General Mathematics Units 3 & 4

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First published in 2025

Insight Publications Pty Ltd
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Insight VCE Revision Questions: General Mathematics Units 3 & 4

ISBN: 9781923016880

Publisher: Robert Beardwood
Project manager: Charlotte Long
Reviewer and writer: Sarah Carmody
Editor: Maryanne Park
Proofreader: Karen Jayne
Cover designer: Melisa Paredes
Internal designer: Bec Yule @ Red Chilli Design
Typesetter: Aptara®, Inc.
Printed by Markono Print Media Pte Ltd

Insight Publications acknowledges the Traditional Custodians of the Country on which we meet and work, the Boonwurrung People of the Kulin Nation. We pay our respects to their Elders past and present, and extend that respect to all Aboriginal and Torres Strait Islander peoples.

Contents

Introduction

Insight's *VCE Revisions Questions: General Mathematics Units 3 & 4* contains questions, worked solutions, mark allocations and tips to help you develop skills for your assessment tasks. The questions cover all areas of study in VCE General Mathematics. A good habit to implement is to test yourself by working through this resource. The process of actively recalling information assists with deeper learning, and you will be able to identify any errors or omissions in your working by comparing your answers with the provided solutions.

Questions are grouped by topics within each area of study, then under the headings 'Exam 1' (multiple-choice questions) and 'Exam 2' (short-answer and extended-answer questions).

Calculator screenshots are included in some worked solutions; for reasons of space, these are all from a TI-Nspire CAS calculator.

By using this resource as part of your study regime throughout the year, you will be prepared for questions you may encounter in your end-of-year VCE exam.

We wish you well with your studies.

The Insight Team

Questions

Area of Study 1 Data analysis – Investigating data distributions

EXAM 1

Use the following information to answer Questions 1–3.

The canteen manager at a secondary school collected students' preferred lunch choices prior to the school athletics carnival.

The number of students from each sporting house (Yellow, Green, Blue and Purple) who selected each lunch option is shown in the two-way table below.

Lunch choice	***Sporting house***			
	Yellow	Green	Blue	Purple
Pizza	82	76	52	51
Pie	76	64	47	49
Salad roll	63	58	54	43
Sushi	71	63	61	48
Total	292	261	214	191

Question 1

The **percentage** of students in Green house who selected a salad roll for their lunch is closest to

A. 5% **B.** 20%

C. 21% **D.** 22%

Question 2

The variables *sporting house* (Yellow, Green, Blue, Purple) and *lunch choice* (pizza, pie, salad roll, sushi) are

A. both nominal variables.

B. both ordinal variables.

C. a nominal variable and an ordinal variable respectively.

D. an ordinal variable and a nominal variable respectively.

Question 3

The most suitable graphical tool to display the data shown in the two-way table would be a

A. histogram.

B. dot plot.

C. scatterplot.

D. segmented bar chart.

Question 4

The stem plot below displays the amount of money spent at the school canteen, per week, by a sample of 34 students.

money spent **key: 3 | 5 = $3.50**

3	2 3 3 4 4 4
3	5 5 6 7
4	
4	6 6 6 7 9
5	1 2 2 3 4
5	5 5 7 9 9
6	0 1 1 2 4
6	7 7 9
7	1

The median amount of money spent at the canteen per week is

A. $2.00

B. $2.20

C. $4.60

D. $5.20

Use the following information to answer Questions 5 and 6.

The weights of female students in Year 9 at a particular secondary school are approximately normally distributed with a mean of 54.6 kg and a standard deviation of 6.2 kg.

Question 5

There are 186 Year 9 females at the secondary school.

The expected number of Year 9 females with weights between 42.2 kg and 73.2 kg is closest to

A. 97

B. 126

C. 177

D. 181

Question 6

A particular Year 9 student, Caterina, has a standardised weight of $z = -1.24$. Her actual weight, rounded to the nearest kilogram, is closest to

A. 46 kg

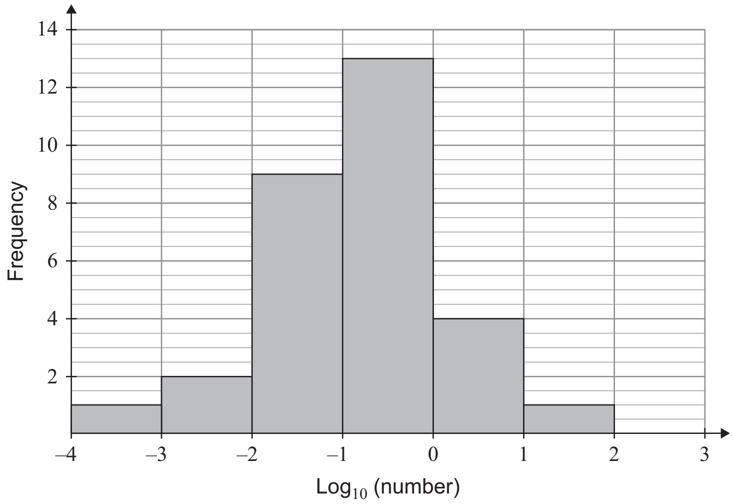
B. 47 kg

C. 50 kg

D. 53 kg

Question 7

The histogram below shows the distribution of a data set that has been plotted on a $\log_{10}$ scale.



Based on the histogram, the percentage of data values less than 0.01 is closest to

A. 3%

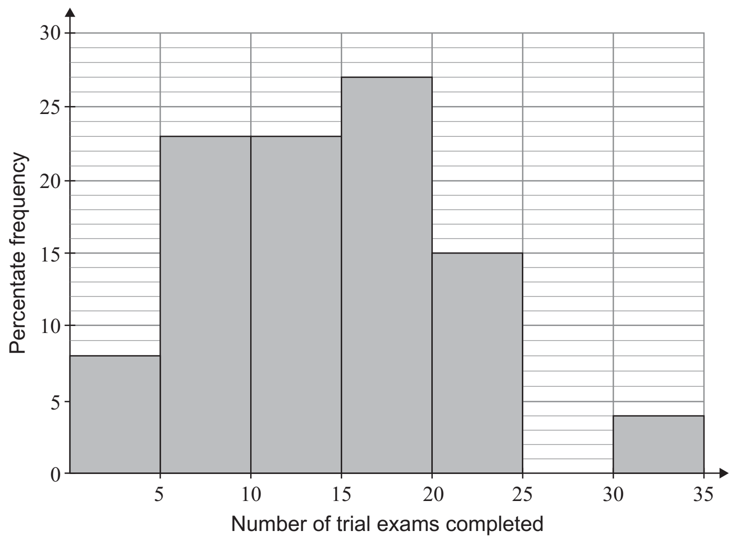
B. 10%

C. 43%

D. 86%

Question 8

The histogram below shows the *number of trial exams completed* by a class of Year 12 students in the final month before their General Mathematics exam.



The median *number of trial exams completed* lies within the interval

A. 0–5

B. 5–10

C. 10–15

D. 15–20

Question 9

The Year 12 teacher ran a revision session at lunchtime, once a week, leading up to the exams. The table below shows the number of students who attended the study session over an eight-week period.

Week	1	2	3	4	5	6	7	8
No. of students attending	13	15	7	12	x	16	17	21

If the mean number of students attending the study sessions is 14.0, the value of x is

A. 3

B. 11

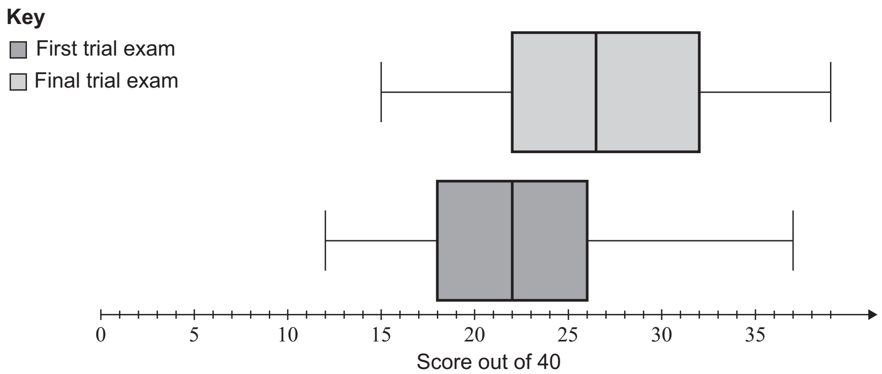
C. 12

D. 13

Use the following information to answer Questions 10–12.

The Year 12 teacher continued to perform statistical analysis on the scores of his Year 12 students. This time the teacher considered the score out of 40 on the students' first multiple-choice trial exam and the score achieved on the final multiple-choice trial exam. The teacher does not give half marks for this exam.

These results are represented in the parallel boxplots below.



Question 10

The variable *score out of 40* can be classified as

A. categorical data.

B. numerical, continuous data.

C. categorical, nominal data.

D. numerical, discrete data.

Question 11

The median *score out of 40* for the Year 12 students' first trial exam is

A. 12

B. 17

C. 22

D. 26

Question 12

The IQR of the boxplot for the Year 12 students' final trial exam is

A. 4.5

B. 5

C. 10

D. 11.5

Use the following information to answer Questions 13 and 14.

The stem plot below displays the *weight,* in kilograms, of the first 25 babies born at a local hospital in 2019.

birth weight **key: 0 | 1 = 0.1**

0	
1	7 9
2	2 4 6 7 8
3	1 2 3 3 4 5 5 6 6 7 9
4	2 3 3 5 7 8
5	1

Question 13

The percentage of babies born with a *weight* between 2.0 and 4.0 kilograms is

A. 4%

B. 8%

C. 36%

D. 64%

Question 14

The median *weight* of the newborn babies, in kilograms, is

A. 3.3

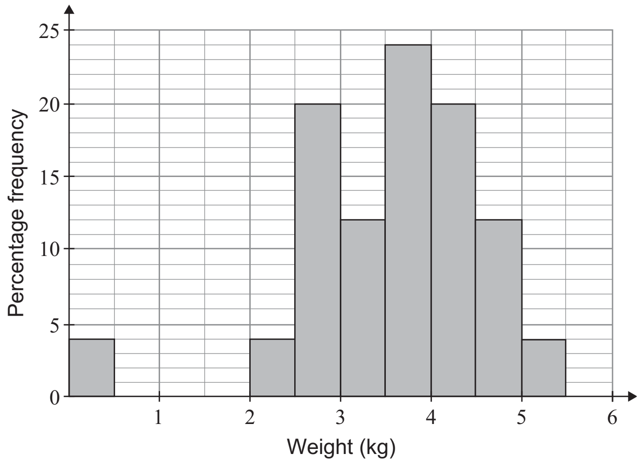
B. 3.4

C. 3.5

D. 3.6

Question 15

The percentage frequency histogram below displays the *weight*, in kilograms, of the next 25 babies born at the local hospital in 2019.



The percentage of babies born who are above the state average of 3.5 kilograms at birth is

A. 38%

B. 40%

C. 50%

D. 60%

Use the following information to answer Questions 16–18.

The birth weight of Victorian babies, born at 39 weeks or later, according to 2018 statistics, is approximately normally distributed with a mean of 3.3 kilograms and a standard deviation of 300 grams.

Question 16

A baby selected at random from the sample used has a standardised birth weight of $z = 0.8$.

The baby's actual birth weight, in kilograms, is

A. 3.54

B. 3.50

C. 3.06

D. 2.64

Question 17

What percentage of babies born in Victoria can be expected to weigh between 2.7 and 3.9 kilograms at birth?

A. 64.0%

B. 66.5%

C. 81.5%

D. 95.0%

Question 18

A sample of 150 babies was chosen for closer studies.

The predicted number of these babies with a birth weight less than 3.0 kilograms or greater than 3.9 kilograms is closest to

A. 17

B. 26

C. 27

D. 28

Question 19

The stem plot below shows the distribution of mathematics *test scores* for a class of 15 students.

test scores **key: 1 | 2 = 12** ***n* = 15**

0	5 6
1	0 1 3
2	3 3 4 5
3	2 5 6
4	9
5	0 0

The percentage of *test scores* that are below 25 is closest to

A. 8%

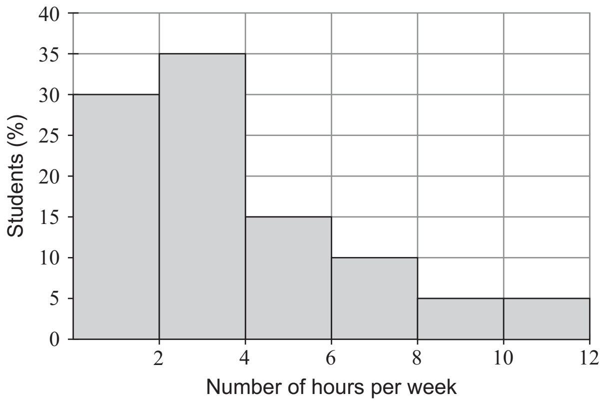
B. 9%

C. 15%

D. 53%

Question 20

The histogram below shows the *number of hours per week* that students participate in cocurricular activities, in a school of 800 students.



The upper quartile (Q_3) falls into which interval?

A. 2 – < 4 hours

B. 4 – < 6 hours

C. 6 – < 8 hours

D. 8 – < 10 hours

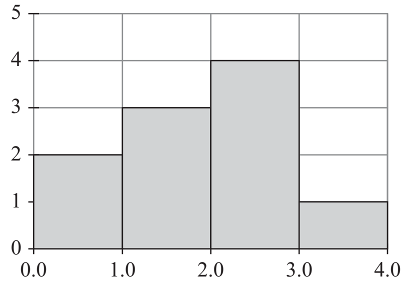
Question 21

The following scores are represented in a histogram with a log scale on the x-axis.

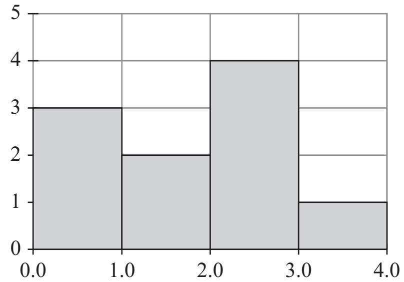
3 7 10 25 87 112 250 812 970 1102

Which one of the log histograms below best represents the data?

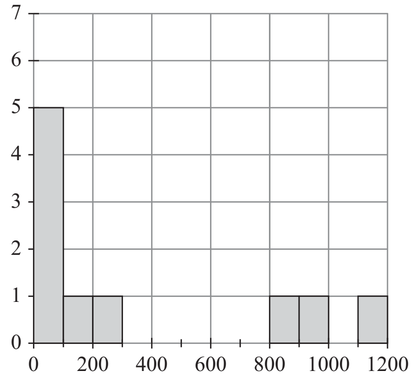
A.



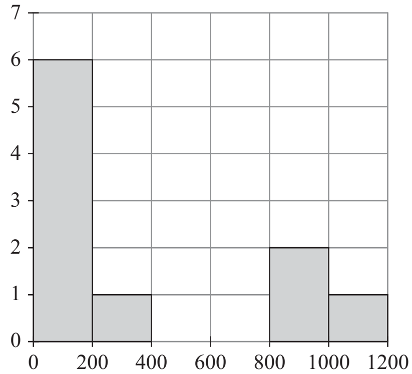
B.



C.



D.



Question 22

The results of a 400-metre race were normally distributed. The mean race time was 138 seconds and had a standard deviation of 6 seconds.

Given that Janet's time was 130 seconds, it can be concluded that

A. she performed in the top 50% but not the top 16%.

B. she performed in the top 16% but not the top 2.5%.

C. she performed in the top 2.5% but not the top 0.15%.

D. she performed in the bottom 50% but not the bottom 16%.

Question 23

An outlier test was conducted on a dataset. An upper fence of 63 was calculated.

If the upper quartile (Q_3) is 45, the value of the lower fence is

A. 12

B. 15

C. 18

D. 33

Question 24

Information relating to the following seven variables was asked for in a questionnaire:

- *name*
- *street address*
- *postcode*
- *occupation*
- *place of birth*
- *number of children*
- *height*.

The number of these variables that are discrete numerical variables is

A. 0

B. 1

C. 2

D. 3

Use the following information to answer Questions 25 and 26.

The annual income of 14 Australians was recorded and is shown in the table below.

Annual income ($)
9550
17 500
27 000
34 800
45 400
53 100
55 750
68 350
102 300
128 000
198 000
212 200
542 000
2 304 000

Question 25

The upper fence is

A. 163 200

B. 198 000

C. 210 000

D. 442 800

Question 26

Which one of the following would be the best way to represent the data?

A. Boxplot

B. Segmented bar chart

C. Two-way frequency table

D. Log scale histogram

Question 27

The distance, in kilometres, that 18 students travel to school from home is recorded and the following summary statistics are calculated.

Min.	First quartile	Median	Third quartile	Max.	Mean	Standard deviation
0.9	3.2	4.7	7.9	12.7	5.4	3.5

One student moves house, which changes her distance travelled from 4.3 km to 4.6 km.

What impact will this have on the statistics shown above?

A. The range will increase.

B. The mean will increase.

C. The interquartile range (IQR) will increase.

D. The median will increase.

Question 28

The stem plot below represents the number of students enrolled in Year 7 over 20 years at a local high school.

key: 9 | 6 = 96 **n = 20**

Stem	Leaf
9	6
10	4
11	2 5 8
12	3 6 7 7 8
13	2 4 4 6 6 6 9
14	3 5 5

The distribution of these data are best described as

A. a positive association.

B. a negative association.

C. positively skewed.

D. negatively skewed.

Question 29

Shoppers were asked to rate the customer service at a store. They were asked to rate the *speed of service* (1 = fast, 2 = average, 3 = slow) and their *likelihood of returning* to the store (yes, no).

These variables are

A. a numerical and ordinal variable respectively.

B. an ordinal and nominal variable respectively.

C. a nominal and ordinal variable respectively.

D. both ordinal variables.

Use the following information to answer Questions 30 and 31.

An exam was undertaken at four schools, with the results normally distributed at each school.

	Number of students	Mean ($\bar{x}$)	Standard deviation (Sx)
School A	120	64	4
School B	165	70	3
School C	112	65	6
School D	90	72	5

Question 30

Mostyn achieved a score of 73 and was informed he was in the top 2.5% of students who undertook the exam in his school.

The school Mostyn attends is

A. School A.

B. School B.

C. School C.

D. School D.

Question 31

76 students from one school achieved a standardised score within one standard deviation of the school mean.

The school they attend is

A. School A.

B. School B.

C. School C.

D. School D.

Question 32

Percentaged segmented bar charts would be an appropriate graphical tool to investigate the association between which of the following pairs of variables?

A. height (in cm) and weight (in kg) of 18-year-old males

B. height of 18-year-old males (in cm) and country of birth (Australia, China and Germany)

C. fitness levels (below average, average, above average) of 18-year-old males and country of birth (Australia, China and Germany)

D. fitness levels (below average, average, above average) and weight (in kg) of 18-year-old males

EXAM 2

Question 1 (5 marks)

A sample of travellers on an evening train was asked how many hours (to the nearest hour) they work in a week. The data were sorted into two categories, based on their age.

The results of this survey are shown in the back-to-back stem plot below.

Key: 1 | 0 = 10 hours

18–24 years	Stem	25–30 years
4 4 3 3	**0**	
9 9 8 8 8 6 6 5	**0**	
4 4 2 2 2 0 0	**1**	0 1 2 2
9 9 9 8 8 7 6 5	**1**	8 9
3 3 2 1 1	**2**	2 3 4
7 7 6 5 5 5 5 5	**2**	5 5 6 8 9
2 2 1	**3**	1 3 3 4
	3	5 5 6 7 8 8
5	**4**	0 0 2 2 4 4
	5	2 2 3 3 4
	5	8

a. What type of data is the variable *hours worked*? 1 mark

b. What percentage of 18–24-year-olds work more than 20 hours a week?
Round your answer to one decimal place. 1 mark

c. Find the median for the data gathered on *hours worked* for those in the 18–24 years category. 1 mark

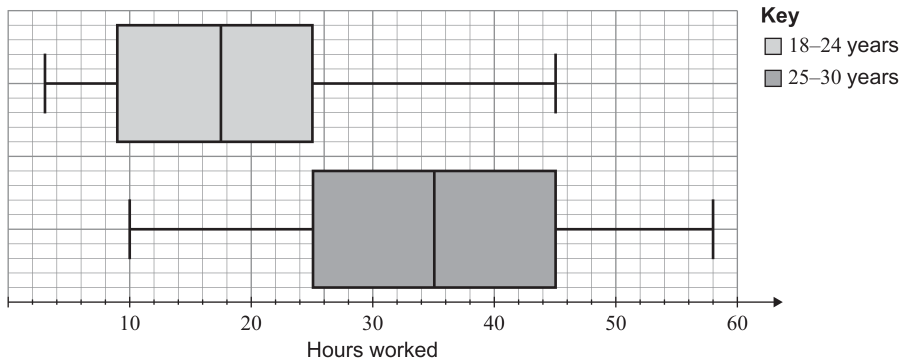
d. The statistician conducting the survey suspects that the data for the 18–24 years age group contain an outlier at 45 hours.

Is the statistician correct?

Explain your answer, showing a calculation to support your argument. 2 marks

Question 2 (3 marks)

A boxplot has been drawn for the *hours worked* data gathered. These boxplots are shown below.



a. Describe the boxplot for the 25–30 years age group, in terms of shape, centre and spread. 1 mark

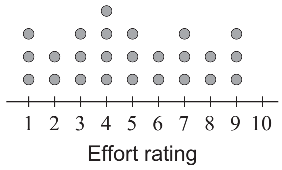
b. Which measure of centre, mean or median, would be most appropriate for a boxplot showing a positive skew?

Explain your answer. 2 marks

Question 3 (3 marks)

The mathematics teacher continued her survey by asking the 25 students to rate their effort in mathematics in previous years of study on a scale from 1 to 10.

The information obtained from the students is displayed in the dot plot below.



a. Find the median *effort rating* given by the students. 1 mark

b. Complete the following statement. 1 mark

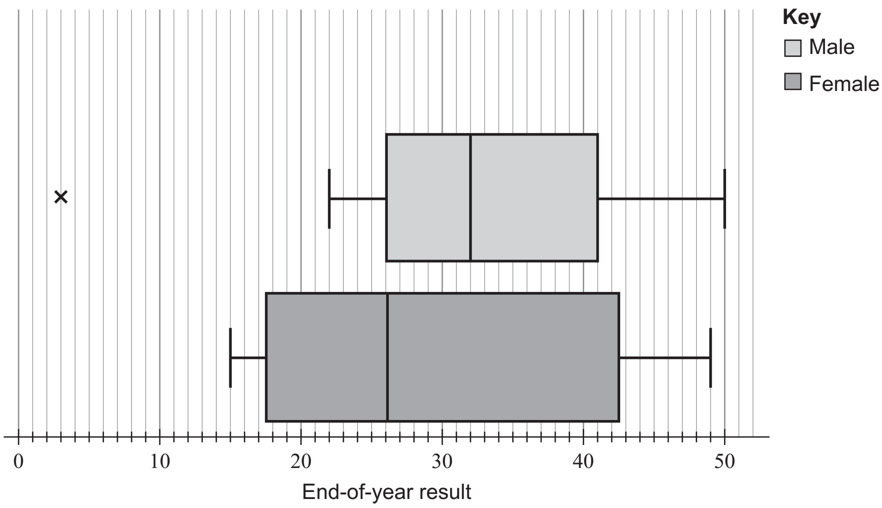
☐ per cent of students gave an *effort rating* between 3 and 7.

c. Is the variable *effort rating* numerical discrete or numerical continuous? 1 mark

Question 4 (5 marks)

End-of-year results for the Year 11 mathematics exam were recorded by the teacher as a score out of 50. The teacher was interested in determining whether *gender* is related to *end-of-year result.*

The students' Year 11 mathematics results are shown in the graph below.



a. Describe the shape of the boxplot for female *end-of-year result*. 1 mark

b. Complete the following five-number summary for male *end-of-year result*. 2 marks

Gender	Minimum	First quartile	Median	Third quartile	Maximum
Male		26	32		50
Female	15	17.5	26	42.5	49

c. Show that the male student who received a result of 3 out of 50 for his *end-of-year result* is an outlier. 2 marks

Question 5 (9 marks)

Table 1

Destination	Flights per month	Flights delayed more than 5 minutes	Minutes delayed
Sydney	186	49	85
Perth	124	31	55
Brisbane	155	41	65
Adelaide	65	17	27
Hobart	94	24	126
Cairns	65	15	285
Darwin	31	10	314
Townsville	54	14	384
Launceston	42	12	235
Uluru	80	21	155
Ballina (Byron)	110	28	43
Sunshine Coast	125	33	62

The data in Table 1 shows a range of scheduled flights, departing from Melbourne Airport, run by a local airline, Airsetter Airlines.

The four variables in this data set are:

- *destination* – name of city
- *flights per month* – total number of flights
- *flights delayed more than 5 minutes* – total number of flights delayed more than 5 minutes
- *minutes delayed* – average minutes delayed in October 2018.

a. How many variables are numerical, discrete variables? 1 mark

A flight is classified as 'delayed' if the departure time is greater than 5 minutes after the scheduled time.

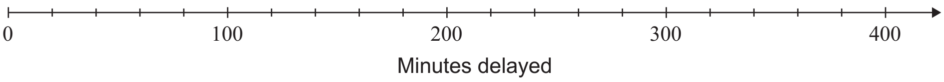
b. What percentage of flights from Melbourne to Townsville was classified as delayed flights?
Round your answer to one decimal place. 1 mark

c. Complete the following five-number summary for the variable *minutes delayed*. 1 mark

Table 2

Minimum	First quartile	Median	Third quartile	Maximum
27	58.5		260	

d. Using the five-number summary, sketch the boxplot for the variable *minutes delayed* on the scale provided below. 2 marks



e. Describe the shape of the distribution of the total *minutes delayed* for the airline. 1 mark

f. Complete the following sentence by writing the correct numerical values in the appropriate boxes. 1 mark

For the distribution of *minutes delayed*, the middle 50% of the data lies between [ ] and [ ] minutes.

g. Write down a calculation to show that the value 27 is **not** an outlier for this data set. 2 marks

Question 6 (4 marks)

Table 1 shows the *day number* and the *number of calories burned* by a gym patron during each 1.5 hour workout for the first 13 days of April 2019.

Table 1

Day number	Number of calories burned	Day number	Number of calories burned
1	630	8	910
2	726	9	879
3	905	10	637
4	558	11	716
5	672	12	847
6	775	13	690
7	821		

a. What type of data is the variable *number of calories burned*? 1 mark

b. Complete the five-number summary for the data *number of calories burned* given in Table 1.

Round your answers to the nearest whole number. 2 marks

Minimum	First quartile	Median	Third quartile	Maximum
558		726	863	

c. Calculate the percentage of days on which the gym patron burned less than 700 calories.

Round your answer to one decimal place. 1 mark

Question 7 (2 marks)

For a particular cycle class, the *number of calories burned* by each gym patron who participates in the class is approximately normally distributed with a mean of 375 calories and a standard deviation of 3 calories.

a. Determine the percentage of patrons who will burn less than 369 calories. 1 mark

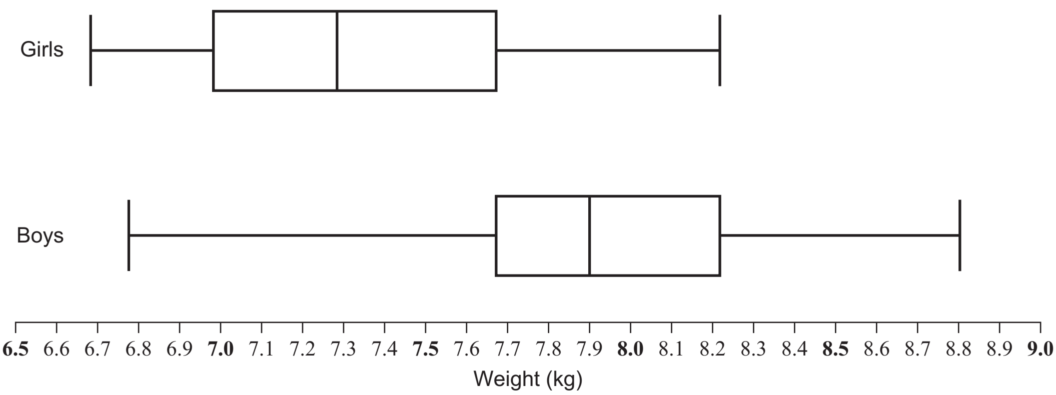
b. 500 patrons participate in the cycle class over a period of one week.

How many of these people can be expected to burn between 375 and 381 calories during the cycle class? 1 mark

Question 8 (4 marks)

A research study is conducted on 2300 babies from birth until they are 12 months old. Of these babies, 1100 were girls and 1200 were boys.

The data on the weight of the babies at six months of age for both boys and girls is shown below.



Weight at 6 months (kg)	Min.	Q_1	Median	Q_3	Max.
girls	6.7	7.0	7.3	7.7	8.2
boys	6.8	7.7	7.9	8.2	8.8

a. How many babies weighed

i. more than 8.2 kg? 1 mark

ii. less than 7.7 kg? 1 mark

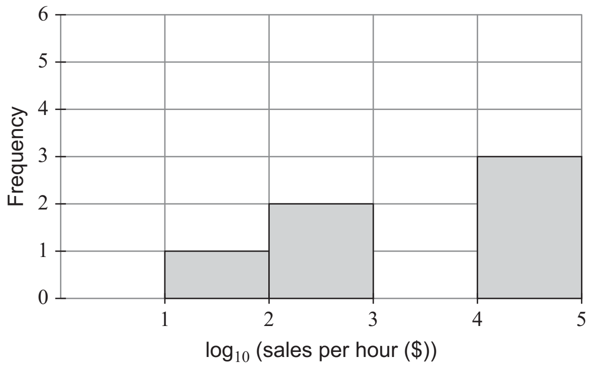
b. It is realised that the boxplot for boys has ignored outliers in the data.

Show that 6.8 is an outlier. 2 marks

Question 9 (1 mark)

After work, Farah visits a homewares store next to the supermarket.

The histogram below shows the distribution of *sales per hour* ($) on a particular day at the homewares store. The histogram has been plotted on a $\log_{10}$ scale.



There were seven hours during this day in which the *sales per hour* were between $100 and $10 000.

Draw a bar on the histogram above to represent the missing information accurately.

(Answer on the histogram above.)

Area of Study 1 Data analysis – Investigating associations between two variables and modelling linear associations

EXAM 1

Use the following information to answer Questions 1 and 2.

The *average number of servings of vegetables* eaten per day by a sample of people is shown below.

The back-to-back stem plot below compares the data for those *under 18 years* and those *18 years or older*.

key: 3 | 1 = 3.1

under 18 years		**18 years or older**
3	0	
8 5 3 3	1	7 8 8
9 5 5 3 1 1	2	3 4 4 6 8
8 7 4 4	3	4 7 7 7 8 9 9
5 2	4	1 3 5 8 8 8
7	5	3 7

Question 1

The percentage of the sample for which the *average number of servings of vegetables* eaten per day was at least 3.5 is closest to

A. 19% **B.** 22%

C. 41% **D.** 46%

Question 2

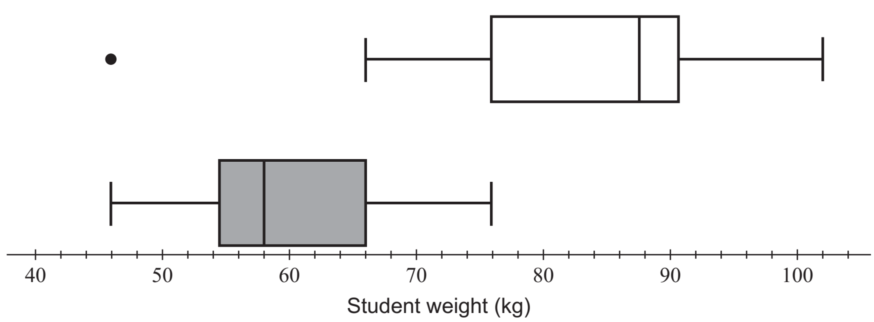
How much higher is the median for *18 years or older* compared to *under 18 years*?

A. 1.2 **B.** 1.6

C. 2.5 **D.** 3.7

Question 3

The following boxplots show the weights of male students in Year 8 (grey) and Year 9 (white).



Which of the following statements is **not** true with regard to the weights of Year 8 and Year 9 male students?

A. 75% of male Year 9 students weigh more than the heaviest male student in Year 8.

B. The distribution of Year 9 male students is negatively skewed with an outlier.

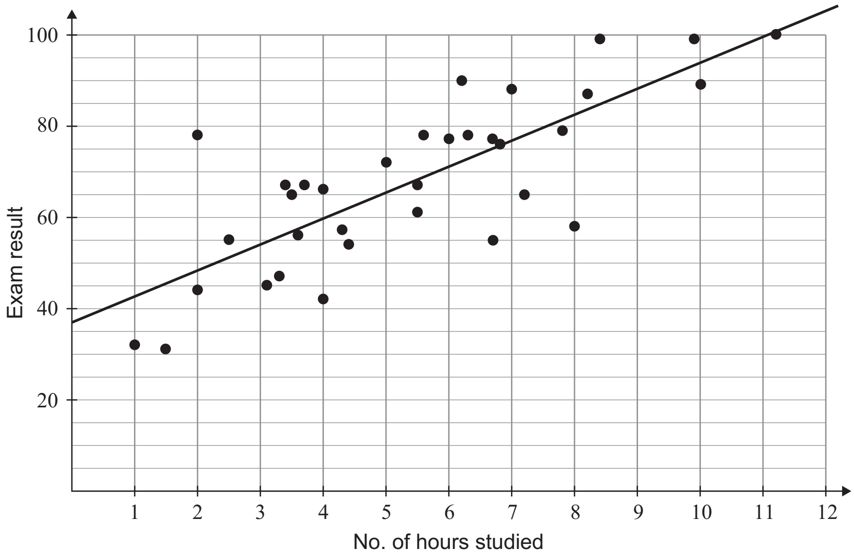
C. The IQR for Year 8 male students is similar to the IQR for Year 9 male students.

D. The mean would be an appropriate measure of centre for the distribution of Year 8 students due to the fact that the distribution does not contain an outlier.

Use the following information to answer Questions 4–6.

The scatterplot below shows the exam results as a percentage (*exam result*) plotted against the number of hours spent studying in the week immediately preceding the exam (*no. of hours studied*).

A least squares regression line has been fitted to the data.



The equation for this least squares regression line is

exam result $= 36.8 + 5.7 \times$ *no. of hours studied*

The Pearson correlation coefficient is $r = 0.7883$.

Question 4

Given the information above, which of the following statements is **not** true?

A. The value of the coefficient of determination is close to 0.62.

B. On average, as the *number of hours studied* increases by 1, the *exam result* increases by 5.7%.

C. 78.8% of the variation in *exam result* can be explained by the variation in *number of hours studied*.

D. Ignoring any outliers, the association between *number of hours studied* and *exam result* can be described as strong, positive and linear.

Question 5

Belinda studied for a total of 7.6 hours in the week preceding her exam and achieved an exam result of 75%. Using the least squares line, the residual value for Belinda's exam result is closest to

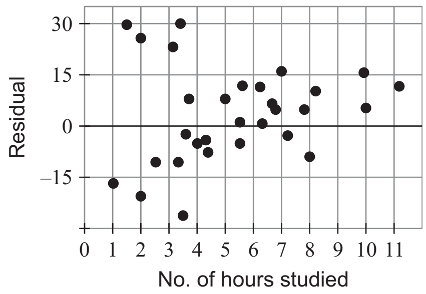
A. –389 **B.** –5

C. 5 **D.** 80

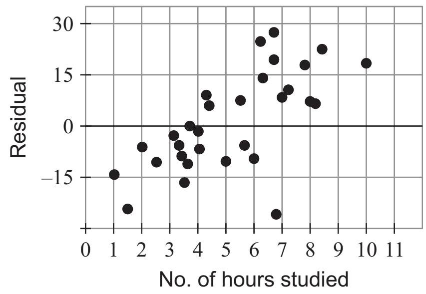
Question 6

Which of the following could be the residual plot for the relationship between *number of hours studied* and *exam result* scores?

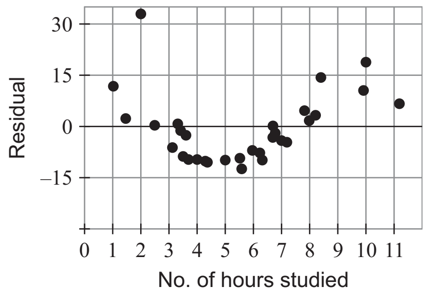
A.



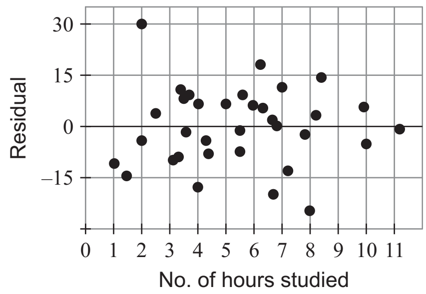
B.



C.



D.

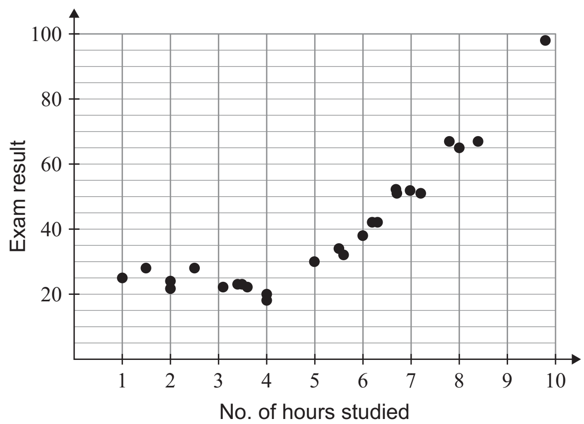


Question 7

The table below gives the exam results and number of hours spent studying for a different group of students.

A non-linear scatterplot for these data is also shown.

No. of hours studied	Exam result	No. of hours studied	Exam result
4	18	5.6	32
5	30	7.8	67
6	38	6.2	42
1	25	3.1	22
2	22	1.5	28
2	24	6.3	42
2.5	28	7.2	51
3.6	22	3.5	23
4	20	6.7	52
7	52	8.4	67
8	65	6.7	52
9.8	98	8.4	67
3.4	23	6.7	51
		5.5	34



A squared transformation will be applied to the variable *no. of hours studied* and can be used to linearise the scatterplot.

The equation of the least squares regression line for this squared transformation is closest to

A. $(\textit{exam result})^2 = 15.4 + 0.76 \times \textit{no. of hours studied}$

B. $\textit{exam result} = 15.4 + 0.76 \times (\textit{no. hours studied})^2$

C. $\textit{exam result} = 15.4 + 0.76 \times \textit{no. of hours studied}$

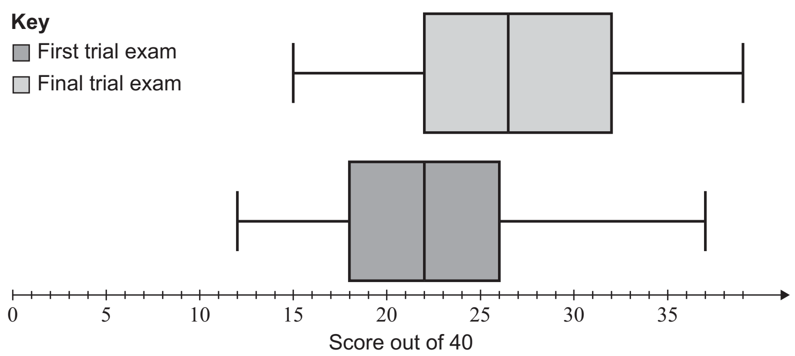
D. $(\textit{no. of hours studied})^2 = 15.4 + 0.76 \times \textit{exam result}$

Question 8

A Year 12 teacher performed a statistical analysis on the scores of his Year 12 students. The teacher considered the score out of 40 on the students' first multiple-choice trial exam and the score achieved on the final multiple-choice trial exam. The teacher does not give half marks for this exam.

These results are represented in the parallel boxplots below.

Student	1	2	3	4	5	6	7	8	9	10	11	12	13
Score on *first trial exam*	18	12	22	13	22	14	14	21	26	20	23	37	19
Score on *final trial exam*	22	15	25	16	25	16	20	30	31	28	25	40	22
Student	14	15	16	17	18	19	20	21	22	23	24	25	26
Score on *first trial exam*	19	23	15	27	24	21	27	16	28	32	24	31	25
Score on *final trial exam*	29	32	21	32	34	24	29	20	36	37	25	34	33



The data displayed in the parallel boxplots supports the contention that, on average, there is an improvement in the students' *score out of 40* between their first trial exam and their final trial exam because

A. the median *score out of 40* has increased from 22 in the first trial exam to 26.5 in the final trial exam.

B. the range of the distribution of *scores out of 40* is 25 for both the first trial exam and the final trial exam.

C. the distribution of *scores out of 40* for the first trial exam is positively skewed, compared to the distribution for the final trial exam, which is approximately symmetric.

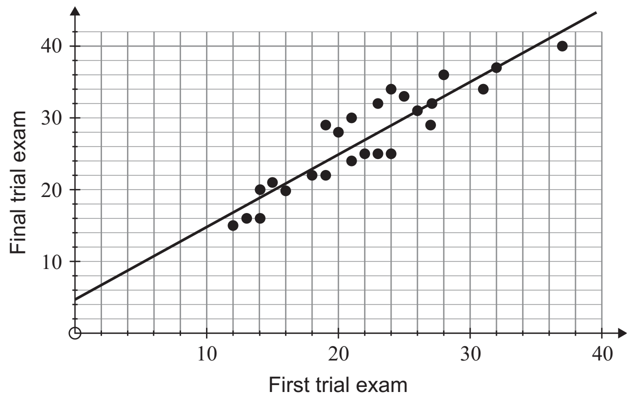
D. the interquartile range for the *scores out of 40* in the first trial exam is greater than that for the final trial exam.

Use the following information in addition to the data provided in Question 8 to answer Questions 9–11.

The Year 12 teacher continued his analysis of the students' trial exams by considering whether an association exists between the students' *first trial exam* and their *final trial exam*.

In order to assist with this, the teacher recorded the data and plotted this on a scatterplot. The value of the Pearson correlation coefficient, r, for the scatterplot is 0.916.

The scatterplot is shown below.



Question 9

The least squares regression equation for the relationship between the *first trial exam* and the *final trial exam* is

A. $\textit{final trial exam} = 4.760 - 1.007 \times \textit{first trial exam}$

B. $\textit{first trial exam} = 4.760 + 1.007 \times \textit{final trial exam}$

C. $\textit{final trial exam} = 1.007 + 4.760 \times \textit{first trial exam}$

D. $\textit{final trial exam} = 4.760 + 1.007 \times \textit{first trial exam}$

Question 10

Which of the following statements is **incorrect**?

A. On average, as the *first trial exam* score increases by 1 mark, the *final trial exam* score decreases by 1.007 marks.

B. A student who achieved a score of 0 marks on their *first trial exam* would achieve a score of approximately 5 marks on their *final trial exam*.

C. 83.91% of the variation in the *final trial exam* score can be explained by the variation in the *first trial exam* score.

D. There is a strong, positive, linear relationship between the *first trial exam* score and the *final trial exam* score for this cohort of Year 12 students.

Question 11

The Year 12 teacher would like to draw a conclusion from the regression analysis, regarding the correlation between his students' *first trial exam* score and their *final trial exam* score.

Which of the following statements is **correct**?

A. If a student achieves a score over 20 marks for their *first trial exam*, they will always achieve a high score in their *final trial exam*.

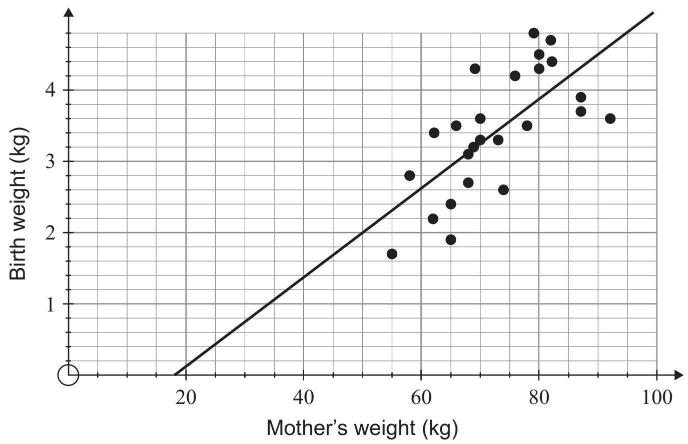
B. A high score in the *first trial exam* will cause a student to get a high score in their *final trial exam*.

C. There is a correlation between *first trial exam* and *final trial exam*. On average, as the *first trial exam* score increased, so did the *final trial exam* score. This suggests that a student with a higher *first trial exam* score will also have a higher *final trial exam* score.

D. On average, there was always an increase in scores between the *first trial exam* and the *final trial exam.*

Use the following information to answer Questions 12–15.

The scatterplot below displays the *birth weight*, in kilograms, of newborn babies, and weight, in kilograms, of their mothers at 39 weeks pregnant. A least squares line has been fitted to the data.



Question 12

Using the least squares line, the predicted value for *birth weight*, when *mother's weight* is 60 kilograms, is approximately

A. 2.6

B. 3.2

C. 3.4

D. 3.6

Question 13

The equation of the least squares line is closest to

A. $\textit{mother's weight} = -1.1203 + 0.063 \times \textit{birth weight}$

B. $\textit{mother's weight} = 1.1203 - 0.063 \times \textit{birth weight}$

C. $\textit{birth weight} = -1.1203 - 0.063 \times \textit{mother's weight}$

D. $\textit{birth weight} = -1.1203 + 0.063 \times \textit{mother's weight}$

Question 14

If the coefficient of determination is 0.6877, the correlation coefficient, *r*, is closest to

A. –0.688

B. –0.473

C. 0.473

D. 0.829

Question 15

Using the least squares line to model the association between *mother's weight* and *birth weight*, the residual value for a baby that is born to a mother weighing 92 kilograms is closest to

A. –2.0 kg

B. –1.1 kg

C. 0.30 kg

D. 2.0 kg

Question 16

The statistical analysis of a set of bivariate data involving variables *x* and *y* resulted in the information displayed in the table below.

Mean	$\bar{x} = 8$	$\bar{y} = 50.53$
Standard deviation	$s_x = 4.47$	$s_y = 18.77$
Correlation coefficient	$r = 0.98$	

Using this information, the equation of the least squares line in the form $y = a + bx$ is

A. $y = -200.18 + 4.1x$

B. $y = 48.66 + 0.23x$

C. $y = 17.6 + 4.1x$

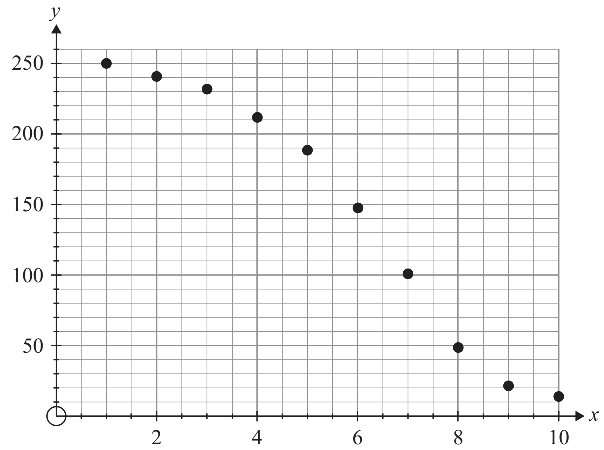
D. $y = 4.1 + 17.6x$

Use the following information to answer Questions 17 and 18.

Jaxon uses the following data to generate the scatterplot below.

x	1	2	3	4	5
y	250	241	232	212	189
x	6	7	8	9	10
y	148	101	49	22	14

The scatterplot shows that the data is non-linear.



Question 17

To linearise the data, Jaxon could apply which of the following sets of transformations?

A. x^2 or $\frac{1}{y}$

B. x^2 or y^2

C. x^2 or $\log y$

D. y^2 or $\log x$

Question 18

Jaxon performs the squared transformation to the variable y.

He then fits a least squares line to the transformed data, with x as the explanatory variable.

Using this line, the predicted value for y when x is equal to 5 is closest to

A. 16 852

B. 361

C. 230

D. 182

Use the following information to answer Questions 19 and 20.

The relationship between the age and price of a car is shown by the equation

$price\ (\$) = 61\,591 - 4164 \times age\ (\text{years})$

The correlation coefficient is −0.9978.

Question 19

Which one of the following statements is **not** true?

A. The car was bought for $61 591 when it was new.

B. 99.78% of the variation in price can be explained by the variation in age.

C. The car depreciates by $4164 for every additional year it ages.

D. There is a strong negative association between the car's age and price.

Question 20

Jose paid $18 000 when the car was 12 years old.

The residual value is

A. −$6377

B. $6377

C. −$11 623

D. $11 623

Use the following information to answer Questions 21 and 22.

The heights of seedlings are measured weekly to track their progress. The data is shown below.

***Age* (weeks)**	0	1	2	3	4	5	6	7	8
***Average height* (cm)**	6.1	6.4	7.5	9.6	12.0	18.2	29.4	46.1	61.2

Question 21

A reciprocal transformation of *average height* is performed.

The equation of the least squares regression line for the transformed data is

A. $\dfrac{1}{age} = 0.168 - 0.021 \times average\ height$

B. $\dfrac{1}{age} = 4.293 + 6.532 \times average\ height$

C. $\dfrac{1}{average\ height} = 4.293 + 6.532 \times age$

D. $\dfrac{1}{average\ height} = 0.168 - 0.021 \times age$

Question 22

A squared transformation of *age* is performed. The equation of the least squares regression line for the transformed data is

$$\text{average height} = 2.43 + 0.856 \times (\text{age})^2$$

At approximately what *age* were the plants when the *average height* was 20 cm?

A. 4.4 weeks

B. 4.5 weeks

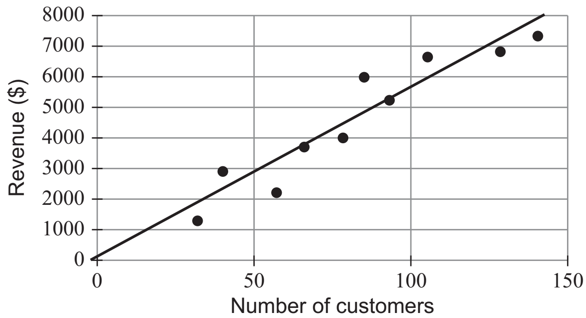
C. 19.6 weeks

D. 20.5 weeks

Use the following information to answer Questions 23–25.

The *number of customers* at a café and the *revenue* received each day is recorded for 10-days and displayed in the scatterplot below.

A least squares line has been fitted to the data.



	Number of customers	Revenue ($)
Mean	82.4	4595
Standard deviation	35.4	2082.3
Correlation coefficient	0.9453	

Question 23

Using the least squares line, when the revenue is at least $200 higher than predicted, the staff receive a small bonus.

How many days did staff receive a bonus over the 10 days?

A. 1

B. 2

C. 3

D. 4

Question 24

What is the coefficient of determination, as a percentage, to three significant figures?

A. 0.894%

B. 0.945%

C. 89.3%

D. 89.4%

Question 25

The y-intercept, a, of the least squares line is best calculated by which one of the following?

A. $0.9453 \times \frac{2082.3}{35.4}$

B. $4595 - 55.60 \times 82.4$

C. $55.60 \times \frac{2082.3}{35.4}$

D. $0.9453 \times \frac{35.4}{2082.3}$

Use the following information to answer Questions 26 and 27.

Data on the cooling of an object in a laboratory freezer was recorded.

These times are shown in the table below.

***Time* (minutes)**	***Temperature* (°C)**
1	210
2	114
3	61
4	45
5	32
6	12
7	8
8	5
9	1.3
10	0.2

Question 26

Which one of the following transformations would be appropriate in order to linearise these data?

A. $\frac{1}{time}, \frac{1}{temp}, \log(time), \log(temp)$

B. $\frac{1}{temp}, \log(temp), time^2$

C. $\log(time), time^2, temp^2$

D. $\frac{1}{time}, \log(time), temp^2$

Question 27

A logarithmic transformation of the explanatory variable is performed.

The equation of the least squares line for the transformed data is

A. $temperature = 182.26 - 203.38 \times time$

B. $\log(time) = 182.26 + 203.38 \times temperature$

C. $temperature = 182.26 - 203.38 \times \log(time)$

D. $\log(temperature) = 182.26 - 203.38 \times \log(time)$

Question 28

Which of the following would **not** represent a positive association?

A. *temperature* and *number of heaters sold*

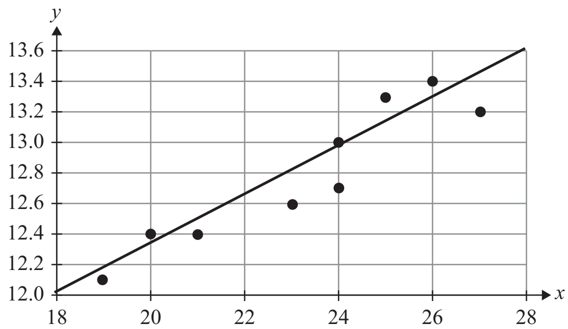
B. *population* and *number of hospitals*

C. *time spent exercising* and *calories burned*

D. *number of cars on the road* and *levels of sound pollution*

Question 29

The scatterplot below shows a least squares line fitted to a scatterplot.



The statistics for the set of data above are shown in the table below.

$\bar{x}$	$\bar{y}$	s_x	s_y	r
23.2	12.8	2.7	0.46	0.9351

The equation of the least squares line is

A. $y = 5.5 - 114.5x$

B. $y = 0.16 + 9.10x$

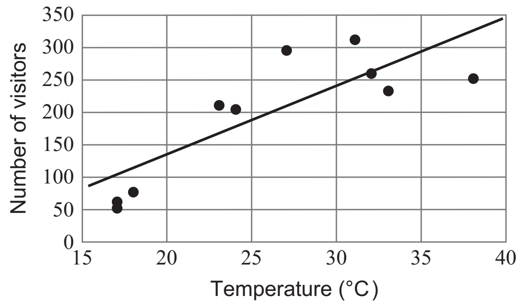
C. $y = -114.5 + 5.5x$

D. $y = 9.10 + 0.16x$

Question 30

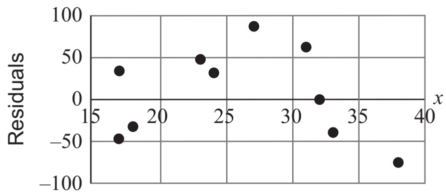
The data below shows the *temperature* and the *number of visitors* to a local pool each day over a 10-day period.

Temperature (°C)	Number of visitors
17	63
23	211
27	294
31	312
33	233
32	260
38	251
24	205
18	77
17	53

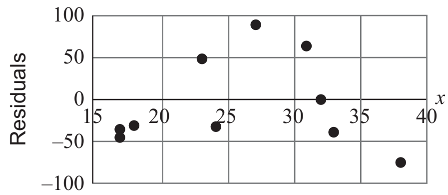


The residual plot that matches the data is

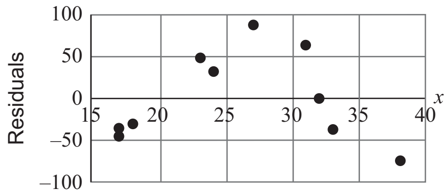
A.



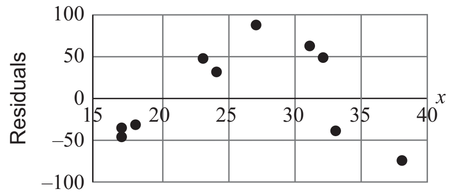
B.



C.



D.



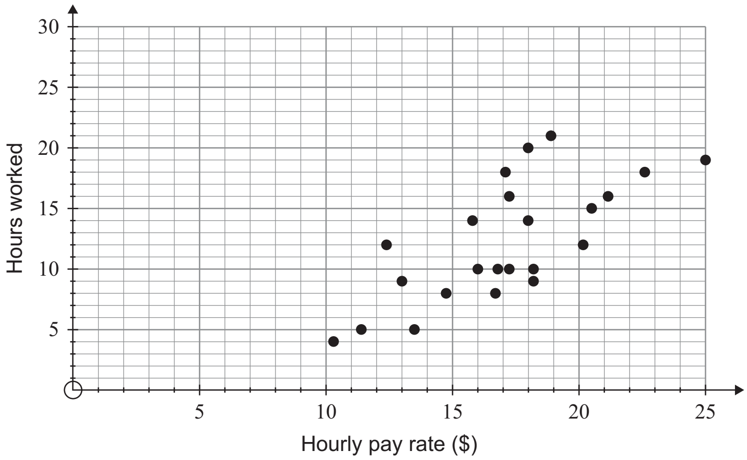
EXAM 2

Question 1 (8 marks)

A small group of train travellers was surveyed on the morning train. Those who specified that they were in the 18–24 years age group were also asked what their *hourly pay rate* was, with the intention of identifying an association between *hours worked* and *hourly pay rate*.

The data are shown in the table below and are also displayed as a scatterplot.

Hourly pay rate ($)	*Hours worked*
11.40	5
18.20	10
10.30	4
12.40	12
13.50	5
16.00	10
13.00	9
14.75	8
16.80	10
16.70	8
18.20	9
25.00	19
17.25	10
15.80	14
20.16	12
21.15	16
22.60	18
18.00	14
20.50	15
17.25	16
18.90	21
17.10	18
18.00	20



a. Use the scatterplot to describe the relationship between *hourly pay rate* and *hours worked* in terms of strength, direction and form. 1 mark

b. Find the value of the Pearson correlation coefficient, rounding your answer to two decimal places. 1 mark

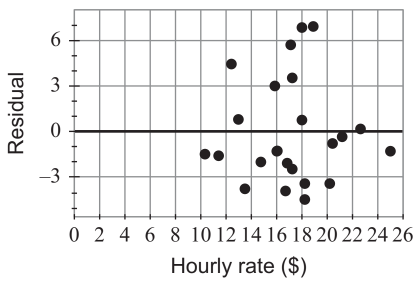
c. Determine the equation of the least squares regression line that can be used to predict the number of *hours worked* from the *hourly pay rate*.

Round the values for the slope and intercept to two significant figures. 2 marks

d. The value of the coefficient of determination for the association between the variables *hourly pay rate* and *hours worked* is 0.5159.

Interpret the coefficient of determination in terms of these variables. 1 mark

e. A residual plot was obtained when the least squares regression line was fitted to the data. This is shown below.



i. Does the residual plot support the assumption that there is a linear association between *hourly pay rate* and *hours worked*?

Explain your answer. 2 marks

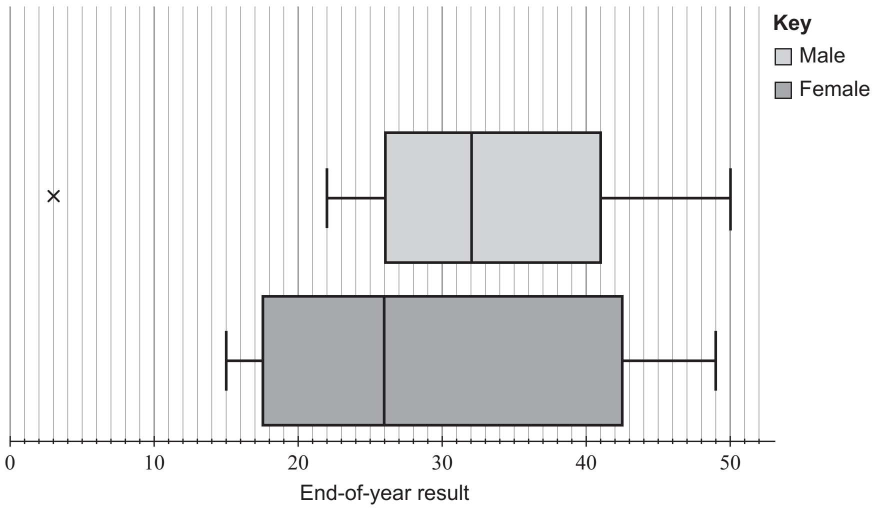
ii. Find the residual value for a train passenger who is paid $25.00 per hour and works 17 hours in a week.

Round your answer to one decimal place. 1 mark

Question 2 (2 marks)

End-of-year results for a Year 11 mathematics exam were recorded by the teacher as a score out of 50. The teacher was interested in determining whether *gender* is related to *end-of-year result*.

The students' Year 11 mathematics results are shown in the graph below.

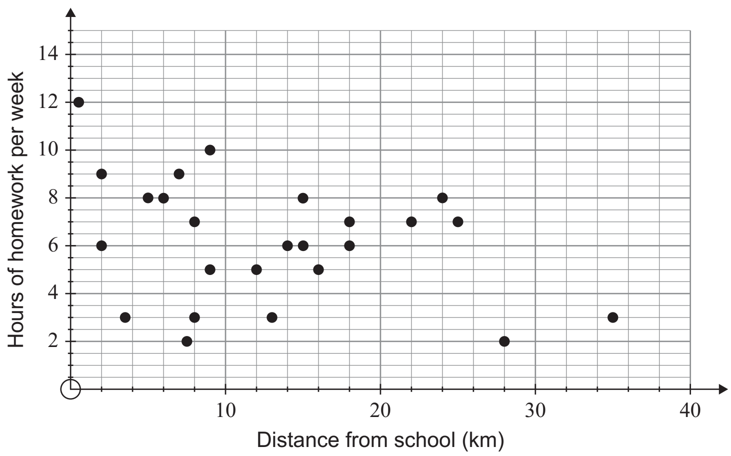


The data shown in the boxplots above suggest that *end-of-year result* is associated with *gender*.

Explain why, quoting the values of an appropriate statistic.

Question 3 (7 marks)

The teacher became concerned about the amount of homework her students were completing, and so began an investigation into the association between *hours of homework per week* and the *distance from school*, in kilometres, for her students. A scatterplot of the data that the teacher gathered is shown below.



The equation of the least squares regression line is

hours of homework per week = 7.46 − 0.0977 × *distance from school*

a. On the scatterplot, draw the least squares regression line. 2 marks

b. Interpret the value of the intercept of the least squares equation in terms of the variables *hours of homework per week* and *distance from school*. 1 mark

c. The value of the coefficient of determination for the scatterplot is 0.1117.

Find the value of the Pearson correlation coefficient, correct to three decimal places. 1 mark

d. The *hours of homework per week* is 3 when the *distance from school* is 8 km.

Determine the residual value if the least squares line is used to predict the *hours of homework per week* for this distance. Round your answer to two decimal places. 1 mark

e. The teacher has a new student in her class who lives 11 kilometres away from school.

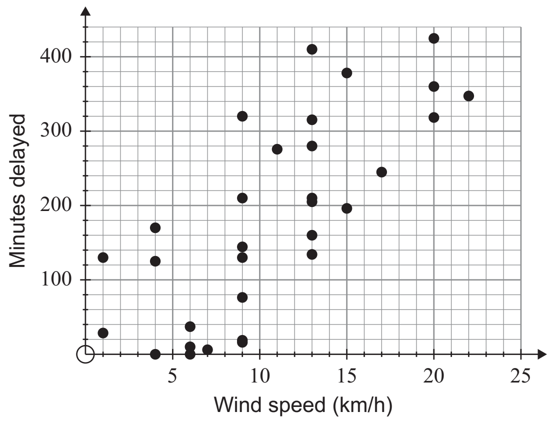
i. Predict the number of *hours of homework per week* that this student can complete, correct to one decimal place. 1 mark

ii. Explain why this is likely a reliable prediction. 1 mark

Question 4 (7 marks)

During October 2018, there was only one scheduled flight between Melbourne and Darwin per day.

The *minutes delayed* and the *wind speed* (km/h) at 9 am in Melbourne for each day in October are plotted on the scatterplot below.



A least squares line is to be fitted to the data with the aim of predicting the number of *minutes delayed* from the *wind speed* on a particular day in Melbourne.

The equation of this line is

$\textit{minutes delayed} = -10.745 + 18.051 \times \textit{wind speed}$

a. Name the explanatory variable in this equation. 1 mark

b. Using the equation of the least squares regression line, show that the number of *minutes delayed* is 242 minutes when the *wind speed* is 14 km/h. 2 marks

c. On a particular day in October, the wind speed was 15 km/h and the flight from Melbourne to Darwin was delayed by 196 minutes.

Determine the residual value when the equation of the least squares line is used to predict the *minutes delayed* when the *wind speed* is 15 km/h.

Round your answer to one decimal place. 2 marks

The value of the correlation coefficient, *r*, is 0.757.

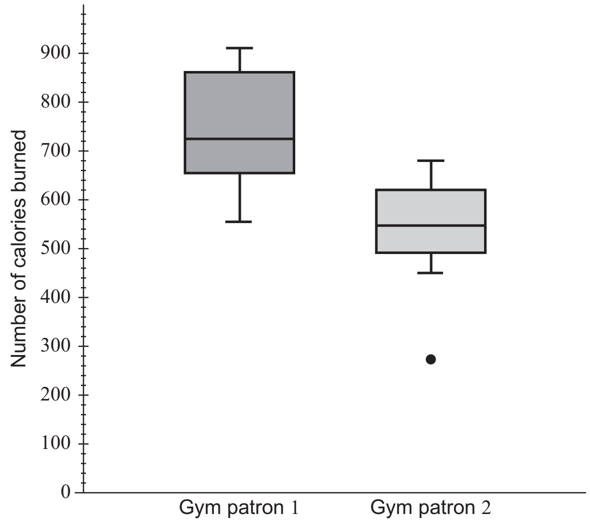
d. Describe the strength of the relationship between *wind speed* and *minutes delayed*. 1 mark

e. What percentage of the variation in the *number of minutes* delayed for flights from Melbourne to Darwin can be explained by the variation in *wind speed* in Melbourne?

Round your answer to the nearest whole number. 1 mark

Question 5 (4 marks)

The parallel boxplots below show the *number of calories burned* by *Gym patron 1* and *Gym patron 2* for a 14-day period. The five-number summary for both gym patrons is also shown.



	Minimum	First quartile (Q_1)	Median	Third quartile (Q_3)	Maximum
Gym patron 1	558	650	720	860	915
Gym patron 2		490	555	615	680

a. What is the minimum number of calories burned by *Gym patron 2*? 1 mark

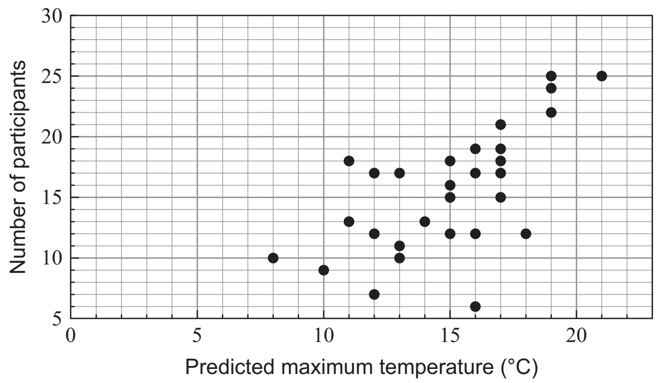
b. On approximately how many days over the 14-day period did *Gym patron 2* burn less calories than the minimum number of calories burned by *Gym patron 1*? 1 mark

c. On a particular day, *Gym patron 2* burned 275 calories.

Explain why this value is an outlier for the data set. 2 marks

Question 6 (8 marks)

The scatterplot below shows the *number of participants* in the morning weights class plotted against the *predicted maximum daily temperature* (°C) on each day during February 2020.



A least squares equation has been fitted to the scatterplot.

The least squares equation is

number of participants = 0.7296 + 1.077 × *predicted maximum temperature*

a. On the scatterplot above, sketch the least squares line using the equation given. 1 mark

b. Interpret the slope of the least squares line in terms of the *predicted maximum temperature* and the *number of participants*. 1 mark

c. Use the equation of the least squares line to find the *predicted maximum temperature* when the *number of participants* is 20.

Round your answer to the nearest degree. 1 mark

d. Show that the residual value is −7 when the *predicted maximum temperature* is 18°C. 1 mark

e. The value of the coefficient of determination for the relationship between the *predicted maximum temperature* and the *number of participants* is 0.4143.

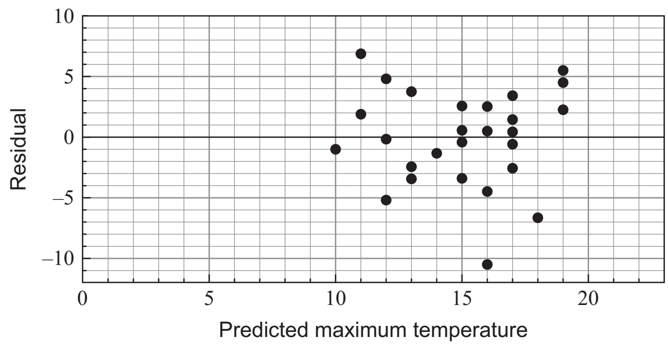
Determine the value of the correlation coefficient, *r*.

Round your answer to two decimal places. 1 mark

f. The mean for *predicted maximum temperature* is 14.97°C.

Calculate the mean for the *number of participants*, rounded to one decimal place. 1 mark

g. The residual plot associated with the least squares line is shown below.



i. There are two residual values missing from the residual plot. Complete the residual plot for a *predicted maximum temperature* of 8°C and 21°C. 1 mark

ii. Does the residual plot confirm the assumption of linearity for the relationship between the *predicted maximum temperature* and the *number of participants*?

Give a reason for your answer. 1 mark

Question 7 (3 marks)

The *median income* ($) for males and females in three age groups is shown in the table below.

	Median income **($)**	
	Sex	
Age **(years)**	**Male**	**Female**
35–44	80 800	52 410
45–54	82 805	54 470
55–64	72 330	50 555

a. Which of the two variables shown, *sex* or *age*, is an ordinal variable? 1 mark

b. Does the data shown support the assumption that the *median income* is associated with *sex*? Refer to the values of an appropriate statistic in your response. 2 marks

Question 8 (3 marks)

A research study is conducted on 1200 babies from birth until they are 12 months old.

It is found that the average weight at birth is 3.25 kg and at 12 months is 9.49 kg. The Pearson correlation coefficient is 0.9364.

A least squares line is fitted to the data.

a. What percentage of the variation in weight can be explained by the variation in age?

Round your answer to one decimal place. 1 mark

b. Write the equation of the least squares line below, using the variables *age* and *weight*.

2 marks

Question 9 (4 marks)

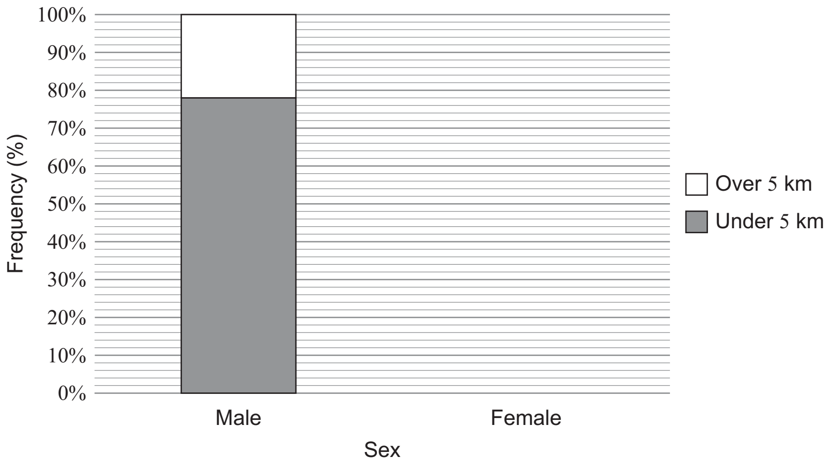
After work, Farah visits a homewares store next to the supermarket.

a. The homewares store records the postcode and sex of its customers over a week.

This information was used to construct the two-way frequency table below.

	Sex		
Distance from store	**Male**	**Female**	**Total**
Under 5 km	45	62	107
Over 5 km	13	35	48
Total	58	97	155

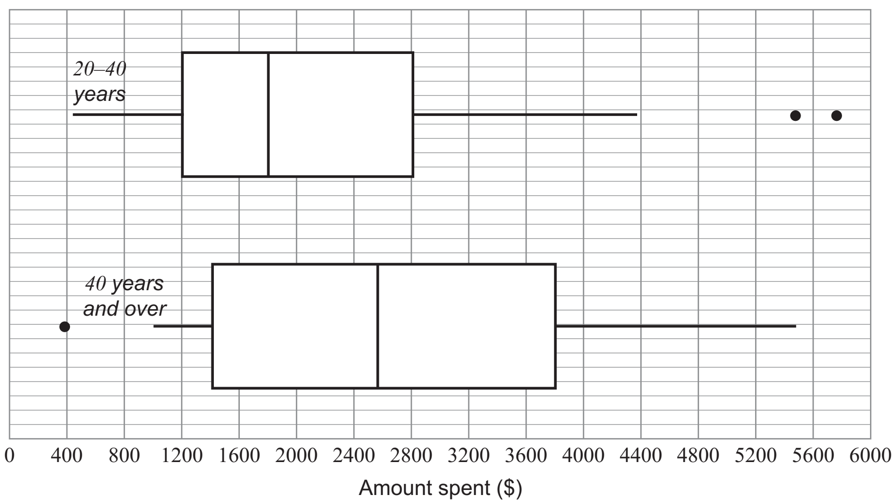
The information from the two-way frequency table is converted into a percentaged segmented bar chart. The data for males has been completed.



Draw the missing column for females on the percentaged segmented bar chart above. 2 marks

(Answer on the percentaged segmented bar chart above.)

b. The boxplots below show the *amount spent* by customers, comparing the age groups *20–40 years* and *40 years and over*.



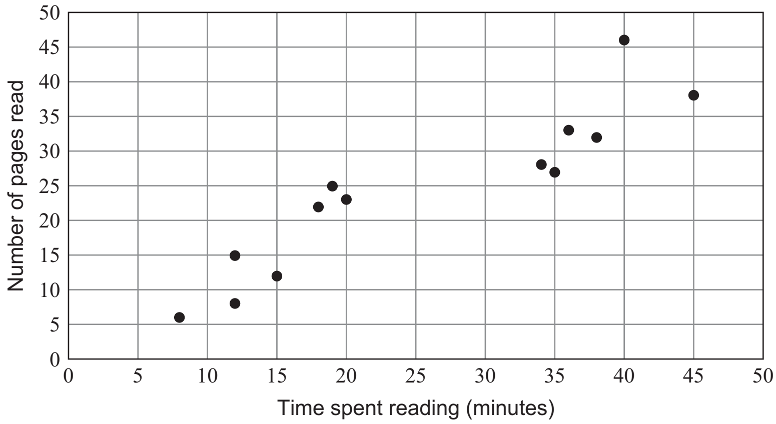
i. Describe the shape of the *amount spent* by those *40 years and over*. 1 mark

ii. There were 52 customers who were between *20–40 years* who spent between \$2800 and \$4400. There were 64 customers who were *40 years and over* who spent between \$1400 and \$3800.

How many customers were there in total? 1 mark

Question 10 (4 marks)

Farah reads novels during her lunch and rest breaks. The amount of *time spent reading* by Farah, in minutes, and the *number of pages read* is shown on the scatterplot below.



A least squares line is fitted to the scatterplot. The equation of this line is

$$\textit{number of pages read} = 2.44 + 0.85 \times \textit{time spent reading}$$

a. Interpret the slope of the least squares line in terms of the *number of pages read* and *time spent reading*. 1 mark

The data from the scatterplot is shown in the table below.

***Time spent reading* (minutes)**	15	35	45	12	8	18	12	34	40	38	19	20	36	
Number of pages read	12	27	38	8	6	22	15	28	46	32	25	23	33	23

b. A value is missing from the table and the scatterplot. The corresponding residual for this value is −0.69.

Determine the missing value. 2 marks

The correlation coefficient for the *time spent reading* and *number of pages read* is 0.9170.

c. Interpret the coefficient of determination in terms of the variables *time spent reading* and *number of pages read*.

Round the percentage to the nearest whole number. 1 mark

Question 11 (4 marks)

The number of staff members rostered on for restocking at the supermarket is measured against the number of items restocked. The data is shown below.

Number of staff	4	5	12	5	4	6	9	15	8	8
Units restocked	220	960	1920	520	190	1470	1790	1990	1840	1760

In order to linearise the data, a $\log_{10}$ transformation is applied and a least squares line is fitted to the transformed equation.

a. Complete an appropriate $\log_{10}$ transformation and write down the equation of the least squares line.

Round the values to four significant figures. 2 marks

b. **i.** A reciprocal transformation is also applied to the data.

The equation of the least squares line fitted to this transformed data is

$$\textit{units restocked} = 2942 - 10\,623 \times \frac{1}{\textit{number of staff}}$$

What is the predicted number of units restocked when 16 staff are rostered on?

Round your answer to the nearest whole number. 1 mark

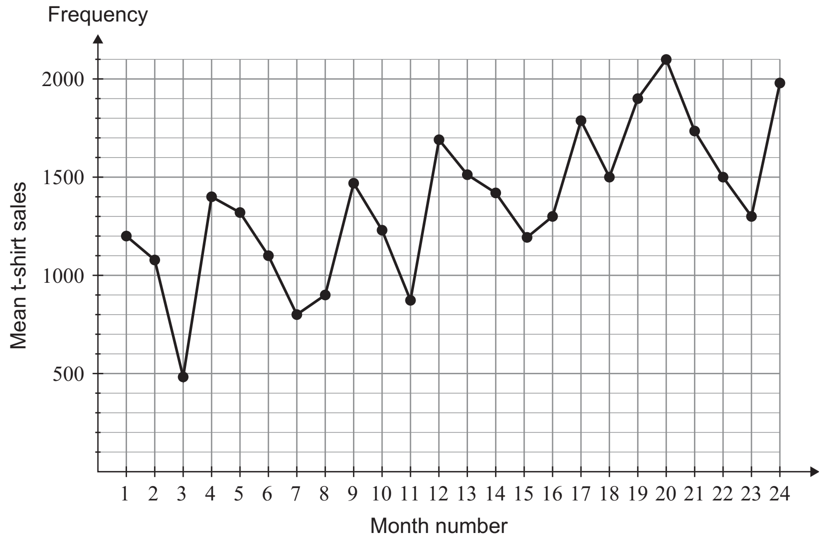
ii. Explain why this prediction may have limited reliability. 1 mark

Area of Study 1 Data analysis – Investigating and modelling time series data

EXAM 1

Question 1

Consider the time series plot below.



The pattern in the time series plot shown above can be best described as having

A. irregular fluctuations only.

B. an increasing trend with seasonality.

C. an increasing trend with irregular fluctuations.

D. an increasing trend with cycles evident.

Use the following information to answer Questions 2–4.

The table below shows the long-term average number of customers using a drive-through coffee booth for each day of the week.

Also shown are the seasonal indices for Monday, Wednesday, Friday, Saturday and Sunday.

	Day of the week						
	Mon.	**Tues.**	**Wed.**	**Thurs.**	**Fri.**	**Sat.**	**Sun.**
Long-term average	1820	1700	1467	1476	1765	1239	1348
Seasonal index	1.18		0.95		1.14	0.80	0.87

Question 2

The seasonal index for Sunday is 0.87.

This tells us that, on average, the number of coffees sold on a Sunday is

A. 87% less than the daily average for coffee sales.

B. 87% more than the daily average for coffee sales.

C. 13% more than the daily average for coffee sales.

D. 13% less than the daily average for coffee sales.

Question 3

The seasonal index for Tuesday is missing.

The correct calculation to find the seasonal index for Tuesday is

A. $1545/1700$

B. $1700 \div 1545$

C. $(7-(1.18-0.95-1.14-0.80-0.87)) \div 2$

D. $1700 \div 10815$

Question 4

In the final week of the school term the actual sales for coffee on a Friday were 1825.

The deseasonalised number of coffee sales for the last Friday of the term was closest to

A. 247

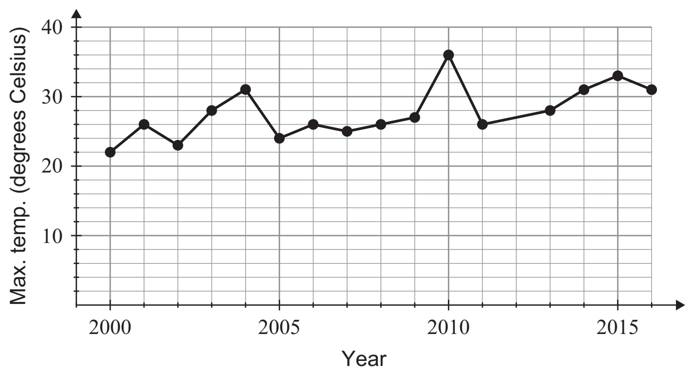
B. 1600

C. 1765

D. 2011

Use the following information to answer Questions 5 and 6.

The graph below shows the maximum daily temperature for October, the month leading up to the mathematics exam, between 2000 and 2016.



Question 5

The time series is best described as having

A. seasonality only.

B. irregular fluctuations only.

C. seasonality with irregular fluctuations.

D. an increasing trend with irregular fluctuations.

Question 6

The five-median smoothed value for 2011 is closest to

A. 26

B. 27

C. 28

D. 31

Use the following information to answer Questions 7 and 8.

A student considered the average minimum temperature for each month in 2017. These are given in the table below:

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	July	Aug.	Sept.	Oct.	Nov.	Dec.
Min. temp. (°C)	28	31	26	22	18	16	15	17	23	27	30	32

Question 7

The seasonal index for the month of May, correct to two decimal places, is closest to

A. 1.80

B. 1.32

C. 1.00

D. 0.76

Question 8

The four-point moving mean with centring is used to smooth the time series data. The smoothed value for August 2017 is closest to

A. 18

B. 19

C. 20

D. 21

Question 9

The number of cars sold monthly by a local car dealership in 2017 are displayed in the table below.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number of cars	125	114	86	75	68	89	120	105	95	78	92	118

Using three-mean smoothing, the smoothed number of cars sold in July is closest to

A. 89

B. 95

C. 104

D. 105

Use the following information to answer Questions 10 and 11.

The seasonal indices for the number of cars sold by the car dealership are shown below.

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	July	Aug.	Sept.	Oct.	Nov.	Dec.
Seasonal index	1.29	1.17		0.77	0.70	0.92	1.24	1.08	0.99	0.80	0.95	1.22

Question 10

The seasonal index for March is

A. 0.13

B. 0.67

C. 0.87

D. 0.93

Question 11

If the deseasonalised predicted value for car sales in July 2019 is 99, the actual value for July 2019 is closest to

A. 80

B. 87

C. 122

D. 123

Question 12

The table below shows the number of cakes sold over 10 days.

Day	1	2	3	4	5	6	7	8	9	10
Sales	15	13	17		21	24	23	18	17	15

A two-point moving mean with centring is used to smooth the time series data above.

If the smoothed sales figure for Day 4 was 19.5, what was the actual sales figure for Day 4?

A. 18

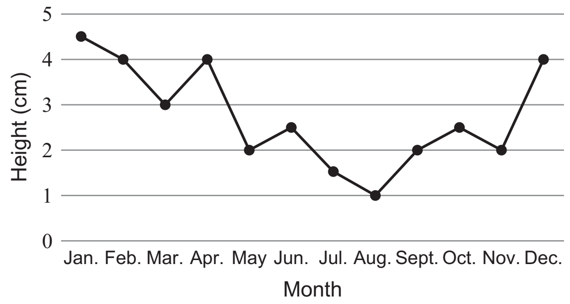
B. 19

C. 20

D. 22

Question 13

A time series plot is shown below.



If five-median smoothing is completed, how many months have a smoothed value of 2?

A. 3

B. 4

C. 5

D. 6

Question 14

The sales figures of plants in a shop were recorded each month. When the seasonal index for the sales in May was calculated, it was determined that the actual sales needed to be increased by 25% in order to correct for seasonality.

What was the seasonal index for May?

A. 0.25

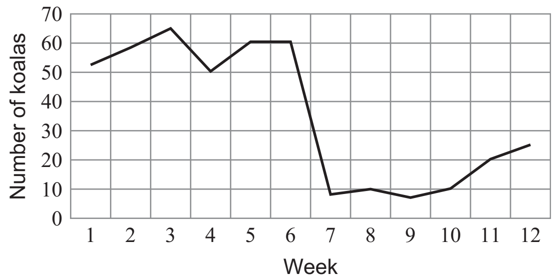
B. 0.75

C. 0.80

D. 1.25

Question 15

The number of koalas observed in a reserve was recorded once a week over the course of 12 weeks. These data are shown in the graph below.



After the sixth week, a bushfire destroyed a section of the reserve.

The impact on the data could best be described as

A. a decreasing trend.

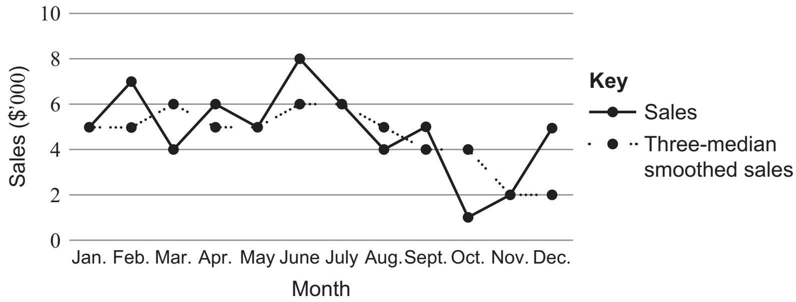
B. a discontinuity.

C. seasonality.

D. smoothed data.

Use the following information to answer Questions 16 and 17.

The sales figures for a store are shown in the time series plot below.



Question 16

Three-median smoothing has been performed; however, there are some errors.

Which months have errors in the smoothed result?

A. May, October

B. January, October, December

C. January, May, October, December

D. January, May, December

Question 17

The seasonal indices for sales data of the store are shown below.

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Seasonal index	0.82	1.04		0.96

The equation of the least squares line fitted to the deseasonalised sales data is

$$\textit{deseasonalised sales}\ (\$'000) = 13.9 + 1.08 \times \textit{quarter number}$$

The actual sales figure for Quarter 3 is closest to

A. $14 525

B. $17 140

C. $20 225

D. $23 355

Question 18

The table below shows the average temperature, in degrees Celsius, for each month in a year.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
29	28	23	20	19	10	7	9	11	16	24	30

The two-mean smoothed average temperature with centring for August is

A. 8

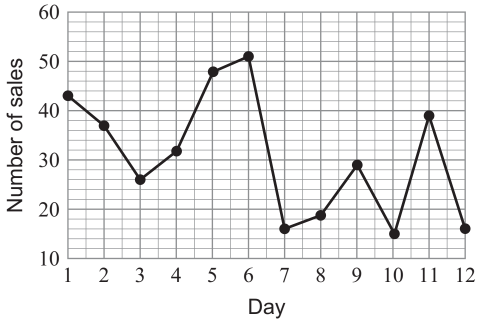
B. 9

C. 10

D. 11

Question 19

The online sales for a bookstore over 12 days are shown in the time series plot below.



The seven-median smoothed *number of sales* value for Day 8 is closest to

A. 29

B. 39

C. 48

D. 51

Question 20

The table below shows two of the seasonal indices for the quarterly sales figures of a business.

Quarter 1	Quarter 2	Quarter 3	Quarter 4
1.07	1.03		

Which one of the following is **not** true about these data?

A. The sales in Quarter 1 are 7% above the average sales.

B. The mean seasonal index for Quarter 3 and Quarter 4 is 0.95.

C. The seasonal index for Quarter 3 could be 0.98.

D. The sales for the year are 400% above the average sales.

Question 21

A business has a seasonal index of 1.20 for Quarter 1.

To correct for seasonality, its sales for Quarter 1 would need to

A. decrease by 20%.

B. decrease by 17%.

C. decrease by 13%.

D. increase by 17%.

EXAM 2

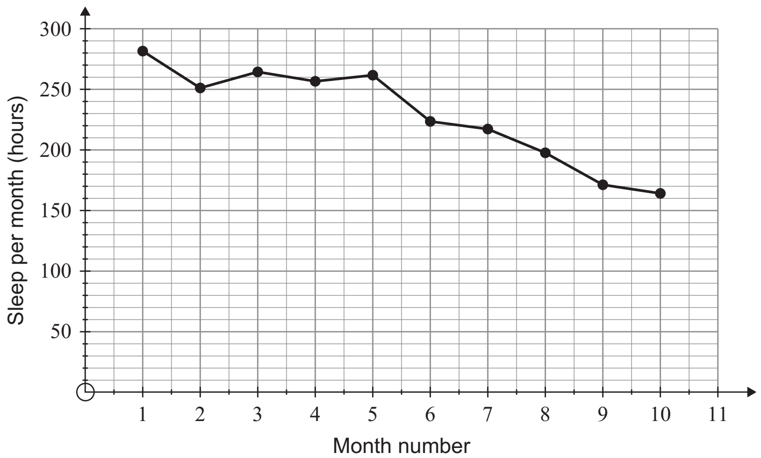
Question 1 (3 marks)

A Year 12 student recorded the amount of sleep that she had in the months leading up to her November exams.

The data she recorded, shown to the nearest hour, are given in the table below.

A time series graph has also been produced.

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.
Month number	1	2	3	4	5	6	7	8	9	10
Sleep per month (hours)	282	252	265	257	262	224	218	198	172	165



a. Describe the time series plot above. 1 mark

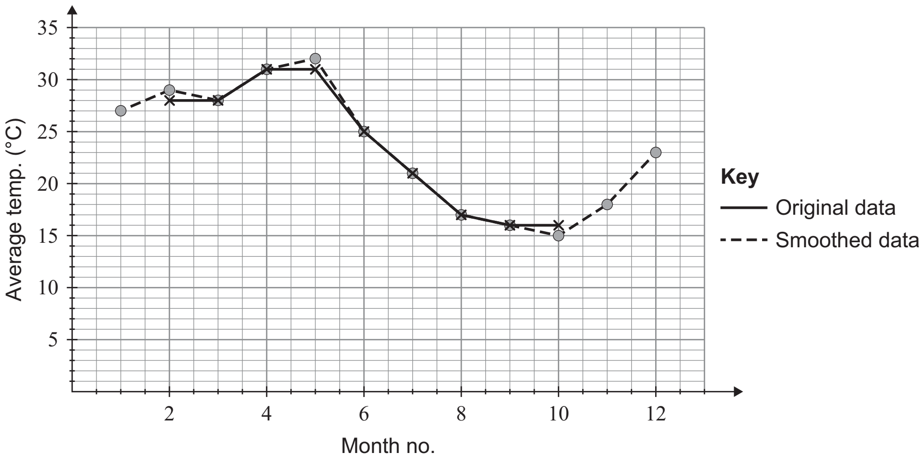
b. Using the data provided, show that the seasonal index for July (Month 7) is 0.95. 2 marks

Question 2 (2 marks)

The following table shows the *average temperature* (°C) of the mathematics classroom, as recorded by the teacher, in the 12 months leading up to the students' final mathematics SAC.

The teacher was concerned that the room may be too hot for the students.

Time	**Oct. 2017**	**Nov. 2017**	**Dec. 2017**	**Jan. 2018**	**Feb. 2018**	**Mar. 2018**	**Apr. 2018**	**May 2018**	**Jun. 2018**	**Jul. 2018**	**Aug. 2018**	**Sept. 2018**
Month no.	1	2	3	4	5	6	7	8	9	10	11	12
Average temp. (°C)	27	29	28	31	32	25	21	17	16	15	18	23



The data are also displayed graphically above.

a. The graph above shows both the original data for *average temperature* and the three-median smoothed data. The three-median smoothed value for August 2018 is missing. Complete the graph for three-median smoothing above. 1 mark

The table below shows the seasonal indices (SI) for each month:

Time	**Oct. 2017**	**Nov. 2017**	**Dec. 2017**	**Jan. 2018**	**Feb. 2018**	**Mar. 2018**	**Apr. 2018**	**May 2018**	**Jun. 2018**	**Jul. 2018**	**Aug. 2018**	**Sept. 2018**
SI	1.13	1.23	1.19	1.33	1.35	1.08	0.89	0.74	0.68	0.64	0.79	0.98

b. Calculate the deseasonalised value for June 2018, correct to one decimal place. 1 mark

Question 3 (2 marks)

Airsetter Airlines wants to investigate the relationship between *month* and *number of delayed flights*.

Table 1 shows the *number of delayed flights* recorded for each month, from October 2017 to October 2018, for flights from Melbourne to Sydney.

Table 1

Month	Oct. '17	Nov. '17	Dec. '17	Jan. '18	Feb. '18	Mar. '18	Apr. '18	May '18	June '18	July '18	Aug. '18	Sept. '18	Oct. '18
Number of delayed flights	45	52	68	85	35	40	62	42	73	58	38	78	49

These data are shown on the time series plot below, along with an incomplete graph for the data that has been smoothed using three-mean smoothing.

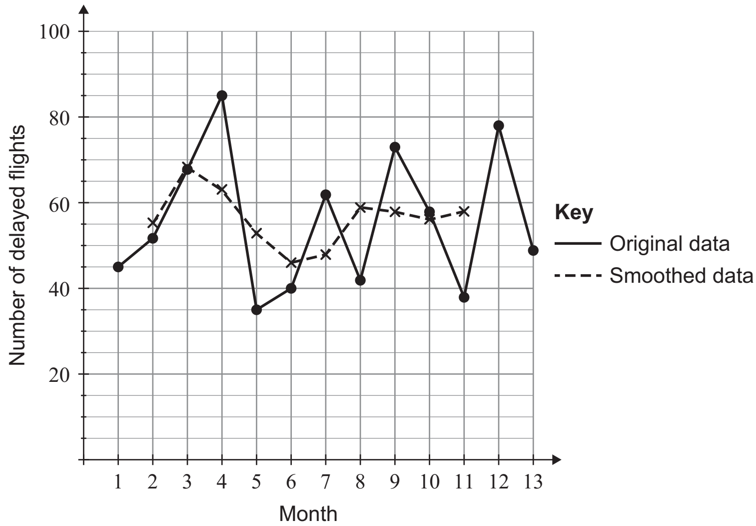


Table 2 shows the three-mean smoothed values for this time series plot.

This table is incomplete.

Table 2

Month	Oct. '17	Nov. '17	Dec. '17	Jan. '18	Feb. '18	Mar. '18	Apr. '18	May '18	June '18	July '18	Aug. '18	Sept. '18	Oct. '18
Number of delayed flights	45	52	68	85	35	40	62	42	73	58	38	78	49
Three-mean smoothing		55	68	63	53	46	48	59	58	56	58		

a. Calculate the three-mean smoothed value for September 2018. 1 mark

b. Plot your answer from **part a.** on the time series plot above. 1 mark

(Answer on the time series plot.)

Question 4 (3 marks)

The gym allows non-members to attend for a casual fee of $12 per visit. The *income* received through these casual fees for each of the four *seasons* in 2017 and 2018 is shown in the time series plot below. Some of the data is missing.

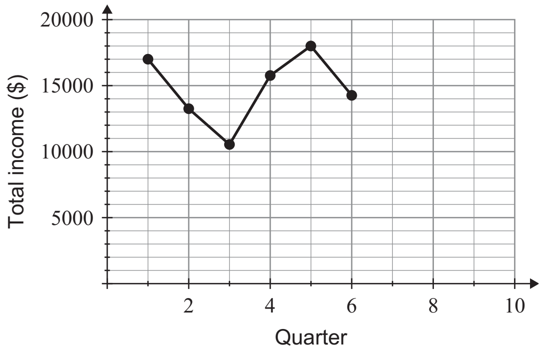


Table 1 below shows the completed data for 2017 and 2018.

Table 1

	Total income ($)			
Year	**Summer**	**Autumn**	**Winter**	**Spring**
2017	17 064	13 236	10 548	15 732
2018	17 976	14 256	11 064	15 876

a. Complete the time series plot above using the data from Table 1. 1 mark

b. The seasonal indices for the four quarters are shown in Table 2 below.

Table 2

	Summer	**Autumn**	**Winter**	**Spring**
Seasonal index	1.21	0.95	0.75	1.09

Table 3 shows the values for Seasons 1–5, which have already been deseasonalised.

Table 3

Season	**Deseasonalised data**	**Season**	**Deseasonalised data**
1	14 102.48	5	14 856.20
2	13 932.63	6	
3	14 064.00	7	
4	14 433.03	8	

Deseasonalise the values for Seasons 6–8 and enter them into the table above. 1 mark

(*Enter values into the table above.*)

c. A least squares line can be used to model the association between the deseasonalised *income* and the *season number*. Use the values in Table 3 on the previous page to find the equation of the least squares line for the deseasonalised income received by the gym through casual fees.

Round the values for the slope and intercept to two decimal places. 1 mark

Question 5 (3 marks)

The amount of screen time Amina had each day over one week is shown below.

Day	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Screen time (hours)	3.3	2.1	3.2	1.5	2.4	4.5	6.1

a. Using the data provided, show that the two-point centred moving mean for Thursday is 2.15. 2 marks

The long-term average number of hours of screen time for each day is shown below.

Day	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.	Weekly average
Screen time (hours)	3.1	2.7	3.8	2.6	2.5	4.4	6.3	3.628
Seasonal index	0.85	0.74			0.69	1.21	1.74	

b. Complete the missing seasonal indices in the table above. 1 mark

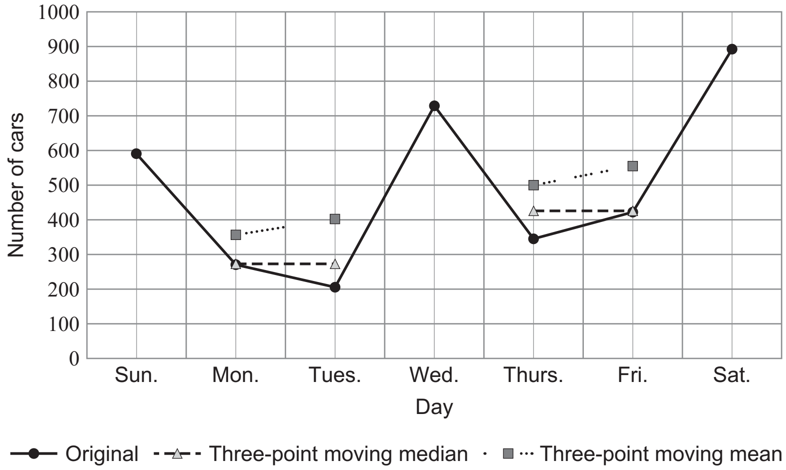
Question 6 (5 marks)

The total number of cars in a carpark is recorded over each day for a week. The results are shown below.

Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number of cars	593	272	208	730	345	425	894

The data is smoothed using both a three-point moving median and three-point moving mean method of smoothing.

Part of the smoothing for both methods has been graphed onto the time series plot below.



a. **i.** Complete the three-point moving median smoothing and the three-point moving mean smoothing on the time series plot above. 2 marks

(Answer on the time series plot above.)

ii. Explain a limitation of using a moving-mean method to smooth data compared to using a moving-median method. 1 mark

b. i. The average number of cars parked each month in a carpark for a year is shown in the table below.

Month	Average number of cars
Jan.	22 100
Feb.	19 700
Mar.	16 400
Apr.	18 400
May	12 500
June	12 900
July	18 800
Aug.	20 300
Sept.	21 500
Oct.	19 700
Nov.	32 400
Dec.	48 200
Average	21 908

Show that the calculation to determine the seasonal index for May is 0.57. 1 mark

ii. In May the following year, the average number of cars was 14 300.

What is the deseasonalised value?

Round your answer to three significant figures. 1 mark

Area of Study 2 Recursion and financial modelling – Recurrence relations, modelling linear and geometric growth and decay, simple interest and flat rate, unit cost depreciation, compound interest investments and loans, reducing balance depreciation

EXAM 1

Question 1

A motorbike was purchased for \$23 200.

After three years, the motorbike has a value of \$16 990.

On average, the motorbike travelled 22 500 kilometres every year during those three years.

The value of the motorbike was depreciated using a unit cost method of depreciation.

The value of the motorbike, in dollars, after n kilometres are travelled, V_n, can be modelled by the rule

A. $V_{n+1} = V_n - 0.092n$

B. $V_{n+1} = 23\,200 - 0.092V_n$

C. $V_n = 23\,200 - 0.092n$

D. $V_n = 23\,200 - 6200n$

Question 2

A printer was purchased for \$1200. After three years, the printer has a value of \$765.

On average, 1740 pages were printed every year.

The value of the printer was depreciated using a unit cost method of depreciation.

The depreciation in the value of the printer, per page printed, is closest to

A. 5 cents.

B. 6 cents.

C. 7 cents.

D. 8 cents.

Question 3

Consider the following recurrence relation.

$A_0 = 3, \qquad A_{n+1} = -2A_n + 3$

The first four terms of this recurrence relation are

A. –3, 9, –15, 33, …

B. 3, –3, 9, –15, …

C. 3, –12, 18, –42, …

D. –12, 18, –42, 78, …

Question 4

Which of the following recurrence relations represents geometric decay?

A. $C_0 = 100, \quad C_{n+1} = 0.84 \times C_n + 100$

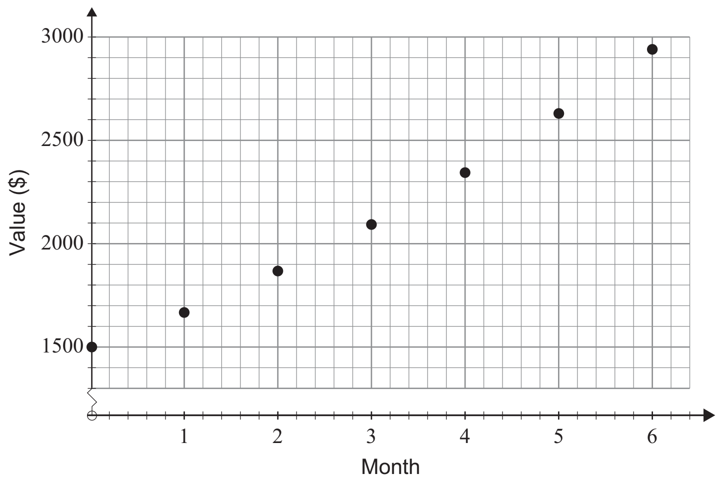
B. $C_0 = 100, \quad C_{n+1} = 1.684 \times C_n$

C. $C_0 = 100, \quad C_{n+1} = 0.684 \times C_n$

D. $C_0 = 100, \quad C_{n+1} = 1.684 \times C_n + 25$

Question 5

Consider the following graph.



Which of the following could be modelled by the graph?

A. A reducing balance loan with regular payments

B. A flat rate loan with regular payments

C. A compound interest account

D. An interest only loan

Question 6

Five years after a car was purchased, it is worth \$26 000. On average, it travels 22 000 kilometres per year.

If it depreciates on a unit cost basis at 14 cents per kilometre, its purchase price was closest to

A. \$10 600 **B.** \$29 080

C. \$41 400 **D.** \$56 000

Question 7

An asset was purchased for \$124 000. It depreciates on a reducing balance basis at a rate of 5.6% per annum.

The value of the asset after six years is shown by the rule

A. $V_6 = 124\,000 - (1.056 \times 6)$

B. $V_6 = 124\,000 - (0.944 \times 6)$

C. $V_6 = 124\,000 - (0.056 \times 6)$

D. $V_6 = 124\,000 \times (0.944)^6$

Question 8

A \$4500 lounge suite is depreciating by 9.4% per annum using the reducing balance method. The value of the lounge suite after n years is represented by which rule?

A. $V_n = 1.094 \times 4500$

B. $V_{n+1} = 0.906V_n$

C. $V_{n+1} = 0.906^n \times 4500$

D. $V_n = 0.906^n \times 4500$

Question 9

When using recursion, which one of the following calculations determines the value of the lounge suite in Question 8 at the end of the third year?

A. $0.906 \times 4077 = 3693.76$

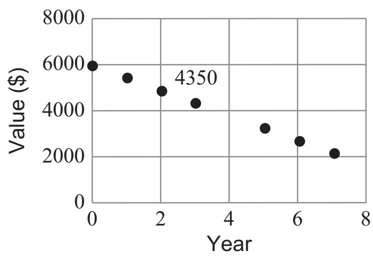
B. $3690 - 405 = 3285$

C. $1.09 \times 4905 = 5346.45$

D. $0.906 \times 3693.76 = 3346.55$

Question 10

The graph below shows the value of an asset that was purchased for \$6000. Its value decreases on a flat rate basis each year.



How much had the asset depreciated by after four years?

A. \$3800

B. \$2200

C. \$1650

D. \$550

Question 11

The following recurrence relation generates a sequence of numbers.

$$T_0 = 6, \qquad T_{n+1} = 3T_n$$

T_5 is equal to

A. 216

B. 486

C. 729

D. 1458

Question 12

An appliance is purchased for \$486. It depreciates using the rule shown below, where n represents the number of time periods.

$$V_n = 486 - 14n$$

The model of depreciation shown is

A. reducing balance depreciation, at a rate of depreciation of 14% per time period.

B. flat rate depreciation, at a rate of depreciation of \$14 per time period.

C. reducing balance loan, at a rate of depreciation of 14% per time period.

D. reducing balance depreciation, at a rate of depreciation of \$14 per time period.

EXAM 2

Question 1 (3 marks)

Casey's first motorbike had a purchase price of \$37 000.

After five years, the value of the motorbike was \$29 600.

a. Let C_n be the value of the motorbike n years after it was purchased.

Assume that the value of the motorbike has depreciated using the **flat rate** method of depreciation.

Write a recurrence relation, in terms of C_n and C_{n+1}, that models the value of the motorbike.

2 marks

b. The motorbike has travelled an average of 4000 km in each of the five years since it was purchased.

Assume that the value of the motorbike has been depreciated using the **unit cost** method of depreciation.

By how much is the value of the motorbike reduced per kilometre travelled? 1 mark

Question 2 (5 marks)

Jonathan purchased a car for $24 990.

The value of Jonathan's car was depreciated using the unit cost method of depreciation.

The value of the car, in dollars, after n kilometres, V_n, can be modelled by the rule shown below.

$$V_n = 24\,990 - 0.0015n$$

a. Show that the value of the car after 20 000 kilometres is $24 960. 1 mark

b. What is the cost, in cents per kilometre, that Jonathan's car is being depreciated? 1 mark

c. If Jonathan was to use the flat rate method of depreciation, it would take him three years to depreciate the car to $23 191.

Find the annual flat rate of depreciation. Give your answer as a percentage. Round your answer to one decimal place. 1 mark

It was suggested to Jonathan that a reducing balance method of depreciation may be more appropriate to depreciate the value of his car.

d. If the car is depreciated at a reducing balance rate of 5.8% per annum, write down a recurrence relation, in terms of A_0, A_{n+1} and A_n, for the value of Jonathan's car, in dollars, after n years. 2 marks

Question 3 (3 marks)

Alison withdraws $5000 from her account to purchase a new photocopier for her business. The value of the photocopier will be depreciated using the reducing balance method. The value of Alison's photocopier, in dollars, after n years, C_n, can be modelled by the recurrence relation shown below.

$$C_0 = 5000, \quad C_{n+1} = R \times C_n$$

a. The annual rate of depreciation for this photocopier is 18%.

What is the value of R in the recurrence relation? 1 mark

b. How much has the photocopier depreciated in value after five years of use? 1 mark

c. The value of the photocopier after one year of use is $4100. Alison's company prints approximately 35 000 pages per month on this photocopier. Alison chooses to calculate the depreciation of her photocopier using unit cost depreciation.

By how much does the photocopier depreciate per page, using this method?

Give your answer in cents to one decimal place. 1 mark

Question 4 (5 marks)

Casey has opened a simple interest savings account in order to save money to buy a new motorbike. Interest is paid monthly into this account. The amount of money in the savings account after n years, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 6000, \quad V_{n+1} = V_n + 216$$

a. How much money did Casey initially deposit in his savings account? 1 mark

b. What is the annual percentage interest rate for this savings account? 1 mark

c. The amount of money in the account after n years, V_n, can also be determined using a rule.

i. Complete the rule by writing the appropriate numbers in the boxes provided below. 1 mark

$V_n = \boxed{} + \boxed{} \times n$

ii. How much money will be in Casey's savings account after five years? 1 mark

Casey decides to compare a compound interest savings account to his savings in the simple interest account. He invests \$6000 at an interest rate of 5.4%, compounding annually. The amount of money in the savings account after n years, V_n, can be modelled by the recurrence relation shown below.

$V_0 = 6000, \qquad V_{n+1} = 1.054 \times V_n$

d. Use recursive calculations to show that the amount of money in Casey's compound interest savings account after two years, V_2, will be \$6665.50. 1 mark

Question 5 (5 marks)

Alison invests her money into a savings account that will pay compound interest every month. The balance of Alison's account, after n months, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 8000, \quad V_{n+1} = 1.00225V_n$$

a. What is the annual interest rate for Alison's savings account? 1 mark

b. Recursion can be used to calculate the balance of the account after each month.

i. Write recursive calculations to show that the balance in the account after two months, V_2, is \$8036.04. 1 mark

ii. After how many months will the balance of Alison's account first exceed \$8500? 1 mark

c. A rule of the form $V_n = a \times b^n$ can be used to determine the balance of Alison's account after n months.

Write the rule for Alison's investment below. 1 mark

d. The value of Alison's investment after 12 months is \$8219 when rounded to the nearest dollar.

What is the equivalent flat rate interest rate, per annum, for Alison's investment?

Round your answer to two decimal places. 1 mark

Question 6 (4 marks)

Jason is a personal trainer who has purchased a large piece of equipment for his home gym.

The value of Jason's equipment is depreciated using the reducing balance method.

The value of the equipment, in dollars, after n years, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 12\,000, \qquad V_{n+1} = 0.92V_n$$

a. Write down a calculation, using the value of the equipment after four years, to show that the value of Jason's equipment after five years, V_5, is $7908.98. 1 mark

b. After how many years will the value of Jason's equipment first fall below $7500? 1 mark

c. Jason has another option for depreciation of his equipment.

He can depreciate the value of his equipment using unit cost depreciation, where a 'unit' is counted each time the equipment is used for a workout.

In the first two years, Jason completed 1400 workouts.

The value of his equipment after this time was $10 850.

i. What is the unit cost of depreciation for Jason's equipment?

Give your answer to the nearest cent. 1 mark

ii. Let V_n be the value of the equipment after n workouts, in dollars.

Write down a rule in terms of V_n that could be used to model the value of the equipment using this unit cost depreciation. 1 mark

Question 7 (4 marks)

Jo invests $80 000 at a simple interest rate of 3.4% per annum.

Her brother Anthony invests the same amount of money in an account earning 3.25% per annum, compounding annually.

a. How much interest does Jo earn each year? 1 mark

b. Write a recurrence relation to model the value of Anthony's investment in terms of V_0, V_{n+1} and V_n. 1 mark

c. What is the value of Anthony's investment after eight years? 1 mark

d. After how many years will Anthony's investment be larger than Jo's? 1 mark

Question 8 (3 marks)

Saskia buys a commercial washing machine for her laundromat business. The value of the washing machine after n years is shown by the equations below:

$$V_0 = 3875 \qquad V_{n+1} = V_n - 410$$

a. **i.** Showing recursive calculations, determine the value of the washing machine after two years. 1 mark

ii. What is the annual flat rate of depreciation?

Write your answer as a percentage rounded to two decimal places. 1 mark

b. An alternative model of depreciation is that the washing machine depreciates by $800 immediately upon purchase, then depreciates at 6% per year on a reducing balance basis.

What is the value of the machine after three years using this method?

Round your answer to the nearest dollar. 1 mark

Area of Study 2 Recursion and financial modelling – Reducing balance loans, annuities, perpetuities, amortisation tables

EXAM 1

Question 1

The value of a reducing balance loan, V_n, after n monthly payments of \$785 have been made, can be determined using the recurrence relation shown below.

$$V_0 = 32\,000, \qquad V_{n+1} = 1.018 \times V_n - 785$$

The value of the loan after four payments have been made is closest to

A. \$31 791

B. \$31 578

C. \$31 361

D. \$31 141

Question 2

The amortisation table for a reducing balance loan is shown below.

The interest rate for this loan is 5.4% per annum, compounding monthly.

The loan is to be repaid with monthly payments of \$1375.

Payment number	Payment	Interest	Principal reduction	Balance of loan
0	0.00	0.00	0.00	275 000.00
1	1375.00	1237.50	137.50	274 862.50
2	1375.00	1236.88	138.12	274 724.38

The value of the principal reduction after the third payment is closest to

A. \$1235.63

B. \$1375.00

C. \$138.74

D. \$139.37

Use the following information to answer Questions 3 and 4.

The value of a reducing balance loan, in dollars, after n months, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 50\,000, \qquad V_{n+1} = 1.00238V_n - 800$$

Question 3

The annual interest rate for this reducing balance loan is closest to

A. 1.24%

B. 1.98%

C. 2.38%

D. 2.86%

Question 4

The value of the reducing balance loan after six months is closest to

A. \$45 319

B. \$45 880

C. \$45 890

D. \$45 918

Question 5

Which one of the following recurrence relations could be used to model the value of a reducing balance loan with regular repayments, R_n, after n months?

A. $R_0 = 300\ 000, \quad R_{n+1} = 0.996 \times R_n - 1250$

B. $R_0 = 300\ 000, \quad R_{n+1} = 0.996 \times R_n + 1250$

C. $R_0 = 300\ 000, \quad R_{n+1} = 1.004 \times R_n + 1250$

D. $R_0 = 300\ 000, \quad R_{n+1} = 1.004 \times R_n - 1250$

Question 6

Four lines of an amortisation table for a reducing balance loan with monthly repayments are shown below.

Repayment number	Payment	Interest	Principal reduction	Balance of loan
13	2700	\$1385.72	\$1314.28	\$348 762.17
14	2700	\$1380.52	\$1319.48	\$347 442.69
15	2700	\$1375.29	\$1324.71	\$346 117.98
16	2700			\$344 788.03

The balance of the loan after repayment number 16 is \$344 788.03.

The interest paid in the sixteenth payment is

A. \$1235.76

B. \$1329.95

C. \$1370.05

D. \$1570.61

Question 7

Jessica invests $10 000 in a savings account earning a rate of 3.8% per annum, compounded monthly. At the end of each month, Jessica adds an additional $250.

A recurrence relation that can be used to model Jessica's investment could be

A. $V_0 = 10\ 000, \quad V_{n+1} = 0.962 \times V_n + 250$

B. $V_0 = 10\ 000, \quad V_{n+1} = 1.038 \times V_n + 250$

C. $V_0 = 10\ 000, \quad V_{n+1} = 0.962 \times V_n - 250$

D. $V_0 = 10\ 000, \quad V_{n+1} = 1.00317 \times V_n + 250$

Use the following information to answer Questions 8 and 9.

Four lines of an amortisation table for a savings account with additional monthly payments of $230.00 are shown below.

The interest rate for this investment is 4.2% per annum, with interest calculated monthly.

Payment number	Payment	Interest	Principal addition	Balance of investment
32	230.00	273.83	503.83	78 740.33
33	230.00	275.59	505.59	79 245.92
34	230.00			79 753.28
35	230.00	279.14	509.14	80 262.42

Question 8

The balance of the investment after payment number 31 was closest to

A. 78 236.50

B. 78 510.33

C. 78 696.50

D. 78 740.33

Question 9

Of the principal addition made in the line for payment number 34, the percentage that was interest is closest to

A. 3%

B. 15%

C. 54%

D. 55%

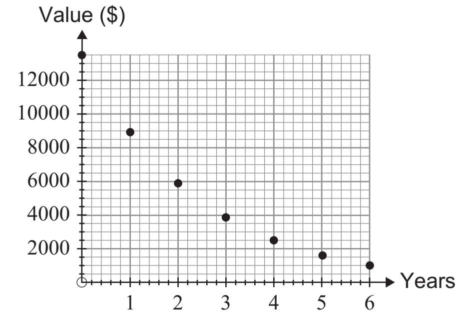
Question 10

Consider the recurrence relation

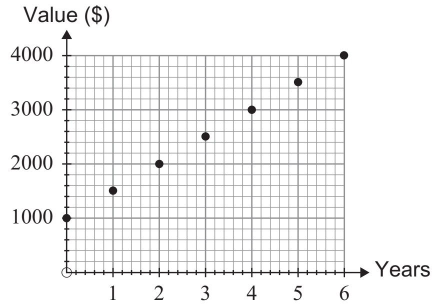
$$V_0 = 1000, \quad V_{n+1} = 1.5V_n + 100$$

A graph that represents the value, in dollars, after n years could be

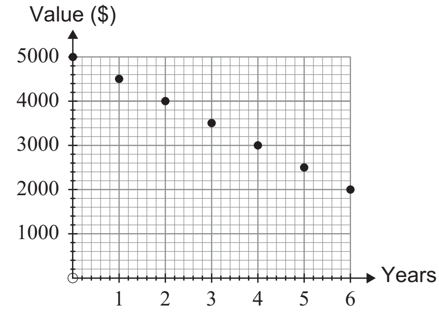
A.



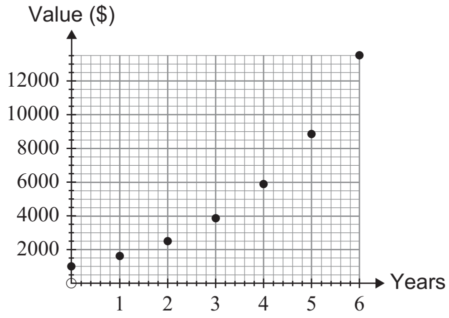
B.



C.



D.



Question 11

The recurrence relation of a financial model is shown below.

$$A_0 = 2500, \quad A_{n+1} = 1.062A_n - 200$$

The value after five time periods is closest to

A. $1500

B. $2185

C. $2245

D. $2303

Use the following information to answer Questions 12 and 13.

Five lines of an amortisation table for a reducing balance loan with quarterly repayments are shown below.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	100 000.00
1	1500.00	1300.00	200.00	99 800.00
2	1500.00	1297.40	202.60	99 597.40
3	1500.00	1294.77	205.23	99 392.17
4	1500.00			99 184.27

Question 12

The annual interest rate is closest to

A. 1.3%

B. 2.0%

C. 5.2%

D. 8.0%

Question 13

The percentage of payment number 4 that goes towards reducing the principal of the loan is

A. 0.21%

B. 1.51%

C. 13.86%

D. 16.10%

Question 14

$80 000 is invested in a perpetuity for seven years, earning 3.7% per annum, compounding monthly.

The monthly payment received from the perpetuity is closest to

A. $2960

B. $1082

C. $964

D. $247

Use the following information to answer Questions 15–17.

Will and Linda are saving for a house, so each invests money into a personal savings account.

Will invests $10 000 in an account earning 4.05% per annum, compounding monthly. He contributes an additional $400 per month.

Linda invests $12 000 in an account earning 4.02% per annum, compounding fortnightly.

Question 15

How many months until Will has doubled his money?

A. 11

B. 22

C. 23

D. 36

Question 16

The amount Will and Linda will have in combined savings at the end of the second year is closest to

A. $13 864

B. $22 904

C. $27 794

D. $33 828

Question 17

Josie has $15 000 invested in the same type of account as Linda.

Josie begins withdrawing money from her account each fortnight.

The maximum she can withdraw without the balance reducing is

A. $20

B. $21

C. $22

D. $23

Question 18

A $24 000 perpetuity is invested for 10 years at a rate of 2.12% per annum, compounding monthly.

The amount of interest earned over 10 years is

A. $3060

B. $5088

C. $24 000

D. $26 640

Use the following information to answer Questions 19 and 20.

Nadia borrowed $700 000 at a rate of 3.3% per annum, compounding monthly. Her repayments are $3724 per month for 25 years.

The first four lines of the amortisation table are shown below.

Payment number	Repayment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	700 000.00
1	3724.00			
2	3724.00	1920.05	1803.95	696 397.05
3	3724.00	1915.09	1808.91	694 588.14

Question 19

The missing amounts in the amortisation table for payment number 1 are

A. $1925, $1799 and $698 201

B. $1926, $1798 and $700 000

C. $1926, $1799 and $698 201

D. $1925, $1798 and $700 000

Question 20

The percentage of the third repayment that contributes to the principal reduction is closest to

A. 0.26%

B. 0.54%

C. 47.2%

D. 48.6%

EXAM 2

Question 1 (6 marks)

In order to purchase his new car, Jonathan had to borrow $15 000.

He was presented with the opportunity to borrow the money from his parents.

The arrangement with his parents can be modelled by the recurrence relation below, which shows the value, in dollars, of his loan, L_n, after n months.

$$L_0 = 15\,000, \quad L_{n+1} = 1.0022L_n - 200$$

a. Find the value, in dollars, of Jonathan's loan after six months. 1 mark

As an alternative, Jonathan can borrow the money from a bank.

The bank will offer Jonathan a $15 000 loan with interest calculated monthly at a rate of 4.5% per annum. Jonathan will be required to make monthly payments to his loan and will repay his loan in seven years.

b. What monthly payment will Jonathan make on his loan? 1 mark

c. How much interest, to the nearest cent, will Jonathan pay in the first two years of his loan? 2 marks

After three years, Jonathan thinks that he is able to increase his monthly repayments to $325.

The interest rate on his loan will remain the same.

d. How much time, to the nearest month, will Jonathan save on his loan under these new conditions? 2 marks

Question 2 (4 marks)

Jason would like to purchase a building in order to open his own gymnasium.

He will borrow $650 000 to make this purchase and will make monthly repayments of $4960.35.

After four years of equal monthly repayments, the balance of Jason's loan will be $535 893.37.

a. What is the annual interest rate for Jason's loan?

Round your answer to one decimal place. 1 mark

b. Show that the total interest that Jason will have paid in the first year of his loan is \$33 177.96. 2 marks

c. After the first four years of his loan, Jason will change the conditions of his loan.

He will make monthly repayments of \$4327.56 and will be charged an annual interest rate of 4.8%, compounding monthly.

Write down a recurrence relation, in terms of G_0, G_{n+1} and G_n that could be used to model the balance of the loan over the next two years. 1 mark

Question 3 (4 marks)

Phil invested money into an annuity investment, earning 4.1% per annum, compounding quarterly. For the first five quarters, he made additional deposits of \$2200.

Parts of the amortisation table are shown below.

Deposit number	Deposit (\$)	Interest (\$)	Principal increase (\$)	Value of investment (\$)
2				
3	2200.00	725.31	2925.31	73 687.19
4	2200.00	755.29	2955.29	76 642.48
5	2200.00	785.59	2985.59	79 628.07
6			3141.19	82 769.26

a. Determine the quarterly interest rate. 1 mark

b. What was the value of the investment after deposit number 2 was made? 1 mark

c. After the fifth quarter, Phil increased the amount of his deposits.

What was the amount of the new deposit? 1 mark

d. For the third quarter, what percentage of the principal increase did the interest contribute to?

Round your answer to one decimal place. 1 mark

Area of Study 2 Recursion and financial modelling – Using the finance solver

EXAM 1

Question 1

Christine invests $4500 into a savings account that pays interest at the rate of 3.25% per annum, compounding quarterly. At the end of each quarter, immediately after the interest has been paid, she adds an additional $500 to her investment.

After two years, the amount of interest that Christine's investment has earned is closest to

A. $136.55

B. $300.95

C. $416.57

D. $685.34

Question 2

Jessica invested $75 000 into a bank account to save for a house. Her goal was to reach $175 000 within 10 years.

For the first five years of the investment, the interest rate was 2.6% per annum, compounding quarterly.

Jessica invested an additional $1000 quarterly.

After five years, Jessica re-evaluated her loan, and found a better savings account with an interest rate of 3.2%, compounding monthly.

For the final five years of Jessica's investment, in order to reach her goal of $175 000, her monthly investment needs to be

A. $1458.33

B. $1138.99

C. $1000.00

D. $767.38

Question 3

Peter borrows $35 000 to buy a motorbike. He takes out a loan for five years at an interest rate of 4.25%, compounding monthly. He makes monthly payments of $648.53.

After two years, Peter receives a pay increase and is able to increase his payments to $750 per month.

Peter can now repay his loan in a shorter time. The amount of time, to the nearest month, he will save is

A. 0 months.

B. 4 months.

C. 5 months.

D. 6 months.

Question 4

Callum borrows \$425 000, in a reducing balance loan, to purchase a new home. Interest is charged at a rate of 5.85% per annum, calculated monthly, over 25 years.

Assuming that Callum will repay his loan in 25 years, his monthly repayment will be

A. \$1505.04

B. \$2071.88

C. \$2699.44

D. \$18 098.33

Use the following information to answer Questions 5 and 6.

Kate borrowed \$425 000 to pay for her house, with the intention of repaying her loan within 10 years.

The interest rate for the first five years was 5.25% per annum, compounding monthly.

Kate made monthly repayments of \$3100.

After five years, the interest rate changed to 4.95%.

Question 5

The amount of interest that Kate has paid in the first five years of her loan is closest to

A. \$6258

B. \$84 910

C. \$101 090

D. \$170 500

Question 6

At the end of the sixth year, Kate still owes \$319 256.

From the end of the sixth year, what will Kate's monthly repayment change to, when rounded to the nearest whole number, in order to repay her loan in 10 years?

A. \$6061

B. \$6720

C. \$6725

D. \$7345

Question 7

Alena invested \$12 000 into an annuity investment for 10 years, with the intention of depositing \$150 per month and earning 4.3% per annum, compounding monthly. However, after two years, she increased her regular deposit to \$220 per month. The interest rate remained the same.

With the increased deposits, how much more than originally expected does the investment grow over the 10 years, to the nearest dollar?

A. \$8004

B. \$10 472

C. \$28 873

D. \$36 877

Question 8

Anh borrowed $415 000 for 30 years at the rate of 3.1% per annum, compounding monthly. The terms of the loan include monthly repayments of $1762.

Anh has an extra $210 per month that she considers adding to her repayments each month.

Determine, to the nearest dollar, how much money would be saved on interest paid over a 10-year period if she increases her monthly repayments by $210.

A. $2100

B. $4299

C. $12 352

D. $18 499

Question 9

Griffin invests $1500 into an investment account, earning interest at 3.7% per annum, compounding fortnightly.

Griffin wishes to add an additional payment each fortnight and has the option of paying $20 or $50.

If he makes additional fortnightly payments of $50, how much more will he have in his account after two years compared to if he contributed only $20 per fortnight?

A. $64

B. $546

C. $1618

D. $2720

Question 10

The recurrence relation for an annuity is shown below, where n is the number of quarters. The interest compounds quarterly.

$$V_0 = 13\,800 \qquad V_{n+1} = 1.0061V_n - 162.50$$

The length of the annuity, in quarters, is closest to

A. 85

B. 91

C. 96

D. 120

Question 11

Nigella is saving for a house deposit.

She invests $4000 in an account earning 2.05% per annum, compounding fortnightly.

Nigella adds an additional $800 per fortnight to the account.

Every 12 months, when Nigella receives a salary increase, she increases the amount of the fortnightly deposit by $50, so that in the second year she is adding $850 per fortnight, and in the third year she is adding $900.

After three years, the balance of her investment is closest to

A. $6561

B. $8073

C. $68 586

D. $72 552

EXAM 2

Question 1 (3 marks)

Katerina has borrowed $45 000 to buy a new car. She will be charged interest at a rate of 7.2% per annum, compounding monthly.

a. For the first year (12 months), Katerina will make monthly repayments of $550.

Find the amount that Katerina will owe on her loan after she has made 12 repayments. 1 mark

b. After four years, Katerina will increase her repayments in order to fully repay her loan in a further two years.

Find the value of Katerina's new repayments, in order to repay the loan during these two years. 2 marks

Question 2 (4 marks)

Jason invests $25 000 in a savings account.

He adds a further monthly payment to his account.

The balance of the account, in dollars, after n months, S_n, can be modelled by the recurrence relation shown below.

$$S_0 = 25\,000 \qquad S_{n+1} = 1.0038\,S_n + 450$$

a. What is the annual contribution that Jason makes to his account? 1 mark

b. What is the annual percentage interest rate for this account? 1 mark

c. When the value of Jason's account exceeds $30 000, he will increase his monthly contribution to $550.

i. What is the balance of the account when this increase occurs?

Round your answer to the nearest dollar. 1 mark

ii. Jason's saving goal is $45 000.

How long will it take him to reach this goal? 1 mark

Question 3 (4 marks)

Samara borrows $310 000 at an interest rate of 3.72% per annum, compounding monthly, for a loan term of 25 years. Her repayments are scheduled to be $1590 per month.

a. Determine the effective interest rate, rounded to two decimal places. 1 mark

b. After the first year, how much interest will she have paid?

Round your answer to the nearest dollar. 1 mark

c. If, instead, Samara makes interest-only repayments for the first 12 months, what will her balance be after the first year? 1 mark

d. Samara still needs to pay off the loan within the original 25-year period, including the first 12 months paying interest-only payments.

How much will she need to repay per month for the remaining 24 years in order to do so?

Round your answer to the nearest dollar. 1 mark

Question 4 (1 mark)

Saskia invests $7076 into an investment account that earns 3.56% per annum, compounding fortnightly. She makes additional payments into the account each fortnight.

What are the additional payments made if the balance is $30 482 six years later?

Round your answer to the nearest dollar.

Question 5 (5 marks)

Saskia's brother Tasman borrows $70 000. The loan is for 10 years, with interest charged at 3.75% per annum, compounding monthly. The repayments are $701 per month.

a. What is the balance of the loan halfway through the loan term of 10 years?

Round your answer to the nearest dollar. 1 mark

b. What is the difference between the interest paid in the first five years and the interest paid in the last five years?

Round your answer to the nearest dollar. Accounting for an adjusted final payment is not required. 1 mark

Tasman has an annuity with a balance of \$52 000, which earns 3.95% per annum, compounding monthly. The annuity is for 12 years.

c. **i.** Tasman wants to change the terms of the annuity so that the withdrawals from it cover the \$701 monthly repayments of the loan for as long as the annuity lasts.

How much must he reduce the length of the annuity by in order to obtain withdrawal payments that sufficiently cover the \$701 monthly loan repayments? 2 marks

ii. Five years after investing the \$52 000, the interest rate of the annuity increases. Two years after the rate change, the balance of the annuity is \$841. The withdrawals of \$701 remain constant throughout this time.

What was the interest rate, as a percentage, after the change?

Round your answer to two decimal places. 1 mark

Area of Study 2 Matrices and their applications

EXAM 1

Question 1

The transpose of $\begin{bmatrix} 2 & 5 & 9 & 11 \\ 3 & 8 & 12 & 15 \end{bmatrix}$ is

A. $\begin{bmatrix} 3 & 2 \\ 8 & 5 \\ 12 & 9 \\ 15 & 11 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 8 & 12 & 15 \\ 2 & 5 & 9 & 11 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 3 \\ 5 & 8 \\ 9 & 12 \\ 11 & 15 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 5 & 9 & 11 \\ 3 & 8 & 12 & 15 \end{bmatrix}$

Question 2

The table below shows the cost of different drink options sold at the school canteen.

Drink choice	Iced tea	Flavoured milk (small)	Flavoured milk (large)	Bottled water
Cost ($)	$3.50	$2.50	$4.00	$1.75

The canteen manager recorded the number of each drink choice sold on a particular day.

These are shown in the table below.

Drink choice	Iced tea	Flavoured milk (small)	Flavoured milk (large)	Bottled water
Number sold	65	156	105	256

The matrix calculation that will find the total amount of money spent on drinks for this particular day is

A. $\begin{bmatrix} 3.50 & 2.50 & 4.00 & 1.75 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 65 & 156 & 105 & 256 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 65 \\ 156 \\ 105 \\ 256 \end{bmatrix}\begin{bmatrix} 3.50 & 2.50 & 4.00 & 1.75 \end{bmatrix}$

D. $\begin{bmatrix} 3.50 & 2.50 & 4.00 & 1.75 \end{bmatrix}\begin{bmatrix} 65 \\ 156 \\ 105 \\ 256 \end{bmatrix}$

Question 3

Let $N = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

The element in row i and column j of N is $n_{i,j}$

The elements in matrix N are determined by the rule $n_{i,j} = 2j + i$

Matrix N is

A. $\begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$

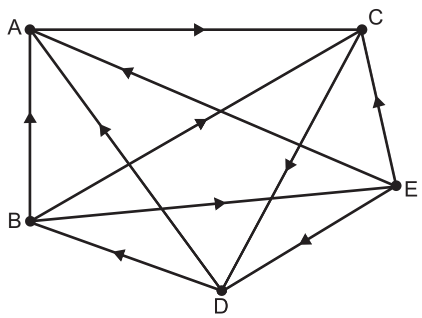
B. $\begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 5 \\ 6 & 4 \\ 8 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 5 & 7 \end{bmatrix}$

Question 4

The graph below represents the results of a round-robin sporting contest, between Adam (A), Benjamin (B), Cameron (C), Dave (D) and Ethan (E).



The winner of this round-robin competition, based on one- and two-step dominance is

A. Adam.

B. Benjamin.

C. Cameron.

D. Dave.

Question 5

The matrix below shows the number of hours that a retail store is open each day (O), for a week of trading, as well as the number of customers (C) who attended the store each day. The store is closed on Sunday.

$$\begin{array}{r c} & \begin{array}{cc} O & C \end{array} \\ \begin{array}{r} \text{Monday} \\ \text{Tuesday} \\ \text{Wednesday} \\ \text{Thursday} \\ \text{Friday} \\ \text{Saturday} \end{array} & \begin{bmatrix} 8 & 2865 \\ 8 & 2254 \\ 8 & 3127 \\ 10 & 4210 \\ 10 & 4367 \\ 5 & 1789 \end{bmatrix} \end{array}$$

On Monday, assuming that the same number of customers entered the store each hour, the number of customers who entered the store in the first hour of trading was closest to

A. 8

B. 358

C. 359

D. 379

Question 6

The following matrix represents the meat dish, beef (B), chicken (C), lamb (L), fish (F) or pork (P), that Abigail will cook on any given day for her family.

$$\begin{array}{c c}
 & \textit{tonight's meal} \\
 & \begin{array}{ccccc} B & C & L & F & P \end{array} \\
 & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\end{array}
\begin{array}{l} B \\ C \\ L \ \textit{tomorrow's meal} \\ F \\ P \end{array}$$

The ‘1’ in row one, column two indicates that if Abigail has chicken one day, she will have beef the next day.

If Abigail wants to cook fish on Monday night for her family, the meat choice that she will make on Friday night the same week is

A. beef.

B. chicken.

C. lamb.

D. pork.

Question 7

Consider the following matrices.

$$A = \begin{bmatrix} 1 & 4 \\ -2 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 3 & 12 \\ 2 & -2 & 5 \\ 6 & 4 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 1 \\ -4 & 7 \end{bmatrix}$$

The number of matrices for which an inverse matrix exists is

A. 0

B. 1

C. 2

D. 3

Question 8

The following matrix represents the prices of different sized towels for sale at a linen store.

$$\begin{bmatrix} 11.95 \\ 18.75 \\ 22.45 \\ 32.99 \\ 38.25 \end{bmatrix} \begin{array}{l} \textit{face washer} \\ \textit{hand towel} \\ \textit{bath mat} \\ \textit{bath towel} \\ \textit{bath sheet} \end{array}$$

In the period leading up to Christmas, the linen store has a sale on towels.

The matrix equation below shows the discounts applied to each size of towel and the resulting 'sale price'.

$$\begin{bmatrix} a & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & e \end{bmatrix} \begin{bmatrix} 11.95 \\ 18.75 \\ 22.45 \\ 32.99 \\ 38.25 \end{bmatrix} = \begin{bmatrix} 10.40 \\ 16.50 \\ 17.51 \\ 24.74 \\ 26.78 \end{bmatrix} \begin{array}{l} \textit{face washer} \\ \textit{hand towel} \\ \textit{bath mat} \\ \textit{bath towel} \\ \textit{bath sheet} \end{array}$$

The value of c in the matrix equation above is closest to

A. 0

B. 0.22

C. 0.78

D. 1.00

Question 9

Matrix G is a 3×2 matrix.

The element in row i and column j of matrix G is $g_{i,j}$, where $g_{i,j} = -2i + j$

The matrix $2G$ is

A. $\begin{bmatrix} -1 & 0 \\ -3 & -2 \\ -5 & -4 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 0 \\ -6 & -4 \\ -10 & -8 \end{bmatrix}$

C. $\begin{bmatrix} 11 & 12 \\ 21 & 22 \\ 31 & 32 \end{bmatrix}$

D. $\begin{bmatrix} -2 & -6 & -10 \\ 0 & -4 & -8 \end{bmatrix}$

Question 10

If $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} + R \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -1 & 3 \end{bmatrix}$, then the matrix R will be

A. $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 11 & 19 \\ 4 & 7 \end{bmatrix}$

C. $\begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Question 11

Belinda works at the local fish-and-chip shop. Her shifts vary between afternoon (*A*) and evening (*E*) shifts.

The matrix below shows the number of hours Belinda works across three weeks for each type of shift.

$$\begin{array}{l} \\ \textit{week 1} \\ \textit{week 2} \\ \textit{week 3} \end{array}\begin{array}{c} \begin{array}{cc} A & E \end{array} \\ \begin{bmatrix} 7 & 13 \\ 5 & 21 \\ 10 & 8 \end{bmatrix} \end{array}$$

The number of evening hours that Belinda works across the three weeks is

A. 20

B. 22

C. 26

D. 42

Question 12

Micah (*M*), Brad (*B*), Leilani (*L*), Riya (*R*) and Saul (*S*) work together in a professional environment.

Micah, Brad and Saul are in positions of leadership, and therefore some of the office staff members report to them.

The matrix below shows the working relationship between the staff members.

$$\begin{array}{c} \\ M \\ B \\ L \\ R \\ S \end{array}\begin{array}{c} \begin{array}{ccccc} M & B & L & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A ‘1’ in the matrix shows that the staff member in that row reports to the staff member in that column.

For example, the ‘1’ in the third row of the second column shows that Leilani reports to Brad.

A ‘0’ in the matrix shows that the staff member in that row works alongside the staff member named in that column but does not report to them.

How many staff members does Brad report to?

A. 0

B. 1

C. 2

D. 3

Question 13

The table below shows the type, number and price of each school notebook purchased by a family with five children at the beginning of the school year.

Notebook	Price	Number
48-page A4	$0.65	12
64-page A4	$0.78	10
128-page A4	$1.16	8
240-page A4	$1.35	6
Musical lined A5	$5.35	10
Journal A5	$12.26	5

The matrix product that displays the total cost of all notebooks purchased is

A. $\begin{bmatrix} 0.65 \\ 0.78 \\ 1.16 \\ 1.35 \\ 5.35 \\ 12.26 \end{bmatrix}\begin{bmatrix} 12 & 10 & 8 & 6 & 10 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 0.65 \\ 0.78 \\ 1.16 \\ 1.35 \\ 5.35 \\ 12.26 \end{bmatrix}\begin{bmatrix} 12 \\ 10 \\ 8 \\ 6 \\ 10 \\ 5 \end{bmatrix}$

C. $\begin{bmatrix} 0.65 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.78 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.35 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.35 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12.26 \end{bmatrix}\begin{bmatrix} 12 & 10 & 8 & 6 & 10 & 5 \end{bmatrix}$

D. $\begin{bmatrix} 12 & 10 & 8 & 6 & 10 & 5 \end{bmatrix}\begin{bmatrix} 0.65 \\ 0.78 \\ 1.16 \\ 1.35 \\ 5.35 \\ 12.26 \end{bmatrix}$

Question 14

The inverse of $\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ is

A. $\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}$

Question 15

The number of eggs produced per day on four farms that have both free-range (*FR*) and caged (*C*) hens is shown in matrix A below.

$$A = \begin{array}{r} \\ farm\ A \\ farm\ B \\ farm\ C \\ farm\ D \end{array}\begin{array}{c} \begin{array}{cc} FR & C \end{array} \\ \begin{bmatrix} 450 & 510 \\ 320 & 470 \\ 605 & 650 \\ 990 & 1020 \end{bmatrix} \end{array}$$

On average, each farm sells the free-range eggs to supermarkets for \$0.32 each and caged eggs for \$0.21 each.

To determine the total revenue of each farm if all eggs are sold in a day, the matrix product is

A. $\begin{bmatrix} 450 & 510 \\ 320 & 470 \\ 605 & 650 \\ 990 & 1020 \end{bmatrix} \times \begin{bmatrix} 0.32 & 0.21 \end{bmatrix}$

B. $\begin{bmatrix} 450 & 510 \\ 320 & 470 \\ 605 & 650 \\ 990 & 1020 \end{bmatrix} \times \begin{bmatrix} 0.32 & 0 \\ 0 & 0.21 \end{bmatrix}$

C. $\begin{bmatrix} 0.32 \\ 0.21 \end{bmatrix} \times \begin{bmatrix} 450 & 510 \\ 320 & 470 \\ 605 & 650 \\ 990 & 1020 \end{bmatrix}$

D. $\begin{bmatrix} 450 & 510 \\ 320 & 470 \\ 605 & 650 \\ 990 & 1020 \end{bmatrix} \times \begin{bmatrix} 0.32 \\ 0.21 \end{bmatrix}$

Question 16

The matrix product *ABC* results in a 4×2 matrix.

If A^2 is defined, and the sum of A and B is defined, the orders of matrices *A, B* and *C*, respectively, are

A. 2×2, 2×2 and 4×2

B. 4×2, 2×2 and 2×2

C. 4×4, 4×2 and 2×2

D. 4×4, 4×4 and 4×2

Question 17

A round-robin competition was played between five competitors. Some of the results are shown below.

- Ingrid and John were defeated by Fred.
- Harriet and John both had exactly three wins each.
- Gloria did not win a game.

The one-step dominance matrix for this round-robin is

A.
$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} F & G & H & I & J \end{array} \\ \textit{winner} \begin{array}{c} F \\ G \\ H \\ I \\ J \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

B.
$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} F & G & H & I & J \end{array} \\ \textit{winner} \begin{array}{c} F \\ G \\ H \\ I \\ J \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

C.
$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} F & G & H & I & J \end{array} \\ \textit{winner} \begin{array}{c} F \\ G \\ H \\ I \\ J \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

D.
$$\begin{array}{cc} & \textit{loser} \\ & \begin{array}{ccccc} F & G & H & I & J \end{array} \\ \textit{winner} \begin{array}{c} F \\ G \\ H \\ I \\ J \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Question 18

A matrix with an order of 3×2 could be which one of the following?

A. A triangular matrix

B. A binary matrix

C. A square matrix

D. A permutation matrix

Question 19

There are different prices for the membership of a gym: $78 per month for a 12-month contract; $88 per month for a six-month contract; and $95 per month for a three-month contract.

In one particular month, there are 124 people on a 12-month contract, 63 on a six-month contract and 13 on a three-month contract.

The matrix product that provides the total amount collected for all membership options during that month is

A. $\begin{bmatrix} 78 \\ 88 \\ 95 \end{bmatrix} \times \begin{bmatrix} 124 & 63 & 13 \end{bmatrix}$

B. $\begin{bmatrix} 124 \\ 63 \\ 13 \end{bmatrix} \times \begin{bmatrix} 78 & 88 & 95 \end{bmatrix}$

C. $\begin{bmatrix} 78 & 88 & 95 \end{bmatrix} \times \begin{bmatrix} 124 \\ 63 \\ 13 \end{bmatrix}$

D. $\begin{bmatrix} 124 & 63 & 13 \end{bmatrix} \times \begin{bmatrix} 95 \\ 88 \\ 78 \end{bmatrix}$

Question 20

Consider the matrix product $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} \times 2 \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Which one of the following is **not** a calculation from the matrix product?

A. $3 \times 2a - 1 \times 2c$

B. $4 \times 2a + 2 \times 2c$

C. $3 \times 2b - 2 \times 2d$

D. $4 \times 2b + 2 \times 2d$

Question 21

Consider the matrix M below.

$$M = \begin{bmatrix} 11 & 17 \\ 23 & 41 \\ 21 & 35 \end{bmatrix}$$

The element M_{21} is

A. 11

B. 17

C. 21

D. 23

Question 22

Matrix E is a 2×3 matrix, and matrix F is a 2×4 matrix.

The order of $E^T \times 2F$ is

A. 2×3

B. 3×4

C. 4×2

D. 2×4

Question 23

A matrix equation is $KX = L$, where $K = \begin{bmatrix} 3 & 2 \\ -6 & -4 \end{bmatrix}$, $X = \begin{bmatrix} a \\ b \end{bmatrix}$ and $L = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

Which one of the following statements is true?

A. $X = K^{-1} \times L$ has a unique solution.

B. K has a determinant of 0.

C. The determinant of K is −24.

D. The inverse of K is $\begin{bmatrix} \frac{1}{6} & \frac{1}{12} \\ \frac{1}{4} & -\frac{1}{8} \end{bmatrix}$.

Question 24

$3M + 2N = \begin{bmatrix} 1 & -2 & 22 \\ 25 & 39 & 12 \end{bmatrix}$

If $N = \begin{bmatrix} -1 & 2 & 5 \\ 2 & 3 & 6 \end{bmatrix}$, matrix M is

A. $\begin{bmatrix} 1 & -2 & 4 \\ 7 & 11 & 0 \end{bmatrix}$

B. $\begin{bmatrix} -1 & -6 & 12 \\ 21 & 33 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 3 & -6 & 12 \\ 21 & 33 & 0 \end{bmatrix}$

D. $\begin{bmatrix} -2 & 4 & 10 \\ 4 & 6 & 12 \end{bmatrix}$

Question 25

Teachers *A, B, C* and *D* are on a roster to supervise after-school activities from Monday to Friday.

The order of supervision follows the transition matrix, T, shown below.

$$\begin{array}{c} \textit{this day} \\ \begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ T = & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array} \textit{next day} \end{array} \end{array}$$

Teacher A supervised first on Monday of the first week.

Which one of the following statements is **not** true?

A. In the second week, Teacher *C* supervises twice.

B. The supervisor on Monday will always supervise on the Friday of the same week.

C. Teacher *D* will supervise on the third Tuesday.

D. The fifth supervision day for Teacher *B* will be on Friday of the third week.

EXAM 2

Question 1 (4 marks)

A postal delivery company charges fees for postage based on the size of the parcel: small (*S*), medium (*M*), large (*L*) and overweight (*O*). Matrix *C* contains the cost of each size parcel.

$$C = \begin{array}{cccc} S & M & L & O \\ \end{array}$$
$$C = \begin{bmatrix} 4 & 5.60 & 7.20 & 9 \end{bmatrix}$$

The number of each type of parcel that was delivered in January 2016 is shown in the matrix P below.

$$P = \begin{bmatrix} 250 \\ 135 \\ 180 \\ 65 \end{bmatrix} \begin{array}{l} S \\ M \\ L \\ O \end{array}$$

a. Write down the order of matrix P. 1 mark

b. **i.** Calculate the matrix product $M = C \times P$. 1 mark

ii. What does matrix M represent? 1 mark

iii. Explain why the matrix product $P \times C$ also exists. 1 mark

Question 2 (2 marks)

A local gymnasium offers members every day (*E*), frequent (*F*) and occasional (*O*) memberships, based on how regularly they attend the gym. Membership is renewed on a monthly basis. The number of members who chose each option for the first three months of 2018 is shown in the matrix G.

$$G = \begin{array}{c} \begin{array}{ccc} E & F & O \end{array} \\ \begin{bmatrix} 125 & 76 & 35 \\ 107 & 95 & 52 \\ 95 & 120 & 43 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{January} \\ \text{February} \\ \text{March} \end{array}$$

a. In total, how many memberships were taken at the gym in February 2018? 1 mark

b. What does the element G_{23} represent? 1 mark

Question 3 (5 marks)

A local produce store sells three types of tomatoes: Roma (*R*), cherry (*C*) and heirloom (*H*). The cost, in dollars, to purchase one kilogram of each type of tomato is shown in matrix A below.

$$A = \begin{bmatrix} 7.98 \\ 9.78 \\ 4.39 \end{bmatrix} \begin{array}{l} R \\ C \\ H \end{array}$$

a. What is the cost of one kilogram of cherry tomatoes? 1 mark

b. Write down the order of matrix A. 1 mark

c. Jack would like to purchase one kilogram of Roma tomatoes and two kilograms of heirloom tomatoes.

The total cost of Jack's purchases can be found by the matrix product $B \times A$.

Write down the row matrix B. 1 mark

$B =$ ______

A local farmer grows all three types of tomatoes, and at the beginning of each season assigns a certain number of square metres to grow each. The number of square metres assigned for each crop in 2018 is shown in matrix C_{2018} below.

$$C_{2018} = \begin{bmatrix} 48 \\ 25 \\ 45 \end{bmatrix} \begin{array}{l} R \\ C \\ H \end{array}$$

Each year, the farmer can expect to lose approximately 10% of each crop due to drought, disease or other factors.

d. Find the expected number of remaining square metres at the start of 2019 for the cherry tomato crop.

Round your answer to the nearest whole number. 1 mark

e. The expected number of square metres remaining for each type of tomato at the start of 2019 can be determined from the matrix product $C_{2019} = k \times C_{2018}$ where k is a scalar value.

Write down the scalar value k. 1 mark

$k =$

Question 4 (3 marks)

A new shopping centre is being built, with two carparks, *A* and *B*, being built around it.

The number of car spaces allocated to non-disabled visitors (*N*), visitors with prams (*P*) and visitors with disability permits (*D*) is shown in matrix S.

$$S = \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 148 & 82 \\ 10 & 4 \\ 12 & 6 \end{bmatrix} \end{array} \begin{array}{c} \\ N \\ P \\ D \end{array}$$

a. Write down the order of matrix S. 1 mark

b. What does the element S_{21} represent? 1 mark

c. Write a matrix product calculation that shows that the total number of parking spaces for carpark *A* is 170 and *B* is 92. 1 mark

Question 5 (2 marks)

The number of trips using a car from the rideshare services Dryve (*D*) and Gogo (*G*) in one night is shown in matrix N below.

$$N = \begin{array}{cc} D & G \\ \left[12\right. & \left.16\right] \end{array}$$

The average fare for these trips was \$11.45 for Dryve and \$9.98 for Gogo.

a. Write down a column matrix, F, to show this information. 1 mark

$F =$ ______________________

b. Calculate the total amount received by the fares for the two services combined. 1 mark

____

____

Question 6 (4 marks)

Kyle manages a hostel for backpackers. It has a combination of single rooms (*S*), twin rooms (*T*), and dormitory rooms. The dormitory rooms are classified as large (*L*) or medium (*M*) sized.

The number of rooms available is shown in the matrix R below.

$$R = \begin{bmatrix} 6 \\ 8 \\ 1 \\ 2 \end{bmatrix} \begin{array}{l} S \\ T \\ L \\ M \end{array}$$

a. How many dormitory rooms are there? 1 mark

____

b. The large room is out of order. The matrix calculation below shows that the number of single, twin and medium rooms is 16 in total.

$T \times R = [16]$

Write down matrix T below. 1 mark

$T =$ ______________________

c. Kyle considers remodelling the hostel.

He proposes increasing the number of rooms, as shown in matrix I below.

$$I = \begin{bmatrix} 9 \\ 14 \\ 2 \\ 4 \end{bmatrix} \begin{matrix} S \\ T \\ L \\ M \end{matrix}$$

The number of people who can be accommodated in each room is shown in matrix P below.

$$\begin{array}{cccc} S & T & L & M \end{array}$$
$$P = \begin{bmatrix} 1 & 2 & 12 & 6 \end{bmatrix}$$

The council regulations require that there is a minimum of one toilet for every nine people in the hostel.

How many toilets are required in Kyle's hostel?

Show the matrix calculation, including a scalar multiplication, to justify your answer. 2 marks

Question 7 (3 marks)

Kyle arranges for games to be provided for guests staying at his house on a Tuesday night.

A round-robin game is played with the guests broken into five teams. The table below outlines the number of direct and indirect wins of each team.

Team	One-step dominance	Two-step dominance
A	2	3
B	3	5
C	1	2
D	2	3
E	2	5

a. From this information, it can be determined with certainty that Team E beat which other team? 1 mark

b. Teams A and B compared their wins and losses and saw that there were only two other teams they had both beaten in common.

Using the information and the table above, determine which teams Team D beat. 2 marks

Area of Study 2 Transition matrices

EXAM 1

Question 1

Families in a small town were asked about their weekly supermarket choices.

Every week, these families choose between The General Store (*G*), Ultramarket (*U*) or Tradingzone (*T*).

No families will miss a weekly shop, and none expressed an interest in leaving the small town.

Which of the following matrices could represent the transition matrix for this information?

A. $$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} G & T & U \end{array} \\ \begin{bmatrix} 0.65 & 0.15 & 0.20 \\ 0.40 & 0.30 & 0.30 \\ 0.22 & 0.64 & 0.10 \end{bmatrix} \end{array} \begin{array}{l} G \\ T \; \textit{next week} \\ U \end{array}$$

B. $$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} G & T & U \end{array} \\ \begin{bmatrix} 0.65 & 0.15 & 0.20 \\ 0.40 & 0.30 & 0.30 \\ 0.22 & 0.64 & 0.10 \end{bmatrix} \end{array} \begin{array}{l} G \\ T \; \textit{next week} \\ U \end{array}$$

C. $$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} G & T & U \end{array} \\ \begin{bmatrix} 0.45 & 0.15 & 0.40 \\ 0.40 & 0.20 & 0.20 \\ 0.15 & 0.65 & 0.25 \end{bmatrix} \end{array} \begin{array}{l} G \\ T \; \textit{next week} \\ U \end{array}$$

D. $$\begin{array}{c} \textit{this week} \\ \begin{array}{ccc} G & T & U \end{array} \\ \begin{bmatrix} 0.65 & 0.13 & 0 \\ 0.10 & 0.23 & 0 \\ 0.25 & 0.64 & 1 \end{bmatrix} \end{array} \begin{array}{l} G \\ T \; \textit{next week} \\ U \end{array}$$

Use the following information to answer Questions 2 and 3.

In Term 1, 2016, on Friday of the first week, 450 students were asked which drink choice, from iced tea (*I*), juice (*J*) or flavoured milk (*M*), they would order to accompany their lunch. The school only serves these choices on Fridays, with students allowed only water for the remaining days of the week.

175 students said that they would choose juice, 215 would choose iced tea and the remaining students selected flavoured milk.

This information is represented in the state matrix, S_1, below.

$$S_1 = \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} \begin{array}{l} I \\ J \\ M \end{array}$$

Assuming the same number of students order a drink from the canteen the following week, the transition matrix for students' drink choice each week is represented by the transition matrix, T, below.

$$T = \begin{array}{c} \textit{this week} \\ \begin{array}{ccc} I & J & M \end{array} \\ \begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix} \end{array} \begin{array}{l} I \\ J \; \textit{next week} \\ M \end{array}$$

Question 2

The number of students who will select juice in the third week is closest to

A. 140

B. 141

C. 142

D. 146

Question 3

In the long term, the number of students who will select iced tea as their drink choice is closest to

A. 209

B. 208

C. 141

D. 140

Use the following information to answer Questions 4 and 5.

A local veterinary clinic runs a dog training school. The school caters for four age groups: puppies (*P*), juniors (*J*), young adults (*Y*) and adults (*A*). Training school starts in August and concludes in November.

The number of dogs enrolled in each training group in August 2017 is shown in the matrix below.

$$A_{2017} = \begin{array}{c} \begin{bmatrix} 25 \\ 17 \\ 21 \\ 14 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} P \\ J \\ Y \\ A \\ L \end{array}$$

At the end of each year, the trainers decide whether the dog can proceed to the next level. Some dogs are required to repeat a class. Some owners decide not to continue with training and leave the program (*L*).

The transition matrix, T, representing the progression of dogs through the training program is shown below.

$$T = \begin{array}{c} \begin{array}{ccccc} P & J & Y & A & L \end{array} \\ \begin{bmatrix} 0.16 & 0 & 0 & 0 & 0 \\ 0.68 & 0.09 & 0 & 0 & 0 \\ 0 & 0.76 & 0.21 & 0 & 0 \\ 0 & 0 & 0.68 & 0.12 & 0 \\ 0.16 & 0.15 & 0.11 & 0.88 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ P \\ J \\ Y \\ A \\ L \end{array}$$

Question 4

The number of dogs that will be enrolled in the adult (A) class for August 2019 is closest to

A. 13

B. 14

C. 15

D. 16

Question 5

In order to maintain the same number of dogs enrolled in each training program every year, the number of puppies enrolled needs to remain consistent. The number of new puppy (P) enrolments required each year is

A. 0

B. 4

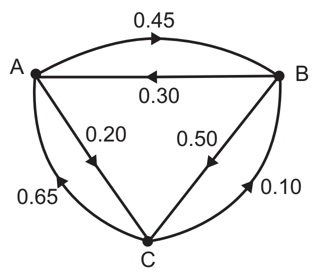
C. 21

D. 24

Question 6

The transition diagram below has been constructed from a transition matrix.

The labelling in the transition diagram is not yet complete.



The corresponding transition matrix, D, is represented by

A. $D = \begin{array}{c} \textit{this year} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.35 & 0.45 & 0.20 \\ 0.30 & 0.20 & 0.50 \\ 0.65 & 0.10 & 0.25 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \text{ next year} \\ C \end{array}$

B. $D = \begin{array}{c} \textit{this year} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.35 & 0.20 & 0.65 \\ 0.45 & 0.30 & 0.10 \\ 0.20 & 0.50 & 0.25 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \text{ next year} \\ C \end{array}$

C. $D = \begin{array}{c} \textit{this year} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.25 & 0.30 & 0.65 \\ 0.45 & 0.20 & 0.10 \\ 0.20 & 0.50 & 0.25 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \text{ next year} \\ C \end{array}$

D. $D = \begin{array}{c} \textit{this year} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.35 & 0.30 & 0.65 \\ 0.45 & 0.20 & 0.10 \\ 0.20 & 0.50 & 0.25 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \text{ next year} \\ C \end{array}$

Use the following information to answer Questions 7–9.

Students in Years 11 and 12 can choose from three levels of mathematics courses: mathematics A (A), mathematics B (B) or mathematics C (C). Alternatively, they can choose not to study mathematics (N). Each term, some students elect to change their course, while others remain in the same course. This is shown in the transition matrix below.

$$T = \begin{array}{c} \\ \\ \end{array} \begin{array}{l} \textit{this term} \\ \begin{array}{cccc} A & B & C & N \end{array} \\ \begin{bmatrix} 0.94 & 0.08 & 0 & 0 \\ 0.04 & 0.88 & 0.16 & 0 \\ 0 & 0.03 & 0.81 & 0 \\ 0.02 & 0.01 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ N \end{array} \textit{next term}$$

The number of students enrolled in each course, or not studying mathematics, in Term 1 of Year 11 is shown below.

$$S_1 = \begin{bmatrix} 120 \\ 70 \\ 25 \\ 15 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ N \end{array}$$

Question 7

The number of students enrolled in each course, or **not** studying mathematics, in Term 3 of Year 12 is closest to

A. $\begin{bmatrix} 118 \\ 70 \\ 23 \\ 18 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ N \end{array}$

B. $\begin{bmatrix} 116 \\ 70 \\ 20 \\ 24 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ N \end{array}$

C. $\begin{bmatrix} 113 \\ 69 \\ 18 \\ 30 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ N \end{array}$

D. $\begin{bmatrix} 112 \\ 68 \\ 17 \\ 33 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ N \end{array}$

Question 8

The school needs 20 or more students enrolled in a class in order for that course to run.

When does mathematics C fall below 20 students?

A. Term 3 of Year 11

B. Term 4 of Year 11

C. Term 1 of Year 12

D. Term 2 of Year 12

Question 9

Students change and leave schools during the two years. Some students leave the school, reducing the number of students in the mathematics courses, while others join the school, increasing the numbers in the mathematics courses. Matrix *B* below shows the number of students each term (from Term 2 of Year 11) who are added or removed from the school, shown by the courses in which they are enrolled.

$$B = \begin{bmatrix} -1 \\ 1 \\ 0 \\ -3 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ N \end{matrix}$$

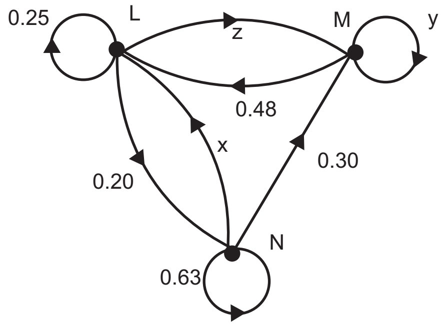
The number of students enrolled in each course, or **not** studying mathematics, in Term 3 of Year 11 is closest to

A. $\begin{bmatrix} 117 \\ 71 \\ 23 \\ 15 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ N \end{matrix}$ **B.** $\begin{bmatrix} 116 \\ 71 \\ 22 \\ 18 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ N \end{matrix}$

C. $\begin{bmatrix} 115 \\ 71 \\ 20 \\ 21 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ N \end{matrix}$ **D.** $\begin{bmatrix} 115 \\ 72 \\ 22 \\ 15 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ N \end{matrix}$

Question 10

The transition diagram below shows the probability of customer movement between three venues. Some of the values are missing from the transitions.



The transition matrix that matches the transition diagram, with missing values included, is

A. $\begin{array}{c} \text{from} \\ \begin{array}{ccc} L & M & N \end{array} \\ \begin{bmatrix} 0.25 & 0.48 & 0.07 \\ 0.55 & 0.52 & 0.30 \\ 0.20 & 0 & 0.63 \end{bmatrix} \end{array} \begin{matrix} L \\ M \\ N \end{matrix} \; \textit{to}$

B. $\begin{array}{c} \text{from} \\ \begin{array}{ccc} L & M & N \end{array} \\ \begin{bmatrix} 0.25 & 0.48 & 0.07 \\ 0.55 & 0.70 & 0.30 \\ 0.20 & 0 & 0.63 \end{bmatrix} \end{array} \begin{matrix} L \\ M \\ N \end{matrix} \; \textit{to}$

C. $\begin{array}{c} \text{from} \\ \begin{array}{ccc} L & M & N \end{array} \\ \begin{bmatrix} 0.25 & 0 & 0.07 \\ 0.55 & 0.52 & 0.30 \\ 0.20 & 0.48 & 0.63 \end{bmatrix} \end{array} \begin{matrix} L \\ M \\ N \end{matrix} \; \textit{to}$

D. $\begin{array}{c} \text{from} \\ \begin{array}{ccc} L & M & N \end{array} \\ \begin{bmatrix} 0.25 & 0.30 & 0.07 \\ 0.55 & 0.70 & 0.30 \\ 0.20 & 0 & 0.63 \end{bmatrix} \end{array} \begin{matrix} L \\ M \\ N \end{matrix} \; \textit{to}$

Question 11

A sample of 500 households is monitored for television-watching habits.

The number of households watching three leading television shows on a Sunday night is recorded on the matrix below. The shows are *I'm a Student, Get Me Out of Here* (*S*), *Masterchess* (*M*) and *The Brick* (*B*).

$$\begin{array}{cc} & \textit{this Sunday} \\ & \begin{array}{ccc} S & M & B \end{array} \\ T = & \begin{bmatrix} 0.81 & 0.08 & 0.06 \\ 0.10 & 0.77 & 0.02 \\ 0.09 & 0.15 & 0.92 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ S \\ M \;\; \textit{next Sunday} \\ B \end{array}$$

It is found that on the third Sunday, 125 households were watching *I'm a Student, Get Me Out of Here*, 212 were watching *Masterchess*, and the rest were watching *The Brick*.

The number of households watching *Masterchess* on the first Sunday is closest to

A. 111

B. 154

C. 163

D. 317

Question 12

A species of insect has a life span of three years.

Consider the Leslie matrix, L, which models the female population distribution of this species.

$$\begin{array}{cc} & \begin{array}{ccc} E & J & A \end{array} \\ L = & \begin{bmatrix} 0 & 2 & 3 \\ 0.7 & 0 & 0 \\ 0 & 0.92 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ E \\ J \\ A \end{array}$$

Which of these statements is **incorrect**?

A. On average, 30% of female eggs are not expected to survive to become juveniles.

B. On average, 92% of female juveniles are expected to survive to become adults.

C. On average, 2 female eggs are expected to be produced by female juveniles.

D. On average, 3 female eggs are not expected to survive to become female adults.

Question 13

The change in the number of people at three stores each week is shown in the transition matrix below.

The number of people at each store in the second week is also shown.

$$T = \begin{bmatrix} 0.05 & 0.81 & 0.17 \\ 0.75 & 0.09 & 0.05 \\ 0.2 & 0.1 & 0.78 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 242 \\ 315 \\ 208 \end{bmatrix}$$

The number of people at each store in the first week is closest to

A. $\begin{bmatrix} 309 \\ 238 \\ 240 \end{bmatrix}$ **B.** $\begin{bmatrix} 378 \\ 243 \\ 134 \end{bmatrix}$

C. $\begin{bmatrix} 372 \\ 240 \\ 130 \end{bmatrix}$ **D.** $\begin{bmatrix} 381 \\ 246 \\ 137 \end{bmatrix}$

EXAM 2

Question 1 (8 marks)

A postal company has four distribution points in a small town: Abbey (A), Buron (B), Cally (C) and Dovan (D). The company is studying the choice made by its customers between the four distribution points in the town. Matrix T, shown below, contains the percentages of customers who are expected to change where they post their parcel from month to month.

$$\begin{array}{cc} & \textit{this month} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ T = & \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array} \textit{ next month} \end{array}$$

Let S_n be the matrix that shows the number of parcels delivered from each distribution centre n months after January 2016. Matrix S_0 below shows the number of parcels delivered from each distribution centre in January 2016.

$$S_0 = \begin{bmatrix} 250 \\ 125 \\ 125 \\ 100 \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array}$$

Matrix S_1 below shows the number of parcels delivered from each distribution centre in February 2016.

$$S_1 = TS_0 = \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} \begin{array}{c} A \\ B \\ C \\ D \end{array}$$

a. Find the value of f, which is missing from the matrix S_1. 1 mark

b. Write a calculation that shows that 134 parcels were expected to be delivered to Cally in February 2016. 1 mark

c. Consider the customers who chose the Dovan distribution centre in January 2016.

i. What percentage of these customers was expected to choose Dovan again in February 2016? 1 mark

ii. How many of these customers are **not** expected to choose Dovan again in February 2016? 1 mark

d. In the long term, how many customers are expected to choose Abbey as their preferred distribution centre? Show matrix calculations to support your answer. 2 marks

In January 2017, the postage company increases its business by extending the zone in which it will collect and deliver parcels. The matrix J_{2017} shows the number of customers who chose each distribution centre in January 2017.

$$J_{2017} = \begin{array}{c} \begin{bmatrix} 350 \\ 225 \\ 270 \\ 240 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ D \end{array} \end{array}$$

The company intends to continue to grow its customer base through increased advertising initiatives. The matrix that contains the number of customers who are expected to choose each distribution centre in February 2017 (F_{2017}) and March 2017 (M_{2017}) can be determined using the following matrix equations

$$F_{2017} = T \times J_{2017} + B \qquad M_{2017} = T \times F_{2017} + B$$

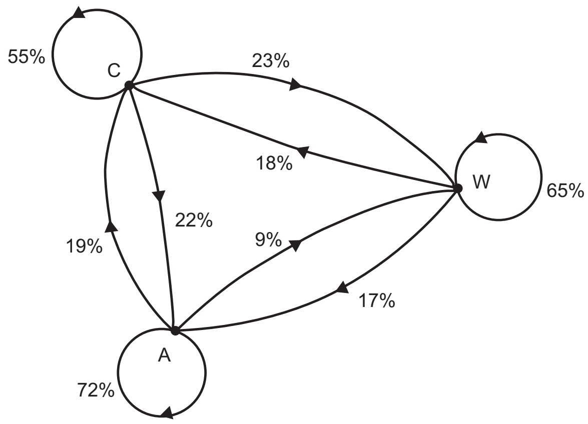
$$\text{where } T = \begin{array}{c} \textit{this month} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \end{array} \textit{ next month} \text{ and } B = \begin{bmatrix} 45 \\ 65 \\ 70 \\ 40 \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ D \end{array}$$

e. Determine the number of customers who are expected to choose the Buron distribution centre in March 2017.

Round your answer to the nearest whole number. 2 marks

Question 2 (3 marks)

Members at a gym are offered a selection of three group classes, which they can attend once a week during their monthly membership. Members can choose from cycle (C), weights (W) and aerobics (A) classes. The transition diagram below shows the way in which the gym members are expected to change their choice of group class from week to week.



a. Of the members who chose aerobics (A) in one week, what percentage are **not** expected to choose aerobics (A) the following week? 1 mark

Matrix S_1 lists the number of gym members who chose each exercise class in the first week of their month membership.

$$S_1 = \begin{array}{c} \begin{bmatrix} 51 \\ 101 \\ 73 \end{bmatrix} \begin{array}{l} C \\ W \\ A \end{array} \end{array}$$

b. How many gym members are expected to choose the weights class (W) in their second week of membership? 1 mark

c. Of the members who chose the cycle class (C) in the first week, how many will remain in the cycle class in the third week of their membership? 1 mark

Question 3 (5 marks)

There are three competing fitness centres in an area: Fit Gym (*F*), Healthy Life (*H*) and Smash It (*S*). Local citizens choose between the three centres at the beginning of each year; although, some people who were gym members choose to leave (*L*) and not sign up to a new centre. The transition matrix T shows the way in which the local citizens are expected to change their choice of fitness centre from year to year.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{array}{cccc} F & H & S & L \end{array} \\ T = & \begin{bmatrix} 0.44 & 0.34 & 0.12 & 0 \\ 0.18 & 0.35 & 0.11 & 0 \\ 0.32 & 0.22 & 0.66 & 0 \\ 0.06 & 0.09 & 0.11 & 1 \end{bmatrix} \begin{array}{c} F \\ H \\ S \\ L \end{array} \textit{next year} \end{array}$$

Let M_1 be the state matrix for the number of citizens expected to choose each fitness centre in 2018.

$$M_1 = \begin{bmatrix} 120 \\ 120 \\ 120 \\ 0 \end{bmatrix} \begin{array}{c} F \\ H \\ S \\ L \end{array}$$

a. How many people will change to a different fitness centre at the beginning of 2019? 1 mark

b. Complete M_3, the state matrix for 2020, below. Round element values to the nearest whole numbers. 1 mark

$$M_3 = \begin{bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{bmatrix} \begin{array}{c} F \\ H \\ S \\ L \end{array}$$

c. How many people will choose Smash It (*S*) in 2022? 1 mark

d. The manager at Healthy Life (*H*) is concerned that the membership numbers are decreasing rapidly. If the number of members falls below 25% of the enrolments in 2018, the fitness centre will be forced to close. The manager believes that he will have to close his fitness centre by 2025.

Is the manager correct in his assumption? Use evidence to support your answer. 2 marks

Question 4 (7 marks)

Once a year, Johanna uses an old family recipe to make homemade sauce using heirloom tomatoes. In order to purchase enough tomatoes for her sauce, she must shop at four different produce suppliers: *A, B, C* and *D*. In 2018, she purchased 3.5 kg from each of the four suppliers. Let S_0 be the state matrix that shows the amount of tomatoes, in grams, purchased by Johanna, from each supplier in 2018.

$$S_0 = \begin{array}{c} \begin{bmatrix} 3500 \\ 3500 \\ 3500 \\ 3500 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \end{array}$$

The quality of the tomatoes at each supplier varies from year to year, and as such Johanna does not always purchase an equal amount of tomatoes from each. The transition matrix T below can be used to predict the weight, in grams, of heirloom tomatoes that Johanna will purchase from each supplier.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ T = & \begin{bmatrix} 0.80 & 0.05 & 0.07 & 0.11 \\ 0.05 & 0.75 & 0.03 & 0.06 \\ 0.10 & 0.12 & 0.86 & 0.05 \\ 0.05 & 0.08 & 0.04 & 0.78 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \end{array} \textit{next year}$$

a. What percentage of Johanna's purchases from supplier *A* in 2018 will be lost to other suppliers in 2019? 1 mark

b. How many grams of tomatoes will be purchased from supplier *C* in 2019? 1 mark

c. Complete the state matrix, S_1, below for the number of tomatoes, in grams, purchased from each supplier in 2019. 1 mark

$$S_1 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} \begin{array}{l} A \\ B \\ C \\ D \end{array}$$

d. How many tomatoes, in grams, is Johanna expected to purchase from supplier *B* in 2021?

Round your answer to the nearest whole number. 1 mark

e. In the long term, what percentage of Johanna's total tomato purchases will come from supplier *D*?

Round your answer to one decimal place. 1 mark

In 2022, supplier *D* informs Johanna that they can only supply her with 2 kilograms of tomatoes due to issues with their own supply. Johanna still requires 14 kilograms of tomatoes in total. Her purchases in 2022 are given by R_0. Johanna's purchases in the following years can be calculated using the matrix equation below.

$$R_{n+1} = T \times R_n + G$$

where

$$R_0 = \begin{array}{c} \begin{bmatrix} 4000 \\ 4000 \\ 4000 \\ 2000 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \end{array} \qquad T = \begin{array}{c} \textit{this year} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.80 & 0.05 & 0.07 & 0.11 \\ 0.05 & 0.75 & 0.03 & 0.06 \\ 0.10 & 0.12 & 0.86 & 0.05 \\ 0.05 & 0.08 & 0.04 & 0.78 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \end{array} \textit{ next year} \qquad G = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

f. How many grams of tomatoes is Johanna expected to purchase from supplier *B* in 2022? 1 mark

g. Johanna has decided that her purchases will remain consistent in the future.

Complete the matrix G, below. 1 mark

$$G = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$$

Question 5 (5 marks)

The company running the summer fair in Princenton (*P*) has other events in nearby towns: Riversville (*R*) and Summertown (*S*). On 1 December only, visitors can buy a pass for the month of December, which allows them entry to all three events for each day during December. 5000 people purchase a monthly pass. The number of monthly pass holders who visit each of the three events is counted daily. The state matrix, V_0, below, shows the number of visitors holding a monthly pass at each event on 1 December.

$$V_0 = \begin{bmatrix} 1500 \\ 1650 \\ 1850 \end{bmatrix} \begin{array}{c} P \\ R \\ S \end{array}$$

a. How many of the monthly pass holders attended the summer event in Riversville (*R*) on 1 December? 1 mark

Let V_n be the state matrix that shows the number of monthly pass holders expected to visit each of the summer events n days after 1 December. The number of visitors expected at each location n days after 1 December can be determined by the matrix recursion relation below.

$V_{n+1} = T \times V_n$ where

$$T = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{c} \textit{today} \\ \begin{array}{ccc} P & R & S \end{array} \\ \begin{bmatrix} 0.1 & 0.45 & 0.35 \\ 0.4 & 0.15 & 0.55 \\ 0.5 & 0.4 & 0.1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ P \\ R \quad \textit{next day} \\ S \end{array}$$

b. Write down the state matrix V_1 below, to show the number of monthly pass holders expected to visit each summer event on 2 December. 1 mark

$$V_1 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} \begin{array}{l} P \\ R \\ S \end{array}$$

c. Write down a calculation to show that, of the 1540 pass holders expected to visit the Princenton (*P*) summer event on 2 December, 90% had visited the Riversville (*R*) or Summertown (*S*) events on 1 December. 1 mark

d. By the end of December, assuming the monthly pass holders attend a summer event each day of the month, the number of visitors at each event will reach a constant total. This will occur when the recurrence relation has reached its steady state.

i. Show that the steady state has been reached at the end of December by showing two consecutive state matrices. 1 mark

ii. How many visitors will visit the summer event in Summertown (*S*) at this time? 1 mark

Question 6 (3 marks)

The summer event at Summertown consists of four children's activities: arts and crafts (*W*), jumping castle (*X*), face painting (*Y*) and musical instruments (*Z*). Each activity allows a maximum of 100 children to participate per hour, beginning at 11 am. On a particular day, 400 child admission tickets are sold. All children enter the event before 11 am, and stay until closing time of 3 pm. Let D_0 be the state matrix that shows the number of children participating in each activity at 11 am. The number of children expected at each activity at midday can be determined by the matrix product shown below.

$$K \times D_0 \qquad \text{where } D_0 = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}\begin{matrix} W \\ X \\ Y \\ Z \end{matrix} \qquad \text{and} \qquad K = \begin{array}{c} \textit{this hour} \\ \begin{array}{cccc} W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 0.24 & 0.40 & 0.20 \\ 0.35 & 0 & 0.25 & 0.30 \\ 0.30 & 0.46 & 0 & 0.50 \\ 0.35 & 0.30 & 0.35 & 0 \end{bmatrix} \end{array}\begin{matrix} W \\ X \\ Y \\ Z \end{matrix} \; \textit{next hour}$$

a. What do the zeros in the transition matrix, K, represent in terms of the summer activity that each child will choose for the next hour? 1 mark

b. At 1 pm each day, additional staff members are rostered on to each activity. At this time, the capacity for each activity at the event in Summertown is raised to 150 participants. State matrix L_n contains the number of visitors at each location n hours after 1 pm, after the capacity is raised to 150.

Matrix L_1 can be determined from the matrix recurrence relation

$$L_1 = K \times L_0 + B \text{ where } L_0 = \begin{bmatrix} 150 \\ 150 \\ 150 \\ 150 \end{bmatrix}\begin{matrix} W \\ X \\ Y, \\ Z \end{matrix}$$

where matrix B shows the required movement to maintain the maximum number of 150 children at each activity.

Find the number of children at each event at 2 pm, prior to any required movement of children between activities. 1 mark

$W =$

$X =$

$Y =$

$Z =$

c. Determine the matrix B. 1 mark

$B =$

Question 7 (5 marks)

As backpackers travel through different cities and locations, they book into different hostels.

The transition matrix, T, below shows the probability that a backpacker will pay for a single room (*S*), a twin room (*T*) or a dormitory style room (*D*) when moving from one hostel to the next.

$$\begin{array}{c} \textit{this hostel} \\ \begin{array}{ccc} S & T & D \end{array} \end{array}$$
$$T = \begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array} \ \textit{next hostel}$$

Kyle's hostel is considered to be the first hostel. There are currently 38 guests at Kyle's hostel and this is represented by the matrix S_0.

$$S_0 = \begin{bmatrix} 5 \\ 12 \\ 21 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array}$$

a. **i.** Complete the following by filling in the boxes provided, showing that there are approximately 19 backpackers staying in a dormitory room at the second hostel. 1 mark

$$\boxed{} \times 5 + \boxed{} \times 12 + \boxed{} \times 21 = 19$$

ii. How many of the 38 backpackers changed their preference of room from Kyle's hostel to the second hostel? 1 mark

A number of the original 38 guests no longer need to book a room and a number of additional travellers need to book one.

The expected number of people booking the different rooms at each hostel can be determined from the matrix recurrence relation below

$$S_{n+1} = T \times S_0 + B$$

where S_0 is the number of guests in the first hostel, and B is the number of guests joining or leaving.

$$\begin{array}{c} \textit{this hostel} \\ \begin{array}{ccc} S & T & D \end{array} \end{array}$$
$$T = \begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array} \ \textit{next hostel}$$

$$S_0 = \begin{bmatrix} 5 \\ 12 \\ 21 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array} \text{ and } B = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} \begin{array}{l} S \\ T \\ D \end{array}$$

b. Interpret the element B_{31} shown in matrix B. 1 mark

c. The third hostel has space for 8 people in single rooms, 12 people in twin rooms and 22 people in dormitory rooms.

Determine whether all the guests expected to book this hostel can be accommodated in the room type they wish to book.

Justify your answer with calculations. 2 marks

Question 8 (3 marks)

The age distribution of a snake population at a particular point in time is as follows:

Age	Number in population
0–3 years (*A*)	500
3–6 years (*B*)	500
6–9 years (*C*)	500

The expected lifespan is 9 years.

The average birth rate for snakes in the 3–6 years age range is 4.1.

The average birth rate for snakes in the 6–9 years age range is 1.5.

The survival rate from 0–3 years to 3–6 years is 70%.

The survival rate from 3–6 years to 6–9 years is 30%.

The birth rate and survival rate are shown in the Leslie matrix, L, below.

$$S_0 = \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix}$$

$$L = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 4.1 & 1.5 \\ 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \end{array}\begin{array}{c} \\ A \\ B \\ C \end{array}$$

a. Using $S_{n+1} = L \times S_n$, determine

i. the age distribution of snakes after one time period of 3 years. 1 mark

ii. the amount the total snake population increases by in that 3-year period. 1 mark

b. After two time periods (6 years), what is the expected number of snakes in the 3–6 years age range? 1 mark

Question 9 (2 marks)

The survival and birth rates of a rodent that lives among Mac's crops are shown below.

The life span of 12 months has been categorised into four age groups: 0−3 months (*A*), 3−6 months (*B*), 6−9 months (*C*) and 9−12 months (*D*).

The Leslie matrix, L, that models the breeding patterns for this rodent population is as follows.

$$L = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1.2 & 1.8 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

The initial state matrix for the rodent population, S_0, is shown below.

$$S_0 = \begin{bmatrix} 80 \\ 90 \\ 50 \\ 30 \end{bmatrix}$$

Using $S_{n+1} = L \times S_n$, determine

a. the number of rodents aged 9–12 months old after one 3-month period. 1 mark

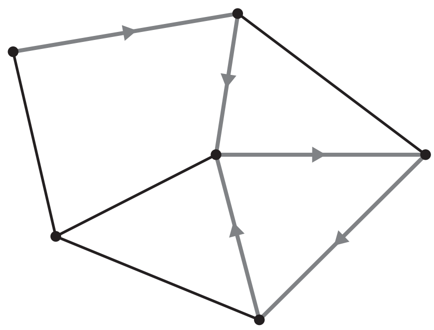
b. the percentage, as a whole number, that the total rodent population increased by in that 3-month period. 1 mark

Area of Study 2 Networks and decision mathematics – Undirected graphs and networks: travelling and connecting problems

EXAM 1

Question 1

Consider the following network.



The navigation through this network is best described as

A. a trail.

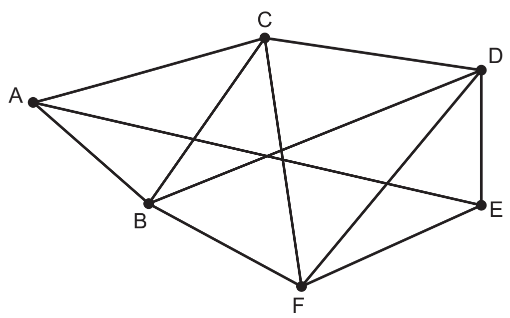
B. a path.

C. a cycle.

D. a circuit.

Question 2

The following graph does not contain an Eulerian circuit.



An edge can be added in order to allow a Eulerian circuit to exist.

This edge should be added between which two vertices?

A. *A* and *C*

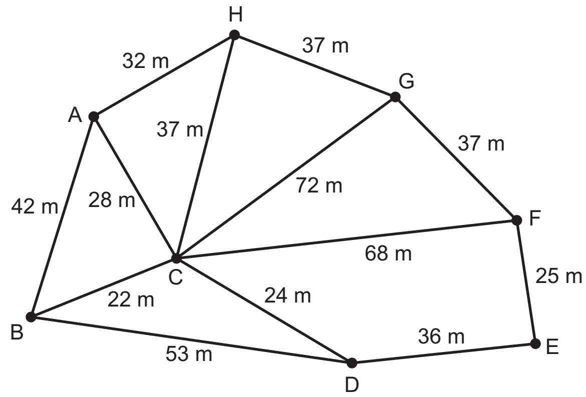
B. *A* and *B*

C. *C* and *E*

D. *E* and *A*

Question 3

Consider the network below.



When using Prim's algorithm to find the minimum spanning tree, which of the following edges is **not** included in the minimum spanning tree?

A. *E–F*

B. *A–H*

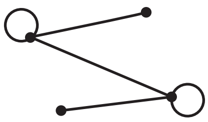
C. *A–C*

D. *C–G*

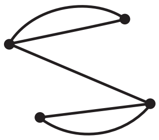
Question 4

Which of the following network diagrams represents a simple graph?

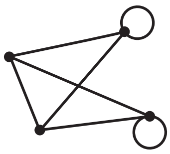
A.



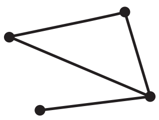
B.



C.

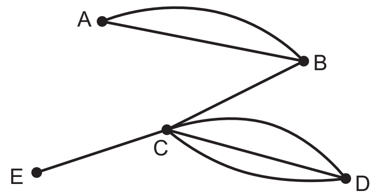


D.



Question 5

Consider the following graph.



The adjacency matrix for this graph is

A.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

B.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

C.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3 & 1 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

D.
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Question 6

For a planar graph with 21 edges and 17 faces, the number of vertices will be

A. 21

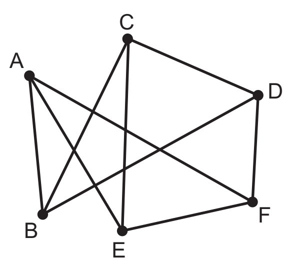
B. 17

C. 6

D. 4

Question 7

Consider the following graph.



The Hamiltonian cycle for this graph is

A. *A – C – D – F – E – B – A*

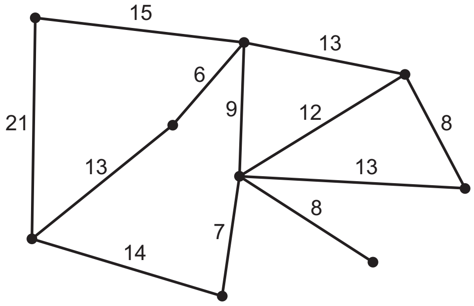
B. *A – B – C – D – E – F – A*

C. *A – F – D – C – E – B – A*

D. *A – F – E – C – D – B – A*

Question 8

Consider the following network.



The length of the minimum spanning tree in this network is

A. 78 **B.** 79

C. 80 **D.** 84

Question 9

A connected planar graph has six edges.

This graph could have

A. four vertices and five faces.

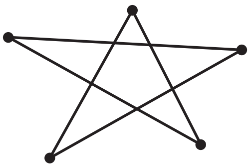
B. four vertices and six faces.

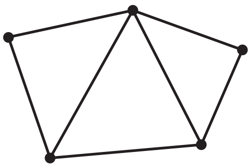
C. four vertices and seven faces.

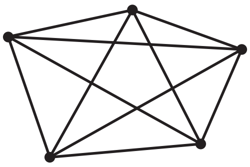
D. four vertices and four faces.

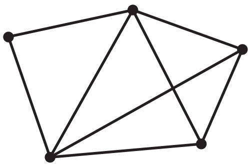
Question 10

Which of the following graphs is **not** a planar graph?

A.


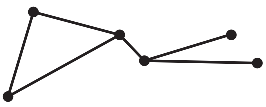
B.


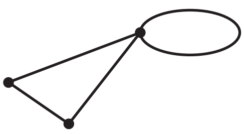
C.


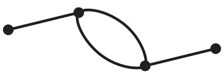
D.


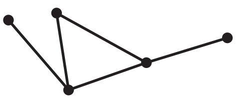
Question 11

Which of the following network diagrams contains a bridge?

A.


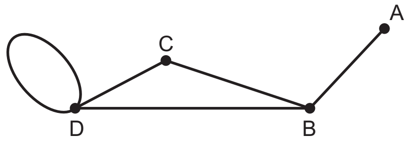
B.


C.


D.


Question 12

The graph below contains an Euler trail.



Which of the following actions will result in the graph no longer containing an Euler trail?

A. Removing the loop at *D*

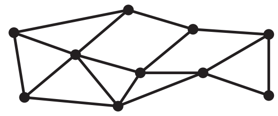
B. Removing the edge *BD*

C. Adding an edge to *AB*

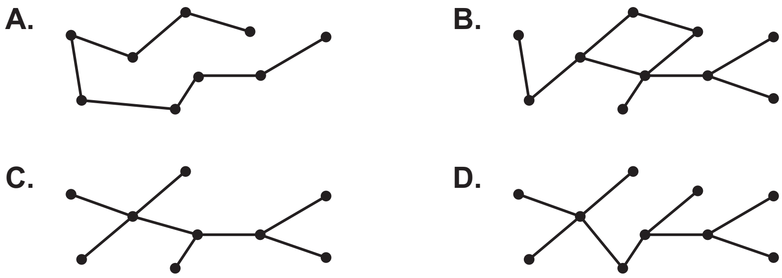
D. Adding an edge to *AC*

Question 13

Consider the network graph shown below.



Which of the following represents a spanning tree derived from this graph?



Question 14

A connected planar graph has five faces and seven edges.

The number of vertices it has is

A. 2

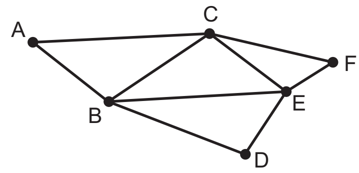
B. 3

C. 4

D. 5

Question 15

Consider the graph below.



Which one of the following is a Hamiltonian path for this graph?

A. *ABCDEF*

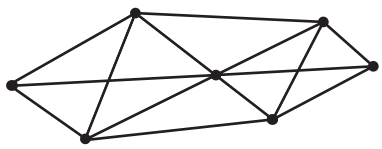
B. *ABDECF*

C. *ABDEBCF*

D. *ACFECBD*

Question 16

Consider the graph below.



Which one of the following is **not** true?

A. It is a planar graph with nine faces.

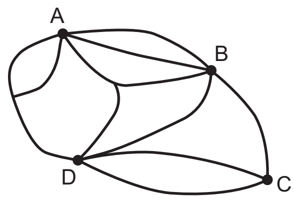
B. It has 14 edges.

C. It has an Eulerian trail.

D. The sum of the degrees is 26.

Question 17

Consider the network of paths below.



Which one of the following adjacency matrices represents this network of paths?

A. $$L = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 3 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 3 & 2 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

B. $$L = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 3 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 3 & 2 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

C. $$L = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

D. $$L = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

Question 18

The minimum time (in days) taken for four people to complete four tasks is shown below.

	Task 1	Task 2	Task 3	Task 4
Ethan	12	10	11	12
Ruby	9	8	8	7
Marcel	5	9	8	6
Louis	11	10	13	10

The four people want to figure out what allocation will allow for the minimum completion time. They consider the following allocations.

I

Person	Task allocated
Ethan	2
Ruby	3
Marcel	1
Louis	4

II

Person	Task allocated
Ethan	3
Ruby	4
Marcel	1
Louis	2

III

Person	Task allocated
Ethan	2
Ruby	4
Marcel	1
Louis	3

Which of these allocations are appropriate in order to ensure the minimum completion time is achieved?

A. Both I and II with a total minimum completion time of 33 days

B. Both I and III with a total minimum completion time of 35 days

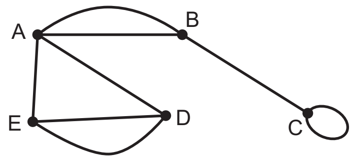
C. I only with a total minimum completion time of 33 days

D. II only with a total minimum completion time of 33 days

Question 19

A connected graph is shown.

Removing which one of the following edges would result in the graph **not** being connected?



A. *AB*

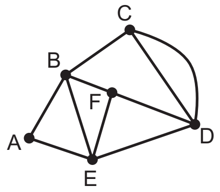
B. *BC*

C. *DE*

D. *AE*

Question 20

Consider the graph below.



Which one of the following is **not** an Eulerian trail for this graph?

A. *F – B – A – E – D – F – B – F – C – D – C*

B. *C – D – C – B – A – E – B – F – E – D – F*

C. *F – B – A – E – B – C – D – F – E – D – C*

D. *C – D – C – B – A – E – D – F – E – B – F*

Question 21

In a complete graph, every vertex is connected to every other vertex by a unique edge.

How many edges does a complete graph with seven vertices have?

A. 6

B. 7

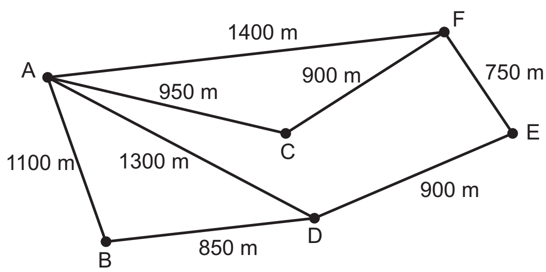
C. 21

D. 24

EXAM 2

Question 1 (4 marks)

A local postal company has six distribution points in a small town: Abbey (*A*), Buron (*B*), Cally (*C*), Dovan (*D*), Ellery (*E*) and Fova (*F*). On the graph below, the distribution centres are shown as vertices *A, B, C, D, E* and *F*. The edges are roads connecting the distribution centres. Distances between distribution centres are also marked on the graph.



a. What is the shortest path from the Abbey distribution centre to Ellery? 1 mark

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b. A postal worker wishes to form a Hamiltonian circuit, starting at Buron, in order to efficiently visit each of the distribution centres.

Write down a possible circuit. 1 mark

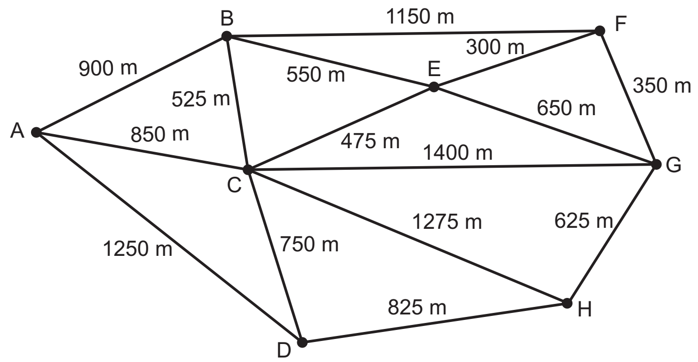
---

c. Explain why it is **not** possible to find an Eulerian circuit for this network. 1 mark

d. If the postal worker wished to form an Eulerian trail through this network, at which distribution centres should he start and finish? 1 mark

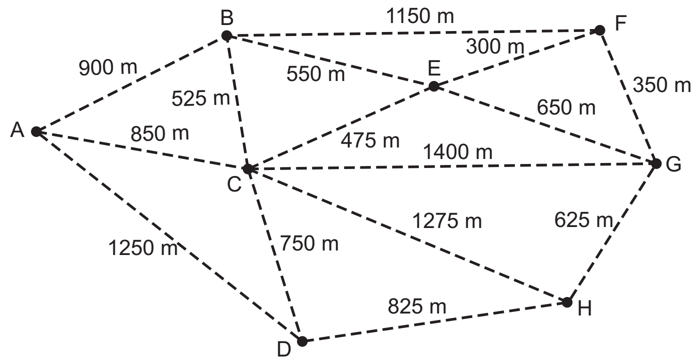
Question 2 (5 marks)

Michelle is planning a fun run to raise money for charity. The graph below shows a series of roads that Michelle is considering using for her running event. Included on the graph are a series of landmarks, *A* to *H*, and the distances between these.



For the safety of all participants, emergency services must be provided with a path to each checkpoint should an injury occur. Emergency services will position themselves at point *A* during the event.

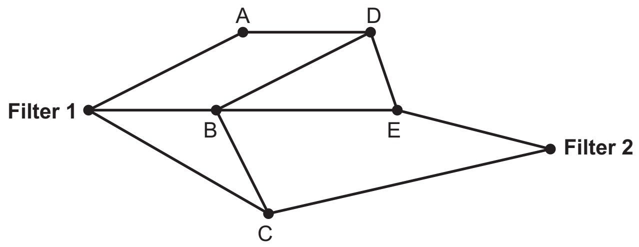
a. On the diagram below, sketch the minimum spanning tree that allows emergency services to connect to each of the checkpoints. 1 mark



b. What is the length of this minimum spanning tree? 1 mark

Question 3 (4 marks)

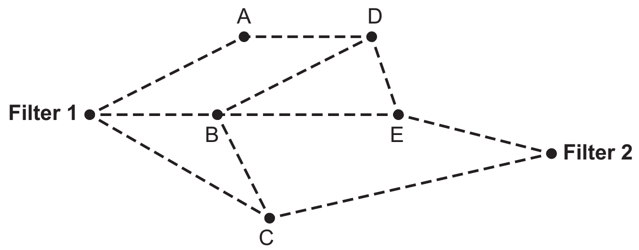
At the end of each day, a juice production line is cleaned thoroughly. The factory has purchased a robot that will clean the tubes automatically. The network of tubes is displayed below with each filter labelled Filter 1, A, B, C, D, E, Filter 2.



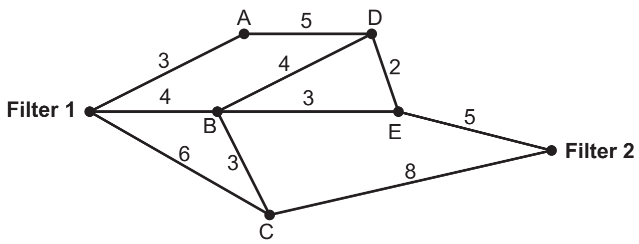
a. How many of the vertices on the graph have degree 3? 1 mark

b. A robot will work its way through the system, checking the function of each filter.

On the diagram below, draw a possible Hamiltonian path for the second robot to follow, starting at Filter 1. 1 mark



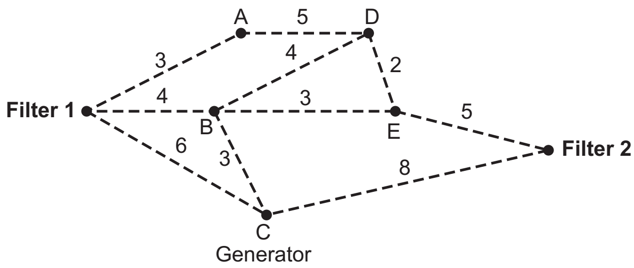
It is sometimes necessary to send the filter repair robot directly to a particular filter. The network below shows the time, in seconds, that it will take the robot to travel between filters.



c. Write down the shortest path that the robot can take between Filter 1 and Filter 2. 1 mark

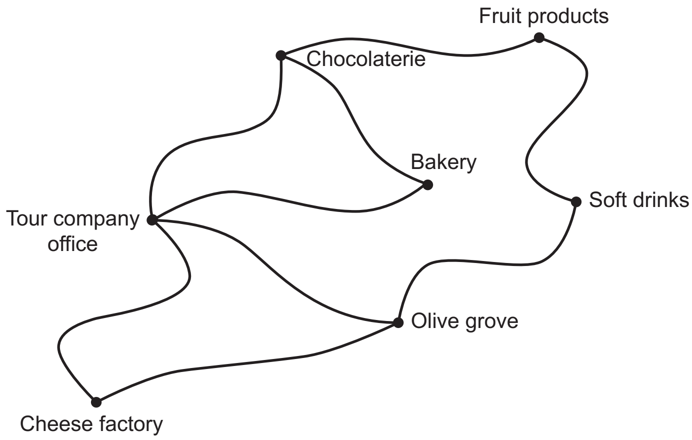
A backup generator has been installed at the factory to allow filters to continue to run in the case of a power outage. The generator is installed at the location of Filter *C*. Wiring must be placed through the tubing, so that the backup power reaches each filter. The weights on the edges indicate the distance, in metres, between each filter.

d. Sketch the minimum spanning tree, representing the wiring from the backup generator on the diagram below. 1 mark



Question 4 (4 marks)

The town of Baysville is well known for its gourmet food, and has developed a gourmet food trail for locals and visitors to take a bicycle tour of the town's best food producers. Along the way, each individual bike track is decorated with artwork by local artists and children from the primary school. Advertisements for local businesses are also on display as riders move along the tracks. The network below shows the food locations represented by vertices. The edges of the network represent the bike tracks between these locations.



a. Which two of the food locations cannot be accessed directly from the tour company's office? 1 mark

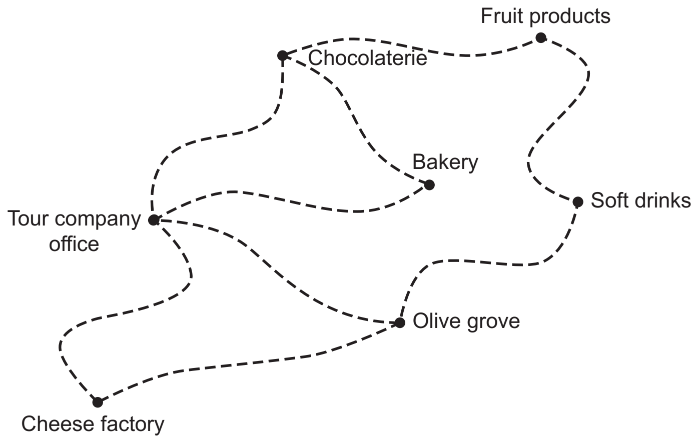
b. The gourmet trail is to start and finish at the tour company office, travelling along each bike track only once.

i. What mathematical name is given to this type of route? 1 mark

ii. Between which two locations can a track be added in order for this gourmet trail route to be possible? 1 mark

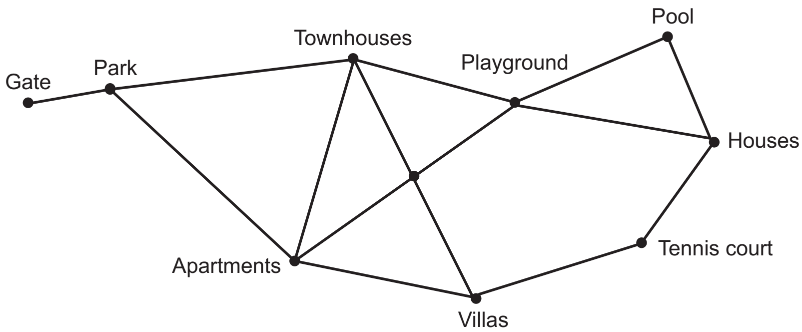
iii. The tour company wishes to create a shorter trail that visits each food location once, and starts and finishes at the tour company office.

Draw one example of this trail on the diagram below. 1 mark



Question 5 (4 marks)

A new housing development is built that includes zones of different houses, plus facilities, as shown in the graph below.



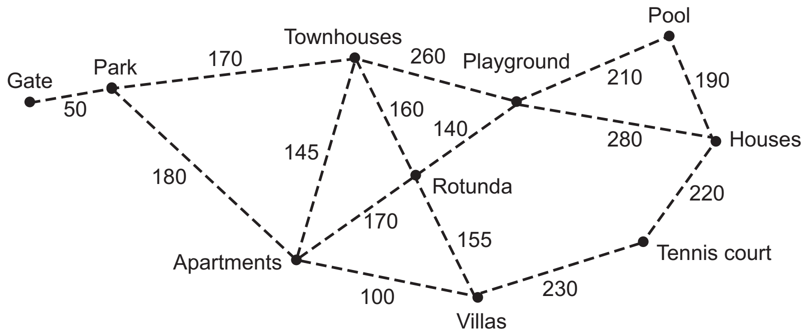
Pip lives in a villa. Every evening, she leaves her villa and walks throughout the development along every road, then returns home to her villa.

a. What is the minimum number of roads she will have to walk along twice? 1 mark

b. Between which locations would extra roads need to be added for Pip to complete her walk without travelling along a road more than once? 1 mark

The diagram below shows the distance, in metres, between each location.

c. Draw the shortest path from the gate to the houses on the diagram below. 1 mark

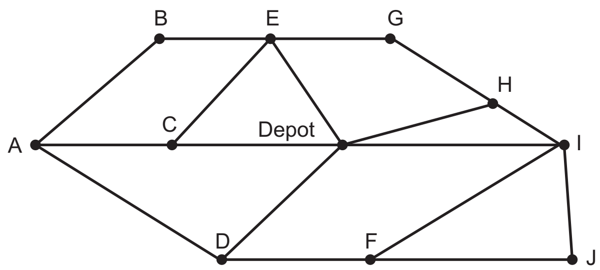


d. When connecting power to each location, a minimum spanning tree is used to minimise wasted cable.

How many metres of cable are saved by using a minimum spanning tree instead of running cable along every road? 1 mark

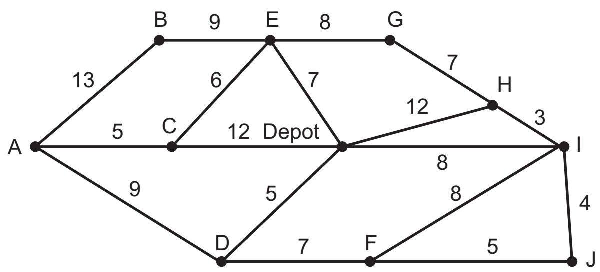
Question 6 (3 marks)

A network of trains is shown below. The towns accessed by the trains are labelled *A* to *J*.



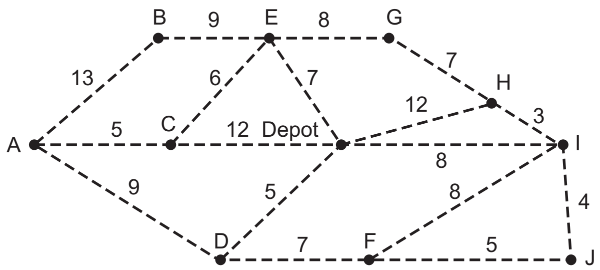
a. How many faces does this network have? 1 mark

The distances, in kilometres, between the towns and the depot are also identified.



b. What is the shortest distance, in kilometres, between the two towns represented by *A* and *H*? 1 mark

A minimum spanning tree is used to connect the stations with new electricity cables in the most efficient way possible.



c. What is the length of the minimum spanning tree, in kilometres? 1 mark

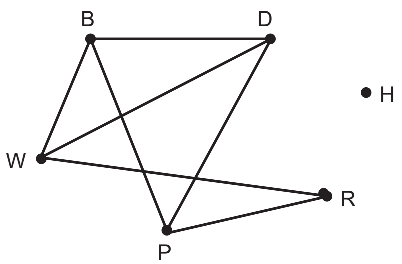
Question 7 (1 mark)

The adjacency matrix below shows the number of connections between the cities Batemans (B), Denman (D), The Heights (H), Rutherford (R), Paterson (P) and Walla (W).

$$\begin{array}{cc} \begin{matrix} B & D & H & R & P & W \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 & 0 \end{bmatrix} & \begin{matrix} B \\ D \\ H \\ R \\ P \\ W \end{matrix} \end{array}$$

The network diagram below is used to represent these connections. However, there are missing edges from the diagram.

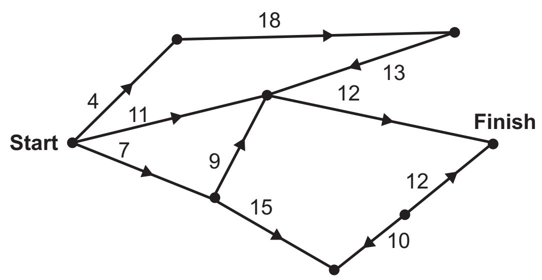
Complete the diagram by drawing the missing edges so it represents the adjacency matrix accurately.



Area of Study 2 Networks and decision mathematics – Directed graphs: flow, matching and critical path analysis

EXAM 1

Question 1

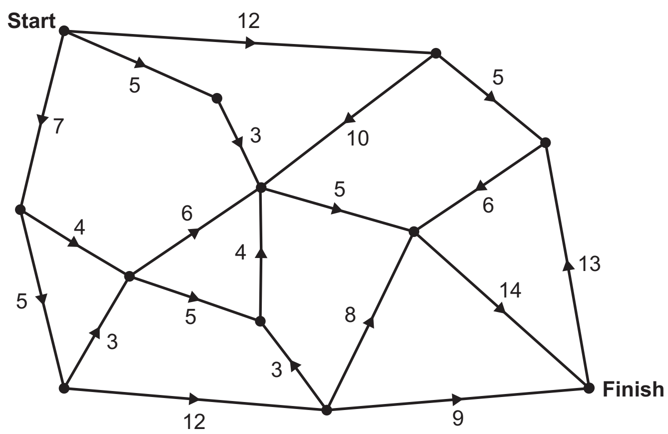


The maximum flow through this network is

A. 12 **B.** 19

C. 22 **D.** 33

Question 2

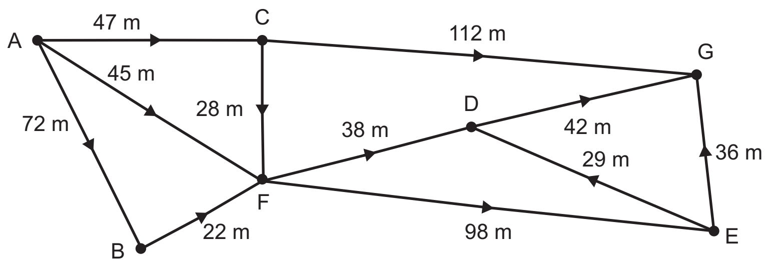


The maximum flow for the directed graph above is

A. 13 **B.** 14

C. 15 **D.** 16

Question 3



Using the graph above, the length of the shortest path from vertex *A* to vertex *D* is

A. 113 m **B.** 94 m

C. 83 m **D.** 66 m

Question 4

Four students, Adam, Bryce, Carey and Donald, are each to be assigned a different job by their teacher in order to complete a project in the minimum time possible.

The table below shows the time, in minutes, that each student would take to complete each of the jobs.

	Adam	Bryce	Carey	Donald
Job 1	38	45	41	42
Job 2	26	22	28	25
Job 3	42	43	38	40
Job 4	31	29	30	32

The minimum time allocation can be achieved only when

A. Adam does Job 1.

B. Bryce does Job 2.

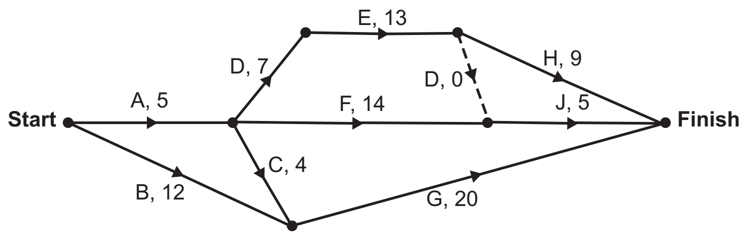
C. Carey does Job 3.

D. Donald does Job 1.

Use the following information to answer Questions 5 and 6.

The directed graph below shows the sequence of activities to complete a project.

All times are in days.



Question 5

Which of the following statements correctly explains the purpose of the dummy activity in the context of this project?

A. Activity *J* cannot start until activity *H* is completed.

B. Activity *J* cannot start until both activities *F* and *E* have been completed; however, only activity *E* is required to be completed for activity *H* to commence.

C. Activity *E* is a predecessor for both activities *F* and *J*.

D. Activity *H* cannot start until both activities *F* and *E* are completed; however, only activity *E* is required to be completed for activity *H* to commence.

Question 6

The completion time for activity *F* is reduced to 12 days.

The length of the critical path for this project, in days, is now

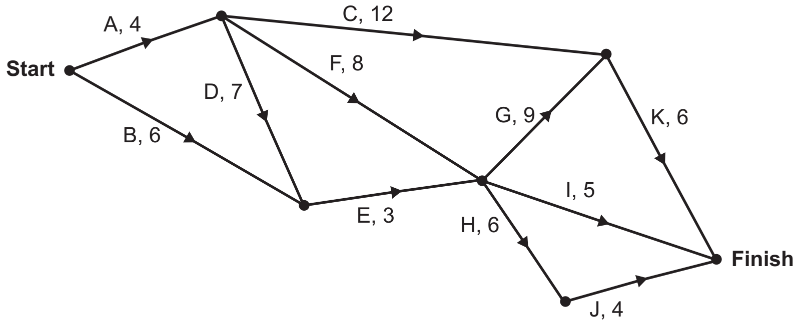
A. 24 **B.** 30

C. 32 **D.** 34

Use the following information to answer Questions 7–9.

The directed graph below shows the sequence of activities required to complete a project.

The time to complete each activity, in days, is also shown.



Question 7

The latest starting time, in days, for activity *G* is

A. 9 **B.** 12

C. 14 **D.** 19

Question 8

The minimum completion time, in days, for the project is

A. 19 **B.** 22

C. 27 **D.** 29

Question 9

The activity with the greatest float time is

A. activity *B.*

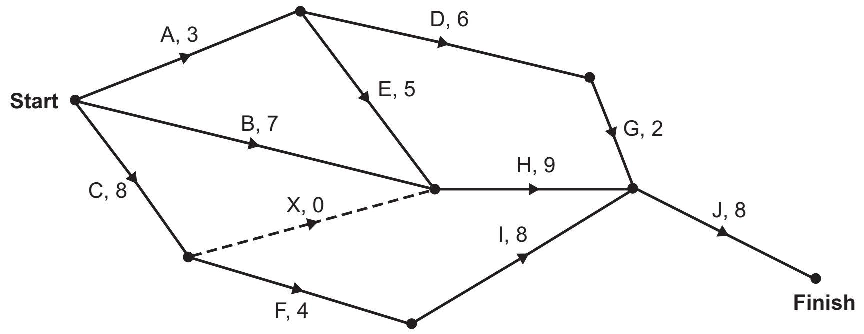
B. activity *C.*

C. activity *H.*

D. activity *I.*

Use the following information to answer Questions 10–12.

The directed network below shows the sequence of 10 activities that are needed to complete a project. The time, in days, that it takes to complete each activity is also shown.



Question 10

Which of the following activities is **not** an immediate predecessor for activity *H*?

A. Activity *B*

B. Activity *C*

C. Activity *D*

D. Activity *E*

Question 11

The latest start time for activity *D* is

A. 6 **B.** 8

C. 9 **D.** 12

Question 12

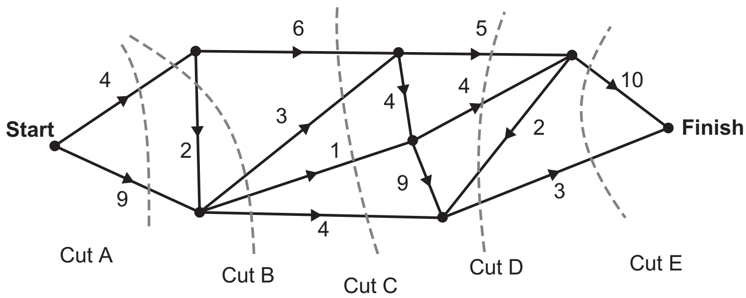
The minimum time in which this project can be completed, in days, is

A. 19 **B.** 24

C. 25 **D.** 28

Use the following information to answer Questions 13 and 14.

The flow of water in litres per minute through a network of pipes is shown below.



Question 13

How many of the cuts shown have a capacity of 14 litres per minute?

A. 0

B. 1

C. 2

D. 3

Question 14

The maximum flow is

A. 8 litres per minute.

B. 11 litres per minute.

C. 13 litres per minute.

D. 14 litres per minute.

Question 15

The minimum time taken (in hours) for four people to complete four tasks is shown below.

	Task 1	Task 2	Task 3	Task 4
Andy	1	3	1	3
Bertha	3	6	3	2
Callum	1	2	2	3
Daisy	5	8	3	4

To minimise the time taken to complete the activities, the optimum allocation of Tasks 1 to 4, in order, is

A. Andy, Bertha, Callum, Daisy.

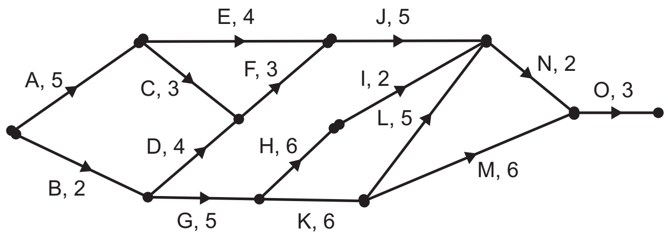
B. Andy, Callum, Daisy, Bertha.

C. Andy, Callum, Bertha, Daisy.

D. Callum, Andy, Daisy, Bertha.

Use the following information to answer Questions 16 and 17.

The directed network below shows the sequence of activities required to complete a project. The time, in weeks, that it takes to complete each activity is also shown.



Question 16

An activity table for the directed network is shown below.

Activity	Duration (weeks)	Immediate predecessor(s)
A	5	–
B	2	–
C	3	*A*
D	4	*B*
E	4	*A*
F	3	*C, D*
G	5	*B*
H	6	*G*
I	2	*H*
J	5	*E, F*
K	6	*G*
L	5	*H, K*
M	6	*H, K*
N	2	*I, J, L*
O	3	*M, N*

It was discovered that a dummy activity was missing from the directed network shown.

Using the activity table above, the missing dummy activity should be connecting

A. the start of activity *H* and the end of activities *L* and *M*.

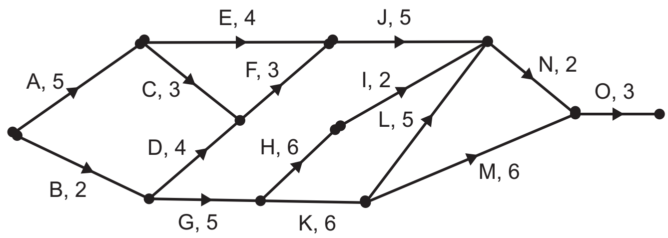
B. the start of activity *H* and the end of activity *K*.

C. the start of activity *H* and the start of activities *L* and *M*.

D. the end of activity *H* and the start of activities *L* and *M*.

Question 17

A decision is made to **not** include the dummy activity in the directed network.



Which one of the following statements about the existing directed network is **not** true?

A. The minimum completion time is 23 weeks.

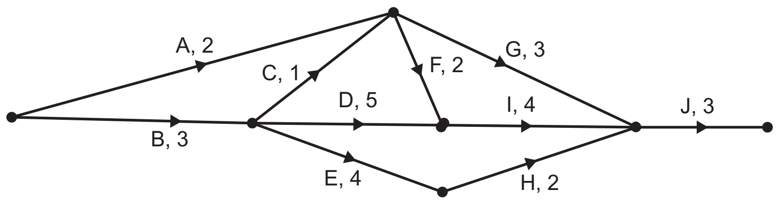
B. The critical path is *B – G – K – L – N – O*.

C. The latest start time of activity *F* is 13 weeks.

D. Increasing the duration of activity *M* to 8 weeks will change the critical path.

Use the following information to answer Questions 18 and 19.

The network below shows the sequence of activities that must be completed in a project, and the time, in weeks, that each activity takes to complete.



Question 18

The minimum completion time for the project is

A. 10 days.

B. 11 days.

C. 13 days.

D. 15 days.

Question 19

Gianni wants to complete the project in 12 days.

To do this he needs to

A. reduce *A* by 2 days and *G* by 1 day.

B. reduce *D* by 3 days.

C. reduce *I* by 3 days.

D. increase *A* by 4 days.

Question 20

A project involves 12 activities, *A* to *L*.

The duration and immediate predecessor(s) are shown in the table below.

Activity	Duration (days)	Immediate predecessor(s)
A	3	–
B	4	*A*
C	2	*A*
D	1	*A*
E	5	*B*
F	4	*D*
G	2	*F*
H	6	*C, E, G*
I	2	*H*
J	1	*H*
K	3	*J*
L	4	*I, K*

Which activity needs to finish after 10 days at the latest in order to not delay the project?

A. *E*

B. *F*

C. *G*

D. *H*

Question 21

A manager is allocating four staff members, Merindah (*M*), Yvonne (*Y*), Jackie (*J*) and Ryder (*R*), to four jobs to ensure the minimum allocation is achieved.

The manager creates a matrix of the data, and then uses the Hungarian algorithm by reducing each row.

The matrix following the *completed row reduction* is shown below.

$$\begin{array}{c c}
\begin{array}{cccc} J_1 & J_2 & J_3 & J_4 \end{array} & \\
\begin{bmatrix} 3 & 4 & 0 & 2 \\ 4 & 3 & 0 & 2 \\ 4 & 1 & 0 & 2 \\ 3 & 2 & 3 & 0 \end{bmatrix} & \begin{array}{c} M \\ Y \\ J \\ R \end{array}
\end{array}$$

The manager realises this is not yet sufficient to determine the minimum allocation and so continues the process of the Hungarian algorithm.

Which of the following tables shows the minimum allocation?

A.

Person	Job
Merindah	3
Yvonne	1
Jackie	2
Ryder	4

B.

Person	Job
Merindah	1
Yvonne	3
Jackie	2
Ryder	4

C.

Person	Job
Merindah	3
Yvonne	2
Jackie	1
Ryder	4

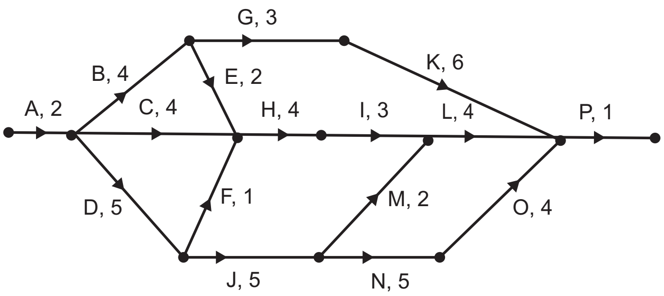
D.

Person	Job
Merindah	1
Yvonne	2
Jackie	3
Ryder	4

Question 22

The directed network below shows the sequence of activities, *A* to *P*, that is required to complete a construction project, and the required time, in weeks, to complete each activity.

The current minimum completion time of the project is 22 weeks, with the critical path *A* – *D* – *J* – *N* – *O* – *P*.



All the activities that are not on the critical path have a float time of two weeks, four weeks, six weeks or x weeks.

What is the value of x?

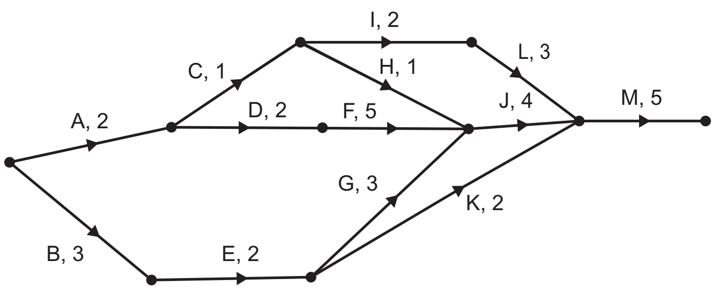
A. 1 **B.** 3

C. 4 **D.** 5

Question 23

The network diagram below shows the sequence of activities, *A* to *M*, for a work program. The duration of each activity is in days.

The critical path is *A* – *D* – *F* – *J* – *M*; it has a minimum completion time of 18 days.



The duration of activity *G* increases, creating a new critical path and a new minimum completion time of 21 days.

What is the new duration, in days, of *G*?

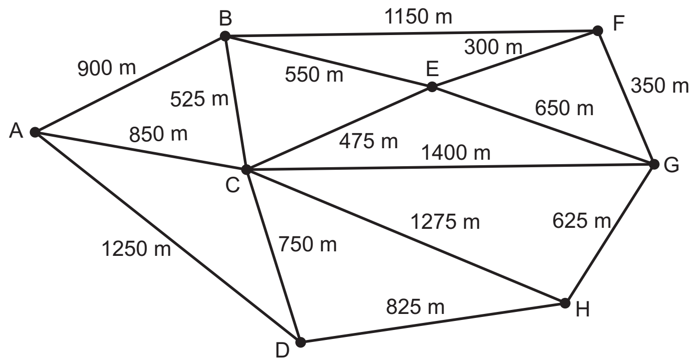
A. 1 **B.** 3

C. 5 **D.** 7

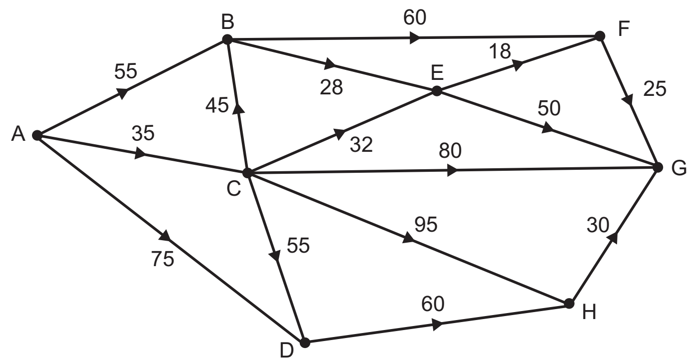
EXAM 2

Question 1 (1 mark)

Michelle is planning a fun run to raise money for charity. The graph below shows a series of roads that Michelle is considering using for her running event. Included on the graph are a series of landmarks, *A* to *H*, and the distances between these.



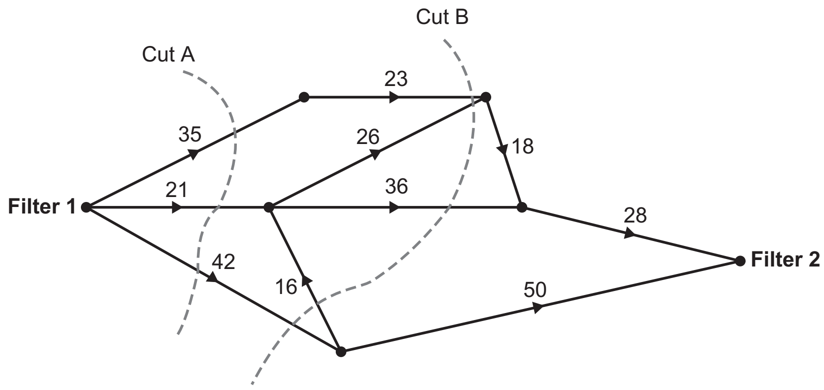
The graph below shows another version of the running course that documents the number of people who can safely run along each road in an hour and the direction they can run in. Michelle is considering an under-12s event before the fun run, the purpose of which would be to get as many children as possible to run through the town wearing coloured T-shirts and costumes.



If the children must start at *A* and finish at *G*, what is the maximum number of children who can run through the town in one hour?

Question 2 (2 marks)

The graph below shows the possible paths that orange juice can be sent through a system of tubes between the first and second major filters at a local factory. The unmarked vertices represent other, smaller filters that the juice also passes through along the way. The weighting of each edge represents the maximum number of litres that can pass through each tube per hour.



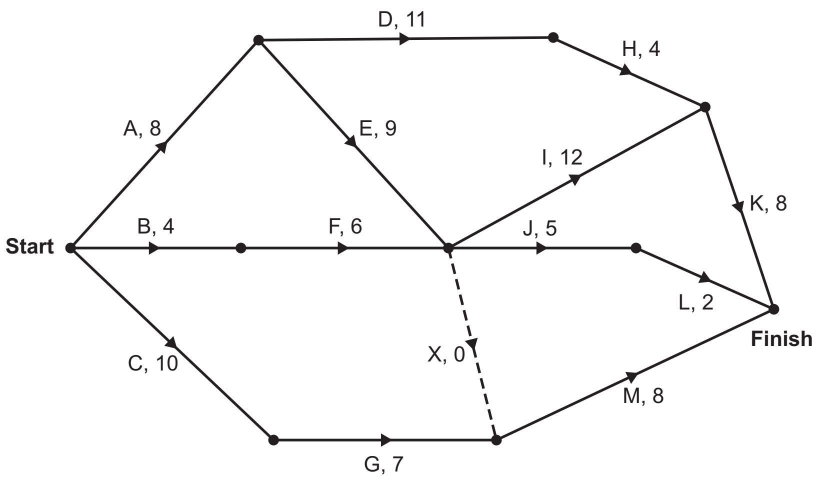
Cut A, shown on the graph above, has a capacity of 98.

a. Write down the capacity of Cut B. 1 mark

b. Determine the maximum number of litres of orange juice that can be sent from Filter 1 to Filter 2 per hour. 1 mark

Question 3 (5 marks)

A new distribution centre will be built by a postage company. This project involves 12 activities, *A* to *L*. The directed network below shows these activities and their completion times, in days.



a. Determine the earliest start time for activity *F*. 1 mark

b. Find the critical path and minimum completion time for the project. 1 mark

c. Which activities have a float time of 7 days? 1 mark

d. Explain the purpose of the dummy activity, X_0, in the context of the project. 1 mark

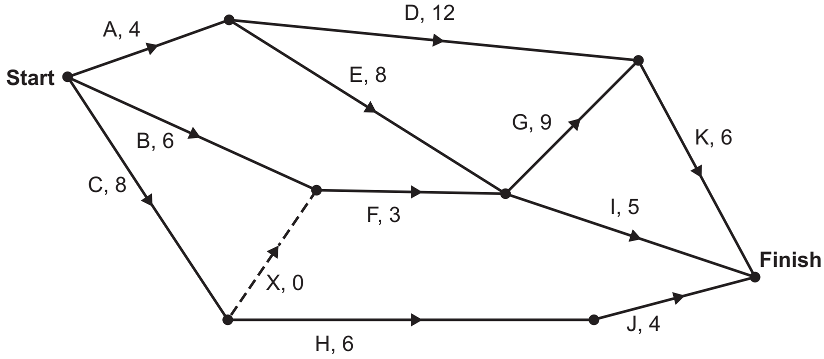
e. The completion times for activities *C*, *E*, *F*, *I* and *M* can each be reduced by one day. The cost of reducing the completion time by one day for these activities is shown in the table below.

Activity	Cost ($)	Activity	Cost ($)
C	2000	*I*	4000
E	4000	*M*	3000
F	1000		

What is the minimum cost to complete the project in the shortest time possible? 1 mark

Question 4 (6 marks)

It was decided that the roads must be re-surfaced prior to a running event. The project involves activities *A* to *K*. The directed graph below shows these activities and their completion time in days.



a. Write down the activities that must be completed before activity *G* can begin. 1 mark

b. Find the minimum completion time for this project and write down the critical path. 2 marks

c. Determine the float time, in days, for activity *H*. 1 mark

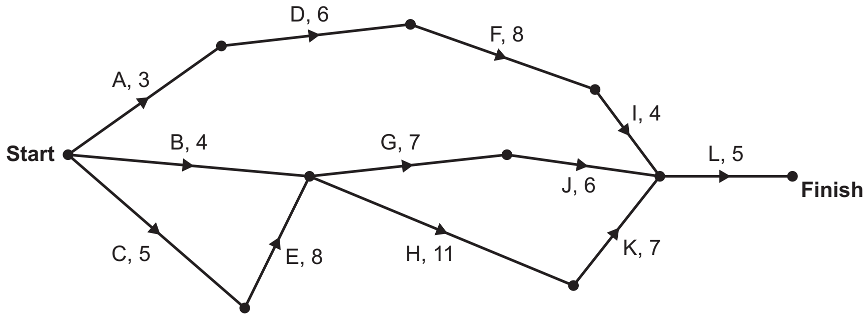
d. The project can be completed earlier if some of the activities are crashed. Activities *E*, *F*, *G* and *I* can be crashed by a maximum of two days. The cost of crashing is \$2000 per activity per day.

i. What is the minimum number of days in which the project can now be completed? 1 mark

ii. What is the minimum cost of completing the project in this time? 1 mark

Question 5 (5 marks)

The factory will install a second system in order to increase the production of orange juice. This project involves 12 activities, *A* to *L*. The directed network below shows these activities and their completion time, in days.



a. Determine the earliest starting time, in days, for activity *J*. 1 mark

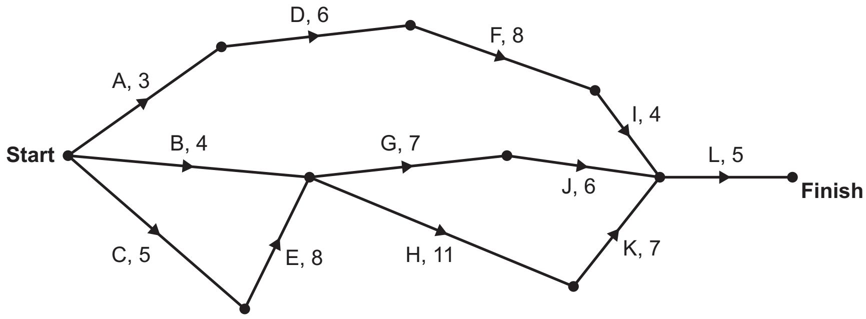
b. The minimum completion time for this project is 36 days.

Write down the critical path. 1 mark

c. How many activities in this network have a float time of 10 days? 1 mark

d. An additional activity, *X*, will be added with duration of three days, an Earliest Start Time (EST) of 13 and a Latest Start Time (LST) of 24.

Draw this additional activity on the directed graph below. 1 mark



e. The completion time of the project can be decreased by reducing the completion time of activities *E* and *K* by one hour each. The additional activity, *X*, will not be included in this new project.

What is the new completion time? 1 mark

Question 6 (3 marks)

Before close of business each day at a bakery, a series of clean-up procedures must be completed by staff in order to leave the bakery clean for the following day. On a particular day, four staff members are rostered on during the clean-up shift. The following table shows the quickest time, in minutes, that each of the staff members can complete the four main tasks.

	Quickest time for completion by each staff member			
Staff member	**Package remaining goods for sale the next day**	**Disinfect all surfaces**	**Deep clean the ovens**	**Sweep and wash all floors**
Jess	24	38	28	42
Adriano	32	45	26	46
Tiana	28	41	32	43
Vish	26	36	29	39

Staff members are allocated one job each in order to minimise the total time taken to close the bakery at the end of the day.

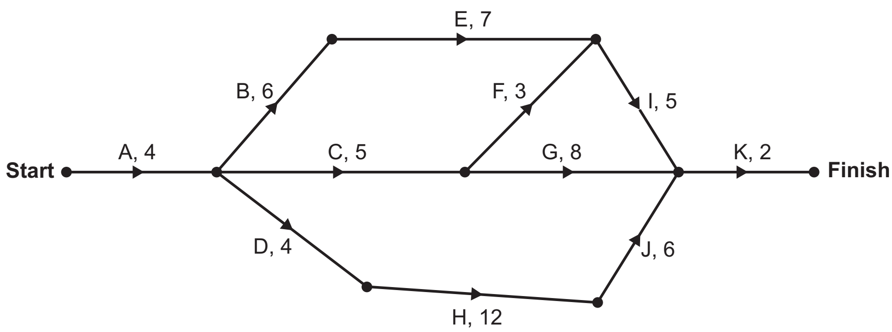
a. To which task should Jess be allocated? 1 mark

b. Packaging remaining goods and deep cleaning the ovens can occur simultaneously, after which the surfaces can be disinfected before the floor is cleaned. Each staff member must contribute to the clean up.

What is the minimum time in which the clean-up tasks can be completed? 2 marks

Question 7 (5 marks)

The Baysville cheese factory is planning to renovate to include a café and make the cheese-making process more visible to visitors. This project involves 11 activities, *A* to *K*. The directed network below shows these activities and their completion time, in weeks.



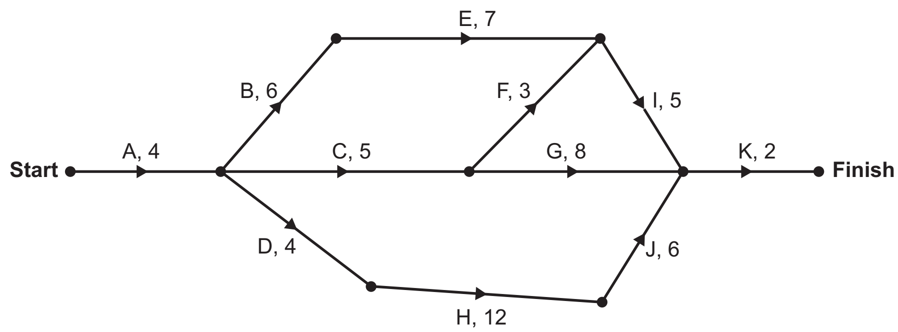
a. What is the minimum completion time, in weeks, for this project? 1 mark

b. How many activities are **not** on the critical path? 1 mark

c. What is the earliest start time for activity *G*? 1 mark

d. After an inspection by a compliance officer, it is recommended that upgrades to the cooling systems in the cheese factory be completed. This new activity, *L*, will take 4 weeks to complete. It has an earliest start time of 8 weeks and a latest start time of 22 weeks.

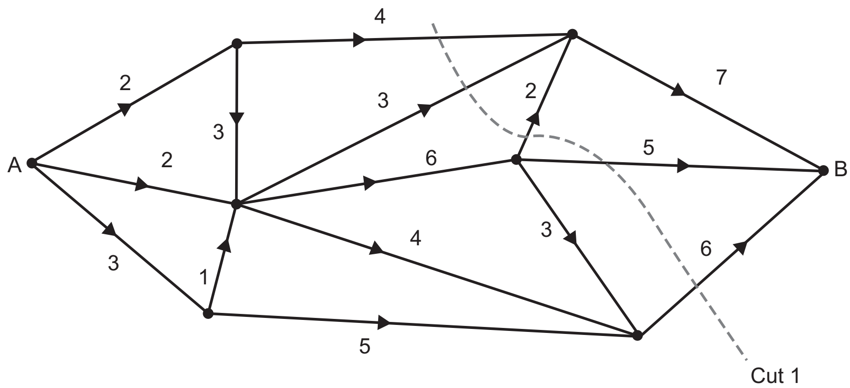
i. Draw this additional activity onto the directed network below. 1 mark



ii. What is the minimum completion time for the project after this activity has been added? 1 mark

Question 8 (3 marks)

The paths between two campsites, *A* and *B*, are shown below. The number of people on each path per minute is shown by the values on the diagram.



a. How many different routes from *A* to *B* include the path that can take a maximum of seven people? 1 mark

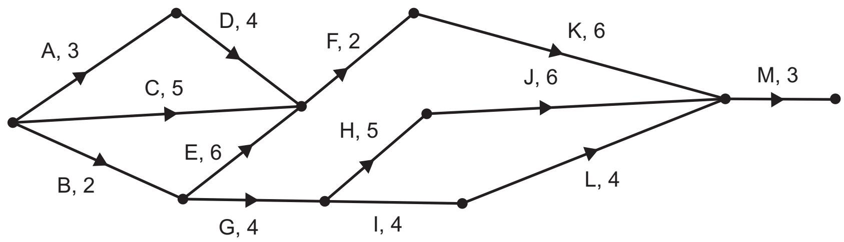
b. What is the capacity of Cut 1? 1 mark

c. Determine the maximum flow per minute. 1 mark

Question 9 (5 marks)

An office building is to be refurbished. The project involves 13 activities, *A* to *M*.

The directed graph below shows the duration of each activity, in weeks.



a. The minimum completion time is 20 weeks.

Write down the critical path. 1 mark

b. There is a delay for activity *I*.

What is the maximum number of weeks it can be delayed without impacting the minimum completion time of the project? 1 mark

c. If the duration of one activity was to be increased by 4 weeks, a second critical path of 20 weeks would be created.

Determine which activity this applies to. 1 mark

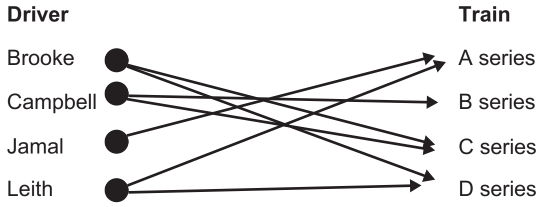
d. It is calculated that any activities that have a duration of five weeks or more can be reduced in time. These activities can only be reduced by a maximum of two weeks each.

Which activities will be reduced and what will the new minimum completion time be? 2 marks

Question 10 (1 mark)

There are four models of trains, which require different accreditation courses to be completed by the train drivers.

The bipartite graph below illustrates the trains that each driver is accredited to drive.



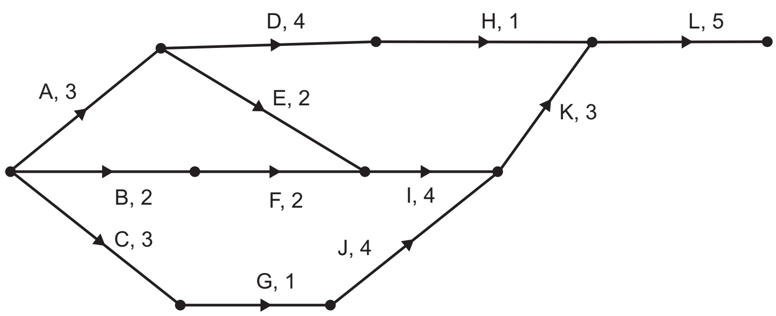
Complete the table below to identify which driver is allocated to each train.

Train	Driver
A	
B	
C	
D	

Question 11 (6 marks)

The train depot is being refurbished.

The project involves 12 activities, *A* to *L*. The network below shows the duration of each activity, in days.

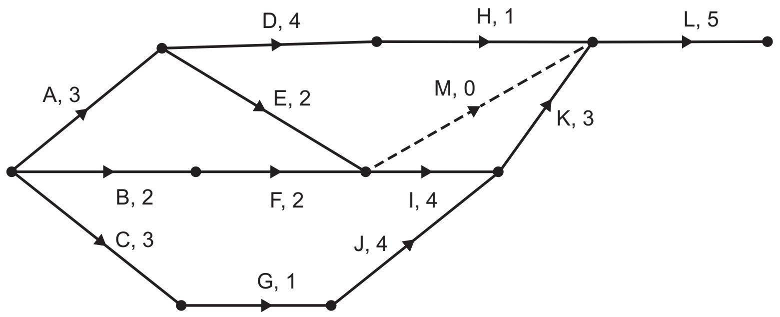


a. Which activities have two immediate predecessors? 1 mark

The minimum completion time is currently 17 days.

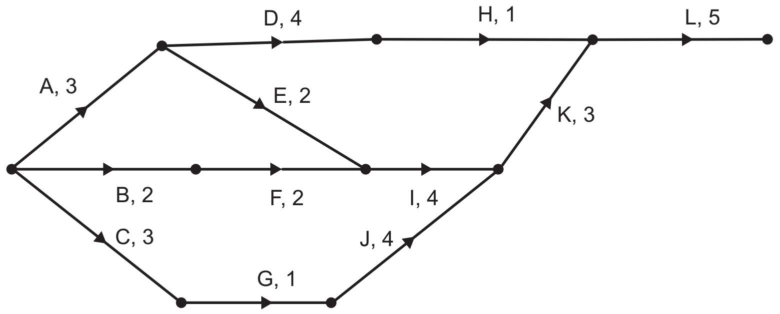
b. Draw the critical path on the diagram above. 1 mark

A dummy activity, *M*, is added, as shown on the diagram below.



c. What is the minimum completion time required, in days, after the dummy activity has been included? 1 mark

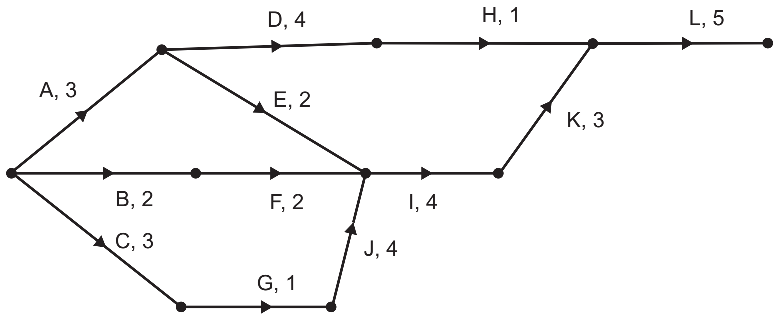
It was determined that the dummy activity was **not** required and so was removed.



d. The project started on Monday 14 January.

What is the latest date on which activity *G* can start? 1 mark

A new manager takes over the project. She makes a change to the order of some activities, which results in a new minimum completion time of 20 days along path *C* – *G* – *J* – *I* – *K* – *L*.



The manager now wants to reduce the length of the project as much as possible.

The maximum reduction for activities that may be reduced is shown in the table below. The cost of reducing the activity is also shown.

Activity	Maximum reduction (days)	Reduction cost per day ($)
A	2	400
D	1	350
J	1	200
K	2	150
L	2	250

e. What is the new minimum completion time, in days, for the project? 1 mark

f. What is the minimum extra cost of completing the project now? 1 mark

Worked solutions

Area of Study 1 Data analysis – Investigating data distributions

EXAM 1

Question 1

Answer: ***D***

Explanatory notes

$\frac{58}{261} \times 100 = 22.22\%$

This is closest to 22%.

Question 2

Answer: ***A***

Explanatory notes

Both variables are categorical variables, as they place students and the lunch choices into categories or groupings.

Within categorical data we have nominal or ordinal data.

Nominal data does not contain an order or hierarchy of any kind.

Ordinal data can be sorted into an order or hierarchy.

In this case, we are not placing either *sporting house* or *lunch choice* into an order or hierarchy; therefore, they are both nominal variables.

Question 3

Answer: ***D***

Explanatory notes

It is not suitable to use a stem plot, histogram, dot plot or scatterplot for categorical data; a segmented bar chart is the most suitable graphical tool as it is the most appropriate display when working with two categorical variables. The other types of graphs that have been suggested are suitable for numerical data.

Question 4

Answer: ***D***

Explanatory notes

The median is found by locating the 'middle' data point, when data values are listed in ascending order.

In this stem plot there are 34 data values; therefore, the 'middle' is between the 17th and 18th data values.

Reading off the stem plot, the middle value is between 5.2 and 5.2. Therefore, the median is 5.2 or $5.20.

TIP

» When finding the middle of two points, add them together and divide by 2.

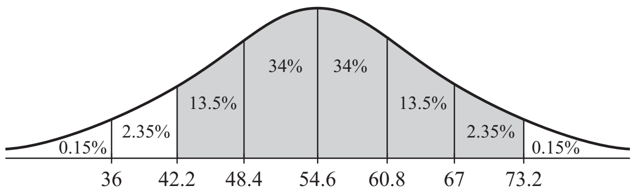
Question 5

Answer: ***D***

Explanatory notes

Describing the weights as ‘approximately normally distributed’ indicates that we should use a bell curve to calculate the percentage of students who fall within this interval.

As seen below, we can then use the bell curve to determine which percentages we need to include.



In this case, we require:

$100\% - 2.5\% - 0.15\% = 97.35\%$

We now need to calculate 97.35% of 186 students:

$\frac{97.35}{100} \times 186 = 181.071$

This is closest to 181.

Question 6

Answer: ***B***

Explanatory notes

$$z = \frac{x - \bar{x}}{s}$$

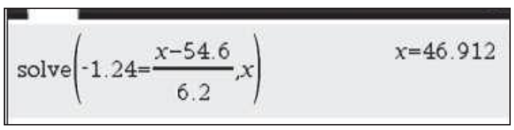
$$-1.24 = \frac{x - 54.6}{6.2}$$

$$-7.688 = x - 54.6$$

$$x = 46.912$$

$$\therefore x = 47 \text{ kg}$$

Your CAS calculator can be used to solve this equation directly from the second line of working above.



Question 7

Answer: ***B***

Explanatory notes

$\log_{10}(0.01) = -2$

The frequency (number) of data less than −2 on the histogram is equal to $1 + 2 = 3$.

Total frequency is equal to $1 + 2 + 9 + 13 + 4 + 1 = 30$. Therefore, the percentage of data less than 0.01 is equal to $\frac{3}{30} \times 100 = 10\%$.

This is closest to 10%.

Question 8

Answer: ***C***

Explanatory notes

In order to find the interval within which the median lies, we must find the ‘middle’ of the histogram. 50% of the data will lie below the median and 50% above.

As this histogram shows the ‘percentage frequency’, we can add the bars of the histogram together to find where 50% lies.

The table below shows the ‘percentage frequency’ for each interval as well as a cumulative total:

Interval	Percentage frequency	Cumulative percentage frequency
0–<5	8	–
5–<10	23	31
10–<15	23	54
15–<20	27	81
20–<25	15	96
25–<30	0	96
30–<35	4	100

50% therefore lies within the interval 10–15. Therefore, option C is correct.

We can see that the intervals represented by options A, B and D are incorrect, as their cumulative percentage frequencies are either lower or higher than the 50% mark.

Question 9

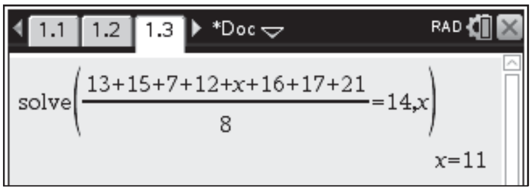
Answer: ***B***

Explanatory notes

Create an equation that shows the sum of the values divided by the number of data values.

$$\frac{13 + 15 + 7 + 12 + x + 16 + 17 + 21}{8} = 14$$

Use your CAS calculator to solve this equation for x.



Question 10

Answer: ***D***

Explanatory notes

The score out of 40 given for the trial exam is numerical, discrete data, as this is recorded as a whole number only.

TIP

» **Recall that numerical, discrete data is given in finite values such as whole numbers. Numerical continuous data is generally recorded on a scale that can take any value on a decimal scale.**

Question 11

Answer: ***C***

Explanatory notes

The median can be found on the boxplot by finding the centre, or 50% mark.

The following is an explanation of the options given in the multiple-choice question:

Option A is the minimum value.

Option B is Q_1.

Option C is the median, the correct answer.

Option D is Q_3.

Question 12

Answer: ***C***

Explanatory notes

The interquartile range, IQR, can be found by subtracting Q_1 from Q_3.

Therefore, the IQR is found by:

$\text{IQR} = Q_3 - Q_1$

$\text{IQR} = 32 - 22$

$\text{IQR} = 10$

Question 13

Answer: ***D***

Explanatory notes

There are 16 babies born with a *weight* between 2.0 and 4.0 kilograms.

$\frac{16}{25} \times 100 = 64\%$

Question 14

Answer: ***C***

Explanatory notes

There are 25 data values, and the median (middle value) will therefore be the thirteenth value, as illustrated below.

birth weight **key: 0 | 1 = 0.1**

0											
1	7	9									
2	2	4	6	7	8						
3	1	2	3	3	4	5	5	6	6	7	9
4	2	3	3	5	7	8					
5	1										

Answer = 3.5

TIP

» **Remember that determining the median of a data set will depend on whether there are an odd or even number of data values. An odd number of data points will result in your median being one of your original data values. An even number of data points will result in your median being between two of your data points.**

Question 15

Answer: ***D***

Explanatory notes

Reading from the graph:

$24 + 20 + 12 + 4 = 60\%$

Question 16

Answer: ***A***

Explanatory notes

$z = \frac{x - \bar{x}}{s}$

$0.8 = \frac{x - 3.3}{0.3}$

$x - 3.3 = 0.24$

A CAS calculator can be used to solve this.

$$\text{solve}\left(0.8 = \frac{x - 3.3}{0.3}, x\right) \qquad x=3.54$$

$x = 3.54$

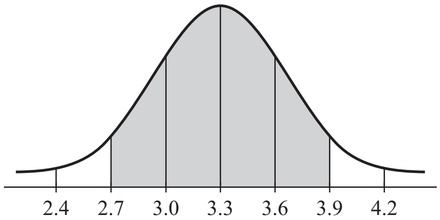
Question 17

Answer: ***D***

Explanatory notes

Use the 68–95–99.7% rule, together with the given standard deviation.

The normal curve is shown below, with relevant sections shaded.



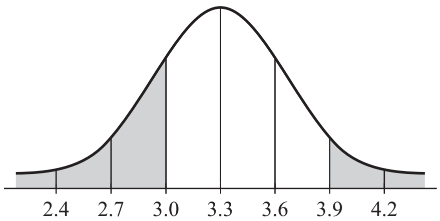
2.7 and 3.9 are both two standard deviations away from the mean. Therefore, $13.5 + 34 + 34 + 13.5 = 95\%$ of the babies born lie between these values.

Question 18

Answer: ***D***

Explanatory notes

The normal curve is shown below, with the required sections highlighted.



The percentage required is $16 + 2.5 = 18.5\%$

Therefore, the predicted number of babies born within these weight ranges will be 18.5% of 150.

$\frac{18.5}{100} \times 150 = 27.75 \approx 28$

Question 19

Answer: ***D***

Explanatory notes

Out of 15 scores in total, 8 scores are below 25.

$\frac{8}{15} \times 100\% = 53.333...\%$

Question 20

Answer: ***B***

Explanatory notes

The upper quartile is the value below which 75% of scores occur and above which 25% occur.

The histogram has a total frequency of 100%, so locating where 75% occurs requires counting the cumulative frequency of each interval:

$0 - < 4$ hours: $30 + 35 = 65\%$

$0 - < 6$ hours: $30 + 35 + 15 = 80\%$

So 75% occurs in the interval $4 - < 6$ hours.

Question 21

Answer: ***A***

Explanatory notes

The score of 10 has a log value of 1, while scores of 100 have a log value of 2, as shown in the screenshot below.

$\log_{10}(10)$	1
$\log_{10}(11)$	1.04139268516
$\log_{10}(100)$	2

So, looking at the log conversions and how the scores provided fit within them gives:

	0 – < 10	10 – < 100	100 – < 1000	1000 – < 10 000	10 000 – < 100 000
Log value	1	2	3	4	5
Scores provided	3, 7	10, 25, 87	112, 250, 812, 970	1102	

So the frequencies of scores in each interval on the log axis should be 2, 3, 4, 1.

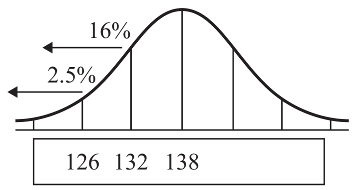
Question 22

Answer: ***B***

Explanatory notes

Using a normal distribution, we can see which segment of the graph 130 seconds falls within. As this is a race, the lower half of the graph represents having a better-than-average time; therefore, Janet is in a segment that is towards the *top*, not *bottom*, of her group.

A time of 130 seconds falls in the interval between 126 seconds and 132 seconds, so is in the fast 16% but not in the fastest 2.5%.



Question 23

Answer: ***B***

Explanatory notes

Working backwards, using the formula to calculate the upper fence:

$Q_3 + 1.5 \times \text{IQR}$

$45 + 1.5 \times \text{IQR} = 63$

$\text{solve}\,(45+1.5 \cdot x=63, x)$	$x=12.$

So the $\text{IQR} = 12$, which means Q_1 is $45 - 12 = 33$

The lower fence is calculated by:

$Q_1 - 1.5 \times \text{IQR}$

$33 - 1.5 \times 12 = 15$

Question 24

Answer: ***B***

Explanatory notes

Number of children is the only numerical discrete variable.

Height is a numerical continuous variable.

Place of birth and *postcode* are not considered numerical; they are categorical as they are groupings.

Question 25

Answer: ***D***

Explanatory notes

When there are 14 scores, the lower quartile is the 4th data value (34 800), and the upper quartile is the 11th data value (198 000).

$\text{IQR} = 198\,000 - 34\,800 = 16\,200$

$\text{Upper fence} = 198\,000 + 1.5 \times 163\,200$

$= \$442\,800$

Question 26

Answer: ***D***

Explanatory notes

Options B and C are incorrect as they are used for categorical data.

Option A is appropriate for numerical data, but the data values provided are too variable for effective representation by these graphs.

A log scale histogram is best used with numerical data with a large range or spread. With scores spread between 9550 and 2 304 000, the analysis of the data would be better if a log scale were applied.

TIP

» **Look at the type of data being used (e.g. numerical and categorical) in order to eliminate options that are not relevant.**

Question 27

Answer: ***B***

Explanatory notes

Increasing the distance will increase the mean.

Option A: The scores of 4.3 and 4.6 are both within the minimum and maximum scores, so the range is not impacted.

Options C and D: Changing the value from 4.3 to 4.6 does not change its position between Q_1 and the median. As a result, the IQR and the median are not impacted.

Question 28

Answer: ***D***

Explanatory notes

This demonstrates a negatively skewed distribution, illustrated by the direction of the 'tail' pointing towards the negative end of the number line.

9	6
10	4
11	2 5 8
12	3 6 7 7 8
13	2 4 4 6 6 6 9
14	3 5 5

TIP

» **Be careful not to confuse positive/negative skew with a positive/negative association – they describe different things. Skewness is best used to describe the shape of the distribution of a single variable, while association describes the relationship between two variables.**

Question 29

Answer: ***B***

Explanatory notes

Speed of service has been measured in categorical groupings, and they are ordered, so it is ordinal.

Likelihood of returning is also categorical, where the options provided (yes, no) are not ordered/ranked, so it is nominal.

Question 30

Answer: ***A***

Explanatory notes

Using the 68–95–99.7% rule, the top 2.5% for each school (two standard deviations or more) is as follows:

A	$64 + 2 \times 4 = 72$
B	$70 + 2 \times 3 = 76$
C	$65 + 2 \times 6 = 77$
D	$72 + 2 \times 5 = 82$

A score of 73 puts Mostyn in the top 2.5% of School A.

OR

A score in the top 2.5% will have a z-score of 2 or greater.

For School A $z = \frac{73 - 64}{4} = 2.25$

For School B $z = \frac{73 - 70}{3} = 1$

For School C $z = \frac{73 - 65}{6} = 1.33$

For School D $z = \frac{73 - 72}{5} = 0.2$

Question 31

Answer: ***C***

Explanatory notes

Within one standard deviation of the mean puts the students within 68% of the total cohort.

$0.68 \times x = 76$ students

$x = 111.7 = 112$ students in total, which is School C

Question 32

Answer: ***C***

Explanatory notes

Percentaged segmented bar charts are used with two categorical variables: in this example, fitness levels are categorised, and country of birth is a categorical variable.

The other options consist of either two numerical variables or one numerical and one categorical variable.

EXAM 2

Question 1a.

Worked solution

The data is discrete numerical.

In this case, the data has been counted as whole numbers, and is therefore discrete.

Mark allocation: 1 mark

- 1 mark for ‘discrete numerical’; ‘numerical’ alone not accepted

Question 1b.

Worked solution

$\frac{17}{44} \times 100 = 38.6363... = 38.6\%$

Mark allocation: 1 mark

- 1 mark for correct answer, rounded accurately to one decimal place

Question 1c.

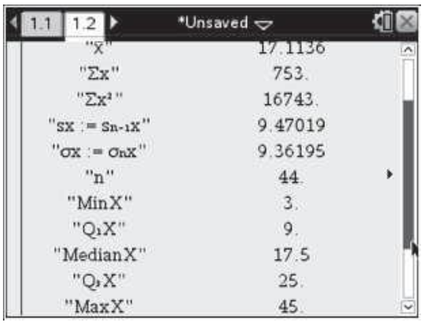
Worked solution

The median is 17.5.

The stem plot can be used to find the median value by counting to the middle.

There are 44 data values, and therefore the median lies between 17 (the 22nd data value) and 18 (the 23rd data values).

Alternatively, the data can be entered into your CAS and a statistical summary obtained.



Mark allocation: 1 mark

- 1 mark for correct answer of 17.5

Question 1d.

Worked solution

Step 1: Find the interquartile range.

$\text{IQR} = Q_3 - Q_1 = 25 - 9 = 16$

Step 2: Use the IQR and Q_3 to find the upper fence.

$\text{upper fence} = Q_3 + 1.5 \times \text{IQR} = 25 + 1.5 \times 16 = 49$

Step 3: Clearly answer the question.

$45 < 49$

∴ it is not an outlier.

The statistician is **not correct**.

Mark allocation: 2 marks

- 1 mark for correct calculation showing that upper fence is 49
- 1 mark for a clear statement that the value of 45 is not an outlier due to it being below the upper fence of 49

Question 2a.

Worked solution

The boxplot is approximately symmetric, with a median at 35 and an IQR of 20.

Mark allocation: 1 mark

- 1 mark for identifying the boxplot as approximately symmetric and stating the numerical values for the median and IQR

Note: If the median is given in conjunction with the range (48), it can be accepted.

Question 2b.

Worked solution

The median would be the most suitable measure of centre. This is the case when data is either skewed or has an outlier.

Mark allocation: 2 marks

- 1 mark for stating median
- 1 mark for explaining that the data is skewed

Question 3a.

Worked solution

The answer can be found with or without your CAS calculator.

<u>Without the CAS calculator:</u>

Write the data values in a list in ascending order.

1 1 1 2 2 3 3 3 4 4 4 4 5 5 5 6 6 7 7 7 8 8 9 9 9

Find the middle number of this list of data.

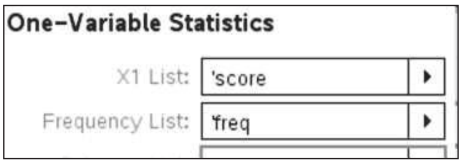
In this case, the middle number is 5. Therefore, the median *effort rating* given is 5.

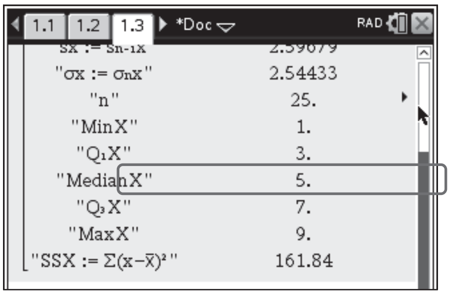
<u>With the CAS calculator (example from TiNspire):</u>

Enter data into a spreadsheet.

A score	B freq	C
1	3	
2	2	
3	3	
4	4	
5	3	

Use the statistics function to find the summary statistics.





Mark allocation: 1 mark

- 1 mark for the correct answer of 5

Question 3b.

Worked solution

There are 9 numbers between 3 and 7 (4, 4, 4, 4, 5, 5, 5, 6, 6).

$$\frac{9}{25} \times 100 = 36\%$$

Mark allocation: 1 mark

- 1 mark for 36%

Question 3c.

Worked solution

The variable is numerical discrete.

The values given in this survey are whole numbers and are therefore discrete.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4a.

Worked solution

The boxplot for female *end-of-year result* is positively skewed.

Mark allocation: 1 mark

- 1 mark for stating positively skewed

Note: Answers that will not be accepted include positive, positive trend, increasing trend and approximately symmetric.

Question 4b.

Worked solution

Minimum = 3

$Q_3 = 41$

These answers should be read directly from the boxplot.

Mark allocation: 2 marks

- 1 mark for minimum of 3
- 1 mark for Q_3 of 41

TIP

» Remember that although the value of 3 is an outlier, it is still a part of the data set. It is, therefore, the minimum.

Question 4c.

Worked solution

$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR}\\ &= 26 - 1.5 \times (41 - 26)\\ &= 26 - 1.5 \times 15\\ &= 3.5\end{aligned}$$

$3 < 3.5$; therefore, it is an outlier.

Mark allocation: 2 marks

- 1 mark for correct calculation of the lower fence
- 1 mark for statement that 3 is below the lower fence and is therefore an outlier

Question 5a.

Worked solution

There are three numerical, discrete variables.

Note: *destination* is not numerical.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5b.

Worked solution

$$\frac{14}{54} \times 100 = 25.9\%$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5c.

Worked solution

Minimum	First quartile	Median	Third quartile	Maximum
27	58.5	**105.5**	260	**384**

Median can be calculated manually, by placing the data in ascending order, as shown below:

27 43 55 | 62 65 85 | 126 155 235 | 285 314 384

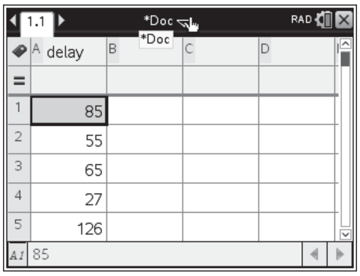
Q_1 Median Q_1

$$85 + \frac{126}{2} = 105.5$$

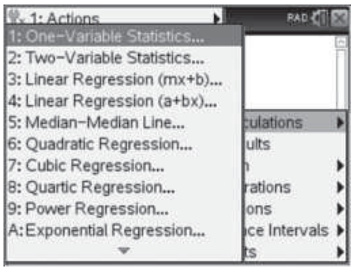
Alternatively, the CAS calculator can be used to calculate the five-number summary.

Data is entered into a spreadsheet, and the summary obtained.

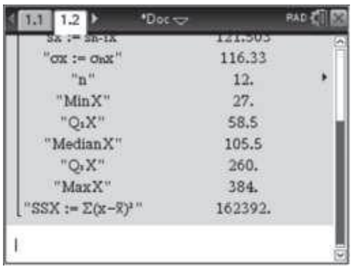
Step 1: Enter into spreadsheet.



Step 2: Obtain one-variable statistics.



Step 3: Read five-number summary from screen.

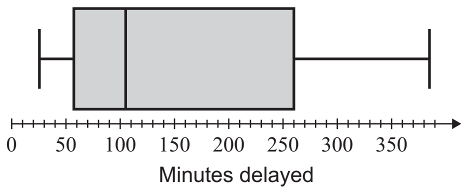


Mark allocation: 1 mark

- 1 mark for both correct answers

Question 5d.

Worked solution



Mark allocation: 2 marks

- 1 mark for accurately marking the median and showing the general shape correctly
- 1 mark for accurately marking Q_1 and Q_3 and accurately plotting whiskers

TIP

» Remember to check that the boxplot does not have an outlier. A quick way to do this is to use your CAS calculator to graph. If the graph does have outliers, remember that your whiskers will go to the last value inside of the lower and/or upper fences.

Question 5e.

Worked solution

positively skewed

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5f.

Worked solution

For the distribution of *minutes delayed*, the middle 50% of the data lies between 58.5 and 260 minutes.

Mark allocation: 1 mark

- 1 mark for correct answers

Question 5g.

Worked solution

Calculate the lower fence using the formula:

$Q_1 - 1.5 \times \text{IQR}$

Lower fence $= 58.5 - 1.5 \times (260 - 58.5) = -243.75$

As 27 is not below the lower fence, it is not an outlier.

Mark allocation: 2 marks

- 1 mark for correct calculation of lower fence
- 1 mark for statement clarifying that 27 is not below the lower fence so is not an outlier

Question 6a.

Worked solution

discrete numerical

Mark allocation: 1 mark

- 1 mark for correct answer

Question 6b.

Worked solution

Using CAS software, the missing values are:

$Q_1 = 654.5$, rounded to 655

Maximum $= 910$

Mark allocation: 2 marks

- 1 mark for each correct answer (up to 2 marks)

Question 6c.

Worked solution

There are five values less than 700.

$$\frac{5}{13} \times 100\% = 38.5\%$$

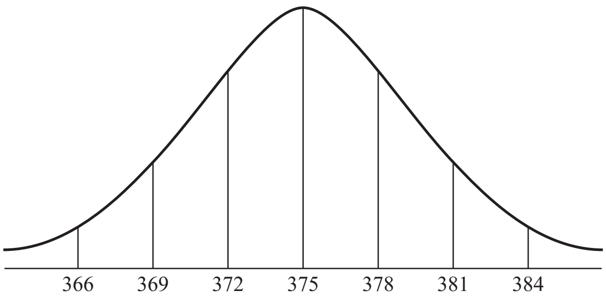
Mark allocation: 1 mark

- 1 mark for correct answer

Question 7a.

Worked solution

Start by drawing a normal distribution curve and calculate the values for ± 1, ± 2 and ± 3 standard deviations either side of the mean, as shown below.



369 calories is two standard deviations away from the mean.

Using the 68–95–99.7% rule, we know that 2.5% of data values lie below this value.

Therefore, 2.5% of patrons will burn less than 369 calories during the cycle class.

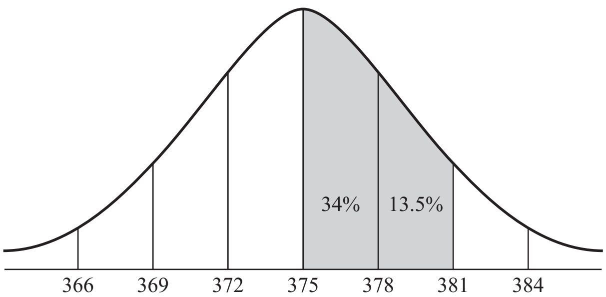
Mark allocation: 1 mark

- 1 mark for answer of 2.5%

Question 7b.

Worked solution

Use the normal distribution curve from part a.



It can be helpful to highlight the areas of the curve that fall within the desired range, as above.

$$34 + 13.5 = 47.5\%$$

$$\frac{47.5}{100} \times 500 = 237.5 \approx 238 \text{ patrons}$$

Mark allocation: 1 mark

- 1 mark for answer of 238

Question 8a.i.

Worked solution

25% of the boys weighed above 8.2 kg.

$0.25 \times 1200 = 300$ babies

Mark allocation: 1 mark

- 1 mark for the correct answer of 300 babies

Question 8a.ii.

Worked solution

75% of the girls and 25% of the boys weighed below 7.7 kg.

$0.75 \times 1100 + 0.25 \times 1200 = 1125$ babies

Mark allocation: 1 mark

- 1 mark for the correct answer of 1125 babies

Question 8b.

Worked solution

Boys: $Q_1 = 7.7$, $Q_3 = 8.2$, IQR $= 0.5$

Lower fence $= 7.7 - 1.5 \times 0.5 = 6.95$

The minimum score of 6.8 is less than the lower fence of 6.95, therefore it is an outlier.

Mark allocation: 2 marks

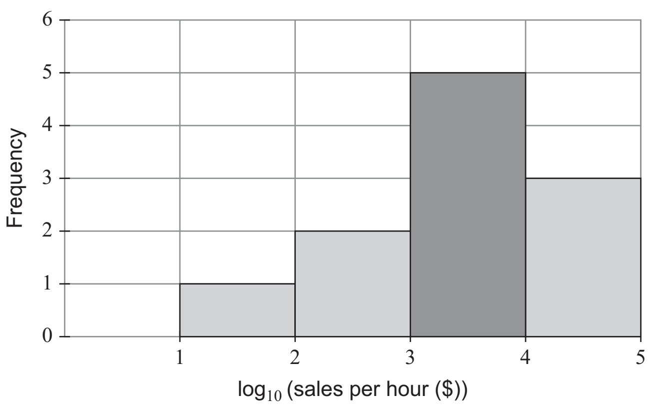
- 1 mark for correctly identifying the fence of 6.95
- 1 mark for stating that 6.8 is lower than 6.95, hence it is an outlier

Question 9

Worked solution

\$100 is represented by $\log_{10}(100) = 2$. \$10 000 is represented by $\log_{10}(10\,000) = 4$.

There is a frequency of 2 already represented in that range, so between 2 and 4 there are 5 remaining.



Mark allocation: 1 mark

- 1 mark for a bar drawn with a frequency of 5 between the values of 3 and 4

Area of Study 1 Data analysis – Investigating associations between two variables and modelling linear associations

EXAM 1

Question 1

Answer: ***D***

Explanatory notes

There are 19 scores that are 3.5 or higher, and there are 41 scores in total.

$\frac{19}{41} \times 100 = 46.34\%$

Option A quotes the raw frequency of 19 instead of converting it to a percentage.

Option B is the number of scores that are 3 or more.

Option C is the total number of scores provided.

Question 2

Answer: ***A***

Explanatory notes

key: 3 | 1 = 3.1

under 18 years		**18 years of older**
3	0	
8 5 3 3	1	7 8 8
9 5 5 3 1 1	2	3 4 4 6 8
8 7 4 4	3	4 7 7 7 8 9 9
5 2	4	1 3 5 8 8 8
7	5	3 7

The median for *under 18 years* is 2.5, and the median for *18 years or older* is 3.7.

$3.7 - 2.5 = 1.2$

Option B uses an incorrect median of 2.1 for *under 18 years*, which may occur if the median was determined by counting in the wrong direction in the stem plot.

Options C and D are only the median values of *under 18 years* and *18 years or older*, so the difference hasn't yet been found.

Question 3

Answer: ***D***

Explanatory notes

Option A is true because the 1st quartile for the Year 9 boxplot is approximately equal to the 3rd quartile for the Year 8 boxplot.

Option B is true because the distribution for Year 8 male students does not contain an outlier and is positively skewed.

Option C is true because when we compare the interquartile range (IQR) for the boxplots, we can see that the IQR for Year 8 students is approximately 12, whereas the IQR for Year 9 students is approximately 15. From this we can conclude that the boxplots have approximately the same IQR.

Option D is not true because it is not suitable to use the mean as a measure of centre for distributions that are either skewed or contain an outlier, or both. In this case, the Year 8 distribution is skewed. This question asks for the statement that is not true, this is therefore our answer.

Question 4

Answer: ***C***

Explanatory notes

Option A is true. We have been given the Pearson correlation coefficient, r, and can square this to find the coefficient of determination, $r^2 = (0.7883)^2 = 0.6214$

Option B is true. This is an interpretation of the slope of our regression equation.

Option C is **not** true. This is an interpretation of the coefficient of determination, r^2; however, the Pearson correlation coefficient, r, has been used in this interpretation. This question asks for the statement that is not true, this is therefore our answer.

Option D is true. An r value of 0.7883 corresponds to a positive, strong relationship. We can observe from the scatterplot that the relationship is linear.

TIP

» **With questions that ask you to determine which answer is not true, make sure you test all answers to see which are true and which one is not true.**

Question 5

Answer: ***B***

Explanatory notes

First, we need to substitute Belinda's study hours into the least squares regression equation:

$\textit{exam result} = 36.8 + 5.7 \times 7.6$

$\textit{exam result} = 80.12$

We now calculate our residual value:

$\textit{residual} = \textit{actual} - \textit{predicted}$

$\textit{residual} = 75 - 80.12$

$\textit{residual} = -5.12$

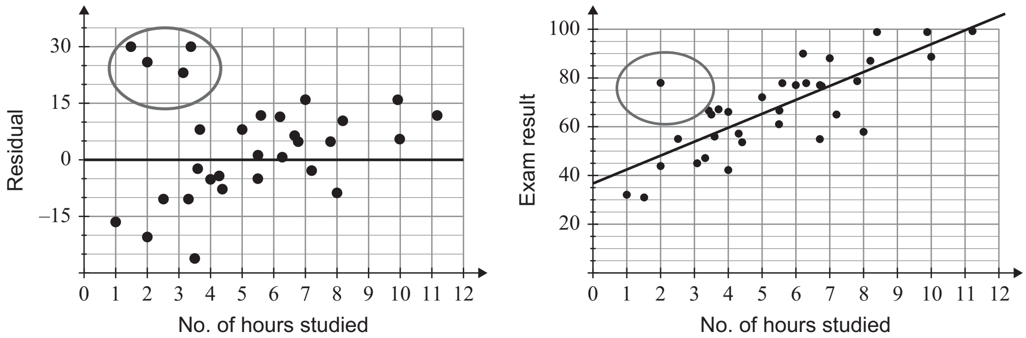
Question 6

Answer: ***D***

Explanatory notes

The scatterplot and residual should maintain a similar shape and pattern, so look for differences in where the values are placed.

Option A shows values between $x = 1$ and $x = 4$ that do not correlate to values on the scatterplot.



Similarly, option B is missing the positive residual value that should be at $x = 2$.

Option C shows the residual plot has a clear pattern, indicating the scatterplot is non-linear, which is incorrect.

Note that a point above the least squares regression line will have a positive residual value, whereas a point below the least squares regression line will have a negative residual value.

Question 7

Answer: ***B***

Explanatory notes

Use your CAS calculator to perform a transformation on the original data.

Option A shows a squared transformation of exam result instead of no. of hours studied.

Option C represents that no transformation of variables has occurred.

Option D has the variable exam result as the explanatory and no. of hours studied as the response variable, which is incorrect.

Question 8

Answer: ***A***

Explanatory notes

Option A is correct. By comparing the medians, we can see that there has been an increase in the measure of centre for the Year 12 students between their first and final trial exams, therefore showing an ‘improvement’.

Option B is incorrect. We should not use the range to compare boxplots, as this only gives us an indication of the difference between the lowest and highest extreme, not what is happening ‘on average’ within the data.

Option C is incorrect. In this case, discussing the shape of the distribution does not allow us to consider whether the students have improved, on average, between their first and final trial exams.

Option D is incorrect. The IQR for the first trial exam is lower than that for the final trial exam: 8 compared to 10.

TIP

» **When comparing boxplots, use either the IQR or the median, as these are not affected by the extreme values. This is especially important if an outlier exists within the distribution or the data is skewed.**

Question 9

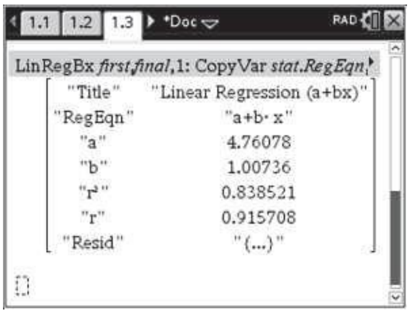
Answer: ***D***

Explanatory notes

The best way to complete this question is to enter the data given in the question into a spreadsheet in your CAS calculator.

You can then use this to calculate the equation of the regression line: $y = a + bx$.

The values given for this calculation, using the TI-Nspire software, are shown below.



Question 10

Answer: ***A***

Explanatory notes

Option A is incorrect. The slope for the regression equation is positive, and therefore the *final trial exam* score **increases** as the *first trial exam* score increases.

Option B is correct. This is an interpretation of the y-intercept for the regression equation.

Option C is correct. This is an interpretation of coefficient of determination, r^2, which is obtained by $(0.916)^2 = 0.839$.

Option D is correct. The Pearson correlation coefficient, given in the question, tells us the strength and direction of the equation. In this case, r is positive, and is between 0.75 and 0.99.

Question 11

Answer: ***C***

Explanatory notes

Option A is incorrect. We cannot make statements that 'guarantee' a result.

Option B is incorrect. We cannot make statements that imply that one variable causes another variable to change.

Option C is correct. There is a correlation, as seen by the regression analysis. The relationship is positive and linear, suggesting that as x increases so does y.

Option D is incorrect. This is a comparison of scores, and does not address a relationship between the two variables. Also, we must ensure that we do not make definitive statements when considering our statistics.

TIP

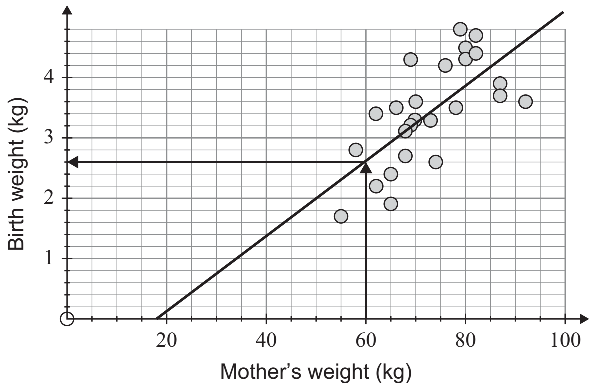
» When conducting data analysis, we must avoid making any statements that imply that a change in one variable will cause a change in the other. Our analysis can only make predictions and discuss trends or associations.

Question 12

Answer: ***A***

Explanatory notes

Reading from the graph:



Answer = 2.6

Question 13

Answer: ***D***

Explanatory notes

Options A and B are incorrect as *mother's weight* is the explanatory variable; in this equation it is written as the response variable.

Option C is incorrect. This equation has both a negative slope and negative y-intercept, but this graph has a positive slope.

Question 14

Answer: ***D***

Explanatory notes

To find the value of r, find the square root of the coefficient: $r = \sqrt{0.6877} = 0.829276$

TIP

» It is important to remember that the square root of a number can either be a positive or a negative. When calculating r from r^2 you must look at the graph or equation that you have been given. If the regression line has a positive gradient (slope), then the value of r will be positive. If it is negative, then r will also be negative.

Question 15

Answer: ***B***

Explanatory notes

Reading from the graph, the actual value for *birth weight* when the *mother's weight* is 92 kilograms is 3.6 kilograms.

The line predicts that the *birth weight* will be $-1.1203 + 0.063 \times 92 = 4.6757$ kilograms.

$\textit{Residual} = \textit{actual value} - \textit{predicted value}$

$= 3.6 - 4.6757$

$= -1.0757$

This is a residual value of −1.1 kilograms.

Question 16

Answer: ***C***

Explanatory notes

$$b = r \times \frac{s_y}{s_x} = 0.98 \times \frac{18.77}{4.47} = 4.11512$$

$$a = \bar{y} - b\bar{x} = 50.53 - 4.115 \times 8 = 17.61$$

Therefore, $y = a + bx$ is $y = 17.6 + 4.1x$

Question 17

Answer: ***B***

Explanatory notes

Option B is correct. These transformations are the only ones that could linearise the data.

Question 18

Answer: ***D***

Explanatory notes

When the y^2 transformation is performed, the equation obtained is $y^2 = 73661.5 - 8115.6x$.

Substitute $x = 5$ and use Solve on the CAS to find the value of y.

solve $(y^2=73661.5-8115.6\cdot5, y)$
$y=-181.888702233$ or $y=181.888702233$

TIP

» When performing a data transformation on the response variable, as above, it is important to note that when finding the least squares line, you will not have an equation in the form of $y = a + bx$. The variable y has been transformed, which means there will be an additional step in your working.

Question 19

Answer: ***B***

Explanatory notes

Option A is referring to the vertical intercept, which is correct with a value of 65 591.

Option B is referring to the coefficient of determination (r^2) in the statement, so the value of $(-0.9978)^2 = 0.9956\ (99.56\%)$ should be used instead.

Option C is referring to the gradient, with a correct value of $4164 used.

Option D is referring to the interpretation of the r-value, which is correct.

TIP

» Make sure you know how to interpret the gradient, vertical intercept, Pearson correlation coefficient and coefficient of determination, as this will help you decipher how each statement is trying to test you.

Question 20

Answer: ***B***

Explanatory notes

Residual value = actual value − predicted value

The actual price paid was $18 000.

The predicted price is calculated by substituting in the age of 12 years:

$61591 - 4164 \times 12 = 11623$

$Residual = 18000 - 11623 = 6377$

Question 21

Answer: ***D***

Explanatory notes

Entering the data into a CAS spreadsheet, then performing statistical analysis of it, using:

X List: the original explanatory variable (*age*)

Y List: the transformed variable (reciprocal values of *average height*)

LinRegBx *a,re*,1: CopyVar *stat.RegEqn,f9*: *st*

"Title"	"Linear Regression (a+bx)"
"RegEqn"	"a+b·x"
"a"	0.168128476359
"b"	-0.020698556223
"r²"	0.979736291121
"r"	-0.989816291602
"Resid"	"{...}"

Options A and D have the correct values for the vertical intercept (*a*) and the gradient (*b*); however, option A has an incorrect variable of $\frac{1}{Age}$ instead of $\frac{1}{Average\ height}$.

TIP

» **Make sure you remember which variable has been transformed and change it in your equation. For example, the log of average height is referred to as log(average height) in the equation.**

Question 22

Answer: ***B***

Explanatory notes

A CAS calculator can be used to solve this, substituting in the value of 20 for the *average height*:

$$\text{solve}\ (20=2.43+0.856\cdot x^2, x)$$
$$x=4.53052987349$$

Question 23

Answer: ***C***

Explanatory notes

The least squares line plots the *predicted* revenue, so any dots above that represent revenue that was higher than predicted. There are three values that are 200 or more units above the line.

Question 24

Answer: ***D***

Explanatory notes

The coefficient of determination is r^2, so

$$r^2 = 0.9453^2$$
$$= 0.89359...$$

Converted to a percentage this is 89.359%, which is then rounded to three significant figures: 89.4%.

Option A is the r^2 value, but it hasn't been converted to a percentage.

Option B is the original r value that has only been rounded.

Option C is if the r value has been rounded to three significant figures first before being squared.

TIP

» **Remember to round after a calculation: rounding too early can result in issues with accuracy.**

Question 25

Answer: ***B***

Explanatory notes

In order to find the vertical intercept, a, the gradient, b, first needs to be found:

$$b = r \times \frac{sy}{sx}$$

$$= 0.9453 \times \frac{2082.3}{35.4}$$

$$= 55.60$$

So to find the vertical intercept, the calculation is:

$$a = \bar{y} - b\bar{x}$$

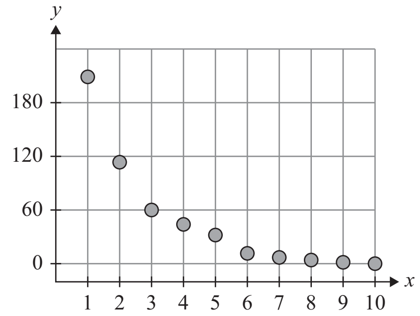
$$= 4595 - 55.60 \times 82.4$$

Question 26

Answer: ***A***

Explanatory notes

The scatterplot for the data is shown below.



Option A lists the transformations that are suitable for a scatterplot of this nature.

Question 27

Answer: ***C***

Explanatory notes

Once the transformation is performed, the coefficients of the least squares line for the new data are shown.

a	182.264…
b	−203.38…

As there was a transformed x variable, the variables in the equation need to reflect the transformation performed.

This changes the equation from $y = a + bx$ to $y = a + b\log(x)$.

Question 28

Answer: ***A***

Explanatory notes

A positive association describes a situation where when one (explanatory) variable increases, the other (response) variable also tends to increase. This is true for all options except option A. As temperature increases, the number of heaters sold would tend to *decrease*.

Question 29

Answer: ***D***

Explanatory notes

$b = 0.9351 \times 0.46 \div 2.7 = 0.1593$

$a = 12.8 - 0.1593 \times 23.2 = 9.10$

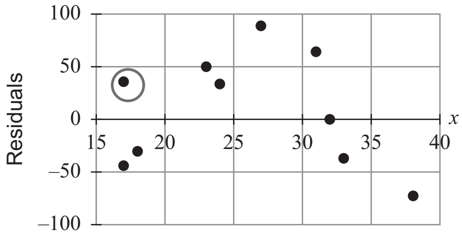
$y = 9.10 + 0.16x$

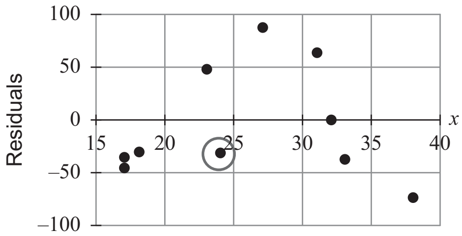
Question 30

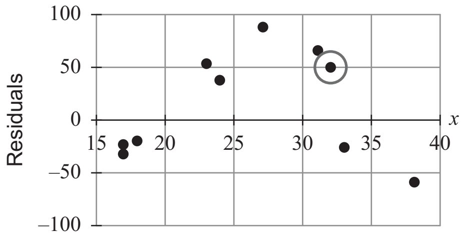
Answer: ***C***

Explanatory notes

The residuals that are in the wrong place for the incorrect options, A, B and D, are shown below:







TIP

» **Visualise the data points and the line of best fit from a scatterplot being tipped so the line becomes horizontal, then inspect which values are not still in the correct position.**

EXAM 2

Question 1a.

Worked solution

Moderate, positive, linear relationship

Mark allocation: 1 mark

- 1 mark for correct answer – all three aspects correct

Question 1b.

Worked solution

$r = 0.718274... = 0.72$

Use your CAS calculator to obtain a statistical summary for the linear regression between *hourly pay rate* and *hours worked*.

The Pearson correlation coefficient, r, will be a part of this summary.

Mark allocation: 1 mark

- 1 mark for stating the value of r, correct rounding included

Question 1c.

Worked solution

The equation obtained when linear regression is performed between *hourly pay rate* and *hours worked* is:

$\textit{hours worked} = -4.857157 + 1.004465 \times \textit{hourly pay rate}$

When rounding to two significant figures:

The intercept:

$-4.857157 \approx -4.9$

The slope:

$1.004465 \approx 1.0$

Therefore, the equation, rounded to two significant figures, is:

$\textit{hours worked} = -4.9 + 1.0 \times \textit{hourly pay rate}$

Mark allocation: 2 marks

- 2 marks for two variables, correct to two significant figures

If this is not the case:

- 1 mark can be awarded if **one coefficient** is correct, and rounded to two significant figures

OR

- 1 mark can be awarded if **both coefficients are correct**, but are not rounded correctly

Question 1d.

Worked solution

The coefficient of determination tells us that, on average, 51.59% of the variation in *hours worked* can be explained by the variation in *hourly pay rate*.

Mark allocation: 1 mark

- 1 mark for correct interpretation

Question 1e.i.

Worked solution

Yes, the residual plot shows no clear pattern, and therefore confirms an assumption of linearity.

Mark allocation: 2 marks

- 1 mark for a clear indication that the assumption is correct
- 1 mark for an explanation specific to the residual plot; for example, random scattering or no pattern

Question 1e.ii.

Worked solution

The predicted value is calculated using the equation obtained earlier (see part c.).

$residual = actual - predicted$

$residual = 17 - (-4.9 + 1 \times 25)$

$residual = 17 - 20.1$

$residual = -3.1$

Mark allocation: 1 mark

- 1 mark for correct answer of −3.1

Question 2

Worked solution

We can see that the median *end-of-year result* differs for males and females. The median for females was 26, compared with the median of 32 for males. This suggests that *end-of-year result* is associated with *gender* because the median is higher for males than for females.

Mark allocation: 2 marks

- 1 mark for a statement that implies a change or difference in median
- 1 mark for numerical values for median quoted accurately for both male and female data

TIP

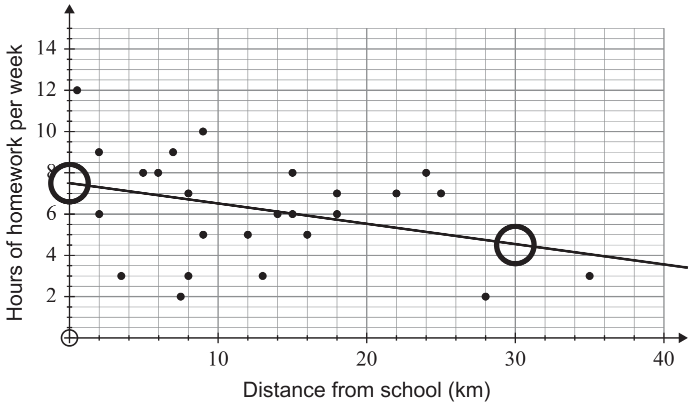
» **Ensure that you use comparative language for this type of question. Use words such as 'differ', 'increase', 'decrease' and 'change'.**

Question 3a.

Worked solution

Below is the scatterplot with line sketched in.

Two points have been circled, as a guide to ensure accuracy of the line.



Mark allocation: 2 marks

- 1 mark for line passing through correct *y*-intercept of 7.46
- 1 mark for line passing through another accurate point; for example, the point at (30,4.5) is circled above

TIP

» When plotting a straight line, identify two points on the line and plot these before joining these points. This can be done by substituting values of x into the equation and plotting them with their corresponding y-values.

Question 3b.

Worked solution

On average, when the *distance from school* is 0 kilometres, the *hours of homework per week* is expected to be 7.46 hours.

Mark allocation: 1 mark

- 1 mark for correct interpretation of intercept, including units for variables and a statement outlining what the value of y is when x is 0 units

Question 3c.

Worked solution

The coefficient of determination is the r^2 value. We can find the square root of this to find the Pearson correlation coefficient, r.

$r = \sqrt{0.1117} = 0.3342 \text{ or } -0.3342$

This relationship is negative and we must therefore select the negative value of r.

The Pearson correlation coefficient, r, is equal to -0.334.

Mark allocation: 1 mark

- 1 mark for correct answer, including the negative

Question 3d.

Worked solution

$residual = actual - predicted$

$residual = 3 - (7.46 - 0.0977 \times 8)$

$residual = -3.68$

Mark allocation: 1 mark

- 1 mark for correct answer

Note: The answer must be a negative.

Question 3e.i.

Worked solution

hours of homework per week

$= 7.46 - 0.0977 \times$ *distance from school*

hours of homework per week $= 7.46 - 0.0977 \times 11$

hours of homework per week $= 6.3853$

hours of homework per week $= 6.4$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3e.ii.

Worked solution

This prediction is interpolation, within the given data range.

Mark allocation: 1 mark

- 1 mark for statement that the data is within the given range

Question 4a.

Worked solution

The explanatory variable is *wind speed*.

Mark allocation: 1 mark

- 1 mark for correct answer

TIP

» **You can usually identify the explanatory variable by looking at the x-axis of your graph. Another way is to consider which variable you will be making a prediction from. In the sentence above, you are told that you are predicting the delay from the wind speed. This suggests that wind speed is the explanatory variable.**

Question 4b.

Worked solution

minutes delayed $= -10.745 + 18.051 \times$ *wind speed*

minutes delayed $= -10.745 + 18.051 \times 14$

minutes delayed $= 241.969 = 242$ minutes

Mark allocation: 2 marks

- 1 mark for substitution into regression equation
- 1 mark for a calculation resulting in correct answer – this must be a number that rounds correctly to 242

Question 4c.

Worked solution

minutes delayed $= -10.745 + 18.051 \times$ *wind speed*

minutes delayed $= -10.745 + 18.051 \times 15$

minutes delayed $= 260$ km/h

residual = *actual* − *predicted*

residual $= 196 - 260$

residual $= -64.0$

Mark allocation: 2 marks

- 2 marks for correct answer of −64.0

If incorrect:

- 1 mark for calculation of predicted value of 260 using least squares equation, or
- 1 mark for showing calculation of (196 − incorrect predicted value).

Question 4d.

Worked solution

This is a strong relationship.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4e.

Worked solution

If $r = 0.757$, square this to find the coefficient of determination, r^2.

$r^2 = (0.757)^2 = 0.573$

Therefore, 57% of the variation in *minutes delayed* can be explained by the variation in *wind speed*.

Mark allocation: 1 mark

- 1 mark for the correct answer

TIPS

» The word 'variation' in conjunction with the phrase 'can be explained by' should help you to identify that this question is asking you to calculate the coefficient of determination.

» Be careful to answer this question in the correct form. It is not asking for r^2 as a decimal, but as a percentage.

Question 5a.

Worked solution

An estimation can be made by reading values from the boxplot – the minimum number of calories burned by *Gym patron 2* is approximately 275.

Mark allocation: 1 mark

- 1 mark for an answer between 270 and 280

Question 5b.

Worked solution

The minimum number of calories burned by *Gym patron 1* is 558. This is slightly higher than the median for *Gym patron 2*, if reading from the boxplot. Therefore, for approximately 50% of the time period, *Gym patron 2* burned less calories than the minimum for *Gym patron 1*.

$\frac{50}{100} \times 14 = 7$ days

Mark allocation: 1 mark

- 1 mark for an answer of 7 days

Question 5c.

Worked solution

The lower fence is given by $Q_1 - 1.5 \times \text{IQR}$.

For *Gym patron 2*, the interquartile range is 125.

Therefore, lower fence $= Q_1 - 1.5 \times \text{IQR} = 490 - 1.5 \times 125 = 302.5$

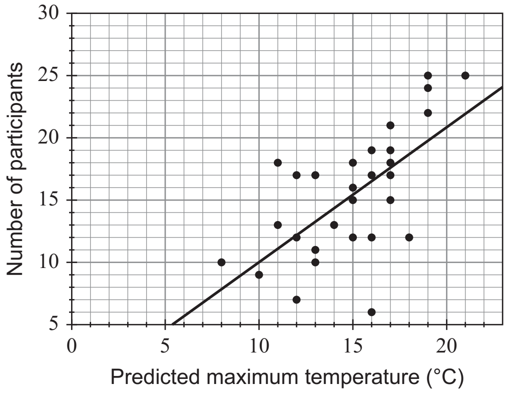
275 is an outlier as it is less than (or falls below) the lower fence.

Mark allocation: 2 marks

- 1 mark for calculation of the lower fence
- 1 mark for statement that 275 is less than (or below) the lower fence

Question 6a.

Worked solution



Mark allocation: 1 mark

- 1 mark for accurate line drawn on scatterplot

TIP

» Usually substituting in x-values from either end assists in plotting the graph; however, if the least squares line does not appear to be reaching the vertical axis on the scatterplot, substitute any other values on the x-axis instead. For example, $(6, 5.7324) \approx (6, 6)$ and $(23, 24.0414) \approx (23, 24)$.

Question 6b.

Worked solution

On average, for every 1°C increase in the *predicted maximum temperature*, it is expected that the *number of participants* will increase by 1.077.

Mark allocation: 1 mark

- 1 mark for statement above or similar

TIP

» When answering a question where you are required to interpret the slope or y-intercept, ensure that you refer to specific variables (instead of 'explanatory variable' and 'response variable') and include units where appropriate.

Question 6c.

Worked solution

$\textit{number of participants} = -0.7296 + 1.077 \times \textit{predicted maximum temperature}$

$20 = -0.7296 + 1.077 \times \textit{predicted maximum temperature}$

Use CAS software to solve this equation.

Therefore, the *predicted maximum temperature* is 19°C when the number of participants is 20.

Mark allocation: 1 mark

- 1 mark for correct answer of 19°C

Question 6d.

Worked solution

According to the scatterplot, when the *predicted maximum temperature* is 18°C, the actual value for *number of participants* is 12.

Using the least squares equation:

$\textit{number of participants} = -0.7296 + 1.077 \times \textit{predicted maximum temperature}$

$\textit{number of participants} = 0.7296 + 1.077 \times 18$

$\textit{number of participants} = 18.6564$

$\textit{number of participants} = 19$

The residual is the difference between the actual value and the predicted value, using the equation:

$\textit{residual} = \textit{actual} - \textit{predicted}$

$= 12 - 19$

$= -7$

Mark allocation: 1 mark

- 1 mark for a calculation that results in a residual value of −7

Question 6e.

Worked solution

$r^2 = 0.4143$

$r = \sqrt{r^2} = 0.64366 \approx 0.64 \text{ or } -0.64$

The positive value of 0.64 is correct as the slope of the graph is also positive.

Mark allocation: 1 mark

- 1 mark for correct answer of 0.64

Question 6f.

Worked solution

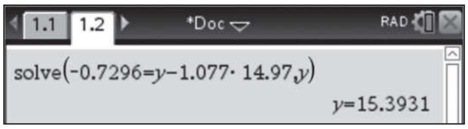
We can use the values for the intercept, the slope and the mean for the *predicted maximum temperature* to find the mean for *number of participants* using the following formula: $a = \bar{y} - b\bar{x}$

The values can be substituted into the equation

$a = \bar{y} - b\bar{x}$

$-0.7296 = \bar{y} - 1.077 \times 14.97$

A CAS calculator can be used to solve for the value of $\bar{y}$, as shown below:



The value for the mean of the *number of participants* is 15.4, when rounded to one decimal place.

Mark allocation: 1 mark

- 1 mark for correct value of 15.4

Question 6g.i.

Worked solution

The residual values can be obtained from the scatterplot, or alternatively calculated using the following:

For $x = 8$

$predicted = -0.7296 + 1.077 \times 8 = 7.8864$

$actual = 10$ (from graph)

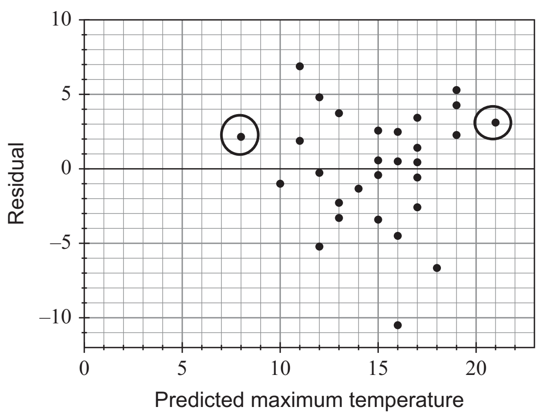
$residual = actual - predicted = 10 - 7.8864 = 2.1136$ (plot 8, 2.1)

For $x = 21$

$predicted = -0.7296 + 1.077 \times 21 = 21.8874$

$actual = 25$ (from graph)

$residual = 25 - 21.8874 = 3.1126$ (plot 21, 3.1)



Mark allocation: 1 mark

- 1 mark for both points plotted correctly

Question 6g.ii.

Worked solution

The residual plot shows no clear pattern (a random arrangement of points) and this confirms the assumption of linearity for the relationship between the *predicted maximum temperature* and the *number of participants*.

Mark allocation: 1 mark

- 1 mark for an answer of ‘yes’ and reference to ‘random points’ or ‘no pattern’

Question 7a.

Worked solution

age

Although *age* can be numerical, in this case it is categorical ordinal, as it uses ordered groupings of 35–44, 45–54 and 55–64.

Mark allocation: 1 mark

- 1 mark for the correct answer of *age*

Question 7b.

Worked solution

Yes, the *median income* differs with *sex*, as shown by the median income of $80 800 for men being significantly higher than $52 410 for women in the 35–44 age group.

Note: Comparing the male and female incomes in the 45–54 or 55–64 age brackets also shows that the *median income* differs with *sex*.

Mark allocation: 2 marks

- 1 mark for confirming that there is an association
- 1 mark for referring to the values across one of the age groups, highlighting that there is a significant difference/change in income between males and females

TIP

» If there are more than two values across the row, ensure each of them is quoted and compared in your response (unless specified otherwise).

Question 8a.

Worked solution

The question is asking for r^2, written as a percentage.

$r^2 = 0.9364^2 = 0.8768$ which is 87.7%

Mark allocation: 1 mark

- 1 mark for the correct answer of 87.7%

Question 8b.

Worked solution

The y-intercept is the *weight* at *age* 0 (birth), which is 3.25.

To find the gradient, CAS can be used:

A age	B weight
0	3.25
12	9.49

a	3.25
b	0.52

or $\frac{y_2 - y_1}{x_2 - x_1} = \frac{9.49 - 3.25}{12 - 0} = 0.52$

$weight = 3.25 + 0.52 \times age$

Mark allocation: 2 marks

- 2 marks for the equation written correctly as $weight = 3.25 + 0.52 \times age$
- 1 mark for the correct coefficients
- 1 mark for the variables correctly written in the form $weight = a + b \times age$

Question 9a.

Worked solution

The percentage conversions are:

62/97 = 64%

35/97 = 36%

Mark allocation: 2 marks

- 2 marks for accurately drawing a column with *under 5 km* segment being 64%, and the *over 5 km* segment being 36%
- 1 mark for drawing an inaccurate column but with segments below 3% in error

Question 9b.i.

Worked solution

The boxplot is positively skewed with an outlier.

Mark allocation: 1 mark

- 1 mark if both pieces of information above are identified
- 0 marks for also including spread (IQR, range) or centre (median) without clearly labelling each component to demonstrate that the shape was specifically known

TIP

» Be careful of further engagement errors, where your answer includes information that hasn't been asked for that may negate your correct answer.

Question 9b.ii.

Worked solution

$20 - 40\ \textit{years}$: $52 + 2\ \textit{outliers} = 25\%$, $54 \times 4 = 216$ customers

$40\ \textit{years and over}$: $64 = 50\%$, $64 \times 2 = 128$ customers

$\textit{Total} = 128 + 216 = 344$ customers

Mark allocation: 1 mark

- 1 mark for the correct answer of 344

Question 10a.

Worked solution

0.85 pages are read for every additional minute spent reading.

OR

For every extra 1 minute spent reading, 0.85 pages are read.

Mark allocation: 1 mark

- 1 mark for a response that includes a reference to the additional 1 unit increase in the explanatory variable (*time spent reading*) and the 0.85 increase in the *number of pages read*

Question 10b.

Worked solution

$\textit{residual} = \textit{actual } y - \textit{predicted } y$

$-0.69 = 23 - y$

$y = 23.69$

$23.69 = 2.44 + 0.85 \times x$

$x = 25$

Mark allocation: 2 marks

- 2 marks for the correct value of 25

OR

- 1 method mark if using the residual formula and then substituting an incorrect predicted value into the equation to find the value of x

Question 10c.

Worked solution

$r = 0.9170$

$r^2 = (0.9170)^2 = 0.8409$

84% of the variation in *number of pages read* can be explained by the variation in *time spent reading*.

Mark allocation: 1 mark

- 1 mark for the sentence shown above

Question 11a.

Worked solution

A log transformation of the explanatory variable (*number of staff*) is the appropriate log transformation:

$\textit{units restocked} = -1551 + 3355 \times \log_{10}(\textit{number of staff})$

Mark allocation: 2 marks

- 2 marks for equation with values and variables written correctly as above
- 1 mark if values are incorrect but the variable $\log_{10}(\textit{number of staff})$ is placed in the equation as the correct transformed (explanatory) variable

Question 11b.i.

Worked solution

$$\begin{aligned}\textit{units restocked} &= 2942 - 10623 \times \frac{1}{16}\\ &= 2278\end{aligned}$$

Mark allocation: 1 mark

- 1 mark for value of 2278

Question 11b.ii.

Worked solution

The value of 16 is outside the range of data provided; therefore it is extrapolation, which may be less reliable.

Mark allocation: 1 mark

- 1 mark for use of the term *extrapolation* to explain why it is less reliable

Area of Study 1 Data analysis – Investigating and modelling time series data

EXAM 1

Question 1

Answer: ***C***

Explanatory notes

There is no repeating pattern within a year (seasonality) or within a regular, longer period of time (cyclical). The fluctuations are random.

Question 2

Answer: ***D***

Explanatory notes

When interpreting a seasonal index, we consider a seasonal index of 1.0 as being on average.

Any value below 1.0 is *below average*. And any value higher than 1.0 is *above average*.

To find the percentage above or below the average, we compare this to 1.0.

For example, for a seasonal index of 0.87, we subtract the index from 1.0:

$1.0 - 0.87 = 0.13$

We can therefore conclude that this is 13% below the average.

Question 3

Answer: ***B***

Explanatory notes

Step 1 – Calculate an average for the week:

$(1820 + 1700 + 1467 + 1476 + 1765 + 1239 + 1348) \div 7 = 1545$

Step 2 – Divide the sales for Tuesday by the weekly average:

$1700 \div 1545 = 1.10$

This matches the calculation in option B.

Question 4

Answer: ***B***

Explanatory notes

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$

$$\text{deseasonalised sales} = \frac{1825}{1.14}$$

$$\text{deseasonalised sales} = 1600$$

Question 5

Answer: ***D***

Explanatory notes

There is not a pattern that can be observed that repeats within the space of a year, so therefore seasonality cannot be considered. Option A is incorrect, as is option C.

The values do not decline over time, so therefore there is not a decreasing trend. Option B is incorrect.

The values appear to be increasing over time, which means that there appears to be an increasing trend. Option D is correct.

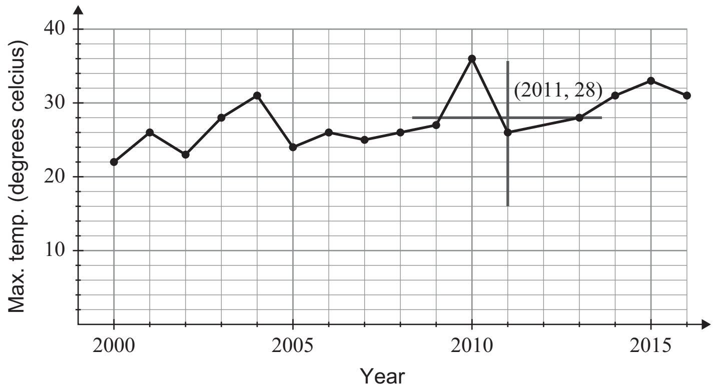
Question 6

Answer: ***C***

Explanatory notes

The five-median smoothed value can be found, graphically, by finding the median of the x values and the median of the y values to decide on a coordinate.

A line can be drawn through the median for the x values and also for the median of the y values. Where these lines cross is the median point. See the graph below.



Question 7

Answer: ***D***

Explanatory notes

Step 1:

Calculate the average temperature across the year.

$$\frac{28 + 31 + 26 + 22 + 18 + 16 + 15 + 17 + 23 + 27 + 30 + 32}{12} = 23.75$$

Step 2:

Find the seasonal index for May by dividing the actual average for May by the yearly average for 2016:

$$\frac{18}{23.75} = 0.757895 \approx 0.76 \text{ (correct to 2 decimal places)}$$

Question 8

Answer: ***B***

Explanatory notes

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Min. temp. (°C)	28	31	26	22	18	16	15	17	23	27	30	32

$$\frac{16+15+17+23}{4} = 17.75 \qquad \frac{15+17+23+27}{4} = 20.5$$

$$\frac{17.75+20.5}{2} = 19.125 \approx 19$$

Question 9

Answer: ***D***

Explanatory notes

Three-mean smoothing using values for June, July and August gives:

$$\frac{89 + 120 + 105}{3} = 104.6667 \approx 105$$

Question 10

Answer: ***C***

Explanatory notes

There are 12 seasons, so the seasonal indices should add to 12.

We are missing one of the indices and can therefore subtract our known values from 12 to find the unknown.

$$12 - 1.29 - 1.17 - 0.77 - 0.70 - 0.92 - 1.24 - 1.08 - 0.99 - 0.80 - 0.95 - 1.22 = 0.87$$

Question 11

Answer: ***D***

Explanatory notes

actual value = deseasonalised value × seasonal index

$$\begin{aligned}\text{actual value} &= 99 \times 1.24 \\ &= 122.76 \approx 123\end{aligned}$$

Question 12

Answer: ***C***

Explanatory notes

Using algebra and a CAS calculator:

$$\left(\frac{1}{2} \times \left(\frac{17 + x}{2} + \frac{21 + x}{2}\right)\right) = 19.5$$

$$x = 20$$

$$\text{solve}\left(\frac{\frac{17+x}{2}+\frac{21+x}{2}}{2}=19.5,x\right) \quad x=20.$$

Or using trial and error, and focusing only on the relevant values:

Day			3	4	5
Sales			17	18	21
Smoothed			17.5	19.5	
Centred				18.5	

Day			3	4	5
Sales			17	19	21
Smoothed			18	20	
Centred				19	

Day			3	4	5
Sales			17	**20**	21
Smoothed			18.5	20.5	
Centred				**19.5**	

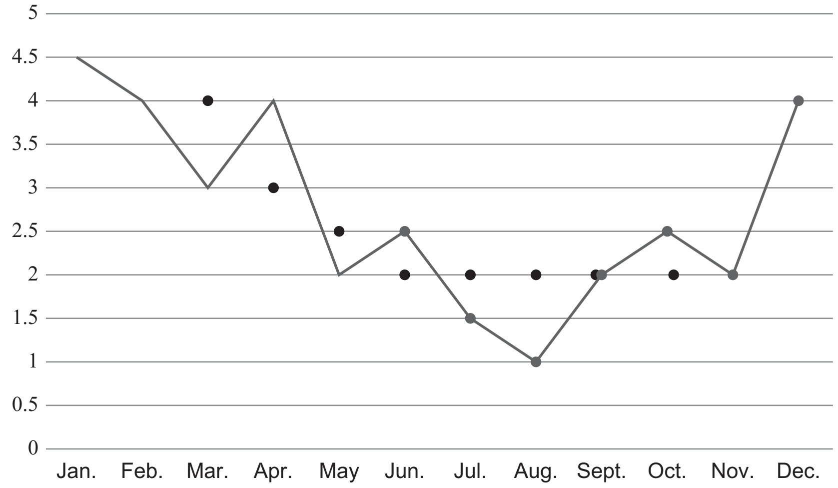
Day			3	4	5
Sales			17	22	21
Smoothed			19.5	21.5	
Centred				20.5	

Question 13

Answer: ***C***

Explanatory notes

The complete five-median smoothing is shown below. There are five months that have a value of 2.



Question 14

Answer: ***C***

Explanatory notes

To correct for seasonality by an increase of 25%:

$$1.25 = \frac{1}{\text{SI}}$$

$$\text{SI} = 0.8$$

Question 15

Answer: ***B***

Explanatory notes

A discontinuity is a form of structural change resulting in a break in the data.

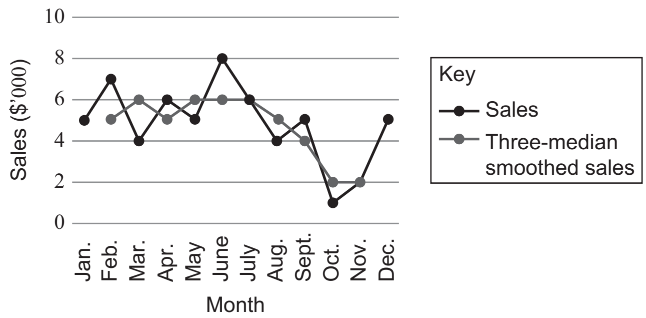
This can be seen in the shift in the data from week 6 to week 7.

Question 16

Answer: ***C***

Explanatory notes

The correct graph should be:



January and December should not have values, as the three-point smoothing creates a shorter line graph as a result. May and October are also incorrectly placed.

Question 17

Answer: ***C***

Explanatory notes

The seasonal index for Quarter 3 is $4 - (0.82 + 1.04 + 0.96) = 1.18$

deseasonalised sales (\$'000) = $13.9 + 1.08 \times 3 = 17.14$ which is \$17 140

actual sales = seasonal index × deseasonalised sales

$= 1.18 \times 17140$

$= 20\,225.20$

Question 18

Answer: ***B***

Explanatory notes

$\frac{7+9}{2} = 8 \quad \frac{9+11}{2} = 10$

Centring: $\frac{8+10}{2} = 9$

Question 19

Answer: ***A***

Explanatory notes

The seven values around Day 8 are:

48, 51, 16, 19, 29, 15, 39

The median of these is 29.

Question 20

Answer: ***D***

Explanatory notes

Option A: A seasonal index of 1.07 means sales are 7% above the average of 1.0 (100%).

Option B: The seasonal indices must add to 4. $Q_1 + Q_2 = 2.10$, which means $Q_3 + Q_4 = 1.90$. This means the mean seasonal index is $1.90 \div 2 = 0.95$.

Option C: It is possible that Q_3 could be 0.98 (in this case Q_4 would be $1.90 - 0.98 = 0.92$).

Option D: The four seasonal indices add up to 4, which represents 400%, but does not represent being *above* the average sales.

Question 21

Answer: ***B***

Explanatory notes

$1/1.20 = 0.83$ or $100/1.20 = 83.3\%$

This means we need to decrease the Q_1 sales by 17% ($100 - 83\% = 17\%$).

EXAM 2

Question 1a.

Worked solution

A decreasing trend only

Mark allocation: 1 mark

- 1 mark for correct answer

Note: A mark will not be awarded if additional information is provided, such as reference to seasonality or variation, with one exception: a mention of random or irregular fluctuations is acceptable.

Question 1b.

Worked solution

Step 1: Find the average for the 10-month period

$$\frac{282 + 252 + 265 + 257 + 262 + 224 + 218 + 198 + 172 + 165}{10}$$

$= 229.5$

Step 2: Find the seasonal index, by dividing the actual figure for July by the average

$$\frac{218}{229.5} = 0.949891 = 0.95$$

The seasonal index for July is 0.95, as required.

Mark allocation: 2 marks

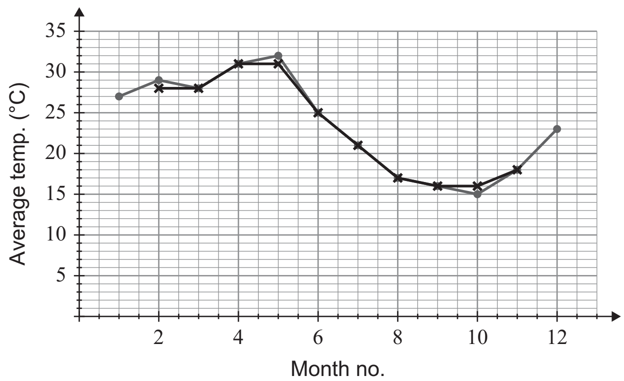
- 1 mark for showing calculation of average
- 1 mark for use of formula: $\text{seasonal index} = \frac{\text{actual}}{\text{average}}$

Question 2a.

Worked solution

Three-median smoothing requires finding the median of three values, written in order.

In this case: 15 18 23



Mark allocation: 1 mark

- 1 mark for correct plot of point (11, 18) and line joining point

Question 2b.

Worked solution

$$\text{deseasonalised} = \frac{\text{actual}}{\text{seasonal index}}$$

$$\text{deseasonalised} = \frac{16}{0.68}$$

$$\text{deseasonalised} = 23.5$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3a.

Worked solution

To find the three-mean smoothed value, use the two values either side of the original September value.

$$\frac{38 + 78 + 49}{3} = 55$$

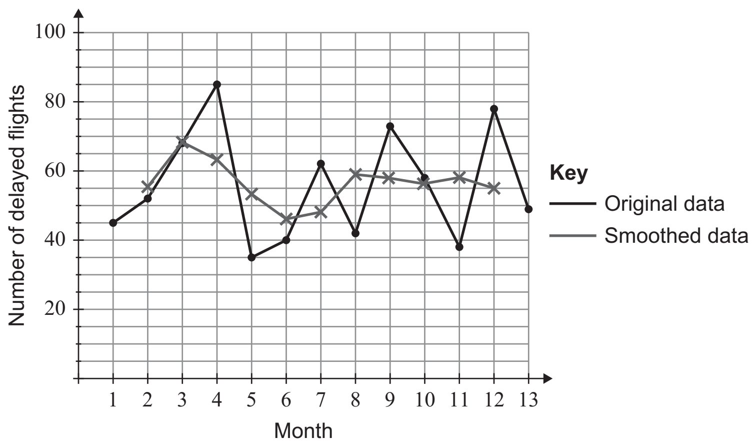
The answer is 55.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3b.

Worked solution



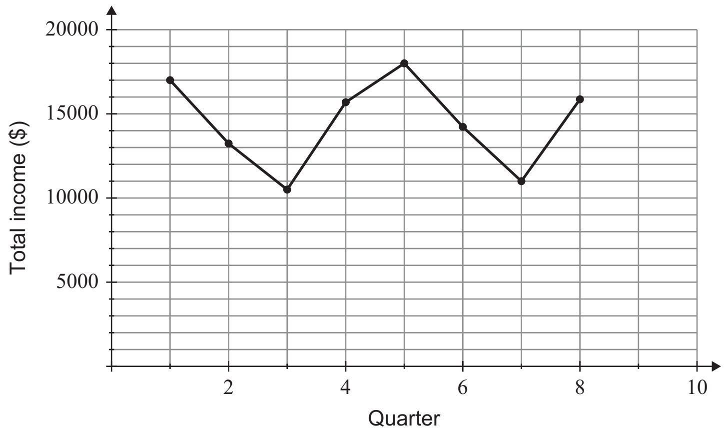
Mark allocation: 1 mark

- 1 mark for correct answer

Note: If your answer to part a. was incorrect you can still get the mark for **part b.** by plotting your value for September 2018 correctly.

Question 4a.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct points connected to the line

Question 4b.

Worked solution

Start by deseasonalising each of the values in Table 1.

All values have been shown below. However, you only need to calculate the last three seasons.

$$\text{deseasonalised value} = \frac{\text{actual value}}{\text{seasonal index}}$$

	Total income ($)			
Year	**Summer**	**Autumn**	**Winter**	**Spring**
2017	$\frac{17064}{1.21} = 14102.48$	$\frac{13236}{0.95} = 13932.63$	$\frac{10548}{0.75} = 14064$	$\frac{15732}{1.09} = 14433.03$
2018	$\frac{17976}{1.21} = 14856.20$	$\frac{14256}{0.95} = 15006.32$	$\frac{11064}{0.75} = 14752$	$\frac{15876}{1.09} = 14565.14$

Start Summer 2017 as 1, Autumn 2017 as 2, etc.

The completed table is shown below:

Season	Deseasonalised data
1	14 102.48
2	13 932.63
3	14 064.00
4	14 433.03
5	14 856.20
6	**15 006.32**
7	**14 752.00**
8	**14 565.14**

Mark allocation: 1 mark

- 1 marks for correct values

Question 4c.

Worked solution

Use the values to find linear equation, $y = a + bx$, using CAS.

deseasonalised income = 13 896.89 + 126.02 × season number

Mark allocation: 1 mark

- 1 marks for correct equation

Question 5a.

Worked solution

$$\frac{\frac{3.2 + 1.5}{2} + \frac{1.5 + 2.4}{2}}{2} = 2.15$$

They may also be calculated separately:

$\frac{3.2 + 1.5}{2} = 2.35$ $\quad$ $\frac{1.5 + 2.4}{2} = 1.95$

Centring: $\frac{2.35 + 1.95}{2} = 2.15$

Mark allocation: 2 marks

- 1 mark for using a correct method (whether or not numbers are correct)
- 1 mark for calculating the correct values and final total

TIP

» 'Show that' questions require you to show working out in order to come to the answer provided; you cannot use the desired solution in your working out to prove it is correct.

Question 5b.

Worked solution

$SI = \frac{actual}{average}$

The missing values are:

3.8/3.628 = 1.047... = 1.05

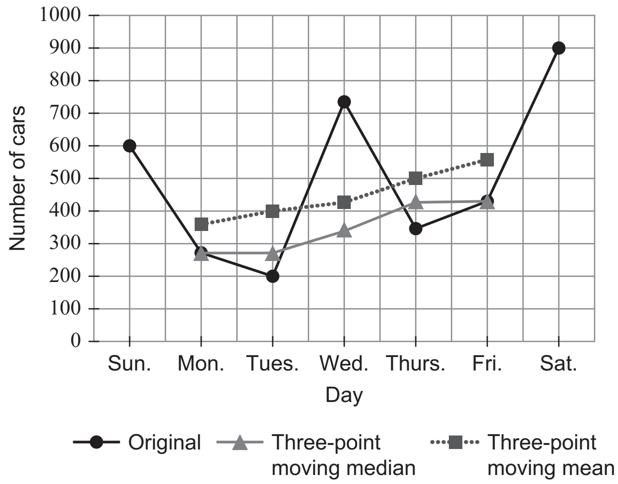
2.6/3.628 = 0.716... = 0.72

Mark allocation: 1 mark

- 1 mark for both values of 1.05 and 0.72

Question 6a.i.

Worked solution



Mark allocation: 2 marks

- 2 marks for both lines finished correctly

OR

- 1 mark if one line only is correct

Question 6a.ii.

Worked solution

The moving mean will be impacted by outliers or extreme values, which do not impact the moving median.

Mark allocation: 1 mark

- 1 mark for identifying that the moving mean will be impacted by outliers, or similar

Question 6b.i.

Worked solution

SI = 12 500/21 908 = 0.57

Mark allocation: 1 mark

- 1 mark for 12 500/21 908

Question 6b.ii.

Worked solution

14 300/0.57 = 25 087.7 = 25 100

Mark allocation: 1 mark

- 1 mark for 25 100

Area of Study 2 Recursion and financial modelling – Recurrence relations, modelling linear and geometric growth and decay, simple interest and flat rate, unit cost depreciation, compound interest investments and loans, reducing balance depreciation

EXAM 1

Question 1

Answer: ***C***

Explanatory notes

Step 1:

Calculate the total depreciation on the motorbike over the three years.

$23\,200 - 16\,990 = 6210$

Step 2:

Divide this by the number of kilometres travelled over the three years to find the cost per kilometre.

$\frac{6210}{22\,500 \times 3} = 0.092$

The motorbike depreciates by $0.092 per kilometre.

Step 3:

Write the information in a rule for the value of the motorbike.

The general rule for unit cost depreciation is:

$V_n = V_0 - nD$

where D is the depreciation per unit and n is the number of units.

Therefore, our rule is:

$V_n = 23\ 200 - 0.092n$

Question 2

Answer: ***D***

Explanatory notes

The printer has depreciated by $1200 - 765 = 435$ total over the three years.

This is equal to $435 \div 3 = 145$ each year.

The cost per page printed is $145 \div 1740 = \$0.08333$ per page.

Question 3

Answer: ***B***

Explanatory notes

$A_0 = 3$

$A_1 = -2 \times 3 + 3 = -6 + 3 = -3$

$A_2 = -2 \times -3 + 3 = 6 + 3 = 9$

$A_3 = -2 \times 9 + 3 = -18 + 3 = -15$

Therefore, the first four terms written in a sequence are:

3, −3, 9, −15, …

Question 4

Answer: ***C***

Explanatory notes

Option A: This represents a combination of geometric decay and linear growth.

Option B: This represents geometric growth only.

Option C: This represents geometric decay only.

Option D: This represents a combination of geometric growth and linear growth.

Question 5

Answer: ***C***

Explanatory notes

The graph shows an increase in the value, and is therefore an investment. This excludes options A, B and D.

Option C is correct because it shows an investment, increasing by slightly more each month. This shows a compounding interest account.

Question 6

Answer: ***C***

Explanatory notes

$22\,000 \times 0.14 \times 5 \text{ years} = \$15\,400 \text{ total depreciation}$

$26\,000 + 15\,400 = \$41\,400 \text{ original price}$

Alternatively, use Solve:

$$\text{solve}(26000 = x - 5 \cdot 0.14 \cdot 22000, x) \qquad x = 41400.$$

Question 7

Answer: ***D***

Explanatory notes

The general form of a rule for reducing balance depreciation is:

$V_n = V_0 \times (1 - r/100)^n$

So:

$V_6 = 124\,000 \times (1 - 5.6/100)^n$

$V_6 = 124\,000 \times (0.944)^6$

Question 8

Answer: ***D***

Explanatory notes

The general rule is in the form $V_n = \left(1 - \frac{r}{100}\right)^n \times V_0$.

This becomes $V_n = \left(1 - \frac{9.4}{100}\right)^n \times 4500$ or $V_n = 0.906^n \times 4500$ as shown in option D.

Options B and C use the wrong notation of V_{n+1}, which is not used in a rule.

Option A incorrectly uses a compounding value of $\left(1 + \frac{9.4}{100}\right)$ instead of $\left(1 - \frac{9.4}{100}\right)$.

TIP

» Look for the difference in notation for a rule and a recurrence relation.

Question 9

Answer: ***D***

Explanatory notes

The recurrence relation for this reducing balance depreciation would be:

$V_0 = 4500, V_{n+1} = 0.906\,V_n$

So after the third year, the value would be as follows:

$V_0 = 4500$

$V_1 = 0.906 \times 4500 = 4077$

$V_2 = 0.906 \times 4077 = 3693.76$

$V_3 = 0.906 \times 3693.76 = 3346.55$

Option A shows the calculation for the value at the end of the second year.

Option B uses the flat rate depreciation method instead of reducing balance depreciation.

Option C uses a compounding decimal, which indicates geometric growth instead of decay.

Question 10

Answer: ***B***

Explanatory notes

After three years, it had depreciated by $(6000 - 4350) = \$1650$, which is \$550 per year.

After four years $\times\ 550 = \$2200$ depreciation.

TIP

» Read the question carefully to help avoid errors that come from making assumptions on what you think the question is going to be. For example, students may assume they are required to find the value after four years of depreciation rather than how much it depreciated by.

Question 11

Answer: ***D***

Explanatory notes

$T_0 = 6$

$T_1 = 3 \times 6 = 18$

$T_2 = 3 \times 18 = 54$

$T_3 = 3 \times 54 = 162$

$T_4 = 3 \times 162 = 486$

$T_5 = 3 \times 486 = 1458$

Question 12

Answer: ***B***

Explanatory notes

The form of the rule shows that it is a flat rate depreciation model, as they are always in the form of $V_n = V_0 - Dn$, where D is the dollar amount of depreciation.

Options A and D: A rule for reducing balance depreciation is of the form $V_n = R^n \times V_0$.

Option C: A rule does not exist for reducing balance loans and this is not relevant for depreciation problems.

EXAM 2

Question 1a.

Worked solution

Annual depreciation is:

$$\frac{37\,000 - 29\,600}{5} = 1\,480$$

Recurrence relation is: $C_0 = 37\,000, \qquad C_{n+1} = C_n - 1\,480$

Mark allocation: 2 marks

- 2 marks for correct recurrence relation, including clear indication of $C_0 = 37\,000$

Note: If relation is not correct, 1 mark may be given for an accurate calculation of the annual depreciation of $1480.

Question 1b.

Worked solution

$1480/4000 = \$0.37$

Therefore, the unit cost depreciation is 37 cents per kilometre.

Mark allocation: 1 mark

- 1 mark for correct answer of $0.37 or 37 cents; in this case, a unit is required

Question 2a.

Worked solution

For this question, you must write calculations to show that the value is $24 960.

Substituting 20 000 into the rule:

$V_n = 24\,990 - 0.0015n$

$V_n = 24\,990 - 0.0015 \times 20\,000$

$V_n = 24\,960$

Mark allocation: 1 mark

- 1 mark for a substitution into the rule

Question 2b.

Worked solution

The unit cost depreciation is $0.0015 according to the rule.

This is written in dollars, and so this must be multiplied by 100 to convert to cents.

$0.0015 \times 100 = 0.15$ cents

Mark allocation: 1 mark

- 1 mark for correct answer, written in cents

Question 2c.

Worked solution

Find the total depreciation over three years:	$24\,990 - 23\,191 = 1799$
Divide this amount by 3 to find the depreciation in one year:	$\frac{1799}{3} = 599.67$
Calculate this amount as a percentage of the original amount:	$\frac{599.67}{24\,990} \times 100 = 2.39964\%$ $\therefore \approx 2.4\%$

Mark allocation: 1 mark

- 1 mark for correct answer

Note: An unrounded answer will *not* be accepted.

Question 2d.

Worked solution

The general relation for reducing balance depreciation is:

$V_0 =$ initial value, $V_{n+1} = R \times V_n$, where $R = 1 - \frac{r}{100}$

In this question we have been given the following: $V_0 = 24\,990$, $r = 5.8\%$ per annum

$$R = 1 - \frac{r}{100}$$

This means that we can work out R: $R = 1 - \frac{5.8}{100}$

$$R = 0.942$$

Therefore, our recurrence relation is: $A_0 = 24\ 990$, $A_{n+1} = 0.942 \times A_n$

Mark allocation: 2 marks

- 1 mark for accurate form of relation, including A_0
- 1 mark for correct value of R included in relation

Question 3a.

Worked solution

$R = 1 - \frac{r}{100}$

$R = 1 - \frac{18}{100}$

$R = 0.82$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3b.

Worked solution

value $= 5000 \times (0.82)^5$

value $= 1853.7$

depreciation $= 5000 - 1853.7$

depreciation $= \$3146.30$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3c.

Worked solution

After one year, the photocopier has depreciated by \$900. In one year, the photocopier produces $35\,000 \times 12 = 420\,000$ pages.

The depreciation per unit is $\frac{900}{420\,000} = \$0.002143$

This is 0.2 cents per copy.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4a.

Worked solution

The initial amount comes from V_0 in the recurrence relation.

This is therefore \$6000.

Mark allocation: 1 mark

- 1 mark for correct answer; dollar sign not required

Question 4b.

Worked solution

The annual interest is \$216.

As a percentage of the initial amount this is:

$\frac{216}{6000} \times 100 = 3.6\%$ per annum

Mark allocation: 1 mark

- 1 mark for correct answer of 3.6%

Question 4c.i.

Worked solution

The first box is V_0 and is therefore 6000.

The second box is the annual interest, and is therefore 216.

Mark allocation: 1 mark

- 1 mark for both values in the correct order, 6000 then 216

Question 4c.ii.

Worked solution

$V_5 = 6000 + 216 \times 5$

$V_5 = \$7080$

Mark allocation: 1 mark

- 1 mark for correct answer; dollar sign not required

Question 4d.

Worked solution

$V_0 = 6000$

$V_1 = 1.054 \times V_0 = 1.054 \times 6000 = 6324$

$V_2 = 1.054 \times V_1 = 1.054 \times 6324 = 6665.496 = 6665.50$

Mark allocation: 1 mark

- 1 mark for showing two calculations, the second of which results in \$6665.50

TIP

» Make sure you show that the previous value obtained is used in the working out to obtain the next value when asked to show recursive calculations

Question 5a.

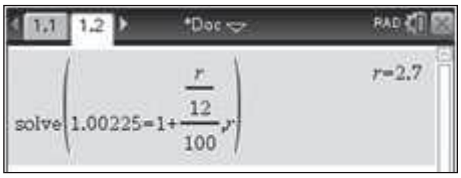
Worked solution

$R = 1 + \frac{(r/12)}{100}$

$1.00225 - 1 = 0.00225$

$0.00225 \times 12 \times 100 = 2.7\%$

Or use the CAS calculator to solve this equation for r.



The annual interest rate is 2.7%.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5b.i.

Worked solution

$V_1 = 1.00225 \times 8000 = 8018$

$V_2 = 1.00225 \times 8018 = 8036.04$

Mark allocation: 1 mark

- 1 mark for both lines of working out showing recursion

Question 5b.ii.

Worked solution

We can continue to use recursion to find when the value of the investment first goes above \$8500.

Using the CAS calculator to solve is another method to find this.

$$\text{solve}(8500=(1.00225)^n \cdot 8000,n)$$
$$n=26.9745773192$$

This will occur after 27 months.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5c.

Worked solution

a = the initial value, therefore $a = 8000$

$b = R$, in this case 1.00225

Therefore, the rule is $V_n = 8000 \times 1.00225^n$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5d.

Worked solution

The interest that Alison's account has earned in 12 months is $8219 - 8000 = 219$.

$$I = \frac{Prt}{100}$$

$$219 = \frac{8000 \times r \times 1}{100}$$

$$\therefore r = \frac{219}{8000} \times 100$$

$$r = 2.74\%$$

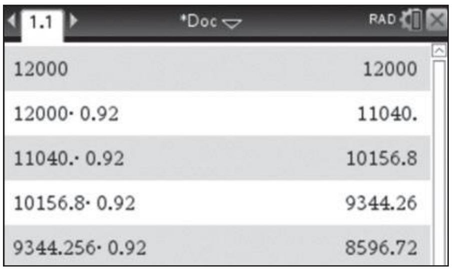
Mark allocation: 1 mark

- 1 mark for correct answer

Question 6a.

Worked solution

First, find the value of the equipment after four years. Use your CAS for this.



The value after four years is $8596.72

$V_5 = 0.92 \times 8596.72 = 7909.98$

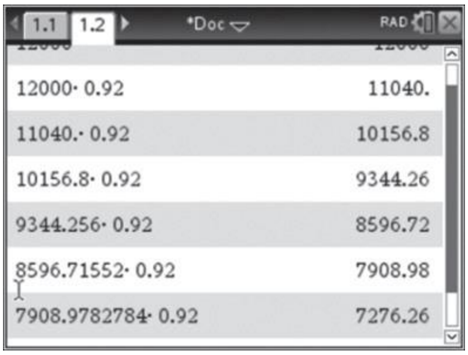
Mark allocation: 1 mark

- 1 mark for calculation shown above

Question 6b.

Worked solution

Using recursion on the CAS or using Solve are two methods that can be used to determine how long it takes.



$\text{solve}(12000\cdot(0.92)^n = 7500, n)$

$n = 5.63677812441$

The value of the equipment first falls below $7500 after six years.

Mark allocation: 1 mark

- 1 mark for correct answer of six years

TIP

» When asked 'how long', you often need to round up your answer to ensure that the time periods are sufficient to reach the value required

Question 6c.i.

Worked solution

The equipment has depreciated by $12\,000 - 10\,850 = \$1150$

The unit cost of depreciation is: $\frac{1150}{1400} = \$0.82$ or 82 cents

Mark allocation: 1 mark

- 1 mark for correct answer of 82 cents

Question 6c.ii.

Worked solution

$V_n = V_0 - nD$

So $V_n = 12\,000 - 0.82n$

Mark allocation: 1 mark

- 1 mark for correct rule

Question 7a.

Worked solution

$0.034 \times 80\,000 = 2720$

Mark allocation: 1 mark

- 1 mark for the correct answer of $2720

Question 7b.

Worked solution

$V_0 = 80\,000\ V_{n+1} = \left(1 + \frac{r}{100}\right) \times V_n$

$V_0 = 80\,000\ V_{n+1} = \left(1 + \frac{3.25}{100}\right) \times V_n$

$V_0 = 80\,000\ V_{n+1} = 1.0325\ V_n$

Mark allocation: 1 mark

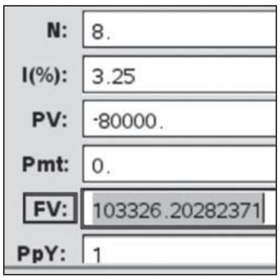
- 1 mark for the correct notation and values

Question 7c.

Worked solution

Using the rule: $V_8 = 1.0325^8 \times 80\,000 = 103\,326.202\ldots$

Using the Financial solver:



Mark allocation: 1 mark

- 1 mark for the correct answer of $103 326.20

Question 7d.

Worked solution

Using recursion to compare the values each year:

80000.	80000.
82720.	82600.
85440.	85284.5
88160.	88056.2…
90880.	90918.0

Alternatively, Jo's investment:

80000	80000
80000+2720	82720
82720+2720	85440
85440+2720	88160
88160+2720	90880

Anthony's investment:

80000	80000
80000 · 1.0325	82600.
82600. · 1.0325	85284.5
85284.5 · 1.0325	88056.24625
88056.24625 · 1.0325	90918.0742531

After four years, Anthony's investment is higher.

Mark allocation: 1 mark

- 1 mark for the correct answer of four years

Question 8a.i.

Worked solution

$V_1 = 3875 - 410 = 3465$

$V_2 = 3465 - 410 = 3055$

Mark allocation: 1 mark

- 1 mark for the calculations shown above, where the answer for one year is used in the working out for the next (the notation '$V_1 =$' is not required)

Question 8a.ii.

Worked solution

$410/3875 \times 100 = 10.58\%$

Mark allocation: 1 mark

- 1 mark for correct answer of 10.58%

Question 8b.

Worked solution

$3875 - 800 = 3075$

6% per annum reducing balance depreciation rate means 94% of the value remains: 0.94

$3075 \times 0.94^3 = 2554.05 = 2554$

OR

Set up the recursion relation $V_0 = 3075$ $V_{n+1} = 0.94 \times V_n$ and run it on your calculator.

Mark allocation: 1 mark

- 1 mark for correct answer of $2554

Area of Study 2 Recursion and financial modelling – Reducing balance loans, annuities, perpetuities, amortisation tables

EXAM 1

Question 1

Answer: ***D***

Explanatory note

$V_0 = 32\,000, \qquad V_{n+1} = 1.018 \times V_n - 785$

Using this recurrence relation:

32000 · 1.018–785	31791.
31791. · 1.018–785	31578.238
31578.238 · 1.018–785	31361.646284
31361.646284 · 1.018–785	31141.1559171

Question 2

Answer: ***C***

Explanatory notes

Calculate the interest earned on the loan:

$$\frac{(5.4/12)}{100} \times 274\,724.38 = 1\,236.26$$

Principal reduction is the difference between the payment and the interest earned:

$1375 - 1236.26 = 138.74$

Question 3

Answer: ***D***

Explanatory notes

The general relation for a reducing balance loan is:

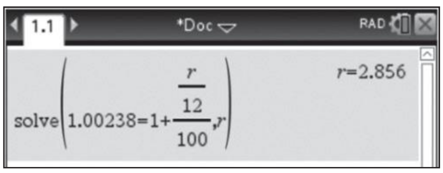
$V_0 =$ the principal, $\qquad V_{n+1} = RV_n - D$

where $R = 1 + \dfrac{r}{100}$

r is the annual interest rate, and in our relation we have a monthly calculation.

Therefore, $R = 1 + \dfrac{r/12}{100}$

In this relation, $R = 1.00238$. Use this to solve for r.



Therefore $r = 2.86\%$

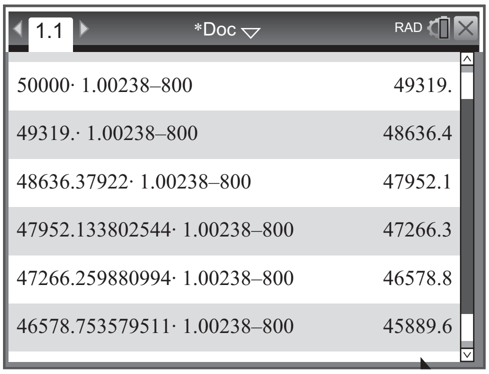
Question 4

Answer: ***C***

Explanatory notes

Use recursion to find the value of the loan after six months.

This is best done using a CAS calculator.



Question 5

Answer: ***D***

Explanatory notes

Options A and B are incorrect. The decimal of 0.996 does not represent compound interest, which is required in reducing balance loans. The value of R must be calculated using:
$R = 1 + \frac{r/n}{100}$ not $R = 1 - \frac{r/n}{100}$.

Option C (and B) is incorrect as the payment is added to balance of the loan, instead of having it subtracted; therefore, it does not reduce the balance of the loan at all.

Question 6

Answer: ***C***

Explanatory notes

The principal reduction between the fifteenth and sixteenth payment is:
$346\,117.98 - 344\,788.03 = 1329.95$

The payment is \$2700; therefore, the difference is the interest paid: $2700 - 1329.95 = 1370.05$

Question 7

Answer: ***D***

Explanatory notes

Compound interest can be modelled using the following general relation:

$V_0 = \text{initial investment}, \qquad V_{n+1} = R \times V_n$

where $R = 1 + \frac{r}{100}$

In this case, the interest is calculated monthly, and so we need to convert the interest to a monthly rate.

$3.8 \div 12 = 0.3\dot{1}\dot{7}$

Therefore:

$R = 1 + \frac{0.317}{100} = 1.003\,17$

We must also remember to add the additional payment to the recurrence relation, therefore leading to the answer:

$V_0 = 10\,000, \qquad V_{n+1} = 1.003\,17 \times V_n + 250$

TIP

» As this is an investment, the payment that Jessica is making is added to the overall value. If this was a loan, we would subtract this payment. When we have compound interest (or a reducing balance loan) we may need to change the annual interest rate. The following information is useful when doing this:

compounding bi-yearly	divide by 2
compounding quarterly	divide by 4
compounding monthly	divide by 12
compounding fortnightly	divide by 26
compounding weekly	divide by 52

Question 8

Answer: ***A***

Explanatory notes

In this amortisation table, the 'balance of investment' is calculated by adding the 'principal addition' to the previous balance.

Therefore:

$$\text{previous balance} + 503.83 = 78\,740.33$$

We can therefore subtract the 'principal addition' from the current balance, to find the previous balance.

$\text{previous balance} = 78\,740.33 - 503.83 = 78\,236.50$

Question 9

Answer: ***D***

Explanatory notes

Step 1:

Calculate the interest earned.

$$79\,245.92 \times \frac{\frac{4.2}{12}}{100} = 277.36$$

Remember, we need to divide the interest rate by 12, as this is an annual interest rate and our interest is calculated monthly.

Step 2:

Calculate the principal addition, by subtracting the previous balance from the current balance.

$79\,753.28 - 79\,245.92 = 507.36$

Step 3:

Calculate the interest as a percentage of the principal addition.

$= \frac{277.36}{507.36} \times 100 = 54.6673\%$

Question 10

Answer: ***D***

Explanatory notes

This recurrence relation is an example of a combination of geometric and linear growth, and we can therefore not consider option A or option C, which both represent decay instead.

Option B is an example of linear growth only, so this is incorrect.

If we substitute the value of one year into the recurrence relation, we can get an answer of $1600.

$V_0 = 1000, \qquad V_1 = 1.5V_0 + 100 = 1.5 \times 1000 + 100 = 1600$

Option D correctly graphs this point.

TIP

» **A linear sequence will always make a straight-line graph; a geometric sequence will make a curved graph.**

Question 11

Answer: ***C***

Explanatory notes

Use your calculator to find the sequence. As it is a financial model, the principal amount of $2500 is not counted as the first term.

2500	2500
2500·1.062−200	2455.
2455.·1.062−200	2407.21
2407.21·1.062−200	2356.45702
2356.45702·1.062−200	2302.55735524
2302.55735524·1.062−200	2245.31591126

Question 12

Answer: ***C***

Explanatory notes

Annual interest rate (for quarterly compounding periods) $= \frac{\text{amount of interest}}{\text{previous balance}} \times 100 \times 4 \text{ quarters}$

$= \frac{1300}{100000} \times 100 \times 4 \text{ quarters} = 5.2\%$

Question 13

Answer: ***C***

Explanatory notes

Principal reduction = $\$99\,392.17 - \$99\,184.27 = \$207.90$

$\frac{207.90}{1500} \times 100 = 13.86\%$

Question 14

Answer: ***D***

Explanatory notes

Use the monthly interest rate:

$\frac{0.037}{12} \times 80\,000 = 246.6666...$

The seven-year time frame is not relevant, as a perpetuity does not change its balance, so the monthly payment remains the same throughout.

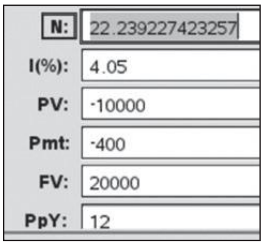
TIP

» Remember that both interest-only loans and perpetuities will retain the same balance throughout.

Question 15

Answer: ***C***

Explanatory notes



As the time taken is more than 22 months (22.239) we need to round up to 23 months.

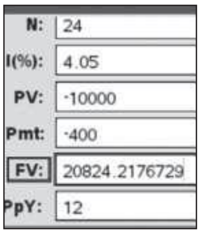
If we rounded to 22 months only, the balance would have fallen slightly short of the required amount.

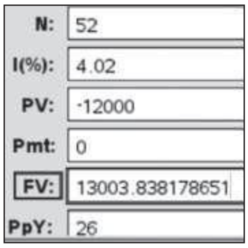
Question 16

Answer: ***D***

Explanatory notes

After two years, Will has \$20 824.22 and Linda has \$13 003.84.





Question 17

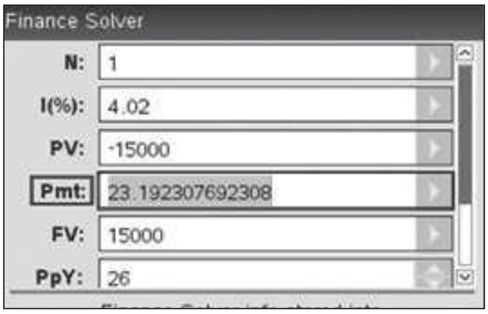
Answer: ***D***

Explanatory notes

This is a perpetuity, so the present value (PV) and future value (FV) need to remain the same after each single withdrawal.

In order to keep it from decaying/reducing, the withdrawal has to be \$23 or less.

As it asks for the maximum amount, \$23 is the correct answer, instead of options A, B or C.



Alternatively:

$$\frac{0.0402}{26} \cdot 15000 \qquad 23.1923076923$$

Question 18

Answer: ***B***

Explanatory notes

$0.0212/12 \times 24\,000 = 42.40$ per month

$42.4 \times 12 \times 10$ years $= 5088$

TIP

» **Remember that interest does not 'compound' in the same way for perpetuities as it does for other financial models – the interest is withdrawn before it can grow the balance of the perpetuity.**

Question 19

Answer: ***A***

Explanatory notes

Using the monthly interest rate, the interest is calculated to be \$1925 on a \$700 000 balance. From there, the principal reduction can be calculated to be \$1799, which means the new balance is \$698 201.

$\frac{0.033}{12} \cdot 700000$	19.25
3724−1925	1799
700000−1799	698201

Question 20

Answer: ***D***

Explanatory notes

$\frac{1808.91}{3724} \cdot 100$	48.5743823845

EXAM 2

Question 1a.

Worked solution

We need to calculate six iterations of our recurrence relation to find the value of Jonathan's loan.

Working using the CAS calculator is shown below.

1.4 1.5 1.6 *Doc	RAD
15000	15000
15000· 1.0022–200	14833.
14833.· 1.0022–200	14665.6
14665.6326· 1.0022–200	14497.9
14497.89699172· 1.0022–200	14329.8
14329.792365102· 1.0022–200	14161.3
14161.317908305· 1.0022–200	13992.5

The value of the loan after six months is \$13 992.47.

Note that the calculator will often round this value and we need to check what the unrounded value is. This can be avoided by ensuring that your float setting on the calculator does not restrict the answer.

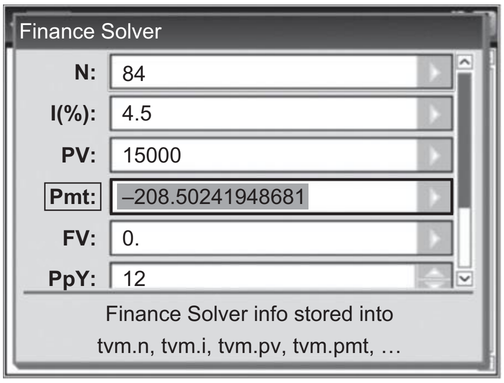
Mark allocation: 1 mark

- 1 mark for correct answer of \$13 992.47, dollar sign not required

Question 1b.

Worked solution

We will need to use the Finance solver in the CAS calculator to find Jonathan's monthly payment.



Jonathan's monthly repayment will be $208.50.

Mark allocation: 1 mark

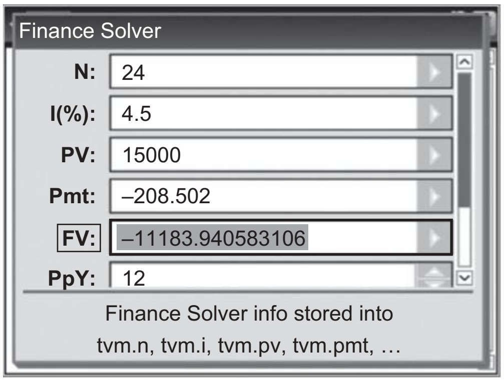
- 1 mark for correct answer

Note: A dollar sign is not required; negative answers are not accepted

Question 1c.

Worked solution

First, we must find the value of Jonathan's loan after two years:



Jonathan still owes $11 183.94 on his loan.

We must now find how much Jonathan has paid off his loan in the first two years:

$15\,000 - 11\,183.94 = 3816.06$

Next, we calculate how much money Jonathan has put into his loan:

$208.50 \times 24 = 5004$

The interest that Jonathan has paid is the difference between the amount of money he has contributed, and the amount that he has paid off the principal of his loan:

$5004 - 3816.06 = 1187.94$

Jonathan has paid $1187.94 interest in the first two years.

Mark allocation: 2 marks

- 2 marks for correct answer

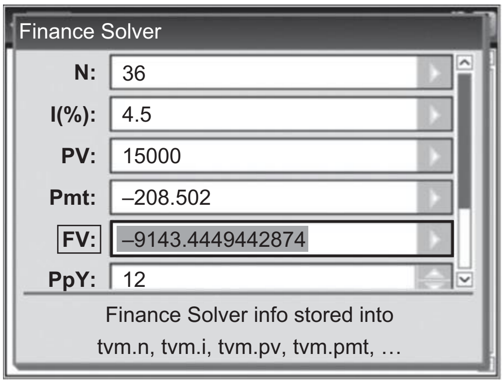
Note: 2 marks can be awarded if the answer to part b. was incorrect but has been used correctly in all parts of this question. This is a consequential mark, so working must be shown.

Note: If the answer is incorrect, 1 mark may be given for a subtraction of any value from 5004 and 1 mark can be given for a correct calculation of the value after two years.

Question 1d.

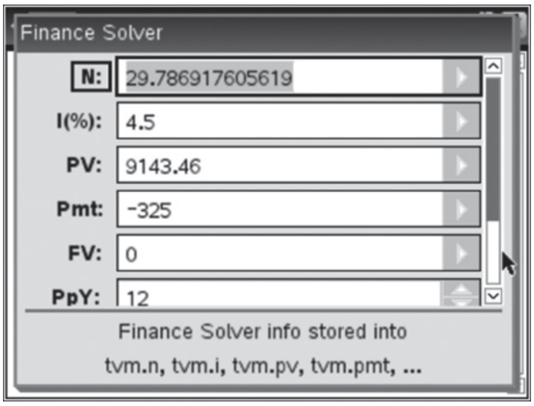
Worked solution

We must first find the value of Jonathan's loan after three years at the original conditions.



Jonathan still owes $9143.44 after three years.

This now becomes our new principal. We also alter our *Pmt* amount and FV becomes 0, as below:



We can see that it will take Jonathan a further 30 months (rounding up) to pay off his loan.

With the original arrangement, Jonathan would have taken another four years, or 48 months, to pay off his loan.

With the new payment amount, he will save 18 months on his loan.

Mark allocation: 2 marks

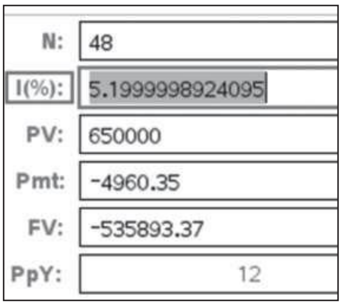
- 2 marks for an answer of 18, with no working required

Note: If incorrect, 1 mark can be given for a correct FV value after three years, if shown and clearly defined.

Question 2a.

Worked solution

Use the finance solver in your CAS to find the annual interest rate.



The annual interest rate is 5.2%.

Mark allocation: 1 mark

- 1 mark for correct answer of 5.2%

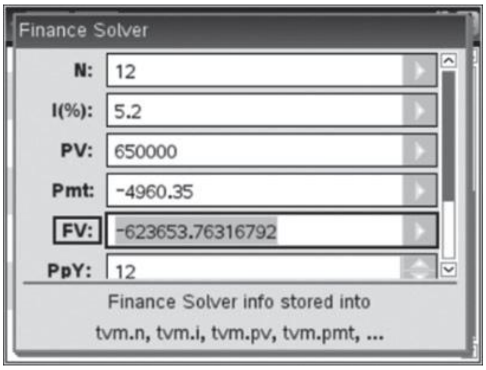
TIP

» **Remember that money going into the bank is a negative value in your finance solver. Money coming out of the bank (into your pocket) is a positive value in your finance solver.**

Question 2b.

Worked solution

Step 1 – Find the value of Jason's loan (FV) after one year. Again, use the finance solver in the CAS.



The value of Jason's loan after one year is \$623 653.76.

Step 2 – Calculate the amount that he has paid into his loan over the year.

$4960.35 \times 12 = 59\,524.20$

Step 3 – Calculate the amount of his loan that Jason has paid off.

$650\,000 - 623\,653.76 = 26\,346.24$

Step 4 – Calculate the amount of interest paid over the year.

interest $= 59\,524.20 - 26\,346.24 = 33\,177.96$

Mark allocation: 2 marks

- 2 marks for the correct value of $33 177.96

OR

- 1 mark for calculating the amount paid into the loan
- 1 mark for calculating the amount paid off the loan

Question 2c.

Worked solution

$G_0 = 535\,893.37,\ G_{n+1} = 1.004\,G_n - 4327.56$

The value of R is found by dividing the annual interest rate by 12.

$R = 1 + \frac{4.8/12}{100}$

Mark allocation: 1 mark

- 1 mark for correct recurrence relation

Question 3a.

Worked solution

$4.1/4 = 1.025$

Mark allocation: 1 mark

- 1 mark for the correct answer of 1.025

Question 3b.

Worked solution

$73\,687.19 - 2925.31 = 70\,761.88$

Mark allocation: 1 mark

- 1 mark for the correct answer of $70 761.88

TIP

» **When dealing with money, round to the nearest cent (2 decimal places) unless stated otherwise.**

Question 3c.

Worked solution

interest $= 79\,628.07 \times 0.01025 = 816.19$

$3141.19 - 816.19 = 2325$

Mark allocation: 1 mark

- 1 mark for a correct answer

Note: This is a consequential mark: any answer showing the calculation of the product of 79 628.07 × answer from part a. will be awarded.

Question 3d.

Worked solution

$$\begin{aligned}\frac{\text{amount of interest}}{\text{principal increase}} \times 100 &= \frac{725.31}{2925.31} \times 100 \\ &= 24.794 \\ &= 24.8\%\end{aligned}$$

Mark allocation: 1 mark

- 1 mark for the correct answer of 24.8%

Area of Study 2 Recursion and financial modelling – Using the finance solver

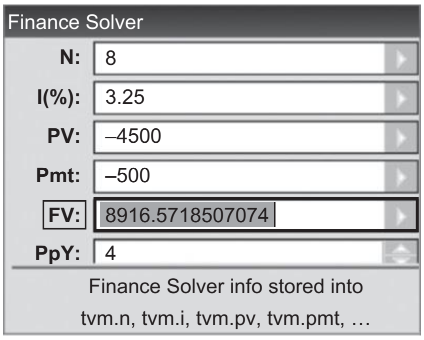
EXAM 1

Question 1

Answer: ***C***

Explanatory notes

Using the finance solver in the CAS calculator, find the value of Christine's investment after two years.



Use this value to find the interest earned on the account, by subtracting the money that Christine has added to the investment herself:

$\text{interest} = \text{value} - \text{principal} - \text{payments}$

$\text{interest} = 8\,916.57 - 4\,500 - 8 \times 500$

$\text{interest} = 416.57$

TIP

» Carefully read the question – do not assume that interest or payments are added monthly. In this case, we have payments and interest paid quarterly. This means that PpY and CpY are 4.

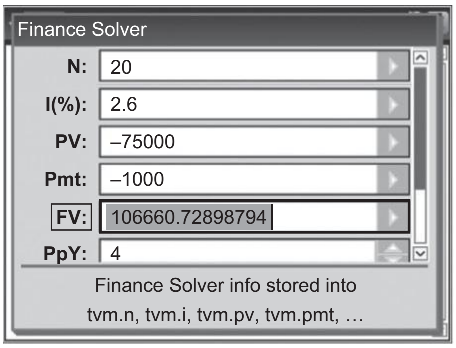
Question 2

Answer: ***D***

Explanatory notes

Step 1 – Calculate the amount in Jessica's savings after five years.

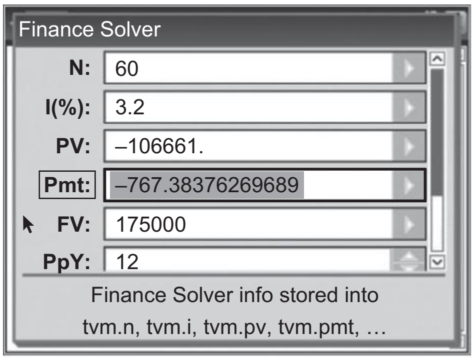
Use your CAS calculator, Finance solver.



The value in Jessica's savings after five years is $106 660.73.

Step 2 – Calculate Jessica’s payments for the final five years.

Move the FV from previous step into PV. Change CpY and PpY to 12.



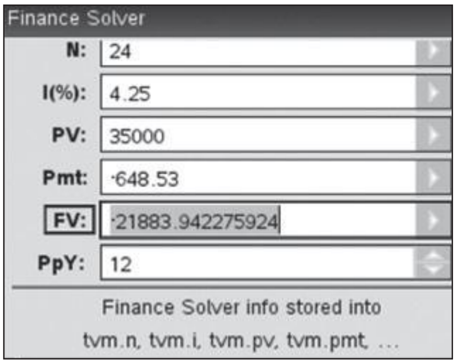
Jessica’s investment will need to be $767.38 per month.

Question 3

Answer: ***C***

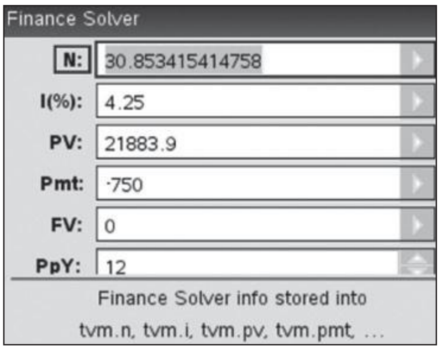
Explanatory notes

Start by calculating the value of Peter’s loan after two years of monthly payments at $648.53.



Now change the payment to $750, and calculate the time that it will take to repay the loan under these new conditions.

Notice that we will move the FV into PV for these new conditions.



It will now take 31 months to repay the remaining balance of this loan.

In total, this is $24 + 31 = 55$ months.

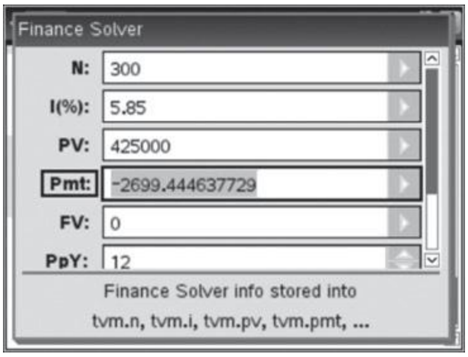
Therefore, Peter has saved 5 months on his loan period.

Question 4

Answer: ***C***

Explanatory notes

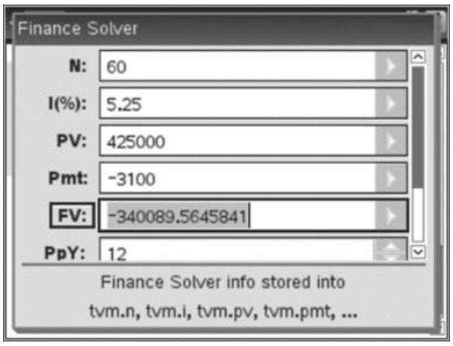
Using the Finance solver in your CAS calculator, find the value of the payment.



Question 5

Answer: ***C***

Explanatory notes



The amount that Kate has paid into her loan is $5 \times 12 \times 3100 = 186\,000$.

The amount that Kate has actually reduced the balance of her loan by is $425\,000 - 340\,089.56 = 84\,910.40$.

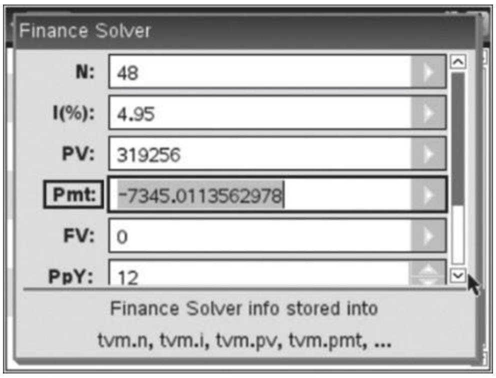
Therefore, the interest that Kate has paid is $186\,000 - 84\,910.40 = 101\,090$.

Question 6

Answer: ***D***

Explanatory notes

Use the finance solver.



With another 4 years to go (from 6 to 10 years), repayments of $7345 are required.

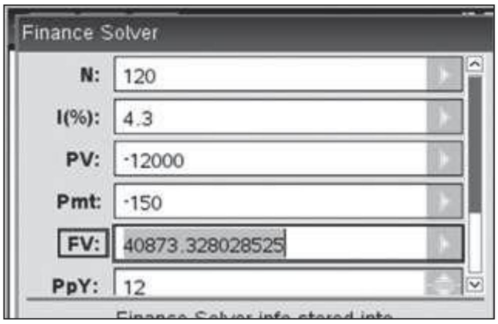
Question 7

Answer: ***A***

Explanatory notes

Original plan:

Value after 10 years: $40 873.33

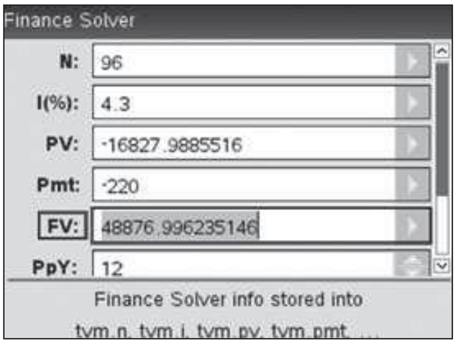


New plan:

Value after two years: $16 827.988…

Value after a further eight years:

$48 876.996…



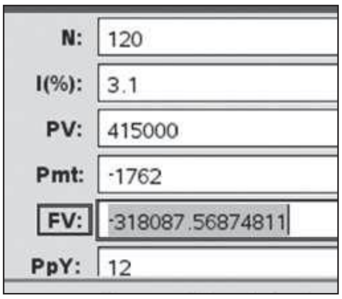
To the nearest dollar, the difference between the two is $8004.

Question 8

Answer: ***B***

Explanatory notes

Maintaining repayments at $1762 will mean the balance of the loan is $318 087.57 after 10 years.

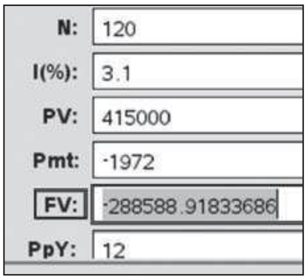


interest paid = amount repaid − amount balance decreased

$= 120 \times \$1762 - (415\,000 - 318\,087.57)$

$= \$114\,527.57$

Making repayments of \$1972 instead will result in the balance being \$288 588.92 after 10 years.



interest paid = amount repaid – amount balance decreased

$= 120 \times \$1972 - (415\,000 - 288\,588.92)$

$= \$110\,228.92$

difference $= \$114\,527.57 - \$110\,228.92$

$= \$4298.65$

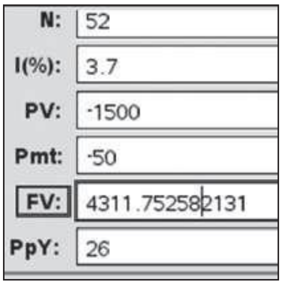
Question 9

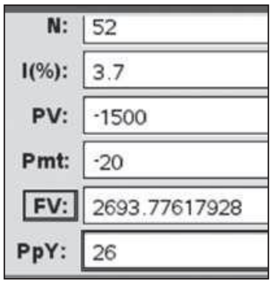
Answer: ***C***

Explanatory notes

$4311.75 - 2693.78 = 1617.97$

\$1618





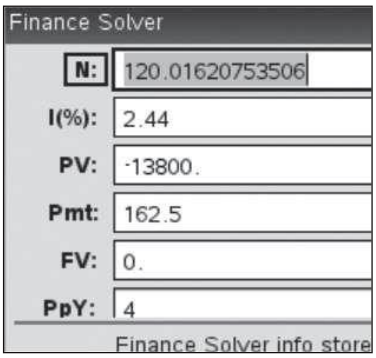
Question 10

Answer: ***D***

Explanatory notes

The value of 0.0061 is for a rate compounding quarterly.

$0.0061 \times 4 \times 100 = 2.44\%$ per annum



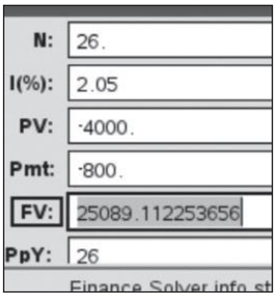
Question 11

Answer: ***D***

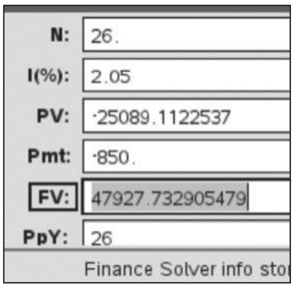
Explanatory notes

Changing the payment each year (26 fortnights), with the updated balance, results in:

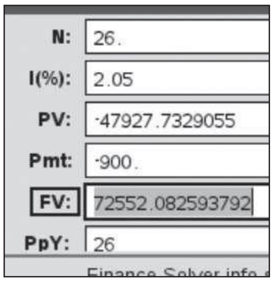
balance after first year:



balance after second year:



balance after third year:

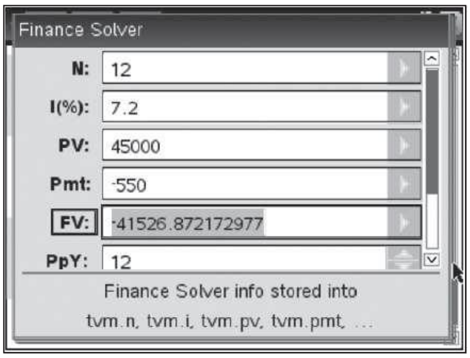


EXAM 2

Question 1a.

Worked solution

Use the finance solver on your CAS calculator.



Katerina will still owe $41 526.87 after 12 months.

Mark allocation: 1 mark

- 1 mark for correct answer

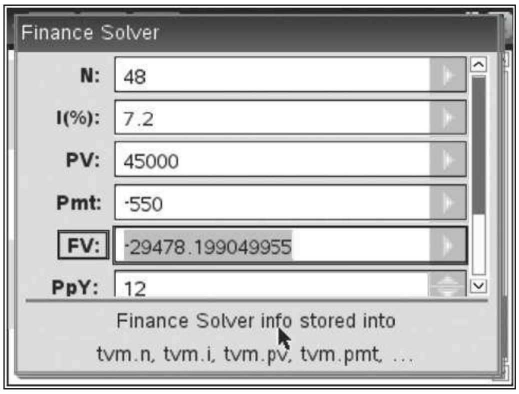
TIP

» Note that there are no specific instructions here on rounding. When we are working with money, unless otherwise specified, give your answer to the nearest cent.

Question 1b.

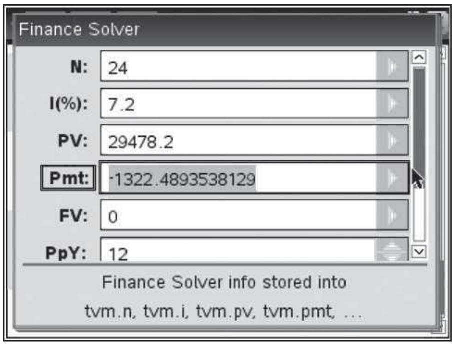
Worked solution

First, we need to find the value of Katerina's loan after four years, with her monthly repayment of $550.



Katerina still owes $29 478.20 after four years.

We must now calculate her payment, in order to repay this fully in a further two years.



Katerina will need to make repayments of $1322.49 per month, in order to repay her loan in a further two years.

Mark allocation: 2 marks

- 2 marks for correct answer of $1322.49

Note: If this is not achieved, 1 mark may be awarded for calculating the value of the loan after four years

Question 2a.

Worked solution

The monthly contribution, as read from the recurrence relation, is $450.

$450 \times 12 = 5400$

The annual contribution is $5400.

Mark allocation: 1 mark

- 1 mark for answer of $5400

Question 2b.

Worked solution

$$R = 1 + \frac{r}{100}$$

$$1.0038 = 1 + \frac{r}{100}$$

Use CAS software to solve for r.

$$\text{solve}\left(1.0038=1+\frac{\frac{r}{12}}{100},r\right) \qquad r=4.56$$

Or $0.0038 \times 12 \times 100 = 4.56$

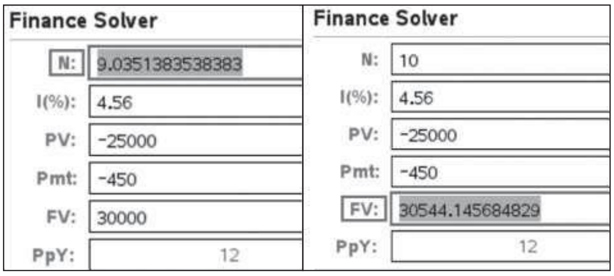
Mark allocation: 1 mark

- 1 mark for answer of 4.56%

Question 2c.i.

Worked solution

Use Finance Solver to find the first time that the value of the investment goes above \$30 000.



The first time that the value of the investment goes above \$30 000 is after 10 months, when the balance is \$30 544.10.

Therefore, the balance is \$30 544, to the nearest dollar.

Mark allocation: 1 mark

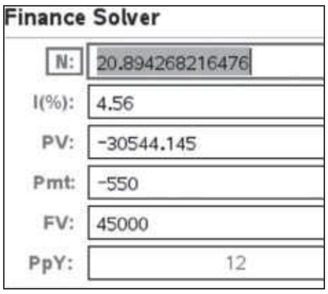
- 1 mark for correctly rounded answer of \$30 544

Question 2c.ii.

Worked solution

From **part c.i.**, we know that it has taken 10 months to reach the value of \$30 544.10.

Using Finance Solver on your CAS calculator, with the new contribution of \$550, it will take another 21 months to reach \$45 000.



$10 + 21 = 31$ months

Mark allocation: 1 mark

- 1 mark for correct answer of 31 months

Question 3a.

Worked solution

Using CAS:

eff (3.72,12)	3.78408599619

Mark allocation: 1 mark

- 1 mark for the correct answer of 3.78%

Question 3b.

Worked solution

N:	12
I(%):	3.72
PV:	310000
Pmt:	-1590
FV:	-302321.96744645
PpY:	12

interest paid = total repayments − amount balance has decreased

$= (1590 \times 12) - (310\,000 - 302\,321.97)$

$= \$11\,401.97$

$= \$11\,402$

Mark allocation: 1 mark

- 1 mark for the correct answer of $11 402

Question 3c.

Worked solution

When making interest-only payments, the balance does not decrease as the payments made are only enough to pay off the interest charged and not the principal, so the balance remains the same. In this case it stays at $310 000.

Mark allocation: 1 mark

- 1 mark for the correct answer of $310 000

Question 3d.

Worked solution

Samara has already had her loan for one year, or 12 months. Therefore, she has 24 years of the 25-year period remaining, which is 288 monthly payments.

Using CAS:

N:	288
I(%):	3.72
PV:	310000
Pmt:	-1629.0163305186
FV:	0
PpY:	12

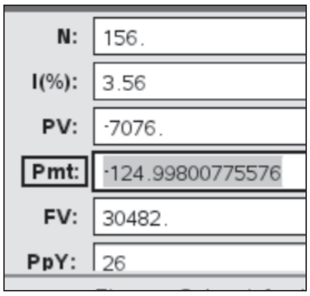
Mark allocation: 1 mark

- 1 mark for the correct answer of $1629

Question 4

Worked solution

\$125

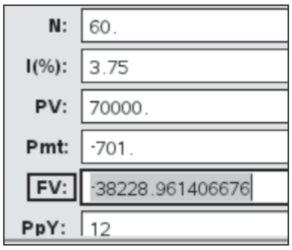


Mark allocation: 1 mark

- 1 mark for correct answer of \$125

Question 5a.

Worked solution



\$38 229

Mark allocation: 1 mark

- 1 mark for correct answer of \$38 229

Question 5b.

Worked solution

First five years:

repayments $= 60 \times 701 = 42\,060$

balance decrease $= 70\,000 - 38\,229 = 31\,771$

interest paid $= 42\,060 - 31\,771 = 10\,289$

Last five years:

repayments $= 60 \times 701 = 42\,060$

balance decrease $= 38\,229$

interest paid $= 42\,060 - 38\,229 = 3831$

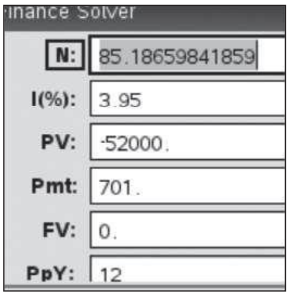
difference $= 10\,289 - 3831 = 6458$ more paid in first five years

Mark allocation: 1 mark

- 1 mark for the value of \$6458

Question 5c.i.

Worked solution



In order to withdraw \$701 per month, the annuity needs to change to be 85 payments of \$701 (86 payments won't cover an entire payment of \$701).

Originally the annuity lasted 144 payments.

$144 - 85 = 59$ less

Mark allocation: 2 marks

- 2 marks for correct answer of 59 months

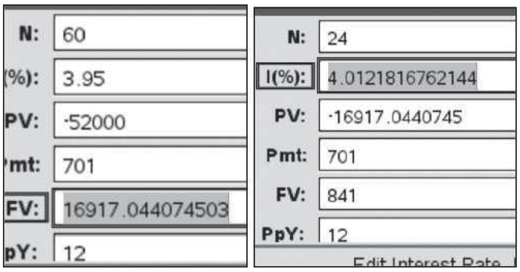
OR

- 1 mark for an answer of 58 if calculations are shown, or for finding a value of 85 or 86 payments

Question 5c.ii.

Worked solution

The balance when the rate changes (after five years) is \$16 917.04, as shown.



Using this balance (PV = 16 917…) and the final value (FV = \$841) for the two-year period (N = 24 months) after the rate change, we can determine that the rate was 4.01%.

Mark allocation: 1 mark

- 1 mark for answer of 4.01%

Area of Study 2 Matrices and their applications

EXAM 1

Question 1

Answer: ***C***

Explanatory notes

The transpose of a matrix is found by 'swapping' the position of all rows and columns; for example, the element in row 1, column 2 would move to row 2, column 1.

This can also be done using your CAS calculator.

Using the Ti-Nspire™ – this can be found through MENU – 7 – 2.

Question 2

Answer: ***D***

Explanatory notes

In order to find a total, we are looking for a matrix multiplication that will result in a 1×1 matrix.

The calculation that will give us this is $\begin{bmatrix} 3.50 & 2.50 & 4.00 & 1.75 \end{bmatrix} \begin{bmatrix} 65 \\ 156 \\ 105 \\ 256 \end{bmatrix}$

If we consider the matrix multiplication being performed,
$3.50 \times 65 + 2.50 \times 156 + 4.00 \times 105 + 1.75 \times 256 = 1485.50$ we obtain the total spent on drinks.

Question 3

Answer: ***D***

Explanatory notes

$$\begin{bmatrix} \text{row 1, column 1} & \text{row 1, column 2} \\ \text{row 2, column 1} & \text{row 2, column 2} \\ \text{row 3, column 1} & \text{row 3, column 2} \end{bmatrix}$$

$$= \begin{bmatrix} i=1, j=1 & i=1, j=2 \\ i=2, j=1 & i=2, j=2 \\ i=3, j=1 & i=3, j=2 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 1 & 2 \times 2 + 1 \\ 2 \times 1 + 2 & 2 \times 2 + 2 \\ 2 \times 1 + 3 & 2 \times 2 + 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 5 & 7 \end{bmatrix}$$

Question 4

Answer: ***B***

Explanatory notes

We must first consider one-step dominance. A matrix is the best way to record this.

$$D_1 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Next, we consider the two-step dominance.

This can be found by squaring the one-step dominance matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 2 & 2 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Adding them together:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 3 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 2 & 0 \end{bmatrix}$$

The sum of the rows shows the total for one- and two-step dominance, revealing Benjamin is the most dominant (8 total):

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & 3 & 2 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 2 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 8 \\ 3 \\ 6 \\ 7 \end{bmatrix}$$

Question 5

Answer: ***B***

Explanatory notes

$\frac{2865}{8} = 358.125 \approx 358$ customers

Question 6

Answer: ***C***

Explanatory notes

The sequence of selections is:

Fish (Mon.) – pork (Tues.) – chicken (Wed.) – beef (Thurs.) – lamb (Fri.)

Question 7

Answer: ***D***

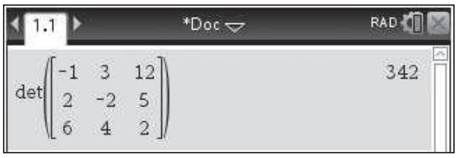
Explanatory notes

An inverse matrix can only exist when the determinant of the matrix does not equal zero. This is only possible for a square matrix and therefore matrix B does not have an inverse.

Both matrices A and D are 2×2 square matrices, and therefore the formula $\det = ad - bc$ can be used.

For matrix A:	**For matrix D:**
$\det = ad - bc$	$\det = ad - bc$
$\det = (1 \times 7) - (4 \times -2)$	$\det = (2 \times 7) - (1 \times -4)$
$\det = 7 - (-8)$	$\det = 14 - (-4)$
$\det = 7 + 8$	$\det = 14 + 4$
$\det = 15$	$\det = 18$

The CAS calculator can be used to find the determinant of matrix C:



The determinant of matrices A, C and D are all non-zero answers, therefore option D is correct.

TIP

» You can only find the determinant, and therefore the inverse, of a square matrix. Finding the determinant is an excellent way to test whether an inverse exists or not. If the determinant is equal to 0, there will not be an inverse.

Question 8

Answer: ***C***

Explanatory notes

The value of c is the percentage discount given on the bath mat.

The discount given on the bath mat is:

$22.45 - 17.51 = 4.94$

This, as a percentage of the original price is $\frac{4.94}{22.45} \times 100 = 22\%$

The decimal that gives us a 22% discount is:

$1 - 0.22 = 0.78$

$\therefore c = 0.78$

Question 9

Answer: ***B***

Explanatory notes

Start by entering the row number and column number into the formula to find each element.

Secondly, multiply this matrix by 2.

Question 10

Answer: ***D***

Explanatory notes

Start with the equation: $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} + R \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -1 & 3 \end{bmatrix}$

Firstly, subtract $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ from both sides:

$$R \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$R \times \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

Next solve the equation, using the inverse of $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$:

$$R = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1} \times \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Question 11

Answer: ***D***

Explanatory notes

Add the elements in the Evening (E) column.

$13 + 21 + 8 = 42$

Question 12

Answer: ***B***

Explanatory notes

The number of staff members who Brad reports to can be found by adding the elements in row 2. He only reports to one person (Saul).

Question 13

Answer: ***D***

Explanatory notes

A 1×1 matrix is required to provide a total. Option D is only the option that does this.

In option D we have 1×6 and a 6×1 matrix. These will multiply to give a 1×1 matrix, as shown below.

$$[12 \quad 10 \quad 8 \quad 6 \quad 10 \quad 5]\begin{bmatrix} 0.65 \\ 0.78 \\ 1.16 \\ 1.35 \\ 5.35 \\ 12.26 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \times 0.65 + 10 \times 0.78 + 8 \times 1.16 \\ +6 \times 1.35 + 10 \times 5.35 + 5 \times 12.26 \end{bmatrix} = [147.78]$$

Question 14

Answer: ***B***

Explanatory notes

$$\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

or:

Determinant $= ad - bc = 4 \times 2 - 3 \times 3 = -1$

$$\frac{1}{-1}\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = -1\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

Question 15

Answer: ***D***

Explanatory notes

Option A is not defined ($(4 \times 2) \times (1 \times 2)$ is not possible).

Option B will produce a 4×2 matrix.

Option C is not defined ($(2 \times 1) \times (4 \times 2)$ is not possible).

Option D will produce a 4×1 matrix → each farm's total revenue.

TIP

» Always think about what the answer should look like to help you decide in which order to multiply matrices. For example, you can foresee if a matrix product will result in a 3×1 matrix or a 1×1 matrix; knowing what you need as an end result can help you save time before you start multiplying.

Question 16

Answer: ***D***

Explanatory notes

If A^2 is defined, A must be a square matrix. As the sum of A and B is defined, A and B must be of the same order. Given it is known that the final product will be a 4×2 matrix, the outside numbers will be 4 and 2.

Therefore, $A \times B \times C = (4 \times 4) \times (4 \times 4) \times (4 \times 2)$

Question 17

Answer: ***A***

Explanatory notes

If John had three wins but lost to Fred, then he defeated the other three players: Gloria, Harriet and Ingrid.

If Gloria lost all games, then Fred, Harriet, Ingrid and John all had wins against her.

Since John beat Harriet, Harriet's three wins were against Gloria, Ingrid and Fred.

Option A is correct as it illustrates this information.

Option B is incorrect as it has a win for John against himself.

Option C is incorrect as it shows a loss for Harriet against Gloria, instead of a win.

Option D is incorrect as it shows a win for John against Fred instead of a loss.

Question 18

Answer: ***B***

Explanatory notes

Options A, C and D require a matrix to be a square matrix, such as a 2×2 or 3×3. A binary matrix can be of any order.

Question 19

Answer: ***C***

Explanatory notes

To provide the total amount combined, a 1×1 matrix is needed.

Options A and B will create 3×3 matrices.

Option D has the numbers written in an incorrect order in the second matrix.

Question 20

Answer: ***C***

Explanatory notes

Option C has incorrectly used the scalar multiplication. It should be $3 \times 2b - 1 \times 2d$ instead of $3 \times 2b - 2 \times 2d$.

$$\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$

Question 21

Answer: ***D***

Explanatory notes

The notation M_{21} refers to the value in row 2, column 1, which is 23.

Question 22

Answer: ***B***

Explanatory notes

E^{T} means that matrix E has been transposed, and the transpose has an order of 3×2.

$2F$ still results in a matrix of the same order, 2×4.

$(3 \times 2) \times (2 \times 4)$ produces 3×4.

TIP

» **Transposing a matrix results in the order of the matrix being 'reversed': the number of columns becomes the number of rows, and vice versa.**

Question 23

Answer: ***B***

Explanatory notes

K has a determinant of $(3 \times -4) - (2 \times -6) = 0$. As a result, no inverse exists, meaning all options except option B are incorrect.

Question 24

Answer: ***A***

Explanatory notes

$$3M + 2\begin{bmatrix} -1 & 2 & 5 \\ 2 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 22 \\ 25 & 39 & 12 \end{bmatrix}$$

$$3M + \begin{bmatrix} -2 & 4 & 10 \\ 4 & 6 & 12 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 22 \\ 25 & 39 & 12 \end{bmatrix}$$

$$3M = \begin{bmatrix} 1 & -2 & 22 \\ 25 & 39 & 12 \end{bmatrix} - \begin{bmatrix} -2 & 4 & 10 \\ 4 & 6 & 12 \end{bmatrix} = \begin{bmatrix} 3 & -6 & 12 \\ 21 & 33 & 0 \end{bmatrix}$$

$$M = 1/3 \times \begin{bmatrix} 3 & -6 & 12 \\ 21 & 33 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 4 \\ 7 & 11 & 0 \end{bmatrix}$$

Question 25

Answer: ***D***

Explanatory notes

Following the transition matrix for Monday to Friday, the roster for supervision would be:

Week	Monday	Tuesday	Wednesday	Thursday	Friday
1	*A*	*C*	*B*	*D*	*A*
2	*C*	*B*	*D*	*A*	*C*
3	*B*	*D*	*A*	*C*	*B*
4	*D*	*A*	*C*	*B*	*D*

The fifth day of supervision for Teacher *B* would be on Thursday of the fourth week.

EXAM 2

Question 1a.

Worked solution

Matrix P has 4 rows and 1 column and therefore the order of P is 4×1.

Mark allocation: 1 mark

- 1 mark for correct answer of 4×1

Question 1b.i.

Worked solution

$$[4.00 \quad 5.60 \quad 7.20 \quad 9.00] \times \begin{bmatrix} 250 \\ 135 \\ 180 \\ 65 \end{bmatrix}$$

$= [3637]$

Mark allocation: 1 mark

- 1 mark for correct matrix

Question 1b.ii.

Worked solution

Matrix M represents the total amount that the postal company charged for parcels for the month of January in 2016.

Mark allocation: 1 mark

- 1 mark for correct statement as above or equivalent

TIP

» Note that this matrix is not showing the amount charged for each type of parcel; this is a combined total for all parcels. You should be clear on this in your explanation.

Question 1b.iii.

Worked solution

The matrix product $P \times C$ exists because the number of columns in matrix P is equal to the number of rows in matrix C.

Mark allocation: 1 mark

- 1 mark for correct statement as above

TIP

» When explaining why a matrix product is or is not defined, be sure to compare the number of columns of the first matrix to the number of rows of the second.

Question 2a.

Worked solution

254 memberships were taken out in February. We add the numbers across row 2:
$107 + 95 + 52 = 254$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 2b.

Worked solution

The number of occasional memberships taken at the gym in February was 52.

Mark allocation: 1 mark

- 1 mark for correctly identifying the element

Question 3a.

Worked solution

$9.78 per kg

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3b.

Worked solution

The order of the matrix is 3×1.

Explanatory notes

This is because the matrix has 3 rows and 1 column.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3c.

Worked solution

$B = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$

Explanatory notes

To find a total, we would require a matrix of order 1×1. As matrix A is a 3×1 matrix, matrix B must be a 1×3 matrix.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3d.

Worked solution

$\frac{10}{100} \times 25 = 2.5$

$25 - 2.5 = 22.5 \approx 23$

There will be 23 square metres of cherry tomatoes remaining.

Mark allocation: 1 mark

- 1 mark for correct answer as a whole number

Question 3e.

Worked solution

$k = 0.9$

If 10% is lost, then 90% (0.9) remains.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4a.

Worked solution

The order is always rows × columns. This matrix is a 3×2 matrix.

Mark allocation: 1 mark

- 1 mark for the correct answer of a 3×2 matrix

Question 4b.

Worked solution

It represents that there are 10 parking spaces allocated to visitors with prams in carpark *A*.

Mark allocation: 1 mark

- 1 mark for a reference to carpark *A*, and visitors with prams

Question 4c.

Worked solution

Pre-multiplying with a row matrix of ones will sum the columns.

$$[1 \quad 1 \quad 1] \times \begin{bmatrix} 148 & 82 \\ 10 & 4 \\ 12 & 6 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct matrix product

Question 5a.

Worked solution

$$\begin{bmatrix} 11.45 \\ 9.98 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for the correct matrix written in this form

Question 5b.

Worked solution

As we need the total combined, this requires a 1×1 matrix, so $N \times F$:

$$[12 \quad 16] \cdot \begin{bmatrix} 11.45 \\ 9.98 \end{bmatrix} \qquad [297.08]$$

Mark allocation: 1 mark

- 1 mark for the correct answer of $297.08

Question 6a.

Worked solution

1 large + 2 medium = 3 dormitory rooms

Mark allocation: 1 mark

- 1 mark for correct answer of 3

Question 6b.

Worked solution

Calculating the sum of all the rooms except the large room:

$$[1 \quad 1 \quad 0 \quad 1] \cdot \begin{bmatrix} 6 \\ 8 \\ 1 \\ 2 \end{bmatrix} \qquad [16]$$

Mark allocation: 1 mark

- 1 mark for correct answer of $T = [1 \quad 1 \quad 0 \quad 1]$

Question 6c.

Worked solution

Dividing by 9 can be achieved by a scalar multiplication using 1/9.

$$\frac{1}{9} \cdot [1 \quad 2 \quad 12 \quad 6] \cdot \begin{bmatrix} 9 \\ 14 \\ 2 \\ 4 \end{bmatrix} \qquad [9.44444444444]$$

OR

$$\frac{1}{9} \cdot [1 \quad 1 \quad 1 \quad 1] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 14 \\ 2 \\ 4 \end{bmatrix} \qquad [9.44444444444]$$

The answer needs to be rounded up to 10 in order to meet the requirement that there be at least one toilet for every nine people. Rounding to nine toilets will mean the regulations are only met for 81 people.

Mark allocation: 2 marks

- 1 mark for answer of 10
- 1 mark for showing calculation, including scalar multiplication using 1/9

TIP

» Consider the wording of questions when rounding for 'at least', or a 'maximum' or 'minimum' amount. You may need to round down or round up in order to satisfy these requirements.

Question 7a.

Worked solution

Team E beat two teams; those two teams had 5 wins combined. That means that Team E had to beat a team that had 2 wins and a team that had 3 wins, as there are no teams that had 4 wins or 5 wins (to make the combination of 4 and 1 or 5 and 0 wins). The only team that had 3 wins was Team B.

Mark allocation: 1 mark

- 1 mark for correct answer of Team B

Question 7b.

Worked solution

Team E beat Team B.

Team B had 3 wins and lost to Team E, so the wins were against Teams A, C and D.

Teams A and D each had a two-step dominance of 3. From this it can be deduced that they each had a win against a team with only 1 win. Therefore, Teams A and D both beat Team C.

That leaves Team E as the only team that Team C beat.

Team A must have beaten Team D, not Team E, as it had 2 wins in common with Team B.

Team D must have beaten Team E to make up its second win.

$$\begin{array}{c} \boldsymbol{A} \\ \boldsymbol{B} \\ \boldsymbol{C} \\ \boldsymbol{D} \\ \boldsymbol{E} \end{array}\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Mark allocation: 2 marks

- 2 marks for identifying that Team D beat Teams C and E (only)

OR

- 1 mark if Team C is identified with one other incorrect team

OR

- 0 if more than 2 wins are identified

Area of Study 2 Transition matrices

EXAM 1

Question 1

Answer: ***D***

Explanatory notes

Each of the columns of a transition matrix, where the total number does not change, should add to 1.

Option D is the only matrix in which each column adds to 1.

TIP

» In this matrix, 1 means that when a family has chosen to shop at Ultramarket, they will not change their selection. They will continue to return to Ultramarket.

Question 2

Answer: ***C***

Explanatory notes

$$\begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix}^2 \times \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} = \begin{bmatrix} 206.857 \\ 142.349 \\ 100.795 \end{bmatrix}$$

The number of students selecting juice, rounded to the nearest whole number, is equal to 142.

Question 3

Answer: ***A***

Explanatory notes

To find the long-term prediction, we need to find the steady state matrix.

$$\begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix}^{30} \times \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} = \begin{bmatrix} 208.698 \\ 140.423 \\ 100.879 \end{bmatrix}$$

To ensure this is the steady state, we find a consecutive matrix to confirm that our state is unchanging.

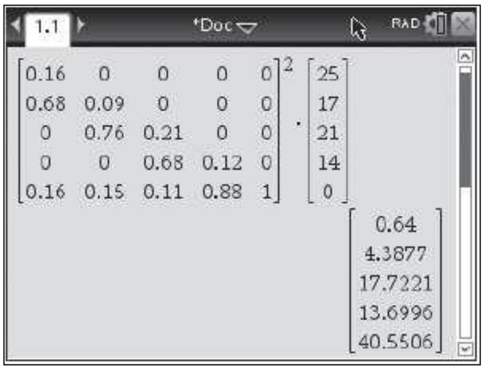
$$\begin{bmatrix} 0.65 & 0.24 & 0.39 \\ 0.15 & 0.54 & 0.33 \\ 0.20 & 0.22 & 0.28 \end{bmatrix}^{31} \times \begin{bmatrix} 215 \\ 175 \\ 60 \end{bmatrix} = \begin{bmatrix} 208.698 \\ 140.423 \\ 100.879 \end{bmatrix}$$

The number of students selecting iced tea, in the long term, is closest to 209.

Question 4

Answer: ***B***

Explanatory notes

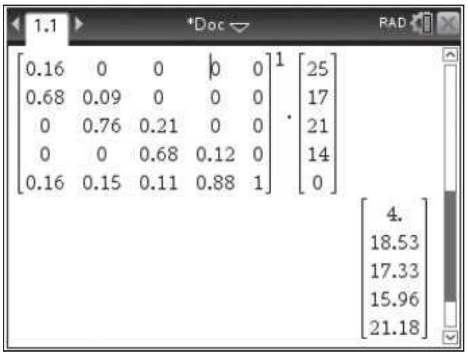


Question 5

Answer: ***C***

Explanatory notes

As there were originally 25 puppies, and after one year there are only 4 puppies remaining, we need to add an additional 21 puppies each year.



Question 6

Answer: ***D***

Explanatory notes

From the transition diagram, enter the following elements in the transition matrix.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{array}{ccc} A & B & C \end{array} \\ D = & \begin{bmatrix} & 0.30 & 0.65 \\ 0.45 & & 0.10 \\ 0.20 & 0.50 & \end{bmatrix} \begin{array}{l} A \\ B \ \textit{next year} \\ C \end{array} \end{array}$$

The columns in the transition matrix should add to 1.

Therefore, subtract the known values from 1 to obtain the remaining element.

Column *A*: $1 - 0.45 - 0.20 = 0.35$

Column *B*: $1 - 0.30 - 0.50 = 0.20$

Column *C*: $1 - 0.65 - 0.10 = 0.25$

Now complete the matrix.

$$\begin{array}{cc} & \textit{this year} \\ & \begin{array}{ccc} A & B & C \end{array} \\ D = & \begin{bmatrix} 0.35 & 0.30 & 0.65 \\ 0.45 & 0.20 & 0.10 \\ 0.20 & 0.50 & 0.25 \end{bmatrix} \begin{array}{l} A \\ B \text{ next year} \\ C \end{array} \end{array}$$

Question 7

Answer: ***D***

Explanatory notes

S_1 represents Term 1 Year 11, so S_7 is Term 3 Year 12 – this is 6 transitions.

$$\begin{bmatrix} 0.94 & 0.08 & 0 & 0 \\ 0.04 & 0.88 & 0.16 & 0 \\ 0 & 0.03 & 0.84 & 0 \\ 0.02 & 0.01 & 0 & 1 \end{bmatrix}^6 \cdot \begin{bmatrix} 120 \\ 70 \\ 25 \\ 15 \end{bmatrix}$$

=

$$\begin{bmatrix} 111.649342648 \\ 67.9377364582 \\ 17.2615213056 \\ 33.1513995878 \end{bmatrix}$$

Question 8

Answer: ***C***

Explanatory notes

It falls below 20 after 4 transitions. This corresponds to Term 1 of Year 12:

$$\begin{bmatrix} 0.94 & 0.08 & 0 & 0 \\ 0.04 & 0.88 & 0.16 & 0 \\ 0 & 0.03 & 0.84 & 0 \\ 0.02 & 0.01 & 0 & 1 \end{bmatrix}^4 \cdot \begin{bmatrix} 120 \\ 70 \\ 25 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 114.2150656 \\ 69.5020032 \\ 19.0570944 \\ 27.2258368 \end{bmatrix}$$

$$\begin{bmatrix} 0.94 & 0.08 & 0 & 0 \\ 0.04 & 0.088 & 0.16 & 0 \\ 0 & 0.03 & 0.84 & 0 \\ 0.02 & 0.01 & 0 & 1 \end{bmatrix}^3 \cdot \begin{bmatrix} 120 \\ 70 \\ 25 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 115.54304 \\ 70.0576 \\ 20.18496 \\ 24.2144 \end{bmatrix}$$

TIP

» **Make sure you count how many transitions there are from one state matrix to the required state matrix.**

Question 9

Answer: ***D***

Explanatory notes

$$S_1 = \begin{bmatrix} 120 \\ 70 \\ 25 \\ 15 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.94 & 0.08 & 0 & 0 \\ 0.04 & 0.88 & 0.16 & 0 \\ 0 & 0.03 & 0.84 & 0 \\ 0.02 & 0.01 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 120 \\ 70 \\ 25 \\ 15 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \\ -3 \end{bmatrix} \begin{bmatrix} 117.4 \\ 71.4 \\ 23.1 \\ 15.1 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.94 & 0.08 & 0 & 0 \\ 0.04 & 0.88 & 0.16 & 0 \\ 0 & 0.03 & 0.84 & 0 \\ 0.02 & 0.01 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 117.4 \\ 71.4 \\ 23.1 \\ 15.1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 115.068 \\ 72.224 \\ 21.546 \\ 15.162 \end{bmatrix}$$

Question 10

Answer: ***A***

Explanatory notes

The missing values are shown with the correct value and in the correct spot in option A.

Options B to D have these and other numbers misplaced.

TIP

» **Some incorrect matrices can quickly be eliminated by checking if the columns do not add up to 1.**

Question 11

Answer: ***D***

Explanatory notes

Calculating values for the first week by using the values for the third week may be completed in the following ways:

$$\begin{bmatrix} 0.81 & 0.08 & 0.06 \\ 0.1 & 0.77 & 0.02 \\ 0.09 & 0.15 & 0.92 \end{bmatrix}^{-2} \cdot \begin{bmatrix} 125 \\ 212 \\ 163 \end{bmatrix}$$

$$\begin{bmatrix} 111.317384914 \\ 316.862105673 \\ 71.8205094129 \end{bmatrix}$$

OR

$$\text{solve}\left(\begin{bmatrix} 0.81 & 0.08 & 0.06 \\ 0.1 & 0.77 & 0.02 \\ 0.09 & 0.15 & 0.92 \end{bmatrix}^2 \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 125 \\ 212 \\ 163 \end{bmatrix}, x\right)$$

$x = 111.317384914$ and $y = 316.862105673$

The number of households watching *Masterchess* was 317.

Question 12

Answer: ***D***

Explanatory notes

The element in L_{13} represents the reproduction rate for female adults *producing* eggs. It represents that, on average, 3 female eggs will be produced by female adults.

Question 13

Answer: ***D***

Explanatory notes

$$\begin{bmatrix} 0.05 & 0.81 & 0.17 \\ 0.75 & 0.09 & 0.05 \\ 0.2 & 0.1 & 0.78 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 242 \\ 315 \\ 208 \end{bmatrix}$$

$$\begin{bmatrix} 381.276501767 \\ 246.411219081 \\ 137.312279152 \end{bmatrix}$$

EXAM 2

Question 1a.

Worked solution

$$S_1 = TS_0 = \begin{array}{c} \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix} \end{array}\begin{array}{c} A \\ B \\ C \\ D \end{array} = \begin{bmatrix} 152.5 \\ 132 \\ 133.75 \\ 181.75 \end{bmatrix}\begin{array}{c} A \\ B \\ C \\ D \end{array} = \begin{bmatrix} 153 \\ 132 \\ 134 \\ 182 \end{bmatrix}\begin{array}{c} A \\ B \\ C \\ D \end{array}$$

Therefore, $f = 132$

Mark allocation: 1 mark

- 1 mark for correct answer of 132

Question 1b.

Worked solution

The answer for Cally, which is represented by g in the matrix S_1, is found by multiplying row 3 in matrix T, with column 1 of matrix S_0. Therefore, we can find this answer through the following calculation.

$0.13 \times 250 + 0.11 \times 125 + 0.62 \times 125 + 0.10 \times 100 = 133.75 = 134$

Mark allocation: 1 mark

- 1 mark for correct calculation, in full, as above

Note: A mark will not be awarded if $S_1 = TS_0$ is given as an answer.

Question 1c.i.

Worked solution

We read this from the transition matrix, T. 0.68 customers who chose Dovan this month will choose it next month. Therefore, 68%.

Mark allocation: 1 mark

- 1 mark for 68% as an answer

Note: The proportion, 0.68, is not accepted.

Question 1c.ii.

Worked solution

0.68 of customers will choose Dovan again in February; therefore, 0.32 will choose a different distribution centre. In January 2016, 100 customers chose Dovan. $0.32 \times 100 = 32$ customers choosing another distribution centre.

Mark allocation: 1 mark

- 1 mark for 32

Question 1d.

Worked solution

To find the long term, we must find the steady state matrix. We find this by calculating two consecutive matrices with identical elements.

$$S_{50} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix}^{50} \times \begin{bmatrix} 250 \\ 125 \\ 125 \\ 100 \end{bmatrix} = \begin{bmatrix} 110.183 \\ 115.693 \\ 134.297 \\ 239.827 \end{bmatrix}$$

$$S_{51} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix}^{51} \times \begin{bmatrix} 250 \\ 125 \\ 125 \\ 100 \end{bmatrix} = \begin{bmatrix} 110.183 \\ 115.693 \\ 134.297 \\ 239.827 \end{bmatrix}$$

Therefore, in the long term, 110 customers will choose Abbey as their preferred distribution centre.

Mark allocation: 2 marks

- 1 mark for 110 customers
- 1 mark for showing two consecutive matrices, as a proof of steady state

Question 1e.

Worked solution

$$F_{2017} = T \times J_{2017} + B$$

$$F_{2017} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \times \begin{bmatrix} 350 \\ 225 \\ 270 \\ 240 \end{bmatrix} + \begin{bmatrix} 45 \\ 65 \\ 70 \\ 40 \end{bmatrix}$$

$$F_{2017} = \begin{bmatrix} 287.1 \\ 297.1 \\ 331.65 \\ 389.15 \end{bmatrix}$$

$M_{2017} = T \times F_{2017} + B$

$$M_{2017} = \begin{bmatrix} 0.45 & 0.12 & 0.08 & 0.15 \\ 0.16 & 0.54 & 0.14 & 0.07 \\ 0.13 & 0.11 & 0.62 & 0.10 \\ 0.26 & 0.23 & 0.16 & 0.68 \end{bmatrix} \times \begin{bmatrix} 287.1 \\ 297.1 \\ 331.65 \\ 389.15 \end{bmatrix} + \begin{bmatrix} 45 \\ 65 \\ 70 \\ 40 \end{bmatrix}$$

$$M_{2017} = \begin{bmatrix} 294.752 \\ 345.042 \\ 384.542 \\ 500.665 \end{bmatrix}$$

In March 2017, 345 customers choose Buron as their distribution centre.

Mark allocation: 2 marks

- 2 marks for an answer of 345

Note: If this is not achieved, 1 mark can be allocated for a correct calculation of the matrix for February 2017.

Question 2a.

Worked solution

72% of members who chose the aerobics class in the first week will choose it again the next week. This means that $100\% - 72\% = 28\%$ will not choose the aerobics class the following week.

Mark allocation: 1 mark

- 1 mark for correct answer of 28

Note: A percentage sign is not required.

Question 2b.

Worked solution

23% of members choosing C will change to $W = 0.23 \times 51 = 11.73 \approx 12$. 9% of members choosing A will change to $W = 0.09 \times 73 = 6.57 \approx 7$. 65% of members choosing W will continue with $W = 0.65 \times 101 = 65.65 \approx 66$. In total, 85 members will choose the weights class in the second week.

Mark allocation: 1 mark

- 1 mark for correct answer of 85

Question 2c.

Worked solution

55% of members will continue with the cycle class from one week to next. From week one to week two: $0.55 \times 51 = 28.05 \approx 28$. From week two to week three: $0.55 \times 28 = 15.4 \approx 15$.

Thus, 15 of the original members who started the cycle class will remain in the cycle class during the third week.

Mark allocation: 1 mark

- 1 mark for correct answer of 15

Question 3a.

Worked solution

We must consider the people who change from this year to next, for each fitness centre, i.e. [change from F] + [change from H] + [change from S]

$(0.18 \times 120 + 0.32 \times 120) + (0.34 \times 120 + 0.22 \times 120) + (0.12 \times 120 + 0.11 \times 120)$

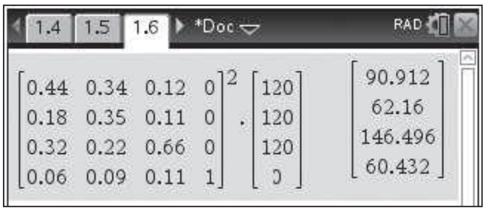
$= 154.8 \approx 155$

Mark allocation: 1 mark

- 1 mark for correct answer of 155

Question 3b.

Worked solution



$$M_3 = \begin{array}{c} \begin{bmatrix} 91 \\ 62 \\ 146 \\ 60 \end{bmatrix} \end{array}\begin{array}{l} F \\ H \\ S \\ L \end{array}$$

Mark allocation: 1 mark

- 1 mark for correct matrix M_3, with all elements correct

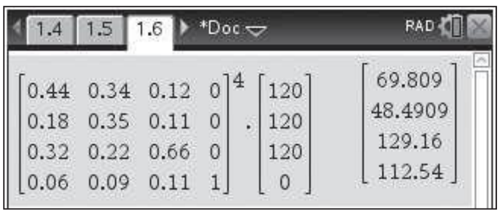
TIP

» You need to check the initial state matrix very carefully. As you have been given M_1, you only require 2 transitions to get M_3. You can use the equation $M_n = T^{n-1} \times M_1$ If you had been given M_0, you would need to do 3 transitions. You can use the equation $M_n = T^n \times M_0$

Question 3c.

Worked solution

The year 2022 is year 5. Therefore, we are finding M_5, which is 4 transitions.



129 people will choose Smash It in 2022.

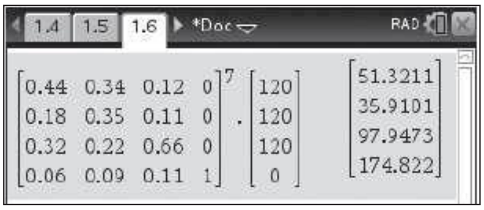
Mark allocation: 1 mark

- 1 mark for correct answer of 129

Question 3d.

Worked solution

The year 2025 is year 8. Therefore, we are finding M_8.



There will be 36 members at the Healthy Life (H) fitness centre in 2025. $\frac{36}{120} \times 100 = 30\%$. The manager is not correct, the membership has not fallen below 25% of its original number.

Mark allocation: 2 marks

- 1 mark for a statement that the manager is incorrect
- 1 mark for a calculation showing that the number is 30% of the original

Question 4a.

Worked solution

20%

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4b.

Worked solution

$0.10 \times 3500 + 0.12 \times 3500 + 0.86 \times 3500 + 0.05 \times 3500$

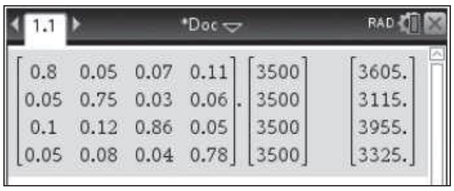
$= 3955$ grams

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4c.

Worked solution



$$S_1 = \begin{array}{c} \begin{bmatrix} 3605 \\ 3115 \\ 3955 \\ 3325 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \end{array}$$

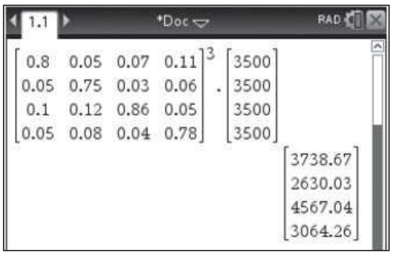
Mark allocation: 1 mark

- 1 mark for correct answer

Question 4d.

Worked solution

The year 2021 can be calculated as the state matrix, S_3. We can find this by calculating $S_3 = T^3 \times S_0$.



2630 grams of tomatoes will be purchased at supplier *B* by Johanna in 2021.

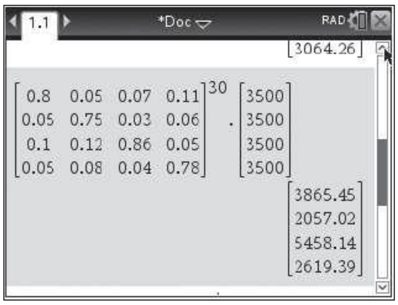
Mark allocation: 1 mark

- 1 mark for correct answer

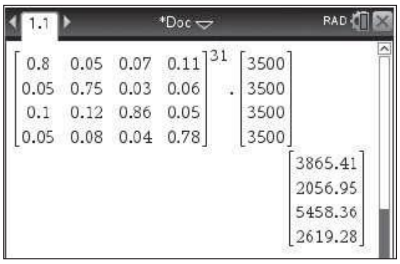
Question 4e.

Worked solution

$S_{30} = T^{30} \times S_0$



$S_{31} = T^{31} \times S_0$



As $S_{30} = S_{31}$, a steady state has been reached. 2619.28 grams of tomatoes will be purchased at supplier *D* in the long term. As a percentage of the total purchase, this is:

$$\frac{2619.28}{14000} \times 100 = 18.7091\%$$

$$\approx 18.7\%$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4f.

Worked solution

4000 grams

Mark allocation: 1 mark

- 1 mark for correct answer

Question 4g.

Worked solution

$$\begin{bmatrix} 0.8 & 0.05 & 0.07 & 0.11 \\ 0.05 & 0.75 & 0.03 & 0.06 \\ 0.1 & 0.12 & 0.86 & 0.05 \\ 0.05 & 0.08 & 0.04 & 0.78 \end{bmatrix} \cdot \begin{bmatrix} 4000 \\ 4000 \\ 4000 \\ 2000 \end{bmatrix} \quad \begin{bmatrix} 3900. \\ 3440. \\ 4420. \\ 2240. \end{bmatrix}$$

In order for her purchases to remain consistent the state matrix must remain the same.

$$G = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 560 \\ -420 \\ -240 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5a.

Worked solution

1650

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5b.

Worked solution

$V_{n+1} = T \times V_n$

A CAS calculator can be used:

$$\begin{bmatrix} 0.1 & 0.45 & 0.35 \\ 0.4 & 0.15 & 0.55 \\ 0.5 & 0.4 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 1500 \\ 1650 \\ 1850 \end{bmatrix} \quad \begin{bmatrix} 1540. \\ 1865. \\ 1595. \end{bmatrix}$$

The state matrix is $V_1 = \begin{array}{c} \begin{bmatrix} 1540 \\ 1865 \\ 1595 \end{bmatrix} \end{array}\begin{array}{c} P \\ R \\ S \end{array}$

Mark allocation: 1 mark

- 1 mark for correct matrix with all elements

Question 5c.

Worked solution

Using the transition matrix, T, we can see that 0.45 of people who visited Riversville on 1 December will visit Princenton the next day, and 0.35 of people who visited Summertown on 1 December will visit Princenton the next day. Therefore, $0.45 \times 1650 + 0.35 \times 1850 = 1390$. As a percentage of the 1540 people who are expected to visit Princenton on 2 December:

$$\frac{1390}{1540} \times 100 = 90.26\% \approx 90\%$$

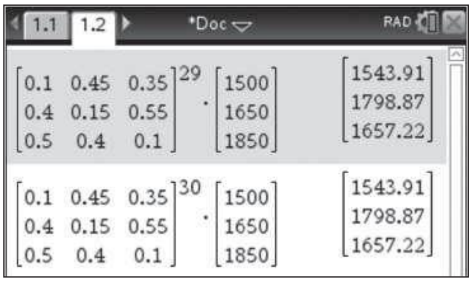
Mark allocation: 1 mark

- 1 mark for calculation

Question 5d.i.

Worked solution

The steady state matrix is reached when 2 consecutive state matrices contain the same elements. As December has 31 days, and 1 December is V_0, we need to find V_{29} and V_{30}.



The steady state matrix has been reached.

Mark allocation: 1 mark

- 1 mark for writing down two consecutive matrices that have equal values

Question 5d.ii.

Worked solution

The number of daily visitors to the Summertown event by the end of December is 1657.

Mark allocation: 1 mark

- 1 mark for correct answer of 1657 people

Question 6a.

Worked solution

No child does the same activity for two hours in a row (in consecutive hours).

Mark allocation: 1 mark

- 1 mark for statement above or similar

Question 6b.

Worked solution

$$K \times L_0 = \begin{bmatrix} 0 & 0.24 & 0.40 & 0.20 \\ 0.35 & 0 & 0.25 & 0.30 \\ 0.30 & 0.46 & 0 & 0.50 \\ 0.35 & 0.30 & 0.35 & 0 \end{bmatrix} \times \begin{bmatrix} 150 \\ 150 \\ 150 \\ 150 \end{bmatrix} = \begin{bmatrix} 126 \\ 135 \\ 189 \\ 150 \end{bmatrix}$$

Therefore, $\begin{aligned} W &= 126 \\ X &= 135 \\ Y &= 189 \\ Z &= 150 \end{aligned}$

Mark allocation: 1 mark

- 1 mark for all values correct

Question 6c.

Worked solution

$$B = \begin{bmatrix} 24 \\ 15 \\ -39 \\ 0 \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for matrix with all elements correct

Question 7a.i.

Worked solution

The quantity choosing a dormitory room is calculated using the percentages in the bottom row and their respective values in S_0: 6% of 5 + 24% of 12 + 76% of 21.

$$T = \begin{array}{c} \begin{array}{ccc} S & T & D \end{array} \\ \begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \end{array} \begin{array}{c} \\ S \\ T \\ D \end{array}$$

$0.06 \cdot 5 + 0.24 \cdot 12 + 0.76 \cdot 21$	19.14

Mark allocation: 1 mark

- 1 mark for correct answers of 0.06, 0.24 and 0.76 (in order)

Question 7a.ii.

Worked solution

Calculating how many did *not* change:

$0.92 \cdot 5 + 0.62 \cdot 12 + 0.76 \cdot 21$	28.

$38 - 28 = 10$ people who did change

Mark allocation: 1 mark

- 1 mark for correct answer of 10

Question 7b.

Worked solution

Four additional backpackers will require a dormitory room.

Mark allocation: 1 mark

- 1 mark for a similar statement

Question 7c.

Worked solution

Recursive calculations to show the number of people for the second and then third hostel would be required as it is a recurrence relation.

$$\begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 12 \\ 21 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 7.01 \\ 9.85 \\ 23.14 \end{bmatrix}$$

$$\begin{bmatrix} 0.92 & 0.14 & 0.13 \\ 0.02 & 0.62 & 0.11 \\ 0.06 & 0.24 & 0.76 \end{bmatrix} \cdot \begin{bmatrix} 7.01 \\ 9.85 \\ 23.14 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 8.8364 \\ 8.7926 \\ 24.371 \end{bmatrix}$$

The answer shows that there are not enough single and dormitory rooms: 9 people are expected for 8 single rooms, and 24 people are expected for only 22 dormitory rooms.

Mark allocation: 2 marks

- 2 marks for identifying there is 1 single room short, and 2 dormitory rooms short

OR

- 1 mark if it is only identified that there are not enough single or dormitory rooms, without values to justify it

Question 8a.i.

Worked solution

The age distribution after one time period (3 years) is:

$$\begin{bmatrix} 0 & 4.1 & 1.5 \\ 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix} \qquad \begin{bmatrix} 2800 \\ 350 \\ 150 \end{bmatrix}$$

So there are 2800 snakes in the 0–3 years age range, 350 snakes in the 3–6 years age range, and 150 snakes in the 6–9 years age range.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 8a.ii.

Worked solution

The total population increases from 1500 to $(2800 + 350 + 150)$ 3300, so the amount the population increases by is 1800.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 8b.

Worked solution

1960 snakes are expected in category B (3–6 years) as seen below.

$$\begin{bmatrix} 0 & 4.1 & 1.5 \\ 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix} \qquad \begin{bmatrix} 2800. \\ 350. \\ 150. \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4.1 & 1.5 \\ 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2800. \\ 350. \\ 150. \end{bmatrix} \qquad \begin{bmatrix} 1660. \\ 1960. \\ 105. \end{bmatrix}$$

Or

$$\begin{bmatrix} 0 & 4.1 & 1.5 \\ 0.7 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}^2 \cdot \begin{bmatrix} 500 \\ 500 \\ 500 \end{bmatrix} \qquad \begin{bmatrix} 1660. \\ 1960. \\ 105. \end{bmatrix}$$

Mark allocation: 1 mark

- 1 mark for correct answer

Question 9a.

Worked solution

The age distribution after one time period (3 months) is shown:

$$\begin{bmatrix} 0 & 1.2 & 1.8 & 0 \\ 0.6 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 80 \\ 90 \\ 50 \\ 30 \end{bmatrix} \qquad \begin{bmatrix} 198. \\ 48. \\ 36. \\ 15. \end{bmatrix}$$

So there are 15 rodents aged 9–12 months.

Mark allocation: 1 mark

- 1 mark for answer of 15

Question 9b.

Worked solution

The total population has increased from 250 to 297 (that is, $198 + 48 + 36 + 15$). This increase is $47/250 \times 100 = 18.8\%$

Mark allocation: 1 mark

- 1 mark for answer of 19%

TIP

» **Remember with a Leslie matrix, the form is different to a regular transition matrix. In a 3×3 Leslie matrix, the reproduction rate will be represented in the first two rows and the survival rate will be represented in the 3rd row. The columns will not add to 1.**

Area of Study 2 Networks and decision mathematics – Undirected graphs and networks: travelling and connecting problems

EXAM 1

Question 1

Answer: ***A***

Explanatory notes

A trail is a walk that contains no repeated edges.

The walk shown repeats vertices, but not edges.

Note: A path is a walk with no repeated edges or vertices. A cycle is a path (no repeated edges or vertices) that starts and ends at the same vertex. A circuit is a trail (no repeated edges) that starts and ends at the same vertex.

Question 2

Answer: ***D***

Explanatory notes

A Eulerian circuit is possible when all vertices in the network have a degree that is an even number.

In this network, vertex *A* has a degree of 3 and vertex *E* also has a degree of 3. Therefore, they have an 'odd degree'.

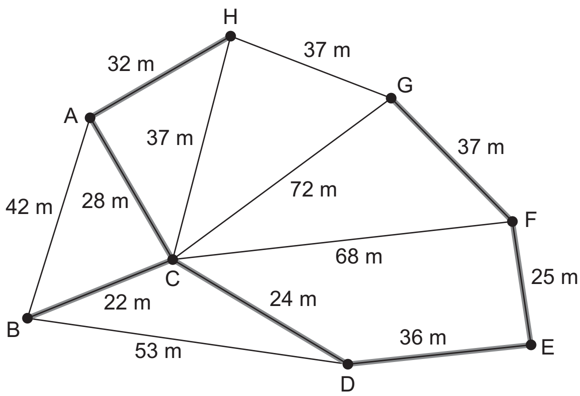
If we were to add an edge between these vertices, they would both have an even degree, therefore making a Eulerian circuit possible.

Question 3

Answer: ***D***

Explanatory notes

The minimum spanning tree is shown on the network below.



As we can see, the edge between *G* and *C* is not used in the minimum spanning tree.

Question 4

Answer: ***D***

Explanatory notes

A simple graph has no loops or multiple edges.

Options A and C contain a loop.

Option B contains multiple edges.

TIP

» **Multiple edges occur when there is more than one edge connecting the same set of vertices.**

Question 5

Answer: ***D***

Explanatory notes

The adjacency matrix will not only record connections between vertices, but also the number of edges connecting each set of vertices. This means that the answer cannot be option B, as this contains only '1s' and '0s'.

There are no loops in the diagram, so this means that the centre diagonal must be all '0s', as no vertex is connected back to itself. This means that the answer is not option A or option C.

Finally, there is one connection between vertex *C* and vertex *E*. The only correct answer, therefore, is option D.

Question 6

Answer: ***C***

Explanatory notes

Use Euler's formula: $v - e + f = 2$

For this question:

$v = ?$

$e = 21$

$f = 17$

Therefore:

$$\begin{aligned} v - 21 + 17 &= 2 \\ v - 4 &= 2 \\ v &= 6 \end{aligned}$$

Question 7

Answer: ***D***

Explanatory notes

A Hamiltonian cycle must go through each vertex exactly once and return to its beginning.

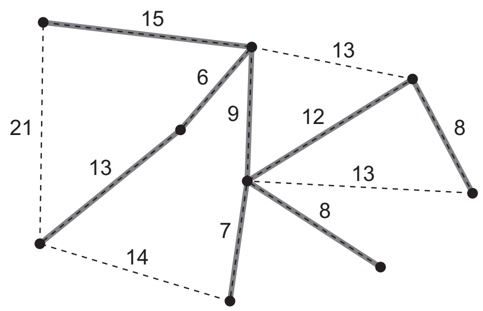
There must, however, be an edge to connect vertices in order to 'travel' through the network.

Option A is incorrect – no edge exists between vertices *A* and *C*. Option B is incorrect – no edge exists between vertices *D* and *E*. Option C is incorrect – no edge exists between vertices *E* and *B*. Option D is correct.

Question 8

Answer: ***A***

Explanatory notes



The length of the minimum spanning tree is:

$15 + 6 + 13 + 9 + 7 + 8 + 12 + 8 = 78$

TIP

» It is important to know the process of finding the minimum spanning tree (Prim's algorithm).

Question 9

Answer: ***D***

Explanatory notes

Using Euler's formula, $v - e + f = 2$, test each option by substituting values for v, e and f.

Option A is incorrect. $4 - 6 + 5 = 3$, not equal to 2

Option B is incorrect. $4 - 6 + 6 = 4$, not equal to 2

Option C is incorrect. $4 - 6 + 7 = 5$, not equal to 2

Option D is correct. $4 - 6 + 4 = 2$

Question 10

Answer: ***C***

Explanatory notes

A planar graph can be re-drawn with no edges crossing.

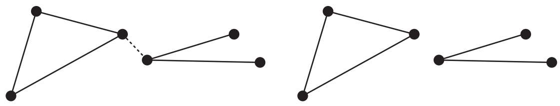
It is not possible to do this with option C.

Question 11

Answer: ***A***

Explanatory notes

Removing a bridge results in a network that is no longer connected.



Question 12

Answer: ***C***

Explanatory notes

Adding an edge to *AB* will make all the degrees of the vertices even (producing an Eulerian circuit), whereas an Eulerian trail needs to have two odd vertices and the rest to be even.

Question 13

Answer: ***D***

Explanatory notes

A tree needs to reach all vertices without creating a loop or circuit.

Option B contains a circuit, and options A and C are each missing a vertex.

Also, this network has 10 vertices and so a tree derived from it must have nine edges (one less edge than the number of vertices).

Question 14

Answer: ***C***

Explanatory notes

Using $V + F - E = 2$,

$V + 5 - 7 = 2$

$V = 4$

Question 15

Answer: ***B***

Explanatory notes

A Hamiltonian path needs to connect to each vertex once only.

Option A is incorrect because there is no edge connecting *C* and *D*.

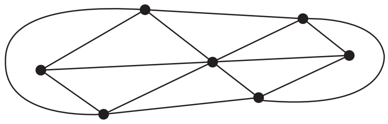
Options C and D both have repeating vertices.

Question 16

Answer: ***D***

Explanatory notes

Option A is true because the graph can be rearranged so that no edges are crossing, revealing nine faces.



Option B is true, it has 14 edges. They may be counted better in the planar image above.

Option C is true. There are two vertices that are odd and the rest are even, making this an Eulerian trail.

Option D is not true. The sum is 28, not 26.

Question 17

Answer: ***A***

Explanatory notes

Option B doesn't recognise the loop to *A*.

Option C doesn't include all three paths between *A* and *D*.

Option D doesn't include the loop at *A* or the three paths between *A* and *D*.

Question 18

Answer: ***A***

Explanatory notes

Completing a row reduction then column reduction will produce:

$$\begin{array}{c} E \\ R \\ M \\ L \end{array}\begin{bmatrix} 2 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 4 & 2 & 1 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

Matching up the people with the possible tasks:

E: 2 or 3

R: 3 or 4

M: 1

L: 2 or 4

This can provide the allocation for both I and II.

Question 19

Answer: ***B***

Explanatory notes

Removing *BC* will leave vertex *C* unconnected to the rest of the graph.

Question 20

Answer: ***A***

Explanatory notes

F – B – A – E – D – ***F – B*** *– F – C – D – C*

The part in bold repeats edge *FB*, which has already been used and connects two vertices (*F* and *C*) that do not have a direct connection.

Question 21

Answer: ***C***

Explanatory notes

For a complete graph, with n vertices:

Edges $= \frac{n(n-1)}{2} = \frac{7\times6}{2} = 21$

Alternatively, the complete graph could be drawn and the edges counted.

EXAM 2

Question 1a.

Worked solution

Possible paths (by inspection) from *A* to *E*:

A – F – E $1400 + 750 = 2150$

A – C – F – E $950 + 900 + 750 = 2600$

A – D – E $1300 + 900 = 2200$

A – B – D – E $1100 + 850 + 900 = 2850$

Therefore, the shortest path is *A – F – E.*

Mark allocation: 1 mark

- 1 mark for correct path; can also be written as *AFE*

TIP

» **Read the question carefully. Note that the length is not what is being requested here.**

Question 1b.

Worked solution

B – A – C – F – E – D – B

OR

B – D – E – F – C – A – B

Mark allocation: 1 mark

- 1 mark for either of the Hamiltonian circuits listed above

Question 1c.

Worked solution

In order for an Eulerian circuit to exist, all vertices must be of an even degree. Vertices *F* and *D* have a degree of 3, and are therefore odd.

Mark allocation: 1 mark

- 1 mark for correct statement as above, or similar

Question 1d.

Worked solution

Start at *D* and finish at *F*.

OR

Start at *F* and finish at *D*.

Mark allocation: 1 mark

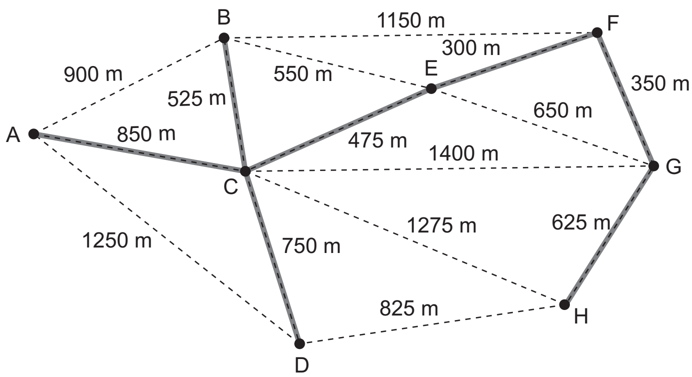
- 1 mark for one of the correct combinations above

TIP

» Remember that an Eulerian trail will start and finish at the odd vertices.

Question 2a.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct minimum spanning tree

Question 2b.

Worked solution

525 + 850 + 475 + 750 + 350 + 625 + 300 = 3875 metres

Mark allocation: 1 mark

- 1 mark for correct length

Note: This is an answer that results from your answer to **part a.** If your answer to this question is incorrect, you will need to check that your minimum spanning tree is correct.

Question 3a.

Worked solution

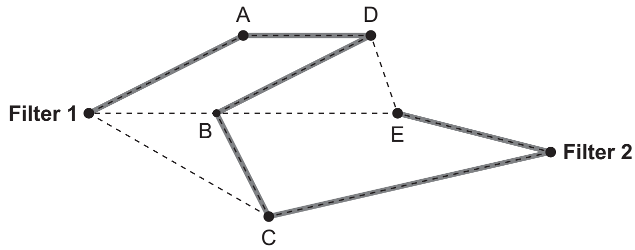
4 vertices have degree 3: Filter 1, *D*, *C* and *E*.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3b.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct answer

Note: Other possible solutions exist.

TIP

» Remember, a Hamiltonian path must pass through each vertex exactly once. A circuit would start and end at the same point.

Question 3c.

Worked solution

Possible paths include:

Filter 1 – *A* – *D* – *E* – Filter 2 $= 3 + 5 + 2 + 5 = 15$

Filter 1 – *B* – *D* – *E* – Filter 2 $= 4 + 4 + 2 + 5 = 15$

Filter 1 – *B* – *E* – Filter 2 $= 4 + 3 + 5 = 12$

Filter 1 – *C* – *B* – *D* – *E* – Filter 2 $= 6 + 3 + 4 + 2 + 5 = 20$

Filter 1 – *C* – *B* – *E* – Filter 2 $= 6 + 3 + 3 + 5 = 17$

Filter 1 – *C* – Filter 2 $= 6 + 8 = 14$.

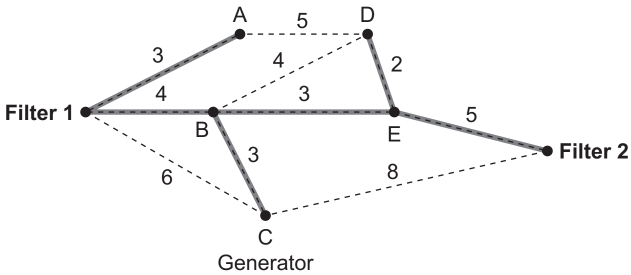
The shortest path is: Filter 1 – *B* – *E* – Filter 2.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 3d.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct answer

Question 4a.

Worked solution

fruit products and soft drinks

Mark allocation: 1 mark

- 1 mark for correct answer of both locations

Question 4b.i.

Worked solution

An Eulerian circuit requires all edges to be used once only, starting and returning to the same vertex.

Mark allocation: 1 mark

- 1 mark for Euler circuit or Eulerian circuit

Question 4b.ii.

Worked solution

All vertices must have an even number degree for this circuit to be possible.

Therefore, a path should be added between the chocolaterie and the olive grove.

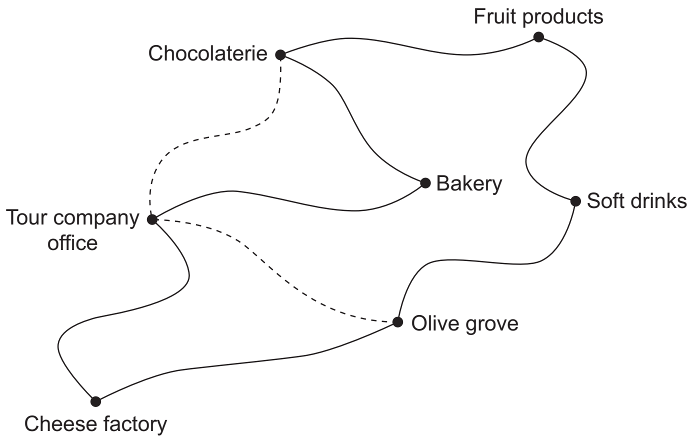
Mark allocation: 1 mark

- 1 mark for a path between chocolaterie and olive grove

Question 4b.iii.

Worked solution

Note: We are looking at a Hamiltonian cycle here. One example is shown below.



Mark allocation: 1 mark

- 1 mark for correct Hamiltonian cycle, visiting each vertex exactly once

Question 5a.

Worked solution

3 roads: between the gate – park, and the houses – tennis court – villas

Mark allocation: 1 mark

- 1 mark for the correct answer of 3

Question 5b.

Worked solution

2 roads are needed between gate – park and houses – villas (or a combination of those 4 locations)

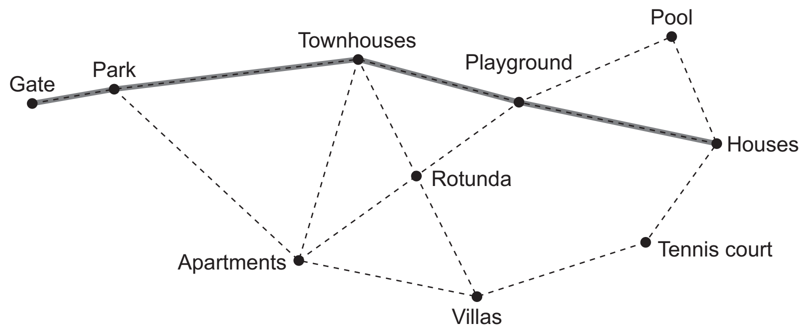
Note: Gate has a degree of 1 and park, houses and villas all have a degree of 3; all other locations have a degree of an even number. In order to produce an Eulerian circuit, as described, all vertices need to have an even degree.

Mark allocation: 1 mark

- 1 mark for identifying that two extra roads are needed between a combination of gate, park, houses and villas

Question 5c.

Worked solution



The shortest path is 760 metres, as shown above.

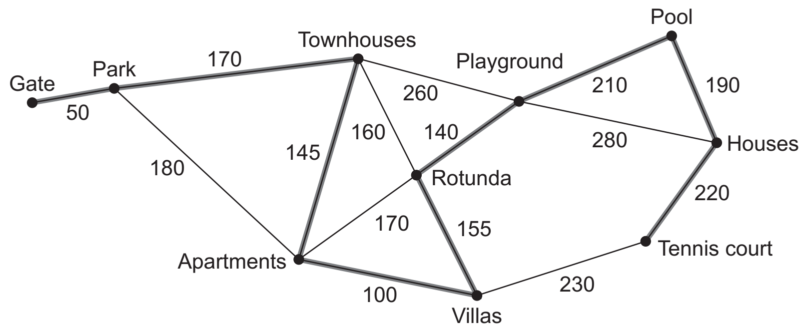
Mark allocation: 1 mark

- 1 mark for the line

Question 6d.

Worked solution

The minimum spanning tree is shown below.



The edges not included in the minimum spanning tree: $180 + 170 + 160 + 260 + 280 + 230 = 1280$ m

1280 m of cable is saved if the minimum spanning tree is used instead of running cable along every road.

Mark allocation: 1 mark

- 1 mark for the correct answer of 1280 m

Question 6a.

Worked solution

The network has 8 faces. This includes the space around the outside.

Mark allocation: 1 mark

- 1 mark for correct answer of 8

Question 6b.

Worked solution

A – *D* – Depot – *I* – *H* = 25 km

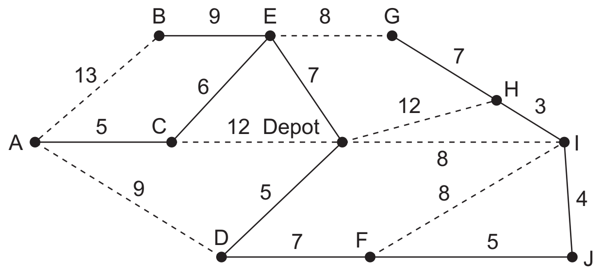
Mark allocation: 1 mark

- 1 mark for correct answer of 25

Question 6c.

Worked solution

The minimum spanning tree is drawn below.



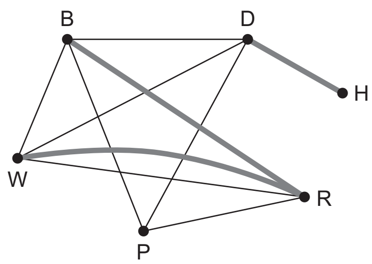
The length of the minimum spanning tree is 58 kilometres.

Mark allocation: 1 mark

- 1 mark for correct answer of 58 km

Question 7

Worked solution



Mark allocation: 1 mark

- 1 mark for all three edges added

Area of Study 2 Networks and decision mathematics – Directed graphs: flow, matching and critical path analysis

EXAM 1

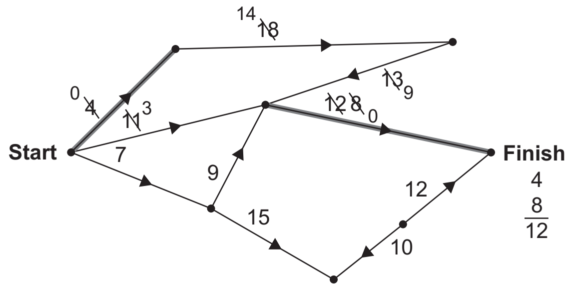
Question 1

Answer: ***A***

Explanatory notes

Maximum flow can be found using cuts, or by using a method of 'tracking' the flow through the graph.

Below is the working for the 'tracking' method. There is more than one way to track through.

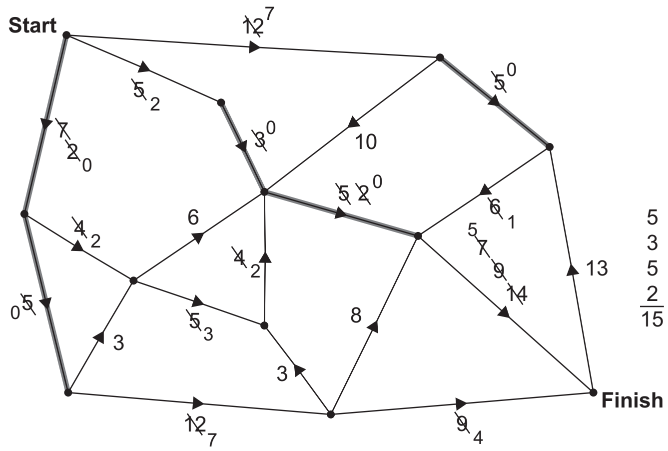


Question 2

Answer: ***C***

Explanatory notes

One way of finding the maximum flow is to use 'tracking' through the directed graph, as shown below. Cuts can also be used.



Question 3

Answer: ***C***

Explanatory notes

We can use trial and error to find the shortest distance between *A* and *D*.

Alternatively, we can use Dijkstra's algorithm to identify the path is *A* – *F* – *D*, which is 83 metres.

	B	*C*	*F*	*G*	*E*	*D*
A	72	47	$\boxed{45}$	X	X	X
F	67	$\boxed{47}$	$\boxed{45}$	X	X	83
C	$\boxed{67}$	$\boxed{47}$	$\boxed{45}$	159	X	83
B	$\boxed{67}$	$\boxed{47}$	$\boxed{45}$	159	X	$\boxed{83}$

Question 4

Answer: ***A***

Explanatory notes

Using the Hungarian algorithm to find the minimum allocation:

Step 1 – Row reduction

(Subtract the smallest number in each row from the other numbers in the row.)

	Adam	**Bryce**	**Carey**	**Donald**
Job 1	0	7	3	4
Job 2	4	0	6	3
Job 3	4	5	0	2
Job 4	2	0	1	3

Step 2 – Column reduction

(Subtract the smallest number in each column from the other numbers in the column.)

	Adam	**Bryce**	**Carey**	**Donald**
Job 1	0	7	3	2
Job 2	4	0	6	1
Job 3	4	5	0	0
Job 4	2	0	1	1

Step 3 – Determine the minimum number of lines required to cover all zeros in the table.

	Adam	**Bryce**	**Carey**	**Donald**
Job 1	0	7	3	2
Job 2	4	0	6	1
Job 3	4	5	0	0
Job 4	2	0	1	1

Only three lines are required; however, four have jobs. We therefore require an additional step.

Step 4 – Find the smallest uncovered number.

Subtract this from each uncovered number.

Add this to any number underneath a point where the lines cross.

	Adam	Bryce	Carey	Donald
Job 1	0	7	3	2
Job 2	3	0	5	0
Job 3	4	5	0	0
Job 4	1	0	0	0

Step 5 – Repeat Step 3, finding the minimum number of lines required to cover the zeros.

	Adam	Bryce	Carey	Donald
Job 1	0	4	3	2
Job 2	3	0	6	0
Job 3	4	5	0	0
Job 4	1	0	1	0

The minimum number of lines is now equal to the number of jobs.

We can do our allocations.

Step 6 – Allocate jobs to students, using zeros as a guide.

Option 1:

	Adam	Bryce	Carey	Donald
Job 1	0	8	3	2
Job 2	3	0	5	0
Job 3	4	6	0	0
Job 4	1	0	0	0

Option 2:

	Adam	Bryce	Carey	Donald
Job 1	0	8	3	2
Job 2	3	0	5	0
Job 3	4	6	0	0
Job 4	1	0	0	0

In both allocations, Adam must do Job 1.

Question 5

Answer: ***B***

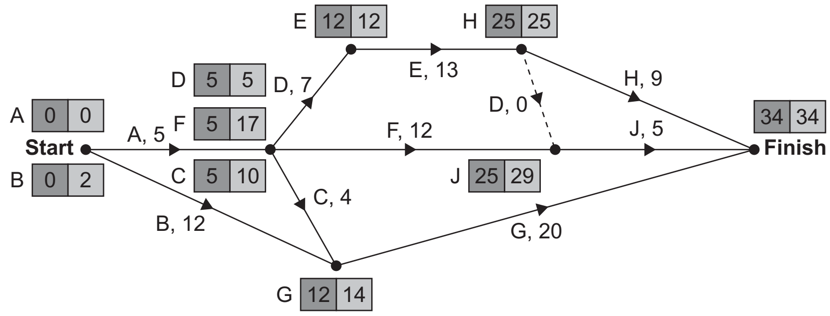
Explanatory notes

A dummy activity is required to indicate that both activity *E* and activity *F* are predecessors of activity *J*; however, only activity *E* is required for activity *H*.

Question 6

Answer: ***D***

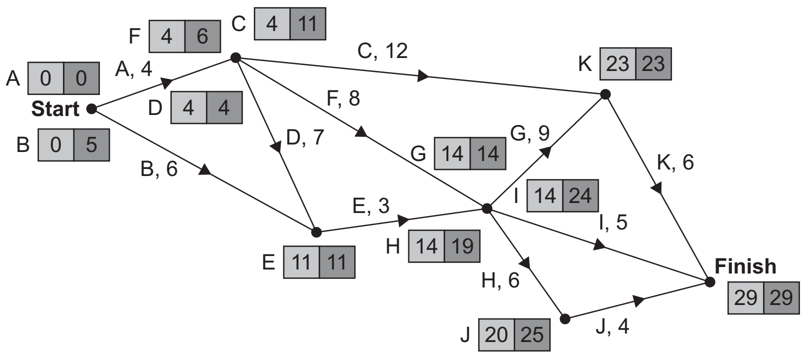
Explanatory notes



Question 7

Answer: ***C***

Explanatory notes



Question 8

Answer: ***D***

Explanatory notes

Refer to diagram in explanatory notes for Question 7.

Question 9

Answer: ***D***

Explanatory notes

Using the working from the explanatory notes for Question 8, float times are:

$B = 5$ $C = 11$ $F = 2$

$H = 5$ $I = 10$ $J = 5$

TIP

» **Only activities that are not on the critical path will have a float time. This can also be called a slack time. Any activity on the critical path must have a float time of 0, and therefore cannot be delayed.**

Question 10

Answer: ***C***

Explanatory notes

Activities *B*, *C* and *E* must all occur before activity *H*.

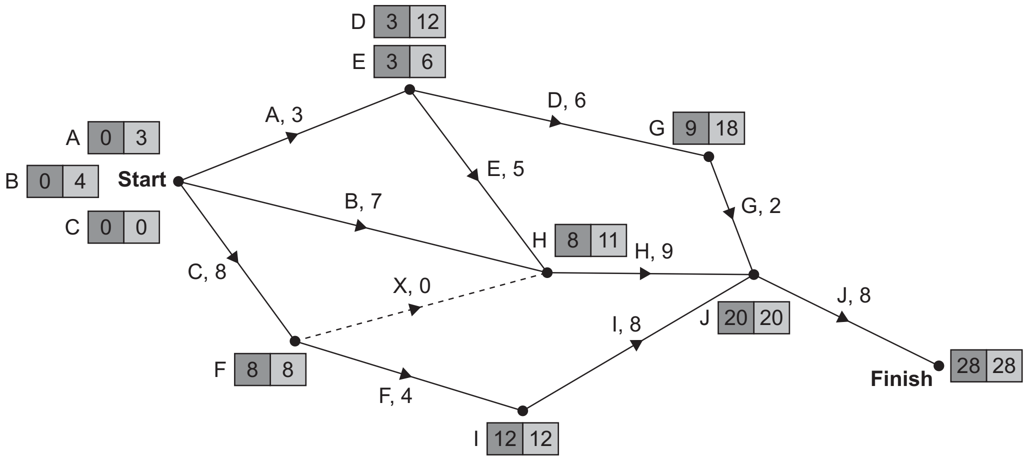
Activity *D* does not have any effect on activity *H*.

Question 11

Answer: ***D***

Explanatory notes

See working below.



TIP

» Remember that, although they do not have a weighting, 'dummy' activities indicate that a certain activity must be completed before another.

Question 12

Answer: ***D***

Explanatory notes

The diagram can also be used to find the critical path and the minimum completion time.

The critical path is comprised of all activities that have a 0 float time – that is, there is no difference between the EST and LST.

The activities that make up the critical path are: *C* – *F* – *I* – *J*.

The length of this path is 28; therefore, the minimum completion time is 28 days.

Question 13

Answer: ***B***

Explanatory notes

Cut A: $4 + 9 = 13$

Cut B: $4 + 3 + 1 + 4 = 12$ (2 is a double cut)

Cut C: $6 + 3 + 1 + 4 = 14$

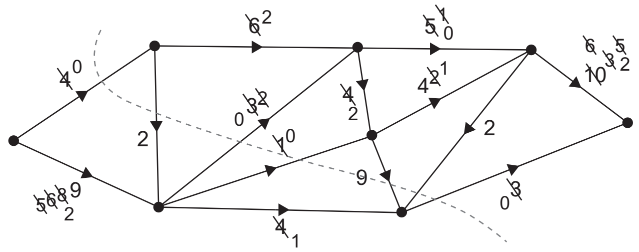
Cut D: $5 + 4 + 3 = 12$ (2 is a double cut)

Question 14

Answer: ***B***

Explanatory notes

Using the process of finding cuts, there is one cut of 11 litres per minute $(4 + 3 + 1 + 3)$.



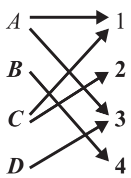
Question 15

Answer: ***B***

Explanatory notes

After row reduction: $\begin{bmatrix} 0 & 2 & 0 & 2 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 1 & 2 \\ 2 & 5 & 0 & 1 \end{bmatrix}$

After column reduction: $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 3 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 2 & 4 & 0 & 1 \end{bmatrix}$



Andy must complete Task 1.

Daisy must complete Task 3.

Callum must complete Task 2.

Bertha must complete Task 4.

Question 16

Answer: ***D***

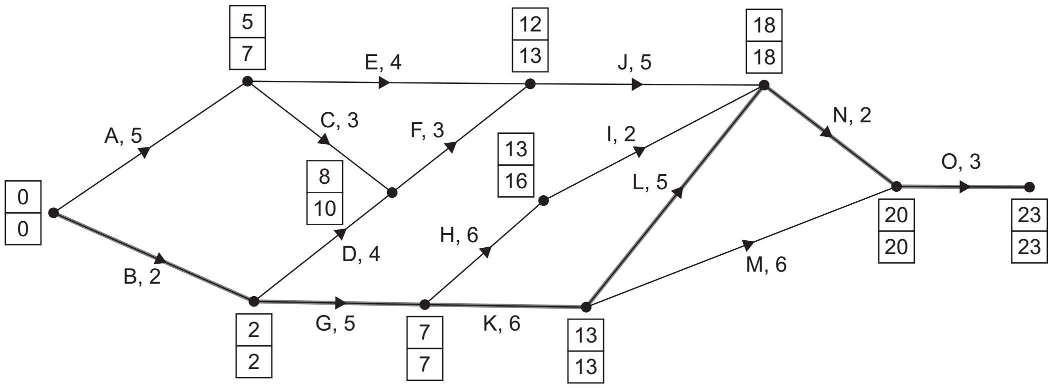
Explanatory notes

An immediate predecessor of an activity must be completed before the activity. *H* needs to be completed before *L* and *M* begin as well as before *I* begins; but *L* and *M* also need *K* to be completed before they can commence. The dummy will join at the end of *H* and the start of *L* and *M*.

Question 17

Answer: ***C***

Explanatory notes



The critical path is shown above, with a minimum completion time of 23 weeks, so options A and B are correct.

If *M* was increased to 8:

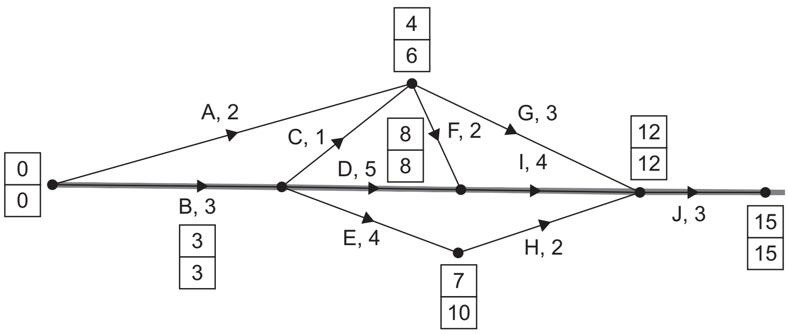
$13 + 8 = 21$, so the critical path would change to *B* – *G* – *K* – *M* – *O*, so option D is correct.

The latest *finishing* time of *F* is 13 weeks. Latest *start* time of *F* is $13 - 3 = 10$ weeks, so option *C* is incorrect.

Question 18

Answer: ***D***

Explanatory notes



The earliest starting time (EST) and latest starting time (LST) are shown above in the format

EST
LST

The critical path *BDIJ* takes 15 days, meaning the whole project will be completed in a minimum of 15 days.

Question 19

Answer: ***C***

Explanatory notes

Using the minimum completion time of 15 days, to complete it in 12 days Gianni needs to reduce the activities in the critical path by 3 days to make it 12 days.

BDIJ is the critical path, so reducing *I* by 3 days will reduce the critical path to 12 days.

Option A is incorrect because activities *A* and *G* are not on the critical path, so will not affect the minimum completion time of 15.

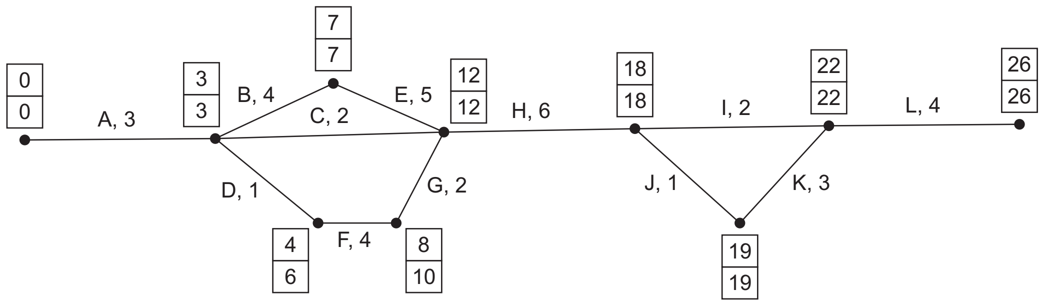
Option B is incorrect because the critical path *BDIJ* will reduce to 12; however, the path *BCFIJ* will still remain at 13 days so will become the new critical path/completion time.

Option D is incorrect because increasing an activity that is not on the critical path will not achieve the desired effect of reducing the time of the critical path.

Question 20

Answer: ***B***

Explanatory notes



Option A: Activity *E* starts at 7 days at the latest and has an activity duration of 5 days, which means it finishes at 12 days at the latest.

Option B: Activity *F* starts at 6 days at the latest and has an activity duration of 4 days, which means it finishes at 10 days at the latest.

Option C: Activity *G* starts at 10 days at the latest and has an activity duration of 2 days, which means it finishes at 12 days at the latest.

Option D: Activity *H* starts at 12 days at the latest and has an activity duration of 6 days, which means it finishes at 18 days at the latest.

Question 21

Answer: ***B***

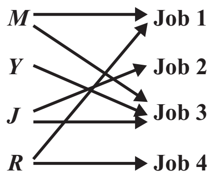
Explanatory notes

The next step in the process is *reducing the columns*, which is the subtraction of the lowest value in each column from all other values in that column:

$$\begin{bmatrix} 3 & 4 & 0 & 2 \\ 4 & 3 & 0 & 2 \\ 4 & 1 & 0 & 2 \\ 3 & 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 & 2 \\ 1 & 2 & 0 & 2 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 3 & 0 \end{bmatrix}$$

-3 -1 -0 -0

The possible allocation is shown in the bipartite graph below. This illustrates that Yvonne needs to do 3, Ryder needs to do 4, and Jackie needs to do 2. This leaves Merindah to do 1.



Question 22

Answer: ***B***

Explanatory notes

The float times for all activities not on the critical path are shown in the table below. Activity *M* has a float of 3.

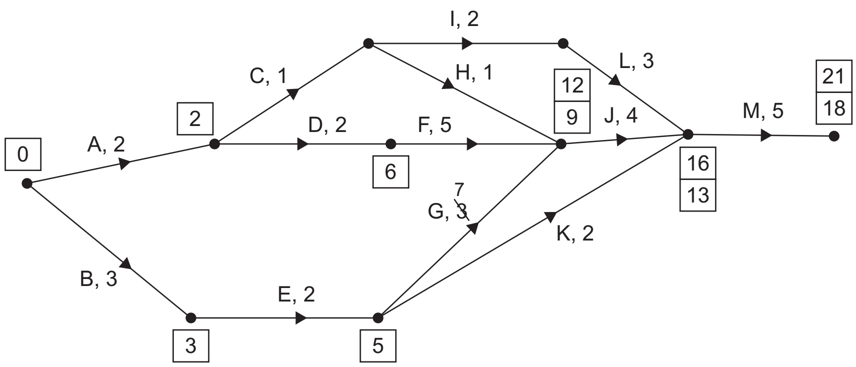
Activity	EST	LST	Float
B	2	4	2
C	2	6	4
E	6	8	2
F	7	9	2
G	6	12	6
H	8	10	2
I	12	14	2
K	9	15	6
L	15	17	2
M	12	15	3

Question 23

Answer: ***D***

Explanatory notes

The diagram shows some of the current ESTs labelled and, when *G* is increased to 7 days, the resulting new ESTs.



Or, identify all paths through the network.

A – D – F – J – M 18 days – critical path

A – C – I – L – M 13 days

A – C – H – J – M 13 days

B – E – G – J – M 17 days – new completion time of 21 days; *G* has increased by 4 days

B – E – K – M 12 days

If the duration of *G* is increased only one path is changed.

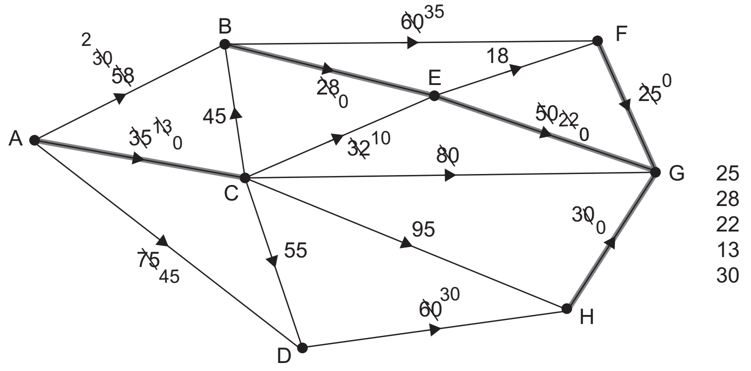
EXAM 2

Question 1

Worked solution

118 children

Below is an example of using the plotting method. Other methods for reaching the final answer exist.



Mark allocation: 1 mark

- 1 mark for correct answer of 118

Question 2a.

Worked solution

$23 + 26 + 36 + 42 = 127$

Mark allocation: 1 mark

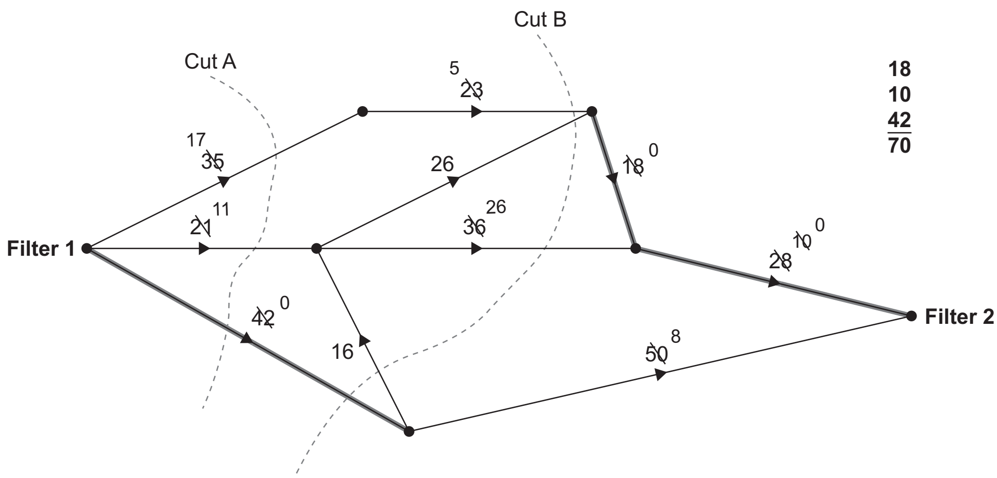
- 1 mark for correct answer

Note: The backwards edge, with a weight of 16, is not included in the total for the cut.

Question 2b.

Worked solution

The maximum flow is 70 litres per hour. One way to calculate this is to use tracking through the network.



Mark allocation: 1 mark

- 1 mark for correct answer

Question 3a.

Worked solution

The earliest start time for activity *F* is 4 days. The predecessor for *F* is Activity *B*, which takes 4 days.

Mark allocation: 1 mark

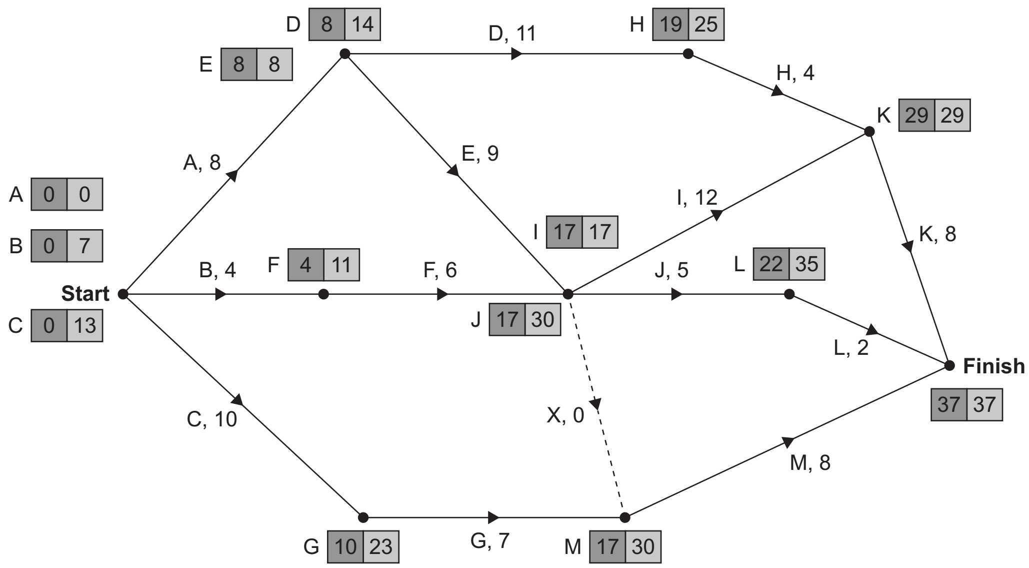
- 1 mark for an answer of 4

Question 3b.

Worked solution

A – E – I – K

Completion time is 37 days.



Mark allocation: 1 mark

- 1 mark for both the critical path and completion time

Question 3c.

Worked solution

B	EST = 0	LST = 7	Float = 7
F	EST = 4	LST = 11	Float = 7

Mark allocation: 1 mark

- 1 mark for stating both *B* and *F*

Question 3d.

Worked solution

The dummy activity is required as activities *E*, *F* and *G* are all predecessors for activity *M*. However, only activity *E* and *F* are required for activity *J*.

Mark allocation: 1 mark

- 1 mark for statement above, or equivalent

Question 3e.

Worked solution

The minimum cost is \$8000. The project can now be completed in 35 days. Only activities *E* and *I* will affect the minimum completion time for the project, as these form the critical path. If we cut *C*, *F* or *M*, the minimum time will not go below 35 days, therefore there is no need to pay for this.

Mark allocation: 1 mark

- 1 mark for \$8000

TIP

» When 'crashing' a project, start by considering activities that are on the critical path. As this is the largest path through the network, this is the way to reduce the overall completion time. Once this has been done, look at activities not on the critical path, and see if these will influence completion time.

Question 4a.

Worked solution

A, *B*, *C*, *E* and *F* are all predecessors of activity *G*.

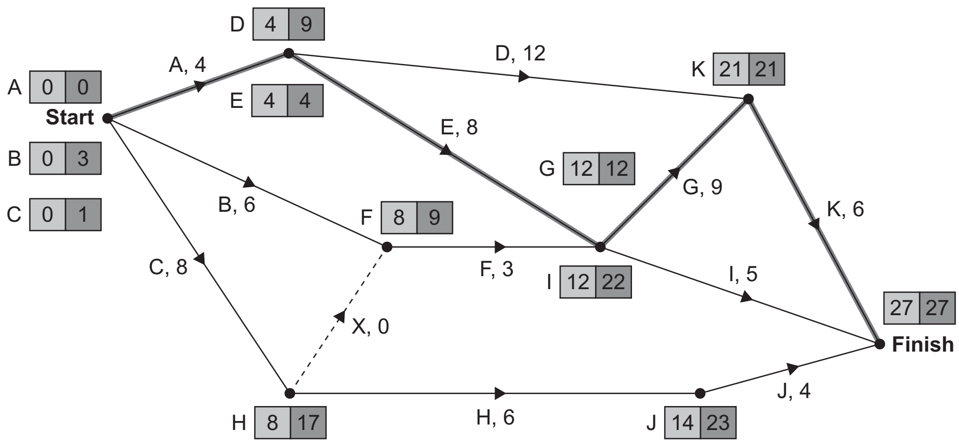
Mark allocation: 1 mark

- 1 mark for correct list of activities

Question 4b.

Worked solution

Minimum completion time is 27 days. Critical path is *A* – *E* – *G* – *K*.



Mark allocation: 2 marks

- 1 mark for correct critical path
- 1 mark for correct minimum completion time

Question 4c.

Worked solution

Float time $= \text{LST} - \text{EST} = 17 - 8 = 9$

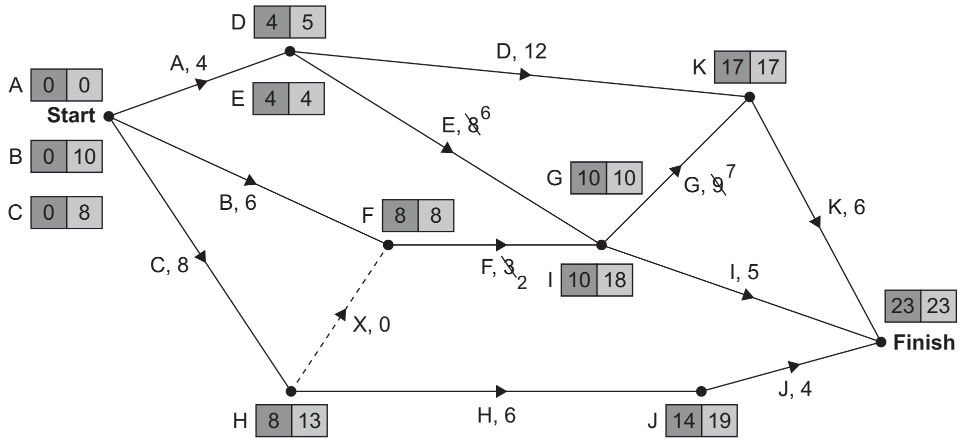
Mark allocation: 1 mark

- 1 mark for correct answer of 9

Question 4d.i.

Worked solution

We start by crashing the activities that are on the critical path, *E* and *G*.



We also need to crash activity *F* by 1 day to ensure that this does not create a new critical path (*C* – *F* – *G* – *K*). The minimum completion time is now 23 days.

Mark allocation: 1 mark

- 1 mark for correct answer of 23

Question 4d.ii.

Worked solution

The cost is calculated by:

Cutting *E* by 2 days = \$4000

Cutting *G* by 2 days = \$4000

Cutting *F* by 1 day = \$2000

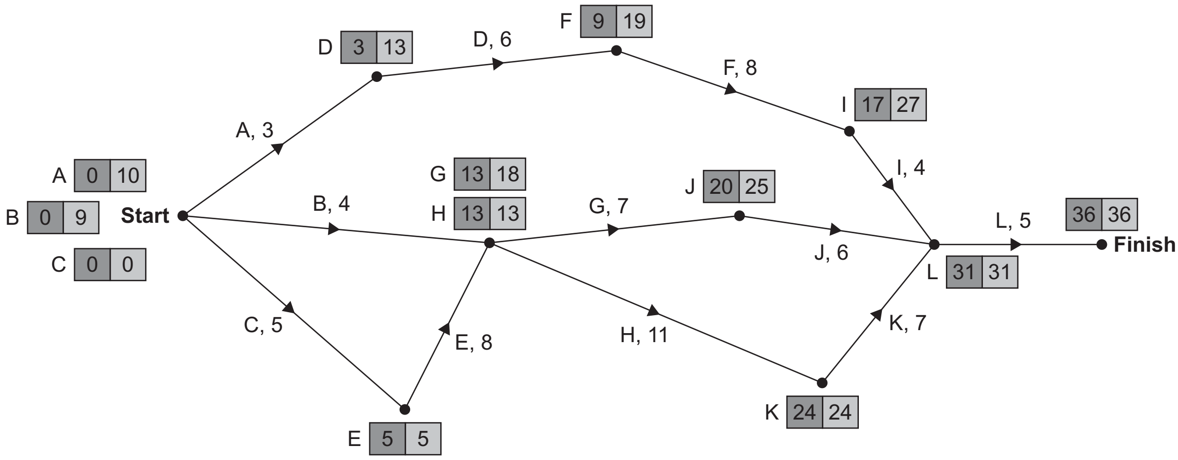
Total cost is $4000 + 4000 + 2000 = \$10\,000$

Mark allocation: 1 mark

- 1 mark for correct answer of \$10 000

Question 5a.

Worked solution



The earliest start time for activity *J* is 20.

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5b.

Worked solution

Critical path is *C* – *E* – *H* – *K* – *L*.

Note: See worked solution for **part a**. above.

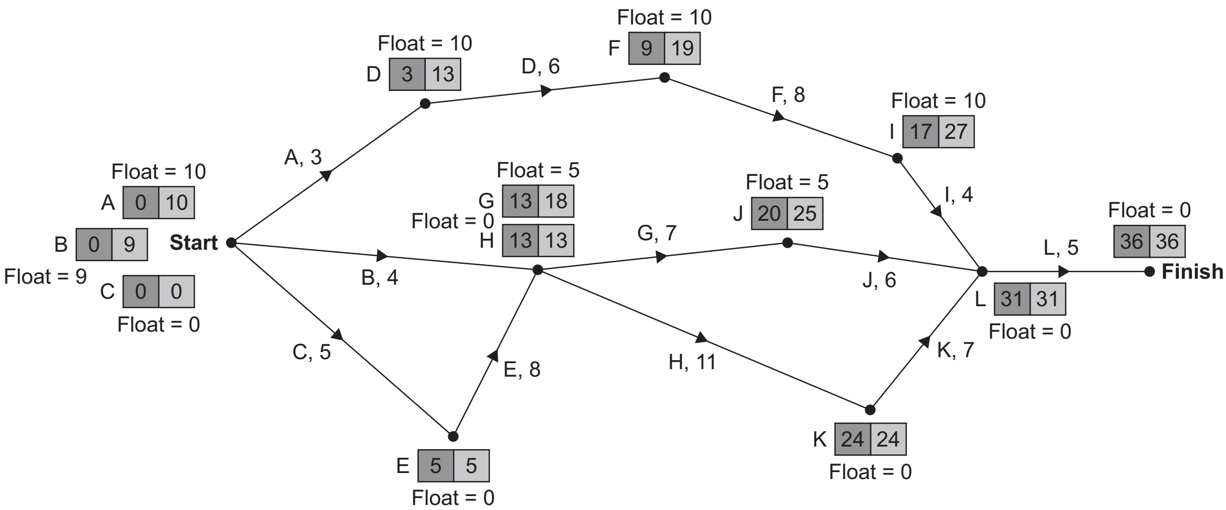
Mark allocation: 1 mark

- 1 mark for correct answer

Question 5c.

Worked solution

Float times shown on network below.



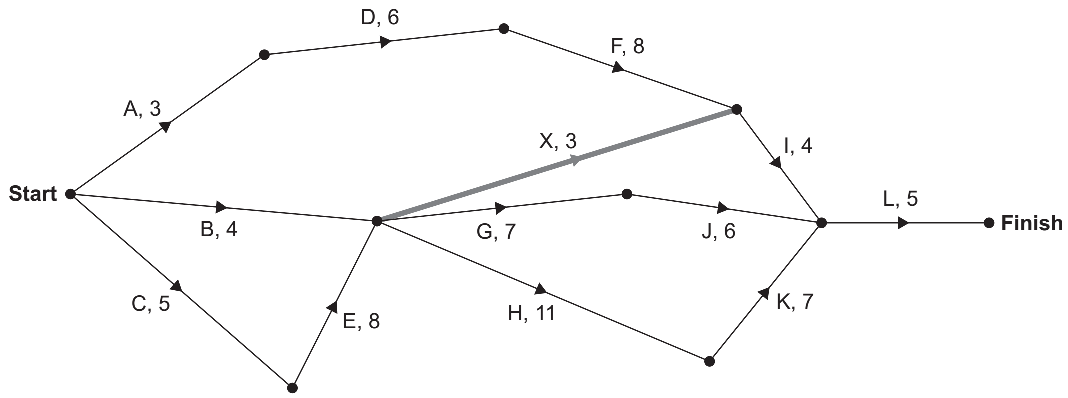
Four activities have a float time of 10 days (*A*, *D*, *F*, *I*).

Mark allocation: 1 mark

- 1 mark for correct answer

Question 5d.

Worked solution

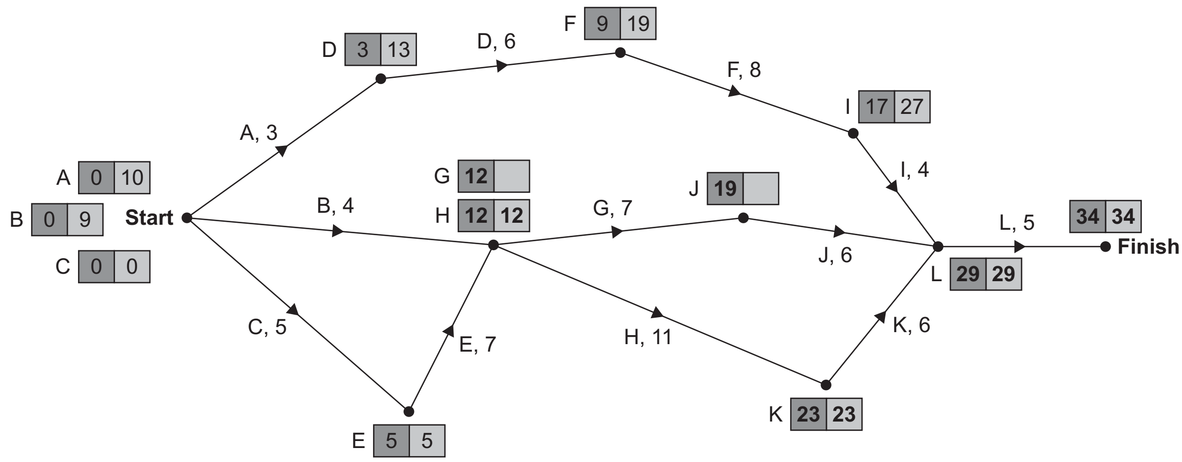


Mark allocation: 1 mark

- 1 mark for correct answer

Question 5e.

Worked solution



34 days

Mark allocation: 1 mark

- 1 mark for correct answer

Question 6a.

Worked solution

The Hungarian algorithm should be used to solve this problem.

Step 1 – row reduction

Subtract the lowest number in each row from all numbers in the row.

~~24~~ 0	~~38~~ 14	~~28~~ 4	~~42~~ 18
~~32~~ 6	~~45~~ 19	~~26~~ 0	~~46~~ 20
~~28~~ 0	~~41~~ 13	~~32~~ 4	~~43~~ 15
~~26~~ 0	~~36~~ 10	~~29~~ 3	~~39~~ 13

Step 2 – column reduction

Subtract the lowest number in each column from the other numbers. A column with a zero in it does not need this step.

~~24 0~~	~~14 4~~	~~28 4~~	~~18 5~~
~~32 6~~	~~19 9~~	~~26 0~~	~~20 7~~
~~28 0~~	~~13 3~~	~~32 4~~	~~15 2~~
~~26 0~~	~~10 0~~	~~29 3~~	~~13 0~~

Step 3 – use lines to cross out zeros

If the number of lines required equals the number of tasks to be allocated, then the process is complete. This is not the case, so we must complete another step.

~~24~~ 0	~~14~~ 4	~~28~~ 4	~~18~~ 5
~~32~~ 6	~~19~~ 9	~~26~~ 0	~~20~~ 7
~~28~~ 0	~~13~~ 3	~~32~~ 4	~~15~~ 2
~~26~~ 0	~~10~~ 0	~~29~~ 3	~~13~~ 0

Step 4 – find the lowest number that is uncovered by a line, and subtract it from all uncovered numbers

~~24~~ 0	~~4~~ 2	~~4~~ 2	~~5~~ 3
~~32~~ 6	~~19~~ 9	~~26~~ 0	~~20~~ 7
~~28~~ 0	~~3~~ 1	~~4~~ 2	~~2~~ 0
~~26~~ 0	~~10~~ 0	~~29~~ 3	~~13~~ 0

Step 5 – use lines to cross out zeros

~~24~~ 0	~~4~~ 2	~~4~~ 2	~~5~~ 3
~~32~~ 6	~~19~~ 9	~~26~~ 0	~~20~~ 7
~~28~~ 0	~~3~~ 1	~~4~~ 2	~~2~~ 0
~~26~~ 0	~~10~~ 0	~~29~~ 3	~~13~~ 0

Four lines are now required to cross out the zeros. It is now possible to allocate tasks.

Step 6 – allocate tasks

~~24~~ 0	4 2	4 2	~~5~~ 3
~~32~~ 6	~~19~~ 9	~~26~~ 0	~~20~~ 7
~~28~~ 0	~~3~~ 1	4 2	~~2~~ 0
~~26~~ 0	~~10~~ 0	~~29~~ 3	~~13~~ 0

Final allocation:

Staff member	Clean-up task
Jess	package remaining goods for sale the next day
Adriano	deep clean the ovens
Tiana	sweep and wash all floors
Vish	disinfect all surfaces

Mark allocation: 1 mark

- 1 mark for assigning Jess to package remaining goods for sale the next day

Question 6b.

Worked solution

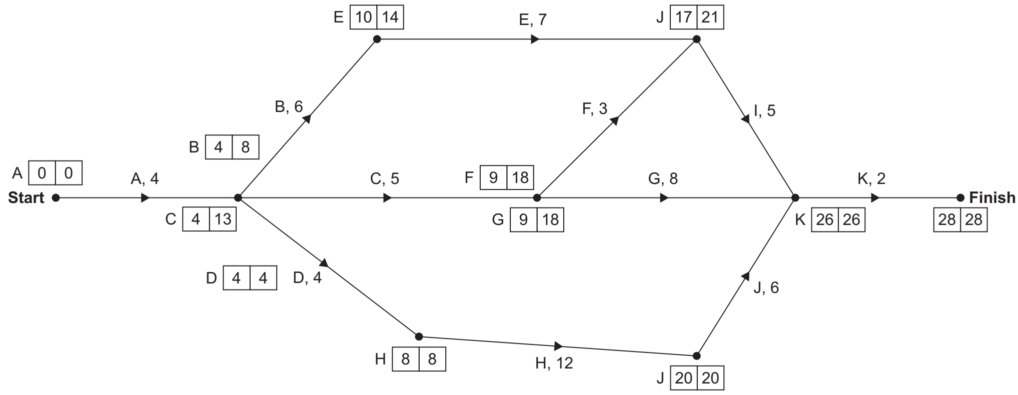
Jess and Adriano can complete their tasks simultaneously. Jess will take 24 minutes to package the goods, and Adriano will take 26 minutes to deep clean the ovens. Therefore, after 26 minutes, Vish can disinfect the surfaces. This will take him 36 minutes. Finally, Tiana can sweep and wash the floors. The minimum time is therefore $26 + 36 + 43 = 105$ minutes.

Mark allocation: 2 marks

- 2 marks for correct time allocation and final time of 105 minutes

Question 7a.

Worked solution



As can be seen from the diagram above, the minimum completion time for this project is 28 weeks.

Mark allocation: 1 mark

- 1 mark for correct answer of 28 weeks

TIP

» Although the above diagram cannot count as working, it is a very useful way to plot your way through the network to find the earliest start time (EST) and latest start time (LST) for each activity.

Question 7b.

Worked solution

Activities with no float time (LST – EST) lie on the critical path. All others do not. As can be seen from the diagram in **part a**., the critical path is *A* – *D* – *H* – *J* – *K*, so there are 6 activities that do not lie on the critical path (*B*, *C*, *E*, *F*, *G*, *I*).

Mark allocation: 1 mark

- 1 mark for correct answer of 6

Question 7c.

Worked solution

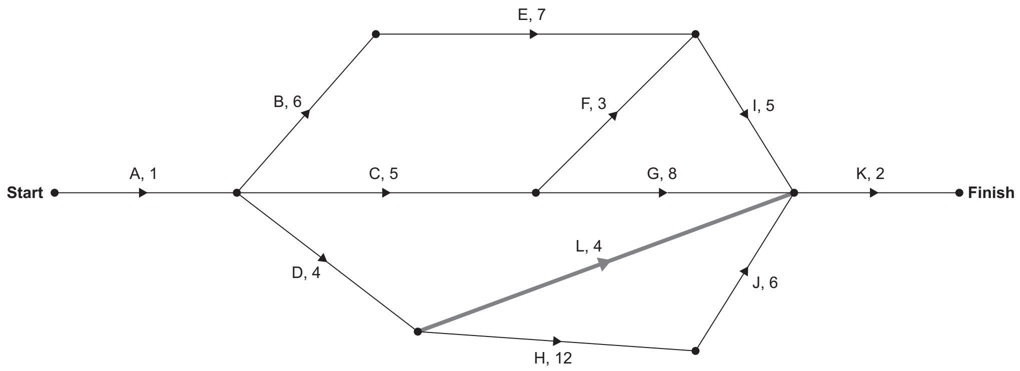
This can again be found on the diagram in **part a**. The earliest start time for activity *G* is 9 weeks.

Mark allocation: 1 mark

- 1 mark for correct answer of 9

Question 7d.i.

Worked solution



Mark allocation: 1 mark

- 1 mark for correct line with label *L*, 4

Question 7d.ii.

Worked solution

The minimum completion time for the project won't change.

Mark allocation: 1 mark

- 1 mark for answer of no change or 28 weeks

Question 8a.

Worked solution

There are seven routes:

2 – 4 – 7

2 – 3 – 3 – 7

2 – 3 – 6 – 2 – 7

2 – 3 – 7

2 – 6 – 2 – 7

3 – 1 – 3 – 7

3 – 1 – 6 – 2 – 7

Mark allocation: 1 mark

- 1 mark for the correct answer of seven routes

Question 8b.

Worked solution

The capacity is $4 + 3 + 2 + 5 + 6 = 20$ people.

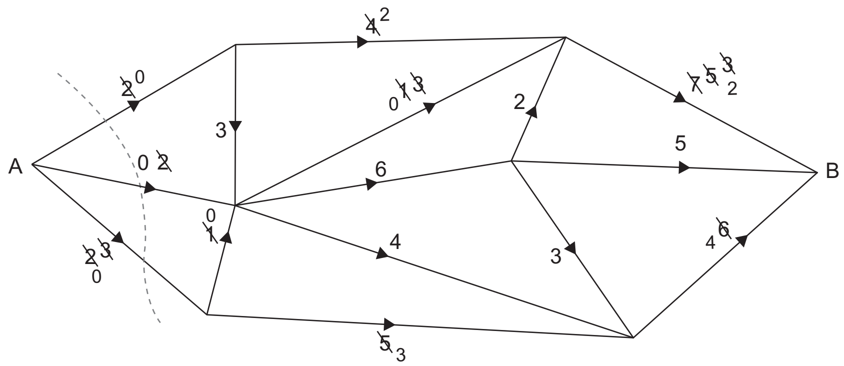
Mark allocation: 1 mark

- 1 mark for the correct answer of 20 people

Question 8c.

Worked solution

Using the process of finding cuts, there is one cut of capacity 7 per minute $(2+2+3)$.



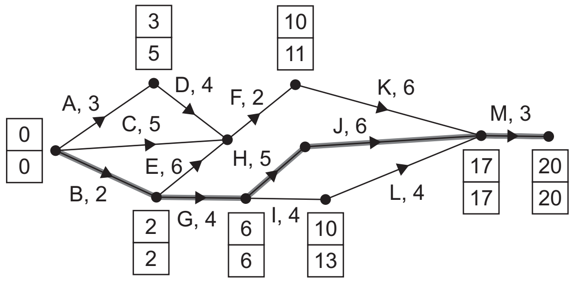
Mark allocation: 1 mark

- 1 mark for the correct answer of 7

Question 9a.

Worked solution

The earliest start times (EST) and latest finishing times (LFT) are shown below, determining the critical path of *B – G – H – J – M*.



Activity	Time	EST	LFT	LST	Float
A	3	0	8	5	5
B	2	0	2	0	0
C	5	0	12	7	7
D	4	3	12	8	5
E	6	2	12	6	4
F	2	8	11	9	1
G	4	2	6	2	0
H	5	6	11	6	0
I	4	6	13	9	3
J	6	11	17	11	0
K	6	10	17	11	1
L	4	10	17	13	3
M	3	17	20	17	0

Mark allocation: 1 mark

- 1 mark for the correct answer of *B – G – H – J – M*

Question 9b.

Worked solution

The latest start time (LST) for activity *I* is LFT – duration $= 13 - 4 = 9$

$\text{Float} = \text{LST} - \text{EST} = 9 - 6 = 3 \text{ weeks}$

Mark allocation: 1 mark

- 1 mark for the correct answer of 3 weeks

Question 9c.

Worked solution

If activity *C* is increased by four weeks, *C* – *F* – *K* – *M* could also become a critical path of 20 weeks.

Mark allocation: 1 mark

- 1 mark for the correct answer of activity *C*

Question 9d.

Worked solution

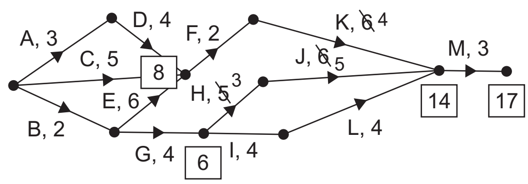
B – *G* – *I* – *L* – *M* is unable to be changed so will have to remain at 17 weeks, so the other paths must also match that.

Either *H* or *J* can only be reduced by one week instead of the maximum two-week reduction in order for the path *B* – *G* – *H* – *J* – *M* to remain 17 weeks in duration.

So activities *H*, *J* and *K* will be reduced, resulting in a new completion time of 17 weeks.

Alternatively:

Reducing both *E* and *K* by one week each will also result in *B* – *E* – *F* – *K* – *M* taking 17 weeks.



Mark allocation: 2 marks

- 1 mark for identifying activities *H*, *J* and *K* or *E* and *K*
- 1 mark for identifying 17 weeks as the new completion time

Question 10

Worked solution

Train	Driver
A	Jamal
B	Campbell
C	Brooke
D	Leith

Mark allocation: 1 mark

- 1 mark for correct allocation as shown above

Question 11a.

Worked solution

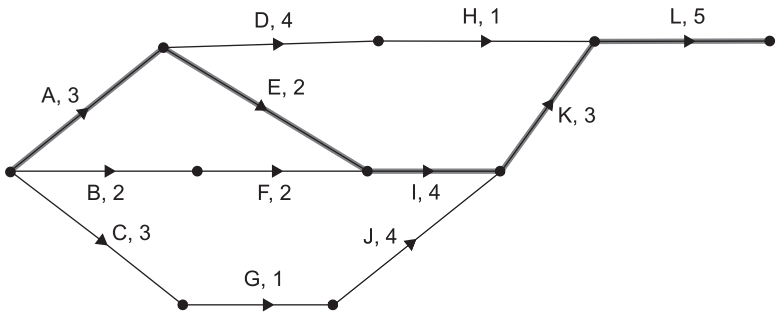
I, *K* and *L* all have two activities that need to be completed before they can begin.

Mark allocation: 1 mark

- 1 mark for correct answer of *I*, *K* and *L*

Question 11b.

Worked solution



Mark allocation: 1 mark

- 1 mark for correctly tracing path *A* – *E* – *I* – *K* – *L*

Question 11c.

Worked solution

17 days, as the dummy has not impacted the minimum completion time

Mark allocation: 1 mark

- 1 mark for correct answer of 17

Question 11d.

Worked solution

Friday 18 January

The EST for activity *G* is three days, and the LST is four days.

This means the earliest activity *G* can start is after three days (when activity *C* is completed): Thursday 17 January; but the *latest* it can start is after four days: Friday 18 January.

Mark allocation: 1 mark

- 1 mark for correct answer of Friday 18 January

Question 11e.

Worked solution

Activities that may be reduced that are on the critical path are: *J*, *K*, *L*.

All three activities can be reduced without changing the direction of the critical path.

The changes result in a new minimum completion time of 15 days.

Mark allocation: 1 mark

- 1 mark for correct answer of 15 days

Question 11f.

Worked solution

$J \times 1 = 200$

$K \times 2 = 150 \times 2 = 300$

$L \times 2 = 250 \times 2 = 500$

Total extra cost $= \$1000$

Mark allocation: 1 mark

- 1 mark for correct answer of $1000

