• Chapter One •

Percentages

Overview

The term ‘per cent’ means ‘out of 100’ and is indicated with the symbol %.

A percentage can therefore also be written as a fraction with the denominator **100**.

This chapter builds on the work covered in Year 7 on decimals and percentages in which learners converted between percentages, decimals and fractions. In Year 8, this topic continues with conversions between percentages, decimals and fractions, and then looks at how percentages are used to find quantities, and how quantities can be expressed as percentages. The applications of percentages to topics such as discount and interest, and calculating percentage increases and decreases are also covered.

Percentages have important uses in many subject areas. For example, percentages are used in agriculture to measure fertilisers, to budget, and to calculate available feed. They are also widely used in science, home economics, business studies and many other areas that affect our everyday lives.

Solomon Islanders frequently apply the concept of percentages in their lives, although they may not use the term directly. For example, percentages are used indirectly when sharing foods in the village, catching different types of fish, and harvesting and selling garden products.

Contents

Chapter skills

This chapter covers the following skills:

- Learning the definition of a percentage
  Per cent means ‘out of 100’
- Converting percentages to fractions and decimals
- Converting fractions and decimals to percentages
- Finding percentages of quantities
- Expressing a quantity as a percentage
- Applying percentages to situations of discount and interest
- Calculating percentage increase and decrease
- Percentage change

$$= \frac{\text{Increase or decrease in a quantity}}{\text{original quantity}} \times 100$$

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 1A Introducing percentages	Learner’s Book 1 • Exercise 1A, pages 4, 5, 6
2	• 1B Percentages to fractions • 1C Fractions to percentages	Learner’s Book 1 • Exercise 1B, page 7 Learner’s Book 1 • Exercise 1C, page 8
3	• 1D Percentages to decimals • 1E Decimals to percentages	Learner’s Book 1 • Exercise 1D, page 9 Learner’s Book 1 • Exercise 1E, page 10
4–5	• 1F Finding percentages of quantities	Learner’s Book 1 • Exercise 1F, page 11
6–7	• 1G Expressing a quantity as a percentage	Learner’s Book 1 • Exercise 1G, page 12
8–9	• 1H Percentage increase and decrease	Learner’s Book 1 • Exercise 1H, page 10
10	• 1I Exploring percentages	Learner’s Book 1 • Learning task 1I, pages 14, 15
11–13	• 1J Discount	Learner’s Book 1 • Exercise 1J, pages 16, 17
14	• 1K Exploring shopping	Learner’s Book 1 • Learning task 1K, pages 18, 19
15	• Test	Teacher’s Guide • Chapter 1 Test

General learning outcomes

Learners should:

Introducing percentages

8.1.1 Understand percentage is a fraction with a special denominator of 100. Per cent means 'out of 100'. (U)

8.1.2 Know how to express percentages by shading boxes and other quantities. (K)

Percentages to fractions

8.1.3 Know how to express percentages as fractions in their simplest form. (K)

Fraction to percentages

8.1.4 Know how to change fractions to percentages. (K)

Percentages to decimals

8.1.5 Know how to convert percentages to decimals. (K)

Decimals to percentages

8.1.6 Know how to convert decimals to percentages. (K)

Finding percentages of quantities

8.1.7 Know how to use percentages to find unknown quantities. (K)

Expressing a quantity as a percentage

8.1.8 Understand that a quantity can be expressed as percentage of another quantity. (U)

Percentage increase and decrease

8.1.9 Know how to apply formula to calculate percentage increase and decrease. (K)

Exploring percentages

8.1.10 Understand how percentages are used to compare quantities and express changes in numbers. (U)

Discounts

8.1.11 Understand that discounts occur when prices on items are reduced by a certain percentages. (U)

8.1.12 Know how to calculate discounts by subtracting the discounted amount from the original price. (K)

Exploring shopping

8.1.13 Understand how percentages are used in the selling and buying of merchandise goods. (U)

1A • Introducing percentages

LB1 pages 4–6

Specific learning outcomes

Leaners should be able to:

8.1.1.1 Define percentage and identify the symbol %.

8.1.2.1 Use diagrams to represent percentages.

8.1.2.2 Express quantities as percentages and fractions.

Teaching points

1. Explain what a percentage is, and give the symbol used to represent it.
2. Use shaded parts of given diagrams to represent fractions, decimals and percentages of a whole.
3. Convert fractions and decimals to percentages, and vice versa.
4. Apply the percentages of quantities to real life applications, including percentage increase and decrease, and discount.

Learner difficulties and remedies

Difficulty

Understanding the concept of a percentage, which is the numerator of a special fraction that has a denominator of 100.

Remedy

- Explain to learners that a percentage is a special fraction that must always be out of 100.

Suggested teaching approach

- Review the work on percentages that was covered in Year 7.
- Explain the concept of percentages. Also, show the symbol that is used for percentages.
  - Percentage: A special fraction with a denominator of 100.
  - Symbol: %
- Use the activities in the LB to help in the explanation of percentages. Colouring in percentages of shapes should help learners to visualise different percentages.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The term 'per cent' means 'out of 100'. A per cent is a 'part out of every hundred' or 'for every hundred'.

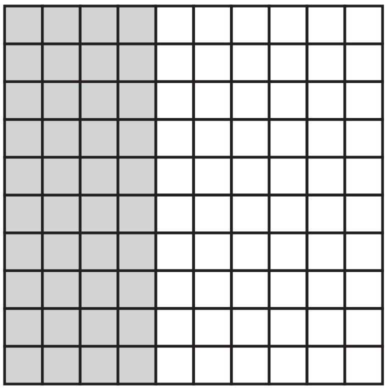
The symbol for per cent is %.

Examples

1 What does 50% mean?

Thinking	**Working**
50% means 50 out of 100 or $\frac{50}{100}$.	We know that $\frac{50}{100}$ simplifies to $\frac{1}{2}$, so we use the terms 50% and one-half interchangeably.

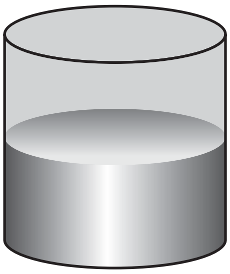
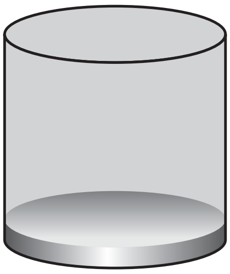
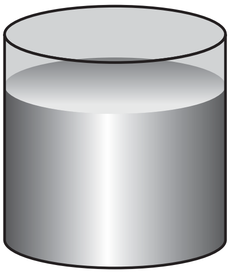
2 What percentage of the following grid is shaded?



Thinking	**Working**
40 out of 100 of the squares are shaded.	40 per cent or 40% of the grid is shaded.

3 Draw containers to demonstrate the following percentages.

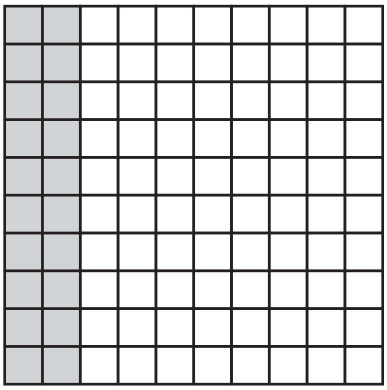
a 50% full **b** 10% full **c** 75% full

Thinking	Working
a 50 parts out of 100 is $\frac{50}{100}$ or $\frac{1}{2}$, so shade half.	**a** 
b 10 parts out of 100 is $\frac{10}{100}$ or $\frac{1}{10}$, so shade one-tenth of the container.	**b** 
c 75 parts out of 100 is $\frac{75}{100}$ or $\frac{3}{4}$, so shade three-quarters of the container.	**c** 

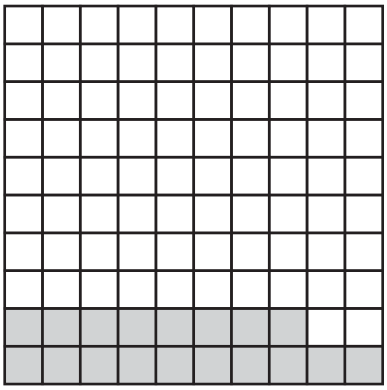
Activity 1A

1 What percentage of the whole square is shaded in each of the diagrams?

a

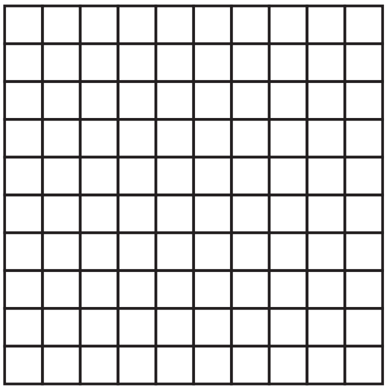


b

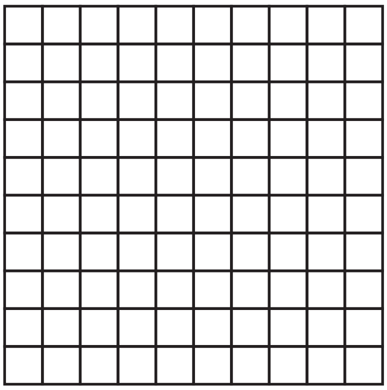


2 Shade these grids to show these percentages

a 75%

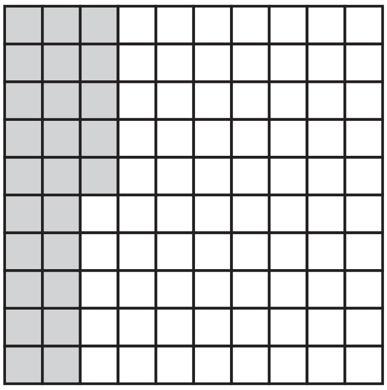


b 6%

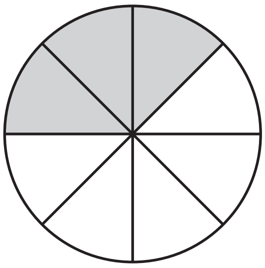


3 Babalu has worked out the percentage of the areas shaded as follows.

a 24% shaded



b 30% shaded



Explain Babalu's mistake in his calculations for each shape.

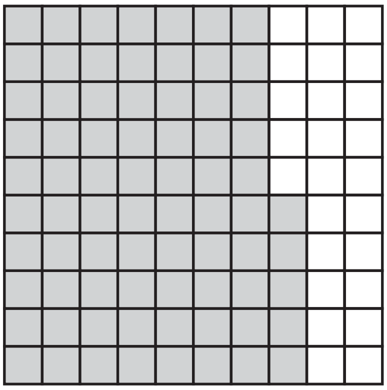
Answers 1A

1 **a** 20%

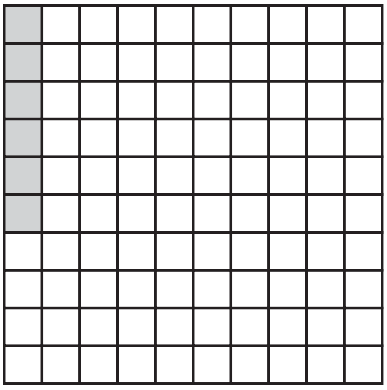
  b 18%

2 Shade these grids to show these percentages

  a 75%



  b 6%



3 **a** Babalu has assumed that the large square is a 10 × 10 grid with 24 shaded squares.

$\frac{24}{100} = 24\%$ is incorrect

In fact it is a 8 × 8 square, so there are 64 squares.

$\frac{24}{64} = 37{\cdot}5\%$

  b Babalu has assumed that the circle has been divided into 10 sectors.

$\frac{3}{10} = 30\%$ is incorrect

In fact it has 8 sectors.

$\frac{3}{8} = 37{\cdot}5\% = 30\%$

1B • Percentages to fractions

LB1 page 7

Specific learning outcomes

Learners should be able to:

8.1.3.1 Change percentages into fractions in their lowest form.

Teaching points

1 Change percentages to fractions by writing the percentage values over 100, and then simplifying. To do this, divide the numerator and denominator by the largest common factor to reduce the fraction to its simplest form.

Learner difficulties and remedies

Difficulty

Simplifying fractions.

Remedy

- When simplifying fractions, look for a largest common number (LCN) that can cancel both the numerator and the denominator. The new equivalent fraction should be the simplified fraction after the cancellation.
- Provide additional examples and exercises on simplifying fractions to help learners understand the concept.

Difficulty

Changing mixed number percentages into fractions.

Remedy

- Follow these steps to change mixed number percentages into fractions.
  - Multiply the whole number by the denominator.
  - Add the product of the two numbers to the numerator.
  - Write the sum of the two numbers over the denominator of the original fraction to create the new improper fraction.
  - Divide the improper fraction by 100.
  - Simply where possible.

Difficulty

Converting a percentage that is more than 100 to a fraction.

Remedy

- Write the percentage as a fraction out of 100.
- Simplify the fraction. It should be an improper fraction.
- Rewrite the improper fraction as mixed number.

Suggested teaching approach

- Solomon Islanders have many names. A person could have three, four, or up to seven names, but they are still the same person. This is similar when changing a percentage to a fraction. The quantity is the same, but is simply expressed in a different form.
- Start the unit with the following activity to remind learners that a person can have different names, just as a quantity can be expressed in different ways: percentage, fraction or decimal.
  - Ask learners how many names they have.
  - What is the cultural significance of their names?
- Stress the fact that a percentage is actually a special fraction out of 100, but instead of expressing it as a fraction, the percentage symbol is used.
- When simplifying fractions, use a common number to cancel both the numerator and denominator.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Percentages can be thought of as special fractions that have a denominator of 100. We use this idea to change percentages to fractions, and then simplify if possible.

Examples

1 Write the following percentages as fractions in simplest form.

a 16%

Thinking	**Working**
1 Divide the value of the percentage by 100. **2** Write the division as a fraction and simplify if possible.	$16\% = 16 \div 100$ $= \frac{16}{100}$ $= \frac{4}{25}$

b 120%

Thinking	**Working**
1 Divide the value of the percentage by 100. **2** Write the division as a fraction and simplify if possible.	$120\% = 120 \div 100$ $= \frac{120}{100}$ $= \frac{6}{5}$ $= 1\frac{1}{5}$

2 Write the following fractional percentages as fractions in simplest form.

a $\frac{1}{4}\%$

Thinking	**Working**
1 Divide the value of the percentage by 100. **2** Perform the division (recall that to divide by a fraction, you must multiply by its inverse). **3** Write the answer, simplifying if possible.	$\frac{1}{4}\% = \frac{1}{4} \div 100$ $= \frac{1}{4} \div \frac{100}{1}$ $= \frac{1}{4} \times \frac{1}{100}$ $= \frac{1}{400}$

b $6\frac{2}{3}\%$

Thinking	**Working**
1 Divide the value of the percentage by 100. **2** Write both values as improper fractions so that the division can be performed. **3** Perform the division (recall that to divide by a fraction, you must multiply by its inverse). **4** Write the answer, simplifying if possible.	$6\frac{2}{3}\% = 6\frac{2}{3} \div 100$ $= \frac{20}{3} \div \frac{100}{1}$ $= \frac{20}{3} \times \frac{1}{100}$ $= \frac{20}{300}$ $= \frac{2}{30}$ $= \frac{1}{15}$

Activity 1B

1 Write the following percentages as fractions in simplest form.

a 17% **b** 48% **c** 9%
d 65% **e** 58% **f** 76%
g 117% **h** 129% **i** 240%
j 315% **k** 138% **l** 360%

2 Write the following fractional percentages as fractions in the simplest form.

a $\frac{1}{2}\%$ **b** $\frac{1}{5}\%$ **c** $\frac{3}{7}\%$
d $\frac{3}{8}\%$ **e** $\frac{1}{10}\%$ **f** $\frac{2}{3}\%$
g $\frac{5}{6}\%$ **h** $\frac{2}{9}\%$ **i** $2\frac{1}{4}\%$
j $6\frac{1}{2}\%$ **k** $5\frac{3}{5}\%$ **l** $8\frac{3}{4}\%$

3 Write these fractions in their simplest form.

a $12\frac{1}{2}\%$ **b** $16\frac{2}{3}\%$ **c** $2\frac{1}{8}\%$

4 Of the leaners from Marara CHS, Tanagai, 76% passed their Year 9 Annual Mathematics National Examination.

a What fraction of the Year 9 learners passed the examination?

b What fraction did not pass the examination?

Answers 1B

1 **a** $\frac{17}{100}$ **b** $\frac{12}{25}$ **c** $\frac{9}{100}$
d $\frac{13}{20}$ **e** $\frac{29}{50}$ **f** $\frac{19}{25}$
g $1\frac{17}{100}$ **h** $1\frac{29}{100}$ **i** $2\frac{2}{5}$
j $3\frac{3}{20}$ **k** $1\frac{19}{50}$ **l** $3\frac{3}{5}$

2 **a** $\frac{1}{200}$ **b** $\frac{1}{500}$ **c** $\frac{3}{700}$
d $\frac{3}{800}$ **e** $\frac{1}{1000}$ **f** $\frac{1}{150}$
g $\frac{1}{120}$ **h** $\frac{1}{450}$ **i** $\frac{9}{400}$
j $\frac{13}{200}$ **k** $\frac{7}{125}$ **l** $\frac{7}{80}$

3 **a** $\frac{1}{8}$ **b** $\frac{1}{6}$ **c** $\frac{17}{800}$

4 **a** $\frac{19}{25}$ **b** $\frac{6}{25}$

1C • Fractions to percentages

LB1 page 8

Specific learning outcomes

Learners should be able to:

8.1.4.1 Convert fractions to percentages by multiplying by 100.

8.1.4.2 Change mixed numbers to percentages by first changing the mixed numbers to improper fractions and multiplying by 100.

Teaching points

1 Convert fractions to percentages by finding an equivalent fraction that has 100 as the denominator. This is the same as multiplying the fraction by $\frac{100}{1}$. For example:

$$\frac{9}{25} = \frac{36}{100} = 36\%$$

$$\frac{9}{\cancel{25}} = \frac{9}{\cancel{25}} \times \frac{\cancel{100}^{4}}{1}\% = 36\%$$

2 Change mixed numbers to an improper fraction before converting to a percentage.

Learner difficulties and remedies

Difficulty

Changing mixed number fractions to percentages

Remedy

- The following two methods can be used to change mixed numbers to percentages.
  - Change mixed numbers into improper fraction, and then multiply by 100. Simplify where possible, and then calculate the percentage.
  - Change the mixed number into decimal, and then multiply it by 100.

Suggested teaching approach

- Explain to learners that percentages are fractions with a denominator of 100. But instead of writing it as fraction, the percentage sign % is used to show it is 'per 100' or 'out of 100'.
- Show learners how to change fractions to percentages.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Percentages can be used to compare numbers. It is easier to use percentages rather than fractions because percentages are all 'out of 100'.

Some percentages are used frequently and it is useful to automatically recall their fraction equivalent. If you don't already know them, try to learn from the table given below.

Fraction	Percentage
$\frac{1}{2}$	50%
$\frac{1}{4}$	25%
$\frac{3}{4}$	75%
$\frac{1}{3}$	$33.\dot{3}\%$
$\frac{2}{3}$	$66.\dot{6}\%$
$\frac{1}{5}$	20%
$\frac{1}{10}$	10%
$\frac{1}{20}$	5%
$\frac{1}{50}$	2%
$\frac{1}{100}$	1%

With the exception of $\frac{1}{3}$ and $\frac{2}{3}$, writing the fractions in the above table as percentages is straightforward. The denominators are factors of 100, so we can convert the fraction to a percentage by:

- multiplying the denominator by a factor that gives 100
- multiplying that same factor to the numerator to give the corresponding percentages.
- writing the numerator with a percentage sign (to show it is out of 100).

For example:

$$\underset{\times 25}{\overset{\times 25}{\frac{3}{4} = \frac{75}{100}}} = 75\% \qquad \underset{\times 20}{\overset{\times 20}{\frac{1}{5} = \frac{20}{100}}} = 20\%$$

For fractions that have denominators other than 2, 5, 10, 25 or 50 (or fractions that can be simplified to these denominators), we need to use another method.

Converting fractions to percentages

- To write a fraction to a percentage, multiply the fraction by 100% (or $\frac{100}{1}$).

There are two methods we can use to multiply a fraction by 100%.

- **Fraction multiplication**: Multiply the fraction by the percentage, as shown in Method 1 of the following Worked Example.
- **Convert the fraction to a decimal**: Convert the fraction to decimal first then multiply by 100%, as shown in Method 2 of the following Worked Example.

If we multiply a fraction or a decimal by 100%, the value of the fraction or decimal is not changed, it is just written in a different form. It is an equivalent fraction or decimal.

Examples

1 Write the following fractions as percentages. Write any answers that are not whole numbers in both fraction and exact decimal form.

a $\frac{7}{8}$ **b** $\frac{11}{15}$

Method 1: Fraction multiplication

Thinking	Working
a **1** Multiply the fraction by 100% by writing 100% as an improper fraction, then performing the multiplication. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 4.)	**a** $\frac{7}{8} \times 100\%$ $= \frac{7}{{}_2\cancel{8}} \times \frac{{}^{25}\cancel{100}}{1}\%$ $= \frac{175}{2}\%$
2 Simplify the answer.	$= 87\frac{1}{2}\%$ or 87·5%
b **1** Multiply the fraction by 100% by writing 100% as an improper fraction, then performing the multiplication. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 5.)	**b** $\frac{11}{15} \times 100\%$ $= \frac{11}{{}_3\cancel{15}} \times \frac{{}^{20}\cancel{100}}{1}\%$ $= \frac{220}{3}$
2 Simplify the answer.	$= 73\frac{1}{3}\%$ or 73·3%

Method 2: Convert to a decimal, and then multiply

Thinking	Working
a Convert the fraction to decimal form first, then multiply the decimal by 100%.	**a** $\frac{7}{8} \times 100\%$ $= 7 \div 8 \times 100\%$ $= 0{\cdot}875 \times 100\%$ $= 87{\cdot}5\%$ or $87\frac{1}{2}\%$
b Convert the fraction to decimal form first, then multiply the decimal by 100%. (When Working with recurring decimals, write out a couple of decimal places so you can place the decimal point in the correct position.)	**b** $\frac{11}{15} \times 100\%$ $= 11 \div 15 \times 100\%$ $= 0{\cdot}7333\ldots \times 100\%$ $= 73{\cdot}3\%$ or $73\frac{1}{3}\%$

2 Write the given fraction as percentage: $3\frac{1}{2}$

Method 1: Fraction multiplication

Thinking	Working
1 Write the mixed number as an improper fraction. Write 100% as an improper fraction, then multiply the two fractions. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 2.)	$3\frac{1}{2} \times 100\%$ $= \frac{7}{{}_{1}\cancel{2}} \times \frac{{}^{50}\cancel{100}}{1}\%$ $= \frac{350}{1}\%$
2 Simplify the answer.	$= 350\%$

Method 2: Convert to a decimal, and then multiply

Thinking	Working
Convert the mixed number or decimal form first, then multiply the decimal by 100%	$3\frac{1}{2} \times 100\%$ $= 3{\cdot}5 \times 100\%$ $= 350\%$

Activity 1C

1 Write the following fractions as percentages. Write any answers that are not whole numbers in both fraction and exact decimal form.

a $\frac{9}{100}$ **b** $\frac{1}{10}$ **c** $\frac{7}{50}$

d $\frac{19}{20}$ **e** $\frac{3}{5}$ **f** $\frac{1}{4}$

g $\frac{3}{2}$ **h** $\frac{16}{8}$ **i** $\frac{32}{40}$

j $\frac{56}{80}$ **k** $\frac{9}{16}$ **l** $\frac{14}{32}$

m $\frac{73}{80}$ **n** $\frac{1}{3}$ **o** $\frac{2}{3}$

p $\frac{1}{6}$ **q** $\frac{2}{9}$ **r** $\frac{13}{60}$

s $\frac{55}{66}$ **t** $\frac{23}{90}$

2 Write the following as percentages. Give any answers that are not whole numbers in exact decimal form.

a $1\frac{1}{4}$ **b** $1\frac{2}{5}$ **c** $3\frac{1}{5}$

d $5\frac{1}{2}$ **e** $2\frac{3}{8}$ **f** $2\frac{3}{4}$

g $5\frac{7}{10}$ **h** $4\frac{5}{8}$ **i** $4\frac{2}{3}$

j $5\frac{1}{3}$ **k** $1\frac{7}{9}$ **l** $7\frac{5}{6}$

3 Copy and complete the table of commonly used fractions, decimals and percentages.

Percentage	Fraction	Decimal
5%		
		0·1
	$\frac{1}{8}$	
20%		
		0·25
	$\frac{1}{3}$	
40%		
		0·5
	$\frac{3}{5}$	
		$0{\cdot}\dot{6}$
75%		
	$\frac{4}{5}$	
100%		

4 Adomea estimated that $\frac{7}{10}$ of the crowd at a soccer match at Lawson Tama were Kossa soccer club supporters. What percentage is this?

5 Rebeka needs to score at least 80% on her mathematics exam to pass the year. She knows that the exam has 40 questions. How many questions does she needs answer correctly to pass the exam?

Answers 1C

1 **a** 9% **b** 10%

c 14% **d** 95%

e 60% **f** 25%

g 150% **h** 200%

i 80% **j** 70%

k 56·25% or $56\frac{1}{4}\%$ **l** 43·75% or $43\frac{3}{4}\%$

m 91·25% or $91\frac{1}{4}\%$ **n** $33.\dot{3}\%$ or $33\frac{1}{3}\%$

o $66.\dot{6}\%$ or $66\frac{2}{3}\%$ **p** $16.\dot{6}\%$ or $16\frac{2}{3}\%$

q $22.\dot{2}\%$ or $22\frac{2}{9}\%$ **r** $21.\dot{6}\%$ or $21\frac{2}{3}\%$

s $83.\dot{3}\%$ or $83\frac{1}{3}\%$ **t** $25.\dot{5}\%$ or $25\frac{5}{9}\%$

2 **a** 125% **b** 140% **c** 320%

d 550% **e** 237·5% **f** 275%

g 570% **h** 462·5% **i** $466{\cdot}\dot{6}\%$

j $533{\cdot}\dot{3}\%$ **k** $177{\cdot}\dot{7}\%$ **l** $783{\cdot}\dot{3}\%$

3

Percentage	Fraction	Decimal
5%	$\frac{1}{20}$	0·05
10%	$\frac{1}{10}$	0·1
12·5%	$\frac{1}{8}$	0·125
20%	$\frac{1}{5}$	0·2
25%	$\frac{1}{4}$	0·25
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0{\cdot}\dot{3}$
40%	$\frac{2}{3}$	0·4
50%	$\frac{1}{2}$	0·5
60%	$\frac{3}{5}$	0·6
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0{\cdot}\dot{6}$
75%	$\frac{3}{4}$	0·75
80%	$\frac{4}{5}$	0·8
100%	1	1

4 $\frac{7}{10} \times \frac{100}{1} = \frac{70}{1}\% = 70\%$

5 80% of 40 = 0·8 × 40 = 32

Rebecca needs to get at least 32 questions correct.

1D • Percentages to decimals

LB1 page 9

Specific learning outcomes

Learners should be able to:

8.1.2.3 Place percentages in ascending or descending order.

8.1.5.1 Change or convert percentages to decimals by dividing the numerator by 100.

Teaching points

1 Change a percentage to an equivalent decimal value using a simple division.

Learner difficulties and remedies

Difficulty

Cancelling or simplifying a fraction that is out of 100.

Remedy

- Encourage learners to practise their times tables.
- Identify the largest common number (LCN) to cancel both the numerator and the denominator.

Suggested teaching approach

- To change a percentage to a decimal you simply divide by 100, because 'per cent' means 'out of 100', and then remove the percentage sign.
- To divide by 100 the decimal point is moved two places to the left, because 100 has two zeroes.
- Encourage learners to understand the two processes above, rather than learn the steps by rote.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

To convert percentages to decimals, learners should follow these steps.

- **Decimal percentages**: Divide the value of the percentage by 100 by moving the decimal point two places to the left.
- **Fraction percentages**: Convert the fraction to a decimal by writing as an equivalent fraction out of 100. Then divide the value of the decimal percentage by 100 by moving the decimal point two places to the left.

Examples

1 Write the following percentages as decimals.

a 23% **b** 171% **c** 3·49%

Thinking	Working
a Divide the value of the percentage by 100 by moving the decimal point two places to the left.	**a** 23% = 23 ÷ 100 = 0·23
b Divide the value of the percentage by 100 by moving the decimal point two places to the left.	**b** 171% = 171 ÷ 100 = 1·71
c Divide the value of the percentage by 100 by moving the decimal point two places to the left. Fill in any empty place value columns with zeroes.	**c** 3·49% = 3·49 ÷ 100 = 0·0349

2 Write the following fractional percentages as decimals.

a $\frac{1}{2}\%$ **b** $3\frac{1}{5}\%$

Thinking	Working
a **1** Write the fraction value as a decimal value.	**a** $\frac{1}{2}\% = 0{\cdot}5\%$
2 Divide the decimal value by 100.	0·5 ÷ 100 = 0·005
b **1** Write the mixed number value as a decimal value.	**b** $3\frac{1}{5}\% = 3{\cdot}2\%$
2 Divide the decimal value by 100.	3·2 ÷ 100 = 0·032

Activity 1D

1 Write the following percentages as decimals.

a 80% **b** 27% **c** 40%
d 15% **e** 66% **f** 92%
g 3% **h** 5% **i** 110%
j 265% **k** 700% **l** 304%
m 0·5% **n** 7·2% **o** 40·5%
p 6·25%

2 Write the following fractional percentages as decimals.

a $\frac{1}{4}\%$ **b** $\frac{1}{8}\%$ **c** $\frac{7}{10}\%$
d $\frac{2}{5}\%$ **e** $2\frac{1}{2}\%$ **f** $10\frac{3}{4}\%$
g $37\frac{1}{5}\%$ **h** $18\frac{4}{5}\%$ **i** $25\frac{1}{3}\%$
j $5\frac{2}{3}\%$ **k** $2\frac{5}{9}\%$ **l** $23\frac{5}{6}\%$

3 Place the following in ascending order.

a 20%, $\frac{1}{4}$, 0·02, $\frac{2}{5}$
b 0·45, $\frac{4}{5}$, 4·5%, $\frac{5}{4}$
c $\frac{3}{10}$, 0·03, $\frac{1}{3}$, 33%
d 0·72, $7\frac{1}{2}\%$, 7·2%, $\frac{7}{2}$

4 Employment figures show that 28% of those employed are tradespeople. What fraction of the employed workforce are tradespeople?

Answers 1D

1 **a** 0·8 **b** 0·27 **c** 0·4
d 0·15 **e** 0·66 **f** 0·92
g 0·03 **h** 0·05 **i** 1·1
j 2·65 **k** 7 **l** 3·04
m 0·005 **n** 0·072 **o** 0·405
p 0·0625

2 **a** 0·0025 **b** 0·001 25 **c** 0·007
d 0·004 **e** 0·025 **f** 0·1075
g 0·372 **h** 0·188 **i** $0{\cdot}25\dot{3}$
j $0{\cdot}05\dot{6}$ **k** $0{\cdot}02\dot{5}$ **l** $0{\cdot}238\dot{3}$

3 **a** Express each number as a percentage.

20%, $\frac{1}{4} = \frac{1}{4} \times \frac{100}{1} = 25\%$

$0{\cdot}02 = 0{\cdot}02 \times 2\%$

$\frac{2}{5} = \frac{2}{5} \times \frac{100}{1} = 40\%$

In order: 2%, 20%, 25%, 40%

Correct order is: 0·02, 20%, $\frac{1}{4}$, $\frac{2}{5}$

b Express each number as a percentage.

$0{\cdot}45 = 0{\cdot}45 \times 100\% = 45\%$

$\frac{4}{5} = \frac{4}{5} \times \frac{100}{1} = 80\%$

4·5%, $\frac{5}{4} = \frac{5}{4} \times \frac{100}{1} = 125\%$

In order: 4·5%, 45%, 80%, 125%

Correct order is: 4·5%, 0·45, $\frac{4}{5}$, $\frac{5}{4}$

c Express each number as a percentage.

$\frac{3}{10} = \frac{3}{10} \times \frac{100}{1}\% = 30\%$

$0{\cdot}03 = 0{\cdot}03 \times 100\% = 3\%$

$\frac{1}{3} = \frac{1}{3} \times \frac{100}{1}\% = 33{\cdot}3\%$, 33%

In order: 3%, 30%, 33%, 33·3%

Correct order is: 0·03, $\frac{3}{10}$, 33%, $\frac{1}{3}$

d Express each number as a percentage.

$0{\cdot}72 = 0{\cdot}72 \times 100\% = 72\%$

$7\frac{1}{2}\% = 7{\cdot}5\%$, 7·2%

$\frac{7}{2} = \frac{7}{2} \times \frac{100}{1}\% = 350\%$

In order: 7·2%, 7·5%, 72%, 350%

Correct order is: 7·2%, $7\frac{1}{2}\%$, 0·72, $\frac{7}{2}$

4 $28\% = \frac{28}{100} = \frac{7}{25}$

1E • Decimals to percentages

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Specific learning outcomes

Learners should be able to:

8.1.6.1 Change decimals to percentages by multiplying the decimal number by 100.

Teaching points

1 A decimal can be written as a percentage by multiplying the decimal number by 100, and then adding the percentage sign.

Learner difficulties and remedies

Difficulty

Moving the decimal point the correct number of places when converting decimals to percentages.

Remedy

- When converting decimal numbers to percentages, multiply the decimal number by 100 because 'per cent' means 'out of 100'.
- Move the decimal point two places to the right because 100 has two zeroes.

Suggested teaching approach

- To change a decimal numbers to a percentage you simply multiply by 100, because 'per cent' means 'out of 100', and then add the percentage sign.
- To multiply by 100 move the decimal point two places to the right, because 100 has two zeroes.
- Encourage learners to understand the two processes above, rather than learn the steps by rote.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Numbers can be written in different forms that have the same value. The following table shows some common decimals with their corresponding percentages.

Decimal	Percentage
0·5	50%
0·25	25%
0·75	75%
0·3	33·3%
0·6	66·6%
0·2	20%
0·1	10%
0·05	5%
0·02	2%
0·01	1%

Examples

1 Find the percentage equivalent for each of the following.

a 0·8 **b** 1·65 **c** 0·032

Thinking	Working
a Multiply by 100%. Show this by moving the decimal point two places to the right. Fill in empty place value columns with zeroes.	**a** 0·08 = 0·8 × 100% = 80%
b Multiply by 100%. Show this by moving the decimal point two places to the right.	**b** 1·65 = 1·65 × 100% = 165%
c Multiply by 100%. Show this by moving the decimal point two places to the right.	**c** 0·0032 = 0·0032 × 100% = 3·2%

Activity 1E

1 Find the percentage equivalent for each of the following.

a 0·9
b 0·4
c 0·8
d 0·6
e 0·17
f 0·47
g 0·82
h 0·53
i 0·051
j 0·438
k 0·007
l 0·342
m 9·2
n 5·1
o 2·02
p 9·01

Answers 1E

1 **a** 90%
b 40%
c 80%
d 60%
e 17%
f 47%
g 82%
h 53%
i 5·1%
j 43·8%
k 0·7%
l 34·2%
m 920%
n 510%
o 202%
p 901%

1F • Finding percentages of quantities

LB1 page 11

Specific learning outcomes

Learners should be able to:

8.1.7.1 Calculate percentages by changing the 'of' sign to '×' then evaluate by multiplying.

Teaching points

1 Find the percentage of a quantity by multiplying it by the equivalent fraction with a denominator 100. Change the *of* sign to multiplication × sign, and then complete the calculation.

Learner difficulties and remedies

Difficulty

Using mixed-number percentages to find quantities.

Remedy

- Change the mixed number to an improper fraction, and then multiply it by $\frac{1}{100}$ to change it to a decimal percentage.

Suggested teaching approach

- To calculate a percentage of a quantity, multiply the quantity by the equivalent fraction with a denominator 100. Change the *of* sign to multiplication × sign, and then complete the calculation.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Finding percentages of an amount is a useful skill. Percentages can be written as fractions out of 100, so we can use fraction multiplication to find a percentage of an amount. Fractions are often easier to work with when performing calculations by hand if they have common factors that cancel. See Worked Example 1.

Alternatively, we can multiply the amount by the decimal equivalent of the percentage. Decimal percentages can be found by multiplying by 100 and moving the decimal point two places to the right. See Worked Example 2.

Remember that the word 'of' can be interpreted mathematically as multiplication.

To find a percentage of an amount:

1 Replace 'of' in the expression with ×.

2 Convert the percentage to a fraction or a decimal.

3 Perform the multiplication and simplify your answer.

Examples

Find the following, rounding answers to two decimal places where necessary.

a 12% of $350

b 5·2% of 30 kg

Method 1: Fraction multiplication

Thinking	Working
a **1** Substitute '×' for 'of' and omit any units.	**a** 12% of $350 = 12% × 350
2 Write the percentage as a fraction, and the amount as an improper fraction with a denominator of 1. Simplify the multiplication by cancelling common factors. (Here, we have divided by 10 and 5, then by 2.)	$= \frac{12}{{}_{2}\cancel{10}\,\cancel{0}} \times \frac{{}^{7}\cancel{35}\,\cancel{0}}{1}$ $= \frac{{}^{6}\cancel{12}}{{}_{1}\cancel{2}} \times \frac{7}{1}$
3 Simplify, then perform the multiplication.	= 6 × 7
4 State your answer, including any units.	= $42
b **1** Substitute '×' for 'of'.	**b** 5·2% of 30 kg = 5·2% × 30
2 Write the percentage as a fraction, and the amount as an improper fraction with a denominator 1. Simplify by cancelling common factors. (Here, we have divided by 10, then by 4.)	$= \frac{{}^{13}\cancel{52}}{{}_{22}\cancel{100}\,\cancel{0}} \times \frac{3\cancel{0}}{1}$
3 Simplify, then perform the multiplication and write the answer.	$= \frac{13}{25} \times 3$ $= \frac{39}{25}$
4 Write your answer in the required form with the correct units.	= 1·56 kg

Method 2: Decimal multiplication

Thinking	Working
a **1** Substitute '×' for 'of'.	**a** 12% of $350 = 12% × 350
2 Write the percentage as a decimal.	= 0·12 × 350
3 Perform the multiplication and write the answer.	= $42
b **1** Substitute '×' for 'of'.	**b** 5·2% of 30 kg = 5·2% × 30
2 Write the percentage as a decimal.	= 0·052 × 30
3 Perform the multiplication and write the answer with the correct units.	= 1·56 kg

Activity 1F

1 Find the following, rounding answers to two decimal places where necessary.

a 6% of $200
b 20% of $150
c 64% of $50
d 80% of $25.50
e 75% of $279.95
f 120% of $35.74
g 150% of 30 kg
h 160% of 25 kg
i 90% of 4 kg
j 23·2% of 40 mm
k 42·5% of 70 mm
l 15·7% of 60 mm
m 3·4% of 50 L
n 9·3% of 500 L
o 0·6% of 3000 L

2 Nick answered 85% of the questions correctly in a test containing 80 questions. How many questions did Nick answers correctly?

3 About 65% of the mass of an adult human is water. Pongi weighs 64 kg. How much of Pongi's mass is water, to the nearest kilogram?

Answers IF

1 **a** $12
b $30
c $32
d $20.40
e $206.96
f $42.89
g 45 kg
h 40 kg
i 3·6 kg
j 9·28 mm
k 29·75 mm
l 9·42 mm
m 1·7 L
n 46·5 L
o 18 L

2 85% of 80 questions
= 0·85 × 80
= 68 questions

3 65% of 64
= 0·65 × 64
= 41·6 kg

1G • Expressing a quantity as a percentage

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Specific learning outcomes

Learners should be able to:

8.1.8.1 Express one quantity as a percentage of another by writing it as a fraction, and then multiplying it by 100.

Teaching points

1 Express a quantity as a percentage of another quantity by writing it as a fraction, and then multiplying by 100. Make sure the quantities are in the same units.

Learner difficulties and remedies

Difficulty

Deciding which number is the numerator and which is the denominator, when expressing one quantity as a percentage of another.

Remedy

- The number of parts out of the total is the numerator. The total number is the denominator.
- Often the first number given is the numerator, and the second number given is the denominator.

Difficulty

Changing decimal numbers including dollars and cents into whole numbers so they are easier to simplify or cancel.

Remedy

- Change both quantities into the same unit. If the quantities are in dollars and cents, then change them into cents so that they are whole numbers.

Difficulty

Solving the word problems with percentages.

Remedy

- Do more practical exercises on worded problem solving, especially where quantities involve units, so that learners gain more experience.

Suggested teaching approach

- Identify which of the two quantities is the whole (denominator) and which is the number of parts to be expressed as a percentage of the whole (numerator). Write the two quantities as a fraction, multiply it by 100 to hange it to a percentage, and then add a percentage sign.
- Make sure that the quantities to be changed into a fraction are in the same unit.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Percentages are a very useful tool that can be used to compare quantities because they are always 'out of 100.' We use them in many real-life situations such as sports and business.

When writing one quantity as a percentage of another:

1 Make sure that both amounts are measured in the same unit.
2 Write it as **fraction** with the 'part amount' as the numerator and the 'whole amount' as the denominator.
3 Convert the fraction to a percentage.

Examples

1 Express the first amount as a percentage of the second. Give your answers as (i) a fraction (ii) an exact decimal.

a 23, 40

b 35, 30

Method 1: Multiply the fraction by 100%

Thinking	Working
a 1 Write a fraction with the first amount as the numerator and the second amount as the denominator.	**a** $\frac{23}{40}$
2 Multiply by $\frac{100}{1}\%$, simplifying the multiplication by cancelling common factors. (Here, the common factor is 20.)	$\frac{23}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$
3 Perform the simplified multiplication.	$= \frac{115}{2}\%$
4 Write your answer as a mixed number (fractional form), and as the equivalent decimal.	**i** $57\frac{1}{2}\%$ **ii** 57·5%
b 1 Write a fraction with the first amount as the numerator and the second amount as the denominator.	**b** $\frac{35}{30}$
2 Multiply by $\frac{100}{1}\%$, simplifying the multiplication by cancelling common factors. (Here, common factors of 5, then 2, are cancelled.)	$\frac{{}^{7}\cancel{35}}{{}_{6}\cancel{30}} \times \frac{100}{1}\%$ $= \frac{7}{{}_{3}\cancel{6}} \times \frac{{}^{50}\cancel{100}}{1}\%$
3 Perform the simplified multiplication.	$= \frac{350}{3}\%$
4 Write your answer as a mixed number (fractional form), and as the equivalent decimal.	**i** $116\frac{2}{3}\%$ **ii** $116.\dot{6}\%$

Method 2: Convert to a decimal, then multiply by 100%

Thinking	Working
a **1** Write a fraction with the first amount as the numerator and the second amount as the denominator.	**a** $\frac{23}{40}$
2 Convert the fraction to a decimal by performing the division. (A calculator could be used for this step.) Keep all of the decimal places (do not round off).	$= 0{\cdot}575$
3 Multiply the decimal value by 100%.	$0{\cdot}575 \times 100\%$
4 Write your answer in decimal form. Convert any decimal remainder to a fraction, and write in fractional form.	**i** $57\frac{1}{2}\%$ **ii** $57{\cdot}5\%$

2 Express the first amount as a percentage of the second: 40 cents, $5.

Method 1: Fraction multiplication

Thinking	Working
1 Write both quantities in the smaller unit.	40 cents, 500 cents
2 Write a fraction with the first amount as the numerator, and the second amount as the denominator, multiplied by $\frac{100}{1}\%$.	$\frac{40}{5\cancel{00}} \times \frac{1\cancel{00}}{1}\%$
3 Simplify by cancelling common factors. (Here, we have divided by 100, then by 5.)	$= \frac{\overset{8}{\cancel{40}}}{\underset{1}{\cancel{5}}} \times \frac{1}{1}\%$ $= \frac{8}{1} \times \frac{1}{1}\%$ $= 8\%$
4 Write the answer.	40 cents is 8% of $5.

Method 2: Convert to a decimal, and then multiply

Thinking	Working
1 Write both quantities in the smaller unit.	40 cents, 500 cents
2 Write a fraction with the first amount as the numerator, and the second amount as the denominator.	$\frac{40}{500}$
3 Convert this fraction to a decimal by performing the division.	$= 0{\cdot}08$
4 Multiply by 100%.	$= 0{\cdot}08 \times 100\%$ $= 8\%$
5 Write the answer.	40 cents is 8% of $5.

Activity 1G

1 Express the first amount as a percentage of the second. Give your answers in (i) fractional and (ii) exact decimal form.

a 13, 50 **b** 16, 20
c 27, 45 **d** 42, 48
e 35, 80 **f** 15, 18
g 54, 96 **h** 15, 27
i 26, 22 **j** 72, 66
k 86, 55 **l** 90, 70

2 Express the first amount as a percentage of the second. Round your answer to two decimal places where necessary.

a 25 cents, $4 **b** 40 m, 2 km
c 7 mm, 2 cm **d** 30 seconds, 4 minutes
e 750 g, 2·5 kg **f** 5 days, 4 weeks
g 80 kg, 1 tonne **h** 85 mL, 2 L
i 600 mm, 7 m **j** 50 minutes, 40 hours

3 Craig received 38 marks out of 40 for a maths test. Find his result as a percentage.

4 In an election for house captain, Jessica received 324 votes. If 540 learners voted, what percentage of them voted for Jessica?

5 Karen planted 30 coconuts in her new coconut plantation, but only 19 survived after a year. What percentage of the new coconuts survived?

6 Lara has a job that pays $160 per week. She receives a pay rise of $24 per week. Express Lara's pay rise amount as a percentage of her original pay.

Answers 1G

1 **a** **i** 26% **ii** 26%
b **i** 80% **ii** 80%
c **i** 60% **ii** 60%
d **i** $87\frac{1}{2}\%$ **ii** $87{\cdot}5\%$
e **i** $43\frac{3}{4}\%$ **ii** $43{\cdot}75\%$
f **i** $83\frac{1}{3}\%$ **ii** $83{\cdot}\dot{3}\%$
g **i** $56\frac{1}{4}\%$ **ii** $56{\cdot}25\%$
h **i** $55\frac{5}{9}\%$ **ii** $55{\cdot}\dot{5}\%$
i **i** $118\frac{2}{11}\%$ **ii** $118{\cdot}\dot{1}\dot{8}\%$
j **i** $109\frac{1}{11}\%$ **ii** $109{\cdot}\dot{0}\dot{9}\%$
k **i** $156\frac{31}{55}\%$ **ii** $156{\cdot}5\dot{6}\dot{3}\%$
l **i** $128\frac{4}{7}\%$ **ii** $128{\cdot}\overline{571428}\%$

2 **a** 6·25% **b** 2%
c 35% **d** 12·5%
e 30% **f** 17·86%
g 8% **h** 4·25%
i 8·57% **j** 2·08%

3 38 out of 40

$= \frac{38}{40} \times \frac{100}{1}\%$

$= \frac{38}{2} \times \frac{5}{1}\%$

$= \frac{19}{1} \times \frac{5}{1}\%$

$= 95\%$

4 324 out of 540

$= \frac{324}{540} \times \frac{100}{1}\%$

$= \frac{324}{27} \times \frac{5}{1}\%$

$= \frac{108}{9} \times \frac{5}{1}\%$

$= \frac{12}{1} \times \frac{5}{1}\%$

$= 60\%$

5 19 out of 30

$= \frac{19}{30} \times \frac{100}{1}\%$

$= \frac{190}{3}\%$

$= 63{\cdot}\dot{3}$

6 Pay rise is \$24 (out of \$160)

$= \frac{24}{160} \times \frac{100}{1}\%$

$= \frac{3}{20} \times \frac{100}{1}\%$

$= \frac{3}{1} \times \frac{5}{1}\%$

$= 15\%$

1H • Percentage increase and decrease

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Specific learning outcomes

Learners should be able to:

8.1.9.1 Calculate the percentage increase or decrease, using the formula:

$$\text{Percentage change} = \frac{\text{Increase or decrease in a quantity}}{\text{original quantity}} \times 100$$

Teaching points

1. Use the formula to calculate the percentage increase or decrease.
2. Use real-life word problems to help learners appreciate the reason for the calculation.

Learner difficulties and remedies

Difficulty

Deciding whether the change is an increase or a decrease.

Remedy

- Use the information given in the question to find out whether it is an increase or decrease.

Difficulty

Calculating a percentage increase or a decrease.

Remedy

- Write the amount of increase or decrease as the numerator and the original quantity as the denominator. Multiply the fraction by 100, and then add the percentage sign.

Suggested teaching approach

- Identify the quantity that is to be increased or decreased because that is the amount that undergoes the percentage change.
- Show examples of how to calculate a percentage change, including a percentage increase and a decrease. To express an increase or decrease as a percentage we calculate the amount of increase or decrease, write it as a fraction of the original amount, and then convert it to a percentage.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Calculating a percentage increase or decrease

A percentage can be used to represent a change in a quantity. We can calculate the result of a percentage increase or decrease by using the fact that the original amount represents 100%, which has a decimal equivalent of 1·0. See Worked Example 1.

To increase an amount by a given percentage:

1. Add the percentage to 100%.
2. Write this percentage as a decimal scale factor. For example: Increase of 32% = 132% = 1·32
3. Multiply this decimal by the amount to be increased.

To decrease an amount by a given percentage:

1. Subtract the percentage from 100%.
2. Write this percentage as a decimal scale factor. For example: Decrease of 16% = 84% = 0·84
3. Multiply this decimal by the amount to be increased or decreased.

Expressing an increase or a decrease as a percentage

An increase or a decrease can be described as the percentage change of an original amount. To calculate this, we express the amount of increase or decrease as a fraction of the original amount, and convert it to a percentage. See Worked Example 2.

Examples

1. Increase or decrease the following amounts by the given percentages. Give answers in decimal form, rounding to two decimal places where necessary.
   - **a** Increase \$2700 by 22%.
   - **b** Decrease 163 kg by 45%.

Thinking	Working
a **1** Add the percentage increase to 100%. Write this new percentage as a decimal scale factor.	**a** 100% + 22% = 122% = 1·22
2 Multiply the scale factor by the original amount to find the new amount.	1·22 × 2700
3 Write the answer.	= $3294
b **1** Subtract the percentage decrease from 100%. Write this new percentage as a decimal scale factor.	**b** 100% – 45% = 55% = 0·55
2 Multiply the scale factor by the original amount to find the new amount.	0·55 × 163
3 Write the answer.	= 89·65 kg

2 Andrew earns $950 per week as an electrician. He receives a pay increase of $32 per week. Write Andrew's pay increase as a percentage of his original pay, correct to one decimal place.

Thinking	Working
1 Write the amount of the increase as a fraction of the original amount.	$\frac{32}{950} \times 100\%$
2 Convert the fraction to a percentage, rounding to the specified number of decimal places.	= 0·033 68… × 100% = 3·4% (1 d.p.)

Activity 1H

1 Increase or decrease the following amounts by the given percentages. Give answers in decimal form, rounding to two decimal places where necessary.

a Increase 1500 by:

i 15% **ii** 25%
iii 45% **iv** 120%

b Decrease 2200 by:

i 10% **ii** 40%
iii 66% **iv** 95%

c Increase $83.75 by:

i 23% **ii** 340%
iii 3·5% **iv** $3\frac{2}{3}$%

d Decrease 5·2 km by:

i 6% **ii** 74%
iii 2·8% **iv** $1\frac{3}{4}$%

2 **a** Georgia earns $1253 per month Working as a receptionist. She receives a pay increase of $49 per month. Write Georgia's pay increase as a percentage of her original pay, correct to one decimal place.

b Jayden earns $67 982 per year Working as a lawyer. He receives a pay increase of $2850 per year. Write Jayden's pay increase as a percentage of his original pay, correct to one decimal place.

c This month, the Edwards family have used 13 kL (kilolitres) less water than last month. If they used 190 kL last month, calculate the reduction in this month's water usage as a percentage of last month's usage, correct to one decimal place.

3 Samani's monthly rent is set to increase by 15%. If he currently pays $750 a month, what will he pay after the increase?

4 Phyllis improves her maths test result by 20%. All tests are out of 60 marks.

a If her first result was 27 marks, what was her next result?

b If Phyllis continues to improve by this percentage, how many more tests must she take to achieve over 90%?

5 Melissa is 165 cm tall, and her younger sister Laura is 150 cm tall.

a Calculate Melissa's height as a percentage increase of Laura's height, to the nearest per cent.

b Calculate Laura's height as a percentage decrease of Melissa's height, to the nearest per cent.

6 **a** Calculate the number of girls in your class as a percentage of the whole class.

b Calculate the increase in this percentage if an extra girl joins the class.

Answers 1H

1 **a** **i** 1725 **ii** 1825
iii 2175 **iv** 3300
b **i** 1980 **ii** 1320
iii 748 **iv** 110
c **i** $103.01 **ii** $368.50
iii $86.68 **iv** $86.82
d **i** 4·89 km **ii** 1·35 km
iii 5·05 km **iv** 5·11 km

2 **a** 3·9% **b** 4·2%
c 14·4% **d** 6·8%

3 $750 × 1·15 = $862.50

4 **a** Increase of 20% so multiply by 1·2:
27 × 1·2 = 32·4
The result was 32 out of 60.

b 90% of 60 = 0·9 × 60 = 54
32 × 1·2 = 38·4
38 × 1·2 = 4646 × 1·2 = 55
Phyllis needs three more tests.

5 **a** Difference in height is:
(165 – 150) cm = 15 cm
$\frac{15}{150} \times 100\% = 10\%$

b $\frac{15}{165} \times 100\% = 9\%$ (nearest %)

6 Learners will provide their own solutions for this question.

1I • Exploring percentages

LB1 pages 14–15

1K • Exploring shopping

LB1 pages 18–19

Specific learning outcomes

Learners should be able to:

8.1.10.1 Calculate percentages for practical applications of percentages and compare the changes in percentages and numbers.

8.1.13.1 Calculate costs, selling prices, discounts, increases and decreases using percentages. Compare discounts offered in shops using percentages.

Teaching points

1. Find percentages used in various practical applications.
2. Calculate costs, selling prices, discounts, increases and decreases using percentages.
3. Use percentages to compare discounts that are offered by different shops.

Suggested teaching approach

- Learners complete **Learning task 1I** on pages 14 and 15, and **Learning tasks 1K** on pages 18 and 19 of the LB.

1J • Discount

LB1 pages 16–17

Specific learning outcomes

Learners should be able to:

8.1.11.1 Define discount.

8.1.12.1 Calculate the discounted price for various items.

Teaching points

1. Explain the meaning of the term 'discount' using examples suggested by the class.
2. Calculate the discounted price and/or the amount saved on various advertised items.

Learner difficulties and remedies

Difficulty

Understanding that discounts can be expressed as a percentage as well as a fixed quantity.

Remedy

- Explain to learners that discounts are normally given as a percentage rather than the fixed value. This means that a discount is given to all quantities that is in proportion to the cost. For example, compare discounts of $10 and 10% for a $50 item and a $5000 item. Show examples of sales in shops that offer percentage discounts such as 10% sale, or 50% off.

Difficulty

Understanding the difference between the two values that are multiplied: the percentage discount and the amount to be discounted.

Remedy

- Take the percentage discount and divide by 100 then multiply it with amount to be discounted.

Difficulty

Dealing with word problems.

Remedy

- Explain the words and terms that are used in maths, especially percentage problems, so that learners are familiar with the language used in worded problems.
- Show learners how to identify the important information in a worded question, and then set up the correct mathematical calculation.
- Do more practical work on how to solve worded discount problems.

Suggested teaching approach

- Use practical examples: Bring in some goods from the shop them calculate the price after various percentage discounts.
- Point out the difference between an actual discount of a fixed amount, and a percentage discount. Show learners how to calculate the specific amount of the discount from the original amount and using the percentage discount.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Businesses that involve with selling goods are called retailers. They either buy or manufacture goods with the aim of selling them at a higher price than it cost to buy or manufacture. The aim is to make extra money, which is known as a **profit**. The price tag on the goods is called the **selling price**. Sometimes retailers may sell their product for less than they paid for the goods. This reduction in price is called a **discount.** It is often expressed as percentages.

Examples

A headphone set with a selling price of $350 is discounted by 25%. Calculate the discounted amount and the discounted selling price, rounding your answer to the nearest cent.

Method 1: Find and subtract the discount amount

Thinking	Working
1 Calculate the dollar amount of the discount.	25% of $350 = 0·25 × 350 = $87.50
2 Subtract the discount amount from the original price to find the sale price.	Sale price = $350 – $87.50 = $262.50

Method 2: Calculate the remaining percentage

Thinking	Working
1 Subtract the percentage discount from 100%.	100% – 25% = 75%
2 Find the remaining percentage value of the price.	Sale price = 75% of \$350 = 0·75 × 350 = \$262.50

Activity 1J

1 Calculate these discounts

a A laptop with a selling price of \$4500 is discounted by 20%. Calculate the new selling price.

b A big \$100 birthday cake is discounted by 25%. Find the new selling price.

c The airfare on Solomon Airlines to Bellona Island, Renbel Province, is \$1200. A 4% discount is offered. How much is new airfare?

d In a recent essay writing competition, 30% of people who entered were female. Of the 20 people were chosen as finalists, 40% were female. How many females were finalists?

2 Two refrigerators are on sale.

Refrigerator A costs \$1300 with a 35% discount.

Refrigerator B costs \$1150 with a 20% discount.

Calculate which of the two refrigerators will be cheaper after the discounts. Show your calculations and explain what you are Working out at each stage.

3 A \$470 watch is discounted to \$350. Calculate:

a the discount

b the discount as a percentage, correct to one decimal place.

Answers 1J

1 **a** \$3600

b \$75

c \$1152

d 8

2 Refrigerator A:

\$1300 with a 35% discount
= 1300 × 0·65
= \$845

Refrigerator B:

\$1150 with a 20% discount
= 1150 × 0·80
= \$920

Refrigerator A is cheaper after the discounts.

3 **a** \$470 – \$350 = 120

b $\frac{120}{470} \times 100 = 25.5319$

The discount is 25·5% of the original price.

• Chapter Two •

Length and Perimeter

Overview

The perimeter is the total length of the boundary of a shape. It is usually measured using metric units such as km, m, cm or mm. Sides of shapes that are equal in length are indicated by markings such as | and || in diagrams.

This chapter covers the topics of measuring lengths and distances accurately, converting between metric units of lengths, calculating the perimeter of various shapes and finding the circumference of circles. Length and perimeter are important measurements in such subjects as science, agriculture, business studies, social studies and technology. In practical subjects, it is necessary to make accurate measurements when cutting materials such as wood, metal or fabric and when calculating the costs of materials.

Solomon Islander navigators have their own ways of communicating the distances between or around islands. Carpenters and agricultural workers frequently use equivalent measurements when discussing how to build houses, or to design a new plantation.

Contents

Chapter skills

This chapter covers the following skills:

- Estimating the length of lines and objects by using known lengths
- Using measuring instruments accurately
- Using common prefixes of units and converting between units.
- Units of length
  1 km = 1000 m
  1 m = 100 cm = 1000 mm
  1 cm = 10 mm
- Calculating the perimeters of shapes including polygons and circles.
- Perimeter of the shape is the distance around the outside of the shape
- Perimeter of square = $4a$
  Perimeter of rectangle = $2(a + b)$
  Perimeter of rhombus = $4a$
  Perimeter of parallelogram = $2(a + b)$
- Circumference of circle: $C = \pi D$ or $C = 2\pi r$
- Estimating uncertainty in measurements

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 2A Estimating lengths	Learner's Book 1 • Exercise 2A, pages 30, 31
2–4	• 2B Measuring lengths • 2C Converting units of length	Learner's Book 1 • Exercise 2B, pages 32, 33 Learner's Book 1 • Exercise 2C, page 34, 35
5	• 2D Working with lengths	Learner's Book 1 • Exercise 2D, pages 36, 37
6–7	• 2E Perimeters of polygons	Learner's Book 1 • Exercise 2E, pages 38, 39
8	• 2F Perimeter of special quadrilateral	Learner's Book 1 • Exercise 2F, page 41
9–10	• 2G Exploring the circumference of a circle • 2H Perimeters of circles	Learner's Book 1 • Learning task 2G, page 42 Learner's Book 1 • Exercise 2H, pages 44, 45
11–12	• 2I Perimeter of a composite shapes	Learner's Book 1 • Exercise 2I, pages 46, 47
13	• 2J Errors in measurements	Learner's Book 1 • Exercise 2J, Pages 48, 49
14	• Test	Teacher's Guide: • Chapter 2 Test

General learning outcomes

Learners should:

Estimate lengths

8.2.1 Know how to estimate lengths using known values. (K)

Measuring lengths

8.2.2 Appreciate how measuring instruments such as ruler, tape measure can be used to give accurate measurements. (a)

Converting units of length

8.2.3 Know how to convert one metric unit of length from one to another. (K)

Working with lengths

8.2.4 Understand that lengths and distances can be measured using different units. (U)

8.2.5 Know how to add and subtract lengths of given shapes and objects with the 'same' and 'different' units. (K)

Perimeters of polygons

8.2.6 Understand the perimeter of a polygon is the distance around the shape and is found by adding the lengths of all the sides together. (U)

Perimeters of special quadrilateral

8.2.7 Know how to calculate the perimeter of quadrilateral shapes with the same or different units. (K)

Exploring the circumference of a circle

8.2.8 Understand that the distance around the circle is known as the perimeter or circumference. (U)

8.2.9 Know how a circumference of a circle is determined and calculated. (K)

8.2.10 Understand the term diameter and how it is determined. (U)

Perimeter of circles

8.2.11 Know how to calculate the circumference of a circle using a formula that uses diameter, radius and pi (π). (K)

Perimeter of composite shapes

8.2.12 Know how to calculate the perimeter of composite shapes. (K)

Errors in measurements

8.2.13 Understand how errors occur during the process of measurement can be identified and calculated. (U)

2A • Estimating lengths

LB1 Pages 30–31

Specific learning outcomes

Learners should be able to:

8.2.1.1 Estimate the lengths of different objects by comparing them to known lengths.

Teaching points

1 Estimate the lengths of various shapes, objects and distances using known or given measurements and values.

Learner difficulties and remedies

Difficulty

Estimating lengths and distances.

Remedy

- Learners should be familiar with the actual lengths and distances of a millimetre (mm), centimetre (cm), metre (m) and kilometre (km). Knowing these measurements would enable learners to estimate almost accurate lengths and distances.

Suggested teaching approach

- Show learners the actual lengths and distances that are used in our metric system such as millimetre, centimetre, metre and kilometre.
- Identify the sign that is used for approximately equal to ≈.
- Bring small objects into the class and have learners estimate the lengths. Then go outside so that learners can estimate longer distances. Where possible measure the lenghs to check learner's estimates.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The skill of estimating lengths and distances can be developed by having learners first record their guesses and then check their guesses with accurate measurements with a ruler or tape measure. When the estimating is based on known measurements such as body measurements or against an identified measurement such as the height of a door, learners quickly improve their skills.

Examples

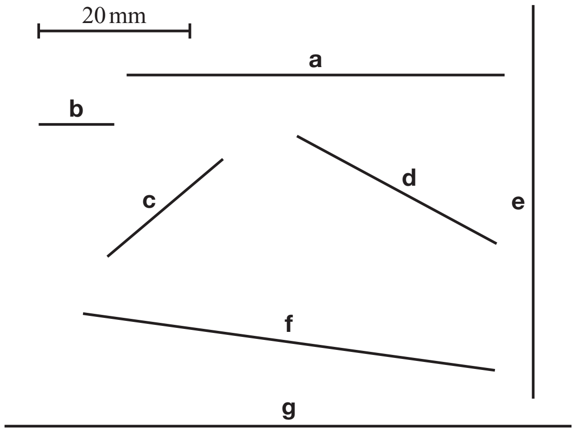
The length of an exercise book is about 290 mm. Estimate the width.

Thinking	Working
1 290 mm is close to 300 mm.	300 mm
2 The width looks as though it is about $\frac{2}{3}$ of the height.	$\frac{2}{3}$
3 Multiply the estimated length by the given fraction that represent the width of the exercise book.	$300 \times \frac{2}{3} = 200$
4 Estimate the width.	200 mm (Note: measuring the width accurately gives 206 mm).

Activity 2A

1 Identify a list of lengths and distances in the school grounds and have groups of 3 or 4 learners use body measurements to estimate the lengths and distances in metric units. They can then use instruments such as rulers, tape measures and trundle wheels to check their estimates. Both estimates and measurements can be recorded in a table, and the results of different groups compared for accuracy.

2 Estimate these lengths in millimetres. Do not use a ruler. A line segment that is exactly 20 mm has been shown as a guide.



3 Estimate the length of each of these objects. Choose your answer from this list of lengths: 10 cm, 20 cm, 40 cm, 80 cm.

a the width of your foot (at its widest point)

b the length of a computer keyboard

c the width of a door

d the length of a telephone handpiece

Answers 2A

1 Learners' answers will vary.

2 **a** 50 mm **b** 10 mm

c 20 mm **d** 30 mm

e 52 mm **f** 55 mm

g 75 mm

3 Learners' answers will vary.

2B • Measuring lengths

LB1 Pages 32–33

Specific learning outcomes

Learners should be able to:

8.2.2.1 Use rulers and tapes to accurately measure the lengths of various objects and shapes.

Teaching points

1 Use a variety of measuring instruments such as a ruler and a tape-measure to accurately measure lengths on diagrams.

Learner difficulties and remedies

Difficulty

Using measuring instruments accurately.

Remedy

- Show learners where to place the start of the scale and where to measure to on the measuring instrument.
- Using and reading instruments such as a trundle wheel, which makes a sound to indicate each metre.

Suggested teaching approach

- Show learners how to use equipment such as a ruler, a trundle wheel and a tape measure to measure lengths and distances.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

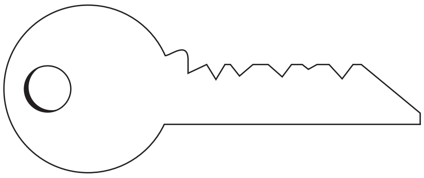
Additional notes

Rulers and tapes can be used to measure lengths and distances. Place the zero (0) on the scale next to one end of the object and the read the scale at the other end as accurately as possible. Learners may need a demonstration of how to measure the straight and curve distances required by Question 3.

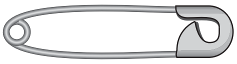
Activity 2B

1 Divide the learners into groups of 3 or 4. Give each group a rope and a metre ruler. Each group is told to measure the distance around a particular building as accurately as possible, first clockwise, and then anticlockwise. Groups report their results to the class and discuss the reasons for any differences in the measurements.

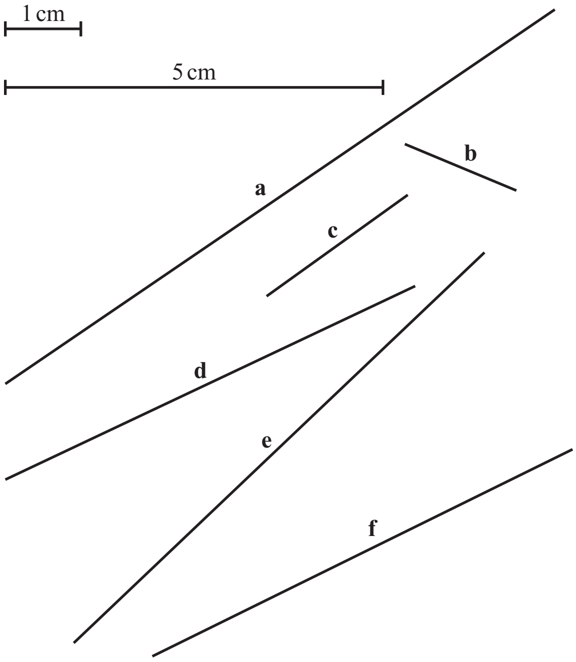
2 Use a ruler to measure the length of this key in millimetres.



3 Use a ruler to measure the length of this safety pin in centimetres.



4 Here are six different line segments. Follow the steps to complete the table:



i First, write your estimate of the length (in cm) for each line segment

ii Next, use a ruler to measure the actual length (in cm) of each line and write it in the table.

	Estimate of length (cm)	Actual measured length (cm)
a		
b		
c		
d		
e		
f		

Answers 2B

1 Learners' own answers.

2 55 mm

3 3 cm

4 Here are six different line segments. Follow the steps to complete the table:

  i First, write your estimate of the length (in cm) for each line segment

  ii Next, use a ruler to measure the actual length (in cm) of each line and write it in the table.

	Estimate of length (cm)	Actual measured length (cm)
a	Learners' own estimates.	8·8
b	Learners' own estimates.	1·6
c	Learners' own estimates.	2·3
d	Learners' own estimates.	6·0
e	Learners' own estimates.	7·5
f	Learners' own estimates.	6·2

2C • Converting units of length

LB1 Pages 34–35

Specific learning outcomes

Learners should be able to:

8.2.3.1 Identify standard units for lengths and their corresponding values in metric units:
1 kilometre (km) = 1000 metres (m)
1 metre (m) = 100 centimetre (cm)
1 centimetre = 10 millimetres (mm)

8.2.4.1 Change metric units of length from one to another using the appropriate conversion factors.

Teaching points

1 Convert one metric unit of length into other metric units using a conversion factor.

Learner difficulties and remedies

Difficulty

Converting one metric unit of length into other metric units.

Remedy

- Identify the units of the original amount.
- Identify the units that you are converting into.
- Decide which conversion factor corresponds to the two sets of units.
- Use the conversion factor to convert the original amount.

Suggested teaching approach

- Identify the metric units in the original amount and in the converted amount.
- Decide which conversion factor would be used for the conversion.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

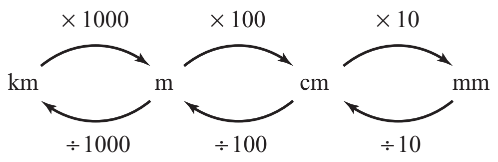
Additional notes

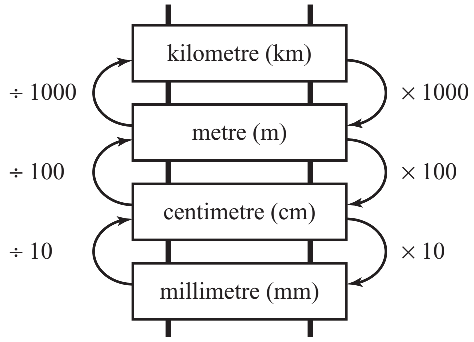
One of the advantages of using the Metric Unit system is the ability of one metric unit to be converted from one unit to another using the given conversion factors that are multiples of ten. A length is the distance between two or more points. It can be measured in any units but the main ones that are used in this unit are; millimetres (mm), centimetres (cm), metres (m) and kilometres (km). The units: mm, cm, m and km are all metric units. The prefix milli- means one-thousandth, centi- means one hundredth, and kilo- means one thousand. Examples of other uses for these Latin prefixes are the names: millipede, century, kilobyte, etc.

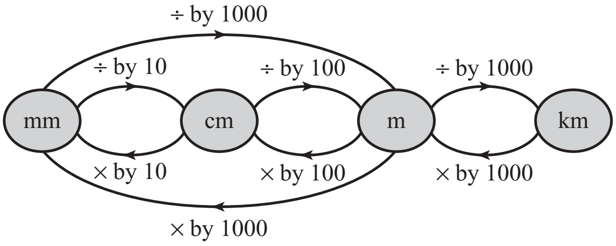
It is important to remember that converting between units of lengths is *not* the same as converting between units of area and volume. Although 1 m is equal to 100 cm, 1 m^2 is *not* equal to 100 cm^2.

The following diagrams show the relationships between all the metric units of lengths; mm, cm, m and km. They are all conversion factors that can be used to convert or change one metric unit from one to another.

Units of length:







Examples

1 Change 45 millimetres (mm) to centimetres (cm): [**mm to cm**]

Thinking	Working
1 Write the metric unit of length that is to be changed to centimetres.	45 mm; change to cm
2 Find the conversion factor.	1 cm = 10 mm
3 Change of metric unit from **small to big**, so **divide** by the conversion factor, which is 10.	45 ÷ 10 = 4·5
4 Give the answer with the correct unit.	4·5 cm

2 Change from 850 millimetres (mm) to metres (m): **[mm to m]**

Thinking	Working
1 Write the metric unit of length that is to be changed to centimetres.	850 mm; change to m
2 Find the conversion factor.	1 m = 1000 mm
3 Change of metric unit from **small to big**, so **divide** by the conversion factor, which is 1000.	850 ÷ 1000 = 0·85
4 Give the answer with the correct unit.	0·85 m

3 Change from 1·2 metres (m) to centimetres (cm): **[m to cm]**

Thinking	Working
1 Write the metric unit of length that is to be changed to centimetres.	120 cm; change to m
2 Find the conversion factor.	1 m = 100 cm
3 Change of metric unit from **big to small**, so **multiply** by the conversion factor, which is 100.	1·2 × 100 = 120
4 Give the answer with the correct unit.	120 cm

Activity 2C

1 Change these lengths (given in mm) to cm.

a 80 mm **b** 12 mm

2 Change these lengths (given in cm) to mm.

a 6 cm **b** 4·2 cm

3 Change these lengths (given in cm) to m.

a 400 cm **b** 80 cm

4 Change these lengths (given in m) to km.

a 1350 m **b** 14 800 m

c 500 m

5 Copy and complete these so that the lengths are changed correctly.

a 4 m = ___ cm **b** 60 cm = ___ mm

c 100 000 m = ___ km **d** 2 km = ___ m

e 0·8 m = ___ mm **f** 44 mm = ___ cm

g 830 mm = ___ m **h** 900 cm = ___ m

6 Say whether the measurements in each pair are equal. Write yes or no.

a 80 m, 800 cm **b** 9000 mm, 9 m

c 50 cm, 5 mm **d** 1000 cm, 10 m

Answers 2C

1 **a** 8 cm **b** 1·2 cm

2 **a** 60 cm **b** 42 cm

3 **a** 4 cm **b** 0·8 cm

4 **a** 1·35 m **b** 14·800 m

c 0·5 m

5 **a** 4 m = 400 cm **b** 60 cm = 600 mm

c 100 000 m = 100 km **d** 2 km = 2000 m

e 0·8 m = 800 mm **f** 44 mm = 4·4 cm

g 830 mm = 0·30 m **h** 900 cm = 0·9 m

6 Say whether the measurements in each pair are equal. Write yes or no.

a no **b** yes

c no **d** yes

2D • Working with lengths

LB1 Pages 36–37

Specific learning outcomes

Learners should be able to:

8.2.5.1 Identify different units that are used to measure objects.

8.2.6.1 Add and subtract lengths with same and or different units by first converting them to the same unit.

Teaching points

1. Converting between the most common metric units used for length.
2. Adding lengths is always done in one unit, so convert units so that all measurements are in the same metric unit before any addition.

Learner difficulties and remedies

Difficulty

Converting units so that all measurements are in the same metric unit.

Remedy

- Know how to change one metric unit from one to another using the conversion table.
- Given the units to be converted, use the corresponding conversion factor.

Suggested teaching approach

- Identify the metric units that are concerned.
- Use the table or conversion factor to change the metric units so that they are in the same unit.
- When converting from a big to to small unit, you multiply. When converting from a small to big unit, you divide.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Before adding measurements with different units, convert all the units so that they are all in one particular unit before doing the calculations.

Hint: Working in the smaller units is often best to avoid having to use decimals.

Examples

Add 5·7 cm and 85 mm.

Thinking	Working
1 Choose to convert to the smaller unit, millimetres. 1 cm = 10 mm	5·7 cm = 5·7 mm
2 Add the measurements in millimetres.	57 + 85 = 142
3 Give the answer with the correct unit.	142 mm

Activity 2D

1 Give the answers in metres.

a 6 km + 700 m
b 2250 m + 1·75 km
c 1 m + 30 cm
d 65 cm + 45 cm
e 12 m – 75 cm
f 3300 m – 2·7 km

2 The height of the Anthony Saru building at Point Cruz, Honiara is 2230 m. What is this height in kilometres (km)?

3 A piece of wood is measuring 3·6 m. It is cut into four equal lengths. What is the length of each piece in millimetres (mm)?

4 Dioko used his bicycle to travel to school and back every day (Monday to Friday) for 2 weeks. The total distance covered is 54 km. How far does Dioko live from school in metres?

Answers 2D

1 **a** 6700 m
b 4000 m
c 1·3 m
d 1·1 m
e 12·75 m
f 6000 m

2 2·23 km

3 900 mm

4 2700 m

2E • Perimeters of polygons

LB1 Pages 38–39

Specific learning outcomes

Learners should be able to:

8.2.7.1 Define the terms ‘perimeter’ and ‘polygon’.
8.2.7.2 Caalculate the perimeter of polygon shapes.

Teaching points

1. The perimeter is the distance around a shape.
2. A polygon is a many-sided shape with all sides straight lines.
3. Calculate the perimeter of given polygons ensuring that the units of each side are the same before adding.

Learner difficulties and remedies

Difficulty

Remembering what the term perimeter means.

Remedy

- Define the perimeter of shapes as the distance around the shape. It is calculated by adding the sides together.

Difficulty

Remembering what the term polygon means.

Remedy

- Explain that polygons are flat shapes with many straight sides.

Difficulty

Finding missing lengths of sides in given polygon shapes.

Remedy

- Use given measurements for the other sides of the shape to find the missing sides.

Suggested teaching approach

- Identify the given polygon shapes and state whether they are regular.
- Find the measurements for all given sides of the shape and state whether regular.
- Make sure that the measurements for the sides of the shape have the same unit, and then add them together to find the perimeter.
- For regular shapes, use a formula to finding the perimeter.
- Marks given on the sides of polygon shapes must be explained their usage
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The **perimeter** is the distance along the boundary of a shape.

The word perimeter comes from two Latin words: *peri*, meaning ‘around’ and *metron*, meaning ‘measure’.

To find the perimeter of a shape, we simply add up the lengths of the sides (first making sure they are all in the same units).

For example, the perimeter of a soccer pitch is 105 + 68 + 105 + 68 = 346 m. Because the pitch is rectangular in shape, it has 2 pairs of equal sides: 2 **lengths** and 2 **widths**.

The perimeter of the pitch is:

$$\begin{aligned} P &= 2 \times 105 + 2 \times 68 \\ &= 210 + 136 \\ &= 346\text{ m} \end{aligned}$$

If we let the pronumerals l and w represent the length and the width, we can write a formula for the perimeter of a rectangle: $P = 2l + 2w$. This formula can also be written as $P = 2(l + w)$: Add the length and the width, then multiply by 2.

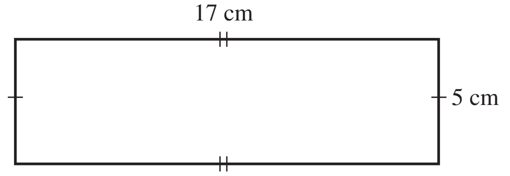
Using this formula, the perimeter is:

$$\begin{aligned} P &= 2(105 + 68) \\ &= 2 \times 173 \\ &= 346\text{ m} \end{aligned}$$

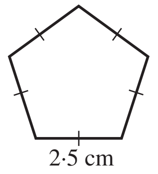
Examples

Use a formula to calculate the perimeter of each of the following shapes.

a



b

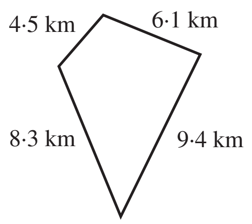


Thinking	**Working**
a **1** Write the formula for the perimeter of a rectangle.	**a** $P = 2l + 2w$
2 Substitute values for length and width.	$= 2 \times 17 + 2 \times 5$
3 Add the products.	$= 34 + 10$
4 Write the total length, including the unit.	= 44 cm
b **1** Let *s* be the length of a side. Write a formula that multiplies *s* by the number of equal sides.	**b** $P = 5s$
2 Substitute the side length.	= 5 × 2·5
3 Write the product with units of length.	= 12·5 cm

Activity 2E

1 Find the perimeter of each of the following shapes.

a



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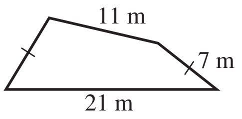

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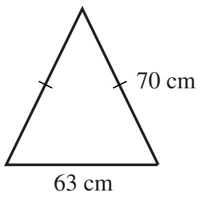

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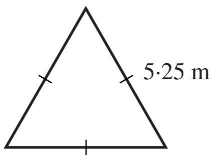

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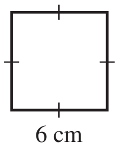

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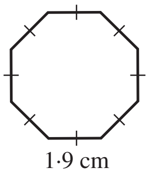
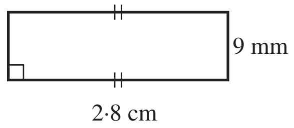


2 Find the perimeter of each shape below.

a



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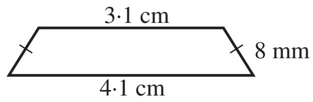

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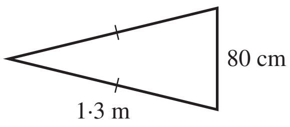

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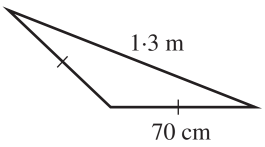

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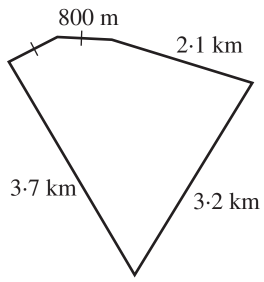

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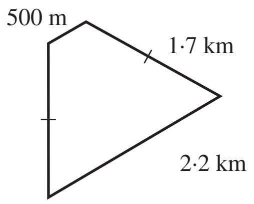
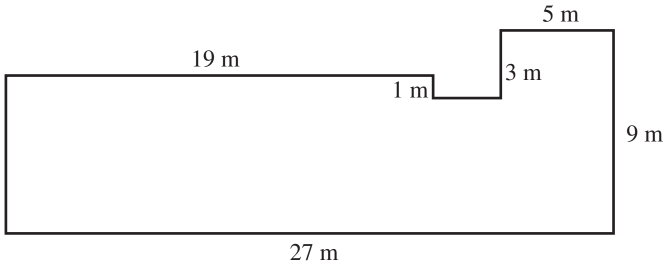
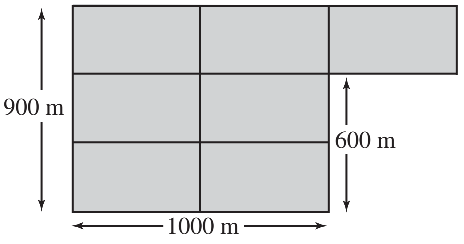


3 The roof plan for a new house appears here. Use it to calculate the length of guttering required if guttering is to be attached along each edge of the roof.



4 A farmer needs to fence a new property. The fencing needs to go along the boundary of the property, and also divide it up into rectangles, as shown.



a What is the total length of fence required?

b If the type of fence used by the farmer costs $5.75 per metre, what will it cost to fence the property?

c If the fencing contractor can build the fence at a rate of 20 metres per hour, how long will the job take?

5 **a** Find the side length of a square if it has the same perimeter as a rectangle of length 19 cm and width 11 cm.

b Find the width of a rectangle is its length is 28 cm and it has the same perimeter as a square of side 20 cm.

Answers 2E

1 **a** 28·3 km **b** 46 m **c** 203 cm
d 15·75 m **e** 24 cm **f** 15·2 cm

2 **a** P = 2(9 mm + 28 mm) = 74 mm
b P = (2 × 8 mm) + 41 mm + 31 mm = 88 mm
c P = 80 cm + (2 × 130 cm) = 340 cm
d P = (2 × 70 cm) + 130 cm = 270 cm
e P = 2·1 km + 3·2 km + 3·7 km + 0·8 km = 10·6 km
f P = (2 × 1·7 km) + 2·2 km = 6·1 km

3 $P = (27 \times 2) + 9 + 1 + 3 + 7 = 74\,\text{m}$

4 **a** $P_{\text{outside}} = 900 + 1000 + 500 + 300 + 500 + 600 + 1000$
$= 4800\,\text{m}$

$P_{\text{inside}} = 900 + 2 \times 1000 + 300 = 3200\,\text{m}$

$P_{\text{total}} = 4800 + 3200 = 8000\,\text{m}$

b $C = 5{\cdot}75 \times 8000 = \$46\,000$

c $8000\,\text{m} \div 20\,\text{m/h} = 400$ hours

5 **a** $P = 2 \times (19 + 11) = 60 \div 4 = 15\,\text{cm}$

b $2 \times 28 + 2 \times w = 20 \times 4$
$2w = 80 - 56$
$w = 24 \div 2$
$w = 12\,\text{cm}$

2F • Perimeter of special quadrilaterals

LB1 Pages 40–41

Specific learning outcomes

Learners should be able to:

8.2.8.1 Calculate the perimeter of quadrilateral shapes with different units.

Teaching points

1 Calculate the perimeter of different quadrilaterals: square, rectangle, rhombus and parallelogram.

Learner difficulties and remedies

Difficulty

Finding missing measurements for the sides of given shapes

Remedy

- Use given measurements for the sides to find the missing side lengths.

Suggested teaching approach

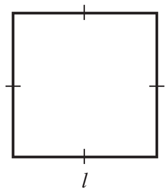
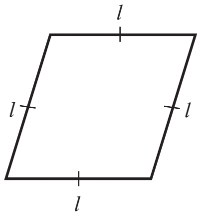
- Explain how to calculate the perimeter of a quadrilateral.
- Identify the properties of given quadrilaterals to help in the calculation of their perimeters.
- Explain how to find the perimeters of compound shapes.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

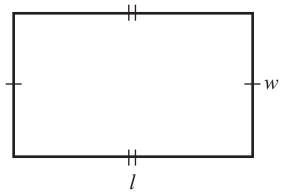
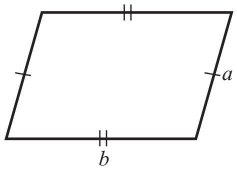
Additional notes

Some quadrilaterals have special properties that opposite sides are equal. We can use formulas to find perimeters, using the same pronumeral for sides of equal length.

Any pronumeral can be used; however, we frequently use the letters *l* (length) and *w* (width) for rectangles, and *b* (base) for distances along the bottom of some shapes, such as triangles or parallelograms. See below for further examples.

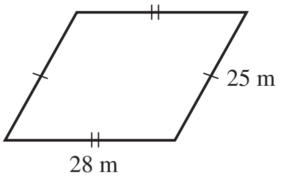
If there is no convenient formula to use, we simply add all the side lengths, using multiplication by 2 to simplify the calculation when equal lengths are added.

Square	Rhombus
 $P = 4l$	 $P = 4l$

Rectangle	Paralleogram
 $P = 2l + 2w$ $P = 2(l + w)$	 $P = 2a + 2b$ $P = 2(a + b)$

Examples

Find the perimeter of the given shape.

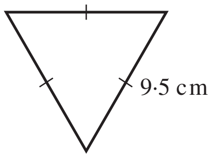


Thinking	Working
1 Find the measurements for sides of shape.	length = 28 m width = 25 m
2 Use the formula to find the perimeter	$P = 2l + 2w$ $= (2 \times 28) + (2 \times 25)$ $= 56 + 50$ $= 106$
3 Give the perimeter with the correct unit.	$= 106\,\text{m}$

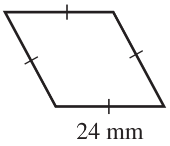
Activity 2F

1 Use a formula to find the perimeter of these shapes:

a

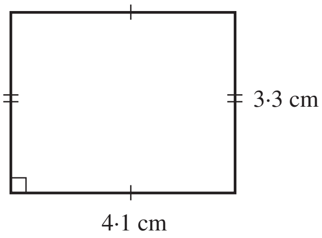


b



c <please supply>

d



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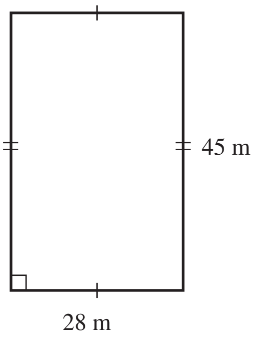

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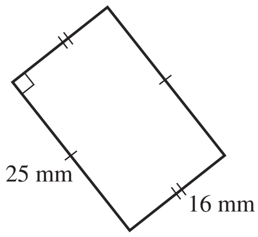
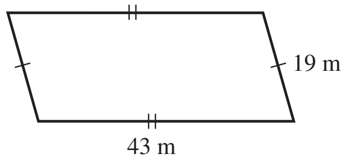
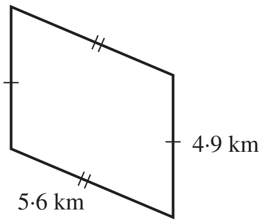


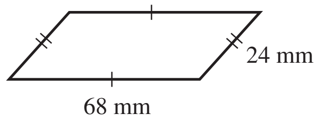
g



h



i



Answers 2F

1 **a** 28·5 cm **b** 96 mm **c** 7·2 cm
d 14·8 cm **e** 146 m **f** 82 mm
g 124 m **h** 21 km **i** 184 mm

2G • Exploring the circumference of a circle

LB1 Pages 42

Specific learning outcomes

Learners should be able to:

8.2.9.1 Define circumference of a circle.
8.2.10.1 Measure the circumferences of circular objects by using ropes and tapes.
8.2.10.2 Write a general statement in words connecting the diameter of the circle to the circumference.
8.2.11.1 Define diameter of a circle.
8.2.11.2 Divide the circumference by the diameter of regular objects to identify the common relationship between them.

Teaching points

1. Identify the circumference and diameter of a circle.
2. Measure the circumferences of circular shapes using a rope.
3. Find the relationship between the circumference and diameter.

Suggested teaching approach

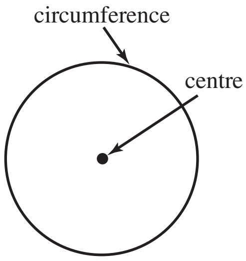
- Learners complete **Learning task 2G** on page 42 of the LB in groups of 3 or 4, using string or rope and a ruler.

Additional notes

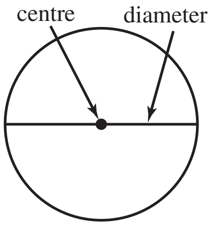
In order to calculate perimeters and areas of circles and shapes that have circular parts, we need to understand how the circumference, diameter and radius of a circle are related. In the following activity, we will investigate and establish these relationships.

The following diagrams illustrate the terms the circumference, radius (*r*), diameter (*d*):

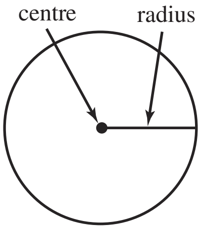
The perimeter of a circle is a called the **circumference.**



The **diameter** of a circle is any straight line from the circumference through the centre to the circumference at the other side.



The **radius** of a circle is any straight line from the centre to the circumference.



Activity 2G

This activity can be used to develop the relationships between the circumference, radius, and diameter of a circle.

Equipment required: ruler, string, circular objects such as cups, plates, lids, drink bottles, wheels, scientific calculator (optional)

Instructions

1. Collect six circular-shaped objects of different sizes.
2. Label them from A to F.
3. For each object, use a ruler to measure and record the radius and diameter of each circle.
4. Lay the string on top of the circumference of the circle. Mark the beginning and end of the circumference on the string, then hold it straight against a ruler and read off the length of the circumference. (A classmate can help you do this reasonably accurately.)

5 Record your measurements in the table given below.

Objects	Radius r (cm)	Diameter d (cm)	Circumference C (cm)	$\frac{C}{d}$	$\frac{C}{d}$

Results

1 Use your completed table from above to describe any patterns or connections you can see between the numbers in the 'radius', 'diameter' and 'circumference' columns.

2 To see the connections more clearly, add two columns to the end of your table, with the headings $\frac{C}{d}$ and $\frac{C}{d}$.
Calculate these two unit ratios for each circle that you measured, round off to two decimal places, and record your results in the new columns. These unit ratios give us a scale factor for the length of the circumference compared to the diameter or radius.

3 Use your completed table and the unit ratios you calculated in the table**,** copy and complete the following sentences.

- **a** The circumference of a circle is approximately ______________ times the size of its diameter.
- **b** The circumference of a circle is approximately _______________ times the size of its radius.
- **c** The length of the diameter of a circle is _____________ the length of its radius.

4 Based on the answers or the results that you have in your table above answer the following questions.

5 If a circle has a diameter of 4 m, predict the lengths of the:

- **a** radius
- **b** circumference.
  - **i** Predict what will happen to: (a) the diameter (b) the circumference when the radius of the circle is doubled.
  - **ii** The exact value of the ratio $\frac{\text{circumference}}{\text{diameter}}$ is represented by the symbol π (the Greek letter Pi). π is an irrational number, which means that if written as a decimal, it is a nonterminating, non-recurring decimal.

2H • Perimeters of circles

LB1 Pages 43–45

Specific learning outcomes

Learners should be able to:

8.2.12.1 Define radius and arc length.

8.2.12.2 Calculate the perimeter of any circles using: $C = \pi D$ or $C = 2\pi r$

8.2.12.3 Find different arc lengths for circles.

Teaching points

1 Explain the terms radius and arc length.

2 Use the formulas: $C = \pi D$ or $C = 2\pi r$ to calculate the circumference of a circle.

3 Use the formula to find the arc length of a circle as a fraction of the circumference.

Suggested teaching approach

- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Learner difficulties and remedies

Difficulty

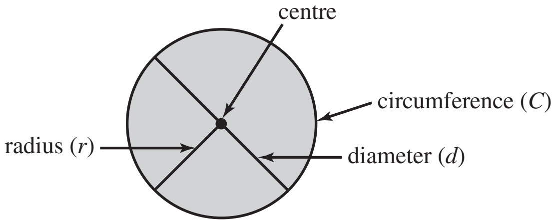
Using formulas to find the circumference and arc length of a given circle.

Remedy

- Explain and define the terms: diameter, radius, and arc length.
- Show learners how to use the radius and the diameter of a circle to find its circumference and and any arc length.

Additional notes

The circumference of a circle is the perimeter of a circle. It is the distance around the outside of the circle.



For any circle:

- the length of the diameter is twice the length of the radius:
  $d = 2r$
- the length of the circumference C is a bit more than three times the diameter:
  $C \approx 3d$
  (Remember that the ≈ sign means 'approximately equal to'.)

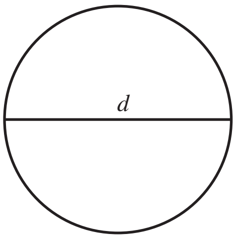
The exact formula for circumference is given by $C = \pi d$, where π (the Greek letter *pi*) is an irrational number (a non-terminating, non-recurring decimal).

Calculators give an approximate value of π of 3·141 592 654. Before the widespread use of calculators it was common to use $\pi \approx 3\frac{1}{7}$ or $\frac{22}{7}$ or 3·14 (correct to two decimal places).

Formulas for calculating the circumference of a circle.

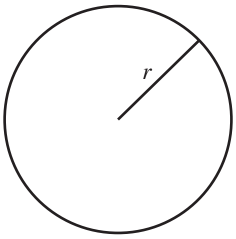
$C = \pi d$

d is the length of the diameter



$C = 2\pi r$

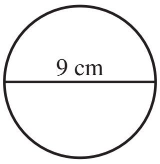
r is the length of the radius



Examples

1 Calculate the circumference of the following circles correct to two decimal places.

a



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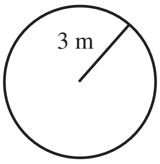
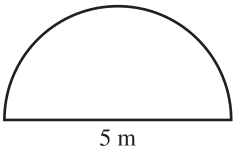


Thinking	**Working**
a **1** Choose the formula that involves diameter.	**a** $C = \pi d$
2 Substitute the value for d into the formula.	$C = \pi \times 9$
3 Use the π button on your calculator to evaluate.	$\approx 28{\cdot}274\,333\,88$
4 Round the answer to two decimal places and write it with the correct unit.	$C = 28{\cdot}27$ cm (2 d.p.)
b **1** Choose the formula that involves radius.	**b** $C = 2\pi r$
2 Substitute the value for r into the formula.	$C = 2 \times \pi \times 3$
3 Use the π button on your calculator to evaluate.	$\approx 18{\cdot}849\,555\,92$
4 Round the answer to two decimal places and write it with the correct unit.	$C = 18{\cdot}85$ m (2 d.p.)

2 Find the perimeter of each of the following shapes correct to two decimal places.

a



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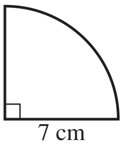


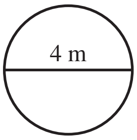
Note: The perimeters of shapes may involve both curved sections and straight sections. If the curved sections are parts of circles, we can calculate their length as a fraction of the circumference and then find the total length of the perimeter.

Thinking	**Working**
a **1** Identify the parts that make up the perimeter (half of a circumference and a straight line equal to the diameter). Use the diameter formula for circumference to write a formula for perimeter.	**a** $P = \frac{C}{2} + d$ where $C = \pi d$ $P = \frac{\pi d}{2} + d$
2 Substitute for d.	$= \frac{\pi \times 5}{2} + 5$
3 Evaluate the curved length, checking that it is a reasonable value. Leave the answer as a long, unrounded decimal.	$\approx 7{\cdot}853\ldots + 5$
4 Add the straight length to find the total and write the answer rounded to the correct number of decimal places with the correct units.	$= 12{\cdot}85$ m (2 d.p.)
b **1** Identify the parts that make up the perimeter (a quarter of a circumference and two straight lines equal to the radius). Use the radius formula for circumference to write a formula for perimeter.	**b** $P = \frac{C}{4} + d$ where $C = 2\pi r$ $P = \frac{2\pi r}{4} + d$
2 Substitute for r.	$= \frac{2 \times \pi \times 7}{4} + 2 \times 7$
3 Evaluate the curved length and the total of the lines. Leave the answer as a long, unrounded decimal.	$\approx 10{\cdot}995\ldots + 14$
4 Add the straight lengths to find the total and write the answer rounded to the correct number of decimal places with the correct units.	$= 25{\cdot}00$ cm (2 d.p.)

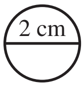
Activity 2H

1 Calculate the circumference of the following circles correct to two decimal places.

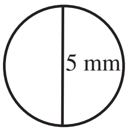
a



b



c



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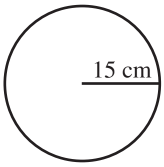

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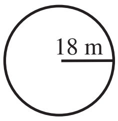

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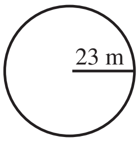

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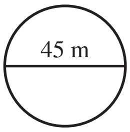

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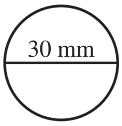

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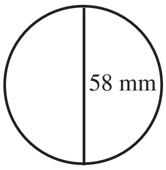

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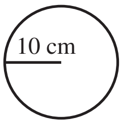

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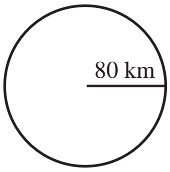

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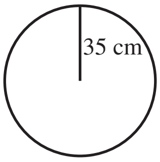
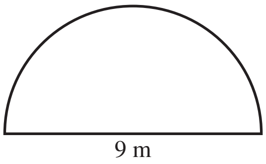


2 Find the perimeter of each of the following shapes correct to two decimal places.

a



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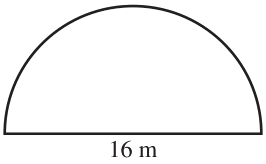

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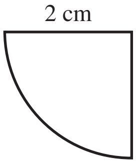

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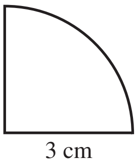

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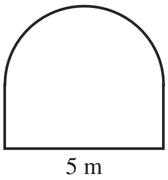

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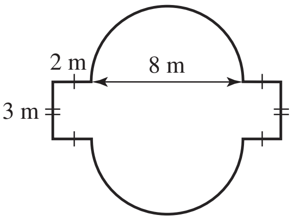
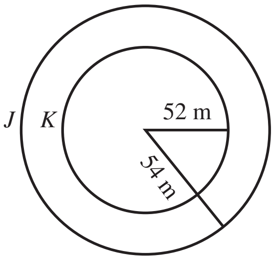
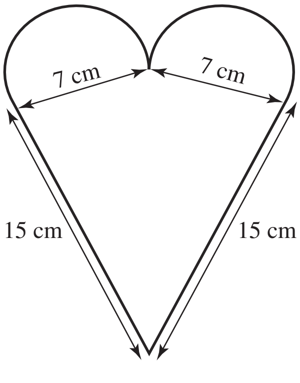


3 A bicycle wheel has a diameter of 60 cm. How far would it move if it is rolled through four revolutions (complete turns)? Give your answer in metres, correct to two decimal places.

4 Jane and Kate run once around the circular track shown. Jane runs along path *J*, while Kate takes path *K*. How much further than Kate does Jane run? Give your answer correct to two decimal places.



5 A box of chocolates is shaped like a heart as shown. Find the perimeter of the box correct to the nearest centimetre.



6 Romano found the circumference of a circle, but lost his working and could only remember part of the answer. His answer began with the digits 314 followed by some zeroes. Romano's friend said he could tell that the diameter must have been approximately a power of 10, such as 100 or 1000. Explain how her friend knew this.

Answers 2H

1 **a** 12·57 m **b** 6·28 cm
c 15·71 mm **d** 94·25 cm
e 113·10 m **f** 144·51 m
g 141·37 m **h** 94·25 mm
i 182·21 mm **j** 62·83 cm
k 502·65 km **l** 219·91 cm

2 **a** 23·14 m **b** 41·13 m
c 7·14 cm **d** 10·71 cm
e 16·85 m **f** 39·13 m

3 $P = d\pi$
$= 60 \times \pi \times 4$
$= 753{\cdot}98$ cm
$= 7{\cdot}54$ m

4 $C = 2\pi r = 2 \times \pi \times 54 = 339{\cdot}292$ m;
$C = 2\pi r = 2 \times \pi \times 52 = 326{\cdot}725$ m;
Difference $= 339{\cdot}29 - 326{\cdot}73$
$= 12{\cdot}57$ m

5 $C = \pi d = \pi \times 7 + 15 + 15 = 51{\cdot}9911$ cm
Perimeter = 52 cm

6 The circumference of a circle is $d\pi$ and is approximately 3·14. So if the circumference was 314 00… . (3·14 × 10… , then the diameter must be a power of 10.

2I • Perimeter of composite shapes

LB1 Pages 46–47

Specific learning outcomes

Learners should be able to:

8.2.13.1 Explain the term 'composite shape'.
8.2.13.2 Calculate the perimeter of composite shapes.

Teaching points

1 Define the term composite shape.
2 Calculate the perimeter of composite shapes.

Learner difficulties and remedies

Difficulty

Identifying the shapes that make up the composite shape and deciding which are the corresponding formulas to calculate the perimeter.

Remedy

- Show learners different polygon shapes and their properties.
- Know which formula to use for various shapes that make up the composite shape.

Suggested teaching approach

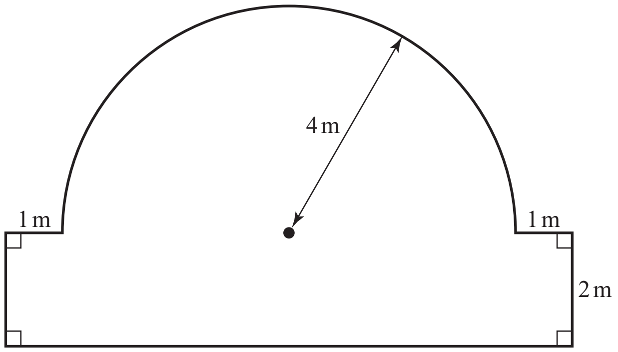
- Identify the different shapes that make up given composite shapes.
- Identify the corresponding formulas that are used to find the perimeter of given composite shapes.
- Identify the formula to find arc length of given circles.
- Use 360° to find the arc length of given circles.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A composite shape is the one that is made up several polygon shapes or parts of a circle. Some shapes have both curved and straight sides. To calculate the perimeter, find the length of each part first and make sure that all lengths are in the same units and then add them up.

Examples

Calculate the perimeter of the composite shape given below.



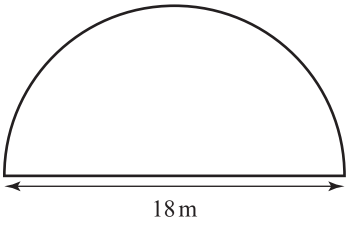
The shape is a semi-circle placed on top of a rectangle.

Thinking	Working
1 Find the base of the rectangle.	1 + 1 + diameter of semicircle = 1 + 1 + 8 = 10 m
2 The perimeter of the curved part is one half of the circumference of a circle with a diameter of 8 m.	$0{\cdot}5 \times \pi \times 8 = 12{\cdot}57$ m
3 the perimeter of the straight part = top + left side + right side + bottom	1 + 1 + 2 + 2 + 10 = 16 m
4 Find the perimeter of the whole shape.	= 12·57 + 16 = 28·57 m

Activity 2I

1 Calculate the perimeter of each of these shapes correct to two decimal places.

a



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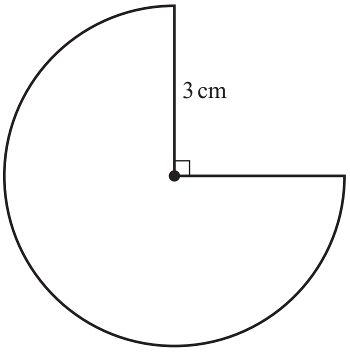
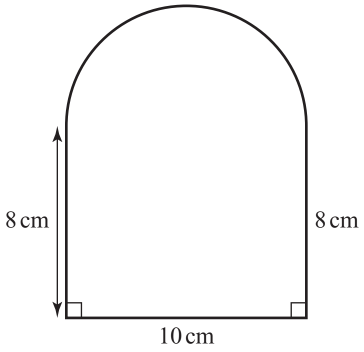
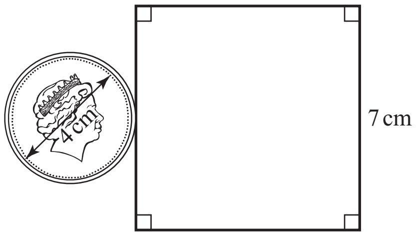


c



2 A coin is rolled once around a square with sides of 7 cm. The diameter of the coin is 4 cm. Calculate the distance travelled by the centre of the coin when the coin has returned to where it started.



Answers 2I

1 **a** 46·3 m
b 20·1 cm
c 41·7 cm

2 $7 \times 4 + 0{\cdot}25 \times 2\pi d \times 4$
= 28 + 12·56
= 40·6 cm

2J • Errors in measurements

LB1 Pages 48–49

Specific learning outcomes

Learners should be able to:

8.2.13.3 Calculate the error that may arise when measurements are given as percentages of other units.

Teaching points

2 Find and calculate errors that occur in the process of measuring length and time.

Suggested teaching approach

- Learners complete **Exercise 2J** on pages 48 and 49 of the LB.

• Chapter Three •

Polygons and Parallel Lines

Overview

Plane shapes are two-dimensional shapes drawn on a flat surface or a plane.

Polygons are plane shapes with straight sides. (When we use the term 'sides', we are referring to straight lines. A circle is not a polygon.) A polygon is a **many-angled** shape. The word polygon is made up of two Greek words *poly* (meaning many) and *gon* (meaning angle).

Regular polygons have all sides of equal length and all angles of equal size. The number of sides gives the name of the polygon.

Parallel lines are straight lines that are always the same distance apart when drawn on a plane.

Polygons and parallel lines are both found in the natural world as well as in objects constructed by humans. For example, polygons can be found in honeycombs and mineral crystals. Architectural designs of buildings involve polygons and parallel lines. In the Solomon Islands, many of the patterns used in designs for woven baskets and mats are based on polygon shapes.

Contents

Chapter skills

This chapter covers the following skills:

- Naming angles
- Classifying angles:
  - Acute
  - Right
  - Obtuse
  - Straight
  - Reflex
  - Revolution or perigon
- Identifying complementary angles
  Add to 90°
- Identifying supplementary angles
  Add to 180°
- Calculating angles and parallel lines
  - Vertically opposite
  - Corresponding
  - Alternate
  - Co-interior
- Classifying triangles
  Side properties
  - Scalene
  - Isosceles
  - Equilateral

  Angle properties
  - Acute
  - Right
  - Obtuse

  Angle sum is 180°
- Classifying quadrilaterals
  - Square
  - Rectangle
  - Parallelogram
  - Rhombus
  - Kite
  - Trapezium

  Angle sum is 360°
- Classifying polygons
  Angle sum = (n – 2) × 180
- Applying geometric bisections

Teaching plan

Lessons	Chapter sections	Class work and home work
1–2	• 3A Angle: Revision	Learner's Book 1 • Exercise 3A, pages 63, 64, 65
3–4	• 3B Angles and parallel lines	Learner's Book 1 • Exercise 3B, pages 67, 68, 69
5	• 3C Properties of triangles	Learner's Book 1 • Exercise 3C, pages 72, 73
6–7	• 3D Properties of quadrilaterals	Learner's Book 1 • Exercise 3D, pages 75, 76
8	• 3E Properties of polygons	Learner's Book 1 • Exercise 3E, pages 78, 79
9	• 3F Exploring geometrical bisections	Learner's Book 1 • Learning task 3F, page 81
10	• Test	Teacher's Guide • Chapter 3 Test

General learning outcomes

Leaners should:

Angle: Revision

8.3.1 Understand how alphabetical letters are used to give names to angles produced with straight lines. (U)

8.3.3 Know that angles are named and classified according to their sizes. (U)

8.3.3 Know how to calculate missing angles. (K)

Angles and parallel lines

8.3.4 Understand that angles are formed by parallel and transversal lines when they cross each other. (U)

8.3.5 Know how to name different angle types and identify their properties. (K)

8.3.6 Know how to find missing angles formed by parallel and transversal lines. (K)

Properties of triangles

8.3.7 Understand that there are different triangles with different names determine by their side and angle properties. (U)

8.3.8 Understand that there are exterior angles that are attached to the sides of any triangles. (U)

Properties of quadrilaterals

8.3.9 Understand quadrilaterals are polygons with four corners and four sides. (U)

8.3.10 Know how to identify and distinguish between the six (6) main quadrilaterals. (K)

8.3.11 Know that the sum of interior angles of a quadrilateral is equal to 360°. (K)

Properties of polygons

8.3.12 Understand polygon shapes and their properties. (U)

8.3.13 Know that quadrilateral shapes can be divided or split into triangles where properties for triangles can be used to find total interior angles in any polygon. (K)

8.3.14 Know how to use the formula to find the interior angle sum of a quadrilateral. (K)

Exploring geometric bisections

8.3.15 Know how to use a compass to construct lines, bisect lines, angles, etc. (K)

3A • Angles: Revision

LB1 Pages 62–65

Specific learning outcomes

Learners should be able to:

8.3.1.1 Name angles using letters as labels for lines and vertices.

8.3.1.2 Draw angles and name them.

8.3.2.1 Name and classify angles according to their size.

8.3.3.1 Calculate missing angles using the properties of complementary, supplementary, etc.

Teaching points

1. Use capital letters to label lines and vertices.
2. Draw angles and name each type of angle.
3. Name individual angles according to the sizes of the angles. Name angle pairs according to their properties.
4. Find unknown angles using the properties of known complementary and supplementary angle pairs.

Learner difficulties and remedies

Difficulty

Distinguishing between the different angle names, and angle properties such as supplementary and complementary angles.

Remedy

- Define the names given to the different types of angles and use diagrams to explain the differences in their properties.
- Distinguish between the names for angles at a point and for pairs of angles. Angles at a point are classified according to their sizes (acute, right, obtuse, straight, reflex and revolution). Pairs of angles are named according to their relationship (adjacent, vertically opposite, complementary and supplementary).
- Understand that there are three ways of labelling angles: using the size of the angle in degrees if known (30°, ¬, 164° etc.); using a lower case letter (*a*, *b*, *c*, etc.) if the size is unknown; or using the capital letters (*A*, *B*, *C*, etc.) at the ends of lines and vertices.

Difficulty

Finding the size of an unknown angle identified with a pronumeral.

Remedy

- Explain that an unknown angle can be calculated using the properties of an angle pair. Identify whether the unknown angle is supplementary, complementary, adjacent or vertically opposite to a known angle. For example, if an unknown angle is complementary to an angle of 26°, then its size can be calculated as $90° - 26° = 64°$.
- Provide activities that have numerals and pronumerals, and learners have to use angle properties to calculate the unknown angles.

Difficulty

Interpreting the symbols used to represent angles.

Remedy

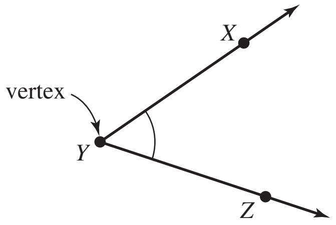
- Identify the different signs and symbols that are used to represent angles such as $\angle$ and $\wedge$. Distinguish between the less than sign (<) and the angle symbol ($\angle$).

Suggested teaching approach

- Use sticks to demonstrate the different types of angles at a point such as a right angle and a straight angle. Then demonstrate examples of angle pairs such as complementary and supplementary angles.
- Explain the properties of the different types of angles and their corresponding names.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Use the **Examples** and **Activity** section at the end of this unit if learners require additional exercises.

Additional notes

An angle is formed when two lines (or rays, or line segments) touch or intersect at a point, known as the vertex. An angle is named using the symbol ∠ and the letters that mark the three points that form the angle. The letter at the vertex is written in the centre. This angle is named $\angle XYZ$ or $\angle ZYX$.

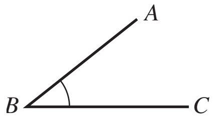


Angles at a point

Acute angle

$0^\circ < \angle ABC < 90^\circ$

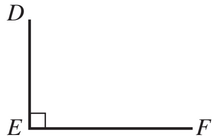
Angle ABC is greater than 0°, but less than 90°.



Right angle

$\angle DEF = 90^\circ$

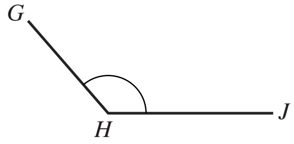
Angle DEF is exactly 90°.



Obtuse angle

$90^\circ < \angle GHJ < 180^\circ$

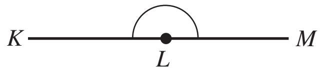
Angle GHJ is greater than 90° but less than 180°.



Straight angle

$\angle KLM = 180^\circ$

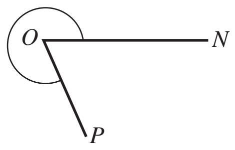
Angle KLM is exactly 180°.



Reflex angle

$180^\circ < \angle NOP < 360^\circ$

Angle NOP is greater than 180° but less than 360°.

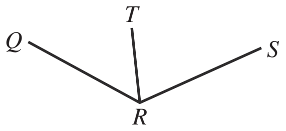


Revolution

The angle is exactly 360°.

Pairs of angles

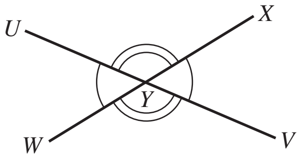
Adjacent angles have a common vertex and a common arm.



$\angle QRT$ *and* $\angle TRS$ are a pair of adjacent angles.

The arm TR is common to both angles.

Vertically opposite angles are equal angles opposite each other when two lines intersect.



The lines UV and WX intersect at Y.

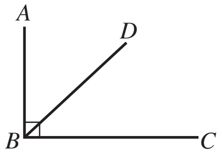
$\angle UYW$ and $\angle XYV$ are a pair of vertically opposite angles.

$\angle UYW = \angle XYV$

$\angle UYX$ and $\angle WYV$ are a pair of vertically opposite angles.

$\angle UYX = \angle WYV$

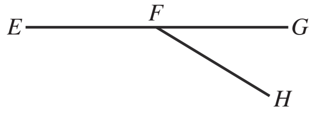
Complementary angles are a pair of angles that add to 90°.



$\angle ABD + \angle CBD = 90^\circ$

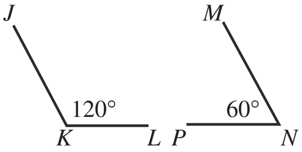
$\angle ABD$ and $\angle CBD$ are complementary angles.

Supplementary angles are a pair of angles that add to 180°.



$\angle EFH + \angle GFH = 180^\circ$

$\angle EFH$ and $\angle GFH$ are supplementary angles.



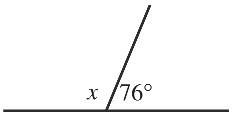
$\angle JKL$ and $\angle MNP$ are supplementary angles.

Examples

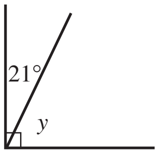
a What is the complement of the angle 14°?

b What is the supplement of the angle 53°?

c Find the value of the angle x.



d Find the value of angle y.

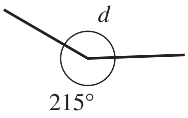


Thinking	Working
a Complementary angles add to 90°. Subtract the known angle from 90° to find the unknown.	**a** $90° - 14° = 76°$ The complement of 14° is 76°.
b Supplementary angles add to 180°. Subtract the known angle from 180° to find the unknown.	**b** $180° - 53° = 127°$ The supplement of 53° is 127°.
c x and 76° are supplementary angles. Write an equation that shows this, and then solve it.	**c** $x + 76° = 180°$ (supplementry angles) $x = 104°$
d y and 21° are complementary angles. Write an equation that shows this, and then solve it.	**d** $y + 21° = 90°$ (complementary angles) $y = 69$

Activity 3A

1 Find the values of the pronumerals in each diagram. Give a reason for each answer.

a



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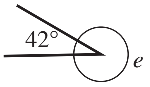
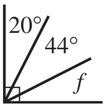
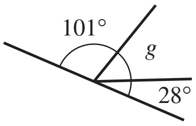


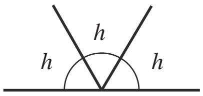
c



d



e



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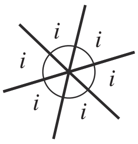

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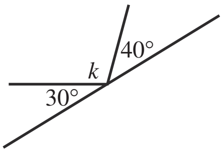

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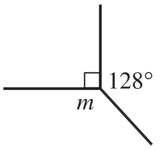

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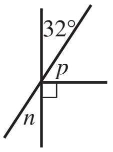
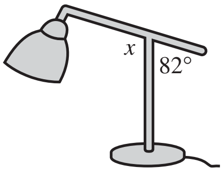


2 Find the missing angle in each of the following.

a

b



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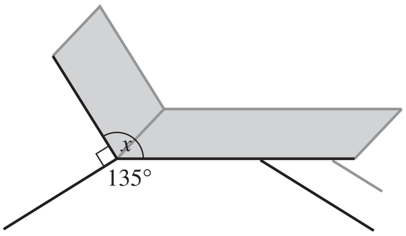


Answers 3A

1 **a** $d = 145°$, angle of revolution

b $e = 318°$, angle of revolution

c $f = 26°$, right angle

d $g = 51°$, straight angle

e $h = 60°$, straight angle

f $i = 60°$, angle of revolution

g $k = 110°$, straight angle

h $m = 142°$, angle of revolution

i $n = 32°$, vertically opposite angle, $p = 58°$, straight angle

2 **a** $x = 180° - 115° = 65°$

b $x = 180° - 82° = 98°$

c $x = 360° - 135° - 90° = 135°$

3B • Angles and parallel lines

LB1 Pages 66–69

Specific learning outcomes

Learners should be able to:

8.3.4.1 Define and identify transversal and parallel lines.

8.3.5.1 Name different types of angles: *vertically opposite*; *corresponding*; *alternate* and *co-interior* or *allied angles*.

8.3.6.1 Find the size of missing angles using the relationships between the angles and parallel lines.

Teaching points

1. Explain the terms ‘transversal’ and ‘parallel lines’.
2. Name the four different types of angles that are created when a transversal cuts a pair of parallel lines.
3. Find unknown angles using the properties of angle pairs: Vertically opposite, alternate, corresponding and co-interior angles.

Learner difficulties and remedies

Difficulty

Identifying and naming the different types of angle pairs that are created when a transversal intersects a pair of parallel lines.

Remedy

- Use diagrams to identify, explain and distinguish between the properties of the angles that are produced with a pair of parallel lines and a transverse line.

Difficulty

Calculating unknown angles represented by a pronumerals using the properties of angles on parallel lines.

Remedy

- Explain that an unknown angle can be calculated using the properties of an angle pair on parallel lines. Identify whether the unknown angle is vertically opposite, co-interior, alternate or corresponding to a known angle. For example, if an unknown angle and an angle of 53° are co-interior, then the size of the unknown angle can be calculated as $180° - 53° = 127°$.

Suggested teaching approach

- Identify the following angle pairs that are produced with a pair of parallel lines and transversal line: corresponding, vertically opposite, alternate, co-interior angles.
- Explain and distinguish between the properties of these pairs of angles.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Use the **Examples** and **Activity** section at the end of this unit if learners require additional exercises.

Additional notes

Angles on parallel lines

A **transversal** is a line that intersects two or more other lines. When the lines that the transversal intersects are parallel:

- pairs of **corresponding angles** are equal
- pairs of **alternate angles** are equal
- pairs of **co-interior angles** are supplementary.

AB and CD are parallel lines. We write this as $AB \parallel CD$.

The transversal XY intersects AB at point E and CD at point F.

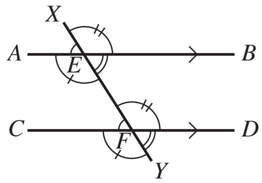
Corresponding angles lie on the same side of the transversal and in the same position above or below the parallel lines. Each of the four angles formed at E has an equal corresponding angle at F.

$\angle AEX = \angle CFE$

$\angle AEF = \angle CFY$

$\angle FEB = \angle YFD$

$\angle DFE = \angle BEX$



Alternate angles lie on opposite sides of the transversal. Two pairs of equal alternate angles are formed.

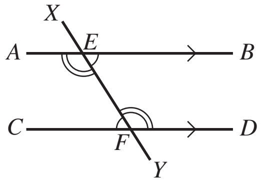
$\angle AEF = \angle EFD$

$\angle BEF = \angle EFC$

Co-interior angles lies on the same side of the transversal, inside the parallel lines. They are supplementary angles and are sometimes called 'allied' angles. Two pairs of co-interior angles are formed.

$\angle AEF + \angle EFC = 180^\circ$

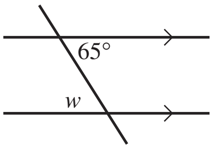
$\angle BEF + \angle EFD = 180^\circ$



Examples

Find the value of the pronumerals in each diagram. Give reasons for your answer.

a



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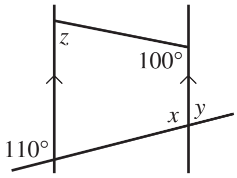

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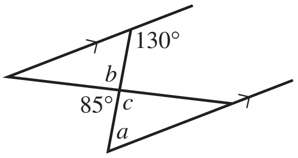
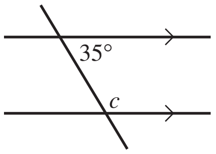


Thinking	**Working**
a Identify the relationship between the known and the unknown angle. Write this in brackets next to your answer as a reason for your answer.	**a** $w = 65^\circ$ (alternate angles on parallel lines)
b **1** Identify relationships between known and unknown angles. Write these in brackets next to your answers as reasons for your answers.	**b** $x = 110^\circ$ (corresponding angles on parallel lines) $z + 100^\circ = 180^\circ$ (co-interior angles) $z = 80^\circ$
2 Once you have found the value of some unknowns, use them to determine others. (Here, we use the value of x to find y.)	$x + y = 180^\circ$ (supplementry angles) $110^\circ + y = 180^\circ$ $y = 70^\circ$
c **1** Identify relationships between known and unknown angles. Write these in brackets next to your answers as reasons for your answers.	**c** $a + 130^\circ = 180^\circ$ (co-interior angles) $a = 50^\circ$
2 Once you have found the value of some unknowns, use them to determine others. (Here, we find the value of c and use it to find b.)	$c + 85^\circ = 180^\circ$ (supplementry angles) $c = 95^\circ$ $b = 95^\circ$ (vertically opposite angles)

Activity 3B

1 Find the value of the pronumerals in each diagram. Give reasons for your answer.

a



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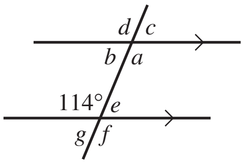

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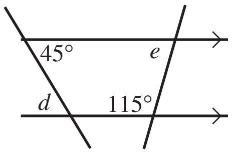

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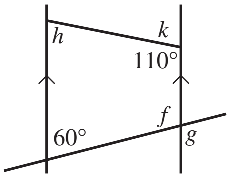
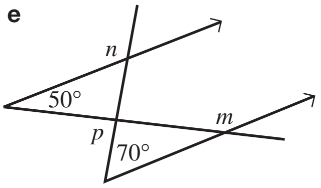


e



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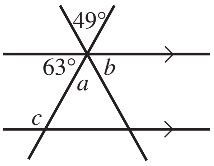
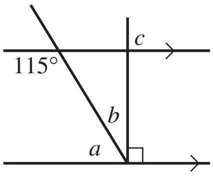


2 Find the value of the pronumerals in each diagram. Give reasons for your answer.

a



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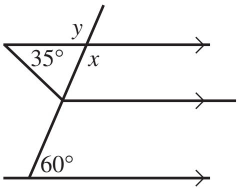
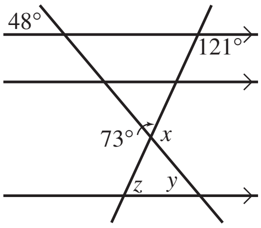
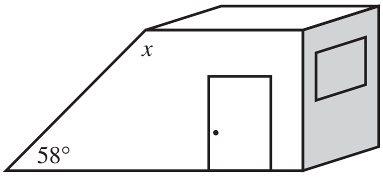


c



3 Find the value of *x*, giving reasons for your answer.

a



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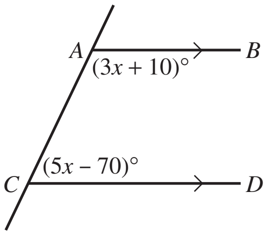


Answers 3B

1 a $c = 145°$, co-interior angles are supplementary

b $a = 114°$, alternate angles are equal
$b = 66°$, co-interior angles are supplementary
$c = 66°$, vertically opposite angles are equal
$d = 114°$, corresponding angles are equal
$e = 66°$, supplementary angles
$f = 114°$, vertically opposite angles are equal
$g = 66°$, supplementary angles

c $d = 45°$, alternate angles are equal
$e = 65°$, co-interior angles are supplementary

d $f = 120°$, co-interior angles are supplementary
$g = 120°$, vertically opposite angles are equal
$h = 70°$, co-interior angles are supplementary
$k = 70°$, alternate angles are equal

e $m = 130°$, co-interior angles are supplementary
$n = 110°$, supplementary
$p = 50° + 70° = 120°$, exterior angle of a triangle

f $a = 49°$, vertically opposite angles are equal
$b = 68°$, straight angle
$c = 117°$, co-interior angles are supplementary

2 a $a = 65°$, co-interior angles are supplementary
$b = 25°$, straight angle
$c = 90°$, corresponding angles are equal

b $x = 120°$, co-interior angles are supplementary
$y = 120°$, vertically opposite angles are equal

c $x = 107°$, supplementary angles
$y = 48°$, corresponding angles are equal
$z = 59°$, co-interior angles are supplementary

3 a $x = 180° – 58° = 122°$, co-interior angles are supplementary

b

$$\begin{aligned} 3x + 10 + 5x - 70 &= 180° \\ 8x - 60 &= 180° \\ 8x &= 240° \\ x &= 30° \end{aligned}$$

co-interior angles are supplementary

3C • Properties of triangles

LB1 Pages 70–73

Specific learning outcomes

Learners should be able to:

8.3.7.1 Identify the ‘side’ and ‘angle’ properties of different triangles.
8.3.7.2 Name different triangles according to their ‘side’ and ‘angle’ properties.
8.3.7.3 Measure angles using protractors.
8.3.8.1 Identify the *exterior angle* properties of any triangle.
8.3.8.2 Find the size of exterior angles using the *exterior angle properties.*

Teaching points

1. Name triangles according their side properties: scalene, isosceles and equilateral.
2. Name triangles according to their angle properties: acute, right, and obtuse.
3. Measure angles using protractors.
4. Identify the exterior angle properties of triangles.
5. Find the sizes of angles using the external angle properties for triangles.

Learner difficulties and remedies

Difficulty

Identifying the properties of the different types of triangles and naming them correctly.

Remedy

- Name triangles according to their side properties (scalene, isosceles, equilateral) and their angle properties (acute, right, obtuse).
- Identify and distinguish the differences between the side and angle properties to assist learners to understand the differences in these triangles.

Difficulty

Calculating an unknown angle in a triangle.

Remedy

- Remember that the angle sum of a triangle is 180°.

Suggested teaching approach

- Show the differences and similarities in a range of examples of different scalene, isosceles and equilateral triangles.
- Identify the properties of the sides of these triangles.
- Show the differences and similarities in a range of examples of different acute-, obtuse- and right-angled triangles.
- Identify the properties of the angles in these triangles.
- Show that the angle sum for any triangle is 180°.
- Clearly distinguish between the interior and exterior angles of a triangle and show that an exterior angle is equal to the sum of the two opposite interior angles.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Use the **Examples** and **Activity** section at the end of this unit if learners require additional exercises.

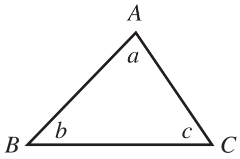
Additional notes

Angle sum of a triangle

A triangle is a three-sided plane shape.

The sum of the angles of a triangle is 180°.

$\angle BAC + \angle ABC + \angle BCA = 180°$ or $a + b + c = 180°$

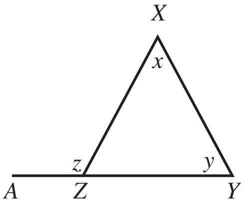


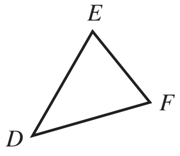
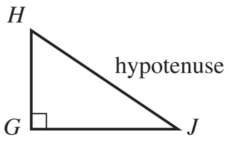
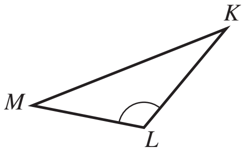
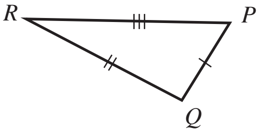
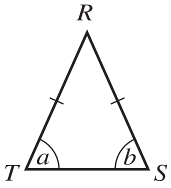
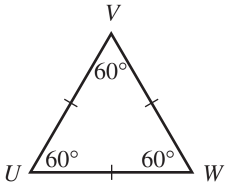
Angle sum of a triangle

An exterior angle is produced when any side of a shape is extended. The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

YZ is extended to A to form the exterior angle $\angle AZX$.

$\angle AZX = \angle ZXY + \angle XYZ$ or $x = x + y$

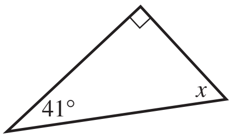


Shape	Definition and properties
Acute-angled triangle  	All angles are acute. $\angle DFE < 90°$, $\angle FED < 90°$, $\angle EDF < 90°$
Right-angled triangle  	One angle equals 90°. $\angle HGJ = 90°$ The side opposite the right angle is called the **hypotenuse**.
Obtuse-angled triangle  	One angle is obtuse. $\angle KLM > 90°$ The other two angles are acute. $\angle LKM < 90°$ $\angle KML < 90°$
Scalene triangle  	All sides and angles are unequal. PQ, QR and RP are different lengths.
Isosceles triangle  	One pair of sides are equal in length. $RS = RT$ The angles opposite the equal sides are also equal in size. $\angle RST = \angle RTS$ or $a = b$
Equilateral triangle  	All sides are equal in length. $UV = VW = WU$ All angles are equal to 60°. $\angle UVW = \angle VWU = \angle WUV = 60°$

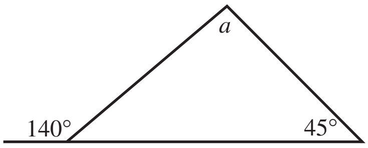
Examples

Find the value of the pronumerals in the shapes below.

a



b



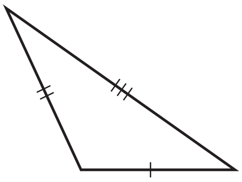
Thinking	Working
a **1** Use the angle properties of the shape to form an equation. Give a reason in brackets. **2** Solve the equation.	**a** $x + 41 + 90 = 180$ (angle sum of a triangle is 180°) $x + 131 = 180$ $x = 180 - 131$ $x = 49°$
b As the exterior angle is equal to the sum of the two interior opposite angles, substitute the known angles into the rule and calculate the unknown angle.	**b** $140 = a + 45$ $a = 140 - 45$ $a = 95°$

Activity 3C

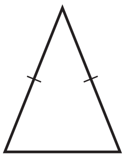
1 For each of the following triangles:

- **(i)** give its side name
- **(ii)** give its angle name
- **(iii)** describe the triangle with an angle and a side name.

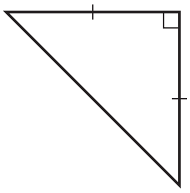
a



b



c



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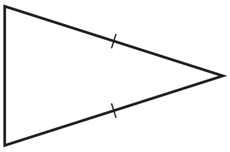

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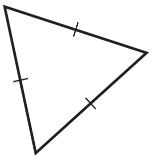

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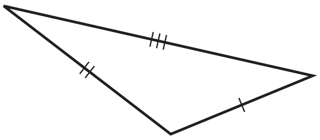
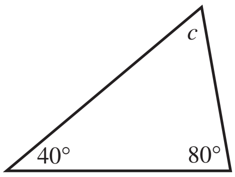


2 Find the size of the angle labelled *c* in each of the following triangles.

a



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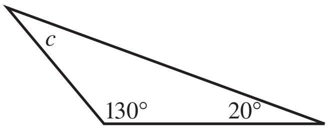
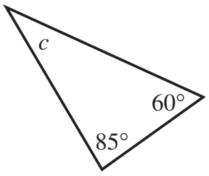


c



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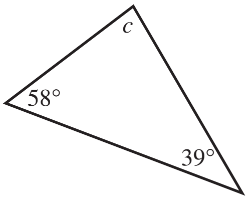

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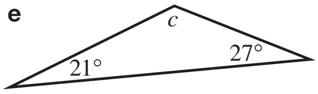

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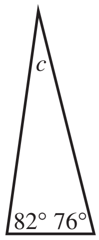
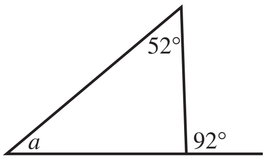
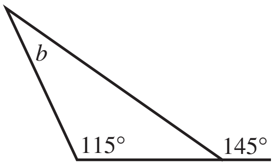


3 Calculate the value of the pronumeral in each of the following.

a



b



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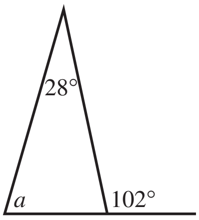

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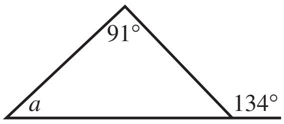

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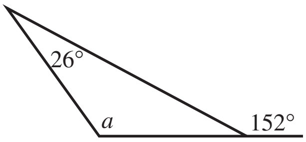
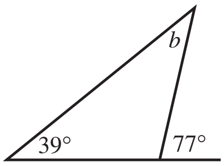
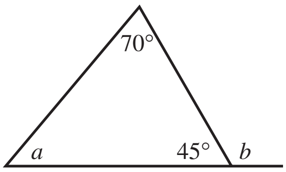


f

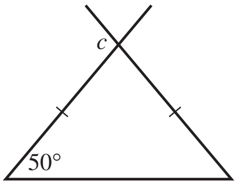


4 Find the unknown angle or angles (represented by the pronumeral) in each of the following diagrams. You may need to use your knowledge of angle properties.

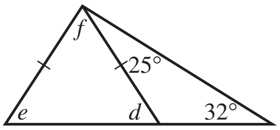
a



b



c



Answers 3C

1 a **i** scalene
ii obtuse
iii an obtuse-angled scalene triangle

b **i** isosceles
ii acute
iii an acute-angled isosceles triangle

c **i** isosceles
ii right
iii a right-angled isosceles triangle

d **i** isosceles
ii acute
iii an acute-angled isosceles triangle

e **i** equilateral
ii acute
iii an acute-angled equilateral triangle

f **i** scalene
ii obtuse
iii an obtuse-angled scalene triangle

2 a 60° **b** 30°
c 35° **d** 83°
e 132° **f** 22°

3 a 40° **b** 30°
c 74° **d** 43°
d 126° **f** 38°

4 a $a = 65°$, $b = 135°$
b $c = 100°$
c $d = 57°$, $e = 57°$, $f = 66°$

3D • Properties of quadrilaterals

LB1 Pages 74–76

Specific learning outcomes

Learners should be able to:

8.3.9.1 Define a quadrilateral.

8.3.10.1 Name the six main quadrilaterals and identify their properties: *rectangle; square; kite; parallelogram; rhombus; trapezium.*

8.3.10.2 Identify the properties of particular quadrilaterals.

8.3.11.1 Find the size of the missing angles in a quadrilateral using the angle sum of a quadrilateral.

8.3.13.1 Calculate the total angles in a quadrilateral using the angle-sum of a triangle.

Teaching points

1. Explain that a quadrilateral is the name of any four-sided polygon.
2. Identify the six main quadrilateral shapes by their properties: rectangle, square, kite, parallelogram, rhombus, trapezium.
3. Find missing angles using the properties of the special quadrilaterals.

Learner difficulties and remedies

Difficulty

Identifying the six main quadrilaterals, and distinguishing between them on the basis of their properties.

Remedy

- Name the six special quadrilaterals according to their properties (square, rectangle, rhombus, parallelogram, trapezium, kite).

Difficulty

Calculating an unknown angle in a quadrilateral.

Remedy

- Remember that the angle sum of a quadrilateral is 360°.

Suggested teaching approach

- Draw a diagram of each of the six special quadrilaterals. Mark in their properties to distinguish each quadrilateral.
- Identify the differences and similarities between these quadrilaterals.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Use the **Examples** and **Activity** section at the end of this unit if learners require additional exercises.

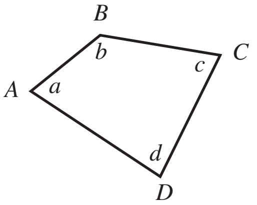
Additional notes

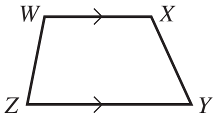
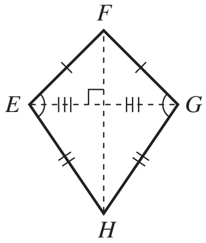
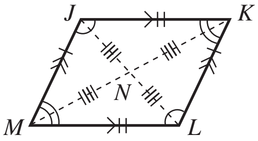
Angle sum of a quadrilateral

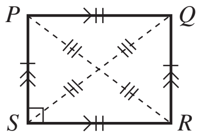
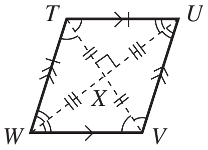
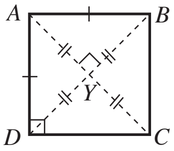
A quadrilateral is a four-sided plane shape.

The sum of the angles of a quadrilateral is 360°.

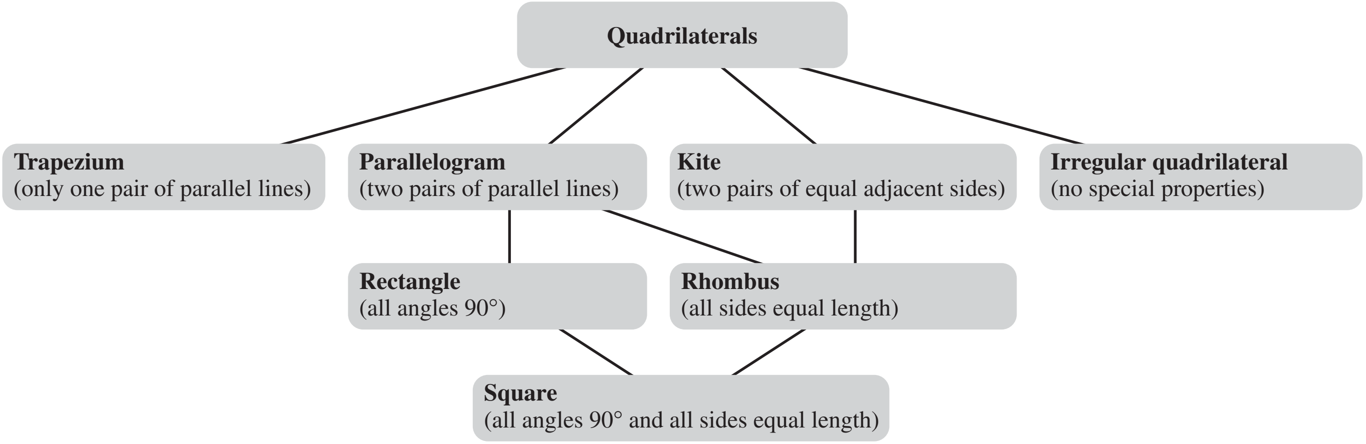
$a + b + c + d = 360°$ or $\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°$



Shape	Definition	Properties
Trapezium 	A trapezium is a quadrilateral with only one pair of opposite sides parallel. *WXYZ* is a trapezium in which $WX \parallel ZY$.	
Kite 	A kite is a quadrilateral with two pairs of equal adjacent sides. *EFGH* is a kite in which $EF = FG$ and $EH = HG$.	• A kite has one pair of equal opposite angles: $\angle FEH = \angle FGH$ • The diagonals are perpendicular: $FH \perp EG$ • The diagonal that passes through the equal angles is **bisected** (cut in half) by the other diagonal.
Parallelogram 	A parallelogram is a quadrilateral with both pairs of opposite sides parallel. *JKLM* is a parallelogram in which $JK \parallel ML$ and $JM \parallel KL$.	A parallelogram has: • both pairs of opposite sides are equal: $JK \parallel ML$ and $JM \parallel KL$. • both pairs of opposite angles are equal: $\angle MJK = \angle MLK$ and $\angle JML = \angle JKL$ • the diagonals bisect each other: $JN = NL$ and $MN = NK$

Shape	Definition	Properties
Rectangle 	A rectangle is a parallelogram with a right angle. *PQRS* is a rectangle in which $\angle PSR = 90°$.	A rectangle has all the properties of a parallelogram, plus: • each angle is 90°: $\angle PSR = \angle SRQ = \angle RQP = \angle QPS = 90°$ • the diagonals are equal in length: $PR = QS$
Rhombus 	A rhombus is a parallelogram with two equal adjacent sides. *TUVW* is a rhombus in which $TU = TW$.	A rhombus has all the properties of a parallelogram, plus: • all sides are equal: $TU = TW = WV = UV$ • the diagonals bisect the angles: $\angle WTU = \angle VTU$, $\angle WVT = \angle TVU$ $\angle TWU = \angle UWV$, $\angle TUW = \angle WUV$ • the diagonals bisect each other at right angles: $TX = XV$, $WX = XU$ $\angle WXT = \angle TXU = \angle UXV = \angle VXW = 90°$
Square 	A square is a rectangle with one pair of equal adjacent sides. *ABCD* is a square in which $AB = AD$.	A square has all the properties of a rectangle, plus: • all sides are equal: $AB = AD = DC = BC$ • the diagonals are equal in length: $AC = BD$ • the diagonals bisect the angles of the square: $\angle DAC = \angle CAB$, $\angle DCA = \angle ACB$, $\angle ABD = \angle DBC$, $\angle ADB = \angle BDC$ • the diagonals bisect each other at right angles: $\angle AYD = \angle DYC = \angle CYB = \angle BYA = 90°$
Alternative definition of a square	A square is a rhombus with a right angle. *ABCD* is a square in which $\angle ADC = 90°$.	A square has all the properties of rhombus, plus: • all angles are right angles: $\angle ADC = \angle DCB = \angle CBA = \angle BAD = 90°$ • the diagonals are equal: $AC = BD$

The quadrilateral family

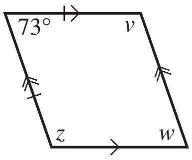


From the above diagram, we can see that:

- rectangles, rhombuses and squares are special types of parallelograms
- a rhombus can also be classified as a special type of kite
- a square can also be classified as a special type of rectangle.

Examples

Find the value of the pronumerals in the shape.

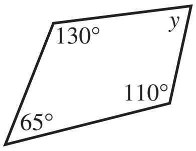


Thinking	Working
1 Identify the shape.	A parallelogram with one pair of adjacent sides equal is a rhombus.
2 Form an equation, with reason.	$v + 73^\circ = 180^\circ$ (co-interior angles on parallel lines)
3 Solve the equation.	$v = 107^\circ$
4 Repeat for the other angles.	$w = 73^\circ$ (opposite angles of a rhombus are equal) $z = 107^\circ$ (opposite angles of a rhombus are equal)
5 List all answers.	$v = 107^\circ$, $w = 73^\circ$, $z = 107^\circ$

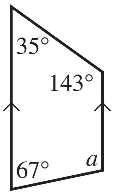
Activity 3D

1 Find the values of the pronumerals in each shape.

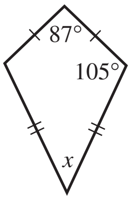
a



b

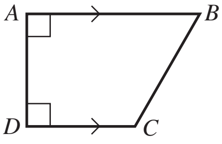


c

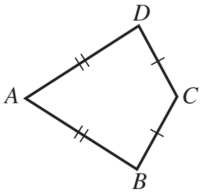


2 What is the most appropriate name for the quadrilateral *ABCD* in each diagram?

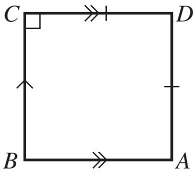
a



b



c



Answers 3D

1 **a** $x = 55^\circ$

b $a = 115^\circ$

c $x = 63^\circ$

2 **a** trapezium

b kite

c square

3E • Properties of polygons

LB1 Pages 77–79

Specific learning outcomes

Learners should be able to:

8.3.12.1 Define the term 'polygon' and identify particular polygon shapes.

8.3.12.2 Identify and name polygon shapes according to the number of sides.

8.3.12.2 Calculate the total angles in a quadrilateral using the angle sum of a triangle.

8.3.14.1 Calculate the total angles in any polygon shape using the formula:
Total angle sum = $(n - 2) \times 180^\circ$

Teaching points

1 Explain the meaning of the term 'polygon'.

2 Name polygon shapes according to their properties (the number of sides).

3 Use the angle sum of a triangle, which is 180°, to find the angle sum in a given quadrilateral by dividing it into triangles.

4 Use the following formula to find the angle sum in any given polygon.
Total angle sum = $(n - 2) \times 180^\circ$

Learner difficulties and remedies

Difficulty

Using the angle sum of a triangle to find the angle sum of any given polygon.

Remedy

- Explain that any regular polygon can be divided into triangles by drawing lines from one vertex in the shape to each of the other vertices. We can then use the angle sum of a triangle (180°) to find the sum of the interior angles in any polygon.

Suggested teaching approach

- Define the term polygon.
- Demonstrate how the angle sum of a triangle can be used to find the sum of interior angles of any regular polygon.
- Derive a formula that can be used to find the sum of interior angles of any given polygon shape using the angle sum of a triangle.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Use the **Examples** and **Activity** section at the end of this unit if learners require additional exercises.

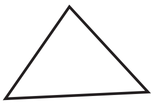
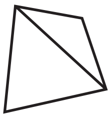
Additional notes

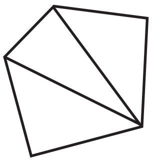
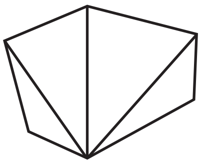
Polygons

A polygon is a plane shape with many sides. (When we use the term 'sides', we are referring to straight lines. A circle is not a polygon.) A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, a pentagon is a polygon with five sides, and so on. A regular polygon has all its sides and angles equal. An equilaterial triangle is a regular three-sided polygon and a square is a regular four-sided polygon. There are no special names for other regular polygons – a regular five-sided figure is simply called a regular polygon.

Angle sum of a polygon

We can find the angle sum of these polygons by dividing each shape into triangles as shown.

Triangle	Quadrilateral
 Angle sum = 180° This is also: Angle sum = (3 – 2) × 180°	 Angle sum = 2 × 180° = 360° Angle sum = (4 – 2) × 180°

Pentagon	Hexagon
 Angle sum = 3 × 180° = 540° Angle sum = (5 – 2) × 180°	 Angle sum = 4 × 180° = 720° Angle sum = (6 – 2) × 180°

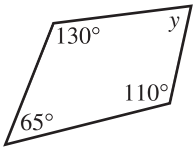
The sum of the angles in a polygon with n sides = $(n - 2) \times 180^\circ$

The **interior angles** (angles inside the shape) of a regular polygon are equal.

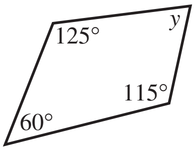
Examples

Find the value of the pronumerals in each shape.

a



b

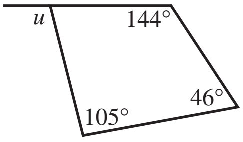


Thinking	Working
a 1 Use the angle properties of the shape to form an equation. Give a reason in brackets.	**a** $x + 41^\circ + 90^\circ = 180^\circ$ (angle sum of a triangle is 180°)
2 Solve the equation.	$x + 131^\circ = 180^\circ$ $x = 180^\circ - 131^\circ$
b 1 Use the angle properties of the shape to form an equation. Give a reason in brackets.	**b** $y + 115^\circ + 60^\circ + 125^\circ = 360^\circ$ (angle sum of a quadrilateral)
2 Solve the equation.	$y + 300^\circ = 360^\circ$ $y = 360^\circ - 300^\circ$ $y = 60^\circ$

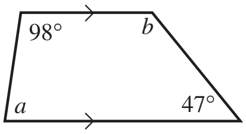
Activity 3E

1 Find the value of the pronumerals in each shape.

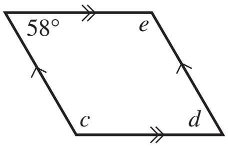
a



b



c



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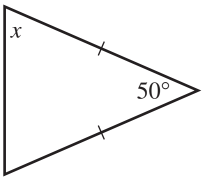
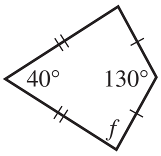
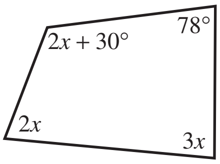


e



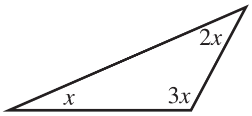
f



2 Sketch a hexagon (a six-sided polygon) and use it to answer the following.

- **a** How many triangles can it be divided into?
- **b** What is the angle sum of a triangle?
- **c** Use you answers to (a) and (b) to determine the sum of the angles in a hexagon.

3 The three angles in a triangle are x, $2x$ and $3x$. What is the size of each angle?



Answers 3E

1 a $a = 180^\circ - 98^\circ = 82^\circ$, $b = 180^\circ - 47^\circ = 133^\circ$

b $c = 180^\circ - 58^\circ = 122^\circ$, $d = 58^\circ$, $e = 122^\circ$

c $x = (180^\circ - 50^\circ) \div 2 = 65^\circ$

d $f = (360^\circ - 130^\circ - 40^\circ) \div 2 = 95^\circ$

e
$$\begin{aligned}(2x + 30) + 2x + 3x + 78 &= 360\\ 7x + 108 &= 360\\ 7x &= 252\\ x &= 36^\circ\end{aligned}$$

f
$$\begin{aligned}u &= 180^\circ - (360^\circ - 144^\circ - 46^\circ - 105^\circ)\\ &= 180^\circ - 65^\circ\\ &= 115^\circ\end{aligned}$$

2 a 4

b 180°

c 720°

2 30°, 60°, 90°

3F • Exploring geometric bisections

LB1 Pages 80–81

Specific learning outcomes

Learners should be able to:

8.3.15.1 Use a compass to:

- bisect angles with straight edge
- bisect a line segment
- identify the centre of a circle
- find the circumference of a circle.

Teaching points

1 The accurate use of compasses and a straight edge to bisect angles and line segments.

2 The use of angle bisectors in a triangle to locate the incentre.

3 Bisecting the sides of a triangle to locate the circumcentre.

4 Creating other geometric designs using these constructions.

Learner difficulties and remedies

Difficulty

Understanding the term bisect.

Remedy

- Explain that to bisect means to divide in half. Bisecting an angle or line segment requires drawing equal arcs. It is important that the opening of a compass is not changed between drawing the first arc and drawing the second arc.

Difficulty

Understanding the terms ‘incentre’ and ‘circumcentre’.

Remedy

- These terms relate to the centres of the circles that are drawn as the final stage of the constructions.

  The **incentre** is the centre of the circle that fits **inside** the triangle, so that the circumference touches each of the three sides.

  The **circumcentre** is the centre of the circle that fits **outside** the triangle, so that all three vertices touch the circumference.

Suggested teaching approach

- Have learners work through **Learning Task 3F** in the LG on page 81 to consolidate the skills used when creating angles with a compass and a straight edge.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.

• Chapter Four •

Algebra Symbols

Overview

Algebra is a mathematical language that uses letters and symbols to communicate general rules and to solve problems. It gives us a symbolic way to describe and generalise patterns that occur in the real world. Engineers use algebra to calculate stress forces on bridges, architects use it to design environmentally friendly buildings, nurses use it to calculate the correct doses of medicine, and accountants use it to calculate how much tax needs to be paid. This chapter will enable learners to continue to develop their skills in manipulating algebraic expressions and apply formulas to solve problems.

Contents

Chapter skills

This chapter covers the following skills:

- Working with mathematical expressions
- Finding equivalent forms for various mathematical expressions
- Using the distributive law

  $a(b + c) = ab + ac$
- Evaluating mathematical expressions
- Deducing a formula which models a particular situation
- Verifying a deduced formula
- Listing values derived from a formula in sequence form

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 4A Algebraic expressions	Learner's Book 1 • Exercise 4A, pages 94, 95
2	• 4B Adding and subtracting like terms	Learner's Book 1 • Exercise 4B, pages 96, 97
3–4	• 4C Multiplying and dividing terms	Learner's Book 1 • Exercise 4C, pages 99, 100
5	• 4D Expanding brackets	Learner's Book 1 • Exercise 4D, pages 101, 102
6–7	• 4E Evaluating algebraic expressions	Learner's Book 1 • Exercise 4E, pages 103,104
8–9	• 4F Formulas in mathematics	Learner's Book 1 • Exercise 4F, page 105, 106, 107
10	• Test	Teacher's Guide • Chapter 4 Test

General learning outcomes

Learners should:

Algebraic expressions

8.4.1 Understand algebraic expressions as mathematic statements made up of terms, coefficients, and constants. (U)

8.4.2 Know how to find values for pronumerals that are given in maths statements. (K)

8.4.3 Know how to identify terms, coefficients and constants in an expression. (K)

8.4.4 Know how to write algebraic expressions for given statements of quantities. (K)

8.4.5 Know how to group terms in a given expression. (K)

Adding and subtracting like terms

8.4.6 Know how to simplify expressions by adding and subtracting like terms. (K)

Multiplying and dividing terms

8.4.7 Understand that terms in an expression can be multiplied and divided with other terms. (U)

Expanding brackets

8.4.8 Understand that brackets in an expression can be expanded using the 'Distributive Law'. (U)

Evaluating algebraic expressions

8.4.9 Understand that expressions can be evaluated by substituting values to replace pronumerals in an expression. (U)

Formulas in mathematics

8.4.10 Understand that a formula is a rule that allows us to make calculations to find values for variables in an equation. (U)

8.4.11 Know how to use various formulas appropriately to evaluate and solve maths problems. (K)

4A • Algebraic expressions

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Specific learning outcomes

Learners should be able to:

8.4.1.1 Define the term 'algebraic expression'. Expression: a mathematical statement that is made up of terms, coefficients, constants separated by addition and subtraction signs.

8.4.1.2 Explain the words: terms; pronumerals; coefficients; like terms; unlike terms; constants.

8.4.2.1 Evaluate pronumerals in mathematical statements.

8.4.3.1 Identify terms, coefficients and constants in expressions.

8.4.4.1 Write algebraic expressions for given statements that represent quantities.

8.4.5.1 Group like terms in expressions.

Teaching points

1. Provide examples and counter-examples to explain the following terms: algebraic expression, terms, pronumerals, coefficient, like terms, unlike terms and constants.
2. Find the values for letters (pronumerals) that are used in algebra to represent unknown quantities.
3. Identify the terms, coefficient and constant in expressions.
4. Write algebraic expressions for statements that represent quantities.
5. Identify groups of like terms within a list of algebraic terms.

Learner difficulties and remedies

Difficulty

Understanding terms that are used in algebra and distinguishing their meaning in mathematics from common usage.

Remedy

- Explain or define to learners the terms that are used in algebra using specific examples.

Suggested teaching approach

- Its important that learners know the definitions or meanings of the terms such as algebraic expression, terms, pronumerals, coefficient, like terms, unlike terms and constants, because their common usage is sometimes different from how we use them in mathematics.
- Explain to learners that in algebra we use letters (pronumerals) to represent unknown quantities. We then set up equations using the pronumerals using the information that is known. We can then solve the equation to find the unknown quantity.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Algebraic language

Learners need to understand the language of algebra to solve problems. In Year 7 they were introduced to many new algebraic words and conventions.

A **pronumeral** (or variable) describes an unknown amount in an algebraic expression or equation and is represented by a letter or a symbol. For example, the average amount of water used by a Year 8 learner per day is a variable that can be represented by a symbol such as a, x, α or β.

Note that to avoid confusion, the pronumeral x is written in *italics* to distinguish it from the multiplication sign ×.

A **term** can be:

- a number or a variable by itself: 7, x
- a number multiplied by a variable: $12c$, $5k$
- different variables multiplied together: mn, abc
- a variable multiplied by itself a number of times: a^3, b^2
- the product of a number and several variables: $3xy$, $4a^2b$.

An **expression** is formed when terms are added or subtracted; for example, $3x + 4y$.

A **coefficient** of x is a number by which the variable is multiplied. The coefficient is written in front of any variable and includes any negative sign in front of the number. 8 is the coefficient of $8x$ and $^-6$ is the coefficient of ^-6y. Note that the coefficient of x is 1 and the coefficient ^-x is $^-1$.

A **constant** is a term that does not contain any variable factors. In the expression $3x + 6$ the constant term is 6.

An **equation** joins two expressions with an equals sign: $3x + 6 = 2x - 4$.

Algebraic conventions

A convention is a rule agreed to by mathematicians.

- A multiplication sign is not required between variables and numbers or other variables ($5 \times b = 5b$ and $a \times d \times f = adf$).
- Division can be shown as a fraction ($3a \div b = \frac{3a}{b}$).
- Indices show variables being multiplied by themselves ($c \times c = c^2$).
- Numbers are written before a variable ($a \times 4 = 4a$).
- Variables are written in alphabetical order ($y \times z \times x = xyz$).

Examples

1 For $6a + 3b - 4 = 2ab - c + 7$:

- **a** identify whether this is an equation or an expression
- **b** write down the coefficient of b
- **c** write down all the terms in the equation or expression
- **d** list the variables
- **e** write down any constants.

Thinking	Working
a Look for an equals sign. If there is one, it is an equation. If there is not, it is an expression.	**a** It is an equation because it has an equals sign.
b Look for the number in front of the b. This is the coefficient of b.	**b** 3 is the coefficient of b.

Thinking	Working
c Look for parts of the equation separated by addition or equals signs. (Subtraction is the addition of a negative term ($^-4 = + (^-4)$).	**c** $6a$, $3b$, $^-4$, $2ab$, ^-c and 7
d Look for the letters or symbols in the equation.	**d** a, b and c are variables
e Look for numbers that are by themselves.	**e** $^-4$ and 7 are constants

2 Rewrite the following using algebraic conventions.
$t \times m \times 6 \times m \div 5$

Thinking	Working
1 Replace the division sign with a fraction bar.	$\frac{t \times m \times 6 \times m}{5}$
2 Rewrite with the numbers in the multiplication at the start.	$= \frac{6 \times t \times m \times m}{5}$
3 Remove multiplication signs.	$= \frac{6tmm}{5}$
4 Write in alphabetical order.	$= \frac{6mmt}{5}$
5 Use indices to show variables being multiplied by themselves. Note that the steps can be done in a different order as long as they are all completed.	$= \frac{6m^2t}{5}$

3 Pete has b blue pens and r red pens. Sue has three times as many blue pens as Pete. She has 2 fewer red ones than Pete.

- **a** Write an expression for the number of pens Pete has altogether.
- **b** If Pete has 16 pens to start with, write an equation to show this information.
- **c** Write an expression for the number of blue pens Sue has.
- **d** Write an expression for the number of red pens Sue has.
- **e** If Sue has 28 pens altogether, write an equation to show this information.
- **f** If Pete loses 4 of his pens, write an expression in terms of b and r to show how many pens Pete has now.

Thinking	Working
a Identify the variables (b and r) that need to be used and the operation needed to be performed on them (+).	**a** $b + r$
b Equate the expression in part a to the information given.	**b** $b + r = 16$
c Decide what needs to be done to the first variable ($3 \times b$).	**c** $3b$
d Decide what needs to be done to the second variable ($r\,^-2$).	**d** $r - 2$
e Use the information in parts c and d to form an expression ($3b + r - 2$) and equate to the information given.	**e** $3b + r - 2 = 28$
f Identify what operation needs to be performed on the expression in part a and write a new expression.	**f** $b + r - 4$

Activity 4A

1 For each of the following:

- **i** $3d + 4f - 3bd - 3 + 7b$
- **ii** $5g - 3r + 5rf = 7 - 3b$
- **iii** $b + f - 3bf + 8$

- **a** identify whether this is an equation or an expression
- **b** write down the coefficient of b
- **c** write down all the terms in the equation or expression
- **d** list the variables
- **e** write down any constants.

2 Rewrite the following using algebraic conventions.

- **a** $d \times 4 \times c$
- **b** $2 \times t \div 7$
- **c** $k \times g \times 4 \times k$
- **d** $6 \div (f \times g)$

3 Nelly has w watches and r rings. Donna has two times as many watches and two fewer rings than Nelly.

- **a** Write an expression for the number of watches and rings Nelly has altogether.
- **b** If Nelly has 8 watches and rings to start with, write an equation to show this information.
- **c** Write an expression for the number of watches Donna has.
- **d** Write an expression for the number of rings Donna has.
- **e** If Donna has 14 watches and rings altogether, write an equation to show this information.
- **f** If Nelly loses two of her rings, write an expression in terms of w and r to show how many watches and rings Nelly has now.

4 Write the following without division and multiplication signs or brackets. (Do not simplify the algebraic fractions.)

- **a** $x \div 6$
- **b** $h \div 9$
- **c** $6 \times a \div 11$
- **d** $15 \div (3 \times r)$
- **e** $21 \div (12 \times v)$
- **f** $4 \times s \div 19$
- **g** $8 \div x - u \div 6$
- **h** $h \div 5 + 4 \div i$
- **i** $c \times u \div 5 + 9 \times y$
- **j** $q \div (7 \times c) - g \times h \div 4$
- **k** $v \times z \div 6 - 8 \div (f \times s)$
- **l** $3 \div (t \times r) + 6 \times w \div (y \times z)$
- **m** $4 \times h \times b \div (2 \times r)$
- **n** $6 \times c \times a \div (5 \times e \times u)$

5 Andrew has y number of pencils in his pencil case.

- **a** Dina has $y + 7$ pencils in her pencil case. What does this mean?
- **b** Simon has $2y$ pencils in his pencil case. What does this mean?
- **c** Suppose Cindy has $2y - 2$ pencils in her pencil case. Does she have more or fewer pencils than Simon?

6 A basket contains some vegetables. There are t tomatoes.

- **a** There are twice as many pawpaw as tomatoes. Write an expression for the number of pawpaw in terms of t.
- **b** The number of cucumbers is five less than the number of tomatoes. Write an expression for the number of cucumbers in terms of t.
- **c** The number of beans is four more than the number of tomatoes. Write an expression for the number of beans in terms of t.

7 Lauren takes h hours and m minutes to finish her mathematics project. Tim takes twice as long as Lauren to complete his project. Write the time Tim takes in minutes as an expression.

8 Ruth has been answering some questions involving algebra, where k represents the variable in the expression.

- **a** What could k represent?
- **b** In this case, what would $2k$ represent?
- **c** What would $k + 4$ represent?
- **d** What would $\frac{k}{2}$ represent?

Answers 4A

1 a **i** expression **ii** equation
iii expression
b **i** 7 **ii** $^{-}3$
iii 1
c **i** $3d$, $4f$, $^{-}3bd$, $^{-}3$, $7b$ **ii** $5g$, $^{-}3r$, $5rf$, 7, $^{-}3b$
iii b, f, $^{-}3bf$, 8
d **i** b, d, f **ii** b, f, g and r
iii b and f
e **i** $^{-}3$ **ii** 7
iii 8

2 a $4cd$ **b** $\frac{2t}{7}$
c $4gk^2$ **d** $\frac{6}{fg}$

3 a $w + r$ **b** $w + r = 8$
c $2w$ **d** $r - 2$
e $2w + r - 2 = 14$ **f** $w + r - 2$

4 a $\frac{x}{6}$ **b** $\frac{h}{9}$
c $\frac{6a}{11}$ **d** $\frac{15}{3r}$
e $\frac{21}{12v}$ **f** $\frac{4s}{19}$
g $\frac{8}{x} - \frac{u}{6}$ **h** $\frac{h}{5} + \frac{4}{i}$
i $\frac{cu}{5} + 9y$ **j** $\frac{q}{7c} - \frac{gh}{4}$
k $\frac{vz}{6} - \frac{8}{fs}$ **l** $\frac{3}{rt} + \frac{6w}{yz}$
m $\frac{4bh}{2r}$ **n** $\frac{6ac}{5eu}$

5 a Dina has 7 more pencils than Andrew.
b Simon has twice as many pencils as Andrew.
c Cindy has 2 fewer pencils than Simon.
d $y - 3$ **e** $y + 10$
f $2y - 3$ **g** $2y + 1$

6 a $p = 2t$
b $c = t - 5$
c $b = t + 4$

7 Time Lauren (in minutes) = $60h + m$
Time Tim (in minutes) = $2(60h + m) = 120h + 2m$

8 a k represents unknown quantity.
b Twice the value of k.
c Four more than k.
d Half of k.

4B • Adding and subtracting like terms

LB1 Pages 96–97

Specific learning outcomes

Learners should be able to:

8.4.6.1 Simplify expressions by adding and subtracting like terms.
Like terms: same bases and same powers.

Teaching points

1 Simplify expressions by adding and subtracting like terms.

Learner difficulties and remedies

Difficulty

Identifying like terms when adding and subtracting like terms.

Remedy

- To identify like terms, ensure that the pronumerals (letters) in each term are written in alphabetic order.
- Explain to learners that addition and subtraction of terms can only be done if the letters are the same.

Suggested teaching approach

- Explain to learners that terms can only be added or subtracted if the all the letters are the same and they are like terms.
- Do not worry about the coefficient or numbers in front of the letters. Just look for the letters or pronumerals.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Like terms

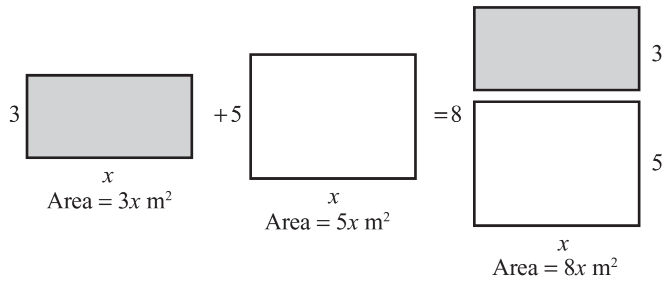
Algebraic terms that have exactly the same variables are called like terms.

We can simplify algebraic expressions by collecting like terms. We collect like terms by adding their coefficients.

$3x$ and $5x$ are like terms and can be simplified by adding the coefficients of x.

$3x + 5x = 8x$

This can be demonstrated by adding two rectangles of the same length, x m, together if one has a width of 3 m and the other as a width of 5 m.



Unlike terms

$3x + 4y$ are unlike terms and cannot be simplified.

Like terms can be simplified by adding their coefficients.

Unlike terms cannot be simplified.

Care must be taken when several variables are involved. To help us identify like terms, we write terms with more than one variable in alphabetical order.

- $5ab$ and $2ac$ are not like terms, as $ab \neq ac$. ($\neq$ means is not equal to.)
- $3xy$ and $5xyz$ are not like terms, as $xy \neq xyz$.
- $6x$ and $4x^2$ are not like terms, as $x \neq x^2$.

However:

- $3xy$ and $5yx$ are like terms as $5yx = 5xy$.
- $11x^2$ and $5x^2$ are like terms.
- $3p^3q^4$ and $^-5p^3q^4$ are like terms.

Examples

1 In each of the following, write down whether the pair of terms is like or unlike.

a $5t$ and $3t$

b $2xy$ and $2x$

c $6m^2n$ and $4mn^2$

d $3xy^2$ and $7y^2x$

Thinking	**Working**
a Do the terms have exactly the same variables? (Both terms have t as their only variable.)	**a** $5t$ and $3t$ are like terms.
b Do the terms have exactly the same variables? (The first term has $x \times y$, but the second term only has x.)	**b** $2xy$ and $2x$ are unlike terms.
c Do the terms have exactly the same variables? (The first term has $m \times m \times n$ whereas the second term has $m \times n \times n$.)	**c** $6m^2n$ and $4mn^2$ are unlike terms.
d Do the terms have exactly the same variables? (The first term has $x \times y \times y$ and the second term has $y \times y \times x$. So, it has exactly the same variables, just in a different order.)	**d** $3xy^2$ and $7y^2x$ are like terms.

2 Simplify the following expressions, if possible, and then check your answers by substituting the value for the variables given in brackets.

a $5x + 3y - x + 7y$ $(x = 1, y = 2)$

b $7x^2 + 4xy + 8yx - 2x$ $(x = 3, y = 2)$

Thinking	**Working**
a **1** Rearrange the expression so that all the like terms are grouped together. Remember to keep the sign in front of each term with that term.	**a** $5x + 3y - x + 7y$ $= 5x - x + 3y + 7y$
2 Collect like terms.	$= 4x + 10y$
3 Substitute the given values into both expressions.	$5x + 3y - x + 7y$ $= 5 \times 1 + 3 \times 2 - 1 + 7 \times 2$ $= 5 + 6 - 1 + 14$ $= 24$ $4x + 10y$ $= 4 \times 1 + 10 \times 2$ $= 24$
4 Check that both expressions give the same value.	As both expressions equal 24, the simplification is correct.
b **1** Rearrange the expression so that all the like terms are grouped together. Reorder variables in the terms if necessary to make the like terms look the same. Remember to keep the sign in front of each term with that term. Note that $7x^2$ and ^-2x are unlike terms.	**b** $7x^2 + 4xy + 8yx - 2x$ $= 7x^2 - 2x + 4xy + 8yx$ $= 7x^2 - 2x + 4xy + 8xy$
2 Collect like terms.	$7x^2 - 2x + 12xy$
3 Substitute the given values into both expressions.	$7x^2 + 4xy + 8yx - 2x$ $= 7 \times 3^2 + 4 \times 3 \times 2 + 8 \times 2 \times 3 - 2 \times 3$ $= 63 + 24 + 48 - 6$ $= 129$ $7x^2 - 2x + 12xy$ $= 7 \times 3^2 - 2 \times 3 + 12 \times 3 \times 2$ $= 63 - 6 + 72$ $= 129$
4 Check that both expressions give the same value.	As both expressions equal 129, the simplification is correct.

Activity 4B

1 In each of the following, write down whether the pair of terms is like or unlike.

a $4w$ and $6w$

b $7u$ and $7w$

c $7t$ and $6i$

d $3e$ and $5e$

e $6d$ and $3de$

f yx and $33xy$

g $9xyz$ and $5xyz$

h $6x$ and $52xyz$

i $6xy$ and $8yx$

j $2xyz$ and $4zyx$

k $4x^2y$ and $7x^2y$

l $8xy^2$ and $12yx^2$

2 Simplify the following expressions, if possible, then check your answers by substituting the value for the variables given in brackets.

a	$6a + 11a$	$(a = 1)$
b	$15f + 6f$	$(f = 6)$
c	$7v - 2v$	$(v = 4)$
d	$5v - 8v$	$(v = 2)$
e	$^{-}7w + 17w$	$(w = {}^{-}2)$
f	$^{-}12d + 15d$	$(d = {}^{-}3)$
g	$8t + 7t + 2d + 5d$	$(d = 1, t = 3)$
h	$40w + 50w + 100v + 34v$	$(v = 2, w = 1)$
i	$6a + 5b + 4a - 3b$	$(a = 2, b = 1)$
j	$18f - 5f + 6g - 3g$	$(f = 1, g = 4)$

3 Simplify these expressions:

a $3hdw + 6hd + 7d - 4hd + 8 - 2d$

b $4x + 15 + 6x + 7xyz - 7 - 3xyz$

c $5x^2 - 6y - 13x^2 - 4y + 2$

d $7x^3 - y - 12x^3 - 5y + xy$

e $28ab + 40 + 30a - 10ab - 6a + 15b$

f $9s + 40st + 5 - 100st - 5s + 15$

g $^{-}9b - 4b^3 + ab - b^3 + 8b - 6ab$

h $6a^3 - 8a^2 - 4a - 5a - 4a^3 - 61$

4 **a** Which of the following expressions cannot be simplified by collecting like terms? Simplify the remaining expressions

i $5x^2 + 4x$

ii $3x^2 + 2x^2$

iii $2xy + 6yx$

iv $4xy^2 + 5x^2y$

v $5xy + xy$

vi $4xy - xy$

5 Lindy was simplifying $7xy - xy$ and wrote: $7xy - xy = 7$.

a What mistake did Lindy make?

b Give Lindy a suggestion to help her avoid this mistake in the future.

Answers 4B

1 **a** like — **b** unlike
c unlike — **d** like
e unlike — **f** like
g like — **h** unlike
i like — **j** like
k like — **l** unlike

2 **a** $17a$, 17 — **b** $21f$, 126
c $5v$, 20 — **d** $^{-}3v$, $^{-}6$
e $10w$, $^{-}20$ — **f** $3d$, $^{-}9$
g $15t + 7d$, 52 — **h** $90w + 134v$, 358
i $10a + 2b$, 22 — **j** $13f + 3g$, 25
k $^{-}4b$, $^{-}20$ — **l** $5b$, 20
m $12d + 9$, 45 — **n** $12f + 5$, 41
o $7r - 3u$, $^{-}1$ — **p** $^{-}7m - 11n$, $^{-}37$
q $3f + 2fg$, 28 — **r** $20j + 6ij$, 224

3 **a** $3dhw + 2dh + 5d + 8$ — **b** $4xyz + 10x + 8$
c $^{-}8x^2 - 10y + 2$ — **d** $^{-}5x^3 - 6y + xy$
e $18ab + 24a + 15b + 40$ — **f** $^{-}60st + 4s + 20$
g $^{-}5b^3 - 5ab - b$ — **h** $2a^3 - 8a^2 - 9a - 61$

4 **a** **i** unlike terms, cannot be simplified

ii like terms, can be simplified

iii like terms, can be simplified

iv unlike terms, cannot be simplified

v like terms, can be simplified

vi like terms, can be simplified

b **ii** $3x^2 + 2x^2 = 5x^2$

iii $2xy + 6xy = 8xy$

v $5xy + xy = 6xy$

vi $4xy - xy = 3xy$

5 **a** Lindy took the *xy* from 7*xy*, instead of taking 1 lot of *xy* from 7 lots of *xy.*

b She should remember that pronumerals with no number in front of them actually have an ‘invisible 1’ in front of them.

4C • Multiplying and dividing terms

LB1 Pages 98–100

Specific learning outcomes

Learners should be able to:

8.4.7.1 Simplify terms and expressions by dividing and multiplying them with other terms using the following rules:

Same sign = Positive

$p \times t = pt$ $\quad$ $^{-}p \times {}^{-}t = pt$

$p \div t = pt$ $\quad$ $^{-}p \div {}^{-}t = pt$

Different signs = Negative

$^{-}a \times b = {}^{-}ab$ $\quad$ $a \times {}^{-}b = {}^{-}ab$

$^{-}a \div b = {}^{-}ab$ $\quad$ $a \div {}^{-}b = {}^{-}ab$

Teaching points

1 Simplify terms and expressions by multiplying and dividing terms.

Learner difficulties and remedies

Difficulty

Simplifying terms and expressions by multiplying and dividing terms, and understanding that multiplication signs are omitted and division signs are converted to fractions.

Remedy

- Multiply and divide coefficients in algebraic terms using the same rules that apply to directed numbers.

Suggested teaching approach

- When multiplying algebraic terms with numerals and pronumerals:
  - multiply numerals just like you multiply ordinary numbers
  - delete the multiplication sign then just write the letters or pronumerals together. The order of the letters does not change their value (although they usually are written in alphabetical order).
- When dividing algebraic terms with numerals and pronumerals:
  - cancel numerals just like you divide ordinary numbers
  - cancel letters if they are the same.

- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The **associative law** tells us that the order of multiplication is not important.

$5 \times 6 \times 2 = 5 \times 2 \times 6$

When multiplying algebraic terms, we can rearrange the multiplication to group numbers and pronumerals separately. Variables are written in alphabetical order.

$$\begin{aligned} 5a \times 2 &= 5 \times a \times 2 \\ &= 5 \times 2 \times a \\ &= 10a \end{aligned} \qquad \begin{aligned} 2d \times 3c &= 2 \times d \times 3 \times c \\ &= 2 \times 3 \times c \times d \\ &= 6cd \end{aligned}$$

When multiplying terms:

- multiply the numbers
- then multiply the variables
- write the variables in alphabetical order.

Examples

1 Simplify:

a $6a \times 3$ **b** $5a \times - 2c \times b \times 3$

Thinking	Working
a Multiply the numbers ($6 \times 3 = 18$) and multiply this product by the variable (a).	**a** $6a \times 3 = 18a$
b Multiply the numbers ($5 \times {}^{-}2 \times 3 = {}^{-}30$) and multiply this product by the product of the variables ($a \times c \times b = abc$), writing them in alphabetical order.	**b** $5a \times {}^{-}2c \times b \times 3 = {}^{-}30abc$

When dividing algebraic terms:

- write as a fraction first
- then simplify the fraction by dividing both the numerator and the denominator by the **Highest Common Factor (HCF)** of the numerator and the denominator
- common factors can be numbers and/or variables.

2 Simplify: $\frac{{}^{-}15ef}{10ef}$

Thinking	Working
1 Identify the HCF of the numerator and the denominator (HCF = $5f$), then divide the HCF into both the numerator and the denominator.	$\frac{{}^{-3}\cancel{15}}{{}_{2}\cancel{10}} \times \frac{e\,{}^{1}\cancel{f}}{{}_{1}\cancel{f}}$
2 Write the simplified expression.	$= -\frac{3e}{2}$

3 Simplify: $14ab \div 7bc$

Thinking	Working
1 Write as a fraction.	$14ab \div 7bc = \frac{14ab}{7bc}$
2 Identify the HCF of the numerator and the denominator ($7b$), then divide it into both the numerator and the denominator.	$= \frac{{}^{2}\cancel{14}a\,{}^{1}\cancel{b}}{{}_{1}\cancel{7}\,{}_{1}\cancel{b}c}$
3 Write the simplified expression.	$= \frac{2a}{c}$

Activity 4C

1 Simplify:

a $5 \times 3a$ **b** $6 \times 2a$ **c** $9g \times 2$

d $7 \times 8z$ **e** $3z \times 12$ **f** $9p \times 6$

g $11e \times 6f$ **h** $9m \times 8n$ **i** $12j \times 11k$

j $6gh \times k \times 3$ **k** $8gh \times 3 \times 2k$ **l** $6b \times 5eh \times 3$

m $4 \times {}^{-}5y$ **n** ${}^{-}3 \times {}^{-}7y$ **o** ${}^{-}2a \times {}^{-}8$

p $11a \times {}^{-}4b$ **q** ${}^{-}3r \times {}^{-}6uq$ **r** ${}^{-}7u \times 8rq$

s ${}^{-}4j \times {}^{-}2 \times 10k$ **t** $10j \times {}^{-}2k \times 3$ **u** $8 \times {}^{-}u \times 3v \times 2$

2 Simplify:

a $\frac{14a}{7}$ **b** $\frac{12b}{3}$ **c** $\frac{63e}{9}$

d $\frac{15f}{5f}$ **e** $\frac{10x}{12x}$ **f** $\frac{x}{15x}$

g $\frac{15ab}{5}$ **h** $\frac{24ef}{8e}$ **i** $\frac{26ab}{13a}$

j $\frac{ef}{8e}$ **k** $\frac{7mn}{28m}$ **l** $\frac{11ef}{77f}$

m $\frac{{}^{-}22ab}{11ab}$ **n** $\frac{{}^{-}6ab}{3ba}$ **o** $\frac{{}^{-}16ef}{{}^{-}4ef}$

p $\frac{{}^{-}9h}{{}^{-}45gh}$ **q** $\frac{{}^{-}7g}{{}^{-}77gh}$ **r** $\frac{{}^{-}9i}{{}^{-}36ij}$

3 Simplify:

a $20c \div 5$ **b** $40d \div 4$ **c** $12g \div 6$

d $60h \div 10h$ **e** $18x \div 27x$ **f** $20x \div 22x$

g $18cd \div 9$ **h** $56cd \div 8$ **i** $30gh \div 6$

j $gh \div 3g$ **k** $45gh \div 18g$ **l** $50gh \div 16g$

m $8cd \div {}^{-}4c$ **n** ${}^{-}50ab \div 30a$ **o** $60cd \div {}^{-}12d$

p $12ab \div {}^{-}8ba$ **q** ${}^{-}ij \div {}^{-}6ij$ **r** ${}^{-}11ij \div {}^{-}ji$

Answers 4C

1 a $15a$ **b** $12a$ **c** $18g$

d $56z$ **e** $36z$ **f** $54p$

g $66ef$ **h** $72mn$ **i** $132jk$

j $18ghk$ **k** $48ghk$ **l** $90beh$

m ${}^{-}20y$ **n** $21y$ **o** $16a$

p ${}^{-}44ab$ **q** $18qru$ **r** ${}^{-}56qru$

s $80jk$ **t** ${}^{-}60jk$ **u** ${}^{-}48uv$

2 a $2a$ **b** $4b$ **c** $7e$

d 3 **e** $\frac{5}{6}$ **f** $\frac{1}{15}$

g $3ab$ **h** $3f$ **i** $2b$

j $\frac{f}{8}$ **k** $\frac{n}{4}$ **l** $\frac{e}{7}$

m ${}^{-}2$ **n** ${}^{-}2$ **o** 4

p $\frac{1}{5g}$ **q** $\frac{1}{11h}$ **r** $\frac{1}{4j}$

3 **a** $4c$ **b** $10d$ **c** $2g$

d 6 **e** $\frac{2}{3}$ **f** $\frac{10}{11}$

g $2cd$ **h** $7cd$ **i** $5gh$

j $\frac{h}{3}$ **k** $\frac{5h}{2}$ **l** $\frac{25h}{8}$

m $^{-}2d$ **n** $-\frac{5b}{3}$ **o** $^{-}5c$

p $-\frac{3}{2}$ **q** $\frac{1}{6}$ **r** 11

4D • Expanding brackets

LB1 Pages 101–102

Specific learning outcomes

Learners should be able to:

8.4.8.1 Expand brackets by multiplying those terms outside by those terms inside the brackets using the distributive law.

8.4.8.2 Write algebraic expressions using brackets.

Teaching points

1. Expand a single bracket using distributive Law.
2. Write algebraic expressions for mathematical statements using brackets.

Learner difficulties and remedies

Difficulty

Understanding that terms outside the brackets are multiplied by each term inside the brackets.

Remedy

- Multiply terms that are outside the brackets with those inside the brackets, one at a time.

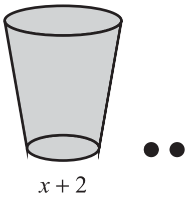
Suggested teaching approach

- Learners multiply the numbers outside the brackets by each term inside the brackets.
- Provide learners with a variety of exercise on expanding brackets, involving both positive and negative numbers.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

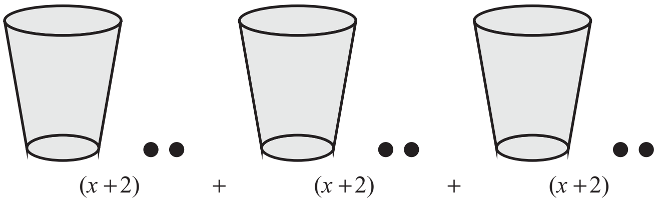
Additional notes

What do we mean by $3(x + 2)$?

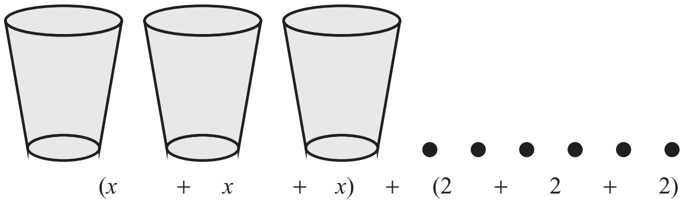
We can show what this expression means by letting x be the number of counters in a cup, so that $x + 2$ is the number of counters in the cup plus two more.



If we have $3(x + 2)$, this represents 3 lots of $(x + 2)$ of $3 \times (x + 2)$ and can be shown by 3 cups with x counters in each and 2 more counters with each cup.



This can be arranged to give 3 cups, each with x counters, and 6 more more counters. This gives a total of $3x = 6$ counters.



There fore we can write:

$3 (x + 2) – 3x + 6$

Expanding is the process of removing brackets using the distributive law. We have seen this before with numbers.
For example:

$$\begin{aligned} 3 \times 102 &= 3 \times (100 + 2) \\ &= 3 \times 100 + 3 \times 2 \\ &= 300 + 6 \\ &= 306 \end{aligned}$$

Remember that variables in algebra are simply representing unknown numbers, so the distributive law must work in algebra if it works with numbers.

So, we can use the distributive law to expand algebraic expressions.

$$\begin{aligned} \overset{\curvearrowright\curvearrowright}{3(x + 2)} &= 3 \times x + 3 \times 2 \\ &= 3x + 6 \end{aligned}$$

Expanding using the distributive law:

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

Examples

1 Use the distributive law to find 24×99.

Thinking	**Working**
1 Write one number in the product as the sum or difference of two numbers that are easy to multiply by $(99 = 100 – 1)$.	24×99 $= 24 \times (100 – 1)$
2 Expand the brackets.	$= 24 \times 100 – 24 \times 1$
3 Simplify.	$= 2400 – 24$ $= 2376$

2 Expand each of the following expressions.

a $5(a-2)$ **b** $2a(a+3b)$ **c** ${}^{-}3(a-b)$

Thinking	Working
a 1 Multiply each term in the bracket by the term in front of the bracket.	**a** $5(a-2)$ $= 5 \times a - 5 \times 2$
2 Simplify.	$= 5a - 10$
b 1 Multiply each term in the bracket by the term in front of the bracket.	**b** $2a(a+3b)$ $= 2a \times a + 2a \times 3b$
2 Simplify.	$= 2a^2 + 6ab$
c 1 Multiply each term in the bracket by the term in front of the bracket, making sure to change signs if necessary.	**c** ${}^{-}3(a-b)$ $= {}^{-}3 \times a - 3 \times {}^{-}b$
2 Simplify.	$= {}^{-}3a + 3b$

Expanding and collecting like terms

After an expression is expanded using the distributive law, it may contain like terms. These can be collected and the expression simplified.

1 Simplify the following expression by expanding the brackets and collecting like terms.

a $3(r-4) - 8r$ **b** $5(a+6b) + a(9-10a)$

Thinking	Working
a 1 Expand all brackets.	**a** $3(r-4) - 8r$ $= 3r - 12 - 8r$
2 Simplify by collecting like terms.	$= {}^{-}5r - 12$
b 1 Expand all brackets.	**b** $5(a+6b) + a(9-10a)$ $= 5a + 30b + 9a - 10a^2$
2 Simplify by collecting like terms.	$= 14a + 30b - 10a^2$

2 Simplify the following expressions by expanding the brackets and collecting like terms.

a $8d - (2d+3)$ **b** $7(a-3b) - 5(a+3b)$

c $3m(3n-4t) - 4n(2m-t)$

Thinking	Working
a 1 Rewrite the expression, inserting a 1 and a × sign in front of the brackets.	**a** $8d - (2d+3)$ $= 8d - 1 \times (2d+3)$
2 Expand the brackets, taking care to change signs where required.	$= 8d - 2d - 3$
3 Simplify by collecting like terms.	$= 6d - 3$
b 1 Expand all brackets, taking care to change signs where required.	**b** $7(a-3b) - 5(a+3b)$ $= 7a - 21b - 5a - 15b$
2 Simplify by collecting like terms.	$= 2a - 36b$
c 1 Expand all brackets, taking care to change signs where required.	**c** $3m(3n-4t) - 4n(2m-t)$ $= 9mn - 12mt - 8mn + 4nt$
2 Simplify by collecting like terms.	$= mn - 12mt + 4nt$

Activity 4D

1 Use the distributive law to find:

a 21×101 **b** 17×102 **c** 16×101

d 23×98 **e** 41×99 **f** 37×97

g 54×11 **h** 38×1010 **i** 64×1100

2 Expand each of the following expressions.

a $5(x+12)$ **b** $2(s-5)$ **c** $12(a+5)$

d $s(4+t)$ **e** $l(6-r)$ **f** $a(2+h)$

g $3(4x+5)$ **h** $9(6x-1)$ **i** $2(3h+4)$

j $2m(n+r)$ **k** $3p(r-z)$ **l** $6p(r+t)$

m $5t(t+2)$ **n** $2p(5p-7)$ **o** $4m(4+2m)$

p $3m(5m+7n)$ **q** $4k(p-5k)$ **r** $10x(2x-5y)$

3 Simplify the following expressions by expanding the brackets and collecting like terms.

a $5(7-d) + 11d$ **b** $8(3-e) - 12e$

c $6n(m-4) - 12nm$ **d** $4x(y-2) - 7xy$

e $5t(7-4k) - 40tk$ **f** $9v(5-10w) - 50v$

g $6m(7n-4) + 4(8mn-4m)$ **h** $2a(5b-8) + 2(6a-12ab)$

i $9x(5-y) + 3(10x-4xy)$ **j** $3t(4-8z) + 6(7tz-20t)$

4 Simplify the following expressions by expanding the brackets and collecting like terms.

a $3x - 4(x+2)$ **b** $5x - (2x+2)$

c $8 - 3(x+2)$ **d** $14 - (3x-7)$

e $5x - 3(5x-2)$ **f** $7x - 3(x-2)$

g $5(x+2) - 3(2x+1)$ **h** $4(3x-2) - 12(x+5)$

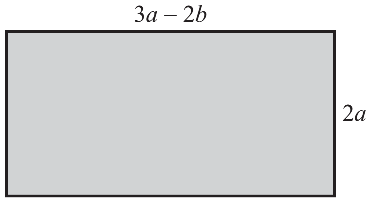
i $3(3x+4) - 5(7x-3)$ **j** $5(x-3) - (5x-1)$

k $5(2x-3y) - 3(3x+4y)$ **l** $4(3x-5y) - 2(5x+2y)$

m $c(3d+4e) - 2c(d+5e)$ **n** $d(5e+2f) - 4d(e+4b)$

o $m(2n-3t) - t(2m-n)$ **p** $a(2p-q) - q(2p-3a)$

5 A rectangle has width $2a$ metres and length $3a - 2b$ metres. Write the expression for the area and expand the brackets.



6 Michael was simplifying by expanding and collecting like terms. His working is shown below.

$$3(2x-1) - (5x-2) = 6x - 3 - 5x - 2$$
$$= x - 5$$

a Identify the mistake that Michael made.

b Correct his mistake by completing the question.

c Suggest how Michael can avoid making this mistake in the future.

Answers 4D

1 a 2121 **b** 1734 **c** 1616

d 2254 **e** 4059 **f** 3589

g 594 **h** 38 380 **i** 70 400

2 a $5x + 60$ **b** $2s - 10$ **c** $12 + 60$

d $4s + st$ **e** $6l - 1r$ **f** $2a + ah$

g $12x + 15$ **h** $54x - 9$ **i** $6h + 8$

j $2mn + 2mr$ **k** $3pr - 3pz$ **l** $6pr + 6pt$

m $5t^2 + 10t$ **n** $10p^2 - 14p$ **o** $16m + 8m^2$

p $15m^2 + 21mn$ **q** $4kp - 20k^2$ **r** $20x^2 - 50xy$

3 **a** $35 + 6d$ **b** $24 - 20e$
c $^{-}24n - 6mn$ **d** $^{-}8x - 3xy$
e $35t - 60kt$ **f** $^{-}5v - 90vw$
g $74mn - 40m$ **h** $^{-}14ab - 4a$
i $75x - 21xy$ **j** $^{-}108t + 18tz$

4 **a** $^{-}x - 8$ **b** $3x - 2$
c $2 - 3x$ **d** $21 - 3x$
e $^{-}10x + 6$ **f** $4x + 6$
g $^{-}x + 7$ **h** $^{-}68$
i $^{-}26x + 27$ **j** $^{-}14$
k $x - 27y$ **l** $2x - 24y$
m $cd - 6ce$ **n** $de + 2df - 16bd$
o $2mn - 5mt + nt$ **p** $2ap - 2pq + 2aq$

5 Area of the rectangle
= length × width
$= 2a(3a - 2b)$
$= 6a^2 - 4ab$

6 **a** When he multiplied out the second bracket, he didn't properly apply the negative number rule, which says that $^{-}1 \times {}^{-}2 = {}^{+}2$.

b $3(2x - 1) - (5x - 2)$
$= 6x - 3 - 5x + 2$
$= x - 1$

c Ensure that the sign outside a set of brackets is multiplied by each term in the brackets. A good strategy would be, whenever there is a minus outside the brackets, write a 1 between the minus and the bracket. Also remember that when multiplying two negative numbers, the answer will be positive.

4E • Evaluating algebraic expressions

LB1 Pages 103–104

Specific learning outcomes

Learners should be able to:

8.4.9.1 Define the term 'evaluate'.

8.4.9.2 Substituting values into expressions.

Teaching points

1 Explain the term evaluate using examples.

2 Find the value for given expressions by replacing the letters in the expressions with corresponding values, and then simplifying the result.

Learner difficulties and remedies

Difficulty

Substituting numbers to replace letters in the expression, and then evaluating the result.

Remedy

- Check the understanding of the instructions, work through a few examples with the class as a whole, and provide questions as practice.
- Check that learners are substituting negative numbers in correctly. You may need to revise multiplication of directed numbers.

Suggested teaching approach

- Make sure that learners are substituting numbers into pronumerals and then calculating the answer correctly.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Variables in an algebraic expression have unknown values. If we want to know the value for an expression when the variables have certain values, we can replace the variables in the expression with those values. This process is called **substitution.**

In the expression $3x + 2y$, x and y are variables. When $x = 5$ and $y = 1$, the expression can be evaluated by substituting 5 for x and 1 for y so:

$3x + 2y = 3 \times (5) + 2 \times (1)$
$= 17$

Because the variables can vary, different values for the variable will give different values for the expression. When $x = {}^{-}3$ and $y = 7$, the expression can be evaluated by substituting $^{-}3$ for x and 7 for y so:

$3x + 2y = 3 \times ({}^{-}3) + 2 \times (7)$
$= 5$

Examples

1 Evaluate the expression $2b - a$ when $a = 3$ and $b = 10$.

Thinking	Working
1 Substitute the values for the variables, taking care to insert multiplication signs where necessary.	When $a = 3$ and $b = 10$ $2b - a = 2 \times 10 - 3$
2 Evaluate.	$= 20 - 3$ $= 17$

Negative numbers

When substituting negative numbers, it is useful to place brackets around the number and its sign.

2 Evaluate the expression $2x^2 - 3y$ when $x = {}^{-}3$ and $y = {}^{-}1$.

Thinking	Working
1 Substitute the values for the variables, taking care to place brackets around the negative numbers and to insert multiplication signs where necessary.	When $x = {}^{-}3$ and $y = {}^{-}1$ $2x^2 - 3y = 2 \times ({}^{-}3)^2 - 3 \times ({}^{-}1)$
2 Evaluate.	$= 2 \times 9 + 3$ $= 18 + 3$ $= 21$

3 Use the rule $y = 3x$ to complete the following table.

x	$^{-}1$	0	2
y			

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>1 Substitute each of the <i>x</i>-values into the rule to find the <i>y</i>-value.</td><td>When $x = {}^{-}1$, $y = 3x$
$= 3 \times (^{-}1)$
$= {}^{-}3$
When $x = 0$, $y = 3x$
$= 3 \times (0)$
$= 0$
When $x = 2$, $y = 3x$
$= 3 \times (2)$
$= 6$</td></tr>
<tr><td>2 Complete the table using the <i>y</i>-values calculated.</td><td><table><tr><td><i>x</i></td><td>$^{-}1$</td><td>0</td><td>2</td></tr><tr><td><i>y</i></td><td>$^{-}3$</td><td>0</td><td>6</td></tr></table></td></tr>
</table>

Activity 4E

1 Evaluate the following expressions when $a = 3$ and $b = 6$.

a $a + 2b$ **b** $3a + 2b$ **c** $2a + 5b$
d ab **e** $3ab$ **f** $10ab - a$
g $8b - 16a$ **h** $10ab - b$ **i** $6b - ab$
j $12a^2$ **k** $2b^3$ **l** $4a^3$
m $a^2 - b^3$ **n** $3a^3 - 2b^2$ **o** $2a^5 - 2b^3$
p $\frac{2a^2}{b}$ **q** $\frac{a^3}{b^2}$ **r** $\frac{144a^2}{3b^3}$
s $a^2 - b - \frac{a}{3}$ **t** $a^3 + 2b^2 - \frac{b}{2}$ **u** $3b^2 + \frac{a^5}{2} - 5$

2 Evaluate the following expressions when $x = {}^{-}2$ and $y = {}^{-}5$.

a $x + y$ **b** $2x + 12y$ **c** $10x + 5y$
d $y - 4x$ **e** $2y - 3x$ **f** $2x - 3y$
g $^{-}5y - 3 + x$ **h** $8 - 6y + 2x$ **i** $7y - 4 - 4x$
j $y^2 - x^2$ **k** $x^3 + y^3$ **l** $^{-}5y^2$

3 Use the rules given to complete the following tables.

a $y = x - 2$

x	$^{-}2$	0	$^{-}4$	$^{-}8$
y				

b $y = x - 4$

x	$^{-}5$	$^{-}3$	$^{-}2$	$^{-}7$
y				

c $y = x - 9$

x	$^{-}7$	$^{-}1$	$^{-}5$	$^{-}9$
y				

d $y = 4x - 1$

x	1	2	3	4
y				

e $y = 2x - 4$

x	$^{-}2$	0	1	2
y				

f $y = 3x + 14$

x	$^{-}4$	$^{-}7$	$^{-}20$	11
y				

4 Evaluate the following expressions for $m = 3$ and $n = {}^{-}2$.

a $2(3m - n)$ **b** $4n(m - 3)$ **c** $m(6 + 3n)$
d $6m(8 - n)$ **e** $3m(n - 2)$ **f** $6n(3m - 4)$
g $\frac{m+n}{3}$ **h** $\frac{2m+n}{5}$ **i** $\frac{6m-n}{4}$
j $\frac{2m+n}{n}$ **k** $\frac{5(m+2n)}{5m}$ **l** $\frac{3(m-4n)}{4n}$

Answers 4E

1 a 15 **b** 21 **c** 36
d 18 **e** 54 **f** 177
g 0 **h** 174 **i** 18
j 108 **k** 432 **l** 108
m $^{-}207$ **n** 9 **o** 54
p 3 **q** $\frac{3}{4}$ **r** 2
s 2 **t** 96 **u** $\frac{449}{2}$, $224\frac{1}{2}$

2 a $^{-}7$ **b** $^{-}64$ **c** $^{-}45$
d 3 **e** $^{-}4$ **f** 11
g 20 **h** 34 **i** $^{-}31$
j 21 **k** $^{-}133$ **l** $^{-}125$

3 a $y = x - 2$

x	$^{-}2$	0	$^{-}4$	$^{-}8$
y	$^{-}4$	$^{-}2$	$^{-}6$	$^{-}10$

b $y = x - 4$

x	$^{-}5$	$^{-}3$	$^{-}2$	$^{-}7$
y	$^{-}9$	$^{-}7$	$^{-}6$	$^{-}11$

c $y = x - 9$

x	$^{-}7$	$^{-}1$	$^{-}5$	$^{-}9$
y	$^{-}16$	$^{-}10$	$^{-}14$	$^{-}18$

d $y = 4x - 1$

x	1	2	3	4
y	3	7	11	15

e $y = 2x - 4$

x	$^{-}2$	0	1	2
y	$^{-}8$	$^{-}4$	$^{-}2$	0

f $y = 3x + 14$

x	$^{-}4$	$^{-}7$	$^{-}20$	11
y	2	$^{-}7$	$^{-}46$	47

g $y = -6x$ <no questions for answers g to o>

x	$^{-}3$	$^{-}2$	$^{-}1$	$^{-}5$
y	18	12	6	30

h $y = {}^{-}5x - 3$

x	$^{-}1$	0	3	5
y	2	$^{-}3$	$^{-}18$	$^{-}28$

i $y = 3 - 4x$

x	$^{-}3$	$^{-}1$	2	4
y	15	7	$^{-}5$	$^{-}13$

j $y = \frac{x}{2}$

x	8	6	12	20
y	4	3	6	10

k $y = \frac{x}{3} - 2$

x	15	9	12	30
y	3	1	2	8

l $y = \frac{x}{4} - 1$

x	12	−4	8	−24
y	2	−2	1	−7

m $y = x^2$

x	0	1	2	3
y	0	1	4	9

n $y = 2x^3 - 1$

x	−2	0	2	4
y	−17	−1	15	127

o $y = x^2 + 3x + 1$

x	0	2	4	8
y	1	11	29	89

4 **a** 22 **b** 0 **c** 0

d 180 **e** −36 **f** −60

g $\frac{1}{3}$ **h** $\frac{4}{5}$ **i** 5

j −2 **k** $-\frac{1}{3}$ **l** $-\frac{33}{8}$

4F • Formulas in mathematics

LB1 Pages 105–107

Specific learning outcomes

Learners should be able to:

8.4.10.1 Define the term ‘formula’.

8.4.11.1 Use various formulas to solve practical problems.

Teaching points

1 Explain what a formula is using examples.

2 Use formulas to find answers to real-life practical questions.

Learner difficulties and remedies

Difficulty

Interpreting the context of the word problems.

Remedy

- Check that learners can understand the language in word questions and can visualise the situation.

Difficulty

Using formulas to solve problems.

Remedy

- Provide more word problems in algebra that require the use of formulas so that learners gain more practice and become more competent.

Suggested teaching approach

- Monitor learner’s progress in this section because a lot of learners find this section difficult.
- Explain the words and terms that are commonly used in algebra so that learners are familiar with the terms when they come across them in problems.
- Explain to learners which formula to use for a given question, and which values need to be substituted into the pronumerals.
- Provide learners with more word problems so that they gain experience and confidence.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Formulas (or formulae) are algebraic equations that are used in practical situations. Formulas are used to calculate one quantity from others. You have already used formulas to find the perimeter and area of simple shapes.

A rectangle has a length of l and a width of w.

The perimeter is by the formula: $P = 2(l + w)$ or $P = 2l + 2w$

The area is given by the formula: $A = lw$

Examples

The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures in degrees Celsius, C, to degrees in Fahrenheit, F. Use this formula to convert 29°C into degrees Fahrenheit.

Thinking	Working
1 Write the formula.	$F = \frac{9C}{5} + 32$
2 Substitute the value or values given ($C = 29$).	$= \frac{9(29)}{5} + 32$
3 Evaluate.	$= 84{\cdot}2°\text{F}$

Activity 4F

1 The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures in degrees Celsius, C, to degrees in Fahrenheit, F. Use this formula to convert the following degrees Celsius into degrees Fahrenheit.

a 10°C **b** 5°C **c** 35°C

2 The formula $C = \frac{5F - 160}{9}$ converts degrees Fahrenheit, F, to degrees Celsius, C. Find the value of C when F is:

a 32°F **b** 41°F **c** 50°F

3 The distance travelled by a moving object travelling at a constant speed can be calculated using the formula $d = st$ where:

d = distance travelled in metres
s = speed of object in metres/second
t = time in seconds that the object is in motion.

Calculate the value of d when:

a $s = 8, t = 4$ **b** $s = 4\frac{1}{2}, t = 1\frac{1}{2}$

Answers 4F

1 **a** 50°F **b** 41°F **c** 95°F

2 **a** 0°C **b** 5°C **c** 10°C

3 **a** 32 m **b** 6·75 m

• Chapter Five •

Rates and Ratios

Overview

A ratio is a comparison of two or more amounts of the same type, written using a ':' symbol.

The order in which a ratio is stated must be made clear. If the ratio of teachers to learners in a class is 1 : 20, then the ratio of learners to teachers is 20 : 1. An understanding of ratio is important for completing many tasks such as a baker making a cake, a builder mixing concrete, a photographer enlarging an image or a cartographer drawing a map. All these tasks require skills in working with ratios and scale factors.

Rates enable us to compare quantities or how an amount changes over time. Rates include the fuel used by a car in L/km or the population increase of a city in people/ year.

Ratios should be written in simplest form, using whole numbers. Each quantity in the ratio must be expressed in the same units, although the units are not written as part of the ratio. If a ratio contains fractions or decimals, all parts of the ratio should be multiplied by the same number to convert it to an equivalent whole number ratio.

Contents

Chapter skills

This chapter covers the following skills:

- Recognising and writing ratios in their simplest form
- Using ratios to determine the quantities or amounts required
- Sharing amounts according to ratios
- Identifying the quantities connected in a rate
- Interpreting everyday rates
- Reading information from rate graphs
- Comparing rates
- Working with speed, distance and time

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 5A Ratio and proportion	Learner's Book 1 • Exercise 5A, pages 118, 119
2	• 5B Simplifying ratios	Learner's Book 1 • Exercise 5B, page 121
3	• 5C Finding quantities using ratios	Learner's Book 1 • Exercise 5C, page 122
4	• 5D Exploring ratios in triangles	Learner's Book 1 • Learning task 5D, page 123
5–6	• 5E Sharing using ratios	Learner's Book 1 • Exercise 5E, pages 124, 125
7–8	• 5F Rates	Learner's Book 1 • Exercise 5F, pages 126, 127
9–10	• 5G Using rates: speed, distance and time	Learner's Book 1 • Exercise 5G, pages 128, 129
11–12	• 5H Graphs and time	Learner's Book 1 • Exercise 5H, pages 130–134
13	• 5I Scale diagrams	Learner's Book 1 • Exercise 5I, pages 135, 136 and 137
14	• 5J Exploring maps	Learner's Book 1 • Exercise 5J, pages 138,139
15	• Test	Teacher's Guide • Chapter 5 Test

General learning outcomes

Learners should:

Ratio and proportion

8.5.1 Understand what is ratio and its practical application in terms of the relationship between quantities. (U)

Simplifying ratios

8.5.2 Know that ratios can be written as fraction and can be reduced to its simplest form. (K)

8.5.3 Know how to simplify ratios of quantities that have different units. (U)

8.5.4 Know how to simplify fractional ratios. (K)

Finding quantities using ratios

8.5.5 Know how to use ratios to find unknown quantities. (K)

Exploring ratios in triangles

8.5.6 Know how to divide lines into different parts or segments using ratios. (K)

8.5.7 Know how to express the lengths of lines within a triangle as ratios. (K)

Sharing using ratios

8.5.8 Understand that ratios can be used to share quantities, not necessary equal. (U)

Rates

8.5.9 Understand rates and its application. (U)

8.5.10 Know how to calculate the rates of different quantities and use them to find other quantities.

Using rates: speed, distance and time

8.5.11 Understand the use of rates with regards to speed, distance and time. (U)

8.5.12 Know how to calculate speed, time and distances.

Graphs and time

8.5.13 Interpret information from time graphs. (U)

Scale diagrams

8.5.14 Know that ratio can be used to reduce the size of real measurements to map measurement using scales as the factor to given the map and real measurement. (K)

Exploring maps

8.5.15 Know that the use of scales in maps is good but does not give detailed information about the map like contours. (K)

5A • Ratio and proportion

LB1 Pages 118–119

Specific learning outcomes

Learners should be able to:

8.5.1.1 Define ratio and proportion.

8.5.1.2 Correctly interpret the symbol that is used to represent ratio: ':'.

8.5.1.3 Write or express quantities as ratios.

Teaching points

1. Explain the terms ratio and proportion using examples.
2. Interpret the symbol that is used for ratio.
3. Write quantities in given ratios.

Learner difficulties and remedies

Difficulty

Understanding the meaning of the terms ratio and rates.

Remedy

- Clearly explain and give more examples to clarify the definitions of the two terms.

Difficulty

Writing quantities as a ratio.

Remedy

- Count the number of each quantity and separate them using the : symbol, which represents a ratio.
- The order in which the ratio is given must correspond to the order of the given quantities.

Suggested teaching approach

- Explain that a ratio is another way of comparing two or more quantities using the : symbol for ratio.
- There are two ways of comparing quantities: as a ratio or as a fraction.
- Explain to learners that writing or expressing quantities in ratio notation must correspond to the order of the information is given. If 'girls' comes first in the information, then the first part of the ratio would represent girls and the second part would represent boys as Girl : Boys.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **ratio** is a comparison of two or more amounts or quantities of the same type, written using a **:** symbol.

The order in which a ratio is stated must be made clear. If the ratio of teachers to learners in a class is 1 : 20, then the ratio of learners to teachers is 20 : 1.

Suppose that an animal shelter has 13 dogs, 11 cats and 6 birds.

A **part : part** ratio compares separate parts.

The ration of the number of cats to the number of dogs is 11 : 13.

The ratio of the number of cats to birds and to dogs is 11 : 6 : 13. (Note that the order of the numbers in the ratio is the same as the written order of the animals.)

A **part : whole** ratio compares a part (or parts) to a whole.

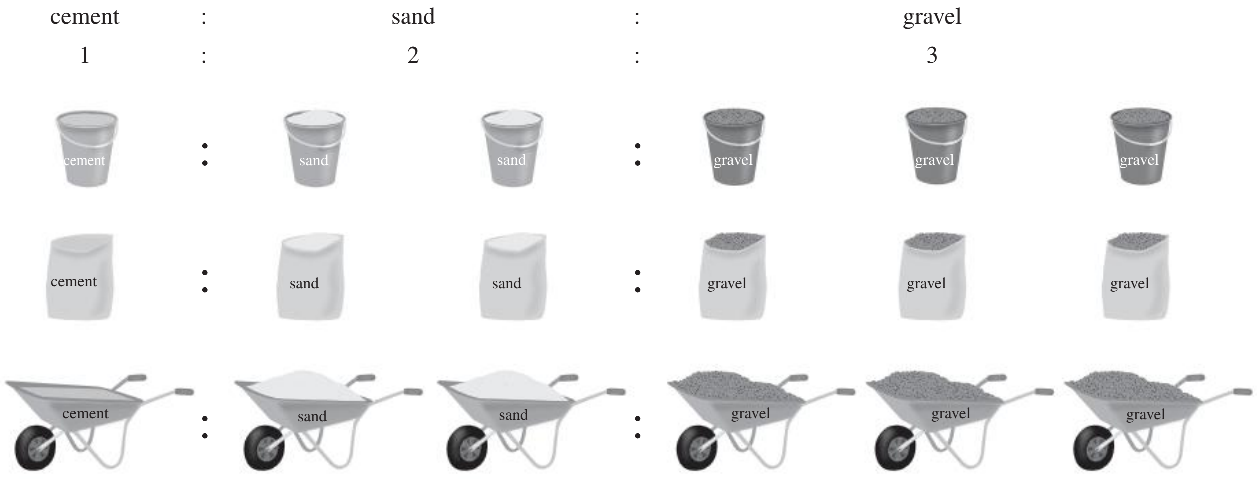
The ratio of the number of cats to the total number of animals is 11 : 30.

The ratio of the number of dogs to the total number of animals is 13 : 30.

Examples

A common ratio used for making concrete is to mix **1 part cement** with **2 parts sand** and **3 parts gravel**. We can write this ratio as **cement : sand : gravel** = **1 : 2 : 3.**

The actual measured amounts of these ingredients don't matter. It is the ratio, or proportion of each ingredient compared with the others, that is important. We could measure them out using buckets, bags or wheelbarrows and still get the correct mixture (provided the buckets, bags or wheelbarrows are all the same size).



Activity 5A

1 Complete these ratio statements:

- **a** A drink mix has 2 parts cordial to 5 parts of water.
  cordial : water = ___________________
- **b** A class has 18 girls and 11 boys.
  girls : boys = ___________________
- **c** A car park has 4 spaces for disabled drivers, and 75 other spaces.
  disabled : other = ___________________
- **d** A real-estate agency has five salaried employees and six on commission.
  commission : salaried = ___________________
- **e** For every 3 dogs a veterinary clinic sees, they see 8 cats.
  cats : dogs = ___________________

2 A greengrocer has nine boxes of apples, seven boxes of bananas and four boxes of mangoes.

- **a** Write a ratio that compares the number of boxes of mangoes to the number of boxes of apples, in simplest form.
- **b** Write a ratio that compares the number of boxes of mangoes to the total number of boxes of fruit, in simplest form.
- **c** Write the number of boxes of mangoes as a percentage of the total number of boxes of fruit.
- **d** Write the number of boxes of bananas as a fraction of the number of boxes of apples.

Answers 5A

1 **a** 2 : 5 **b** 18 : 11
c 4 : 75 **d** 6 : 5
e 8 : 3
2 **a** 4 : 9 **b** 1 : 5
c 20% **d** $\frac{7}{9}$

5B • Simplifying ratios

LB1 Pages 120–121

Specific learning outcomes

Learners should be able to:

8.5.2.1 Expressing ratios of quantities as fractions in their simplest form.
8.5.3.1 Simplify ratios of quantities that are given in different units.
8.5.4.1 Simplify fractional ratios by using a common number to remove all the denominators.

Teaching points

1. Write ratios as fractions and then simplify them.
2. Simplify ratios with different units.
3. Simplify ratios with fractions or decimals.

Learner difficulties and remedies

Difficulty

Converting metric units.

Remedy

- Revise the conversion factors in the metric ladder and change one unit to another.
- Know the values of each length and distances in the metric system (1 cm = 10 mm etc.).

Difficulty

Simplifying fractional ratios.

Remedy

- Identify the highest common factor to cancel the denominator in a fractional ratios or to simplify each part of a ratio.

Suggested teaching approach

- Always look for a Lowest Common Factor (LCF) that can be used to simplify each parts of the ratio.
- Explain to learners how ratios are used to compare quantities. Ratios can be expressed as numbers, measurements with metric units or fractional ratios.
- Show learners how to do convert a metric unit to another.
- Remind learners that units of quantities used in ratios are the same. If they are different then you have to convert them to one unit.
- When converting metric units, it's better to change to units that do not have any decimals.
- When simplifying a fractional ratio convert it to an equivalent whole number ratio. To do this, multiply by the Lowest Common Factor (LCF) to cancel the denominator and give a ratio of whole numbers.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Ratios are used to compare two or more quantities using the ratio symbol called 'is to' with the symbol ':'. Ratios can also be expressed in fraction form.

If the ratio of cats to dogs is 11 : 13, then the number of cats is $\frac{11}{13}$ of the number of dogs.

Ratios can also be simplified in a similar way to fractions: by dividing each number in the ratio by a common factor. If we divide by the highest common factor (HCF), we obtain the ratio in **simplest form**.

For example, the HCF of the numbers in the ratio 4 : 12 is 4.

$$\begin{array}{lccc} & 4 & : & 12 \\ & \div 4 \downarrow & & \downarrow \div 4 \\ = & 1 & : & 3 \end{array}$$

To write a ratio in simplest form, divide each number in the ratio by the highest common factor (HCF).

Ratios should be written in simplest form, using whole numbers only. Each quantity in the ratio must be expressed in the same units, but the units are not written in the ratio.

If a ratio contains fractions or decimals, multiply all parts in the ratio by the same number to convert them to an equivalent whole number ratio.

Examples

1 A box of chocolates has seven plain, five nut and three soft-centred chocolates.

a Write a ratio that compares the number of nut chocolates to the number of plain chocolates, in simplest form.

b Write a ratio that compares the number of soft-centred chocolates to the total number in the box, in simplest form.

c Write the number of soft-centred chocolates as a fraction of the number of plain chocolates.

Thinking	Working
a Write the ratio in words, in the correct order, with the numbers underneath. Simplify by dividing by a common factor, if possible. (Here, there are no common factors—the ratio is already in simplest form.)	**a** nut : plain = 5 : 7
b **1** Find the number of parts in the whole (the total number of chocolates).	**b** 7 + 5 + 3 = 15
2 Write the ratio in words, in the correct order, then write the numbers underneath. Simplify by dividing by the HCF. (Here, the HCF is 3.)	soft-centred : total = 3 : 15 = 1 : **5** (÷3)
c Write the part: whole ratio as a fraction, then convert the fraction to a percentage.	**c** $\frac{1}{5}$ $= \frac{20}{100}$ = 20

2 Write the ratio 25 m : 1500 cm in simplest form.

Thinking	Working
1 Are the units the same? If not, write both quantities in the smaller unit.	25 m : 1500 cm = 2500 cm : 1500 cm
2 Remove the units.	= 2500 : 1500
3 Divide both numbers by the HCF (500).	= 5 : 3

3 Write the ratio $2\frac{1}{2}$: 1 : 3 in simplest form.

Thinking	Working
1 Write any mixed numbers as improper fractions.	$2\frac{1}{2} : 1 : 3$ $= \frac{5}{2} : 1 : 3$
2 Multiply by the denominator to turn the fraction into a whole number. Multiply the other numbers in the ratio by the denominator as well to maintain the ratio.	$= \frac{5}{2} \times 2 : 1 \times 2 : 3 \times 2$ $= 5 : 2 : 6$

Activity 5B

1 Write each of the following ratios in simplest form.

a 2 : 8 **b** 3 : 15 **c** 4 : 16
d 4 : 10 **e** 12 : 34 **f** 180 : 150
g 20 : 300 **h** 500 : 4000 **i** 225 : 325
j 25 : 10 : 5 **k** 24 : 4 : 8 **l** 33 : 15 : 27
m 3 : 12 : 6 **n** 36 : 4 : 16 **o** 670 : 500 : 215

2 Write each of the following ratios in simplest form.

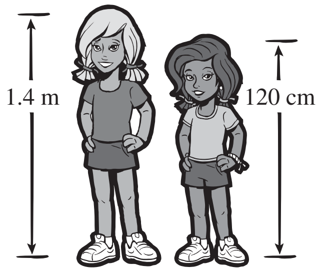
a 5 cm : 3 mm **b** 75 cm : 1·5 m
c 5 km : 600 m : 300 m **d** 8000 m : 5 km : 4·5 km
e 60 cents : $2 **f** $3.50 : 45 cents
g 650 g : 3·5 : kg **h** 6 L : 500 mL
i 45 seconds : 2 minutes **j** 480 kg : 3 tonnes
k 8 months : 1 year **l** 2·5 L : 2000 mL
m 8 hours : 180 minutes **n** 10 minutes : 20 seconds
o 5400 g : 6 kg : 1·2 kg **p** 4 days : 4 weeks : 1 fortnight
q 80 cm : 25 m : 20 cm **r** $19 : $20 : 60 cents

3 Write each of the following ratios in simplest form.

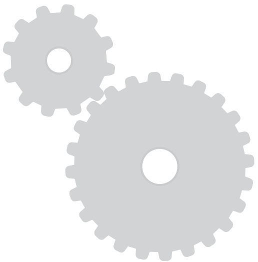
a $3\frac{1}{2}$: 1 **b** 4 : 3
c 4 : 2 **d** 5: 2
e 3: 2 **f** 4 : 3
g 5 : 2 **h** 7: 6
i 5 : 3 **j** 3 : 9
k 6 : 9 **l** 6 : 6

4 Express each of the stated ratios in the simplest form.

a The ratio of the taller girl's height to the shorter girl's height.

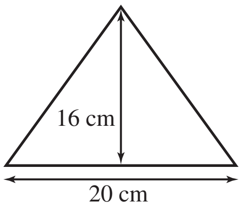


b The ratio of the teeth on the small gear to the big gear.



c The ratio of the larger can's mass to the smaller can's mass.

d The ratio of the triangle's height to its base length.



Answers 5B

1 **a** 1 : 4 **b** 1 : 5 **c** 1 : 4
d 2 : 5 **e** 6 : 17 **f** 6 : 5
g 1 : 15 **h** 1 : 8 **i** 9 : 13
j 5 : 2 : 1 **k** 6 : 1 : **l** 11 : 5 : 9
m 1 : 4 : 2 **n** 9 : 1 : 4 **o** 134 : 100 : 43
2 **a** 50 : 3 **b** 1 : 2 **c** 50 : 6 : 3
d 16 : 10 : 9 **e** 3 : 10 **f** 70 : 9
g 13 : 70 **h** 12 : 1 **i** 3 : 8
j 4 : 25 **k** 2 : 3 **l** 5 : 4
m 8 : 3 **n** 30 : 1 **o** 9 : 10 : 2
p 2 : 14 : 7 **q** 8 : 25 : 2 **r** 95 : 100 : 3
3 **a** 7 : 2 **b** 12 : 11 **c** 12 : 7
d 41 : 16 **e** 5 : 3 **f** 5 : 4
g 20 : 9 **h** 31 : 24 **i** 12 : 7
j 23 : 54 **k** 49 : 72 **l** 9 : 10
4 **a** 7 : 6 **b** 1 : 2 **c** 11 : 5
d 4 : 5

5C • Finding quantities using ratios

LB1 Page 122

Specific learning outcomes

Learners should be able to:

8.5.5.1 Find missing numbers and quantities using given ratios.

Teaching points

1 Finding a missing number given two equal ratios.

Learner difficulties and remedies

Difficulty

Using ratios to find unknown quantities and to solve problems.

Remedy

- Show learners different approaches and steps of how to find missing quantities using ratios.
- Use cross multiplication to find unknown quantities.
- Multiply each part in a ratio by a common factor to increase it or decrease it and then find the unknown parts of the ratio.

Suggested teaching approach

- Use different methods or approaches to find missing quantities.
  - – Common factor
  - – Unit ratios
  - – Fractions
  - – Cross multiplication
- When solving word problems, the ratios must correspond to the information given in the questions.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Ratios can be used to calculate unknown quantities using information provided about the other parts in the ratio.

Missing quantities can be calculated using the four different methods: common factor, unit ratios, fractions and cross multiplication.

Examples

Method 1: Common factor

1 Find the value of the unknown, n, if $n:24 = 9:4$.

Thinking	Working
1 Write the ratio with the unknown value underneath the ratio with the two known values.	$9:4$ $= \quad n:24$
2 Consider the side of the ratios in which both values are known (the right-hand side). Use these values to find the multiplier that turns the first ratio into the second. (Here, we multiply 4 by 6 to get 24.)	$\begin{tikzcd} 9:4 \arrow[d, bend right, "\times 6"'] \arrow[d, bend left, "\times 6"] \\ n:24 \end{tikzcd}$
3 Because the ratios are equivalent, multiplying the known value on the other side of the ratio (9) by the same multiplier (6) gives the value of the unknown (n).	$n = 9 \times 6$
4 Write the answer.	$n = 54$

Method 2: Unit ratios

2 Find the value of b if $3:b = 5:7$

Thinking	Working
1 Write the ratio containing the unknown value underneath the ratio with the two known values.	$5:7$ $= \quad 3:b$
2 Divide both numbers in the top ratio by the number that is above the known value in the second ratio (in this case 5, which is above 3). This gives a unit ratio. Show the division using arrows.	$\begin{tikzcd} 5:7 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 1:\frac{7}{5} \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 3:\frac{7}{5} \times 3 \end{tikzcd}$ $3:b$
3 Now, multiply the unit ratio to obtain the known amount in the second ratio. (Here, we multiply by 3.)	
4 Use the sequence of operations used to turn the first ratio into the second to find b.	$b = 7 \div 5 \times 3$ $b = 4{\cdot}2$

Method 3: Fractions

3 Find the value of b if $3:b = 5:7$

Thinking	Working
1 Write each ratio as a fraction, ensuring that the pronumeral is a numerator. (This means that the second number in the other ratio, 7, is also a numerator.) Because the ratios are equivalent, the fractions are also equivalent.	$5:7$ $= 3:b$ $\frac{b}{3} = \frac{7}{5}$
2 Solve the equation for b. (Here, we multiply both sides by 3.)	$b = \frac{7}{5} \times 3$
3 Write the answer.	$b = 4{\cdot}2$

Method 4: Cross multiplication

4 The ratio of Matai's height to Boso's height is 9 : 10. If Boso is 152 cm tall, find Matai's height. Write your answer to the nearest centimetre.

Thinking	Working
1 Write the information given in the form of ratio.	Matai : Boso $9:10$ $x:14 = 52$
2 Write the ratio.	$9:10$ $x:152$
3 Cross multiply the parts of the two ratios.	$\begin{tikzcd} 9:10 \arrow[d, phantom, "\times"] \\ x:152 \end{tikzcd}$
4 Solve for x to find the missing value.	$\frac{\cancel{10}x}{\cancel{10}} = \frac{136\cancel{8}}{\cancel{10}}$ $x = 136{\cdot}8$
5 Write the answer with the correct unit.	$x = 137$ cm (nearest cm)

Activity 5C

Use any of the four methods to calculate quantities and missing values in the following questions.

1 In a private school the ratio of boys to girls is 8 : 7. If there are 400 boys at the school, how many girls are there?

2 An ice-cream recipe mixes cream and milk in the ratio 2 : 5 before adding the flavouring.

- **a** Find the amount of milk needed if 6 litres of cream is used.
- **b** Find the amount of cream needed if 12 litres of milk is used.

3 Find the value of each unknown:

a $21:g = 7:3$ **b** $8:h = 1:7$ **c** $i:72 = 5:8$
d $a:15 = 2:5$ **e** $b:6 = 5:3$ **f** $c:18 = 5:3$
g $13:2 = p:18$ **h** $5:3 = u:300$ **i** $4:7 = v:700$
j $2:9 = 24:r$ **k** $18:5 = 180:t$ **l** $11:6 = 330:w$

4 Find the value of each unknown:

a $a:44 = 3:8$ **b** $c:32 = 2:5$ **c** $b:20 = 2:9$
d $i:10 = 11:4$ **e** $h:12 = 9:8$ **f** $g:30 = 7:4$
g $6:x = 10:1$ **h** $2:x = 5:11$ **i** $3:x = 2:7$
j $2:x = 8:13$ **k** $4:x = 8:15$ **l** $4:x = 5:9$

5 A jam manufacturer sells jam in 250 g and 450 g jars. The prices for each jar are to be in the same ratio as the jar sizes. If the price of the larger jar is $3.69, what should be the price of the smaller jar?

Answers 5C

1 350

2 **a** 15 L **b** 4·8 L

3 **a** 9 **b** 56 **c** 45

d 6 **e** 10 **f** 30

g 117 **h** 500 **i** 400

j 108 **k** 50 **l** 180

4 **a** 16·5 **b** 12·8 **c** 4·44

d 27·5 **e** 13·5 **f** 52·5

g 0·6 **h** 4·4 **i** 10·5

j 3·25 **k** 7·5 **l** 7·2

5
$$\frac{x}{250} = \frac{3{\cdot}69}{450}$$
$$x = \frac{3{\cdot}69}{500} \times 450$$
$$= 2{\cdot}05$$

The price of the smaller jar is $2.05.

5D • Exploring ratios in triangles

LB1 Page 123

Specific learning outcomes

Learners should be able to:

8.5.6.1 Divide lines into different parts using given ratios.

8.5.7.1 Identify special ratios for the lengths of lines constructed inside triangles.

Teaching points

1 Use ratios to divide a line into different parts.

2 Construct triangles according to ratios of the sides.

Suggested teaching approach

- Learners complete **Learning task 5D** on page 123 of the LB.

Activity 5D

1 Use a compass and ruler to construct triangles (where possible) with sides according to the ratios below. Identify the triangles by name: equilateral, isosceles, right-angled or scalene.

a 3 : 4 : 5 **b** 2 : 2 : 3

c 1 : 4 : 8 **d** 1 : 1 : 1

e 7 : 9 : 11 **f** 5 : 12 : 13

Answers 5D

1 **a** right-angled **b** iscoseles

c scalene **d** This triangle is not possible.

e equilateral **f** right-angled

5E • Sharing using ratios

LB1 Pages 124–125

Specific learning outcomes

Learners should be able to:

8.5.8.1 Share quantities according to given ratios.

8.5.8.2 Solve problems using given ratios.

Teaching points

1 Use given ratios to share quantities.

Learner difficulties and remedies

Difficulty

Identifying parts of the ratio that can be used to share quantities.

Remedy

- Tell learners that the ratio must correspond with the given information about the parts that make up the ratio. If Fred comes before John, then the first part of the ratio must apply to Fred.

Difficulty

Following the correct steps when sharing quantities in given ratios.

Remedy

- Explain the steps to follow in using given ratios to share quantities: Add the parts of the ratio to get the total parts, then divide it by the amount to be shared, and then multiply the result by each part of the ratio.

Suggested teaching approach

- Explain the steps that must be followed in sharing quantities:
  - Identify the quantities to be shared and the ratio to be used to share the quantities.
  - Add each part of the ratio to find the total parts of the ratio.
  - Divide the total number into the amount or quantities to be shared.
  - Multiply the answer by each part of the ratio to find the shared amount that is represented by each parts of the ratio.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

To split or share quantities in a certain ratios, it is important to look at the fraction of each person receives or the allocation of the sharing of the quantities.

To share an amount in a given ratio:

1 Add the values in the ratio to find the total number of parts.

2 Divide the amount by the total number of parts to find the size of one part.

3 Multiply the values in the ratio by the size of one part.

Examples

Method 1: Find the size of one part

1 Share 45 in the ratio 2 : 7.

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts = 2 + 7 = 9
2 Divide the full amount by the number of parts to find the size of one part.	Size of one part = $\frac{45}{9}$ = 5
3 Multiply the ratio by the size of one part.	2 : 7 = 5 × 2 : 5 × 7 = 10 : 35
4 Check that the new amounts add up to the original amount.	10 + 35 = 45 ✓

Method 2: Find the fraction

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts = 2 + 7 = 9
2 Express each part as a fraction by using the part as the numerator and the total as the denominator.	$\frac{2}{9}$ $\frac{7}{9}$
3 To find each part, multiply the full amount by the fraction.	$\frac{2}{9} \times 45 = 10$ $\frac{7}{9} \times 45 = 35$
4 Check that the new amounts add up to the original total.	10 + 35 = 45 ✓

Method 1: Find the size of one part

2 Jack, Andrew and Patrick share 24 coconuts among themselves in the ratio 4 : 3 : 1. How many coconuts does each person receive?

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts = 4 + 3 + 1 = 8
2 Divide the amount by the number of parts to find the size of one part.	Size of one part = $\frac{24}{8}$ = 3
3 Multiply the ratio by the size of one part.	4 : 3 : 1 = 4 × 3 : 3 × 3 : 1 × 3 = 12 : 9 : 3
4 Check that the new amounts add up to the original total.	12 + 9 + 3 = 24 ✓
5 State the answer.	Jack receives 12 coconuts. Andrew receives 9 coconuts. Patrick receives 3 coconuts.

Method 2: Find the fraction

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts = 4 + 3 + 1 = 8
2 Express each part as a fraction by using the part as the numerator and the total as the denominator.	$\frac{4}{8}$ $\frac{3}{8}$ $\frac{1}{8}$
3 To find each part, multiply the amount by the fraction.	$\frac{4}{8} \times 24 = 12$ $\frac{3}{8} \times 24 = 9$ $\frac{1}{8} \times 24 = 3$
4 Check that the new amounts add up to the original total.	12 + 9 + 3 = 24 ✓
5 State the answer.	Jack receives 12 coconuts. Andrew receives 9 coconuts. Patrick receives 3 coconuts.

Activity 5E

1 Share the following amounts in the ratios stated in the brackets.

a 30 (2 : 3)
b 20 (3 : 1)
c 42 (5 : 1)
d 39 (2 : 11)
e 84 (16 : 5)
f 38 (12 : 7)
g 56 (4 : 3 : 1)
h 42 (2 : 4 : 1)
i 54 (2 : 7 : 9)
j 33 (2 : 5 : 4)
k 50 (2 : 3 : 5)
l 96 (1 : 7 : 4)

2 **a** Kathy has a basket with 45 mangoes that she has been instructed to share with her little brother in the ratio 3 : 2. How many mango fruits does each child receive?

b Mrs Ethel makes cassava pudding to sell in the school bazaar. She divides the pudding into 84 equal slices, and then places the pieces in three separate containers in the ratio 2 : 7 : 3. How many pieces of cassava pudding should she place in each container?

3 How many parts are in the ratio 10 : 5 : 3?

4 After 21 matches, the win : loss ratio of KOSSA FC team is 5 : 2. How many matches has the team won, and how many have they lost?

5 The instructions on a container of orange juice concentrate say to mix with water in the ratio 1 : 4. If Boseto wishes to make 2 L of juice according to the instructions, how many mL of concentrate and water should he mix?

6 Two-stroke motor fuel is made by mixing 1 part oil with 24 parts petrol. Anastasia wishes to mix up 5 litres of two-stroke fuel. How much oil and petrol does she need?

7 The ratio of boys to girls at Marara CHS is 7 : 8. If there are 300 learners, how many of each gender are there?

Answers 5E

1 **a** 12, 18
b 15, 5
c 35, 7
d 6, 33
e 64, 20
f 24, 14
g 28, 21, 7
h 12, 24, 6
i 6, 21, 27
j 6, 15, 12
k 10, 15, 25
l 8, 56, 32

2 **a** 27 mangoes, 18 mangoes
b 14 : 49 : 21

3 18

4 $\frac{5}{7} \times 21 = 15$ wins, $\frac{2}{7} \times 21 = 6$ losses

5 Ratio is 1 : 4 so there are 5 parts.

$\frac{2000}{5} = 400$

We need 400 mL of juice and 1600 mL of water.

6 Ratio is 1 : 24 so there are 25 parts.

$\frac{5000}{25} = 200$

We need 200 mL of oil and 4800 mL of petrol.

7 Ratio is 7 : 8 so there are 15 parts.

$\frac{300}{15} = 20 \cdot 7 \times 20 = 140$ boys

$8 \times 20 = 160$ girls

5F • Rates

LB1 Pages 126–127

Specific learning outcomes

Learners should be able to:

8.5.9.1 Define the term 'rate'.
Rate: a measure of how one quantity changes with respect to another.
8.5.10.1 Calculate rates of given quantities.
8.5.10.2 Use given rates to find other quantities.

Teaching points

1 Explain the meaning of the term rates using several real-life examples.

2 Calculate the rates of different quantities, and then use them to find other rates.

Learner difficulties and remedies

Difficulty

Understanding what a rate is and applying rates to real-life situations.

Remedy

- Explain the definition of a rate is and provide examples of practical rates found in real life.

Difficulty

Using units in rates.

Remedy

- Identify which units are used in the rate. Sometimes a unit needs to be converted into another metric unit to answer the question.
- Use examples such as selling coconuts or mangoes. Find the cost of one coconut and then increase the number to find the cost of 10 coconuts or 100 coconuts etc.

Suggested teaching approach

- Define the term rate.
- Use practical examples that are familiar to the learners.
- Explain to the learners that the use of units in a rate is very important in most practical applications.
- Explain two approaches that can be used to find rates:
  - – Find the unit rate first (what one is equal to) then use it to find other information.
  - – Use cross multiplication.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **rate** is a way of comparing two quantities that use different units. Frequently, the word *per* is used meaning 'for each' and has the symbol / or the letter 'p'. For example, speed is a rate that is written as 10 metres per second, 10 m/s or 10 mps.

A rate is similar to a unit ratio. It represents a certain number of a first quantity for every one of a second quantity. So it is the number of the first quantity for every 1 of the second quantity.

These two methods can be used to calculate different rates.

The unitary method

This method looks at how 'per unit' or one unit can be calculated in relation to other quantities. Calculating a number per 'unit' (where a 'unit' might be litres, kilograms, boxes or people) is a useful method for solving problems. The first part of the method is similar to calculating a unit ratio where the per unit is calculated first before it can be used to calculate other quantities.

If 30 L of petrol costs $45, then we can calculate the cost of 50 L by first finding the cost of 1 litre.

$45 for 30 L
= $1.5 for 1 litre (÷ 30)
= $75 for 50 litres (× 50)

The cross-multiplication method

In the cross-multiplication method, the numbers are arranged in the ratio format. Let the unknown quantity be *x*. Write the ratio with *x* first. Then write the ratio with the given numbers underneath. Make sure that the values with matching units are in corresponding positions in the ratios. Then cross-multiply to find an equation for *x* that can be solved.

Examples

5 kg of potatoes costs $12.99. How much would 16 kg cost?

The unitary method

Thinking	Working
1 Calculate the price per unit (per kilo).	$12.99 for 5 kg ÷ 5 ÷ 5 = $2.598 for 1 kg
2 Multiply the price per unit by the number of units (16 kg.)	× 16 × 16 = $41.57 for 16 kg

Cross-multiplication method

Thinking	Working
1 Write the information given in the form of ratio.	\$12.99 : 5 kg
2 Write the ratio.	12.99 : 5 x : 16
3 Cross multiply the parts of the two ratios.	x : 16 $\times$ 12.99 : 5
4 Solve for x to find the missing value.	$5 \times x = 16 \times 12.99$ $5x = 207.84$ $x = 41.57$
5 Write the answer with the correct unit.	x = \$42.57 (nearest cent)

Activity 5F

1 Match each rate in the first box with the most likely rate in the second box.

Speed of a car	8 L/100 km
Wage rate for a supermarket employee	100 km/hr
Petrol consumption	\$1800/m^2
Cost of building a house	\$18/hour

2 A cleaner earns \$12 per hour. How much will he be paid for 5 hours work?

3 Water is boiling in a saucepan. The water level drops by 85 mL in 5 minutes.
- **a** Express the water loss as a rate per minute in simplest form.
- **b** The saucepan boils dry in 40 minutes. How much water was in the saucepan initially?

4 A truck has a diesel consumption of 24 litres per 100 km.
- **a** How much diesel will it use on a 350 km journey?
- **b** On another journey it used 18 litres. Estimate how far it travelled.

5 A team of baggage handlers can load an aircraft at a rate of 15 pieces per minute.
- **a** How many pieces can be loaded in 5 minutes?
- **b** How long will it take to load an aircraft with 240 pieces of baggage?
- **c** How many pieces can be loaded in 60 minutes?
- **d** Express this rate in ‘pieces per hour’.

6 Rolls of curtain material sell for \$15/m.
- **a** How much will 4 m sell for?
- **b** How much will 11 m sell for?
- **c** Calculate the length of curtain material that sells for \$112.50.

Answers 5F

1

Speed of a car	100 km/hr
Wage rate for a supermarket employee	\$18/hour
Petrol consumption	8 L/100 km
Cost of building a house	\$1800/m^2

2 \$60

3 **a** 17 mL/minute **b** 680 L

4 **a** 84 L **b** 75 km

5 **a** 75 **b** 16 minutes **c** 900
d 900 pieces/hour

6 **a** \$60 **b** \$165 **c** 7·5 m

5G • Using rates: speed, distance and time

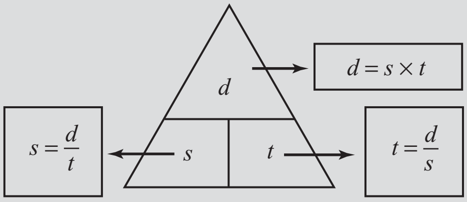
LB1 Pages 128–129

Specific learning outcomes

Learners should be able to:

8.5.11.1 Define speed:
Speed: the rate at which the distance changes with respect to time. It is measured in units of distance divide by time.

8.5.11.2 Use the given diagram to calculate the rates of speed, distances and time.



Teaching points

1 Find the meaning of the following terms: speed, distance and time.

2 Calculate the rates of speed, distances and time using the given formula.

Learner difficulties and remedies

Difficulty

Choosing which of the three formulas to use, and identifying the units.

Remedy

- Which formula to use is determined by which quantity is required. If a question asks for speed, then use the speed formula. If a distance is required, then use distance formula If the units used in a question are kilometres per hour but it ask for metres per second, then learners have to convert the unit of time from hours to seconds, and the unit of distance from kilometres to metres.

Difficulty

Converting metric units of time and distances.

Remedy

- Remind learners of the metric unit ladder and the steps for converting units.

Suggested teaching approach

- Explain to the learners the three areas where rates are being used or calculated: speed, distance and time.
- Show learners the three different formulas that are used.
- Show to learners how to use these formulas to calculate various rates and how they can be practically applied in real-life situations.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.

- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Speed is a rate that compares distance and time. It is used to describe how fast something is travelling. For example, a car could be travelling at 60 kilometres per hour (kph or km/h), or a cheetah could be running at 25 metres per second (m/s or mps).

The three formulas that link speed, time and distance are:

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{or} \quad s = \frac{d}{t}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} \quad \text{or} \quad t = \frac{d}{s}$$

$$\text{distance} = \text{speed} \times \text{time} \quad \text{or} \quad d = s \times t$$

Examples

Calculate the average speed for the following situations. Express your answers correct to two decimal places where necessary.

a a car that travels 360 km in 4 hours

b a female athlete who sprinted 100 m in 10·49 seconds

Thinking	Working
a **1** Write the formula for calculating average speed.	**a** $s = \frac{d}{t}$ $d = 360$ km $t = 4{\cdot}5$ h
2 Substitute the values for distance and time.	$s = \frac{360}{4{\cdot}5}$
3 Evaluate. Note that as the original units are km and hours, the speed unit is km/h.	Average speed = 80 km/h
b **1** Write the formula for calculating average speed.	**b** $s = \frac{d}{t}$ $d = 100$ m $t = 10{\cdot}49\,s$
2 Substitute the values for distance and time.	$s = \frac{100}{10{\cdot}49}$
3 Evaluate, rounding your answer to the stated number of decimal places. Note that as the original units are meters and seconds, the speed unit is m/s.	Average speed = 9·53 m/s

Activity 5G

1 Calculate the average speed for the following. Express your answers correct to two decimal places where

  a 80 kilometres travelled in 2 hours

  b 200 kilometres travelled in 4 hours

  c 160 kilometres travelled in 2·4 hours

  d 150 kilometres travelled in 3·9 hours

  e 1120 kilometres travelled in 12 hours

2 A bus takes 5 hours to travel 370 km. What is the average speed?

3 A cyclist covers 10 km in half an hour. What is the average speed?

4 Caroline walks at a steady speed of 4 km/hr for 2 hours. How far has she walked?

5 Emily walked 22 km at a constant speed of 4 km/h. How long did it take her?

6 Linda's averaged 80 km/h in her car for 3 hours. How far did she travel?

7 In May 2010 the following world records stood. All events are athletics except for the 100 m swimming.

	100 m	**100 m swimming**	**400 m**	**5000 m**	**Marathon (42 km)**
Men	9·58 s	46·91 s	43·18 s	12 min 37·35 s	2 h 3 min 59 s
Women	10·49 s	52·07 s	47·60 s	14 min 11·15 s	2 h 15 min 25 s

  a What is the average speed of the men's 100 m running record in m/s to one decimal place?

  b If the men could maintain the speed calculated in **a**, how long would it take to cover the 42 km of the full marathon? Answer to the nearest minute.

  c What is the average speed in m/s for both the women's 100 m and 5000 m running events, to two decimal places?

  d In which event is the ratio of the men's time to the women's time closest to 1 : 1?

  e Complete the following sentence. (Write the answer correct to one decimal place.) Men can run 100 m ________ times faster than they can swim 100 m.

Puzzle

A train is exactly 1 km long. It is travelling at 60 km/h or 1 km/minute. How long will it take to completely cross over a bridge that is 1 km long?

Answers 5G

1 **a** 40 km/h **b** 50 km/h

  c 66·67 km/h **d** 38·46 km/h

  e 93·33 km/h

2 74 km/h

3 20 km/h

4 8 km

5 $\frac{22}{4} = 5.5$ hours

6 80 × 3 = 240 km

7 **a** 10·4 m/s

  b 4038 seconds = 67 minutes (neatest minute)

  c 100 m: 9·53 m/s
  5000 m: 5·87 m/s

  d In the marathon the ratio of men to women is 1·0922 : 1, which is closest to 1 : 1.

  e Men can run 100 m 4·9 times faster than they can swim 100 m.

5H • Graphs and time

LB1 Pages 130–134

Specific learning outcomes

Learners should be able to:

8.5.13.1 Interpret time graphs.

Teaching points

2 Interpret information displayed in distance–time graphs.

Learner difficulties and remedies

Difficulty

Reading the scales and axes labels of distance–time graphs.

Remedy

- Know that in a distance–time graph, the horizontal axis always represents time and the vertical axis represents distance.
- Interpret the significance of a rising, falling and a horizontal section of a distance–time graph.

Difficulty

Locating points on graph.

Remedy

- Go to the horizontal axis first, and then the vertical axis.

Suggested teaching approach

- Make sure learners know how to read the graph. The time scale is on the horizontal axis and the distance travelled is shown on the vertical axis.
- Explain different parts of the graph and what they actually represent.
- Show how to locate points on the graph, using horizontal then vertical (h, v).
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

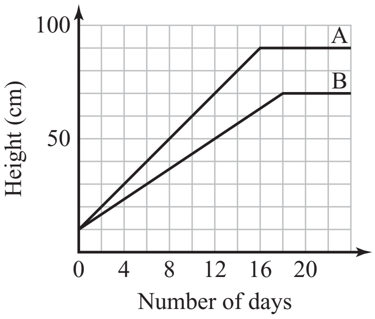
Line graphs show how one quantity changes in respect with another.

A distance–time graph shows the progress of a journey travelled or a distance an object has moved.

The time scale is on the horizontal axis and the distance travelled is shown on the vertical axis.

Examples

1 The graph given below shows what happened to the height of two tomato plants after they were planted in a garden. One receives fertiliser and the other did not.



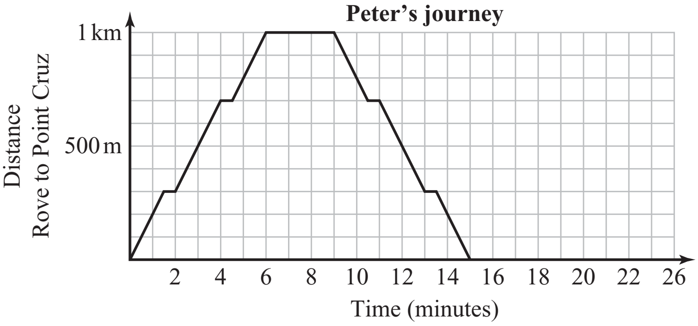
a What height were the tomato plants when planted?

b Which plant grew faster?

c How does the graph show when the plants stopped growing?

Thinking	Working
a Read the values from the vertical axes at the when the number of days is 0. Include the correct units.	a They were both 10 cm high.
b Look for the steepest line. The graph of plant A is steeper than B.	b Plant A grew faster.
c Look for a horizontal line. The lines graphs are horizontal after days 16 and 18.	c Plant A stopped growing after 16 days, and plant B stopped after 18 days.

2 The graph below shows Peter's morning walk. He started at the Police Club in Rove and walked a kilometre to Point Cruz then returned back to the Police Club in Rove.



a How long are each of Peter stops on his way to Point Cruz?

b How long does it take Peter to reach Point Cruz?

c How long does it take for Peter to return to the Police Club at Rove?

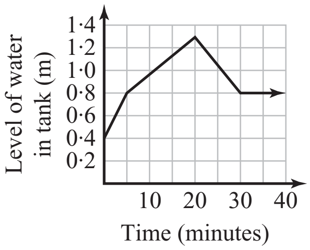
d Does Peter walk faster on his way to Point Cruz or his return to the Police Club at Rove?

Solution

Thinking	Working
a Read the time elapsed for each of the horizontal lines on the first part of the journey. Here, the distance is not changing. Include the correct units.	a Peter makes two stops each of 0·5 minutes or 30 seconds.
b Read the time elapsed to reach 1 km. Include the correct units.	b 6 minutes
c Read the time elapsed for the entire journey. Include the correct units.	c 15 minutes
d Both trips cover 1 km in 6 minutes. The graphs going up and down are equally steep.	d The speeds are the same for both journeys.

Activity 5H

1 Sue decides to add some water to a storage tank. When the tank looks full, she walks away. She doesn't realise that there is a hole in the side of the tank that causes the tank to leak.

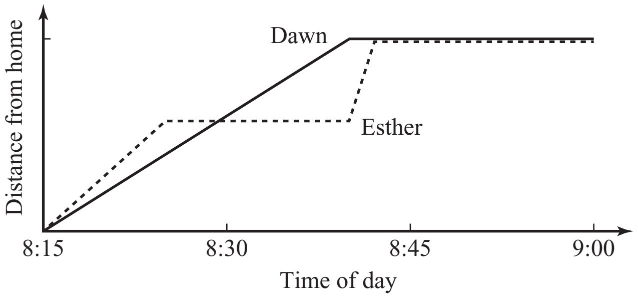


a What is the level of water in the tank when she starts filling it?

b How long does it take to fill the tank?

c At what level is the hole in the tank? Explain by referring to the graph.

2 The Thompson twins (Dawn and Esther) both left home at 8:15 am to walk to school. School starts at 8:45 am. One of the twins stopped at a friend's place on the way.



a Which twin stopped at a friend's place, and for how long?

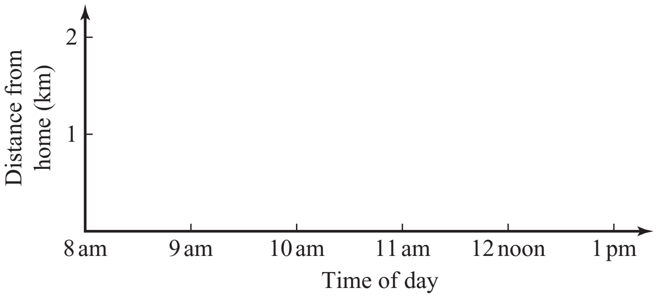
b Which twin walked faster? Explain how the graph shows this.

3 This is what happened to Michael this morning:

He left home at 8:30 am, walked 2 km to school and arrived at school at 9 am. At 10 am he went to a 1-hour dentist's appointment. The dentist is halfway between school and home. He arrived back at school at 11:30 am. At 12:15 pm he felt sick, and was collected by one of his parents at 12:30 pm. The drive home took 5 minutes.

On this diagram, draw a distance–time graph to show Michael's distance from home between 8 am and 1 pm.

Michael's miserable morning



Answers 5H

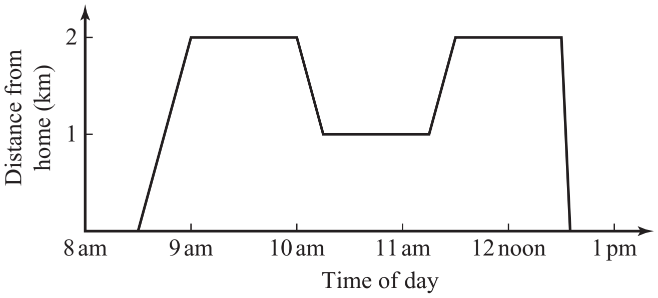
1 **a** 0·4 L **b** 20 minutes **c** 1·3 L

2 **a** Ester stopped for 30 minutes.

b Ester walked faster because her section of the graph is steeper.

3

Michael's miserable morning



5I • Scale diagrams

LB1 Pages 135–137

Specific learning outcomes

Learners should be able to:

8.5.14.1 Use scale ratios to calculate actual measurements for maps and plans.

Teaching points

1 Use a scale of a map, expressed as a ratio, to find the map measurements, and real or actual measurements on the ground.

Learner difficulties and remedies

Difficulty

Using a scale factor in maps and plans to calculate the real lengths.

Remedy

- To find the real measurement, multiply the scale factor by the measurement on the map.

Difficulty

Converting metric units of distances measured on a map or plan.

Remedy

- Remind learners of the metric ladder to convert units.

Suggested teaching approaches

- Revise ratios before doing this unit.
- Show learners maps and scale drawings that include a ratio (e.g. 4 : 5). The first number in the ratio represents the distance on the map. The second is the corresponding distance is in real life.
- If the ratio is a unit ratio (e.g. 1 : 1000), the larger value in the scale ratio is called the scale factor. A scale factor of 1000 means that in real life a length is 1000 times longer than in the diagram.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Scale factors

Consider two tall buildings in Honiara. The Anthony Saru Building is 45 m tall and the Tongs Building is 30 m tall.

The ratio of their heights can be written directly as 45 : 30. This is correct, but it does not really give us a 'feel' for their relative sizes. The Anthony Saru Building is taller, but how many times taller? Writing the ratio of their heights as a unit ratio makes the comparison more meaningful.

To write a unit ratio, divide both numbers in the ratio by the second number.

Anthony Saru Building : Tongs Building

45 : 30

÷30 : ÷30

1·5 : 1

The unit ratio tells us that the Anthony Saru Building is 1·5 times, the height of the Tongs Building. Because 1·0 = 100%, we can also say that the Anthony Saru Building's height is 150% of the height of the Tongs Building or that its height is 50% taller than the height of the Tongs Building.

1·5 is the scale factor we multiply the height of the Tongs Building by to get the height of the Anthony Saru:

30 m × 1·5 = 45 m

Dividing the height of the Anthony Saru Building by 1·5 gives us the height of the Tongs Building:

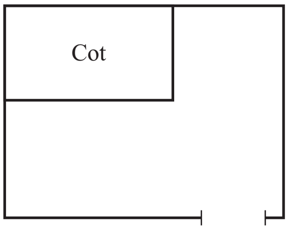
45 m ÷ 1·5 = 30

- To write the ratio $a : b$ as a unit ratio, calculate a b and express the ratio as $\frac{a}{b}$: 1 (divide both of the numbers in the ratio by b).
- The quotient $\frac{a}{b}$ is known as the scale factor. It tells us the size of a in terms of how many times the size of b it is. It is the number we must multiply b by to get a.
- If a is greater than b ($a > b$), then $\frac{a}{b}$ is greater than 1, and more than 100%.
- If a is smaller than b ($a < b$), then $\frac{a}{b}$ is less than 1, and less than 100%.

Scale drawings

Scale drawings use ratios to present accurate information about distances and still have a diagram of a convenient size. Maps and building plans are two examples of scale drawings.

A scale drawing is a plan that shows lengths in proportion of those in real life. This is a scale drawing of a bedroom. The scale is 1 : 75. This means 1 cm on the drawing represents 75 cm in the bedroom.



A scale drawing contains a ratio; for example, 1 : 1 000 000. The first number in the ratio is the distance on the diagram. The second is how many times bigger the distance is in real life.

As with other ratios, scale ratios should be written without units and in simplest form. For example, a plan of a house may use a scale of 5 mm to represent 1 metre or 5 mm : 1 m. To convert this to a scale ratio, we express both values in the same units, and simplify:

e.g. 5 mm : 1 m

= 5 mm : 1000 mm

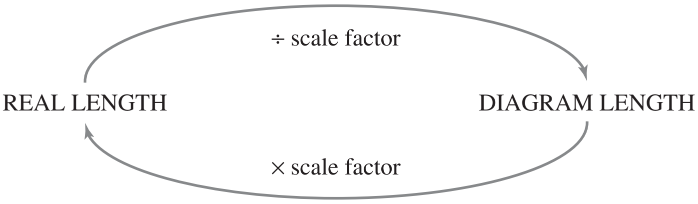
= 5 : 1000

= 1 : 200

If the scale ratio is a unit ratio, the larger value in the scale ratio is called the scale factor.

A scale factor of 200 means that in real life the object is 200 times as big as in the diagram. Another way to think of this is to imagine that the object was shrunk by a factor of 200 to make it fit onto a page.

To write a ratio as a scale factor, convert to appropriate smaller units and simplify by cancelling common factors.



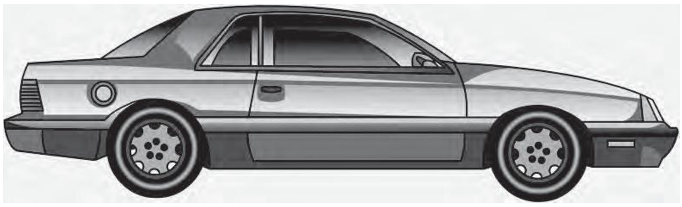
When very small objects are enlarged to a scale, the operations in the diagram above are reversed.

Examples

1 Convert 2 mm : 1 m to a scale ratio and underline the scale factor.

Thinking	**Working**
1 Write both quantities in the smaller units (mm).	2 mm : 1 m = 2 mm : 1000 mm
2 Simplify by dropping the units and any common factors. Underline the scale factor.	= 1 : <u>500</u>

2 The diagram below uses the scale of 1 : 80. Find the real length of the car in metres.



Thinking	**Working**
1 Measure the length of the car (in mm) on the diagram (65 mm).	Diagram length = 65 mm
2 Multiply the diagram length by the scale factor to get the real length.	Real length = 65 mm × 80 = 5200 mm
3 Convert your answer to metres.	= 5·2 m

3 The height of a tower is 70 m. How high should it be drawn on a diagram with a scale of 1:200? Answer in millimetres.

Thinking	**Working**
1 Divide the real length by the scale factor to find the diagram height.	70 m ÷ 200 = 0·35 m
2 Convert the diagram height to mm.	= 350 mm

Activity 5I

1 Convert the following to scale ratios and underline the scale factor.

a 1 cm:6 m
b 1 mm:40 cm
c 2 mm:50 cm
d 5 cm:3 m
e 5 mm:4 m
f 2 mm:1 m
g 4 cm:1 m
h 4 mm:50 cm
i 1 cm:1 km
j 1 cm:500 m
k 5 cm:2 km
l 4 cm:1 km

2 A council plan is drawn to a scale of 1:100. Find the real length in metres for each of the following diagram lengths.

a 2 cm
b 3 cm
c 5 cm
d 8 mm
e 23 mm
f 19 mm
g 40·3 cm
h 92·5 cm
i 1·2 m

3 A two-storey house is to be drawn to scale. Various lengths in the house are given below. How long should each length be drawn on a diagram with a scale of 1:400? Answer in millimetres.

a 2 m
b 4 m
c 8 m
d 40 cm
e 40 m
f 80 m
g 320 cm
h 1200 mm
i 5200 mm
j 480 cm
k 0·8 m
l 0·24 m

4 A map is drawn to the scale of 1:2000. Find the real distances in metres for each of the following lengths measured on the map.

a 3 cm
b 5 cm
c 10 cm
d 2·6 cm
e 38·4 cm
f 43 mm
g 9 mm
h 17·9 mm
i 51 mm

5 Are the following scale ratios? Answer Yes or No.

a 4:7000
b 6:120 000
c 1:400
d 1:500 000
e 65:500
f 1 cm:17 mm

6 In the ratio 1:3500, the scale factor is:

7 Find the real length in metres for each of the following diagram lengths. The scale ratio is given in brackets in each case.

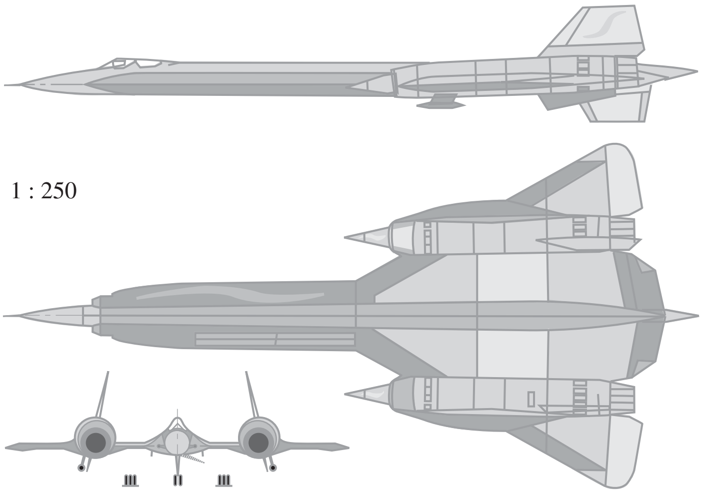
a 7 cm (1:100)
b 5 mm (1:1000)
c 4 cm (1:2000)
d 8 cm (1:500)
e 6 mm (1:50)
f 40 mm (1:3000)
g 21 mm (1:300 000)
h 13 cm (1:400 000)
i 8·8 cm (1:2000)

8 Find the diagram length in millimetres for each of the following real lengths. The scale ratio is given in brackets in each case.

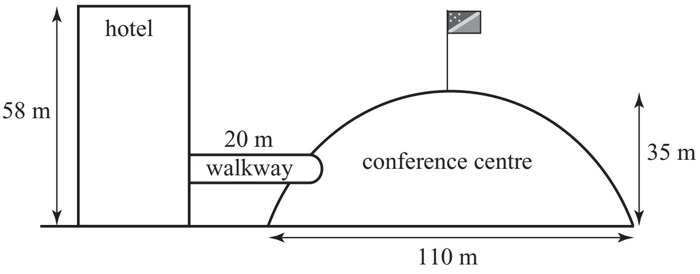
a 6 m (1:300)
b 16 m (1:2000)
c 700 m (1:200)
d 16 km (1:4000)
e 12 km (1:500 000)
f 180 m (1:600 000)
g 50 cm (1:20)
h 400 cm (1:500)
i 36 m (1:400)

9 An engineer's drawing of an aircraft if shown below. Find, to the nearest metre:

a the length of the aircraft
b the wing span.

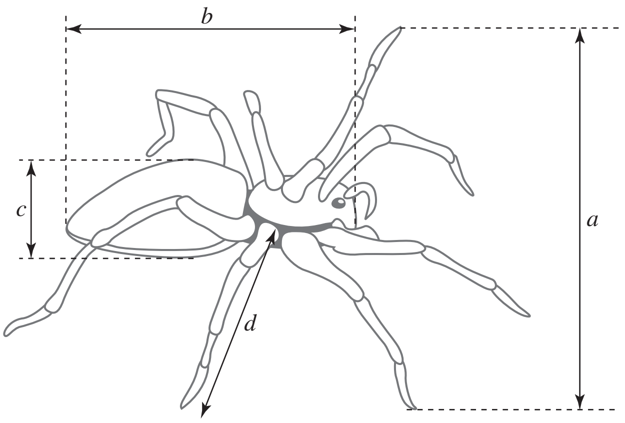


10 A developer has a proposal for a city project. A section of her rough plans for the project appears below. If a scale model is to be made of the proposed development, what will be the lengths on the model of the following if a scale of 1:2000 is used? Answer in millimetres.



a the height of the hotel
b the length of the conference centre
c the height of the conference centre
d the length of the connecting walkway.

11 Scale drawings aren't always smaller than the real thing. The diagram below is of a spider, drawn to a scale of 2:1 (i.e. the real size of the spider is 0·5 or $\frac{1}{2}$ the diagram size). Find the real length of each distance marked with a variable on the diagram. (Answer in mm.)



Answer 5I

1 **a** 1 : 600 **b** 1 : 400 **c** 1 : 250
d 1 : 60 **e** 1 : 800 **f** 1 : 500
g 1 : 25 **h** 1 : 125 **i** 1 : 100 000
j 1 : 500 000 **k** 1 : 400 000 **l** 1 : 2500

2 **a** 2 m **b** 3 m **c** 5 m
d 0·8 m **e** 2·3 m **f** 1·9 m
g 40·3 m **h** 92·5 m **i** 120 m

3 **a** 5 mm **b** 10 mm **c** 20 mm
d 1 mm **e** 100 mm **f** 200 mm
g 8 mm **h** 3 mm **i** 13 mm
j 12 mm **k** 2 mm **l** 0·6 mm

4 **a** 60 m **b** 100 m **c** 200 m
d 52 m **e** 768 m **f** 86 m
g 18 m **h** 35·8 m **i** 102 m

5 **a** No **b** No **c** Yes
d Yes **e** No **f** No

6 D

7 **a** 7 m **b** 5 m **c** 80 m
d 40 m **e** 0·3 m **f** 120 m
g 630 m **h** 5200 m **i** 176 m

8 **a** 20 mm **b** 8 mm **c** 3500 mm
d 4000 mm **e** 240 mm **f** 3 mm
g 25 mm **h** 8 mm **i** 90 mm

9 **a** 92 mm × 250 = 23 000 mm
= 23 m
b 46 mm × 250 = 11 500 mm
= 11·5 m

10 **a** 58 000 ÷ 2000 = 29 mm
b 1100 000 mm ÷ 2000 = 55 mm
c 35 000 mm ÷ 2000 = 17·5 mm
d 200 000 ÷ 2000 = 10 mm

11 **a** 25 mm **b** 19 mm
c 6·5 mm **d** 13·5 mm

5J • Exploring maps

LB1 Pages 138–139

Specific learning outcomes

Learners should be able to:

8.5.15.1 Define the term ‘contour line’.
8.5.15.2 Draw cross-section diagrams from contours on a map.
8.5.15.3 Estimate heights of key features of a map using cross-section diagrams.

Teaching points

1. Explain what contour lines are on a map.
2. Show how to draw a cross-sectional (landform graph) from a map.
3. Find a distance on the map using a contour line and the map scale.
4. Show how the cross-sections of given maps can be used to find distances between points in the map.

Learner difficulties and remedies

Difficulty

Interpreting the contours on the map.

Remedy

- Use clay to build a model of a hill that can be sliced in half so learners can see the elevations change along the cross-section.
- Read the scales vertically and horizontally to calculate the distances.
- Have the learners tell a story about the journey according to the information on the map and the cross-section diagrams.

Suggested teaching approach

- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Learners complete **Learning task 5J** on pages 138 and 139 in the LB.

Activity 5J

Challenge learners to make a model of an island that has just one mountain out of clay on a piece of cardboard. Have them draw a rectangle around their model and draw a scale along each edge of the rectangle so that 1 cm represents 10 m.

Draw the same-sized rectangle on another piece of cardboard. Sketch a map showing the outline of the island on the second cardboard. Mark the position of highest point of the hill on the map, and then sketch two contour lines that match the clay model.

Ask classmates to check the accuracy of the map with the model. Display both the map and model in the classroom.

• Chapter Six •

Statistics

Overview

The study of statistics is the collection, interpretation and presentation of data. Surveys and studies are used to collect data such as the colour of houses, the number of learners in Year 8, and their average age. In today's world, it is important that we can make sense of statistical data from many sources. For example, we can check tables to see the schedules of the ships that go to the islands and back, or consult a calendar to determine the right time to harvest gardens. We make decisions every day based on statistics and probabilities for information collected and their occurrences. In this chapter, learners will develop skills so that they can understand how to collect data, present it in useful forms, and interpret it accurately.

Contents

Chapter skills

This chapter covers the following skills:

- Collecting numerical data, discrete and continuous, by using class intervals
- Drawing histograms and column graphs and finding the mode
- Finding the median, range and interquartile range of a set of data
  - – The media is the middle value when the data is written as an ordered list.
  - – The range is the difference between the highest and lowest values.
  - – The lower quartile is Q_1 is the middle value of the lower half.
  - – The upper quartile or Q_3 is the middle value of the upper half.
  - – The interquartile range (IQR) = $Q_3 - Q_1$.
- Drawing stem-and-leaf plots
- Finding the mean or average of a set of data
  - – Let n stand for the number of values.
  - – The notation Σx means the sum of the values.
  - – To calculate the mean (or average we use) $x = \frac{\Sigma x}{n}$.
- Comparing the mean and median
- Finding the mean absolute difference
  - – The mean absolute difference is $\frac{\Sigma|x - \bar{x}|}{n}$.

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 6A Collecting numerical data	Learner's Book 1 • Exercise 6A, pages 151, 152
2	• 6B Column graphs and histograms	Learner's Book 1 • Exercise 6B, pages 155, 156
3	• 6C The median, range, and interquartile range (IQR)	Learner's Book 1 • Exercise 6C, page 159
4	• 6D Stem-and-leaf plots	Learner's Book 1 • Exercise 6D, pages 161, 162
5	• 6E The mean	Learner's Book 1 • Exercise 6E, pages 164, 165
6	• 6F Mean absolute difference	Learner's Book 1 • Exercise 6F, page 166
7	• 6G Comparing the mean and the median	Learner's Book 1 • Exercise 6G, page 167
8–9	• 6H Practical data analysis	Learner's Book 1 • Exercise 6H, page 169
10	• Test	Teacher's Guide • Chapter 1 Test

General learning outcomes

Learners should:

Collecting numerical data

8.6.1 Understand numerical data and the two types of numerical data: Discrete and continuous numerical data. (U)

8.6.2 Know how to tabulate collected data into a frequency table. (K)

Column graphs and histogram

8.6.3 Understand that column and histogram graphs can be used to represent and display collected numerical data. (U)

8.6.4 Know how to construct column and histogram graphs. (K)

8.6.5 Know that frequency of data indicate that some scores appear many times more than others. (K)

The median, range and interquartile range (iqr)

8.6.6 Understand that data collected can be analysed by looking at the median, range and interquartile range (IQR). (U)

Stem-and-leaf plots

8.6.7 Know how to display data using stem-and-leaf plots. (K)

The mean

8.6.8 Understand how average can be determined in a given set of scores .(U)

8.6.9 Know how to find the mean of a set of scores. (K)

Mean absolute difference

8.6.10 Know how to find the scores around the mean regardless of whether negative or positive set of scores. (K)

8.6.11 Know how to calculate the mean absolute values of set of scores. (K)

Comparing the mean and the median

8.6.12 Understand the difference between the mean and median as both are appropriate measure of the centre of a set of scores. (U)

Practical data analysis

8.6.12 Understand how statistics can be used to analyse collected data in practical situations.

6A • Collecting numerical data

LB1 Pages 150–152

Specific learning outcomes

Learners should be able to:

8.6.1.1 Define and identify discrete numerical data and continuous numerical data.

8.6.2.1 Record data in frequency tables.

Teaching points

1 Explain the term numerical data and provide a variety of examples.

2 Distinguish between discrete numerical data and continuous numerical data with examples.

3 Display collected information (data) in a frequency table and draw some conclusions.

Learner difficulties and remedies

Difficulty

Tabulating data into frequency tables.

Remedy

- Show learners how to use the three columns: score, tally and frequency for both discrete and continuous data.
- Provide more exercises on how to tabulate data in a frequency table.

Difficulty

Identifying numerical data, and distinguishing between discrete and continuous data.

Remedy

- Provide counting and measuring tasks to generate numerical data, discrete and continuous data.

Suggested teaching approach

- Define the following terms: data, numerical data, discrete data and continuous data.
- Explain to learners the features of the frequency table such as tally and frequency, and describe how they are used.
- Show learners how to enter the numerical data into a frequency table using the tally and frequency columns.
- Explain how continuous data is used in a frequency table. Take note that some learners might take the interval sign in the continuous data as a minus sign.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Types of data

There are two different types of statistical data: categorical and numerical. **Categorical data** is when information is sorted into groups of the same type. For example, eye colour of learners is categorical because the data collected would be the number of learners with blue eyes, brown eyes, green eyes, etc. Another example is the favourite fruit of a group of learners because the data collected would include fruits such as apples, bananas, papaya, mangoes, oranges.

Numerical data involves recording the data as numbers.

Discrete numerical data involves counting and usually results in distinct values as whole numbers. Examples of this are the number of points scored by a basketball player over a season or the number of emails received each day.

Continuous numerical data involves measurements and are not usually restricted to whole numbers, but can include fractional or decimal values that lie between whole numbers. Examples of continuous data are the weights of newborn babies at a hospital or the volume of water in a dam each day. The accuracy of the measurements depends on the measuring equipment used.

If a taxi driver had four hires in a day and a total of $78.90 in fares was collected, the number of hires would be an example of discrete data and the fare total would be an example of continuous data.

Grouped data

When a large amount of numerical data is spread out over many values, it is often grouped class intervals.

- Discrete class intervals are used for discrete data: 0–9, 10–19, 20–29.
- Continuous class intervals are used for continuous data: 0–<10, 10–<20, 20–<30.

A class interval is found by dividing the range into a number of equally sized intervals, ideally to have between five and 10 columns in the frequency column graph. For example, if a discrete data set has values ranging from 10 to 75, the ange is 65. If we use a class interval of 10, we would get seven columns with intervals 10–19, 20–29, … 70–79.

Frequency tables

Any data that has been collected can be recorded in a frequency table. Here, each time a piece of data occurs, it is recorded in the tally column. Data are actually counted in the tally column and their results are written in the frequency column. Frequency means the number of times an event occurs.

Examples

1 For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

a How many television sets are in your house?

b What is the height of the tallest person in your family?

c How far do you have to travel to get to school?

d What is your favourite dessert?

Thinking	**Working**
a Are you counting, measuring or putting things in categories? (Here, we are counting the number of televisions.)	**a** Discrete data
b Are you counting, measuring or putting things in categories? (Here, we are measuring height.)	**b** Continuous data
c Are you counting, measuring or putting things in categories? (Here, we are measuring distance.)	**c** Continuous data
d Are you counting, measuring or putting things in categories? (Here, we are putting things in categories.)	**d** Categorical data

2 A Year 7 class of 25 learners was surveyed about the number of brothers and sisters they have. The results were:

2, 3, 5, 0, 1, 1, 2, 0, 3, 1, 4, 0, 1, 1, 3, 2, 2, 2, 1, 1, 0, 1, 1, 2, 4

a Construct a frequency table showing this information.

b Is this data categorical, discrete or continuous?

Thinking	**Working**
a **1** Construct a table with three columns that shows the item being surveyed, the tally and the frequency.	**a** Frequency table
2 Enter the data into the tally column. Start at the left of the list of numbers and work to the right. (Don't count all the 1s, then all the 2s etc.) Notice 𝍸 is used to represent 5.	
3 Enter these tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.	
b Is the data categorical, discrete or continuous?	**b** It is discrete data.

Number of brothers and sisters	Tally	Frequency
0	||||	4
1	𝍸 ||||	9
2	𝍸 |	6
3	|||	3
4	||	2
5	|	1
	Total	25

3 The Marist basketball team keeps track of the points they score on the way to the Honiara Basketball League Championship. The point totals are:

73, 84, 68, 45, 52, 44, 45, 52, 66, 42, 43, 40, 53, 47, 82, 76, 42, 57, 65, 81, 80, 40, 56, 72, 74, 83, 41, 66, 76, 75, 68, 81, 82, 79, 58, 81, 78, 80, 78, 76

Group the results and then construct a frequency table showing this information.

Thinking	**Working**
1 Decide on how to group the data by subtracting the smallest score from the highest to find the range. Then divide this number by 2, 5, 10 etc. until you get an interval size that gives between five and 10 groups. (If you get a decimal number, round to the nearest whole number.)	Range = 84 – 40 = 44 44 ÷ 5 = 8·8 Number of groups = $\frac{44}{5}$ = 8·8 So, we have nine groups with an interval size of 5.
2 Construct a table with three columns. Write the grouped values in the first column. Be careful with your groupings so that you don't miss values or include them twice. (In this example, we will use 40–44, 45–49, 50–54 and so on.)	
3 Use the tally column to help with the count. Remember to go through the data list systematically from the first piece of data to the last, tallying as you go.	
4 Enter the tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.	

Number of points scored	Tally	Frequency
40–44	𝍸 ||	7
45–49	|||	3
50–54	|||	3
55–59	|||	3
60–64		0
65–69	𝍸	5
70–74	|||	3
75–79	𝍸 ||	7
80–84	𝍸 ||||	9
	Total	40

Activity 6A

1 For each of the following, state whether the data from a survey asking these questions is *categorical*, *discrete* or *continuous.*

a How do you travel to school?

b What colour hair do you have?

c How much time did you spend at the beach last summer?

d What is your height?

e How many cars passed by the school in a day?

f What is your favourite book?

g How many trees are in your front yard?

2 A Year 8 class of 25 learners was surveyed about the number of pets they own. The results were:

0, 0, 2, 1, 1, 1, 0, 6, 9, 4, 3, 0, 1, 3, 2, 1, 1, 4, 5, 4, 3, 2, 1, 4, 7

a Construct a frequency table showing this information.

b Is this data categorical, discrete or continuous?

3 A group of girls was surveyed about how many hours of television they watched on a normal week night. The results were:

3, 2, 1, 1, 0, 2, 2, 0, 4, 2, 3, 0, 1, 2, 2

a Construct a frequency table showing this information.

b How many girls were surveyed?

4 A Year 8 class was surveyed about how much money they had in their pockets at that time.

The results were:

$1.20, $0.75, $1.00, $2.55, $1.50, $4.00, $0.00, $0.00, $0.80, $2.40, $1.25, $0.00, $3.60, $4.40, $4.30, $2.90, $1.80, $2.35, $2.20, $0.50, $0.40, $2.50, $1.50, $1.75

a Group the results into intervals of 50c and then construct a frequency table showing this information.

b How many learners were surveyed?

5 A class was surveyed about the number of hours spent exercising over a weekend. The results were as follows:

1, 3, 2, 1, 1, 2, 0, 2, 0, 1, 1, 2, 6, 3, 4, 2, 1, 0, 2, 8, 3, 5, 1, 2, 0, 2

a Construct a frequency table of this data.

b How many learners were surveyed?

c How many learners did not exercise at all?

d How many learners spent more than 2 hours exercising?

e What number of hours was spent exercising by the majority of the members of the class?

f What is the greatest number of hours spent exercising by a learner?

6 Sixty 13-year-old learners were tested to find their pulse rate (heart beat) when resting.

The following beats per minute were measured.

70, 68, 68, 76, 79, 68, 76, 55, 55, 60, 60, 94, 72, 65, 64, 93, 71, 62, 67, 82, 76, 65, 77, 82, 81, 59, 74, 74, 67, 68, 78, 76, 63, 82, 81, 82, 74, 70, 66, 63, 84, 81, 69, 84, 79, 71, 70, 54, 68, 64, 78, 58, 84, 61, 75, 72, 73, 71, 91, 66

a Construct a frequency table using class intervals of 51–60, 61–70 etc.

b Construct a frequency table using class intervals of 51–55, 56–60, 61–65 etc.

c Which table shows the spread of results more clearly? Give a brief reason to support your answer.

Answers 6A

1 **a** categorical **b** categorical

c continuous **d** continuous

e discrete **f** categorical

g discrete

2 **a**

Number of pets	Tally	Frequency
0	||||	4
1	𝍸 ||	7
2	|||	3
3	|||	3
4	||||	4
5	|	1
6	|	1
7	|	1
8		0
9	|	1
	Total	25

b discrete

3 **a**

Watching TV in a week (hours)	Tally	Frequency
0	|||	3
1	|||	3
2	𝍸 |	6
3	||	2
4	|	1
	Total	15

b 15

4 **a**

Pocket money ($)	Tally	Frequency
0·00–0·49	||||	4
0·50–0·99	|||	3
1·00–1·49	|||	3
1·50–1·99	||||	4
2·00–2·49	|||	3
2·50–2·99	|||	3
3·00–3·49		0
3·50–3·99	|	1
4·00–4·49	|||	3
	Total	24

b 24

5 **a**

Exercising in a weekend (hours)	Tally	Frequency
0	||||	4
1	𝍸 ||	7
2	𝍸 |||	8
3	|||	3
4	|	1
5	|	1
6	|	1
7		0
8	|	1
	Total	26

b 26

c 4

d 7

e 2 hours

f 8 hours

6 **a**

Pulse rate (beats/min)	Frequency
51–60	7
61–70	21
71–80	19
81–90	10
91–100	3
Total	60

b

Pulse rate (beats/min)	Frequency
51–55	3
56–60	4
61–65	8
66–70	13
71–75	10
76–80	9
81–95	10
86–90	0
91–95	3
Total	60

c Table **b** shows a more detailed spread of the data.

6B • Column graphs and histograms

LB1 Pages 153–156

Specific learning outcomes

Learners should be able to:

8.6.3.1 Identify the properties of column graphs and histograms.

8.6.4.1 Appropriately display collected data using column graphs and histograms.

8.6.4.2 Define and identify the modal score in a set of scores.

Teaching points

1. Describe a column graph and a histogram, and identify the differences between them.
2. Collect data and display it using a column graph or a histogram as appropriate.
3. Find the modal scores of a set of scores using graphs.

Learner difficulties and remedies

Difficulty

Constructing column and histogram graphs

Remedy

- Show learners the steps to construct a column graph and a histogram as shown in the additional notes. Point out the differences between a column graph (for discrete data) and a histogram (for continuous data).

Difficulty

Labelling the axes: which data goes on which axes?

Remedy

- The frequency always goes on the vertical axis.

Suggested teaching approach

- Define terms such as horizontal lines (bars) and vertical lines (columns).
- Follow the steps for constructing a column graph and a histogram.
- Identify the features of these two types of graphs and show how they are used for discrete (column graph) and continuous data (histogram). Identify the differences and similarities of these two graphs.
- Remind learners that the vertical and horizontal axes must be labelled and the graph should have a title.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Column graphs

Column graphs are used to represent illustrate discrete data that have been counted, such as the number of fish that were caught during the fishing trip. Column graphs are sometimes also called bar graphs.

Features of a column graph:

- Columns are used to represent discrete data or information collected.
- The columns are usually drawn vertically.
- The columns have spaces or gaps between them.

Drawing a column graph

All column graphs should include:

1. title
2. categories or a numerical scale with equally spaced intervals (gaps) between the numbers on the horizontal axis
3. a numerical scale with equally spaced intervals (gaps) between the numbers on the vertical axis

4 a label for the horizontal axis that explains the variable represented
5 a label for the vertical axis that shows the frequency
6 a small gap between each column (usually half a column width) to show that this is categorical or discrete (counted) data.

Histograms

Histograms used to represent continuous data. This is data that represents quantities such as measurements or weights. The columns have no space between them and each column represents a range of values rather that a single value. It is displayed in a graph called a histogram with grouped continuous data. Each group is represented by a column. The columns do not have gaps in between each other. The frequency is shown on the vertical axis. The horizontal axis is labelled with the measurement units.

Features of a histogram:

- Columns are used to represent data or information collected.
- Columns are drawn vertically.
- Columns have **no** spaces or gaps between them.

Drawing a histogram

All histograms should include:

1 a title
2 a continuous scale on the horizontal axis
3 a numerical scale with equally spaced intervals between the numbers on the vertical axis
4 a label for the horizontal axis that explains the variable represented
5 a label for the vertical axis that shows the frequency
6 no gap between each column
7 a half-column width before the first column even if the first set of data starts at zero.

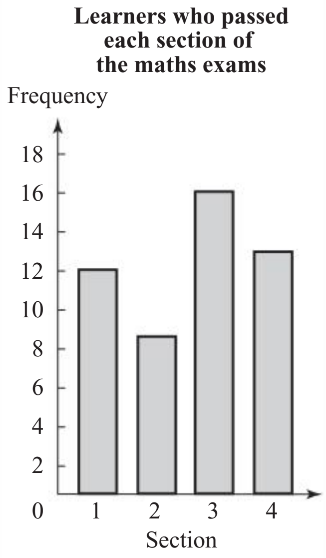
Examples

1 Year 8 learners sat a mathematics examination that had four sections. The results given in the table below shows the number of learners who passed each section.

a Construct a column graph to represent the data.

b Label the axes and show the scales for the graph.

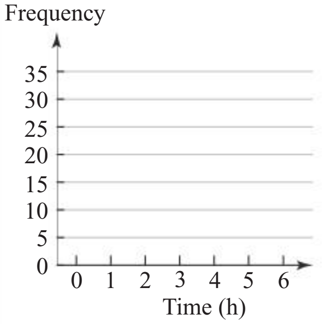
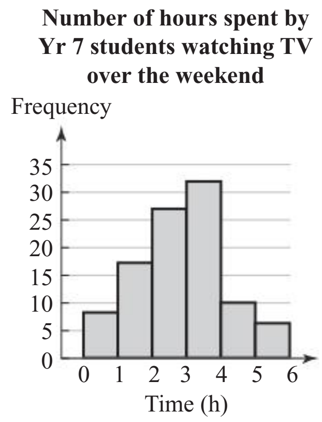
Section	Frequency
1	12
2	9
3	16
4	13

Thinking	Working
1 Find a suitable vertical scale. Use the largest frequency (16) and divide it by 10 because we want a maximum of 10 divisions on the vertical axis. Round your answer to the nearest whole number.	$\frac{16}{10} = 1{\cdot}6$ ≈ 2
2 Draw a frequency column graph using the frequency table given, labelling the axes and naming the graph.	

2 100 Year 7 learners were surveyed about the number of hours they spent watching TV over a weekend. The data is presented below.

Draw a histogram to represent the data given below.

Hours spent watching TV over the weekend	Number of learners
0–<1	8
1–<2	17
2–<3	27
3–<4	32
4–<5	10
5 or more hours	6

Thinking	Working
1 Draw axes; scale and label the *x*- and *y*-axes appropriately.	
2 Ensure rectangles each have the same width on the *x*-axis but with no gaps between the *x*-axis categories. Add a title.	

Activity 6B

1 The frequency table below shows the colour of cars passing the school in a particular timed period. Draw a column graph to represent this information.

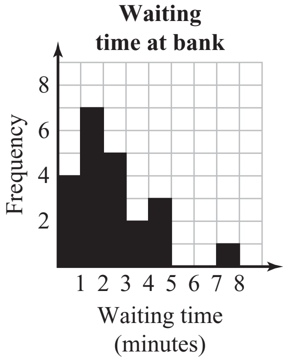
Colour	White	Red	Green	Blue	Silver	Other
Frequency	42	6	5	12	18	17

2 The owner of a car dealership wants a graph of the number of cars sold plotted against the value of the cars sold in the last year. The data is presented below.

Value of car	Number of cars sold
Under $100 000	25
$100 000–<$200 000	73
$200 000–<$300 000	132
$300 000–<$400 000	43
$400 000–<$500 000	18
$500 000 or more	9

Draw a histogram to represent this data.

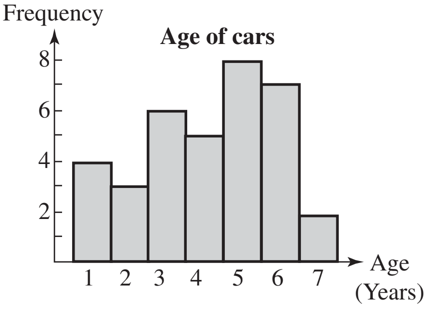
3 Peta carried out a survey to find out how long some customers had to wait in a queue at a bank. This histogram shows the results.



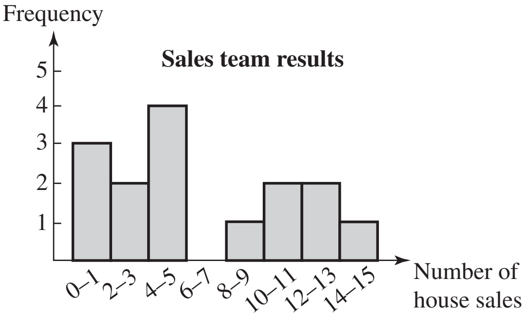
- **a** How many customers had to wait less than a minute?
- **b** How many customers had to wait between 3 and 6 minutes?
- **c** How many customers were surveyed altogether?
- **d** Explain why you cannot work out how many customers waited more than 2·5 minutes from this information.

4 Construct a frequency table for each of the following graphs.

a



b



Answers 6B

1 <<insert Figure 6.6>> <to be supplied>

2 <<insert figure 6.7>> <to be supplied>

3 **a** 4 **b** 5 **c** 22

d The waiting times are shown in intervals of whole minutes, so it is not possible to break down the data into intervals of 0·5 minutes.

4 **a**

Age (years)	Frequency
1	4
2	3
3	6
4	5
5	8
6	7
7	2

b

Number of house sales	Frequency
0–1	3
2–3	2
4–5	4
6–7	0
8–9	1
10–11	2
12–13	2
14–15	1

6C • The median, range and IQR

LB1 Pages 157–159

Specific learning outcomes

Learners should be able to:

8.6.5.1 Explain the terms: median, range and interquartile range (IQR).

8.6.5.2 Calculate the median, range and IQR of a set of scores.

Teaching points

1. Explain the following terms: median, range, and interquartile range (IQR).
2. Calculate the median, range, and IQR.

Learner difficulties and remedies

Difficulty

Identifying the mean, median, range and interquartile range (IQR).

Remedy

- Show the mean, median, range and IQR to learners, and demonstrate the differences between them using examples.

Difficulty

Finding all the quartiles of a set of numbers or scores and calculating the interquartile range (IQR).

Remedy

- Use scores or sets of numbers to identify the four quartiles.
- Know how to calculate the median, range and IQR using formulas.

Suggested teaching approach

- Review the definitions of median and range that were covered in Year 7.
- Show learners how to order a set of data and then find the four quartiles. Point out the difference between finding the middle score of even and odd data.
- Calculate the interquartile range IQR.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The **median** is one of the measures of centre. It is the middle value of an ordered data set. To calculate the median, we arrange the data set in ascending order, then count the number of data values.

- If the number of values is **odd**, select the middle value.
- If the number value is **even**, select the two middle values. Add them together and divide by two to get the middle number of the median.

The median is the middle number when the numbers in a set are placed in order.

The median of 47, 55, 43, 68, and 61 is 55.

If there is an even number of values, the median is halfway between the two middle values.

The median of 4, 8, 3 and 9 is 6, because when in order the numbers are 3, 4, 8, 9, and 6 is halfway between 4 and 8.

Measures of spread

Both the range and interquartile range are measures of the spread of the set of data. To make sense of the data we collect, in addition to the measure of the centre, we measure the spread of the data using range and interquartile range.

The **range** is the difference between the highest or largest score or number and the smallest lowest score or number.

Range = highest value – lowest value

Data sets can be divided into four equal parts, called **quartiles**.

The median is the middle of the set. It divides the set into a top half and a bottom half.

- If the number of values is **odd**, do not include the median data point in the top half and bottom half.
- If the number of values is **even**, the median is the midpoint between the two middle values, so all data points are included in the top half and bottom half.

The **lower quartile** (Q_1) is the middle value of the bottom half. The **upper quartile** (Q_3) is the middle value of the top half.

The **interquartile range** or **IQR** measures the spread between the two quartiles.

Interquartile range = $Q_3 - Q_1$

Examples

1 For the following data set, calculate the:

a range

b lower quartile

c lower quartile

d upper quartile

e interquartile range

1, 1, 4, 6, 8, 9, 10, 11, 14, 17, 18, 24

Thinking	Working
a Range = highest value – lowest value	**a** Range = 24 – 1 = 23
b The median is the middle value, which is the midpoint of 9 and 10.	**b** Median = 9·5
c The lower quartile is the median of 1, 1, 4, 6, 8 and 9.	**c** $Q_1 = 5$
d The upper quartile is the median of 10, 11, 14, 17, 18, and 24.	**d** $Q_3 = 15·5$
e Interquartile range = $Q_3 - Q_1$	IQR = 15·5 – 5 = 10·5

2 For each of the following data sets calculate the:

a median

b range

Set 1: 1, 1, 2, 3, 4, 4, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 30

Set 2: 1, 1, 2, 3, 4, 4, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 16

Thinking	Working
a The median is the middle value.	**a** Set 1: 1 1 2 3 4 4 4 5 **6** 7 8 9 10 12 14 16 30 Median = 6 Set 2: 1 1 2 3 4 4 5 **6** 7 8 9 10 12 14 16 16 Median = 6
b Range = highest value – lowest value	**a** Set 1: Range = 30 – 1 = 29 Set 2: Range = 16 – 1 = 15

Activity 6C

1 Calculate the median for each of these sets of numbers.

a 5, 15, 15, 20, 30, 50, 75

b 8, 8, 9, 4, 7, 6, 5, 4

2 Write down the range for each set of numbers.

a 7, 9, 13, 14, 19, 23

b 604, 593, 447, 608, 591, 453, 618, 539, 512

3 Work out the lower quartile, median, upper quartile and interquartile range for each set of numbers.
   a 2, 3, 5, 5, 6, 7, 8, 8, 8, 8, 9, 9, 9, 10, 11, 11, 12
   b 29, 31, 34, 42, 45, 49, 53, 56

4 These are the times (in seconds) taken by 14 athletes to run 400 m:
   59, 66, 61, 66, 60, 61, 71, 59, 58, 70, 64, 69, 63, 63
   a Calculate the range of these times.
   b Find lower quartile.
   c Find the upper quartile.
   d Write down the interquartile range.

Answers 6C

1 **a** 20 **b** 6·5
2 **a** 16 **b** 171
3 **a** 5·5, 8, 9·5, 4 **b** 32·5, 43·5, 51, 18·5
4 58, 59, 59, 60, 61, 61, 63, 63, 64, 66, 69, 66, 70, 71
   a 71 – 58 = 13
   b Lower quartile: 58, 59, 59, 60, 61, 61, 63
   $Q_1 = 60$
   c Upper quartile: 63, 64, 66, 69, 66, 70, 71
   $Q_3 = 69$
   d Interquartile range $= Q_3 - Q_1$
   $= 69 - 60$
   $= 39$

6D • Stem-and-leaf plots

LB1 Pages 160–162

Specific learning outcomes

Learners should be able to:
8.6.1.1 Define and identify the properties of a stem-and-leaf plot.
8.6.1.2 Order data in a stem-and-leaf plot.

Teaching points

1 Distinguish the stem and leaf parts for an example of a stem-and-leaf plot.
2 Organise data using a stem-and-leaf plot and then re-order the leaves to make an ordered stem-and-leaf plot.

Learner difficulties and remedies

Difficulty

Using a stem-and-leaf plot to represent a set of data.

Remedy

- Use the idea of tree and leaves to explain this new concept.
- If the data involve two-digit numbers, divide each given number into units and tens-value digits. The tens are the stems of a tree and the units are the leaves of three tree.

Difficulty

Finding the median of a set of scores using the stem-and-leaf plot.

Remedy

- Use a stem-and-leaf plot to represent the set of scores.
- Find the middle number when the scores are arranged in an ordered stem-and-leaf plot. Highlight the differences between odd and even data.

Suggested teaching approach

- This is a new concept at this level so work through this section thoroughly with learners.
- Use the concept of a tree to teach the concept of stem-and-leaf plot. Learners are all familiar with trees. Trees have stems and leaves therefore use it to explain the stem-and-leaf plot.
- Two-digit numbers are divided into units and tens digits. Unit digits go in the leaves and the tens digits go in the stem.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

When there is a large amount of numerical data it is often convenient to combine several pieces of data into a group. **Stem-and-leaf plots** are useful because they can group data without losing individual data values.

A stem-and-leaf plot consists of two parts, a **stem** and a **leaf**. Each piece of data must be divided into two parts so it can be plotted. For example, a score of 23 can be written with the tens numerals as the stem and the unit digit as the leaf.

stem	leaf
2	3

The smallest place value is used as the leaf with the remaining part of the number as the stem.

Further values that share the same stem can be added to the graph 23, 36, 24, 27, 23, 32.

stem	leaf
2	3 4 7 3
3	6 2

A stem-and-leaf plot can be used to find statistics by first ensuring the numbers in the leaf section are re-organised into ascending order to construct an ordered stem-and-leaf plot.

stem	leaf
2	3 3 4 7
3	2 6

A stem-and-leaf plot should always include a key. Note that a decimal point is not used in the plot.

stem	leaf
34	3 3 4 7
35	2 6

Key: 34 | 7 = 34·7

Another example of a stem-and-leaf plot is as shown.

stem	leaf
53	3 3 4 7
54	2 6

Key: 53 | 7 = 537

It is important that there are enough stems to spread the data. When data values are very close together, each division of the stem is divided into two parts, a lower and an upper section. The lower part contains the digits 0 to 4 in the leaf and the upper part contains the digits 5 to 9 in the leaf. As an example of this, consider the following data set.

21, 32, 41, 22, 25, 29, 33, 42, 33, 44, 21, 28, 37, 40, 39, 36, 24, 32, 44, 30, 34

If we used the tens value as the stem we would have only three stems (2, 3 and 4). To spread out the data we can use:

2_L for the lower values: 20, 21, 22, 23, 24

2_U for the upper values 25, 26, 27, 28, 29

stem	leaf
2L	1 1 2 4
2U	5 8 9
3L	0 2 2 3 3 4
3U	6 7 9
4L	0 1 2 4 4

Key: 3 | 2 = 32

Examples

A class obtained the following percentage results (rounded to the nearest whole number) in a science test.

69, 54, 91, 42, 73, 75, 81, 96, 88, 76, 65, 83, 74, 76, 82, 61, 57, 64, 58, 79, 83, 90, 73, 60, 51

Express the following data as an ordered stem-and-leaf plot with a class interval of 10.

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>
1 Decide upon the stem you will use for your stem-and-leaf plot. This will be the key for your stem-and-leaf plot.
2 List the stems down the page in ascending order. In this case, the tens digit is the stem (4, 5, 6, 7, 8, 9). Write the key you are using beside the stem-and-leaf plot.
3 Work through the data in the order given. List the leaf of each individual result in the appropriate row. The leaf is the units digit.
</td><td>
<table>
<tr><th>stem</th><th>leaf</th></tr>
<tr><td>4</td><td>2</td></tr>
<tr><td>5</td><td>4 7 8 1</td></tr>
<tr><td>6</td><td>9 5 1 4 0</td></tr>
<tr><td>7</td><td>3 5 6 4 6 9 3</td></tr>
<tr><td>8</td><td>1 8 3 2 3</td></tr>
<tr><td>9</td><td>1 6 0</td></tr>
</table>
Key: 5 | 4 = 54
</td></tr>
<tr><td>
4 Order the data in the leaf part of the stem-and-leaf plot, putting the lowest value closest to the stem.
</td><td>
<table>
<tr><th>stem</th><th>leaf</th></tr>
<tr><td>4</td><td>2</td></tr>
<tr><td>5</td><td>1 4 7 8</td></tr>
<tr><td>6</td><td>0 1 4 5 9</td></tr>
<tr><td>7</td><td>3 3 4 5 6 6 9</td></tr>
<tr><td>8</td><td>1 2 3 3 8</td></tr>
<tr><td>9</td><td>0 1 6</td></tr>
</table>
Key: 5 | 4 = 54
</td></tr>
</table>

Activity 6D

1 Construct a stem-and-leaf plot for these scores in a golf tournament:

77, 82, 71, 69, 80, 105, 91, 84, 88, 73, 71, 77, 81, 85, 92, 85, 76, 68, 92, 83

2 A class obtained the following percentage results (rounded to the nearest whole number) in a science test. Express each of the following data as an ordered stem-and-leaf plot with a class interval of 10.

a 57, 48, 42, 41, 31, 22, 37, 30, 23, 28, 34, 52

b 25, 44, 15, 29, 37, 26, 33, 35, 14, 8, 2, 27, 30, 42, 34, 21, 18, 3

3 This stem-and-leaf plot shows a set of pulse rates taken from a class of 32 learners after a 5-minute 'step' test. The figures were taken over a 1-minute period as soon as the test finished.

Pulse rate (beats/min)

7	8
8	2 5
9	6 7
10	0 4 6 6 9
11	1 2 2 8
12	2 5 6 7 7 9
13	0 0 2 5 5 5 6
14	3 4 7
15	
16	2 4

a What was the mode pulse rate?

b Work out the median pulse rate.

c Work out each of these:

- **i** the range
- **ii** the lower quartile
- **iii** the upper quartile
- **iv** the interquartile range

4 This 'back-to-back' stem-and-leaf plot shows the number of points scored for and against the school's basketball team in its 12 games last season.

For		Against
	7	3
	6	2 2 5
9 8 2 2 1	5	1 5 7 7
8 7 7 3 1 0	4	8 9
5	3	6
	2	9

a Calculate the median score for and the median score against.

b The diagram gives very little information about the number of games won and lost.

- **i** Explain why the team must have won at least one game.
- **ii** What is the greatest number of games the team could have won?

Answers 6D

1 a

stem	leaf
6	8 9
7	1 1 3 6 7 7
8	0 1 2 3 4 5 5 8
9	1 2 2
10	5

Key: 6 | 8 = 68

2 a

stem	leaf
2	2 3 8
3	0 1 4 7
4	1 2 8
5	2 7

Key: 2 | 3 = 23

b

stem	leaf
0	2 3 8
1	4 5 8
2	1 5 6 7 9
3	0 3 4 5 7
4	2 4

Key: 2 | 1 = 21

3 **a** 135 **b** 125·5

c **i** 86 **ii** 106

iii 135 **iv** 29

4 **a** For: 40·5, Against: 56

b **i** The lowest score against the team is 29. All scores for the team are greater than 29, so they must have won that game.

ii The team could have won at least 8 games (all except when the other team scored 73, 62, 62, 65 points).

6E • The mean

LB1 Pages 163–165

Specific learning outcomes

Learners should be able to:

8.6.7.1 Define the term 'mean'.

8.6.7.2 Explain the formula for calculating the mean of a set of scores.

8.6.8.1 Calculate the mean or average of a set of scores.

Teaching points

1 Explain that a 'mean' is the average of a set of values, and provide some examples in different contexts.

2 identify the formula and symbols that are used to calculate the mean.

3 Calculate the mean of sets of scores.

Learner difficulties and remedies

Difficulty

Understanding the difference between the mean and the median as a measure of centre.

Remedy

- Identify and distinguish the two terms.
- Show how they are calculated and what those two values represent.
- Provide examples of calculating the mean and median from data sets so that learners are clear and learners would confident in working with them.

Suggested teaching approach

- Use the formula to calculate the mean of a set of scores.
- The mean and the median are measures of centre. Explain to learners that the mean represents the 'average' score of a set of data. The median is the middle score. Extreme scores influence the mean, but not the median.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The mean is one of the ways to measuring the centre of set of scores. The mean is the arithmetic of a set of data.

$$\text{Mean} = \bar{x} = \frac{\text{sum of the scores}}{\text{number of scores}} = \frac{\sum x}{n}$$

The mean of 5, 8, 19 and 32 is:

$$\bar{x} = \frac{5 + 8 + 19 + 32}{4} = \frac{64}{4} = 16$$

The mean gives us information about the sum of the numbers.

The mean weight of the 8 forwards in the school rugby team is 71·5 kg. What is their total weight?

Total weight = number of values × mean = 8 × 71·5 = 572 kg

The mean can also be calculated from a frequency table.

To calculate the mean from a frequency table we add a third column. This is headed 'number × frequency' or $x \times f$.

Examples

1 Frangipani Ice-cream shop recorded the number of customers they have during the last 12 days in the month of June.

72, 39, 57, 82, 36, 42, 64, 54, 62, 48, 74, 42

Find the mean for the set of scores.

Thinking	Working
1 To find the mean, add all the data values and divide by the number of data values.	$\bar{x} = \frac{\sum x}{n}$ $= \frac{72 + 39 + 57 + 82 + 36 + 42 + 64 + 54 + 62 + 48 + 74 + 42}{12}$
2 Simply your answer.	$= \frac{672}{12}$ $= 56$

2 The table below shows the number of taxi hires from the Kings Taxi Service in Honiara. Find the mean for the data.

Number of hires x	Frequency f	$x \times f$
3	1	3
4	6	24
5	5	25
6	7	42
7	9	63
8	0	0
9	2	18
Total	30	175

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>1 To find the mean from a frequency table, we add all the scores in the third column with the heading ($x \times f$). Then find the total of the ($x \times f$).</td><td>
<table>
<tr><th>Number of hires x</th><th>Frequency f</th><th>$x \times f$</th></tr>
<tr><td>3</td><td>1</td><td>3</td></tr>
<tr><td>4</td><td>6</td><td>24</td></tr>
<tr><td>5</td><td>5</td><td>25</td></tr>
<tr><td>6</td><td>7</td><td>42</td></tr>
<tr><td>7</td><td>9</td><td>63</td></tr>
<tr><td>8</td><td>0</td><td>0</td></tr>
<tr><td>9</td><td>2</td><td>18</td></tr>
<tr><td>Total</td><td>30</td><td>175</td></tr>
</table>
</td></tr>
<tr><td>2 Divide it by ($x \times f$) by the total of the column (x).</td><td>$\bar{x} = \frac{\sum xf}{\sum f}$
$= \frac{175}{30}$
$= 5{\cdot}83$ (2 d.p.)</td></tr>
</table>

Activity 6E

1 Calculate the mean for each of these sets of numbers:
   - **a** 5, 15, 15, 20, 30, 50, 75
   - **b** 8, 8, 9, 4, 7, 6, 5, 4

2 Jordan recorded the points he won in a board game he played with several of his friends. The points he earned in the last 15 games are listed below.

23, 37, 42, 36, 45, 52, 25, 65, 47, 37, 54, 36, 50, 48, 33

Find the mean of this data set.

3 Anna recorded the number of points the Makira netball team scored during the Telekom netball championship that held in Bellona, Rennell and Bellona Province in the month of November, 2013.

The points her team earned in 18 games are listed below.

43, 45, 56, 26, 36, 44, 64, 54, 67, 34, 42, 63, 51, 29, 41, 30, 55, 39

Find the mean of score of this data set.

4 Four consecutive numbers add up to 34. (Note: consecutive numbers follow each other in sequence, for example 15, 16, 17, 18, 19).
   - **a** What is the mean of the numbers?
   - **b** What are the numbers?

Answers 6E

1	**a** 30	**b** 6·375
2	42	
3	45·5	
4	**a** 8·5	**b** 7, 8, 9, 10

6F • Mean absolute difference

LB1 Page 166

Specific learning outcomes

Learners should be able to:

8.6.9.1	Explain the term 'mean absolute difference'.
8.6.9.2	Explain the symbol that is used for absolute values: $\lvert -a \rvert = a$.
8.6.10.1	Calculate the mean and the mean absolute difference for given set of scores.
8.6.11.1	Explain the term 'absolute value'.

Teaching points

1 Explain what the mean absolute difference is, and its symbol.

2 Find the mean absolute difference of given data.

Learner difficulties and remedies

Difficulty

Calculating the mean absolute difference of a set of data or scores. This is a new concept at this level.

Remedy

- Use the approach that was done for the calculation of the mean to explain this concept. This is similar to the way in which the mean of a set of scores was calculated using the formula, but different in a sense that all differences (whether positive or negative) are all taken to be positive.
- Know the symbol or sign that is used for 'mean absolute difference'.

Suggested teaching approach

- This is a new concept at this level so introduce it slowly and thoroughly.
- The difference between the 'mean' and the 'mean absolute difference' is that the 'differences' are taken as positive regardless as to whether they are positive or negative.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

One way of describing the spread of a set of numbers is to use the 'mean absolute difference'.

- First calculate the mean. We use the symbol $\bar{x}$ for the mean.
- Then, for each data item, calculate the absolute difference between it and the mean. This means we subtract, and ignore the sign.
- Finally, calculate the mean of the absolute differences.

The symbol for the absolute difference between a data item x and the mean $\bar{x}$ is $|x - \bar{x}|$. For example, $|7 - 2{\cdot}4| = 4{\cdot}6$ and $|1 - 2{\cdot}4| = 1{\cdot}4$.

The absolute difference between a data value and the mean is always positive, regardless of whether the data item is above or below the mean.

Examples

1 Calculate the mean absolute deviation of these scores of some of the weight of fish that are sold in the market.
8, 9,4, 7·8, 7,8, 8·6

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>1 Calculate the mean.</td><td>$\bar{x} = \frac{8 + 9{\cdot}4 + 7{\cdot}8 + 7{\cdot}8 + 8{\cdot}6}{5}$
$= \frac{41{\cdot}6}{5}$
$= 8{\cdot}32$</td></tr>
<tr><td>2 Use a table to find the differences and absolute differences.</td><td>
<table>
<tr><th></th><th>Difference</th><th>Absolute difference</th></tr>
<tr><td>8 – 8·32</td><td>−0·32</td><td>0·32</td></tr>
<tr><td>9·4 – 8·32</td><td>1·08</td><td>1·08</td></tr>
<tr><td>7·8 – 8·32</td><td>−0·52</td><td>0·52</td></tr>
<tr><td>7·8 – 8·32</td><td>−0·52</td><td>0·52</td></tr>
<tr><td>8·6 – 8·32</td><td>0·28</td><td>0·28</td></tr>
<tr><td></td><td>Total</td><td>2·72</td></tr>
</table>
</td></tr>
<tr><td>3 Find the mean absolute difference.</td><td>Mean absolute difference
$= \frac{2{\cdot}72}{5}$
$= 0{\cdot}544$</td></tr>
</table>

Activity 6F

1 Calculate the mean absolute difference of this set of numbers. Show your working.

3, 8, 5, 8, 6, 0, 7, 3

2 This table gives the mean daily minimum and maximum temperatures for Ballarat for each month of the year.

Month	Mean temperature	
	Daily minimum	Daily maximum
Jan	10·8	24·9
Feb	11·4	25·0
Mar	9·9	22·1
Apr	7·4	17·5
May	5·8	13·5
Jun	4·0	10·7
Jul	3·2	10·0
Aug	3·7	11·4
Sep	4·9	13·8
Oct	6·2	16·5
Nov	7·7	19·4
Dec	9·4	22·5

a Calculate the mean absolute difference for the mean daily temperatures. Working can be done on a separate sheet of paper.

- **i** the minimum
- **ii** the maximum

b Describe one conclusion you can make based on your answers to part **a**.

Answers 6F

1 $\bar{x} = \frac{3 + 8 + 5 + 8 + 6 + 0 + 7 + 3}{8} = \frac{40}{8} = 5$

	Difference	Absolute difference
3 – 5	−2	2
8 – 5	3	3
5 – 5	0	0
8 – 5	3	3
6 – 5	1	1
0 – 5	−5	5
7 – 5	2	2
3 – 5	−2	2
	Total	18

Mean absolute difference $= \frac{18}{8} = 2{\cdot}25$

2 **a** **i** $\bar{x} = 7{\cdot}0\dot{3}$

Mean absolute difference = 2·4

ii $\bar{x} = 17{\cdot}275$

Mean absolute difference = 4·625

b The daily maximum temperatures are more spread. There is about a 4-degrees difference on average. The daily minimum temperatures are more consistent. There is only about a 2-degrees difference on average.

6G • Comparing the mean and the median

LB1 Pages 167

Specific learning outcomes

Learners should be able to:

8.6.11.2 Identify which of mean or median provides the best measure for the centre of a set of scores.

8.6.11.3 Find mean and median of a set of given scores.

Teaching points

1 Both mean and median are measures of centres. The mean takes into account all the data, so is influenced by extreme values. The median is the middle score, so is not affected by extreme values.

2 Find the mean and median of given scores in different contexts and choose which is the most appropriate.

Learner difficulties and remedies

Difficulty

Deciding which measure of the centre of a data sample is best for a particular context.

Remedy

- Understanding the differences in calculating the mean and median.
- Identify and distinguish these two terms and how they are used in a set of numbers or scores.
- Show how they are calculated and what those two values represent.
- Provide examples of clacialting and interpreting the mean and median so that learners are clear about the differences and similarities, and are confident in working with them.

Suggested teaching approach

- Explain how the mean is calculated from sets of data, and from grouped frequency tables using the formula. Contrast this with how the median is calculated.
- Show how these two measures of centre are used in different contexts.
- Explain when it is best to use the mean and when to use the median.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- An **Activity** is provided at the end of this unit if learners require additional material.

Additional notes

We use three **measures of centre**: the mean, median and mode.

The **mean** is the arithmetic average of a data set:

The **median** is the middle value of an ordered data set. If there are two middle values, the median is the average of these two values.

The **mode** is the value that occurs most often in a data set. There may be no mode. If there are two modes, the data set is bimodal. If there are more than two modes, the data set is multimodal, but the mode is not of any use in these cases. A data set of this type is often considered to have no mode.

Activity 6G

1 **a** For each of the following data sets calculate the mean and the median.

Set 1: 8, 9, 9, 10, 10, 10, 10, 10, 14, 15, 16, 18, 19, 20, 22

Set 2: 8, 9, 9, 10, 10, 10, 10, 10, 14, 15, 16, 18, 19, 20, 22

b What effect did the change in data values from Set 1 to Set 2 have on each of the sample statistics you have calculated?

2 Stella recorded the number of points her netball team scored this season. The points her team earned in 18 games are listed below:

44, 44, 56, 26, 36, 44, 64, 54, 67, 34, 42, 63, 51, 29, 41, 30, 55, 39

a Calculate the mean.

b Find the median.

c Find the mode.

3 In the 2014 Honiara Rugby season, the Avaiki Rugby team, in all games including the finals, scored the following points:

12, 24, 10, 12, 23, 24, 12, 8, 22, 34, 17, 38, 20, 20, 19, 18, 32, 44, 18, 22, 44, 20, 18, 38, 12

a Calculate the mean and median number of points scored for the season.

b Which of these statistics better represents their scores for the season?

Answers 6G

1 **a** Set 1: mean = 13·33, median = 10
Set 2: mean = 12, median = 10

b The mean decreased from 13·33 to 12. The median was unchanged.

2 **a** 45·5

b 44

c 44

3 mean = 22·44, median = 20
The mean takes into account of the scores, even the extreme scores such as 44. The median only gives an idea of the middle of the scores.

6H • Practical data analysis

LB1 Pages 168–169

Specific learning outcomes

Learners should be able to:

8.6.13.1 Collect data through various means and activities.

8.6.13.2 Analyse collected data with regards to the mean, range and IQR.

Teaching points

1. Learners collect data.
2. The data is analysed according to statistics that best match the question. This might require calculations of the mean, range and IQR, etc.

Learner's difficulties and remedies

Difficulty

Applying the concepts that have been taught. In some situations learners may not fully understood the concepts taught regardless of the fact they could able to solve some of the problems or questions at hand.

Remedy

- Teaching must be holistic to ensure that learners understand what they learnt, rather than memorising the steps and approaches that are used to solve those problems.

Suggested teaching approaches

- Have the learners plan and complete their own statistical investigation following the instructions in **Exercise 6H** on pages 168 and 169 in the LB to consolidate what has been taught throughout this chapter.

• Chapter Seven •

Transformations

Overview

The term ‘to transform’ means to change the appearance or position of an object. There are several types of transformations that can be described mathematically. This chapter covers three transformations that change an object’s position but not size, and one transformation that changes an object’s size.

The transformation called a translation describes how a shape or object may move in a straight line. A reflection describes how a shape or object might be turned over in the way an image appears in a mirror. A rotation describes how a shape or object might turn about a central point. The shape and orientation of an object that is translated, reflected or rotated is unchanged.

An enlargement increases the lengths of all sides in a shape in the same proportion from a fixed point called the centre of enlargement. This is rather like blowing up a balloon where the centre point is at the lips of the person doing the blowing up.

The opposite of enlarge is reduce, so a transformation of a shape or object that gets proportionally smaller can be called a reduction. Other transformations exist such as a stretch, a shear and a glide reflection, but these are not included in the curriculum at this level.

Transformations feature in nature and in most cultural objects. The designs of patterns that appear on material frequently incorporate translations, reflections, rotations and enlargements, and manufacturers use tessellations to repeat designs over a length of cloth. Carvers and architects also apply transformations to create designs in their plans.

Contents

Chapter skills

This chapter covers the following skills:

- Identifying transformations
  - Translation
  - Reflection
  - Rotation
  - Enlargement
- Finding congruent shapes
- Exploring tessellations
- Reducing and enlarging shapes using a grid
- Flipping, sliding and turning in 3D
- Using mathematical language for transformations
- Producing patterns using transformations
- Looking for line and rotational symmetry

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 7A Transformations	Learner’s Book 1 • Exercise 7A, page 180
2–3	• 7B Translation	Learner’s Book 1 • Exercise 7B, pages 181–183
4–5	• 7C Reflection	Learner’s Book 1 • Exercise 7C, pages 184, 185
6–7	• 7D Rotation	Learner’s Book 1 • Exercise 7D, pages 186, 187
8–9	• 7E Enlargements and reductions	Learner’s Book 1 • Exercise 7E, pages 189, 190
10	• 7F Exploring similar areas and volumes	Learner’s Book 1 • Exercise 7F, page 191
11–12	• 7G Congruent shapes	Learner’s Book 1 • Exercise 7G, pages 192, 193
13–14	• 7H Exploring tessellations	Learner’s Book 1 • Exercise 7H, pages 194, 195
15	• Test	Teacher’s Guide • Chapter 7 Test

General learning outcomes

Learners should:

Transformations

8.7.1 Understand that transformations change an object's shape, position or size. (U)

8.7.2 Understand that there are four methods that can be used to transform an object or shape. (U)

Translation

8.7.3 Understand translation in terms of its movements. (U)

8.7.4 Describes the movement of an object that is being translated without changing the size and shape of the object. (U).

Reflection

8.7.5 Understand reflection in terms of the image, mirror line (line of reflection) and line of symmetry. (U)

8.7.6 Know how to reflect objects using the mirror line. (K)

8.7.7 Know how to identify the line of symmetry in shapes. (K)

Rotation

8.7.8 Understand that objects can change their positions through rotation. (U)

Enlargements and reductions

8.7.9 Understand that shapes or objects can either be enlarged or reduced with a given scale factor. (U)

8.7.10 Know how to enlarge and reduce shapes and objects using grids. (K)

8.7.11 Know how to enlarge and reduce shapes and objects using scale factor. (K)

Exploring similar areas and volumes

8.7.12 Understand that areas or volumes of shapes can be increased or decreased with given scale factors. (U)

8.7.13 Know that the enlarged and reduced areas and volumes of shapes create a relationship between old and new amounts. (K)

Congruent shapes

8.7.14 Understand that if two shapes have the same lengths of sides and same size angles then they are congruent. (U)

Exploring tessellations

8.7.15 Understand the term tessellation. (U)

8.7.16 Use the transformations translation, reflection, rotation and enlargement to create tessellations. (K)

7A • Transformations

LB1 Page 180

Specific learning outcomes

Learners should be able to:

8.7.1.1 Define the term 'transformation'.

8.7.1.2 Identify four types of transformations of an object: translation, reflection, rotation and enlargement.

Teaching points

1. Explain what the term transformation means.
2. Explain the terms object and image, and the letter notation used to indicate transformations of the points on objects.
3. Distinguish between four particular transformations: translation, reflection, rotation and enlargement.

Learner difficulties and remedies

Difficulty

Understanding the concept of transformation in relation to the four main methods: translation, reflection, rotation and enlargement.

Remedy

- Identify the four transformations.
- Define the four transformations, and describe their similarities and differences using examples.
- Show how to use the four methods to transform objects.

Suggested teaching approach

- This is a new concept to learners at this level so it's important that teachers work thoroughly through the examples.
- Define the four methods of transformation with the learners and distinguish between them using examples to illustrate the similarities and differences. In particular point out which transformations produce congruent shapes (translation, reflection, rotation), which produces shapes with a different size (enlargement), and which produces shapes with the same size and orientation (translation).
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **transformation** changes a shape (or object) or a set of points according to a particular rule. A transformation may move, turn, flip or change the shape of a figure. Three transformations covered in this chapter move a shape but retain its original shape and size. These are called translation, reflection and rotation. One transformation changes the size of the shape or object, and is called an enlargement.

The original shape is called the **object** and the transformed shape is called the **image** of the transformation. To identify the type of transformation it is often necessary to check the labels of each angle on the figure and its image.

The angles on the object are usually labelled in capital letters A, B, C …

The matching angles on the image will be labelled A', B', C' …

If a shape is transformed further, the angles would be labelled A', B', C' …

There are three types of transformations in which the shape does not change:

- translation
- reflection
- rotation

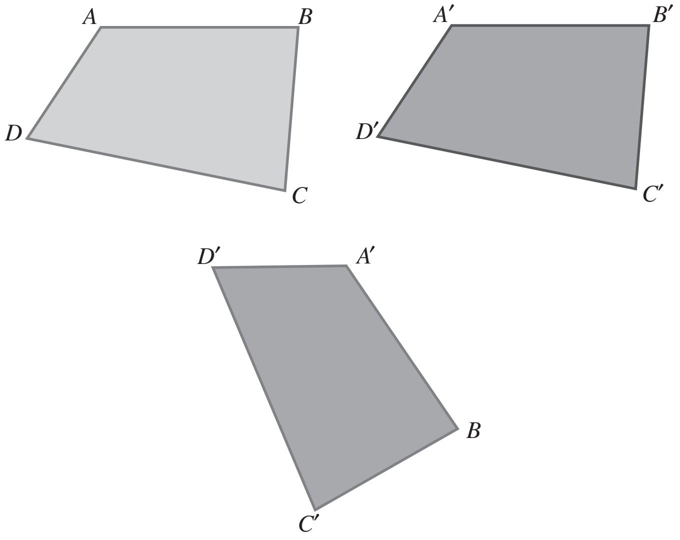
Each type of transformation moves a figure in a particular way. Transformations can be combined to make a complex move and can be reversed to move the image back to its original position and orientation.

Examples

The shape *ABCD* given below has been transformed.

The first image has been translated to the right. It is in a different position but its shape and orientation are unchanged.

The second image has been rotated. The orientation is different but its shape is unchanged.



Activity 7A

Take the class to a church to find examples of transformations in woven mat designs, pictures and carvings. Look particularly for examples of frieze patterns.

For example, this frieze pattern uses translation and reflections to repeat the design along its length around the top and bottom of this Solomon Island craft item.

<<insert Figure 7.2>>

7B • Translation

LB1 Pages 181–183

Specific learning outcomes

Learners should be able to:

8.7.3.1 Define the term 'translation'.

8.7.4.1 Describe the translations of objects using 'up, down, left and right'.

8.7.4.2 Use mathematical language to describe a translation of an object.

Teaching points

1 Explain that a translation is a transformation that moves a shape of object in a straight line.

2 Describe the translation of objects as a combination of up, down, left and right movements.

3 Describe the translation of objects by describing the movement of object.

Learner difficulties and remedies

Difficulty

Moving the objects up, down, and sideways right or left.

Remedy

- Remind learners of the properties of a translation.
- Use practical examples to show learners how to translate the objects.

Suggested teaching approach

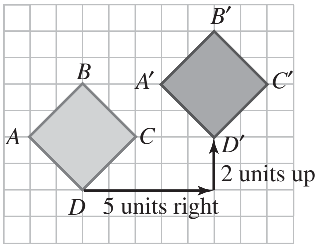
- Define a translation using the terms up, down, left and right.
- Explain all the properties of translation showing learners that the size and the orientation of an object is unchanged.
- Explain that when an object moves through translation, every point in the object moves the same distance in the same direction.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

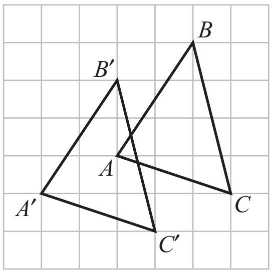
In a translation, a shape is moved or slides across a surface. When a figure undergoes translation, the image is moved without turning or changing shape. It is simply moved a number of units left or right, or a number of units up or down, or a combination of both.

Side lengths and angle stay the same size. Each point in the object moves the same distance and in the same direction.

The example given below shows a figure that has been translated 5 units to the right and 2 units up. This translation can be written as **[5, 2]**.



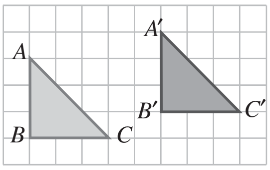
The diagram shows a triangle that has been translated 2 units to the left and 1 unit down. The image of *A* is *A*′.

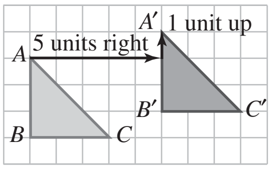


- Vertices of an image are denoted by a dash. *A*′ or *B*′. We say '*A* dash' or '*A* prime'.
- To distinguish between the original figure and the translated image, use a different colour or broken lines.
- Translations can be reversed by doing the opposite horizontal and vertical moves. For example, the reverse of a translation of 4 units left and 3 units down is 4 units right and 3 units up.

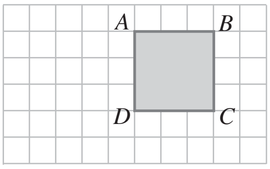
Examples

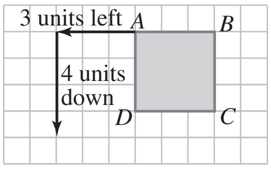
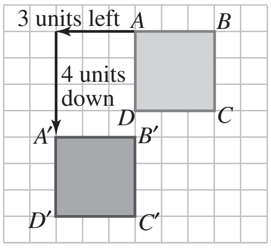
1 Describe the translation shown in the diagram.



Thinking	**Working**
1 Start by selecting one of the vertices and count the units in the horizontal and vertical directions to get to the corresponding translated vertex.	
2 Write the translation in words or as an ordered pair.	5 units right and 1 unit up or [5, 1].

2 Copy the following onto grid paper and draw the resulting image after the translation: 3 units left and 4 units down.



Thinking	**Working**
1 Start by selecting one of the vertices (*A*) and move according to the given translation. **2** Label this vertex with a letter using image notation (*A′*).	
3 Repeat with the remaining vertices and then draw the resulting image in a different colour. **4** Label all vertices of the image using image notation.	

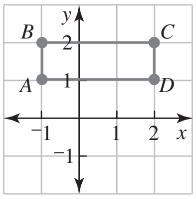
If our shape is drawn on the **Cartesian plane**, we can identify the coordinates of the vertices and then find the coordinates of the transformed shape after a given transformation.

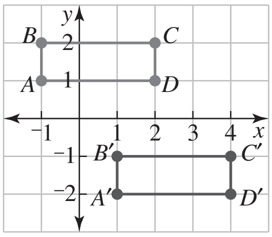
3 **a** Write the coordinates of each vertex on the shape below.

b Copy the shape onto grid paper and draw the resulting image after the translation of 2 units to the right and 3 units down.

c Write the coordinates of the translated vertices.

d Explain how the coordinates of the image could be found without drawing the shape.

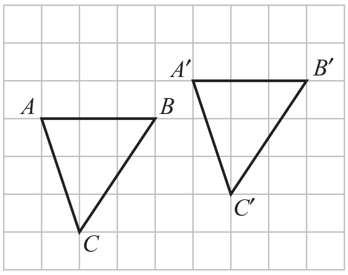


Thinking	**Working**
a Use the *x*- and *y*-axes to identify the coordinates of each of the vertices.	**a** $A = (-1, 1)$ $B = (-1, 2)$ $C = (2, 2)$ $D = (2, 1)$
b Translate each vertex by the given translation, labelling each vertex with image notation. (Here, we move each point 2 units right and 3 units down. This is a translation of [2, −3].)	**b** 
c Identify the coordinates of each vertex on the resulting image.	**c** $A' = (1, -2)$ $B' = (1, -1)$ $C' = (4, -1)$ $D' = (4, -2)$
d Look for the connection between the original coordinates, the transformation and the image coordinates.	**d** Adding 2 to the *x*-coordinate of the original and subtracting 3 from the *y*-coordinate of the original will give the image coordinates.

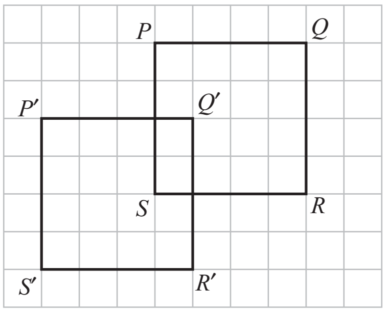
Activity 7B

1 Describe the translation of each shape. Describe how far and which way the shape has been translated. Use language like (up, down, right, left).

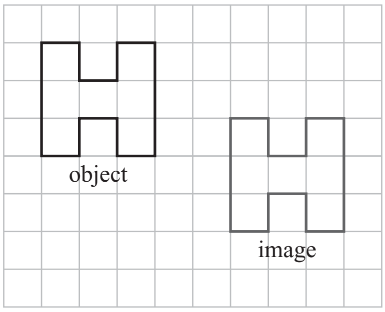
a



b

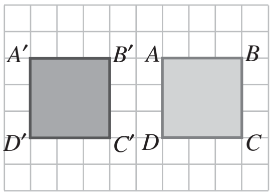


c

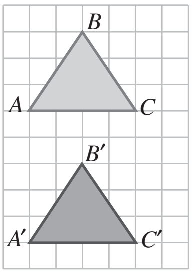


2 Describe the translation shown in each of the following diagrams.

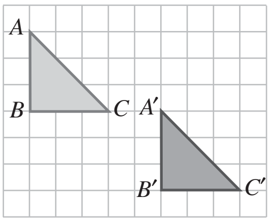
a



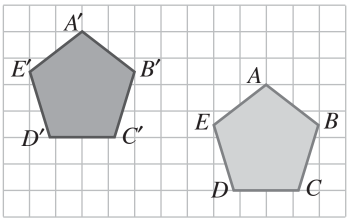
b



c

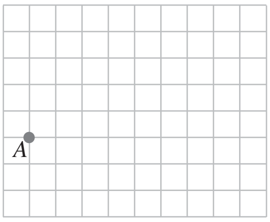


d

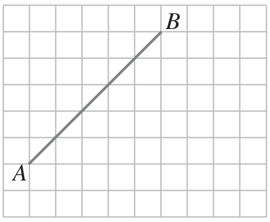


3 Copy each of the following onto grid paper and draw the resulting image after the translation.

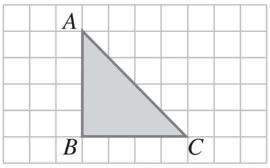
a 6 units right and 2 units up



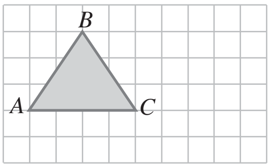
b 6 units left and 6 units down



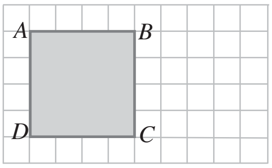
c 6 units left and 1 unit down



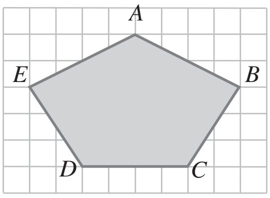
d 1 unit right and 6 units down



e [7, −5]



f [−6, 4]

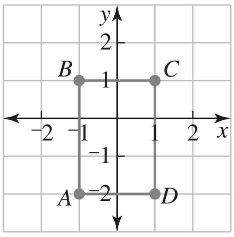


4 **a** Write the coordinates of each vertex on the given shape.

b Copy the shape onto grid paper and draw the resulting image after translation of 1 unit to the left and 2 units up.

c Write the coordinates of the translated vertices.

d Explain how the coordinates of the image could be found without drawing the shape.



Answers 7B

1 **a** 4 units right and 1 unit up

b 3 units left and 2 units down

c 5 units right and 2 units down

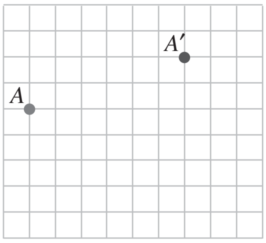
2 **a** 5 units left

b 5 units down

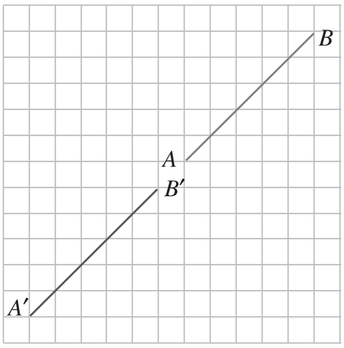
c 5 units right and 3 units down

d 7 units left and 2 units up

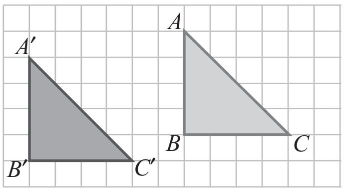
3 **a**



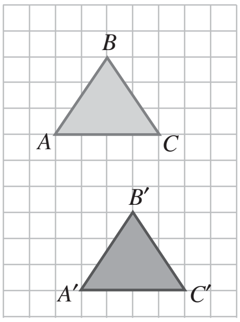
b



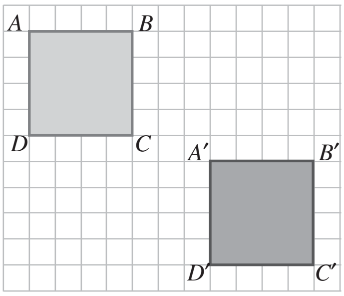
c



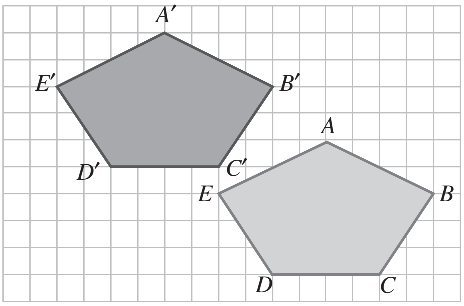
d



e

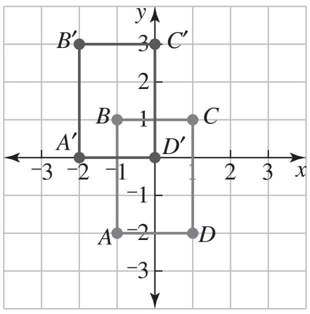


f



4 **a** $A(^-1, ^-2)$, $B(^-1,1)$, $C(1, 1)$, $D(1, ^-2)$

b



c $A'(^-2, 0)$, $B'(^-2, 3)$, $C'(0, 3)$, $D'(0,0)$

d Subtracting 1 from the x-coordinates of the original and adding 2 to the y-coordinates of the original.

7C • Reflection

LB1 Pages 184–185

Specific learning outcomes

Learners should be able to:

8.7.5.1 Define the term ‘reflection’.

8.7.5.2 Identify the properties of reflections.

8.7.6.1 Reflect objects and shapes in a mirror line and identify their properties.

8.7.7.1 Identify any lines of symmetry in objects and shapes.

Teaching points

1. Explain what a reflection in a mirror line does to an object, and identify its properties.
2. Explore reflections of given objects and shapes using mirror lines for different orientations.
3. Show the lines of symmetry in given objects and shapes.

Learner difficulties and remedies

Difficulty

Locating or writing the image of the reflected object.

Remedy

- Explain the properties of reflection to enable learners to reflect object using mirror line.
- Identify the mirror line in shapes undergoing a reflection.
- Point out that when an object is reflected, the shape does not change. The distance from the mirror line to each point in the object is the same as the distance from the mirror line to each corresponding point in the image.

Suggested teaching approach

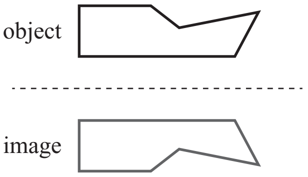
- Explain and define a reflection.
- Identify and explain the properties of reflection.
- Demonstrate how to reflect an image showing that the distance of each point in the object from the mirror line is the same distance of the image of each point from the mirror line.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

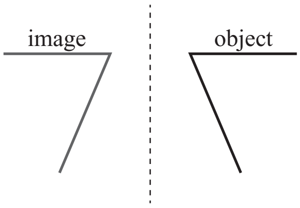
A **reflection** is a transformation that creates an image of the original figure in the same way that a mirror creates a reflection of an object. A reflection is therefore often called a mirror image. It is a reversed or ‘flipped’ version of the original object about a **line of reflection**. A reflected image has the same shape and size as the original but its position has changed and its orientation is reversed.

The term ‘orientation’ means whether the order of the angle labels change from clockwise to anticlockwise as a result of the transformation.

Each point on a reflected image is the same distance from the line of reflection as the original, but on the opposite side of the mirror. The distance is the **perpendicular distance**, which is measured by a line that makes a right angle with the line of reflection.



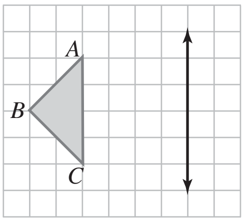
To reflect a figure, reflect each vertex on the figure and then join the points in order.

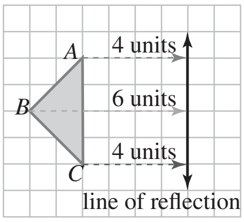
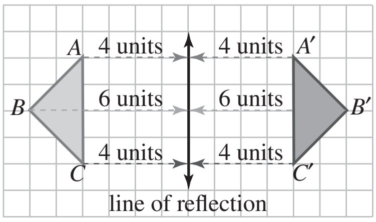


- A reflection always flips the original, so the order in which the vertices are labelled in the image will be opposite to that in the original figure.
- A reflection is as far from one side of the line of reflection as the original figure is from the other side of it.
- When reflected, the properties of the shape will remain the same. The size of the side lengths and angles remain unchanged.

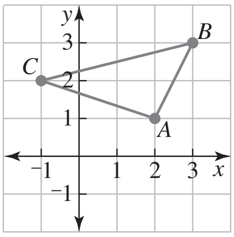
Examples

1 Copy the following figure onto grid paper and draw the resulting image when the triangle is reflected in the line of reflection shown.



Thinking	**Working**
1 Measure the perpendicular distance between each vertex and the line of reflection.	
2 Use the distance between the vertices on the line of reflection to plot the points on the opposite side of the line of reflection and reproduce the original figure. **3** Label the image vertices with image notation.	

2 **a** **i** Write down the coordinates of each vertex on the given shape.

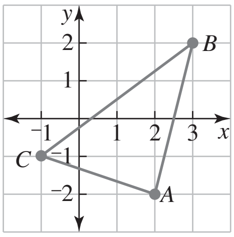


ii Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

iii Write the coordinates of the reflected vertices.

iv Explain how the coordinates of the image could be found without drawing the shape.

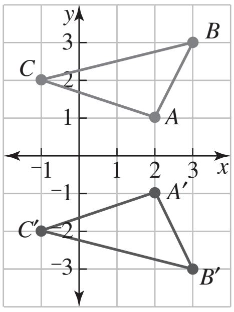
b **i** Write down the coordinates of each vertex on the given shape.

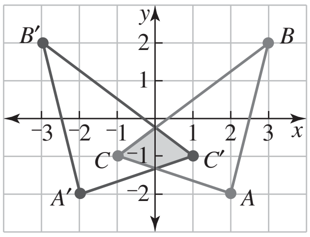


ii Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

iii Write the coordinates of the reflected vertices.

iv Explain how the coordinates of the image could be found without drawing the shape.

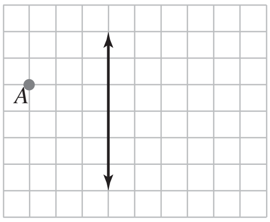
Thinking	**Working**
a **1** Use the x- and x-axes to identify the Cartesian coordinates of the points.	**a** $A = (2, 1)$ $B = (3, 3)$ $C = (-1, 2)$
2 Reflect each point in the x-axis and join them in order to form the reflected shape, labelling each vertex with image notation.	
3 Identify the coordinates of each vertex on the resulting image.	$A' = (2, -1)$ $B' = (3, -3)$ $C' = (-1, -2)$
4 Look for the connection between the coordinates of the original and the image.	The x-coordinate of the image stays the same as that of the original. The y-coordinate of the image is the negative of that of the original.

Thinking	Working
b **1** Use the x- and y-axes to identify the Cartesian coordinates of the points.	**b** $A = (2, -2)$ $B = (3, 2)$ $C = (-1, -1)$
2 Reflect each point in the y-axis and join them in order to form the reflected shape, labelling each vertex with image notation.	
3 Identify the coordinates of each vertex on the resulting image.	$A' = (-2, -2)$ $B' = (-3, 2)$ $C' = (1, -1)$
4 Look for the connection between the coordinates of the original and the image.	The x-coordinate of the image is the negative of that of the original. The y-coordinate of the image stays the same as that of the original.

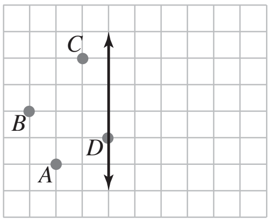
Activity 7C

1 Copy the following figures onto grid paper and draw the resulting image when each is reflected in the line of reflection show.

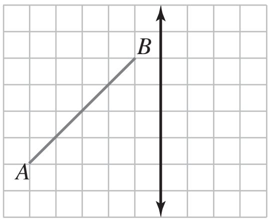
a



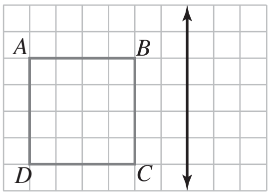
b



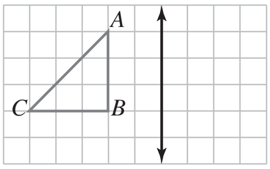
c



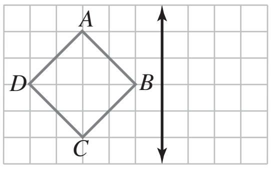
d



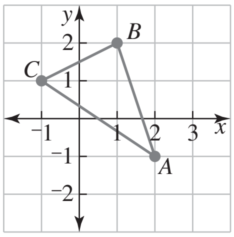
e



f



2 **a** **i** Write down the coordinates of each vertex on the given shape.

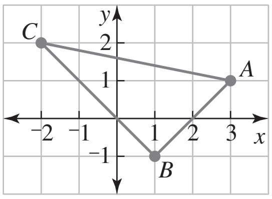


ii Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

iii Write the coordinates of the reflected vertices.

iv Explain how the coordinates of the image could be found without drawing the shape.

b **i** Write down the coordinates of each vertex on the given shape.



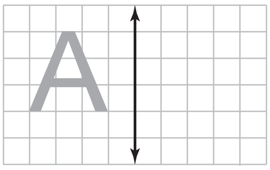
ii Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

iii Write the coordinates of the reflected vertices.

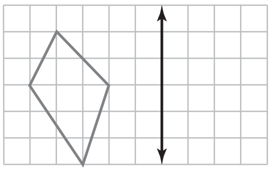
iv Explain how the coordinates of the image could be found without drawing the shape.

3 Copy each of the following figures onto grid paper, then draw the reflection of each in the line of reflection showsn.

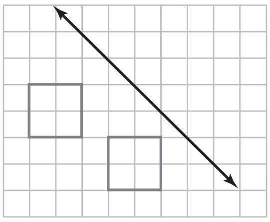
a



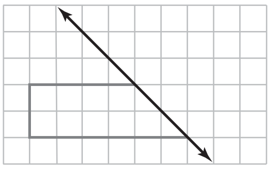
b



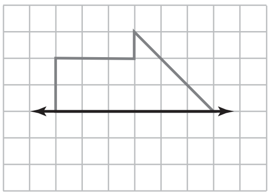
c



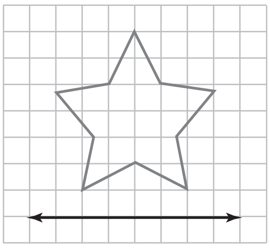
d



e



f



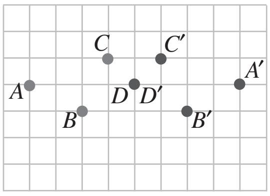
4 **a** Plot the points $(-1, 5)$, $(3, 1)$ and $(2, -1)$ on a Cartesian plane and label them A, B and C.

b Join the points to form triangle ABC, then reflect the shape first in the x-axis and then in the y-axis.

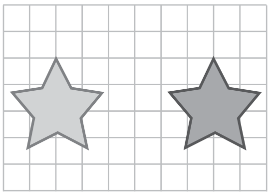
c Write the coordinates of the points after the second reflection.

5 Copy each of the following onto grid paper and indicate where the line of reflection should be placed to produce the given image.

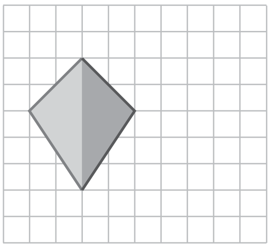
a



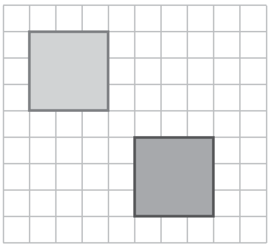
b



c

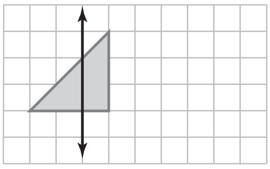


d

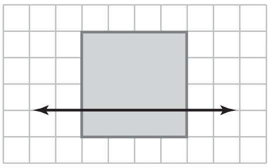


6 Copy each of the following onto grid paper and draw the reflected images. The image may be on both sides of the line of reflection.

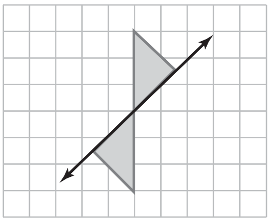
a



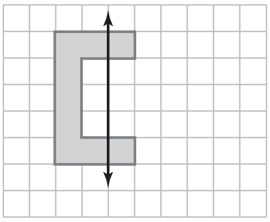
b



c

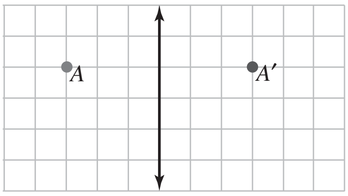


d

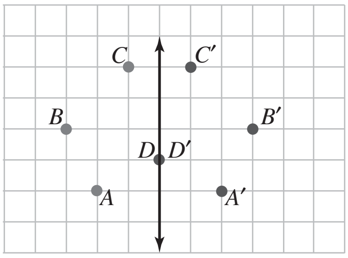


Answers 7C

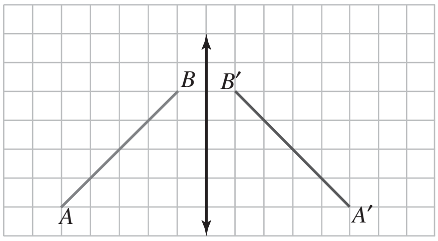
1 a



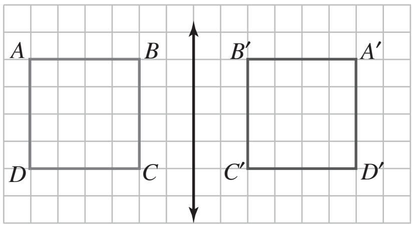
b



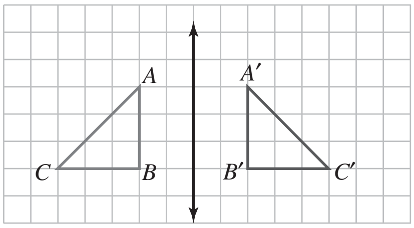
c



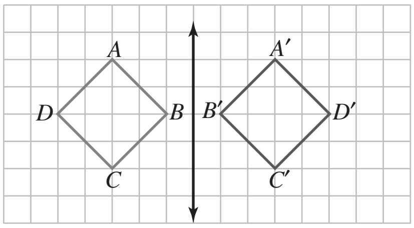
d



e

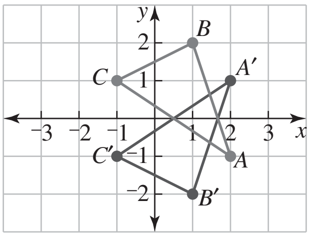


f



2 a i $A(2, -1)$, $B(1, 2)$, $C(-1, 1)$

ii

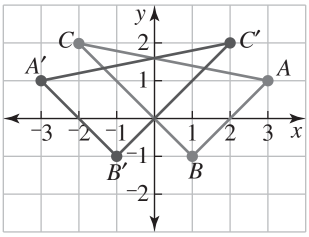


iii $A'(2, 1)$, $B'(1, -2)$, $C'(-1, 1)$

iv The x-coordinates of the image are the same as the original but the y-coordinates of the image are the negative of the original.

b i $A(3, 1)$, $B(1, -1)$, $C(-2, 2)$

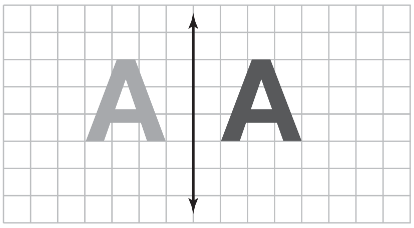
ii



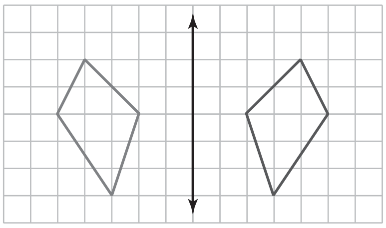
iii $A'(-3, 1)$, $B'(-1, 1)$, $C'(2, 2)$

iv The x-coordinates of the image are the negative of the original but the y-coordinates of the image are the same as the original.

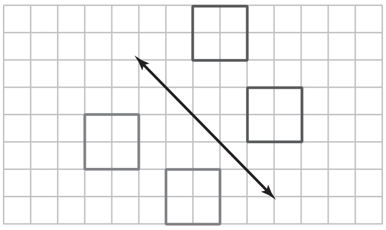
3 a



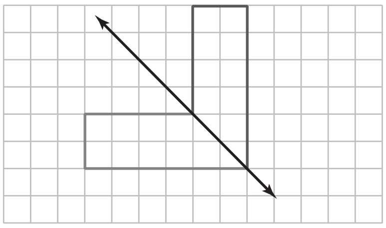
b



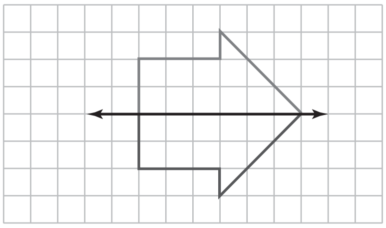
c



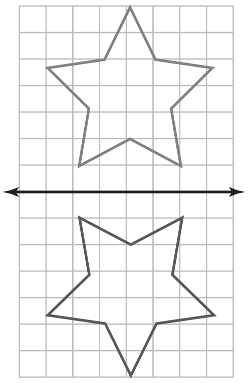
d



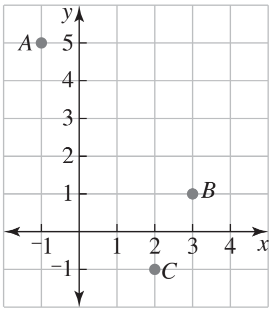
e



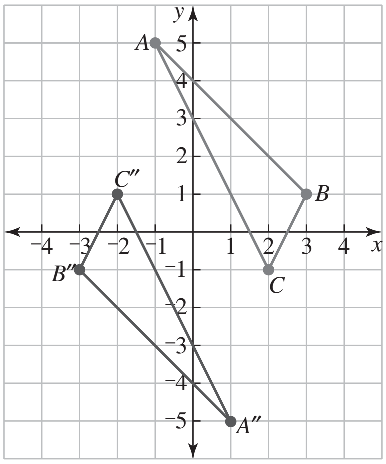
f



4 a



b

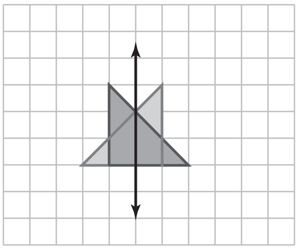


c $A'(1, {}^-5)$, $B'({}^-3, {}^-1)$, $C'({}^-2, 1)$.

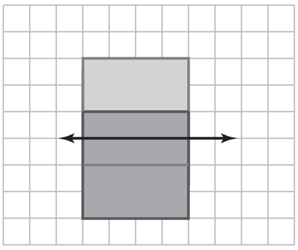
5 a <insert Figures 7.26e-h> b

c d

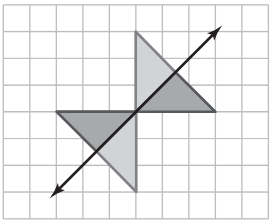
6 a



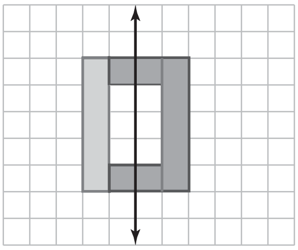
b



b



b



7D • Rotation

LB1 Pages 186–187

Specific learning outcomes

Learners should be able to:

8.7.8.1 Define the term ‘rotation’.

8.7.8.2 Identify properties of rotations: centre (point) of rotation; directions: clockwise or anticlockwise; and size of angle in a turn.

8.7.8.3 Rotate an object given its centre of rotation, and the direction and angle of its turn.

Teaching points

1. Explain what rotation is.
2. Identify the properties of rotation.
3. Rotate objects and shapes with given centre of rotations, direction of rotation and the number of degrees for the rotation.

Learner difficulties and remedies

Difficulty

Rotating shapes with given centre of rotation, direction and size of the turn.

Remedy

- Explain the properties of rotation.
- Demonstrate how to rotate objects and shapes with given centre of rotations, direction and size of the rotation.
- Identify the centre of rotation.
- Identify the direction of the rotation and by how many degrees.

Suggested teaching approach

- Explain to learners all the properties of rotation.
- Demonstrate how to find the centre of rotation and the direction of the rotation because this is vital for the proper rotation of given objects.
- Show learners how to use protractors to rotate objects.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A transformation of a figure that is **rotated** it around a fixed point is called a **rotation**. The fixed point is called the **centre of rotation** and is labelled with the letter *O*.

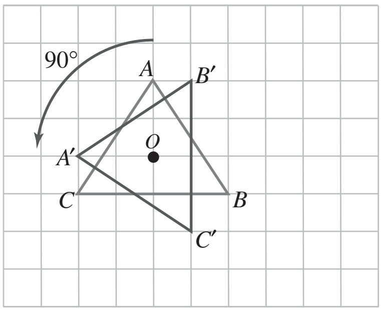
When rotating any figure you need to know:

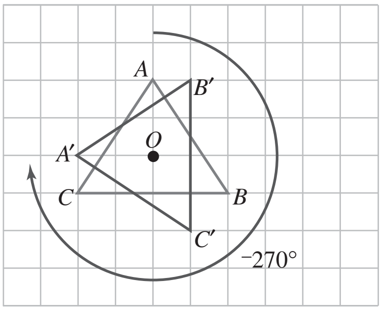
- the location of the centre of rotation
- the size of the angle of rotation
- the direction of the rotation (clockwise ↷ or anticlockwise ↶).

The centre of rotation can be located inside or outside the figure and is the only point that does not rotate. The size of the angle of rotation is generally in multiples of 30° or 45°. Common angles of rotation are 30°, 45°, 60°, 90°, 180° and 270°. A rotation of 360° will rotate the image of the figure through a full revolution about the centre of rotation and will return the figure to its original position.

- The vertices of a rotated image are labelled in the same order as the original figures.
- Rotations of 360°, 720° and other multiples of 360° result in the image in the same position as the original figure.
- Two rotations that result in an image in the same position will always add to 360°. For example, 90° clockwise is equivalent to 270° anticlockwise because 90° + 270° = 360°.
- A rotation in an anticlockwise direction is a positive rotation.
- A rotation in a clockwise direction is a negative rotation.
- The same steps are used when the centre of rotation is outside the shape.
- We can find the coordinates of an image after a rotation of 90°, 180° or 270° in a clockwise or anticlockwise direction about the origin using a Cartesian plane.

The following two shapes given below have shown how they have been rotated 90^0 anticlockwise and 270° clockwise.

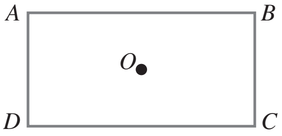


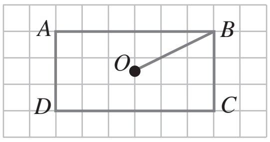
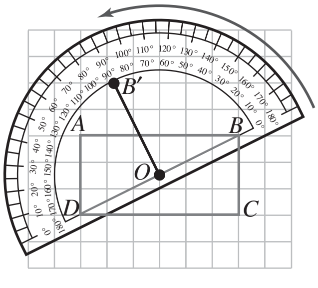
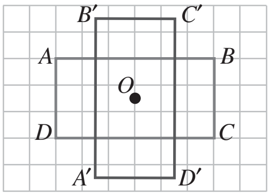


A clockwise rotation of 180^0 is the same as an anticlockwise rotation of 180°.

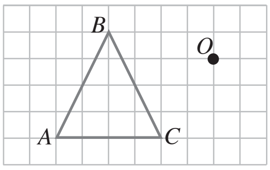
Examples

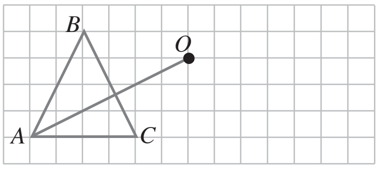
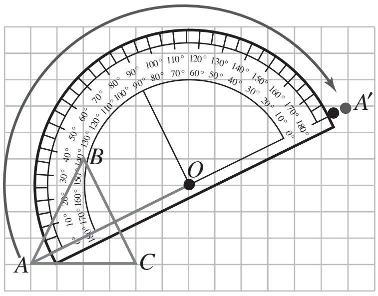
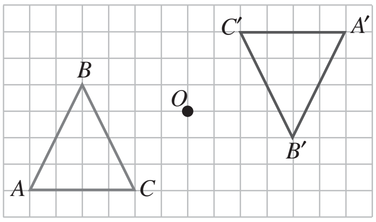
1 Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 90° in an anticlockwise direction (90°) about *O*.

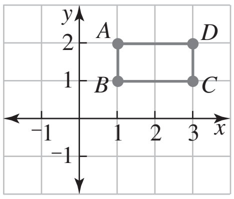


Thinking	**Working**
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.	
2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex. Plot the resulting point. The new point must be the same distance from *O* as the original point.	
3 Continue the process with the other vertices and connect the resulting points to produce the original figure in its rotated position.	

2 Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 180° in a clockwise direction (–180°) about *O.*



Thinking	**Working**
1 Select a key point, such as a vertex, and join this point to the centre of rotation using a straight line.	
2 Place the centre of a protractor on the centre of rotation, with the chosen vertex on the base line. Measure the given angle of rotation in the given direction from this vertex and plot a point. Extend *AO* through this point and mark the point *A′* so that *OA′* is the same length as *OA*.	
3 Continue the process with the other vertices and connect the resulting points to reproduce the original figure in its rotated position.	



3 **a** Write the coordinates of each vertex on the given shape and copy the shape onto grid paper

b **i** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

ii Write the coordinates of the rotated vertices.

iii Explain how the coordinates of the image could be found without drawing the shape.

c **i** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

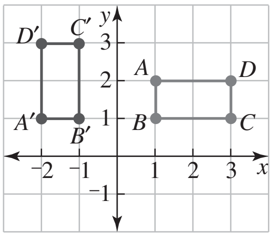
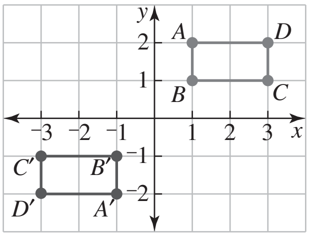
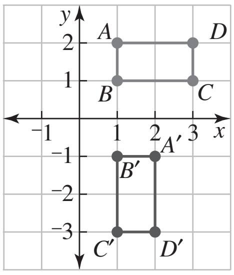
ii Write the coordinates of the rotated vertices.

iii Explain how the coordinates of the image could be found without drawing the shape.

d **i** Draw the resulting image after the original is rotated 270° in an anticlockwise direction.

ii Write the coordinates of the rotated vertices.

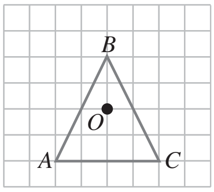
iii Explain how the coordinates of the image could be found without drawing the shape.

Thinking	**Working**
a Use the *x*- and *y*-axes to identify the coordinates of each of the vertices.	**a** $A = (1, 2)$ $B = (1, 1)$ $C = (3, 1)$ $D = (3, 2)$
b **1** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 90° in an anticlockwise direction about the origin.)	**b** 
2 Identify the coordinates of each vertex on the resulting image.	$A' = (^-2, 1)$ $B' = (^-1, 1)$ $C' = (^-1, 3)$ $D' = (^-2, 3)$
3 Look for the connection between the original coordinates, the transformation and the image coordinates.	The *x*-coordinates of the image vertices are the negative of the *y*-coordinates of the original vertices and the *y*-coordinates of the image vertices are the *x*-coordinates of the original vertices.
c **1** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point by 180° in an anticlockwise direction about the origin.)	**c** 
2 Identify the coordinates of each vertex on the resulting image.	$A' = (^-1, ^-2)$ $B' = (^-1, ^-1)$ $C' = (^-3, ^-1)$ $D' = (^-3, ^-2)$
3 Look for the connection between the original coordinates, the transformation and the image coordinates.	All the coordinates of the image vertices are the negative of the original coordinates of the vertices.
d **1** Rotate each vertex around the origin by the given rotation, labelling each vertex with image notation. (Here, we rotate each point in an anticlockwise direction by 270° or $^-90°$.)	**d** 
2 Identify the coordinates of each vertex on the resulting image.	$A' = (2, ^-1)$ $B' = (1, ^-1)$ $C' = (1, ^-3)$ $D' = (2, ^-3)$
3 Look for the connection between the original coordinates, the transformation and the image coordinates.	The *x*-coordinates of the image vertices are the *y*-coordinates of the original vertices and the *y*-coordinates of the image vertices are the negative of the *x*-coordinates of the original vertices.

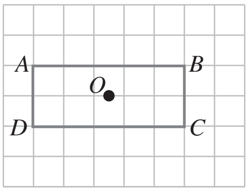
Activity 7D

1 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

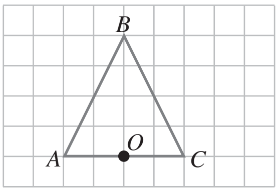
a Rotate 180° in an anticlockwise direction (180°) about *O*.



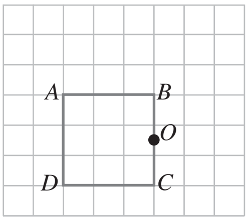
b Rotate 270° in an anticlockwise direction (270°) about *O*.



c Rotate 180° in a clockwise direction (−180°) about *O.*

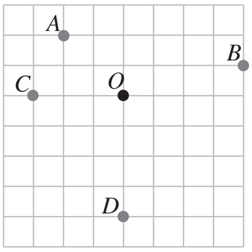


d Rotate 90° in a clockwise direction (−90°) about *O*.

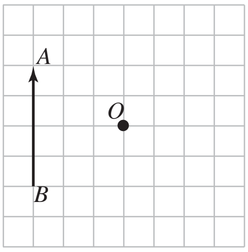


2 Copy each of the following onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

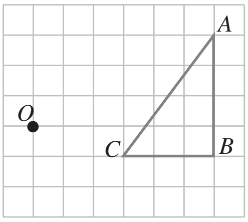
a Rotate each of the points 90° in a clockwise direction (−90°) about *O*.



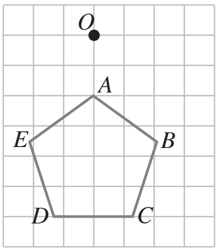
b Rotate 270° in an anticlockwise direction (270°) about *O*.



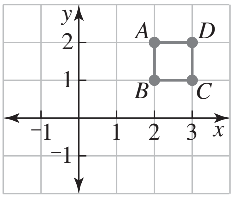
c Rotate 180° in a clockwise direction (−180°) about *O*.



d Rotate 90° in an anticlockwise direction (90°) about *O*.



3 **a** Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



b **i** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

ii Write the coordinates of the rotated vertices.

iii Explain how the coordinates of the image could be found without drawing the shape.

c **i** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

ii Write the coordinates of the rotated vertices.

iii Explain how the coordinates of the image could be found without drawing the shape.

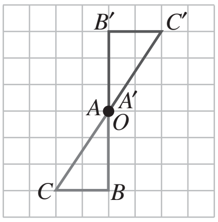
d **i** Draw the resulting image after the original is rotated 270° in an anticlockwise direction about the origin.

ii Write the coordinates of the rotated vertices.

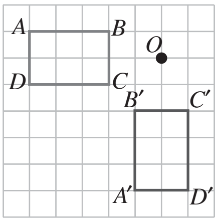
iii Explain how the coordinates of the image could be found without drawing the shape.

4 Find the rotation (the size of the rotation and the direction) that has taken place to produce the following images. Identify more than one rotation that will achieve the same result.

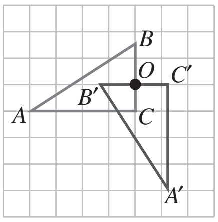
a



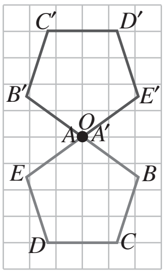
b



c

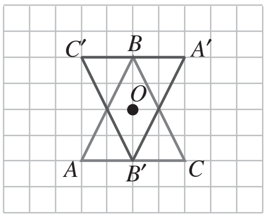


b

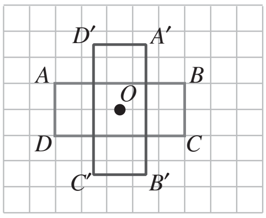


Answers 7D

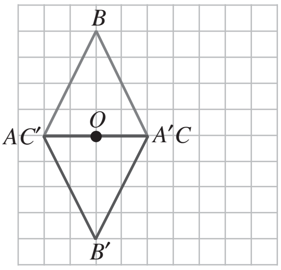
1 a



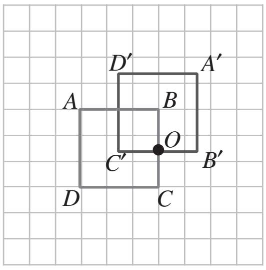
b



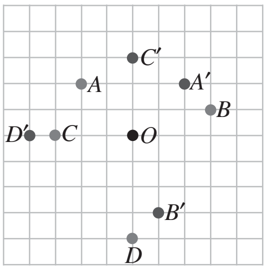
c



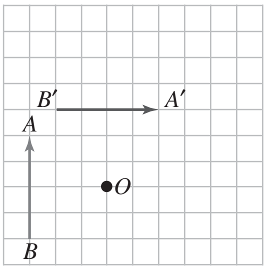
d



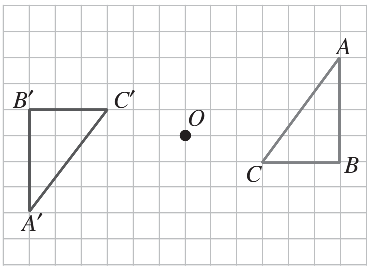
2 a



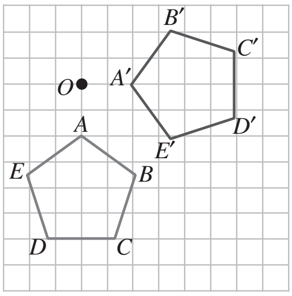
b



c

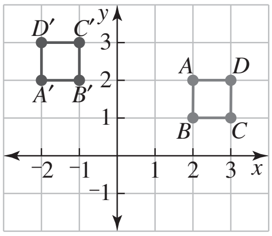


d



3 a $A(2, 2)$, $B(2, 1)$, $C(3, 1)$, $D(3, 2)$

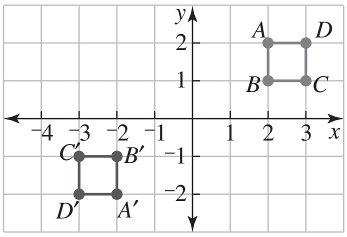
b i



ii $A'(-2, 2)$, $B'(-1, 2)$, $C'(-1, 3)$, $D'(-2, 3)$

iii The *x*-coordinates of the image are the negative of the *y*-coordinates of the original vertices and the *y*-coordinates of the image are the *x*-coordinates of the original vertices.

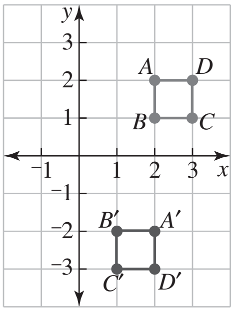
c i



ii $A'(-2, -2)$, $B'(-2, 1)$, $C'(-3, -1)$, $D'(-3, -2)$

iii All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

d i



ii $A'(2, -2)$, $B'(1, -2)$, $C'(1, -3)$, $D'(2, -3)$

4 a 180° in an clockwise direction about *O*.
180° in an anticlockwise direction about *O*.

b 90° in an anticlockwise direction about *O*.
270° in an clockwise direction about *O*.

c 90° in an anticlockwise direction about *O*.
270° in an clockwise direction about *O*.

d 180° in an clockwise direction about *O*.
180° in an anticlockwise direction about *O*.

7E • Enlargements and reductions

LB1 Pages 188–190

Specific learning outcomes

Learners should be able to:

8.7.9.1 Define enlargement and reduction.
Enlargement: Increase the size of an object.
Reduction: Decrease the size of an object.

8.7.10.1 Enlarge shapes or objects on a variety of grids.

8.7.11.1 Find the scale factors for the enlargement or reduction of objects.

8.7.11.2 Enlarge shapes or objects by given scale factors.

Teaching points

1. Explain the definitions of enlargement and reduction.
2. Demonstrate to learners how to enlarge and reduce shapes using a grid.
3. Find the scale factor used for the enlargement and reduction.
4. Enlarge and reduce shapes using given scale factors.

Learner difficulties and remedies

Difficulty

Using a scale factor to enlarge shapes or objects.

Remedy

- Explain to learners what scale factor is, and how it is used.
- Remind learners that scale factors do not have units, but they must consider the units of the measurements that they are enlarging.

Difficulty

Identifying the centre of enlargement.

Remedy

- Demonstrate to learners how to find the centre of enlargement.

Suggested teaching approach

- Define an enlargement.
- Identify all the properties of enlargement using examples to illustrate the centre of enlargement, and the scale factor.
- Show learners how to enlarge objects and shapes using grid.
- Do examples on the board to show learners how to enlarge shapes and objects using the scale factor with the given centre of enlargement.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

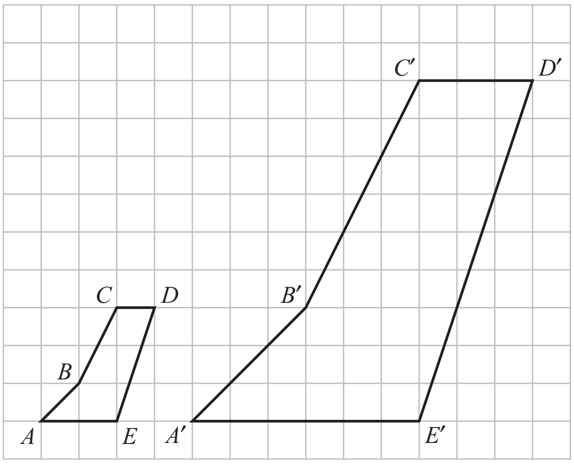
Additional notes

An **enlargement** or **reduction** changes the size of an object, making it either larger or smaller. The shape of the object stays the same.

We use labels like *A, B, C* for the original object and *A′, B′, C′* for the transformed image.

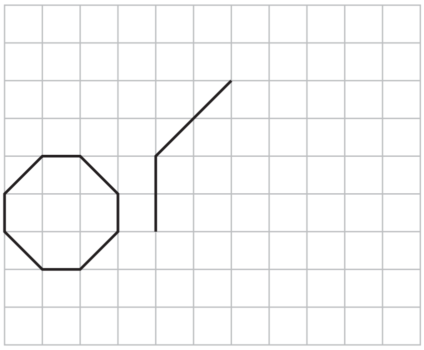
Examples

1 The diagram shows an enlargement in which the sides of the image are three times as long as the sides of the object. We say the scale factor is 3 in this example.

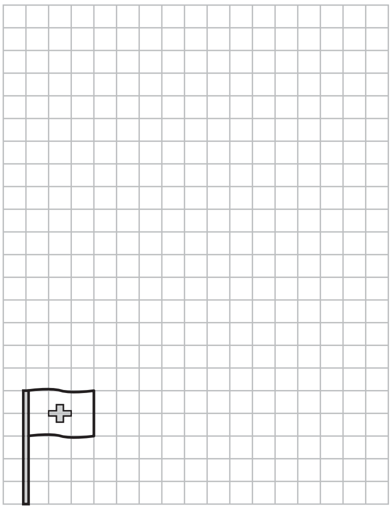


Activity 7E

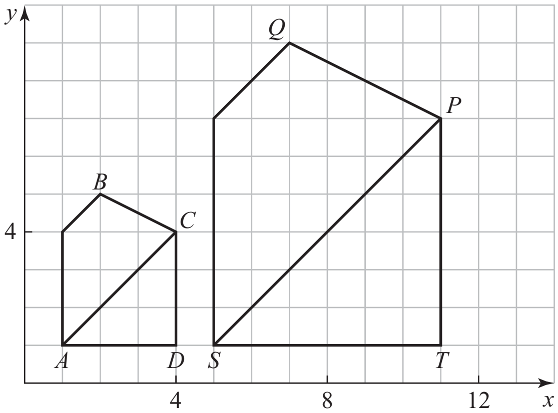
1 Complete the drawing on the right of the octagon to show the image of the octagon when it is enlarged by a scale factor of 2.



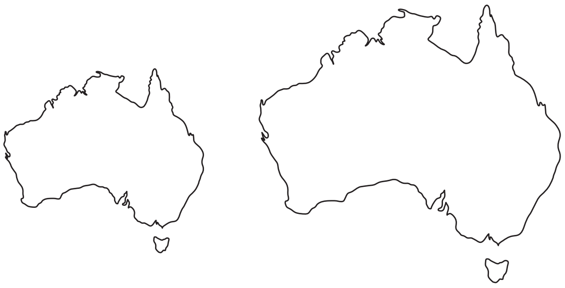
2 Next to this flag, draw the image when it is enlarged by a scale factor of 3.



3 Work out the scale factor in this enlargement.



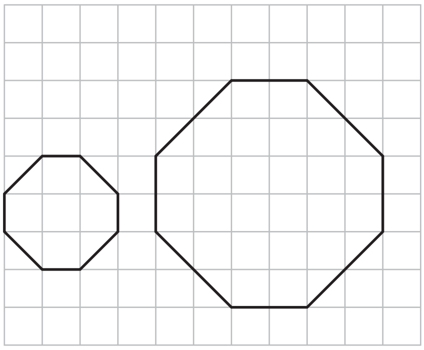
4 Make some measurements with a ruler and the use the division key on a calculator to estimate the scale factors for these enlargements.



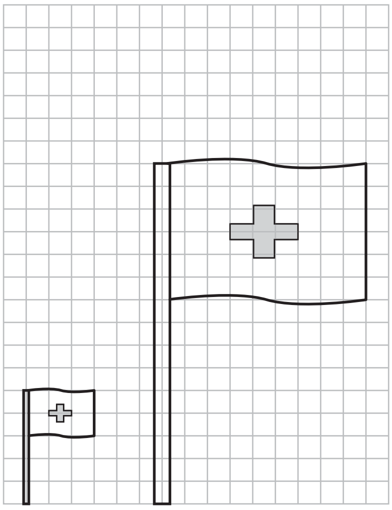
5 A photocopier has a setting that says '× 1·414'. It is used to enlarge a diagram of a rectangle measuring 80 mm by 60 mm. What would be the dimensions of the rectangle in the enlargment?

Answers 7E

1



2



3 Scale figure is 2.

4 Scale figure is 1·5.

5 113 mm by 85 mm

7F • Exploring similar areas and volumes

LB1 Page 191

Specific learning outcomes

Learners should be able to:

8.7.12.1 Find areas and volumes of shapes.

8.7.12.2 Use scale factors to calculate areas and volumes enlarged by given scale factors.

8.7.13.1 Identify the relationship between the scale factors for length and volume.

Teaching points

1. Calculate the areas of shapes and the volumes of solid prisms.
2. Enlarge the shapes and prisms according to a given scale factor, and then compare the areas and volumes of the original and enlarged prisms.
3. Find the relationships between the side lengths, areas and volumes of original and enlarged shapes.

Suggested teaching approach

- Learners complete **Learning Task 7F** on page 191 of the LB.

Additional notes

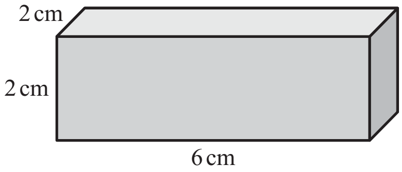
The scale factor for enlargements and reductions in Exercise 7E were all in relation to changes in lengths

The deductions to be made from Learning Task 7F are that:

The scale factor of the enlargement of an area of a shape is the square of the enlargement for length.

And, the scale factor of the enlargement of a 3D object is the cube of the enlargement for length.

Examples



The scale factor of the enlargement of each edge of the prism is 2.

The area of the front surface of the prism enlarges from 12 cm^2 to 48 cm^2.

Scale factor is 2^2 or 4.

The volume of the prism increases from 24 cm^3 to 192 cm^3.

Scale factor of 2^3 or 8.

Activity 7F

Challenge the learners to find one pair of everyday objects that can be found in different sizes. Examples include water bottles, toothpaste cartons, food tins or jars.

1. Measuring the heights of the small and large items.
2. Calculate the scale factor for length by dividing the smaller height into the larger one.
3. Now compare the measurements for the area of one side of the object or, if circular, the area of the labels on the two objects.
   - Is the scale factor for area the square of the scale factor for the height?
   - If so, the objects are related, so the larger object is a true enlargement of the smaller object.
4. Now fill the smaller object with water or sand. Pour the water or sand into the larger object.
   - How many times can you do this until the large object is full? For a true enlargement, it should be the cube of the scale factor of the heights.

7G • Congruent shapes

LB1 Pages 192–193

Specific learning outcomes

Learners should be able to:

8.7.14.1 Define the term 'congruent'.

8.7.14.2 Compare the properties of shapes and objects to identify whether they are congruent.

8.7.14.3 Identify the effects of combinations of transformations.

Teaching points

1 Use examples to show the meaning of the term congruent: angles and side lengths remain the same.

2 Explain that while similar shapes become larger or smaller by a given scale factor, congruent shapes do not change size.

3 The transformations that have congruent object and image pairs are translations, reflections and rotations.

4 Note in a reflection and rotation the orientation of an object and image is changed, but they are still congruent shapes.

Learner difficulties and remedies

Difficulty

Identifying shapes that are congruent.

Remedy

- Explain the properties of congruent shapes.
- Check the shapes are congruent by measuring the side lengths of the shapes and equivalent angles.

Suggested teaching approach

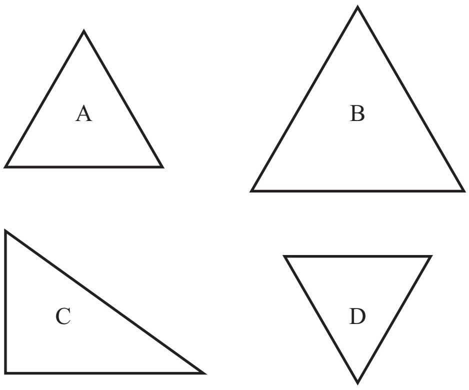
- Identify the properties of congruent shapes.
- Explain to learners what would make shapes similar or congruent; two figures or objects are congruent if they have the same shape and size.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Two figures are congruent if they are the same shape and size. All matching sides are the same length and all matching angles are the same size. Congruent shapes may be in different orientations if they have undergone a reflection or a rotation. For these transformations the shape and size is unchanged, so the figures are congruent.

Examples

Which of the following shapes are congruent?

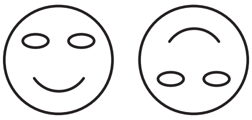


The only pair of congruent triangles is A and D.

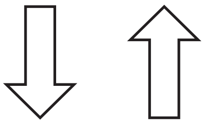
Activity 7G

1 Find four pairs of congruent shapes below.

a

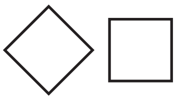


b

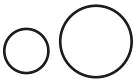


c

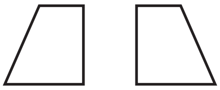
d



e



f



2 Describe whether each of the four shapes on the left in each congruent pair in Question **1** has been translated, reflected or rotated to give the image shape on its right.

Answers 7G

1 a, b, d, f

2 **a** rotated **b** rotated

d rotated **f** reflected

7H • Exploring tessellations

LB1 Pages 194–195

Specific learning outcomes

Learners should be able to:

8.7.15.1 Define the term ‘tessellation’.

8.7.16.2 Use the transformations: rotation, translation and reflection to create tessellation.

Teaching points

1 Explain that a tessellation of a flat surface is the tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps

2 Provide pictures to show simple examples of tessellations such as hexagons in a honeycomb and rectangles on a brick wall.

3 Demonstrate how any quadrilateral will tessellate by tracing around a template on the board

4 Use the transformations translation, rotation and reflection to create patterns using tessellations

Suggested teaching approach

- Learners complete **Learning Task 7H** on pages 194 and 195 of the LB.

Activity 7H

Challenge the learners to identify further examples of tessellations to be found within the classroom, on cultural items such as weaving and carving, and in nature. Share their findings with the class.

• Chapter Eight •

Equations and Inequations

Overview

Equations and inequations are topics in mathematics that use algebraic language to describe and generalise patterns that occur in the real world, and to solve problems. Chapter 4 introduced algebraic expressions. In this chapter, learners explore situations where two expressions may be compared. When the two expressions are true for the same pronumeral values the two expressions are considered equal, and this is shown with the use of the equals symbol (=). Sometimes one expression may be greater than (>) or smaller than (<) the other for pronumeral values. Occasionally one expression may be greater than or equal to (≥) or less than or equal to (≤) another expression.

We can use equations and inequations to develop formulas to describe where and when birds migrate, or how a bridge supports the weight of cars on it, or how oceans move so that we can predict tides. Modelling relationships using algebra allows us to identify patterns and make predictions from these patterns.

This chapter will enable learners to extend their skills in using and manipulating algebraic expressions, and to solve equations and inequations.

Contents

Chapter skills

This chapter covers the following skills:

- Solving equations by inspection
- Solving equations by flow charts
- Solving equations by inverse operations
- Using equations to model real-world problems
- Solving inequations

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 8A Solving equations by inspection	Learner's Book 1 • Exercise 8A, pages 4, 5
2	• 8B Solve equations with flow charts	Learner's Book 1 • Exercise 8B, pages 6, 7
3	• 8D Solving equations using inverse operations	Learner's Book 1 • Exercise 8C, pages 8, 9
4	• 8D Solving two-three steps equations	Learner's Book 1 • Exercise 8D, pages 10, 11
5–6	• 8E Inequations	Learner's Book 1 • Exercise 8E, pages 12–15
7	• Test	Teacher's Guide • Chapter 8 Test

General learning outcomes

Leaners should:

Equations

8.8.1 Understand that equations are algebraic statements using pronumerals to represent missing numbers with an equals sign that can be evaluated. (U)

Solving equations by inspection

8.8.2 Understand that an equation with one unknown can be easily solve to make the statement true. (U)

Solving equation with flow charts

8.8.3 Know how to find the unknown using flow chart. (K)

Solving equation using inverse operations

8.8.4 Know how to perform 'inverse operations' of an equation to find the value of an unknown. (K)

Solving two- and three-step equations

8.8.5 Know how to solve for pronumerals using the two- and three-step method. (K)

Inequations

8.8.6 Understand that inequation refers to the relationships between numbers, groups of numbers, variable or groups of variables when they are compared. (U)

8.8.7 Know how to use the inequality symbols: <, >, ≥, ≤ to express algebraic statements. (K)

8.8.8 Understand that the number line can be used to represent inequation statements. (U)

8.8.9 Know how to apply the 'inverse operation' procedures to inequations to make the statement true. (K)

8A • Solving equations by inspection

LB2 Pages 4–5

Specific learning outcomes

Learners should be able to:

8.8.1.1 Define the term 'equation'.

Equation: two expressions that are equal for some values of the pronumeral(s).

8.8.2.1 Solve simple equations by inspection.

Teaching points

1. Find the missing value that is represented by the letters in simple equations using inspection method.
2. Simple equations may be 'guessed' using inspection but it is still worth using words that will help to solve more complex equations in future.
3. Encourage learners to check their answer by substituting the pronumeral value mentally into the original equation. This is a useful habit to develop for all future work in algebra.

Learner difficulties and remedies

Difficulty

Understanding why alphabetical letters are used in equations to represent missing numbers or values.

Remedy

- Alphabetical letters are used to represent unknown numbers or values quanities in problems.

Difficulty

Finding the unknown value represented by the pronumeral in an equation.

Remedy

- Use trial and error to find the value of the pronumeral.
- Use inspection to find the number that is represented by the pronumeral in a one-step equation.

Suggested teaching approach

- Explain to learners that when solving an equation, you have to find the value or number that is missing, and is represented by the alphabetical letter or pronumeral in the equation.
- Show learners how to solve an equation by inspection.
- To solve by inspection you find the missing number in the equation by looking at the equation. This involves mentally substituting a number into the equation and multiplying, dividing or adding to find the correct answer.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

An equation is a mathematical sentence that contains two expressions connected by an equals sign. The equals sign tells us that the two expressions have the same value. Equations are used when some information is unknown. Unknown amounts are represented by pronumerals (or variables), which are letters or symbols.

For example, $x + 2 = 7$ is an equation where the pronumeral x represents a variable such as length. Finding the value that makes the equation a true number sentence is called **solving** the equation. We can solve the equation $x + 2 = 7$ to obtain the **solution** $x = 5$.

We **solve** an equation by finding the unknown value that makes an equation a true number sentence. By substituting $y = 3$ into $2 \times 4 = 5 + y$ we get a true number sentence, so the **solution** to $2 \times 4 = 5 + y$ is $y = 3$.

If y is any other value, we would get a false number sentence.

Equations are useful in solving everyday problems by providing a shorthand way of writing information. There are many methods that can be used to solve different types of equations.

Solving by inspection

Many equations involve only one operation (+, –, ×, ÷) and are sometimes called one-step equations.

For example, $3x = 9$, $= 7$, $x + 3 = 11$, $4 - x = 1$ are one-step equations.

These equations can be solved by simply looking at the equation and guessing the value of x.

When we do this, we need to check that our solution will make a true number sentence. This method is called **solving by inspection**.

Here are some more **one-step equations** and their solutions:

For $k - 4 = 5$, the solution is $k = 9$.
Check: $9 - 4 = 5$ is a true number sentence.

For $5a = 25$, the solution is $a = 5$.
Check: $5 \times 5 = 25$ is a true number sentence.

Examples

1. Write an equation for each of the following. Use the letter in brackets to represent the variable.
   - **a** Seven is added to a number to give a result of ten. (*a*)
   - **b** A number is multiplied by five, then four is subtracted to give a result of sixteen. (*b*)

Thinking	**Working**
a Write an expression containing the variable showing the operations that have been performed on it and equate this expression to the unknown amount.	**a** $a + 7 = 10$
b **1** Write an expression containing the variable showing the operations that have been performed on it and equate this expression to the unknown amount.	**b** $b \times 5 - 4 = 16$
2 Simplify.	$5b - 4 = 16$

To check that the value obtained for a variable is the solution, we substitute the value into each side of the equation. If the left-hand side (LHS) is equal to the right-hand side (RHS), we know that we have correctly solved the equation.
We know that $x = 5$ is the solution for $x + 2 = 7$ because $5 + 2 = 7$ is a true number sentence.

Check by substitution whether the value given in the brackets is the solution for the equation $\frac{x + 4}{6} = 2$ ($x = 14$). (Does it make the equation true?) Answer yes or no.

Thinking	**Working**
1 Substitute the x-value into the left-hand side of the equation. ($x = 14$).	$\text{LHS} = \frac{x+4}{6}$ $= \frac{14+4}{6}$ when $x = 14$
2 Simplify.	$= \frac{18}{6}$ $= 3$ $\neq$ RHS
3 Check whether the left-hand side of the equation equals the right-hand side.	No, the value given is not the solution.

Activity 8A

1 Write an equation for each of the following. Use the letter in brackets to represent the variable.

- **a** Eight is added to a number to give a result of twelve. (a)
- **b** Four is subtracted from a number to give a result of sixteen. (b)
- **c** Nine times a number gives a result of sixty-three. (c)
- **d** The sum of eleven and a number is zero. (f)
- **e** Seven is added to three times a number to give a result of ten. (u)
- **f** A number is multiplied by two, then seven is added to give a result of thirteen. (v)
- **g** Nine is added to a number, then the result is divided by seven to give six. (x)
- **h** The sum of six and a number is multiplied by eight to give a result of zero. (z)

2 Check by substitution whether the value given in the brackets is the solution for each of the following equations. (Does it make the equation true?) Answer yes or no.

a $m + 2 = 9$ ($m = 7$)
b $l - 2 = {}^{-}9$ ($l = {}^{-}11$)
c $n - 6 = 3$ ($n = 9$)
d $10 - p = 4$ ($p = 14$)
e $5q = 55$ ($q = 11$)
f $\frac{r}{8} = 9$ ($r = 72$)
g $6s - 7 = 23$ ($s = 4$)
h $3a + 5 = {}^{-}10$ ($a = {}^{-}5$)
i $3(u - 8) = 6$ ($u = 6$)
j $4(5 - v) = 12$ ($v = 8$)
k $\frac{w+3}{4} = 8$ ($w = 29$)
l $\frac{6-x}{7} = {}^{-}4$ ($x = 34$)

3 Solve these equations by inspection.

a $x + 5 = 2$ **b** $x - 6 = {}^{-}3$
c $x + 4 = {}^{-}1$ **d** $x - 2 = {}^{-}2$
e $x + 12 = 7$ **f** $16 - x = 17$

4 Solve these equations by inspection.

a $12x = {}^{-}36$ **b** ${}^{-}2y = {}^{-}4$
c ${}^{-}3x = 18$ **d** ${}^{-}11y = {}^{-}88$
e ${}^{-}y = {}^{-}18$ **f** ${}^{-}x = 4$

5 Solve these equations by inspection.

a $-\frac{x}{4} = 12$ **b** $\frac{x}{8} = {}^{-}3$
c $-\frac{x}{5} = {}^{-}3$ **d** $\frac{48}{x} = {}^{-}24$
e $-\frac{50}{y} = {}^{-}10$ **f** $-\frac{100}{x} = 4$

6 Four people each won a share of \$132 from one lotto ticket. Suppose the prize was \$$x$. Write this as an equation.

7 The cost of a torch with batteries is \$12. The torch costs \$8 more than the batteries. How much do the batteries cost?

Answers 8A

1 **a** $8a = 12$ **b** $b - 4 = 16$
c $9c = 63$ **d** $f + 11 = 0$
e $3u + 7 = 10$ **f** $2v + 7 = 13$
g $\frac{x+9}{7} = 6$ **h** $8z + 6 = 0$

2 **a** yes **b** no
c yes **d** no
e yes **f** yes
g no **h** yes
i no **j** no
k yes **l** yes

3 **a** $x = {}^{-}3$ **b** $x = 3$
c $x = {}^{-}5$ **d** $x = 0$
e $x = {}^{-}5$ **f** $x = {}^{-}4$

4 **a** $x = {}^{-}3$ **b** $y = 2$
c $x = {}^{-}6$ **d** $y = 8$
e $y = 18$ **f** $x = {}^{-}4$

5 **a** $x = {}^{-}48$ **b** $y = {}^{-}24$
c $x = 15$ **d** $y = {}^{-}2$
e $y = 5$ **f** $x = {}^{-}25$

6 $\frac{x}{4} = 132$

7
$$\begin{aligned} b + (b + 8) &= 12 \\ 2b + 8 &= 12 \\ 2b &= 4 \\ b &= 2 \end{aligned}$$

The batteries cost is \$2.

8B • Solving equations with flow charts

LB2 Pages 6–7

Specific learning outcomes

Learners should be able to:

8.8.3.1 Solve equations with one unknown using the ‘flow chart’ method.

Teaching points

1 Display a one-step equation as a flow chart.
2 Reverse the flow chart and use the inverse operation
3 Write the solution for the value of the pronumerals.
4 Check the solution works for the original equation.

Learner difficulties and remedies

Difficulty

Using the flow chart to find the missing values.

Remedy

- Draw boxes and then insert the information that is given into the boxes, including the operations or signs.
- Work backwards using opposite operations to multiplication, division, addition and subtraction to find the missing number.

Suggested teaching approach

- Identify the equation to be solved.
- Draw boxes for the equation and then write the operations inside the boxes.
- Use the opposite operations to find the missing number.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Some equations are difficult to solve by inspection because the solution is not a whole number, or more than one operation is involved.

For example: $x + 3\frac{1}{4} = 7\frac{1}{5}$, $3x = 157$, $x - 3{\cdot}75 = 21{\cdot}8$ or $\frac{x}{7} = 53\frac{1}{9}$ are all one-step equations but their solutions are not easy to see.

$2x + 5 = 11$, $3x - 4 = 31$, $\frac{5x}{4} = 5$, $\frac{x + 6}{8} = 2$ are all two-step equations.

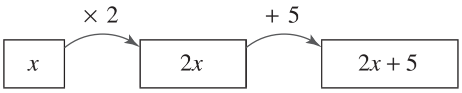
We need to use other methods to solve these equations.

Using flow charts

Flow charts were used to build expressions. They can also be used to solve equations. To use a flow chart correctly, we need to apply the order of operations that were used to build the expression.

For the expression $2x + 5$:

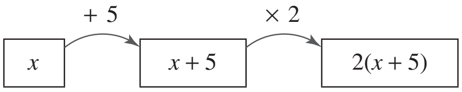
- x is first multiplied by 2 to give $2x$
- 5 is then added to give $2x + 5$.



This expression is $2(x + 5)$, which is quite different to $2x + 5$.

For the expression $2(x + 5)$:

- 5 is added to x to give $x + 5$.
- this is all multiplied by 2 to give $2(x + 5)$.
  Note that brackets are used when more than one terms is multiplied by a factor.



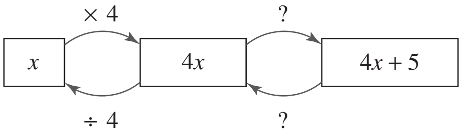
This expression is $2(x + 5)$, which is quite different from $2x + 5$.

Note: The order in which we apply the operation is important in building expressions.

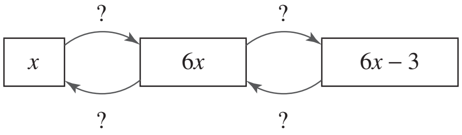
Examples

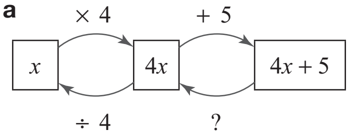
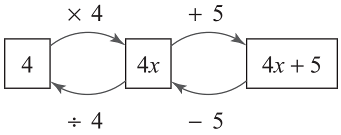
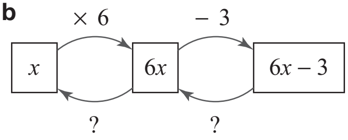
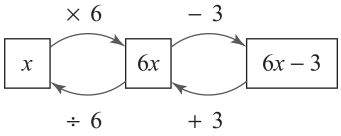
1 Copy and complete the following simplified flow charts to show the order of the operations needed to build and undo the expression.

a



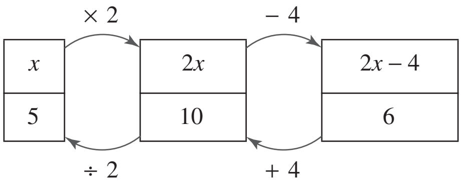
b



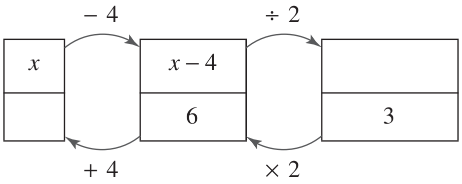
Thinking	**Working**
a 1 Above the top arrow write the missing operation needed to build the expression.	**a** 
2 Below the bottom arrow write the missing operation that will undo this expression.	
b 1 Above the top arrows write the missing operation needed to build each expression.	**b** 
2 Below the bottom arrows write the missing operation needed to undo this expression.	

2 For each equation in the following flow charts, write down the equation to be solved and then find the solution. Complete the flow chart first if necessary.

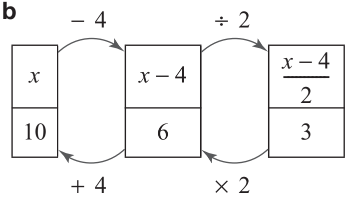
a



b



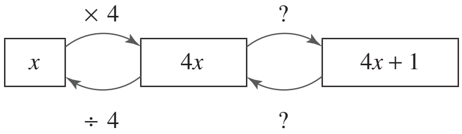
Thinking	**Working**
a 1 The expression in the last box on the right and the number in the box underneath it form the equation.	**a** Equation is $2x - 4 = 6$
2 The solution is the number under x in the first box on the left of the flow chart.	$x = 5$

Thinking	**Working**
b **1** Complete the missing parts of the flow chart.	**b** 
2 The expression in the last box on the right and the number in the box underneath it form the equation.	$\frac{x-4}{2} = 3$
3 The solution is the number under *x* in the first box on the left of the flow chart.	$x = 10$

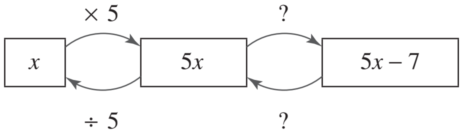
Activity 8B

1 Copy and complete the following simplified flow charts to show the order of operations needed to build and undo the expression.

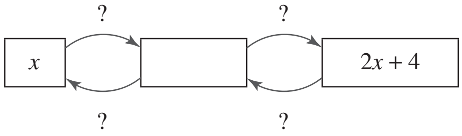
a



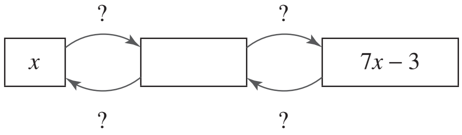
b



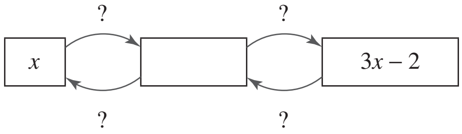
c



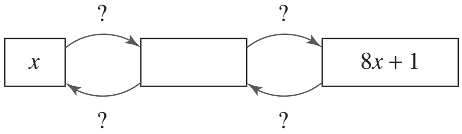
d



e

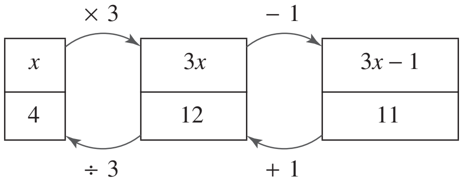


f

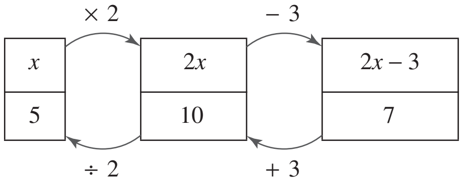


2 Write down (i) the equation to be solved and (ii) the solution to the equation shown in each of the following flow charts. Complete the flow chart first if necessary.

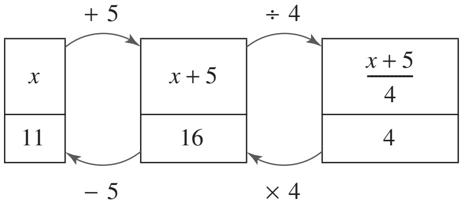
a



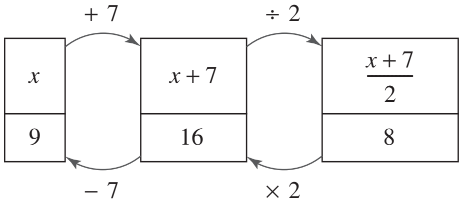
b



c



d



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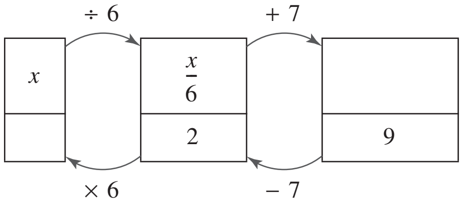

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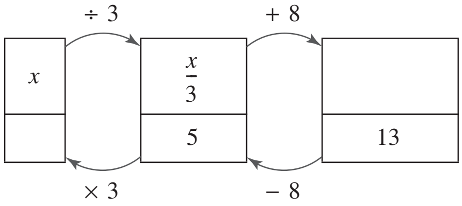

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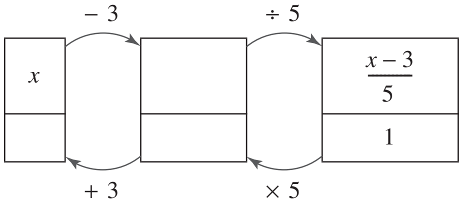

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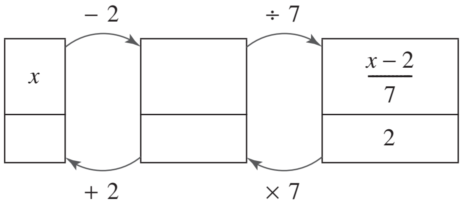

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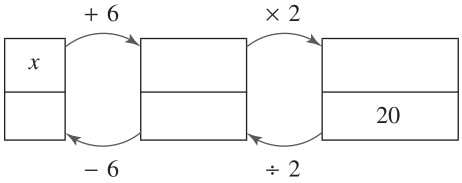

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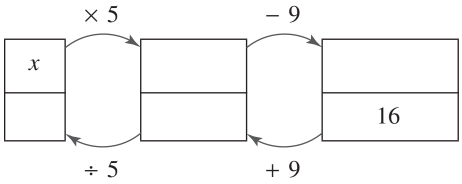

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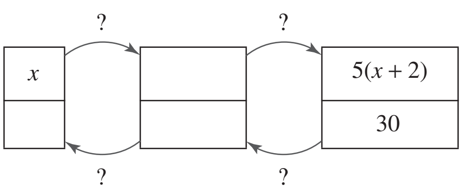

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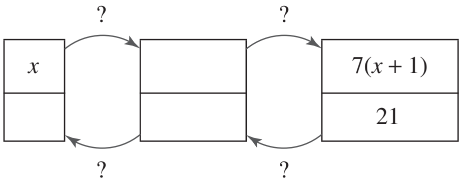


3 Draw a flow chart and use backtracking to solve each of the following equation.

a $5x + 3 = 8$ **b** $7x + 3 = 24$

c $2x + 3 = 11$ **d** $2x - 7 = 3$

e $3x - 2 = 10$ **f** $8x - 11 = 13$

4 Draw a flow chart and use backtracking to solve each of the following equations. Check your solutions by substitution.

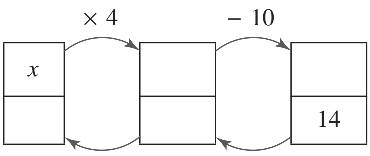
a $\frac{x}{4} + 5 = 7$ **b** $\frac{x}{2} + 6 = 11$

c $\frac{x}{7} + 3 = 5$ **d** $\frac{x}{2} - 2 = 1$

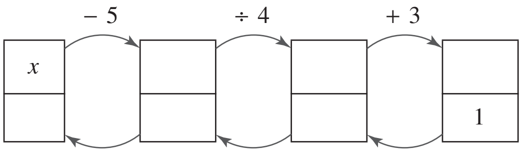
e $\frac{x}{5} - 1 = 2$ **f** $\frac{x}{2} - 4 = 2$

5 Copy and complete the following flow charts. Write the equation and use the flow chart to find the solution for x.

a

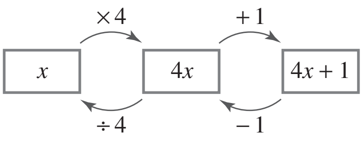


b

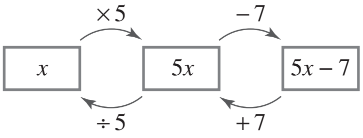


Answers 8B

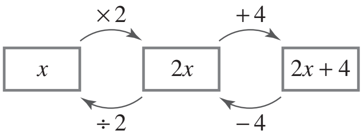
1 a



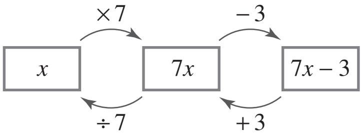
b



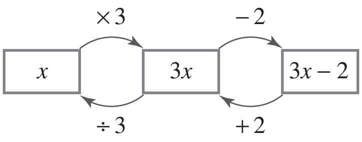
c



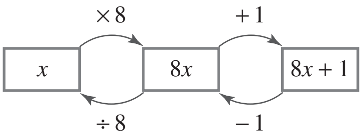
d



e



f



2 a **i** $3x - 1 = 11$ **ii** $x = 4$

b **i** $2x - 3 = 7$ **ii** $x = 5$

c **i** $\frac{x+5}{4} = 4$ **ii** $x = 11$

d **i** $\frac{x+7}{2} = 8$ **ii** $x = 9$

e **i** $\frac{x}{6} + 7 = 9$ **ii** $x = 12$

f **i** $\frac{x}{3} + 8 = 13$ **ii** $x = 15$

g **i** $\frac{x-3}{5} = 1$ **ii** $x = 8$

h **i** $\frac{x-2}{7} = 2$ **ii** $x = 16$

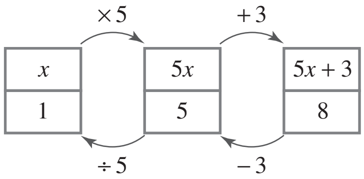
i **i** $2(x + 6) = 20$ **ii** $x = 4$

j **i** $5x - 9 = 16$ **ii** $x = 5$

k **i** $5(x + 2) = 30$ **ii** $x = 4$

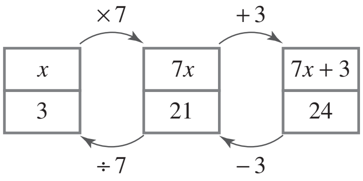
l **i** $7(x + 1) = 21$ **ii** $x = 2$

3 a



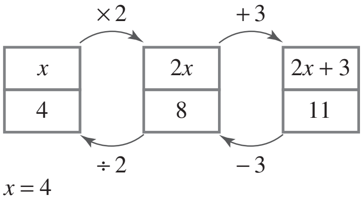
$x = 1$

b



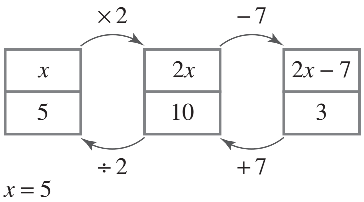
$x = 3$

c



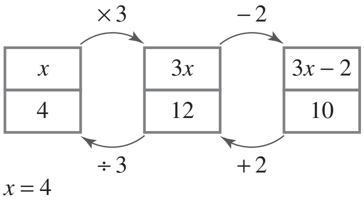
$x = 4$

d



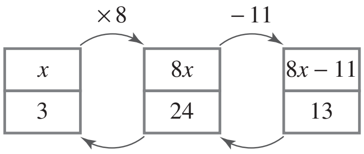
$x = 5$

e



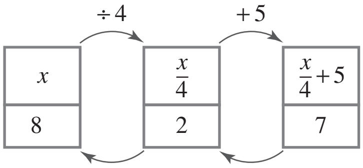
$x = 4$

f



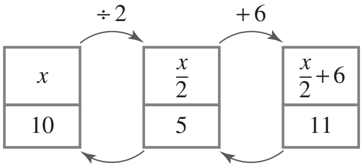
$x = 3$

4 a



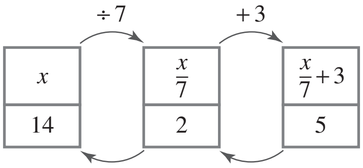
$x = 8$

b



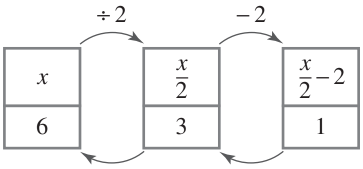
$x = 10$

c



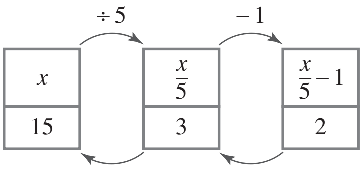
$x = 14$

d



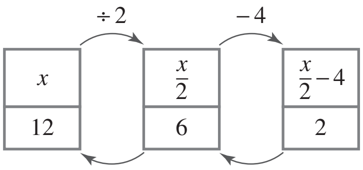
$x = 6$

e



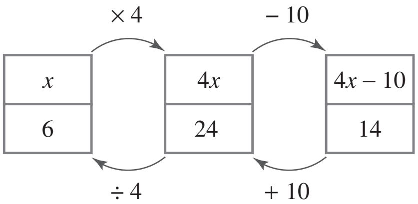
$x = 15$

f



$x = 12$

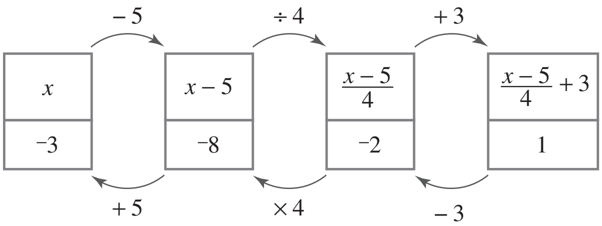
5 a



$4x - 10 = 14$

$x = 6$

b



$\frac{x-5}{4} - 3 = 1$

$x = {}^-6$

8C • Solving equations using inverse operations

LB2 Pages 8–9

Specific learning outcomes

Learners should be able to:

8.8.4.1 Solve equations using the 'inverse operations' method.

Teaching point

1 Solve equations using inverse operations in a flow chart.

2 Remind learners that (+, –) are inverse operations and (×, ÷) are inverse operations.

3 To solve an equation using inverse operations, apply the opposite operation in the opposite order.

Learner difficulties and remedies

Difficulty

How to use the inverse operations to find the missing numbers and solve the equations

Remedy

- Identify the given operation and then use the opposite operation to find the missing number. Remind learners that (+, –) are inverse operations and (×, ÷) are inverse operations.
- Solve those equations algebraically. To solve an equation using inverse operations, apply the opposite operation in the opposite order.

Suggested teaching approach

- Identify the pronumerals in the equation.
- Identify the signs for the operations then use the inverse operations to solve the equations.
- Solve those equations algebraically using backtracking. Undo each operation using the inverse (opposite) operation in the opposite order that was used to build the equation.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

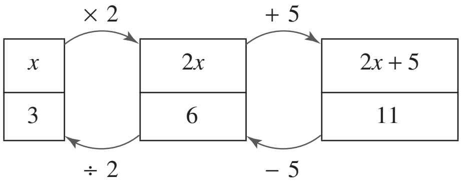
Additional notes

Inverse operations

To solve equations using flow charts, we undo operations used to build the equation by applying inverse operations (opposite operations) in the reverse order. We move backwards along the flow chart. It also referred to a 'Backtracking'. You work backward using the flow chart.

Backtracking

To solve $2x + 5 = 1$, we set up the following flow chart.



1. Draw a flow chart for the expression on the LHS of the equation.
2. Add boxes underneath the flow chart boxes and arrow signs in the opposite direction.
3. Write the inverse operations on the arrow signs.
4. Write the number on the RHS of the equation in the box on the right-hand side of the flow chart underneath the expression.
5. Complete the flow chart using the inverse operations.
6. The number under x is the solution.

This method of solving equations is called **backtracking**. Backtracking along this flow chart gives us the solution $x = 3$ to the equation $2x + 5 = 11$.

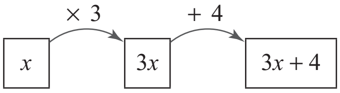
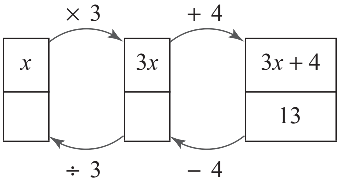
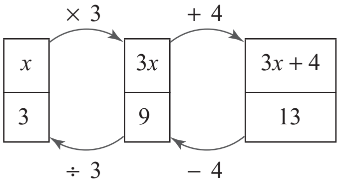
When backtracking, we undo an operation by using the inverse (opposite) operation.

+ and – are inverse operations

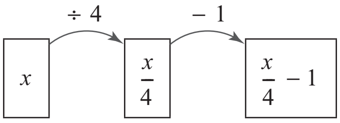
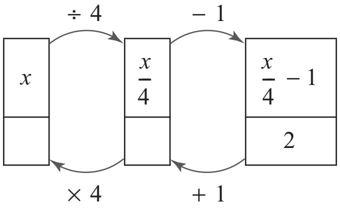
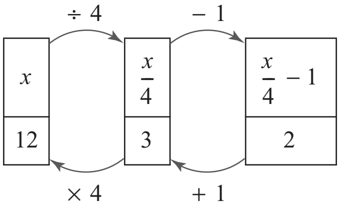
× and ÷ are inverse operations

Examples

1 Draw a flow chart and use backtracking to solve the equation: $3x + 4 = 13$.

Thinking	**Working**
1 Build the expression on the LHS of the equation with a flow chart. Start with x and identify the operations needed. (× 3, + 4)	
2 Write the value from the RHS of the equation under the last box on the right of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (÷ 3, – 4)	
3 Perform the required operations to obtain a value for x.	
4 State the solution.	$x = 3$

2 Draw a flow chart and use backtracking to solve the equation $\frac{x}{4} - 1 = 2$. Check your solution by substitution.

Thinking	**Working**
1 Build the expression on the LHS of the equation with a flow chart. Start with x and identify the operations needed. (÷ 4, – 1)	
2 Write the value from the RHS of the equation under the right-hand box of the built expression. Identify the operations needed to undo the equation and write them underneath with reverse arrows. (× 4, + 1)	
3 Perform the required operations to obtain a value for x.	
4 State the solution.	$x = 12$
5 Check the solution by substituting your answer into the left-hand side of the equation. If the left-hand side equals the right-hand side of the equation, you have found the solution.	Check: LHS $= \frac{x}{4} - 1$ $= \frac{12}{4} - 1$ $= 3 - 1$ $= 2$ = RHS

Activity 8C

1 Use inverse operations to solve each of the following equations. Check your answers by substitutions to the equations.

a $3(x + 1) = 12$ **b** $2(x + 3) = 16$ **c** $2(x + 5) = 14$

d $5(x - 6) = 10$ **e** $6(x - 4) = 18$ **f** $3(x - 5) = 3$

2 Use backtracking to solve each of the following equations. Check your solutions by substitution.

a $\frac{x+8}{3} = 3$ **b** $\frac{x+5}{2} = 4$ **c** $\frac{x+10}{7} = 2$

d $\frac{x-2}{5} = 1$ **e** $\frac{x-3}{4} = 2$ **f** $\frac{x-5}{2} = 4$

3 Use backtracking to solve each of the following equations. Check your solutions by substitution.

a $2x + 3 = 13$ **b** $3x + 2 = 11$ **c** $5x + 6 = 11$

d $5x - 3 = 17$ **e** $4x - 1 = 7$ **f** $8x - 9 = 7$

g $2(x + 7) = 20$ **h** $4(x - 3) = 8$ **i** $6(x - 1) = 12$

j $\frac{x}{2} + 3 = 8$ **k** $\frac{x}{7} + 10 = 12$ **l** $\frac{x}{3} - 2 = 3$

m $\frac{x+2}{4} = 2$ **n** $\frac{x-4}{5} = 2$ **o** $\frac{x-3}{2} = 2$

4 Solve each of the following three-step equations using backtracking. Check your solutions by substitution.

a $\frac{2x+1}{3} = 3$ **b** $\frac{3x+7}{2} = 11$ **c** $\frac{5x-2}{4} = 7$

d $\frac{3x}{5} + 6 = 9$ **e** $\frac{2x}{7} + 8 = 12$ **f** $\frac{7x}{2} - 10 = 4$

g $\frac{2(x+4)}{5} = 6$ **h** $\frac{3(x+2)}{4} = 3$ **i** $\frac{2(x-3)}{7} = 6$

Answers 8C

1 **a** $x = 3$ **b** $x = 5$ **c** $x = 2$
d $x = 8$ **e** $x = 7$ **f** $x = 6$
2 **a** $x = 1$ **b** $x = 3$ **c** $x = 4$
d $x = 7$ **e** $x = 11$ **f** $x = 13$
3 **a** $x = 5$ **b** $x = 3$ **c** $x = 1$
d $x = 4$ **e** $x = 2$ **f** $x = 2$
g $x = 3$ **h** $x = 5$ **i** $x = 3$
j $x = 10$ **k** $x = 14$ **l** $x = 15$
m $x = 6$ **n** $x = 14$ **o** $x = 7$
4 **a** $x = 4$ **b** $x = 5$ **c** $x = 6$
d $x = 5$ **e** $x = 14$ **f** $x = 4$
g $x = 11$ **h** $x = 2$ **i** $x = 24$

8D • Solving two- and three-step equations

LB2 Pages 10–11

Specific learning outcomes

Learners should be able to:

8.8.5.1 Solve equations requiring two- and three-steps.

Teaching points

1 Solve two- and three-step equations using inverse operations.

2 Remind learners that (+, –) are inverse operations and (×, ÷) are inverse operations.

3 To solve an equation using inverse operations, apply the opposite operation in the opposite order.

Learner difficulties and remedies

Difficulty

Determining the order of the steps required to solve two- and three-step equations.

Remedy

- Use inverse operations to solve the two- and three-step equations. Reverse the order using inverse operations to determine the solution to the equation.
- Use the rule 'whatever you do to the left-hand side (LHS) do it to the right-hand side (RHS)' to solve given equations.

Suggested teaching approach

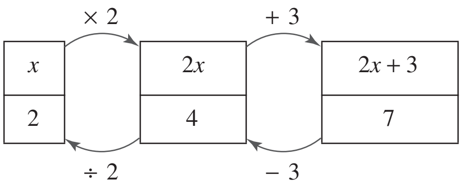
- Identify the pronumeral and the order of the operations applied to build the equation.
- Apply the opposite operations in the opposite order to 'undo' the equation and find the value of the pronumeral.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The inverse operations with the balance method

We can now solve equations that involve two to three steps using inverse operations with the balance method. Up until now, we have used flow charts to show us both the order and the operations we need to use to solve equations. Here, we will use the same information, but show it in a different way without drawing boxes.

Consider $2x + 3 = 7$.



Notice that the three boxes contain three equivalent equations. We have moved backwards through the flow chart and used inverse (opposite) operations (– 3, ÷ 2) to solve the equation

For $2x + 3 = 7$, $x = 2$.

Let's now show the same steps using the inverse operations with the balance method. The order of operations in the flow chart tells us how our equation was built. By moving backwards and performing the inverse operation, we will undo our equation to find the solution.

$2x + 3 = 7$

$2x + 3 \mathbf{- 3} = 7 \mathbf{- 3}$ (subtract 3 from both sides)

$2x = 4$

$\frac{2x}{\mathbf{2}} = \frac{4}{\mathbf{2}}$ (divide both sides by 2)

$x = 2$

Examples

Solve the equation $3x - 7 = 11$ using the balance method. Check your solution by substitution.

Thinking	Working
1 Write the equation. Identify the last operation to be performed on the LHS of it. This is the first operation to be undone. (– 7)	$3x - 7 = 11$
2 Use the inverse operation (+ 7) on both sides of the equals sign and simplify your equation.	$3x - 7 + \mathbf{7} = 11 + \mathbf{7}$ $3x = 18$
3 Identify the next operation to be undone and apply the inverse operation (÷ 3). If one side of the equation is now the variable by itself, you have found the solution. Otherwise, continue the process until you do have the variable by itself.	$\frac{3x}{\mathbf{3}} = \frac{18}{\mathbf{3}}$ $x = 6$
4 Check the solution by substitution.	Check: LHS $= 3x - 7$ $= 3 \times 6 - 7$ $= 18 - 7$ $= 11$ $=$ RHS

Activity 8D

1 Solve each of the following equations using two to three steps operations and balance of equations.

a $3x + 5 = 8$ **b** $5x + 1 = 21$ **c** $7x + 3 = 17$

d $2x + 1 = 11$ **e** $2x + 5 = 19$ **f** $2x + 10 = 8$

g $3x - 4 = 20$ **h** $5x - 1 = 4$ **i** $6x - 2 = 16$

j $4x - 3 = 17$ **k** $2x - 3 = 7$ **l** $4x - 1 = 31$

m $\frac{2x}{3} = 8$ **n** $\frac{2x}{7} = 2$ **o** $\frac{5x}{2} = 20$

p $\frac{x + 5}{3} = 5$ **q** $\frac{x + 1}{3} = 7$ **r** $\frac{x + 4}{5} = 3$

s $2(x - 5) = 14$ **t** $3(x + 2) = 24$ **u** $7(x - 2) = 35$

Answers 8D

1 **a** $x = 1$ **b** $x = 4$ **c** $x = 2$

d $x = 5$ **e** $x = 7$ **f** $x = -1$

g $x = 8$ **h** $x = 1$ **i** $x = 3$

j $x = 5$ **k** $x = 5$ **l** $x = 8$

m $x = 12$ **n** $x = 7$ **o** $x = 8$

p $x = 10$ **q** $x = 20$ **r** $x = 11$

s $x = 12$ **t** $x = 6$ **u** $x = 7$

8E • Inequations

LB2 Pages 12–15

Specific learning outcomes

Learners should be able to:

8.8.6.1 Define the term ‘inequation’.
Inequation: an expression where a pronumeral represents a range of values.

8.8.7.1 Use inequality symbols: <, >, ≥, and ≤ to write relationships between expressions.

8.8.8.1 Use a number line to represent the inequations.

8.8.9.1 Solve inequation problems by applying inverse operation procedures.

8.8.9.2 Use equations and inequations to solve word problems.

Teaching points

1. Explain the term inequation and include simple examples.
2. Use the inequality symbols <, >, ≥, and ≤ to write and solve the relationships between expressions.
3. Use number lines to represent the inequations.
4. Solve inequations by using the inverse operation method.
5. Solve word problems involving inequations.

Learner difficulties and remedies

Difficulty

Understanding what the symbols <, >, ≥, and ≤ mean.

Remedy

- Explain clearly what the symbols <, >, ≥, and ≤ represent using examples from real life.
- Explain the differences between an equation and an inequation.
- Show how to use the inequality symbols using relationships between two or more quantities.

Difficulty

Interpreting the symbols <, >, ≥, and ≤ using the number line.

Remedy

- Use the number line to demonstrate how inequality symbols are used.
- Show the range of numbers could be used with the inequality symbol to make the statement true.

Difficulty

Solving problems involving inequations.

Remedy

- Use the same steps as solving two- to three-steps equations.

Suggested teaching approach

- Show learners the inequality symbols <, >, ≥, and ≤. Use examples from real life to illustrate each inequality symbol.
- Explain to learners that when they solve inequations, the process is the same as for equations.
- A solution involving an inequality symbol <, >, ≥, and ≤ does not represent a unique solution like the solution to an equation with an equals sign. The inequality gives a range of values.
- Also to take note that when you divide or multiply an inequality by a negative number to solve for the pronumeral, reverse the direction of the inequality symbol.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

An **inequation** is a statement indicating that there is an inequality between two values. The inequation signs such as <, >, ≥, and ≤ are used to make the comparison between two values or quantities to show which of these values are bigger or smaller when they are compared.

The following are the interpretations of the inequality symbols:

- The notation $a \neq b$ means that *a* is **not equal to** *b*. It does not say that one is greater than the other, we just know that they are not equal.
- The notation $a < b$ means that *a* is **less than** *b*.
- The notation $a > b$ means that *a* is **greater than** *b*. In either case, *a* is not equal to *b*.

There are two types of inequality relations that are also used:

- The notation $a \leq b$ means that *a* is **less than or equal to** *b* (or, equivalently, **not greater than** *b*, or **at most** *b*).
- The notation $a \geq b$ means that *a* is **greater than or equal to** *b* (or, equivalently, **not less than** *b*, or **at least** *b*)

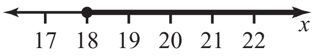
Solving inequations

Solving an inequation means finding the value or range of values for the pronumerals that make the number sentence true. A **solution** of an inequation is usually a range of values, and drawing the values on a number line helps learners interpret meaning visually.

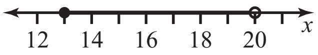
As with equations, it is a good practice to check the solution of an inequation by substituting a value back into the original inequation to see if it makes a true statement.

Examples

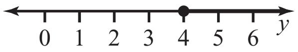
1 If x is the age at which a person can vote in the National Election in Solomon Islands, write an inequation for x and illustrate the solution on a number line.

Thinking	Working
1 Write the statement into the inequation format	$x \ge 18$
2 Draw the number line to show the inequality statement using the number line. Note that a 'filled in' or 'closed' end circle indicates that the end number is included.	

2 If x is the age at which you are called a teenager, write an inequation for x and show the solution on a number line. The teenage age is between 13 to 20. The x has to be 13 or more but less than 20.

Thinking	Working
1 Write the statement into the inequation format	Reading from the centre to the left and then to the right, we have $13 \le x < 20$
2 Draw the number line to show the inequality statement using the Number line. Note that a 'filled in' or 'closed' end circle indicates that the end number is included.	

3 A number y is multiplied by 3, then 2 is subtracted to give a result that is 10 or more. Write the inequation and solve it, showing the solution on a number line.

Thinking	Working
1 Write the statement into the inequation format	$3y - 2 \ge 10$
2 Solve the inequation algebraically by taking one term at a time. Remove 2	$3y - 2 \ge 10$
3 Remove 3 that stays with the y	$3y \ge 12$ $\frac{3y}{3} \ge \frac{12}{3}$
4 The answer is	$y \ge 4$
5 Use the number line to represent the inequalities	
6 Check the solution works for $y = 5$	$3 \times 5 - 2 = 15 - 2 = 13$
Substitute $y = 5$ into $3y - 2 \ge 10$	Yes! $13 \ge 10$

Activity 8E

1 Write each of the following statements as an inequation.

- **a** The number of goals g by which I expect my football team to win next weekend is at least 5 but no more than 8.
- **b** The mark m that I expect to get on my next mathematics test is more than 60% but no more than 100%.
- **c** Learners in Year 7 are at least 11 but no more than 13 years old.
- **d** To get a discount ski ticket, a person must be less than 15 or greater than 70 years of age.

2 Solve the following inequations.

a $x + 9 < 12$ **b** $z - 13 \le 8$ **c** $5z > 15$

d $7m > 42$ **e** $3p - 2 \ge 4$ **f** $6q + 5 \le 11$

g $5 + 4n \ge 6$ **h** $8 - 2c \le 10$ **i** $3 - 7h \ge 4$

3 Solve the following inequations, showing appropriate setting out.

a $3x + 2 > 17$ **b** $5y - 7 \le 8$ **c** $2h - 4 \ge 0$

d $\frac{b}{2} + 3 > 9$ **e** $\frac{2k}{3} + 9 < 13$ **f** $\frac{2m + 4}{3} > 6$

4 Write an inequation for each of the following statements and then solve it using a number line.

- **a** I think of a number n, multiply it by 6, then add 2 to get a result greater than 50.
- **b** Four times a certain number y divided by 12 gives a result greater than 3.
- **c** A certain number q divided by 9, plus 8, gives a result of 10 or more.
- **d** Twice a certain number p divided by 8, minus 1, gives a result of 1 or less.

5 When I add d dollars to my bank account, which had a balance of $100, the new balance is between $125 and $148. Write an inequation for d and solve it.

Answers 8E

1 **a** $5 \le g \le 8$ **b** $60 < m \le 100$ **c** $11 \le a \le 13$

d $a < 15$ or $a > 70$

2 **a** $x < 3$ **b** $z \le 21$ **c** $z > 3$

d $m > 6$ **e** $p \ge 2$ **f** $q \le 1$

g $n \ge 0{\cdot}25$ **h** $c \ge 1$ **i** $h \le -\frac{1}{7}$

3 **a** $x > 5$ **b** $y \le 3$ **c** $h \ge 2$

d $b > 12$ **e** $k < 6$ **f** $m > 7$

4 **a** $n > 16$ **b** $y > 9$ **c** $q \ge 18$

d $p \le 8$

5 $128 \le d + 100 \le 148$

$28 \le d \le 48$

I had between $28 and $48 in my account.

• Chapter Nine •

Indices

Overview

Throughout history many civilisations have devised different ways of recording numbers. While our standard Arabic numerals have proved ideal for most everyday counting and calculations, they can be very awkward for dealing with exceptionally large or small numbers. Mathematicians invented some alternative ways of representing numbers to overcome this problem using powers. If a number is written to a power, that number is multiplied by itself.

6 to the power 5 is $6^5 = 6 \times 6 \times 6 \times 6 \times 6 = 7776$

The closest distance from the Earth to the Sun is 147 000 000 km. This can be written more concisely as $1{\cdot}47 \times 10^8$ km using indices. The thickness of a human hair can vary from 0·000 04 m to 0·000 25 m. This range can be written more conveniently as 4×10^{-5} m to $2{\cdot}5 \times 10^{-5}$ m.

Contents

Chapter skills

This chapter covers the following skills:

- Representing numbers and expressions in index form
- Working with the index laws as applied to numbers and algebraic expressions
- Working with, and graphing, simple exponential relations in modelling situations
- Exploring scientific notation
- Exploring negative powers

Teaching plan

Lessons	Chapter sections	Class work and home work
1–2	• 9A Index numbers	Learner's Book 1 • Exercise 9A, pages 26 and 27
3–4	• 9B Exploring index laws	Learner's Book 1 • Exercise 9B, pages 28, 29
5–6	• 9C Index laws with algebraic terms	Learner's Book 1 • Exercise 9C, pages 30, 31
7–8	• 9D Managing brackets: other index laws	Learner's Book 1 • Exercise 9D, pages 32, 33
9	• 9E Exploring scientific notation	Learner's Book 1 • Exercise 9E, pages 34, 35
10	• Test	Teacher's Guide • Chapter 9 Test

General learning outcomes

Learners should:

Index numbers

8.9.1 Understand how an index number is formed. (U)

8.9.2 Know how to express numbers in an index and expanded notations. (K)

Exploring index laws

8.9.3 Understand that there are laws that must be used when dealing with indices. (U)

Index laws with algebraic terms

8.9.4 Know how to simplify indices using the four index laws algebraically. (K)

Managing brackets: other index laws

8.9.5 Know how to use and apply three extra index laws that involve brackets. (K)

Exploring scientific notation

8.9.6 Understand that very big and very small numbers can be simplified by expressing them in scientific notation. (U)

8.9.7 Know how to write very big and small numbers in scientific notation. (K)

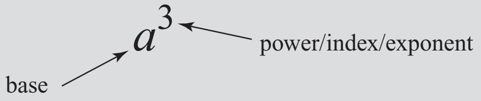
9A • Index numbers

LB2 Pages 26–27

Specific learning outcomes

Learners should be able to:

8.9.1.1 Identify the terms 'base', 'power', 'index' and 'exponent'.



8.9.2.1 Write numbers in index notation.

8.9.2.2 Expand index numbers into expanded form.

8.9.2.3 Expand and evaluate numbers given in index form.

Teaching points

1 Write numbers in index form.
2 Use examples to explain the terms, index, power, and base.
3 Revise the multiplication of directed numbers.
4 Expand and evaluate index numbers.

Learner difficulties and remedies

Difficulty

Understanding how an index number is formed.

Remedy

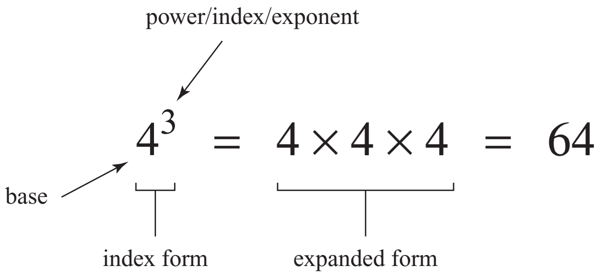
- Explain to learners that numbers can be written in many different ways or notations. One of them is using index notation.
- When the same is multiplied by itself then it can be expressed using index notation with a base and a power or index.
- When an index number is expanded you multiply the base by itself. The index number tells you the number of times to multiply.

Suggested teaching approach

- Show learners examples of index numbers. Explain its features and properties: it has a base and a power or index number.
- Show how to write numbers in index notation.
- Show how to expand index numbers. Multiply the base number by itself according to the index number.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Index numbers have a **base** and **power** or **index**. The base is the number raised to the power or index.



When expanded index numbers show repeated multiplication. $6 \times 6 \times 6$ can be simplified or re-written as 6^3. The value of 6^3 is 216.

Examples

Index form

1 Write the following in index form.

a $7 \times 7 \times 7 \times 7$ **b** $2 \times 2 \times 2 \times 5 \times 5$

Thinking	Working
a Count the number of times the base appears in the string of multiplications and use this number as the index. (Here, there are four 7s multiplied together, so 4 is the index.)	**a** $7 \times 7 \times 7 \times 7$ $= 7^4$
b Count the number of times each base appears in the string of multiplications and use these numbers as the index of each base. (Here, we have the product of three 2s and two 5s.)	**b** $2 \times 2 \times 2 \times 5 \times 5$ $= 2^3 \times 5^2$

Expanded form

2 Write the following in expanded form.

a 8^5 **b** $3^2 \times 4^3$

Thinking	Working
a Use the index to determine the number of times the base will appear in the string of multiplications. (Here, an index of 5 means there are five 8s multiplied together.)	**a** 8^5 $= 8 \times 8 \times 8 \times 8 \times 8$
b Use the index of each base to determine the number of times each base will appear in the string of multiplications. (Here, the index of 3 is 2 and the index of 4 is 3, so we have two 3s and three 4s multiplied together.)	**b** $3^2 \times 4^3$ $= 3 \times 3 \times 4 \times 4 \times 4$

3 Evaluate index number $(-2)^7$.

Thinking	Working
1 Use the index to find out how many times the base will multiply itself.	$(-2)^7$ the power is 7
2 Expand the Index number according to the value given by the power.	$(-2)^7$ $= {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2 \times {}^{-}2$
3 Evaluate to find the answer.	$= {}^{-}128$

4 Solve $5^x = 625$.

Thinking	Working
1 Finding unknown power can be done through trial and error. Try one and if does not work then try the other.	$5^x = 625$
2 Try and multiply 5 by itself a number of times.	$5 \times 5 = 25$ $5 \times 5 \times 5 = 125$ $5 \times 5 \times 5 \times 5 = 625 = 5^4$
3 $5^4 = 625$	$x = 4$

Activity 9A

1 Write the following in index form.

a $2 \times 2 \times 2 \times 2$
b $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
c $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
d $11 \times 11 \times 11 \times 11 \times 11 \times 11$
e $13 \times 13 \times 13 \times 13 \times 13 \times 13$
f $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
g $5 \times 5 \times 5 \times 5 \times 6 \times 6 \times 6$
h $2 \times 2 \times 2 \times 3 \times 3$
i $4 \times 4 \times 4 \times 4 \times 10 \times 10 \times 10 \times 10$
j $7 \times 7 \times 7 \times 7 \times 7 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
k $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
l $11 \times 11 \times 11 \times 13 \times 13 \times 13 \times 13 \times 13 \times 13$
m $6 \times 6 \times 6 \times 6 \times 7 \times 7 \times 7 \times 9 \times 9$
n $12 \times 14 \times 14 \times 14$

2 Write the following in expanded form.

a 5^7 **b** 4^6 **c** 7^3
d 9^6 **e** 10^4 **f** 3^7
g $2^3 \times 5^4$ **h** $3^7 \times 11^5$ **i** $9^5 \times 7^5$
j $2 \times 4^6 \times 6^4$ **k** $3^4 \times 7^2 \times 12$ **l** $9^3 \times 11 \times 17^2$

3 Express the following in index form and then evaluate:

a base 1, index 3 ________________
b base 2, index 9 ________________
c base 9, index 2 ________________
d base 10, index 4 ________________

4 Evaluate these index numbers:

a $(^-2)^5$ ________________________________
b $(^-1)^4$ ________________________________
c $(^-10)^3$ ________________________________
d $(^-9)^2$ ________________________________
e $^-2^4$ ________________________________
f $^-5^3$ ________________________________
g $^-(^-2)^8$ ________________________________

5 Solve these equations to find out the value of *x*.
Hint: Try multiplying the base by itself a number of times.

a $8^x = 512$ ____________________
b $2^x = 32$ ____________________
c $10^x = 10\,000\,000$ _____________
d $(^-4)^x = 16$ __________________

6 Use a 'trial and error' process to solve these equations.

a $x^5 = 243$
b $x^3 = 729$
c $x^4 = 1296$
d $x^2 = 62\,500$

Answers 9A

1 **a** 2^4 **b** 4^7 **c** 8^8
d 11^6 **e** 13^6 **f** 9^9
g $5^4 \times 6^3$ **h** $2^3 \times 3^2$ **i** $4^4 \times 10^4$
j $7^5 \times 8^6$ **k** $2^3 \times 3^4 \times 5^2$ **l** $11^3 \times 13^6$
m $6^4 \times 7^3 \times 9^2$ **n** 12×14^3

2 **a** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
b $4 \times 4 \times 4 \times 4 \times 4 \times 4$
c $7 \times 7 \times 7$
d $9 \times 9 \times 9 \times 9 \times 9 \times 9$
e $10 \times 10 \times 10 \times 10$
f $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
g $2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
h $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 \times 11 \times 11$
i $9 \times 9 \times 9 \times 9 \times 9 \times 7 \times 7 \times 7 \times 7 \times 7$
j $2 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 6 \times 6 \times 6 \times 6$
k $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 12$
l $9 \times 9 \times 9 \times 11 \times 17 \times 17$

3 **a** $1^3 = 1$ **b** $2^9 = 512$
c $9^2 = 81$ **d** $10^4 = 10\,000$

4 **a** $^-32$ **b** 1
c $^-1000$ **d** 81
e $^-16$ **f** $^-125$
g $^-256$

5 **a** $x = 3$ **b** $x = 5$
c $x = 7$ **d** $x = 2$

6 **a** $x = 3$ **b** $x = 9$
c $x = 6$ **d** $x = 250$

9B • Exploring index laws

LB2 Pages 28–29

Specific learning outcomes

Learners should be able to:

8.9.3.1 Identify some of the laws that are used to manage indices:

1 Same base when multiplying, add powers.
2 Same base when dividing, subtract powers.
3 Square root of numbers is the same as a base with the power of $\frac{1}{2}$.

Teaching points

1 Identify the laws that are used to evaluate index numbers of the same base when multiplied and divided.

2 Know how to simplify and evaluate indices using the index laws when a base is raised to a power of a power.

3 Show why the square root of numbers is the same as a base to the power of $\frac{1}{2}$. For example, $16^{\frac{1}{2}} = \sqrt{16} = 4$.

Suggested teaching approach

- Learners complete **Learning task 9B** on page 28 of the LB to discover the index rules for themselves.

9C • Index Laws with algebraic terms

LB2 Pages 30–31

Specific learning outcomes

Learners should be able to

8.9.4.1 Simplify indices using index laws.

$a^n \times a^p = a^n + {}^p$
$a^n \div a^p = a^n + {}^p$
$b^1 = b$
$b^0 = 1$

Teaching points

2 Check the understanding of the four index laws using a variety of examples.

3 Apply the index laws to simplify index expressions.

Learner difficulties and remedies

Difficulty

Understanding index notation.

Remedy

- Explain to learners that index notation is just a short form way of writing repeated multiplication. For example, $3 \times 3 \times 3 \times 3$ is written as 3^4 in index notation.

Difficulty

Simplifying index expressions using the index laws.

Remedy

- Look for the base. Only terms with the **same base** can be simplified using the index laws.
- When the bases are the same, use the index laws to simplify them.

Suggested teaching approach

- Know when and where to apply those index laws.
- When simplifying using the index laws, look for the base and the operation. The index laws apply only when the bases are the same. The operation will determine which index law to use.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Multiplying index numbers

There are two ways to calculate the value of $2^3 \times 2^5$.

Method 1

Evaluate each part separately and find the product. To evaluate a number or an expression, calculate its value term by term.

$2^3 = 8$ and $2^5 = 32$

$$\begin{aligned} 2^3 \times 2^5 &= 8 \times 32 \\ &= 256 \end{aligned}$$

Method 2

Write each number in expanded form and count the number of times the base appears. Rewrite the product in index form and use a calculator to find its value.

$$\begin{aligned} 2^3 \times 2^5 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^8 \\ &= 256 \end{aligned}$$

We will use this second method to investigate multiplying index numbers further.

Did you notice that the index of the final product is equal to the sum of the indices of the numbers being multiplied?

$$\begin{aligned} &3^2 \times 3^4 \\ &= 3^{2+4} \\ &= 3^6 \end{aligned}$$

$$\begin{aligned} &4^3 \times 4^4 \\ &= 4^{3+4} \\ &= 4^7 \end{aligned}$$

We can state this pattern as a rule that we can use.

Rule 1 When multiplying index numbers with the same base, keep the base and add the indices.

If the numbers do not all have the same base, the indices cannot be added. $2^3 \times 3^2$ cannot be written more simply in index form.

Dividing index numbers

There are two ways to calculate the value of $5^8 \div 5^5$.

Method 1

Evaluate each part separately and find the quotient using a calculator.

$$\begin{aligned} 5^8 \div 5^5 &= 390\,625 \div 3125 \\ &= 125 \end{aligned}$$

Method 2

We could write each number in expanded form and divide by cancelling common factors before simplifying the indices.

$$\begin{aligned} 5^8 \div 5^5 &= \frac{{}^{1}\cancel{5} \times {}^{1}\cancel{5} \times {}^{1}\cancel{5} \times {}^{1}\cancel{5} \times {}^{1}\cancel{5} \times 5 \times 5 \times 5}{{}_{1}\cancel{5} \times {}_{1}\cancel{5} \times {}_{1}\cancel{5} \times {}_{1}\cancel{5} \times {}_{1}\cancel{5}} \\ &= \frac{5 \times 5 \times 5}{1} \\ &= 5^3 \\ &= 125 \end{aligned}$$

Did you notice that the index of the final product is equal to the difference in the indices of the numbers being divided?

$$\begin{aligned} 5^8 \div 5^5 &= 5^{8-5} \\ &= 5^3 \end{aligned}$$

Rule 2 When dividing index numbers with the same base, keep the base and subtract the indices.

Index numbers to a power of 1

There are two ways to calculate the value of $3^5 \div 3^5$.

Method 1

$$\begin{aligned} 3^6 \div 3^5 &= \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 3}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

Method 2

Use the rule for dividing index numbers, which is to keep the base and subtract the indices. This simplifies the index number.

$$\begin{aligned} 3^6 \div 3^5 &= 3^{6-5} \\ &= 3^1 \end{aligned}$$

Therefore, $3^1 = 3$.

This is true for any base, not just 3, so we can write $a^1 = a$.

Rule 3 Any base to the power of 1 is equal to the base.

The zero power

There are two ways to calculate the value of $5^3 \div 5^3$.

Method 1

Use the rule for dividing index numbers, which is to keep the base and subtract the indices. This simplifies the index number.

$$5^3 \div 5^3 = 5^{3-3}$$
$$= 5^0$$

Method 2

Write in expanded form and cancel common factors.

$$5^3 \div 5^3 = \frac{5^3}{5^3} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5}} = 1$$

Therefore, $5^0 = 1$.

This is true for any base, not just 3, so we can write $a^0 = 1$.

Rule 4 Any number raised to the power of zero equals one. (The exception is zero 0^0 is undefined.)

Note the difference between 'simplify' and 'evaluate'.
To simplify an expression involving index numbers, we apply the rules for multiplying and dividing them, writing it in a simpler form.

To evaluate an expression involving index numbers, we find the value of the answer as an actual number.

Examples

1 Simplify the following.

a $3^7 \times 3^5$ **b** $2^3 \times 3^5 \times 2^4 \times 3^6$

Thinking	Working
a Are the bases all the same? If yes, keep the base and add the indices.	**a** $3^7 \times 3^5$ $= 3^{7+5}$ $= 3^{12}$
b Are the bases all the same? If not, group the numbers that have the same base and add their indices. Write the answer as a product of the index numbers with different bases.	**b** $2^3 \times 3^5 \times 2^4 \times 3^6$ $= 2^3 \times 2^4 \times 3^5 \times 3^6$ $= 2^{3+4} \times 3^{5+6}$ $= 2^7 \times 3^{11}$

2 Simplify the following.

a $3^9 \div 3^4$ **b** $(2^{11} \times 5^4) \div (2^9 \times 5^3)$

Thinking	Working
a Are the bases the same? (Yes.) If yes, write the base (3) and subtract the indices.	**a** $3^9 \div 3^4$ $= 3^{9-4}$ $= 3^5$
b **1** Are the bases the same? (No.) If not, rewrite the division as a fraction. Subtract the indices of numbers with the same base. **2** Write the answer as a product of index numbers with different bases.	**b** $(2^{11} \times 5^4) \div (2^9 \times 5^3)$ $= \frac{2^{11} \times 5^4}{2^9 \times 5^3}$ $= 2^{11-9} \times 5^{4-3}$ $= 2^2 \times 5$

Activity 9C

1 Simplify the following.

a $5^2 \times 5^3$ **b** $3^7 \times 3$

c $6^4 \times 6^2$ **d** $4^5 \times 4^2$

e $9^2 \times 9^7$ **f** $17^5 \times 17^6$

g $63^4 \times 63^{12}$ **h** $4^3 \times 5^2 \times 4^6 \times 5^3$

i $2^7 \times 6^3 \times 2^3 \times 64$ **j** $7^3 \times 8^4 \times 7 \times 8^8$

k $11^2 \times 9^2 \times 11^5 \times 9^5$ **l** $10^{10} \times 3^8 \times 10 \times 3^7$

2 Simplify, if possible, and then evaluate the following.

a $(-2)^2 \times -(2)^5$ **b** $(-6)^7 \times (-6)^2$

c $(-3)^4 \times (-3)^5$ **d** $(-7)^2 \times (-7)^2$

e $-5^2 \times 3^3$ **f** $-(2)^2 \times -4^2$

g $-6^2 \times (-8)^4$ **h** $-7^2 \times -9^2$

3 Simplify, the following.

a $7^4 \div 7^2$ **b** $3^9 \div 3^4$

c $2^8 \div 2^5$ **d** $13^4 \div 13^2$

e $17^6 \div 17^3$ **f** $25^8 \div 25^5$

g $(-2)^5 \div (-2^3)$ **h** $(-4)^4 \times (-4)^3$

i $(-6)^3 \div -(6)^2$ **j** $(3^4 \times 2^6) \div (3^2 \times 2^3)$

k $(6^5 \times 4^3) \div (6^3 \times 4^2)$ **l** $(9^{11} \times 13^5) \div (9^9 \times 13^4)$

m $(5^{12} \times 7^6) \div 7^5$ **n** $(11^2 \times 8^3) \div 11 \times 8$

o $(2^7 \times 3^5 \times 4^2) \div (2^5 \times 4)$

4 Simplify the multiplications first, then the divisions, and then evaluate each of the following.

a $\frac{8^2 \times 8^3}{8^4}$ **b** $\frac{5^5 \times 5^4}{5^6}$

c $\frac{1^{13} \times 1^5}{1^9}$ **d** $\frac{(-4)^2 \times (-4)^5}{(-4)^4}$

e $\frac{(-3)^3 \times (-3)^2}{-3}$ **f** $\frac{(-2)^5 \times (-2)^3}{-2^2}$

5 Evaluate the following.

a 7^0 **b** 20^0

c $\left(\frac{5}{6}\right)^0$ **d** $\left(-3\frac{1}{2}\right)^0$

e $(-0{\cdot}5234)^0$ **f** 3×5^0

g $2^2 \times 6^0$ **h** -3^0

i -6×5^0 **j** $3 \times (-6)^0$

6 **a** Evaluate each index number separately, then find the product or quotient.

i $4^5 \times 4^3$ **ii** $7^5 \div 7^2$

iii $(9^2 \times 9^3) \div 9^4$ **iv** $(5^3 \times 5^4) \div (5^4 \times 5^3)$

b Repeat part a, this time using the rules for multiplying and/or dividing index numbers to simplify, then evaluate the result.

c Which process is more efficient—the one used in part a, or in part b?

7 **a** Write the value of each of the following.

i 1^5 **ii** 1^7

iii 1^{15} **iv** 0^4

v 0^8 **vi** 0^{11}

b What conclusions can you draw from this about:

i powers of 1 **ii** powers of 0?

7 **a** State whether the value of the following will be positive or negative.

i $(-4)^3$ **ii** -4^3

iii $(-6)^6$ **iv** -6^6

b Explain how brackets may affect the signs of the answers for parts **i** and **ii** and for **iii** and **iv**.

c Find the value of the following.

i $-3^3 \times 4^2$ **ii** $(-2)^4 \times (-3)^2$
iii $-2^5 \times -5^2$ **iv** $(-4)^3 \times (-5)^2$
v $-2^3 \times (2^4)$ **vi** $-2^3 \times (-2)^4$

Answers 9C

1 a 5^5 **b** 3^8 **c** 6^6
d 4^7 **e** 9^9 **f** 17^{11}
g 63^{16} **h** $4^9 \times 5^5$ **i** $2^{10} \times 6^7$
j $7^4 \times 8^{12}$ **k** $11^7 \times 9^7$ **l** $10^{11} \times 3^{15}$

2 a $(-2)^7 = -128$ **b** $(-6)^9 = -10\,077\,696$
c $(-3)^9 = -19\,683$ **d** $(-7)^4 = 2401$
e $-25 \times 27 = -675$ **f** $4 \times -16 = -64$
g $-36 \times 4096 = -147\,456$ **h** $-49 \times -81 = 3969$

3 a 7^2 **b** 3^5 **c** 2^3
d 13^2 **e** 17^3 **f** 25^3
g $(-2)^2$ **h** -4 **i** -6
j $3^2 \times 2^3$ **k** $6^2 \times 4$ **l** $9^2 \times 13$
m $5^{12} \times 7$ **n** 11×8^2 **o** $2^2 \times 3^5 \times 4$

4 a 8 **b** 1 **c** 1
d -64 **e** 81 **f** -64

5 a 1 **b** 1 **c** 1
d 1 **e** 1 **f** 3
g 4 **h** -1 **i** -6
j 3

6 a **i** $4^5 \times 4^3 = 1024 \times 64 = 65\,536$
ii $7^5 \div 7^2 = 16\,807 \div 49 = 343$
iii $(9^2 \times 9^3) \div 9^4 = (81 \times 729) \div 6561 = 9$
iv $(5^3 \times 5^4) \div (5^3 \times 5^4) = (125 \times 625) \div (625 \times 125)$
$= 78\,125 \div 78\,125 = 1$

b **i** $4^5 \times 4^3 = 4^8 = 65\,536$
ii $7^5 \div 7^2 = 7^3 = 343$
iii $(9^2 \times 9^3) \div 9^4 = 9^5 \div 9^4 = 9$
iv $(5^3 \times 5^4) \div (5^3 \times 5^4) = 5^7 \div 5^7 = 5^0 = 1$

c The process used in b is more efficient.

7 a **i** 1 **ii** 1
iii 1 **iv** 0
v 0 **vi** 0

b **i** 1 raised to any positive whole number power is equal to 1.
ii 0 raised to any positive whole number power is equal to 0.

7 a **i** Negative **ii** Negative
iii Positive **iv** Negative

b A negative number when raised to an odd power will give a negative answer regardless of the brackets, as seen in i and ii. A negative number raised to an even power will give a positive answer when enclosed in brackets, as seen in iii, but if not enclosed in brackets will remain negative, as seen in iv.

c **i** $-27 \times 16 = -432$ **ii** $16 \times 9 = 144$
iii $-32 \times -25 = 800$ **iv** $-64 \times 25 = -1600$
v $-8 \times 16 = -128$ **vi** $-8 \times 16 = -128$

9D • Managing brackets: Other index laws

LB2 Pages 32–33

Specific learning outcomes

Learners should be able to:

8.9.5.1 Expand the bracket with indices and simplify using the index laws with brackets.

$$(b^n)^p = b^{n \times p}$$
$$(a \times b)^n = a^n \times b^n$$
$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

Teaching points

1 Expand brackets with indices and simplify them using the index laws.

Learner difficulties and remedies

Difficulty

Remembering which rule to use when index numbers are raised to a power.

Remedy

- When raising a number in index form to a power, keep the base and multiply the indices.

Difficulty

Remembering which rule to use when a product or quotient of factors are raised to a power.

Remedy

- When a product or quotient of factors in brackets has been raised to a power, raise each factor in the brackets to that power.

Suggested teaching approach

- Use the index law for index numbers when bases are raised to a power of a power.
- Use all the index laws to simply expressions with index numbers.
- Remind learners of the difference between evaluating and simplifying index numbers.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Raising an index form to a power

$(7^2)^3$ is 7^2 multiplied by itself three times.

$$\begin{aligned}(7^2)^3 &= 7^2 \times 7^2 \times 7^2 \\ &= 7^{2+2+2} \\ &= 7^6\end{aligned}$$

$$\begin{aligned}(7^2)^3 &= (7 \times 7) \times (7 \times 7) \times (7 \times 7) \\ &= 7^6\end{aligned}$$

Using the pattern here we can see that:

$$(7^2)^3 = 7^{2\times 3}$$
$$= 7^6$$

This is true for any base, not just 7, so we can write $(a^n)^p = a^{n\times p}$.

Rule 5 When raising a number in index form to a power, keep the base and multiply the indices

Raising a product to a power

How could we calculate $(4\times 7)^3$?

We could find the product in brackets first, then raise it to the power.

$$(4\times 7)^3 = (28)^3$$
$$= 21\,952$$

We could raise each factor in the product to the power.

$$(4\times 7)^3 = (4\times 7)\times(4\times 7)\times(4\times 7)$$
$$= 4\times 4\times 4\times 7\times 7\times 7$$
$$= 4^3\times 7^3$$
$$= 64\times 343$$
$$= 21\,952$$

Using the pattern here we can see that:

$$(4\times 7)^3 = 4^3\times 7^3$$

This is true for all bases, not just 4 and 7, so we can write $(a\times b)^n = a^n\times b^n$.

Rule 6 If a product in brackets has been raised to a power, each factor in the brackets is raised to that power.

Raising a quotient to a power

What does $\left(\frac{2}{5}\right)^3$ simplify to? We have a base of $\frac{2}{5}$ and an index of 3.

$$\left(\frac{2}{5}\right)^3 = \frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}$$
$$= \frac{2\times 2\times 2}{5\times 5\times 5}$$
$$= \frac{2^3}{5^3}$$
$$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$$

Each number in the brackets has been raised to the power of 3.

Using the pattern here we can see that:

$$\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$$

This is true for all bases, not just 4 and 7, so we can write:

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

or

$$(a\div b)^p = a^p\div b^p$$

Rule 7 If a quotient in brackets has been raised to a power, each number in the brackets is raised to that power.

Examples

1 Simplify $(2^3)^5$

Thinking	Working
Keep the base and multiply the indices.	$(2^3)^5$ $= 2^{3\times 5}$ $= 2^{15}$

2 Expand the brackets in the following.

a $(2\times 3)^4$

b $\left(\frac{5}{7}\right)^2$

Thinking	Working
a Raise every factor in the brackets to the power outside the brackets.	a $(2\times 3)^4$ $= 2^4\times 3^4$
b Raise every number in the brackets to the power outside the brackets.	b $\left(\frac{5}{7}\right)^2$ $= \frac{5^2}{7^2}$

Activity 9D

1 Simplify the following.

a $(9^6)^2$

b $(14^7)^3$

c $(6^8)^8$

d $(10^5)^5$

e $(4^2)^3$

f $(5^2)^0$

g $(2^4)^5$

h $(7^0)^2$

2 Expand the brackets in the following.

a $(4\times 3)^2$

b $(2\times 5)^3$

c $(3\times 2)^5$

d $(4\times 7)^4$

e $(7\times 10)^3$

f $(8\times 9)^1$

g $(2\times 3\times 1)^6$

h $(4\times 5\times 6)^2$

i $\left(\frac{3}{5}\right)^3$

j $\left(\frac{2}{9}\right)^2$

k $\left(\frac{10}{13}\right)^3$

l $\left(\frac{12}{13}\right)^2$

m $\left(\frac{1}{2}\right)^5$

n $\left(\frac{1}{10}\right)^3$

o $\left(\frac{1}{11}\right)^6$

p $\left(\frac{1}{12}\right)^2$

3 Use a combination of two or more of the rules for working with index numbers to simplify the following. Leave your answers in index form (do not evaluate).

a $(8^2\times 7^3)^2$

b $\left(\frac{6^2}{7^2}\right)^3$

c $\frac{(2\times 4)^5}{2}$

d $\frac{(5^3)^2}{5^2}$

e $\frac{3^4\times 4^7}{9^2\times 3^4}$

f $\frac{3^7\times 4^0}{3^5\times 4^5}$

g $\frac{(7\times 8)^4}{7^2}$

h $\frac{(8\times 9\times 10)^3}{(9\times 8)^3}$

i $\frac{(4^3\times 5)^4}{(5^2\times 4)^2}$

j $(2\times 3)^4\times\left(\frac{1}{3}\right)^2$

k $\left(\frac{3}{2}\right)^4\times\left(\frac{2}{3}\right)^3$

l $\left(\frac{1}{3^5}\right)^2\times\left(\frac{2}{3}\right)^2$

4 Evaluate: $\left(\frac{3^3\times 2^5}{3^2\times 2^3}\right)^2$

5 Use the appropriate index law to find the value of x in each of the following.

a $10^{12}\div 10^5 = 10^x$

b $(5^6)^3 = 5^x$

c $(6^x)^2 = 6^{20}$

d $\frac{7^4\times 7^x}{7^{11}} = 1$

e $\frac{(3^4)^x}{3^{12}} = 1$

f $\frac{(5^6)^x\times(5^x)^2}{5^4} = 5^4$

Answers 9D

1 **a** 9^{12} **b** 14^{21}

c 6^{64} **d** 10^{25}

e 4^6 **f** 1

g 2^{20} **h** 1

2 **a** $4^2 \times 3^2$ **b** $2^3 \times 5^3$

c $3^5 \times 2^5$ **d** $4^4 \times 7^4$

e $7^3 \times 10^3$ **f** 8×9

g $2^6 \times 3^6$ **h** $4^2 \times 5^2 \times 6^2$

i $\frac{3^3}{5^3}$ **j** $\frac{2^2}{9^2}$

k $\frac{10^3}{13^3}$ **l** $\frac{12^2}{13^2}$

m $\frac{1^5}{2^5}$ **n** $\frac{1^3}{10^3}$

o $\frac{1^6}{11^6}$ **p** $\frac{1^2}{12^2}$

3 **a** $8^4 \times 7$ **b** $\frac{6^{2\times3}}{7^{2\times3}} = \frac{6^6}{7^6}$

c $2^4 \times 4^5$ **d** $\frac{5^{3\times2}}{5^2} = \frac{5^6}{5^2} = 5^4$

e $\frac{4^7}{9^2}$ **f** $\frac{3^2}{4^5}$

g $\frac{7^4 \times 8^4}{7^2} = 7^2 \times 8^4$ **h** $\frac{8^3 \times 9^3 \times 10^3}{9^3 \times 8^3} = 10^3$

i $\frac{4^{12} \times 5^4}{5^4 \times 4^2} = 4^{10}$ **j** $2^4 \times 3^4 \times \frac{1^2}{3^2} = 2^4 \times 3^2$

k $\frac{3^4}{2^4} \times \frac{2^3}{3^3} = \frac{3}{2}$ **l** $\frac{1^2}{3^{10}} \times \frac{2^2}{3^2} = \frac{2^2}{3^{12}}$

5 **a** $10^7 = 10^x, x = 7$ **b** $5^{18} = 5^x, x = 18$

c $6^{2x} = 6^{20}, x = 10$ **d** $\frac{7^{4+x}}{7^{11}} = \frac{7^{11}}{7^{11}} = 1, x = 7$

e $\frac{3^{4x}}{3^{12}} = \frac{3^{12}}{3^{12}} = 1, x = 3$ **f** $\frac{5^{8x}}{5^4} = \frac{5^8}{5^4} = 5^4, x = 1$

9E • Exploring scientific notation

LB2 Pages 34–35

Specific learning outcomes

Learners should be able to:

8.9.6.1 Define scientific notation:

Scientific notation: numbers that are written as a product of a decimal number between 1 and 10, and an integral power of 10.

8.9.7.1 Convert large ordinary numbers to scientific notation.

8.9.7.2 Convert small decimal numbers (less than 1) to scientific notation.

Teaching points

1. Explain, with examples, numbers that are written in scientific notation.
2. Write large numbers in scientific notation with positive powers of 10.
3. Write very small numbers less than 1 in scientific notation with negative powers of 10.

Suggested teaching approach

- Learners complete **Learning Task 9E** on pages 34 and 35 of the LB to investigate scientific notation.

• Chapter Ten •

The Coordinate Plane

Overview

Coordinate geometry uses the axes on a Cartesian plane to locate points, lines, curves and shapes and to find relationships between them.

- The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.
- The point of intersection of these axes is called the origin, which has the coordinates (0, 0).
- All points can be described by an ordered pair known as coordinates (x, y) as measured from the origin.

In this chapter coordinate geometry will be used to explore linear relationships.

In the Solomon Islands, people have used the positions of stars, the Moon, the Sun and surrounding islands to travel from villages to villages and from island to island. They were unknowingly using coordinates to locate villages and islands from a distance.

Contents

Chapter skills

This chapter covers the following skills:

- Plotting points on a set of Cartesian axes
- Plotting sets of points that follow simple mathematical rules
- Developing and graphing simple linear and non-linear relations that model real-life situations
- Performing reflections about the line $y = x$
- Graphing straight lines using the gradient and y-intercept method

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 10A Plotting points on the cartesian plane	Learner's Book 1 • Exercise 10A, pages 47, 48
2	• 10B Linear patterns and simple rules	Learner's Book 1 • Exercise 10B, page 50
3	• 10C Combination rules	Learner's Book 1 • Exercise 10C, pages 52, 53
4–5	• 10D Plotting a line using a rule	Learner's Book 1 • Exercise 10D, pages 54, 55 and 56
6	• 10E Horizontal and vertical lines	Learner's Book 1 • Exercise 10E, pages 57, 58
7	• 10F Using the x- and y-intercepts	Learner's Book 1 • Exercise 10F, page 59
8	• 10G Gradients of straight lines	Learner's Book 1 • Exercise 10G, pages 61, 62
9	• 10H The gradient and the y-intercept	Learner's Book 1 • Exercise 10H, page 64
10	• 10I Finding equations of lines	Learner's Book 1 • Exercise 10I, page 65
11	• Test	Teacher's Guide • Chapter 10 Test

General learning outcomes

Learners should:

Plotting points on the plane

8.10.1 Understand the Cartesian plane with regards to the axes, sides and coordinates. (U)

8.10.2 Know how to name and plot coordinates of given objects or points on the coordinate plane. (K)

Linear patterns and simple rules

8.10.3 Understand that groups of points with linear alignment have rules that relates x- and y-values. (U)

Combination rules

8.10.4 Understand that the value of y can be determined by computing x-values with other values. (U)

8.10.5 Know how to find the rule that relates the x- and y-values that are given in tables and graphs. (K)

Plotting a line using a rule

8.10.6 Know how to plot graphs from given equations. (K)

8.10.7 Know how to use the equations (rules) to find missing values then plot the points on the coordinate plane. (K)

Horizontal and vertical lines

8.10.8 Understand that lines that run horizontal and vertical lines or graphs have single values equated to them. (U)

8.10.9 Know how to find the equations and sketch graphs of horizontal and vertical lines. (K)

Using the x- and y-intercepts

8.10.10 Understand that the most vital points be used to graph linear equations are x- and y-intercepts. (U)

Gradients of straight lines

8.10.11 Understand gradient and how it is determined. (U)

8.10.12 Know how to calculate the gradient of a straight line graph. (K)

The gradient and *y*-intercept

8.10.13 Know the general equation for straight line and identify the gradient and *y*-intercept. (K)

8.10.14 Know how to sketch linear graphs. (K)

Finding equations of lines

8.10.15 Know how to find the equations of given linear graphs. (K)

10A • Plotting points on the Cartesian plane

LB2 Pages 46–48

Specific learning outcomes

Leaners should be able to:

8.10.1.1 Identify and name key components of the Cartesian plane: *x*- and *y*-axes, and origin.

8.10.2.1 Plot coordinate pairs of points and objects on the coordinate plane.

Teaching points

1 Identify the key features of the Cartesian plane: axes, quadrants and the origin.

2 Plot coordinate points on a coordinate plane.

Learner difficulties and remedies

Difficulty

Seeing the relationship between the numbers along the *x*-axis and numbers on *y*-axis.

Remedy

- Explain to learners that in a coordinate plane two sets of numbers are used: an *x*-axis (horizontal line) and a *y*-axis (vertical line).

Difficulty

Plotting points on the Cartesian plane.

Remedy

- To plot a coordinate point, plot the first number in the pair on the *x*-axis, and the second number on the *y*-axis.
- To write a coordinate point use the rule that the *x*-coordinate comes first and *y*-coordinate comes second in the brackets.
- To locate a point, identify the *x*-coordinate and draw a vertical line through the point on the *x*-axis. Then identify the *x*-coordinate and draw a horizontal line through the point on the *y*-axis. The two lines will intersect in the Cartesian plane at the coordinate point.

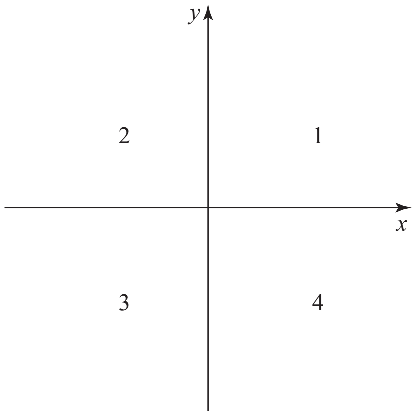
Suggested teaching approach

- When naming points on the coordinate plane:
  - identify the point
  - write the coordinates in brackets with *x* first and *y* second.
- When plotting coordinate points:
  - identify the pair of numbers to be plotted on the coordinate plane
  - plot the point at the intersection of a vertical line through the point on the *x*-axis and a horizontal line through the point on the *y*-axis.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A Cartesian plane is made up of two number lines that intersect at right angles. The horizontal number line is generally called the *x*-axis and the vertical is usually called the *y*-axis. We use *x*- and *y*-coordinates in the form (*x*, *y*) to locate the positions of points in the plane.

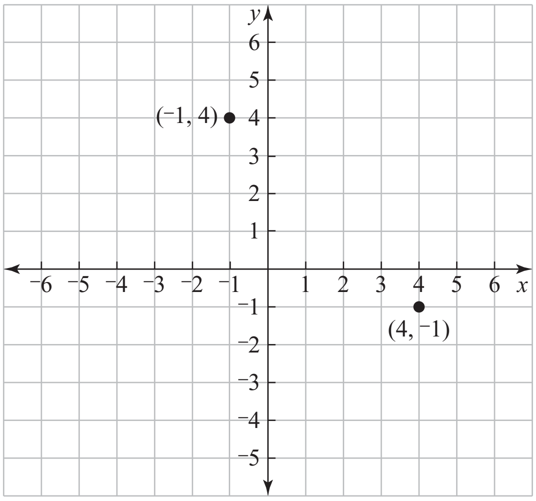
The Cartesian plane is also called the coordinate plane. The two axes divide the plane into four quadrants, numbered anticlockwise from the top right-hand side as shown.



The points on a Cartesian or coordinate plane are named using the numbers on the *x*-axis and the *y*-axis as a coordinate point (*x, y*). The first number comes from the *x*-axis, which is the horizontal line. The second number comes form *y*-axis, which is the vertical line. The order of the coordinates is important. (2, 7) is a different point to (7, 2).

Examples

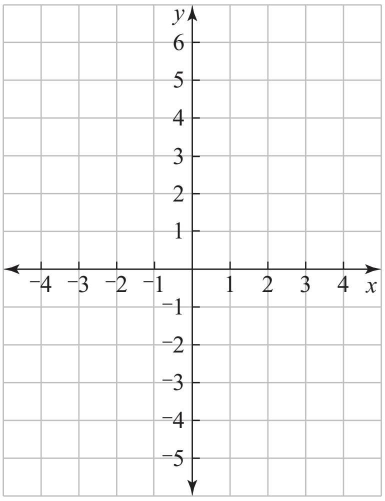
Plot the coordinate points (4, −1) and (−1, 4) on the same set of axes.

Thinking	Working
1 Identify the coordinates that need to be plotted.	(4, −1) and (−1, 4)
2 Find the *x*-coordinate and the *y*-coordinate.	(4, −1) *x*-coordinate = 4 *y*-coordinate = −1 (−1, 4) *x*-coordinate = −1 *y*-coordinate = 4
3 Plot and label the points on a Cartesian plane.	

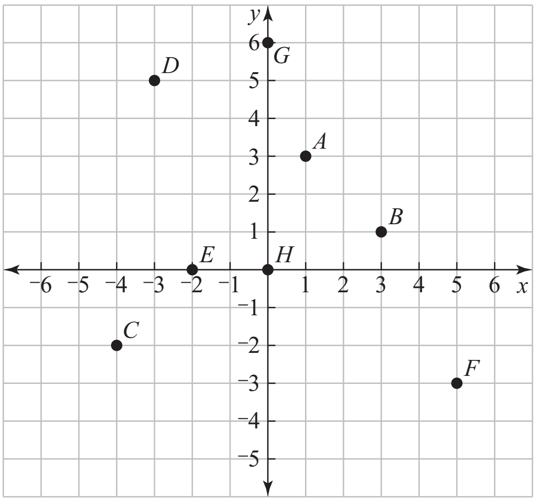
Activity 10A

1 Plot these points on the axes provided:

a (3, 2)
b (1, 6)
c (−3, 4)
d (−4, 2)
e (3, 0)
f (0, −5)



2 Write down the coordinates of the points marked *A*–*H*.



3 a Plot all nine points on a piece of graph paper.

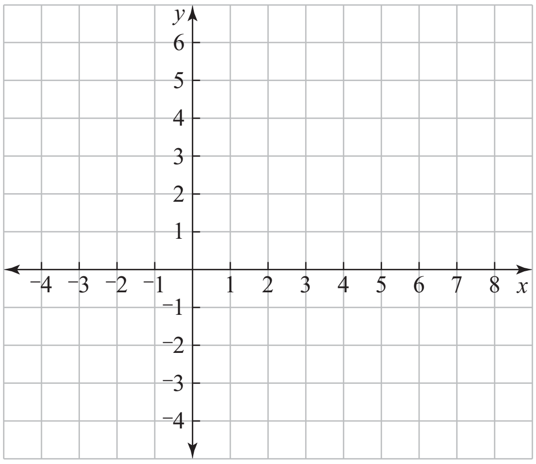
A = (3, 4) *B* = (2, 2) *C* = (7, 2)
D = (−2, 4) *E* = (−3, 2) *F* = (−4, 0)
G = (−1, 0) *H* = (3, 0) *I* = (1, −4)

b Which set of three points forms a right-angled triangle?

c Which set forms an isosceles triangle?

d Which set of three points lie in a straight line?

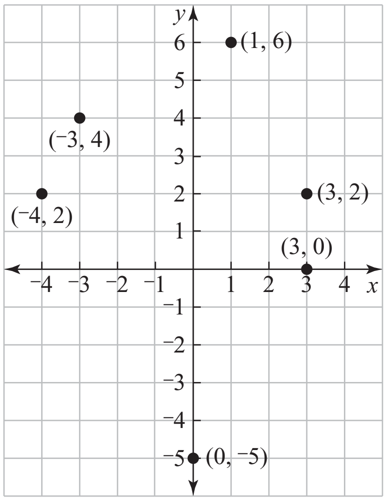
4 Plot these points and join them in the order listed. Join the last point to the first also: (2, 3) (6, 1) (2, −1) (0, −5) (−2, −1) (−6, 1) (−2, 3) (0, 7)



What shape does this give?

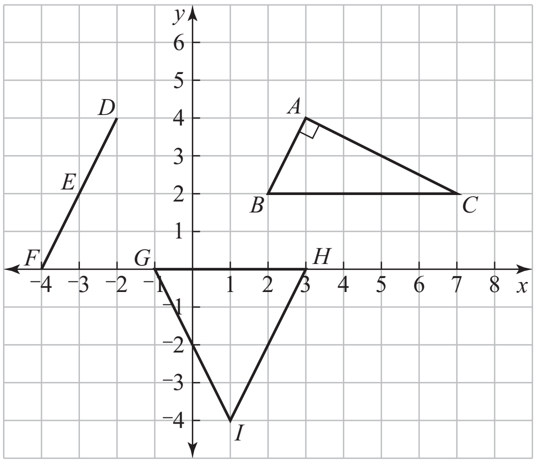
Answers 10A

1



2 $A = (1, 3)$ $B = (3, 1)$ $C = (4, -2)$
$D = (-3, 5)$ $E = (-2, 0)$ $F = (5, -3)$
$G = (0, 6)$ $H = (0, 0)$

3 **a**



b A, B and C

c G, H and I

d D, E and F

4 A four-pointed star.

10B • Linear patterns and simple rules

LB2 Pages 49–50

Specific learning outcomes

Learners should be able to:

8.10.3.1 List the coordinates of points that are in linear alignment.

8.10.3.2 Deduce a rule or equation for a a linear set of coordinates.

8.10.3.3 Deduce a rule or equation from a set of coordinates in tables.

Teaching points

1 Plot points that are in a linear alignment on a Cartesian plane.

2 Identify the coordinates of the plotted points.

3 Identify the rule for a set of points in a linear alignment.

Learner difficulties and remedies

Difficulty

Identifying the rule that relates all the coordinate points on a straight line (linear graph).

Remedy

- Find the rule by asking questions about what change in x would result in a particular change in y.
- Find the y-value when $x = 0$. Then look at how y changes as x increases by 1.

Suggested teaching approach

- To find a rule for a linear relationship see what change in x results in a particular change in y. Look for the y-value when $x = 0$.
- Use other numbers to try and find the rule.
- Make sure that the rule applies to all coordinate points.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **linear graph** is a straight line graph that shows the relationship between two variables. It can be drawn if we know the rule that connects the two variables. This rule shows a **linear relationship**. The rule or equation is used to complete a table of values that shows the x- and y-coordinates of some points on the line. These are plotted on the **Cartesian plane** and connected with a line.

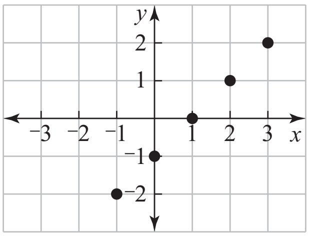
Some simple rules link the x- and y-coordinates to give a straight line graph such as $y = x$, $y = x + 2$ or $y = 2x$.

Examples

1 **a** Use a table of values to display the points (3, 2), (2, 1), (1, 0), (0, −1), (−1, −2).

b Use the table of values to draw a graph of the relationship.

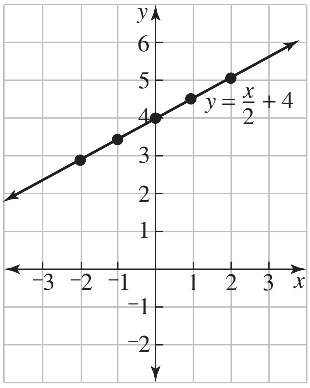
c Write down the rule that relates x to y.

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>a In a table of values, write the <i>x</i>-values in the first row and the <i>y</i>-values in the second row.</td><td><table><tr><td>x</td><td>−1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y</td><td>−2</td><td>−1</td><td>0</td><td>1</td><td>2</td></tr></table></td></tr>
<tr><td>b Draw and label the Cartesian plane, plot the points, connect them with a straight line.</td><td>
</td></tr>
<tr><td>c Each <i>y</i>-value is one less than its corresponding <i>x</i>-value.</td><td>The points all lie of the line with the rule <i>y</i> = <i>x</i> – 1.</td></tr>
</table>

2 a Copy and complete the following table of values for the rule $y = \frac{x}{2} + 4$ for values of x in the range −2 to 2.

x	−2	−1	0	1	2
y					
(x, y)					

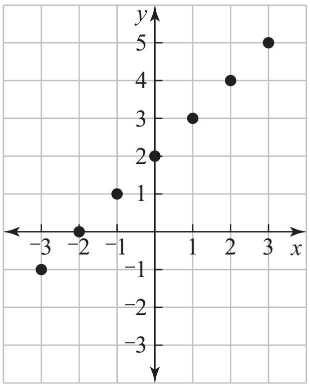
b Use the table of values to draw a graph of the relationship.

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>a 1 Substitute each value of <i>x</i> into the rule to find the corresponding value of <i>y</i>.</td><td>a $y = \frac{x}{2} + 4$
$y = \frac{-2}{2} + 4$ when $x = -2$
$= 3$
$y = \frac{-1}{2} + 4$ when $x = -1$
$= 3{\cdot}5$
$y = \frac{0}{2} + 4$ when $x = 0$
$= 4$
$y = \frac{1}{2} + 4$ when $x = 1$
$= 4{\cdot}5$
$y = \frac{2}{2} + 4$ when $x = 2$
$= 5$</td></tr>
<tr><td>2 Write the <i>y</i>-values in the second row and enter the ordered pairs in the third row.</td><td><table><tr><th>x</th><th>y</th><th>(x, y)</th></tr><tr><td>−2</td><td>3</td><td>(−2, 3)</td></tr><tr><td>−1</td><td>3·5</td><td>(−1, 3·5)</td></tr><tr><td>0</td><td>4</td><td>(0, 4)</td></tr><tr><td>1</td><td>4·5</td><td>(1, 4·5)</td></tr><tr><td>2</td><td>5</td><td>(2, 5)</td></tr></table></td></tr>
<tr><td>b Draw and label the <i>x</i> and <i>y</i> Cartesian axes, plot the points, connect them with a straight line and label the graph with the rule.</td><td>
</td></tr>
</table>

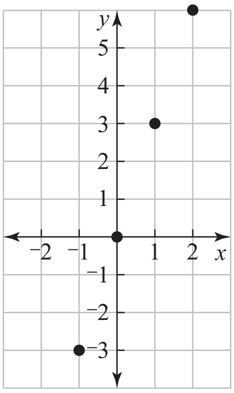
Activity 10B

1 For each diagram make a list of the coordinates of the plotted points. Then write down the rule that relates x to y.

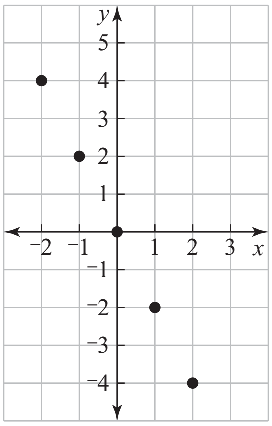
a



b



c



2 a Draw a graph to display the points (−2, 4), (−1, 2), (0, 0), (1, −2), (2, −4).

b Write down the rule that relates x to y.

3 Write down the rule that relates x to y for each table of values.

a

x	−2	−1	0	1	2
y	6	7	8	9	10

b

x	−2	−1	0	1	2
y	−5	−4	−3	−2	−1

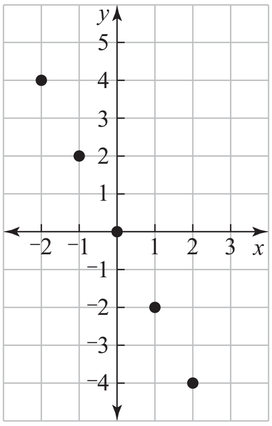
c

x	−2	−1	0	1	2
y	6	3	0	−3	−6

Answers 10B

1 a $y = x + 2$ **b** $y = 3x$

2 a



b $y = -2x$

3 a $y = x + 8$ **b** $y = x - 3$ **c** $y = -3x$

10C • Combination rules

LB2 Pages 51–53

Specific learning outcomes

Learners should be able to:

8.10.4.1 Find the rule to relate x-values to y-values by multiplying the x-value by a number then either add or subtract by a constant.

8.10.5.1 Find the rules that relate values of y to x in the given in tables.

8.10.5.2 Find the rule or equations for given linear graphs.

8.10.4.1 Identify patterns of coordinates to determine values for m and c for linear equations in the form $y = mx$.

8.10.5.1 Deduce the linear equations from tables of coordinates.

8.10.5.2 Find the rule or equation from a given graph.

Teaching points

1. Find the rules that relate the x- and y-coordinates through the use of the four operations.
2. Identify a rule for a set of coordinates in a linear pattern, given a table of coordinates.
3. Find the rule for a given linear graph.

Learner difficulties and remedies

Difficulty

Identifying the rule for linear patterns of coordinates that relates the x- and y-coordinates.

Remedy

- Use trial and error to find the rule. First look for the y-value when $x = 0$. Then look at how y changes as x increases by 1.

Suggested teaching approach

- Use trial and error to find the rule for a linear relationship. First look for the y-value when $x = 0$. Then look at how y changes as x increases by 1.
- See what change in x results in a particular change in y. Look for the y-value when $x = 0$.
- Use other numbers to try and find the rule.
- Make sure that the rule is satisfied so that all the ordered pairs make true statements when they are substituted into the rule.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

We can identify linear relationships and find their rule using any of the following methods.

1 Using ordered pairs

If we have a list of ordered pairs we can look for a pattern.

(1, 8), (2, 15), (3, 22), (4, 29), (5, 36), (6, 43)

The value of y increases by 7 as the x-value increases by 1, so $7x$ is a term in our rule 1 needs to be added to complete our rule.

Rule: $y = 7x + 1$

2 Using a graph

The ordered pairs (⁻2, ⁻13), (0, 1) and (2, 15) plotted on a Cartesian plane can be connected with a straight line. They form a linear relationship.

The y-value increases by 14 when the x-value increases by 2. That means that the y-value increases by 7 as the x-value increases by one. So, we need $7x$ in our rule, and 1 is added to find the y-value.

Rule: $y = 7x + 1$.

3 Satisfying the rule

Satisfying the rule means that the ordered pairs make true statements when they are substituted into the rule. By substituting (⁻2, ⁻13), (0, 1) and (2, 15) into $y = 7x + 1$, we can see that all points satisfy this rule.

$^-13 = 7 \times {}^-2 + 1$

$1 = 7 \times 0 + 1$

$15 = 7 \times 2 + 1$

Sometimes, when given a list of ordered pairs (or a table of values) only one operation has been performed on x to obtain y. Finding the rule is very easy. Be careful to check that the rule applies to all points.

Sometimes rules involve two steps. The y-value is worked out by multiplying the x value by a certain number m and after that step some other constant number c is added or subtracted.

The multiplying number m is given by the increase between each of the successive y-values.

Examples

1 Find the rule for each of the following sets of points.

a (0, 3), (1, 4), (2, 5), (3, 6), (4, 7)

b (⁻3, ⁻6), (1, 2), (4, 8), (6, 12)

Thinking	Working
a **1** Look for a pattern. Can we perform one operation on x to obtain y? Yes (+ 3).	**a** 3 has been added to each x-value to obtain y.
2 Write the rule.	$y = x + 3$
3 Check the rule by substituting one x-value from the list into the equation to obtain the given y-value.	Check: Using the point (4, 7) $y = x + 3$ $= 4 + 3$ when $x = 4$ $= 7$ Our rule is correct.
b **1** Look for a pattern. Can we perform one operation on x to obtain y? Yes (× 2).	**b** Each x-value has been multiplied by 2 to obtain y.
2 Write the rule.	$y = 2x$
3 Check the rule by substituting one x-value from the list into the equation to obtain the given y-value.	Check: Using the point (4, 8) $y = 2x$ $= 2 \times 4$ when $x = 4$ $= 8$ Our rule is correct.

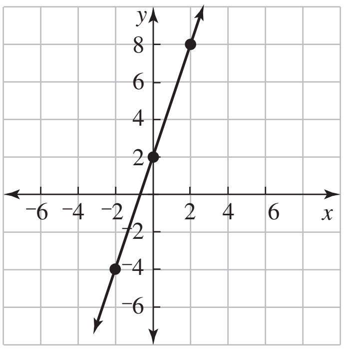
When more than one operation has been performed on x to obtain y, we cannot see the relationship so easily. We need to look for another pattern.

2 Find the rule for the following table of values.

x	−3	1	5	9
y	−24	−4	16	36

Thinking	Working
1 Look for a pattern. Can we perform one operation on x to obtain y? No. (Subtracting 21 or multiplying by 8 works for the first point, but not for the others.)	We cannot perform one operation on x to obtain y.
2 Look for another pattern. By how much is the x-value increasing? By how much is the y-value increasing? Use this information to find the amount by which y is increasing as x increases by 1. This is the coefficient of x.	y increases by 20 as x increases by 4, so y increases by 5 as x increases by 1. We need $5x$ in our rule.
3 What number needs to be added or subtracted to find the y-value? This is the constant in the rule.	9 needs to be subtracted.
4 Write the rule.	$y = 5x - 9$
5 Check the rule by substituting one x-value from the table into the rule to obtain the corresponding y-value.	Check: use the values $x = 5$, $y = 16$ $y = 5x - 9$ $= 5 \times 5 - 9$ $= 25 - 9$ $= 16$ Our rule is correct.

3 Find the rule that describes the following relationship.



Thinking	Working
1 List at least three points that lie on the line.	(−2, −4), (0, 2), (2, 8)
2 Look for a pattern. By how much is the x-value increasing? By how much is the y-value increasing? Use this information to find the amount by which y is increasing as x increases by 1. This number is the coefficient of x.	We cannot perform one operation on x to obtain y. y increases by 6 as x increases by 2, so y increases by 3 as x increases by 1. We need $3x$ in our rule.
3 After the x-value is multiplied by the coefficient found in step 2 (3), what number needs to be added or subtracted to find the corresponding y-value?	2 needs to be added.
4 Write the rule.	$y = 3x + 2$
5 Check the rule by substituting one x-value from the list into the equation to obtain the given y-value.	Check: Using the point (2, 8) $y = 3x + 2$ $= 3 \times 2 + 2$ when $x = 2$ $= 6 + 2$ $= 8$ Our rule is correct.

Activity 10C

1 Find the rule for each of the following sets of points.

a (0, −1), (1, 0), (2, 1), (3, 2), (4, 3)

b (2, 3), (3, 4), (4, 5), (6, 7), (7, 8)

c (−3, 3), (−1, 5), (2, 8), (3, 9), (5, 11)

d (−2, −5), (0, −3), (2, −1), (5, 2), (6, 3)

e (1, 1), (3, 3), (4, 4), (7, 7), (10, 10)

f (−1, −4), (0, 0), (1, 4), (2, 8), (3, 12)

g (−2, −10), (0, 0), (2, 10), (4, 20), (6, 30)

h (−2, 2), (−1, 1), (0, 0), (1, −1), (2, −2)

2 Find the rule for the following tables of values.

a

x	−7	−7	1	4
y	−6	−4	2	5

b

x	−2	3	6	7
y	−5	0	3	4

c

x	−4	0	5	7
y	8	0	−10	−14

d

x	−2	−1	5	6
y	−14	−7	35	42

e

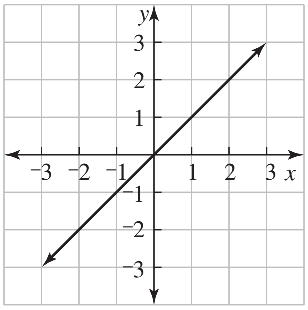
x	−6	−3	0	6
y	2	1	0	−2

f

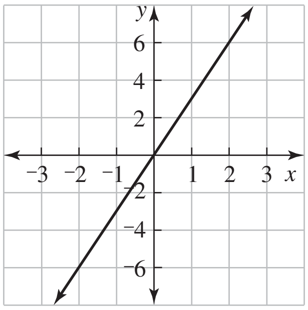
x	−5	0	10	20
y	−1	0	2	4

3 Find the rule that describes each of the following relationships.

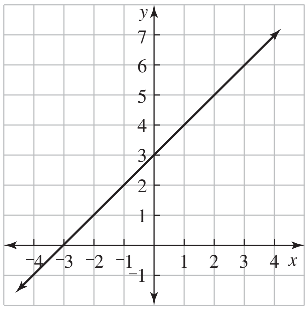
a



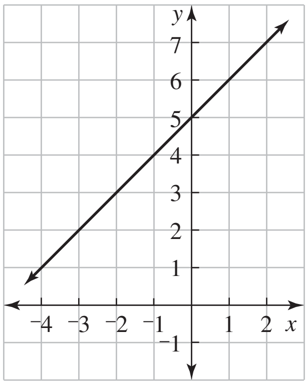
b



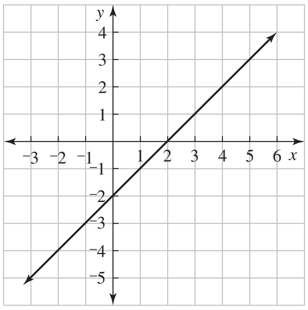
c



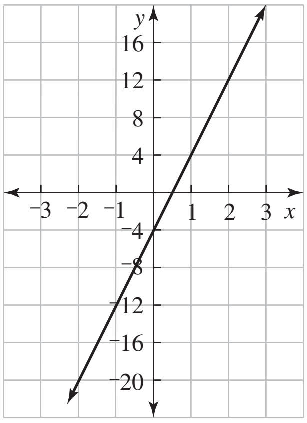
d



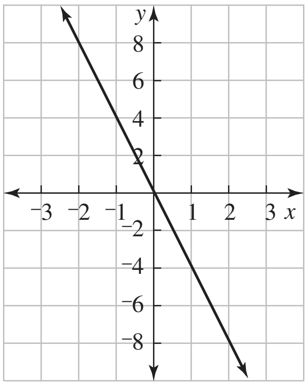
e



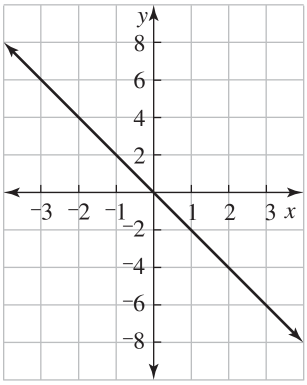
f



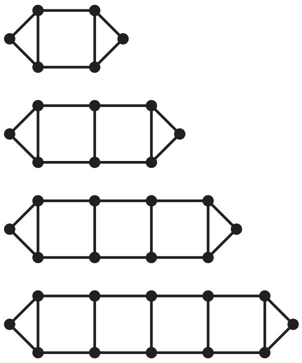
g



h



4 a Write a rule for the number of dots (*D*) in terms of the numbers of squares (*S*) in the pattern below.



b Explain how you found the coefficient of *S* in your rule.

c Explain how you found the amount to add on in your rule.

d Use your rule to find the number of dots needed to construct 8 squares with this pattern.

Answers 10C

1 a $y = x - 1$
b $y = x + 1$
c $y = x + 6$
d $y = x - 3$
e $y = x$
f $y = 4x$
g $y = 5x$
h $y = {}^-x$

2 **a** $y = x + 1$ **b** $y = x - 3$

c $y = {}^{-}2x$ **d** $y = 7x$

e $y = -\frac{x}{3}$ **f** $y = \frac{x}{5}$

3 **a** $y = x$ **b** $y = 3x$

c $y = x + 3$ **d** $y = x + 5$

e $y = x - 2$ **f** $y = 8x - 4$

g $y = -4x$ **h** $y = -2x$

4 **a** $D = 2S + 4$

S	1	2	3	4
D	6	8	10	12

b D increases by 2 as S increases by 1, so S is multiplied by 2.

c To obtain D, 4 needs to be added to each S value after doubling.

d $D = 2 \times 8 + 4$
$= 16 + 4$
$= 20$ dots

10D • Plotting lines using a rule

LB2 Pages 54–56

Specific learning outcomes

Learners should be able to:

8.10.6.1 Define a linear relation.

8.10.6.2 Plot graphs by substituting x-values into the equation to find the corresponding y-values.

8.10.7.1 Complete given tables using given rules then plot the points on a set of Cartesian axes.

Teaching points

1. Explain that a linear relation is a rule or equation of a straight line graph.
2. Plot graphs by evaluating tables with the x- and y-coordinates.
3. Complete tables using given rules, and then plot the graph on the Cartesian plane.

Learner difficulties and remedies

Difficulty

Plotting the coordinate pairs from a given equation.

Remedy

- Start by substituting $x = 0$ into the equation as this will identify where the line crosses the vertical axis.
- Substitute two further values for x. Plot the points. If all three points line up as in a straight line the substitutions are likely to be correct and the graph can be drawn and labelled with its equation.

Suggested teaching approach

- Substitute x-values to a given rule or equation to find the corresponding y-values. Write in a table of values.
- Plot the coordinate points on a coordinate plane.
- Label the axes with the variables and check whether units are involved.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Many situations with two variables demonstrate a linear relationship if they increase or decrease at a constant rate. For example, a daily rate of pay for work done, a steady flow rate of a liquid, or the speed of a vehicle traveling at a constant speed all have constant rates of change.

In mathematical word problems that involve linear relationships we need to choose a suitable pronumeral to represent a variable. In measurement, we used h to represent the height and V to represent the volume. When time is a variable we usually represent it by t.

Examples

1 For each statement, define each of the two variables involved.

a The volume of water in litres in a water tank increases during a day of heavy rain.

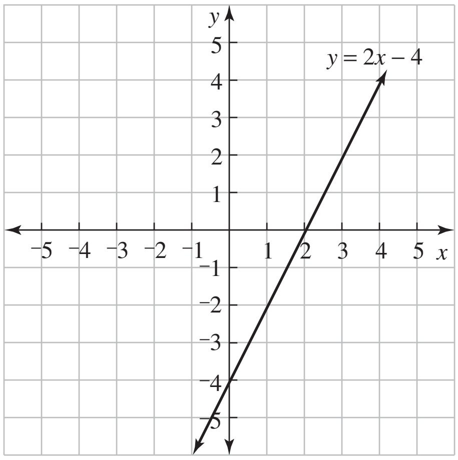
b The average speed of a car as it travels over a distance measured in kilometres.

Thinking	Working
a **1** Identify the two variables. **2** Define each variable.	**a** volume and time Let V be the volume of water in the tank in litres. Let t be the time during which the tank is filling in hours.
b **1** Identify the two variables. **2** Define each variable.	**b** distance and speed Let d be the distance travelled (km). Let s be the speed the car travelled (km/hr).

2 Define all the variables you are using in the following situation and then write a linear equation to represent the total cost.
The cost of a building project at a school consists of a \$1000 fixed cost to cover the materials used and a variable cost of the labour, which is calculated by a fixed rate of \$30 per hour.

Thinking	Working
1 Identify and define the variables.	Let C be the cost of the project and h be the number of hours of labour.
2 Identify the fixed costs.	Cost of materials = \$1000 Hourly rate of labour = \$30
3 Write the equation.	$C = 30h + 1000$

3 For the graph with the rule $y = 2x - 4$:



a state the x-intercept and write the coordinates of the point

b state the y-intercept and write the coordinates of the point

c state whether the gradient of the line is positive or negative.

Thinking	Working
a The x-intercept is where the graph cuts the x-axis. Write the coordinate of the point.	**a** The x-intercept is 2: (2, 0).
b The y-intercept is where the graph cuts the y-axis. Write the coordinate of the point.	**b** The y-intercept is −4: (0, −4).
c The graph slopes upwards from left to right.	**c** The gradient is positive.

4 A tap is left dripping at a constant rate. The number of litres of water wasted is shown in the table below.

Number of hours the tap is left dripping, *d* (h)	0	2	6	11
Number of litres of water wasted, *w* (L)	0	10	30	55

a Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of hours tap was left dripping, d' on the x-axis and 'Number of litres wasted, w' on the y-axis.

b Is this relationship linear?

c From your graph, decide how much water would be wasted in:

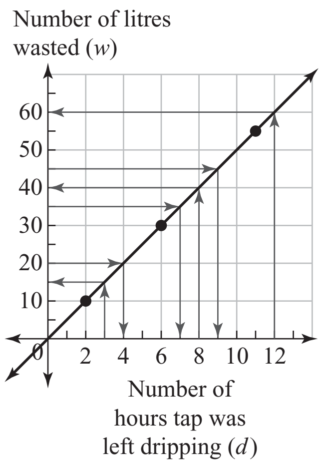
- **i** 3 hours
- **ii** 8 hours
- **iii** 12 hours.

d From your graph, decide how long the tap had been dripping if the number of litres wasted was:

- **i** 20 litres
- **ii** 35 litres
- **iii** 45 litres.

e Is it sensible to join the points in this situation?

f Write the rule that shows the relationship between the number of hours the tap is left dripping, d, and the number of litres of water wasted, w.

Thinking	Working
a Draw and label your x- and y-axes and plot the points. Join them in order with straight lines. Make sure your scale will allow you to read off the values required.	**a** 
b Decide whether all points lie on the same straight line. If so, the graph is linear.	**b** Yes, this is a linear graph.
c Read off the y-coordinate of the point on the line that has the given x-coordinate. This is your answer.	**c** 15 litres 40 litres 60 litres
d Read off the x-coordinate of the point on the line that has the given y-coordinate. This is your answer.	**d** 4 hours 7 hours 9 hours
e Do we measure or count the variables? Variables that can be measured are continuous. In between values exist, so a linear graph is sensible.	**e** Because both time and volume are continuous variables, it is sensible to use a linear graph.
f Look for a pattern between the number of hours the tap is left dripping (d) and the number of litres of water wasted (w).	**f** $w = 5d$

Activity 10D

1 i Copy and complete each of the following tables of values for the rules given for values of x in the range −2 to 2.

ii Use the table of values to draw a graph of the relationship.

a $y = x + 1$

x	−2	−1	0	1	2
y	−1				
(*x*, *y*)	(−2, 1)				

b $y = \dfrac{x}{2}$

x	−2	−1	0	1	2
y	−1				
(*x*, *y*)	(−2, 1)				

c $y = 3x - 1$

x	−2	−1	0	1	2
3x	−6				
3x − 1	−7				
(x, y)	(−2, −7)				

d $y = -x - 4$

x	−2	−1	0	1	2
−x	2				
−x − 4	−2				
(x, y)	(−2, −2)				

2 Plot the graphs for each of the following linear relationships using only three points.

a $y = -2x$

b $y = -3x + 9$

3 **a** Draw the graph of each linear equation on the same Cartesian plane for values of x from −2 to 2.

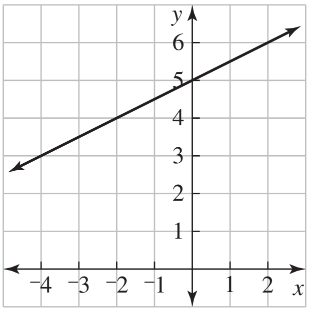
i $y = \frac{1}{2}x$

ii $y = 2x$

iii $y = 3x$

b What effect does the coefficient of x have on the steepness of the line?

4 Find any three points that lie on this line.



5 The battery in Mr Kabolo's car is fully charged. It delivers a current of 30 amps. Unfortunately he leaves the car lights on. The current in the battery drops by 5 amps every hour.

a Complete this table:

Time (hours)	Battery current (amps)
0	30
1	25
2	
3	

b Mr Kabolo's daughter Brianna believes there is a rule that helps explain the relationship between the battery current (B) and the time (t) for which the battery is left on. Which of these is the most suitable rule?

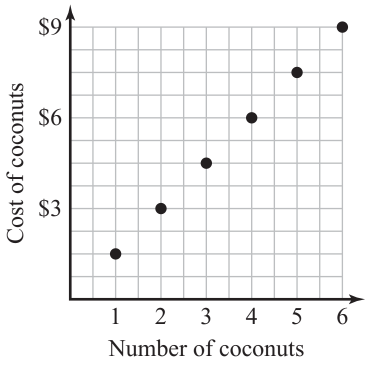
A $B = 30 - t$

B $B = 30t$

C $B = 30 - 5t$

D $B = 5t + 25$

6 The graph below shows the cost of coconuts.



a What is the cost of 2 coconuts?

b What is the cost of 6 coconuts?

c What is the cost of 1 coconut?

d How many coconuts can you buy for $7.50?

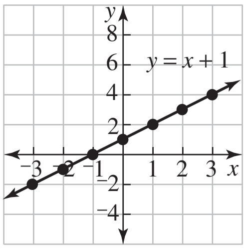
e One of the points on this graph has coordinates (4, 6). Explain what the values 4 and 6 tell you?

Answers 10D

1 **a** **i**

x	−2	−1	0	1	2
y	−1	0	1	2	3
(x, y)	(−2, −1)	(−1, 0)	(0, 1)	(1, 2)	(2, 3)

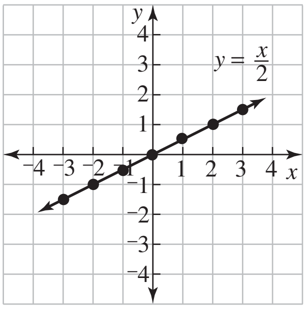
ii



b **i**

x	−2	−1	0	1	2
y	−1	−0·5	0	0·5	1
(x, y)	(−2, −1)	(−1, −0·5)	(0, 0)	(1, 0·5)	(2, 1)

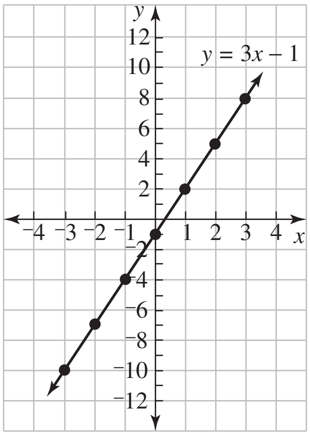
ii



c **i**

x	−2	−1	0	1	2
3x	−6	−3	0	3	6
3x – 1	−7	−4	−1	2	5
(x, y)	(−2, −7)	(−1, −4)	(0, −1)	(1, 2)	(2, 5)

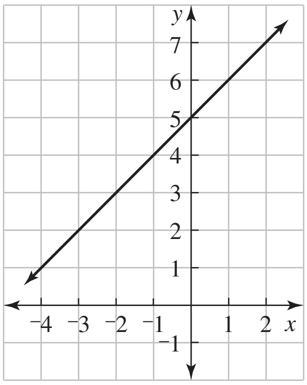
ii



d **i**

x	−2	−1	0	1	2
−x	2	1	0	−1	−2
−x – 4	−2	−3	−4	−5	−6
(x, y)	(−2, −2)	(−1, −3)	(0, −4)	(1, −5)	(2, −6)

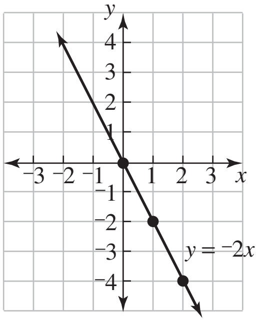
ii



2 **a**

x	−1	0	1
y	2	0	−2

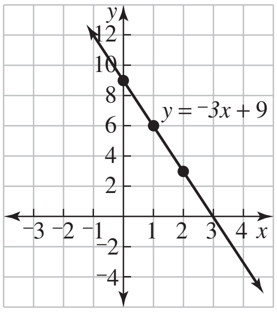
ii



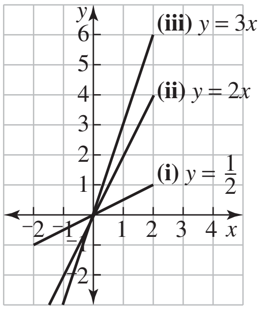
b

x	−1	0	1
y	12	9	6

ii



3 **a**



d **i**

x	−2	−1	0	1	2
$\mathbf{\frac{1}{2}x}$	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	−2
2x	−4	−2	0	2	−6
3x	−6	−3	0	3	(2, −6)

b The larger the coefficient, the steeper the line for positive coefficients. But, for negative coefficients, the smaller the coefficient the steeper the line.

4 **a** (−4, 3), (−2, 4), (0, 5), (2, 6)

5 **a**

Time (hours)	Battery current (amps)
0	30
1	25
2	20
3	15

b The correct rule is (D) $B = 30 - 5t$.

6 **a** \$3

b \$9

c \$1.50

d 5 coconuts

e 4 coconuts cost \$6.

10E • Horizontal and vertical lines

LB2 Pages 57–58

Specific learning outcomes

Learners should be able to:

8.10.8.1 Identify the properties of horizontal and vertical linear graphs.

8.10.9.1 Deduce the equations of vertical and horizontal linear graphs.

Teaching points

1. Identify the properties of horizontal and vertical graph.
2. Find equations of horizontal and vertical graphs.
3. Sketch horizontal and vertical lines.

Learner difficulties and remedies

Difficulty

Expecting that the graph of $x = 2$ runs parallel to the x-axis. This is incorrect.

Remedy

- Explain to learners that x-values are plotted on the horizontal line but the graphs of $x = c$ are vertical. This is because only the y-values change; the x-values are always 2. Similarly graphs of $y = c$ are always horizontal.

Suggested teaching approach

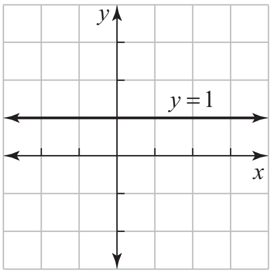
- Identify all the properties of vertical and horizontal linear graphs.
- Find the equation of given vertical and horizontal linear graphs by using the properties of those graphs.
- Sketch vertical and horizontal lines from their rules.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

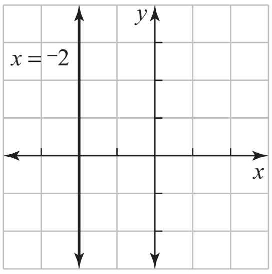
Horizontal lines

The graph of $y = c$ is a horizontal line parallel to the x-axis that passes through the point $(0, c)$, This point is the y-intercept.

Horizontal line graphs where $c \neq 0$ are the only linear graphs that do not have an x-intercept.



A horizontal line has a gradient of zero.



Vertical lines

The graph of $x = a$ is a vertical line parallel to the y-axis that passes through the point $(a, 0)$. This point is the x-intercept.

Vertical line graphs where $a \neq 0$ are the only linear graphs that do not have a y-intercept.

A vertical line has a gradient that cannot be evaluated. We say the gradient is undefined.

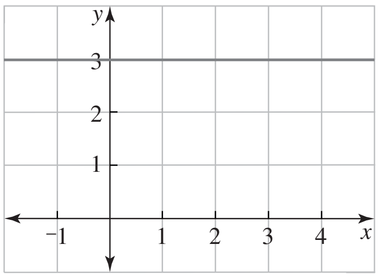
Examples

1 **a** Plot the points in the following table of values and join using a straight line.

b Describe the line and state its gradient and any intercept.

c Find the rule for the line.

x	0	1	2	3	4
y	3	3	3	3	3

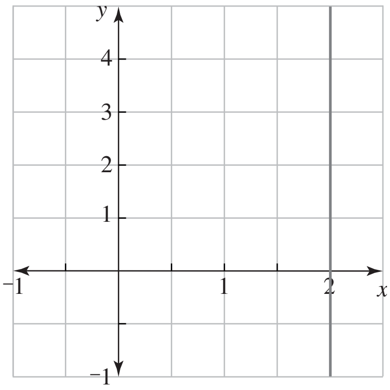
Thinking	Working
a Draw and label your x- and y-axes and plot the points. Join them in order with straight lines.	
b Describe the line and then read the y-coordinate when $x = 0$.	The graph is a horizontal line passing through $y = 3$. The line is parallel to the x-axis. The gradient of the line is 0 (the coefficient of x). The y-intercept is 3: (0, 3)
c State the rule.	$y = 3$

2 **a** Plot the points in the following table of values and join using a straight line.

b Describe the line and state its gradient and any intercept.

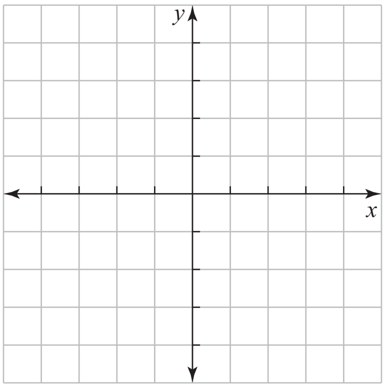
c Find the rule for the line.

x	2	2	2	2	2
y	1	2	3	4	5

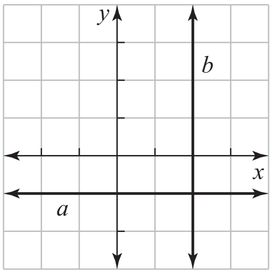
Thinking	Working
a Draw and label your x- and y-axes and plot the points. Join them in order with straight lines.	
b Describe the line and then read the x-coordinate when $y = 0$.	The graph is line parallel to the y-axis and passes through the point $x = 2$. This gradient is the steepest possible for a straight line. We say the gradient is undefined. The x-intercept is 2.
c State the rule.	$x = 2$

Activity 10E

1 Draw the graphs of $y = {}^-3$ and $x = 1$ on the same diagram.



2 Write down the rule for each of these lines.



3 Say whether these statements are true or false.

- **a** The graph of $x = 3$ is a horizontal line.
- **b** The line $x = 2$ is parallel to the x-axis.
- **c** $y = 0$ is the same as the x-axis.

4 Write down the equation of the line that:

- **a** is parallel to the x-axis and passes through (1, 9)
- **b** is parallel to the y-axis and passes through (⁻2, 4).

5 For each table of values below:

- **i** plot the line
- **ii** decide whether the relationship is linear
- **iii** find the gradient
- **iv** decide whether the line is parallel to an axis
- **v** state any intercepts.

a $y = 3$

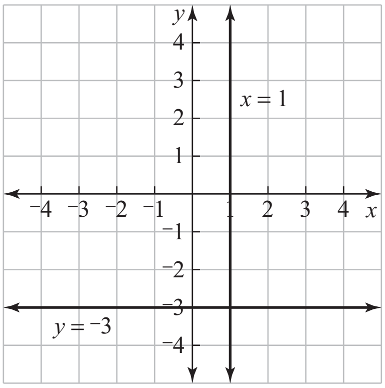
x	0	1	2
y	3	3	3
(x, y)	(0, 3)	(1, 3)	(2, 3)

b $y = {}^-5$

x	0	1	2
y	⁻5	⁻5	⁻5
(x, y)	(0, ⁻5)	(1, ⁻5)	(2, ⁻5)

Answers 10E

1

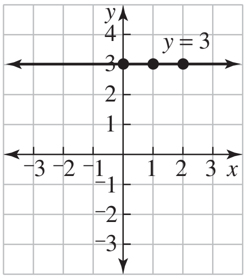


2 a $y = {}^-1$ **b** $x = 2$

3 a false **b** false **c** true

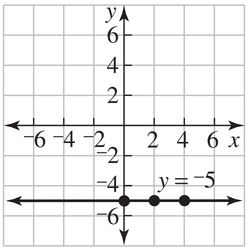
4 a $y = 9$ **b** $x = {}^-2$

5 a i



- **ii** The relationship is linear because the graph is a straight line.
- **iii** Zero gradient.
- **iv** Parallel to the x-axis.
- **v** No x-intercept, y-intercept is 3.

b i



- **ii** The relationship is linear because the graph is a straight line.
- **iii** Zero gradient.
- **iv** Parallel to the x-axis.
- **v** No x-intercept, y-intercept is ⁻5.

10F • Using the *x*- and *y*-intercepts

LB2 Page 59

Specific learning outcomes

Learners should be able to:

8.11.10.1 Sketch or plot linear graphs by finding the x- and y-intercepts.

Teaching points

1 Sketch linear graphs by finding the x- and y-intercepts.

Learner difficulties and remedies

Difficulty

Understanding why in the given equation of $y = mx + c$, you make $y = 0$ and $x = 0$ when finding where the graph cuts the axes.

Remedy

- Explain to learners that the x-intercept of a graph is the point where the line crosses the x-axis at $y = 0$. The y-intercept of a graph is the point where the line crosses the y-axis at $x = 0$.

Difficulty

Solving the rule or equation algebraically.

Remedy

- Remind learners how to use inverse operations to solve an equation for a pronumeral.
- Solve the linear equation when $y = 0$ and $x = 0$ to find where the graph cuts the axes.

Suggested teaching approach

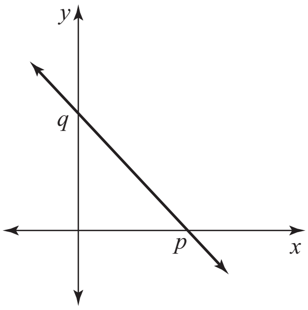
- Find where the linear graph cuts the y-axis first because it is usually easier. The y-intercept of a graph is the point where the line crosses the y-axis at $x = 0$. In the general equation $y = mx + c$, c indicates where the graph cuts the y-axes. If $c = 0$ then the graph passes through the origin.
- The x-intercept of a graph is the point where the line crosses the x-axis. Substitute $y = 0$ and solve algebraically to find the x-intercept.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

An intercept is the name given to the point where a graph cuts an axis. Most straight lines have two intercepts: an x-intercept and a y-intercept.

x-intercept = $(p, 0)$

y-intercept = $(0, q)$

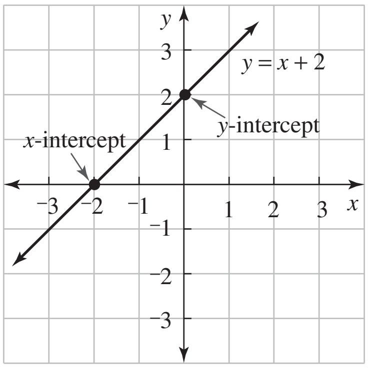


To work out intercepts from the equation of a line:

- For the x-intercept, substitute $y = 0$.
- For the y-intercept, substitute $x = 0$.

The x-intercept of a graph is the point where the line crosses the x-axis.

The y-intercept of a graph is the point where the line crosses the y-axis.

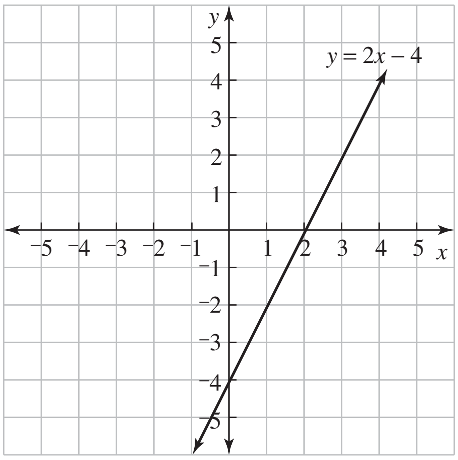


Examples

For the graph with the rule $y = 2x - 4$:

a state the x-intercept and write the coordinates of the point

b state the y-intercept and write the coordinates of the point



Thinking	**Working**
a The x-intercept is where the graph cuts the x-axis. Write the coordinate of the point.	**a** The x-intercept is 2; it has the coordinates (2, 0).
b The y-intercept is where the graph cuts the y-axis. Write the coordinate of the point.	**b** The y-intercept is $^-4$; it has the coordinates $(0, ^-4)$.

Activity 10F

1 Work out the y-intercepts for the lines that have these equations by substituting $x = 0$:

a $y = x + 5$ **b** $x + 4y = 8$

c $2x - 3y = 6$ **d** $x + 2y - 4 = 0$

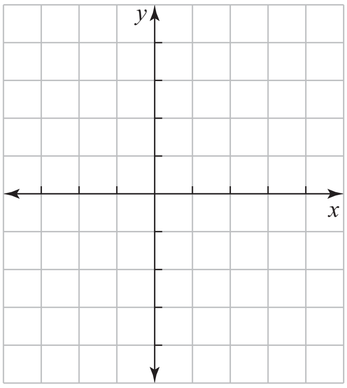
2 Work out the x-intercepts for the lines that have these equations by substituting $y = 0$.

a $y = 2x - 8$ **b** $y = 3x + 6$

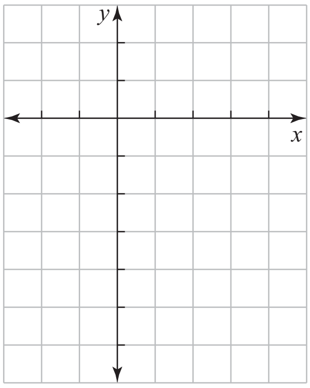
c $y = {^-x} - 7$ **d** $y = \frac{1}{2}x + 4$

3 Work out both the x- and y-intercepts for these lines and then draw the line by joining the intercepts.

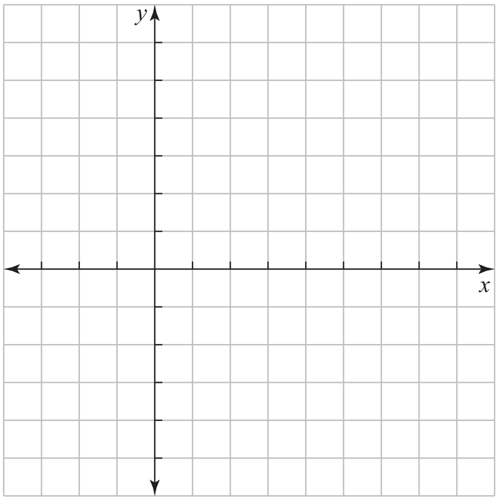
a $x + 2y = 4$



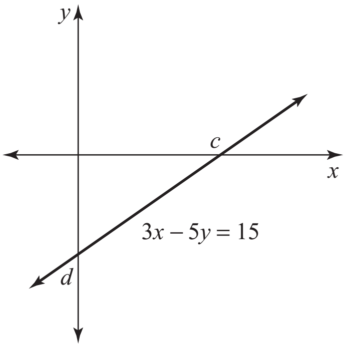
b $2x - y = 6$



c $2x + 5y = 10$



4 Write down the values of c and d.



Answers 10F

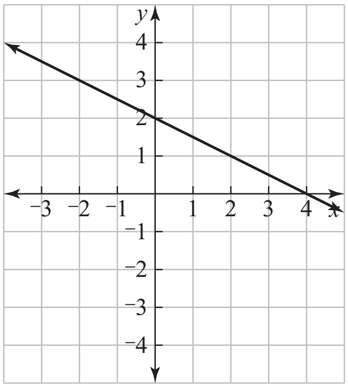
1 **a** (0, 5) **b** (0, 2)

c (0, 3) **d** (0, 2)

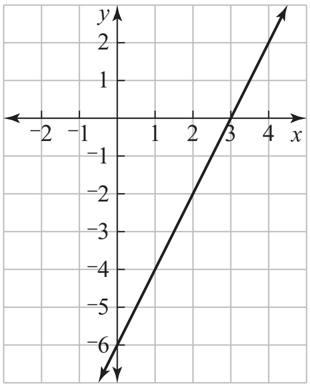
2 **a** (4, 0) **b** (⁻2, 0)

c (⁻7, 0) **d** (⁻8, 0)

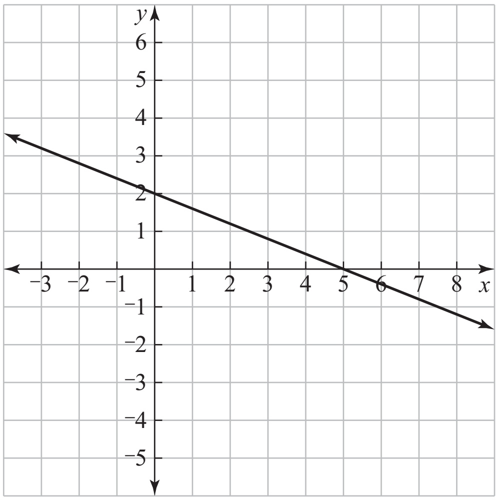
3 **a** $x + 2y = 4$



b $2x - y = 6$



c $2x + 5y = 10$



4 $c = 5$, $d = {}^-3$

10G • Gradients of straight lines

LB2 Pages 60–62

Specific learning outcomes

Learners should be able to:

8.10.11.1 Define the term 'gradient'.

8.10.11.2 Determine a gradient by dividing the rise by the run between two convenient points on the line.

Formula: $m = \frac{\text{rise}}{\text{run}}$

8.10.12.1 Calculate gradient of given graphs, either positive or negative.

Teaching points

1. Explain that the gradient of a line on a graph is a measure of its steepness.
2. Find the gradient of a line joining two points on a graph by first calculating the rise and the run.
3. Identify whether the gradient of given graph is positive (uphill) or negative (downhill), by reading the graph from left to right.
4. The gradient of a horizontal line is 0. The gradient of a vertical line is undefined.

Learner difficulties and remedies

Difficulty

Finding the gradient of the graph.

Remedy

- Pick two points on the linear graph, construct a right-angled triangle below the line then use the $\frac{\text{rise}}{\text{run}}$ to find the gradient of the line.
- From the first point count how many units you go up and how many units you run along to reach the next point. The gradient of the linear graph is $\frac{\text{rise}}{\text{run}}$.

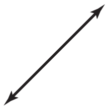
Suggested teaching approach

- Explain to learners that gradient of a graph refers to slope or steepness of the graph.
- Pick two points on the linear graph. From the first point count how many units you go up and how many units you run along to reach the next point. The gradient of the linear graph is $\frac{\text{rise}}{\text{run}}$.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

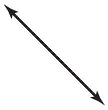
Additional notes

The **gradient** tells us about the slope or steepness of a line.

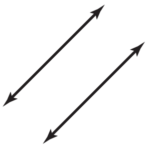
This line has a positive gradient as it slopes upwards from left to right.



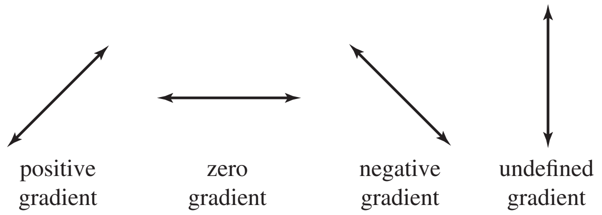
This line has a negative gradient as it slopes downwards from left to right.



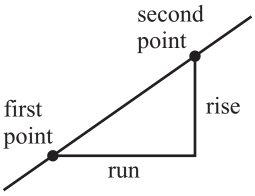
Lines with the same gradient are parallel.

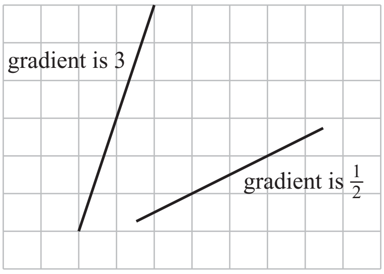


The gradient of a line describes the slope or steepness of the line.



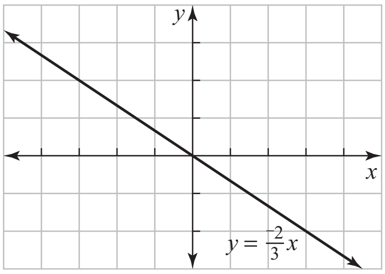
The **gradient** of a line is a number that tells us how steep it is. It is measured by describing how much the line rises vertically for every 1 unit it runs horizontally. It helps to think of the gradient as a fraction: $\frac{\text{rise}}{\text{run}}$.





Lines that slope downwards from left to right have a **negative** gradient.

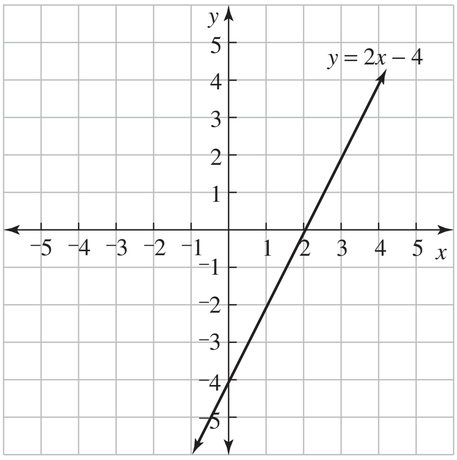
The $y = mx$ rule also applies when m is negative.



For this line $\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$ because for every 3 units the line runs along it goes *down* 2 units.

Examples

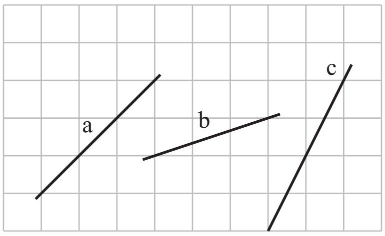
For the graph with the rule $y = 2x - 4$ state whether the gradient of the line is positive or negative.



Thinking	Working
The graph slopes upwards from left to right	The gradient is positive.

Activity 10G

1 Write down the gradient of these lines.



2 Draw lines that have these gradients.

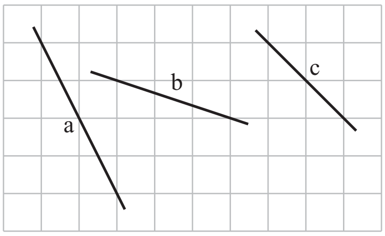
a 4

b $\frac{2}{3}$

3 Arrange these gradients in order from least steep to most steep: 3, $\frac{1}{2}$, 2, $\frac{1}{5}$, 1

4 What is the gradient of a horizontal line?

5 Write down the gradients of these lines.



6 Draw lines that have these gradients

a −3

b $-\frac{3}{4}$

7 Arrange these gradients in order from least steep to most steep: −4, $-\frac{1}{3}$, −2, $-\frac{1}{5}$, −1

Answers 10G

1 **a** 1 **b** $\frac{1}{3}$ **c** $\frac{1}{2}$

2 **a** Line should rise up 4 units and run along 1 unit.

b Line should rise up 2 units and run along 3 units.

3 $\frac{1}{5}$, $\frac{1}{2}$, 1, 2, 3

4 $m = 0$

5 **a** −2 **b** $-\frac{1}{3}$ **c** −1

6 **a** Line should go down 3 units and run along 1 unit.

b Line should go down 3 units and run along 4 units.

7 $-\frac{1}{5}$, $-\frac{1}{3}$, −1, −2, −4

10H • The gradient and the *y*-intercept

LB2 Pages 63–64

Specific learning outcomes

Learners should be able to:

8.10.13.1 Identify the equation for straight lines that is the form of $y = mx + c$, then find gradient and the *y*-intercept.

8.10.14.1 Sketch linear graphs by finding the gradient and the *y*-intercept.

Teaching points

1 Find the gradient and *y*-intercept of linear equations that are given in the form of $y = mx + c$.

2 Notice that parallel lines have the same gradient, or *m*-value.

3 Sketch a linear graph given the gradient and *y*-intercept.

Learner difficulties and remedies

Difficulty

Sketching a linear graph using the *y*-intercept and the gradient.

Remedy

- Recognise the constant value *c* in an equation $y = mx + c$ is the *y*-intercept.
- Recognise that *m* in an equation $y = mx + c$ is the gradient.
- To graph linear equations, plot the *y*-intercept first, then use the gradient to plot the next point, and then draw a straight line through the two points.

Suggested teaching approach

- Explain that the general equation for a straight line is given in the form of $y = mx + c$, where *m* is the gradient and *c* is the *y*-intercept.
- To sketch the linear graph using the *y*-intercept and gradient method, first find the *y*-intercept, which is given by *c* in the general equation. Then use the rise over run in the gradient from *m* to locate the next point. Join the two points with a straight line.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Any straight line can be expressed using the rule $y = mx + c$.

m is the gradient of the line.

c is the *y*-intercept of the line.

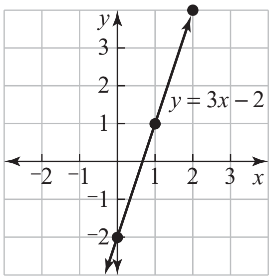
This information can be used to draw graphs of lines.

Mark in the point given by *c* on the *y*-axis.

Through this point draw a line with a gradient of *m*.

Examples

Use the gradient and y-intercept method to sketch the graph for the equation $y = 3x - 2$.

Thinking	Working
1 Use the equation $y = 3x - 2$ to identify the y-intercept.	y-intercept is 2: (0, 2)
2 Use the gradient to locate the second point.	$m = \frac{\text{rise}}{\text{run}} = \frac{3}{1}$ From (0, 2) run along 1 and rise up 3 The next point is (1, 5).
3 Join the two points using a straight line.	

Activity 10H

1 Write down the gradients of these lines.

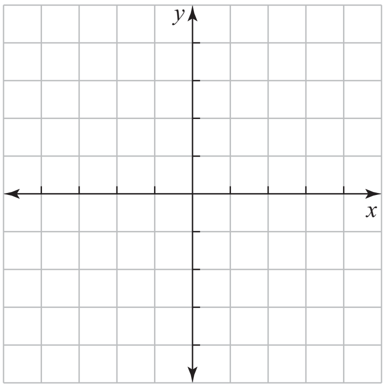
a $y = 3x - 1$ **b** $y = \frac{2}{5}x + 6$ **c** $y = {}^{-}2x + 3$

2 Write down the y-intercept for each of the lines in Question **1**.

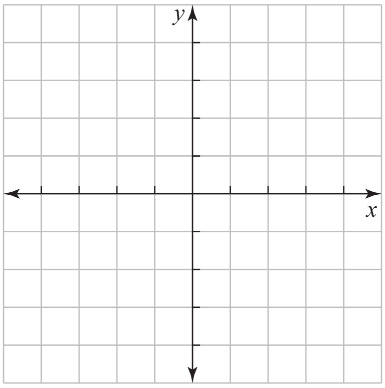
a __________ **b** __________ **c** __________

3 Draw the graphs for the lines with these rules

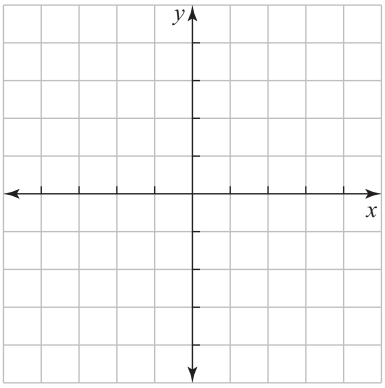
a $y = 2x - 1$



b $y = \frac{1}{2}x + 2$



c $y = {}^{-}3x + 2$

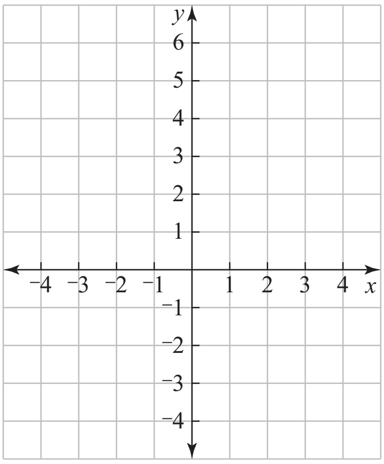


4 Draw each of these three lines on the *same* grid:

$y = 2x - 3$

$y = \frac{1}{2}x + 2$

$y = 2x + 1$



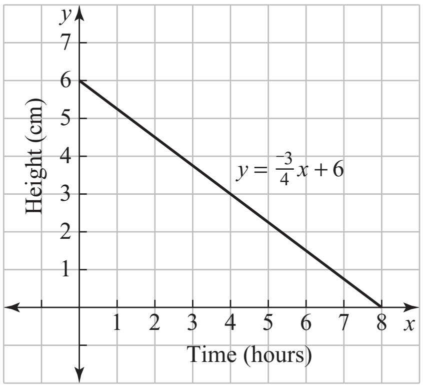
5 Explain how you can tell from the rules for the lines:

a whether two lines are parallel

b whether one line is steeper than another

c whether a line cuts the y-axis above or below (0, 0).

6 A candle burns slowly at a steady rate until it is completely melted. The rule for the height y (cm) of the candle x (hours) after it starts burning is $y = -\frac{3}{4}x + 6$.



a what is the y-intercept of the line? What does it represent in this situation?

b Complete this statement: Every hour, the height of the candle ______ by _____ cm.

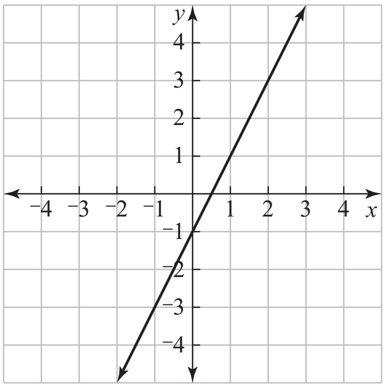
c Explain why the line cannot be extended beyond (8, 0).

Answers 10G

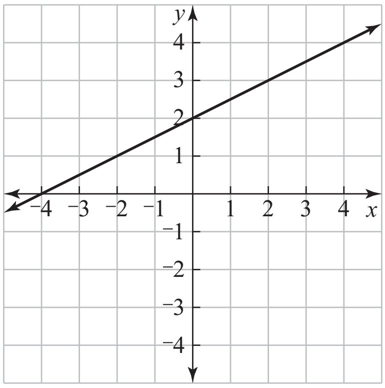
1 **a** 3 **b** $\frac{2}{5}$ **c** ⁻2

2 **a** ⁻1 **b** 6 **c** 3

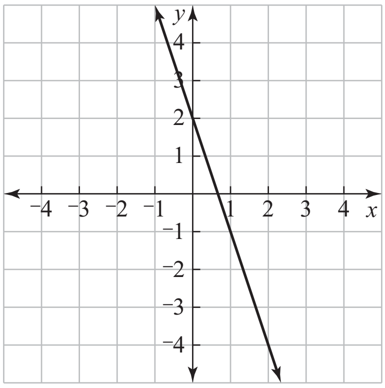
3 a



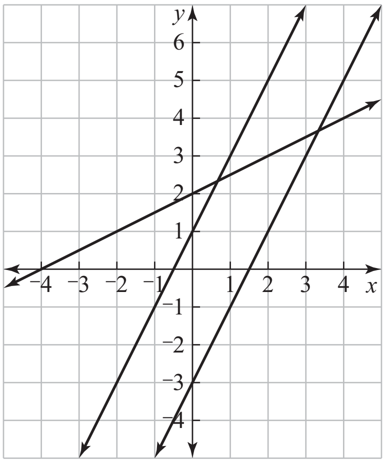
b



c



4



5 a Parallel lines have the same gradient. In rules of the form $y = mx + c$, m (the coefficients of x) will be the same.

b The value of m (the coefficient of x) will be greater if the gradient is steeper.

c If a line cuts the y-axis above (0, 0), then $c > 0$.
If a line cuts the y-axis at (0, 0), then $c = 0$.
If a line cuts the y-axis below (0, 0), then $c < 0$.

6 a $c = 6$ This means that the initial height of the candle is 6 cm.

b Every hour, the height of the candle decreases by $\frac{3}{4}$ cm.

c If the graph extended beyond 8 hours, the height of the candle would be negative, which is not possible.

10I • Finding equations of lines

LB2 Page 65

Specific learning outcomes

Learners should be able to:

8.10.15.1 Find linear equations by substituting values to m and c in the equation $y = mx + c$.

8.10.15.2 Write equations for given linear graphs by finding the gradients and y-intercepts.

Teaching points

1. Write the equation of a linear graph by substituting m and c in the linear equation $y = mx + c$.
2. Find equations for given graphs using the gradient and y-intercepts identified in the graphs.

Learner difficulties and remedies

Difficulty

Identifying the y-intercept and the gradient from a graph in order to find its equation.

Remedy

- Identify the y-intercept where the graph crosses the y-axis. This is c in the general equation $y = mx + c$.
- Calculate the gradient by identifying two points on the graph and then using $m = \frac{\text{rise}}{\text{run}}$. This is m in the general equation $y = mx + c$.
- Insert these two values into the general equation for a line graph to find the equation of the graph.

Suggested teaching approach

- Show learners how to identify the y-intercept, where the graph crosses the y-axis. This is c in the general equation $y = mx + c$.
- Show learners how to calculate the gradient by identifying two points on the graph, and then using $m = \frac{\text{rise}}{\text{run}}$. This is m in the general equation $y = mx + c$.
- Insert these two values into the general equation for a line graph to find the equation of the graph.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The gradient and y-intercept can be used to find the equations for given linear graphs and incomplete equations that are given in the form of $y = mx + c$.

The gradient of a line tells us how steep the line or graph is. It is found using $m = \frac{\text{rise}}{\text{run}}$. This is m in the general equation $y = mx + c$.

The y-intercept is where the line cut the vertical or y-axis. This is c in the general equation $y = mx + c$.

Examples

1 Write the equations for each with the given gradient and y-intercepts in the general form $y = mx + c$.

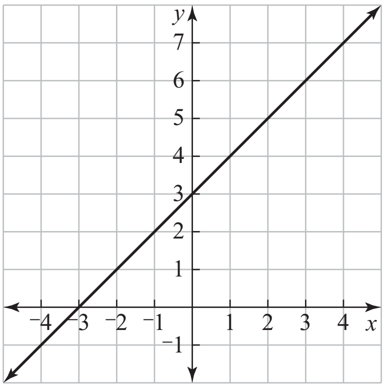
a $m = 4, c = 7$

b $m = \frac{1}{4}, c = {}^{-}2$

c $m = -\frac{2}{3}, c = \frac{1}{2}$

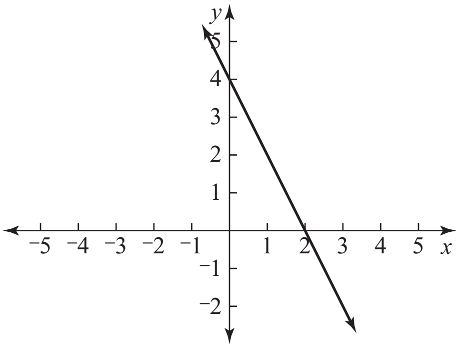
Thinking	Working
a **1** The general equation for linear graph is $y = mx + c$.	$y = mx + c$
2 Replace the letters in the general equation with the values.	$m = 4, c = 7$
3 Write the linear equation.	$y = 4x + 7$
b **1** The general equation for linear graph is $y = mx + c$.	$y = mx + c$
2 Replace the letters in the general equation with the values.	$m = \frac{1}{4}, c = {}^{-}2$
3 Write the linear equation.	$y = \frac{1}{4}x + {}^{-}2$
c **1** The general equation for linear graph is $y = mx + c$.	$y = mx + c$
2 Replace the letters in the general equation with the values.	$m = -\frac{2}{3}, c = \frac{1}{2}$
3 Write the linear equation.	$y = -\frac{2}{3}x + \frac{1}{2}$

2 Find the equation for the given linear graph.



Thinking	Working
1 Find where the graph cuts the y-axis.	y-intercept = 3
2 Find the gradient.	Gradient = 1
3 Substitute into the general equation to give the equation for the linear graph.	$y = mx + c$
4 Write the equation.	$y = x + 3$

3 Find the equation for the given linear graph.



Thinking	Working
1 Find where the graph cuts the y-axis.	y-intercept = 4
2 Find the gradient.	Gradient = ${}^{-}2$
3 Substitute into the general equation to give the equation for the linear graph.	$y = mx + c$
4 Write the equation.	$y = {}^{-}2x + 4$

Activity 10I

1 Write down the gradient and vertical intercept for each of the lines with these rules:

Rule	Gradient	y-intercept
a $y = 4x + 5$		
b $y = 3x - 1$		
c $y = x + 6$		
d $y = 0{\cdot}5x + 1$		
e $y = 6x - 4$		
f $y = \frac{2}{3}x$		
g $y = 8 - x$		

2 Write down the general rule for each line with the given gradient and y-intercept:

a $m = 2$, y-intercept = 4

b $m = 6$, y-intercept = ${}^{-}1$

c $m = {}^{-}2$, y-intercept = 0

d $m = {}^{-}1$, y-intercept = 2

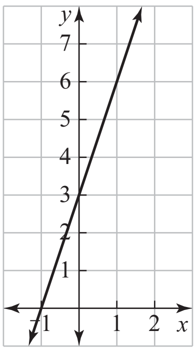
3 For each of these line graphs write:

i the gradient

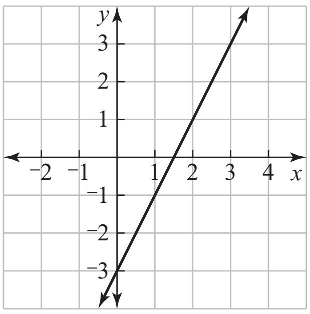
ii the y-intercept

iii the general rule

a



b

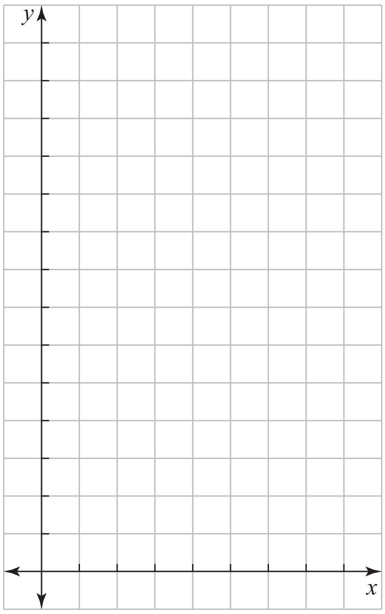


4 Given below is the data that is used by a grass cutter to cut grass around Naha area in Honiara. Here is some data for the amount of fuel, in litres, it uses.

Time in hours (x)	1	2	5	8
Amount of fuel in litres (y)	1·5	3	7·5	12

a How much fuel does the grass cutter use per hour?

b Use the table to plot the four points on a graph with x- and y-axes. Join the four points to make a line.



c Use the graph to estimate how long the grass cutter has been running if it has used 4 litres of fuel?

d State the gradient and the intercept of the line?

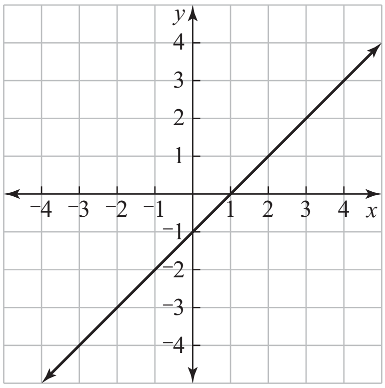
e Write down the equation of the line?

f Assuming that the fuel consumption continues at steady rate, use the equation to find:

- **i** the amount of fuel used after 12 hours
- **ii** the time taken to use 36 L of fuel.

5 Find the equation for each of the following graphs

a



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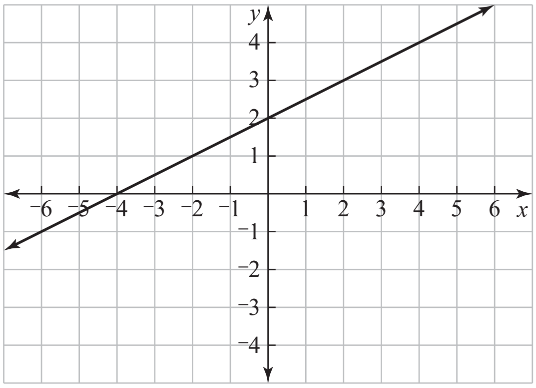


Answers 10I

1

Rule	Gradient	y-intercept
a $y = 4x + 5$	4	5
b $y = 3x - 1$	3	−1
c $y = x + 6$	1	6
d $y = 0{\cdot}5x + 1$	0·5	1
e $y = 6x - 4$	6	−4
f $y = \frac{2}{3}x$	$-\frac{2}{3}$	0
g $y = 8 - x$	−1	8

2 **a** $y = 2x + 4$

b $y = 6x - 1$

c $y = -2x$

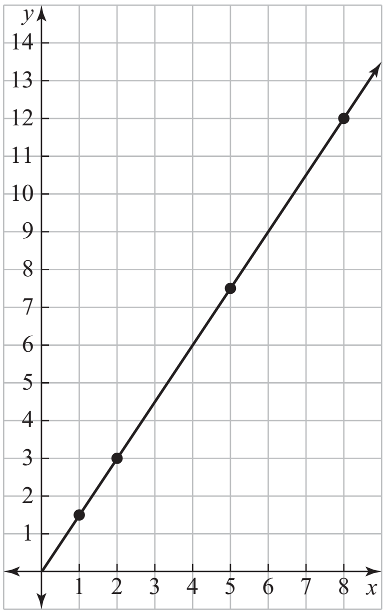
d $y = 2 - x$

3

	a	b
Gradient	3	2
y-intercept	−1	−3
Rule	$y = 3x - 1$	$y = 2x - 3$

4 **a** 1·5 L/h

b



c 3 hours

d $m = 1{\cdot}5$, y-intercept $= 0$

e $y = 1{\cdot}5x$

f **i** 18 L

ii 24 hours

5 **a** $y = x - 1$

b $y = \frac{x}{2} + 2$

• Chapter Eleven •

Pythagoras

Overview

Pythagoras of Samos (who lived approximately 580–496 BCE) was a Greek philosopher and famous teacher of mathematics who is credited with devising the famous theorem that is named after him. He may have become aware that a group of Egyptian engineers known as the ‘rope stretchers’ were already using a practical tool that made use of one special property of right-angled triangles. They used a circle of rope with 12 knots tied at regular intervals around the circle. Pegging the rope to the ground at intervals of 3, 4 and 5 knots produced a right-angled triangle, which was used to ensure that building foundations or walls were constructed accurately. It is a useful mathematical tool applied by builders, carpenters, engineers and surveyors to determine the straight-line distance between two points or to check whether an angle is a right angle. It is also used in the audio-visual industry to determine the height and width of TV and projection screens.

Pythagoras’ theorem is a mathematical statement that can be proved to be true in many different ways. It has fascinated learners and mathematicians for many centuries.

Contents

Chapter skills

This chapter covers the following skills:

- Identifying the hypotenuse and adjacent sides in a right-angled triangle
- Understanding and writing the mathematical connection between the sides of a right-angled triangle in algebraic form
- Using Pythagoras’ theorem to find side lengths in right-angled triangles Pythagoras’ theorem: $h^2 = a^2 + b^2$
- Recognising some Pythagorean triads
- Applying Pythagoras’ theorem to practical situations
- Using Pythagoras’ theorem to find sides in shapes containing right-angled triangles
- Finding the lengths between points plotted on a Cartesian coordinate system
- Using Pythagoras’ theorem to find missing lengths in three-dimensional problems.

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 11A Exploring Pythagoras’ theorem • 11B Stating Pythagoras’ theorem	Learner’s Book 1 • Exercise 11A, pages 76, 77 Learner’s Book 1 • Exercise 11B, page 78
2–3	• 1C Finding the length of the hypotenuse	Learner’s Book 1 • Exercise 11C, page 79–82
4	• 11D Pythagorean triads	Learner’s Book 1 • Exercise 11D, page 83
5–6	• 1E Finding the length of a perpendicular side	Learner’s Book 1 • Exercise 11E, pages 84–86
7–8	• 11F Pythagoras’ theorem in exact form	Learner’s Book 1 • Exercise 11F, pages 87, 88
9–10	• 11G Composite shapes	Learner’s Book 1 • Exercise 11G, pages 90, 91
11	• 11H Coordinate geometry	Learner’s Book 1 • Exercise 11H, page 92
12–13	• 11I Pythagoras in three dimensions	Learner’s Book 1 • Learning task 11I, pages 94, 95
14	• 11J Exploring areas of triangles	Learner’s Book 1 • Exercise 11J, pages 96, 97
15	• Test	Teacher’s Guide • Chapter 1 Test

General learning outcomes

Learners should:

Exploring Pythagoras’ theorem

8.11.1 Understand Pythagoras’ theorem and how it is formed. (U)

Stating Pythagoras’ theorem

8.11.2 Recognise Pythagoras’ theorem and how it is used to relate the three sides of the right-angled triangle. (S)

Finding the length of the hypotenuse

8.11.3 Know how to use Pythagoras’ theorem to find the length of the hypotenuse. (K)

Pythagorean triads

8.11.4 Understand how Pythagorean triads are formed and how they are applied using Pythagoras' theorem. (U)

Finding the length of a perpendicular side

8.11.5 Know how to use Pythagoras' theorem to find the lengths of any of the two shorter sides of the right-angled triangle. (U)

Pythagoras' theorem in exact form

8.11.6 Know how to make irrational numbers to be in an exact form by making them to be in their 'square root' surd form. (K)

8.11.7 Know how to find the missing sides of the right-angled triangle leaving the answer in exact form (surds). (K)

Composite shapes

8.11.8 Understand that single shape can be made up of many different shapes. (U)

8.11.9 Know how to find lengths of composite shapes. (K)

Coordinate geometry

8.11.10 Understand that Pythagoras' theorem can be used to find the distance between two points on a linear graph. (U)

Pythagoras in three dimensions

8.11.11 Know how to calculate the dimensions of the cross-sections of solids that reveal right-angled triangles. (K)

Exploring areas of triangle

8.11.12 Investigate different approaches that can be used to find the area of a square by breaking the shape into small triangles. (S)

11A • Exploring Pythagoras' theorem

LB2 Pages 76–77

Specific learning outcomes

Learners should be able to:

8.11.1.1 State Pythagoras' theorem and its components: $h^2 = a^2 + b^2$

8.11.1.2 Measure the lengths of the sides of various right-angle triangles and apply them to Pythagoras' theorem.

Teaching points

1 State Pythagoras' theorem $h^2 = a^2 + b^2$ is true by measuring the sides of given right-angled triangles.

2 Measure the lengths of the three sides of given right-angled triangle and relate them to Pythagoras' theorem.

Suggested teaching approach

- Learners complete **Learning task 11A** on pages 76 and 77 of the LB.

Activity 11A

Learners explore further loops of cotton to find identify enlargements of the 3 : 74 : 5 triangle

1 Working in pairs, tie the cotton into a loop that is exactly 24 cm. Use the points of three pencils to make the loop into a triangle with the distance between two of the pencil points, 10 cm. Adjust the position of the third pencil so that one angle looks like a right angle. Mark the positions of the pencil points on your book, and draw in the triangle and measure its sides. You should discover the smaller sides are 6 cm and 8 cm.

2 Working in pairs, tie the cotton into a loop that is exactly 36 cm. Use the points of three pencils to make the loop into a triangle with the distance between two of the pencil points, 15 cm. Adjust the position of the third pencil so that one angle looks like a right angle. Mark the positions of the pencil points on your book, and draw in the triangle and measure its sides. What are the measurements of the smaller sides of the triangle this time?

3 Find the dimensions of one more right-angled triangle that is an enlargement of the two previous triangles.

11B • Stating Pythagoras' theorem

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Specific learning outcomes

Learners should be able to:

8.11.2.1 Explain Pythagoras theorem in terms sides of the right-angled triangle: $h^2 = a^2 + b^2$

8.11.2.2 Write equations for given triangles that relate all three sides of the right-angled triangle.

Teaching points

1 State Pythagoras' theorem $h^2 = a^2 + b^2$.

2 Measure the lengths of the three sides of given right-angled triangle and relate them to Pythagoras' theorem.

Learner difficulties and remedies

Difficulty

Understanding the relationship between the two shorter sides and the hypotenuse in a right-angled triangle.

Remedy

- Establish that the longest side of a right-angled triangle is opposite the right angle.
- Explain to learners the relationship between the three sides of the right-angled triangle: when the lengths of the two shortest sides are squared individually and added together, their sum is equal to the square of the length for the longest side.

Suggested teaching approach

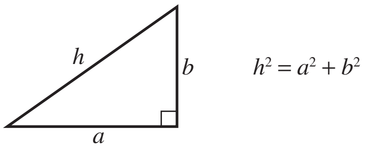
- This is a new concept at this level so explain each unit clearly to learners before moving on in the chapter.
- Explain Pythagoras' theorem $h^2 = a^2 + b^2$.
- Use a variety of examples to demonstrate how Pythagoras' theorem can be applied in right-angled triangles to find unknown sides. Learners should be able to substitute different pronumerals and numbers into the theorem.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

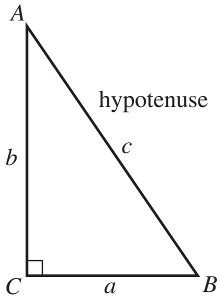
A triangle that contains a 90° angle (a right angle) is called a **right-angled triangle.** The longest side of a right-angled triangle, which is opposite the right angle, is called the **hypotenuse.**

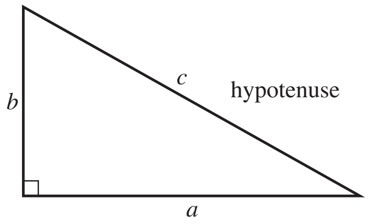
When labelling triangles, we use capital letters for the vertices and lower case letters for the side lengths.

So, triangle *ABC* on the right, which can be written as ΔABC**,** has vertices *A*, *B* and *C* with sides *a*, *b* and *c* opposite the vertices that have the same letter. It is usual to label the hypotenuse as side *c* and the two shorter sides as *a* and *b*. (It does not matter which side is *a* and which is *b*.)

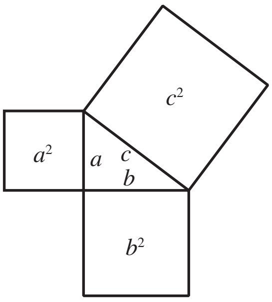


Pythagoras' theorem: For any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.

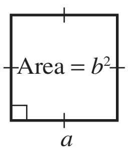


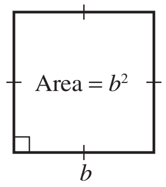


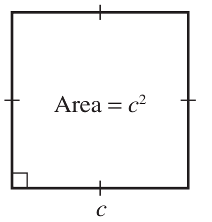
Note that a **theorem** is a mathematical statement that can be shown to be true. A **proof** is a step-by-step argument that demonstrates the truth of a mathematical theorem.



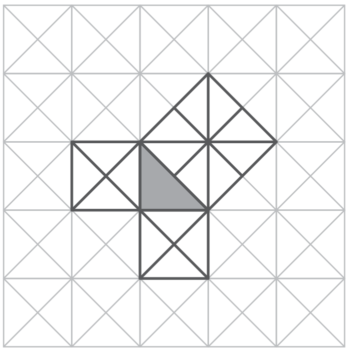
The area of a square of side *a* is a^2. In a right-angled triangle, the areas of the squares drawn onto the two shorter sides add to give the area of the square drawn on the hypotenuse.







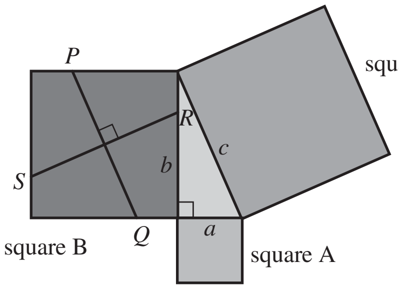
Pythagoras' theorem can be demonstrated visually. If we take an isosceles right-angled triangle and place it onto square grid paper that has been divided into small identical triangular tiles, the combined area of the squares on the two shorter sides is eight small triangles, the same number of triangles covered by the square on the hypotenuse. This shows only that the theorem works for one particular type of right-angled triangle. For the theorem to be shown as true, we must prove that it works for all right-angled triangles.



Demonstrating Pythagoras' theorem using areas

1 On a piece of grid paper, draw any right-angled triangle, and label the sides *a*, *b* and *c*.

2 Construct a square on each side of the triangle as shown below. Label the squares *A*, *B* and *C* to correspond to the sides *a*, *b* and *c.*

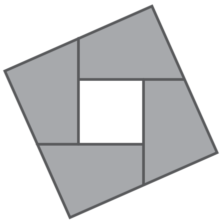


3 Find the centre of square *B*. Rule a line *PQ* through the centre, so that it is parallel to the hypotenuse of the triangle.

4 Rule a second line, *RS*, perpendicular (at right angles) to *PQ* that passes through the centre of the square.

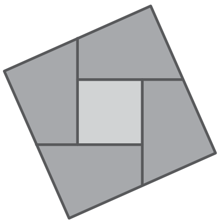
5 Cut square *B* into its four pieces. Then, cut out square *A* and square *C*.

6 Rearrange the four pieces of square B as shown.

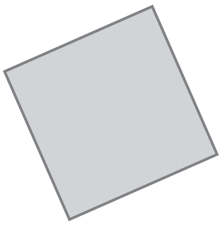


7 Place square A in the centre. It should fit exactly.

The total area of this square
= area of square A + area of square B
= $a^2 + b^2$



8 Compare the area of this square to the area of square C, c^2. The two squares should be identical in area, demonstrating that $c^2 = a^2 + b^2$.



The converse of Pythagoras’ theorem

A converse is a reversal of a proved theorem.

Because Pythagoras’ theorem is only true for right-angled triangles, the converse of Pythagoras’ theorem can be used to check whether or not a triangle is right-angled. That is, if the sum of the squares of the two shorter sides is equal to the square of the longer side, the triangle is right-angled.

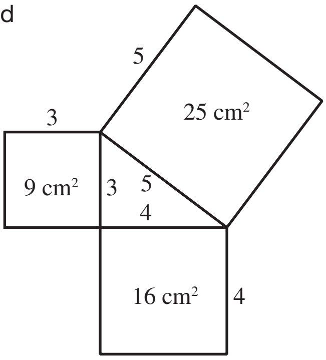
Consider a triangle of sides 3 cm, 4 cm, and 5 cm.

We can show this is a right-angled triangle using the converse of Pythagoras’ theorem.

Here, $a = 3$, $b = 4$ and $c = 5$.

Now: $c^2 = 5^2$
$= 25$ and
$a^2 + b^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$

As $c^2 = a^2 + b^2$ holds for this triangle, we can say that this is a right-angled triangle.



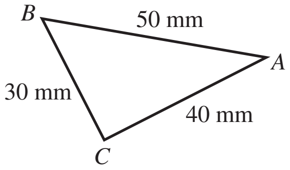
If Pythagoras’ theorem holds, then the triangle must be a right-angled triangle. This statement is known as the ‘converse’ of Pythagoras’ theorem.

Builders use the converse of Pythagoras’ theorem to make sure building frames and walls are right-angled. They call it ‘the 3-4-5 principle: Can you see why?

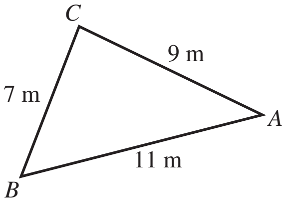
Examples

Use the converse of Pythagoras’ theorem to determine whether each of the following is a right-angled triangle. If it is, state which angle is the right angle.

a



b



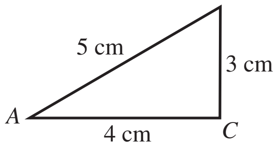
Thinking	**Working**
a 1 Identify the longest side as c and the other two sides as a and b.	**a** $a = 30$ mm $b = 40$ mm $c = 50$ mm
2 Evaluate the left-hand side of Pythagoras’ theorem, c^2.	$c^2 = 50^2$ $= 2500$
3 Evaluate the right-hand side of Pythagoras’ theorem, $a^2 + b^2$.	$a^2 + b^2 = 30^2 + 40^2$ $= 900 + 1600$ $= 2500$
4 Does $c^2 = a^2 + b^2$? If so, the triangle is right-angled.	$c^2 = a^2 + b^2$, so ΔABC is a right-angled triangle.
5 The right angle is the angle opposite the hypotenuse, c.	$\angle ABC$ is the right angle.

Thinking	**Working**
b 1 Identify the longest side as c and the other two sides as a and b.	**b** $a = 7$ m $b = 9$ m $c = 11$ m
2 Evaluate c^2.	$c^2 = 11^2$ $= 121$
3 Evaluate $a^2 + b^2$.	$a^2 + b^2 = 7^2 + 9^2$ $= 49 + 81$ $= 130$
4 Does $c^2 = a^2 + b^2$? If so, the triangle is right-angled.	$c^2 \neq a^2 + b^2$, so ΔABC is not a right-angled triangle.

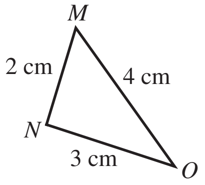
Activity 11B

1 Use the converse of Pythagoras’ theorem to determine whether each of the following triangles is a right-angled triangle. If it is, state which angle is the right angle.

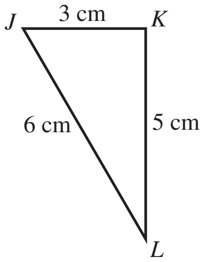
a



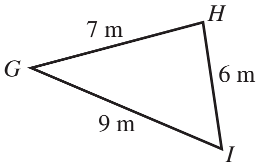
b



c



d



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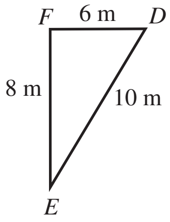

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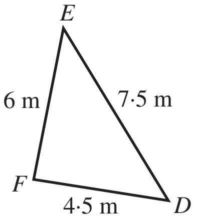

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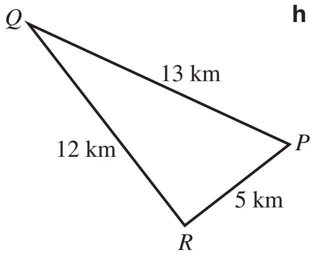

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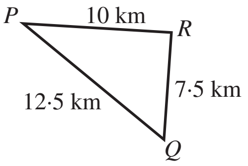
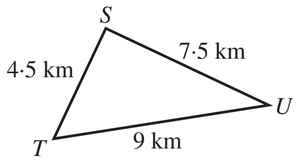
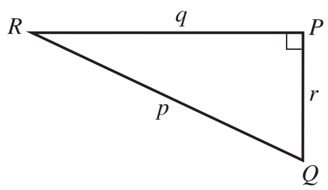


i



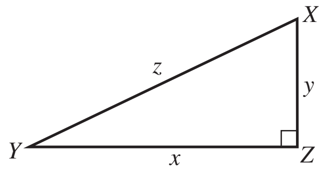
2 For each of the following triangles, select the correct statement of Pythagoras' theorem from those provided.

a



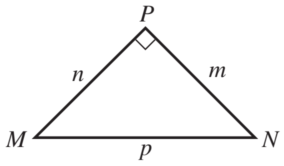
A $p^2 = q^2 + r^2$
B $q^2 = p^2 + r^2$
C $r^2 = p^2 + q^2$
D $q^2 - p^2 = r^2$

b



A $z^2 + y^2 = x^2$
B $x^2 + z^2 = y^2$
C $x^2 - y^2 = z^2$
D $z^2 = x^2 + y^2$

c



A $n^2 + p^2 = m^2$
B $p^2 + m^2 = n^2$
C $n^2 - m^2 = p^2$
D $n^2 + m^2 = p^2$

Answers 11B

1 **a** ∠*ACB* is a right angle **b** no right angle
c no right angle **d** no right angle
e ∠*EFD* is a right angle **f** ∠*EFD* is a right angle
g ∠*QRP* is a right angle **h** ∠*PRQ* is a right angle
i no right angle

2 **a** A **b** D **c** D

11C • Finding the length of the hypotenuse

LB2 Pages 79–82

Specific learning outcomes

Learners should be able to:

8.11.3.1 Identify and define the term 'hypotenuse'.
8.11.3.2 Calculate the length of the hypotenuse using Pythagoras' theorem.

Teaching points

1. Explain that the longest side of a right-angled triangle is called the hypotenuse.
2. The hypotenuse is also the side opposite the right angle in the triangle.
3. Calculate the length of the hypotenuse using Pythagoras' theorem.

Learner difficulties and remedies

Difficulty

Calculating the length of the hypotenuse.

Remedy

- Remind learners that the hypotenuse is the side opposite the right angle.
- Work though more examples using Pythagoras' theorem to calculate the length of the hypotenuse.
- Set more exercises on finding the length of the hypotenuse.

Difficulty

Calculating square roots when applying Pythagoras' theorem.

Remedy

- Explain to learners that the square root is the opposite of the square of a number.
- Show learners how to find square roots of numbers using a calculator.
- Have a table of squares and square roots for learners to refer to complete their calculations.

Suggested teaching approach

- Explain how the three sides in Pythagoras' theorem are related.
- Use Pythagoras' theorem to find the hypotenuse: $h^2 = a^2 + b^2$
- Define and explain square roots and how they are used in the Pythagoras' theorem to find the length of the hypotenuse of the right-angled triangle.

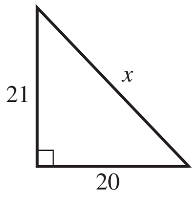
  Note that a square root of a number is the opposite of squaring a number.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

If we know the lengths of the two shorter sides of a right-angled triangle, we can use Pythagoras' theorem to find the length of the hypotenuse.

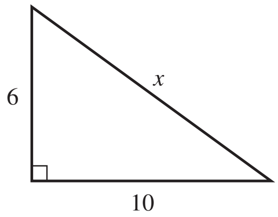
Examples

1 Calculate the length of the hypotenuse in the following triangle.



Thinking	Working
1 State Pythagoras' theorem and define the side lengths *a*, *b*, and *c*, letting *c* be the unknown length of the hypotenuse, *x*.	$c^2 = a^2 + b^2$ $a = 20, b = 21, c = x$
2 Substitute the values for *a*, *b* and *c* into Pythagoras' theorem.	$x^2 = 20^2 + 21^2$
3 Simplify the equation.	$x^2 = 400 + 441$ $= 841$
4 Solve for *x* by finding the square root of both sides of the equation.	$x = \sqrt{841}$ $x = 29$ units

2 Find the length of the hypotenuse in the following right-angled triangle. Give your answer as a square root.

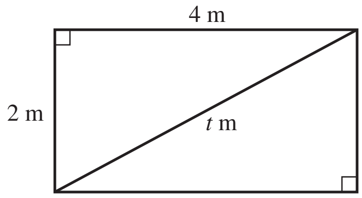


Thinking	**Working**
1 State Pythagoras' theorem, and define the side lengths.	$c^2 = a^2 + b^2$ $a = 6,\ b = 10,\ c = x$
2 Substitute the values for *a*, *b* and *c* into Pythagoras' theorem.	$x^2 = 6^2 + 10^2$
3 Simplify the equation.	$x^2 = 36 + 100$ $= 136$
4 Solve for *x* by finding the square root of both sides of the equation. Leave the answer as a square root (surd form).	$x = \sqrt{136}$

Pythagoras' theorem may be applied to other shapes provided they contain right-angled triangles. Sometimes, it is necessary to identify the required right-angled triangle by carefully adding a line or lines to the diagram.

3 Use Pythagoras' theorem to find the value of the unknown length in each of the following diagrams. Round your answers to two decimal places, if necessary.

a



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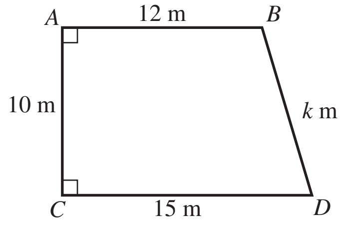
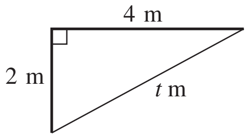
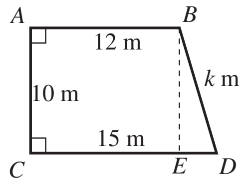
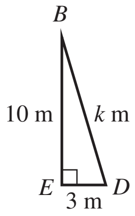
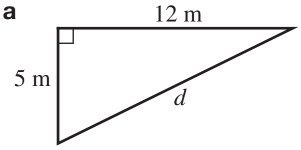


Thinking	**Working**
a 1 Identify a right-angled triangle and define the side lengths.	**a** Let $a = 2,\ b = 4,\ c = t$. 
2 State Pythagoras' theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $t^2 = 2^2 + 4^2$
3 Simplify the equation.	$= 4 + 16$ $= 20$
4 Solve for *t* by finding the square root of both sides. Use a calculator and round to two decimal places, with the correct units.	$t = \sqrt{20}$ $\approx 4{\cdot}472\,135\,955$ $t = 4{\cdot}47$ m (2 d.p.)
b 1 Draw a line from point *B* down to the line *CD* to form a right-angled triangle *BED*.	**b** 
2 Redraw the right-angled triangle and define the side lengths.	$c = k$ $a = 10$ $b = ED$ $= 15 - 12$ $= 3$ 
3 State Pythagoras' theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $c^2 = 10^2 + 3^2$
4 Simplify the equation.	$= 100 + 9$ $= 109$
5 Solve for *k* by finding the square root of both sides of the equation. Use a calculator round to two decimal places, with the correct units.	$k = \sqrt{109}$ $\approx 10{\cdot}440\,306\,51$ $k = 10{\cdot}44$ m (2 d.p.)

Activity 11C

1 Calculate the length of the hypotenuse in the following triangles.

a



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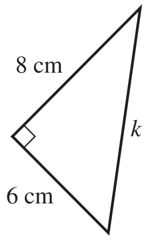

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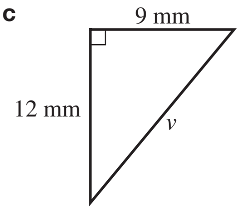

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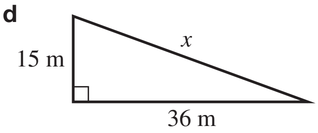

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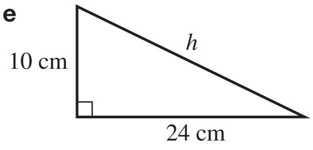

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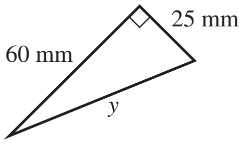
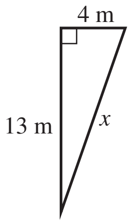


2 Find the length of the hypotenuse in the following right-angled triangle. Leave your answer as a square root.

a



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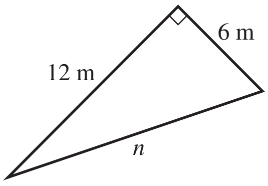

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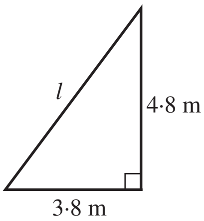

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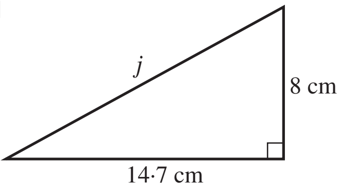

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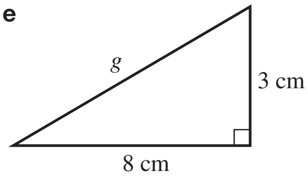

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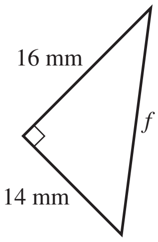
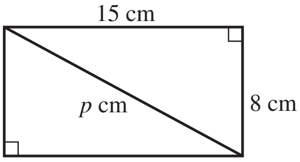


3 Use Pythagoras' theorem to find the value of the unknown length in each of the following diagrams. Round your answers to two decimal places, if necessary.

a



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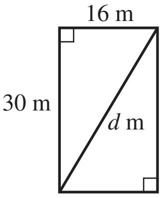

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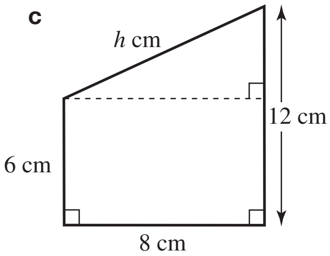

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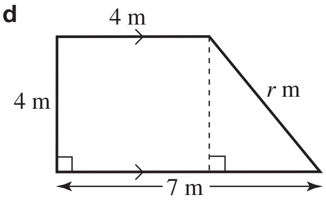

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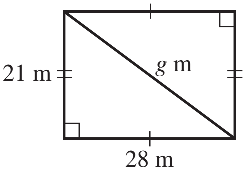

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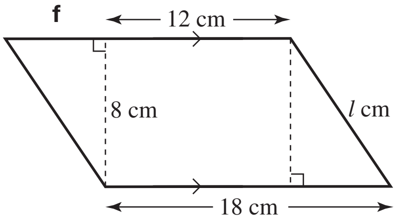

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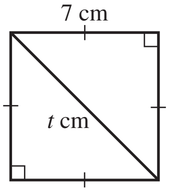

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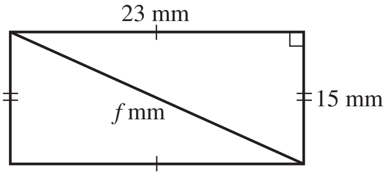

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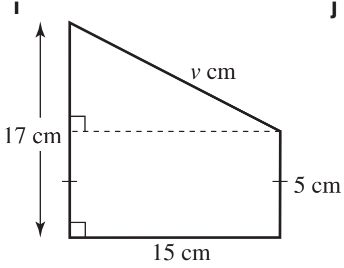

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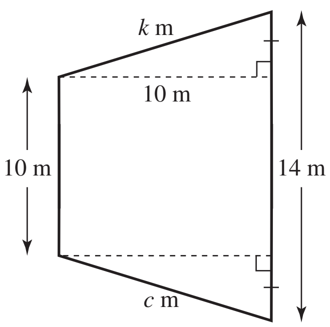

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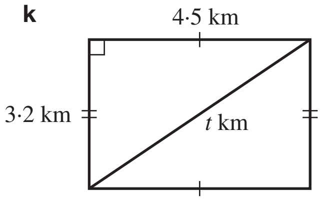

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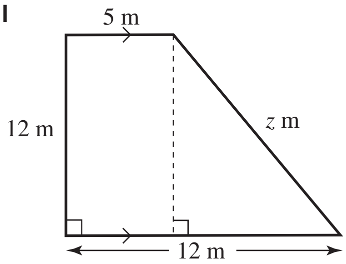
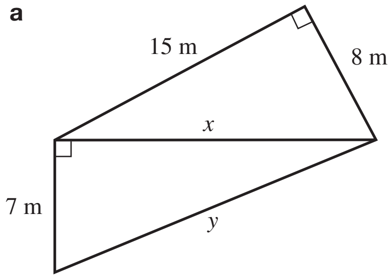


4 Find the unknown lengths in the following diagrams.

a



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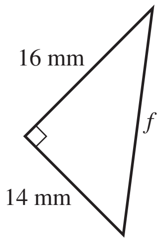


Answers 11C

1 **a** 13 m **b** 10 cm **c** 15 mm
d 39 m **e** 26 cm **f** 65 mm

2 **a** $\sqrt{185}$ m **b** $\sqrt{180}$ m **c** $\sqrt{37\cdot48}$ m
d $\sqrt{280\cdot09}$ cm **e** $\sqrt{73}$ cm **f** $\sqrt{452}$ mm

3 **a** 17 cm **b** 34 cm **c** 10 cm
d 5 m **e** 35 m **f** 10 cm
g 9·90 cm **h** 27·46 mm **i** 19·21 cm
j 10·20 m **k** 5·52 km **l** 13·89 m

4 **a** $x = 17$ m, $y = \sqrt{338}$ m
b $x = 13$ cm, $y = \sqrt{194}$ cm

11D • Pythagorean triads

LB2 Page 83

Specific learning outcomes

Learners should be able to:

8.11.4.1 Define a Pythagorean triad.

8.11.4.2 Identify sets of three numbers that form Pythagorean triads: 3, 4, 5; 9, 12, 15 etc.

Teaching points

1. Explain that a Pythagorean triad is a set of three whole numbers that conform to the rule $h^2 = a^2 + b^2$.
2. Find sets of numbers that make up triads.

Learner difficulties and remedies

Difficulty

Understanding what a triad is.

Remedy

- Explain to learners that Pythagorean triads are sets of three exact numbers that satisfy Pythagoras' theorem.
- Show that there are no decimal approximations in the square roots when finding the hypotenuse of a right-angled triangle when the sides lengths are Pythagorean triads.

Difficulty

Identifying numbers that are Pythagorean triads.

Remedy

- Recognise sets of three numbers written in ascending order, where the sum of the squares of the first two equals the square of the third number.
- Show learners that many of the triads are multiples of triads. For example:

  Multiples of 3, 4, 5 are Pythagorean triads: 6, 8, 10 and 9, 12, 15 etc.

  Multiples of 5, 12, 13 are Pythagorean triads: 10, 24, 26 and 15, 36, 39 etc.

Suggested teaching approach

- Explain the term triad and how it is used in Pythagoras' theorem.
- Three exact numbers must be used in triads. They cannot be decimal approximations that have been rounded.
- Identify triad numbers that can be used in Pythagoras' theorem. The common one is 3, 4 and 5.
- To find others triads, multiply a triad by 2, 3, 4, and 5.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A group of three positive whole numbers (a, b, c) that satisfy Pythagoras' theorem is called a **Pythagorean triad**. They are also called Pythagorean triples. Some common Pythagorean triads are shown in the table below.

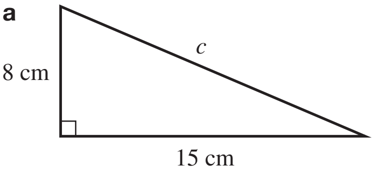
(3, 4, 5)	(8, 15, 17)	(12, 35, 37)
(5, 12, 13)	(9, 40, 41)	(16, 63, 65)
(7, 24, 25)	(11, 60, 61)	(20, 21, 29)

Multiples of a Pythagorean triads also satisfy Pythagoras' theorem. For example, if we multiply each number in (3, 4, 5) by 2, we have (6, 8, 10) and $6^2 + 8^2 = 10^2$. Dividing Pythagorean triads by a number also gives a set of numbers that satisfy Pythagoras' theorem. For example, dividing (7, 24, 25) by 2 gives (3·5, 12, 12·5) and $3{\cdot}5^2 + 12^2 = 12{\cdot}5^2$. The numbers in a Pythagorean triad need to be exact; it is not good enough for them to be rounded off to the required value.

Examples

Using your knowledge of common Pythagorean triads, state the value of the unknown side in each of the following triangles.

a



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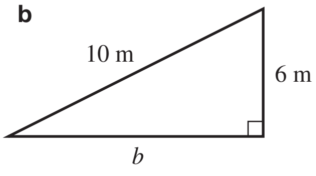
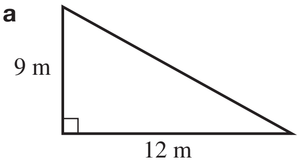


Thinking	Working
a **1** Check if the values of the shorter sides match a common triple, (a, b, c).	**a** $a = 8$, $b = 15$, $c = ?$
2 State the answer.	$c = 17$ cm
b **1** Check if the values of the hypotenuse and one of the shorter sides match a common triple, (a, b, c).	**b** $a = 6$ and $c = 10$ $= 3 \times 2$ $= 5 \times 2$ (6, ?, 10) is a multiple (× 2) of the common triple (3, 4, 5).
2 Find the missing number in the triple by multiplying the common triple number by the same multiple.	$b = 4 \times 2$
3 State the answer.	$b = 8$ m

Activity 11D

1 Using your knowledge of common Pythagorean triads, find the value of the missing sides in each of the following.

a



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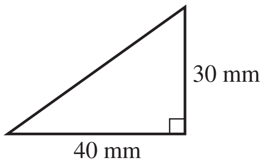

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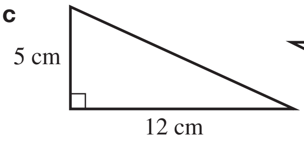

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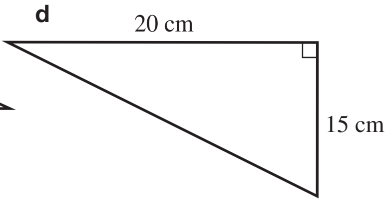

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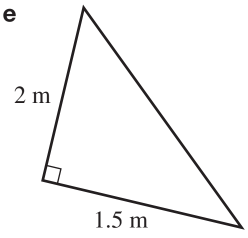

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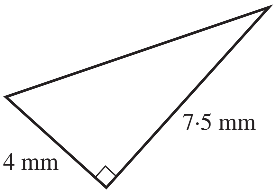

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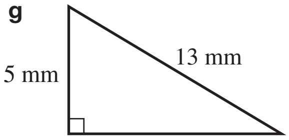

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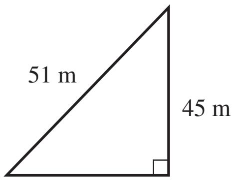

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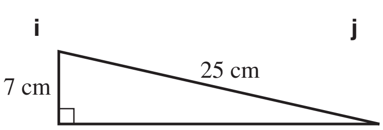

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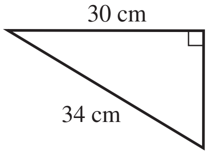

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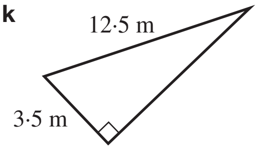

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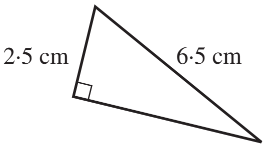


2 Find the value of c, using known Pythagorean triads.

a $a = 10$, $b = 24$ **b** $a = 15$, $b = 36$
c $a = 9$, $b = 12$ **d** $a = 27$, $b = 36$
e $a = 18$, $b = 80$ **f** $a = 50$, $b = 120$
g $a = 25$, $b = 60$ **h** $a = 500$, $b = 1200$
i $a = 80$, $b = 84$

3 Determine which of the following are Pythagorean triples.

a (6, 8, 10) **b** (24, 45, 51)
c (14, 48, 50) **d** (25, 60, 80)
e (10, 16, 28) **f** (20, 48, 52)
g (10, 18, 22) **h** (12, 50, 53)

Answers 11D

1 **a** 15 m **b** 50 mm **c** 13 cm
d 25 cm **e** 2·5 m **f** 8·5 mm
g 12 mm **h** 24 m **i** 24 cm
j 16 cm **k** 12 m **l** 6 cm

2 **a** 26 **b** 39 **c** 15
d 45 **e** 82 **f** 130
g 65 **h** 1300 **i** 116

3 **a** 2(3, 4, 5) yes **b** 3(8, 15, 17) yes
c 2(7, 24, 25) yes **d** 5(5, 12) not 5(13) no
e 2(5, 8, 14) no **f** 4(5, 12, 13) yes
g 2(5, 9, 11) no **h** $12^2 + 50^2 = 2644$ but $53^2 = 2809$ no

11E • Finding the length of a perpendicular side

LB2 Pages 84–86

Specific learning outcomes

Learners should be able to:

8.11.5.1 Find the length of the missing sides of given triangles using Pythagoras' theorem.

8.11.5.2 Solve practical questions using Pythagoras' theorem.

Teaching points

1. Calculate the length of a perpendicular side of a right-angled triangle given the hypotenuse and one other side.
2. Solve practical questions by first drawing and labelling a right-angled triangle.

Learner difficulties and remedies

Difficulty

Rearranging Pythagoras' theorem to find the perpendicular side or the two shorter sides.

Remedy

- Explain to learners how to rearrange the Pythagoras' theorem using algebra knowledge.
- Use inverse operations to rearrange the formula to make one of the two shorter sides the subject of the equation.
- Solve the formula using algebra techniques. Remind learners to include units if given in the question.

Suggested teaching approach

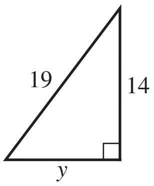
- Remind learners of Pythagoras' theorem $h^2 = a^2 + b^2$.
- Explain to learners how algebra techniques can be used to rearrange the formula.
- Learners rearrange the formula and make the shorter side the subject.
- Solve to find the length of the perpendicular side or a shorter side.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

The length of one of the shorter sides of the right-angled triangle can be found if we are given the lengths of the hypotenuse and the other side.

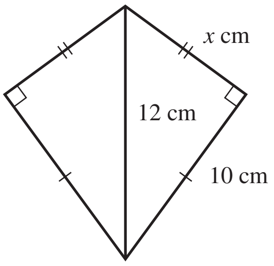
Examples

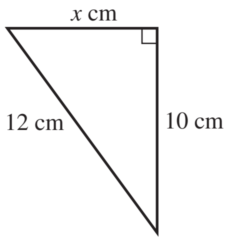
1 Fine the value of the unknown shorter side in this right-angled triangle. Give your answer as an exact value.



Thinking	Working
1 State Pythagoras' theorem and define the side lengths.	$c^2 = a^2 + b^2$ Let $a = y$, $b = 14$, $c = 19$
2 Substitute the values into Pythagoras' theorem.	$19^2 = y^2 + 14^2$
3 Rewrite so that the unknown is isolated on one side of the equation. (Here, 14^2 has been subtracted from both sides.)	$y^2 = 19^2 - 14^2$
4 Simplify the equation.	$= 361 - 196$ $= 165$
5 Solve for y by finding the square root of both sides of the equation. Leave your answer as a square root.	$y = \sqrt{165}$

2 Find the value of the variable, correct to two decimal places, in the following diagram.

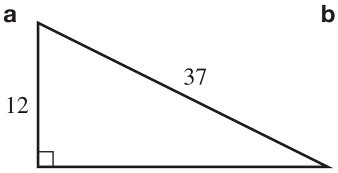


Thinking	**Working**
1 Draw a diagram of the appropriate right-angled triangle.	
2 State Pythagoras' theorem and define the side lengths.	$c^2 = a^2 + b^2$ Let $a = 10$, $b = x$, $c = 12$
3 Substitute the values into Pythagoras' theorem.	$12^2 = 10^2 + x^2$
4 Rewrite so that the unknown is isolated on one side of the equation. (Here, 10^2 has been subtracted from both sides).	$x^2 = 12^2 - 10^2$
5 Simplify the equation.	$= 144 - 100$ $= 44$
6 Solve for x by finding the square root of both sides of the equation. Use a calculator and write it correct to two decimal places.	$x = \sqrt{44}$ $= 6{\cdot}6332495$ $x = 6{\cdot}63$ m (2 d.p.)

Activity 11E

1 Find the value of the unknown shorter side in these right-angled triangles.

a



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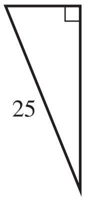

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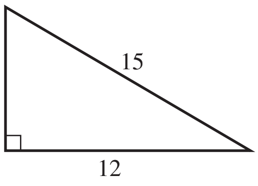

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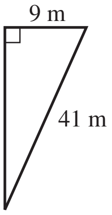

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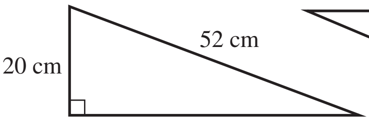

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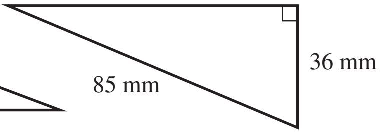
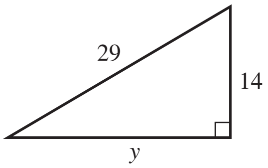


2 Find the value of the unknown shorter side in these right-angled triangles. Leave your answer as a square root.

a



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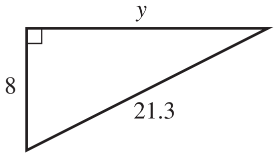

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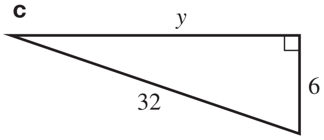

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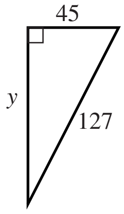
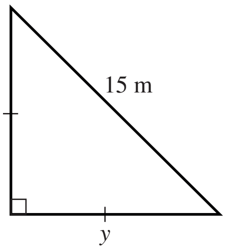
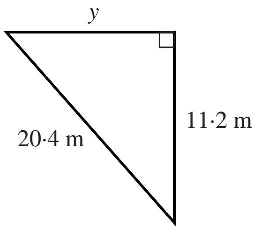


e

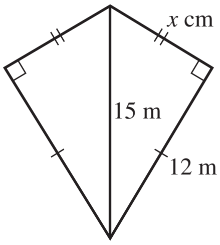


f

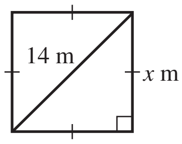


3 Find the value of the unknown side(s) in each of the following diagrams. Round to two decimal places, if necessary.

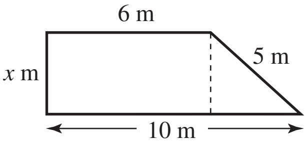
a



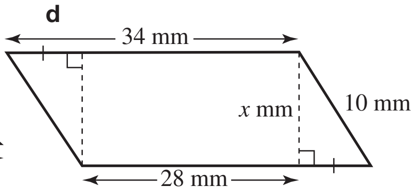
b



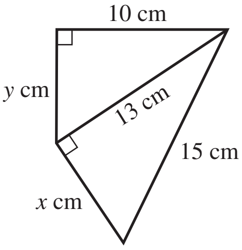
c



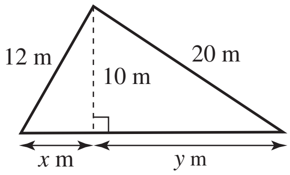
d



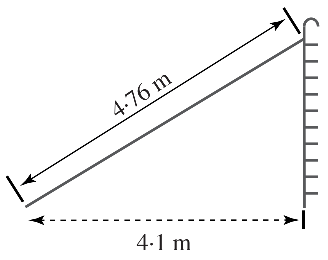
e



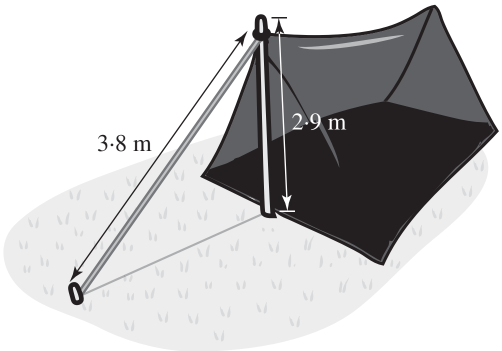
f



4 A playground slide has a length of 4·76 m and it covers a horizontal distance of 4·1 m on the ground. What is the height of the slide, correct to two decimal places?

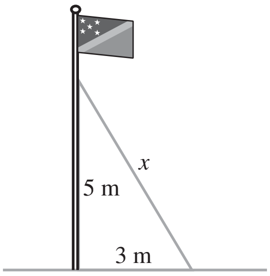


5 A ‘guy rope’ is attached to the top of a tend pole and pegged into the ground so that the rope is tight. If the rope is 3·8 m long, and the pole is 2·9 m long, how far from the base of the pole should the rope be pegged if it is to be tight?

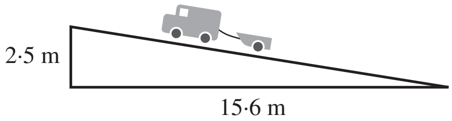


6 Safety regulations for ladders state that the distance that a ladder can be placed from a wall is no closer than one-quarter of its length and no further than half its length. If a ladder is 3·2 m long, give three heights that it can reach up the wall.

7 **a** A support wire is attached 5 m up a flagpole. The other end of the wire is attached to the ground 3 m from the base of the flagpole. What is the length of the wire?



b A boat ramp is to be built that will cover a horizontal distance of 15·6 m while descending a vertical distance of 2·5 m. What is the length of the ramp?



Answers 11E

1 **a** 35 **b** 24 **c** 9
d 40 m **e** 48 cm **f** 77 mm

2 **a** $\sqrt{645}$ **b** $\sqrt{389{\cdot}69}$ **c** $\sqrt{988}$
d $\sqrt{14\,104}$ **e** $\sqrt{112{\cdot}5}$ m **f** $\sqrt{290{\cdot}72}$ m

3 **a** $x = 9$ m **b** $x = 9{\cdot}90$ m
c $x = 3$ m **d** $x = 8$ mm
e $x = 7{\cdot}48$ cm, $y = 8{\cdot}31$ cm **f** $x = 6{\cdot}63$ m, $y = 17{\cdot}32$ m

4 $4{\cdot}76^2 = 4{\cdot}1^2 + h^2$
$h^2 = 4{\cdot}76^2 - 4{\cdot}1^2$
$= 5{\cdot}8476$
$h = 2{\cdot}42$ m

5 $3{\cdot}8^2 = 2{\cdot}9^2 + x^2$
$x^2 = 3{\cdot}8^2 - 2{\cdot}9^2$
$= 6{\cdot}03$
$x = 2{\cdot}46$ m

6 Sample answers
$h^2 + 0{\cdot}8^2 = 3{\cdot}2^2$ (longest distance)
$h^2 = 3{\cdot}2^2 - 0{\cdot}8^2$
$= 9{\cdot}6$
$h = 3{\cdot}10$
$h^2 + 1{\cdot}6^2 = 3{\cdot}2^2$ (shortest distance)
$h^2 = 3{\cdot}2^2 - 1{\cdot}6^2$
$= 7{\cdot}68$
$h = 2{\cdot}77$

7 **a** 5·83 m
b 15·80 m

11F • Pythagoras’ theorem in exact form

LB2 Pages 87–88

Specific learning outcomes

Learners should be able to:

8.11.6.1 Define and identify the term ‘irrational number’.
8.11.6.2 Define numbers in exact form as a surd or a square root.
8.11.7.1 Calculate the length of a missing side of a right-angle triangle expressing the answer in exact form.

Teaching points

1. Explain that irrational numbers cannot be written as a fraction or as a terminating decimal. Examples of are $\sqrt{2}$, $\sqrt{156}$, $\sqrt[3]{7}$ and π.
2. Exact form numbers use the square root surd form such as $\sqrt{2}$ instead of a decimal approximation 1·41 (2 d.p.).
3. Use Pythagoras’ theorem to calculate the length of a missing sides expressing the answer in exact form.

Learner difficulties and remedies

Difficulty

Understanding irrational numbers or surds.

Remedy

- Remind learners that Pythagorean triads perfect square numbers so they have some clear understanding of numbers that have exact square roots.
- Explain to learners that non-triad numbers can be written as a square root (surd) rather than using a calculator to find its decimal approximation and rounding to a given number of decimal places. Answers in surd form are exact.

Suggested teaching approach

- Identify irrational numbers. Show that square roots of numbers that are not perfect squares are irrational. We call them surds.
- Show that exact answers are left as square roots or surds.
- Explain that when learners use a calculator to evaluate a surd they get a decimal approximation, not an exact answer. The number of decimal places goes on forever. These are irrational numbers. We usually round these numbers to a given number of decimal places.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Irrational numbers are numbers that cannot be written as a fraction or as a terminating decimal. Examples of are $\sqrt{2}$, $\sqrt{156}$, $\sqrt[3]{7}$ and π.

$\sqrt{2} = 1{\cdot}414\,213\,562\ldots$

There are an infinite number of decimal places. The square roots of numbers that are not perfect squares are irrational numbers. We call them surds.

We can write irrational numbers as an exact value or as a decimal approximation.

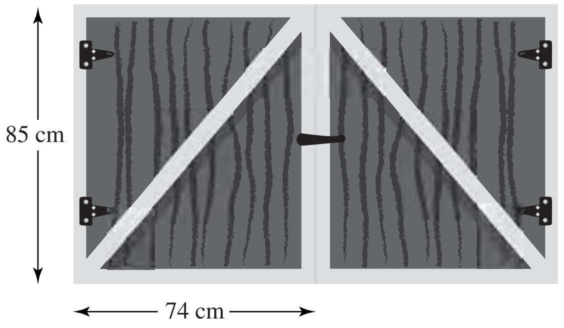
Exact: $\sqrt{2}$

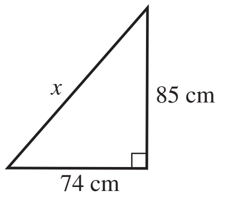
Decimal approximation: 1·41 (2 d.p.)

When applying Pythagoras' theorem to calculate the length of a missing sides, answers can be expressed in exact form (surd) or as a decimal approximation.

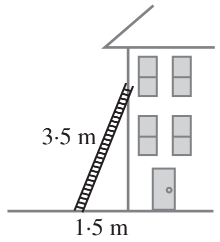
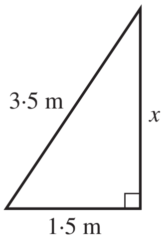
Examples

1 What is the length of one of the diagonal supports of the frame shown? State your answer as an exact value and correct to two decimal places.



Thinking	**Working**
1 Draw a diagram of the appropriate triangle. Label the right angle, the given measurements and the unknown side.	
2 Use Pythagoras' theorem to find the value of the variable.	$c^2 = a^2 + b^2$ $x^2 = 74^2 + 85^2$ $= 5476 + 7225$ $= 12\,701$ $x = \sqrt{12\,701}$ (exact) $= 112{\cdot}70$ (2 d.p.)
3 State the answer correct to two decimal places.	The support is approximately 112·70 cm long.

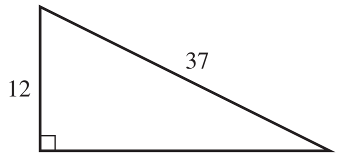
2 A ladder 3·5 m long is leaning against a vertical wall with the base 1·5 m from the bottom of the wall on horizontal ground. How high up the wall does the ladder reach? State your answer as an exact value and correct to two decimal places.

Thinking	**Working**
1 Draw a labelled diagram with all measurements indicated.	
2 Draw a diagram of the appropriate triangle.	
3 Use Pythagoras' theorem to find the value of the variable.	$c^2 = a^2 + b^2$ Let $a = 1{\cdot}5$, $b = x$, $c = 3{\cdot}5$ $3{\cdot}5^2 = 1{\cdot}5^2 + x^2$ $x^2 = 3{\cdot}5 - 1{\cdot}5^2$ $= 12{\cdot}25 - 2{\cdot}25$ $= 10$ $x = \sqrt{10}$ (exact) $= 3{\cdot}16$ (2 d.p.)
4 State the answer.	The ladder will reach 3·16 m up the wall.

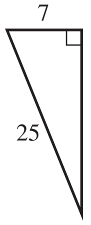
Activity 11F

1 For each of the following triangles, calculate the length of the unknown side. Express the answer as a surd, and then round to two decimals places where necessary.

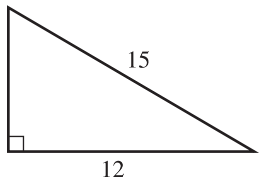
a



b

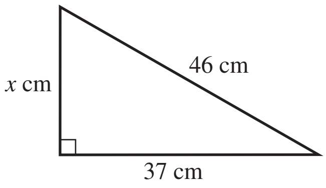


c

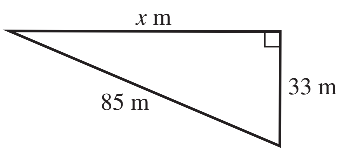


2 Use Pythagoras' theorem to find the value of x. Write your answer correct to two decimal places.

a



b



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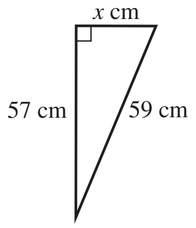
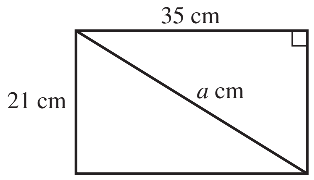
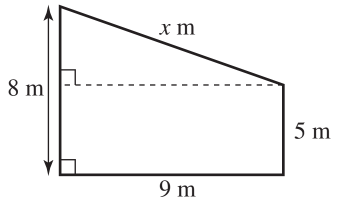


3 Use Pythagoras' theorem to find the value of the unknown in each of the following diagrams. Write your answer correct to two decimal places, if necessary.

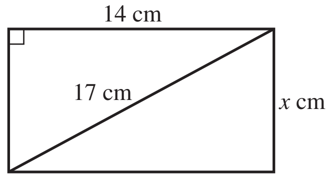
a



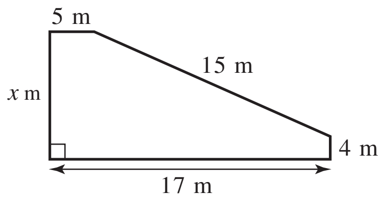
b



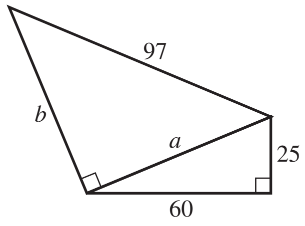
c



d



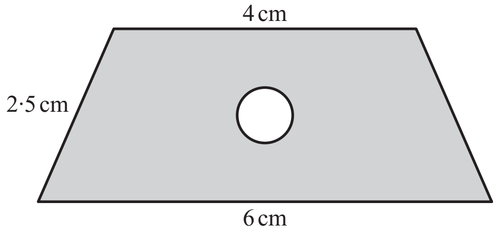
4 Calculate the exact values of *a* and *b*.



5 Sharon is 20 m due south of a target. She throws a ball that stops 16 m due east of a target. Draw a diagram to represent the information and then calculate how far the ball travelled. Express your answer as a surd and then round to one decimal place.

6 A safety blade is a symmetrical shape with the dimensions shown. Calculate the height of the blade correct to one decimal place.

7 A ladder 4 m long is placed against a wall so that the foot of the ladder is 1·5 m away from the wall. Draw a diagram to represent the information and then calculate, correct to one decimal place, how far up the wall the ladder will reach.



Answers 11F

1 **a** $60^2 = x^2 + 45^2$

$x^2 = 60^2 - 45^2 = 1575$

$x = \sqrt{1575} = 39{\cdot}69$ m

b $37^2 = 35^2 + y^2$

$y^2 = 37^2 - 35^2 = 144$

$y^2 = \sqrt{144} = 12$ m

c $6{\cdot}1^2 = 4{\cdot}3^2 + z^2$

$z^2 = 6{\cdot}1^2 - 4{\cdot}3^2 = 18{\cdot}7^2$

$z = \sqrt{18.72} = 4{\cdot}33$ m

2 **a** 27·33 cm **b** 78·33 m **c** 15·23 cm

3 **a** 40·82 **b** 9·49 **c** 9·64

d 13

4 $a^2 = 60^2 + 25^2 = 4225$

$a = \sqrt{4225} = 65$

$a^2 + b^2 = 97^2$

$b^2 = 97^2 - 4225$

$b = \sqrt{5186} = 72$

5 $a^2 = 16^2 + 20^2 = 656$

$a = \sqrt{656} = 25{\cdot}6$

The ball has travelled 25·6 m.

6 $2{\cdot}5^2 = 1^2 + x^2$

$x^2 = 2{\cdot}5^2 - 1^2 = 5{\cdot}25$

$y^2 = \sqrt{5{\cdot}25} = 2{\cdot}29$

The blade is 2·3 cm in height.

7 $4^2 = 1{\cdot}5^2 + x^2$

$x^2 = 4^2 - 1{\cdot}5^2 = 13{\cdot}75$

$y^2 = \sqrt{13{\cdot}75} = 3{\cdot}7$

The ladder reaches 3·7 m up the wall.

11G • Composite shapes

LB2 Pages 89–91

Specific learning outcomes

Learners should be able to:

8.11.8.1 Define a composite shape and identify examples of composite shapes.

8.11.9.1 Find missing lengths indicated by pronumerals in given composite shapes.

Teaching points

1. Explain that composite shapes are shapes that can be cut into triangles and quadrilaterals in order to calculate properties such as the lengths of sides, angles, area and perimeter.
2. Calculate missing lengths that represented by pronumerals.

Learner difficulties and remedies

Difficulty

Identifying those shapes that make up the composite shape and using Pythagoras' theorem to find missing sides.

Remedy

- Know the shapes that make up the composite shape.
- Use Pythagoras' theorem correctly to find missing sides.

Suggested teaching approach

- Identify those shapes that made up the composite shape.
- Use Pythagoras' theorem to find the missing sides.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

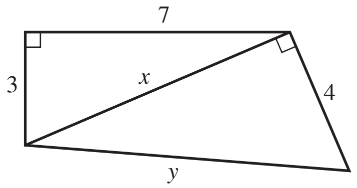
Additional notes

Composite shapes are shapes that are made up of different standard geometric shapes. Ideally, for calculation purposes the shape is best divided into a minimum number of rectangles and right-angled triangles. Both the natural environment and human made objects are made up of composite shapes.

Pythagoras' theorem can be used to find unknown or missing lengths by breaking the composite shapes into different components.

Examples

Find the unknown lengths in the following diagram. Give your answers in exact form (surd form).

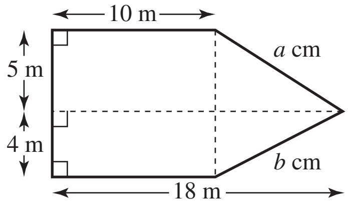


Thinking	Working
1 Identify the right-angled triangle where two of the three sides are known and define the side lengths.	$a = 3$ $b = 7$ $c = x$
2 State Pythagoras' theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $x^2 = 3^2 + 7^2$
3 Simplify the equation.	$= 9 + 49$ $= 58$
4 Solve for *x* by finding the square root of both sides. Leave your answer as a square root (surd form).	$x = \sqrt{58}$
5 Identify the second right-angled triangle and define the side lengths using your answer for *x*.	$a = x = \sqrt{58}$ $b = 4$ $c = y$
6 State Pythagoras' theorem, then substitute in the values. Use the exact surd value determined previously.	$c^2 = a^2 + b^2$ $y^2 = \left(\sqrt{58}\right)^2 + (4)^2$ $y^2 = 58 + 16$
7 Simplify the equation.	$= 74$
8 Solve for *y* by finding the square root of both sides. Leave your answer as an exact value.	$y = \sqrt{74}$

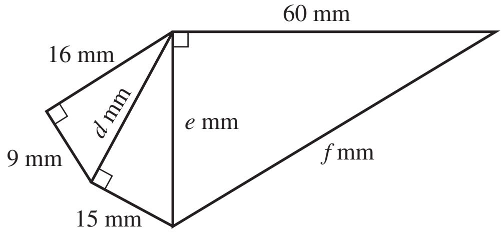
Activity 11G

1 Find the exact value of the variables in the following diagrams. Give each answer as an exact value and correct to two decimal places.

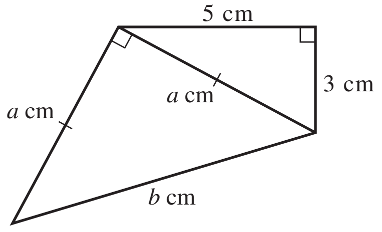
a



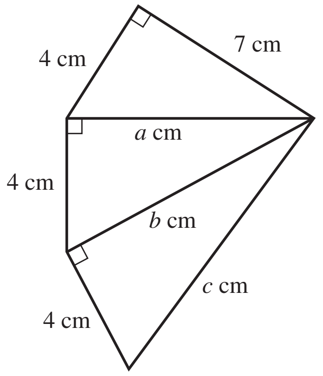
b



c



d



Answers 11G

1 **a** $a^2 = 5^2 + (18 - 10)^2$
$= 89$
$a = \sqrt{89}$
$= 9{\cdot}43$ (2 d.p.)
$b^2 = 4^2 + 8^2$
$= 80$
$b = \sqrt{80}$
$= 8{\cdot}94$ (2 d.p.)

b $d^2 = 9^2 + 16^2$
$= 337$
$d = \sqrt{337}$
$= 18{\cdot}36$ (2 d.p.)
$e^2 = 15^2 + 337$
$= 562$
$e = \sqrt{562}$
$=23{\cdot}71$ (2 d.p.)
$f^2 = 60^2 + 562$
$= 4162$
$f = \sqrt{4162}$
$= 64{\cdot}51$ (2 d.p.)

c $a^2 = 3^2 + 5^2$
$= 34$
$a = \sqrt{34}$
$= 5{\cdot}83$ (2 d.p.)
$b^2 = a^2 + a^2$
$= 81$
$b = \sqrt{68}$
$=8{\cdot}25$ (2 d.p.)

d $a^2 = 7^2 + 4^2$
$= 65$
$a = \sqrt{65}$
$= 8{\cdot}06$ (2 d.p.)
$b^2 = 4^2 + 65$
$= 81$
$b = \sqrt{81}$
$=9$
$c^2 = 9^2 + 4^2$
$= 97$
$c = \sqrt{97}$
$= 9{\cdot}85$ (2 d.p.)

11H • Coordinate geometry

LB2 Page 92

Specific learning outcomes

Learners should be able to:

8.11.10.1 Calculate the lengths of line segments between two points on a graph using the Pythagoras' theorem.

Teaching points

1 Find the length of the line segment between two points coordinates on a Cartesian plane using Pythagoras' theorem.

Learner difficulties and remedies

Difficulty

Identifying the coordinates of the line segments.

Remedy

- Find the coordinates of the two points at the ends of the line segments on a Cartesian plane where *x* comes first then *y* in the coordinate point.

Difficulty

Finding the lengths of the two shorter sides of the triangle after the line segment is given.

Remedy

- Identify the two end points that made up the line segment.
- Count the number of units in the lengths of the two shorter sides.
- Use Pythagoras' theorem to find the length of the third segment, which is the hypotenuse of the right-angled triangle.

Suggested teaching approach

- Find the coordinates of the two points that made up the line segment.
- Identify the right-angled triangle formed with the given line segment as a hypotenuse.
- Find the lengths of the two shorter sides of the right-angled triangle on the line segment.
- Use the Pythagoras' theorem to find the length of the line segment (hypotenuse) of the triangle.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Coordinate geometry uses the axes on a Cartesian plane to locate points and lines on a graph.

- The horizontal axis is known as the *x*-axis and the vertical axis is known as the *y*-axis.
- The point of intersection of these axes is called the **origin**, which has the coordinates (0, 0).
- All points can be described by an ordered pair known as **coordinates** (*x*, *y*) as measured from the origin.
- An **interval** is the line segment formed when two points are joined.

The line segment formed when two points are joined can become the hypotenuse of a right-angled triangle.

We use **coordinate geometry** to find relationships between points, such as the distance between two points. Coordinate geometry allows us to use algebraic techniques to solve geometric problems.

The distance formula

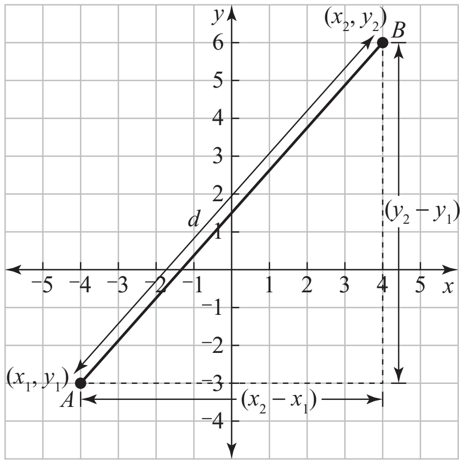
To find the distance between two points, we use the Cartesian coordinate system because the horizontal and vertical axes are at right angles to each other.

In the diagram below we have two points (x_1, y_1) and (x_2, y_2). To find the distance between these two points, we connect them with a straight line. If the two points are $A(x_1, y_1)$ and $B(x_2, y_2)$, the horizontal distance is given by $(x_2 - x_1)$, the difference in the x-coordinates and the vertical distance is given by $(y_2 - y_1)$, the difference in the y-coordinates.

- The difference between the x-coordinates $(x_2 - x_1)$ gives the length of one side of a right-angled triangle.

- The difference between the x-coordinates $(x_2 - x_1)$ gives the length of one side of a right-angled triangle.

- The difference between the y-coordinates $(y_2 - y_1)$ gives the length of another side.
- These two lines form two of the sides of a right-angled triangle, with the distance between the two points, d, the length of the hypotenuse.
- We use Pythagoras' theorem to find the distance between these two points.



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Examples

Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form.

a (−4, 6) and (2, −2)

b (−2, 1) and (2, −4)

c (3, 5) and (5, 2)

Thinking	Working
a **1** Identify the two points.	**a** Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$
2 Write the distance formula and substitute in the relevant values. (Here $x_1 = -4$, $x_2 = 2$, $y_1 = 6$, $y_2 = -2$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2 - -2)^2 + (-2 - 6)^2}$ $= \sqrt{6^2 + (-8)^2}$ $= \sqrt{36 + 64}$ $= \sqrt{100}$
3 Evaluate and include units.	= 10 units
b **1** Identify the two points.	**b** Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (2, -4)$
2 Write the distance formula and substitute in the relevant values. (Here $x_1 = -2$, $x_2 = 2$, $y_1 = 1$, $y_2 = -4$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2 - -2)^2 + (-4 - 1)^2}$ $= \sqrt{4^2 + (-5)^2}$
3 Evaluate, leaving the answer in exact surd form and including units.	$= \sqrt{41}$ units
c **1** Identify the two points.	**c** Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (5, 2)$
2 Write the distance formula and substitute in the relevant value. (Here $x_1 = 3$, $x_2 = 5$, $y_1 = 5$, $y_2 = 1$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 3)^2 + (2 - 5)^2}$ $= \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{4 + 9}$
3 Evaluate by simplifying the surd value, leaving the answer in exact surd form and including units.	$= \sqrt{13}$ units

Activity 11H

1 Find the distance between the following Cartesian coordinates, giving your answers in exact form.

a (2, 5) and (5, 9)

b (−4, 2) and (8, −3)

c (−4, 13) and (4, −2)

d (−6, 34) and (4, 10)

e (−4, 5) and (4, −1)

f (−11, −4) and (13, 3)

g (2, −5) and (3, −1)

h (−1, 3) and (5, 2)

i (4, −3) and (−2, 2)

j (−3, −5) and (−1, −7)

k (3, −4) and (−1, 2)

l (2, −7) and (6, 1)

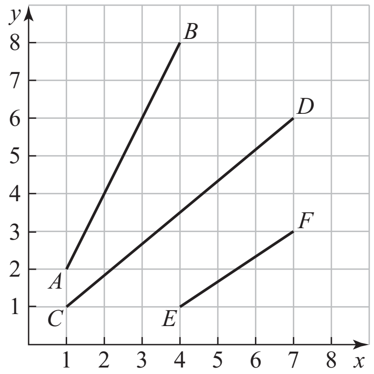
2 Find the length of the following line segments. Give the answers in exact form.

i *A*

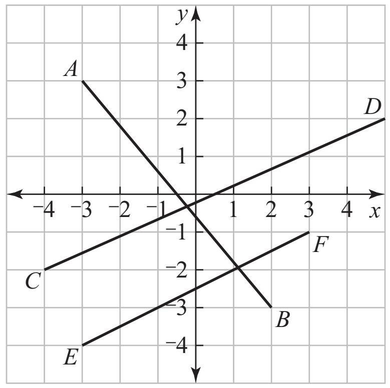
ii *CD*

iii *EF*

a



a



Answers 11H

1 **a** 5
b 13 units
c 17 units
d 26 units
e 10 units
f 25 units
g $\sqrt{17}$ units
h $\sqrt{37}$ units
i $\sqrt{61}$ units
j $\sqrt{8}$ units
k $\sqrt{52}$ units
l $\sqrt{80}$ units

2 **a** **i** $\sqrt{45}$ units
ii $\sqrt{61}$ units
iii $\sqrt{13}$ units
b **i** $\sqrt{61}$ units
ii $\sqrt{97}$ units
iii $\sqrt{45}$ units

11I • Pythagoras' theorem in three dimensions

LB2 Pages 93–95

Specific learning outcomes

Learners should be able to:

8.11.11.1 Calculate the dimensions of any given cross-sections of solids using Pythagoras' theorem.

Teaching points

1 Calculate the sides of given cross-sections of solids using Pythagoras' theorem.

Learner difficulties and remedies

Difficulty

Calculating unknown sides in given cross-sections of solids using Pythagoras' theorem.

Remedy

- Use given measurements in the solids to find the unknown side lengths for right-angled triangles produced in the cross-sections of the solids.
- Know how to use the Pythagoras' theorem to find the hypotenuse of a triangle that is produced in the cross-sections of the solid.

Suggested teaching approach

- Identify the 3D shapes.
- Identify the right-angled triangle in the solid that includes the unknown side.
- Find the side lengths of the other two sides in the right-angled triangle.
- Use the Pythagoras' theorem to find the length of the unknown side.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

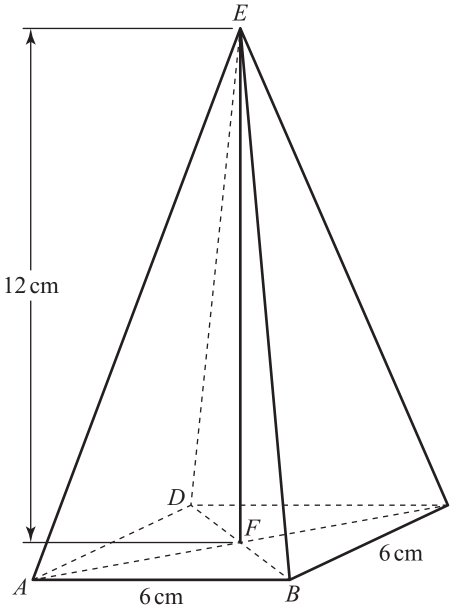
Additional notes

We can use the Pythagoras' theorem in some 3D shapes to calculate the length of given lines or sides of given shapes. This requires identifying a suitable right-angled triangle then Pythagoras' theorem can be used to calculate the missing side.

Examples

Find the length of the line segments in exact form.

a *AC*
b *AF*
c *BD*
d *BE*

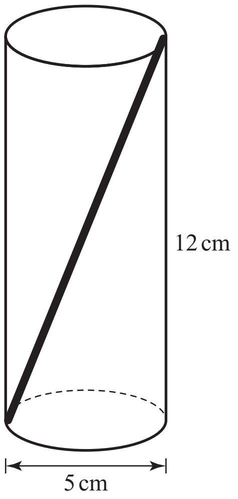


Thinking	**Working**
a Write Pythagoras' theorem and then substitute in values from the right-angled triangle that includes the unknown side. Evaluate and then leave the answer in exact form including units.	$AC^2 = AB^2 + BC^2$ $AC^2 = 6^2 + 6^2$ $AC^2 = 36 + 36 = 72$ $AC = \sqrt{72}$ cm
b *F* is the midpoint of *AC*, so $AF = \frac{1}{2}AC$.	$AF = \frac{1}{2}AC = \frac{\sqrt{72}}{2}$
c *AC* and *BD* are diagonals of the base, so *BD* = *AC*.	$BD = AC = \sqrt{72}$ cm
d Write Pythagoras' theorem and then substitute in values from the right-angled triangle that includes the unknown side. Evaluate and then leave the answer in exact form including units.	$BE^2 = BF^2 + FE^2$ $BE^2 = \left(\frac{\sqrt{72}}{2}\right)^2 + 12^2$ $= \frac{72}{4} + 144$ $= 18 + 144 = 162$ $BE = \sqrt{162}$ cm

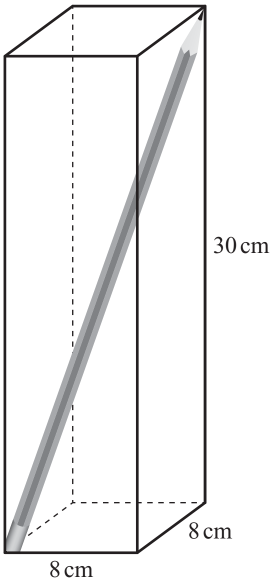
Activity 11I

1 Find the length of the longest object that can be placed into these containers, expressing the answers to the nearest centimetre.

a



b



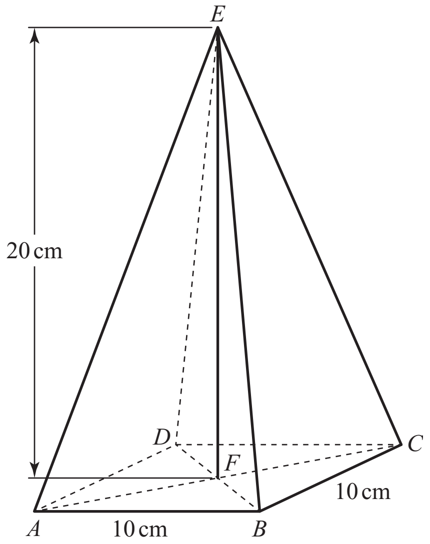
2 Find the length of the lines, expressing the answers in exact form.

a *AC*

b *AF*

c *BD*

d *BE*



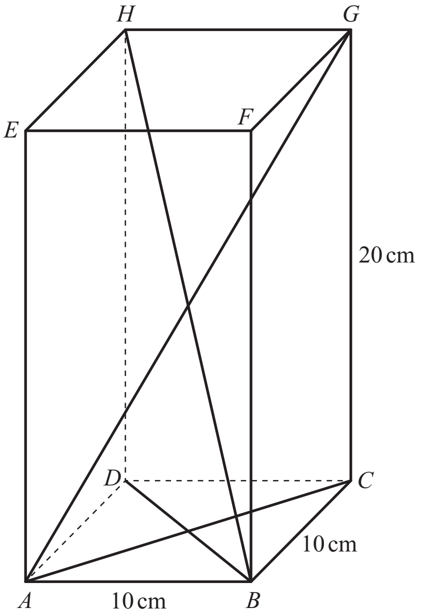
3 Find the length of the sides expressed:

- **i** in exact form
- **ii** correct to two decimal places

a *AC*

b *AF*

c *BH*



Answers 11I

1 a 13 cm **b** 32 cm

2 a $\sqrt{200}$ cm **b** $\frac{\sqrt{200}}{2}$ cm

c $\sqrt{200}$ cm **d** $\sqrt{450}$ cm

3 a i $\sqrt{200}$ cm **ii** 14·14 cm

b i $\sqrt{500}$ cm **ii** 22·36 cm

c i $\sqrt{600}$ cm **ii** 24·59 cm

11J • Exploring areas of triangles

LB2 Pages 96–97

Specific learning outcomes

Learners should be able to:

8.11.11.2 Investigate composite areas of squares and triangles using formulas.

Teaching points

1. This task is a visual proof of Pythagoras' theorem based on the area formulas of triangles and squares.
2. Learners need to remember the formula of triangle $A = \frac{1}{2}bh$ and to follow each step of the task as shapes are cut out and rearranged in special ways.

Suggested teaching approach

- Learners can work in pairs to complete **Learning Task 11J** on pages 96 and 97 of the LB.

• Chapter Twelve •

Probability

Overview

The probability of an event occurring is the chance of that outcome occurring. Examples of events in probability are an earthquake occuring, tossing a head with a coin, rolling a 4 on a die or it raining on a particular day.

Some events are more likely to happen than others. Mathematicians like to apply numbers as measures of the likelihood of particular events happening. Probabilities range from 0, for an impossible event, to 1, for a certain event. The probability of obtaining a 7 on a normal die is 0, whereas the probability of getting a number less than 7 is 1.

Any possible result of an event is called an outcome. When tossing a coin, there are two possible outcomes: obtaining a head or obtaining a tail. A successful outcome is the result that is desired, such as getting a head on a coin toss. The probability of an event, Pr(event), can be written as a decimal, fraction or percentage. Probabilities can be determined by experiments and by collecting data over a long period of time, but they are often determined theoretically for equally likely events.

Contents

Chapter skills

This chapter covers the following skills:

- Using set notation
  - $\in$ means is 'an element of'
  - $\notin$ means is 'not an element of'
  - ξ represents the universal set
  - $\cap$ denotes the intersection of two sets
  - $\cup$ denotes the union of two sets
  - ϕ or {} represents an empty or null set
- Using probability notation

  $\Pr(A) = \dfrac{n(\text{A})}{n(\varepsilon)}$ or

  $$\Pr(A) = \frac{\text{number of elements } A}{\text{number of elements in the universal set}}$$

  The probability of the complement of A is $\Pr(A')$ and $\Pr(A') = 1 - \Pr(A)$
- Drawing and interpreting Venn diagrams
- Displaying sample spaces using tree diagrams
- Using information statistics to predict the likelihood of an event
- Simulating experiments to estimate probabilities

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 12A Exploring sets	Learner's Book 1 • Exercise 12A, page 110
2–4	• 12B Sets and venn diagrams	Learner's Book 1 • Exercise 12B, pages 113, 114
5–6	• 12C Sets and probability	Learner's Book 1 • Exercise 12C, pages 116, 117
6–8	• 12D Sample space grid	Learner's Book 1 Exercise 12D, pages 119, 120
9–10	• 12E Tree diagrams	Learner's Book 1 • Exercise 12E, page 122
11–12	• 12F Exploring a pack of cards	Learner's Book 1 • Exercise 12F, page 123
13–14	• 12G Estimating proportion using sampling	Learner's Book 1 • Exercise 12G, page 124
15	• Test	Teacher's Guide • Chapter 12 Test

General learning outcomes

Learners should:

Exploring sets

8.12.1 Understand that sets are used to group items together. (U)

8.12.2 Know how to group elements or objects with certain criteria using circles. (K)

Sets and Venn diagrams

8.12.3 Understand sets as a method of grouping objects or elements. (U)

8.12.4 Know how to write sets using 'curly brackets' and capital letters to name them. (K)

8.12.5 Understand 'elements' and how they are represented in sets and the symbols used. (U)

8.12.6 Know that a set that includes all possible outcomes is known as universal set. (K)

8.12.7 Understand that some elements may not be shown in the group set. (U)

8.12.8 Understand that a set that does not have any elements in it is called the empty set or null set. (U)

8.12.9 Understand that when two sets have elements that are common we say that they intersect. (U)

8.12.10 Understand that when two sets are combined together we refer to them as union. (U)

Sets and probability

8.12.11 Understand how to find the probability of an outcome in a given set. (U)

8.12.12 Know how to find the probabilities of an outcome using set notation. (K)

Sample space grids

8.12.13 Understand that grid can be used to represent the total outcome or sample space of an event. (U)

8.12.14 Know how to calculate probabilities of compound events using a grid. (K)

Tree diagrams

8.12.15 Know how to use a 'tree diagram' to display a sample space for compound events of three or more events. (U)

Exploring a deck of cards

8.12.16 Know how to calculate various probabilities from a deck of cards. (K)

Estimating proportions using sampling

8.12.17 Understand how a sampling method is used to estimate quantities from a large group of quantities. (U)

12A • Exploring sets

LB2 Page 110

Specific learning outcomes

Learners should be able to:

8.12.1.1 Group objects together according to given criteria.

8.12.2.1 Group elements or objects into different classes or groups using circles.

Teaching points

1. Group objects and elements together in sets.
2. Display grouped objects with circles on Venn diagrams.

Suggested teaching approach

- Learner complete **Learning task 12A** page 110 of the LB. This should assist learners to identify what a set is and how it is used to group elements.

12B • Sets and Venn diagrams

LB2 Pages 111–114

Specific learning outcomes

Learners should be able to:

8.12.3.1 Define the term 'sets'.

8.12.3.2 Group together elements and put them together as sets.

8.12.4.1 Write sets of elements by using the curly brackets and name sets using alphabetical letters.

8.12.5.1 Define and identify elements of sets.

8.12.5.2 Identify the symbol for element: $\in$

8.12.6.1 Define the term 'universal set' and identify its symbol: ξ

8.12.7.1 Define the term 'complement of a set' and identify its symbol.

8.12.7.2 Identify elements that are complement to the main set.

8.12.8.1 Define empty or null set and identify its symbol: { } or ∅

8.12.9.1 Define 'intersection' and identify its symbol: $\cap$

8.12.9.2 Find elements that are intersected in given sets.

8.12.10.1 Define 'union' and identify its symbol: $\cup$

8.12.10.2 Find sets that are in union.

Teaching points

1. Explain that set is a collection of elements.
2. Write sets with curly brackets { } and use capital letters to name the set.
3. Identify the elements of sets and its symbol.
4. Explain what the universal set is and show its symbol.
5. Explain what the complement of a set is and its use.
6. Explain what a null set is and show its symbol.
7. Explain what the intersection of a set is, its symbol, and how to find elements that are in the intersection.
8. Explain what the union of a set is, its symbol, and how to find elements that are in the union.

Learner difficulties and remedies

Difficulty

Understanding the two approaches that are presented in probability: theoretical probability and practical probability.

Remedy

- Note the difficulty that is stated above.
- Explain clearly the differences and similarities in theoretical and practical probability.
- Use examples to show the differences and similarities in these two approaches.

Difficulty

Understanding sets and their components and symbols.

Remedy

- Define sets: A group of items of the same type that are grouped together.
- Curly brackets are used to group items or element of the same type.
- Identify all the symbols that are used in sets.
- Use set notation to represent elements in sets.

Difficulty

Understanding other parts of sets such as the universal set, complement of a set, empty or null set, intersection and union.

Remedy

- Define all the terms that are used in sets such as universal set, complement of a set, empty or null set, intersection and union.
- Use the correct set notation to represent the universal set, complement of a set, empty or null set, intersection and union.
- Show learners where to use them in the application of sets.

Suggested teaching approach

- Define sets and all its other components such as; curly brackets, elements, universal set, complement, null set, intersection, union set.
- Identify all the symbols that are used in sets.
- Show how to use curly brackets and alphabetical letters to name the elements of sets, the universal set and the complement of a set.
- Explain what a null set is and show its symbol and its usage.
- Explain what an intersection is in sets and show its symbol.
- Show how to find elements that are in the intersection of two or more sets.
- Explain what is a union in sets and show its symbol.
- Show how to find elements that are in the union of two or more sets.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Sometimes, we want to know the probabilities involved when two different events, such as tossing a coin and rolling a die, occur within the same experiment. We might want to know the probability of both events occurring, of one or the other event occurring, of one event occurring given that the other event has occurred or the probability of neither event occurring. Venn diagrams help us to calculate these probabilities.

Sets

A set is a collection of items called elements. A set can be described in words, or listed.

$S = \{\text{seasons of the year}\}$
$\quad = \{\text{Summer, Autumn, Spring, Winter}\}$

The symbol $\in$ means 'is an element of' or belongs to.

The symbol $\notin$ means 'is not an element of' or does not belong to.

March $\in \{\text{months of the year}\}$
Friday $\quad \{\text{months of the year}\}$

The symbol n is used for the number of elements in a set.

$D = \{\text{days of the week}\}$
$n(D) = 7$

The universal set is the set of all possible elements under consideration. We use the symbol ε for the universal set (ε is the Greek letter, epsilon).

The universal set for numbers on a six-sided die is {1, 2, 3, 4, 5, 6}.

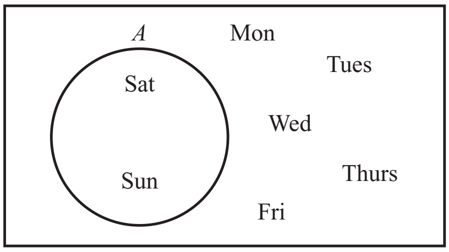
Venn diagrams

Venn diagrams can be used to represent sets.

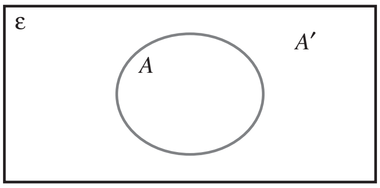
Universal set = D = {days of the week}

A = {weekend days}

A' = {days during the working week}

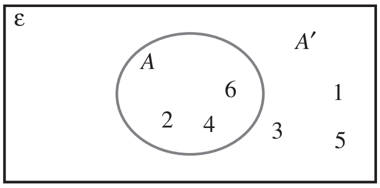


- A Venn diagram is usually represented by a rectangle that contains all the possible outcomes in the universal set, ε.



- The outcomes of an event are contained in a circle with a letter name, such as A.
- A', the complement of A, lies outside the circle.

Consider rolling a die.



Then, the universal set ξ = {1, 2, 3, 4, 5, 6} and all the outcomes are written inside the rectangle.

If A represents the outcomes of rolling an even number, then A = {2, 4, 6} and these outcomes are contained in a circle labelled A.

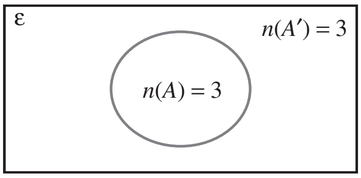
The complement of A, is the set A'. It contains all the elements from the universal set that are not in the set.

A', the complement of A, would be rolling an odd number.

A' = {1, 3, 5} and these outcomes would be outside circle A.

$n(A) = 3$

$n(A') = 3$



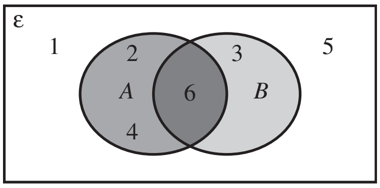
Unions and intersections

We may often want to represent two different events in a Venn diagram. The outcomes of the second event can be contained in another circle labelled *B*, for example.

If we again consider rolling a die, where:

$\xi = \{1, 2, 3, 4, 5, 6\}$
$A = \{2, 4, 6\}$
$B = \{\text{multiples of } 3\}$.

A and *B* will overlap as they both contain the number 6.

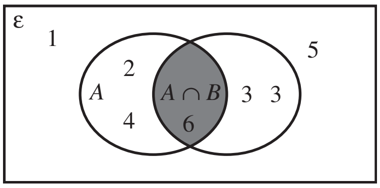


$A \cup B$ represents the union of *A* and *B* and includes all the outcomes in *A* **or** *B*.

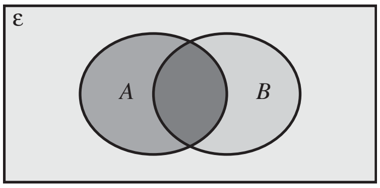
$A \cup B = \{2, 3, 4, 6\}$

$A \cap B$ represents the intersection of *A* and *B* and includes the outcomes that are in both *A and B*.

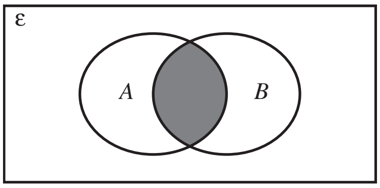
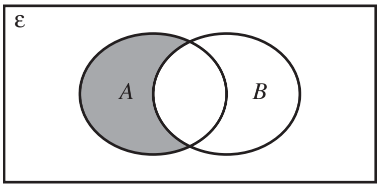
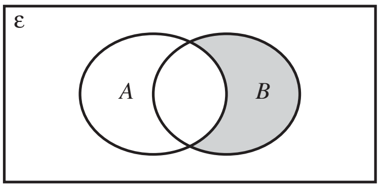
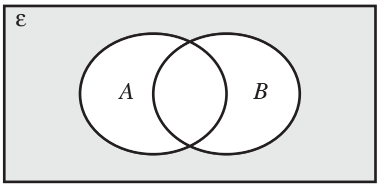
$A \cap B = \{6\}$



Consider the following diagram.



There are four distinct areas in this diagram.

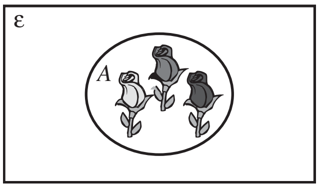
1 The area enclosed by the overlap of *A* and *B*: $A \cap B$ • is known as the intersection of *A* **and** *B* • represents the outcomes in which *A* **and** *B* both occur.	 ε A B
2 The area enclosed by the remainder of *A*: $A \cap B'$ • contains the outcomes in *A* not in the overlap with *B* • represents the outcomes in which *A* occurs but *B* does not.	 ε A B
3 The area enclosed by the remainder of *B*: $A' \cap B$ • contains the outcomes in *B* not in the overlap with *A* • represents the outcomes in which *B* occurs but *A* does not.	 ε A B
4 The area outside both circles: $A' \cap B'$ • contains the outcomes in the universal set not in either *A* or *B* • represents the outcomes in which neither *A* nor *B* occurs.	 ε A B

Summary of probability notation

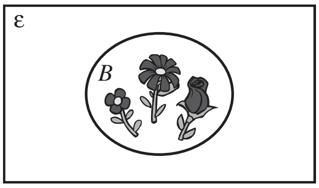
ξ	universal set, all possible outcomes.
A	The set of elements in event *A.*
A′	Complement of *A* The set of elements that are in the universal set but not in set *A.*
$A \cap B$	The intersection of sets *A* and *B*. It contains outcomes that are in *A* **and** *B* (common to *A* and *B*).
$A \cup B$	Th union of sets *A* and *B*. It contains outcomes that are in *A* **or** *B* (some outcomes belong to both *A* and *B*).
n(*A*)	The number of elements in set *A*.
n(ξ):	The number of elements in the universal set.
{ } **or** ∅	The empty set is a set with no elements.

For example:

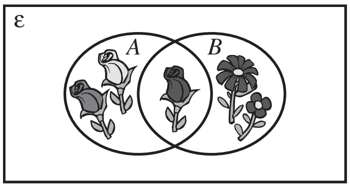
A = {roses}



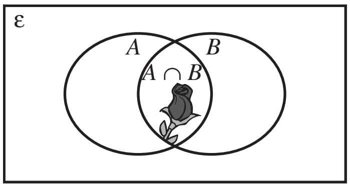
B = {red flowers}



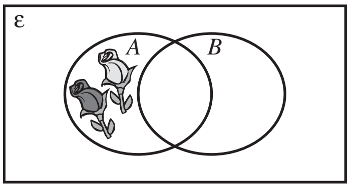
$A \cup B$ = {roses of any colour **or** any red flowers}



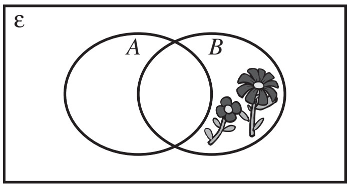
$A \cap B$ = {flowers that are roses **and** that are red}
= {red roses}



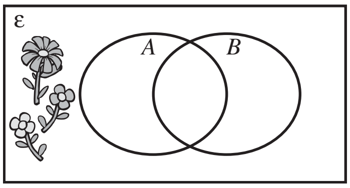
$A \cap B'$ = {flowers that are roses **and** are not red}
= {roses of any colour except red}



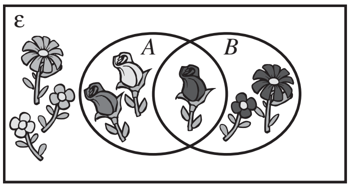
$A' \cap B$ = {flowers that are not roses **and** that are red}
= {red flowers that are not roses}



$A' \cap B'$ = {flowers that are not roses **and** are not red}
= {any flower that is not red or a rose}

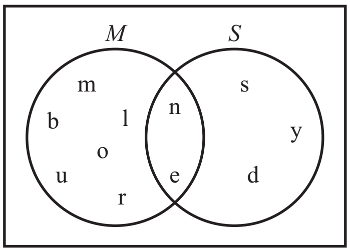


ξ = the universal set
$= A \cap B + A \cap B' + A' \cap B + A' \cap B'$

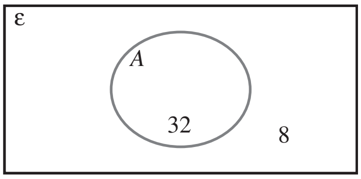


Examples

1 **a** List the two sets:
M = letters used to write Sydney and Melbourne have e and n in common.
S = letters used to write Sydney and Melbourne have e and n in common.
b Draw a Venn diagram to represent the two sets.
c List the set $M \cup S$.
d How many elements are in the set $M \cup S$?
e List the set $M \cap S$.
f How many elements are in the set $M \cap S$?

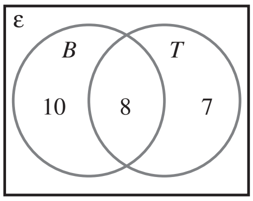
Thinking	**Working**
a Write the letters in each of the word in a set (only write a letter once in each set).	$M = \{\text{m, e, l, b, o, u, r, n}\}$ $S = \{\text{s, y, d, n, e}\}$
b Draw a Venn diagram to represent the sets. Write the letters that are in both M **and** S (n and e) in the overlap.	
c The combination of all the elements in either one **or** both of two sets is	$M \cup S$ = {b, d, e, l, m, n, o, r, s, u, y}
c There are 11 elements in the union.	$n(M \cup S) = 11$
d The intersection contains the letters n and e.	$M \cap S = \{\text{e, n}\}$
e There are two elements in the intersection.	$n(M \cap S) = 2$

2 For this Venn diagram that shows the number of people in a group, find:
a $n(A)$
b $n(A')$
c $n(\xi)$



Thinking	**Working**
a Write the number inside the circle as the number in the required set (A).	**a** $n(A) = 32$
b Write the number outside the circle as the number in the complement of the required set (A').	**b** $n(A') = 8$
c Write the sum of these numbers as the number in the universal set (ξ).	**c** $n(\xi) = n(A) + n(A')$ $= 32 + 8$ $= 40$

2 In a Year 8 class of 25 learners, all of whom enjoy watching either basketball or tennis or both basketball and tennis, 18 enjoy watching basketball and 15 enjoy watching tennis. This information is shown in the Venn diagram below. Write down the number of elements in the following sets.
a $n(B)$
b $n(T)$
c $n(B')$
d $n(T')$
e $n(B \cap T)$
f $n(B \cup T)'$

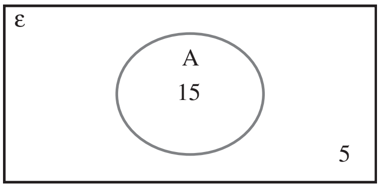


Thinking	Working
Write the number in each of the sets defined, including the universal set and the intersection.	**a** $n(B) = 18$ **b** $n(T) = 15$ **c** $n(B') = 7$ **d** $n(T') = 10$ **e** $n(B \cap T) = 8$ **f** $n(B \cup T)' = 0$

Activity 12B

1 For this Venn diagram that shows the number of people in a group, write down:

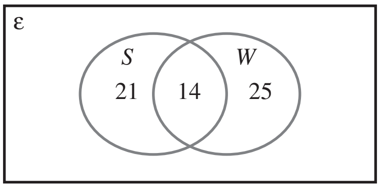
a $n(A)$ **b** $n(A')$ **c** $n(\varepsilon)$



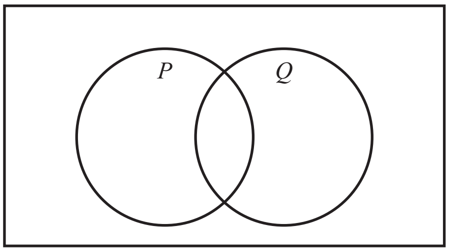
2 In a survey of 60 music learners at a school, all of whom play either a string instrument, a wind instrument or both, 35 play a string instrument and 39 play a wind instrument. This information is shown in the Venn diagram at right. If a learner is chosen at random, find the probability that they:

a play both a string instrument and a wind instrument

b only play a wind instrument.



3 Shade the following sets on each of three separate Venn diagrams that look like this:



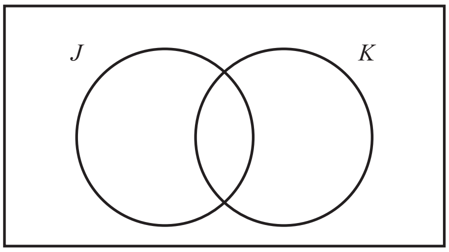
a $P \cap Q$ **b** $P \cup Q$ **c** P'

4 Place the numbers from these sets in the correct places in the Venn diagram.

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$J = \{2, 3, 5, 7, 11\}$

$K = \{1, 3, 5, 7, 9, 11\}$



5 ξ = {numbers from 1 to 40}

J = {even numbers}

K = {factors of 40} (numbers that divide exactly into 40)

a List K.

b Write down $n(K')$.

c List $J \cap K$.

6 ξ = {numbers from 1 to 10}

M = {even numbers}

N = {factors of 9}

a List N.

b List N'.

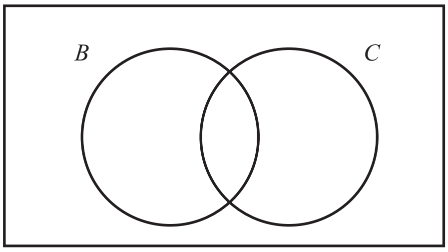
c Write down $M \cap N$ as simply as possible.

7 Cameron takes the bus to school 5 days a week. He also travels in the family's car on Tuesdays and Wednesdays and both days of the weekend.

a Represent this information in the Venn diagram.

B = days on the bus

C = days in the car



b Which days does Cameron travel in the car but not on the bus?

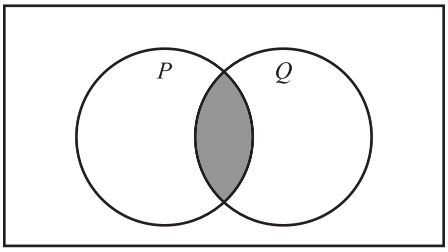
c Write down in your own words a description (not a list) of what $C \cap T$ represents.

Answers 12B

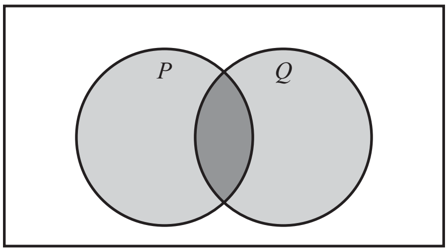
1 a 15 people **b** 5 people **c** 20 people

2 a $\frac{7}{30}$ **b** $\frac{5}{12}$

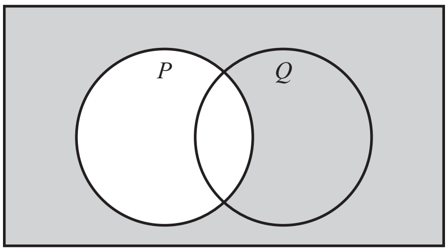
3 a



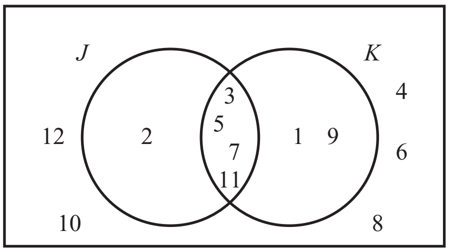
b



c



4



5 **a** $K = \{1, 2, 4, 5, 8\ 10, 20, 40\}$

b $n(K') = 8$

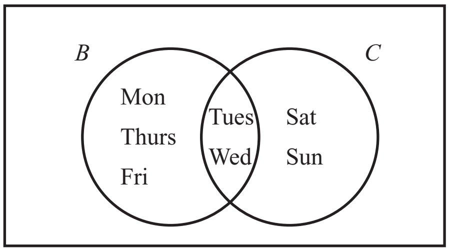
c $J \cap K = \{2, 4, 8\ 10, 20, 40\}$

6 **a** $N = \{1, 3, 9\}$

b $N' = \{2, 4, 5, 6, 7, 8, 10\}$

c $M \cap N = \{2, 4, 6, 8, 10\} = M$

7 **a**



b Saturday and Sunday

c $B \cap C$ = {days of the week when Cameron travels on the bus **and** in the family car}

12C • Sets and probability

LB2 Pages 116–118

Specific learning outcomes

Learners should be able to:

8.12.11.1 Find the probability of an outcome that is given in sets.

8.12.11.2 Calculate the probability of an outcome from sets using the formula: $\Pr(A) = \frac{n(A)}{n(\xi)}$

Teaching points

1. Calculating the probability of a particular outcome from a set of equally likely outcomes.
2. Equally likely means each of the outcomes has the same chance of being selected. A head and a tail have the same chance for each toss of a coin.
3. The term 'randomly selected' means the method of selection does not favour any particular outcome unfairly.
4. Calculate the probability of an outcome in sets using the formula:

   $\Pr(A) = \frac{n(A)}{n(\xi)}$
5. Remind learners that the probability of an event must lie between 0 (impossible) and 1 (certain).

Learner difficulties and remedies

Difficulty

Finding the probability of events in a set.

Remedy

- Remind learners that sets are used to group elements of the same type.
- Probability tells us the chance of an event to occur specifically those elements that are being grouped together using sets. To calculate the probability of an outcome in sets use the formula $\Pr(A) = \frac{n(A)}{n(\xi)}$.
- Know when and where to use sets, and when and where to use probability.

Suggested teaching approach

- Define sets and explain how to use sets to group items, objects or elements of the same type.
- Define probability and explain how to find the theoretical probability of an event.
- Show how to use sets to find the probability that an element in a specific set would occur.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

If each element in a set is equally likely to occur or be chosen, then we can calculate the probability from the fraction.

$$\Pr(A) = \frac{\text{number of elements in set } A}{\text{number of elements in the universal set}}$$

This can be simplified to:

$$\Pr(A) = \frac{n(A)}{n(\xi)}$$

The probability of an event A and its complementary event A' add to 1.

$$\Pr(A) + \Pr(A') = \frac{1}{3} + \frac{2}{3} = 1$$

Examples

1. Peter was selling raffle tickets during the Bishop Epalle School Bazaar. A raffle has 40 tickets, numbered 1–40. Work out the probability that the winning ticket includes the numeral 1.

Thinking	Working
1 Write all the elements for the 40 tickets in numbers from 1 to 40 into curly brackets. Count the number of elements in the set.	ξ = {numbers from 1 to 40} $n(\xi) = 40$
2 Write the set of numbers with 1 into curly brackets. Count the number of elements in the set.	$A = \{1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31\}$ $n(A) = 13$
3 Use formula to calculate the probability that the number comes out would have numeral 1 in it.	$\Pr(A) = \frac{n(A)}{n(\xi)}$
4 Write the probability as a fraction.	$= \frac{13}{40}$

2 A six-sided die is tossed. What is the probability that the number obtained is not a multiple of 3.

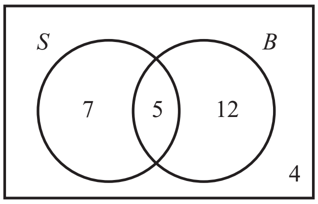
Thinking	**Working**
1 Write all the elements for the six-sided die.	$\varepsilon = \{1, 2, 3, 4, 5, 6\}$ $n(\varepsilon) = 6$
2 Write the numbers in the set that are multiple of 3,	$M = \{3, 6\}$
3 Write the numbers in the set that are NOT the multiple of 3 (complement of 3)	$M' = \{1, 2, 4, 5\}$ $n(M') = 4$
4 Use formula to calculate the probability that the number comes out would have numeral 1 in it.	$\Pr(M') = \frac{n(M')}{n(\varepsilon)}$
4 Write the probability as a fraction.	$= \frac{4}{6} = \frac{2}{3}$

Activity 12C

1 A letter is chosen at random from the 26 letters of the alphabet. Calculate these probabilities:
- **a** Pr(letter chosen is in the word Perth)
- **b** Pr(letter chosen is in the word Melbourne)

2 A dartboard has 20 spaces, numbered 1–20. If a dart is equally likely to hit any one of these spaces, work out these probabilities:
- **a** Pr(number is less than 8)
- **b** Pr(number is odd)
- **c** Pr(number includes the digit 2)
- **d** Pr(number is multiple of 3)
- **e** Pr(number is not a multiple of 4)

3 This Venn diagram shows the number of learners in a class who have a brother (*B*) or a sister (*S*).



- **a** How many learners are in the class?
- **b** How many learners have a brother?
- **c** How many learners have a sister but not a brother?
- **d** What is the probability that a learner has a brother but not a sister?
- **e** What is the probability that a learner has neither a brother nor a sister?

4 In a class of 32 learners, 12 are the oldest child in their family. What is the probability that a learner is *not* the oldest child in their family?

5 A survey of 50 salespeople showed that in the last year 35 had travelled on Qantas, 15 had travelled on Virgin and 12 had travelled on both airlines.
- **a** If Q = {travellers on Qantas} and V = {travellers on Virgin}, find:
  - **i** $\Pr(Q)$
  - **ii** $\Pr(Q \cap V)$
  - **iii** $\Pr(V')$
- **b** What is the probability a salesperson in the survey had not flown on either airline?

Answers 12C

1 **a** $\frac{5}{26}$ **b** $\frac{8}{26} = \frac{4}{13}$

2 **a** $\frac{7}{20}$ **b** $\frac{10}{20} = \frac{1}{2}$

c $\frac{3}{20}$ **d** $\frac{16}{20} = \frac{4}{5}$

3 **a** 28 **b** 17

c 7 **d** $\frac{7}{28} = \frac{1}{4}$

e $\frac{4}{28} = \frac{1}{7}$

4 $\frac{20}{32} = \frac{5}{8}$

5 **a** **i** $\frac{35}{50} = \frac{7}{10}$

ii $\frac{12}{50} = \frac{6}{25}$

iii $\frac{35}{50} = \frac{7}{10}$

b $\frac{12}{50} = \frac{6}{25}$

12D • Sample space grids

LB2 Pages 119–120

Specific learning outcomes

Learners should be able to:

8.12.12.1 Define and identify a sample space for compound events.

8.12.12.2 Use a simple grid to represent the total sample space of two events.

8.12.13.1 Calculate the probabilities of compound events using a grid.

Teaching points

1 A sample space grid is used to represent all the possible outcomes for two events.

2 The total number of points on the grid is the denominator for the probability fraction using the formula $\Pr(A) = \frac{n(A)}{n(SS)}$.

3 The number of points on the grid that meet the desired outcome(s), will become the numerator of the probability fraction.

4 While two dice might be thrown at the same time, to distinguish the outcome on one die with that of the other it is best to either label one as a red die and the other a blue die, or nominate one as the ‘first die’ and the other the ‘second die’ to ensure all combinations are counted.

Learner difficulties and remedies

Difficulty

Constructing a grid for two compound events to show the sample space.

Remedy

- Explain to learners what a sample space is.
- Combine different sample spaces of various single events to make up the sample space for a compound event.
- Count the numbers of elements in a sample space for the compound event.

Suggested teaching approach

- Explain how to use grids to find the sample spaces of given events both single and compound.
- Grids can be simplified to show the possible outcomes on the top and side of the grids.
- Elements in the sample space are represented by the intersections of the lines in the grid.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Sample space

The sample space is the complete list of all the possible outcomes of a probability experiment, such as rolling a die, tossing a coin, or selecting a card from a pack. To determine the probability of a particular event, it is useful to list the sample space. When the outcomes of an experiment are equally likely, we can use the sample space to determine the probability of a particular event occurring. A favourable outcome is one where the event for which we are calculating the probability occurs.

Theoretical probability

To calculate a theoretical probability, we need to know the sample space. A sample space is a list of all possible outcomes and the outcomes must all be equally likely. Successful outcomes must be some or all of the outcomes listed in the sample space.

The sample space from tossing a coin is a head (H) and a tail (T).

If a successful outcome is tossing a head, there is only one successful outcome possible out of two outcomes in the sample space.

$\Pr(H) = \frac{1}{2}$

The sample space from rolling a normal die is 1, 2, 3, 4, 5, 6.

If a successful outcome is rolling a 2, then there is only one successful outcome possible out of six outcomes in the sample space.

$\Pr(2) = \frac{1}{6}$

When more than one outcome is considered a success, we can calculate the probability by counting the number of successes overall or calculating each probability separately and adding them.

For example, finding the probability of rolling a 1 or a 2 on a die can be calculated in two ways. There are two successful outcomes possible out of six outcomes in the sample space.

$\Pr(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$

We could have calculated this by considering the probability of each event separately and adding them.

$$\Pr(1) = \frac{1}{6} \text{ and } \Pr(2) = \frac{1}{6} \text{ so } \Pr(1 \text{ or } 2) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Calculating probability

$$\Pr(\text{event}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

All outcomes must be equally likely.

Examples

1 Find the probability of:

a rolling an odd number with a normal die

b selecting a vowel (a, e, i, o, u) from a bag containing 26 identical pieces of paper, each containing a different letter of the alphabet.

Thinking	**Working**
a **1** List the sample space and count the number in the sample space. **2** Identify the favourable outcomes and count the number of favourable outcomes. **3** Find the probability using the rule $\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ And simplify the fraction if possible.	**a** Sample space:1, 2, 3, 4, 5, 6 There are 6 outcomes in the sample space. Favourable outcomes are: 1, 3 and 5. There are 3 favourable outcomes. $\Pr(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$
b **1** List the sample space and count the number in the sample space. **2** Identify the favourable outcomes and count the number of favourable outcomes. **3** Find the probability using the rule $\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ and simplify the fraction if possible.	**b** Sample space: *a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z* There are 26 outcomes in the sample space. Favourable outcomes are: *a, e, i, o, u* There are 5 favourable outcomes. $\Pr(\text{vowel}) = \frac{5}{26}$

2 Alvin has a normal pack of 52 playing cards and he selects one card from the pack.

- **a** List the sample space.
- **b** Find the probability of that card being:
  - **i** a diamond
  - **ii** a king or a queen.

Thinking	**Working**
a **1** List the sample space.	**a** ♥: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ♦: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ♣: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ♠: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K (♥ = Hearts, ♦ = Diamonds, ♣ = Clubs, ♠ = Spades, A = Ace, J = Jack, Q = Queen, K = King)
b **i** **1** Count the number of successful outcomes in the sample space. Count the total number of outcomes. **2** Calculate the probability by using Pr(event) = $\frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	**b** Number of successful outcomes = 13 Total number of outcomes = 52 Pr(diamond) = $\frac{13}{52}$ $= \frac{1}{4}$
ii **1** Count the number of successful outcomes. **2** Calculate the probability by using Pr(event) = $\frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	Number of ways of selecting a king or a Queen = 8 Pr(King or Queen) = $\frac{8}{52}$ $= \frac{2}{13}$

3 In Katherine's class there are 24 learners. Five were from Malaita Province, three from Temotu Province, six from Western Province, two from Isabel, five from Makira, one from Renbell, one from Choisel and one from Central Province. A member of the class is selected at random. What is the probability that a learner chosen at random were from:

- **a** Western Province?
- **b** Makira Province?
- **c** Central Province?

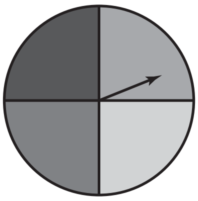
Thinking	**Working**
a **1** Write the number from Western Province. **2** Write the total number of outcomes. **3** Use the formula to find the probability as fraction then simplify.	**a** 6 Total = 24 Pr(Western Prov) = $\frac{6}{24}$ $= \frac{1}{4}$
b **1** Write the number from Makira Province. **2** Write the total number of outcomes. **3** Use the formula to find the probability as fraction.	**b** 5 Total = 24 Pr(Makira Province) = $\frac{5}{24}$ $= \frac{5}{24}$
c **1** Write the number from Central Province **2** Write the total number of outcomes **3** Use the formula to find the probability as fraction.	**c** 1 Total = 24 Pr(Central Province) = $\frac{1}{24}$ $= \frac{1}{24}$

Activity 12D

1 For each of the following:

- **i** list the sample space
- **ii** find the probability of the event occurring.

- **a** tossing a coin and getting a head
- **b** rolling a normal die and getting a 4
- **c** spinning the spinner shown and it points to blue
- **d** choosing an ace of hearts out of a normal pack of 52 playing cards
- **e** choosing a red marble from a bag containing one red, one blue, one green and two orange marbles



2 Find the probability of:

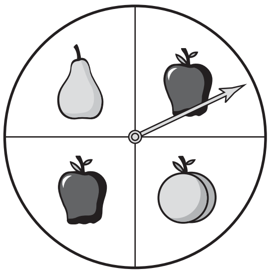
- **a** rolling an even number with a normal die
- **b** choosing an *a* from the letters of the word *mathematics*
- **c** getting a 4 on a roll of a six-sided die
- **d** drawing a card from a standard pack and getting a diamond
- **e** drawing a card from a standard pack and getting a king
- **f** drawing a card from a standard pack and getting a black 4
- **g** drawing a card from a standard pack and getting a red 7
- **h** drawing a card from a standard pack and getting the 9 of spades.

3 Philip has a pack of 52 playing cards and he selects one card from the pack. Find the probability of:

- **a** a 7
- **b** an ace or a 10.

4 Find the probability of the following events occurring.

a rolling a normal die and getting a 3 or a 6
b rolling a normal die and getting an odd number
c spinning the following spinner and it pointing to apple



Answers 12D

1 **a** **i** {head, tail} **ii** $\frac{1}{2}$
b **i** {1, 2, 3, 4, 5, 6} **ii** $\frac{1}{6}$
c **i** {red, blue, green, yellow} **ii** $\frac{1}{4}$
d **i**

A♥	A♦	A♠	A♣
2♥	2♦	2♠	2♣
3♥	3♦	3♠	3♣
4♥	4♦	4♠	4♣
5♥	5♦	5♠	5♣
6♥	6♦	6♠	6♣
7♥	7♦	7♠	7♣
8♥	8♦	8♠	8♣
9♥	9♦	9♠	9♣
10♥	10♦	10♠	10♣
J♥	J♦	J♠	J♣
Q♥	Q♦	Q♠	Q♣
K♥	K♦	K♠	K♣

ii $\frac{1}{52}$
e **i** {red, blue, green, orange, orange}
ii $\frac{1}{5}$

2 **a** $\frac{1}{2}$ **b** $\frac{2}{11}$ **c** $\frac{1}{6}$
d $\frac{1}{4}$ **e** $\frac{1}{13}$ **f** $\frac{1}{26}$
g $\frac{1}{26}$ **h** $\frac{1}{52}$

3 **a** $\frac{1}{13}$ **b** $\frac{2}{13}$

4 **a** $\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

12E • Tree diagrams

LB2 Pages 121–122

Specific learning outcomes

Learners should be able to:

8.12.14.1 Use tree diagram to find probabilities of events.
8.12.14.2 Construct tree diagrams to display sample spaces of compound events then calculate their probabilities.

Teaching points

1. Find the probabilities of given events using a tree diagram.
2. While a sample space grid provides a useful way of recording the outcomes of two events, use a tree diagram for a combination of three or more events.

Learner difficulties and remedies

Difficulty

Drawing tree diagrams to represent the sample space from given information.

Remedy

- Count the number of events.
- Identify the sample spaces for each of the events.
- Draw a tree diagram where each branch in the tree represents the possible outcomes in an events.

Suggested teaching approach

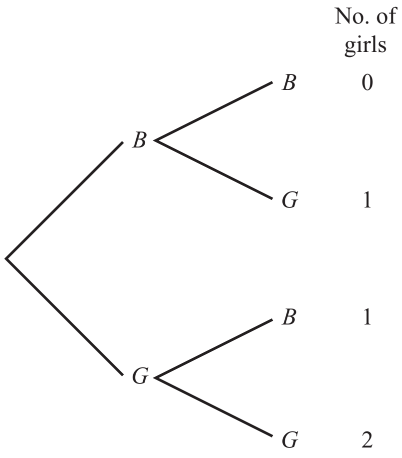
- Show that a tree diagram can be used for three or more events. Each branch in the tree represents the possible outcomes in one of the events.
- Find the sample spaces for each of the events.
- Draw the tree diagram and write the probability of each outcome on the branch.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **tree diagram** shows the different outcomes when a process involving change or making choice is repeated.

Examples

This tree diagram shows the possibilities when a couple have two children. There are four possible results and we assume they are equally likely. The tree shows that couple with two children is more likely to have one girl than two girls.



Calculate the probability of having:

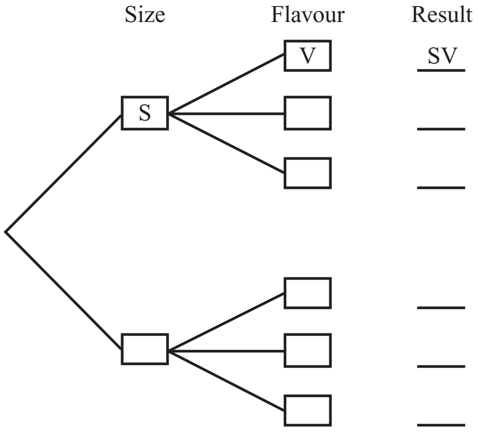
a 1 girl **b** 2 girls

Thinking	**Working**
a There are two branches in the tree diagram for 1 girl: BG and GB	Pr(1 girl) = $\frac{2}{4} = \frac{1}{2}$
b There is only one branch in the tree diagram for 2 girls: GG	Pr(2 girls) = $\frac{1}{4}$

Activity 12E

1 A small shop bells ice-creams. There are three flavours (vanilla, raspberry and butterscotch) and two sizes (single and double).

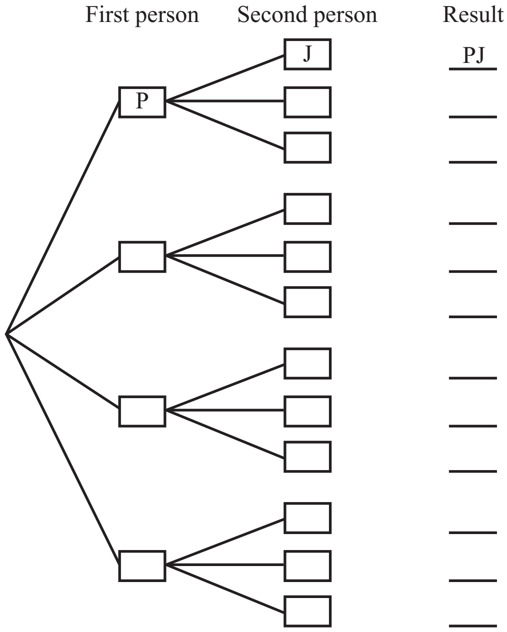
a Complete this tree diagram to show the different possibilities:



b How many different types of ice-cream can be ordered?

c If the ice-creams can also be be chocolate-dipped, calculate the number of different possibilities.

2 Two boys (Paul and John) and two girls (Rachel and Sally) have been given two tickets for a concert. They have to decide who will go by picking the tickets from a basket. The tree diagram given below shows the different outcomes.



a Complete the diagram.

b How many possibilities are there?

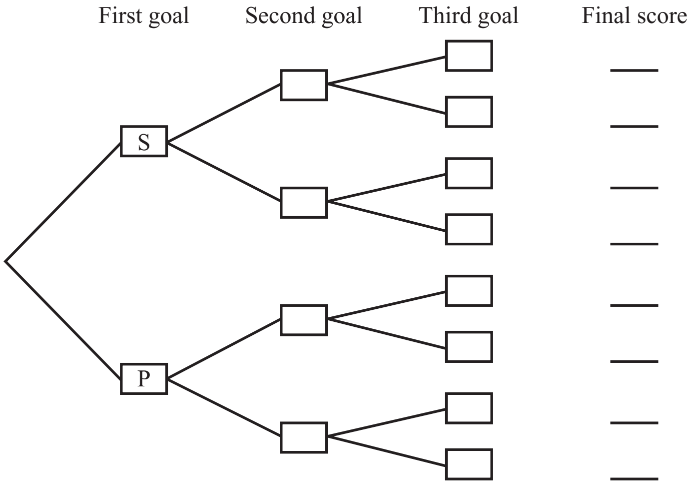
c How many of the possible results include Rachel?

d What is the probability that Rachel will go to the concert?

e Explain in terms of probability whether it is likely or unlikely that exactly one boy *and* one girl end up going.

3 Three goals were scored in the annual staff versus learners hockey game.

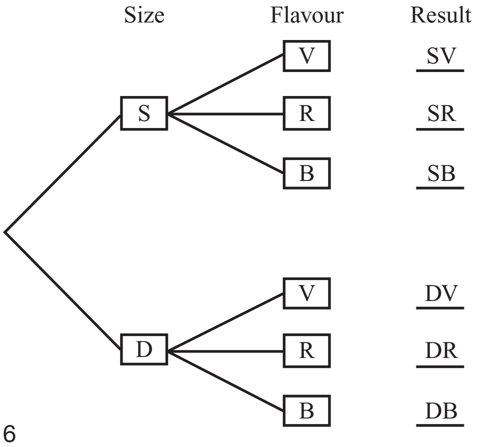
a Complete the tree diagram to show the different possibilities. At each stage, use S or P to represent the team that scored first, second and third. Assume that each team is equally likely to score a goal.



b What final score to you think was more likely: 2–1 or 3–0?

Answers 12E

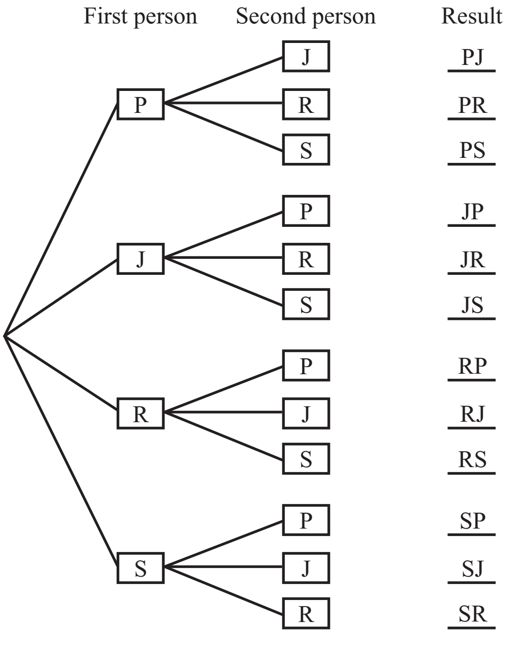
1 a



b 6

c 12

2 a



b 12

c 6

d $\frac{6}{12} = \frac{1}{2}$

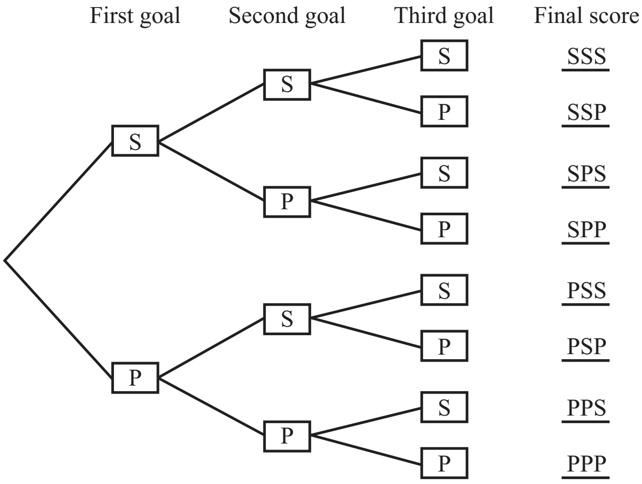
e $\Pr(BB) = \frac{2}{12} = \frac{1}{6}$

$\Pr(GG) = \frac{2}{12} = \frac{1}{6}$

$\Pr(BG) = \frac{8}{12} = \frac{4}{6}$

There is a $\frac{2}{3}$ (likely) chance that a boy and a girl end up going.

3 a



b There are two braches on the tree diagram with a score of 3–0: SSS, PPP

$\Pr(3–0) = \frac{2}{8} = \frac{1}{4}$

There are six branches on the tree with a score of 2–1: SSP, SPS, PSS, SPP, PPS, PSP

$\Pr(2–1) = \frac{6}{8} = \frac{3}{4}$

$\Pr(BG) = \frac{8}{12} = \frac{4}{6}$

It is more likely that the score is 2–1 than 3–0.

12F • Exploring a pack of cards

LB2 Page 123

Specific learning outcomes

Learners should be able to:

8.12.15.1 Find the probability of events occurring with playing cards.

Teaching points

1. This topic is best taught with packs of cards for the learners to use.
2. Check the learners are familiar with the different types of cards by name and picture.
3. Demonstrate how it is possible to draw a card at random.
4. Find the probability of an event using a standard pack of playing cards.

Suggested teaching approach

- Learners complete **Learning task 12F** on page 123 of the LB and **Activity 12F** given below.

Additional notes

Playing cards are often used in games of chance.

There are 52 cards:

26 red cards: 13 hearts and 13 diamonds

26 black cards: 13 clubs and 13 spades

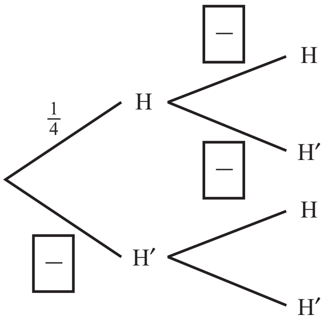
The 13 cards in a suit are A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. The ace, jack, queen and king are not numbered.

Activity 12F

1 If one card is drawn from a pack of cards at random, write down these probabilities:

- **a** Pr(red)
- **b** Pr(club)
- **c** Pr(8 of spades)
- **d** Pr(not a diamond)
- **e** Pr(an ace)
- **f** Pr(a numbered card)
- **g** Pr(a card with a number greater than 6)
- **h** Pr(a black 4 or 5)

2 This tree diagram shows the possible outcomes when two cards are drawn from a pack of 52, with replacement. The diagram shows that the probability of getting a heart at each stage is $\frac{1}{4}$.



- **a** Add the missing probabilities of the the three boxes.
- **b** By multiplying the probabilities along each branch, or otherwise, calculate the probability that both cards are hearts.

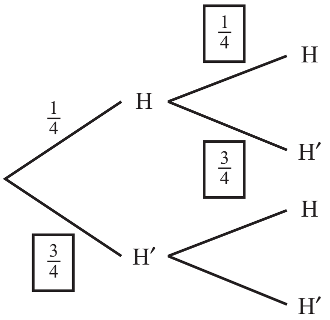
Answers 12F

1 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{52}$

d $\frac{3}{4}$ **e** $\frac{1}{13}$ **f** $\frac{10}{13}$

g $\frac{4}{13}$ **h** $\frac{1}{13}$

2 a



b $\frac{1}{16}$

12G • Estimating proportions using sampling

LB2 Pages 124–125

Specific learning outcomes

Learners should be able to:

8.12.16.1 Estimate probabilities using simulations.

8.12.16.2 Find the probabilities of events occurring in a sample proportional to the total quantities, groups, or population.

Teaching points

1 Find probabilities of an event occurring in a sample space in relation to the total or whole population.

Learner difficulties and remedies

Difficulty

Estimating probabilities that occur in the whole population from a sample of collected data.

Remedy

- Understand the difference between the sample population and the whole population.
- Remind learners how to use ratios and fractions to increase or decrease quantities in a given proportion.

Suggested teaching approach

- Explain the differences between sample population and total population.
- Show how sample population is used to estimate other quantities in the identified population. You may need to revise the use of ratios and fractions to increase or decrease quantities in a given proportion.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

To gather information about a **population** we take a small part of the group called a **sample**, rather than examining all members of the group. We can then **estimate** the quantity in the population using the proportion or probability from the sample.

For example, if a police patrol has found that 153 of a sample of 1000 cars they have stopped do not have a current roadworthy certificate, then they can use this **relative frequency** to estimate that the probability is about 0·15.

$$\text{Relative frequency} = \frac{\text{number counted}}{\text{sample size}}$$
$$= \frac{153}{1000} \approx 0{\cdot}15$$

The expected number of times the event occurs can be predicted using the number of times the process is repeated (trials) and the probability of the event.

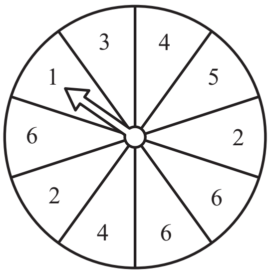
Expected number = number of trials × probability of event

The expected number can be used to estimate the numbers in a population based on the numbers found in a sample of that population. In the case of the police patrol, the expected number of cars in the population of 560 000 without a current roadworthy certificate is:

$$\text{Expected number} = 560\,000 \times 0{\cdot}15$$
$$= 84\,000$$

Examples

1 If this pointer were spun 200 times, how many times would you expect it to land on 4?



Thinking	Working
1 Find the probability of a 4 on a single spin.	$\Pr(4) = \frac{2}{10} = \frac{1}{5}$
2 Find the expected number in 200 trials.	Expected number = number of trials × probability $= 200 \times \frac{1}{5}$ = 40

Activity 12G

1 A medical centre has 1200 patients. A sample of 50 showed 27 females and 23 males.

  a What is the proportion of females in the sample? Give your answer as a percentage.

  b Estimate the total number of female patients at the medical centre.

2 A fair coin is tossed 12 times. How many times would you expect it to come up heads?

3 The pointer in the previous Example is spun 90 times. How many times would you expect it to stop on the number 6?

4 Every time Tom lands a dart on one of the numbered spaces (1–20) on a dartboard he records the result. Altogether he hits the numbered spaces 100 times. Each number is equally likely. What is the expected number of times he will land on a prime number?

  Hint: Prime numbers have exactly two factors (itself and 1), e.g. 13 is a prime number, 1 and 18 are not.

Answers 12G

1 **a** $\frac{27}{50} = 54\%$

  b $\frac{27}{50} \times 1200 = 648$

2 $\frac{1}{2} \times 12 = 6$

3 $\frac{3}{10} \times 90 = 27$

4 Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19

  $\frac{8}{20} \times 100 = 40$

• Chapter Thirteen •

Area and Volume

Overview

An area is the amount of surface inside a plane (flat) shape or within a boundary. It is measured in 'square' units: square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) or square kilometres (km^2). When we find the area of a shape, we are counting the number of whole squares and parts of whole squares that would cover or fit exactly to the surface of the shape or boundary. To aid area calculations, formulas are used for standard shapes such as triangles rectangles, quadrilaterals and circles.

Volume is the term used to describe the amount of space within a shape. Since this involves three dimensions, and is therefore measured in cubic' units: cubic centimetres cm^3, cubic metres m^3. To aid volume calculations, formulas are used for standard solids such as cubes, prisms and cylinders. Capacity is a measure of how much liquid would fill an object and its units are millilitre (mL) or litre (L).

These concepts are widely used in many subjects. For example, in technology we may need to find the area covered by the top of a table, or in agriculture we need to know the area to be used for a garden.

Solomon Islanders apply the concepts of area and volume for making gardens, building houses and many other projects.

Contents

Chapter skills

This chapter covers the following skills:

- Recognising common metric units of volume
- Obtaining areas by counting squares in order to develop new rules for the area of regular shapes
- Calculating the area of a triangle or one-half of the area of a suitable rectangle
- Calculating areas of shapes based on rectangles and triangles
- Finding areas using formulas
- Area of rectangle = length × width
- Area parallelogram = base × height
- Area triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
- Area trapezium = $\frac{1}{2}(a + b) \times \text{height}$
- Area circle = πr^2
- Total surface area of a solid is the sum of the area of each of its faces
- Finding volume using a formula: volume of prism = area base × height
- Finding volumes of shapes based on rectangular prisms

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 13A Using grids to find area • 13B Area of rectangles	Learner's Book 1 • Exercise 13A, pages 136, 137 Learner's Book 1 • Exercise 13B, pages 139–141
2–3	• 13C Area of parallelograms	Learner's Book 1 • Exercise 13C, pages 142, 143
4	• 13D Area of triangles	Learner's Book 1 • Exercise 13D, pages 145, 146
5–6	• 13E Areas of trapeziums	Learner's Book 1 • Exercise 13E, pages 147, 148
7	• 13F Areas of irregular shapes	Learner's Book 1 • Exercise 13F, pages 149–151
8–9	• 13G Area of circles	Learner's Book 1 • Exercise 13G, pages 152–154
10–11	• 13H Areas of compound shapes	Learner's Book 1 • Exercise 13H, pages 155, 156
12–13	• 13I Total surface area of prisms	Learner's Book 1 • Learning task 13I, pages 157–158
14	• 13J Volumes of prisms	Learner's Book 1 • Exercise 13J, pages 159–161
15	• Test	Teacher's Guide • Chapter 13 Test

General learning outcomes

Learners should:

Using grids to find area

8.13.1 Understand area and how it is determined using grids on regular and irregular shapes. (U)

Areas of rectangles

8.13.2 Know how to use the formula to calculate the area of various rectangles. (K)

Areas of parallelograms

8.13.3 Understand that a parallelogram is made from a rectangle with same base and height. (U)

8.13.4 Know how to calculate the area of various parallelograms. (K)

Areas of triangles

8.13.5 Know how to determine the area of various triangles. (K)

Areas of trapeziums

8.13.6 Understand how the formula to calculate the area of any trapezium shape is derived. (U)

8.13.7 Know how to find the area of trapeziums (K)

Areas of irregular shapes

8.13.8 Understand that not all shapes are regular, some are irregular and don't fit exactly into the square grid. (U)

Area of circles

8.13.9 Know how to use the formula to find the area of any given circle. (K)

Areas of compound shapes

8.13.10 Understand that compound shapes are made up of standard shapes. (U)

8.13.11 Know how to apply different formulas to calculate the areas of compound shapes. (K)

Total surface areas of prisms

8.13.12 Understand what is a prism. (U)

8.13.13 Know what is the 'surface area' of a prism. (U)

8.13.14 Know how to calculate the total surface area of a prism. (K)

Volumes of prisms

8.13.15 Understand volume and how it is determine. (U)

8.13.16 Know how to use formula to calculate the area of given solid. (K)

13A • Using grids to find area

LB2 Pages 136–137

Specific learning outcomes

Learners should be able to:

8.13.1.1 Define the term 'area'.

8.13.1.2 Estimate areas of given shapes by counting the number of unit squares that would fit inside the shapes.

Teaching points

1. Explain that area is the term used to describe the amount of space within a two-dimensional shape.
2. Estimate the area of given shapes by counting the number of unit squares that would cover the shapes.

Learner difficulties and remedies

Difficulty

Estimating the area of incomplete squares on a grid.

Remedy

- First count the whole squares within the shape and then look for any squares that appear to add up to one whole square. Finally, add any remaining squares by estimating the fraction of the square that is shaded. Cross off each square as it is counted.

Suggested teaching approach

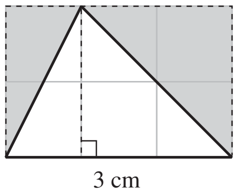
- Explain to learners that when estimating areas of shapes using a grid, we count the number of square boxes that are covered by the given shape.
- When some of the squares in the grid are not covered fully by the given shape, then learners have to estimate the area shaded in that square.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A square grid can be used to calculate the area of any given regular or irregular flat shape. If the shape does not completely cover all of the squares, then the grid can be used to estimate the area. This can be done by counting the number of squares that make up the shape. Some of the squares may not be completely covered but those incomplete squares can be paired to produce complete squares, which can then be counted.

Examples

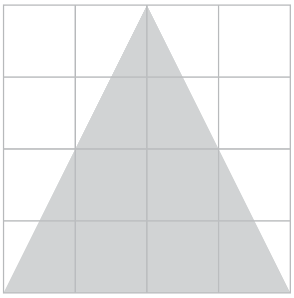
Estimate the area of given flat triangle in white using grids.

Thinking	Working
1 Identify the irregular shape inside the grid.	
2 Count the number of complete squares that are in the shape.	1
3 Count the number of incomplete squares that are in the shape then estimate the make-up number of squares they would make.	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=2$
4 Add the complete squares and the estimated incomplete squares to find the total area.	$1+2=3$ The estimated area of the shape is $3\,\text{cm}^2$.

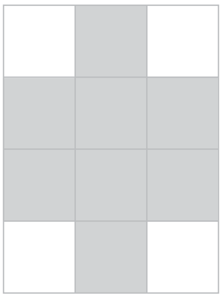
Activity 13A

1 The following shapes have been draw on centimetre-grid paper. Find the shaded area in each case.

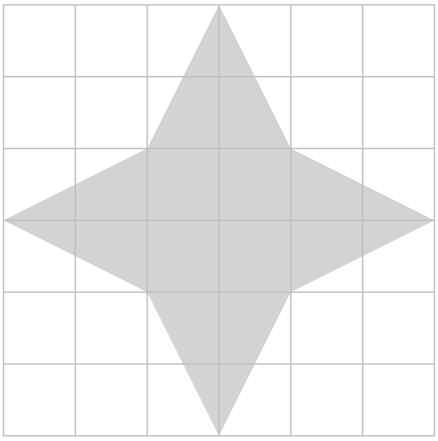
a



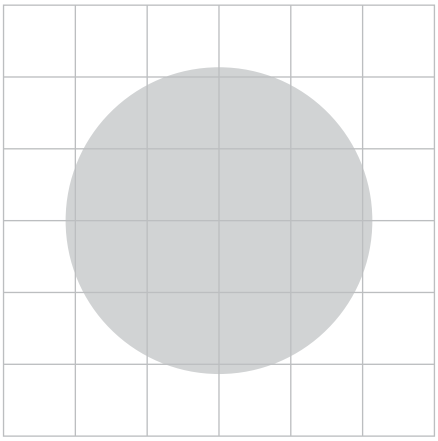
b



c



d



Answers 13A

1 **a** 8 cm^2

b 8 cm^2

c 12 cm^2

d 14 cm^2

13B • Areas of rectangles

LB2 Pages 138–141

Specific learning outcomes

Learners should be able to:

8.13.2.1 Identify a rectangle and its properties.

8.13.2.2 Use the formula to calculate the area of rectangles: Area = base × height

Teaching points

1. Revise the properties of rectangles.
2. Calculate the area of given shapes using the area formulas: Area = base × height or Area = length × width

Learner difficulties and remedies

Difficulty

Converting metric units to find the area of given shapes.

Remedy

- Help learners to identify which unit needs to be converted.
- Revise the metric ladder of conversion factors.

Difficulty

Identifying which shapes to calculate the area, especially in compound shapes.

Remedy

- Revise different shapes and their properties.
- Know the formula that would be used to find the areas of shapes found in compound shapes.

Suggested teaching approach

- Explain that area of given shapes can be calculated in two ways:
  - – using a square grid to count squares
  - – using a formula
- Explain to learners how to solve word problems using the given information given to find the area of rectangles.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Consider this rectangle of length 6 cm and width 3 cm.

The rectangle contains 3 rows of 6 squares. Its area is:

3 cm × 6 cm = 18 cm^2

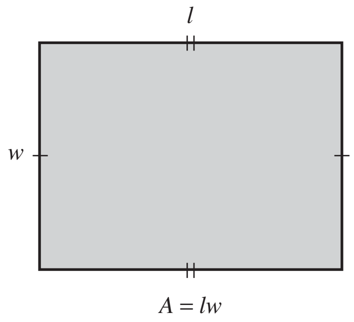
The area A of a rectangle is length l multiplied by width w.

This formula can be written as $A = lw$.

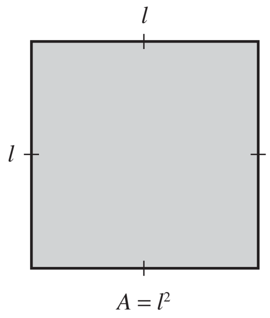
When we find the area of a shape or boundaries including rectangle, we are counting the number of whole squares and parts of whole squares that would cover or fit exactly to the surface of a shape or boundary.

The length and width of a square are equal, so the area of a square can be found using the formula $A = l^2$.

a



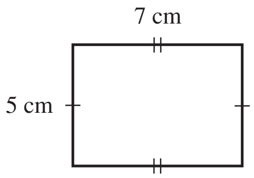
a



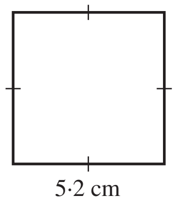
Examples

Calculate the area of the following shapes.

a



b

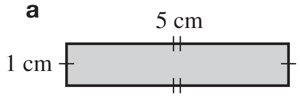


Thinking	**Working**
a **1** Write the formula for the area of a rectangle. **2** Identify *l* and *w*, and substitute their values into the formula. **3** Evaluate, writing the answer with the correct units.	**a** $A = lw$ $A = 7 \times 5$ $A = 35\,\text{cm}^2$
b **1** Write the formula for the area of a square. **2** Identify *l* and substitute its value into the formula. **3** Evaluate, writing the answer with the correct units.	**b** $A = l^2$ $A = 5{\cdot}2 \times 5{\cdot}2$ $A = 27{\cdot}04\,\text{cm}^2$

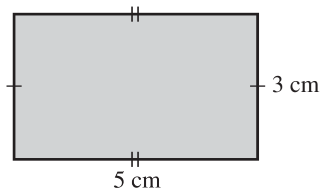
Activity 13B

1 Calculate the area of the following shapes.

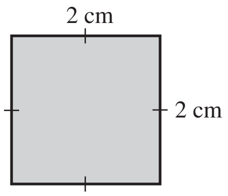
a



b



c



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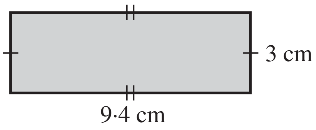
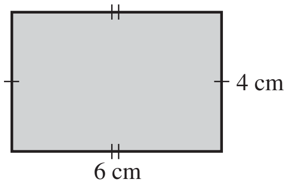
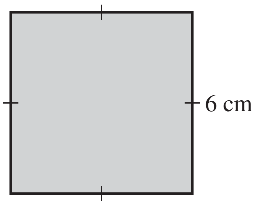


e



f



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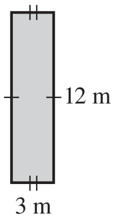
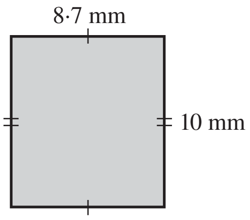
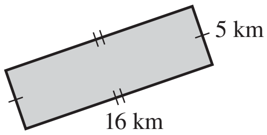


h



i



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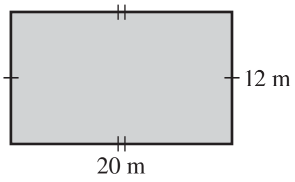

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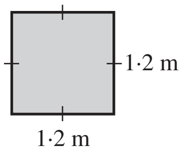
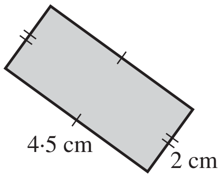
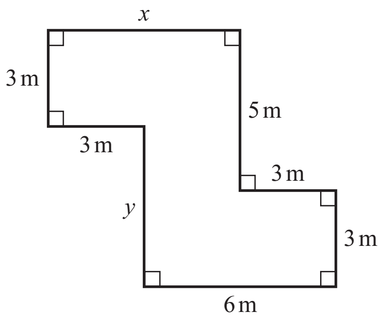


l

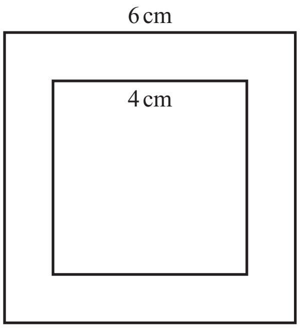


2 **a** How long are the two sides marked *x* and *y*?

b Add dashed lines to the diagram to divide into three rectangles and then calculate the area of the whole shape.



3 The diagram shows that a square with sides of 4 cm has been removed from a square with sides of 6 cm. Calculate the area of the remaining part.



4 Teresa and William have been asked to paint the front of a fence. The fence is rectangular, with a length of 24 m and a height of 2 m. 1 litre of fence paint covers about 6 m^2.
  a Calculate the area to be painted.
  b How many litres of paint will they need?

5 A rectangular swimming pool is 50 m long by 16 m wide. The surrounding area is to be tiled to a distance of 2 m from the edge of the pool on all sides. Calculate the area of tiling needed.

6 A school hockey field is rectangular and measures 112 m by 49 m. Calculate its area:
  a exactly
  b to the nearest 100 m^2

Answers 13B

1 **a** 5 cm^2 **b** 15 cm^2 **c** 4 cm^2
d 28·2 cm^2 **e** 24 cm^2 **f** 36 cm^2
g 36 m^2 **h** 87 mm^2 **i** 80 km^2
j 240 m^2 **k** 1·44 m^2 **l** 9 cm^2
2 **a** $x = 6$ m, $y = 5$ m **b** 42 m^2
3 20 cm^2
4 **a** 48 m^2 **b** 8 L
5 136 m^2
4 **a** 5488 m^2 **b** 5500 m^2

13C • Areas of parallelograms

LB2 Pages 142–143

Specific learning outcomes

Learners should be able to:

8.13.3.1 Identify a parallelogram and its properties.
8.13.4.1 Use the formula to calculate the area of a parallelogram:
Area = base × height

Teaching points

1. Show how a parallelogram has the same area as a rectangle on the same base and with the same height.
2. Note that the perpendicular height of the parallelogram is involved in calculating the area, not the lengths of the sloping side.
3. Calculate the area of given parallelogram shapes using the area formula for parallelogram.
4. Show the properties of parallelogram shapes.

Learner difficulties and remedies

Difficulty

Identifying the parallelogram shape and its properties.

Remedy

- Use the properties of the parallelogram to identify the shape of parallelograms.

Difficulty

Using the formula to calculate the area of parallelograms.

Remedy

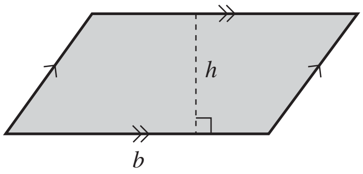
- Know the formula used to calculate the area of parallelogram and substitute in the correct values.
- Remind learners that the perpendicular height of the parallelogram is used not the length of the sloping side.

Suggested teaching approach

- Show learners the properties of a parallelogram.
- Develop the formula for finding the area of a parallelogram.
- Show how to use the formula and provide a variety of examples.
- Identify the measurements that can be used to find the area of a given parallelogram.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **parallelogram** is a four-sided shape that has two pairs of parallel sides. The pairs of parallel sides are marked by > and >> symbols. The parallelogram has a **base**, *b*, and a **height** *h*, which is **perpendicular** to the base. Perpendicular means ‘at right angles to’. This is shown by the symbol for the right angle ∟.

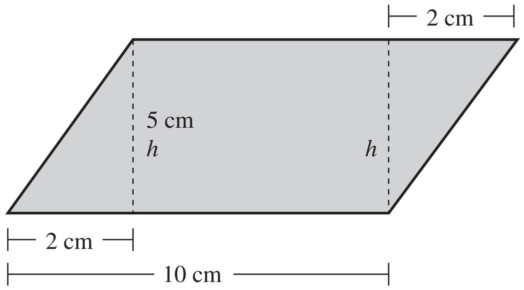


Developing a formula for area

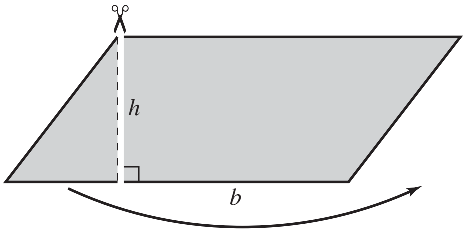
We can find a formula for calculating the area of a parallelogram by cutting and rearranging it into the familiar shape of a rectangle.

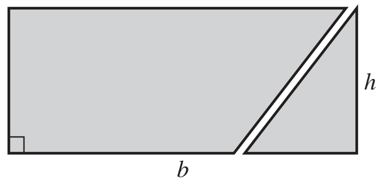
Construct a parallelogram by ruling a horizontal line 10 cm long and labelling it *b*. At one end of the line, and 2 cm in from the other end, measure two perpendicular heights of 5 cm. Mark them with dotted lines and label them *h*. Use the marked heights to rule a second 10 cm line parallel to the first. Make sure that this line starts 2 cm in from one end, and extends 2 cm past the other end of the first line.

Draw in the two shorter sloping sides of the parallelogram, which are parallel.



Transform your parallelogram into a rectangle by cutting down one of the dotted lines marking the height and rearranging the two pieces formed into a rectangle, as shown below.



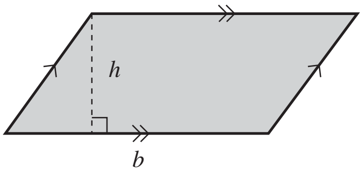


The base and height of the parallelogram become the length and width of a rectangle, and so we can multiply them together to find the area.

The area A of a parallelogram is base b multiplied by width w.

This formula can be written as $A = bh$.

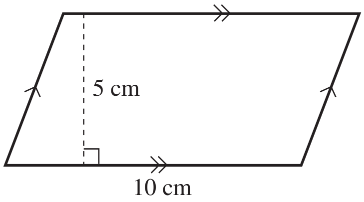
Sides that are marked with the same number of > symbols are parallel.



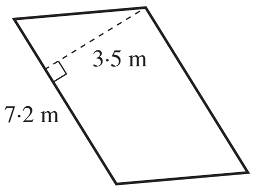
Examples

Find the area of each of the following parallelograms.

a



b

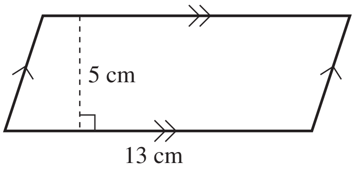


Thinking	Working
a **1** Write the formula for the area of a parallelogram. **2** Identify b and h, and substitute their values into the formula. **3** Evaluate, writing the answer with the correct units.	**a** $A = bh$ $A = 10 \times 5$ $A = 50\,\text{cm}^2$
b **1** Write the formula for the area of a parallelogram. **2** Identify b and h, and substitute their values into the formula. **3** Evaluate, writing the answer with the correct units.	**b** $A = bh$ $A = 7{\cdot}2 \times 3{\cdot}5$ $A = 25{\cdot}2\,\text{m}^2$

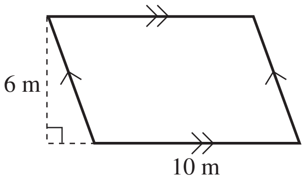
Activity 13C

1 Find the area of each of the following parallelograms.

a



b



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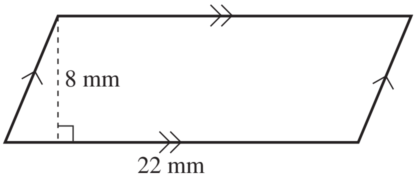
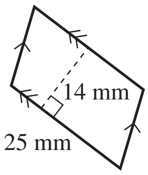
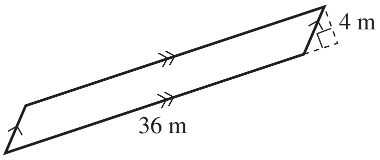


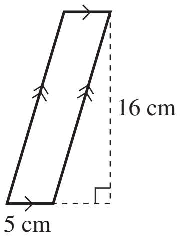
d



e

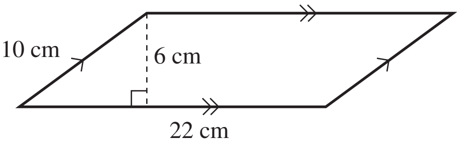


f

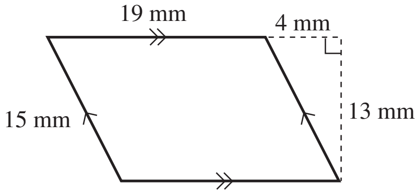


2 **a** Find the area of this parallelogram, in cm^2.

b Find the perimeter, in cm.



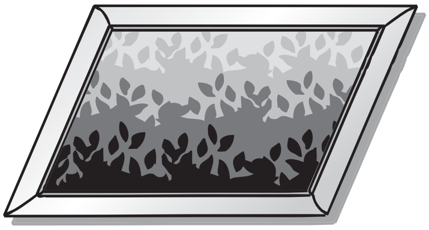
3 **a** Find the height of this parallelogram.



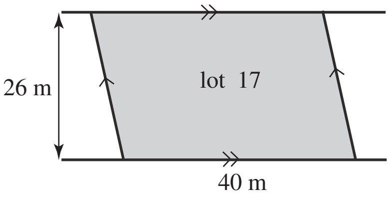
b Find the area of a parallelogram with a base length of 14 m, a sloping side length of 5 m and a height of 4 m.

4 A parallelogram with a base length of 12 cm has an area of 90 cm^2. What is the height of the parallelogram?

5 The picture frame shown has a height of 15 cm and encloses an area of 300 cm^2. What is the base of length of the picture frame?

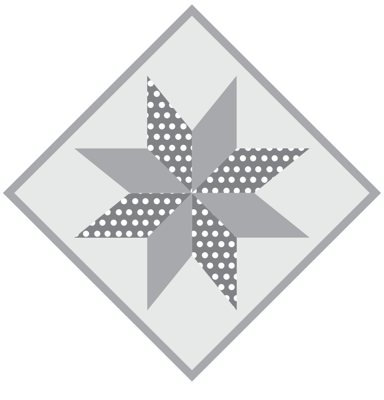


6 A block of land appears on a council map as shown. Calculate its area in square metres.



7 **a** How many identical parallelograms are shown in the quilt pattern below? (Ignore the different colours)

b If a single parallelogram has a base length of 8·5 cm and a height of 6·5 cm, calculate the area of the material required for all of the parallelograms on the quilt.



Answers 13C

1 **a** 65 cm^2 **b** 60 m^2 **c** 176 mm^2
d 350 mm^2 **e** 144 m^2 **f** 80 cm^2

2 **a** 132 cm^2 **b** 64 cm

3 **a** 13 mm **b** 56 m^2

4 90 cm^2 ÷ 12 cm = 7·5 cm height

5 300 cm^2 ÷ 15 cm = 20 cm base length

6 26 m × 40 m = 1040 m^2

7 **a** 8 (There is more to the quilt than the parallelograms that make up the central portion.)
b 8 × 8·5 × 6·5 = 442 cm^2

13D • Areas of triangles

LB2 Pages 144–146

Specific learning outcomes

Learners should be able to:

8.13.5.1 Identify a triangular shape and its properties.

8.13.5.2 Use the formula to calculate the area of any triangle**:**

$$\text{Area} = \frac{1}{2}\,\text{base} \times \text{height}$$

Teaching points

1. Demonstrate how any triangle can be drawn by halving a parallelogram but retaining the base.
2. Note the importance of identifying the perpendicular height, especially when the triangle is differently oriented.
3. Calculate the areas of given triangles using the formula.

Learner difficulties and remedies

Difficulty

Identifying which measurements to use for the base and heights, especially when measurements for slanted sides are given.

Remedy

- Do more work on identifying which sides and measurements to use with the formula. Help learners to identify the perpendicular height and the base. It is sometimes useful to rotate the triangle so that the base is horizontal. The height is then the perpendicular height from the base to the vertex of the triangle.
- Provide more exercises on finding areas of various triangles using the formula.

Suggested teaching approach

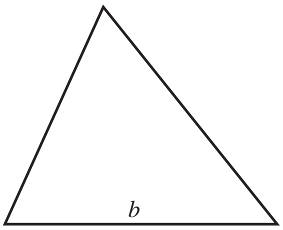
- Develop the formula for finding the area of a triangle.
- Show how to use the formula and provide a variety of examples.
- Identify which side is the base and which is the height in a range of triangles with different orientations.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

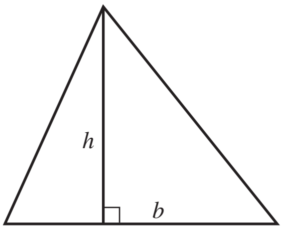
How do you find the area of a triangle? When faced with a new problem, mathematicians often try to use what they already know to solve it.

To find the area of a triangle, we will use a shape we already know how to find the area of the rectangle. Can triangles be 'rearranged' to form rectangles?

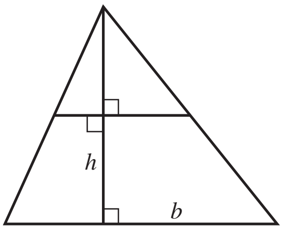
1. Use a ruler and a pencil to draw a triangle. So it is easy to work with, the side lengths should be at least 10 cm long. Label one side as the base, *b*.



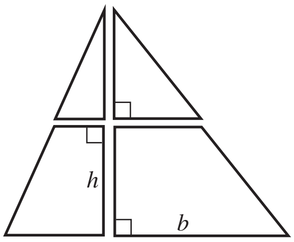
2. Rule a line from the apex (the point at the top) of the triangle to the base. The line should be perpendicular (at an angle of 90°) to the base. Label this line *h*, the height of the triangle.



3. Mark the point halfway up this line and use it to rule a line parallel to the base.



4. Cut along these two inside lines to create four pieces.

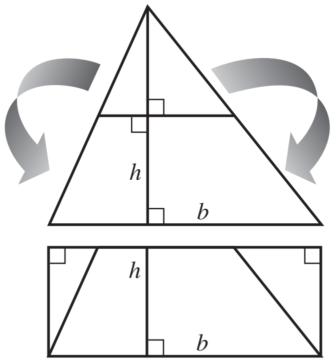


Now:

- Use the four pieces to form a rectangle.
- Reform the triangle.
- Keep moving between the rectangle and the triangle until you are able to do it easily.
- What is the simplest way to move from the triangle to the rectangle? Write down the steps involved, then share them with a friend or the class.

Did you find that the simplest way was to rotate the two top pieces until they touched the bottom two?

We can find the area of the rectangle formed by finding the length and the width and multiplying them.



The length of the rectangle is the length of the base of the triangle, *b*.

The width of the rectangle is half the height of the triangle, $\frac{1}{2}h$, or $\frac{h}{2}$, (Can you see why? Look back at the instructions for step 3.)

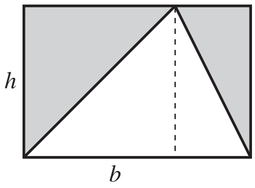
So, instead of writing $A = \text{length} \times \text{width}$, we can write:

$A = b \times \frac{h}{2}$, or $A = \frac{bh}{2}$ for the area of the rectangle, which is also the area of our original triangle.

This rule will work for all triangles. You can check this by repeating steps 1–4 with differently shaped triangles, then rearranging them into rectangles.

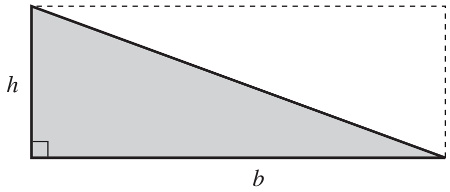
Another way of looking at it

Another way of showing that the area of a triangle is equal to $\frac{bh}{2}$ is to draw a rectangle around the triangle, touching the points of the triangle, like this:

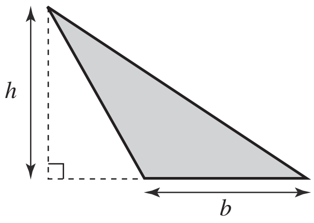


The area of this rectangle is $b \times h$, or bh. We can see by using symmetry that the two smaller, shaded triangles together are equal to the original unshaded triangle. This means that the original triangle occupies half the area of the rectangle, or $\frac{bh}{2}$.

It is easy to see that the area of a triangle is half the area of the enclosing rectangle if you consider a right-angled triangle.



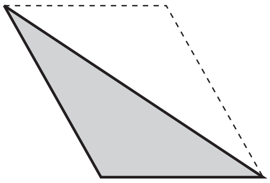
For some triangles, such as this one, the perpendicular height is measured outside of the triangle. We can find the height by extending the base, then drawing a perpendicular line from this extension to the opposite vertex. These triangles also follow the rule $A = \frac{bh}{2}$.



We can draw another identical triangle to form an enclosing parallelogram.

The area of the parallelogram is bh.

One triangle forms half of the parallelogram, so its area is $\frac{bh}{2}$.



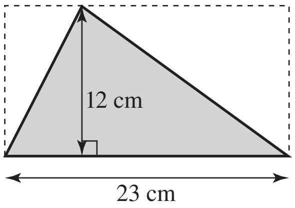
The area of a triangle is equal to half of the product of the base and the perpendicular heigh.

$$A = \frac{bh}{2}$$

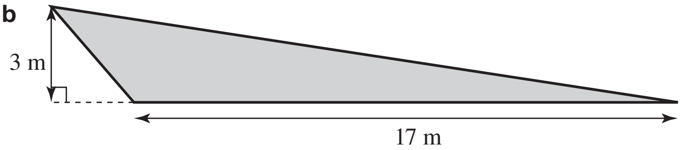
Examples

Calculate the area of the following triangles.

a



b

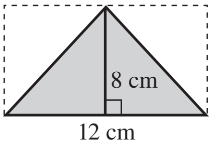


Thinking	Working
a **1** Write the formula for the area of a triangle. **2** Identify b and h, and substitute their values into the formula. **3** Evaluate, writing the answer with the correct units.	**a** $A = \frac{bh}{2}$ $A = \frac{23 \times 12}{2}$ $A = 138\,\text{cm}^2$
b **1** Write the formula for the area of a triangle. **2** Identify b and h, and substitute their values into the formula. **3** Evaluate, writing the answer with the correct units.	**b** $A = \frac{bh}{2}$ $A = \frac{17 \times 3}{2}$ $A = 25{\cdot}5\,\text{m}^2$

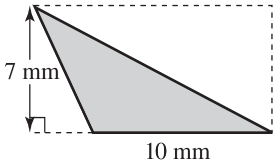
Activity 13D

1 Calculate the area of the following triangles.

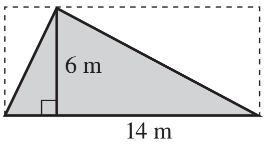
a



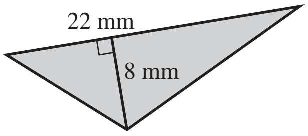
b



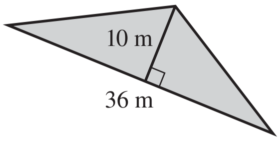
c



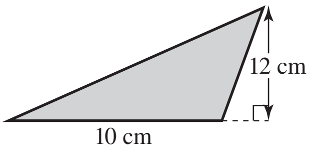
d



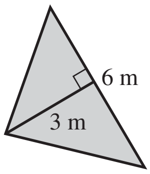
e



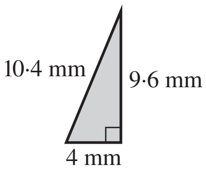
f



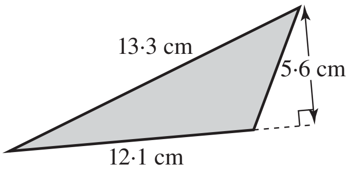
g



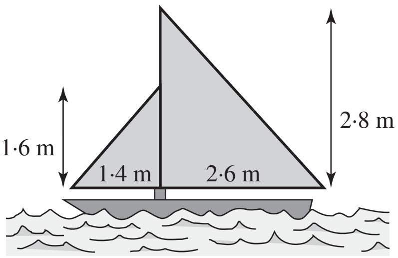
h



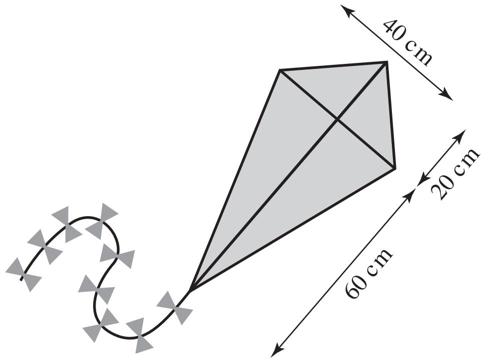
i



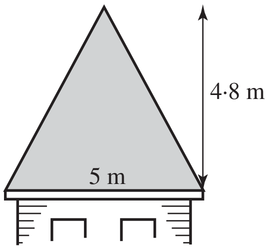
2 Find the total area of the sails on this yacht.



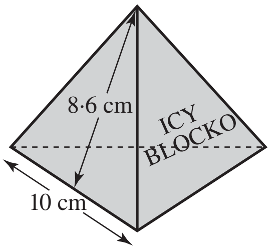
3 Find the area of paper required to build the kite shown.



4 The label on a tin of paint claims the contents will cover 4 square metres. How many tins would be needed to paint the triangular roof section pictured?



5 Find the total area of the four identical triangular faces of the container of frozen drink (a tetrapak) shown.



Answers 13D

1 **a** $48\,cm^2$
  b $35\,mm^2$
  c $42\,m^2$
  d $88\,mm^2$
  e $180\,m^2$
  f $60\,cm^2$
  g $9\,m^2$
  h $19{\cdot}2\,mm^2$
  i $33{\cdot}88\,cm^2$

2 $\frac{1}{2} \times 1{\cdot}6 \times 1{\cdot}4 + \frac{1}{2} \times 2{\cdot}8 \times 2{\cdot}6 = 4{\cdot}76\,m^2$

3 $\frac{1}{2} \times 60 \times 40 + \frac{1}{2} \times 20 \times 40 = 1600\,cm^2$

4 $\frac{1}{2} \times 4{\cdot}8 \times 5 = 12\,m^2$, $12 \div 4 = 3$ tins of paint

5 $\frac{1}{2} \times 8{\cdot}6 \times 10 = 43\,cm^2 \times 4 = 172\,cm^2$

13E • Areas of trapeziums

LB2 Pages 147–148

Specific learning outcomes

Learners should be able to:

8.13.6.1 Identify a trapezium and its properties.

8.13.6.2 Derive the formula to calculate the area of a trapezium.

8.13.7.1 Use the formula to calculate the areas of trapezium:

Area $= \frac{1}{2}(a + b) \times$ Height

Teaching points

1 Describe a trapezium as a quadrilateral with one pair of parallel sides.

2 Demonstrate how a trapezium can be transformed into a rectangle equal in area.

3 Use the formula to find areas for given trapeziums.

Learner difficulties and remedies

Difficulty

Identifying the shape of a trapezium and its properties.

Remedy

- Ask learners to describe a trapezium in words.

Difficulty

Identifying and substituting the correct measurements into the formula.

Remedy

- Know the different parts of the formula.
- Check they substitute the correct value into the formula and make accurate calculations.
- Remember the appropriate units for the answer.
- Provide more exercises and practical work.

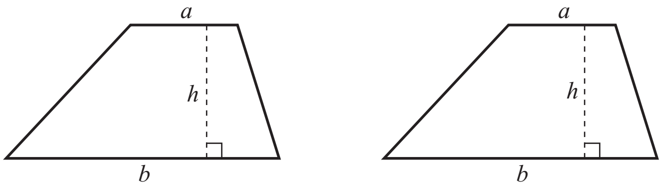
Suggested teaching approach

- Develop the formula for finding the area of a trapezium.
- Show them how to use the formula using a variety of examples.
- Show and identify corresponding measurements that would be inserted to the formula to find the area of trapeziums.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

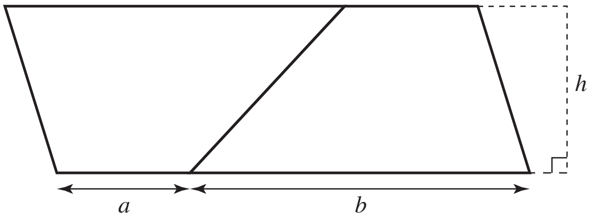
Additional notes

A **trapezium** is a quadrilateral with only one pair of opposite sides parallel.

The two trapeziums below are identical.



If the first shape is rotated around 180° and joined to the other, they form a parallelogram, as shown below.



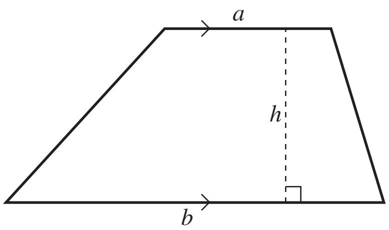
The area of this parallelogram is

(base length) × height $= (a + b) \times h$
$= (a + b)h$

However, this parallelogram contains two of the same trapezium, so the area of one trapezium is half of this amount,

$\frac{(a + b) \times h}{2}$ or $\frac{1}{2}(a + b) \times h$

Area of a trapezium

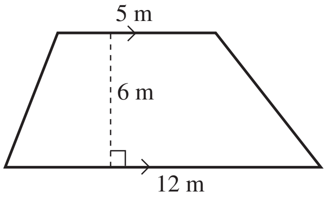


$A = \frac{1}{2}(a + b)h$ or $\frac{(a + b)h}{2}$ where a and b = the lengths of the parallel sides and h = height perpendicular to a and b.

The area of a trapezium equals half the sum of the lengths of the parallel sides multiplied by the perpendicular height.

Examples

Find the area of the following trapezium.

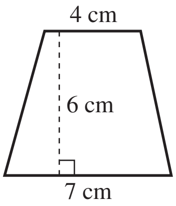


Thinking	Working
1 Write the formula for the area of a trapezium. **2** Substitute the values for *a*, *b* and *h*. **3** Evaluate, remembering the order of operations. Write your answer with the correct units of area.	$A = \frac{1}{2}(a + b)h$ $= \frac{1}{2} \times (5 + 12) \times 6$ $= 8{\cdot}5 \times 6$ $= 51\ \text{m}^2$

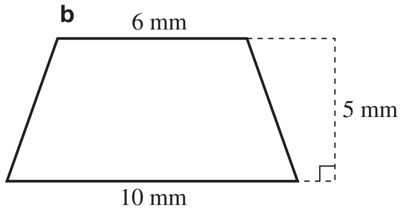
Activity 13E

1 Find the area of each of the following trapeziums.

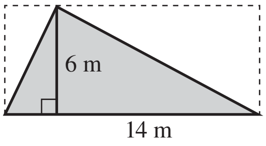
a



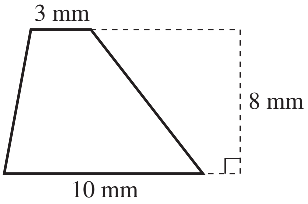
b



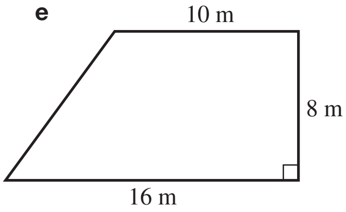
c



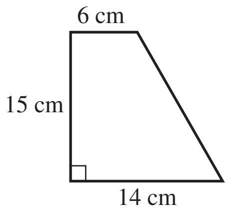
d



e

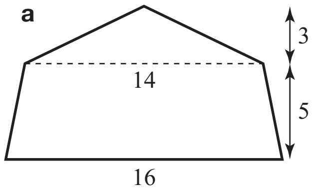


f

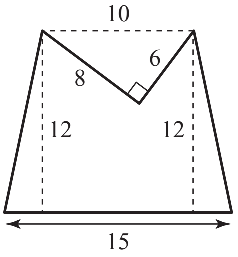


2 Calculate the areas of each of these shapes. All measurements are in metres.

a



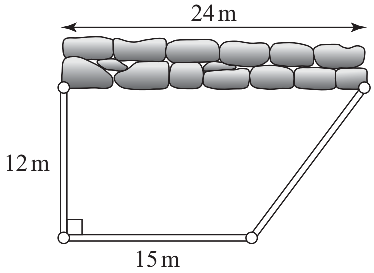
b



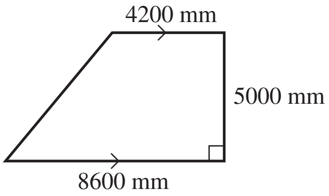
3 A farmer builds fences three sides of an enclosure in a trapezium shape. The fourth side is an existing stone wall.

a Calculate the area of the enclosure.

b The perimeter of the enclosure is 66 m. Calculate the length of fencing used.



4 Sam wants to pave an area using slate costing $32 per square metre. The area he wishes to pave is shaped as shown.



a Find the area to be paved in m^2.

b What will be the total cost of the slate?

Answers 13E

1 **a** $33\ \text{cm}^2$ **b** $40\ \text{mm}^2$ **c** $40\ \text{m}^2$

d $52\ \text{mm}^2$ **e** $104\ \text{m}^2$ **f** $150\ \text{cm}^2$

2 **a** $96\ \text{m}^2$ **b** $126\ \text{m}^2$

3 **a** $234\ \text{m}^2$ **b** 42 m

4 **a** $A = \frac{1}{2}(a + b)$, $A = \frac{1}{2}(4{\cdot}2 + 8{\cdot}6) \times 5$, $A = 32\ \text{m}^2$

b $C = 32 \times 32 = \$1024$

13F • Areas of irregular shapes

LB2 Pages 149–151

Specific learning outcomes

Learners should be able to:

8.13.8.1 Estimate the areas of irregular shapes by combining parts of the shape to make unit squares.

Teaching points

1. Use a square grid to estimate the area of an irregular shape.
2. Count the whole squares first, then any half-squares, and then match other part-squares that make the equivalent of a whole square in area.
3. Apply the technique to a variety of real images.

Learner difficulties and remedies

Difficulty

Estimating the area of incomplete squares on a grid so the areas of irregular shapes can be determined.

Remedy

- Have learners share their estimations for the areas of irregular shapes and check each other's accuracy when their answers differ.

Suggested teaching approach

- Identify the irregular shape.
- Count the number of complete squares then estimate the remaining part-squares.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

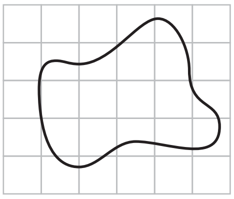
Additional notes

Area of irregular shapes can be estimated using square grids. This can be done by counting the number of squares that would fit exactly into the irregular shape. Irregular shapes do not fit exactly onto square grids because of their curves and bumps so areas are estimated. Count the whole squares first, and then any half-squares, and then match other part-squares that make the equivalent of a whole square in area.

Examples

Estimate the area of the irregular shape given below by counting the number of squares that are inside the shape.

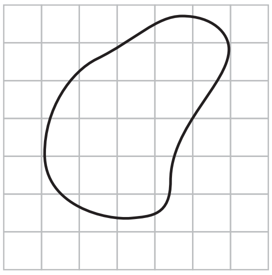
Note: If more than half of the square is in the shape, then include it to the count.

Thinking	Working
1 Identify the irregular shape inside the grid.	
2 Count the number of complete squares in the shape.	8
3 Count the number of incomplete squares that are in the shape then estimate the make-up number of squares they would make	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=4$
4 Add the complete squares and the estimated Incomplete squares to find the total area.	Estimate area is 12 cm^2.

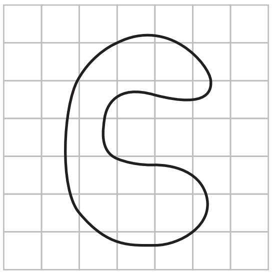
Activity 13F

1 Estimate the area of these shapes by including or excluding squares

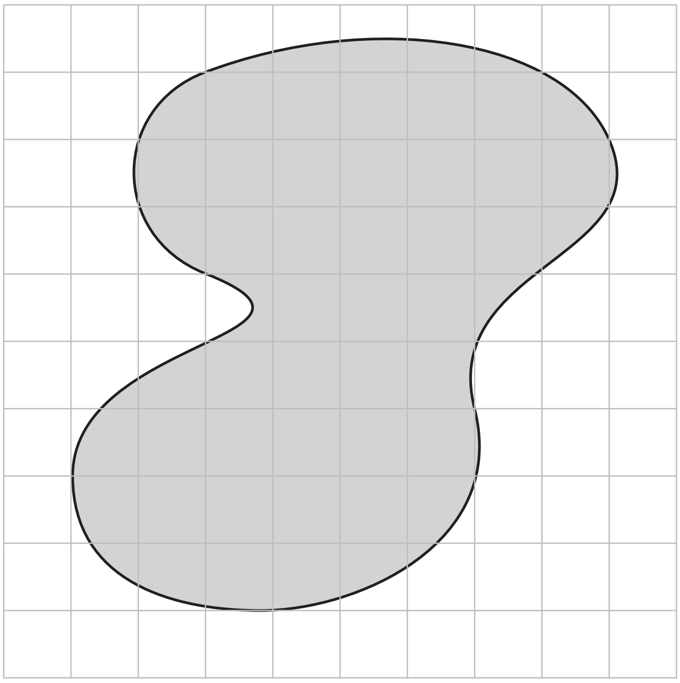
a



b



2 Use the grid to estimate the area of the lake. The squares on the grid represent square kilometres.



Answers 13F

1 **a** 15 cm^2

b 12 cm^2

2 45 km^2

13G • Area of circles

LB2 Pages 152–154

Specific learning outcomes

Learners should be able to:

8.13.9.1 Calculate the area of various circles using the formula: Area = πr^2

Teaching points

1 Link this topic to the previous one by calculating the area of a circle by tracing onto a grid.

2 Show how to use the formula with appropriate approximations for π.

Learner difficulties and remedies

Difficulty

Identifying the radius and the diameter in the formula to find area of a circle.

Remedy

- Identify and distinguish between radius and diameter.
- Show when and where to use radius and diameter distinctively.

Difficulty

Using the formula with π to find area of the circle.

Remedy

- Show how to calculate values involving π, either using a calculator or using an approximation.
- Do some exercises on how to find the area of circles.

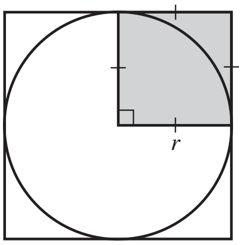
Suggested teaching approach

- Explain the formula for calculating the area of a circle: $A = \pi r^2$
- Explain that the diameter is twice the radius.
- Show how to make calculations involving π, either using a calculator or using an approximation such as 3·14.
- Explain the interchanging use of diameter and radius.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

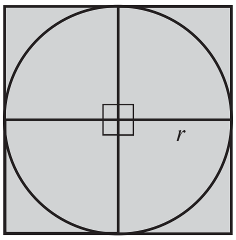
Additional notes

To find an approximation for the area of a circle, we can first consider the circle enclosed by a square whose side length is the same as the diameter of the circle. (The sides of the square just touch the circle at one point on each side.) This square can be divided into four identical smaller squares.

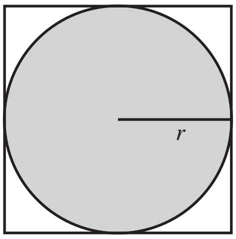
The area of the shaded square is r^2.



The area of the square that encloses the circle is $4r^2$ ($r^2 \times 4$).

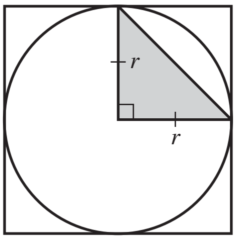


The area of the circle is clearly less than $4r^2$.

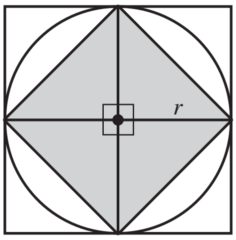


Now we consider a square drawn inside the circle, whose vertices touch the circumference. This square can be divided into four identical smaller triangles.

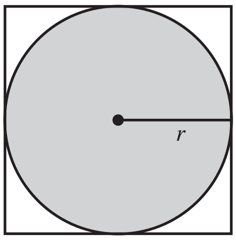
The area of the shaded triangle is $\frac{r^2}{2}$.



The area of the square inside the circle is $4 \times \frac{r^2}{2} = 2r^2$.



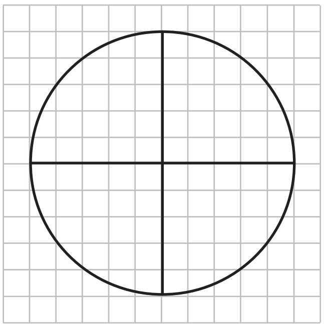
The area of the circle is clearly more than $2r^2$.



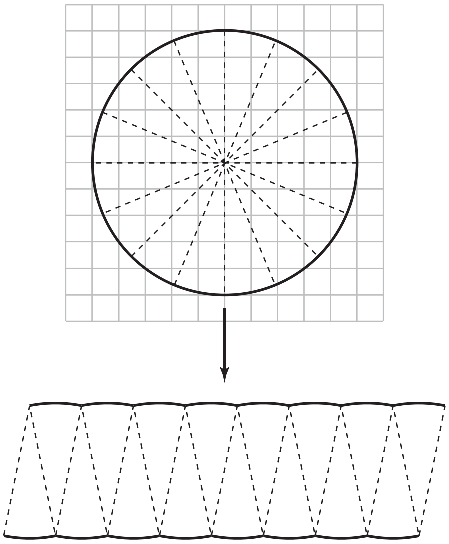
So, the area of the circle is greater than $2r^2$, but less than $4r^2$. We can write this mathematically as $2r^2 <$ Area of circle $< 4r^2$.

To develop a more accurate rule for finding the area of a circle, we can cut the circle into pieces and rearrange them to form a shape we know how to find the area of.

1 Use a compass to draw three circles of the same size on centimetre grid paper. A radius of 4 or 5 cm is a convenient size. Divide the circle into four quarters. Estimate the area by counting the squares in a quarter and multiplying by 4. Write down your estimate for the area of your circle.



2 Divide the circle into sixteenths by dividing the quarters in half, then in half again. Cut them out and glue them onto plain paper or into your workbook as shown below, making sure that you alternate the curved edge between the top and bottom.



3 You should notice that the shape formed by the sectors of the circle resembles a rectangle. Write down the rule for finding the area of a rectangle.

4 The circumference of the original circle has been chopped up into the small curved edges along the top and bottom sides of the rectangle, so the length of the rectangle, l, is equal to half the circumference, $2\pi r$.

We can write: $l = \frac{\cancel{2}\pi r}{\cancel{2}} = \pi r$

5 The two shorter sides of the rectangle correspond to the length of the radius in the original circle. We can write w (width) $= r$.

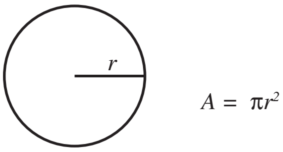
6 Write down the rule for the area of the rectangle again. Now, replace the two dimensions in the formula with $l = \pi r$ and $w = r$.

$$\begin{aligned} A &= l \times w \\ &= \pi r \times r \\ &= \pi r^2 \end{aligned}$$

7 Use the formula $A = \pi r^2$ to calculate the area of the circle. Is the calculated area greater or less than the area you found by counting squares?

We stated earlier that the area of a circle would be between $2r^2$ and $4r^2$ (the 'inside' and 'outside' squares shown at the beginning of this section). As π has an approximate value of 3·141 592 654, $A = \pi r^2$ will give such an area.

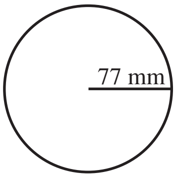
Area of a circle



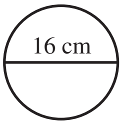
Examples

1 Find the area of the following circles, correct to two decimal places.

a



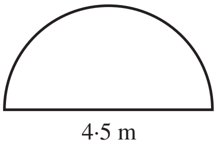
b



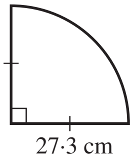
Thinking	Working
a 1 Write the formula. 2 Substitute for *r*. 3 Evaluate the formula. 4 Round the answer to the specified number of decimal places and include units of area.	**a** $A = \pi r^2$ $= \pi \times 77^2$ $= 18\,626{\cdot}502\ldots$ $\approx 18\,626{\cdot}50\,\text{mm}^2$ (2 d.p.)
b 1 Write the formula. 2 Halve the diameter to get the radius. 3 Substitute for *r*. 4 Evaluate the formula. 5 Round the answer to the specified number of decimal places and include units of area.	**b** $A = \pi r^2$ $d = 16\,\text{cm}$, so $r = 8\,\text{cm}$ $\therefore A = \pi \times 8^2$ $= 201{\cdot}061\ldots$ $\approx 201{\cdot}06\,\text{cm}^2$ (2 d.p.)

2 Find the area of the following shapes, correct to two decimal places.

a



b

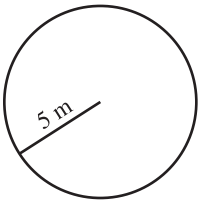


Thinking	Working
a 1 Modify the area formula to show the fraction of the whole circle being calculated (in this case, one half). 2 Find the radius by halving the diameter. 3 Substitute into the formula. 4 Evaluate the formula. 5 Round and write the answer, including the correct units.	**a** $A = \dfrac{\pi r^2}{2}$ $d = 4{\cdot}5\,\text{m}$, so $r = 2{\cdot}25\,\text{m}$ $A = \dfrac{\pi \times 2{\cdot}25^2}{2}$ $= 7{\cdot}952\,156\ldots$ $= 7{\cdot}95\,\text{m}^2$ (2 d.p.)
b 1 Modify the area formula to show the fraction of the whole circle being calculated (in this case, one quarter). 2 Substitute into the formula. 3 Evaluate the formula. 4 Round and write the answer, including the correct units.	**b** $A = \dfrac{\pi r^2}{4}$ $A = \dfrac{\pi \times 27{\cdot}3^2}{4}$ $= 585{\cdot}349\,39\ldots$ $= 585{\cdot}35\,\text{cm}^2$ (2 d.p.)

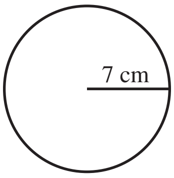
Activity 13G

1 Find the area of the following circles, correct to two decimal places.

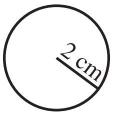
a



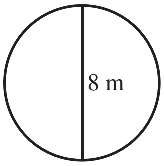
b



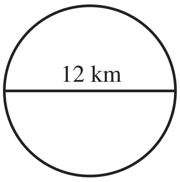
c



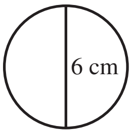
d



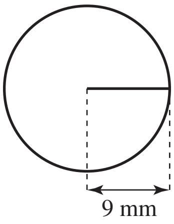
e



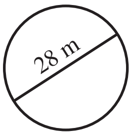
f



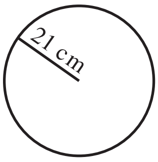
g



h

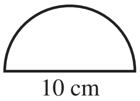


i

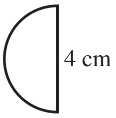


2 Find the area of the following shapes, correct to two decimal places.

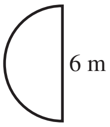
a



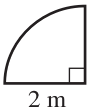
b



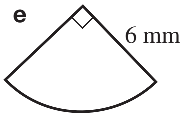
c



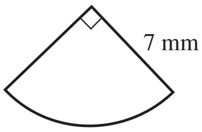
d



e



f



3 Calculate the area of each of the following circles, correct to the nearest cm^2.

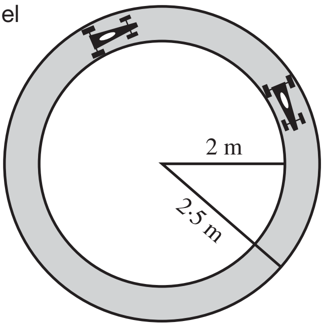
a a circle of radius 35 cm

b a circle of diameter 68 7cm

4 a The area of a circle with a diameter of 25 m if closest to:

b If a circle has a radius of 75 mm, then its area, correct to two decimal places is?

5 Find the area of a circular model racetrack of outer radius 2·5 m and inner radius of 2 m. Give your answer correct to two decimal places.



Answers 13G

1 a 78·54 m^2 **b** 153·94 cm^2 **c** 12·57 cm^2

d 50·27 m^2 **e** 113·10 km^2 **f** 28·27 cm^2

g 254·47 mm^2 **h** 615·75 m^2 **i** 1385·44 cm^2

2 a 39·27 cm^2 **b** 6·28 cm^2 **c** 14·14 m^2

d 3·14 m^2 **e** 28·27 mm^2 **f** 38·48 mm^2

3 a 3848 cm^2 **b** 3632 cm^2

4 a 491 m^2: C **b** 17 671·46 mm^2: C

5 $A = \pi(2{\cdot}5^2) - \pi(2^2) = 7{\cdot}07\,\text{m}^2$

13H • Areas of compound shapes

LB2 Pages 155–156

Specific learning outcomes

Learners should be able to:

8.13.10.1 Define and identify compound shapes. Compound shapes: shapes that made up of many different shapes.

8.13.10.2 Identify a compound shape and the different shapes that make it compound.

8.13.11.1 Use formulas for each different shapes to calculate the area of compound shapes.

Teaching points

1. Check learners' understanding of the term compound shape.
2. Identify strategies for calculating the area of compound shapes.
3. Use formulas to calculate the area of the simple shapes in the compound shapes.
4. Note when it is appropriate to add the areas of individual shapes and when it is necessary to subtract.

Learner difficulties and remedies

Difficulty

Identifying which shapes make up the compound shape.

Remedy

- Revise the properties of basic shapes that could be part of a compound shape.
- Know the formulas for calculating the areas of basic shapes in compound shapes.

Difficulty

Know the steps to follow when finding the area of compound shapes.

Remedy

- Identify different shapes that make up the compound shape.
- Use the formulas to find their areas and add them to get the total area of the compound shape.

Suggested teaching approach

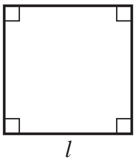
- Identify different shapes that make up the compound shapes.
- Identify the formulas that are needed to find the areas of the simple shapes.
- Add the areas (or subtract them) to find the area for the compound shape.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A composite shape is one made up of simpler shapes.

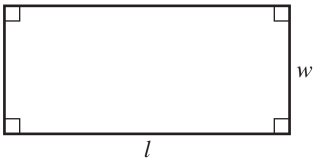
If we can find the areas of the simpler shapes, these can be added to calculate the total area.

Area formula summary



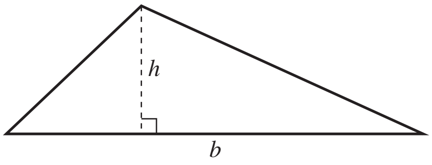
Square

$A = l^2$



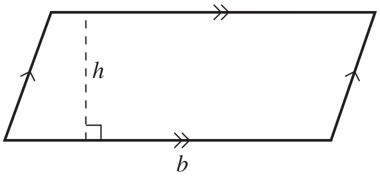
Rectangle

$A = lw$

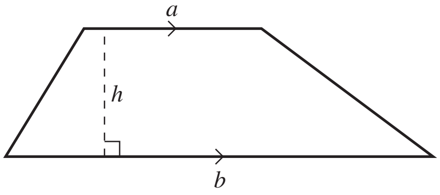


Triangle

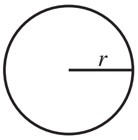
$A = \frac{1}{2}bh$



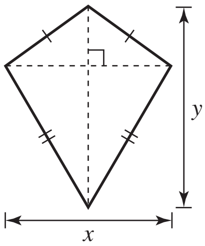
Parallelogram
$A = bh$



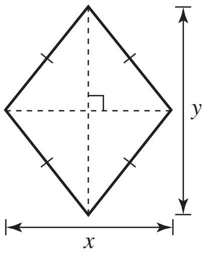
Trapezium
$A = \frac{1}{2}(a + b)h$



Circle
$A = \pi r^2$



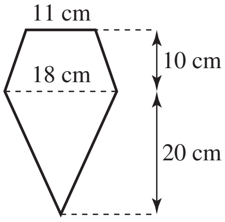
Kite
$A = \frac{1}{2}xy$



Rhombus
$A = \frac{1}{2}xy$

Examples

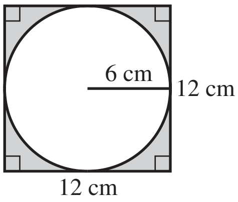
1 Find the area of this composite shape.



Thinking	**Working**
1 Identify the individual shapes that make up the total area.	$A_{\text{total}} = A_{\text{trapezium}} + A_{\text{triangle}}$
2 Write the formulas for each of the individual shapes. Use different pronumerals for the different heights (*h* for the trapezium and *H* for the triangle).	$= \frac{1}{2}(a + b)h + \frac{1}{2}bH$
3 Substitute the known lengths into the formulas. (Remember that multiplying by $\frac{1}{2}$ is the same as dividing by 2.)	$= \frac{1}{2}(11 + 18) \times 10$ $+ \frac{1}{2} \times 18 \times 20$
4 Evaluate the total area, writing your answer with the correct units.	$= 325\,\text{cm}^2$

Areas of parts of shapes can be found by subtracting the areas that are not included from the overall area.

2 Find the shaded area in the following shape, correct to two decimal places.

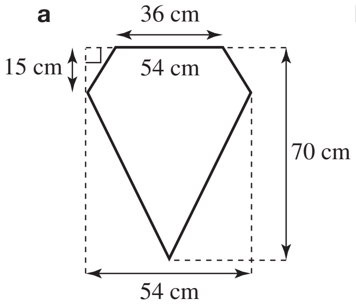


Thinking	**Working**
1 Write the shaded area as the subtraction of the area of one shape from the area of the other.	$A_{\text{shaded}} = A_{\text{square}} + A_{\text{circle}}$
2 Write the formulas for each of the individual shapes.	$= l^2 - \pi r^2$
3 Substitute the known lengths into the formula.	$= 12^2 - \pi \times 6^2$
4 Evaluate the difference, writing your answer with the correct units.	$= 144 - 36\pi$ $= 30{\cdot}90\,\text{cm}^2$

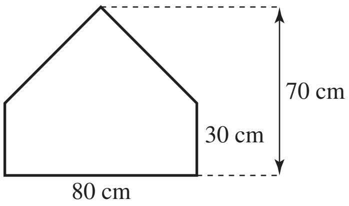
Activity 13H

1 Find the area of these composite shapes, correct to two decimal places where appropriate.

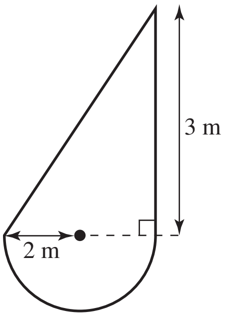
a



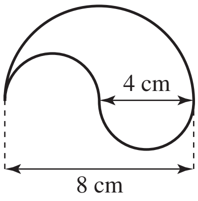
b



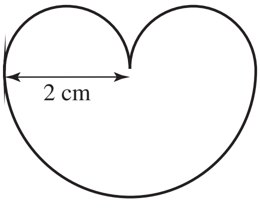
c



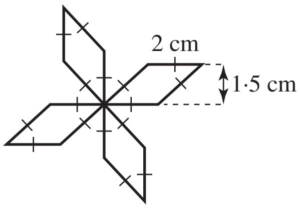
d



e

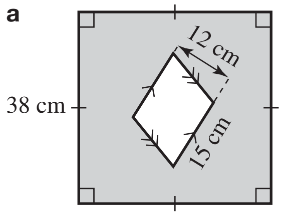


f

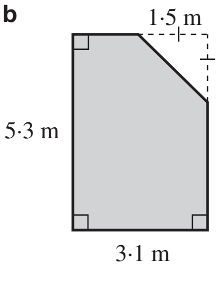


2 Find the shaded area in each of the following shapes, correct to two decimal places where appropriate.

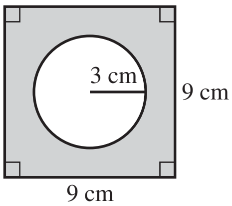
a



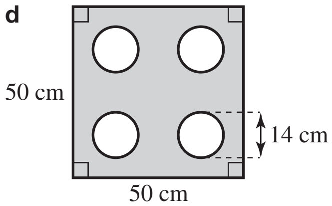
b



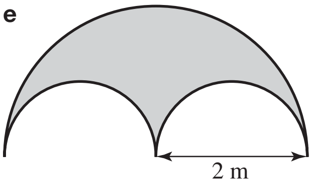
c



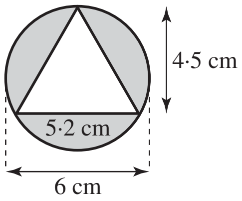
d



e



f



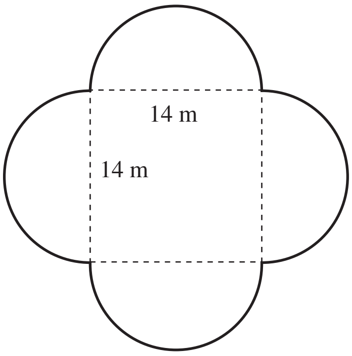
3 The area of the following shape, correct to the nearest m^2, is:

A 196 m^2

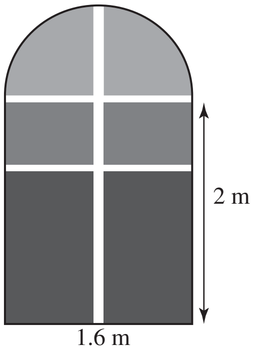
B 308 m^2

C 504 m^2

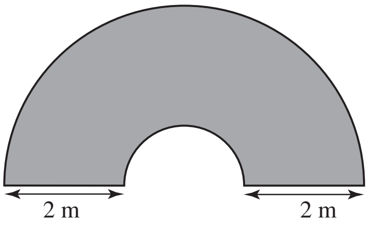
D 1232 m^2



4 A stained-glass church window is shaped as shown. What area of glass, correct to two decimal places, does it contain? (Ignore the parts of the frame, between the glass areas).

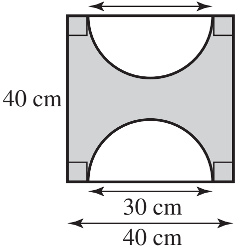


5 Michael needs to buy some pine-bark for a garden bed built around a semicircle of diameter 2 m. What area does he need to cover? Give your answer correct to two decimal places.

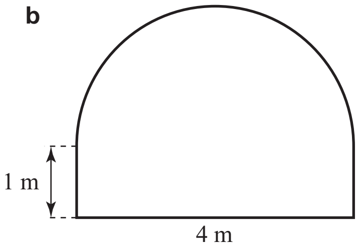


6 Find the perimeter and area of each shape, correct to two decimal places.

a



b



Answers 13H

1 **a** 2160 cm^2 **b** 4000 cm^2
c 12·28 m^2 **d** 25·13 cm^2
e 9·42 cm^2 **f** 12 cm^2
2 **a** 1264 cm^2 **b** 15·31 m^2
c 52·73 cm^2 **d** 1884·25 cm^2
e 3·14 m^2 **f** 16·57 cm^2
3 C 504 m^2
4 4·21 m^2
5 12·57 m^2
6 **a** 194·25 cm; 893·14 m^2
b 12·28 m; 10·28 m^2

13I • Total surface area of prisms

LB2 Pages 157–158

Specific learning outcomes

Learners should be able to:

8.13.12.1 Identify a prism and its properties.
8.13.13.1 Identify the shapes that make up the surface area of prisms.
8.13.14.1 Calculate the surface area of each faces of prisms.
8.13.14.2 Calculate the total surface area of prisms.

Teaching points

1. Identify the main properties of a prism as being a solid object that has a constant cross-section along its length or height.
2. Draw a net of the prism showing the shape of each face as if it is formed out of flattened cardboard.
3. Calculate the area of all the faces that make up the prism.
4. Calculate the total surface area that of the prism.

Learner difficulties and remedies

Difficulty

Identifying the polygon shapes that make up the faces of the prism.

Remedy

- Revise the names of various polygon shapes and their corresponding properties.

Difficulty

Calculating the total surface area of prisms.

Remedy

- Sketch the net of the solid identifying the polygon shapes. Note this can often be done in several different ways, although the number of surfaces and their shapes will remain the same.
- Use the correct formulas to calculate the areas of different faces, and then add them together to get the total area of the prism.

Suggested teaching approach

- Identify the numbers of faces and the polygon shapes that make up the faces of the prism.
- Identify the corresponding formulas and steps that must be used to find the area of the given polygon shapes that made up the faces of the prism
- Add the areas of various faces to find the total area of the surface area of given prism.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

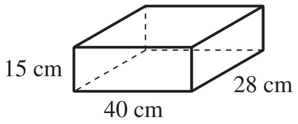
The surface area of a prism (a solid with straight sides) is the sum of all the areas of the faces of the shape.

To find the surface area of any prisms, find the area of each face, and then add them together.

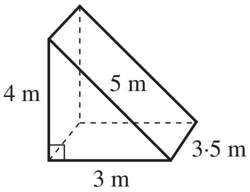
Examples

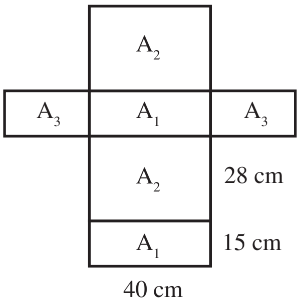
Draw the nets of the following polyhedral and use them to calculate their surface area.

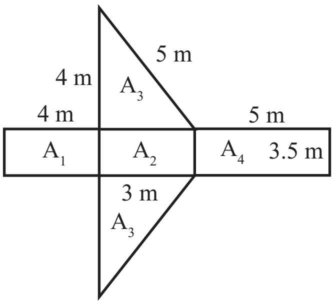
a



b

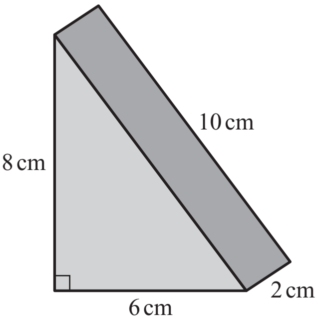


Thinking	**Working**
a **1** To draw the net, imagine cutting along the edges of the prism and 'flattening' it out. Draw each face as a flat shape. Check that your net has the same number of faces as the solid. Label; each of the different areas A_1, A_2, A_3 … etc. Label identical faces in the same way.	**a** 
2 Write a formula that shows the surface area (SA) as the total of the individual areas.	$SA = 2A_1 + 2A_2 + 2A_3$ $SA = 2l_1w_1 + 2l_2w_2 + 2l_3w_3$
3 Substitute the values of the different dimensions and evaluate each different area.	$= 2 \times (40 \times 15) + 2 \times (40 \times 28) + 2 \times (28 \times 15)$ $= 1200 + 2240 + 840$
4 Find the total area, writing your answer with area units.	$= 4280\,\text{cm}^2$

Thinking	**Working**
b **1** To draw the net, imagine cutting along the edges of the prism and 'flattening' it out. Draw each face as a flat shape. Check that your net has the same number of faces as the solid. Label each of the different areas A_1, A_2, A_3 … etc. Label identical faces in the same way.	**b** 
2 Write a formula that shows the surface area (SA) as the total of the individual areas.	$SA = A_1 + A_2 + 2A_3 + A_4$ $= l_1w_1 + l_2w_2 + 2\frac{bh}{2} + l_3w_3$
3 Substitute the values of the different dimensions and evaluate each different area.	$= (3·5 \times 4) + (3 \times 3·5) + 2 \times \left(\frac{3 \times 4}{2}\right) + (5 \times 3·5)$ $= 14 + 10·5 + 12 + 17·5$
4 Find the total area, writing your answer with area units.	$= 54\,\text{cm}^2$

Activity 13I

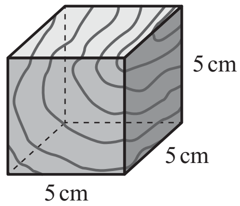
1 Calculate the surface area of this shape.



2 This wooden block has six identical square faces.

a What is the area of one face?

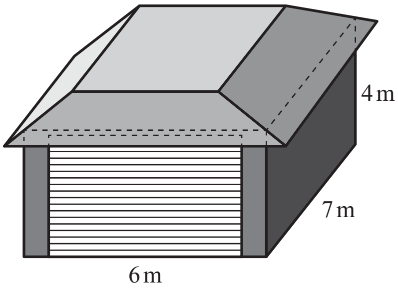
b What is the total area of all 6 faces?



3 The four outside walls of this garage are to be painted. This includes the roller door at the front but not the roof. Each litre of the paint chosen covers $5\,\text{m}^2$.

a What is the total area to be painted?

b How many litres of paint are needed?



4 A film-maker needs to make fake gold bars as props for a movie. This is done by painting 800 blocks of wood with gold paint. A litre of paint covers 9 m^2 (= 900 000 cm^2). Each block of wood measures 12 cm by 8 cm by 6 cm.
   - **a** What is the surface area of one block?
   - **b** How many 1-litre cans of paint will be needed?

Answers 13H

1 96 cm^2

2 **a** 25 cm^2 **b** 150 cm^2

3 **a** 104 m^2 **b** 20·8 L

4 **a** 432 cm^2 **b** 4 cans

13J • Volume of prisms

LB2 Pages 159–161

Specific learning outcomes

Learners should be able to:

8.13.15.1 Define the term 'volume'.

8.13.16.2 Calculate the volume of various prisms and cuboids by using the formula:
Volume = Area of base × Height
$V = L \times W \times H$

Teaching points

1. Explain that a volume is a measure of the amount of space within a solid object.
2. Find volume of various prisms and cuboids using the volume formula.
3. The term capacity is also a measure of volume but in terms of litres rather than metres. It is best explained as a measure of how much liquid would fill the solid object.

Learner difficulties and remedies

Difficulty

Identifying the shapes that make up the front face of the prism.

Remedy

- Know and identify all the polygon shapes and their properties.

Difficulty

Finding the volume of given prisms.

Remedy

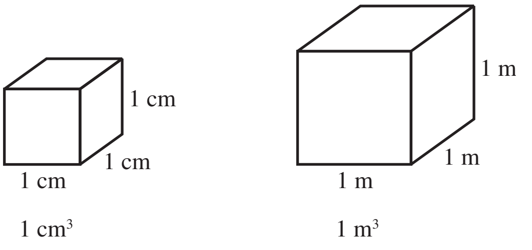
- Identify the shape that makes up the front face of the prism.
- Identify the formula to use to calculate the area of the front face of the prism.
- Find the area of the front face.
- Multiply the area of the front face by the height to find the volume of any given prism.

Suggested teaching approach

- Identify the shape that make up the front face of the prism.
- Identify the formula to use to calculate the area of the front face of the prism.
- Find the area of the front face.
- Multiply the area of the front face by the height to find the volume of any given prism.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

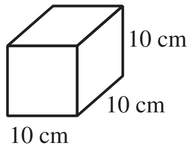
Volume is the amount of space taken up by three-dimensional objects, and is measured in units such as cubic centimetres (cm3) and cubic metres (m3).



(Not to scale)

Volume is a measurement of the number of whole and parts of whole cubes that could be fitted into a solid (without any gaps).

Capacity is the term used for the volume of space inside a container.

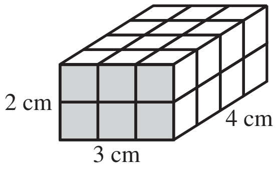


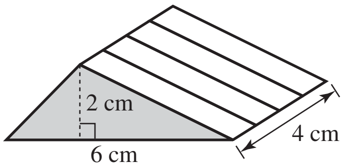
A litre (L) is the space inside a 10 cm × 10 cm × 10 cm cube.

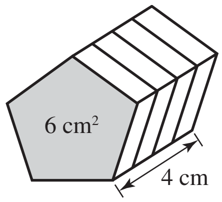
A millilitre (mL) is the space inside a cubic centimetre and a kilolitre (kL) is the space inside a cubic metre.

A **prism** is a solid that has a **uniform cross-section.** This means that the shape on one of the ends of the prism continues unchanged, throughout the height of the prism. This shape is called the base of the prism. The height of the prism is the perpendicular distance from the base to the top. The base of a prism is a polygon.

Below are three prisms: a rectangular-based prism, a triangular-based prism and a pentagonal-based prism.

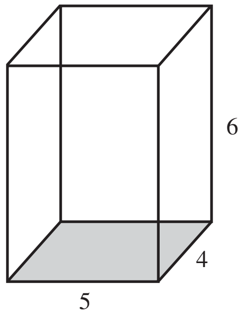






Finding the volume of a rectangular prism

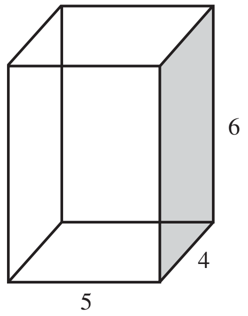
Recall that to find the volume of a rectangular prism, we multiply together the length (*L*), width (*W*) and height (*H*). Using our definition of a prism, we can see that any one of the faces of the rectangular prism can act as a base.



$V = lwH$

$V = 5 \times 4 \times 6$

$= 120 \text{ cm}^3$



$V = lwH$

$V = 6 \times 4 \times 5$

$= 120 \text{ cm}^3$

If we look at the formula $V = L \times W \times H$, we can see that multiplying *L* by *W* gives us the area of the rectangular base: $A = lw$. So, we can write the formula:

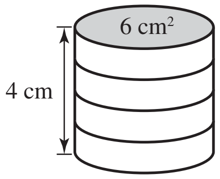
$V = L \times W \times H$

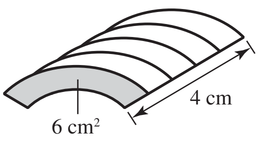
$V = AH$

where A = the area of the base.

Because they all have a uniform cross-section, the volume of all prisms can be calculated in the same way: find the area of the base and multiply it by the height.

A general formula for the volume of solids with a uniform cross-section





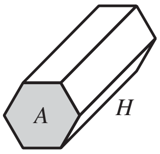
There are other solids that are not prisms that have a uniform cross-section. A solid with a circular uniform cross-section is called a **cylinder.** The solid shown here with the odd-shaped curvy base has no special name, but does have a uniform cross-section.

The rectangular, triangular and pentagonal prisms shown in blue on the previous page and the cylinder and the odd-shaped solid shown here all have the same volume, as they all have the same cross-sectional area (6 cm^2) and height (4 cm).

The formula for the volume of solids with a uniform cross-section:

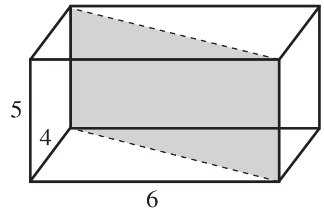
$V = AH$

where A = area of the cross-section, H = height (*H* is perpendicular to *A*)



Triangular prisms

If we make a diagonal cut from corner to corner across the base and up through the height of a rectangular prism, we create a triangular prism that has half the volume of the rectangular prism.



We can write the volume of this triangular prism as:

$V = \frac{1}{2} \times 6 \times 4 \times 5$ or $V = \frac{6 \times 4 \times 5}{2}$

$= 60 \text{ cm}^3$ $= 60 \text{ cm}^3$

We could also use the formula $V = AH$ to find the volume, but, as the shape of the base of the prism is a triangle, its area is given by $A = \frac{1}{2}bh$, where b = length of base and h = perpendicular height to the base.

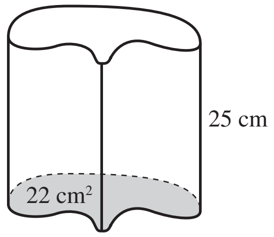
Substituting in our formula for *A*, we get: $V = \frac{1}{2}bhH$.

Note that we use *h* to represent the height of the triangular base and *H* to represent the height of the prism.

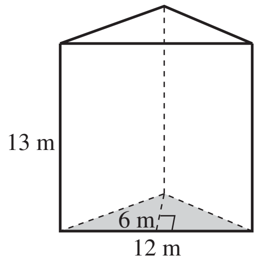
Examples

1 Find the volume of each solid. Give answers correct to two decimal places, where appropriate.

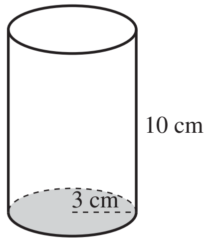
a



b



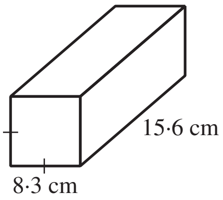
c



Thinking	**Working**
a 1 Write the formula.	**a** $V = AH$
2 Substitute the values for *A* and *H* into the formula. Note that the base area *A* is already given.	$= 22 \times 25$
3 Evaluate, and write the answer with the appropriate units of volume.	$= 550 \text{ cm}^3$
b 1 Write the general formula.	**b** $V = AH$
2 Substitute the area formula for the triangular base.	$= \frac{1}{2}bh \times H$
3 Substitute the values for *b*, *h* and *H*.	$A = \frac{1}{2} \times 12 \times 6 \times 13$
4 Evaluate the volume and write the answer with the appropriate units.	$= 468 \text{ m}^3$

Thinking	**Working**
c **1** Write the general formula.	**c** $V = AH$
2 Substitute the area formula for the circular base.	$= \pi r^2 \times H$
3 Substitute the values for r and H.	$= \pi \times 3^2 \times 10$
4 Evaluate the volume. Round the answer correct to two decimal places and write it with the correct units.	$= 282{\cdot}74\,\text{cm}^3$ (2 d.p.)

2 Find the capacity of the container, in litres. Round your answer correct to one decimal place.

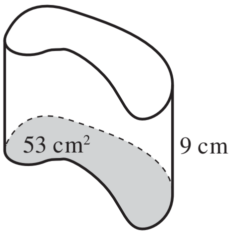


Thinking	**Working**
1 Write the formula for volume.	$V = AH$
2 Substitute the formula for A, the area of the square base.	$= l^2 \times H$
3 Substitute the values	$= 8{\cdot}3^2 \times 15{\cdot}6$
4 Evaluate the volume, including the units.	$= 1074{\cdot}684\,\text{cm}^3$
5 Convert the answer to mL. ($1\,\text{mL} = 1\,\text{cm}^3$)	Capacity $= 1074{\cdot}684\,\text{mL}$
6 Convert mL to L by dividing by 1000.	$= 1{\cdot}074\,684\,\text{L}$
7 Round the answer to one decimal place.	$= 1{\cdot}1\,\text{L}$ (1 d.p.)

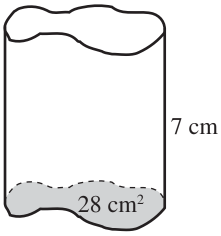
Activity 13J

1 Find the volume of each solid. Give answers correct to two decimal places, where appropriate.

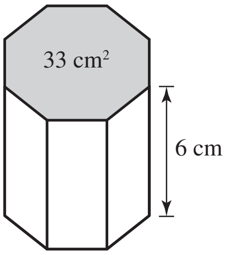
a



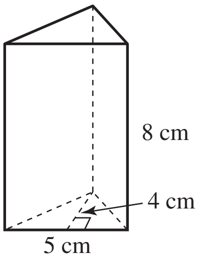
b



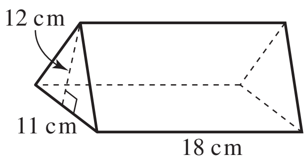
c



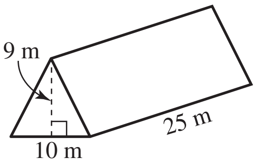
d



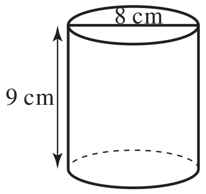
e



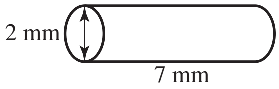
f



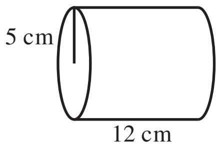
g



h

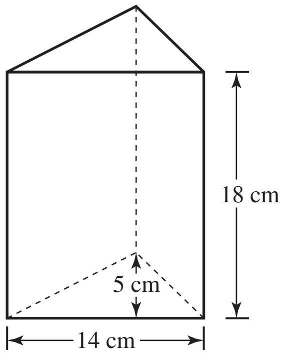


i

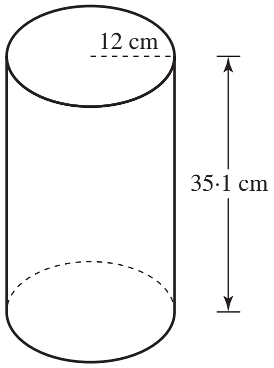


2 Find the capacity of the following containers, in millilitres. Round your answers correct to one decimal place where necessary.

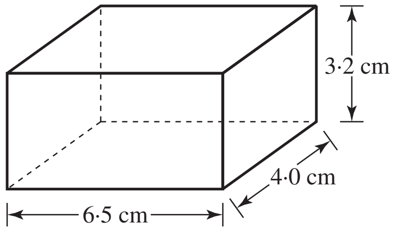
a



b

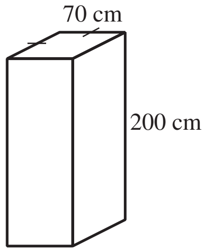


c

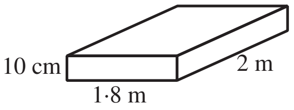


3 Find the volume of each of the following shapes (i) in cm^3 and (ii) in m^3. For the cylinders, round your answers correct to two decimal places.

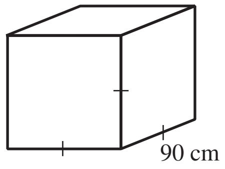
a



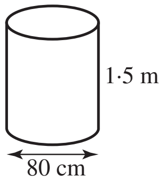
b



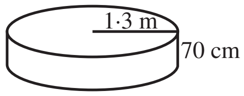
c



d



e



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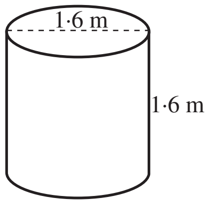
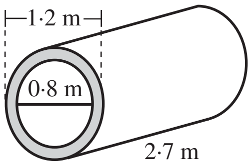
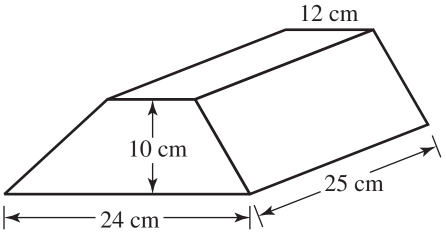


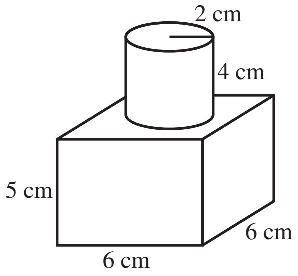
4 Find the volume of the hollowed-out cylinder, correct to two decimal places.



5 Find the capacity of this prism in litres.



6 Find the capacity of the model aircraft fuel tank.



Answers 13J

1 **a** 447 cm^3 **b** 196 cm^3 **c** 198 cm^3
d 80 cm^3 **e** 1188 cm^3 **f** 1125 m^3
g 452·39 cm^3 **h** 21·99 mm^3 **i** 942·48 cm^3

2 **a** 630·0 mL **b** 15 878·9 mL **c** 83·2 mL

3 **a** **i** 9 800 000 cm^3 **ii** 0·98 m^3
b **i** 3 600 000 cm^3 **ii** 0·36 m^3
c **i** 729 000 cm^3 **ii** 0·729 m^3
d **i** 753 982·24 cm^3 **ii** 0·75 m^3
e **i** 3 716 504·11 cm^3 **ii** 3·72 m^3
f **i** 216 990·88 cm^3 **ii** 3·22 m^3

4 $\text{Volume}_{\text{total}} = \text{Volume}_{\text{big cylinder}} - \text{Volume}_{\text{small cylinder}}$
$= \pi(0{\cdot}6)^2(2{\cdot}7) - \pi(0{\cdot}4)^2(2{\cdot}7)$
$= 1{\cdot}70\ \text{m}^3$

5 $V = AH$
$= \frac{1}{2}(a + b)\ h \times H$
$= (12 + 24)10 \times 25$
$= 4500\ \text{cm}^3$
$= 4{\cdot}5\ \text{L}$

6 $\text{Volume}_{\text{total}} = \text{Volume}_{\text{cylinder}} - \text{Volume}_{\text{rectangular prism}}$
$= \pi(2)^2(4) - 5 \times 6 \times 6$
$= 230{\cdot}27\ \text{cm}^3$
$= 230{\cdot}27\ \text{mL}$

• Chapter Fourteen •

Polyhedra and Networks

Overview

The world that we live in today are all made of three dimensional objects such as buildings, trees, stones, animals, human beings etc. Three dimensional (3D) objects or shapes can be viewed from various positions such as from the top, front and side that can be drawn as two-dimensional elevations. However, 3D objects can also be drawn on triangular or isometric dot paper. Isometric drawings show three surfaces as if viewing a corner of the object. Perspective drawings are even more natural ways of viewing a solid object where nearer edges are represented larger than edges that are further away.

A 3D shape can look different when viewed from different perspectives. A top view, or bird's eye view is what you see when looking directly down on the shape, a side view is what you see when looking directly at the side and the front view is what you see when looking at the front.

A network is the name given to the way mathematicians classify polygons and polyhedral according to their various properties such the numbers of vertices, faces and edges.

Contents

LB2 page

Chapter skills

This chapter covers the following skills:

- Using polyhedra definitions
- Drawing 3D shapes using isometric drawing
- Visualising and drawing three-dimensional shapes
- Drawing one-point and two-point perspectives
- Defining polyhedra
- Recognising and drawing nets of selected polyhedra
- Using networks to show vertices, paths, traversability and the shortest path
- Constructing a solid from its net and verifying Euler's rule

Teaching plan

Lessons	Chapter sections	Class work and home work
1	• 14A Solid shapes	Learner's Book 1 • Exercise 14A, pages 175, 176
2–3	• 14B Nets for solid shapes	Learner's Book 1 • Exercise 14B, pages 177–179
4	• 14C Drawing 3D shapes	Learner's Book 1 • Exercise 14C, pages 181, 182
5	• 14D Perspective drawings	Learner's Book 1 • Exercise 14D, page 186
6–7	• 14E Polyhedron definitions	Learner's Book 1 • Exercise 14E, page 188
8–9	• 14F Networks and their applications	Learner's Book 1 • Exercise 14F, pages 190, 191
10	• Test	Teacher's Guide • Chapter 14 Test

General learning outcomes

Learners should:

Solid shapes

8.14.1 Know that solid shapes (3D) can be drawn or represented on two-dimensional planes and using dotted lines to represent any hidden edges. (U)

8.14.2 Know that solid shapes are grouped into three categories: prism, pyramids and those with circular shapes. (K)

Nets for solid shapes

8.14.3 Understand how two-dimensional nets are cut and folded to form solid shapes. (U)

8.14.4 Know how to draw and identify nets of solid shapes. (K)

Drawing 3d shapes

8.14.5 Understand that three-dimensional shapes have lengths, widths and heights.(U)

8.14.6 Know how to use draw 3D solids on dotted grids to visualise three views. (K)

Perspective drawings

8.14.7 Know how to draw and design two-dimensional drawings to represent depth of three-dimensional objects. (K)

Polyhedron definitions

8.14.8 Know how to identify shapes that are polyhedra. (K)

8.14.9 Know the five regular polyhedra with faces that are congruent regular polygons with the same number of faces meeting at each vertex. (K)

8.14.10 Know how to draw a planar network for three-dimensional shapes. (K)

8.14.11 Understand what an order of vertex is. (U)

Networks and their applications

8.14.12 Understand that a network is formed when vertices and edges are drawn together. (U)

8.14.13 Know how to determine the number of edges (degrees) at a vertex and whether it is odd or even. (K)

14A • Solid shapes

LB2 Pages 174–176

Specific learning outcomes

Leaners should be able to:

8.14.1.1 Define and identify solid shapes: cube, pyramid, triangular-based prism, tetrahedron, cylinder, cone, cuboid and sphere.

8.14.2.1 Identify three categories of solid shapes and their corresponding shapes: prism, pyramids and solids with circular shapes.

8.14.2.2 Define the terms ‘prism’ and ‘pyramid.

8.14.2.3 Sketch the various solid shapes.

Teaching points

1. Explain what solid shapes are and identify their properties.
2. Show three different categories of solid shapes: prisms, pyramids and solids with circular shapes (cone, cylinder and sphere).
3. Draw various solid shapes.

Learner difficulties and remedies

Difficulty

Understanding the differences between a prism and a pyramid.

Remedy

- Know the properties of prisms and pyramid to distinguish the two and able to identify them
- Discuss which properties are common for cones and pyramids, and for cylinders and prisms.

Suggested teaching approach

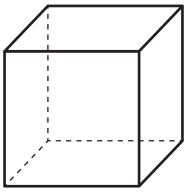
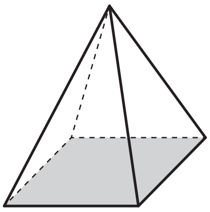
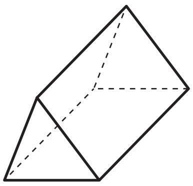
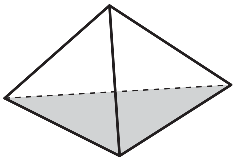
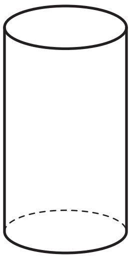
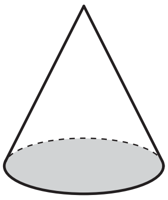
- Define and distinguish between prisms and pyramids.
- Identify their properties.
- Recognise prisms and pyramids in other objects in real life.
- Draw and sketch pyramids, prisms and solid shapes with circular edges and surfaces.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **solid** is a three-dimensional (3D) shape or object.

- A **prism** has a polygon base and a regular cross-section (any cut parallel to the base gives an identical shape to the base).
- A **pyramid** has a polygon base but all other faces are triangles, so the shape tapers to a point.
- A **polyhedron** is a three-dimensional shape with faces that are polygons.
- Other solid shapes have circular edges or surfaces such as a cone, cylinder and sphere.

Some examples of solid shapes:

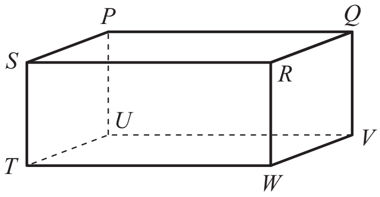
Cube	Pyramid
	
Triangular prism	**Tetrahedron**
	
Cylinder	**Cone**
	

Prisms and pyramids are made up of flat, two-dimensional 2D shapes called faces. Two faces meet at a line called an edge.

Three or more faces meet at a vertex. The plural of vertex is vertices.

Examples

Draw a cuboid labelled *P* to *W*. How many faces, edges and vertices are on a cuboid?

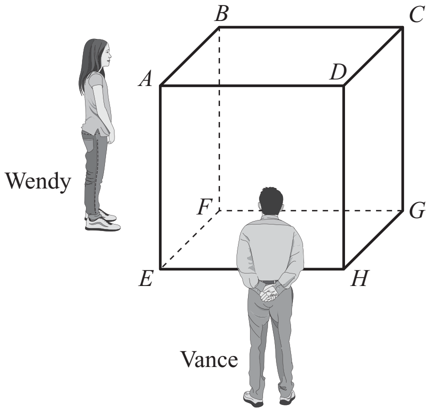
Thinking	Working
1 Draw a cuboid with external solid lines and internal dashed lines. 2 Label the vertices *P* to *W*. *PQRS* is a face. *RW* is an edge. *W* is a vertex.	 A cuboid has 6 faces, 12 edges and 8 vertices.

2 Show.

Activity 14A

1. What shape would make a good model for these objects?
   - **a** the hole in a pencil sharpener
   - **b** a brick
   - **c** a die (faces numbered 1 to 6)
   - **d** can of spaghetti
   - **e** a tennis ball
2. Draw a prism that has a pentagon (5 sides) for its cross-section.

3 Vance is standing in front of a large cube. Wendy is to the left of Vance, and is facing another face of the cube.

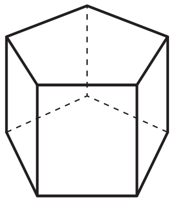


Which of the labels *A, B, C, D, E, F, G, H* would be marked on the side that Wendy sees?

Answers 14A

1 **a** cone
  b cuboid
  c cube
  d cylinder
  e sphere

2



3 *ABEF*

14B • Nets for solid shapes

LB2 Pages 177–179

Specific learning outcomes

Learners should be able to:

8.14.3.1 Cut sheets of cardboard to produce solid shapes.
8.14.3.3 Identify the five regular polyhedra and their corresponding nets.
8.14.4.1 Draw nets for a variety of prisms and pyramids.
8.14.4.2 Cut out the nets to produce solid shapes.

Teaching points

1. Draw the nets of solids on cardboard, cut them out and then fold to make a model of each shape.
2. Show five regular polygons and their nets.
3. Draw and sketch various prism and pyramids.

Learner difficulties and remedies

Difficulty

Constructing nets for given solid shapes.

Remedy

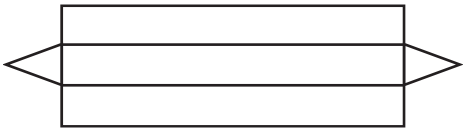
- Search for suitable templates in other books or on the internet that can be printed and cut out directly, then folded and glued to make the solids.
- Show learners different prisms and pyramids, and familiarise them with different solid shapes and their nets.
- Sketch and draw more solid shapes.

Suggested teaching approach

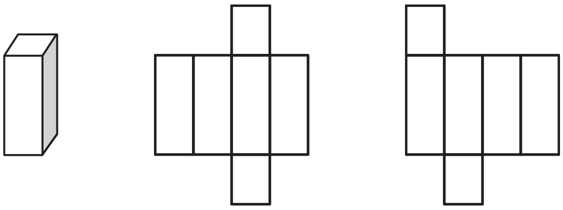
- Collect boxes or objects that are pyramids and prisms, and unfold them to show their nets.
- Assist learners to draw nets of other prisms, pyramids and other solids.
- Identify nets of other solids and 3D shapes and draw their nets.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

Boxes and containers come in many different shapes and sizes. Boxes can be made by drawing an outline of the faces on a flat surface, and then folding the outline into the three-dimensional solid.

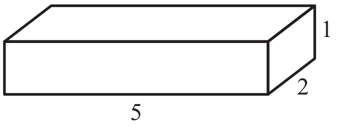


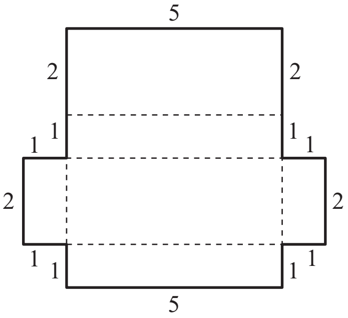
A net is the 2D outline of a solid that shows all of the faces. A net can be folded to make the 3D solid. Solids can have more than one net.



Examples

Draw a net for this cuboid (measurement in cm).

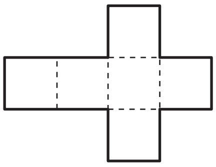


Thinking	Working
1 Count the number of faces that the cuboid has **2** Draw out the net of the cuboid using Darker lines **3** Use the dotted lines to draw in the lines that would be used to fold the shape with.	

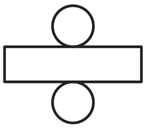
Activity 14B

1 Name the solids that have these nets.

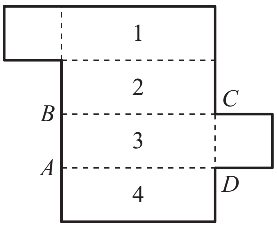
a

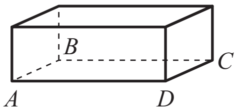


b



2 When this net is folded, it makes the cuboid shown here. The four rectangles on the net have been numbered.





a Give the number of the rectangle that ends up on:

i the bottom	**ii** the top
iii the front	**iv** the back.

b When the net is folded, two points end up directly above *A*. Mark these two points on the net with a cross.

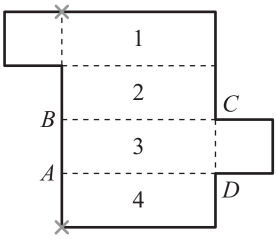
3 Draw a net that could be used to make each of the following solids.

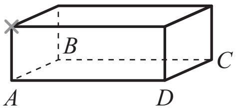
a a cone	**b** a triangular prism
c tetrahedron	**d** square-based pyramid

Answers 14B

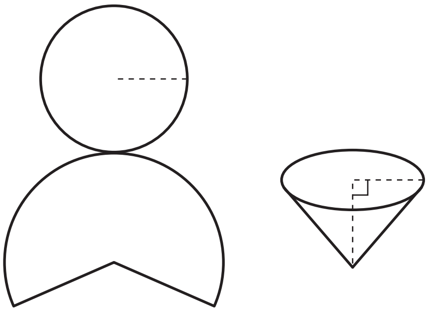
1 a cube	**b** cylinder
2 a i 3	**ii** 1
iii 4	**iv** 2

b

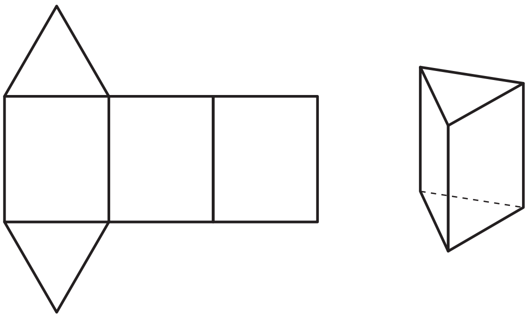




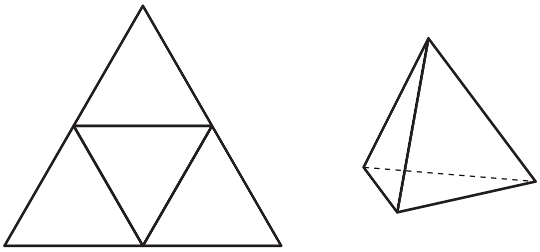
3 a



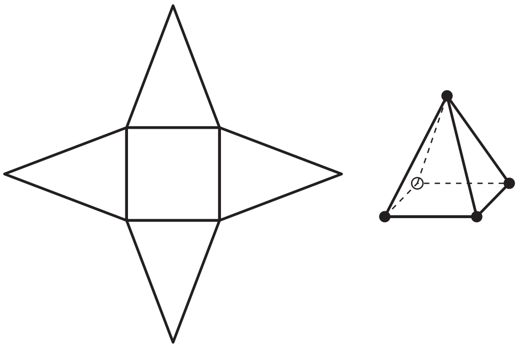
b



c



d



14C • Drawing 3D shapes

LB2 Pages 180–182

Specific learning outcomes

Learners should be able to:

8.14.5.1 Identify the lengths, width and heights of various solids.

8.14.6.1 Identify three views that can be produced by drawing cubes onto dotted paper.
Isometric drawings: Shows corners
Oblique drawings: shows a front view
Orthogonal drawings: Shows three separate views

8.14.6.2 Draw cubes and composite solids on dot paper.

Teaching points

1. Identify the lengths, widths and heights of various solids.
2. Distinguish between the three types of drawings for solid shapes using stacks of cubes: isometric, oblique and orthogonal.
3. Draw cubes and composite solids with 3D views.

Learner difficulties and remedies

Difficulty

Drawing 3D shapes on the 2D plane.

Remedy

- Use isometric dots to assist in drawing the 3D shapes on 2D planes.

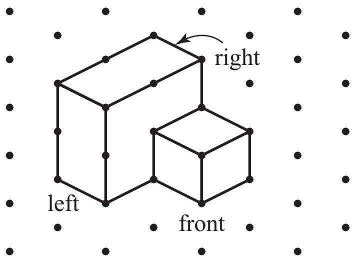
Suggested teaching approach

- Use isometric dots to construct and sketch three-dimensional solids as two-dimensional repesentations of the shapes.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A plan view shows the heights of different parts of a solid when viewed from above.

Isometric drawing



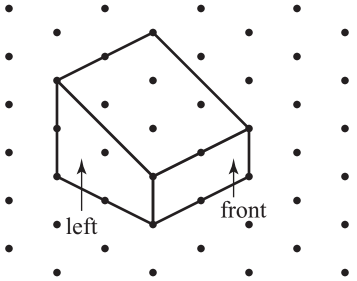
Plan view

2	2
	1

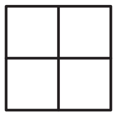
It is also possible to have views from the front, or either side (left or right). These show a profile of the shape, from one side only.

Isometric drawing

a

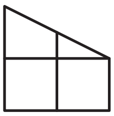


b



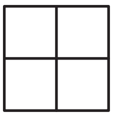
top

c



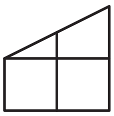
left

d



front

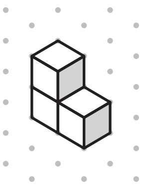
e

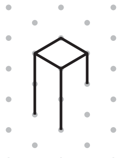
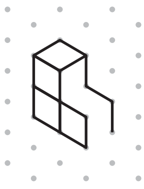
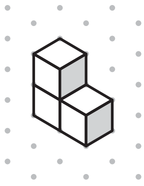


right

Examples

Use isometric dot paper to copy the following shape.

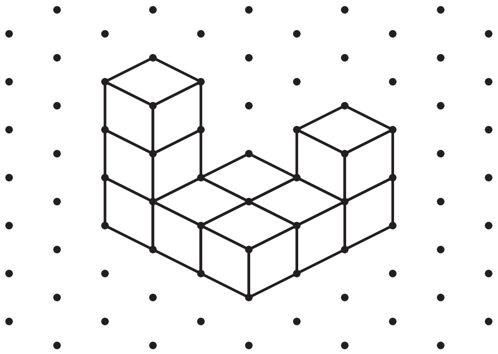


Thinking	Working
1 Join the points of a parallelogram that will represent the top face. Draw in all the necessary vertical lines from the corners of the face.	
2 Draw in all the lines from the bottom of the verticals you have just drawn in step 1 that represent horizontal lines. Draw in all additional required vertical lines from the ends of the lines you have just drawn.	
3 Join the ends of the lines you have just drawn. To create the illusion of 3D, choose one direction (front, side or top) and shade in all faces you would see from that direction.	

Activity 14C

1 Write numbers in each of the squares to show a plan view for this shape.

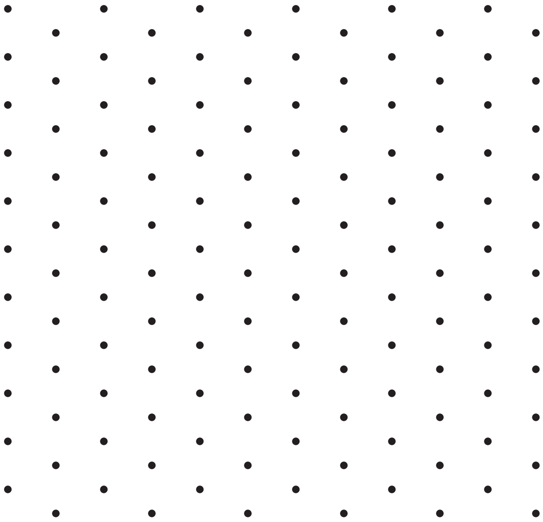
Isometric drawing



2 Here are some plan views of office buildings. Draw a sketch of each building on the isometric grids.

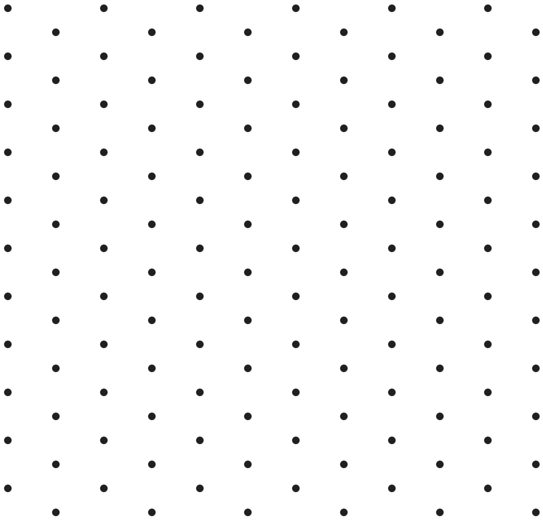
a

1	1	2	2
1	1	1	1

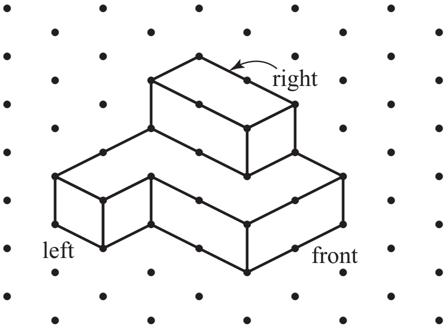


b

2	3	2
1	2	1
1	1	1

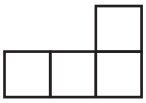


3 Look carefully at this shape.

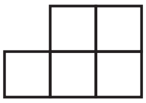


Here are four different views. Decide whether each view is from the top, front, left or right.

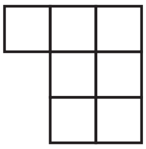
a



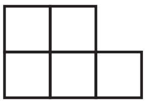
b



c



d

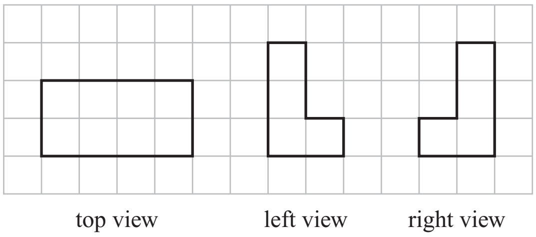


4 Draw the front, top, left and right views of the shape that fits this plan view.

3	1	1	1
1	1	1	2

<<insert Figure 14.24>>

5 A toddler has piled up a number of cubes. Here are three different views.



a It is impossible to tell exactly how many cubes have been used to make this shape. Name another view that would give you enough information to work out the number of cubes used.

b What is the smallest number of cubes that could have been used?

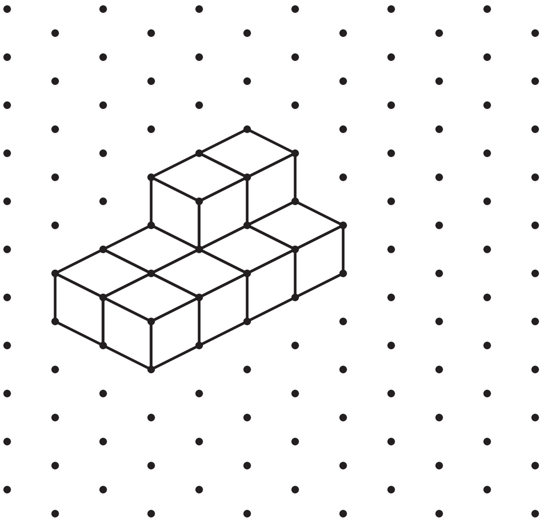
c What is the largest number of cubes that could have been used?

Answers 14C

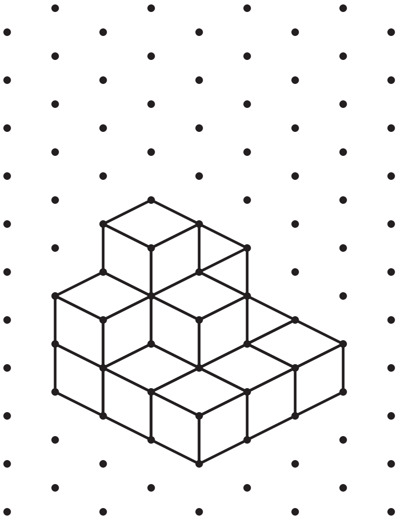
1

3		
1	1	
1	1	2

2 a



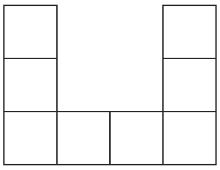
b



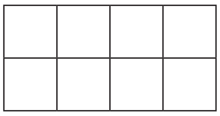
3 a front **b** right

c top **d** left

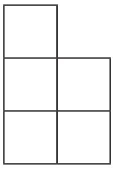
4 a front



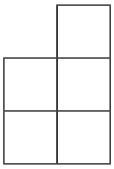
b top



c left



d right



5 a The front view is needed to see the middle two stacks of blocks.

b If the middle two stacks of blocks are one block high, then 12 blocks are needed.

c If the middle two stacks of blocks are three block high, then 16 blocks are needed.

14D • Perspective drawings

LB2 Pages 183–186

Specific learning outcomes

Learners should be able to:

8.14.7.1 Draw perspective drawings of solids using one- and two-point perspectives.

Teaching points

1. Show pictures that demonstrate objects in perspective.
2. Draw shapes and pictures using one- and two-point perspective drawings.

Suggested teaching approach

- Learners complete **Exercise 14D** on page 183 of the LB.

Activity 14D

Have the learners work outside the classroom and sketch a one or two-point perspective drawing of their classroom block. Present their results to the class and identify any vanishing points and centre lines in their picture.

14E • Polyhedra definitions

LB2 Pages 187–188

Specific learning outcomes

Learners should be able to:

8.14.3.2 Define the terms ‘polyhedron’ and ‘polyhedra’.

8.14.8.1 Identify the properties of polyhedra.

8.14.8.2 Identify and define different parts of polyhedron shapes: edge, vertex, face.

8.14.9.1 Name the five regular polyhedra called Platonic solids.

8.14.10.1 Define the term ‘planar network’.

8.14.10.2 Draw planar network for a cube.

8.14.11.1 Explain the term ‘order of vertex’.

8.14.11.2 State the order for each vertex and the total order for a network.

Teaching points

1. Explain that a polyhedron is any solid that has polygon faces.
2. Identify the different parts of a polyhedron: edge, vertex, and face.
3. Show models of the five regular polyhedron shapes, called Platonic solids.
4. Explain that planar network is a two-dimensional diagram of a three-dimensional object showing all vertices, edges and regions
5. Draw planar network for three-dimensional shapes.
6. Explain that an order of vertex is the total number of edges connecting at a vertex.
7. State the order for vertex and the total order for a given network.

Learner difficulties and remedies

Difficulty

Understanding the definition of a polyhedron.

Remedy

- Define and explain the term polyhedron (or polyhedra) using examples.

Difficulty

Understanding the concept of network and planar network.

Remedy

- Explain thoroughly network, planar network and order of vertex (lines or edges that touch a corner or point).
- Provide some examples and exercises on the order of a vertex.

Suggested teaching approach

- This concept is new to learners so it's important to explain the concept of planar network, especially the order of vertices in a network.
- Identify polyhedral shapes and name different parts of the shapes such as faces, edges and vertex.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A polyhedron is a three-dimensional shape with faces that are all polygons. All edges are straight lines. The plural of polyhedron is polyhedra.

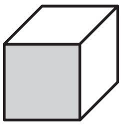
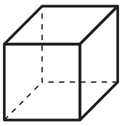
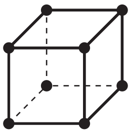
- Platonic solids are polyhedra with faces that are all identical regular polygons. There are only five Platonic solids: tetrahedron, cube, octahedron, dodecahedron, icosahedron.
- Archimedean solids consist of two or more regular polygons but are not prisms. There are 13 Archimedean solids.

Platonic solids can repeseted by 2D planar networks in which the edges do not cross. The planar networks can be analysed to determin the number of:

- faces or regions (don't forget to include the region outside of the lines of shape)
- edges (draw the network so tht the egdes do not cross)
- vertices

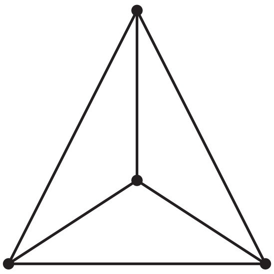
Examples

1 Show how many faces, edges and vertices on a cube.

Face	Edge	Vertex
The flat part of a solid.  A cube has 6 faces.	The line segment where two faces meet.  A cube has 12 edges.	A corner where several edges and faces meet.  A cube has 8 vertices.

2 Draw a planar network of a tetrahedron and then state the number of faces, edges and vertices.

A tetrahedon has 4 faces (or regions including the outside), 6 edges and 4 vertices.



Activity 14E

1 A regular polyhedron is one in which all the faces are the same shape and size.

Complete these sentences. Use four of the words from this list: {square, tetrahedron, rectangle, equilateral triangle, cube, rhombus}.

A __________________ has 6 faces, each one of which is a(n) _______________. A __________________ has 4 faces, each one of which is a(n) _______________.

2 Complete this table to show the number of faces, edges and vertices for each shape.

Name of solid	Number of faces	Number of edges	Number of vertices
Cuboid			
Pentagonal pyramid			
Hexagonal prism			

3 Name a solid that has 5 faces, 8 edges and 5 vertices.

4 A cylinder has 3 faces, 2 edges and no vertices.

a Write down the number of faces, edges and vertices for a cone.

Faces: ___________________

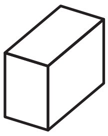
Edges: ___________________

Vertices: ___________________

b Are cylinders and cones polyhedra?

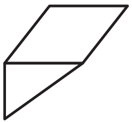
5 Copy and complete these diagrams to show the hidden edges.

a



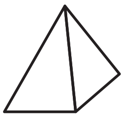
cuboid

b



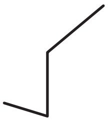
triangular prism

c

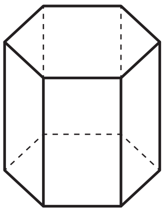


pyramid

6 Quinta has started to draw a cuboid. She was interrupted after drawing three edges. Copy the diagram and complete it to show where the other nine edges would go, including the hidden ones.



7 This hexagonal prism (cross-section has 6 edges) has 8 faces. Is is possible to paint it with fewer than 8 colours so that no two adjoining faces are the same colour? Explain.



Answers 14E

1 A cube has 6 faces, each one of which is a square.
A tetrahedron has 4 faces, each one of which is a equilateral triangle.

2

Name of solid	Number of faces	Number of edges	Number of vertices
Cuboid	6	12	8
Pentagonal pyramid	6	10	6
Hexagonal prism	8	18	12

3 square-based pyramid

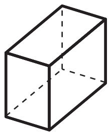
4 **a** Faces: 2

Edges: 1

Vertices: 0

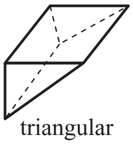
b Cylinders and cones are not polyhedral because their faces are not all polygons.

5 **a**



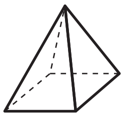
cuboid

b



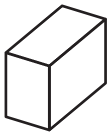
triangular prism

c



pyramid

6 **a**



7 The mininum number of colours is three. The maximum number of faces that meet at any vertex is three.

14F • Networks and their applications

LB2 Pages 189–191

Specific learning outcomes

Learners should be able to:

8.14.12.1 Define the following terms: network, vertex (node), edge (path), traversal and region.

8.14.13.1 Identify network vertices that have odd or even degrees.

8.14.13.2 Find the shortest path for networks given as diagrams or maps.

8.14.13.3 Draw and construct networks with given vertices and edges.

Teaching points

1 Define the following terms with some real-life examples: network, vertices (nodes), edges (path), transversal and region

2 Find the number of degrees for a network and state whether a vertex has **odd** or **even** degrees.

3 Find the routes for given diagrams or maps.

4 Draw or construct a network with given vertices and edges.

Learner difficulties and remedies

Difficulty

Understanding a network with nodes, paths and regions.

Remedy

- Explain the different parts of a network and assist learners in reading and constructing networks.

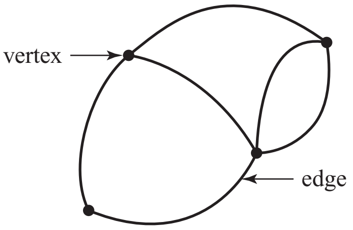
Suggested teaching approach

- Explain and define the terms network, vertices (nodes), edges (path), transversal and region.
- Also explain the concepts of odd and even degrees in a network.
- Sketch and draw a variety of networks and maps.
- Use the **Learner difficulties and remedies** section to assist you in teaching this unit. It highlights the difficulties that some learners face, and gives you ideas about how to rectify these problems.
- Some **Examples** follow and an **Activity** is provided at the end of this unit. Use these if learners require additional material.

Additional notes

A **network** diagram represents a number of possible pathways between different points. Sometimes the 'pathways' are called **edges** and the 'points' are called **services.**

This network has 6 edges and 4 vertices.



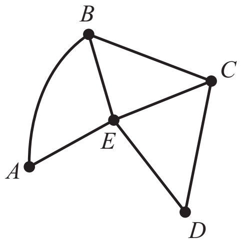
An **odd** vertex has an odd number of edges connected.
An **even** vertex has an even number of edges connected.

The degree of a vertex is the number of edges connected to it.

C is an odd vertex.

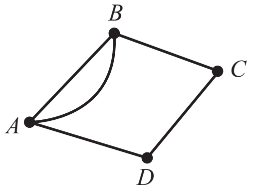
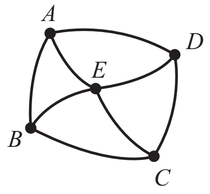
E is an even vertex.

The degree of *D* is 2.



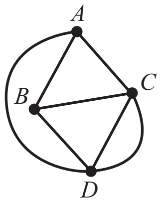
A network is **traversable** if each edge can be traced, once only, without lifting pen from paper. In some networks it is also possible to start and finish at the same vertex.

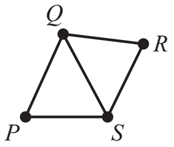
Examples

This network is traversable. One possible path is *A* to *B* to *C* to *D* to *A* to *B*. However, it is not possible start and finish at the same vertex.	
This network is *not* traversable.	

Activity 14F

1





a Make a list of the even vertices in these networks.

b Make a list of the odd vertices in these networks.

2 Draw a network that has:

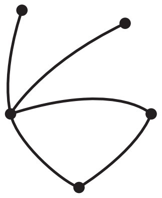
a 2 odd and 1 even vertices

b all odd vertices

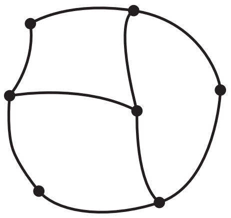
c all even vertices

3 Decide whether these networks are traversable. Answer yes or no.

a

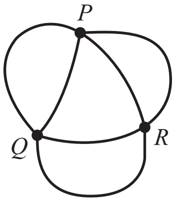


b



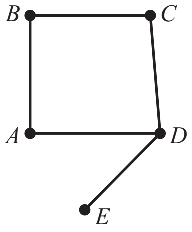
________________ ________________

4 This network can be traversed so that you start and finish at the same vertex. Places labels 1–6 on the six edges to show how this can be done.

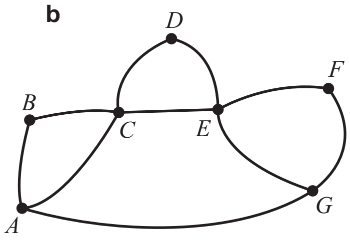


5 The networks drawn here are traversable. Write down the two points at which you must start and finish to traverse them.

a



b



________________ ________________

Answers 14F

1 **a** Even: *C*, *D*, *P*, *R*

b Odd: *A*, *B*, *Q*, *S*

2 Learners' own diagrams.

a Network should have 2 odd and 1 even vertices.

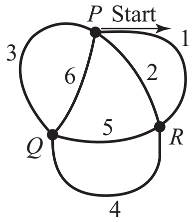
b Network should have all odd vertices.

c Network should have all even vertices.

3 **a** yes

b no

4



5 **a** *D* and *E*

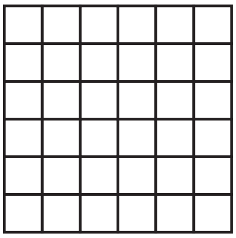
b *A* and *G*

• Chapter 1 Test •

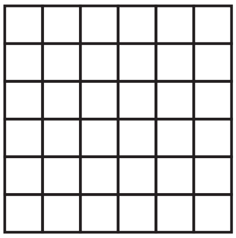
Percentages

1 Shade these grids to show the given percentages.

a 75%



b 40%



2 Write the given percentages in order from largest to smallest.

53%, 7%, 49%, 17%, 70%

____

3 Write the following percentages as fractions. Simplify if possible.

a 17% ____________

b $12\frac{1}{2}\%$ ____________

c $2\frac{1}{2}\%$ ____________

4 Change the following fractions to percentages, correct to two decimal places if necessary.

a $\frac{3}{25}$ ____________

b $\frac{5}{6}$ ____________

5 Change these percentages to decimals.

a 29% ____________

b 69·2% ____________

6 Change these decimals to percentages.

a 0·01 ____________

b 0·653 ____________

7 Work out the following.

a 32% of 600

b 5% of 200

8 Calculate the following increases as percentages.

a 60 is increased to 90

b 35 is increased to 42

9 Calculate the following decreases as percentages.

a 40 is decreased to 38

b If 400 is decreased to 150

10 A Year 8 class has 23% of learners from Temotu Province. What percentage are from a province other than Temotu Province?

11 The blood types of learners in a school were recorded. Write these percentages as decimals.

a 39% have type O blood ____________

b 2% have type AB blood ____________

12 A fence that is 1·5 m high is raised to a new height of 1·8 m.

a Calculate the increased distance in metres.

b What is the percentage increase in height?

13 A chocolate cake originally selling at $350 was discounted by 25%. What was the amount of the discount in dollars?

14 A watch worth $155 was discounted to $42.

a What was the amount of the discount in dollars?

b What was the discount as a percentage?

15 MV Maetalau sailed to Auki with 200 passengers. 30% were females. 20 passengers did not pay their fares. 40% of those who did not pay their fares were females. How many females did not pay their fares?

Answers

<Answers to come at 1pp>

Length and Perimeter

1 Convert the following lengths of metres (m) to millimetres (mm).

a 80 m ____________

b 1350 m ____________

2 Convert these measurements from centimetres (cm) to metres (m).

a 120 cm ____________

b 14 900 cm ____________

c 4 cm ____________

d 0·91 cm ____________

3 Change the following as specified.

a 3 m to cm ____________

b 60 cm to mm ____________

c 10 000 cm to km ____________

d 8300 mm to km ____________

4 Give the answers to the following in metres.

a 6 km + 700 m

b 1 m + 30 cm

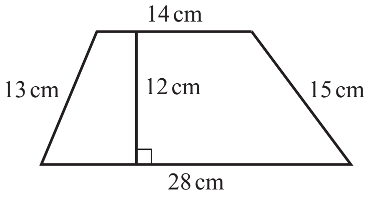
c 3300 m + 2·7 km

5 The height of the Panatina Pavilion is 35 m. What is the height in km?

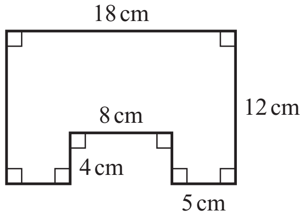
6 A piece of wood measuring 3·6 m is cut into four equal lengths. What is the length of each piece in mm?

7 Calculate the perimeter of each of these figures.

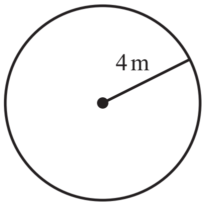
a



b



8 Calculate the following for the circle given below.

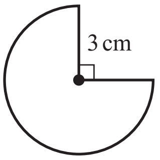


a diameter

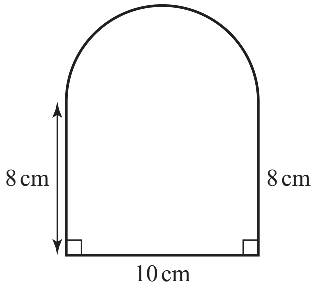
b circumference

9 Find the perimeter of the given composite shapes.

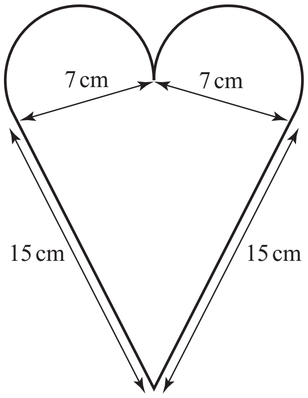
a



b



10 A box of chocolates is shaped like a heart as shown. Find the perimeter of the box correct to the nearest mm.



Answers

<Answers to come at 1pp>

• Chapter 3 Test •

Polygons and Parallel Lines

1 Define the following angles.

a alternate angles

b corresponding angles

c co-interior angles

d vertically opposite angles

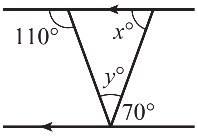
2 Explain the following terms.

a complementary angles

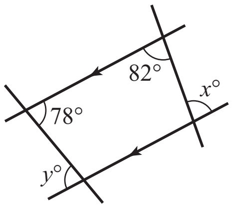
b supplementary angles

3 Find the size of the unknown marked angles

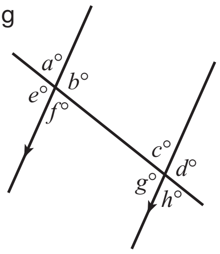
a



b

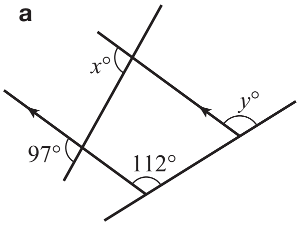


4 Which angle is corresponding to angle *d*°?

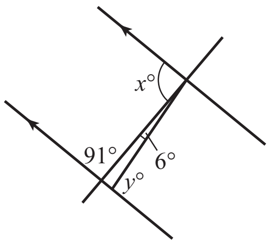


5 Find the sizes of the angles marked *x*° and *y*° in each diagram.

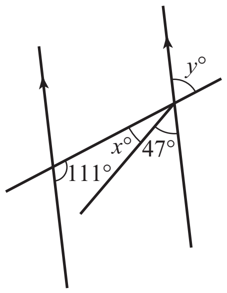
a



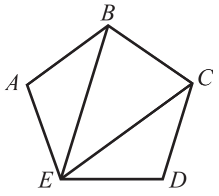
b



c



6 *ABCDE* is a regular pentagon. It can be split into three isosceles triangles by drawing in two diagonals.

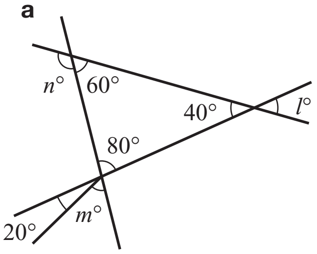


a Name the acute triangle(s).

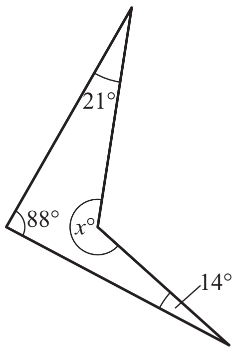
b Name the obtuse triangle(s).

7 Calculate the missing angles that are marked with the alphabetical letters.

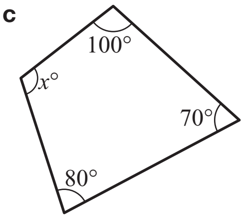
a



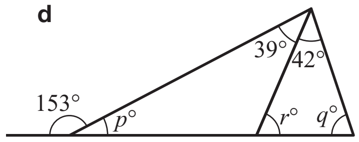
b



c

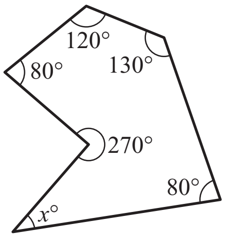


d



8 Name any three (3) properties of a quadrilateral.

9 Use the formula for the angle sum of a polygon to find the missing angle.



Answers

<Answers to come at 1pp>

• Chapter 4 Test •

Algebra Symbols

1 Write down the coefficient of x in each the following.

a $6x - 2$

b $\frac{x}{2} + 4$

2 Simplify the following expressions.

a $2t + 4p$

b $3q \times r$

c $24r \div 8y$

3 Write each of the following statements using mathematical symbols.

a add 4 to x times 12

b 3 less than twice x

4 Write an algebraic expression for each of the following statements.

Yvonne has x dollars in her wallet. How much would she have if she:

a spent $30

b received $60

c spent half of her money.

5 Collect like terms and simplify.

a $x + 6x - 3x$

b $p + 2p + 4p$

6 Simplify the following.

a $5k - 3y + 2k - 6y + k$

b $^{-}11x - 7 + 8x - 5x + 1$

7 Collect like terms and simplify.

a $4pq - 3qp + pq$

b $3ab - 2bc + 7ac + 7ba + 4ca - cb$

8 Simplify each of the following.

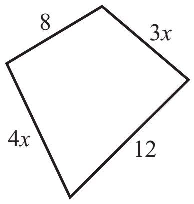
a $x^2 + 8x + 2x^2$

b $3x + 5x^2 - 11x + 2x^2 - x$

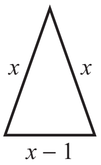
c $4x^2 + 5x - 3 - 5x^2 + x + 2$

9 Write an expression for the perimeter of the given shape.

a



b



10 Express the following as simply as possible.

a $30x$ $^{-}5$ <please check this line>

b $20 \times {}^{-}x \div {}^{-}2$

c $3xy \times {}^{-}8x \div 18y$

11 Expand and simplify.

a $18x - 3(2x + 7)$

b $6 - (x - 2)$

c $3x + 2(1 - 2x)$

12 Expand and simplify each of these expressions.

a $5(x + 3) + 2(x + 6)$

b $^{-}2(3x - 1) + 3(x - 5)$

c $3(2x + 5) - 2(x + 1)$

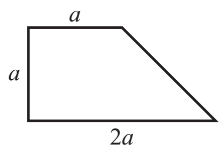
13 Evaluate the given expressions if $x = 3$, $y = 4$ and $z = {}^{-}2$.

a $5x - 7y + 4z$

b $x^2 + y^2$

c $\frac{2x + y}{5}$

14 **a** Add two lines to this shape to show how it can be split into three identical parts.



b Use the formula for the area of a triangle ($A = \frac{1}{2} \times \text{base} \times \text{height}$) to write down an expression for each of the parts.

c Write down an expression for the area of the whole shape.

Answers

<Answers to come at 1pp>

• Chapter 5 Test •

Ratios and Rates

1 Define the following terms.

a ratios

____

b rates

____

2 Write these ratios in their simplest form.

a 6 : 10 ____________

b 65 : 195 ____________

c 250 : 350 ____________

3 Write these ratios in their simplest form.

a \$45 : \$30 ____________

b 20 kg : 30 kg ____________

c 8 hours : 12 hours ____________

4 Simplify the following ratios.

a 2 m : 250 cm

b 4 km : 3500 m

c $\frac{1}{3} : \frac{1}{2}$

d $4 : \frac{2}{5}$

e $\frac{1}{2} : 2\frac{1}{3}$

5 In a school the ratio of boys to girls is 8 : 7. If there are 100 boys at the school, how many girls are there?

6 An ice-cream recipe mixes cream and milk in the ratio 2 : 5 before adding flavouring.

a How much milk is needed if 6 litres of cream is used?

b How much cream is needed if 12 litres of milk is used?

7 If 80 tomato seeds are planted, then the ratio of the seeds that germinate to the seeds that do not germinate is 7 : 13. How many seeds germinate?

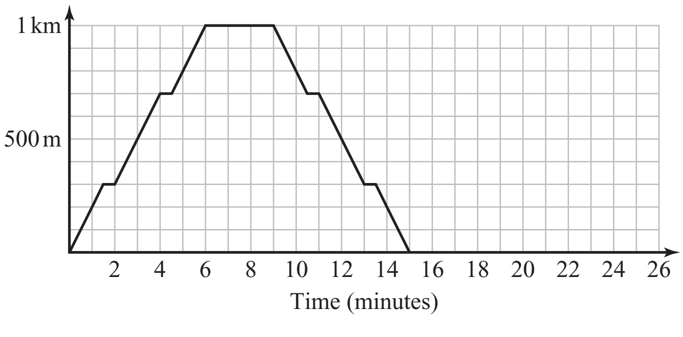
8 A class decides to buy maths teacher a gift at the end of the year. Girls and boys contributed in the ratio 5 : 1. If the gift cost \$180, how much did the boys contribute?

9 Bruce, Cathy and Philip paid \$7, \$10 and \$8 respectively for a raffle ticket. The ticket won a prize of \$600, which must be shared among the three according to the amount spent on the tickets. How much would Bruce receive?

10 A cyclist covers 10 km in half an hour. What is the average speed in km/h?

11 Harry runs for half an hour at an average speed of 16 km/h and then walked for 2 hours at 5 km/h. What was the total distance covered?

12 Peter walked to a park 1 km from his home. He then rested for 3 minutes before walking home. The graph below represents his journey.



a How long did after leaving home did Peter reach the park?

b For how long did Peter stop each time when walking to the park?

c How long did it take Peter to complete the journey and return home?

13 A council plan is drawn to a scale of 1 : 100. Find the real length in metres for each of the following diagram lengths.

a 23 mm

b 40·3 cm

c 1·2 m

14 The scale ratio for a housing estate plan is 1 : 900. A rectangular block measures 3 cm by 2·4 cm on the plan.

a Find the area of the block on the plan in cm^2.

b Find the actual dimensions of the block in cm.

Answers

<Answers to come at 1pp>

• Chapter 6 Test •

Statistics

1 Define the following terms and state the differences and similarities between them.

a discrete data

b continuous data

2 Give any two properties of a column graph.

3 Define the following terms.

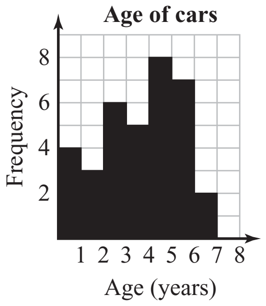
a range

b median

c mean

4 Identify any two properties of stem-and-leaf plot.

5 Complete the frequency table for the given graph.

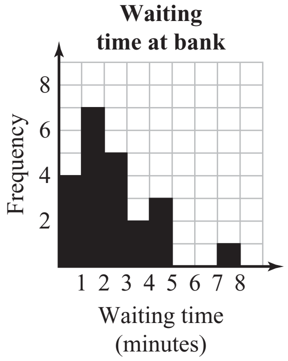


Age (years)	Frequency

6 Draw a frequency column graph to represent the results using a suitable scale.

Number of goals scored	Frequency
10	4
11	3
12	6
13	3
14	2
15	0
16	4

7 Pita carried out a survey to find out how long customers waited in a queue at a bank. This graph shows the results.



a How many customers had to wait less than a minute?

b How many customers had to wait between 3 and 6 minutes?

c How many customers were surveyed altogether?

8 The following frequency table shows the times taken, in minutes, by learners at a particular school to complete a cross-country race.

Time (min)	Frequency
20–<22	2
22–<24	12
24–<26	27
26–<28	15
28–<30	8
30–<32	5
32–<34	1

a Calculate the mean time taken by the learners to complete the cross-country race.

b Draw a histogram to represent the results.

c Five learners did not complete the race within the time limit. How would their results affect the mean time taken?

d How might you record their times in order to include them in the histogram?

9 The times (in seconds) taken by 14 athletes to run 400 m are shown below.

59, 66, 61, 66, 60, 61, 71, 59, 58, 70, 64, 69, 63, 63

a Calculate the range of these times.

b What is the lower quartile and the upper quartile?

c Calulate the interquartile range.

10 This stem-and-leaf plot shows a set of pulse rates taken from a class of 32 learners after a 5-minute 'step' test. The figures were taken over a 1-minute period as soon as the test finished.

Stem	Leaf
7	8
8	2 5
9	6 7
10	0 4 6 6 9
11	1 2 2 8
12	2 5 6 7 7 9
13	0 0 2 5 5 5 6
14	3 4 7
15	
16	2 4

Calculate each of the following for the set of data.

a mode ______________

b median ______________

c range ______________

d lower quartile ______________

e upper quartile ______________

f interquartile range ______________

Answers

<Answers to come at 1pp>

• Chapter 7 Test •

Transformations

1 Define the term transformation.

____

2 Name the four types of transformations.

____

____

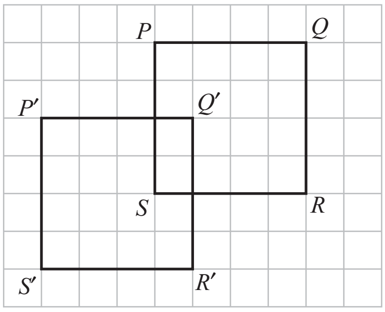
____

____

____

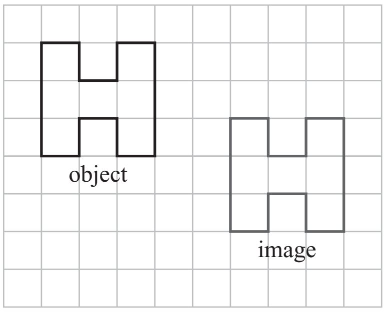
3 Describe each translation below giving the direction and the number of units.

a



____

b

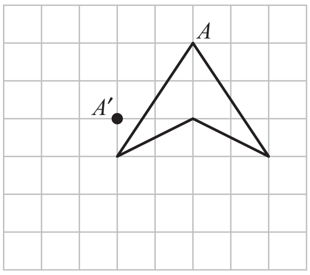


____

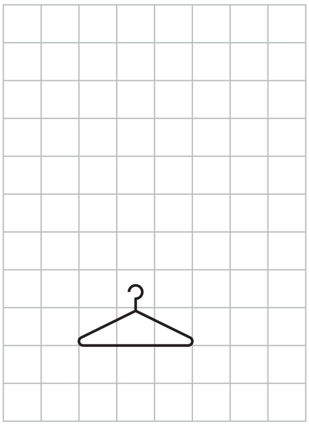
4 **a** Describe the translation of point *A* to *A′* in the given diagram.

____

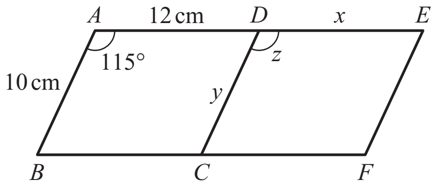
b Draw the image of the shape translated below.



5 Translate the coat hanger 1 unit to the left and 4 units up.



6 The diagram shows a translation in which *A* moves to *D*.



a Which point is the image of *B*? ______________

b Which point is the image of *D*? ______________

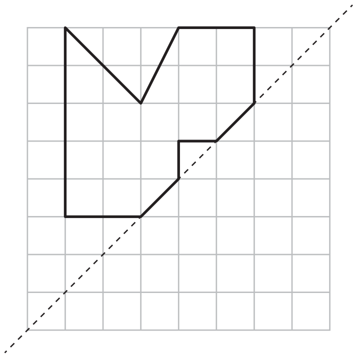
c Write down the values of *x*, *y* and *z*? ______________

____

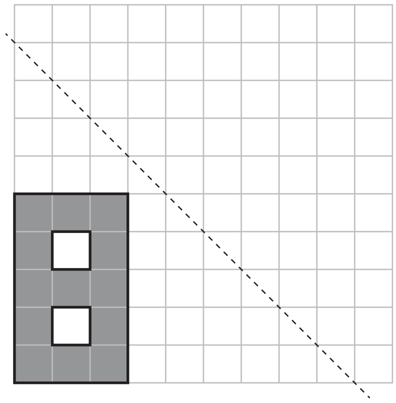
____

7 Draw the image of each shape when reflected in the mirror line.

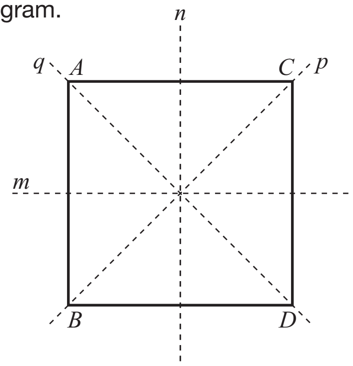
a



b



8 Consider the following diagram.



a How many mirror lines are the diagram?

____

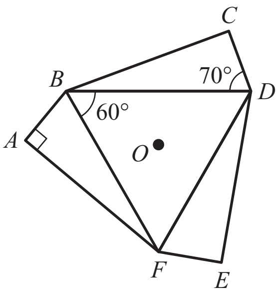
b What is the image of *Q* when reflected in *q*?

____

c If the image of *A* is *C*, then in which line was it reflected?

____

9 This shape maps onto itself when turned through 120°.



a What is the centre of rotation?

____

b Write down the image of these points when the shape is rotated through 120° anticlockwise.

i *C* ______________

ii *F* ______________

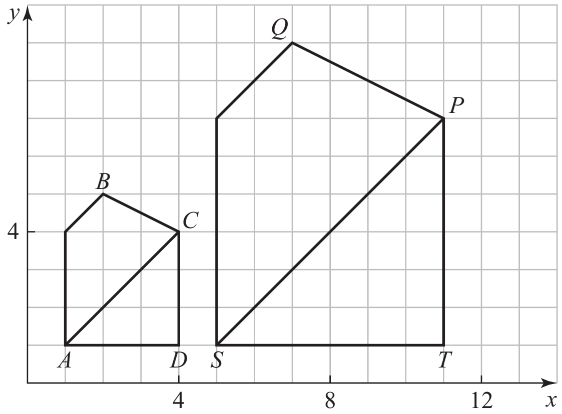
c What are the other two angles through which it will rotate onto itself?

____

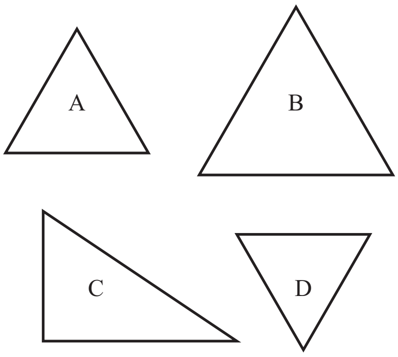
____

____

10 Find the scale factor of the enlargement given below.



11 Which of the following shapes are congruent?



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Answers

<Answers to come at 1pp>

• Chapter 8 Test •

Equations and Inequations

1 Solve the following equations by inspection.

a $16 - x = 7$ **b** $^{-}11y = ^{-}88$

c $-\frac{x}{4} = 12$ **d** $\frac{48}{x} = ^{-}24$

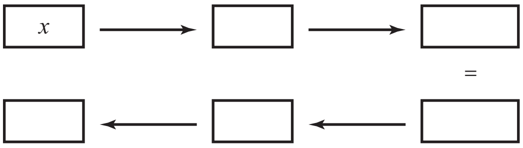
2 Write the equation for the following statement. (You don't have to solve it.)

When 8 is subtracted from a number the result is $^{-}19$.

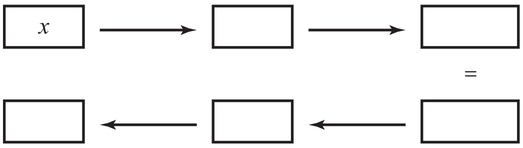
____

3 Solve these equations by writing in the correct symbols in each flow chart and reversing the flow.

a $10x - 2 = ^{-}12$



b $2x + 19 = ^{-}11$



4 Solve each of the following equations.

a $^{-}6x = ^{-}12$ **b** $^{-}x = 14$

c $\frac{x}{2} = ^{-}12$ **d** $-\frac{x}{3} = 9$

5 Solve the following.

a $10x + 6 = ^{-}74$ **b** $^{-}x + 8 = 13$

6 Find the value of the pronumerals.

a $\frac{3x}{4} = ^{-}6$ **b** $\frac{x}{4} - 9 = ^{-}7$

7 Solve the given equations.

a $4(x - 2) = 12$ **b** $6x(3x + 1) = 42$

c $4(x + 1) - 2(x - 3) = 12$

8 The cost of a torch is $12. The torch costs $8 more than the batteries. How much do the batteries cost?

9 If x is multiplied by $^{-}8$ and 15 is added, then the result is $^{-}1$. Find the value of x.

10 Tevita took 39 eggs to make Christmas cakes. He made four (4) cakes, each with the same number of eggs. Three (3) eggs were not used because they were rotten.

a Circle the equation that represents the number of eggs used for each cake.

A $3x + 4 = 39$ **B** $4x + 3 = 39$

C $4x - 3 = 39$ **D** $3x - 4 = 39$

b Solve the equation to find the number of eggs in each cake.

11 Gladys has n CDs. She then gives away 5 CDs to her friends. When she doubles the number of her remaining CDs, she now has 48. How many CDs did Gladys have originally?

12 Solve these inequations and represent the answer on a number line.

a $^{-}3x < 12$

b $^{-}2x \geq ^{-}6$

13 Solve the following inequations.

a $5x < {}^-15$

b $x + 4 > 10$

c $\frac{x}{2} \geq 6$

d $3x + 8 \geq 2$

e $\frac{4x}{7} \leq {}^-8$

f $2(3x - 4) \leq {}^-14$

g ${}^-2x + 1 > {}^-5$

h $30 - 4x \leq 18$

i $3 - \frac{x}{2} < {}^-4$

j $2(3 - x) < 6$

Answers

<Answers to come at 1pp>

• Chapter 9 Test •

Indices

1 Write the following in index form.

a $8 \times 8 \times 8 \times 8$ ____________

b $^-3 \times {}^-3 \times {}^-3 \times {}^-3 \times {}^-3$ ____________

2 Express the followings in expanded form then evaluate.

a 9^4 **b** 6^5

3 Evaluate:

a $(^-7)^4$ **b** $^-(^-9)^3$

4 Solve each equation to find the value of *x*.

a $8^x = 512$ **b** $2^x = 32$

c $10^x = 1\,000\,000$ **d** $(^-4)^x = 16$

5 Simplify each of the following.

a w^1 ____________ **b** p^0 ____________

6 Simplify each of the following.

a $4 \times 2r \times r$ **b** $5t \times y \times 3t \times 2y$

7 Simplify these expressions.

a $p^7 \times p \times p^4$ **b** $d^5 \times d \times d^3 \times d$

8 Simplify these expressions.

a $5k \times 6tk^2$ **b** $12p^3 \times 8r^2p^2$

c $\frac{p^{12}}{p^3}$ **d** $y^8 \div y^5$

e $\frac{6x^7}{3x^5}$ **f** $\frac{2x^9}{x^3}$

9 Expand and simplify the following.

a $(3x^2)^4$ **b** $(3x^2y^3)^2$

c $\frac{(2x^3)^4}{(4x)^2}$ **d** $\frac{(6x^2y^3)^2}{(2xy)^3}$

10 Write each number in standard form.

a 36·2 **b** 600

c 135·5 **d** 617 000

e 0·0045 **f** 0·011 03

g 0·000 000 007 81

11 Write the following as numbers.

a $3{\cdot}6 \times 10^0$ **b** $6{\cdot}312 \times 10^2$

c $2{\cdot}07 \times 10^3$ **d** $1{\cdot}8 \times 10^{-3}$

e $9{\cdot}03 \times 10^{-2}$

12 A 1 kg bag of rice contains about 500 000 grains. Write this number in standard form.

Answers

<Answers to come at 1pp>

• Chapter 10 Test •

The Coordinate Plane

1 Plot these points on the set of Cartesian axes provided.

a (3, 2)

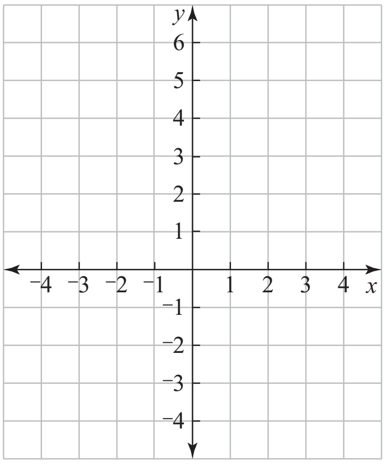
b (1, 6)

c (−3, 4)

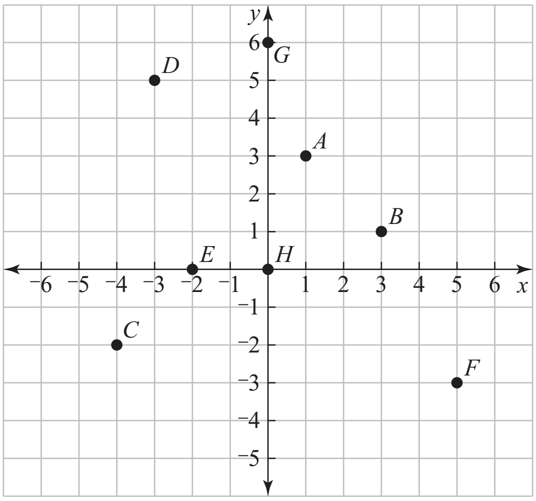
d (−4, −2)

e (3, 0)

f (0, −4)



2 Write down the coordinates of the points marked *A* to *H*.



A ____________ *B* ____________

C ____________ *D* ____________

E ____________ *F* ____________

G ____________ *H* ____________

3 Write down the rule that relates *x* to *y* for each set of points.

a ______________________

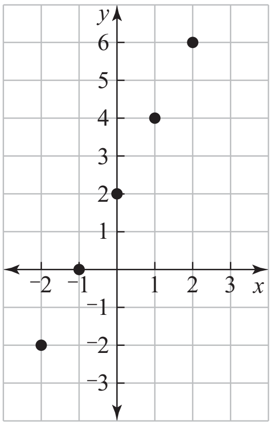
x	−2	−1	0	1	2
y	6	7	8	9	10

b ______________________

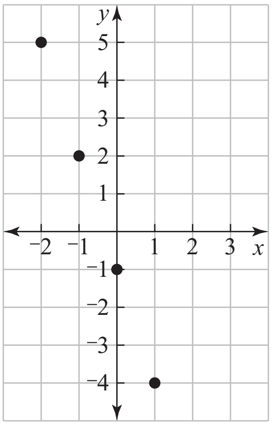
x	−2	−1	0	1	2
y	7	9	11	13	15

4 For each diagram make a list of the coordinates of the plotted points. Then write down the rule that relates *x* to *y*.

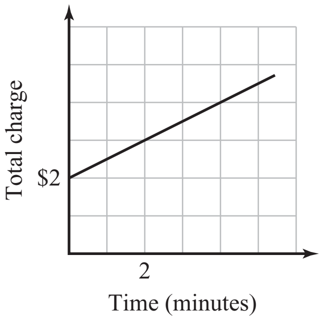
a



b



5 The graph below gives information about how the charge in dollars a phone call to Fiji is related to the number of minutes.



a What is the charge for a 4-minute call?

____

b Complete the statement: There is a fixed charge of

$ ____________ to start the call and then each minute

costs $____________.

c Write down the rule for this line in the form of $y = mx + c$.

____

d How much would it cost to make a phone call to Fiji for exactly 1 hour?

____

6 Complete this table using the rule $y = 2x + 3$.

x	**y (calculated from $y = 2x + 3$)**	**Coordinates (x, y)**
11		
22		
33		
44		
55		

7 Say whether these statements are true or false.

a The graph of $x = 3$ is a horizontal line. ____________

b The line $x = 2$ is parallel to the x-axis. ____________

c $y = 0$ is the same as the x-axis. ____________

8 Find the x-intercept for each equation.

a $y = 2x - 8$ **b** $y = {}^{-}x - 7$

c $2x - 3y = 6$ **d** $x + 2y - 4 = 0$

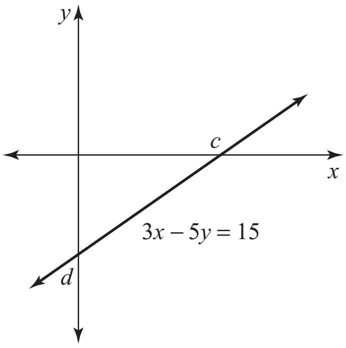
9 Find the x- and y-intercepts for each equation.

a $2x - y = 6$ **b** $2x + 5y = 10$

10 Write down the coordinate points for c and d in the following set of axes.

____

____



11 Find the gradient for the given equations.

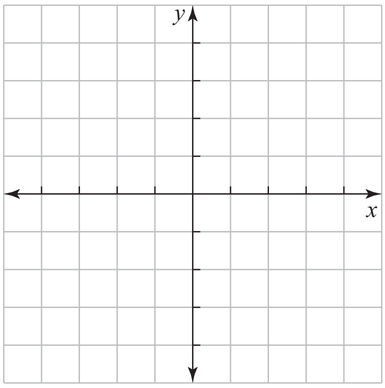
a $y = 3x - 1$ ____________

b $y = \frac{2x}{5} + 6$ ____________

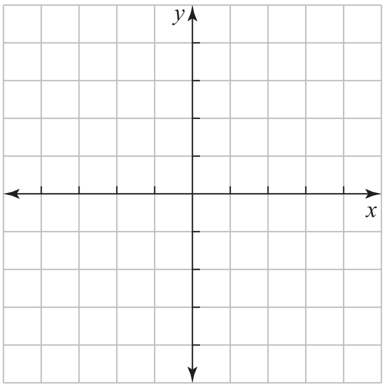
c $y = {}^{-}2x + 3$ ____________

12 Graph each of the given equations.

a $y = 2x - 1$



b $y = \frac{x}{2} + 2$



13 Write down the general rule for each line with the given gradient and y-intercept:

a $m = 2$, y-intercept $= 4$

____

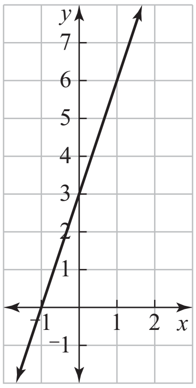
b $m = 6$, y-intercept $= {}^{-}1$

____

14 For each of the following line graphs write down the:

- **i** gradient
- **ii** y-intercept
- **iii** general rule

a

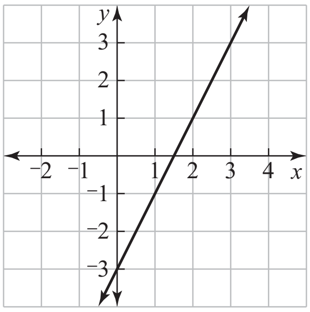


____

____

____

b



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____

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Answers

<Answers to come at 1pp>

• Chapter 11 Test •

Pythagoras

1 Fill in the missing word(s) in the following sentences.

a The longest side of a right-angled triangle is called

the ____________.

b For any ____________, the square length of the

____________is equal to the sum of the squares of the lengths of the two shorter sides.

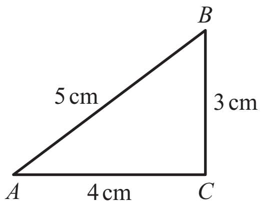
c The equation $c^2 = a^2 + b^2$ is known as

____

____

2 State whether each of the following is a right-angled triangle. Give reasons for each answer.

a



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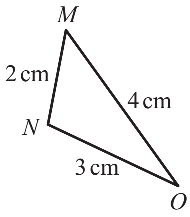
b



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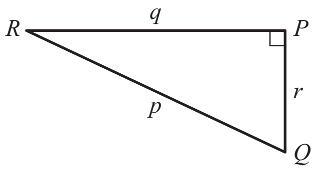
3 For each of the following triangles, select the correct statement of Pythagoras' theorem.

a



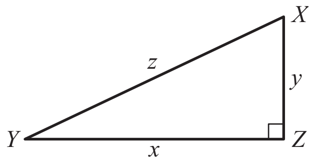
A $p^2 = q^2 + r^2$

B $q^2 = p^2 + r^2$

C $r^2 = p^2 + q^2$

D $q^2 - p^2 = r^2$

b



A $z^2 + y^2 = x^2$

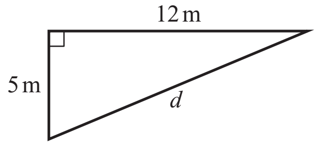
B $x^2 + z^2 = y^2$

C $x^2 - y^2 = z^2$

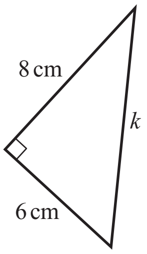
D $z^2 = x^2 + y^2$

4 Calculate the length of the hypotenuse in each of the following triangles.

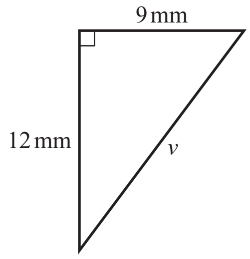
a



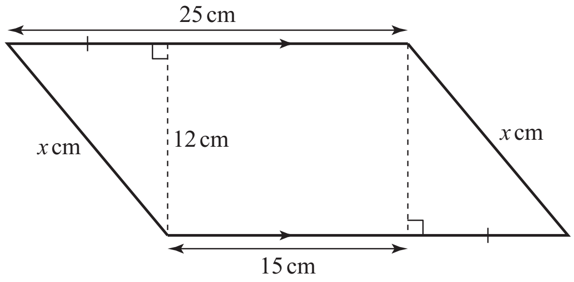
b



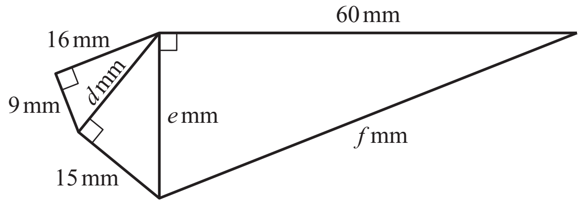
c



5 Find the value of x in the diagram.

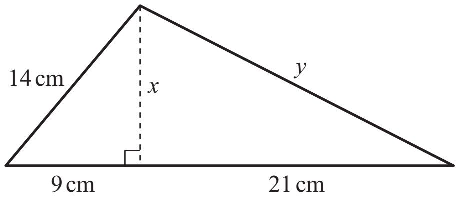


6 Find the exact value of the variables in the following diagrams. For any surd values, also give the answer correct to two decimal places.

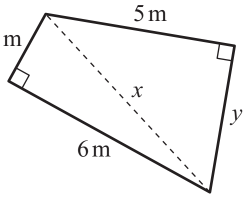


7 Find the exact values of the variables in the following diagrams.

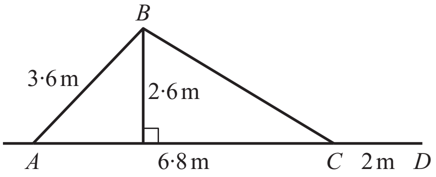
a



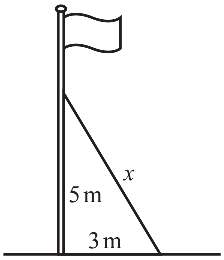
b



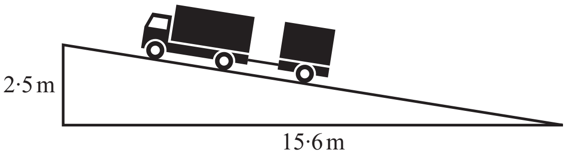
8 A playground slide is 2·6 m high. The ladder is 3·6 m long. The distance from the base of the slide, *C*, to the base of the ladder, *A*, is 6·8 m. The end of the slide, *CD*, is 2 m. Find the total length of the slide from *B* to *D*.



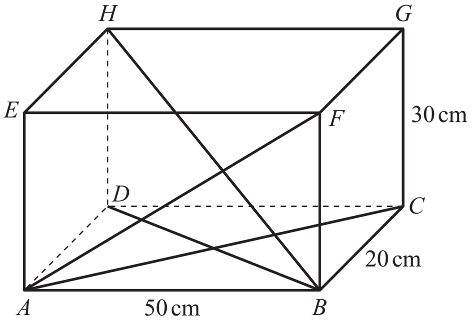
9 A support wire is attached 5 m up a flagpole. The other end of the wire is attached to the ground 3 m from the base of the flagpole. What is the length of the wire?



10 A ramp covers a horizontal distance of 15·6 m and a vertical distance of 2·5 m. What is the length of the ramp?



11 Find the length of the lines $\overline{AC}$, $\overline{AF}$, $\overline{BD}$ and $\overline{BH}$ in the following, expressing the answers in exact form.



Answers

<Answers to come at 1pp>

• Chapter 12 Test •

Probability

1 Define the following terms.

a sets

b universal set

c complement of a set

d empty or null set

e intersection of sets

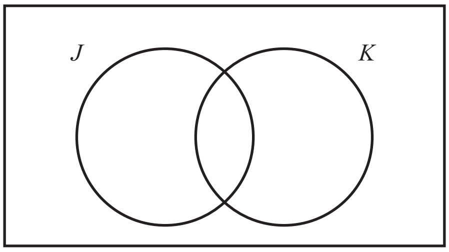
f union of sets

2 Place the numbers from the following sets in the Venn diagram.

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$J = \{2, 3, 5, 7, 11\}$

$K = \{1, 3, 5, 7, 9, 11\}$



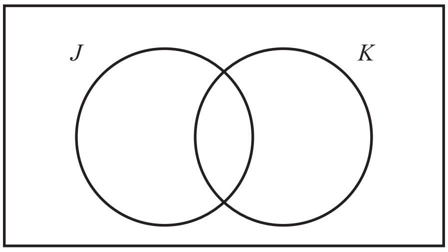
3 **a** Place the numbers from the following sets in the Venn diagram.

ξ = {numbers from 1 to 40}

J = {even numbers}

K = {factors of 40}

Note: Factors of 40 divide into 40 with no remainder.



b List the set K.

c Write down $n(K')$.

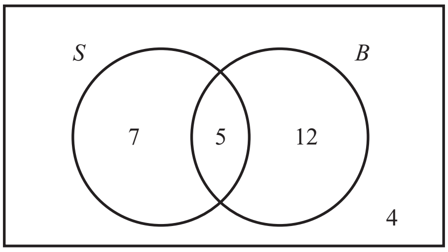
d List the set $J \cap K$.

4 A letter is chosen at random from the 26 letters of the alphabet. Calculate these probabilities that the letters letter chosen are in the following names.

a Pr(*KAKABONA*)

b Pr(*MAROVO*)

5 The Venn diagram below shows the number of learner in a class who have a brother (B) or a sister (S).



a How many learners are in the class?

b How many learners have a brother?

c How many learners have a sister but not a brother?

d What is the probability that a learner has a brother but not a sister?

e What is the probability that a learner has neither a brother nor a sister?

6 In a class of 32 learners, 12 are the oldest child in their family. What is the probability that a learner is **not** the oldest child in the family?

7 A survey had been carried on people who have travelled to and from Isabel Province during 2014. 35 had travelled by boat (B), 15 travelled by plane (P), and 12 travelled on both boat and plane.

a Represent this information in a Venn diagram.

b Find the following probabilities.

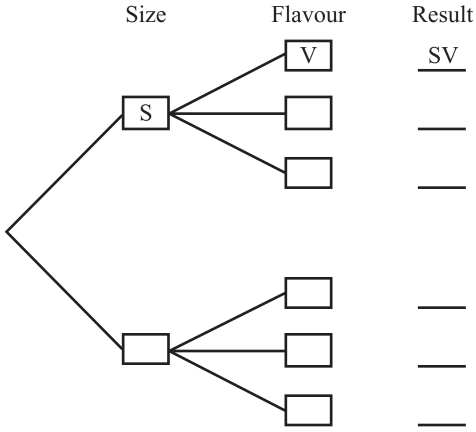
i Pr(B)

ii Pr($B \cap P$)

iii Pr(P')

8 The Frangipani Ice-Cream shop sells three (3) ice-cream flavours: vanilla, raspberry and butterscotch. They come in two (2) sizes: single and double.

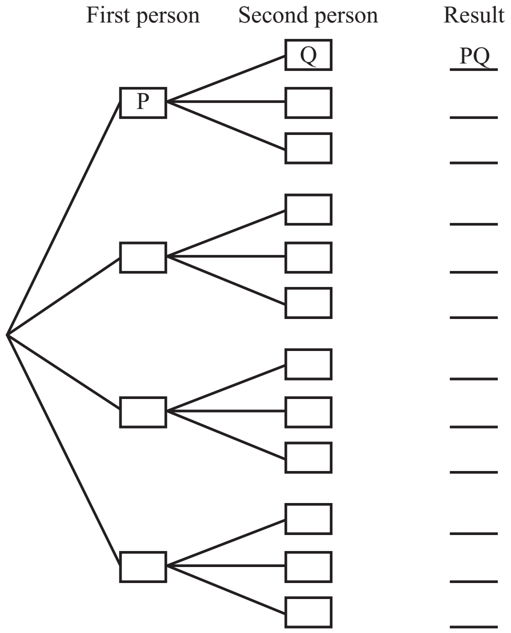
a Complete this tree diagram to show the different possibilities.



b How many different types of ice-creams can be ordered?

c Single size ice-creams can also be chocolate-dipped. Add this information to the tree diagram and find the number of different possibilities now.

9 Paul and Gideon (both boys) and Rosa and Gladys (both girls) have been given two tickets for a rock concert. They have to decide who goes by drawing lots. This tree diagram shows the different possibilities



a Complete the diagram.

b How many possibilities are there?

c How many of the possibilities results include Rosa?

d Explain, in terms of probability, whether is it likely or unlikely that exactly one boy and one girl end up going?

Answers

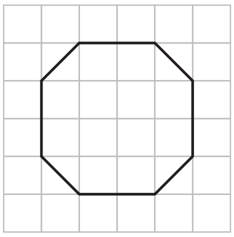
<Answers to come at 1pp>

• Chapter 13 Test •

Area and Volume

1 Work out the area of these shapes by counting squares and parts of squares. Each square represents 1 cm^2.

a



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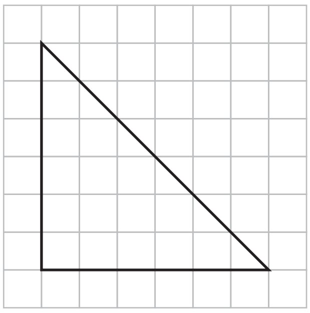
b



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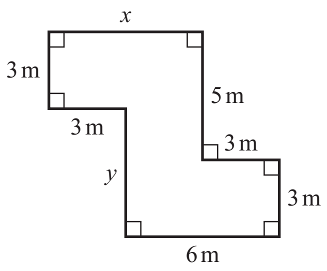
2

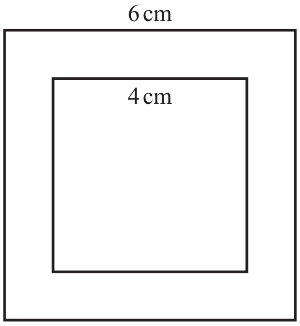


a Find the lengths of sides *x* and *y*?

b Add dashed lines to divide the shape into three rectangles.

c Calculate the area of the whole shape.

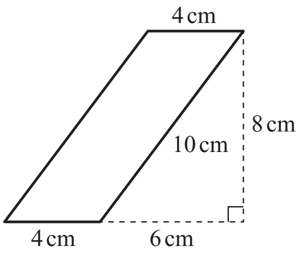
3 A frame has a square with sides of 4 cm removed from a square with sides of 6 cm. Calculate the area of the frame.



4 Work out the areas of these parallelograms. Choose the area units carefully; some will be in cm^2 and others in m^2.

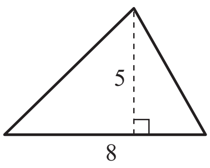
a

b

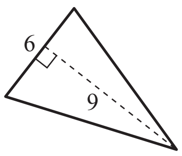


5 Calculate the areas of these triangles. All the measurements are in cm^2.

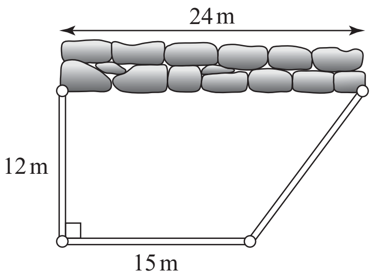
a



b



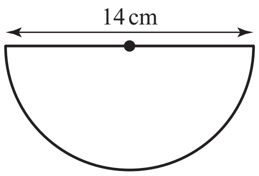
5 A farmer forms an enclosure by building a fence on three sides joining an existing stone wall. The enclosure is a trapezium.



a Calculate the area of the enclosure.

b If the perimeter of the enclosure is 66 m, calculate the length of the fence.

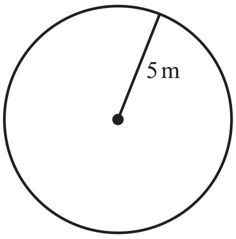
6 A semicircle is shown.



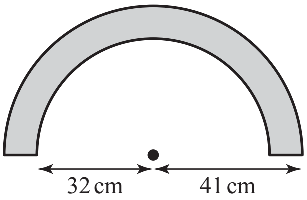
a What is the radius of this semicircle?

b Calculate the area of the semicircle.

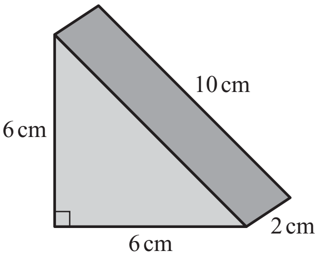
7 Calculate the area of the circle. Give your answer correct to the nearest m^2.



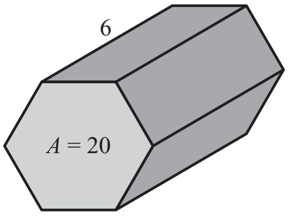
8 Semicircular concrete tree-rings are made with an inside radius of 32 cm and an outside radius of 41 cm. What are of ground does one of these rings cover?



9 Calculate the surface area of the given shape.

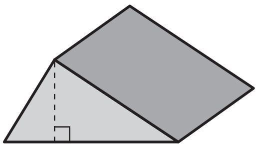


10 Calculate the volume of this prism. The lengths are in cm, and the cross-section area is in cm^2.



11 A prism has a triangular cross-section, with a base of 6 cm and a height of 4 cm. The length of the prism from end to end is 8 cm.

a Add three measurements in the correct positions to this diagram.



b Calculate the volume of the prism.

Answers

<Answers to come at 1pp>

• Chapter 14 Test •

Polyhedra and Networks

1 Define the following terms.

a polyhedron

____

b edge

____

c vertex

____

d face

____

e transverse

____

f network

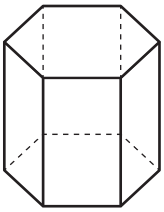
____

2 Explain the term traversable network.

____

3 Name the solid shapes given below.

a



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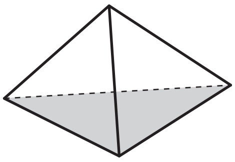
b



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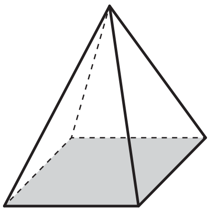
c



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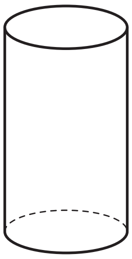
d



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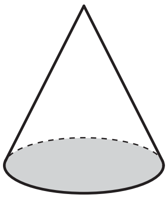
e



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4 What shape would make a good model for these objects?

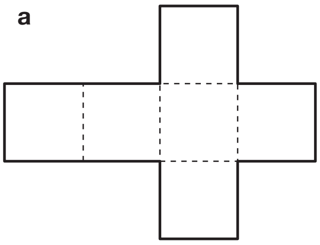
a the hole in a pencil sharpener _____________

b a brick _____________

c a six-sided die _____________

5 Name the solids that have these nets.

a



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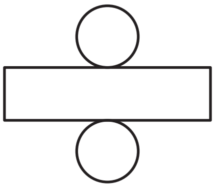
b



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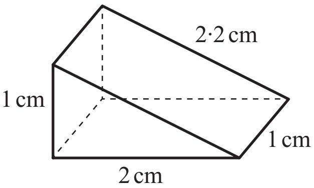
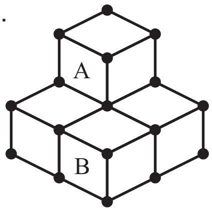
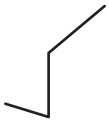
6 Draw a net for this solid.



7 A shape is made up of five small cubes. Draw the result if both cubes marked A and B are removed.



8 Mary started to draw a cuboid but was interrupted after drawing three edges. Copy and complete the diagram to show the other nine edges. Draw the internal edges using dashed lines.



9 Draw a networks that has:

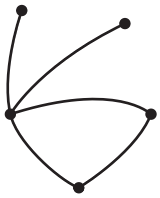
a 2 odd and 1 even vertices

b all odd vertices

c all even vertices.

10 Decide whether these networks are traversable. Answer yes or no.

a



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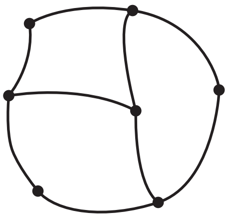
b



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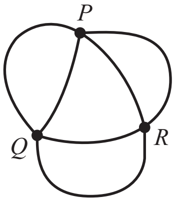
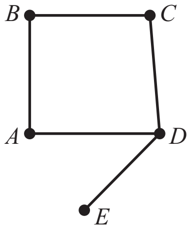
11 This network can be traversed so that you start and finish at the same vertex. Place labels 1 to 6 on the six edges to show how this can be done.



12 The networks drawn here are traversable. Write down the two points at which you must start and finish to traverse them.

a



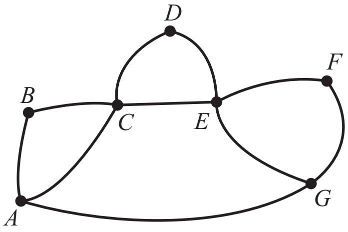
Start

____

Finish

____

b



Start

____

Finish

____

Answers

<Answers to come at 1pp>