending on the size of the electroscope, it could cause injury if dropped.	Be sure to place the electroscope properly on the bench top.

MATERIALS

- an electroscope
- rabbit fur
- perspex rod
- balloon

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PROCEDURE

PART A

1. Set up the electroscope. Wrap the rabbit fur around the perspex rod, and quickly move the fur up and down the rod. This will cause an electron transfer from the rabbit fur to the rod, making the rod negatively charged.
2. Remove the rabbit fur, and bring the charged end of the perspex rod in close proximity to the conduction plate of the electroscope.
3. Observe what happens to the leaves of the electroscope.

PART B

1. Blow up the balloon and rub it rapidly on your head until you notice that hairs are sticking up to the balloon.
2. Turn on a tap so it has a light stream of water coming from it.
3. Bring the part of the balloon you were just rubbing on your head in close proximity to the water stream without actually touching it to the water and observe what happens.

DISCUSSION QUESTIONS

1. From your knowledge of electrons, explain why the electroscope leaves and the water move when a charged object comes close.
2. In terms of Part A, do you think you would have noticed anything different if the rod was positively charged instead of negatively charged? Explain your answer.

SECTION REVIEW 7.1

REMEMBERING

1. Contrast the difference between an attractive force and a repulsive force.
2. State the units for charge.
3. State the formula used to determine the force exerted on a charge from another charge.

UNDERSTANDING

4. Show that the permittivity of free space has a magnitude of 8.84×10^{-12}.

APPLYING

5. A charged particle of 8 C is separated from a charged particle of −7 C by a distance of 2 m. Calculate the force that these two particles exert on each other and state whether the force is attractive or repulsive.
6. Two charges of 9 μC are separated by a distance of 30 cm. Calculate the force of repulsion that these charges exert on each other.
7. Calculate the force on an electron due to a helium nucleus containing two protons and two neutrons at a separation distance of 8×10^{-11} m.
8. Calculate the force on the helium nucleus from Question **7** due to an electron. Compare it with the force on the electron due to the helium nucleus.

7.2 Solving problems: Coulomb's law

Solving problems with Coulomb's law is more complicated than simply putting numbers into a formula. When experimental data is obtained for a known relationship, the data can be manipulated in order to find an unknown constant in the experiment. Additionally, formulas can be algebraically manipulated to acquire a deeper understanding of the relationships between variables.

Algebra: Relationships between variables

It is possible to determine how much the force on a charged particle changes, even if we do not know specific values. For example, we can determine how the force on a charged particle changes if the distance between it and another charged particle increases or decreases by a given factor. Additionally, we can determine how the force of a charged particle changes if the magnitude of the charge on the other charged particle increases or decreases by a given factor.

Consider the following example. Two point charges q and Q exert a force F on each other when separated by a distance r. This can be represented as:

$$F = \frac{kqQ}{r^2}$$

Now, the distance separating these two charges has doubled. This can be represented as:

$$F_{2r} = \frac{kqQ}{(2r)^2}$$

The denominator can be simplified as:

$$F_{2r} = \frac{kqQ}{4r^2}$$

This can be further separated as:

$$F_{2r} = \frac{1}{4}\frac{kqQ}{r^2}$$

By using our initial information that $F = \frac{kqQ}{r^2}$, we see that:

$$F_{2r} = \frac{1}{4}F$$

This shows that as we double the distance r between q and Q (i.e. increase the distance by a factor of 2), we quarter the force the charged particles initially exerted on each other.

Graphing: relationships between variables

Relationships between variables are not always simple. In your study of physics you have been exposed to a range of different proportionalities. These include direct proportionalities, squared proportionalities and inverse proportionalities.

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WORKED EXAMPLE 7.2.1

A charge q is separated from a charge Q by some distance r.

1 How does the force on q change if the magnitude of the charge on Q is doubled?

2 How does the force on q change if q and Q remain the same, but the distance between them decreases by a factor of 3?

ANSWER

1 State the equation:

$$F = \frac{kqQ}{r^2}$$

and $F_{2Q} = \frac{kq2Q}{r^2}$

Separating the 2 from the numerator we obtain:

$$F_{2Q} = 2 \times \frac{kqQ}{r^2}$$

and substituting in $F = \frac{kqQ}{r^2}$, we can conclude that:

$$F_{2Q} = 2 \times F$$

When the charge Q doubles in magnitude, the force exerted on q doubles as well.

2 $F = \frac{kqQ}{r^2}$

And $F_{\frac{r}{3}} = \frac{kqQ}{\left(\frac{r}{3}\right)^2}$

The denominator can be simplified as:

$$F_{\frac{r}{3}} = \frac{kqQ}{\frac{r^2}{9}}$$

which can be further simplified as follows:

$$F_{\frac{r}{3}} = 9\left(\frac{kqQ}{r^2}\right)$$

Substituting $F = \frac{kqQ}{r^2}$ into the equation, we can conclude that:

$$F_{\frac{r}{3}} = 9F$$

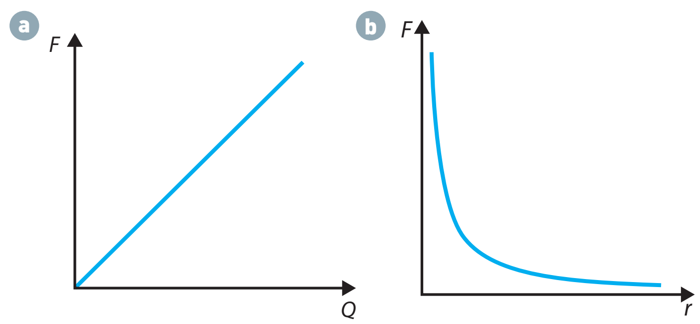
When the separation distance between q and Q is decreased by a factor of 3, the force is 9 times stronger.

This quantitatively verifies what we stated as the second law of electrostatics: that the force exerted between two point charges is directly proportional to the product of their strengths and inversely proportional to the square of the distance between them.

An excellent way to determine the relationship between variables is to graph them. If a linear function is obtained when the two variables are plotted against each other, the relationship is direct. If a parabolic function is obtained, the relationship is squared. Sometimes it is hard to determine the type of function from a set of data; however, using a spreadsheet program to draw a regression line and find an appropriate function describing that line can be very helpful.

Experimentally, if q is separated by a fixed distance from Q and the charge on Q continues to increase in magnitude, we will obtain a graph similar to that in Figure 7.2.1a. If q and Q are fixed in charge and magnitude, and we increase the distance between them, we obtain a graph similar to that in Figure 7.2.1b.

FIGURE 7.2.1 (**a**) A linear function. This function has a constant gradient, showing that the factor by which *Q* changes will directly affect the force *F* in the same fashion. (**b**) An inverse-squared function. This function shows that as the separation distance is increased, the force *F* between the two charged particles decreases dramatically.



In Coulomb's law, we can deduce the following relationships:

$$F \propto q$$

$$F \propto Q$$

$$F \propto \frac{1}{r^2}$$

These are found from the theoretical relationship of Coulomb's law. It is important to note that relationships are stated between variables, not constants. It does not make sense to say that $F \propto k$, as constants are used to help equate variables.

Manipulating data with Coulomb's law

When collecting data, it is paramount to manipulate the data in order to find constants which equate the variables tested. At any given time, only one variable can be changed (the independent variable, usually plotted on the x axis) to test its effect on another variable (the dependent variable); all other variables in the experiment remain constant.

The effect of changing *q* on *F*

As one point charge changes in magnitude, the other point charge and the distance between the point charges remains constant. It is possible to plot F against q in order to determine the type of relationship between F and q.

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WORKED EXAMPLE 7.2.2

Two point charges A and B are separated by a distance of 50 cm. Point charge A has a fixed charge of 2 mC, and point charge B can have its charge varied. The force on point charge A was measured as the charge on point charge B increased and the data recorded in the table below.

Charge on B (C)	Force on A (N)
0.002	1.44×10^7
0.004	2.88×10^7
0.006	4.32×10^7
0.008	5.76×10^7
0.010	7.20×10^7

Plot the force on A against the charge on B to determine the type of relationship between *F* and *q*.

ANSWER

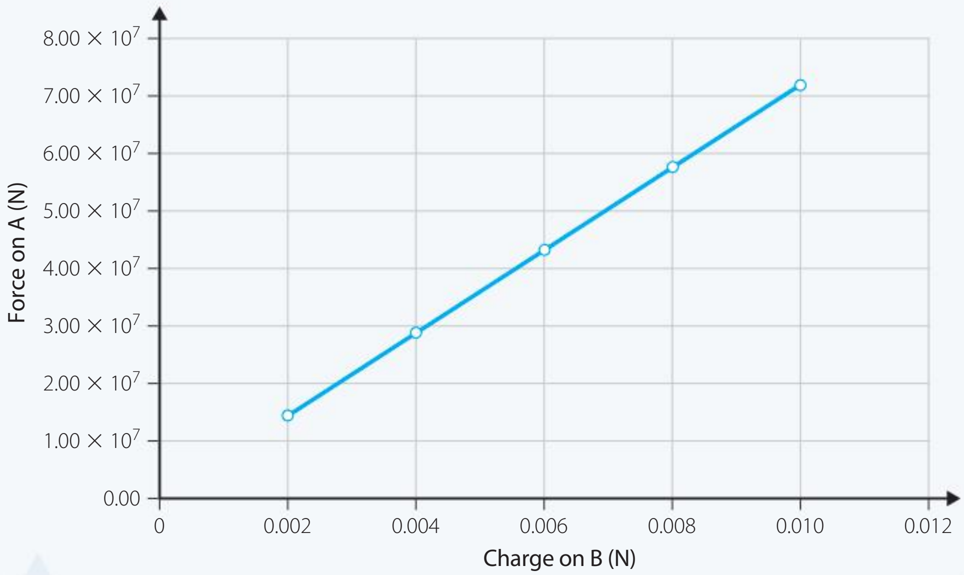

FIGURE 7.2.2

Note that the relationship between *F* and *q* is directly proportional. As *q* increases, *F* increases proportionally. This is indicated by the linear trend line.

The effect of changing *r* on *F*

As both point charges remain constant but the distance between them changes, it is possible to plot the force *F* that the point charges exert on each other against the separation distance *r*, to determine the type of relationship between *F* and *r*. This relationship is reinforced when the data is organised linearly.

The inverse-squared law

When graphing data, there are many different types of curves that look quite similar, and it is very hard to distinguish between them if a relationship is being looked for with no prior knowledge (Figure 7.2.3, page 130). This is where the process of manipulating data to demonstrate a linear relationship becomes helpful.

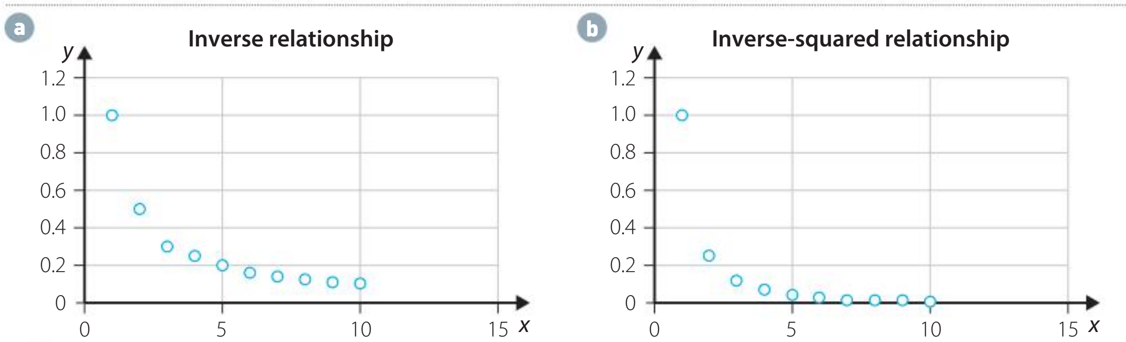

FIGURE 7.2.3 (**a**) An inverse relationship. (**b**) An inverse-squared relationship. Note how similar these curves look and how it may be difficult to distinguish between these relationships when no prior knowledge of the relationship or equation is known.

WORKED EXAMPLE 7.2.3

The following data was collected when like charges of 5 μC were separated by an increasing distance *r*. By manipulating data to show a linear relationship, find an experimental value for Coulomb's constant, *k*.

F (N)	*r* (m)
2250	0.01
563	0.02
250	0.03
141	0.04
90	0.05
63	0.06
46	0.07
35	0.08
28	0.09
23	0.10

ANSWER

From theory, we know that the relationship between *F* and *r* is an inverse-squared relationship. Making a new column for $\frac{1}{r^2}$ we obtain the following:

F (N)	*r* (m)	$\frac{1}{r^2}$ (m^{-2})
2250	0.01	10 000
563	0.02	2500
250	0.03	1111
141	0.04	625
90	0.05	400
63	0.06	278
46	0.07	204
35	0.08	156
28	0.09	123
23	0.10	100

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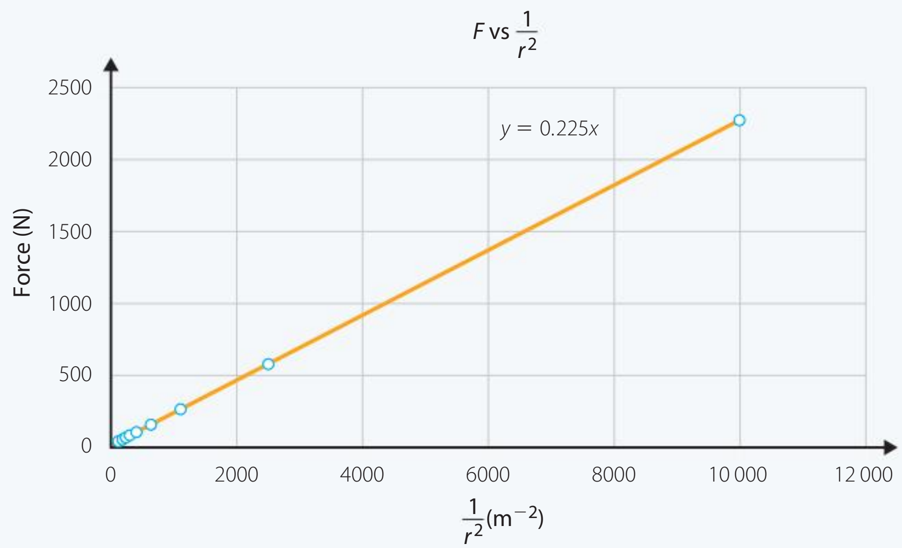
1 Separating Coulomb's law, we can see that:

$$F = \frac{kqQ}{r^2}$$

$$F = kqQ\frac{1}{r^2}$$

$$y = mx$$

2 By plotting F against $\frac{1}{r^2}$, the gradient will be equal to kqQ:



3 We can find k as follows:

Gradient $= m = 0.225\,\text{N m}^2$

$0.225 = kqQ$

$$k = \frac{0.225}{qQ}$$

$$k = \frac{0.225}{5\times10^{-6}\times5\times10^{-6}}$$

$\therefore\ k = 9\times10^{9}\,\text{N m}^2\,\text{C}^2$

As expected.

SECTION REVIEW 7.2

UNDERSTANDING

1 State the relationship between force and distance for two charged particles Q and q.

APPLYING

2 A charge of 4 mC is separated from a charge of −2 mC by 2.5 m. Calculate the force these particles exert on each other.

3 Calculate the distance between charges of 7 μC and 12 μC when the force they exert on each other is 1.89 N of repulsion.

4 Two charges of the same charge and magnitude are separated by a distance r. How does the force of repulsion change when the distance between these charges is tripled?

5 Two unlike charges, one double the charge of the other, are separated by a distance *r*. By what factor does the force of attraction between them change when the distance between them is quartered?

6 For two point charges *Q* and *q*, how does the force on *Q* change when *q* is halved if the separation distance remains the same?

7 For two point charges *Q* and *q*, how does the force on *Q* change when the distance between *Q* and *q* is increased by a factor of 5?

ANALYSING

8 The following data is collected from an experiment in which two point charges of –50 μC are separated by a distance *r*. The force exerted by each charge was measured with a Coulomb meter at increasing separation distances as shown in the table below.

r (m)	*F* (N)
0.05	9001
0.10	2250
0.15	992
0.20	566
0.25	365
0.30	251
0.35	180
0.40	147
0.45	120
0.50	92

Plot *F* on the *y* axis against $\frac{1}{r^2}$ on the *x* axis and use the gradient to determine Coulomb's constant *k*.

SYNTHESISING

9 The following data was collected from an experiment in which two point charges of 3.5 μC and –5.5 μC respectively were separated by a distance *r*. The force exerted by each charge on the other was measured with a Coulomb meter at increasing separation distances as shown in the table below.

r (mm)	*F* (N)
5	7000
10	1742
15	203
20	431
25	281
30	192
35	137
40	106
45	89
50	70

Graphically analyse the data to determine an experimental value for *k* and ε_0. Identify any anomalies.

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7.3 Electric fields

All objects with electric charge emanate an **electric field** around themselves. This field depends on the size of the charge, and the distance from the charged object. In section 7.2, you used Coulomb's law to calculate the magnitude of the force that charged particles exerted on each other. The reason these particles exert either an attractive or repulsive force on each other is due to their electric field.

electric field the field due to an electric charge, which applies a force to other electric charges

Electric field strength

Electric fields emanate outwards from charged objects or particles. The electric field at a point is defined as the force per unit charge that acts on a small positive test charge at that point (Figure 7.3.1).

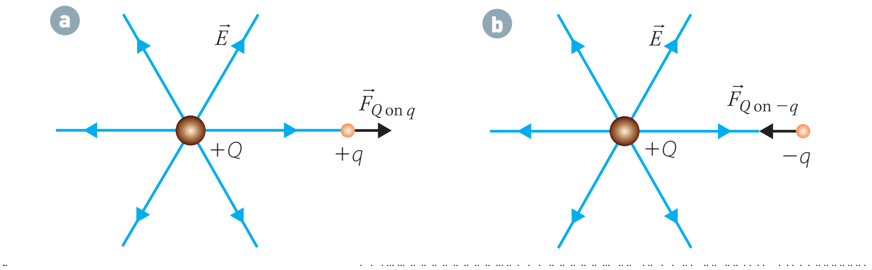

FIGURE 7.3.1 (**a**) A large positive charge repels a small positive test charge. (**b**) A large positive charge attracts a small negative test charge.

The electric field strength at any point can be determined as follows:

$$E = \frac{F}{q}$$

Force is a vector, which means that the electric field is also a vector quantity and has both magnitude and direction. An electric field exerts a force in the direction of the field on a positive charge, and in the opposite direction on a negative charge. Table 7.3.1 shows the approximate electric field strength and direction of different appliances and electric sources.

KEY FORMULA

Electric field strength

$$E = \frac{F}{q}$$

Where:

E = electric field strength (N C^{-1})

F = force acting on test charge q (N)

q = charge of the test object in the field (C)

TABLE 7.3.1 Some electric field strengths

ELECTRIC FIELD DUE TO ...	APPROXIMATE FIELD STRENGTH (N C^{-1})
a hairdryer, 20 cm away	4
a thunderstorm	50, upwards
Earth's fair weather field	100, downwards
high voltage overhead power lines, 30 m away	10–1000
an old electric blanket, 10 cm away	2000

WORKED EXAMPLE 7.3.1

A battery is connected across a piece of copper wire resulting in an electric field of $3.0\,\text{N}\,\text{C}^{-1}$ in the wire. What is the force on an electron in this wire?

ANSWER

1 We know that $E = 3.0\,\text{N}\,\text{C}^{-1}$ and that $q = {}^{-}1.6 \times 10^{-19}\,\text{C}$.

$$E = \frac{F}{q}$$

$$F = Eq$$

$$F = 3.0 \times -1.6 \times 10^{-19}$$

$$F = -4.8 \times 10^{-19}\,\text{N}$$

2 The force is in the direction opposite to that of the field.

KEY FORMULA

Electric field strength when q is unknown

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Where:

Q = large charge (C)

r = distance in metres (m) between q and Q

$\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\,\text{N}\,\text{m}^2\,\text{C}^2$; also known as Coulomb's constant, k

Electric field strength simplified:

$$E = \frac{kQ}{r^2}$$

The electric field strength can also be found if the force is unknown, but the charge emanating from the electric field is known, as well as the distance the test charge is from the source. The greater the distance between the test charge and the source of the electric field, the smaller the force the test charge will experience, and hence the smaller in magnitude the electric field is at that point.

As

$$E = \frac{F}{q} \quad (1)$$

And

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad (2)$$

We can substitute (2) into (1) to obtain the following:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\cancel{q}Q}{r^2} \times \frac{1}{\cancel{q}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

The electric field from Q at an infinite distance away from a test charge q is $0\,\text{N}\,\text{C}^{-1}$. From this logic, if r is large enough, the electric field from Q is approximated to be zero.

WORKED EXAMPLE 7.3.2

What is the electric field strength on a test charge q if it is placed at a distance of 5.5 m from a source charge of $4.6\,\mu\text{C}$?

ANSWER

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E = 9 \times 10^9 \times \frac{4.6 \times 10^{-6}}{5.5^2}$$

$$E = 1369\,\text{N}\,\text{C}^{-1}$$

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It is important to note that in all calculations between a large charge Q and test charge q, the field of Q is so much larger than that of q that the field of q is not represented in diagrams.

Electric field lines

Fields can be represented by field diagrams. An electric field diagram uses lines with arrows to show the direction of the force on a small positively charged test particle. This model is called the **electrostatic field model**.

To draw a field diagram, start by considering the force on a test charge at various points. Start with a single positive charge and think about what happens when you put a small positive test charge close to it. Like charges repel, so the test charge will be accelerated away from the positive charge. By looking at these force vectors in Figure 7.3.2, we are able to extrapolate these force vectors into **electric field lines** that demonstrate the direction a positive test charge will move when placed in close proximity to the source. Conversely, if we apply the same logic with the same force vectors with a positive test charge being acted on by a large negative charge, we obtain the field lines pointing in the opposite direction (Figure 7.3.3).

electrostatic field model
the model that assigns an electric field to stationary charges; it is this field that exerts forces on other charges

electric field lines
net lines of force pointing in the direction a positive test charge will move when placed in the electric field due to a charge Q

7.3.1 Electric field lines

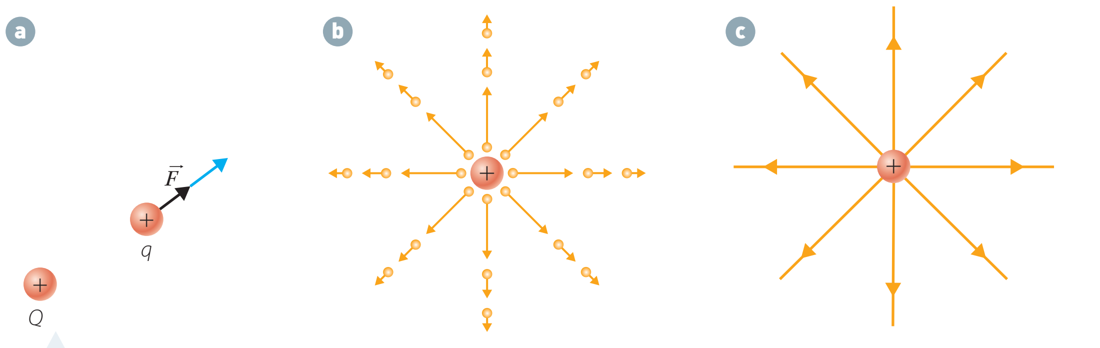

FIGURE 7.3.2 (**a**) A test charge is accelerated away from the positive source charge. (**b**) The size of the vector decreases as the distance between test charge and source charge increases. (**c**) Join up vectors to obtain electric field lines. Note that field lines always point *away* from a positive charge.

Field lines of two close charges

A positive and negative charge are attracted to each other. Coulomb's law allows us to calculate the magnitude of this attraction, and field lines allow us to visually represent how this attraction occurs.

Electric field lines:

- point in the direction of the force acting on a positively charged particle due to the field
- never cross, as they represent *net* lines of force
- begin on positive point charges and end on negative point charges
- field strength is proportional to the density of the field lines.

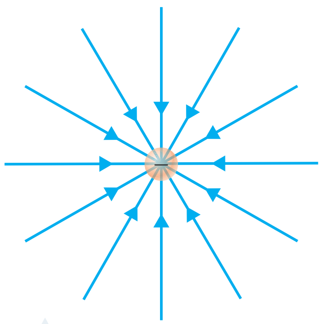

FIGURE 7.3.3 Field line diagram for a negative point charge. Note that the field lines always point *towards* a negative charge.

Uniform electric fields

uniform electric field electric field that has the same magnitude and direction at all points

A **uniform electric field** is one that has the same magnitude and direction at all points. A uniform electric field exists close to any large, flat, uniform distribution of charge. On a small scale, a uniform electric field can be created by two charged parallel plates. This arrangement of charged plates is called a capacitor (Figure 7.3.4).

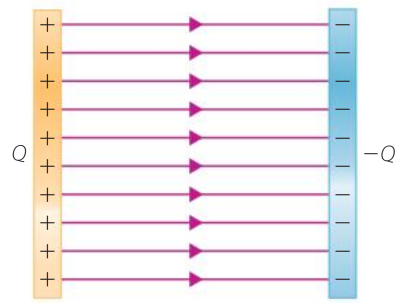

FIGURE 7.3.4 Uniform electric field formed between the plates of a parallel plate capacitor.

On a larger scale, we can look at the field very close to the surface of a large charged sphere, such as the dome of a van de Graaff generator. In this case, the field is approximately uniform. You have seen and used this useful approximation many times before for the gravitational field of Earth. Every time you write $F = mg$ you are making the approximation that Earth is flat. Most of the time this is perfectly reasonable. Close to the surface of Earth the gravitational field is effectively constant.

This applies to Earth's electric field as well. You will be familiar with the effects of Earth's electric field in stormy weather (Figure 7.3.5). In sunny weather, Earth has an electric field of approximately $100\,NC^{-1}$, pointing downwards. Although the field varies over the surface of Earth, it can be treated as uniform for distances of a few hundred metres or less. The force due to Earth's electric field on a small charged particle (e.g. an electron) is many orders of magnitude greater than the force on it due to the gravitational field. For neutral objects such as humans, the gravitational force is much greater.

7.3.2 Earth's vertical electric field

Electrical potential energy

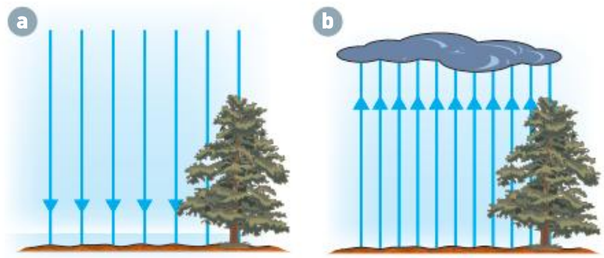

FIGURE 7.3.5 Earth's electric field in (**a**) sunny weather, and (**b**) stormy weather. Note that in stormy weather the field changes direction and is stronger.

Just as the gravitational field does work on an object falling or being lifted in a gravitational field, the electric field does work on a charged object in an electric field. For example, when a pencil falls from a height h, the gravitational field does work on the pencil, and the pencil accelerates towards the ground. The same happens to a charged object when placed in an electric field. The charge will accelerate in the direction of the field. In both of these cases, the work done on the pencil or charge is equal to the change in potential energy of the system.

electrical potential energy potential energy stored in an electric field. The change in potential energy of an object is also the work done on that object by the electric field

Although we refer to these objects having potential energy when they are initially stationary in the field, this is not strictly true. Rather, the objects in the system have potential energy. We could say Earth and the pencil combined have the potential energy, or even the gravitational field and the pencil. Potential energy exists whenever a force acts between objects, and hence a small isolated charge cannot be said to have potential energy. It only has potential energy because of its interaction with an electric field, which is due to yet another charged object.

7.3.3 Electrical potential energy

7.3.4 Electrical potential energy video

Hence, fields are not only a way of exerting a force at a distance: *fields also store energy*. The gravitational field stores gravitational potential energy and the electric field stores **electrical potential energy**.

electrical potential potential energy per unit charge in an electric field

When considering a charge in an electric field, it is important to consider the **electrical potential** (or just 'potential' of that charge). Electrical potential has the same relationship to potential energy that the field has to force. The field is defined as force per unit charge $E = \dfrac{F}{q}$, and the potential is defined as the potential energy per unit charge $V = \dfrac{U}{q}$. The electrical potential, V, is measured in JC^{-1}, also known as volts.

potential difference the difference in potential between two points in an electric field; the work done per charge

The difference in potential for a charge at different points in the field is called the **potential difference** or, more commonly, voltage. The change in potential energy when a charged particle is displaced in a

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field is equal to the work done on the object. More specifically, the potential difference for a displaced charge in an electric field is equal to the work done per unit charge during the displacement.

KEY FORMULA

Electrical potential

$$V = \frac{U}{q}$$

Where:

V = electrical potential of a charge (V)

U = potential energy (J)

q = magnitude of the charge in the field (C)

POTENTIAL DIFFERENCE

$$\Delta V = \frac{\Delta U}{q}$$

Where:

ΔV = potential difference between two points (V)

ΔU = change in potential energy (J), also known as the work done on the charge

q = magnitude of the charge moving in the field (C)

WORKED EXAMPLE 7.3.3

An alpha particle of 3.2×10^{-19} C moves at a distance of 1.0 m along Earth's fair weather field lines. The field is 100 V m^{-1}, so the particle passes through a potential difference of 100 V. What is the work done on the alpha particle?

ANSWER

$$\Delta V = \frac{\Delta U}{q} = \frac{W}{q}$$

$W = q\Delta V$

$W = 3.2 \times 10^{-19}\,\text{C} \times 100\,\text{V}$

$W = 3.2 \times 10^{-17}\,\text{J}$

The magnitude of the work done is 3.2×10^{-17} J as the alpha particle is moving with the field.

Positive and negative potential difference

A positively charged object released from rest in an electric field will be accelerated in the direction of the field. The force exerted on the object by the field acts to increase its kinetic energy. From conservation of energy, we know this kinetic energy must come from somewhere. It comes from a decrease in potential energy. Hence the positive charge has moved from a point of higher electrical potential to one of lower electrical potential, so ΔV must be negative.

If a negatively charged object moves from higher to lower potential so that ΔV is negative, then the change in potential energy, ΔU, is positive. This can only happen if the charge has some initial kinetic energy, or if some external force is doing work on the system. This is shown in Figure 7.3.6, page 138. This is the case for an electron in a circuit passing through a battery. Table 7.3.2, page 138, summarises the changes in potential and energy when a charge moves in a field. The field will do work on the charge when a positive charge moves with the field or a negative charge moves against the field. Work must be done on the system to make a positive charge move against the field or a negative charge move with the field.

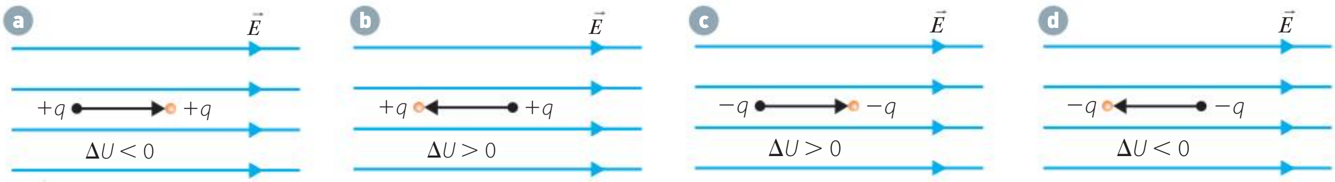

FIGURE 7.3.6 Kinetic energy changes when a charge moves in an electric field. (**a**) A positive charge moving in the direction of the field experiences an increase in kinetic energy (E_k), and a decrease in potential energy, U, hence the potential difference is negative. (**b**) A positive charge moving against the direction of the field experiences a decrease in E_k, an increase in U, and hence the potential difference is positive. (**c**) A negative charge moving in the direction of the field experiences a decrease in E_k, an increase in U, and hence the potential difference is positive. (**d**) A negative charge moving against the direction of the field experiences an increase in E_k, a decrease in U, and hence the potential difference is negative. Note that if the change in potential is positive, the potential difference is also positive and vice versa.

TABLE 7.3.2 Changes in potential energy for a charge moving in an electric field

CHARGE	MOVEMENT *WITH* OR *AGAINST* THE FIELD LINES	CHANGE IN POTENTIAL	CHANGE IN POTENTIAL ENERGY	WORK DONE *BY* OR *ON* THE FIELD
+	With	Negative (decrease)	Negative (decrease)	By
+	Against	Positive (increase)	Positive (increase)	On
–	With	Positive (increase)	Positive (increase)	On
–	Against	Negative (decrease)	Negative (decrease)	By

WORKED EXAMPLE 7.3.4

An electron in an X-ray machine is accelerated through a potential difference of 100 kV before colliding with a target and emitting X-rays. What is the energy of the electron just before it hits the target?

ANSWER

If the electron accelerates from rest, before it hits the target all its potential energy will have converted into kinetic energy.

So:

$\Delta U = q\Delta V$

$\Delta U = 1.6 \times 10^{-19}\,\text{C} \times 100\,\text{kV}$

$\therefore \Delta U = 1.6 \times 10^{-14}\,\text{J}$

This is the energy the electron has just before it hits the target.

The zero of potential energy

The changes in the potential and potential energy of a system can be positive or negative, depending on how we define the **zero of potential energy**. Electrostatics uses the same convention as gravity in this circumstance, so that the zero of potential energy is when the components of the system are infinitely separated, and all the charges that make up the system are very far apart.

zero of potential energy when all charges in the system are infinitely far apart; any other arrangement will have positive or negative potential energy

Consider the case of two positive charges very far apart. This arrangement has zero potential energy. If we want to bring them closer together into some final arrangement, we have to do work on them because they will repel each other. Work is applied to the system so the final potential energy is

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positive. Now consider a positive and negative charge very far apart so that their arrangement has zero potential energy. These charges attract each other, and if we release them from very far apart they will move towards each other. This causes an increase in kinetic energy and hence a decrease in potential energy. A system of a positive and a negative charge at any separation less than infinity has negative potential energy.

SECTION REVIEW 7.3

REMEMBERING

1 Draw an electric field diagram for a positive charge and a negative charge.

2 State the difference between electrical potential and electrical potential energy.

UNDERSTANDING

3 Explain how a change in potential energy for a charge moving in an electric field can be concluded to be positive or negative.

4 Compare a uniform electric field and a non-uniform electric field (such as a field produced from a point charge).

APPLYING

5 A current of 1.0 A flows through a 12 V battery for 1.0 s. What is the change in potential energy of the battery?

6 What is the electric field strength of a proton at a distance of 2 cm?

7.4 Solving problems: electric field strength

Solving problems involving electric field strength requires an understanding of the scenario at hand. The electric field due to a point charge varies with the size of the charge, and decreases with the distance squared from the charge. Additionally, electric field strength can be determined by considering the force that one point charge Q exerts on a second point charge q.

WORKED EXAMPLE 7.4.1

Two conducting spheres each of charge 6.0 μC are hanging by insulating thread from a beam.

1 Find the electric field strength experienced by each sphere if they are 10 cm apart.

2 Determine how the electric field strength would change if the spheres had double the charge but remained 10 cm apart.

ANSWER

1 $E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$

$E = 9.0 \times 10^9\ \text{N m}^2\ \text{C}^{-1} \times \frac{6.0 \times 10^{-6}\ \text{C}}{0.1^2}$

$E = 5.4 \times 10^6\ \text{N C}^{-1}$

2 $E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$

$E_{2Q} = \frac{1}{4\pi\varepsilon_0} \times \frac{2Q}{r^2}$

$E_{2Q} = 2E$

The field would double if the charges remained at the same distance apart but the spheres had double the charge.

WORKED EXAMPLE 7.4.2

If a charge of 4.6 μC is placed 11 m from a test charge q, what is the electric field strength the test charge experiences?

ANSWER

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

$$E = 9 \times 10^9\ \text{N}\,\text{m}^2\,\text{C}^{-2} \times \frac{4.6 \times 10^{-6}\ \text{C}}{\left(11\,\text{m}^2\right)^2}$$

$$E = 342\ \text{N}\,\text{C}^{-1}$$

SECTION REVIEW 7.4

APPLYING

1 Calculate the electric field strength 20 cm from a charge of 25 μC.

2 The electric field strength 12 cm from a charge is $1.0 \times 10^{-3}\ \text{N}\,\text{C}^{-1}$. Determine the size of the charge.

3 The force experienced by an electron in an electric field is 100 N. What is the magnitude of the electric field?

4 At what distance from an object carrying a charge of 4.0×10^{-8} C would the electric field strength be $2.3 \times 10^{6}\ \text{N}\,\text{C}^{-1}$?

ANALYSING

5 Two positive charges of 10 μC and 20 μC are situated 50 cm apart in air. Determine the electric field strength midway between these charges.

7.5 Solving problems: charge in an electric field

Electric charges create electric fields, and electric fields store electrical potential energy. If a test charge is placed in an electric field, work will be done on or by the test charge. This is governed by the potential difference between two points in the electric field. Charges placed in electric fields can be initially stationary and move according to the field, or they can have some initial velocity.

Even though charges can be considered stationary when placed in electric fields, the fact that they have electrical potential energy means that they will end up moving. Charges have mass and in an electric field they experience a force, so they will behave according to Newton's laws of motion. Newton's first law tells us that an object experiencing a net force will accelerate. Newton's second law quantifies this acceleration as $a = \frac{F}{m}$. From the definition of an electric field $E = \frac{F}{q}$, we can see that $F = Eq$; therefore, $a = \frac{Eq}{m}$. This is the acceleration a charge q will experience when in an electric field, due to the electrical potential.

KEY FORMULA

$$a = \frac{Eq}{m}$$

Where:

a = acceleration of the test charge q ($m\,s^{-2}$, a vector quantity)

E = electric field strength ($N\,C^{-1}$, a vector quantity)

q = charge of the test charge q (C)

m = mass of the test charge (kg)

WORKED EXAMPLE 7.5.1

A positive calcium ion (Ca^{2+}) with mass of 6.7×10^{-26} kg experiences an acceleration of $4.8 \times 10^{13}\,m\,s^{-2}$ as it moves through a channel in a cell membrane. What is the electric field in the membrane?

ANSWER

$$a = \frac{Eq}{m}$$

$$E = \frac{am}{q}$$

$$E = \frac{4.8 \times 10^{13}\ m\,s^{-2} \times 6.7 \times 10^{-26}\ kg}{3.2 \times 10^{-19}\ C}$$

$$E = 1.0 \times 10^{7}\,N\,C^{-1}$$

The electric field is in the direction of the acceleration.

WORKED EXAMPLE 7.5.2

Consider a sodium ion (Na^{+}) of mass 3.82×10^{-26} kg travelling through a channel in a cell membrane. If the electric field strength in the membrane is $1.0 \times 10^{7}\,N\,C^{-1}$, what acceleration does the sodium ion experience?

ANSWER

$$a = \frac{Eq}{m}$$

$$a = \frac{1.0 \times 10^{7}\ N\,C^{-1} \times 1.6 \times 10^{-16}\ C}{3.82 \times 10^{-26}\ kg}$$

$$a = 4.19\,m\,s^{-2}$$

SECTION REVIEW 7.5

APPLYING

1 It takes 155 J of work to bring a charge of +11 C from infinity to a positively charged conductive sphere. What is the electrical potential of this sphere?

2 What is the electrical potential at a distance of 2 cm from a charge of 5.0×10^{-7} C?

3 An electron moves in an electric field from a point at which the potential is 1.5 V to a point at which the potential is 3.5 V.

- a Determine the change in the potential energy of the electron.
- b Is this change positive or negative?
- c Where does the energy come from or go to?

4 What is the acceleration of a sodium ion, Na^+, of mass 3.82×10^{-26} kg in an electric field of 1.0×10^{7} N C^{-1}?

ANALYSING

5 What is the speed of a particle that has been accelerated from rest through a potential difference of 1000 V when the particle is:

- a an electron?
- b a proton?
- c an alpha particle?

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Attractive force
   b Electrical potential
   c Electrical potential energy
   d Electric field
   e Electric field lines
   f Electrostatic field model
   g Potential difference
   h Repulsive force
   i Uniform electric field
   j Zero of potential energy

2 State the quantities in electrostatics that can be used to determine electric field strength.

CATEGORY QUESTIONS

3 Compare test charges in an electric field with electrons moving in a closed circuit.

4 Explain how to determine whether work is done on or by the field when charges move in electric fields.

5 Explain why it is useful to assume the electric field at Earth's surface is uniform.

ELABORATION QUESTIONS

6 Explain in terms of electric fields why like charges repel and unlike charges attract each other.

7 Why do charges accelerate in electric fields?

8 If fields could only do work on charges, and the electric field could *not* have work done on it, what would this mean for our current understanding of potential and Newton's laws?

EVIDENCE QUESTIONS

9 The cells in your body have an electric field in their membranes and this field is maintained by pumps in the cell wall. These pumps move ions through the membrane, so that the outside of the cell is positively charged compared to the inside of the cell. The electric field points into the cell and has a magnitude of approximately $10\,000\,000\,\mathrm{N\,C^{-1}}$. Explain why pumps are needed to move these ions if there is such a strong electric field in the cell.

10 During storms, people are advised to stay indoors for safety. Explain how electric field changes at Earth's surface during a storm and how a lightning strike occurs.

END-OF-CHAPTER EXAM

End-of-chapter test

1 An attractive force would be observed in which of the following scenarios?

A An electron placed near an electron.

B A proton placed near an electron.

C A proton placed near a proton.

D An alpha particle placed near a proton.

2 A charge q is separated from a charge Q by some distance r. The force experienced between them is 1500 N. Which of the following must be true?

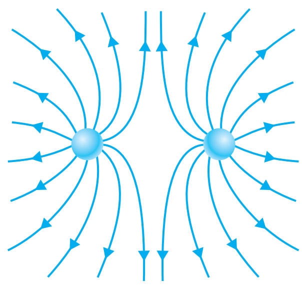
A q and Q are attracted to each other.

B q and Q have the same magnitude of charge.

C q and Q have like charges.

D q and Q have a magnitude of charge smaller than 1 C.

3 The field diagram below shows the interaction of two fields. These fields are due to:



A two positive charges.

B two negative charges.

C two neutrally charged spheres.

D a positive and negative charge.

4 Kinetic energy changes when a charge moves in an electric field. If the kinetic energy of the charge decreases, then:

A the charge must be positive.

B the charge must be negative.

C the change in potential difference must be positive.

D the change in potential difference must be negative.

5 Which of the following are the units for an electric field?

A N C^{-1}

B V m^{-1}

C E

D kg mC

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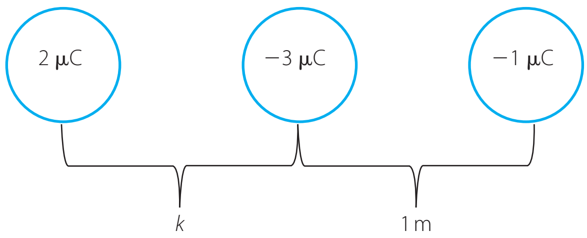
6 If an electron is moving with the field lines, does this mean the electrical potential of the electron is increasing or decreasing?

7 Determine the net force on the middle sphere in the illustration below.



8 What work needs to be done to move a charge of 2.0×10^{-14} C across a potential difference of 6.0×10^{2} V?

9 Two protons in a molecule are 3.8×10^{-10} m apart. Find the magnitude of the electric field due to one of the protons at this distance.

10 How is electrical potential related to the electrical potential energy?

11 Does a positive force between two charged particles mean there is an attractive or repulsive force between them?

12 What arrangement must two charges have in order to have zero potential energy?

13 Explain why the potential difference changes when a charged particle is moving in an electric field.

14 What does it mean for an electric field to have work done on it?

15 The table below shows how the electrostatic force changes between two 10 μC spheres for increasing separation distances *r*.

r (cm)	*F* (N)
5	3.60
10	0.95
15	0.40
20	0.22
25	0.15
30	0.10
35	0.05

Manipulate the data to obtain a linear relationship, and use that to determine an experimental value for the constant *k* from the gradient.

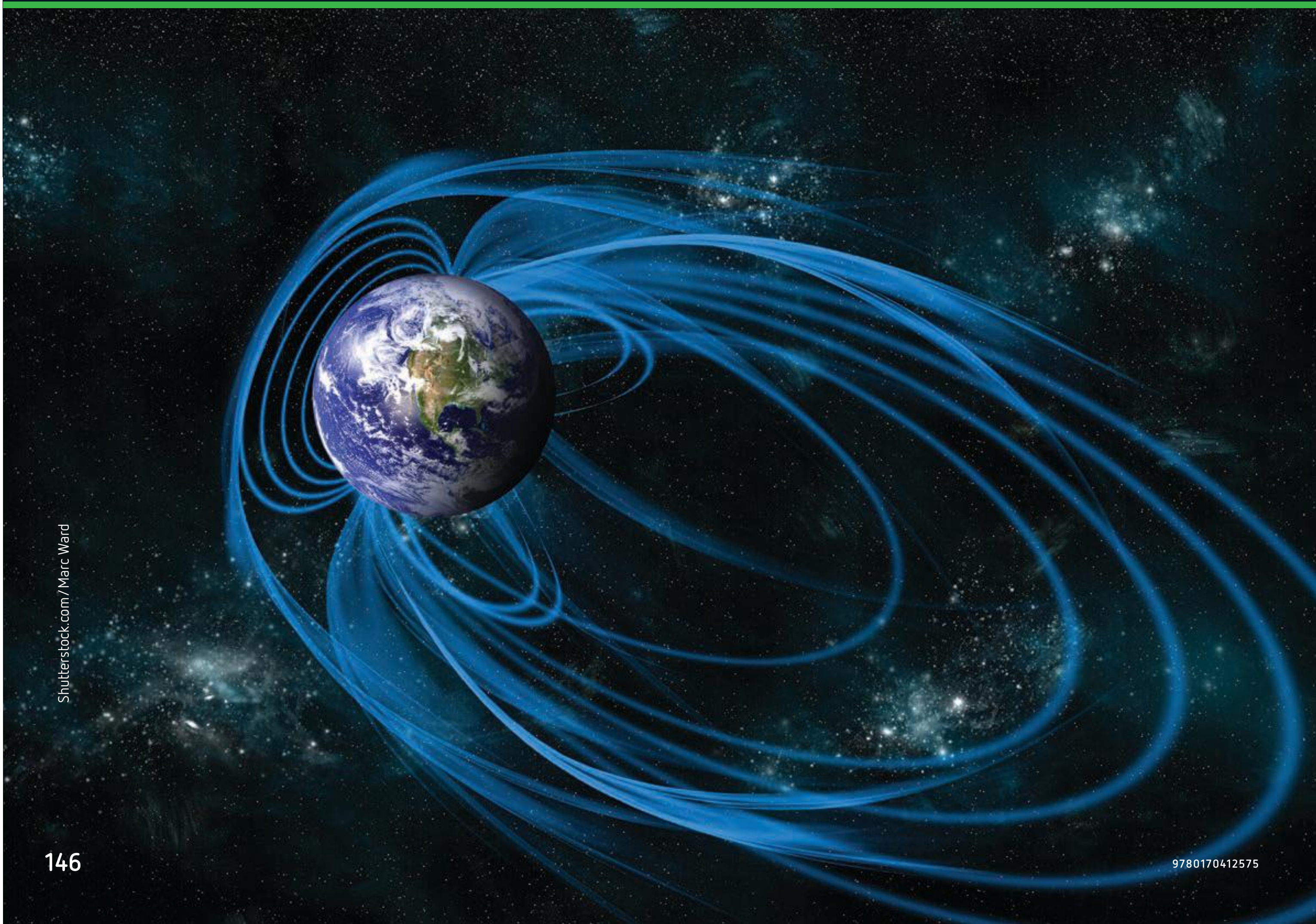
8 MAGNETIC FIELDS

Introduction

Magnetic fields are important to us in many ways. Earth's magnetic field has been used for navigation by organisms such as pigeons, sharks, bees and bacteria for millions of years, and humans have used it for thousands of years. The properties of magnetic materials such as magnetite and iron in Earth's crust have only been explained in the last couple of hundred years. It is the properties and behaviour of fundamental particles such as electrons that give rise to the magnetic behaviour of metals such as iron. In order to understand magnetism, scientists need to extend the previous model of the electromagnetic field.

Stimulus question

Are electricity and magnetism independently occurring phenomena?



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8.1 Magnetic fields

A **magnetic field** is the field created by moving charges, including charges in magnetic materials (such as iron). The strength of a magnetic field depends on the source. If the magnetic field is produced from charges within a material (such as iron), the field depends on how much *alignment* there is within the atom. If the magnetic field is produced from moving charges, or a current, the strength is proportional to the current.

magnetic field
field created by moving charges, including charges in magnetic materials

8.1.1 Magnetic field basics

8.1.2 What are magnetic fields?

Electron alignment

Metals, for example iron, cobalt, nickel and ores such as magnetite, have natural magnetic properties. An object that is magnetised has its internal **magnetic domains** aligned. Each metal substance is abundant in electrons, and each electron can be thought of as having a north and south **magnetic pole** (Figure 8.1.1). This means that all the small magnets within each material must point in the same direction in order for a magnet to be formed. If the metal maintains these magnetic properties, it is referred to as a permanent magnet.

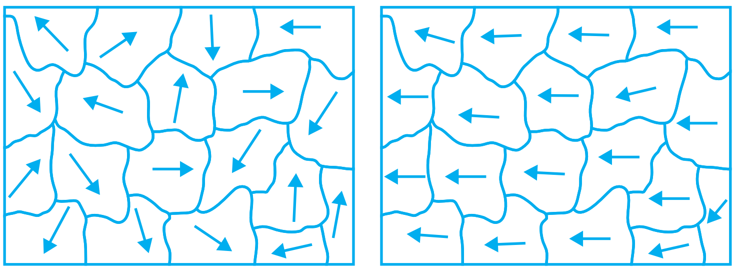

FIGURE 8.1.1 (**a**) Randomly aligned domains. The material is not magnetised as a result. (**b**) Domains are lined up. This material would be magnetised.

magnetic domain
region within a magnetic material where the magnetic properties point in the same direction

magnetic pole
point where magnetic field lines go out of or come in to

magnet
a substance that has the majority of its magnetic domains aligned. Magnets produce magnetic fields

Magnets are magnetic substances that have a large majority of their domains aligned. They exhibit magnetic fields, and can, in turn, affect other magnetic materials nearby. Materials can be classified by the effect that nearby magnets have on them. It is important to note that if a magnetised material is cut in half, there would still be a north and south pole because each of the individual domains can be thought of as miniature magnets. The strength of a magnetic field of a magnet is dependent on how many of the internal domains are aligned.

diamagnetic
materials weakly repelled by nearby magnets; examples include bismuth and copper

paramagnetic
materials weakly attracted to nearby magnets; examples include aluminium and rare earth ions. Paramagnetic substances do not retain permanent magnetism

Types of magnetic materials

Michael Faraday (1791–1867) classified materials based on how they were influenced by nearby magnets. Materials can be **diamagnetic**, **paramagnetic** or **ferromagnetic**. Diamagnetic materials are weakly repelled by nearby magnets, paramagnetic materials are weakly attracted by nearby magnets, and ferromagnetic materials are strongly attracted to nearby magnets.

ferromagnetic
materials that are strongly attracted to nearby magnets; examples include iron, nickel and cobalt. Ferromagnetic substances can retain permanent magnetism and this can be induced by other very strong magnets

SECTION REVIEW 8.1

REMEMBERING

1 Define the following term.
   a Magnet
   b Magnetic field

UNDERSTANDING

2 Explain how a magnetic field is formed.

3 Compare diamagnetic, paramagnetic and ferromagnetic materials.

8.1.3 Types of magnets

8.2 Representing magnetic fields

north pole
the pole of a magnet where the field lines start; they are drawn exiting the north pole

south pole
the pole of a magnet where the field lines end; they are drawn entering the south pole

If a material is magnetised, it will have a specific **north pole** and **south pole**. Magnetic field lines are defined from this property. Similar, but not the same as electric field lines, magnetic field lines are drawn coming *out* from the north end of a magnet, and *in* to the south end (Figure 8.2.1).

When a north pole of one magnet and a south pole of another magnet are brought in close proximity, they will *attract* and their field lines will align (Figure 8.2.2a). When two like poles are brought into close proximity, they *repel* and their field lines will push away from each other (Figure 8.2.2b).

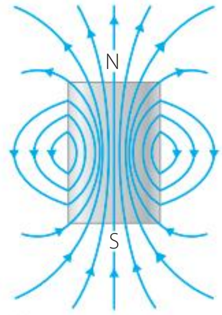

FIGURE 8.2.1 Magnetic field lines coming out of a bar magnetic. A bar magnet is a bar of metal that has all its domains aligned. Note that the arrows point *out* from the north pole, and *in* to the south pole.

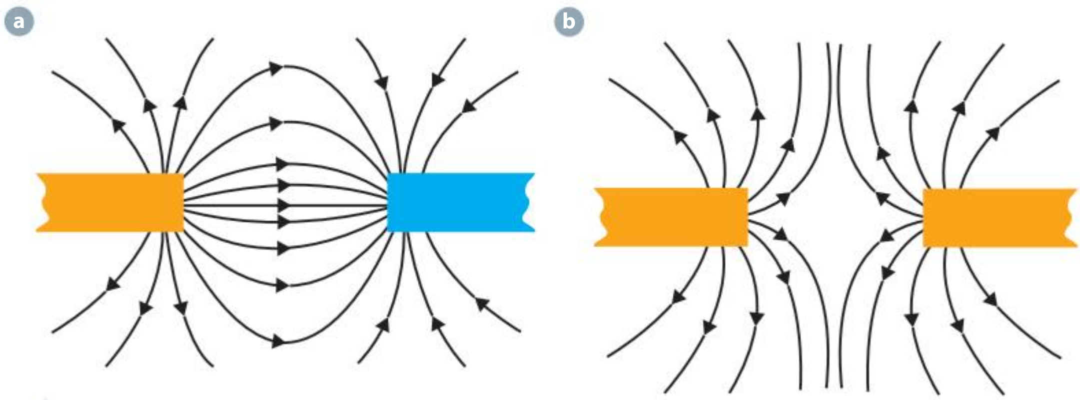

FIGURE 8.2.2 (**a**) Two unlike poles (north and south) brought together and attracting due to their field lines lining up. (**b**) Two like poles brought together and repelling. Note the similarity between these field lines and the electric field lines in the previous chapter when like charges and unlike charges are brought in close proximity.

PRACTICAL ACTIVITY 8.2.1

Visualising magnetic field lines

AIM

To observe the magnetic field lines of different magnets in two and three dimensions.

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Dropped magnets can damage equipment and toes.	Ensure magnets are placed on the benchtop and are moved across the surface of the benchtop.
Inhaled iron filings can cause respiratory problems.	Ensure iron filings are in a case and the room is well ventilated.

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» MATERIALS

- bar magnet
- horseshoe magnet
- case of iron filings
- six very small compasses (or free iron filings)

METHOD

1 Scatter the small compasses (or iron filings) on a flat surface, around both a bar magnet and a horseshoe magnet.
2 Observe how the compasses or iron filings align according to where they are around the magnets. Slightly shift the compasses around and notice their re-alignment to the field.
3 Sketch lines according to how you think the field lines look for both magnets and compare the lines for the bar magnet and the horseshoe magnet.
4 Place the case of iron filings above the north or south end of the bar magnet and rotate the magnet on the surface. Note how the iron filings move in three dimensions around different parts of the bar magnet. Repeat this step with the horseshoe magnet.

DISCUSSION QUESTIONS

1 Why do the iron filings act the way they do when brought into close proximity to the magnets?
2 Compare and contrast the field lines between the north and south pole of the bar magnet and horseshoe magnet.

Earth's magnetic field

Earth is filled with many magnetic substances in its crust, mantle and molten core. Combined, these materials give Earth a magnetic field that many animals can sense and use to migrate. The magnetic poles of Earth are not the same place as the geographical poles.

The magnetic pole closest to the north geographic pole is near Hudson Bay, Canada. The north compass needle points to the pole on Earth's surface, so it is actually the *south magnetic pole*. The magnetic pole closest to the south geographic pole is in Antarctica and about 2000 km from the south geographic pole. The south compass needle points to this pole on Earth's surface, so it is actually the *north magnetic pole* (Figure 8.2.3). The historical naming of the magnetic pole which the compass needle points to as 'north' was done in approximately 1600 CE, well before the nature of Earth's magnetic field was fully understood.

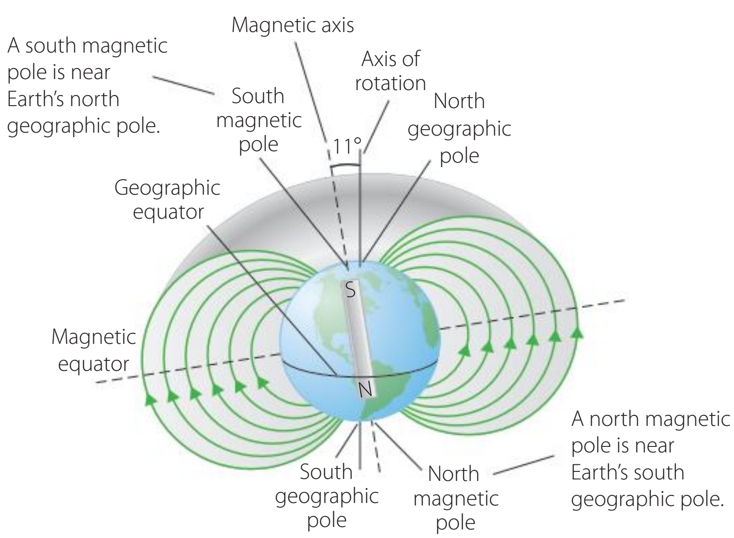

FIGURE 8.2.3 Earth's magnetic field. Notice that the geographic north pole is closer to the magnetic south pole, and conversely the geographic south pole is closer to the magnetic north pole. This is a two-dimensional representation. The field lines are all over Earth's surface as well as in its centre.

8.2.1 Magnetic pole reversal happens all the time

8.2.2 Why Earth's magnetic poles could be about to swap places – and how it would affect us

Wandering poles

Earth's magnetic poles 'wander' on a daily and annual basis. Explore this phenomenon, and suggest whether the geographic south pole and magnetic south pole will ever be at the same location.

Aurora Borealis and Aurora Australis

Earth's magnetic field forms a shield that keeps out high-energy charged particles from space (mainly from the Sun). These particles would otherwise be extremely damaging to organisms on Earth. The few particles that manage to leak in through Earth's magnetic field follow helical paths towards the magnetic poles. When these high-energy charged particles reach the atmosphere, they collide with air molecules, which are then ionised. The energy released when the atoms and molecules re-form are the auroras – the Aurora Borealis or Northern Lights, and the Aurora Australis or Southern Lights (Figure 8.2.4).

8.2.3 The Northern and Southern Lights

8.2.4 Aurora

8.2.5 What is an aurora?

FIGURE 8.2.4 The Aurora Australis.

SECTION REVIEW 8.2

REMEMBERING

1 What pole do magnetic field lines come out of for diagrammatic purposes?

UNDERSTANDING

2 Consider three magnets at the corners of an equilateral triangle with their north poles facing inwards. Draw the magnetic field lines in this situation.

3 Compare the magnetic fields of bar magnets and horseshoe magnets.

APPLYING

4 If two magnets with different magnetic field strengths come into close proximity, which magnet will move more towards the other? Explain your answer.

ANALYSING

5 Research ferromagnetism. What is the difference between a ferromagnetic material and an anti-ferromagnetic material? Name two examples of each.

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8.3 Moving charge and magnetic fields

B field
a magnetic field

Magnetic fields emanate from magnets where the field lines point from north to south. Magnetic fields are also formed from moving charges. If a current is flowing through a wire, a magnetic field is formed around the wire.

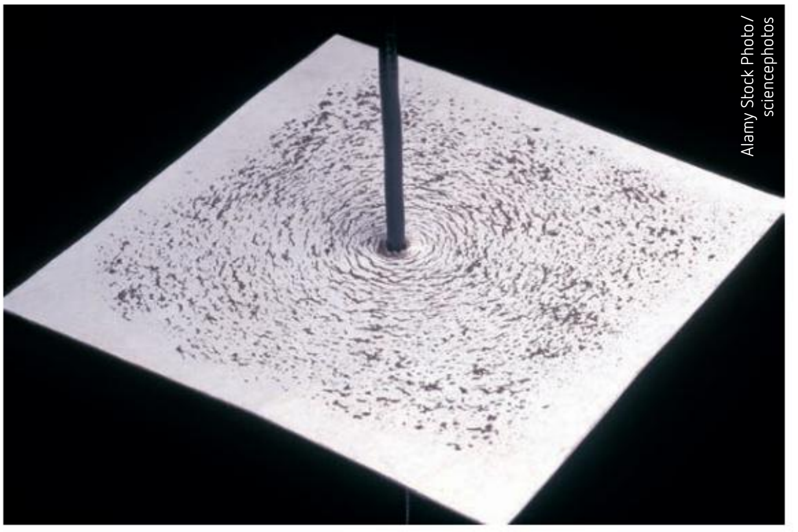
A magnetic field, also known as a **B field**, has a corresponding strength. In a natural magnet, this strength is dependent on how many of the domains align. In the case of current moving through a wire, the B field strength depends on the magnitude of the current through the wire, and how far away from the wire the B field is being measured. These observations were made by Jean-Baptiste Biot and Felix Savart, who performed many experiments with magnets and current-carrying wires. They found that for a point P some distance from a long, straight current-carrying wire:

- the magnetic field is perpendicular to both the direction of the current and to a line between the wire and P
- the magnitude of the field is inversely proportional to the distance from the wire to P (Figure 8.3.1)
- the magnitude of the field is proportional to the current.

Mathematically, this is modelled as follows:

$$B = \frac{\mu_0 I}{2\pi r}$$

Table 8.3.1 summarises the strengths of some magnetic fields. Most magnetic fields are measured in micro- or millitesla, as the tesla (T) is a very large unit. Only in the last few decades have scientists been able to create magnetic fields larger than a few tesla.



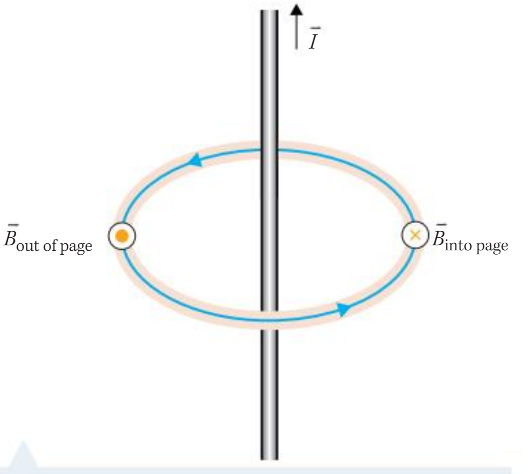


FIGURE 8.3.1 A current-carrying wire will cause a magnetic field to be induced around the wire in concentric circles about the wire. Note that the field is less defined further away from the wire as it is weaker.

KEY FORMULA

Magnetic field strength from current-carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

Where:

B = magnetic field strength distance r from the current-carrying conductor (T)

μ_0 = permeability of free space ($4\pi \times 10^{-7}$ T m A^{-1})

I = current travelling through the conductor (A)

r = perpendicular distance from the current-carrying conductor (m)

SYLLABUS LINK

Chapter 9 of *Nelson QScience Units 1 & 2* discusses that charge is conserved at all points in an electrical circuit.

TABLE 8.3.1 Some typical magnetic field strengths

SOURCE OF FIELD	APPROXIMATE MAGNITUDE (T)
Earth (surface)	3×10^{-5} to 6×10^{-5}
Typical fridge magnet (surface)	5×10^{-3}
Modern rare earth magnet (surface)	1
Medical MRI system (inside)	3
Strong research facility field	100
Neutron star	$>10^{8}$

WORKED EXAMPLE 8.3.1

Calculate the magnetic field strength at a perpendicular distance of 10 cm from a long wire carrying a current of 10 A.

ANSWER

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{4\pi \times 10^{-7} \times 10\,\text{A}}{2\pi \times 0.1}$$

$$B = 2 \times 10^{-5}\,\text{T}$$

The strength of the magnetic field is 2×10^{-5} T. It is typical to expect a magnetic field to be in this order of magnitude.

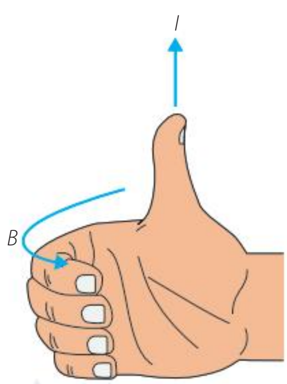


FIGURE 8.3.2 Maxwell's screw rule shows the direction of the *B* field around a current-carrying conductor. Point your right thumb in the direction of conventional current, and your fingers will wrap around in the direction of the *B* field.

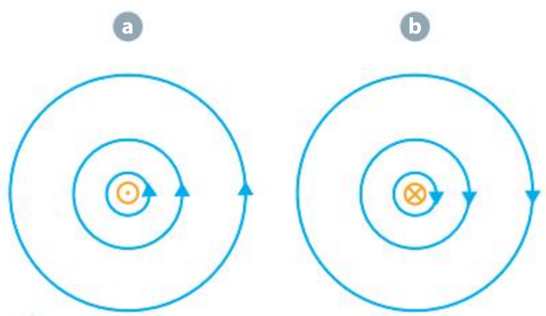


FIGURE 8.3.3 (**a**) Conventional current is shown coming out of the page, denoted by a circle with a dot in the middle. This means the thumb points up, and the field is formed anticlockwise according to the direction of the fingers. (**b**) Conventional current is shown going into the page, denoted by a circle with a cross in the middle. This means the thumb points down, and the field is formed clockwise according to the direction of the fingers. For ease of memory think of an arrow. When it is coming towards you, you just see its point or dot. When it is going way from you, you see its cross feathers on the shaft.

Right-hand rule

Determining which way the *B* field is moving around a current-carrying conductor depends on the direction of the current. An easy way to determine this is with the right-hand rule for current-carrying conductors. This is also known as Maxwell's screw rule. In this case, your right thumb points in the direction of conventional current flow (opposite to electron flow), and your fingers wrap around the wire in the direction of the *B* field (Figure 8.3.2).

In order to show this in a scientific diagram, we need to introduce some new symbols. As the *B* field acts in three dimensions, we need to identify symbols for into the page and out of the page. For Figure 8.3.2, we can represent the *B* field around the wire as shown in Figure 8.3.3.

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WORKED EXAMPLE 8.3.2

A long wire is carrying a current directly upwards, out of the page.

1 Draw the magnetic field lines due to the current as seen from above.

2 Draw arrows on your diagram showing the direction of the magnetic field lines.

ANSWER

Remember the following when drawing magnetic field lines:

- The density of the field lines is an indication of the field strength.
- Currents produce field lines that form concentric circles about the current.
- The direction of the field is given by the right-hand rule called Maxwell's screw rule.

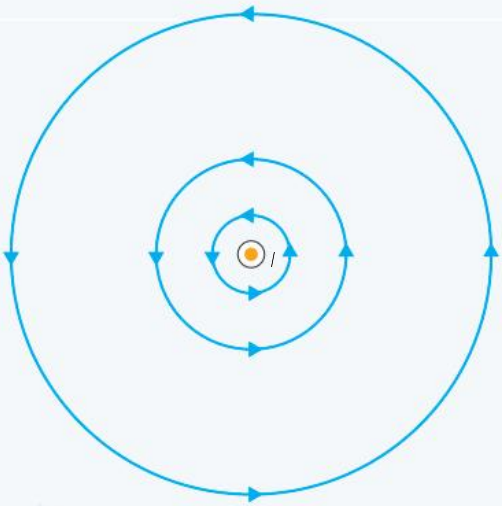


FIGURE 8.3.4

PRACTICAL ACTIVITY 8.3.1

B field about a current-carrying wire

AIM

To observe the *B* field around a current-carrying wire.

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
High current can cause sparks if a short-circuit occurs.	Ensure voltage on the DC power supply is controlled.

MATERIALS

- 12 V DC power pack
- very long wire (looped several times to mimic a large current)
- insulated platform
- set of small compasses or iron filings
- resistor

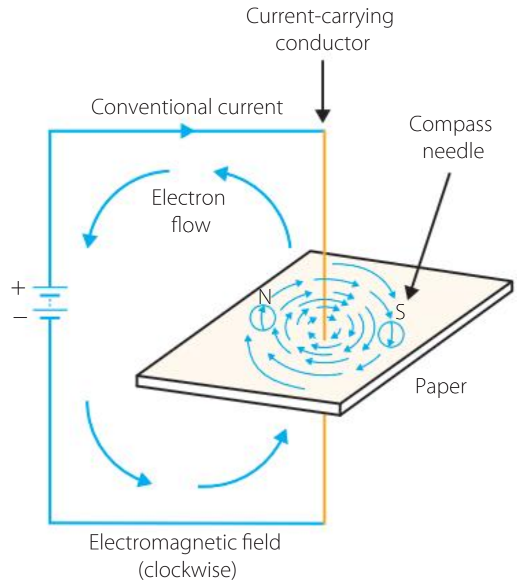
METHOD

1 Set up the circuit as shown in Figure 8.3.5.

2 Place the small compasses or iron filings on the platform, and notice the field lines close to the wire and further away from the wire.

3 Change the direction of the current. Observe the compass needles change direction.

FIGURE 8.3.5 Experimental set-up to observe magnetic field about a current-carrying conductor



SECTION REVIEW 8.3

REMEMBERING

1 Define 'magnetic field strength'.

2 How does magnetic field strength vary with distance from a long, straight current-carrying wire?

UNDERSTANDING

3 Describe two sources of magnetic fields.

4 Explain how the right-hand rule can be used to find the direction of the *B* field around a wire.

5 Draw the magnetic field lines due to a wire carrying current directly downwards, as seen from above.

APPLYING

6 What is the magnetic field strength at a distance of 1.0 cm from a wire carrying a current of 1.0 A?

7 At what distance from a wire carrying a current of 12 A is the field strength 1.0 mT?

8 What current is necessary in a long, straight wire to produce a magnetic field strength of 50 mT at a distance of 1 cm?

9 Find the *B* field strength at distances 20 cm, 40 cm, 60 cm, 80 cm and 1.0 m from a wire carrying 10 A of current.

10 Plot a graph of field as a function of distance from the wire for the previous question.

8.4 Solenoids and electromagnets

solenoid
a coil of current-carrying wire that creates a large uniform field within the coil

electromagnet
a magnet with a north and a south pole formed by a current in a solenoid

A current-carrying wire creates a magnetic field. If we want to create a large field, we use lots of wires. One way to do this is to coil a single wire into a **solenoid**, or large coil.

Each loop of the wire creates a magnetic field (Figure 8.4.1b). Inside the coils these fields add up to give a large and approximately uniform magnetic field. The more loops or turns of a wire, the greater the field. In a tightly wound solenoid, the internal field lines are straight and parallel. Outside the coil the field lines are more spread out. The result is an extremely useful device called an **electromagnet**, as the field lines are now synonymous to a bar magnet with a north and a south pole. Solenoids are used in transformers, magnetic switches and many other applications where large magnets are required. Electromagnetics have a huge advantage over permanent magnets in that their magnetic fields can be switched on and off with the current.

8.4.1 Solenoid

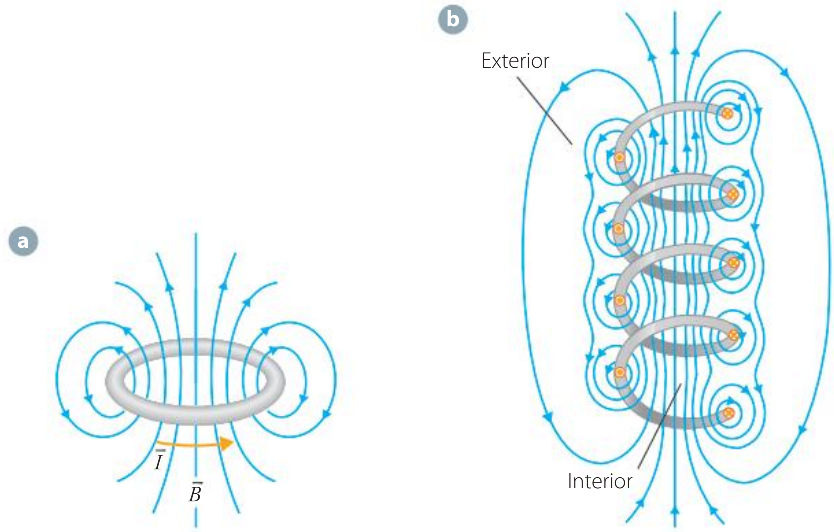


FIGURE 8.4.1 (**a**) Magnetic field due to a single loop of wire. (**b**) Magnetic field due to a solenoid. Note that the field lines resemble a bar magnet.

B field in a solenoid

The *B* field that is produced from a solenoid is dependent on a few variables:

- the magnitude of the current flowing through the wire
- the number of turns (coils) in the solenoid.

This is modelled mathematically as:

$$B = \mu_0 n I$$

KEY FORMULA

Magnetic field produced by a solenoid

$B = \mu_0 n I$

Where:

B = strength of the field inside the solenoid (T)

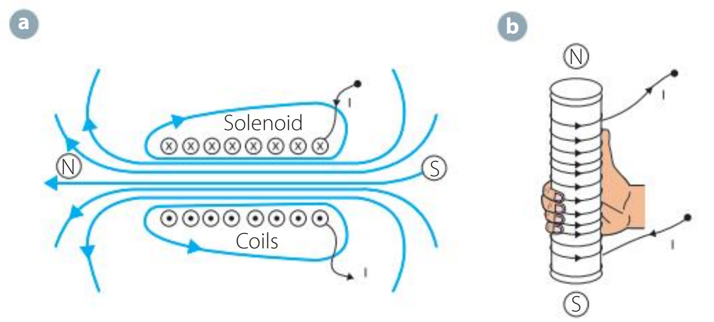
μ_0 = permeability of free space ($4\pi \times 10^{-7}$ T m A^{-1})

n = number of turns in the solenoid per metre

I = current travelling through the solenoid (A)

Additionally, if the direction of the current through the coils is known, the magnetic field direction can be found using an adaptation of Maxwell's screw rule. If the fingers curl in the direction the conventional current is flowing in the coil, the thumb points in the direction of the north-pole of the electromagnet (Figure 8.4.2).

FIGURE 8.4.2 **(a)** A solenoid with the cross-section of the current in the coils, and the magnetic field lines. **(b)** The right-hand rule being used to determine the magnetic field direction in a solenoid.



WORKED EXAMPLE 8.4.1

A solenoid with a current of 3 A travelling through it produces a B field of 0.2 mT inside the coil. Determine how many turns per metre are in the solenoid.

ANSWER

$$B = \mu_0 n I$$

$$n = \frac{B}{\mu_0 I}$$

$$n = \frac{2 \times 10^{-4}\,\text{T}}{4\pi \times 10^{-7}\,\text{TmA}^{-1} \times 3\,\text{A}}$$

$n \approx 53$ turns per metre

PRACTICAL ACTIVITY 8.4.1

Investigating properties of an electromagnet

AIM

To determine factors that change the strength and poles of an electromagnet.

RISK ASSESSMENT

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Strong magnetic fields may cause damage to nearby electronics or cause nearby magnetic materials to move undesirably.	Ensure all magnetic materials and electronic devices are not on the lab bench with the solenoid.

MATERIALS

- DC power supply and connecting cables
- solenoid
- rheostat
- bar magnet

»

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METHOD

1 Connect the solenoid and rheostat in series and attach to a DC power supply.
2 Beginning from the lowest voltage, turn on the power supply and bring the bar magnet close to the solenoid. Observe what happens.
3 Steadily increase the voltage, while keeping the resistance constant. Observe the interaction the solenoid has with the bar magnet.
4 Change the direction of the current by switching the terminals of the DC power supply. Observe how the interaction the solenoid has with the bar magnet changes.

DISCUSSION QUESTIONS

1 What would happen if the bar magnet was to be pulled inside the solenoid?
2 Explain why an electromagnet is considered more useful than a bar magnet.

PRACTICAL ACTIVITY 8.4.2

Make your own electromagnet

AIM

To make an electromagnet.

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Nails can pierce skin.	Handle with care.
Holding the wire to the battery terminals could possibly cause burns.	Ensure sticky tape is used to keep the wires on the battery terminals.

RISK ASSESSMENT

MATERIALS

- 2 m of fine, insulated wire
- small battery (AA)
- large steel nail
- sticky tape
- paperclips

METHOD

1 Check the nail is not magnetised by trying to pick up a paperclip with it.
2 Wrap the wire tightly around the nail, always winding in the same direction. Leave enough wire free at each end to connect the battery.
3 Sticky tape each end of the wire to a battery terminal.
4 Now try to pick up a paperclip with your nail (electromagnet).

DISCUSSION QUESTIONS

1 What was the maximum number of paperclips you could pick up? How do you think this number could be increased?

2 Does the magnet still work when the battery is disconnected? Suggest a reason for this.

PRACTICAL ACTIVITY 8.4.3

Force on a current-carrying wire in an external magnetic field

AIM

To observe the force on a current-carrying wire in an external magnetic field.

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Strong permanent magnets can be damaged if not set at a fixed distance. They can also cause pinching of skin.	Ensure magnets are kept with keepers betwween them and are at a large enough distance apart that they will not fly together.
Alligator clips can be sharp.	Handle sharp objects with care.

MATERIALS

- DC power supply
- long, light conductive material (such as a long piece of aluminium foil)
- two alligator clips
- rheostat
- large horseshoe magnet (large *B* field) or two neodymium magnets
- three retort stands, boss heads and clamps

Connecting wires

METHOD

1 Set up the circuit as shown in Figure 8.4.3.

2 Turn on the power supply and observe what happens to the aluminium sheet.

3 Reverse the terminals of the DC power supply and notice what happens to the aluminium sheet when the current is reversed.

DISCUSSION QUESTIONS

1 What variables determine the amount by which the aluminium sheet will move?

2 Describe how you could determine the north and south pole of the magnet if it was not indicated on the magnet.

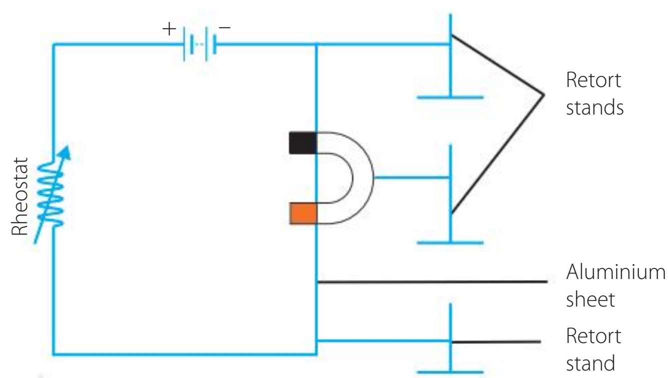

FIGURE 8.4.3 Experimental set-up of current-carrying conductor in a magnetic field

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PRACTICAL ACTIVITY 8.4.4

Force on parallel current-carrying wires

AIM

To observe how two parallel current-carrying wires behave when in close proximity.

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Crocodile clips, pins and scissors can pierce skin.	Handle sharp objects with care.

RISK ASSESSMENT

MATERIALS

- two long, thin wires
- two retort stands, boss heads and clamps
- two alligator clips
- DC power supply
- rheostat
- ammeter
- connecting wires

METHOD

1 Set up the circuit with the rheostat in series with the long thin connecting wires hanging close together between the retort stands. Ensure the long wires are connected in parallel so the current flows through them in the same direction (Figure 8.4.4a).

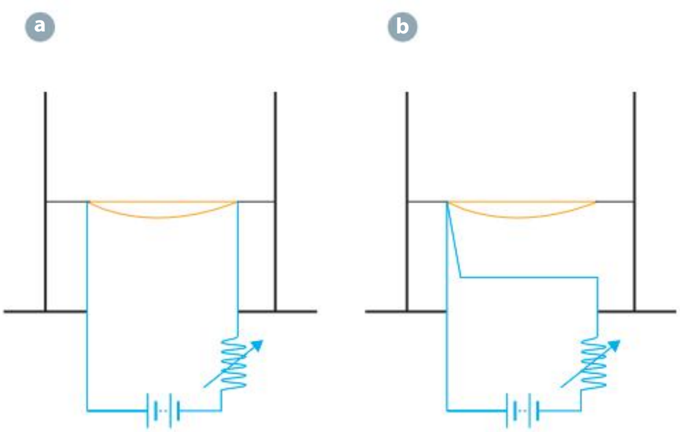

FIGURE 8.4.4 Long thin wires connected in (**a**) parallel, (**b**) in series.

2 Turn on the DC power supply and observe how the wires move.

3 Turn off the power supply and reconnect the wires in series, but keep them hanging parallel. The current should now be flowing in opposite directions (Figure 8.4.4b).

4 Turn on the DC power supply and observe how the wires move.

DISCUSSION QUESTIONS

1 Why does changing the current change the direction of the forces on the wires?

2 Draw the magnetic field lines about each of the wires in both of these scenarios.

SECTION REVIEW 8.4

REMEMBERING

1 Define the term 'electromagnet'.

UNDERSTANDING

2 Explain how the magnetic field lines in a solenoid can be found from the magnetic field lines around a single loop of wire carrying current.

APPLYING

3 Find the magnetic field strength in a solenoid of 50 turns per metre, carrying a current of 80 mA.

4 The *B* field in a solenoid of 100 turns is found to be 60 μT. Determine the current through the coil.

ANALYSING

5 Explore why electromagnets have a much wider application to services than bar magnets.

8.5 Solving problems: magnetic fields

The *B* field around bar magnets and electromagnets depends on a number of variables. Each of these variables needs to be carefully considered when solving problems regarding the magnetic field strength. The variables that affect magnetic fields from magnets and electromagnets are:

- distance from magnet or electromagnet
- magnitude of current in wire or solenoid
- number of turns in a coil (electromagnet only).

As long as these variables are known, it is possible to determine the magnetic field at a given location. Additionally, if the *B* field and two of the above three variables are known, it is possible to calculate the unknown third variable.

WORKED EXAMPLE 8.5.1

At what distance from a wire carrying a current of 15 A is the field 1.0 μT? If this same wire is then coiled into a solenoid with 25 turns over 50 cm, what is the *B* field strength inside the coil?

ANSWER

$$B = \frac{\mu_0 I}{2\pi r}$$

$$r = \frac{\mu_0 I}{2\pi B}$$

$$r = \frac{4\pi \times 10^{-7} \times 15\,\text{A}}{2\pi \times 1.0 \times 10^{-6}\,\text{T}}$$

$r = 3\,\text{m}$

3 m is the distance from the wire where the field is 1.0 μT

If this wire was then coiled:

$$B = \mu_0 n I$$

$$B = 4\pi \times 10^{-7} \times \frac{25}{0.5} \times 15$$

$B = 9.42 \times 10^{-4}\,\text{T}$ or $B = 94.2\,\text{mT}$

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WORKED EXAMPLE 8.5.2

If a coil with 0.5 A of current and 100 turns has a B field of 7.6×10^{-5} T inside it, what must be the length of the coil?

ANSWER

$B = \mu_0 nI$ where n is number of turns per metre (n/L)

$$B = \frac{\mu_0 nI}{L}$$

$$L = \frac{\mu_0 nI}{B}$$

$$L = \frac{4\pi \times 10^{-7} \times 100 \times 0.5\,\text{A}}{7.6 \times 10^{-5}\,\text{T}}$$

$L = 0.826$ m

SECTION REVIEW 8.5

UNDERSTANDING

1 What is the effect on the magnetic field within a solenoid if:

a current is increased?

b the current direction is reversed?

c the number of turns per metre is decreased?

APPLYING

2 What is the magnetic field a distance of 10 mm from a wire carrying a current of 1.6 A?

3 How large a current is necessary to produce a field of 0.15 T a distance of 2.0 cm from a long, straight wire?

4 If a solenoid of 125 turns per metre has a B field of 0.5 T, what is the current through the coil?

5 If a coil with 0.34 A of current and 50 turns has a B field of 1.5×10^{-5} T inside it, what is the length of the coil?

8.6 Force on particles in a magnetic field

Scientists can measure the magnetic field by its effect on moving charges, such as a current in a wire. Experiments using lengths of current-carrying wire in magnetic fields show that the **magnetic force** on the wire depends on four things:

magnetic force the force that a magnetic field exerts on a moving charge or current

1 the magnitude of the current carried, I

2 the strength of the magnetic field, B

3 the angle between the direction of the current and the field, θ

4 the length of the wire in the magnetic field, L.

These observations can be summarised mathematically as $F = BIL\sin\theta$. This equation uses the magnitude of vector quantities, and so the resulting force calculated is the magnitude of the force acting on the wire. This equation tells us that the force is zero if L and B are in the same direction (Figure 8.6.1a) and that the force on a current-carrying wire is a maximum when it is perpendicular to the external field B (Figure 8.6.1b and c). Otherwise, the force exerted on the wire is between 0 N and the maximum force experienced, governed by the angle θ the wire makes with the field. The closer θ is to 0 (i.e. the wire is lying in the same direction as the magnetic field), the smaller the magnetic force the wire experiences due to the external field.

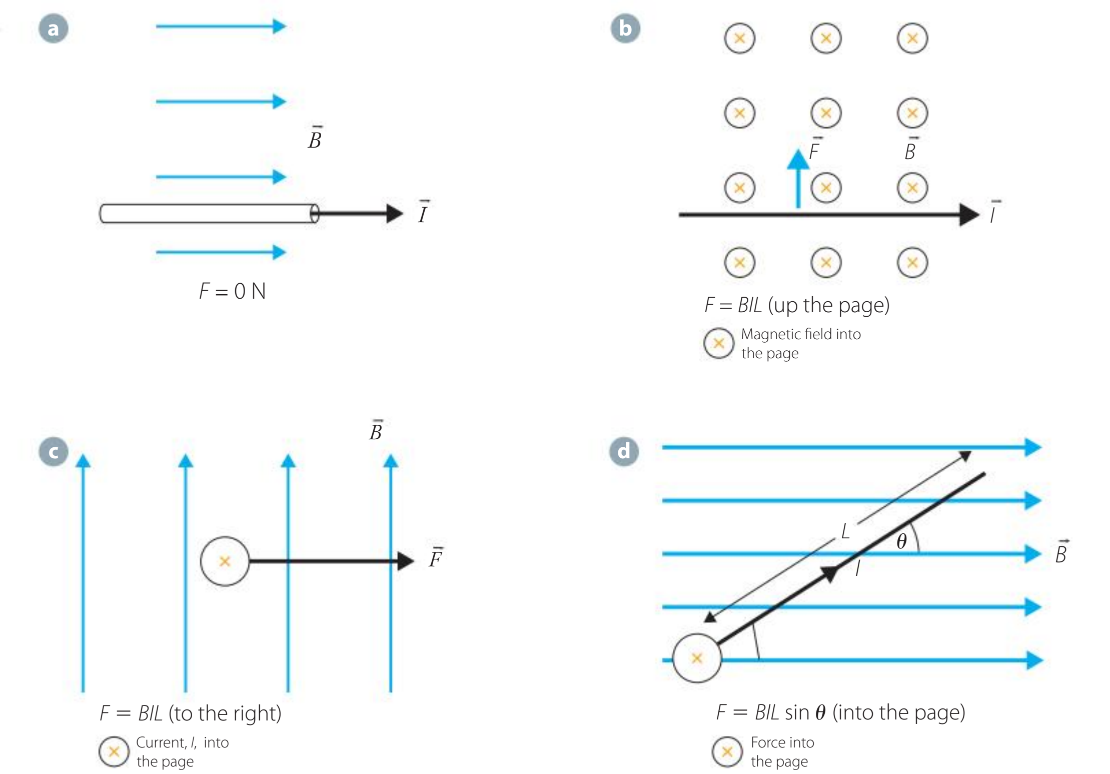

FIGURE 8.6.1 The force a wire of length L experiences in an external B field when **(a)** the wire is parallel to the field, **(b)** the wire is perpendicular to the field, **(c)** the wire is perpendicular to the field with a different orientation, **(d)** the wire has an angle θ to the field where $\theta \neq 0$ or 90.

8.6.1 Magnetic force on a charge

KEY FORMULA

Force on a current-carrying conductor

$F = BIL \sin\theta$

Where:

F = magnetic force on the wire (N)

I = current flowing through the wire (A)

B = magnitude of the external magnetic field (T)

θ = angle the wire makes with the external magnetic field (°)

This can also be rearranged to express magnetic field strength as follows:

$$B = \frac{F}{IL\sin\theta}$$

Right-hand rule for currents and charges in external *B* fields

To find the direction of a force on a current-carrying wire (or charges) in external B fields, we need to use the right-hand rule. This rule is slightly different from Maxwell's screw rule that has been mentioned previously in this chapter. There are many ways to use your right hand for this rule.

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Hold out your right hand flat, and move your hand so that your thumb points in the direction of conventional current and your fingers point in the direction of the B field. Your palm then points in the direction in which the magnetic force will push the wire (Figure 8.6.2). Try this for Figure 8.6.1b and c above and note that your palm will point in the direction in which the force is indicated on each diagram. As the right-hand rule is the result of a vector cross product, there are different ways in which it can be applied, but results will all be the same.

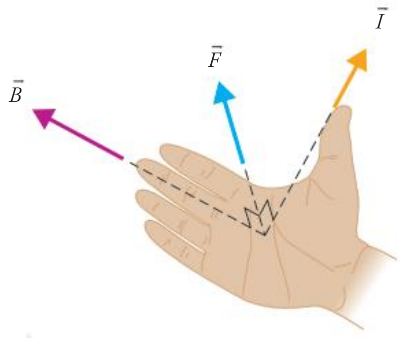

FIGURE 8.6.2 The right-hand rule for determining the direction the magnetic force acts on a current-carrying wire in an external B field.

WORKED EXAMPLE 8.6.1

An overhead power line carrying a current of 1000 A is in a magnetic field of 30 μT. The field is perpendicular to the wire. What force per unit length does the wire experience?

ANSWER

$F = BIL \sin\theta$

$\frac{F}{L} = 30 \times 10^{-6}\,\text{T} \times 1000\,\text{A} \times \sin 90°$

$F = 0.030\,\text{N m}^{-1}$

This is quite a large force considering how long power lines are. However, power line current is alternating, so the force also alternates direction, rather than pulling on the power line in a single direction.

SECTION REVIEW 8.6

REMEMBERING

1 State the equation to find the force on a current-carrying wire in an external B field.

2 State the orientation of a current-carrying wire to the B field if no force is exerted on it from the field.

UNDERSTANDING

3 State the direction in which the magnetic force acts on a wire if the current is carried in the wire from left to right, and the magnetic field is coming out of the page.

APPLYING

4 A 5 m long current-carrying wire is at an angle of 30° to a magnetic field. It carries a current of 30 A and experiences a force of 0.02 N. What is the strength of the magnetic field?

5 A straight wire 2.1 m long and carrying a current of 0.85 A has a force of 5.0×10^{-2} N exerted on it by a uniform magnetic field at right angles to the wire. What is the magnitude of the magnetic field?

ANALYSING

6 A wire is carrying a large current directly upwards, out of the page.

a Draw the magnetic field lines, as seen from above, due to this current.

b Consider a second vertical current-carrying wire close to the first wire. If the current in the second wire is also upwards, what is the direction of the force on this second wire? Draw it on your diagram.

c What is the direction of the force on the first wire due to the current in the second wire? Draw it on your diagram.

d How would your answers for parts b and c change if the current in the second wire was going downwards?

7 What total force would be exerted on a span of wire 200 m long if the wire carries a current of 1000 A and is in a magnetic field of 20 μT? How does this force compare with the gravitational force acting on the wire if the wire has a linear density of 750 kg km^{-1}?

8.7 Force on moving particles in a magnetic field

A current is a collection of charges moving in the same direction. A current experiences a force in a magnetic field, so we can expect that a single moving charge will also experience a force. The magnitude of the magnetic force that a charged particle experiences is written mathematically as $F = qvB\sin\theta$.

KEY FORMULA

Force on a moving charged particle

$$F = qvB\sin\theta$$

F = magnitude of the force on the moving charge (N)

q = magnitude of the charge moving in the magnetic field (C)

v = magnitude of the velocity of the moving charge ($m\,s^{-1}$)

B = strength of the external magnetic field (T)

θ = angle between the vectors v and B (°)

The force is again perpendicular to both the magnetic field and the velocity of the charged particle. Using the right-hand rule allows us to find the direction of the magnetic force on a charged particle when it enters a magnetic field. In Figure 8.6.2, the thumb pointed in the direction of conventional current. This time, the thumb points in the direction a positive charge will move in the field, your fingers point in the direction of the B field, and your palm will again point in the direction of the magnetic force. If you are considering the force on a negatively charged particle, remember to point your thumb initially in the opposite direction as negative charges will travel in the opposite direction to positive charges in a magnetic field (Figure 8.7.1)

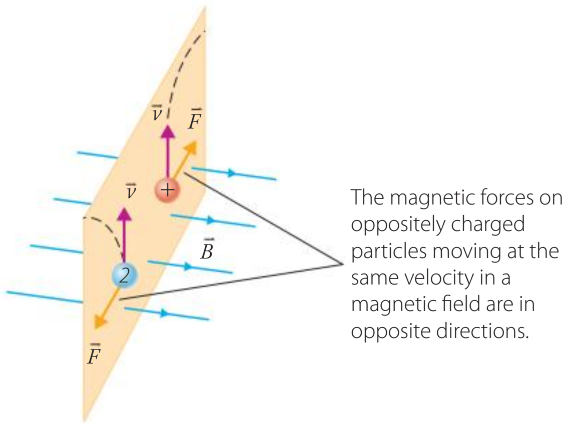

FIGURE 8.7.1 The forces on positive and negatively charged particles in a magnetic field. Note that they are in opposite directions. The dashed lines show the path the particles will take in this field.

WORKED EXAMPLE 8.7.1

An alpha particle enters Earth's magnetic field at a velocity of $55\,000\,\text{m s}^{-1}$. The local field strength is $40\,\mu\text{T}$. What is the range of possible accelerations of the alpha particle?

ANSWER

First, consider that $F = ma$ and that $F = qvB\sin\theta$, where $-90° \leq \theta \leq 90°$

Now, $F = ma$

$F = qvB\sin\theta$

$\therefore ma = qvB\sin\theta$

Hence, $a = \dfrac{qvB\sin\theta}{m}$

where a will have a range of values depending on the value of $\sin\theta$. The maximum magnitude a can have is when $\theta = 90$, i.e. when $\sin\theta = 1$. So:

$$a = \frac{qvB}{m}$$

$$a = \frac{2 \times 1.6 \times 10^{-19}\,\text{C} \times 55\,000\,\text{m s}^{-1} \times 40 \times 10^{-6}\,\text{T}}{6.6 \times 10^{-27}\,\text{kg}}$$

$$a = 1.1 \times 10^{8}\,\text{m s}^{-2}$$

The acceleration of the alpha particle can have any value between $-1.1 \times 10^{8}\,\text{m s}^{-2}$ and $1.1 \times 10^{8}\,\text{m s}^{-2}$ depending on the angle between the direction of the velocity and the magnetic field.

Paths of particles in magnetic fields

Fast-moving charged particles experience large forces, even in small magnetic fields. These forces result in large accelerations, meaning that the direction of the velocity of the charged particle is changing. The path of a charged particle in a uniform magnetic field depends on the angle between the initial velocity and the field. The path may be a straight line, a circle, or a helix.

If the particle enters the field with velocity perpendicular to the B field, it will experience the maximum magnetic force. This will cause the charged particle to follow a circular path, as per the right-hand rule (Figure 8.7.2a). If instead the particle has initial velocity with a component in the direction of the field, this component of velocity is not altered by the field. However, the perpendicular component is altered by the acceleration due to the field (from the magnetic force acting on the particle). In this case, the particle follows a helical path, with the axis of the helix in the direction of the field (Figure 8.7.2b). If the particle enters the field with velocity parallel to the field, no force will act on the charged particle and it will travel in a straight line (Figure 8.7.2c).

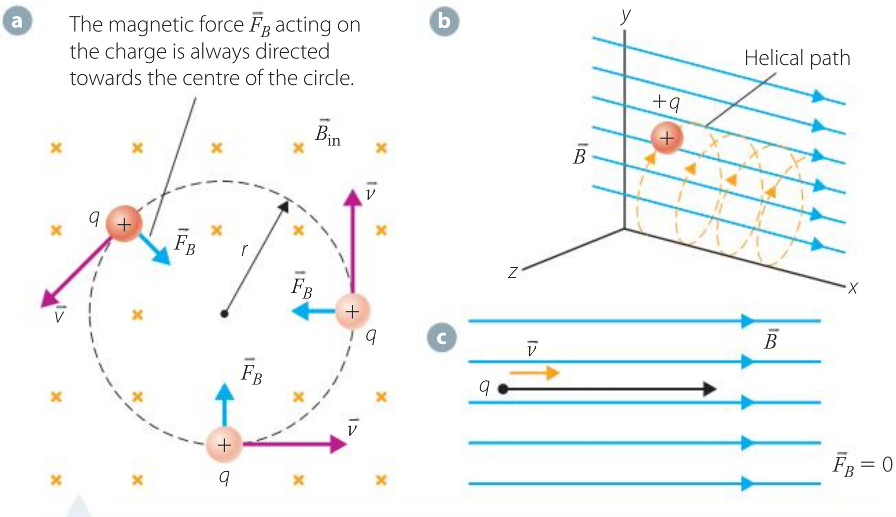

FIGURE 8.7.2 In a uniform B field, the path of a charged particle is (**a**) circular when it enters in a direction perpendicular to the field, (**b**) helical when it enters at an angle to the field strictly between 0 and 90°, and (**c**) not deflected when travelling parallel to the field.

WORKED EXAMPLE 8.7.2

A magnetic field points in the positive z direction. Draw the path of an electron in the field with an initial velocity that is:

1 in the positive x direction
2 in the negative x direction
3 in the positive z direction.

ANSWER

Use the right hand rule to solve these problems. In all cases, the B field is coming out of the page, and your thumb should point in the opposite direction to that in which the electron is travelling as it is negatively charged. Your palm then points in the direction the electron will move. As the electron is coming in perpendicular to the field in the first two scenarios, it will follow a circular path in these two cases. In the third scenario, the electron's path will not be altered because it is travelling parallel to the magnetic field.

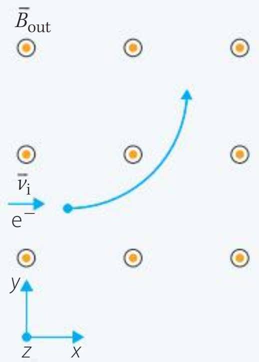

FIGURE 8.7.3 An electron entering the B field in the positive x direction.

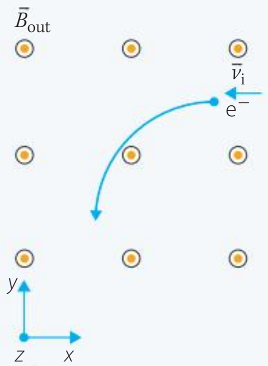

FIGURE 8.7.4 An electron entering the B field in the negative x direction.

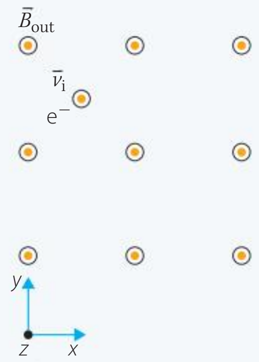

FIGURE 8.7.5 An electron enters the B field in the positive z direction.

INQUIRING FURTHER

Research the mass spectrometer. Explain how the mass spectrometer uses an understanding of charges in moving fields to determine the composition of samples. Why are mass spectrometers so important to forensic scientists?

SECTION REVIEW 8.7

REMEMBERING

1 State how the forces on moving negatively and positively charged particles differ in a magnetic field.

UNDERSTANDING

2 Which of the possible paths in Figure 8.7.6 is not possible for a charged particle entering a region of uniform magnetic field?

FIGURE 8.7.6 (**a**) A helical path; (**b**) A path with no deflection; (**c**) A path with a right angle

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3 Determine the initial direction of the deflection of the charged particles in Figure 8.7.7 as they enter the magnetic fields shown.

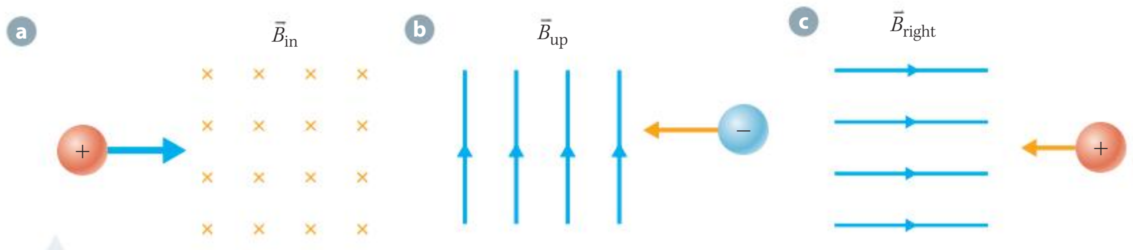


FIGURE 8.7.7

APPLYING

4 What is the minimum magnitude of a magnetic field necessary to apply a force of 1×10^{-12} N to an electron moving at a speed of 500 km s^{-1}?

5 A *B* field is uniform in the negative *z* direction. A proton enters this field from the positive *x* direction with velocity 55 m s^{-1}. What is the magnitude and direction of the force this proton experiences?

6 What acceleration would an electron entering a field of 20 μT experience at a speed of 55 000 m s^{-1} if it is travelling:

- a perpendicular to the field?
- b parallel to the field?
- c at an angle of 45° to the field?

ANALYSING

7 How can the motion of a moving charged particle be used to distinguish between an electric field and a magnetic field? Give a specific example.

8 A proton and an electron enter a uniform magnetic field. The particles are travelling at the same speed perpendicular to the field. Draw a diagram showing their paths and explain the differences of the paths each charged particle takes.

9 Sketch the path of a proton as it enters a *B* field from the positive *x* direction, if the *B* field is going into the page.

8.8 Mandatory practicals

PRACTICAL ACTIVITY 8.8.1

Strength of a magnet

AIM

To determine how the strength of a magnetic field changes with distance from a bar magnet.

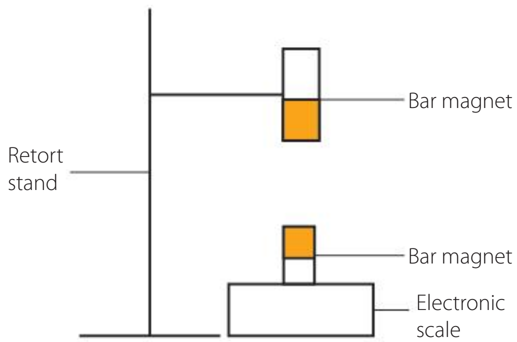
» MATERIALS

- two bar magnets
- electronic scale
- retort stand, boss head and clamp
- ruler

METHOD

1 Set up the materials as shown below in Figure 8.8.1. Ensure that the like ends of the magnets are pointed towards each other.

FIGURE 8.8.1 Experimental set-up to determine how magnetic fields exert different forces at different distances.



2 Beginning with a separation distance of 30 cm, record the mass of the bar magnet on the scale.
3 Decrease the separation distance by 5 cm and record the mass of the bar magnet in a suitable table. Repeat the procedure to obtain at least four more readings.
4 Convert all masses to weight in newtons and plot a graph of force against distance.

DISCUSSION QUESTIONS

1 What happened to the mass of the bar magnet on the scale as the bar magnet in the retort stand was brought closer?
2 Why do you think this mass changed in this way?
3 Suggest what would happen if the magnetic field between the magnets was attractive instead of repulsive.
4 What kind of relationship exists between magnetic force and separation distance? Compare this to other forces you have studied so far.
5 Manipulate the data to draw a linear graph. Determine the relationship between magnetic force and separation distance.

PRACTICAL ACTIVITY 8.8.2

A current balance

AIM

To measure the magnetic force on a current balance and find the magnetic field strength in a solenoid. »

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RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Strong magnetic fields may cause damage to nearby electronics or cause nearby magnetic materials to move undesirably.	Ensure all magnetic materials and electronic devices are not on the lab bench with the solenoid.
Crocodile clips, pins and scissors can pierce skin.	Handle sharp objects with care.

RISK ASSESSMENT

MATERIALS

- air core solenoid
- current balance (to make: thin, stiff plastic or cardboard; stiff conducting wire; copper or zinc sheet; pin; fine sandpaper, scissors, sticky tape)
- two short pieces of wire (mass known)
- two DC power supplies
- two ammeters
- two rheostats
- two switches
- crocodile clips and leads

METHOD

MAKING THE CURRENT BALANCE APPARATUS

1 Cut a rectangle from the cardboard or plastic so that half will fit into the solenoid and half is outside.
2 Attach a small pin to the middle of one of the short sides of the rectangle, overhanging the outside end (Figure 8.8.2)
3 Make a rectangular half loop of conducting wire, to sit near the edges of the rectangle that goes into the solenoid. Make sure it is attached to the rectangle.
4 Cut two supports out of the metal sheet, bend them and attach them to the end of the solenoid as shown in Figure 8.8.2. Use the crocodile clips to connect them to the current balance circuit. Note that they should not make any electrical contact with the solenoid.
5 Bend the ends of the rectangular half loop so that they sit on the metal supports.
6 Use sandpaper to clean the metal and ensure a good electrical connection.

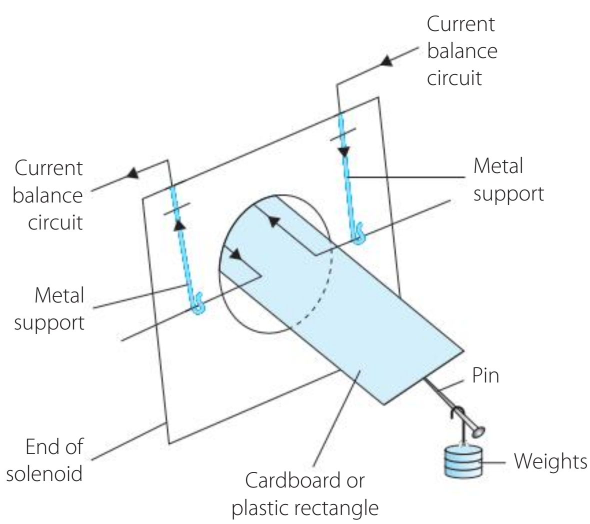
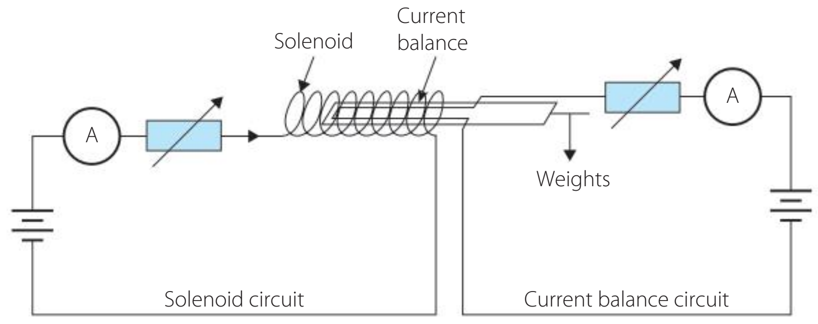


FIGURE 8.8.2 Diagram demonstrating how to set up the current balance inside the solenoid.

7 Measure the length of the current element that is perpendicular to the magnetic field of the solenoid (the short end of the rectangle).

8 Balance the current balance by hanging pieces of wire or small weights over the pin.

FIGURE 8.8.3 Diagram of circuit connections to connect the solenoid and current balance respectively.



FORCE ON WIRES DUE TO AN EXTERNAL FIELD

1 Connect the balance and the solenoid circuits as shown in Figure 8.8.3.

2 Turn on both circuits and observe what happens to the current balance – it needs to act as a seesaw with the inner end being pushed down by the magnetic force. Alter the apparatus accordingly.

3 Adjust the number of weights and their positions on the current balance until the current balance is parallel with the laboratory bench.

4 Record the current in the solenoid, the current in the current balance, the distance from the pivot to the current element, and the distance from the pivot to the balancing weights. Weigh the masses that were added to make the current balance parallel to the bench and record this value.

ANALYSIS OF RESULTS

1 Calculate:

- a gravitational force on balancing masses
- b torque by weight force on current balance (product of gravitational force and distance from masses to pivot point)
- c magnetic force on current balance in the solenoid
- d magnetic field in the solenoid.

DISCUSSION QUESTIONS

1 Comment on the quality of your data and how this affects your results.

2 How could you improve the quality of your data?

3 Redesign the experiment so that you can experimentally determine the relationship between the current in the solenoid and the strength of the magnetic field in the solenoid. Hint: You will need to use the magnetic force the field exerts on the current balance!

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Diamagnetic
   b Electromagnet
   c Electromagnetic field model
   d Ferromagnetic
   e Magnet
   f Magnetic domain
   g Magnetic field
   h North pole
   i Paramagnetic
   j Right-hand rule
   k Solenoid
   l South pole

2 State the factors that determine the strength of a magnetic field.

CATEGORY QUESTIONS

3 Explain how the right-hand rule and Maxwell's screw rule can be used to find the direction of a force in scenarios including moving charged particles in an external *B* field, and current-carrying wires in an external *B* field.

4 Explain how coils of wire create an electromagnet when current is travelling through the wire.

5 Compare the models used to draw electric fields and magnetic fields.

ELABORATION QUESTIONS

6 Why do charged particles move when they enter magnetic fields?

7 Why are some materials better magnets than others?

8 How can the path of a charged particle entering a magnetic field be predicted?

EVIDENCE QUESTIONS

9 Magnetic fields are used in mass spectrometers to determine the atoms or molecules that are in a sample of material. Explore the uses of mass spectrometers, and explain how the mass spectrometer works.

10 Research the Australian synchrotron, and explain how it uses magnetic fields to accelerate electrons to velocities close to the speed of light. What is the purpose of this?

END-OF-CHAPTER EXAM

End-of-chapter test

1 A magnetic field exists around a current-carrying wire. At 2 cm from the wire, the field strength is 6.0×10^{-5} T. The field is 3.0×10^{-5} T at:

A 4 cm from the wire.

B 3 cm from the wire.

C 1 cm from the wire.

D 0.5 cm from the wire.

2 A ferromagnetic material is:

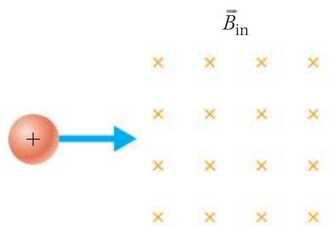
A strongly attracted to nearby magnets.

B weakly attracted to nearby magnets.

C weakly repelled from nearby magnets.

D not affected by nearby magnets.

3 What is the direction of the magnetic force acting on the following charged particle as it enters the *B* field below?



A Into the page

B Out of the page

C Up

D Down

4 As the distance *r* from a current-carrying wire increases, the *B* field at *r*:

A increases linearly.

B increases exponentially.

C decreases exponentially.

D decreases inversely.

5 Are magnetic field lines drawn coming out of the north pole, or going into the north pole?

6 Does increasing the number of turns per metre increase or decrease the *B* field within a solenoid?

7 An electron having a charge of 1.6×10^{-19} C and velocity of 2.8×10^{3} m s^{-1} enters a magnetic field of strength 20 μT at an angle of 30° to the field lines. Calculate the force and acceleration experienced by the electron as a result of interaction with the field.

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8 A current balance is set up in a solenoid with *B* field 0.20 T. The current through the wire in the current balance is 3.6 A and 3 cm of the wire is perpendicular to the field in the solenoid. The side length of the current balance is 12 cm long. What is the magnitude of force the current balance experiences in this magnetic field?

9 When is the magnetic force on a moving charged particle zero?

10 Why do iron filings follow magnetic field lines near magnets?

11 Explain how an electromagnet can be created.

12 Are electricity and magnetism independently occurring phenomena? Explain.

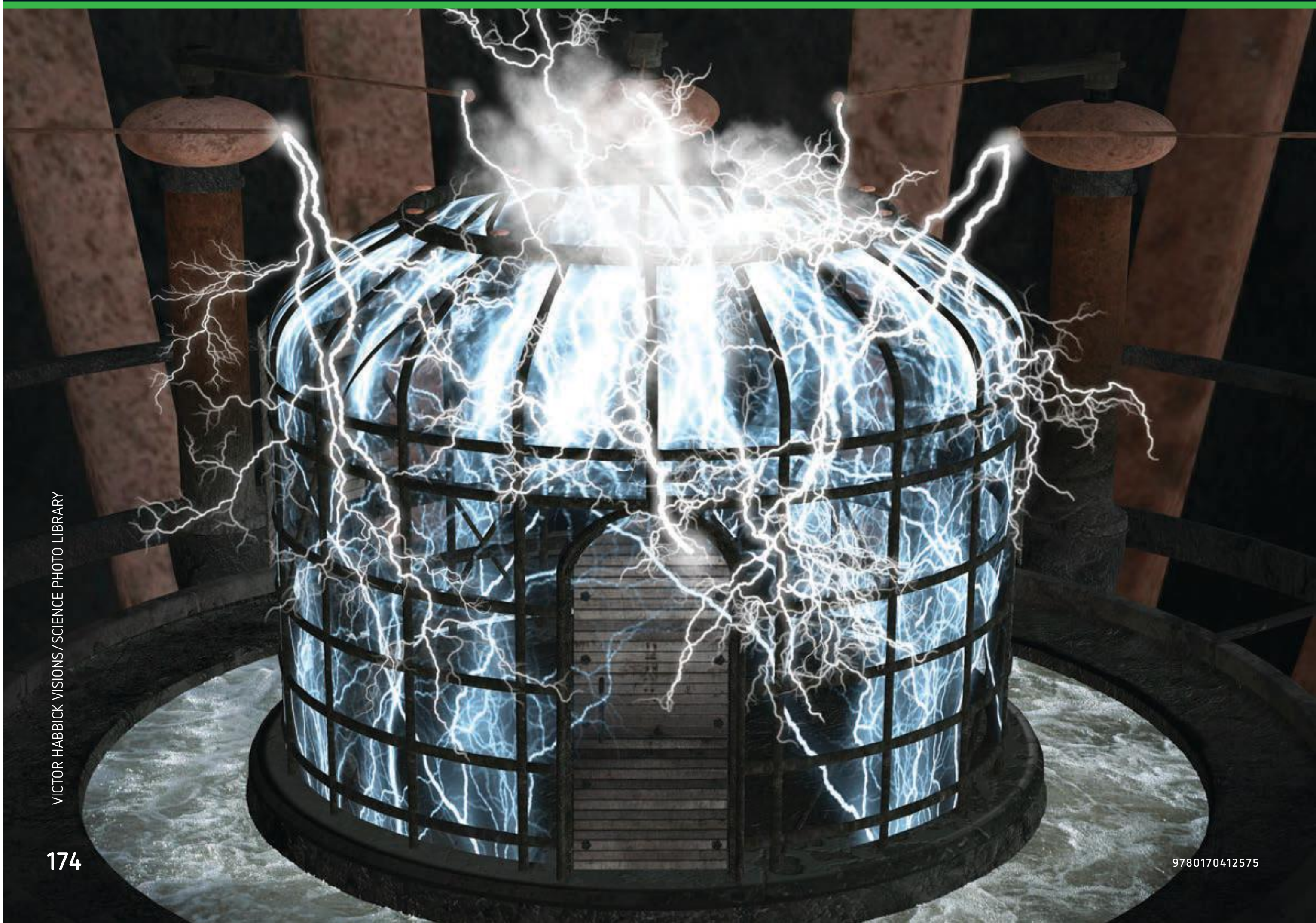
9 ELECTROMAGNETIC INDUCTION

Introduction

In the previous chapter, we discovered that there was an intrinsic link between electricity and magnetism in that an electric current in a conductor can create a magnetic field. It was a logical step for scientists to ask the question whether the opposite was true: can a magnetic field produce an electric current? The work of Joseph Henry and Michael Faraday independently showed in 1820 that this was indeed possible through the process of electromagnetic induction. This is an example of a principle of symmetry that many physicists hold as being a fundamental property of nature. The impact of the discovery of electromagnetic induction cannot be understated as it resulted in the development, among other things, of the electric generator and motor.

Inquiry questions

- How can a magnetic field produce a current?
- What factors impact on the size of the current produced?
- What is alternating current?
- How is electricity produced and transported for use in the home?



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9.1 Electromagnetic induction

electromagnetic induction the production of an electromotive force (emf) or a voltage in an electrical conductor due to its dynamic interaction with a magnetic field

electromotive force (emf) a difference in potential that tends to give rise to an electric current

induced current a current that is produced due to the presence of an electromotive force

Electromagnetic induction is the production of an electric field by a changing magnetic field. An electric field in a region means that there is an **electromotive force (emf)** acting between points in that region. If this emf acts upon free charge carriers, then a current will be generated. This current is referred to as an **induced current**. This is the basis of electromagnetic induction used in electricity generation.

Faraday quantitatively investigated the size of the current produced by moving a magnet near a conducting wire using a device like that shown in Figure 9.1.1. During this investigation, Faraday discovered that not only did the current depend on the strength of the magnet used, but it also depended on the area enclosed by the loop and the angle at which the two are orientated to each other. These three factors are combined in the concept of magnetic flux.

To understand how the process works, it is important to investigate magnetic flux.

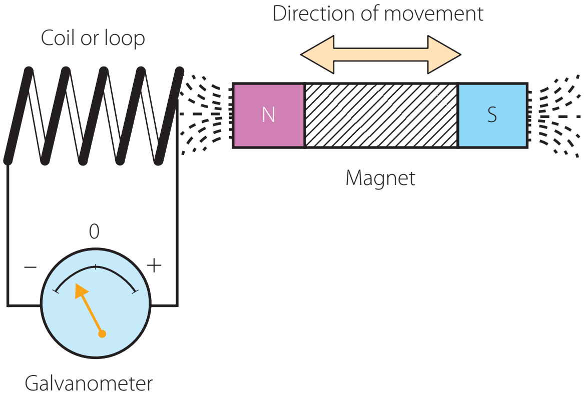


FIGURE 9.1.1 A current is induced when a magnet is moved relative to a coil of wire connected to a circuit.

Magnetic flux

magnetic flux a measurement of the total magnetic field that passes through a given area; has the unit weber (Wb)

magnetic flux density (*B*) the strength of a magnetic field per unit area

Magnetic flux is a measurement of the total magnetic field passing through a given area. It is directly proportional to the number of magnetic field lines passing through the defined area, which is referred to as the **magnetic flux density (*B*)**.

To derive a formula for magnetic flux, first consider a uniform magnetic field passing through an area, A, as shown in Figure 9.1.2. We choose the direction of the vector $\vec{A}$ to be perpendicular to the area, as this gives us a unique way of representing the area. The magnitude of the vector $\vec{A}$ is the area A, in units of m^2. The magnetic flux density, $\vec{B}$, is the number of field lines crossing the area and depends on the directions of both $\vec{B}$ and $\vec{A}$. The magnetic flux, Φ, for a uniform magnetic field through a loop of area A is defined as:

$$\Phi = B_{\perp}A = BA\cos(\theta)$$

KEY FORMULA

$\Phi = B_{\perp}A = BA\cos(\theta)$

Where:

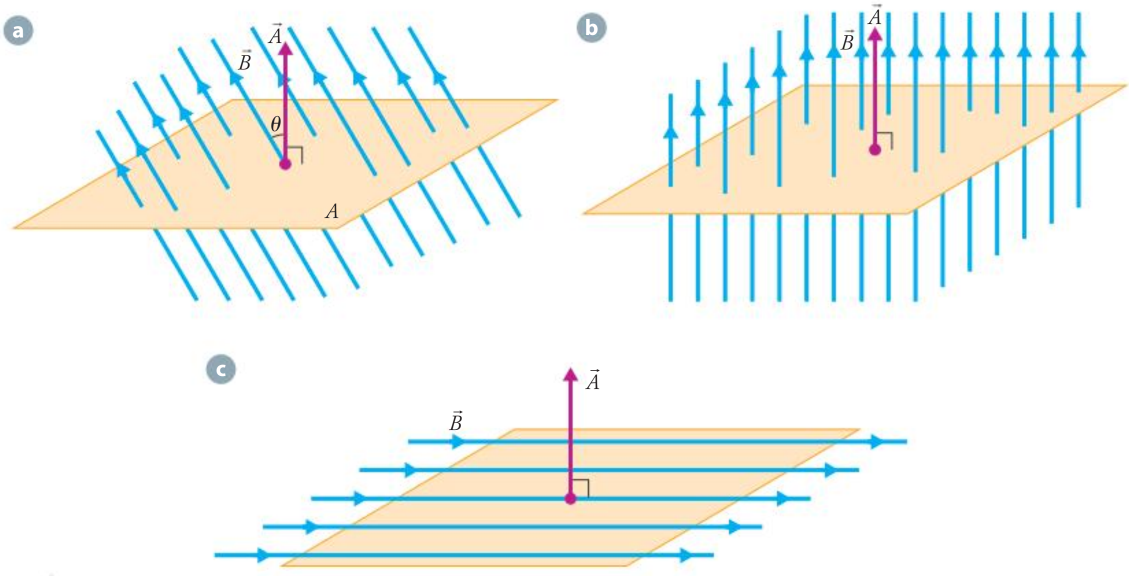
Φ = magnetic flux (Wb)

B = magnetic field strength (T)

A = area of the surface (m^2)

θ = angle between the magnetic field lines and a normal to the surface (°)

The magnetic flux, Φ, has units of $T\,m^2$ or the weber, Wb, after Wilhelm Weber, who co-invented the first electromagnetic telegraph.



9.1.1 What is magnetic flux?

9.1.2 Flux and magnetic flux

FIGURE 9.1.2 (**a**) The flux through the area A depends on the angle, θ, between the area vector $\vec{A}$ and the field, $\vec{B}$: $\Phi = BA\cos\theta$. (**b**) Flux is at a maximum when θ is zero and $\vec{B}$ and $\vec{A}$ are parallel. (**c**) Flux is at a minimum when $\theta = 90°$ and $\vec{B}$ and $\vec{A}$ are perpendicular.

The flux, has maximum amplitude when the field is in the same direction as, or opposite to, the vector $\vec{A}$, when $\vec{B}$ is perpendicular to the surface of the area. The flux is zero when $\vec{A}$ is perpendicular to $\vec{B}$, that is, when $\vec{B}$ is parallel to the surface of the area.

WORKED EXAMPLE 9.1.1

A loop of cross-sectional area $0.050\,\text{m}^2$ is in a uniform magnetic field of magnitude 0.24 T.

1 Draw diagrams showing the loop and field and identifying the angle, θ, between the area vector $\vec{A}$ and the field $\vec{B}$, when the flux is a:

 a minimum

 b maximum.

2 Find the maximum and minimum values of the flux through the loop.

ANSWERS

1 a Identify that flux has a minimum value of $\Phi = BA\cos\theta = 0$ when $\theta = 90°$ and that this is when the loop is parallel to the field.

 b Identify that flux has a maximum value of $\Phi = BA$ when $\theta = 0°$ and that this is when the loop is perpendicular to the field.

2 Apply the magnetic flux formula:
$\Phi = BA\cos\theta$
The maximum flux occurs when the field lines and the surface are perpendicular:
$\Phi_{max} = BA\cos(0°) = BA$

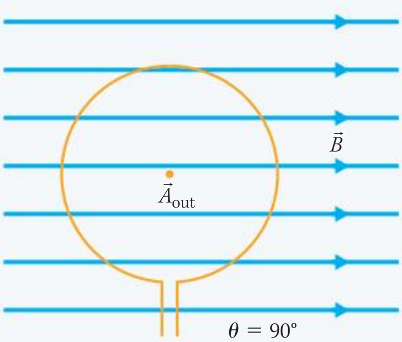


FIGURE 9.1.3 Loop is parallel to the field.

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Insert known values:

$\Phi_{max} = 0.24\,T \times 0.05\,m^2$

Calculate the answer:

$\Phi_{max} = 1.2 \times 10^{-2}\,Wb$

The minimum flux of 0 occurs when the field lines and the surface are parallel:

$\Phi_{max} = BA\cos(90°) = 0$

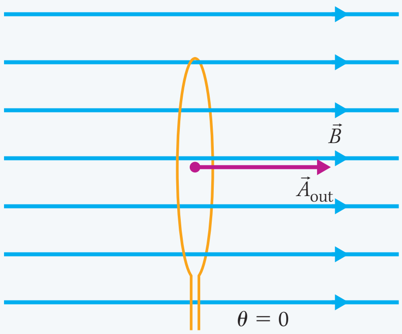

FIGURE 9.1.4 Loop is perpendicular to the field.

INQUIRING FURTHER

The property of the magnetic flux passing through a surface is commonly used in many devices, including some burglar alarms. In one of the simplest of designs, the hermetic reed switch, any change in the magnetic flux through a sensor will trigger the alarm.

Investigate the magnetic reed switch in order to determine when its use would be most appropriate.

SECTION REVIEW 9.1

REMEMBERING

1 Recall the units of magnetic flux.

2 Define 'electromagnetic induction'.

UNDERSTANDING

3 Explain why the magnetic flux through a loop of wire orientated perpendicularly to a uniform magnetic field will decrease as it is rotated to become parallel to the magnetic field.

4 Compare the concepts of magnetic fields and magnetic flux and explain their differences.

APPLYING

5 Calculate the magnetic flux that passes through a loop of wire that has a diameter of 10.0 cm when it is placed perpendicularly within a uniform magnetic field of strength 0.050 T.

6 If the angle formed between the normal to a surface of area 0.25 m^2 and a uniform magnetic field of strength 0.10 T is equal to 35°, determine the magnitude of the magnetic flux passing through the surface.

7 A solenoid produces a magnetic field of 0.25 T in its interior. The field is approximately uniform. What is the radius of the coil, given that the flux through any loop of the solenoid is equal to 5.0 mWb?

ANALYSING

8 A loop of cross-sectional area 0.015 m^2 is in a uniform magnetic field of 0.030 T. Initially the loop is perpendicular to the field lines (Figure 9.1.5). The loop is rotated about an axis parallel to its long sides at a uniform angular velocity of 5 revolutions per second.

   a What is the flux through the loop at $t = 0$ s?

   b Draw a graph of the flux through the loop as a function of time. Mark important features on your graph including the maximum flux and the period.

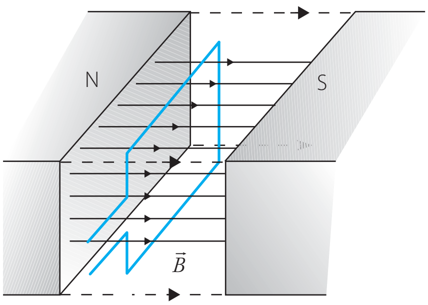

FIGURE 9.1.5

9.2 Faraday's law of induction

induced emf

an emf created by a changing magnetic field

9.2.1 What is Faraday's law?

9.2.2 Faraday's law introduction

9.2.3 What is Faraday's law of induction?

Faraday's experiments showed that when magnetic flux changes with time, an electric field is induced. If there is loop within this field, an **induced emf** is produced within it.

When there is a change in the magnitude of the magnetic flux ($\Delta\Phi$) through a loop of wire over a small timeframe (Δt), the magnitude of the induced emf is given by Faraday's law.

KEY FORMULA

Faraday's law of induction:

The induced emf in a loop of wire is equal to the negative of the change in magnetic flux ($\Delta\Phi$) divided by the change in time (Δt).

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t} = \frac{-(\Phi_f - \Phi_i)}{\Delta t}\ \text{(V)}$$

According to Faraday's law, the induced emf will have units of $\text{T m}^2\,\text{s}^{-1}$, which is the same as the volt, V. The negative sign indicates that the induced emf *opposes* the change in flux.

Once an emf is induced, a current will flow if there are free charge carriers and a path for them to flow along. This is usually achieved by putting a metal coil in the field. This induced current is related to the emf by Ohm's law: $i = \frac{\text{emf}}{R}$, where R is the resistance of the path. The symbol i is used for current, rather than I, as the current may vary with time.

We use the term 'emf' here rather than potential difference for a reason. Because they often do the same thing and have the same unit, volt (V), they are often treated as if they are the same thing, but from the definitions given below, we can see that they are not the same.

- Potential difference is the unique difference in potential energy per unit charge *between any two points* in an electric field.
- Emf is the energy per unit charge available to a charged particle. The induced emf between any two points in a changing magnetic field *does not* have a unique value, but depends on the path between the two points. This is because it depends on the flux enclosed. Different closed paths between two points may contain different fluxes.

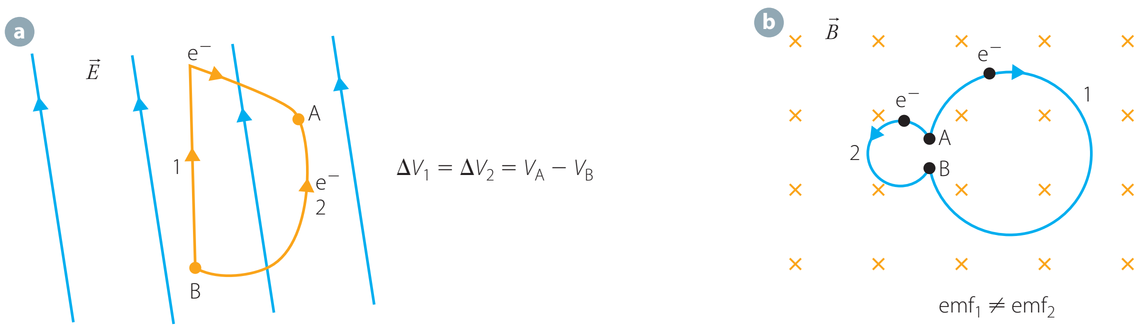

FIGURE 9.2.1 (**a**) Potential difference: an electron moves from point A to point B in an electric field. The change in potential energy of the electron is the same regardless of path taken. (**b**) Emf: an electron passes through two loops in a changing magnetic field. The emf measured across each loop is different because the loops contain different magnetic fluxes, even though they have the same beginning and end points.

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INQUIRING FURTHER

An induction cooktop (Figure 9.2.2) uses electromagnetic induction to transfer electrical energy in a ferromagnetic cooking vessel placed on it. The coil of the cooktop, which is mounted below its surface, carries an alternating current that, in turn, creates an alternating current that passes through the cooking surface and into the base of the vessel. This process creates a changing magnetic flux through the surface of the vessel, and induces a current in its walls. This current flows through the vessel and ultimately produces heat, which is transferred to its contents.

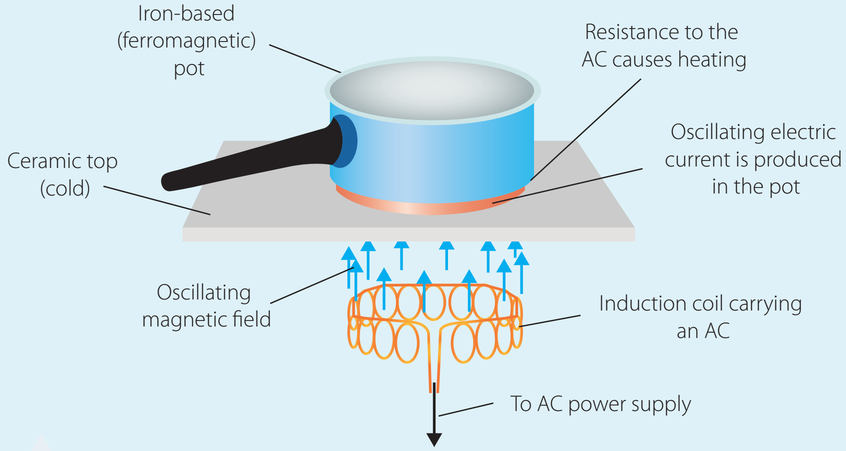

FIGURE 9.2.2 Induction cooking uses electromagnetic induction to transfer energy very effectively to the cooking vessel. This energy is manifested as heat.

Investigate the process of induction cooking to determine its benefits and drawbacks.

SECTION REVIEW 9.2

REMEMBERING

1 Define the concept of a change in magnetic flux.

2 State Faraday's law of electromagnetic induction.

UNDERSTANDING

3 Compare the concept of potential difference with that of electromotive force.

4 Explain how electromagnetic induction results in the formation of an induced current.

5 A loop is placed in a uniform magnetic field and is moved in a straight line at a constant velocity through the field. Explain why no current is induced in the loop.

APPLYING

6 Use a flow diagram to explain how a cast iron pot placed on an induction cooktop is able to heat its contents.

7 Explain why an aluminium pot placed on an induction cooktop is not able to heat its contents.

9.3 Solving problems: Faraday's law

By, substituting the magnetic flux equation into Faraday's law, we can see that:

KEY FORMULA

Faraday's law and the magnetic flux equation combined:

$$\text{emf} = \frac{-(\Delta BA\cos\theta)}{\Delta t} = \frac{-(\Phi_f - \Phi_i)}{\Delta t}$$

Where:

emf = electromagnetic force induced (V)

B = magnetic field strength (T)

A = area of the surface (m^2)

θ = angle between the magnetic field lines and a normal to the surface (°)

Φ = magnetic flux (Wb)

t = time over which the change occurs (s)

Inspecting this equation, we can see that there are three ways to induce an emf:

- a change in the magnetic flux density, B
- a change in the area, A
- a change in the angle, θ, between the area and the field.

In practice, it is usually either the magnetic field or the angle that is varied. For example, when a loop or coil of wire with area A is placed in a field, the flux through the loop can be varied by spinning the loop. This changes the angle, and induces an emf in the loop. The same effect can be achieved by spinning a magnet near the loop. In both cases the flux varies in time. This is used in generators.

We can take any parameter kept constant out of the brackets. For example, if area and angle are kept constant while B is varied, we write:

$$\text{emf} = \frac{-A\cos\theta\Delta B}{\Delta t} = \frac{-A\cos\theta(B_f - B_i)}{\Delta t}$$

An emf is produced by a changing magnetic flux density.

WORKED EXAMPLE 9.3.1

A loop of conducting wire with an area of 0.500 m^2 is placed perpendicularly within a uniform 0.300 T magnetic field. Calculate the induced emf in the coil if it is removed from the magnetic field in 0.100 s.

ANSWER

1 Apply Faraday's law:

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t}$$

2 Insert the magnetic flux equation:

$$\text{emf} = \frac{-\Delta(BA\cos\theta)}{\Delta t}$$

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3 Rearrange the equation for a changing magnetic flux density:

$$\text{emf} = \frac{-A\cos\theta(B_f - B_i)}{\Delta t}$$

4 Insert known values:

$$\text{emf} = \frac{-0.500\,\text{m}^2 \times \cos 0° \times (0\,\text{T} - 0.300\,\text{T})}{0.100\,\text{s}}$$

5 Calculate the answer:
emf = 15.0 V

If the area and field are held constant but the angle is changed use the rearranged equation:

$$\text{emf} = \frac{-BA\Delta\cos\theta}{\Delta t} = \frac{-BA(\cos\theta_f - \cos\theta_i)}{\Delta t}$$

WORKED EXAMPLE 9.3.2

A loop of conducting wire with an area of $0.0250\,\text{m}^2$ is placed perpendicularly within a uniform 0.500 T magnetic field. Calculate the induced emf in the loop if the loop is rotated so that it becomes parallel to the magnetic field in 0.600 s.

ANSWER

1 Apply Faraday's law:

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t}$$

2 Insert the magnetic flux formula:

$$\text{emf} = \frac{-\Delta(BA\cos\theta)}{\Delta t}$$

3 Rearrange the formula for a changing angle:

$$\text{emf} = \frac{-BA(\cos\theta_f - \cos\theta_i)}{\Delta t}$$

4 Insert known values:

$$\text{emf} = \frac{-0.500\,\text{T} \times 0.0250\,\text{m}^2(\cos 90° - \cos 0°)}{0.600\,\text{s}}$$

5 Calculate the answer:
emf = 2.08×10^{-2} V

To generate a larger emf, a coil containing multiple loops of wire is used. Each loop will have an emf induced between its ends, so connecting n loops in series is like connecting n batteries in series. Simply add the emf in all loops. Therefore:

KEY FORMULA

Faraday's law for multiple loops:

$$\text{emf} = \frac{-n\Delta\Phi}{\Delta t} = \frac{-n\Delta(BA\cos\theta)}{\Delta t}$$

Where:

emf = electromagnetic force induced (V)

n = number of loops of wire

B = magnetic field strength (T)

A = area of the surface (m^2)

θ = angle between the magnetic field lines and a normal to the surface (°)

Φ = magnetic flux (Wb)

t = time over which the change occurs (s)

WORKED EXAMPLE 9.3.3

A wire loop of cross-sectional area 0.050 m^2 is in a magnetic field. The loop is perpendicular to the field. The field changes with time as shown.

1 Sketch a graph of flux through the loop as a function of time.

2 Sketch a graph of $\frac{\Delta\Phi}{\Delta t}$ as a function of time.

3 Find the emf induced between the ends of the wire.

4 Find the current induced in the loop when the loop has a resistance of 0.15 Ω.

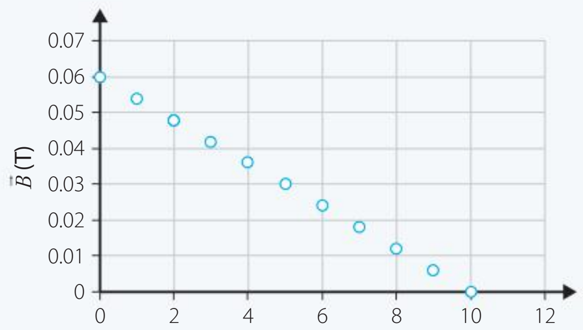

FIGURE 9.3.1 Magnetic field as a function of time

ANSWERS

1 Use $\Phi = BA\cos\theta$, noting that $\theta = 0°$ and $A = 0.05\ m^2$ are given in the question.

2 $\frac{\Delta\Phi}{\Delta t}$ is the gradient of the $\Phi(t)$ graph, which is constant. We find this gradient by taking the rise over run for a section of the $\Phi(t)$ graph.

3 The emf is the negative of the gradient of our $\Phi(t)$ graph, which we can see is –0.3 mV; hence the emf is +0.3 mV.
emf = +0.3 mV

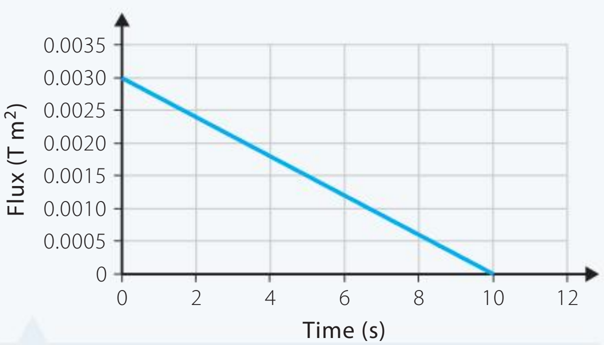

FIGURE 9.3.2 Flux as a function of time

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4 Apply Ohm's law:

$$I = \frac{\text{emf}}{R}$$

Insert known values:

$$I = \frac{-3.0 \times 10^{-4}\ \text{V}}{0.15\ \Omega}$$

Calculate the answer:

$I = +0.002\ \text{A} = +0.2\ \text{mA}$

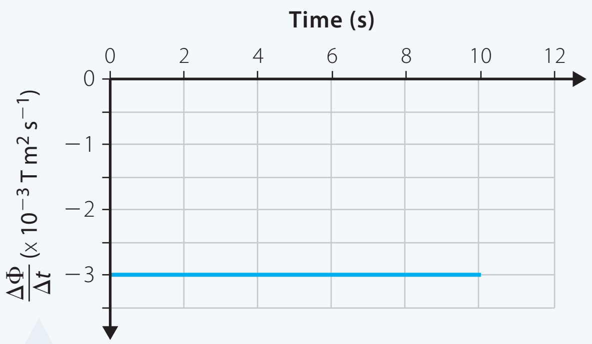

FIGURE 9.3.3 Graph of $\frac{\Delta\Phi}{\Delta t}$ as a function of time

SECTION REVIEW 9.3

REMEMBERING

1 List the three variables that can be altered in order to induce an emf in a loop of wire placed in a magnetic field.

2 For each of the variables listed in your answer to Question **1**, write an equation that can be used to calculate the magnitude of the emf induced if the other two variables are kept constant.

UNDERSTANDING

3 Figure 9.3.4 shows the flux through a loop of wire as a function of time.

- **a** State the time at which the emf produced will be at a minimum.
- **b** State the time at which the emf produced will be at a maximum.
- **c** Sketch the rate of change of flux through the loop using the same time scale.
- **d** Sketch the induced emf across the loop as a function of time using the same time scale.

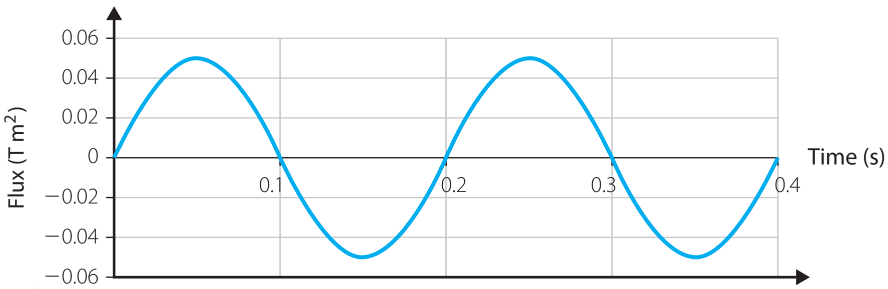

FIGURE 9.3.4

APPLYING

4 A loop of conducting wire with an area of 0.45 m^2 is placed perpendicularly within a uniform 0.10 T magnetic field. Calculate the induced emf in the coil if the loop is removed from the magnetic field in 0.20 s.

5 A loop of conducting wire with an area of 0.250 m^2 is placed perpendicularly within a uniform 0.350 T magnetic field. Calculate the induced emf in the loop if the loop is rotated so that it becomes parallel to the magnetic field in 0.50 s.

6 Calculate the rate at which a uniform magnetic field passing perpendicularly through a loop of area 0.35 m^2 must change in order to produce an emf of 12.0 V.

9.4 Lenz's law

The negative sign in the equation for induced emf indicates the direction of the induced current.

Consider a loop in a magnetic field that is getting stronger with time (Figure 9.4.1). There are two possible directions in which the induced current can flow. The negative sign tells us that the current must flow such that the flux through the loop decreases. Why is this? The short answer is conservation of energy.

The potential energy of the changing magnetic field is transformed into electric potential energy. The result is an electric field that can do work by applying a force. Work is done on any free electrons by the induced electric field. The electrons then flow, giving the induced current. The induced current takes its energy from the changing magnetic flux (via the electric field), and so reduces the rate at which the flux changes.

Consider what would happen if the current acted to produce a further increase in the magnetic flux through the loop. The flux would increase more, giving a bigger induced current, giving a bigger flux and so on. The result would be a 'perpetual motion machine' that made more and more current without any source of energy. This would violate conservation of energy and cannot happen. So the current *must* flow in the other direction and act to decrease the magnetic flux through the loop. Lenz's law is essentially a statement of *conservation of energy*.

> **KEY LAW**
>
> Lenz's law:
>
> An induced emf acts to produce an induced current. The induced current is in the direction that causes a change in magnetic flux that opposes the change in flux which induced the emf.

9.4.1 Lenz's law

a When the magnet is moved towards the stationary conducting loop, a current is induced in the direction shown. The magnetic field lines are due to the bar magnet.

b This induced current produces its own magnetic field directed to the left that counteracts the increasing external flux.

c When the magnet is moved away from the stationary conducting loop, a current is induced in the direction shown.

d This induced current produces a magnetic field directed to the right and so counteracts the decreasing external flux.

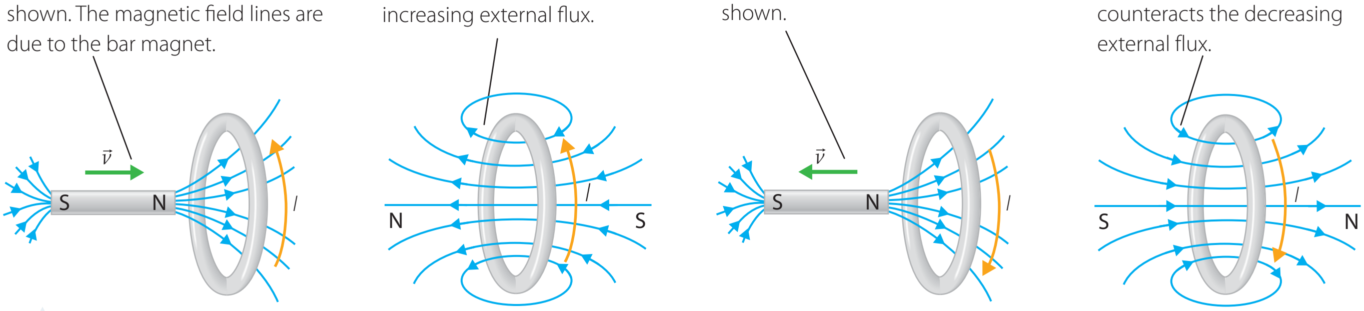

FIGURE 9.4.1 A moving bar magnet induces a current in a conducting loop. The direction of the current is determined by Lenz's law.

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Eddy currents

Induced currents are not only seen in wire loops, but in any material in which there are free charge carriers. If a magnet is moved around over a piece of metal, the changing magnetic field will induce **eddy currents** in the metal. The electrons move in circles in the region where the field is changing. They form loops and spirals of current, like eddies in a cup of tea when you stir it. These eddy currents create magnetic fields that oppose the changing flux from the moving magnet. They act to slow down or brake the magnet. This is called **magnetic braking**.

eddy current
a circular current induced in a conductor due to a changing magnetic field

magnetic braking
braking due to the interaction of eddy currents and an external magnetic field

SECTION REVIEW 9.4

REMEMBERING

1 State Lenz's law.

2 Define 'eddy currents'.

UNDERSTANDING

3 Explain how Lenz's law is consistent with the principle of the conservation of energy.

4 Sketch a flow diagram that summarises Lenz's law for a magnet that is being pushed into a coil of wire.

APPLYING

5 Use Lenz's law to determine the direction in which the induced current will flow in each current-carrying wire in the vicinity of a loop of wire shown in Figure 9.4.2.

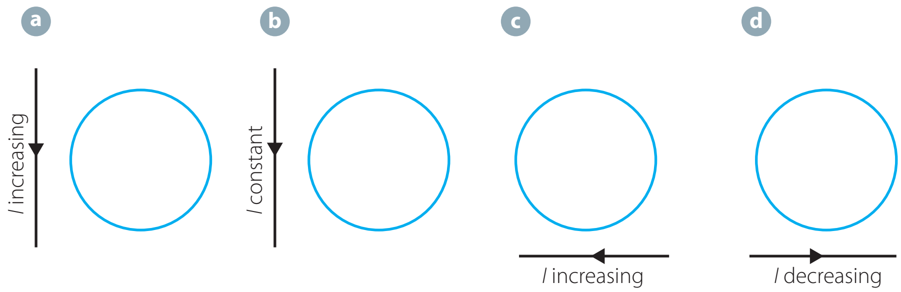

FIGURE 9.4.2

9.5 Production and transmission of alternating current

The production and transmission of **alternating current (AC)** through the use of electrical generators and transformers, rely on the phenomenon of electromagnetic induction.

alternating current (AC)
electrical current that alternates its direction of travel sinusoidally with time

generator
a device used to produce electrical current by electromagnetic induction

Generators

The majority of electricity produced worldwide relies on the operation of an AC **generator**. At a fundamental level, a generator transforms kinetic energy, in the form of the relative movement of coils of wire and magnets into electrical energy through the induction of an emf across the coils to generate a current. The energy required to produce the movement may come from any source. In Australia it is mostly supplied by burning coal or gas. A small fraction comes from the gravitational potential energy of

9.5.1 What is alternating current?

9.5.2 AC circuits: alternating current electricity?

water (hydroelectric power stations) and the kinetic energy of air molecules (wind turbines). Many other nations use nuclear energy. All of these energy sources require generators to produce electricity.

armature
the frame of the rotating part of a motor or generator, holding one or more coils

Figure 9.5.1 shows a very simple AC generator. An alternating current is one that varies between positive and negative values. Typically AC varies sinusoidally. The coil is attached to an **armature** that rotates in the magnetic field between the poles of the two magnets. As the armature rotates, the flux through the loops of the coil varies, causing an emf across the ends of the coil. Each end of the coil is attached to a conducting slip ring that slides against a brush. The brushes are then connected to the external circuit that uses the emf generated.

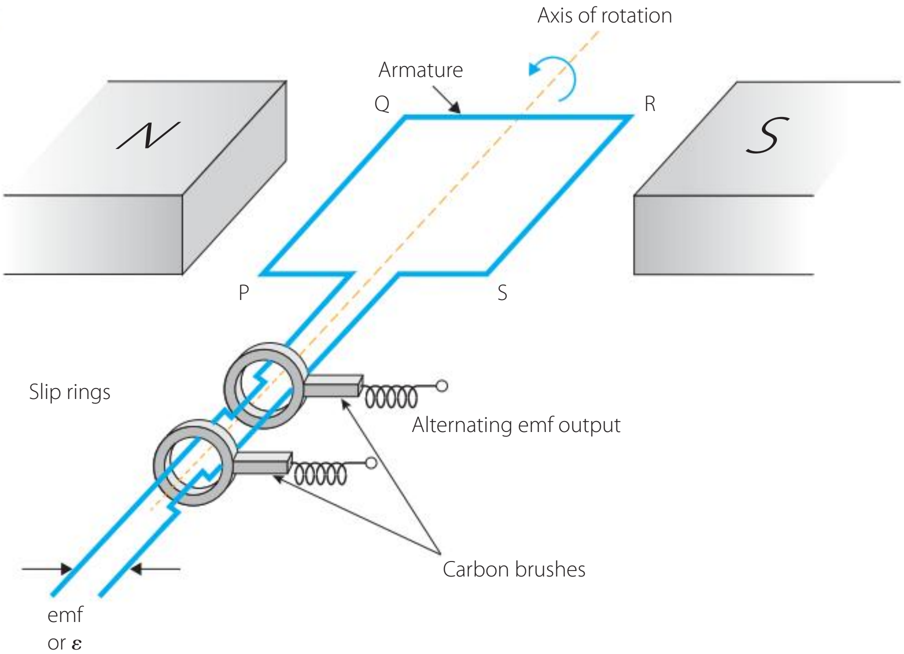

FIGURE 9.5.1 The rotation is circular, so the circular or periodic functions (sine and cosine) represent the loop's motion.

In Figure 9.5.2, the flux vs time graph is a sine curve because the original flux is zero.

The flux varies with the angle, θ, which varies in time such that $\theta(t) = \left(\frac{2\pi}{T}\right)t = 2\pi ft$, where T is the period of rotation. The frequency is $f = \frac{1}{T}$. Hence, the flux as a function of time is given by:

$$\Phi = nBA\sin(2\pi ft)$$

KEY FORMULA

The flux passing through a rotating armature in which a coil with n turns of area A rotates in a magnetic field of magnitude B

$\Phi = nBA\sin(2\pi ft)$

Where:

Φ = magnetic flux (Wb)

n = number of coils of wire

B = magnetic field strength (T)

A = area of the surface (m^2)

f = frequency of rotation (Hz)

t = time (s)

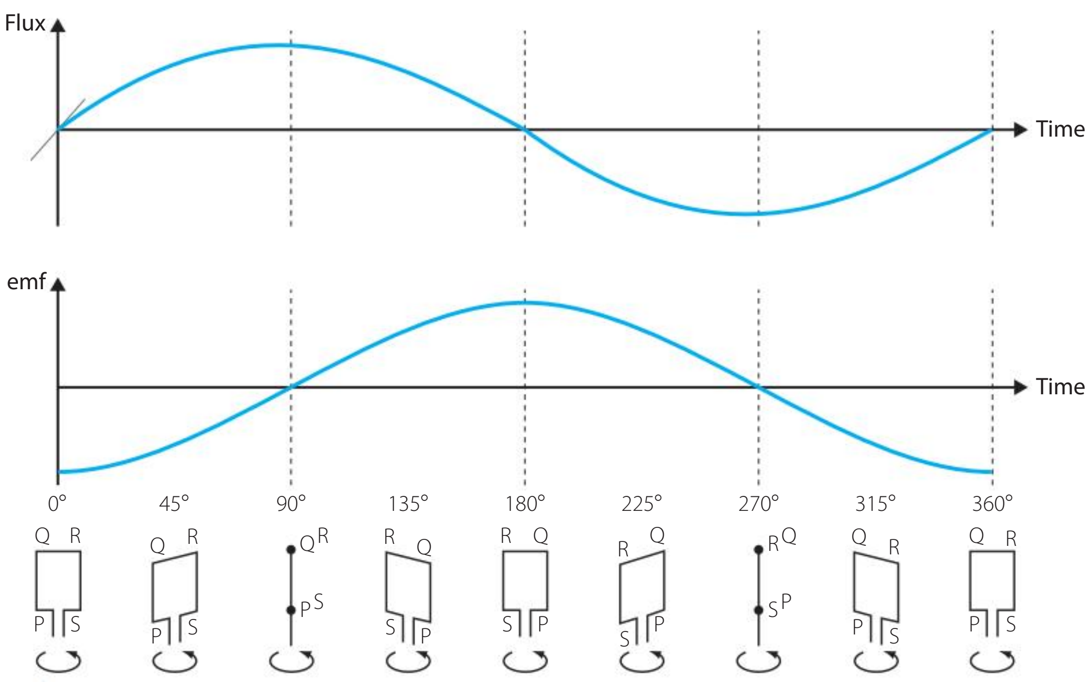

FIGURE 9.5.2 A loop rotates in a circle. The flux through the loop describes a sine curve (top graph). The rate of change of flux – the gradient – also describes a sinusoidal curve, but the emf is the negative of the rate of change of flux (lower graph). The rotation of the coil is shown for each one-eighth of a turn.

The emf is the negative of the gradient of the flux as a function of time.

So the emf is given by emf $= 2\pi fnBA\cos(2\pi ft)$.

KEY FORMULA

The emf produced by a rotating armature in a magnetic field

emf $= 2\pi fnBA\cos(2\pi ft)$

Where:

emf = electromagnetic force induced (V)

f = frequency of rotation (Hz)

B = magnetic field strength (T)

A = area of the surface (m^2)

t = time (s)

This is shown in Figure 9.5.2. Note that when the flux is changing most rapidly, the emf has its maximum values. For a sine curve, the gradient is greatest at $t = 0$, $t = \frac{T}{2}$ and again at the end of each cycle. When the flux is at a peak, at $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, the gradient is momentarily zero, so the emf is zero. The flux and emf have the same frequency.

The maximum emf occurs when $\cos(2\pi ft) = \pm 1$.

The maximum emf can be changed by changing f, n, B or A. If f is changed, the period changes as well as the emf.

Usually the armature has a large coil of wire, as the emf is proportional to the number of loops or turns in the coil. However, the bigger the coil, the heavier it is, so sometimes it is the magnets that are rotated instead of the coil.

KEY FORMULA

$emf_{max} = 2\pi fnBA$

The maximum emf produced by a rotating armature in a magnetic field is equal to 2π times the product of the frequency rotation (f), the number of turns (n), the magnetic flux density (B) and the area of the armature (A).

WORKED EXAMPLE 9.5.1

A square coil of side length 0.10 m is made up of 400 turns. It is rotated at 25 Hz in a magnetic field of magnitude 0.10 T.

1 Find the maximum emf induced.

2 There is a maximum flux through the coil at $t = 0$. Sketch the emf as a function of time.

ANSWERS

1 Apply the maximum emf in an armature equation:

$\text{emf}_{max} = 2\pi fnBA$

Insert known values:

$\text{emf}_{max} = 2\pi \times 25\,\text{Hz} \times 400 \times 0.10\,\text{T} \times (0.10\,\text{m})^2$

Calculate the answer:

$\text{emf}_{max} = 63\,\text{V}$

2 We know from part **a** that the emf varies between −63 V and +63 V. The period of its oscillations is the same as the period of rotation, which is $T = \frac{1}{f} = \frac{1}{25\,\text{Hz}} = 0.04\,\text{s}$.

Note that as we do not know which way the coil is turning, a sketch showing emf starting from zero and decreasing first is also possible.

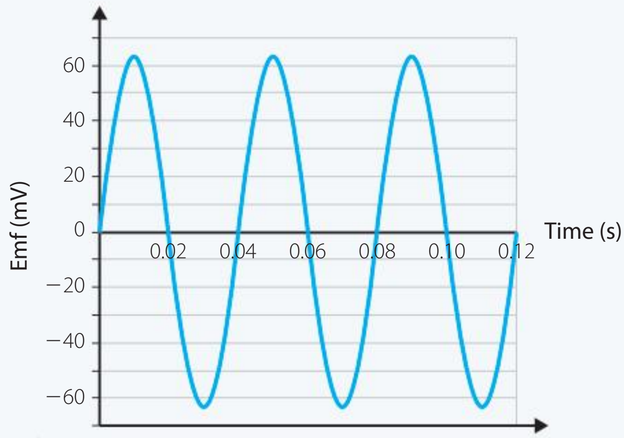

FIGURE 9.5.3 Emf as a function of time

Alternating current

For a sinusoidal potential difference, the relevant quantities are peak potential difference, V_P; peak-to-peak potential difference, V_{P-P}; period, T; and frequency, f. These are shown graphically in Figure 9.5.4.

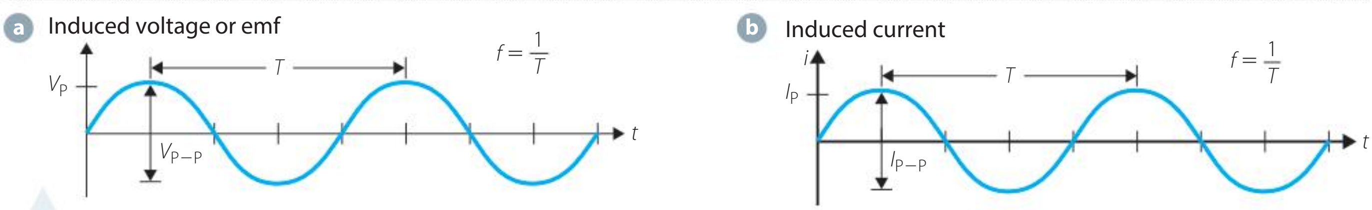

FIGURE 9.5.4 Induced emf gives rise to an induced current. The peak voltage and peak current are proportional to each other and they have the same frequency. (**a**) The induced emf or voltage has a peak value, V_P, that is half the peak-to-peak value, V_{P-P}. It describes one cycle in one period of time, T. (**b**) The induced current, i, has a peak value, I_P, that is half the peak-to-peak value I_{P-P}. It describes one cycle in one period of time, T.

direct current (DC) a current that flows in a single direction

When dealing with **direct current (DC)**, potential difference and current are constant, or at least constantly in the same direction. Values for AC vary between a peak positive value and a peak negative value, oscillating back and forth in each cycle. The average of the AC potential difference over one cycle is zero, yet the AC potential difference obviously delivers energy during that time; that is, it delivers power to a circuit. Power is proportional to the square of the potential difference. If we square the potential difference and find the average, we can get a value for the average power. To convert this to a single

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potential difference that would deliver the same power as the original AC potential difference, we take the square root of this average. The single value of potential difference that we get when we square, average and take the square root is called the **root mean square (rms)** value.

root mean square (rms)
the average AC potential difference or current that produces the same power in a load as a DC potential difference or current of the same magnitude

KEY FORMULA

Root mean square voltage, V_{rms}, as a function of the peak voltage, V_{peak}

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = 0.707V_{peak}$$

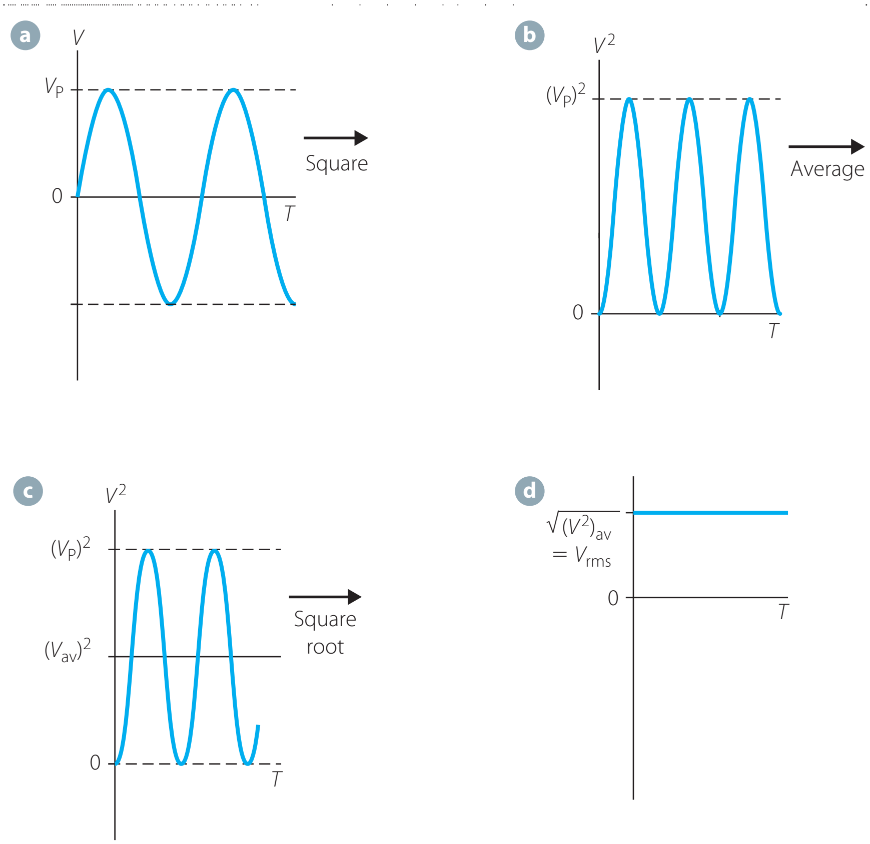

FIGURE 9.5.5 (**a**) A sinusoidal $V(t)$. (**b**) We square this to get $V^2(t)$. This has a peak value of V_P^2. (**c**) The average value of $V^2(t)$, which has the value $\frac{1}{2}V_P^2$. (**d**) Taking the square root of this value gives us the rms value: $V_{rms} = \frac{1}{\sqrt{2}}V_{peak}$.

9.5.3 Alternating current (AC) vs direct current (DC)

For an AC generator, the rms output emf is therefore calculated as:

KEY FORMULA

Root mean square emf of an AC generator

$$emf_{rms} = \frac{emf_{max}}{\sqrt{2}} = \frac{2\pi fnBA}{\sqrt{2}} = \sqrt{2}\pi fnBA$$

The rms potential difference is an average AC potential difference that produces the same power in a resistive component as a constant DC potential difference of the same magnitude. AC systems are usually described using rms values.

Similarly, to calculate the root mean square current of an AC:

KEY FORMULA

Root mean square current, I_{rms}, as a function of the peak current, I_{peak}

$$I_{rms} = \frac{I_{peak}}{\sqrt{2}} = 0.707\, I_{peak}$$

The rms current produced in a generator that is connected to a load of resistance *R* is calculated as:

KEY FORMULA

Root mean square current of an AC generator

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{\sqrt{2}\pi f nBA}{R}$$

The average power of an alternating current can be calculated as:

KEY FORMULA

The average AC power as a function of rms voltage and current and of peak voltage and current

$$P_{av} = V_{rms} I_{rms} = \frac{V_{peak} I_{peak}}{2}$$

WORKED EXAMPLE 9.5.2

If an AC generator that produces a peak voltage of 340 V is connected to an external load with a resistance of 55 Ω, calculate:

1 the rms potential difference produced

2 the rms current produced

3 the average power of the AC.

ANSWERS

1 a Apply the rms voltage equation:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

b Insert known values:

$$V_{rms} = \frac{340\text{ V}}{\sqrt{2}}$$

c Calculate the answer:

$$V_{rms} = 240\text{ V}$$

2 a Apply Ohm's law

$$V_{rms} = I_{rms} R$$

b Rearrange for the unknown, I_{rms}:

$$I_{rms} = \frac{V_{rms}}{R}$$

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c Insert known values:

$$I_{rms} = \frac{240\,V}{55\,\Omega}$$

d Calculate the answer

$I_{rms} = 4.37\,A$

3 a Apply the average power formula:

$P_{av} = V_{rms} I_{rms}$

b Insert known values:

$P_{av} = 240\,V \times 4.37\,A$

c Calculate the answer:

$P_{av} = 1.05\,kW$

Transformers

Most appliances use a transformer to convert the 240 V mains power to a lower potential difference. Some also convert the AC potential difference to DC. The transformer may be inside the device, or it may be a separate plug pack. There are also transformers at electricity substations. These drop the potential difference from the thousands of volts at which electricity is transmitted to the 240 V that is supplied to homes and businesses.

FIGURE 9.5.6 Various transformers

A transformer consists of two solenoids or coils of wire placed near each other so that an alternating current in the primary coil can induce a current in the secondary coil. The link between input and output is by electromagnetic induction; there is no electrical connection.

Solenoids are used because they produce a large and approximately uniform magnetic field inside the coil. The field in the primary solenoid varies sinusoidally with the alternating current in the coil. The coils need to be coupled so that the changing magnetic field in the primary coil causes a changing magnetic flux in the secondary solenoid. There are two ways of doing this. First, the coils can share the same space by placing one within the other. This is sometimes used in cordless appliances such as kettles. The second, and more usual, way is to link the coils using a ferromagnetic core.

The primary coil is wound around one side of an iron core. The current in the primary coil magnetises the whole core, not just the part within the primary coil. The time-varying current in the primary coil causes a time-varying magnetic field inside the secondary core. This creates a time-varying electric field, hence an emf and current in the secondary coil. Figure 9.5.7 shows how this works.

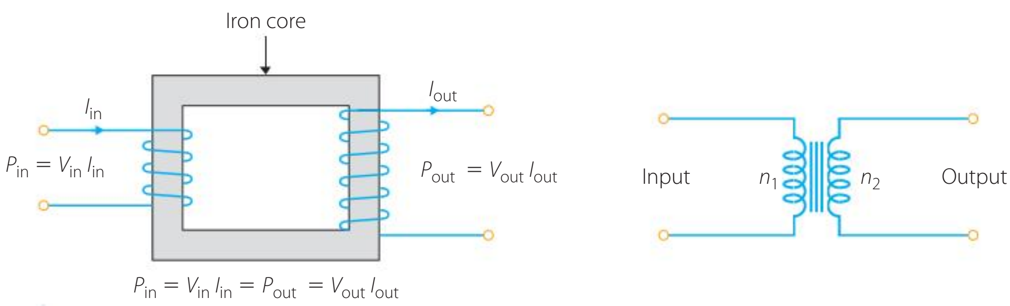


FIGURE 9.5.7 (**a**) A schematic diagram of a transformer and (**b**) its circuit symbol

The flux through any loop is the same for both coils. If the primary coil has n_p turns then:

KEY FORMULA

$$V_p = -n_p \frac{\Delta\Phi}{\Delta t}$$

The alternating potential difference in the primary coil of a transformer (V_p) creates an alternating magnetic field in the iron core, which is equal to the negative of the number of loops in the coil multiplied by the rate of change of the magnetic flux $\left(-n_p \frac{\Delta\Phi}{\Delta t}\right)$.

KEY FORMULA

$$V_s = -n_s \frac{\Delta\Phi}{\Delta t}$$

An alternating magnetic flux through the secondary coil in a transformer $\left(-n_s \frac{\Delta\Phi}{\Delta t}\right)$, produces a potential difference in the coil (V_S).

As $\frac{\Delta\Phi}{\Delta t}$ is the same for both coils:

KEY FORMULA

$$\frac{V_p}{V_s} = \frac{n_p}{n_s}$$

The ratio of the voltages in the two arms of a transformer $\frac{V_p}{V_s}$ is equal to the ratio of the number of coils of each arm $\frac{n_p}{n_s}$.

Assuming that the transformer is 100% efficient, power out = power in or $P_{out} = P_{in}$. We can apply Ohm's law to show that $I_s V_s = I_p V_p$. This can be included in the previous equation to give the full transformer equation:

KEY FORMULA

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} = \frac{n_p}{n_s}$$

The transformer equation: The ratio of the secondary voltage (V_s) to the primary voltage (V_p) is equal to the ratio of the primary current (I_p) to the secondary current (I_s) and is also equal to the ratio of the secondary number of coils (n_s) to the primary number of coils (n_p).

In reality, transformers are not 100% efficient, and a small amount of energy is lost as heat through resistance and eddy currents, although this is usually less than 1% of the total energy transformed.

It is important to realise that transformers only work when an alternating current is passing through the primary coil and would not work if direct current is passing through the coil. This is due to the need for a time-changing magnetic field to produce an emf in the secondary coil. This is one of the primary reasons that AC is widely used today – it can easily be transformed.

A **step-up transformer** ($n_s > n_p$) has a higher emf and lower current on the secondary side. A **step-down transformer** ($n_s < n_p$) has a lower emf and higher current on the secondary side.

step-down transformer a transformer with an output potential difference that is lower than the input potential difference

step-up transformer a transformer with an output potential difference that is higher than the input potential difference

WORKED EXAMPLE 9.5.3

A 120 W, 24 V AC supply is connected to the input terminals of a transformer. The primary coil is wound with 240 turns. The output emf is 72 V. Assume there is no power loss in the transformer.

1 Find the number of turns on the secondary coil.

2 Is this a step-up or step-down transformer?

3 What is the output current?

ANSWERS

1 a Apply the transformer equation:

$$\frac{V_p}{V_s} = \frac{n_p}{n_s}$$

b Rearrange for the unknown:

$$n_s = n_p \frac{V_s}{V_p}$$

c Insert known values:

$$n_s = 240 \times \frac{72\ \text{V}}{24\ \text{V}}$$

d Calculate the answer:

$$n_s = 720$$

2 Step up transformer as $n_s > n_p$

3 a Apply the conservation of energy:

$$P_{in} = P_{out}$$

b Apply Ohm's law:

$$P_{in} = I_s V_s$$

c Rearrange for the unknown:

$$I_s = \frac{P_{in}}{V_s}$$

d Insert known values:

$$I_s = \frac{120\ \text{W}}{72\ \text{V}}$$

e Calculate the answer:

$$I_s = 1.7\ \text{A}$$

SECTION REVIEW 9.5

REMEMBERING

1 Describe the purpose of an electrical generator.

2 Define 'alternating current'.

3 Describe the purpose of a transformer.

4 Describe the difference between a step-up and a step-down transformer.

UNDERSTANDING

5 Why does a rotating coil in a magnetic field produce an alternating emf?

6 Why is an alternating current necessary for a transformer?

APPLYING

7 A rectangular coil of 30 turns and area 100 cm^2 rotates at 1200 revolutions per minute in a uniform magnetic field of flux density 0.50 T.

- **a** Find the frequency of the generated emf.
- **b** Find the maximum emf.
- **c** What is the rms emf?
- **d** Write the equation that gives the emf at any instant.

8 The armature of an AC generator is rotating at a constant speed of 35 revolutions per second in a horizontal field of flux density 1.0 T. The diameter of the cylindrical armature is 24 cm and its length is 40 cm. The generator is connected to a load of resistance 10.0 Ω.

- **a** What is the maximum emf induced in the armature if it has 30 turns?
- **b** What is the rms emf produced by this generator?
- **c** What is the rms current produced by the generator?
- **d** What is the average power output of the generator?

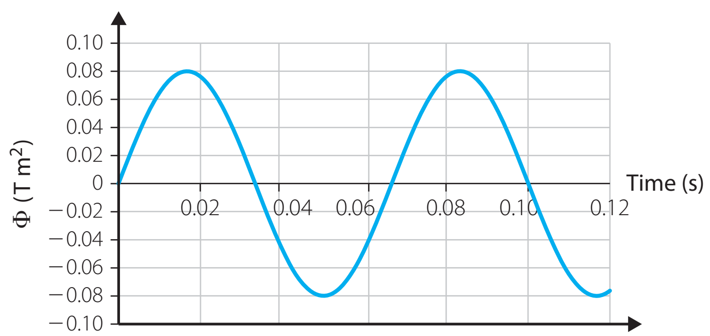
9 A step-up transformer is connected to an AC generator that delivers 8.0 A at 120 V. The ratio of the number of turns in the secondary coil to the number of turns in the primary is 500.

- **a** What is the emf in the secondary coil?
- **b** What is the average power input?
- **c** What is the maximum power output?
- **d** What is the maximum current in the secondary coil?

ANALYSING

10 Figure 9.5.8 shows the magnetic flux as a function of time through each loop of a 30-turn coil in a generator.

FIGURE 9.5.8



- **a** Find the maximum and minimum emf and the period of oscillation of the emf.
- **b** Derive an equation that describes the emf produced as a function of time.
- **c** Draw a graph showing the emf produced as a function of time.
- **d** Mark the rms emf on the graph.

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
  - a Alternating current
  - b Direct current
  - c Eddy currents
  - d Electrical generator
  - e Electric potential
  - f Electromagnetic induction
  - g Electromotive force
  - h Magnetic field
  - i Magnetic flux
  - j Magnetic flux density
  - k Transformer

2 Describe the differences between magnetic fields, magnetic flux and magnetic flux density.

3 Describe the difference between electromotive force and electric potential.

CATEGORY QUESTIONS

4 Describe in detail the process by which an electrical current is induced in a coil of wire that is moving relative to a magnetic field.

5 Explain three different ways in which the magnetic flux through a coil of wire may be changed.

6 Explain Lenz's law.

7 Sketch a flow diagram to show the process of electricity transmission from its initial production until its delivery to a house.

8 Explain why it is necessary to step up the voltage of electricity before it is transmitted long distances.

ELABORATION QUESTIONS

9 Explain what would happen if Lenz's law stated that the magnetic flux change produced by an induced current acted in the same direction as the changing magnetic flux that produced it.

10 List five sources of the mechanical energy that are commonly used to turn the turbine in a commercial electrical generator.

EVIDENCE QUESTIONS

11 Research and explain how the phenomenon of electromagnetic induction can be used as an effective means to reduce the velocity of moving objects.

12 Suggest reasons for the universal adoption of alternating current as the preferred form for electrical energy supply.

END-OF-CHAPTER EXAM

End-of-chapter test

1 The production of an electric field by a changing magnetic field is referred to as:

A electromotive force.
B electrolysis.
C electromagnetic induction.
D electricity.

2 Which of the following does not affect the magnitude of the magnetic flux passing through a surface?

A The composition of the surface.
B The magnitude of the magnetic field.
C The area of the surface.
D The orientation of the surface to the magnetic field.

3 The unit used to describe the magnetic flux is the weber (Wb). Another unit that can be used is:

A Tm.
B Tm^{-1}.
C Tm^{2}.
D Tm^{-2}.

4 The feature of a graph of the magnetic flux passing through a surface as a function of time that gives an indication of the size of the emf as a function of time is:

A the *y* intercept.
B the *x* intercept.
C the area under the line.
D the gradient of the line.

5 Name the type of current produced when a coil of wire is rotated in a uniform magnetic field.

6 If a surface is orientated parallel to a magnetic field, will the magnetic flux through the surface be at a maximum or a minimum?

7 Name the current produced by a magnet that is moved around over a solid piece of metal.

8 State Lenz's law.

9 Calculate the magnetic flux through a loop of area $0.025\,m^2$ that is placed perpendicularly to a uniform magnetic field of $5.0 \times 10^{-2}\,T$.

10 If a loop of area $0.025\,m^2$ that is initially placed perpendicularly to a uniform magnetic field of $5.0 \times 10^{-2}\,T$ is quickly removed from the magnetic field over a period of 0.05 s, calculate the emf that will be produced in the loop.

11 Explain how Lenz's law agrees with the conservation of energy.

12 A square solenoid consisting of 250 loops of wire of side length 12 cm is initially orientated perpendicularly to a uniform magnetic field of 0.10 T. Calculate the emf that would be produced if the solenoid is rotated over a period of 0.20 s until a normal to its surface is at an angle of 25° to the magnetic field lines.

13 The area of a conducting loop is increased from $10.0\,cm^2$ to $25\,cm^2$ over a period of 2.0 s while the loop is in a uniform magnetic field of 2.0 T. Calculate the magnitude of the emf produced if the magnetic field and the loop are perpendicular to each other.

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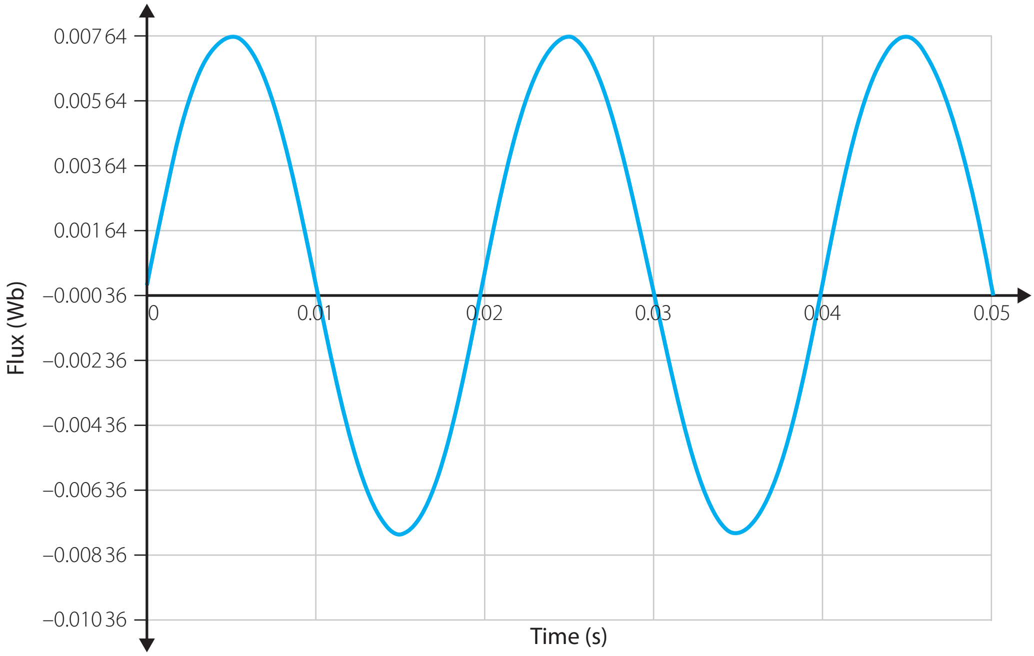
14 The magnetic flux as a function of time graph below is for the 100-turn armature of an AC generator.



a Calculate the frequency of rotation of the armature.

b Calculate the maximum emf produced by the generator.

c Draw a graph of the emf produced by the generator as a function of time.

15 The armature of an AC generator has an area of $2.5 \times 10^{-2}\,m^2$ and rotates at a rate of 50 Hz in magnetic field of strength 0.15 T. It produces a maximum emf of 150 V.

a How many turns must the coil contain?

b What is the rms emf produced?

c What is the rms current produced if the generator is connected to a load of 55 Ω?

d What is the average power output of the generator?

16 A step-up transformer is connected to an AC generator that delivers 10.0 A at 240 V. The ratio of the number of turns in the secondary coil to the number of turns in the primary is 700.

a What is the emf in the secondary coil?

b What is the average power input?

c What is the maximum power output?

d What is the maximum current in the secondary?

10 ELECTROMAGNETIC RADIATION

Introduction

The culmination of electromagnetic theory came in the 19th century with the work of theoretical physicist James Clark Maxwell. Maxwell managed to unify all the known phenomena of electricity and magnetism with the field theory of Michael Faraday into four equations that completely altered the way we view light. By analysing these four equations, Maxwell could describe many of the properties of light, but, most particularly, he was able to predict that light was an electromagnetic wave. This chapter will investigate the implications of this prediction.

Inquiry questions

- What is electromagnetic radiation?
- How do electromagnetic waves propagate through empty space?
- How are electromagnetic waves created?

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10.1 Electromagnetic waves

Maxwell's most important contribution to physics was to take the equations and experimental results of Faraday and others, and unify them into a single theory of electromagnetism. This theory is summed up in four **differential equations**.

differential equations
equations that relate the rate of change of displacement in space to the rate of change of displacement in time

Maxwell's equations: These four equations describe all the known classical phenomena involving electricity and magnetism.

$\oint \vec{E}\, d\vec{A} = \dfrac{q}{\varepsilon_0}$ Gauss's law: Electric fields are created by charges.

$\oint \vec{B}\, d\vec{A} = 0$ Gauss's law in magnetism: There are no isolated magnetic poles (monopoles).

$\oint \vec{B} d\vec{s} = -\dfrac{d\Phi B}{dt}$ Faraday's law: Electric fields are created by changing magnetic fields.

$\oint \vec{B} d\vec{s} = \mu_0 \varepsilon_0 \dfrac{d\Phi E}{dt} + \mu_0 I$ Ampère–Maxwell law: Magnetic fields are created by moving charges (currents) and changing electric fields.

FIGURE 10.1.1 Scottish physicist James Clerk Maxwell united the phenomena of electricity and magnetism into one theory.

10.1.1 Electromagnetic waves

10.1.2 James Clerk Maxwell: The greatest physicist you've never heard of

Electromagnetic waves

When Maxwell combined the second pair of his equations, he made a fundamental discovery. When a changing magnetic field produces an electric field, this electric field must also be changing. This changing electric field will then produce a changing magnetic field that would go on to produce another changing electric field and so on.

Armed with this discovery, Maxwell could show that by manipulating the equations, the result of the interaction of these changing electric and magnetic fields was a wave of coupled, self-sustaining oscillating electric and magnetic fields that travel as transverse waves and can propagate through empty space. The equations went on to show that these **electromagnetic waves** explained all the known phenomena of electricity and magnetism.

electromagnetic waves
waves produced by an oscillating charge resulting in mutually perpendicular electric and magnetic fields

Electromagnetic waves can travel through a vacuum

Electromagnetic (EM) waves propagate as transverse waves consisting of electric and magnetic fields oscillating both at right angles to each other and to the direction of travel (Figure 10.1.2). Because the oscillations of EM waves occur in fields rather than a medium (as is required by mechanical waves), they do not require a medium to travel through and can therefore propagate through empty space.

Even though light had been shown to behave like a wave some 60 years before Maxwell's discovery, there was much debate about the medium through which it travelled. Maxwell's description of EM waves could put to rest the need for the luminous aether – the hypothesised substance that pervaded the universe and provided a medium through which light waves propagated.

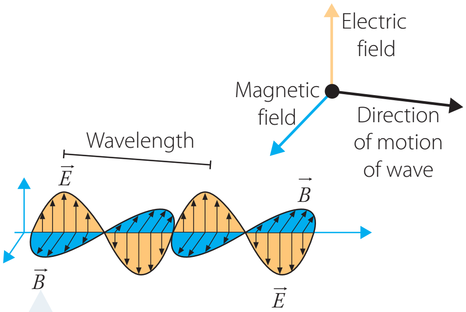

FIGURE 10.1.2 Electromagnetic waves are transverse waves consisting of coupled electric and magnetic field oscillations.

The speed of light

KEY FORMULA

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8\,\text{m s}^{-1}$$

The speed of light in a vacuum (c), is dependent upon the magnetic permeability (μ_0) and the electrical permittivity (ε_0) of the vacuum.

From his equations, Maxwell could predict that the speed of light in a vacuum (c) was inversely proportional to the square root of the product of the magnetic permeability (μ_0) and the electrical permittivity (ε_0) of the vacuum.

This value agreed, within uncertainty, with the experimentally measured value of the speed of light. In addition, the speed depends only on the constants μ_0 and ε_0, which are properties of empty space. This agreement between theory and experiment provided strong support in favour of Maxwell's theories.

For light in a vacuum, $v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8\,\text{m s}^{-1}$, but for light in any other medium, the speed is lower and depends on the permittivity and permeability of the medium.

KEY FORMULA

The speed of light in any medium (v) is dependent upon the magnetic permeability (μ) and the electrical permittivity (ε) of that substance.

$$v = \frac{1}{\sqrt{\varepsilon \mu}}$$

10.1.3 How fast does light travel?

wave equation
a differential equation that describes wave behaviour; its solutions are wave functions that are typically sinusoids

The **wave equation**, $v = f\lambda$, gives us a relationship between the speed, frequency and wavelength for electromagnetic waves, just as it does for mechanical waves.

KEY FORMULA

The wave velocity (v) is directly proportional to the frequency (f) and the wavelength (λ) of the electromagnetic wave.

$$v = f\lambda$$

WORKED EXAMPLE 10.1.1

An antenna for a radio station transmits a signal with frequency of 106.3 MHz. Calculate the wavelength of this electromagnetic wave.

ANSWER

1. Use the wave velocity formula:
   $v = f\lambda$
2. Rearrange for wavelength:
   $\lambda = \frac{v}{f}$
3. Insert known values in the correct units:
   $\lambda = \frac{3.0 \times 10^8\,\text{m s}^{-1}}{106.3 \times 10^6\,\text{Hz}}$
4. Calculate the final value:
   $\lambda = 2.82\,\text{m}$

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The electromagnetic spectrum

Visible light is only one kind of electromagnetic wave. In fact, Maxwell also predicted that a large range of frequencies was possible for electromagnetic waves, well beyond the visible spectrum. Figure 10.1.3 shows the **electromagnetic spectrum**, which shows the full range of EM waves that have been produced or detected. As can be seen, they form a continuous spectrum from long-wavelength, low-frequency radio waves through to short-wavelength and high-frequency gamma rays.

electromagnetic spectrum the family of electromagnetic radiations – radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays – which travel at $3.0 \times 10^8 \, m \, s^{-1}$ in a vacuum

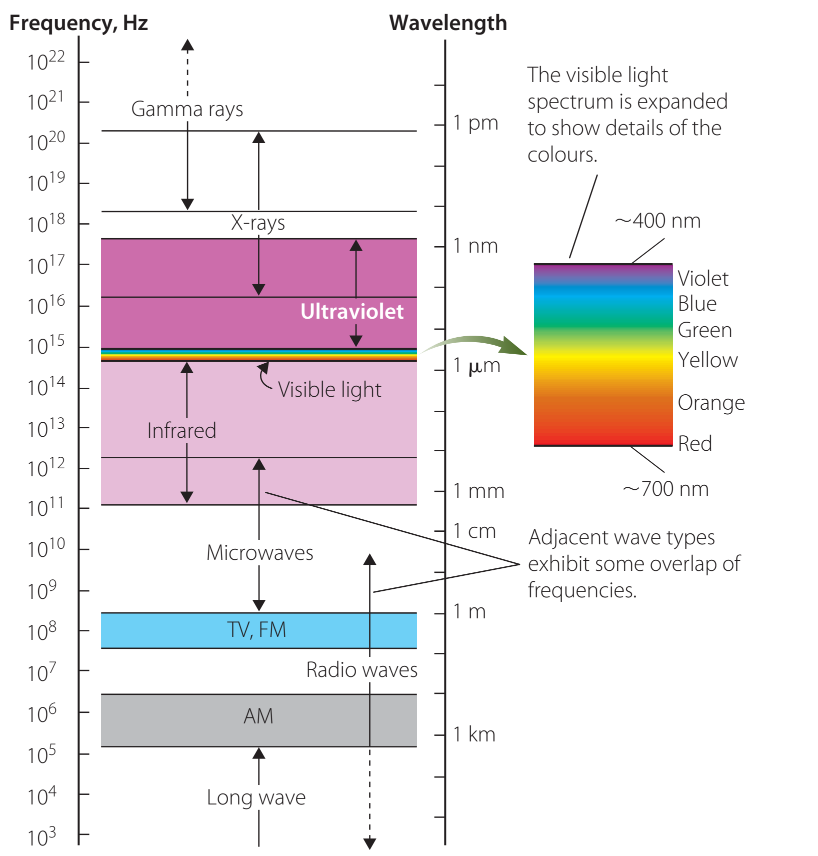

FIGURE 10.1.3 The electromagnetic spectrum showing all EM waves that have been produced or detected ordered according to their wavelengths and frequencies.

SYLLABUS LINK

Chapter 1 of *Nelson QScience Physics Units 1 & 2* discusses that heat can be transferred as infrared radiation. This radiation is made of electromagnetic waves, often in the infrared part of the spectrum.

10.1.4 The electromagnetic spectrum

Wave behaviour of light

Maxwell provided definitive proof that light and all other forms of electromagnetic radiation travels as a wave. This agreed with the fact that all EM waves are known to exhibit wave behaviour including reflection, refraction, diffraction and the principle of superposition, as discussed in Chapter 18 of *Nelson QScience Physics Units 1 & 2.*

Making and detecting electromagnetic waves

According to Maxwell's equations, EM waves consist of oscillating electric and magnetic fields that self-propagate; but for this to occur they require an initial change in one of these fields. What is the source of these initial oscillations?

All charged particles and magnetised materials produce fields in the space around them. These, however, are insufficient to produce EM waves, as the fields are static and do not change with time. The same is true of steady electric currents.

If, however, the current in a wire changes with time, as is the case in alternating current (AC), the wire emits electromagnetic radiation. This can be generalised to the important statement that *whenever a charged particle accelerates, it radiates energy in the form of electromagnetic waves.*

An important application of the fact that accelerating charges produce EM radiation is that of the antenna, which can be used to transmit or receive EM waves.

A transmitting antenna works by being connected to an AC. This current causes charges to oscillate back and forth along the length of the antenna, which creates a magnetic field. The magnetic field is perpendicular to the axis of the antenna and forms loops around the antenna.

The current oscillates with frequency f. The magnetic field also oscillates with this frequency. This time-varying magnetic field creates a time-varying electric field. This electric field also oscillates with frequency f, at right angles to the magnetic field.

These two oscillating fields are the EM wave. The EM wave travels in a direction perpendicular to both the fields that make up the wave, which are themselves mutually perpendicular. This is shown in Figures 10.1.2 and 10.1.4.

The frequency of the wave is f, the same as the frequency of the current, and it travels at speed c in vacuum or very close to this in air. In this way, an EM wave of any frequency can be produced by changing the frequency of the oscillations in the current.

When this travelling electromagnetic wave is incident on free charges, such as electrons in a receiving antenna, it will make them oscillate. The electric field applies a force to the electrons, and an AC with the same frequency, f, is produced. This current can be picked up and converted into another form. For example, a transducer attached to the antenna can convert the current into sound waves. Hence, an antenna can either transmit electromagnetic waves if it is connected to an AC, or receive them and produce an AC. This is how radio, TV and mobile phone antennae work. The length of the antenna depends on the frequency of the waves it must transmit or receive and the current it must carry. Longer antennae are used for lower frequency electromagnetic waves.

SYLLABUS LINK

Chapters 7 and 8 discuss charged particles and magnetised materials.

SYLLABUS LINK

Chapter 12 discusses how the movement of electrons within an atom produces the majority of EM waves that exist in the universe.

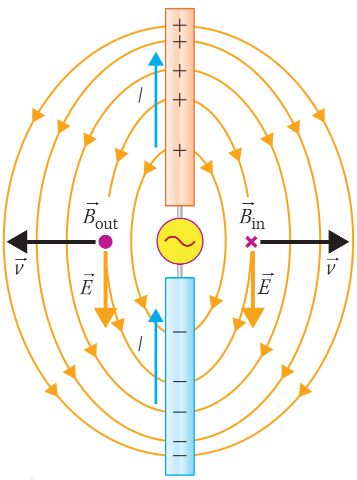

FIGURE 10.1.4 An antenna can transmit EM waves as a result of an AC travelling through it.

Maxwell's legacy

Maxwell's equations were startling, largely due to their ability to describe known phenomena, but also due to their predictive power. However, in many ways the largest impact of Maxwell's work was the scientific process that he followed. He combined existing theories into a new model that made predictions that could be investigated. This was in stark contrast to the usual method of making observations and then attempting to formulate a theory that agreed with these observations. Many theoretical physicists have since followed the same process, including, most notably, Albert Einstein whose famous 'thought experiments' made incredible predictions that are still being shown to be experimentally correct today over a hundred years later.

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INQUIRING FURTHER

The $1.9 billion Square Kilometre Array (SKA) is a gigantic array of radio telescopes, receiving dishes and antennae funded by Australia, Canada, China, Germany, Italy, New Zealand, South Africa, Sweden, the Netherlands and the United Kingdom. It is to be built mostly in Australia and South Africa. It is a massive international project, far beyond the scope of any individual nation.

The SKA will be at least 50 times more sensitive than any existing radio telescope and be able to survey the sky 10 000 times faster. It will collect massive amounts of data, more than the current global internet load.

Research the SKA and answer the following questions.

1 What is astrobiology?

2 Describe briefly what a radio telescope does.

3 Why do you think that large telescopes such as the SKA are typically built in remote areas?

4 Why is it necessary for the SKA project to be run by such a large international collaboration? Consider the positive and negative impacts that such a collaboration might have.

10.1.5 SKA

SECTION REVIEW 10.1

REMEMBERING

1 Name the scientist who unified all the information about electromagnetic phenomena into a single theory, described by four equations.

2 State the range of wavelengths for visible light.

UNDERSTANDING

3 If Figure 10.1.4 represents a transmitting antenna used by a radio station, should you have your car radio antenna orientated horizontally or vertically to listen to this station?

4 When light (or other electromagnetic radiation) travels across a given region, what:

a oscillates?

b is transported?

5 What does a radio wave do to the charges in the receiving antenna to provide a signal for your car radio?

APPLYING

6 The red light emitted by a helium–neon laser has a wavelength of 633 nm.

a What is the frequency of the light waves?

b How long would it take for a signal from this laser to travel from Earth to the Moon and back a distance of 768 000 km?

ANALYSING

7 What experiments can be done to show that light is a wave? Give at least three examples. Hint: Think about what you learnt about wave behaviour in Unit 2 of *Nelson QScience Physics Units 1 & 2*.

8 The human eye is most sensitive to light with a frequency of 5.45×10^{14} Hz, which is in the green-yellow region of the visible electromagnetic spectrum.

a What is the wavelength of this light?

b Why do you think the eye is most sensitive to this wavelength?

9 Mark on an electromagnetic spectrum the wavelengths you have used today.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Antenna
   b Differential equations
   c Electrical permittivity
   d Electromagnetic waves
   e Magnetic permeability
   f Wave equations

2 Recall the speed at which electromagnetic waves propagate in a vacuum.

CATEGORY QUESTIONS

3 Explain the key finding of the last two of Maxwell's equations.

4 Describe the features of an electromagnetic wave.

5 Give two examples, other than visible light, of an electromagnetic wave.

6 List the major influence on the speed at which an electromagnetic wave travels through an object.

7 Explain how an electromagnetic wave is:
   a transmitted from an antenna
   b received from an antenna.

ELABORATION QUESTIONS

8 Describe how Maxwell's equations were able to put to rest the hypothesised 'luminous aether' theory.

9 List the properties of light that were successfully explained by Maxwell's electromagnetic wave theory.

10 Explain why the orientation of a receiving antenna is important for it to be able to pick up incoming electromagnetic waves.

11 Explain why AC must be used in a transmitting antenna, and how information can be encoded by this current.

EVIDENCE QUESTIONS

12 Re-read the section on Maxwell's legacy and research other instances where physicists have developed mathematical theories to describe natural phenomena and later found the evidence to support it.

13 If it is known that waves interact most strongly with objects whose size is comparable to the wavelength of the wave, explain why AM waves (which can carry more information but have a smaller wavelength) are more common in remote locations while FM waves (which carry less information but have a longer wavelength) are commonly used in urban locations.

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END-OF-CHAPTER EXAM

End-of-chapter test

PHYSICS

1 The orientation of oscillations in the electric and magnetic fields in an electromagnetic wave are:

A perpendicular.
B congruent.
C parallel.
D antiparallel.

2 An electromagnetic wave is a:

A transverse wave.
B surface wave.
C longitudinal wave.
D sound wave.

3 Which of the following waves has a wavelength that lies in the range of 1 cm to 1 m?

A Gamma rays
B X-rays
C Microwaves
D Visible light

4 Which of the following waves has the highest frequency?

A Radio waves
B Microwaves
C Infrared waves
D Visible light

5 What is the term given to the speed at which electromagnetic waves propagate through a medium?

6 Does light travel faster or slower in a vacuum than in other substances?

7 What is necessary for the production of an electromagnetic wave?

8 What did the final two equations in Maxwell's equations suggest?

9 Show that if the electrical permittivity of a vacuum is $8.85 \times 10^{-12}\,C^2\,N^{-1}\,m^{-2}$ and its magnetic permeability is $1.257 \times 10^{-6}\,N\,s^2\,C^{-2}$, then the speed of light in a vacuum is $3.0 \times 10^8\,m\,s^{-1}$.

10 If light is travelling at a velocity of $2.54 \times 10^8\,m\,s^{-1}$ through a medium that has an electrical permittivity of $7.86 \times 10^{-12}\,C^2\,N^{-1}\,m^{-2}$, calculate the magnetic permeability of the substance.

11 A mobile phone transmits a signal with a frequency of 1800 MHz.

a Name the part of the electromagnetic spectrum this wave belongs to.

b Determine the wavelength of this signal.

c Calculate how long it takes this wave to travel from Perth to Sydney, a distance of about 3300 km.

d Explain why it actually takes longer than this for a signal to travel from a mobile phone in Perth to a mobile phone in Sydney.

12 Explain the entire process involving the transmission of a radio wave at a radio station to it being picked up by the radio in a car.

» UNIT FOUR

REVOLUTIONS IN MODERN PHYSICS

- Topic 1: Special relativity
- Topic 2: Quantum theory
- Topic 3: The Standard Model

Unit 4: Revolutions in modern physics provides a basis for student examination of relative motion, light and matter, including the limitations of classical physics theories that led to the development of the special theory of relativity and the quantum theory of light and matter. The development of the quantum theory of the atom and the derivation of the Standard Model of particle physics are examined, while technologies such as GPS navigation, lasers, modern electric lighting, medical imaging, quantum computers and particle accelerators are investigated. Student inquiry and analytic skills are developed through experimentation and investigation of a range of phenomena, such as atomic emission and absorption spectra and the photoelectric effect.

UNIT OBJECTIVES

By the end of this unit, students should:

1. describe and explain special relativity, quantum theory and the Standard Model
2. apply understanding of special relativity, quantum theory and the Standard Model
3. analyse evidence about special relativity, quantum theory and the Standard Model
4. interpret evidence about special relativity, quantum theory and the Standard Model
5. investigate phenomena associated with special relativity, quantum theory and the Standard Model
6. evaluate processes, claims and conclusions about special relativity, quantum theory and the Standard Model
7. communicate understandings, findings, arguments and conclusions about special relativity, quantum theory and the Standard Model.

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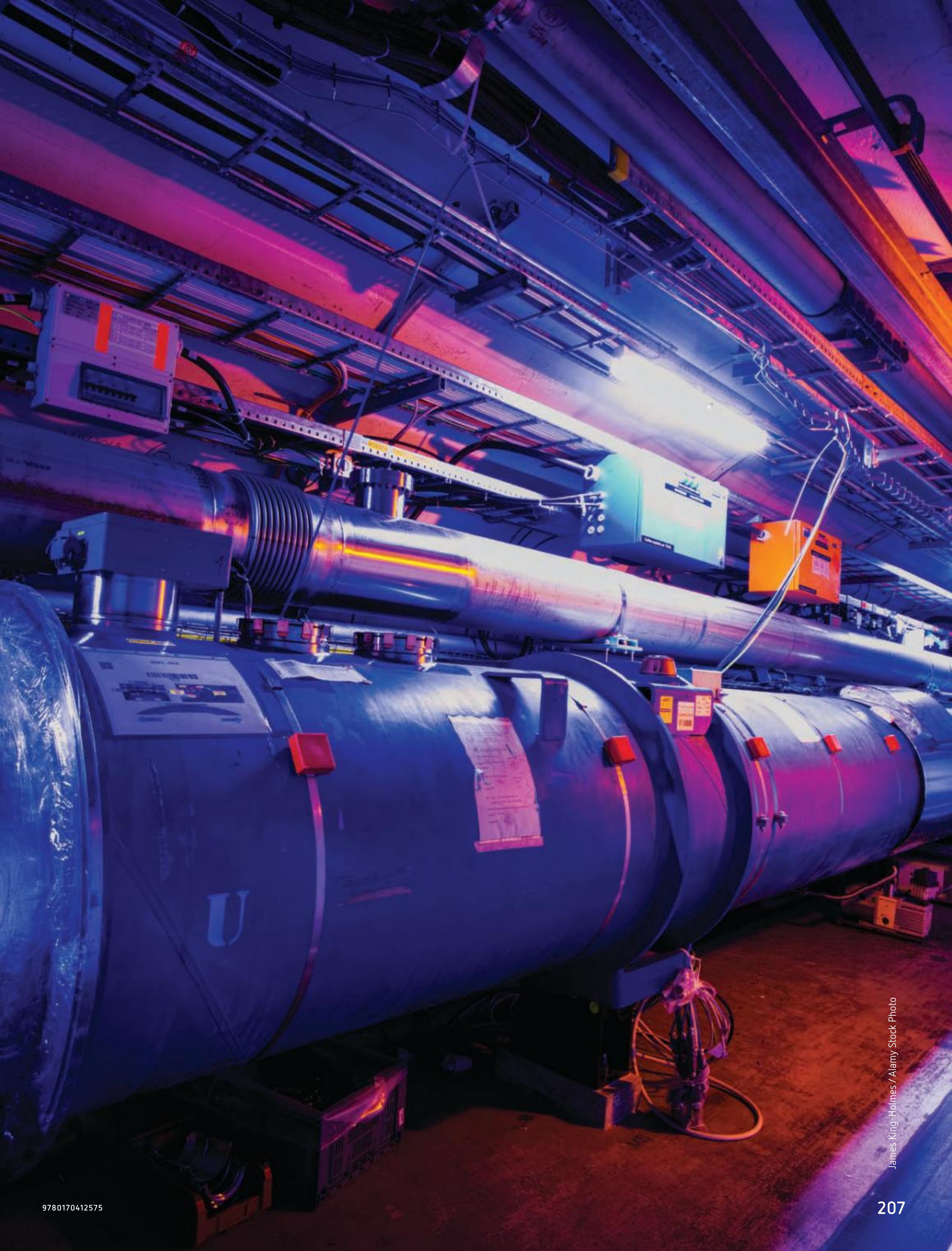

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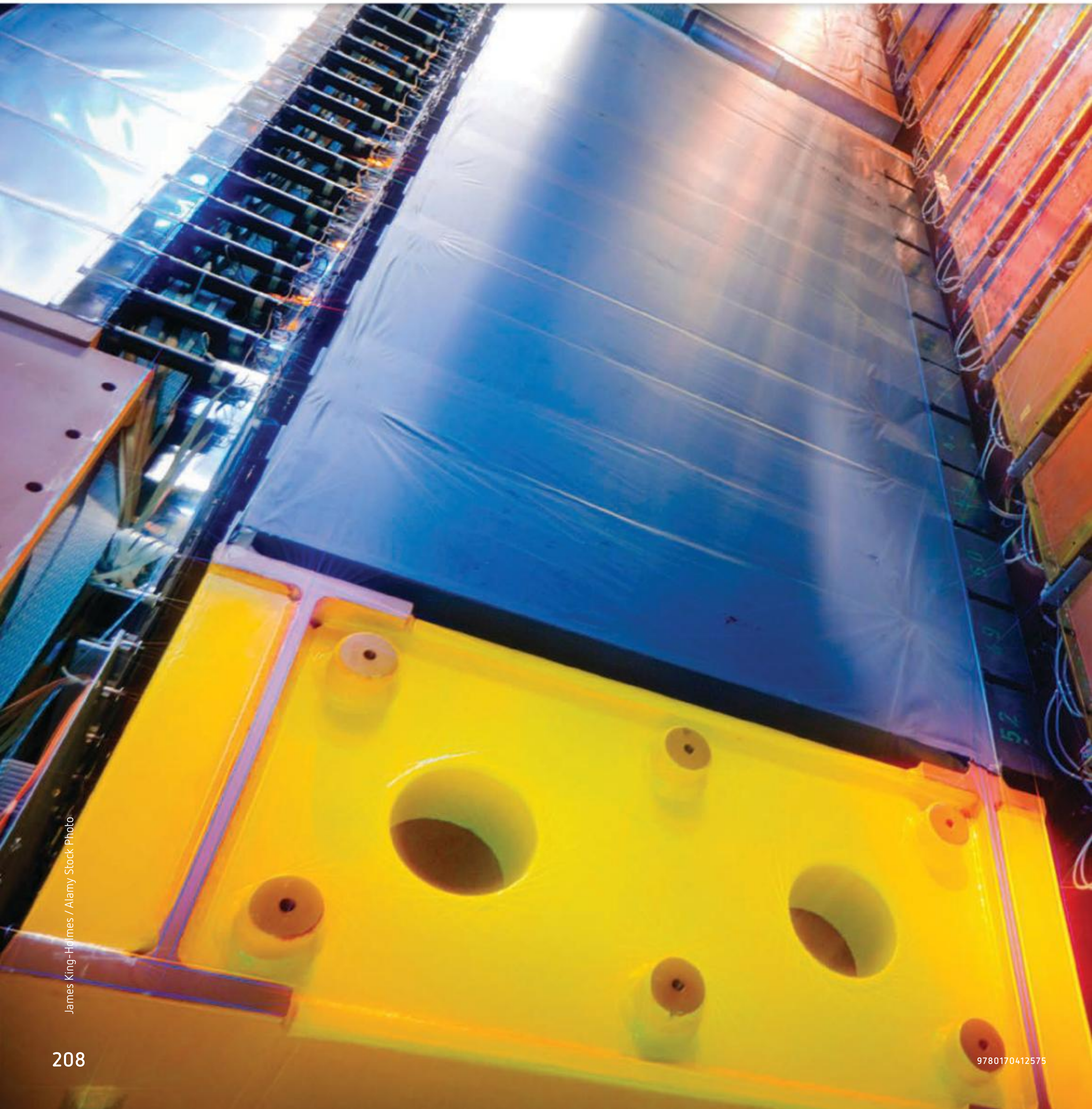
» UNIT FOUR

REVOLUTIONS IN MODERN PHYSICS



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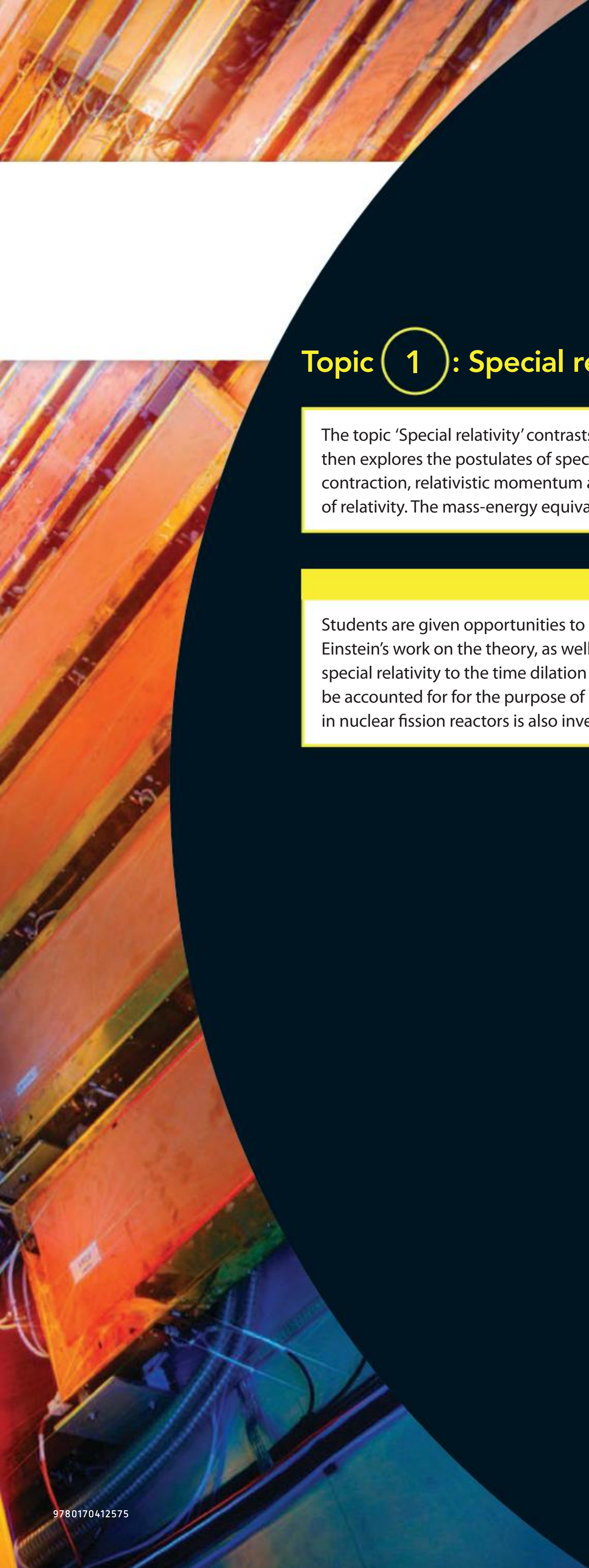

Topic 1: Special relativity

The topic 'Special relativity' contrasts Newtonian physics with inertial frames of reference and then explores the postulates of special relativity. Concepts of simultaneity, time dilation, length contraction, relativistic momentum and paradoxical scenarios are used to explain the phenomena of relativity. The mass-energy equivalence relationship is also explored.

SCIENCE AS A HUMAN ENDEAVOUR

Students are given opportunities to investigate the development of special relativity, including Einstein's work on the theory, as well as the studies of Maxwell and Lorentz. The application of special relativity to the time dilation of orbiting satellites, as well as relativistic effects that need to be accounted for for the purpose of navigation, are explored. The equivalence of mass and energy in nuclear fission reactors is also investigated.

KEY FORMULAS

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$\Delta E = \Delta mc^2$$

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11 SPECIAL RELATIVITY

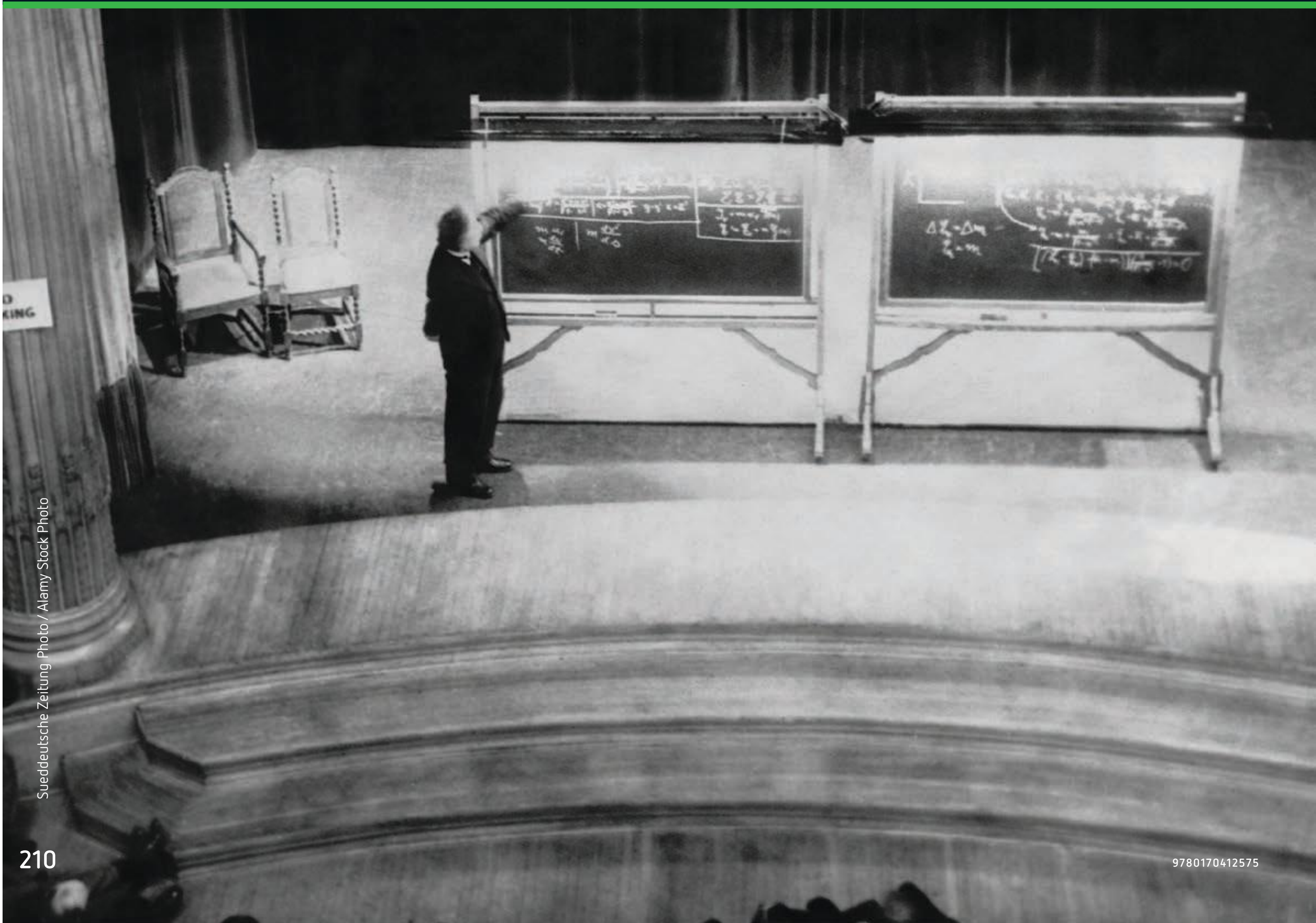
Introduction

Towards the close of the 19th century, most scientists were quite satisfied with the development of mathematical models to explain the results of experiments and observations of the natural world. Newton's earlier work on motion had been thoroughly tested and found to be valid in all circumstances. However, within the space of 30 years there would be a total transformation of these ideas. Small, but significant, discrepancies were beginning to exist between predictions based on Newtonian physics and experimental results. Refinements to the theory of electromagnetism by James Maxwell were a trigger for this change. Albert Einstein then used some powerful ideas and mathematical modelling to convince others that we needed a new way to look at the nature of time, space, matter and energy. Enter special relativity.

Stimulus questions

Why does Newtonian physics fail when modelling scenarios in which objects are moving at close to the speed of light?

Can objects with mass travel at the speed of light?



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11.1 Newtonian physics

Newton's three laws of motion are the foundations of **classical mechanics**. The way macroscopic objects act when coming into contact with each other are predictable based on the known properties of inertia, momentum, force and action–reaction pairs. These descriptions of motion, although applicable to all macroscopic bodies, are unable to describe what happens at the microscopic level. After many experiments, it was determined that there must be a new model of physics to describe all the unknown phenomena. This is known as **quantum physics**, and this new model accounts for all the discrepancies that classical mechanics cannot describe.

11.1.1 Particles and waves: The central mystery of quantum mechanics

11.1.2 Six things everyone should know about quantum physics

Muons in the atmosphere

At this stage, you are probably sceptical about an example that is not explained by classical mechanics. Let us consider **muons**. Muons are particles similar to electrons, but are 200 times greater in mass.

The upper atmosphere is constantly being bombarded by energy from cosmic rays. When these cosmic rays collide with molecules, muons are created from this interaction. These muons are accelerated towards Earth at very high speeds, close to the speed of light. Muons have a half-life of only 2.2 µs, and hence typically decay before reaching the surface of Earth, even at these extremely high speeds.

An experiment carried out on this phenomenon in the early 1960s by David Frisch (1918–91) and James Smith collected data that was unable to be explained by classical mechanics. Frisch and Smith's data showed that of the muons detected in the upper atmosphere, a large majority of them were detected close to Earth's surface, approximately 6.5 km further down. Considering the muon was travelling at almost the speed of light, and only exists for 2.2 µs, the muons should only be able to travel less than 1 km before decaying. This observation was evidence against classical mechanics, and hence falls into the realm of quantum physics. Specifically, this experimental result suggested that the muon has undergone **relativistic effects** – muons do not obey the time and space constraints of Newtonian physics.

classical mechanics
the study of motion in accordance with Newton's laws; it is also known as Newtonian physics

SYLLABUS LINK
Chapter 14 of *Nelson QScience Physics Units 1 & 2* discusses Newton's laws of motion.

quantum physics
the science of very small particles for which classical mechanics fails to explain the interactions that are observed

muon
a particle formed by cosmic rays in the upper atmosphere

relativistic effect
when time and space act differently for one object compared to others

SECTION REVIEW 11.1

REMEMBERING

1 Describe a relativistic effect.

2 Name the scientists who noticed that muons do not obey the rules of Newtonian space and time.

UNDERSTANDING

3 What is the purpose of quantum physics?

4 Explain how muons in the atmosphere are evidence that classical mechanics is not able to describe all motion in the universe.

11.2 Frame of reference and inertial frame of reference

Galileo was one of the earliest thinkers to comment on events observed in different **frames of reference**. In his book, *Dialogues on the Two Chief Systems of the World*, Galileo described a thought experiment in which a sailor drops an object from the mast of a sailing ship moving at a steady velocity. He asked the question: 'Where would the object land relative to the deck of the ship?' In his frame of reference, the sailor would see the object fall straight down parallel to the mast (Figure 11.2.1a); however, a nearby

SYLLABUS LINK
Chapter 18 of *Nelson QScience Physics Units 1 & 2* discusses the speed of light and properties of light.

frame of reference
a framework in which motion of an object is described according to a coordinate system. Frames of reference are observational and can be inertial or non-inertial (accelerating)

observer on land (a different frame of reference) would see the object would follow a parabolic path (Figure 11.2.1b).

Later, Newton would agree with Galileo. He described frames of reference that were stationary or moving at constant velocity as **inertial frames of reference**. Inertial frames of reference do not accelerate. An inertial frame of reference is an ideal situation in which objects at rest remain at rest and objects travelling at a constant velocity remain travelling at a constant velocity. Examples include a spaceship, a table and a cruising aeroplane. Non-inertial frames of reference include merry-go-rounds and aeroplanes taking off or landing (changing velocity).

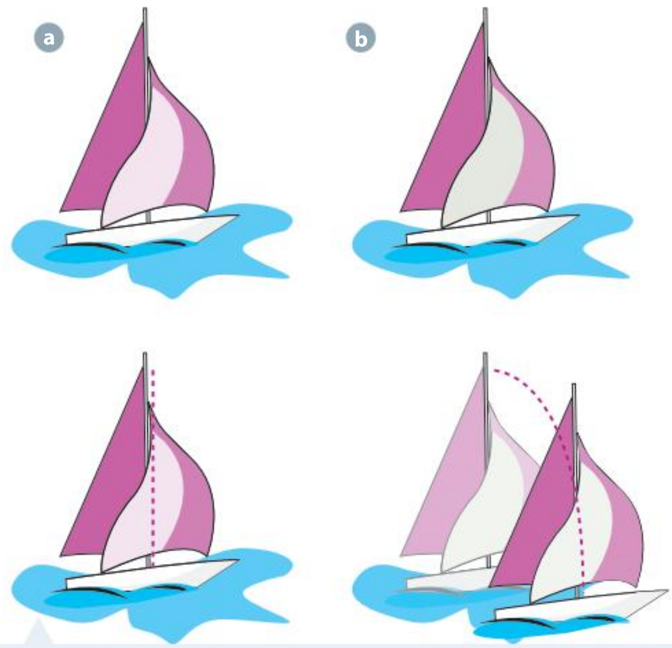

FIGURE 11.2.1 Path of a falling object in the reference frame of (**a**) a sailor on the ship and (**b**) an observer on land.

Galilean transformations

Classical physics, the physics of Galileo and Newton, relies on the 'sensible' idea that inertial coordinate systems are equivalent. That is, there is a set of translation rules to connect measurements in one frame of reference (or coordinate system) to any other reference frame.

Consider Figure 11.2.2. This represents two frames of reference in two dimensions. The *privileged* frame of reference, P, has coordinates (x, y). P is stationary with respect to a moving reference frame, P′, with coordinates (x', y'). The two coordinates can be made to coincide at the same time, but then P′ moves further and further away from P. This motion depends on the relative speed, v, of P′ with respect to P and the time elapsed, Δt, after the two frames coincided.

The time interval in each frame of reference is the same. That is, for classical physics, time intervals are measured in the same way and occur at the same rate. Clocks in all frames of reference are identical in their time keeping. Time is **invariant**. Thus, Δt in P and $\Delta t'$ in P′ are equal – the time interval is denoted as Δt.

inertial frame of reference
one in which Newton's first law applies to a very good approximation, and there is no acceleration. Any departures from the law are negligible; also known as an inertial reference frame

invariant
does not vary; it is the same in all reference frames

Galilean transformation
for a two-dimensional inertial frame of reference, P′, moving at constant speed in the x-direction with respect to another inertial frame, P:
$x = x' + v\Delta t$
$y = y'$

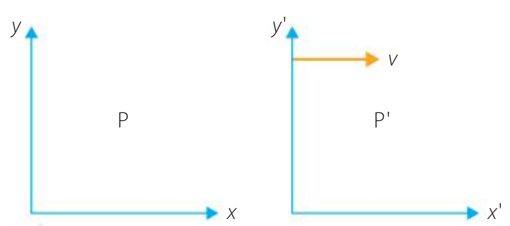

FIGURE 11.2.2 Two inertial frames that are moving relative to each other. P is regarded as the stationary frame.

Suppose an observer located at (0, 0) in frame P sees a rocket some distance y away. It is travelling parallel to the x axis at speed v (Figure 11.2.2). Initially (at $t = 0$ s), the rocket's coordinates in P′ coincide with those in P. But as the rocket travels in P′, its x' coordinates increase in the same time interval Δt, by $v\Delta t$. This is just the distance between the two coordinate systems. Thus, the x coordinates in P relate to the x' coordinates in P′ by the **Galilean transformation**:

$$x = x' + v\Delta t$$

Similar arguments can be used to show that if the rocket is travelling at speed v parallel to the y and y' axes:

$$y = y'$$

The transformation equations rely on the relative speeds of the two frames of reference. If P′ had been chosen as the stationary frame, then P would be travelling at negative speed with respect to P′. This needs to be taken into account in the Galilean transformation equations. Finally, if the rocket was travelling with vector velocity $\vec{v}$, the transformation equations in the x and y directions require the use of the x and y components of the velocity respectively.

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WORKED EXAMPLE 11.2.1

A car travels at 12 m s^{-1} through an intersection. After 10 s, what is the position of the car with respect to the coordinates based on the:

1 intersection?

2 car?

ANSWER

1 Consider the intersection as (0, 0). The intersection is the privileged frame for this question.
$x = 0\,\text{m} + 12 \times 10$
$x = 120\,\text{m}$
The car has travelled 120 m with respect to the intersection.

2 The car has not moved with respect to itself.
$x' = 0\,\text{m}$

SECTION REVIEW 11.2

REMEMBERING

1 Define 'inertial frame of reference'.

2 Write the Galilean transformations.

UNDERSTANDING

3 The path followed by an object dropped from the mast of a moving ship can appear to be both a straight line and a parabola. Explain why.

4 Compare the terms frame of reference and inertial frame of reference.

APPLYING

5 A bus is travelling at 12 m s^{-1} past a stationary pedestrian. After 5.0 s, what is the position of the bus relative to the observer? What is the position of the observer relative to the bus after 6.0 s?

6 A train passes through a station at 30 m s^{-1}. A person on the train walks towards the front of the train at 2 m s^{-1}. After 5.0 s, the person has moved a distance from the common origin on the train and station. What are the coordinates of the person with respect to:

a the station?

b the train?

Remember to choose the stationary frame according to the question.

11.3 The two postulates of special relativity

As well as considering a sailor dropping an object from the mast of a sailing ship, Galileo discussed the situation of a person walking within the cabin of a ship. If the sailing ship moves forwards at a velocity of 5 m s^{-1} relative to Earth, a person moving forwards at a velocity of 1 m s^{-1} relative to the cabin will be moving forwards at a velocity of 6 m s^{-1} relative to Earth. The position and velocity of the person is different in each reference frame.

According to Galileo and Newton, acceleration of a body will be the same in each frame of reference, providing they are inertial frames. In the sailing ship's cabin, for example, the person may have accelerated from 0 to 1 m s^{-1} in 0.5 s in the cabin, which is an acceleration of 2 m s^{-2}. From the perspective of a nearby land-based observer, the person will have accelerated from $5\,m\,s^{-1}$ to $6\,m\,s^{-1}$ in the same time frame, which is again an acceleration of $2\,m\,s^{-2}$.

Principles of classical relativity

relativity principle
the laws of physics are the same in all inertial frames of reference

The **relativity principle** states that the laws of physics are the same in all inertial frames of reference. It can be shown that the laws of motion, including those for the conservation of energy and momentum, are the same in all inertial reference frames. Observers in different inertial frames would record different velocities, and therefore determine different values of energy and momentum. Nevertheless, they would agree that there had been no net change of either energy or momentum. Consequently, they would agree on conservation of energy and conservation of momentum.

This suggests that there is no privileged inertial frame. No inertial reference frame is better than any other. If you are on a train travelling at a steady velocity of $80\,km\,h^{-1}$ west across the Nullarbor, it is quite valid for you to argue that, from your reference frame, you are stationary and Earth is moving at $80\,km\,h^{-1}$ east. Providing your ride is smooth and at steady velocity, there is no experiment you can perform to test whether you are moving or stationary.

Galileo wrote about this in his book, in which he discussed the example of a person observing a range of movements in the cabin of a sailing ship that was sailing steadily at constant velocity. He argued that the person would not be able to tell if the ship was moving or not. He believed that there is no absolute frame of reference against which the velocities of all other frames can be measured. In this he differed from Newton, who believed that the Earth–Sun system could be considered the absolute frame of reference.

KEY CONCEPT

In all inertial reference frames:

1 the laws of physics are the same

2 the speed of light is constant.

However the following, as stated earlier, can be agreed on.

- The laws of motion are the same in all inertial frames of reference.
- The laws of conservation of energy and conservation of momentum apply in all inertial frames of reference.
- All inertial frames are equivalent. All are equally valid.

Special relativity

special relativity
the physics theory regarding the relationship between space and time, which is not explained by Newtonian or Galilean relativity

Albert Einstein (1879–1955) reflected long and hard about what an electromagnetic wave (travelling at the speed of light) would look like if you travelled along with it. He concluded that you would see no change in time; the wave would be stuck in time, but it would continue to oscillate in space. How could something change, yet no time pass? In order to resolve this problem, Einstein decided to investigate the nature of space and time. The result was the theory of **special relativity**. This was a bold step but a well-trodden path to discovery: questioning taken-for-granted assumptions leads to fresh, more powerful ways of understanding the world.

11.3.1 Einstein's theory of special relativity

11.3.2 What's so special about special relativity?

Einstein's theory was based on the following two clear propositions.

1 *First postulate of special relativity*: the laws of physics are the same in all inertial frames of reference – the principle of special relativity.

2 *Second postulate of special relativity*: the speed of light has the same value, c, in all inertial frames. It does not depend on the speed of either the source or the observer.

These postulates contradict a few of the ideas of Newtonian and Galilean relativity presented earlier. The first postulate of special relativity excludes the possibility of there being a privileged reference frame. The second postulate contradicts the idea that, according to different reference frames, different objects would have different velocities (such as the sailor dropping an object on the boat). To elaborate on the second postulate of relativity, light will travel at $3.0 \times 10^8\,m\,s^{-1}$ in a vacuum, regardless of the speed of the observer.

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THE DEVELOPMENT OF SPECIAL RELATIVITY

SCIENCE AS A HUMAN ENDEAVOUR

Albert Einstein's work on the theory of special relativity was built on the studies of Maxwell and Lorentz. Research the development of the special theory of relativity, including the type of work that inspired Einstein, and how subsequent studies have led to further discoveries in physics.

SECTION REVIEW 11.3

REMEMBERING

1 State the two postulates of special relativity.

UNDERSTANDING

2 How did Einstein's postulates differ from the agreed-upon rules of relativity deduced by Galileo and Newton?

3 Compare how the speed of light in a moving inertial reference frame changes with that in a stationary inertial reference frame.

11.4 Measuring motion

In order to measure how motion is considered in different reference frames, you need to consider the velocities of objects relative to each other. It is straightforward to consider the velocity of an object when there is a stationary observer. For example, if a car travelling at $20\,m\,s^{-1}$ east passes a person sitting on a bench, they will observe the car travelling at $20\,m\,s^{-1}$ east. If the person then begins to walk at $2\,m\,s^{-1}$ east, and another car passes them at $20\,m\,s^{-1}$ east, then the **relative motion** of this car will be different. The relative motion of the moving car and moving person depend on whose reference frame is being considered at the time.

relative motion
the motion of a moving object according to a moving observer. When relative motion is being evaluated, one reference frame must always be considered stationary

Relative velocities

It is possible to calculate the velocity of an object in one inertial frame of reference relative to another inertial frame of reference, providing the object is travelling along the same direction. If a person is moving at $2\,m\,s^{-1}$ east, and a car passes them at $20\,m\,s^{-1}$ east, the person would observe the car as travelling at $18\,m\,s^{-1}$ east. In order to consider this in two or three dimensions, this becomes much more complex. To do this, we need to consider vectors and how they compare to the relative motion of an object in different inertial frames of reference.

Take the example of an aeroplane flying from Sydney to Perth. At the cruising altitude of a passenger aircraft, about 10 km, there is nearly always a strong westerly wind that varies from $50\,km\,h^{-1}$ to more than $300\,km\,h^{-1}$. The direction of the aircraft's movement is almost exactly parallel to the direction of the wind. The westbound flight is scheduled at 4 hours 25 minutes (Figure 11.4.1a), while the eastbound flight is 3 hours 50 minutes (Figure 11.4.1b). The aeroplane's velocity, relative to the ground, is higher on the way to Sydney than on the way to Perth.

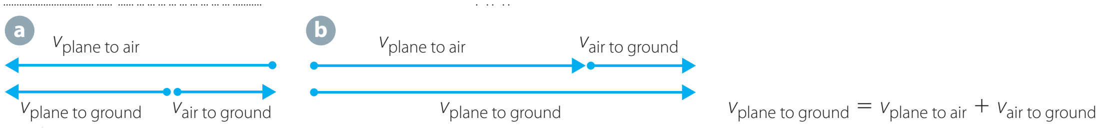

FIGURE 11.4.1 Vector addition showing the velocity of the plane relative to the ground is (**a**) smaller when the plane is travelling against the wind, known as a *head wind* and (**b**) greater when the plane is travelling in the same direction as the wind, known as a *tail wind*.

The velocity of the plane relative to the ground is the velocity of the plane relative to the air *plus* the velocity of the air relative to the ground. More generally, the velocity of A relative to B is the velocity of A relative to C *plus* the velocity of C relative to B. This holds true for vectors in two dimensions.

$$v_{AB} = v_{AC} + v_{CB}$$

KEY FORMULA

$v_{AB} = v_{AC} + v_{CB}$

Where:

v_{AB} = velocity and direction of object A relative to B ($m\,s^{-1}$)

v_{AC} = velocity and direction of object A relative to C ($m\,s^{-1}$)

v_{CB} = velocity and direction of object C relative to B ($m\,s^{-1}$)

WORKED EXAMPLE 11.4.1

An aeroplane is headed due north at a speed of $400\,km\,h^{-1}$. There is a westerly wind (i.e. coming from the west) of $80\,km\,h^{-1}$.

1 Draw a vector diagram to show the velocity of the plane relative to the ground.

2 What is the speed of the plane relative to the ground?

3 What is the resultant velocity of the plane relative to the ground?

ANSWER

1 The vector diagram, Figure 11.4.2, shows a plane travelling north, through a westerly wind. This results as a north-easterly direction of the plane.

2 Using Figure 11.4.2 from part **1** and Pythagoras's theorem:

$v^2 = 400^2 + 80^2$

$v = 408\,km\,h^{-1}$

3 We now know the speed, so we must find the direction. Finding θ can be done with tangent:

$\tan\theta = \frac{80}{400}$

$\theta = \tan^{-1}(0.2)$

$\theta = 11.3°$

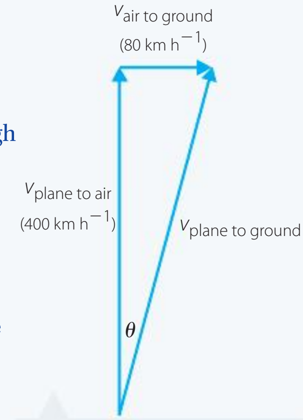

FIGURE 11.4.2

SECTION REVIEW 11.4

REMEMBERING

1 State the formula for calculating relative velocity.

UNDERSTANDING

2 Explain whether the velocity of a moving train is the same in all reference frames.

APPLYING

3 A taxi is travelling at $10\,m\,s^{-1}$ when a person nearby starts to walk towards it at $2\,m\,s^{-1}$. A stationary person on the other side of the street also observes the situation. What is the speed of the person nearby relative to the:

- **a** taxi?
- **b** person on the other side of the road?

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4 The tide is running south at $3.0\,m\,s^{-1}$. At the same time, a yacht is steering at $4.0\,m\,s^{-1}$ directly towards a buoy in the east. What is the velocity of the yacht relative to the shore? Show your answer with a vector diagram.

5 A person rows at $1.0\,m\,s^{-1}$ through the water of a river that is flowing at $0.5\,m\,s^{-1}$. The rower keeps the boat moving perpendicular to the bank.

  a Draw a vector diagram to show the velocity of the rower from the reference frame of a person on the bank.

  b Using your diagram, specify completely the velocity of the boat relative to the bank.

SYNTHESISING

6 Consider relative motion of cars on a highway. A police car is travelling south and wants to know if any of the north-bound traffic is speeding. Explain how they would do this. Compare this method with a police officer standing on the side of the road with a speed camera.

11.5 The concept of simultaneity

When two events happen simultaneously, they are said to happen at the same time. However, what may be considered a simultaneous event in one inertial reference frame may not be considered simultaneous in another reference frame. Measurement of time in different reference frames produces interesting results, not the least being that for simultaneous events. The idea that simultaneous events happen at different times in different reference frames argues against the existence of a universal time frame.

Let us consider a light positioned in the centre of a train carriage (Figure 11.5.1). At each end of the carriage there is a sensor or camera that can detect when the light is on or off. From the perspective of someone inside the carriage, when the light turns on, both sensors will recognise this at the same time, that is, *simultaneously*. Even if the train carriage is moving, an observer inside the carriage will notice that both sensors detect the light turning on at the same time.

Now consider a situation in which the train carriage *is* moving at a very high constant velocity, v, relative to the ground. An observer on the ground (reference frame T) watches the train go past and is able to see the light globe and each sensor (Figure 11.5.2).

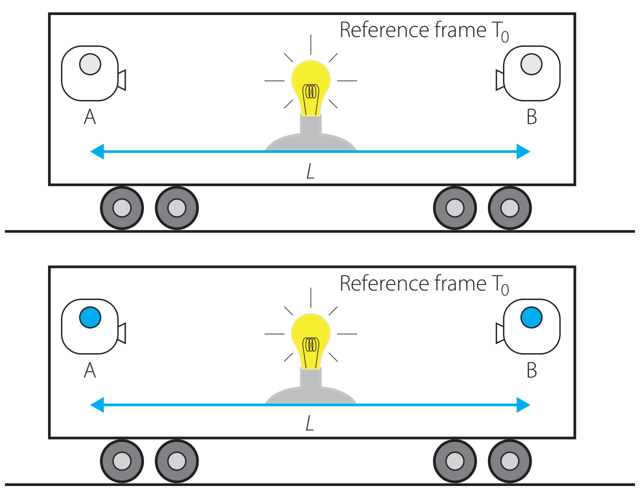

FIGURE 11.5.1 Reference frame T_0, inside a moving train.

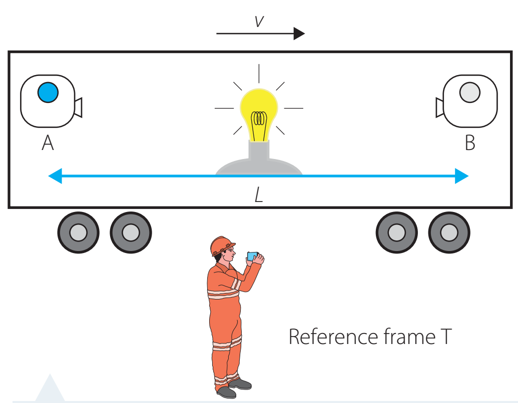

FIGURE 11.5.2 Reference frame T: view from the ground

As the train carriage moves past, the observer in reference frame T notices that the globe, and sensor A are both on, but sensor B is off. In this reference frame, the observer sees sensor A moving forwards to meet the light waves moving towards the globe, so sensor A is triggered first. Sensor B is moving away from the light waves and hence takes longer to detect the light. The simultaneous events in reference frame T_0 are not simultaneous in reference frame T. It is important to note that the speed of light in both reference frames is the same.

simultaneity when two events occur simultaneously in one reference frame and *cannot* occur simultaneously in another reference frame, if the other reference frame is moving relative to the first reference frame

The events described above occur because **simultaneity** depends on the agreement of time measurements, and time is relative. If Einstein's second postulate was not true, then simultaneity could be agreed on across different reference frames. This seems to be against common sense or experience. Because the speed of light is so much greater than ordinary speeds, we do not see any evidence of timing problems in everyday life.

SECTION REVIEW 11.5

REMEMBERING

1 Define 'simultaneity'.

UNDERSTANDING

2 Explain why simultaneous events cannot occur at the same time in different frames of reference when one of the frames of reference is travelling at close to the speed of light.

3 Consider the scenario of the light on a passing train in Figure 11.5.2. If this experiment was to be conducted in normal conditions on Earth, would we notice sensor A turning on before sensor B? Justify your response.

APPLYING

4 Consider two trees, a gum and a pine separated by a distance *d*. A dog is sitting at the centre between these two trees as a storm starts brewing. Each tree is hit by a bolt of lightning simultaneously from the reference frame of the dog. At this moment, a cat runs past the gum tree towards the pine tree at a velocity close to the speed of light. From the cat's reference frame, which tree does the lightning strike first?

11.6 The consequences of constant speed in a vacuum

The theory of special relativity asks us to give up our Newtonian view of space and time, and accept some very strange and puzzling ideas. To illustrate this, we will use a technique that Einstein used himself: simple thought experiments that are based on the two postulates of special relativity. The example in the previous section of the light on a train carriage was an example of a thought experiment: one that we are unable to carry out, but which obeys the laws of special relativity.

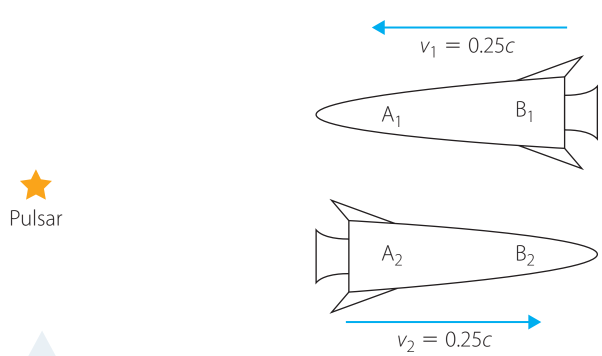

FIGURE 11.6.1 Passing rockets both view light from a pulsar.

Time

Assume two rockets, each travelling at 0.25*c* but in opposite directions, pass each other at the instant they receive a light pulse from a distant pulsar (Figure 11.6.1). Observers on each rocket attempt to measure the speed of this pulse of light by using two light-sensitive cells set on the outside of the rocket (A_1 and B_1, and A_2 and B_2 respectively).

Each rocket uses a timer and the distance between the sensors to measure the speed of light relative to its own reference frame. Both rockets measure the speed of light at $3.00 \times 10^8 \, m\,s^{-1}$ relative to their respective reference frames.

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Galilean relativity would have argued that the rocket heading towards the pulsar would have registered a light speed of 1.25c, and the rocket moving away from the pulsar would have measured a light speed of 0.75c. Of course, in practice this would be a difficult experiment to carry out. However, we can conceive of such a scenario and use logic and Einstein's postulates to relate what each observer would see or measure. In this case, the time taken for the light to travel between sensors A and B on each ship is the same, and each would measure the speed of light as c.

Let us now imagine another experiment using a train, Einstein's favourite thought experiment apparatus. The train is running on a smooth track at high velocity v relative to the ground. On one carriage a pair of mirrors m_1 and m_2 is set up so that a series of light pulses is able to bounce back and forth between them. Fred is standing on the carriage with the mirrors, and Pierre is standing on the ground nearby, watching the train pass (Figure 11.6.2). Both Fred and Pierre have identical, very accurate watches that are capable of measuring very small time increments. They both agree that the distance between the two mirrors is width, w.

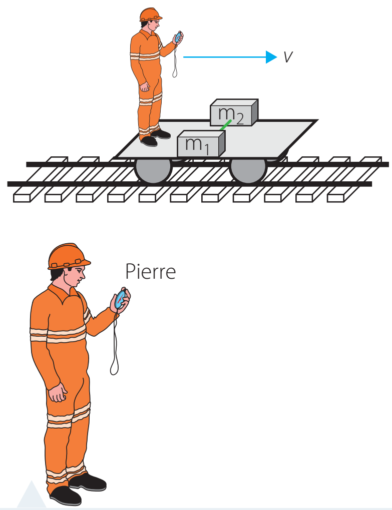

FIGURE 11.6.2 A passing high-speed rail cart

Fred and Pierre both measure the time it takes for the light to travel between the mirrors. Fred sees the situation as a simple path of light between the two mirrors (Figure 11.6.3a), and hence measures the time it takes for the light to travel from m_1 to m_2 and back again as $t_0 = \frac{2w}{c}$. As the train is moving very fast, Pierre sees the situation quite differently. From his viewpoint, the velocity of the mirrors results in the light pulse forming a triangle (Figure 11.6.3b). Because both mirrors are moving with velocity v to the right relative to Pierre, he measures the time for one pulse to move from m_1 to m_2 and back again as time t. One half of the journey forms a right-angled triangle with sides of length w, $\frac{vt}{2}$ and $\frac{ct}{2}$, as c is constant in all reference frames.

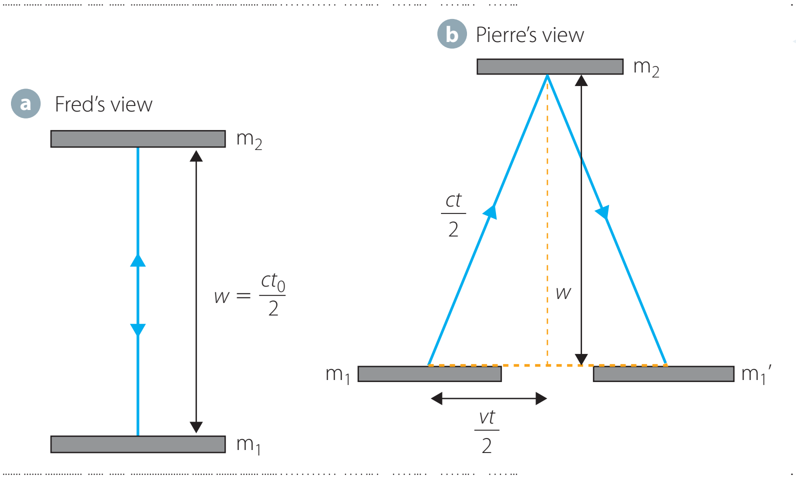

FIGURE 11.6.3 An event as viewed from (**a**) Fred's reference frame and (**b**) Pierre's reference frame.

Using Pythagoras' theorem, we can relate the time interval that Pierre records for this light pulse to happen to the time Fred records as follows:

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{vt}{2}\right)^2 + w^2$$

$$\left(\frac{ct}{2}\right)^2 - \left(\frac{vt}{2}\right)^2 = w^2$$

$$\frac{t^2}{4}(c^2 - v^2) = w^2$$

$$\frac{t^2}{4}\left(1 - \frac{v^2}{c^2}\right) = \frac{w^2}{c^2}$$

$$\frac{t}{2}\sqrt{1 - \frac{v^2}{c^2}} = \frac{w}{c}$$

$$t\sqrt{1 - \frac{v^2}{c^2}} = \frac{2w}{c}$$

$$t\sqrt{1 - \frac{v^2}{c^2}} = t_0$$

$$\therefore t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

proper time the time interval between two events occurring at the same place in an inertial reference frame, as measured by an observer in that inertial frame

time dilation a longer time measured by an observer outside the reference frame in which the event occurs

Fred's measurement t_0 is called the **proper time**, as the measurement from Fred was taken in the frame of reference in which the event was occurring. Pierre, who is observing the carriage moving past him, observes that the time t he recorded is *longer* than the time t_0 Fred recorded. Pierre's time measurement has been dilated. Einstein argued that the postulates of special relativity led to the understanding that observers in different inertial reference frames would not agree on time measurements. **Time dilation** is only significant when inertial reference frames are moving relative to each other at speeds close to the speed of light. Time dilation is not encountered day-to-day, as the velocities that are observed daily, v, are much smaller than c, meaning that $\frac{v^2}{c^2}$ tends to zero, and $t = t_0$. Time dilation is a consequence of events occurring at speeds close to the speed of light, including those travelling at the speed of light.

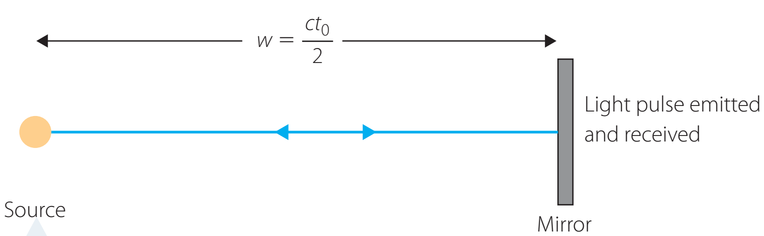

FIGURE 11.6.4 An event as viewed within an inertial reference frame.

Length

If time measurements are different relative to different inertial frames, what happens to the measurements of length? We will now consider a different thought experiment with a moving train (of course).

A moving train travels at a velocity v_0 relative to a station. A source of light in a reference frame emits a short pulse of light that travels to a mirror and back (Figure 11.6.4). An observer in the same reference frame as the mirror and the light source measures the time t_0 for the pulse. If w_0 is the distance from the source to the mirror, the **proper length**, and c is the speed of light, then t_0 is equal to $t_0 = \frac{2w}{c}$.

proper length length measured in an inertial frame of reference in which the object is stationary

On the platform, an observer who is stationary relative to the moving frame of the train measures the time of transit of the light pulse to be t_1, the distance between the source and the mirror to be w and the distance between the location of the source and the mirror to be L_1,as shown in Figure 11.6.5a.

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In this reference frame, the observer notes that during the transit of the light pulse to the mirror, the mirror has moved forwards a distance of $v_0 t$, as shown in Figure 11.6.5b. Then:

$$L_1 = w + v_0 t_1$$

However, L_1 is also equal to ct_1, as the speed of light is the same in all reference frames. Therefore:

$$ct_1 = w + v_0 t_1$$

$$\text{So: } t_1 = \frac{w}{c - v_0}$$

In Figure 11.6.5c, the light pulse bounces off the mirror and travels back to the source in a time measured in this reference frame to be t_2. So:

$$L_2 = w - v_0 t_2$$

However, L_2 is also equal to ct_2, so we have:

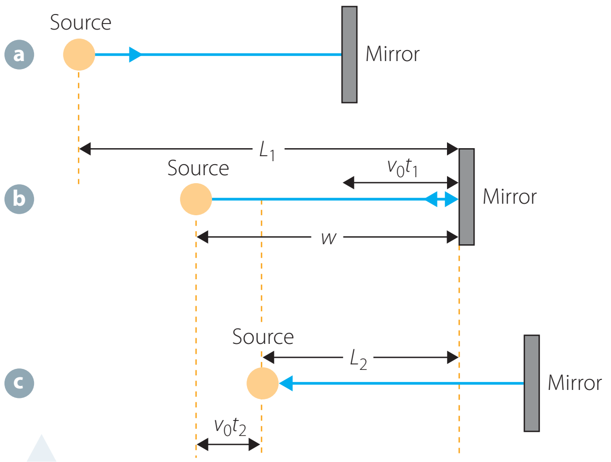

FIGURE 11.6.5 The event as viewed in the stationary frame of reference.

$$ct_2 = w - v_0 t_2$$

$$t_2 = \frac{w}{c + v_0}$$

From the perspective of the observer in the stationary reference frame (on the ground), the total time for the journey is $t_1 + t_2$.

$$t_1 + t_2 = \frac{w}{c - v_0} + \frac{w}{c + v_0}$$

$$t_1 + t_2 = \frac{w(c + v_0) + w(c - v_0)}{c^2 - v_0^2} = \frac{2wc}{c^2 - v_0^2}$$

From time dilation, we know that:

$$t = \frac{t_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

Therefore we have:

$$\frac{2wc}{c^2 - v_0^2} = \frac{t_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$\Rightarrow \frac{2wc}{c^2 - v_0^2} = \frac{2\frac{w_0}{c}}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$\Rightarrow \frac{wc^2}{c^2 - v_0^2} = \frac{w_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$\Rightarrow \frac{w}{1 - \frac{v_0^2}{c^2}} = \frac{w_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}$$

$$\therefore w = w_0\sqrt{1 - \frac{v_0^2}{c^2}}$$

length contraction length measurements are shorter in a reference frame that is moving relative to an inertial frame

This represents **length contraction**. This means that an observer will measure the length of an object moving relative to them to be shorter than when it is at rest. The length of an object at rest is called its proper length. Length contraction is only observed when v is close to c. At ordinary speeds on Earth, $\frac{v}{c}$ becomes zero; therefore, $w = w_0$.

SECTION REVIEW 11.6

REMEMBERING

1. Does time dilation cause a time interval to increase or decrease when an observer is viewing an event moving close to the speed of light?
2. Does length contraction cause the length of an object to appear longer or shorter when an observer is viewing an object moving close to the speed of light?
3. Define 'proper time'.
4. Define 'proper length'.

UNDERSTANDING

5. Explain why time dilation and length contraction are not observed day to day.

11.7 Special relativity definitions

To gain a clear understanding of special relativity, a few terms need to be defined in order to find their quantities mathematically. So far, we have understood that there is a time dilation and length contraction associated with reference frames moving at close to the speed of light. In order to understand these, we have had to define proper time and proper length, quantities that can only be observed when measured in the frame in which the event is occurring.

Time dilation

Lorentz factor factor by which both time dilation and length contracted are affected when v is very close to c

We have seen that time dilation is only significant when inertial reference frames are moving relative to each other at speeds close to the speed of light. As previously derived, time dilation can be expressed mathematically as:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

KEY FORMULA

Time dilation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where:

t = dilated time, measured in the frame of a moving observer (s)

t_0 = proper time, measured by an observer in the inertial reference frame (s)

v = velocity of the inertial reference frame ($m\,s^{-1}$)

c = speed of light ($3 \times 10^8\ m\,s^{-1}$)

Time measurements depend on the frame of reference in which they are made. A clock in an inertial frame that is moving relative to a clock in a second inertial frame will be regarded as running slow. This means that the time interval for the moving clock is greater than the time interval for the other clock.

In the time dilation equation, the reciprocal of the term $\sqrt{1 - \frac{v^2}{c^2}}$ is called the **Lorentz factor**, γ. It is also common to represent the factor $\frac{v}{c}$ as β, such that the Lorentz factor can be simplified to $\frac{1}{\sqrt{1 - \beta^2}}$.

KEY FORMULA

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where:

γ = the Lorentz factor

This allows for time dilation to be expressed as $t = \gamma t_0$. Table 11.7.1 demonstrates how γ, and hence time dilation, changes as v approaches c. Figure 11.7.1 demonstrates this graphically.

TABLE 11.7.1 Approximate values of γ for various values for β.

$\beta = \frac{v}{c}$	γ
0	1
0.0010	1.0000005
0.010	1.00005
0.10	1.005
0.20	1.021
0.50	1.155
0.80	1.667
0.90	2.294
0.94	2.931
0.99	7.089
0.999	22.37

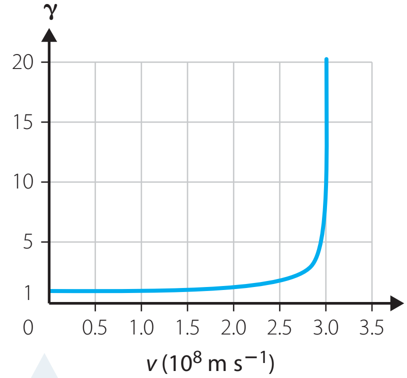

FIGURE 11.7.1 Graph of γ against v.

Proper time interval

Proper time is the time interval between two events occurring at the same place in an inertial frame, as measured by an observer in that inertial frame. Let us refer back to Pierre and Fred from Section 11.6. The time measured by Pierre is not the proper time, because Pierre is not travelling with the mirrors. Now consider what would happen if the mirrors were on the ground with Pierre and the experiment was repeated. Fred could argue that his train was stationary and that Pierre was moving at $-v$ with respect to him. This means he would find that Pierre's clock is slow compared to his. How do we reconcile these two observations?

Time dilation is about measurement in *different* inertial frames; it is a result of relative movement between the frames. The clocks do not physically change. Time dilation is about what an observer in one frame measures about an event in another, and it must be reciprocal because there is no absolute frame of reference.

WORKED EXAMPLE 11.7.1

A pilot in a rocket travelling with a velocity of $0.250c$ presses a button to flash a 'Hello' sign for 5.00 s at a space station as the rocket passes.

1 How long is the flash seen by an observer on the space station?

2 Explain your reasoning.

ANSWER

1 $t = \dfrac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$t = \dfrac{5.00\,\text{s}}{\sqrt{1-\dfrac{(0.250c)^2}{c^2}}}$

$t = \dfrac{5.00\,\text{s}}{\sqrt{0.9375}}$

$t = 5.16\,\text{s}$

This is relativistic time due to the reference frames moving so fast relative to each other.

2 The observer in the space station views the rocket travelling towards it at a velocity of $0.250c$. From this viewpoint, the clock on the rocket will be slow compared to the one in the space station. Any observer regards a clock that is moving relative to their frame of reference as running slow.

relativistic time
time dilation observed due to objects moving at very high speeds relative to each other

Length contraction and proper length

All observers will measure an object moving at relativistic speeds as being shorter or contracted in the direction of relative motion than when the object is at rest. This phenomenon is known as length contraction and is represented as follows: $L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$, or simply $L = \dfrac{L_0}{\gamma}$.

KEY FORMULA

Length contraction

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Where:

L = length of the object measured by an observer who is moving at constant velocity relative to the object's inertial frame (m)

L_0 = proper length, the length of the object at rest (m)

v = velocity of the object relative to the observer (m s^{-1})

c = speed of light ($3 \times 10^8\,\text{m s}^{-1}$)

relativistic length
length contraction due to objects moving at very high speeds relative to each other

Proper length of an object is best measured at rest, or else in the frame of an observer moving with the object being measured. The length contraction or relativistic length is only observed when reference frames are travelling at very high speeds relative to each other.

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WORKED EXAMPLE 11.7.2

1 An observer on the Moon notices a spaceship travelling past at a speed of $2.08 \times 10^8\,\mathrm{m\,s^{-1}}$. The spaceship has a proper length of 120 m. What length will the observer on the Moon measure the spaceship to be?

2 A crewed mission is to be sent to a newly discovered exoplanet 8 light-years away. Their spacecraft will travel at a velocity of $0.5c$ to get there.

a According to the mission crew:

i how far away from Earth is the exoplanet?

ii how long will the journey take?

b According to the mission command on Earth, how long will the journey take?

ANSWER

1

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 120\,\mathrm{m}\sqrt{1-\frac{(2.08\times10^8)^2}{(3.00\times10^8)^2}}$$

$$L = 120\,\mathrm{m}\times 0.72$$

$$L = 86.5\,\mathrm{m}$$

2 a i

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 8\text{ light-years}\sqrt{1-\frac{(0.5c)^2}{c^2}}$$

$$L = 8\text{ light-years}\times 0.866$$

$$L = 6.9\text{ light-years}$$

ii

$$t_{\text{crew}} = \frac{L}{v_{\text{crew}}}$$

$$t_{\text{crew}} = \frac{6.9\text{ light-years}}{0.5c}$$

$$t_{\text{crew}} = 14\text{ years}$$

b

$$t_{\text{Earth}} = \frac{L_0}{v}$$

$$t_{\text{Earth}} = \frac{8\text{ light-years}}{0.5c}$$

$$t_{\text{Earth}} = 16\text{ years}$$

Rest mass

Length and time have been shown to have values that depend on the relative motion of the observers. The third fundamental physical quantity is mass. From energy and momentum analyses, Einstein deduced that the mass of an object will be relative, and will be dependent on its relative speed as follows:

rest mass
also known as proper mass; it is the mass as measured when the mass is stationary in an inertial reference frame. Proper mass never changes

relativistic mass
also known as relativistically corrected mass. The greater the relative velocity, the greater the relativistic mass: $m = \gamma m_0$

$m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$. The term m_0 is known as the **rest mass**, as measured when the object is stationary in an inertial reference frame. m is the measurement of its mass in a reference frame moving in relation to a stationary frame, and is known as the **relativistic mass**.

KEY FORMULA

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where:

m = rest mass (kg)

m_0 = relativistic mass (kg)

v = speed of the moving inertial frame of reference (m s^{-1})

c = speed of light ($3.00 \times 10^8 \text{ m s}^{-1}$)

This means that as the velocity of an object approaches the speed of light, the mass of the object will greatly increase. In fact, there is an ultimate velocity. As v approaches c in size, γ approaches zero, and m will be very large in size. If v was greater than c, then the term would result in the square root of a negative number from $\sqrt{1-\frac{v^2}{c^2}}$, which is an invalid result. This implies that the speed of an object with non-zero rest mass cannot be equal to, or exceed, the speed of light. It appears that the speed of light is the ultimate speed in the physical world.

INQUIRING FURTHER

Tachyons are particles that are said to travel faster than the speed of light. Research tachyons and the current evidence surrounding their existence.

WORKED EXAMPLE 11.7.3

1 What is the relativistically corrected mass of an electron whose speed is measured to be $1.8 \times 10^8 \text{ m s}^{-1}$? Rest mass of an electron is 9.109×10^{-31} kg.

2 At what speed is a particle moving if its relativistic mass is five times larger than its rest mass?

ANSWER

1 $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$m = \frac{9.109 \times 10^{-31} \text{ kg}}{\sqrt{1-\frac{(1.8 \times 10^8 \text{ m s}^{-1})^2}{(3 \times 10^8 \text{ m s}^{-1})^2}}}$$

$m = 1.1 \times 10^{-30}$ kg

2 $m = \gamma m_0$

$\gamma = \dfrac{m}{m_0}$

$\gamma = 5$

$5 = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$1 - \dfrac{v^2}{c^2} = 0.04$

$\dfrac{v^2}{c^2} = 0.96$

$\dfrac{v}{c} = 0.9798$

$v = 2.4 \times 10^8\,\mathrm{m\,s^{-1}}$

Relativistic momentum

Now that we have a definition for relativistic mass, what does this mean about momentum? Special relativity uses the classical equation $p = mv$. However, m is now the relativistic mass whose magnitude depends on the velocity of the mass. This means that the magnitude of the **relativistic momentum** of an object is given by:

$$p_v = \frac{m_0 v}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{p_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

SYLLABUS LINK

Chapter 14 of *Nelson QScience Physics Units 1 & 2* discusses momentum and impulse.

relativistic momentum momentum of particle due to the relativistic mass at high relative speeds

KEY FORMULA

Momentum dilation

$$p_v = \frac{p_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

Where:

p_v = relativistically corrected momentum (N s)

p_0 = momentum calculated from the rest mass in (N s)

v = velocity of the object relative to the observer ($\mathrm{m\,s^{-1}}$)

c = speed of light ($3 \times 10^8\,\mathrm{m\,s^{-1}}$)

WORKED EXAMPLE 11.7.4

What is the relativistic momentum of an electron whose speed is measured to be $2.0 \times 10^8\,\mathrm{m\,s^{-1}}$? Rest mass of an electron is $9.109 \times 10^{-31}\,\mathrm{kg}$.

ANSWER

$$p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p_v = \frac{9.109 \times 10^{-31}\,\mathrm{kg} \times 2.0 \times 10^8\,\mathrm{m\,s^{-1}}}{\sqrt{1 - \frac{(2.0 \times 10^8\,\mathrm{m\,s^{-1}})^2}{(3.0 \times 10^8\,\mathrm{m\,s^{-1}})^2}}}$$

$$p_v = \frac{1.822 \times 10^{-22}\,\mathrm{N\,s}}{0.745}$$

$$p_v = 2.446 \times 10^{-22}\,\mathrm{N\,s}$$

SECTION REVIEW 11.7

REMEMBERING

1 Write the equation for time dilation and length contraction.

2 Write the relativistic equation for mass and momentum.

UNDERSTANDING

3 Explain the relationship between relativistic mass and relativistic momentum.

APPLYING

4 A spacecraft has a proper length of 50 m. It travels past an observer at 0.6*c*. What was its length in the observer's frame of reference?

5 A train carriage travels at 0.75*c*. To an observer outside the carriage, it fits exactly between two markers, P and Q. To an observer on the train, does the carriage appear to fit exactly between P and Q? If not, what can you conclude about events in the different frames of reference?

6 What is the relativistic mass of a proton travelling at 0.65*c*, if its rest mass is 1.673×10^{-27} kg.

7 A spaceship travels at an average velocity of 0.4*c* to an exoplanet 4.4 light-years away.

a What is the distance from Earth to the exoplanet in the frame of reference of the spacecraft?

b What is the difference between the times taken from Earth's perspective and the perspective of the spacecraft?

8 What is the relativistic mass of a proton whose speed is 0.75*c*?

9 A neutron has a relativistic mass that is 2.5 times greater than its rest mass. What is the speed of the neutron?

10 What is the relativistic momentum of a proton whose speed is measured to be 0.1*c*?

ANALYSING

11 A pion has a mean lifetime of 26 ns in Earth's inertial frame of reference. What is its mean lifetime in the pion's frame of reference if it has velocity $v = 0.75c$?

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11.8 Time dilation and length contraction phenomena

Time dilation and experimental evidence

Recall that muons have a mean lifetime of 2.2 μs and travel at close to the speed of light through Earth's atmosphere. This means that the muons are expected to only travel a distance of approximately 656 m during their lifetime.

Frisch and Smith measured an average of 563 muons per hour in the upper detector in the atmosphere. Assuming that other muons were passing the detector at the same rate, and assuming no muons are created between detectors, the number of muons at the second detector a few kilometres down should be almost impossible to count. In fact, they measured 412 muons per hour – far too many to be ignored and close to their predicted count rate based on the relativistic analysis.

This time dilation and length contraction is caused by relativistic effects. This is summarised in Figure 11.8.1. An observer on a moving muon (the muon's frame of reference) and an observer on Earth (Earth's frame of reference) will agree on the following about the event:

- relative speed, 0.995*c*
- number of physical decays of muons
- number of elapsed mean lifetimes.

They will disagree on:

- mean lifetime
- distance between the collectors.

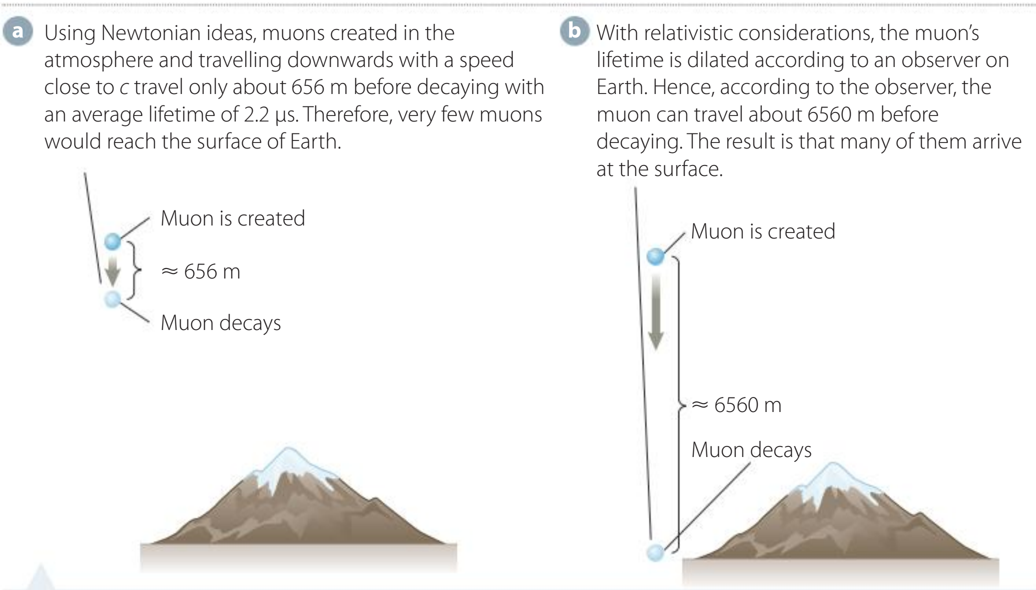

FIGURE 11.8.1 Travel of muons according to an observer **(a)** in the muon's frame of reference and **(b)** on Earth.

In regards to the time dilation observed of the muon:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{2.2\,\mu\text{s}}{\sqrt{1 - \frac{(0.995c)^2}{c^2}}}$$

$$t = 22.0\,\mu\text{s}$$

suggesting that according to an observer on Earth, the muon will not decay until after 22.0 μs and can travel much further, as its half-life has increased almost tenfold.

SCIENCE AS A HUMAN ENDEAVOUR

Satellites are constantly in orbit around Earth. Research how time dilation is relevant to clocks on satellites, and how relativistic effects need to be accounted for for the purpose of navigation. What experimental evidence do satellites provide that support the phenomena of time dilation?

WORKED EXAMPLE 11.8.1

Muons with a mean lifetime of 2.2×10^{-6} s and travelling at 0.999*c* were observed at a height of 10 000 m above Earth (in Earth's reference frame).

1 Use Newtonian ideas to calculate the distance travelled by a muon in one lifetime.

2 Use Newtonian ideas to calculate the number of mean lifetimes that would elapse before the muon reached the ground.

3 Justify why it is unlikely that the muon will reach the ground.

ANSWER

1 $d = vt$

$d = 0.999 \times 3 \times 10^8\,\text{m s}^{-1} \times 2.2 \times 10^6\,\text{s}$

$d = 659\,\text{m}$

2 $n = \frac{10\,000\,\text{m}}{659\,\text{m}}$

$n = 15.2$

3 One mean lifetime is the time in which the detected muons would be expected to have decayed. Using Newtonian ideas, it is unlikely that the muon would last for 15.2 lifetimes.

Length contraction and experimental evidence

Now let's consider the muon decay phenomenon from a position of length contraction. The time dilation is observed from Earth's reference frame. However, length contraction is observed from the reference frame of the muon. Neither reference frame sees the relativistic effects of both time dilation and length contraction.

The proper length in this scenario is measured from Earth's reference frame. This means that, in the muon's frame of reference, length is contracted due to the muon measuring the proper time. The muon sees the length contracted to the original 659 m in this circumstance.

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SECTION REVIEW 11.8

UNDERSTANDING

1 Can one reference frame measure both the proper time and proper length?

2 Explain the experiment used to verify time dilation in muon decay.

APPLYING

3 Muons are formed 3.0 km above Earth. They travel at 0.996c and have a mean lifetime in the muon's rest frame of 2.2 μs. For an observer on a muon, how long does it take for the muon to travel to the ground?

4 Consider the scenario in Question **3**. For an observer on the ground, how many lifetimes will elapse before the muons arrive on the ground?

11.9 Solving problems: time dilations; length contractions; relativistic momentum

When solving problems in special relativity, it is sometimes difficult to determine the proper time, proper length and rest mass to, in turn, calculate the time dilation, length contraction and relativistic momentum. Proper time and proper length are *not* defined in the same frame of reference. As a general rule, the proper time is defined in the frame of reference of the object moving, and proper length is defined by a stationary observer.

Consider again a moving train, whose length is measured from the front of the carriage, F, to the back of the carriage, B. The train moves between points P and Q. The proper time is the time it takes for the train to get from P to Q, as measured by an observer travelling on the train. The proper length of the train is measured by an observer not on the train, as the distance between F and B.

Rest mass is an intrinsic property of an object. The rest mass does not vary under effects of time dilation and length contraction, and can only be measured by an observer in the same frame of reference as the object whose rest mass is being measured. In the case of the moving train above, the rest mass would be the mass as measured by the observer on the train.

Once proper time, proper length and rest mass are known, calculating the relativistic effect is simple based on the formulas derived earlier. Understanding in which frame of reference the proper time and length are measured is also pivotal in understanding simultaneity.

WORKED EXAMPLE 11.9.1

A rapidly moving rail cart (Figure 11.9.1) travelling at speed v approaches a pair of markers.

An observer on the side of the track measures the cart to be L_{AB} in length. As the cart speeds past the markers, the land-based observer notices that the front of the cart coincides with marker B and the rear of the cart coincides exactly with marker A. This means the observer sees these events as simultaneous.

In this reference frame, the cart took 1.2 s for the front of the car to move from A to B.

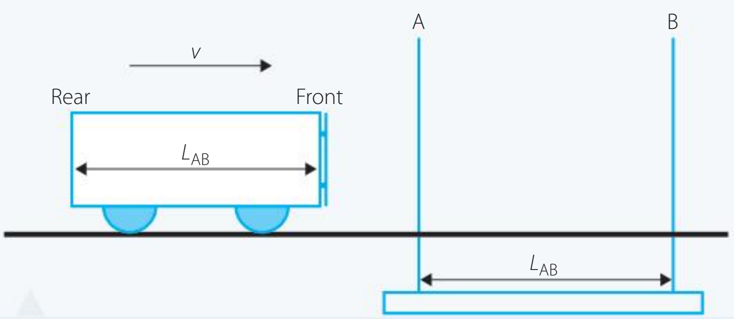

FIGURE 11.9.1 A rapidly moving rail cart as observed from the reference frame of a ground-based observer

For this scenario, define the proper length and proper time, and state which reference frame will observe time dilation, length contraction and relativistic momentum.

ANSWER

There are two frames of reference to consider: the cart-based observer and the ground-based observer. The cart-based observer will measure the proper time, and observe length contraction. As the cart-based observer is stationary relative to the cart, this is the reference frame in which rest mass can be measured.

The ground-based observer measures the proper length L_{AB}, but the 1.2 s they observe is a dilated (longer) time. As rest mass is observed in the other reference frame, the ground-based observer will also perceive the cart as having a larger momentum than it actually does (relativistic momentum).

WORKED EXAMPLE 11.9.2

A tiger is sprinting at $0.87c$ from the base of a tree to catch a deer eating at a nearby shrub. The tiger notes that he caught the deer in just 3.6 s and travelled just 65 m. A nearby elephant observes the tiger to have momentum 9.65×10^{10} N s. For this scenario, determine how the tiger and the elephant each measure:

1. the distance the tiger travels
2. the time it takes for the tiger to catch the deer
3. the momentum of the tiger during its pursuit.

ANSWER

1. From the tiger's perspective, the distance travelled is 65 m. This length is contracted. From the elephant's perspective, the proper length is observed. The proper length is:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_0 = \frac{65\,\text{m}}{\sqrt{1-\frac{(0.87c)^2}{c^2}}}$$

$$L_0 = \frac{65\,\text{m}}{0.493}$$

$$L_0 = 131.8\,\text{m}$$

2. From the tiger's perspective, the pursuit took 3.6 s. This is the proper time. The elephant will observe time dilation. The time the elephant measures for the tiger to catch the deer is:

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{3.6\,\text{s}}{\sqrt{1-\frac{(0.87c)^2}{c^2}}}$$

$$t = \frac{3.6\,\text{s}}{0.493}$$

$$t = 7.30\,\text{s}$$

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3 The elephant measures the momentum of the tiger to be 9.65×10^{10} N s. This momentum is relativistic, as the rest mass is measured from the tiger's reference frame. Therefore, the tiger will measure the proper momentum as follows:

$$p = \frac{p_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p_0 = p\sqrt{1-\frac{v^2}{c^2}}$$

$$p_0 = 9.65 \times 10^{10}\ \text{N s} \times 0.493$$

$$p_0 = 4.76 \times 10^{10}\ \text{N s}$$

And the rest mass can be calculated as:

$$m_0 = \frac{p_0}{v}$$

$$m_0 = \frac{4.76 \times 10^{10}\ \text{N s}}{0.87 \times 3 \times 10^{8}\ \text{m s}^{-1}}$$

$$m_0 = 182.28\ \text{kg}$$

SECTION REVIEW 11.9

UNDERSTANDING

1 Explain how to determine the proper length, proper time and rest mass in a given situation.

APPLYING

2 Electrons in an electron gun are measured to be about 0.5% more massive than electrons at rest. What is the velocity of the electrons in the electron gun?

3 Two identical clocks are synchronised. One clock is sent off in a spaceship travelling with a speed of 0.7*c*. After 49 years on the Earth clock, what is the time on the spaceship, as observed from Earth?

4 A pion, travelling at 0.5*c*, has a relativistically corrected mean lifetime of 26 ns in Earth's frame of reference. Calculate the mean lifetime as measured in the pion's reference frame.

5 The pilot of a non-accelerating spacecraft, moving away from Earth at great speed, celebrates the passing of six birthdays. Earth-bound observers measure this elapsed time to be 10 years. Relative to Earth:

a what is the speed of the spacecraft?

b how far does the spaceship travel over these six birthdays?

11.10 The mass–energy equivalence relationship

To explore the relationship between matter and energy, let us discuss a thought experiment involving the momentum of a photon, p = mv, that we know to be $\frac{E}{c}$. Imagine a stationary spacecraft in distant space.

The pilot of the spacecraft is practising aiming with a laser gun that is capable of firing a single photon or a high-energy beam. The single photon is fired towards the rear of the spacecraft

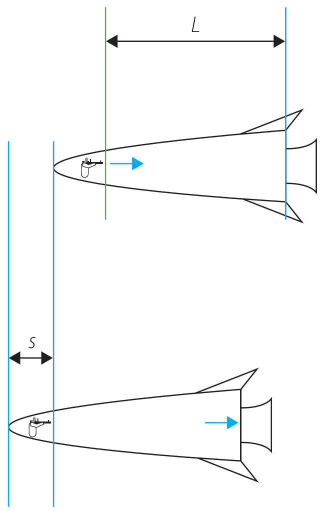

FIGURE 11.10.1 A photon fired towards the rear of a spacecraft. *L* denotes the distance the photon travels; *s* denotes the distance the spacecraft recoils due to conservation of momentum.

(Figure 11.10.1). The photon has momentum $\frac{E}{c}$ (where E is the energy of the photon). Given that momentum is conserved in this isolated system, the pilot and, in turn, the spacecraft (because the pilot is seated in the spacecraft) will recoil with equal but opposite momentum. The distance the craft moves, s, will be equal to vt, the velocity of the craft multiplied by the time the photon takes to reach the rear of the spacecraft.

The magnitude of the momentum the spacecraft receives will be mv, and M is the mass of the spacecraft. Of course, the pilot will probably not notice this movement because of the small size of the momentum of the photon. However, as small as it is, the movement will occur, just as it does when you jump into the air on Earth and Earth recoils with the same momentum. The time it takes for the photon to reach the rear of the craft, t, will be equal to $\frac{L}{c}$, where L is the length of the spacecraft cabin. From this, we can deduce that:

$$Mv = \frac{E}{c}$$

$$\Rightarrow v = \frac{E}{Mc}$$

where v is the velocity the spacecraft recoils with. As $s = vt$, we can express the distance the spacecraft travels as follows:

$$s = vt = \frac{E}{Mc} \times t = \frac{E}{Mc} \times \frac{L}{c}$$

$$\therefore s = \frac{EL}{Mc^2}$$

As the photon reaches the rear of the spacecraft, the momentum will be transferred, causing the craft to stop. There is no net external force applied to the system; the total momentum of the system has not changed, yet the system has moved. It is the photon that has changed position and caused this re-distribution. In this interaction with the photon, the spacecraft has behaved exactly as would be expected if there had been a redistribution of mass (Einstein's 'mass equivalent' of energy hypothesis).

If we suppose the photon to have a relativistically corrected mass, m, then its momentum would be $p = mc$. Thus, the speed of the photon as it travels to the rear of the spacecraft is $c = \frac{L}{t}$. Therefore, we could equate the momentum as follows:

$$\frac{Ms}{t} = \frac{mL}{t}$$

$$s = \frac{mL}{M}$$

We now have two equations for s, and equating these we obtain:

$$\frac{EL}{Mc^2} = \frac{mL}{M}$$

$$\Rightarrow E = mc^2$$

rest energy
defined from the rest mass by $E = m_0c^2$

KEY FORMULA

Mass–energy equivalency

$E = mc^2$

Where:

E = change in energy equivalent to the change in mass (J)

m = change in mass (kg)

c = speed of light ($3 \times 10^8\ \text{m s}^{-1}$)

Of course, this is just a thought experiment and does not necessarily prove anything. However, the above equation has been derived in a variety of situations and, more importantly, has been successfully tested by experimentation.

The energy associated with mass at rest is called the **rest energy** of the mass, and is given more specifically by the equation $E = m_0c^2$. Mass and energy are equivalent, as mass is a manifestation of energy.

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The relativistic total energy of an object that is moving is defined by:

total energy = rest energy + relativistic kinetic energy

where the **relativistic kinetic energy** (specifically the change in the relativistic kinetic energy) is related to a change in mass as follows:

$$\Delta E_k = \Delta mc^2$$

$$\Delta E_k = (m - m_0)c^2$$

$$\Delta E_k = mc^2 - m_0c^2$$

$$\Delta E_k = \gamma m_0c^2 - m_0c^2$$

$$\Delta E_k = (\gamma - 1)m_0c^2$$

where γ is the reciprocal of the Lorentz factor $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.

relativistic kinetic energy defined from the rest mass by $\Delta E_k = (\gamma - 1)m_0c^2$

WORKED EXAMPLE 11.10.1

An electron is accelerated from rest by a potential difference of 500 kV.

1 What speed does the electron obtain?

2 What speed would the electron attain if non-relativistic mechanics were used?

ANSWER

1 First, equate $\Delta E = qV$ to $\Delta E_k = (\gamma - 1)m_0c^2$ and rearrange for γ.

$$qV = (\gamma - 1)m_0c^2$$

$$\gamma = \frac{qV}{m_0c^2} + 1$$

$$\gamma = \frac{1.6 \times 10^{-19}\,C \times 500\,000\,\text{V}}{(9.109 \times 10^{-31}\,\text{kg})(3 \times 10^8\,\text{m s}^{-1})^2} + 1$$

$$\gamma = 1.9784$$

Now, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and v can be found as follows:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma^2 \times (1 - \frac{v^2}{c^2}) = 1$$

$$\gamma^2c^2 - c^2 = v^2\gamma^2$$

$$v = \sqrt{\frac{\gamma^2c^2 - c^2}{\gamma^2}}$$

$$v = \sqrt{\frac{1.9784^2(3 \times 10^8)^2 - (3 \times 10^8)^2}{1.9784^2}}$$

$$v = 2.59 \times 10^8\,\text{m s}^{-1}$$

2 Non-relativistically:

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$v = \sqrt{\frac{2 \times (1.6 \times 10^{-19}\, c) \times 500\,000\,\text{V}}{9.109 \times 10^{-31}\,\text{kg}}}$$

$$v = 4.19 \times 10^8\,\text{m s}^{-1}$$

INQUIRING FURTHER

ENERGY AND MOMENTUM AT THE AUSTRALIAN SYNCHROTRON

The storage ring at the Australian Synchrotron has a radius of about 355 m but it should fit on a kitchen table, from the viewpoint of the particles inside it. The reason has to do with Einstein's relativity. Electrons travel in the storage ring (Figure 11.10.2) at very near the speed of light. For the 3 GeV Australian Synchrotron, electrons travel at 99.999995% the speed of light. This means that their masses must be relativistically corrected. Relativistic corrections become necessary when the speed of the electron is about 0.1*c*.

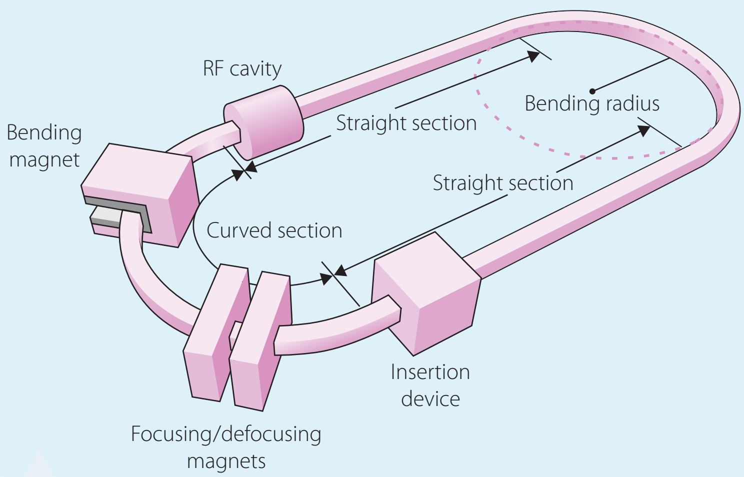

FIGURE 11.10.2 A synchrotron storage ring comprises straight and curved sections.

11.10.1 The Synchrotron

Electrons do not follow a perfect circular path. They travel along straight sections and are then subjected to curved sections. When a 3 GeV electron is affected by bending magnets that produce a 1.3 T field, it follows a circular path. This path has a radius wider than the kitchen table. In the straight sections it is affected by insertion devices called wigglers and undulators. These produce radiation in the nanometre range. This is a result of the combined effect of the relativistically corrected Doppler shift and length contraction in the electron's frame of reference.

1 For the Australian Synchrotron what is:

a the maximum energy available?

b its overall radius?

c the speed of the electrons?

d the magnitude of the magnetic field produced by the bending magnets?

e the value of the Lorentz factor when the electron is travelling at 0.1*c* and maximum speed? (Hint: You will need to use a value for *c* that has enough significant figures.)

2 For the electrons in the storage ring at the Australian Synchrotron, if $\beta = \frac{v}{c}$, what is the value of:

a β?

b v?

3 Copy and complete the table below to compare the kinetic energy and momentum for an electron travelling at $0.99999995c$ according to both classical and relativistic physics.

TABLE 11.10.1

	KINETIC ENERGY (J)	MOMENTUM (N s)
Classical		
Relativistic		

4 On a single set of axes, plot the kinetic energy vs β for kinetic energies up to 1000 keV for both classical and relativistic energies. Use a spreadsheet to produce values of v from values of β; hence, find γ and E_k. Compare and contrast these two graphs.

Special relativity leads to the idea of mass–energy equivalence. Research how this equivalency applies in nuclear fission reactors.

SCIENCE AS A HUMAN ENDEAVOUR

The ultimate speed limit

Only light, massless photons with high energy are able to travel at the speed of light, c. Although these photons are massless, they still have intrinsic momentum capable of causing effects on matter with mass. In all scenarios we have dealt with so far, all particles have been travelling at close to the speed of light, but not at the speed of light. This raises the question: can particles with mass travel at this ultimate speed limit?

Consider a particle with mass travelling through a vacuum. There is no friction. In order for this particle to increase its speed, it must accelerate. In order for acceleration to occur, an unbalanced force must be applied to the particle, and in order for a force to be exerted on the particle, work must be done. In other words, for a particle to move with higher velocity, more energy must be applied. Additionally, the heavier the particle is, the more energy will be required to accelerate it.

Now consider the particle's rest mass m_0. As the particle begins to travel at velocities closer and closer to the speed of light, the mass of the particle becomes relativistically corrected. An outside observer who is applying energy to the particle to make it go faster, now observes the particle getting heavier. No matter how hard they try, as they apply energy to cause the particle to travel faster and faster, the particle will become heavier and heavier in proportion to this increased energy. This leads to an impasse: applying more energy to increase the velocity of an object that is already travelling at close to c, will increase its relative mass, causing the energy input for acceleration to be negated. Quite simply, for an object with mass to travel at the speed of light, it would have infinite mass. Due to the mass–energy equivalency, to move infinite mass would require infinite energy. An impractical reality!

SECTION REVIEW 11.10

REMEMBERING

1 State the mass–energy equivalency equation.

UNDERSTANDING

2 Explain why we need to understand the mass–energy equivalency to solve problems of special relativity.

APPLYING

3 An electron gains 600 keV of energy in an electron gun. What speed does it attain?

4 A positron is the antiparticle of an electron. Both have a rest mass of 9.109×10^{-31} kg. When an electron and a positron collide, they annihilate each other and produce energy in the form of electromagnetic radiation. How much energy is released by the collision?

ANALYSING

5 The ultimate speed is *c*, but there does not appear to be an ultimate kinetic energy, $E = mc^2$. Explain this apparent contradiction.

6 Explain why no object can travel at the speed of light in a vacuum.

11.11 Paradoxical scenarios

A paradox is a statement that negates itself. In other words, it presents a scenario that is self-contradictory in that it is both true, and cannot be true at the same time. A famous example is the Grandfather Paradox. In this scenario, a person travels back in time and kills their grandfather before the conception of their father or mother. This presents a contradiction in that if the grandfather was dead, the time traveller would never have been born to begin with and so could not have gone back in time to kill their grandfather. From this a number of questions are presented, and many are debated.

Not unlike the Grandfather Paradox scenario, special relativity presents its own paradoxes to do with time and space. Both time dilation and length contraction result in realities that are seemingly contradictory, but can be explained by relativistic effects. These include thought experiments known as the twins paradox, flashlights on a train, and the ladder in the barn.

Twins paradox

Identical twins are just that: two people who are identical. They look the same, their DNA is the same and they age with time in the same way, assuming all factors are constant. We will now consider a thought experiment involving identical twins Chris and Taylor. Chris makes a journey into outer space in a high-speed spacecraft, while Taylor remains behind on Earth. When Chris returns back to Earth, he observes that Taylor has aged more.

This poses a problem: each twin sees the other moving relative to their own reference frame and, so paradoxically, each twin should have found the other to have aged less. For example, it can be argued that the clocks on Earth are running slow, or the clocks on the spacecraft are running slow, depending on where you measure proper time. It depends on whose perspective you take for the scenario, Chris's or Taylor's.

Solution to the twins paradox

The easiest way to think about the scenario is to first consider the journey of the travelling twin. Chris's spacecraft needs to change direction at some point in order to arrive back on Earth. This means there are two inertial frames of reference for Chris, as we cannot consider the whole journey as one reference frame due to the change in velocity (acceleration). So we now consider that while Taylor is at rest in the same inertial frame throughout the entire journey, Chris's frame switches from being at rest to velocity away from Earth, to being at rest and again with a velocity towards Earth.

Let's consider the first half of the journey in which Chris is headed outbound on the spacecraft. The twins align their clocks, and Chris heads off at a constant velocity close to the speed of light, say $0.8c$. Chris travels to a planet that is measured to be 10 light-years away from Earth. In Chris's reference frame, length is contracted and he measures only 6 light-years between Earth and the planet. This means Chris measures the trip to the planet to take 7.5 years $\left(\frac{6 \text{ light-years}}{0.8c}\right)$, whereas Taylor would measure the time taken for this part of the journey to be 12.5 years $\left(\frac{10 \text{ light-years}}{0.8c}\right)$.

This has taken the time measurement from each perspective based on length contraction, but hasn't considered how each twin reads the other's clock. When he reaches the planet, Chris's clock reads that 7.5 years have gone by, but when this event is observed by Taylor, Taylor's clock does not read that 12.5 years have gone by. Rather, from Taylor's perspective, it takes 12.5 years for Chris to get to the star, plus an additional 10 years for the light to travel back to Earth for Taylor to observe the event. Hence Taylor's clock will indicate that 22.5 years have gone by since Chris left Earth.

When Chris reaches the star he sees Taylor's clock as it was 10 years ago, and hence Taylor's clock appears to be the one running slow from Chris's reference frame: it reads $12.5 - 10 = 2.5$ years. So Chris and Taylor are each observing the other's clock as running slow, from their reference frame.

This is where the paradox comes in, but it is resolved when considering the return journey. When Chris travels back to Earth, Taylor views Chris's clock as changing from 7.5 years to 15 years in just 2.5 years time. This is because Taylor's clock goes from reading 22.5 years to 25 years from the time it took to observe Chris reaching the planet to when Chris arrived back on Earth. From Chris's perspective, the return journey took 15 years (7.5 years each way), and it was not until after returning to Earth that Taylor's watch could be observed.

The conclusion is that Taylor, the Earth-bound twin, will have a faster clock, and hence he will have aged more. As the twins are reunited, and the length contraction and time dilation is observed from both frames of reference, less time will have elapsed on Chris's clock. This solution only works when considering the travelling twin as moving at speeds close to c, to a very long distance away, and then returning. If there was no return, there would be no way to determine whose clock was in fact 'correct'.

Flashlight on a train

Consider a thought experiment in which, you guessed it, we have a moving train. On this train there are two people, one at each end of a carriage, and an observer on the ground. The train is length L, and moves between markers A and B with velocity v, from west to east. The observer on the ground is standing at marker B. If the person at the rear of the carriage shines a flashlight at the person standing at the front of the carriage, an observer on the carriage sees the light travelling at $v = c$. The person on the ground observes the velocity of the light as travelling at $v + c$. This is paradoxical, as nothing can travel faster than c.

Solution to flashlight on a train

Einstein's second postulate states that the speed of light has the same value, c, in all inertial frames. It does not depend on the speed of either the source or the observer. Therefore, the flashlight on a train scenario is not a simple relative motion problem, as the speed of light is invariant. This problem is solved by considering the laws of special relativity, not the laws of Newtonian mechanics or Galilean relativity.

Ladder in the barn

The third paradoxical scenario that will be considered is one involving a ladder with length L_L and a barn with length L_B with big doors on opposite ends (Figure 11.11.1). When at rest $L_L > L_B$. Consider now

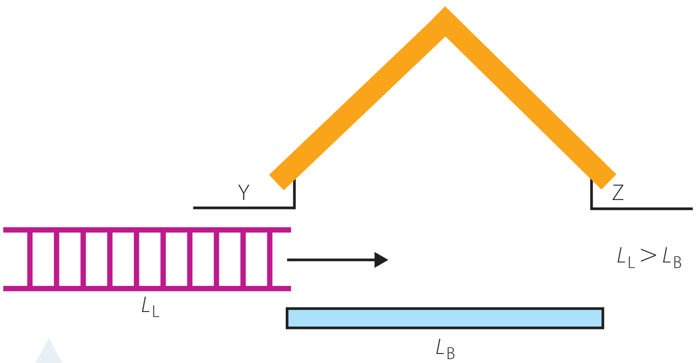

FIGURE 11.11.1 Situation of the ladder in the barn where $L_L > L_B$ when both the garage and ladder are at rest.

that the ladder is moving parallel to the ground at very high speed, through the open barn doors. From length contraction, the ladder will be able to fit inside the garage and there will be a brief moment when both doors could be shut and the ladder can be contained. However, if the reference frame of the ladder is considered, it is the garage that is moving, and the garage that is contracting in length to an even smaller size. This paradox arises from a misunderstanding of simultaneity.

Solution to the ladder in the barn

According to special relativity, events cannot occur simultaneously in both reference frames when the frames of reference have very high relative speeds. As mentioned, if the scenario is considered from the perspective of the ladder moving and the garage stationary, there will be a length contraction of the ladder, and for a moment the ladder will be able to fit in the barn and both the front and the rear of the ladder are inside the barn simultaneously. However, if the situation is considered in which the garage is moving and the ladder is stationary, the garage will undergo a length contraction, and even less of the ladder will be able to fit into the barn than previously.

Now, consider the doors Y and Z on either end of the garage are closed for a brief period of time, in the frame in which the ladder is moving (contracted). This is the event as observed by a stationary observer in the garage (where the ladder is moving and the garage is not). Now let's consider what would happen from the ladder's perspective. As the ladder approaches Z (right-hand door in Figure 11.11.1), it closes, but opens in time for the front of the ladder to head out. At a later time, the back of the ladder passes through door Y, which closes then opens again (still in the reference frame of the garage moving). At no point from this reference frame did the ladder need to fit inside the garage, but each door could still close while the ladder moved through, as simultaneity is relative.

In both reference frames, the ladder can fit inside the barn so that its doors can close, but this does not happen simultaneously in both frames of reference. The doors, from the frame of reference of the ladder, do not open and close at the same time. They do, however, close at the same time according to the outside observer.

SECTION REVIEW 11.11

UNDERSTANDING

1 In your own words explain each of the following paradoxical scenarios.

- **a** Flashlights on a train
- **b** Ladder in the barn
- **c** Twins paradox

APPLYING

2 The twins paradox illustrates the effects of time dilation. Alex and Sam are twins. Alex takes an extra-terrestrial journey that takes him to a distant stellar system and then returns. The average speed of Alex's spacecraft is 0.6*c*. Sam remains on Earth. When Alex returns, Sam is 20 years older. How much has Alex aged according to his space watch?

3 Xavier, Ignatius and Maxine are triplets. Xavier pilots a spacecraft away from Earth towards the centre of the Milky Way for a distance of 7 light-years relative to Earth. Maxine pilots a similar spacecraft in the opposite direction, away from the centre of the Milky Way. Her craft also travels outwards for a distance of 7 light-years. Their average speed is 0.61*c*. Ignatius remains on Earth. Both Maxine and Xavier return at the same time.

- **a** Will Xavier and Maxine appear to be the same age on their return?
- **b** How much older than the travellers will Ignatius be when the travellers return?

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   - a Classical mechanics
   - b Frame of reference
   - c Galilean transformation
   - d Inertial frame of reference
   - e Invariant
   - f Length contraction
   - g Lorentz factor
   - h Muon
   - i Proper length
   - j Proper time
   - k Quantum physics
   - l Relative motion
   - m Relativistic effect
   - n Relativistic kinetic energy
   - o Relativistic length
   - p Relativistic mass
   - q Relativistic momentum
   - r Relativistic time
   - s Relativity principle
   - t Rest energy
   - u Rest mass
   - v Simultaneity
   - w Special relativity
   - x Time dilation

2 What is special relativity and when do we need to consider it in applications?

CATEGORY QUESTIONS

3 Compare and contrast Newtonian mechanics, Galilean relativity and special relativity.

4 Explain time dilation and how it changes with increasing speed.

5 Explain length contraction and how it changes with increasing speed.

ELABORATION QUESTIONS

6 Explain why particles cannot travel at the speed of light in a vacuum.

7 Why is the twins paradox not in fact a paradox?

8 Explain the mass–energy equivalency and how it is derived.

EVIDENCE QUESTIONS

9 Muon decay offers experimental evidence for time dilation. Explain this phenomenon in terms of the evidence collected about muons in Earth's atmosphere.

10 Research ring satellites and how they provide experimental evidence that supports the phenomena of time dilation.

END-OF-CHAPTER EXAM

End-of-chapter test

1 Which of the following is one of the postulates of special relativity?

A Particles can move at the speed of light.

B Proper length and proper time are measured in the same frame of reference.

C The speed of light is the same in all frames of reference.

D The speed of light changes depending on the frame of reference.

2 Ike observes a clock three light-seconds away. Ike is at rest relative to the clock. Which statement is true about the time Ike observes?

A Ike observes the clock as showing the same time as the watch on his wrist.

B Ike observes the time on the clock as different but by an uncertain amount.

C Ike observes the time as three seconds faster than his watch.

D Ike observes the time as three seconds slower than his watch.

3 A particle is travelling at close to c in space, and a floating, stationary observer watches it go by. Which of the following must be true?

A The particle measures the proper time in its reference frame.

B The particle measures the proper length in its reference frame.

C The particle measures the rest mass in its reference frame.

D The particle is massless.

4 A train is moving east at $6\,\text{km h}^{-1}$ and passes a train heading west at $2\,\text{km h}^{-1}$. According to the eastbound train, the oncoming train is travelling at:

A $2\,\text{km h}^{-1}$.

B $4\,\text{km h}^{-1}$.

C $6\,\text{km h}^{-1}$.

D $8\,\text{km h}^{-1}$.

5 State the mass–energy equivalency relationship.

6 An astronaut floating stationary in space notices an oncoming spaceship travelling at $0.1c$. According to the astronaut, the spaceship takes 2 seconds to reach him. Will the spaceship observe this leg as taking longer than two seconds, or less than two seconds?

7 State the two postulates of special relativity.

8 A muon travels at the speed of light down towards Earth. An observer on the muon notices that it decays after $3\,\mu\text{s}$. How long does it take for the muon to decay for an observer on Earth?

9 Consider the muon from the previous question. According to the muon, it travels a distance of 700 km before it decays. How far the does the observer on Earth see the muon travel before it decays?

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10 Barnard's Star in the Milky Way is 6 light-years away from Earth. As measured by a person on Earth, it would take 6 light years to reach this star. A rocket leaves for Barnard's Star at a speed of $v = 0.65c$ relative to Earth. Assume that Earth and Barnard's Star are stationary relative to each other. According to the frame of reference of the rocket, what is the distance between Earth and Barnard's Star?

11 Explain why no object can travel at the speed of light in a vacuum.

12 Explain the concept of simultaneity.

13 Define the following terms.

a Proper length

b Proper time

c Rest mass

» UNIT FOUR

REVOLUTIONS IN MODERN PHYSICS

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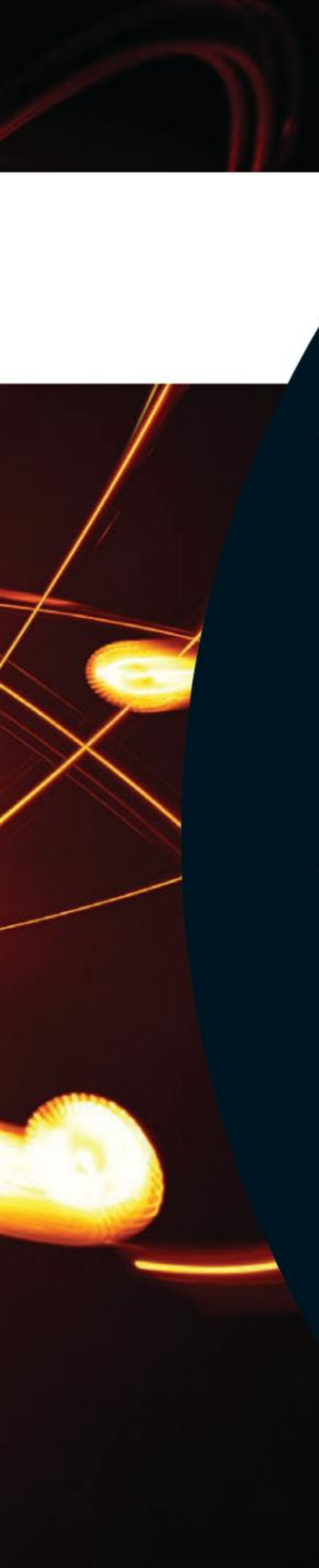

Topic 2: Quantum theory

The topic 'Quantum theory' is fundamental to our understanding of modern physics. Students explore the nature of light, as both a wave and as a particle, including the phenomena of interference as exhibited by Young's double-slit experiment. The wave-particle nature of light is further demonstrated through black-body radiation and photoelectric effect experiments. Students are given opportunities in such experiments to perform multiple investigations to develop their skills in the graphical representation of data, the identification of relationships between variables, and the evaluation of accuracy and precision of data. The nature of the atom and atomic spectra are also explored.

SCIENCE AS A HUMAN ENDEAVOUR

Students should be given opportunities to investigate the development of the model of the atom; the development of scientific theories and the role of new evidence in challenging existing theories; how Earth is used to predict climate patterns; the development of the quantum model; and the relationship between black-body radiation and the greenhouse effect.

KEY FORMULA

$$\lambda_{max} = \frac{b}{T}$$

$$E = hf$$

$$h = 6.626 \times 10^{-34}\ \text{J s}$$

$$E_k = hf - W$$

$$\lambda = \frac{h}{p}$$

$$n\lambda = 2\pi r$$

$$mvr = \frac{nh}{2\pi}$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

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12 QUANTUM THEORY

Introduction

Quantum theory was developed when classical models, such as the wave model of light, were unable to explain experimental results.

In this chapter, the nature of light is explored through the study of such experiments and phenomena as Young's double-slit experiment, black-body radiation, atomic line spectra, the photoelectric effect and de Broglie wavelength.

Stimulus question

Quantum theory, the concept of energy existing in discrete packets, has led to the development of modern electronics, computing and communications technology. How many appliances can you list that are dependent upon quantum theory?

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12.1 The nature of light

Many early scientists conjectured that light had a finite speed. This was first demonstrated by Ole Roemer (1644–1710) at the end of the 17th century. It was finally confirmed by the astronomer James Bradley (1693–1762) in 1728. Bradley measured the difference between where a star was expected to be seen and where it was actually observed. This **stellar aberration** was due to the finite speed of light and the speed of Earth combining to affect the position from which the starlight appeared to come.

stellar aberration the variation in the visible and actual position of a star due to the relative motion of Earth

He determined the speed of light to be about 298 000 km s^{-1} (in modern units). This value was later refined in land-based experiments by Fizeau (1849), Foucault (1852) and Michelson (1879, 1883 and 1926). The standard constant speed of light is now accepted as 299 792 458 m s^{-1}; however, for the purpose of our calculations, we will use 3.00×10^8 m s^{-1}.

INQUIRING FURTHER

The speed of light in a vacuum is 299 792 458 m s^{-1} (approximately 3.00×10^8 m s^{-1}). Recently reported studies have suggested that the speed of light has increased. If the speed of light turns out not to be constant, then Einstein's theory will come under pressure and may be superseded. Conduct research into and write a literature review about these recent studies.

In 1801, Thomas Young (1773–1829) demonstrated the effects of interference for light. This indicated that light had wave-like properties. Young's method was used to determine the wavelengths of visible light. Later, in the same century, James Clerk Maxwell (1831–79) used calculus-based mathematics to bring all the concepts of electricity and magnetism neatly together into four simple relationships. As well as being a triumph of theoretical physics, it also began the transformation of our understanding of light and matter. Maxwell was a giant of physics. Born in Scotland, he was educated at the University of Edinburgh, and Trinity College, Cambridge. It was at Cambridge that he began his most important work. In statistical thermodynamics he developed the Maxwell distribution, which predicted the range of molecular speeds in a gas. In optics, he produced the first colour photograph, and he improved our understanding of colour perception and its link to colour deficiency. In astronomy, Maxwell used mathematical modelling to show that the rings of Saturn were most likely made of small rock particles. Maxwell's greatest work was the unification of electric and magnetic theory into electromagnetism. Einstein's reflections on Maxwell's work led to the theory of relativity.

Maxwell's equations are found within Chapter 10.

12.1.1 James Clerk Maxwell

Maxwell's equations were seen as a great triumph of theoretical physics because they unified the two fields of electricity and magnetism. They are as important to electromagnetism as Newton's laws of motion and the law of universal gravitation are to mechanics. Maxwell realised that his latter two equations meant that an electric field produced by the varying magnetic field would itself be varying. It would therefore produce a varying magnetic field, and so on.

magnetic permeability (μ_0) physical property of a medium associated with magnetism

electrical permittivity(ε_0) a physical property of a medium associated with electricity

Maxwell deduced that these waves could move through space with a fixed velocity, v. The velocity of the electromagnetic wave was only dependent on the constants of **magnetic permeability(μ_0)** and **electrical permittivity(ε_0)**. Using modern values, this turns out to be:

KEY FORMULA

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12})(4\pi \times 10^{-7})}}$$

$$= 2.99 \times 10^8 \text{ m s}^{-1}$$

The speed of electromagnetic radiation in a vacuum was the same as that of light. This result, Maxwell argued, suggested two things.

1 Visible light must be like an electromagnetic wave.
2 The speed of light depends only on the medium.

Initially, scientists were sceptical, but it was not long before Maxwell's view was accepted because of the strength of his mathematical arguments and the accumulation of evidence.

The first experiment to confirm the presence of these electromagnetic waves was conducted by Heinrich Hertz (1857–94) in 1886, seven years after Maxwell's death. Hertz used a high-voltage spark gap to produce some electromagnetic waves that were detected a few metres away. He later showed that these waves could be reflected and refracted, and had a speed of $3.0 \times 10^8\,\mathrm{m\,s^{-1}}$. This was experimental confirmation of predictions based on Maxwell's equations.

We now know that, as well as visible light, the electromagnetic wave spectrum consists of radio waves, microwaves, infrared light, ultraviolet light, X-rays and gamma radiation. They are all produced by the acceleration of charged particles, as predicted by Maxwell's equations.

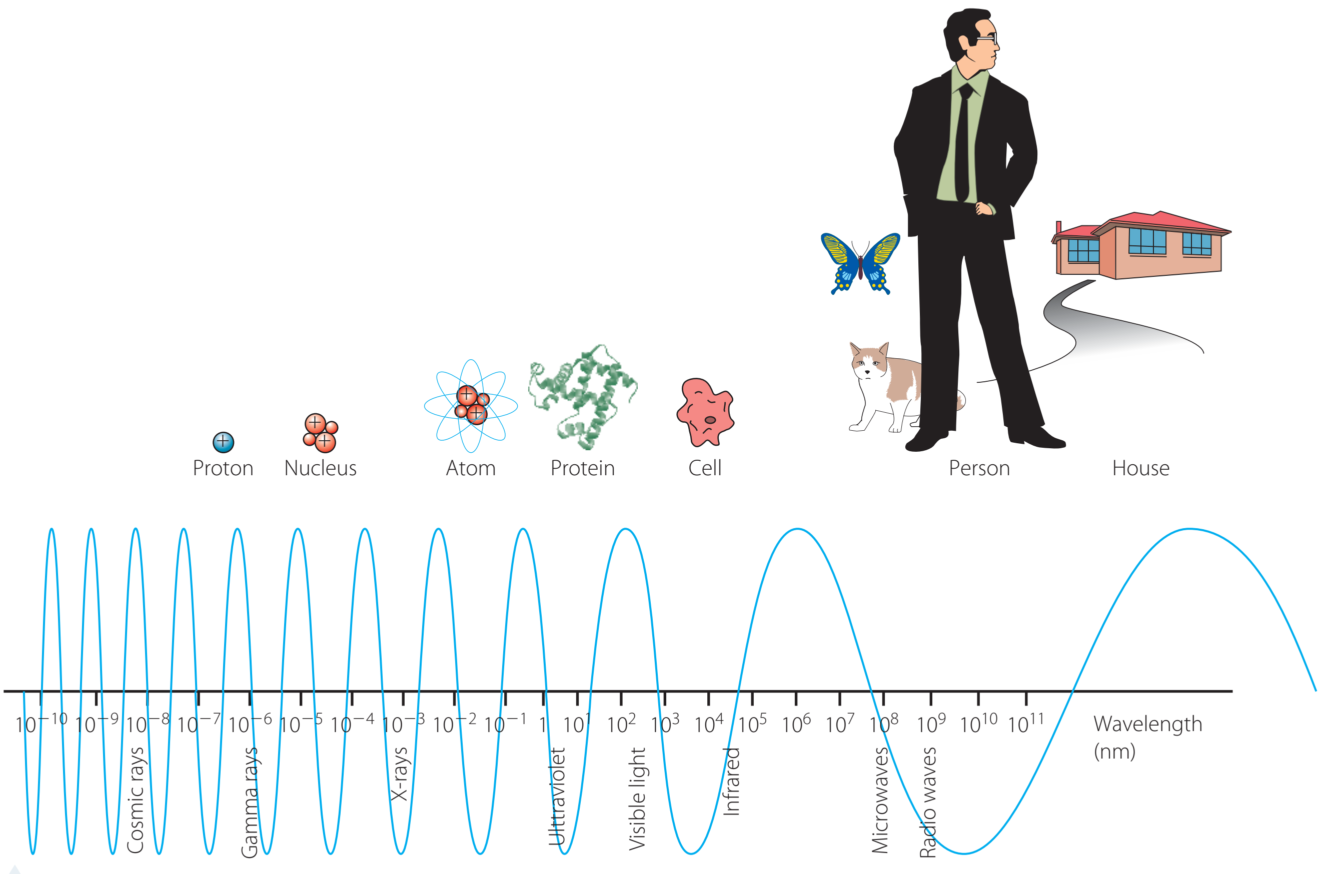

FIGURE 12.1.1 Electromagnetic spectrum

Light as an electromagnetic wave

You have met several scientific models already in your study of physics. A model is generally considered successful when it has both explanatory and predictive power. Newtonian mechanics, the study of forces and motion, is a very powerful model that helps us to understand and predict how objects will behave when forces are exerted upon them. The classical wave model, coupled with the electromagnetic field model, explains many of the behaviours of light. Light is an electromagnetic wave and behaves just like other waves – it reflects, refracts, demonstrates diffraction and interference, just as sound and other mechanical waves do.

Each of the models mentioned above replaced earlier, less successful models. Newtonian mechanics replaced Aristotelian mechanics, and the electromagnetic wave model replaced Newton's particle model

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of light. For hundreds of years it seemed as if these models could explain the behaviour of all physical systems. However, experiments conducted during the scientific revolution of the 17th century started to provide results that could *not* be explained by these models.

You may wonder why it took so long for these experiments to be conducted. Often in science new developments occur or findings are determined following the development of new instruments. Consequently, new or adapted theories need to be constructed, as such improvements in technology allow for more precise, or different sorts, of measurement. Thus, there is a strong relationship between science and technology. If the results of these experiments disagree with the existing models and theories, then new models and theories need to be developed.

Experiments from the late 19th and early 20th century showed results that did not agree with the classical models of light. Gustav Kirchhoff, whose laws are applied when analysing electrical circuits, introduced the term **black-body radiation** in 1859. This refers to the infrared, visible or higher frequency light emitted by an object due to its thermal energy. The **emission spectrum** released by a black body was not able to be explained in terms of the classical models of waves. The classical wave model was also unable to explain the **photoelectric effect** or the phenomenon of **atomic spectral lines**. These mismatches between theory and experiment suggested that physicists needed to rethink the very nature of light itself. This was the beginning of **quantum** mechanics.

The development of the quantum theory was a major revolution in science. It changed the way in which we understand the behaviour of matter and the nature of the universe. It caused divisions among scientists, and vigorous argument and debate. There is still debate among physicists as to the interpretation of aspects of the quantum theory. However, it has been a spectacularly successful theory, arguably having a greater impact on our daily lives than any other theory in science. The quantum theory led to the development of semiconductors and semiconductor devices, including diodes and transistors. All of our modern information and communication technology is based on these developments and you use them constantly. Every time you use a computer, smart phone or television you are using an application of quantum mechanics. It has been estimated that 90% or more of the wealth of the world today is directly related to quantum mechanics!

The oscillating electric and magnetic fields that make up the light wave are coupled. As the electric field varies, it creates a varying magnetic field, which in turn creates a varying electric field, and so on, and thus the wave can propagate through empty space.

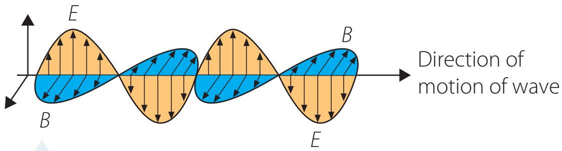

FIGURE 12.1.2 Light travels as a wave with two perpendicular waveforms – the electric field, *E*, and the magnetic field, *B*, which are at 90° to each other. Hence, light is termed an electromagnetic wave.

When the light wave meets a medium other than a vacuum, the electric and magnetic fields interact with the atoms and electrons in the material, slowing the light wave down and causing its path to refract and bend. The electromagnetic wave model of light also explains polarisation. Polarisation is a result of the electric field component of light only being allowed to oscillate in one particular plane.

Maxwell's electromagnetic wave model of light was able to successfully predict and explain all of these behaviours of light, as well as correctly predicting the speed of light. Hence, the wave model of light became the accepted model. It was a very successful model for more than a hundred years, and is still very useful; however, this electromagnetic wave model did not always correctly predict the outcome of experiments. Two experiments in particular showed that a new model was needed. These were experiments with black-body radiation and the photoelectric effect that we will explore later in this chapter.

black-body radiation
the electromagnetic radiation emitted by a black body, with a spectrum characteristic of the temperature of the body

emission spectrum
the spectrum of radiation emitted by an object, for example, black-body radiation or atomic spectra from a discharge tube

photoelectric effect
the ejection of electrons from a surface by incident photons of sufficient energy

atomic spectral lines
an emission or absorption spectrum consisting of discrete lines, characteristic of the energy levels of a particular atom or molecule; also called a line spectrum

quantum
a discrete unit or amount of some physical property, such as energy, charge, mass or angular momentum

SECTION REVIEW 12.1

REMEMBERING

1 Name one contribution to the understanding of light by each of these scientists:
   a Roemer
   b Bradley
   c Young.

2 On what does the speed of electromagnetic radiation depend?

UNDERSTANDING

3 State what Maxwell achieved by his famous equations.

4 Name two phenomena that the wave particle model of light can explain.

5 Draw a representation of an electromagnetic wave. Describe what it means.

APPLYING

6 Describe Hertz's experiment and why it was so important?

ANALYSING

7 How could electromagnetic waves be used to accelerate a small bunch of electrons?

REFLECTING

8 How does scientific knowledge develop and change? In your answer refer to the example of the speed of light and Maxwell's work.

9 Construct a table that lists two examples to support the wave nature and the particle nature of light.

12.2 Young's double-slit experiment

12.2.1 Young's double-slit introduction

In Newton's time (the 17th century) there were two competing models for light – the 'undulatory' or wave model and the 'corpuscular' or particle model. Isaac Newton himself was a proponent of the particle model while at around the same time Christian Huygens was working on his wave model. In the early 19th century experiments such as Young's double-slit experiment (sometimes called the twin-slit experiment) provided convincing evidence that light acts like a wave.

EXPERIMENT 12.2.1

The wave nature of light

When a light wave with wavelength, λ, is passed through a pair of closely spaced narrow slits, it forms an interference pattern. A pattern of maxima and minima resulting from constructive and destructive interference can be observed. If the pattern is observed at some distance L from the slits, where L is much greater than the slit separation d, the distance between any two adjacent maxima is given by the formula:

$$\Delta y = \frac{n\lambda L}{d}, \ L >> d$$

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AIM

To observe the wave interference behaviour of light incident on various arrangements of narrow slits.

MATERIALS

- laser or laser pointer
- slides with two closely spaced slits of known slit separation
- diffraction grating
- projecting screen
- tape measure

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Lasers can cause serious eye damage.	Make sure the laser is always pointing away from people or reflective surfaces. Never look directly into the laser.

RISK ASSESSMENT

PROCEDURE

1. Arrange the laser, a double-slit slide of known separation and the screen as shown in Figure 12.2.1.
2. Measure the distance from the slide to the screen.
3. Turn on the laser and observe the pattern formed on the screen.
4. Measure the distance from the central maximum to the farthest bright spot that you can clearly see. Count the number of spots between the central spot and the one you measure to.
5. Replace the slide with another double-slit slide of known slit separation and observe the pattern formed.
6. Measure the distance from the central maximum to the farthest bright spot that you can clearly see. Count the number of spots between the central spot and the one you measure to.
7. Replace the slide with a third double-slit slide of known slit separation and observe the pattern formed.
8. Measure the distance from the central maximum to the farthest bright spot that you can clearly see. Count the number of spots between the central spot and the one you measure to.
9. Replace the slide with a diffraction grating and observe the pattern formed.

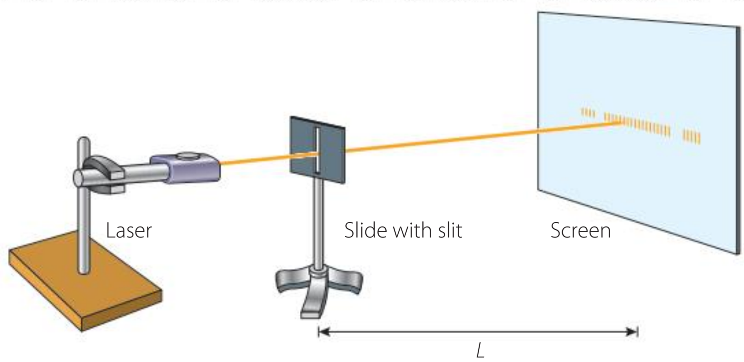

FIGURE 12.2.1 Experimental set-up

RESULTS AND ANALYSIS

1. Sketch each of the patterns you observed. Note the relative brightness of the various bright fringes.
2. For the various double-slit slides, record the number of bright fringes to each side of the central bright fringe. Record the distance from the central bright fringe (maximum) to each successive bright fringe, y, as well as the slit separation, d.

3 Explain how the patterns observed may be generated by constructive and destructive interference.

4 For the double-slit slides, use your measurements of L and y and the known slit separations, d, to calculate the wavelength of the laser light.

DISCUSSION

1 In what way does the interference pattern produced by the double-slit slide differ from the interference pattern produced by the diffraction grating?

2 How does the slit separation affect the distance between adjacent maxima (y values)?

3 Why do we need to use the wave model of light to explain the results of this experiment?

Young's double-slit experiment is one of many experiments that may be explained by the wave model of light. Light waves refract in a similar way to mechanical waves. Unlike mechanical waves, such as sound and water waves, light waves do not require a medium through which to propagate.

The wave model also correctly predicts the diffraction of light by small apertures and around small objects. In the wave model, light propagates as a wave and, hence, is spread out over a region of space, just as a water wave is spread out over a surface. It is this spreading out or delocalisation that allows diffraction and interference to occur. When more than one light wave exists in a region of space, superposition means the waves can add to give points of constructive and destructive interference.

In Young's double-slit experiment, a coherent light source is shone through a pair of narrow, closely spaced slits. The resulting interference pattern is able to be observed on a distant screen. The interference pattern of constructive interference (bright bands) and destructive interference (dark bands) provides a pattern of bright and dark fringes (Figure 12.2.2a). This is due to the path difference between the light waves coming from the two different slits.

The path difference is calculated as shown in Figure 12.2.2b.

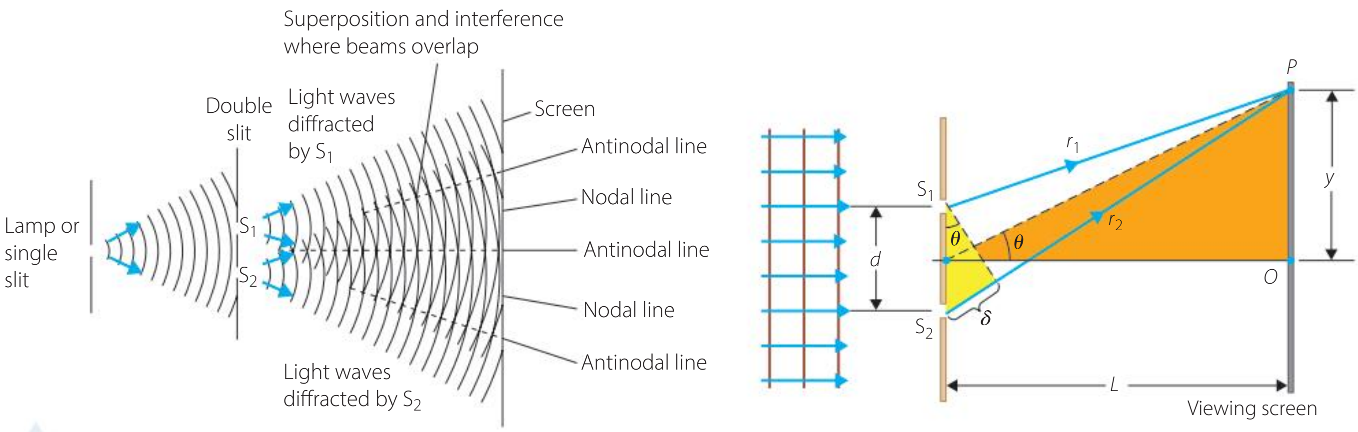

FIGURE 12.2.2 Young's double-slit experiment. (**a**) Bright fringes (constructive interference) occur along nodal lines, as crests interact with crests, while dark fringes (destructive interference) occur along antinodal lines, as crests interact with troughs. (**b**) Bright fringes occur when the path difference, δ, is equal to a multiple of the wavelength of light, i.e. $\delta = n\lambda$. Dark fringes occur when the path difference, δ, differs by exactly one half of the wavelength and the waves are precisely out of phase, i.e. $\delta = (n - ½)\lambda$.

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KEY FORMULA

$$\Delta y = \frac{n\lambda L}{d}$$

Where:

L = distance from the slits (m)

d = slit separation (m)

$L >> d$

λ = wavelength (m)

y = distance between any two adjacent maxima (m)

When the screen is a long way from the slits, the two rays, r_1 and r_2, are approximately parallel. The interference pattern produced at the screen is due to the two rays travelling different distances to reach a given point, P, on the screen. The difference in distance travelled from each source slit is termed the path difference. **Constructive interference** occurs when crests intersect with crests and troughs intersect with troughs. At these points there is an antinode or bright fringe in the pattern. This occurs whenever the path difference is equal to a whole number of wavelengths, $\delta = n\lambda$. At the points where a crest intersects with a trough, **destructive interference** occurs, as the waves are always half a cycle out of phase. This results in a node where the path difference, δ, is equal to a multiple of one half of the wavelength, i.e. $\delta = (n - ½)\lambda$.

constructive interference the superposition of waves where crests intersect with crests and troughs intersect with troughs. It is characterised by antinodes or bright fringes and occurs whenever the path difference is equal to a whole number of wavelengths, $\delta = n\lambda$

destructive interference the superposition of waves where a crest intersects with a trough, due to incoherent wave sources or sources being half a cycle out of phase. This results in a node where the path difference, δ, is equal to a multiple of one half of the wavelength, $\delta = (n - ½)\lambda$

KEY FORMULA

Path difference for constructive interference (points on an antinodal line):

$\delta = n\lambda$

Path difference for destructive interference (points on a nodal line):

$\delta = (n - ½)\lambda$

Where:

δ = path difference (m)

n = number of adjacent maxima from the central maximum (an integer)

λ = wavelength (m)

WORKED EXAMPLE 12.2.1

A coherent light source of wavelength 450 nm is shone through a pair of slits onto a screen. Determine the path difference from the slits to the third bright maximum.

ANSWER

A bright fringe indicates a maximum: that is, a point of constructive interference (an antinodal line). As it is the third bright fringe, then $n = 3$.

$\delta = n\lambda$

$\delta = 3 \times 450 \times 10^{-9}$ m

$\delta = 1.35 \times 10^{-6}$ m

WORKED EXAMPLE 12.2.2

When light of wavelength 528 nm is incident upon a pair of slits 0.05 mm apart there is an interference pattern produced on a screen 2.4 m away.

1 Determine the distance to the third-order bright fringe from the central maximum.

2 How would this differ if the wavelength of light was increased?

ANSWER

1 $\Delta y = \frac{n\lambda L}{d}$

$$\Delta y = \frac{3 \times 528 \times 10^{-9} \times 2.4}{0.05 \times 10^{-3}}$$

$\Delta y = 0.076\,\text{m}$

2 If the wavelength were increased, the numerator of the equation would increase hence the distance between the central maximum and the third-order bright fringe would increase.

SECTION REVIEW 12.2

REMEMBERING

1 State what result (bright fringe or dark fringe) occurs when two wave crests meet on a screen.

2 Contrast a nodal point (dark fringe) with an antinodal point (bright fringe).

UNDERSTANDING

3 Explain how the wave nature of light allows an interference pattern to form in the double-slit experiment.

4 In Young's double-slit experiment, state what happens to the spacing of the light and dark fringes (increases, decreases or stays the same) if:

- a the wavelength of light is decreased
- b the screen is moved closer to the slits
- c the space between the slits is decreased.

APPLYING

5 In a double-slit experiment, light of wavelength 630 nm is incident on a pair of slits spaced a distance of 1.5 mm apart. The screen is a distance 2.0 m from the slits. Describe the positions of the first three bright spots.

6 In a measurement to find the wavelength of a light source, a viewing screen is placed a distance 4.8 m from a pair of slits with a separation 0.030 mm. The first dark fringe is a distance of 4.5 cm from the centre line on the screen.

- a Determine the wavelength of the light.
- b Determine the distance between any two adjacent bright spots.

ANALYSING

7 A pair of slits spaced 0.015 mm apart is illuminated with light of two wavelengths at the same time: $\lambda_1 = 630$ nm and $\lambda_2 = 420$ nm. The viewing screen is a distance 3.0 m from the slits. At what position on the screen, other than at $y = 0$, do the maxima from the two interference patterns first coincide?

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12.3 Wave–particle duality of light

Waves and particles

We have seen that if light is shone through two slits, as in a Young's double-slit experiment, an interference pattern is seen. This is shown in Figure 12.3.1a. In this experiment, light clearly acts like a wave, and produces a pattern of high and low intensity just like a mechanical water wave, such as that shown in Figure 12.3.1b.

Experiments such as this successfully supported Maxwell's electromagnetic wave model of light; however, this wave model did not always correctly predict the outcome of experiments, such as with black-body radiation and the photoelectric effect for which light was found to behave as a particle.

So, is light a particle or a wave? The answer is that it acts like both, and the behaviour you see depends on the experiment you conduct. On its own, neither the wave nor the particle model completely explains the behaviour of light. They are *complementary* models; both are needed, and which one is used depends on the situation. We describe this need for both models as the **wave–particle duality**.

The question then arises: What about things that we know to be particles? If light is a wave and a particle, what about an electron? Or a proton? Could they also have a dual wave–particle nature?

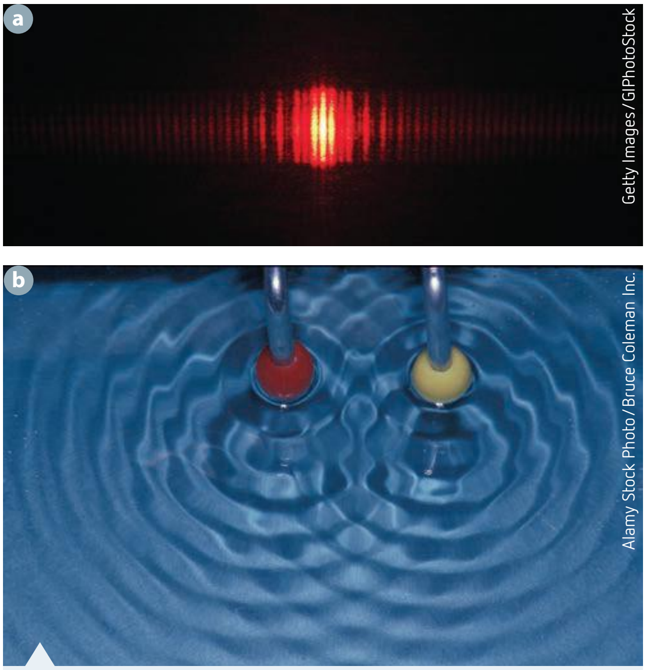

FIGURE 12.3.1 The characteristic interference patterns from (**a**) a double-slit experiment using a LASER and (**b**) two coherent wave sources in water.

wave–particle duality
the dual nature of matter and energy, requiring both the wave and the particle model to completely explain all observed behaviour of matter and energy

The de Broglie wavelength for particles

In 1924 Louis de Broglie (1892–1987) introduced the idea (in his doctoral thesis) that any moving particle has an associated wavelength. The idea was revolutionary in physics, and the implications enormously important; however, many physicists were sceptical at the time.

De Broglie claimed that a particle of mass m, moving at a velocity v, would have an associated wavelength. The wavelength λ, is known as the de Broglie wavelength.

12.3.1 Wave–particle duality

KEY FORMULA

The de Broglie wavelength

$$\lambda = \frac{h}{mv} \text{ or } \lambda = \frac{h}{p}$$

Where:

h = Planck constant, 6.63×10^{-34} J s

$p = m \times v$ = the momentum of the particle (kg m s^{-1}).

WORKED EXAMPLE 12.3.1

1 What is the de Broglie wavelength of an electron travelling at $25\,\mathrm{m\,s^{-1}}$?

2 How fast would a cricket ball with a mass of 160 g have to travel to have the same de Broglie wavelength as the electron in part **1**?

ANSWER

1 $\lambda = \frac{h}{mv}$

$m = 9.11 \times 10^{-31}\,\mathrm{kg}$, $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$

$$\lambda = \frac{6.63 \times 10^{-34}\,\mathrm{J\,s}}{9.11 \times 10^{-31}\,\mathrm{kg} \times 25\,\mathrm{m\,s^{-1}}}$$

$\lambda = 2.9 \times 10^{-5}\,\mathrm{m}$

2 $\lambda = \frac{h}{mv}$

$v = \frac{h}{m\lambda}$

$$v = \frac{6.63 \times 10^{-34}\,\mathrm{J\,s}}{0.16\,\mathrm{kg} \times 2.9 \times 10^{-5}\,\mathrm{m}}$$

$v = 1.4 \times 10^{-28}\,\mathrm{m\,s^{-1}}$

As the momentum of a photon is $p = \frac{E}{c}$, and $E = hf = \frac{hc}{\lambda}$, the momentum of a photon may also be written as $p = \frac{h}{\lambda}$. Hence, de Broglie's equation also allows us to calculate the momentum of a photon: $p = \frac{h}{\lambda}$.

KEY FORMULA

The momentum of a photon

$$p = \frac{h}{\lambda}$$

Where:

h = Planck constant, $6.63 \times 10^{-34}\,\mathrm{J\,s}$

p = momentum of the photon ($\mathrm{kg\,m\,s^{-1}}$)

λ = de Broglie wavelength of the photon

In any interaction between objects, including collisions, both energy and momentum must be conserved. We will see the results of this when we explore phenomena such as the photoelectric effect.

De Broglie waves and the Bohr model

When the idea of the de Broglie wavelength was incorporated into the Bohr model of the atom, it gave a justification for the quantisation of energies. The explanation treats the electrons as standing waves, hence there is an integer number of wavelengths that fit the orbit of the electron around the nucleus. The electrons act like standing waves on a string the exact length of the orbit.

The condition for a stable orbit is then $n\lambda = 2\pi r$.

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KEY FORMULA

The de Broglie wavelength for an electron

$n\lambda = 2\pi r$

Where:

n = an integer

λ = de Broglie wavelength of the electron (m)

r = radius of the orbit (m)

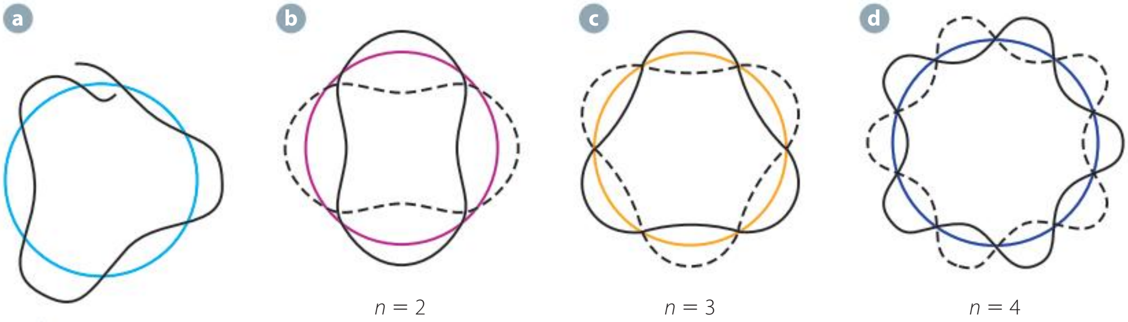

FIGURE 12.3.2 Electrons as standing waves, showing how energy levels correspond to different standing wave modes. (**a**) is not a standing wave and hence not a stable orbit, (**b**), (**c**) and (**d**) are stable orbits.

WORKED EXAMPLE 12.3.2

Calculate the longest three wavelengths of an electron in an orbit of radius 5.3 nm.

ANSWER

$$n\lambda = 2\pi r$$

$$\lambda = \frac{2\pi r}{n}$$

$$\lambda_1 = \frac{2\pi r}{1},\ \lambda_2 = \frac{2\pi r}{2},\ \lambda_3 = \frac{2\pi r}{3}$$

$$\lambda_1 = \frac{2\pi(5.3 \times 10^{-9}\ \text{m})}{1}$$

$$= 3.3 \times 10^{-8}\ \text{m}$$

$$\lambda_2 = \frac{2\pi(5.3 \times 10^{-9}\ \text{m})}{2}$$

$$= 1.7 \times 10^{-8}\ \text{m}$$

$$\lambda_3 = \frac{2\pi(5.3 \times 10^{-9}\ \text{m})}{3}$$

$$= 1.1 \times 10^{-8}\ \text{m}$$

So, what does it mean for a particle or object to have a wavelength? It does *not* mean that it follows a wiggly path, undulating up and down as it travels. What it does mean is that in some sense the particle is delocalised, or spread out, in space, just as a wave is.

Although the idea of electrons as waves gave some physical explanation for the quantisation of energy levels in the Bohr model, it did nothing to address the other failings of the model. As the wave nature of electrons became better understood, better models of the atom were developed. However, the Bohr model was of great historical importance, and can still be used to give a useful model of simple atomic systems.

SCIENCE AS A HUMAN ENDEAVOUR

WAVE–PARTICLE DUALITY SEEN IN CARBON-60 MOLECULES

It is not only electrons that may behave as waves and create interference patterns. Atoms including helium, sodium and even large molecules such as carbon-60 'buckyballs' have been used to produce interference patterns. However, the more mass an object has, the smaller its de Broglie wavelength, and the harder it is to observe its wave behaviour.

The wave–particle theory originally put forward by de Broglie is now the basis of several important technologies. Electron microscopes use the wave nature of electrons to create images of objects too small to be visualised using light. Electron microscopy has been enormously important in medicine and the biological sciences, allowing organisms such as bacteria and viruses to be 'seen' and studied. Neutron diffraction is also carried out at the OPAL reactor in Sydney, where the wave nature of the neutron is used to produce diffraction patterns and subsequently investigate the properties of materials and test for stress damage in machine parts, as well as in other applications.

Electron microscopes

In a scanning electron microscope (SEM) the electrons interact with atoms at the surface of the sample. Some electrons are reflected and some electrons are ejected from atoms at the surface. These are used to create images such as that in Figure 12.3.3b. SEMs can be used to 'see' objects as small as 1 nm. In a transmission electron microscope (TEM) the electrons are passed through a very thin sample and collected by a detector on the other side. The image can be formed by the sample simply absorbing some electrons and allowing others to pass through, like a shadow. The electrons can also produce a diffraction pattern on the other side. TEMs can be used to 'see' objects as small as 0.1 nm.

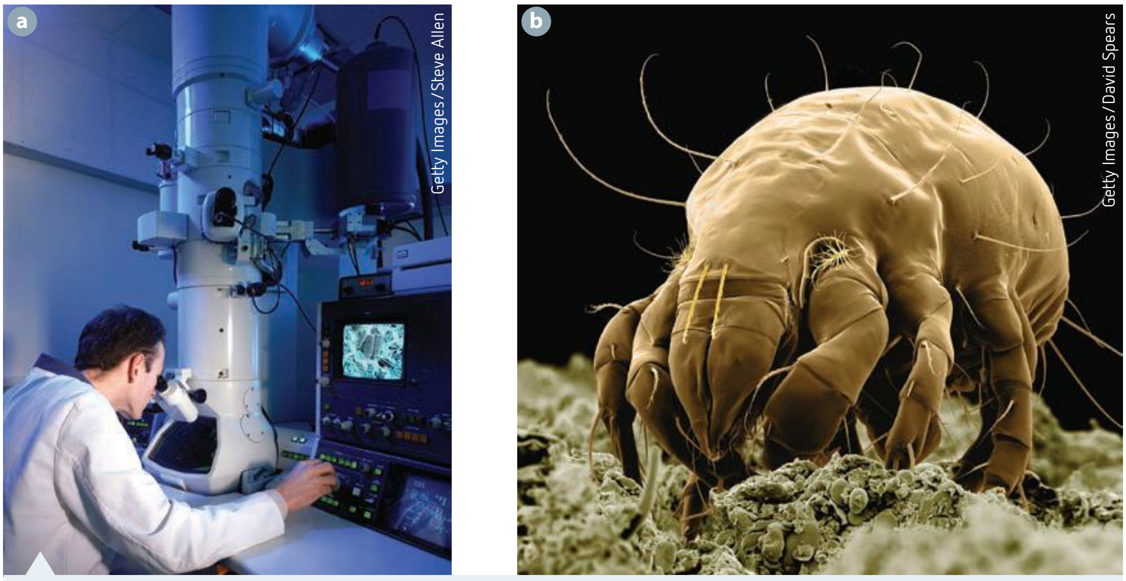

FIGURE 12.3.3 (**a**) An electron microscope; (**b**) A photo taken with an electron microscope of a mite that is less than 1 mm long.

Quantum tunnelling

Quantum tunnelling is another phenomenon that is unexplainable by classical physics. Particles 'tunnel' through barriers they do not have enough energy to 'jump'. The Tonomura experiment, named after Dr Tonomura Akira, was voted by physicists to be the most beautiful experiment ever performed. In this experiment the wave property of electron beams was found to create a vector potential to act on a beam of charged particles.

From the 1920s, what is now known as modern quantum mechanics was developed by physicists including Niels Bohr (1885–1962), Louis de Broglie (1892–1987), Werner Heisenberg (1901–76) and Erwin

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Schrödinger (1887–1961). The term 'modern' is used to distinguish it from earlier quantum mechanics, such as the Bohr model.

The new model that they came up with included the idea of uncertainty. In any experiment, as you know, there will be uncertainties due to various sources including equipment limitations. Heisenberg proposed that there is also an intrinsic uncertainty and that the behaviour of particles is **probabilistic**. In other words, it cannot be predicted with certainty, no matter how much you know about the particles. In classical mechanics the behaviour of all objects including subatomic particles is **deterministic** and completely predictable, once you have enough information. This is a fundamental difference between quantum mechanics and classical mechanics.

In the double-slit experiment, a probability wave associated with each particle passes through both slits. These probability waves interfere on the other side. When we measure which slit the wave passes through, we no longer have the probability wave passing through both slits, so we no longer have an interference pattern. The modern probabilistic model of quantum mechanics correctly predicts the results of these types of experiments.

probabilistic not deterministic, unable to be predicted regardless of how much information is known

deterministic predictable, able to be determined if enough information is available

SECTION REVIEW 12.3

REMEMBERING

1 Define 'probabilistic'.

UNDERSTANDING

2 A photon, an electron and a neutron all have the same wavelength. Rank the velocities, from fastest to slowest, of the three particles with reference to the de Broglie wavelength formula and their relative masses.

3 Explain why we would not notice the wave nature of a cricket ball moving at $25\,m\,s^{-1}$.

APPLYING

4 A bullet of mass 50 g travels at $1.2 \times 10^{3}\,m\,s^{-1}$. Determine its de Broglie wavelength.

5 A beam of electrons with de Broglie wavelength 1.5×10^{-10} m is incident on an electron biprism that behaves like a double slit with a separation of 4.5×10^{-9} m. If the detectors are arranged on a screen a distance 20 cm from the slits, what is the location of:

- a the first interference maximum?
- b the first interference minimum?
- c the second interference maximum?

ANALYSING

6 A photon of energy 3.6×10^{-15} J collides with a stationary electron that is free to move.

- a What is the magnitude of the momentum of the photon before the collision?
- b After the collision, the photon returns along its original path and the electron moves forwards with a momentum of $2.1 \times 10^{-23}\,kg\,m\,s^{-1}$.
  - i What is the de Broglie wavelength of the electron after the collision?
  - ii Is the wavelength of the photon now the same as, greater than, or less than before? Give reasons for your answer.
  - iii Calculate the wavelength of the photon before and after the collision.

REFLECTING

7 Think of all the devices that you have used today that rely on semiconductors. How would your life be different if quantum physics had not been developed?

8 Read about the 'many worlds' interpretation of quantum mechanics. Write a brief summary of this interpretation. Perform research to find out about hidden variables or probabilistic model of quantum mechanics and reflect on how the many worlds interpretation fits with either of these models. Justify this interpretation based on the arguments of one of these two models.

12.4 Black-body radiation

spectrum
the distributed components of light or another wave arranged by frequency (or wavelength)

12.4.1 Black-body radiation

continuous spectrum
a spectrum containing radiation of all wavelengths; for example, a rainbow is composed of the various wavelengths of the visible spectrum

black body
an object with a perfectly absorbing surface that emits radiation with a spectrum that is characteristic of the temperature of the object

All objects continuously radiate energy in the form of electromagnetic waves. At any non-zero temperature, a body emits radiation of all wavelengths, but the distribution or **spectrum** of wavelengths depends on its temperature. If an object is very hot, you can see the light that is being emitted; for example, you can see the glowing coals in a fire or the filament of a light globe. At low temperatures, the wavelengths of the emitted radiation are mainly in the low frequency, infrared region and cannot be seen, although you may still be able to feel the radiation as heat with your skin. Measurements show that hotter objects emit more electromagnetic radiation, and that greater amounts of this radiation is at shorter wavelengths (higher energies). Hence, if you take a piece of metal and heat it up slowly, it will glow a dim red at first, then bright yellow and eventually very bright white.

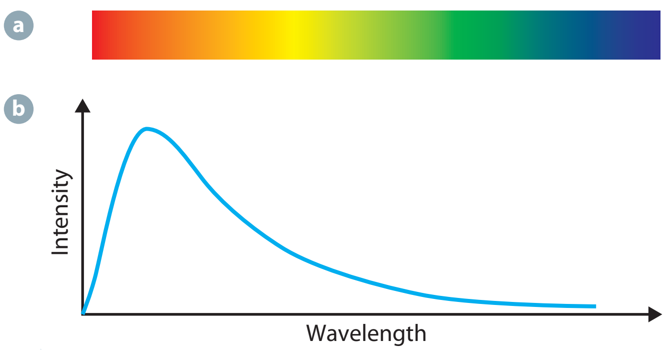

FIGURE 12.4.1 (**a**) A continuous emission spectrum. (**b**) The intensity of emitted radiation is a function of wavelength for a continuous black body spectrum.

Measurement of the intensity of emitted radiation as a function of wavelength shows that it is a continuous distribution of wavelengths from the infrared, through the visible to the ultraviolet. This distribution is called a **continuous spectrum**. The shape of the spectrum depends only on the temperature of the object, and not on any of its other properties. A continuous spectrum is shown in Figure 12.4.1.

What is a black body?

A **black body** is an ideal surface that completely absorbs all wavelengths of electromagnetic radiation incident on it. Hence it is a *black* body. Such a surface will also be a perfect emitter of electromagnetic radiation at all wavelengths. The black-body radiation is characteristic of the temperature of the black body and is mainly in the infrared part of the electromagnetic spectrum when the body is at room temperature.

Although a true black body is only a theoretical concept, it can be closely simulated in a laboratory. Consider a cavity (hollow space) that has the interior walls blackened and which is kept at a constant temperature (Figure 12.4.2). If a small hole is made in the wall of the cavity, it will act like a black-body radiator. Any radiation that falls on the hole from the outside will pass through it. After multiple reflections, the radiation will be absorbed by the interior surfaces. As the cavity is in thermal equilibrium with its surroundings, the interior surfaces will emit radiation at the same rate at which it is absorbed. The radiation that escapes depends only on the temperature of the cavity. It is not affected by the size of the cavity or the material of which it is made.

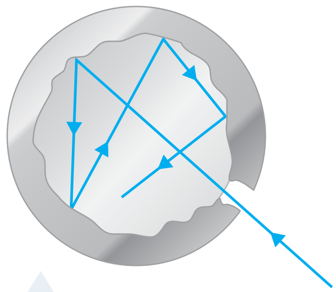

FIGURE 12.4.2 The opening to a cavity is a good approximation of an ideal black body. Note that it is the *opening* to the cavity that is the black body, not the entire hollow object. The hole acts as a perfect absorber.

Remember that this is an idealised object, not a real one. In practice, materials that absorb most of the light incident on them are good approximations of a black body. This is where the term 'black body' comes from – black objects absorb most of the light incident on them, regardless of wavelength.

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The black-body emission spectrum

The black-body model is useful because it allows us to determine the temperature of distant objects. For example, we can estimate the surface temperature of the Sun by measuring its electromagnetic spectrum.

In 1893 Wilhelm Wien (1864–1928) derived a relationship between the position of the peak wavelength at which radiation is emitted and the temperature of a black body. He used the idealised black-body cavity model to derive the relationship, now known as Wien's displacement law or simply as Wien's law. The position of the peak wavelength is given by Wien's law.

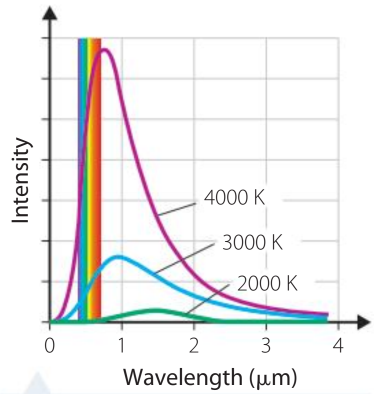

FIGURE 12.4.3 Intensity and distribution of wavelengths of radiation from a black body at different temperatures. Note that the peak in the radiation curve gets higher and shifts to shorter wavelengths as the temperature increases.

KEY FORMULA

$$\lambda_{max} = \frac{b}{T}$$

Where:

λ_{max} = peak wavelength (m)

T = absolute temperature (K)

b = Wien's constant, 2.898×10^{-3} m K

EXPERIMENT 12.4.1

Black-body radiation

AIM

To observe black-body radiation from an incandescent light globe filament as the temperature of the filament changes.

MATERIALS

- 12 V incandescent light globe
- power pack or variable 12 V DC power supply
- spectroscope (optional)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Light globes can get very hot.	Do not touch the globe and make sure that the globe is left to cool before placing it away.

RISK ASSESSMENT

PROCEDURE

1 Make sure the power supply is switched off and that the voltage is turned to zero. Connect the light globe across the terminals of the power supply. Turn on the power supply and turn up the voltage from zero until the globe just starts to glow. Record the colour that the globe appears to glow. If you have access to a spectroscope, record the wavelength of the colour of light emitted.

2 Increase the voltage to the next increment. Observe and record the colour changes and wavelengths of light emitted. Repeat for all increments to the largest safe value.

»

» RESULTS AND ANALYSIS

1 What colour range does the globe glow between the lowest and highest voltages?
2 What wavelengths were observed as the voltage increased?
3 What is happening to the temperature of the filament as the voltage is increased?

DISCUSSION

1 In what way does the increase in temperature affect the light emitted from the light globe?
2 At what voltage is the highest wavelength of light emitted? How does this relate to the energy emitted?

WORKED EXAMPLE 12.4.1

The surface of the Sun has a temperature of approximately 5800 K. If we treat the Sun as a black body, what is the peak wavelength of the radiation emitted? Refer to an electromagnetic spectrum to describe the part of the spectrum to which this wavelength belongs.

ANSWER

$$\lambda_{max} = \frac{b}{T}$$

$$= \frac{2.898 \times 10^{-3}\ \text{m K}}{5800\ \text{K}}$$

$$= 5.0 \times 10^{-7}\ \text{m}$$

This is yellow (visible) light.

WORKED EXAMPLE 12.4.2

The black-body spectrum in Figure 12.4.4 is for the star Antares. What is the surface temperature of Antares?

ANSWER

$$b = 2.898 \times 10^{-3}\ \text{m K}$$

$$\lambda_{max} = \frac{b}{T}$$

$$800 \times 10^{-9}\ \text{m} = \frac{2.898 \times 10^{-3}\ \text{m K}}{T}$$

$$T = \frac{2.898 \times 10^{-3}}{800 \times 10^{-9}}$$

$$T = 3622.5\ \text{K}$$

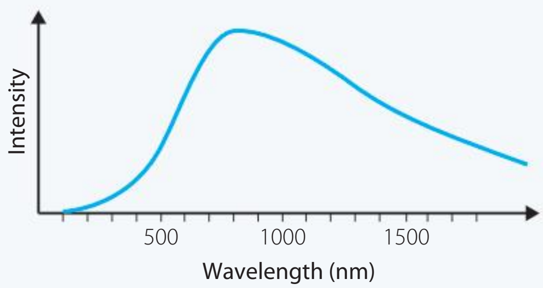

FIGURE 12.4.4

Wien's law was a successful model in that it accurately predicted the position of the peak wavelength. However, there were still two problems. First, there was no theory that explained the shape of the curve. Second, Wien's law was based on an idealised system – a cavity with a small hole. It is difficult to see how this theoretical model could represent the surface of a solid piece of material or of a star such as the Sun.

Classically, it was thought that the thermal radiation originated from oscillating charged particles near the surface of an object. Previously we saw that oscillating charges are a source of electromagnetic

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waves. This is how antennae work. The oscillating charges in the antenna produce an electromagnetic wave of the same frequency as the oscillations.

You may recall that the temperature of a material is a measure of the average kinetic energy of the atoms of that material. In a gas or a liquid the particles are free to move. The higher the temperature, the more kinetic energy the particles have and the faster they move. In a solid material, the atoms are not free to move, so this kinetic energy is observed as vibrations, and the higher the temperature, the higher the frequency of vibration. And as you know, atoms are made up of smaller particles including protons and electrons, which are charged. Therefore, this theory provided the oscillating charges needed to produce the electromagnetic radiation.

Now consider again the ideal model of the black-body cavity. If the atoms on the inside surface are acting as little antennae, we would see standing waves set up between the walls of the cavity. The waves produced by the vibrating atoms in the inside surface would reflect from the opposite surface. If the waves have the right wavelength, a standing wave is set up, just like a standing wave on a string. We call these standing waves **modes of vibration**.

modes of vibration characteristic patterns of oscillation, usually with a discrete set of allowed frequencies.

SCIENCE AS A HUMAN ENDEAVOUR

THE 'ULTRAVIOLET CATASTROPHE'

Classically, all the possible modes of vibration would be equally probable, and the total energy would be divided equally between them all. In any cavity, more short wavelength modes would be able to fit in the cavity. This means more short wavelength radiation should be emitted through the hole. As the temperature of the cavity increased, so should the total energy. As the energy increased, the energy associated with the short wavelengths (ultraviolet, X-rays and gamma rays) would approach infinity. According to this theory, even a regular heater should be emitting dangerous amounts of X-rays and gamma rays!

Figure 12.4.5 shows a comparison of a theoretical spectrum based on this model and a measured spectrum. This mismatch between theory and experiment was called the 'ultraviolet catastrophe'; however, it was only a catastrophe for the theory that predicted it. A new theory was needed to solve these problems. The German physicists Max Planck and Albert Einstein solved the problem by postulating Planck's quanta, that we now know as photons.

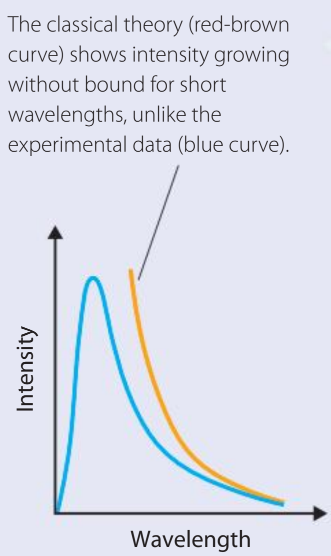

FIGURE 12.4.5 Comparison of the classically predicted and experimental black-body spectra

SECTION REVIEW 12.4

REMEMBERING

1 Define 'continuous spectrum'.

2 Name the scientist who derived the relationship between the position of the peak wavelength emitted and the temperature of a black body.

UNDERSTANDING

3 Describe why a surface may be described as a 'black body'.

4 Sketch a typical 'intensity versus wavelength' graph for a black body. Indicate the peak wavelength.

APPLYING

5 The surface of a star is measured to have a peak wavelength of 4.25×10^{-7} m (425 nm). Determine the surface temperature of the star. Use $b = 2.898 \times 10^{-3}$ m K.

6 A black body is known to have a surface temperature of 4000 K. Use the value of $b = 2.898 \times 10^{-3}$ m K to determine the peak wavelength.

ANALYSING

7 Describe the relationship between the peak wavelength emitted from a black body and its surface temperature.

8 Figure 12.4.6 shows the black-body radiation spectrum for the star Vega.
   a What is the peak wavelength?
   b Calculate the surface temperature of Vega.

9 The filament of an incandescent light globe can be modelled as a black body. A tungsten filament reaches a temperature of 2900 K.
   a What is its peak wavelength?
   b Explain why such light globes emit more radiation in the infrared region than in the visible part of the electromagnetic spectrum.

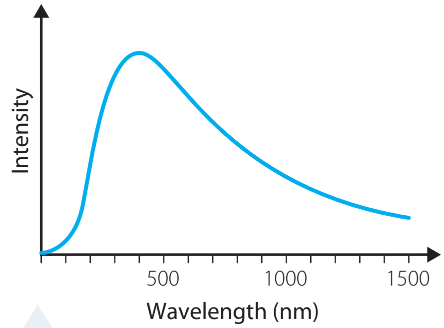

FIGURE 12.4.6

12.5 Planck's quanta and photon characteristics

In 1900 Max Planck (1858–1947) used 'lucky guesswork' (as he called it) to derive a formula that correctly matched the experimentally observed spectrum. Planck proposed that the atoms could only oscillate with **discrete** energies, given by the formula $E = nhf$, where h is now known as the **Planck constant**.

discrete
able to take only specific values, not continuous; for example, a line spectrum is a discrete spectrum

Planck constant
the constant of proportionality between energy and frequency for photons: $h = 6.63 \times 10^{-34}$ J s

KEY FORMULA

$E = nhf$

Where:

n = an integer

f = frequency of oscillation (Hz)

h = Planck's constant = 6.63×10^{-34} J s or 4.14×10^{-15} eV s

Note: Kinetic energy (E_k) and the Planck constant, h, must have consistent units. If E_k is in J then h must be in J s. If E_k is in eV then use h in eV s.

quantised
existing in discrete amounts, not able to be divided into arbitrarily small amounts

This was a radical proposition. It means that the energy of the oscillators is **quantised**, that is, it may only take the discrete values given by the equation, rather than any possible value within a continuous range.

Planck deduced that the oscillators could only emit and absorb electromagnetic radiation (i.e. light) in packets of specific energies. He called these packets of energy 'quanta'. The amount of energy emitted is equal to the amount of energy lost by an oscillator when it goes to a lower energy state. For example, if an oscillator goes from an energy of $E_3 = 3hf$ to $E_2 = 2hf$, the energy lost is $E_3 - E_2 = 3hf - 2hf = hf$.

One quantum of light has energy $E = hf$. The universal wave equation $c = f \times \lambda$ may also be applied, using $f = \frac{c}{\lambda}$, to determine these discrete energies.

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KEY FORMULA

$$E = nhf = nh\frac{c}{\lambda}$$

Where:

n = an integer

f = frequency of oscillation (Hz)

h = Planck's constant = 6.63×10^{-34} J s

$c = 3.00 \times 10^{-8}$ m s^{-1}

λ = wavelength of light (m)

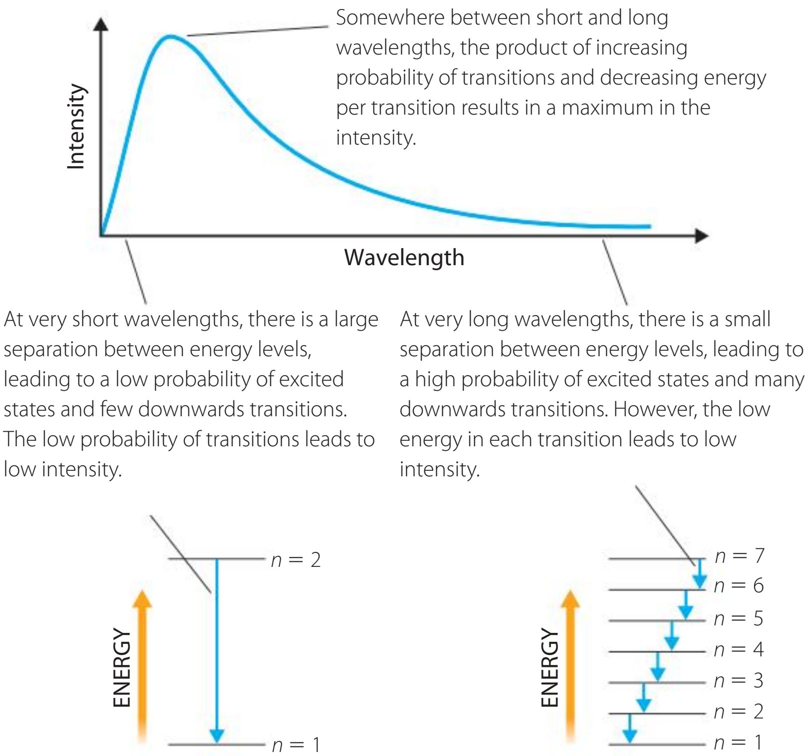

FIGURE 12.5.1 Planck's model for black-body radiation

Planck combined this idea of quantisation with two ideas from classical statistical mechanics. First, the probability of an oscillator having a particular energy decreases as the energy increases. Hence, the probability of an atom being in a higher energy state (termed an excited state) is lower. This means that the intensity of radiation at high frequencies (short wavelengths) is small. Second, the probability of a *change* in energy decreases with the relative gap between energy levels. The relative gap is larger for lower energies, or long wavelengths, so intensity is again low at these wavelengths. In between these extremes, we see the peak intensity observed in the experimental spectra.

The quantisation of energy was such a revolutionary departure from the classical physics of Newtonian mechanics, electromagnetism and thermodynamics that even Planck was reluctant to accept his own idea. Although Planck had discovered a mathematical way of explaining the shape of the black-body spectrum, he was concerned that there was no physical model for how the energy could be in these discrete packets. It was Einstein who put physical meaning to Planck's quantum hypothesis.

WORKED EXAMPLE 12.5.1

A quantum of energy has a wavelength 5.8×10^{-5} m.

1 Determine the frequency of this quantum.

2 Determine the energy of this quantum.

ANSWER

1 $c = f\lambda$

$$f = \frac{c}{\lambda}$$

$$f = \frac{3.00 \times 10^8 \text{ m s}^{-1}}{5.8 \times 10^{-5} \text{ m}}$$

$f = 5.2 \times 10^{12}$ Hz

2 $E = hf$

$E = 6.63 \times 10^{-34}$ J s $\times 5.2 \times 10^{12}$ Hz

$E = 3.4 \times 10^{-21}$ J

WORKED EXAMPLE 12.5.2

Determine the frequency of a quantum of energy with $E = 2.5$ keV.

ANSWER

$E = hf$

$$f = \frac{E}{h}$$

$$f = \frac{2.5 \times 10^3 \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$f = 6.0 \times 10^{17}$ Hz

WORKED EXAMPLE 12.5.3

A packet of energy 'quanta' of 7.96×10^{-19} J is emitted as a photon of light following an electron falling to a lower energy state, from $n = 3$ to $n = 1$.

1 Determine the frequency of light emitted.

2 Calculate the wavelength of the photon.

3 Convert the magnitude of the energy from joules to electron volts.

ANSWER

1 $E = nhf$

7.96×10^{-19} J $= 2 \times 6.63 \times 10^{-34}$ J s $\times f$

$f = 6.00 \times 10^{14}$ Hz

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2 $c = f\lambda$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3.00 \times 10^8 \text{ m s}^{-1}}{6.00 \times 10^{14} \text{ Hz}}$$

$$\lambda = 5.00 \times 10^{-7} \text{ m or } 500 \text{ nm}$$

3 $$E = \frac{7.96 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ J eV}^{-1}}$$

$$E = 4.98 \text{ eV}$$

SCIENCE AS A HUMAN ENDEAVOUR

DR ROBERT COLMAN – BLACK-BODY RADIATION AND CLIMATE CHANGE MODELLING

Dr Robert Colman is head of the Climate Change Processes Team in the Centre for Australian Weather and Climate Research at the Bureau of Meteorology in Melbourne. Dr Colman did a PhD in physics at the University of Melbourne, after studying maths and physics as an undergraduate. He has been a lead author contributing to the United Nations Intergovernmental Panel on Climate Change (IPCC) climate change assessments. The IPCC assesses scientific, technical and socio-economic information on climate change and is currently in its sixth assessment cycle. The IPCC is a massive international collaboration that reviews the work of thousands of scientists, providing rigorous scientific information to decision makers, including governments such as our own.


Courtesy of Dr. Robert Colman

FIGURE 12.5.2 Dr Robert Colman

The Climate Change Processes Team conducts research into the causes of past climate changes and into the changes we can expect in temperature, rainfall and other climate features in the future. The mathematical models used are based on fundamental physical principles, such as the laws of conservation of energy, mass and momentum, as well as a wealth of experimental observations.

Black-body radiation is one of the important physical processes taken into account. The combined Earth–atmosphere system acts as a black body. It absorbs radiation from the Sun, and also radiates energy out into space.

The amount of energy absorbed depends on the **albedo** (reflectivity) of Earth's surface and atmosphere.

albedo the ratio of light reflected by a surface to light incident on it; a surface with an albedo of 1 is perfectly reflective, and an albedo of 0 is perfectly absorbing

The albedo is a measure of how much light is reflected. The more light is reflected, the less energy is absorbed. Snow and sea ice have a much greater albedo – they reflect more light – than rock or ocean. This means that more light is reflected. As polar snow and ice melts, more energy is absorbed by the darker surface beneath. This leads to an increased rate of heating.

The amount of energy radiated by Earth must ultimately match the energy it absorbs from the Sun for the Earth's temperature to remain relatively constant. However, very little radiation from the surface can escape directly to space. Instead, it is mostly absorbed in the atmosphere by water vapour, CO_2 and other 'greenhouse gases'. It is then re-radiated, until it eventually escapes back into space. The altitude of this 'final escape' is about 5 km; it is the temperature of the 'black-body radiator' at this level, not at the surface, that would be seen by a space traveller looking at Earth. Here the atmosphere is much cooler than at the surface, so the atmosphere makes the black-body radiator much less 'efficient' than if the radiation could escape directly from the surface. This is the 'natural' greenhouse effect, and keeps the surface about 33°C warmer than it would otherwise be, at a comfortable 15°C on average.

Human emissions raise the height at which this black-body radiation occurs, because they introduce more greenhouse gases, such as CO_2 and CH_4. This makes Earth an even less efficient radiator, so the whole planet and atmosphere must warm up to restore the balance: greenhouse warming!

The models that Dr Colman and his team use need to include these effects. Climate modelling is a challenging task because there are so many factors that affect the climate. These include changes in the atmosphere, oceans, ice sheets and land surface. Modelling these complex interactions requires programs with millions of lines of code, run on the world's most powerful supercomputers. It also requires major new collaborations between atmospheric physicists, oceanographers, plant biologists, mathematicians and computer scientists.

To test the models, scientists use them to simulate observed features of recent climate and past climate changes. If a model can accurately reproduce measured temperatures and rainfall from the past, it raises confidence that the model will be able to predict temperatures and rainfall in the future. Climate models are a lot like the weather models used to forecast weather, but climate models make projections for decades into the future.

Figure 12.5.3 shows the results from two sets of models compared to measured data for temperature variations in the last hundred years. The black lines show the temperature anomaly. Temperature anomaly is the difference between recently recorded temperatures and long-term average temperatures. The lower set of models, shown in blue, do not include anthropogenic (human-caused) effects. The upper set of models, shown in orange, does include these effects and fits the data much better. This shows that models do not reproduce the observed warming unless the effects of greenhouse gases emitted as a result of human activities are included. Multiple sources of evidence, including results such as this, have convinced climate scientists that humans have significantly changed Earth's climate, and that even larger changes are to come.

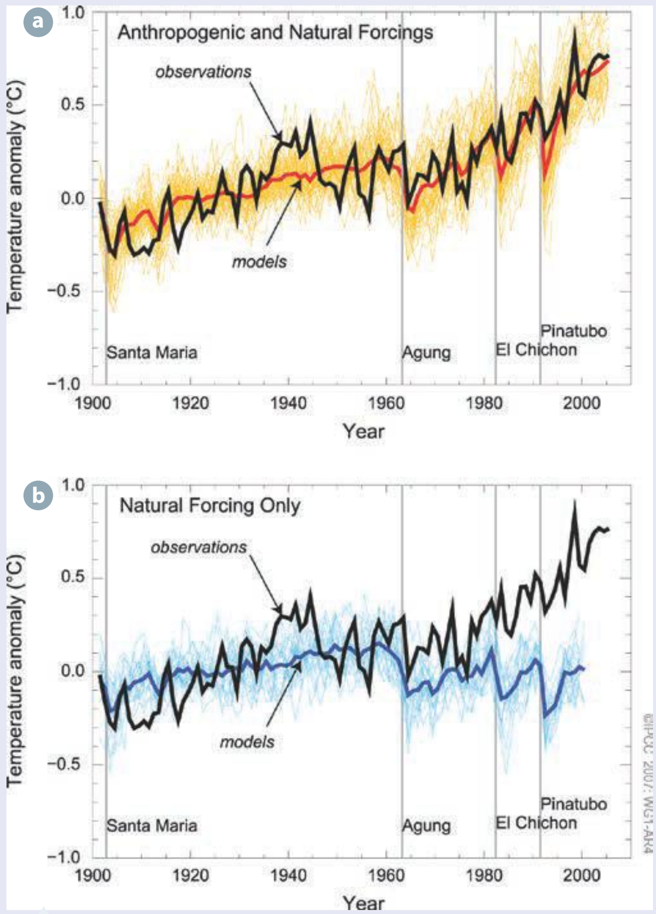

FIGURE 12.5.3 Models of climate change that (**a**) do include anthropogenic effects and (**b**) do not include anthropogenic effects. The black lines are measured data and the coloured lines are simulations from the models. The heavy coloured lines are averages from all the individual models. 'Temperature anomaly' is the difference in global temperature from the long-term mean. Grey vertical lines show when prominent volcanoes erupted.

QUESTIONS

1 Define 'anthropogenic'.

2 If a year is colder than usual, would you expect the temperature anomaly for that year to be positive or negative? What if it was hotter than usual?

3 How have advances in information technology contributed to improved climate modelling?

4 What effect do you think clouds have on the albedo of Earth? How do you think this affects the solar energy absorbed by Earth? What about the energy radiated?

5 What effect do you think large volcanic eruptions might have on the climate?

6 There is still debate in the media and among politicians about the causes and even the existence of climate change. Research and evaluate the arguments put forward by climate-change sceptics. Reflect on your own views on this topic. Write a brief summary of your position in the debate, and include evidence to support your opinions.

SECTION REVIEW 12.5

REMEMBERING

1 Name two classical models in physics.

2 Define 'quantised'.

UNDERSTANDING

3 Show that the units for the Planck constant must be J s.

4 Vega is a blue star and Antares is a red star. Which is hotter? Explain your answer.

APPLYING

5 An atomic oscillator has frequency $f = 6.1 \times 10^{12}$ Hz, and is in the $n = 3$ state.

- **a** What is the energy of this oscillator?
- **b** What frequency of light will be emitted if it transitions to the $n = 2$ state?

6 Determine the frequency and wavelength of a quantum of energy with $E = 1.7 \times 10^{-19}$ J.

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REFLECTING

7 For climate scientists, the evidence of anthropogenic global warming is clear. However, many people, including government and industry leaders, do not accept the evidence. What social, cultural and economic factors might be important in decision and policy making about the climate? Why do you think that some politicians are reluctant to make policy decisions that require action to be taken to mitigate the risks associated with climate change?

12.6 The photoelectric effect

Quantisation and the photoelectric effect

Planck introduced the idea of quantised electromagnetic energy to explain the black-body spectrum. It was already known at the time that matter was quantised. Scientists accepted that matter came in discrete quanta, or atoms, and that atoms combined to form molecules, and so on. In 1897, J.J. Thompson (1856–1940) discovered electrons when he realised that cathode rays were made of tiny negatively charged subatomic particles. So, although it was known that atoms could be broken down into smaller components, these components were themselves quantised into discrete particles. Although the idea of quantisation of matter was already well established while the quantum model was being developed, the idea of quantisation of energy was completely new, as it contradicted the accepted model of light as a wave at the time. The photoelectric effect provided the evidence needed for quantisation of energy to be accepted.

Einstein's explanation of the photoelectric effect gave a physical meaning to the idea of quantisation of energy of electromagnetic radiation. It meant that, in some circumstances, light behaved like particles. The term '**photon**' was introduced in 1926 by the chemist Gilbert Lewis (1875–1946) to describe these particles. Further experiments by Arthur Holly Compton (1892–1962) provided evidence for the existence of photons. Compton scattered single photons from electrons and found that only particular energies were absorbed. Photons are now accepted as particles with zero rest mass, and with energy given by $E = hf$.

photon
a particle or quanta of light, having energy $E = hf$

The photoelectric effect was first observed by Heinrich Hertz in 1887. He observed that when light is shone on a highly polished metal surface, electrons can be emitted from the surface. One of Hertz's assistants, Philipp Lenard (1862–1947), performed experiments to investigate the photoelectric effect in detail. Lenard developed much of the equipment needed to make quantitative measurements of the intensity and energy of the emitted 'cathode rays', as they were called at the time. Other physicists, including Robert Millikan (1868–1953), also investigated the effect.

Their data showed that:

- no electrons were emitted unless the frequency of the light was above some minimum threshold (or critical) frequency, f_0, regardless of the intensity of the light
- the number of electrons (the current), if emitted, was proportional to the intensity. It did not vary with the frequency of the light (as long as the frequency was of the threshold frequency or greater).

When light is shone through the quartz window at the polished metal plate, X, **photoelectrons** are emitted. The photoelectrons are attracted to the positively charged metal plate, Y. The ammeter, A, measures the current of photoelectrons produced – the **photocurrent**.

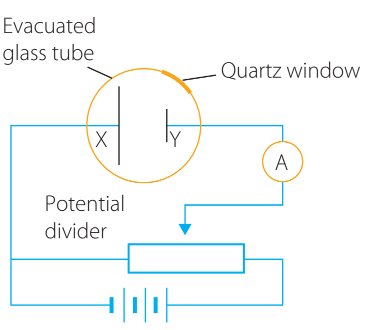

FIGURE 12.6.1 The photoelectric effect apparatus

photoelectron
an electron ejected from a metal surface following absorption of a photon of sufficient energy

photocurrent
the current formed by electrons ejected from a surface by incident photons

threshold frequency, f_0 the minimum frequency of light needed to eject an electron from a metal surface

stopping voltage the reverse bias voltage required to stop the flow of photoelectrons in a photoelectric effect experiment

work function the energy required to eject an electron from a metal surface; effectively, it is the ionisation energy for the bulk material

Using this apparatus, experiments have shown the following.

- A photocurrent is only produced when the frequency of the light is above some minimum value, termed the **threshold frequency, f_0**. This implies a threshold wavelength also, $\lambda_0 = \frac{c}{f_0}$, above which no photocurrent is produced.
- The size of the current (the number of photoelectrons produced) depends on the intensity of the light but not on the frequency, as long as the frequency is above the threshold f_0.
- There is no time delay between light being incident on the metal and photoelectrons being emitted, regardless of intensity.
- Different metals have unique, characteristic threshold frequencies.

The voltage divider is used to vary the potential difference between X and Y. When the potential difference is reversed, the maximum kinetic energy of the emitted photoelectrons can be measured. This is called a reverse bias voltage. In this case, plate Y is negative and repels the photoelectrons.

The reverse bias voltage between X and Y is slowly increased and the current observed until it drops to zero. At this point, the potential difference is equal to the maximum energy per unit charge of the electrons. This potential difference is called the **stopping voltage**, V_s. Hence, the product of the potential difference and the charge on the electron, qV_s, is equal to the maximum kinetic energy of the photoelectrons; that is, $E_{k(max)} = qV_s = eV_s$.

KEY FORMULA

The kinetic energy of a photoelectron

$E_{k(max)} = qV_s = eV_s$

Where:

E_k = kinetic energy (J)

q = charge (C)

V_s = stopping voltage (V)

e = charge on electron (1.60×10^{-19} C)

The photoelectric effect experiment provides the characteristic results that the maximum kinetic energy (measured using $E_k = qV_s$) depends on the frequency of light, but *not* on the intensity, as shown in Figure 12.6.2. Note that different metals have varied work functions, hence different characteristic threshold frequencies. The **work function**, W, is equivalent to the product of the Planck constant and the threshold frequency, $W = hf_0$.

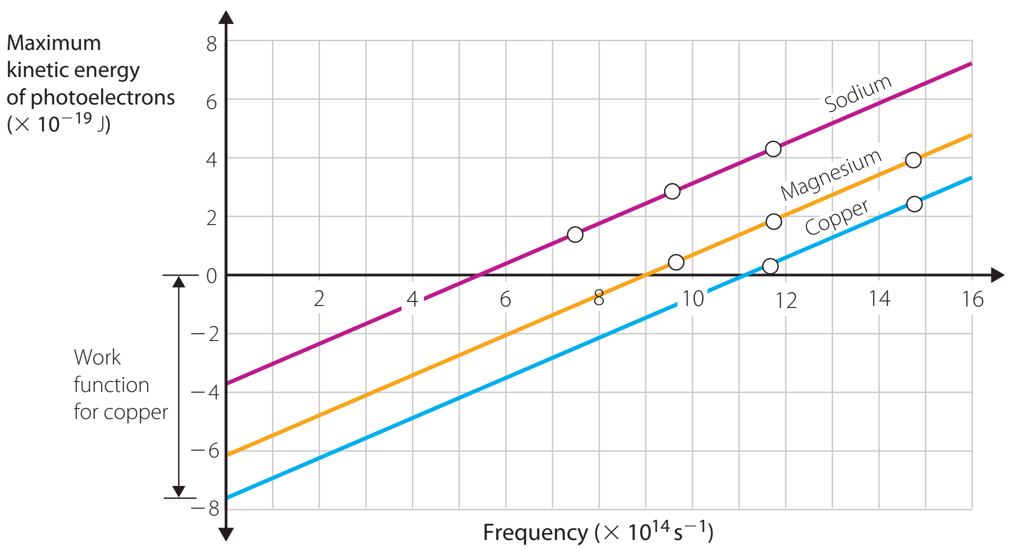

FIGURE 12.6.2 Experimental results for the photoelectric effect. Maximum kinetic energy as a function of frequency for three different metals. The data is extrapolated to determine the work functions for each metal.

The electromagnetic wave model of light previously discussed could not explain the observations of the photoelectric effect. Table 12.6.1 compares the results of these experiments with the predictions from classical electromagnetic wave theory.

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TABLE 12.6.1 Comparison of the photoelectric experimental results with predictions of classical electromagnetic wave theory

PHOTOELECTRIC EXPERIMENTAL OBSERVATION	PREDICTION OF CLASSICAL ELECTROMAGNETIC WAVE THEORY (UNSUPPORTED)
Photocurrent only occurs for frequencies above f_0, and where f_0 is characteristic of the material.	Electrons should be emitted at any frequency, as long as the intensity is high enough.
The size of the current depends on intensity but not frequency.	The current should depend on both intensity and frequency.
There is no time delay between the absorption of light and the emission of a photoelectron at any intensity.	At low intensities, it takes time for enough energy to be absorbed by the atoms. Hence, there should be a delay between the light being turned on and electrons being emitted. The delay should be longer for lower intensities.
The maximum kinetic energy of the electrons depends on the frequency of light but *not* on the intensity.	The kinetic energy should only be related to the intensity, and not to the frequency.

Just as with black-body radiation, a new theory was needed to explain these results. It was Einstein who came up with a new model in 1905. His explanation combined two ideas – the familiar conservation of energy and Planck's recently introduced idea of 'quantisation'. Note, it was for his explanation of the photoelectric effect that Einstein won his Nobel Prize in Physics (and not for the development of relativity, which was deemed too controversial at the time).

Conservation of energy, threshold frequency and work function

You are already familiar with the idea of conservation of energy. Einstein explained the photoelectric effect by saying that electromagnetic radiation, or light, is quantised. When it interacts with matter, such as the metal plate X in Figure 12.6.1, it can only give up its energy in discrete amounts. Each quantum of light has energy $E = hf$, a relationship between energy and frequency first introduced by Planck to explain black-body radiation.

When an electron in the metal plate X (Figure 12.6.1) absorbs a photon, it gains this energy. However, to leave the metal plate requires a given amount of energy; effectively an ionisation energy. Hence, the threshold frequency, which is characteristic of the metal, is a measure of this ionisation energy. This energy is called the work function of the metal, and is given by $W = hf_0$.

Putting this together with the conservation of energy, Einstein determined that $E_{k(max)} = hf - hf_0 = E - W$. The photoelectric equation relates the kinetic energy of the photoelectrons with the incident energy of the photon and the work function of the metal; that is, the maximum kinetic energy of photoelectrons equals the energy of the incident photon less the work function.

Looking again at Figure 12.6.2, we can now see that the gradient of each line must be equal to the Planck constant, h. The extrapolated straight lines of best fit in Figure 12.6.2 cross the y axis at the value of the work function; that is, the y intercept provides the value W. This graphical representation of the experimental data allows us to quickly find values for both the Planck constant, h, and the work function, W, of the metal used.

KEY FORMULA

$E = hf$

Where:

E = energy (J)

h = Planck constant, $h = 6.63 \times 10^{-34}$ J s

f = frequency of incident light (Hz)

KEY FORMULA

$W = hf_0$

Where:

W = work function (J)

h = Planck constant (J s)

f_0 = threshold frequency (Hz)

KEY FORMULA

Photoelectric equation

$E_{k(max)} = hf - hf_0 = E - W$

TABLE 12.6.2 Work functions of various metals

METAL	*W* (eV)	*W* (J)
Sodium (Na)	2.46	3.94×10^{-19}
Aluminium (Al)	4.08	6.53×10^{-19}
Iron (Fe)	4.50	7.20×10^{-19}
Copper (Cu)	4.70	7.52×10^{-19}
Zinc (Zn)	4.31	6.90×10^{-19}
Silver (Ag)	4.73	7.57×10^{-19}
Platinum (Pt)	6.35	1.02×10^{-18}
Lead (Pb)	4.14	6.62×10^{-19}

Note: these are typical values for these metals. Measured values vary depending on whether the metal is a single crystal or polycrystalline and which face of the crystal is illuminated.

KEY CONCEPT

The photoelectric effect

The photoelectric effect is the ejection of electrons from a polished metal surface by incident light. The light must equal a minimum threshold frequency for this to occur.

- The threshold frequency corresponds to a minimum energy, $E = hf_0$.
- The minimum energy corresponds to the work function, W, of the metal.
- The maximum kinetic energy of the photoelectrons is $E_{k(max)} = hf - W$. This is a statement of conservation of energy.
- The photocurrent, which is proportional to the number of photo electrons, depends on the intensity of the light. The intensity is a measure of the number of incident photons.

WORKED EXAMPLE 12.6.1

Using the graph in Figure 12.6.3 find the value of the work function for caesium. Give your answer in eV and J.

ANSWER

$f_0 = 4.4 \times 10^{14}\,\text{Hz}$

$W = hf_0$

$W = 6.63 \times 10^{-34}\,\text{J s} \times 4.4 \times 10^{14}\,\text{Hz}$

$W = 2.9 \times 10^{-19}\,\text{J}$

$$W = \frac{2.9 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-19}\,\text{J eV}^{-1}} = 1.8\,\text{eV}$$

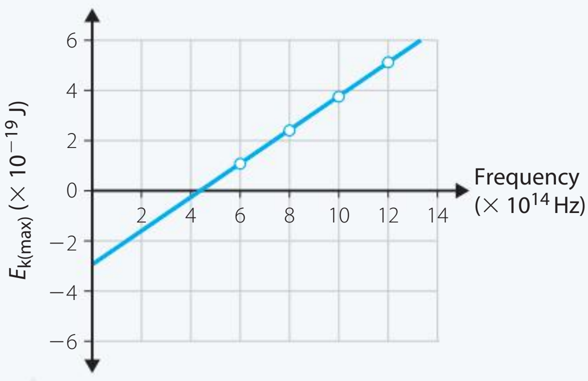

FIGURE 12.6.3

You will have noticed that the formula refers to the *maximum* kinetic energy of the photoelectrons. The photoelectrons have all values of energy *up to* this maximum value. After absorbing the energy *hf* from the incident light, the electrons have energy *hf*. These are the valence electrons that are free to move through the metal and are not bound to any particular atom. It is these conduction electrons that can be ejected as photoelectrons. The electrons that are bound to the atoms cannot gain enough energy to be ejected in this way. Electrons that have absorbed $E = hf$ lose a minimum energy of $W = hf_0$, the work function, to escape the material. This may occur if they are at the very surface and have no interactions with other electrons or nuclei as they escape. However, most of the electrons will lose some of the absorbed energy in collisions, and many do not leave the metal at all. This results in an increase in the temperature of the metal. Hence, there is a continuous spectrum of electron energies from zero to the maximum value kinetic energy value of $E_{k(max)} = hf - W$.

WORKED EXAMPLE 12.6.2

Ultraviolet light of wavelength 200 nm is incident on a polished silver plate. The work function for silver is 4.73 eV.

1 What is the kinetic energy of the fastest moving electrons?

2 What is the kinetic energy of the slowest moving electrons?

3 What is the threshold frequency for silver?

ANSWER

1 $E_{k(max)} = hf - W$

$$f = \frac{c}{\lambda}$$

$$E_{k(max)} = \frac{hc}{\lambda} - W$$

$$E_{k(max)} = \frac{4.14 \times 10^{-15}\ \text{eV s} \times 3.0 \times 10^{8}\ \text{m s}^{-1}}{200 \times 10^{-9}\ \text{m}} - 4.73\ \text{eV}$$

$$E_{k(max)} = 1.5\ \text{eV}$$

2 0 eV

3 $W = hf_0$

$$f_0 = \frac{W}{h}$$

$$f_0 = \frac{4.73\ \text{eV}}{4.14 \times 10^{-15}\ \text{eV s}}$$

$$f_0 = 1.1 \times 10^{15}\ \text{Hz}$$

WORKED EXAMPLE 12.6.3

Find the range of energies of photons in the visible spectrum, in eV. The visible spectrum ranges from blue light with wavelength approximately 400 nm to red light with wavelength approximately 700 nm.

ANSWER

$$E = hf$$

$$f = \frac{c}{\lambda}$$

$$E = \frac{hc}{\lambda}$$

$$E_{max} = \frac{hc}{\lambda_{min}}$$

$$E_{max} = \frac{4.14 \times 10^{-15}\,\text{eV s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{400 \times 10^{-9}\,\text{m}}$$

$$E_{max} = 3.1\,\text{eV}$$

$$E_{min} = \frac{hc}{\lambda_{max}}$$

$$E_{min} = \frac{4.14 \times 10^{-15}\,\text{eV s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{700 \times 10^{-9}\,\text{m}}$$

$$E_{min} = 1.8\,\text{eV}$$

WORKED EXAMPLE 12.6.4

Light of frequency 6.20×10^{14} Hz is incident on a polished sodium surface. The work function for sodium is 3.94×10^{-19} J.

1 Determine the maximum kinetic energy of any ejected photoelectrons.

2 Calculate the threshold frequency for sodium.

ANSWER

1 $E_{k(max)} = hf - W$

$E_{k(max)} = 6.63 \times 10^{-34}\,\text{J s} \times 6.20 \times 10^{14}\,\text{Hz} - 3.94 \times 10^{-19}\,\text{J}$

$E_{k(max)} = 4.11 \times 10^{-19} - 3.94 \times 10^{-19}\,\text{J}$

$E_{k(max)} = 1.71 \times 10^{-20}\,\text{J}$

2 $W = hf_0$

$$f_0 = \frac{W}{h}$$

$$f_0 = \frac{3.94 \times 10^{-19}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}}$$

$f_0 = 5.94 \times 10^{14}\,\text{Hz}$

SECTION REVIEW 12.6

REMEMBERING

1 What principle did Einstein use to explain the energies of photoelectrons?

2 What is a photon?

UNDERSTANDING

3 How is a photoelectron different from any other electron?

4 Explain how you can determine the Planck constant from a graph of frequency against stopping voltage for a photoelectric experiment.

5 It requires more energy to remove an electron from the surface of a polished piece of copper than from a polished piece of lithium.

a Which metal has the larger work function?

b Which metal has the greater threshold frequency?

c Which metal has the greater threshold wavelength?

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APPLYING

6 If a metal has a work function *W*, and is irradiated with incident light of frequency *f*, how is the possible energy of any emitted photoelectrons determined?

7 The graph in Figure 12.6.4 shows the results of a photoelectric experiment using magnesium metal.

a Determine a value for the Planck constant from this graph.

b Determine a value for the work function of magnesium.

c Imagine that silver had been used in this experiment instead of magnesium. Silver has a work function of 4.73 eV. Where on the graph would a line representing silver lie?

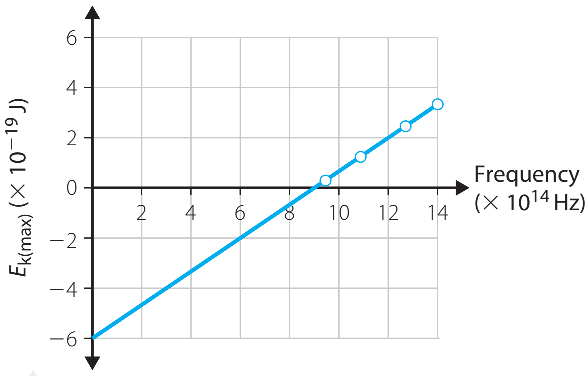

FIGURE 12.6.4

ANALYSING

8 Figure 12.6.5 shows a photoelectric tube with light of frequency *f* and intensity *I* incident on a metal cathode. Electrons emitted from the cathode are collected at the anode. The potential difference (pd) between the anode and cathode is varied, and the resulting photocurrent is measured. Figure 12.6.6 shows the results of this experiment.

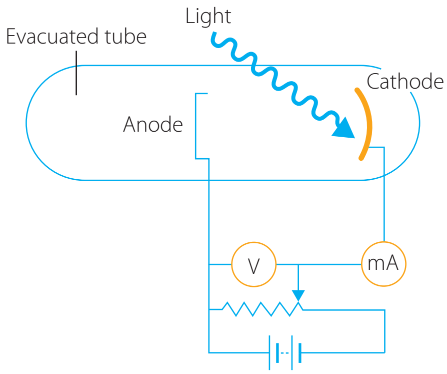

FIGURE 12.6.5

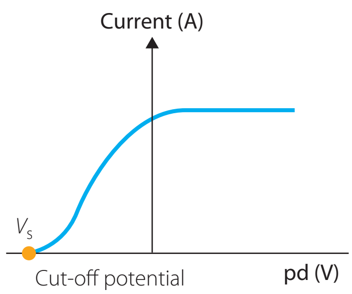

FIGURE 12.6.6

a Why is the photocurrent constant at positive values of potential difference?

b If the frequency of the light is varied, which of the graphs in Figure 12.6.7 represents the relationship between the stopping voltage, V_s, and *f*?

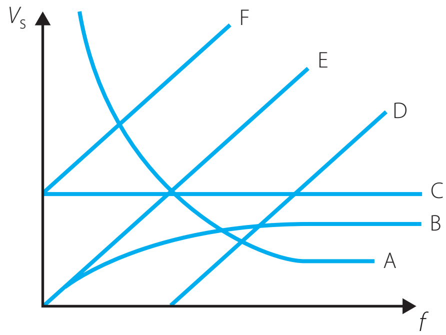

FIGURE 12.6.7

12.7 The model of the atom and atomic spectra

We saw earlier that observations of black-body radiation and the photoelectric effect led to the development of the photon (particle) model of light. At the same time that Planck, Einstein and others were developing this new quantum model of light, other physicists were investigating the nature of atoms.

The Thomson model of the atom

The idea that matter consisted of atoms dates back to ancient Greece, and until the end of the 19th century it was believed that atoms were indivisible. Then in 1897, J.J. Thomson (1856–1940) experimentally observed electrons being emitted from atoms. This established that an atom was not a single homogeneous particle, but consisted of matter – subatomic particles – with positive and negative charges; although, at the time it was not known how the positive and negative parts were arranged.

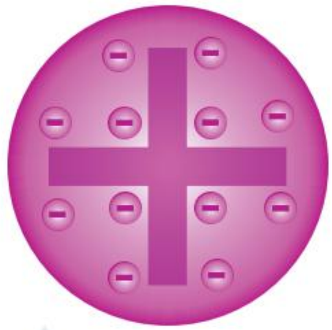

FIGURE 12.7.1 Thomson's plum-pudding model of the atom

Measurements of the mass of electrons showed that almost all of the mass of an atom was associated with the positive part of the atom. Based on this evidence, in 1904 Thomson suggested a model in which the atom consisted of a sphere of positive charge with electrons embedded in it. This model was known as the 'plum-pudding' model of the atom (Figure 12.7.1) , as the electrons were scattered through the positive matter, like raisins scattered through a plum pudding (or chocolate chips in a chocolate chip muffin).

The Rutherford model of the atom

Then, in 1909 Hans Geiger (1882–1945) and Ernest Marsden (1889–1970), who were students working with Ernest Rutherford, performed the alpha-particle scattering experiment in which a thin gold foil was placed in a beam of alpha particles (helium nuclei). When they measured the paths of the scattered alpha particles, they found that most alpha particles passed through the metal foil and were only minimally deflected, if at all; however, about one in 20 000 was scattered backwards toward the source. This result implied the presence of a tiny, very dense and positively charged nucleus. Rutherford later said 'It was as if you fired a 15-inch shell at a sheet of tissue paper and it came back to hit you.'

FIGURE 12.7.2 In a spectroscope, a prism is used to disperse the light, allowing specific elements to be detected by their unique emission spectrum.

Rutherford went on to suggest a planetary model of the atom to explain these observations. This model involved electrons orbiting the nucleus in a circular path under the influence of the electrostatic force. This was analogous to the way planets orbit the Sun under the influence of the gravitational force.

Although Rutherford's model fitted the experimental data from the gold-foil scattering experiment, there were still problems. In this planetary model, the electrons (charged particles) were undergoing centripetal acceleration. This is not a problem for uncharged objects such as planets, but accelerating charged particles emit energy. The electrons should have been acting like those in an antenna and emitting electromagnetic waves. Because energy is always conserved, an electron that was emitting energy as electromagnetic radiation would lose kinetic and potential energy and spiral into the nucleus and the atom would collapse. This was a significant flaw in this model.

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Atomic spectra

None of the classical models of the atom, such as those described above, were able to fully explain or predict the behaviour of atoms. New models and theories were needed. Experimental data showing that the energy in atoms is quantised would prove vital for developing the new quantum mechanical atomic theory. This data came from the measurements of atomic line spectra.

A **spectroscope** uses a prism or diffraction grating to separate, or disperse, light into its component colours. White light disperses into all the colours of the rainbow. We have already seen in the previous chapter that black bodies produce a continuous spectrum. In contrast, when a gas is heated, it produces a spectrum consisting of discrete colours. This is observed through a spectroscope as a pattern of parallel coloured lines and hence is called a **line spectrum**. A heated gas produces an **emission spectrum**. When white light is passed through a cold gas, an **absorption spectrum** is produced. Absorption spectra have the same characteristic lines as emission spectra, but they are dark lines on a continuous coloured background.

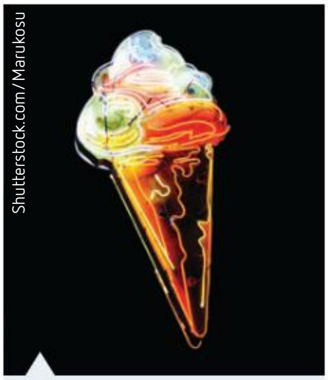


FIGURE 12.7.3 Neon lights are an example of a gas discharge tube.

spectroscope
a device that disperses radiation by energy (or wavelength or frequency) so that a spectrum may be observed and measured

line spectrum
an emission or absorption spectrum consisting of discrete lines that are characteristic of the energy levels of a particular atom or molecule

emission spectrum
the spectrum of radiation emitted by an object; for example, black-body radiation or atomic spectra from a discharge tube

absorption spectrum
the wavelengths (or frequencies or energies) of radiation absorbed by a material

Gustav Kirchhoff (1824–87) and Robert Bunsen (1811–99) had recognised as early as the 1860s that line spectra can be used to identify elements. They discovered two new elements, caesium and rubidium, in 1861 using spectroscopy.

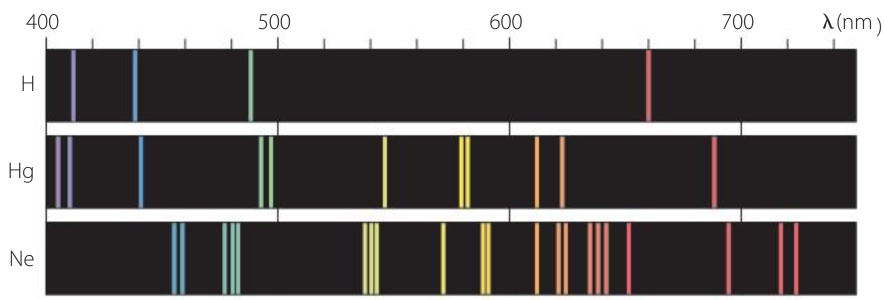


FIGURE 12.7.4 Emission spectra of hydrogen, mercury and neon.

Although Kirchhoff, Bunsen and others had observed and used spectra, there was no theory that explained why the phenomena existed, although it was presumed that the characteristic spectra were related to the internal structure of the atom. To solve this puzzle, the simplest atom, hydrogen, was subject to intense theoretical and experimental investigation.

EXPERIMENT 12.7.1

Observing emission spectra

AIM

To observe the spectra from different light sources using a spectroscope or diffraction grating.

»

MATERIALS

- a simple spectroscope or diffraction grating
- sources of light including filtered sunlight, fluorescent tube, incandescent lamp, light-emitting diode (LED) and various discharge tubes (e.g. sodium lamp)

RISK ASSESSMENT

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Looking directly at the Sun through a diffraction grating can cause eye damage.	Never look directly at the Sun. Only observe sunlight indirectly.
Looking at a LASER light can cause permanent eye damage.	Do not use a LASER as a light source.

PROCEDURE

1 In an otherwise darkened room, look through the spectroscope or diffraction grating(s) at the various light sources.

2 Note your observations for each light source. If you are using a spectroscope, note the wavelength or frequency also.

LIGHT SOURCE	OBSERVATIONS (COLOURS, WAVELENGTHS, FREQUENCIES)
Filtered sunlight	
Incandescent lamp	
Fluorescent lamp	
LED	
Discharge tube (e.g. sodium lamp)	

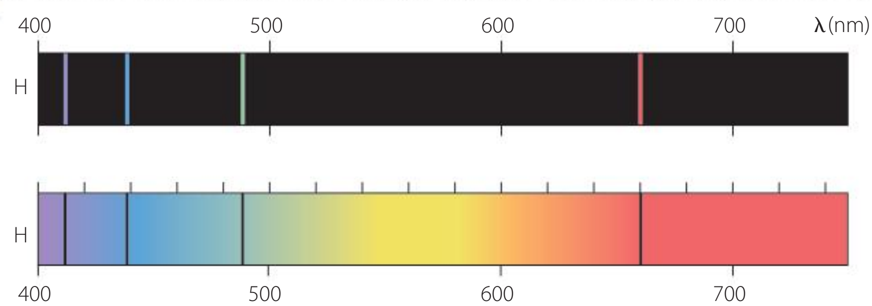
3 Draw a complete visible spectrum for the filtered sunlight. It will include all of the colours of the rainbow.

4 Draw the emission spectrum for the fluorescent lamp. You should be able to see some bright green and purple lines at these specific wavelengths.

The hydrogen spectrum

Hydrogen produces infrared, visible and ultraviolet emission spectra. The emission and absorption spectra of hydrogen are shown in Figure 12.7.5. Dark lines in the absorption spectrum of any element coincide with the bright lines in its emission spectrum.

FIGURE 12.7.5 The emission and absorption spectra of hydrogen.



The Balmer series of spectral lines for atomic hydrogen

In 1855, Johann Balmer derived an empirical formula for the visible series that now bears his name.

Balmer showed that the observed wavelengths were proportional to $\frac{m^2}{(m^2 - n^2)}$ with $n = 2$ and m greater than n.

The other spectral series of hydrogen are named after those who discovered them. The Lyman series is in the ultraviolet region and the Paschen series is in the infrared region of the spectrum. Other series of even longer wavelength are the Brackett series and the Pfund series. These lines have a similar pattern of separation, but with different values for n and m.

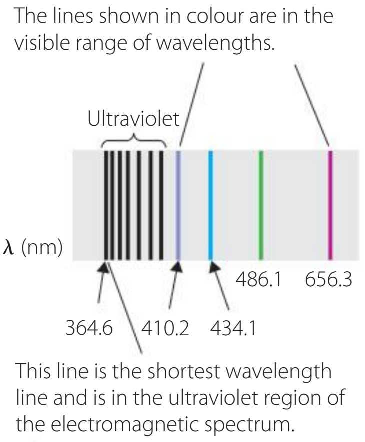


FIGURE 12.7.6 Emission spectra wavelengths for hydrogen, some of which are in the visible region.

TABLE 12.7.1 Lines in the spectral series of hydrogen

SERIES NAME	PART OF SPECTRUM	SHORTEST WAVELENGTH (NM)	LONGEST WAVELENGTH (NM)
Lyman	Ultraviolet	91.1	121.6
Balmer	Visible	364.5	656.3
Paschen	Infrared	820.1	1870 .0

In the 1880s Johannes Rydberg (1854–1919) was working on finding a mathematical description of the line spectra of the alkali metals (lithium, sodium and so on). He read Balmer's work on hydrogen and realised that his own mathematical model and Balmer's were equivalent. Rydberg expressed the relationship as:

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, where λ is the wavelength of the line, n_f and n_i are integers and R is a constant known as the Rydberg constant, $R = 1.097 \times 10^7\,\mathrm{m}^{-1}$.

Rydberg arrived at his formula empirically; that is, by fitting an equation to the observed data. At the time there was no theoretical model of the atom that could predict the relationship between positions of spectral lines, or even the existence of spectral lines. His work was important in that it led to the development of the first quantum mechanical model of the atom – the Bohr model.

WORKED EXAMPLE 12.7.1

For the Brackett series in the far infrared, $n_i = 4$.

1 Find the longest wavelength in the series.

2 Find the shortest wavelength in the series.

ANSWER

1 The longest wavelength corresponds to the smallest energy, which would occur between the $n = 5$ and $n = 4$ states:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\ \text{m}^{-1} \times \left(\frac{1}{4^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{\lambda} = 2.47 \times 10^5\ \text{m}^{-1}$$

$$\lambda = 4050\ \text{nm}$$

2 The shortest wavelength is the largest energy, or when the electron comes from $n = \infty$ to $n = 4$:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_\text{f}^2} - \frac{1}{n_\text{i}^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_\text{f}^2} - \frac{1}{\infty^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_\text{f}^2}\right)$$

$$\lambda = \frac{n_\text{f}^2}{R} = \frac{4^2}{1.097 \times 10^7\ \text{m}^{-1}}$$

$$\lambda = 1460\ \text{nm}$$

The Bohr model of the atom

In 1913, Niels Bohr (1885–1962) combined the concepts of Rutherford's planetary model of the atom and Einstein's photons to predict the observed spectra of hydrogen. To solve the problems of the planetary model, Bohr made several postulates.

KEY CONCEPT

Bohr's postulates

1. An electron in an atom moves in a circular orbit about the nucleus under the influence of the electrostatic attraction of the nucleus.
2. Only certain orbits are stable. Electrons in these orbits do not emit energy.
3. The greater the radius of the orbit, the greater is its energy. Atoms emit radiation when an electron goes from one orbit to another orbit with lower energy. The energy released is:

   $E = E_\text{f} - E_\text{i} = hf$

4. The orbits are characterised by quantised radii, given by

   $$r = \frac{nh}{2\pi m_\text{e} v}$$

   Where:

   r = radius (m)

   m_e = mass of the electron (kg)

   v = velocity

   h = Planck constant

   n = an integer

The postulates were a mixture of classical physics (postulate 1), recently introduced quantum principles (postulate 3) and completely new ideas (postulates 2 and 4).

Bohr's first postulate was drawn directly from the earlier planetary model of the atom. His second postulate simply stated what had been observed – that atoms do not collapse. Bohr's third and fourth postulates distinguish his model as the first quantum model of the atom. Bohr's third postulate states that energies are quantised and may take only a discrete set of values. The relationship between radius and energy is given by classical electromagnetism. The greater the separation between a positive charge (the nucleus) and a negative charge (the electron), the greater the potential energy of the system. The different possible energies are called **energy levels**. Using classical electromagnetism, Bohr showed that the energy of any given level was proportional to $\frac{1}{n^2}$, where n is the integer in the equation for the radius in postulate 4. Hence, $E_n = \frac{k}{n^2}$, where k is a constant.

Bohr's fourth postulate was also based on the quantisation of a physical property, in this case the **angular momentum, *L***, of the electron. Angular momentum is to circular motion what momentum is to linear motion. It is a conserved quantity and is given by $L = mvr$.

KEY FORMULA

$L = mvr$

Where:

L = angular momentum of the object ($kg\,m^2\,s^{-1}$)

m = mass (kg)

v = velocity ($m\,s^{-1}$)

r = orbital radius (m)

By saying that L may only have discrete values,

then: $L = mvr = \frac{nh}{2\pi}$

Where:

h = Planck constant

n = an integer (the quantum number)

In postulate 3, Bohr was saying that only discrete values of r were possible, that is, that only discrete values of energy were allowed. We often refer to these energy levels as electron energies; however, we must remember that this energy belongs to the electron–nucleus system as they are separated charged particles. All isolated atoms of one element will have the same set of energy levels, but specific elements have different sets of energy levels, due to their different nuclei and numbers of electrons.

energy levels the allowed energies of a nucleus–electron system; often referred to as electron energy levels, even though they are characteristic of the atom, not of individual electrons

angular momentum, *L* momentum associated with rotational or orbital motion, $L = mvr$

An atom can make a transition from one level to a lower level by emitting a photon of energy equal to the difference between the levels. This occurs when an electron moves from an orbit further from the nucleus to one closer to the nucleus. In contrast, to move from a lower energy level to a higher energy level requires an electron to absorb energy. The Bohr model was thus able to explain the existence of discrete line spectra, as emission spectra can be explained by electrons moving from higher to lower energy orbits. The energy gap between the energy levels is equal to the energy of the photon emitted when the transition occurs.

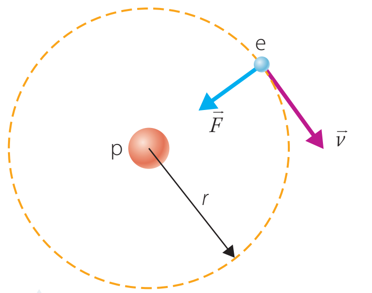


FIGURE 12.7.7 In Bohr's model of the hydrogen atom, the electron occupies discrete orbits.

KEY FORMULA

$E_{gap} = E_f - E_i = hf_{photon}$

and recalling that $E_n = \frac{k}{n^2}$ we can write:

$hf_{photon} = E_f - E_i = k\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, where k is a constant.

If we now use the universal wave equation, $f = \frac{c}{\lambda}$, we can see that:

$$\frac{1}{\lambda} = \frac{k}{h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For hydrogen, this relationship is expressed as the Rydberg equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where R = the Rydberg constant = $1.097 \times 10^7\ m^{-1}$

In fact, Bohr showed that the constant $\frac{k}{h}$ was equal to the Rydberg constant, $R = 1.097 \times 10^7\ m^{-1}$. Bohr's model now not only predicted the existence of line spectra, but quantitatively predicted the positions of the lines for the hydrogen atom.

The hydrogen emission spectrum

ground level the lowest possible energy level of a nucleus–electron system

Bohr's model explained the observed line spectra as resulting from transitions between energy levels in atoms. The lowest wavelength (highest energy) line corresponds to the ionisation energy of electrons in the lowest possible energy level. This level is called the **ground level** of the atom and corresponds to the electron being in the orbit with the smallest radius. Ionisation is the removal of an electron from the atom to an infinite distance, or at least so far away that the electrostatic attraction is negligible.

Energy levels can be represented in two ways:

1 with the ionisation energy being taken as zero (all the energy states then have a negative potential energy), or

2 with the ground state level being taken as zero.

In both representations, the energy difference between levels is the same. We shall generally use the first representation, as this corresponds to the usual convention for choosing the zero of potential energy, as in electromagnetism.

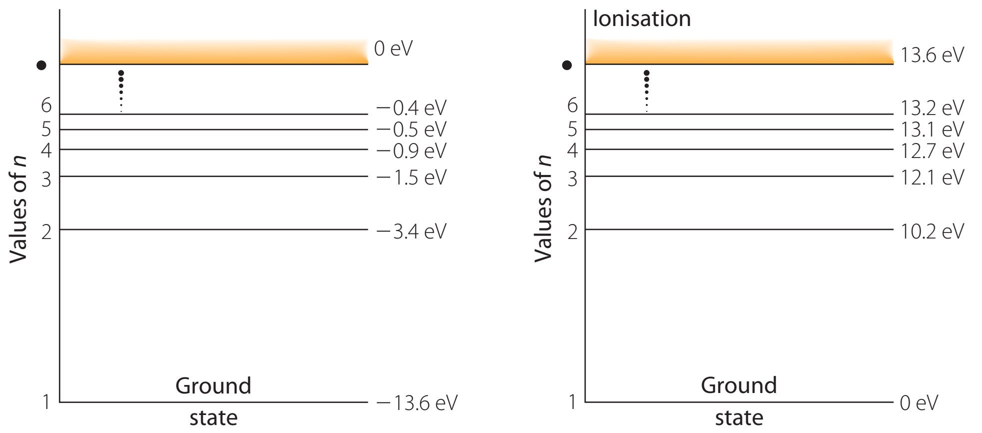


FIGURE 12.7.8 The energy levels of the hydrogen atom have (**a**) the ionisation energy as zero and (**b**) the ground state as zero. Either form is acceptable.

The energy of each level can be deduced from the wavelengths, and hence energies, of the lines in the emission spectra. The highest energy lines for hydrogen are those in the Lyman series in the ultraviolet region. These lines correspond to transitions to the ground state from higher energy levels, hence for these lines $n_f = 1$ and $n_i = 2, 3, 4 \ldots$ The Balmer series, in the visible region, corresponds to transitions to the $n = 2$ level, so $n_f = 2$ and $n_i = 3, 4, 5 \ldots$

Figure 12.7.9 shows the energy levels for hydrogen and the transitions corresponding to these two series of spectral lines.

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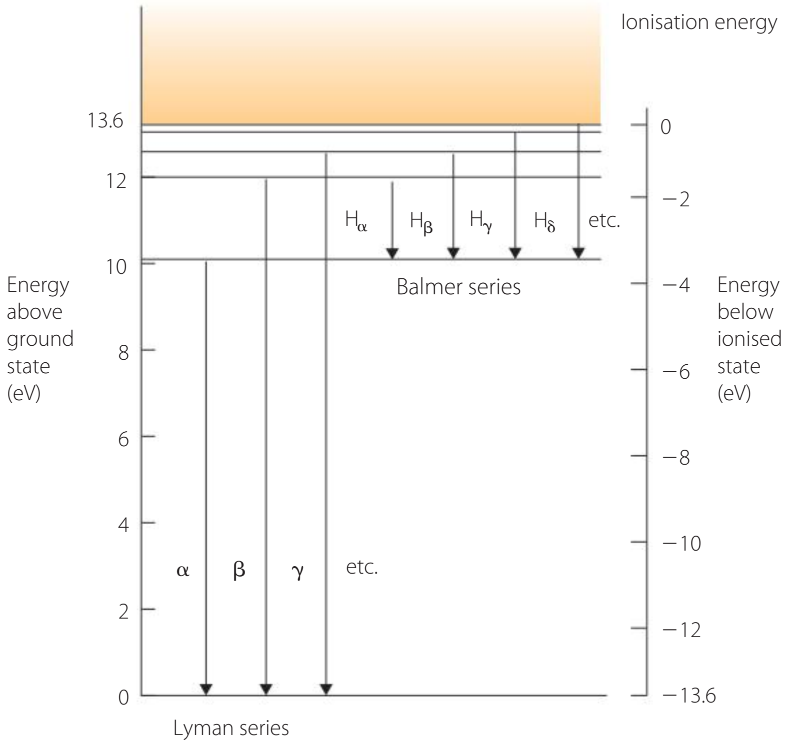

FIGURE 12.7.9 This energy level model for hydrogen shows the transitions corresponding to the Lyman and Balmer emission spectral series.

WORKED EXAMPLE 12.7.2

Refer to Figure 12.7.9 to answer the following questions.

1 Calculate the frequency of the lowest energy line in the Lyman series.

2 Calculate the wavelength corresponding to the highest energy line in the Balmer series.

ANSWER

1 $E_f - E_i = hf$

$$f = \frac{E_f - E_i}{h}$$

$$f = \frac{10.2\,\text{eV} - 0\,\text{eV}}{6.63 \times 10^{-34}}$$

$$f = \frac{1.63 \times 10^{-18}\,\text{J}}{6.63 \times 10^{-34}\,\text{Js}}$$

$$f = 2.46 \times 10^{15}\,\text{Hz}$$

Or, alternatively

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, where R = the Rydberg constant = $1.097 \times 10^7\,\text{m}^{-1}$,

and the electron falls from from $n = 2$ to $n = 1$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda} = 8.23 \times 10^6$$

$\lambda = 1.22 \times 10^{-7}$ m or 122 nm

And since $c = f\lambda$ then $f = \frac{c}{\lambda}$

$$f = \frac{3.00 \times 10^8}{1.22 \times 10^{-7}}$$

$f = 2.46 \times 10^{15}$ Hz

b $E_f - E_i = hf$

13.6 − 0 eV

$$f = \frac{13.6 - 0\text{eV}}{h}$$

$$f = \frac{13.6 \times 1.6 \times 10^{-19}\ \text{J}}{6.63 \times 10^{-34}\ \text{Js}}$$

$f = 3.28 \times 10^{15}$ Hz

And since $c = f\lambda$ then $\lambda = \frac{c}{f}$

$$\lambda = \frac{3.00 \times 10^8}{3.28 \times 10^{15}}$$

$\lambda = 9.15 \times 10^{-8}$ m or 91.5 nm

WORKED EXAMPLE 12.7.3

1. Construct an energy level diagram like that shown in Figure 12.7.9, showing the transitions corresponding to the Paschen series, for which $n_f = 3$ and $E_3 = -1.5$ eV.
2. Calculate the frequency of a photon released in a transition from $n_i = 6$ to $n_f = 3$.

ANSWER

1

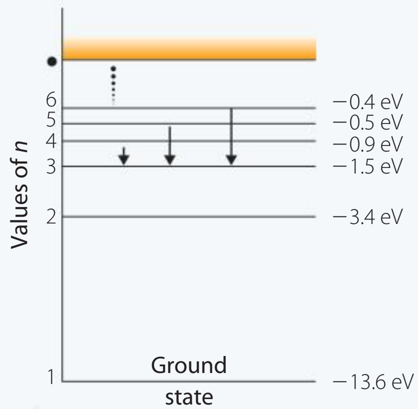

FIGURE 12.7.10

2 $E_f - E_i = hf$

$$f = \frac{E_f - E_i}{h}$$

$$h = 6.63 \times 10^{-34}\,\text{J s} = 4.14 \times 10^{-15}\,\text{eV}$$

$$f = \frac{-0.4\,\text{eV} - (-1.5\,\text{eV})}{4.14 \times 10^{-15}\,\text{eV s}}$$

$$f = 2.66 \times 10^{14}\,\text{Hz}$$

Absorption spectra occur when a photon of exactly the right energy is incident on an atom. A photon with too little or too much energy cannot be absorbed; only a photon with energy corresponding exactly to a gap between energy levels can be absorbed. As a gas, most atoms will be in their ground state; that is, most will have their electrons in their lowest possible energy levels. Hence, the most likely transitions will be from the ground state, $n = 1$, to higher levels. However, as long as the temperature is above absolute zero, there will always be some atoms in an excited state, with electrons in levels $n = 2$, $n = 3$ and so on. That is why in an absorption spectrum we see lines corresponding to all the transitions of the emission spectrum.

As in the emission spectrum, the Balmer series in the absorption spectrum corresponds to electrons going *from* the $n = 2$ state to higher levels.

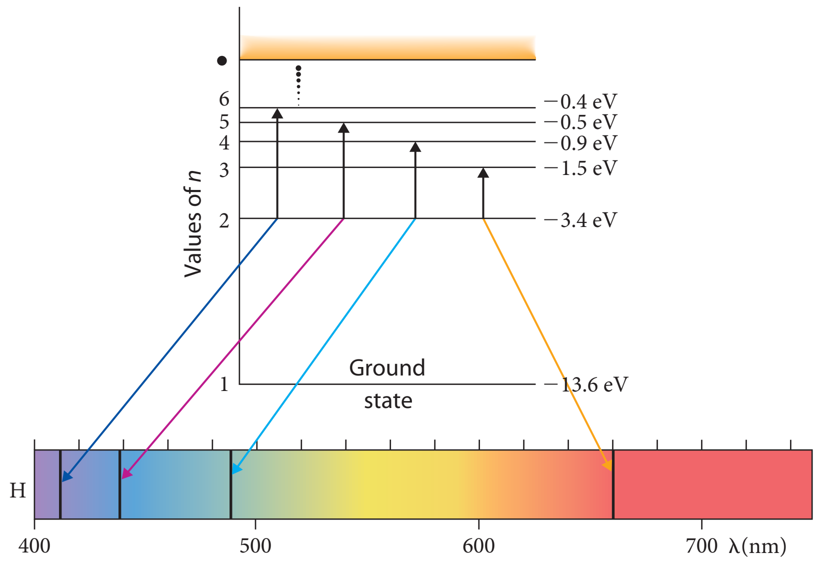

FIGURE 12.7.11 The absorption spectrum of hydrogen and the corresponding transitions are represented on this energy level diagram.

WORKED EXAMPLE 12.7.4

Consider photons of the following energies: 1.9 eV, 10.2 eV, 12.5 eV, 13.6 eV. Which of these could be absorbed by hydrogen gas? Explain your answer.

ANSWER

Referring to the hydrogen energy level diagram, Figure 12.7.11 and the possible energy differences using $E_f - E_i$:

- the 1.9 eV photon could be absorbed (transition from $n = 2$ to $n = 3$)
- the 10.2 eV photon could be absorbed (transition from $n = 1$ to $n = 2$)
- the 12.5 eV photon could not be absorbed (there is no energy gap corresponding to 12.5 eV)
- the 13.6 eV photon could be absorbed (transition from $n = 1$ to $n = \infty$; i.e. atom is ionised)

Spectra of other atomic species

Each type of atom produces a unique line spectrum. The energy levels are unique because each different type of atom has a different number of protons and hence a different nuclear charge. Accordingly, the force exerted by the nucleus on the electrons differs, giving rise to different potential energies at different orbital radii, $r = \frac{nh}{2\pi m_e v}$. There is also a different number of electrons in different atoms. Electrons close to the nucleus 'shield' the outer electrons somewhat from the nuclear charge. This also acts to change the potential energy at the different allowed radii. These effects combine to give a unique fingerprint for each atom in the form of a unique line spectrum. This is extremely useful, as it allows the presence of different types of atoms to be detected. For example, we know from the line spectrum of the Sun that there is a great deal of hydrogen and helium present, as well as larger atoms. Other stars have different characteristic spectra, indicating the presence of other atomic species.

Different molecules also have characteristic spectra. The spectrum of a molecule is not simply the sum of the spectra of the atoms that make up the molecule. This is because when atoms bind together to form a molecule the energy levels change. Hence, it is possible to distinguish between ethanol and methanol by their spectra, even though both contain only carbon, hydrogen and oxygen.

EXPERIMENT 12.7.2

Measuring the Planck constant

Light-emitting diodes (LEDs) produce photons of a particular wavelength when electrons transition from a higher to a lower energy level within the semiconductor material of the LED. Hence, it is possible to measure the Planck constant using the wavelengths of LEDs. The minimum voltage that will cause an LED to emit light is called the threshold voltage, V_t:

$$V_t = \frac{hc}{e\lambda}$$

where h is the Planck constant, 6.63×10^{-34} J s, c is the speed of light, 3.00×10^{8} m s^{-1}, e is the charge on an electron, 1.60×10^{-19} C, and λ is the wavelength of the light produced by the LED.

AIM

To experimentally measure the Planck constant.

»

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MATERIALS

- 5 different-coloured LEDs
- DC power supply
- 1 kV resistor
- 10 kV variable resistor
- voltmeter or multimeter

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
High voltages can damage equipment or cause electric shock.	Only use low voltages. Ask your teacher to check your circuit before you turn on the power supply.
Large currents will damage the LEDs.	Always have a resistor in the circuit with the LED and power supply.

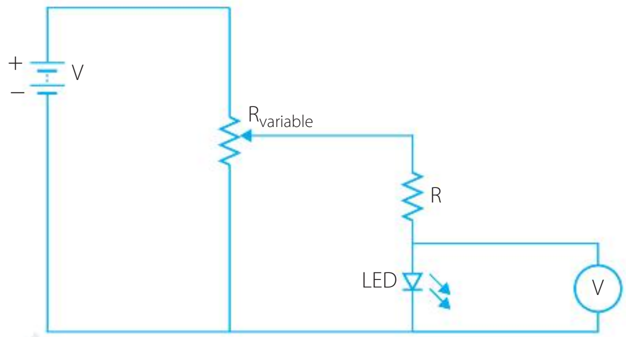

FIGURE 12.7.12 Circuit diagram for measuring the Planck constant.

PROCEDURE

1. Connect the circuit as shown in Figure 12.7.12. When your circuit is set up and has been checked you will need to work in a dark room to make your measurements.
2. Slowly increase the voltage until the LED just starts to glow.
3. Record the voltage at which this occurs.

RESULTS

1. Record your results in a table such as that below.

COLOUR OF LED	WAVELENGTH, λ (M)	$\frac{1}{\lambda}$ (M^{-1})	THRESHOLD VOLTAGE, V_t (V)

2. Sketch a graph of V_t against $\frac{1}{\lambda}$. Be sure to label the axes with their units and provide a title.

ANALYSIS OF RESULTS

1. Determine the gradient of your graph and state its units.
2. Using the gradient, calculate the Planck constant.

DISCUSSION

1. Explain why you plot V_t vs $\frac{1}{\lambda}$ rather than V_t vs λ. What would be the shape of a V_t vs λ graph?
2. Does your value for the Planck constant agree with the accepted value? Determine the percentage difference between the experimental and theoretical values.
3. What could you do to minimise uncertainties in your experiment?

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SCIENCE AS A HUMAN ENDEAVOUR

FLUORESCENCE

Fluorescence is the glowing of substances when illuminated by ultraviolet light. An atom is excited to an energy state several levels above its ground state by absorption of a high-energy ultraviolet photon. The atom can decay (de-excite) in a number of ways, one of which is through a series of smaller steps back down to the ground state. In this case, lower-energy photons are emitted, some of which may be in the visible range. Fluorescent dyes are used in paints and even laundry detergents to make colours brighter in sunlight by converting some of the absorbed ultraviolet light into radiant visible light – making your whites whiter!

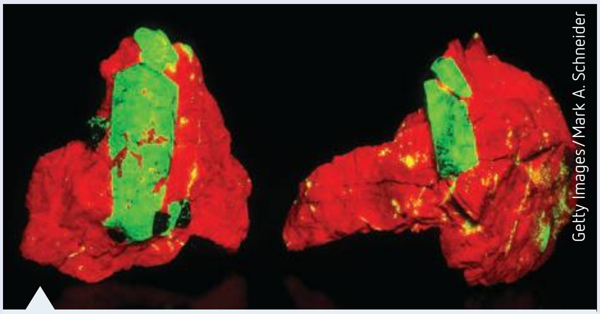

FIGURE 12.7.13 These rocks (willemite and calcite) fluoresce in the visible spectrum when illuminated by an ultraviolet light.

Limitations of the Bohr model

Bohr was able to explain qualitatively and quantitatively the existence and positions of the spectral lines of a hydrogen atom. His estimate of the size of the largest stable radius also agreed closely with the measured size of the hydrogen atom. However, the Bohr model could not predict the spectra of multi-electron atoms, even one as simple as two-electron helium. It also could not explain the different intensities of lines or why some lines split into multiple, closely spaced lines – fine and hyperfine structure – or the magnetically induced Zeeman effect.

Finally, Bohr's model introduced the idea of quantised atomic energy levels, but it did not offer any explanation for why they should be quantised. Successful models have both predictive power and explanatory power. Bohr's model lacked explanatory power, and had limited predictive power. It was superseded by a more comprehensive quantum mechanical model developed by Schrödinger (1887–1961) and Heisenberg (1901–1976). This modern quantum model built on the ideas of de Broglie, described earlier in this chapter.

SECTION REVIEW 12.7

REMEMBERING

1 Name three physicists who contributed to the development of the model of the atom. Briefly describe their contributions.

2 What were the most important successes and limitations of the Bohr model?

3 State Bohr's postulates. What was quantised in Bohr's model of the atom?

UNDERSTANDING

4 The spectral lines for transitions to the $n = 2$ state from higher levels for a particular atom are in the infrared region of the spectrum. Would you expect this atom to have any series of spectral lines in the visible range? If so, to what transitions would they correspond?

5 Explain the difference between an absorption spectrum and an emission spectrum. Why are there lines at the same frequencies and wavelengths?

6 Explain why Bohr's model of the atom is considered the first quantum mechanical model.

APPLYING

7 How many different emission spectral lines would there be as a result of transitions from the −1.52 eV energy level in sodium gas? The sodium energy levels are shown in Figure 12.7.14.

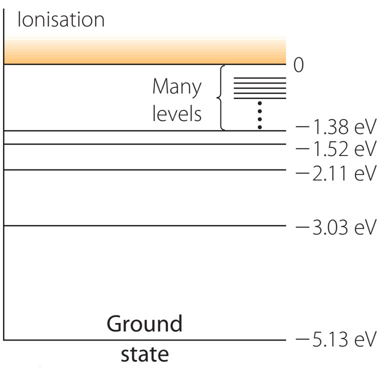

FIGURE 12.7.14 Energy level diagram for sodium

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8 The ground-state energy level of the electron in a hydrogen atom is negative. What is the significance of the negative sign? What is the zero energy level in this model?

APPLYING

9 Figure 12.7.14 shows an energy level diagram for sodium.

a How many possible emission spectral lines are there for transitions from the –2.11 eV level?

b Determine the energy released in each of these transitions.

c What are the wavelengths of the photons emitted in these transitions? If any are in the visible light region, identify their colour.

10 What is the shortest wavelength of photons that can be emitted from hydrogen atoms as they return from excited states to the ground state?

11 Two energy levels within a particular atom are 10 eV and x eV. When an atom of this element returns from the higher level to the lower energy level, radiation of wavelength 450 nm is emitted. What are the possible values of x?

12 What is the wavelength of X-ray photons of energy 3.6×10^4 eV?

13 An atom is excited to its third energy level above the ground state, $n = 4$.

a How many different spectral lines can it emit?

b Which of the energy level transitions will produce the photon of greatest energy?

c Which of the energy level transitions will produce a photon of the longest wavelength?

ANALYSING

14 Hydrogen atoms have only one electron, yet the spectrum of hydrogen contains a large number of lines. Explain how this is possible.

12.8 Mandatory practical

EXPERIMENT 12.8.1

The photoelectric effect

It is possible to observe the photoelectric effect using some very simple equipment.

AIM

To observe the photoelectric effect.

MATERIALS

- electroscope
- polished zinc plate
- steel wool or fine sand paper
- glass rod
- polyethylene rod
- fur or woollen fabric
- ultraviolet light source
- other light sources, for example lasers or torches

RISK ASSESSMENT

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Ultraviolet light can damage your eyes.	Do not look directly at the ultraviolet light source. Turn it off when not in use.

PROCEDURE 1

1 Clean the zinc plate *thoroughly* with the steel wool or sand paper, then place it on the electroscope. Try not to leave any fingerprints on it as you do so.

2 Charge up the glass rod by rubbing it vigorously with the fur or wool.

3 Touch the rod to the zinc plate. You should observe the leaves of the electroscope separate.

4 Time how long it takes for the leaves to fall back together. If it takes more than two minutes, just record your result as 'more than two minutes' and discharge the electroscope by touching it with your hand. Remember to avoid touching the zinc plate.

5 Repeat steps **2–4**, but this time shine one of your light sources on the plate.

6 Repeat steps **2–4** with each of your other light sources.

PROCEDURE 2

7 Charge up the polyethylene rod by rubbing it vigorously with the fur or wool.

8 Use the rod to charge the electroscope.

9 Time how long it takes for the leaves of the electroscope to fall back together.

10 Repeat steps **7–9**, but this time shine a light source on the zinc plate. Repeat this for each light source.

PROCEDURE 3

11 Charge the zinc plate with the polyethylene rod and then shine the ultraviolet light on it from various distances away. This varies the intensity of the light on the plate.

RESULTS

1 Draw a diagram showing your experimental set-up. Label all parts clearly.

2 Record your results in a table, such as the one below. Be sure to include units for your data.

The glass rod becomes positively charged, hence the plate is also positively charged when charged by conduction with the glass rod. The polyethylene rod becomes negatively charged, hence the plate is also negatively charged when charged by conduction with the polyethylene rod.

CHARGE ON PLATE	LIGHT SOURCE USED	TIME TO DISCHARGE (S)

3 If you measured the time to discharge using the ultraviolet light at various distances, that is, you conducted Procedure 3, record your results in an appropriate table, as below.

DISTANCE TO LIGHT (M)	TIME TO DISCHARGE (S)

ANALYSIS OF RESULTS

1 Explain your findings for each case investigated.

2 Plot a graph of time to discharge as a function of distance to light source for Procedure 3. Would you expect this graph to be linear? If not, what shape do you expect?

DISCUSSION

What could you do to improve the accuracy and precision of this experiment?

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Emission spectrum
   b Energy level
   c Quanta

2 Explain the term 'wave–particle duality' and provide examples to illustrate your understanding.

3 Draw an energy level diagram of hydrogen, noting the approximate energies in joules.

CATEGORY QUESTIONS

4 Identify the key differences between classical physics and quantum physics.

5 Describe the features of a black body.

6 Contrast the Rutherford and Bohr models of the atom.

ELABORATION QUESTIONS

7 Determine the de Broglie wavelength of an electron travelling at $15.0\,m\,s^{-1}$. Describe what occurs to the wavelength of an electron as the velocity increases.

8 A polished lead surface of a metal with a work function of 4.25 eV is illuminated with light of wavelength 200 nm. What is the wavelength of the photoelectrons emitted?

9 In a double-slit experiment, light with wavelength 589 nm is used to illuminate twin slits. The pattern produced is observed on a wall 2.00 m from the slits. The 10th interference minimum (dark spot) is found to be at a position 7.26 mm from the central bright spot. Calculate the slit separation.

EVIDENCE QUESTIONS

10 Perform research into the application of quantum mechanics and list six significant devices that exist because of this scientific breakthrough.

11 Predict the outcome of shining light of frequency $f = 4.05 \times 10^{15}$ Hz onto an iron metal surface of work function 7.20×10^{-19} J.

END-OF-CHAPTER EXAM

End-of-chapter test

1 The electromagnetic spectrum consists of several regions. Which of the options below shows electromagnetic regions in order of increasing energy?

A Ultraviolet, infrared, gamma rays and X-rays

B Infrared, gamma rays, X-rays and ultraviolet

C Infrared, ultraviolet, gamma rays and X-rays

D Infrared, ultraviolet, X-rays and gamma rays

2 The Planck constant is equivalent to:

A 6.63×10^{-19} J s.

B 1.60×10^{-19} J s.

C 6.63×10^{-34} J s.

D 1.60×10^{-34} J s.

3 What series of the hydrogen emission spectra emits light in the visible region?

A Lyman series

B Paschen series

C Balmer series

D Rydberg series

4 Which statement regarding wave interference is correct?

A Constructive interference occurs along nodal lines.

B Destructive interference occurs when a crest meets with another crest.

C A crest meeting a trough is an example of destructive interference.

D Antinodal lines always result in a dark fringe.

5 State the term used to describe an electron ejected from a metal surface in the photoelectric effect.

6 Draw a characteristic line graph for the photoelectric effect. Label the axes and any key points on the graph.

7 At what speed does all electromagnetic radiation travel in a vacuum?

8 What is the term given to the minimum energy required to eject an electron from the surface of a metal.

9 Contrast an emission spectrum with an absorption spectrum.

10 A hydrogen atom is in the $n = 4$ energy state. It then returns to the ground state. What are all the possible energies the emitted photon(s) could have? (Refer to Figure 12.7.8.)

11 What is the de Broglie wavelength of an electron travelling at $25\,m\,s^{-1}$?

12 A quantum of energy has a wavelength 4.25×10^{-5} m.

a Determine the frequency of this quantum.

b Determine the energy of this quantum.

c State its energy in eV.

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13 In a double-slit experiment, light with wavelength 426 nm is used to illuminate twin slits that are separated by 0.010 mm. The pattern produced is observed on a wall 2.00 m from the slits. Find the position of:

a the first interference maximum (bright spot)

b the first interference minimum (dark spot)

c the second interference maximum (bright spot).

14 In a measurement to find the wavelength of a light source, a viewing screen is placed a distance 2.8 m from a pair of slits with a separation 0.020 mm. The first dark fringe is a distance of 2.5 cm from the centre line on the screen. Determine the wavelength of the light.

15 Ultraviolet light of wavelength 210 nm is incident on a polished silver plate. The work function for silver is 4.73 eV.

a Calculate the work function of silver in joules.

b Determine the kinetic energy of the fastest moving electrons.

c Determine the threshold frequency for silver.

» UNIT FOUR

REVOLUTIONS IN MODERN PHYSICS

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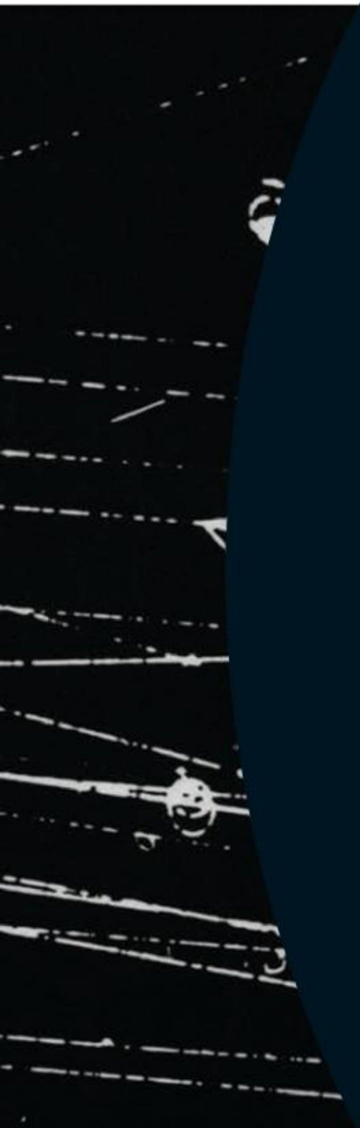

Topic 3: The Standard Model

The topic 'The Standard Model' is a keystone in our understanding of modern physics. In this topic, students explore the range of elementary particles and antiparticles, and understand the reasons for the continuing search for elementary particles in particle physics. The current form of the Standard Model is explored through the consideration of the fundamental forces, as well as the particles leptons, hadrons, mesons, baryons and their quarks. The conservation of lepton and baryon numbers, as well as the symmetry in particle interactions, are also explored.

SCIENCE AS A HUMAN ENDEAVOUR

Students research the Standard Model of Particle Physics, noting contributors to its development. Students are given opportunities to investigate the role of particle accelerators, such as the Large Hadron Collider, and examine interactions between neutrinos and electrons in experimental apparatus such as the Kamioka Neutrino Observatory.

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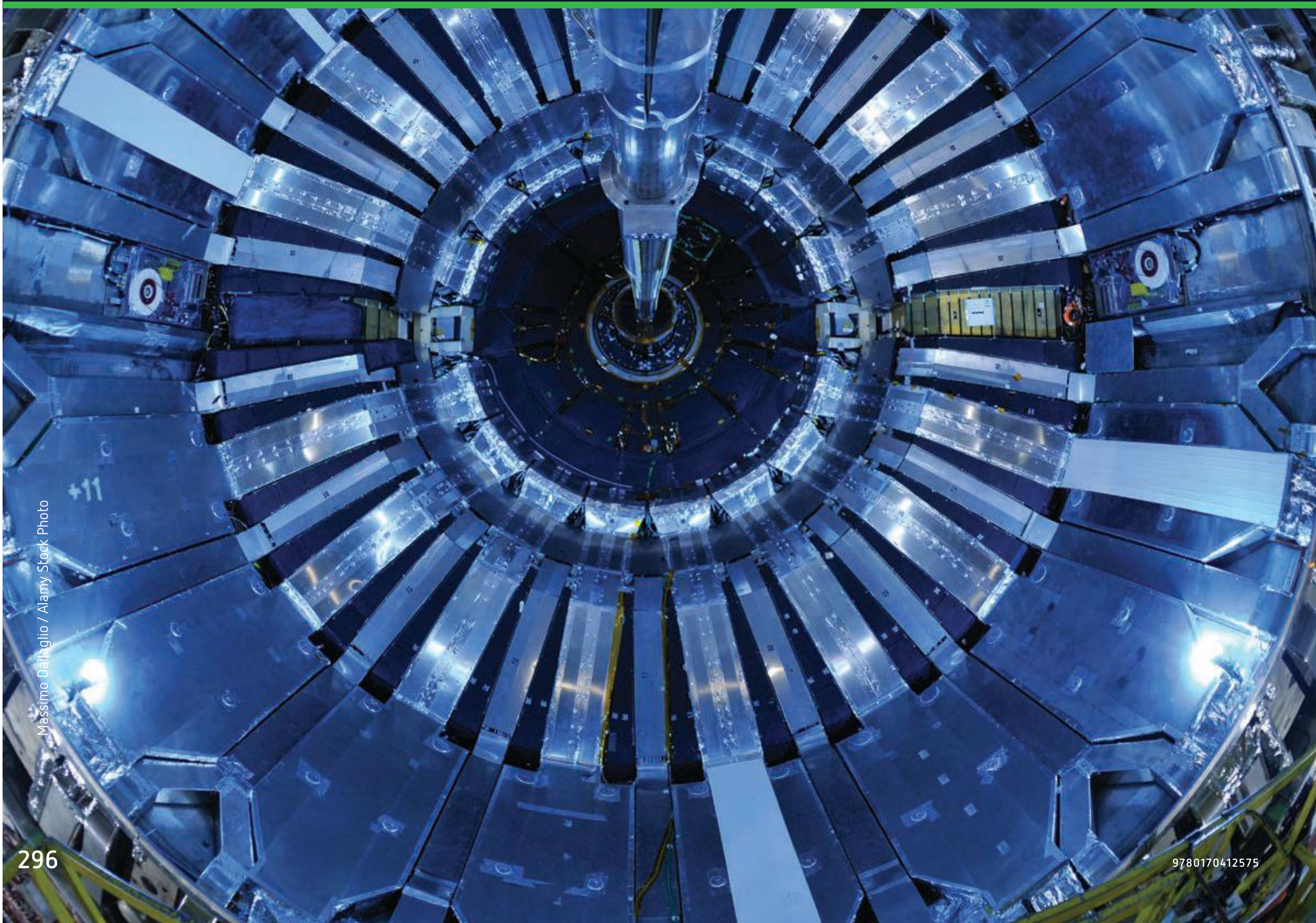
13 THE STANDARD MODEL

Introduction

Over the past century, scientists and engineers working in particle physics have made a series of significant discoveries. These discoveries have led to new theories that describe and predict a plethora of elementary particles and their interactions. Physicists believed that there must be an underlying structure giving rise to the properties and behaviours of these particles. The Standard Model of Particle Physics (the Standard Model for short) is our best current understanding of the universe on the smallest of scales. The model also provides answers to questions about the origins of the biggest of objects, the universe itself.

Stimulus questions

- What are the fundamental building blocks of matter?
- How do we know that elementary particles are indeed elementary?
- What predictions can be made from the details of the Standard Model?
- How do particles interact with each other?
- What are the strengths and limitations of the Standard Model?



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13.1 Elementary particles and antiparticles

One of the characteristics of physics as a science is the belief that complex systems can be understood by examining and understanding the motion and interactions of simpler, component parts. Underlying this belief is another: that, as we break the system into smaller and smaller parts, we will eventually come to a point where there are no internal components. Then we will be dealing with the basic building blocks out of which everything else is constructed.

The idea that there are fundamental building blocks of matter has persisted through the ages, although it has undergone many changes as new discoveries and observations have been made. The ancient Greeks thought that the universe was made out of four basic elements: earth, air, fire and water. Then Democritus introduced the concept of the atom. Later it was found that atoms were made of even smaller components including, electrons, protons and neutrons.

The Standard Model of Particle Physics makes the prediction that these components are themselves made of smaller components called **elementary particles**. Their existence has been supported by experimental evidence gathered from particle colliders.

13.1.1 A brief particle history

13.1.2 What are fundamental particles?

elementary particle a particle whose substructure is unknown

Elementary particles

The model of the atom was largely developed as of 1932. There is a tiny, massive nucleus that contains positively charged protons and neutral neutrons. Surrounding this is a cloud of negatively charged electrons.

A fourth particle, the photon, was also already known by the 1930s. Recall that the photon has no mass and no charge. It does have quantised energy and momentum, which depend on its frequency. The electron and the photon both appear to be elementary particles.

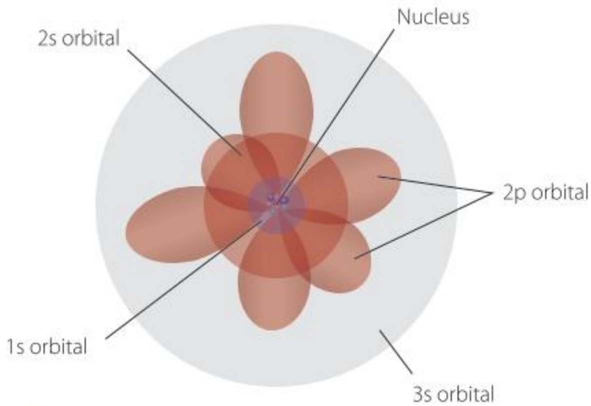

FIGURE 13.1.1 In the modern quantum mechanical model of the atom, the protons and neutrons are contained in the nucleus, surrounded by the electron clouds.

Under the standard model there are three broad classes or families of particles. They are quarks, leptons and gauge bosons. The quarks and leptons represent the most fundamental particles, while the means by which they interact with each other is by exchanging gauge bosons.

Recall that when we are considering the gravitational force between the Sun and Earth, we can treat Earth as if all its mass were concentrated at its centre and use Newton's law of gravitation.

Earth is not really a point mass because it has *internal structure*. When people are close to Earth's surface, the gravitational field no longer appears to be that due to a point mass; the internal structure becomes important. Similarly, a charged object can be treated as a point charge when it interacts with another charged object a long way away, but up close, we must treat it differently.

The electric field of an electron is exactly what we expect for a point charge, suggesting that it has no internal structure and strongly supporting the idea that it is an elementary particle.

SYLLABUS LINK

Chapter 12 discusses the development of the model of the atom.

13.1.3 Positron

13.1.4 Developing the positron

positron the antiparticle of an electron with charge $+e$ and mass m_e

antimatter matter composed of antiparticles, such as positrons, antiprotons and antineutrons

Schrödinger equation a differential equation that describes the wave-like properties of matter existing within a potential field

However, experimental evidence suggests that the neutron is *not* an elementary particle. Although the neutron has no charge, it does have its own intrinsic magnetic field. A magnetic field is the result of moving charged particles or a changing electric field. This indicates that the neutron has some internal structure, and contains charges that add to zero total charge. This suggested to physicists that the neutron is *not* an elementary particle. This suggestion was supported by experimental evidence gained from understanding radioactive decay.

As the proton has a mass very close to that of a neutron and is otherwise a similar particle in its behaviour, the same question was also raised of the proton. Evidence that the proton is not an elementary particle also came from studies of radioactivity.

When a nucleus decays, different types of particles may be emitted. These include β^- and β^+ particles. It was observed that these β particles had the same mass as an electron, and either a positive or negative charge equal to the electron charge.

It turned out that the β^- particle was an electron. When a nucleus emits a β^-, its proton number increases by one and its neutron number decreases by one. Similarly, when a β^+ particle is emitted, a proton is converted to a neutron. Hence, it appeared that neutrons could be converted into protons and vice versa by the emission of these negatively or positively charged electrons, suggesting that neither the neutron nor the proton is an elementary particle.

The positively charged electrons, called **positrons**, that are involved in β^+ decay were discovered in cosmic rays in 1932 by Carl Anderson. Positrons were the first **antimatter** particle to be observed.

By 1932 there were five particles known: three 'normal' matter particles, the electron, proton and neutron; one antimatter particle, the positron; and the photon. Of these, it was already believed that the proton and neutron were not themselves elementary particles, and so the hunt for their component parts began.

13.1.5 Antimatter

13.1.6 What is the Schrödinger equation?

Antimatter

In the 1920s, Paul Dirac developed a relativistic version of the **Schrödinger equation** that could describe many properties of the electron.

However, there appeared to be an issue with this approach: the equation that Dirac had developed to describe the electron had two solutions. One of these solutions gave the wave functions for electrons, the other solution described a particle with the same mass but the opposite charge and magnetic moment. It also predicted that if these two particles should meet, they would both be destroyed, producing a burst of energy. This is called **annihilation**.

annihilation the destructive process resulting when a particle and its antiparticle meet

antiparticle a particle with the same mass and opposite charge and/or spin to a corresponding particle

cloud chamber a chamber containing a supersaturated vapour through which high-energy particles pass, causing vapour trails to be formed and therefore allowing the path of the particles to be visualised

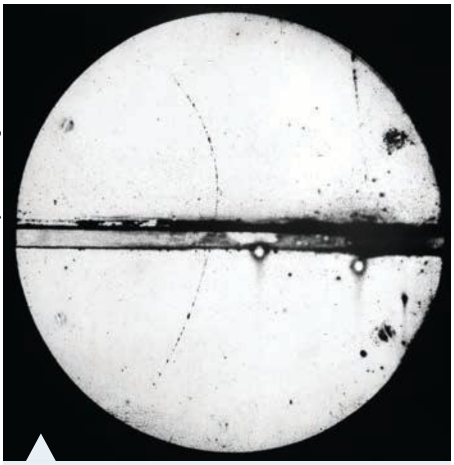
Science Photo Library/Emilio Segre Visual Archives

FIGURE 13.1.2 This cloud chamber photograph shows the track of the first identified positron.

Hence this second particle was the 'anti-electron'. It was this particle, the **antiparticle** to the electron, that Anderson observed in 1932. We now refer to this particle as the positron, to indicate that it has a positive charge, opposite to the negative charge of an electron.

Anderson was using a **cloud chamber** to study cosmic rays. A cloud chamber is a particle detector that detects ionising radiation. It uses a supersaturated vapour, like a layer of fog, in a container or chamber. When a charged particle enters the chamber, it ionises the vapour. The resulting electrically charged vapour particles act as sites for condensation of the vapour. This produces a visible trail of condensation along the path of the particle.

Anderson placed his cloud chamber in a magnetic field. The magnetic field caused the moving charged particles to follow curved paths, just like the ions in a mass spectrometer. This allowed him to distinguish between

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positive and negative charges. Recall that the direction in which a particle's path curves depends on its charge, and that the sharpness of the curve (radius of curvature) depends on the mass.

Anderson noted that some of the tracks in his cloud chamber had electron-like curvature but were deflected in a direction corresponding to a positively charged particle. Anderson had discovered the anti-electron, or positron. Anderson was awarded a Nobel Prize for Physics in 1936 for this discovery.

Dirac's theory went further and suggested that *an antiparticle exists for every particle*. It has subsequently been verified that almost every known elementary particle has a distinct antiparticle.

We represent the antiparticle of a known particle, such as the neutron (n) or electron (e^-), either by placing a bar over the symbol, as in $\bar{n}$ for the antineutron, or by showing that the sign is reversed, as in e^+ for the positron.

INQUIRING FURTHER

PET — AN EXAMPLE OF THE USE OF ANNIHILATION

Electron–positron annihilation is used in the medical diagnostic technique called positron-emission tomography (PET). The patient is injected with a glucose solution containing a radioactive nuclide (tracer) that decays by positron emission, and the glucose is carried throughout the body by the blood. This is useful for locating cancers because glucose is metabolised rapidly in cancerous tumours and the tracer accumulates at those sites, providing a strong signal for a PET detector system. The images produced in a PET scan can also indicate a wide variety of disorders in the brain, including Alzheimer's disease.

Investigate the process by which the positrons emitted by the radioactive glucose undergo annihilation in the body to produce a signal that can be picked up by a PET scanner.

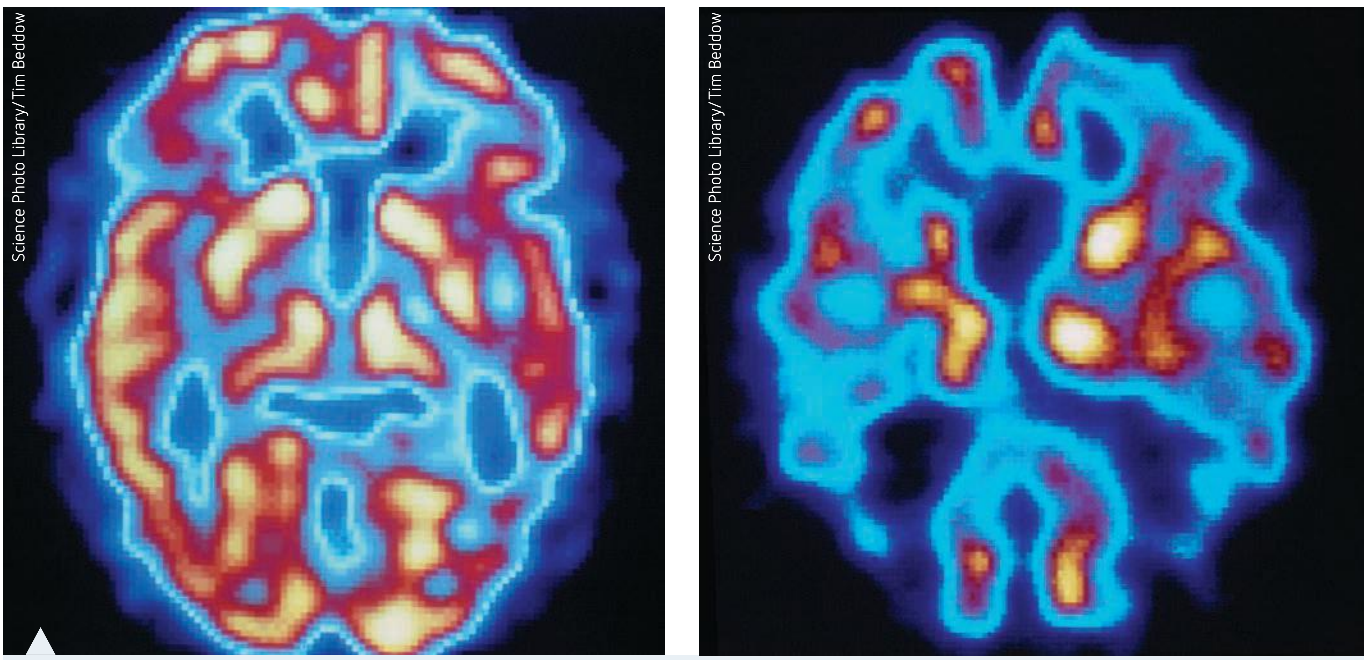

FIGURE 13.1.3 PET scans of the brain of a healthy older person (left) and that of a person with Alzheimer's disease (right). Lighter regions contain higher concentrations of radioactive glucose, indicating increased brain activity.

PRACTICAL ACTIVITY 13.1.1

Build your own cloud chamber and detect cosmic rays

There are very few particle physics experiments that you can do without expensive, and usually high voltage, equipment. However, with some simple equipment you *can* build your own cloud chamber that will detect cosmic rays. You will need some chemicals and dry ice, so you need to be very careful and follow all safety instructions. Typically, it takes about 10 to 20 minutes to detect a high-energy cosmic ray, depending on solar flare activity, so you may also need to be a bit patient after you have built your cloud chamber.

AIM

To build a cloud chamber and detect cosmic rays.

MATERIALS

- a clear glass or clear plastic tank, such as a small fish tank, about 15 cm tall and 15 cm wide by 30 cm long
- a strong source of light such as an overhead or slide projector
- a sheet of metal for a lid for the tank (same size as tank)
- a sheet of cardboard cut to fit the metal lid
- three sheets of felt, 30 cm by 30 cm
- a whole roll of black electrical tape
- foam padding, about 5 cm thick, same dimensions as the tank
- cardboard box just slightly bigger than the clear tank
- glue that is not soluble in alcohol, such as silicon sealant
- isopropyl alcohol (isopropanol) (pure, about 500 mL)
- dry ice, about 500 g
- safety equipment: tongs, gloves, lab coats and safety glasses

RISK ASSESSMENT

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Isopropyl alcohol is toxic and can cause skin irritation. Refer to the materials safety data sheet (MSDS) for more information.	Wear a lab coat, safety glasses and gloves when using isopropyl alcohol. Use in a well-ventilated space or fume cupboard. Dispose of gloves and wash hands thoroughly at the end of the experiment.
Isopropyl alcohol is highly inflammable.	Store isopropyl alcohol away from any sources of heat and ensure bottles are correctly labelled.
The isopropyl alcohol vapour used in the cloud chamber has a low flash point.	Keep chamber well away from all sources of heat and flames.
Dry ice is very cold and can cause cold-burns.	Wear thick gloves and use tongs to handle dry ice.

In your write-up, add any more risks you can think of, as well as ways to manage them.

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PROCEDURE

Building the cloud chamber will take some time. You will probably need a whole lab class to build your cloud chamber and then a second class to test it. Stop at step **6** unless you have ample time to proceed and test your cloud chamber.

1. Fold or roll the felt into strips about 5 cm wide and 30 cm long. Attach these to the inside of the clear container with the glue or sealant to form a ring around the bottom, as shown in Figure 13.1.4. Give the glue or sealant time to dry. (You may want to do this the day before.)

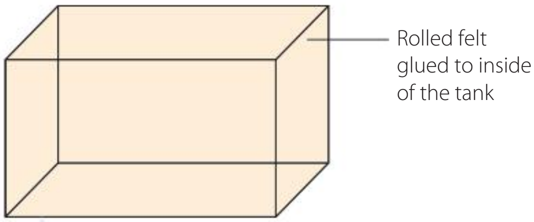

FIGURE 13.1.4 Felt folded and attached to tank as a 'soak zone'.

2. Cover one side of your cardboard sheet with strips of black electrical tape, as neatly as you can. This is to make particle tracks easier to view.
3. Attach the cardboard sheet, tape side out, to the metal lid so that when the lid is in place the black tape faces into the tank.
4. Cut the cardboard box down to about 7 cm tall and place the piece of foam in the bottom of it.
5. Check that your cloud chamber all fits together: the metal lid sits on top of the foam with the black tape facing upwards. The tank then sits upside-down on top of this so the ring of felt padding is at the top.

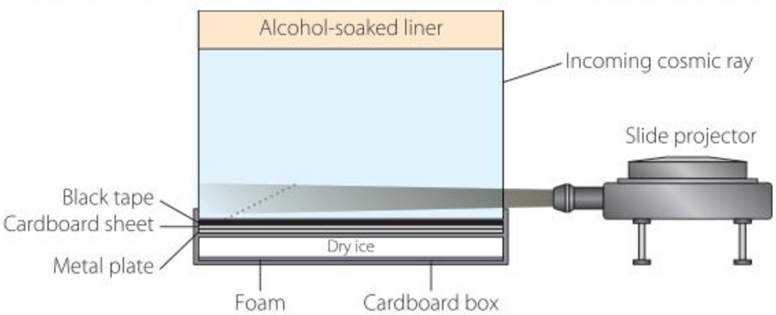

FIGURE 13.1.5 Cloud chamber and light source

It needs to all fit tightly and be well sealed when you are performing your observations. If it is not, air currents will make the particle tracks hard to see.

If it all fits together neatly, then you are ready to add the dry ice and isopropyl alcohol. If not, you will need to make some modifications until it fits neatly together.

6. Remove the tank and metal lid and place a layer of dry ice on top of the foam using tongs. Your teacher may do this step for you.
7. Soak the felt strips with isopropyl alcohol. Your teacher may do this step for you.
8. Seal the metal lid to the tank with tape (more electrical tape, or duct tape).
9. Place the tank upside-down on top of the dry ice so the metal lid is in contact with the dry ice.
10. Arrange the light source so it shines horizontally through the side of the tank. You need a bright light source to clearly illuminate the particle tracks.
11. Turn on the light, and observe your chamber for at least 10 to 20 minutes.

You should see a mist-like fog form inside your chamber. This will be most obvious near the bottom; do not worry if you can only see a thin layer, that will be enough.

RESULTS

1. During the 10 to 20 minutes you observe your chamber, you should see fine tracks forming in your chamber. The tracks, which are the results of the random passage of cosmic rays through the vapour, can form at any time and will last only a very brief time before they disappear, so you will need to watch carefully for the entire period. Draw these tracks and/or photograph or video them.

» 2 Once your chamber is working, you can try putting a strong magnet on one side and observe what happens to the new particle tracks. Note that you will need a *very strong* magnet to obtain any significant curvature of the particle paths.

ANALYSIS OF RESULTS

Record as many tracks as you can. Look for sudden changes of direction in tracks that are straight lines. These could be muon decays, with the incoming track the muon and the outgoing track the electron into which it decays. You may also see tracks that branch like a Y. These are usually due to collisions, for example a muon colliding with an electron and transferring some kinetic energy to it. The stem of the Y is the incoming muon and the two branches are the muon and electron moving off after the collision.

DISCUSSION

Can you identify what is happening in the various tracks that you observed?
You might only be able to hypothesise what the incoming particles were.

SECTION REVIEW 13.1

REMEMBERING

1 Define 'elementary particle'.

2 State the differences between matter particles and their antimatter particles.

3 List the mass and charge of:

- **a** a positron
- **b** an antiproton
- **c** an antineutron.

UNDERSTANDING

4 Explain how a cloud chamber can be used to detect whether an electron or a positron is passing through it.

5 An electron enters a region of uniform magnetic field and curves to the right as shown in Figure 13.1.6. A second charged particle enters the same field but curves to the left. Explain how the particle could be identified as a positron or a proton.

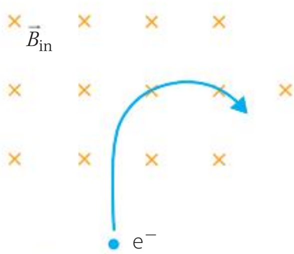

FIGURE 13.1.6 An electron's trajectory curves to the right when it enters the magnetic field.

13.2 Particle physics: the continuing search for elementary particles

13.2.1 How particle accelerators work

particle accelerator
a device in which electric and magnetic fields are used to accelerate beams of particles to high speeds

Following on from the discovery of antiparticles, many other 'new' particles were discovered in many different experiments. Some of these were discovered by analysing the different tracks in cloud chamber experiments, some were found in cosmic-ray experiments like that carried out by Anderson. Other particles were discovered in nuclear decays and still others continue to be found in particle accelerator experiments.

Cosmic rays pass through Earth's atmosphere at random, and their energies vary widely. Hence it is impossible to design well-controlled experiments that use cosmic rays to create new particles.

The energies of the particles emitted in some nuclear decays are well defined, but they are generally low. The equation $E = mc^2$ suggested that if higher energies could be used, more massive particles might be created. Thus, physicists looked for ways to produce beams in which there were more particles with high enough energies to produce, and so enable them to study, new particles. The breakthrough was achieved with the invention of **particle accelerators**.

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From the 1950s onwards, many more particles were discovered in experiments involving high-energy collisions between known particles in these accelerators. The more energy available in the collisions, the higher the number of the new particles produced and the greater their mass.

These new particles are characteristically very unstable and have very short half-lives that range between 10^{-6} s and 10^{-23} s. Their decays produce lighter particles, some of which are also unstable. So, each collision between just two initial particles may result in many outgoing particles, which need to be detected and identified simultaneously. To enable this, physicists and engineers worked together to develop huge, complex apparatus to use at the new particle accelerators. This is an example of the role that technology plays in allowing new experiments, which in turn lead to the development and refinement of theory.

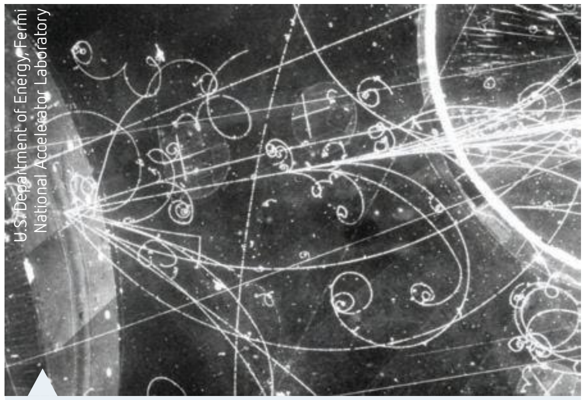

FIGURE 13.2.1 A cloud chamber showing multiple tracks resulting from many particles. Some of the tracks seem to emerge from other tracks, indicating that a particle has decayed into a new particle.

The largest, most powerful and possibly best-known particle accelerator in the world is the Large Hadron Collider (LHC) in Cern, Switzerland. The LHC consists of a 27 km long synchrotron ring built 100 m under the local mountains and uses superconducting electromagnets to accelerate beams of protons up to 7 TeV, approximately 0.999 999 991 times the speed of light. From your previous study you will remember that the kinetic energy of an object is $\frac{1}{2}mv^2$. Even though the mass of the particles is very small, because of the very high velocity, the kinetic energy of the particles is very large.

FIGURE 13.2.2 Inside the tunnel containing the Large Hadron Collider at CERN

When protons collide head on at these colossal energies, a wide variety of particles are created. Seven detector stations are used at various points along this synchrotron ring and detect the properties such as the mass, charge and magnetic spin of these collision products.

The most well-publicised discovery at the LHC was the detection of the Higgs boson, and was the culmination of the work of many scientists and engineers. The story of its detection is discussed further in the Science as a Human Endeavour box at the end of the chapter.

13.2.2 The Large Hadron Collider

13.2.3 The Higgs boson

The particle zoo

So far, several hundred particles have been identified in particle accelerator experiments. The wide variety of properties and behaviours of these new particles led to the term 'particle zoo'. These newly discovered particles included the antiproton, discovered by Emilio Segré and Owen Chamberlain in 1955, and the antineutron, discovered in 1956 by Bruce Cork.

Some of these particles and their properties are listed in Table 13.2.1. The various ways of classifying these particles will be explored in the following sections.

Particle mass

The masses in Table 13.2.1 are given in units of MeV c^{-2}. Recall that mass and energy are related by Einstein's mass equivalence relationship: $E = mc^2$. In particle physics, it is common to give masses in terms of their energy equivalent, rather than in kilograms. $1\text{ MeV}\,c^{-2} = 1.78 \times 10^{-30}$ kg.

TABLE 13.2.1 Some particles and their properties

PARTICLE NAME	SYMBOL	ANTI-PARTICLE	MASS (MeV c^{-2})	B	L_e	L_μ	L_τ	LIFETIME (s)	SPIN
			Leptons						
Electron	e^-	e^+	0.511	0	+1	0	0	Stable	$\frac{1}{2}$
Electron-neutrino	ν_e	$\bar{\nu}_e$	<7 eV c^{-2}	0	+1	0	0	Stable	$\frac{1}{2}$
Muon	μ^-	μ^+	105.7	0	0	+1	0	2.20×10^{-6}	$\frac{1}{2}$
Muon-neutrino	ν_μ	$\bar{\nu}_\mu$	<0.3	0	0	+1	0	Stable	$\frac{1}{2}$
Tau	τ	τ^+	1784	0	0	0	+1	$<4 \times 10^{-13}$	$\frac{1}{2}$
Tau-neutrino	ν_τ	$\bar{\nu}_\tau$	<30	0	0	0	+1	Stable	$\frac{1}{2}$
			Hadrons – mesons						
Pion	π^+	π^-	139.6	0	0	0	0	2.60×10^{-8}	0
	π^0	Self	135.0	0	0	0	0	0.83×10^{-16}	0
Kaon	K^+	K^-	493.7	0	0	0	0	1.24×10^{-8}	0
	$\bar{K}^0_S$	$\bar{K}^0_S$	497.7	0	0	0	0	0.89×10^{-10}	0
	$\bar{K}^0_L$	$\bar{K}^0_L$	497.7	0	0		0	5.2×10^{-8}	0
Eta	η	Self	548.8	0	0	0	0	$<10^{-8}$	0
	η'	Self	958	0	0	0	0	2.2×10^{-10}	0
			Hadrons – baryons						
Proton	p	$\bar{p}$	938.3	+1	0	0	0	Stable	$\frac{1}{2}$
Neutron	n	$\bar{n}$	939.6	+1	0	0	0	614	$\frac{1}{2}$
Lambda	Λ^0	$\bar{\Lambda}^0$	1115.6	+1	0	0	0	2.6×10^{-10}	$\frac{1}{2}$
Sigma	Σ^+	$\bar{\Sigma}^-$	1189.4	+1	0	0	0	0.80×10^{-10}	$\frac{1}{2}$
	Σ^0	$\bar{\Sigma}^0$	1192.5	+1	0	0	0	63×10^{-20}	$\frac{1}{2}$
	Σ^-	$\bar{\Sigma}^+$	1197.3	+1	0	0	0	1.5×10^{-10}	$\frac{1}{2}$
Delta	Δ^{++}	$\bar{\Delta}^{--}$	1230	+1	0	0	0	6×10^{-24}	$\frac{3}{2}$
	Δ^+	$\bar{\Delta}^-$	1231	+1	0	0	0	6×10^{-24}	$\frac{3}{2}$
	Δ^0	$\bar{\Delta}^0$	1232	+1	0	0	0	63×10^{-24}	$\frac{3}{2}$
	Δ^-	$\bar{\Delta}^+$	1234	+1	0	0	0	6×10^{-24}	$\frac{3}{2}$
Xi	Ξ^0	$\bar{\Xi}^0$	1315	+1	0	0	0	2.9×10^{-10}	$\frac{1}{2}$
	Ξ^-	$\bar{\Xi}^+$	1321	+1	0	0	0	1.64×10^{-10}	$\frac{1}{2}$
Omega	Ω^-	Ω^+	1672	+1	0	0	0	0.82×10^{-10}	$\frac{3}{2}$

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Particle lifetime

The lifetimes of the particles given in Table 13.2.1 are the mean lifetimes, t_{mean}. Recall from your studies of radioactivity that unstable nuclei have a half-life that determines how likely they are to decay. In a large population of nuclei with a half-life $t_{\frac{1}{2}}$, one half of the nuclei will decay in a time $t_{\frac{1}{2}}$.

The mean lifetime is related to the half-life by: $t_{mean} = \dfrac{t_{\frac{1}{2}}}{\ln 2} \approx 1.44 t_{\frac{1}{2}}$.

We are using the symbol t_{mean} rather than τ to denote the mean lifetime, to avoid confusion with the τ (tau) particle. You may see lifetimes written as τ in other sources.

Particle spin

Spin is another important property of the particles listed in Table 13.2.1.

A particle's spin is a complex magnetic feature resulting from it having its own magnetic moment. Spin is a measure of its intrinsic magnetic field. Initially this magnetic moment was thought to be due to the particle's rotational motion or spin, hence the name. A modern understanding of quantum mechanics suggests that they are not in fact spinning, but the name remains.

Like other properties of particles, spin is quantised; that is, it can take only specific discrete values. Some particles have spins with integer values, others have half-integer values of spin. These values relate to the magnetic moment and hence magnetic fields of the particles. Particles with non-zero spin have a magnetic moment and hence their own magnetic field.

Particles can be classified according to their spin. Particles with half-integer spin, $s = \frac{1}{2}, \frac{3}{2}$, and so on, are called fermions. As can be seen in Table 13.2.1, leptons and baryons have half-integer spins and are classified as fermions.

Fermions obey the Pauli exclusion principle. The Pauli exclusion principle states that any two fermions in the same quantum system cannot have identical sets of quantum numbers. Electrons in an atom are an example of this – no two electrons in any given atom can have identical quantum numbers.

Particles with integer spin, $s = 0, 1, 2, \ldots$ are called bosons. Mesons and photons are bosons. Bosons do not obey the exclusion principle.

Fermions and bosons play different roles in the Standard Model, as we shall see in the following sections.

The other particle properties, such as baryon number (B) and lepton number (L), are discussed in the next chapter.

The search for order

If you look at Table 13.2.1 you will note that the particles listed are classified into two different types: leptons and hadrons.

In fact, almost all particles can be divided into leptons and hadrons. Leptons, such as electrons, generally have small mass. Hadrons, such as protons and neutrons, generally have large mass and are themselves composed of particles called quarks.

The following sections will investigate the properties of these categories together with one more that does not appear in Table 13.2.1.

spin
a quantum property of particles that results from them having their own magnetic moment and therefore magnetic field

fermions
particles with half-integer spin ($s = \frac{1}{2}, \frac{3}{2} \ldots$), that obey the Pauli exclusion principle

leptons
a family of elementary particles that include electrons, taus, muons, their neutrinos and all of their antiparticles

baryons
a family of heavy subatomic particles, such as neutrons and protons, which contain composite structures made up of three quarks

13.2.4 January 1925: Wolfgang Pauli announces the exclusion principle

Pauli exclusion principle
quantum mechanical principle that two fermions in the same quantum system cannot have identical sets of quantum numbers; e.g. no two electrons can be in the same shell or orbital around an atom and have the same energy

bosons
particles with integer spin ($s = 0, 1, 2, \ldots$); these particles do not obey the Pauli exclusion principle

mesons
a family of heavy subatomic particles that contain composite structures made up of one quark and an antiquark

hadron
a family of elementary particles with a large mass consisting of mesons and baryons

quark
a type of elementary particle (along with leptons and gauge bosons)

SECTION REVIEW

13.2

REMEMBERING

1 Describe how a particle accelerator can raise the energy of a beam of particles.
2 Define 'particle spin'.
3 State the two families to which particles can belong if they are being classified according to their spin. State the value of the spin for each family.

UNDERSTANDING

4 A synchrotron with a constant magnetic field can hold a particle in orbit but is unable to change its speed. Explain why this is the case. Draw a diagram to help explain your answer.
5 Explain the impact that the Pauli exclusion principle has on the allowed states of fermions.

APPLYING

6 If the mass of an electron is 9.11×10^{-31} kg, convert this into units of MeV c^{-2}.
7 Consider a proton with an energy of 4 TeV in the LHC.
   a What is the energy of this particle in joules?
   b How fast would a mosquito, with a mass of 3 mg, need to fly to have the same amount of energy? Comment on your answer.
8 Use Table 13.2.1, page 304, to describe the following particles according to their category, symbol, antiparticle, mass, lifetime and spin.
   a Neutron
   b Proton
   c Electron
   d Tau
   e Tau-neutrino

REFLECTING

9 What is your intuitive reaction to the idea that there may be hundreds, if not thousands of elementary particles? Critique the idea that for a theory to be correct it should be simple and elegant. What arguments can you find for or against this idea?

13.3 Gauge bosons and the fundamental forces of nature

gauge boson force carrying particles that mediate particle interactions through the four fundamental forces

The Standard Model of Particle Physics consider quarks and leptons to be the elementary particles that make up all *ordinary* matter. The distinction between the two is based on an understanding of how they interact with the forces of nature.

The photon is not listed in Table 13.2.1 The photon is known as a **gauge boson**. This is the third category of particles and they act as force carriers.

There are four forces believed to be responsible for all interactions. You have learned about these forces before in your study of physics. They are the gravitational force, the electromagnetic force, the strong nuclear force and the weak nuclear force. Each of these forces is mediated by a gauge boson.

In this particle exchange model of interactions, the basic process is that one particle emits a gauge boson that is subsequently absorbed by another elementary particle. This interaction will be further developed in Chapter 14.

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Electromagnetic force

Recall that electrons and other charged particles interact via the electric and magnetic fields. Recall also that the electromagnetic field consists of particles called photons. The electromagnetic force can therefore be pictured in terms of the exchange of photons between electrically charged particles. The electromagnetic force is said to be mediated by photons.

The idea that electromagnetic interactions could be mediated by photon exchange led physicists to question whether other types of interaction might be modelled in the same way.

Strong nuclear force

The discovery that atomic nuclei are composed of protons and neutrons led to the introduction of a new type of force. The protons in any nucleus should strongly repel one another due to their positive charges. So, there must be some other force acting to stop the nucleus flying apart. We call the force that holds the nucleus together the strong nuclear force, or the nuclear force, because it must be strong to overcome the proton–proton repulsion and it acts between components of the nucleus.

The first theory attempting to explain the nature of the attractive force between nucleons was proposed in 1935 by Japanese physicist Hideki Yukawa. Yukawa used the idea of gauge bosons to explain the strong nuclear force through the pi mesons (π). These particles were discovered in cosmic radiation in 1947.

The later development of quark theory led to the understanding that the strong nuclear force is in fact not fundamental, but rather arises from interactions between quarks. The force that leads to these interactions is termed the **strong force**.

The strong force is responsible for the attractive force that exists between quarks and only has a very short range of about 10^{-15} m (about the size of a nucleus). Its gauge boson is the **gluon**, so named because it 'glues' particles together.

strong force
the attractive force that acts between quarks, holding them together; it is mediated by gluons

gluon
the gauge boson that mediates the strong force

Weak nuclear force

The weak nuclear force is involved in nuclear decay. The exchange particles for the weak nuclear force, **W and Z bosons**, have since been detected.

In 1979 Sheldon Glashow, Abdus Salam and Steven Weinberg won the Nobel Prize in Physics for developing a theory that unifies the electromagnetic and weak interactions. This **electroweak theory** postulates that the weak and electromagnetic interactions have the same strength when the particles involved have very high energies. Because of the mass difference between photons and the W and Z bosons, the electromagnetic and weak forces are quite distinct at low energies, but become similar at very high energies, when the rest energy is negligible relative to the total energy. This behaviour, as one goes from high to low energies, is called *symmetry breaking* because the forces are similar, or symmetric, at high energies but are very different at low energies.

W and Z bosons
the particles that mediate the weak nuclear force

electroweak theory
theory that combines the electromagnetic and weak interactions

13.3.1 The Nobel Prize in 1979

Gravitational gauge bosons

Finally, the gravitational force is a long-range force that has a strength of only about 10^{-39} times that of the strong force. Although this is the force that holds the planets, stars and galaxies together, its effect on elementary particles is negligible. The gravitational force is thought to be mediated by field particles called **gravitons**. The gravitational force is not part of the Standard Model, and the graviton has not yet been detected; however, there are several major international research facilities, including the Australian International Gravitational Research Centre in Western Australia, trying to detect gravity waves or gravitons.

graviton
the hypothetical gauge boson of the gravitational force

13.3.2 ARC Centre of Excellence for Gravitational Wave Discovery

The different forces and their gauge bosons are listed in Table 13.3.1.

TABLE 13.3.1 Forces and their gauge bosons

INTERACTIONS	RELATIVE STRENGTH	RANGE OF FORCE	MEDIATING FIELD PARTICLE	MASS OF FIELD PARTICLE (GeV c^{-2})
Strong	1	Short (1 fm)	Gluon	0
Electromagnetic	10^{-2}	∞	Photon	0
Weak	10^{-5}	Short (10^{-3} fm)	W, Z^0 bosons	80.4, 80.4, 91.2
Gravitational	10^{-39}	∞	Graviton	0

SECTION REVIEW 13.3

REMEMBERING

1 Which forces are included in the Standard Model of Particle Physics?

2 Name the gauge bosons associated with each of the following forces:

a electromagnetic

b strong

c weak

d gravitational.

3 What is the electroweak interaction?

UNDERSTANDING

4 Explain why the force that holds protons and neutrons together is called the strong nuclear force.

5 Explain the basic process behind the particle exchange model of interactions.

APPLYING

6 Calculate the gravitational and electrostatic forces acting between two protons that are 1 fm apart. What does this say about the size of the strong nuclear force acting between them?

13.4 Leptons

As we have seen, in the 1950s there were a huge number of particles known. Broadly, these could be classified as particles with mass – leptons and hadrons – and those without mass, which include exchange particles such as photons.

Leptons are mostly particles with very small mass, such as the electron. We have already noted that electrons appear to be elementary particles because they show no signs of having internal structure. Because the heavier leptons and their antiparticles appear to be identical to electrons and positrons in all respects except for mass, it appears that they too are elementary particles. This leads physicists to believe that the whole family of leptons, including neutrinos, are elementary particles.

Leptons (from the Greek 'leptos', meaning small or light) are particles that do not interact by means of the strong nuclear force, but they do interact via the gravitational, electromagnetic and weak forces. All leptons have spin $\frac{1}{2}$. Unlike hadrons, which have size and structure, leptons appear to be truly elementary, meaning that they have no structure and are point-like.

Unlike hadrons, the number of known leptons is small. Currently, scientists believe that there are only six leptons (plus their antiparticles): the electron, the muon and the tau, plus a neutrino associated with each.

The neutrino was first predicted in 1930 by Dirac. At this time, radioactive beta decay had been observed experimentally, but momentum and energy did not appear to be conserved in these decays.

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Hence, Dirac proposed the existence of a very light, uncharged particle that could carry the unaccounted-for energy and momentum.

Current experiments indicate that neutrinos have a small, but non-zero, mass. Direct experimental evidence for the neutrino associated with the tau was announced by the Fermi National Accelerator Laboratory (Fermilab) in 2000.

SCIENCE AS A HUMAN ENDEAVOUR

The Kamioka Neutrino Observatory is located 1 km under Mount Kamioka in Japan. The Super-Kamiokande detector consists of a huge steel cylinder, 41 m tall and 39 m in diameter that contains 50 000 tonnes of highly purified water.

When neutrinos from sources such as the Sun or supernovae enter the water, they can interact with electrons or nuclei in the water molecules. These interactions produce particles that travel faster than the speed of light in water. (Note that this is much slower than the speed of light in vacuum.) This produces shock waves in the water, which are seen as Cherenkov radiation – a blue-green glow in a cone shape. This cone of light shows up as a ring, incident on detectors on the cylinder structure. The information from the detectors is analysed to determine the type of particle that interacted with the water.

Research the Super-Kamiokande neutrino detector in order to determine why the facility is located deep underground.

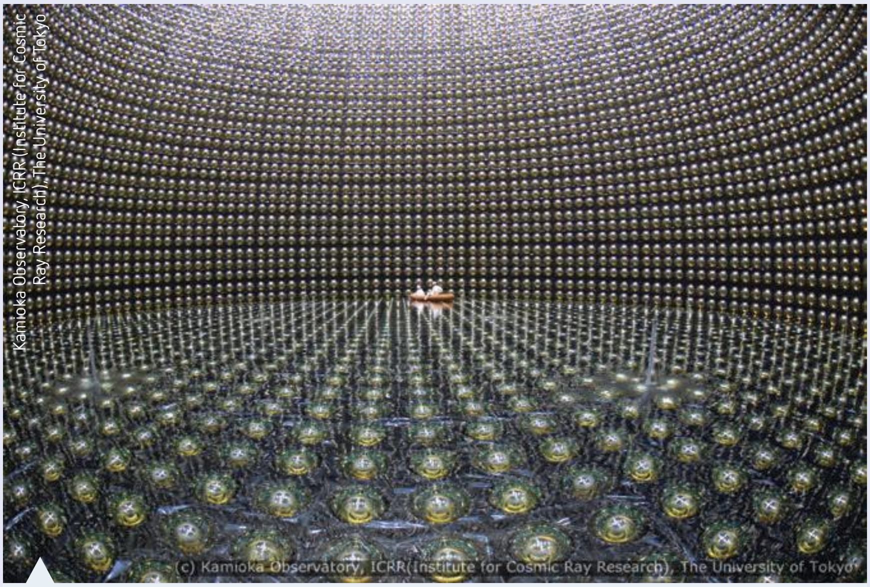

FIGURE 13.4.1 The Super-Kamiokande neutrino detector with multiple detectors, which are used to visualise any Cherenkov radiation forming from the interaction of passing neutrinos with the purified water.

SECTION REVIEW 13.4

REMEMBERING

1 Recall the six types of leptons. Give the names and symbols of each.

2 Recall the forces that are felt by the six types of leptons.

UNDERSTANDING

3 Explain why leptons are considered to be elementary.

4 List the six leptons in order of increasing mass.

13.5 Hadrons: mesons, baryons and their quarks

Hadrons mostly have a large mass, and are further divided into mesons and baryons based on their mass and spin. Families of particles with similar mass but different electric charge are seen for hadrons but not leptons.

Hadrons interact with each other via all four of the fundamental forces of nature.

In addition to their high masses, many of the properties of hadrons raised questions with physicists about whether they were in fact fundamental particles or not. Some of these properties include the:

- existence of groups of hadrons with similar mass but different charge
- ways in which hadrons decay, often to other lighter hadrons
- ways particular hadrons are produced in particular reactions
- magnetic moments of uncharged particles such as the neutron.

Many experiments since their discovery have indeed shown that hadrons are not in fact elementary particles, but rather are made up from constituent particles called quarks.

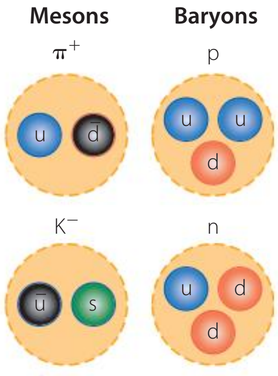

FIGURE 13.5.1 The quark composition of two mesons and two baryons. Mesons consist of a quark–antiquark pair while baryons consist of three quarks.

Quarks

In 1963, Murray Gell-Mann and George Zweig independently proposed a model for the substructure of hadrons. According to their model, all hadrons are composed of two or three elementary constituents called quarks. Figure 13.5.1 is a pictorial representation of the quark compositions of several hadrons.

This early quark model had three types of quarks, designated by the symbols u, d and s, which are given the arbitrary names up, down and strange. The various types of quarks are called **flavours**.

flavours
the six classifications of quark types: up, down, strange, charm, top and bottom

Each quark has an associated antiquark of opposite charge, baryon number, strangeness, charm, topness and bottomness.

The compositions of all hadrons known when Gell-Mann and Zweig presented their model can be completely specified by the following three simple rules.

- A meson consists of one quark and one antiquark.
- A baryon consists of three quarks.
- An antibaryon consists of three antiquarks.

WORKED EXAMPLE 13.5.1

Determine whether these quark combinations are possible particles or not.

1 $\mathrm{d\bar{d}}$

2 dd

3 $\mathrm{\bar{u}d\bar{d}}$

4 $\mathrm{\bar{u}\bar{d}\bar{d}}$

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ANSWERS

1 Possible (meson – it contains a quark–antiquark pair)

2 Not possible

3 Not possible

4 Possible (antibaryon – it contains three antiquarks)

A meson consists of a quark–antiquark pair. A baryon consists of three quarks, and an antibaryon of three antiquarks.

Table 13.5.1 lists the properties of quarks.

TABLE 13.5.1 Properties of the six quarks

NAME	SYMBOL	SPIN	CHARGE	BARYON NUMBER	STRANGENESS	CHARM	BOTTOMNESS	TOPNESS
Up	u	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	0
Down	d	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0	0	0	0
Strange	s	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	−1	0	0	0
Charmed	c	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	+1	0	0
Bottom	b	$\frac{1}{2}$	$\frac{1}{3}e$	$\frac{1}{3}$	0	0	+1	0
Top	t	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	+1

Unlike any particles we have seen before, quarks carry a fractional electric charge. The u, d and s quarks have charges of $+\frac{2}{3}e$, $-\frac{1}{3}e$ and $-\frac{1}{3}e$ respectively, where e is the electron charge ($e = 1.60 \times 10^{-19}$ C). Quarks have spin $\frac{1}{2}$, which means that they are classified as fermions and have their own intrinsic magnetic moment; and therefore, magnetic field. When quarks combine to form a particle, the charge of the particle is the arithmetic sum of the charges of its quarks. The spin of the particle is the sum of the spins of its quarks. However, spin is a vector, so we need to be more careful in how we add the spins. Both the magnitude and the direction of spin are quantised.

Although the original quark model was highly successful in classifying particles into families, some discrepancies occurred between its predictions and certain experimental decay rates. Consequently, several physicists proposed a fourth quark flavour in 1967. The fourth quark, designated c, was assigned a property called charm. A *charmed* quark has charge $+\frac{2e}{3}$, just as the up quark does, but its charm distinguishes it from the other three quarks. This introduces a new quantum number C, representing charm. The new quark has charm C = +1, its antiquark has charm of C = −1; all other quarks have C = 0.

Evidence that the charmed quark exists began to accumulate in the 1970s, when a series of heavy mesons with long lifetimes were discovered. The existence of these new mesons provided firm evidence for the fourth quark flavour.

Further developments in particle physics led to more elaborate quark models and the prediction of two new quarks, top (t) and bottom (b). To distinguish these quarks from the others, quantum numbers called *topness* and *bottomness* (with allowed values +1, 0, −1) were assigned to all quarks and antiquarks (Table 13.5.1).

Although no isolated quark has ever been observed experimentally, the quark model does describe the properties of mesons and baryons.

Mesons

The name 'meson' means middle-sized. Several mesons have masses in the range between the masses of the electron and the proton, although mesons having masses greater than that of the proton have been found. Mesons all have zero or integer spin (0, 1, …); and therefore, are bosons.

No stable meson has ever been observed. All mesons decay. Some decays produce lighter (but themselves unstable) mesons, but all meson decay chains ultimately produce electrons, positrons, neutrinos and photons.

As previously discussed, each meson is constructed from one quark and one antiquark. Table 13.5.2 lists some representative mesons and their constituents.

TABLE 13.5.2 Quark composition of some mesons

		ANTIQUARKS									
		$\bar{b}$		$\bar{c}$		$\bar{s}$		$\bar{d}$		$\bar{u}$	
QUARKS	B	Y	($\bar{b}b$)	B_c^-	($\bar{c}b$)	$\bar{B}_S$	($\bar{s}b$)	$\bar{B}_{D^0}$	($\bar{d}b$)	B^-	($\bar{u}b$)
	C	B_c^{-1}	($\bar{b}c$)	$\frac{J}{Y}$	($\bar{c}c$)	D_S^+	($\bar{s}c$)	D^+	($\bar{d}c$)	D^0	($\bar{u}c$)
	S	B_S^0	($\bar{b}s$)	D_S^-	($\bar{c}s$)	ϕ	($\bar{s}s$)	$\bar{K}_0$	($\bar{d}s$)	K^-	($\bar{u}s$)
	D	B_D^0	($\bar{b}d$)	D^-	($\bar{c}d$)	K^0	($\bar{s}d$)	π^0	($\bar{d}d$)	π^-	($\bar{u}d$)
	U	B^+	($\bar{b}u$)	$\bar{D}_0$	($\bar{c}u$)	K^+	($\bar{s}u$)	π^+	($\bar{d}u$)	π^0	($\bar{u}u$)

Baryons

The name 'baryon' means 'heavy' in Greek. Baryons have masses equal to or greater than the proton mass. Their spin is always a half-integer value $\left(\frac{1}{2}, \frac{3}{2}, \ldots\right)$ and so they are fermions. Protons and neutrons are baryons. Protons are the only stable baryon. All others decay in such a way that the end products of the decay chain include a proton.

Baryons are composed of three quarks, and antibaryons of three antiquarks. Table 13.5.3 lists the quark composition of some baryons.

Note: Some baryons have the same quark composition; for example, the p and Δ^+ baryons have the same composition (uud), as do the n and Δ^0 baryons (udd). In these cases, the Δ particles are considered to be excited states of the proton and neutron.

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TABLE 13.5.3 Quark composition of some baryons

PARTICLE	SYMBOL	QUARK COMPOSITION
Proton	p	uud
Neutron	n	udd
Lambda	Λ^0	uds
Sigma	Σ^+	uus
	Σ^0	uds
	Σ^-	dds
Delta	Δ^{++}	uuu
	Δ^+	uud
	Δ^0	udd
	Δ^-	ddd
Xi	Ξ^0	uss
	Ξ^-	dss
Omega	Ω^-	sss

SECTION REVIEW 13.5

REMEMBERING

1 Recall the combination of quarks that is required to produce a meson.

2 Recall the combination of quarks that is required to produce a baryon.

3 Recall the fundamental forces experienced by quarks.

UNDERSTANDING

4 Explain why the name meson, meaning middle sized, is appropriate.

5 Explain the evidence that supported the idea that hadrons were not elementary particles.

APPLYING

6 Which of the following quark combinations is a possible particle? Explain your answer.

- a ud
- b udd
- c $\bar{u}dd$
- d $\bar{u}u\bar{d}$
- e $ud\bar{d}$

7 Create a diagrammatic representation of the D^- meson.

8 Create a diagrammatic representation of the Δ^- baryon.

13.6 The Standard Model today

The Standard Model: a summary

Particle physicists now believe that all matter is made up of two types of elementary particles: quarks and leptons, as well as the force carriers that are needed for them to interact. There appears to be a symmetry between the quarks and leptons, in that there are six types of each, along with their antiparticles.

Quarks combine to form mesons and baryons, which are both considered to be hadrons. Mesons are made up of two quarks, as shown in Table 13.5.2. Baryons are made up of three quarks each, as shown in Table 13.5.3.

13.6.1 The Standard Model

In addition to these particles, there are also the exchange particles associated with the four fundamental forces, as shown in Table 13.3.1.

Although the details of the Standard Model are complex, its essential ingredients are summarised in Figure 13.6.1. This diagram shows that quarks participate in all the fundamental forces and that leptons participate in all except the strong force.

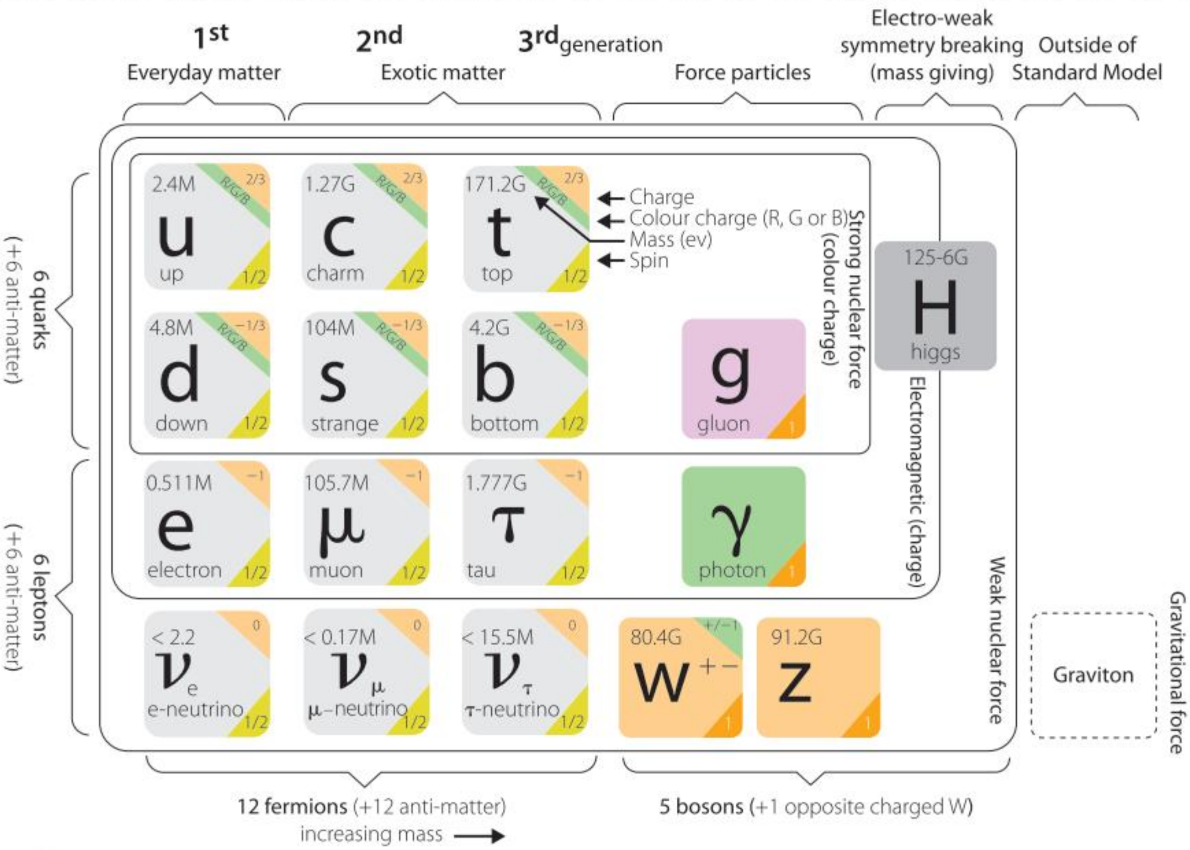

FIGURE 13.6.1 The 'periodic table' of the elementary particles of the Standard Model of Particle Physics

Note that the Standard Model *does not* include the gravitational force at present. However, gravity is included in Figure 13.6.1 because physicists hope to eventually incorporate this force into a **unified theory**.

unified theory
any theory that demonstrates how fundamental forces can be united, and explains the mechanism by which they become distinct, for example the electroweak theory

Limitations of the Standard Model

The Standard Model has helped us make sense of the huge number of particles. It has provided a sort of 'periodic table' to help us understand particle properties as arising from their underlying quark compositions.

The Standard Model has also been very successful in explaining the origin and nature of three of the four fundamental forces – the strong, weak and electromagnetic. However, the Standard Model does not include the gravitational force.

The inclusion of the Higgs boson in the Standard Model gives a mechanism by which particles have mass. Particles acquire mass because of interactions with Higgs bosons. As we have seen, mass is the property that causes the gravitational field; and therefore, the gravitational force. However, the Standard Model does not explain how the gravitational force is mediated. Many physicists believe that the gravitational field is mediated by massless exchange particles called gravitons. This is consistent with

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the way other fields are mediated by exchange particles (Table 13.3.1), but the Standard Model does not include the graviton; therefore, the Standard Model is incomplete.

Gravity is the force that is most significant in the large-scale structure and evolution of the universe. Theories such as the Big Bang theory base their predictions and explanations of cosmological phenomena largely on our current understanding of gravity as described by the general theory of relativity. For example, the predicted existence of **dark energy** and **dark matter** is based on observed gravitational effects. The development of a more fundamental explanation for the gravitational field, or a **grand unified theory (GUT)** that explains all four forces, is therefore of great importance not only to particle physicists but also to cosmologists.

The structure and rate of expansion of the universe can be explained by the presence of dark matter and dark energy. Dark matter neither radiates nor reflects energy – hence it is dark. Its presence is inferred from gravitational effects. Cosmologists believe that dark matter and dark energy consist of subatomic particles of a type as yet undiscovered. The Standard Model does not predict such a particle – there is a mismatch between current theories in cosmology and the Standard Model of Particle Physics.

Finally, the Standard Model in its current form has been found to be consistent with experimental results over the last 50 to 60 years, with one exception. The Standard Model predicts that the neutrino should be massless.

In 1998, at the Super-Kamiokande neutrino detector in Japan, it was discovered that neutrinos can change from one type to another. This implies that they have mass.

Astronomical observations including 'galactic lensing' imply a neutrino mass of about $0.2\,\text{eV}\,c^{-2}$ to $1.5\,\text{eV}\,c^{-2}$. The masses of the various types of neutrinos are not yet known, other than that they are very, very small. However, there is general agreement that neutrinos are not massless. Various theories have been put forward modifying the Standard Model to allow for non-zero neutrino masses.

dark energy
energy that is predicted from the increasing rate of expansion of the universe, but which is not identifiable as any currently known energy form

dark matter
matter that is postulated to explain gravitational effects but which is not observable by the emission or reflection of light

grand unified theory (GUT)
a theory that unites all four fundamental forces in a single model and explains the symmetry-breaking mechanism that caused them to separate into the four distinct forces we now know. There is as yet no widely accepted GUT

13.6.2 Standard Model and neutrino mass

Mass and the Higgs boson

One of the questions raised by the Standard Model is why all the field particles except the W and Z bosons are massless. Or, put another way, why do the W and Z bosons have mass, and in fact what is the origin of the mass of all the other massive elementary particles?

To resolve this problem, a hypothetical particle called the Higgs boson, which provides a mechanism for breaking the electroweak symmetry, was proposed. When particles interact via the Higgs field, they gain mass from this interaction. The field particle is the Higgs boson. The Standard Model modified to include the Higgs boson provides a logically consistent explanation of the massive nature of the W and Z bosons.

The Higgs boson was named for Peter Higgs, one of a group of physicists who proposed its existence in 1964. The existence of the Higgs boson was confirmed in March 2013 by CERN (the European Organization for Nuclear Research) after a particle with properties corresponding to those predicted for the Higgs boson was observed in July 2012.

Discovery of the Higgs boson

As we have seen above, the mass of the Z and W bosons that mediate the electroweak force is something of a mystery in the Standard Model. Related to this is the question as to how particle masses arise and what determines the mass of a given particle.

The theories behind this are extremely complicated mathematically and well beyond the scope of this text. They deal with fundamental symmetries in nature, and are referred to as *gauge theories*. They are based on quantum field theories, in which exchange particles are the mechanism by which particles interact and reactions occur.

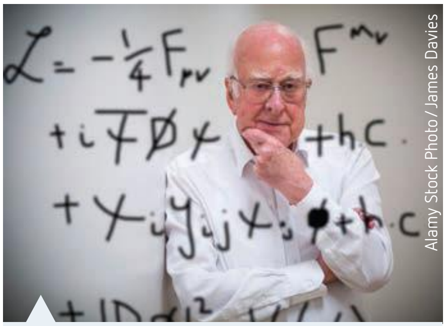

FIGURE 13.6.2 Peter Higgs developed the mechanism that predicted the existence of the Higgs boson concurrently and independently of Englert and Brout, and Guralnik, Hagen and Kibble.

In 1964, British physicist Peter Higgs proposed the existence of a field, now called the Higgs field, with which particles with mass interact. It is this field that gives them the property of mass. The field is composed of particles, now called Higgs bosons.

In the same year, and published in the same journal (*Physical Review Letters*), were two other papers that also described a mechanism by which particles acquired mass via interactions with a field. The first paper was by Francois Englert and Robert Brout, who were working in Brussels, and the other was by Gerald Guralnik, Carl Hagen and Tom Kibble, a collaboration of British (Kibble) and American (Guralnik and Hagen) physicists working in London.

Although the popular literature generally only mentions Higgs, all three papers are considered of equal importance in the development of the relevant theory by particle physicists. Many physicists, including Higgs himself, have argued that the name 'Higgs boson' is inappropriate because it does not acknowledge the contribution made by the others. However, given the widespread and long-term use of the name, it is unlikely to change now.

Remember that for a model to be considered a good one, it must *explain existing phenomena* and *produce testable predictions*. The model developed by Englert, Brout, Higgs, Guralnik, Hagen and Kibble explained why the weak force has a short range, while the electromagnetic force has an infinite range. It also explained why particles have mass. The model predicted the existence of a gauge boson – the Higgs boson – as well as its approximate mass. The detection of this particle is an important test of the theory.

However, detecting a Higgs boson is not a simple task. It requires massive energies of colliding particles to produce a reaction in which isolated Higgs bosons can be detected. It also requires massive computing power to analyse the data generated in such collision events. The Large Hadron Collider (LHC) was constructed, among other purposes, to search for the Higgs boson.

In July 2012, almost 50 years after its prediction, the detection of a particle with the properties and approximate mass of the Higgs boson was announced by physicists at CERN. It is interesting to note that the announcement *did not* claim that the particle found was the Higgs boson. Such a finding is of enormous importance in confirming the Standard Model and in understanding the nature of matter, forces and energy – the basic concepts in physics. This explains why researchers were hesitant to make such a significant claim without further careful analysis and measurement. Nonetheless, the announcement caused enormous excitement.

Eight months later, in March 2013, a press release from CERN stated that 'the measured interactions of the new particle with other particles strongly indicates that it is a Higgs boson'. The same press release (see weblink) quotes a spokesperson for one of the experiment teams as saying: 'The preliminary results with the full 2012 data set are magnificent and to me it is clear that we are dealing with a Higgs boson though we still have a long way to go to know what kind of Higgs boson it is.'

In 2013, the Nobel Prize for physics was awarded to Peter Higgs and Francois Englert (Robert Brout died in May 2011, and the Nobel Prize is not awarded posthumously). The citation by the Nobel society states that the award was given 'for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider'.

Although many physicists believe that Guralnik, Hagen and Kibble should also be recognised, the Nobel Prize is awarded to a maximum of three people.

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The Standard Model and cosmology

As you have seen, the Standard Model of particle physics deals with elementary particles, which are the smallest building blocks of matter. In contrast, cosmology deals with the large-scale structure of the universe, such as galaxies and nebulae. So, it is not immediately obvious how the two are connected.

To understand the current structure of the universe, and predict its future, we need to understand the evolution of the universe. The currently accepted theory of how the universe began, the Big Bang theory, states that the universe began as an infinitesimally small and dense singularity about 14 billion years ago. This singularity exploded, forming all the elementary particles described by the Standard Model of particle physics. The interaction of these particles, which led to the formation of other particles – atoms, molecules and eventually stars and galaxies – is governed by the fundamental forces.

Hence, to understand the evolution of the universe, we need to start with particle physics. But to understand where the particles came from in the first place, we need to use cosmology: the two are intimately connected.

The Big Bang theory is an expansion theory of the universe. It says that the universe was once much smaller and has been expanding since some time in the past. There are other expansion theories in cosmology, as well as steady state theories (which say that the universe is not expanding) and even oscillatory theories which say that the universe is oscillating in size and happens to be expanding at the moment. However, the Big Bang theory is currently the most widely accepted cosmological theory.

There are two main experimental observations that support the Big Bang theory. The first is the **redshift** of light from distant galaxies, described in the next section. If we assume that this means that all points in space are moving away from us and extrapolate backwards in time, then the universe must once have been much smaller. The second important piece of evidence is the existence of the **cosmic microwave background radiation**, which is discussed later.

In addition to the redshift and cosmic microwave background radiation, the different characteristic spectra of distant stars and the structure of distant galaxies, compared to close ones, implies that the universe is changing with time; and therefore, not in a steady state. This argument is based on the idea that looking at distant galaxies is equivalent to looking backwards in time. This is because light takes a finite time to travel a given distance. If a star is 1000 light-years away, then the light from that star we observe today left the star 1000 years ago and what we are actually observing is what that star looked like 1000 years ago.

redshift
the observed shift to longer wavelength of spectral lines in distant stars

cosmic microwave background radiation
the observed radiation coming from all points in space corresponding to radiation from a black body at 3 K; it is believed to come from an earlier, much hotter stage of the evolution of the universe

The expanding universe

In 1912 Vesto Melvin Slipher reported that most galaxies are receding from Earth at speeds of up to several million kilometres per hour. Slipher used Doppler shifts in spectral lines to measure the velocities of various galaxies.

Recall that at this time spectra had already been used to identify specific atomic species. Slipher observed spectra from distant galaxies that had the same line spacings as known species, but with the frequency of each line shifted by a fixed amount.

When the source of a wave is moving, the frequency of the wave detected by an observer is shifted. A higher frequency is detected if the source of the wave is travelling towards the observer. A lower frequency is detected if the source is travelling away from the detector. You may have noticed this effect when an ambulance or police car goes past you with its siren on. Initially, as the vehicle approaches, you hear a higher frequency. As the vehicle passes you, the frequency drops and you hear a lower frequency as it moves away.

The spectral lines from distant galaxies that Slipher observed were consistently shifted to lower frequencies. The shift varied for different galaxies, but in each case the shift was towards a lower frequency (or longer wavelength). This is called a redshift, because red is at the lower frequency end of the spectrum. From these shifts Slipher deduced that the galaxies were moving away from us.

Subsequent observations by other astronomers consistently showed that the spectra of stars in all observed galaxies were redshifted. It appeared that everything was moving away from us!

Not only did observation show that all galaxies are moving away from us, it also seems that the more distant the galaxy, the greater the redshift. This implies that the further away a galaxy is, the faster it is moving.

In the late 1920s, Edwin P. Hubble put forward the theory that the entire universe is expanding. Observations showed that the speeds at which galaxies are receding from Earth increase in direct proportion to their distance from us.

This is possible if the entirety of space–time is expanding, with all points getting further away from each other. So the universe must have started out much smaller.

The current theory in cosmology, the Big Bang theory, says that it started with an explosion of all matter from a single point, a singularity.

The Big Bang theory and the evolution of the universe

According to the Big Bang theory, the universe erupted from an infinitely dense singularity about 14 billion years ago.

For the first few moments after the Big Bang the universe was at such extremely high energy (temperature) that all matter was contained in a quark–gluon plasma.

It is thought that in these first moments all four fundamental forces were unified; the strong, electroweak and gravitational forces were joined to form a single force.

The evolution of the four fundamental forces from the Big Bang to the present is shown in Figure 13.6.3.

Then, about 10^{-35} s after the Big Bang, when the temperature had dropped to about 10^{29} K, symmetry breaking occurred for gravity. This symmetry breaking meant that the properties of the gravitational force became distinct from those of the other forces. At this time, the strong and electroweak forces remained unified.

It was a period when particle energies were so great that very massive particles as well as quarks, leptons and their antiparticles existed. For some reason not yet understood, far more matter than antimatter particles formed. The amount of matter far exceeds the amount of anti-matter in our universe to this day. This is not explained by the Standard Model of Particle Physics, or by the Big Bang theory.

Then the universe rapidly expanded and cooled, and the strong and electroweak forces became distinct.

The universe continued to cool and, at approximately 10^{-10} s after the Big Bang, the electroweak force split into the weak force and the electromagnetic force.

After a few minutes, protons and neutrons condensed out of the plasma. For half an hour, the universe underwent thermonuclear detonation, exploding as a hydrogen bomb and producing most of the helium nuclei that now exist. The universe continued to expand, and its temperature dropped.

Until about 700 000 years after the Big Bang, the universe was dominated by radiation. High-energy radiation prevented matter from forming neutral hydrogen atoms because collisions would instantly ionise any atoms formed. Photons were continuously scattered from the vast numbers of free electrons, resulting in a universe that was opaque to radiation.

This is an important point in the history of the universe. Astronomers claim that when you look through a telescope into space, you are looking into the past. This is because light takes a finite time to travel through space. Hence, the further away you look into space, the further back in time you are seeing, because the light has taken a long time to reach you.

No matter how far away you look with an optical telescope, you cannot see further back than the time when the universe was 700 000 years old. This is because the universe was opaque to light before then, so no light can reach us from that time. The development of more powerful radio telescopes, such as the

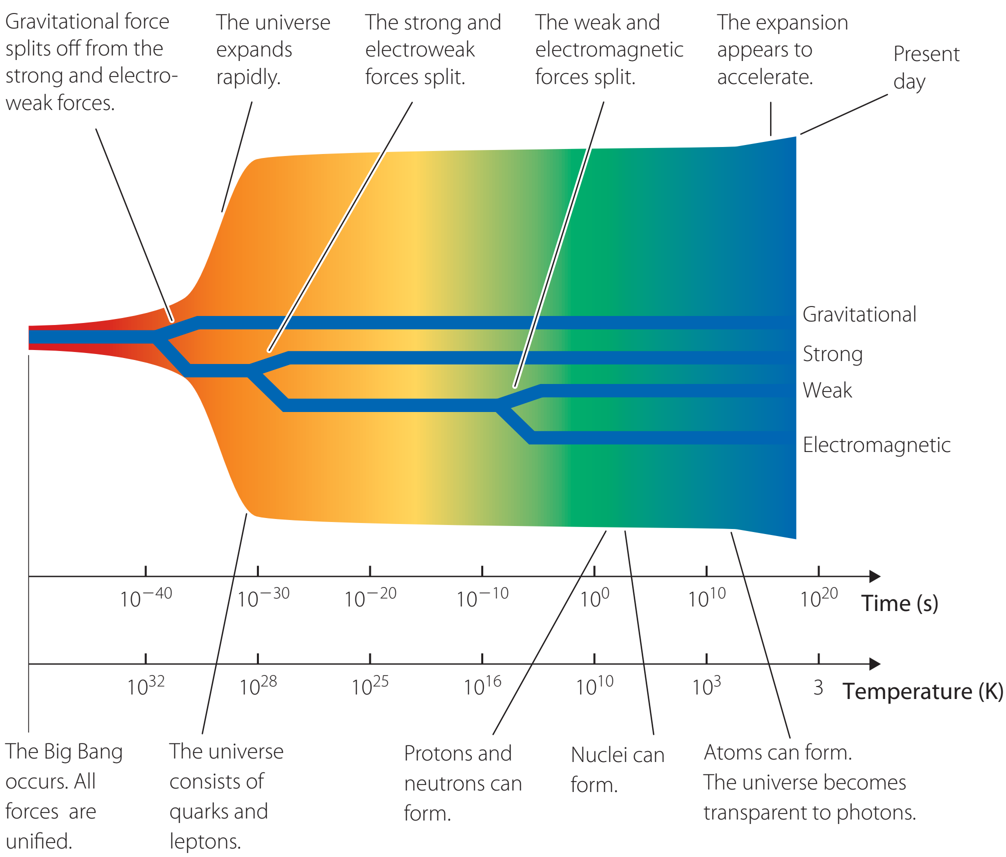

FIGURE 13.6.3 The Big Bang and the evolution of the fundamental forces from one original fundamental force

Square Kilometre Array (SKA) in Western Australia, that can detect much longer wavelengths will be able to analyse signals from beyond this time.

The 700000-year-old universe had expanded and cooled to approximately 3000 K and protons could now bind to electrons to form neutral hydrogen atoms. Now that atoms existed as the main state of matter, far more wavelengths of radiation were *not* absorbed by atoms than were absorbed, and the universe suddenly became transparent to photons.

Radiation no longer dominated the universe, and clumps of neutral matter steadily grew: first atoms, then molecules, gas clouds, stars and finally galaxies.

The universe has continued to expand and cool since the Big Bang.

One prediction of the Big Bang theory is that residual radiation from the Big Bang should currently be observable. This was predicted in 1948 by Ralph Alpher and Robert Herman, although their work was largely ignored at the time.

This radiation comes from the time when the universe was at a temperature of about 3000 K and photons were first able to pass through the matter in the universe. We would expect a radiation spectrum from the early universe to look like that of a 3000 K black-body curve.

The universe has since expanded and cooled and the predicted radiation spectrum now corresponds to a 3 K black-body curve. This background radiation should have a peak intensity at a wavelength of a few millimetres, which is in the microwave region of the spectrum. Hence this residual radiation is known as 'cosmic microwave background radiation'.

Cosmic microwave background radiation was first observed in 1965 by Arno Penzias and Robert Wilson. The measurement of the radiation was important in establishing the Big Bang theory as the accepted theory in cosmology.

In 1965, Arno A. Penzias and Robert W. Wilson of Bell Laboratories were testing a sensitive microwave receiver for satellite communications. A faint background hiss was interfering with their experiments. They noticed that the hiss was the same regardless of the direction they pointed the antenna. They cooled the microwave detector and went outside to chase a flock of pigeons out of the horn-shaped antenna, but the signal remained.

In a casual conversation with colleagues they realised that what they had taken to be interference caused by pigeons was actually the residual radiation from the Big Bang.

Subsequent measurements confirmed that the radiation they measured corresponded to that of a black body at 2.7 K. Penzias and Wilson were awarded the Nobel Prize for their discovery in 1978.

The Big Bang theory gives us a model for the beginning of the universe. But what will happen to the universe in the future? Will it continue to expand and cool forever, or will it at some time begin to contract again? What happens will depend on the amount of mass in the universe, and on the rate at which the universe is expanding.

If there is enough mass, then eventually the gravitational force will cause the expansion to slow down and stop, and then reverse it. Gravity will pull all the galaxies, stars and planets back together into a 'big crunch'. If there is not enough mass, or if the rate of expansion is too great, then the universe will continue to expand and cool forever.

In 1998 observations of the apparent brightness and the redshift of supernovae were used to measure their distance and speed of recession from Earth. These observations led astronomers to the conclusion that the expansion rate is increasing.

To explain this acceleration, physicists proposed the existence of dark energy, which is energy possessed by the vacuum of space. The theory is that in the early universe, gravity dominated over the dark energy. As the universe expanded and the gravitational force between galaxies became smaller because of the great distances between them, the dark energy became comparatively more important. The dark energy results in an effective repulsive force that causes the expansion rate to increase. This is a similar mechanism to that which causes hot gases to expand.


Alamy Stock Photo/cbphoto

FIGURE 13.6.4 The Anglo-Australian Telescope at the Siding Spring Observatory, NSW

Other observations carried out between 2006 and 2011 at the Anglo-Australian Telescope at the Siding Spring Observatory have provided strong evidence that the expansion rate is indeed increasing. The new results imply that dark energy is likely to account for about 72% of the energy in the universe!

The dark energy may be acting to increase the rate of expansion of the universe. If the balance between dark energy and gravity favours dark energy, then the universe will continue to expand forever. Based on the observable mass of the universe, dark energy should win out. However, based on observable gravitational effects, there appears to more than just visible matter in the universe.

'Dark matter' was first postulated by Jan Oort in 1932. Recall from your study of gravity and orbital motion that the orbital velocity of a planet depends on the mass of the star about which it orbits. Similarly, the orbital velocity of stars about the galactic centre depends on the mass in the galaxy. Oort observed that the measured velocity of stars in the Milky Way did not correspond to that predicted by the observable mass in the Milky Way. The observable mass is that calculated from the objects that we can see – mainly stars.

Observations by other astronomers also provided evidence that the total mass in galaxies, including our own, is far greater than that due to visible matter. The 'missing matter' needed to explain these observations was given the name 'dark matter' because we cannot see it.

Cosmologists now theorised that most of the matter in the universe is this mysterious dark matter.

The nature of both dark energy and dark matter remains a mystery that many physicists hope to solve.

Limitations of the Big Bang theory

The Big Bang theory is the model of the evolution of the universe that is most widely accepted by cosmologists today. However, as with all the models and theories we have examined so far, it has limitations.

One of the predictions based on the combination of particle theory and the Big Bang theory is the creation of magnetic monopoles (isolated magnets with only one magnetic pole). However, no magnetic monopoles have ever been observed.

INQUIRING FURTHER

SEARCH FOR A GRAND UNIFIED THEORY

There have been many attempts to compose a theory that combines the findings of quantum mechanics and the Standard Model of Particle Physics with the predictions made by the general theory of relativity.

One of the most popular current attempts to describe the universe as we know it is string theory in which matter is not made up of the point-like particles described by the Standard Model, but rather by tiny one-dimensional strings vibrating in a universe of eleven dimensions.

One of the many predictions of string theory is that of a multiverse in which every possible configuration of matter exists, and the reason that the universe exists in the state in which we observe it is because that is the configuration of the universe in which we live.

Research some of the variants of string theory and discover how they are closer to being a true grand unified theory than most competing models.

SECTION REVIEW 13.6

REMEMBERING

1 Name the three types of fundamental particles.

2 What causes the redshift in observed matter?

3 What is dark matter and why is it given this name?

UNDERSTANDING

4 Describe three limitations of the Standard Model.

5 Why are we not able to see further back in time than when the universe was approximately 700 000 years old?

6 Why is the detection of the Higgs boson of such importance to the Standard Model?

7 Why could the Higgs boson not be detected with earlier particle physics experiments?

APPLYING

8 List the particles that you would normally expect to be within your body. Include the gauge bosons.

9 The Higgs boson detected has a mass between 125 GeV c^{-2} and 126 GeV c^{-2}. What is this in kilograms? Give your answer in the form (__ ± __) × 10⁻__ kg.

10 The construction of the LHC required the cooperation of many universities around the world. Why do you think this was needed and what ramifications did this have?

11 Use the information on black-body radiation provided in previous chapters to find the peak wavelength radiated by:

- **a** a black body at 3000 K (the temperature of the universe 700 000 years after the Big Bang)
- **b** a black body at 3 K (the universe today).

REFLECTING

12 If you were in charge of the Nobel committee for physics, to whom would you have awarded a Nobel Prize for the prediction of the Higgs boson? Justify your answer.

13 Why do you think a unified theory that incorporates all four forces is important to both particle physicists and cosmologists? How has your understanding of the way in which different areas of physics interact changed? Draw a concept map to illustrate your understanding.

14 The Higgs boson was called 'the God particle' by Leon Lederman. Perform research to find out why. Comment on Higgs's response that it was embarrassing and 'the kind of misuse ... which I think might offend some people'.

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Cloud chamber
   b Dark energy
   c Dark matter
   d Electroweak theory
   e Elementary particle
   f Grand unified theory
   g Particle accelerator
   h Unified theory

2 Explain the basic process behind the particle-exchange model of force interactions.

3 Detail the two categories of particles that make up all of ordinary matter and name the particles within each category.

CATEGORY QUESTIONS

4 Describe the differences between leptons, hadrons and gauge bosons.

5 Describe the differences between fermions and bosons.

6 Describe the differences between mesons and baryons.

7 Describe the four fundamental forces in terms of the particles they act upon and the particles that mediate their interactions.

ELABORATION QUESTIONS

8 Elaborate on the details of the electroweak theory and suggest how this was the first successful step in the development of a unified theory.

9 Explain the process by which particles are said to gain mass.

10 Suggest reasons why the Standard Model is said to be successful.

11 Explain what would be needed by any theory claiming to be a grand unified theory.

EVIDENCE QUESTIONS

12 Explain the evidence that exists that suggests that electrons are fundamental particles while protons and neutrons are not.

13 Provide reasons to support the suggestion that the Standard Model is incomplete.

14 Explain how the Standard Model has been able to support the postulates of the Big Bang theory.

END-OF-CHAPTER EXAM

End-of-chapter test

1 Which of the following particles is classified as a gauge boson?

A Proton
B Quark
C Neutron
D Photon

2 Which of the following particles is classified as a lepton?

A Proton
B Quark
C Electron
D Photon

3 Which of the following particles is classified as a hadron?

A Proton
B Quark
C Electron
D Photon

4 The correct symbol for a positron is:

A p^+.
B p^0.
C e.
D e^+.

5 To which class of particles does the Pauli exclusion principle apply?

A Leptons
B Bosons
C Mesons
D All of the above

6 Which of the following forces is mediated by the Z^0 boson?

A Strong
B Weak
C Electromagnetic
D Gravitational

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7 Which of the following forces is not experienced by leptons?

A Strong

B Weak

C Electromagnetic

D Gravitational

8 Recall the correct term for the antimatter version of a proton.

9 Recall the correct classification of all particles that are composed of two or three quarks.

10 Recall the term for the quantum property of atoms that is related to their intrinsic magnetic moment.

11 Recall the term given for the destructive process that occurs when a particle meets its antiparticle.

12 Calculate the mass, in kg, of the following particles:

a π

b Ω.

13 Explain the differences between fermions and bosons.

14 Explain the differences between leptons and hadrons.

15 Explain the differences between the strong nuclear force and the strong force.

16 Explain the differences between mesons and baryons.

17 Determine whether these quark combinations are possible or not. For those that are possible, identify that particle.

a cd

b $\bar{u}ud$

c $\bar{u}\bar{s}\bar{s}$

d $c\bar{c}$

14 PARTICLE INTERACTIONS

Introduction

The fundamental particles, also known as elementary particles, are the building blocks of the universe. In the previous chapter, we saw that these elementary particles are grouped according to specific properties. At this fundamental level, these particles have interactions that can only be represented theoretically, as we are unable to see what is actually happening. The search for order in the universe is never ending, and at the particle level we need to consider which components of the building blocks are conserved.

Stimulus question

What is the significance of symmetry in particle interactions?

Getty Images / Anton Petrus

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14.1 Lepton number and baryon number

Leptons and baryons are both particles with unique characteristics. They are both categorised as fermions, which obey the Pauli exclusion principle.

Lepton number

A lepton is a particle that does not interact by means of the strong nuclear force, but it does interact via the gravitational, electromagnetic and weak forces. There are three varieties of leptons: the electron (e), the muon (μ) and the tau (τ), each with an associated neutrino (ν). There are three quantum numbers called **lepton numbers**, L_e, L_μ and L_τ, associated with these particles. The electron and the electron neutrino (ν_e) have electron lepton number $L_e = +1$, and the antileptons e^+ and $\overline{\nu}_e$ have $L_e = -1$. All other particles have $L = 0$. Table 13.2.1 (page 304) lists the lepton numbers of each type of lepton. L_μ and L_τ are also equal to 1, and the antileptons to these have lepton number -1. The lepton notation used depends on the particles in the reaction.

lepton number quantum number associated with each lepton, antilepton and non-leptonic particle

The lepton number is a quantum number associated with each lepton and its neutrino (+1), and each antilepton and antineutrino (−1), to help describe the absence of reactions that created new particles. The lepton numbers are conserved during a reaction.

Baryon number

A baryon is a relatively heavy fermion with half-integer spin. Protons and neutrons are examples of baryons. Every particle can be assigned a **baryon number**: $B = +1$ for all baryons, $B = -1$ for all antibaryons; $B = 0$ for all other particles. Table 13.2.1 (page 304) lists the baryon number for each type of particle. Baryon number is used to describe the annihilation and creation of particles at a very small scale. Experimental results show that whenever a baryon is created in a decay or nuclear reaction, an antibaryon is also created. Baryons and antibaryons can annihilate, just like electrons and positrons do. The baryon numbers are conserved during a reaction.

baryon number quantum number associated with each baryon, antibaryon and non-baryonic particles

SECTION REVIEW 14.1

REMEMBERING

1 State the lepton number for each lepton and antilepton.

2 Define 'baryon number'.

3 State the baryon numbers for baryons, antibaryons and other particles.

UNDERSTANDING

4 Explain the difference between lepton number and baryon number.

14.2 The conservation of lepton number and baryon number

The laws of conservation of energy, linear momentum, angular momentum, spin and electric charge provide us with a set of rules that all processes and interactions must obey. In addition, we are confined to a new set of conservation rules: the **law of conservation of lepton number** and the **law of conservation of baryon number** in any reaction or decay.

law of conservation of lepton number each of the lepton numbers L_e, L_μ and L_τ is a conserved quantity

law of conservation of baryon number whenever a nuclear reaction or decay occurs, the sum of the baryon numbers before the process must equal the sum of the baryon numbers after the process

14.2.1 Particle interactions and conservation laws

Decay of the neutron with lepton and baryon conservation

Consider the decay of a neutron:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Before the decay, the lepton electron number is $L_e = 0$; after the decay it is $0 + 1 + (-1) = 0$. This shows that lepton electron number is conserved. The reason we use the L_e notation is because this decay is concerned with the electron.

Consider again the decay of the neutron. On the left-hand side, the neutron has baryon number $B = +1$. On the right-hand side, the proton has baryon number $B = +1$ and the electron and antineutrino have baryon number $B = 0$. As baryon number is conserved, this is an allowed process for neutrons.

neutrino oscillation
a phenomenon in which a neutrino with a given lepton association (e, μ or τ) can later be measured to have switched to another neutrino type (e, μ or τ); lepton number is still conserved in this instance

When baryon or lepton number is different

The decay of a proton into a positron and a neutral pion satisfies the conservation of energy, momentum and charge. It does not, however, satisfy the law of conservation of baryon number. Hence, this decay has never been observed.

If lepton number is not conserved in a reaction, it means that the process is not possible. It has been found that lepton number is conserved in all reactions between particles; however, neutrinos have been observed to change from one type to another. This is called **neutrino oscillation**.

WORKED EXAMPLE 14.2.1

Use the law of conservation of lepton number and baryon number to determine if the following reactions are possible.

1 $p + n \rightarrow p + p + n + \bar{p}$

2 $\mu^{-} \rightarrow e^{-} + \bar{\nu}_e + \nu_\mu$

3 $p + n \rightarrow p + p + \bar{p}$

ANSWER

1 All these particles are baryons, so lepton number will be conserved.
Baryon number before the reaction:
$B = (+1) + (+1) = +2$
Baryon number after the reaction:
$B = (+1) + (+1) + (+1) + (-1) = +2$
Both baryon and lepton number are conserved; therefore, the reaction is allowed.

2 These particles are all leptons, so baryon number will be conserved.
Lepton numbers before the reaction:
$L = +1$
Lepton number after the reaction:
$L = +1 + (-1) + (+1) = +1$
The lepton numbers are conserved, and on this basis the decay is possible.

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3 All these particles are baryons so lepton number will be conserved.
Baryon number before the reaction:
$B = (+1) + (+1) = +2$
Baryon number after the reaction:
$B = (+1) + (+1) + (-1) = +1$
The total baryon number is reduced and not conserved. From this, we can conclude that the reaction cannot occur.

Reaction diagrams

A reaction diagram provides a pictorial representation of a particle interaction. Reaction diagrams show the particles that exist before the reaction to the left and those that result from the reaction to the right. In reaction diagrams, the horizontal axis represents time, with the positive *x* direction pointing to the right.

Figure 14.2.1a shows a reaction diagram for the decay of a neutron into a proton, an electron and an electron antineutrino. Note that on the diagram each particle is represented by an arrow, and the direction of the arrows matches the direction of time, where time is on the horizontal axis. For antiparticles, the direction of the arrow is reversed. This allows us to distinguish between particles and antiparticles. The arrows on reaction diagrams *do not* represent trajectories. Straight lines represent matter and wiggly lines represent massless **exchange particles**, such as photons. An example of a reaction diagram in which only photons are produced in a reaction is depicted in Figure 14.2.1b.

14.2.2 Scientific background: neutrino oscillations

exchange particle a particle carrying force which is responsible for behaviour during other particle interactions. Sometimes exchange particles are the result of a particle interaction, such as the case with electron-positron annihilation.

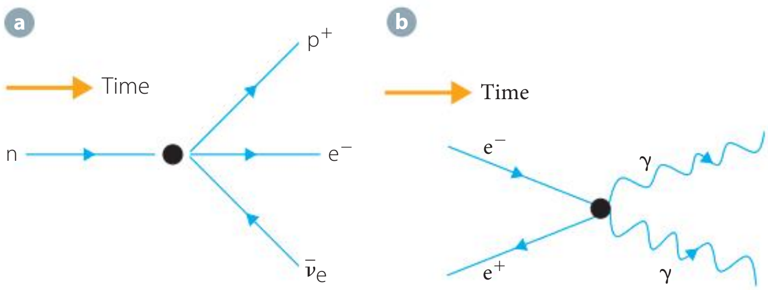

FIGURE 14.2.1 **(a)** A simple reaction diagram showing a neutron decaying to a proton, electron and an electron antineutrino. **(b)** Positron–electron annihilation results in two photons being produced.

INQUIRING FURTHER

Different exchange particles cause different exchange forces when colliding. Particles are collided together not to see what becomes annihilated, but rather to see what can be created. What are these different types of exchange particles? When and how were they discovered? Where are physicists currently observing these particles and forces, and why?

WORKED EXAMPLE 14.2.2

Draw a reaction diagram for the process $\pi^+ + p \rightarrow K^+ + \Sigma^+$.

ANSWER

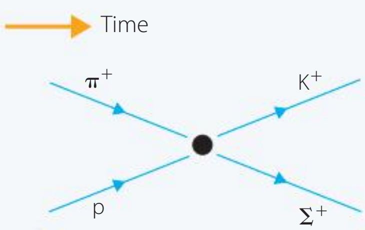

FIGURE 14.2.2 Pion and proton reaction diagram

SECTION REVIEW 14.2

REMEMBERING

1 List four quantities that must be conserved in all reactions.

APPLYING

2 Is the decay $\pi^+ + p \rightarrow K^+ + \Sigma^+$ allowed?

3 Is the reaction $p + e^{-1} \rightarrow n + v_e$ allowed?

4 Is lepton number conserved in the reaction $\pi^+ \rightarrow \mu^+ + v_e + v_\mu$?

5 Show that baryon number is conserved in the reaction $p + p \rightarrow p + p + \pi^0$.

6 Draw the reaction diagram for a negatively charge tau to a tau neutrino, an electron and an anti-electron neutrino.

7 Draw the reaction diagram for a proton and neutron to a neutron, antiproton and two protons.

8 Draw the reaction diagram for the reactions:

a $\Omega^- \rightarrow \Lambda^0 + K^-$

b $\pi^- \rightarrow \mu^- + \overline{v}_\mu$

ANALYSING

9 The following reaction is forbidden. Determine which conservation law is violated.

$p + \overline{p} \rightarrow \mu^+ + e^-$

10 Consider the reaction $\pi^- + \_\_ \rightarrow K^0 + \Lambda^0$. What properties must the missing particle have if the reaction is allowed? Give an example of such a particle.

14.3 Feynman diagrams

Feynman diagram a diagram that models exchange particles and exchange forces over time in space, when particles come into close proximity to each other

exchange force strong, electromagnetic, weak or gravitational force associated with the exchange particles gluons, photons, Q and Z particles and gravitons respectively; for example, an exchange of photons between electrons produces an electromagnetic force

Feynman diagrams are used to show more detail regarding the behaviour of various particles and **exchange forces** in a given interaction. Feynman diagrams use specific rules to visually represent the behaviour of particles during this time.

- Time is measured in the positive y direction.
- Space is measured in the positive x direction.
- Solid straight lines with upwards arrows represent particles.
- Solid straight lines with downwards arrows represent antiparticles.
- Wavy lines represent electromagnetic force from exchange particles (photon).
- Helical lines represent strong force from exchange particles (gluon).
- Dashed lines represent weak force from exchange particles (boson).
- Charge is always conserved during the interaction.
- Baryon number is always conserved during the interaction.

Figure 14.3.1 shows a labelled Feynman diagram of electron–electron interaction. It is important to recognise that in all Feynman diagrams forwards in time is in the upwards direction. The arrows indicate only what type of particle is interacting during each given scenario.

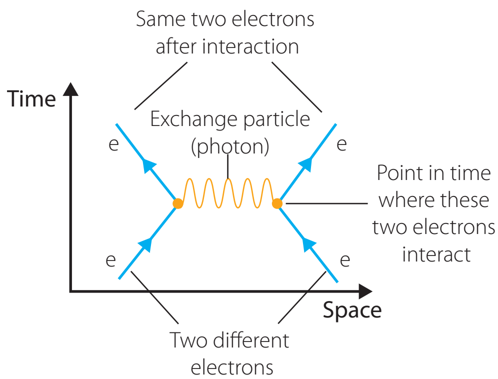

FIGURE 14.3.1 Labelled Feynman diagram. The exchange particle (a photon in this case) happens at the same instant the two electrons come close together. This exchange causes the electrostatic repulsion of the electrons. Note: The arrows do not note a trajectory; they indicate that the electron is a particle moving in time.

14.3.1 Let's draw Feynman diagrams

14.3.2 Feynman diagrams for beginners

14.3.3 How Feynman diagrams almost saved space

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Electron–electron interaction

When two electrons interact with each other, there is an electrostatic repulsion between them. This repulsion is due to the exchange of a photon between the electrons. After this interaction, the electrons still exist as they did originally (Figure 14.3.2).

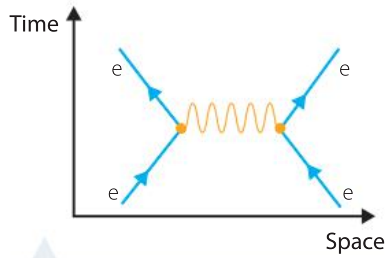

FIGURE 14.3.2 Feynman diagram of an electron–electron interaction

Electron–positron interaction

When an electron and a positron interact they will annihilate, resulting in two high-energy photons. During this process, mass is not conserved, but both lepton and baryon number are conserved. As a result, it is an allowed interaction. The interaction of the electron and positron is a particle interaction, denoted by the solid line in Figure 14.3.3.

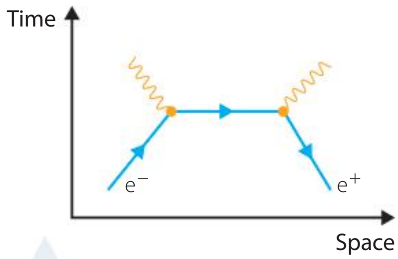

FIGURE 14.3.3 Feynman diagram depicting an electron–positron annihilation

Chapter 7 of *Nelson QScience Physics Units 1 & 2* discusses neutrons decaying into a proton and an electron.

Neutron decay

Neutrons are generally stable within a nucleus. However, when a neutron does become unstable it decays into a proton, an electron and an electron antineutrino (Figure 14.3.4). After this decay, there is a weak interaction between the new proton and the electron and antineutrino pair. The electron, due to its mass, has the ability to then leave with some velocity. This is what happens in radioactive nuclides during β^- decay.

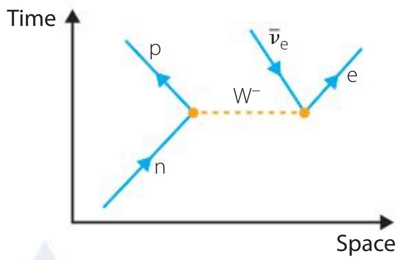

FIGURE 14.3.4 β^- decay of a neutron

WORKED EXAMPLE 14.3.1

A proton and an electron interact with a weak exchange particle to produce a neutron and a neutrino. Represent this interaction with a Feynman diagram.

ANSWER

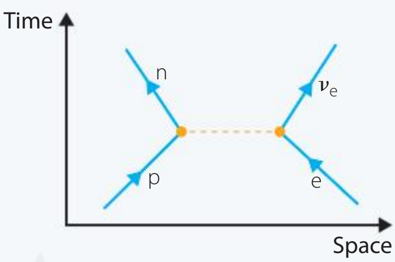

FIGURE 14.3.5 Feynman diagram of proton–electron interaction

SCIENCE AS A HUMAN ENDEAVOUR

THE STANDARD MODEL AND ITS CONTRIBUTORS

The Standard Model of Particle Physics describes fundamental forces. Research the Standard Model and create a timeline of notable physicists who contributed to its development. Be sure to include those who not only discovered particles, but also those who derived mathematical expressions and visual representations to explain their interactions. During this activity, note down all the exchange particles and their associated forces. When complete, create a poster with all of this information to help summarise and visualise the Standard Model and its contributors.

SECTION REVIEW 14.3

REMEMBERING

1 What do wavy lines in a Feynman diagram represent?

2 State the difference in using upwards and downwards arrows on a Feynman diagram.

UNDERSTANDING

3 Why is it important to show the forces acting between particles?

4 Explain how Feynman diagrams are more specific than reaction diagrams.

APPLYING

5 A proton and neutron interact weakly. A proton and neutron are the result of this interaction. Draw a Feynman diagram for this scenario.

6 Describe the interaction taking place in Figure 14.3.6.

7 Draw the Feynman diagram of β^- decay.

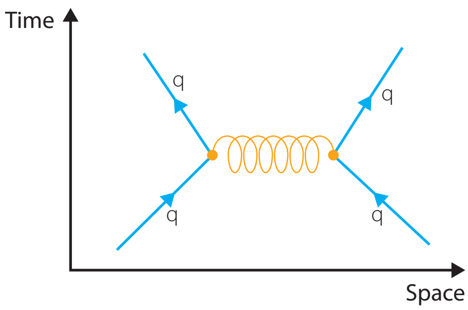

FIGURE 14.3.6

14.4 Symmetry in particle interactions

symmetry the invariance of physical laws under transformations such as translation, reflection or rotation in time or space

symmetry breaking a change in the behaviour of a physical system or the laws of physics that govern its behaviour when a symmetry operation such as a translation, reflection or rotation in time or space takes place

time reversal when reactions are reversed in time

The idea of **symmetry** in physics is a very important one. Physicists believe that the laws of physics will correctly describe physical phenomena under various transformations – such as reversal of direction in space or time, and rotations. These are called symmetries. There are three symmetries in particular that give us the means of predicting possible particle interactions. These are time-reversal symmetry, charge-reversal symmetry and crossing symmetry.

If we apply any of these symmetries to an allowed reaction, then the resulting reaction is also allowed under the conservation laws that have been discussed. This does not mean that the reaction is likely to take place, just that it is allowed. In general, the probability of a new reaction occurring will be very different from the probability of the reaction from which it was derived. In some cases the new reaction does not in fact occur. The reason for this **symmetry breaking** is a matter of ongoing theoretical and experimental research.

Time-reversal symmetry

Recall that the horizontal direction on a reaction diagram represents time. If this diagram is then reflected, swapping left to right, the direction of time is reversed. This means that the reaction occurs in the reverse order. If all the conservation laws previously described were obeyed by the original reaction, then the new process will also obey all the conservation laws. This process is called **time reversal**.

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Figure 14.4.1 shows time reversal applied to electron capture. The first process (Figure 14.4.1a) demonstrates the scenario in which an electron is captured by a proton, resulting in a neutron and an electron neutrino. This process occurs naturally in some nuclei. If time reversal is then applied to this process, the reaction becomes $n + \nu_e \rightarrow p + e$. This reaction still obeys all conservation laws, and is hence an allowed reaction.

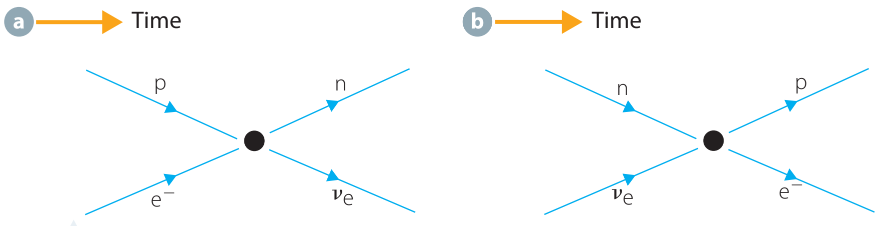

FIGURE 14.4.1 Time reversal applied to electron capture. (**a**) An electron is captured by a proton. (**b**) A neutron reacts with a neutrino to form a proton and electron.

If **time-reversal symmetry** is applied to a known allowed reaction, then the new reaction generated is theoretically possible. However, this new reaction may not be experimentally realisable because of the requirement for conservation of energy and momentum, or because of the extremely low probability of simultaneously combining all the reacting particles. So, although it may seem as though time reversal can be applied to any reaction, this is not always viable.

time-reversal symmetry when an allowed reaction is written such that it runs in the opposite direction in time; the new reaction is also allowed in that it does not break any of the known conservation laws

Consider a reaction in which the mass of the product is less than the mass of the reactants. An application of time reversal to this reaction would mean that the mass of the products would be greater than the mass of the reactants. The only way this is possible is if the reacting particles have enough kinetic energy to convert into mass to create the new particles, or if there is some other source of energy available. This may be a particle that gives up energy but does not otherwise change in the reaction. An extreme example is electron–positron pair annihilation. You start with two particles of mass that annihilate to give photons with no mass. Applying time reversal to this process requires photons with enough energy and an exact momentum to interact such that an electron–positron pair is produced. The process of electron–positron pair production from high energy photons is not simply the reverse of electron–positron annihilation. It involves a complex interaction with a nucleus to allow for conservation of momentum and energy.

Now consider the probability of particular reactions occurring under time reversal. An example is the decay reaction depicted in Figure 14.4.2a of a positive muon turning into a positron, an antimuon neutrino and an electron neutrino. This process has been experimentally observed. Applying time reversal to this reaction (Figure 14.4.2b) creates a reaction between a positron, antimuon neutrino and electron neutrino. Although this reaction is theoretically possible and it does not break any conservation laws, the probability of finding all three particles close enough together to react like this is negligible. This is particularly the case with short-lived exotic particles such as muons and neutrinos that only interact very weakly with matter. It is practically impossible to simultaneously collide three or more particles.

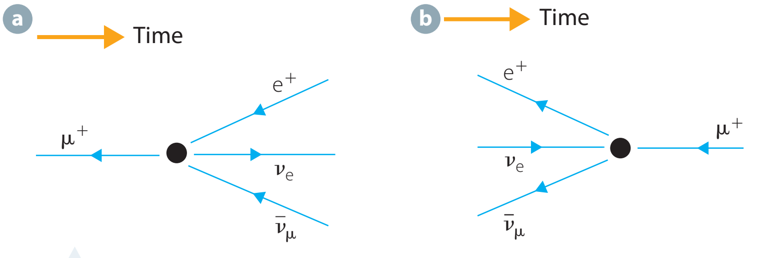

FIGURE 14.4.2 (**a**) A positive muon decays into a positron, antimuon neutrino and an electron neutrino. (**b**) Time reversal is applied to the process in part a. This reaction is not observed due to its extremely low probability.

WORKED EXAMPLE 14.4.1

What is the result of applying time reversal to the β^- decay of a neutron? Do you think the time-reversed reaction is likely to occur?

ANSWER

$$n \rightarrow p + \beta^- + \bar{\nu}_e$$

Applying time reversal yields:

$$p + \beta^- + \bar{\nu}_e \rightarrow n$$

This reaction is unlikely to occur due to the very low probability of having all three particles in such close proximity that they would collide to form a neutron.

Charge-reversal symmetry

charge-reversal symmetry
if all particles in an allowed reaction are replaced with their antiparticles (which have opposite charge), the new reaction is also allowed under known conservation laws

Charge-reversal symmetry says that if the charges on all particles in a reaction are reversed, then this new reaction is also possible in that it does not violate any conservation laws. Strictly speaking, charge reversal is used to refer to swapping all particles for their antiparticles, even those that are electrically neutral such as neutrons and neutrinos. As with time-reversal symmetry, applying charge-reversal symmetry produces a reaction that does not violate conservation principles, but may also be so unlikely as to rarely or never occur naturally.

Consider the decay of a muon: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. Under charge reversal, this becomes $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$. This decay also occurs.

WORKED EXAMPLE 14.4.2

Apply charge-reversal symmetry to the decay of a negative kaon, shown in Figure 14.4.3 to draw a reaction diagram for the decay of a positive kaon.

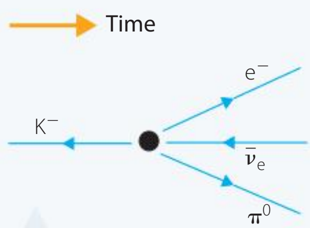

FIGURE 14.4.3

ANSWER

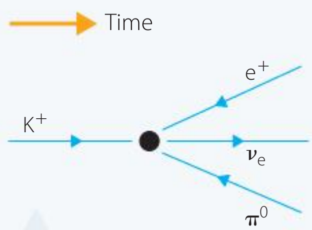

FIGURE 14.4.4

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Crossing symmetry

Crossing symmetry is the third type of symmetry that can be applied to predict new reactions. In crossing symmetry, one particle is taken and crossed to the other side of the reaction, and converted to its antiparticle. Crossing symmetry, as with time-reversal and charge-reversal symmetry, predicts reactions that do not violate conservation principles. However, this does not mean that the reactions predicted actually occur. Whether a reaction can occur, or the probability of its occurrence, depends on the energy available, the mass differences of the particles and the conservation of other properties, such as angular momentum. As with other symmetries, crossing-symmetry reactions may be theoretically possible, but have a negligible probability of ever occurring.

crossing symmetry if a particle in an allowed reaction is crossed to the other side of the reaction and replaced with its antiparticle, the new reaction is also allowed under known conservation principles provided enough energy is available

WORKED EXAMPLE 14.4.3

A proton captures an electron to form a neutron and an electron neutrino.

1 Draw the reaction diagram for this process.

2 Apply crossing symmetry to the electron and draw the resulting reaction diagram.

ANSWER

1

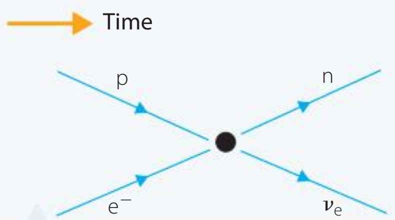

FIGURE 14.4.5 $p + e^- \rightarrow n + \nu_e$

2

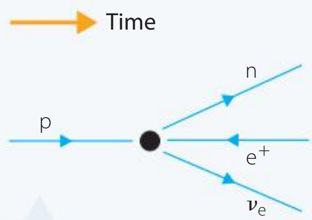

FIGURE 14.4.6 New reaction after crossing the electron: $p \rightarrow n + \nu_e + e^+$. This is an observed reaction.

PRACTICAL ACTIVITY 14.4.1

Pipe-cleaner reaction diagrams

AIM

To make reaction diagrams using pipe cleaners and then apply the three different symmetry operations to generate new reactions.

MATERIALS

- pipe cleaners of different colours
- thread
- large bead or ring

»

RISK ASSESSMENT

RISK ASSESSMENT

POSSIBLE RISKS	MINIMISATION STRATEGIES
Sharp ends can pierce skin.	Ensure pipe cleaners are handled with care and that the sharp end is not touched.

METHOD

1 With different coloured pipe cleaners representing different particles and the ring representing a reaction, apply the rules for drawing reaction diagrams to making a pipe-cleaner diagram. Twist one end of each pipe cleaner into the bead or ring, and bend the pipe cleaner so it is either coming into or going out of the reaction.

2 Sketch the diagram and write down what it represents.

3 Apply one of the symmetry operations to your pipe cleaners. Do this by bending the 'particles' to come into the reaction from different directions. Remember that when you do this, a particle is coming in from the opposite direction in time, so you need to convert it into its antiparticle.

See how many different reactions you can make.

4 Sketch and write down all these new reactions.

SECTION REVIEW 14.4

REMEMBERING

1 List the three types of symmetries that can be applied to generate new reactions.

2 When crossing symmetry is applied to move a particle from one side of a reaction to the other, what else must be done?

UNDERSTANDING

3 Why are many of the reactions predicted by a charge reversal not observed, even though they are possible?

APPLYING

4 A negative sigma, Σ^-, commonly decays by the reaction $\Sigma^- \rightarrow n + \pi^-$. Apply a charge-reversal symmetry to predict the decay process for a positive sigma.

5 Apply time reversal to the reaction of electron–positron annihilation. Write the reaction and draw the reaction diagram. Explain why this reaction does not occur without other particles being involved.

6 Apply time reversal to the reaction shown in Figure 14.4.7. Do you think the new reaction is likely to occur?

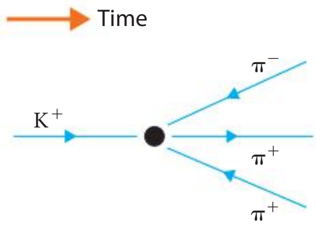

FIGURE 14.4.7 Reaction diagram for kaon decay

ANALYSING

7 Consider the decay of a neutron $n \rightarrow p + e + \bar{\nu}_e$. By applying crossing symmetry, time reversal and then a second symmetry operation to this reaction, we can arrive at the process of β^+ decay. Draw the series of reaction diagrams for these symmetry operations, starting with the neutron decay reaction as given, and ending with β^+ decay.

8 Write two reactions by applying crossing symmetry to the reaction for electron capture. Do you think these reactions are likely to occur? You are welcome to apply this symmetry to any of the reactions you have seen in the chapter so far.

SYNTHESISING

9 Some of the reactions that have been looked at are very unlikely to occur, so why are physicists so interested in these particle interactions? Research the current experiments being undertaken at the Large Hadron Collider to help find an answer to this question.

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Define the following terms.
   a Baryon number
   b Charge-reversal symmetry
   c Crossing symmetry
   d Exchange forces
   e Exchange particles
   f Feynman diagram
   g Law of conservation of baryon number
   h Law of conservation of lepton number
   i Lepton number
   j Neutrino oscillation
   k Symmetry
   l Symmetry breaking
   m Time-reversal symmetry
   n Time reversal

2 State the three kinds of particle symmetries.

CATEGORY QUESTIONS

3 Explain the difference between reaction diagrams and Feynman diagrams.

4 Explain how to determine if the lepton number in a reaction is conserved.

5 What does it mean if the baryon number in a reaction is not conserved?

ELABORATION QUESTIONS

6 Why is the reaction produced by applying symmetry to a reaction sometimes not viable?

7 How do you apply charge reversal to a neutrally charged particle?

EVIDENCE QUESTION

8 Research the exchange particle called a graviton. What is it? Is there currently any evidence of its existence?

END-OF-CHAPTER EXAM

End-of-chapter test

1 In Feynman diagrams antiparticles are represented by:

A straight lines with the arrows in the direction of time.

B straight lines with the arrows in the opposite direction of time.

C dashed lines.

D wavy lines.

2 The particle interaction indicated by the reaction below diagram is:

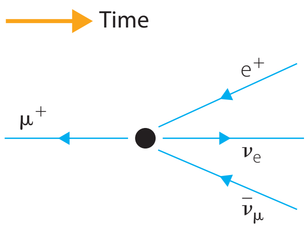

FIGURE 14.5.1

A an electron, electron neutrino and antimuon antineutrino reacting to create a muon.

B a positron, electron neutrino and antimuon antineutrino reacting to create a muon.

C an antimuon decaying into an electron, electron neutrino and muon antineutrino

D an antimuon decaying into a positron, electron neutrino and muon antineutrino.

3 If charge reversal is applied to an electron neutrino, the particle obtained is:

A a positron neutrino.

B an electron antineutrino.

C a positron antineutrino.

D an electron neutrino.

4 When crossing a particle in a reaction diagram, what must it convert into?

5 Are all reactions under all symmetries theoretically possible?

6 Apply charge reversal to the reaction $\tau^- \rightarrow e^- + \nu_\tau + \bar{\nu}_e$.

7 Apply crossing symmetry to the electron in the reaction $\tau^- \rightarrow e^- + \nu_\tau + \bar{\nu}_e$. Draw a reaction diagram for the resulting reaction. Do you think this reaction is likely to occur?

8 Show that lepton number is conserved in the reaction $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$.

9 Show that baryon number is conserved in the decay of a neutron: $n \rightarrow p + e^- + \bar{\nu}_e$.

10 Draw the Feynman diagram for the decay of a neutron.

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UNITS 3 & 4 PRACTICE EXAM

MULTIPLE-CHOICE QUESTIONS

QUESTION 1

What work needs to be done to move a charge of 3.0×10^{-14} C across a potential difference of 9.0×10^{2} V?

A 7.2×10^{11} J

B 2.7×10^{-11} J

C 2.7×10^{11} J

D 2.7×10^{-11} eV

QUESTION 2

A magnetic field exists around a current-carrying wire. At a distance of 4.0 cm from the wire, the field strength is measured to be 6.0×10^{-5} T. As the sensor is moved away from the wire the field strength is found to decrease to 3.0×10^{-5} T while the current is unchanged. What is the new distance between the wire and the sensor?

A 4.0 cm from the wire.

B 2.0 cm from the wire.

C 10.0 cm from the wire.

D 8.0 cm from the wire.

QUESTION 3

An inclined plane is at an angle of 40° from the horizontal. Find the components of the acceleration due to gravity parallel to the plane and perpendicular to the plane, respectively.

A 7.5 m s^{-2}; 6.3 m s^{-2}

B 0.75 m s^{-2}; 0.63 m s^{-2}

C 0.63 m s^{-2}; 0.77 m s^{-2}

D 6.3 m s^{-2}; 7.5 m s^{-2}

QUESTION 4

Which of the following forces is not experienced by leptons?

A Strong nuclear

B Weak nuclear

C Electromagnetic

D Gravitational

QUESTION 5

The mechanism that provides the centripetal acceleration that satellites experience in orbit is:

A force tension.

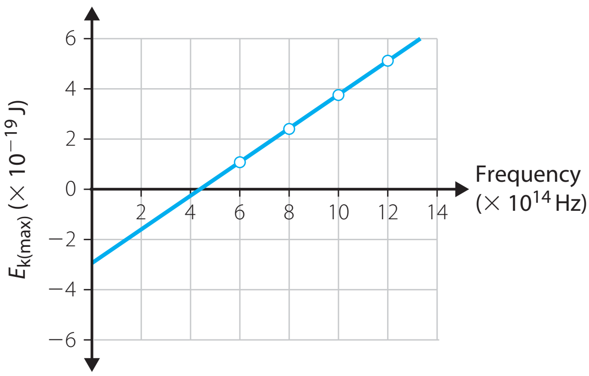
B mass.

C force friction.

D force gravity.

QUESTION 6

Using the graph below and Planck's constant, find the value of the work function for caesium in eV and J.



A 2.85×10^{-19} eV; 1.78 J

B 1.78 eV; 2.85×10^{-19} J

C 2.85 eV; 1.78 J

D 1.78×10^{-19} eV; 2.85 J

QUESTION 7

A charge q is separated from a charge $Q = +3.00 \times 10^{-6}$ C by a distance of 6.00×10^{-5} m. The force experienced between them is measured to be $+4.50 \times 10^{-1}$ N. Which value of q satisfies these conditions?

A -6.0×10^{-14} C

B $+6.0 \times 10^{-9}$ C

C $+6.0 \times 10^{14}$ C

D $+6.0 \times 10^{-4}$ C

QUESTION 8

A rollercoaster car turns to the right on a horizontal track with a velocity of 10 m s^{-1}. At this point the track has a radius of curvature of 25 m. Determine the magnitude and direction of the force applied by the seat on a 75 kg person seated in the car.

A 300 N left; 735 N vertically up

B 300 N right; 735 N vertically up

C 300 N right; nil vertically

D 300 N left; nil vertically

QUESTION 9

Determine the altitude of a satellite in orbit around Earth when its orbital speed is 5.0×10^3 m s^{-1}. Use $r_{Earth} = 6.37 \times 10^6$ m and $M_{Earth} = 5.97 \times 10^{24}$ kg.

A 6.37×10^6 m

B 1911 km

C 9.56×10^6 m

D 1.59×10^7 m

QUESTION 10

Determine the de Broglie wavelength of an electron travelling at 35.5 m s^{-1}.

A 2.05×10^{-5} m

B 0.025 m

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C 40 m

D 205 nm

QUESTION 11

A 40 g object takes 12 s to revolve 10 times around a fixed point at a distance of 25 cm. The speed of the object would be found to be:

A 0.76 m s^{-1}.

B 0.65 m s^{-1}.

C 1.31 m s^{-1}.

D 0.131 m s^{-1}.

QUESTION 12

A photon has energy 4.5 eV. What is its energy, wavelength and frequency, respectively?

A 3.60×10^{-19} J; 2.76 nm; 1.09×10^{15} Hz

B 7.20 J; 2.76×10^{-7} m; 2.18×10^{15} Hz

C 7.20×10^{-19} J; 5.52×10^{-7} m; 0.545×10^{15} Hz

D 7.20×10^{-19} J; 2.76×10^{-7} m; 1.09×10^{15} Hz

QUESTION 13

Which statement regarding wave interference patterns is *incorrect*?

A Constructive interference occurs along anti-nodal lines.

B Destructive interference occurs when a wave crest meets with a wave trough.

C Anti-nodal lines always result in a dark fringe.

D A crest meeting a trough is an example of destructive interference.

QUESTION 14

A rope is tied to an 80 kg box that is sitting stationary on an inclined plane making a slope of 22°. The friction between the box and the slope is measured to be 125 N. The tension in the rope is:

A 169 N.

B 602 N.

C 294 N.

D 851 N.

QUESTION 15

The maximum height reached by a rocket launched at 25° to the horizontal with an initial velocity of 35 m s^{-1} is:

A 51.34 m.

B 62.5 m.

C 0.75 m.

D 11.16 m.

QUESTION 16

State the term given for the destructive process that occurs when a particle meets its antiparticle.

A Electron pairing

B Annihilation

C Antiparticle release

D Neutral bosons

QUESTION 17

An observer views a clock that is positioned a significant distance away. The observer, who is initially at rest relative to the clock, views an event that takes 4.0 s. Later, the same event takes place, but this time whilst the clock is moving at 0.6c relative to the observer. What length of time would the event take now, according to the observer?

A 3.2 s

B 5.8 s

C 5.0 s

D 3.0 s

QUESTION 18

A force vector has a magnitude of 35 N and a direction of 60° above the horizontal. The magnitude of the vector's horizontal and vertical components are, respectively?

A 30.3 N; 17.5 N

B 0 N; 30.3 N

C 17.5 N; 0 N

D 17.5 N; 30.3 N

QUESTION 19

Which of the following statements is a postulate of Einstein's theory of special relativity?

A Particles can move at the speed of light.

B Proper length and proper time are measured in the same frame of reference.

C The speed of light is the same in all frames of reference.

D The speed of light changes depending on the frame of reference.

QUESTION 20

What would be required to happen to the orbital velocity of a satellite if it were moved to an Earth orbit of lesser altitude?

A The satellite would need to speed up.

B The satellite would need to be slowed down.

C The satellite's velocity would remain constant.

D The satellite would need to exceed the escape velocity of Earth.

SHORT-RESPONSE QUESTIONS

QUESTION 1

Titan, a moon of Saturn, has an orbital radius of 1.22×10^6 km. It takes Titan 15 days and 22 hours to revolve around Saturn. Use this data and the orbital velocity formula to determine the mass of Saturn.

QUESTION 2

Determine the gravitational field strength on the surface of a planet with a mass of 3.8×10^{25} kg and a radius of 4250 km.

QUESTION 3

A muon travelling towards Earth at 0.7c is known to have a mean lifetime of 3.0 ns when observed under laboratory conditions. How long does it take for the muon to decay as viewed by an observer on the surface of the Earth?

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QUESTION 4

Determine the initial velocity of a projectile that travels a horizontal distance of 250 m in 4.0 s after being launched at an angle of 15° above the horizontal.

QUESTION 5

Two stars are known to be at a distance of 16 light-years from each other. An observer travels from one star toward the other at a velocity of one half of the speed of light. According to the observer within the spacecraft, what is the distance that they measure between the stars?

QUESTION 6

Calculate the magnetic flux through a wire loop of area 0.035 m^2 placed perpendicularly within a uniform magnetic field of intensity 8.0×10^{-2} T.

QUESTION 7

Explain why a wave model of light is needed to understand the interference pattern produced in the double-slit experiment.

QUESTION 8

Recall the leptons.

QUESTION 9

To what velocity must a satellite be propelled if it is to maintain an orbital radius of 10 000 km around Earth? Use mass of Earth as 5.97×10^{24} kg.

QUESTION 10

Describe the factors that may have an effect on the magnitude of magnetic flux passing through a surface.

QUESTION 11

Describe the difference between particles classified as leptons and as hadrons.

QUESTION 12

With what minimum velocity must an object be thrown vertically to reach a height of 22.0 m above its release point?

QUESTION 13

State the two postulates of Einstein's theory of special relativity.

QUESTION 14

The area of a conducting loop is increased from 10.0 cm^2 to 25.0 cm^2 over a period of 0.40 s while the loop lies in a perpendicular, uniform magnetic field of 1.8×10^2 T. Calculate the magnitude of the emf produced.

QUESTION 15

Cosmic background radiation has a spectrum similar to that produced by a black body at 3.1 K. Determine the peak wavelength of this radiation.

QUESTION 16

a Calculate the time period, *T*, for a single oscillation of a pendulum on Earth, given a length of 0.60 m.

b Determine how the period would differ if the experiment were conducted on a planet with a greater acceleration due to gravity.

QUESTION 17

A satellite orbits the Sun with a radius of 1.85×10^{10} m. Determine its orbital speed, given the mass of the Sun is 2.0×10^{30} kg.

QUESTION 18

The electromagnetic spectrum consists of several regions. Place the six major regions of the spectrum in order of increasing energy: ultraviolet, infrared, gamma rays and X-rays, visible light, radio waves.

QUESTION 19

A 14.5 kg object is raised a height of 1.10 m to be placed onto a bench.

a Calculate the force applied in raising the mass at a constant velocity.

b Calculate the work done in raising the mass.

c State the maximum kinetic energy the object may attain if dropped from the height of the bench.

QUESTION 20

A 65 kg skier accelerates from rest at $2.30\ \text{m s}^{-2}$ down a 45° slope.

a Calculate the gravitational force acting on the skier down the slope.

b Calculate the total resistance force acting on the skier.

c Calculate the speed of the skier after they have travelled a distance of 50 m down the slope.

COMBINATION-RESPONSE QUESTIONS

QUESTION 1

The table below shows how the electrostatic force changes between two +10 μC spheres for increasing separation distances *r*.

Separation distance, *r* (m)	Electrostatic force, *F* (N)
0.05	360.0
0.10	95.0
0.15	40.0
0.20	22.0
0.25	15.0
0.30	10.0
0.35	5.0

Manipulate the data to obtain a linear relationship, and use that to determine an experimental value for the constant *k* (Coulomb's constant) from the gradient of your linear graph.

QUESTION 2

In a double-slit experiment light with wavelength 489 nm is used to illuminate twin slits positioned a distance *d* metres apart. A characteristic interference pattern is observed on a wall 1.20 m from the slits where the third interference minimum (dark spot) is found to be at a point 1.55 mm from the central bright spot. Use this information to calculate the slit separation.

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QUESTION 3

A ball is kicked off the ground with a velocity of 12.0 m s^{-1} and at an angle of 18.0° with the level ground. It lands on the pitch an unknown distance away. Calculate:

a the time of flight of the ball

b the maximum height reached above the ground

c the vertical and horizontal velocities of the ball at the maximum height

d the total distance (range) travelled by the ball before striking the ground.

QUESTION 4

An object of mass 2.20 kg is whirled on the end of a 1.20 m long string in a vertical circle at constant speed. The string will break if the force exceeds 300 N. Determine the maximum velocity that the mass may be whirled before the string breaks.

QUESTION 5

The table below shows data collected in a photoelectric experiment. Complete the table and plot an appropriate graph to determine:

a Planck's constant

b the work function for this metal.

Wavelength (nm)	Frequency ($\times 10^{14}$ Hz)	KE_{max} of photoelectrons (eV)	KE_{max} of photoelectrons (J)
595		0.67	
520		0.98	
460		1.35	
412		1.63	

QUESTION 6

A current balance is set up in a solenoid with *B* field 0.15 T. A current of 2.4 A runs through a 2.5 cm length of wire perpendicular to the field within the solenoid. Determine the magnitude of force the current balance experiences in the magnetic field.

QUESTION 7

Explain the concept of simultaneity.

QUESTION 8

Draw the Feynman diagram for the β^- decay of a neutron.

QUESTION 9

Calculate the gravitational force acting between two planets of mass 3.85×10^{25} kg and 5.29×10^{24} kg respectively. The planets are at an average mean distance of 920 000 km.

QUESTION 10

A step-up transformer is connected to an AC generator that delivers 10.0 A at 240 V. The ratio of the number of turns in the secondary coil to the number of turns in the primary coil is 850.

a What is the emf in the secondary coil?

b What is the average power input?

c What is the maximum power output?

d What is the maximum current in the secondary circuit?

ANSWERS

CHAPTER 1: GRAVITY AND MOTION

1.1 SECTION REVIEW

REMEMBERING

1 Tip-to-tail; parallelogram

2 Careful scale drawing and measurement using ruler and protractor.

3 a Magnitude increases

b Magnitude decreases

c Change of direction

4 a $A_x = |\vec{A}|\cos\theta$; $A_y = |\vec{A}|\sin\theta$; θ relative to positive x-axis

b $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

UNDERSTANDING

5 Tip-to-tail addition produces a triangle – resultant completes the triangle from tail of first addend to tip of second addend; parallelogram addition produces a parallelogram: both tails start at same point – resultant is diagonal from start to completion of parallelogram; both methods require careful measurements using ruler and protractor.

6 Subtraction is the addition of the negative.

7 Without a consistent scale, resultant length and angle cannot be measured correctly.

8 All triangles used in calculations must be right-angled.

APPLYING

9 a $\vec{C} = \vec{A} + \vec{B}$

b $\vec{C} = \vec{B} - \vec{A}$

c $\vec{A} = \vec{C} - \vec{B}$

d $\vec{C} = 2\vec{A} + 3\vec{B}$

ANALYSING

10 a $R_x = P_x + Q_x$; $R_y = P_y + Q_y$

b $R_x = P_x - Q_x$; $R_y = P_y - Q_y$

c $R_x = 2P_x - 3Q_x$; $R_y = 2P_y - 3Q_y$

1.2 SECTION REVIEW

REMEMBERING

1 See pages 10–11

2 See pages 11–12

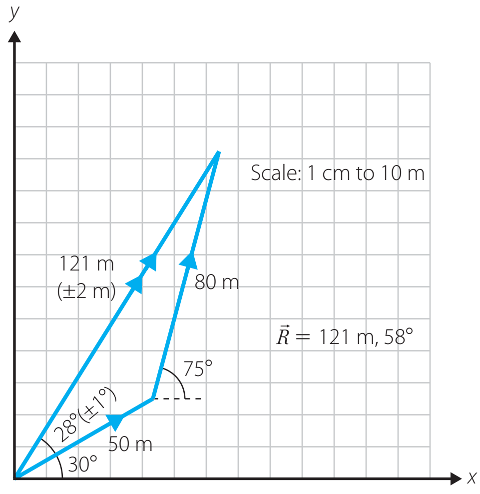
3 See pages 12–13

UNDERSTANDING

4 See Figure 1.1.6, page 9

APPLYING

5 a



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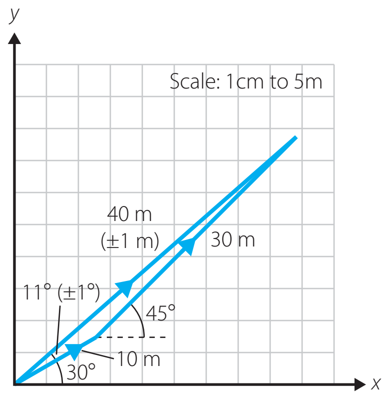
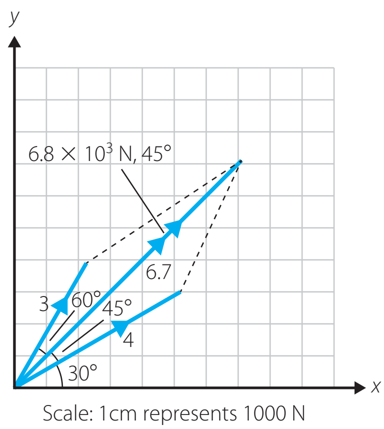


6 a



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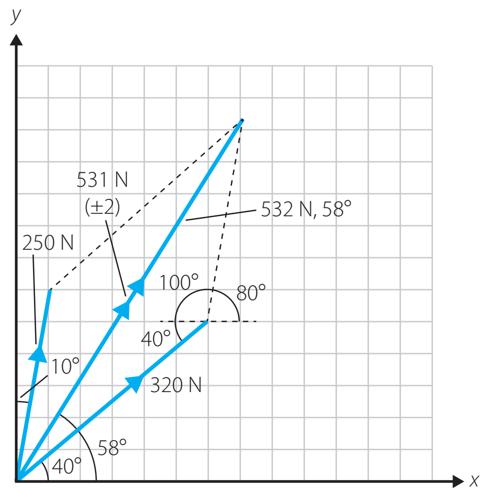
b



Scale: 1cm represents 50 N

ANALYSING

7 a $R_x = 2A_x - B_x$

$R_x = 2 \times 15\,\text{m s}^{-1} \times \cos 30° - 25\,\text{m s}^{-1} \times \cos 60°$

$R_x = 12.48\,\text{m s}^{-1}$

$R_y = 2A_y - B_y$

$R_y = 2 \times 15\,\text{m s}^{-1} \times \sin 30° - 25\,\text{m s}^{-1} \times \sin 60°$

$R_y = {}^{-}6.65\,\text{m s}^{-1}$

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{\left(13.48\,\text{m s}^{-1}\right)^2 + \left({}^{-}6.65\,\text{m s}^{-1}\right)^2}$

$R = 15\,\text{m s}^{-1}$

$\theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

$\theta = \tan^{-1}\left(\dfrac{{}^{-}6.65\,\text{m s}^{-1}}{13.48\,\text{m s}^{-1}}\right)$

$\theta = {}^{-}26.3°$

$\theta = 360° - 26.3°$

$\theta = 334°$

b $R_x = 2A_x - B_x$

$R_x = 2 \times 35\,\text{N} \times \cos 120° - 25\,\text{N} \times \cos 45°$

$R_x = {}^{-}52.7\,\text{N}$

$R_y = 2A_y - B_y$

$R_y = 2 \times 35\,\text{N} \times \sin 120° - 25\,\text{N} \times \sin 45°$

$R_y = 42.9\,\text{N}$

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{\left({}^{-}52.7\,\text{N}\right)^2 + \left(42.9\,\text{N}\right)^2}$

$R = 68\,\text{N}$

$\theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

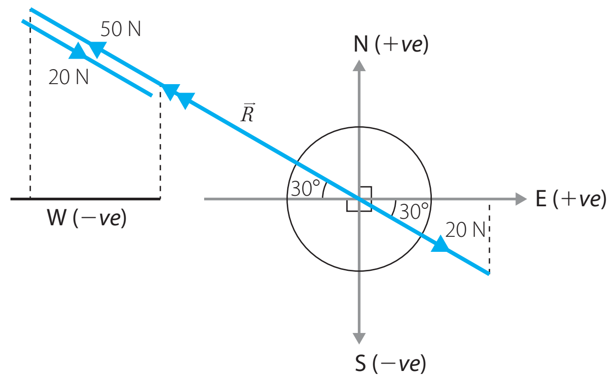
$\theta = \tan^{-1}\left(\dfrac{42.9\,\text{N}}{{}^{-}52.7\,\text{N}}\right)$

$\theta = {}^{-}39.2°$

$\theta = 180° - 39.2°$

$\theta = 141°$

8 a



Scale: 1 cm represents 5 N

Take north and east as positive:

$R_x = P_x + Q_x$

$R_x = 20\,\text{N} \times \cos 30° + {}^{-}50\,\text{N} \times \cos 30°$

$R_x = {}^{-}26\,\text{N}$

$R_y = P_y + Q_y$

$R_y = {}^{-}20\,\text{N} \times \sin 30° + 50\,\text{N} \times \sin 30°$

$R_y = 15\,\text{N}$

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{\left({}^{-}26\,\text{N}\right)^2 + \left(15\,\text{N}\right)^2}$

$R = 30\,\text{N}$

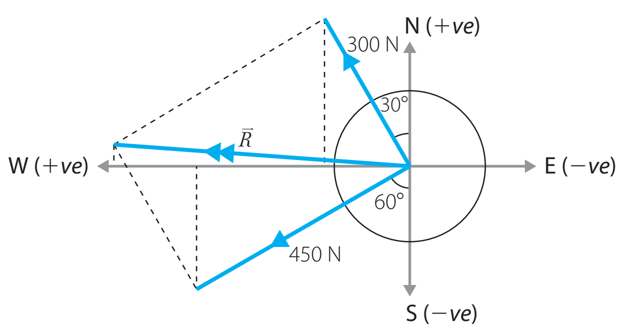
$\theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$

$\theta = \tan^{-1}\left(\dfrac{15\,N}{{}^{-}26\,N}\right)$

$\theta = {}^{-}30°$

$\theta_{\text{true}} = 300°$

b



Scale: 1 cm represents 50 N

Take north and west as positive:

$R_x = P_x + Q_x$

$R_x = 300\,\text{N} \times \cos 60° + 450\,\text{N} \times \cos 30°$

$R_x = 540\,\text{N}$

$R_y = P_y + Q_y$

$R_y = 300\,\text{N} \times \sin 60° + {}^{-}450\,\text{N} \times \sin 30°$

$R_y = 34.8\,\text{N}$

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(540\,\text{N})^2 + (34.8\,\text{N})^2}$

$R = 541\,\text{N}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$\theta = \tan^{-1}\left(\frac{34.8\,\text{N}}{541\,\text{N}}\right)$

$\theta = 3.7°$

$\Rightarrow$ N86°W

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a Distance and time (speed and acceleration are derived from distance and time)

b Mass, distance and time (speed and acceleration, hence force, work-energy, impulse-momentum are derived from distance and time combined with the effect of mass)

2 Kinematics: velocity (change of displacement) and (vector) acceleration (change of velocity)

Dynamics: the sum of forces in different directions causes acceleration.

3 Motion can be resolved into rectangular components. These are independent of each other.

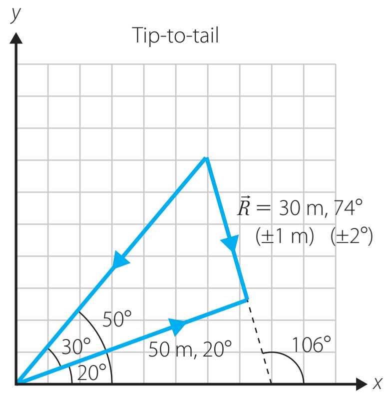
CATEGORY QUESTIONS

4 a Use a Cartesian grid. Tip-to-tail (direct addition) is simplest; parallelogram (easier to place addends but must produce tip-to-tail construction); method of components (use tip-to-tail or parallelogram on axis system, then add resolutes – diagram can become quite complex)

b $\vec{R} = \vec{C} - 2\vec{D}$

$\vec{R} = (50\,\text{m}, 20°) - 2 \times (30\,\text{m}, 50°)$

$\vec{R} = (50\,\text{m}, 20°) + 2 \times ({}^{-}30\,\text{m}, 50°)$



Scale: 1 cm represents 10 m

c Use trigonometrical ratios to find *x*- and *y*-resolutes, add corresponding resolutes to find *x*- and *y*-components of the resultant; use Pythagoras to calculate magnitude; use trigonometry to find angle.

d $\vec{R} = \vec{C} - 2\vec{D}$

$\vec{R} = (50\,\text{m}, 20°) - 2 \times (30\,\text{m}, 50°)$

$\vec{R} = (50\,\text{m}, 20°) + 2 \times ({}^{-}30\,\text{m}, 50°)$

$R_x = C_x - 2D_x$

$R_x = 50\,\text{m} \times \cos 20° + {}^{-}60\,\text{m} \times \cos 50°$

$R_x = 8.42\,\text{m}$

$R_y = C_y + Q_y$

$R_y = 50\,\text{m} \times \sin 20° + {}^{-}60\,\text{m} \times \sin 50°$

$R_y = {}^{-}28.9\,\text{m}$

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(8.42\,\text{m})^2 + ({}^{-}28.9\,\text{m})^2}$

$R = 30\,\text{m}$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$\theta = \tan^{-1}\left(\frac{{}^{-}28.9\,\text{m}}{8.42\,\text{m}}\right)$

$\theta = {}^{-}73.8°$

e The calculational method is more likely to be accurate. Where the number of significant figures in the data exceeds the number of significant figures it is possible to draw using a ruler and protractor.

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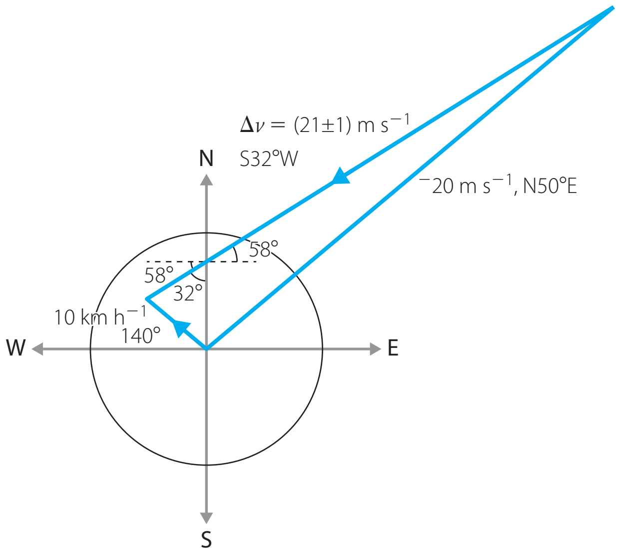
ELABORATION QUESTIONS

5 By scale drawing:



then:

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a_{av} = \frac{(21 \pm 1)\,\mathrm{ms}^{-1}}{25\,\mathrm{s}}$$

$$a_{av} = (0.84 \pm 0.04)\,\mathrm{ms}^{-2}$$

6 a

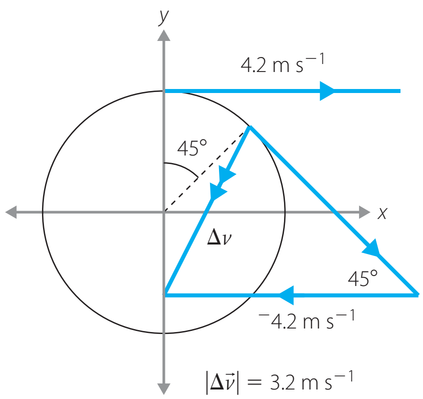
$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi \times 10\,\mathrm{m}}{4.2\,\mathrm{ms}^{-1}}$$

$$T = 15\,\mathrm{ms}^{-1}$$

b



c

$$a_{av} = \frac{\Delta v}{\Delta t}$$

$$a_{av} = \frac{3.2\,\mathrm{ms}^{-1}}{\left(\dfrac{15\,\mathrm{ms}^{-1}}{8\,\mathrm{s}}\right)}$$

$$a_{av} = 1.7\,\mathrm{ms}^{-2}$$

EVIDENCE QUESTIONS

7 a Student answers will vary.

b Student answers will vary.

8 Student answers will vary.

END-OF-CHAPTER EXAM

1 B

2 C

3 A

4 C

5 D

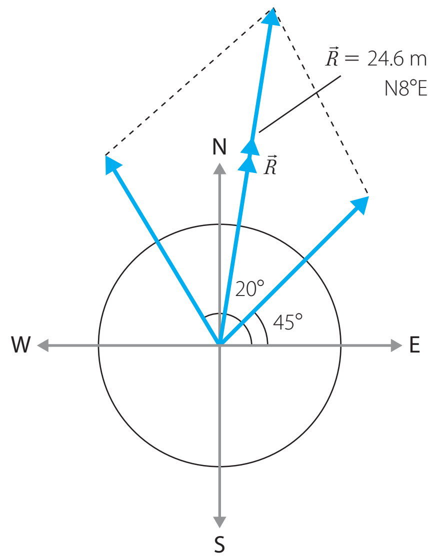
6 Rectangular

7 Positive

8 Add *x*-components of the addends; add *y*-components of the addends

9 Magnitude × 4 and direction reversed

10



Scale: 1 cm represents 5 cm

11 a

$$a_{\parallel} = g\sin\theta$$

$$a_{\parallel} = 9.8\,\mathrm{ms}^{-2} \times \sin 30°$$

$$a_{\parallel} = 4.9\,\mathrm{ms}^{-2}$$

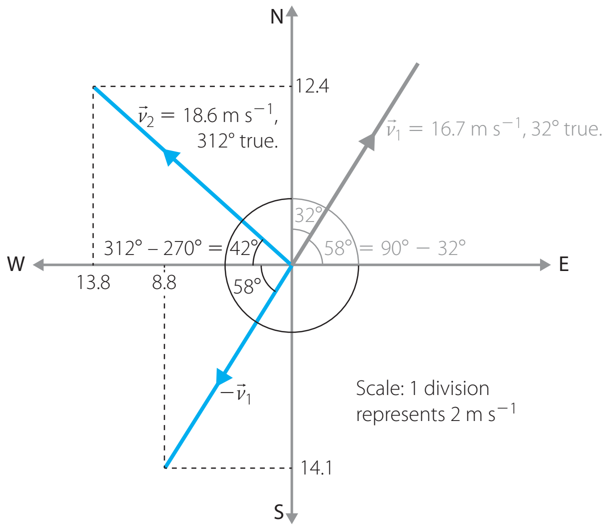
b

$$a_{\perp} = g\cos\theta$$

$$a_{\perp} = 9.8\,\mathrm{ms}^{-2} \times \cos 30°$$

$$a_{\perp} = 8.5\,\mathrm{ms}^{-2}$$

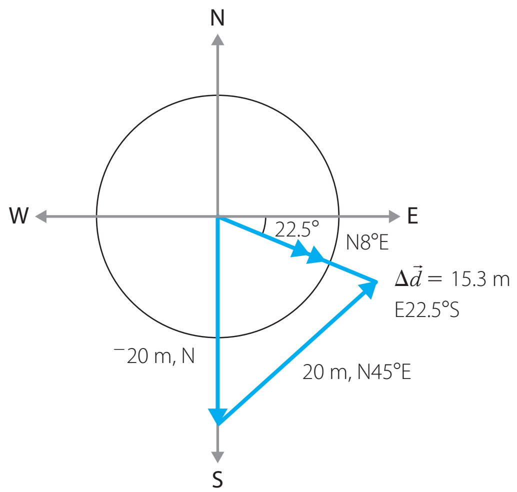
12



13 $\vec{v}_{av} = \dfrac{\Delta\vec{x}}{\Delta t} = k \times \Delta\vec{x},\ k = \dfrac{1}{\Delta t}$ = scalar multiplier

14 a P(20 m, N0°E); P(20 m, N45°E)

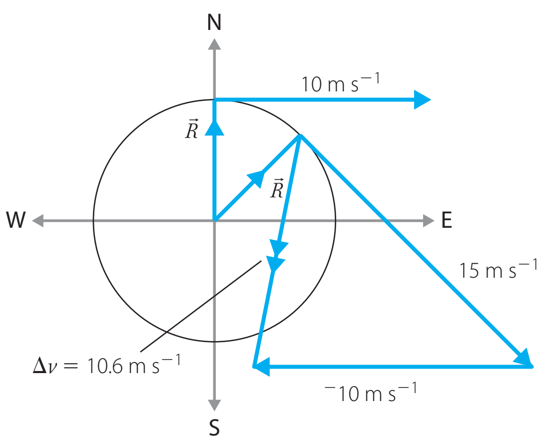
b



$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{15.3\text{ m}}{2\text{s}} = 7.65\text{ m s}^{-1}$$

Scale: 1 cm represents 5 m

c



$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{10.6\text{ m s}^{-1}}{2.0\text{ s}} = 5.3\text{ m s}^{-2}$$

Scale: 1 cm represents 2.5 m s^{-1}

CHAPTER 2: PROJECTILE MOTION

2.1 SECTION REVIEW

REMEMBERING

1 a See Figure 2.1.1, page 20

b $u_x = |\vec{u}|\cos\theta;\ u_y = |\vec{u}|\sin\theta$

2 See key formula boxes, pages 20–1

UNDERSTANDING

3 Field lines are approximately parallel; over relatively small vertical distances the differences in $|\vec{g}|$ are negligible.

4 Perpendicular resolutes have no component in the direction of parallel resolutes.

APPLYING

5 a Horizontal component: $u_x = 17\text{ m s}^{-1}$; vertical component: $u_y = 10\text{ m s}^{-1}$

b Horizontal component: $u_x = 11\text{ m s}^{-1}$; vertical component: $u_y = 11\text{ m s}^{-1}$

c Horizontal component: $u_x = 1.2\text{ m s}^{-1}$; vertical component: $u_y = 2.1\text{ m s}^{-1}$

6 a $|\vec{v}_{top}| = u_x = |\vec{u}|\cos\theta$; horizontal

$|\vec{v}_{top}| = 12\text{ m s}^{-1} \times \cos 70°$

$|\vec{v}_{top}| = 4.1\text{ m s}^{-1}$

b 9.8 m s^{-2}

c $y = 4.0\text{ m};\ u_y = |\vec{u}|\sin\theta = 12\text{ m s}^{-1} \times \sin 70° = 11.3\text{ m s}^{-1}$;

$v_y = 0\text{ m s}^{-1};\ g = {}^{-}9.8\text{ m s}^{-2};\ t = ?$

$y = \frac{1}{2}gt^2 + u_y t$

$4.9t^2 - 11.3t + 4.0 = 0$

$t = -\dfrac{{}^{-}11.3}{2 \times 4.9} \pm \dfrac{\sqrt{(11.3)^2 - 4 \times 4.9 \times 4.0}}{2 \times 4.9}$

$t = 1.15 \pm 0.71\text{ s}$

$t = 0.44\text{ s and }1.9\text{ s}$

7 $R = \dfrac{|\vec{u}|^2 \sin 2\theta}{g}$

$R = \dfrac{(300\text{ m s}^{-1})^2 \sin(2 \times 35°)}{9.8\text{ m s}^{-2}}$

$R = 8629\text{ m} = 8.6\text{ km}$

ANALYSING

8 *suvat* uses slightly different symbols and are written in a different order. When the order of terms are aligned, the equations are of the same form.

9 $y = 25\text{ m};\ u_y = |\vec{u}|\sin\theta = u\sin 60°$;

$v_y = 0\text{ m s}^{-1};\ g = {}^{-}9.8\text{ m s}^{-2};\ t = ?$

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$u_y = \sqrt{2gy}$

$u \sin 60° = \sqrt{2 \times 9.8\,\text{m s}^{-2} \times 25\,\text{m}}$

$u = \dfrac{\sqrt{2 \times 9.8\,\text{m s}^{-2} \times 25\,\text{m}}}{\sin 60°}$

$u = 25.6\,\text{m s}^{-1}$

REFLECTING

10 Vertically: maximum height is reduced by increased downward acceleration due to air resistance. Horizontally: range is reduced by opposing deceleration due to air resistance.

2.2 SECTION REVIEW

REMEMBERING

1 a Projectile motion near Earth; air resistance is negligible; velocity has rectangular components (x, y).

b See key formula, page 24

2 See key formula, page 25

UNDERSTANDING

3 a $v_y = gt + u_y$

b $\theta = \tan^{-1}\left(\dfrac{v_y}{u_x}\right)$

4 Both motions have zero initial speed in the vertical direction; both are subject to the same gravitational acceleration.

5 Launch and landing are on the same horizontal plane; see derivation, page 25.

APPLYING

6 Horizontally:

$u_x = v_x = u\cos\theta$

$v_x = 50\,\text{m s}^{-1} \times \cos 50°$

$v_x = 32\,\text{m s}^{-1}$

Vertically:

$y = ?;\ u_y = u\sin\theta = 50\,\text{m s}^{-1} \times \sin 50° = 38\,\text{m s}^{-1};$

$v_y = ?;\ g = {}^{-}9.8\,\text{m s}^{-2};\ t = 5.0\,\text{s}$

$v_y = gt + u_y$

$v_y = {}^{-}9.8\,\text{m s}^{-2} \times 5.0\,\text{s} + 38\,\text{m s}^{-1}$

$v_y = {}^{-}10.7\,\text{m s}^{-1}$

Together:

$v_{(x,y)} = \sqrt{v_x^2 + v_y^2}$

$v_{(x,y)} = \sqrt{\left(32\,\text{m s}^{-1}\right)^2 + \left({}^{-}10.7\ \text{m s}^{-1}\right)^2}$

$v_{(x,y)} = 33.7\,\text{m s}^{-1}$

$\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

$\theta = \tan^{-1}\left(\dfrac{{}^{-}10.7\,\text{m s}^{-1}}{32\,\text{m s}^{-1}}\right)$

$\theta = -18.4°$

7

LAUNCH SPEED (m s^{-1})	SMALLEST ANGLE TO THE HORIZONTAL (°)	LARGEST ANGLE TO THE HORIZONTAL (°)	RANGE
20	30	60	35.3
30	41	39	91
22	38	52	48

ANALYSING

8 a The paintball shot and the bag fall with the same acceleration ($g = 9.8\,\text{m s}^{-2}$)

b Vertically:

$y = 10\,\text{m};\ u_y = u\sin\theta = 0\,\text{m s}^{-1};\ v_y = ?;$

$g = 9.8\,\text{m s}^{-2};\ t = ?$

$y = \dfrac{1}{2}gt^2$

$t = \sqrt{\dfrac{2y}{g}}$

$t = \sqrt{\dfrac{2 \times 10.0\,\text{m}}{9.8\,\text{m s}^{-2}}}$

$t = 1.4\,\text{s}$

Horizontally:

$u_x = v_x = u = \dfrac{x}{t}$

$u = \dfrac{12.0\,\text{m}}{1.4\,\text{s}}$

$u = 8.4\,\text{m s}^{-1}$

2.3 SECTION REVIEW

REMEMBERING

1 See page 27

2 See Figure 2.2.4, page 24

UNDERSTANDING

3 a $\left(v_y\right)_{\text{inst}} = 0$; constant field, $g = 9.8\,\text{m s}^{-2}$

b $\left(v_y\right)_{\text{inst}} = u_x$ no air resistance; constant field, $g = 9.8\,\text{m s}^{-2}$

4 Measure changes of displacement from the launch position.

5 a Horizontal component of projectile motion; no air resistance; constant gravitational field.

b When a projectile is passing the point of projection on its return.

$v_y = gt + u_y$

$t = \dfrac{v_y - u_y}{g},\ g < 0$

$t = \dfrac{2u_y}{g},\ v_y = {}^{-}u_y$

No air resistance; constant gravitational field

c horizontal launch velocity; no air resistance; constant gravitational field

APPLYING

6 a Vertically:

$s_y = ?;\ u_y = u\sin\theta = 40\,\text{m s}^{-1} \times \sin 30° = 20\,\text{m s}^{-1};$

$v_y = 0;\ g = {}^{-}9.8\,\text{m s}^{-2};\ t = ?$

$v_y^2 = 2gy + u_y^2$

$s_y = \dfrac{v_y^2 - u_y^2}{2g}$

$s_y = \dfrac{0 - (20\,\text{m s}^{-1})^2}{2 \times {}^{-}9.8\,\text{m s}^{-2}}$

$s_y = 20.4\,\text{m}$

b Vertically:

$u_y = u\sin\theta = 40\,\text{m s}^{-1} \times \sin 30° = 20\,\text{m s}^{-1};$

$v_y = 0;\ g = {}^{-}9.8\,\text{m s}^{-2};\ t = ?$

$v_y = gt + u_y$

$t = \dfrac{v_y - u_y}{g}, g < 0$

$t = \dfrac{0 - 20\,\text{m s}^{-1}}{{}^{-}9.8\,\text{m s}^{-2}}$

$t = 2.0\,\text{s}$

c $g = {}^{-}9.8\,\text{m s}^{-2}$

d Horizontally:

$u_x = v_x = u\cos\theta$

$v_x = 40\,\text{m s}^{-1} \times \cos 30°$

$v_x = 34.6\,\text{m s}^{-1}$

Vertically:

$s_y = ?;\ u_y = u\sin\theta = 40\,\text{m s}^{-1} \times \sin 30° = 20\,\text{m s}^{-1};$

$v_y = 0;\ g = {}^{-}9.8\,\text{m s}^{-2};\ t = 1.0\,\text{s}$

$v_y = gt + u_y$

$v_y = {}^{-}9.8\,\text{m s}^{-2} \times 1.0\,\text{s} + 20\,\text{m s}^{-1}$

$v_y = 10.2\,\text{m s}^{-1}$

Together:

$v_{(x,y)} = \sqrt{u_x^2 + v_y^2}$

$v_{(x,y)} = \sqrt{(34.6\,\text{m s}^{-1})^2 + (10.2\,\text{m s}^{-1})^2}$

$v_{(x,y)} = 36\,\text{m s}^{-1}$

$\theta = \tan^{-1}\left(\dfrac{v_y}{u_x}\right)$

$\theta = \tan^{-1}\left(\dfrac{34.6\,\text{m s}^{-1}}{10.2\,\text{m s}^{-1}}\right)$

$\theta = 74°$

7 $R = \dfrac{u^2 \sin 2\theta}{g}, 0 < \theta < 90°$

$R = \dfrac{(50\,\text{m s}^{-1})^2 \times \sin(2 \times 75°)}{9.8\,\text{m s}^{-2}}$

$R = 128\,\text{m}$

ANALYSING

8 $|\vec{v}| = \sqrt{u_x^2 + v_y^2}$

Horizontally:

$u_x = u\cos\theta$

$u_x^2 = u^2\cos^2\theta$

Vertically:

$v_y = gt + u_y$

$v_y^2 = (gt + u_y)^2$

$v_y^2 = (gt)^2 + (u_y)^2 + 2gtu_y$

But $u_y = u\sin\theta$

$v_y^2 = (gt)^2 + (u\sin\theta)^2 + 2gtu\sin\theta$

$v_y^2 = gt(gt + 2u\sin\theta) + u^2\sin^2\theta$

$u_x^2 + v_y^2 = u^2\cos^2\theta + gt(gt + 2u\sin\theta) + u^2\sin^2\theta$

$u_x^2 + v_y^2 = u^2(\sin^2\theta + \cos^2\theta) + gt(gt + 2u\sin\theta)$

$u_x^2 + v_y^2 = u^2 + gt(gt + 2u\sin\theta)$

$|\vec{v}| = \sqrt{u^2 + gt(gt + 2u\sin\theta)}$

9 Vertically:

$s_y = {}^{-}30\,\text{m};\ u_y = |\vec{u}|\sin\theta = 70\,\text{m s}^{-1} \times \sin 30° = 35\,\text{m s}^{-1};$

$v_y = ?;\ g = {}^{-}9.8\,\text{m s}^{-2};\ t = ?$

$s_y = \dfrac{1}{2}gt^2 + u_y t$

$4.9\,\text{m s}^{-2}t^2 - 35\,\text{m s}^{-1}t - 30\,\text{m} = 0$

$t = -\dfrac{{}^{-}35\,\text{m s}^{-1}}{2 \times 4.9\,\text{m s}^{-2}} \pm \dfrac{\sqrt{(35\,\text{m s}^{-1})^2 - 4 \times 4.9\,\text{m s}^{-2} \times {}^{-}30\,\text{m}}}{2 \times 4.9\,\text{m s}^{-2}}$

$t = 3.57\,\text{s} \pm 4.34\,\text{s}$

$t = 7.93\,\text{s}$

Horizontally:

$u_x = u\cos\theta = \dfrac{x}{t}$

$x = ut\cos\theta$

$x = 70\,\text{m s}^{-1} \times 7.93\,\text{s} \times \cos 30°$

$x = 480\,\text{m}$

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CHAPTER REVIEW QUESTIONS

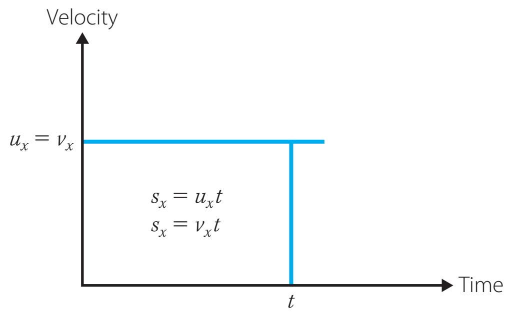
DETAIL QUESTION

1 Upwards: $x > 0$ above launch position; $g = {}^{-}9.8\,\mathrm{m\,s^{-2}}$.

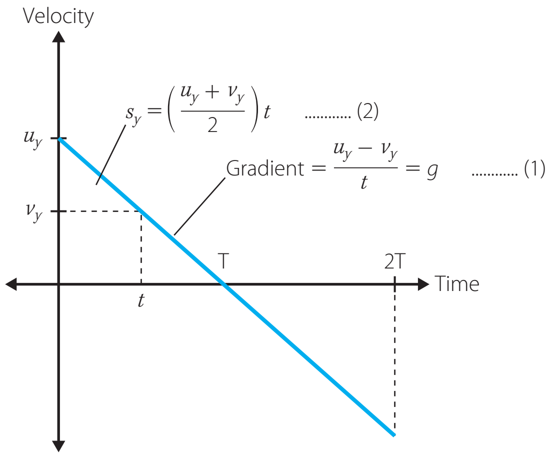
Downwards: $x > 0$ below launch position; $g = {}^{+}9.8\,\mathrm{m\,s^{-2}}$.

CATEGORY QUESTIONS

2 a



b



Using equations numbered (1) and (2) from the graph above:

$$g = \frac{v_y - u_y}{t} \ldots\ldots(1)$$

$$v_y = gt + u_y \ldots\ldots(3)$$

$$s_y = \frac{v_y + u_y}{2} \times t \ldots\ldots(2)$$

$$(1)\times(2)\;\; s_y \times g = \left(\frac{v_y - u_y}{t}\right)\times\left(\frac{v_y + u_y}{2}\times t\right)$$

$$2s_y \times g = v_y^2 - u_y^2$$

$$v_y^2 = 2gs_y + u_y^2 \quad (4)$$

$$(3)\text{ into }(2)\; s_y = \frac{\left(gt + u_y\right) + u_y}{2} \times t$$

$$s_y = \frac{gt + 2u_y}{2} \times t$$

$$s_y = \frac{1}{2}gt^2 + u_y \times t \quad (5)$$

ELABORATION QUESTIONS

3 Horizontally:

$$u_x = u\cos\theta = \frac{x}{t}$$

$$t = \frac{x}{u\cos\theta}$$

Vertically:

$$u_y = u\sin\theta$$

$$y = \frac{1}{2}gt^2 + u_y t$$

$$y = \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right) + u\sin\theta\left(\frac{x}{u\cos\theta}\right)$$

$$y = \left(\frac{g}{2u^2\cos^2\theta}\right)x^2 + x\tan\theta$$

For given initial conditions in $g, u\theta$:

$$y = Kx^2 + cx,\text{ where } K = \frac{g}{2u^2\cos^2\theta},\; c = \tan\theta$$

$\Rightarrow$ parabola

4 Horizontally:

$$u_x = u\cos\theta = \frac{x}{t}$$

$$x = ut\cos\theta$$

Vertically:

$$u_y = u\sin\theta$$

$$v_y = gt + u_y$$

$$v_y = gt + u\sin\theta$$

$$t = \frac{v_y - u\sin\theta}{g}$$

$$x = u \times \frac{v_y - u\sin\theta}{g} \times \cos\theta$$

$$v_y^2 = 2gy + u_y^2$$

$$v_y^2 = 2gH + 2u^2\sin^2\theta$$

$$v = \sqrt{\frac{2gH + 2u^2\sin^2\theta - u\sin\theta}{g} \times \cos\theta}$$

$$x = \frac{u\cos\theta}{g} \times \sqrt{2gH + 2u^2\sin^2\theta} - u\sin\theta$$

g, H are given, $x = f(u, \theta)$

EVIDENCE QUESTIONS

5 Scenario is unrealistic:

Horizontally:

$$u_x = v_x = 250\,\mathrm{km\,h^{-1}} \times 10^3\,\mathrm{m\,km^{-1}} \times \frac{1\,\mathrm{h\,s^{-1}}}{3600}$$

$$v_x = 69.4\,\mathrm{m\,s^{-1}}$$

$\Rightarrow$ Unrealistically high horizontal component of landing velocity.

From horizontal data:

$v_x = \frac{x}{t}$

$t = \frac{x}{v_x}$

$t = \frac{400\,\text{m}}{69.4\,\text{m s}^{-1}}$

$t = 5.76\,\text{s}$

Vertically:

$y = 200\,\text{m};\ u_y = 0\,\text{m s}^{-1};\ v_y = ?;\ g = {}^{+}9.8\,\text{m s}^{-2};$

$t = 5.76\,\text{s}$

$v_y = gt + u_y$

$v_y = {}^{+}9.8\,\text{m s}^{-2} \times 5.76\,\text{s}$

$v_y = 56.4\,\text{m s}^{-1}$

$v_y = 56.4\,\text{m s}^{-1} \times 10^{-3}\,\text{km m}^{-1} \times 3600\,\text{s h}^{-1}$

$v_y = 203\,\text{km h}^{-1}$

⇒ Unrealistically high vertical component of landing velocity.

From horizontal data:

$t = 5.76\,\text{s}$ (see above)

Vertically:

$y = 200\,\text{m};\ u_y = 0\,\text{m s}^{-1};\ v_y = ?;\ g = {}^{+}9.8\,\text{m s}^{-2};\ t = ?$

$y = \frac{1}{2}gt^2 + u_y t$

$y = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2y}{g}}$

$t = \sqrt{\frac{2 \times 200\,\text{m}}{{}^{+}9.8\,\text{m s}^{-2}}}$

$t = 6.4\,\text{s}$

Combining horizonal and vertical components:

$v_{(x,y)} = \sqrt{u_x^2 + v_y^2}$

$v_{(x,y)} = \sqrt{(250\,\text{km h}^{-1})^2 + (203\,\text{km h}^{-1})^2}$

$v_{(x,y)} = 322\,\text{km h}^{-1}$

$\theta = \tan^{-1}\left(\frac{v_y}{u_x}\right)$

$\theta = \tan^{-1}\left(\frac{203\,\text{km h}^{-1}}{250\,\text{km h}^{-1}}\right)$

$\theta = 39°$

⇒ Unrealistically high landing speed and angle.

Vertically:

$s_y = 200\,\text{m};\ u_y = 0\,\text{m s}^{-1};\ v_y = ?;\ g = {}^{+}9.8\,\text{m s}^{-2};\ t = ?$

$s_y = \frac{1}{2}gt^2 + u_y t$

$s_y = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2y}{g}}$

$t = \sqrt{\frac{2 \times 200\,\text{m}}{{}^{+}9.8\,\text{m s}^{-2}}}$

$t = 6.4\,\text{s}$

⇒ Time needed to reach drop zone is less than the time needed to fall vertically – the load misses the target.

6 Students' realistic answers will vary according to secondary data collected. (Launch angle < 45° – typically 38–42° – depends on athlete as well as projectile motion analysis).

END-OF-CHAPTER EXAM

1 C

2 C

3 C

4 A

5 B

6 a Vertically:

$s_y = u_y t + \frac{1}{2}gt^2$

$s_y = 75\,\text{m};\ u_y = 0\,\text{m s}^{-1};\ g = 9.8\,\text{m s}^{-2};\ t = ?$

$t^2 = \frac{75\,\text{m} \times 2}{9.8\,\text{m s}^{-2}}$

$t = \sqrt{\frac{75\,\text{m} \times 2}{9.8\,\text{m s}^{-2}}}$

$t = 3.9\,\text{s}$

b Vertically:

$y = 75\,\text{m};\ u_y = 0\,\text{m s}^{-1};\ v_y = ?;\ g = {}^{+}9.8\,\text{m s}^{-2};\ t = 3.9\,\text{s}$

$v_y^2 = 2gy + u_y^2$

$v_y^2 = 2gy$

$v_y^2 = 2 \times {}^{+}9.8\,\text{m s}^{-2} \times 75\,\text{m}$

$v_y^2 = 1470\,\text{m}^2\,\text{s}^{-2}$

$|\vec{v}| = u_x^2 + v_y^2$

$|\vec{v}| = \sqrt{(25\,\text{m s}^{-1})^2 + 1470\,\text{m}^2\,\text{s}^{-2}}$

$|\vec{v}| = 46\,\text{m s}^{-1}$

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c $\theta = \tan^{-1}\left(\frac{v_y}{u_x}\right)$

$\theta = \tan^{-1}\left(\frac{45.7\,\text{m s}^{-1}}{25\,\text{m s}^{-1}}\right)$

$\theta = 61°$

7 $u_x = u\cos\theta = \frac{x}{t}$

$u = \frac{x}{t\cos\theta}$

$u = \frac{235\,\text{m}}{1.76\,\text{s} \times \cos 15°}$

$u = 139\,\text{m s}^{-1}$

8 a $R = \frac{u^2 \sin 2\theta}{g}, 0 < \theta < 90°$

$R = \frac{(30.0\,\text{m s}^{-1})^2 \times \sin(2 \times 42°)}{9.8\,\text{m s}^{-2}}$

$R = 91.3\,\text{m}$

$u_x = u\cos\theta = \frac{x}{t}$

$t = \frac{x}{u\cos\theta}$

$t = \frac{91.3\,\text{m}}{30.0\,\text{m s}^{-1} \times \cos 42°}$

$t = 4.1\,\text{s}$

b $h = \frac{u_y^2}{2g}$

$h = \frac{(u\sin\theta)^2}{g}$

$h = \frac{(30.0\,\text{m s}^{-1} \times \sin 42°)^2}{2 \times 9.8\,\text{m s}^{-2}}$

$h = 20.6\,\text{m}$

c $u_h = u_x = u\cos\theta$

$u_h = 30.0\,\text{m s}^{-1} \times \cos 42°$

$u_h = 22.3\,\text{m s}^{-1}$

d $9.8\,\text{m s}^{-2}$, downwards

9 Constant acceleration due to gravity (magnitude and vertical direction); no air resistance.

10 Recognise that the minimum launch velocity coincides with both objects landing on the ground at the same time.

Geometrically:

$\tan\theta = \frac{24.7\,\text{m}}{42.8\,\text{m}}$

$\theta = 30°$

$R = \frac{u^2 \sin 2\theta}{g}$

$u = \sqrt{\frac{Rg}{\sin 2\theta}}$

$u = \sqrt{\frac{42.8\,\text{m} \times 9.8\,\text{m s}^{-2}}{\sin(2 \times 30°)}}$

$u = 22.0\,\text{m s}^{-1}$

$u_x = \frac{R}{t}$

$t = \frac{R}{u_x} = \frac{42.8\,\text{m}}{22.0\,\text{m s}^{-2} \times \cos 30°}$

$t = 2.2\,\text{s}$

11 **Scenario A:**

When $s_y = 0, \theta = 30°$

$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{(30.0\,\text{m s}^{-1})^2 \times \sin(2 \times 30°)}{9.8\,\text{m s}^{-2}}$

$R = 79.5\,\text{m}$

When $s_y = 0, \theta = 60°$

$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{(30.0\,\text{m s}^{-1})^2 \times \sin(2 \times 60°)}{9.8\,\text{m s}^{-2}}$

$R = \frac{(30.0\,\text{m s}^{-1})^2 \times \sin(120°)}{9.8\,\text{m s}^{-2}}$

$R = \frac{(30.0\,\text{m s}^{-1})^2 \times \sin(180° - 120°)}{9.8\,\text{m s}^{-2}}$

$R = 79.5\,\text{m}$

When $s_y = 8.0\,\text{m}, \theta = 30°$

$u_y = 30\,\text{ms}^{-1} \times \sin 30° = 15\,\text{ms}^{-1}$ $v_y = ?$ $a = {}^{-}9.8\,\text{ms}^{-2}$ $t = ?$

$$s_y = \frac{1}{2}gt^2 + u_y t$$

$$8.0\,m = \frac{1}{2} \times {}^{-}9.8\,\text{ms}^{-2} \times t^2 + 15\,\text{ms}^{-1} \times t$$

$$4.9t^2 - 15\,t + 8.0 = 0$$

$$t = -\frac{{}^{-}15}{2 \times 4.9} \pm \frac{\sqrt{({}^{-}15)^2 - 4 \times 4.9 \times 8.0}}{2 \times 4.9}$$

$$t = 1.53 \pm 0.84$$

$$t = 0.69\,\text{s} \;\text{ or }\; t = 2.4\,\text{s}$$

$$s_x = u_x t$$

$$s_x = 30\,\text{ms}^{-1} \times \cos 30° \times 0.69\,\text{s} \;\text{ or }\; s_x = 30\,\text{ms}^{-1} \times \cos 30° \times 2.4\,\text{s}$$

$$s_x = 17.9\,\text{m} \;\text{ or }\; s_x = 62.4\,\text{m}$$

When $s_y = 8.0\,\text{m}, \theta = 60°$

$u_y = 30\,\text{ms}^{-1} \times \sin 60° = 25.98\,\text{ms}^{-1}$ $v_y = ?$ $a = {}^{-}9.8\,\text{ms}^{-2}$ $t = ?$

$$s_y = \frac{1}{2}gt^2 + u_y t$$

$$8.0\,\text{m} = \frac{1}{2} \times {}^{-}9.8\,\text{ms}^{-2} \times t^2 + 25.98\,\text{ms}^{-1} \times t$$

$$4.9t^2 - 25.98\,t + 8.0 = 0$$

$$t = -\frac{{}^{-}25.98}{2 \times 4.9} \pm \frac{\sqrt{({}^{-}25.98)^2 - 4 \times 4.9 \times 8.0}}{2 \times 4.9}$$

$$t = 2.65 \pm 2.32$$

$$t = 0.33\,\text{s} \;\textit{ or }\; t = 4.97\,\text{s}$$

$$s_x = u_x t$$

$$s_x = 30\,\text{ms}^{-1} \times \cos 60° \times 0.33\,\text{s} \;\text{ or }\; s_x = 30\,\text{ms}^{-1} \times \cos 60° \times 4.97\,\text{s}$$

$$s_x = 4.92\,\text{m} \;\text{ or }\; s_x = 74.6\,\text{m}$$

When $s_y = {}^{-}10\,\text{m}$

$u_y = 30\,\text{ms}^{-1} \times \sin 30° = 15\,\text{ms}^{-1}$ $v_y = ?$ $a = {}^{-}9.8\,\text{ms}^{-2}$ $t = ?$

$$s_y = \frac{1}{2}gt^2 + u_y t$$

$${}^{-}10\,\text{m} = \frac{1}{2} \times {}^{-}9.8\,\text{ms}^{-2} \times t^2 + 15\,\text{ms}^{-1} \times t$$

$$4.9t^2 - 15\,t + {}^{-}10 = 0$$

$$t = -\frac{{}^{-}15}{2 \times 4.9} \pm \frac{\sqrt{({}^{-}15)^2 - 4 \times 4.9 \times {}^{-}10}}{2 \times 4.9}$$

$$t = 1.53 \pm 2.09$$

$$t = 3.62\,\text{s}$$

$$s_x = u_x t$$

$$s_x = 30\,\text{ms}^{-1} \times \cos 30° \times 3.62\,\text{s}$$

$$s_x = 94.2\,\text{m}$$

When $s_y = {}^{-}10\,\text{m}$

$u_y = 30\,\text{ms}^{-1} \times \sin 60° = 25.98\,\text{ms}^{-1}$ $v_y = ?$ $a = {}^{-}9.8\,\text{ms}^{-2}$ $t = ?$

$$s_y = \frac{1}{2}gt^2 + u_y t$$

$${}^{-}10\,\text{m} = \frac{1}{2} \times {}^{-}9.8\,\text{ms}^{-2} \times t^2 + 25.98\,\text{ms}^{-1} \times t$$

$$4.9t^2 - 15\,t + {}^{-}10 = 0$$

$$t = -\frac{{}^{-}25.98}{2 \times 4.9} \pm \frac{\sqrt{({}^{-}25.98)^2 - 4 \times 4.9 \times {}^{-}10}}{2 \times 4.9}$$

$$t = 2.65 \pm 3.01$$

$$t = 5.66\,\text{s}$$

$$s_x = u_x t$$

$$s_x = 30\,\text{ms}^{-1} \times \cos 60° \times 5.66\,\text{s}$$

$$s_x = 84.9\,\text{m}$$

Comparing results:

	LANDING POSITION RELATIVE TO LAUNCH (m)			
Launch angle (°)	$^{-}10$	0	8.0	8.0
30	94.2	79.5	17.9	62.4
Launch angle (°)	84.9	79.5	4.92	74.6

Scenario B:

Horizontally:

$$s_x = D = u_x t$$

$$D = u\cos\theta \times t$$

$$t = \frac{D}{u\cos\theta} \qquad (1)$$

Vertically:

$s_y = h$ $u_y = u\sin\theta$ $v_y = ?$ $g = {}^{-}9.8\,\text{ms}^{-2}$ $t = \dfrac{D}{u\cos\theta}$ [*subst*.(1)]

$$s_y = \frac{1}{2}gt^2 + u_y t$$

$${}^{-}4.9t^2 + u\sin\theta \times t = h$$

$$4.9 \times \left(\frac{D}{u\cos\theta}\right)^2 - u\sin\theta \times \frac{D}{u\cos\theta} = {}^{-}h$$

$$\frac{4.9}{u^2} \times \frac{D^2}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta} \times D + h = 0$$

$$\frac{4.9}{u^2}D^2 - \cos^2\theta\frac{\sin\theta}{\cos\theta} \times D + h\cos^2\theta = 0$$

$$\frac{4.9}{u^2}D^2 - \sin\theta\cos\theta \times D + h\cos^2\theta = 0$$

$$\frac{9.8}{u^2}D^2 - 2\sin\theta\cos\theta \times D + 2h\cos^2\theta = 0$$

$$\frac{9.8}{u^2}D^2 - \sin 2\theta \times D + 2h\cos^2\theta = 0$$

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Given conditions:

$D = 18\,\text{m} \quad \theta = 45^\circ \quad h = {}^{-}2$

$\frac{9.8^{2}}{u^{2}} \times 18^{2} - \sin(2\times 45^\circ)\times 18 + 2\times {}^{-}2 \times \cos^{2} 45^\circ = 0$

$\frac{3175.2}{u^{2}} - 18 - 2 = 0$

$\frac{3175.2}{u^{2}} = 20$

$u = \sqrt{\frac{3175.2}{20}}$

$u = 12.6\,\text{m}\,\text{s}^{-1}$

Finding the optimal angle of launch:

Assume:

$u = 12.6\,\text{m}\,\text{s}^{-1} \quad h = {}^{-}2.00\,\text{m}$

$\frac{9.8}{u^{2}} D^{2} - \sin 2\theta \times D + 2h\cos^{2}\theta = 0$

$\frac{9.8}{(12.6)^{2}} D^{2} - \sin 2\theta \times D + 2\left({}^{-}2\right)\cos^{2}\theta = 0$

$\frac{9.8}{158.76} D^{2} - \sin 2\theta \times D - 4\cos^{2}\theta = 0$

$D^{2} - 16.2\sin 2\theta \times D - 64.8\cos^{2}\theta = 0$

$D = -\frac{{}^{-}16.2\sin 2\theta}{2} \pm \frac{\sqrt{\left({}^{-}16.2\sin 2\theta\right)^{2} - 4\times 1\times {}^{-}64.8\cos^{2}\theta}}{2}$

$D = 8.1\sin 2\theta + \sqrt{65.61\sin^{2} 2\theta + 64.8\cos^{2}\theta} \quad (D > 0, \Delta > 8.1\sin 2\theta)$

To find θ such that D is a maximum:

a differentiate to find turning point

b CAS calculator

c spreadsheet.

$\theta = 42^\circ$

CHAPTER 3: INCLINED PLANES

3.1 SECTION REVIEW

REMEMBERING

1 a Force by surface on object, parallel to the surface.

b Force by surface on object, perpendicular to the surface.

2 a $\vec{N} + \vec{w} = 0$

b $\vec{N} + \vec{w} = m\vec{a}$

c $\vec{w} = m\vec{g}$

3 a $m\vec{g}\sin\theta - \vec{f} = m\vec{a}$

b $\vec{N} - m\vec{g}\cos\theta = 0$

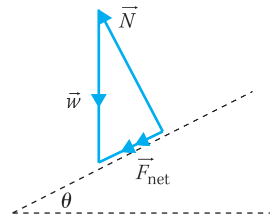
UNDERSTANDING

4 Friction is an electrostatic force between two interacting surfaces.

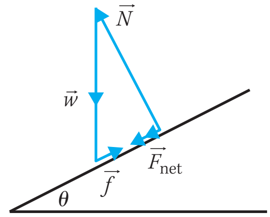
5 Weight force applied to mass draws the mass into the surface, thereby decreasing the distance between interacting particles and increasing the effect of the electrostatic force.

6 Static friction increases to a maximum at sliding. Sliding friction is approximately constant and less than the maximum static friction.

7 a



b



APPLYING

8 a $\Sigma F_{\parallel} = N - mg\cos\theta = 0$

b $\Sigma F_{\parallel} = f_{s} - mg\sin\theta = 0$

c $f_{s} - mg\sin\theta = 0$

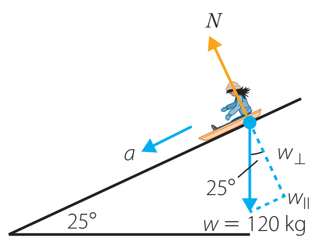
$\sin\theta = \frac{f}{mg}$

$\theta = \sin^{-1}\left(\frac{f}{mg}\right)$

$\theta = \sin^{-1}\left(\frac{15\,\text{N}}{3.0\,\text{kg}\times 9.8\,\text{N}\,\text{kg}^{-1}}\right)$

$\theta = 31^\circ$

9



a $\Sigma F_{\parallel} = N - mg\cos\theta = 0$

$\Sigma F_{\perp} = 0$

$N - mg\cos\theta = 0$

$N = mg\cos\theta$

$N = 120\,\text{kg}\times 9.8\,\text{m}\,\text{s}^{-2}\times\cos 25^\circ$

$N = 1.1\times 10^{3}\,\text{N}$

b $\Sigma F_{\parallel} = w_{\parallel}$

$\Sigma F_{\parallel} = mg\sin\theta$

$\Sigma F_{\parallel} = 120\,\text{kg}\times 9.8\,\text{m}\,\text{s}^{-2}\times\sin 25^\circ$

$\Sigma F_{\parallel} = 497\,\text{N}$

$\Sigma F_{\parallel} = ma$

$$a = \frac{\Sigma F_{\|}}{m}$$

$$a = \frac{497\,\text{N}}{120\,\text{kg}}$$

$$a = 4.1\,\text{m s}^{-2}$$

c $a = 4.1\,\text{m s}^{-2}$; $d = 120\,\text{m}$; $u = 0\,\text{m s}^{-1}$; $v = ?$

$$v^2 = u^2 + 2ad$$

$$v^2 = 0 + 2 \times 4.1\,\text{m s}^{-2} \times 120\,\text{m}$$

$$v = 31.4\,\text{m s}^{-1}$$

ANALYSING

10 a The maximum static friction occurs at the point where an object begins to slide. Once overcome, the kinetic friction is less, but not predictable from this data.

b $\Sigma F = mg\sin\theta - f_s = 0$

$$f_s = mg\sin\theta$$

$$f_s = 20\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 30°$$

$$f_s = 98\,\text{N}$$

3.2 SECTION REVIEW

REMEMBERING

1 See page 43

2 a See Figure 3.2.1, page 43

b See Figure 3.2.2, page 44

c See Figure 3.2.3, page 44

3 a $\Sigma F_{\perp} = N - w_{\perp} = 0$

b $\Sigma F_{\|} = w_{\|} = mg\sin\theta = ma$

c $\Sigma F_{\|} = w_{\|} - f_k = mg\sin\theta - f_k = ma$

UNDERSTANDING

4 Normal force is perpendicular to the plane, so has no component down the plane; therefore, the component of the weight force parallel to the plane and the kinetic friction are the only two forces with components parallel to the plane.

5 A mass already in motion at constant speed will continue in that motion (no acceleration) unless a net external force is applied (Newton's first law). If the acceleration is zero, the net force parallel to the slope is zero – the forces down the slope equal the forces up the slope (Newton's second law).

APPLYING

6 a $w_{\|} = mg\sin\theta$

$$w_{\|} = 100\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 20°$$

$$w_{\|} = 335\,\text{N}$$

b $N = w_{\perp}$

$$N = mg\cos\theta$$

$$N = 100\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \cos 20°$$

$$N = 921\,\text{N}$$

c $mg\sin\theta = ma$

$$a = g\sin\theta$$

$$a = 9.8\,\text{m s}^{-2} \times \sin 20°$$

$$a = 3.35\,\text{m s}^{-2}$$

7 $\Sigma F_{\|} = w_{\|} - f_k = 0$

$$w_{\|} - f_k = 0$$

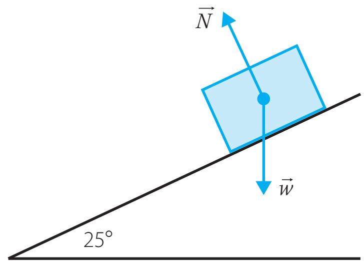
$$f_k = mg\sin\theta$$

$$f_k = 400\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 5°$$

$$f_k = 342\,\text{N}$$

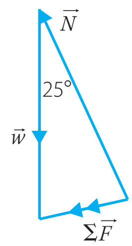
ANALYSING

8 a i



ii

iii



b $w_{\|} - f_s = 0$

$$f_s = mg\sin\theta$$

$$f_s = 2.0\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 25°$$

$$f_s = 8.3\,\text{N}$$

9 a $\Sigma F_{\|} = ma$

$$w_{\|} + F_{ext} = ma$$

$$F_{ext} = ma - w_{\|}$$

$$F_{ext} = ma - mg\sin\theta$$

$$F_{ext} = 60\,\text{kg} \times 3.2\,\text{m s}^{-2} - 60\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 15°$$

$$F_{ext} = 40\,\text{N}$$

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b $\Sigma F_{\parallel} = ma$

$F_{ext} - w_{\parallel} = ma$

$F_{ext} = ma + w_{\parallel}$

$F_{ext} = ma + mg\sin\theta$

$F_{ext} = 60\,\text{kg} \times 3.2\,\text{m s}^{-2} + 60\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 15°$

$F_{ext} = 344\,\text{N}$

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 See Figure 3.1.4, page 38 and Figure 3.2.4, page 44. Compare F_{net} with and without friction along the surface.

2 Static friction: forces hold object in place. Kinetic friction is less because the movement breaks the forces more readily.

CATEGORY QUESTIONS

3 Untrue. Only two of four criteria for a Newton's third law pair are satisfied. They are equal and opposite, but act on the same (not different) object and are of different types (electrostatic or gravitational).

ELABORATION QUESTIONS

4 Static friction:

$\sin\theta = \dfrac{145\,\text{cm}}{320\,\text{cm}}$

$\sin\theta = 0.453125$

$\theta = 27°$

$w_{\parallel} - f_s = 0$

$f_s = mg\sin\theta$

$f_s = 150\,\text{kg} \times 9.8\,\text{m s}^{-2} \times 0.453125$

$f_s = 666\,\text{N}$

Kinetic friction:

$\sin\theta = \dfrac{150\,\text{cm}}{320\,\text{cm}}$

$\sin\theta = 0.46875$

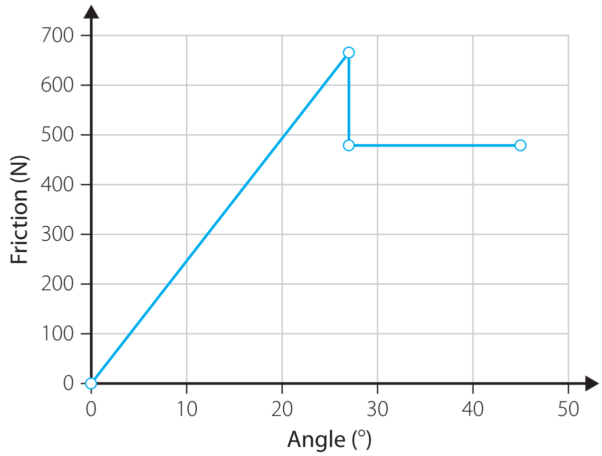
$\theta = 28°$

$w_{\parallel} - f_k = ma$

$f_k = mg\sin\theta - ma$

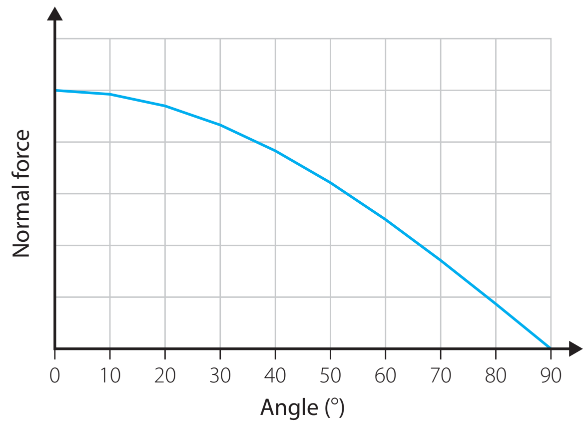
$f_k = 150\,\text{kg} \times 9.8\,\text{m s}^{-2} \times 0.46875 - 150\,\text{kg} \times 1.4\,\text{m s}^{-2}$

$f_k = 479\,\text{N}$



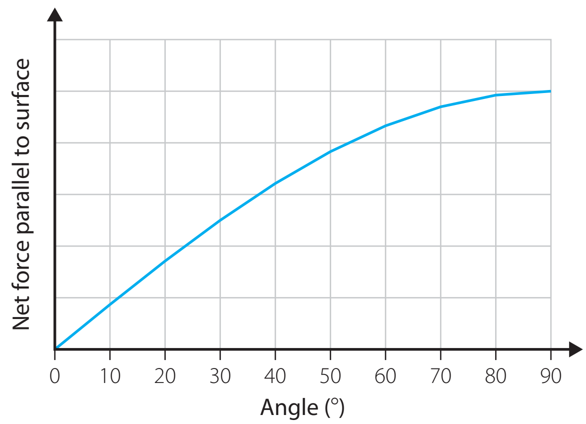
5 a i $N - mg\cos\theta = 0$

$N = mg\cos\theta$



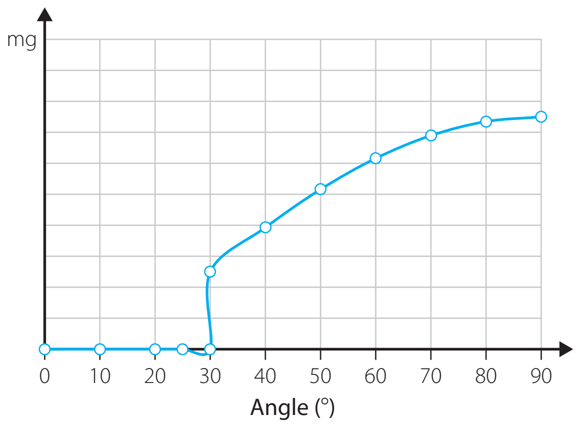
ii $\Sigma F_{\parallel} = w_{\parallel}$

$\Sigma F_{\parallel} = mg\sin\theta$



b Normal force is always perpendicular to friction, so no effect (same as 5ai above).

However, for friction-affected surfaces, the net force along the surface is affected by static friction – no net force up to a particular angle, depending on the interacting surfaces. Kinetic friction, usually considered to be constant, reduces the frictionless force by a constant amount once static friction has been overcome.



EVIDENCE QUESTION

6 Student answers will vary.

END-OF-CHAPTER EXAM

1 A

2 C

3 C

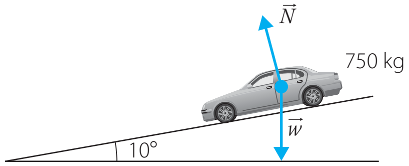
4 A

5 C

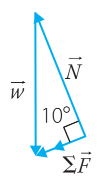
6 Friction points up the slope if an object is sliding down the slope and down the slope if the object is being pulled up the slope.

7 Static friction between two surfaces at rest rises to a maximum value as the external force on one object increases. Kinetic friction occurs when two surfaces slide relative to each other. It is approximately constant and less than static friction.

8 a



b



c $F_{\perp} = mg\cos\theta$

$F_{\perp} = 750\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \cos 10°$

$F_{\perp} = 7238\,\text{N}$

9 a $\Sigma F = w_{||} - f_k = ma$

$f_k = w_{||} - ma$

$f_k = mg\sin\theta - ma$

$f_k = 90\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 60° - 90\,\text{kg} \times 2.70\,\text{m s}^{-2}$

$f_k = 521\,\text{N}$

b $s = 100\,\text{m}$ $u = 0$ $v = ?$

$a = 2.70\,\text{m s}^{-2}$ $t = ?$

$v^2 = u^2 + 2as$

$v = \sqrt{u^2 + 2as}$

$v = \sqrt{2 \times 2.70\,\text{m s}^{-2} \times 100\,\text{m}}$

$v = 23.2\,\text{m s}^{-1}$

10 a $N = mg\cos\theta$

$N = 35\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \cos 30°$

$N = 2.97 \times 10^2\,\text{N}$

b $a = \frac{\Delta v}{\Delta t}$

$a = \frac{20\,\text{m s}^{-1} - 0\,\text{m s}^{-1}}{5.0\,\text{s}}$

$a = 4.0\,\text{m s}^{-2}$

$\Sigma F = ma$

$\Sigma F = 35\,\text{kg} \times 4.0\,\text{m s}^{-2}$

$\Sigma F = 140\,\text{N}$

c $\Sigma F = w_{||} - f_k = ma$

$f_k = w_{||} - \Sigma F$

$f_k = mg\sin\theta - \Sigma F$

$f_k = 35\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \sin 30° - 140\,\text{N}$

$f_k = 31.5\,\text{N}$

CHAPTER 4: CIRCULAR MOTION

4.1 SECTION REVIEW

REMEMBERING

1 a Number of revolutions in one second.

b Time taken for one revolution.

2 a $f = \frac{1}{T}$

b $v = \frac{2\pi r}{T}$

c $v = 2\pi rf$

UNDERSTANDING

3 For uniform motion, the fraction, *n*, of the circumference travelled in the same fraction, *n*, of the period is given by:

$v = \frac{n \times 2\pi r}{n \times T} = \frac{2\pi r}{T}$

APPLYING

4 a $f = \frac{1}{50\,\text{s}} = 0.02\,\text{Hz}$

b $f = \frac{1}{0.34\,\text{s}} = 2.94\,\text{Hz}$

c $f = \frac{1}{2.8 \times 10^{-3}\,\text{s}} = 357\,\text{Hz}$

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5 a $v = \frac{2\pi r}{T}$

$v = \frac{2\pi \times 0.37\,\text{m}}{2.9 \times 10^{-2}\,\text{s}}$

$v = 80\,\text{m s}^{-1}$

b $v = \frac{2\pi r}{T}$

$v = \frac{2\pi \times 1.2\,\text{m}}{0.3\,\text{s}}$

$v = 25\,\text{m s}^{-1}$

c $v = \frac{2\pi r}{T}$

$v = \frac{2\pi \times 1.5 \times 10^{11}\,\text{m}}{365.25\,\text{d} \times 24\,\text{h d}^{-1} \times 60\,\text{min h}^{-1} \times 60\,\text{s min}^{-1}}$

$v = 2.99 \times 10^{4}\,\text{m s}^{-1}$

ANALYSING

6 a Displacement of Jupiter relative to Earth at two different times (relative speed of Jupiter); radius and period for point J.

b $V_{\text{Jupiter}} = \frac{\Delta D_{\text{Jupiter}}}{\Delta t}$ (relative speed of Jupiter away from Earth); $v_J = \frac{2\pi r_J}{T_J}$ (rotational speed of J towards Earth); $V_J = V_{\text{Jupiter}} - v_J$ (total speed of point J).

4.2 SECTION REVIEW

REMEMBERING

1 See page 50

2 See page 51

UNDERSTANDING

3 a The number of events in a time interval is divided into the time for each event.

b $v = \frac{2\pi r}{T}$

$v = 2\pi r \times \frac{1}{T}$

$f = \frac{1}{T}$

$v = 2\pi r f$

4 a $v = \frac{2\pi r}{T}$ and $v = 2\pi r f$

$r = \frac{vT}{2\pi}$ $\quad r = \frac{v}{2\pi f}$

b $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v}$

c $v = 2\pi r f$

$f = \frac{v}{2\pi r}$

APPLYING

5 a $T = \frac{1}{f}$

$T = \frac{1}{12\,\text{Hz}}$

$T = 83\,\text{ms}$

b $T = \frac{1}{f}$

$T = \frac{1}{4.9 \times 10^{3}\,\text{Hz}}$

$T = 0.20\,\text{ms}$

c $T = \frac{1}{f}$

$T = \frac{1}{2.5 \times 10^{14}\,\text{Hz}}$

$T = 4 \times 10^{-15}\,\text{ms}$

6 a $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v}$

$T = \frac{2\pi \times 35\,\text{m}}{4.2 \times 10^{7}\,\text{m s}^{-1}}$

$T = 5.2\,\mu\text{s}$

b $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v}$

$T = \frac{2\pi \times 5.6 \times 10^{7}\,\text{m}}{4.1 \times 10^{3}\,\text{m s}^{-1}}$

$T = 8.6 \times 10^{4}\,\text{s}$

7 a $v = 2\pi r f$

$r = \frac{v}{2\pi f}$

$r = \frac{3.0 \times 10^{5}\,\text{m s}^{-1}}{2\pi \times 3.0 \times 10^{5}\,\text{s}^{-1}}$

$r = 0.16\,\text{m}$

b $v = \frac{2\pi r}{T}$

$r = \frac{vT}{2\pi}$

$r = \frac{4.5\,\text{m s}^{-1} \times 28\,\text{d} \times 24\,\text{h d}^{-1} \times 60\,\text{min h}^{-1} \times 60\,\text{s min}^{-1}}{2\pi}$

$r = 1.7 \times 10^{6}\,\text{m}$

ANALYSING

8 An object describes a total of 2π radians (one circle) – a measure of angle – in one period, T.

9 $v_A = \dfrac{2\pi r_A}{T}$ and $v_B = \dfrac{2\pi r_B}{T}$

$r_B = 2r_A$ and $T_A = T_B$

$$\frac{v_A}{v_B} = \frac{\frac{2\pi}{T} r_A}{\frac{2\pi}{T} \times 2r_A}$$

$$\frac{v_A}{v_B} = \frac{1}{2}$$

REFLECTING

10 Student answers will vary.

4.3 SECTION REVIEW

REMEMBERING

1 a Centre-seeking

b Constant tangential speed; therefore, constant angular speed.

2 a $a = \dfrac{v^2}{r} = \dfrac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$

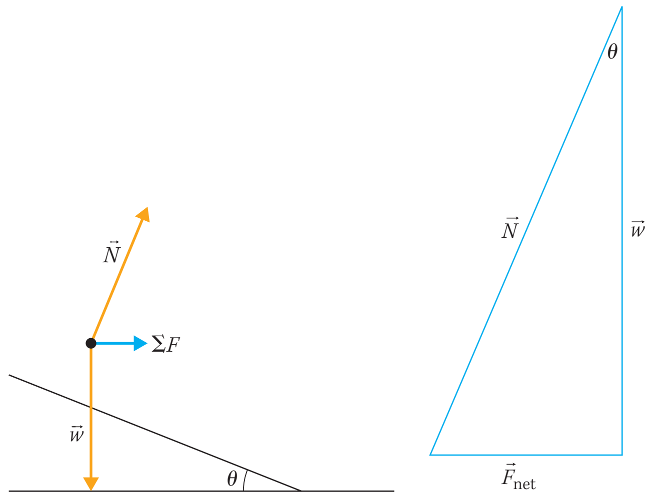
Instantaneous rate of change of velocity is pointed to the centre of a circle for uniform circular motion.

b $\Sigma F = ma = m \times \dfrac{v^2}{r} = m \times \dfrac{4\pi^2 r}{T^2} = m \times 4\pi^2 r f^2$

Instantaneous sum of all forces are centre-directed along the line of the instantaneous centripetal acceleration for uniform circular motion

3 Electrostatic, magnetic, gravitational, friction and tension

4



UNDERSTANDING

5 Centripetal force is a net force comprising the sum of real forces.

6 $\Sigma F = w - N = \dfrac{mv^2}{r}$; $N = mg - \dfrac{mv^2}{r} = 0$; $v = \sqrt{gr}$ (See Figure 4.3.14, page 58)

APPLYING

7

m (kg)	v (m s^{-1})	r (m)	a (m s^{-2})	ΣF (N)
1.0	2.0	0.55	$a = \dfrac{v^2}{r}$ $a = \dfrac{(2.0\,\text{m s}^{-1})^2}{0.55\,\text{m}}$ $a = 7.3\,\text{m s}^{-2}$	$\Sigma F = ma$ $\Sigma F = 1.0\,\text{kg} \times 7.3\,\text{m s}^{-2}$ $\Sigma F = 7.3\,\text{N}$
400	20	150	$a = \dfrac{v^2}{r}$ $a = \dfrac{(20\,\text{m s}^{-1})^2}{150\,\text{m}}$ $a = 2.67\,\text{m s}^{-2}$	$\Sigma F = ma$ $\Sigma F = 400\,\text{kg} \times 2.67\,\text{m s}^{-2}$ $\Sigma F = 1.1 \times 10^3\,\text{N}$
1.5×10^3	28	50	$a = \dfrac{v^2}{r}$ $a = \dfrac{(28\,\text{m s}^{-1})^2}{50\,\text{m}}$ $a = 15.7\,\text{m s}^{-2}$	$\Sigma F = ma$ $\Sigma F = 1.5 \times 10^3\,\text{kg} \times 15.7\,\text{m s}^{-2}$ $\Sigma F = 2.4 \times 10^4\,\text{N}$

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ANALYSING

8 $\left|\Sigma F(\text{by motorcycle on road})\right|$

$= \left|\Sigma F(\text{by road on motorcycle})\right| = m\frac{v^2}{r}$

$\left|\Sigma F(\text{by motorcycle on road})\right| = 400\,\text{kg} \times \frac{\left(25\,\text{m s}^{-1}\right)^2}{80\,\text{m}}$

$\left|\Sigma F(\text{by motorcycle on road})\right| = 3.1\,\text{kN}$

$\Sigma F = \left|\vec{N} + m\vec{g}\right| = \frac{mv^2}{r}$

9 a i $\Sigma F = m\omega^2 r = ma$

$a = \omega^2 r$

ii $a = \omega^2 r$

$a = \frac{4\pi^2 r}{T^2}$

$a = \left(\frac{2\pi}{T}\right)^2 r$

$\omega^2 = \left(\frac{2\pi}{T}\right)^2$

$\omega = \frac{2\pi}{T}$

$v = \frac{2\pi r}{T} = \frac{2\pi}{T} r$

$v = \omega r$

b Angular velocity

REFLECTING

10 Student answers will vary. Net force and centripetal force must be shown to be equivalent. Real forces: normal, tension, gravitational, electrostatic and magnetic

4.4 SECTION REVIEW

REMEMBERING

1 a See list, page 60

b See list, page 61

UNDERSTANDING

2 a $a = \frac{v^2}{r}$

$v^2 = ar$

$v = \sqrt{ar}$

b $\Sigma F = m\frac{4\pi^2 r}{T^2}$

$T^2 = m\frac{4\pi^2 r}{\Sigma F}$

$T = \sqrt{m\frac{4\pi^2 r}{\Sigma F}}$

$T = 2\pi\sqrt{\frac{mr}{\Sigma F}}$

APPLYING

3 a $a = \frac{v^2}{r}$

$a = \frac{\left(100\,\text{km h}^{-1} \times 10^3\,\text{m km}^{-1} \times \frac{1}{60}\text{h min}^{-1} \times \frac{1}{60}\text{min s}^{-1}\right)^2}{120\,\text{m}}$

$a = 6.43\,\text{m s}^{-2}$

b $\left|\Sigma F(\text{by road on car})\right| = \left|\Sigma F(\text{by car on road})\right| = ma$

$\left|\Sigma F(\text{by car on road})\right| = 1.2 \times 10^3\,\text{kg} \times 6.43\,\text{m s}^{-2}$

$\left|\Sigma F(\text{by car on road})\right| = 7.7 \times 10^3\,\text{N}$

4 a $\Sigma F = mg\tan\theta$

$\Sigma F = 75\,\text{kg} \times 9.8\,\text{m s}^{-2} \times \tan 42°$

$\Sigma F = 662\,\text{N}$

b $\Sigma F = m\frac{v^2}{r} = 662\,\text{N}$

$r = \frac{75\,\text{kg} \times \left(18\,\text{m s}^{-1}\right)^2}{662\,\text{N}}$

$r = 37\,\text{m}$

5 $\Sigma F = ma$

$\Sigma F = N - mg = m\frac{v^2}{r}$

$N = mg + m\frac{v^2}{r}$

$N = 50\,\text{kg} \times 9.8\,\text{m s}^{-2} + 50\,\text{kg} \times \frac{(12\,\text{m s}^{-1})^2}{28\,\text{m}}$

$N = 747$

6 $mg - N = m\frac{v^2}{r}$

when $N = 0$

$g = \frac{v^2}{r}$

$v = \sqrt{gr}$

$v = \sqrt{9.8\,\text{m s}^{-2} \times 34\,\text{m}}$

$v = 18\,\text{m s}^{-1}$

7 $N - mg = m\frac{v^2}{r}$

$N = mg + m\frac{v^2}{r}$

$N = 82\,\text{kg} \times 9.8\,\text{m s}^{-2} + 82\,\text{kg} \times \frac{\left(35\,\text{m s}^{-1}\right)^2}{45\,\text{m}}$

$N = 3.0 \times 10^3\,\text{N}$

ANALYSING

8 **a** $25r^2 = \frac{4\pi^2 r}{T^2}$

$T^2 = \frac{4\pi^2 r}{25}$

$T = \frac{2\pi}{5} r^{-\frac{1}{2}}$

b $T = \frac{2\pi}{5} r^{-\frac{1}{2}}$

$f = \frac{1}{T}$

$f = \frac{5\sqrt{r}}{2\pi}$

$f = \frac{5\sqrt{40\,\text{m}}}{2\pi}$

$f = 5.0\,\text{Hz}$

9 At speed, $v = \sqrt{gr}$, the normal is zero and the vehicle has no contact with the road; therefore, no ability to push on the road so that the road can provide a (reaction) friction force to help steer the car.

10 The velocity is attached (at right angles) to the end of the radius vector; thus, the velocity vector moves around the circle with the same frequency as the radius vector.

11 The velocity vectors form a circle of radius, v; hence,

$a = \frac{\text{circumference}}{T}$

$a = \frac{2\pi v}{T}$

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Newton's first law states that a body will travel at constant velocity unless forced to change by changing speed or changing direction or changing both speed and direction simultaneously. For uniform circular motion, only direction changes such that the object describes a circle.

2 Newton's third law states that forces act in pairs such that:

$F(\text{by person on surface}) = -F(\text{by surface on person})$

In order to run around a corner, a person must push outwards so that an inwards directed force is applied by the surface to change their direction.

3 The outwards force applied to the bitumen (liquid) surface by cars causes the bitumen surface to slide over the underlying base of the road.

CATEGORY QUESTIONS

4 Centripetal force is always the net of real forces.

5 Constant speed on a circle. Equations relate to measured variables. Equations involving period/frequency must be repetitive motion.

6 All equations are the same: $\Sigma F = \frac{mv^2}{r}$

Horizontal circle: $\Sigma F = \frac{mv^2}{r}$

Banked track: $\Sigma F = mg\tan\theta = \frac{mv^2}{r}$

Vertical circle: $\Sigma F = |\vec{N} + m\vec{g}| = \frac{mv^2}{r}$

ELABORATION QUESTIONS

7 **a** $\frac{GM_E}{r^2} = \frac{v^2}{r}$

$v = \sqrt{\frac{GM_E}{r_E + r_s}}$

$v = \sqrt{\frac{6.67 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \times 5.98 \times 10^{24}\,\text{kg}}{(6.37 \times 10^6\,\text{m} + 2.02 \times 10^7\,\text{m})}}$

$v = 3.75 \times 10^3\,\text{m}\,\text{s}^{-1}$

b $\frac{F_{orbit}}{F_{surface}} = \frac{m\frac{GM_E}{r^2}}{mg}$

$\frac{F_{orbit}}{F_{surface}} = \frac{\frac{GM_E}{(r_E + r_s)^2}}{g}$

$\frac{F_{orbit}}{F_{surface}} = \frac{\frac{6.67 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \times 5.98 \times 10^{24}\,\text{kg}}{(6.37 \times 10^6\,\text{m} + 2.02 \times 10^7\,\text{m})^2}}{9.8\,\text{m}\,\text{s}^{-2}}$

$\frac{F_{orbit}}{F_{surface}} = 0.051$

8 **a** At top, marble travels under the loop:

$mg + N = m\frac{v^2}{r}$

When $N = 0$, the marble falls down.

$v(\text{highest})_{\text{min}} = \sqrt{gr}$

Energy transfers (see Q Science Physics, Chapter 15, Section 15.3)

$\Delta E_T = 0$

$E_k(\text{lowest position}) + E_P(\text{lowest}) = E_k(\text{highest}) + E_P(\text{highest})$

$\frac{1}{2}mv(\text{lowest})^2_{\text{min}} + 0 = \frac{1}{2}mv(\text{highest})^2_{\text{max}} + mg \times 2r$

$v(\text{lowest})^2_{\text{min}} = v(\text{highest})^2_{\text{max}} + 4gr$

$v(\text{lowest})_{\text{min}} = \sqrt{\sqrt{gr}^2 + 4gr}$

$v(\text{lowest})_{\text{min}} = \sqrt{5gr}$

Range of speeds for the marble to stay on the track while completing a full loop:

$v > \sqrt{5gr}$

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b At top, marble travels over the track:

$$mg - N = m\frac{v^2}{r}$$

When $N = 0$, the marble leaves the track.

$$v(\text{highest})_{\text{max}} = \sqrt{gr}$$

Energy transfers (see QPhysics 11, Chapter 15, Section 15.3)

Maximum speed at lowest position:

$$\Delta E_T = 0$$

$$E_k(\text{lowest position}) + E_P(\text{lowest}) = E_k(\text{highest}) + E_P(\text{highest})$$

$$\frac{1}{2}mv(\text{lowest})^2_{\text{max}} + 0 = \frac{1}{2}mv(\text{highest})^2_{\text{max}} + mg \times 2r$$

$$v(\text{lowest})^2_{\text{max}} = v(\text{highest})^2_{\text{max}} + 4gr$$

$$v(\text{lowest})_{\text{max}} = \sqrt{\sqrt{gr}^2 + 4gr}$$

$$v(\text{lowest})_{\text{max}} = \sqrt{5gr}$$

Minimum speed at lowest position:

$$\Delta E_T = 0$$

$$E_k(\text{lowest position}) + E_P(\text{lowest}) = E_k(\text{highest}) + E_P(\text{highest})$$

$$\frac{1}{2}mv(\text{lowest})^2_{\text{max}} + 0 = 0 + mg \times 2r$$

$$v(\text{lowest})^2_{\text{max}} = 4gr$$

$$v(\text{lowest})_{\text{max}} = 2\sqrt{gr}$$

Range of speeds for marble to stay on the track while completing a full loop:

$$2\sqrt{gr} < v < \sqrt{5gr}$$

9 Qualitatively: at a critical speed, the ground must supply a force on the leg greater than the force needed to break the bone. Student quantitative answers will vary depending on the source of data.

10 Sum of three vectors: student answers will vary depending on the source of data.

END-OF-CHAPTER EXAM

1 D

2 A

3 A

4 C

5 C

6 Centre-seeking rate of change of velocity of an object *cf* net force on a single object and directed towards the centre of a circle.

7 Newton's third law: force by Emily's hand on the string is equal and opposite to the force by the string on Emily's hand.

8 $a = 2\pi rf$

$$a = 2\pi \times 0.30\,\text{m} \times 33.3\,\text{min}^{-1} \times \frac{1}{60}\,\text{min s}^{-1}$$

$$a = 1.05\,\text{m s}^{-2}$$

9 **a** $\Sigma F(\text{on motorcyclist}) = m\frac{v^2}{r}$

$$\Sigma F(\text{on motorcyclist}) = 65\,\text{kg} \times \frac{\left(\frac{100}{3.6\,\text{m s}^{-1}}\right)^2}{85\,\text{m}}$$

$$\Sigma F(\text{on motorcyclist}) = 590\,\text{N}$$

b By Newton's third law:

$$|\Sigma F(\text{by road on motorcyclist and rider})| = |\Sigma F(\text{by motorcyclist and rider on road})|$$

$$|\Sigma F(\text{by motorcyclist and rider on road})| = m\frac{v^2}{r}$$

$$|\Sigma F(\text{by motorcyclist and rider on road})| = (350\,\text{kg} + 65\,\text{kg}) \times \frac{\left(\frac{100}{3.6\,\text{m s}^{-1}}\right)^2}{85\,\text{m}}$$

$$|\Sigma F(\text{by motorcyclist and rider on road})| = 3.8 \times 10^3\,\text{N}$$

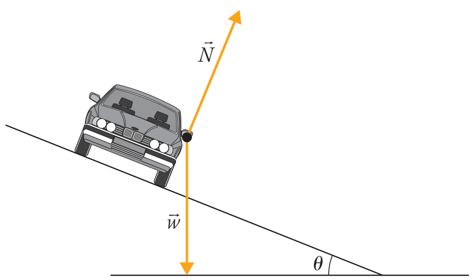
10 $T - mg = m\frac{v^2}{r}$

$$v = \sqrt{(T - mg) \times \frac{r}{m}}$$

$$v = \sqrt{(200\,\text{N} - 2.5\,\text{kg} \times 9.8\,\text{m s}^{-2}) \times \frac{0.75\,\text{m}}{2.5\,\text{kg}}}$$

$$v = 7.25\,\text{m s}^{-1}$$

11 **a**



b **i** $a = g\tan\theta$

$$a = 9.8\,\text{m s}^{-2} \times \tan 12°$$

$$a = 2.1\,\text{m s}^{-2}$$

ii $a = \frac{v^2}{r} = 2.1\,\text{m s}^{-2}$

$$v = \sqrt{2.1\,\text{m s}^{-2} \times 265\,\text{m}}$$

$$v = 23.5\,\text{m s}^{-1}$$

CHAPTER 5: GRAVITATIONAL FORCE AND FIELD

5.1 SECTION REVIEW

REMEMBERING

1 a Aristotle – introduced the term 'gravitas' and the concept of objects naturally falling toward Earth.

b Einstein – developed general relativity, extending our understanding of gravity.

c Galileo – utilised an empirical method to conduct experiments in gravity, ascertaining that falling objects accelerate uniformly.

d Newton – developed the law of universal gravitation that related force, mass and distance, obeying an inverse-square law.

2 Gravitas

3 Force gravity and light intensity.

UNDERSTANDING

4 Aristotle's method was philosophical and theoretical. Galileo's method of scientific exploration was empirical, relying on experimental observations.

5 Halley, Hooke and Newton each recognised the elliptical orbits of planets, as described by Kepler, linking their paths to the force applied by the Sun that Newton later termed gravity.

APPLYING

6 Newton's 'universal' law of gravitation applies to all masses throughout the universe.

ANALYSING

7 a Increases

b Decreases

REFLECTING

8 The development of the model of gravity has been refined over centuries with each contributing scientist building on the understandings of those before them with emerging discoveries made by new equipment, technologies or approaches.

Aristotle (384–322 BCE) 'Gravitas' provided a heaviness to pull objects straight down	Copernicus (1472–1543) Heliocentrism (sun-centred Universe)	Galileo (1564–1642) Empirical method based on experimental observation; uniform acceleration due to gravity
Brahe (1546–1601) Observational data	Kepler (1571–1630) Mathematical interpretations of data	Newton (1643–1727) Force gravity; Universal law of Gravitation

5.2 SECTION REVIEW

REMEMBERING

1 $\Delta E_{total} = 0, \Delta E_k = -\Delta E_p$

2 Gravitational potential energy is transformed into kinetic energy.

3 a Gravitational potential energy – the energy stored in a system due to its position within a gravitational field.

b Kinetic energy – the energy attributed to a moving object.

c Potential energy – energy stored in a system.

UNDERSTANDING

4 The object does not change its vertical position or its potential.

5 Falling objects accelerate due to the gravitational field. The change in kinetic energy is a result of the work done by the system.

APPLYING

6 5.88×10^4 J

7 a 0 J. Kinetic energy can be termed 'moving energy'.

b 4 m

8 a 1764 J

b $v = 7.67\ \text{m s}^{-1}$

9 a 5.00×10^5 J

b 3.92×10^5 J

c 8.92×10^5 J

10 4.41×10^6 J

ANALYSING

11 GPE = *mgh*. *g* for an asteroid is much lower than that of Earth.

12 a 108.16 m

b $12.6\ \text{m s}^{-1}$

c $v = 46.04\ \text{m s}^{-1}$

5.3 SECTION REVIEW

REMEMBERING

1 Force weight is measured in Newtons and is dependent upon gravitational acceleration. Mass is measured in kilograms and is the amount of matter in an object.

2 Mars → Mercury → Uranus → Venus → Saturn → Earth → Neptune → Jupiter

UNDERSTANDING

3 Weight is dependent on gravitational acceleration whereas mass is not.

APPLYING

4 11 760 N; 1956 N

5 F_w Mars = 295.2 N; F_w Venus = 711.2 N

6 1966.8 N

7 24 725.4 N; 27 753 N

8 1161 N

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ANALYSING

9 The radius of Earth is large, so large that heights of buildings, even mountains above the surface are insignificant.

10 Gravitational fields exist around all masses and gravitational attraction occurs regardless of contact.

5.4 SECTION REVIEW

REMEMBERING

1 $m\,s^{-2}$

2 $v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

UNDERSTANDING

3 v_y indicates the vertical component of velocity.

4 The acceleration due to gravity is the cause of acceleration near Earth's surface. $a_y = g$ as gravity acts vertically.

5 The vertical velocity at the maximum height is equal to $0\,m\,s^{-1}$.

APPLYING

6 $t = 6.12\,s$

7 $s = 15.76\,m$

8 $u = 10.38\,m\,s^{-1}$

9 $g = 1.36\,m\,s^{-2}$

ANALYSING

10 a $s = 45.92\,m$

b $t_f = 6.12\,s$

11

g ($m\,s^{-2}$)	ALTITUDE (km)
9.81	0.0
9.51	100
7.33	1000
1.49	10 000

REFLECTING

12 g is dependent not only on mass but also the radius.

13 An object falling to Earth will take the same time to reach the ground, regardless of whether it also has a horizontal velocity.

5.5 SECTION REVIEW

REMEMBERING

1 Gravitational field strength is determined using $g = \frac{Gm_1}{r^2}$ and is measured in $m\,s^{-2}$. Gravitational force is determined using $F = \frac{Gm_1m_2}{r^2}$, and is measured in N.

2 $g = \frac{Gm_1}{r^2}, F = \frac{Gm_1m_2}{r^2}$

3 Mass and radius

UNDERSTANDING

4 The mass of the planet (M) only is required to determine the field.

5 Newton's third law of equal and opposite reactions reflects the force of attraction acting similarly on both objects.

6 Both force gravitation and light intensity weaken with distance; twice the distance leads to one quarter the magnitude.

APPLYING

7 $\frac{1}{16}F$

8 2.160×10^{10} m from Earth

9 $a = 1.62\,m\,s^{-2}$

ANALYSING

10 Gravitational acceleration acts at a distance; however, it follows the inverse square law. When distances are large in magnitude then the resultant force approaches zero.

11 3.42×10^8 m from Earth

REFLECTING

12

Mass		Weight		Gravitational field	
The amount of matter in an object, measured in kg		The force applied to an object in a gravitational field, measured in N		$g = \frac{Gm}{R^2}$	
	Weightlessness		Normal force		
	The specific case when force of weight is apparently zero due to a lack of reaction or normal force, such as an object in orbit		The equal and opposite reaction force that opposes force weight and that acts perpendicular to a surface		Gravitation force
		Apparent weight			$F = \frac{GmM}{R^2}$
		The unbalanced resultant force due to the sum of force weight and addition forces, such as force upthrust			

13 As the distance in the formula is squared this variable has a larger effect than the mass.

14 The centre of mass is a point where it may be assumed that the total mass is concentrated. This is particularly useful for dealing with planetary motion.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a Gravitational field – the gravitational field strength around any mass is determined by the distance from the centre of mass and the mass itself. The force applied to any mass within the gravitational field is, in turn, determined by the strength of the gravitational field.

b Gravitational potential energy – the potential energy associated with the interaction of objects via the gravitational force. The potential energy is stored in the gravitational field.

c Newtonian constant of gravitation – $6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$

2 Gravitational field is determined by input of the values for the gravitational constant G, the mass of the larger body (planet) and the distance from the centre of mass into the formula $g = \frac{GM}{R^2}$. The gravitational field is measured in $\text{m}\,\text{s}^{-2}$. Gravitational force, in contrast, also requires the value of the mass of the object itself, and measures the force applied to the mass due to the gravitational field. The gravitational force is measured using $F = \frac{GmM}{R^2}$ and is measured in N. Force gravitational may be determined using Newton's second law: $F = mg$.

3 $a = g\sin\theta$, $a \propto \sin\theta$

CATEGORY QUESTIONS

4 g = acceleration due to gravity, $\frac{m}{s^2}$ or $9.80\,\text{m}\,\text{s}^{-2}$; G = universal gravitational constant, $6.67 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$

5 $F = \frac{Gm_1m_2}{r^2}$ represents the product of the two masses as well as the inverse square of the radius.

6 Cavendish used a torsion balance and large, known spherical masses to measure the force of attraction between the known large and smaller masses.

ELABORATION QUESTIONS

7 The force is reduced by a factor of 100.

8 3.06 s

9 a $g = 1.42\,\text{N}\,\text{kg}^{-1}$

b $g = 1.42\,\text{m}\,\text{s}^{-2}$

c 71 J

d 71 J

EVIDENCE QUESTIONS

10 Galileo measured the period of time for a sphere to roll down an inclined plane of known length, noting the period of time for a range of intervals. He was able to conclude that the time period required to travel the full length of the incline was double the time period for it to travel one quarter of the length, that is, $s \propto t^2$ and that an object released from rest accelerated uniformly.

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11 $F = \frac{G2mm}{(3R)^2}$ the force will change by a factor of $\frac{2}{9}$.

END-OF-CHAPTER EXAM

1 D

2 B

3 D

4 C

5 $N\,m^2\,kg^{-2}$

6 $W = Fs = mgh$

7 $12.51\,N\,kg^{-1}$

8 1764 J

9 Mass represents the amount of matter in a body, measured in kg. Weight is a force, dependent on both mass and acceleration due to gravity.

10 a $T = 1.099$ s

b $T = 2.696$ s

11 240.5 N

12 a 13.72 N

b 16.46 J

c 16.46 J

13 $Fg = \frac{Gm_1m_2}{R^2}$ force gravitation is dependent not only on the masses of objects but also varies inversely with distance squared. The Moon is significantly closer to Earth.

14 1.25×10^{28} N

15 $16.57\,m\,s^{-1}$

CHAPTER 6: ORBITAL MOTION

6.1 SECTION REVIEW

REMEMBERING

1 a–d Galileo and Copernicus were both early supporters of the heliocentric model of the solar system, placing the Sun at the centre of the motion of planets. Kepler performed calculations based on Brahe's work, developing his laws of planetary motion. Newton further refined our understandings, developing the universal law of gravitation.

2 Concentric – denoting circles that share the same centre.

UNDERSTANDING

3 Epicycles, or circles on circles, describe the apparent retrograde (backward) motion of planets as viewed using a geocentric model of the solar system.

APPLYING

4 The geocentric model centred on Earth with all planets and the Sun revolving around it. The heliocentric model placed the Sun at its centre.

ANALYSING

5 The scientific method is experimental in nature, consisting of measurement, observation, experimentation and the independent testing of hypotheses. Each new discovery has led to further testing of hypotheses and experimentation.

REFLECTING

6 Scientific knowledge changes with more refined data, improved models and new equipment for observation. The modification of the geocentric model to the heliocentric model as the prevailing model of the solar system is one such example.

6.2 SECTION REVIEW

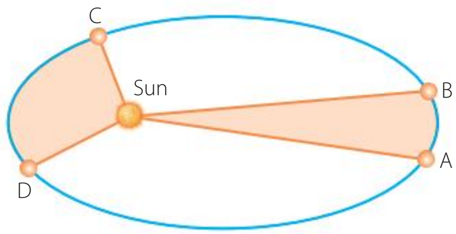
REMEMBERING

1 Kepler's first law (the law of ellipses): all planets move in elliptical orbits with the Sun at one focus.

Kepler's second law (the law of equal areas): a line that connects a planet to the Sun sweeps out equal areas in equal time intervals.

Kepler's third law (the law of periods): the square of the period of a planet's orbit is proportional to the cube of its mean orbital distance.

2



UNDERSTANDING

3 Kepler's law of periods is applicable to all planetary systems where objects orbit the same body. The unique $\frac{T^2}{R^3}$ value is the same constant value for all revolving bodies of the system.

4 Conservation of angular momentum.

APPLYING

5 Orbital radius Europa = 6.68×10^8 m

Orbital radius Ganymede = 1.06×10^9 m

Orbital radius Callisto = 1.87×10^9 m

6 The average $\frac{T^2}{R^3}$ for A and B = 1.935×10^{-19} and for exoplanet X = 1.841×10^{-19}. The $\frac{T^2}{R^3}$ ratios vary by less than 10%; therefore, exoplanet X may be considered part of this system.

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REFLECTING

7 Mean orbital distance is the average radius of orbit. Planets travel in elliptical paths; therefore, an average radius must be used.

8 Comets have highly eccentric, elliptical paths compared to planets with closer to circular paths around the Sun.

6.3 SECTION REVIEW

REMEMBERING

1 a $F_c = \frac{mv^2}{r}$

b $F_g = \frac{Gm_1m_2}{r^2}$

2 The astronomical unit is the mean Earth–Sun distance: 1.50×10^{11} m

3 $c = 3.00 \times 10^8\ \text{m s}^{-1}$

UNDERSTANDING

4 1.0 Mpc = 3.09×10^{22} m; 1.0 ly = 9.47×10^{15} m

The mega parsec is approximately 3 million times larger than the light year.

5 4.18×10^7 ly

6 Kepler's third law, the law of periods, and the $\frac{T^2}{R^3}$ constant may be used to determine whether a body may belong to a given system.

APPLYING

7 $T = 5424$ s or 90.4 min

8 $T = 9.26 \times 10^8$ s or 29.4 years

9 a 2.18×10^{20} km

b 7.05 Mpc

c 7.05×10^6 pc

ANALYSING

10 $T = 50.9$ days

11 $T = 13\,873$ s or 3.85 h

12 3.61×10^7 m or 36 100 km

REFLECTING

13 The AU is a larger scale and more convenient measure.

14 The period decreases as the radius decreases: $T^2 \propto R^3$

15 No, the $\frac{T^2}{R^3}$ constant may only be applied to systems orbiting the same mass.

6.4 SECTION REVIEW

REMEMBERING

1 $v_{\text{orbit}} = \sqrt{\frac{GM}{r}}$

2 $v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$

UNDERSTANDING

3 $v_{\text{orbit}}^2 \propto \frac{1}{\text{radius}}$; as radius increases v_{orbit} decreases.

4 $F_{\text{centripetal}} = F_{\text{gravitational}}$ for objects in orbit, hence they experience weightlessness.

APPLYING

5 $v_{\text{orbit}} = 6.78 \times 10^3\ \text{m s}^{-1}$

6 $F_g = 32\,334$ N

7 $T = 24\,791$ s or 6 h 53 min

ANALYSING

8 $v_{\text{escape}} = 4977\ \text{m s}^{-1}$

9 $F_g = 210.7\ \text{N} \Rightarrow g = 0.22\ \text{m s}^{-2}$

10 $R = 9.01 \times 10^6$ m or 9014 km

REFLECTING

11 LEO advantages – accessibility for repair and replacement.

LEO disadvantages – greater atmospheric friction and necessary propulsion.

12 $v_{\text{escape}} > v_{\text{orbit}}$ because the escape velocity must overcome the gravitational attraction of the body entirely.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 Kepler's first law (the law of ellipses): all planets move in elliptical orbits with the Sun at one focus.

Kepler's second law (the law of equal areas): a line that connects a planet to the Sun sweeps out equal areas in equal time periods.

Kepler's third law (the law of periods): the square of the period of a planet's orbit is proportional to the cube of its mean orbital distance. The unique $\frac{T^2}{R^3}$ value is the same constant value for all revolving bodies of the system.

2 $v_{\text{orbit}} = \sqrt{\frac{Gm}{r}}$; $v_{\text{escape}} = \sqrt{\frac{2Gm}{R}}$

The escape velocity is larger than the orbit velocity due to it requiring the craft to escape the gravitational pull of the body entirely.

3 1.0 AU = 1.50×10^{11} m; 1.0 Mpc = 3.09×10^{22} m; 1.0 ly = 9.47×10^{15} m

CATEGORY QUESTIONS

4 The gravitational force acting between masses provides the acceleration to keep satellites in orbit.

5 An increase in radius decreases the velocity of orbit in accordance with the orbit velocity formula.

6 Satellites placed in LEO are more accessible from Earth to aid in visit and repair.

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ELABORATION QUESTIONS

7 A lesser radius of orbit leads to a greater force of gravitational acceleration.

8 32 668 m s^{-1}

9 22 554 N

10 5.27 h

11 4126 m s^{-1}

EVIDENCE QUESTIONS

12 8.804×10^8 s

13 1.58 m s^{-2}

END-OF-CHAPTER EXAM

1 A

2 C

3 A

4 D

5 Altitude $= 9.56 \times 10^6$ m

6 $M_{Saturn} = 5.68 \times 10^{26}$ kg

7 $F_g = 1.98 \times 10^{20}$ N

8 $T = 5.95 \times 10^7$ s or 16 536 h or 688 days.

9 a Kepler's first law (the law of ellipses): all planets move in elliptical orbits with the Sun at one focus.

Kepler's second law (the law of equal areas): a line that connects a planet to the Sun sweeps out equal areas in equal time intervals.

Kepler's third law (the law of periods): the square of the period of a planet's orbit is proportional to the cube of its mean orbital distance.

b Empirical laws are derived from observation or experiment.

10 Heliocentric – Sun centred; Geocentric – Earth centred.

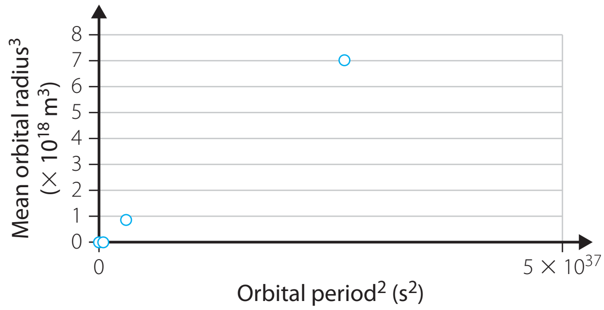
11 Orbital velocity: the specific velocity of an object for it to continue orbiting the mass at a given altitude.

Orbital acceleration: the acceleration due to gravity that keeps a satellite in orbit about a mass.

Escape velocity: the velocity required to escape the gravitational pull of a planetary body.

12 The $\frac{T^2}{R^3}$ constant is 2.64×10^{-19} s^2 m^{-3}. This can also be shown graphically.

PLANET	MEAN ORBITAL RADIUS (m)	ORBITAL PERIOD (s)	MEAN ORBITAL RADIUS^3 (m^3)	ORBITAL PERIOD^2 (s^2)
Earth	1.50×10^{11}	3.15×10^7	3.38×10^{33}	9.92×10^{14}
Jupiter	7.78×10^{11}	3.74×10^8	4.71×10^{35}	1.40×10^{14}
Saturn	1.43×10^{12}	9.29×10^8	2.92×10^{36}	8.63×10^{17}
Uranus	2.98×10^{12}	2.65×10^9	2.65×10^{37}	7.02×10^{18}



The $\frac{T^2}{R^3}$ values for each planet in the solar system range from 2.97×10^{-19} to 2.65×10^{-19}. These values are very close. The points on the graph of radius cubed versus period squared also describe a linear relationship, supporting Kepler's third law.

13 A larger radius of orbit leads to a smaller acceleration as they are inversely proportional, $a_c \propto \frac{1}{R}$.

14 $v_{orbit} = 9.43 \times 10^4$ m s^{-1}

15 The models of the solar system have been revised following improved precision, experimentation and observation. Simpler explanations, such as that of the heliocentric model, have replaced previous, more complicated models, such as the geocentric model that required epicycles to describe planetary motion.

CHAPTER 7: ELECTROSTATICS

7.1 SECTION REVIEW

REMEMBERING

1 Attractive force causes two unlike charges to move together whereas repulsive forces cause two like charges to move apart.

2 C – Coulombs

3 Coulomb's law: $F = \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{r^2}$

UNDERSTANDING

4 $k = \frac{1}{4\pi\varepsilon_0}$

$$\varepsilon_0 = \frac{1}{4\pi k}$$

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9}$$

$$\varepsilon_0 = 8.84 \times 10^{-12}$$

As expected.

APPLYING

5 $F = \frac{kqQ}{r^2}$

$F = \frac{9\times10^9 \times 8 \times -7}{2^2}$

$F = -1.26\times10^{11}$ N; therefore, the force is repulsive.

6 $F = \frac{kqQ}{r^2}$

$F = \frac{9\times10^9 \times 9\times10^{-6} \times 9\times10^{-6}}{(0.3)^2}$

$F = 8.1$ N

7 $F = \frac{kqQ}{r^2}$

$F = \frac{9\times10^9 \times 3.2\times10^{-19} \times -1.6\times10^{-19}}{\left(8\times10^{-11}\right)^2}$

$F = \frac{-4.608\times10^{-8}\text{ N}}{\left(8\times10^{-11}\right)^2}$

$F = -7.2\times10^{-8}$ N

8 $F = -7.2\times10^{-8}$ N in the opposite direction. The force of attraction is equal in magnitude and opposite in direction.

7.2 SECTION REVIEW

UNDERSTANDING

1 Force is inversely proportional to distance.

APPLYING

2 $F = \frac{kqQ}{r^2}$

$F = \frac{9\times10^9 \times -2\times10^{-3} \times 4\times10^{-3}}{(2.5)^2}$

$F = \frac{-72\,000}{6.25}$

$F = -1.15\times10^4$ N

3 $F = \frac{kqQ}{r^2}$

$r = \sqrt{\frac{kqQ}{F}}$

$r = \sqrt{\frac{9\times10^9 \times 7\times10^{-6} \times 12\times10^{-6}}{1.89}}$

$r = \sqrt{\frac{0.756}{1.89}}$

$r = 0.63$ m

4 $F = \frac{kqQ}{r^2}$

$F_{3r} = \frac{kqQ}{(3r)^2}$

$F_{3r} = \frac{1}{9}\times\frac{kqQ}{r^2}$

$F_{3r} = \frac{1}{9}F_r$

The force will decrease by a factor of 9.

5 $F_r = \frac{kqQ}{r^2}$

$F_{\frac{r}{4}} = \frac{kqQ}{\left(\frac{r}{4}\right)^2}$

$F_{\frac{r}{4}} = \frac{1}{\left(\frac{1}{4}\right)^2}\times\frac{kqQ}{r^2}$

$F_{\frac{r}{4}} = 16\times F_r$

The force will increase by a factor of 16.

6 The force will also halve.

7 $F_r = \frac{kqQ}{r^2}$

$F_{5r} = \frac{kqQ}{(5r)^2}$

$F_{5r} = \frac{1}{5^2}\times\frac{kqQ}{r^2}$

$F_{5r} = \frac{1}{25}F_r$

The force decreases by a factor of 25.

ANALYSING

8 The gradient becomes kqQ. Students should determine gradient to be approximately 22.5 N m^{-2} and obtain k of approximately 9×10^9 Nm^2 C^{-2}.

SYNTHESISING

9 With anomaly excluded, the gradient is approximately equal to 0.1749 N m^{-2}. The gradient is equal to $\frac{qQ}{4\pi\varepsilon_0}$. Students should obtain value for ε_0 as approximately 8.8×10^{-12}.

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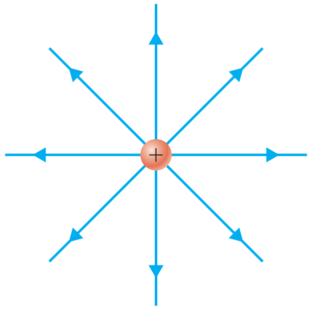
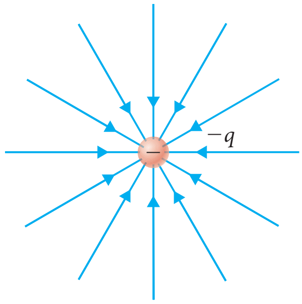
7.3 SECTION REVIEW

REMEMBERING

1





2 Electric potential is the potential energy per unit of charge, whereas electric potential energy is how much potential energy is stored in the electric field.

UNDERSTANDING

3 A change in potential energy is positive if a charge in the field is positive and moving against the field lines *or* if the charge is negative moving in the direction of the field lines. A change in potential energy is negative if a charge in the field is positive and is moving *with* the field lines *or* if the charge is negative moving against the field lines. A change in potential energy is positive if work is done on the field, and negative if it is done by the field.

4 A uniform electric field has field lines pointing in the same direction and is the same strength at every point, whereas a non-uniform electric field has field lines pointing in different directions.

APPLYING

5 $q = It$

$q = 1\,\text{C}$

$\Delta U = q\Delta V$

$\Delta U = 1 \times 12$

$\Delta U = 12\,\text{J}$

6 $E = \dfrac{kQ}{r^2}$

$E = \dfrac{9 \times 10^9 \times 1.6 \times 10^{-19}}{(0.02)^2}$

$E = 3.6 \times 10^{-6}\,\text{N}\,\text{C}^{-1}$

7.4 SECTION REVIEW

APPLYING

1 $E = \dfrac{kQ}{r^2}$

$E = \dfrac{9 \times 10^9 \times 25 \times 10^{-6}}{(0.2)^2}$

$E = 5.63 \times 10^6\,\text{N}\,\text{C}^{-1}$

2 $E = \dfrac{kQ}{r^2}$

$Q = \dfrac{Er^2}{k}$

$Q = \dfrac{1.0 \times 10^{-3} \times 0.12^2}{9 \times 10^9}$

$Q = 1.6 \times 10^{-15}\,\text{C}$

3 $E = \dfrac{\text{F}}{q}$

$E = \dfrac{100}{1.6 \times 10^{-19}}$

$E = 6.25 \times 10^{20}\,\text{N}\,\text{C}^{-1}$

4 $E = \dfrac{kQ}{r^2}$

$r = \sqrt{\dfrac{kQ}{E}}$

$r = \sqrt{\dfrac{9 \times 10^9 \times 4 \times 10^{-8}}{2.3 \times 10^6}}$

$r = \sqrt{\dfrac{360}{2.3 \times 10^6}}$

$r = 0.0125\,\text{m}$

ANALYSING

5 Consider the 10 μC charge on the left, and the 20 μC charge on the right. Work out the electric field due to each of the charges half way between them:

$E_{10C} = \dfrac{kQ}{r^2}$

$E_{10C} = \dfrac{9 \times 10^9 \times 10 \times 10^{-6}}{0.25^2}$

$E_{10C} = 1.44 \times 10^6$ N to the right

$E_{20C} = \dfrac{kQ}{r^2}$

$E_{20C} = \dfrac{9 \times 10^9 \times 20 \times 10^{-6}}{0.25^2}$

$E_{20C} = 2.88 \times 10^6$ N to the left

From vector addition, this means that the electric field strength half way between the charges is 1.44×10^6 N to the left.

7.5 SECTION REVIEW

APPLYING

1 $U = qV$

$V = \frac{U}{q}$

$V = \frac{155}{11}$

$V = 14.1\,\text{V}$

2 $E = \frac{kQ}{r^2}$

$E = \frac{9 \times 10^9 \times 5.0 \times 10^{-7}}{(0.02)^2}$

$E = 1.13 \times 10^7\ \text{V m}^{-1}$

$V = E \times L$

$V = 1.13 \times 10^7 \times 0.02$

$V = 2.25 \times 10^5\ \text{V}$

3 a $\Delta U = q\Delta V$

$\Delta U = 1.6 \times 10^{-19} \times 2$

$\Delta U = 3.9 \times 10^{-19}\ \text{J}$

b The change is positive

c The electric field

4 $a = \frac{Eq}{m}$

$a = \frac{1 \times 10^{-7} \times 1.6 \times 10^{-19}}{3.82 \times 10^{-26}}$

$a = 0.42\,\text{m s}^{-2}$

ANALYSING

5 For all of the following:

$\Delta U = q\Delta V$

$\frac{1}{2}mv^2 = q\Delta V$

$v = \sqrt{\frac{2q\Delta V}{m}}$

a $v = \sqrt{\frac{2q\Delta V}{m}}$

$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1000}{9.11 \times 10^{-31}}}$

$v = 1.8 \times 10^7\ \text{m s}^{-1}$

b $v = \sqrt{\frac{2q\Delta V}{m}}$

$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1000}{1.67 \times 10^{-27}}}$

$v = 4.38 \times 10^5\ \text{m s}^{-1}$

c $v = \sqrt{\frac{2q\Delta V}{m}}$

$v = \sqrt{\frac{2 \times 2 \times 1.6 \times 10^{-19} \times 1000}{4 \times 1.67 \times 10^{-27}}}$

$v = 3.10 \times 10^5\ \text{m s}^{-1}$

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a Attractive force: when two particles of unlike charge move towards each other

b Electric potential: the difference in potential between two points in an electric field. The work done per charge

c Electric potential energy: potential energy stored in an electric field. The change in potential energy of an object is also the work done on that object by the electric field

d Electric field: the field due to electric charge, which applies a force to electric charges

e Electric field lines: net lines of force pointing in the direction a positive test charge will move when placed in the electric field due to a charge Q.

f Electrostatic field model: the model that assigns an electric field to stationary charges; it is this field that exerts forces on other charges

g Potential difference: the difference in potential between two points in an electric field. The work done per charge

h Repulsive force: when two particles of like charge move away from each other

i Uniform electric field: electric field which has the same magnitude and direction at all points

j Zero of potential energy: when all charges in the system are infinitely far apart. Any other arrangement may have positive or negative potential energy

2 F, q, Q and r

CATEGORY QUESTIONS

3 Test charges in their own field have an electric field emanating radially from them if positive, or towards them if negative. When placed in an electric field, the direction of the external field will depend on the direction the positive or negative test charge moves respectively. In a circuit, electrons are negatively charged and moving towards the positive terminal in a circuit. These electrons are moving and creating current. Work is being done for these electrons to move.

4 A change in potential energy is positive if work is done on the field, and negative if it is done by the field. If work is done on the field, it means that the test charge is going the *opposite* way to what is expected.

5 Over a few hundred metres it is useful to assume Earth's electric field is uniform as it is approximately uniform. This way predictions can be made about how charges will move close to Earth's surface.

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ELABORATION QUESTIONS

6 The electric field interaction with unlike charges is what causes the attraction, as the field lines connect as they are going in the same direction. The electric field interaction with like charges causes repulsion as the field lines are going in opposite directions and do not connect.

7 Charges move in electric fields as they are attracted or repelled from a source. This attraction/repulsion is non-uniform and causes a test charge to accelerate. For example, if a negative test charge was put into an electric field with a large positive source, it would move against the field lines towards the source. As the distance decreases as the test charge moves towards the source, the force of attraction is now greater, and it moves towards the positive charge with a greater force. This process continues and causes acceleration.

8 This would mean that negative charges can only move against the field lines, and positive charges with the field lines. This means change in electric potential would always be negative. If this is true, then there is no other force can interact with the charge in the field. This skews our understanding of force, as an object will move based on the net force exerted on it.

EVIDENCE QUESTIONS

9 If the electric field is very large and negative on the inside of the cell, it is possible that the positive ions need to move against the field to get into the cell. This means that an external force (the pump) needs to help the ions achieve this. Additionally, if the negatively charged ions need to be transported out of the cell, work will also need to be done by them with the aid of the pump.

10 Usually Earth is considered negatively charged compared to the atmosphere, meaning that the electric field normally points towards Earth. When a storm cloud comes over, the storm cloud is filled with many free electrons, meaning that the electric field direction changes to point from Earth's surface towards the cloud. This causes electrons to move towards the bottom of the cloud, leaving the top of the cloud positive. These free electrons then have work done on them by the field and want to move towards Earth in order to neutralise. This discharge is called lightning. As the electrons are moving very fast by the time they reach Earth, they have a lot of energy which comes out in the form of light and heat, which is why people should take cover. Note: storm clouds can also discharge themselves with other storm clouds.

END-OF-CHAPTER EXAM

1 B

2 C

3 A

4 C

5 B

6 Increasing

7 $F = \frac{kqQ}{r^2}$

$$F = \frac{9\times10^9 \times 2\times10^{-6} \times -3\times10^{-6}}{(1)^2}$$

$F = -0.054\text{ N left}$

$$F = \frac{kqQ}{r^2}$$

$$F = \frac{9\times10^9 \times -1\times10^{-6} \times -3\times10^{-6}}{(1)^2}$$

$F = 0.027\text{ N left}$

$F_{net} = 0.054\text{ N} + 0.027\text{ N}$

$F_{net} = 0.081\text{ N left}$

8 $W = qV$

$W = 2.0\times10^{-14} \times 6.0\times10^2$

$W = 1.2\times10^{-11}\text{ J}$

9 $E = \frac{kQ}{r^2}$

$$E = \frac{9\times10^9 \times 1.6\times10^{-19}}{\left(3.8\times10^{-10}\right)^2}$$

$E = 9.97\times10^9\text{ N C}^{-1}$

10 Electric potential is the potential energy per unit of charge, whereas electric potential energy is how much potential energy is stored in the electric field.

11 Repulsive

12 They must be infinitely far apart.

13 Potential difference depends on the change in energy of a charge moving in a field. If it is a negative potential difference, it is gaining KE, but losing PE. If it has a positive potential difference, it is losing KE but gaining PE. This is due to the law of conservation of energy.

14 It means a charge is going against what the field 'wants' it to do.

15 The gradient becomes kQ. Students should determine the gradient to be approximately 22.5 N m^{-2} and calculate k to be approximately $9\times10^9\text{ N m}^2\text{ C}^{-2}$ when plotting F against the inverse of r squared.

CHAPTER 8: MAGNETIC FIELDS

8.1 SECTION REVIEW

REMEMBERING

1 a A magnetic material which has the majority of its magnetic domains aligned. Magnets have magnetic fields, which in turn affect other substances nearby.

b The field created by moving charges, including charges in magnetic materials.

UNDERSTANDING

2 By having all magnetic domains aligned. This causes a magnetic field to form, where the field lines are drawn out from the south pole and into the north pole.

3 Diamagnetic materials are weakly repelled by nearby magnets, paramagnetic materials are weakly attracted by nearby magnets, and ferromagnetic materials are strongly attracted by nearby magnets.

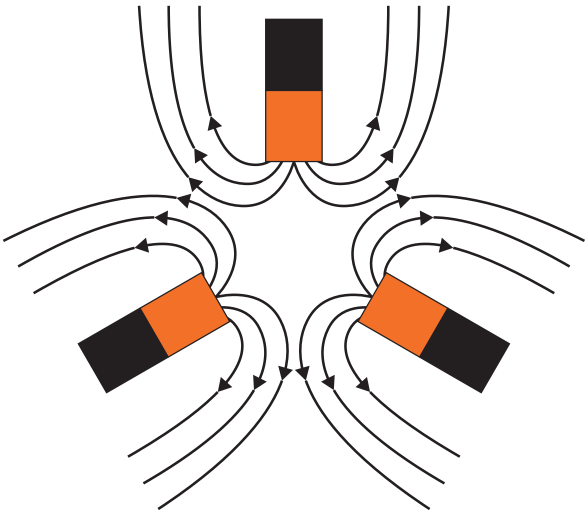
8.2 SECTION REVIEW

REMEMBERING

1 North

UNDERSTANDING

2



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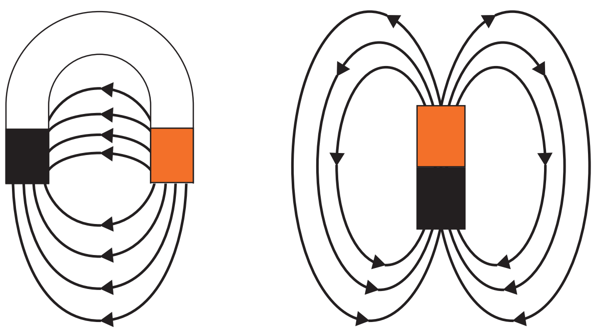


APPLYING

4 It depends which magnet has the stronger magnetic field. The magnet with the weaker magnetic field will move more towards the one with the stronger magnetic field.

ANALYSING

5 Ferromagnetic materials are attracted to magnets and can be turned into permanent magnets. Examples include iron, cobalt, nickel, gadolinium and terbium (students may have other examples). Antiferromagnetic materials occur only at particular temperatures and have no magnetic properties at all. Examples include some transition metals including chromium and nickel oxide.

8.3 SECTION REVIEW

REMEMBERING

1 $B = \frac{\mu_0 I}{2\pi r}$

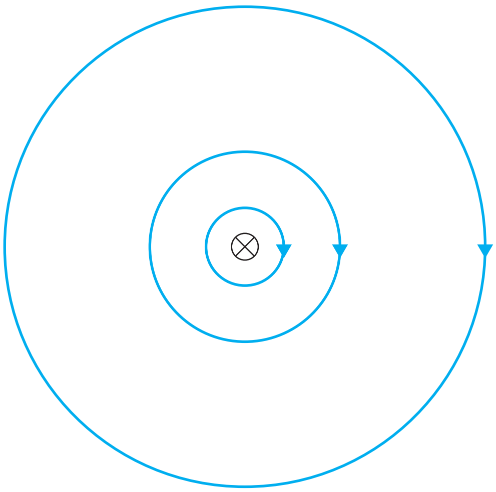
2 Magnetic field strength is inversely proportional to the distance away from a current-carrying wire.

UNDERSTANDING

3 Permanent magnet and a current-carrying wire

4 Point right thumb in the direction of the conventional current in the wire, the fingers will curl in the direction of the magnetic field about the wire.

5



APPLYING

6 $B = \frac{\mu_0 I}{2\pi r}$

$B = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.01}$

$B = 2 \times 10^{-5}$ T

7 $B = \frac{\mu_0 I}{2\pi r}$

$r = \frac{\mu_0 I}{2\pi B}$

$r = \frac{4\pi \times 10^{-7} \times 12}{2\pi \times 1 \times 10^{-3}}$

$r = 2.4 \times 10^{-3}$ m

8 $B = \frac{\mu_0 I}{2\pi r}$

$I = \frac{2\pi r B}{\mu_0}$

$I = \frac{2\pi \times 0.01 \times 50 \times 10^{-3}}{4\pi \times 10^{-7}}$

$I = 2.5 \times 10^3$ A

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9 $B = \frac{\mu_0 I}{2\pi r}$

$B = \frac{1}{r} \times \frac{4\pi \times 10^{-7} \times 10}{2\pi}$

$B = \frac{2 \times 10^{-6}}{r}$

$B_{0.2} = 1 \times 10^{-5}$ T

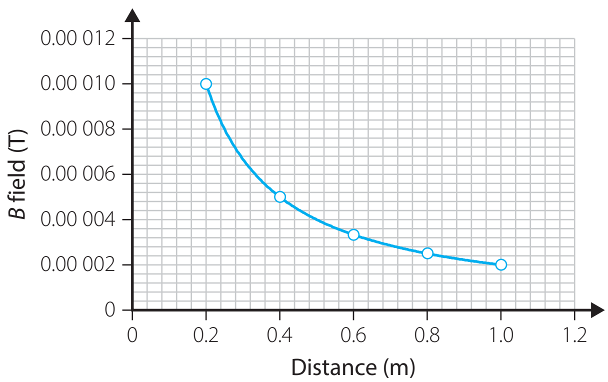
$B_{0.4} = 5 \times 10^{-6}$ T

$B_{0.6} = 3.3 \times 10^{-6}$ T

$B_{0.8} = 2.5 \times 10^{-6}$ T

$B_{1.0} = 2.0 \times 10^{-6}$ T

10



8.4 SECTION REVIEW

REMEMBERING

1 A magnet produced due to a current-carrying wire curled up.

UNDERSTANDING

2 With Maxwell's screw rule. Using your right hand, curl your fingers in the direction of the current running through the solenoid, and your thumb will point in the direction of the north pole.

APPLYING

3 $B = \mu_0 nI$

$B = 4\pi \times 10^{-7} \times 50 \times 80 \times 10^{-3}$

$B = 5.03 \times 10^{-6}$ T

4 $B = \mu_0 nI$

$I = \frac{B}{\mu_0 n}$

$I = \frac{60 \times 10^{-6}}{4\pi \times 10^{-7} \times 100}$

$I = 0.477$ A

ANALYSING

5 They can be altered in their magnetic strength based on the current and size. This is very useful for applications as the same solenoid can produce differing strength electromagnets.

8.5 SECTION REVIEW

UNDERSTANDING

1 a The magnetic field increases proportionally.
b The north pole and south pole switch ends.
c The magnetic field decreases proportionally.

APPLYING

2 $B = \frac{\mu_0 I}{2\pi r}$

$B = \frac{4\pi \times 10^{-7} \times 1.6}{2\pi \times 10 \times 10^{-3}}$

$B = 3.2 \times 10^{-5}$ T

3 $B = \frac{\mu_0 I}{2\pi r}$

$I = \frac{2\pi r B}{\mu_0}$

$I = \frac{2\pi \times 0.02 \times 0.15}{4\pi \times 10^{-7}}$

$I = 1.5 \times 10^{4}$ A

4 $B = \mu_0 nI$

$I = \frac{B}{\mu_0 n}$

$I = \frac{0.5}{4\pi \times 10^{-7} \times 125}$

$I = 3183$ A

5 $B = \frac{\mu_0 NI}{L}$

$L = \frac{\mu_0 NI}{B}$

$L = \frac{4\pi \times 10^{-7} \times 50 \times 0.34}{1.5 \times 10^{-5}}$

$L = 1.42$ m

8.6 SECTION REVIEW

REMEMBERING

1 $F = BIL \sin\theta$

2 The wire would be parallel to the B field if no force is exerted on it.

UNDERSTANDING

3 Downwards

APPLYING

4 $F = BIL \sin\theta$

$B = \frac{F}{IL \sin\theta}$

$B = \frac{0.02}{30 \times 5 \times \sin 30°}$

$B = \frac{0.02}{75}$

$B = 2.67 \times 10^{-4}$ T

5 $F = BIL \sin\theta$

$B = \dfrac{F}{IL \sin\theta}$

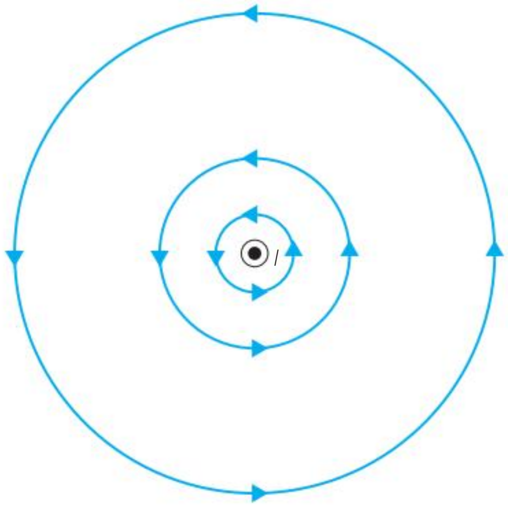
$B = \dfrac{5 \times 10^{-2}}{0.85 \times 2.1 \times \sin 90°}$

$B = \dfrac{5 \times 10^{-2}}{1.785}$

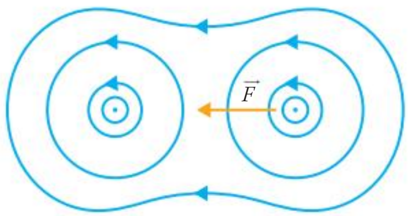
$B = 2.8 \times 10^{-2}$ T

ANALYSING

6 a



b



c The force would point to the right (towards the second wire).

d The forces would be repulsive, and would push away from each other (both force vectors would point outwards).

7 $F_B = BIL$

$F_B = 20 \times 10^{-6} \times 1000 \times 200$

$F_B = 4$ N

$F_G = mg$

$F_G = (750 \times 0.2) \times 9.81$

$F_G = 1470$ N

The force due to gravity is much greater than the force due to the B field acting on the wire.

8.7 SECTION REVIEW

REMEMBERING

1 They will experience a force in the opposite direction to each other. For example, if a positive charge experienced an upward force, a negative charge travelling on the same path would experience a downward force.

UNDERSTANDING

2 c

3 a Upwards

b Out of the page

c No force

APPLYING

4 $F = qvB$

$B = \dfrac{F}{qv}$

$B = \dfrac{1 \times 10^{-12}}{1.6 \times 10^{-19} \times 500\,000}$

$B = 12.5$ T

5 $F = qvB$

$F = 1.6 \times 10^{-19} \times 55B$

$F = 8.8 \times 10^{-18}\, B$ downwards

6 a $a = \dfrac{F}{m}$

$a = \dfrac{qvB}{m}$

$a = \dfrac{1.6 \times 10^{-19} \times 55\,000 \times 20 \times 10^{-6}}{9.11 \times 10^{-31}}$

$a = \dfrac{1.76 \times 10^{-19}}{9.11 \times 10^{-31}}$

$a = 1.93 \times 10^{11}$ m s^{-2}

b $a = 0$ m s^{-2}

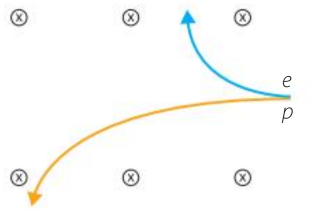
c $a = 1.93 \times 10^{11} \times \sin 45$

$a = 1.37 \times 10^{11}$ m s^{-2}

ANALYSING

7 If travelling in an electric field, the particle will end up moving with or against the field after it initially aligns. If travelling in a magnetic field, the particle will follow a helical path.

8



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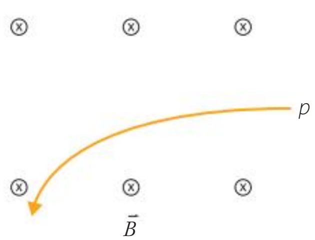

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CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a–l See glossary

2 The strength of the current producing it, how far away from the wire the field strength is being measured and the number of turns in the solenoid per metre.

CATEGORY QUESTIONS

3 The right-hand rule determines the direction of the force as follows: if your thumb points in the direction of the positive charge (or the current direction), fingers point in the direction of the external magnetic field, then the palm pushes in the direction that a positive charge will move. Maxwell's screw rule determines which direction the *B* field about a current-carrying conductor as follows: if you give a 'thumbs up/down' where your thumb points in the direction of current in a wire, then your fingers wrap in the direction of the *B* field about the wire.

4 When one wire is looped in a circle with a current flowing through it, the field lines loosely look like that of a bar magnet. When more and more of the same wire is looped in the same way (making a solenoid) the resulting *B* field when a current is passed through the wire is synonymous to a bar magnet, but with a strength that can be altered. This is an electromagnet.

5 The models are very similar and use the same arrows and notations. In the electric field model, field lines point from positive to negative. In the magnetic field model, the field lines point from north to south.

ELABORATION QUESTIONS

6 The *B* field caused by a moving charge (current) interacts with the external *B* field which causes a force on the particle.

7 Due to their intrinsic properties, typically to do with their electrons, will depend on whether they are good magnets or not.

8 With the right-hand rule.

EVIDENCE QUESTIONS

9 Uses for mass spectrometers include forensic analysis and medical uses. They work by separating ions of different mass/charge ratio to a collector grid by passing them through a magnetic field. This way, all ions in a compound can be analysed.

10 The strength of the fields is constantly increasing, causing the electrons to travel very fast as a large force is applied to them. The magnetic fields are also used to steer electrons in the synchrotron. The purpose of this is to understand how particles react at this speed and how their properties change to further understand our universe.

END-OF-CHAPTER EXAM

1 A

2 A

3 C

4 D

5 Out of the north pole.

6 Increase

7 $F = qvB\sin\theta$

$F = 1.6\times10^{-19}\times2.8\times10^{3}\times20\times10^{-6}\sin30°$

$F = 8.96\times10^{-21}\sin30°$

$F = 4.48\times10^{-21}\ \text{N}$

$a = \dfrac{F}{m}$

$a = \dfrac{4.48\times10^{-21}}{9.11\times10^{-31}}$

$a = 4.92\times10^{9}\ \text{m s}^{-2}$

8 $F = BIL$

$F = 0.2\times3.6\times0.03$

$F = 0.0216\ \text{N}$

9 When it enters the field parallel to the magnetic field.

10 They are aligning with the magnetic field as the iron filings have magnetic domains that are easily affected by strong external fields. Iron is ferromagnetic meaning that it is strongly affected by nearby magnets.

11 By running a current through a solenoid (a coil of wire).

12 No. Electricity causes magnetic fields and moving magnetic fields also induce electricity. This is due to the magnetic properties of electrons.

CHAPTER 9: ELECTROMAGNETIC INDUCTION

9.1 SECTION REVIEW

REMEMBERING

1 The units of magnetic flux are T m^2; also called Weber, Wb.

2 Electromagnetic induction is the process of changing a magnetic field such that an electric field (or emf) is created or induced. This can result in an induced current if there are free charge carriers present.

UNDERSTANDING

3 The magnetic flux is proportional to the number of field lines passing through a surface. When a loop of wire is orientated perpendicularly to a magnetic field, all of the magnetic field lines pass through the surface. When the surface is rotated, the field lines no longer all pass through the surface. When the surface is parallel, no field lines will pass through the surface.

4 A magnetic field is an indication of the magnetic force created by a moving charge or by a magnetic material. Magnetic flux is a measure of the amount of a magnetic field passing through a surface

APPLYING

5 Apply the magnetic flux formula:

$\Phi = BA\cos\theta$

Insert known values:

$\Phi = 0.05 \times \left(\pi \times 0.1^2\right) \times \cos 0°$

Calculate the answer:

$\Phi = 1.5708 \times 10^{-3}$ Wb

Give the answer the correct number of significant digits and units:

$\Phi = 1.6 \times 10^{-3}$ Wb

6 Apply the magnetic flux formula:

$\Phi = BA\cos\theta$

Insert known values:

$\Phi = 0.1 \times 0.25 \times \cos 35°$

Calculate the answer:

$\Phi = 0.02048$ Wb

Give the answer the correct number of significant digits and units:

$\Phi = 2.0 \times 10^{-2}$ Wb

7 Apply the magnetic flux formula:

$\Phi = BA\cos\theta$

$\cos\theta = 1$, since the field and surface are perpendicular:

$\Phi = BA$

$A = \pi r^2$, since the area is a circle:

$\Phi = B\left(\pi r^2\right)$

Rearrange for the unknown:

$r = \sqrt{\dfrac{\Phi}{B\pi}}$

Insert known values:

$r = \sqrt{\dfrac{5 \times 10^{-3}}{0.25 \times \pi}}$

Calculate the answer:

$r = 0.07979$ m

Give the answer the correct number of significant digits and units:

$r = 8.0$ cm

ANALYSING

8 a Apply the magnetic flux formula:

$\Phi = BA\cos\theta$

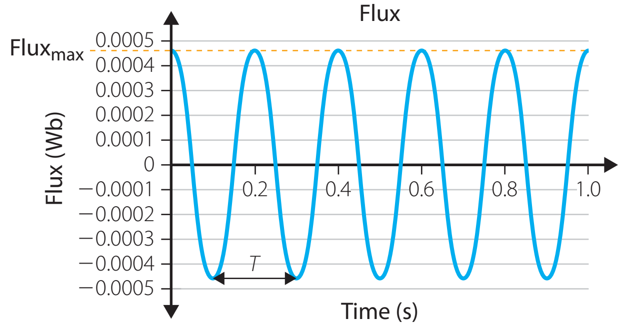
Insert known values:

$\Phi = 0.03 \times 0.015 \times \cos 0°$

Calculate the answer with the correct number of significant digits and units:

$\Phi = 4.5 \times 10^{-4}$ Wb

b



9.2 SECTION REVIEW

REMEMBERING

1 A change in magnetic flux occurs when the amount a magnetic field passing through a surface alters over a period of time

2 When the magnetic flux passing through a loop of wire changes with time, an induced emf is produced in the loop. The induced emf is equal to the negative of the change of magnetic flux divided by the change in time

UNDERSTANDING

3 Potential difference is the difference in potential energy per unit charge between two points in an electric field and is independent of the path between the points. The emf is the energy per unit charge available to a charged particle and is dependent upon the path between the points

4 Electromagnetic induction results in the formation of an induced emf which, if there are free charge carriers and a path for them to flow along, will result in the movement of these charges. This is known as an induced current

5 No current will be induced in the loop of wire because the current is proportional to the emf produced. This in turn is proportional to the rate of change of flux. The rate of change of flux is zero because the area of the loop, the magnetic field strength and the angle between the magnetic field and a normal to the surface of the coil are all constant. If the rate of change of flux is zero, then the emf produced must also be zero. Therefore, the induced current must also be zero.

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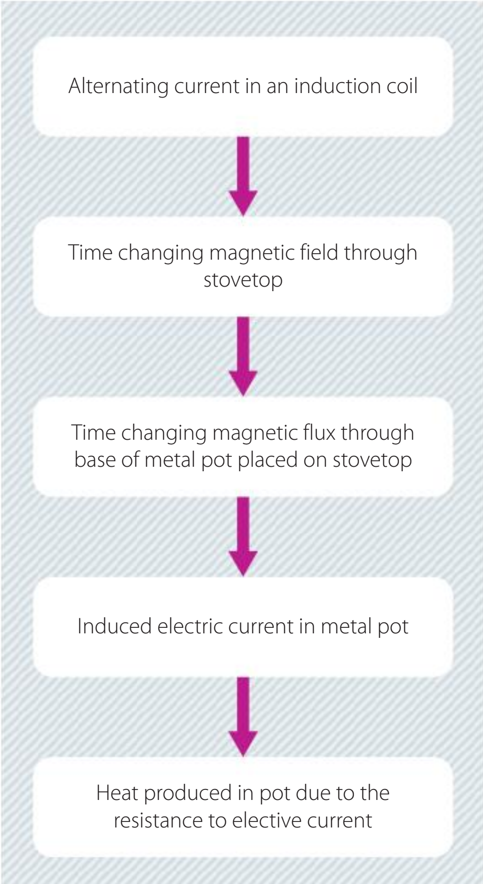
APPLYING

6



7 Aluminium is non-magnetic so it will not respond as well to induction as others. In addition, the current required to heat the contents of the pan would be significantly higher.

9.3 SECTION REVIEW

REMEMBERING

1 Magnetic field strength, the area of the surface and the angle between the normal to the surface and the magnetic field

2 Magnetic field strength: $\text{emf} = -A\cos\theta\frac{\Delta B}{\Delta t}$

The area of the surface: $\text{emf} = -B\cos\theta\frac{\Delta A}{\Delta t}$

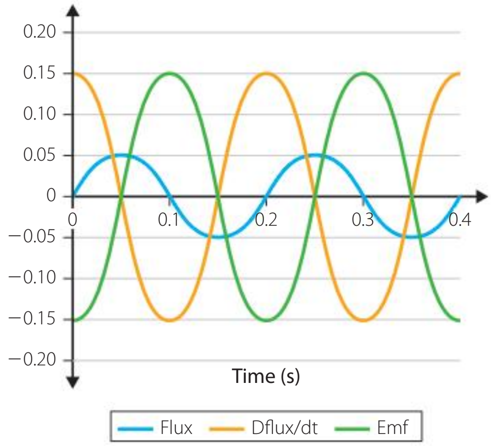
The angle between the normal to the surface and the magnetic field: $\text{emf} = -BA\frac{\Delta\cos\theta}{\Delta t}$

UNDERSTANDING

3 a The emf will be at a minimum when the gradient of the graph = 0, this occurs at $t = 0.05$ s, 0.15 s, 0.25 s and 0.35 s.

b The emf will be at a maximum at the point of maximum gradient, this occurs at $t = 0$ s, 0.1 s, 0.2 s, 0.3 s and 0.4 s.

c



d Answer included on graph in part c.

APPLYING

4 Apply Faraday's law:

$$\text{emf} = -\frac{\Delta\Phi}{\Delta t}$$

Bring the constant values outside of the delta operator:

$$\text{emf} = -A\cos\theta\frac{\Delta B}{\Delta t}$$

Insert known values:

$$\text{emf} = -0.45\,\text{m}^2 \times \cos 0 \times \frac{(0 - 0.1\,\text{T})}{0.2\,\text{s}}$$

Calculate the answer with correct number of significant digits and units:

emf = 0.23 V

5 Apply Faraday's law:

$$\text{emf} = -\frac{\Delta\Phi}{\Delta t}$$

Bring the constant values outside of the delta operator:

$$\text{emf} = -BA\frac{\Delta\cos\theta}{\Delta t}$$

Insert known values:

$$\text{emf} = -0.35\,\text{T} \times 0.25\,\text{m}^2 \times \frac{(\cos 90° - \cos 0°)}{0.5\,\text{s}}$$

Calculate the answer with correct number of significant digits and units:

emf = 0.18 V

6 Apply Faraday's law:

$$\text{emf} = -\frac{\Delta\Phi}{\Delta t}$$

Bring the constant values outside of the delta operator:

$$\text{emf} = -A\cos\theta\frac{\Delta B}{\Delta t}$$

Rearrange for the unknown value:

$$\frac{\Delta B}{\Delta t} = \frac{\text{emf}}{A\cos\theta}$$

Insert known values:

$$\frac{\Delta B}{\Delta t} = \frac{12\,\text{V}}{0.35\,\text{m}^2 \times \cos 0°}$$

Calculate the answer with correct number of significant digits and units:

$$\frac{\Delta B}{\Delta t} = -34\,\text{T}\,\text{s}^{-1}$$

9.4 SECTION REVIEW

REMEMBERING

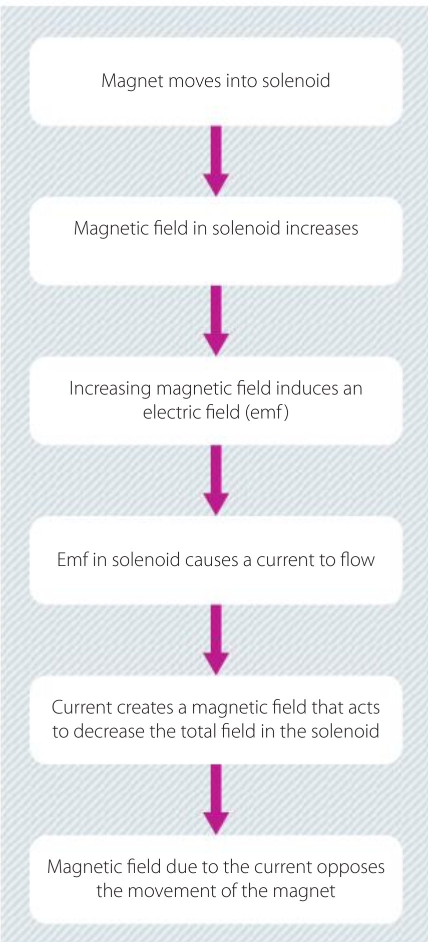
1 A current created by an induced emf from a changing flux flows in a direction that causes a magnetic flux change that opposes the original change in flux.

2 A circular current induced in a conductor due to a changing magnetic field.

UNDERSTANDING

3 If Lenz's law was altered so the direction of induced current flowed in the opposite direction, that is a direction which caused a magnetic flux change in the same direction of the original change in flux, the induced emf would compound and increase even more, resulting in ever increasing potential energy available to free charges. This would violate the principle of the conservation of energy.

4



APPLYING

5 a Clockwise

b No current produced (constant current indicates no changing flux).

c Anti-clockwise

d Clockwise

9.5 SECTION REVIEW

REMEMBERING

1 An electric generator converts kinetic energy (from the rotation of the armature) into electrical energy (through the induction of a current in the armature).

2 A current that varies with time between positive and negative values (directions).

3 A transformer is a device used to alter the voltage and current.

4 A step-up transformer increases voltage and decreases current. A step-down transformer decreases voltage and increases current.

UNDERSTANDING

5 A coil of wire rotating in a magnetic field experiences a changing flux due to the changing orientation between the magnetic field and the area inside the coil. According to Faraday's law, this induces an emf. Because the orientation between the field and the coil alternates, so will the emf.

6 A transformer requires an alternating current as it relies on a *changing* flux to pass through the coils. A direct current would create a constant flux through the coils.

APPLYING

7 a Convert the frequency to SI units:

$$f = 1200 \text{ rpm} \times \frac{1 \text{ min}}{60 \text{ s}} = 20 \text{ Hz}$$

b Apply the maximum emf equation:

$\text{emf}_{max} = 2\pi f n B A$

Insert known values:

$\text{emf}_{max} = 2\pi \times 20\,\text{Hz} \times 30 \times 0.5\,\text{T} \times 0.01\,\text{m}^2$

Calculate answer with correct number of significant digits and units:

$\text{emf}_{max} = 19\,\text{V}$

c Apply the root mean square equation:

$\text{emf}_{RMS} = 0.707 \times \text{emf}_{max}$

Insert known values:

$\text{emf}_{RMS} = 0.707 \times 19\,\text{V}$

Calculate answer with correct number of significant digits and units:

$\text{emf}_{RMS} = 13\,\text{V}$

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d Use the emf equation:

$\text{emf} = \text{emf}_{\text{max}} \cos(2\pi ft)$

Insert known values:

$\text{emf} = 19 \times \cos(2\pi \times 20t)$

Simplify the answer:

$\text{emf} = 19\cos(40\pi t)$

8 a Apply the maximum emf equation:

$\text{emf}_{\text{max}} = 2\pi fnBA$

Insert known values recalling $A = w \times l$:

$\text{emf}_{\text{max}} = 2\pi \times 35\,\text{Hz} \times 30 \times 1\,\text{T} \times \left(0.24 \times 40\ \text{m}^2\right)$

Calculate answer with correct number of significant digits and units:

$\text{emf}_{\text{max}} = 633\,\text{V}$

b Apply the root mean square equation:

$\text{emf}_{\text{RMS}} = 0.707 \times \text{emf}_{\text{max}}$

Insert known values

$\text{emf}_{\text{RMS}} = 0.707 \times 633\ \text{V}$

Calculate answer with correct number of significant digits and units

$\text{emf}_{\text{RMS}} = 448\ \text{V}$

c Apply Ohm's law:

$I_{\text{RMS}} = \dfrac{\text{emf}_{\text{RMS}}}{R}$

Insert known values:

$I_{\text{RMS}} = \dfrac{448\,\text{V}}{100\,\Omega}$

Calculate the answer to the correct number of significant figures:

$I_{\text{RMS}} = 4.49\,\text{A}$

d Apply the average power equation:

$P_{\text{AVG}} = \text{emf}_{\text{RMS}} \times I_{\text{RMS}}$

Insert known values:

$P_{\text{AVG}} = 448\ \text{V} \times 4.48\ \text{A}$

Calculate the answer to the correct number of significant figures:

$P_{\text{AVG}} = 2.01 \times 10^3\ \text{W}$

9 a Apply the transformer equation:

$\dfrac{V_S}{V_P} = \dfrac{n_S}{n_P}$

Rearrange for the unknown variable:

$V_S = V_P \dfrac{n_S}{n_P}$

Insert known variables:

$V_S = 120\,\text{V} \times 500$

Calculate the answer to the correct number of significant figures:

$V_S = 6.0 \times 10^3\ \text{V}$

b Apply the average power equation:

$P_{\text{AVG}} = V_{\text{RMS}} I_{\text{RMS}}$

Insert the known variables:

$P_{\text{AVG}} = 120\ \text{V} \times 8\ \text{A}$

Calculate the answer to the correct number of significant figures:

$P_{\text{AVG}} = 960\ \text{W}$

c Assuming the transformer is 100% efficient:

$P_{\text{OUT}} = P_{\text{IN}}$

Insert known values:

$P_{\text{OUT}} = 960\ \text{V}$

d Apply the average power equation:

$P_{\text{AVG,OUT}} = V_{\text{RMS,S}} \times I_{\text{RMS,S}}$

Rearrange for the unknown variable:

$I_{\text{RMS,S}} = \dfrac{P_{\text{AVG,OUT}}}{V_{\text{RMS,S}}}$

Insert known values:

$I_{\text{RMS,S}} = \dfrac{960\ \text{W}}{60\,000\ \text{V}}$

Calculate the answer to the correct number of significant figures:

$I_{\text{RMS,S}} = 0.016\ \text{A}$

ANALYSING

10 a Apply the emf_{max} equation:

$\text{emf}_{\text{max}} = 2\pi fnBA$

Insert known values, where $f = \dfrac{\text{cycles}}{s} = \dfrac{1.5}{0.1\ \text{s}} = 15\ \text{Hz}$, and $BA = \Phi_{\text{max}} = 0.08\ \text{Wb}$:

$\text{emf}_{\text{max}} = 2\pi \times 15\ \text{Hz} \times 30 \times 0.08\ \text{Wb}$

Calculate the answer to the correct number of significant figures:

$\text{emf}_{\text{max}} = 230\ \text{V}$

Apply the period equation:

$\text{emf}_{\text{min}} = 0\,\text{V}$

$T = \dfrac{1}{f}$

Insert known values:

$T = \dfrac{1}{15\ \text{Hz}}$

Calculate the answer to the correct number of significant figures:

$T = 0.067\ \text{s}$

b Use the emf equation:

$\text{emf} = \text{emf}_{max}\cos(2\pi ft)$

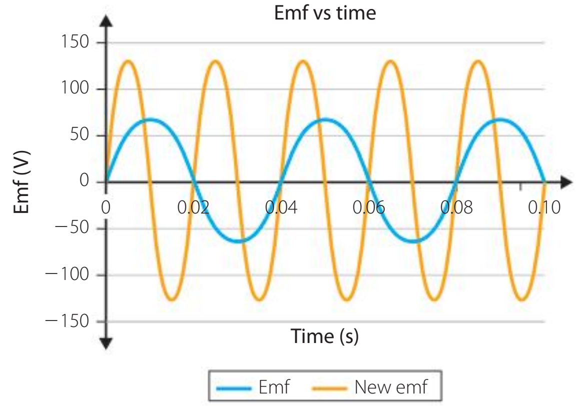
Insert known values:

$\text{emf} = 230\times\cos(2\pi\times 15t)$

Simplify the answer:

$\text{emf} = 230\cos(30\pi t)$

c



d Answer included on graph in part c.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a–k See glossary

2 Magnetic field is an indication of the strength of the magnetic force, magnetic flux is an indication of the amount of the magnetic field passing through an area and magnetic flux density is the magnetic field per unit area.

3 Potential difference is the difference in potential energy per unit charge between two points in an electric field and is independent of the path between the points, whereas the emf is the energy per unit charge available to a charged particle and is dependent upon the path between the points.

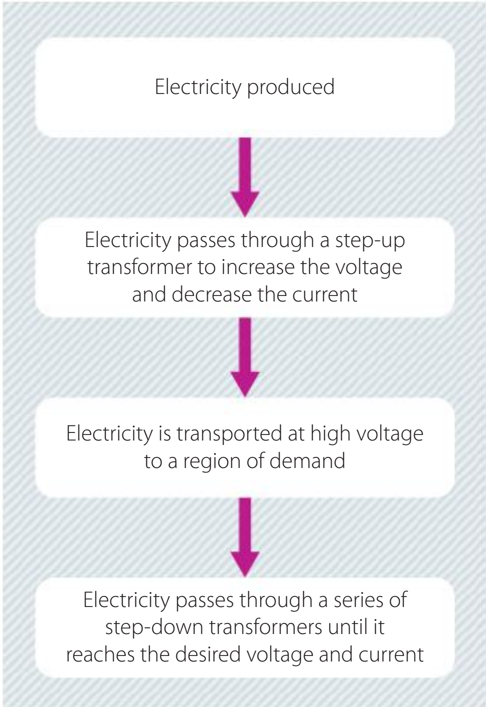
CATEGORY QUESTIONS

4 When the amount of a magnetic field passing through a coil (the magnetic flux) is changing, it results in the induction of an electromotive force inside of the surface in accordance with Faraday's Law. The emf causes the free electrons in the coil to move, resulting in a current.

5 The magnetic flux through a coil of wire may be changed by altering the magnetic field strength, changing the area of the coil of wire or by changing the orientation of the coil relative to the magnetic field.

6 Lenz's law states that the current induced by a changing magnetic flux will flow in a direction that produces a magnetic flux that changes in a direction that opposes the original magnetic flux.

7



8 The amount of power lost during transmission increases with current. In order to decrease the current it passes through a step-up transformer, which increases the voltage but decreases the current.

ELABORATION QUESTIONS

9 If Lenz's law was altered so the direction of induced current flowed in the opposite direction, that is a direction which caused a magnetic flux change in the same direction of the original change in flux, the induced emf would compound and increase even more, resulting in an ever increasing potential energy available to free charges. This would violate the principle of the conservation of energy.

10 Answers may include: burning of coal, gas or bio fuel; wind; nuclear reactions; waves; falling water; or geothermal activity.

EVIDENCE QUESTIONS

11 Answers may vary but should include mention of induction braking.

12 Answers may vary but should include mention of ease of changing voltage and current.

END-OF-CHAPTER EXAM

1 C

2 A

3 C

4 D

5 Alternating current (AC)

6 Maximum

7 Eddy current

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8 An induced emf acts to produce an induced current. The induced current is in the direction that causes a magnetic flux change that opposes the change in flux, which induced the emf.

9 Apply the magnetic flux formula:

$\Phi = BA\cos\theta$

Insert known values:

$\Phi = 0.05\ \text{T} \times 0.025\ \text{m}^2 \times \cos 0°$

Calculate the answer with the correct number of significant digits and units:

$\Phi = 1.3 \times 10^{-3}\ \text{Wb}$

10 Apply Faraday's law:

$$\text{emf} = -\frac{\Delta\Phi}{\Delta t}$$

Bring the constant values outside of the delta operator:

$$\text{emf} = -A\cos\theta\frac{\Delta B}{\Delta t}$$

Insert known values

$$\text{emf} = -0.025\ \text{m}^2 \times \cos 0° \times \frac{(0 - 0.05\ \text{T})}{0.05\ \text{s}}$$

Calculate the answer with correct number of significant digits and units

$\text{emf} = 0.025\ \text{V}$

11 If Lenz's law was altered so the direction of induced current flowed in the opposite direction, that is a direction which caused a magnetic flux change in the same direction of the original change in flux, the induced emf would compound and increase even more, resulting in an ever increasing potential energy available to free charges. This would violate the principle of the conservation of energy.

12 Apply Faraday's law:

$$\text{emf} = -\frac{\Delta\Phi}{\Delta t}$$

Bring the constant values outside of the delta operator:

$$\text{emf} = -nBA\frac{\Delta\cos\theta}{\Delta t}$$

Insert known values:

$$\text{emf} = -250 \times 2\ \text{T} \times (0.12\ \text{m})^2 \times \frac{\cos 65° - \cos 0°}{0.2\ \text{s}}$$

Calculate the answer with correct number of significant digits and units:

$\text{emf} = 21\ \text{V}$

13 Apply Faraday's law:

$$\text{emf} = -\frac{\Delta\Phi}{\Delta t}$$

Bring the constant values outside of the delta operator:

$$\text{emf} = -B\cos\theta\frac{\Delta A}{\Delta t}$$

Insert known values:

$$\text{emf} = -2\ \text{T} \times \cos 0 \times \frac{\left(0.0025\,\text{m}^2 - 0.001\ \text{m}^2\right)}{2\,\text{s}}$$

Calculate the answer with correct units, sign (magnitude) and number of significant digits:

$\text{emf} = 1.5 \times 10^{-3}\ \text{V}$

14 a Apply the frequency equation:

$$f = \frac{1}{T}$$

Insert the period as taken from the graph:

$$f = \frac{1}{0.02\ \text{s}}$$

Calculate the answer with correct number of significant digits and units:

$f = 50\ \text{Hz}$

b Apply the emf_{max} equation:

$\text{emf}_{max} = 2\pi fnBA$

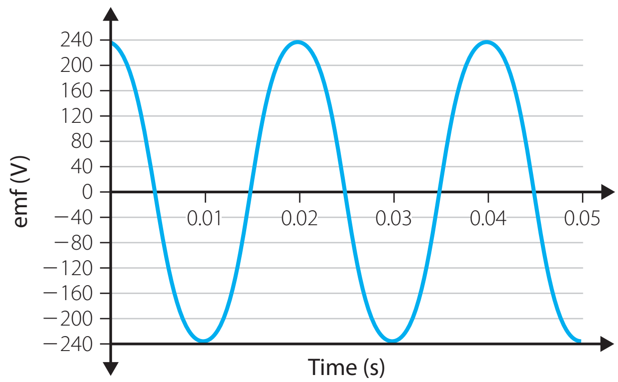
Insert known values, where $BA = \Phi_{max} = 0.00764\ \text{Wb}$:

$\text{emf}_{max} = 2\pi \times 50\ \text{Hz} \times 100 \times 0.00764\ \text{Wb}$

Calculate the answer to the correct number of significant figures:

$\text{emf}_{max} = 240\ \text{V}$

c



15 a Apply the emf_{max} equation:

$\text{emf}_{max} = 2\pi fnBA$

Rearrange for the unknown variable:

$$n = \frac{\text{emf}_{max}}{2\pi fBA}$$

Insert known values:

$$n = \frac{150\ \text{V}}{2\pi \times 50\ \text{Hz} \times 0.15\ \text{T} \times 2.5 \times 10^{-2}\ \text{m}^2}$$

Calculate the answer to the correct number of significant figures

$n = 130$

b Apply the emf_{RMS} equation:

$$\text{emf}_{RMS} = \frac{\text{emf}_{max}}{\sqrt{2}}$$

Insert known values:

$$\text{emf}_{RMS} = \frac{150}{\sqrt{2}}$$

Calculate the answer to the correct number of significant figures:

$\text{emf}_{RMS} = 110\ \text{V}$

c Apply Ohm's law:

$$I_{RMS} = \frac{\text{emf}_{RMS}}{R}$$

Insert known values:

$$I_{RMS} = \frac{110\ \text{V}}{55\ \Omega}$$

Calculate the answer to the correct number of significant figures:

$I_{RMS} = 2.0\ \text{A}$

d Apply the average power equation:

$P_{AVG} = \text{emf}_{RMS} \times I_{RMS}$

Insert known values:

$P_{AVG} = 110\ \text{V} \times 2.0\ \text{A}$

Calculate the answer to the correct number of significant figures:

$P_{AVG} = 220\ \text{W}$

16 a Apply the transformer equation:

$$\frac{V_S}{V_P} = \frac{n_S}{n_P}$$

Rearrange for the unknown variable:

$$V_S = V_P \frac{n_S}{n_P}$$

Insert known variables:

$V_S = 240\ \text{V} \times 700$

Calculate the answer to the correct number of significant figures:

$V_S = 1.7 \times 10^5\ \text{V}$

b Apply the average power equation:

$P_{AVG} = V_{RMS} \times I_{RMS}$

Insert the known variables:

$P_{AVG} = 240\ \text{V} \times 10\ \text{A}$

Calculate the answer to the correct number of significant figures:

$P_{AVG} = 2400\ \text{W}$

c Assuming the transformer is 100% efficient:

$P_{OUT} = P_{IN}$

Insert known values:

$P_{OUT} = 2400\ \text{V}$

d Apply the average power equation:

$P_{AVG,OUT} = V_{RMS,S} \times I_{RMS,S}$

Rearrange for the unknown variable:

$$I_{RMS,S} = \frac{P_{AVG,OUT}}{V_{RMS,S}}$$

Insert known values:

$$I_{RMS,S} = \frac{2400\ \text{W}}{1.7 \times 10^5\ \text{V}}$$

Calculate the answer to the correct number of significant figures:

$I_{RMS,S} = 0.014\ \text{A}$

CHAPTER 10: ELECTROMAGNETIC RADIATION

10.1 SECTION REVIEW

REMEMBERING

1 James Clerk Maxwell

2 Approximately 400–700 nm

UNDERSTANDING

3 The receiving antenna should be orientated the same way as the transmitting antenna. This ensures that the electric field is a maximum along the direction of the antenna, so it can accelerate the electrons in the antenna most effectively. As the transmitting antenna in Figure 10.1.4 is vertical, a car radio antenna should also be vertical to receive a signal from this radio station.

4 **a** It is the magnitudes of the electric and magnetic fields that oscillate.

b Energy and momentum are transported by the fields.

5 The radio wave applies an electric field; and therefore, a force to the electrons in the antenna. The force has the same frequency as the field. As the electrons are free to move, they are accelerated by the field and an AC current of the same frequency as the radio wave is produced. This signal is translated into sound.

APPLYING

6 **a** Apply the wave equation:

$$f = \frac{c}{\lambda}$$

Substitute in known values:

$$f = \frac{3.0 \times 10^8\ \text{m s}^{-1}}{633 \times 10^{-9}\ \text{m}}$$

Calculate the answer with the correct number of significant figures and units

$f = 3.0 \times 10^{14}\ \text{Hz}$

b Apply the distance equation:

$d = vt$

Rearrange for the required variable:

$$t = \frac{d}{v}$$

Insert known values, recalling that the Earth-Moon distance is 384 400 km and it is a round trip:

$$t = \frac{768\,800\,000\ \text{m}}{3.00 \times 10^8\ \text{m s}^{-1}}$$

Calculate the answer with the correct number of significant figures and units:

$t = 2.6\,\text{s}$

ANALYSING

7 You have seen that waves reflect such that the angle of incidence is equal to the angle of reflection. Waves refract and change direction and wavelength when they enter a new medium. Waves interfere and produce interference patterns with maxima and minima. Any experiment, such as

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observing interference patterns, creating standing waves, measuring angles of refraction, etc can be used to show that light is a wave.

8 a Use the speed of light equation:

$$c = f\lambda$$

Rearrange for the required variable:

$$\lambda = \frac{c}{f}$$

Insert known values:

$$\lambda = \frac{3.0 \times 10^8 \text{ m s}^{-1}}{5.45 \times 10^{14} \text{ Hz}}$$

Calculate the answer with the correct number of significant figures and units:

$$\lambda = 550\,\text{nm}$$

b This is a yellow colour and approximately the peak wavelength radiated by our sun. It makes sense that human eyes have evolved to be most sensitive to the light most available in our environment.

9 Student answers will vary. They should have a lengthy list, probably including light from microwave ovens, TVs and mobile phones. In winter, they may also include heaters.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a–f See glossary

2 $c = 3.0 \times 10^8\,\text{m s}^{-1}$

CATEGORY QUESTIONS

3 Maxwell's last two equations showed that a time-changing electric field would produce a time-changing magnetic field which would in turn produce a time-changing electric field and so on. This showed that electromagnetic waves were self-sustaining waves that did not require a medium through which to travel.

4 Electromagnetic waves are coupled transverse waves that consist of coupled electric and magnetic oscillations that do not need a medium through which to travel.

5 Answers may vary but should include two of the following: radio waves, microwaves, infrared radiation, ultraviolet radiation, X-rays or gamma rays.

6 The electrical permittivity and the magnetic permeability of the material.

7 a An oscillating current is passed through an antenna and the movement of the electrons produces oscillating electric and magnetic fields.

b The oscillating electric and magnetic fields of an incoming electromagnetic wave cause the electrons in the antenna to move; thereby producing an electric current.

ELABORATION QUESTIONS

8 Maxwell's equations showed that electromagnetic waves did not require a medium through which to travel.

9 Answers may vary but should include: the wave properties of light, the speed of light and the medium through which they travelled.

10 The orientation is important, so the incoming electromagnetic wave can accelerate the electrons in the antenna most effectively. The receiving antenna should be orientated the same way as the transmitting antenna. This ensures that the electric field is a maximum along the direction of the antenna.

11 The electrons in direct current do not accelerate which is the requirement for the production of electromagnetic waves. Information can be encoded in the wave by altering the amplitude (AM – amplitude modulation) or by altering the frequency (FM – frequency modulation) of the current in the antenna.

EVIDENCE QUESTIONS

12 Answers may vary but may include: quantum mechanics, Einstein's relativity, Bose-Einstein condensates, etc.

13 Ideally, radio waves will carry as much information as possible in order to ensure the integrity of the sound played on the radio. FM waves are the most appropriate for this purpose. However, the shorter wavelength of FM waves (1–10 m) means they are more likely to interfere with everyday objects of a similar size, such as houses and trees; and therefore, be unsuitable for long distance broadcast. AM waves, although being less useful to encode information, have a much longer wavelength (300 m – 1 km) and as such are less likely to encounter objects of a similar size that would disrupt their travel.

END-OF-CHAPTER EXAM

1 C

2 A

3 C

4 D

5 c – the speed of light

6 Faster

7 An accelerating charge

8 That light was a self-sustaining transverse wave consisting in oscillations in the electric and magnetic fields.

9 Apply the electromagnetic wave velocity equation:

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

Insert known values:

$$c = \frac{1}{\sqrt{8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1}\text{m}^{-2} \times 1.257 \times 10^{-6} \text{ N s}^2 \text{ C}^{-2}}}$$

Calculate the answer with the correct number of significant figures and units:

$$c = 3.00 \times 10^8 \text{ m s}^{-1}$$

10 Apply the electromagnetic wave velocity equation:

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

Rearrange for the required variable:

$$\mu = \frac{1}{\varepsilon \times c^2}$$

Insert known values:

$$\mu = \frac{1}{7.86 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\text{m}^{-2} \times \left(3.0 \times 10^8\right)^2}$$

Calculate the answer with the correct number of significant figures and units:

$$\mu = 1.41 \times 10^{-6}\ \text{N s}^2\ \text{C}^{-2}$$

11 a Microwave

b Use the speed of light equation:

$$c = f\lambda$$

Rearrange for the required variable:

$$\lambda = \frac{c}{f}$$

Insert known values:

$$\lambda = \frac{3.0 \times 10^8\ \text{m s}^{-1}}{1800 \times 10^6\ \text{Hz}}$$

Calculate the answer with the correct number of significant figures and units:

$$\lambda = 17\ \text{cm}$$

c Apply the distance equation:

$$d = vt$$

Rearrange for the required variable:

$$t = \frac{d}{v}$$

Insert known values, recalling that the Earth-Moon distance is 384 400 km and it is a round trip:

$$t = \frac{3\,300\,000\ \text{m}}{3.00 \times 10^3\ \text{m s}^{-1}}$$

Calculate the answer with the correct number of significant figures and units

$$t = 11\ \text{ms}$$

d The electromagnetic wave needs to pass through the circuitry of the phone, the transmitting and receiving antenna and possibly a satellite too.

12 The required sound is converted to an electric current through the process of electromagnetic induction inside of a microphone or audio playing device. This electric current is encoded within the oscillations of a carrier current. This modulated wave is then passed through a transmitting antenna that causes the electrons to oscillate. These accelerating charges produce the radio wave that travels to the antenna of the car. The oscillating electric field of the radio waves cause the electrons in the car's antenna to oscillate; and therefore, produce an electric current. This current is decoded by the radio into sound waves by the speakers in the car.

CHAPTER 11: SPECIAL RELATIVITY

11.1 SECTION REVIEW

REMEMBERING

1 When time and space act differently for one object compared to others.

2 Frisch and Smith

UNDERSTANDING

3 To describe the physics of the very small, where classical mechanics fails to do so.

4 Muons should decay much higher in the atmosphere than they actually do. As we can observe them travelling much further, and can detect them close to Earth's surface, this means the muon lifetime is not described by classical mechanics, but rather has undergone relativistic effects.

11.2 SECTION REVIEW

REMEMBERING

1 One in which Newton's first law applies to a very good approximation, and there is no acceleration. Any departures from the law are negligible. Also known as an inertial reference frame.

2 $x = x' + v\Delta t$

$y = y'$

UNDERSTANDING

3 It depends on the reference frame of the observer. To an observer travelling on the ship, the ball will fall in a straight line. An observer on land will observe a parabola.

4 A frame of reference is where the observer is according to the event. An inertial frame of reference is not accelerating.

APPLYING

5 a Position of the bus according to the observer after 5 seconds: 60m

b Position of the observer relative to the bus after 6 seconds: −72 m.

5 a (5,160) b (5,10)

11.3 SECTION REVIEW

REMEMBERING

1 *First postulate of special relativity*: the laws of physics are the same in all inertial frames of reference – the principle of special relativity.

Second postulate of special relativity: the speed of light has the same value, *c*, in all inertial frames. It does not depend on the speed of either the source or the observer.

UNDERSTANDING

2 That the speed of light is constant in all reference frames. This differed from Galileo and Newton as they suggested that it would travel faster or slower depending on the frame of reference of the observer.

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3 It does not. The speed of light is constant in all inertial reference frames.

11.4 SECTION REVIEW

REMEMBERING

1 $v_{AB} = v_{AC} + v_{CB}$

UNDERSTANDING

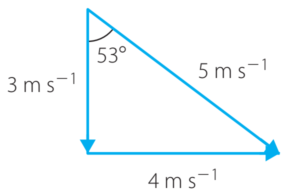
2 The velocity of the train depends on the reference frame of the observer. If an observer is on the train, the train will have a relative velocity of $0\,m\,s^{-1}$. If an observer is on the platform and the train passes, the train will have a relative velocity to the observer of v.

APPLYING

3 a $12\,m\,s^{-1}$

b $2\,m\,s^{-1}$

4



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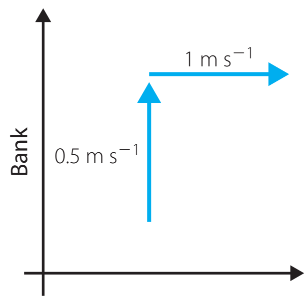

b $v = 1.12\,m^{-1}$ relative to the bank at an angle of 63.43°.

SYTHESISING

6 In a police car travelling head on to oncoming traffic, the relative velocity of all the traffic would be the addition of the police car's velocity and that of the traffic. In order to rectify this, the speed camera would need to take into account the speed of the police car at the time of the reading. This differs from the stationary police officer who is not moving relative to the traffic, and therefore does not need to amend the instrument when assessing the speed of the cars.

11.5 SECTION REVIEW

REMEMBERING

1 When two events occur simultaneously in one reference frame they *cannot* occur simultaneously in another reference frame, if the other reference frame is moving relative to the first reference frame.

UNDERSTANDING

2 Simultaneity occurs because the measurement of time in the two different reference frames are not agreed upon, hence the timing will not be agreed upon either.

3 No. On Earth, trains travel much slower than the speed of light, and the relativistic effects would not be observed.

APPLYING

4 The pine tree. The light travels towards the cat and the cat is running away from the light at the gum tree. As the cat is travelling close to the speed of light, this is what it will observe.

11.6 SECTION REVIEW

REMEMBERING

1 According to an outside observer the time interval will increase (because they do not measure the proper time).

2 According to the outside observer the length will be shorter.

3 The time interval between two events occurring at the same place in an inertial reference frame, as measured by an observer in that inertial frame.

4 Length measured in an inertial frame of reference in which the object is stationary.

UNDERSTANDING

5 Day to day the effects of time dilation and length contraction are not observed as objects move nowhere near fast enough to observe these effects.

11.7 SECTION REVIEW

REMEMBERING

1 $t = \dfrac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$

2 $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$p_v = \dfrac{p_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

UNDERSTANDING

3 They are directly proportional to each other.

APPLYING

4 $L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$

$L = 50 \times \sqrt{1-\dfrac{(0.6c)^2}{c^2}}$

$L = 50 \times \sqrt{0.64}$

$L = 40\,m$

5 No it does not, as the observer on the train measures the proper length, which is longer than the distance between P and Q. The events are viewed differently according to the frame of reference.

6 $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m = \frac{1.673\times 10^{-27}}{\sqrt{1-\frac{(0.65c)^2}{c^2}}}$$

$$m = \frac{1.673\times 10^{-27}}{\sqrt{0.5775}}$$

$$m = 2.20\times 10^{-27}\text{ kg}$$

7 a $$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 4.4\sqrt{1-\frac{(0.4c)^2}{c^2}}$$

$$L = 4.4\times\sqrt{0.84}$$

$$L = 4\text{ ly}$$

b $$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{0.4\times 4.4}{\sqrt{0.84}}$$

$t = 1.92$ years

time difference $= 1.92 - 1.76$

$= 0.16$ years

8 $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1-\frac{(0.75c)^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{0.4375}}$$

$$m = 1.51m_0$$

9 $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$2.5m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2.5}$$

$$\frac{v^2}{c^2} = 0.84$$

$$v = 0.92c$$

10 $$p = \frac{p_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$p = \frac{p_0}{\sqrt{1-\frac{(0.1c)^2}{c^2}}}$$

$$p = \frac{p_0}{\sqrt{0.99}}$$

$$p = 1.005\,p_0$$

ANALYSING

11 $$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$$

$$t_0 = 26\text{ ns}\sqrt{1-\frac{(0.75c)^2}{c^2}}$$

$$t_0 = 26\text{ ns}\times\sqrt{0.4375}$$

$$t_0 = 17.20\text{ ns}$$

11.8 SECTION REVIEW

UNDERSTANDING

1 No

2 Detectors were placed in the upper and lower atmosphere to detect the number of muons that did not decay within their mean lifetime. As many more were detected than expected, it suggested that something more was at play than simple classic mechanics.

APPLYING

3 $$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_0 = \frac{3\text{ km}}{\sqrt{1-\frac{(0.996c)^2}{c^2}}}$$

$$L_0 = \frac{3\text{ km}}{\sqrt{0.007\,984}}$$

$$L_0 = 33.57\text{ km}$$

4 $$t = \frac{2.2\,\mu\text{s}}{\sqrt{1-\frac{(0.966c)^2}{c^2}}}$$

$$t = \frac{2.2\,\mu\text{s}}{\sqrt{0.007987}}$$

$$t = 24.62\,\mu\text{s}$$

$$\text{lifetimes} = \frac{24.62}{2.2}$$

$$\text{lifetimes} = 11.19$$

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11.9 SECTION REVIEW

UNDERSTANDING

1 Proper length is measured when the object is at rest. Proper time is measured in the reference frame of the event occurring, and rest mass is measured when an object is at rest.

APPLYING

2 $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$1.005 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$1-\frac{v^2}{c^2} = \left(\frac{1}{1.005}\right)^2$

$v = c \times \sqrt{1-\left(\frac{1}{1.005}\right)^2}$

$v = 0.30c$

3 $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$t = \frac{49\text{ years}}{\sqrt{1-\frac{(0.7c)^2}{c^2}}}$

$t = \frac{49\text{ years}}{\sqrt{0.51}}$

$t = 68.61\text{ years}$

4 $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$

$t_0 = 26\,\text{ns}\sqrt{1-\frac{(0.5c)^2}{c^2}}$

$t_0 = 26\,\text{ns} \times \sqrt{0.75}$

$t_0 = 22.5\,\text{ns}$

5 a $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$10 = \frac{6}{\sqrt{1-\frac{v^2}{c^2}}}$

$1-\frac{v^2}{c^2} = \left(\frac{6}{10}\right)^2$

$v = c \times \sqrt{1-\left(\frac{6}{10}\right)^2}$

$v = 0.8c$

b $d = vt$

$d = 0.8 \times 10$

$d = 8\,\text{ly}$

11.10 SECTION REVIEW

REMEMBERING

1 $E = m_0c^2$

UNDERSTANDING

2 As objects travel close to the speed of light, their mass changes. This means that the mass–energy equivalency needs to be amended accordingly.

APPLYING

3 $qV = (\gamma - 1)m_0c^2$

$\gamma = \frac{qV}{m_0c^2} + 1$

$\gamma = \frac{1.6 \times 10^{-19}\,\text{C} \times 6000\,\text{V}}{\left(9.109 \times 10^{-31}\,\text{kg}\right) \times \left(3 \times 10^8\,\text{m s}^{-1}\right)^2} + 1$

$\gamma = 1.0117$

Now:

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\gamma^2 \times \left(1-\frac{v^2}{c^2}\right) = 1$

$\gamma^2c^2 - c^2 = v^2\gamma^2$

$v = \sqrt{\frac{\gamma^2c^2 - c^2}{\gamma^2}}$

$v = \sqrt{\frac{1.0117^2 \times \left(3 \times 10^8\right)^2 - \left(3 \times 10^8\right)^2}{1.0117^2}}$

$v = 4.55 \times 10^7\,\text{m s}^{-1}$

4 $E = mc^2$

$E = (2 \times 9.109 \times 10^{-31}) \times (3 \times 10^8)^2$

$E = 1.64 \times 10^{-13}$ J

$E = 1.02$ MeV

ANALYSING

5 As an object travels closer to the speed of light, its relativistic mass increases accordingly. This means that the faster an object is moving, the heavier it is. As the object is getting heavier, more energy is required to move it.

6 As an object approaches the speed of light, it will become infinitely heavy, meaning it will slow down. This means an object with mass will never reach the speed of light, not even in a vacuum.

11.11 SECTION REVIEW

UNDERSTANDING

1 a Student answers will vary

b Student answers will vary

c Student answers will vary

2 $t_0 = t\sqrt{1 - \frac{v^2}{c^2}}$

$t_0 = 20\sqrt{1 - \frac{(0.6c)^2}{c^2}}$

$t_0 = 20 \times 0.8$

$t_0 = 16$ years

3 a Yes

b According to Earth, both travellers are gone for 0.61×7 years. This is 8.54 years and is measured as the dilated time. Proper time (or time they were actually gone) is:

$t_0 = t\sqrt{1 - \frac{v^2}{c^2}}$

$t_0 = 8.54\sqrt{1 - \frac{(0.61c)^2}{c^2}}$

$t_0 = 8.54 \times 0.7924$

$t_0 = 6.77$ years

This means that Ignatius will be 8.54 – 6.77 = 1.77 years older than the travellers upon their return.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a–x See glossary

2 The physics of motion when objects move close to the speed of light.

CATEGORY QUESTIONS

3 Student answers will vary, but should include discussion of speed of light, inertial and non-inertial reference frames, and relative velocities.

4 Time dilation occurs when objects move close to the speed of light. The proper time is measured outside of the reference frame of the event occurring. The greater the speed of the reference frame the event is happening in, the greater the time dilation observed.

5 Length contraction occurs when objects move close to the speed of light. The proper length of an object is measured when it is at rest or in the frame of reference of the object moving. As speed increases, observers not in the frame of reference of the moving object will notice the object to be shorter than at slower speeds.

ELABORATION QUESTIONS

6 As an object approaches the speed of light, it will become infinitely heavy, meaning it will slow down. This means an object with mass will never reach the speed of light, not even in a vacuum.

7 As it can be explained with the laws of special relativity.

8 $E = m_0c^2$ is the mass–energy equivalency derived from the momentum of a photon and the momentum of a moving mass interacting.

EVIDENCE QUESTIONS

9 Muons are detected much closer to the surface of Earth than they should be according to their half-life. This means that the muons are measuring a shorter time than the observers on Earth. As a result of this, they are travelling much further than they should before they decay. This only happens as muons move close to the speed of light.

10 Student answers will vary.

END-OF-CHAPTER EXAM

1 C

2 D

3 A

4 D

5 $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

6 More than two seconds.

7 *First postulate of special relativity*: the laws of physics are the same in all inertial frames of reference – the principle of special relativity.

Second postulate of special relativity: the speed of light has the same value, *c*, in all inertial frames. It does not depend on the speed of either the source or the observer.

8 Much shorter than 3 μs.

9 Shorter than 700 km.

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10 $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

$$L_0 = \frac{6\,\text{ly}}{\sqrt{1-\frac{(0.65c)^2}{c^2}}}$$

$$L_0 = \frac{6\,\text{ly}}{\sqrt{0.5775}}$$

$$L_0 = 7.90\,\text{ly}$$

11 As an object approaches the speed of light, it will become infinitely heavy, meaning it will slow down. This means an object with mass will never reach the speed of light, not even in a vacuum.

12 When two events occur simultaneously in one reference frame and *cannot* occur simultaneously in another reference frame, if the other reference frame is moving relative to the first reference frame. This is due to time measurements being different according to different observers.

13 a Proper length is measured when the object is at rest.

b Proper time is measured in the reference frame of the event occurring.

c Rest mass is measured when an object is at rest.

CHAPTER 12: QUANTUM THEORY

12.1 SECTION REVIEW

REMEMBERING

1 a Roemer – demonstrated that light had a finite speed.

b Bradley – using stellar aberration confirmed that light had a finite speed.

c Young – demonstrated wave-like properties of light, such as interference.

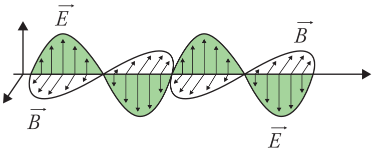
2 Magnetic permeability and electrical permittivity.

UNDERSTANDING

3 Maxwell related electricity and magnetism using calculus.

4 Interference; diffraction; reflection; refraction.

5



The electromagnetic wave is the combination of the electric field and magnetic field travelling at right angles to each other.

APPLYING

6 Hertz confirmed the presence of electromagnetic waves using a high-voltage spark. Hertz's experiments confirmed the predictions of Maxwell's equations.

ANALYSING

7 The changing electric and magnetic fields influence charged particles, such as electrons.

REFLECTING

8 Scientific knowledge, models and theories evolve as developments occur or more precise or different measurements are made. The speed of light has been determined with greater precision over time due to improvements in experimental apparatus. Maxwell's predictions based on mathematical equations successfully supported the electromagnetic wave model of light giving further validation for this model.

9

EVIDENCE FOR THE WAVE NATURE OF LIGHT	EVIDENCE FOR THE PARTICLE NATURE OF LIGHT
Refraction	Black body radiation
Polarisation	Photoelectric effect

12.2 SECTION REVIEW

REMEMBERING

1 Bright fringe (constructive interference)

2 Nodal points occur where a wave crest meets with a wave trough leading to destructive interference and a dark fringe. Antinodal points occur where wave crests coincide leading to constructive interference and a bright fringe.

UNDERSTANDING

3 The interference pattern observed in the double-slit experiment is a result of constructive and destructive interference as coherent waves of light meet with varying path differences, either cancelling or reinforcing each other.

4 a Decreases

b Decreases

c Increases

APPLYING

5 8.40×10^{-4} m; 1.68×10^{-3} m; 2.52×10^{-3} m

6 a $\lambda = 5.63 \times 10^{-7}$ m

b $y = 0.09$ m

ANALYSING

7 0.252 m. The second maximum of $\lambda = 630$ nm coincides with the third maximum of 420 nm.

12.3 SECTION REVIEW

REMEMBERING

1 The inherent uncertainty in measurement of the motion of a particle where its position is unable to be predicted precisely.

UNDERSTANDING

2 Photon → electron → neutron

3 Its mass is so large that the de Broglie wavelength is imperceptible.

4 $\lambda = 1.105 \times 10^{-35}$ m s^{-1}

5 a 6.67×10^{-3} m

b 3.33×10^{-3} m

c 1.33×10^{-2} m

ANALYSING

6 a $p = 1.20 \times 10^{-23}$ kg m s^{-1}

b i $\lambda = 3.16 \times 10^{-11}$ m

ii The momentum before and after the collision is conserved.

iii $\lambda = 7.37 \times 10^{-11}$ m

REFLECTING

7 Computer and transistor-based devices that rely on semiconductors would not be possible, changing the nature of society drastically.

8 The 'many worlds interpretation' of quantum mechanics suggests many possible alternative histories and futures branching at each event and resulting in different and separate branches of the universe. This interpretation supports the probabilistic model of quantum mechanics as every outcome of every event exists in its own 'world'.

12.4 SECTION REVIEW

REMEMBERING

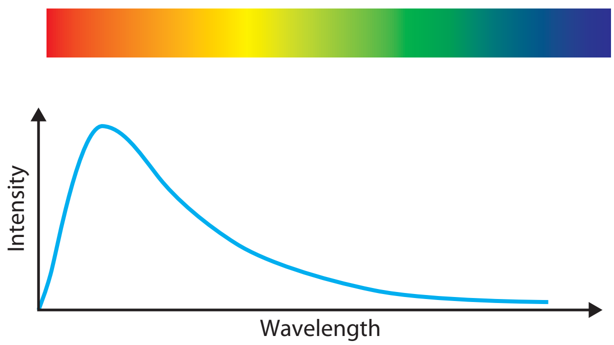
1 The spectrum representing all wavelengths of electromagnetic radiation, including the visible section.

2 Wilhelm Wien

UNDERSTANDING

3 A black body is an ideal surface that absorbs all wavelengths of electromagnetic radiation incident upon it.

4



The intensity of emitted radiation is a function of wavelength for a continuous black body spectrum.

APPLYING

5 6819 K

6 7.245×10^{-7} m or 725 nm

ANALYSING

7 $\lambda_{max} = \frac{b}{t}$: as the surface temperature increases, the peak wavelength decreases.

8 a $\lambda_{max} = 420$ nm or 4.20×10^{-7} m

b $T = 6900$ K

9 a 9.99×10^{-7} m

b The peak wavelength is beyond the 390–700 nm range of the visible section of the electromagnetic spectrum; therefore, the majority of radiation is emitted in the infrared region.

12.5 SECTION REVIEW

REMEMBERING

1 Newtonian mechanics, electromagnetism and thermodynamics are examples of classical physics.

2 Quantised – a term used to describe energy existing in discrete amounts.

3 $E = hf$, $\therefore h = \frac{E\ (\text{J})}{f\ (\text{Hz})}$ or J s.

4 Blue light has a higher frequency than red light; therefore, higher energy. Therefore, Vega is hotter than Antares.

APPLYING

5 a 1.21×10^{-20} J or 0.076 eV

b $E_3 - E_2 = 4.04 \times 10^{-21}$ J

6 $f = 2.56 \times 10^{14}$ Hz, $\lambda = 1.17 \times 10^{-6}$ m

REFLECTING

7 Student answers will vary

12.6 SECTION REVIEW

REMEMBERING

1 Einstein explained the energies of photoelectrons using the conservation of energy and quantisation.

2 A photon is a particle or quanta of light, having energy $E = hf$.

UNDERSTANDING

3 Photoelectrons are electrons ejected from a metal surface due to an incident quanta of energy (photon).

4 The gradient of a kinetic energy versus frequency graph represents Planck's constant, h. The stopping voltage, V_{stop}, can be used to determine the kinetic energy as KE = qV_{stop}.

5 a Copper

b Copper

c Lithium

APPLYING

6 KE = $hf - W$

7 a h = gradient = 8.21×10^{-34} J s

b $W = hf_0 = 7.39 \times 10^{-19}$ J or 4.62 eV (experimental)

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c The line representing silver would be parallel to that of magnesium although its x-intercept differs, reflecting its differed work function.

ANALYSING

8 a The positive potential difference accelerates electrons toward it; however, the photocurrent remains constant as the number of electrons ejected is dependent upon the light intensity.

b D

12.7 SECTION REVIEW

REMEMBERING

1 Thomson – established that an atom was not a single homogeneous particle but a collection of sub-atomic particles.

Rutherford – determined the presence of a very small, very dense and positively charged nucleus.

Bohr – postulated that electrons orbit the nucleus in stable, discrete orbits.

2 The Bohr model of the atom combined Rutherford's planetary model of the atom and Einstein's photons to predict the observed spectra of hydrogen. Bohr's model introduced the idea of quantised atomic energy levels but did not offer an explanation why. Successful models have both predictive and explanatory power.

3 Bohr's postulates:

- An electron in an atom moves in a circular orbit about the nucleus under the influence of the electrostatic attraction of the nucleus.
- Only certain orbits are stable. Electrons in these orbits do not emit energy.
- The greater the radius of the orbit, the greater is its energy. Atoms emit radiation when an electron goes from one orbit to another orbit with lower energy. The energy released is: $E = E_f - E_i = hf$
- The orbits are characterised by quantised radii, given by:

$$r = \frac{nh}{2\pi m_e v}$$

Where r is the radius in m, m_e is the mass of the electron in kg, v is its velocity, h is Planck's constant and n is an integer. The electron orbits had quantised radii and energies.

UNDERSTANDING

4 If spectral lines for the $n = 2$ state are typically in the infra-red region it is unlikely to produce the spectral lines in the visible region for the very largest transitions.

5 Absorption spectrum – the wavelengths or frequencies of radiation absorbed by a material.

Emission spectrum – the spectrum of radiation emitted by an object such as a black body or a star.

Dark lines in an absorption spectrum coincide with the bright lines in an emission spectrum as they represent the same energy level differences.

6 Bohr's model of the atom represented discrete energy levels for electrons and quanta of electromagnetic (light) energy.

APPLYING

7 6

8 The negative sign indicates that energy is emitted as the electron makes the transition down to this lower level. The zero energy level in this model is at infinity.

APPLYING

9 a 3

b $E_3 - E_1 = 3.02\ \text{eV} = 4.83 \times 10^{-19}\ \text{J}$

$E_3 - E_2 = 0.92\ \text{eV} = 1.47 \times 10^{-19}\ \text{J}$

$E_2 - E_1 = 2.10\ \text{eV} = 3.36 \times 10^{-19}\ \text{J}$

c $\lambda_{3-1} = 4.12 \times 10^{-7}\ \text{m or } 412\ \text{nm}$; (Violet)

$\lambda_{3-2} = 1.35 \times 10^{-6}\ \text{m or } 1353\ \text{nm}$; (Infra-red)

$\lambda_{2-1} = 5.92 \times 10^{-7}\ \text{m or } 592\ \text{nm}$; (Orange)

10 $E = 13.6\ \text{eV} = 2.18 \times 10^{-18}\ \text{J}$

$x = 9.14 \times 10^{-8}\ \text{m or } 91.4\ \text{nm}$

11 12.76 eV or 7.24 eV

12 $3.45 \times 10^{-11}\ \text{m}$

13 a 6

b $E_{4 \to 1}$

c Longest wavelength has lowest frequency and least energy and occurs between the two closest energy levels. $2.50 \times 10^{-8}\ \text{m},\ 5.00 \times 10^{-8}\ \text{m},\ 7.50 \times 10^{-8}\ \text{m}$.

ANALYSING

14 The single electron, when excited, may take multiple energy levels; and therefore, multiple energy transitions.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

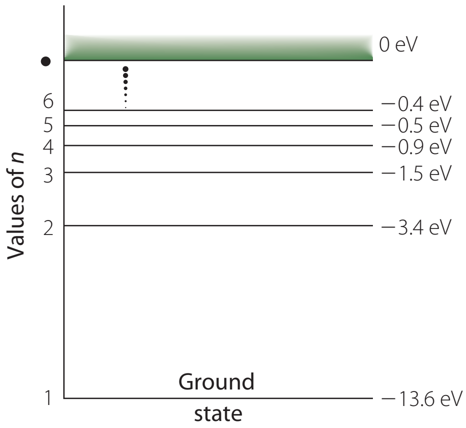
1 a Emission spectrum – a spectrum of the electromagnetic radiation emitted from a source.

b Energy level – the discrete energy orbits of electrons about the nucleus of an atom.

c Quanta – a discrete amount of energy, e.g. photons.

2 Wave–particle duality is the concept that light and matter exhibit both properties of waves and of particles on the quantum scale. These properties are exhibited through experiments such as two source interference (wave) and the photoelectric effect (particle).

3



The energy levels of an atom may be stated in eV or J. Either form is acceptable.

ENERGY (eV)	ENERGY (J)
−0.4	-6.40×10^{-20}
−0.5	-8.00×10^{-20}
−0.9	-1.44×10^{-19}
−1.5	-2.40×10^{-19}
−3.4	-5.44×10^{-19}
−13.6	-2.18×10^{-18}

CATEGORY QUESTIONS

4 Classical physics preceded quantum physics and is mostly concerned with Newtonian mechanics. Quantum physics was developed at the turn of the 20th century and is based largely on the concept of energy existing in discrete packets.

5 A black body is a perfect absorber of all wavelengths of electromagnetic radiation. A black body is also a perfect emitter.

6 Rutherford's model of the atom was a 'planetary model' with a central positive nucleus surrounded by orbiting electrons with a significant distance between.

Bohr's model of the atom accepted Rutherford's model, further detailing the specific, stable orbits of electrons and their quantised energies.

ELABORATION QUESTIONS

7 4.85×10^{-5} m. As the velocity increases the de Broglie wavelength decreases.

8 6.32×10^{-7} m

9 $d = 1.541 \times 10^{-3}$ m

EVIDENCE QUESTIONS

10 Mobile phones, global positioning systems (GPS), microwaves, laptop computers, televisions and magnetic resonance imaging (MRI) devices

11 Photoelectrons will be produced with energies up to 1.97×10^{-18} J.

END-OF-CHAPTER EXAM

1 D

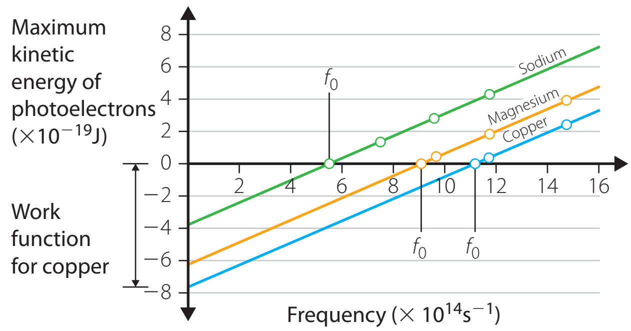
2 C

3 C

4 C

5 Photoelectron

6



7 3.00×10^{8} m s^{-1}

8 Work function

9 An emission spectrum indicates lines corresponding to specific differences in energy levels that are characteristic to the atom. An absorption spectrum is the full spectrum but with these specific energy levels subtracted or absorbed.

10 12.7 eV, 12.1 eV, 10.2 eV, 2.5 eV, 1.9 eV and 0.6 eV

11 2.91×10^{-5} m

12 a 7.059×10^{12} Hz

b 4.68×10^{-21} J

c 0.02925 eV

13 a 0.0852 m

b 0.0426 m

c 0.1704 m

14 3.57×10^{-7} m

15 a 7.52×10^{-19} J

b 1.95×10^{-19} J

c 1.13×10^{15} Hz

CHAPTER 13: THE STANDARD MODEL

13.1 SECTION REVIEW

REMEMBERING

1 An elementary particle is a particle whose substructure is unknown.

2 Particles and antiparticles have the same mass but opposite charges and/or spin.

3 a Positron: mass = 0.511 MeV c^{-2}, charge = +1

b Antiproton: mass = 938.3 MeV c^{-2}, charge = −1

c Antineutron: mass = 939.6 MeV c^{-2}, charge = 0

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UNDERSTANDING

4 A cloud chamber shows the trail left by particles travelling through it as they ionise the gas inside the chamber. If the chamber is in a magnetic field, particles with the same mass but opposite charge will follow paths with opposite curvature. Therefore, electrons and positrons will deflect in opposite directions.

5 Because protons and positrons have the same charge (+1), they will deflect in the same direction in a magnetic field. The radius of the curvature of this deflection is however mass dependent. Because the proton has a significantly larger mass than the positron, the radius of its curvature will be significantly larger than that of the positron.

13.2 SECTION REVIEW

REMEMBERING

1 A particle accelerator uses magnetic and electric fields to increase the velocity of beam charged particles and therefore increase their energy

2 Particle spin is a quantum property of particles that results from them having their own magnetic moment and therefore magnetic field

3 Fermions have half integer spin, Bosons have integer spin

UNDERSTANDING

4 Recall that $F = qvB\sin\theta$, and the direction of the force is given by the right-hand rule. The force is always perpendicular to the velocity and hence to the displacement, hence the field cannot do work, $W = Fs\cos\theta$, on the particle because $\theta = 90°$ so $\cos\theta = 0$. As no work is done, there is no change in kinetic energy and no change in speed.

5 The Pauli exclusion principle states that no two fermions can simultaneously exist in the same quantum state.

APPLYING

6 Apply the conversion factor:

$$m\left(\text{MeV c}^{-2}\right) = m\left(\text{kg}\right) \times \frac{1\ \text{MeV c}^{-2}}{1.78 \times 10^{-30}\ \text{kg}}$$

Insert known values:

$$m\left(\text{MeV c}^{-2}\right) = 9.11 \times 10^{-31}\ \text{kg} \times \frac{1\ \text{MeV c}^{-2}}{1.78 \times 10^{-30}\ \text{kg}}$$

Calculate the answer with the correct units and number of significant digits:

$$m\left(\text{MeV c}^{-2}\right) = 0.511\ \text{MeV c}^{-2}$$

7 a Apply the conversion factor:

$$E\ (\text{J}) = E\ (\text{eV}) \times \frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}$$

Insert known values recalling that $1\ \text{TeV} = 1 \times 10^{12}\ \text{eV}$:

$$E\ (\text{J}) = 4 \times 10^{12}\ \text{eV} \times \frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}$$

Calculate the answer with the correct units and number of significant digits:

$$E\ (\text{J}) = 6 \times 10^{-7}\ \text{J}$$

b Apply the kinetic energy equation:

$$\text{KE} = \frac{1}{2}mv^2$$

Rearrange for the unknown variable:

$$v = \sqrt{\frac{2\text{KE}}{m}}$$

Insert known values:

$$v = \sqrt{\frac{2 \times 6.4 \times 10^{-7}\ \text{J}}{0.003\ \text{kg}}}$$

Calculate the answer with the correct units and number of significant digits:

$$v = 0.02\ \text{m s}^{-1}$$

8 a Category = baryon; symbol = n; antiparticle = $\overline{n}$; mass = 939.6 MeV c^{-2}; lifetime = 614; spin = $\frac{1}{2}$

b Category = baryon; symbol = p; antiparticle = $\overline{p}$; mass = 938.3 MeV c^{-2}, lifetime = stable; spin = $\frac{1}{2}$

c Category = lepton; symbol = e^-; antiparticle = e^+; mass = 0.511 MeV c^{-2}; lifetime = stable; spin = $\frac{1}{2}$

d Category = lepton; symbol = τ ; antiparticle = τ^+; mass = 1784 MeV c^{-2}; lifetime = $< 4 \times 10^{13}$ s; spin = $\frac{1}{2}$

e Category = lepton; symbol = ν_τ , antiparticle = $\overline{\nu_\tau}$; mass = $<$ 30 MeV c^{-2}; lifetime = stable; spin = $\frac{1}{2}$

REFLECTING

9 Student answers will vary

13.3 SECTION REVIEW

REMEMBERING

1 The electromagnetic force, the strong nuclear force and the weak nuclear force

2 a Photon

b Gluon

c W and Z bosons

d Hypothetical Graviton

3 The electroweak interaction is a theory that combines the electromagnetic and weak interactions when particles have very high energies.

UNDERSTANDING

4 It is called the strong force because it must be large in magnitude in order to overcome the electrostatic repulsion of protons in a nucleus.

5 One particle emits a gauge boson that is subsequently absorbed by another particle.

■ APPLYING

6 Apply Coulomb's law to calculate the electrostatic force:

$$F_E = k\frac{Q_1Q_2}{r^2} = 9.0\times10^9\ \text{N m}^2\ \text{C}^{-2}\frac{(1.6\times10^{-19}\ \text{C})^2}{(1\times10^{-15})^2} = 230\ \text{N}$$

Apply the universal gravitation equation to calculate the electrostatic force:

$$F_E = G\frac{m_1m_2}{r^2} = 6.67\times10^{-11}\ \text{N m}^2\ \text{kg}^{-2}\frac{(1.67\times10^{-27}\ \text{kg})^2}{(1\times10^{-15})^2}$$

$$= 1.9\times10^{-34}\ \text{N}$$

The electrostatic force is large and repulsive, while the gravitational force is attractive but negligible in comparison. Hence, the strong nuclear force must provide at least a 230 N attractive force to overcome the repulsion due to the electrostatic force.

13.4 SECTION REVIEW

■ REMEMBERING

1 Electrons (e^-), electron-neutrino (ν_e), muon (μ^-), muon-neutrino (ν_μ), tau (τ), tau-neutrino (ν_τ)

2 Leptons interact via the electromagnetic, weak nuclear and gravitational forces.

■ UNDERSTANDING

3 They are considered elementary because they have a small mass, show no signs of an internal structure and their anti-particles are identical except for their charge.

4 Electron-neutrino → muon-neutrino → electron → tau-neutrino → muon → electron

13.5 SECTION REVIEW

■ REMEMBERING

1 Mesons are constructed of a quark and an antiquark.

2 Baryons are composed of three quarks.

3 Quarks interact via the strong, weak and electromagnetic forces.

■ UNDERSTANDING

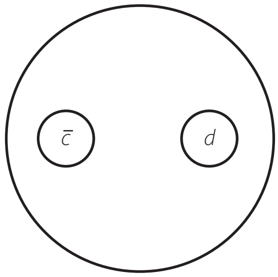
4 Mesons have masses in between that of electrons and protons.

5 a Groups of hadrons with similar mass but different charge.
  b The decay of hadrons into lighter hadrons.
  c The production of hadrons from reactions.
  d Uncharged particles having magnetic moments.

■ APPLYING

6 a Not possible
  b Possible (baryon)
  c Not possible
  d Possible (anti-baryon)
  e Not possible

7



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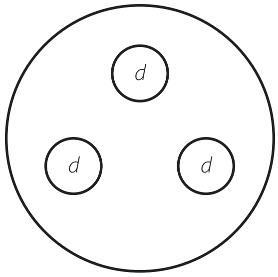

13.6 SECTION REVIEW

■ REMEMBERING

1 Quarks, leptons and bosons

2 The overall expansion of the universe.

3 Dark matter (so called because it is currently invisible to our observation technologies), is the hypothetical matter postulated to account for the lack of agreement with the measured mass of galaxies and their orbital velocity.

■ UNDERSTANDING

4 The inability to incorporate gravity, the prediction that neutrinos have zero mass and the prediction of magnetic monopoles.

5 The universe was opaque to light prior to then.

6 The discovery of the Higgs boson supported the Higgs mechanism (based on the Standard Model) that seeks to explain the origins of mass.

7 Because the technology to create the conditions necessary to produce the particle, as well as the computing power to analyse the data were not available prior to the LHC.

■ APPLYING

8 All quarks, leptons and gauge bosons

9 Apply the conversion factor:

$$125\times10^9\ \text{eV c}^{-2}\times\frac{1.78\times10^{-36}\ \text{kg}}{1\ \text{eV}} = 2.225\times10^{-25}\ \text{kg}$$

$$126\times10^9\ \text{eV c}^{-2}\times\frac{1.78\times10^{-36}\ \text{kg}}{1\text{eV}} = 2.22428\times10^{-25}\ \text{kg}$$

Apply the average equation:

$$m_{AVG} = \frac{2.225\times10^{-25}\ \text{kg} + 2.22428\times10^{-25}\ \text{kg}}{2}$$

$$= 2.2339\times10^{-25}\ \text{kg}$$

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Apply the confidence interval equation:

$$\Delta m_{\text{AVG}} = \frac{2.225\times10^{-25}\ \text{kg} - 2.224\,28\times10^{-25}\ \text{kg}}{2}$$
$$= 1.78\times10^{-27}\ \text{kg}$$

Give the answer in the correct format with correct number of significant digits:

$m_{\text{AVG}} = (2.23 \pm 0.018)\times10^{-25}\ \text{kg}$

10 Student answers will vary

11 a Apply the black-body equation

$$\lambda = \frac{b}{T}$$

Insert known values:

$$\lambda = \frac{2.898\times10^{-3}\ \text{m K}^{-1}}{3000\ \text{K}}$$

Calculate the answer with the correct units and number of significant digits:

$\lambda = 1\times10^{-6}\ \text{m}$

b Apply the black body equation:

$$\lambda = \frac{b}{T}$$

Insert known values:

$$\lambda = \frac{2.898\times10^{-3}\ \text{m K}^{-1}}{3\ \text{K}},$$

Calculate the answer with the correct units and number of significant digits:

$\lambda = 1\times10^{-3}\ \text{m}$

REFLECTING

12 Student answers will vary

13 Student answers will vary

14 Student answers will vary

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a–h See glossary

2 A particle emits a gauge boson which is absorbed by another particle

3 Quarks: up, down, charm, strange, top and bottom. Leptons: electron, electron-neutrino, muon, muon-neutrino, tau and tau-neutrino

CATEGORY QUESTIONS

4 Leptons and hadrons are both particles of ordinary matter whereas gauge bosons are particles that mediate forces. Leptons and hadrons are differentiated according to mass due to the fact that leptons are elementary but hadrons are made up of quarks.

5 Fermions are particles with half-integer spin whereas bosons are particles with integer spin.

6 Mesons are particles composed of one quark and one antiquark, whereas baryons are composed of three quarks.

7 Strong force acts upon quarks through the exchange of gluons.

Electromagnetic force acts upon quarks and leptons through the exchange of photons.

Weak force acts upon quarks and leptons through the exchange of W and Z bosons.

Gravity acts upon quarks and leptons, theoretically through the exchange of the gravitons.

ELABORATION QUESTIONS

8 The electroweak theory is the postulates that the electromagnetic and weak nuclear forces are equal in magnitude when particles are of a high enough energy. Grand unified theories attempt to combine all four forces under one explanation. This was the first successful attempt to do so.

9 Mass is a result of the breaking of the electroweak symmetry through the exchange of a Higgs boson. In this process, particles interact with the Higgs field and gain mass.

10 Student answers will vary but should include its predictive power.

11 Student answers will vary but should include mention of the ability to explain gravity in terms of particle interactions and unify all forces under one common origin.

EVIDENCE QUESTIONS

12 The fact that the electric and magnetic fields of an electron are close to what we would expect of a point charge suggests that it is an elementary particle. Neutrons are likely to be composed of smaller particles as they have a magnetic field despite being neutral. Because protons are similar in mass to a neutron, it is also likely that it is not elementary.

13 Student answers will vary but should include: inability to incorporate gravity, the prediction that neutrinos have zero mass and the prediction of magnetic monopoles.

14 Student answers will vary but should include the predictions of various radiations at different epochs in the history of the universe.

END-OF-CHAPTER EXAM

1 D

2 C

3 A

4 D

5 A

6 B

7 A

8 Antiproton

9 Hadrons

10 Spin

11 Annihilation

12 a Apply the conversion factor:

$$m\,(\text{kg}) = m\,(\text{MeV c}^{-2}) \times \frac{1.78 \times 10^{-30}\ \text{kg}}{1\ \text{MeV c}^{-2}}$$

Insert known values:

$$m\,(\text{kg}) = 135\ \text{MeV c}^{-2} \times \frac{1.78 \times 10^{-30}\ \text{kg}}{1\ \text{MeV c}^{-2}}$$

Calculate the answer with the correct units and number of significant digits:

$$m\,(\text{kg}) = 2.4 \times 10^{-28}\ \text{kg}$$

b Apply the conversion factor:

$$m\,(\text{kg}) = m\,(\text{MeV c}^{-2}) \times \frac{1.78 \times 10^{-30}\ \text{kg}}{1\ \text{MeV c}^{-2}}$$

Insert known values:

$$m\,(\text{kg}) = 1672\ \text{MeV c}^{-2} \times \frac{1.78 \times 10^{-30}\ \text{kg}}{1\ \text{MeV c}^{-2}}$$

Calculate the answer with the correct units and number of significant digits:

$$m\,(\text{kg}) = 3.0 \times 10^{-27}\ \text{kg}$$

13 Fermions are particles with half-integer spin whereas bosons are particles with integer spin.

14 Leptons and hadrons are differentiated according to mass due to the fact that leptons are elementary but hadrons are made up of quarks.

15 The strong nuclear force is the force that acts between particles in the nucleus, namely the protons and the neutrons. The strong force is the more fundamental force that acts between the quarks that compose the neutrons and protons.

16 Mesons are particles composed of one quark and one antiquark, whereas baryons are composed of three quarks.

17 a Not possible

b Not possible

c Possible (anti-boson)

d Possible (meson)

CHAPTER 14: PARTICLE INTERACTIONS

14.1 SECTION REVIEW

REMEMBERING

1 Each lepton number has lepton number 1, and each antilepton has lepton number −1.

2 Baryon number is used to describe annihilation and creation of particles on a very small scale.

3 1, −1 and 0 respectively

UNDERSTANDING

4 A lepton number helps describe the absence of reactions which created new particles, whereas baryon number helps describe annihilation and creation.

14.2 SECTION REVIEW

REMEMBERING

1 Energy, momentum, lepton number, baryon number.

APPLYING

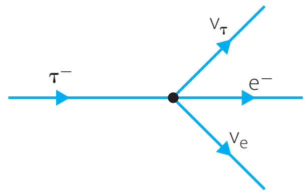
2 Yes, baryon number is conserved.

3 Yes, lepton and baryon number are conserved.

4 No

5 $1 + 1 = 1 + 1 + 0$

6



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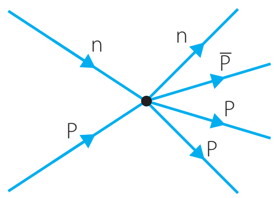

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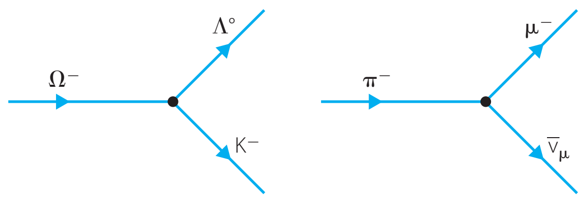

ANALYSING

9 The law of conservation of lepton number is violated.

10 The missing particle must be a baryon. An example is a proton or neutron.

14.3 SECTION REVIEW

REMEMBERING

1 Electromagnetic force from exchange particles (photons).

2 Upward arrows are particles. Downward arrows represent antiparticles.

UNDERSTANDING

3 These forces are the reason interactions happen, so it is important to include them in representations.

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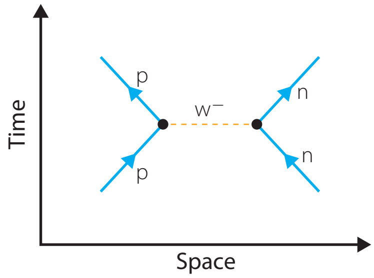
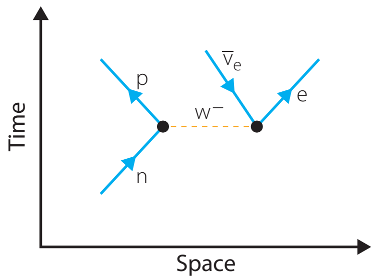
4 They show the forces that are causing the reaction to occur.

5



6 Two quarks interact strongly (gluon) and two quarks are maintained after this interaction.

7



14.4 SECTION REVIEW

REMEMBERING

1 Time reversal, charge reversal and crossing symmetry.

2 The particle that is crossed must be replaced with its antiparticle.

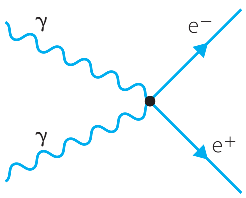
UNDERSTANDING

3 The likelihood of the two reactants being in such close proximity for this to occur is extremely unlikely.

APPLYING

4 $\Sigma^+ \rightarrow \bar{n} + \pi^+$

5 $2\gamma \rightarrow e^- + e^+$

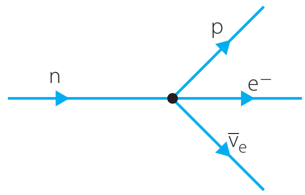


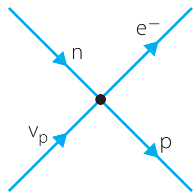
Without other particles, these two gamma photons will not collide in order to create an electron and a positron.

6 $\pi^+ + \pi^+ + \pi^- \rightarrow K^+$. The new reaction is unlikely to occur as the chance of having two pions and one antipion together is very remote.

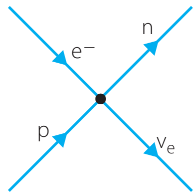
ANALYSING

7 In order to obtain positron decay, the electron antineutron needs to be crossed, then time reversal needs to be applied, then finally the electron needs to be crossed.

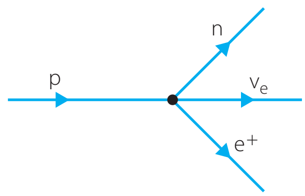




Crossing symmetry

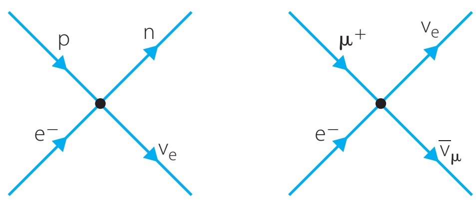


Time reversal



Crossing symmetry

8



SYNTHESISING

9 These interactions and the possibility of them happening is one of the many parts of the universe we are trying to understand. Physicists are interested in these interactions as when something very unlikely happens, it does help build evidence to an existing theory of how we currently understand the universe.

CHAPTER REVIEW QUESTIONS

DETAIL QUESTIONS

1 a–n See glossary

2 Time reversal, charge reversal and crossing symmetry

CATEGORY QUESTIONS

3 Reaction diagrams show the reactants and products in particle interactions, but Feynman diagrams also show the exchange particle/force that causes the interaction to happen.

4 By adding the number of lepton numbers on the left side of the reaction and ensuring they are equal to the addition of lepton numbers on the right side of the reaction. If this is not equal, the reaction is impossible.

5 The reaction is not possible.

ELABORATION QUESTIONS

6 The likelihood of the reactants being in such close proximity for this to occur is extremely unlikely.

7 By converting it to its antiparticle.

EVIDENCE QUESTION

8 A graviton is an exchange particle which interacts with gravitational force. There is still more evidence required to prove their existence. They are still considered hypothetical. Gravitational waves were first observed in 2016.

END-OF-CHAPTER EXAM

1 B

2 C

3 B

4 Its antiparticle

5 Yes

6 $\tau^+ \rightarrow e^+ + \bar{\nu}_\tau + \nu_e$

7 $\tau^- + e^+ \rightarrow \nu_\tau + \bar{\nu}_e$

$\tau^- + \nu_\tau \rightarrow e^- + \bar{\nu}_e$

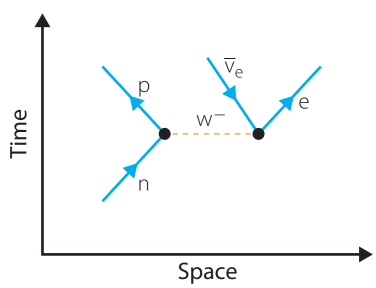
$\tau^- + \nu_e \rightarrow e^- + \nu_\tau$

The most likely reaction to occur is the first stated above. The other two are unlikely.

8 $1 = 1 - 1 + 1$

9 $1 = 1 + 0 + 0$

10



UNITS 3 & 4 PRACTICE EXAM

MULTIPLE-CHOICE QUESTIONS

1 B

2 D

3 D

4 A

5 D

6 B

7 D

8 B

9 C

10 A

11 C

12 D

13 C

14 A

15 D

16 B

17 C

18 D

19 C

20 A

SHORT-RESPONSE QUESTIONS

1 5.68×10^{26} kg

2 $140\,m\,s^{-2}$ or $140\,N\,kg^{-1}$

3 4.20 ns

4 $64.7\,m\,s^{-1}$

5 13.85 ly

6 2.8×10^{-3} Wb

7 Interference patterns result from constructive and destructive combinations of waves.

8 Electron, electron neutrino, muon, muon neutrino, tau, tau neutrino

9 $6.31 \times 10^3\,m\,s^{-1}$

10 Area, current and angle of inclination

11 Leptons (and quarks) are the basic building blocks of matter. Leptons includes the electron, electron neutrino, muon, muon neutrino, tau and tau neutrino. Leptons and hadrons experience the weak interaction. Hadrons include mesons and baryons and interact via the strong force.

12 $20.77\,m\,s^{-1}$

13 The laws of physics are the same in all inertial reference frames. The speed of light in a vacuum has the same value, *c*, in all inertial frames of reference.

14 0.675 V

15 9.34×10^{-4} m

16 a $T = 1.55$ s

b The period would increase.

17 $8.49 \times 10^4\,m\,s^{-1}$

18 Radio waves; infrared; visible light; ultraviolet; X-rays; gamma rays

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19 a 142.1 N

b 156.3 J

c 156.3 J

20 a 450.4 N

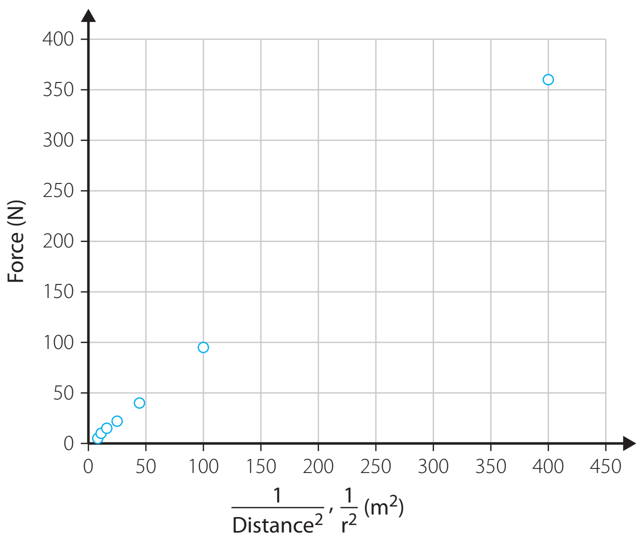
b −300.9 N

c $15.2\,\text{m}\,\text{s}^{-1}$

COMBINATION-RESPONSE QUESTIONS

1

SEPARATION DISTANCE, r (m)	ELECTROSTATIC FORCE, *F* (N)	$\frac{1}{\text{separation distance}^2}, \frac{1}{r^2}$ (m^{-2})
0.05	360.0	400
0.10	95.0	100
0.15	40.0	44.44
0.20	22.0	25
0.25	15.0	16
0.30	10.0	11.11
0.35	5.0	8.16



Gradient = 9.02×10^{-1}; $k = 9.02 \times 10^{9}$

2 $D = 9.46 \times 10^{-4}\,\text{m}$

3 a 0.757 s

b 0.702 m

c $v_V = 0\,\text{m}\,\text{s}^{-1}$; $v_H = 11.41\,\text{m}\,\text{s}^{-1}$

d 8.639 m

4 $12.3\,\text{m}\,\text{s}^{-1}$

5

WAVELENGTH (nm)	FREQUENCY ($\times 10^{14}$ Hz)	KE_{max} OF PHOTOELECTRONS (eV)	KE_{max} OF PHOTOELECTRONS (J)
595	5.042	0.67	1.072×10^{-19}
520	5.769	0.98	1.568×10^{-19}
460	6.521	1.35	2.160×10^{-19}
412	7.282	1.63	2.608×10^{-19}

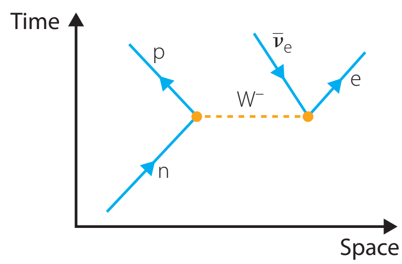
a $h = 6.958 \times 10^{-34}\,\text{J}\,\text{s}$

b $W = 2.314 \times 10^{-19}\,\text{J}$

6 $F = 9.0 \times 10^{-3}\,\text{N}$

7 When two events occur simultaneously, they are said to happen at the same time. However, what may be considered a simultaneous event in one inertial reference frame may not be considered simultaneous in another reference frame. The idea that simultaneous events happen at different times in different reference frames argues against the existence of a universal time frame. Events that appear simultaneous in one reference frame T_0 are not simultaneous in reference frame T as time is relative.

8



9 $F_g = 1.60 \times 10^{22}\,\text{N}$

10 a 204 000 V

b 2400 W

c 2400 W

d 0.012 A

GLOSSARY

A

absorption spectrum the wavelengths (or frequencies or energies) of radiation absorbed by a material

albedo the ratio of light reflected by a surface to light incident on it; a surface with an albedo of 1 is perfectly reflective, and an albedo of 0 is perfectly absorbing

alternating current (AC) electrical current that alternates its direction of travel sinusoidally with time

angle of inclination angle, θ, relative to the horizontal; $0° < \theta < 90°$

angular momentum, L momentum associated with rotational or orbital motion, $L = mvr$

annihilation the destructive process resulting when a particle and its antiparticle meet

antimatter matter composed of antiparticles, such as positrons, antiprotons and antineutrons

antiparticle a particle with the same mass and opposite charge and/or spin to a corresponding particle

apparent weightlessness the experience of having no normal force exerted on you; this occurs during free-fall

armature the frame of the rotating part of a motor or generator, holding one or more coils

astronomical unit (AU) a unit of measure equivalent to Earth's mean orbital radius about the Sun (1.50×10^{11} m)

atomic spectral lines an emission or absorption spectrum consisting of discrete lines, characteristic of the energy levels of a particular atom or molecule; also called a line spectrum

attractive force when two particles of unlike charge move towards each other

B

***B* field** a magnetic field

baryon number quantum number associated with each baryon, antibaryon and non-baryonic particles

baryons a family of heavy subatomic particles, such as neutrons and protons, which contain composite structures made up of three quarks

black body an object with a perfectly absorbing surface that emits radiation with a spectrum that is characteristic of the temperature of the object

black-body radiation the electromagnetic radiation emitted by a black body, with a spectrum characteristic of the temperature of the body

bosons particles with integer spin (s = 0, 1, 2, …); these particles do not obey the Pauli exclusion principle

C

centre of mass the average position of the mass in an object or group of objects. It is the point at which the gravitational force can be modelled as acting when the object is in a gravitational field

centripetal centre-seeking; directed towards the centre

centripetal force in uniform circular motion, the sum of real forces that points towards the centre of the circle

charge-reversal symmetry if all particles in an allowed reaction are replaced with their antiparticles (which have opposite charge), the new reaction is also allowed under known conservation laws

classical mechanics the study of motion in accordance with Newton's laws; also known as Newtonian physics

cloud chamber a chamber containing a supersaturated vapour through which high-energy particles pass, causing vapour trails to be formed and therefore allowing the path of the particles to be visualised

component two or more vectors into which a vector can be resolved

concentric spheres spheres that share a common centre

conservation of baryon number whenever a nuclear reaction or decay occurs, the sum of the baryon numbers before the process must equal the sum of the baryon numbers after the process

conservation of lepton number each of the lepton numbers L_e, L_μ and L_τ is a conserved quantity

constructive interference the superposition of waves where crests intersect with crests and and troughs intersect with troughs. It is characterised by antinodes or bright fringes and occurs whenever the path difference is equal to a whole number of wavelengths, $\delta = n\lambda$

continuous spectrum a spectrum containing radiation of all wavelengths; for example, a rainbow is composed of the various wavelengths of the visible spectrum

cosmic microwave background radiation the observed radiation coming from all points in space corresponding to radiation from a black body at 3 K; it is believed to come from an earlier, much hotter stage of the evolution of the universe

Coulomb's law the second law of electrostatics; states that the force exerted between two point charges is directly proportional to the product of their electric charges, inversely proportional to the square of the distance between them and inversely proportional to the permittivity of the surrounding medium

crossing symmetry if a particle in an allowed reaction is crossed to the other side of the reaction and replaced with its antiparticle, the new reaction is also allowed under known conservation principles provided enough energy is available

D

dark energy energy that is predicted from the increasing rate of expansion of the universe, but which is not identifiable as any currently known energy form

dark matter matter that is postulated to explain gravitational effects but which is not observable by the emission or reflection of light

destructive interference the superposition of waves where a crest intersects with a trough, due to incoherent wave sources or sources being half a cycle out of phase. This results in a node where the path difference, δ, is equal to a multiple of one half of the wavelength, $\delta = (n - \frac{1}{2})\lambda$

deterministic predictable, able to be determined if enough information is available

diamagnetic materials weakly repelled by nearby magnets; examples include bismuth and copper

differential equations equations that relate the rate of change of displacement in space to the rate of change of displacement in time

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direct current (DC) a current that flows in a single direction

discrete able to take only specific values, not continuous; for example, a line spectrum is a discrete spectrum

E

eddy current a circular current induced in a conductor due to a changing magnetic field

electric field the field due to an electric charge, which applies a force to other electric charges

electric field lines net lines of force pointing in the direction a positive test charge will move when placed in the electric field due to a charge Q

electrical permittivity (ε_0) a physical property of a medium associated with electricity

electrical potential potential energy per unit charge in an electric field

electrical potential energy potential energy stored in an electric field. The change in potential energy of an object is also the work done on that object by the electric field

electromagnet a magnet with a north and a south pole formed by a current in a solenoid

electromagnetic induction the production of an electromotive force (emf) or a voltage in an electrical conductor due to its dynamic interaction with a magnetic field

electromagnetic spectrum the family of electromagnetic radiations – radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays – which travel at $3.0 \times 10^8 \, \text{m s}^{-1}$ in a vacuum.

electromagnetic waves waves produced by an oscillating charge resulting in mutually perpendicular electric and magnetic fields

electromotive force (emf) a difference in potential that tends to give rise to an electric current

electrostatic field model the model that assigns an electric field to stationary charges; it is this field that exerts forces on other charges

electroweak theory theory that combines the electromagnetic and weak interactions

elementary particle a particle whose substructure is unknown

ellipse a regular, curved shape that is a conic section (formed by cutting a cone obliquely); the path of satellites in orbit around larger bodies

emission spectrum the spectrum of radiation emitted by an object, for example, black-body radiation or atomic spectra from a discharge tube

empirical methods the central tenet of the scientific method whereby hypotheses are tested using observation and experimentation.

energy levels the allowed energies of a nucleus–electron system; often referred to as electron energy levels, even though they are characteristic of the atom, not of individual electrons

epicycles smaller circles whose centre is on the radius of larger circles; used by Ptolemy to describe the motion of planets

escape velocity the minimum velocity required for an object to escape the gravitational field of a planet or other large mass

exchange force strong, electromagnetic, weak or gravitational force associated with the exchange particles gluons, photons, Q and Z particles and gravitons respectively; for example, an exchange of photons between electrons produces an electromagnetic force

exchange particle a particle carrying force which is responsible for behaviour during other particle interactions. Sometimes exchange particles are the result of a particle interaction, such as the case with electron-positron annihilation.

F

fermions particles with half-integer spin ($s = \frac{1}{2}, \frac{3}{2}$ ...), that obey the Pauli exclusion principle

ferromagnetic materials that are strongly attracted to nearby magnets; examples include iron, nickel and cobalt. Ferromagnetic substances can retain permanent magnetism and this can be induced by other very strong magnets

Feynman diagram a diagram that models exchange particles and exchange forces over time in space, when particles come into close proximity to each other

field the means by which action-at-a-distance forces are exerted

first law of electrostatics like charges repel and unlike charges attract

flavours the six classifications of quark types: up, down, strange, charm, top and bottom

frame of reference a framework in which motion of an object is described according to a coordinate system. Frames of reference are observational and can be inertial or non-inertial (accelerating)

free-fall falling with the acceleration g, the local gravitational field strength

frequency number of times a circular motion is completed in a time period

G

Galilean transformation for a two-dimensional inertial frame of reference, P′, moving at constant speed in the x-direction with respect to another inertial frame, P: $x = x' + v\Delta t, y = y'$

gauge boson force carrying particles that mediate particle interactions through the four fundamental forces

generator a device used to produce electrical current by electromagnetic induction

geocentric model a superseded model of the solar system with the Sun, Moon and planets revolving about Earth at its centre

geostationary satellite a satellite positioned directly above a point on the equator; they have periods of approximately 24 hours

geosynchronous satellite a satellite that completes one orbit of Earth in the same time as Earth completes one revolution; they have periods of approximately 24 hours.

gluon the gauge boson that mediates the strong force

grand unified theory (GUT) a theory that unites all four fundamental forces in a single model and explains the symmetry-breaking mechanism that caused them to separate into the four distinct forces we now know . There is as yet no widely accepted GUT

gravitas Aristotelian idea about the 'heaviness' of objects made of earth that allowed them to fall in straight lines towards Earth

gravitational field the field that mediates the gravitational force between all objects with mass; the field surrounding all objects with mass: $g = \frac{GM}{r^2}$

gravitational potential energy the potential energy associated with the interaction of objects via the gravitational force; the potential energy is stored in the gravitational field

graviton the hypothetical gauge boson of the gravitational force

ground level the lowest possible energy level of a nucleus–electron system

H

hadron a family of elementary particles with a large mass consisting of mesons and baryons

heliocentric model a current model of the solar system with the Sun (Helios) at its centre and all planets revolving about it. Closely associated with the work of Nicolaus Copernicus and Galileo Galilei

I

induced current a current that is produced due to the presence of an electromotive force

induced emf an emf created by a changing magnetic field

inertial frame of reference one in which Newton's first law applies to a very good approximation, and there is no acceleration. Any departures from the law are negligible; also known as an inertial reference frame

invariant does not vary; it is the same in all reference frames

inverse-square law describes a relationship in which the dependent variable is proportional to the square of the inverse of the independent variable

K

kinetic (or sliding) friction force that impedes motion once motion has begun

L

length contraction length measurements are shorter in a reference frame that is moving relative to an inertial frame

lepton number quantum number associated with each lepton, antilepton and non-leptonic particle

leptons a family of elementary particles that include electrons, taus, muons, their neutrinos and all of their antiparticles

light-year (ly) a measure of the distance that light would travel in one year (9.47×10^{15} m)

line spectrum an emission or absorption spectrum consisting of discrete lines that are characteristic of the energy levels of a particular atom or molecule

Lorentz factor factor by which both time dilation and length contracted are affected when v is very close to c

low Earth orbit (LEO) a satellite orbit within the range of approximately 250 km to 1000 km above Earth's surface

M

magnet a magnetic material that has the majority of its magnetic domains aligned. Magnets have magnetic fields that, in turn, affect other substances nearby

magnetic braking braking due to the interaction of eddy currents and an external magnetic field

magnetic domain a region within a magnetic material (such as iron) where the magnetic properties point in the same direction

magnetic field the field created by moving charges, including charges in magnetic materials

magnetic flux a measurement of the total magnetic field that passes through a given area; has the unit weber (Wb)

magnetic flux density the strength of a magnetic field per unit area

magnetic force the force that a magnetic field exerts on a moving charge or current

magnetic permeability (μ_0) physical property of a medium associated with magnetism

magnetic pole magnetic north or south pole, a point where magnetic field lines go out of or come in to

mean orbital radius the average radius of orbit of one massive object about another, e.g. the Earth revolving about the Sun

megaparsec (Mpc) the distance subtended by an angle of one arcsecond $\times 1 \times 10^6$ (3.09×10^{22} m)

mesons a family of heavy subatomic particles that contain composite structures made up of one quark and an antiquark

modes of vibration characteristic patterns of oscillation, usually with a discrete set of allowed frequencies

muon a particle formed by cosmic rays in the upper atmosphere

N

negligible any value or variation in a value that is too small to be taken into account

neutrino oscillation a phenomenon in which a neutrino with a given lepton association (e, μ or τ) can later be measured to have switched to another neutrino type (e, μ or τ); lepton number is still conserved in this instance

north pole the pole of a magnet where the field lines start; they are drawn exiting the north pole

O

orbit a regularly repeated elliptical path of one object about another massive object, such as a planet about a sun

orbit velocity the precise velocity required for an object to continue to orbit a mass at a given altitude

P

parallelogram method vector addition in which the tail of each vector is connected at the same position; the vectors are used as adjacent sides of a parallelogram and the resultant is the diagonal that starts at the tails of the vectors being added

paramagnetic materials weakly attracted to nearby magnets; examples include aluminium and rare earth ions. Paramagnetic substances do not retain permanent magnetism

particle accelerator a device in which electric and magnetic fields are used to accelerate beams of particles to high speeds

Pauli exclusion principle quantum mechanical principle that two fermions in the same quantum system cannot have identical sets of quantum numbers; e.g. no two electrons can be in the same shell or orbital around an atom and have the same energy

period time taken for an object undergoing circular motion to complete one revolution

photocurrent the current formed by electrons ejected from a surface by incident photons

photoelectric effect the ejection of electrons from a surface by incident photons of sufficient energy

photoelectron an electron ejected from a metal surface following absorption of a photon of sufficient energy

photon a particle or quanta of light, having energy $E = hf$

Planck constant the constant of proportionality between energy and frequency for photons: $h = 6.626 \times 10^{-34}$ J s

positron the antiparticle of an electron with charge $+e$ and mass m_e

potential difference the difference in potential between two points in an electric field; the work done per charge

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potential energy energy stored in a system due to the interaction of components in the system via forces; energy stored in a field. Potential energy gives a system the ability to do work

probabilistic not deterministic, unable to be predicted regardless of how much information is known

proper length length measured in an inertial frame of reference in which the object is stationary

proper time the time interval between two events occurring at the same place in an inertial reference frame, as measured by an observer in that inertial frame.

Q

quantised existing in discrete amounts, not able to be divided into arbitrarily small amounts

quantum a discrete unit or amount of some physical property, such as energy, charge, mass or angular momentum

quantum physics the science of very small particles for which classical mechanics fails to explain the interactions that are observed

quark a type of elementary particle (along with leptons and gauge bosons)

R

rectangular components components that are at right-angles to each other; perpendicular components

redshift the observed shift to longer wavelength of spectral lines in distant stars

relative motion the motion of a moving object according to a moving observer. When relative motion is being evaluated, one reference frame must always be considered stationary

relativistic effect when time and space act differently for one object compared to others

relativistic kinetic energy defined from the rest mass by $\Delta E_k = (\gamma - 1)m_0c^2$

relativistic length length contraction due to objects moving at very high speeds relative to each other

relativistic mass also known as relativistically corrected mass. The greater the relative velocity, the greater the relativistic mass: $m = \gamma m_0$

relativistic momentum momentum of particle due to the relativistic mass at high relative speeds

relativistic time time dilation observed due to objects moving at very high speeds relative to each other

relativity principle the laws of physics are the same in all inertial frames of reference

repulsive force when two particles of like charge move away from each other

resolute component of a vector

rest energy defined from the rest mass by $E = m_0c^2$

rest mass also known as proper mass; it is the mass as measured when the mass is stationary in an inertial reference frame. Proper mass never changes

rolling friction force that acts to oppose motion and that rotates an object as it rolls along a slope

root mean square (rms) the average AC potential difference or current that produces the same power in a load as a DC potential difference or current of the same magnitude

S

satellite a natural (e.g. moon) or synthetic (e.g. GPS or communications satellite) body that orbits a significantly larger mass

scalar multiplier positive or negative number that can change the magnitude and/or the direction of a vector

Schrödinger equation a differential equation that describes the wave-like properties of matter existing within a potential field

second law of electrostatics Coulomb's law

simultaneity when two events occur simultaneously in one reference frame and *cannot* occur simultaneously in another reference frame, if the other reference frame is moving relative to the first reference frame

solenoid a coil of current-carrying wire that creates a large uniform field within the coil

south pole the pole of a magnet where the field lines end; they are drawn entering the south pole

special relativity the physics theory regarding the relationship between space and time, which is not explained by Newtonian or Galilean relativity

spectroscope a device that disperses radiation by energy (or wavelength or frequency) so that a spectrum may be observed and measured

spectrum the distributed components of light or another wave arranged by frequency (or wavelength)

spin a quantum property of particles that results from them having their own magnetic moment; and therefore, magnetic field

static friction force that impedes motion up to the point where motion begins

stellar aberration the variation in the visible and actual position of a star due to the relative motion of Earth

step-down transformer a transformer with an output potential difference that is lower than the input potential difference

step-up transformer a transformer with an output potential difference that is higher than the input potential difference

stopping voltage the reverse bias voltage required to stop the flow of photoelectrons in a photoelectric effect experiment

strong force the attractive force that acts between quarks, holding them together; it is mediated by gluons

symmetry the invariance of physical laws under transformations such as translation, reflection or rotation in time or space

symmetry breaking a change in the behaviour of a physical system or the laws of physics that govern its behaviour when a symmetry operation such as a translation, reflection or rotation in time or space takes place.

T

threshold frequency, f_0 the minimum frequency of light needed to eject an electron from a metal surface

time dilation a longer time measured by an observer outside the reference frame in which the event occurs

time reversal when reactions are reversed in time

time-reversal symmetry when an allowed reaction is written such that it runs in the opposite direction in time; the new reaction is also allowed in that it does not break any of the known conservation laws

U

unified theory any theory that demonstrates how fundamental forces can be united, and explains the mechanism by which they become distinct, for example the electroweak theory

uniform electric field electric field that has the same magnitude and direction at all points

W

W and Z bosons particles that mediate the weak nuclear force

wave equation a differential equation that describes wave behaviour; its solutions are wave functions that are typically sinusoids

wave–particle duality the dual nature of matter and energy, requiring both the wave and the particle model to completely explain all observed behaviour of matter and energy

weight the gravitational force that acts on an object, $F_w = mg = \frac{GMm}{r^2}$

work function the energy required to eject an electron from a metal surface; effectively, it is the ionisation energy for the bulk material

work, *W* energy transferred due to the action of a force: $W = Fs$

Z

zero of potential energy when all charges in the system are infinitely far apart; any other arrangement will have positive or negative potential energy

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Key

element name
atomic number
symbol
atomic weight*

* standard atomic weight based on 12 C.
( ) indicates mass number of longest-lived isotope
For higher-precision values for atomic masses, visit ciaaw.org

s-block
p-block
d-block transition metals
d-block lanthanides and actinides

He gas at room temperature
Br liquid at room temperature
Tc synthetic (does not occur naturally)
Li solid at room temperature

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
hydrogen **1** **H** 1.008																	helium **2** **He** 4.003
lithium **3** **Li** 6.941	beryllium **4** **Be** 9.012											boron **5** **B** 10.81	carbon **6** **C** 12.01	nitrogen **7** **N** 14.01	oxygen **8** **O** 16.00	fluorine **9** **F** 19.00	neon **10** **Ne** 20.18
sodium **11** **Na** 22.99	magnesium **12** **Mg** 24.31											aluminium **13** **Al** 26.98	silicon **14** **Si** 28.09	phosphorus **15** **P** 30.97	sulfur **16** **S** 32.06	chlorine **17** **Cl** 35.45	argon **18** **Ar** 39.95
potassium **19** **K** 39.10	calcium **20** **Ca** 40.08	scandium **21** **Sc** 44.96	titanium **22** **Ti** 47.87	vanadium **23** **V** 50.94	chromium **24** **Cr** 52.00	manganese **25** **Mn** 54.94	iron **26** **Fe** 55.85	cobalt **27** **Co** 58.93	nickel **28** **Ni** 58.69	copper **29** **Cu** 63.55	zinc **30** **Zn** 65. 38	gallium **31** **Ga** 69.72	germanium **32** **Ge** 72.63	arsenic **33** **As** 74.92	selenium **34** **Se** 78.97	bromine **35** **Br** 79.90	krypton **36** **Kr** 83.80
rubidium **37** **Rb** 85.47	strontium **38** **Sr** 87.62	yttrium **39** **Y** 88.91	zirconium **40** **Zr** 91.22	niobium **41** **Nb** 92.91	molybdenum **42** **Mo** 95.95	technetium **43** **Tc** (98)	ruthenium **44** **Ru** 101.1	rhodium **45** **Rh** 102.9	palladium **46** **Pd** 106.4	silver **47** **Ag** 107.9	cadmium **48** **Cd** 112.4	indium **49** **In** 114.8	tin **50** **Sn** 118.7	antimony **51** **Sb** 121.8	tellurium **52** **Te** 127.6	iodine **53** **I** 126.9	xenon **54** **Xe** 131.3
caesium **55** **Cs** 132.91	barium **56** **Ba** 137.33	**57–71** **lanthanides**	hafnium **72** **Hf** 178.5	tantalum **73** **Ta** 180.9	tungsten **74** **W** 183.8	rhenium **75** **Re** 186.2	osmium **76** **Os** 190.2	iridium **77** **Ir** 192.2	platinum **78** **Pt** 195.1	gold **79** **Au** 197.0	mercury **80** **Hg** 200.6	thallium **81** **Tl** 204.4	lead **82** **Pb** 207.2	bismuth **83** **Bi** 209. 0	polonium **84** **Po** (209)	astatine **85** **At** (210)	radon **86** **Rn** (222)
francium **87** **Fr** (223)	radium **88** **Ra** (226)	**89–103** **actinides**	rutherfordium **104** **Rf** (267)	dubnium **105** **Db** (268)	seaborgium **106** **Sg** (271)	bohrium **107** **Bh** (270)	hassium **108** **Hs** (269)	meitnerium **109** **Mt** (278)	darmstadtium **110** **Ds** (281)	roentgenium **111** **Rg** (282)	copernicium **112** **Cn** (285)	nihonium **113** **Nh** (286)	flerovium **114** **Fl** (289)	moscovium **115** **Mc** (289)	livermorium **116** **Lv** (293)	tennessine **117** **Ts** (294)	oganesson **118** **Og** (294)

lanthanum **57** **La** 138.9	cerium **58** **Ce** 140.1	praseodymium **59** **Pr** 140.9	neodymium **60** **Nd** 144.2	promethium **61** **Pm** (145)	samarium **62** **Sm** 150.4	europium **63** **Eu** 152.0	gadolinium **64** **Gd** 157.3	terbium **65** **Tb** 158.9	dysprosium **66** **Dy** 162.5	holmium **67** **Ho** 164.9	erbium **68** **Er** 167.3	thulium **69** **Tm** 168.9	ytterbium **70** **Yb** 173.0	lutetium **71** **Lu** 175.0
actinium **89** **Ac** (227)	thorium **90** **Th** 232.0	protactinium **91** **Pa** 231.0	uranium **92** **U** 238.0	neptunium **93** **Np** (237)	plutonium **94** **Pu** (244)	americium **95** **Am** (243)	curium **96** **Cm** (247)	berkelium **97** **Bk** (247)	californium **98** **Cf** (251)	einsteinium **99** **Es** (252)	fermium **100** **Fm** (257)	mendelevium **101** **Md** (258)	nobelium **102** **No** (259)	lawrencium **103** **Lr** (266)

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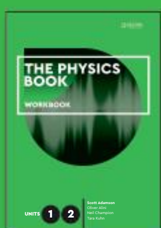

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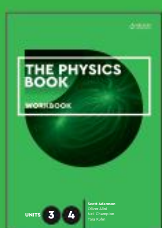

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