MATHEMATICS

# OXFORD MATHS 7

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# CONTENTS

Introducing Oxford Maths 7-10 Victorian Curriculum ..... vi

**Chapter 1** Whole numbers ..... 2

**1A** Place value ..... 4
**1B** Adding whole numbers ..... 11
**1C** Subtracting whole numbers ..... 17
**1D** Multiplying whole numbers ..... 23
Checkpoint ..... 28
**1E** Long multiplication ..... 29
**1F** Dividing whole numbers ..... 33
**1G** Indices and square roots ..... 39
**1H** Order of operations ..... 45
Chapter review ..... 51

**Chapter 2** Factors and primes ..... 54

**2A** Number sequences ..... 56
**2B** Divisibility rules ..... 60
**2C** Multiples and the lowest common multiple ..... 64
Checkpoint ..... 68
**2D** Factors and the highest common factor ..... 69
**2E** Prime and composite numbers ..... 74
**2F** Prime factorisation ..... 78
Chapter review ..... 86

**Chapter 3** Fractions and ratios ..... 88

**3A** Fractions ..... 90
**3B** Equivalent fractions ..... 96
**3C** Ordering fractions ..... 102
**3D** Mixed numbers and improper fractions ..... 108
Checkpoint ..... 112
**3E** Adding and subtracting fractions ..... 113
**3F** Multiplying fractions ..... 119
**3G** Dividing fractions ..... 125
**3H** Ratios ..... 130
Chapter review ..... 137

**Chapter 4** Decimals and percentages ..... 140

**4A** Decimals ..... 142
**4B** Ordering and rounding decimals ..... 148
**4C** Adding and subtracting decimals ..... 153
**4D** Multiplying decimals ..... 158
Checkpoint ..... 163
**4E** Dividing a decimal by a whole number ..... 164

**4F** Dividing a decimal by a decimal ..... 169
**4G** Percentages ..... 173
**4H** Fractions, decimals and percentages ..... 179
**4I** Calculating percentages ..... 185
Chapter review ..... 191

**Chapter 5** Integers and the Cartesian plane ..... 194
**5A** Negative numbers ..... 196
**5B** Adding and subtracting positive numbers ..... 202
**5C** Adding and subtracting negative numbers ..... 206
Checkpoint ..... 211
**5D** The Cartesian plane ..... 212
**5E** Interpreting graphs ..... 219
Chapter review ..... 227

Semester 1 review ..... 230

AMT explorations 1 ..... 234

**Chapter 6** Algebra ..... 236
**6A** Variables ..... 238
**6B** Writing formulas ..... 244
**6C** Terms, expressions and equations ..... 249
**6D** Order of operations in algebra ..... 254
Checkpoint ..... 259
**6E** Substitution ..... 260
**6F** Simplifying ..... 265
**6G** Solving equations by inspection ..... 270
**6H** Solving equations using inverse operations ..... 276
Chapter review ..... 284

**Chapter 7** Angles ..... 286
**7A** Classifying angles and lines ..... 288
**7B** Angles at a point ..... 297
**7C** Angles and parallel lines ..... 304
Checkpoint ..... 312
**7D** Classifying triangles ..... 314
**7E** Classifying quadrilaterals ..... 320
Chapter review ..... 329

**Chapter 8** Shapes and objects ..... 334
**8A** Polygons ..... 336
**8B** 3D objects ..... 343
**8C** Plans and elevations ..... 350
Checkpoint ..... 355

**8D** Translations ..... 357
**8E** Rotations and reflections ..... 364
**8F** Combined transformations ..... 370
**8G** Line and rotational symmetry ..... 374
Chapter review ..... 381

**Chapter 9 Length, area and volume ..... 386**
**9A** Length and perimeter ..... 388
**9B** Area of a rectangle ..... 395
**9C** Area of a parallelogram ..... 401
Checkpoint ..... 407
**9D** Area of a triangle ..... 409
**9E** Surface area ..... 414
**9F** Volume and capacity ..... 420
Chapter review ..... 428

**Chapter 10 Statistics and probability ..... 432**
**10A** Collecting and classifying data ..... 434
**10B** Summary statistics ..... 440
**10C** Constructing and interpreting graphs ..... 446
**10D** Stem-and-leaf plots ..... 456
Checkpoint ..... 462
**10E** Describing probability ..... 464
**10F** Theoretical probability ..... 470
**10G** Experimental probability ..... 476
Chapter review ..... 483

**Chapter 11 Computational thinking ..... 486**
**11A** Printing information to the computer screen ..... 488
**11B** Variables in coding ..... 491
**11C** Asking a user for input ..... 495
**11D** Conditions in code ..... 499

Semester 2 review ..... 504
AMT explorations 2 ..... 512
NAPLAN practice ..... 514
STEAM projects ..... 528
Answers ..... 536
Glossary ..... 622
Index ..... 630
Acknowledgements ..... 635

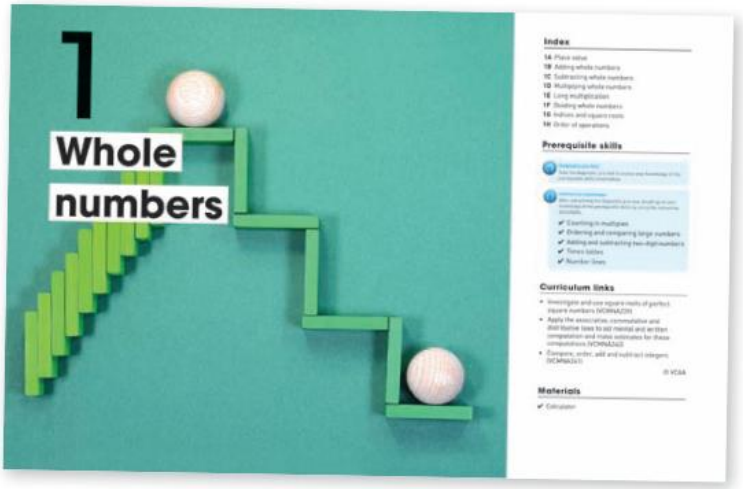
CONTENTS

# Introducing Oxford Maths 7–10

*Oxford Maths 7-10 Victorian Curriculum* utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

## Key features of Student Books

- Complete access to all digital resources available on Student obook pro.
- Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- STEAM projects encourage inter-disciplinary thinking.
- Semester reviews provide an opportunity to revise key concepts from each semester.
- NAPLAN practice allows students to revise numeracy skills for the National Assessment Program.




**Each chapter opens with:**

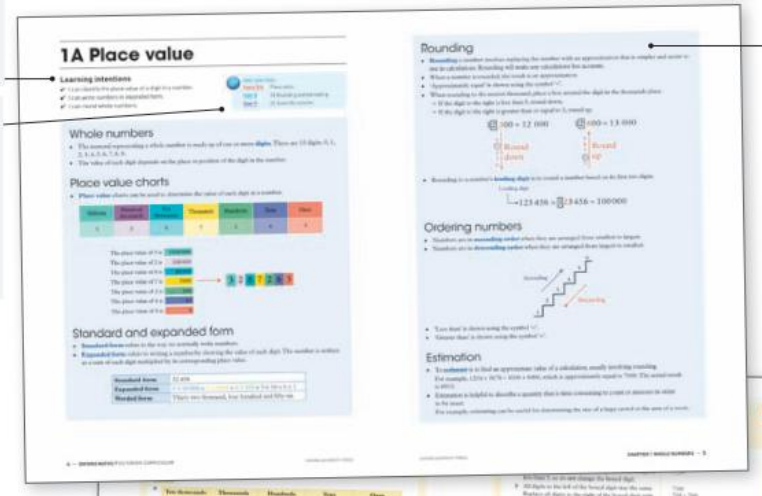
- **Prerequiste skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the VCAA mathematics syllabus.
- **Materials** used to complete the exercises.

**Learning intentions**

- Signpost the foundational skills being developed in each section.

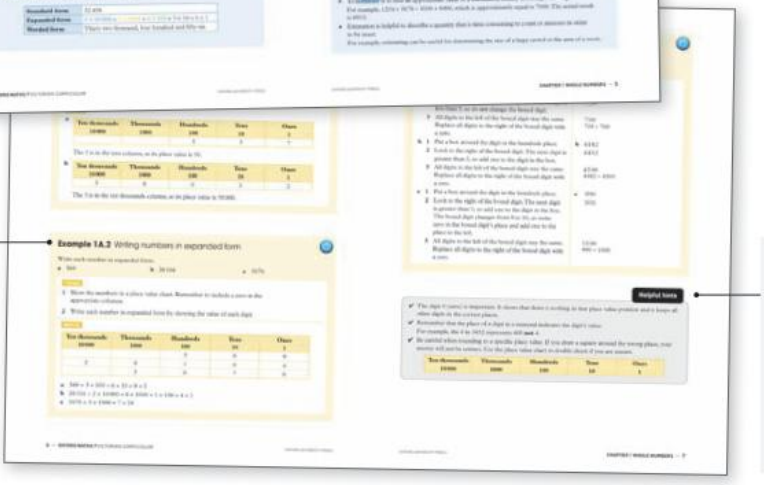
**Inter-year links**

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.




**New theory**

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life, and aid understanding.




**Worked examples**

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

**Helpful hints**

- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

### Differentiated learning pathways

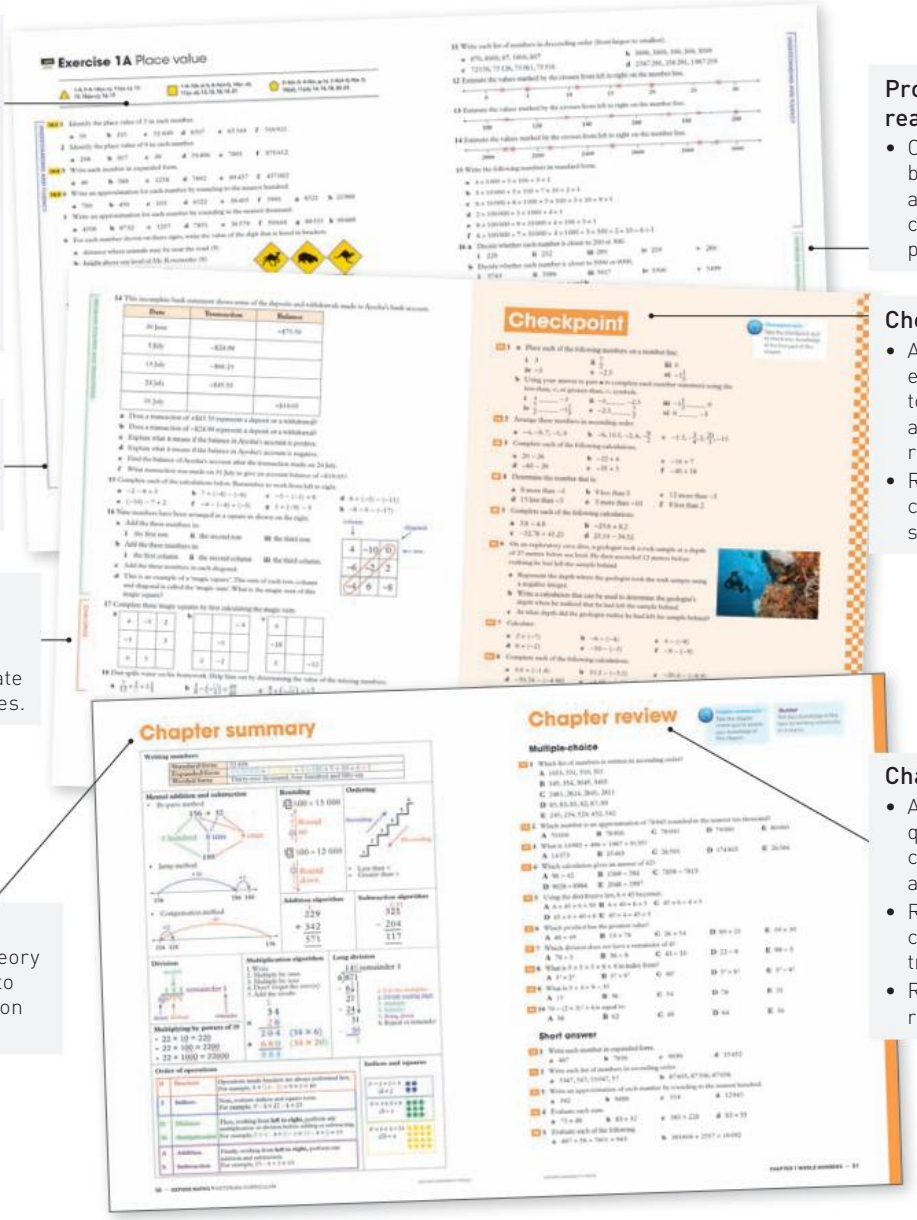
- Each exercise is separated into three pathways, tailoring for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every chapter.

### Understanding and fluency

- Basic exercises dedicated to practising key concepts.

### Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.




### Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

### Checkpoint

- A section in the middle of each chapter dedicated to summarising key skills and encouraging memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

### Chapter summary

- Condenses all the theory from each section into one accessible revision page.

### Chapter reviews

- Additional practice questions to further consolidate understanding at the end of each chapter.
- Reference to an online chapter review quiz to track results.
- Reference to Quizlet test to revise new terminology.

### Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.

### Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem-solve using design thinking.

### Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

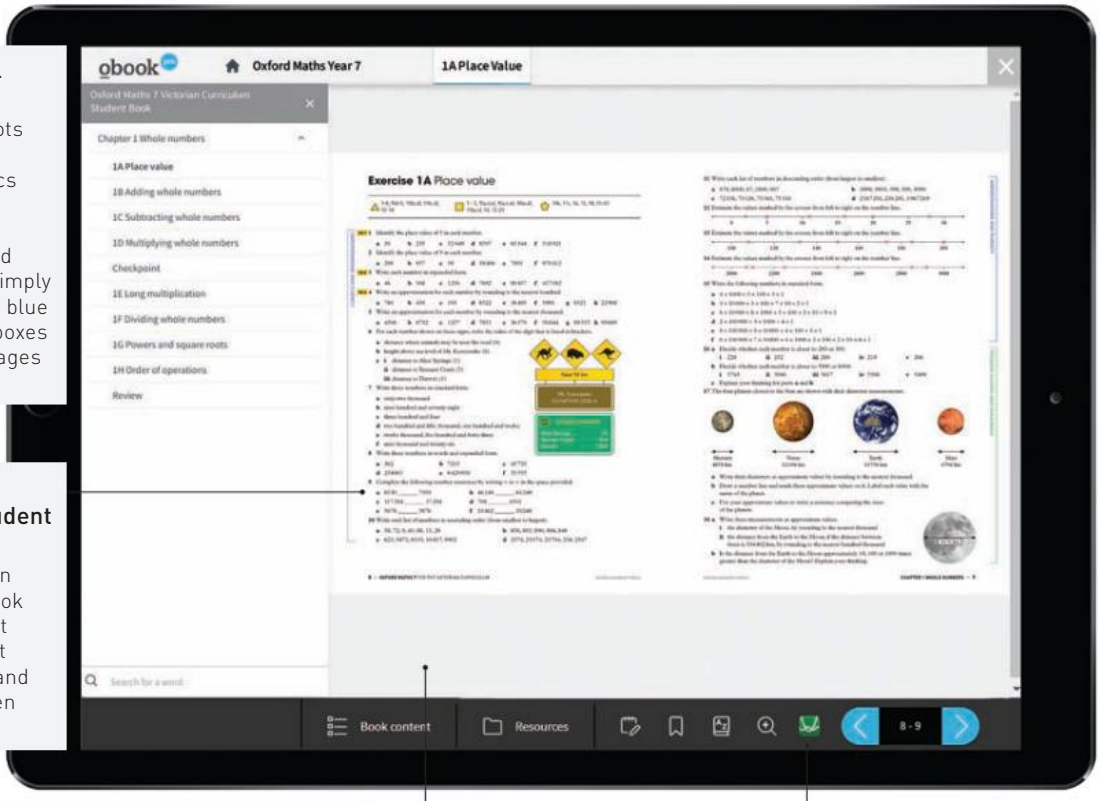
## Key features of Student obook pro

obook pro

- Student obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- It offers a complete digital version of the Student Book with interactive note-taking, highlighting and bookmarking functionality, allowing students to revisit points of learning.
- A complete ePDF of the Student Book is also available for download for offline use and read-aloud functionality.

### Integrated digital resources

- Integrated hotspots allow students to access diagnostics tests, quizzes, interactive skill sheets, videos and inter-year links simply by clicking on the blue digital resource boxes throughout the pages of the book.




### Complete digital version of the Student Book

- The digital version of the Student Book is true to the print version, making it easy to navigate and transition between print and digital.

### Toolbar features

- Notes can be added and saved to the text by simply selecting and highlighting.
- Bookmarks can be saved to any page.
- *Australian Concise Oxford Dictionary* can provide immediate definitions to any word within the text.

### Desmos integration

- Our partership with Desmos allows students to access a suite of calculator tools as they read through the text, providing convenient graphical support as well as the opportunity to investigate plane geometry and Cartesian coordinates.

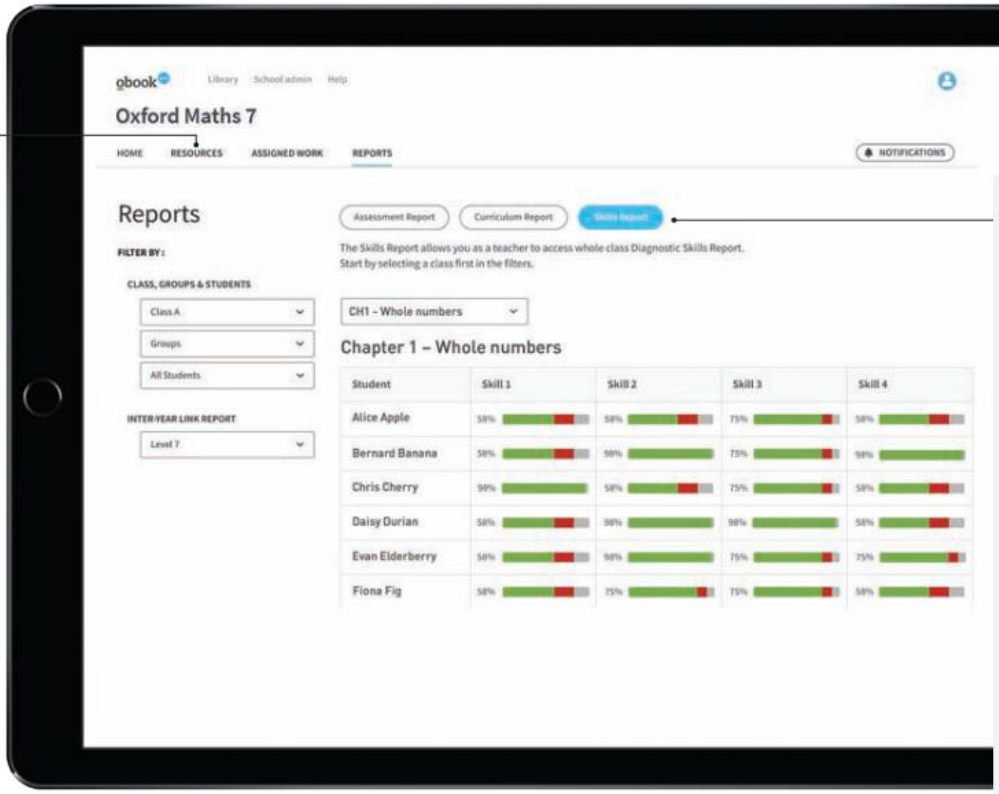
## Benefits for students

- Integrated *Australian Concise Oxford Dictionary* look-up feature
- Targeted instructional videos for every worked example question
- Groundwork resources to support assumed knowledge
- Interactive assessments to consolidate understanding
- Auto-marked practice exam question sets
- Integrated Quizlet sets, including real-time online quizzes with live leaderboards
- Access to online assessment results to track progress.

## Key features of Teacher obook pro

obook pro

- Teacher obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- Each chapter and topic of the Student Book is accompanied by full teaching support, including assessment reporting, worked solutions, chapter tests, detailed teacher notes and lesson plans.
- Teachers can use their Teacher obook pro to share notes and easily assign resources or assessments to students, including due dates and email notifications.




### Learning pathway reports

- Teachers are provided with clear and tangible evidence of student learning progress through innovative reports.
- Assessment reports directly show how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding.
- Curriculum reports summarise student performance against specific curriculum content descriptors and curriculum codes.
- Skill reports indicate the students' understanding of a specific skill in mathematics.

### Additional resources

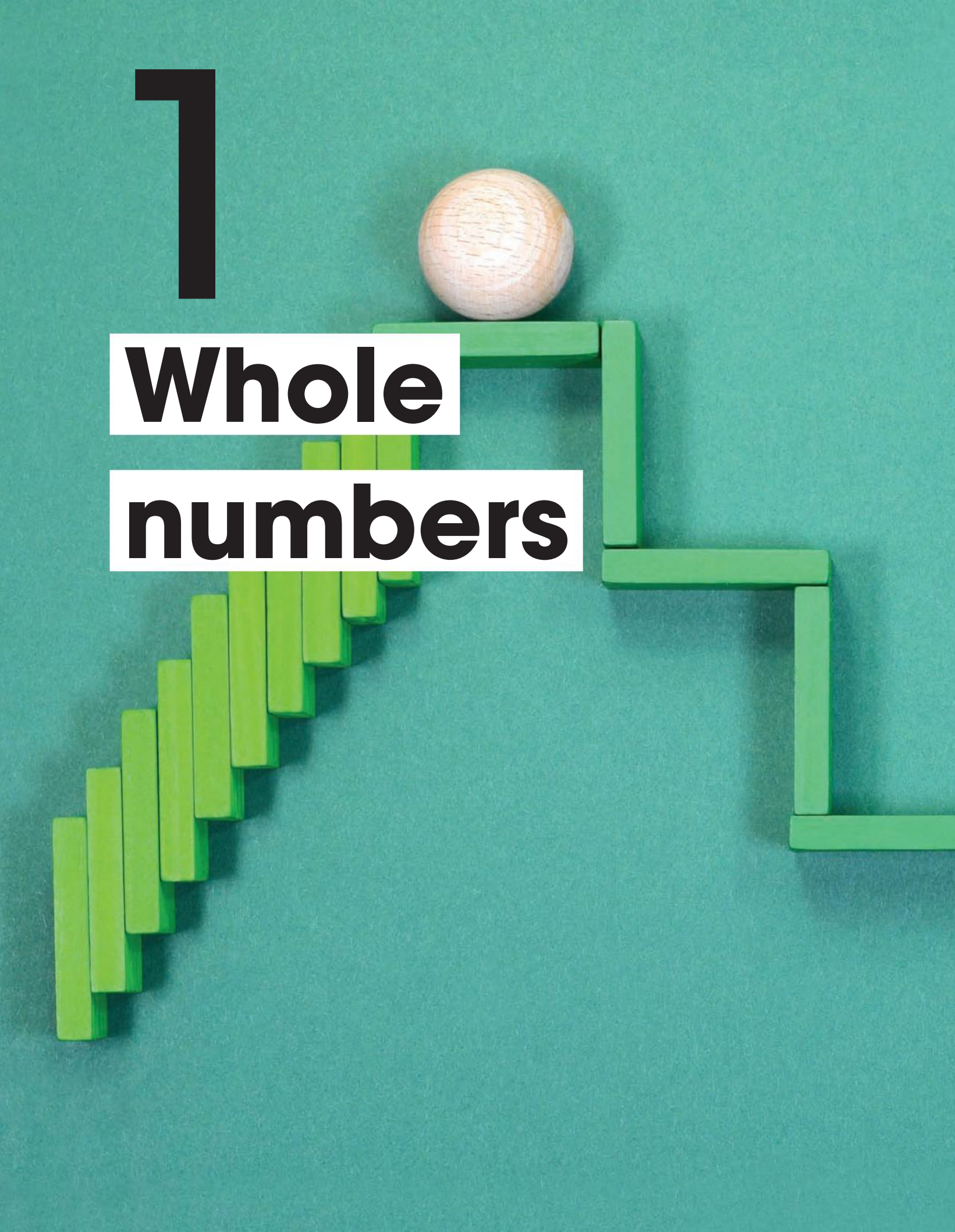
- Each chapter of the Student Book is accompanied by additional interactive skillsheets, worksheets, investigations and topic quizzes to help students progress.

## Benefits for teachers

- Diagnostic pre-tests and chapter tests that track students' progress against Study Design key knowledge, providing detailed learning pathway reports that differentiate each student's ability in each skill
- Assign reading and assessments to students either individually, or in groups – administration is taken care of!
- Ability to set-up classes, monitor student progress and graph results
- Worked solutions for every Student Book question
- Detailed teacher notes, teaching programs and lesson plans.

# 1

# Whole numbers



## Index


**1A** Place value
**1B** Adding whole numbers
**1C** Subtracting whole numbers
**1D** Multiplying whole numbers
**1E** Long multiplication
**1F** Dividing whole numbers
**1G** Indices and square roots
**1H** Order of operations


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Counting in multiples
- ✔ Ordering and comparing large numbers
- ✔ Adding and subtracting two-digit numbers
- ✔ Times tables
- ✔ Number lines

## Curriculum links

- Investigate and use square roots of perfect square numbers (VCMNA239)
- Apply the associative, commutative and distributive laws to aid mental and written computation and make estimates for these computations (VCMNA240)
- Compare, order, add and subtract integers (VCMNA241)

© VCAA

## Materials

✔ Calculator

# 1A Place value

## Learning intentions

- ✔ I can identify the place value of a digit in a number.
- ✔ I can write numbers in expanded form.
- ✔ I can round whole numbers.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Place value |
| Year 8 | 1A Rounding and estimating |
| Year 9 | 2E Scientific notation |

## Whole numbers

- The numeral representing a whole number is made up of one or more **digits**. There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- The value of each digit depends on the place or position of the digit in the number.

## Place value charts

- **Place value** charts can be used to determine the value of each digit in a number.

| Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|
| 3 | 2 | 8 | 7 | 2 | 6 | 3 |

| | |
|---|---|
| The place value of 3 is | 3 000 000 |
| The place value of 2 is | 200 000 |
| The place value of 8 is | 80 000 |
| The place value of 7 is | 7000 |
| The place value of 2 is | 200 |
| The place value of 6 is | 60 |
| The place value of 3 is | 3 |

→ 3 2 8 7 2 6 3

## Standard and expanded form

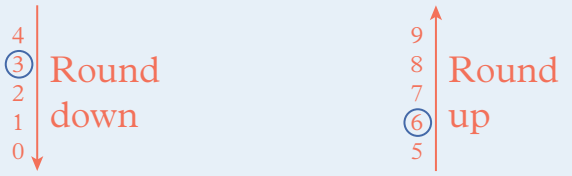
- **Standard form** refers to the way we normally write numbers.
- **Expanded form** refers to writing a number by showing the value of each digit. The number is written as a sum of each digit multiplied by its corresponding place value.

| | |
|---|---|
| **Standard form** | 32 456 |
| **Expanded form** | $3 \times 10\,000 + 2 \times 1000 + 4 \times 100 + 5 \times 10 + 6 \times 1$ |
| **Worded form** | Thirty-two thousand, four hundred and fifty-six |

# Rounding

- **Rounding** a number involves replacing the number with an approximation that is simpler and easier to use in calculations. Rounding will make any calculations less accurate.
- When a number is rounded, the result is an approximation.
- 'Approximately equal' is shown using the symbol '≈'.
- When rounding to the nearest thousand, place a box around the digit in the thousands place.
  - → If the digit to the right is less than 5, round down.
  - → If the digit to the right is greater than or equal to 5, round up.

$$1\boxed{2}\,300 \approx 12\,000 \qquad 1\boxed{2}\,600 \approx 13\,000$$



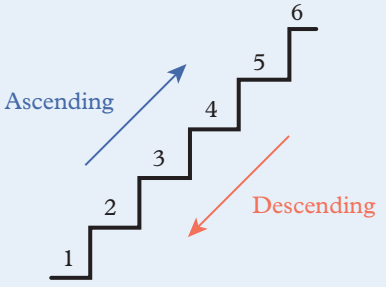

- Rounding to a number's **leading digit** is to round a number based on its first two digits.

Leading digit

$$123\,456 = \boxed{1}23\,456 \approx 100\,000$$

# Ordering numbers

- Numbers are in **ascending order** when they are arranged from smallest to largest.
- Numbers are in **descending order** when they are arranged from largest to smallest.




- 'Less than' is shown using the symbol '<'.
- 'Greater than' is shown using the symbol '>'.

# Estimation

- To **estimate** is to find an approximate value of a calculation, usually involving rounding.
  For example, 1234 + 5678 ≈ 1000 + 6000, which is approximately equal to 7000. The actual result is 6912.
- Estimation is helpful to describe a quantity that is time consuming to count or measure in order to be exact.
  For example, estimating can be useful for determining the size of a large crowd or the area of a room.

## **Example 1A.1** Identifying the place value of a digit in a number

Identify the place value of the 5 in each number.

**a** 357 **b** 58 032

**THINK**

1. Show the numbers in a place value chart. Remember to include a zero in the appropriate columns.
2. Consider the place value of the 5.

**WRITE**

**a**

| Ten thousands 10 000 | Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 |
|---|---|---|---|---|
| | | 3 | 5 | 7 |

The 5 is in the tens column, so its place value is 50.

**b**

| Ten thousands 10 000 | Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 |
|---|---|---|---|---|
| 5 | 8 | 0 | 3 | 2 |

The 5 is in the ten thousands column, so its place value is 50 000.

## **Example 1A.2** Writing numbers in expanded form

Write each number in expanded form.

**a** 369 **b** 28 104 **c** 5070

**THINK**

1. Show the numbers in a place value chart. Remember to include a zero in the appropriate columns.
2. Write each number in expanded form by showing the value of each digit.

**WRITE**

| Ten thousands 10 000 | Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 |
|---|---|---|---|---|
| | | 3 | 6 | 9 |
| 2 | 8 | 1 | 0 | 4 |
| | 5 | 0 | 7 | 0 |

**a** $369 = 3 \times 100 + 6 \times 10 + 9 \times 1$

**b** $28\,104 = 2 \times 10\,000 + 8 \times 1000 + 1 \times 100 + 4 \times 1$

**c** $5070 = 5 \times 1000 + 7 \times 10$

## **Example 1A.3** Rounding a number

Write an approximation for each number by rounding to the nearest hundred.

**a** 719 **b** 4482 **c** 990

| THINK | WRITE |
|---|---|
| **a** **1** Put a box around the digit in the hundreds place. | **a** $\boxed{7}19$ |
| **2** Look to the right of the boxed digit. The next digit is less than 5, so do not change the boxed digit. | $\boxed{7}19$ |
| **3** All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero. | $\boxed{7}00$<br>$719 \approx 700$ |
| **b** **1** Put a box around the digit in the hundreds place. | **b** $4\boxed{4}82$ |
| **2** Look to the right of the boxed digit. The next digit is greater than 5, so add one to the digit in the box. | $4\boxed{4}82$ |
| **3** All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero. | $4\boxed{5}00$<br>$4482 \approx 4500$ |
| **c** **1** Put a box around the digit in the hundreds place. | **c** $\boxed{9}90$ |
| **2** Look to the right of the boxed digit. The next digit is greater than 5, so add one to the digit in the box. The boxed digit changes from 9 to 10, so write zero in the boxed digit's place and add one to the place to the left. | $\boxed{9}90$ |
| **3** All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero. | $1\boxed{0}00$<br>$990 \approx 1000$ |

**Helpful hints**

- ✔ The digit 0 (zero) is important. It shows that there is nothing in that place value position and it keeps all other digits in the correct places.
- ✔ Remember that the place of a digit in a numeral indicates the digit's value.
  For example, the 4 in 3452 represents 400 **not** 4.
- ✔ Be careful when rounding to a specific place value. If you draw a square around the wrong place, your answer will not be correct. Use the place value chart to double check if you are unsure.

| Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|
| 10 000 | 1000 | 100 | 10 | 1 |

ANS p536

# Exercise 1A Place value

1–5, 7–9, 10(a, c), 11(a, c), 12, 13, 15(a–c), 16, 17

1–6, 7(b, d, f), 8–9(d–f), 10(c, d), 11(c, d), 13, 15, 18, 19, 21

2–3(e, f), 4–5(c, g, h), 7–8(d–f), 9(e, f), 10(d), 11(d), 14, 15, 18, 20–23

UNDERSTANDING AND FLUENCY

1A.1 **1** Identify the place value of 5 in each number.

**a** 56 **b** 235 **c** 52 649 **d** 8507 **e** 65 344 **f** 516 921

**2** Identify the place value of 9 in each number.

**a** 298 **b** 957 **c** 39 **d** 59 406 **e** 7891 **f** 970 412

1A.2 **3** Write each number in expanded form.

**a** 46 **b** 568 **c** 1256 **d** 7692 **e** 89 457 **f** 457 082

1A.3 **4** Write an approximation for each number by rounding to the nearest hundred.

**a** 780 **b** 450 **c** 103 **d** 6522 **e** 38 405 **f** 5991 **g** 8521 **h** 22 968

**5** Write an approximation for each number by rounding to the nearest thousand.

**a** 4506 **b** 8732 **c** 1257 **d** 7851 **e** 36 579 **f** 50 644 **g** 89 555 **h** 99 689

**6** For each number shown on these signs, write the value of the digit that is listed in brackets.

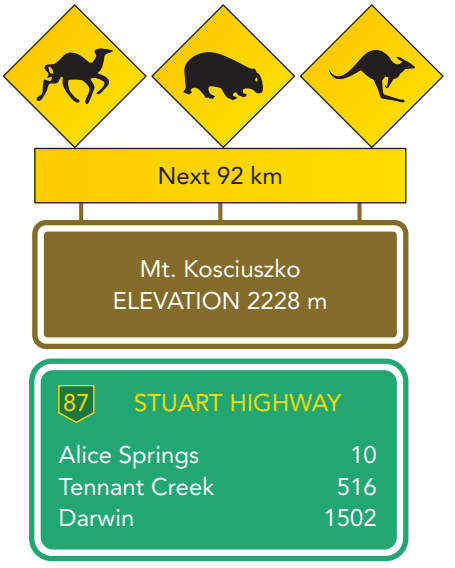
**a** distance where animals may be near the road (9)

**b** height above sea level of Mt. Kosciuszko (8)

**c** **i** distance to Alice Springs (1)

**ii** distance to Tennant Creek (5)

**iii** distance to Darwin (1)




**7** Write these numbers in standard form.

**a** sixty-two thousand

**b** nine hundred and seventy-eight

**c** three hundred and four

**d** two hundred and fifty thousand, one hundred and twelve

**e** twelve thousand, five hundred and forty-three

**f** nine thousand and twenty-six

**8** Write these numbers in words and expanded form.

**a** 362 **b** 7215 **c** 45 733

**d** 234 601 **e** 6 420 058 **f** 55 555

**9** Complete the following number sentences by writing < or > in the space provided.

**a** 8530 _____ 7503 **b** 46 249 _____ 64 249

**c** 317 294 _____ 37 294 **d** 709 _____ 4503

**e** 5678 _____ 5876 **f** 10 462 _____ 10 248

**10** Write each list of numbers in ascending order (from smallest to largest).

**a** 58, 72, 9, 40, 88, 15, 28 **b** 856, 805, 890, 806, 846

**c** 625, 9472, 6105, 10 417, 9902 **d** 2374, 23 074, 23 704, 234, 2347

**11** Write each list of numbers in descending order (from largest to smallest).

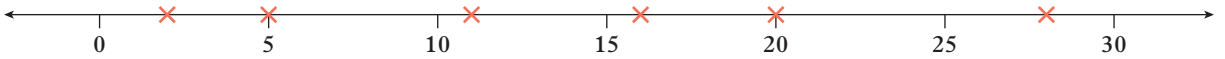
**a** 870, 8000, 87, 1800, 807

**b** 3999, 3909, 399, 309, 3099

**c** 72 156, 75 126, 75 561, 75 516

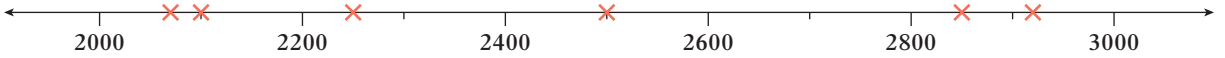
**d** 2 567 291, 256 291, 1 967 219

**12** Estimate the values marked by the crosses from left to right on the number line.




**13** Estimate the values marked by the crosses from left to right on the number line.

**14** Estimate the values marked by the crosses from left to right on the number line.




**15** Write the following numbers in standard form.

**a** $4 \times 1000 + 5 \times 100 + 3 \times 1$

**b** $3 \times 10\,000 + 3 \times 100 + 7 \times 10 + 2 \times 1$

**c** $6 \times 10\,000 + 8 \times 1000 + 5 \times 100 + 3 \times 10 + 9 \times 1$

**d** $2 \times 100\,000 + 3 \times 1000 + 4 \times 1$

**e** $8 \times 100\,000 + 9 \times 10\,000 + 4 \times 100 + 3 \times 1$

**f** $6 \times 100\,000 + 7 \times 10\,000 + 4 \times 1000 + 3 \times 100 + 2 \times 10 + 6 \times 1$

**16 a** Decide whether each number is closer to 200 or 300.

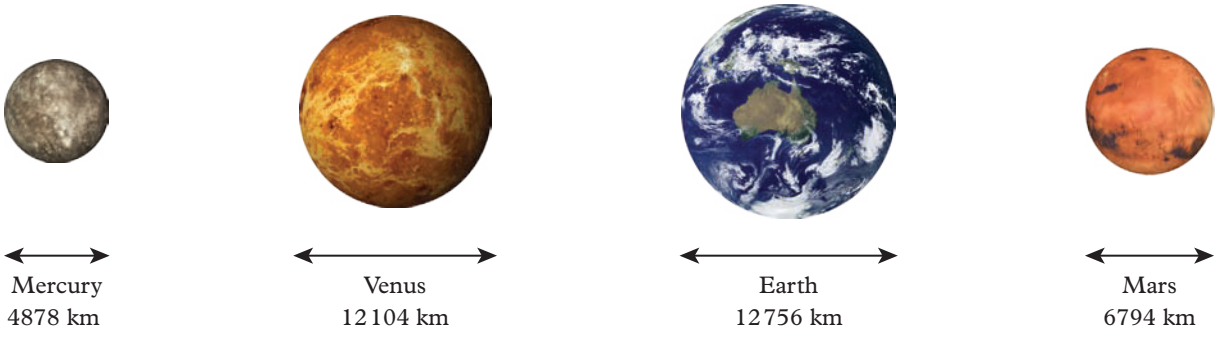
**i** 228 **ii** 252 **iii** 280 **iv** 219 **v** 266

**b** Decide whether each number is closer to 5000 or 6000.

**i** 5743 **ii** 5086 **iii** 5617 **iv** 5508 **v** 5499

**c** Explain your thinking for parts **a** and **b**.

**17** The four planets closest to the Sun are shown with their diameter measurements.




**a** Write their diameters as approximate values by rounding to the nearest thousand.

**b** Draw a number line and mark these approximate values on it. Label each value with the name of the planet.

**c** Use your approximate values to write a sentence comparing the sizes of the planets.

**18 a** Write these measurements as approximate values.

**i** the diameter of the Moon, by rounding to the nearest thousand

**ii** the distance from the Earth to the Moon, if the distance between them is 334 402 km, by rounding to the nearest hundred thousand

**b** Is the distance from the Earth to the Moon approximately 10, 100 or 1000 times greater than the diameter of the Moon? Explain your thinking.

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

CHALLENGE

**19** **a** How many different two-digit numbers can you make from 3 and 5 if you cannot repeat digits? List them in ascending order.

**b** How many different two-digit numbers can you make from 3 and 5 if you can repeat digits? List them in descending order.

**20** **a** How many different two-digit numbers can you make from 2, 4 and 7 if you cannot repeat digits? List them in descending order.

**b** How many different two-digit numbers can you make from 2, 4 and 7 if you can repeat digits? List them in ascending order.

**21** Consider the digits 1, 3, 6 and 9. Use these digits to write:

**a** the largest four-digit number without repeating any digits

**b** the smallest four-digit number if digits can be repeated

**c** the largest even number without repeating any digits

**d** all the four-digit numbers between 3620 and 6350 if no digits can be repeated

**22** Consider the place value chart. The digit in the thousands column is currently missing.

| Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | * | 0 | 2 | 9 |

**a** When rounded to the nearest hundred thousand, what will the number be?

**b** What digit, or digits, placed in the thousands column will give the number 1 230 000 when rounded to the nearest ten thousand?

**c** If the digit in the thousands column is 9, and 760 000 is added to the number in the table, determine the number rounded to the nearest ten thousand?

**d** If the digit in the thousands column is 9, and 760 000 is subtracted from the number in the table, determine the number rounded to the nearest ten thousand?

**23** A multi-billionaire designed a special new lock for his safe. The lock, shown below, is reset to a different four numbers every 2 minutes. To crack the code, you must round each number so that the sum of the digits is closest to 20.

For example, 48922 would be rounded to 48900 as 4 + 8 + 9 = 21, and rounding to the nearest hundred in this case gives the closest digit sum to 20.

Can you crack the code in the given time limit?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Place value

Interactive skillsheet Rounding integers

Investigation Roman numerals

Topic quiz 1A

# 1B Adding whole numbers

## Learning intentions

✔ I can use mental strategies to add whole numbers.
✔ I can use the addition algorithm for large numbers.

Inter-year links
Years 5/6 Adding whole numbers
Year 8 1B Adding and subtracting whole numbers

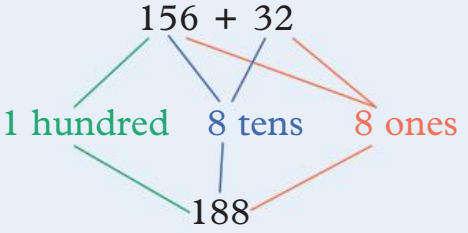
## Addition properties

| Addition (+) |
|---|
| Sum |
| Plus |
| More than |
| Increase |
| Total |
| Together |

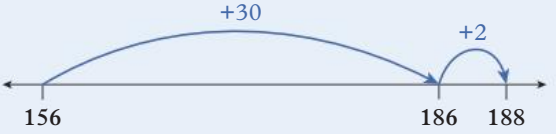
- The **associative law of addition** states that regardless of how the numbers are grouped, the answer does not change.
  For example, $(2 + 4) + 6 = 2 + (4 + 6)$.
- The **commutative law of addition** states that the order in which numbers are added does not change the answer.
  For example, $2 + 4 = 4 + 2$.

## Mental addition strategies

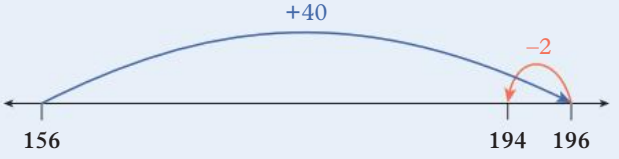
- The **by-parts method** involves adding the digits in each place value separately.
  For example, $156 + 32 = 100 + (50 + 30) + (6 + 2)$
  $= 188$




- The **jump method** involves breaking down one of the given numbers, then adding each part of the number in stages.
  For example, $156 + 32 = 156 + 30 + 2$
  $= 188$




- The **compensation method** involves rounding one number to make the calculations easier. The amount required to round the number is then added or subtracted depending on if the number was rounded up or down.
  For example, $156 + 38 = 156 + 40 - 2$
  $= 194$



# Addition algorithm

- In the addition algorithm, the numbers are written one under the other with the digits lined up according to their place value.
  - → Always start by adding the digits in the ones column first, followed by the tens column, then the hundreds column and so on.

$$\begin{array}{r} {\scriptstyle 1}\phantom{0} \\ 229 \\ +\ 342 \\ \hline 571 \\ \hline \end{array} \longleftarrow \text{answer}$$

Ones column: 9 + 2 = 11, carry the 1
Tens column: 2 + 4 + (1) = 7
Hundreds column: 2 + 3 = 5

- Regrouping occurs when there is a result of 10 or more in one column.
  - → For example, 11 is the same as 1 ten and 1 one, so a result of 11 is the same as placing 1 in the tens column and 1 in the ones column. The 1 ten is represented by the orange 1 as it has been regrouped from the ones column.

## **Example 1B.1** Using mental strategies to add two numbers

Work out each sum using the mental strategy given in brackets.

**a** 78 + 21 (by-part method)
**b** 36 + 77 (jump method)
**c** 67 + 58 (compensation method)

**THINK**

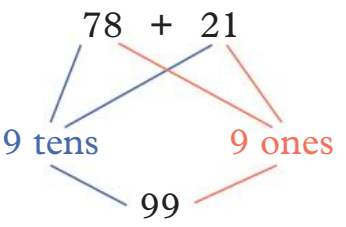
**a** **1** Add the tens together.
**2** Add the ones together.
**3** Combine the two answers to get the final answer.

**b** **1** Choose a starting number, then break down the second number into tens and ones.
**2** Add the tens to the starting number followed by the ones.

**c** **1** Choose a starting number, then round the second number to the nearest 10.
**2** Add the rounded number to the starting number, then add or subtract the amount needed to round the second number.

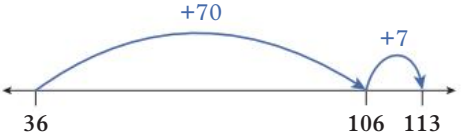
**WRITE**

**a**



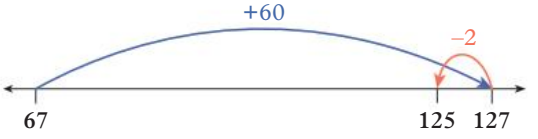

78 + 21 is 99.

**b**




36 + 77 is 113.

**c**




67 + 58 is 125.

## **Example 1B.2** Using the addition algorithm to add two numbers

Calculate 634 + 891 using the addition algorithm.

$$\begin{array}{r} 634 \\ +\ 891 \\ \hline \end{array}$$

**THINK**

1. Add the digits in the ones column: 4 + 1 = 5.
2. Add the digits in the tens column: 3 + 9 = 12.

   Write 2 in the tens column of the answer and write 1 above the hundreds column.
3. Add the digits in the hundreds column: 1 + 6 + 8 = 15.

**WRITE**

$$\begin{array}{r} {\scriptstyle 1}\phantom{0} \\ 634 \\ +\ 891 \\ \hline 1525 \end{array}$$

## **Example 1B.3** Using the addition algorithm to add three numbers

Use the addition algorithm to calculate 3108 + 547 + 1619.

**THINK**

1. Set out the addition problem by lining up digits according to place value.
2. Add the digit in the ones column: 8 + 7 + 9 = 24.

   Write 4 in the ones column of the answer and write 2 above the tens column.
3. Add the digits in the tens column: 2 + 0 + 4 + 1 = 7.
4. Add the digits in the hundreds column: 1 + 5 + 6 = 12.

   Write 2 in the hundreds column of the answer and write 1 above the thousands column.
5. Add the digits in the thousands column: 1 + 3 + 0 + 1 = 5.

**WRITE**

$$\begin{array}{r} {\scriptstyle 1}\ {\scriptstyle 2}\phantom{00} \\ 3108 \\ 547 \\ +\ 1619 \\ \hline 5274 \end{array}$$

**Helpful hints**

- ✔ Don't forget that addition is commutative – reordering sums can make them a lot easier! For example, 12 + 123 455 = 123 455 + 12.
- ✔ Make sure that your place value columns are well aligned so that you add together the correct digits.
- ✔ When you are adding multiple large numbers together, it can be helpful to estimate your answer using rounding to ensure you have the right answer. You can compare your two results and easily see if you have gone way off track.

ANS p536

# Exercise 1B Adding whole numbers

1–12, 14, 16, 20(a) | 1–8, 13, 15, 17, 18 | 1(d), 2(d), 3(d), 4(g, h), 7(e–h), 8(d–f), 9–12, 17–20

UNDERSTANDING AND FLUENCY

1B.1 **1** Use the by-part method to work out each sum. Draw the corresponding diagram to support your answer.

**a** 458 + 102 **b** 75 + 23 **c** 34 + 57 **d** 49 + 810

**2** Use the jump method to work out each sum. Draw the corresponding diagram to support your answer.

**a** 103 + 46 **b** 112 + 13 **c** 21 + 94 **d** 66 + 22

**3** Use the compensation method to work out each sum. Draw the corresponding diagram to support your answer.

**a** 48 + 56 **b** 274 + 27 **c** 26 + 58 **d** 79 + 89

**4** Use any mental strategy to work out each sum.

**a** 50 + 30 **b** 120 + 60 **c** 360 + 20 **d** 25 + 13
**e** 16 + 61 **f** 56 + 34 **g** 65 + 35 **h** 123 + 345

**5** Use any mental strategy to work out each sum.

**a** 20 + 40 + 10 **b** 500 + 100 + 300 **c** 18 + 12 + 50 **d** 73 + 7 + 20
**e** 52 + 11 + 27 **f** 35 + 29 + 35 **g** 115 + 205 + 80 **h** 203 + 203 + 203

1B.2 **6** Calculate each sum using the addition algorithm.

**a** $\begin{array}{r} 243 \\ +\ 715 \\ \hline \phantom{0} \end{array}$ **b** $\begin{array}{r} 538 \\ +\ 154 \\ \hline \phantom{0} \end{array}$ **c** $\begin{array}{r} 758 \\ +\ 461 \\ \hline \phantom{0} \end{array}$ **d** $\begin{array}{r} 697 \\ +\ 805 \\ \hline \phantom{0} \end{array}$

**7** Use the addition algorithm to calculate each of the following.

**a** 641 + 478 **b** 157 + 296 **c** 2438 + 5160 **d** 3762 + 1489
**e** 2175 + 485 **f** 96 + 5743 **g** 16 407 + 782 **h** 8009 + 35 714

1B.3 **8** Use the addition algorithm to calculate each of the following.

**a** 385 + 461 + 723 **b** 749 + 218 + 837 **c** 927 + 56 + 614
**d** 869 + 21 + 70 **e** 2764 + 8293 + 451 **f** 7605 + 246 + 38

**9** Evaluate the following without using a calculator.

**a** 23 748 + 61 392 + 53 709 **b** 82 407 + 9385 + 411
**c** 34 715 + 306 937 + 8256 **d** 947 + 600 411 + 85 103

**10** Evaluate the following without using a calculator.

**a** 28 + 7640 + 459 + 7 + 834 + 2406 **b** 92 762 + 547 + 85 + 2942 + 6073

**11** Check your answers to questions **9** and **10** with a calculator.

**12** You decide to train for a local cycling race. On the first weekend you cycle 32 km, on the second you cycle 45 km and on the third you cycle 59 km. To find the total distance covered over the three weekends, you can write the calculation in a number of ways.

**a** Decide if the following statements are true or false.

**i** 32 + 45 + 59 = (3 + 4 + 5) + (2 + 5 + 9)

**ii** 32 + 45 + 59 = 30 + 40 + 60 + 2 + 5 − 1

**iii** 32 + 45 + 59 = 59 + (30 + 40) + (2 + 5)

**b** List two other ways to write the calculation.

**c** Calculate the total distance covered.

**13** **a** Identify two two-digit numbers that add to 53.

**b** Identify two three-digit numbers that add to 386 and meet these conditions.

**i** both numbers are odd

**ii** both numbers are even

**c** Identify three whole numbers that add to 5207. Suggest another set of three numbers that add to the same total.

**14** Riley is going to make enchiladas for his family for dinner. The recipe says that it will take 20 minutes preparation time and 55 minutes baking time.

**a** How long will it take for Riley to have dinner ready?

**b** Riley starts preparing the meal at 5:15 pm. What time will it be ready for the family to eat?

**15** The Tour de France is an annual international cycling race that finishes in Paris, France. In 2011, Cadel Evans won the race, following the itinerary shown.

**a** Answer these questions without using a calculator.

**i** How far did the competitors ride in the first five days of the race?

**ii** How far did the competitors ride before their first rest day?

**iii** How far did the cyclists ride between leaving Gap and finishing in Paris?

**iv** What was the total distance covered in the Tour de France?

**b** Check your answers using a calculator.

**c** Research the lastest Tour de France. What total distance was covered?

| Date in 2011 | Start and finish | Distance |
|---|---|---|
| 2 July | Passage du Gois → Mont des Alouettes | 192 km |
| 3 July | Les Essarts → Les Essarts (team time trial) | 23 km |
| 4 July | Olonne-sur-Mer → Redon | 198 km |
| 5 July | Lorient → Mûr-de-Bretagne | 172 km |
| 6 July | Carhaix → Cap Fréhel | 165 km |
| 7 July | Dinan → Lisieux | 226 km |
| 8 July | Le Mans → Châteauroux | 218 km |
| 9 July | Aigurande → Super-Besse Sancy | 189 km |
| 10 July | Issoire → Saint-Flour | 208 km |
| 11 July | Le Lioran Cantal | rest |
| 12 July | Aurillac → Carmaux | 158 km |
| 13 July | Blaye-les-Mines → Lavaur | 168 km |
| 14 July | Cugnaux → Luz-Ardiden | 211 km |
| 15 July | Pau → Lourdes | 152 km |
| 16 July | Saint-Gaudens → Plateau de Beille | 169 km |
| 17 July | Limoux → Montpellier | 192 km |
| 18 July | Département de la Drôme | rest |
| 19 July | Saint-Paul-Trois-Châteaux → Gap | 163 km |
| 20 July | Gap → Pinerolo | 179 km |
| 21 July | Pinerolo → Galibier Serre-Chevalier | 200 km |
| 22 July | Modane Valfréjus → Alpe-d'Huez | 110 km |
| 23 July | Grenoble → Grenoble (time trial) | 42 km |
| 24 July | Créteil → Paris Champs-Élysées | 95 km |

PROBLEM SOLVING AND REASONING

**16** To enter the stadium for a soccer match, patrons used one of four gates. The number of people who passed through the turnstiles of each gate is shown.

**a** Estimate the number of people who attended the match by first rounding the numbers at each gate to the nearest thousand.

| Gate | Number of people |
|---|---|
| A | 8759 |
| B | 9042 |
| C | 10 365 |
| D | 11 008 |

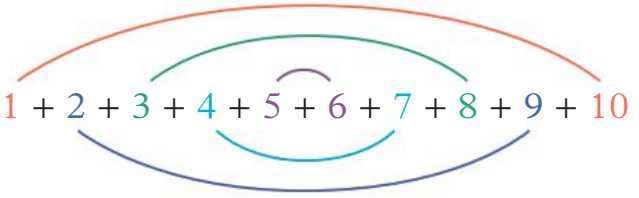
**b** Calculate the exact number of people who attended the match.

**c** Which answer would a sports commentator be more likely to use when reporting on the match?

**17** Consider adding the numbers from 1 to 10.

**a** Add the numbers in order: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ?

**b** This calculation can also be complete by first pairing the numbers. Write the smallest number and the largest number together, then the second smallest and second largest together and so on.




Complete this calculation for adding the numbers from 1 to 10.

(1 + 10) + (2 + 9) + (3 + ___) + (4 + ___) + (5 + ___)
= ___+___+___+___+___
= ___

**c** Which strategy did you find easier? Justify your answer.

**18** Use the strategy of grouping numbers in suitable pairs to add the numbers from 1 to 20.

**19 a** Use the strategy of grouping numbers in suitable pairs to add these numbers.

**i** from 1 to 9 **ii** from 1 to 19

**b** Explain what is different about using this strategy with an odd number of numbers.

**c** Use a suitable strategy to add these numbers.

**i** from 2 to 8 **ii** from 3 to 17 **iii** from 5 to 25

CHALLENGE

**20 a** Identify two consecutive numbers that add to 53.

**b** Identify three consecutive numbers that add to 114.

**c** Identify four consecutive numbers that add to 190.

**d** What strategy did you use to find the sets of consecutive numbers?

Check your Student obook pro for these digital resources and more:

Worksheet
Adding numbers

Investigation
Abracadabra with magic squares

Topic quiz
1B

# 1C Subtracting whole numbers

## Learning intentions

✔ I can use mental strategies to subtract whole numbers.
✔ I can use the subtraction algorithm for large numbers.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Subtracting whole numbers |
| Year 8 | 1B Adding and subtracting whole numbers |

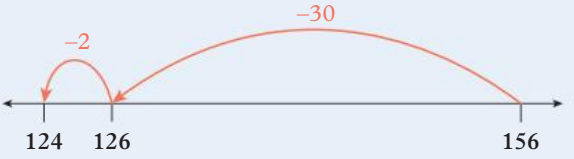
## Subtraction properties

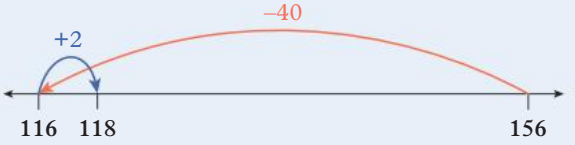
- Subtraction of two or more numbers is not associative. This means that two numbers cannot be grouped and subtracted before subtracting from the other number. Order is important.
  For example, (8 – 5) – 2 = 1 and 8 – (5 – 2) = 5.
- Subtraction of two or more numbers is not commutative. This means that two or more numbers cannot be subtracted in any order and achieve the same result.
  For example, 7 – 4 is not the same as 4 – 7.

| Subtraction (–) |
|---|
| Difference |
| Take away |
| Less than |
| Fewer |
| Reduce |
| Minus |

## Mental subtraction methods

- The jump method and the compensation method can also be applied to subtraction.
  - → Jump method for subtraction
    For example, 156 – 32 = 156 – 30 – 2
    = 124
  - → Compensation method for subtraction
    For example, 156 – 38 = 156 – 40 + 2
    = 118




## Subtraction algorithm

- In the subtraction algorithm, the number to be subtracted is written under the other number with the digits lined up according to their place value.
  - → If the subtraction in a particular column cannot be done, take a value from the next column to the left. This is the opposite of the carrying process in the addition algorithm. For example, 1 ten is also 10 ones and can be moved into the ones column.
  - → Always start by subtracting the digits in the ones column first, followed by the tens column, then the hundreds column and so on.

```
   1 11
   3 2 1
 – 2 0 4
 -------
   1 1 7
```

← answer

Ones column: Take 10 from the tens column
11 – 4 = 7
Tens column: 1 – 0 = 1
Hundreds column: 3 – 2 = 1

## Example 1C.1 Using mental strategies to subtract two numbers

Work out each difference using the mental strategy given in brackets.

**a** 81 – 56 (jump method)

**b** 78 – 49 (compensation method)

**THINK**

**a** **1** Choose a starting number, and then break down the second number into tens and ones.

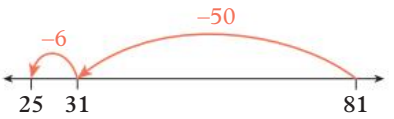
**2** Subtract the tens from that number followed by the ones.

**b** **1** Choose a starting number, then round the second number to the nearest 10.

**2** Subtract the rounded number from the starting number, then add or subtract the amount needed to round the second number.

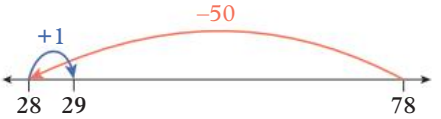
**WRITE**

**a**




81 – 56 is 25.

**b**




78 – 49 is 29.

## Example 1C.2 Using the subtraction algorithm to subtract two numbers

Calculate 426 – 281 using the subtraction algorithm.

$$\begin{array}{r} 426 \\ -\ 281 \\ \hline \\ \hline \end{array}$$

**THINK**

**1** Subtract the digits in the ones column: 6 – 1 = 5.

**2** Subtract the digits in the tens column: 2 – 8.

Take 1 from the hundreds column by reducing the hundreds to 3 and increasing the tens as 12. Subtract the digits in the tens column: 12 – 8 = 4.

**3** Subtract the digits in the hundreds column: 3 – 2 = 1.

**WRITE**

$$\begin{array}{r} {\scriptstyle 3\ 12} \\ \cancel{4}\cancel{2}6 \\ -\ 281 \\ \hline 145 \\ \hline \end{array}$$

## Example 1C.3 Using the subtraction algorithm to subtract numbers with different numbers of digits

Use the subtraction algorithm to calculate 788 – 27.

**THINK**

**1** Set out the subtraction problem by lining up digits according to place value.

**2** Subtract the digits in the ones column: 8 – 7 = 1.

**3** Subtract the digits in the tens column: 8 – 2 = 6.

**4** Subtract the digits in the hundreds column: 7 – 0 = 7. Recall that if there is no digit in a place then it has a value of zero.

**WRITE**

$$\begin{array}{r} 788 \\ -\ 27 \\ \hline 761 \\ \hline \end{array}$$



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**Helpful hints**

- ✔ Make sure that your place value columns are well aligned so that you subtract the correct numbers. See example to the right.

$$\begin{array}{r} 654 \\ -\ 321 \\ \hline 333 \\ \hline \end{array}$$

- ✔ Subtraction is not commutative. Unlike with addition problems, you cannot change the order of the numbers around the sign and achieve the same result.
  For example, 12 – 4 is not the same as 4 – 12.
- ✔ Read each question carefully. You may find some subtraction questions easy to calculate in your head. Don't find yourself performing lots of regrouping and end up with an answer as easy to find as 1!
  For example, 101–99 requires lots of work in the subtraction algorithm, but you might be able to see that the answer is 2 straight away!

ANS p537

# Exercise 1C Subtracting whole numbers

△ 1–6, 7(d, e, f), 9–12, 14, 17 □ 1–4, 6(a–d), 7–10, 11(a–c), 13–15, 18 ⬠ 1(d), 2(d), 4(g, h), 6(g, h), 8, 11, 13, 15, 16, 18, 19

UNDERSTANDING AND FLUENCY

1C.1 **1** Use the jump method to work out each difference. Draw a corresponding diagram to support your answer.

**a** 60 – 20 **b** 78 – 26 **c** 65 – 52 **d** 184 – 96

**2** Use the compensation method to work out each difference. Draw a corresponding diagram to support your answer.

**a** 85 – 19 **b** 69 – 28 **c** 129 – 67 **d** 281 – 48

**3** Use any mental strategy to work out each difference.

**a** 60 – 40 **b** 300 – 200 **c** 170 – 30 **d** 250 – 40
**e** 710 – 20 **f** 178 – 58 **g** 270 – 35 **h** 393 – 281

**4** Use any mental strategy to work out each problem.

**a** 50 + 20 – 10 **b** 700 + 100 – 300 **c** 14 + 16 – 20 **d** 98 + 12 – 1
**e** 570 + 25 – 80 **f** 36 – 12 + 8 **g** 63 – 41 + 20 **h** 400 – 50 + 85

1C.2 **5** Calculate each difference using the subtraction algorithm.

**a** $\begin{array}{r} 58 \\ -\ 25 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 87 \\ -\ 46 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 624 \\ -\ 359 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 406 \\ -\ 134 \\ \hline \\ \hline \end{array}$

**6** Use the subtraction algorithm to calculate each of the following.

**a** 56 – 14 **b** 81 – 36 **c** 456 – 234 **d** 938 – 651
**e** 624 – 185 **f** 3846 – 1724 **g** 7508 – 5631 **h** 8135 – 6479

1C.3 **7** Use the subtraction algorithm to calculate each of the following.

**a** 6327 – 215 **b** 5962 – 647 **c** 2475 – 728
**d** 7836 – 908 **e** 4025 – 462 **f** 5002 – 431

**8** Determine the difference without using a calculator.

**a** 36 274 – 28 093 **b** 508 246 – 137 651
**c** 211 537 – 36 409 **d** 81 752 – 8362

UNDERSTANDING AND FLUENCY

**9** Evaluate without using a calculator.

**a** 58 945 – 45 120 – 1205

**b** 101 101 – 45 751 – 458 – 2378

**10** Check your answers to question **9** with a calculator.

**11** For each calculation:

**i** estimate the answer by first rounding each number to its leading digit

**ii** use pen and paper to work out the exact answer

**iii** use a calculator to check the result you obtained for part **ii**.

**a** 61 + 48 – 77

**b** 254 – 123 + 448

**c** 4708 – 369 + 532

**d** 29 071 + 8275 – 17 466

**e** 77 – 25 + 89 – 60 + 41

**f** 809 + 1252 – 754 – 36

PROBLEM SOLVING AND REASONING

**12** The longest river in the world is the Nile in Africa, with a length of 6650 km. The longest river in Australia is the Darling River, with a length of 2740 km. To find the difference in length between these two rivers, you can write the calculation in a number of ways.

**a** Decide if the following statements are true or false.

**i** 6650 – 2740 = 6000 – 2000 – 600 – 700 – 50 – 40

**ii** 6650 – 2740 = 6650 – 2000 – 700 – 40

**iii** 6650 – 2740 = 6650 – 3000 – 260

**iv** 6650 – 2740 = 6700 – 2700 – 60

**b** List two other ways to write the calculation.

**c** Calculate the difference between the two river lengths.

**13 a** Identify two two-digit numbers that have a difference of 43.

**b** Identify two numbers that have a difference of 168 and meet these conditions.

**i** both numbers are odd

**ii** both numbers are even

**c** Identify two three-digit numbers that have a difference of 286 and meet these conditions.

**i** one number must be more than twice the other

**ii** one number must be triple the other

**14** Josh is comparing the distance to travel by plane from Melbourne to Rome using two different routes. One journey stops at Hong Kong to refuel, while another stops at Singapore.

| Flight sector | Distance by air |
|---|---|
| Melbourne–Hong Kong | 7435 km |
| Melbourne–Singapore | 6064 km |
| Hong Kong–Rome | 9307 km |
| Singapore–Rome | 10 048 km |

**a** Determine the total flight distance from Melbourne to Rome if the plane stops at Hong Kong on the way.

**b** Determine the total flight distance from Melbourne to Rome if the plane stops at Singapore on the way.

**c** Which flight distance is the shortest and by how much?

**15** The 162-storey building named Burj Khalifa, located in Dubai, United Arab Emirates, was completed in 2010.

**a** How does the height of this building compare with other structures? Find the difference in height between Burj Khalifa and each of the structures shown in this table.

| Structures | Date completed | Height |
|---|---|---|
| Washington Monument (Washington DC, USA) | 1884 | 169 m |
| Eiffel Tower (Paris, France) | 1889 | 300 m |
| Empire State Building (New York, USA) | 1931 | 381 m |
| Sydney Tower (Sydney, Australia) | 1981 | 309 m |
| Petronas Towers (Kuala Lumpur, Malaysia) | 1998 | 452 m |
| Taipei 101 (Taipei, Taiwan) | 2003 | 509 m |
| Q1 (Gold Coast, Australia) | 2005 | 323 m |
| Eureka Tower (Melbourne, Australia) | 2006 | 297 m |

**b** Which two structures could have their heights added to give a result closest to the height of Burj Khalifa?

**c** Which three structures could have their heights added to give a result closest to the height of Burj Khalifa?

**16** The highest mountain in the world is Mount Everest, which has a height above sea level of 8848 m.

**a** How does this compare to Mount Kosciuszko, the highest mountain in Australia, at 2228 m above sea level?

**b** Compare the height of Mount Everest with the three mountains in our solar system listed below.

| Mountain | Location | Height above surface |
|---|---|---|
| Mons Huygens | The Moon | 4700 m |
| Maxwell Montes | Venus | 11 000 m |
| Olympus Mons | Mars | 21 171 m |

**c** Determine the difference in height between the mountains listed for Mars and the Moon.

**d** Determine the difference in height between Mount Kosciuszko and Maxwell Montes.

PROBLEM SOLVING AND REASONING

**17** There are 191 people travelling on a train. At the first station, 52 people leave the train and another 48 board the train. At the following station, 69 people get off the train and 75 get on. How many people are now on the train?

**18** In May 2000, five planets in our solar system reached a point in their orbit around the Sun where they were roughly positioned in a straight line on the same side of the Sun (as viewed from the Sun). Earth was positioned on the opposite side of the Sun. The average distance from the Sun for each planet is shown on the right.

**a** Use this information to find the distance between these planets.

**i** Jupiter and Saturn

**ii** Mercury and Saturn

**iii** Venus and Mercury

**b** If the Sun has a diameter of about 1 392 000 km, what was the distance between Jupiter and Earth?

**c** In June 2010, Uranus, Jupiter and Mercury were roughly lined up on one side of the Sun while Venus, Mars and Saturn were lined up on the other side. At this time, what was the approximate distance between:

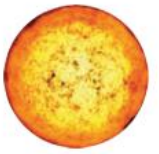
**i** Jupiter and Mercury?

**ii** Saturn and Venus?

**iii** Jupiter and Saturn?

**d** The film *War of the Worlds* is based on the idea that Martians travelled to Earth when the distance between Mars and Earth was the shortest. About what distance would the Martian spacecraft have travelled?

Earth
149 600 000 km



Sun

Mercury
57 909 000 km

Venus
108 200 000 km

Mars
227 940 000 km

Jupiter
778 400 000 km

Saturn
1 423 600 000 km

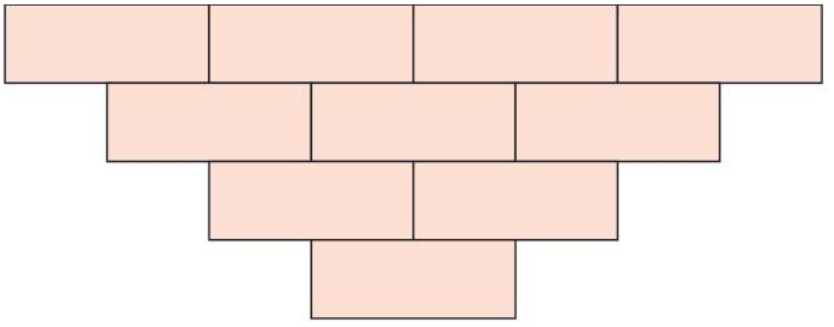
CHALLENGE

**19** Consider the diagram below.

You are to place the numbers 1 to 10 in each of the boxes in the diagram. Each box may contain one number, which should be equal to the difference of the two boxes above. There is more than one solution to this problem.

**a** Explain why the number 10 must be placed in the top row.

**b** How many different solutions can you find? Symmetrical solutions are considered to be the same.



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**Interactive skillsheet** Adding and subtracting whole numbers

**Investigation** Discoveries across the globe

**Topic quiz** 1C

# 1D Multiplying whole numbers

## Learning intentions

- ✔ I can use mental strategies to multiply whole numbers.
- ✔ I can calculate the result of multiplying two numbers together.
- ✔ I can multiply by powers of 10.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Multiplying whole numbers |
| Year 8 | 1C Multiplying and dividing whole numbers |

## Multiplication properties

| Multiplication (×) |
|---|
| Product |
| Times |
| Multiply |
| Groups of |

- Multiplication is repeated addition which can be reduced to multiplication of factors resulting in a **product**.

$$\underbrace{3 + 3}_{\text{repeated addition}} = \underbrace{2 \times 3}_{\text{factors}} = \underbrace{6}_{\text{product}}$$

- The **associative law of multiplication** states that regardless of how the numbers are grouped, the answer does not change.
  For example, $(3 \times 5) \times 2 = 3 \times (5 \times 2)$.
- The **commutative law of multiplication** states that the order in which numbers are multiplied does not change the answer.
  For example, $14 \times 2 = 2 \times 14$.
- The **distributive law** states that multiplication can be 'distributed' across a bracket. This means that large numbers can be broken down into a group of smaller numbers, which can then be multiplied separately.

$$\begin{aligned} 5 \times 32 &= 5 \times (30 + 2) \\ &= 5 \times 30 + 5 \times 2 \\ &= 150 + 10 \\ &= 160 \end{aligned}$$

## Short multiplication

- Short multiplication involves multiplying a large number by a one-digit number.

$$\begin{array}{r} {\scriptstyle +2}\phantom{7} \\ 37 \\ \times \quad 4 \\ \hline 148 \end{array}$$

1. Multiply 4 by 7 to get 28
2. Write the 8 in the ones column below
3. Add the 2 to the tens column above
4. Multiply 4 by 3 to get 12
5. Add the 2 tens to the 12 and get 14
6. Write 14 below

# Multiplying by powers of 10

- Multiplying a number by 10 increases each digit's place value by 1 column. Move the whole number one place value space to the left and insert a zero into the ones column.
- Multiplying a number by 100 increases each digit's place value by 2 columns. Move the whole number two place value spaces to the left and insert two zeros into the tens and ones columns.
- Multiplying a number by 1000 increases each digit's place value by 3 columns. Move the whole number three place value spaces to the left and insert three zeros into the ones, tens and hundreds columns.

$22 \times 10 = 220$
$22 \times 100 = 2200$
$22 \times 1000 = 22000$

## Example 1D.1 Using mental strategies to multiply numbers

Work out each product using the multiplication strategy given in brackets.

**a** $8 \times 16$ (the distributive law)

**b** $4 \times 8 \times 5$ (associative law)

**THINK**

**a** **1** Break the second number into tens and ones: $16 = 10 + 6$.

**2** Multiply each part of the second number by the first number: $8 \times 10 + 8 \times 6$.

**3** Add the results together: $80 + 48 = 128$.

**b** **1** Use brackets to group pairs of numbers that are easy to multiply together.

**2** Multiply the grouped numbers together: $8 \times 5 = 40$.

**3** Multiply the result by the third number: $4 \times 40 = 160$.

**WRITE**

**a** $8 \times 16 = 8 \times (10 + 6)$

$= 8 \times 10 + 8 \times 6$

$= 80 + 48$

$= 128$

**b** $4 \times 8 \times 5 = 4 \times (8 \times 5)$

$= 4 \times 40$

$= 160$

## Example 1D.2 Multiplying by a one-digit number

Use short multiplication to calculate $157 \times 3$.

**THINK**

**1** Set out the multiplication problem by lining up digits according to their place value.

**2** Multiply 3 by the 7 in the ones column. $3 \times 7 = 21$, put the 1 in the ones column below and put the 2 in the tens column above the 5.

**3** Multiply 3 by the 5 in the tens column. $3 \times 5 = 15$, add the extra 2 to get 17. Put the 7 in the tens column below and put the 1 in the hundreds column above the 1.

**4** Multiply 3 by the 1 in the hundreds column. $3 \times 1 = 3$, add the extra 1 to get 4. Put the 4 in the hundreds column below.

**WRITE**

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 157 \\ \times\ \ 3 \\ \hline 471 \\ \hline \end{array}$$

## Example 1D.3 Multiplying by 10, 100 or 1000

Calculate each product.

**a** $62 \times 10$ **b** $153 \times 100$ **c** $27 \times 1000$

**THINK**

**a** To multiply by 10, move 62 one place value space to the left and insert a zero into the empty space.

**b** To multiply by 100, move 153 two place value spaces to the left and insert two zeros into the empty spaces.

**c** To multiply by 1000, move 27 three place value spaces to the left and insert three zeros into the empty spaces.

**WRITE**

**a** 62_
$62 \times 10 = 620$

**b** 153_ _
$153 \times 100 = 15\,300$

**c** 27_ _ _
$27 \times 1000 = 27\,000$

**Helpful hints**

✔ Don't forget to keep the place value of the digits when using the distributive law.
For example,
$32 \times 3 = 30 \times 3 + 2 \times 3$
$32 \times 3 \neq 3 \times 2 + 2 \times 3$

✔ When you multiply by 10, you're not 'adding a zero.' Instead you're moving the digit one place to the left and then inserting a zero in the ones column. Think, 'insert a zero'.

ANS p538

# Exercise 1D Multiplying whole numbers

△ 1–5, 6–7(a–d), 8–10, 12, 13(a, b) □ 1–7, 9–11, 13, 14 ⬠ 2–4, 6, 7, 11–16

**1** Use your times tables knowledge to answer each product.

**a** $7 \times 4$ **b** $9 \times 2$ **c** $12 \times 11$ **d** $6 \times 7$

1D.1 **2** Use the distributive law to work out each product. Break up the larger number into tens and ones and multiply them separately with the smaller number.

**a** $15 \times 3$ **b** $14 \times 8$ **c** $5 \times 19$ **d** $7 \times 16$

**3** Use the associative law to work out each product. Use brackets to group pairs of numbers that are easy to multiply together.

**a** $9 \times 3 \times 2$ **b** $2 \times 6 \times 7$ **c** $8 \times 4 \times 9$ **d** $3 \times 5 \times 11$

1D.2 **4** Use short multiplication to calculate each product.

**a** 74 × 8 **b** 93 × 2 **c** 45 × 3 **d** 26 × 7

**e** 192 × 5 **f** 804 × 9 **g** 532 × 6 **h** 281 × 4

1D.3 **5** Calculate each product.

**a** 5 × 10 **b** 32 × 10 **c** 639 × 10 **d** 48 × 100

**e** 7 × 100 **f** 103 × 100 **g** 51 623 × 10 **h** 2784 × 1000

**6** Calculate the product using any multiplication strategy.

**a** 4 × 3 × 10 **b** 6 × 7 × 10 **c** 2 × 10 × 9 **d** 15 × 4 × 100

**e** 8 × 5 × 1000 **f** 41 × 3 × 1000 **g** 356 × 1000 × 2 **h** 125 × 1000 × 4

**7** Calculate each product. You can use the strategy of multiplying by 10 or 100 or 1000. Remember that 60 is the same as 10 × 6 or 6 × 10 (see the commutative law).

**a** 8 × 60 **b** 4 × 30 **c** 12 × 70 **d** 93 × 40

**e** 217 × 50 **f** 6345 × 20 **g** 52 × 800 **h** 428 × 300

**8** Copy this table into your workbook.

| First number × second number | Product | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the product |
|---|---|---|---|---|
| 10 × 10 | | | | |
| 100 × 10 | | | | |
| 1000 × 10 | | | | |
| 10 000 × 10 | | | | |

**a** Write your answer to each multiplication in the product column.

**b** Complete each row by writing the number of zeros in the first number, the second number and the product.

**c** Can you see a pattern? Explain how this pattern provides a quick method of doing multiplications like this.

**d** Use this method to calculate each product.

**i** 10 × 100 **ii** 100 × 100 **iii** 100 × 1000 **iv** 1000 × 1000

**9 a** To find 4000 × 100, the calculation can be written as 4 × 1000 × 100 or 1000 × 100 × 4.

**i** Calculate 1000 × 100.

**ii** Multiply this result by 4 to obtain your final answer.

**b** Use this strategy to calculate each of these products.

**i** 300 × 10 **ii** 700 × 100 **iii** 6000 × 100 **iv** 2000 × 1000

**v** 100 × 50 **vi** 1000 × 400 **vii** 10 000 × 8000 **viii** 100 × 9000

**10 a** One way to calculate 300 × 20 is to calculate 3 × 100 × 2 × 10 or 3 × 2 × 100 × 10.

**i** Calculate 3 × 2.

**ii** Calculate 100 × 10.

**iii** Multiply the results you found in parts **i** and **ii** and write your answer to 300 × 20.

**b** Use this strategy to calculate each product.

**i** 400 × 20 **ii** 3000 × 30 **iii** 200 × 600 **iv** 9000 × 500

**v** 70 × 800 **vi** 600 × 4000 **vii** 30 000 × 7000 **viii** 800 × 20 000

**11** Calculate each product.

**a** 40 × 10 × 200 **b** 60 × 900 × 3000 **c** 700 × 20 × 400 **d** 5000 × 300 × 80

**12** From a part-time job, James has saved $37 each week. To find the amount he has saved after 28 weeks, you can write the calculation in a number of ways.

**a** Decide if the following statements are true or false.

**i** $37 \times 28 = 37 \times (20 + 8)$

**ii** $37 \times 28 = (30 + 7) \times (20 + 8)$

**iii** $37 \times 28 = 28 \times 37$

**iv** $37 \times 28 = 37 \times 20 \times 37 \times 8$

**v** $37 \times 28 = 28 \times 30 + 28 \times 7$

**b** List two other ways to write the calculation.

**c** Perform the calculation to find the total amount that James has saved.

**13** Rory enjoys rowing with her team on the lake. She knows that for each full stroke, they glide 5 metres and for each half stroke they glide 3 metres.

**a** If the team takes 230 full strokes, how far will they glide?

**b** If the team takes 358 half strokes, how far will they glide?

**c** If the lake is only 1000 metres long, how many full strokes does the team need to take to cross the lake 3 times? Justify your answer.

**14** The train to Melbourne from Frankston has 6 carriages. During peak hour, all 88 seats per carriage are full.

**a** How many seats are there on the train?

**b** If there are 166 passengers standing in each carriage during peak hour, how many passengers are standing in the entire train?

**c** How many people are there on the train during peak hour?

**d** If another carriage containing 166 standing passengers and 88 sitting passengers was added to the train, how many people would there be on the 7-carriage train?

**15** A local electrician is ordering light bulbs for a new apartment complex. Each apartment needs 43 light bulbs.

**a** If there are 30 apartments in the complex, how many light bulbs does the electrician need to order?

**b** It takes approximately 4 minutes to install a light bulb. How long will it take the electrician to install all the light bulbs in the complex? Give your answer in minutes.

**c** Show that it takes the electrician one hour to install 15 light bulbs.



**16** Multiplication can be represented visually in many different ways. One visual representation of $15 \times 3$ is:

$$15 \times 3 = 3 \begin{array}{|c|c|} \hline 10 & 5 \\ \hline 30 & 15 \\ \hline \end{array} = 45$$

Draw five different visual representations of $24 \times 6$ by splitting up the factors of 24 into columns and the factors of 6 in rows.

Check your Student obook pro for these digital resources and more:

Investigation
Multiplying magic

Worksheet
Multiplying by 10, 100 or 1000

Topic quiz
1D

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

1A **1** Write each number in words and expanded form.

**a** 836 **b** 9654 **c** 629 000

1A **2** Write each number in standard form.

**a** Four hundred and nineteen

**b** Ninety-six thousand, two hundred

**c** Three hundred and twenty-five thousand, six hundred and fifty-six

1A **3** Write an approximation for each number by rounding to the nearest thousand.

**a** 9234 **b** 25 872 **c** 135 896

1B **4** Use any mental strategy to calculate each sum.

**a** 70 + 40 + 4 **b** 87 + 39 + 23 **c** 118 + 232 + 250

1B **5** Use the addition algorithm to calculate each of the following.

**a** 654 + 387 **b** 6826 + 8347 **c** 34 763 + 365

1B **6** The first AFLW game was held on Friday, 3 February 2017 at Princes Park. There was a crowd of 24 568 in attendance. The national television audience for the game was 896 000. What was the total number of people who saw the game?

1C **7** Use any mental strategy to calculate each difference.

**a** 350 – 60 **b** 586 – 46 **c** 287 – 143

1C **8** Use the subtraction algorithm to calculate each of the following.

**a** 964 – 732 **b** 4863 – 568 **c** 8256 – 4737

1C **9** The original books in the Harry Potter series have the following number of pages:

The Philosopher's Stone – 223
The Chamber of Secrets – 251
The Prisoner of Azkaban – 317
The Goblet of Fire – 636
The Order of the Phoenix – 766
The Half-Blood Prince – 607
The Deathly Hallows – 607

**a** What is the total number of pages in the series?

**b** What is the difference in pages between the longest and the shortest books?

1D **10** Use the distributive law to calculate each product.

**a** 13 × 7 **b** 6 × 18 **c** 23 × 8

1D **11** Calculate the product using any multiplication strategy.

**a** 536 × 9 **b** 437 × 40 **c** 675 × 700

1D **12** The list of ingredients for making 24 pancakes is to the right.

Gayi is planning to make pancakes for all the Year 7 students at her school. To have enough ingredients for one pancake each, Gayi needs six times the ingredients.

3 cups milk
2 eggs
4 teaspoons vanilla extract
4 cups self-raising flour
1 cup sugar
50 g butter

**a** How many Year 7 students are there at Gayi's school?

**b** How much of each ingredient does Gayi need?



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# 1E Long multiplication

## Learning intentions

- ✔ I can use long multiplication to multiply a two-digit number by a two-digit number.
- ✔ I can use long multiplication to multiply a three-digit number by a two-digit number.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Multiplying whole numbers |
| Year 8 | 1C Multiplying and dividing whole numbers |

## Long multiplication

- In long multiplication, the digits are lined up according to place value.
- The process of long multiplication uses the distributive law to multiply the first number by the digits of the second number separately and then find their sum.

Write →

$$\begin{array}{rrl} & {}^{2}34 & \\ \times & 26 & \\ \hline & 204 & (34\times6) \\ + & 680 & (34\times20) \\ \hline & 884 & \end{array}$$

Multiply by ones →

Multiply by tens →
Don't forget the zero(s)

Add the results →

### Example 1E.1 Using long multiplication to multiply a two-digit number by a two-digit number

Calculate the product using long multiplication.

$$\begin{array}{r} 68 \\ \times\ 37 \\ \hline \end{array}$$

**THINK**

1. Set up the multiplication problem by lining up the digits according to their place value.
2. Multiply $7 \times 8 = 56$. Write the 6 in the ones column below and write the 5 in the tens column above the 6.

   Multiply $7 \times 6 = 42$. Add the extra 5 to get 47. Write the 7 in the tens column and the 4 in the hundreds column below.
3. Start a new line and multiply $3 \times 8 = 24$. Write the 4 in the tens column below and the 2 in the hundreds column next to the 6.
4. Write a zero in the ones column as the next step is to multiply by tens and there will be zero ones.
5. Multiply $3 \times 6 = 18$. Add the extra 2 to get 20 hundreds. Write the 0 in the hundreds column and the 2 in the thousands below.
6. Add the results of the products and write the answer.

**WRITE**

$$\begin{array}{rr} & {}^{2\ 5} \\ & 68 \\ \times & 37 \\ \hline & 476 \\ + & 2040 \\ \hline & 2516 \end{array}$$

$68 \times 37 = 2516$

## Example 1E.2 Using long multiplication to multiply a three-digit number by a two-digit number

Calculate the product using long multiplication.

$$\begin{array}{r} 543 \\ \times\ \ 86 \\ \hline \end{array}$$

**THINK**

1. Set out the multiplication problem by lining up the digits according to their place value.
2. Multiply by ones: 543 × 6 = 3258
3. Multiply by tens: 543 × 80 = 43 440
4. Add the results of the products and write the answer underneath.

**WRITE**

$$\begin{array}{r} {\scriptstyle 3\ 2} \\ {\scriptstyle 2\ 1} \\ 543 \\ \times\ \ 86 \\ \hline 3258 \\ 43440 \\ \hline 46698 \end{array}$$

543 × 86 = 46 6968

**Helpful hints**

- ✔ Take care to line up digits by place value when setting up long multiplication problems to avoid making errors when adding the products together.
- ✔ Don't forget to write down the zeros! When multiplying by a multiple of 10, don't forget to insert a zero in the ones column.
- ✔ When multiplying a three-digit number by a three-digit number, insert three zeros in the third line of your algorithm. In the first line you are multiplying by ones, in the second line you are multiplying by tens, and in the third line you are multiplying by hundreds.

$$\begin{array}{r} 12 \\ \times\ 34 \\ \hline 48 \\ 360 \\ \hline 408 \end{array}$$

ANS p539

# Exercise 1E Long multiplication

△ 1–8, 9a(i, ii), 10–13 | □ 1, 2(e–h), 3, 4(c, d), 5(e–h), 6, 7(e–h), 8–10, 12, 14, 16 | ⬠ 2(g, h), 3, 5(e–h), 6, 7(e–h), 8, 9, 12, 13, 15–18

UNDERSTANDING AND FLUENCY

1E.1 **1** Calculate each product using long multiplication.

**a** $\begin{array}{r} 94 \\ \times\ 27 \\ \hline \end{array}$ **b** $\begin{array}{r} 13 \\ \times\ 11 \\ \hline \end{array}$ **c** $\begin{array}{r} 25 \\ \times\ 18 \\ \hline \end{array}$ **d** $\begin{array}{r} 37 \\ \times\ 22 \\ \hline \end{array}$

**2** Multiply the following pairs of numbers using long multiplication.

**a** 38 × 15 **b** 62 × 24 **c** 46 × 32 **d** 85 × 73

**e** 65 × 48 **f** 56 × 81 **g** 99 × 18 **h** 24 × 92

**3** Check your answers to question **2** using a calculator.

1E.2 **4** Calculate each product using long multiplication.

**a** $\begin{array}{r} 133 \\ \times\ 19 \\ \hline \end{array}$ **b** $\begin{array}{r} 205 \\ \times\ 34 \\ \hline \end{array}$ **c** $\begin{array}{r} 147 \\ \times\ 56 \\ \hline \end{array}$ **d** $\begin{array}{r} 281 \\ \times\ 64 \\ \hline \end{array}$

**5** Multiply the following pairs of numbers using long multiplication.

**a** 415 × 23 **b** 283 × 54 **c** 123 × 37 **d** 231 × 56

**e** 782 × 49 **f** 506 × 81 **g** 1059 × 18 **h** 2654 × 42

**6** Check your answers to question **5** using a calculator.

**7** Multiply the following pairs of numbers using long multiplication. Hint: Your third multiplication line should end with two zeros. This is because you are multiplying by hundreds.

**a** 346 × 125 **b** 865 × 347 **c** 624 × 253 **d** 937 × 625

**e** 538 × 124 **f** 361 × 253 **g** 497 × 516 **h** 702 × 281

**8** Check your answers to question **7** using a calculator.

**9** **a** Estimate these products by first rounding each number to the leading digit of each number, then multiplying.

**i** 591 × 82 **ii** 2175 × 93 **iii** 7856 × 304 **iv** 63 019 × 5647

**b** Check how close your estimations are to the exact result by performing the calculations.



**10** An average adult human heart beats around 72 times in a minute.

**a** How many times does it beat in one hour?

**b** How many times does it beat in one day?

**11** A school has 25 students in each of its 32 classes.

**a** How many students are enrolled at this school?

**b** On a particular day, three students are away from 15 of the classes and two students are away from 11 of the classes. How many students are at school on this day?

**12** Over the school holidays, a team of eight teenagers deliver take-away menus to homes near a pizza restaurant. How many menus are delivered in a week if each teenager visits 46 homes each day?

**13** Galápagos tortoises move extremely slowly, covering a distance of about 260 m in 1 hour.

**a** What distance could a Galápagos tortoise travel in 4 hours?

**b** Compare this result with the distance that a human could walk in 4 hours, assuming humans walk about 4500 m per hour.

**c** The longest lifespan on record belongs to a male Galápagos tortoise kept in a British military fort for 154 years.

Assume there are 365 days in a year. Calculate how long he lived in:

**i** months **ii** days **iii** hours

**iv** minutes **v** seconds

**14** **a** Assume there are 365 days in a year. Calculate how old you will be at your next birthday in:

**i** months **ii** days **iii** hours

**iv** minutes **v** seconds

**b** Estimate how many hours there are between now and your next birthday.

PROBLEM SOLVING AND REASONING

**15** Earth travels a distance of about 2 575 200 km each day. Assuming there are 365 days in a year, estimate, by rounding to the leading digits in your calculations, the distance Earth travels in one complete orbit around the Sun.

**16** In 2017, around 392 000 hectares of vegetation was cleared in Queensland. When 392 000 hectares of vegetation is cleared, it is estimated that 45 million animals, including koalas, birds and reptiles, die from loss of habitat.

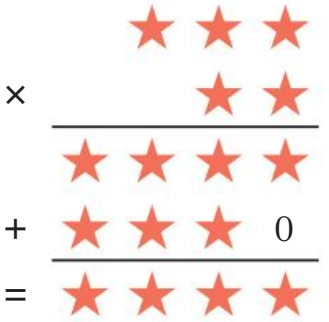
**a** If 392 000 hectares of vegetation is cleared every year from 2018–22, what is the total area of land that will be cleared over this time period?

**b** Estimate how many animals will die if this occurs.

**c** If 392 000 hectares of vegetation is cleared in 2020 and the area of vegetation cleared doubles each year, what is the total area of land that will be cleared from the start of 2020 to the end of 2 022?

CHALLENGE

**17** Fill in the stars using only the numbers 1, 2, 3, 4, 5 and 6 to solve the following problem. There may be more than one answer.




**18** Siân and Ajarné are building a model of a horse-riding arena they've seen in a picture. In the picture, the arena is a flat and level rectangle that is 6 cm long and 4 cm wide with a fence around the perimeter.

**a** Draw a picture of the arena and label the sides lengths.

**b** Determine the area of the picture of the arena. Recall that the area of a rectangle is the length multiplied by the width.

Siân and Ajarné decide to build their model arena on a piece of cardboard. They have a square piece of cardboard with a length of 25 cm.

**c** What is the dimensions of the largest arena that can be built on the cardboard if they want the ratio of the length and width of the rectangle to be the same?

**d** Determine the area of the model arena.

The girls decide to build the fence out of matchsticks. Each matchstick is 2mm wide.

**e** How many matchsticks will they need? Recall that there are 10 millimetres in 1 centimetre.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying whole numbers

Topic quiz
1E

# 1F Dividing whole numbers

## Learning intentions

✔ I can calculate quotients using short division.
✔ I can calculate quotients using long division.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Dividing whole numbers |
| Year 8 | 1C Multiplying and dividing whole numbers |

## Division

| Division (÷) |
|---|
| Quotient |
| Divide |
| Shared |
| Distributed |

- The **dividend** is the number that is divided by another number.
- The **divisor** is the number that divided the dividend.
- The **quotient** is the whole number result after the division has occurred.
- The **remainder** is the part of the dividend that is left over when the number is not exactly divisible by the divisor.

$$91 \div 6 = 15 \text{ remainder } 1$$

dividend divisor quotient remainder

- One number is said to be exactly divisible by another number when there is no remainder (the remainder is zero).
  For example, 91 is not exactly divisible by 6, but 36 is exactly divisible by 6.

## Short division

- Always start by dividing the leading digit of the dividend by the divisor. This is different from addition, subtraction and multiplication, where the calculation starts with the ones digits.

quotient

$$\begin{array}{r} 1\ 4\ 5 \\ 6\overline{)8^{2}7^{3}1} \end{array} \text{ remainder } 1$$

divisor dividend remainder

## Long division

- Long division is the same process as short divsion, however, all the steps are recorded. The same six steps listed are repeated until the answer is found.

Step 1: Divide
Step 2: Multiply
Step 3: Subtract
Step 5: Bring down
Step 6: Repeat or remainder

$$\begin{array}{r} 1 \\ 6\overline{)8\ 7\ 1} \\ -\ 6\phantom{\ 1} \\ 2\ 7\phantom{\ 1} \end{array} \qquad \begin{array}{r} 1\ 4\phantom{\ 1} \\ 6\overline{)8\ 7\ 1} \\ -\ 6\phantom{\ 1} \\ 2\ 7\phantom{\ 1} \\ -\ 2\ 4\phantom{\ 1} \\ 3\ 1\phantom{\ 1} \end{array} \qquad \begin{array}{r} 1\ 4\ 5 \text{ remainder } 1 \\ 6\overline{)8\ 7\ 1} \\ -\ 6\phantom{\ 1} \\ 2\ 7\phantom{\ 1} \\ -\ 2\ 4\phantom{\ 1} \\ 3\ 1\phantom{\ 1} \\ -\ 3\ 0\phantom{\ 1} \\ 1\phantom{\ 1} \end{array}$$

## **Example 1F.1** Using short division

Use short division to calculate each quotient and remainder.

**a** $4\overline{)98}$ **b** $7\overline{)1459}$

**THINK**

**a** **1** How many 4s in 9? Write 2 above the 9 on the quotient line. Work out the remainder. $4 \times 2 = 8$, so remainder is $9 - 8 = 1$. Write the 1 next to the 8 to get 18.

**2** How many 4s in 18? Write 4 above the 8 on the quotient line. Work out the remainder. $4 \times 4 = 16$, so remainder is $18 - 16 = 2$.

**b** **1** How many 7s in 1? Write a zero above the 1. Underline 1 and 4.

**2** How many 7s in 14? Write 2 above the 4 on the quotient line.

**3** How many 7s in 5? Write a zero above the 5. Underline 5 and 9.

**4** How many 7s in 59? Write 8 above the 9 on the quotient line. Work out the remainder. $7 \times 8 = 56$, so remainder is $59 - 56 = 3$.

**WRITE**

**a**
$$\begin{array}{r} 2\,4 \\ 4\overline{)9^{1}8} \end{array} \text{ remainder } 2$$

$98 \div 4 = 24$ remainder 2

**b**
$$\begin{array}{r} 0208 \\ 7\overline{)1459} \end{array} \text{ remainder } 3$$

$1459 \div 7 = 208$ remainder 3

## **Example 1F.2** Using long division

Use long division to calculate $6492 \div 19$.

**THINK**

**1** List multiples of 19.

**2** Divide: How many 19s in 6? Write a zero above the 6. Underline 6 and 4.
How many 19s in 64? Write 3 above 4 on the quotient line.
Multiply: $19 \times 3 = 57$
Subtract: Remainder is $64 - 57 = 7$
Bring down 9 and write it beside the remainder of 7.
Then, the next number for dividing 79.

**3** Divide: How many 19s in 79? Write 4 above 9 on the quotient line.
Multiply: $19 \times 4 = 76$
Subtract: Remainder is $79 - 76 = 3$
Bring down 2 and write it beside the remainder of 3.
Then, the next number for dividing 32.

**4** Divide: How many 19s in 32? Write 1 above 2 on the quotient line.
Multiply: $19 \times 1 = 19$
Subtract: Remainder is $32 - 19 = 13$

**5** Check and write the answer.

**WRITE**

$$\begin{array}{r} 0\,3\phantom{\,9\,2} \\ 19\overline{)6\,4\,9\,2} \\ -\,5\,7\phantom{\,9\,2} \\ \hline 7\,9\phantom{\,2} \end{array}$$

$$\begin{array}{r} 0\,3\,4\phantom{\,2} \\ 19\overline{)6\,4\,9\,2} \\ -\,5\,7\phantom{\,9\,2} \\ \hline 7\,9\phantom{\,2} \\ -\,7\,6\phantom{\,2} \\ \hline 3\,2 \end{array}$$

$$\begin{array}{r} 0\,3\,4\,1 \text{ remainder } 13 \\ 19\overline{)6\,4\,9\,2} \\ -\,5\,7\phantom{\,9\,2} \\ \hline 7\,9\phantom{\,2} \\ -\,7\,6\phantom{\,2} \\ \hline 3\,2 \\ -\,1\,9 \\ \hline 1\,3 \end{array}$$

$6492 \div 19 = 341$ remainder 13

**Helpful hints**

- ✔ Remember that division starts from the left, dividing the largest place value digit first.
- ✔ Remainders and decimals are not the same thing! For example, remainder 2 is not the same as 0.2.
- ✔ Remember that 0341 = 341, as a zero in the first place value doesn't change the value of the number. However, 308 is not the same as 38. In this case, the zero shows that there are zero tens and keeps the 3 in the hundreds column and the 8 in the ones column.
- ✔ When dividing, the remainder needs to be less than the divisor.
- ✔ When using long division, it can be helpful to have the list of steps handy until you are confident in the skill.

Step 1: Divide
Step 2: Multiply
Step 3: Subtract
Step 5: Bring down
Step 6: Repeat or remainder

ANS p539

# Exercise 1F Dividing whole numbers

△ 1–4, 5(a–d), 6(a, b), 7–10, 12–14, 16(a, b)

□ 1–4, 5(a, c, e, g), 6, 7, 9–11, 15, 16(a, b), 18

⬠ 3(g, h), 4(c, d), 5(e–h), 6, 7, 10, 11, 13, 17–20

**1** For each division, identify:

**i** the dividend **ii** the divisor

**iii** the quotient **iv** the remainder

**a** $9 \div 2 = 4$ remainder 1 **b** $17 \div 7 = 2$ remainder 3 **c** $30 \div 5 = 6$

1F.1 **2** Use short division to calculate each quotient and remainder.

**a** $2\overline{)13}$ **b** $5\overline{)23}$ **c** $3\overline{)17}$ **d** $10\overline{)25}$

**e** $8\overline{)38}$ **f** $4\overline{)26}$ **g** $9\overline{)66}$ **h** $6\overline{)48}$

**3** Use short division to calculate each quotient and remainder.

**a** $538 \div 4$ **b** $756 \div 6$ **c** $172 \div 3$ **d** $1229 \div 5$

**e** $877 \div 5$ **f** $489 \div 7$ **g** $668 \div 8$ **h** $1234 \div 4$

**4** Another way to write a division calculation is as a fraction.
For example, $57 \div 3$ is the same as $\frac{57}{3}$. The horizontal line between the two numbers replaces the division sign. Perform each division using short division.

**a** $\frac{63}{9}$ **b** $\frac{1470}{6}$ **c** $\frac{658}{7}$ **d** $\frac{1251}{3}$

1F.2 **5** Use long division to calculate each quotient and remainder.

**a** $542 \div 21$ **b** $739 \div 18$ **c** $884 \div 26$ **d** $798 \div 12$

**e** $465 \div 22$ **f** $881 \div 41$ **g** $7462 \div 35$ **h** $1234 \div 56$

**6** Use long division to calculate each quotient and remainder.

**a** $1658 \div 43$ **b** $4509 \div 45$ **c** $15\,789 \div 13$ **d** $12\,023 \div 80$

**7** Evaluate each of the following.

**a** $\frac{272}{17}$ **b** $\frac{3350}{25}$ **c** $\frac{47\,136}{32}$ **d** $\frac{36\,088}{52}$

**8** Copy this table.

| First number ÷ second number | $\frac{\text{First number}}{\text{second number}}$ | Quotient | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the quotient |
|---|---|---|---|---|---|
| 10 ÷ 10 | $\frac{10}{10}$ | | | | |
| 100 ÷ 10 | $\frac{100}{10}$ | | | | |
| 1000 ÷ 10 | $\frac{1000}{10}$ | | | | |
| 10 000 ÷ 10 | $\frac{10\,000}{10}$ | | | | |
| 100 ÷ 100 | $\frac{100}{100}$ | | | | |
| 1000 ÷ 100 | $\frac{1000}{100}$ | | | | |
| 10 000 ÷ 100 | $\frac{10\,000}{100}$ | | | | |

**a** Write your answer to each division in the 'Quotient' column.

**b** Complete each row of the table by writing the number of zeros in the first number, the second number and the quotient.

**c** Can you see a pattern? Explain how this pattern provides a shortcut for dividing by multiples of 10.

**d** Use this method to work out the following quotients.

**i** 10 000 ÷ 10 **ii** 100 000 ÷ 100 **iii** 1000 ÷ 1000 **iv** 10 000 ÷ 1000

**9 a** To work out 5000 ÷ 100, the calculation can be written as $\frac{5000}{100}$.

**i** Calculate 1000 ÷ 100.

**ii** Multiply this result by 5 to obtain your final answer.

**b** Use this strategy to evaluate the quotient of each of the following.

**i** 200 ÷ 10 **ii** 600 ÷ 100 **iii** 9000 ÷ 100 **iv** 4000 ÷ 1000

**v** 800 ÷ 10 **vi** 3000 ÷ 100 **vii** 50 000 ÷ 1000 **viii** 6000 ÷ 10

**10 a** One way to evaluate 600 ÷ 20 is to write the calculation as $\frac{600}{20}$.

**i** Calculate 6 ÷ 2.

**ii** Calculate 100 ÷ 10.

**iii** Multiply the results obtained in parts **i** and **ii** and write your answer to 600 ÷ 20.

**b** Use this strategy to perform each division.

**i** $\frac{900}{30}$ **ii** $\frac{8000}{20}$ **iii** $\frac{1200}{60}$ **iv** $\frac{25\,000}{500}$

**v** 6000 ÷ 300 **vi** 80 000 ÷ 4000 **vii** 70 000 ÷ 700 **viii** 1600 ÷ 80

**11 a** Estimate the quotient of each of the following by first rounding each number to the first digit of each number before dividing. (Hint: use your strategy from question **10**.)

**i** 627 ÷ 33 **ii** 5940 ÷ 18 **iii** 3852 ÷ 214 **iv** 83 490 ÷ 3795

**b** Use a calculator to check how close your estimations are to the exact result.

**12** Georgia is helping to arrange a birthday party for her younger brother. She has 165 sweets to share among 15 party bags. To work out the number of sweets in each party bag, you can describe the calculation in a number of ways.

**a** Which of the following number sentences correctly describes the information above?

**A** Divide 165 by15.

**B** How many times does 165 go into 15?

**C** Evaluate 166 ÷ 15.

**D** Find the quotient when 15 is divided by 165.

**E** Find the product of 165 and 15.

**b** List one other way to write the calculation.

**c** Determine the number of sweets in each party bag.

**13** Chris can type (on his computer) at a rate of 68 words each minute.

**a** How long would it take him to type a message of 272 words?

**b** Explain how you could check the answer to this division problem using multiplication.

**14** Charlotte swam 1500 m in a 50 m swimming pool. How many laps of the pool did she complete?

**15** The ancestor of the common wombat and the southern hairy-nosed wombat is the giant wombat (diprotodons), a marsupial that disappeared from Australia around 40 000 years ago. On average, giant wombats were 3 m in length and 180 cm in height, with a mass of 2000 kg. The mass of an average common wombat is 37 kg. The photo shows a hairy-nosed wombat and its mass.

**a** How many common wombats, when their weights are added together, would be approximatively equivalent to one giant wombat?

**b** How many hairy-nosed wombats, when their weights are added together, would be approximatively equivalent to one giant wombat?

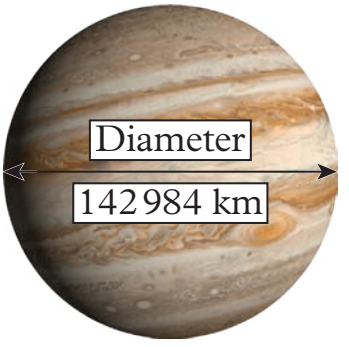
**16** Isobel and Connor are helping to make food parcels for a charity organisation. There are 1350 bags of rice to be shared among 84 food parcels.

**a** How many bags of rice will be in each food parcel?

**b** How many bags of rice will be left over?

**c** Determine the number of food parcels that could be made up with an equal share of the bags of rice so that no bags are left over. There are many solutions to this question.

**17** Earth orbits the Sun in about 365 days, while the planet Jupiter completes an orbit around the Sun in about 4333 Earth days. Earth has a diameter of approximately 12 756 km.




**a** Estimate the number of times that Earth would complete a full orbit around the Sun for each one of Jupiter's orbits. First round each number to its leading digit.

**b** If Earth was drawn to the same scale as Jupiter is shown in the photo, approximately how many Earths would fit across the equator of Jupiter?

CHALLENGE

**18** An apartment building has twelve apartments that are numbered from 110 to 121. The manager has twelve key cards for the apartments numbered from 1 to 12.

The manager has forgotten which key card is for which apartment but knows that each apartment number is divisible by the number of its key card.

Determine which key cards pair with each apartment.

**19** Below are the numbers 1 to 9. Using only the operations +, −, × and ÷, find at least one way to make 100 using all the digits.

**1 2 3 4 5 6 7 8 9**

**+ − × ÷**

**= 100**

**20** The following is a long division problem showing all the working out steps and the result. There is no remainder. When this book was printed, the numbers were replaced with letters! Fortunately, each letter represents the same number throughout the problem. Construct the original problem.

```
        I A E D H E
A B ) C D F G G C
    − A B
        B F
      − G C
        A A G
      − A I B
              J G
            − J I
                G C
              − G C
                  I
```

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- Interactive skillsheet — Dividing whole numbers
- Interactive skillsheet — Multiplying and dividing by powers of 10
- Investigation — Lolly bags
- Topic quiz — 1F

# 1G Indices and square roots

## Learning intentions

- ✔ I can convert numbers between index and expanded form.
- ✔ I can evaluate numbers in index form.
- ✔ I can evaluate the square root of a number.

**Inter-year links**

| | |
|---|---|
| Year 8 | 4A Indices |
| Year 9 | 2A Indices |

## Index notation

- An **index** or **exponent** of a number indicates how many times a number is multiplied by itself. The number that is being multiplied is called the **base**. A number is in **index form** when it is written as a base with an index.

index/exponent

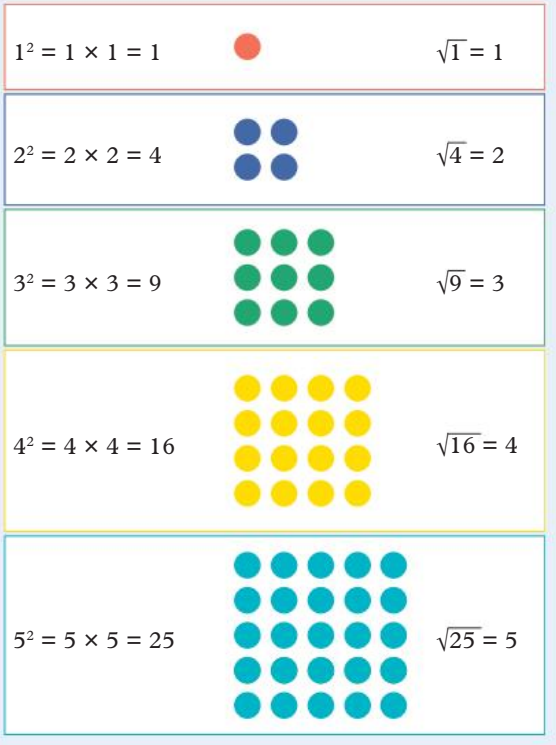
base → $2^3 = 2 \times 2 \times 2 = 8$

index form  expanded form  basic numeral

→ In the example above, the numbers in index form read '2 to the power of 3'. Similarly, $3^4$ is read as '3 to the power of 4'.

## Perfect square numbers

- **Perfect square** numbers are numbers that can be represented in a square pattern. They can also be expressed as the product of two equal whole numbers.
  For example, $3 \times 3 = 9$, so 9 is a perfect square number.
- To **square** a number is to multiply it by itself or 'raise it to the power of 2'.
  For example, squaring 3, or raising 3 to the power of 2, can be written numerically as:
  $3^2 = 3 \times 3 = 9$.




## Square roots

- To find the **square root** of a number, find the number that when squared (raised to the power of 2) results in the original number.
  For example, $\sqrt{9} = 3$ because when 3 is multiplied by itself the result is 9.
- The function of a square root of a number is denoted by the radical symbol $\sqrt{\ }$ .

## **Example 1G.1** Converting numbers to index form

Write each repeated multiplication in index form.

**a** $8 \times 8 \times 8 \times 8 \times 8 \times 8$

**b** $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$

**THINK**

**a** Identify the base and the index. The base is the number that is repeatedly multiplied, 8. The index represents the number of times the base appears in expanded form, 6.

**b** Identify the base and the index. Notice that there are two different bases. Write the first base, 3, with its index, 2, multiplied by the second base, 4, with its index, 5.

**WRITE**

**a** $8 \times 8 \times 8 \times 8 \times 8 \times 8$
$= 8^6$

**b** $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$
$= 3^2 \times 4^5$

## **Example 1G.2** Calculating the value of a number in index form

Write each of the following in expanded form and calculate its value.

**a** $4^3$

**b** $2^4 \times 3^2$

**THINK**

**a** **1** Identify the base and the index. The base is 4 and the index is 3, so 4 is multiplied by itself 3 times.

**2** Perform the multiplication.

**b** **1** Identify the base and the index. In this case, there are two bases, so each base needs to be expanded separately.

**2** Perform the multiplication.

**WRITE**

**a** $4^3 = 4 \times 4 \times 4$

$= 16 \times 4$
$= 64$

**b** $2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 4 \times 4 \times 9$
$= 16 \times 9$
$= 144$

## **Example 1G.3** Calculating the square root

Calculate the value of each of the following square roots.

**a** $\sqrt{16}$

**b** $\sqrt{144}$

**THINK**

Use the times tables or square numbers to determine which number multiplied by itself is equal to the number under the square root symbol.

**WRITE**

**a** $4 \times 4 = 16$
So, $\sqrt{16} = 4$

**b** $12 \times 12 = 144$
So, $\sqrt{144} = 12$

**Helpful hints**

✔ Don't confuse indices and multiplication!

For example, $2^3 = 2 \times 2 \times 2$
$2^3 \neq 2 \times 3$

✔ Write your indices carefully – they should be smaller than the base and sit on the right shoulder of the base. Your $4^2$ should look different to your 42!

$4^2 \neq 42$

ANS p540

# Exercise 1G Indices and square roots

△ 1–3, 4–7(b, d, f, h), 8, 9(1st, 2nd columns), 10, 11(a, b), 12, 13, 15, 17

□ 1–3, 4–7(f, g, h), 8, 9(1st, 3rd columns), 10–14, 16, 18, 19, 21

⬠ 1(e, f), 3(e, f), 4–7(f, g, h), 8(c, d), 9(f, h, i), 10, 11(c, d), 14, 17, 20, 22–24

UNDERSTANDING AND FLUENCY

**1G.1** **1** Write each repeated multiplication in index form.

**a** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
**b** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
**c** $11 \times 11 \times 11$
**d** $20 \times 20 \times 20 \times 20 \times 20$
**e** $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
**f** $300 \times 300 \times 300 \times 300$

**2** How would you read each answer obtained in question **1**? Write each answer using words.

**3** Write each repeated multiplication in index form.

**a** $7 \times 7 \times 7 \times 9 \times 9 \times 9 \times 9$
**b** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2$
**c** $3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
**d** $8 \times 8 \times 8 \times 8 \times 13 \times 13$
**e** $2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7$
**f** $19 \times 19 \times 23 \times 23 \times 23 \times 23 \times 31$

**1G.2** **4** Write each of the following in expanded form and calculate its value.

**a** $3^2$ **b** $2^5$ **c** $7^3$ **d** $5^2$ **e** $9^4$ **f** $4^3$ **g** $1^5$ **h** $6^3$

**5** Calculate the value of each of the following.

**a** $2^4$ **b** $3^4$ **c** $7^2$ **d** $5^3$ **e** $11^3$ **f** $12^2$ **g** $10^3$ **h** $16^2$

**6** Write each product in expanded form and calculate its value.

**a** $2^3 \times 4^2$
**b** $3^4 \times 5^2$
**c** $8^2 \times 10^3$
**d** $3^5 \times 1^4$
**e** $2^6 \times 7^2$
**f** $10^2 \times 3^3$
**g** $9^4 \times 11^2$
**h** $6^2 \times 3^2$

**1G.3** **7** Calculate the value of each of the following square roots.

**a** $\sqrt{9}$
**b** $\sqrt{36}$
**c** $\sqrt{64}$
**d** $\sqrt{1}$
**e** $\sqrt{49}$
**f** $\sqrt{121}$
**g** $\sqrt{144}$
**h** $\sqrt{4008}$

**8** Without using a calculator, complete each statement.

**a** $11^2 = 121$ so $\sqrt{121} =$ _____
**b** $35^2 = 1225$ so $\sqrt{1225} =$ _____
**c** $61^2 = 3721$ so $\sqrt{___} = 61$
**d** $298^2 = 88\,804$ so $\sqrt{___} = 298$

**9** Evaluate each of the following by first working out the value of any numbers written in index form.

**a** $3^4 + 2^3$
**b** $10^2 + 5^2$
**c** $8^2 - 2^6$
**d** $4^3 - 1^9$
**e** $10^2 \div 5^2$
**f** $6^4 \div 3^3$
**g** $4^2 + 2^4 + 3^3$
**h** $9^2 + 2^2 - 1^5$
**i** $10^2 \times 3^2 \times 2^2$
**j** $4 \times 2^3 \times 5$
**k** $7^2 \times 8 \times 9^2$
**l** $9^3 \times 6^2 \times 8^1$

**10** Which number is bigger: $3^5$ or $5^3$?

**11** Arrange the numbers in each list from smallest to largest. You may like to use a calculator to help you.

**a** $3^2$, $4^5$, $2^3$, $5^4$ **b** $7^6$, $6^7$, $1^{50}$, $50^3$ **c** $3^{10}$, $9^4$, $6^5$, $10^3$ **d** $8^3$, $3^8$, $4^6$, $12^2$

**12** State if each of the following is true or false.

**a** $12 < \sqrt{121} < 13$ **b** $8 < \sqrt{16} < 10$ **c** $7 < \sqrt{64} < 9$ **d** $11 < \sqrt{144} < 13$

**13** The square root of a non-perfect square number is not a whole number. We can, however, find two consecutive whole numbers that the square root is between. For example, what is $\sqrt{10}$? Recall that $3^2 = 9$ and $4^2 = 16$. Since 10 is between the perfect squares 9 and 16, $\sqrt{10}$ is between 3 and 4. This is useful for estimating square roots.

State if each of the following is true or false.

**a** $2 < \sqrt{7} < 3$ **b** $5 < \sqrt{26} < 6$ **c** $7 < \sqrt{30} < 8$ **d** $9 < \sqrt{80} < 10$

**14** Write the closest whole numbers that each square root lies between.

**a** $__ < \sqrt{15} < __$ **b** $__ < \sqrt{33} < __$ **c** $__ < \sqrt{40} < __$

**d** $__ < \sqrt{99} < __$ **e** $__ < \sqrt{120} < __$ **f** $__ < \sqrt{214} < __$

**15** **a** Complete this table.

| Index form | Expanded form | Basic numeral | Number of zeros in the basic numeral |
|---|---|---|---|
| $10^1$ | 10 | 10 | 1 |
| $10^2$ | $10 \times 10$ | 100 | |
| $10^3$ | | | |
| $10^4$ | | | |
| $10^5$ | | | |

**b** Can you see a pattern? This pattern provides a quick method for finding the value of numbers in index form with a base of 10. Describe this method.

**c** Use this method to evaluate the following.

**i** $10^8$ **ii** $10^6$ **iii** $10^{15}$ **iv** $10^{10}$

**d** Use the pattern in the table to help you deduce the value of $10^0$.

**e** Write each number in index form using a base of 10 and the appropriate index.

**i** 100 **ii** 10 000 **iii** 1 **iv** 10 000 000

**16** Explain how finding the square of a number and finding the square root of a number are related.

**17** After 1 week, the weight of a baby mouse is double its weight at birth. For the next few weeks, its weight continues to double each week.

**a** What number do you multiply by to double a quantity?

**b** After 1 week, the mouse would have a weight that is twice as big as the birth weight. After 2 weeks, the weight would be $2 \times 2$ or 4 times as big as the birth weight. Write each number in index form.

**c** In index form, write the number of times the weight is bigger than the birth weight after three weeks.

**d** How much bigger is the baby mouse after five weeks?

**18** A colony of bacteria grows very quickly and triples its size each day.

| | Size of colony (in millions) | | |
|---|---|---|---|
| **Number of days** | **Expanded form** | **Index form** | **Number of bacteria (in millions)** |
| 1 | 3 | $3^1$ | 3 |
| 2 | $3 \times 3$ | $3^2$ | 9 |
| 3 | $3 \times 3 \times 3$ | | |
| 4 | | | |
| 5 | | | |

**a** Complete the table above.

**b** What is the size of the colony after:

**i** 6 days?

**ii** 1 week?

**iii** 2 weeks?

**c** How long does it take for the bacteria colony to be:

**i** 27 times larger than day 1?

**ii** 243 times larger than day 1?

**iii** 6561 times larger than day 1?

**19** Dianna sends a text message to four of her friends. Each friend forwards it to another four people, who each then send it to another four people.

**a** How many text messages in total have been sent?

**b** Explain how indices can be used to solve this problem.

**20** Over the summer school holidays, Taylor is offered a part-time job for which he will earn \$100 per week. The job is available for 4 weeks. Taylor decides to discuss a different payment plan with his prospective boss. 'How about paying me only \$5 in the first week and then in each of the other weeks paying me five times as much as the week before.' The boss thinks this new plan might save her money. Which payment plan do you think the boss should go with? Use calculations to justify your answer.

**21** Arrange the numbers 1 to 16 in pairs so that each pair adds up to a perfect square.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16



**22** Interesting patterns emerge when we investigate square numbers. Consider the difference between two consecutive square numbers. Recall that consecutive means one following directly after another.

**a** The square numbers are 1, 4, 9, 16, 25…. Find difference between the first 10 pairs of consecutive square numbers, starting with 1 and 4. Record your results in a table.

**b** What do all these differences have in common?

**c** Consecutive numbers in a sequence can be written as $n$ and $n + 1$ where $n$ is a whole number. Write a number sentence showing the difference between two consecutive square numbers using $n$ and $n + 1$.

**d** Using algebra, we can simplify the number sentence to $2 \times n + 1$. Explain how $2 \times n + 1$ relates to part **b**.

**23** An evil genius has discovered a new toxic serum that, when consumed, decreases a person's weight by taking the square root of their current weight, every hour on the hour.
The evil genius tests the serum on four brave volunteers.

**a** At the time they consume the serum, Person A weighs 99 kg, Person B weighs 142 kg, Person C weighs 56 kg, and Person D weighs 82 kg. Without using a calculator, approximate the weight of each person after the first hour has past.

**b** Without using a calculator, approximate the weight of each person after the second hour has past. Remember, the serum will decrease each person's weight by the square root of their new weight, not their original weight.

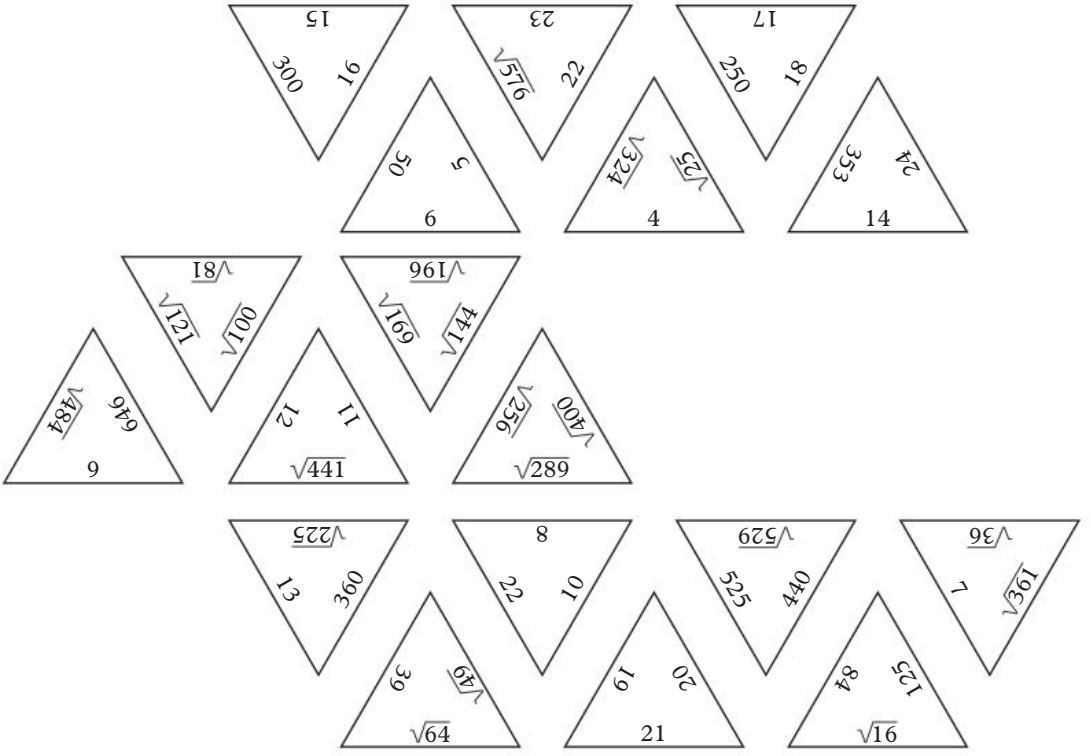
**c** Without using a calculator, approximate the weight of each person after the third hour has past.

**d** Estimate how much each person will weigh after 10 hours?

**e** Will the weight of each person ever reach 0 kg?

**f** Determine a mathematical serum formula that could hypothetically decease a person's weight to 0 kg after 13 hours.

**24** Consider the following triangles. On the edges are square roots and basic numerals. Match the square roots to their result. Your finished puzzle should look like a diamond.
You may like to draw the triangles and cut them out so that you can move them around easily. For an extra hard challenge, do not use a cut-out version of the triangle to help you.




Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Indices
- Interactive skillsheet — Squares and square roots
- Investigation — Patterns and squared numbers
- Topic quiz — 1G

# 1H Order of operations

## Learning intentions

- ✔ I can order operations in calculations involving two or more operations.
- ✔ I can use the BIDMAS acronym to solve problems involving order of operations.

Inter-year links

| | |
|---|---|
| Years 5/6 | Order of operations |
| Year 8 | 1H Order of operations |

## Order of operations

- Operations at the same ranking are performed in order from left to right.
- Where there is more than one set of brackets in the problem, perform the operations inside the innermost brackets first.

  **BIDMAS** can be used to help remember the order of operations.

| | | |
|---|---|---|
| B | **Brackets** | Operations inside brackets are always performed first.<br>For example, $5 \times (10 - 2) = 5 \times 8 = 40$ |
| I | **Indices** | Next, evaluate indices and square roots<br>For example, $3^3 - 4 = 27 - 4 = 23$ |
| D<br>M | **Division**<br>**Multiplication** | Then working from **left to right,** perform any multiplication or division before adding or subtracting.<br>For example, $5 \times 3 - 4 + 8 \div 4 = 15 - 4 + 2 = 13$ |
| A<br>S | **Addition**<br>**Subtraction** | Finally, working from **left to right,** perform any addition and subtraction.<br>For example, $15 - 4 + 2 = 13$ |

### Example 1H.1 Order of operations

Without using a calculator, evaluate each of the following.

**a** $3 + 8 \div 2 - \sqrt{4}$ **b** $7 \times 2^3 - 9 \times 5$

**THINK**

**a** 1 Indices and square roots are worked out before ×, ÷, + and −. Calculate $\sqrt{4} = 2$.

2 Division is performed before addition. Calculate $8 \div 2 = 4$.

3 Working from left to right, perform the addition then the subtraction.

**b** 1 Indices and square roots are worked out before ×, ÷, + and −. Calculate $2^3 = 8$.

2 Multiplication is performed before subtraction so, working from left to right, first calculate $7 \times 8 = 56$.

3 Perform the next multiplication. Calculate $9 \times 5 = 45$.

4 Perform the subtraction.

**WRITE**

**a** $3 + 8 \div 2 - \sqrt{4}$
$= 3 + 8 \div 2 - 2$
$= 3 + 4 - 2$
$= 5$

**b** $7 \times 2^3 - 9 \times 5$
$= 7 \times 8 - 9 \times 5$
$= 56 - 9 \times 5$
$= 56 - 45$
$= 11$

## **Example 1H.2** Order of operations with brackets

Without using a calculator, evaluate each of the following.

**a** $16 \div (10 - 2) + 3 \times 7$ **b** $8 + [12 \div (9 - 5)]^2 - 1$

**THINK**

**a** **1** Perform any operations inside brackets first. Calculate $10 - 2 = 8$.

**2** Perform division and multiplication before addition. Working from left to right, first calculate $16 \div 8 = 2$.

**3** Perform multiplication before addition. Calculate $3 \times 7 = 21$.

**4** Perform the addition.

**b** **1** There are two sets of brackets. Perform the operation in the innermost set of brackets first. Calculate $9 - 5 = 4$.

**2** Perform the operation in the remaining set of brackets. Calculate $12 \div 4 = 3$.

**3** Apply indices before addition and subtraction. Calculate $3^2 = 9$.

**4** Working from left to right, perform the addition then the subtraction.

**WRITE**

**a** $16 \div (10 - 2) + 3 \times 7$
$= 16 \div 8 + 3 \times 7$
$= 2 + 3 \times 7$
$= 2 + 21$
$= 23$

**b** $8 + [12 \div (9 - 5)]^2 - 1$
$= 8 + [12 \div 4]^2 - 1$
$= 8 + 3^2 - 1$
$= 8 + 9 - 1$
$= 17 - 1$
$= 16$

**Helpful hints**

BIDMAS

- ✔ Remember to use BIDMAS and not to just work from left to right.
- ✔ Remember to always complete any operations in brackets first!
- ✔ Calculators don't always get the order of operations correct. If you are using a basic calculator, you will need to type in the calculation in a way that follows the BIDMAS rules.

| Addition (+) | Subtraction (−) | Multiplication (×) | Division (÷) |
|---|---|---|---|
| Sum | Difference | Product | Quotient |
| Plus | Take away | Times | Divide |
| More than | Less than | Multiply | Shared |
| Increase | Fewer | Groups of | Distributed |
| Total | Reduce | | |
| Together | Minus | | |

ANS p542

# **Exercise 1H** Order of operations

△ 1–3(1st, 2nd columns), 4–5(1st column), 6(a–c), 7–10, 14 □ 1, 2–3(3rd column), 4–5(2nd column), 6(a, c, e), 7, 9, 11, 12, 14–16 ⬠ 1(i–l), 2–3(3rd column), 4(g, h), 5–6(d, e, f), 7, 9, 12, 13, 16–18

**1H.1** **1** Without using a calculator, evaluate each of the following.

**a** $6 + 15 \div 3$ **b** $9 - 4 \times 2$ **c** $7 + 3 \times 6 - 4$ **d** $12 \div 4 + 7 \times 8$

**e** $4 \times 4 + 6 \div 2$ **f** $2 + 9 \times 4 \div 6$ **g** $10 \times 5 \div 5 + 2$ **h** $12 \div 3 + 4 - 8$

**i** $7 + 25 \div 5 \times 3 + 2$ **j** $16 - 8 \div 4 \times 2 + 5$ **k** $6 \times 4 - 10 \times 2$ **l** $9 \times 3 - 6 \times 3 \div 2$

2 Following the correct order of operations, complete the calculations below.

**a** $10 \times 6 - 5 \times 9$ **b** $8^2 - 5 \times 3 + 11$ **c** $21 - 3^2 \times 2 + 1$

**d** $4 \times 3^3 - 6^2 \times 2 + 1$ **e** $\sqrt{9} + 9 \times 8 - 2^3$ **f** $34 - 3^3 + 2 \times 4^2$

**g** $\sqrt{144} \div 12 + 2^2 - 5$ **h** $22 - 2 \times 11 + 4^2 \div \sqrt{16}$ **i** $7 - 3^2 + \sqrt{4} \times 3^3$

**j** $84 \div 7 \times 2^2 - 5 + \sqrt{36}$ **k** $4 \times \sqrt{49} + 6^2 - \sqrt{81}$ **l** $5 \times \sqrt{100} - \sqrt{9} \div 3 \times 7^2$

1H.2 3 Without using a calculator, evaluate each of the following. Remember to start with the innermost bracket.

**a** $28 \div (12 - 5) + 2 \times 6$ **b** $(9 + 3) \div (15 - 13)$ **c** $4^3 - 10 \times (3 + 1)$

**d** $\sqrt{64} \div 4 + (2 \times 3) + 7$ **e** $\sqrt{100} \times 5 - 2 \times (2 + 3)$ **f** $3 \times [8 + (2 \times 9 - 4)] + 20$

**g** $[2 \times (3 + 4 - 5) + 4] \div 4$ **h** $3 - 22 \div (4 + 7) + 2^3$ **i** $8 \div [2 \times (4 - 3) + 2] \times \sqrt{16}$

**j** $2^3 + (5 - 3) \times 9 - \sqrt{4}$ **k** $4^3 - [7 - 5 \times (3 + 8)] + 3$ **l** $(5^2 - 5) \times (3^2 + 3) \times (4^2 - 4)$

4 For each calculation:

**i** find an estimate of the answer by first rounding each number to its leading digit

**ii** use a calculator to work out the exact answer

**a** $57 + 216 \times 4$ **b** $41 + 572 \div 286$

**c** $10^2 - 33 \times 3 + 82$ **d** $3852 \div 963 + 78 \times 29$

**e** $51 \times 4 \div 17 + 626$ **f** $5328 \times 2 - 81 \times 38$

**g** $\sqrt{49} + 756 \div 42 \times 6 - 3^2$ **h** $2765 + 429 - 57 \times 5 \div 19 + 12$

5 For each calculation:

**i** find an estimate of the answer by first rounding each number to its leading digit

**ii** use a calculator to work out the exact answer

**a** $69 \times (28 + 11) - 51 \times 2$ **b** $(67 + 31) \div (12 - 5)$

**c** $815 \times (61 - 49) + 17 \times 3$ **d** $19 \times (9 - 4) + 3 \times (52 + 27)$

**e** $\sqrt{100} \times 7 + 2 \times (3^2 + 42)$ **f** $[13 \times (7 - 5) + 28] \times 2$

6 For each of the following:

**i** write the problem using numbers and symbols. Use brackets where necessary

**ii** work out the result

**a** 10 divided by 2, then subtracted from the product of 11 and 4

**b** The quotient of 12 and 4, then multiplied by 2 to the power of 3

**c** 9 to the power of 2 take away 3 to the power of 3, all doubled

**d** 9 subtracted from 17 then multiplied by square root 4, all shared between 2

**e** The product of square root 16 and 6, then divided by 2 to the power of 3

**f** 3 less than the difference of 14 and 7

7 The average of a set of scores is calculated as follows:

average = (total sum of the scores) ÷ (number of scores)

For example, the average of 9, 10 and 14 is found by working out $(9 + 10 + 14) \div 3$.

**a** Use the correct order of operations to find the average of 9, 10 and 14.

**b** Find the average of each set of numbers.

**i** 15, 18 and 24 **ii** 3, 5, 6, 8, 9 and 11

**iii** 278 and 356 **iv** 7, 7, 11 and 15

**v** 50, 51, 52, 52 and 60 **vi** 50, 20, 30, 40, 60, 50 and 30

**8** Shaz makes scooters and skateboards. She has made eight scooters and eight skateboards but needs to order wheels first. If each scooter has two wheels and each skateboard has four wheels, how many wheels does Shaz need to order?

**9** Ines is in charge of transporting four Year 7 classes to a local farm on a school excursion. She decides that they will hire coaches. Each class consists of 21 students and 2 teachers. One coach can carry 48 people including the driver.

**a** If each coach is driven by one of the teachers, how many coaches does Ines need to hire?

**b** How many spare seats will there be?

**c** The cost of hiring the coaches is shared evenly across all the students and teachers. If it costs $598 to hire one coach for a day, how much would each person need to pay?

**10** Kyle goes down to the store to buy ingredients for his cake. He buys 3 cartons of 12 eggs, but 4 eggs from each carton break on the way home. He returns to the store and buys another carton of 12 eggs. How many eggs in total does Kyle have that are not broken?

**11** Jun is purchasing all the tennis balls for her club for the season. Each tennis student at the club needs 8 balls for them to use during practice. At the busiest time of the day there are 4 classes running. There is a class with 2 students, a class with 3 students, a class with 1 student, and a class with 5 students. How many tennis balls should Jun purchase?

**12** Sam is organising food for a movie night. He buys enough party pies for each attendee to have 4 pies. He buys 96 pies, but then finds out that 2 people cannot come to the movie night. How many people will attend the movie night?

**13** In Australian Rules football, each team scores goals and behinds. Each goal has a value of six points and each behind has a value of one point. During a match, the scoreboard usually shows the number of goals, behinds and points both teams have scored. This photo, taken by a football fan, does not show the number of points.

**a** Write statements to show the total points scored by each team. Use the numbers and operations needed to calculate the total:

**i** points scored by the Sharks

**ii** points scored by the visitors

**b** Calculate the number of points each team scored.

**c** At this stage of the match, which team is winning and by how much?

**14** You and a group of friends go to the cinema. At the cinema kiosk, you order eight medium soft drinks, four jumbo containers of popcorn and seven ice creams.

**a** Write a mathematical calculation to show how to work out the total cost of the food and drink.

**b** Calculate the total cost.

**15** Jordan completed a worksheet on the order of operations. Identify the errors in Jordan's working.

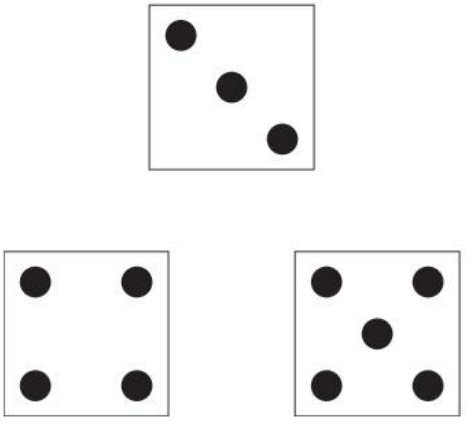
**a** $5 \times 7 - 2 \times (3-1)^2 - 1 = 35 - 2 \times 2^2 - 1$
$= 35 - 4^2 - 1$
$= 35 - 16 - 1$
$= 15$

**b** $(3+4)^2 - 2^2 \times (5-2) = 3 + 4^2 - 2^2 \times 3$
$= 3 + 16 - 4 \times 3$
$= 15 \times 3$
$= 45$



**16** In the late 1780s, students in a class were asked to add the whole numbers from 1 to 100. A student named Johann Carl Friedrich Gauss (who later became a famous mathematician) correctly answered the question in less than one minute by using the strategy of adding pairs of numbers. Try this calculation yourself. Try to find the answer in less than one minute.

**17** Edmund tells Roneet to roll three dice and whatever numbers come up he will be able to make all the numbers from 1 to 20 using only BIDMAS. Roneet rolls the three dice and the numbers that come up are: 3, 4 and 5. Use 3, 4 and 5 once only and BIDMAS to make the numbers 1 to 20. You can use squares and square roots.



**18** Simplify the following problems.

**a** $\dfrac{45}{8(9-8)-3} + \dfrac{2(4)^2 - 5\sqrt{4}}{100 \div 50}$

**b** $\dfrac{(6-5)^2 + (7-4)^2}{\sqrt{25} - \sqrt{9}}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Order of operations

Investigation
Four 4s

Topic quiz
1H

# Chapter summary

## Writing numbers

| | |
|---|---|
| **Standard form** | 32 456 |
| **Expanded form** | 3 × 10 000 + 2 × 1000 + 4 × 100 + 5 × 10 + 6 × 1 |
| **Worded form** | Thirty-two thousand, four hundred and fifty-six |

## Mental addition and subtraction

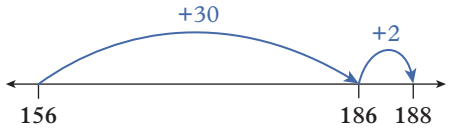
- By-parts method

156 + 32

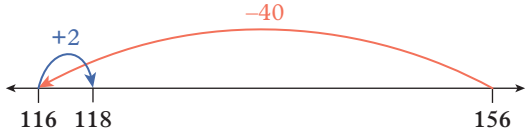
1 hundred  8 tens  8 ones

188

- Jump method




- Compensation method




## Rounding

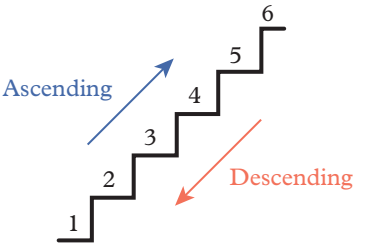
1$\boxed{2}$ 600 ≈ 13 000

9 8 7 (6) 5 — Round up

1$\boxed{2}$ 300 ≈ 12 000

4 (3) 2 1 0 — Round down

## Ordering




- Less than <
- Greater than >

## Addition algorithm

```
  1
  229
+ 342
  571
```

## Subtraction algorithm

```
  1 11
  3 2 1
– 2 0 4
  1 1 7
```

## Division

quotient

```
  1 4 5  remainder 1
6)8 2 7 3 1
```

divisor  dividend  remainder

## Multiplying by powers of 10

- 22 × 10 = 220
- 22 × 100 = 2200
- 22 × 1000 = 22000

## Multiplication algorithm

1. Write
2. Multiply by ones
3. Multiply by tens
4. Don't forget the zero(s)
5. Add the results

```
  2
  3 4
× 2 6
  2 0 4   (34 × 6)
+ 6 8 0   (34 × 20)
  8 8 4
```

## Long division

```
  145 remainder 1
6)871
– 6
  27
– 24
  31
– 30
  1
```

1. List the multiples
2. Divide leading digit
3. Multiply
4. Subtract
5. Bring down
6. Repeat or remainder

## Order of operations

| | | |
|---|---|---|
| **B** | **Brackets** | Operations inside brackets are always performed first. For example, 5 × (10 – 2) = 5 × 8 = 40 |
| **I** | **Indices** | Next, evaluate indices and square roots. For example, $3^3 - 4 = 27 - 4 = 23$ |
| **D** | **Division** | Then, working from **left to right,** perform any multiplication or division before adding or subtracting. For example, 5 × 3 – 4 + 8 ÷ 4 = 15 – 4 + 2 = 13 |
| **M** | **Multiplication** | |
| **A** | **Addition** | Finally, working from **left to right,** perform any addition and subtraction. For example, 15 – 4 + 2 = 13 |
| **S** | **Subtraction** | |

## Indices and squares

$2^2 = 2 \times 2 = 4$
$\sqrt{4} = 2$

$3^2 = 3 \times 3 = 9$
$\sqrt{9} = 3$

$4^2 = 4 \times 4 = 16$
$\sqrt{16} = 4$



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# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

1A **1** Which list of numbers is written in ascending order?

**A** 1053, 531, 510, 501
**B** 345, 354, 3045, 3405
**C** 2461, 2614, 2641, 2611
**D** 85, 83, 81, 82, 87, 89
**E** 245, 254, 524, 452, 542

1A **2** Which number is an approximation of 78 945 rounded to the nearest ten thousand?

**A** 70 000 **B** 78 900 **C** 78 000 **D** 79 000 **E** 80 000

1B **3** What is 14 985 + 486 + 1987 + 9135?

**A** 14 373 **B** 25 483 **C** 26 593 **D** 174 805 **E** 26 566

1C **4** Which calculation gives an answer of 42?

**A** 96 – 42 **B** 1569 – 584 **C** 7859 – 7813
**D** 9026 – 8984 **E** 2048 – 1997

1D **5** Using the distributive law, 6 × 45 becomes:

**A** 6 × 40 + 6 × 50 **B** 6 × 40 + 6 × 5 **C** 45 × 6 + 4 × 5
**D** 45 × 6 + 40 × 6 **E** 45 × 4 + 45 × 5

1E **6** Which product has the greatest value?

**A** 48 × 49 **B** 13 × 78 **C** 26 × 54 **D** 89 × 21 **E** 59 × 30

1F **7** Which division does *not* have a remainder of 4?

**A** 79 ÷ 5 **B** 36 ÷ 8 **C** 43 ÷ 10 **D** 22 ÷ 6 **E** 99 ÷ 5

1G **8** What is 5 × 5 × 5 × 8 × 8 in index form?

**A** $3^5 \times 2^8$ **B** $5^3 + 8^2$ **C** $40^5$ **D** $5^3 \times 8^2$ **E** $5^3 - 8^2$

1H **9** What is 5 + 4 × 9 – 3?

**A** 15 **B** 38 **C** 54 **D** 78 **E** 21

1H **10** $70 - (2 \times 3)^2 + 4$ is equal to:

**A** 38 **B** 62 **C** 49 **D** 64 **E** 34

## Short answer

1A **1** Write each number in expanded form.

**a** 467 **b** 7856 **c** 9030 **d** 15 452

1A **2** Write each list of numbers in ascending order.

**a** 5347, 547, 53 047, 57 **b** 87 605, 87 506, 87 056

1A **3** Write an approximation of each number by rounding to the nearest hundred.

**a** 392 **b** 9488 **c** 554 **d** 12 945

1B **4** Evaluate each sum.

**a** 75 + 49 **b** 83 + 32 **c** 561 + 224 **d** 85 + 33

1B **5** Evaluate each of the following.

**a** 467 + 56 + 7801 + 943 **b** 383 604 + 2557 + 16 092

1B **6** Find two three-digit whole numbers that add to 478 if:

**a** both numbers are odd **b** both numbers are even.

1C **7** Evaluate each of the following differences.

**a** 9564 – 5381 **b** 371 625 – 38 047

**c** 17 659 – 9816 **d** 101 011 – 59 678

1C **8** Evaluate each of the the following.

**a** 4895 – 1625 – 325 **b** 978 – 486 – 239

1D **9** Use a mental strategy to evaluate each of the following.

**a** $38 \times 11$ **b** $41 \times 21$ **c** $5 \times 8 \times 7$

**d** $72 \times 14$ **e** $2 \times 48 \times 4$ **f** $91 \times 5 \times 2$

1D **10** Calculate each product.

**a** $68 \times 7$ **b** $91 \times 6$ **c** $546 \times 3$

**d** $741 \times 9$ **e** $2458 \times 8$ **f** $17853 \times 4$

1E **11** Use a strategy to evaluate each of the following.

**a** $389 \times 100$ **b** $412 \times 6000$ **c** $3400 \times 200$

**d** $500 \times 300 \times 40$ **e** $25 \times 48 \times 4$ **f** $931 \times 50 \times 3$

1E **12** Determine the value of each of the following.

**a** $542 \times 37$ **b** $456 \times 78$ **c** $6135 \times 29$

**d** $9856 \times 11$ **e** $25 \times 96 \times 2$ **f** $122 \times 43 \times 5$

1F **13** Evaluate each of the following.

**a** $850 \div 6$ **b** $\frac{68445}{4}$

**c** $\frac{7521}{3}$ **d** $567204 \div 7$

1F **14** Evaluate each of the following.

**a** $45987 \div 11$ **b** $18423 \div 16$

**c** $\frac{5648}{23}$ **d** $\frac{78525}{45}$

1F **15** Use a strategy to evaluate each of the following.

**a** $7000 \div 10$ **b** $80000 \div 2000$

**c** $15000 \div 30$ **d** $210000 \div 700$

1G **16** Write each in expanded form and calculate its value.

**a** $2^6$ **b** $5^3 \times 3^2$ **c** $2^3 \times 7^2$ **d** $6^2 \times 4^3$

1G **17** Write 10 000 in index form with a base of 10.

1G **18** Write each of these in index form and evaluate.

**a** seven squared **b** the square of six

**c** the square of 19 **d** 8 squared

1G **19** Calculate the value of each of these square roots.

**a** $\sqrt{25}$ **b** $\sqrt{81} + 4^2$

**c** $\sqrt{64} - \sqrt{16}$ **d** $\sqrt{100} + \sqrt{9} - \sqrt{49} - \sqrt{36}$

1H **20** Use the correct order of operations to perform each of the following calculations.

**a** $7^2 - 6 \times 4 + 3$ **b** $8 \times (19 - 13) + 2 \times \sqrt{36}$

**c** $4 + 3 \times (5 - 2)^2$ **d** $16 - 2 \times (3 + 4) + 9$

1H **21** Evaluate each of the following.

**a** $6 \times [12 - (8 - \sqrt{4})]$ **b** $100 - [5 \times (3 - 2)]^2$

**c** $[6^3 \div (3^2 - \sqrt{9})] \div 9$ **d** $(3 \times 3)^2 - (2 \times 4)^2 + \sqrt{49}$

## Analysis

**1** Saina gives her dad the following question:

$20 + 10 \div 5 \times 6$

'Got it!', says Dad. 'The answer is 36.'

'Uh-uh, Dad, you're wrong', says Saina.

**a** What did Saina's dad do wrong?

**b** What is the correct answer?

**2** Jordan wrote a list of the type and quantity of food he needed for his birthday party. He carefully noted the price per item during his shopping trip.

**a** Calculate how much Jordan spent on each item.

**b** How much did he spend in total?

**c** If he paid with two $50 notes, how much change did he get?

**d** What is the cheapest item on the list?

**e** Which item did he spend the least money on? Is this different from the answer to part **d**? Explain why or why not.

**f** Each bag of Allen's mixed lollies has 40 lollies. How many lollies in total does Jordan have?

**g** Jordan wants lolly bags for himself and his friends with at least 10 lollies in each bag. What is the maximum number of friends he can invite?

**h** Jordan adds Freddo frogs to the lolly bags. How many Freddo frogs does each friend get? How many Freddo frogs are left over?

| Item | Quantity | Price per item |
|---|---|---|
| Party Pie 24 pack | 4 | $4 |
| Allen's mixed lollies | 3 | $3 |
| Chips 100 g | 4 | $2 |
| M&Ms 250 g | 2 | $4 |
| Freddo frogs 20 pack | 2 | $5 |
| Cocktail sausages 2 kg pack | 1 | $6 |
| Frozen pizza | 1 | $10 |
| Mudcake | 3 | $4 |

**i** Jordan and his friends need to be divided into equal groups for games. Using your answer from part **g**, what size could the groups be if there must be at least three people in each?

**3** Tom and Kate are elite supercar drivers. They are competing in the next Melbourne 400 championships. Four races are held over four days and each race is 100 km long.

**a** If there are 19 laps in each race, approximate the distance of each lap in kilometres.

**b** If Tom has completed 246 km, which day of the event is it?

**c** There are 16 turns in the track. Over the four days, how many turns will each driver make?

**d** Lap times are usually around 2 minutes each. On Day 1 of the race, Kate's incredibly consistent lap time was 1 minute and 56 seconds, and Tom's consistent lap time was 2 minute and 6 seconds. Determine each driver's total time for one day's race. Give your answer in minutes and seconds.

**e** On the second day of the race day, Kate's lap time was a little faster at 1 minute 49 seconds but she took an extra long pit stop time of 3 minutes. Was Kate's total time shorter on Day 1 or Day 2? State the difference between the two times.

**f** For each car, pit stop teams can change between hard tyres and soft tyres. Hard tyres are not as fast as soft tyres, but they last longer on the track. Soft tyres provide the driver with more grip on the road so that they can go faster, but they don't last long before the driver must change the tyres. During a pit stop, the tyre must be changed from soft to hard, or from hard to soft.

On day three, Tom completes his first two laps on hard tyres at his previous consistent time of 2 minutes and 6 seconds. His pit stop team then changes his tyres in 17 seconds, and on the soft tyres his lap time drops by 12 seconds for 8 laps. After these 8 laps, Tom needs to change his tyres again, and on the new set of hard tyres he completes 5 laps at a consistent lap time of 2 minutes and 1 second, with the pit stop taking 15 seconds.

With 4 laps to go, Tom can change his tyres again. Another pit stop will take 15 seconds, and he will complete each lap on the new soft tyres in 1 minute and 55 seconds. Should Tom pit stop again and, if so, how much quicker will he complete the race?

# 2 Factors and primes

## Index

**2A** Number sequences
**2B** Divisibility rules
**2C** Multiples and the lowest common multiple
**2D** Factors and the highest common factor
**2E** Prime and composite numbers
**2F** Prime factorisation

## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Counting in multiples
- ✔ Ordering and comparing large numbers
- ✔ Times tables
- ✔ Multiplying and dividing whole numbers
- ✔ Indices

## Curriculum links

- Investigate index notation and represent whole numbers as products of powers of prime numbers (VCMNA238)

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# 2A Number sequences

## Learning intentions

✔ I can find the rule between terms in a sequence.
✔ I can find more terms in a sequence.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Number sequences |
| Year 8 | 1D Multiples, factors, squares and cubes |

## Number sequences

- A **sequence** is an ordered list of numbers with a set rule.
- A **term** is an individual number in a sequence.
- A **rule** is a description of the relationship between each term and the next term in a sequence.

Sequence: 2, 4, 6, 8, ...
Rule: +2

+2 +2 +2

1st term, 2nd term, 3rd term, 4th term, continue on

## Finding the rule for a sequence

- To find the rule for a sequence, consider all possible operations including:
  - → addition
  - → subtraction
  - → multiplication
  - → division
  - → indices
  - → square roots.

### Example 2A.1 Identifying the rule in a sequence

Identify the rule in words for each sequence.

**a** 3, 6, 9, 12, ...
**b** 136, 94, 52, 10, ...
**c** 2, 4, 8, 16, 32, ...

**THINK**

1. Look for a pattern between the numbers in the sequence.
2. Check the pattern or rule works with all values.
3. Describe the pattern in a sentence.

**WRITE**

**a** Add three

+3 +3 +3

3, 6, 9, 12, ...

Each term is 3 more than the term before.

**b** Subtract 42

−42 −42 −42

136, 94, 52, 10, ...

Each term is 42 less than the term before.

**c** Multiply by 2

×2 ×2 ×2 ×2

2, 4, 8, 16, 32, ...

Each term is double the value of the term before.


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## **Example 2A.2** Generating terms in a sequence

Generate the next two terms in each sequence.

**a** 11, 22, 33, 44, __, __.

**b** 800, 400, 200, 100, __, __.

**THINK**

**a** **1** Identify the rule for the sequence. Each term is 11 more than the term before.

**2** Add 11 to 44 for the first new term, then add 11 again for the second new term.

**b** **1** Identify the rule for the sequence. Each term is divided by 2 to find the next term.

**2** Divide 100 by 2 for the first new term, then divide by 2 again for the second new term.

**WRITE**

**a** +11 +11 +11 +11 +11

11, 22, 33, 44, ___, ___, ...

44 + 11 = 55 and 55 + 11 = 66

The next two terms in the sequence are 55 and 66.

**b** ÷2 ÷2 ÷2 ÷2 ÷2

800, 400, 200, 100, ____, ____, ...

100 ÷ 2 = 50 and 50 ÷ 2 = 25

The next two terms in the sequence are 50 and 25.

**Helpful hints**

✔ You may think that sequences *always* increase as you move to the right, but sequences can *decrease* as well. For example, 24, 12, 6, 3, …

ANS p543

# **Exercise 2A** Number sequences

1–7, 9, 10, 11(a, b), 12, 13 | 1(d–f), 2(d–f), 3–6, 9–12, 14, 15 | 1(d–f), 2(d–f), 4, 6, 9–12, 14–16

2A.1 **1** Identify the rule in words for each sequence.

**a** 4, 6, 8, 10, 12, …

**b** 9, 18, 27, 36, 45, …

**c** 5, 10, 15, 20, 25, …

**d** 24, 20, 16, 12, 8, …

**e** 810, 270, 90, 30, 10, …

**f** 4, 16, 64, 256, 1024, …

**2** Generate the first three terms of the following sequences.

**a** starting at 8, multiply by 3 to get the next term

**b** starting at 500, divide by 5 to get the next term

**c** starting at 23, subtract 7 to get the next term

**d** starting at 1000, add 200 to get the next term

**e** starting at 25, multiply by 2 then subtract 20 to get the next term

**f** starting at 18, add 6 then divide by 2 to get the next term

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

2A.2 **3** Generate the next two terms in each sequence.

**a** 6, 12, 18, 24, _____, _____

**b** 7, 14, 21, 28, _____, _____

**c** 78, 72, 66, 60, _____, _____

**d** 1408, 704, 352, 176, _____, _____

**e** 91, 78, 65, 52, _____, _____

**f** 4, 16, 64, 256, ____, _____

**4** Identify the 6th term in each sequence.

**a** 4, 8, 12, …

**b** 6, 11, 16, …

**c** 99, 93, 87, …

**d** 76, 61, 46, …

**e** 5, 10, 20, …

**f** 4096, 1024, 256, …

**5** Find the missing terms in the following sequences.

**a** 5, 12, ___, 26, ___, 40, 47, 54, …

**b** 53, 45, 37, ___, 21, 13, ___, …

**c** ___, 15, 24, 33, 42, 51, ___, …

**d** 300, ___, 250, 225, 200, ___, 150

**6** Find the missing terms in the following sequences.

**a** 4, 8, ___, 32, 64, 128, ___, 512, …

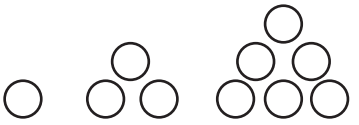
**b** 4000, ___, 1000, 500, ___, 125, …

**c** ___, 15, 45, 135, 405, ___, …

**d** 729, 243, ___, ___, 9, 3, …

**7** **a** Draw the next term in this sequence.

**b** What is the rule for this sequence?

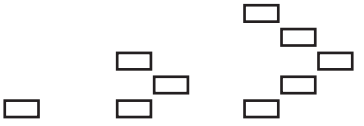


**8** **a** Draw the next term in this sequence.

**b** What is the rule for this sequence?

**9** **a** Draw the next term in this sequence.

**b** What is the rule for this sequence?



**10** **a** Draw the next term in this sequence.

**b** What is the rule for this sequence?

PROBLEM SOLVING AND REASONING

**11** **a** The fifth term in a sequence is 35. If the rule for this sequence is add 5, what is the first term?

**b** The eighth term in a sequence is 90. If the rule for this sequence is subtract 12, what is the third term?

**c** The sixth term in a sequence is 192. If the rule for this sequence is multiply by 2, what is the first term?

**d** The sixth term in a sequence is 81. If the rule for this sequence is divide by 3, what is the second term?

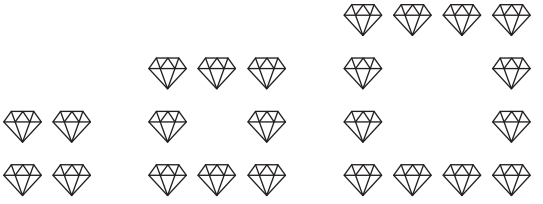
**12** Trinh has a collection of diamonds. She arranges the diamonds in a pattern as shown here.

**a** Draw the next term in this sequence.

**b** Write down the sequence that represents the number of diamonds in each term.

**c** Identify the rule in words for the sequence.

**d** How many diamonds would be in the 10th term of this pattern?



**e** Trinh only has enough diamonds for exactly 6 terms in the sequence. How many diamonds does she have?

PROBLEM SOLVING AND REASONING

**13** Chao planted a small tree in his backyard on New Years's Day. The tree was 60 cm tall when he planted it. He measured how tall the tree was at the start of each month. Over the year, Chao found that the tree grew by 8 cm each month. How tall was the tree on New Year's Day of the following year?

**14** Naisha made a video of herself performing a new dance. She liked it so much that the following day she shared the video with four of her friends. Naisha's friends were impressed with her video so the next day each of her friends shared it with four new people.

Determine the following if this sharing pattern continues in this way.

**a** How many people will see Naisha's video for the first time on the 7th day after she created it?

**b** How many people in total will see the video in the 7 days, including Naisha?

**15** Saina's dad asks her, 'I am thinking of a sequence that begins 4, 16, … What is the rule for the sequence, Saina?' She replies, 'Don't be silly Dad. You need at least three terms of a sequence to find the rule!'

Who is correct, Saina or her dad? Give a reason for your answer.

CHALLENGE

**16** Arvin works in a fruit and vegetable shop. He is given the task of arranging 200 apples on a display shelf. Arvin decides that he will arrange the apples in rows. Each row of apples will have two fewer apples than the previous row.

**a** Find a pattern for the 200 apples that follows the rule: subtract 2.

**b** How many rows will there be in the display?

**c** How many apples will there be in the bottom row?

**d** How many apples will there be in the top row?

**e** Is there more than one pattern that has a total of 200 apples and follows the rule subtract 2? Why or why not?

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Number sequences

**Investigation** The cost of stationery

**Topic quiz** 2A

# 2B Divisibility rules

## Learning intentions

✔ I can determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10.

**Inter-year links**

**Year 8** 1C Multiplying and dividing whole numbers

## Divisibility

- If the remainder is zero, then we can say that the dividend is exactly divisible, or divisible by the divisor. For example, 108 ÷ 9 = 12, so 108 is exactly divisible by 9, but 91 is not exactly divisible by 6.

dividend

$$\frac{108}{9} = 12$$

divisor quotient

91 ÷ 6 = 15 remainder 1

dividend divisor quotient remainder

## Divisibility rules

- We can use the divisibility rules to test if one number is divisible by another.

| | | |
|---|---|---|
| ÷2<br>The last digit of the number is even. | ÷3<br>The sum of all the digits in the number is divisible by 3. | ÷4<br>The number made by the last two digits is divisible by 4. |
| ÷5<br>The last digit is 5 or 0. | ÷6<br>The number is divisible by 2 and 3. | ÷7<br>There is no easy trick for 7. |
| ÷8<br>The number made by the last three digits is divisible by 8. | ÷9<br>The sum of all the digits in the number is divisible by 9. | ÷10<br>The last digit is 0. |



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## **Example 2B.1** Using the divisibility rules

Check if the number 845 026 is divisible by each of the following and provide a reason for using/not using the divisibility rules.

**a** 2 **b** 3 **c** 4

**THINK**

**a** Check if the last digit of the number is even.

**b** Check if the sum of all the digits in the number is divisible by 3.

**c** Check if the number made by the last two digits is divisible by 4.

**WRITE**

**a** 845 02$\boxed{6}$

Since the last digit is even, 845 026 is divisible by 2.

**b** $8 + 4 + 5 + 0 + 2 + 6 = 25$

Since 25 is not exactly divisible by 3, 845 026 is not divisible by 3.

**c** 845 0$\boxed{26}$

Since 26 is not exactly divisible by 4, 845 026 is not divisible by 4.

## **Example 2B.2** Determining divisibility of numbers

Determine if the first number is divisible by the second number.

**a** 1218 and 8 **b** 43 938 and 6

**THINK**

**a** To determine if the number is divisible by 8, check if the number made by the last three digits is divisible by 8.

**b** To determine if the number is divisible by 6, check if the number is divisible by 2 and 3.

**WRITE**

**a** 1$\boxed{218}$

2 7 remainder 2

8 )21 5 8

Since 218 is not exactly divisible by 8, 1218 is not exactly divisible by 8.

**b** 43 93$\boxed{8}$

Since the last digit is even, it is divisible by 2.

$4 + 3 + 9 + 3 + 8 = 27$

Since 27 is divisible by 3, 43 938 is divisible by 3. Since 43 938 is divisible by 2 and 3, it is also divisible by 6.

**Helpful hints**

✔ Be careful when checking for divisibility. If your calculations are wrong, you might come to the wrong conclusion.

ANS p544

# Exercise 2B Divisibility rules

1–7, 9, 11, 12 | 2, 4–9, 12–16 | 2, 4–6, 9, 10, 12–17

UNDERSTANDING AND FLUENCY

2B.1 **1** Check if the number 546 is divisible by each of the following and provide a reason for using/not using the divisibility rules.

**a** 2 **b** 3 **c** 4 **d** 5
**e** 6 **f** 8 **g** 9 **h** 10

**2** Check if the number 8640 is divisible by each of the following and provide a reason for using/not using the divisibility rules.

**a** 2 **b** 3 **c** 4 **d** 5
**e** 6 **f** 8 **g** 9 **h** 10

**3** Check if the number 65 190 is divisible by:

**a** 2 **b** 3 **c** 4 **d** 5
**e** 6 **f** 8 **g** 9 **h** 10

**4** Check if the number 321 272 is divisible by:

**a** 2 **b** 3 **c** 4 **d** 5
**e** 6 **f** 8 **g** 9 **h** 10

2B.2 **5** Determine if the first number is divisible by the second number.

**a** 315 814 and 4 **b** 516 696 and 8 **c** 274 752 and 6
**d** 26 535 and 9 **e** 367 168 and 8 **f** 74 074 and 6

**6** State whether the following statements are true or false.

**a** 432 is divisible by 6 and 8 **b** 816 is divisible by 3 and 4
**c** 7265 is divisible by 2 and 5 **d** 6561 is divisible by 8 and 9

PROBLEM SOLVING AND REASONING

**7** The Year 7 captains have organised a sausage sizzle. They have ordered 1050 sausages. They want to make sure each student can have an equal number of sausages with no left-overs.

**a** Use the divisibility tests to determine whether the captains can provide 2, 3 or 4 sausages per person without wastage.

**b** Determine which of these would be the best option for a class of 350 students.

**8** Anais has 208 pipe cleaners. She plans to use all of them to make legs for either grasshoppers or octopuses to sell at the school fair. Given that grasshoppers have 6 legs and octopuses have 8 legs, which should she choose to have no legs left over?

**9** Yanni orders chocolate chips for his cookie business and receives 7398. He wants to guarantee that his customers receive either 8 or 9 chocolate chips in each cookie, while also having no chocolate chips left over. How many chocolate chips should go in each cookie?

**10** An Australian soft toy company is exporting 1662 toy quokkas all over the world. They can send the toy quokkas in boxes of 6 or 9. To avoid having any toy quokkas left over, which should they choose?

**11** Hayley knows how to test for divisibility of 2, 3, 4 and 5. Explain how she could use two of these divisibility tests to check whether 14 805 is divisible by 15.

**12** Scott tells you that if a number is divisible by 4 and 6 then it will also be divisible by 24. Is Scott correct? Give a reason for your answer.

**13** Invent a divisibility test for 12. Explain why your test works.

**14 a** Are there 118 440 minutes in an exact number of hours?

**b** Give a different divisibility test that would check this.

**15** There is a divisibility test for 11.

→ Subtract the last digit from the number made by the remaining digits.
→ If that number is divisible by 11, then the original number is also divisible by 11.
→ Repeat the process if you want to assess its divisibility based on a smaller number.

For example:
10 571
1057–1 = 1056
105–6 = 99 which is divisible by 11, therefore 10 571 is divisible by 11.

Check to see if these numbers are divisible by 11.

**a** 53 411

**b** 962 016

**16** Find numbers that go in the boxes to make this five-digit number divisible by 450.

373□□

**17** Design a box for macarons according to these rules:

**1** The box contains 35 macarons in lines in a single layer in a geometric shape without any spaces between them.

**2** There are 4 different colours of macarons, 9 of each colour, available to use.

**3** No macaron can be directly next to a macaron of the same colour, but diagonally is allowed.

**18** A teacher writes a mystery number on a white board. They then ask their class to list the divisors of the number, one divisor per student.

The first student says, "The number is divisible by 2."
The second student says, "The number is divisible by 3."
The third student says, "The number is divisible by 4."
…
The 30th student says, "The number is divisible by 31."
The teacher then tells the class, "Two students were wrong when they made their claim. These students spoke one after the other."
Which two students spoke incorrectly?

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Divisibility rules

**Investigation** No remainders left over

**Topic quiz** 2B

PROBLEM SOLVING AND REASONING

CHALLENGE

# 2C Multiples and the lowest common multiple

## Learning intentions

✔ I can find multiples of a number.

✔ I can find the lowest common multiple of two numbers.

Inter-year links

Years 5/6 Multiples and factors

Year 8 1D Multiples, factors, squares and cubes

## Multiples

- The **multiples** of a given whole number result from multiplying that number by another whole number.
  For example, the multiples of 3 are 3, 6, 9, 12, 15, 18, …
- There are infinitely many multiples of any given number.
- Multiples greater than zero can be generated using the times tables. The multiples of the red numbers are shown in blue below.

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

## Lowest common multiple

- **Common multiples** are multiples that appear in the multiples list for two or more different whole numbers.
  For example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 … and multiples of 4 are 4, 8, 12, 16, 20, 24 … so the first three common multiples of 3 and 4 are 12, 24 and 36.
- The **lowest common multiple (LCM)** is the lowest (first) multiple that is common in the multiples list of two or more numbers.
  For example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 … and multiples of 4 are 4, 8, 12, 16, 20, 24 … so the LCM of 3 and 4 is 12.
- To find the LCM of 2 and 3:
  1. List all multiples of 2.
  2. List all multiples of 3.
  3. Circle the common multiples of 2 and 3.
  4. Find the lowest common multiple.

multiples of 2: 2, 4, 6, 8, 10, 12 …

multiples of 3: 3, 6, 9, 12, 15, 18 …

LCM: 6

## **Example 2C.1** Finding multiples

List the first four multiples of each number.

**a** 4 **b** 7

**THINK**

**a** For the first four multiples, multiply 4 by 1, 2, 3 and 4.

**b** For the first four multiples, multiply 7 by 1, 2, 3 and 4.

**WRITE**

**a** $4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
The first four multiples of 4 are 4, 8, 12 and 16.

**b** $7 \times 1 = 7$
$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
The first four multiples of 7 are 7, 14, 21 and 28.

## **Example 2C.2** Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of 12 and 30.

**THINK**

1. List the multiples of 12.
2. List the multiples of 30.
3. Circle the multiples that are the same in the two lists.
4. Find the lowest number in the list of common multiples.

**WRITE**

multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 …
multiples of 30: 30, 60, 90, 120, 150, 180 …
The LCM is 60.

**Helpful hints**

- ✔ Some people incorrectly think that times tables are finite. But we can multiply beyond 9.
- ✔ Lowest common multiples are not necessarily 'low' in value, they can be quite large numbers.
- ✔ There is no such thing as a 'highest common multiple' as any given number has infinitely many multiples.

ANS p545

# Exercise 2C Multiples and the lowest common multiple

1(b, d, f, h), 2–8, 9(b, d, f, h), 10–15 | 1(b, c, f, h), 2, 4, 6, 7(a, c, e, f), 8, 9, 12, 14, 16–18 | 3, 5, 7(e, g), 8, 9(c, e, h), 12, 16–20

UNDERSTANDING AND FLUENCY

2C.1 **1** List the first four multiples of each number.

**a** 10 **b** 6 **c** 9 **d** 5 **e** 11 **f** 30
**g** 25 **h** 13 **i** 40 **j** 22 **k** 14 **l** 31

**2** List all the multiples of 8 between 23 and 65.

**3** List all the multiples of 7 between 50 and 100.

**4** Find the first four common multiples of 4 and 5.

**5** Find the first six common multiples of 3 and 4.

**6** **a** List the multiples of 5 and 6 up to 60.
**b** Identify the common multiples of 5 and 6 in the lists from part **a**.
**c** What is the lowest common multiple of 5 and 6?

2C.2 **7** Find the lowest common multiple (LCM) of each pair of numbers.

**a** 2 and 5 **b** 4 and 7 **c** 6 and 9 **d** 8 and 12
**e** 5 and 8 **f** 7 and 10 **g** 15 and 20 **h** 8 and 6

**8** Find the LCM of each group of numbers.

**a** 2, 3 and 4 **b** 5, 6 and 20 **c** 3, 4 and 9 **d** 4, 6 and 8
**e** 5, 3 and 2 **f** 30, 9 and 2 **g** 6, 7 and 8 **h** 2, 3, 4 and 5

**9** Find the LCM of each pair of numbers.

**a** 12 and 8 **b** 14 and 4 **c** 16 and 5 **d** 12 and 11
**e** 18 and 16 **f** 24 and 8 **g** 20 and 80 **h** 50 and 7

**10** Find the multiple of 8 that is closest to each number.

**a** 85 **b** 110 **c** 220 **d** 300

**11** Find the multiple of 14 that is closest to each number.

**a** 50 **b** 130 **c** 210 **d** 330

**12** Find the multiple of 45 that is closest to each number.

**a** 120 **b** 230 **c** 310 **d** 430

**13** Sunny and Jai have made the same number of cupcakes for a fundraising bake sale. Sunny cooked her cupcakes in batches of 9 and Jai cooked his in batches of 11. What is the smallest possible number of cupcakes that each of them could have baked?

**14** Daquan has piano lessons on every fifth day and cricket training on every third day. If he has both piano lessons and cricket training on August 5, on what date will he next have both a piano lesson and cricket training?

**15** Every fourth paling of this fence is painted blue.

**a** The fence is made up of 84 palings in total. How many palings are painted blue?

**b** The fence is to be extended to 128 palings in total. How many more palings will need to be painted blue?

**16** According to the timetable, a particular train is expected to depart from the station every 12 minutes. The first train of the day departs at 5:12 am.

**a** List all the times this train should depart from the station in the first hour of service.

**b** What is the departure time of the:

**i** the third train of the day?

**ii** the sixth train of the day?

**c** What number train will depart from the station at 8:24 am? Explain your strategy for determining the solution.

**17** Kiara is planning to purchase a new hat and water bottle for all Year 2 students at a school in Papua New Guinea. The hats are sold in packs of 8 and the water bottles are sold in packs of 13. What is the smallest number of packs of hats and water bottles that she can purchase so that there will be exactly one hat per water bottle?

**18** Batman, Superman and Wonder Woman decide to race through a city to determine who is the fastest. Batman uses the speed booster in the Batmobile to pass 7 blocks per boost. Superman leaps through the air at 3 blocks per jump. Wonder Woman runs 5 blocks at a time, then flies 5 blocks at a time, and then repeats this process.

**a** If they all land at the finish line at the same time, what is the shortest possible distance of the race in blocks?

**b** How many speed boosters did Batman use in the race?

**c** How many leaps did Superman make in the race?

**d** How many blocks did Wonder Woman run and how many blocks did she fly?

**19** Do the numbers 3 and 4 have a highest common multiple? Give a reason for your answer.

**20 a** Find three consecutive numbers such that the first number is a multiple of 3, the second is a multiple of 4, and the third is a multiple of 5. Describe the general rule used to determine these sets of numbers.

**b** Find four consecutive numbers such that the first number is a multiple of 6, the second is a multiple of 7, the third is a multiple of 8, and the fourth is a multiple of 9. Describe the general rule used to determine these sets of numbers.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Lowest common multiple

**Investigation** Neon signs

**Topic quiz** 2C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

2A **1** Identify the rule in words for each sequence.

**a** 5, 7, 9, 11, 13, ….
**b** 12, 24, 36, 48, …
**c** 3, 6, 12, 36, …

2A **2** Generate the first three terms of the following sequences.

**a** starting at 9, multiply by 4 to get the next term
**b** starting at 280, subtract 20 to get the next term
**c** starting at 36, add 12 to get the next term
**d** starting at 2000, divide by 5 to get the next term

2A **3** **a** Draw the next term in this sequence.

**b** What is the rule for this sequence?
**c** How many triangles would be in the fifth term?

2A **4** Theo builds a tower using 5 blocks. Each day he adds 4 blocks to the tower. How many blocks make up the tower on the sixth day?

2B **5** Check if the number 157 464 is divisible by:

**a** 2 **b** 3 **c** 4 **d** 5
**e** 6 **f** 8 **g** 9 **h** 10

2B **6** Determine if the first number is divisible by the second number.

**a** 319 999 and 9 **b** 816 966 and 8 **c** 653 184 and 6

2B **7** State whether the following statements are true or false.

**a** 30 240 is divisible by 9 and 8
**b** 34 565 is divisible by 4 and 5
**c** 50 625 is divisible by 15

2B **8** A book company have printed 39 366 copies of their latest book called *Swings and Roundabouts*. They have boxes that hold 9 books and boxes that hold 8 books. Use divisibility tests to find which boxes they will use to ship the books if they don't want to have any books left over.

2C **9** List the first five multiples of each number.

**a** 7 **b** 12 **c** 15 **d** 24

2C **10** **a** List all the multiples of 6 between 1 and 100.
**b** List all the multiples of 9 between 1 and 100.
**c** List all the common multiples of 6 and 9 between 1 and 100.
**d** What is the LCM of 6 and 9?

2C **11** Find the LCM of each pair of numbers.

**a** 3 and 5 **b** 6 and 8 **c** 7 and 12

2C **12** Amiya has netball training on every fourth day and work on every fifth day. If she has both netball training and work on Monday:

**a** how many days until she next has both on the one day?
**b** on what day of the week will she next have them both?

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# 2D Factors and the highest common factor

## Learning intentions

✔ I can find all the factors of a number.

✔ I can find the highest common factor of two or more numbers.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Multiples and factors |
| Year 8 | 1D Multiples, factors, squares and cubes |
| Year 9 | 3C Factorising using the HCF |

## Factors

- The **factors** of a whole number are the whole numbers which divide exactly into that number.
  For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- The divisibility rules are helpful when looking for factors of large numbers.

| ÷2 | ÷3 | ÷4 | ÷5 | ÷6 |
|---|---|---|---|---|
| The last digit of the number is even. | The sum of all the digits in the number is divisible by 3. | The number made by the last two digits is divisible by 4. | The last digit is a 5 or 0. | The number is divisible by 2 and 3. |

- **Factor pairs** are two numbers that when multiplied give a certain product. It is often easier to find factors in pairs.
  For example, the factor pairs that give a product of 12 are 1 and 12, 2 and 6, 3 and 4.

## Highest common factor

- A **common factor** is a number that divides exactly into two or more whole numbers.
  For example, 12 and 9 have a common factor of 3.
- The **highest common factor (HCF)** is the greatest factor that is common to two or more given numbers.
  For example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12, so the HCF of 8 and 12 is 4.
- To find the HCF of 16 and 24:
  1. List all factors of 16.
  2. List all factors of 24.
  3. Circle the common factors of 16 and 24.
  4. Find the highest common factor.

factors of 16: 1, 2, 4, 8, 16

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

HCF: 8

## **Example 2D.1** Finding factor pairs

Complete the following factor pairs.

**a** 15 = 1 × 15 or 3 × ____

**b** 18 = 1 × ____ or 2 × ____or 3 × ____

**THINK**

**a** Divide the number by the given factor to find the other half of the factor pair.

**b** Divide the number by each of the given factors to find the other halves of the factor pairs.

**WRITE**

**a** 15 ÷ 3 = 5
15 = 1 × 15 or 3 × 5

**b** 18 ÷ 1 = 18
18 ÷ 2 = 9
18 ÷ 3 = 6
18 = 1 × 18 or 2 × 9 or 3 × 6

## **Example 2D.2** Finding factors

Find all the factors of each number.

**a** 48

**b** 16

**THINK**

**a** **1** 1 × 48 = 48, so place the 1 on the left and the 48 on the right.

**2** 2 × 24 = 48, so place the 2 after 1 on the left and the 24 on the right, just before 48.

**3** 3 × 16 = 48, so place the 3 on the left and the 16 on the right.

**4** 4 × 12 = 48, so place the 4 on the left and the 12 on the right.

**5** 5 is not a factor of 48 as 48 is not divisible by 5.

**6** 6 × 8 = 48, so place the 6 on the left and the 8 on the right.

**7** 7 is not a factor of 48 as 48 is not divisible by 7.

**8** The next possible factor is 8 which has already appeared in the list, so all the factors have been found.

**b** **1** 1 × 16 = 16 and 2 × 8 = 16, so place them in the table.

**2** Using the divisibility rule for 3, 1 + 6 = 7, which is not divisible by 3, so 3 is not a factor of 16.

**3** 4 × 4 = 16, which is a perfect square. Only write 4 once.

**4** The next possible factor is 8 which has already appeared in the list, so all the factors have been found.

**WRITE**

**a**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | 48 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | | | | | | | | 24 | 48 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | | | | | | 16 | 24 | 48 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | | | | 12 | 16 | 24 | 48 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 6 | | 8 | 12 | 16 | 24 | 48 |

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

**b**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | | | 8 | 16 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 4 | | 8 | 16 |

Factors of 16 are 1, 2, 4, 8, 16.



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## **Example 2D.3** Finding the highest common factor

Find the highest common factor (HCF) of 24 and 36.

**THINK**

1 List all the factors of 24 and all the factors of 36.

2 Circle the common factors: the factors that appear in both lists.

3 Identify the highest common factor: the common factor with the greatest value.

**WRITE**

factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
The HCF is 12.

**Helpful hints**

- ✔ Be careful not to mix up HCF and LCM!
- ✔ Make sure you use a systematic method for finding all the factors; it is easy to miss one or two.
- ✔ Each factor should only appear once in your list, so the factors of perfect squares should only be recorded once.
  For example, 4 × 4 = 16 gives a single factor of 4.

ANS p546

# **Exercise 2D** Factors and the highest common factor

△ 1–3, 4(d–i), 5(a, d, g), 6–8, 10(c–f), 11(c–f), 12–14, 17
□ 3, 4(d–i), 5(b, e, h), 6(d, e, f), 9, 10(e–h), 11(e–h), 12, 15–19
⬠ 3(d, e, f, i, l), 4(d, h, k), 5(c, i), 6(d, f), 10(g, h), 11(g, h), 12(b, d), 15, 19–22

2D.1 **1** Complete the following factor pairs.

**a** 20 = 1 × 20 or 4 × ____
**b** 24 = 1 × ____ or 2 × ____ or 3 × ____
**c** 32 = 1 × ____ or ____ × 16 or ____ × ____
**d** 28 = 1 × ____ or ____ × 14 or 4 × ____
**e** 49 = 1 × ____ or ____ × 7
**f** 50 = ____ × 50 or 2 × ____ or 5 × ____

2D.2 **2** Find all the factors of each number.

**a** 15 **b** 18 **c** 9 **d** 12 **e** 10 **f** 14

**3** Find all the factors of each number.

**a** 20 **b** 24 **c** 32 **d** 28 **e** 49 **f** 50
**g** 6 **h** 66 **i** 56 **j** 81 **k** 27 **l** 100

**4** Find all the factors of each number. Use the divisibility tests to check if a number is a factor.

**a** 44 **b** 54 **c** 60 **d** 64 **e** 75 **f** 105
**g** 120 **h** 132 **i** 200 **j** 98 **k** 168 **l** 231

UNDERSTANDING AND FLUENCY

**5** Use your answers to question **3** to find the common factors of each set of numbers.

**a** 6 and 20 **b** 81 and 27 **c** 50 and 100
**d** 56 and 100 **e** 6 and 81 **f** 32 and 56
**g** 6, 24 and 27 **h** 20, 28 and 32 **i** 28, 49 and 56

**6** Find the factors between 10 and 30 of each of the following numbers.

**a** 26 **b** 55 **c** 42 **d** 63 **e** 70 **f** 36

**7** **a** List the factors of 18.
**b** List the factors of 36.
**c** List the common factors of 18 and 36.
**d** What is the highest common factor (HCF) of 18 and 36?

**8** **a** List the factors of 21.
**b** List the factors of 45.
**c** List the common factors of 21 and 45.
**d** What is the HCF of 21 and 45?

**9** **a** List the factors of 12.
**b** List the factors of 30.
**c** List the common factors of 12 and 30.
**d** What is the HCF of 12 and 30?

2D.3 **10** Find the HCF of each pair of numbers.

**a** 8 and 24 **b** 15 and 27 **c** 24 and 42 **d** 5 and 20
**e** 36 and 32 **f** 45 and 30 **g** 63 and 35 **h** 50 and 100

**11** Find the HCF of each group of numbers.

**a** 4, 8 and 12 **b** 10, 25 and 30 **c** 16, 24 and 32 **d** 15, 6 and 27
**e** 42, 36 and 18 **f** 20, 30 and 50 **g** 38, 26 and 14 **h** 9, 6, 12 and 18

**12** Find the LCM and the HCF of each group of numbers.

**a** 12 and 30 **b** 10 and 35 **c** 4, 12 and 16 **d** 2, 9 and 18

PROBLEM SOLVING AND REASONING

**13** Sophie is thinking of an odd number that is a factor of 24 and is not 1. What number is she thinking of?

**14** Max is arranging 18 pieces of sushi on a plate for his guests. He would like to arrange them so that there are the same number of pieces in each row.

**a** Suggest one way that Max could arrange the sushi on a plate. You may like to present your answer using a diagram.
**b** How many possible arrangements are there? Describe the arrangements that are possible.

**15** Archie has 24 tickets to the drive-in and Veronica has 42 tickets to the school dance. They want to sell them in packs that contain the same number of each type of ticket. What is the largest number of identical packs they can make?

**16** For a school fete, Hanna and Josh are deciding how to divide up a batch of 120 cookies so that there is an equal number of cookies in each bag. Suggest at least three ways this could be done.

**17** Describe the differences between multiples and factors.

**18** **a** What is the highest common factor of 18 and 45?

**b** What is the 'lowest common factor' of 18 and 45?

**c** Explain why the 'lowest common factor' of two or more numbers is not particularly useful. Consider the lowest common factors of a few different pairs of numbers.

**19** Tobias is making bunches of balloons for his little brother's birthday party. He has 56 purple balloons, 64 green balloons and 104 blue balloons. He wants all the bunches to be identical.

**a** What is the greatest number of bunches Tobias can make if he uses every balloon?

**b** How many of each colour will be in each bunch?

**c** How many balloons will be in each bunch?

**20** Find the smallest whole number that has only:

**a** one factor **b** two factors **c** three factors **d** four factors **e** five factors **f** six factors



**21** The HCF of two numbers is 16. The smaller of the two numbers is 64 less than the larger number. Both numbers are less than 100. Determine two possible numbers that fit this description.

**22** An abundant number is a number that is smaller than the sum of its factors (excluding 1 and itself). The number 12 is the first abundant number. Can you find all the abundant numbers between 1 and 60?

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Highest common factor

**Worksheet** Finding the highest common factor

**Topic quiz** 2D

# 2E Prime and composite numbers

## Learning intentions

✔ I can identify if a number is prime or composite.
✔ I can find the prime factors of positive whole numbers.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Prime and composite numbers |
| Year 8 | 5G Factorising |
| Year 9 | 3C Factorising using the HCF |

## Prime numbers

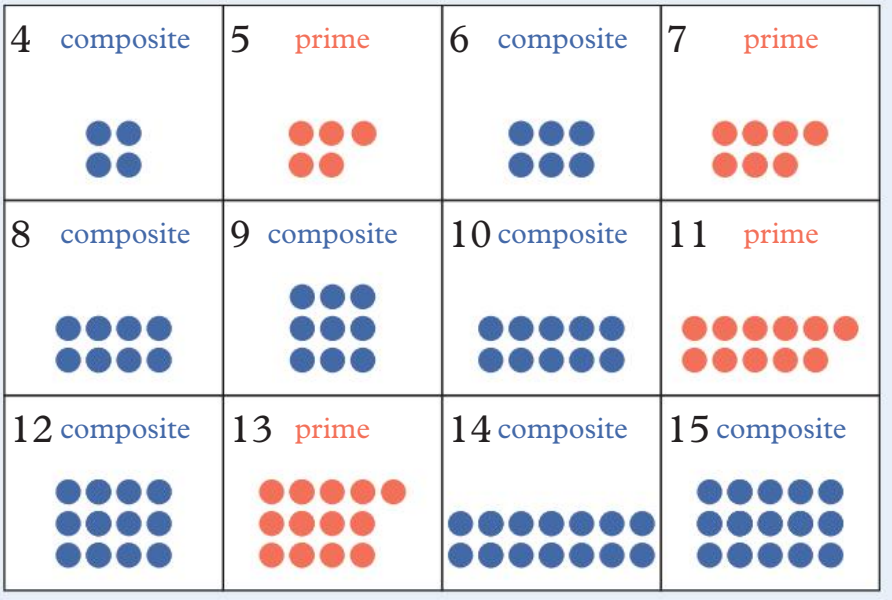
- A **prime number** is a positive whole number greater than 1 that has exactly two factors: itself and 1. For example, the only factors of 7 are 1 and 7, so 7 is a prime number.
- The divisibility rules can be used to quickly check if a number is a prime. For example, if a number (other than 2) is even, then it is divisible by 2 and not a prime number.

## Composite numbers

- A **composite number** is a positive whole number that is exactly divisible by at least one whole number other than itself and 1.
- All composite numbers can be written as the product of **prime factors**, factors that are prime numbers.
- 1 is neither a prime nor a composite because it only has one factor.
- 0 is neither a prime nor a composite because it is not a positive number.

$6 = 2 \times 3$

composite number — prime factors





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## **Example 2E.1** Identifying prime and composite numbers

State whether each number is prime or composite. Give a reason for your answer.

**a** 19 **b** 48 **c** 57

**THINK**

**a** Determine if 19 has any factors other than 1 and itself.

**b** Determine if 48 has any factors other than 1 and itself. 48 is an even number, so it is divisible by 2.

**c** Determine if 57 has any factors other than 1 and itself. 57 is divisible by 3.

**WRITE**

**a** 19 is a prime number because it only has two factors: 1 and itself.

**b** 48 is a composite number since it has more than two factors.

**c** 57 is a composite number since it has more than two factors.

## **Example 2E.2** Finding prime factors

List the prime factors of:

**a** 40 **b** 16

**THINK**

**a** List all the factors of 40. Identify the prime numbers.

**b** List all the factors of 16. Identify the prime numbers.

**WRITE**

**a** Factors: 1, 2, 4, 5, 8, 10, 20, 40
The prime factors of 40 are 2 and 5.

**b** Factors: 1, 2, 4, 8, 16
The only prime factor of 16 is 2.

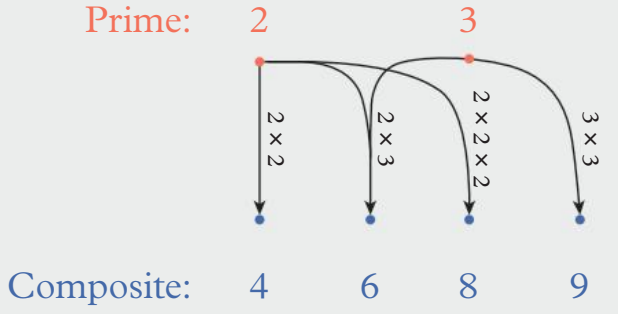
**Helpful hints**

✔ Although most even numbers are composite numbers, not all odd numbers are primes. For example, 21 is an odd number but it has four factors, 1, 3, 7 and 21, so it is not prime.

✔ Remember that 0 and 1 are neither prime nor composite.

✔ 2 is the only even number that is also a prime number.

✔ All composite numbers can be written as the product of prime factors. Here are the first two prime numbers. Can you see how they make up the first four composite numbers?



ANS p547

# Exercise 2E Prime and composite numbers

1–7, 8(a, b), 9(a, b), 10–13 | 3–4, 5(g–l), 6(g–l), 7–9, 12–16 | 5(j, k, l), 6(j, k, l), 7, 8(c, d), 9(c, d), 12–13, 15, 17–18

UNDERSTANDING AND FLUENCY

**1** The prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

**a** List all the factors of each of the prime numbers from 1 to 20.

**b** How many factors does each prime number have?

**2** The composite numbers from 1 to 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 and 20.

**a** List all the factors of each of the composite numbers from 1 to 20.

**b** How many factors does each composite number have?

**c** Do composite numbers have more, fewer or the same number of factors as prime numbers?

**3** List all the factors of each number.

**a** 24 **b** 49 **c** 73 **d** 125 **e** 37 **f** 82

**4** Use your answers to question **3** to decide whether each number is prime or composite.

2E.1 **5** State whether each number is prime or composite. Give a reason for your answer.

**a** 14 **b** 28 **c** 27 **d** 47 **e** 38 **f** 79

**g** 18 **h** 43 **i** 77 **j** 59 **k** 131 **l** 87

2E.2 **6** List the prime factors of:

**a** 4 **b** 10 **c** 18 **d** 35 **e** 16 **f** 20

**g** 27 **h** 40 **i** 42 **j** 56 **k** 33 **l** 64

PROBLEM SOLVING AND REASONING

**7** Describe the difference between prime numbers and composite numbers.

**8** Find two prime numbers that add to give each of the following numbers.

**a** 24 **b** 36 **c** 82 **d** 144

**9** Find two prime numbers that multiply to give each of the following numbers.

**a** 21 **b** 55 **c** 26 **d** 115

**10** Bobby is writing out the factors of 162 in factor pairs, but he only wants to record pairs that contain composite numbers. Which two of the following factor pairs contain only composite numbers?

**A** $2 \times 81$ **B** $3 \times 54$ **C** $6 \times 27$ **D** $1 \times 162$ **E** $9 \times 18$

**11** **a** Write down all the prime numbers between 10 and 30.

**b** What do these numbers have in common?

**c** Are all prime numbers odd? Use an example to justify your answer.

**12** Explain why the number 1 is a special number that is neither prime nor composite.

**13** Answer true or false to each statement. Give a reason for your answer.

**a** All even numbers are prime numbers.

**b** All odd numbers are prime numbers.

**c** The two smallest prime numbers are 2 and 3.

**d** All composite numbers are even numbers.

**e** The product of two prime numbers is always even.

**f** The product of two prime numbers is always a composite number.

**g** The number 0 is neither a prime number nor a composite number since it is even.

**h** Between 1 and 30, there are more composite numbers than prime numbers.

**14** Zeke is the head of a manufacturing business that packages mugs into boxes of 30. He wants to pack the mugs in a single rectangular layer.

The dimensions of the arrangement shown are 2 by 6.

**a** What are the dimensions of all the rectangles Zeke can make with his 30 mugs?

**b** Why would he choose to pack the mugs in groups of 30 and not 29 or 31?

**c** If Zeke wanted to pack boxes of 36, list all the possible arrangements he can choose from if he decided that he could stack them. Use a systematic approach to determine all possible options.

**15** A pair of prime numbers that differ by 2 are called twin primes. For example, 3 and 5 are twin primes, as $5 - 3 = 2$.

**a** List the three other pairs of twin primes that occur between 1 and 20.

**b** How many pairs of twin primes occur between 21 to 100? List all the twin primes between 21 and 100.

**16** What is the largest three-digit prime number?

**17** How many three-digit prime numbers can you make with all three numbers 1, 3 and 5?

**18** In a particularly long school, every Year 7 locker is in one long row. One day, all the Year 7 classes lined up at the front locker (Locker 1) and the Maths teachers gave the following instructions:

→ The first student in line must open every locker
→ The second student in line must close every second locker (locker 2, 4, 6, 8, etc.)
→ The third student in line must open/close every third locker (locker 3, 6, 9, 12, etc.)
→ The fourth student in line must open/close every fourth locker (locker 4, 8, 12, 16, etc.)
→ … and so on until the very last student (student 220).

Where it says open/close, it means if the locker was closed, the student had to open it, and if the locker was open, the student had to close it.

After each of the 220 students in Year 7 took their turn opening and closing the lockers, how many lockers were left open? Hint: Try to spot a pattern in the number of lockers which will be open.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Prime and composite numbers

**Investigation** Encryption codes

**Topic quiz** 2E

# 2F Prime factorisation

## Learning intentions

- ✔ I can express whole numbers as a product of prime factors.
- ✔ I can find the lowest common multiple of two numbers using prime factorisation.
- ✔ I can find the highest common factor of two numbers using prime factorisation.

**Inter-year links**

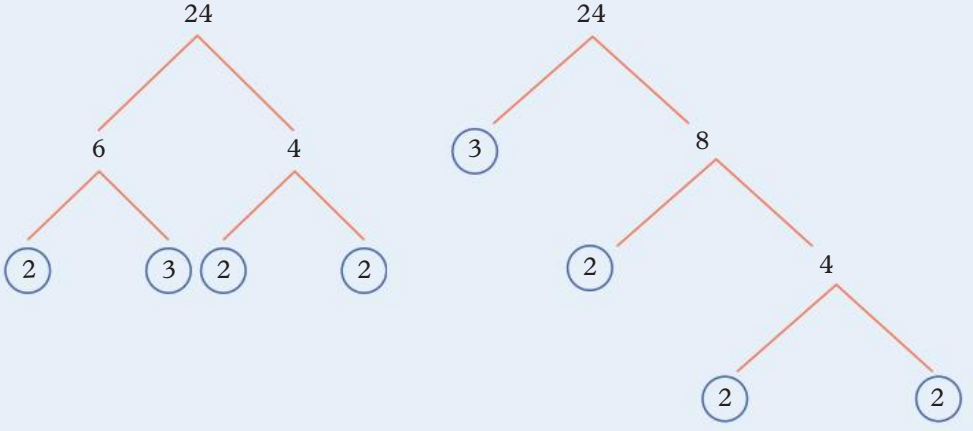
| | |
|---|---|
| Year 8 | 5G Factorising |
| Year 9 | 3C Factorising using the HCF |

## Prime factorisation

- **Prime factorisation** is the product of the primes that make up a whole number.
  For example, $2 \times 3$ is the product of the primes that make up 6, so $2 \times 3$ is the prime factorisation of 6.
- The Fundamental Theorem of Arithmetic states:
  1. Every number greater than 1 is either a prime number or can be written as a product of prime numbers.
     For example, $24 = 2^3 \times 3$.
  2. There is only one prime factorisation for every composite number.
     For example, $24 = 2^3 \times 3$ is the unique factorisation for 24.

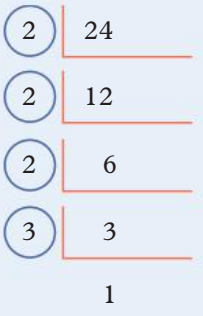
## Factor trees

- Prime factorisation can be performed using **factor trees**. In factor trees, composite numbers are broken down into factor pairs until prime numbers are found.
- To find the prime factorisation of a number using factor trees:
  1. Write the initial number at the top of the tree.
  2. On the branches splitting the initial number in two, write a factor pair that when multiplied give the initial number.
  3. Repeat set two for any composite factors until the numbers at the ends of the branches are all composite.
  4. Write the initial number as a product of its prime factors. Simplify the products using indices and write the bases in ascending order.



# Repeated division

- Prime factorisation can also be performed using repeated division. In repeated division, the initial composite number is divided by primes until the result is 1.
- To find the prime factorisation of a number using repeated division:
  1. Divide the initial number by its lowest prime factor.
  2. Divide the quotient by its lowest prime factor.
  3. Repeat the division process until the quotient is 1.
  4. Write the initial number as a product of its prime factors. Simplify the products using indices and write the bases in ascending order.




## Example 2F.1 Finding prime factorisation using factor trees

Use a factor tree to find the prime factorisation of each number.

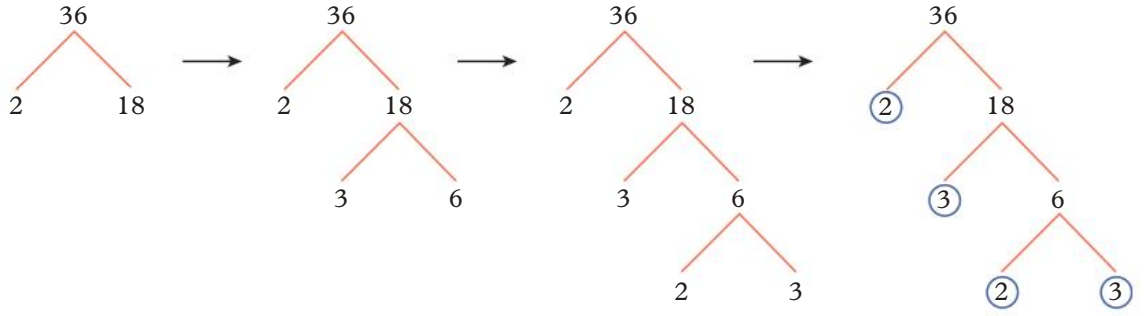
**a** 36 **b** 72

**THINK**

1. Split the composite number into two smaller factors. If no factors are obvious, divide by its lowest prime factor to identify a factor pair. The lowest prime factor of even numbers is always 2.
2. Find a factor pair of any resulting composite factors. Repeat the process until all the factors are prime.
3. Write the initial composite number as a product of its prime factors. Remember to simplify the products using indices and place the bases in ascending order: 2, 3, 5, ...

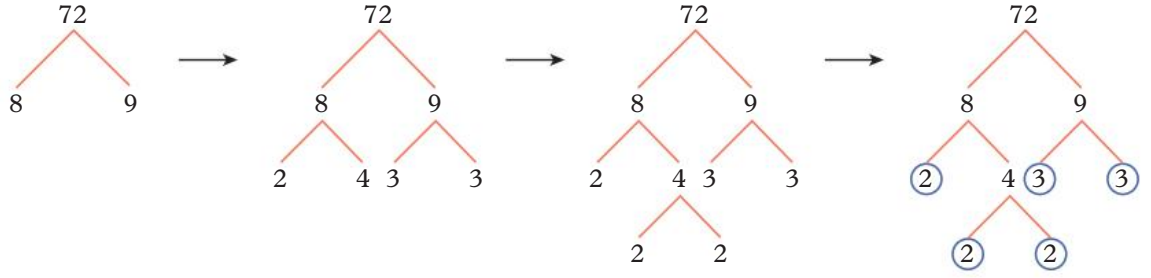
**WRITE**

**a**




$36 = 2 \times 3 \times 2 \times 3$
$= 2^2 \times 3^2$

**b**




$72 = 2 \times 2 \times 2 \times 3 \times 3$
$= 2^3 \times 3^2$

## **Example 2F.2** Finding prime factorisation using repeated division

Use repeated division to find the prime factorisation of each number.

**a** 12 **b** 210

**THINK**

1. Divide the composite number by its lowest prime factor. The divisibility rules can be used to quickly identify a prime factor.
2. Divide the resulting composite number by a prime factor. Repeat the process until the quotient is 1. Only divide the quotient by prime numbers.
3. Write the initial composite number as a product of its prime factors. Simplify the products using indices and place the bases in ascending order: 2, 3, 5, …

**WRITE**

**a**

$$\begin{array}{r|l} 2 & 12 \\ \hline & 6 \end{array} \longrightarrow \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \longrightarrow \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \longrightarrow \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$12 = 2 \times 2 \times 3$
$\quad = 2^2 \times 3$

**b**

$$\begin{array}{r|l} 2 & 210 \\ \hline & 105 \end{array} \longrightarrow \begin{array}{r|l} 2 & 210 \\ \hline 5 & 105 \\ \hline & 21 \end{array} \longrightarrow \begin{array}{r|l} 2 & 210 \\ \hline 5 & 105 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \longrightarrow \begin{array}{r|l} 2 & 210 \\ \hline 5 & 105 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \longrightarrow \begin{array}{r|l} 2 & 210 \\ \hline 5 & 105 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$210 = 2 \times 3 \times 5 \times 7$

**Helpful hints**

- ✔ When finding factor pairs for your factor trees, remember that the numbers in a factor pair must multiply together to produce the composite number.
- ✔ When factorising using repeated division, ONLY divide composites using prime numbers.
- ✔ When using the repeated division method, you can determine some factor pairs by dividing the initial composite number by each of its factors.

ANS p547

# Exercise 2F Prime factorisation

△ 1–2, 3(b, d, h), 4, 5, 6–7(1st, 2nd column), 8–11, 14(a, b)

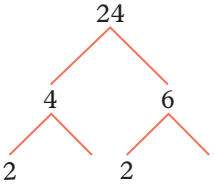
□ 1(d–f), 2(d–h), 3(d–h), 4, 5(b, c), 6–7(3rd column), 9–10, 12, 14–15, 17

⬠ 1(d, f), 2(c, h, j), 3(c, h, j), 4, 5(c), 6–7(4th column), 9, 13–14, 16–20

UNDERSTANDING AND FLUENCY

**1** Complete each factor tree and show each composite number as a product of prime factors.

**a**

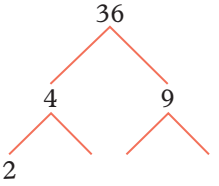



The prime factors of 24 are 2 and ____.

$24 = 2 \times 2 \times$ ____ $\times$ ____

$= 2^3 \times$ ____

**b**

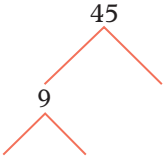



The prime factors of 36 are ____ and ____.

$36 = 2 \times$ ____ $\times$ ____ $\times$ ____

$= 2^2 \times$ ____

**c**

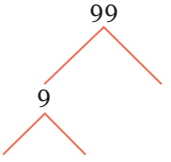



The prime factors of 45 are ____ and 5.

45 = ____ $\times$ 3 $\times$ ____

= ____ $\times$ ____

**d**

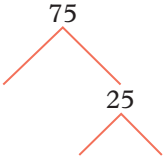



The prime factors of 99 are 3 and ____.

99 = 3 $\times$ ____ $\times$ ____

= ____ $\times$ ____

**e**

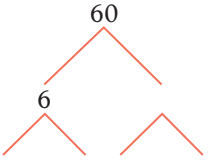



The prime factors of 75 are ____ and ____.

75 = ____ $\times$ ____ $\times$ ____

= 3 $\times$ ____

**f**




The prime factors of 60 are ____, ____ and ____.

60 = ____ $\times$ ____ $\times$ ____ $\times$ ____

= ____ $\times$ ____ $\times$ ____

2F.1 **2** Use a factor tree to find the prime factorisation of each number.

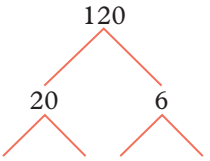
| | | |
|---|---|---|
| **a** 26 | **b** 32 | **c** 50 |
| **d** 63 | **e** 78 | **f** 58 |
| **g** 84 | **h** 92 | **i** 144 |

**3** Use a factor tree to find the prime factorisation of each number.

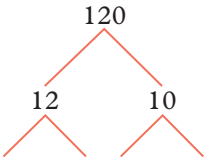
| | | |
|---|---|---|
| **a** 48 | **b** 96 | **c** 135 |
| **d** 182 | **e** 168 | **f** 285 |
| **g** 244 | **h** 416 | **i** 450 |

**4** **a** Complete each factor tree for the composite number 120. Remember to continue until all of the prime factors are found.

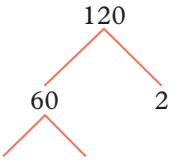
**i**




**ii**




**iii**




**b** Write the prime factorisation for each tree. Do the intital factors change the result of each prime factorisation?

**5** Complete each repeated division and show each composite number as a product of prime factors.

**a**

| | |
|---|---|
| 2 | 80 |
| 2 | 40 |
| 2 | 20 |
| □ | 10 |
| □ | □ |
| | 1 |

The prime factors of 80 are 2 and ____.

80 = 2 × 2 × ____ × ____ × ____

= $2^4$ × ____

**b**

| | |
|---|---|
| 5 | 225 |
| □ | 45 |
| 3 | □ |
| □ | □ |
| | 1 |

The prime factors of 225 are ___ and ____.

225 = 3 × 3 × ____ × ____

= $3^2$ × ____

**c**

| | |
|---|---|
| 2 | 570 |
| □ | □ |
| □ | □ |
| 19 | 19 |
| | 1 |

The prime factors of 570 are 2, ___, ___ and ____.

570 = 2 × ____ × ____ × ____

= ____ × ____ × ____ × ____

2F.2 **6** Use repeated division to find the prime factorisation of each number.

| | | | |
|---|---|---|---|
| **a** 28 | **b** 56 | **c** 44 | **d** 85 |
| **e** 132 | **f** 52 | **g** 250 | **h** 90 |
| **i** 72 | **j** 220 | **k** 100 | **l** 400 |

**7** Use repeated division to find the prime factorisation of each number.

| | | | |
|---|---|---|---|
| **a** 228 | **b** 560 | **c** 324 | **d** 845 |
| **e** 343 | **f** 512 | **g** 154 | **h** 960 |
| **i** 740 | **j** 1140 | **k** 798 | **l** 1716 |

**8** Decide if the following products are the prime factorisation of each composite number. Use the Fundamental Theorem of Arithmetic to justify your answer.

**a** $24 = 2 \times 2 \times 2 \times 3$

**b** $36 = 2^2 \times 3^2$ and $36 = 2^3 \times 3$

**c** $19 = 2 \times 17$

**d** $100 = 4 \times 5^2$

**9** What is the smallest number that has:

**a** four different prime factors?

**b** five prime factors?

**c** six different prime factors?

**10** Erin is making tickets for her International Women's Day trivia night. She wants to cut square tickets that are the same size from a sheet of paper which measures 240 cm by 330 cm. What is the largest size the square tickets can be if Erin uses all the paper? Use each number's prime factorisation to help you.

**11** A chocolate cake mixture has been baked in a large rectangular container. The chef wishes to cut the cake into 36 individual serves.

**a** Describe how prime factors can be used to divide the cake into 36 equal pieces.

**b** Draw a diagram to show a way of cutting the cake using prime factors.

**c** Draw a diagram to show one more way of cutting the cake into 36 equal pieces.

**12** Using only the product of the prime factors 2, 3 and 5 make the following numbers.

**a** 100 **b** 180 **c** 1000

**13** Using only the product of the prime factors 3, 5 and 7 make the following numbers.

**a** 315 **b** 441 **c** 1575

**14** We can use the prime factorisation to find the HCF of a pair of numbers.

**1** First, find the prime factorisation of each number.

**2** Circle the prime factors that are common to both numbers.

**3** Multiply them together to find the HCF.

For example,

$$24 = 2 \times 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$\text{HCF} = 2 \times 3$$
$$= 6$$

Use prime factorisation to find the HCF of each pair of numbers.

**a** 20 and 42 **b** 35 and 45 **c** 72 and 64 **d** 84 and 96

**15** Ming has made 45 purple (blueberry) and 60 green (pistachio) macarons. She sorts them into bags so that each bag has the same number of purple macarons and the same number of green macarons with none of either colour left over.

**a** Use prime factors to find the HCF of 45 and 60.

**b** What is the smallest number of macarons that could be in each bag?

**16** The school library has been given 55 science fiction and 75 fantasy books. They plan to sort the books into groups that will be given as prizes in a reading competition. Each prize is to have the same number of science fiction books and the same number of fantasy books, with none left over. How many books will be in each prize? Use prime factorisation to help you determine the answer.

PROBLEM SOLVING AND REASONING

**17** We can use the prime factorisation of numbers to find the LCM of a pair.

1. First, find the prime factorisation of each number. Write it out using index notation.
2. Circle the highest index of each prime factor listed.
3. Multiply them together to find the LCM.

For example,

$$72 = 2^3 \times 3^2$$
$$60 = 2^2 \times 3 \times 5$$
$$\text{LCM} = 2^3 \times 3^2 \times 5$$
$$= 360$$

Use prime factorisation to find the LCM of each pair of numbers.

**a** 20 and 42 **b** 35 and 45 **c** 75 and 64 **d** 84 and 96

**18** Two dairy farmers, Yonas and Samara, are transporting their entire herds between paddocks. Yonas is moving his cows in groups of 21 and Samara is moving her cows in groups of 34. If each farmer has the same number of cows, what is the smallest number of cows in each herd?

**19** Two different buses leave the city at midday for the Yarra Valley. The first bus makes a stop every 25 minutes and the second bus makes a stop every 30 minutes. After they leave the city, when will the buses next stop at the same time?

CHALLENGE

**20** Some four-digit numbers can be written as the product of a three-digit number and a two-digit number using each of the digits 1 to 9 exactly once. Use prime factorisation to help find the two factors in these products.

| | | | |
|---|---|---|---|
| | | | |
| × | | | |
| 4 | 3 | 9 | 6 |

| | | | |
|---|---|---|---|
| | | | |
| × | | | |
| 5 | 3 | 4 | 6 |

| | | | |
|---|---|---|---|
| | | | |
| × | | | |
| 5 | 7 | 9 | 6 |

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Prime factorisation

**Investigation** Pizza with a topping of primes

**Topic quiz** 2F

# Chapter summary

## Sequences

Sequence: 2, 4, 6, 8, …
Rule: +2

+2 +2 +2

1st term, 2nd term, 3rd term, 4th term, continue on

- To find the rule for a sequence, consider all possible operations including:
  - addition
  - subtraction
  - multiplication
  - division
  - indices
  - square roots.

## Lowest common multiple

multiples of 2: 2, 4, 6, 8, 10, 12 …
multiples of 3: 3, 6, 9, 12, 15, 18 …
LCM: 6

## Highest common factor

factors of 16: 1, 2, 4, 8, 16
factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
HCF: 8

## Divisibility

| | | |
|---|---|---|
| ÷2<br>The last digit of the number is even. | ÷3<br>The sum of all the digits in the number is divisible by 3. | ÷4<br>The number made by the last two digits is divisible by 4. |
| ÷5<br>The last digit is 5 or 0. | ÷6<br>The number is divisible by 2 and 3. | ÷7<br>There is no easy trick for 7. |
| ÷8<br>The number made by the last three digits is divisible by 8. | ÷9<br>The sum of all the digits in the number is divisible by 9. | ÷10<br>The last digit is 0. |

## Prime numbers

$6 = 2 \times 3$

composite number; prime factors

- 1 is neither a prime nor a composite because it only has one factor.
- 0 is neither a prime nor a composite because it is not a positive number.

## The Fundamental Theorem of Arithmetic

1 Every number greater than 1 is either a prime number or can be written as a product of prime numbers.

For example,

$24 = 2^3 \times 3$

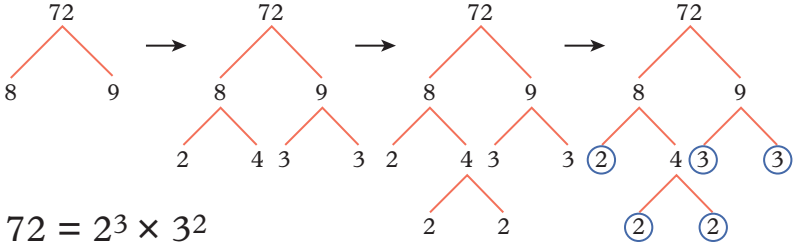
2 There is only one prime factorisation for every composite number.

For example,

$24 = 2^3 \times 3$

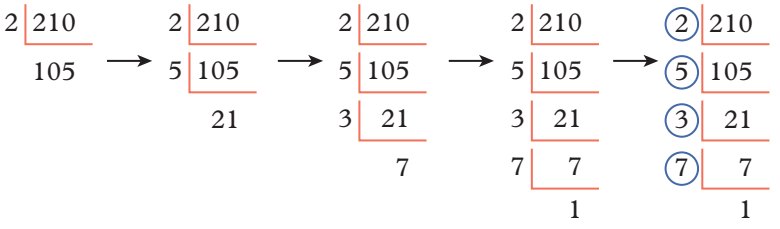
is the *unique* factorisation for 24.

## Factor trees




$72 = 2^3 \times 3^2$

## Repeated division




$210 = 2 \times 3 \times 5 \times 7$

# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

2A **1** The missing term in the sequence 6, 12, 18, ___, 30, 36, … is

**A** 24 **B** 6 **C** 42 **D** 50 **E** 15

2A **2** The fourth term in a sequence is 16. If the rule for this sequence is subtract 6, then the first term is

**A** 6 **B** 22 **C** 28 **D** 34 **E** 40

2B **3** A number that is divisible by 3 and 5 is

**A** 65 **B** 125 **C** 99 **D** 55 **E** 135

2B **4** A number that is divisible by 3 and 8 is

**A** 9072 **B** 243 **C** 846 **D** 4096 **E** 531

2C **5** A number that is *not* a common multiple of 6 and 16 is

**A** 24 **B** 48 **C** 96 **D** 144 **E** 192

2C **6** The lowest common multiple of 9 and 5 is

**A** 90 **B** 45 **C** 30 **D** 14 **E** 135

2D **7** What is the highest common factor of 10 and 25?

**A** 5 **B** 10 **C** 25 **D** 50 **E** 15

2D **8** Which of the following is *not* a factor of 60?

**A** 4 **B** 8 **C** 12 **D** 15 **E** 20

2E **9** Which number is *not* a prime number?

**A** 11 **B** 25 **C** 37 **D** 53 **E** 5

2E **10** Which one of the following is a prime number?

**A** 24 **B** 29 **C** 39 **D** 63 **E** 55

2F **11** What is the prime factorisation of 60?

**A** $3 \times 4 \times 5$ **B** $2^2 \times 15$ **C** $2^2 \times 3 \times 5$ **D** $2 \times 3 \times 10$ **E** $2 \times 3 \times 5 \times 6$

2F **12** Which one of the following composite numbers has the most prime factors?

**A** 20 **B** 24 **C** 30 **D** 36 **E** 40

## Short answer

2A **1** Find the missing terms in these sequences.

**a** 14, 18, ___, 26, ____, 34, …
**b** 75, ___, 65, 60, 55, 50, …
**c** ___, 10, 20, 40, 80, ___, …
**d** 176, 88, 44, ___, ___, …

2A **2** On Monday Sasha notices 5 ants in her kitchen, on Tuesday she sees 10 ants and on Wednesday there are 20 ants. If this pattern continues, how many ants will be in Sasha's kitchen the following Monday?

2B **3** Check if the number 13 824 is divisible by:

**a** 2 **b** 4 **c** 6 **d** 8

2B **4** The Year 7 captains have organised a cupcake fund raiser. They have ordered 2625 cupcakes. They want to make sure each student can have an equal number of cupcakes with no left-overs.

**a** Use the divisibility tests to determine whether the captains can provide 2, 3, 4, 5, 6 or 7 cupcakes per person without wastage.

**b** Determine which of these would be the best option for a class of 375 students.

2C **5** Write all the multiples of 4 between 22 and 45.

2C **6** There are 100 houses in a street. The houses are numbered from 1–100. All the houses that are a multiple of 6 have a dog. All the houses that are multiples of 8 have a cat. Which houses have both a dog and a cat?

2C **7** Find the lowest common multiple of each group of numbers.

**a** 12 and 18 **b** 6, 9 and 15

2D **8** List all the factors of 80.

2D **9** Susie has 24 soft toys. She arranges them on her bed in rows so that there are the same number of toys in each row. Suggest three different ways Susie could arrange her toys.

2D **10** Find the highest common factor of each group of numbers.

**a** 12 and 18 **b** 6, 9 and 15

2D **11** Find the first three numbers, excluding 1, that have:

**a** a factor of 7

**b** four different factors

**c** an odd number of factors.

2E **12** Find two different prime numbers that add to give each of the following numbers.

**a** 14 **b** 21 **c** 26 **d** 30

2E **13** Find two prime numbers that multiply to give each of the following numbers.

**a** 15 **b** 33 **c** 34 **d** 58

2F **14** **a** Is 40 a prime or composite number? Give a reason to support your answer.

**b** Draw a factor tree and list the prime factors of 40.

**c** Write 40 as a product of its prime factors in index notation.

2F **15** An op shop has 48 shirts and 36 jumpers to give away. The staff sort clothes into bags so that each bag has the same number of shirts and jumpers without any left over.

**a** Use prime factors to find the HCF of 36 and 48.

**b** How many bags will there be if the least number of items are placed in each bag?

**c** How many pieces of clothing in total will be in each bag?

## Analysis

Tahlia is an animal photographer. She has 280 photos of mammals and 168 photos of birds. Tahlia plans to make a book from her photos.

**a** How many photos does Tahlia have in total?

**b** Tahlia wants to have either exactly 6 or 8 photos on every page. Which number does she choose so that she has no photos left over?

**c** How many pages are in her book?

**d** How many photos of mammals should Tahlia put on each page if she wants to make sure all pages in the book contain the same number of mammal photos?

**e** How many photos of birds should Tahlia put on each page if she wants to make sure all pages in the book contain the same number of bird photos?

**f** Every fifth page of the book has photos of sea mammals and every sixth page has birds of prey. Which pages would contain photos of both sea mammals and birds of prey?

**g** Every fourth page has photos of big cats. Which pages would contain photos of big cats, sea mammals and bird of prey?

# 3

# Fractions and ratios

## Index


**3A** Fractions
**3B** Equivalent fractions
**3C** Ordering fractions
**3D** Mixed numbers and improper fractions
**3E** Adding and subtracting fractions
**3F** Multiplying fractions
**3G** Dividing fractions
**3H** Ratios


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Visual representations of fractions
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying and dividing whole numbers
- ✔ Highest common factor
- ✔ Number lines

## Curriculum links

- Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (VCMNA242)
- Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (VCMNA243)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)
- Express one quantity as a fraction of another, with and without the use of digital technologies (VCMNA245)
- Recognise and solve problems involving simple ratios (VCMNA249)

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# 3A Fractions

## Learning intentions

- ✔ I can identify fractions from images, statements and number lines.
- ✔ I can express one quantity as a fraction of another.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Fractions |
| Year 8 | 2A Fractions |

## Fractions

- A **fraction** is a part or a portion of a whole, or multiple wholes.

$\frac{4}{5}$

the **numerator** shows the number of equal parts of the whole

the **denominator** shows the total number of parts in one whole

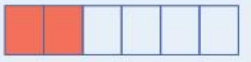
- **Proper fractions** have a **numerator** that is less than the **denominator**.

  Some examples are $\frac{3}{5}, \frac{2}{6}, \frac{11}{15}$.

Proper fraction

Numerator < Denominator

$\frac{2}{6}$



- **Improper fractions** have a numerator that is greater than or equal to the denominator.

  Some examples are $\frac{13}{6}, \frac{9}{5}, \frac{31}{8}$.

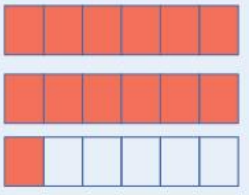
- **Mixed numbers** consist of both a whole number and a proper fraction.

  For example, $2\frac{1}{6} = 2 + \frac{1}{6}$.

Improper fraction

Numerator ≥ Denominator

$\frac{13}{6}$



Mixed number

$2\frac{1}{6}$

- Whole numbers can be written as fractions:
  - → where the denominator is 1

    For example, $5 = \frac{5}{1}, 2 = \frac{2}{1}, 100 = \frac{100}{1}$.
  - → or the denominator divides into the numerator without a remainder

    For example, $\frac{2}{2} = 1, \frac{9}{3} = 3, \frac{28}{7} = 4$.

## Fractions on a number line

- Fractions can be written on a number line.
  - → Number lines can be used to compare the size of numbers.

## Fractions of a set

- Fractions can show the proportion or number of a group of objects within a larger set of objects.
  - → A **set** is a defined group of objects or numbers.

    For example, here is a set of 5 counters. There are 2 blue counters so $\frac{2}{5}$ of the counters are blue.

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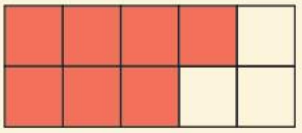
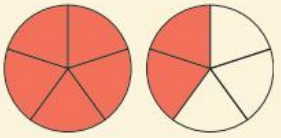
## **Example 3A.1** Identifying fractions from a diagram

**a** The rectangle represents a whole. What fraction is shaded of the whole shape?



**b** Each circle represents a whole. What fraction is shaded of the whole shape?



**THINK**

1. Count the number of equal parts in the whole shape.
2. Count the number of shaded parts in the shape.
3. Write the number of shaded parts as the numerator, and the number of parts in each whole as the denominator.

**WRITE**

**a** The shape is cut into 10 parts.
The shape has 7 shaded parts.
The fraction of the shape that is shaded is $\frac{7}{10}$.

**b** The shape is cut into 5 parts. There are two shapes.
There are 7 shaded parts.
The fraction of the shape that is shaded is $\frac{7}{5}$.

## **Example 3A.2** Identifying fractions from a number line

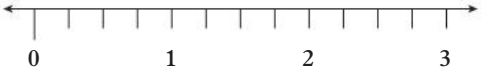
Draw the fractions $\frac{3}{4}$, $\frac{5}{4}$, $2\frac{1}{4}$ on a number line.

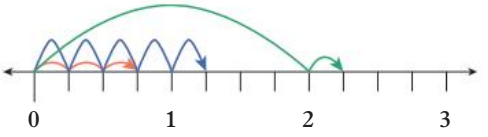
**THINK**

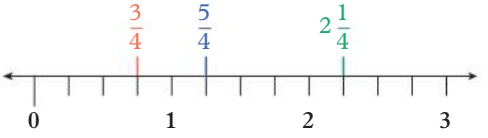
1. Draw a number line from 0 to 3, making sure the smallest and largest fractions are between the numbers on the scale.

   Divide the number line into segments of the denominator. Divide each section between two whole numbers into 4 parts.
2. Find the location of the fractions on the number line by counting up to the fraction from 0.
3. Mark and label the fractions on the number line.

**WRITE**



## **Example 3A.3** Identifying fractions from a set

Identify the fraction that is described by the following statement.

17 correct answers on a quiz containing 25 questions.

| THINK | WRITE |
|---|---|
| **1** Determine the total number of objects in the set. There are 25 questions in the quiz. | There are 25 questions in the quiz. |
| **2** Determine the number of objects in the group within the set. There are 17 correct answers in the quiz. | There are 17 correct answers in the quiz. |
| **3** Write the number of objects in the group within the set as the numerator, and the number of objects in the set as the denominator. | $\frac{17}{25}$ of the answers to the quiz were correct. |

**Helpful hints**

- ✔ Although fractions are made up of a numerator and a denominator, a fraction is just one value.
- ✔ Be careful when selecting the important information in a question. Try to determine the total number of objects in the set first. Make sure you write this as the denominator.

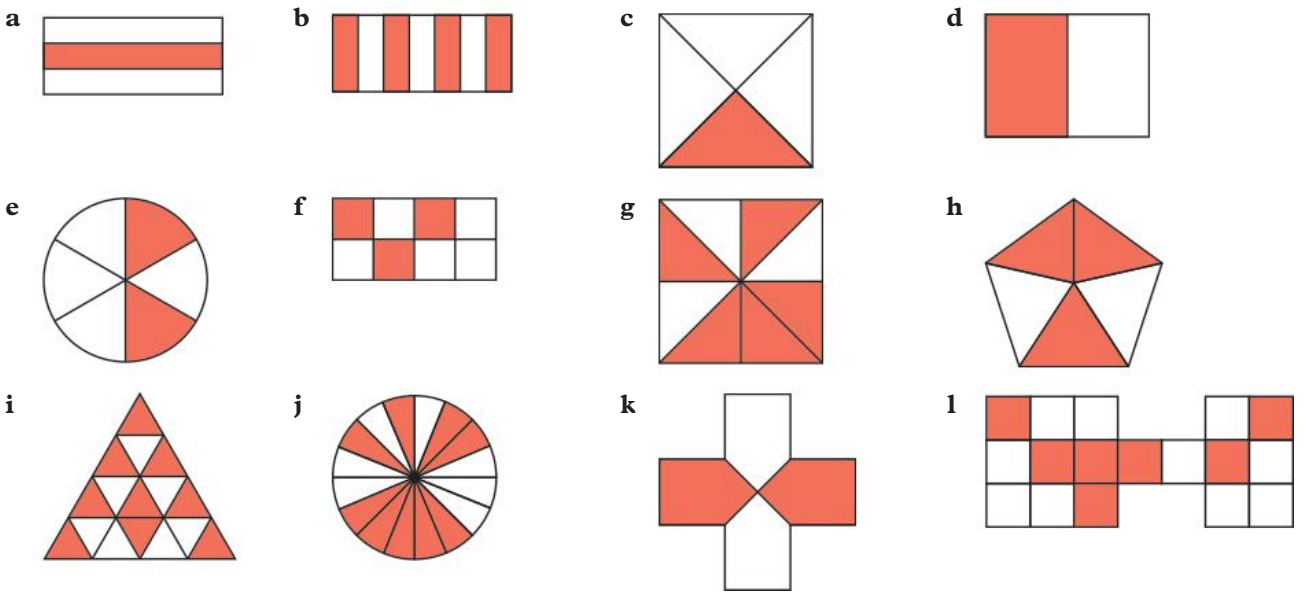
ANS p549

# **Exercise 3A** Fractions

△ 1(1st, 2nd column), 2–9, 10(b, d, f), 12–14, 15(a, b, c) ▢ 1(3rd, 4th column), 2, 4–5, 7–9, 10, 12, 15–17, 18(a, b, c) ⬠ 1(4th column), 2, 5(e–h), 8, 10, 12, 15, 18–20

UNDERSTANDING AND FLUENCY

**3A.1** **1** What fraction of each shape is shaded?

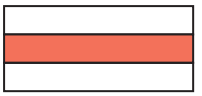


**2** What fraction of each shape in question **1** is *not* shaded?

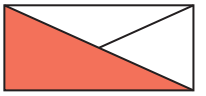
**3** Which diagram correctly displays that $\frac{1}{3}$ of the shape has been shaded? Provide a reason for your selection.

**A**

**B**



**C**



**4** Consider the following fractions: $\frac{5}{2}, 2\frac{3}{5}, \frac{7}{8}, 1\frac{5}{9}, \frac{17}{2}, \frac{16}{31}, \frac{29}{15}, 12\frac{1}{7}, \frac{13}{22}$.

**a** Which of the above fractions are proper fractions?

**b** Which of the above fractions are improper fractions?

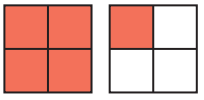
**c** Which of the above fractions are mixed numbers?

**5** If one whole shape represents 1 whole, write the number that is represented by the shading in each diagram as:

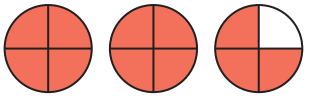
**i** an improper fraction

**ii** a mixed number.

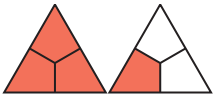
**a**



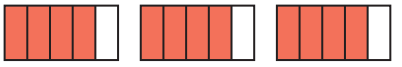
**b**



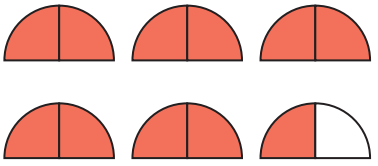
**c**



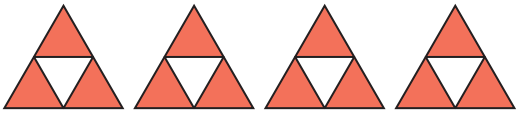
**d**



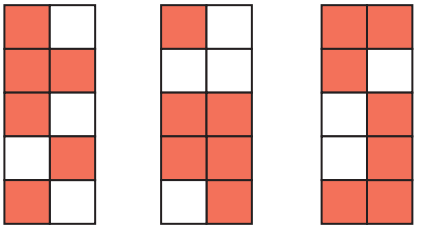
**e**



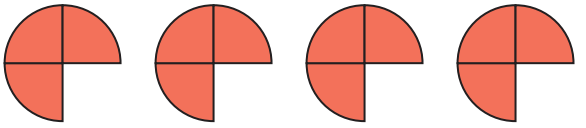
**f**



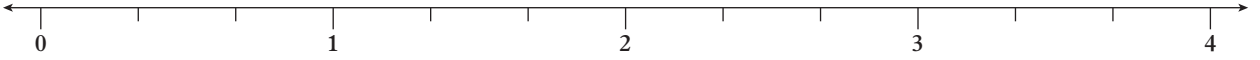
**g**



**h**



**6** The number line below has been divided into four equal intervals from 0 to 4. Each interval has been further subdivided into three equal parts.



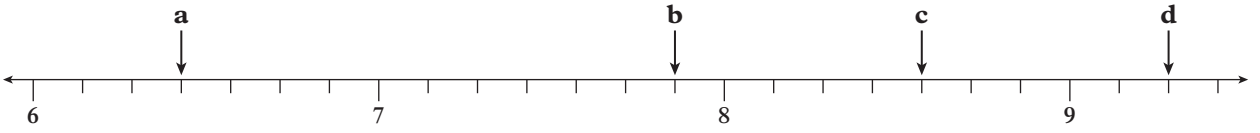

**a** What fraction of an interval is each part?

**b** Label each mark on the scale with the value it represents.

3A.2 **7** Draw the fractions $\frac{2}{5}, \frac{5}{5}, \frac{8}{5}, \frac{13}{5}$ and $\frac{17}{5}$ on a number line.

**8** Draw the fractions $7\frac{3}{4}, \frac{36}{4}, 9\frac{2}{4}, \frac{33}{4}$ and $\frac{29}{4}$ on a number line.

**9** Write down the values shown for **a–d** on the number line.



UNDERSTANDING AND FLUENCY

**3A.3** **10** Identify the fraction that is described by each statement.

**a** 12 correct answers on a quiz containing 23 questions

**b** The number of months consisting of 31 days in a year

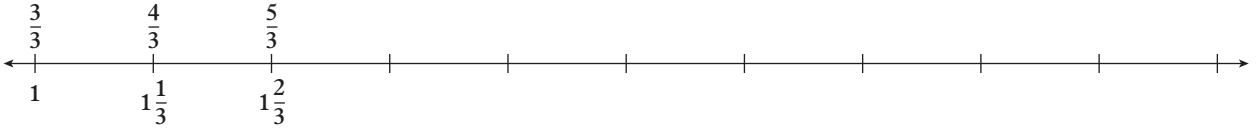
**c** \$23 change from a \$50 note

**d** A basket contains 41 balls, where 8 are red

**e** 17 episodes of a 22-episode series have been watched

**f** 21 slices of pizza where each slice is one-eighth of a pizza

**11** The number line below has been divided into equal intervals. The first three points have been labelled using improper fractions and mixed numbers.




**a** Complete the marked number line using both improper fractions and mixed numbers.

**b** Write down the fraction that is:

**i** the number that is $\frac{1}{3}$ to the right of $\frac{5}{3}$

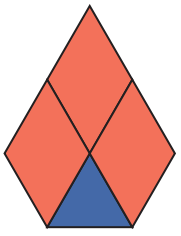
**ii** the improper fraction that is $\frac{2}{3}$ to the left of 4

**iii** the mixed number that is 1 whole to the right of $\frac{7}{3}$.

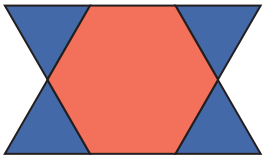
PROBLEM SOLVING AND REASONING

**12** If the whole shape is 1 whole, determine the fraction of each colour. Hint: Can you find the blue triangles inside the other shapes?

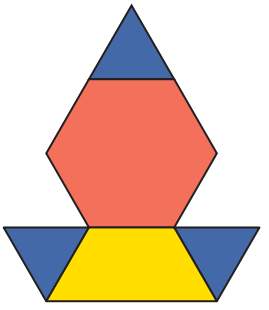
**a**



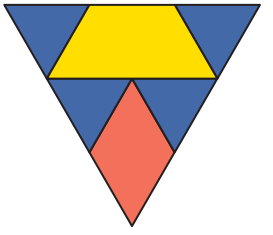
**b**



**c**



**d**



**13** What is the difference between a proper and an improper fraction?

**14** What is the difference between an improper fraction and a mixed number?

**15** There are 60 minutes in 1 hour. What fraction of an hour is:

**a** 30 minutes?

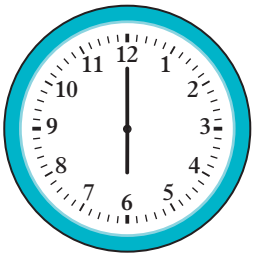
**b** 15 minutes?

**c** 17 minutes?

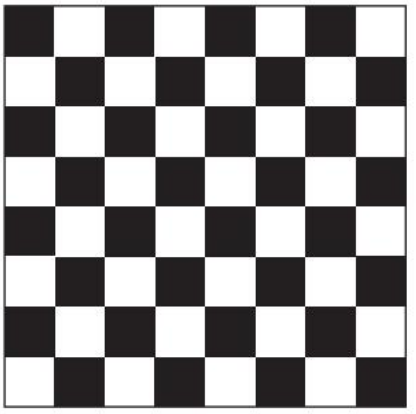
**d** 31 minutes?

**e** 119 minutes (as an improper fraction)?

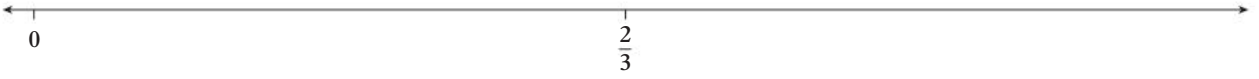
**f** 311 minutes (as a mixed number)?



**16** Determine what fraction of the shape is shaded. Explain your reasoning.



**17 a** Here is a number line with 0 and $\frac{2}{3}$ marked. Copy the number line and mark where 1 should be positioned. Explain how you positioned 1.




**b** Here is a number line with 0 and $\frac{5}{4}$ marked. Copy the number line and mark where 1 should be positioned. Explain how you positioned 1.

**18** Setting up a number line for large denominators is hard to do accurately without grid paper or a ruler. We can make it easier for denominators that are not prime numbers by dividing it into parts, then dividing those into more parts. To divide a number line into twelfths we can divide into halves, then quarters, then twelfths.

**a** Draw a number line from 0 to 1. Mark $\frac{6}{12}$ halfway between 0 and 1.

**b** Mark $\frac{3}{12}$ halfway between 0 and $\frac{6}{12}$, and mark $\frac{9}{12}$ halfway between $\frac{6}{12}$ and 1.

**c** Mark thirds between each marking. These are the remaining twelfths.

A number line can be marked for other denominators in similar ways.

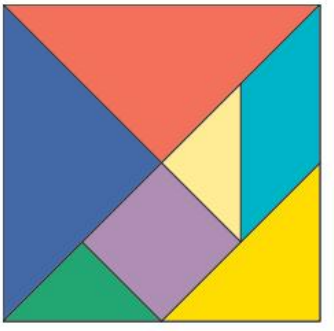
**d** By repeatedly halving, mark a number line between 2 and 3 into sixteenths.

**e** Explain how you could mark a number line from 0 to 1 into fifteenths.



**19** I am a fraction. My denominator has exactly three factors, and my numerator is the cube of a prime number. The difference between my denominator and my numerator is 1. What fraction am I?

**20** The puzzle on the right is called a Tangram. It is made up of two large triangles, one medium size triangle, two small triangles, one square and one parallelogram.



**a** What fraction of the whole Tangram is the small triangle?

**b** What fraction of the whole Tangram is the medium triangle?

**c** What fraction of the whole Tangram is the large triangle?

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Identifying fractions

**Investigation** Apple pies

**Topic quiz** 3A

# 3B Equivalent fractions

## Learning intentions

- ✔ I can recognise equivalent fractions on a fraction wall and number line.
- ✔ I can identify and find equivalent fractions.
- ✔ I can simplify fractions.

**Inter-year links**

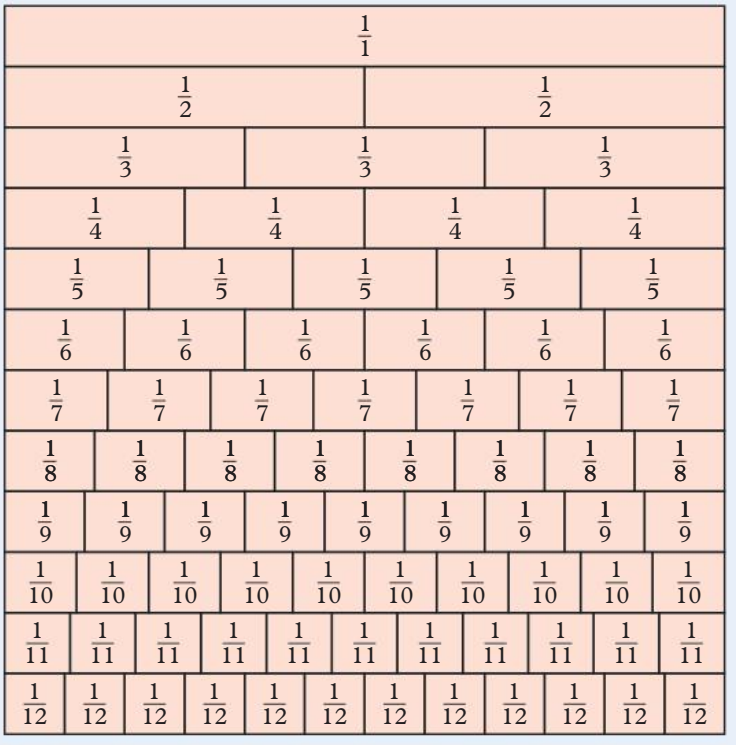
| | |
|---|---|
| Years 5/6 | Equivalent fractions |
| Year 8 | 2A Fractions |

## Equivalent fractions

- **Equivalent fractions** are fractions that have the same numerical value. They mark the same point on the number line.

  For example, $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{6}{12}$ all mark the same point on the number line, so they are equivalent fractions.
- A fraction is equivalent to 1 if the numerator and denominator are the same.

  For example, $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots, \frac{10}{10}, \ldots, \frac{43}{43}, \ldots, \frac{101}{101}$ are all equivalent to 1.
- A **fraction wall** is a visual representation to help compare and identify fractions, set out in the form of a wall.




- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.

$$\frac{2}{3} \overset{\times 5}{\underset{\times 5}{=}} \frac{10}{15} \qquad \frac{12}{10} \overset{\div 2}{\underset{\div 2}{=}} \frac{6}{5}$$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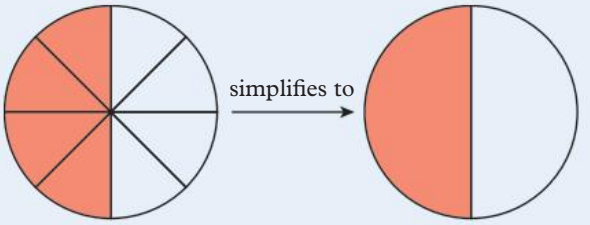
# Simplifying fractions

- Fractions in their **simplest form** have a highest common factor of 1 between the numerator and the denominator.
  For example, $\frac{3}{4}$ is in its simplest form, as the highest common factor (HCF) between 3 and 4 is 1.




- To find an equivalent fraction in its simplest form:
  → cancel out common factors until the only common factor between the numerator and denominator is 1.

  For example, $\frac{36}{72} = \frac{\cancel{36}^{18}}{\cancel{72}^{36}}$
  $= \frac{\cancel{18}^{3}}{\cancel{36}^{6}}$
  $= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$
  $= \frac{1}{2}$.

  → find the HCF of the numerator and the denominator and then divide the numerator and denominator by the HCF.

  For example, the HCF of 36 and 72 is 36, so $\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}}$
  $= \frac{1}{2}$.

- For mixed numbers, only simplify the fraction part, the whole number stays the same.

  For example, $2\frac{12}{24} = 2\frac{\cancel{12}^{1}}{\cancel{24}^{2}}$
  $= 2\frac{1}{2}$.

## Example 3B.1 Writing equivalent fractions

Complete each set of equivalent fractions.

**a** $\frac{14}{8} = \frac{\quad}{4}$

**b** $\frac{5}{13} = \frac{15}{\quad}$

**THINK**

1. Find a relationship between the two known numerators and denominators. Consider if the equivalent fraction has been found by using multiplication or division.
2. Use the same relationship to obtain the missing numerator or denominator of the second fraction.

**WRITE**

**a** $\frac{14}{8} = \frac{\quad}{4}$ (÷2)

$\frac{14}{8} = \frac{7}{4}$ (÷2)

**b** $\frac{5}{13} = \frac{15}{\quad}$ (×3)

$\frac{5}{13} = \frac{15}{39}$ (×3)

## **Example 3B.2** Simplifying fractions using repeated division

Write each fraction in its simplest form using repeated division.

**a** $\frac{80}{144}$ **b** $3\frac{54}{72}$

**THINK**

**a** **1** Find a common factor of 80 and 144. They are both even numbers so 2 is a common factor.

**2** Divide both the numerator and the denominator by 2.

**3** Find another common factor and repeat the process. Repeat until the only common factor is 1.

**b** **1** Find a common factor of 54 and 72. They both have digit sums that are multiples of 3, so 3 is a common factor.

**2** Leave the whole number as it is and divide both the numerator and the denominator by 3.

**3** Find another common factor and repeat the process. Repeat until the only common factor is 1.

**WRITE**

**a** $\frac{80}{144} = \frac{\cancel{80}^{40}}{\cancel{144}^{72}}$

$= \frac{\cancel{40}^{20}}{\cancel{72}^{36}}$

$= \frac{\cancel{20}^{10}}{\cancel{36}^{18}}$

$= \frac{\cancel{10}^{5}}{\cancel{18}^{9}}$

$= \frac{5}{9}$

**b** $3\frac{54}{72} = 3\frac{\cancel{54}^{18}}{\cancel{72}^{24}}$

$= 3\frac{\cancel{18}^{3}}{\cancel{24}^{4}}$

$= 3\frac{3}{4}$

## **Example 3B.3** Simplifying fractions using the HCF

Write each fraction in its simplest form using the HCF.

**a** $\frac{21}{49}$ **b** $2\frac{45}{65}$

**THINK**

**a** **1** Find the highest common factor (HCF) of the numerator and the denominator.

**2** Divide both the numerator and the denominator by the HCF.

**b** **1** Find the HCF of the numerator and the denominator of the fraction component.

**2** Leave the whole number as it is and divide both the numerator and the denominator by the HCF.

**WRITE**

**a** Factors of 21: 1, 3, 7, 21

Factors of 49: 1, 7, 49

HCF = 7

$\frac{21}{49} = \frac{\cancel{21}^{3}}{\cancel{49}^{7}}$

$= \frac{3}{7}$

**b** Factors of 45: 1, 3, 5, 9, 15, 45

Factors of 65: 1, 5, 13, 65

HCF = 5

$2\frac{45}{65} = 2\frac{\cancel{45}^{9}}{\cancel{65}^{13}}$

$= 2\frac{9}{13}$

**Helpful hints**

- ✔ Remember that 'equivalent' is another way of saying 'equal to'.
- ✔ Don't be fooled by fractions that have odd or prime numbers; sometimes they can be simplified. For example, $\frac{3}{15} = \frac{1}{5}$.
- ✔ Use the divisibility rules from Chapter 2 to help you find common factors. The larger the common factor, the faster you'll find the equivalent fraction in its simplest form.

ANS p550

# Exercise 3B Equivalent fractions

△ 1(a, c, e), 2–9, 11–13 | □ 3–6, 7(b, d), 8, 10, 11, 13–15(a, b) | ⬠ 3(e, f), 4(b, d, f), 5(b, d, f), 7(d), 8, 11, 13–17

**1** Using the fraction wall in the theory, determine all of the fractions on the wall equivalent to:

**a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{3}{12}$ **d** $\frac{2}{5}$ **e** $\frac{2}{3}$ **f** $\frac{7}{7}$

3B.1 **2** Complete each set of equivalent fractions.

**a** $\frac{4}{9} = \frac{}{36}$ **b** $\frac{24}{8} = \frac{12}{}$ **c** $2\frac{5}{11} = 2\frac{}{33}$

**d** $\frac{42}{70} = \frac{}{10}$ **e** $\frac{144}{48} = \frac{}{12}$ **f** $3\frac{2}{8} = 3\frac{}{72}$

**g** $\frac{2}{3} = \frac{}{9} = \frac{}{90}$ **h** $\frac{5}{6} = \frac{}{18} = \frac{20}{}$ **i** $\frac{2}{5} = \frac{10}{} = \frac{}{15}$

**3** Complete each set of equivalent fractions.

**a** $5\frac{1}{13} = 5\frac{}{39} = 5\frac{5}{}$

**b** $\frac{}{10} = \frac{2}{20} = \frac{}{30} = \frac{8}{}$

**c** $\frac{8}{} = \frac{24}{21} = \frac{}{35} = \frac{}{77}$

**d** $3\frac{3}{5} = 3\frac{}{30} = 3\frac{9}{} = 3\frac{}{70}$

**e** $\frac{1}{12} = \frac{6}{} = \frac{8}{} = \frac{}{144} = \frac{15}{} = \frac{}{240}$

**f** $\frac{2}{3} = \frac{}{6} = \frac{10}{} = \frac{}{30} = \frac{}{75} = \frac{400}{}$

3B.2 **4** Write each fraction in its simplest form using repeated division.

**a** $\frac{10}{16}$ **b** $\frac{28}{36}$ **c** $\frac{63}{56}$ **d** $2\frac{12}{30}$ **e** $5\frac{33}{36}$ **f** $6\frac{15}{75}$

3B.3 **5** Write each fraction in its simplest form using the HCF.

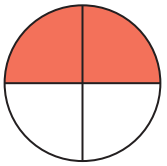
**a** $\frac{11}{33}$ **b** $\frac{30}{48}$ **c** $\frac{116}{94}$ **d** $7\frac{20}{45}$ **e** $4\frac{56}{64}$ **f** $3\frac{16}{54}$

**6** List the fractions that are equivalent.

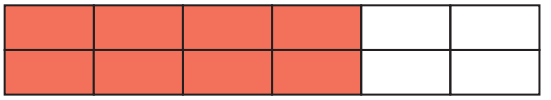
| | | | | |
|---|---|---|---|---|
| $\frac{1}{3}$ | $\frac{5}{10}$ | $\frac{12}{15}$ | $\frac{6}{18}$ | $\frac{17}{25}$ |
| $\frac{30}{45}$ | $\frac{16}{48}$ | $\frac{18}{54}$ | $\frac{20}{60}$ | $\frac{52}{156}$ |

**7** What fraction of each shape is shaded? Simplify your answers.

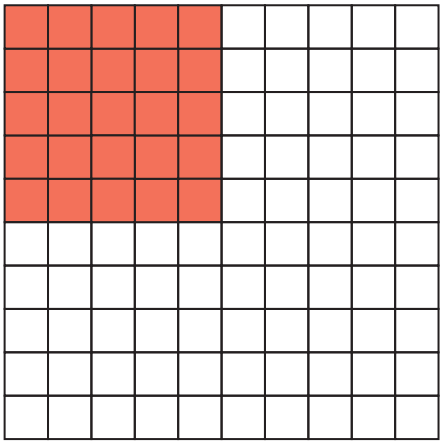
**a**



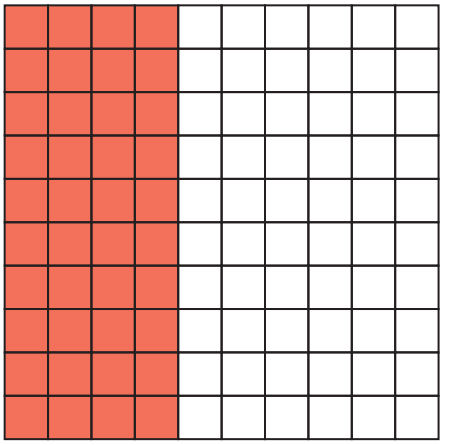
**b**



**c**



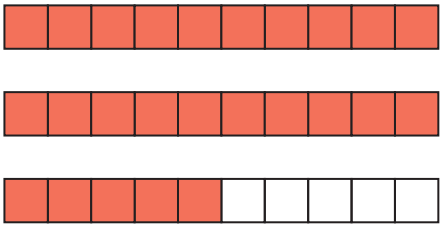
**d**



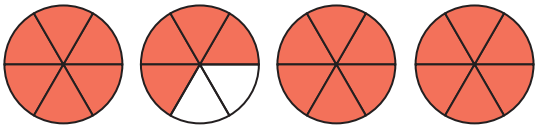
**8** If each shape represents 1 whole, write the number that is represented by the shading in each diagram as:

**i** a simplified improper fraction **ii** a simplified mixed number.

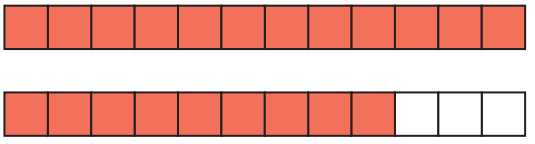
**a**



**b**



**c**





**9** Ingrid is making toy wallabies. She has 60 buttons to use as eyes. So far she has made 13 wallabies. What fraction of the buttons does she have left? Write your answer in its simplest form.

**10** Larni scored 22 of her team's 34 goals. What fraction of the team's goals did Larni score? Write your answer in its simplest form.

**11** Cristiano spent 75 minutes of a 90-minute football game on the field. What fraction of time, in its simplest form, did he spend off the field?

**12** Find the values of the missing numbers.

**a** $\frac{27}{18} = \frac{\quad}{50}$ **b** $\frac{120}{\quad} = \frac{48}{54}$ **c** $\frac{33}{39} = \frac{143}{\quad}$

**13** Keiynan is $\frac{5}{6}$ of the way through his 300-page book. If he reads 10 more pages, what fraction is he through the book? Write your answer in its simplest form.

**14** The Thomson Reservoir is the largest water storage reservoir in Melbourne. When it is full, it can hold enough water to fill the Melbourne Cricket Ground (MCG) 628 times.

**a** If there was only enough water to fill the MCG 471 times, what fraction of the reservoir contains water?

**b** A state of emergency is declared when the reservoir's water level is below $\frac{1}{4}$. How many MCGs could be filled at that water level?

**15** We can simplify a fraction by dividing the numerator and the denominator by the highest common factor (HCF). Therefore, we can also use simplifying of fractions to find the HCF. By multiplying all the numbers that divide the numerator and denominator to simplify the fraction, the result is the HCF.

For example, $\frac{80}{144} = \frac{\cancel{80}^{40}}{\cancel{144}^{72}} = \frac{\cancel{40}^{20}}{\cancel{72}^{36}} = \frac{\cancel{20}^{10}}{\cancel{36}^{18}} = \frac{\cancel{10}^{5}}{\cancel{18}^{9}} = \frac{5}{9}$.

The HCF of 80 and 144 is $2 \times 2 \times 2 \times 2 = 16$.

Find the HCF of the following numbers by simplifying fractions.

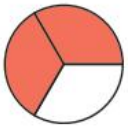
**a** 42 and 315 **b** 252 and 270 **c** 3024 and 3780

**16** When simplifying by using repeated division, especially when the HCF is not obvious or not easy to determine, a useful strategy is to divide by prime numbers until there is a remainder. That is, divide by 2 until one or both of the numbers are not even, then divide by 3 until the digit sum of one or both of the numbers is not a multiple of 3, then divide by 5 until one or both the numbers do not end in a 0 or a 5, and so on.

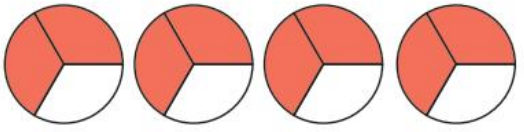
Simplify the following fractions using repeated division.

**a** $\frac{48\,600}{4800}$ **b** $\frac{121\,500}{202\,500\,000}$ **c** $\frac{196}{2058}$ **d** $\frac{161\,051}{2310}$

**17** Here is a diagram showing $\frac{2}{3}$ of the whole circle shaded.



Here are four more of the same diagram. However, the fraction shaded depends on what is considered the whole.



Simplifying where possible, determine what fraction is shaded if:

**a** one whole is considered as one full circle

**b** one whole is considered as two full circles

**c** one whole is considered as four full circles

**d** one whole is considered as $\frac{1}{3}$ of a circle

**e** one whole is considered as $\frac{2}{3}$ of a circle

**f** one whole is considered as $\frac{4}{3}$ of a circle.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Equivalent fractions

**Investigation** Quilt design

**Topic quiz** 3B

PROBLEM SOLVING AND REASONING

CHALLENGE

# 3C Ordering fractions

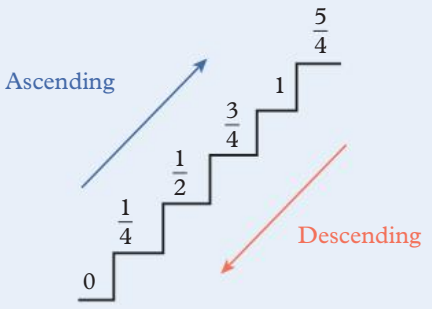
## Learning intentions

- ✔ I can compare fractions with the same or different denominators.
- ✔ I can order fractions in ascending and descending order.
- ✔ I can order and compare fractions using equivalent fractions.

**Inter-year links**
**Year 8** 2A Fractions

## Ordering fractions

- 'Less than' is shown using the symbol '<'.
- 'Greater than' is shown using the symbol '>'.
- **Unit fractions** are fractions that have a numerator of 1. The larger the denominator of a unit fraction, the smaller the fraction.

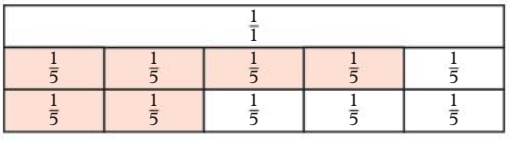


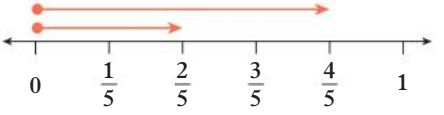

## Ordering and comparing methods

**Same denominator**

To compare $\frac{2}{5}$ and $\frac{4}{5}$, think about having 2 parts, each one-fifth of a whole, and 4 of the same sized parts of the same sized whole.

$\frac{2}{5} < \frac{4}{5}$

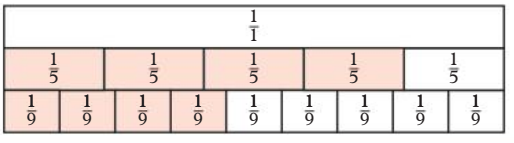


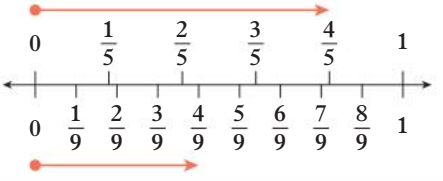




**Same numerators**

To compare $\frac{4}{5}$ and $\frac{4}{9}$, remember that $\frac{1}{5}$ is greater than $\frac{1}{9}$, so $\frac{4}{5}$ is 4 large parts compared to the 4 smaller parts of $\frac{4}{9}$.

$\frac{4}{9} < \frac{4}{5}$





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# Ordering and comparing fractions using equivalent fractions

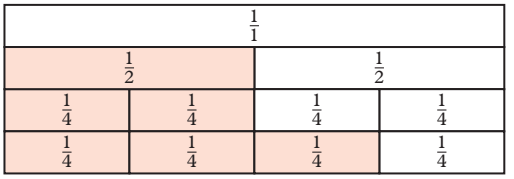
- A **common denominator** is a common multiple of the denominators of two or more fractions.

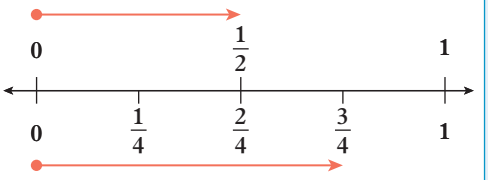
**Common denominator**

To compare $\frac{1}{2}$ and $\frac{3}{4}$, change $\frac{1}{2}$ to an equivalent fraction with a denominator of 4. Then $\frac{1}{2}$ can be easily compared with $\frac{3}{4}$.

$\frac{1}{2} = \frac{2}{4}$

$\frac{2}{4} < \frac{3}{4}$




- The **lowest common denominator (LCD)** is the lowest common multiple of the denominators in two or more fractions.

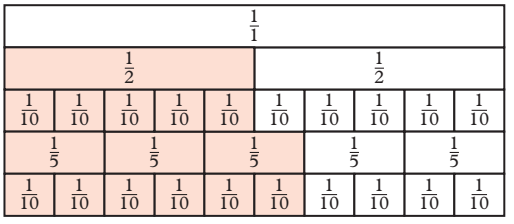
**Lowest common denominator**

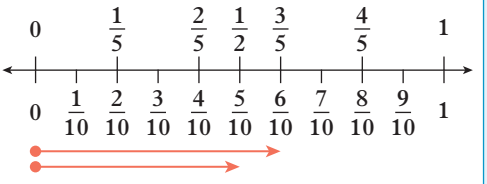
To compare $\frac{1}{2}$ and $\frac{3}{5}$, find the LCD and multiply the numerator and denominator of each fraction to get an equivalent fraction. Then compare the fractions with the same denominator.

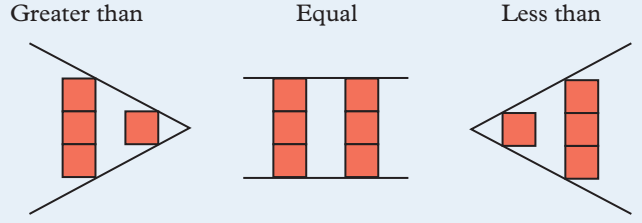
The LCD of 2 and 5 is 10.

$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$

$\frac{3}{5} \overset{\times 2}{=} \frac{6}{10}$




| Greater than | Equal | Less than |
|---|---|---|
| $3 > 1$ | $3 = 3$ | $1 < 3$ |
| 3 is greater than 1 | 3 is equal to 3 | 1 is less than 3 |

## **Example 3C.1** Comparing fractions with the same denominator

Which fraction is greater: $\frac{5}{8}$ or $\frac{7}{8}$? Write your answer as an inequality.

**THINK**

Check if the fractions have the same denominator, same numerator, or neither. These fractions have the same denominator, so $\frac{5}{8}$ is less than $\frac{7}{8}$ as it has fewer of the same parts of the whole.

| $\frac{1}{1}$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

**WRITE**

$\frac{5}{8} < \frac{7}{8}$

## **Example 3C.2** Comparing fractions with the same numerator

Which fraction is greater: $\frac{5}{8}$ or $\frac{5}{6}$? Write your answer as an inequality.

**THINK**

Check if the fractions have the same denominator, same numerator, or neither. These fractions have the same numerator, so $\frac{5}{6}$ is greater than $\frac{5}{8}$ as it has 5 greater parts of $\frac{1}{6}$ compared to 5 smaller parts of $\frac{1}{8}$.

| $\frac{1}{1}$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | | |
| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

**WRITE**

$\frac{5}{8} < \frac{5}{6}$

## **Example 3C.3** Comparing fractions with different denominators and numerators

Which fraction is greater: $\frac{5}{7}$ or $\frac{11}{14}$? Write your answer as an inequality.

**THINK**

1. Compare fractions by converting one fraction to an equivalent fraction that has either the same denominator or the same numerator as the other fraction. Since 14 is a multiple of 7, find an equivalent fraction to $\frac{5}{7}$ with a denominator of 14.
2. Compare the fractions with the same denominator and write the answer using the original fractions.

**WRITE**

$$\frac{5}{7} \overset{\times 2}{=} \frac{10}{14}$$

$\frac{10}{14} < \frac{11}{14}$ therefore, $\frac{5}{7} < \frac{11}{14}$.

## Example 3C.4 Ordering fractions by finding the lowest common denominator

Which fraction is greater: $\frac{5}{6}$ or $\frac{7}{9}$? Write your answer as an inequality.

**THINK**

1 Find the LCM of the denominators by listing the multiples of 6 and 9. Identify the lowest multiple that appears in both lists.

2 For each fraction, find the equivalent fraction with a denominator equal to the LCM.

6 multiplied by 3 is 18, so multiply the numerator and denominator of $\frac{5}{6}$ by 3.

9 multiplied by 2 is 18, so multiply the numerator and denominator of $\frac{7}{9}$ by 2.

3 Compare the fractions with the same denominator and write the answer using the original fractions.

**WRITE**

Multiples of 6: 6, 12, 18, 24, …

Multiples of 9: 9, 18, 27, 36, …

The LCM is 18.

So, 18 is the LCD.

$$\frac{5}{6} \overset{\times 3}{=} \frac{15}{18} \qquad \frac{7}{9} \overset{\times 2}{=} \frac{14}{18}$$

$\frac{15}{18} > \frac{14}{18}$ therefore, $\frac{5}{6} > \frac{7}{9}$

**Helpful hints**

- ✔ Remember to consider both the numerator and the denominator of fractions before comparing fractions.
- ✔ Multiplying two denominators together will find a common denominator but not always the lowest common denominator.

ANS p551

# Exercise 3C Ordering fractions

△ 1–4, 5(a, c), 6(a, b), 7–11, 13, 14, 15(a, b)

□ 3(b, d, f), 5(b, d), 6(c, d), 7, 8, 9(b, c), 10(b, c), 11, 12, 15–17

⬠ 5(b, d), 6(c, d), 8, 9(c, d), 10(c, d), 11, 12, 15–18

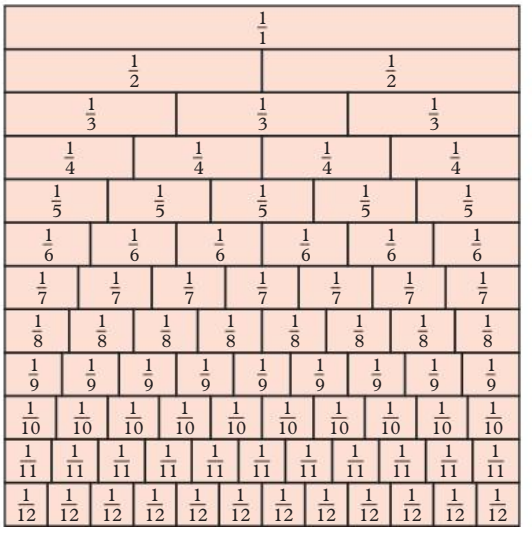
1 Use the fraction wall to decide which fraction is greater.

a $\frac{2}{3}$ or $\frac{5}{8}$

b $\frac{7}{10}$ or $\frac{9}{12}$

c $\frac{2}{12}$ or $\frac{1}{5}$

d $\frac{5}{6}$ or $\frac{7}{8}$



3C.1 **2** Identify the greater fraction in each pair. Write your answer as an inequality, with the greater fraction first.

**a** $\frac{3}{6}$ or $\frac{5}{6}$ **b** $\frac{7}{5}$ or $\frac{9}{5}$ **c** $\frac{93}{15}$ or $\frac{32}{15}$

**d** $1\frac{3}{8}$ or $1\frac{7}{8}$ **e** $3\frac{9}{20}$ or $3\frac{11}{20}$ **f** $\frac{108}{4}$ or $\frac{180}{4}$

3C.2 **3** Identify the greater fraction in each pair. Write your answer as an inequality, with the greater fraction first.

**a** $\frac{2}{3}$ or $\frac{2}{4}$ **b** $\frac{16}{44}$ or $\frac{16}{45}$ **c** $\frac{77}{100}$ or $\frac{77}{12}$

**d** $2\frac{5}{6}$ or $2\frac{5}{7}$ **e** $4\frac{13}{20}$ or $4\frac{13}{15}$ **f** $\frac{55}{17}$ or $\frac{55}{30}$

**4** **a** Draw a number line and mark the position of each of these fractions.

$$\frac{12}{7}, \frac{10}{7}, \frac{15}{7}, \frac{5}{7}, \frac{21}{7}, \frac{32}{7}, \frac{1}{7}, \frac{31}{7}$$

**b** Using the number line, list the fractions in ascending order (smallest to largest) using the less than sign '<'. What do you notice?

**5** Write each list of fractions in ascending order (smallest to largest) using the less than sign.

**a** $\frac{8}{11}, \frac{2}{11}, \frac{15}{11}, \frac{5}{11}$ **b** $\frac{16}{7}, \frac{15}{7}, \frac{6}{7}, \frac{14}{7}$ **c** $\frac{10}{5}, \frac{10}{13}, \frac{10}{15}, \frac{10}{3}$ **d** $\frac{77}{59}, \frac{77}{101}, \frac{77}{70}, \frac{77}{89}$

**6** Write each list of fractions in descending order (largest to smallest).

**a** $\frac{1}{5}, 1\frac{2}{5}, 3\frac{2}{5}, 1\frac{4}{5}, 2\frac{1}{5}, \frac{4}{5}$ **b** $\frac{6}{5}, \frac{6}{2}, \frac{6}{6}, \frac{6}{4}, \frac{6}{3}, \frac{6}{8}$ **c** $\frac{67}{67}, \frac{1}{67}, \frac{100}{67}, \frac{76}{67}, \frac{33}{67}, \frac{33}{67}$ **d** $\frac{38}{40}, \frac{38}{60}, \frac{38}{2}, \frac{38}{37}, \frac{38}{19}, \frac{38}{59}$

3C.3 **7** Which fraction in each pair is greater? Write your answer as an inequality.

**a** $\frac{3}{6}$ or $\frac{5}{12}$ **b** $\frac{7}{10}$ or $\frac{30}{40}$ **c** $\frac{93}{15}$ or $\frac{32}{5}$

**d** $\frac{43}{48}$ or $\frac{7}{8}$ **e** $1\frac{9}{20}$ or $1\frac{55}{100}$ **f** $\frac{23}{2}$ or $\frac{180}{14}$

3C.4 **8** Determine which fraction is greater by finding their lowest common denominator (LCD). Write your answer as an inequality.

**a** $\frac{5}{6}$ or $\frac{8}{9}$ **b** $\frac{10}{20}$ or $\frac{11}{14}$ **c** $2\frac{15}{24}$ or $2\frac{14}{16}$

**d** $\frac{12}{7}$ or $\frac{13}{5}$ **e** $\frac{6}{17}$ or $\frac{5}{9}$ **f** $\frac{17}{16}$ or $\frac{18}{17}$

**9** Write each list of fractions in ascending order.

**a** $\frac{7}{5}, \frac{2}{5}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{16}{5}$ **b** $\frac{5}{8}, \frac{11}{4}, \frac{2}{6}, \frac{7}{6}, \frac{5}{4}, \frac{9}{8}$ **c** $\frac{2}{9}, \frac{4}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{3}, \frac{9}{6}$ **d** $\frac{8}{15}, \frac{5}{4}, \frac{6}{3}, \frac{11}{10}, \frac{9}{20}, \frac{10}{6}$

**10** Write each list of fractions in descending order.

**a** $\frac{3}{2}, \frac{8}{5}, \frac{7}{6}, \frac{10}{6}, \frac{5}{2}, \frac{12}{5}$ **b** $\frac{1}{8}, \frac{11}{12}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}, \frac{2}{6}$ **c** $\frac{10}{7}, \frac{9}{8}, \frac{3}{4}, \frac{3}{2}, \frac{7}{8}, \frac{6}{7}$ **d** $\frac{2}{9}, \frac{1}{3}, \frac{5}{7}, \frac{4}{3}, \frac{11}{7}, \frac{13}{9}$

**11** Complete each number statement using the less than, <, or greater than, >, symbols.

**a** $\frac{1}{3}$ _____ $\frac{1}{4}$ **b** $\frac{2}{5}$ _____ $\frac{5}{2}$

**c** $4\frac{5}{6}$ _____ $\frac{23}{3}$ **d** $\frac{42}{5}$ _____ $7\frac{6}{7}$

**e** $\frac{7}{9}$ _____ $\frac{6}{7}$ **f** $8\frac{5}{6}$ _____ $\frac{49}{5}$

**12** Without changing denominators complete each number statement using the less than, <, or greater than, >, symbols.

**a** $\frac{12}{13}$ _____ $\frac{13}{14}$ **b** $\frac{4}{5}$ _____ $\frac{3}{4}$

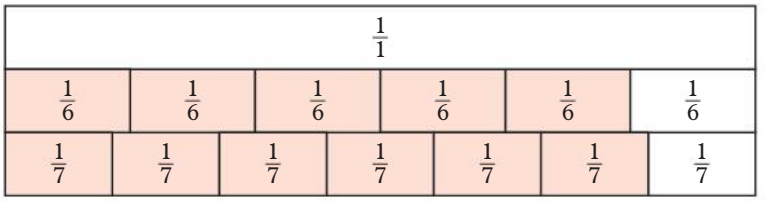
**c** $\frac{1}{6}$ _____ $\frac{2}{7}$ **d** $\frac{3}{9}$ _____ $\frac{3}{10}$

**e** $\frac{4}{19}$ _____ $\frac{4}{17}$ **f** $\frac{5}{16}$ _____ $\frac{5}{26}$

**13** Philip was comparing two of his test results. He scored $\frac{22}{25}$ on the first test and $\frac{45}{50}$ on the second. On which test did Philip perform better?

**14** Zach and Melanie are comparing their results after an afternoon of archery. From 35 attempts, Zach scored 20 bullseyes. Melanie scored 24 bullseyes from 40 attempts. Who hit the bullseye on the archery target with more accuracy? Justify your answer.

**15** We can compare fractions that both have a numerator that is one away from a whole. The fraction with a larger denominator has smaller parts, so will be missing a smaller part.




$\frac{5}{6} < \frac{6}{7}$

Write the inequality that compares the following pairs of fractions.

**a** $\frac{5}{6}, \frac{8}{9}$ **b** $\frac{17}{16}, \frac{18}{17}$ **c** $\frac{100}{101}, \frac{99}{100}$ **d** $\frac{48}{47}, \frac{47}{46}$

**16** The following inequalities and explanations may or may not be correct. Write the correct inequalities and give explanations.

**a** $\frac{5}{8} > \frac{5}{7}$ since $8 > 7$

**b** $\frac{70}{34} > \frac{9}{10}$ since $70 > 9$

**c** $\frac{17}{5} < \frac{23}{5}$ since $17 < 23$

**17** Sophie and Tilly are practising shooting goals in netball. Sophie shoots 29 goals from 40 attempts and Tilly shoots 25 goals from 30 attempts. Sophie says, 'I don't want to make you feel bad Tilly, but I am obviously the most accurate shooter since I got 29 goals and you only got 25.' Tilly replies, 'Well I don't want to make you feel bad about your maths and shooting skills, but I am the most accurate!' Which student is correct? Justify your answer.

**18** Place in ascending order.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\frac{21}{5}$ | $\frac{15}{4}$ | $\frac{17}{8}$ | $\frac{25}{6}$ | $\frac{60}{9}$ | $\frac{11}{2}$ | $\frac{10}{3}$ |

**19** Place in ascending order.

| | | | | |
|---|---|---|---|---|
| $\frac{98}{99}$ | $\frac{997}{999}$ | $\frac{9996}{9999}$ | $\frac{99995}{99999}$ | $\frac{999994}{999999}$ |

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Ordering fractions

**Investigation** Drinking duo

**Topic quiz** 3C

# 3D Mixed numbers and improper fractions

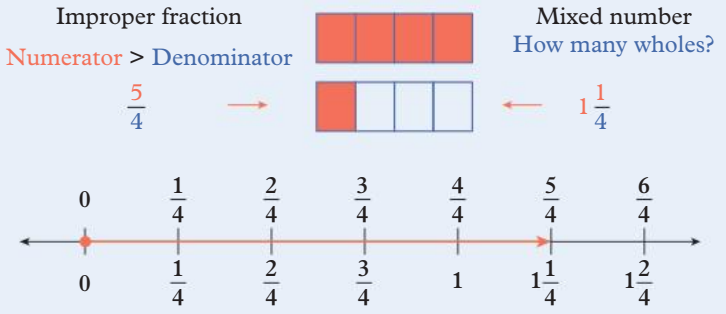
## Learning intentions

- ✔ I can identify mixed numbers and improper fractions on a number line.
- ✔ I can convert improper fractions to mixed numbers.
- ✔ I can convert mixed numbers to improper fractions.

Inter-year links
Year 8 2A Fractions

## Mixed numbers and improper fractions

- Mixed numbers can be written as improper fractions. Improper fractions can also be written as mixed numbers.
- Mixed numbers consist of both a whole number and a proper fraction.
  For example, $1\frac{1}{4} = 1 + \frac{1}{4}$.




## Improper fraction to mixed number

- Improper fractions show how many parts of equal size are present. Converting improper fractions to mixed numbers allows us to see how many wholes can be created from those equal parts and how many parts are left over.
- To convert an improper fraction to a mixed number:
  1. Divide the numerator by the denominator.
  2. Write down the whole number answer and place the remainder over the denominator.
  3. Simplify the fraction if possible.

$$\frac{13}{4} = 13 \div 4 = 3 \text{ remainder } 1$$
$$= 3\frac{1}{4}$$

## Mixed number to improper fraction

- Mixed numbers consist of a whole number and a fraction. Converting mixed numbers to improper fractions allows us to see how many equal parts there are in the whole number and the fraction.
  1. Multiply the denominator by the whole number and add the numerator.
  2. Write the result over the denominator.

$$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$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## **Example 3D.1** Converting improper fractions to mixed numbers

Convert the improper fraction $\frac{26}{6}$ to a mixed number.

**THINK**

1. Divide the numerator by the denominator.
2. Write down the whole number answer and place the remainder over the denominator.
3. Simplify the fraction.

**WRITE**

$\frac{26}{6} = 4$ remainder 2

$4\frac{2}{6} = 4\frac{\cancel{2}^{1}}{\cancel{6}_{3}}$

$= 4\frac{1}{3}$

## **Example 3D.2** Converting mixed numbers to improper fractions

Convert the mixed number $3\frac{2}{7}$ to an improper fraction.

**THINK**

1. Multiply the denominator by the whole number and add the numerator.
2. Write the result over the denominator.

**WRITE**

$3\frac{2}{7} = \frac{(7 \times 3) + 2}{7}$

$= \frac{23}{7}$

**Helpful hints**

- ✔ An improper fraction can be simplified before or after writing it as a mixed number.
- ✔ It can be easier to simplify after writing an improper fraction as a mixed number as the numerator will be much smaller.
  For example, $\frac{12}{8} = 1\frac{\cancel{4}^{1}}{\cancel{8}_{2}} = 1\frac{1}{2}$ or $\frac{\cancel{12}^{3}}{\cancel{8}_{2}} = \frac{3}{2} = 1\frac{1}{2}$.

ANS p552

# **Exercise 3D** Mixed numbers and improper fractions

1–3, 4(a, b), 5–6, 7(a, c, e, g), 8–11 | 1, 2, 4(c, d), 6, 7, 9, 10, 12, 14 | 1(e, f, k, l), 2(e, f, k, l), 4(d), 7, 9, 12–15

3D.1 **1** Convert each improper fraction to a mixed number.

**a** $\frac{7}{5}$ **b** $\frac{26}{3}$ **c** $\frac{25}{9}$ **d** $\frac{43}{10}$ **e** $\frac{75}{8}$ **f** $\frac{88}{13}$

**g** $\frac{22}{7}$ **h** $\frac{17}{4}$ **i** $\frac{49}{6}$ **j** $\frac{50}{11}$ **k** $\frac{67}{5}$ **l** $\frac{111}{2}$

**3D.2** **2** Convert each mixed number to an improper fraction.

**a** $1\frac{2}{9}$ **b** $3\frac{1}{6}$ **c** $5\frac{8}{9}$ **d** $2\frac{6}{11}$ **e** $10\frac{4}{5}$ **f** $8\frac{3}{4}$

**g** $2\frac{3}{7}$ **h** $12\frac{1}{3}$ **i** $4\frac{5}{8}$ **j** $5\frac{3}{10}$ **k** $7\frac{1}{12}$ **l** $2\frac{4}{15}$

**3** State whether the following statements are true or false.

**a** To convert an improper fraction to a mixed number, divide the numerator by the denominator, write down the whole number answer and place the remainder over the denominator.

**b** An improper fraction is smaller than 1.

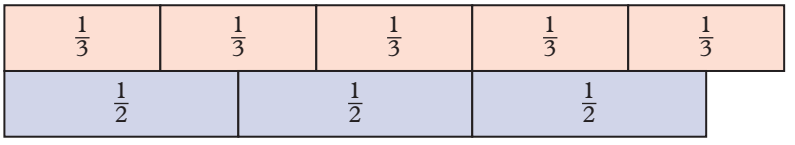
**c** A mixed number can have an improper fraction part.

**d** $\frac{45}{18} \neq 2\frac{8}{16}$

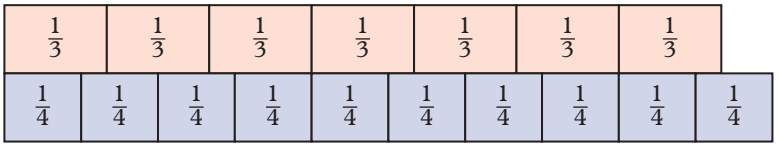
**e** An improper fraction can be simplified before or after converting it to a mixed number.

**4** Write an inequality that compares the two fractions shown in each diagram. Provide your answers as mixed numbers and improper fractions.

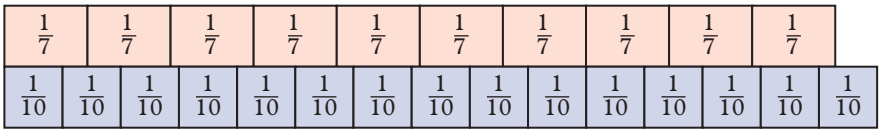
**a**



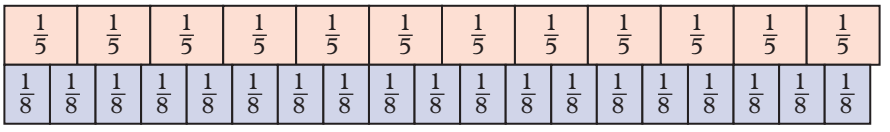

**b**




**c**




**d**




**5** Convert each fraction to an improper fraction (if necessary) and use the symbols < or > to make a true statement.

**a** $2\frac{4}{7}$ _____ $\frac{17}{7}$ **b** $8\frac{2}{3}$ _____ $\frac{28}{3}$ **c** $\frac{38}{11}$ _____ $2\frac{6}{11}$ **d** $9\frac{4}{5}$ _____ $\frac{51}{5}$

**6** **a** Copy the number line and divide each interval into three equal parts. Label each of these parts using mixed numbers.

**b** Convert each fraction to a mixed number, simplifying where necessary, and then mark its position on the number line.

**i** $\frac{23}{3}$ **ii** $\frac{20}{3}$ **iii** $5\frac{2}{6}$ **iv** $7\frac{4}{6}$ **v** $\frac{24}{4}$ **vi** $\frac{12}{3}$

**c** Arrange the fractions in ascending order.

**7** Convert both fractions to mixed numbers and use the < or > symbols to make a true statement.

**a** $\frac{28}{3}$ _____ $\frac{17}{5}$ **b** $\frac{49}{6}$ _____ $\frac{38}{4}$ **c** $\frac{13}{2}$ _____ $\frac{26}{9}$ **d** $\frac{28}{5}$ _____ $\frac{61}{10}$

**e** $\frac{42}{7}$ _____ $\frac{63}{9}$ **f** $\frac{27}{6}$ _____ $\frac{15}{4}$ **g** $\frac{32}{5}$ _____ $\frac{63}{8}$ **h** $\frac{21}{2}$ _____ $\frac{41}{4}$

**8** Leica buys 8 packets of trading cards to share equally with four of her friends.

**a** Write a mixed number to represent how many packets Leica and each of her friends receive.

**b** If each packet has 20 trading cards, how many cards does each person receive?

**9** Sava is on a week-long camping trip with five friends. He has bought nine packets of marshmallows to toast on the campfire each night.

**a** Write a mixed number to represent how many packets of marshmallows is allocated to each person.

**b** If each packet contains 10 marshmallows, how many does each person receive?

**10** The following conversions have mistakes. Explain the mistake and correct the calculation.

**a** $3\frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$ **b** $\frac{14}{5} = 3\frac{1}{5}$ **c** $6\frac{2}{3} = \frac{2}{6 \times 2 + 3} = \frac{2}{15}$ **d** $3\frac{4}{7} = \frac{7}{25}$

**11** Darius is a chocaholic. His favourite type of chocolate comes in a block of 24 pieces. One day Darius ate $2\frac{2}{3}$ blocks of chocolate. Determine how many pieces of chocolate Darius ate.

**12** Asha wrote the following calculation to convert $2\frac{4}{7}$ to an improper fraction: $2\frac{4}{7} = 2 + \frac{4}{7} = \frac{14}{7} + \frac{4}{7} = \frac{14+4}{7} = \frac{18}{7}$

**a** Use this method to write the following mixed numbers as improper fractions.

**i** $3\frac{2}{5}$ **ii** $1\frac{6}{17}$ **iii** $9\frac{1}{2}$ **iv** $9\frac{7}{10}$

**b** Explain how the method for converting from a mixed number to an improper fraction in the worked examples relates to the alternative method.

The reverse also works for writing an improper fraction as a mixed number: $\frac{18}{7} = \frac{14+4}{7} = \frac{14}{7} + \frac{4}{7} = 2 + \frac{4}{7} = 2\frac{4}{7}$

**c** Use this method to write the following improper fractions as mixed numbers.

**i** $\frac{8}{3}$ **ii** $\frac{17}{2}$ **iii** $\frac{11}{6}$ **iv** $\frac{47}{10}$

**d** Explain how the method for converting from an improper fraction to a mixed number in the worked examples relates to the alternative method.

**13** Georgina, Roisin and Vanessa decide to order large pizzas for a class lunch. Each pizza will be cut into eight equal slices. The girls estimate that each person will eat two slices and there will be a maximum of 22 people.

**a** Write the number of pizzas required as:

**i** an improper fraction **ii** a mixed number.

**b** Will each person be able to receive the estimated two slices of pizza if five large pizzas have been ordered?

**c** The girls' budget does not allow them to order more than five pizzas. What is the minimum number of equal slices each pizza can be cut into, if everyone is to receive two slices?

**d** What fraction of a pizza will be left over if the pizzas are cut in the way suggested in part **c**?

**e** On the day of the lunch, three people were away and $4\frac{5}{8}$ of the pizzas were eaten. If each pizza was cut into eight slices, how many slices were eaten?



**14** Write $\frac{69\,615}{31\,122}$ as a simplified mixed number.

**15** There is a mixed number $a\frac{b}{c}$, where $a$ represents a whole number, $b$ represents the numerator and $c$ represents the denominator of a proper fraction. After converting this mixed number into an improper fraction, the fraction becomes $\frac{ab}{c}$. For example, $9\frac{3}{10} = \frac{93}{10}$. In this case $a = 9$, $b = 3$, $c = 10$. List more fractions like this.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Mixed numbers and improper fractions

**Investigation** Sandwich fractions

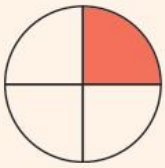
**Topic quiz** 3D

# Checkpoint

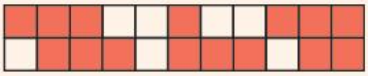
**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

3A **1** What fraction of each shape is shaded?

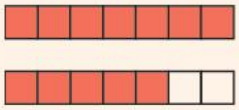
**a**



**b**



**c**



3A **2** Write the fractions indicated on each number line. Used mixed numbers where required.

**a**

**b**

3A **3** Write the fraction represented by each statement.

**a** Lachlan has collected 17 of the 20 trading cards in a set.
**b** Sal has surveyed 51 of the 100 people required.
**c** Katharine made 16 attempts at kicking a goal and got 15 in.

3B **4** Complete each set of equivalent fractions.

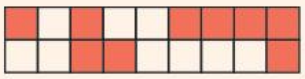
**a** $\frac{12}{5} = \frac{}{15}$ **b** $\frac{20}{} = \frac{4}{7}$ **c** $\frac{18}{27} = \frac{}{3} = \frac{}{6}$

3B **5** Simplify the following fractions.

**a** $\frac{28}{56}$ **b** $\frac{88}{55}$ **c** $7\frac{40}{48}$

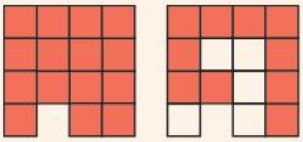
3B **6** Write the simplified fraction that the shaded section represents.

**a**



**b**

**c**



3C **7** Write the correct inequality to compare each pair of fractions.

**a** $\frac{87}{121}, \frac{78}{121}$ **b** $\frac{55}{8}, \frac{55}{9}$ **c** $\frac{3}{4}, \frac{5}{6}$

3C **8** Write the following lists of fractions in ascending order.

**a** $\frac{9}{17}, 1\frac{5}{17}, \frac{20}{17}, \frac{1}{17}$ **b** $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}, \frac{7}{12}$ **c** $5\frac{1}{8}, 5\frac{1}{3}, 5\frac{1}{4}, 5\frac{1}{6}$ **d** $\frac{15}{9}, \frac{13}{4}, \frac{17}{6}, \frac{19}{12}$

3D **9** Write these mixed numbers as simplified improper fractions.

**a** $2\frac{3}{5}$ **b** $6\frac{5}{7}$ **c** $12\frac{35}{40}$

3D **10** Write these improper fractions as simplified mixed numbers.

**a** $\frac{19}{3}$ **b** $\frac{67}{8}$ **c** $\frac{180}{21}$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# 3E Adding and subtracting fractions

## Learning intentions

- ✔ I can add and subtract fractions with the same denominator.
- ✔ I can add and subtract fractions with different denominators.
- ✔ I can add and subtract mixed numbers.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Adding and subtracting fractions |
| Year 8 | 2B Adding and subtracting fractions |

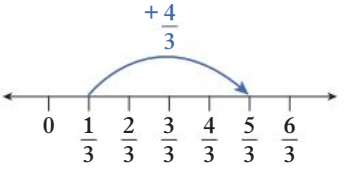
## Adding and subtracting like and unlike fractions

- **Like fractions** have the same denominators and **unlike fractions** have different denominators. For example, $\frac{2}{3}$ and $\frac{4}{3}$ are like fractions, whereas $\frac{2}{3}$ and $\frac{5}{4}$ are unlike fractions.

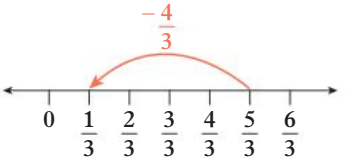
**Adding and subtracting like fractions**

1. Add or subtract the numerators and keep the same denominator.
2. Simplify the fraction.

$$\frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$



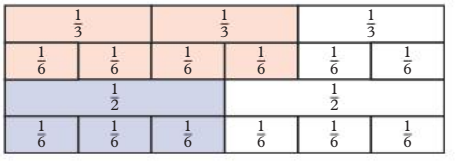

$$\frac{5}{3} - \frac{4}{3} = \frac{1}{3}$$



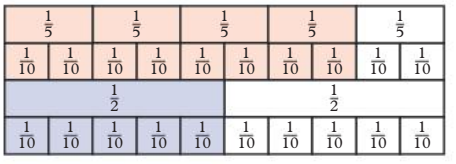

**Adding and subtracting unlike fractions**

1. Convert into equivalent fractions with a common denominator.
2. Add or subtract the numerators and keep the same denominator.
3. Simplify the fraction.

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$$




$$\frac{4}{5} - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$$



# Adding and subtracting mixed numbers

| **Adding and subtracting mixed numbers** | |
|---|---|
| **1** Convert mixed numbers into improper fractions.<br>**2** Convert the improper fractions into equivalent fractions with a common denominator.<br>**3** Add or subtract the numerators and keep the same denominator.<br>**4** Convert the answer into a mixed number and simplify if possible. | $2\frac{1}{4} + 3\frac{1}{5} = \frac{9}{4} + \frac{16}{5}$<br>$= \frac{45}{20} + \frac{64}{20}$<br>$= \frac{109}{20}$<br>$= 5\frac{9}{20}$ |
| | $2\frac{3}{4} - 1\frac{1}{2} = \frac{11}{4} - \frac{3}{2}$<br>$= \frac{11}{4} - \frac{6}{4}$<br>$= \frac{5}{4}$<br>$= 1\frac{1}{4}$ |

- Before adding and subtracting fractions, it can be helpful to estimate the answer. Consider if the answer will be greater than or equal to $\frac{1}{2}$, 1, 2 etc. For mixed numbers, you can estimate by adding or subtracting just the whole number parts.

## **Example 3E.1** Adding and subtracting like fractions

Find the result of each of these.

**a** $\frac{3}{8} + \frac{7}{8}$ **b** $\frac{9}{14} - \frac{2}{14}$ **c** $\frac{1}{5} + \frac{3}{5} - \frac{2}{5}$

**THINK**

**a** **1** Add the numerators and keep the same denominator.

**2** Simplify the fraction by dividing the numerator and denominator by the HCF.

**b** **1** Subtract the numerators and keep the same denominator.

**2** Simplify the fraction by dividing the numerator and denominator by the HCF.

**c** **1** Remember the order of operations. Addition and subtraction are both the last operation to be applied, so work from left to right.

**2** Simplify the fraction. The HCF of 2 and 5 is 1, so the fraction is already in its simplest form.

**WRITE**

**a** $\frac{3}{8} + \frac{7}{8} = \frac{10}{8}$
$= \frac{\cancel{10}^{5}}{\cancel{8}_{4}}$
$= \frac{5}{4}$

**b** $\frac{9}{14} - \frac{2}{14} = \frac{7}{14}$
$= \frac{\cancel{7}^{1}}{\cancel{14}_{2}}$
$= \frac{1}{2}$

**c** $\frac{1}{5} + \frac{3}{5} - \frac{2}{5} = \frac{4}{5} - \frac{2}{5}$
$= \frac{2}{5}$

## **Example 3E.2** Adding and subtracting unlike fractions

Find the result of each of these.

**a** $\frac{5}{6} + \frac{1}{2}$ **b** $\frac{11}{5} - \frac{6}{4}$

**THINK**

**a** 1 Convert into equivalent fractions with a common denominator. The LCM of 6 and 2 is 6. Change $\frac{1}{2}$ to an equivalent fraction with a denominator of 6.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

**b** 1 Convert into equivalent fractions with a common denominator. The LCM of 5 and 4 is 20. Change both fractions to equivalent fractions with a denominator of 20.

2 Subtract the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

**WRITE**

**a**

$$\frac{1}{2} \overset{\times 3}{=} \frac{3}{6}$$

$$\frac{5}{6} + \frac{1}{2} = \frac{5}{6} + \frac{3}{6}$$

$$= \frac{\cancel{8}^{4}}{\cancel{6}_{3}}$$

$$= \frac{4}{3}$$

**b**

$$\frac{11}{5} \overset{\times 4}{=} \frac{44}{20} \qquad \frac{6}{4} \overset{\times 5}{=} \frac{30}{20}$$

$$\frac{11}{5} - \frac{6}{4} = \frac{44}{20} - \frac{30}{20}$$

$$= \frac{\cancel{14}^{7}}{\cancel{20}_{10}}$$

$$= \frac{7}{10}$$

## **Example 3E.3** Adding and subtracting mixed numbers

Find the result of each of these.

**a** $1\frac{4}{9} + 2\frac{2}{9}$ **b** $4\frac{1}{3} - 2\frac{1}{4}$

**THINK**

**a** 1 Convert the mixed numbers into improper fractions.

2 Add the numerators and keep the same denominator.

3 Convert the answer into a mixed number and simplify if possible.

**b** 1 Convert the mixed numbers into improper fractions.

2 Re-write the improper fractions with a common denominator. The LCM multiple of 3 and 4 is 12.

3 Subtract the numerators and keep the same denominator.

4 Convert the answer into a mixed number and simplify if possible.

**WRITE**

**a**

$$1\frac{4}{9} + 2\frac{2}{9} = \frac{13}{9} + \frac{20}{9}$$

$$\frac{13}{9} + \frac{20}{9} = \frac{33}{9}$$

$$= 3\frac{\cancel{6}^{2}}{\cancel{9}_{3}} \quad (33 \div 9 = 3 \text{ remainder } 6)$$

$$= 3\frac{2}{3}$$

**b**

$$4\frac{1}{3} - 2\frac{1}{4} = \frac{13}{3} - \frac{9}{4} \qquad \frac{13}{3} \overset{\times 4}{=} \frac{52}{12} \qquad \frac{9}{4} \overset{\times 3}{=} \frac{27}{12}$$

$$\frac{13}{3} - \frac{9}{4} = \frac{52}{12} - \frac{27}{12}$$

$$= \frac{25}{12} \quad (25 \div 12 = 2 \text{ remainder } 1)$$

$$= 2\frac{1}{12}$$

**Helpful hints**

- ✔ Only add or subtract the numerators of fractions, the denominator stays the same.
  For example, $\frac{5}{6} - \frac{3}{2} \neq \frac{5-3}{6-2}$.
- ✔ Sometimes you only need to find an equivalent fraction of one of your fractions for their denominators to be the same.
  For example, $\frac{1}{4} - \frac{1}{8} = \frac{2}{8} - \frac{1}{8}$.

ANS p552

# Exercise 3E Adding and subtracting fractions

△ 1(a ,d, g), 2(a, d, g), 3–4(1st, 2nd columns), 5–8, 9(a, b), 10–12, 16(a, c)

□ 3(b, d, f, g), 4–5, 7–9, 12, 13, 15, 16(b, c, d), 17(a)

⬠ 3–4(3rd column), 5, 7(g, h), 8–9, 12, 14, 15, 16(b, d), 17–18

UNDERSTANDING AND FLUENCY

**3E.1** **1** Find the result of each sum. Simplify and give your answer as a mixed number, if necessary.

**a** $\frac{1}{5} + \frac{2}{5}$ **b** $\frac{4}{7} + \frac{2}{7}$ **c** $\frac{5}{12} + \frac{3}{12}$

**d** $\frac{5}{18} + \frac{6}{18}$ **e** $\frac{3}{37} + \frac{16}{37}$ **f** $\frac{9}{23} + \frac{16}{23}$

**g** $\frac{15}{11} + \frac{2}{11} + \frac{8}{11}$ **h** $\frac{2}{13} + \frac{4}{13} + \frac{5}{13}$ **i** $\frac{23}{100} + \frac{7}{100} + \frac{14}{100}$

**2** Find the result of each difference. Simplify and give your answer as a mixed number, if necessary.

**a** $\frac{11}{7} - \frac{5}{7}$ **b** $\frac{25}{21} - \frac{8}{21}$ **c** $\frac{19}{13} - \frac{5}{13}$

**d** $\frac{13}{15} - \frac{3}{15}$ **e** $\frac{26}{27} - \frac{25}{27}$ **f** $\frac{21}{33} - \frac{5}{33}$

**g** $\frac{48}{17} - \frac{13}{17} - \frac{10}{17}$ **h** $\frac{55}{3} - \frac{47}{3} - \frac{7}{3}$ **i** $\frac{21}{8} - \frac{2}{8} - \frac{15}{8}$

**3E.2** **3** Find the result of each of these. Simplify your answer.

**a** $\frac{2}{3} + \frac{1}{6}$ **b** $\frac{2}{12} + \frac{5}{6}$ **c** $\frac{1}{2} - \frac{3}{8}$

**d** $\frac{3}{4} - \frac{3}{16}$ **e** $\frac{2}{15} + \frac{4}{5}$ **f** $\frac{4}{30} + \frac{7}{10}$

**g** $\frac{5}{8} - \frac{5}{48}$ **h** $\frac{3}{26} + \frac{1}{13}$ **i** $\frac{17}{23} - \frac{1}{69}$

**4** Find the result of each of these.

**a** $\frac{1}{2} + \frac{2}{3}$ **b** $\frac{3}{5} - \frac{1}{4}$ **c** $\frac{3}{8} + \frac{5}{6}$

**d** $\frac{5}{9} - \frac{1}{2}$ **e** $\frac{3}{4} + \frac{2}{3}$ **f** $\frac{1}{3} - \frac{1}{5}$

**g** $\frac{4}{7} + \frac{1}{2}$ **h** $\frac{5}{6} - \frac{3}{5}$ **i** $\frac{6}{7} - \frac{1}{3}$

**5** Find the result of each of these.

**a** $\frac{2}{3} + \frac{11}{12} + \frac{3}{4}$ **b** $\frac{3}{5} + \frac{13}{20} + \frac{12}{15}$ **c** $\frac{7}{8} + \frac{5}{3} - \frac{5}{12}$

**d** $\frac{5}{6} - \frac{1}{2} + \frac{3}{8}$ **e** $\frac{21}{5} + \frac{8}{3} - \frac{13}{10}$ **f** $\frac{8}{5} - \frac{11}{8} + \frac{1}{4}$

**6** **a** Match the following diagrams with the correct calculation. Determine which diagram and which calculation does not have a match.

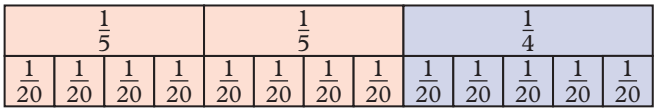
**b** Write the answer to each calculation. Use the diagrams to assist you.

**c** Write a calculation and its answer for the diagram without a match.

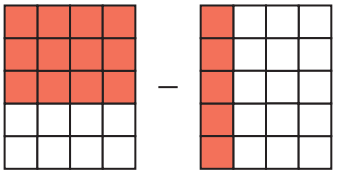
**d** Draw a diagram (fraction wall or number line) for the calculation without a match.

**i** $\frac{2}{5} - \frac{1}{4}$ **ii** $\frac{3}{5} + \frac{1}{4}$ **iii** $\frac{2}{5} + \frac{1}{4}$ **iv** $\frac{3}{5} - \frac{1}{4}$

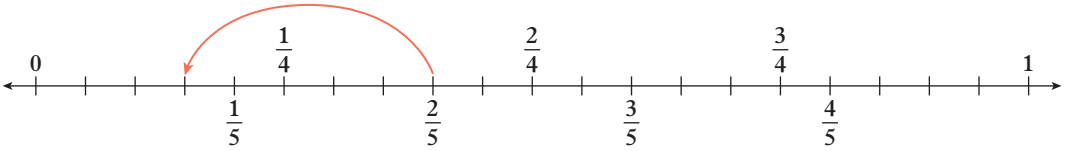
**A**



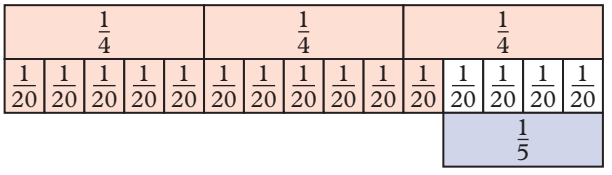

**B**



**C**




**D**




3E.3 **7** Find the result of each of these.

**a** $3\frac{2}{5} + 4\frac{1}{5}$ **b** $6\frac{4}{5} - 3\frac{2}{5}$ **c** $2\frac{8}{11} + 1\frac{5}{11}$ **d** $8\frac{1}{9} - 4\frac{4}{9}$

**e** $12\frac{2}{3} - 2\frac{1}{3} - 8\frac{2}{3}$ **f** $3\frac{2}{5} + 4\frac{1}{5} + 1\frac{4}{5}$ **g** $6\frac{5}{12} + 2\frac{1}{12} - 7\frac{11}{12}$ **h** $3\frac{1}{8} + 2\frac{5}{8} - 1\frac{7}{8}$

**8** Find the result of each of these.

**a** $2\frac{2}{3} + 1\frac{1}{2}$ **b** $4\frac{4}{7} - 1\frac{2}{3}$ **c** $3\frac{2}{5} + 2\frac{1}{4}$ **d** $5\frac{2}{3} - 3\frac{4}{5}$

**e** $6\frac{5}{6} - 4\frac{3}{10}$ **f** $3\frac{2}{14} + 2\frac{2}{21}$ **g** $1\frac{5}{6} + 5\frac{1}{2}$ **h** $4\frac{1}{6} - 2\frac{7}{24}$

**9** Find the result of each of these.

**a** $\frac{8}{3} + 5 + 1\frac{1}{8}$ **b** $1\frac{1}{7} + 2\frac{1}{2} - 3\frac{5}{14}$ **c** $\frac{28}{5} - 4\frac{1}{7} + 1\frac{1}{2}$ **d** $3\frac{5}{12} - \frac{10}{5} + \frac{17}{6}$

**10** Dean completed a 10 km run in $\frac{5}{7}$ of an hour, while Mark completed the run in $\frac{3}{4}$ of an hour.

**a** Which runner was faster?

**b** What was the difference between the two times?

**11** A factory worker is employed to make T-shirts. On average, each T-shirt takes $3\frac{1}{2}$ minutes for cutting, $2\frac{4}{5}$ minutes for sewing and $4\frac{1}{6}$ minutes for finishing. What is the total time required to make one T-shirt?

PROBLEM SOLVING AND REASONING

**12** A group of friends organises to catch up for lunch and decides to order pizza. Each person estimates what fraction of a pizza he or she can eat. Luisa can eat $\frac{1}{3}$, Declan $\frac{1}{2}$, Nisa $\frac{3}{8}$, Tomas $\frac{2}{3}$ and Christopher $\frac{3}{2}$ of a pizza.

**a** How many full pizzas will need to be ordered?

**b** What fraction of a pizza will be left over?

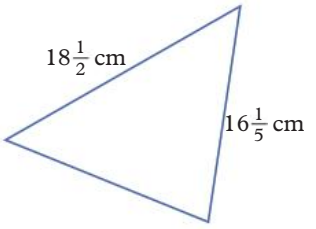
**13** Laila is planning to travel overseas at the end of the year and needs to save money, so she devises a budget. She decides to allocate $\frac{2}{5}$ of her weekly pay to living expenses, $\frac{1}{3}$ to rent, $\frac{1}{8}$ to going out and the remainder to savings.

**a** Calculate what fraction of Laila's pay will go towards living expenses, rent and going out.

**b** What fraction of her pay will go towards savings? Explain your answer.

**14** The total length around the outside of the triangle on the right is $49\frac{2}{5}$ cm.

The length measurements of two of the sides are known. What is the length of the unknown side?




**15** While on camp, students were required to complete a 21 km bushwalk over three days. They covered $8\frac{2}{5}$ km on the first day and $7\frac{1}{3}$ km on the second day.

**a** What distance will they need to walk on the third day?

**b** How much further did they walk on the first day than on the second?

**16** Complete each of these patterns by filling in the gaps. Explain how the patterns work.

**a** $\frac{2}{7}, \frac{5}{7}, \frac{8}{7}$, _____, _____, _____, _____

**b** $10, 11\frac{2}{5}, 12\frac{4}{5}$, _____, _____, _____, _____

**c** $\frac{19}{3}, \frac{16}{3}, \frac{13}{3}$, _____, _____, _____, _____

**d** $12\frac{14}{15}, 10\frac{10}{15}, 8\frac{6}{15}$, _____, _____, _____

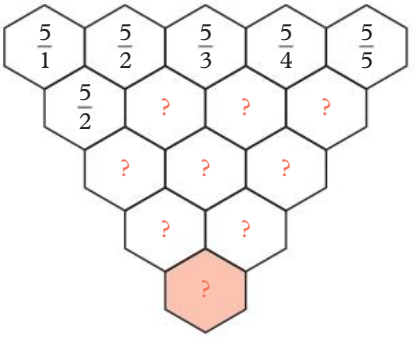
**17** Use a single pair of brackets to make each equation true.

**a** $\frac{1}{2} - \frac{5}{8} - \frac{2}{5} = \frac{11}{40}$

**b** $\frac{19}{28} - \frac{3}{7} + \frac{3}{14} = \frac{1}{28}$

CHALLENGE

**18** Replace each question mark with the difference between the two fractions above it. What will the number at the bottom be?




Repeat this process with $\frac{9}{1}, \frac{9}{2}, \frac{9}{3}, \frac{9}{4}, \frac{9}{5}, \frac{9}{6}, \frac{9}{7}, \frac{9}{8}, \frac{9}{9}$ across the top. What will the number at the bottom of this triangle be?

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Adding and subtracting fractions

**Investigation** Concoction cups

**Topic quiz** 3E

# 3F Multiplying fractions

## Learning intentions

- ✔ I can multiply fractions by whole numbers.
- ✔ I can multiply fractions by fractions.
- ✔ I can multiply fractions with cancelling.
- ✔ I can multiply mixed numbers.

Inter-year links
Year 8 2C Multiplying and dividing fractions

## Multiplying fractions

- When multiplying fractions, the denominators do not need to be the same.
- To multiply two fractions together:

  **1** Multiply the numerators together.

  **2** Multiply the denominators together.

  **3** Simplify the fraction.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

- When multiplying mixed numbers, convert the mixed numbers into improper fractions, multiply across, and give the result as a mixed number.

## Cancelling

- When multiplying fractions, **cancelling** or dividing out common factors can simplify the process.
  → Cancelling can only be performed diagonally and vertically, not horizontally.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}^{1}}{\cancel{3}^{1} \times 4} = \frac{\cancel{2}^{1}}{\cancel{4}^{2}} = \frac{1}{2}$$

Diagonally



Vertically

## Multiplying by whole numbers

- Whole numbers can be written as fractions with a denominator of 1. For example, $8 = \frac{8}{1}$.
- The number 1 can be written as a fraction where the numerator and denominator are equal.

  For example, $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{100}{100} \ldots$
- When multiplying any number or fraction by 1, the result remains the same.
- When asked to find a fraction 'of' a whole number or fraction, replace the 'of' with '×'.

  For example, $\frac{2}{3}$ of 20 is the same as $\frac{2}{3} \times 20$.

## **Example 3F.1** Multiplying fractions using repeated addition

Use repeated addition to calculate $6 \times \frac{1}{2}$.

**THINK**

1. Write the multiplication as a sum of the fractions.
2. Add the numerators.
3. Simplify the fraction.

**WRITE**

$$6 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$
$$= \frac{6}{2}$$
$$= 3$$

## **Example 3F.2** Multiplying fractions with simplifying

Calculate the result of $\frac{3}{5} \times \frac{2}{6}$.

**THINK**

1. Multiply the numerators together, and then multiply the denominators together.
2. Simplify the fraction. 6 and 30 have a common factor of 6.

**WRITE**

$$\frac{3}{5} \times \frac{2}{6} = \frac{3 \times 2}{5 \times 6}$$
$$= \frac{\cancel{6}^{1}}{\cancel{30}^{5}}$$
$$= \frac{1}{5}$$

## **Example 3F.3** Multiplying fractions with cancelling

Calculate the result of $\frac{8}{5} \times \frac{25}{12}$.

**THINK**

1. Look diagonally and vertically for any common factors between the numerators and denominators.

   Cancel the 8 and 12 by dividing both numbers by 4. Then cancel the 25 and 5 by dividing both numbers by 5.
2. Multiply the numerators together, and then multiply the denominators together.

**WRITE**

$$\frac{8}{5} \times \frac{25}{12} = \frac{\cancel{8}^{2}}{\cancel{5}^{1}} \times \frac{\cancel{25}^{5}}{\cancel{12}^{3}}$$
$$= \frac{2 \times 5}{1 \times 3}$$
$$= \frac{10}{3}$$

## **Example 3F.4** Multiplying mixed numbers

Calculate the result of $1\frac{2}{3} \times 2\frac{7}{10}$.

**THINK**

1. Convert each mixed number into an improper fraction.
2. Look diagonally and vertically for any common factors between the numerators and denominators.

   Cancel the 5 and 10 by dividing both numbers by 5. Then cancel the 3 and 27 by dividing both numbers by 3.
3. Multiply the numerators together, and then multiply the denominators together.
4. Convert the improper fraction to a mixed number.

**WRITE**

$$1\frac{2}{3} \times 2\frac{7}{10} = \frac{5}{3} \times \frac{27}{10}$$

$$= \frac{\cancel{5}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}^{2}}$$

$$= \frac{1 \times 9}{1 \times 2}$$

$$= \frac{9}{2}$$

$$= 4\frac{1}{2}$$

## **Example 3F.5** Multiplying fractions with unit conversions

Calculate $\frac{1}{5}$ of 2 hours. Give your answer in minutes.

**THINK**

1. Convert the number of hours into minutes. There are 60 minutes in 1 hour.
2. Replace 'of' with '×' and cancel any common factors.
3. Write the whole number as a fraction with 1 as the denominator.
4. Cancel out the common factor of 5.
5. Multiply the numerators together, and then multiply the denominators together. Write the answer including the units.

**WRITE**

$\frac{1}{5}$ of 2 hours $= \frac{1}{5}$ of 120 minutes

$$= \frac{1}{5} \times 120$$

$$= \frac{1}{5} \times \frac{120}{1}$$

$$= \frac{1}{\cancel{5}^{1}} \times \frac{\cancel{120}^{24}}{1}$$

$$= 24 \text{ minutes}$$

**Helpful hints**

- ✔ Remember when multiplying and dividing fractions you do not need to write the fractions with a common denominator.
- ✔ Always write whole numbers as fractions before multiplying.
- ✔ Always convert mixed numbers to improper fractions before multiplying.
- ✔ Cancelling out common factors of the numerators and denominators before multiplying means you are simplifying before multiplying (which is often easier since you are working with smaller numbers).

ANS p553

# Exercise 3F Multiplying fractions

1–6, 7(a–f), 9(a, c, f, h), 10–13, 15, 17(a) | 2(d, e, g, h), 3–4, 5(e–h), 6, 7(g–l), 8–10, 12, 14, 16, 17 | 2–4(g–i), 5(g, h), 6(d, e, f), 7(g–l), 9–10, 12, 14, 16–18

UNDERSTANDING AND FLUENCY

3F.1 **1** Use repeated addition to calculate the following products.

**a** $3 \times \frac{2}{5}$ **b** $4 \times \frac{2}{9}$ **c** $6 \times \frac{1}{6}$ **d** $2 \times \frac{11}{3}$

3F.2 **2** Calculate each of these products. Leave your answer as an improper fraction if applicable.

**a** $\frac{1}{5} \times \frac{3}{2}$ **b** $\frac{2}{3} \times \frac{1}{7}$ **c** $\frac{11}{3} \times \frac{2}{5}$

**d** $\frac{1}{3} \times \frac{4}{9}$ **e** $\frac{3}{8} \times \frac{7}{2}$ **f** $\frac{11}{13} \times \frac{6}{7}$

**g** $\frac{1}{3} \times \frac{7}{15}$ **h** $\frac{3}{10} \times \frac{9}{7}$ **i** $\frac{9}{2} \times \frac{9}{5}$

3F.3 **3** Calculate each of these products.

**a** $\frac{8}{9} \times \frac{1}{4}$ **b** $\frac{2}{3} \times \frac{6}{11}$ **c** $\frac{3}{5} \times \frac{10}{13}$

**d** $\frac{1}{8} \times \frac{4}{17}$ **e** $\frac{7}{9} \times \frac{12}{35}$ **f** $\frac{20}{27} \times \frac{9}{15}$

**g** $\frac{3}{4} \times \frac{18}{24}$ **h** $\frac{9}{35} \times \frac{20}{21}$ **i** $\frac{14}{28} \times \frac{8}{13}$

3F.4 **4** Calculate each of these products. Give your answer as a mixed number if necessary.

**a** $\frac{3}{5} \times 1\frac{4}{11}$ **b** $5\frac{2}{4} \times \frac{6}{7}$ **c** $5\frac{3}{5} \times 3\frac{1}{4}$

**d** $4\frac{2}{3} \times \frac{6}{2}$ **e** $\frac{7}{9} \times 3\frac{5}{7}$ **f** $5\frac{2}{3} \times 2\frac{2}{5}$

**g** $1\frac{3}{20} \times \frac{8}{5}$ **h** $3\frac{1}{3} \times 4\frac{1}{8}$ **i** $2\frac{4}{9} \times 3\frac{3}{16}$

**5** Calculate each of these products.

**a** $\frac{3}{8} \times 24$ **b** $\frac{5}{11} \times 66$ **c** $\frac{12}{15} \times 4$ **d** $5 \times \frac{16}{25}$

**e** $9 \times \frac{1}{24}$ **f** $6 \times \frac{5}{36}$ **g** $\frac{8}{3} \times 15$ **h** $9 \times \frac{4}{81}$

**6** Calculate:

**a** $\frac{2}{5}$ of $\frac{1}{8}$ **b** $\frac{3}{4}$ of $\frac{3}{7}$ **c** $\frac{2}{11}$ of $\frac{7}{9}$

**d** $\frac{9}{4}$ of $\frac{5}{11}$ **e** $2\frac{7}{13}$ of 26 **f** $5\frac{1}{2}$ of $3\frac{1}{3}$

3F.5 **7** Calculate each fraction amount in the units shown in the brackets.

**a** $\frac{1}{4}$ of 3 hours (minutes) **b** $\frac{2}{3}$ of 1 year (months) **c** $\frac{4}{5}$ of \$80 (dollars)

**d** $\frac{6}{13}$ of the year (weeks) **e** $\frac{2}{5}$ of 2 minutes (seconds) **f** $\frac{3}{4}$ of 100 L (litres)

**g** $\frac{3}{7}$ of 2 weeks (days) **h** $\frac{9}{4}$ of \$200 (dollars) **i** $\frac{2}{3}$ of $6\frac{1}{2}$ hours (hours)

**j** $\frac{7}{5}$ of $11\frac{1}{4}$ km (km) **k** $\frac{5}{6}$ of 10 years (months) **l** $\frac{4}{7}$ of \$2.80 (cents)

**8** Identify whether the following statements are true or false.

**a** When multiplying a whole number by a proper fraction, the answer is always smaller than the whole number.

**b** When multiplying a whole number by an improper fraction or mixed number, the answer is always smaller than the whole number.

**9** When multiplying three fractions together, start by multiplying the first two fractions together, then multiply the result by the third fraction. Calculate:

**a** $\frac{5}{9} \times \frac{9}{10} \times \frac{12}{17}$ **b** $\frac{3}{8} \times \frac{15}{26} \times \frac{8}{9}$ **c** $\frac{28}{5} \times \frac{11}{6} \times \frac{24}{7}$ **d** $1\frac{1}{12} \times \frac{8}{3} \times \frac{15}{4}$

**e** $\frac{22}{13} \times 10\frac{2}{5} \times 2\frac{1}{11}$ **f** $4\frac{1}{2} \times 5\frac{2}{5} \times \frac{21}{8}$ **g** $\frac{12}{5} \times 7\frac{1}{2} \times 3\frac{1}{4}$ **h** $6\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{2}{7}$

**10** Calculate the following. Hint: Remember the rules for the order of operations.

**a** $\frac{5}{8} + \frac{1}{2} \times \frac{3}{4}$ **b** $\frac{2}{3} \times \frac{4}{5} + \frac{8}{15}$ **c** $\frac{13}{6} - \frac{5}{3} \times \frac{5}{4}$ **d** $\frac{3}{8} + \frac{9}{4} \times \frac{11}{6} - \frac{8}{3}$

**11** Toby has a garage sale where he hopes to sell 450 used books he has collected. He manages to sell $\frac{7}{9}$ of the books. How many books is he left with?

**12** Hilaire works at the local swimming pool as a lifeguard. Her shift begins at 3:45 pm and ends at 6:15 pm.

**a** Write the hours Hilaire works in one shift as:

**i** a mixed number **ii** an improper fraction.

**b** If Hilaire has the same shift five nights a week, calculate the total number of hours she works as a mixed number.

**13** While correcting his students' homework, a teacher found three different answers to the question:

'Find $2\frac{4}{9} \times 3\frac{3}{8}$ in its simplest form'. The answers given were: $6\frac{12}{72}$, $8\frac{1}{4}$ and $6\frac{12}{17}$.

**a** Which answer is correct?

**b** Comment on the error that is likely to have been made in each case.

**14** A store takes $\frac{2}{5}$ off the price of its televisions during the stocktake sales. If a television normally costs \$6320, what will it cost during the stocktake sale?

**15** Amie manages to complete $\frac{3}{8}$ of her $5\frac{1}{2}$ km run before stopping. How far did she run prior to stopping?

**16** Multiplying a number by a unit fraction is the same as dividing the number by the denominator of the unit fraction.

For example, $\frac{1}{2} \times 10 = \frac{10}{2}$
$= 10 \div 2$

**a** Write the following multiplications as divisions.

**i** $\frac{1}{3} \times 33$ **ii** $\frac{1}{8} \times 40$ **iii** $\frac{1}{101} \times 55$ **iv** $\frac{1}{7} \times \frac{4}{9}$

**b** Write the following divisions as multiplications.

**i** $56 \div 7$ **ii** $90 \div 10$ **iii** $92 \div 35$ **iv** $\frac{8}{11} \div 5$

For any fraction, we can use the associative property to do the multiplication in a different order.

For example, $\frac{3}{2} \times 10 = 3 \times \left(\frac{1}{2} \times 10\right)$
$= 3 \times \frac{10}{2}$
$= 3 \times (10 \div 2)$
$= 3 \times 5$
$= 15$

**c** Calculate the following using this method.

**i** $\frac{3}{8} \times 16$ **ii** $\frac{8}{5} \times 35$

**iii** $\frac{12}{7} \times 63$ **iv** $\frac{7}{6} \times \frac{18}{5}$

**17 a** Calculate each of the following products. Determine if the product is less than both fractions, greater than both fractions or between both fractions.

**i** $\frac{2}{3} \times \frac{4}{5}$ **ii** $3 \times \frac{1}{7}$ **iii** $\frac{9}{4} \times \frac{11}{3}$ **iv** $\frac{5}{6} \times 2\frac{5}{6}$

**v** $\frac{8}{3} \times \frac{3}{4}$ **vi** $\frac{1}{9} \times \frac{1}{7}$ **vii** $3\frac{1}{3} \times \frac{10}{3}$ **viii** $\frac{5}{3} \times 4$

**b** When will the product of two fractions be a value less than both fractions?

**c** When will the product of two fractions be a value greater than both fractions?

**d** When will the product of two fractions be a value between both fractions?

**e** What value does one of the fractions need to be so that the product is the same value as one of the fractions?

CHALLENGE

**18** Saina's dad showed her the following multiplication:

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$

Dad said, 'Wow, that took me ages to work out!'

Saina said, 'No Dad, this is really easy!'

How did Saina turn this into an easy calculation?

**19** Consider the following multiplication pattern:

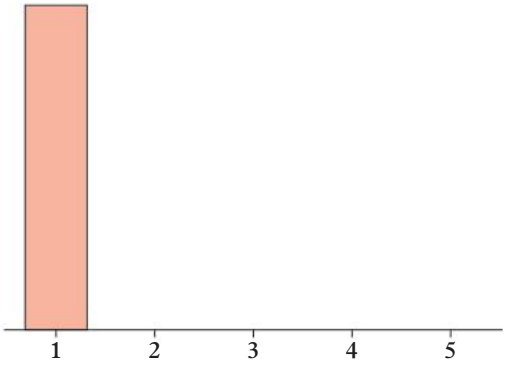
$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \ldots$

**a** Determine the value of the number by multiplying up to the number $\frac{1}{5}$.

**b** Simplify the products in the table below.

| **Column** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Value** | $\frac{1}{1}$ | $\frac{1}{1} \times \frac{1}{2}$ | $\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3}$ | $\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$ | $\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}$ |

**c** Complete the column graph below with the value of each column calculated in part **b**. For example, the value of column 1 is 1. Use that as one whole to determine the size of the other columns.




**d** As more fractions are multiplied in the pattern, does the result become greater or smaller?

**e** Estimate the whole number that the pattern will 'almost' produce but never actually give as a result.

Check your Student obook pro for these digital resources and more:

**Worksheet** Multiplying fractions

**Investigation** All things equal

**Topic quiz** 3F

# 3G Dividing fractions

## Learning intentions

- ✔ I can identify the reciprocal of a fraction.
- ✔ I can divide proper and improper fractions.
- ✔ I can divide mixed numbers.

**Inter-year links**
Year 8 — 2C Multiplying and dividing fractions

## The reciprocal of a fraction

- When a fraction is multiplied by its **reciprocal**, the result is 1.

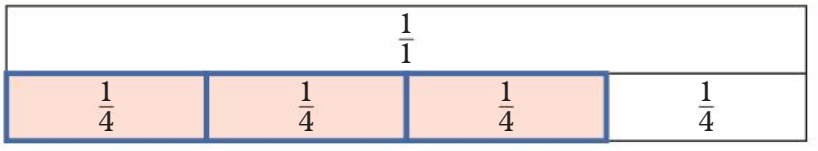
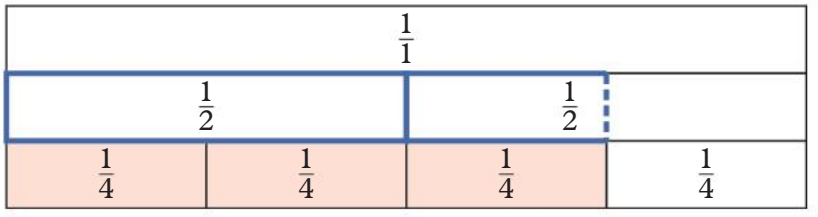
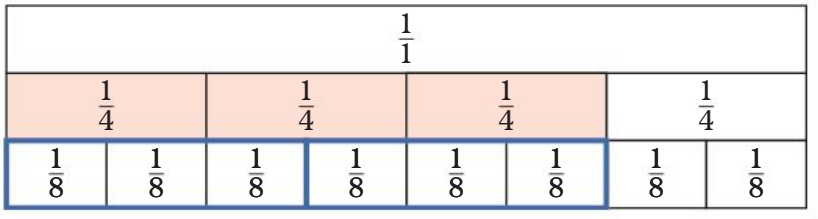
  For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ as $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.
- To find the reciprocal of a fraction, invert the fraction by swapping the numerator and the denominator.

Reciprocal

$$\frac{3}{4} \quad \frac{4}{3}$$

## Dividing fractions by fractions

- Dividing fractions can be thought of as asking: 'How many of one fraction fits into another fraction?'

| | | |
|---|---|---|
| $\frac{3}{4} \div \frac{1}{4} = 3$ | How many $\frac{1}{4}$ are there in $\frac{3}{4}$? |   |
| $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$ | How many $\frac{1}{2}$ are there in $\frac{3}{4}$? |   |
| $\frac{3}{4} \div \frac{3}{8} = 2$ | How many $\frac{3}{8}$ are there in $\frac{3}{4}$? |   |

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction. Keep the first fraction the same, and then multiply by the reciprocal of the second fraction.

  For example, $\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$.

# Dividing fractions by whole numbers

- Dividing by a whole number is the same as multiplying by the reciprocal of the whole number.
- A whole number can be written as a fraction with denominator 1, so the reciprocals of whole numbers are the unit fractions.

Reciprocal

$$5 = \frac{5}{1} \quad \frac{1}{5}$$

# Dividing fractions by mixed numbers

- When dividing fractions by mixed numbers, start by converting the mixed numbers into improper fractions, then divide the fractions by multiplying by the reciprocal. Provide the result as a mixed number or proper fraction.

For example, $1\frac{2}{3} \div 2\frac{1}{4} = \frac{5}{3} \div \frac{9}{4}$

$= \frac{5}{3} \times \frac{4}{9}$

$= \frac{20}{27}$

## Example 3G.1 Finding the reciprocal of fraction

Write the reciprocal of each value.

**a** $\frac{3}{7}$ **b** $\frac{1}{8}$ **c** $2$ **d** $2\frac{3}{4}$

**THINK**

**a** Invert the fraction. Swap the numerator and the denominator.

**b** Invert the fraction. Simplify the result.

**c** Write the whole number as a fraction with a denominator of 1. Invert the fraction.

**d** Write the mixed number as an improper fraction. Invert the improper fraction.

**WRITE**

**a** $\frac{7}{3}$

**b** $\frac{8}{1} = 8$

**c** $2 = \frac{2}{1}$. The reciprocal is $\frac{1}{2}$.

**d** $2\frac{3}{4} = \frac{11}{4}$. The reciprocal is $\frac{4}{11}$.

## Example 3G.2 Dividing fractions by fractions

Calculate $\frac{1}{7} \div \frac{3}{5}$.

**THINK**

1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Multiply the numerators together and then multiply the denominators together.

**WRITE**

$\frac{1}{7} \div \frac{3}{5} = \frac{1}{7} \times \frac{5}{3}$

$= \frac{1 \times 5}{7 \times 3}$

$= \frac{5}{21}$

## **Example 3G.3** Dividing fractions by whole numbers

Calculate $\frac{4}{5} \div 6$.

**THINK**

1. Keep the first fraction the same and multiply by the reciprocal of the second fraction. $6 = \frac{6}{1}$ so the reciprocal is $\frac{1}{6}$.
2. Cancel the 4 and 6 by dividing both numbers by 2.
3. Multiply the numerators together and then multiply the denominators together.

**WRITE**

$$\frac{4}{5} \div 6 = \frac{4}{5} \times \frac{1}{6}$$

$$= \frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{6}^{3}}$$

$$= \frac{2 \times 1}{5 \times 3}$$

$$= \frac{2}{15}$$

## **Example 3G.4** Dividing mixed numbers

Calculate $4\frac{3}{8} \div 2\frac{6}{7}$.

**THINK**

1. Convert each mixed number to an improper fraction.
2. Keep the first fraction the same and multiply by the reciprocal of the second fraction.
3. Cancel the 35 and 20 by dividing each by 5.
4. Multiply the numerators together and then multiply the denominators together.
5. Change the improper fraction to a mixed number.

**WRITE**

$$4\frac{3}{8} \div 2\frac{6}{7} = \frac{35}{8} \div \frac{20}{7}$$

$$= \frac{35}{8} \times \frac{7}{20}$$

$$= \frac{\cancel{35}^{7}}{8} \times \frac{7}{\cancel{20}^{4}}$$

$$= \frac{7 \times 7}{8 \times 4}$$

$$= \frac{49}{32}$$

$$= 1\frac{17}{32}$$

**Helpful hints**

- ✔ The phrase 'Keep Change Flip' can help you remember how to turn a division problem into a multiplication problem.
- ✔ You can change any whole number into a fraction with a denominator of 1.
- ✔ Always simplify your fractions!

ANS p554

# Exercise 3G Dividing fractions

1–11 | 4–7, 9–14 | 5–7, 9, 11–16

UNDERSTANDING AND FLUENCY

**1** Rewrite each division problem in the form 'How many ____ are there in ____?'

**a** $2 \div \frac{1}{8}$ **b** $4 \div \frac{1}{3}$ **c** $\frac{1}{2} \div \frac{1}{12}$ **d** $\frac{1}{3} \div \frac{1}{9}$

**2** Write the following statements as a division.
Then, use the fraction wall to find the quotient.

**a** How many $\frac{1}{9}$ are there in $\frac{2}{3}$?

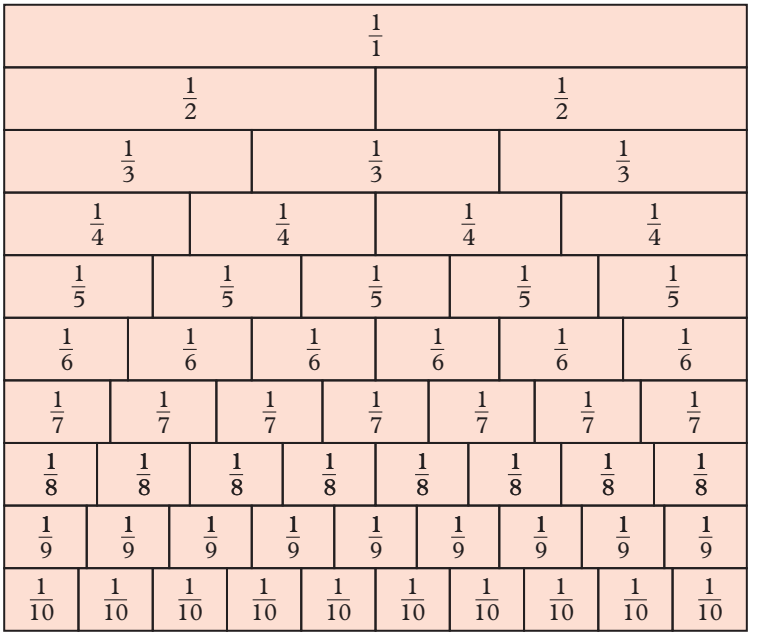
**b** How many $\frac{1}{5}$ are there in $\frac{3}{10}$?

**c** How many $\frac{2}{5}$ are there in $\frac{9}{10}$?

**d** How many $\frac{1}{3}$ are there in $\frac{5}{6}$?

**e** How many $\frac{3}{8}$ are there in 1?

**f** How many $\frac{2}{9}$ are there in $\frac{4}{6}$?




3G.1 **3** Write the reciprocal of each of these.

**a** $\frac{65}{31}$ **b** $\frac{11}{3}$

**c** $\frac{2}{5}$ **d** $\frac{27}{4}$

**e** $\frac{1}{100}$ **f** 8

**g** $\frac{34}{1}$ **h** $\frac{65}{31}$

**i** $4\frac{2}{7}$ **j** $9\frac{1}{8}$

**4** Using the fact that dividing by a number is equivalent to multiplying by its reciprocal, write each of these divisions as multiplications. Do not perform the multiplication.

**a** $\frac{4}{7} \div \frac{5}{9}$ **b** $\frac{8}{3} \div \frac{1}{9}$ **c** $\frac{3}{5} \div 4$ **d** $6 \div \frac{3}{10}$ **e** $9\frac{3}{8} \div \frac{3}{2}$

**f** $\frac{8}{3} \div 6\frac{7}{10}$ **g** $5\frac{1}{5} \div 7\frac{4}{9}$ **h** $3 \div 2\frac{3}{8}$ **i** $4\frac{5}{8} \div 10$ **j** $21 \div 32$

3G.2 **5** Calculate each of the quotients. Write your answer as a mixed fraction.

**a** $\frac{3}{8} \div \frac{2}{5}$ **b** $\frac{1}{12} \div \frac{1}{11}$ **c** $\frac{3}{4} \div \frac{3}{10}$ **d** $\frac{1}{9} \div \frac{4}{17}$ **e** $\frac{1}{10} \div \frac{4}{25}$

**f** $\frac{6}{7} \div \frac{8}{21}$ **g** $\frac{16}{15} \div \frac{12}{5}$ **h** $\frac{14}{3} \div \frac{7}{11}$ **i** $\frac{4}{13} \div \frac{20}{3}$ **j** $\frac{15}{27} \div \frac{11}{21}$

3G.3 **6** Calculate each of the quotients.

**a** $\frac{2}{9} \div 6$ **b** $\frac{3}{4} \div 9$ **c** $\frac{5}{2} \div 10$ **d** $\frac{8}{9} \div 2$ **e** $4 \div \frac{7}{5}$

**f** $6 \div \frac{1}{7}$ **g** $14 \div \frac{7}{6}$ **h** $12 \div \frac{8}{9}$ **i** $5 \div \frac{10}{13}$ **j** $18 \div \frac{21}{5}$

3G.4 **7** Calculate each of the quotients.

**a** $3\frac{1}{2} \div \frac{4}{5}$ **b** $1\frac{4}{7} \div \frac{5}{2}$ **c** $6\frac{1}{3} \div 1\frac{1}{4}$ **d** $7\frac{1}{5} \div \frac{22}{3}$

**e** $1\frac{2}{7} \div \frac{15}{21}$ **f** $2\frac{7}{9} \div \frac{4}{18}$ **g** $\frac{13}{4} \div 2\frac{6}{10}$ **h** $3\frac{3}{5} \div 1\frac{4}{5}$

**i** $4 \div 1\frac{1}{5}$ **j** $2\frac{1}{3} \div 5$ **k** $9\frac{1}{11} \div 10$ **l** $7\frac{3}{5} \div 19$

**8** A batch of a chocolate pudding recipe requires $\frac{1}{6}$ kg of sugar. How many batches of chocolate pudding can be made from a 1 kg packet of sugar?

**9** Over the school holidays, Brianna worked a total of 44 hours stacking shelves at the supermarket. If she only worked $5\frac{1}{2}$ hour shifts, how many shifts did she work?

**10** Find the reciprocal of each answer from question **3**. Compare these to the original numbers. What can you conclude about finding the reciprocal of a number?

**11** Rhys shares $3\frac{3}{8}$ pizzas equally between eight friends and himself. What fraction of a pizza will each person receive?

**12** A new housing estate is to be developed on $43\frac{1}{5}$ hectares of land. If each block measures $\frac{3}{5}$ hectares, how many blocks will there be?

**13** Gianluca must pack 15 kg of apples into plastic bags that can hold a maximum of $1\frac{1}{3}$ kg.

**a** How many $1\frac{1}{3}$ kg bags can he fill?

**b** How many kilograms of apples will be left over?

**14** Priya and Rosa run a cafe that serves high tea on Sunday afternoons. Their menu states that each customer receives 4 finger sandwiches. Usually Priya and Rosa cut whole sandwiches into 3 fingers and the customers receive 4 fingers.

**a** Write the fraction of a whole sandwich each customer receives as an improper fraction.

**b** This Sunday they make 36 whole sandwiches. How many customers can they serve?

**c** They quickly realise they have more customers than expected, but they don't have enough ingredients to make any more sandwiches so decide to cut each sandwich into 5 fingers instead of 3. How many customers could they serve with 4 fingers now?

**d** What fraction of a whole sandwich does each customer receive in this case?

**15** Sava has started an online business selling backpacks that he makes from recycled T-shirts. He needs $1\frac{1}{3}$ T-shirts to make 1 backpack.

**a** How many T-shirts does he need to make 30 backpacks?

**b** At one of his favourite op-shops Sava finds 15 T-shirts that are perfect for his backpacks. How many backpacks will he be able to make from these T-shirts?



**16** Consider the number line below. If all the intervals are the same size, state whether the following are true or false.

**a** $A = 14$

**b** $B > 15$

**c** $C = 15\frac{3}{4}$

**d** $D < 17\frac{1}{2}$

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Multiplying and dividing fractions

**Investigation** Paper cuts

**Topic quiz** 3G

# 3H Ratios

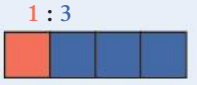
## Learning intentions

- ✔ I can compare quantities using ratios.
- ✔ I can find and simplify equivalent ratios.
- ✔ I can divide quantities into a ratio.

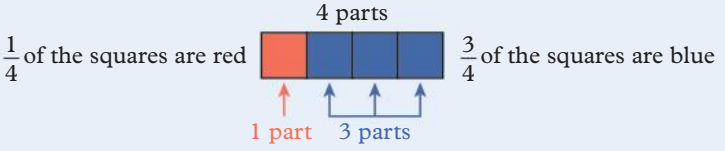
Inter-year links
Year 8 3D Ratios

## Ratios

- A **ratio** is a comparison of two or more quantities of the same kind.
  - → Ratios are shown by a colon (:) meaning 'compared with' and showing the relationship between the equal parts.
    For example, the ratio 1 : 3 reads '1 part compared with 3 parts' or '1 to 3.'



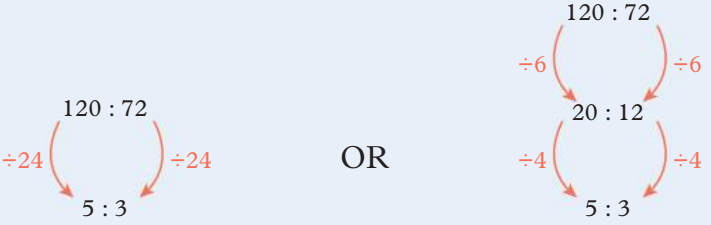

- A ratio must be written in the order of the given worded description.
  For example, one part cordial to five parts water is written as 1 : 5.
- Ratios can be viewed as fractions where the numerator is the number in the ratio and the denominator is the total number of parts.



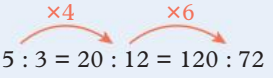

- Before writing a ratio, the numbers must be in the same unit of measurement.

## Simplifying ratios

- A ratio is in its simplest form when the only common factor in the ratio is 1. Ratios can be simplified by:
  - → dividing the numbers in the ratio by the highest common factor,
  - → or by repeated division of common factors.

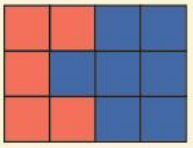



- Numbers in a ratio can be multiplied or divided by a value to create an equivalent ratio.



## **Example 3H.1** Determining ratios from a diagram

What is the ratio of red squares to blue squares?



**THINK**

1. Count the red squares and count the blue squares.
2. Order the numbers as they appear in the question, red then blue.
3. Write the ratio with a colon.

**WRITE**

There are 5 red and 7 blue squares.

Red : Blue

5 : 7

## **Example 3H.2** Simplifying ratios

Write the ratio 35 : 56 in its simplest form.

**THINK**

1. Find the highest common factor (HCF) of each number in the ratio.
2. Simplify by dividing each number in the ratio by the HCF.
3. Write the ratio.

**WRITE**

The HCF of 35 and 56 is 7.

35 : 56

÷7 ÷7

= 5 : 8

35 : 56 = 5 : 8

## **Example 3H.3** Writing ratios with a change of units

Write 6 cm to 42 mm as a ratio in its simplest form.

**THINK**

1. Convert the quantities to the same unit. Write them in the smaller unit, mm. Recall that 1 cm = 10 mm.
2. Write the comparison as a ratio in the order given.
3. Simplify by dividing each number in the ratio by the HCF. The HCF of 60 and 42 is 6.

**WRITE**

6 cm to 42 mm = 60 mm to 42 mm

= 60 : 42

÷6 ÷6

= 10 : 7

## **Example 3H.4** Dividing a quantity into a ratio

Divide 20 into the ratio 2 : 3.

| THINK | WRITE |
|---|---|
| **1** Find the total number of parts in the ratio. | Number of parts = 2 + 3<br>= 5 |
| **2** Divide the quantity by the total number of parts. | 20 ÷ 5 = 4 |
| **3** Multiply 4 by each part of the ratio. | 4 × 2 = 8<br>4 × 3 = 12<br>The ratio 2 : 3 divides 20 into 8 and 12. |

**Helpful hints**

- ✔ Keep the order of the numbers in the ratio the same as the order of the worded comparison. It tells you the order of the quantities you are comparing.
- ✔ Ratios are not written in size order, only in comparison order.
- ✔ Remember to convert the units of measurement, ensuring that they are equal, before writing a ratio. If you need help remembering the equal measurements, check out the table to the right.

| |
|---|
| 1 cm = 10 mm |
| 1 m = 100 cm |
| 1 km = 1000 m |
| 1 kg = 1000 g |
| 1 L = 1000 mL |

ANS p555

# **Exercise 3H** Ratios

△ 1–7, 8(a, e, g), 9(a–d), 10–11, 14, 16, 18 ▢ 2–6, 7(d–f), 8–9, 11, 13, 15, 17, 18, 20 ⬠ 5–6, 7(d–f), 8–9, 11, 13, 15, 19–22

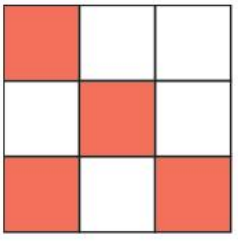
UNDERSTANDING AND FLUENCY

3H.1 **1** Write each comparison as a ratio of:

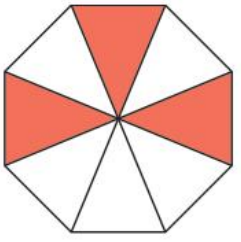
**i** the number of shaded parts to the number of non-shaded parts

**ii** the number of shaded parts to the total number of parts.

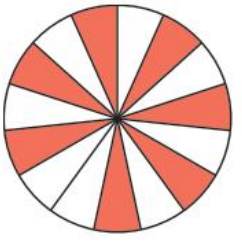
**a**



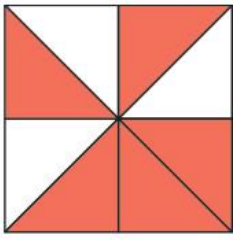
**b**



**c**



**d**



**2** Write each comparison as a ratio of:

**a** the number of red jelly beans compared to the number of yellow jelly beans

**b** the number of orange jelly beans compared to the total number of jelly beans

**c** the number of pink jelly beans compared to the number of yellow jelly beans compared to the number of blue jelly beans

**d** the number of red and blue jelly beans compared to the number of pink and purple jelly beans.

**3** Write each comparison as a ratio in the given order.

**a** 132 cm to 207 cm **b** 16 kg to 35 kg

**c** 18 min to 11 min **d** \$85 to \$121

**4** Write each comparison as a ratio in the given order.

**a** Kayla spent 5 hours on Facebook and 1 hour doing homework.

**b** Australia won three gold, four silver and nine bronze medals in the World Swimming Championships.

**c** Twenty-eight students tried out for the volleyball team while 23 students tried out for the netball team.

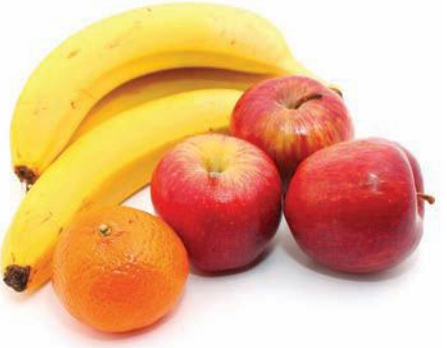
**d** To make a drink of cordial, use one part cordial to four parts water.

3H.2 **5** Write each ratio in its simplest form.

**a** 21 : 6 **b** 12 : 18 **c** 52 : 64 **d** 36 : 96

**e** 88 : 48 **f** 40 : 45 **g** 65 : 13 **h** 22 : 132

**6** A fruit bowl contains apples, oranges and bananas and is shown in the image to the right. There are three bananas, three apples and one orange.



Write each comparison as a ratio in its simplest form.

**a** the number of apples to the number of oranges

**b** the number of apples to the number of bananas

**c** the number of oranges to the number of bananas to the number of apples

**7** Complete the patterns of equivalent ratios.

**a** 3:2
6: 4
15:__

**b** 4:5
__: 25
16:__

**c** __: 210
10:70
2:__

**d** 144:__
24:9
8:__

**e** __: 121
150:__
15:11

**f** 147:__
__: 63
7:9

3H.3 **8** Write each comparison as a ratio in its simplest form.

**a** 42 mm to 7 cm **b** 2 km to 400 m **c** 50c to \$3

**d** 5 weeks to 15 days **e** 500 g to 3.6 kg **f** 335 m to 0.6 km

**g** 55 seconds $3\frac{1}{2}$ minutes **h** 2 h 35 min to 45 minutes

3H.4 **9** Divide the following amounts into the given ratio.

**a** 33 in 5 : 6 **b** 35 in 3 : 2 **c** 70 in 2 : 5 **d** 132 in 2 : 1

**e** 119 in 9 : 8 **f** 195 in 2 : 13 **g** 2304 in 8 : 1 **h** 1554 in 5 : 9

**10** Max scored $\frac{87}{100}$ (or 87 out of 100) for his Geography test.

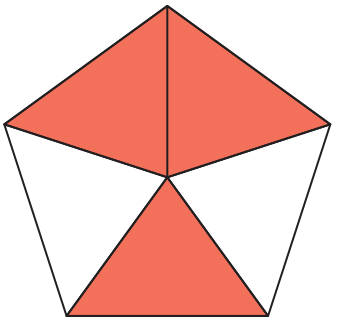
Write a ratio for:

**a** the marks received compared to the total number of marks

**b** the marks lost compared to the total number of marks

**c** the marks received compared to the marks lost.

**11** Look at the pentagon shown.



**a** What is the ratio of shaded sections to non-shaded sections?

**b** What is the ratio of non-shaded sections to the total number of sections?

**c** What is the ratio of shaded sections to the total number of sections?

**d** Can any of the ratios in parts **a–c** be written as fractions? Explain your reasoning.

**e** Can any of the ratios in parts **a–c** *not* be written as fractions? Explain your reasoning.

**f** True or false: Ratios can be written as fractions when comparing the ratio of a part to its whole.

**12** In a bag of 28 red and blue marbles, 16 marbles are red.

**a** How many marbles are blue?

**b** Write the ratio of red to blue marbles in its simplest form.

**13** Jesse and James bought a $13 lottery ticket together. James was a bit short on cash, so Jesse paid $8 and James paid the rest. Against all odds, they won $26 million dollars. They decide to share the money in the same ratio that they paid for the ticket. How much money do Jesse and James each get?

**14** A breakfast cereal mix is made using 8 kg of oats, 4 kg of dried fruit and 12 kg of wheat flakes.

**a** Write the ratio of oats to dried fruit to wheat flakes in its simplest form.

**b** Write each ingredient as a fraction of the whole mix.

**15** Geoff mixes 4 L of blue paint with 5 L of yellow paint to create a shade of green for his studio.

**a** Write the ratio of the amount of blue paint to the amount of green paint.

**b** How much green paint does Geoff have?

**c** What fraction of the new shade is the blue paint?

**d** A year later, Geoff notices marks on the wall that won't wipe off. He decides to repaint that area. If he originally bought a 2 L tin of blue paint and a 6 L tin of yellow paint, how much of the same shade of green paint can Geoff make?

**16** Last week, Shannen completed extra chores around home and earned $32. This week, she earned $14 less. Write as a ratio, in its simplest form, the amount of last week's pocket money to this week's pocket money.

**17** The table shows the attendance at each performance of the local school production.

| Performance | Number of people attending |
|---|---|
| Thursday evening | 486 |
| Friday evening | 549 |
| Saturday matinee | 390 |
| Saturday evening | 520 |

**a** Find the total attendance at the school production over the three days.

**b** Write as a ratio in its simplest form:

**i** the number attending Thursday's performance to the number attending Friday's performance

**ii** the number attending Saturday's matinee performance to the number attending Saturday's evening performance.

**c** If the venue can seat a maximum of 550 people, write as a ratio in its simplest form the total attendance to the total possible attendance over the three days.

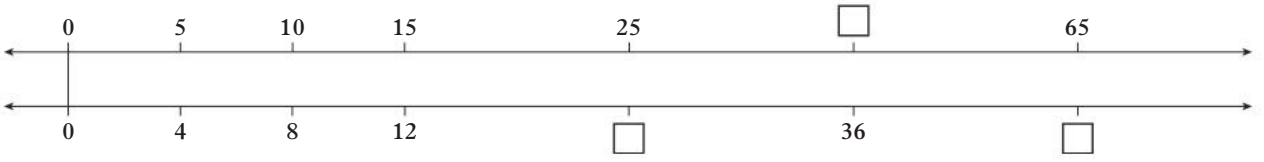
**18** Is each pair of ratios equivalent? Give a reason for your answer.

**a** 7 : 8 and 21 : 24 **b** 20 : 32 and 5 : 8 **c** 63 : 18 and 9 : 2 **d** 30 : 45 and 90 : 60

**19** A two-ingredient dough recipe uses $1\frac{1}{4}$ cups of self-raising flour to 1 cup of Greek yoghurt and makes four servings. How many cups of self-raising flour and Greek yoghurt are needed to make 10 servings? Write your answers using mixed numbers.

**20** The ratio of apple juice to soda water for a fruit punch recipe is 5 : 3. If the recipe requires 750 mL of apple juice, how much soda water is required?

**21** Below is a double number line that is being used to show the relationship between the ratio 5 : 4 and ratios that are equivalent to it.




This can also be shown in a table. See below.

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 5 | 10 | 15 | 25 | | 65 |
| 0 | 4 | 8 | 12 | | 36 | |

**a** Complete the double number line and ratio table.

**b** There are two calculations that produce the equivalent ratios 5 : 4, 10 : 8 and 15 : 12.

We can see that $5 \times 3 = 15$ and $4 \times 3 = 12$, but also $5 + 10 = 15$ and $4 + 8 = 12$ .

Explain why both pairs of calculations move between the equivalent ratios 5 : 4 and 15 : 12.

**22** A bag has a number of green and orange marbles in it. The ratio of green to orange marbles is 5 : 8. When four green marbles are taken out of the bag, the ratio changes to 3 : 5. Find the number of marbles originally in the bag.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Visual representations of ratios

**Interactive skillsheet** Simplifying ratios

**Interactive skillsheet** Dividing a quantity into a ratio

**Investigation** Quilt patterns

**Topic quiz** 3H

# Chapter summary

## Types of fractions

- Proper fractions
  $\frac{3}{5}, \frac{2}{6}, \frac{11}{15}, \frac{3}{3}$
- Improper fractions
  $\frac{13}{6}, \frac{9}{5}, \frac{31}{8}$
- Mixed numbers
  $2\frac{1}{6}, 1\frac{2}{3}, 4\frac{5}{6}$

## Equivalent fractions

$$\frac{2}{3} \overset{\times 5}{\underset{\times 5}{=}} \frac{10}{15} \qquad \frac{12}{10} \overset{\div 2}{\underset{\div 2}{=}} \frac{6}{5}$$

- The HCF of 36 and 72 is 36,

$$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}} = \frac{1}{2}$$

## Improper fractions to mixed numbers

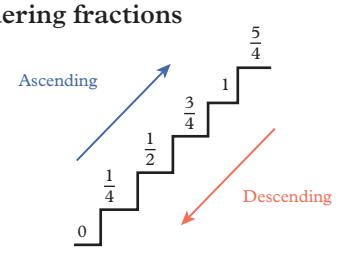
$$\frac{13}{4} = 13 \div 4 = 3 \text{ remainder } 1$$

$$= 3\frac{1}{4}$$

## Mixed numbers to improper fractions

$$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$

## Ordering fractions




- To compare $\frac{1}{2}$ and $\frac{3}{5}$ find the LCD and multiply the numerator and denominator of each fraction to get an equivalent fraction.

## Multiplying fractions

1. Multiply the numerators together.
2. Multiply the denominators together.
3. Simplify the fraction.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

## Reciprocal

$$\frac{3}{4} \quad \frac{4}{3} \qquad 5 = \frac{5}{1} \quad \frac{1}{5}$$

## Dividing fractions

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.
- Keep the first fraction the same, and multiply by the reciprocal of the second fraction.

$$\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$$

## Simplifying ratios

$$120 : 72 \xrightarrow{\div 24} 5 : 3$$

## Add and subtract like fractions

1. Add or subtract the numerators and keep the same the denominator.
2. Simplify the fraction.

$$\frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

## Add and subtract unlike fractions

1. Convert into equivalent fractions with a common denominator.
2. Add or subtract the numerators and keep the same the denominator.
3. Simplify the fraction.

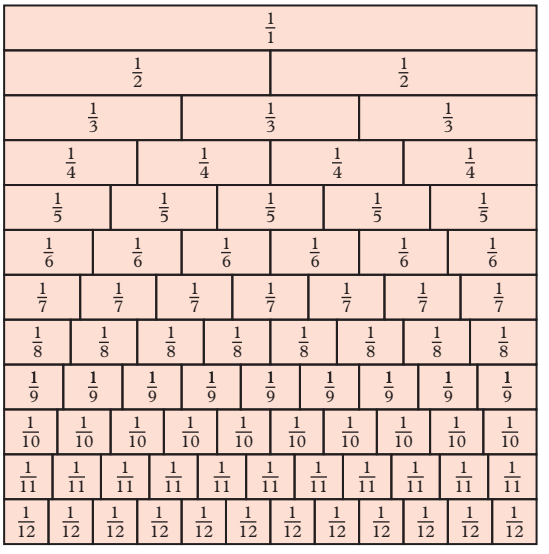
$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$$

## Add and subtract mixed numbers

1. Convert mixed numbers into improper fractions.
2. Convert the improper fractions into equivalent fractions with a common denominator.
3. Add or subtract the numerators and keep the same the denominator.
4. Convert the answer into a mixed number.
5. Simplify the fraction.

$$2\frac{1}{4} + 3\frac{1}{5} = \frac{9}{4} + \frac{16}{5} = \frac{45}{20} + \frac{64}{20} = \frac{109}{20} = 5\frac{9}{20}$$

## Fraction wall





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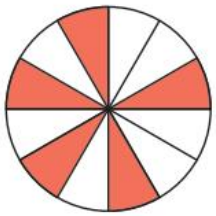
# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

3A **1** What fraction of the shape is coloured red?



**A** 5 **B** $\frac{5}{10}$ **C** $\frac{7}{12}$ **D** $\frac{5}{12}$ **E** 7

3A **2** A worksheet has 29 questions on it. If you have completed 17 of the questions, what fraction of the worksheet is complete?

**A** 17 **B** $\frac{29}{17}$ **C** 29 **D** $\frac{1}{2}$ **E** $\frac{17}{29}$

3B **3** Which of the following is an equivalent fraction to $\frac{2}{5}$?

**A** $\frac{3}{5}$ **B** $\frac{6}{15}$ **C** $\frac{2}{10}$ **D** $\frac{1}{10}$ **E** $\frac{5}{15}$

3B **4** Simplify $\frac{120}{195}$.

**A** 15 **B** $\frac{8}{13}$ **C** $\frac{40}{65}$ **D** $\frac{24}{39}$ **E** $\frac{1}{1.625}$

3C **5** $1\frac{2}{6}, \frac{2}{3}, 1\frac{1}{2}, \frac{5}{12}$ in descending order is

**A** $1\frac{1}{2}, 1\frac{2}{6}, \frac{2}{3}, \frac{5}{12}$ **B** $\frac{5}{12}, \frac{2}{3}, 1\frac{2}{6}, 1\frac{1}{2}$ **C** $1\frac{2}{6}, 1\frac{1}{2}, \frac{5}{12}, \frac{2}{3}$ **D** $\frac{2}{3}, \frac{5}{12}, 1\frac{1}{2}, 1\frac{2}{6}$ **E** $\frac{5}{12}, 1\frac{2}{6}, \frac{2}{3}, 1\frac{1}{2}$

3D **6** Convert $3\frac{4}{7}$ to an improper fraction.

**A** $\frac{7}{7}$ **B** $\frac{4}{10}$ **C** $\frac{16}{7}$ **D** $\frac{25}{7}$ **E** $2\frac{11}{7}$

3E **7** What is $\frac{2}{5} + \frac{1}{3}$ equal to?

**A** $\frac{3}{8}$ **B** $\frac{2}{15}$ **C** $\frac{1}{2}$ **D** $\frac{11}{15}$ **E** $\frac{1}{15}$

3E **8** What is $1\frac{1}{2} + 2\frac{5}{6} - 1\frac{2}{3}$?

**A** $2\frac{2}{3}$ **B** 4 **C** $2\frac{4}{5}$ **D** 6 **E** $\frac{3}{8}$

3E, 3F **9** Which of the following pairs of fractions has a lowest common denominator that is not equal to the product of the denominators?

**A** $\frac{4}{12}, \frac{6}{7}$ **B** $\frac{3}{2}, \frac{2}{3}$ **C** $\frac{5}{8}, \frac{11}{12}$ **D** $\frac{14}{1}, \frac{2}{5}$ **E** $\frac{5}{18}, \frac{9}{35}$

3F **10** What is $\frac{2}{7}$ of $\frac{8}{9}$?

**A** $\frac{16}{63}$ **B** $\frac{10}{16}$ **C** $\frac{6}{2}$ **D** $\frac{18}{56}$ **E** $\frac{74}{63}$

3F **11** What is $\frac{14}{5} \times \frac{3}{21} \times \frac{25}{30}$ in its simplest form?

**A** 3 **B** $\frac{5}{3}$ **C** $\frac{3}{5}$ **D** $\frac{1}{3}$ **E** $\frac{1}{2}$

3G **12** What is $\frac{7}{20} \div \frac{14}{5}$ equal to?

**A** $\frac{1}{2}$ **B** $\frac{1}{8}$ **C** $\frac{7}{15}$ **D** $\frac{21}{25}$ **E** $\frac{49}{50}$

3H **13** Written as a ratio, what is 20 cm to 1 m?

**A** 20 : 1 **B** 20 : 10 **C** 20 : 100 **D** 20 : 1000 **E** 5 : 1

3H **14** What is a mixture of 100 mL cordial to 1.4 L of water written as a ratio in its simplest form?

**A** 100 : 1.4 **B** 100 : 1400 **C** 1 : 14 **D** 101.4 **E** 5 : 70

3H **15** Dario and Barbosa divide \$150 in the ratio 3 : 2. Dario will get

**A** \$5 **B** \$60 **C** \$30 **D** \$90 **E** \$50

## Short answer

3A **1** Mark the following fractions on a number line between the given values.

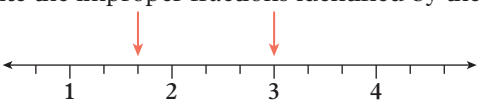
**a** $1\frac{3}{4}$ on a number line from 0 to 2

**b** $\frac{7}{12}$ on a number line from 0 to 1

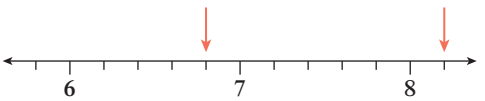
**c** $\frac{16}{3}$ on a number line from 2 to 6

3A **2** Write the improper fractions identified by the arrows.

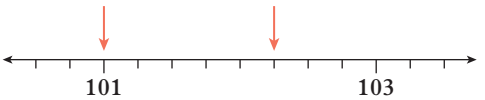
**a**




**b**




**c**




3B **3** For each of the following fractions, write an equivalent fraction using the given information.

**a** $\frac{2}{5}$ with a denominator of 25

**b** $\frac{9}{8}$ with a numerator of 27

**c** $\frac{3}{7}$ with a numerator of 21

**d** $\frac{50}{120}$ with a denominator of 24

3B **4** Simplify each fraction.

**a** $\frac{16}{20}$ **b** $\frac{5}{45}$ **c** $\frac{50}{100}$ **d** $\frac{21}{49}$

3C **5** Place these fractions in ascending order.

$\frac{1}{4}, \frac{2}{7}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}, \frac{11}{20}$

3D **6** Complete this table.

| | Mixed number | Improper fraction |
|---|---|---|
| **a** | $2\frac{3}{8}$ | |
| **b** | | $\frac{11}{2}$ |
| **c** | | $\frac{17}{9}$ |
| **d** | $5\frac{1}{3}$ | |

3E **7** Calculate:

**a** $\frac{1}{9} + \frac{4}{9}$ **b** $\frac{7}{11} - \frac{5}{11}$ **c** $\frac{2}{3} - \frac{1}{2}$ **d** $\frac{3}{10} + \frac{2}{5}$

3E **8** Calculate:

**a** $2\frac{5}{7} + 3\frac{1}{7}$ **b** $2\frac{1}{5} - 1\frac{2}{5}$ **c** $1\frac{3}{8} - \frac{1}{4}$ **d** $1\frac{4}{5} + 2\frac{2}{3}$

3F **9** Calculate:

**a** $\frac{2}{5} \times \frac{4}{7}$ **b** $\frac{3}{10} \times \frac{5}{9}$ **c** $\frac{2}{3} \times \frac{10}{11}$ **d** $\frac{12}{5} \times \frac{4}{9}$

3F **10** Calculate:

**a** $\frac{2}{3} \times 1\frac{4}{5}$ **b** $2\frac{3}{7} \times \frac{1}{5}$ **c** $2\frac{1}{4} \times 3\frac{2}{3}$ **d** $1\frac{3}{8} \times 4\frac{1}{3}$

3G **11** Calculate:

**a** $\frac{3}{4} \div \frac{2}{3}$ **b** $\frac{25}{12} \div \frac{5}{3}$ **c** $\frac{49}{24} \div \frac{7}{16}$ **d** $\frac{1}{4} \div \frac{1}{3}$

3G **12** Calculate:

**a** $3\frac{4}{7} \div 2\frac{1}{7}$ **b** $\frac{8}{9} \div 1\frac{1}{3}$ **c** $3\frac{3}{5} \div \frac{6}{15}$ **d** $2\frac{3}{5} \div 1\frac{9}{15}$

3H **13** Use the photo of the Smarties to write each comparison as a ratio.

**a** the number of pink Smarties to the number of yellow Smarties

**b** the number of yellow Smarties to the number of green Smarties

**c** the number of orange Smarties to the total number of Smarties

**d** the number of orange and pink Smarties to the number of green Smarties

3H **14** Write as ratios in their simplest form.

**a** 24 : 36 **b** 15 : 20

**c** 75c to \$1.20 **d** 8 weeks to 4 days

3H **15** The ratio of snakes to lizards in a reptile zoo is 7 : 5. If there are 40 lizards, how many snakes are there in the zoo?

## Analysis

**1** A survey of 300 families found that 150 have one pet and 100 have two pets.

**a** What fraction of the families have one pet? Write this in its simplest form.

**b** What fraction of the families have two pets? Write this in its simplest form.

**c** Write a ratio for the number of families with one pet compared to those with two pets.

**d** If $\frac{1}{12}$ of the families have three or more pets, how many families is this?

**e** What fraction of the families do not have pets? How many families is this?

**f** A news reporter said that 5 in every 10 people have one pet, and 3 in every 10 people have two pets. Are they correct? Use your knowledge of equivalent fractions to support your answer.

**2** Lili, Camila and Madelaine each bought half a cake in their favourite flavour.

**a** What fraction of a whole cake do they have altogether?

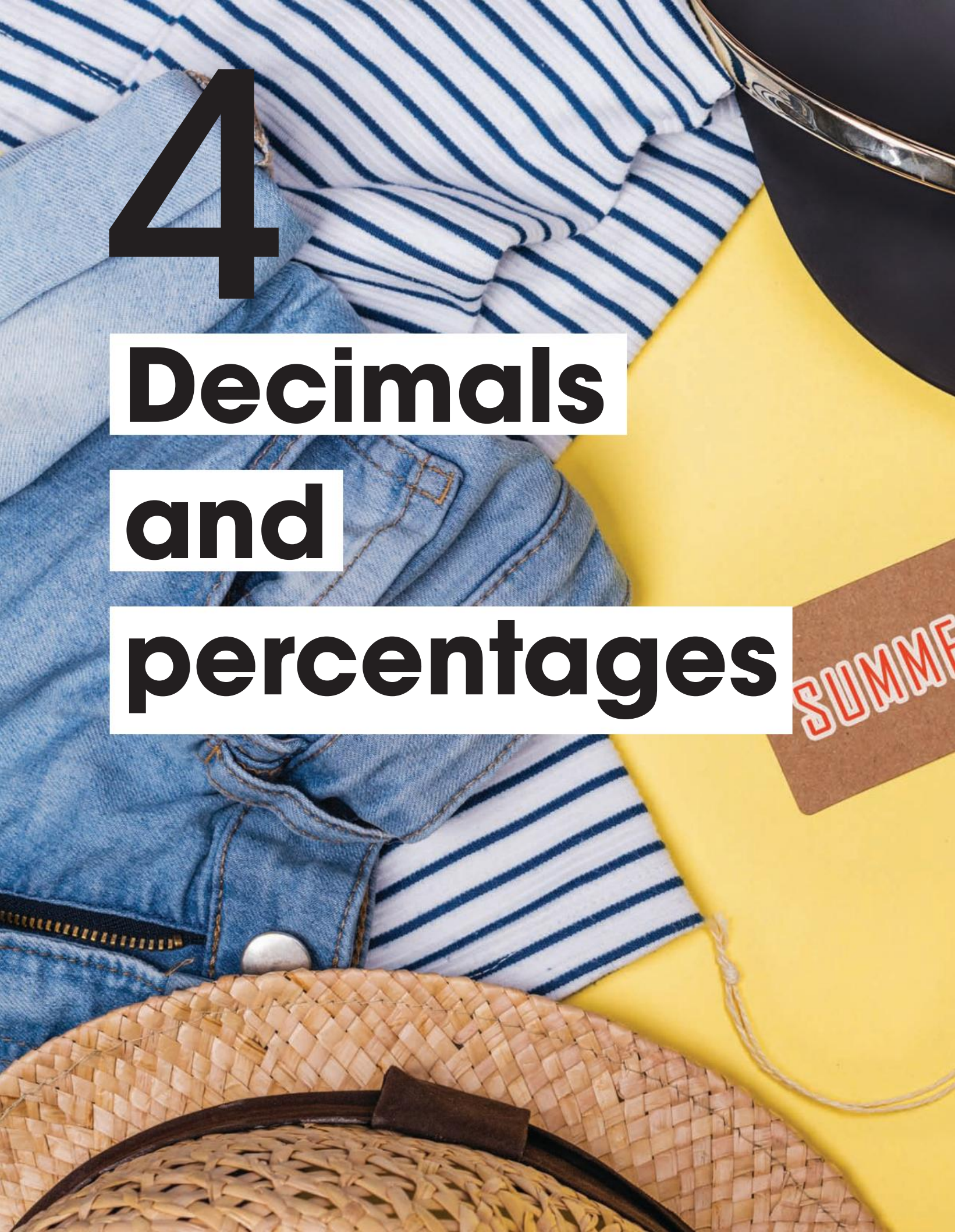
**b** Lili ate $\frac{1}{3}$ of her half-cake, Camila ate $\frac{3}{5}$ of her half-cake, and Madelaine ate $\frac{7}{15}$ of her half-cake. What fraction of a whole cake have they eaten in total?

**c** What fraction of a whole cake do they have left?

**d** What fraction of the amount of cake they started with do Lili, Camila and Madelaine have left?

**e** Their friend Casey says he can eat $\frac{3}{4}$ of the leftover cake. How much of the total cake does Casey think he can eat?

**f** If Casey does eat $\frac{3}{4}$ of the leftover cake and Lili, Camila and Madelaine cannot ever eat more cake than they did in part **b**, who would be able to eat the rest of the cake tomorrow?




# 4 Decimals and percentages

## Index

**4A** Decimals
**4B** Ordering and rounding decimals
**4C** Adding and subtracting decimals
**4D** Multiplying decimals
**4E** Dividing a decimal by a whole number
**4F** Dividing a decimal by a decimal
**4G** Percentages
**4H** Fractions, decimals and percentages
**4I** Calculating percentages

## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Place value
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying whole numbers
- ✔ Dividing whole numbers
- ✔ Equivalent fractions

## Curriculum links

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (VCMNA244)
- Round decimals to a specified number of decimal places (VCMNA246)
- Connect fractions, decimals and percentages and carry out simple conversions (VCMNA247)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (VCMNA248)
- Recognise and solve problems involving simple ratios (VCMNA249)
- Investigate and calculate 'best buys', with and without digital technologies (VCMNA250)

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## Materials

- ✔ Calculator
- ✔ Ruler

# 4A Decimals

## Learning intentions

- ✔ I can identify the place value of a digit in a decimal number.
- ✔ I can convert a decimal to a fraction.
- ✔ I can convert a fraction to a decimal.

**Inter-year links**

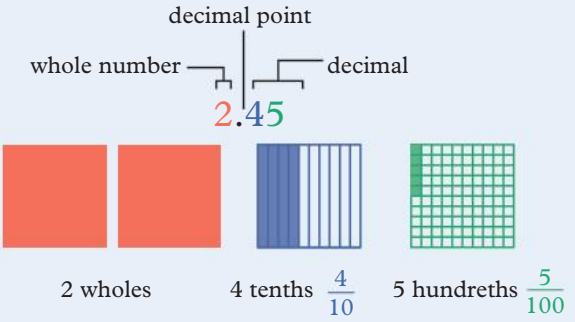
| | |
|---|---|
| Years 5/6 | Fractions and decimals |
| Year 8 | 2D Decimals |

## Decimals

- A **decimal number** is a number with a whole number part and a fraction part that are separated by a **decimal point**. The fraction part of a decimal is a '**decimal fraction**', but we usually just refer to it as a decimal.
- A decimal is a useful way of writing fractions with denominators of 1, 10, 100, 1000, etc.

  For example,

  $\frac{2}{1} = 2$, $\frac{2}{10} = 0.2$, $\frac{2}{100} = 0.02$, $\frac{2}{1000} = 0.002$.




## Value of decimals

- The value of each digit depends on the place or position of the digit in the number.

  For example, the decimal number 12.3456 is shown in the place value table.

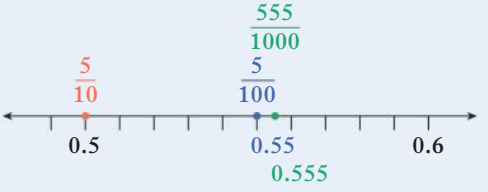
| Tens 10 | Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|
| 1 | 2 | . | 3 | 4 | 5 | 6 |

  → Each place value to the right is 10 times smaller than the place before.

  → Each place value to the left is 10 times larger than the place before.

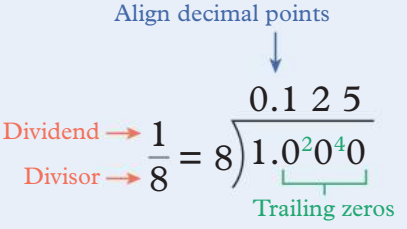
- Understanding the place value of the digit is important when converting between fractions and decimals.

| | | |
|---|---|---|
| 0.5 | 5 tenths | $\frac{5}{10}$ |
| 0.55 | 55 hundredths | $\frac{55}{100}$ |
| 0.555 | 555 thousandths | $\frac{555}{1000}$ |




## Fractions to decimals

- To convert from a fraction to a decimal, **trailing zeros** are added following digits after the decimal point in the dividend until there are repeating digits or there is no remainder. Trailing zeros do not change the value of the number.



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## Example 4A.1 Writing the place value of digits

What is the place value of the digit 9 in each number?

**a** 5.7839 **b** 0.981 **c** 12.693

**THINK**

1 Draw a place value table and place the digits into the chart according to their position relative to the decimal point.

2 Determine which column the number 9 is in to determine its place value.

**WRITE**

| Tens 10 | Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|
| | 5 | . | 7 | 8 | 3 | 9 |
| | 0 | . | 9 | 8 | 1 | |
| 1 | 2 | . | 6 | 9 | 3 | |

**a** Nine ten-thousandths or $\frac{9}{10\,000}$

**b** Nine tenths or $\frac{9}{10}$

**c** Nine hundredths or $\frac{9}{100}$

## Example 4A.2 Writing a fraction as a decimal when the denominator is a power of 10

Write the following fractions as decimals.

**a** $\frac{7}{10}$ **b** $\frac{13}{100}$ **c** $2\frac{81}{100}$

**THINK**

**a** Place the numerator in the place value column indicated by the denominator. There are 7 tenths, so place the 7 in tenths column.

**b** There are 13 hundredths, so place the 1 in the tenths column and the 3 in the hundredths column.

**c** There are 2 ones and 81 hundredths, so place the 2 in the ones column, the 8 in the tenths column and the 1 in the hundredths column.

**WRITE**

| Tens 10 | Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|
| | 0 | . | 7 | | | |
| | 0 | . | 1 | 3 | | |
| | 2 | . | 8 | 1 | | |

**a** $\frac{7}{10} = 0.7$ **b** $\frac{13}{100} = 0.13$ **c** $2\frac{81}{100} = 2.81$

## **Example 4A.3** Writing a decimal as a fraction

Write the following decimals as fractions.

**a** 1.6 **b** 0.42 **c** 12.098

**THINK**

1. Identify any whole numbers to the left of the decimal.
2. Place the decimal number as the numerator of the fraction. Use the place value of the last digit to determine the denominator.
3. Simplify the fraction by finding common factors to the numerator and denominator.

**WRITE**

**a** $1.6 = 1\frac{6}{10}$

$= 1\frac{\cancel{6}^{3}}{\cancel{10}_{5}}$

$= 1\frac{3}{5}$

**b** $0.42 = \frac{42}{100}$

$= \frac{\cancel{42}^{21}}{\cancel{100}_{50}}$

$= \frac{21}{50}$

**c** $12.098 = 12\frac{98}{1000}$

$= 12\frac{\cancel{98}^{49}}{\cancel{1000}_{500}}$

$= 12\frac{49}{500}$

## **Example 4A.4** Writing a fraction as a decimal using short division

Write $\frac{5}{8}$ as a decimal.

**THINK**

1. As the denominator of the fraction is not a power of 10, write the fraction as a division problem with a decimal point after the numerator.
2. Work out the division using short division. Include trailing zeros after the decimal point until you can divide exactly without a remainder. Align the decimal points.

**WRITE**

$\frac{5}{8} = 5 \div 8$

$$\begin{array}{r} 0.6\ 2\ \ 5 \\ 8\overline{)5.0^{2}0^{4}0} \end{array}$$

$\frac{5}{8} = 0.625$

**Helpful hints**

- ✔ Decimals and fractions are often thought of as two separate skills. They are actually two different ways of writing the same thing!
- ✔ If you have trouble determining the place value of a digit, you can draw the place value table to help you.

| Tens 10 | Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|
| 1 | 2 | . | 3 | 4 | 5 | 6 |

ANS p556

# Exercise 4A Decimals

1–6, 7(1st, 2nd columns), 8–10, 11(1st, 2nd columns), 12, 14, 16, 17

1(g–l), 2, 4, 5, 7, 9–12, 13(b–d), 15–19, 20(a, b)

1(i–k), 2(a, b, f), 7, 9–11, 13, 15, 18–21

UNDERSTANDING AND FLUENCY

4A.1 **1** What is the place value of the digit 8 in each number?

| | | | |
|---|---|---|---|
| **a** 28.61 | **b** 304.128 | **c** 927.835 | **d** 16.0058 |
| **e** 84.761 | **f** 62.1852 | **g** 821.793 | **h** 71.085 |
| **i** 0.0018 | **j** 3.8024 | **k** 8236.12 | **l** 183.049 |

**2** Write each of these as a decimal number. Refer to a place value chart to help you.

**a** $5000 + 700 + 80 + 5 + \frac{1}{10} + \frac{9}{100} + \frac{5}{1000}$

**b** $400 + 90 + 6 + 0.2 + 0.08 + 0.003$

**c** 6 thousandths

**d** 5 ones, 7 tenths and 3 thousandths

**e** 3 tens, 6 ones and 7 hundredths

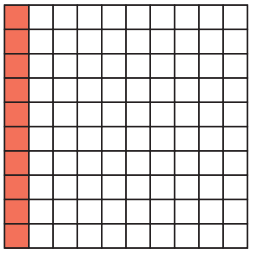
**f** 4 tens, 1 tenth and 7 ten-thousandths

**3** How many decimal places does each decimal have?

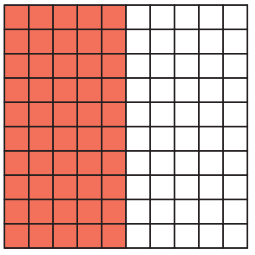
| | | | |
|---|---|---|---|
| **a** 6.214 | **b** 0.388 82 | **c** 16.03 | **d** 198.413 020 |
| **e** 28 | **f** 11.2 | **g** 2.000 102 84 | **h** 53.0 |

**4** Describe the decimal shown in each hundredths grid.

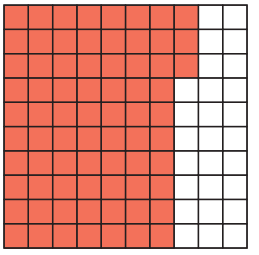
**a**



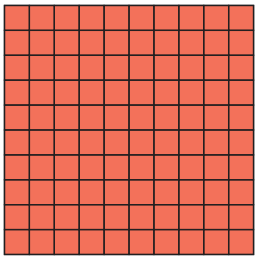
**b**



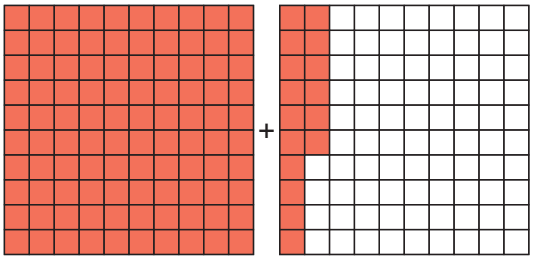
**c**



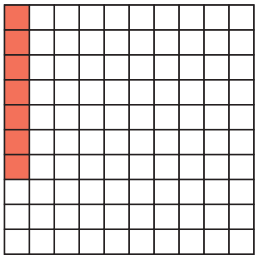
**d**



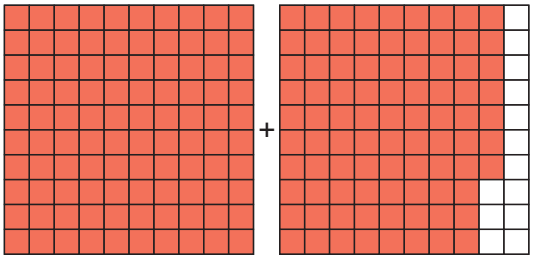
**e**



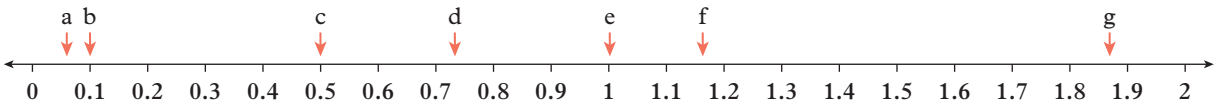
**f**



**g**



**5** Match each decimal from question **4** to the equivalent place on the number line below.




**4A.2** **6** Write each fraction as a decimal. You may like to refer to a place value chart to help you.

**a** $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{33}{100}$ **d** $\frac{531}{1000}$

**e** $\frac{27}{100}$ **f** $\frac{11}{100}$ **g** $\frac{2978}{10\,000}$ **h** $\frac{804}{1000}$

**7** Write each fraction as a decimal.

**a** $\frac{3}{100}$ **b** $\frac{17}{100}$ **c** $\frac{9}{10}$ **d** $\frac{2010}{10\,000}$

**e** $\frac{375}{10\,000}$ **f** $\frac{156}{10\,000}$ **g** $\frac{7}{100}$ **h** $\frac{921}{1000}$

**i** $\frac{42}{1000}$ **j** $\frac{3}{10\,000}$ **k** $\frac{1981}{100\,000}$ **l** $\frac{86}{10\,000}$

**4A.3** **8** Write each decimal as a fraction with a denominator of 100, and then simplify.

**a** 0.25 **b** 0.50 **c** 0.35 **d** 0.88

**e** 0.23 **f** 0.08 **g** 0.12 **h** 0.95

**9** Write each decimal as a fraction in its simplest form.

**a** 0.2 **b** 0.7 **c** 0.13 **d** 0.26

**e** 0.009 **f** 0.426 **g** 0.512 **h** 0.720

**i** 0. 0004 **j** 1.56 **k** 2.375 **l** 3.28

**10** Write each decimal as a mixed number in its simplest form.

**a** 5.83 **b** 3.902 **c** 6.45 **d** 8.35

**e** 2.0008 **f** 3.9 **g** 12.88 **h** 1.706

**4A.4** **11** Write each fraction as a decimal.

**a** $\frac{7}{8}$ **b** $\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $\frac{9}{8}$

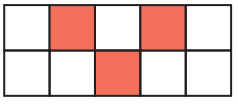
**e** $\frac{57}{4}$ **f** $\frac{15}{8}$ **g** $6\frac{3}{5}$ **h** $7\frac{1}{4}$

**i** $\frac{3}{20}$ **j** $\frac{1}{25}$ **k** $\frac{7}{50}$ **l** $\frac{9}{40}$

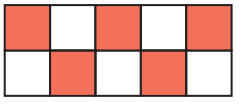
**12** For each rectangle, write the area that is shaded as a:

**i** fraction **ii** decimal.

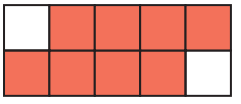
**a**



**b**



**c**



**13** When the denominator of a fraction is a power of 10, the fraction can be written as a decimal without performing division; for example, $\frac{7}{10} = 0.7$. Convert the given fractions into tenths, hundredths or thousandths. Then write each equivalent fraction as a decimal (see example).

For example, $\frac{3}{5} = \frac{3}{5} \times \frac{2}{2}$

$= \frac{6}{10}$

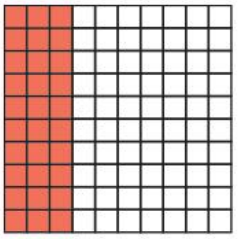
$= 0.6$

**a** $\frac{1}{5}$ **b** $\frac{3}{4}$ **c** $\frac{7}{20}$

**d** $\frac{4}{5}$ **e** $\frac{11}{50}$ **f** $\frac{2}{4}$

**g** $\frac{13}{20}$ **h** $\frac{1}{2}$ **i** $\frac{19}{50}$

**14** Look at the grid shown on the right. Three of the ten columns are shaded. The fraction that has been shaded is $\frac{3}{10}$.



**a** Write the fraction as a decimal.

**b** How many small squares are there in total? How many of these are shaded?

**c** Write the number of squares that are shaded as a fraction.

**d** Explain why your answers for parts **a** and **c** are the same.

**15** Jacey bought some lollipops for a car trip: two raspberry, two apple, one cola, two watermelon and three strawberries and cream.

**a** How many lollipops are there in total?

**b** Write the number of raspberry lollipops as a fraction of the total number of lollipops.

**c** Write your answer to part **b** as a decimal.

**d** Write the number of each of the other lollipops as both a fraction and a decimal of the total number of lollipops.

**16** You are offered 0.3 of this block of chocolate or a piece that is $\frac{1}{4}$ of the block of chocolate. Which portion is larger? Show how you worked this out.

**17** Tim wrote that $\frac{42}{1000} = 0.42$. Can you help explain where he might have gone wrong?

**18** Salwa is making a crocheted doily for a project at school. She has 4.25 m and $1\frac{4}{5}$ m of yarn.

**a** Convert one of the measurements so that they are both in the same form.

**b** How much yarn does Salwa have?

**c** If she needs a total of 6 m of yarn, does she have enough?

**19** Look at the section between 7 cm and 8 cm on your ruler. It is divided into tenths. So each small mark represents $\frac{1}{10}$ of 1 cm or 0.1 cm.

**a** If you magnified the section between 7.3 cm and 7.4 cm to see it divided into tenths, each small mark would represent $\frac{1}{10}$ of $\frac{1}{10}$ cm. What is this as a decimal number?

**b** If you magnified the section between 7.30 cm and 7.31 cm to see it divided into tenths, each small mark would represent $\frac{1}{10}$ of $\frac{1}{100}$ cm. What is this as a decimal number?

**c** If you repeated this process another two times, what would each of the small marks represent?

**20** Write each of the following as a decimal number.

**a** $3400 + \frac{710}{10} + \frac{49}{100}$

**b** $\frac{37}{10000} + \frac{230}{10} + \frac{54}{100}$

**c** $\frac{283}{10} + \frac{1002}{100} + \frac{987}{1000}$

**d** five million ten thousandths

**e** ninety-nine thousand tenths, ninety-nine hundred hundredths, ninety-nine thousandths

**f** twelve hundred forty-seven tenths, nine thousand five hundred and twenty-nine hundredths

**21** Consider this list of fractions: $3\frac{7}{9}$, $3\frac{11}{12}$, $3\frac{8}{10}$, $3\frac{162}{200}$, $3\frac{7}{8}$. Which fraction is closest to 3.82?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Place value with decimals

**Interactive skillsheet** Fractions and decimals

**Investigation** Target race

**Topic quiz** 4A

# 4B Ordering and rounding decimals

## Learning intentions

✔ I can compare and order decimals.
✔ I can round decimals to the appropriate decimal place.

**Inter-year links**

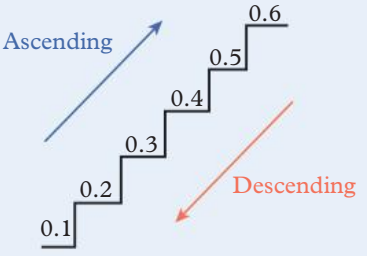
| | |
|---|---|
| Years 5/6 | Ordering decimals |
| Year 8 | 2D Decimals |
| Year 9 | 1A Calculator skills |

## Ordering numbers

- To order decimal numbers, consider the digits from left to right.
  For example, 5.673 and 5.689 have the same whole number and tenth values. 8 hundredths is larger than 7 hundredths so 5.673 < 5.689.

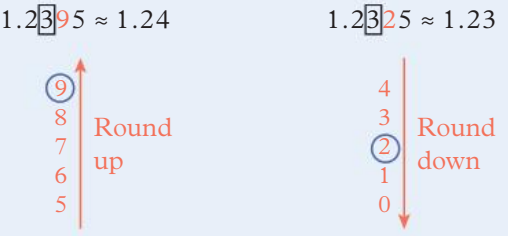
| Ones | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ |
|---|---|---|---|---|
| 5 | . | 6 | 7 | 3 |
| 5 | . | 6 | 8 | 9 |

- Decimals can be represented on the number line.
  → To represent tenths, separate the line into ten parts between two whole numbers.




## Rounding

- Rounding a number involves replacing the number with an approximation that is simpler and easier to use in calculations, but which will also make any calculations less accurate.
  → 'Approximately equal' is shown using the symbol '≈'.
- When rounding 1.2345,
  → to the nearest tenth or one **decimal place**: 1.$\boxed{2}$345 = 1.2
  → to the nearest hundredth or two decimal places: 1.2$\boxed{3}$45 = 1.23
  → to the nearest thousandth to three decimal places: 1.23$\boxed{4}$5 = 1.235




- **Accuracy** describes the closeness of a measurement to a given exact measurement. The higher the degree of accuracy, the closer the measurement is to the given exact measurement.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## **Example 4B.1** Ordering decimals

Arrange the following decimals in ascending order.

3.14, 4.13, 1.34, 3.41

| THINK | WRITE |
|---|---|
| **1** Group the numbers by comparing the whole number parts. There are one 4, two 3s and one 1. | 1.34, 3.14, 4.13<br>3.41 |
| **2** Compare the tenth values of numbers with the same whole number part. 4 tenths are larger than 1 tenth. | 1.34, 3.14, 4.13<br>3.41 |
| **3** Write the decimals in ascending order, smallest to largest. | 1.34, 3.14, 3.41, 4.13 |

## **Example 4B.2** Rounding decimals

Round 0.429 63 to:

**a** two decimal places

**b** three decimal places

**c** four decimal places.

| THINK | WRITE |
|---|---|
| **a 1** Draw a box around the second decimal place. | **a** 0.4$\boxed{2}$963 |
| **2** Look at the digit to the right of the box. The digit to the right is 9. Add 1 to the digit in the box. | 0.4$\boxed{2}$963 |
| **3** Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same. | 0.4$\boxed{3}$<br>0.42963 ≈ 0.43 |
| **b 1** Draw a box around the third decimal place. | **b** 0.42$\boxed{9}$63 |
| **2** Look at the digit to the right of the box. The digit to the right is 6. Add 1 to the digit in the box. | 0.42$\boxed{9}$63 |
| **3** As the boxed digit changes from 9 to 10, write zero in the boxed digit's place and add one to the place to the left. | 0.43$\boxed{0}$ |
| **4** Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same. As three decimal places are requested, keep the trailing zero in the final answer. | 0.42963 ≈ 0.430 |
| **c 1** Draw a box around the fourth decimal place. | **c** 0.429$\boxed{6}$3 |
| **2** Look at the digit to the right of the box. The digit to the right is 3. Do not change the digit in the box. | 0.429$\boxed{6}$3<br>0.429$\boxed{6}$ |
| **3** Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same. | 0.42963 ≈ 0.4296 |

**Helpful hints**

- ✔ You may think that 0.000 19 is greater than 0.2 because there are more digits shown. Remember to compare from left to right. As 0 tenths is smaller than 2 tenths, 0.000 19 < 0.2.
- ✔ Make sure you round to the appropriate decimal places.
  For example, if a chip packet costs \$1 and I have \$1.50, don't tell me that I can buy 1.5 chip packets, I can only buy 1!
- ✔ When asked for a specific number of decimal places, always include them, even if the last digits are trailing zeros.
  For example, 1.002 rounded to two decimal places is 1.00, not 1.

ANS p557

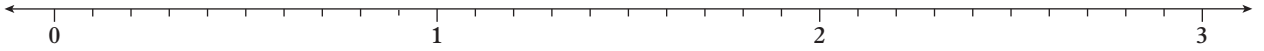
# Exercise 4B Ordering and rounding decimals

△ 1–8, 10–12, 15, 17(a, e) | □ 2, 4–9, 11, 13, 16, 17, 18(a, b) | ⬠ 2(d, f, h), 4, 5, 7–9, 13, 14, 16, 17(a, c, e), 18, 19

UNDERSTANDING AND FLUENCY

**1** Copy the number line below and mark the following numbers on it.

2.5, 1.8, 0.1, 0.8, 1.5, 1.0




**2** Insert < (less than) or > (greater than) between each pair of decimals to make the statement true. Remember to compare each equivalent place value column of each number. Use the place value chart to help you.

**a** 4.73 ___ 3.473 **b** 3.21 ___ 3.24 **c** 9.6841 ___ 9.684 09

**d** 0.017 ___ 0.107 **e** 55.03 ___ 55.003 **f** 12.218 ___ 12.2108

**g** 6.9891 ___ 6.8999 **h** 1378.5 ___ 1378.499 **i** 800.78 ___ 800.778

4B.1 **3** Arrange the following decimals in ascending order.

**a** 9.10, 1.91, 1.90, 9.01

**b** 5.55, 5.5, 5.05, 0.55

**c** 4.567, 45.67, 4.0567, 4.5067

**d** 1, 0.001, 0.100 001, 0.101

**4** Write each list of decimals in ascending order.

**a** 0.641, 0.461, 0.146, 0.328, 0.823, 0.238

**b** 2.341, 2.143, 2.431, 4.132, 1.234, 3.412

**c** 38.162, 36.218, 38.261, 36.812, 36.182, 32.861

**d** 18.4251, 18.2145, 18.4125, 18.1524, 18.4152, 18.2154

**5** Write each list in the order written in brackets.

**a** 0.000 04, 0.0003, 0.000 030, 0.00040, $\frac{2}{10\,000}$, $\frac{21}{100\,000}$ (descending)

**b** 0.81, 0.801, $\frac{81}{10}$, 0.8, 8.0, $\frac{80\,001}{100\,000}$ (ascending)

**c** $\frac{1}{2}$, 0.51, $\frac{501}{100}$, 0.0510, $\frac{1}{5}$, 0.15 (ascending)

**d** $3 + \frac{2}{10} + \frac{6}{100}$, $3 + \frac{25}{100}$, $3 - \frac{25}{100}$, 3.75, 3.026, $3 + \frac{6}{10} + \frac{2}{100}$ (descending)

4B.2 **6** Round 3.546 52 to:

**a** three decimal places **b** four decimal places **c** two decimal places.

**7** Round each of these decimals to the number of decimal places indicated in the brackets.

**a** 82.631 (2) **b** 59.111 78 (3) **c** 0.263 418 (4)
**d** 0.266 66 (2) **e** 15.0003 (3) **f** 2.175 498 7 (6)
**g** 0.218 638 2 (3) **h** 165.545 47 (4) **i** 43.806 251 (5)

**8** Round 168.246 588 to the following.

**a** the nearest hundred **b** the nearest ten **c** the nearest whole number
**d** the nearest tenth **e** the nearest hundredth **f** the nearest thousandth

**9** Draw this number line. Each division on the number line should be 1 cm apart.

**a** Mark the decimal numbers 23.15, 23.3, 23.45, 23.55, 23.7 and 23.95 on the number line.
**b** Which whole number is closest to each of the decimal numbers less than 23.5?
**c** Which whole number is closest to each of the decimal numbers greater than 23.5?
**d** Explain how you would round decimals to the nearest whole number.

**10** During an athletics carnival, Mark threw a javelin the following distances:

22.81 m, 21.98 m, 21.89 m and 22.76 m.

**a** What was Mark's shortest throw?
**b** What was Mark's longest throw?
**c** Explain how you compared the distances to decide which is largest and which is smallest.

**11** Consider the decimal numbers 12.021, 12.201 and 12.210.

**a** Write these numbers in expanded form. You may like to write them in a place value chart first to help you.
**b** What does the zero represent in each number?
**c** What would happen if you wrote each number without its zero? For example, if you wrote 12.021 as 12.21.
**d** Zeros between non-zero digits, such as the zero in 12.021, are placeholder zeros. Why is it important that these zeros are never left out?
**e** Zeros at the end of a decimal (such as the zero in 12.210) are trailing zeros. Does it matter if these are left out? Explain your answer.
**f** Write the three numbers in ascending order (from smallest to largest).

**12** Mia had to round 0.726 53 to three decimal places. Explain why her answer of 0.726 is incorrect.

**13** Another means of removing decimal places is called truncating. To truncate a decimal, we drop all decimal places after a given place value. For example, truncating 0.123 45 after four decimal places is 0.1234.

**a** Truncate the following decimals to the given number of decimal places.
**i** 5.418 52 to one decimal place and two decimal places
**ii** 3.0014 to one decimal place and three decimal places
**iii** 2.999 99 to two decimal places and one decimal place
**iv** 0.003 124 1 to two decimal places and five decimal places.
**b** Round each of the numbers in part **a** to the same number of decimal places.
**c** When does rounding and truncating give the same answer?
**d** When does rounding and truncating give different answers?

PROBLEM SOLVING AND REASONING

**14** State the smallest number of decimal places you can round each pair of numbers to and be able to correctly state which is larger using the rounded numbers.

**a** 0.1234 and 0.1324

**b** 4.3567 and 4.4456

**c** 0.000 234 158 and 0.000 023 415 8

**d** 9.999 99 and 9.999 399

**15** Explain why the following statements are incorrect. Give the correct statement.

**a** $0.0002 > 0.1$

**b** $0.001 < 0.001\,00$

**c** $0.05 = 0.5$

**d** $12.34 < 5.678$

**16** Tyler has given the following reasons to justify the inequalities he has written. Decide whether each inequality is correct and whether his reason is correct.

**a** $\frac{1}{8} > \frac{1}{5}$ as $\frac{1}{8}$ is equal to 0.8 and $\frac{1}{5}$ is equal to 0.5 and $0.8 > 0.5$

**b** $0.34 < 0.6789$ as 34 is less than 6789

**c** $0.04 > 0.0391$ as $\frac{4}{100} > \frac{3.91}{100}$

**17** Consider the following situations. State how the bold number could be rounded and why.

**a** Ronnie's dad got 32 L of petrol at 129.9 cents/L. It cost **$42.2175** and Ronnie's dad paid using a savings card.

**b** Ronnie's mum got 30 L of petrol at 133.4 cents/L. It cost **$40.02** and Ronnie's mum paid using cash.

**c** A hiker has to turn **32.2851°** clockwise and walk 20 m.

**d** A rocket has to turn **32.2851°** clockwise and travel 20 000 km.

**e** The height of a person is **1.592 412 m**.

**f** The length of a bug is **1.592 412 cm**.

CHALLENGE

**18** In Australia most things are measured using the metric system. This means that we can think of smaller units, like m, in terms of larger units, like km, by rounding to the correct number of decimal places.

**a** Round 58.348 cm to the nearest mm. Write your answer in cm.

**b** Round 98.2124 L to the nearest mL. Write your answer in L.

**c** Round 214.192 751 km to the nearest m. Write your answer in km.

**d** Round 214.192 751 km to the nearest cm. Write your answer in km.

**19** State the number of decimal places each of the following is asking to be rounded to.

**a** A number in centimetres rounded to the nearest millimetre.

**b** A number in litres rounded to the nearest millilitre.

**c** A number in kilometres rounded to the nearest metre.

**d** A number in kilometres rounded to the nearest centimetre.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Rounding decimals

**Interactive skillsheet** Ordering decimals

**Investigation** Decimals in the library

**Topic quiz** 4B

# 4C Adding and subtracting decimals

## Learning intentions

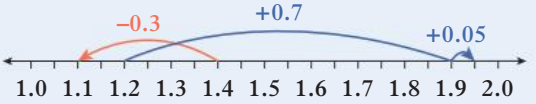
✔ I can add decimal numbers.
✔ I can subtract decimal numbers.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Adding and subtracting decimals |
| Year 8 | 2E Adding and subtracting decimals |

## Adding and subtracting decimals

- Number lines are helpful to visualise the addition and subtraction of decimals. For example, the diagram shows $1.2 + 0.75 = 1.95$ and $1.4 - 0.3 = 1.1$.




- Decimals can be added or subtracted in the same way as whole numbers using the vertical algorithm or using fractions.
- Addition and subtraction of decimals is related to fractions.

| Using fractions | Using the vertical algorithm |
|---|---|
| $3.02 + 4.15 = 3 + \frac{2}{100} + 4 + \frac{15}{100}$<br>$= 7 + \frac{17}{100}$<br>$= 7.17$ | $\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \end{array}$ |

- When adding or subtracting decimals, the digits must be positioned in their correct place value columns and the decimal points must be lined up.
- Blank spaces in the columns may be replaced with additional zeros to assist working out.

For example, 3.2 + 3.002 can be written as $\begin{array}{r} 3.200 \\ +\ 3.002 \\ \hline 3.202 \end{array}$

### Example 4C.1 Using vertical addition to add decimals

Use vertical addition to calculate 13.21 + 3.089.

| THINK | WRITE |
|---|---|
| **1** Arrange the numbers vertically. Ensure the decimal points and the digits are lined up. | $\begin{array}{r} 13.21 \\ +\ 3.089 \\ \hline \end{array}$ |
| **2** Replace blank spaces with additional zeros so that each number contains the same number of decimal places. | $\begin{array}{r} 13.210 \\ +\ 03.089 \\ \hline \end{array}$ |
| **3** Add the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation. | $\begin{array}{r} 13.210 \\ +\ 03.089 \\ \hline 16.299 \end{array}$ |

## Example 4C.2 Using vertical subtraction to subtract decimals

Use vertical subtraction to calculate 29.43 – 16.785.

| THINK | WRITE |
|---|---|
| **1** Arrange the numbers vertically. Ensure the decimal points and the digits are lined up. | $\begin{array}{r} 29.43 \\ -\ 16.785 \\ \hline \end{array}$ |
| **2** Replace any blank spaces with additional zeros so that each number contains the same number of decimal places. | $\begin{array}{r} 29.430 \\ -\ 16.785 \\ \hline \end{array}$ |
| **3** Subtract the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation. | $\begin{array}{r} \overset{8}{\cancel{2}}\overset{13}{\cancel{9}}.\overset{12}{\cancel{4}}\overset{10}{\cancel{3}}\cancel{0} \\ -\ 16.785 \\ \hline 12.645 \end{array}$ |

**Helpful hints**

✔ Make sure that the decimal point in each number is lined up one under the other when adding or subtracting decimals.

For example, $\begin{array}{r} 100000.3 \\ -\ 12.000003 \\ \hline \end{array}$

✔ Remember to consider the place value of the digits in decimal fractions. Otherwise, you could make a mistake like 0.5 + 0.6 = 0.11 instead of 0.5 + 0.6 = 1.1.

ANS p558

# Exercise 4C Adding and subtracting decimals

1–12, 15–17, 20 | 3–10, 13–14, 16, 18, 19, 22 | 5, 8–10, 16, 19–23

UNDERSTANDING AND FLUENCY

**1** For each sum below:

**i** show the expression on a number line **ii** calculate the sum.

**a** 0.5 + 0.3 **b** 1.3 + 0.7 **c** 0.8 + 0.6 **d** 1.2 + 0.6

**2** For each difference below:

**i** show the expression on a number line **ii** calculate the difference.

**a** 0.8 – 0.3 **b** 2 – 0.4 **c** 1.8 – 0.9 **d** 1.1 – 0.5

**3** Estimate the result of each calculation by rounding each number to the nearest 10.

**a** $\begin{array}{r} 38.62 \\ +\ 49.59 \\ \hline \end{array}$ **b** $\begin{array}{r} 132.61 \\ +\ 183.86 \\ \hline \end{array}$ **c** $\begin{array}{r} 16.345 \\ +\ 175.928 \\ \hline \end{array}$ **d** $\begin{array}{r} 54.756 \\ +\ 237.309 \\ \hline \end{array}$

**e** $\begin{array}{r} 9.58 \\ 117.371 \\ +\ 22.65 \\ \hline \\ \hline \end{array}$ **f** $\begin{array}{r} 46.2863 \\ +\ 1152.93 \\ 9 \\ \hline \\ \hline \end{array}$ **g** $\begin{array}{r} 120.146 \\ 11.06 \\ +\ 813.705 \\ \hline \\ \hline \end{array}$ **h** $\begin{array}{r} 8.05 \\ 261.841 \\ +\ 7832.5 \\ \hline \\ \hline \end{array}$

4C.1 **4** Work out each calculation in question **3** and check how the answers compare with your estimates.

**5** Use vertical addition to calculate each of these.

**a** 12.582 + 9.81 + 13.62 **b** 1482 + 37.31 + 0.487

**c** 48.799 + 3.218 + 14 **d** 3826.1 + 0.4326 + 28.571

**e** 0.9453 + 128.62 + 40.1 + 8.3279 **f** 12.063 + 0.004 + 3.904

**6** Estimate the result of each calculation by rounding each number to the nearest 10.

**a** $\begin{array}{r} 79.84 \\ -\ 23.61 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 163.88 \\ -\ 129.16 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 487.6 \\ -\ 92.7 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 33.7 \\ -\ 10.48 \\ \hline \\ \hline \end{array}$

**e** $\begin{array}{r} 1492.6 \\ -\ 15.92 \\ \hline \\ \hline \end{array}$ **f** $\begin{array}{r} 172 \\ -\ 58.73 \\ \hline \\ \hline \end{array}$ **g** $\begin{array}{r} 23.078 \\ -\ 7.49 \\ \hline \\ \hline \end{array}$ **h** $\begin{array}{r} 2031.703 \\ -\ 798.0912 \\ \hline \\ \hline \end{array}$

4C.2 **7** Work out each calculation in question **6** and check how the answers compare with your estimates.

**8** Use vertical subtraction to find each difference.

**a** 172.91 – 48.984 **b** 16.872 – 9.385

**c** 2014.23 – 1998.76 **d** 414.9 – 389.76

**e** 17 – 10.325 **f** 14.008 – 3.67

**9** Check your answers to questions **5** and **8** with a calculator.

**10** Without using a calculator, evaluate each of the following. Remember to follow the order of operations (BIDMAS).

**a** 12 + 629.1 + 34.75 – 392.7

**b** 68.419 – 23.8 + 17.75

**c** 37.941 – 37.899 + 15.371

**d** 648 – (58.63 – 28.21)

**e** 192.62 – (154 + 38.39)

**f** (84.7 + 93.2) – (128.8 – 109.6)



**11** Angelo needs the following lengths of fencing for repairs on his farm: 15.36 m, 28.42 m, 5.8 m and 17.1 m. State the total fencing length Angelo requires.

**12** Shanti bought five items priced at \$3.69, \$14.99, \$7.25, \$10.59 and \$5.99.

**a** Estimate the total cost of the five items.

**b** Use your estimate from part **a** to estimate how much change she should receive from \$50.

**c** Find the sum of the five items and hence the exact amount of change she should receive.

**d** How accurate was your estimate?

**13** Natasha scored 8.3, 9.1, 8.5, 7.9 and 8.8 in her gymnastics competition.

**a** Calculate Natasha's total score.

**b** Monique competed in the same gymnastics competition and received scores of 8.6, 7.5, 9.2, 8.3 and 8.5. Calculate Monique's total score.

**c** Who received the higher score and what was the difference in their totals?

**14** Jonathan, Karamo and Tan competed in the high jump championships, clearing heights of 1.43 m, 1.39 m and 1.38 m, respectively. If the school record for the high jump was 1.52 m, find the difference between each competitor's jump and the record.

**15** Look at the following calculations and describe the error(s) each student made.

**a** $\begin{array}{r} 1\,2.7 \\ +\ 3.4\,1 \\ \hline 4.6\,8 \end{array}$ **b** $\begin{array}{r} 9.8 \\ -\ 4.47 \\ \hline 5.47 \end{array}$ **c** $\begin{array}{r} 6.00 \\ -\ 1.25 \\ \hline 5.75 \end{array}$ **d** $\begin{array}{r} 5.2 \\ -\ 1.7 \\ \hline 3.5 \end{array}$

**16** Performing an estimate of the calculation (usually mentally) before using the algorithm can help avoid careless mistakes where the answer is significantly different to the estimate. For each of the following calculations complete the following.

**i** Write the calculation with the numbers each rounded to their nearest whole number.

**ii** Calculate an estimate using the rounded numbers and state whether your estimate should be higher or lower than the actual answer.

**iii** Use a calculator to determine the actual answer.

**iv** State a reason why someone could get these calculations wrong when doing them by hand.

**a** 9.58 + 1.67 **b** 15.02 – 9.18 **c** 14 – 2.85 **d** 24.658 – 2.984 + 6.182

**17** Matthew is planning to paint four bedrooms over a long weekend. He needs these amounts of paint: 4.2 L, 3.89 L, 2.5 L and 1.9 L.

**a** How many litres of paint will Matthew need?

**b** If the paint comes in 5 L tins, how many tins will he need to buy?

**18** Melbourne's average total monthly rainfall for the month of September is 65.7 mm. In the past year, it rained on five days during the month of September with daily totals of 28.7 mm, 12.8 mm, 1.3 mm, 0.9 mm and 20.98 mm. Comment on how this past year's monthly total compares with the average monthly total for September.

**19** Adrian is a sales rep and spends much of his time on the road travelling from one country town to the next. He is required to record the car's odometer readings at the end of each day, as shown in this log sheet.

| Day | Odometer reading | Distance travelled (km) |
|---|---|---|
| Start | 28 071.5 | 0 |
| Day 1 | 28 258.7 | |
| Day 2 | 28 407.3 | |
| Day 3 | 28 657.1 | |
| Day 4 | 28 801.9 | |
| Day 5 | 29 053.4 | |

**a** Complete the log sheet. You can use your calculator.

**b** What was the odometer reading at the beginning of Adrian's travels?

**c** Which day did Adrian travel the least distance? How far did he travel on this day?

**d** Which day did Adrian travel the greatest distance? How far did he travel on this day?

**e** Find the total distance Adrian travelled in the five days.

**20** When travelling domestically you usually need to make sure that your checked luggage is less than 20 kg. Elio is checking in three bags, weighing 4.5 kg, 13.79 kg and 3.8 kg.

**a** What is the total weight of his checked luggage?

**b** How much weight will he need to remove to stay under 20 kg?

**21** Elisa's monthly bank statement is shown below. An opening balance of $328.56 states how much money is in Elisa's account at the beginning of the month.

| Date | Transaction details | Debit (−) | Credit (+) | Balance |
|---|---|---|---|---|
| 28 Feb | Opening balance | | | 328.56 CR |
| 05 Mar | Personal cheque 531287 | | 125.42 | |
| 08 Mar | FJ Jewellers | 58.99 | | |
| 09 Mar | Athletique Gym Membership | 65.00 | | |
| 16 Mar | ABWDL Smith St | 250.00 | | |
| 23 Mar | Officeworks | 28.53 | | |
| 30 Mar | Interest | | 1.78 | |
| 30 Mar | Monthly fee | 5.00 | | |
| 31 Mar | Closing balance | | | |

**a** Calculate the following.

**i** The balance after each transaction

**ii** The total debits

**iii** The total credits

**iv** The final balance at the end of the month

**b** Elisa wishes to purchase a drone, retailing for $199. Comment on whether she will have enough money in her account at the end of the month.

**22** Calculate 100 – 0.009 245 – 0.524 13 – 0.140 907 – 12.380 718.

**23** An alternative to rounding a decimal number is to truncate the number by dropping all the decimal places after a given place value and not changing the final digit. For example, 12.345 truncated to 2 decimal places is 12.34.

**a** Truncate the following to the number of decimal places in the brackets.

**i** 462.2718 (1)

**ii** 1.2793 (3)

**iii** 0.009149 (2)

**iv** 12.64920 (2)

**b** Estimate 7.21824 + 2.742 + 17.68276 by:

**i** rounding each number to 0 decimal places

**ii** truncating each number to 0 decimal places.

**c** Evaluate 7.21824 + 2.742 + 17.68275. Give your answer:

**i** rounded to 3 decimal places

**ii** truncated to 3 decimal places.

**d** Which estimate is the better estimate of the actual result: the rounded or the truncated estimate?

**e** When will the rounded and truncated estimates be the same?

**f** The sum of 3 numbers is estimated using two different methods; one by rounding each number to 1 decimal place and the other by truncating each number to 1 decimal place. Which estimate, if any, will always be greater than or equal to the other estimate? Explain your answer.

**g** What is the maximum possible difference between the 2 estimates described in part **f**?

**h** What is the maximum possible difference between the sum of 3 numbers rounded to 1 decimal place and the sum of the same 3 numbers truncated to 2 decimal places?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Adding and subtracting decimals

**Investigation** Takeaway decimals

**Topic quiz** 4C

# 4D Multiplying decimals

## Learning intentions

✔ I can multiply decimals.
✔ I can multiply a decimal by a power of 10.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Multiplying decimals |
| Year 8 | 2F Multiplying and dividing decimals |
| Year 9 | 1A Calculator skills |

## Multiplying decimals

- Decimals can be multiplied in the same way as whole numbers.
- The result of the product of two or more decimal numbers has the same number of decimal places as the total number of decimal places in the original problem.
- To multiply decimal numbers:
  1. Count the decimal places in the problem.
  2. Multiply the decimals as whole numbers.
  3. Write the answer with this same number of decimal places. Add zeros where necessary.

$$\begin{array}{r} 2.75 \leftarrow \text{two decimal places} \\ \times 0.03 \leftarrow \text{two decimal places} \end{array} = 4 \text{ decimal places}$$

## Multiplying by powers of 10

- Multiplying a number by 10 increases the place value of each digit by 1 column. Move the decimal point one place value space to the right and insert a zero where necessary. $5.23 \times 10 = 52.3$
- Multiplying a number by 100 increases the place value of each digit by 2 column. Move the decimal point two place value spaces to the right and insert a zero where necessary. $5.23 \times 100 = 523.0$
- Multiplying a number by 1000 increases the place value of each digit by 3 column. Move the decimal point three place value spaces to the right and insert a zero where necessary. $5.23 \times 1000 = 5230.0$

### Example 4D.1 Multiplying decimals

Find each product.

**a** $0.7 \times 0.03$ **b** $0.12 \times 6$

**THINK**

1. Count the decimal places in the problem.
2. Multiply the decimals as whole numbers.
3. Write the answer with this same number of decimal places. Add zeros where necessary.

**WRITE**

**a** $0.7 \times 0.03$ There are three decimal places in total.
$7 \times 3 = 21$
$0.021$
$0.7 \times 0.03 = 0.021$

**b** $0.12 \times 6$ There are two decimal places in total.
$12 \times 6 = 72$
$0.72$
$0.12 \times 6 = 0.72$

## **Example 4D.2** Multiplying decimals using long multiplication

Use long multiplication to evaluate 18.6 × 4.9.

**THINK**

1. Estimate the result by rounding to the nearest whole number.
2. Count the decimal places in the problem. There are two decimal places in total.
3. Multiply the decimals as whole numbers using long multiplication.
4. Write the answer with this same number of decimal places. Add zeros where necessary.
5. Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.

**WRITE**

18.6 × 4.9 ≈ 19 × 5
= 95

18.6 × 4.9

3 2
7 5

```
   186
×   49
  1674
+ 7440
  9114
```

91.14

18.6 × 4.9 = 91.14

## **Example 4D.3** Multiplying by 10, 100 or 1000

Find each product.

**a** 6.245 × 10 **b** 2.27 × 100 **c** 15.3 × 1000

**THINK**

**a** When multiplying by 10, move the decimal point one place value space to the right.

**b** When multiplying by 100, move the decimal point two place value spaces to the right. Don't include the decimal point if there are no digits other than zero after it.

**c** When multiplying by 1000, move the decimal point three place value spaces to the right. Add zeros where necessary.

**WRITE**

**a** 6.245
6.245 × 10 = 62.45

**b** 2.27
2.27 × 100 = 227

**c** 15.300
15.3 × 1000 = 15 300

**Helpful hints**

✔ When multiplying by a power of ten, move the decimal point to the **right**.

✔ Don't try to line up the decimal point in the algorithm for multiplying. It won't give the correct result. Instead, remember to count the decimal places and use the total in the answer.
0.1 2 3 × 0.4 5 = Answer has 5 decimal places

✔ Multiplying with decimals may not make a number greater! The product may be greater than both factors, between the factors, or smaller than both factors.

ANS p559

# Exercise 4D Multiplying decimals

1–3, 4–5(1st, 2nd columns), 6–11, 12(a, b, e), 13(a, b), 17

2, 4–5(2nd, 3rd columns), 6, 7(g–l), 8, 9(a, b), 12, 13, 14(a, b), 15, 18

2(g–i), 5, 7(3rd column), 8, 9, 12–14, 16, 18–20

UNDERSTANDING AND FLUENCY

**1** How many decimal places will there be in the answer to each calculation?

**a** $1.56 \times 2.489$ **b** $0.4896 \times 1.356$ **c** $12.364\,29 \times 3.8$

**d** $15.6308 \times 46.75$ **e** $0.0056 \times 46.21$ **f** $0.167 \times 0.007$

4D.1 **2** Use your knowledge of times tables and decimal places to find each product.

**a** $0.2 \times 0.6$ **b** $0.3 \times 0.8$ **c** $0.6 \times 0.7$

**d** $0.08 \times 0.9$ **e** $0.004 \times 0.9$ **f** $0.7 \times 0.7$

**g** $1.2 \times 0.8$ **h** $0.006 \times 0.04$ **i** $0.0001 \times 0.007$

**3** Find each product.

**a** $3.8 \times 7$ **b** $9.8 \times 4$ **c** $0.63 \times 5$

**d** $1.14 \times 8$ **e** $4.0008 \times 9$ **f** $3.28 \times 6$

**g** $2.759 \times 3$ **h** $51.4 \times 7$ **i** $19.86 \times 4$

4D.2 **4** Use long multiplication to evaluate the following.

**a** $24.7 \times 2.8$ **b** $63.4 \times 7.2$ **c** $128.4 \times 9.8$

**d** $13.64 \times 2.1$ **e** $15.78 \times 8.3$ **f** $148.42 \times 6.5$

**g** $298.61 \times 0.16$ **h** $13.213 \times 1.84$ **i** $52.475 \times 42.9$

**5** Evaluate without using a calculator.

**a** $1.37 \times 24$ **b** $2.41 \times 36$ **c** $13.21 \times 15$

**d** $8.64 \times 72$ **e** $92.84 \times 0.002$ **f** $186.71 \times 0.04$

**g** $0.05 \times 248$ **h** $0.006 \times 617$ **i** $20.03 \times 52$

**6** Check your answers to questions **4** and **5** with a calculator.

4D.3 **7** Find each product.

**a** $6 \times 1000$ **b** $42 \times 100$ **c** $0.1284 \times 10$

**d** $15.721 \times 100$ **e** $0.0004 \times 1000$ **f** $0.0702 \times 100$

**g** $6.1 \times 10$ **h** $15.0 \times 10$ **i** $0.31 \times 1000$

**j** $64.813 \times 100$ **k** $4.8 \times 100\,000$ **l** $128.468 \times 1\,000\,000$

**8** Multiplying by 60 is the same as multiplying by 6 and then multiplying by 10, as shown in the example.

For example, $4.86 \times 60 = 4.86 \times 6 \times 10$
$= 29.16 \times 10$
$= 291.6$

Use this strategy to determine each product.

**a** $12.98 \times 700$ **b** $37.2 \times 8000$ **c** $43.62 \times 2400$

**d** $106.2 \times 320$ **e** $6.5 \times 15\,000$ **f** $0.27 \times 45\,000$

**9** Evaluate each of the following without using a calculator.

**a** $(2.387 + 4.21 - 2.7) \times 3.1$

**b** $6.85 \times 7.2 - (4.5 + 8.2)$

**c** $14.85 - (6.57 + 12.35) \times 0.1$

**d** $62.87 \times 0.002 + 17.8 \times 0.02$

**10** Look at the calculations in the first column of this table.

| Original calculation | Estimated answer | Answer |
|---|---|---|
| 38 × 21 | 40 × 20 = 800 | 798 |
| 3.8 × 21 | | |
| 38 × 2.1 | | |
| 3.8 × 2.1 | | |
| 380 × 210 | | |

**a** What do the calculations have in common?

**b** How are these calculations different?

**c** Complete the second column by estimating the answer. Round each number to its leading digit (first non-zero digit). The first one has been done for you.

**d** Complete the third column by using the digits 798 and a decimal point to create a number close to the estimated answer obtained in part **c**.

**e** What is the relationship between the total number of decimal places in the original calculation and the number of decimal places in the answer?

**11** Barry purchases 58.7 L of unleaded petrol at a cost of $1.32 per litre.

**a** Estimate the total cost of the petrol.

**b** Calculate the total cost of the petrol rounded to two decimal places.

**c** Find the difference between the estimated cost and the actual cost of the petrol purchased.

**12** Calculate the cost of each of these items, rounding your answers to two decimal places.

**a** 0.75 kg of cherries at $12.99 per kg

**b** 1.56 kg of Granny Smith apples at $3.90 per kg

**c** 2.78 kg of bananas at $3.50 per kg

**d** 1.2 kg of grapes at $6.90 per kg

**e** Three punnets of strawberries at $2.99 per punnet

**f** 2.89 kg oranges at $3.49 per kg

**13** For each of the following calculations:

**i** estimate the product by rounding both numbers to the leading digit

**ii** calculate the actual product using a calculator

**iii** find the difference between the estimation and the actual product.

**a** 6.1295 × 9.1463

**b** 7561.2 × 256.94

**c** 0.004 127 × 1295.291

**d** 0.850 005 × 0.006 24

**14** We can see where to place the decimal point in a decimal multiplication problem by writing the calculation using fractions.

For example, $12.3 \times 0.02 = \frac{123}{10} \times \frac{2}{100}$

$= \frac{246}{1000}$

$= 0.246$

Write the following calculation as fractions before multiplying to determine the products as decimals.

**a** 0.07 × 0.003

**b** 200.3 × 0.005

**c** 1.74 × 6.21

**d** 9.99 × 2.21

PROBLEM SOLVING AND REASONING

**15** The local gym offers its clients the following payment plans for a yearly membership.

Option 1: one payment of $645

Option 2: monthly instalments of $59.45

**a** Determine how much option 2 costs for a year.

**b** Which option is the cheapest?

**c** How much money is saved if the cheaper option is chosen?

**16 a** Calculate the value of each of the following.

**i** $0.2^2$ **ii** $0.4^2$ **iii** $0.25^2$ **iv** $0.008^2$ **v** $1.2^2$

**b** Comment on the number of decimal places in the numbers to be squared and the number of decimal places in the answer. What interesting point do you observe?

**17** The magnification on a microscope is 1000×. This means that objects appear 1000 times larger than they actually are. How wide would an insect's eye that is 0.008 mm in diameter appear when viewed through this microscope?

**18** Colin is a plumber. He charges an initial fee of $120 and an additional $36.80 per hour.

**a** The Grant family needs a pipe to be unblocked. If this job takes Colin a total of 3.75 hours, how much will he charge?

**b** The Torcasios family requires a new hot water system to be installed. If this job takes Colin a total of 0.75 hours, how much will he charge?

CHALLENGE

**19** The grid method can also be used for multiplying decimal numbers.

For example, $5.76 \times 21.823 = (5 + 0.7 + 0.06) \times (20 + 1 + 0.8 + 0.02 + 0.03)$.

| × | **20** | **1** | **0.8** | **0.02** | **0.003** |
|---|---|---|---|---|---|
| **5** | 100 | 5 | 4.0 | 0.10 | 0.015 |
| **0.7** | 14 | 0.7 | 0.56 | 0.014 | 0.0021 |
| **0.06** | 1.2 | 0.06 | 0.048 | 0.0012 | 0.00018 |
| **Total** | 115.2 | 5.76 | 4.608 | 0.1152 | 0.01728 |

Therefore, $5.76 \times 21.823 = 115.2 + 5.76 + 4.608 + 0.1152 + 0.01728 = 125.70048$

Use the grid method to calculate the following products.

**a** $8.12 \times 0.45$ **b** $1.285 \times 8.21$ **c** $3.127 \times 19.21$

**20** Use the digits 0–9 only once to write a product as close to 7.9 as possible.

□□.□□□ × □.□□□□

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Multiplying decimals

**Investigation** Burning joules

**Topic quiz** 4D

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

4A **1** What is the place value of the digit 5 in each number?

**a** 81.475 **b** 9.5024 **c** 6.3915 **d** 12.0561

4A **2** Write the following decimals as fractions in its simplest form.

**a** 0.17 **b** 5.3 **c** 0.15 **d** 9.125

4A **3** Write the following fractions as decimals.

**a** $\frac{71}{100}$ **b** $\frac{12}{5}$ **c** $\frac{5}{8}$ **d** $11\frac{7}{20}$

4B **4** Complete the statements by filling in the missing symbol: >, =, or <.

**a** 1.3 ____ 0.6 **b** 0.1 ____ 0.10 **c** 0.07 _____ 0.5 **d** 31.2729 _____ 31.8

4B **5** Arrange the following in the order written in brackets.

**a** 17.80, 1.708, 1.087, 7.108, 7.180 (ascending)
**b** 1.05689, 15.689, 15.986, 1.5689, 1.5698 (descending)
**c** 0.7, 0.08, $\frac{9}{10}$, $\frac{18}{100}$, 0.20 (ascending)
**d** 1.4, $\frac{3}{2}$, $\frac{34}{20}$, $\frac{11}{5}$, 1.80 (descending)

4B **6** Round the following numbers to the decimal places written in brackets.

**a** 6.472 912 (2 decimal places)
**b** 32.1825 (3 decimal places)
**c** 0.003 047 2 (3 decimal places)
**d** 99.987 192 (1 decimal place)

4C **7** Calculate the following.

**a** 2.1 + 3.8 **b** 8.7 – 6.5
**c** 5.28 + 7.99 **d** 15.12 – 8.54

4C **8** Evaluate the following calculations.

**a** 47.91 + 38.27 **b** 203.4 + 87.49
**c** 83.47 – 21.36 **d** 421.8 – 39.72

4D **9** Calculate the following products.

**a** 5.91 × 10 **b** 0.00412 × 100
**c** 0.124 × 1000 **d** 52.82 × 0.1

4D **10** Calculate the following products.

**a** 0.06 × 0.009 **b** 2.3 × 0.8
**c** 19.3 × 0.45 **d** 20.8 × 6.6

4D **11** Estimate each of the calculations by rounding each number to the nearest whole number.

**a** 3.48 – 1.28 + 9.19 **b** 9.14 + 10.4 × 0.6
**c** 37.1 × (15.2 + 4.8) **d** (17.7 – 8.9) × (0.05 + 0.18)

4D **12** Evaluate each of the calculations in question **11**.

# 4E Dividing a decimal by a whole number

## Learning intentions

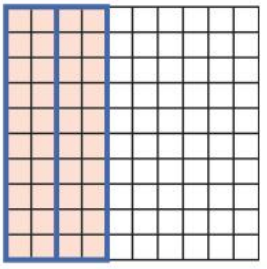
- ✔ I can divide a decimal by a whole number.
- ✔ I can divide a decimal by a power of 10.
- ✔ I can use the unitary method to solve problems.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Dividing a decimal by a whole number |
| Year 8 | 2F Multiplying and dividing decimals |

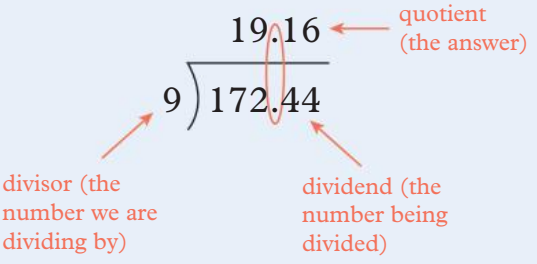
## Dividing decimals

- 0.4 ÷ 2 is asking: 'How large are the groups when 4 tenths is divided into 2 equal groups?'



0.4 is shown on the hundreds square.
0.4 is grouped into 2 groups of 20 hundredths.
So, there are 2 tenths or 0.2 in each group.
0.4 ÷ 2 = 0.2

- When dividing a decimal by a whole number, use the same method used for dividing whole numbers: short division.
- Align the decimal point in the answer with the dividend.




- Trailing zeros are added following digits after the decimal point in the dividend until there is no remainder.

## Dividing by powers of 10

- Dividing a number by 10 decreases the place value of each digit by 1 column. Move the decimal point one place value space to the left and insert a zero where necessary.

$$5.23 \times 10 = 52.3$$

- Dividing a number by 100 decreases the place value of each digit by 2 columns. Move the decimal point two place value spaces to the left and insert a zero where necessary.

$$5.23 \times 100 = 523.0$$

- Dividing a number by 1000 decreases the place value of each digit by 3 columns. Move the decimal point three place value spaces to the left and insert a zero where necessary.

$$5.23 \times 1000 = 5230.0$$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# Rates

A **rate** is a comparison between two or more different quantities. It is a measure of how a quantity changes for each unit of another quantity.

For example, $1.20 per 1.5 kg of flour is a rate.

## Example 4E.1 Dividing a decimal by a whole number

Evaluate 16.5 ÷ 6.

**THINK**

1. Evaluate the division in the same way as for whole numbers, using short division.
2. Align the decimal point and the digits that have the same place value.
3. Add trailing zeros after the last digit in the dividend to obtain an exact decimal answer without a remainder.

**WRITE**

$$\begin{array}{r} 2.\ 7\ 5 \\ 6\overline{)16.^{4}5^{3}0} \end{array}$$

16.5 ÷ 6 = 2.75

## Example 4E.2 Dividing by 10, 100 or 1000

Evaluate each quotient.

**a** 12.5 ÷ 10 **b** 20.08 ÷ 100 **c** 3.14 ÷ 1000

**THINK**

**a** When dividing by 10, move the decimal point one place value space to the left.

**b** When dividing by 100, move the decimal point two place value spaces to the left. Include trailing zeros where necessary.

**c** When dividing by 1000, move the decimal point three place value spaces to the left. Include trailing zeros where necessary.

**WRITE**

**a** 12.5
12.5 ÷ 10 = 1.25

**b** 20.08
20.08 ÷ 100 = 0.2008

**c** 003.14
3.14 ÷ 1000 = 0.00314

**Helpful hints**

✔ When dividing by a power of ten, move the decimal point to the **left**.

✔ When adding trailing zeros after the decimal, you are not changing the number. You are just writing a zero that was already implicitly there.

ANS p560

# Exercise 4E Dividing a decimal by a whole number

1(1st column), 2, 3–4(1st column), 5–8, 10, 11, 14, 17, 18 | 2, 3–4(2nd column), 5, 6, 9, 12–15, 19–21 | 2(g–l), 3–4(2nd column), 6, 13, 14, 16, 20, 22–24

UNDERSTANDING AND FLUENCY

**1** Evaluate each of the following using short division.

**a** $4\overline{)24.92}$ **b** $5\overline{)68.605}$ **c** $8\overline{)72.816}$
**d** $7\overline{)161.98}$ **e** $9\overline{)100.035}$ **f** $3\overline{)4.9194}$
**g** $9\overline{)26.2953}$ **h** $5\overline{)20.75}$ **i** $7\overline{)24.57}$
**j** $6\overline{)0.1536}$ **k** $4\overline{)0.4832}$ **l** $2\overline{)1.45608}$

4E.1 **2** Perform each division using short division. Remember to use trailing zeros if needed.

**a** $15 \div 6$ **b** $27 \div 4$ **c** $63 \div 5$
**d** $17.4 \div 4$ **e** $12.3 \div 6$ **f** $2.031 \div 5$
**g** $4.83 \div 8$ **h** $234.3 \div 6$ **i** $56.23 \div 4$
**j** $9.55 \div 8$ **k** $6.87 \div 6$ **l** $25.9 \div 8$

**3** Calculate the cost of one item in each case.

**a** Seven exercise books cost \$8.96 **b** Four tyres cost \$870
**c** 12 pens cost \$15.36 **d** 10 icy-poles cost \$7.50
**e** 15 gym sessions cost \$177 **f** Eight movie passes for \$58
**g** Five chocolates cost \$6.50 **h** Five soft drinks cost \$9

**4** Calculate the cost of 1 kg in each case.

**a** 6 kg of oranges cost \$23.28 **b** 5 kg of nectarines cost \$24.90
**c** 3 kg of mangos cost \$11.94 **d** 6 kg of scotch fillet costs \$71.82
**e** 2 kg of capsicums cost \$13.96 **f** 4 kg of apples cost \$31.92
**g** 12 kg firewood cost \$20.40 **h** 7 kg chicken wings for \$29.89

4E.2 **5** Evaluate each quotient.

**a** $678.543 \div 1000$ **b** $953 \div 100$ **c** $288.94 \div 10$
**d** $15.721 \div 100$ **e** $293.4 \div 1000$ **f** $70.2 \div 100$
**g** $68\,942.1 \div 1000$ **h** $1509.5 \div 10\,000$ **i** $4.36 \div 100$

**6** Dividing by 900 is the same as dividing by 9 and then dividing by 100.

For example, $5342.4 \div 900 = (5342.4 \div 9) \div 100$
$= 593.6 \div 100$
$= 5.936$

Use this strategy to calculate the following.

**a** $393.946 \div 700$ **b** $231\,120 \div 200$
**c** $16\,790.4 \div 30$ **d** $1248.9 \div 500$
**e** $307\,152 \div 5000$ **f** $8192.5 \div 200$
**g** $937\,800 \div 10\,000$ **h** $5620.23 \div 600$

**7** A group of Year 7 students decides to divide the $558.75 they have raised throughout the year equally between three charities. How much will each charity receive?

**8** Eight tickets to an indie music concert cost a total of $1087.20. What was the cost of each ticket?

**9** Four friends agree to contribute equally to buying a birthday present for their friend Winston. If the gift is $98.60, how much should each person pay?

**10** Milk can be bought in different sizes. At a supermarket, a 1 L container of milk costs $2.97 and a 2 L container costs $4.98.

**a** Calculate the cost for 1 L if you buy the 2 L container of milk.

**b** Compare this cost to the cost of the 1 L container of milk. Which option is the best buy? Explain why it is better value.

**11** Pizza can be bought by the slice or as a whole pizza that is equivalent to six slices. Compare the cost per slice. Which option is better value and by how much?

**12** Lorena has the option of buying a weekly travel ticket for $13.75 or a daily ticket costing $2.95.

**a** How much does her weekly ticket cost per day if she uses it five days a week?

**b** How does the price of a daily ticket compare with that of a weekly ticket? Which is the better option?

**c** Are there occasions when buying daily tickets would be better than buying a weekly ticket? Explain.

**13** Customers can buy single eggs or a carton of a dozen eggs at a health food store.

**a** Compare the cost per egg in each case. Which option is better value?

**b** How much would customers save per egg if they bought the better value option?

**c** Why do you think the store offers both options?

**14** Calculate the cost of 100 g in each case. Where appropriate, round to the nearest cent.

**a** 300 g of leg ham costs $11.19

**b** 500 g of cheese costs $7.80

**c** 900 g of green beans costs $4.76

**d** 200 g of aniseed rings cost $2.99

**15** Simon bought a 500 g pack of tomatoes for $3.98. Julia bought three tomatoes on the right, which weighed a total of 300 g.

**a** Calculate the amount paid by each person for 100 g of tomatoes.

**b** Who made the best buy?

**16** A recipe requires 800 g of pasta sauce. Katie considers two options:

| | |
|---|---|
| Firenze pasta sauce | 400 g jar for $3.73 |
| Milano pasta sauce | 500 g jar for $4.09 |

**a** Compare the value of 100 g of each product.

**b** How many jars should Katie buy for her recipe? Which brand of sauce should Katie buy if she wants value for money? Explain.

PROBLEM SOLVING AND REASONING

**17** The first division prize of $6 428 212 in Lotto is to be shared among 18 winners.

**a** How much will each winner receive exactly?

**b** Explain how you were able to calculate this.

**18** A barrel of 100 L of cordial has been made to share among 180 students. State whether each student would receive more or less than 1 L. Explain your reasoning.

**19** Finn has the option of three different summer jobs: picking grapes, delivering pamphlets or packing boxes. He tries each of them out. He earns $51.20 picking grapes for 8 hours, $46.40 delivering pamphlets for 6 hours, and $32.50 packing boxes for 4 hours. Which job pays the best by the hour?

**20** The unitary method is a useful technique for solving problems by first finding the value of a single item, and then multiplying to find the value of a group of items.

Roland would like to buy 10 mangos. His friend Brianna bought three mangos for $6.30.

**a** How much did Brianna pay for one mango?

**b** Use your answer to part **a** to find how much Roland should pay for 10 mangos.

**c** Explain how this problem uses both division and multiplication.

**21** Work out the cost of 10 avocados in each situation.

**a** 8 avocados cost $5.60

**b** 16 avocados cost $9.60

**c** 32 avocados cost $20.80

**d** 16 avocados cost $13.60

**22** Work out each of these using a calculator. Where appropriate, round to the nearest cent.

**a** 24 scones cost $29.76. Calculate the cost of 10 scones.

**b** 15 kiwi fruits cost $31.80. Calculate the cost of 12 kiwi fruits.

**c** 150 g of salami costs $5.60. Calculate the cost of 100 g of salami.

**d** 375 g of sugar costs $2.56. Calculate the cost of 100 g of sugar.

**23** A 175 g bag of crinkle cut chips can be bought for $4.45. Tubes of Surfwave chips come in two sizes: 150 g for $4.27 and 190 g for $5.34. Blue Stone chips come in 185 g bags for $4.16.

**a** Use a calculator to work out the value of 100 g of each product.

**b** Which is the best buy?

**c** What other factors could influence what you buy?

CHALLENGE

**24** Ava walks 8 km in 1 hour and 19 minutes, Caleb walks 6 km in 1 hour and 7 minutes, and Isaac walks 10 km in 1 hour 44 minutes.

**a** Who is the fastest walker?

**b** What is this person's pace in kilometres per hour?

Check your Student obook pro for these digital resources and more:

**Worksheet** Dividing decimals by a whole number

**Investigation** Best buys – pasta sauce

**Topic quiz** 4E

# 4F Dividing a decimal by a decimal

## Learning intentions

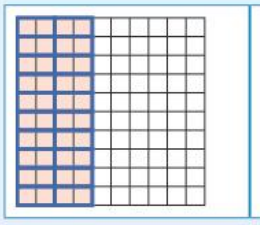
✔ I can divide a decimal by a decimal.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Dividing a decimal by a whole number |
| Year 8 | 2F Multiplying and dividing decimals |

## Dividing a decimal by a decimal

- 0.4 ÷ 0.02 is asking: 'How many 2 hundredths are in 4 tenths?'



0.4 is shown on the hundreds square.
0.4 is grouped in to 20 groups of 2 hundredths.
So, there are 20 groups of 2 hundredths in 4 tenths.
0.4 ÷ 0.02 = 20

- When you divide by a decimal, you can create an equivalent problem by moving the decimal point to the right or multiplying by $\frac{10}{10}, \frac{100}{100}, \frac{1000}{1000}, \ldots$
- To divide a decimal by a decimal:
  1. Convert the divisor to a whole number by moving the decimal point to the right.
  2. Move the decimal point the same number of spaces to the right on the dividend.
  3. Perform short division and add trailing zeros where necessary. Make sure the decimal points are aligned.

Dividend Divisor

$1.2 \div 0.4 = 12 \div 4 = 3$

### Example 4F.1 Dividing by a decimal

Evaluate each division problem.

**a** 45 ÷ 1.2 **b** 16.482 ÷ 0.04

**THINK**

1. Convert the divisor to a whole number by moving the decimal point to the right. Add a zero where necessary.
2. Move the decimal point the same number of spaces to the right of the dividend.
3. Perform short division and add trailing zeros where necessary. Make sure the decimal points are lined up.

**WRITE**

**a** 45.0 ÷ 1.2 = 450 ÷ 12

```
      3 7. 5
12)45 9 0.6 0
```

45 ÷ 1.2 = 37.5

**b** 16.482 ÷ 0.04 = 1648.2 ÷ 4

```
    412.05
4)1648.20
```

16.482 ÷ 0.04 = 412.05

**Helpful hints**

- ✔ Moving the decimal point of the dividend and the divisor one place to the right is the same as multiplying by $\frac{10}{10}$.
- ✔ Remember that trailing zeros should be added to the dividend until you can divide exactly, without a remainder.
- ✔ Dividing with decimals may not make a number smaller! The quotient may be larger than or smaller than the dividend.

ANS p560

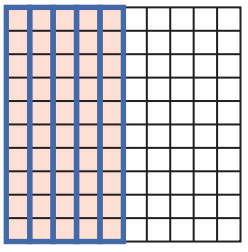
# Exercise 4F Dividing a decimal by a decimal

△ 1–10, 14, 17 ▢ 4–7, 10–13, 15–19 ⬠ 5–7, 11, 13, 15, 17–20

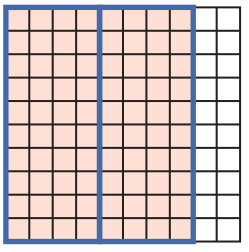
UNDERSTANDING AND FLUENCY

**1** Write an equation describing a division of a decimal by a decimal in each diagram. Use the example in the blue box on the previous page to help you.

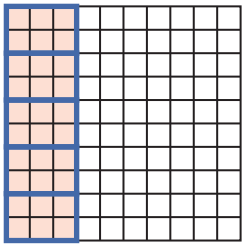
**a**



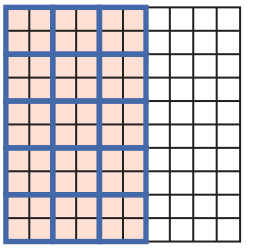
**b**



**c**



**d**



**2** Determine answers to your equation in question **1**. Use the diagrams to help you.

**3** Write an equivalent division problem for each of these so that you are dividing by a whole number.

**a** 62.356 ÷ 0.8 **b** 9.84 ÷ 0.008 **c** 19.5674 ÷ 0.06
**d** 0.462 ÷ 0.3 **e** 0.042 ÷ 0.0004 **f** 0.005 ÷ 0.02
**g** 46.2085 ÷ 0.00009 **h** 0.056 ÷ 0.2 **i** 2.108 ÷ 0.005

**4** Use your knowledge of dividing by a decimal and your times tables to calculate each of the following.

**a** 1.8 ÷ 0.2 **b** 5.4 ÷ 0.9 **c** 4.2 ÷ 0.6
**d** 18 ÷ 0.09 **e** 0.021 ÷ 0.007 **f** 320 ÷ 0.4
**g** 7.2 ÷ 0.08 **h** 0.0006 ÷ 0.00002 **i** 28.4 ÷ 0.004

4F.1 **5** Evaluate each division problem.

**a** 3.86 ÷ 0.2 **b** 0.843 ÷ 0.003 **c** 5.929 ÷ 0.07
**d** 71.82 ÷ 0.6 **e** 6.512 ÷ 0.04 **f** 0.9265 ÷ 0.005
**g** 27.32 ÷ 0.0008 **h** 42.39 ÷ 0.9 **i** 7.296 ÷ 0.02

**6** Evaluate each division problem. Remember to use trailing zeros if needed.

**a** 9.524 ÷ 0.8 **b** 1.735 ÷ 0.04 **c** 32.475 ÷ 0.006
**d** 0.041 ÷ 0.05 **e** 6.37579 ÷ 0.002 **f** 51.374 ÷ 0.08
**g** 0.00753 ÷ 0.00004 **h** 1.36947 ÷ 0.0006 **i** 0.0573 ÷ 0.004

7 Following the correct order of operations, complete the calculations below.

**a** $(12.6 \div 0.003) \times 3.8$

**b** $(8.34 - 2.18) \div 0.28$

**c** $1.4^2 + 8.38 \div 0.004$

**d** $\sqrt{(0.0048 \div 0.03) \times 100}$

**e** $\sqrt{(3.2 \times 5.1) - (0.16 \times 2)} \div 0.008$

**f** $2.3(1.08 \times 7.23) - (4.206 \div 1.2)^2$

8 Lacey has \$8.40 to buy muffins.

**a** Write a division calculation to calculate the number of muffins she could buy with this money.

**b** Write an equivalent division problem so that you are dividing by a whole number.

**c** Calculate the number of muffins Lacey can buy.

9 How many 0.375 L bottles can be filled from a 32.25 L water dispenser?

10 Using the cost shown on each item:

**i** write a division problem to calculate the number of each item you could buy with \$12.00

**ii** write an equivalent division problem so you are dividing by a whole number

**iii** calculate the number of each item you could buy.

**a**

**b**

**c**

**d**

11 **a** Calculate:

**i** $200 \div 40$ **ii** $200 \div 4$ **iii** $200 \div 0.4$ **iv** $200 \div 0.04$

**b** What overall pattern do you notice as the divisor becomes smaller?

**c** What in particular do you notice about the answers when you are dividing by a decimal number less than 1?

**d** Is the following statement true or false?

When dividing by decimal numbers less than 1, the smaller the divisor, the larger the answer.

12 Some of the division problems (A–H) listed below produce the same answer.

**A** $840\,000 \div 12\,000$ **B** $84\,000 \div 1200$ **C** $84\,000 \div 120$ **D** $84 \div 1200$

**E** $840 \div 12$ **F** $84 \div 12$ **G** $8.40 \div 0.12$ **H** $8.4 \div 12$

**a** Which division problems shown produce the same answer?

**b** Explain why they give the same answer.

13 Use your calculator to perform the first division, and then use the result to perform the second division by hand.

**a i** $7\,654\,394 \div 6158$ **ii** $0.765\,439\,4 \div 6.158$

**b i** $4\,309\,524\,297 \div 45\,721$ **ii** $43\,095.242\,97 \div 45.721$

**c i** $1\,510\,963\,552 \div 76\,192$ **ii** $151\,096.3552 \div 76\,192\,000$

**d i** $5\,072\,488\,775 \div 139$ **ii** $507.248\,877\,5 \div 1\,390\,000$

PROBLEM SOLVING AND REASONING

**14** Hamish considers two options when buying dog food:

- 1.2 kg bag for \$2.88
- 0.8 kg bag for \$2.20

**a** Divide \$2.88 by 1.2 to calculate the cost per kg for the 1.2 kg bag.

**b** Divide \$2.20 by 0.8 to calculate the cost per kg for the 0.8 kg bag.

**c** Which is the best buy?

**15** As part of an awareness-raising campaign for childhood cancer, a gold ribbon is to be cut into lengths and sold to supporters to wear.

**a** How many pieces 0.15 m long can be cut from a 21.7 m spool of ribbon?

**b** Will any ribbon be left over? If so, how much?

**16** One litre of petrol costs \$1.54. How many litres of petrol were purchased if the cost was \$70.07?

**17** Describe how you can decide which bottle of drink is the better value for money. Is the 2 L bottle of orange mineral water or the 1.5 L bottle of cola the better buy?

CHALLENGE

**18** Place a decimal point in 324 and 405 so that when the larger decimal is divided by the smaller decimal the result is 8. Is there more than one answer? Explain.

**19** Use the digits 0–9 only once to write a quotient as close to 25 as possible. Explain the process you used to solve this.

$$\square\square\square.\square\square \div \square.\square\square\square\square$$

**20** This question considers the most famous decimal number.

**a** Evaluate the following calculation.

$$\frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9}$$

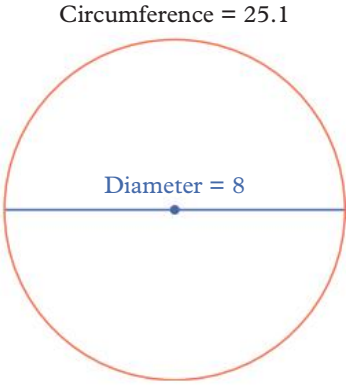
**b** Write the result as a decimal correct to two decimal places.

**c** Determine the pattern in the calculation from part **a**.

**d** Determine the next two terms in the calculation if it continued to follow the pattern.

**e** Calculate the new result with the two extra terms. Write your answer as a decimal and round your result to two decimal places.

Consider the circle to the right. The circumference of a circle is the perimeter (out edge length). The diameter of a circle is the length from one side of the circle to the other that runs through the centre.




**f** Divide the circumference by your result from part **e**. Round your result to two decimal places.

**g** Multiply the diameter by your result from part **e**. Round your result to two decimal places.

**h** Describe the relationship between the number from part **e** and the diameter and circumference of a circle.

**i** Explain what would happen to the relationship between the circumference and the diameter if more terms were added to the calculation in part **a** and then the new value was used instead.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Dividing decimals

**Investigation** Dealing with the dollar

**Topic quiz** 4F



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# 4G Percentages

## Learning intentions

✔ I can determine percentages from images and written scenarios.

✔ I can convert between fractions and percentages.

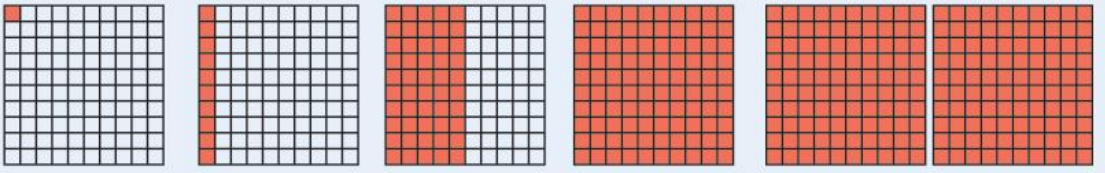
**Inter-year links**

**Years 5/6** Fractions, decimals and percentages

**Year 8** 3A Percentages, decimals and fractions

## Percentages

- A **percentage** is a proportion of a whole that has been divided into 100 parts.



1% 10% 50% 100% 200%

- The term '**per cent**' means 'per hundred' or 'out of 100'. The symbol for percentage is %.
  - → 75% also means $\frac{75}{100}$.
  - → 50% also means $\frac{50}{100}$ or exactly half.
  - → 200% also means $\frac{200}{100}$ or exactly 2.
- To write a fraction as a percentage, find an equivalent fraction with a denominator of 100.

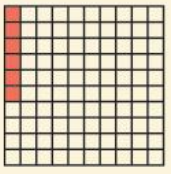
  For example, $\frac{9}{25} = \frac{36}{100}$
  $= 36\%$
- To write a decimal as percentage, multiply the decimal by 100.

  For example, $0.65 \times 100 = 65\%$.

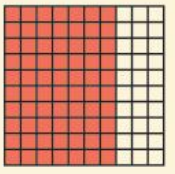
### Example 4G.1 Finding the percentage of a shaded figure

What percentage of these hundred squares has been shaded?

**a**



**b**



**THINK**

1 Count the number of sections that are shaded out of 100.

2 Write the shaded sections as a percentage.

**WRITE**

**a** There are 6 squares shaded out of 100.
6%

**b** There are 70 squares shaded out of 100.
70%

## Example 4G.2 Writing a fraction with a denominator of 100 as a percentage

Write the following fractions as a percentage.

**a** $\frac{19}{100}$ **b** $\frac{150}{100}$

**THINK**

The denominator of the fraction is 100, so write the digits of the numerator as the percentage.

**WRITE**

**a** $\frac{19}{100} = 19\%$

**b** $\frac{150}{100} = 150\%$

## Example 4G.3 Converting a decimal to a percentage

Write the following decimals as percentages.

**a** 0.75 **b** 0.2

**THINK**

To convert a decimal to a percentage, multiply by 100%. 'Move' the decimal point two places to the right and insert a place-holding zero in the empty space if necessary.

**WRITE**

**a** $0.75 \times 100 = 75\%$

**b** $0.20 \times 100 = 20\%$

## Example 4G.4 Writing a percentage as a fraction

Write each percentage as a fraction in its simplest form.

**a** 48% **b** 125%

**THINK**

To write a percentage to a fraction, write the percentage as a fraction with a denominator of 100. Simplify the fraction.

**WRITE**

**a** $48\% = \frac{48}{100}$

$= \frac{\cancel{48}^{12}}{\cancel{100}_{25}}$

$= \frac{12}{25}$

**b** $125\% = \frac{125}{100}$

$= \frac{\cancel{125}^{5}}{\cancel{100}_{4}}$

$= \frac{5}{4}$

**Helpful hints**

- ✔ Remember that one whole is equal to 100%. If you have a percentage that is greater than 100%, then you have more than one whole.
- ✔ Like decimals, percentages are also another way of writing fractions, although percentages represent a fraction with a denominator of 100.
- ✔ Remember that
  - → 6% refers to 6 hundredths
  - → 60% refers to 60 hundredths
  - → 600% refers to 600 hundredths
- ✔ Percentages are meaningless without context. You should ask yourself, 'this is a percentage of what?'

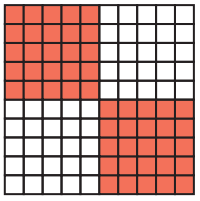
ANS p561

# Exercise 4G Percentages

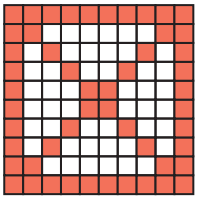
△ 1–3, 4–7(1st, 2nd columns), 8–12, 14, 17 ▢ 3, 4–7(2nd, 3rd columns), 9, 11, 13, 15–19 ⬠ 3–7(2nd, 3rd columns), 9, 11, 13, 15, 18–20

4G.1 **1** What percentage of each square has been shaded?

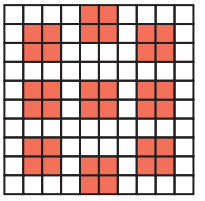
**a**



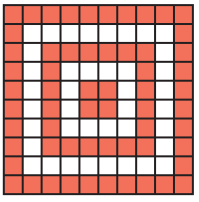
**b**



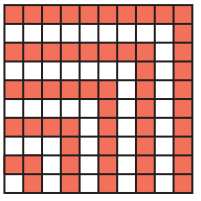
**c**



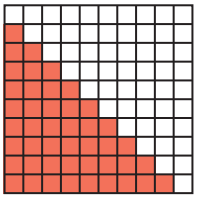
**d**



**e**



**f**



**2** Write the percentage of each square in question **1** that is unshaded.

4G.2 **3** Write each fraction as a percentage.

**a** $\frac{23}{100}$ **b** $\frac{78}{100}$ **c** $\frac{35}{100}$ **d** $\frac{99}{100}$

**e** $\frac{13}{100}$ **f** $\frac{7}{100}$ **g** $\frac{82}{100}$ **h** $\frac{4}{100}$

**4** Write each percentage as a fraction with a denominator of 100.

**a** 7% **b** 19% **c** 43% **d** 57%

**e** 3% **f** 63% **g** 11% **h** 91%

**i** 79% **j** 21% **k** 99% **l** 33%

4G.3 **5** Write each decimal as a percentage.

**a** 0.23 **b** 0.8 **c** 0.13 **d** 0.67

**e** 0.6 **f** 1.7 **g** 2.09 **h** 3.14

**i** 0.79 **j** 1.91 **k** 0.04 **l** 0.55

4G.4 **6** Write each percentage as a fraction in its simplest form.

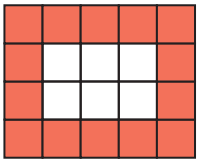
**a** 64% **b** 18% **c** 27%
**d** 80% **e** 5% **f** 95%
**g** 10% **h** 45% **i** 60%
**j** 75% **k** 35% **l** 88%

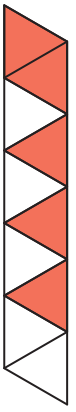
**7** Write each fraction as a percentage by first converting to an equivalent fraction with a denominator of 100.

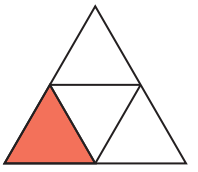
**a** $\frac{9}{20}$ **b** $\frac{11}{50}$ **c** $\frac{7}{4}$
**d** $\frac{17}{25}$ **e** $1\frac{3}{5}$ **f** $\frac{5}{25}$
**g** $\frac{12}{20}$ **h** $\frac{46}{50}$ **i** $\frac{60}{50}$
**j** $2\frac{1}{4}$ **k** $\frac{160}{200}$ **l** $\frac{40}{200}$

**8** Complete these tasks for the shapes below.

**i** Count the number of sections that are shaded and the total number of sections.
**ii** Write the proportion that is shaded as a fraction.
**iii** Find an equivalent fraction that has a denominator of 100.
**iv** Write this as a percentage.
**v** Write this as a decimal.

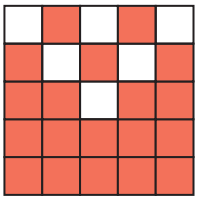
**a**


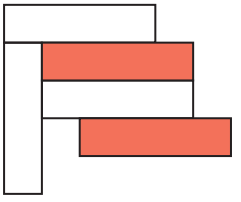
**b**


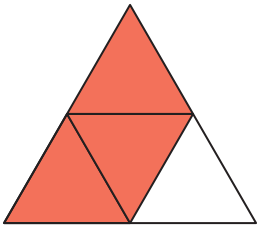
**c**


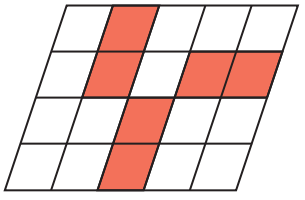
**9** Complete these tasks for the shapes below.

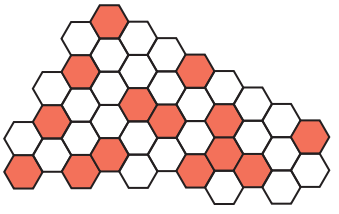
**i** What percentage of each figure has been shaded?
**ii** Write your percentage as a decimal.

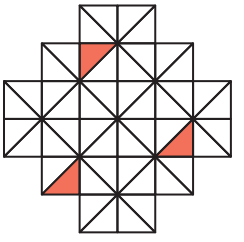
**a**


**b**


**c**


**d**


**e**


**f**


**10** Match each jug with the percentage by which it has been filled.

**a**

**b**

**c**

**d**

**A** 20% **B** 5% **C** 85% **D** 45%

**11** Write a percentage to describe each situation by first finding an equivalent fraction that has a denominator of 100.

**a** Four students out of 10 have blonde hair

**b** Eighteen people out of 20 come to a party

**c** Two people out of 50 miss the train

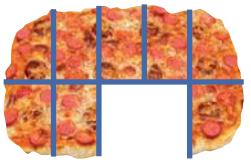
**d** One person out of five walks to school

**12 a** Write the percentage eaten from each of the three pizzas (assume all pieces are equal size).

**b** Write the portion of pizza not eaten from each tray as a percentage.

**c** Write a brief explanation of how you calculated as a percentage the portion of pizza that was eaten.

**i**



**ii**

**iii**

**13** Write, as a simplified ratio, the amount consumed or watched to the amount remaining.

**a** Gerard drinks 60% of a box of 30 cans of lemonade.

**b** Jayne reads 75% of a series of 12 books.

**c** Jordan eats 45% of a muesli bar.

**d** Robert watches 18% of a movie.

**14** While downloading songs to his iPhone on Spotify, Geoff noticed a horizontal bar on the screen. The bar's shading changed according to what percentage of the song had downloaded.

**a** Draw a rectangle 10 cm long and 1.5 cm wide to represent the horizontal bar.

**b i** Why do you think you were instructed to draw the rectangle 10 cm long?

**ii** What will each 1 cm represent along the rectangle?

**iii** What will each 1 mm represent along the rectangle?

**c** Using the horizontal bar from part **a**, show how the bar would look when 15% of a song has been downloaded.

**d** Draw four rectangles of the same dimensions as part **a** to show these percentages.

**i** 85% **ii** 38% **iii** 23% **iv** 96%

**15** Find the percentage that remains in each situation. Remember to add the portions of the ratio together to determine the whole.

**a** The ratio of homework problems Renee completes to the problems Renee does not complete is 6 : 4.

**b** The ratio of M&M's Elise eats to the M&M's Elise does not eat is 35 : 15.

**c** The ratio of levels on a game Xavier completes to the levels Xavier does not complete is 7 : 13.

**d** The ratio of spare batteries Peter uses to spare batteries Peter does not use is 1 : 3.

PROBLEM SOLVING AND REASONING

**16** Reuben is having difficulty understanding how it can be that these two containers are both 50% full, yet the volume of liquid in each is different. Explain the relationship between the volume and the percentage of liquid in the containers.

**17** Percentages can be more than 100%. This can only happen when it is possible to have more than a whole.

**a** If 200% means double the whole amount, what does 300% mean?

**b** To write 100% as a number, first write it as a fraction: $100\% = \frac{100}{100} = 1$.

Use this method to write each percentage as a number.

**i** 200% **ii** 300% **iii** 400% **iv** 150%

**18** A hamburger from Greasy Joe has about 450% of the daily recommended fat intake.

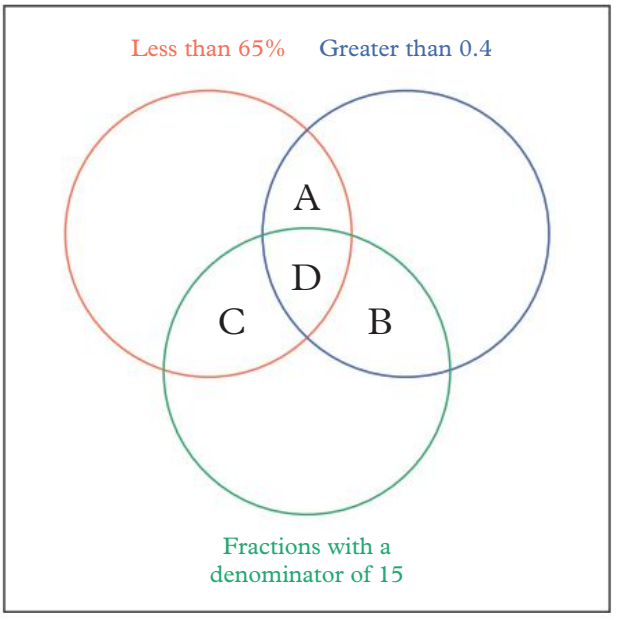
**a** Write this percentage as a number.

**b** If you decide to get chips with the burger, the meal would have 5.25 times the amount of the daily recommended fat. Write this amount as a percentage.

CHALLENGE

**19** Lauren fills her glass of water to the brim. She empties out 20% of the water. She decides there's now too little water and adds 20% of the amount of water that is in the glass. Explain why the glass is not filled to the brim.

**20** Below is a Venn diagram. Where two circles overlap, we consider the statement to be true for both groups.




**a** Determine if there are any values that can appear in A. If so, state the values or the region.

**b** Determine if there are any values that can appear in B. If so, state the values or the region.

**c** Determine if there are any values that can appear in C. If so, state the values or the region.

**d** Determine if there are any values that can appear in D. If so, state the values or the region.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Percentages

**Investigation** Percentages with punch

**Topic quiz** 4G

# 4H Fractions, decimals and percentages

## Learning intentions

✔ I can convert between fractions, decimals and percentages.

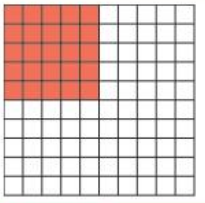
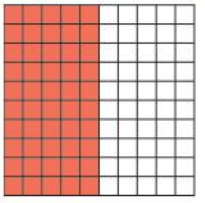
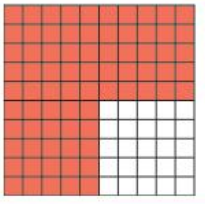
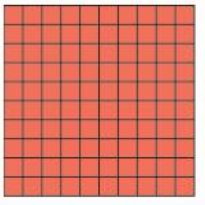
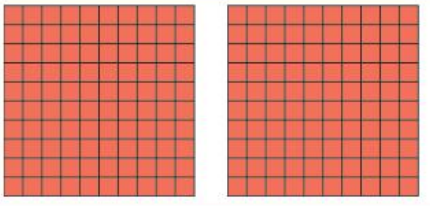
✔ I can choose between fractions, decimals and percentages to solve problems.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Fractions, decimals and percentages |
| Year 8 | 3A Percentages, fractions and decimals |
| Year 9 | 1A Calculator skills |

## Fractions, decimals and percentages

- Fractions, decimals and percentages are different ways of representing the same value.

| Percentage | Fraction | Decimal | |
|---|---|---|---|
| 25% | $\frac{25}{100} = \frac{1}{4}$ | 0.25 |  |
| 50% | $\frac{50}{100} = \frac{1}{2}$ | 0.5 |  |
| 75% | $\frac{75}{100} = \frac{3}{4}$ | 0.75 |  |
| 100% | $\frac{100}{100} = 1$ | 1 |  |
| 200% | $\frac{200}{100} = 2$ | 2 |  |

# Converting between fractions, decimals and percentages

| | Fraction | Decimal |
|---|---|---|
| **Percentage to …** | Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100} = \frac{3}{20}$ | Divide the percentage by 100. $23.4\% = \frac{23.4}{100} = 0.234$ |

| | Percentage | Decimal |
|---|---|---|
| **Fraction to …** | Write the fraction as an equivalent fraction with a denominator of 100. $\frac{7}{25} \xrightarrow{\times 4} \frac{28}{100} = 28\%$ | Divide the numerator by the denominator. Add trailing zeros as required. Align decimal points. Dividend → 1, Divisor → 8: $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ = 0.125 (Trailing zeros) |

| | Percentage | Fraction |
|---|---|---|
| **Decimal to …** | Multiply the decimal by 100. $0.75 \times 100 = 75\%$ | Place the decimal as the numerator of the fraction and the denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100}$ |

## Example 4H.1 Converting from a percentage

Fill in the blanks of the following table.

| Percentage | Fraction | Decimal |
|---|---|---|
| 40% | | |

**THINK**

1. To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100. Simplify the fraction.
2. To convert to a decimal, divide the percentage by 100.

**WRITE**

| Percentage | Fraction | Decimal |
|---|---|---|
| 40% | $\frac{\cancel{40}^{2}}{\cancel{100}_{5}} = \frac{2}{5}$ | $40\% = 0.4$ |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## **Example 4H.2** Converting from a decimal

Fill in the blanks of the following table.

| Percentage | Fraction | Decimal |
|---|---|---|
| | | 0.86 |

**THINK**

1 To convert a decimal to a percentage, multiply by 100%.

2 To convert a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point in the decimal fraction part of the number. Simplify the fraction.

**WRITE**

| Percentage | Fraction | Decimal |
|---|---|---|
| 0.86 = 86% | $\frac{\cancel{86}^{43}}{\cancel{100}_{50}} = \frac{43}{50}$ | 0.86 |

## **Example 4H.3** Converting from a fraction

Fill in the blanks of the following table.

| Percentage | Fraction | Decimal |
|---|---|---|
| | $\frac{1}{25}$ | |

**THINK**

1 Start by finding an equivalent fraction with a denominator of 100.

2 To convert a fraction to a percentage, use the equivalent fraction with a denominator of 100, and write the numerator as the percentage.

3 To convert to a decimal, use the equivalent fraction $\frac{4}{100}$, which is equal to 4 hundredths.

**WRITE**

$$\frac{1}{25} = \frac{1}{25} \times \frac{4}{4}$$
$$= \frac{4}{100}$$

| Percentage | Fraction | Decimal |
|---|---|---|
| $\frac{4}{100} = 4\%$ | $\frac{1}{25}$ | $\frac{4}{100} = 0.04$ |

**Helpful hints**

- ✔ Don't forget to simplify your fractions.
- ✔ Remember that sometimes it is easier to convert from a fraction to a decimal when the fraction has a denominator of 100, before you have simplified your answer.
- ✔ Remember that 6% is equal to '6 out of one hundred', which is '6 hundredths' and equal to 0.06.

ANS p562

# Exercise 4H Fractions, decimals and percentages

△ 1–8, 10, 12 □ 4–7, 9–11, 13–14 ⬠ 5–7, 9, 11, 13–15

UNDERSTANDING AND FLUENCY

4H.1 **1** Complete the table to show the equivalent forms of each amount.

| Percentage | Fraction | Decimal |
|---|---|---|
| | $\frac{1}{2}$ | |
| | | 0.25 |
| 75% | | |
| | $\frac{3}{10}$ | |
| | | 0.125 |
| 62.5% | | |
| | $\frac{1}{5}$ | |
| | | 0.4 |
| 60% | | |

4H.1 **2** Write each percentage as a fraction and a decimal. Simplify your fractions where possible.

**a** 46% **b** 13% **c** 99%
**d** 25% **e** 20% **f** 50%
**g** 5% **h** 8% **i** 1%

4H.2 **3** Write each decimal as a percentage and a fraction. Simplify your fractions where possible.

**a** 0.28 **b** 0.88 **c** 0.15
**d** 0.62 **e** 0.45 **f** 0.72
**g** 0.09 **h** 0.04 **i** 0.3

4H.3 **4** Write each fraction as a percentage and a decimal.

**a** $\frac{51}{50}$ **b** $\frac{17}{20}$ **c** $\frac{14}{5}$
**d** $\frac{9}{4}$ **e** $\frac{13}{25}$ **f** $\frac{21}{2}$
**g** $\frac{405}{1000}$ **h** $\frac{83}{20}$ **i** $\frac{7121}{100}$

**5** Write each percentage as a decimal and a fraction. Simplify your fractions where possible.

For example, $12.3\% = 0.123 = \frac{123}{1000}$.

**a** 23.84% **b** 19.65% **c** 46.7%

**d** 3.09% **e** 567.4% **f** 0.467%

**g** 12.895% **h** 73.28% **i** 200.5%

**6** Write each decimal as a percentage and a fraction. Simplify your fractions where possible.

**a** 0.518 **b** 9.02 **c** 0.105

**d** 0.7 **e** 21.5 **f** 8.41

**g** 7.1 **h** 0.009 **i** 0.1765

**7** Write each fraction as a percentage and a decimal. Remember you can use division to convert a fraction to a decimal.

For example, $\frac{2}{3} = 3\overline{)2.00}$

**a** $\frac{3}{8}$ **b** $\frac{3}{40}$ **c** $\frac{3}{80}$

**d** $\frac{3}{32}$ **e** $\frac{17}{8}$ **f** $\frac{91}{40}$

**g** $9\frac{37}{80}$ **h** $11\frac{29}{32}$ **i** $\frac{9}{80}$



**8** Eclectus parrots are found in north-eastern Australia. The male is green and the female is red and blue.

**a** Write the number of male parrots pictured as a fraction of the total number of parrots.

**b** What percentage of the group is:

**i** male? **ii** female?

**c** Write each answer to part **b** as a decimal.

**9** Converting a fraction to a percentage can also be done by finding the fraction of 100%.

For example, $\frac{3}{5} = \frac{3}{5}$ of $100\% = \frac{3}{5} \times 100\% = \frac{300}{5}\% = 60\%$.

Use this method to convert the following fractions to percentages.

**a** $\frac{17}{25}$ **b** $\frac{21}{10}$ **c** $\frac{43}{20}$ **d** $\frac{7}{8}$

**10** Consider the following numbers:

| | | |
|---|---|---|
| 109% | $1\frac{9}{10}$ | 0.38 |
| $\frac{38}{200}$ | 19% | 20% |
| $\frac{38}{20}$ | 1.9 | $\frac{19}{95}$ |
| 38% | 1.09 | $\frac{19}{5}$ |
| 3.8 | 190% | 0.2 |

**a** Determine which values are equivalent.

**b** For each value list, there should be a fraction, decimal and percentage showing the equivalent value. Identify the values that are missing a fraction, decimal or percentage representation, and write the value in that form.

**c** Order the decimal form of all the values in descending order.

PROBLEM SOLVING AND REASONING

**11** The elements that make up the Sun are listed in the table below.

**a** Write the percentage of hydrogen as a fraction and a decimal.

**b** If Fe and Mg are the only metals, find how much of the Sun is made of metal and write it as a:

**i** percentage **ii** decimal **iii** fraction.

**c** What do the percentages of the listed elements add to? Why do you think this might be?

| Element | Percentage of total mass |
|---|---|
| H (hydrogen) | 71.0% |
| He (helium) | 27.1% |
| O (oxygen) | 0.97% |
| C (carbon) | 0.40% |
| Si (silicon) | 0.099% |
| N (nitrogen) | 0.096% |
| Mg (magnesium) | 0.076% |
| Ne (neon) | 0.058% |
| S (sulphur) | 0.040% |
| Fe (iron) | 0.014% |

**12** Eliza had cycled 496 km of the gruelling 620 km Great Victorian Bike Ride. What percentage of the ride did she still have to complete?

**13** Laura and Rosie were comparing their working of a particular problem. They were required to convert $17\frac{1}{8}\%$ to a decimal. Their setting out is slightly different; however, both methods produce the same result. Comment on whether the answer and setting out of each is correct.

Laura

$$\begin{aligned} 17\tfrac{1}{8}\% &= 17.125\% \\ &= \frac{17.125}{100} \\ &= \frac{17.125}{100} \times \frac{1000}{1000} \\ &= \frac{17125}{100\,000} \\ &= 0.171\,25 \end{aligned}$$

Rosie

$$\begin{aligned} 17\tfrac{1}{8}\% &= 17.125\% \\ &= \frac{17.125}{100} \\ &= 17.125 \div 100 \\ &= 0.171\,25 \end{aligned}$$

CHALLENGE

**14** What are the possible denominators, less than 100, of a simplified fraction such that the decimal and percentage forms have a fixed number of decimal places?

For example, the fraction $\frac{1}{2}$ is equivalent to 0.5 and 50%. They have a fixed number of decimal places. But $\frac{1}{3}$ is equivalent to 0.333... and 33.33...% where both have a recurring decimal of 3.

**15** Without using the same digit more than once, fill in the spaces to make a correct statement.

$$\frac{\square\square}{\square\square} = \square\square.\square\ \%$$

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Fractions, decimals and percentages

**Investigation** Dominations

**Topic quiz** 4H

# 4I Calculating percentages

## Learning intentions

✔ I can express one quantity as a percentage of another.
✔ I can find a percentage of quantities.

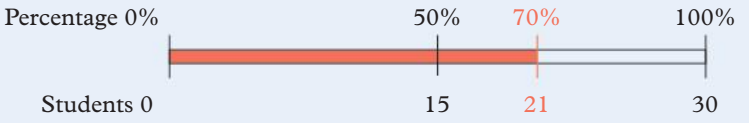
**Inter-year links**

| | |
|---|---|
| Years 5/6 | Calculating percentages |
| Year 8 | 3B Calculating percentages |
| Year 9 | 1C Mark-ups and discounts |

## Expressing a quantity as a percentage of another

- Percentages can be used in a social context to compare numbers and to represent fractions and ratios.
- To express one quantity as a percentage of another, write the equivalent fraction out of 100.
  For example, in a class of 30, 21 students have a sister.

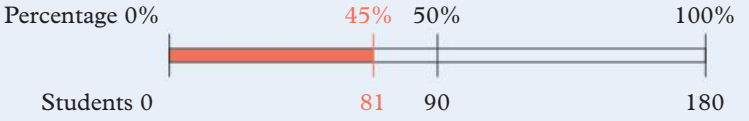
$$\frac{21}{30} \overset{\div 3}{=} \frac{7}{10}$$
$$= \frac{70}{100}$$
$$= 70\%$$




## Percentage of a quantity

- To calculate a percentage of a quantity, write the percentage as a decimal, and multiply by the quantity.
  → Recall that when asking for a fraction or percentage 'of' an amount, replace the 'of' with the multiplication symbol, '×'.
- For example, 45% of 180 students like ice-cream more than frozen yoghurt.

$$45\% \text{ of } 180 = \frac{45}{100} \times \frac{180}{1}$$
$$= 81 \text{ students}$$




### Example 4I.1 Expressing a quantity as a percentage of another

Express 60 as a percentage of 150.

**THINK**

1. Express the amount as a fraction of the total and simplify.
2. To convert a fraction to a percentage, write the fraction with a denominator of 100.
3. Write your answer.

**WRITE**

$$\frac{60}{150} \overset{\div 30}{=} \frac{2}{5}$$
$$= \frac{40}{100}$$
$$= 40\%$$

60 is 40% of 150.

## Example 4I.2 Calculating a percentage of a quantity

Calculate 12% of \$160.

| THINK | WRITE |
|---|---|
| 1 Convert the percentage to a decimal. | 12% of \$160 = 0.12 × \$160 |
| 2 Multiply by the quantity. | = \$19.20 |

**Helpful hints**

✔ You can use fractions or decimals to find the percentage of a quantity. Both methods will give you the same answer.

For example, 21 out of 30 as a percentage is $\frac{21}{30}$ of 100% $= \frac{21}{\cancel{30}^{3}} \times \frac{\cancel{100}^{10}}{1}$

$= \frac{\cancel{21}^{7}}{\cancel{3}^{1}} \times \frac{10}{1}$

$= 7 \times 10$

$= 70$

ANS p563

# Exercise 4I Calculating percentages

1–7, 9, 10, 12, 14, 18(a, b) | 1–8, 11, 13, 15, 16, 18–21 | 1–5(2nd column), 7, 11, 13, 15, 16, 18–22

UNDERSTANDING AND FLUENCY

4I.1 **1** Express the first number as a percentage of the second.

**a** 15 out of 20 **b** 25 out of 20
**c** 3 out of 5 **d** 3 out of 6
**e** 100 out of 50 **f** 90 out of 16
**g** 37 out of 40 **h** 7 out of 8

**2** Calculate the percentage of each quantity.

**a** 5% of 3900 **b** 10% of 120
**c** 20% of 520 **d** 50% of 80
**e** 15% of 50 **f** 90% of 20
**g** 60% of 65 **h** 75% of 16

4I.2 **3** Calculate the percentage of each amount.

**a** 12% of \$40 **b** 17% of \$450
**c** 11% of \$500 **d** 15% of \$1326
**e** 8% of \$8750 **f** 9% of \$1870
**g** 22% of \$6750 **h** 25% of \$35.80

**4** Calculate each amount. Write your answer as a decimal.

**a** 2.5% of 180 **b** 5.5% of 70

**c** 75.8% of 120 **d** 20.4% of 150

**e** 42.8% of 65 **f** 17.5% of 1650

**g** 5.3% of 960 **h** 16.8% of 150

**i** 85.2% of 95 **j** 17.6% of 120

**k** 12.5% of 99 **l** 10.2% of 78

**5** Calculate the percentage of each amount.

**a** 110% of 40 **b** 105% of 70

**c** 300% of 15.2 **d** 250% of 160

**e** 120% of 8.9 **f** 460% of 98

**g** 165% of 90 **h** 220% of 140

**6** Look at the percentage problems and your answers to question **5**.

**a** What do you notice about each percentage compared to those in question **4**?

**b** Compare each answer to the corresponding original amount.

**7** Meredith is impressed at how quickly her mother can mentally calculate percentages. One of her mother's strategies is shown below.

To obtain 10%, divide the amount by 10.
This is because 10% can be written as $\frac{1}{10}$.
So 10% of 72 is 7.2 (since $72 \div 10 = 7.2$).

You can extend this idea to find other values. For example, to find 20% of an amount, find 10% and then double the value. Use this logic to copy and complete these sentences.

**a** To find 5% of an amount, first obtain 10% and then _____ the value.

**b** To find 30% of an amount, first obtain 10% and then _____ the value.

**c** To find 40% of an amount, first obtain 10% and then _____ the value.

**8** A survey showed 92% of commuters were not satisfied with public transport.

**a** If 6500 commuters took part in the survey, how many people were not satisfied with the public transport system?

**b** How many of the 6500 commuters were satisfied with the system?

**9** Lachlan scored 18 out of 25 on his first test and 23 out of 30 for his next test.

**a** Calculate what percentage he scored for his first test.

**b** Calculate what percentage he scored for his second test.

**c** Which test did Lachlan perform better on? Explain your answer.

**10** A laptop has a recommended price of $2199. A store has a sale offering 30% off.

**a** Calculate the value of the discount.

**b** Subtract this from the original price to find the sale price of the item.

**c** If you could only spend $1500 on a new laptop, would you be able to buy this?

**11** A camping store has discounted its tents by 20%.

**a** Calculate the discount on a tent that normally retails at $570.

**b** What is the sale price of the tent?

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

**12** Sevi visits two stores for a new enclosure for her pet snake. 'Ripper Reptiles' has enclosures that were originally priced at \$600 discounted by 18%. 'Enclosed' stocks them for \$650 discounted by 25%.

**a** What is the value of the discount at each store?

**b** What is the new price of the enclosure at each store?

**c** Which store should Sevi buy the enclosure from?

**13** The ratio 8 : 5 has some parts either added or subtracted so that the ratio becomes 1 : 1 after simplifying. (Some parts have been added or subtracted from the total.)

**a** If the first part of the ratio is altered, what percentage of the original first part is the new first part?

**b** If the second part of the ratio is altered, what percentage of the original second part is the new second part?

**14** Wally, Artemis and Megan plan to share a bag of lollies. Wally claims 55%, Artemis claims $\frac{1}{4}$, and Megan is happy with the rest.

**a** What proportion of the lollies does Megan get as a decimal?

**b** How many lollies do each get if there are 460 lollies in the bag?

**c** Write the following ratios:

**i** The lollies Wally receives to the lollies he does not receive.

**ii** The lollies Wally receives to the lollies Artemis receives.

**iii** The lollies Artemis receives to the lollies Megan receives.

**d** If you and one of these three people could share the entire bag of lollies, but the other person would keep their initial share, who would give you the closest to a 1 : 1 ratio? Explain your answer.

**15** If we know the amount a percentage is equal to, we can work out the original total.

For example, 12 is 30% of what number?

$$30\% = \frac{3}{10} = \frac{3 \times 4}{10 \times 4} = \frac{12}{40}$$

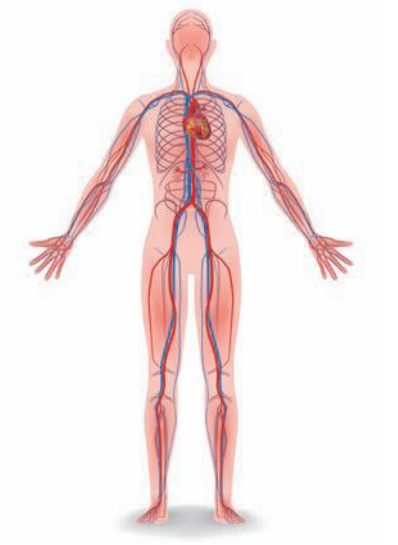
so 12 is 30% of 40.

**a** 36 is 50% of what number?

**b** 162 is 90% of what number?

**c** 126 is 210% of what number?

**d** 333 is 37% of what number?



**16** Angus learns that the human body contains 65% water, 18% protein, 10% fat, 6% minerals and vitamins and 1% carbohydrate. Find the mass of each component in people of the following masses.

**a** 45 kg **b** 63 kg **c** 98 kg

**17** A food store decides the prices of items by marking up (or increasing) the wholesale price by 25% and then rounding up to the next 5 cents. Calculate the final price of the items with these wholesale prices.

**a** \$2.99 per kg of tomatoes

**b** \$1.02 per 1L of orange juice

**c** \$5.85 per bag of lollies

**d** \$7.34 per chocolate block

**e** \$1.37 per noodle cup

**f** 95 cents per apple

**18** Many items that you buy include a goods and services tax (GST). This means that 10% is added to the cost of most goods and services.

The items listed have not been taxed yet. For each item:

**i** find the amount of GST to be added

**ii** find the price it will be sold for.

**a** $1199 laptop

**b** $209 calculator

**c** $499 orthotics

**d** $2.99 packet of biscuits

**e** $29.99 T-shirt

**f** $54 backpack

**19** From question **18**, what do you notice about the amount of GST in relation to the original price? Write a sentence describing a shortcut to finding 10% of a price.

**20 a** Calculate 75% of 250% of 120% of 95% of 80.

**b** Calculate what percentage the answer from part **a** is out of 80.

**21** A rectangle has its length reduced to 80% of its original length and its width reduced to 40% of its original width. Determine what percentage the area of the reduced rectangle is of the area of the original rectangle.

**22** A demolition company is tasked with removing a skyscraper on a tight deadline. The company claim they can remove 17% of the total building's height each day.

**a** If the building is 247 m tall, how tall will the building be after the first day?

**b** How long will it take to remove the entire building?

After an investigation, the company was found to be guilty of false claims. Instead of 17% of the total building's height per day, the company actually remove 17% of the remaining height each day.

**c** Until this new claim, will the company remove the building in more or less time than their original promise?

**d** How long will it take to remove the building according to the new claim?

The company update their claim to reflect the investigation, but add a line claiming they will remove the remaining height of the building in one day if the height is less than 20 m.

**e** How long will it take to remove the building under the company's new claim?

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Calculating percentages

**Investigation** Calculating the football ladder

**Topic quiz** 4I

# Chapter summary

## Place value table

| Ten thousands 10 000 | Thousands 1000 | Hundreds 100 | Tens 10 | Ones 1 | . | Tenths $\frac{1}{10}$ | Hundredths $\frac{1}{100}$ | Thousandths $\frac{1}{1000}$ | Ten-thousandths $\frac{1}{10\,000}$ |
|---|---|---|---|---|---|---|---|---|---|

## Adding and subtracting decimals

- When adding or subtracting decimals, the digits must be positioned in their correct place value columns and the decimal points must be lined up.

$$\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \end{array}$$

- Blank spaces in the columns may be replaced with additional zeros to assist working out.

$$\begin{array}{r} 3.200 \\ +\ 3.002 \\ \hline 3.202 \end{array}$$

## Rounding

$1.23\boxed{3}95 \approx 1.24$ — Round up (9, 8, 7, 6, 5)

$1.23\boxed{3}25 \approx 1.23$ — Round down (4, 3, 2, 1, 0)

## Multiplying by 10s

$5.23 \times 10 = 52.3$

$5.23 \times 100 = 523.0$

$5.23 \times 1000 = 5230.0$

## Dividing by 10s

$5.23 \div 10 = 0.523$

$5.23 \div 100 = 0.0523$

$5.23 \div 1000 = 0.00523$

## Multiplying decimals

1. Count the decimal places in the problem.
2. Multiply the decimals as whole numbers.
3. Write the answer with this same number of decimal places. Add zeros where necessary.

$2.75$ ← two decimal places
$\times 0.03$ ← two decimal places
= four decimal places

$$\begin{array}{r} {}^{2\,1} \\ 275 \\ \times\ \ 3 \\ \hline 825 \end{array}$$

$2.75 \times 0.03 = 0.0825$

## Dividing by a whole number

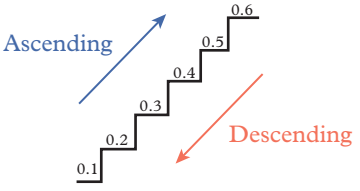
$$\begin{array}{r} 19.16 \\ 9\overline{)172.44} \end{array}$$

quotient (the answer)

divisor (the number we are dividing by)

dividend (the number being divided)

## Ordering




## Dividing by a decimal

1. Convert the divisor to a whole number by moving the decimal point to the right.
2. Move the decimal point the same number of spaces to the right on the dividend.
3. Perform short division and add trailing zeros where necessary. Make sure the decimal points are aligned.

Dividend Divisor

$1.2 \div 0.4 = 12 \div 4 = 3$

## Converting between fractions, decimals and percentages

| | Fraction | Decimal |
|---|---|---|
| **Percentage to …** | Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100} = \frac{3}{20}$ | Divide the percentage by 100. $23.4\% = \frac{23.4}{100} = 0.234$ |

| | Percentage | Decimal |
|---|---|---|
| **Fraction to …** | Write the fraction as an equivalent fraction with a denominator of 100. $\frac{7}{25} \xrightarrow{\times 4} \frac{28}{100} = 28\%$ | Divide the numerator by the denominator. Add trailing zeros as required. Align decimal points. Dividend → 1, Divisor → 8: $8\overline{)1.0^{2}0^{4}0}$ = 0.125. Trailing zeros |

| | Percentage | Fraction |
|---|---|---|
| **Decimal to …** | Multiply the decimal by 100. $0.75 \times 100 = 75\%$ | Place the decimal as the numerator of the fraction and the denominator 10, 100, 1000 … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100}$ |



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# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

4A **1** What is $\frac{1}{5}$ written as a decimal?

**A** 0.2 **B** 0.5 **C** 1.5 **D** 0.15 **E** 5.1

4A **2** What is 0.26 as a fraction in its simplest form?

**A** 0.26 **B** $\frac{2}{6}$ **C** $\frac{26}{100}$ **D** 26% **E** $\frac{13}{50}$

4A **3** Which of the following does not equal 3.71?

**A** $3 + \frac{7}{10} + \frac{1}{100}$ **B** $\frac{37}{10} + \frac{1}{100}$ **C** $\frac{30}{10} + \frac{70}{100} + \frac{10}{1000}$

**D** $\frac{301}{100} + \frac{700}{1000}$ **E** $\frac{371}{1000}$

4B **4** What is 13.054 872 6 rounded to three decimal places?

**A** 13.0 **B** 13.054 **C** 13.055 **D** 13.0549 **E** 13.055 000 0

4B **5** Which statement is false?

**A** 12.345 > 12.309
**B** 0.0006 > 0.000 03
**C** 48.2189 < 48.950
**D** 22.4 > 22.056
**E** 183.506 < 183.099

4C **6** Which has the smallest answer?

**A** 1.568 + 6.238 − 4.22
**B** 1.456 + 0.853 + 1.0899
**C** 10.9856 − 7.53 + 2.2
**D** 11.236 − 4.2689 − 3.0501
**E** 1.451 + 1.254 + 0.684

4D **7** What is 6.04 × 2.1?

**A** 1.2684 **B** 12.684 **C** 126.84 **D** 1268.4 **E** 0.12684

4E **8** What is 8.21 ÷ 4?

**A** 2.0525 **B** 2.21 **C** 4.21 **D** 32.84 **E** 2.525

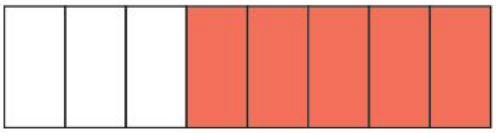
4E **9** If 10 apples cost \$9.85, how much would six apples cost?

**A** \$0.985 **B** \$1.64 **C** \$5.91 **D** \$16.42 **E** \$59.10

4F **10** Which has the largest answer?

**A** 23 ÷ 0.4 **B** 48 ÷ 1.2 **C** 60.5 ÷ 1.1 **D** 15 ÷ 0.3 **E** 28.5 ÷ 0.5

4G **11** What percentage of this figure is shaded?



**A** $\frac{5}{8}$% **B** 5% **C** 50% **D** 62.5% **E** $\frac{3}{8}$%

4G **12** What is 48% as a fraction in its simplest form?

**A** $\frac{48}{100}$ **B** $\frac{24}{50}$ **C** 0.48 **D** $\frac{12}{25}$ **E** $\frac{6}{12.5}$

4G **13** What is $\frac{9}{20}$ written as a percentage?

**A** 9% **B** $\frac{9}{20}$% **C** 45% **D** 180% **E** $\frac{45}{100}$

4H **14** The fraction $\frac{7}{8}$ can also be written as:

**A** 78% **B** 7.8 **C** 0.75 **D** 87.5% **E** 114.3%

4H **15** What is 64.5% written as a decimal?

**A** 0.645 **B** 64.5 **C** 0.0645 **D** 64 500 **E** 6.45

4H **16** What is 0.1265 written as a percentage?

**A** 0.1265% **B** 0.001 265% **C** 1265% **D** 126 500% **E** 12.65%

4I **17** What is 25% of 220?

**A** 5500 **B** 245 **C** 195 **D** 55 **E** 880

## Short answer

4A **1** What is the place value of the digit 7 in each number?

**a** 10.578 **b** 2.075 **c** 0.000 517 **d** 16.794

4A **2** Write each fraction as a decimal.

**a** $\frac{87}{100}$ **b** $\frac{117}{1000}$ **c** $\frac{3}{10}$ **d** $\frac{7}{100}$

4A **3** Write each fraction as a decimal.

**a** $\frac{16}{25}$ **b** $\frac{19}{50}$ **c** $\frac{15}{20}$ **d** $\frac{3}{8}$

4B **4** For each number:

**i** write the number of decimal places

**ii** round to two decimal places.

**a** 1589.231 57 **b** 12.709 66 **c** 0.9286 **d** 152.875 964 3

4B **5** Place this list in descending order.

12.5089, 12.8905, 18.0958, 12.0985, 10.9852

4C **6** Calculate:

**a** 23.25 + 84.15 + 13.06

**b** 1.025 + 3.508 + 4.999

**c** 14.562 − 12.869

**d** 5.239 − 2.506 + 1.5555

4D **7** Calculate:

**a** 12.63 × 100 **b** 15.3298 × 10 **c** 4.2 × 50 **d** 3.047 × 200

4D **8** Calculate:

**a** 0.7 × 0.3 **b** 0.9 × 0.04 **c** 1.1 × 0.08 **d** 0.08 × 0.04

4D **9** Calculate:

**a** 25.8 × 0.1 **b** 46.8 × 0.01 **c** 6.5 × 7.2 **d** 4.23 × 1.9

4E **10** Calculate:

**a** 78.94 ÷ 10 **b** 1256.359 ÷ 100

**c** 124.56 ÷ 10 **d** 189 765.6 ÷ 1000

4E **11** Calculate:

**a** 18.654 ÷ 2 **b** 94.56 ÷ 6 **c** 148.63 ÷ 5 **d** 327.45 ÷ 8

4E **12** Evie considers two packs of cheese: 500 g for \$7.80 and 700 g for \$12.60.

**a** Calculate the value of 100 g of each product.

**b** Which product is the best buy? Explain.

4F **13** Calculate:

**a** 6.2 ÷ 0.5 **b** 0.75 ÷ 0.2 **c** 8.62 ÷ 0.4 **d** 0.0081 ÷ 0.05

4G **14** Write each percentage as a fraction:

**i** with a denominator of 100

**ii** in its simplest form.

**a** 49% **b** 22% **c** 75% **d** 5%

4G **15** What percentage of a pie remains if Caleb eats 23% and Hogan eats 31%?

4H **16** Write each decimal as a fraction in its simplest form.

**a** 0.75 **b** 0.18 **c** 0.489 **d** 0.003

4H **17** Write each decimal as a percentage.

**a** 0.56 **b** 0.08 **c** 1.79 **d** 0.2748

4H **18** Write each percentage as a decimal.

**a** 78% **b** 46.5% **c** 9% **d** 10.679%

4I **19** Write each number as a percentage.

**a** $\frac{3}{5}$ **b** $\frac{7}{8}$ **c** 1 **d** $2\frac{3}{4}$

4I **20** Calculate each amount.

**a** 10% of 450 **b** 30% of 500 **c** 45% of 300

**d** 85% of 650 **e** 6% of 1499 **f** 41% of 347

4I **21** Calculate each amount.

**a** 272% of 550 km **b** 34.75% of 950 g

**c** 5.8% of $120 **d** 16.2% of 450 L

## Analysis

Rob, Carina, Belinda and Mario want to buy Alicia a bunch of white roses for her birthday.

**a** Rob has $10.35, Carina has $8.25, Belinda has $9.90 and Mario has $10.80. How much money do they have altogether?

**b** Their local florist, Fantastic Flowers, has white roses for $2.99 each. How many can they buy? How much change will they have left over?

Fantastic Flowers has a special offer, that if you buy a dozen white roses you get a 25% discount.

**c** How much would a dozen white roses cost without the discount?

**d** How much would a dozen white roses cost with the discount?

Mario bargains with the storekeeper and manages to buy 18 white roses for a 37.5% discount.

**e** Convert this percentage to a decimal.

**f** How much would 18 white roses cost without the discount?

**g** What is 37.5% of this amount?

**h** How many decimal places does this number have? Round this number to two decimal places.

**i** Find out how much Mario paid for the flowers and the amount of money left.

**j** If the change was divided evenly between Rob, Carina, Belinda and Mario, how much would they each get? (Round your answer to the nearest cent.)

# 5 Integers and the Cartesian plane

## Index

**5A** Negative numbers
**5B** Adding and subtracting positive numbers
**5C** Adding and subtracting negative numbers
**5D** The Cartesian plane
**5E** Interpreting graphs

## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Number lines
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying whole numbers
- ✔ Dividing whole numbers
- ✔ Adding and subtracting fractions

## Curriculum links

- Compare, order, add and subtract integers (VCMNA241)
- Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (VCMNA242)
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (VCMNA255)
- Investigate, interpret and analyse graphs from real life data, including consideration of domain and range (VCMNA257)

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# 5A Negative numbers

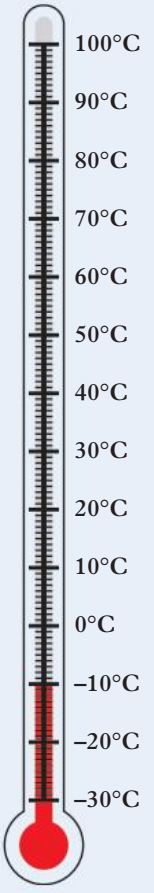
## Learning intentions

- ✔ I can represent positive and negative numbers on a number line.
- ✔ I can use negative numbers to represent real-life situations.
- ✔ I can compare the value of positive and negative numbers.
- ✔ I can arrange positive and negative numbers in ascending and descending order.

Inter-year links

| | |
|---|---|
| Year 5/6 | Negative numbers |
| Year 8 | 1E Negative integers |

## Negative numbers




- **Negative numbers** are less than zero in value.
  - → Negative numbers always include a negative sign, –, placed directly before the first digit. For example, 'negative ten' is written as –10.
  - → Positive numbers, which are greater than zero in value, can be represented using a positive sign, +. The positive sign can also be left out. For example, 'positive ten' or 'ten' is written as +10 or 10.
- Although the negative sign and the minus sign look the same, the function of the symbol depends on its context.

| Example | Function of '–' | Read as |
|---|---|---|
| –8 | Indicates the number is less than zero in value | 'negative eight' |
| 2 – 8 | Indicates the operation of subtraction | 'two minus eight' |
| –8 + 2 | Indicates the number is less than zero in value | 'negative eight plus two' |

- In the real world, negative numbers are used to represent quantities that lie on a scale that goes below zero.
  For example, temperatures, sea level and bank balances.

## Integers

- In maths, different types of numbers are divided into sets. The set of **integers** includes:
  - → positive whole numbers
  - → negative whole numbers
  - → zero.
- Integers can be shown on a number line.
  - → The numbers on a number line increase in value going from left to right.
  - → The closer a number is to 0, the smaller the **magnitude** of the number.
  - → On a horizontal number line, negative numbers are located to the left of the zero because they are less than zero in value.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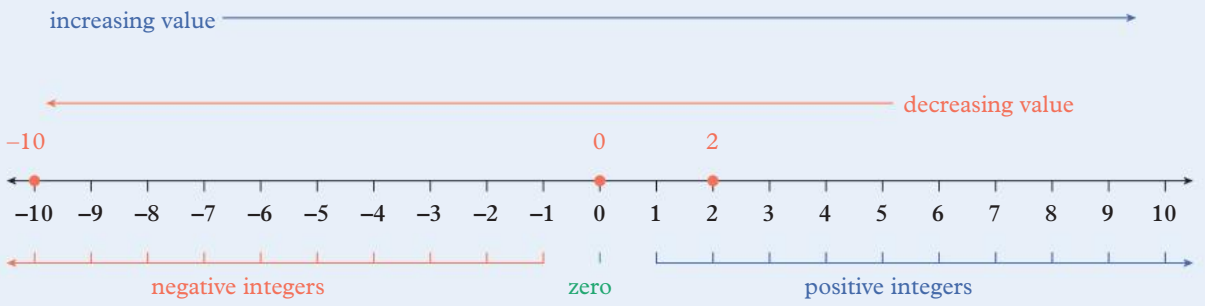
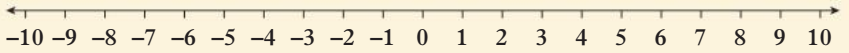



- Fractions and decimals can also be shown on a number line.

## Example 5A.1 Comparing positive and negative numbers

Locate the following numbers on the number line in order to complete each number statement using the less than, <, or greater than, >, symbols.

**a** 5 _____ –8 **b** –3 _____ $-\frac{1}{3}$




**THINK**

**a** **1** Locate each number on the number line. 5 is positive and located to the right of the zero. –8 is negative and located to the left of the zero.

**2** Compare the value of the numbers. 5 is located farther to the right on the number line, so 5 is greater than –8.

**3** Complete the statement using the correct symbol.

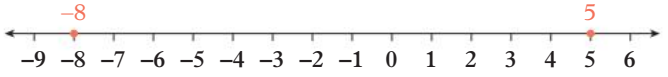
**b** **1** Locate each number on the number line. Both numbers are negative and located to the left of the zero. $-\frac{1}{3}$ is located a third of the distance between 0 and –1 from 0.

**2** Compare the value of the numbers. $-\frac{1}{3}$ is located farther to the right on the number line, so $-\frac{1}{3}$ is greater in value than –3, or –3 is less than $-\frac{1}{3}$.

**3** Complete the statement.

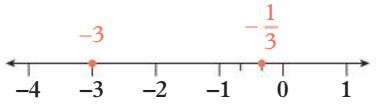
**WRITE**

**a**




$5 > -8$

**b**




$-3 < -\frac{1}{3}$

## Example 5A.2 Ordering positive and negative numbers

Arrange these numbers in ascending order.

$6, -\frac{7}{2}, -1$

**THINK**

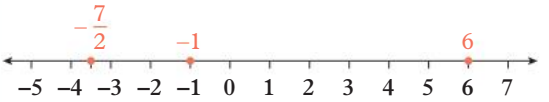
1 Locate the numbers on the number line. To locate the improper fraction on the number line, convert the improper fraction into a mixed number. $-3\frac{1}{2}$ is located halfway between $-4$ and $-3$.

2 Arranging the numbers in 'ascending order' involves ordering the numbers from smallest to largest in value. The numbers increase in value reading left to right along the number line.

**WRITE**

$-7 \div 2 = -3$ remainder 1

$-\frac{7}{2} = -3\frac{1}{2}$




$-\frac{7}{2}, -1, 6$

**Helpful hints**

✔ Although the negative sign is identical to the minus sign, remember that the meaning of the symbol depends on its context!
For example, $-8 - 2$ is read as negative eight minus two.

ANS p564

# Exercise 5A Negative numbers

1, 2–3(1st, 2nd columns), 4–5(a–d), 6–8, 10–12, 16 | 2–3(2nd, 3rd columns), 4–5(d–f), 9, 11, 14–17, 19(a) | 2–3(g–i), 4–5(e, f), 9(d–f), 13, 17–20

UNDERSTANDING AND FLUENCY

**1** **a** Place each of the following numbers on a number line.

**i** 2 **ii** 4 **iii** 0 **iv** $-5$ **v** $-2$ **vi** $-1$

**b** Which is the larger number in each of the following pairs of numbers?

**i** 2 and 4 **ii** 0 and $-5$ **iii** $-2$ and 4

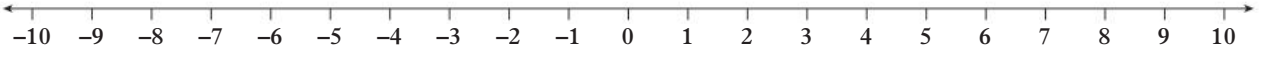
**iv** $-2$ and $-5$ **v** 4 and $-1$ **vi** $-1$ and $-2$

**c** Which is the smaller number in each of the following pairs of numbers?

**i** 1 and 3 **ii** 0 and 1 **iii** $-1$ and 5

**iv** $-2$ and $-3$ **v** 3 and $-1$ **vi** $-4$ and $-3$

5A.1 **2** Locate the following numbers on the number line in order to complete each number statement using the less than, <, or greater than, >, symbols.




**a** $-4$ _____ 6 **b** 0 _____ $-3$ **c** $-5$ _____ $-9$

**d** $-7$ _____ $-\frac{1}{7}$ **e** $-2$ _____ $-2\frac{2}{3}$ **f** $-\frac{5}{4}$ _____ $-2$

**g** $-4\frac{2}{5}$ _____ $-\frac{20}{5}$ **h** $-\frac{7}{2}$ _____ $-3\frac{2}{3}$ **i** $-\frac{13}{3}$ _____ $-\frac{21}{5}$

**3** Complete each number statement using the less than, <, or greater than, >, symbols.

**a** $-40$ _____ $30$ **b** $0$ _____ $-22$ **c** $-65$ _____ $-55$

**d** $78$ _____ $-\frac{100}{2}$ **e** $-\frac{127}{10}$ _____ $-\frac{328}{100}$ **f** $-150\frac{79}{190}$ _____ $0$

**g** $248\frac{23}{25}$ _____ $-300\frac{67}{70}$ **h** $-\frac{120}{7}$ _____ $15\frac{5}{7}$ **i** $-\frac{143}{9}$ _____ $-\frac{95}{6}$

5A.2 **4** Arrange these numbers in ascending order.

**a** $4, -6, 6, -2, 0$ **b** $-13, 10, -3, 4, -\frac{5}{2}$

**c** $-1, -\frac{2}{3}, 1, \frac{10}{3}, -12$ **d** $16, -5\frac{2}{3}, 10, -5, 5, -\frac{9}{4}$

**e** $-20, 25\frac{35}{40}, 25, -20\frac{4}{5}, -20.75, 25\frac{17}{20}$ **f** $-1, -\frac{12}{13}, 0.6, 1\frac{5}{6}, -\frac{12}{10}, -1.25$

**5** Arrange these numbers in descending order.

**a** $2.1, -5, 4.8, -6, 8$ **b** $-35, -46.2, -12, 43, 80.8$

**c** $-4.7, 8.9, -12.5, 6.5, -2.3$ **d** $7.6, -4\frac{1}{3}, 15, -4.5, 12.5, -\frac{11}{2}$

**e** $-50.3, -36\frac{25}{45}, 48.9, -45\frac{4}{5}, -48.9, \frac{12}{5}$ **f** $-11, -\frac{120}{11}, -0.11, -11\frac{5}{6}, -\frac{110}{12}, -11.25$

**6** List all the integers between $-5$ and $7$ (not including $-5$ and $7$).

**7** List five fractions between $-1$ and $1$ in ascending order.

**8** List five mixed numbers between $-5$ and $5$ in descending order.



**9** Write an integer to represent each of the following situations.

**a** The temperature at a ski resort is 3 degrees below zero.

**b** You have \$360 in the bank.

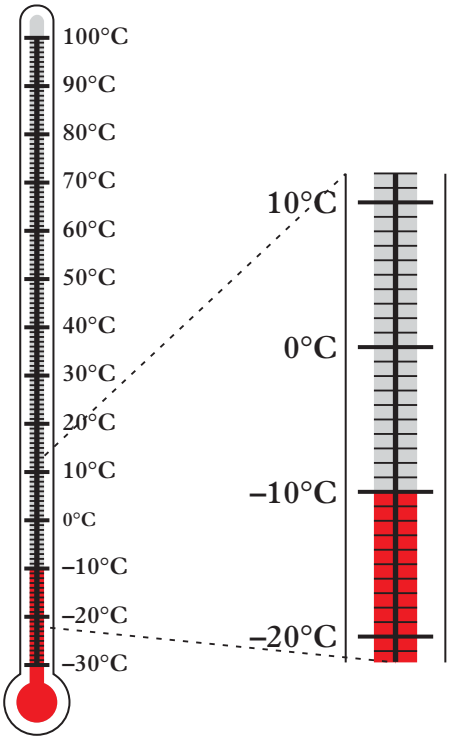
**c** The lift stops at the second floor below the ground floor.

**d** Lake Eyre is 15 m below sea level.

**e** The top of Mt Kosciuszko is 2230 m above sea level.

**f** Your bank account is overdrawn by \$28.

**10** Some thermometers have scales that show positive and negative temperatures.




**a** What is the temperature shown on this thermometer?

**b** Determine whether each of the temperatures listed below is higher or lower than the temperature shown on the thermometer.

**i** 8°C

**ii** 0°C

**iii** −15°C

**iv** −8°C

**c** How can you tell whether a number is larger or smaller on the thermometer scale?

**11** A group of King penguins and Gentoo penguins is living in a specially designed enclosure at the Melbourne Aquarium. The temperature of their environment is kept between −10°C and 2°C.

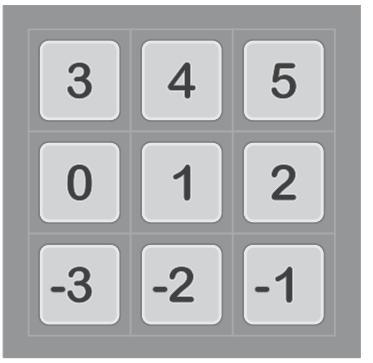
**a** Show these two temperatures on a number line.

**b** Which temperature is lower?

**c** The pool contains salt water kept at a temperature of −1°C. List the three temperatures in ascending order.

**d** If the temperature of the air in the enclosure is −9°C, would it feel warmer or cooler to a penguin leaving the pool?

**12** The panel of buttons in the lift of a multi-storey school is shown below.




**a** What is the highest level you can travel to in the lift?

**b** What is the lowest level you can travel to in the lift?

**c** How many levels does the school have?

**d** Which button most likely corresponds to ground level?

**e** Describe the number of floors you move between, and in what direction, if you enter the lift at ground level and press the button labelled '3'.

**f** Describe the number of floors you move between, and in what direction, if you enter the lift at ground level and press the button labelled '−2'.

**g** Describe the number of floors you move between, and in what direction, if you enter the lift at level −1 and press the button labelled '3'.

**h** What does the negative sign represent in this problem?

**13** Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

**a** Katie checks her bank balance and sees that she has $100 in her account. Write this amount as a positive integer.

**b** Katie's bank allows her account to be overdrawn. This means that she can withdraw more money than she actually has in her account. If she withdraws $120 at an ATM, how much does she owe the bank?

**c** Write her new bank balance as a negative integer.

**14** In financial terminology, you are said to be 'in the black' if you have money and 'in the red' if you owe money. Some students either have money in their accounts or owe money to a bank in the following amounts.

| Student | Amount |
|---|---|
| Paulo | $85.50 |
| Kamilla | $98.05 |
| Amad | $135.98 |
| Jessica | $172.20 |

**a** Write each student's bank balance as a positive or a negative number.

**b** Who has the most money in the bank?

**c** Who owes the most money to the bank?

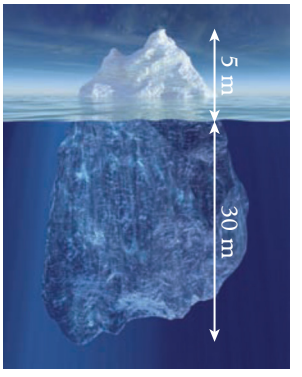
**d** List the bank balances in order of increasing value. Explain your reasoning.

**15** Zero is sometimes described as a 'reference point', with the sign of a number indicating its direction from zero. Raj's school is located 2 kilometres from his house. Raj's mum is a doctor and works at a hospital located 9 kilometres from their house, in the opposite direction to Raj's school.

**a** Locate Raj's school and the hospital where Raj's mum works on the number line below.

**b** What is the distance between Raj's school and the hospital where Raj's mum works?

**16** In 1912, the luxury liner RMS *Titanic* sank on its maiden voyage after its hull was damaged by an iceberg, which lay mostly under water. If the surface of the water, referred to as 'sea level', is assigned a value of 0 metres, complete the following with reference to the figure on the right.




**a** Represent the height of the part of the iceberg that is located above sea level using a positive integer.

**b** Represent the length of the iceberg that lies under water using a negative integer.

**c** What is the total height of this iceberg?

**d** The wreck of the *Titanic* now lies on the sea floor, 3800 m below sea level. Write this value as a negative integer.

PROBLEM SOLVING AND REASONING

**17** Gannets are large seabirds that fish by diving into the sea and pursuing their prey underwater. For this question take the surface of the water (sea level) to be at a height of 0 metres.

**a** A gannet is flying $25\frac{1}{2}$ metres above sea level. Represent the gannet's height using a positive mixed number.

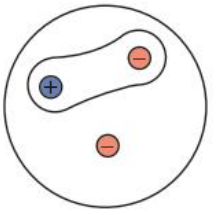
**b** A squid is swimming $12\frac{1}{3}$ metres below sea level. Represent the squid's depth using a negative mixed number.

**c** A shark is swimming at $25\frac{9}{10}$ metres below the squid. How far below sea level is the shark? Explain your answer in words, or with a diagram.

**d** Is the gannet or the shark closer to the squid? Use a number line to justify your answer.

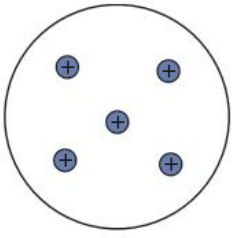
**18** When something is electrically charged, it contains an imbalance of positive and negative electric charges. The total electric charge of an object can be determined by pairing as many positive and negative charges as possible and then counting the remaining positive or negative charges that are not paired.

For example, the object represented by the diagram to the right has a charge of −1, because there is one negative charge that is not paired with a positive charge.

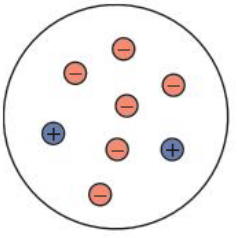


**a** For each diagram below, pair as many positive and negative charges as possible and then determine the electric charge of each object.

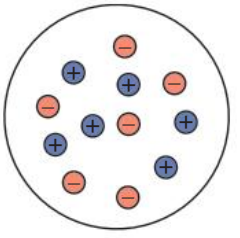
**i**



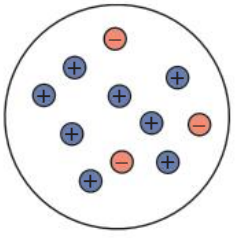
**ii**



**iii**



**iv**



**b** Which diagrams have the same electric charge? Explain how this is possible when they have different numbers of positive and negative charges.

CHALLENGE

**19 a** I have a single-digit negative integer and a single-digit positive integer that add together to give a positive integer. What numbers could they be? Justify your answer.

**b** I have a single-digit negative integer and a two-digit positive integer. If I subtract one integer from the other, I get a negative integer. What numbers could they be? Justify your answer.

**20** The coldest temperature that is physically possible is called 'absolute zero'. When measured in degrees Celsius absolute zero is approximately −273°C.

The Celsius scale was invented as a way of measuring temperature so that 0°C is the freezing temperature of water. Scientists often use another scale for measuring temperature called the Kelvin scale, which is the same as Celsius, except 0 Kelvin (0 K) is set at absolute zero.

**a** Is it physically possible for the temperature to be negative Kelvin? Justify your answer.

**b** Draw a number line to represent the Kelvin scale from 0 K to 300 K.

**c** Mark your number line at the point that corresponds to 0°C. Write this temperature in Kelvin.

**d** Use your number line to convert the following temperatures from the Celsius scale (°C) to the Kelvin scale (K).

**i** −273°C **ii** −10°C **iii** 27°C

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Ordering positive and negative numbers

Investigation
Weather at Thredbo

Topic quiz
5A

# 5B Adding and subtracting positive numbers

## Learning intentions

- ✔ I can add and subtract positive integers.
- ✔ I can solve problems involving addition and subtraction of positive integers.

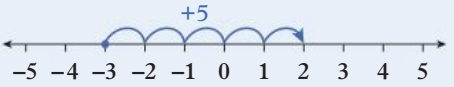
**Inter-year links**

| | |
|---|---|
| Year 5/6 | Negative numbers |
| Year 8 | 1F Adding and subtracting integers |

## Adding positive integers

- Adding a positive integer increases the value of the number, which corresponds to moving right on the number line.

  For example, to add 5, move 5 steps to the right:

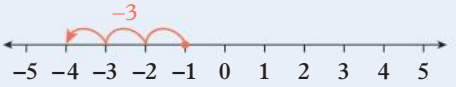



$-3 + 5 = 2$

## Subtracting positive integers

- Subtracting a positive integer decreases the value of the number, which corresponds to moving left on the number line.

  For example, to subtract 3, move 3 steps to the left:




$-1 - 3 = -4$

### Example 5B.1 Adding and subtracting positive integers

Use the number line to calculate:

**a** $-10 + 4$ **b** $5 - 7$

**THINK**

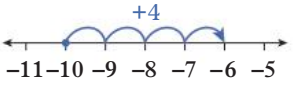
**a** **1** Draw a number line. Make sure the first number in the calculation is included on the number line.

**2** Start at the first number, then consider whether the second number will increase or decrease the value of the first number. Adding 4 will increase the value of the first number, so move 4 steps to the right.

**3** Write down the answer.

**WRITE**

**a**



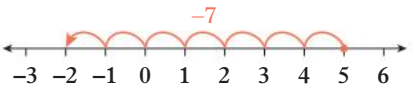

$-10 + 4 = -6$

**b** **1** Draw a number line.

**2** Start at the first number, then consider whether the second number will increase or decrease value. Subtracting 7 will decrease the value, so move 7 steps to the left.

**3** Write down the answer.

**b**




$5 - 7 = -2$

## Example 5B.2 Writing addition and subtraction problems

The temperature inside Helen's house is −4°C. She turns on the heater and, after half an hour, the temperature of the house has increased by 15°C.

**a** Write a calculation that can be used to determine the temperature of the house after half an hour.

**b** What will the temperature of the house be after half an hour?

**THINK**

**a** **1** Identify the starting temperature.

**2** Identify the change in temperature and include the appropriate sign. An increase of 15°C means add 15, or +15.

**3** Write the calculation by combining the starting temperature and the change in temperature.

**b** Determine the answer by completing the calculation.

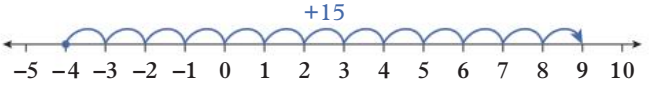
**WRITE**

**a** Starting temperature: −4°C

Change in temperature: +15°C

$-4 + 15$

**b**




$-4 + 15 = 9$

After half an hour, the house will be 9°C.

**Helpful hints**

✔ Avoid making assumptions about the sign of the answer; subtraction can produce positive numbers and addition can produce negative numbers.

For example, $-5 + 3 = -2$ and $5 - 3 = 2$.

✔ Take care when crossing over 0!

For example, $3 - 5 = -2$.

ANS p565

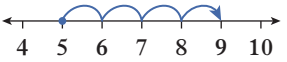
# Exercise 5B Adding and subtracting positive numbers

1–4, 5(a–c), 6(a–d), 7, 8, 11, 13, 14 | 2–3(2nd, 3rd columns), 4, 5(d–f), 6(a, c, d, e), 8, 10, 12, 13, 15 | 3(g–i), 5–6(d–f), 9, 12–16

UNDERSTANDING AND FLUENCY

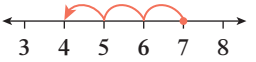
**1** Complete each number sentence using the diagrams provided.

**a**



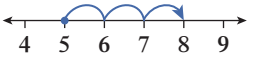

5 + 4 = _____

**b**



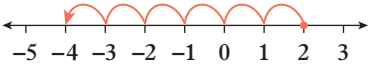

7 − 3 = _____

**c**



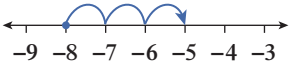

_____ + 3 = _____

**d**



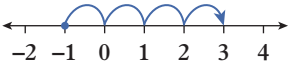

+2 − _____ = _____

**e**



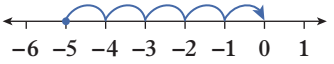

−8 + 3 = _____

**f**



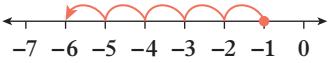

−1 + _____ = _____

**g**




_____ + _____ = _____

**h**




_____ − _____ = _____

5B.1 **2** Use the number line to calculate the following.

**a** 2 + 5 **b** −3 + 4 **c** −9 + 7
**d** 7 − 2 **e** −4 − 6 **f** −2 + 3
**g** 5 − 9 **h** −7 + 7 **i** −8 + 5

**3** Complete each of the following calculations. Hint: Try visualising the calculation on a long number line.

**a** 10 − 15 **b** −18 + 5 **c** −11 + 6
**d** −50 − 30 **e** −68 + 9 **f** −55 + 28
**g** 62 − 72 **h** −77 + 79 **i** 123 − 255

**4** Determine the number that is:

**a** 5 more than −3 **b** 7 less than 2 **c** 6 more than −6
**d** 8 less than −1 **e** 4 more than −5 **f** 3 less than 1

**5** Recall the rules for adding and subtracting decimals to complete each of the following calculations without using a calculator.

**a** 1.8 − 3.5 **b** −15.4 + 7.9 **c** −23.62 + 30.41
**d** 10.17 − 22.95 **e** −3.812 + 12.055 **f** 5.66 − 17.125

**6** Recall the rules for adding and subtracting fractions and mixed numbers to complete each of the following calculations without using a calculator.

**a** $-\frac{7}{11}+\frac{10}{11}$ **b** $3\frac{3}{5}-4\frac{1}{5}$ **c** $\frac{30}{21}-\frac{35}{7}$
**d** $-\frac{21}{5}+\frac{8}{3}$ **e** $-3\frac{5}{9}-6\frac{2}{3}$ **f** $\frac{13}{20}-\frac{14}{15}$

**7** What number, when added to each of the following numbers, will give 0?

**a** −4
**b** 7
**c** 1
**d** −10

8 Without performing each calculation, decide whether the result of the following will be a positive integer, a negative integer or zero.

**a** $-12 + 9$ **b** $-30 + 20$ **c** $25 - 18$

**d** $-45 + 45$ **e** $-29 - 15$ **f** $-19 + 24$

9 Describe how you tell by looking at the numbers in a sum whether the result will be positive or negative or zero.

5B.2 10 Party pies are stored in a freezer at −18°C. To serve them, the party pies are heated up by 98°C.

**a** Write a calculation that can be used to determine the temperature of the pies after they have been heated up.

**b** What will the temperature of the party pies be after they have been heated up?

11 A dolphin is swimming 2 m below the surface of the water and then leaps up a vertical distance of 3 m. How high above the water's surface does the dolphin reach?

12 The temperature at 6 am is −8.25°C. One hour later the temperature has increased by 5.3°C.

**a** Write a calculation that can be used to determine the temperature at 7 am.

**b** What will be the temperature at 7 am?

13 Geetika owes her sister $50.

**a** Represent this amount as a negative integer.

**b** Geetika pays back $35. Write a sum to represent this problem.

**c** After paying back $35, how much does Geetika owe her sister?

14 Oscar has overdrawn his bank account by $26.95.

**a** Represent this amount as a negative number.

**b** Oscar deposits $40 into his account. Write a sum to determine Oscar's new account balance.

**c** What is the balance of Oscar's account after he deposits the $40?

15 Mauna Kea in Hawaii is the 'tallest' mountain in the world with a total height of 10 210 metres. However, 6005 metres of Mauna Kea is located below sea level, so Mount Everest gets the title of 'highest' mountain in the world with a total height of 8848 metres, all of which is located above sea level.

**a** What is the height of the Mauna Kea above sea level?

**b** How much higher above sea level is the peak of Mount Everest than the peak of Mauna Kea?

**c** If the base of both mountains were located at sea level, how much higher would the peak of Mauna Kea be than the peak of Mount Everest?

16 Two identical 4-sided dice are rolled, and the two numbers are added together. The list of all possible results is given below.

−6, −4, −2, 0, 2, 4, 6

What are the numbers on the dice? Explain how you solved this problem.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Worksheet** Adding integers using counters

**Investigation** Modelling addition and subtraction

**Topic quiz** 5B

# 5C Adding and subtracting negative numbers

## Learning intentions

- ✔ I can add and subtract negative integers.
- ✔ I can solve problems involving addition and subtraction of negative integers.

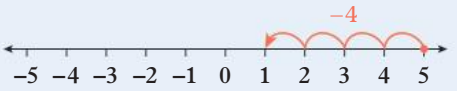
**Inter-year links**

| | |
|---|---|
| Year 5/6 | Negative numbers |
| Year 8 | 1F Adding and subtracting integers |

## Adding negative integers

- Adding a negative integer decreases the value of the number and corresponds to moving left on the number line.

  For example, to add −4, move 4 steps to the left:

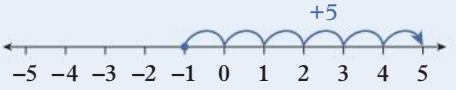



$5 + (-4) = 5 - 4$
$= 1$

## Subtracting negative integers

- Subtracting a negative integer increases the value of the number and corresponds to moving right on the number line.

  For example, to subtract −5, move 5 steps to the right:




$-1 - (-5) = -1 + 5$
$= 4$

## Adding and subtracting negative numbers

- Adding a negative number is equivalent to subtracting a value. $+(-) = -$
- Subtracting a negative number is equivalent to adding a value. $-(-) = +$

→ Note that brackets are being used to separate the operation and the negative sign of the second number.

minus sign

$-1 - (-5)$

negative signs

## Example 5C.1 Adding and subtracting negative integers

Use the number line to calculate:

**a** $3 + (-4)$

**b** $-6 - (-2)$

**THINK**

**a** **1** Simplify the calculation. Adding a negative number is equivalent to subtracting a value, so replace $+(-)$ with $-$.

**2** Draw a number line.

**3** Start at the first number, then subtract the value by moving left.

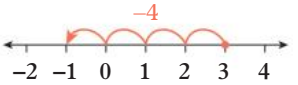
**b** **1** Simplify the calculation. Subtracting a negative number is equivalent to adding a value, so replace $-(-)$ with $+$.

**2** Draw a number line.

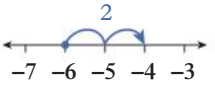
**3** Start at the first number, then add the value by moving right.

**WRITE**

**a** $3 + (-4) = 3 - 4$
$= -1$




**b** $-6 - (-2) = -6 + 2$
$= -4$




## Example 5C.2 Writing problems involving addition and subtraction of negative integers

A scuba diver is swimming at a depth of 8 metres below sea level when they spot a leatherback turtle swimming at a depth of 13 metres below sea level.

**a** Represent the depth of the diver and the depth of the turtle as negative integers.

**b** Write a difference calculation that can be used to determine the difference between the depth of the diver and the depth of the turtle.

**c** What is the difference between the depth of the turtle and the depth of the diver?

**THINK**

**a** Identify the depth of the diver and the turtle.

**b** To calculate the difference between the depth of the diver and the turtle, subtract the depth of the turtle from the depth of the diver.

**c** Determine the answer by simplifying, then completing the calculation.

**WRITE**

**a** Diver: $-8$ metres
Turtle: $-13$ metres

**b** $-8 - (-13)$

**c** $-8 - (-13) = -8 + 13$
$= 5$

The difference between the depth of the turtle and the depth of the diver is 5 metres.

**Helpful hints**

- ✔ Write your calculations out in full and double-check any changes in the sign to avoid making errors in the arithmetic.
- ✔ When simplifying calculations involving negative numbers, only combine the signs located *between* the numbers.

  The sign of the first number indicates the starting point of the calculation on the number line.

start to the left of 0

$$-1 - (-5) = -1 + 5$$
$$- (-) = +$$

ANS p566

# Exercise 5C Adding and subtracting negative numbers

△ 1–3, 4–8(1st, 2nd columns), 10, 12, 15(a–d)

□ 1–2(c, d), 3–5(b, d, f, g), 6–7(b, c, e), 8, 9(a, b), 11, 14, 15(a–d), 16

⬠ 1(c, d), 3(g, h), 5–8(3rd column), 9(c, d), 13, 15–18

UNDERSTANDING AND FLUENCY

**1** Simplify each of the following calculations by combining the operation with the negative sign to write an equivalent calculation.

**a** $-3 - (-5)$ **b** $6 + (-5)$ **c** $-2 + (-4)$ **d** $7 - (-1)$

**2** Complete each of the number sentences below using a plus or minus sign to write an equivalent calculation.

**a** $-3 + (-6) = -3$ __ $6$ **b** $1 - (-3) = 1$ __ $3$

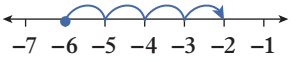
**c** $8 - (-9) = 8$ __ $9$ **d** $-5 + (-7) = -5$ __ $7$

**3** Complete each number sentence using the diagrams provided.

**a**

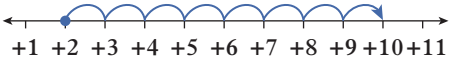
$7 + (-4) =$ _____

**b**



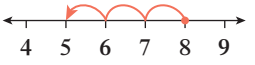

$-6 - (-4) =$ _____

**c**



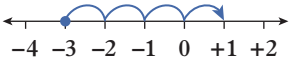

$2 - (-8) =$ _____

**d**



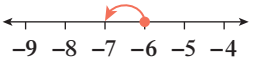

$8 +$ _____ $=$ _____

**e**



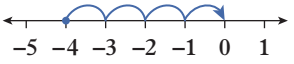

_____ $- (-4) =$ _____

**f**



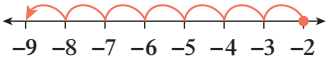

$-6 + (-1) =$ _____

**g**




_____ $-$ _____ $=$ _____

**h**




_____ $+$ _____ $=$ _____

5C.1 **4** Use the number line to calculate:

**a** $1 + (-8)$ **b** $-2 - (-3)$ **c** $1 - (-5)$

**d** $6 + (-5)$ **e** $-3 - (-1)$ **f** $-7 - (-7)$

**g** $4 - (-1)$ **h** $5 - (-5)$ **i** $-6 - (-9)$



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**5** Complete each of the following calculations.

**a** $20 + (-7)$ **b** $-24 - (-13)$ **c** $-14 + (-15)$

**d** $-13 + (-10)$ **e** $38 - (-27)$ **f** $-82 - (-14)$

**g** $21 + (-43)$ **h** $-115 - (-44)$ **i** $250 - (-50)$

**6** Recall the rules for adding and subtracting decimals to complete each of the following calculations without a calculator.

**a** $2.4 + (-1.3)$ **b** $8.8 - (-11.3)$ **c** $-16.33 - (-12.54)$

**d** $-25.01 + (-7.85)$ **e** $-5.123 - (-15.683)$ **f** $9.736 + (-12.044)$

**7** Recall the rules for adding and subtracting fractions and mixed numbers to complete each of the following calculations without using a calculator.

**a** $\frac{7}{12} - \left(-\frac{5}{12}\right)$ **b** $1\frac{3}{7} + \left(-3\frac{1}{7}\right)$ **c** $-\frac{13}{6} + \left(-\frac{15}{18}\right)$

**d** $-12\frac{5}{6} - \left(-3\frac{3}{18}\right)$ **e** $\frac{17}{4} + \left(-\frac{23}{5}\right)$ **f** $-\frac{5}{12} - \left(-\frac{22}{7}\right)$

**8** Without performing each calculation, decide whether the result of the following will be a positive integer, a negative integer, or zero.

**a** $13 + (-8)$ **b** $25 - (-35)$ **c** $-18 - (-15)$

**d** $-33 - (-19)$ **e** $53 + (-53)$ **f** $-26 + (-42)$

**9** What integer should be subtracted from each of these to give a result of 0?

**a** $-5$ **b** $+2$ **c** $-8$ **d** $7$

5C.2 **10** Food in the freezer is frozen to a temperature of −15°C, while food in the main section of the fridge is cooled to a temperature of 4°C.

**a** Write a difference calculation that can be used to determine the difference between the temperature in the main section and the temperature in the freezer.

**b** What is the difference in temperature between the two sections of the fridge?

**11** At Mount Buller, the daily maximum and minimum temperatures were recorded over a week.

| | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|---|
| **Maximum temperature (°C)** | 7 | 3 | 8 | 2 | −1 | 4 | 5 |
| **Minimum temperature (°C)** | 1 | −2 | −1 | −3 | −5 | −2 | 0 |

**a** Calculate the difference between the minimum and maximum temperatures for each day of the week.

**b** Which day had the biggest range of temperature?

**12** Ayesha has $74 in her bank account.

**a** Represent Ayesha's account balance as a positive integer.

**b** Ayesha withdraws some money from her account. Her account balance is now −$31. Write a difference calculation that can be used to determine the amount of money Ayesha withdrew from her account.

**c** How much money did Ayesha withdraw from her account?

**13** The depth of a cave is determined by measuring the distance from its highest entrance to the lowest accessible point in the cave. Turquoise Cave has a depth of 56.83 metres and Amethyst Cave has a depth of 75.29 metres.

**a** Represent the depth of each cave using negative numbers.

**b** Subtract the depth of Turquoise Cave from Amethyst Cave.

**c** Subtract the depth of Amethyst Cave from Turquoise Cave.

**d** Explain why your answers to parts **b** and **c** are different.

**e** Which answer represents the difference in the depth of each cave? Explain.

PROBLEM SOLVING AND REASONING

**14** This incomplete bank statement shows some of the deposits and withdrawals made to Ayesha's bank account.

| Date | Transaction | Balance |
|---|---|---|
| 30 June | | +\$75.50 |
| 5 July | −\$24.99 | |
| 13 July | −\$68.23 | |
| 24 July | +\$45.55 | |
| 31 July | | +\$19.05 |

**a** Does a transaction of +\$45.55 represent a deposit or a withdrawal?

**b** Does a transaction of −\$24.99 represent a deposit or a withdrawal?

**c** Explain what it means if the balance in Ayesha's account is positive.

**d** Explain what it means if the balance in Ayesha's account is negative.

**e** Find the balance of Ayesha's account after the transaction made on 24 July.

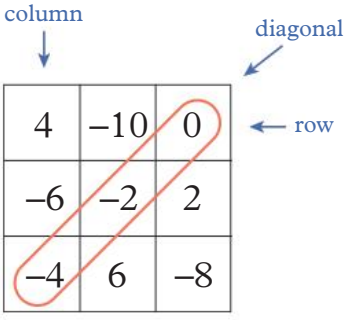
**f** What transaction was made on 31 July to give an account balance of +\$19.05?

**15** Complete each of the calculations below. Remember to work from left to right.

**a** $-2 - 9 + 3$ **b** $7 + (-4) - (-6)$ **c** $-5 - (-1) + 8$ **d** $6 + (-3) - (-11)$

**e** $(-10) - 7 + 2$ **f** $-4 - (-4) + (-5)$ **g** $1 + (-9) - 3$ **h** $-8 - 6 - (-17)$

**16** Nine numbers have been arranged as a square as shown on the right.




**a** Add the three numbers in:

**i** the first row **ii** the second row **iii** the third row.

**b** Add the three numbers in:

**i** the first column **ii** the second column **iii** the third column.

**c** Add the three numbers in each diagonal.

**d** This is an example of a 'magic square'. The sum of each row, column and diagonal is called the 'magic sum'. What is the magic sum of this magic square?

CHALLENGE

**17** Complete these magic squares by first calculating the magic sum.

**a**

| | | |
|---|---|---|
| 4 | −3 | 2 |
| −1 | | 3 |
| 0 | 5 | |

**b**

| | | |
|---|---|---|
| | | −4 |
| | −1 | |
| 2 | −2 | |

**c**

| | | |
|---|---|---|
| 6 | | |
| −18 | | |
| 3 | | −12 |

**18** Datt spills water on his homework. Help him out by determining the value of the missing numbers.

**a** $\frac{7}{12} + \frac{3}{\square} = 1\frac{1}{3}$ **b** $\frac{5}{8} - \left(-\frac{\square}{5}\right) = \frac{49}{40}$ **c** $\frac{8}{5} + \left(-\frac{\square}{15}\right) = 1\frac{1}{5}$

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Adding and subtracting with negative numbers

**Worksheet** Subtracting integers using counters

**Investigation** Slide rule

**Topic quiz** 5C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

5A **1** **a** Place each of the following numbers on a number line.

**i** 3 **ii** $\frac{7}{2}$ **iii** 0
**iv** −3 **v** −2.5 **vi** $-1\frac{1}{3}$

**b** Using your answer to part **a** to complete each number statement using the less-than, <, or greater-than, >, symbols.

**i** 3 _____ −3 **ii** −3_____ −2.5 **iii** $-1\frac{1}{3}$ _____ 0
**iv** $\frac{7}{2}$ _____ $-1\frac{1}{3}$ **v** −2.5_____ $\frac{7}{2}$ **vi** 0 _____ −3

5A **2** Arrange these numbers in ascending order.

**a** −4, −9, 7, −1, 0 **b** −6, 10.5, −2, 6, $-\frac{9}{2}$ **c** −1.5, $-\frac{3}{4}$, 2, $\frac{20}{3}$, −15

5B **3** Complete each of the following calculations.

**a** 20 − 26 **b** −22 + 6 **c** −16 + 7
**d** −60 − 20 **e** −35 + 5 **f** −40 + 18

5B **4** Determine the number that is:

**a** 8 more than −4 **b** 9 less than 5 **c** 12 more than −3
**d** 15 less than −5 **e** 5 more than −10 **f** 8 less than 2

5B **5** Complete each of the following calculations.

**a** 3.6 − 4.8 **b** −23.6 + 8.2
**c** −32.76 + 45.23 **d** 23.14 − 34.52

5B **6** On an exploratory cave dive, a geologist took a rock sample at a depth of 27 metres below sea level. He then ascended 12 metres before realising he had left the sample behind.

**a** Represent the depth where the geologist took the rock sample using a negative integer.
**b** Write a calculation that can be used to determine the geologist's depth when he realised that he had left the sample behind.
**c** At what depth did the geologist realise he had left his sample behind?

5C **7** Calculate:

**a** 2 + (−7) **b** −6 − (−4) **c** 4 − (−9)
**d** 8 + (−2) **e** −10 − (−5) **f** −9 − (−9)

5C **8** Complete each of the following calculations.

**a** 3.6 + (−1.8) **b** 10.2 − (−5.5) **c** −20.4 − (−8.9)
**d** −50.34 − (−4.98) **e** −4.68 − (−18.45) **f** 4.68 + (−24.28)

5C **9** Calculate:

**a** $\frac{3}{7}-\left(-\frac{2}{7}\right)$ **b** $2\frac{5}{6}+\left(-1\frac{1}{6}\right)$ **c** $-\frac{4}{5}+\left(-\frac{9}{10}\right)$
**d** $-5\frac{3}{8}-\left(-4\frac{1}{8}\right)$ **e** $\frac{15}{2}+\left(-\frac{7}{5}\right)$ **f** $-\frac{12}{5}-\left(-\frac{20}{3}\right)$

5C **10** A submarine is travelling at a depth of 12.2 metres below sea level. It descends a further 3.7 metres, and then ascends 5.4 metres.

**a** Represent this problem using a calculation.
**b** What is the final depth of the submarine?

# 5D The Cartesian plane

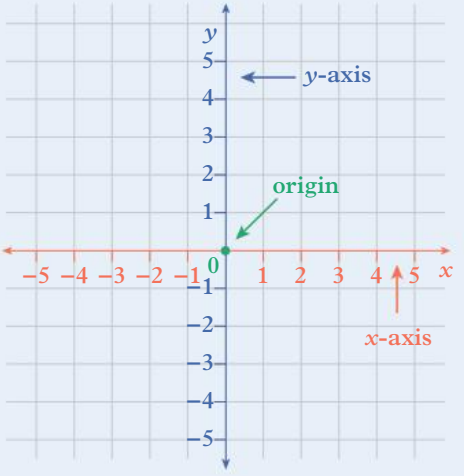
## Learning intentions

- ✔ I can plot and identify coordinate points on the Cartesian plane.
- ✔ I can plot coordinate points from a table of integer values.
- ✔ I can recognise linear relationships.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | The Cartesian plane |
| Year 8 | 6D Plotting linear and non-linear relationships |
| Year 9 | 4B Plotting linear relationships |

## The Cartesian plane




- The **Cartesian plane** is made up of two number lines that cross at right angles and intersect at zero.
  - → The horizontal number line is called the ***x*-axis**.
  - → The vertical number line is called the ***y*-axis**.
  - → The point at which the two axes intersect at zero is called the **origin**.

## Cartesian coordinates

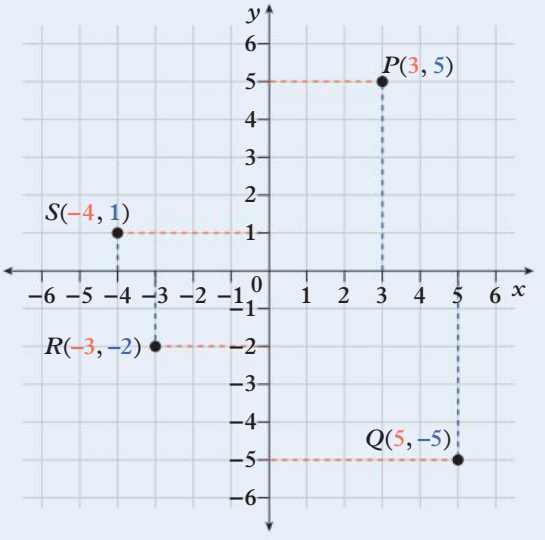
- **Cartesian coordinates** describe the position of a point on the Cartesian plane.
- The ***x*-coordinate** describes the point's horizontal distance from the origin.
- The ***y*-coordinate** describes the point's vertical distance from the origin.
- The *x*-coordinate is always listed first, followed by a comma, and then the *y*-coordinate. Coordinates are enclosed in brackets.

(−4, 1)

*x*-coordinate – horizontal distance from the origin

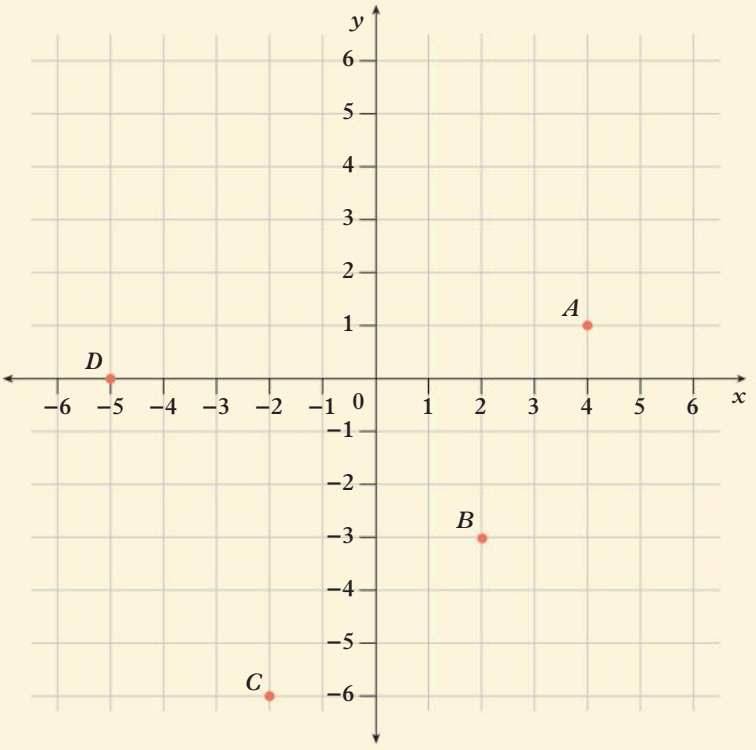
*y*-coordinate – vertical distance from the origin

- The sign of each coordinate indicates its location relative to the origin.
  - → A positive *x*-coordinate indicates that the point is right of the origin.
  - → A negative *x*-coordinate indicates that the point is left of the origin.
  - → A positive *y*-coordinate indicates that the point is above the origin.
  - → A negative *y*-coordinate indicates that the point is below the origin.
- For example, the coordinates above describe the position of a point located:
  - → 4 units to the left of the origin
  - → 1 unit above the origin.



## **Example 5D.1** Writing coordinate points

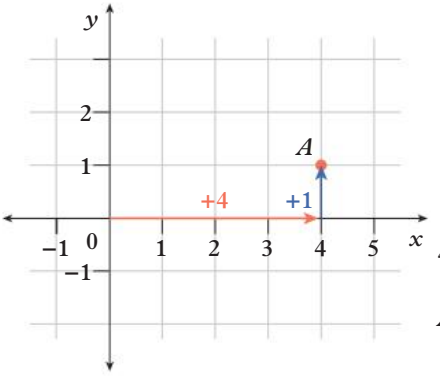
Write the coordinates of points *A*–*D* shown on the Cartesian plane below.




**THINK**

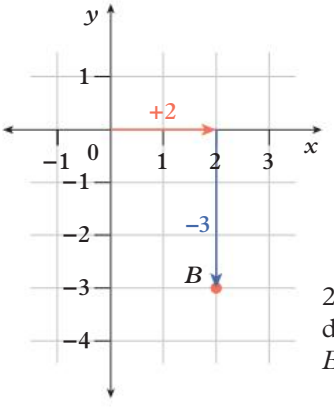
Point *A* is located 4 units to the right of the origin, so its *x*-coordinate is +4, or simply '4', and 1 unit up, so its *y*-coordinate is +1, or simply '1'.

**WRITE**



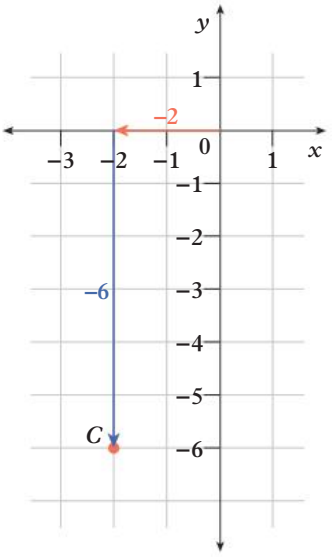

4 units right, 1 unit up
*A*(4,1)

Point *B* is located 2 units to the right of the origin, so its *x*-coordinate is 2, and 3 units down, so its *y*-coordinate is −3.



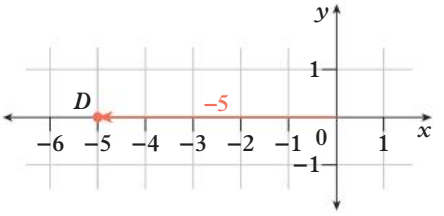

2 units right, 3 units down
*B*(2, −3)

Point $C$ is located 2 units to the left of the origin, so its $x$-coordinate is $-2$, and 6 units down, so its $y$-coordinate is $-6$.




2 units left, 6 units down
$C(-2,-6)$

Point $D$ is located 5 units to the left of the origin, so its $x$-coordinate is $-5$. As point $D$ is located on the $x$-axis, its $y$-coordinate is 0.




5 units left
$D(-5,0)$

## Example 5D.2 Plotting points on the Cartesian plane

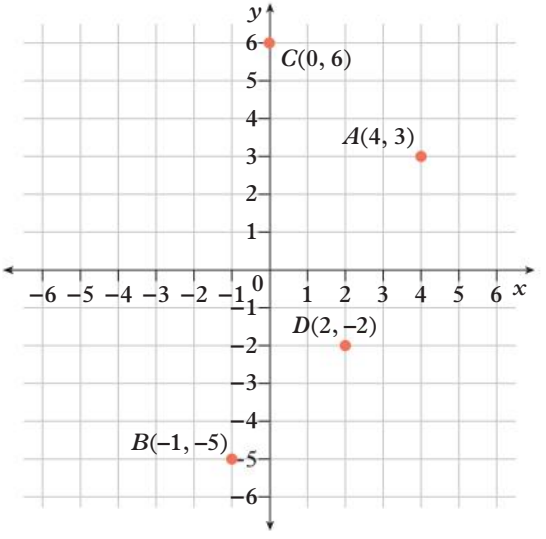
Plot and label the following points on the Cartesian plane.

**a** $A(4,3)$ **b** $B(-1,-5)$ **c** $C(0,6)$ **d** $D(2,-2)$

**THINK**

1. Draw a Cartesian plane, with the $x$- and $y$-axes drawn to include the largest coordinates.
2. Mark each point with a dot and label the dot with its letter and coordinates.
   - **a** Point $A$ has an $x$-coordinate of 4 and a $y$-coordinate of 3. Start at the origin, move 4 units to the right and 3 units up.
   - **b** Point $B$ has an $x$-coordinate of $-1$ and a $y$-coordinate of $-5$. Start at the origin, move 1 unit to the left and 5 units down.
   - **c** Point $C$ has an $x$-coordinate of 0 and a $y$-coordinate of 6. As the $x$-coordinate is 0, stay on the $y$-axis and move 6 units up.
   - **d** Point $D$ has an $x$-coordinate of 2 and a $y$-coordinate of $-2$. Start at the origin, move 2 units to the right and 2 units down.

**WRITE**



## Example 5D.3 Plotting points from a table of values and recoginising linear relationships

Consider the table of values to the right.

**a** List the coordinates of each point.

**b** Plot the points on the Cartesian plane.

**c** State whether or not the points lie on a straight line.

| ***x*-coordinate** | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| ***y*-coordinate** | −1 | 0 | 1 | 2 | 3 | 4 | 5 |

**THINK**

**a** Write out the coordinates for each point. List the *x*-coordinate first, followed by a comma, and then the *y*-coordinate.

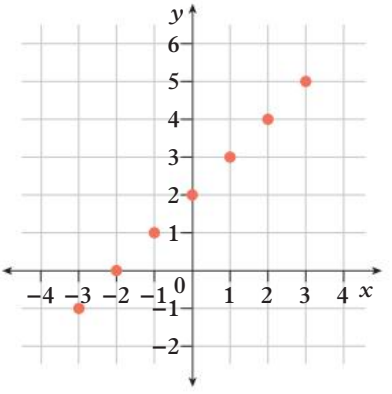
**b** Draw a Cartesian plane, and then plot each point on the plane.

**c** Examine the pattern formed by the points. Joining the points with a smooth line shows that the points lie on a straight line.

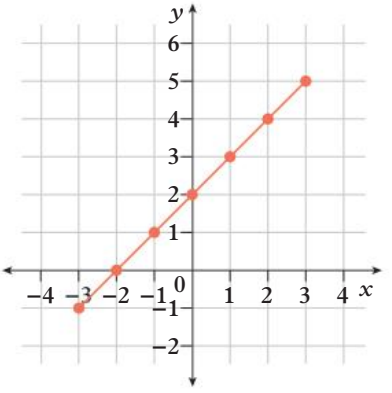
**WRITE**

**a** (−3,−1), (−2,0), (−1,1), (0,2), (1,3), (2,4), (3,5)

**b**




**c**




The points form a straight line.

**Helpful hints**

✔ Take care when reading and writing coordinates – the *x*-coordinate is always listed first, followed by the *y*-coordinate.

✔ Remember that the sign of the *x*- and *y*-coordinates indicates the position of each coordinate relative to the origin.

| | **Position relative to the origin** | |
|---|---|---|
| **Sign** | + | – |
| ***x*-coordinate** | Right | Left |
| ***y*-coordinate** | Up | Down |

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# Exercise 5D The Cartesian plane

1–6, 7(a), 8, 9(a, b), 11, 12, 14(a, b) | 3–8, 9(c, d), 10, 12–14 | 6, 7(b), 9(c, d), 10, 12, 13, 15

UNDERSTANDING AND FLUENCY

**1** Match the letter used to label each of the following coordinates on the Cartesian plane.

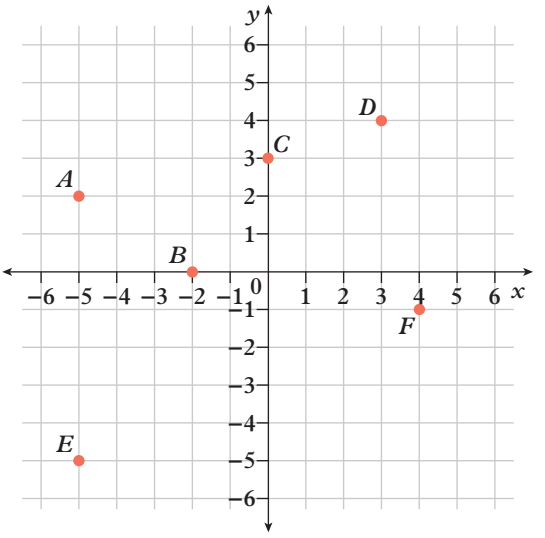
**a** (3, 4) **b** (−5, 2) **c** (4, −1)

**d** (−5, −5) **e** (0, 3) **f** (−2, 0)

**2** Which point in question **1** is:

**a** on the $x$-axis?

**b** on the $y$-axis?




5D.1 **3** Write the coordinates of the points $A$–$H$ shown on the Cartesian plane.

**4** List the points in question **4** that have:

**a** the same $x$-coordinate

**b** the same $y$-coordinate

**5** Write the coordinates of the origin.

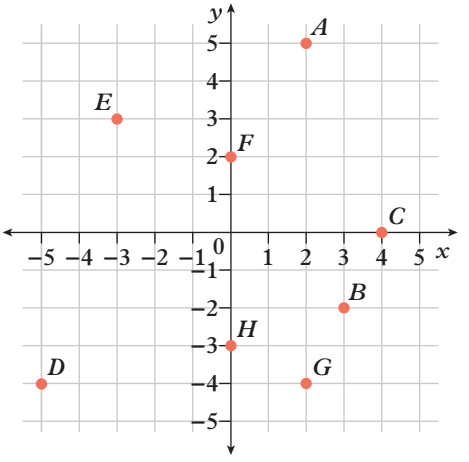
5D.2 **6** Plot and label the following points on the Cartesian plane.

**a** $A(2, -3)$ **b** $B(-4, -4)$

**c** $C(-4, 3)$ **d** $D(-1, 0)$

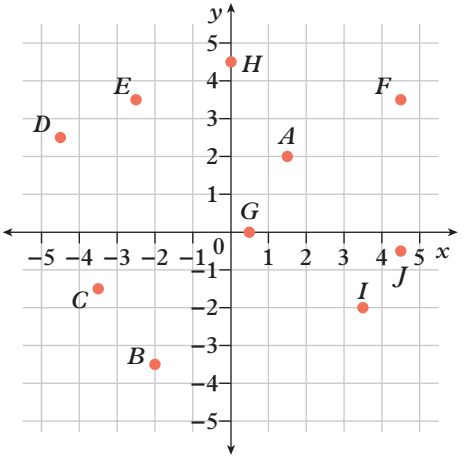
**e** $E(0, -2)$ **f** $F(-3, 2)$

**g** $G(5, -4)$ **h** $H(0, 5)$

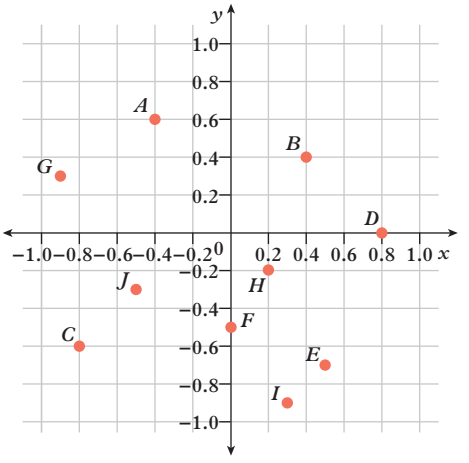



**7** Write the coordinates of the points $A$–$J$ shown on each of the Cartesian planes below.

**a**




**b**



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**8** For each of the following sets of coordinates complete the following.

**i** Draw a Cartesian plane using an axes scale that is suitable for each set of points.

**ii** Label each point with its coordinates.

**a** (−0.2, 0.4), (1.8, −0.6), (0.6, 0), (−1.2, −1.4), (0, 1.6), (2, 0.8)

**b** $\left(1\frac{1}{3},-2\frac{1}{3}\right),\left(0,-1\frac{2}{3}\right),\left(-\frac{2}{3},3\right),\left(-2\frac{2}{3},-1\frac{1}{3}\right),\left(2\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{3},0\right)$

5D.3 **9** For each of the tables of values below complete the following.

**i** List the coordinates for each point.

**ii** Plot the points on the Cartesian plane.

**iii** State whether or not the points lie on a straight line.

**a**

| ***x*-coordinate** | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| ***y*-coordinate** | −5 | −4 | −3 | −2 | −1 | 0 | 1 |

**b**

| ***x*-coordinate** | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| ***y*-coordinate** | 4 | −1 | −4 | −5 | −4 | −1 | 4 |

**c**

| ***x*-coordinate** | −3 | −3 | −3 | −3 | −3 | −3 | −3 |
|---|---|---|---|---|---|---|---|
| ***y*-coordinate** | −3 | −2 | −1 | 0 | 1 | 2 | 3 |

**d**

| ***x*-coordinate** | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|
| ***y*-coordinate** | −1.5 | 1 | 2.5 | 3 | 2.5 | 1 | −1.5 | −5 |

**10** Describe the location on the Cartesian plane of points with the following characteristics.

**a** A $x$-coordinate of 0.

**b** A $y$-coordinate of 0.

**c** Both an $x$- and a $y$-coordinate of 0.

**11** Ting is an entomologist – a scientist who studies insects. She sets up an observation post in the forest and records the insects she observes on a Cartesian plane. The observation post is located at the origin.

**a** List the coordinates of the:

**i** dragonfly **ii** butterfly

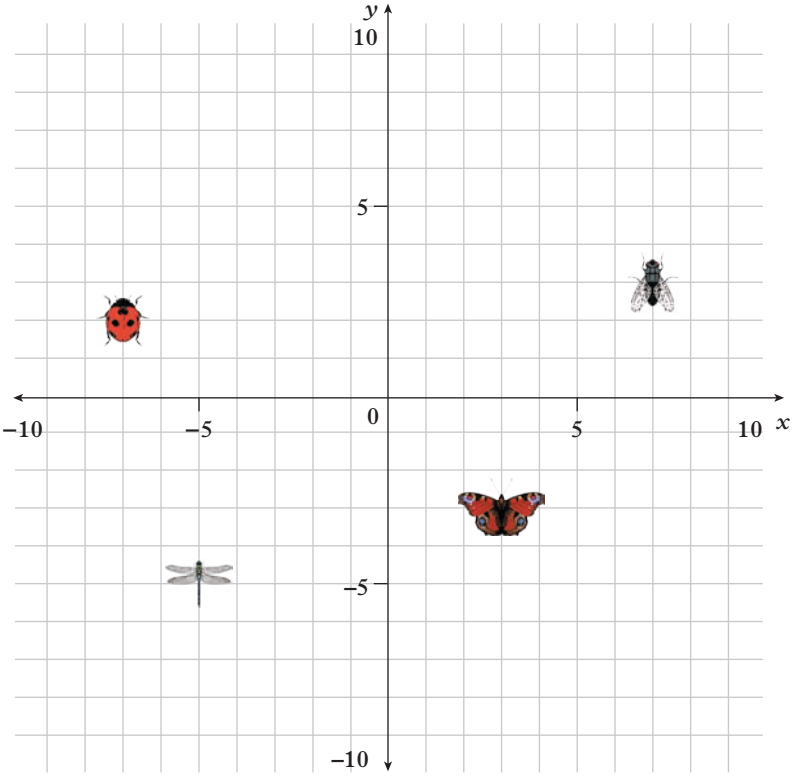
**iii** ladybird **iv** fly.

**b** Which insect is closest to the observation post?

**c** Which insect is furthest away from the observation post?

**d** Which two insects are separated by the greatest horizontal distance?

**e** Which two insects are separated by the greatest vertical distance?

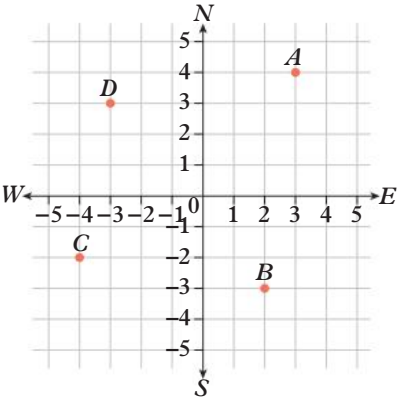



UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

**12** Oliver creates a map of his house and the houses of his four best friends: Alex ($A$), Bian ($B$), Chi ($C$) and Diaz ($D$).




**a** If Oliver's house is located at the origin:

**i** which of his friends lives furthest east of Oliver?

**ii** which of his friends lives furthest north of Oliver?

**b** If 1 square represents 1 kilometre:

**i** how far west does Diaz live from Oliver?

**ii** how far south does Chi lives from Oliver?

**c** If Bian moves 5 kilometres west and 2 kilometres south of his current home, what are the new coordinates of Bian's house?

**13** The temperature at a ski resort was recorded every 2 hours starting at midnight. The results are shown below.

| **Time (number of hours after midnight)** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Temperature (°C)** | −4 | −5 | −6 | −3 | −1 | 3 | 4 | 7 | 5 | 4 | 0 | −2 |

**a** Draw a Cartesian plane and label the horizontal axis as 'Time' and the vertical axis as 'Temperature'.

**b** Decide on the scale to use for your horizontal axis. Will you need negative numbers on the scale? Explain why or why not. Mark your scale on the time axis.

**c** Decide on the scale to use for the vertical axis. Mark your scale on the temperature axis.

**d** List the coordinates for each point. Remember that the horizontal coordinate is always written first.

**e** Plot the points listed in the table of values.

**f** What was the coldest recorded temperature? At what time of the day was this temperature recorded?

**g** What was the warmest recorded temperature? At what time of the day was this temperature recorded?

CHALLENGE

**14** Draw a Cartesian plane with both axes extending from −5 to 5.

**a** Plot the points $A(-3, 4)$ and $B(-3, -5)$.

**b** What is the distance between points $A$ and $B$?

**c** Plot the point $C(2, -5)$.

**d** What is the distance between points $B$ and $C$?

**e** Find the coordinates of point $D$ so that the points $A$, $B$, $C$ and $D$ form the corners of a rectangle.

**15** Plot two points with the coordinates $(-2, 3)$ and $(4, 3)$ on the Cartesian plane.

**a** What is the distance between the points?

**b** If a line joining these two points forms the side of a square, list the coordinates of the other two corners of the square.

**c** Part **b** has two possible answers. List the second possible answer.

**d** What is the vertical distance between the points listed in parts **b** and **c**?

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** The Cartesian plane

**Investigation** Fishy business

**Topic quiz** 5D

# 5E Interpreting graphs

## Learning intentions

✔ I can interpret and analyse graphs of real-life data.

Inter-year links

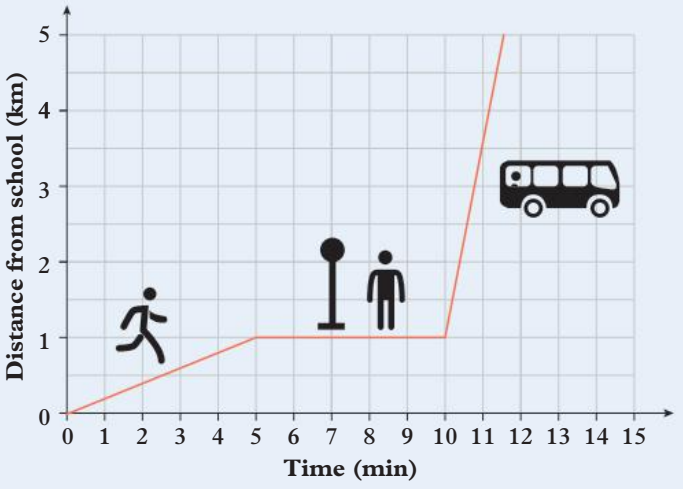
| | |
|---|---|
| Years 5/6 | Interpreting data |
| Year 8 | 6F Finding linear equations |
| Year 9 | 4E Determining linear equations |

## Interpreting graphs

- Graphs on the Cartesian plane show the relationship between two quantities.
- The **gradient** or slope of a graph indicates how quickly one quantity is changing compared to the other.

## Distance–time graphs

- **Distance–time graphs**, or **travel graphs**, show the relationship between distance covered over time when travelling at a certain speed.
  - → Time is shown in the horizontal axis.
  - → Distance is shown on the vertical axis.



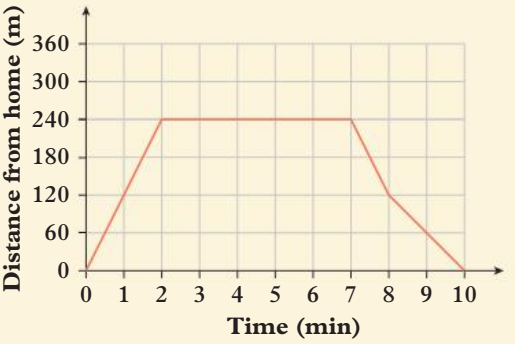

- The gradient of a distance–time graph indicates how much distance is covered per unit of time, or the **speed** of travel. The steeper the slope, the bigger the change in distance overtime, or the greater the speed of travel.
  - → On a distance–time graph, a horizontal line indicates that there is no change in distance over time, or that the person or object is not moving, or **stationary**, with a speed of zero.
- Speed is defined as distance travelled per unit time.
  - → Common measures of speed include metres per second and kilometres per hour.

    For example, if a car travels 60 kilometres in 1 hour, its speed can be calculated:

$$\text{speed} = \frac{60 \text{ kilometres}}{1 \text{ hour}}$$
$$= 60 \text{ kilometres per hour}$$

## **Example 5E.1** Interpreting distance–time graphs

Laura walks to a cafe near her house for a coffee, and then walks home again. The distance–time graph on the right shows her distance from her house from the time she leaves for the cafe to the time she returns home.




**a** How far was Laura from home at:

**i** 0 minutes? **ii** 2 minutes? **iii** 8 minutes?

**b** What does the horizontal line indicate about the speed at which Laura was travelling over that time interval?

**c** Where was Laura located 4 minutes into her walk?

**d** How far is the cafe from Laura's house?

**e** How long did it take Laura to:

**i** walk to the cafe? **ii** walk home from the cafe?

**THINK**

**a** Time is shown on the horizontal axis and distance is shown on the vertical axis. Locate each time on the horizontal axis and trace up to the corresponding point on the graph, then across to read the distance off the vertical axis.

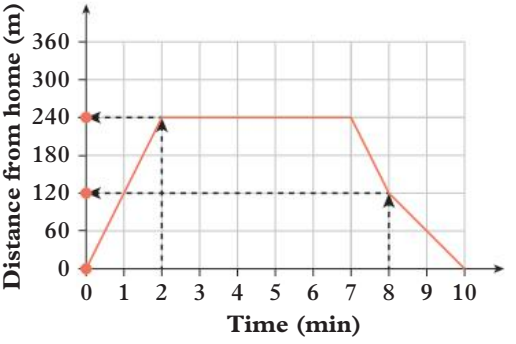
**b** A horizontal line indicates that there is no change in distance with respect to time. This means that Laura was stationary, or not moving, with a speed of zero over that time interval.

**c** Locate the time on the horizontal axis and trace up to the corresponding point on the graph. At 4 minutes, the graph is horizontal meaning that Laura is stationary, so we can assume that she was located at the cafe.

**d** As the horizontal line represents the time at which Laura is at the cafe, trace from the horizontal line across to the vertical axis to read the distance from Laura's house to the cafe.

**WRITE**

**a**

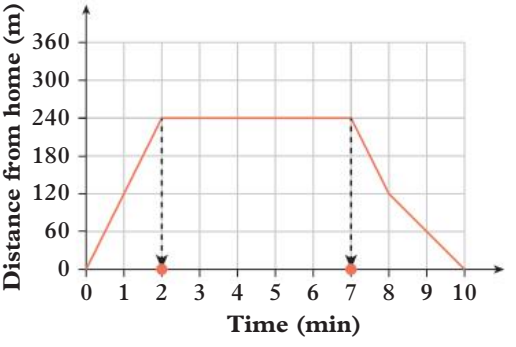



**i** 0 m **ii** 240 m **iii** 120 m

**b** The horizontal line indicates that Laura was stationary with a speed of zero over that time interval.

**c** 4 minutes into her walk, Laura was located at the cafe.

**d** The cafe is 240 m from Laura's house.



**e i** As Laura started walking at 0 minutes, trace from the start of the horizontal line down to the horizontal axis to read the time it took Laura to walk to the cafe.

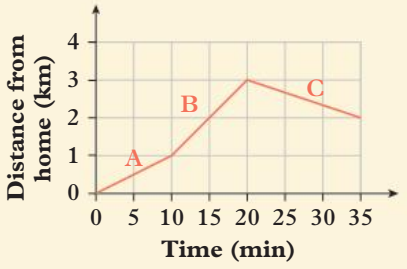
**ii** Trace from the end of the horizontal line down to the horizontal axis to read the time at which Laura started to walk home. The journey takes 10 minutes in total, so subtract the time from when she started to walk home from the total time for the duration of her walk home.

**e i** 2 minutes

**ii** 10 – 7 = 3 minutes

## **Example 5E.2** Calculating speed from a distance–time graph

The distance–time graph below shows Daniel's distance from his house over the duration of a jog.




Calculate Daniel's speed during interval B.

**THINK**

1. Determine the duration of interval B by tracing from the graph along the horizontal axis. Interval B starts at 10 minutes and ends at 20 minutes.
2. Determine the distance travelled in interval B by tracing from the graph across to the vertical axis. At the start of interval B, Daniel is 1 km from his house and at the end of the interval, he is 3 km from his house.
3. Calculate the speed by dividing the distance travelled by the time taken to travel that distance.

**WRITE**

duration = 20 – 10
= 10 minutes

distance = 3 – 1
= 2 km

$$\text{speed} = \frac{2 \text{ km}}{10 \text{ minutes}}$$
= 0.20 km per minute

**Helpful hints**

- ✔ Try reading distance–time graphs as you would a story – consider the significance of any distinctive features like horizontal lines and changes in steepness.
- ✔ Read axes labels carefully – take note of the scale used and the units.

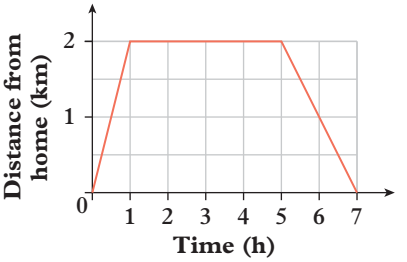
ANS p568

# Exercise 5E Interpreting graphs

1–6, 9 | 2, 4, 5, 7, 8, 10, 11 | 4, 5, 7, 8, 10–12

UNDERSTANDING AND FLUENCY

5E.1 **1** The graph below represents Taisha's journey to and from school on a school day.




**a** What is the distance between Taisha's home and school?

**b** How long was Taisha at school?

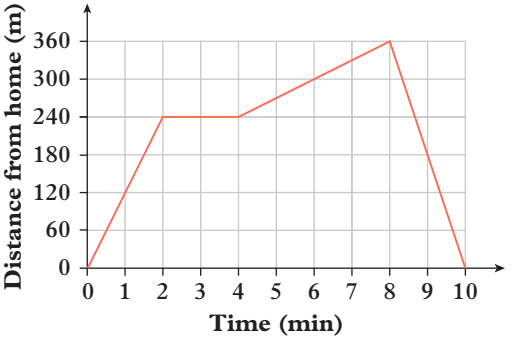
**c** How long did it take Taisha to:

**i** walk from home to school?

**ii** walk from school to home?

**d** According to the graph, what was the total distance she walked to and from school?

**2** Levi leaves home, walks to a local shop to buy a bag of prawn crackers, then walks to his friend's house. Levi and his friend then jog back to Levi's house together.



**a** How far was Levi from home at:

**i** 0 minutes?

**ii** 1 minute?

**iii** 8 minutes?

**b** What does the horizontal line indicate about the speed at which Levi was travelling over that time interval?

**c** Where was Levi located:

**i** 3 minutes into his walk?

**ii** 8 minutes into his walk?

**iii** 10 minutes into his walk?

**d** How far are the following locations from Levi's house?

**i** The shop **ii** Levi's friend's house

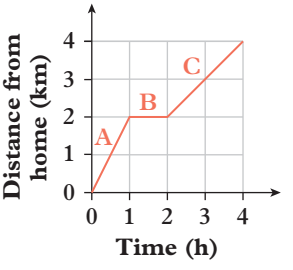
**e** How long did it take Levi to:

**i** walk to the shop?

**ii** walk from the shop to his friend's house?

**iii** jog home from his friend's house?

5E.2 **3** The distance–time graph on the right shows Sarah's journey from her home to her local thrift shop after stopping to visit her friend along the way.




Calculate Sarah's speed during:

**a** interval A **b** interval B **c** interval C



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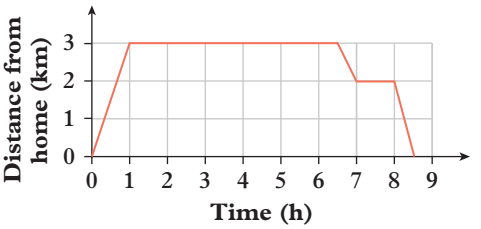
**4** Nina walks from her home to school. On the way home, she stops for a dental appointment. The graph on the right represents her journey.




**a** How far is Nina's school from her house?

**b** If Nina left home at 7:30 am, at what time did she leave school to walk to the dentist?

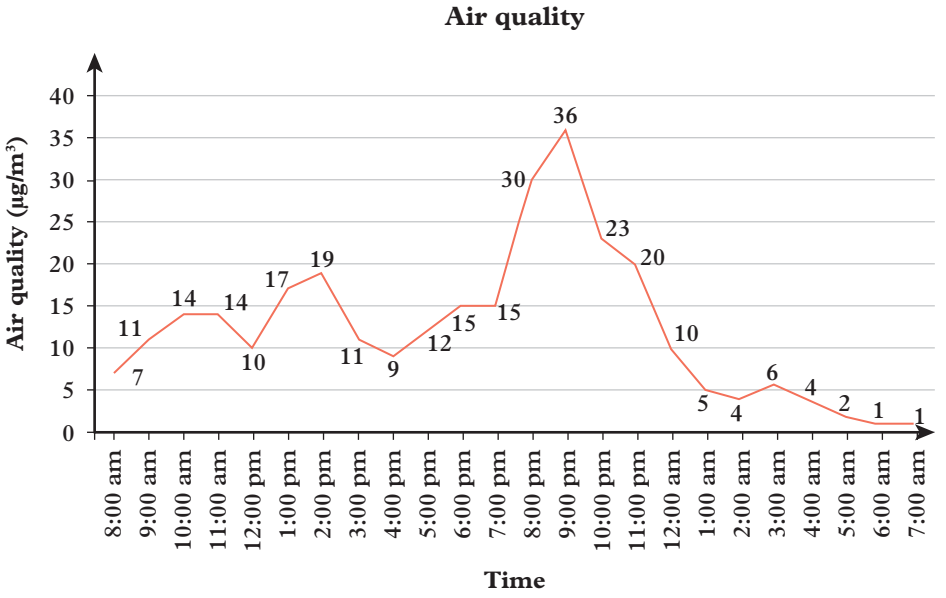
**c** How far did she travel from school to get to the dental clinic?

**d** How long was Nina's dental appointment?

**e** What time did Nina arrive home?

**f** During which part of her journey did Nina walk the fastest? What was her speed?

**5** The graph below shows the air quality on a particular July day in Melbourne. Air quality is measured in micrograms, μg, of particulate matter per cubic metre, $m^3$, of air – the higher the value of the air quality, the worse the quality of air.




**a** When was the air quality at its worst on this particular day?

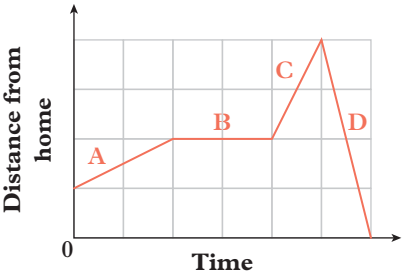
**b** What was the air quality at 3.00 pm?

**c** Find two different times when the air quality was the same at both times.

**d** Over what time period was the air quality improving at the fastest rate?



**6** Alex's journey to and from school is shown below.




**i** Answer true or false to each statement.

**ii** Use the graph to explain why each statement is true or false.

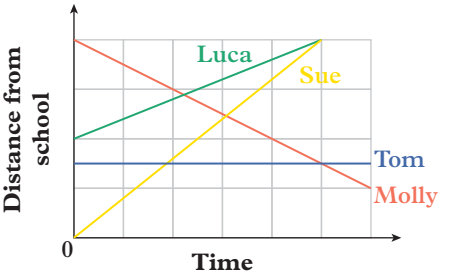
**a** Section A shows that Alex is walking home.

**b** Section B shows that Alex is stationary.

**c** Section C shows Alex walking slower than she did in section A.

**d** Section D shows Alex walking faster than she did in section C.

**7** This graph shows the travels of four students.

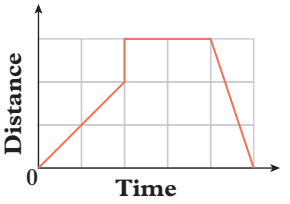



**a** Which student is stationary or not moving?

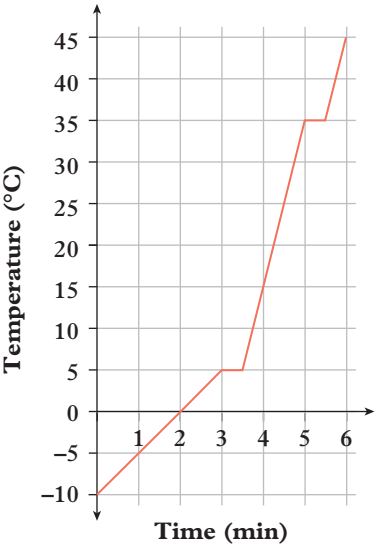
**b** Which student is travelling the fastest?

**c** Which student is travelling in an opposite direction to two of the other students?

**8** Does the graph below represent a journey that could really take place? Explain why or why not.




**9** Frozen soup is heated in a microwave oven. At different stages, the heating is paused so that the soup can be stirred. The graph on the right shows the temperature change of the soup over the first 6 minutes of heating.




**a** What was the temperature of the soup when it was first placed in the microwave?

**b** What was the temperature of the soup after 1 minute?

**c** When was the soup first stirred?

**d** The soup was stirred again at 5 minutes. What was its temperature at this time?

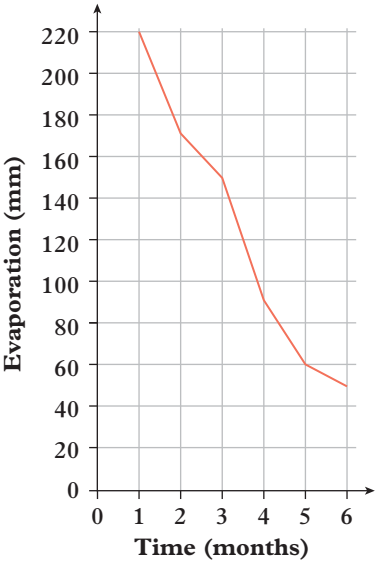
**e** In which time interval(s) on the graph was the temperature of the soup constant?

**f** What does the slope of this graph represent?

**g** In which time interval(s) was the temperature of the soup increasing the fastest? Explain.

**h** The soup needs to be heated to 65°C before it is ready to eat. Estimate how much longer the soup needs to be in the microwave to reach this temperature.

**10** Farmers and water authorities use information about the evaporation rate of water to manage water storage effectively. The graph on the right shows data for evaporation recorded in Albany, Western Australia, over the first six months of a year. The evaporation values, in millimetres, are obtained using a standard-sized container and equipment that measures the change in the water level due to evaporation.




**a** What was the total amount of evaporation in January, month 1?

**b** What was the total amount of evaporation in March?

**c** What does the slope of this graph represent?

This table shows the amount of evaporation for the second half of the year.

| **Month** | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|
| **Evaporation (mm)** | 59 | 67 | 84 | 106 | 150 | 199 |

**d** Copy the graph above and extend the horizontal axis to include all the remaining months of the year.

**e** Plot the evaporation values for July to December on your graph and join each point with a line.

**f** Which month of the year had the highest evaporation? What was the amount of evaporation in this month?

**g** Which month of the year had the lowest evaporation? What was the amount of evaporation in this month?

**h** Based on your answers to parts **f** and **g**, list two factors that affect the amount of evaporation.

**11** Jeanne is a diving instructor. She uses GPS trackers to record the depths of scuba divers, and how long they are underwater.

Jeanne graphs the information from the GPS tracker on a particular day. To draw her graph, she takes note of the following information:

- The deck of the boat is 1.5 metres above the water
- The water is 20 metres deep
- Scuba divers can be in the water for up to 45 minutes

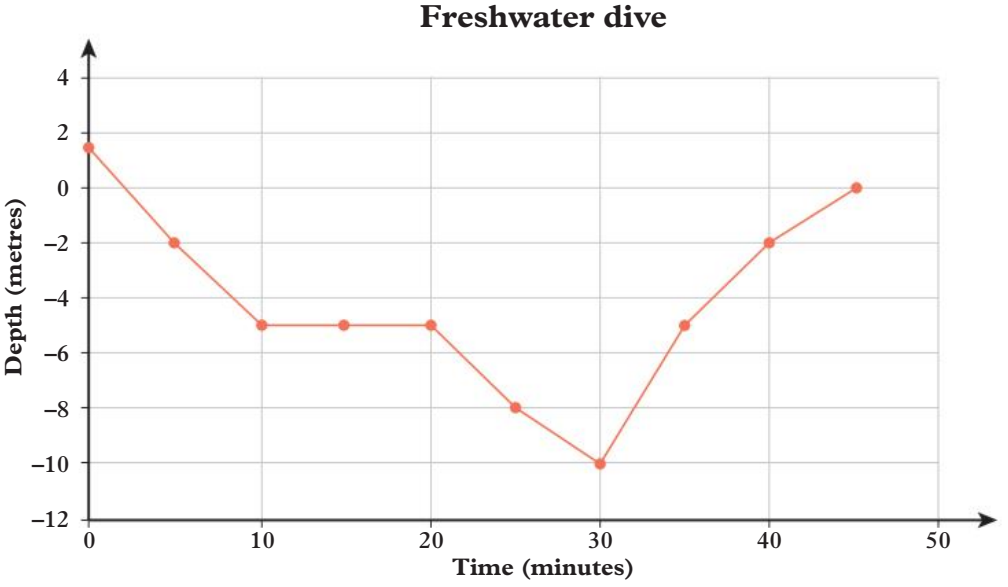
If time is shown on the horizontal axis and depth is shown on the vertical axis:

**a** what range of values should Jeanne use for the horizontal axis?

**b** what range of values should Jeanne use for the vertical axis?

**c** does either axis require negative numbers? Explain your answer.

On another day, Jeanne produces this graph for a diver who dives to the bottom of a lake.




**d** How long did it take the diver to reach the bottom of the lake from the deck of the boat?

**e** Did the diver take longer to reach the bottom of the lake or return to the surface of the water? Justify your answer.

**f** At one stage, the diver stops to take some measurements. At what depth did the diver make these measurements, and how long did the diver take to make the measurements?

**12** One morning at 9 am you begin to climb a high mountain, taking a walking track that leads to the peak. It is a hard walk and you have to stop and rest several times along the way.

You camp overnight on the peak of the mountain, then at 9 am the next morning, you begin to walk down the same walking track. Because you are walking downhill, it is easier and you reach the bottom of the mountain in much less time than it took you to reach the peak. Is there a single point along the track which you pass at exactly the same time on both days? Use a graph to help answer the question.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

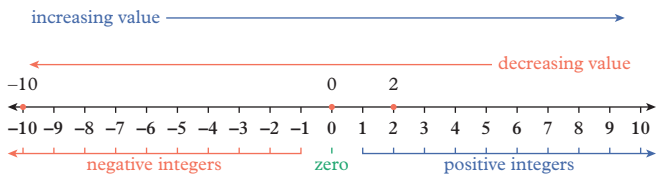
**Interactive skillsheet** Distance–time graphs

**Investigation** Comparing temperature

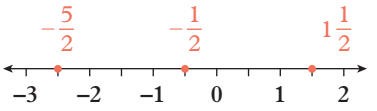
**Topic quiz** 5E

# Chapter summary

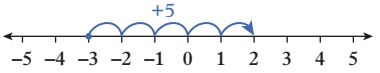
## Integers



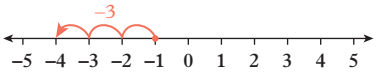

## Negative fractions




## Adding and subtracting positive integers

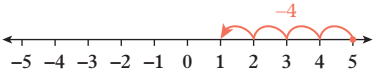



$-3 + 5 = 2$

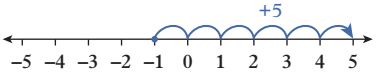



$-1 - 3 = -4$

## Adding and subtracting negative integers




$5 + (-4) = 5 - 4$
$= 1$

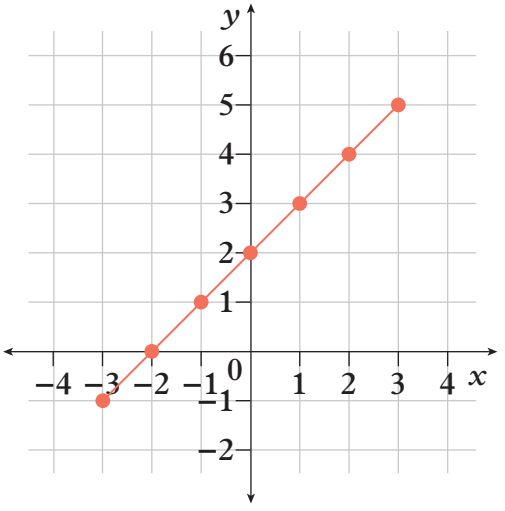



$-1 - (-5) = -1 + 5$
$= 4$

## Linear relationships

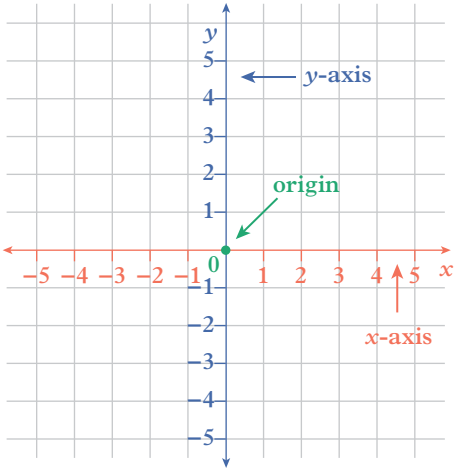
| *x*-coordinate | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| *y*-coordinate | –1 | 0 | 1 | 2 | 3 | 4 | 5 |

(–3, 1), (–2, 0), (–1, 1), (0, 2), (1, 3), (2, 4), (3, 5)




## Cartesian coordinates

## The Cartesian Plane




## Negative sign notation

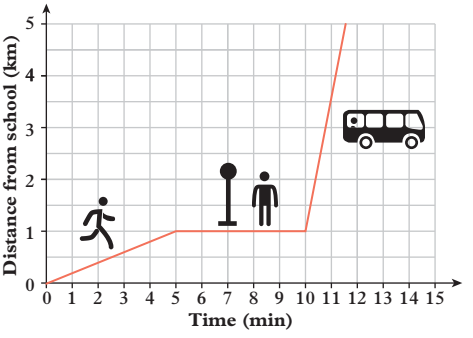
$+(-) = -$

$-(-) = +$

## Speed

$$\text{speed} = \frac{60 \text{ kilometres}}{1 \text{ hour}}$$
$= 60$ kilometres per hour
$= 60$ km/h

## Distance–time graph



# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

5A **1** Which integer lies between −3 and 3?

**A** −5 **B** −1.5 **C** −1 **D** 1.5 **E** 5

5A **2** Which number is less than $-3\frac{3}{4}$ in value?

**A** $-4\frac{1}{3}$ **B** $-\frac{5}{2}$ **C** −1 **D** 2.5 **E** 5

5A **3** Which number is greater than $-\frac{26}{5}$ in value?

**A** −3.5 **B** −7.5 **C** −10.5 **D** −6 **E** −65

5B **4** Which of these calculations gives a positive result?

**A** 0 − 12 **B** −12 + 12 **C** 12 − 24
**D** −12 + 24 **E** −12 − 24

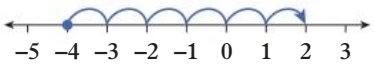
5B **5** −4 + (−6) − (−5) =

**A** −10 **B** −5 **C** 5 **D** 7 **E** 15

5C **6** Which integer when added to 5 gives −3?

**A** −8 **B** −2 **C** 2 **D** 8 **E** −3

5C **7** Which calculation best represents this diagram?

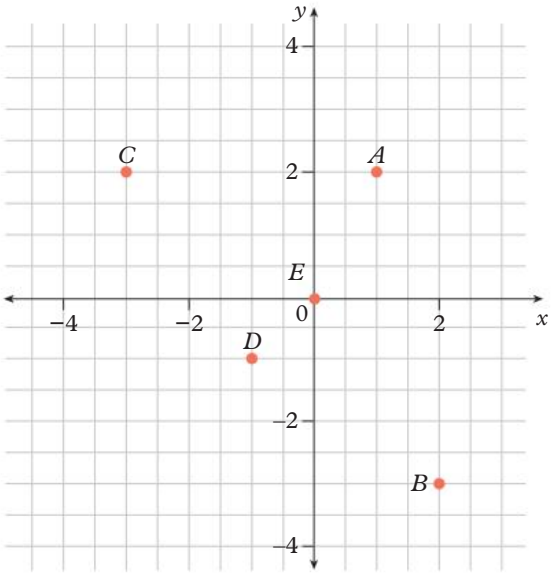



**A** −4 − 6 = 2 **B** −4 + 2 = −2 **C** −4 − 2 = −6 **D** −4 + (−6) = 2 **E** −4 − (−6) = 2

5C **8** Which two integers give a sum of −9 and a difference of 1?

**A** −9 and 1 **B** −4 and 5 **C** 4 and 5 **D** 4 and −5 **E** −4 and −5

5D **9** Which coordinates match the coordinates for point C on the Cartesian plane below?




**A** (1, 2) **B** (2, −3) **C** (0, 0) **D** (−3, 2) **E** (−1, 1)

5D **10** Which of the following coordinates is located on the $x$-axis?

**A** (0, 0) **B** (0, −1) **C** (0, 1) **D** (1, 1) **E** (−1, −1)

5E **11** Anthony takes his car out for a drive. The distance–time graph for his drive is shown below.

The car is stationary from:

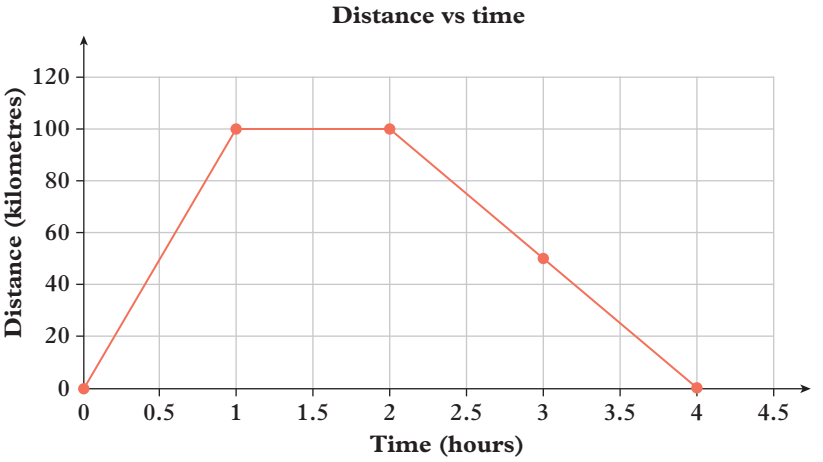
**A** 0 to 1 hours

**B** 1 to 2 hours

**C** 2 to 3 hours

**D** 3 to 4 hours

**E** The car is never stationary




## Short answer

5A **1** Decide whether each statement is true or false.

**a** $-3 > +3$ **b** $-12 < -5$ **c** $0 > -7$ **d** $1 < -1$

5A **2** Arrange each list in ascending order.

**a** 25, −5, 0, 12, −10, 2 **b** −3, 9, 6, −6, 0, −12

5A **3** Arrange each list in descending order.

**a** $1, \frac{9}{5}, -2, -\frac{9}{10}, 0, 2\frac{2}{5}$ **b** $5\frac{1}{4}, -4.75, -\frac{15}{4}, 5.5, \frac{7}{2}, -2\frac{3}{4}$

5A **4** Write an integer to represent each of these situations.

**a** Death Valley in California, USA, is 86 m below sea level.

**b** You have \$615 in the bank.

**c** The top of Mt Everest is 8848 m above sea level.

**d** Your bank account is overdrawn by \$35.

5B **5** Use the number line to calculate:

**a** $1 - 9$ **b** $-7 + 8$ **c** $-8 - 6$ **d** $-10 + 9$

5B **6** A scuba diver is 5.5 metres below sea level. If she dives a further 2 metres down, then rises by 4 metres, what is her final position?

5C **7** Use the number line to calculate:

**a** $-6 + (-3)$ **b** $(-4) - (-5)$ **c** $2 + (-4)$ **d** $(+11) - (+3)$

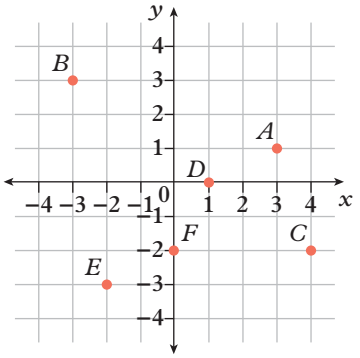
5C **8** You have \$25 in your bank account. The bank mistakenly deducts \$40 from your account. They realise their mistake, but then make a second deduction of \$35. What is the final balance of your bank account?

5D **9** **a** Complete this table of values.

| | | | | | | |
|---|---|---|---|---|---|---|
| ***x*-coordinate** | 0 | 1 | 2 | | 4 | |
| ***y*-coordinate** | 3 | 4 | | 6 | 7 | |
| **coordinates** | (0, 3) | (1, 4) | (2, 5) | (3, 6) | | (5, 8) |

**b** Draw a Cartesian plane with a suitable scale on each axis and plot each point shown in the table. Label each point with its coordinates.

5D **10** Write the coordinates of the points *A*–*F* shown on the Cartesian plane below.



5D **11** Plot and label these points on the Cartesian plane.

**a** $A(-4, -2)$ **b** $B(-1, 4)$ **c** $C(0, 3.5)$ **d** $D\left(3\frac{1}{2}, -5\frac{1}{2}\right)$

5E **12** Jung walks from his home to school. On his way home from school, he stops at a shop with friends. The graph on the right represents his journey.

**a** How far does Jung walk from home to school?

**b** How long did Jung stop at the shop?

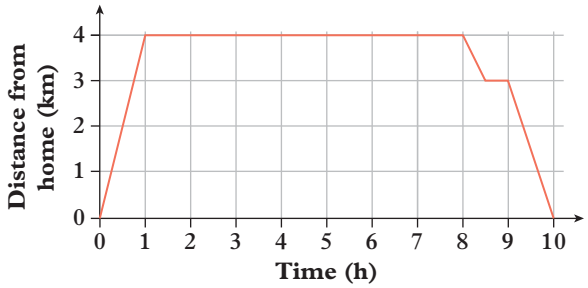
**c** How long did it take Jung to walk:

**i** from home to school?

**ii** from school to the shop?

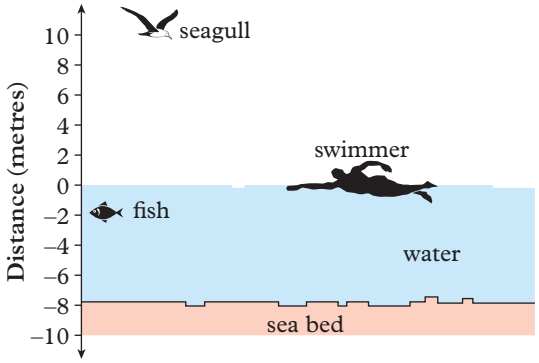
**iii** from the shop to home?

**d** What was Jung's fastest walking speed?




## Analysis

Refer to the scale shown with the diagram below to answer each of the questions that follow.



**a** Represent the position of each of the following using an integer in metres.

**i** The seagull

**ii** The fish

**iii** The swimmer

**iv** The sea bed

**b** Calculate the vertical distance between the:

**i** seagull and fish

**ii** sea bed and fish.

**c** A sting ray is swimming at three times the depth at which the fish is swimming. Represent its position using an integer.

The seagull spots the fish under the water and dives towards it. This table of values shows the height of the seagull after spotting the fish.

| **Time (seconds)** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| **Height (metres)** | 10 | 8.5 | 5 | 2.5 | 0 | −1.5 | −2.5 | −1.5 | 0.5 |

**d** The seagull's height above the water when it spots the fish can be represented by the coordinate point (0, 10). List the values in the table as a set of nine coordinate points.

**e** Plot these points on a Cartesian plane and label the axes using the title of each row of the table.

**f** If you join the points with a smooth line, do the points form a straight line or a curve?

**g** What does the slope of this graph represent?

**h** Describe the seagull's movements in the first 8 seconds after it spots the fish.

**i** If the fish was located 2.75 metres below the surface of the water 6 seconds after the seagull dived, did the seagull catch the fish?

# Semester 1 review

## Short answer

**1** Write the following numbers in the appropriate expanded form.

**a** 4817 **b** 19 030 **c** $6^4$ **d** $51^3$

**2** Evaluate the following.

**a** $\sqrt{36}$ **b** $99 \times 240$ **c** $345 + 698$

**d** $3^4$ **e** $1000 - 248$ **f** $3234 \div 7$

**3** **a** Round 135 678 to the nearest:

**i** ten

**ii** ten thousand

**iii** hundred thousand.

**b** Round 57.658 214:

**i** to the nearest ten

**ii** to the nearest tenth

**iii** correct to three decimal places

**iv** correct to two decimals places

**v** to the nearest whole number.

**4** Calculate the following sums.

**a** $859 + 472$ **b** $\frac{5}{4} + \frac{3}{8}$ **c** $53.7 + 2.41$

**d** $(-15) + (-8)$ **e** $\left(-\frac{9}{2}\right) + \left(+\frac{7}{9}\right)$ **f** $(+0.67) + (-0.43)$

**5** Calculate the following differences.

**a** $859 - 472$ **b** $\frac{5}{4} - \frac{3}{8}$ **c** $53.7 - 2.41$

**d** $(-15) - (-8)$ **e** $\left(-\frac{9}{2}\right) - \left(+\frac{7}{9}\right)$ **f** $(+0.67) - (-0.43)$

**6** Calculate the following products.

**a** $375 \times 73$ **b** $\frac{8}{3} \times \frac{9}{16}$ **c** $2.7 \times 0.41$ **d** $5\frac{1}{2} \times 8\frac{2}{3}$

**7** Calculate the following quotients.

**a** $498.4 \div 8$ **b** $\frac{15}{4} \div \frac{3}{2}$ **c** $10.2 \div 0.12$ **d** $4\frac{3}{4} \div 2\frac{3}{8}$

**8** Write an integer, fraction, ratio or percentage to answer each of the following questions.

**a** Two-fifths of a class of 25 students said their family did not own a pet. How many of the students' families did not own a pet?

**b** Out of 20 power saws, 15 were faulty. What fraction of the power saws were faulty?

**c** 120 students were on a camp and 45% preferred to eat breakfast before 7:30 am. How many students preferred to eat breakfast before 7:30 am?

**d** This year, a company made \$70 million in sales – up from \$50 million last year. Write this year's sales as a percentage of last year's sales for the company.

**e** Sara was sharing lollies between her younger brother Vishnu and herself in the ratio 3 : 2. If she gave herself 15 lollies, how many lollies did she start with?

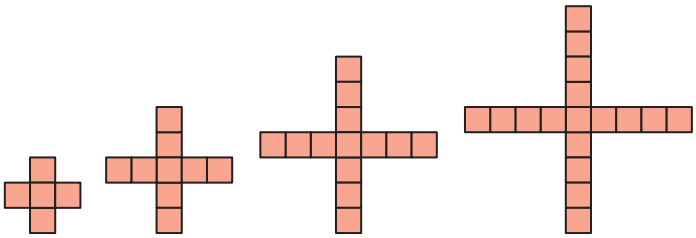
**f** On a Monopoly board, there are 22 properties, four railroads and two utilities available for purchase. Write the ratio of the number of properties to the total number of railroads and utilities.

**9** Calculate the highest common factor and the lowest common multiple for each of the following pairs of numbers.

**a** 32 and 80 **b** 24 and 40 **c** 21 and 98 **d** 210 and 462

**10** Describe the following sequences in words.

**a**



**b** 7, 1, −5, −11 ... **c** 3, 6, 12, 24 ... **d** $\frac{150}{7}, \frac{30}{7}, \frac{6}{7}, \frac{6}{35}$ ...

**11** Decide whether the following numbers are prime or composite.

**a** 51 **b** 47 **c** 227 **d** 221

**12** Use the divisibility rules to determine whether the following are true or false.

**a** 6 is a factor of 9252 **b** 4 is a factor of 3166

**c** 4427 is a multiple of 9 **d** 7265 is a multiple of 5

**13** Evaluate each of the following.

**a** $\begin{array}{r} 479 \\ +\ 342 \\ \hline \end{array}$ **b** $\begin{array}{r} 416 \\ -\ 387 \\ \hline \end{array}$ **c** $\begin{array}{r} 257 \\ \times\ 63 \\ \hline \end{array}$ **d** $7\overline{)2394}$

**14** Evaluate each of the following.

**a** $8 + 4 \times 7 - 3$ **b** $8 \div 4 \times 2 - 3 + 9$ **c** $\frac{2}{3} - \frac{5}{12} \div \frac{7}{4}$

**d** $5.2 \times 0.7 - 1.5$ **e** $(-5) - (+2) - (-8) + (-4)$ **f** $\frac{2}{5} \times 4.8 - (-3.81)$

**15** Simplify the following.

**a** $8\frac{4}{16}$ **b** $\frac{45}{18}$ **c** 55 : 75 **d** 91 : 35

**16** Consider the table of values below.

| ***x*-coordinate** | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| ***y*-coordinate** | −7 | −4 | −1 | 2 | 5 | 8 | 11 |

**a** List the coordinates of each point.

**b** Plot the points on the Cartesian plane

**c** State whether or not the points lie on a straight line.

**17** At 7 am the temperature was −1°C, at 1 pm the temperature was 18°C, and at 7 pm the temperature was −4°C.

**a** How much did the temperature increase from 7 am to 1 pm?

**b** How much did the temperature decrease from 1 pm to 7 pm?

**18** Complete the table below. Where appropriate, express all fractions as improper fractions and round your answers to two decimal places.

| **Percentage** | **Fraction** | **Decimal** |
|---|---|---|
| 35% | | |
| | $\frac{31}{4}$ | |
| 4.5% | | |
| | $\frac{68}{7}$ | |
| | $\frac{14}{25}$ | |
| | | 6.42 |
| | $\frac{13}{5}$ | |
| | | 8.931 |

**19** Order the following lists of numbers in ascending order.

**a** $\frac{9}{5}, \frac{8}{6}, \frac{5}{6}, \frac{9}{8}$ **b** 0.5, 5.5, 0.05, 5.0 **c** 5, −3, −4, 1 **d** $-\frac{11}{25}, -0.41, -\frac{2}{5}, -\frac{3}{7}$

**20** Write the following numbers as the product of their prime factors.

**a** 56 **b** 45 **c** 216 **d** 350

**21** Calculate the following powers.

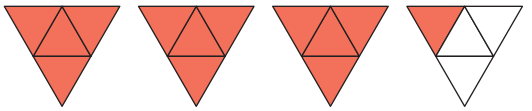
**a** $7^3$ **b** $2^8$ **c** $3^4$ **d** $17^2$

**22** Plot the following sets of numbers on a single number line.

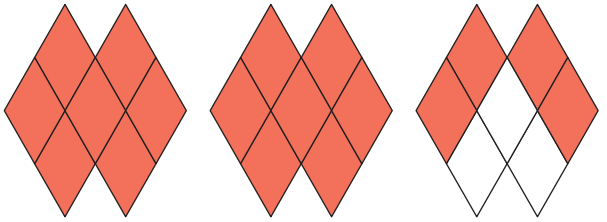
**a** 2, −3, 5, −1 **b** $\frac{3}{2}, \frac{3}{4}, -\frac{1}{2}, -\frac{5}{4}$ **c** −0.1, 0.05, 0.10, −0.2

**23** If each composite shape below is considered to be a whole, express the shaded portion of each diagram in the form specified in brackets.

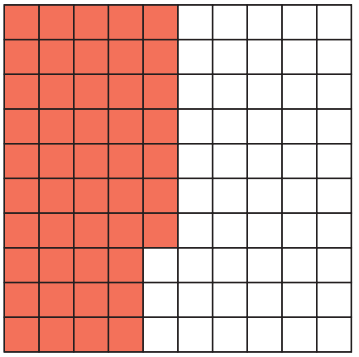
**a** (mixed number)



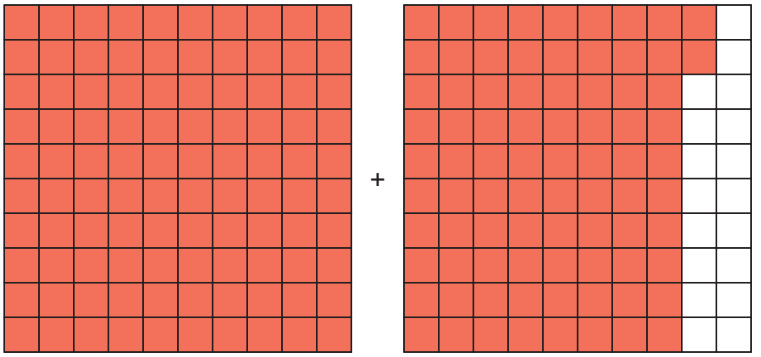
**b** (improper fraction)



**c** (percentage)



**d** (percentage)



**24 a** How many common multiples of 8 and 12 are between 0 and 100?

**b** How many common factors do 120 and 140 share?

**25** A car has an indicator that blinks every 2 seconds. The car in front of it has an indicator that blinks every 3 seconds. If both indicators begin to blink at the same time, how many times will they blink at the same time within a minute?

**26 a** What is the smallest number divisible by 2, 3, 4, 5, 6, 7, 8, 9 and 10?

**b** What is the smallest number you can multiply 350 by so that it is a square number?

## Analysis

1 A gardener plants different coloured roses at one metre spacings to create a display. From the left-most edge of the flowerbed, a white rose (w) is planted every 2 m, a red rose (r) is planted every 3 m, and a pink rose (p) is planted in all the gaps where a white or red rose is not used. Where a white rose and a red rose would both be placed, a yellow rose (y) is used instead.

**a** Complete the sequence of the first 12 rose colours.

p, w, r, __, __, __, __, __, __, __, __, __, __

**b** Describe the rule for planting yellow roses.

**c** Describe the rule for planting pink roses.

The gardener continues the pattern until the 50th metre.

**d** What is the colour of the last rose?

**e** How many yellow roses are used?

2 A boutique bakery makes pastry packs of cannolis, eclairs and macarons. Each pack contains the same combination of pastries. The bakery makes 30 cannolis, 45 eclairs and 105 macarons a day to put into pastry packs.

**a** How many packs can be made? All pastries must be used.

**b** How many of each pastry is included in each pack?

**c** What fraction of the pack is macarons?

It costs the bakery $1.20 to make a cannoli, $0.80 to make an eclair and $0.65 to make a macaron.

**d** How much does it cost to make all of the cannolis, eclairs and macarons for the pastry packs for a day?

Each cannoli sells for $3.30, each eclair sells for $3.30 and each macaron sells for $2.60.

**e** How much do all of the packs made in a day sell for?

**f** How much does each pack sell for?

3 Jason walks to and from the park. Jason's distance from his home on his way to the park is shown on the distance–time graph below. He stops partway to the park to meet up with his friend Kate.

**a** How far does Jason live from Kate?

**b** How far is the park from Jason's house?

**c** How far is the park from Kate's house?

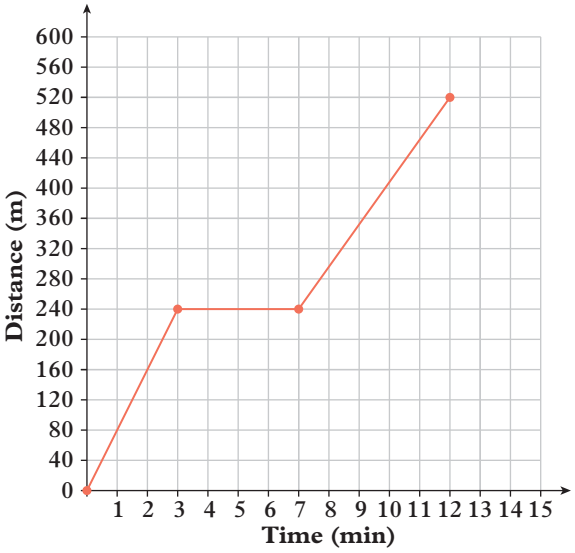
**d** How long did it take Jason to get to the park?

**e** How long did Jason have to wait for Kate?

After an hour at the park, Jason and Kate head home separately. Jason walks directly home. It takes him eight minutes to get home.

**f** Sketch a distance–time for Jason's trip home.

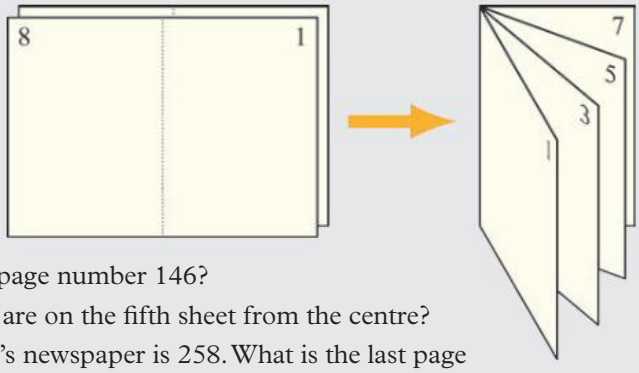
**g** How fast, in m/min, was did Jason walk home?



# EXPLORATIONS 1

## 1 Newspapers

A newspaper is made of several A2 sheets folded in the middle so that each sheet makes 4 numbered pages. For example, two sheets make 8 pages numbered as shown.




**a** Friday's newspaper has 64 pages. One sheet has the page number 11 on it. What are the other three page numbers on this sheet?

**b** Saturday's newspaper is larger with 220 pages. What are the other three page numbers on the sheet that has page number 146?

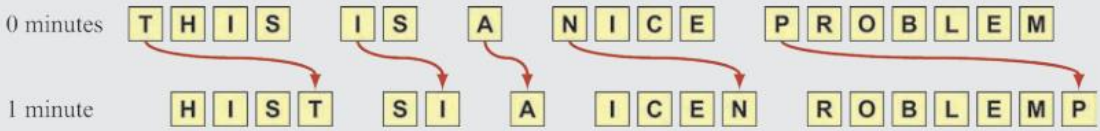
**c** Sunday's paper has 96 pages. What four page numbers are on the fifth sheet from the centre?

**d** The sum of the page numbers on one sheet in Monday's newspaper is 258. What is the last page number in this newspaper?

**e** Sometimes, to get the right number of pages, a single half-sheet of two pages needs to be used somewhere in the newspaper. Tuesday's newspaper has such an insert, and one of its pages is numbered 53. Anh opened the newspaper with its pages facing up. He noticed that there were twice as many sheets below the insert as there were above it. Find the last page number in this newspaper and explain why there is only one answer.

## 2 Word Spins

You are making sentences from a collection of alphabet tiles. After one minute, you pick up the first letter of each word and place it at the end, as in the example shown.




As you continue to do this each minute, the whole collection of letters moves along from left to right but the length of each 'word' stays the same.

**a** How many minutes does it take for the sentence 'THIS IS A NICE PROBLEM' to return to normal?

**b** How many minutes does it take for the sentence 'MATHS HOTSHOTS CAN SOLVE HARDER STUFF' to return to normal?

**c** Make up your own sentence that will take exactly 2 hours and 12 minutes to return to normal.

**d** Using exactly 20 alphabet tiles, make up a sentence that takes the shortest possible time to return to normal.

**e** Using exactly 20 alphabet tiles, make up a sentence that takes the longest possible time to return to normal.

**f** In a variation of the game, not only do you move the first letter to the end of each word, but at the same time you also move the entire first word to the end of the sentence. Does this affect any of your answers for parts **a** to **e**?

## 3 Copy of a Copy

The school photocopier has seven buttons that reduce, maintain or enlarge the area of a printed image:

**a** Mr Tashibo wants to make a smaller copy of a Japanese alphabet poster. He runs the original through the copier at 80% and then runs the copy through at 75%. What is the effective percentage factor overall? Could he have achieved the same result using two different buttons?

**b** Ms Prash wants to enlarge a map by a factor of 200% but notices that this is not one of the options. Using the copier no more than three times, what is the closest she can get to achieving this?

**c** One day, Dr Richo notices that three out of the seven buttons are not working but, by using copies of copies as necessary, it is still possible to achieve all seven factors from the remaining four buttons. What could those four buttons be? Find all possibilities.

**d** Is it possible to achieve all seven factors if only three of the seven buttons are still working? Explain.

**e** A different copier has five buttons, including the standard 100%. Using copies of copies as necessary, each of the factors 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20% and 10% can be achieved. What percentages could be on the other four buttons?

## 4 Ts and Rs

We define two numerical operations labelled T and R.

The effect of T is to add 1 to a number. For example, if we apply the operation T to the number 2 three times in a row, we obtain 3, then 4, then 5.

The effect of R is to find the negative reciprocal of a number – that is, divide 1 by that number and change it from positive to negative, or vice versa. For example, if we apply the operation R to 2 we obtain $-\frac{1}{2}$, and if we apply the operation R to $-\frac{3}{2}$ we obtain $\frac{3}{2}$. Note that R can never be applied to the number 0.

The operations T and R can be combined. For example, we can turn 0 into $\frac{2}{5}$ by successfully applying the operations T T T R T T R T:

$$0 \xrightarrow{T} 1 \xrightarrow{T} 2 \xrightarrow{T} 3 \xrightarrow{R} -\frac{1}{3} \xrightarrow{T} \frac{2}{3} \xrightarrow{T} \frac{5}{3} \xrightarrow{R} -\frac{3}{5} \xrightarrow{T} \frac{2}{5}.$$

**a** Starting with 2, list the numbers produced by successively applying the operations T R R T R T R T R.

**b** Find a sequence of operations that turns $\frac{3}{4}$ into $\frac{2}{3}$.

**c** Find a sequence of operations that turns 3 into 0.

**d** Explain how any positive integer can be turned into 0.

**e** Explain how 0 can be turned into any negative integer.

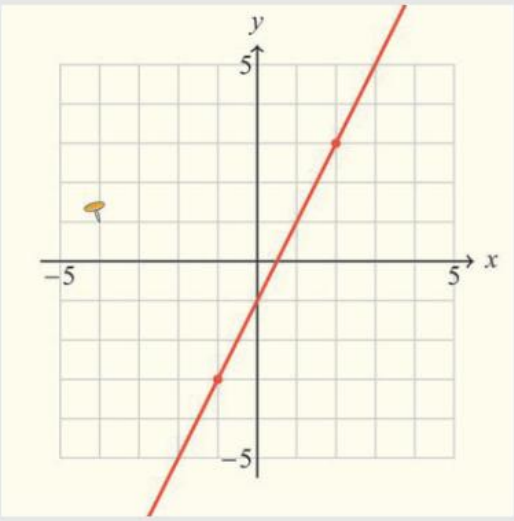
**f** Can any integer be turned into any other integer?

## 5 Fold and Pierce

**a** On a piece of paper, Sarafina draws a number plane and plots the points (–1, –3) and (2, 3). She then draws a line through the two points, all the way to the edge of the paper, as shown.

Sarafina carefully folds the paper along the line and places a pin through the point (–4, 1).

Which other point does the pin go through?




**b** Sarafina unfolds the paper and draws a line through the two pin holes. She then folds the paper twice, once along the original line and once along the new line, and places the pin at (3, 0).

Which other points does the pin go through?

**c** On a new piece of paper, Sarafina draws another Cartesian plane and draws a line through a point $P$ on the $x$-axis and a point $Q$ on the $y$-axis. She folds along the line and then places a pin, which goes through (4, –1) and (2, 5).

What are the coordinates of $P$ and $Q$?

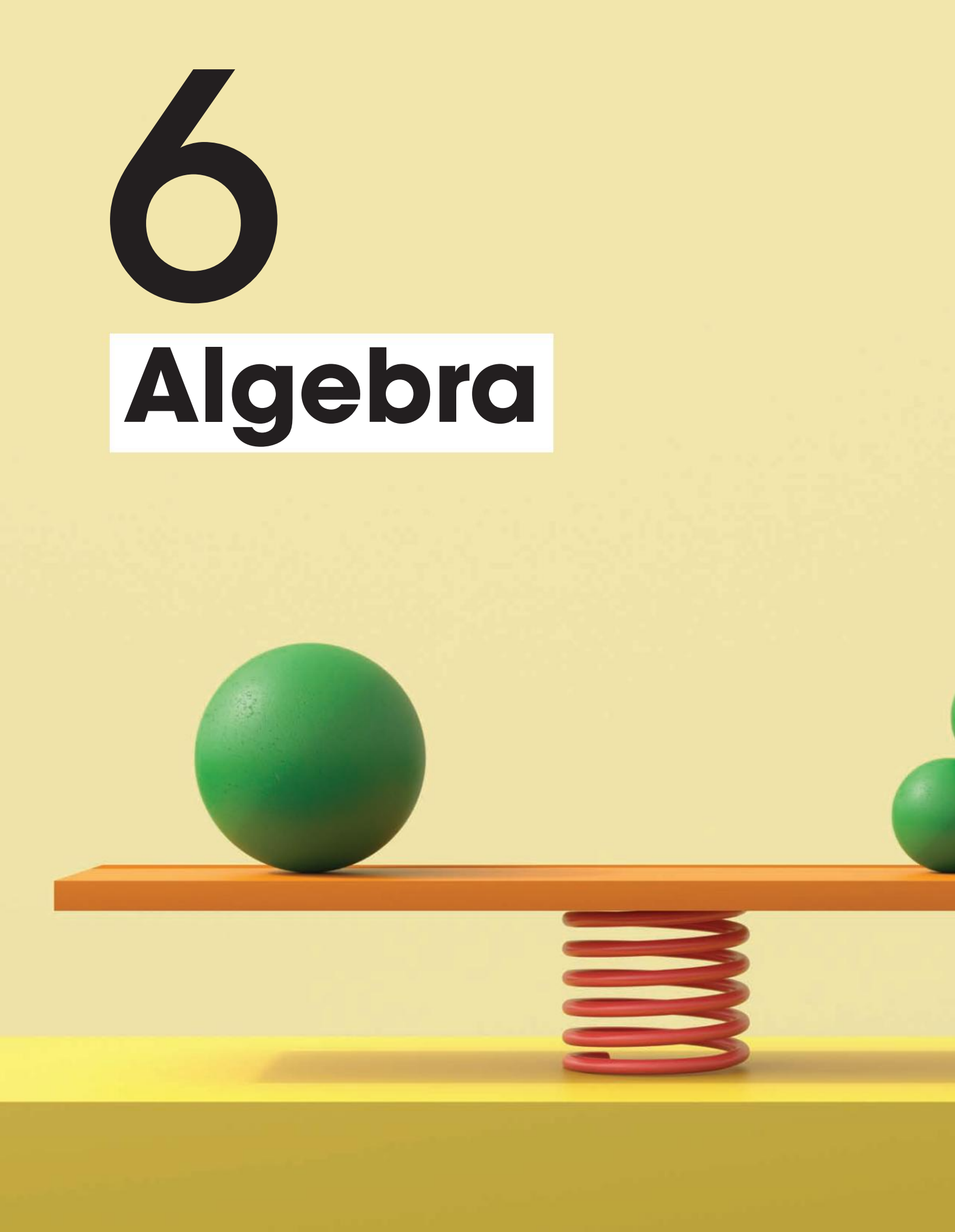
Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

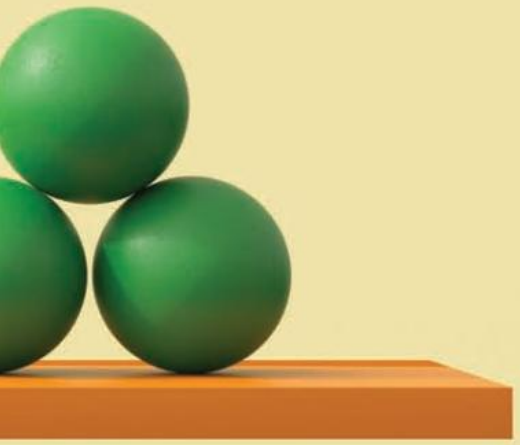
AUSTRALIAN MATHS TRUST

Problemo POWERED BY AUSTRALIAN MATHS TRUST

# 6

# Algebra





## Index


**6A** Variables
**6B** Writing formulas
**6C** Terms, expressions and equations
**6D** Order of operations in algebra
**6E** Substitution
**6F** Simplifying
**6G** Solving equations by inspection
**6H** Solving equations using inverse operations


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Times tables
- ✔ Order of operations
- ✔ Number sequences
- ✔ Reading tables and graphs

## Curriculum links

- ✔ Introduce the concept of variables as a way of representing numbers using letters (VCMNA251)
- ✔ Create algebraic expressions and evaluate them by substituting a given value for each variable (VCMNA252)
- ✔ Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (VCMNA253)
- ✔ Solve simple linear equations (VCMNA256)

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# 6A Variables

## Learning intentions

- ✔ I can identify variables.
- ✔ I can represent variables using a table of values.
- ✔ I can use rules to complete tables of values.

**Inter-year links**

| | |
|---|---|
| Year 5/6 | Number sequences |
| Year 8 | 5A Pronumerals |

## Unknowns and pronumerals

- A **pronumeral** is a letter or symbol that is used in place of a number.
- In algebra, the **unknown** number is represented using a pronumeral.

  For example, consider the incomplete sum:

  ? + 3 = 5

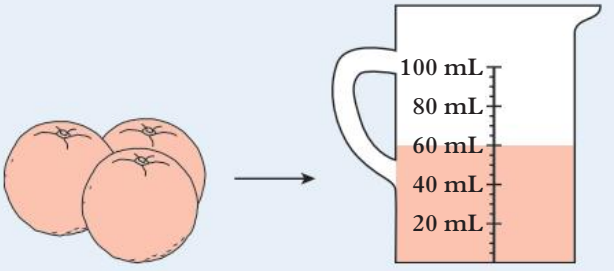
  We can re-write the incomplete sum above using the pronumeral '$x$' as:

  $x + 3 = 5$

## Variables

- A **variable** is a quantity that can have different values. Variables can be represented using pronumerals.

  For example, the number of oranges you put into a juicer affects the volume of orange juice that comes out. In this example, both the *number of oranges* and the *volume of juice* are variables.




- A **rule** describes the relationship between two or more variables.

  For example, the relationship between the above variables can be described by the rule *one orange makes 20 millilitres of juice*, or:

  *volume of juice* = 20 × *number of oranges*

- A **formula** is a rule written using pronumerals.

  The above rule can be re-written as a formula by representing *volume of juice*, in mL, with the pronumeral $v$ and the *number of oranges* with the pronumeral $n$:

  $v = 20 \times n$

  The relationship between two variables can be represented by:

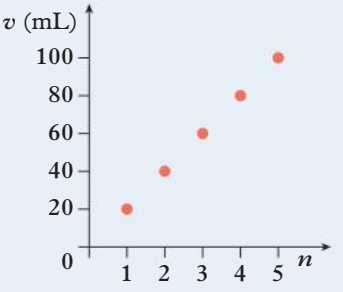
→ a rule

$v = 20 \times n$

→ a table of values

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $v$ (mL) | 20 | 40 | 60 | 80 | 100 |

→ a graph.



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## Example 6A.1 Identifying variables

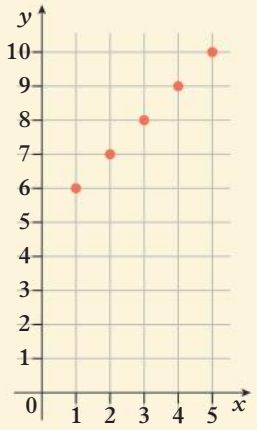
Identify the variables in the following relationships.

**a** distance travelled = 20 × time spent travelling

**b**

| Age (years) | 10 | 11 | 12 | 13 |
|---|---|---|---|---|
| Height (cm) | 140 | 148 | 154 | 160 |

**c**




**THINK**

**a** Look for the quantities that can take different values.

**b** In tables, variables are labelled at the start of a row or column.

**c** In graphs, variables are labelled on the axes. Remember that variables can be represented using pronumerals.

**WRITE**

**a** Distance travelled and time spent travelling

**b** Age and height

**c** $x$ and $y$

## Example 6A.2 Using a rule to find the values of a variable

The cost of one avocado is \$4. Use this information to complete the table below.

| Number of avocados | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Total cost (\$) | | | | | |

**THINK**

**1** One avocado costs \$4, therefore two avocados cost 2 × \$4 = \$8 and so on.

**2** Write the total cost values in the table.

**WRITE**

$1 \times 4 = 4 \quad 2 \times 4 = 8 \quad 3 \times 4 = 12 \quad 4 \times 4 = 16 \quad 5 \times 4 = 20$

| Number of avocados | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Total cost (\$) | 4 | 8 | 12 | 16 | 20 |

## **Example 6A.3** Using a formula to find the values of a variable

Use the given formula to find the values of $y$ in the following table.

$y = x + 6$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**THINK**

1. Interpret the formula.
2. Apply the formula to each value of $x$.
3. Write the values of $y$ in the table.

**WRITE**

Add 6 to the value of $x$ to get $y$.

$1 + 6 = 7$ $2 + 6 = 8$ $3 + 6 = 9$ $4 + 6 = 10$ $5 + 6 = 11$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 7 | 8 | 9 | 10 | 11 |

**Helpful hints**

✔ A variable is a number or quantity, not an object.
In the formula:
$v = 30 \times n$
the pronumeral '$n$' represents the *number* of oranges, not one single orange.

✔ An unknown has a fixed value, but variables can take different values.
For example, in the incomplete sum $x + 3 = 5$, the unknown value of $x$ must be $x = 2$ for the sum to be correct.
On the other hand, I can choose to put $n = 5$ or even $n = 100$ oranges into a juicer!

ANS p572

# **Exercise 6A** Variables

1–5, 7, 8, 9(a–d), 10(a, c), 11, 12 | 1(e–i), 3, 4, 6, 7(b, d), 9(c–f), 10(b, d), 11, 13, 14 | 1(g–i), 4, 6, 9(d–f), 10(b, d), 13–15

6A.1 **1** Identify the variables in the following relationships.

**a** A runner's heart rate and the speed at which they run

**b** wage = \$16 × number of hours spent working

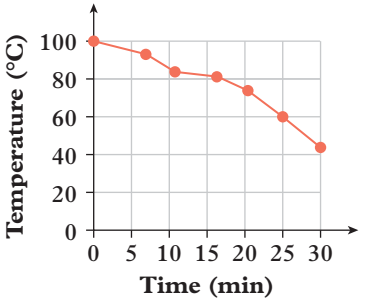
**c**

| **Mass of chicken (kg)** | 0.500 | 1.00 | 1.50 | 2.00 | 2.50 |
|---|---|---|---|---|---|
| **Time to roast (min)** | 30 | 60 | 90 | 120 | 150 |

**d**

| $x$ | −1 | −2 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 1 | 0 | 2 | 3 | 4 |

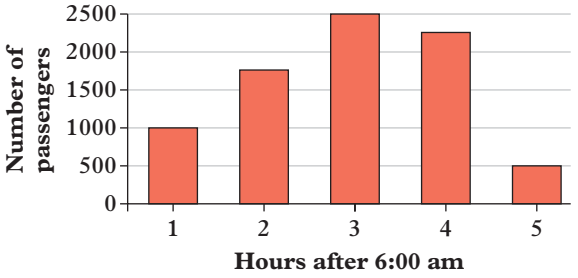
**e**



**f** area = 2 × length

**g** $y = x - 5$

**h**



**i** $V = \frac{n+1}{5}$

6A.2 **2** The cost of one banana is $2. Use this information to complete the table below.

| **Number of bananas** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Total cost ($)** | | | | | |

**3** On a rainy day, the amount of water in a rainwater tank increases by 10 L every hour. Assuming the tank is initially empty, use this information to complete the table below.

| **Time (h)** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Volume of water in the tank (L)** | | | | | |

**4** Anna's car uses 9.2 L of petrol for every 100 km travelled. If her petrol tank initially contains 50 L of petrol, use this information to complete the table below.

| **Distance travelled (km)** | 0 | 100 | 200 | 300 | 400 | 500 |
|---|---|---|---|---|---|---|
| **Volume of petrol in the tank (L)** | 50 | | | | | |

**5** The cost of a child's movie ticket is $15. If $n$ is the number of children and $c$ is the total cost in dollars, draw a table that shows the total cost of buying 1, 2, 3, 4 and 5 child movie tickets.

**6** Eli is watching a movie and trying to make his popcorn last as long as possible. He starts the movie with 150 pieces of popcorn and eats two pieces every minute.

If $t$ is the time in minutes from the start of the movie and $p$ is the number of pieces of popcorn left in Eli's bucket, draw a table that shows the number of pieces of popcorn left in Eli's bucket at 20-minute intervals over the first 100 minutes of the movie.

**7** Use the given rule to find the values of the unknown variable in each of the following tables.

**a** Add 5 to each value of $x$.

| $x$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** Subtract 7 from each value of $x$.

| $x$ | 7 | 12 | 17 | 22 | 27 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**c** Multiply each value of $a$ by 2.

| $a$ | 10 | 11 | 13 | 16 | 20 |
|---|---|---|---|---|---|
| $b$ | | | | | |

**d** Divide each value of $m$ by 3.

| $m$ | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| $n$ | | | | | |

**8** Match each rule **a–d** with its corresponding formula **A–D**.

**a** Multiply $x$ by 7 to get $y$.

**b** Add 7 to $x$ to get $y$.

**c** Multiply $x$ by 7 then add 7 to get $y$.

**d** Subtract 7 from $x$ to get $y$.

**A** $y = x + 7$

**B** $y = x - 7$

**C** $y = x \times 7$

**D** $y = x \times 7 + 7$

6A.3 **9** Use the given formula to find the values of $y$ in each of the following tables.

**a** $y = x + 3$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** $y = x - 2$

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**c** $y = 4 \times x$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**d** $y = 12 \div x$

| $x$ | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**e** $y = 2 \times x - 3$

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**f** $y = x^2$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**10** For each of the following formulas, draw up a table to represent the relationship between $x$ and $y$. Complete the table by selecting five values for $x$ and finding the corresponding value of $y$.

**a** $y = 10 \times x$

**b** $y = x \div 3$

**c** $y = x - 8$

**d** $y = 3 \times x + 7$



**11** Hira works at a cafe and receives a 50-cent tip from every customer she serves.

**a** Use this information to complete the table below.

| **Number of customers** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Amount in tips earned ($)** | | | | | |

**b** What are the two variables in this relationship?

**c** How much money will Hira earn if she serves:

**i** five customers?

**ii** six customers?

**iii** 12 customers?

**d** How many customers would Hira have to serve to earn:

**i** $2 in tips?

**ii** $10 in tips?

**iii** $100 in tips?

**12** The relationship between two variables can be represented by a rule, a table of values or a graph. Each representation has different advantages. Match each representation **a–c** to its corresponding property **A–C**.

**a** A rule — **A** can be used to visually identify any patterns in the relationship between two variables.

**b** A table of values — **B** can be used to calculate the value of an unknown variable.

**c** A graph — **C** provides a sample of values for each variable.

**13** A pasta machine flattens and stretches one piece of dough to produce a new piece that is longer. One piece of dough is 10 cm long when it goes into the pasta machine. It comes out of the machine 30 cm long. Another 20 cm piece goes into the machine and comes out 60 cm long. A third 30 cm piece goes into the machine and comes out 90 cm long.

**a** Write a sentence describing the relationship between the length of dough that goes into the machine and the length of dough that comes out of the machine.

**b** Complete the table below.

| **Length of dough in (cm)** | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| **Length of dough out (cm)** | | | | | | |

**c** Calculate the length of dough:

**i** coming out of the machine if the length going in is 100 cm

**ii** going into the machine if the length coming out is 240 cm.

**14 a** Consider the calculation below:

$x - 7 = 13$

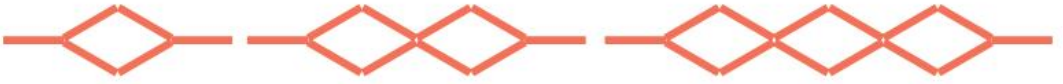
Is $x$ a variable? Explain your reasoning.

**b** Decide whether the pronumeral(s) in each of the following calculations are variables or represent a specific unknown number.

**i** $5 \times p = 10$ **ii** $2 \times a + 1 = 15$ **iii** $y = x - 5$

**15** The formula $y = 4x + 2$ represents a relationship that relates to designs of a rollerblading track. Each track is made up of a different number of identical concrete slabs.

**Aerial plan of some possible tracks**



**one diamond** **two diamonds** **three diamonds**

**a** What does $x$ represent?

**b** What does $y$ represent?

**c** Describe the design of the largest rollerblading track that could be made using 22 concrete slabs.

**d** Design a new rollerblading track using a different shape. Write a formula to represent your design.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Variables

Investigation
Which internet provider is best?

Topic quiz
6A

# 6B Writing formulas

## Learning intentions

- ✔ I can identify patterns.
- ✔ I can write formulas from a rule, a table of values and worded problems.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 5A Pronumerals |

## Writing algebraic formulas

- Algebraic formulas are a powerful way to make **generalisations**. In maths, generalising involves writing a rule or formula that describes a pattern or a relationship between two or more variables. In algebraic formulas:
  - → Products are simplified by leaving out the multiplication sign and placing the number before the pronumeral. $x \times 7 = 7 \times x = 7x$
  - → When a pronumeral is multiplied by 1, the 1 is left out. $x \times 1 = 1 \times x = x$
  - → Quotients are represented using fractions. $x \div 7 = \frac{x}{7}$

### Example 6B.1 Recognising patterns

Describe the pattern in the following sequence.
1, 3, 9, 27, 81, …

| THINK | WRITE |
|---|---|
| **1** Look for a pattern between the numbers in the sequence. | ×3 ×3 ×3 ×3<br>1, 3, 9, 27, 81, ... |
| **2** Describe the pattern in a sentence. | Each term is 3 times the term before. |

### Example 6B.2 Writing a formula from a table of values

Use the pronumerals in the table to write a formula describing the relationship between the variables.

| $x$ | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 | 7 |

| THINK | WRITE |
|---|---|
| **1** Look for a pattern between the two variables. | Each $y$ value is obtained by subtracting 2 from $x$. |
| **2** Check the pattern works with all values. | $5 - 2 = 3$<br>$6 - 2 = 4$<br>$8 - 2 = 6$<br>$9 - 2 = 7$ |
| **3** Write the formula using the pronumerals in the table. | The formula is $y = x - 2$. |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## **Example 6B.3** Writing a formula from a worded problem

A packet of lollies is divided among 4 people. If there are $p$ lollies in the packet, write a formula for the number of lollies, $n$, each person receives.

| THINK | WRITE |
|---|---|
| **1** Identify the variables in the problem. | $n$ = number of lollies each person receives<br>$p$ = number of lollies in the packet |
| **2** Think about which operation is taking place. | The number of lollies in the packet, $p$, must be *divided* between 4 people. |
| **3** Write a formula for the relationship between the two variables. The number of lollies each person receives, $n$, is equal to the number of lollies in the packet, $p$, divided by 4. | $n = p \div 4$ |
| **4** Simplify any products or quotients. | $n = \frac{p}{4}$ |

**Helpful hints**

You can use any letter or symbol you like to represent a variable. The number of oranges going into a juicer, for example, can be represented by any letter or symbol: $o$, $x$ or even a ☺!

ANS p573

# **Exercise 6B** Writing formulas

△ 1–4, 5(a–c), 7–9, 11–13, 15 □ 2, 4, 5(b, d, e, f), 6, 9, 10, 12, 13, 15, 16, 17(a–c), 18(a, b), 19 ⬠ 2(c, d), 5(d–f), 10, 12, 14–16, 17(d–f), 18(d–f), 19, 20

6B.1 **1** Describe the pattern in each of the following sequences.

**a** 0, 4, 8, 12, 16, …
**b** 1, 12, 23, 34, 45, …
**c** 2, 20, 200, 2000, 20 000, …
**d** 48, 24, 12, 6, 3, …

**2** Write the next three terms in the following sequences.

**a** 5, 12, 19, 26, 33, …
**b** 92, 86, 80, 74, 68, …
**c** 1, 4, 9, 16, 25, …
**d** 1, 2, 6, 24, 120, …

**3** Use each table to complete the rule for the relationship between the set of $x$ and $y$ values.

**a** Rule: add _____ to $x$ to get $y$.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 13 | 14 | 15 | 16 | 17 |

**b** Rule: divide each value of $x$ by _____ to get $y$.

| $x$ | 4 | 8 | 12 | 16 | 20 |
|---|---|---|---|---|---|
| $y$ | 1 | 2 | 3 | 4 | 5 |

UNDERSTANDING AND FLUENCY

**4** Complete the formula for each relationship from the table of values.

**a** $y = x +$ _____

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 8 | 9 | 10 | 11 | 12 |

**b** $y = x -$ _____

| $x$ | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $y$ | 1 | 2 | 3 | 4 | 5 |

**c** $n =$ _____ $\times m$

| $m$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $n$ | 10 | 15 | 20 | 25 | 30 |

**d** $f = e \div$ _____

| $e$ | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| $f$ | 1 | 2 | 3 | 4 | 5 |

6B.2 **5** Use the pronumerals in each table to write a formula describing the relationship between the variables.

**a**

| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $y$ | 5 | 6 | 7 | 8 | 9 |

**b**

| $p$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $t$ | 0 | 3 | 6 | 9 | 12 |

**c**

| $c$ | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| $d$ | 6 | 7 | 8 | 9 | 10 |

**d**

| $k$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $n$ | 14 | 28 | 42 | 56 | 70 |

**e**

| $x$ | 6 | 10 | 18 | 26 | 30 |
|---|---|---|---|---|---|
| $y$ | 3 | 5 | 9 | 13 | 15 |

**f**

| $m$ | 4 | 9 | 11 | 16 | 19 |
|---|---|---|---|---|---|
| $h$ | 8 | 13 | 15 | 20 | 23 |

6B.3 **6** Your friend takes 3 pens out of their pencil case. If there were originally $p$ pens in your friend's pencil case, write a formula for the number of pens, $n$, left in your friend's pencil case.

**7** Natasha has $x$ dollars in her wallet. She receives \$20 as a gift. Write a formula for the amount of money, $t$, Natasha now has in her wallet.

**8** A recipe for scrambled eggs requires 2 eggs per person. Write a formula for the number of eggs, $n$, needed to make scrambled eggs for $p$ people.

**9** At the start of each year, Ying Ying is 5 years older than her little sister.

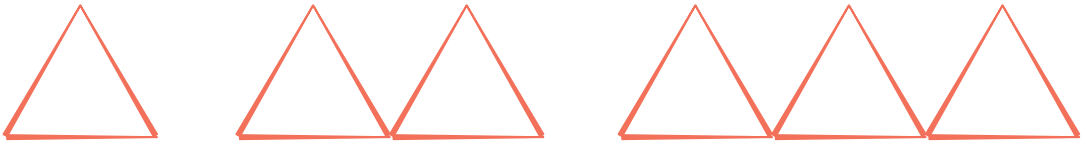
**a** Identify the variables in this problem and select a pronumeral to represent each variable.

**b** Use your pronumerals to write a formula that represents the relationship between the girls' ages.

**10** There are twice as many dogs than people at the park. Using pronumerals, write a formula to represent this relationship.

PROBLEM SOLVING AND REASONING

**11** A pattern can be seen when shapes containing different numbers of triangles are made using toothpicks.



**1 triangle** **2 triangles** **3 triangles**

**a** Complete the table below to show the relationship between the number of triangles and the number of toothpicks needed to make the shape.

| **Number of triangles** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Number of toothpicks** | | | | | | |

**b** Write a formula for the relationship between the two variables.

**c** Use the formula to find:

**i** the number of toothpicks needed to make a shape containing 120 triangles

**ii** the number of triangles that could be made with 237 toothpicks.

**12** Tayla goes for a run and her fitness tracker informs her that she has taken 6 minutes to run 1 kilometre.

**a** Assuming Tayla continues to run at the same speed, complete the table below.

| **Time (min)** | 6 | 12 | 18 | 24 | 30 |
|---|---|---|---|---|---|
| **Distance (km)** | | | | | |

**b** Write a formula for the relationship between the distance that Tayla will cover and the time she spends running.

**c** What distance would you expect Tayla to cover in one hour?

**13** The formula $c = 5n$ relates to the cost in dollars, $c$, of buying a certain number of burgers, $n$.

**a** How much does one burger cost?

**b** How much will 10 burgers cost?

**14** The cost of hiring a car is made up of a daily rate plus a set fee for insurance. The total cost of hiring a car, $c$, can be calculated using the formula $c = 25d + 110$.

**a** What variable does the pronumeral $d$ represent in this formula?

**b** How much does the insurance cost?

**c** Excluding the set fee for insurance, how much does it cost to hire the car per day?

**d** How much will it cost to hire the car for one week?

**15** Complete the formula for each relationship from the table of values. Notice that each rule involves more than one operation acting on the variable, so finding the formula can be more difficult.

**a** $y = 2x +$ ____

+1 +1 +1 +1

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 7 | 9 | 11 | 13 |

+2 +2 +2 +2

**b** $y = 5x -$ ____

+1 +1 +1 +1

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 3 | 8 | 13 | 18 | 23 |

+5 +5 +5 +5

**c** $y = 3x +$ ____

+1 +1 +1 +1

| $x$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $y$ | 16 | 19 | 22 | 25 | 28 |

+3 +3 +3 +3

**16** Look at the numbers written above and below each table of values in question **15**.

**a** How do these numbers relate to the values of $x$ and $y$?

**b** How do the numbers below the $y$ values relate to the formulas?

**c** Explain how the numbers below the table can help you work out the formula in more difficult cases.

PROBLEM SOLVING AND REASONING

**17** Use the strategy of finding differences in question **15** to work out the formula for each relationship.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 8 | 11 | 14 | 17 |

**b**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 1 | 5 | 9 | 13 | 17 |

**c**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 8 | 10 | 12 | 14 | 16 |

**d**

| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $y$ | 22 | 29 | 36 | 43 | 50 |

**e**

| $x$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $y$ | 4 | 7 | 10 | 13 | 16 |

**f**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 6 | 11 | 16 | 21 | 26 |

**18** For each of the following tables:

**i** complete the missing $x$ and $y$ values

**ii** write a formula for the relationship between $x$ and $y$.

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | | 10 | | | 25 |

**b**

| $x$ | 3 | 5 | 7 | | | |
|---|---|---|---|---|---|---|
| $y$ | | | | 54 | 66 | 78 |

**c**

| $x$ | 10 | 15 | | | | |
|---|---|---|---|---|---|---|
| $y$ | | | | | 6 | 7 |

**d**

| $x$ | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|
| $y$ | 21 | | | | 5 |

**e**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 1 | | | 7 | |

**f**

| $x$ | 2 | 4 | 6 | | | |
|---|---|---|---|---|---|---|
| $y$ | | | | 6 | 10 | 14 |

CHALLENGE

**19** Consider the incomplete table below.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | | | 18 | | |

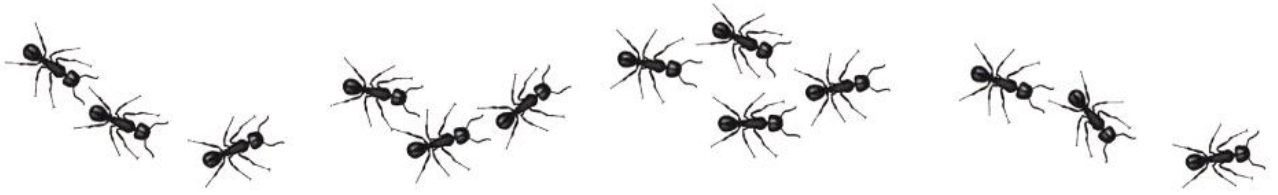
Is it possible to write a formula for the relationship between $x$ and $y$ based on the information in this table? Why or why not?

**20** Leo is watching ants crawling out of an ant hill. In the first minute, one ant leaves the ant hill. In the second minute, three ants leave the ant hill. In the third minute, five ants leave the ant hill.

**a** Assuming the pattern continues, construct a table to show how many ants will leave the ant hill each minute of the first 10 minutes that Leo spends watching the hill.

**b** Write a formula for the relationship between these variables.

**c** If Leo spends a whole hour watching the ant hill, how many ants will leave the ant hill in the sixtieth minute?



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6B



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# 6C Terms, expressions and equations

## Learning intentions

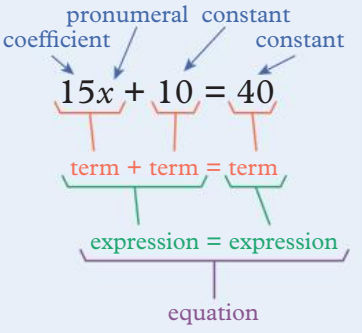
- ✔ I can identify and write terms, expressions and equations.
- ✔ I can write expressions and equations to represent worded problems.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 5A Pronumerals |

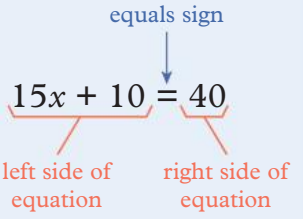
## Terms

- In algebra, a **term** is a single pronumeral, a single number, or the product of a number and one or more pronumerals.
  For example, $p$, $6a$, $5mn$, $-3x$, $\frac{1}{2}v$, 2.
- The **coefficient** of a pronumeral is the number by which that pronumeral is being multiplied.
  For example, the coefficient of $3x$ is 3, the coefficient of $x$ is 1, and the coefficient of $-\frac{1}{2}x$ is $-\frac{1}{2}$.
- A **constant term** is a term without any pronumerals.
  For example, 2, 75, −10.




## Expressions and equations

- An **expression** is either a single term or the sum or difference of two or more terms. For example, $x$, $a - 5$, $4f + 5gh + 6$.
- An **equation** is a mathematical statement that shows equivalence between the expression on the left side of the equation and the expression on the right side of the equation. Two expressions are **equivalent** when they have the same value.
  For example, $y = 10x$, $x + 3 = 5$, $10p + 2 = 11q - 7$.
  → Equations always have an equal to sign, whereas expressions do not.
  → A formula is a special type of equation that describes the relationship between two or more variables.
  For example, $y = 10x$ and $10p + 2 = 11q - 7$ are both formulas.
  → The operations in equations can be described using a range of vocabulary.




| Addition (+) | Subtraction (−) | Multiplication (×) | Division (÷) |
|---|---|---|---|
| • Sum | • Difference | • Product | • Quotient |
| • Plus | • Take away | • Times | • Divide |
| • More than | • Less than | • Multiply | • Shared |
| • Increase | • Fewer | • Groups of | • Distributed |
| • Total | • Reduce | | |
| • Together | • Minus | | |

## **Example 6C.1** Identifying terms, coefficients and constants

Consider the expression $3a - b + 7$.

**a** Determine the number of terms in the expression.

**b** Identify any variables in the expression.

**c** Identify the coefficient of each term that contains a variable.

**d** List any constant terms.

**THINK**

**a** Count the number of terms. The terms are separated by a + or − sign.

**b** Identify the pronumerals in the expression.

**c** Identify the number by which each variable is being multiplied.
As $3a = 3 \times a$, the coefficient of $3a$ is 3.
As $b = -1 \times b$, the coefficient of $b$ is $-1$.

**d** List the terms that do not contain a pronumeral.

**WRITE**

**a** There are three terms: $3a$, $-b$ and 7.

**b** The variables are $a$ and $b$.

**c** The coefficient of $3a$ is 3.
The coefficient of $b$ is $-1$.

**d** The constant term is 7.

## **Example 6C.2** Writing expressions

Write an expression for each statement.

**a** the sum of $a$ and 3

**b** twice the value of $c$

**THINK**

**a** Think about which operation is taking place. Finding 'the sum' means to add, so the sum of $a$ and 3 is $a$ plus 3.

**b** Finding twice the value of a number is to multiply by 2. Twice the value of $c$ is 2 times $c$. Simplify the product by leaving out the multiplication sign and placing the number first.

**WRITE**

**a** $a + 3$

**b** $2 \times c = 2c$

## **Example 6C.3** Writing equations with one unknown

Write an equation for each statement. Use $x$ for the unknown number.

**a** 10 less than a number is equal to 0.

**b** A quarter of a number is equal to 28.

**THINK**

**a** '10 less than a number' means subtract 10 from the unknown number, $x$, to get 0.

**b** 'A quarter of a number' means divide the unknown number, $x$, by 4 to get 28. Remember division is written as a fraction.

**WRITE**

**a** $x - 10 = 0$

**b** $x \div 4 = 28$
$\frac{x}{4} = 28$

## Example 6C.4 Writing formulas

Write a formula for each statement.

**a** $x$ divided by 3 is equal to $y$.

**b** $y$ is 5 less than $x$.

**THINK**

**a** Think carefully about the order of variables in the formula. '$x$ is divided by 3' means divide $x$ by 3 to get $y$.

Remember that quotients are written as fractions.

In formulas, the unknown with a coefficient of 1 is placed on the left-hand side of the equation.

**b** '$y$ is 5 less than $x$' means $y$ is equal to $x$ minus 5.

**WRITE**

**a** $x \div 3 = y$

$\frac{x}{3} = y$

$y = \frac{x}{3}$

**b** $y = x - 5$

**Helpful hints**

- ✔ The coefficient of $x$ is 1, not 0.
- ✔ As well as acting as an operator, minus signs can be read as negative signs and are therefore a property of the variables which they precede.

  For example, in the expression $5 - 3x$, the coefficient of $x$ is $-3$.
- ✔ Remember subtraction and division are not commutative, that is:

  $x - 3$ is not the same as $3 - x$

  $\frac{x}{3}$ is not the same as $\frac{3}{x}$

ANS p574

# Exercise 6C Terms, expressions and equations

△ 1–8, 9(a–f), 10(a–e), 12, 13

□ 1(2nd column), 2, 3(e–h), 5, 7, 8, 9(e–h), 10, 11, 13, 15, 16

⬠ 1(2nd column), 2(h–l), 3(g, h), 8–11, 14, 16–21

6C.1 **1** For each expression:

**i** Determine the number of terms in the expression.

**ii** Identify any variables in the expression.

**iii** Identify the coefficient of each term that contains a variable.

**iv** List any constant terms.

**a** $5x + 6$ **b** $2a + 4b + 3$ **c** $m + 4$

**d** $2 + 11k$ **e** $4c + 7d$ **f** $13y$

**g** $f + 5$ **h** $6x + w + 1 + 3y$ **i** $8p + 10 + m$

UNDERSTANDING AND FLUENCY

**2** Answer true or false for each statement. For the statements that are false, provide a reason for your answer.

**a** $y = 3x + 2$ is an expression
**b** $4d$ is a term
**c** $7k - 3$ is an expression
**d** the constant in $5a + 3$ is 5
**e** $m = n + 9$ is a formula
**f** the coefficient of $x$ is 1
**g** $8n = 24$ is an expression
**h** the coefficient of $6y$ is 6
**i** the constant in $4c + 5$ is 5
**j** $8 + 3a$ is an equation
**k** $6x + 2$ has two terms
**l** $5 - 2y$ is an expression

**3** State whether each of these is an expression or an equation.

**a** $3x + 2 = 5$
**b** $x - 6$
**c** $8 = 4 + y$
**d** $a + b + c = 11$
**e** $5 - 2abc$
**f** $d = 3f + 7$
**g** $k = 8m$
**h** $p + q + 2r + 5$

**4** Which of the equations in question **3** is also a formula?

6C.2 **5** Write an expression for each statement.

**a** the sum of $a$ and 7
**b** $b$ divided by 2
**c** 5 less than $c$
**d** 3 times the value of $d$
**e** $e$ subtracted from 8
**f** twice the value of $f$
**g** 4 divided by $g$
**h** 9 more than $h$
**i** the sum of $k$, $m$ and $n$

**6** How many terms are there in each expression for question **5**?

**7** Write an expression for each statement. Use the pronumeral $x$ to represent the unknown number.

**a** 8 more than a number
**b** 15 less than a number
**c** a number is multiplied by 3
**d** a number is divided by 7

**8** Write an expression for each statement. Use the pronumeral $n$ to represent the unknown number.

**a** a number is multiplied by 4 and then 5 is added to the result
**b** a number is divided by 3 and then 7 is added to the result
**c** a number is doubled and then 9 is subtracted from the result
**d** a number is multiplied by 6 and then 1 is subtracted from the result

6C.3 **9** Write an equation for each statement. Use $x$ for the unknown number.

**a** 5 more than a number is equal to 11
**b** 3 less than a number is equal to 4
**c** a number multiplied by 6 is equal to 18
**d** a number divided by 2 is equal to 5
**e** twice a number is equal to 24
**f** a number is subtracted from 10 and the result is 8
**g** a number tripled plus 1 is equal to 7
**h** two-fifths of a number minus 8 is equal to 0

6C.4 **10** Write an equation for each statement.

**a** $x$ divided by 6 is equal to $y$
**b** the sum of $x$ and 11 is equal to 20
**c** $y$ is 9 less than $x$
**d** $b$ is 130 more than $a$
**e** $q$ is 10 times the value of $p$
**f** a quarter of $d$ is equal to $c$
**g** $m$ is double the value of $n$
**h** $f$ is three-tenths of the value of $g$

**11** List the similarities and differences between equations and formulas.

**12** Teresa has $n$ Easter eggs. Write an expression to represent each situation below.

**a** Her friend has 10 more eggs.

**b** Her cousin has three times as many eggs.

**c** Her younger brother has half as many eggs.

**d** Her sister has four fewer eggs.

**13** Conor receives $p$ dollars per week as pocket money.

**a** How much pocket money does he receive:

**i** in a fortnight? **ii** in a year?

**b** If he also earns $w$ dollars per week for mowing his neighbour's lawn, how much does he receive each week?

**14** The cost of hiring a canoe is \$15 per hour plus an extra booking fee of \$10. The total cost of hiring a canoe, $c$, can be represented using the equation $c = 15n + 10$.

**a** Identify all the variables in this equation.

**b** How many terms are there in the expression on the right side of the equation?

**c** What does the constant term represent?

**15** Each week, your family buys 10 apples, seven bananas and six oranges from the local fruit market. The cost varies each week.

**a** Identify all the variables in this problem.

**b** How many terms would you expect there to be in an expression for the total weekly cost of fruit? Explain your reasoning.

**c** Choose a pronumeral to represent the cost of each type of fruit.

**d** Write an expression for the cost of:

**i** the apples **ii** the bananas **iii** the oranges.

**e** Write an expression for the total weekly cost of the fruit.

**f** Let $w$ = the total weekly cost of the fruit. Use your answer to part **e** to write a formula for this relationship.

**16** Your family buys 500 grams of chicken and 300 grams of cheese from the deli every week. The cost varies each week. Write a formula for the relationship between the total weekly cost of chicken and cheese and the price, per gram, of chicken and cheese.

**17** Entrance to the museum costs \$10 per student. Students who also wish to attend an optional show at the planetarium need to pay an extra \$5. Write a formula for the total cost of admissions to the museum, including the additional planetarium costs, for a large group of Year 7 students.

**18** Describe a real-life situation that could be represented by each of these expressions.

**a** $a + 4$ **b** $20 - b$ **c** $6c$ **d** $\frac{d}{5}$

**19** If $y$ represents an unknown number, write an expression for the sum of the number multiplied by itself and 3. (Hint: use index form)

**20** Which expression in each pair of expressions represents the larger number?

**a** $x + 3$ and $x + 5$ **b** $x + 1$ and $x - 1$ **c** $x - 7$ and $x - 4$

**d** $x - 2$ and $x$ **e** $2x + 7$ and $2x + 4$ **f** $3x - 6$ and $3x + 1$

**21** For what values of $x$ is $3x < x + 1$?

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6C

PROBLEM SOLVING AND REASONING

CHALLENGE

# 6D Order of operations in algebra

## Learning intentions

✔ I can write expressions and equations using grouping symbols.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 5A Pronumerals |
| Year 9 | 3A Simplifying |

## Order of operations

- The order of operations also applies to algebraic expressions and equations.

| | | |
|---|---|---|
| **B** | **Brackets** | Operations inside grouping symbols are always performed first. |
| **I** | **Indices** | Next, simplify powers and square roots. |
| **D**<br>**M** | **Division**<br>**Multiplication** | Then working from left to right, perform any multiplication or division. |
| **A**<br>**S** | **Addition**<br>**Subtraction** | Finally, working from left to right, perform any addition and subtraction. |

## Grouping symbols

- Operations inside **grouping symbols** are always performed first. Grouping symbols include brackets, the fraction line in a quotient, and the root sign.

  For example:

  → brackets

  $$2(7-3) = 2 \times (7-3) = 2 \times 4 = 8$$

  brackets

  $$2(7-3)$$

  → fraction line

  $$\frac{7-3}{2} = \frac{4}{2} = 2$$

  $$\frac{7-3}{2}$$

  fraction line

  → root sign.

  $$2\sqrt{7-3} = 2 \times \sqrt{7-3} = 2 \times \sqrt{4} = 2 \times 2 = 4$$

  $$2\sqrt{7-3}$$

  root sign

- The multiplication sign between numbers and brackets is often left out.

  For example, $2 \times (x+1) = 2(x+1)$.



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# Indices

- Repeated multiplication of variables can be written in index form.
  For example, $a \times a \times a = a^3$.
- Indices, or powers, only apply to the number or pronumeral immediately to the left of the power. Brackets can be used to apply the power to a whole term.
  Compare $2x^2 = 2 \times x^2 = 2 \times x \times x$ with $(2x)^2 = 2x \times 2x = 4x^2$.

## **Example 6D.1** Writing expressions using grouping symbols

Write an expression for each statement, using brackets where appropriate.

**a** Add 2 to $a$, then multiply the result by 3

**b** The sum of $a$ and $b$ all divided by 5

**c** $a$ is multiplied by 4 and the result is squared

**d** 7 less than half of $a$

| THINK | WRITE |
|---|---|
| **a** **1** Think about which operation is taking place. 'Add 2' means +2.<br>**2** Look for words that refer to the order of operations. '*Then* multiply *the result* by 3' means the multiplication should be performed *after* the addition. Group $a + 2$ using brackets, then multiply the brackets by 3.<br>**3** Simplify the product by placing the number first and removing the multiplication sign. | **a** $a + 2$<br>$(a + 2) \times 3$<br>$= 3(a + 2)$ |
| **b** **1** Finding 'the sum' means to add, so the sum of $a$ and $b$ is $a$ plus $b$.<br>**2** '*All* divided by 5' means the whole sum should be divided by 5. Group the terms together using brackets, then divide the whole expression by 5.<br>**3** Simplify the quotient by rewriting the expression as a fraction. The fraction line is a grouping symbol, so the brackets are no longer necessary. | **b** $a + b$<br>$(a + b) \div 5$<br>$= \frac{a + b}{5}$ |
| **c** **1** 'Multiplied by 4' means 4 times $a$. Simplify the product.<br>**2** '*The result* is squared' means the whole term should be squared. Brackets are required to square the whole term, including the coefficient. | **c** $a \times 4 = 4a$<br>$(4a)^2$ |
| **d** '7 less' means subtract 7 from 'half of $a$', which is $a$ divided by 2. Simplify the quotient by writing it as a fraction. | **d** $a \div 2 - 7$<br>$= \frac{a}{2} - 7$ |

## **Example 6D.2** Writing formulas using grouping symbols

Jasmine is doorknocking on her street to raise money for the Australian Red Cross. Her parents donate $x$ dollars before she leaves the house. At the first house she visits, she collects \$10. At the second house she visits, her generous neighbour triples the amount of money in her collection tin.

**a** Identify the variables in the problem.

**b** Write an expression for the amount of money in Jasmine's tin after she leaves the first house.

**c** Write a formula for the total amount of money in Jasmine's collection tin, $t$, after she visits the second house.

| THINK | WRITE |
|---|---|
| **a** Identify the variables in the problem. | **a** $x$ = the initial amount of money in Jasmine's collection tin when she leaves the house<br>$t$ = the total amount of money in Jasmine's collection tin after she visits the second house |
| **b** Jasmine collects \$10 at the first house, so the amount of money in her tin is equal to her parents' donation, $x$, plus the \$10 she collects at the first house. | **b** $x + 10$ |
| **c** **1** After Jasmine visits the second house, the total amount of money in the tin is 3 times the amount of money that was already in the tin. Brackets are necessary to group $x + 10$, the amount of money that was already in the tin. Simplify the product by placing the number first and removing the multiplication sign. | **c** $(x + 10) \times 3$<br>$= 3(x + 10)$ |
| **2** Write the formula by writing an equation for the relationship between the two variables. Equate the total amount of money in the tin, $t$, with the expression in terms of $x$. | The formula is $t = 3(x + 10)$. |

**Helpful hints**

✔ Always consider the order of operations dealing with algebraic expressions

BIDMAS

✔ The position of grouping symbols can change the order of operations – take care when placing brackets, fractions and roots!

For example,

$2x + 1$ is not the same as $2(x + 1)$

$\frac{x+1}{2}$ is not the same as $\frac{x}{2} + 1$

$2\sqrt{x+1}$ is not the same as $2\sqrt{x} + 1$

ANS p575

# Exercise 6D Order of operations in algebra

1–8, 10 | 1(d, h), 2(2nd column), 4, 5(e–i), 7, 9, 12–14 | 2(2nd column), 4(2nd column), 5(h, i), 9, 11, 13–16

UNDERSTANDING AND FLUENCY

**1** Rewrite each expression using standard algebraic notation.

**a** $7 \times (x + 6)$ **b** $2 \times (x - 3)$ **c** $(x + 1) \times 5$ **d** $(x - 2) \times 8$

**e** $(x + 4) \div 7$ **f** $(x - 5) \div 11$ **g** $(9 + x) \div 2$ **h** $(1 - x) \div 6$

6D.1 **2** Write an expression for each statement using brackets where appropriate.

**a** add 1 to $a$, then multiply the result by 10

**b** subtract 9 from $a$, then divide the result by 5

**c** $a$ is first divided by 3, then 6 is added

**d** the sum of $a$ and 21 is all divided by 7

**e** the product of 7 and $a$ is all squared

**f** 12 less than $a$ squared

**g** 4 less than the square root of $a$

**h** the square root of the product of 10 and $a$

**3** The number sentence '7 less than half of $a$' is represented by the expression:

$\frac{a}{2} - 7$

Re-write the number sentence to describe the expression below.

$\frac{a - 7}{2}$

**4** Write number sentences to describe the order of operations in the following expressions and equations.

**a** $y = 7x + 10$ **b** $y = \frac{x + 12}{9}$

**c** $6 - \sqrt{x}$ **d** $q = \frac{p}{3} - 8$

**e** $\sqrt{2s - 1}$ **f** $s = t^2 - 1$

**g** $b = \frac{1}{2}(a^2 - 1)$ **h** $n = (m + 7)^2 - 16$

**5** What is the first operation performed on $x$ in each of the following expressions?

**a** $7(3x + 4)$ **b** $\frac{x - 5}{2} + 9$ **c** $5(x + 6) - 1$

**d** $\frac{3x - 2}{7}$ **e** $9\sqrt{\frac{x}{8} + 3}$ **f** $4(x^2 - 3) + 5$

**g** $\frac{x^2 + 7}{5} - 2$ **h** $3(8x - 1)^2$ **i** $\sqrt{x^2 + 12}$

6D.2 **6** An empty bus collects $p$ passengers from the terminal and travels towards the first stop. At the first stop, 4 passengers get off. The number of passengers on the bus is doubled after picking people up at the second stop.

**a** Identify the variables in the problem.

**b** Write an expression for the number of passengers on the bus after it leaves the first stop.

**c** Write a formula for the total number of passengers on board the bus, $t$, after the second stop.

**7** Ada receives $m$ dollars for her birthday. She spends \$15 on a paint set and half of the remaining money on a canvas.

**a** Identify the variables in the problem.

**b** Write an expression for the amount of money in Ada's wallet after she buys a paint set.

**c** Write a formula for the amount of money Ada has left, $r$, after buying the paint set and canvas.

PROBLEM SOLVING AND REASONING

**8** A gardener divides a packet of seeds among four pots. He plants $n$ seeds in each pot, then adds 3 more seeds to each pot. Write a formula for the total number of seeds planted, $s$.

**9** Kai has $x$ dollars in his wallet. His mum gives him another \$30 before he goes out. Kai gives half of his money to the cashier at the cinema and receives \$7 change. Write a formula for the amount of money Kai has left, $y$, after visiting the cinema.

**10** Oscar has some friends over for a hotpot. They each put 3 dumplings in the hotpot. While Oscar and his friends are chatting, his little brother takes 5 of the dumplings for himself. Write a formula for the number of dumplings each person will now get to eat.

**11** Suqi, Max and Nas sell hats at a craft market each week. A tenth of the money they make at the market goes to tax, \$75 goes to stall hire, and the remainder is divided equally between the three friends. Write a formula for the amount of money each person receives.

**12** Celeste purchases equipment for her swimming club and bathers for each swimmer in the club. The equipment costs \$450 and each pair of bathers costs \$50. Write a formula for the total amount that each swimmer needs to pay if the cost of equipment and bathers is divided evenly among them.

**13** A market garden has tomato plants arranged in rows of equal length. There are twice as many tomato plants in each row as there are rows. Write a formula for the total number of tomato plants in the garden.

**14** Ananya visits her friend Sophie. She drives to and from Sophie's house at a speed of 50 km/h and spends 2 hours at Sophie's house. If Ananya is out for a total of $t$ hours, write a formula to find how far Ananya lives from Sophie.

**15** A square sports ground has light poles placed 3 metres apart along each side of the ground, starting at each corner of the ground.

- **a** Write a formula for the number of light poles placed along one side of the ground.
- **b** Write a formula for the number of light poles placed around the whole ground.
- **c** Explain why your answer to part **b** is not 4 times your answer to part **a**.

CHALLENGE

**16** Lena is selling tickets to her school play. She sells 8 more adult tickets than child tickets and 3 times as many senior tickets as she does child tickets. If child tickets are \$8, senior tickets are \$11 and adult tickets are \$16, write an expression for the total amount of money Lena makes from tickets sales using only one variable.

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6D



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# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

6A **1** Identify the variables in the following relationships.

**a** The number of rooms in a hotel and the total number of guests that can stay in the hotel.

**b** The time it takes to drive from Melbourne to Ballarat and the speed of the car.

6A **2** The Blue Cup Cafe makes a profit of $2 on every cup of coffee they sell.

**a** Use this information to complete the table below.

| **Number of cups of coffee** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Profit ($)** | | | | | |

**b** What are the two variables in this relationship?

**c** How much profit will The Blue Cup Cafe make if it sells:

**i** five cups of coffee? **ii** 15 cups of coffee?

**d** How many cups of coffee would The Blue Cup Cafe have to sell to make a profit of:

**i** $8? **ii** $16?

6B **3** Use the pronumerals in each table to write a formula describing the relationship between the variables.

**a**

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $y$ | 6 | 12 | 18 | 24 | 30 |

**b**

| $m$ | 10 | 12 | 14 | 16 | 18 |
|---|---|---|---|---|---|
| $n$ | 4 | 6 | 8 | 10 | 12 |

6B **4** Remy scores a total of 15 goals in $n$ soccer games. If he scored the same number of goals in each game, write a formula for the number of goals Remy scores in each game, $g$.

6C **5** For each expression:

**i** determine the number of terms in the expression

**ii** identify any variables in the expression

**iii** identify the coefficient of each term that contains a variable

**iv** list any constant terms.

**a** $10x + 2$ **b** $\frac{1}{2}f$ **c** $a + 5b - 12$ **d** $x - y$

6C **6** Write an expression for each statement.

**a** the sum of $t$ and 6 **b** 11 less than $k$

**c** 6 times the value of $b$ **d** $h$ divided by 9

6C **7** Write an equation for each statement.

**a** Half of $x$ is equal to $y$. **b** 9 more than $b$ is equal to $c$.

**c** 20 less than $m$ is equal to $n$. **d** 6 times $p$ is equal to $q$.

6D **8** Write an expression for each statement using brackets where appropriate.

**a** 7 times the square root of the sum of $a$ and 6 **b** $b$ less than 8 all divided by 3

**c** $c$ is divided by 9, then 6 is added to the result **d** the sum of $d$ and 4 all squared

6D **9** A teacher sets up four rows of $n$ tables for a class test. She then realises she hasn't set up enough tables so adds two more tables to each row. Write an expression for the total number of tables set up for the test.

6D **10** Daniel is preparing fruit baskets for the apartments in his building. He is preparing one fruit basket for each apartment. Originally, there are $p$ pieces of fruit in each basket, but Daniel decides to take one piece of fruit out of each basket to keep for himself. If there are three residents in each apartment, write an equation for the number of pieces of fruit each resident will receive, $n$.

# 6E Substitution

## Learning intentions

✔ I can substitute values into and evaluate expressions and equations.

Inter-year links

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 5B Substitution |

## Substitution

- **Substitution** involves replacing each pronumeral in an algebraic expression with a number.
- To **evaluate** an algebraic expression, substitute the known values of each pronumeral into the expression and work out the value of the expression.
  → It is important to follow the correct order of operations when evaluating an expression.
- Substitution can be used to find the value of an unknown variable when the values of all the other variables in an equation are known.

Let $x = 5$

$x^2 + \frac{x}{3} = 5^2 + \frac{5}{3}$

### Example 6E.1 Evaluating expressions containing one pronumeral

Evaluate each expression by substituting 4 for $x$.

**a** $x + 2$ **b** $3x - 1$

**THINK**

**a** Replace $x$ with 4 and complete the addition.

**b** Replace $x$ with 4 and complete the operations in the correct order. Remember that $3x = 3 \times x$ and multiplication is performed before subtraction.

**WRITE**

**a** $x + 2 = 4 + 2$
$= 6$

**b** $3x - 1 = 3 \times 4 - 1$
$= 12 - 1$
$= 11$

### Example 6E.2 Evaluating expressions containing more than one pronumeral

Evaluate each expression by letting $b = 3$ and $c = 5$.

**a** $7(c - b)$ **b** $\frac{b + c}{2}$

**THINK**

**a** 1 Replace $c$ with 5 and $b$ with 3.
2 Perform the subtraction inside the brackets first, then the multiplication.

**WRITE**

**a** $7(c - b) = 7(5 - 3)$
$= 7 \times 2$
$= 14$

**b** **1** Replace $b$ with 3 and $c$ with 5.

**2** Remember the fraction line acts like a pair of brackets and groups $b$ and $c$ together. Perform the addition inside the fraction first, then the division.

**b** $\frac{b+c}{2} = \frac{3+5}{2}$

$= \frac{8}{2}$

$= 4$

## Example 6E.3 Using formulas

Find the value of $y$ if $x = 3$.

**a** $y = \frac{x}{3}$

**b** $y = \frac{1}{3}(15 - x)$

**THINK**

**a** To find the value of $y$ substitute $x = 3$ into the expression on the right-hand side of the equation and evaluate.

**b** **1** Substitute $x = 3$ into the right-hand side of the equation and evaluate. Remember that $\frac{1}{3}(15 - x) = \frac{1}{3} \times (15 - x)$.

**2** Perform the subtraction inside the brackets first, and then the multiplication.

**WRITE**

**a** $y = \frac{x}{3}$

$= \frac{3}{3}$

$y = 1$

**b** $y = \frac{1}{3}(15 - x)$

$= \frac{1}{3} \times (15 - 3)$

$= \frac{1}{3} \times 12$

$= \frac{12}{3}$

$y = 4$

**Helpful hints**

✔ Multiplication signs are left out of algebraic terms and expressions. The implied operation between a coefficient and a pronumeral, a coefficient and a set of brackets, or two pronumerals is *multiplication*. For example,

$2x = 2 \times x$

$2(x + 1) = 2 \times (x + 1)$

$xy = x \times y$

✔ Always double-check for any changes in the sign when substituting negative numbers into an expression or equation. For example, when substituting $x = -2$ into the expression $10 - 2x$.

$10 - 2x = 10 - 2 \times -2$

$= 10 + 4$

$= 14$

ANS p575

# Exercise 6E Substitution

1–6, 7(1st, 2nd columns), 8, 9–10(b, d, g, i), 11(a, c, f, g), 12, 14, 15 | 4–6, 7(2nd, 3rd columns), 8, 9(f, h, j), 10, 11(c, e, h), 13, 16, 17 | 5(e, f), 6(d–f), 7(2nd, 3rd columns), 10(f–j), 11(f, g, h), 16–20

UNDERSTANDING AND FLUENCY

**6E.1** **1** Evaluate each expression by substituting 6 for $x$.

**a** $x + 5$ **b** $x - 2$ **c** $3x$
**d** $\frac{x}{2}$ **e** $5x + 8$ **f** $21 - x$

**2** Evaluate each expression by substituting 5 for $y$.

**a** $y + 12$ **b** $y - 1$ **c** $8y$
**d** $\frac{y}{5}$ **e** $3y - 2$ **f** $5 - y$

**3** Evaluate $3k + 5$ by substituting each of these values for $k$.

**a** $k = 1$ **b** $k = 3$ **c** $k = 4$
**d** $k = 7$ **e** $k = 0$ **f** $k = \frac{1}{3}$

**4** Evaluate $4m - 5$ by substituting each of these values for $m$.

**a** $m = 2$ **b** $m = 5$ **c** $m = 20$
**d** $m = 1$ **e** $m = 0$ **f** $m = \frac{1}{4}$

**5** Evaluate each expression by substituting in the given value.

**a** $x + 10, \quad x = -1$ **b** $3a, \quad a = -2$
**c** $5g - 3, \quad g = -1$ **d** $\frac{y}{7}, \quad y = -7$
**e** $9 - k, \quad k = -5$ **f** $12 - 2m, \quad m = -4$

**6** Evaluate each expression by substituting in the given value.

**a** $\frac{2}{3}x, \quad x = \frac{1}{3}$ **b** $10q, \quad q = \frac{1}{5}$
**c** $n + \frac{1}{2}, \quad n = \frac{1}{2}$ **d** $s - \frac{1}{4}, \quad s = \frac{2}{3}$
**e** $\frac{5}{2}b - 3, \quad b = \frac{1}{6}$ **f** $2y + \frac{3}{4}, \quad y = \frac{1}{4}$

**6E.2** **7** Evaluate each expression by letting $a = 2$, $b = 3$ and $c = 6$.

**a** $a + 4$ **b** $b + c$ **c** $5c$
**d** $3a - c$ **e** $7a + b$ **f** $\frac{c}{b}$
**g** $6(a + c)$ **h** $\frac{a + b}{5}$ **i** $3(c - a)$
**j** $4b + 3c$ **k** $\frac{c + 4}{a}$ **l** $8a - 2b$

**8** The term $ab$ means $a \times b$. Rewrite the following terms using the multiplication symbol, $\times$.

**a** $2ab$ **b** $a^2$ **c** $a^2b$

**9** Evaluate the following expressions by first rewriting the following terms using multiplication symbols, and then letting $m = 3$ and $n = 5$.

**a** $m^2$ **b** $2n^3$
**c** $m^2 + 5$ **d** $n^2 - 8$
**e** $4m^2$ **f** $2(n^2 + 3)$
**g** $9mn$ **h** $3n(m^2 + 1)$
**i** $\sqrt{3m}$ **j** $\sqrt{2n + 6}$

**6E.3** **10** Find the value of $y$ if $x = 4$.

**a** $y = 19 - 3x$ **b** $y = \frac{1}{4}x$ **c** $y = \frac{3}{2}x + 7$ **d** $y = \frac{1}{5}(x + 6)$ **e** $y = \frac{2}{3}(x - 7)$

**f** $y = \frac{4}{x}$ **g** $y = \frac{28}{x} + 3$ **h** $y = 10 - x^2$ **i** $y = 2x^2 - 40$ **j** $y = 3\sqrt{x}$

**11** Find the value of $p$ for each of these $t$ values using the formula $p = 6t - 2$.

**a** $t = 1$ **b** $t = 9$ **c** $t = -2$ **d** $t = 6.4$

**e** $t = 5.3$ **f** $t = 0.8$ **g** $t = -\frac{1}{2}$ **h** $t = \frac{2}{3}$

**12** At the school canteen, a salad roll costs \$5 and a can of drink costs \$2.

**a** Write an expression for the cost of $r$ salad rolls and $d$ cans of drink.

**b** Evaluate the expression to find the cost of three salad rolls and five cans of drink.

**c** Find the cost of eight salad rolls and seven cans of drink.

**13** **a** Write a formula that can be used to calculate the number of months, $m$, in a given number of years, $y$.

**b** Use your formula to find the number of months in:

**i** 5 years **ii** a decade

**iii** a century **iv** 80 years.

**14** The number of legs on $n$ tarantulas can be represented by the pronumeral $l$.

**a** Write a formula for the relationship between $l$ and $n$.

**b** Find the value of $l$ for each of the following values of $n$.

**i** 2 **ii** 5

**iii** 8 **iv** 11

**c** Use the formula to calculate how many legs there would be on 231 spiders.

**15** The relationship between average speed, $s$, distance, $d$, and time, $t$, can be represented by the formula $s = \frac{d}{t}$.

**a** Calculate the value of $s$ for these $d$ and $t$ values.

**i** $d = 10, t = 2$ **ii** $d = 36, t = 9$

**iii** $d = 6.3, t = 3$ **iv** $d = 22, t = 4$

**b** Find the average speed (in metres per second) of a car that travels 100 metres in 5 seconds.

**16** In most countries of the world, temperature is measured in degrees Celsius. Some countries, like the USA, use degrees Fahrenheit.
A formula can be used to convert from one temperature unit to the other. Let $C$ represent the temperature in degrees Celsius and $F$ represent the temperature in degrees Fahrenheit.

**a** Use the formula $C = \frac{5}{9}(F - 32)$ to convert these temperatures to degrees Celsius.

**i** 59°F **ii** 86°F

**iii** 32°F **iv** 95°F

**b** Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures to degrees Fahrenheit.

**i** 20°C **ii** 100°C

**iii** 25°C **iv** 60°C

**c** If the temperature in New York is 41°F, what is it in degrees Celsius?

**d** Which temperature is higher: 50°F or 15°C? Justify your answer.

PROBLEM SOLVING AND REASONING

**17** The perimeter of a two-dimensional shape is equal to the total length of the shape's outside edge.

**a** **i** Write a formula for the relationship between the total perimeter of a square, $p$, and the length of one side of the square, $s$.

**ii** Use the formula to calculate the perimeter of a square with a side length of 2 cm.

A regular pentagon is a two-dimensional shape that has five sides of equal length.

**b** **i** Write a formula for the relationship between the total perimeter of a pentagon, $p$, and the length of one side of the pentagon, $s$.

**ii** Use the formula to calculate the perimeter of a regular pentagon with a side length of 2 cm.

A regular polygon is a many-sided shape that has sides of equal length.

**c** Write a formula for the relationship between the perimeter of a polygon, $p$, that has $n$ sides, and the length of one side of that polygon, $s$.

**18** Alex has started working at the local pizza parlour. Alex's normal rate of pay is \$15 per hour, but on Sundays he earns \$18 per hour.

**a** Write a formula for the amount of money Alex earns in a week.

**b** In his first week, Alex works 40 hours from Monday to Friday and 3 hours on Sunday. Use your formula to determine how much Alex earned in this week.

**c** At the beginning of the next week, Alex works for 5 hours on Sunday. How long would he have to work at the normal rate to earn the same amount of money as in the previous week? Give your answer to the nearest hour.

**d** Alex does some research and discovers that it is illegal to pay him less than \$21 per hour on Sunday. He brings this up with his manager, who agrees to the minimum rate of pay as well as paying him for the previous 2 weeks of stolen wages. Calculate the amount of money Alex is owed.

**19** Consecutive numbers are whole numbers that follow immediately after each other. For example, 1, 2 and 3 are consecutive numbers.

**a** Write a formula that can be used to calculate the product of two consecutive numbers.

**b** Use your formula to find the product of two consecutive numbers if one of the numbers is 23.

**c** Explain why there is more than one answer to part **b**.

CHALLENGE

**20** Consider an unknown two-digit number. Adding the sum of its digits to the product of its digits gives the original number.

**a** Use algebra to find the two-digit number.

**b** There is more than one solution to this problem. Find them all and explain how you know you have found all the solutions.

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Topic quiz
6E

# 6F Simplifying

## Learning intentions

✔ I can identify like terms.
✔ I can simplify expressions by collecting like terms.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 5C Adding and subtracting algebraic terms |
| Year 9 | 3A Simplifying |

## The Laws of Arithmetic

- All the Laws of Arithmetic apply to algebraic terms and expressions, this means:

| | Arithmetic | Algebra |
|---|---|---|
| **The associative law** | $(3 + 5) + 2 = 3 + (5 + 2)$<br>$(3 \times 5) \times 2 = 3 \times (5 \times 2)$ | $(a + b) + c = a + (b + c)$<br>$(a \times b) \times c = a \times (b \times c)$ |
| **The commutative law** | $3 + 2 = 2 + 3$<br>$3 \times 2 = 2 \times 3$ | $a + b = b + a$<br>$a \times b = b \times a$ |

## Like terms

- **Like terms** are terms which contain exactly the same pronumerals.
  → Like terms can have different coefficients.
  → Following the associative law, the order of pronumerals can be different in two like terms.

| Examples of pronumeral(s) | Examples of like terms |
|---|---|
| $x$ | $x, 5x, \frac{1}{2}x, 0.5x, \ldots$ |
| $ab$ | $ba, -3ab, \frac{1}{3}ba, 1.3ab, \ldots.$ |
| $m^2$ | $2m^2, -m^2, \frac{10}{3}m^2, 5.8\,m^2, \ldots$ |

## Simplification

- Following the commutative law, algebraic expressions can be rearranged to group like terms together.
- To **simplify** an algebraic expression, or 'collect like terms', add or subtract the coefficients of like terms.
  **1** Circle the like terms in the expression.
  **2** Group the like terms together.
  **3** Add and subtract the coefficients of each set of like terms.
  → Constant terms should also be grouped and combined.

$$5a + 6b - 2a + 3b + a - b$$
$$= 5a - 2a + a + 6b + 3b - b$$
$$= 4a + 8b$$

## Example 6F.1 Identifying like terms

Identify and list the like terms in the following groups of terms.

**a** $5a, 5b, 25a, a, \frac{3}{2}b, 0.5a$

**b** $-ab, a, -a^2, 7a, 5ba, 13a^2$

**THINK**

**a** Circle the like terms, or terms that contain exactly the same pronumerals.

The value of the coefficients do not affect whether or not two or more terms are like terms, so $5a$, $25a$, $a$ and $0.5a$ are like terms because they all contain the pronumeral '$a$'. Similarly, $5b$ and $\frac{3}{2}b$ are like terms.

**b** Circle the like terms.

According to the commutative law, $ab = ba$, so although the order of the pronumerals is different, $-ab$ and $5ba$ are like terms. Remember that $a^2 = a \times a$, so $a$ and $a^2$ are not like terms. This means there are two more pairs of like terms: $a$ and $7a$, and $-a^2$ and $13a^2$.

**WRITE**

**a** $5a, 5b, 25a, a, \frac{3}{2}b, 0.5a$

The two sets of like terms in the list are:

$5a$, $25a$, $a$ and $0.5a$

$5b$ and $\frac{3}{2}b$

**b** $-ab, a, -a^2, 7a, 5ba, 13a^2$

The three sets of like terms in the list are:

$-ab$ and $5ba$

$a$ and $7a$

$-a^2$ and $13a^2$

## Example 6F.2 Simplifying expressions

Simplify each expression by collecting the like terms.

**a** $15a + a - 7a + 3a$

**b** $5a + 3b - c + 16d$

**c** $2ab + b^2 + 5 - ba + 6b - 10b + ab - 2$

**THINK**

**a** 1 Circle the like terms in the expression.

2 Group the like terms together. In this case, all the terms are like terms, so no grouping is required.

3 Add and subtract the coefficients of each set of like terms. Remember that $a = 1 \times a$, so the coefficient of '$a$' is 1.

**b** Identify any like terms in the expression. In this case, there are no like terms, so this expression cannot be simplified any further.

**c** 1 Circle the like terms in the expression. Remember that $b^2 = b \times b$, so $b$ and $b^2$ are *not* like terms. The constants should also be identified.

2 Group the like terms together. The constants should also be grouped.

3 Add and subtract the coefficients of each set of like terms. Remember that the minus sign in front of the $ba$ term is a property of that term, so the coefficient of $ba$ is $-1$. Because $b^2$ has no like terms, it cannot be simplified any further. The constants should also be combined.

**WRITE**

**a** $15a + a - 7a + 3a$

$15a + a - 7a + 3a = 12a$

**b** $5a + 3b - c + 16d$

**c** $2ab + b^2 + 5 - ba + 6b - 10b + ab - 2$

$2ab + b^2 + 5 - ba + 6b - 10b + ab - 2$

$= 2ab - ba + ab + 6b - 10b + b^2 + 5 - 2$

$= 2ab - 4b + b^2 + 3$



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**Helpful hints**

- ✔ For two terms to be like terms, the pronumeral components of the terms must be *exactly* the same, but the order of the pronumerals can vary.
- ✔ Note that:
  - → $ab$ and $ba$ are like terms
  - → $a^2$ and $a$ are *not* like terms
- ✔ In algebraic expressions, the plus and minus signs can also be read as positive or negative coefficients. The signs can therefore been seen as 'belonging' to the terms to the right of them.
  For example, in the expression $6x + y - 2x + 5y$, the minus sign can also be viewed as a negative sign belonging to the $2x$ term, so the coefficient of the second $x$ term is $-2$.

ANS p576

# Exercise 6F Simplifying

△ 1–11, 13, 17 | □ 1–2(2nd column), 3–4(e–h), 5(c, d), 6(c, d), 9, 10, 12, 14, 16, 19 | ⬠ 1(g, h), 3(g, h), 6(c, d), 9, 10, 12, 15, 16, 18–20

6F.1 **1** Identify and list the like terms in the following groups of terms.

**a** $3a, 2c, 10a, 15b, d, 9a$

**b** $x, 2y, 2x, -2x, 0.5x, 0.5y$

**c** $7pq, 18q, -5p, \frac{1}{3}pq, -\frac{1}{3}q, qp$

**d** $8m^2, m^2, 9m, 12mn, -m, 5.56m^2$

**e** $11b, 3ab, -ab, 6ba, 9ba, \frac{3}{2}b$

**f** $9xyz, 3xy, 13xy, zyx, -yx, 7x$

**g** $f, f^2, f^3, f^4, f^5, f^6$

**h** $g^2h, 4gh, \frac{1}{2}gh^2, 10gh^2, 0.01g^2h, hg$

**2** Answer true or false to each statement. Explain why each false statement is false.

**a** $5p$ and $9p$ are like terms.

**b** $\frac{1}{3}k$ and $k$ are like terms.

**c** $0.7a$ and $0.7d$ are like terms.

**d** $-xy$ and $xy$ are like terms.

**e** $2a$ and $3ac$ are like terms.

**f** $8abc$ and $-4acb$ are like terms.

**g** $2p$ and $p^2$ are like terms.

**h** $\frac{5}{3}x^3$ and $11x^3$ are like terms.

**3** Write three more examples of terms that are like terms with each of the following.

**a** $4y$

**b** $7a$

**c** $-9n$

**d** $\frac{1}{2}ab$

**e** $-10gh$

**f** $0.3kmn$

**g** $5a^2b$

**h** $t^5$

**4** Simplify each pair of like terms by adding or subtracting the coefficients of each term.

**a** $12a + 6a$ **b** $a + 10a$ **c** $5b - 4b$ **d** $3m - m$

**e** $7ab + 9ab$ **f** $13fg + 7gf$ **g** $3a^2 - 2a^2$ **h** $25x^3 + 16x^3$

**5** Recall the rules for adding and subtracting negative numbers to simplify the following expressions.

**a** $5y - 10y$ **b** $-2t + 7t$ **c** $-4p - 9p$ **d** $8ab - 9ab$

**6** Recall the rules for adding and subtracting fractions to simplify the following expressions.

**a** $\frac{2}{3}b - \frac{1}{3}b$ **b** $\frac{3}{5}q + \frac{2}{5}q$ **c** $\frac{1}{4}sp - \frac{1}{5}sp$ **d** $\frac{5}{6}r + \frac{2}{9}r$

UNDERSTANDING AND FLUENCY

**7** Use the associative law to simplify the following expressions.

**a** $16a + 18a + 2a$ **b** $33x^2 + 25x^2 + 15x^2$

**c** $4a \times 8b \times 5c$ **d** $15x \times 4x \times 100x$

**8** Use the commutative law to rearrange, and then simplify the following expressions.

**a** $7m + 65m + 13m$ **b** $-10y^2 + 6y^2 + 7y^2$

**c** $2m \times 4n \times 15n$ **d** $35a \times 3b \times 2c$

6F.2 **9** Simplify each expression by collecting the like terms.

**a** $10p - 3p + 5p$

**b** $6a + 4ab - 2ab$

**c** $7f - f + 1 + 3f + 3$

**d** $6m + 5 - 3m^2 - 2 + 11m$

**e** $7x + 8y - 3x - 2y$

**f** $21i - 6j - 6i + 5j + 7$

**g** $9k^2 - 2 + 11k - 10 - 5k + 6k^2$

**h** $12 - g^2 + 2g^3 + 5g^2 + 3 - g^3$

**10** Simplify each expression by collecting the like terms.

**a** $11a^2 + 13ab^2 - 4ab^2 + 2a^2$

**b** $2wx^2 - 2 - 2w^2 + 5w - wx^2 - 5$

**c** $4a^2 - 7a^2b + 6b^2 + 2a^2b - a^2 + 9$

**d** $3xy + 6 + 7x^2y - 9xy^2 - 5 - xy + 4xy^2$

**11** Consider the expression $5m + 4k + 9m + k$.

**a** Evaluate the expression by substituting $m = 2$ and $k = 7$ into the expression.

**b** Simplify the expression by adding the like terms.

**c** Check your answer to part **b** by substituting $m = 2$ and $k = 7$ into the simplified expression.

**d** Which expression was easier to evaluate? Why?

**12** Consider the expression $4ab + 3a^2b + 9ba + 2ba^2$.

**a** Simplify the expression by collecting like terms.

**b** Evaluate the expression when $a = 6$ and $b = 9$.

PROBLEM SOLVING AND REASONING

**13** An architect is drawing up the plans for three hotels. The hotels will have $x$ rooms on each level. The first hotel will have 4 levels, the second will have 8 and the third will have 15.

**a** Write an expression for the total number of rooms in all three hotels.

**b** Find the total number of rooms if the hotels will have 20 rooms on each level.

**14** A restaurant serves finger sandwiches as part of its high tea set menu. The chef wants to develop a formula for the number of full-size sandwiches she needs to prepare. One full-size sandwich is cut into 3 finger sandwiches. Each customer receives 6 finger sandwiches. The restaurant has $x$ tables that seat 2 customers, $y$ tables that seat 4 customers and $z$ tables that seat 6 customers.

**a** If all of the tables in the restaurant are fully occupied:

**i** write a formula for the number of finger sandwiches, $m$, required for the high tea service

**ii** write a formula for the number of full-size sandwiches, $n$, the chef must prepare.

**b** If the restaurant has 10 tables for 2 customers, 8 tables for 4 customers and 3 tables for 6 customers, how many full-size sandwiches need to be prepared to serve all the customers?

**15** Zoha arranges the books in her bookshop in shelves. At the start of the week, each shelf contains $n$ books. By the end of the week, Zoha has sold one-third of one shelf, three-quarters of a second shelf and no books from the remaining shelves.

**a** If Zoha has 6 shelves altogether:

**i** write a formula for the number of books in the bookshop at the start of the week, $s$

**ii** write a formula for the number of books in the bookshop at the end of the week, $t$.

**b** If each shelf holds 36 books, how many books did Zoha sell during the week?

**16** Think of a whole number and then complete the following operations.

**1** Add three
**2** Double it
**3** Add four
**4** Divide by two
**5** Take away the original number

Show that no matter which number you choose, the result will always be 5.

**17** Consider the following attempts made by a student to simplify each expression.

For each calculation:

**i** identify and describe the student's mistake
**ii** determine the correct answer.

**a** $2x - 5x + 3x = -6x$

**b** $x^2 + x^2 = x^4$

**c** $5x^2y + 7yx^2 - 3xy^2 = 5x^2y + 4xy^2$

**18** State whether the following mathematical statements are true or false. Explain why each statement is true or false using the Laws of Arithmetic where appropriate. Show calculations where appropriate.

The Laws of Arithmetic:

→ The *associative law* states that regardless of how numbers are grouped in sums and products, the answer does not change.

→ The *commutative law* states that the order in which numbers are added or multiplied does not change the answer.

→ The *distributive law* states that multiplication can be 'distributed' across a bracket.

**a** $a + b = b + a$

**b** $fgh = ghf$

**c** $a \times 1 = a$

**d** $a + 0 = 0$

**e** $2(a + b) = 2ab$

**f** $2 + (3 + y) = y + 5$

**g** $2(3 \times y) = 5y$

**h** $0(a + b) = 0$

**19** Does $(a - b) - c = a - (b - c)$? Explain your reasoning.

**20** Simplify the expression: $-3.2x - \frac{2}{3}x + \frac{5}{4}y + 0.99y - \frac{11}{6}x - 51.2y - y$.

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6F

# 6G Solving equations by inspection

## Learning intentions

✔ I can solve simple equations by inspection.

✔ I can solve simple equations using the guess, check and improve method.

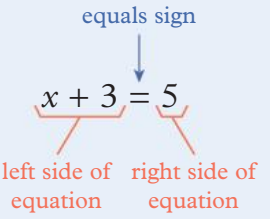
**Inter-year links**

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 6B Solving equations using inverse operations |
| Year 9 | 4A Solving linear equations |

## Solving equations

- To **solve** an equation, find the **solution** to that equation.
- To solve an equation, find the solution to that equation. For example, the solution to the equation $x + 3 = 5$ is $x = 2$ because $2 + 3 = 5$.
  - → When solving equations, the equals sign should be viewed as an indication of equivalence. That is, an indication that the left-hand side of an equation is equal in value to the right-hand side of the equation.

    *What value of x will make the left-hand side of the equation equal to the right-hand side of the equation?*




## Linear equations

- A **linear equation** with two variables is an equation where both variables are raised to the power of 1. For example, $y = x + 3$.
  - → In order for a linear equation with two variables to have a single unique solution, the value of one of the variables needs to be specified.

## Solution methods

- **Solving by inspection** involves mentally identifying the solution to an equation.
- The **guess, check and improve** method involves using trial-and-error when it is difficult to predict the solution.
- To solve equations by inspection:
  1. Examine the equation and identify a number that will make the equation a true statement.
  2. Check if the number is the solution to the equation.
- To solve equations using the guess, check and improve method:
  1. Guess a possible solution.
  2. Check if the number is the solution to the equation.
  3. Improve your guess and repeat steps 1 and 2 until the solution is found.
- To check if a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.

  For example, $x = 2$ is a solution to the equation $x + 3 = 5$ because $2 + 3 = 5$ is a true statement.



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## **Example 6G.1** Checking a solution

Determine if $x = 11$ is the solution to $x - 4 = 7$.

**THINK**

1. Write the equation.
2. Substitute $x = 11$ into the expression on the left-hand side (LHS) of the equation and evaluate.
3. Determine whether the statement is true or false by comparing the value of the left-hand side (LHS) to the value of the right-hand side (RHS) of the equation.
4. Write your answer.
   $11 - 4 = 7$ is a true statement, so $x = 11$ is the solution.

**WRITE**

$x - 4 = 7$

$$\begin{aligned} \text{LHS} &= x - 4 \\ &= 11 - 4 \\ &= 7 \end{aligned}$$

RHS = 7

LHS = RHS

$x = 11$ is the solution to $x - 4 = 11$.

## **Example 6G.2** Solving equations by inspection

Solve each equation by inspection.

**a** $a + 8 = 14$ **b** $4x = 20$ **c** $\frac{m}{5} = 9$

**THINK**

**a**
1. Consider what number, when added to 8, gives 14.
2. Check the solution.
3. Write the solution to the equation.

**b**
1. Consider what number, when multiplied by 4, gives 20.
2. Check the solution.
3. Write the solution to the equation.

**c**
1. Consider what number, when divided by 5, gives 9.
2. Check the solution.
3. Write the solution to the equation.

**WRITE**

**a** $a + 8 = 14$

Let $a = 6$

$$\begin{aligned} \text{LHS} &= a + 8 \\ &= 6 + 8 \\ &= 14 \end{aligned}$$

RHS = 14

LHS = RHS

$a = 6$

**b** $4x = 20$

Let $x = 5$

$$\begin{aligned} \text{LHS} &= 4x \\ &= 4 \times x \\ &= 4 \times 5 \\ &= 20 \end{aligned}$$

RHS = 20

LHS = RHS

$x = 5$

**c** $\frac{m}{5} = 9$

Let $m = 45$

$$\begin{aligned} \text{LHS} &= \frac{m}{5} \\ &= \frac{45}{5} \\ &= 9 \end{aligned}$$

RHS = 9

LHS = RHS

$m = 45$

## Example 6G.3 Solving equations using the 'guess, check and improve' method

Use the 'guess, check and improve' method to solve $2x + 6 = 20$.

**THINK**

1. Set up a table to show each guess, the value of the left-hand side (LHS) of the equation and the value of the right-hand side (RHS) of the equation.
   The first row of the table has been completed for a guess of $x = 3$.
2. Compare the value of the LHS and the value of the RHS and adjust your next guess accordingly. In this case, the LHS = 12 which is lower than the RHS = 20. The solution must therefore be higher than the first guess of $x = 3$.
3. Repeat step 2 until the value of the LHS = RHS.
4. Write the solution.

**WRITE**

| Guess $x$ | LHS $2x + 6$ | RHS | LHS = RHS? |
|---|---|---|---|
| 3 | 12 | 20 | no |

| Guess $x$ | LHS $2x + 6$ | RHS | LHS = RHS? |
|---|---|---|---|
| 3 | 12 | 20 | no |
| 8 | 22 | 20 | no |
| 7 | 20 | 20 | yes |

$x = 7$ is the solution to $2x + 6 = 20$.

## Example 6G.4 Using equations to solve worded problems

Jamie and Emma both collect basketball cards. Jamie has collected 8 more cards than Emma.

**a** Write an expression for the number of cards that Jamie has in his collection if Emma has $x$ cards in her collection.

**b** If Jamie has 30 cards in his collection, write an equation that can be used to find how many cards Emma has in her collection.

**c** Solve the equation by inspection to find the value of $x$.

**d** How many cards does Emma have in her collection?

**THINK**

**a** Think about which operation is required. Jamie has 8 *more* cards than Emma, so Jamie has $x$ plus 8 cards.

**b** Write an equation containing the variable $x$. If Jamie has 30 cards in his collection, the number of cards Emma has in her collection, $x$, plus 8 equals 30.

**c** Consider what number, when added to 8, gives 30. Check your solution.

**d** Emma has $x$ cards in her collection.

**WRITE**

**a** $x + 8$

**b** $x + 8 = 30$

**c** $x + 8 = 30$

Let $x = 22$

$$\begin{aligned} \text{LHS} &= x + 8 \\ &= 22 + 8 \\ &= 30 \end{aligned}$$

RHS = 30

LHS = RHS

$x = 22$

**d** Emma has 22 cards in her collection.



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**Helpful hints**

- ✔ Try to think of the equals sign, =, as an indication that the left-hand side of an equation is equal in value to the right-hand side of the equation, rather than a command to 'work out the answer'.
- ✔ Remember to define your pronumerals before writing equations to represent a worded problem.

ANS p577

# Exercise 6G Solving equations by inspection

△ 1, 2, 3(1st column), 4, 5(a, b), 6, 8, 10, 12(a–f), 14–16, 19

□ 2(d–i), 3(2nd column), 5–9, 11, 12(e–h), 13, 15, 16, 18, 21

⬠ 2(g–i), 3(n–r), 5(d), 8(c, d), 9, 11, 12(f, g, h), 17, 18, 20, 22, 23

**1** Substitute $x = 4$ into the expression on the left-hand side of each equation. Does this substitution make a true statement?

**a** $x + 9 = 13$ **b** $x - 3 = 2$ **c** $9 - x = 1$

**d** $5x = 20$ **e** $\frac{x}{2} = 2$ **f** $3x = 7$

6G.1 **2** Determine if the value given in brackets for each pronumeral is the solution to the equation.

**a** $m - 8 = 5$ ($m = 13$) **b** $7p = 28$ ($p = 3$) **c** $k + 5 = 14$ ($k = 8$)

**d** $11 - n = 7$ ($n = 4$) **e** $\frac{t}{4} = 6$ ($t = 24$) **f** $9y = 28$ ($y = 3$)

**g** $6 + c = 7$ ($c = 1$) **h** $f - 4 = 4$ ($f = 6$) **i** $\frac{x}{5} = 1$ ($x = 5$)

6G.2 **3** Solve each equation by inspection.

**a** $a + 7 = 12$ **b** $b - 1 = 5$ **c** $2c = 16$

**d** $\frac{d}{3} = 6$ **e** $5e = 50$ **f** $\frac{f}{8} = 2$

**g** $g - 6 = 3$ **h** $h + 4 = 11$ **i** $10 - i = 6$

**j** $7j = 56$ **k** $3 + k = 13$ **l** $l - 5 = 6$

**m** $\frac{20}{m} = 5$ **n** $\frac{x}{6} = 9$ **o** $x + 11 = 11$

**p** $9 = p - 2$ **q** $33 = 3q$ **r** $3 = \frac{t}{4}$

**4** Complete this table to solve $4x + 11 = 35$ using the 'guess, check and improve' method.

| Guess $x$ | LHS $4x + 11$ | RHS | LHS = RHS? |
|---|---|---|---|
| 3 | 23 | 35 | no |
| 9 | 47 | 35 | no |
| 5 | | 35 | |
| 7 | | 35 | |
| | | | |

6G.3 **5** Use the 'guess, check and improve' method to solve each equation.

**a** $2x + 7 = 29$ **b** $3x - 5 = 19$ **c** $6x - 4 = 74$ **d** $7x + 9 = 121$

**6** Which solution will make $4x + 3 = 11$ a true statement?

**A** $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$ **E** $x = 5$

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

**7** What value of $x$ will make $2x + 1 = 7$ a true statement?

**8** Solve each equation by inspection.

**a** $2x - 3 = 1$ **b** $5x + 2 = 22$ **c** $4x - 1 = 11$ **d** $9 - 2x = 7$

**9** Write an equation that has a solution of $x = 6$ and includes:

**a** addition

**b** multiplication

**c** subtraction and division

**d** brackets, addition and subtraction.

**10** Consider the linear equation $y = 3x + 5$. Find the value of:

**a** $y$ when $x = 1$ **b** $y$ when $x = 12$ **c** $x$ when $y = 11$ **d** $x$ when $y = 20$

**11** Consider the linear equation $10a - 2b = 20$. Determine the value of:

**a** $b$ when $a = 4$ **b** $a$ when $b = 5$ **c** $b$ when $a = 2$ **d** $a$ when $b = 25$

**12** For each of the following statements:

**i** Write an equation. Use $x$ to represent the unknown number.

**ii** Solve the equation using any method.

**iii** Check your solution by substituting the solution back into the equation.

**a** When 6 is added to a number, the result is 13.

**b** When a number is multiplied by 9, the result is 54.

**c** 7 less than a number is equal to 5.

**d** Twice a number is 15.

**e** A number is multiplied by 2, then 5 is added to give 23.

**f** 8 less than a number all times 3 is equal to 24.

**g** The sum of one quarter of a number and 2 is equal to 20.

**h** Half of the sum of a number and 3 is equal to 13.

PROBLEM SOLVING AND REASONING

6G.4 **13** T-shirts are on sale for \$15 each.

**a** Write an expression for the cost of $n$ T-shirts.

**b** Write an equation to represent how many T-shirts you can buy with \$60.

**c** Solve the equation by inspection to find the value of $n$.

**d** How many T-shirts can you buy with \$60?

**14** Nikita has \$5 and buys a sausage in bread at a fundraising event.

**a** If a sausage in bread costs $s$ dollars, write an expression for the amount of change Nikita receives.

**b** Nikita receives \$3 change. Write an equation that can be used to find the cost of a sausage in bread.

**c** Solve the equation by inspection to find the value of $s$.

**d** How much is a sausage in bread?

**15** A bus has 17 unoccupied seats and a total seating capacity of 48.

**a** Select and define a pronumeral to represent the unknown in this problem.

**b** Write an equation to represent this problem.

**c** Solve the equation by inspection.

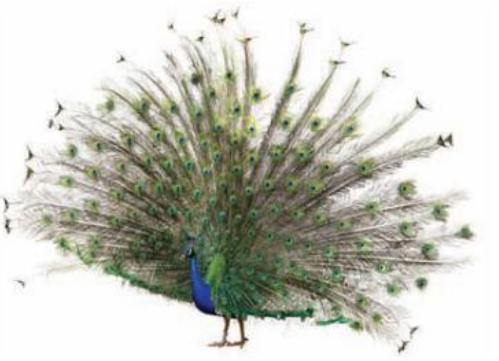
**d** Assuming there is one person in each of the occupied seats, how many people are on board the bus?

**16** James earns \$12 an hour working at the local deli. He earns a total of \$336 in January.

**a** Select and define a pronumeral to represent the unknown in this problem.

**b** Use the pronumeral from part **a** to write an equation for the relationship between the total number of hours James worked, and the total amount of money he earned in January.

**c** Use the equation from part **b** to determine how many hours he worked in January.



**17** There are twice as many peacocks at the zoo as there are giraffes.

**a** Write an equation to represent this problem.

**b** If there are 8 peacocks at the zoo, use the equation from part **a** to determine how many giraffes there are at the zoo.

**18** There are $x$ cars and $y$ motorbikes in a car park. Altogether, there are 15 vehicles and 42 wheels in the car park.

**a** Write an equation to represent the total number of vehicles in the car park.

**b** Write an equation to represent the total number of wheels in the car park.

**c** Use the 'guess, check and improve' method to determine how many cars and motorbikes are in the car park.

**19** In a volleyball tournament, Eleanor plays 4 more games than Jing. They play a total of 12 games. If Eleanor played $x$ games and Jing played $y$ games, complete the following.

**a** Write an equation to represent the relationship between the number of games Eleanor played and the number of games Jing played.

**b** Write an equation to represent the total number of games they played.

**c** Use the 'guess, check and improve' method to determine how many games Eleanor and Jing each played.

**20** Identify the most efficient method to solve each of the following equations. Justify your selection by referring to the operations in each equation.

**a** $10 - x = 2$

**b** $5(x + 3) = 4x$

**21** Describe one advantage and one disadvantage of:

**a** solving equations by inspection.

**b** the 'guess, check and improve' method for solving equations.

**22** Solve each equation by inspection.

**a** $\frac{x}{2} + 3 = 4$

**b** $\frac{x}{3} - 1 = 3$

**c** $\frac{x + 1}{4} = 2$

**d** $\frac{x - 2}{6} = 1$

**23** Consider the equation $x^2 + 10 = 7x$.

**a** Use the 'guess, check and improve' method to solve the equation.

**b** There are two possible solutions to this equation. Find a second solution to this equation.

**c** Compare this equation to the equation in question **3**. All the equations in question **3** have only one solution. What feature of the equation above generates a second solution?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

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Solving equations by inspection

Investigation
Everyday groceries

Topic quiz
6G

# 6H Solving equations using inverse operations

## Learning intentions

✔ I can solve equations using inverse operations.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Writing formulas |
| Year 8 | 6C Solving equations with the unknown on both sides |
| Year 9 | 4A Solving linear equations |

## Equivalent equations

- **Equivalent equations** are equations that have the same solution.
  For example, the following equations are equivalent because they all have the solution, $x = 1$.

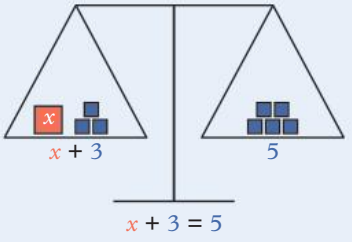
$$2x + 3 = 5 \quad (-3)$$
$$2x = 2 \quad (\div 2)$$
$$x = 1$$

- The simplest equivalent equation to any equation is the solution, in which the pronumeral is by itself on one side of the equation.
  For example, the solution to the equivalent equations listed above is represented by the equation $x = 1$.

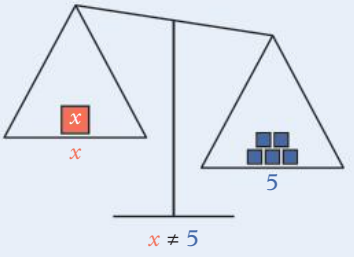
## The balance model

- In the balance model, an equation is viewed as a set of balanced scales with each scale representing one side of the equation.
  To 'solve' an equation using the balance model:
  1. Remove items from the scale containing the pronumeral block until there is a single pronumeral block by itself on one side of the scales.
  2. Make the same changes to the other scale to re-balance the scales. With only one pronumeral on one scale, the scales represent the simplest equivalent equation, or the solution to the equation.

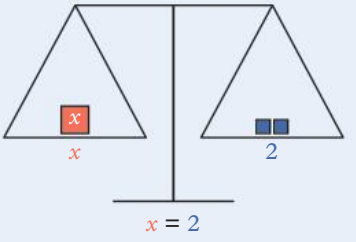
For example, the diagram below represents the equation $x + 3 = 5$.




To get $x$ by itself, we remove three blue blocks from the left-hand scale.




To re-balance, we also remove three from the right-hand scale.




Once these changes have been made, it is clear that one $x$ block has the same mass as 2 blue blocks, therefore $x = 2$.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# Inverse operations

To algebraically 'add or remove blocks' from an equation, we use inverse operations.

- **Inverse operations**, or opposite operations, reverse the effect of another operation.

| Operation | Inverse operation |
|---|---|
| +3 | −3 |
| −3 | +3 |
| ×3 | ÷3 |
| ÷3 | ×3 |

- Inverse operations are used to manipulate equivalent equations until we obtain the simplest equivalent equation – the solution, in which the pronumeral, $x$, is by itself on one side of the equation.

  To solve equations algebraically using inverse operations:

  1. Identify the operation(s) acting on the pronumeral.
  2. Identify the inverse operation(s) required to reverse the operation(s) and get $x$ by itself on one side of the equation.
  3. Apply the inverse operations to both sides of the equation or 'do the same to both sides' to maintain equivalence.

  For example, solving this equation using inverse operations:

$$x + 3 = 5$$
$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

- For equations involving more than one operation, inverse operations must be performed in the reverse order to BIDMAS.

## Example 6H.1 Solving one-step equations using inverse operations

Solve the one-step equations below using inverse operations.

**a** $x - 7 = 11$

**b** $4x = 64$

**THINK**

**a** 1 Identify the operation acting on the pronumeral.

2 Identify and apply the inverse operation to both sides of the equation. The opposite of subtracting 7 is adding 7 to $x$.

3 Simplify both sides of the equation by completing any arithmetical operations.

4 Write the solution to the equation.

**b** 1 Identify the operation.

2 Identify and apply the inverse operation. The opposite of multiplying by 4 is dividing by 4.

3 Simplify both sides of the equation.

4 Write the solution.

**WRITE**

**a** The operation acting on $x$ is −7.

The inverse operation to −7 is +7.

$x - 7 + 7 = 11 + 7$

$x + 0 = 18$

$x = 18$

**b** The operation acting on $x$ is ×4.

The inverse operation to ×4 is ÷4.

$\frac{4x}{4} = \frac{64}{4}$

$\frac{\cancel{4}x}{\cancel{4}} = 16$

$x = 16$

## Example 6H.2 Solving two-step equations using inverse operations

Solve the two-step equations below using inverse operations.

**a** $2x + 5 = 11$ **b** $\frac{x}{3} - 4 = 11$

**THINK**

**a** **1** Apply the inverse operations in the reverse order to BIDMAS. First, identify any addition/subtraction acting on the $x$ term. 5 is being added to the $x$ term, so subtract 5 from both sides of the equation and simplify.

**2** Second, identify any multiplication/division acting on $x$. $x$ is being multiplied by 2, so divide both sides of the equation by 2 and simplify to get $x$ by itself on the left-hand side of the equation and obtain the solution.

**b** **1** Apply the inverse operations in the reverse order to BIDMAS. 4 is being subtracted from the $x$ term, so add 4 to both sides of the equation and simplify.

**2** $x$ is being divided by 3, so multiply both sides of the equation by 3 and simplify to obtain the solution.

**WRITE**

**a**
$$2x + 5 = 11$$
$$2x + 5 - 5 = 11 - 5$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$\frac{\cancel{2}x}{\cancel{2}} = 3$$
$$x = 3$$

**b**
$$\frac{x}{3} - 4 = 11$$
$$\frac{x}{3} - 4 + 4 = 11 + 4$$
$$\frac{x}{3} = 15$$
$$\frac{x}{3} \times 3 = 15 \times 3$$
$$\frac{\cancel{3}x}{\cancel{3}} = 45$$
$$x = 45$$

## Example 6H.3 Solving two-step equations involving grouping symbols using inverse operations

Solve the two-step equations below using inverse operations.

**a** $3(x - 5) = 9$ **b** $\frac{x + 13}{7} = 2$

**THINK**

**a** **1** Apply the inverse operations in the reverse order to BIDMAS. Note that the '−5' is contained within brackets, and is therefore considered a bracketed operation, not an addition/subtraction. As there is no addition/subtraction, begin by identifying any multiplication/division and apply the inverse operation. The bracketed expression is being multiplied by 3, so divide both sides of the equation by 3 and simplify.

**2** There are no longer any operations acting on the brackets, so they can be removed. 5 is being subtracted from $x$, so add 5 to both sides of the equation and simplify to obtain the solution.

**WRITE**

**a**
$$3(x - 5) = 9$$
$$\frac{3(x - 5)}{3} = \frac{9}{3}$$
$$\frac{\cancel{3}(x - 5)}{\cancel{3}} = 3$$
$$(x - 5) = 3$$
$$x - 5 + 5 = 3 + 5$$
$$x = 8$$

**b** **1** Apply the inverse operations in reverse order to BIDMAS. Note that the '+13' is contained within a fraction and the fraction line acts like a bracket. The '+13' is therefore considered a bracketed operation, not an addition/subtraction. The bracketed operation is being divided by 7, so multiply both sides of the equation by 7 and simplify.

**2** 13 is being added to $x$, so subtract 13 from both sides of the equation and simplify to obtain the solution.

**b**

$$\frac{x + 13}{7} = 2$$

$$\frac{x + 13}{7} \times 7 = 2 \times 7$$

$$\frac{\cancel{7}(x + 13)}{\cancel{7}} = 14$$

$$(x + 13) = 14$$

$$x + 13 - 13 = 14 - 13$$

$$x = 1$$

**Helpful hints**

- ✔ To find the solution, the pronumeral does not have to be on the left-hand side of the equation – as long as the pronumeral is by itself on one side of the equation, you've found the solution!
- ✔ Remember that in equations such as $1 - 2x = 3$, the coefficient of $x$ is $-2$. The negative sign must also be taken into account when applying inverse operations.

ANS p578

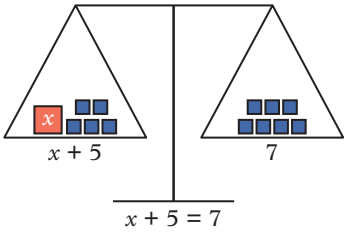
# Exercise 6H Solving equations using inverse operations

△ 1–5, 6–9(2nd, 3rd columns), 10(a, c), 11(c–f), 12, 13, 14(a, b), 18

□ 3–5, 6(g–l), 7(e–i), 8(f–i), 9(i–p), 10(b, d), 11(e–h), 13, 15, 16, 19, 21(a, b)

⬠ 7(g–i), 8(g–i), 9(i–p), 11(e–h), 14, 16, 17, 19–22

**1** Consider the diagram on the right. Each blue block has a mass of 1 g. The mass of the $x$ block is unknown.



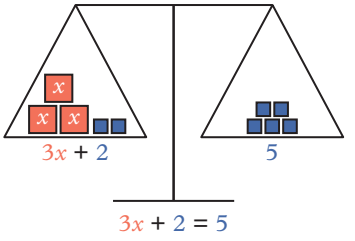

**a** What change do you need to make to the left-hand scale to get $x$ by itself?

**b** What change should you make to the right-hand scale to keep the scales balanced?

**c** Draw a new diagram showing the scales after the changes are made in parts **a** and **b**.

**d** What is the mass of the $x$ block?

**2** Consider the diagram on the right. Each blue block has a mass of 1 g. The mass of the $x$ block is unknown.




**a** What two changes do you need to make to the left-hand scale to get $x$ by itself? Hint: Remember you need to be able to make the same change to both the left and the right-hand scales. Begin by removing the blue blocks from the left-hand scale.

**b** What two changes should you make to the right-hand scale to keep the scales balanced?

**c** Draw a new diagram showing the scales after the changes are made in parts **a** and **b**.

**d** What is the mass of the $x$ block?

UNDERSTANDING AND FLUENCY

**3** Identify the inverse operation to each of the operations listed below.

**a** $+5$ **b** $-3$ **c** $\times 8$ **d** $\div 2$

**4** Solve each equation by completing the incomplete calculations below.

**a**
$$a + 5 = 12$$
$$a + 5 - __ = 12 - __$$
$$a = __$$

**b**
$$b - 3 = 8$$
$$b - 3 + __ = 8 + __$$
$$b = __$$

**c**
$$8c = 32$$
$$\frac{8c}{__} = \frac{32}{__}$$
$$c = __$$

**d**
$$\frac{d}{3} = 5$$
$$\frac{d}{3} \times __ = 5 \times __$$
$$d = __$$

**5** Apply the inverse operation(s) given in the brackets to both sides of the following equations to solve each equation.

**a** $x + 7 = 12$ $(-7)$ **b** $a - 9 = 1$ $(+9)$

**c** $4p = 24$ $(\div 4)$ **d** $\frac{s}{6} = 5$ $(\times 6)$

**e** $10 + y = 33$ $(-10)$ **f** $\frac{1}{2}f = 45$ $(\div \frac{1}{2})$

**g** $-d = 17$ $(\div -1)$ **h** $3x + 1 = 10$ $(-1, \text{ then } \div 3)$

**6H.1** **6** Solve the one-step equations below using inverse operations.

**a** $x + 8 = 13$ **b** $x - 4 = 2$ **c** $3x = 24$

**d** $\frac{x}{5} = 6$ **e** $7x = 56$ **f** $x - 6 = 11$

**g** $\frac{x}{3} = 4$ **h** $x + 1 = 8$ **i** $33x = 11$

**j** $x + 15 = 4$ **k** $-\frac{x}{10} = 18$ **l** $23 - x = 50$

**6H.2** **7** Solve the two-step equations below using inverse operations.

**a** $4x + 3 = 27$ **b** $\frac{x}{5} + 1 = 3$ **c** $2x - 5 = 1$

**d** $\frac{x}{3} - 4 = 2$ **e** $\frac{x}{4} - 1 = 6$ **f** $7x + 1 = 5$

**g** $2 - 3x = 20$ **h** $\frac{x}{9} + 12 = 2$ **i** $5x + 2 = 2$

**6H.3** **8** Solve the two-step equations below using inverse operations.

**a** $2(x + 3) = 16$ **b** $\frac{x - 4}{5} = 3$ **c** $3(x - 2) = 27$

**d** $\frac{x + 3}{7} = 2$ **e** $\frac{x - 1}{6} = 1$ **f** $\frac{x + 2}{3} = 6$

**g** $4(7 - x) = 12$ **h** $\frac{5 - x}{4} = 8$ **i** $6(x + 15) = 18$

**9** Solve each of the following equations using inverse operations.

**a** $9x + 4 = 13$ **b** $\frac{a}{2} + 5 = 12$ **c** $7(x + 1) = 49$

**d** $\frac{x + 4}{3} = 5$ **e** $\frac{p}{6} - 3 = 4$ **f** $6(d - 4) = 30$

**g** $3f - 7 = 17$ **h** $\frac{z - 8}{7} = 3$ **i** $11x + 3 = 47$

**j** $\frac{t}{5} + 8 = 14$ **k** $\frac{q + 6}{4} = 1$ **l** $\frac{2y}{7} = 3$

**m** $3(8 - x) = 6$ **n** $\frac{5c}{4} = 15$ **o** $4n - 8 = 0$

**10** Consider the linear equation $y = \frac{x}{2} + 3$. Find the value of:

**a** $y$ when $x = 2$ **b** $y$ when $x = 1$ **c** $x$ when $y = 5$ **d** $x$ when $y = 0$

**11** Solve the following linear equations by first substituting the bracketed values into each equation.

**a** $y = 3x - 10$ $(y = 2)$

**b** $y = 2(x + 7)$ $(y = 20)$

**c** $v = \frac{n}{4} - 13$ $(v = 7)$

**d** $x + 2y = 4$ $(x = 4)$

**e** $3a - 5b = 3$ $(a = 6)$

**f** $m = \frac{n + 5}{8}$ $(m = 0)$

**g** $y = \frac{9 - x}{3}$ $(y = 5)$

**h** $1 - p - \frac{q}{3} = 5$ $(q = 1)$

**12** There are $x$ mice in a cage. The pet-shop owner moves more mice into the cage so that there is now double the number of mice for sale. Monique buys 3 mice, leaving 15 mice in the cage.

**a** Write an equation to represent this problem.

**b** Solve this equation to determine how many mice were originally in the cage.

**13** An animal shelter provides care to injured wildlife before they are released back into their natural habitat. There are $n$ animals in the shelter on Monday. On Wednesday, 3 animals are brought in, half of the animals in the shelter are released on Friday, then 5 more animals are brought in on Saturday.

**a** Write an expression for the number of animals in the shelter at the end of Saturday.

**b** If there were 3 animals in the shelter on Monday, use the expression from part **a** to calculate the number animals in the shelter at the end of Saturday.

**14** To solve each of the following problems, write an equation to represent the problem and solve the equation using inverse operations. Show all steps of your working.

**a** A restaurant has an outdoor terrace it sets up with tables during summer. Each table seats four people. How many tables should the restaurant set up if 64 people book for Sunday lunch?

**b** The cost of hiring a tennis court in the evening is \$6 plus \$15 per hour. For how many hours can you hire the tennis court if you have \$81?

**c** You have \$20 and buy a drink for \$3. The cost of a standard pizza is \$9. Extra toppings are \$2 each. How many extra toppings could you get on a pizza?

**d** Alisa is saving to buy a phone that costs \$369. She is able to save \$24 per week. If she currently has \$105, in how many weeks can she buy the phone?

**15** You have some money in your wallet and your older sister gives you \$5 for cleaning her room. Since you are saving money to buy new runners, your mother agrees to match 'dollar for dollar' the amount you have. That is, she will give you extra money so that you now have twice the amount of money you had. You just have enough money to buy runners for \$136.

**a** Write an equation to represent this problem.

**b** Solve the equation to determine how much money you originally had in your wallet.

PROBLEM SOLVING AND REASONING

**16** Yuhan is planning to take a taxi to the airport. A taxi ride costs \$1.90 per minute in addition to a flag fall, or fixed starting price, of \$6.20.

**a** Write an equation for the total cost of a taxi ride in terms of the length of the taxi ride.

**b** According to Google maps, it should take Yuhan 32 minutes to get to the airport from his house. What is the total cost of a 32-minute taxi ride?

**c** Yuhan's taxi gets stuck in traffic and the taxi ride ends up costing him \$80. Based on this cost, determine how long it took Yuhan to get to the airport to the nearest minute.

**17** A chemistry student is boiling water using a Bunsen burner. The Bunsen burner increases the temperature of the water by 14.5°C per minute. The water initially measures 18°C.

**a** Write an equation for the temperature of the water in terms of the length of time spent over the Bunsen burner.

**b** What will the temperature of the water be after 5 minutes?

**c** How long will it take for the water to reach 100°C? Give your answer to the nearest minute.

**18** Ting has a packet of 45 stickers. She gives some friends 5 stickers each and keeps 15 stickers for herself. Use algebra to determine how many of Ting's friends received stickers.

**19** Ravi is a maths tutor and earns \$900 in June. This is \$100 less than twice the amount Ravi earned in May. Use algebra to determine how much money Ravi earned in May.

**20** Liam has a bag of lollies for his birthday party. Once his friends arrive, he hands out party bags with 3 lollies in each and keeps 12 lollies for himself.

**a** Write an equation for the initial number of lollies in Liam's bag in terms of the number of friends at Liam's party.

**b** Find the minimum number of lollies in Liam's bag initially. Justify your answer.

**c** It is possible that Liam's bag contained more lollies than this minimum number. What other possible solutions can you find? Explain.

**d** What is the pattern for the range of solutions you found?

CHALLENGE

**21** Solve the following equations for $x$.

**a** $\frac{8x}{3} + 1 = 9$

**b** $5(x + 3) - 2 = 33$

**c** $\frac{3(5x - 2)}{7} + 6 = 18$

**d** $4\left(\frac{x}{3} + 1\right) - 5 = 19$

**22** Find the value of $b$ in the following equation using any method.

$a + 3b = a + b + 5$

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Solving equations using inverse operations

**Investigation** A balancing act

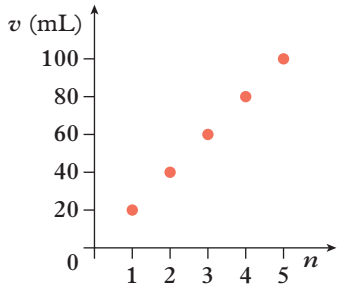
**Topic quiz** 6H

# Chapter summary

## Variables

$v = 20 \times n$

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $v$ (mL) | 20 | 40 | 60 | 80 | 100 |




## Terms, expressions and equations

pronumeral, constant, coefficient, constant

$$15x + 10 = 40$$

term + term = term

expression = expression

equation

## Substitution

Let $x = 5$

$$x^2 + \frac{x}{3} = 5^2 + \frac{5}{3}$$

## Algebraic notation

$x \times 7 = 7 \times x = 7x$

$x \times 1 = 1 \times x = x$

$x \div 7 = \frac{x}{7}$

## Grouping symbols

brackets

$2(7 - 3)$

$\frac{7 - 3}{2}$ fraction line

$2\sqrt{7 - 3}$ root sign

## Order of operations

| | | |
|---|---|---|
| **B** | **Brackets** | Operations inside grouping symbols are always performed first. |
| **I** | **Indices** | Next, simplify powers and square roots. |
| **D** **M** | **Division** **Multiplication** | Then, working from left to right, perform any multiplication or division. |
| **A** **S** | **Addition** **Subtraction** | Finally, working from left to right, perform any addition and subtraction. |

## Like terms

| Examples of pronumeral(s) | Examples of like terms |
|---|---|
| $x$ | $x, 5x, \frac{1}{2}x, 0.5x, \ldots$ |
| $ab$ | $ba, -3ab, \frac{1}{3}ab, 1.3ab, \ldots$ |
| $m^2$ | $2m^2, -m^2, \frac{10}{3}m^2, 5.8m^2, \ldots$ |

## Simplification

$5a + 6b - 2a + 3b + a - b$

$= 5a - 2a + a + 6b + 3b - b$

$= 4a + 8b$

## Solving equations by inspection

$a + 8 = 14$

Let $a = 6$.

LHS $= a + 8$

$= 6 + 8$

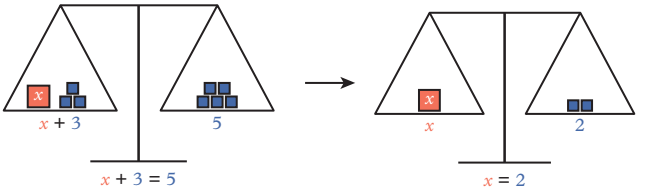
$= 14$

RHS $= 14$

LHS = RHS

So, $a = 6$.

## The balance method




## The guess, check and improve method

$2x + 6 = 20$

| Guess $x$ | LHS $2x + 6$ | RHS | LHS = RHS? |
|---|---|---|---|
| 3 | 12 | 20 | no |
| 8 | 22 | 20 | no |
| 7 | 20 | 20 | yes |

## Inverse operations

| Operation | Inverse operation |
|---|---|
| $+3$ | $-3$ |
| $-3$ | $+3$ |
| $\times 3$ | $\div 3$ |
| $\div 3$ | $\times 3$ |

$3(x - 5) = 9$

$\frac{3(x - 5)}{3} = \frac{9}{3}$

$\frac{\cancel{3}(x - 5)}{\cancel{3}} = 3$

$(x - 5) = 3$

$x - 5 + 5 = 3 + 5$

$x = 8$

BIDMAS

# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

6A **1** Which table of values matches the rule $y = 3x$?

**A**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 5 | 6 | 7 | 8 | 9 |

**B**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 2 | 3 | 4 | 5 | 6 |

**C**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 4 | 6 | 8 | 10 | 12 |

**D**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 6 | 9 | 12 | 15 | 18 |

**E**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | $\frac{2}{3}$ | 1 | $\frac{4}{3}$ | $\frac{5}{3}$ | 2 |

6B **2** Which formula describes the relationship between the two variables in the table?

| $a$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $b$ | 8 | 9 | 10 | 11 | 12 |

**A** $b = a + 6$ **B** $b = 4a$ **C** $b = a + 8$ **D** $b = 6 - a$ **E** $b = 2a$

6C **3** Which expression best represents the statement 'the sum of $a$, $b$ and $c$'?

**A** $abc$ **B** $a + bc$ **C** $ab + c$ **D** $a - b - c$ **E** $a + b + c$

6D **4** I think of a number, multiply it by 4, then subtract 6 to get a result of 12. The equation that best represents this series of operations is:

**A** $4x - 6 = 12$ **B** $4(x - 6) = 12$ **C** $4x + 6 = 12$ **D** $4x = 12 - 6$ **E** $4x - 24 = 12$

6D **5** The sum of a number and 6 is squared and the result is doubled. Which expression correctly represents this number sentence?

**A** $2n^2 + 6$ **B** $2(n^2 + 6)$ **C** $(2n)^2 + 6$ **D** $2(n + 6)^2$ **E** $(2n + 6)^2$

6E **6** If $a = 4$, $b = 3$ and $c = 1$, what does the expression $2a + b - c$ equal?

**A** 26 **B** 10 **C** 8 **D** 12 **E** 4

6F **7** When the expression $8g - 4gh + 4g + 10hg$ is fully simplified, the result is:

**A** $12g + 6gh$ **B** $4g + 14gh$ **C** $4g - 4gh$ **D** $12g - 4gh + 10hg$ **E** $4g - 4gh + 10hg$

6G **8** Which one of the following equations is $x = 7$ a solution to?

**A** $x + 1 = 7$ **B** $2x = 14$ **C** $x + 1 = 6$ **D** $7x = 1$ **E** $3x + 1 = 21$

6H **9** The value of $x$ the equation $5(x - 7) = 30$ is:

**A** $-1$ **B** 10 **C** 12 **D** 13 **E** 18

6H **10** What is the solution to $\frac{x - 6}{2} = 2$?

**A** $x = 2$ **B** $x = 4$ **C** $x = 8$ **D** $x = 10$ **E** $x = 16$

## Short answer

6A **1** Gerald is collecting worms for his compost. He starts collecting worms at 1:00 pm. in the afternoon. He collects 5 worms every 30 minutes.

**a** Identify the variables in this problem.

**b** Use this information to complete the table below.

| **Time** | 1:00 pm | 1:30 pm | 2:00 pm | 2:30 pm | 3:00 pm |
|---|---|---|---|---|---|
| **Total number of worms collected** | | | | | |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6B **2** Write a formula for the relationship between the two variables in each of the tables below.

**a**

| $x$ | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| $y$ | 6 | 7 | 8 | 9 | 10 |

**b**

| $m$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $n$ | 3 | 5 | 7 | 9 | 11 |

6C **3** For each expression, identify:

**i** the number of terms
**ii** any variables used
**iii** the coefficient of each term containing a variable
**iv** any constant terms.

**a** $7x + 3y$ **b** $a + 3$ **c** $5d + 2c + 9$ **d** $4 + e + 2t + 6s$

6C **4** Write an equation for each statement.

**a** A number multiplied by 7 equals 21.
**b** A number divided by 3 equals 6.
**c** 7 more than a number equals 10.
**d** 2 fewer than a number equals 17.

6D **5** Write an expression for each statement using grouping symbols where appropriate.

**a** Add 7 to $x$ then multiply the result by 3
**b** The sum of $p$ and 5 all divided by 13
**c** Half of $c$ all squared
**d** 3 less than $v$ squared

6E **6** Evaluate each expression by substituting $x = 1$, $y = 4$ and $z = 2$.

**a** $x + y + z$ **b** $2x + y$ **c** $3(y - z)$ **d** $\frac{xy}{z}$

6E **7** Find the value of $b$ if $a = 3$ is substituted into each formula.

**a** $b = 3a + 5$ **b** $b = 4a - 7$ **c** $b = \frac{a}{3} + 1$ **d** $b = 2(a + 6)^2$

6F **8** Simplify the following expressions by collecting the like terms.

**a** $6x - 5x$
**b** $7p + 3p^2 - p$
**c** $11st + 9t - 5t - 7st$
**d** $13mn - 3m^2 + 5nm + 3m + 7m^2$

6G **9** Solve each equation by inspection.

**a** $4a = 28$ **b** $b - 10 = 21$ **c** $c + 8 = 11$ **d** $\frac{d}{2} = 5$

6H **10** Use the guess-and-check method to find the value of $x$ in the equation below.

$$\frac{2(x - 10)}{5} = 4$$

6H **11** Solve each equation using inverse operations.

**a** $x + 9 = 21$
**b** $\frac{x}{4} = 12$
**c** $3x + 5 = 17$
**d** $2(x - 1) = 24$
**e** $\frac{x + 10}{3} = 2$
**f** $9 - \frac{x}{5} = 20$

6H **12** Check your answers to question **11** by substituting the solutions back into each equation.

## Analysis

Santo is a session musician who earns money by playing back-up guitar in recording sessions. He charges \$250 per hour, plus a flat rate of \$300 for each recording session.

**a** Identify the variables in this problem.
**b** Select and define pronumerals to represent the variables in this problem.
**c** Write a formula for the relationship between these variables.
**d** Use the formula to determine how much Santo will earn from a 5-hour recording session.

Santo also plays at music festivals. For music festivals, he charges \$325 per hour with no flat-rate fee.

**e** Write a formula that can be used to calculate how much Santo will earn from a given music festival.
**f** Santo plays at a music festival and earns \$1950. How long did he play at the festival?
**g** Santo is asked to play at a recording session and a festival on the same day. He decides he will take whichever job pays more. The music festival will pay Santo for six hours and the recording session will pay him for seven hours. Which job will Santo choose?
**h** If Santo only works one job, how many hours would he have to work in each job to earn the same amount of money?

# 7

# Angles

## Index

**7A** Classifying angles and lines
**7B** Angles at a point
**7C** Angles and parallel lines
**7D** Classifying triangles
**7E** Classifying quadrilaterals

## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Comparing angles less than 180°
- ✔ Units of measurement
- ✔ Classifying shapes

## Curriculum links

- Classify triangles according to their side and angle properties and describe quadrilaterals (VCMMG262)
- Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral (VCMMG263)
- Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (VCMMG264)
- Investigate conditions for two lines to be parallel and solve simple numerical problems using reasoning (VCMMG265)

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## Materials

- ✔ Protractor
- ✔ Compass
- ✔ Ruler
- ✔ Calculator

# 7A Classifying angles and lines

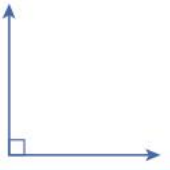
## Learning intentions

- ✔ I can identify and name line segments, rays, lines, parallel lines and perpendicular lines.
- ✔ I can measure and draw angles using a protractor.
- ✔ I can identify and classify acute, right, obtuse, straight, reflex and revolution angles.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Understanding angles |
| Year 8 | 7A Angles |
| Year 9 | 7A Angles and lines |

## Lines

| Object | Definition | Example |
|---|---|---|
| **Point** | A location in space that has no length or width. Points are represented using a dot labelled with a capital letter. | |
| **Line segment** | A line joining two points. Line segments are named using a bar and the two end points labels. | Name: $\overline{AB}$ |
| **Ray** | A line that starts at a point and continues to infinity. Rays are named using an arrow, which indicates the direction of the ray. | Name: $\overrightarrow{AB}$ |
| **Line** | A line that continues to infinity in both directions. Lines are named using a double-headed arrow. | Name: $\overleftrightarrow{AB}$ |
| **Perpendicular lines** | Two lines that meet to form a right angle. Right angles are labelled with a small box where the lines meet. |  |
| **Parallel lines** | Two lines that have the same slope and are always the same distance apart so that they never meet. Parallel lines are labelled with matching arrowheads. | |

## Angles

- When two lines or rays meet, they form an **angle**.
  - → Angles are marked using a portion of a circle called an **arc**.
  - → The two lines or rays are called the **arms** of the angle.
  - → The point at which the two lines meet is called the **vertex**.
- Angles can be named using:
  - → a pronumeral, letter or symbol that stands in for a number. In the following example the pronumeral $b$ is standing in for the size of the angle.
  - → an angle symbol followed by the capital letter labels at each point of the angle. The letter that labels the vertex should be placed in the middle of the name.



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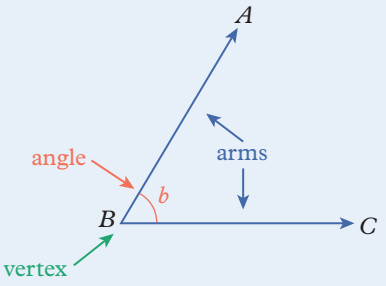
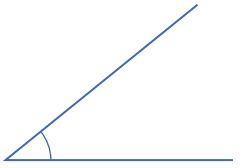
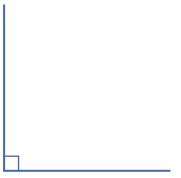
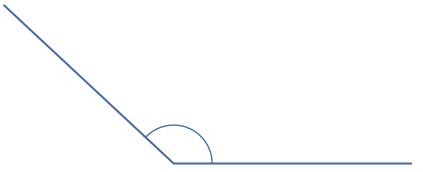
For example, in the diagram below $b = \angle ABC = 60°$.

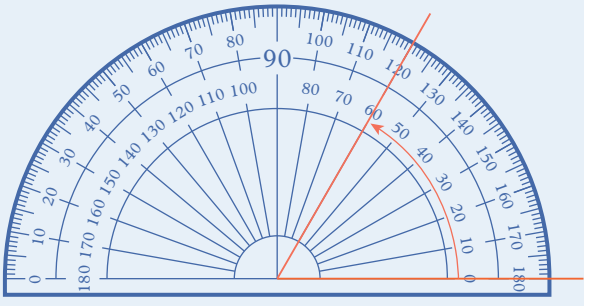



- Angles are classified according to their size.

| Classification | Definition | Example |
|---|---|---|
| **Acute angle** | Between 0° and 90° |  |
| **Right angle** | 90° |  |
| **Obtuse angle** | Between 90° and 180° |  |
| **Straight angle** | 180° | |
| **Reflex angle** | Between 180° and 360° | |
| **Revolution** | 360° | |

# Measuring angles using a protractor

- Angles are commonly measured in **degrees**, °.
  For example, there are 360° in a full revolution.
- To measure an angle using a protractor:
  1. Place the centre cross of the protractor over the vertex of the angle.
  2. Place the zero line, that reads 0°, on one arm of the angle.
  3. Read the scale from 0° to the other arm of the angle.
     For example, this angle is 60°. This measurement is taken from the inner scale because the horizontal arm of the angle is on the right-hand side.

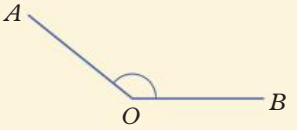


## **Example 7A.1** Naming angles and lines

Name the objects below using standard mathematical notation.

**a**

**b**




**THINK**

**a** Identify the object. The line extends from point $P$ to infinity, so this is a ray. Rays are denoted by an arrow symbol. The arrow symbol should indicate that the line extends from point $P$ in the direction of $Q$.

**b** Identify the object. The circular marker at the vertex, $O$, indicates that this is an angle. Angles are denoted using an angle symbol, $\angle$.

The vertex label, $O$, must be placed in the middle of the name.

**WRITE**

**a** $\overrightarrow{PQ}$

**b** $\angle AOB$ or $\angle BOA$

## **Example 7A.2** Measuring and classifying angles

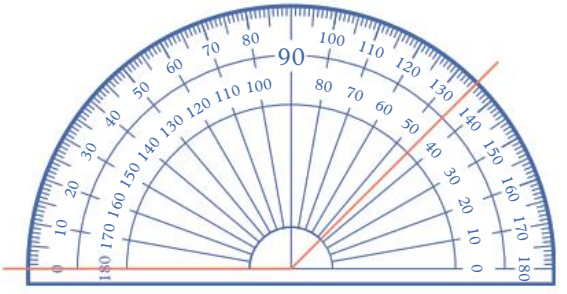
Measure and classify this angle.

**THINK**

**1** Measure the angle using a protractor. Because the horizontal arm of the angle is on the left-hand side, use the scale that begins from 0° on the left-hand side of the protractor.

**2** Classify the angle. As 135° is greater than 90° but less than 180°, this is an obtuse angle.

**WRITE**



The angle is 135°.

This is an obtuse angle.

## **Example 7A.3** Drawing acute angles using a protractor

Use a protractor to draw an angle of 60°.

**THINK**

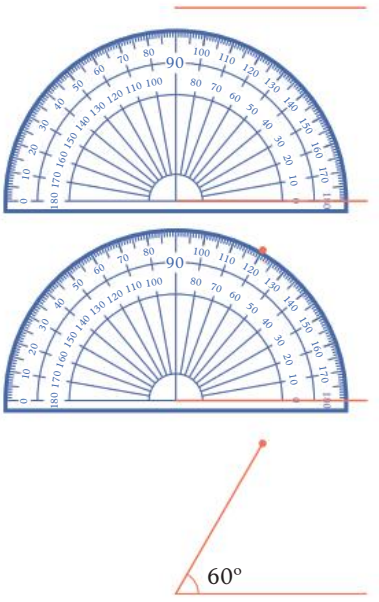
1 Begin by drawing a horizontal line segment.

2 Place the protractor over the segment so that the centre cross aligns with one end of the segment, and the segment lies under either the protractor's left- or right-hand zero line.

3 Draw a point at the 60° mark on your protractor. Because the segment is aligned with the right-hand zero line, use the inner scale to measure 60°. If the segment is aligned with the left-hand zero line, the outer scale should be used.

4 Using a ruler, join the end of the horizontal segment with the dot, and label the angle.

**WRITE**

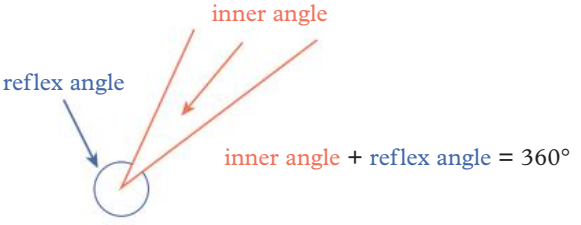



## **Example 7A.4** Drawing reflex angles using a protractor

Use a protractor to draw an angle of 240°.

**THINK**

1 A reflex angle cannot be drawn directly using a 180° protractor as shown in example 7A.3. First determine the size of the acute or obtuse angle *inside* the reflex angle by subtracting the reflex angle from a full revolution, 360°.

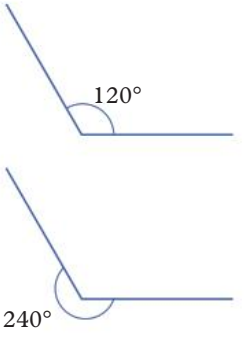



2 Use a protractor to draw the inner angle.

3 Mark the reflex angle on the underside of the obtuse angle.

**WRITE**

inner angle = 360° − 240°
= 120°



**Helpful hints**

- ✔ When reading a protractor, remember to read from 0° and use the correct scale – the outer or inner scale. To ensure you use the correct scale, it can be helpful to visually determine whether an angle is acute, obtuse or reflex before measuring it with a protractor.
- ✔ When drawing angles, think carefully about which side of the line you should place the arc on – incorrect placement of the arc can turn an acute or obtuse angle into a reflex angle and vice versa!
- ✔ Mathematical diagrams are not necessarily drawn to scale. When working with mathematical diagrams, the labels should be your first point of reference. Measuring devices like protractors should only be used when a diagram is marked as 'drawn to scale', or as directed by the question.

ANS p579

# Exercise 7A Classifying angles and lines

△ 1–4, 5(a–c), 6–8, 9(a–c), 10(a, b), 11(a, c), 13(a–c)

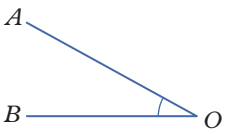
□ 1–4, 5(b, d, e), 6–8, 9(b–e), 10–11(c, d), 14–16

⬠ 1–3, 5(d–f), 6(c, d), 8–11(c, d), 15–17

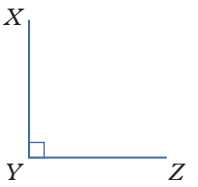
UNDERSTANDING AND FLUENCY

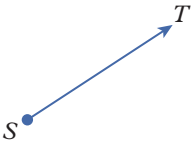
7A.1 **1** Name the objects below using standard mathematical notation.

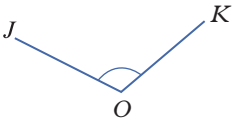
**a**

**b**



**c**

**d**


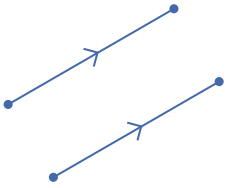

**e**


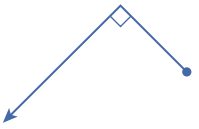

**f**


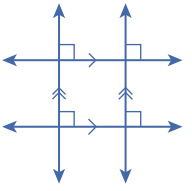

**g**

**h**

**2** Use mathematical terms to describe these diagrams.

**a**


**b**


**c**


**3** Draw a diagram of the following arrangements. Use standard mathematical symbols to illustrate the relationship between the lines in each diagram.

**a** two parallel rays

**b** a ray perpendicular to a segment

**c** five rays that radiate out from the same point

**d** two parallel lines with a line perpendicular to them

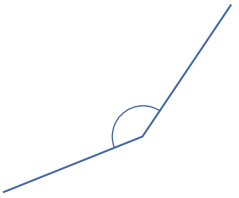
**4** Consider the following angles.

**a** By visual inspection, order each angle from largest to smallest.

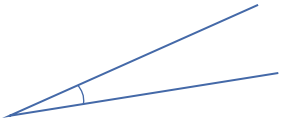
**b** Match each angle to one of the following descriptions: 90°, 180°, 360°, between 0° and 90°, between 90° and 180°, between 180° and 360°.

**c** Classify each angle.

**i**



**ii**



**iii**

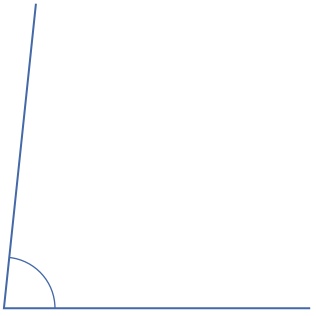
**iv**

**v**

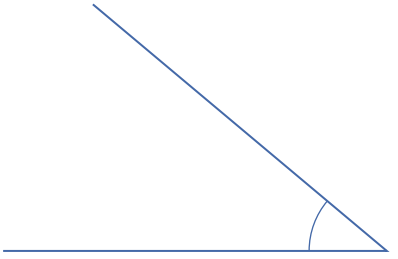
**vi**

7A.2 **5** Measure and classify each of the angles below.

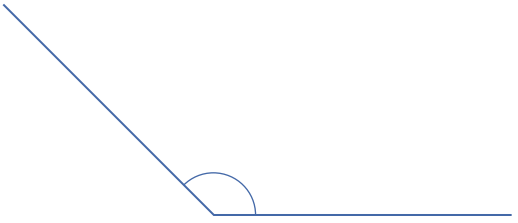
**a**



**b**

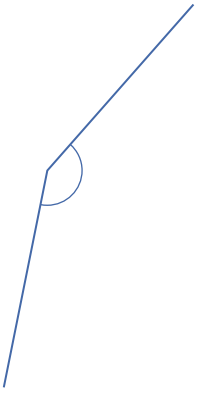


**c**

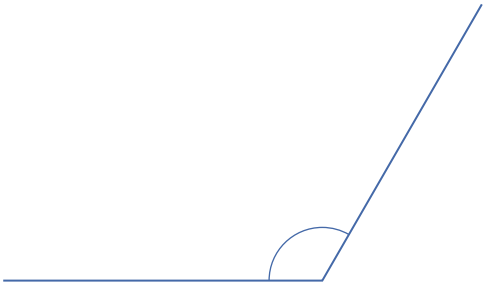


**d**

**e**

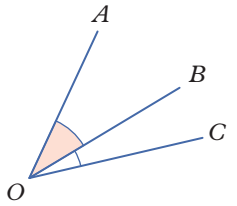


**f**

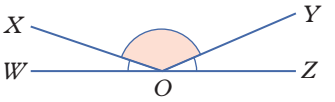


**6** Name and classify the shaded angle in each of the following diagrams.

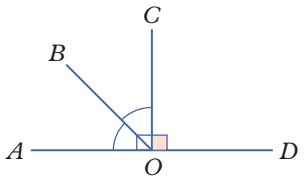
**a**



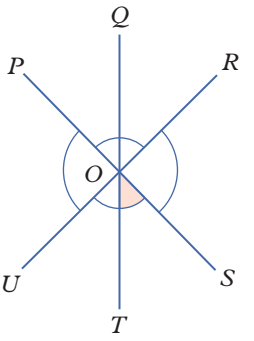

**b**



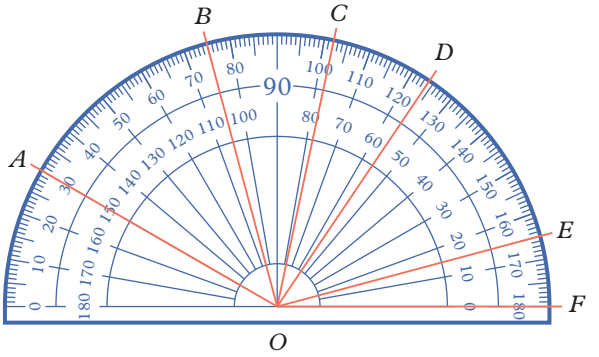

**c**




**d**




**7** Consider the diagram below. Read from the inner scale because the horizontal arm of the angle is aligned with the right-hand zero line, $\angle AOF = 150°$.




**a** What is the size of $\angle EOF$?

**b** What is the size of $\angle DOF$?

**c** Classify $\angle COF$.

**d** Name all the acute angles.

**e** Name all the obtuse angles.

**8** **i** Without a protractor, use a ruler to draw an estimate of each of the angles listed below.

**ii** Now use a protractor to measure each of your drawings. Write the actual size of each angle next to each drawing. How many of your estimates were within 10° of the actual measurement?

**a** 80° **b** 42° **c** 173° **d** 10°

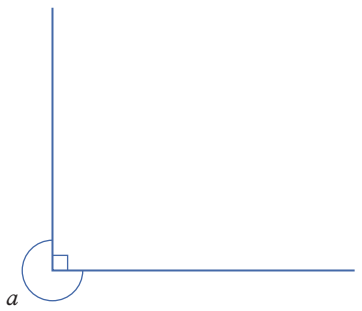
7A.3 **9** Use a protractor to draw the following angles.

**a** 45° **b** 120° **c** 165°

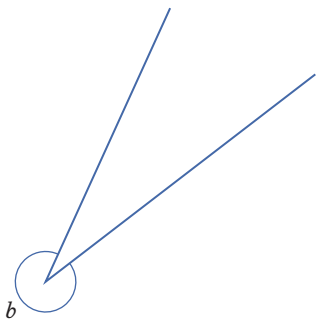
**d** 9° **e** 94° **f** 112°

**10** Find the size of the reflex angle by first measuring the acute or obtuse angle.

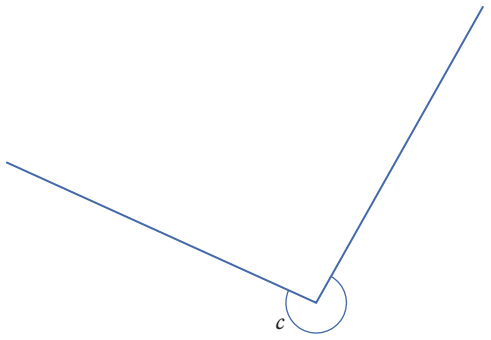
**a**



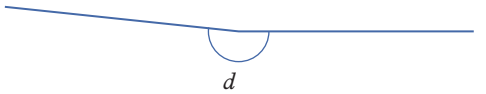

**b**




**c**



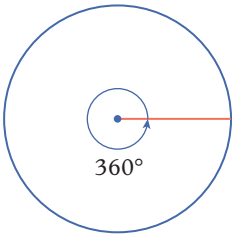

**d**




7A.4 **11** Use a protractor to draw the following angles.

**a** 270° **b** 200° **c** 335° **d** 185°

**12** Over 3000 years ago, Babylonian astronomers estimated that there were 360 days in a year and so they divided circles into 360 degrees (360°).
We still use this system today.




**a** How many right angles make up a straight angle?

**b** How many right angles make up a revolution?

**c** How many straight angles make up a revolution?

**13** Classify the angles produced by adding together the following pairs of angles.

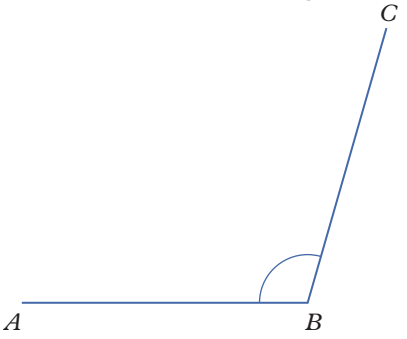
**a** two right angles

**b** two straight angles

**c** a right angle and a straight angle

**d** two obtuse angles

**14** A distracted student measures the angle below as $\angle ABC = 74°$.
What mistake did they make and what is the actual size of the angle?

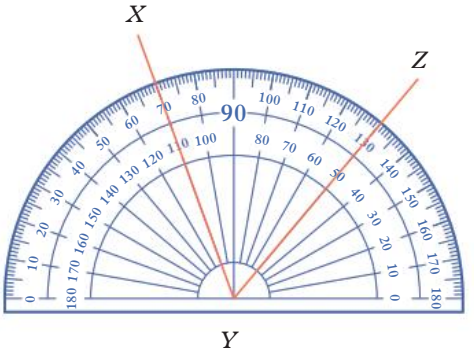



UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

**15** Determine the size of $\angle XYZ$ without taking any further measurements.

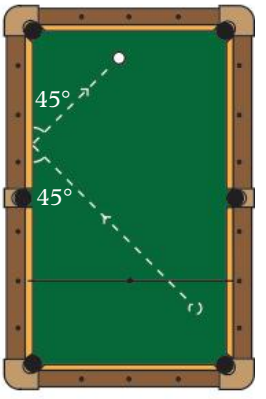



**16** Angles are also used to measure the amount of turn when a person or object changes direction. A plane is flying from Adelaide to Broome. When it is halfway there, bad weather forces it to change direction and land in Perth. Use your protractor to measure the acute angle the plane must turn through to fly towards Perth.

CHALLENGE

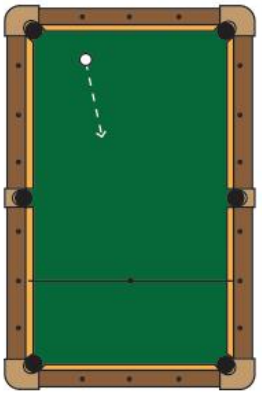
**17** Pool players make use of angles when planning their shots. They know that when a ball bounces off the side of a table it rebounds at the same angle.

The ball on the pool table shown at right hits the side of the table at an angle of 45° and bounces off at an angle of 45°.




Use your knowledge of drawing angles to show which pocket the white ball will end up in in each of the scenarios below.

**a**



**b**

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Classifying angles

**Investigation** Mini-golf challenge

**Topic quiz** 7A

# 7B Angles at a point

## Learning intentions

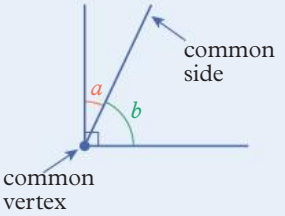
- ✔ I can identify complementary, supplementary and vertically opposite angles.
- ✔ I can find unknown angles using complementary and supplementary relationships.
- ✔ I can find unknown angles at a point.

**Inter-year links**

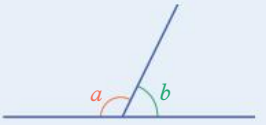
| | |
|---|---|
| Years 5/6 | Understanding angles |
| Year 8 | 7A Angles |
| Year 9 | 7A Angles and lines |

## Pairs of angles

- **Adjacent angles** share a common side and a common vertex, but do not overlap.
  For example, angles $a$ and $b$ are adjacent in each of the diagrams shown.
- **Complementary angles** are two angles that form a right angle or add to make 90°.
  $a$ is the *complement* of $b$ because $a + b = 90°$.

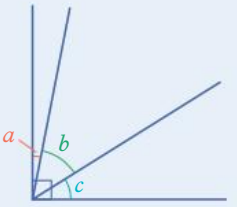



- **Supplementary angles** are two angles that add to 180°.
  $a$ is the *supplement* of $b$ because $a + b = 180°$.
- Complementary and supplementary angles are not necessarily adjacent.

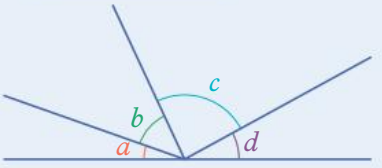



## Angles at a point

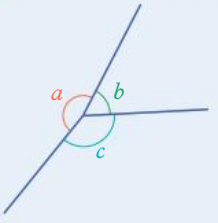
- Angles at a point that form a right angle add to 90°.
  For example, $a + b + c = 90°$.



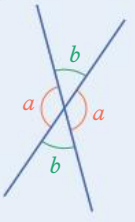

- Angles at a point that form a straight angle add to 180°.
  For example, $a + b + c + d = 180°$.




- Angles at a point that form a revolution add to 360°.
  For example, $a + b + c = 360°$.



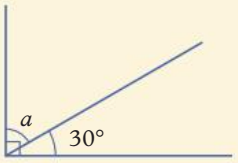

- Where two or more straight lines meet at a point, they form **vertically opposite angles**, or 'X' angles, that are equal in size.
  For example, $a = a$ and $b = b$.



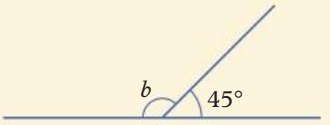
## **Example 7B.1** Finding complementary and supplementary angles

Find the size of the unknown angle in each of the diagrams below.

**a**




**b**




**THINK**

**a** **1** Identify the relationship between the unknown and known angles. $a$ and 30° are adjacent complementary angles.

**2** Calculate the size of the unknown angle. Complementary angles add to 90°, so $a = 90° - 30°$.

**b** **1** Identify the relationship between the angles. $b$ and 45° are adjacent supplementary angles.

**2** Calculate the size of the unknown angle. Supplementary angles add to make a straight angle, 180°, so $b = 180° - 45°$.

**WRITE**

**a** $a = 90° - 30°$
$= 60°$

**b** $b = 180° - 45°$
$= 135°$

## **Example 7B.2** Finding the complement and the supplement

**a** Find the complement of 42°.

**b** Find the supplement of 42°.

**THINK**

**a** Complementary angles add to make a right angle, 90°. Subtract 42° from 90° to find the complement of 42°.

**b** Supplementary angles add to make a straight angle, 180°. Subtract 42° from 180° to find the supplement of 42°.

**WRITE**

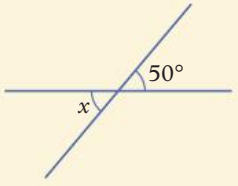
**a** $90° - 42° = 48°$
48° is the complement of 42°.

**b** $180° - 42° = 138°$
138° is the supplement of 42°.

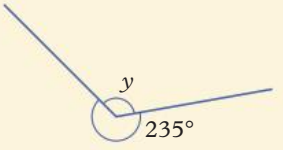
## **Example 7B.3** Finding unknown angles at a point

Find the size of the unknown angle in each of the diagrams below.

**a**




**b**




**THINK**

**a** Identify the relationship between the unknown and known angles. $x$ is vertically opposite to 50° and vertically opposite angles are equal in size.

**b** Identify the relationship between the angles. Angles around a point add to 360°. Subtract 235° from 360°.

**WRITE**

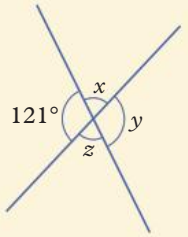
**a** $x = 50°$

**b** $y = 360° - 235°$
$y = 125°$

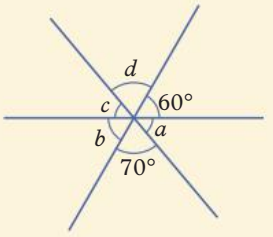
## **Example 7B.4** Finding multiple unknown angles at a point

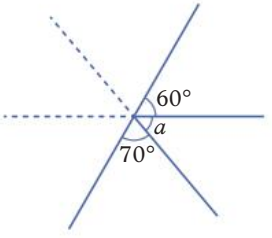
Find the size of each of the unknown angles in the diagrams below.

**a**




**b**




| THINK | WRITE |
|---|---|
| **a** Identify any relationships between unknown and known angles. | **a** |
| **1** $y$ is vertically opposite to 121° and vertically opposite angles are equal in size. | $y = 121°$ |
| **2** $x$ and 121° are supplementary angles, so subtract 121° from 180° to calculate $x$. | $x = 180° - 121°$<br>$= 59°$ |
| **3** $z$ and $x$ are vertically opposite and vertically opposite angles are equal in size. | $z = 59°$ |
| **b** Identify any relationships between the angles. | **b** |
| **1** $b$ is vertically opposite to 60°, $d$ is vertically opposite to 70° and vertically opposite angles are equal. | $b = 60°$<br>$d = 70°$ |
| **2** 70°, $a$ and 60° form a straight angle, 180°. Subtract 70° and 60° from 180° to calculate $a$. | $a = 180° - 70° - 60°$<br>$= 50°$ |
|   | |
| **3** $c$ is vertically opposite to $a$ and vertically opposite angles are equal in size. | $c = 50°$ |

### Helpful hints

- ✔ Try not to confuse complementary and supplementary angles, complementary angles add to 90° and supplementary angles add to 180°.
- ✔ Complementary and supplementary angles do not have to be adjacent – they are just pairs of angles that add up to 90° and 180°, respectively.
- ✔ Vertically opposite angles are angles that sit opposite each other where two lines meet, they do not have to be arranged vertically!

ANS p580

# Exercise 7B Angles at a point

1–4, 6–8, 9(a–c), 10(a, c), 11(a, b), 15

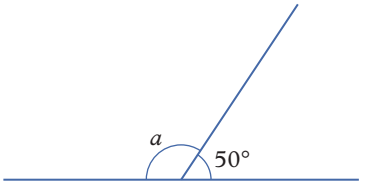
1(d–f), 2(b, d), 3, 5, 6(c–f), 8, 9(c–e), 10(b–d), 11–13, 16

1(e, f), 2(c, d), 5, 6(d, e, h), 8(e, f), 9(d–f), 11, 14, 16, 17

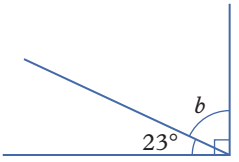
UNDERSTANDING AND FLUENCY

7B.1 **1** Find the size of the unknown angle in each of the diagrams below.

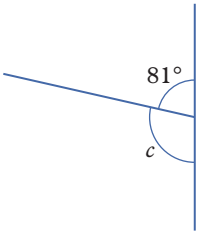
**a**



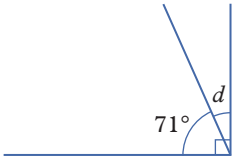

**b**



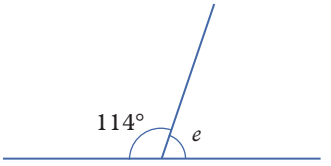

**c**



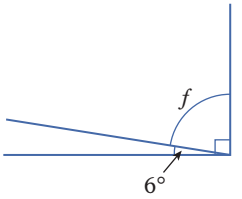

**d**




**e**

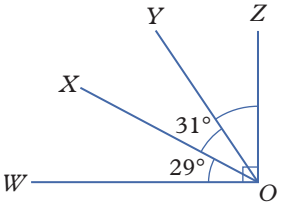



**f**

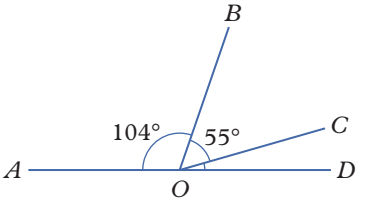



**2** Calculate the size of the angle named in brackets in each of the diagrams below.

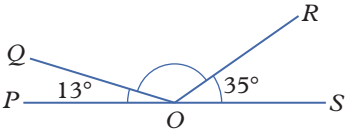
**a** ($\angle YOZ$)



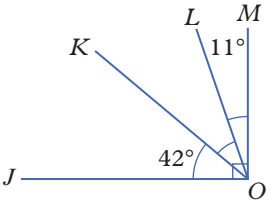

**b** ($\angle COD$)




**c** ($\angle QOR$)




**d** ($\angle KOL$)




**3** Name the two pairs of adjacent angles in each of the diagrams in question **2**.

7B.2 **4** **a** Find the complement of these angles.

**i** 23° **ii** 77° **iii** 88° **iv** 9°

**b** Find the supplement of these angles.

**i** 158° **ii** 21° **iii** 100° **iv** 97°

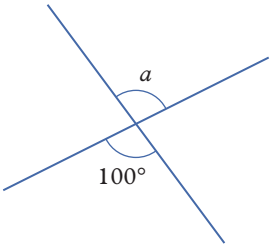
**5** For each angle listed below, find the angle that is its:

**i** complement **ii** supplement.

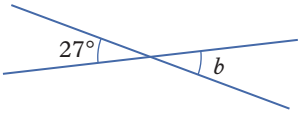
**a** 46° **b** 12° **c** 85° **d** 27°

**7B.3** **6** Find the size of the unknown angle in each of the diagrams below.

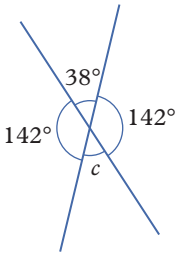
**a**



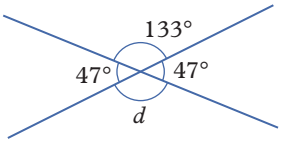

**b**



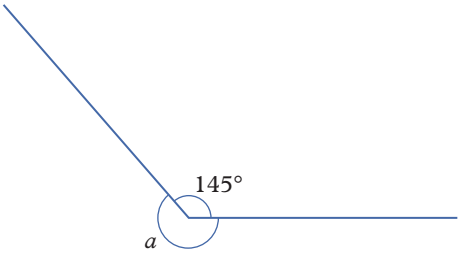

**c**



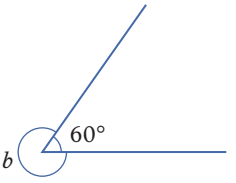

**d**



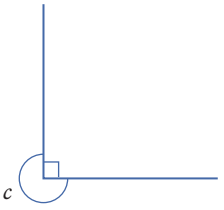

**e**



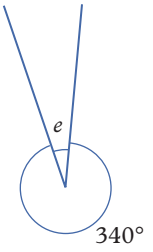

**f**




**g**



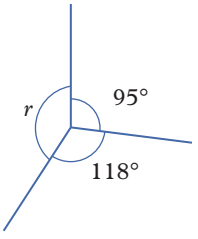

**h**



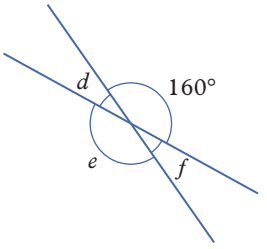

**7** Are the pairs of angles in each diagram in question **6e–h** adjacent? Explain your answer.

**7B.4** **8** Find the size of each of the unknown angles in the diagrams below.

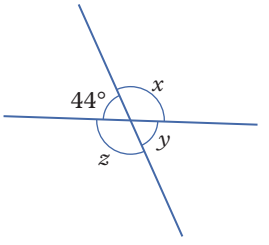
**a**



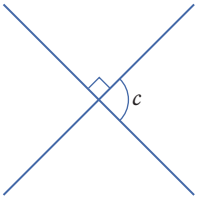

**b**



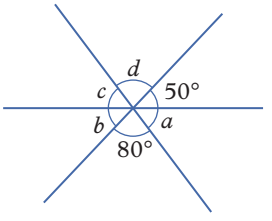

**c**



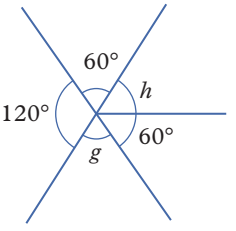

**d**




**e**



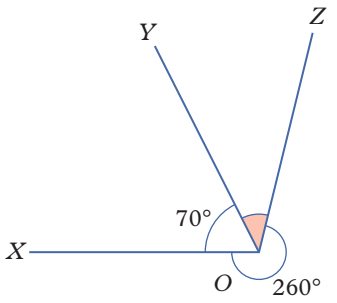

**f**



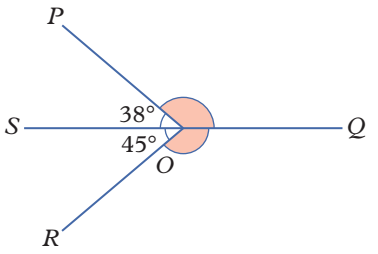
UNDERSTANDING AND FLUENCY

**9** Name and determine the size of each of the shaded angles in the diagrams below.

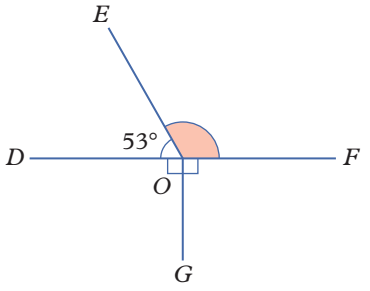
**a**



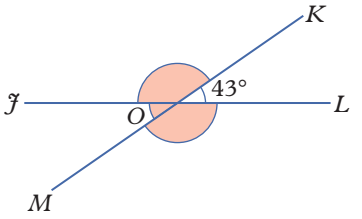

**b**



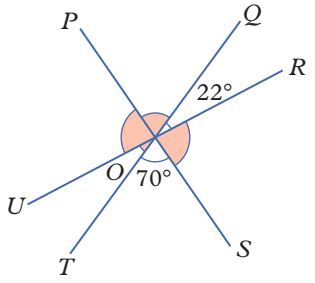

**c**




**d**




**e**




PROBLEM SOLVING AND REASONING

**10** For the each of the turns described below:

**i** find the size of the remaining angle each person needs to turn through

**ii** state whether the pairs of angles are complementary, supplementary or add to make a full revolution.

**a** Peter turns 56° in his car in an attempt to make a right-hand turn.

**b** Gloria leaves her house but forgets something and turns through 87° in order to turn back.

**c** Michelle completes 146° of a pirouette.

**d** Leigh completes $\frac{2}{3}$ of a left-hand turn on his bicycle.

**11** Chelsea, Kane and Sara are sharing a large round pizza. Chelsea eats one piece, Kane eats four pieces and Sara eats three pieces.

**a** How many pieces do they need to cut the pizza into?

**b** If every piece is equal in size, what is the smallest angle in each piece of pizza?

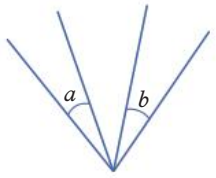
The next weekend Chelsea's friend Ben joins them for dinner, so they decide to order two pizzas.
This time Chelsea eats one piece, Kane eats four pieces, Sara eats two pieces and Ben eats three pieces.
All the pizza is eaten.

**c** How many pieces is this in total?

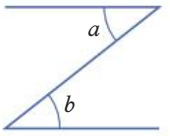
**d** What is the angle size of each piece of pizza? Remember that there are two pizzas to share and every price is equal in size.

**12** Adjacent angles share a common side and a common vertex, but do not overlap. Explain why $a$ and $b$ are not adjacent angles in each of the diagrams below.

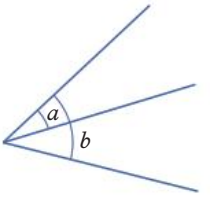
**a**




**b**

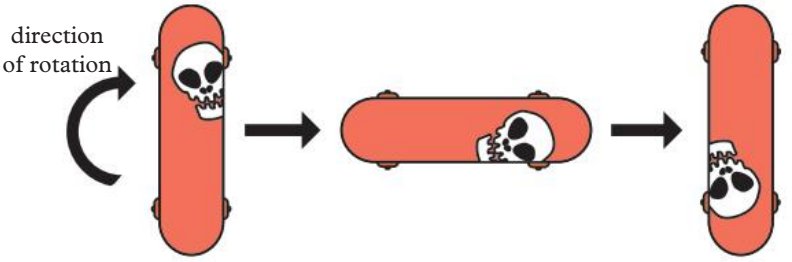



**c**




**13** An aerial freestyle skier launches herself into the air but is knocked off balance by a gust of wind. She estimates that she has already turned through 415°. To land safely she needs to have turned through a multiple of 180°. How much further does she need to rotate if she is to land safely?

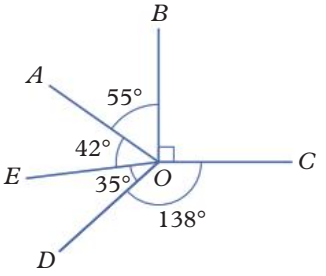
**14** Skateboarding, snowboarding and wakeboarding tricks often involve rotation through the air. A '180 aerial' means the rider spins through 180° and then lands with the board facing the opposite direction.




Show the start and end position of a rider's board after the following aerial rotations.

**a** 360° **b** 540° **c** 720°

**15** Consider the diagram on the right. Determine whether the following statements are true or false. Give a reason for your answer.




**a** $\angle BOC = 90°$

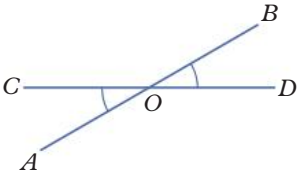
**b** $\angle AOB$ and $\angle DOE$ are complementary

**c** $\angle EOA$ is the supplement of $\angle COD$

**d** $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$

**16** Can two acute angles be supplementary? Explain.

**17** Use the diagram below and your knowledge of supplementary angles to explain why vertically opposite angles must be equal in size.




Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Angles at a point

**Investigation** Sharing the poles

**Topic quiz** 7B

PROBLEM SOLVING AND REASONING

CHALLENGE

# 7C Angles and parallel lines

## Learning intentions

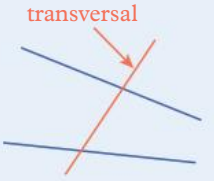
- ✔ I can identify parallel and transversal lines.
- ✔ I can identify alternate, corresponding and co-interior angles.
- ✔ I can find unknown angles by identifying alternate, corresponding and co-interior angles.
- ✔ I can determine if two lines are parallel using alternate, corresponding and co-interior angles.

**Inter-year links**

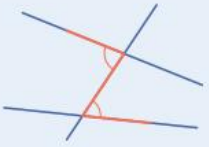
| | |
|---|---|
| Years 5/6 | Understanding angles |
| Year 8 | 7A Angles |
| Year 9 | 7A Angles and lines |

## Transversals

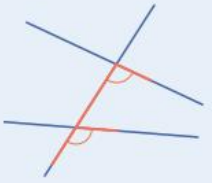
- A **transversal** is a straight line that crosses two or more lines. Transversals create a number of angles when they cross a set of lines:



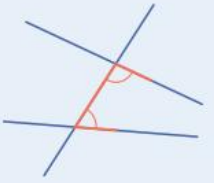

  - → **Alternate angles**, or 'Z' angles, are on alternate, or opposite, sides of the transversal.
  - → **Corresponding angles**, or 'F' angles, are on the same side of the transversal and are both either above or below each of the parallel lines.
  - → **Co-interior angles**, or 'C' angles, are on the same side of the transversal between the parallel lines.



*alternate angles*



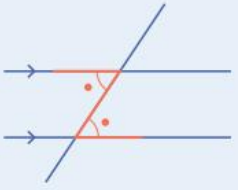
*corresponding angles*



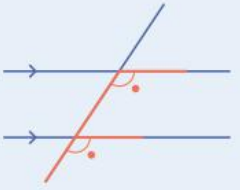
*co-interior angles*

## Transversals and parallel lines

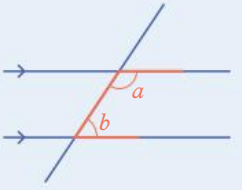
- When a transversal crosses a pair of parallel lines:
  - → alternate angles, or 'Z' angles, are equal in size
  - → corresponding angles, or 'F' angles, are equal in size
  - → co-interior angles, or 'C' angles, are supplementary; that is, they add to 180°



*alternate angles*



*corresponding angles*




*co-interior angles*

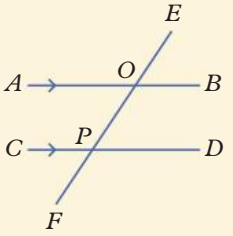
$a + b = 180°$

- These pairs of angles can be measured to determine if two lines are parallel – if any of the relationships listed above are true, then the lines crossed by the transversal are parallel.

- Transversal lines can form acute, obtuse or right angles with parallel lines.

## Example 7C.1 Identifying alternate, corresponding and co-interior angles

Consider the diagram below and identify the angle that is:




**a** alternate to $\angle AOF$.

**b** corresponding to $\angle CPE$.

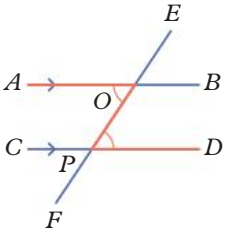
**THINK**

**a** Alternate angles are on opposite sides of the transversal and lie inside the two angles made by the letter 'Z'. Draw a 'Z' that incorporates $\angle AOF$. The other angle inside this 'Z' is $\angle EPD$, so $\angle AOF$ and $\angle EPD$ are alternate angles.

**b** Corresponding angles are on the same side of the transversal and lie inside the two angles made by the letter 'F'. Note that the F is upside down and reversed. Draw an 'F' that encloses the $\angle CPE$. The other angle inside this 'F' is $\angle AOE$, so $\angle CPE$ and $\angle AOE$ are corresponding angles.

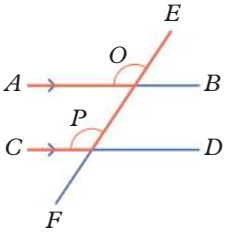
**WRITE**

**a**




$\angle EPD$ is alternate to $\angle AOF$.

**b**




$\angle AOE$ is corresponding to $\angle CPE$.

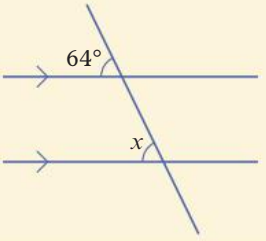
## Example 7C.2 Finding unknown alternate, corresponding and co-interior angles

For each of the following diagrams:

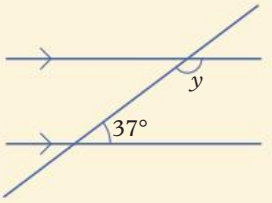
**i** Name the relationship between the unknown and known angles.

**ii** Find the size of the unknown angle.

**a**




**b**




**THINK**

**a i** Identify the relationship between the unknown and known angles. $x$ and 64° lie inside the two angles of the letter 'F', so they are corresponding angles.

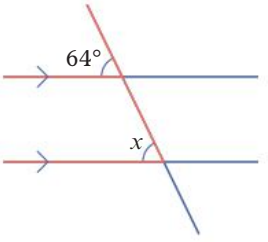
**ii** Corresponding angles formed by parallel lines are equal in size.

**b i** Identify the relationship between the unknown and known angle. $y$ and 37° lie inside the two angles of the letter 'C', so they are co-interior angles.

**ii** Co-interior angles formed by parallel lines are supplementary, that is, they add to 180°, so $y = 180° - 37°$.

**WRITE**

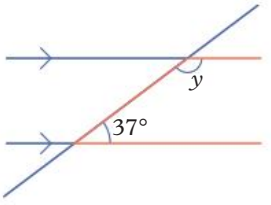
**a**




**i** $x$ and 64° are corresponding angles.

**ii** $x = 64°$

**b**

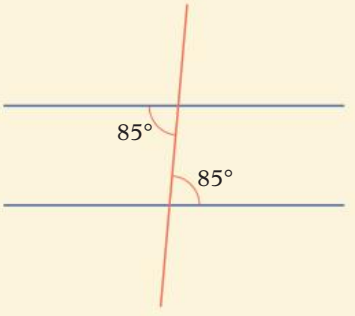


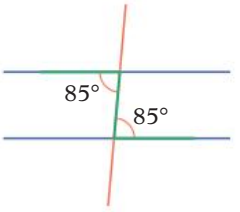

**i** $y$ and 37° are co-interior angles.

**ii** $y = 180° - 37°$
$= 143°$

## Example 7C.3 Determining if two lines are parallel using alternate, corresponding and co-interior angles

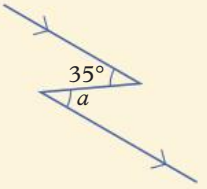
Decide whether the two blue lines in each of the diagrams below are parallel. Explain your reasoning.

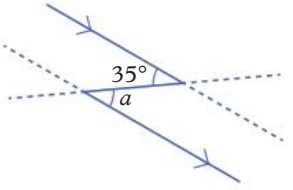


| THINK | WRITE |
|---|---|
| **1** Identify the relationship between the labelled angles. The labelled angles lie inside the two angles of the letter 'Z', so they are alternate angles. |  |
| **2** Determine whether the relationship between the angles holds. Alternate angles formed by parallel lines are equal in size. The alternate angles are equal in size, so the blue lines are parallel. | |
| **3** Write the answer and explanation. | The blue lines are parallel because the alternate angles are both 85°. |

## Example 7C.4 Finding unknown alternate, corresponding and co-interior angles in complex diagrams

Find the size of the unknown angle below. Explain how you obtained your answer.




| THINK | WRITE |
|---|---|
| **1** Identify and highlight any parallel and transversal lines in the diagram. |  |
| **2** Identify the relationship between the unknown and known angles. $a$ and 35° lie inside the angles of the letter 'Z', so they are alternate angles. | $a$ and 35° are alternate angles. |
| **3** Write the answer and provide an explanation. | $a$ = 35° because alternate angles formed by parallel lines are equal in size. |

**Helpful hints**

- ✔ Remember to look for the letter to determine which pair of angles you might be dealing with:
  - Alternate angles form a 'Z'.
  - Corresponding angles form an 'F'.
  - Co-interior angles form a 'C'.
- ✔ Take care to identify the nature of the relationship between a pair of angles – when two lines are parallel, alternate and corresponding angles are equal in size, while co-interior angles are supplementary!

ANS p580

# Exercise 7C Angles and parallel lines

1–3, 5, 6, 7(a–d), 8(a–c), 9(b, e, g), 10

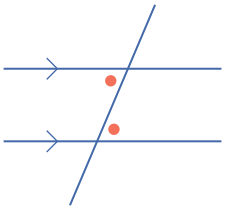
1(e–i), 2, 3(a, e, f), 4(a, b), 5(c, d), 6(f–i), 7(d–g), 8(b, d, e), 9(d–g), 11, 13

3, 4, 5(a), 6(f–i), 7(f, g, h), 8(d–f), 9(d–h), 12, 14

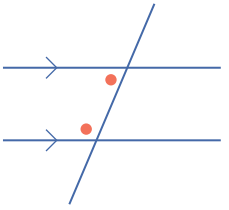
UNDERSTANDING AND FLUENCY

**1** Identify each pair of angles as alternate, corresponding or co-interior.

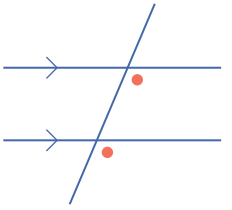
**a**



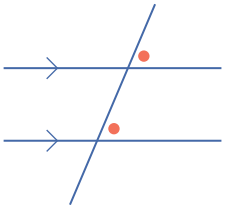
**b**



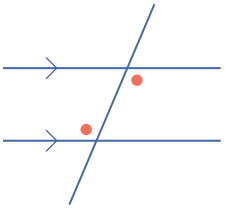
**c**



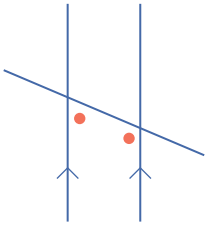
**d**



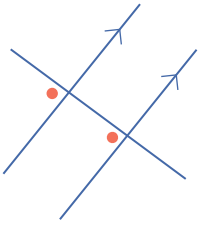
**e**



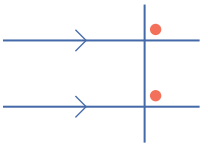
**f**



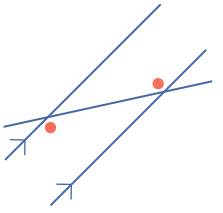
**g**



**h**



**i**



**2** Use your knowledge of angles and parallel lines to state whether the pairs of angles in question **1** are equal in size or supplementary.

7C.1 **3** Consider the diagram below and identify the angle that is:

**a** corresponding to $\angle EOB$

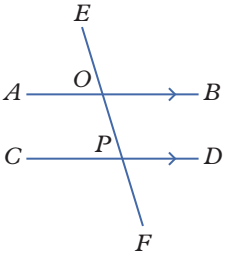
**b** alternate to $\angle CPE$

**c** corresponding to $\angle AOE$

**d** co-interior to $\angle AOF$

**e** co-interior to $\angle EPD$

**f** alternate to $\angle BOF$

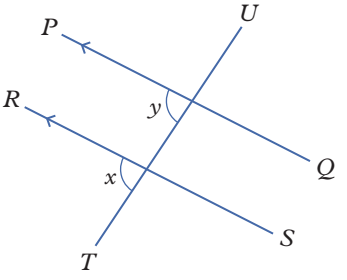



**4** In the diagram on the right, identify and name all the pairs of:

**a** alternate angles

**b** co-interior angles

**c** corresponding angles




**5** Use the diagram given in question **3** to identify an angle that is:

**a** vertically opposite to $\angle AOF$

**b** supplementary to $\angle FPD$

**c** vertically opposite to $\angle EPD$

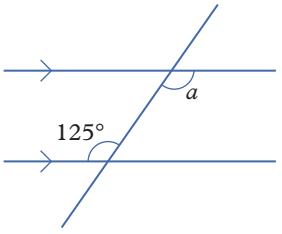
**d** supplementary to $\angle CPE$

**7C.2** **6** For each of the following diagrams:

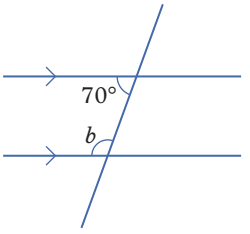
**i** name the relationship between the unknown and known angles

**ii** find the size of the unknown angle.

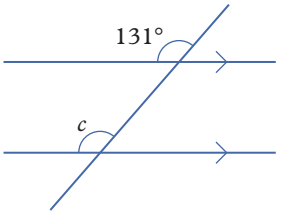
**a**



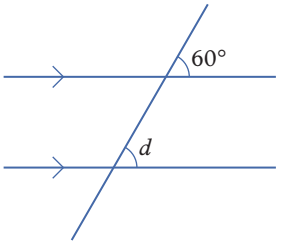

**b**



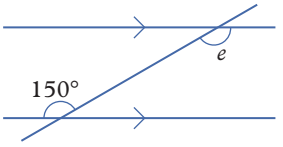

**c**



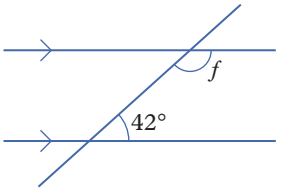

**d**



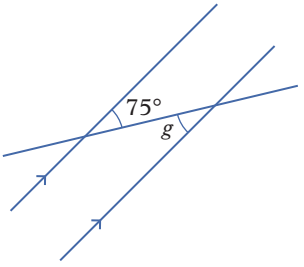

**e**



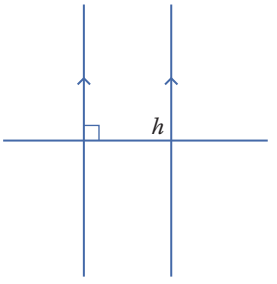

**f**



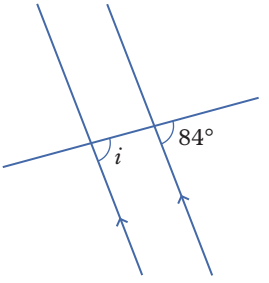

**g**




**h**

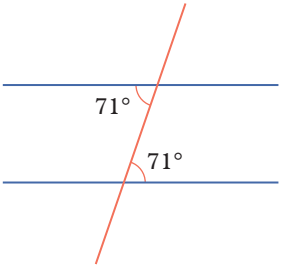



**i**

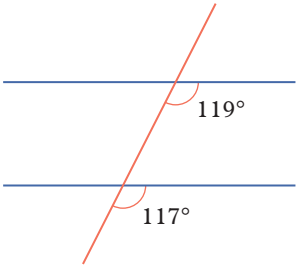



**7C.3** **7** Decide whether the blue lines in each of the diagrams below are parallel. Explain your reasoning by referring to the relationship between the angles in the diagram.

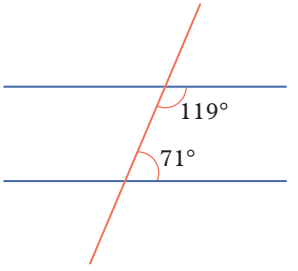
**a**



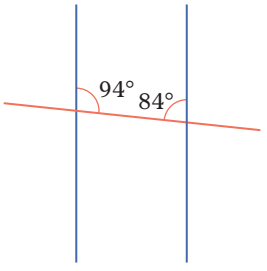

**b**



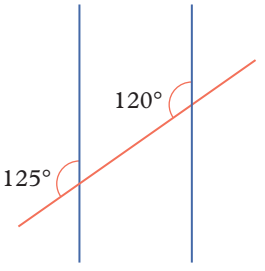

**c**



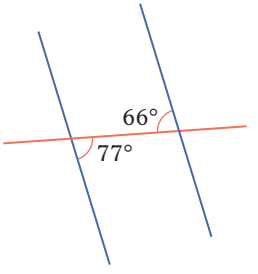

**d**



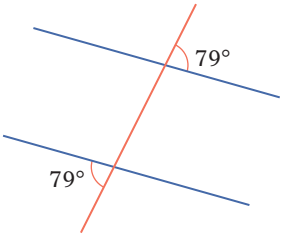

**e**



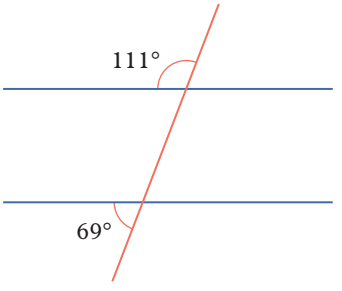

**f**




**g**




**h**

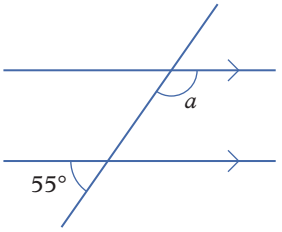


**8** For each of the following diagrams:

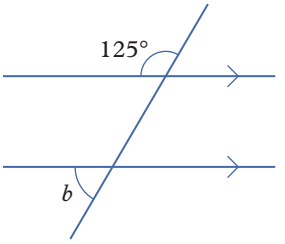
**i** name the relationship between the unknown and known angles

**ii** find the size of the unknown angle.

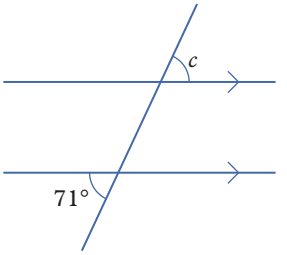
**a**



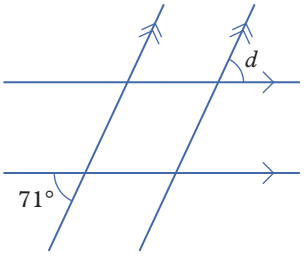

**b**



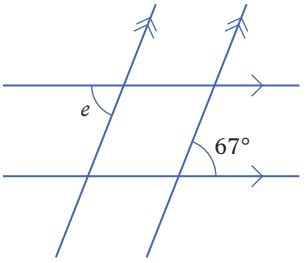

**c**



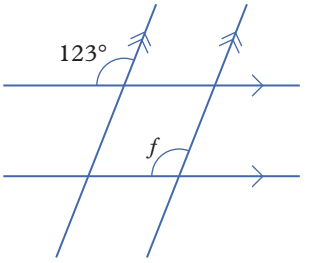

**d**




**e**

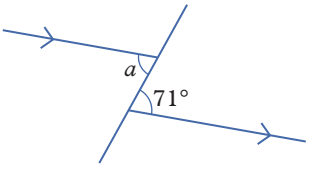



**f**

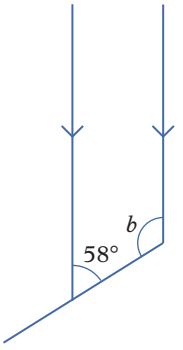



7C.4 **9** Find the size of each of the unknown angles. Explain how you obtained your answer.

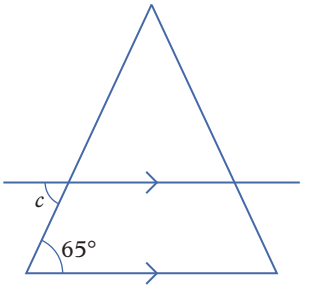
**a**



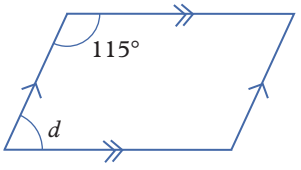

**b**



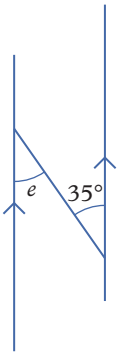

**c**



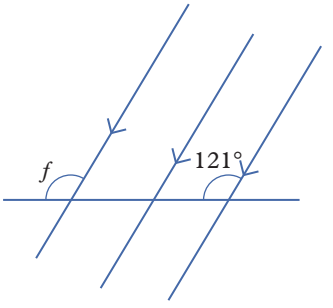

**d**



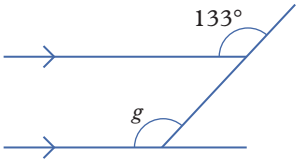

**e**



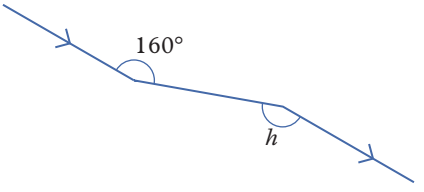

**f**



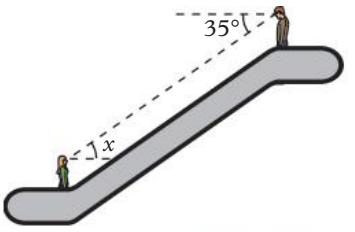

**g**



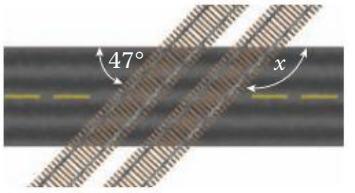

**h**



**10** Cameron is standing on top of the escalators, looking down at Sarah. If Cameron is looking down at an angle of 35°, at what angle is Sarah looking up at him?




**11** A road crosses a pair of railway lines as shown in the diagram. If a driver approaches the railway lines at an angle of 47° as shown, at what angle does the car leave the tracks?

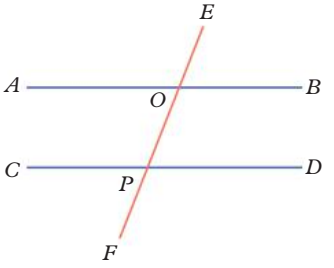



**12** Amal is putting up some paintings over a straight crack in the wall. She knows that the top painting is parallel with the edge of the ceiling.

- **a** Find the size of the angles marked $a$, $b$ and $c$ if the paintings are parallel with one another.
- **b** Amal measures angle $c$ and finds that it is 66°. Are the paintings parallel with one another? Explain your reasoning.
- **c** By how much will she have to move the bottom painting in order to make it parallel to the top one?

**13** A student draws the diagram below right and makes the following measurements with their protractor:

$\angle AOF = 81°$ $\quad$ $\angle FPD = 99°$

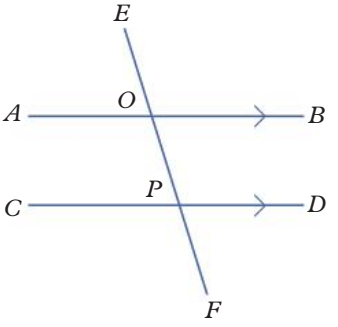



- **a** Assuming the student's measurements are accurate, are the two blue lines in the diagram parallel? Use the student's measurements to justify your answer.
- **b** For the two blue lines to be parallel, $\angle AOF = \angle EPD$. This relationship is referred to as a condition that must be satisfied for the two blue lines to be parallel. Write two more conditions that could be used to show the two blue lines to be parallel.



**14** A mathematical proof is an argument that shows or proves that a statement is true. For example, the proof below shows that alternate angles formed by a transversal crossing parallel lines are equal in size.

$\angle FOB$ and $\angle CPE$ are alternate angles.
$\angle AOE = \angle FOB$ since vertically opposite angles are equal in size.
$\angle AOE = \angle CPE$ since corresponding angles on parallel lines are equal in size.
Therefore, $\angle FOB = \angle CPE$.




- **a** Use the diagram to prove that co-interior angles are supplementary.
- **b** Use the diagram to show that when a transversal runs through a pair of parallel lines:
  - **i** all the acute angles formed by the transversal are equal in size
  - **ii** all the obtuse angles formed by the transversal above are equal in size
  - **iii** if any one of the angles formed by the transversal is a right angle, then all other angles formed must be right angles.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Angles and parallel lines

**Investigation** The streets of New York

**Topic quiz** 7C

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

7A **1** Match the following labels to the corresponding diagram.

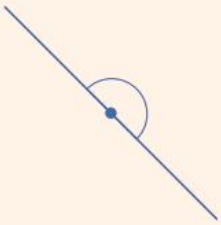
**a** acute **b** obtuse **c** reflex
**d** right **e** straight **f** revolution

**A**

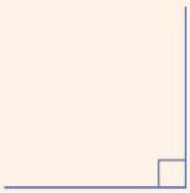
**B**

**C**

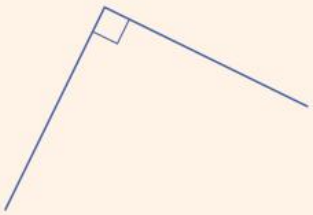
**D**



**E**



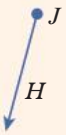
**F**



7A **2** Identify and name the following objects below using standard mathematical notation.

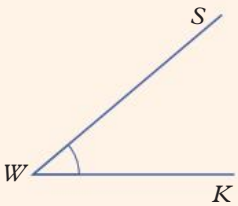
**a**

**b**



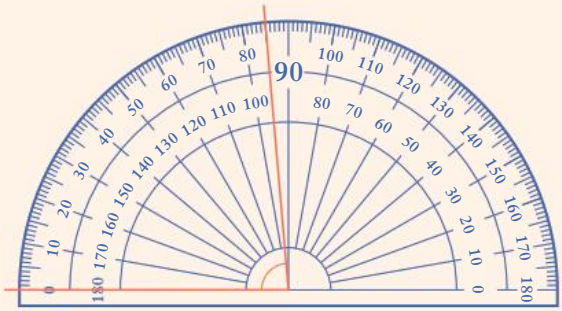

**c**

**d**

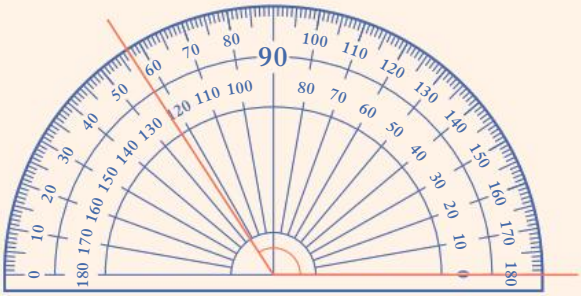



7A **3** State the size of the following angles.

**a**



**b**



7B **4** Decide if the following angles add to form:

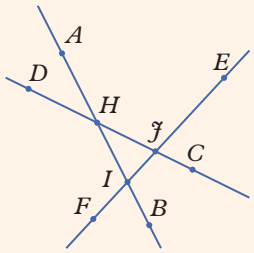
**A** a right angle **B** a straight angle **C** a revolution
**D** a reflex angle **E** an obtuse angle

**a** 33° and 147° **b** 160° and 200° **c** 55°, 92° and 43° **d** 21°, 37° and 32°

7B **5** Calculate the complement and supplement of the following angles.

**a** 19° **b** 37° **c** 89°

7B **6** If $\angle AHD = 37°$, $\angle HIJ = 70°$ and $\angle EJH = 106°$ in the diagram to the right, determine the size of the following angles.

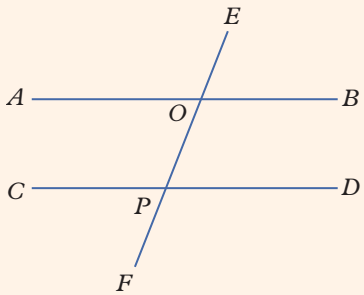



**a** $\angle AHJ$ **b** $\angle IJC$ **c** $\angle FIB$

7C **7** Determine whether the following statements are true or false.

**a** Two angles that form a straight angle are supplementary.
**b** Vertically opposite angles are equal in size.
**c** A transversal is a straight line that crosses two or more other lines.
**d** Corresponding angles are supplementary.
**e** Co-interior angles add to 90°.

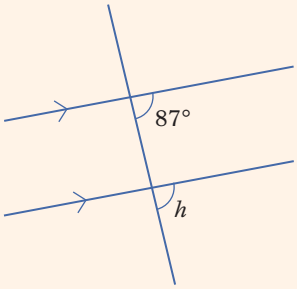
7C **8** State whether the angle pairs are alternate, corresponding, co-interior, or vertically opposite.



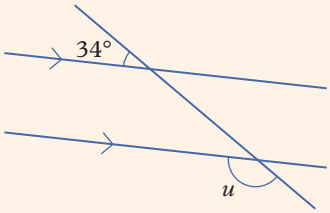

**a** $\angle FOB$ and $\angle EPD$ **b** $\angle AOF$ and $\angle EPD$
**c** $\angle AOE$ and $\angle FOB$ **d** $\angle CPF$ and $\angle AOF$

7C **9** Determine the values of the pronumerals in each of the diagrams below.

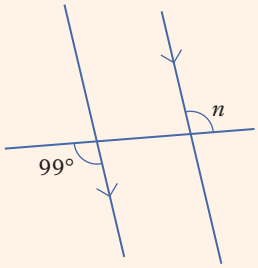
**a**




**b**

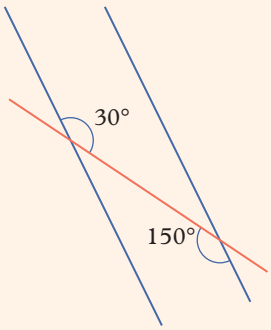



**c**

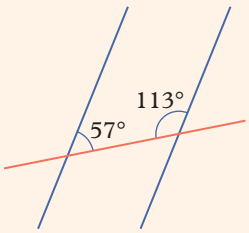



7C **10** Decide if the blue lines in each of the diagrams are parallel. Give a reason for your answer.

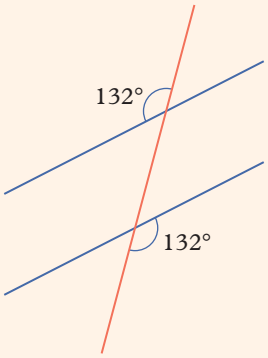
**a**



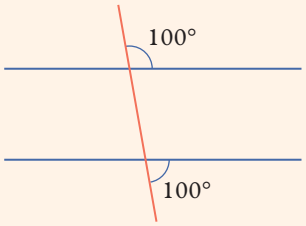

**b**




**c**




**d**



# 7D Classifying triangles

## Learning intentions

- ✔ I can identify and classify equilateral, isosceles, scalene and right-angled triangles.
- ✔ I can find unknown angles using the sum of angles in a triangle.

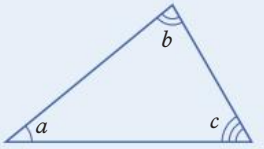
**Inter-year links**

| | |
|---|---|
| Year 8 | 7B Triangles |
| Year 9 | 7B Pythagoras' theorem |

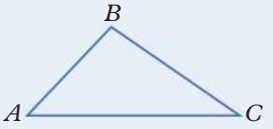
## Triangles

- A **triangle** is a two-dimensional (2D) shape with three straight sides.
- The **interior** or **internal angles** in a triangle add to 180°.

  $a + b + c = 180°$

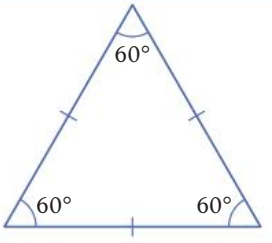
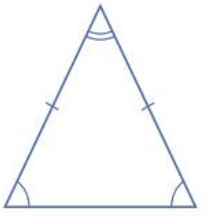
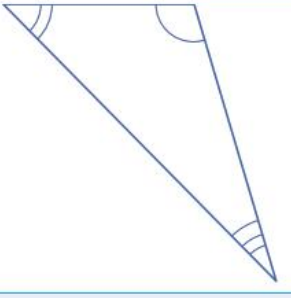



- Triangles are named using a triangle symbol, followed by capital letters placed at each vertex or corner of the triangle. For example, $\triangle ABC$.




## Classifying triangles

- Triangles can be classified by their side lengths and interior angles. In geometric diagrams:
  - → sides of equal length are marked using matching hatch marks or dashes.
  - → angles of equal size are marked using the same number of arcs.

| Type of triangle | Properties | Example |
|---|---|---|
| **Equilateral triangle** | • three sides of equal length<br>• three identical internal angles of 60° |  |
| **Isosceles triangle** | • two sides of equal length<br>• two internal angles that are equal in size |  |
| **Scalene triangle** | • three sides of three different lengths<br>• three different internal angles |  |



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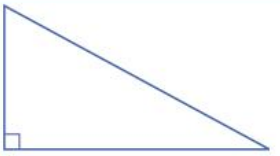
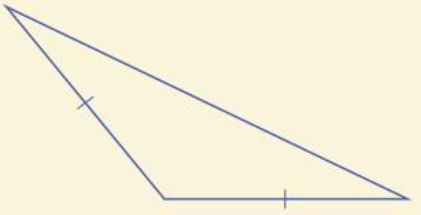
| Type of triangle | Properties | Example |
|---|---|---|
| **Right-angled triangle** | • two perpendicular sides, or one right angle, 90° |  |

## Example 7D.1 Classifying triangles

Classify the triangle below.



**THINK**

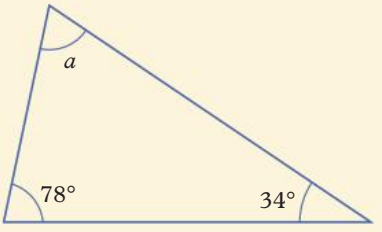
Examine the sides of the triangle to determine how many sides are equal in length. Two sides of the triangle are equal in length and two angles are equal in size, so it must be an isosceles triangle.

**WRITE**

Isosceles triangle

## Example 7D.2 Finding an unknown angle in a triangle

Find the size of the unknown angle in the triangle below.




**THINK**

The internal angles in a triangle add to 180°. Subtract each of the known angles from 180° for the third angle, $a$.

**WRITE**

$a = 180° - 78° - 34°$
$= 68°$

**Helpful hints**

✔ Try not to confuse the values of common angles – the sum of all internal angles in a triangle is 180°, *not* 360°!

ANS p582

# Exercise 7D Classifying triangles

1–4, 5(a, b), 7, 10, 14(a)

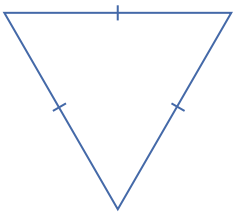
1(a, c, d, e, g), 2(b, d, f), 3(c–f), 4, 5, 6(a, b), 7, 8, 11, 12(a, b, e, f), 14(a–c)

1(e–i), 3, 4, 5, 6(c, d), 7, 9, 11, 12(c, e ,f), 13, 14(d, e, f), 15

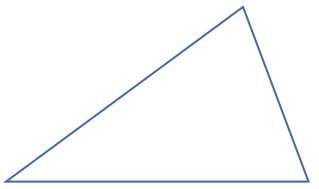
UNDERSTANDING AND FLUENCY

7D.1 **1** Classify each of the triangles below.

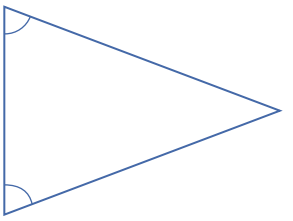
**a**



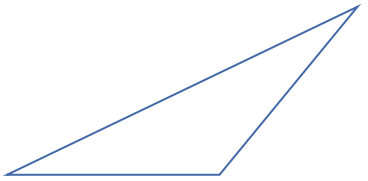
**b**



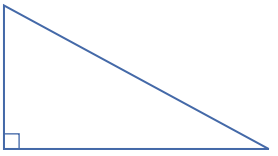
**c**



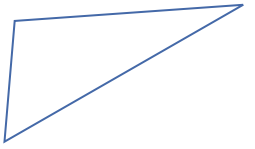
**d**



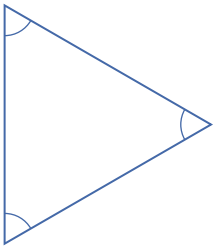
**e**



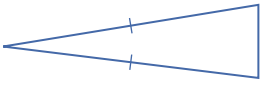
**f**



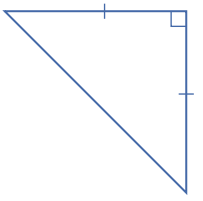
**g**



**h**

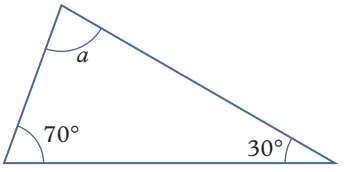


**i**

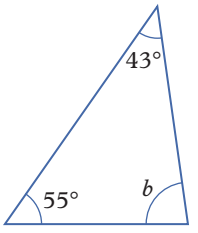


7D.2 **2** Find the size of the unknown angle in each of the triangles below.

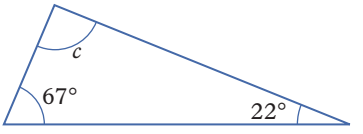
**a**



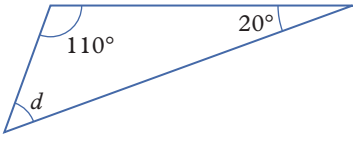

**b**



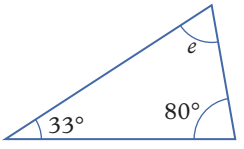

**c**



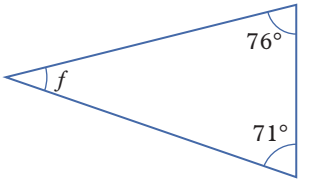

**d**




**e**

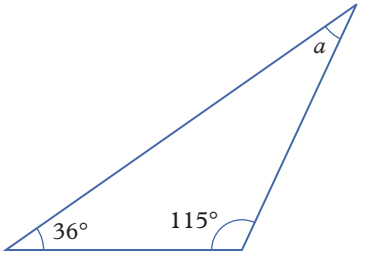



**f**

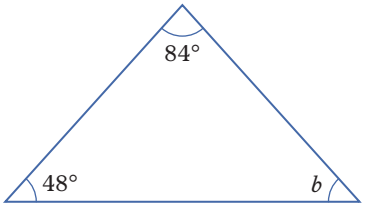



**3** Classify each of the following triangles by first finding the unknown angle.

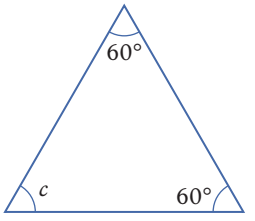
**a**



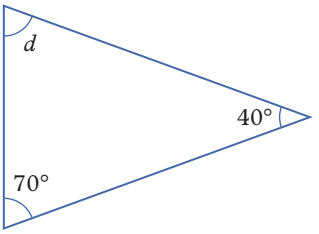

**b**



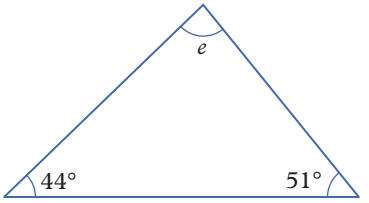

**c**



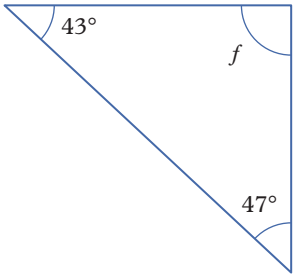
**d**




**e**

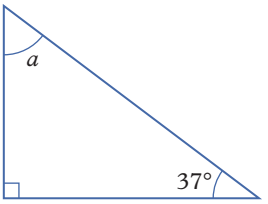



**f**

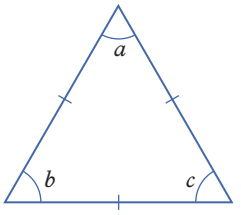



**4** Use your knowledge of the properties of a triangle to find the size of each labelled unknown angle.

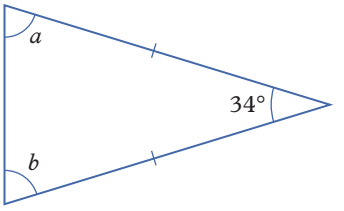
**a**



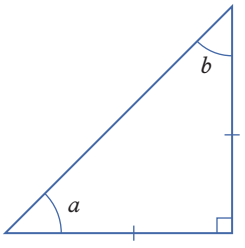

**b**



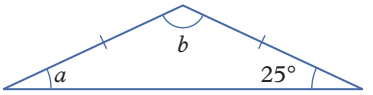

**c**



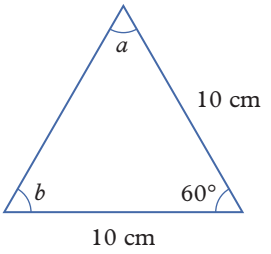

**d**




**e**

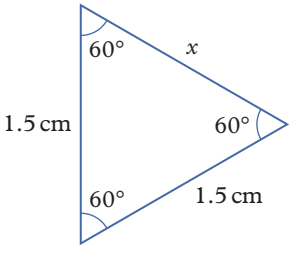



**f**

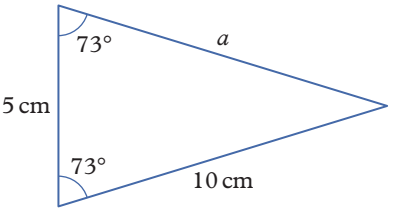



**5** Use your knowledge of triangle properties to find the size of each labelled unknown side length.

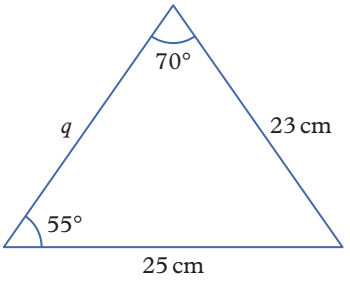
**a**



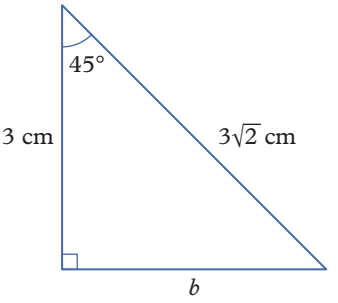

**b**




**c**




**d**




**6** Draw a diagram of the following triangles, labelling all the internal angles. Include letter labels and hatch marks where appropriate. You are not required to draw the angles using a protractor.

**a** $\triangle ABC$, where $\angle ABC = 145°$ and $\angle ACB = 25°$

**b** $\triangle PQR$, where $\angle PQR = 60°$ and $\angle PRQ = 60°$

**c** $\triangle XYZ$, where $\angle XYZ = 23°$ and $\angle YXZ = 67°$

**d** $\triangle JKL$, where $\angle KJL = 50°$ and $\angle JLK = 50°$

**7** **a** Decide whether it is possible to draw a triangle with each of the following sets of interior angles.

**i** 90°, 60°, 30° **ii** 100°, 35°, 35° **iii** 60°, 60°, 60°

**iv** 5°, 10°, 165° **v** 90°, 90°, 20° **vi** 13°, 13°, 154°

**b** Classify the triangles that can be drawn.

**8** Is it possible for a right-angled triangle to also be an isosceles triangle? Justify your answer.

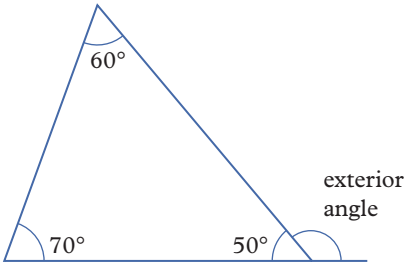
**9** Which one of the four types of triangle can have an obtuse angle? Explain your reasoning.

**10** **a** Complete the table below.

| Type of triangle | Number of sides of equal length | Number of internal angles that are equal in size |
|---|---|---|
| Equilateral | | |
| Isosceles | | |
| Scalene | | |

**b** Using the table from part **a**, describe the relationship between the number of sides of equal length and number of angles that are equal in size in a triangle.

**11** The triangle below has had one of its sides extended and an exterior angle marked in.




**a** Why do you think this angle is called an 'exterior' angle?

**b** Use your knowledge about the sum of angles on a straight line to find the size of the exterior angle.

**c** Add the two opposite interior angles together (shown in red). What do you find?

**d** Copy the triangle and extend the other two sides in the anti-clockwise direction in order to mark in the other two exterior angles.

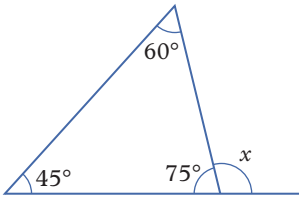
**e** Determine the size of the other two exterior angles.

**f** For each of these exterior angles, find the sum of the 'opposite' two interior angles. How do these relate to the size of each exterior angle?

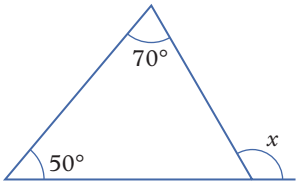
**g** Write a sentence to describe the relationship between an exterior angle and the two interior angles opposite that exterior angle. Explain this relationship with reference to your knowledge of supplementary angles.

**12** Use your findings from question **11** to calculate the size of angle $x$ for these triangles.

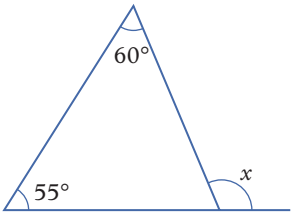
**a**



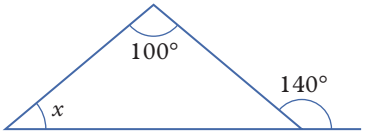

**b**



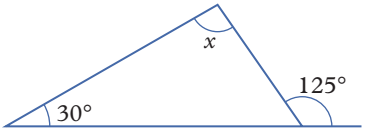

**c**



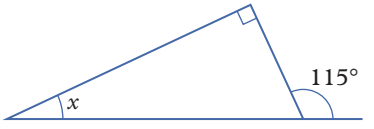

**d**




**e**




**f**



**13** Look at the triangle in question **11**.

**a** What is the sum of the exterior angles of this triangle?

**b** Select two more triangles from question **12** and determine:

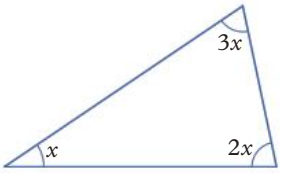
**i** the size of each of the exterior angles

**ii** the sum of the exterior angles.

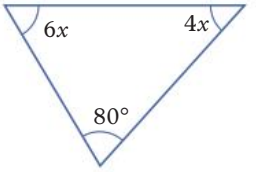
**c** Write a sentence describing your findings regarding the sum of the exterior angles of a triangle.

**14** Use your knowledge of triangle properties and solving algebraic equations to find the value of the pronumerals in these triangles.

**a**

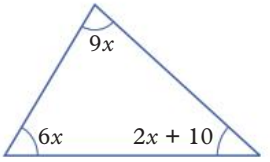



**b**

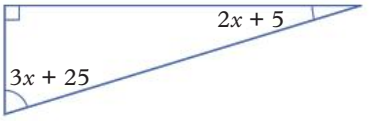



**c**

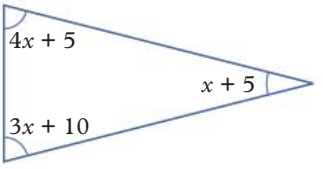
**d**



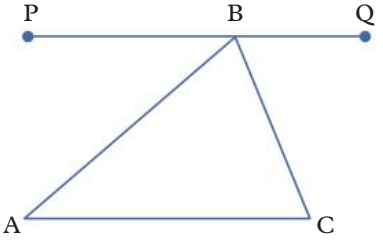

**e**




**f**




**15** Consider the diagram below in which $\overline{PQ}$ is parallel to the base of $\overline{AC}$.




**a** Recall the relationships between angles created by transversals discussed in section 7C and classify the following pairs of angles as alternate, corresponding or co-interior.

**i** $\angle PBA$ and $\angle BAC$

**ii** $\angle QBC$ and $\angle ACB$

**b** Use the diagram to explain why

$\angle PBA + \angle ABC + \angle QBC = 180°$ .

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Triangle properties

**Investigation** Triangles are all around

**Topic quiz** 7D

# 7E Classifying quadrilaterals

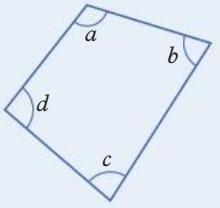
## Learning intentions

- ✔ I can identify and classify squares, rectangles, rhombuses, parallelograms, kites and trapeziums.
- ✔ I can find unknown angles using the sum of angles in a quadrilateral.

Inter-year links
Year 8 7G Quadrilaterals

## Quadrilaterals

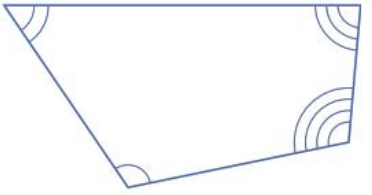
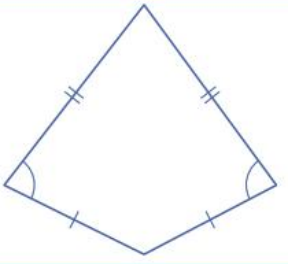
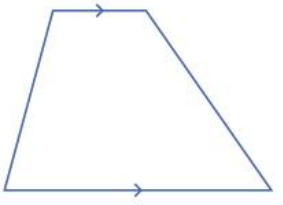
- A **quadrilateral** is a two-dimensional (2D) shape with four straight sides.
- The interior or internal angles in a quadrilateral add to 360°.

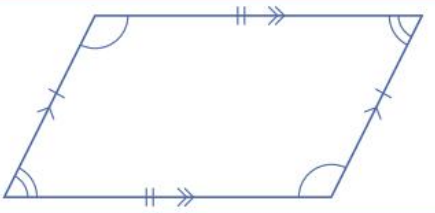
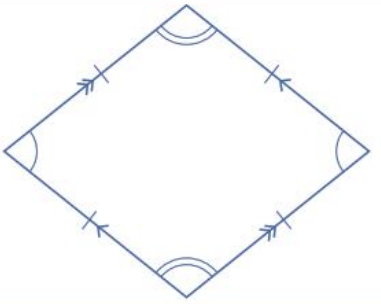
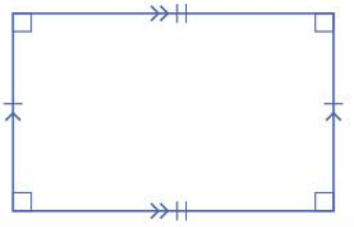
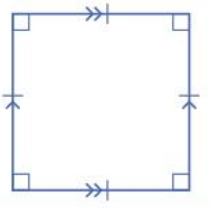



$a + b + c + d = 360°$

## Classifying quadrilaterals

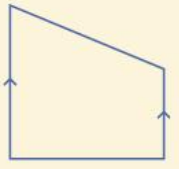
- Quadrilaterals can be classified by their side lengths and interior angles.

| Type of quadrilateral | Properties | Example |
|---|---|---|
| **Irregular quadrilateral** | • four different internal angles |  |
| **Kite** | • two pairs of sides that are equal in length<br>• one pair of opposite angles that are equal in size<br>• no parallel sides |  |
| **Trapezium** | • at least one pair of parallel sides |  |

| Type of quadrilateral | Properties | Example |
|---|---|---|
| **Parallelogram** | • two pairs of parallel sides<br>• two pairs of opposite angles that are equal in size<br>• opposite sides equal |  |
| Three special types of parallelogram are: | | |
| **Rhombus** | All parallelogram features plus<br>• four sides of equal length |  |
| **Rectangle** | All parallelogram features plus<br>• four right angles, 90° |  |
| **Square** | All parallelogram features plus<br>• four sides of equal length<br>• four right angles, 90° |  |

## **Example 7E.1** Classifying quadrilaterals

Classify the quadrilateral below.



**THINK**

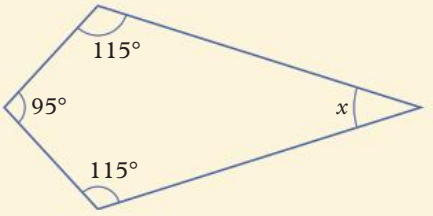
Examine the sides and angles of the quadrilateral to identify its special properties. The quadrilateral has one pair of parallel sides, so it must be a trapezium.

**WRITE**

Trapezium

## **Example 7E.2** Finding an unknown angle in a quadrilateral

Find the size of the unknown angle in the quadrilateral below.




**THINK**

The internal angles in a quadrilateral add to 360°. Subtract each of the known angles from 360° for the fourth angle, $x$.

**WRITE**

$x = 360° - 115° - 115° - 95°$
$= 35°$

**Helpful hints**

✔ Take care when reading diagram notation, remember:
- → Sides of equal length are marked using the same number of hatch marks.
- → Parallel sides are marked using the same number of arrowheads.
- → Angles of equal size are marked using the same number of arcs.

ANS p583

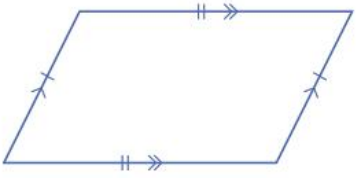
# **Exercise 7E** Classifying quadrilaterals

1–4, 5(a–c), 6(a, b, f, h), 7, 8, 11, 12 | 1–3, 4(b–e), 5(b–e), 6(b–g), 9, 10(a, b), 11, 13(a–e), 14(a–c) | 1, 2, 4(b, e, f), 5(d–f), 6(g–i), 10(c), 11, 13–15

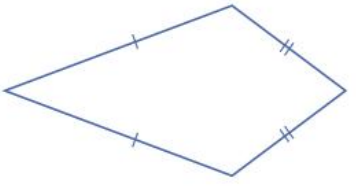
UNDERSTANDING AND FLUENCY

7E.1 **1** Classify each of the quadrilaterals.

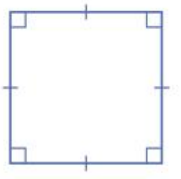
**a**



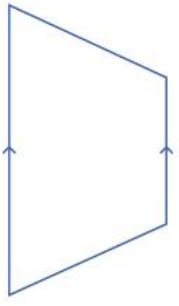
**b**



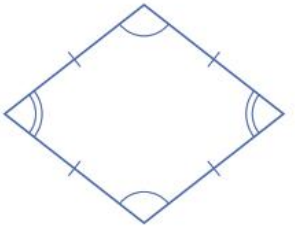
**c**



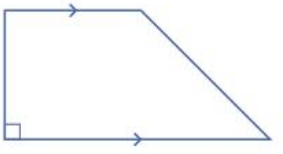
**d**



**e**



**f**

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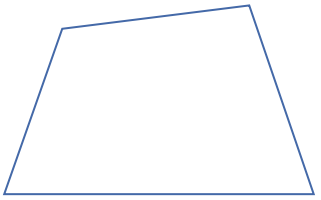
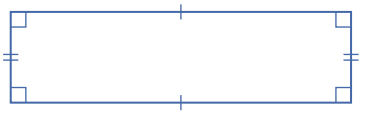
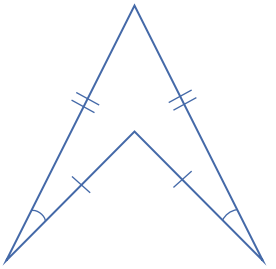
**g**



**h**



**i**



**2** Match the following descriptions to one of the six special quadrilaterals.

**a** all sides are equal in length, two pairs of opposite angles that are equal in size

**b** one pair of opposite sides is parallel

**c** two pairs of opposite sides are parallel and equal in length, all internal angles are 90°

**d** two pairs of opposite sides that are parallel and equal in length; two pairs of opposite angles that are equal in size

**e** all sides are equal in length; all angles are 90°

**f** two pairs of adjacent sides equal in length, one pair of opposite angles is equal

**3** Decide whether each statement about the quadrilaterals is true or false.

**a** A rhombus has exactly four sides that are equal in length.

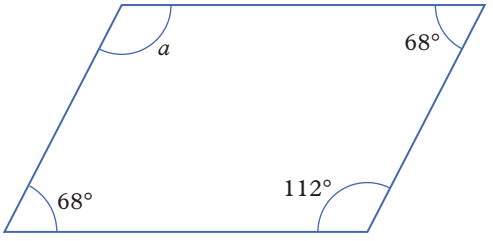
**b** A kite has two pairs of opposite angles that are equal in size.

**c** A trapezium has at least one pair of parallel sides.

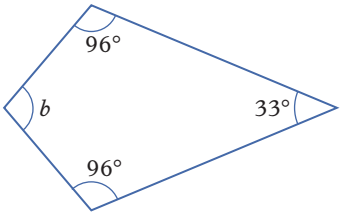
**d** Irregular quadrilaterals have exactly four sides that are of different lengths.

7E.2 **4** Find the size of the unknown angle in each of the quadrilaterals below.

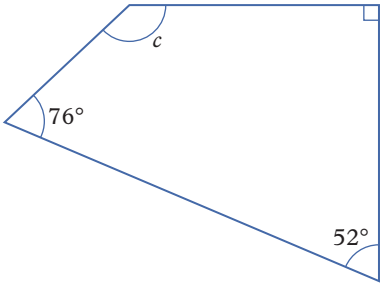
**a**



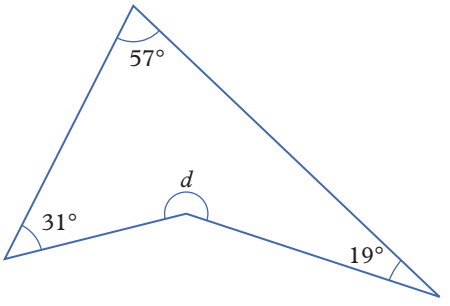

**b**



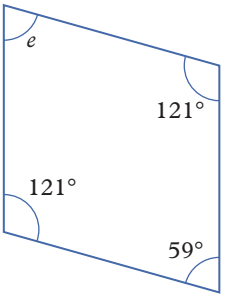

**c**



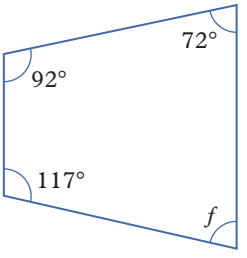

**d**




**e**



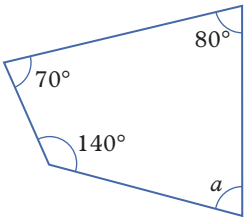

**f**



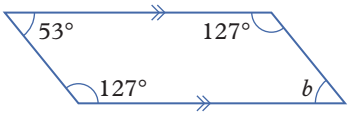
UNDERSTANDING AND FLUENCY

**5** Classify each of the following quadrilaterals, and then find the unknown angle.

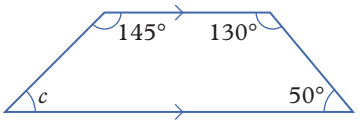
**a**



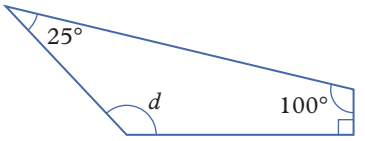

**b**



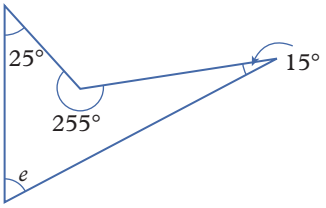

**c**



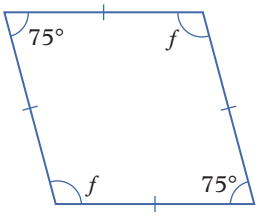

**d**




**e**



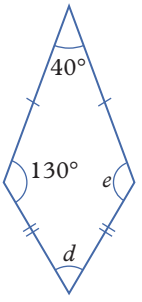

**f**



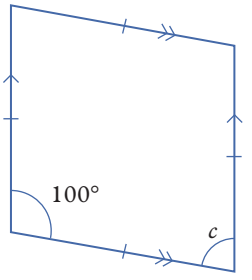

PROBLEM SOLVING AND REASONING

**6** Use your knowledge of quadrilateral properties to find the size of each labelled unknown angle.

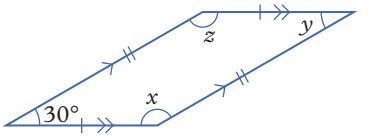
**a**



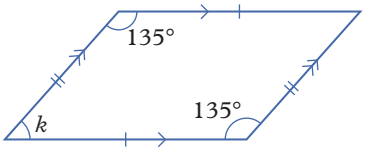

**b**




**c**

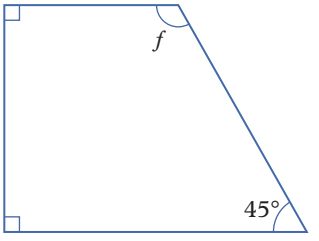



**d**

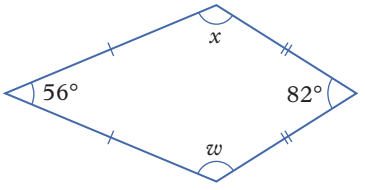



**e**

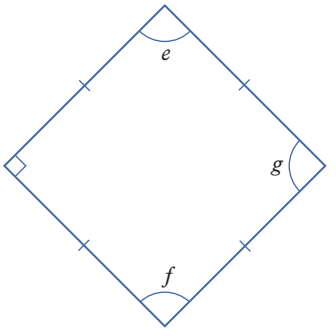
**f**



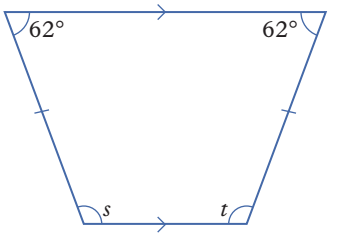

**g**



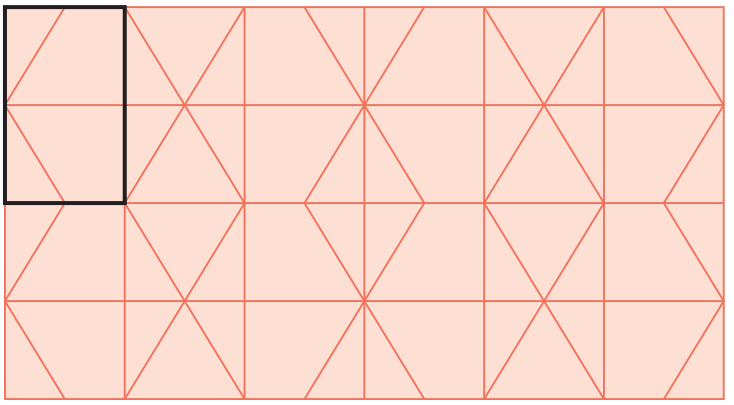

**h**




**i**

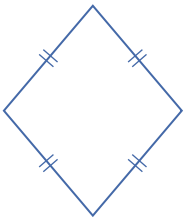


**7** Identify and list the different types of quadrilaterals in this grid. Your list should include any larger, composite shapes, like the rectangle outlined below.

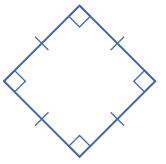


**8** These shapes are commonly referred to as 'diamonds'. Classify each shape using their mathematical names.

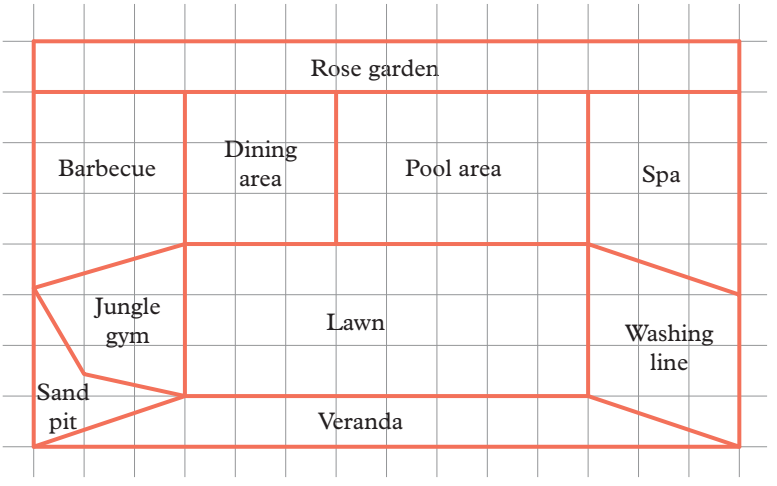
**a**



**b**

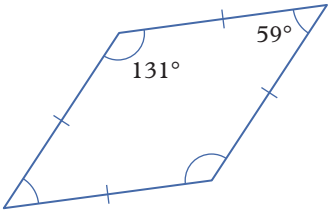


**9** The Singh family is renovating their backyard. The backyard is rectangular, and their plans for the backyard involve a variety of quadrilaterals. Identify which type of quadrilateral each area of the plan makes, giving the most specific name possible.

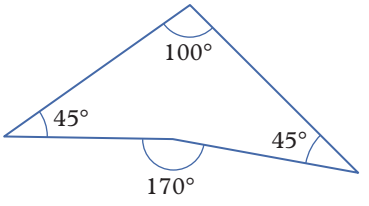



**10** Each of the following diagrams is incorrectly labelled. Identify the incorrect label(s) and explain how you know the labelling is incorrect in each case.

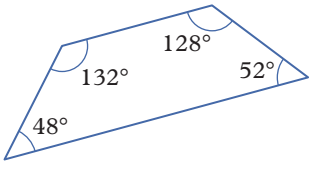
**a** Rhombus




**b** Kite




**c** Irregular quadrilateral



**11** The following descriptions could be referring to more than one type of quadrilateral. List all the types of quadrilaterals that fit each description.

**a** All interior angles are right angles.

**b** All sides are equal in length.

**c** Two pairs of sides are equal in length.

**d** At least one pair of sides are parallel.

**e** Two pairs of opposite sides are equal in length.

**12** Some of the special quadrilaterals are related. Answer the following questions and explain your answer by referring to the special features of each type of quadrilateral.

**a** Is a square a rectangle?

**b** Is a rectangle a square?

**c** Is a rectangle a parallelogram?

**d** Is a square a rhombus?

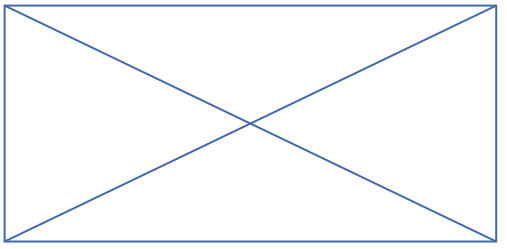
**13 a** Draw a rectangle into your workbook and draw a diagonal line joining two opposite corners of the rectangle. How many triangles have you formed?

**b** What is the sum of the angles in a triangle?

**c** Complete the following statement:

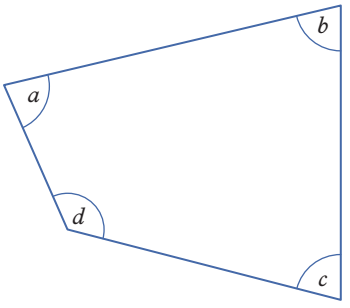
The sum of angles in a rectangle = 2 × _____° = _____°

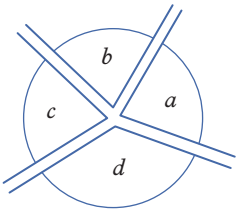
**d** A friend adds another diagonal line to the rectangle as shown below. This forms four triangles. Explain why it is not correct to say that the angle sum of a rectangle is 4 × 180° = 720°.



**e** How can the calculation in part **d** be corrected to show the true sum of angles in a rectangle?

Another way to demonstrate the angle sum of a quadrilateral is to arrange the internal angles of a quadrilateral as shown in the diagrams below.



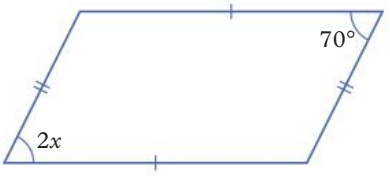




**f** What type of angle is formed by rearranging the internal angles of a quadrilateral as shown above?

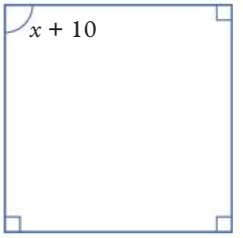
**g** How many degrees are in this type of angle? Does this match the angle sum you found in part **c**?

**14** Use your knowledge of quadrilateral properties and basic algebra to find the value of the pronumerals in these quadrilaterals.

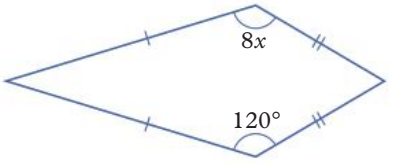
**a**



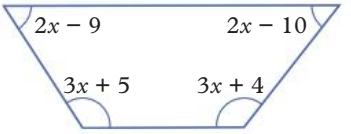

**b**



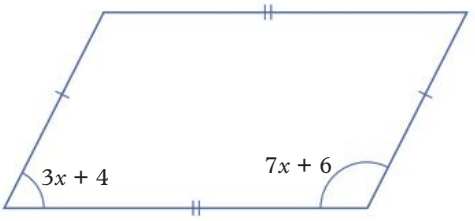

**c**



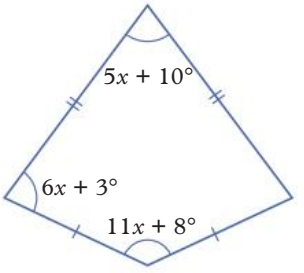

**d**



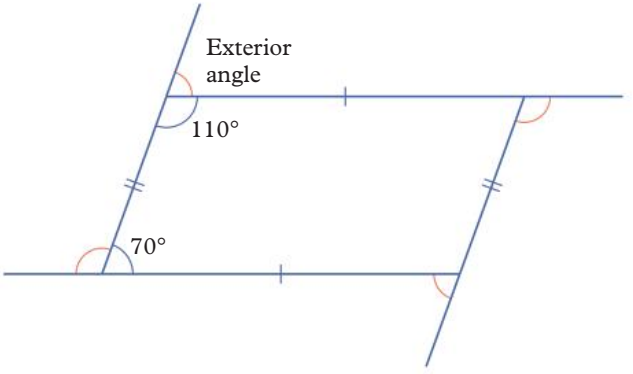

**e**




**f**




**15** Exterior angles can be found for quadrilaterals as well as triangles.




**a** Calculate the four exterior angles of the above parallelogram.

**b** Add these angles together. What do you find? How does this compare to the sum of exterior angles for a triangle?

**c** Repeat the activity for at least two different quadrilaterals on this page. Write a sentence about what you find.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Quadrilateral properties

**Investigation** Quadrilaterals everywhere

**Topic quiz** 7E

# Chapter summary

## Lines

point $A$

line segment $\overline{AB}$

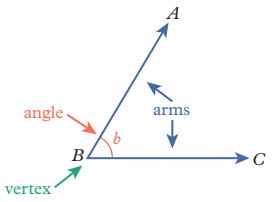
ray $\overrightarrow{AB}$

line $\overleftrightarrow{AB}$

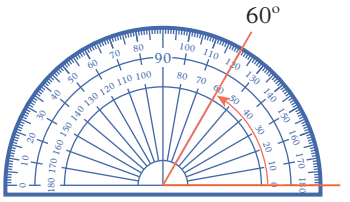
perpendicular lines

parallel lines

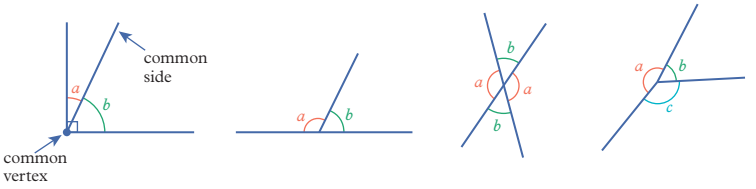
## Angle terminology




## Measuring angles




## Angles around a point



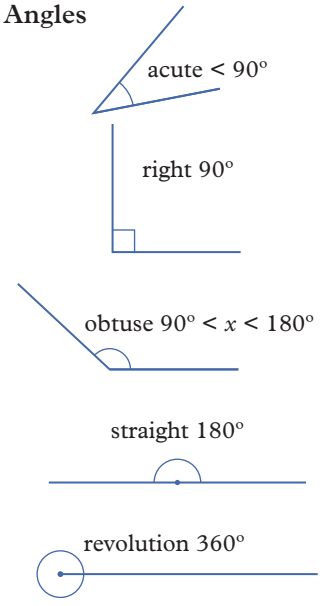

complementary $a + b = 90°$

supplementary $a + b = 180°$

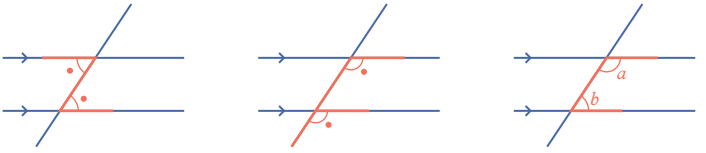
vertically opposite

revolution $a + b + c = 360°$

## Angles




## Angles and parallel lines



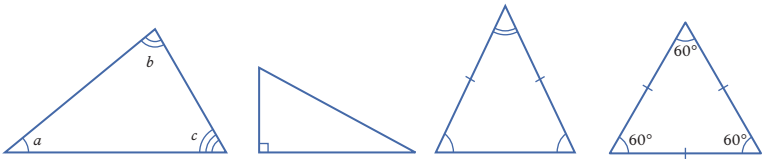
alternate '$Z$'

corresponding '$F$'

co-interior '$C$'
$a + b = 180°$

## Triangles

$a + b + c = 180°$




scalene

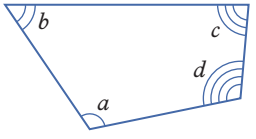
right-angled

isosceles

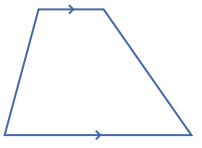
equilateral

## Quadrilaterals

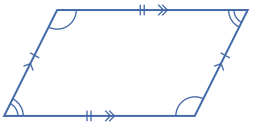
$a + b + c + d = 360°$



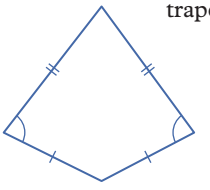
irregular quadrilateral



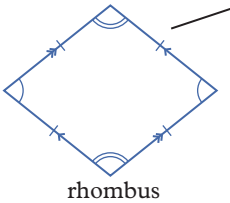
trapezium



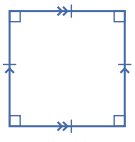
parallelogram



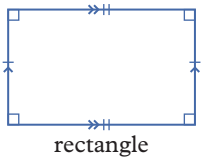
kite



rhombus



square



rectangle

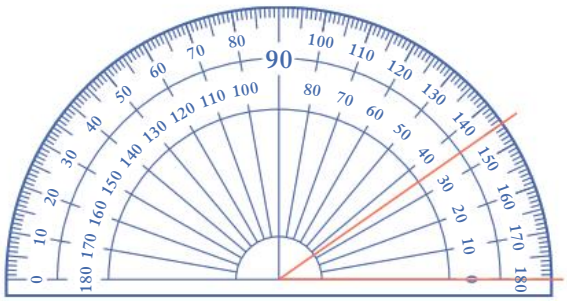
# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

7A **1** What is this angle closest to?

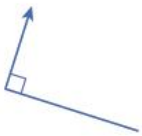


**A** 30° **B** 35° **C** 45° **D** 145° **E** 150°

7A **2** In which of the following lists are the types of angles listed in order of decreasing size?

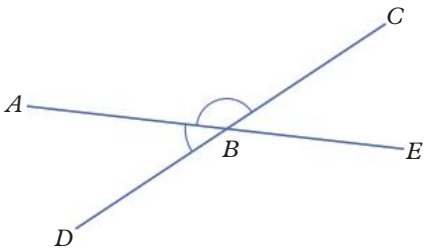
**A** acute, obtuse, reflex **B** right, straight, revolution **C** reflex, obtuse, straight
**D** obtuse, straight, acute **E** revolution, obtuse, right

7A **3** The diagram below is best described as:



**A** a ray parallel to a line segment. **B** two parallel rays.
**C** a line segment perpendicular to a ray. **D** two perpendicular line segments.
**E** two complementary lines.

7A **4** Which of the following angles is not marked with an arc?

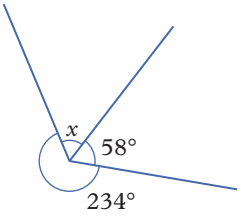



**A** $\angle ABC$ **B** $\angle DBA$ **C** $\angle CBA$ **D** $\angle DBE$ **E** $\angle ABD$

7B **5** Two angles that add to 90° are called:

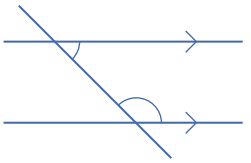
**A** acute angles. **B** right angles. **C** perpendicular angles.
**D** complementary angles. **E** supplementary angles.

7B **6** What is the size of angle $x$ on the diagram?



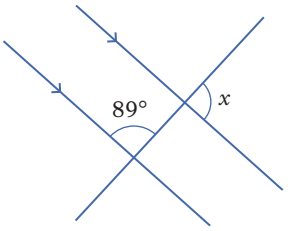

**A** 42° **B** 68° **C** 122° **D** 292° **E** 360°

7C **7** The angles marked in the diagram below are:



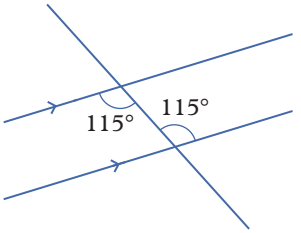
**A** equal in size.
**B** corresponding and complementary.
**C** co-interior and complementary.
**D** corresponding and supplementary.
**E** co-interior and supplementary.

7C **8** The value of $x$ in the diagram below is:



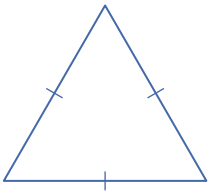

**A** 1° **B** 89° **C** 91° **D** 101° **E** 271°

7C **9** Based only on the information in the diagram, the lines are parallel because:



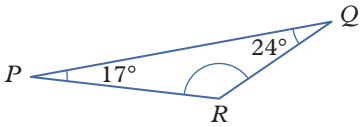

**A** the alternate angles are supplementary.
**B** the corresponding angles are equal.
**C** the co-interior angles are equal.
**D** the alternate angles are equal.
**E** the corresponding angles are supplementary.

7D **10** The triangle below is:



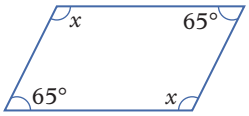
**A** a scalene triangle.
**B** an isosceles triangle.
**C** an equilateral triangle.
**D** a right-angled triangle.
**E** an obtuse triangle.

**11** In the diagram below, $\angle QRP$ is equal to:




**A** 17° **B** 139° **C** 41° **D** 24° **E** 319°

7E **12** The quadrilateral below is:

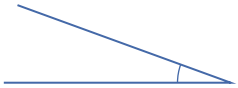



**A** a kite. **B** a parallelogram. **C** a rhombus.
**D** a rectangle. **E** a trapezium.

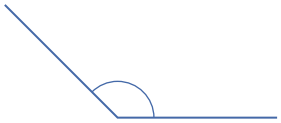
## Short answer

7A **1** Measure and classify the angles below as acute, right, obtuse, straight, reflex or a revolution.

**a**



**b**



**c**

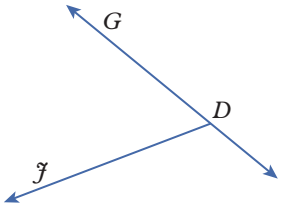
**d**

7A **2** Draw each of the following angles using a protractor.

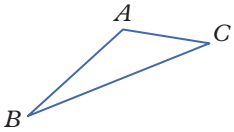
**a** 40° **b** 115° **c** 200°

7A **3** Identify and name all of the objects in the following diagrams.

**a**

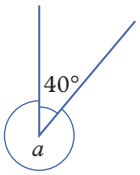



**b**

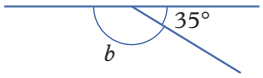



7B **4** Find the size of the unknown angles in each of the diagrams below.

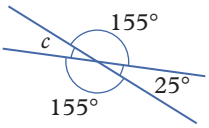
**a**



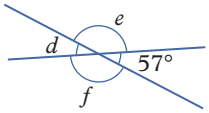

**b**



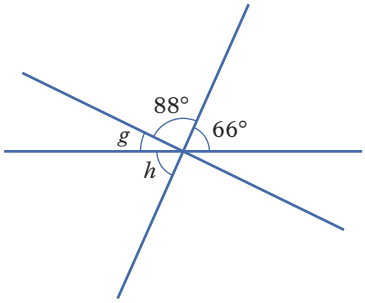

**c**



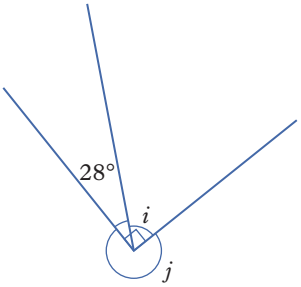

**d**




**e**




**f**



**7B** **5** **a** Find the complementary angle to:

**i** 23° **ii** 79°

**b** Find the supplementary angle to:

**i** 9° **ii** 127°

**7C** **6** Use this diagram to find:

**a** the angle corresponding to $c$

**b** the alternate angle to $e$

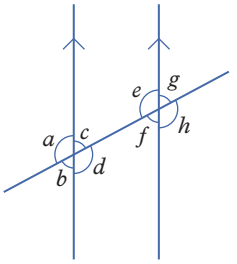
**c** the co-interior angle to $f$

**d** If $f = 77°$, find:

**i** $b$

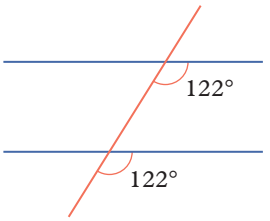
**ii** $d$

**iii** $a$

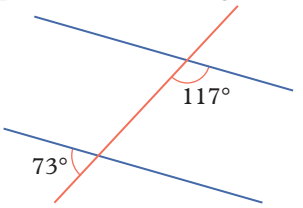



**7C** **7** Decide whether the blue lines in each diagram are parallel. Explain your reasoning.

**a**

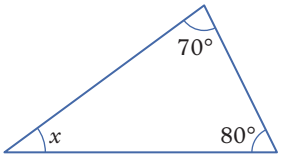



**b**

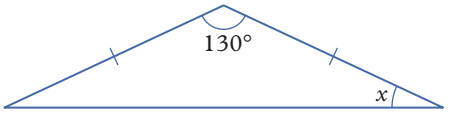



**7D** **8** Classify each of the triangles below and hence find the value of $x$.

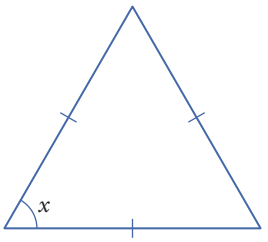
**a**



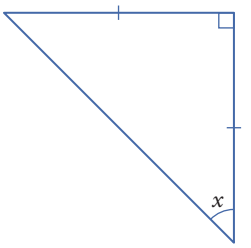

**b**




**c**

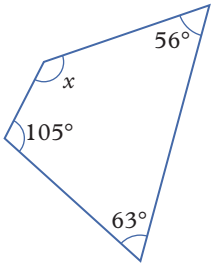



**d**

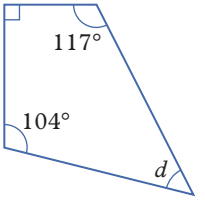



**7E** **9** Find the value of the pronumerals in each diagram.

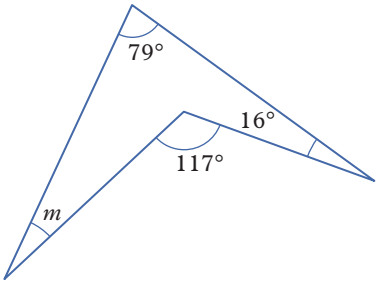
**a**



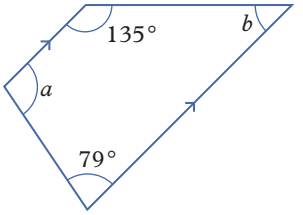

**b**




**c**

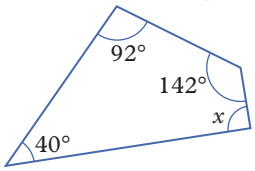



**d**

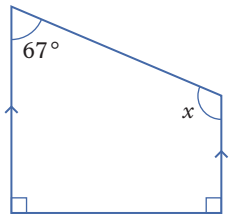


7E **10** Classify each of the quadrilaterals below and hence find the value of $x$.

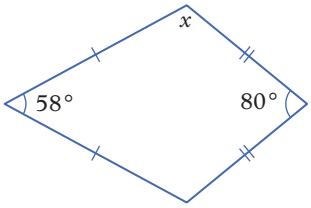
**a**



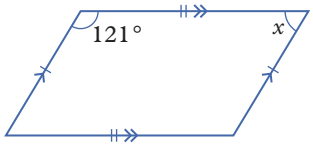

**b**




**c**




**d**



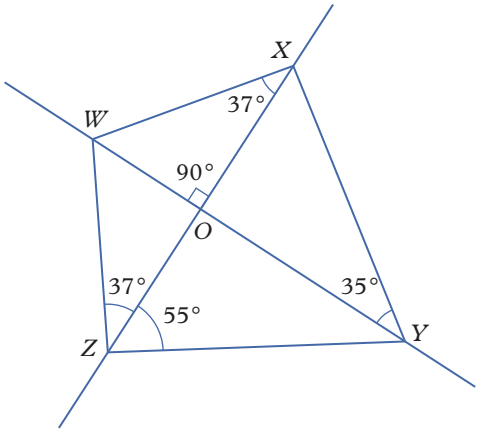

7E **11** Consider the following diagram where $\angle WXZ = 37°$, $\angle WZX = 37°$, $\angle OZY = 55°$ and $\angle OYX = 35°$.

**a** Find the size of the following angles.

**i** $\angle XWZ$ **ii** $\angle YOZ$

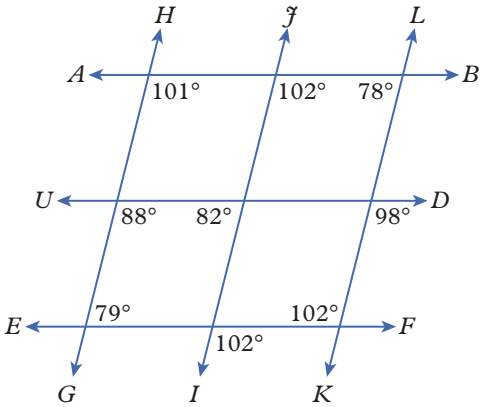
**iii** $\angle OYZ$ **iv** $\angle XOY$

**b** Classify the quadrilateral $WXYZ$.

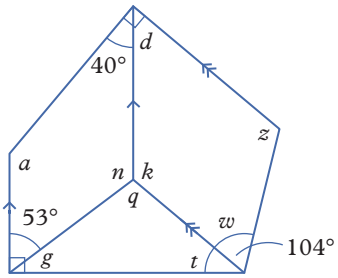



## Analysis

**1** Identify and name the parallel lines in the diagram below.




**2** The irregular pentagon below has been broken into two different quadrilaterals and a triangle. Find the value of all the pronumerals.



# 8

# Shapes and objects

## Index


**8A** Polygons
**8B** 3D objects
**8C** Plans and elevations
**8D** Translations
**8E** Rotations and reflections
**8F** Combined transformations
**8G** Line and rotational symmetry


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Classifying shapes
- ✔ Classifying angles
- ✔ Classifying triangles
- ✔ The Cartesian plane

## Curriculum links

- Draw different views of prisms and solids formed from combinations of prisms (VCMMG260)
- Describe translations, reflections across an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates. Identify line and rotational symmetries (VCMMG261)

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## Materials

- ✔ Ruler
- ✔ Isometric dot paper

# 8A Polygons

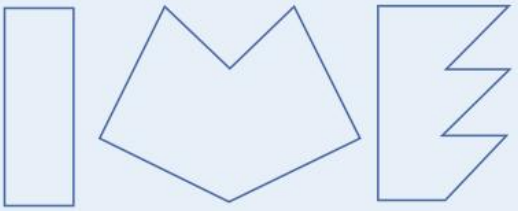
## Learning intentions

- ✔ I can classify polygons as regular or irregular.
- ✔ I can classify polygons as convex or concave.
- ✔ I can describe and name polygons.

Inter-year links
Years 5/6 2D shapes
Year 8 7B Triangles

## Polygons

- A **polygon** is a closed shape with straight sides.



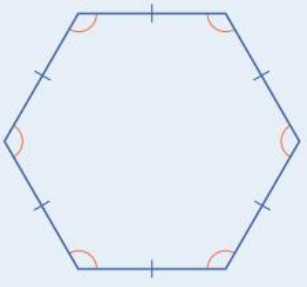
Polygons

Non-polygons

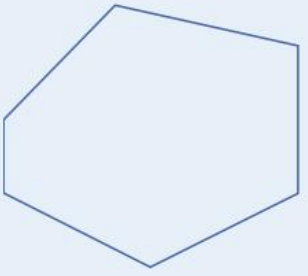
- Polygons are named according to how many sides or angles they have.

| Greek/Latin prefix | Number of sides | Name of shape |
|---|---|---|
| penta- | 5 | pentagon |
| hexa- | 6 | hexagon |
| hepta- | 7 | heptagon |
| octa- | 8 | octagon |
| nona- | 9 | nonagon |
| deca- | 10 | decagon |
| dodeca- | 12 | dodecagon |
| poly- | Many | polygon |

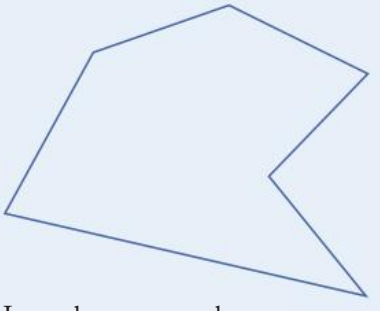
- Triangles (three sides) and quadrilaterals (four sides) are also polygons.
- **Regular polygons** have all sides equal in length and all angles equal in size, otherwise the polygon is an **irregular polygon**.
- **Concave polygons** have at least one interior reflex angle (greater than 180°). **Convex polygons** have all interior angles less than 180°. All regular polygons are convex polygons.



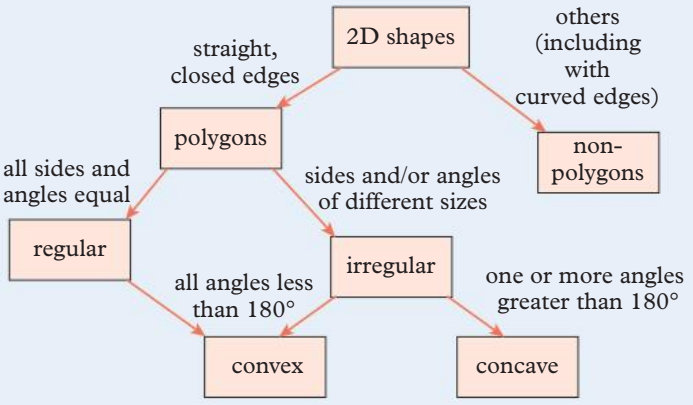
Regular, convex hexagon



Irregular, convex hexagon

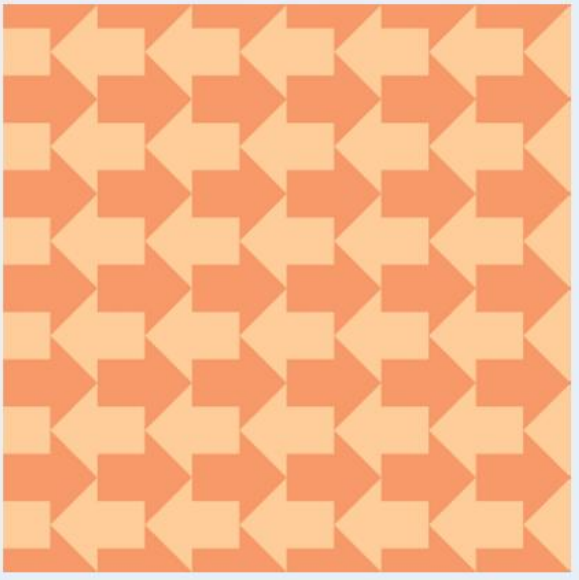


Irregular, concave hexagon




## Tessellations

- A **tessellation** is a pattern of shapes that has no overlaps or gaps.

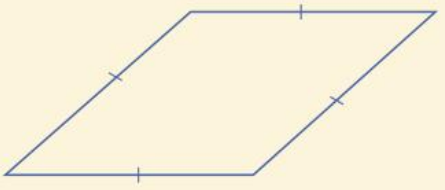


### **Example 8A.1** Describing polygons

Decide if this polygon is:

**a** regular or irregular

**b** convex or concave.



**THINK**

**a** Examine the sides and angles. The angles are not equal in size, so the polygon is irregular.

**b** Examine the internal angles. There are no internal angles larger than 180°, so the polygon is convex.

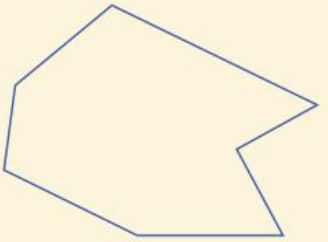
**WRITE**

**a** This polygon is irregular.

**b** This polygon is convex.

## **Example 8A.2** Naming polygons

Describe and name this shape.



**THINK**

1 Examine the sides of the shape. All sides are straight, so it is a polygon.

All sides are not equal in length, so it is irregular. It has seven sides, so it is a heptagon.

2 Examine the angles of the shape. There is a reflex angle, so it is concave.

**WRITE**

The shape is an irregular, concave heptagon.

**Helpful hints**

- ✔ For a shape to be a polygon all sides must be straight! Some common shapes, such as circles, are not polygons.
- ✔ An equilateral triangle is a regular polygon as all angles are equal in size and all side lengths are the same length.

ANS p584

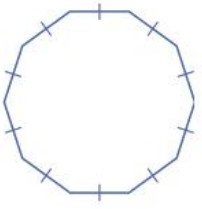
# **Exercise 8A** Polygons

1–4, 5(a, b, d), 6–9, 12 | 1(b, e, f, i), 2(d–f), 3(d–f), 5, 7–9, 10(a, b), 11–13 | 1(b, e, f, i), 2(d–f), 3(e, f), 5(d–f), 9, 10, 13, 14, 15(c, d), 16

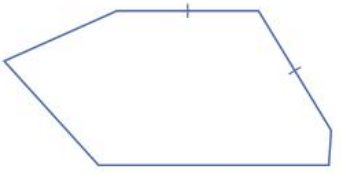
UNDERSTANDING AND FLUENCY

1 Identify whether each shape is a polygon or a non-polygon.

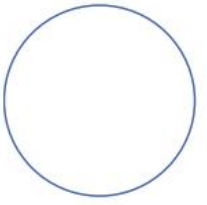
a



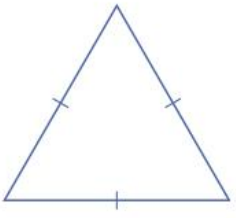
b



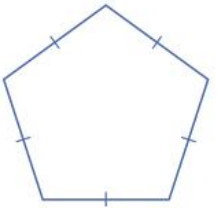
c



d



e



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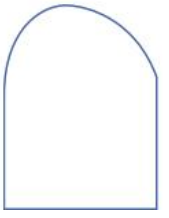



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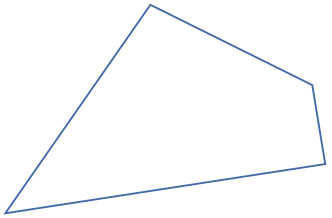
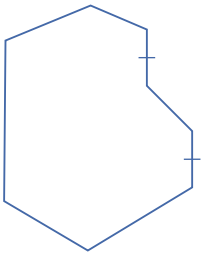
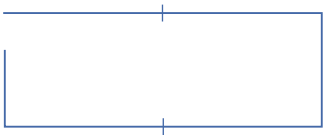
**g**



**h**



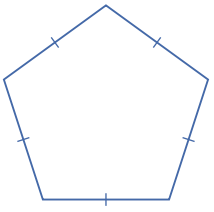
**i**



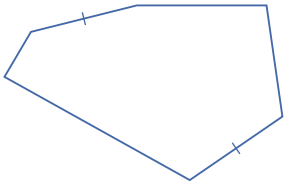
**8A.1** **2** Decide if each polygon is:

**i** regular or irregular **ii** convex or concave.

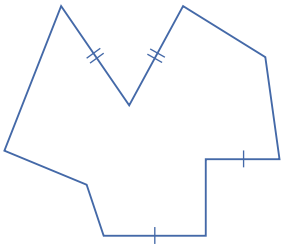
**a**



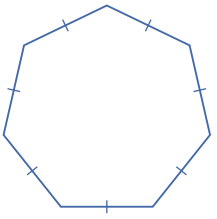
**b**



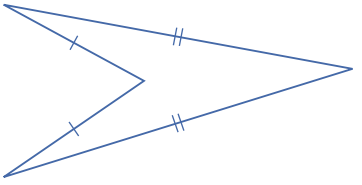
**c**



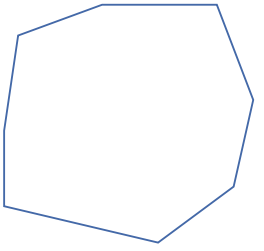
**d**



**e**

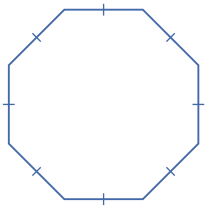


**f**

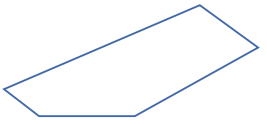


**8A.2** **3** Describe and name each shape.

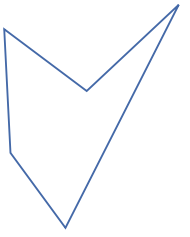
**a**



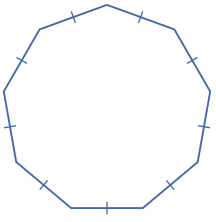
**b**



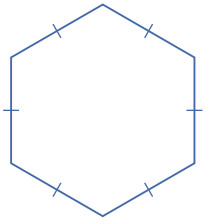
**c**



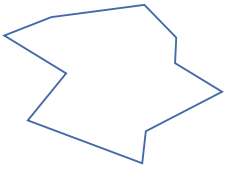
**d**



**e**



**f**



**4** Consider each polygon only in question **1**.

**i** State if the polygon is regular or irregular.

**ii** Write the number of sides the polygon has.

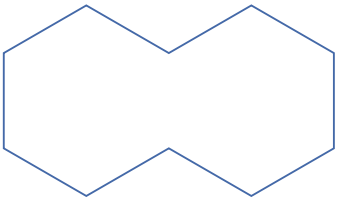
**iii** Does the polygon have an internal reflex angle?

**iv** Give the polygon a name using your answers to parts **i**, **ii** and **iii**.

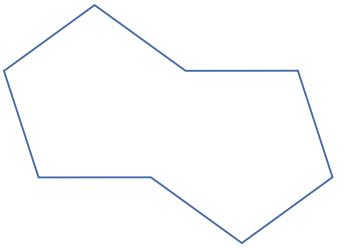
**5** Use your knowledge of describing polygons to name each of the following shapes.

**a**

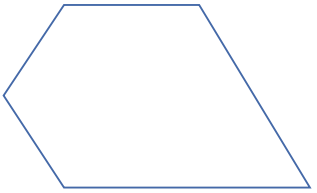
**b**



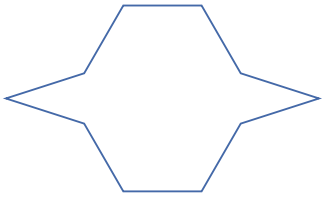
**c**



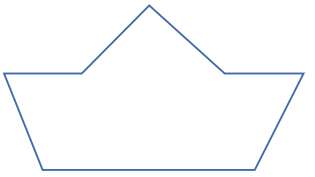
**d**



**e**

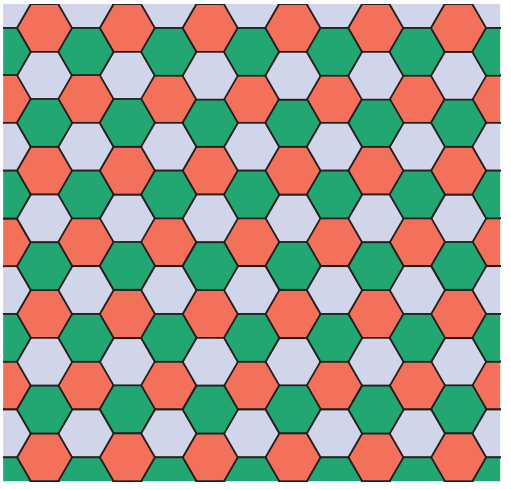


**f**



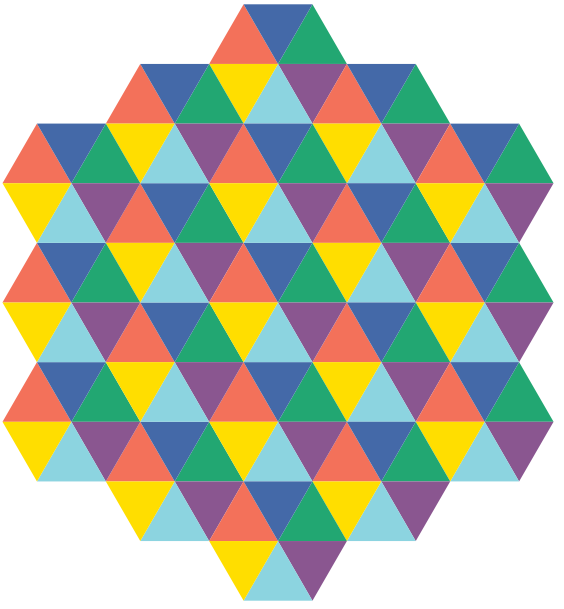
**6** The shape that makes up the tessellation in the image below is:

**A** a pentagon
**B** a hexagon
**C** a square
**D** a triangle
**E** an octagon

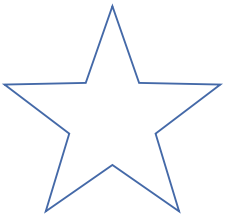


**7** The shape that makes up the tessellation in the image below is:

**A** a pentagon
**B** a hexagon
**C** a square
**D** a triangle
**E** an octagon



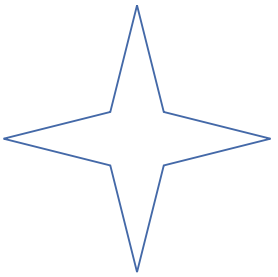
**8** A familiar type of concave polygon is the star. The star on the right can be called either a five-pointed star or a concave decagon.



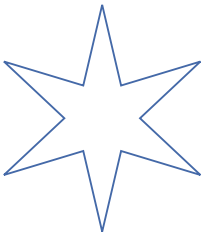
**a** Explain both methods for naming the star.

**b** Name the following star shapes according to each method.

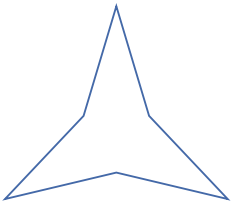
**i**



**ii**

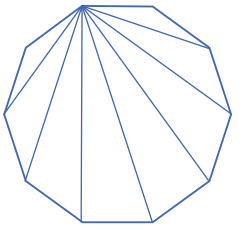


**iii**



**c** What relationship can you see between the two methods for naming stars?

**9** To calculate the interior angle sum of any polygon, divide the shape into triangles as shown in this decagon.



**a** What property of triangles makes it useful to do this?

**b** How many sides does a decagon have?

**c** How many triangles are in the decagon?

**d** Use your answer from part **c** to calculate the interior angle sum in a decagon.

**10** Consider each shape below and use the triangle method from question **9** to find:

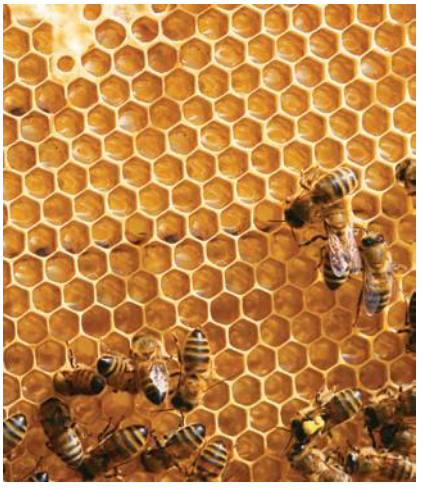
**i** the number of triangles each shape can be divided into

**ii** the interior angle sum in each shape.

**a** a pentagon **b** a hexagon **c** a heptagon **d** a nonagon

**11** Are triangles and quadrilaterals considered to be polygons? Explain your answer.

**12** Look at these photographs of a honeycomb and of the outer wall of the National Aquatic Center (known as the Water Cube) in Beijing.



**a** What shapes are in these two photos?

**b** What is the same and what is different about these two patterns?

PROBLEM SOLVING AND REASONING

**13** The two images shown in question **12** are examples of tessellations. There are three main types of tessellations:

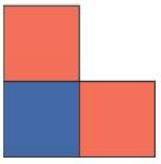
→ regular tessellations, which are made from identical, regular polygons
→ semi-regular tessellations, which are made from a number of regular polygons
→ irregular tessellations, which are made from any number of different shapes.

**a** Describe the types of tessellations shown in question **12**.

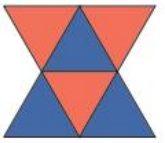
Aside from the hexagonal pattern shown in question **12**, there are only two other regular tessellations: using a square and using an equilateral triangle.

**b** Complete these two other regular tessellations by adding another 10 of the repeating shape.

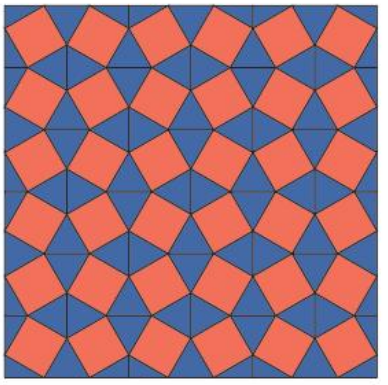
**i**



**ii**



**14** Here is an example of a semi-regular tessellation. What polygons are included in this tessellation?



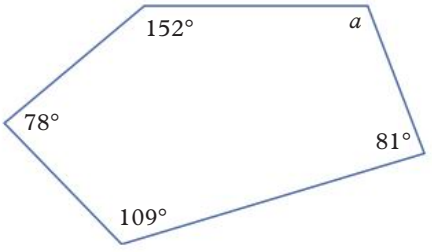
CHALLENGE

**15** Use your findings from question **9** to find the size of an individual internal angle in each of these shapes if they are regular polygons.

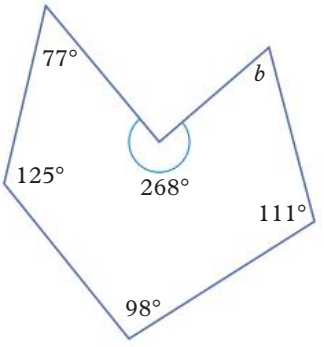
**a** pentagon **b** hexagon **c** heptagon **d** nonagon

**16** Find the size of each unknown angle in these irregular polygons.

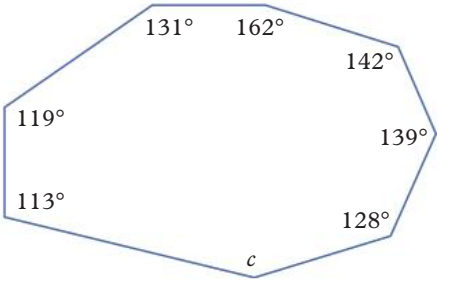
**a**




**b**




**c**




Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Polygons

**Investigation** Constructing regular polygons

**Topic quiz** 8A

# 8B 3D objects

## Learning intentions

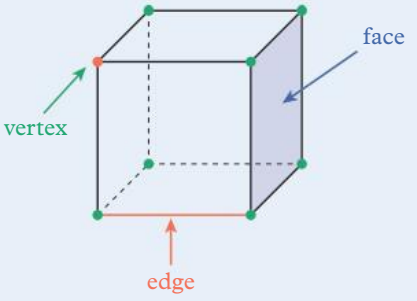
✔ I can draw right prisms.
✔ I can identify and name 3D objects.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | 3D objects |
| Year 8 | 8F Volume of prisms |
| Year 9 | 6C Volume and capacity |

## 3D objects

- A **face** is a flat surface on a three-dimensional (3D) object.
  - → An **edge** is a line segment that joins two faces.
  - → A **vertex** (plural: vertices) is a corner point where three or more edges meet.

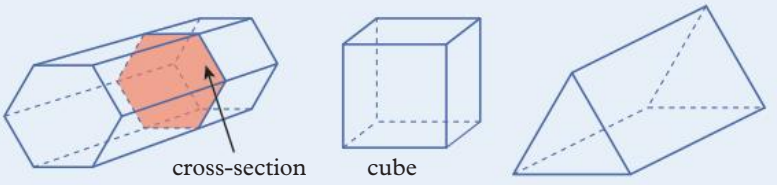



- A **cross-section** of a 3D object is the 2D shape obtained when cutting the object.
- A **polyhedron** (plural: polyhedra) is a 3D object with all faces being polygons.
  - → All polyhedra have straight edges.
  - → **Cylinders**, **cones** and **spheres** are examples of 3D objects that are not polyhedra.

**Non-polyhedra**

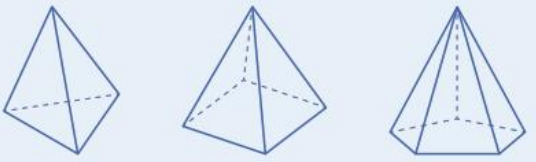
- A **prism** is a polyhedron with matching ends that are the same shape and size. The matching ends in a prism are joined by straight edges. In a **right prism**, the angles between the matching ends and the other faces are right angles.
  - → Prisms have a consistent cross-section throughout their length. That is, all cross-sections of the prism parallel to the matching ends are the same shape and size.
  - → Prisms are named after the shape at each matching end. For example, if the end shape is a hexagon it is a hexagonal prism.
  - → The other faces (not end faces) of a right prism are rectangles.

**Right prisms**



- A **pyramid** is a polyhedron with a **base** and triangular faces joining the base which all meet at a single point.
  - → Pyramids are named after the shape of the base. For example, if the base is a pentagon it is a pentagonal-based pyramid.

**Pyramids**

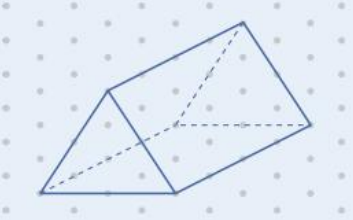


triangular-based pyramid | square-based pyramid | pentagonal-based pyramid

## Drawing prisms

- Isometric or square dot paper can be used to draw 3D shapes.
  - → Dashed lines should be used to show hidden edges.

Triangular prism



### Example 8B.1 Using a 2D shape to draw a prism

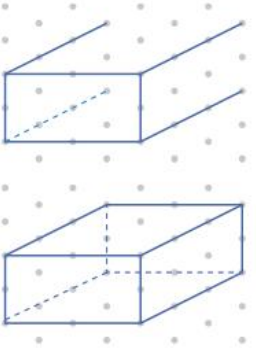
Use isometric dot paper to draw a prism with the 2D shape as one of the matching ends.

Rectangular prism

**THINK**

1. Draw a line from each vertex using the dots on the paper as a guide. Make sure each line is the same length and that all three lines are parallel. Show the hidden edge as a dashed line.
2. Complete the prism by drawing two horizontal lines and two vertical lines that join the ends of the lines that have just been drawn. Show any hidden edges as dashed lines.

**WRITE**





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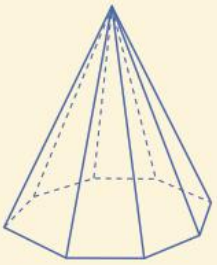
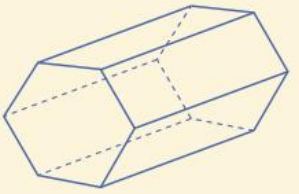
## **Example 8B.2** Identifying and naming 3D objects

Name the following 3D objects.

**a**



**b**



| THINK | WRITE |
|---|---|
| **a** **1** Determine whether the shape is a polyhedron. All of the faces are flat and all of the edges are straight, so this object is a polyhedron. | **a** This is an octagonal-based pyramid. |
| **2** Determine whether the shape is a prism, pyramid or other polyhedron. This object has a base with the triangular faces meeting at a point, so it is a pyramid. | |
| **3** Name the object using the shape of the base. | |
| **b** **1** Determine whether the shape is a polyhedron. All of the faces are flat and all of the edges are straight, so this object is a polyhedron. | **b** This is a hexagonal prism. |
| **2** Determine whether the shape is a prism, pyramid or other polyhedron. This shape has matching ends and a consistent cross-section throughout its length, so it is a prism. | |
| **3** Name the object using the shape at the matching ends. | |

**Helpful hints**

- ✔ The matching ends of a prism must be polygons. Many people mistakenly assume that cylinders are prisms as they have a consistent cross-section throughout their length. However, the matching ends of a cylinder are not polygons.
- ✔ When using isometric dot paper to draw 3D objects, the paper should be held so there are vertical columns of dots.

ANS p585

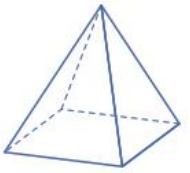
# Exercise 8B 3D objects

1, 2, 3(a, c, e), 4–6, 7(d–f), 8–10, 13, 15 | 2, 3(c–f), 4, 7–9, 12, 14, 16, 17 | 3(e, f), 4(g–i), 7(d–f), 10, 11, 14, 16–19

UNDERSTANDING AND FLUENCY

Grids to complete the Exercise 8B Isometric Drawings can be found in the Exercise 8B BLM.

**1** Look at this square-based pyramid.



**a** How many faces does it have?

**b** How many edges does it have?

**c** How many vertices does it have?

**2** Look at this pentagonal pyramid.

**a** How many faces does it have?

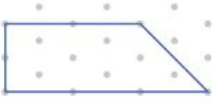
**b** How many edges does it have?

**c** How many vertices does it have?

8B.1 **3** Use isometric dot paper to draw a prism with the 2D shape as one of the matching ends.

**a** Rectangular prism

**b** Trapezium prism



**c** Triangular prism

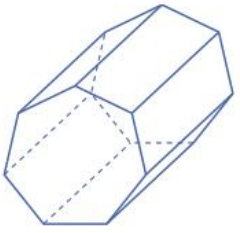
**d** Parallelogram prism

**e** Pentagonal prism

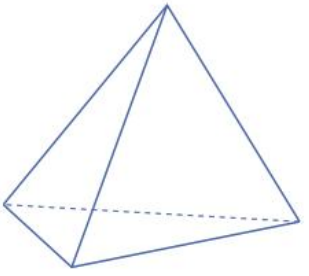
**f** Quadrilateral prism

**4** Identify whether the following shapes are polyhedra or non-polyhedra.

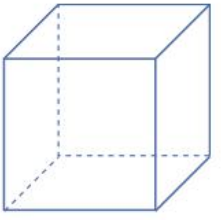
**a**



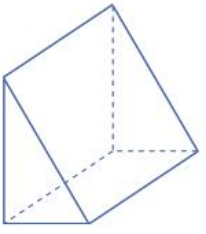
**b**



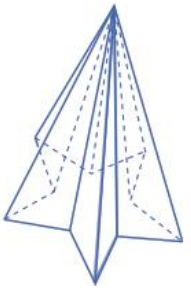
**c**



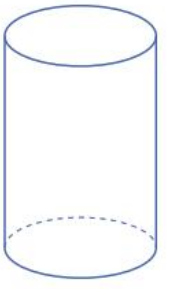
**d**



**e**



**f**





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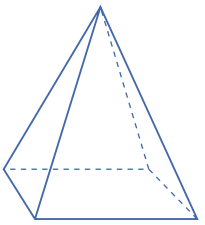
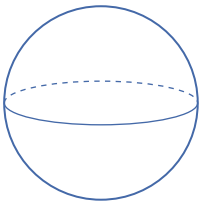
**g**



**h**



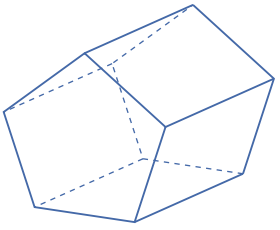
**i**

**5** What 2D shape forms the base or matching end of each prism and pyramid in question **4**?

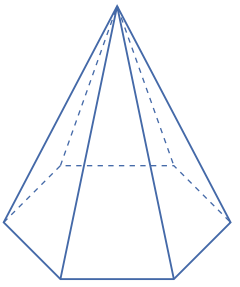
**6** Use your answers to questions **1** to **3** to name the prisms and pyramids in question **4**.

**8B.2** **7** Name the following 3D objects.

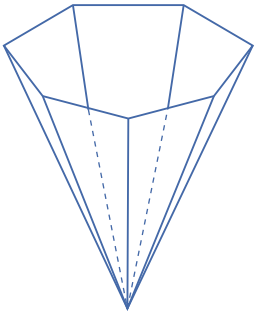
**a**



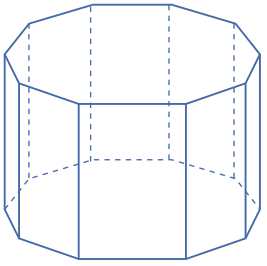
**b**



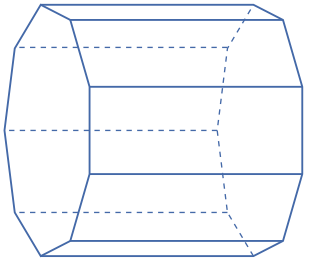
**c**



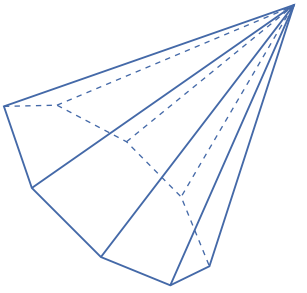
**d**



**e**



**f**



**8** Look at each object and give its mathematical name.

**a**

**b**

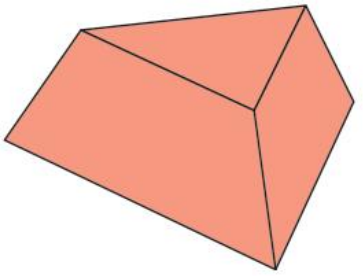
**c**

**9** Use isometric dot paper to draw the blocks in this picture.

UNDERSTANDING AND FLUENCY

**10** Use isometric dot paper to draw the wooden block in this picture.

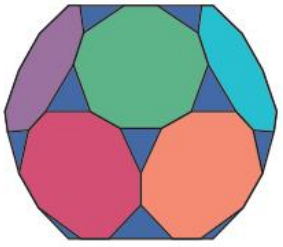
**11** Use isometric dot paper to draw the block in this picture.



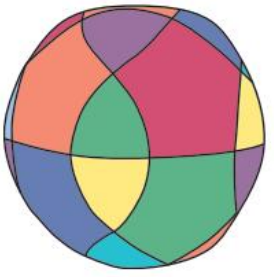
PROBLEM SOLVING AND REASONING

**12** Decide whether each 3D object is a polyhedron. Explain your answers.

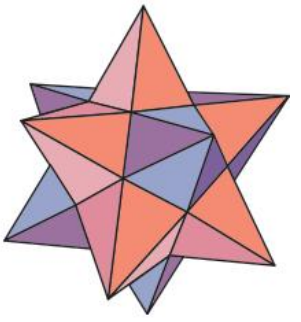
**a**



**b**



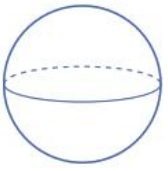
**c**



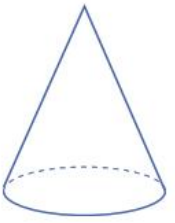
**13** Consider these three 3D objects.

**a** Name each object.

**i**



**ii**



**iii**

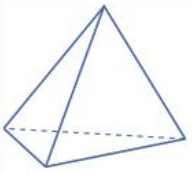
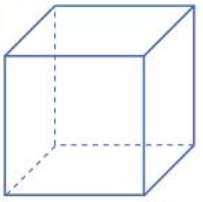
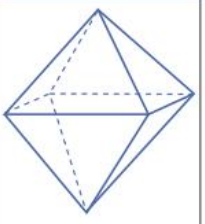
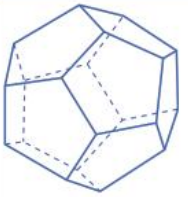
**b** Explain why they are not polyhedra.

**14** Is a soccer ball a polyhedron? Explain your answer.

**15** Look at all the polyhedra listed in question **4**. Create a table listing the number of faces ($F$), the number of edges ($E$) and the number of vertices ($V$) for each object.

**16** There is a group of special polyhedra named the Platonic solids. These polyhedra are all regular. This means that all their faces are the same shape, their edges are the same length and the vertices meet at the same angles.

**a** Copy and complete the table below.

| | **Tetrahedron** | **Cube (Hexahedron)** | **Octahedron** | **Dodecahedron** | **Icosahedron** |
|---|---|---|---|---|---|
| **Image** |  |  |  |  | |
| **Number of faces (*F*)** | | | | 12 | 20 |
| **Name of polygon on face** | | | | | |

**b** Add two rows to the table and find the number of vertices ($V$) and the number of edges ($E$) for each Platonic solid.

**c** Add another row to the table and calculate $F + V - E$.

**d** What do you find?

**17** Euler's rule states that for any polyhedron, $F + V - E = 2$.

**a** Is this true for all the Platonic solids above?

**b** Investigate Euler's rule further by finding $F + V - E$ for all the polyhedra in question **4**. What do you find?

**18** Use Euler's rule to find the number of faces on each polyhedron, given the object has:

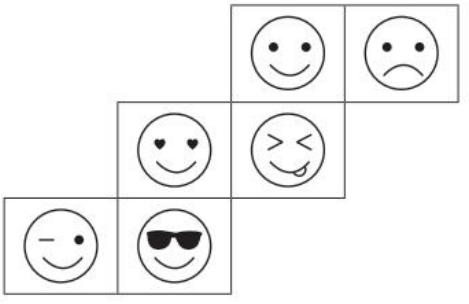
**a** 10 edges, 4 vertices

**b** 6 edges, 3 vertices

**c** 18 edges, 6 vertices

**d** 20 edges, 10 vertices

**19** The net below is folded into a cube. It is placed on a table with the face on top. Which face is at the bottom?



Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Prisms

**Investigation** Mathematical drawing

**BLM** Exercise 8B

**Worksheet** Drawing 3D objects

**Topic quiz** 8B

PROBLEM SOLVING AND REASONING

CHALLENGE

# 8C Plans and elevations

## Learning intentions

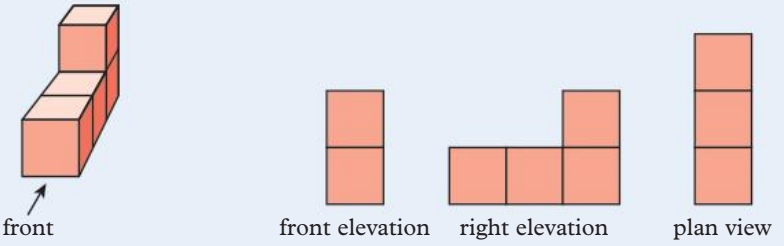
- ✔ I can draw elevations of prisms.
- ✔ I can draw elevations of 3D objects formed from combinations of prisms.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | 3D objects |
| Year 8 | 7F Constructing triangles |
| Year 9 | 6B Surface area |

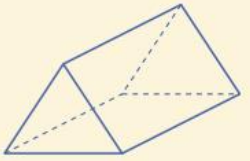
## Plans

- **Plans** and **elevations** are 2D drawings of a 3D object from a single view.
- A plan view shows what a 3D object looks like when viewed from directly above the object. An elevation shows what a 3D objects looks like when viewed from the front or side of the object.
- A set of a plan and elevations consists of a front elevation, a side elevation (either left or right) and a plan view.
- A 3D object can be drawn from a set of a plan and elevations.




### Example 8C.1 Drawing a plan and elevations of prisms

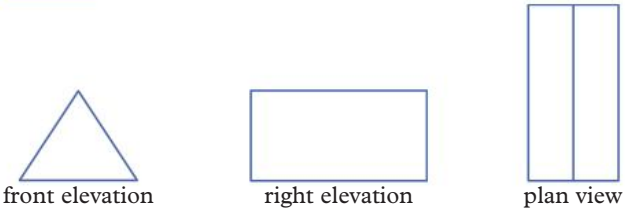
Draw a set of a plan and elevations for this prism.



**THINK**

1. Picture viewing the object from the front. From this angle the view is a triangle. Draw the triangle as the front elevation.
2. Picture viewing the object from the right-hand side. From this angle the view is a rectangle. Draw the rectangle as the right elevation.
3. Picture viewing the object from above. From this angle the view is a rectangle split in two. Draw this as the plan view.

**WRITE**





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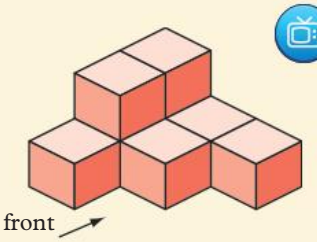
## Example 8C.2 Drawing a plan and elevations of 3D objects

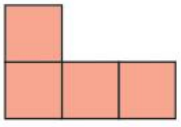
Draw a set of a plan and elevations for this object.



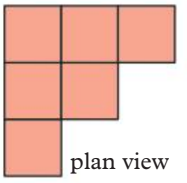

**THINK**

1. Picture viewing the object from the front, as indicated by the arrow. Three cubes are visible on the bottom and only one on the top. Draw this as the front elevation.
2. Picture viewing the object from the right-hand side. Three cubes are visible on the bottom and two on the top. Draw this as the right elevation.
3. Picture viewing the object from above. Three rows of cubes are visible: three in the top row, two in the next row and one in the bottom row. Draw this as the plan view.

**WRITE**


front elevation

right elevation


plan view

**Helpful hints**

Do not try to include perspective in your views! For example, when looking directly at a sloping edge (as in the right elevation of Example 8C.1), the slope will not be visible from that view.

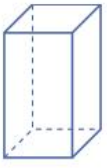
ANS p586

# Exercise 8C Plans and elevations

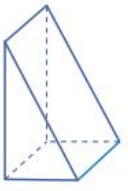
1–4, 5(a, b), 6–8, 11(a) | 1, 2, 3(d–f), 4–6, 8, 9, 11(a, b, c) | 3(d–f), 4, 5(c, d), 6, 9, 11(d), 12, 13

8C.1 **1** Draw a set of a plan and elevations for these prisms.

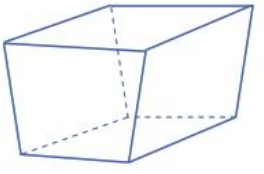
**a**



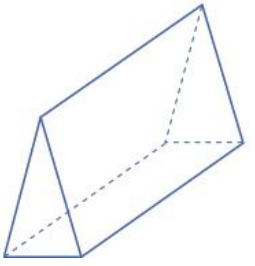
**b**



**c**



**d**



**2** Draw a set of a plan and elevations for these objects. Do not include the writing on the packaging or the dots on the die.

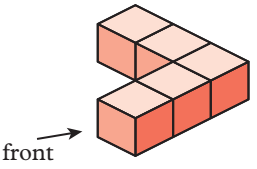
**a**

**b**

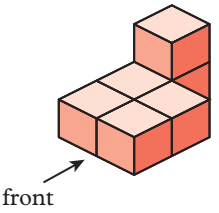
**c**

**8C.2** **3** Draw a set of a plan and elevations for each object.

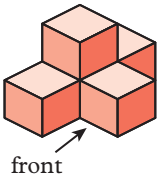
**a**



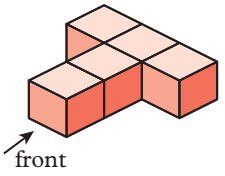

**b**



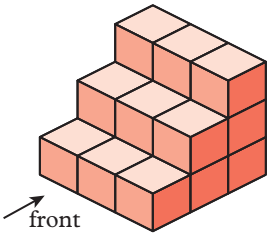

**c**



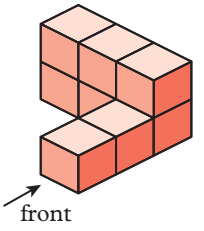

**d**




**e**

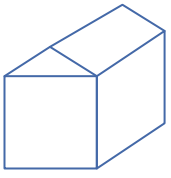



**f**

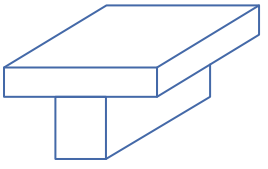



**4** Draw a set of a plan and elevations for each of these objects.

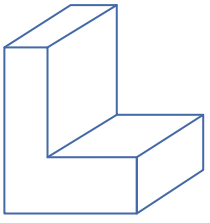
**a**



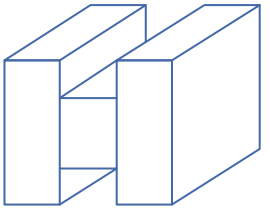
**b**



**c**

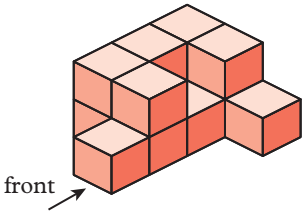


**d**

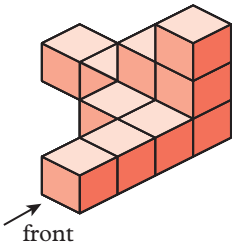


**5** Draw a set of a plan and elevations for each of these more complex objects.

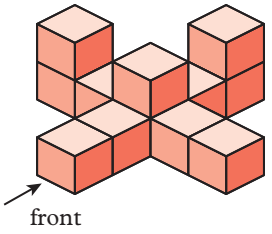
**a**



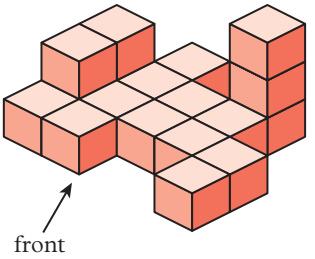

**b**




**c**

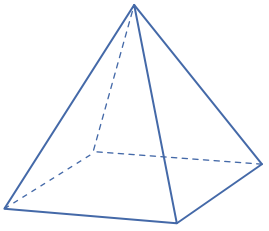



**d**

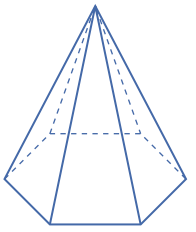



**6** Draw a set of a plan and elevations for each of these pyramids.

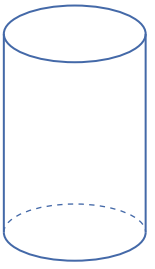
**a**



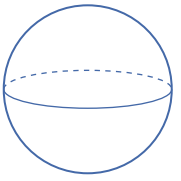
**b**



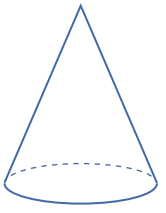
**7** Explain why the side elevation of an upright cylinder is a rectangle.



**8** What shapes are the front elevation, side elevation and plan view of a sphere?



**9** Draw a set of a plan and elevations for this cone.



UNDERSTANDING AND FLUENCY

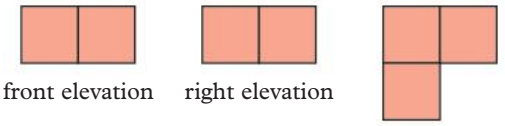
PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

**10** Draw a set of a plan and elevations for this tent.

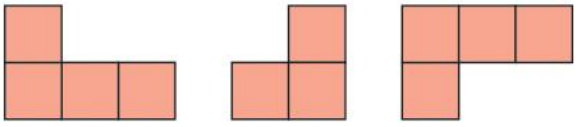
**11** Use each set of a plan and elevations to construct the matching 3D object. Draw the objects on isometric or square dot paper.

**a**



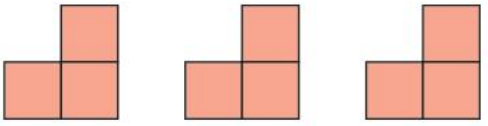
front elevation right elevation plan view

**b**



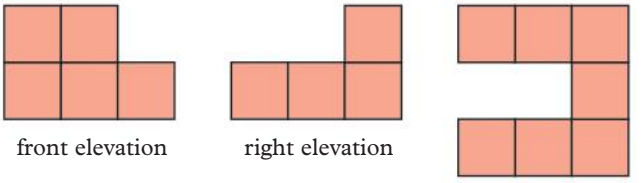
front elevation right elevation plan view

**c**



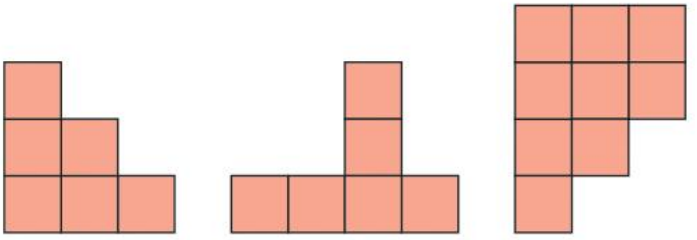
front elevation right elevation plan view

**d**



front elevation right elevation plan view

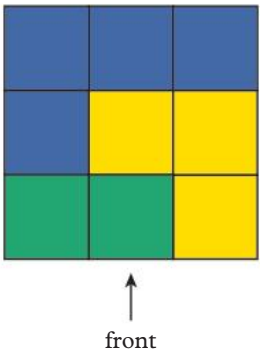
**12** Use this set of a plan and elevations to construct the matching 3D object. Draw the object on isometric or square dot paper.



front elevation right elevation plan view

CHALLENGE

**13 a** An apartment block has the following plan view. The blue squares represent three stories high. The green squares represent two stories high. The yellow squares represent one story high. Draw the front and right elevations.




**b** Draw a 3D version of the apartment block on isometric dot paper.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Plans and elevations

**Investigation** Designs for furniture

**Worksheet** Plans and 3D objects

**Topic quiz** 8C

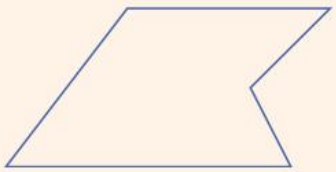
# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

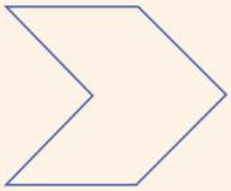
8A **1** For each of the following polygons:

**i** determine if it is regular or irregular
**ii** determine if it is concave or convex
**iii** name the polygon.

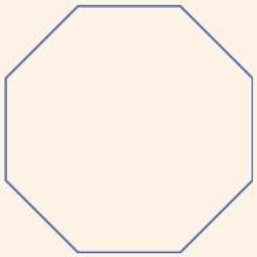
**a**



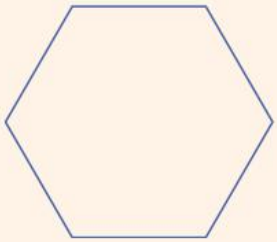
**b**



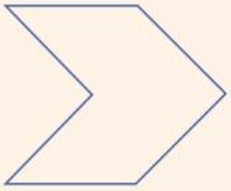
**c**



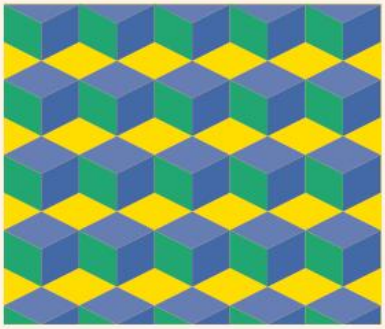
**d**



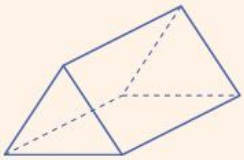
8A **2** Draw a tessellation using the following shape.



8A **3** Identify the 2D shape used in this tessellation.

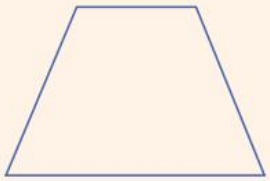


8B **4** Look at this triangular prism.



**a** How many faces does it have?
**b** How many edges does it have?
**c** How many vertices does it have?

8B **5** Use isometric dot paper to draw a right prism with this 2D shape as one of the matching ends.



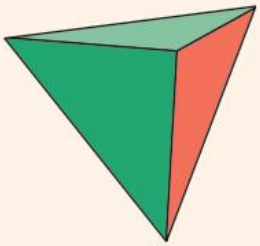
8B **6** Look at each object and give its mathematical name.

**a**

**b**

**c**

8B **7** Use isometric dot paper to draw this 3D object.



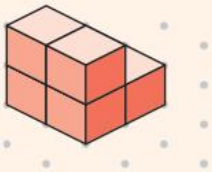
8C **8** Draw a set of a plan and elevations for these objects.

**a**

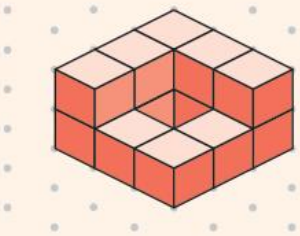
**b**

8C **9** Draw a set of a plan and elevations for each object.

**a**

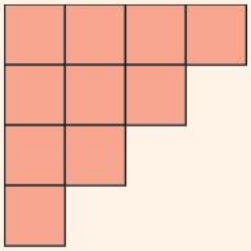


**b**

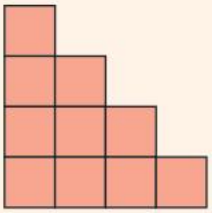


**c**

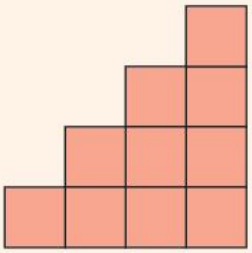
8C **10** Use each set of a plan and elevations to construct the matching 3D object. Draw the objects on isometric or square dot paper.



plan view



front elevation



right elevation

# 8D Translations

## Learning intentions

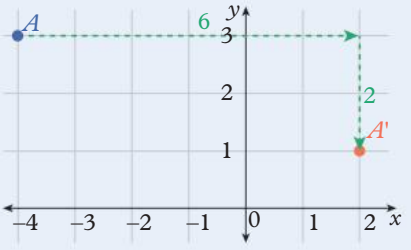
✔ I can describe and perform translations on the Cartesian plane.

**Inter-year links**

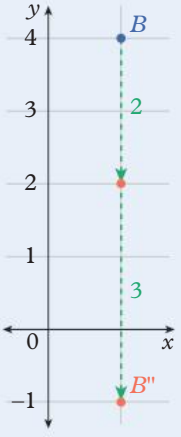
| | |
|---|---|
| Years 5/6 | Transformations |
| Year 8 | 7C Transformations |

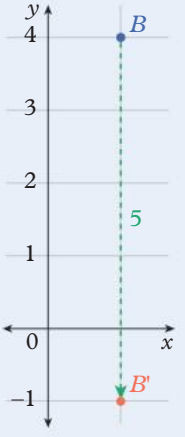
## Translations

- A **translation** is a transformation which moves a point, line or shape without turning or changing the size. Translations in the Cartesian plane are recorded by counting how many units the object slides up or down and left or right.
  - → Translations along the $x$-axis slide the shape left or right.
  - → Translations along the $y$-axis slide the shape up or down.
  - → Point $A$ has been translated 6 units to the right along the $x$-axis and 2 units down the $y$-axis.



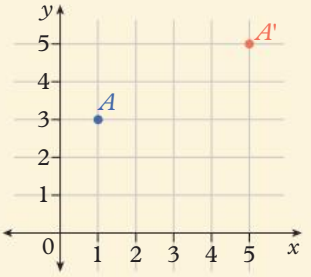

- A transformed object (also called an **image**) should always be given a name that indicates that it is a transformation – usually by adding a small dash to the top right of the original name. For example, the image of point $A$ is called $A'$ ('A prime').
  - → If an image is translated twice, label the second translation with two dashes; for example, $A''$.
- Multiple translations along the same axis can be combined into a single translation.
  - → For example, a translation of 2 units right followed by a translation of 1 unit left is the same as a translation 1 unit right.
  - → For example, a translation of 2 units down followed by a translation of 3 units down is the same as a translation of 5 units down.



## **Example 8D.1** Identifying a translation

Identify the translation shown in the following diagram.

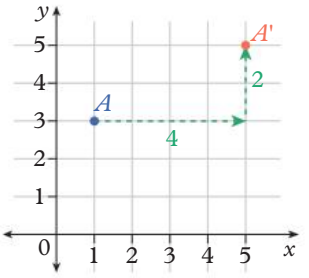



**THINK**

1 Count the number of units the point has been translated right or left.

2 Count the number of units the point has been translated up or down.

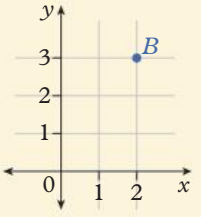
**WRITE**




The point has been translated 4 units right and 2 units up.

## **Example 8D.2** Performing a translation

Translate this point 3 units to the right and 4 units down.



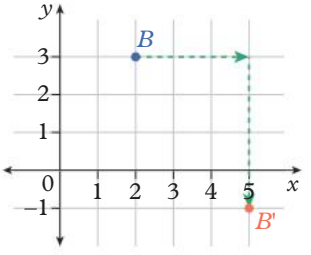

**THINK**

1 Count 3 units to the right.

2 Count 4 units down.

4 Label the translated image $B'$ to show that it is a transformation of the original point.

**WRITE**



## Example 8D.3 Finding coordinates after a translation

A point $P$ is at $(-3, 4)$ on the Cartesian plane. It is translated 6 units down and 10 units to the right. What are the coordinates of the translated point $P'$?

**THINK**

1. If translating to the right, add the units to the $x$-coordinate. If translating to the left, subtract the units from the $x$-coordinate.
2. If translating up, add the units to the $y$-coordinate. If translating down, subtract the units from the $y$-coordinate.

**WRITE**

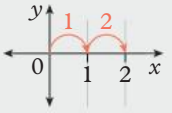
$-3 + 10 = 7$

$4 - 6 = -2$

The coordinates of $P'$ are $(7, -2)$.

**Helpful hints**

When identifying or performing a translation, do not start counting at the starting position. Start counting at the position of the first square beyond the shape.




ANS p589

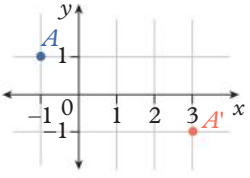
# Exercise 8D Translations

| | | |
|---|---|---|
| 1, 2, 3–5(a–d), 6–9, 10(a, b), 11(b) | 1–3(d–f), 4(e–h), 5(d–f), 6, 7(e, f), 8, 10(c, d, e), 11(a–c), 12(a, b) | 2(e, f), 3(d–f), 4(e–h), 5(f), 6, 8(c), 10–11(e, f), 12(c), 13, 14 |

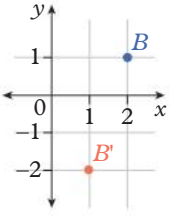
Grids to complete the Exercise 8D translations can be found in the Exercise 8D BLM.

8D.1 **1** Identify the translation shown in the following diagrams.

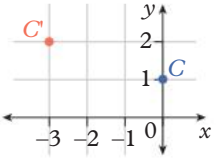
**a**



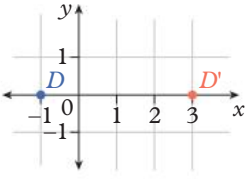

**b**



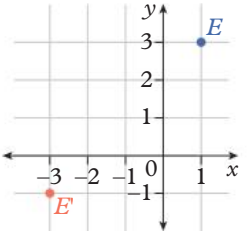

**c**



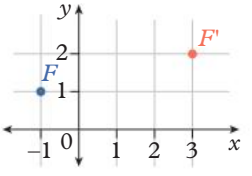

**d**




**e**



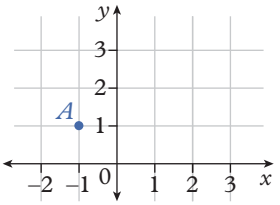

**f**



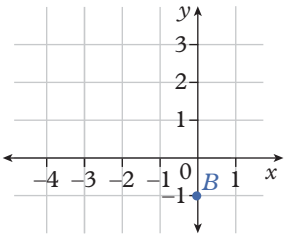

UNDERSTANDING AND FLUENCY

**8D.2** **2** Translate the points by the given amount.

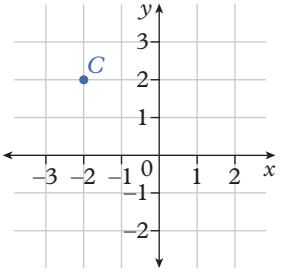
**a** 2 units right and 1 unit up



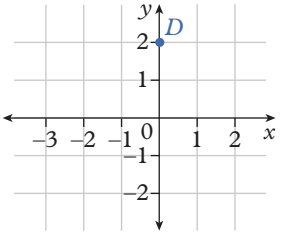

**b** 4 units left and 3 units up



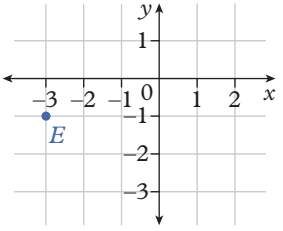

**c** 4 units down



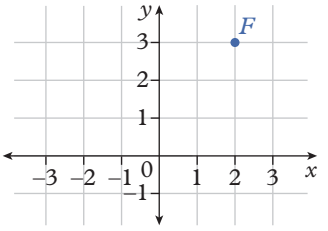

**d** 3 units left




**e** 4 units right and 2 units down




**f** 3 units left and 1 unit down




**8D.3** **3** **a** Point $Q$ is at (2, 1) on the Cartesian plane. It is translated 7 units to the left and 4 units down. What are the coordinates of the translated point $Q'$?

**b** Point $R$ is at (–5, 0) on the Cartesian plane. It is translated 2 units to the right and 5 units up. What are the coordinates of the translated point $R'$?

**c** Point $S$ is at (2, –4) on the Cartesian plane. It is translated 12 units to the left and 4 units up. What are the coordinates of the translated point $S'$?

**d** Point $T$ is at (1, 7) on the Cartesian plane. It is translated 1 unit to the right and 9 units down. What are the coordinates of the translated point $T'$?

**e** Point $U$ is at (–3, –6) on the Cartesian plane. It is translated 8 units to the left and 4 units down. What are the coordinates of the translated point $U'$?

**f** Point $V$ is at (8, –4) on the Cartesian plane. It is translated 3 units to the left and 13 units up. What are the coordinates of the translated point $V'$?

**4** Describe the translation required to move the first point to the second point.

**a** $A(2, 1)$ to $A'(5, 5)$ **b** $B(-3, 3)$ to $B'(0, 4)$ **c** $C(5, 2)$ to $C'(1, -1)$

**d** $D(-2, 0)$ to $D'(2, -2)$ **e** $E(4, -1)$ to $E'(6, -3)$ **f** $F(-2, -3)$ to $F'(3, 3)$

**g** $G(2, -4)$ to $G'(-4, 2)$ **h** $H(0, 6)$ to $H'(-3, -4)$

**5** Which single translation is equal to these combined translations?

**a** 2 units right and 5 units up, followed by 3 units right and 2 units down

**b** 5 units right and 1 unit up, followed by 4 units left and 6 units down

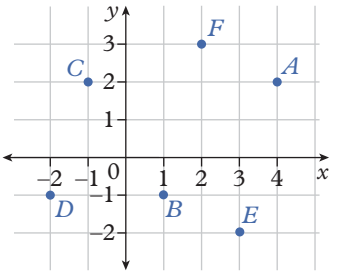
**c** 1 unit left and 2 units down, followed by 2 units left and 8 units down

**d** 4 units right and 3 units up, followed by 9 units right and 3 units up

**e** 2 units right and 4 units down, followed by 2 units right and 4 units up

**f** 7 units left and 1 unit down, followed by 1 unit left and 10 units up

**6** Look at the following Cartesian plane.




**a** Which point has been translated 1 unit right and 4 units up from point *B*?

**b** Which point has been translated 1 unit left and 4 units down from point *A*?

**c** Describe these translations.

**i** point *B* to point *C*

**ii** point *A* to point *B*

**iii** point *D* to point *A*

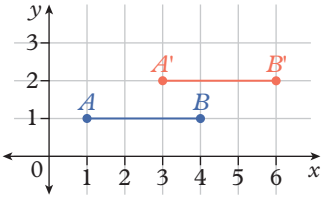
**iv** point *F* to point *E*

**v** point *C* to point *F*

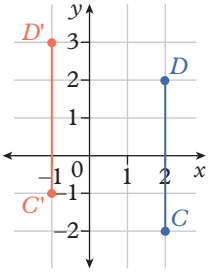
**vi** point *E* to point *B*

**7** Identify the translations shown in the following diagrams.

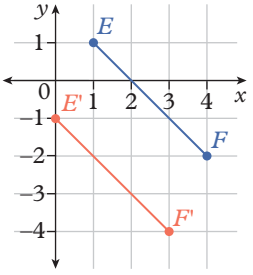
**a**



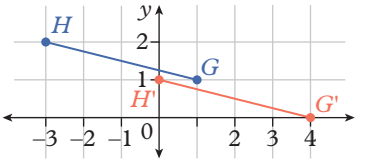

**b**




**c**

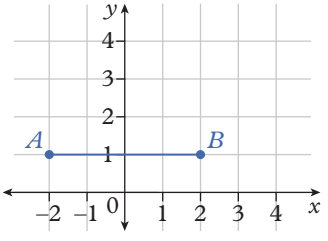



**d**

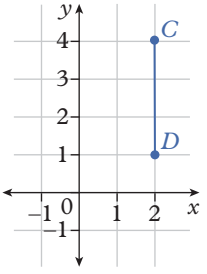



**8** Translate these line segments by the given amounts.

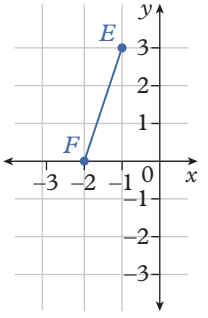
**a** 2 units right and 3 units up



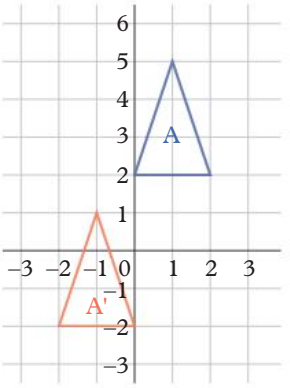

**b** 3 units left and 2 units down




**c** 1 unit right and 3 units down



**9** What translation is shown in the following diagram?




**A** 4 units up and 2 units right

**B** 2 units up and 4 units right

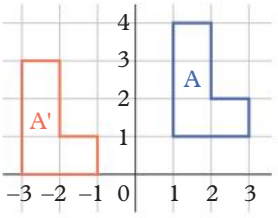
**C** 4 units down and 2 units right

**D** 4 units down and 2 units left

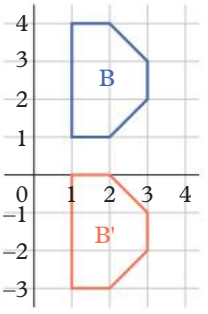
**E** 2 units down and 4 units left

**10** Identify the translations shown in the following diagrams.

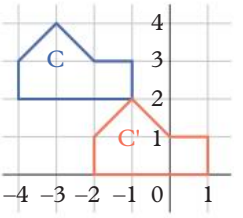
**a**



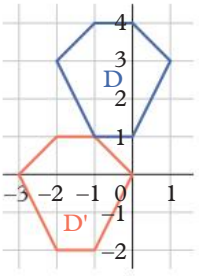

**b**



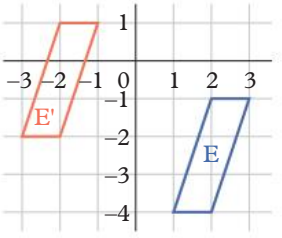

**c**



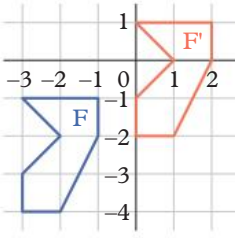

**d**




**e**

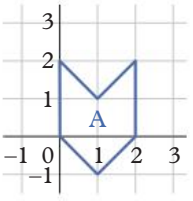



**f**

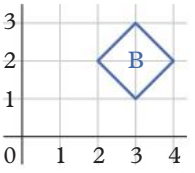



**11** Translate the shapes by the given amount.

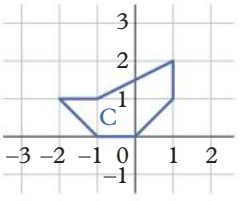
**a** 4 units up and 3 units left



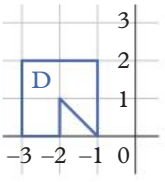

**b** 4 units down and 5 units left



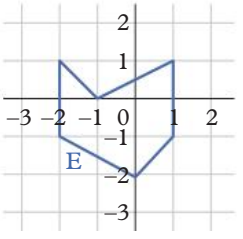

**c** 1 unit down and 1 unit right



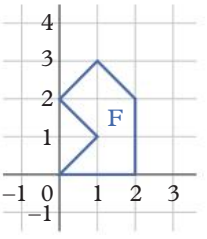

**d** 5 units up and 5 units right



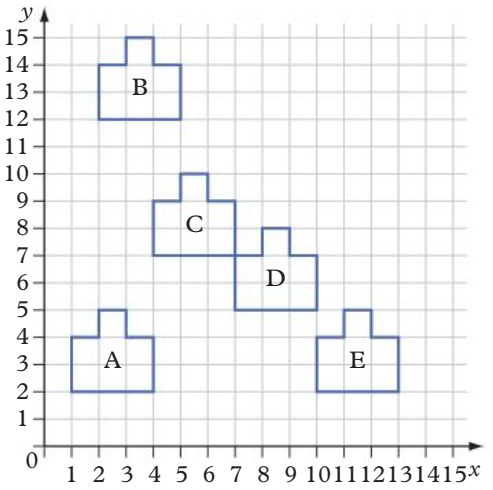

**e** 3 units down and 2 units right




**f** 2 units up and 2 units right



**12** Look at the following shapes on a Cartesian plane.




**a** Which shape has been translated 3 units right and 5 units up from shape A?

**b** Jordan thinks that shape E has been translated 6 units right from shape A. Can you explain where Jordan may have gone wrong?

**c** Describe these translations.

**i** shape B to shape C

**ii** shape B to shape E

**iii** shape D to shape A

**iv** shape E to shape B

**v** shape D to shape C

**vi** shape A to shape E

**13** The figure in question **12** consists of an original shape and four mirror images. It was assumed that shape A was the original shape. Use the clues below to find the real original shape and the final mirror image.

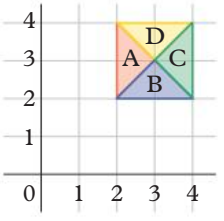
Clue 1: The first image has been translated 3 units to the left and 2 units up from the original figure.

Clue 2: The second image has been translated 6 units to the right and 5 units down from the first image.

Clue 3: The third image has been translated 8 units to the left and 10 units up from the second image.

Clue 4: The final image has been translated 1 unit to the left and 10 units down from the third image.

**14** Using only translations of the four triangles turn this square into a parallelogram which is not a square.




Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Translations

**BLM** Exercise 8D

**Topic quiz** 8D

PROBLEM SOLVING AND REASONING

CHALLENGE

# 8E Rotations and reflections

## Learning intentions

✔ I can perform reflections across an axis on the Cartesian plane.

✔ I can perform rotations of multiples of 90° on the Cartesian plane.

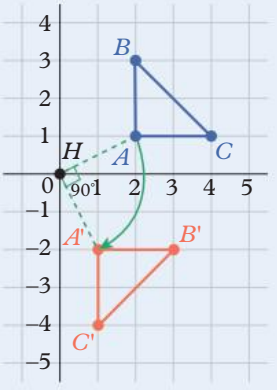
**Inter-year links**

**Years 5/6** Transformations

**Year 8** 7C Transformations

## Rotations

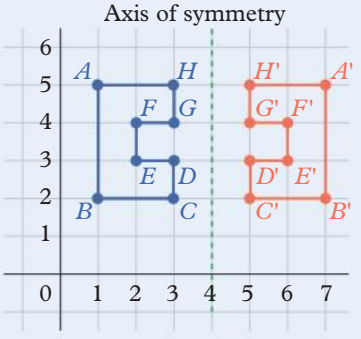
- A **rotation** turns a shape about a fixed point. To perform a rotation, three details are required:
  1. The centre of rotation.
     - Usually the origin or a coordinate.
  2. The angle of the rotation.
     - Between 0° and 360°.
  3. The direction of the rotation.
     - Clockwise or anticlockwise.




- When performing a rotation, identify a single point to concentrate on. This is called the **point of focus**.
  → After rotating one point you may be able to determine the position of the other rotated points in relation to this point. If not, proceed by rotating a second point.

## Reflections

- **Reflections** are performed across an **axis of symmetry**. Each point in the image must be the same distance from the axis of symmetry as the corresponding original point.




- As with rotations, start by reflecting a single point (the point of focus), and continue to reflect single points until you can confidently draw the whole reflected shape.



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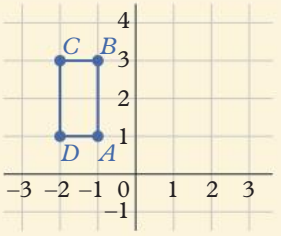
## **Example 8E.1** Performing a rotation on the Cartesian plane

Rotate this shape 90° clockwise around the origin, $O$, and give the coordinates of the image.

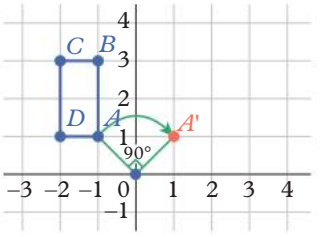


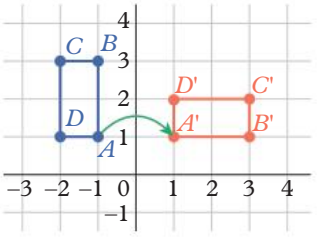

**THINK**

1. Choose a point of focus, $A$.
2. Rotate $A$ by 90° clockwise about the origin. Label the rotated point $A'$.
3. Choose a second point to rotate, $B$. Rotate $B$ to 90° clockwise about the origin and label the rotated point $B'$.
4. If you can, draw the remaining points of the rotated shape. If needed, rotate a second point from the original shape. The rotated shape must be the same shape and size as the original shape.
5. Give the coordinates of the image.

**WRITE**

Point $A$ is the point of focus.

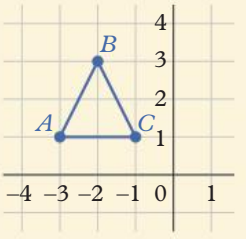



The coordinates of the image are $A'(1, 1)$, $B'(3, 1)$, $C'(3, 2)$ and $D'(1, 2)$.

## **Example 8E.2** Performing a reflection on the Cartesian plane

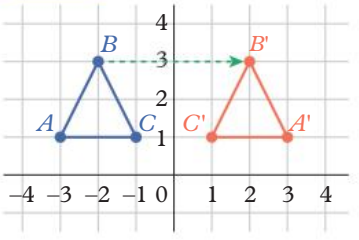
Reflect this shape across the $y$-axis and give the coordinates of the image.




**THINK**

1. Choose a point of focus, $B$.
2. Reflect $B$ across the $y$-axis. Label the reflected point $B'$.
3. Reflect the other points across the $y$-axis and draw the reflected shape.
4. Give the coordinates of the image.

**WRITE**




The coordinates of the image are $A'(3, 1)$, $B'(2, 3)$ and $C'(1, 1)$.

**Helpful hints**

Remember to give your image a name that identifies it as a transformation. For example, for an object B, the image is named B'.

ANS p590

# Exercise 8E Rotations and reflections

△ 2(e, f), 3(d–f), 4(e–h), 5(f), 6, 8(c), 10–11(e, f), 12(c), 13, 14

□ 1–4, 5(c, d, e), 6–8, 11, 14, 15(a–c)

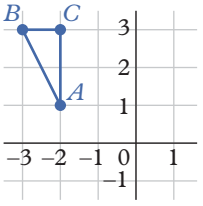
⬠ 1(b, d), 2, 3(b, d), 4, 5(e, f), 6, 7(d), 10, 15–17

UNDERSTANDING AND FLUENCY

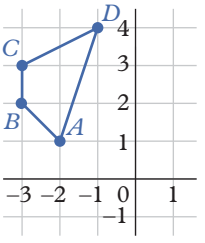
Grids to complete the Exercise 8E rotations and reflections can be found in the Exercise 8E BLM.

8E.1 **1** Rotate each of the shapes 90° clockwise around the origin, *O*, and give the coordinates of the images.

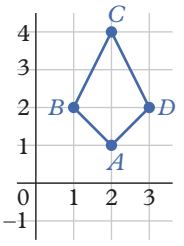
**a**



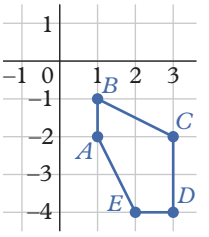

**b**




**c**



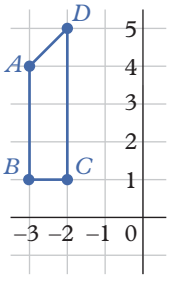

**d**



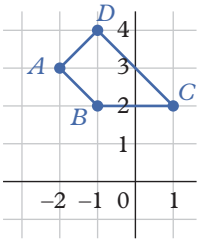

**2** Rotate each of the shapes in question **1** 90° anticlockwise around the origin, *O*, and give the coordinates of the images.

8E.2 **3** Reflect each of the shapes across the *y*-axis and give the coordinates of the images.

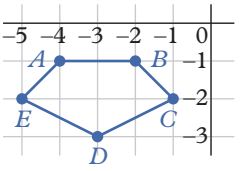
**a**



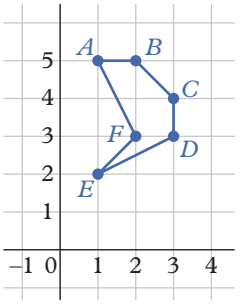

**b**




**c**



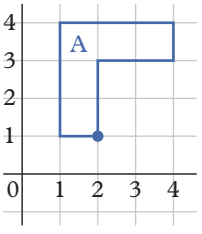

**d**



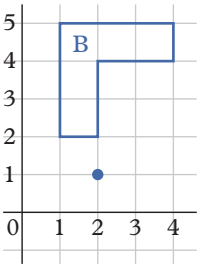

**4** Reflect each of the shapes in question **3** across the *x*-axis and give the coordinates of the images.

**5** Rotate each shape 90° clockwise around the given point.

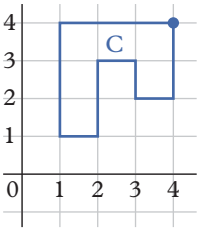
**a**



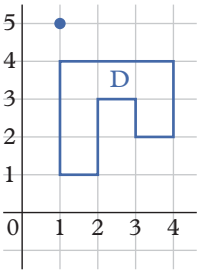

**b**



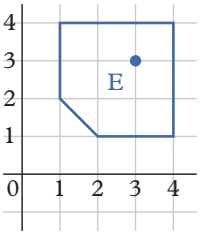

**c**



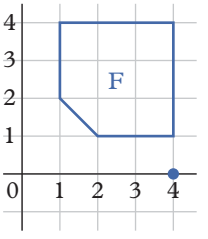

**d**




**e**



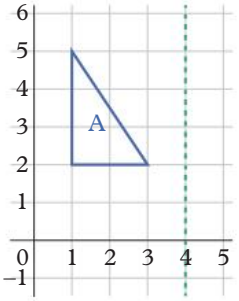

**f**



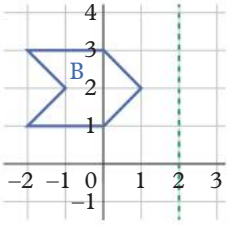
**6** Rotate the shapes in question **5** 180° clockwise around the given point.

**7** Reflect each of these shapes across the axis of symmetry shown.

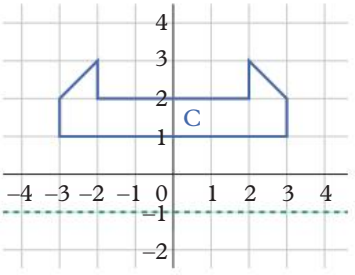
**a**



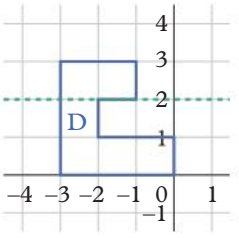

**b**




**c**

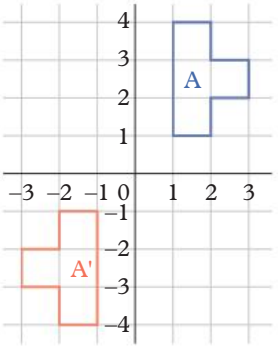



**d**

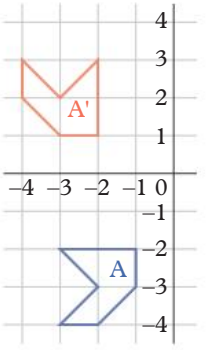



**8** Each of these shapes has been rotated clockwise around the origin. By how many degrees have they been rotated?

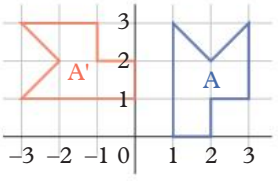
**a**



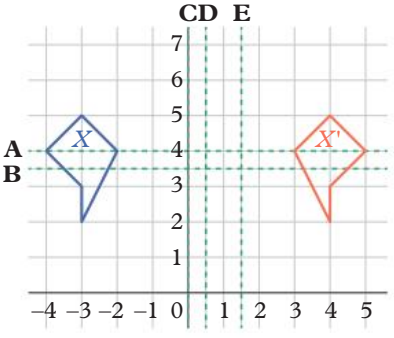

**b**



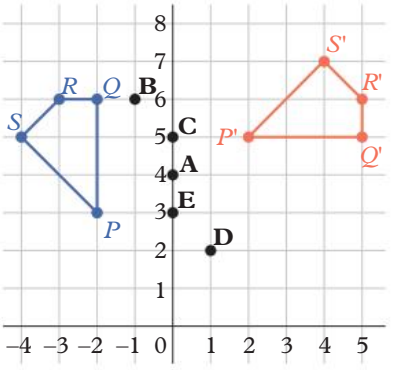

**c**



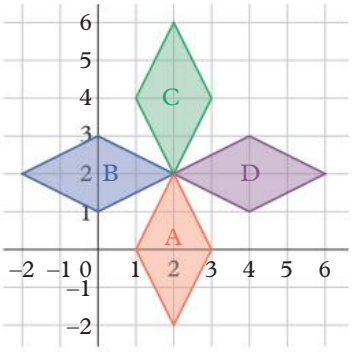

**9** Which of the given lines has the shape been reflected across?



**10** Which of the given points has the shape been rotated 90° clockwise around?

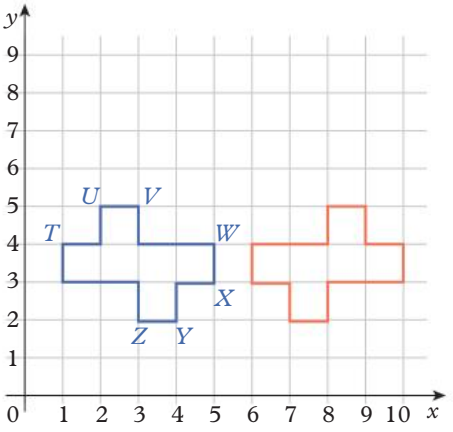



**11** This figure is made up of a number of rotations of one shape. Describe the transformations for the shapes B, C and D given that they are rotations of shape A.

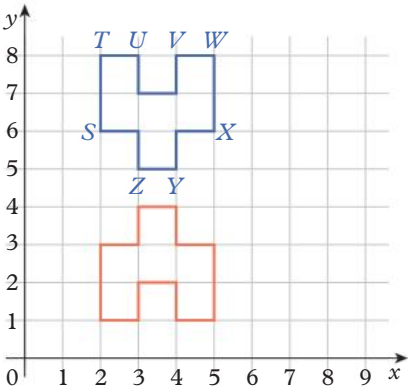



**12** Write the coordinates of the point $V'$ for each of these reflections.

**a**



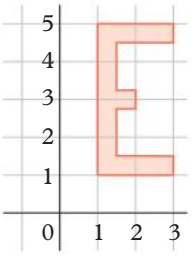

**b**



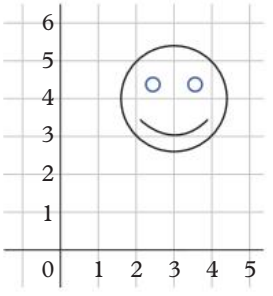



**13** For each of these shapes, one half of the figure has been reflected in an axis of symmetry to create the full shape. Draw the axis of symmetry that has been used.

**a**



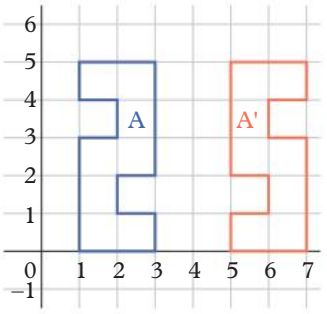
**b**



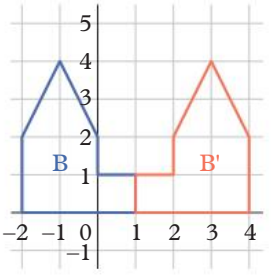
**14** Explain why a rotation of 90° clockwise can also be described as rotation of 270° anticlockwise.

**15** Draw the axis of symmetry for these reflections.

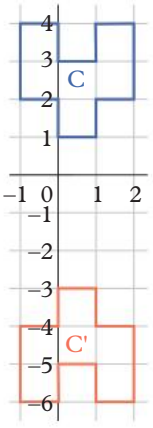
**a**



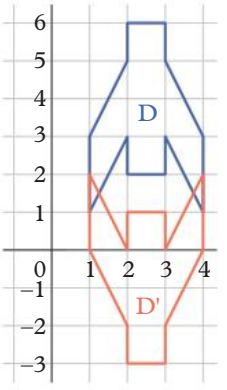

**b**




**c**




**d**




**16 a** Plot the following coordinates on graph paper and join them to make a shape.
(2, 2), (6, 2), (4, 5)

**b** Reflect the shape across the *y*-axis. What are the new coordinates?

**c** Reflect the original shape across the *x*-axis. What are the new coordinates?

**d** Rotate the original shape by 180° around the origin. What are the new coordinates?

**17** Draw a Cartesian plane to 10 on both the *x* and *y* axes and complete the following.

**a** Plot the following two quadrilaterals on the same Cartesian plane.
Vertices of first quadrilateral: (8, 2), (10, 4), (8, 6) and (6, 4)
Vertices of second quadrilateral: (4, 10), (2, 8), (4, 6) and (6, 8)

**b** Draw the line of symmetry between the two quadrilaterals.

**c** What do you notice about the pairs of coordinates that are reflections of each other in this line?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Rotations

**Interactive skillsheet** Reflections

**BLM** Exercise 8E

**Investigation** Tetris

**Topic quiz** 8E

# 8F Combined transformations

## Learning intentions

✔ I can perform combined transformations.

**Inter-year links**

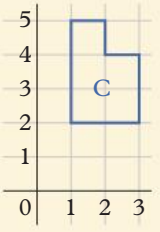
| | |
|---|---|
| Years 5/6 | Transformations |
| Year 8 | 7C Transformations |
| Year 9 | 6D Dilations and similar figures |

## Combined transformations

- Translations, rotations and reflections (across straight lines) are all examples of **isometric transformations**. Isometric transformations do not change the shape or size of a shape or object.
- When performing combined transformations, the order in which the transformations are performed is important. For example, a translation followed by a reflection will probably not result in the same image as the same reflection followed by the same translation.

### Example 8F.1 Performing a combined transformation

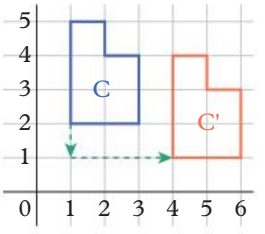
Translate this shape 3 units right and 1 unit down, and then reflect the translated shape in the $x$-axis.

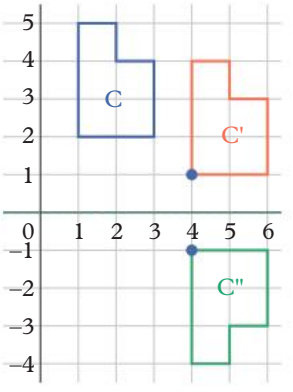



**THINK**

1. Select a point of focus on shape C.
2. Translate the point of focus 3 units to the right and 1 unit down.
3. Redraw the shape starting from the point of focus.
4. Label the new shape C' to show that it is a transformation of the original shape.
5. Reflect the point of focus of C' in the $x$-axis.
6. Reflect the other points of C' in the $x$-axis and draw the reflected shape.
7. Label the final shape C".

**WRITE**



ANS p594

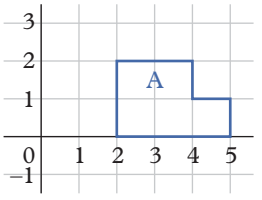
# Exercise 8F Combined transformations

1–3, 5(a–c), 6, 8 | 1–3, 5(d–f), 7, 8 10 | 1, 4, 5(d–f), 7, 9–11

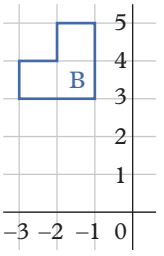
Grids to complete the Exercise 8F transformations can be found in the Exercise 8F BLM.

8F.1 **1** Perform the following combined transformations.

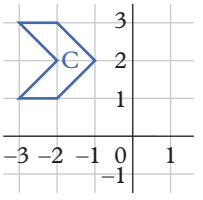
**a** Translate 1 unit left and 3 units up, and then reflect across the *y*-axis.



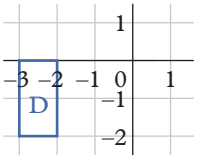

**b** Translate 3 units right and 3 units down, and then reflect across the *x*-axis.



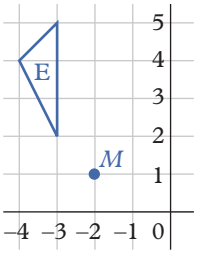

**c** Reflect across the *y*-axis, and then translate 3 units down.



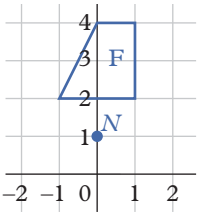

**d** Reflect across the *x*-axis, and then translate 3 units right and 2 units up.




**e** Rotate 180° clockwise about the point *M*, and then translate 2 units right and 1 unit down.




**f** Rotate 90° anticlockwise around the point *N*, and then reflect across the *y*-axis.



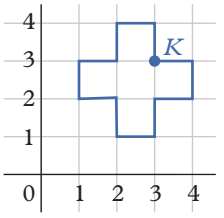

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

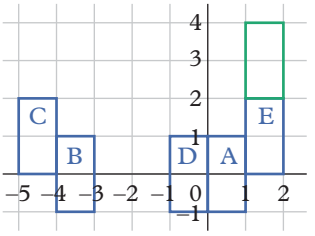
**2** Where will the coordinates of point $K$ be after these two transformations?

→ Reflection across the $y$-axis

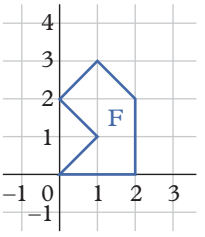
→ Translation of 3 units right and 2 units up




**3** Which is the correct final position of the green rectangle after it has been translated 2 units left and 3 units down, before being reflected across the $y$-axis?




**4** **a** Translate this shape by 3 units right and 3 units up, before reflecting the translated image across the $y$-axis.

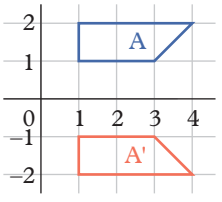



**b** Perform this combined transformation in the opposite order; that is, reflect the shape across the $y$-axis, and then translate the reflected image 3 units right and 3 units up.

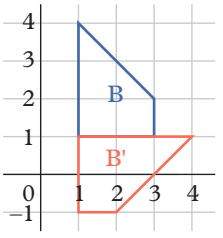
PROBLEM SOLVING AND REASONING

**5** Identify each transformation shown as a single translation, rotation or reflection.

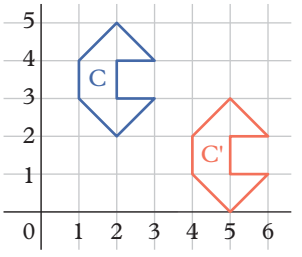
**a**



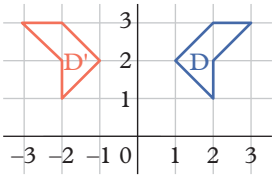

**b**



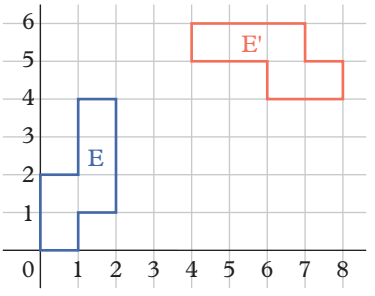

**c**



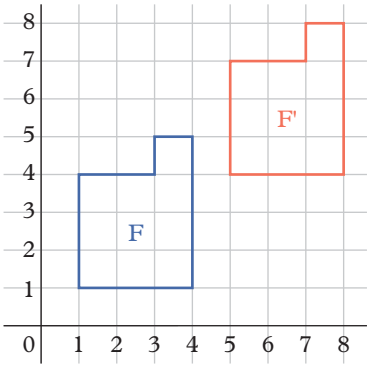

**d**



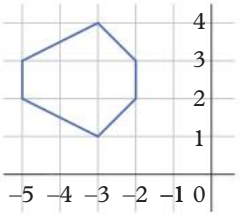

**e**



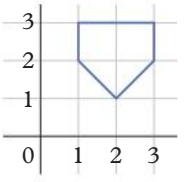

**f**



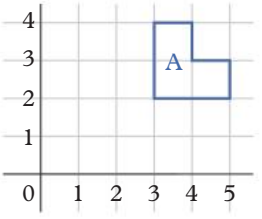
**6** This shape is reflected across the $y$-axis and then reflected across the $x$-axis. What single transformation would produce the same result?




**7** This shape is rotated by 180° around the origin, $O$, and then reflected across the $x$-axis. What single transformation would produce the same result?

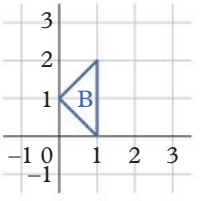



**8** Transform this shape according to the steps listed. Draw each shape.




Translate 3 units to the left and 4 units down, then rotate 90° clockwise from the bottom right corner, then reflect across the $y$-axis. Label the final image A'''.

**9** Transform this shape according to the steps listed. Draw each shape.



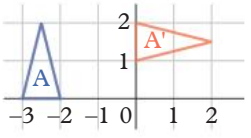

Translate 2 units to the right and 2 units up, then rotate 90° anticlockwise from the bottom vertex, then reflect across the $x$-axis. Label this image B'''.

**10** Explain how two successive reflections can result in the same result as a horizontal translation.

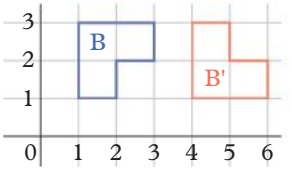


**11** A rotation clockwise around the origin followed by a translation have been performed in the following diagrams. State the transformations.

**a**




**b**




Check your Student obook pro for these digital resources and more:

**BLM** Exercise 8F

**Investigation** Alien walk

**Topic quiz** 8F

# 8G Line and rotational symmetry

## Learning intentions

✔ I can identify lines of symmetry.

✔ I can identify and determine angles of rotational symmetry.

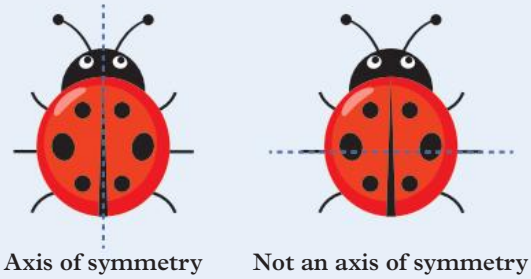
Inter-year links

Years 5/6 Symmetry

Year 8 7D Congruence

## Line symmetry

- An object has **line symmetry** if it can be divided into two halves that are mirror reflections of each other.
- The line about which an object is symmetrical is called an axis of symmetry (or line of symmetry).
- The **order of line symmetry** is the number of axes of symmetry for a shape.
  For example, this ladybird has one axis of symmetry, so the order of line symmetry is 1.

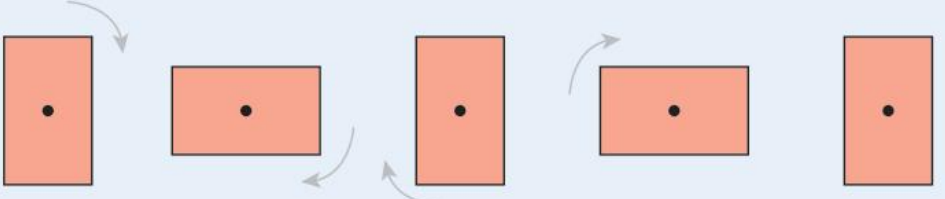


Axis of symmetry  Not an axis of symmetry

- Objects can have none, one or many axes of symmetry.

## Rotational symmetry

- An object has **rotational symmetry** if it fits onto exactly itself after being rotated less than 360°.
- The **order of rotational symmetry** is the number of times an object fits exactly onto itself when being rotated through 360°.
- Objects with no rotational symmetry have an order of rotational symmetry of 1.



Original shape  90 degrees  Original = 180 degrees  270 degrees  Original = 360 degrees

All rectangles have an order of rotational symmetry of 2.



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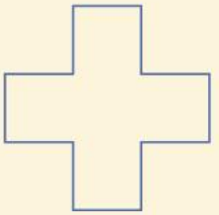
## **Example 8G.1** Finding the order of line symmetry

What is the order of line symmetry for this shape?



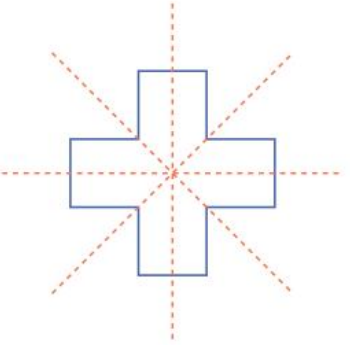
**THINK**

1 Draw all axes of symmetry. Consider horizontal, vertical, diagonal axes of symmetry. Each side of the axis must be a mirror image.

2 Count the number of axes of symmetry.

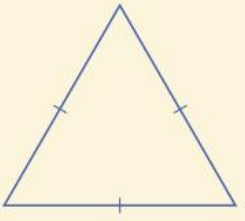
**WRITE**

The order of line symmetry for this shape is four.



## **Example 8G.2** Finding the order of rotational symmetry

What is the order of rotational symmetry for this shape?



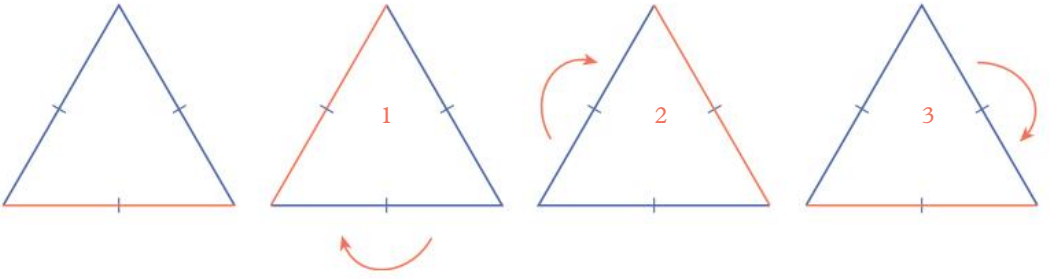
**THINK**

1 Rotate the triangle until it looks exactly the same as when you started.

2 Count the number of times the shape looks exactly the same in a 360° rotation.

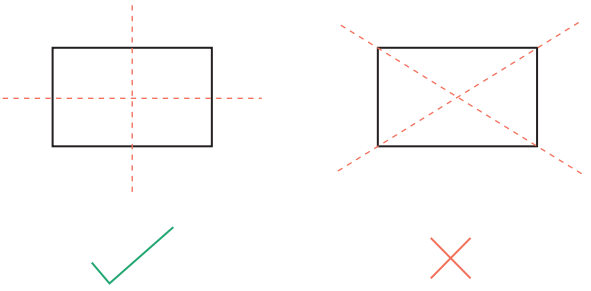
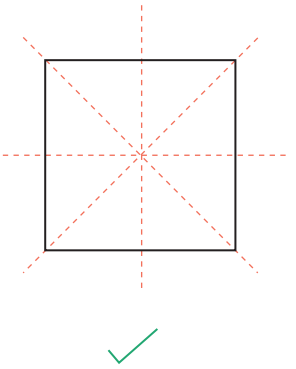
**WRITE**

The order of rotational symmetry for this shape is 3.



**Helpful hints**

Be careful with diagonal lines of symmetry! Many people mistakenly think that the diagonals of rectangles and parallelograms are axes of symmetry when they are not.

| Rectangles | Squares |
|---|---|
|  |  |

ANS p595

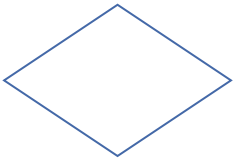
# Exercise 8G Line and rotational symmetry

1, 2, 4, 7, 8, 10, 11 | 1, 3, 4, 6–9, 12, 14, 16 | 1, 4, 7, 8, 13, 15, 17–19

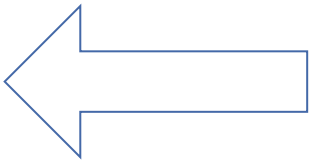
UNDERSTANDING AND FLUENCY

**8G.1** **1** What is the order of line symmetry for each shape?

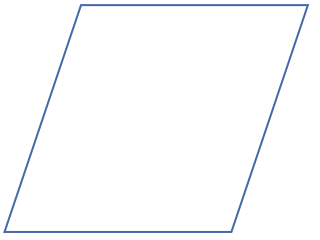
**a**



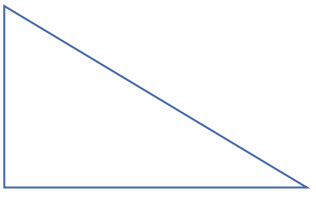
**b**



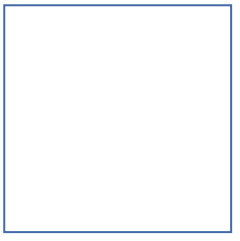
**c**



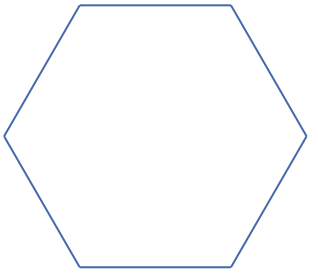
**d**



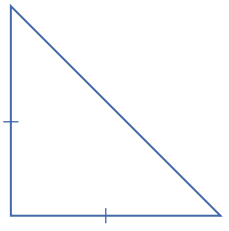
**e**



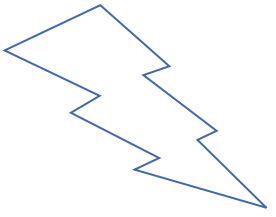
**f**



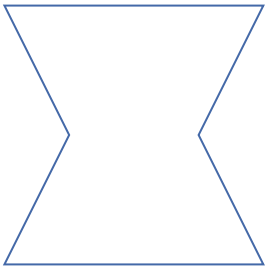
**g**



**h**



**i**



**2** Consider the 26 letters of the alphabet in upper case.

**a** Find the axis or axes of symmetry for each letter (if they exist). Consider the letter O to be a circle.

**b** Which letters have an order of line symmetry of 0?

**c** Which letters have an order of line symmetry of 2 or more?

**3** What is the order of line symmetry for each object?

**a**

**b**

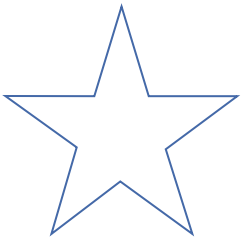
**c**

**d**

**e**

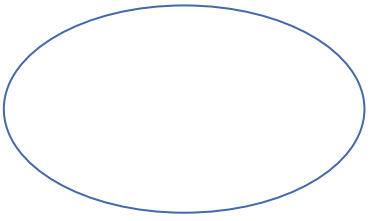
**8G.2** **4** What is the order of rotational symmetry for each shape?

**a**

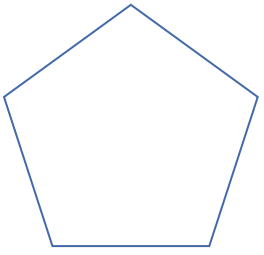


**b**

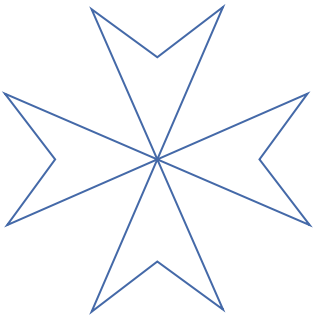
**c**



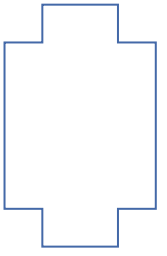
**d**



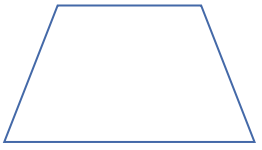
**e**



**f**

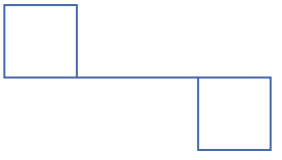


**g**



**h**

**i**



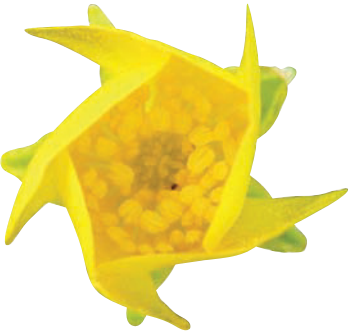
UNDERSTANDING AND FLUENCY

**5** Find the order of rotational symmetry for the figures below.

**a**

**b**

**c**



**6** What letters of the alphabet (in upper case) have an order of rotational symmetry that is 2 or more? List these with their orders of rotational symmetry. Consider the letter O to be a circle.

**7** Draw a shape that has order of line symmetry of:

**a** 2 **b** 4

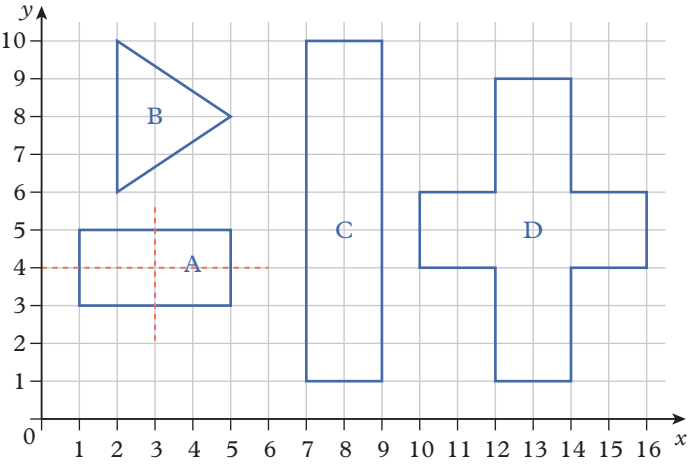
**8** Draw a shape that has order of rotational symmetry of:

**a** 2 **b** 4

PROBLEM SOLVING AND REASONING

**9** John claims that because an equilateral triangle has an order of line symmetry of 3, as well as an order of rotational symmetry of 3, line symmetry and rotational symmetry are the same. Explain why he is incorrect, using an example in your explanation.

**10** The Cartesian plane can be used to help identify vertical and horizontal lines of symmetry. Shape A has a vertical line of symmetry that runs through 3 on the $x$-axis and a horizontal line of symmetry that runs through 4 on the $y$-axis. Find the lines of symmetry for the other three shapes.




**11** An example of an uppercase word which has symmetry is 'CHECKBOOK'.

**a** What kind of symmetry does 'CHECKBOOK' have?

**b** Write three other words with this kind of symmetry.

**12** Another example of an uppercase word which has an axis of symmetry is 'MUM'.

**a** What kind of symmetry does 'MUM' have?

**b** Does this symmetry also exist in lowercase letters?

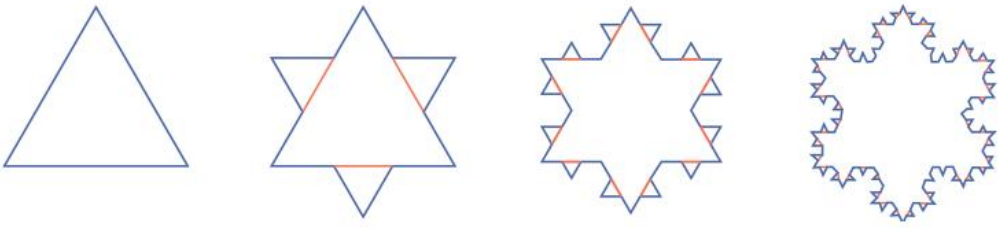
**c** Write three other words with this kind of symmetry.

**13** What kind of symmetry does the word 'SWIMS' have? Find three other words with this kind of symmetry.

**14 a** What is the order of line symmetry of a circle? Explain your answer.

**b** What is the order of rotational symmetry of a circle? Explain your answer.

**15** The Koch snowflake is a fractal curve. It is built up in a sequence of stages, beginning with an equilateral triangle.



**a** How many lines of symmetry does the first triangle have?

**b** How many lines of symmetry does the fourth snowflake have?

**c** What can you say about the number of lines of symmetry as the snowflake gets more sides? Explain your answer.

**16** Use centimetre graph paper to complete the following.

**a** Plot the following coordinates on graph paper.

(5, 2), (5, 8), (3, 8)

**b** Add one pair of coordinates to make a quadrilateral that has a vertical line of symmetry.

**17** Use centimetre graph paper to complete the following.

**a** Plot the following coordinates on graph paper.

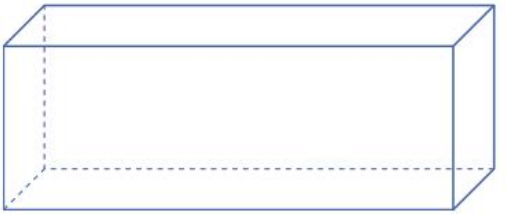
(2, 2), (5, 4), (2, 8)

**b** Add one pair of coordinates to make a quadrilateral that has a horizontal line of symmetry.

**18** Symmetry can also be applied to 3D objects, although 3D objects have planes of symmetry instead of axes of symmetry. Consider this rectangular prism.

**a** How many axes of symmetry does a rectangle have?

**b** Draw a diagram of a rectangular prism showing all the planes of symmetry.



**19 a** How many planes of symmetry does a cube have?

**b** Draw a diagram showing all the planes of symmetry of a cube.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Line symmetry

**Interactive skillsheet** Rotational symmetry

**Investigation** Flags of the world

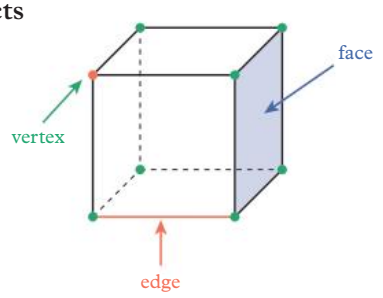
**Topic quiz** 8G

# Chapter summary

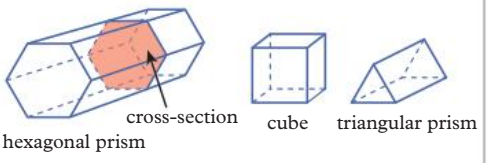
## Polygons

polygons non-polygons

## 3D objects



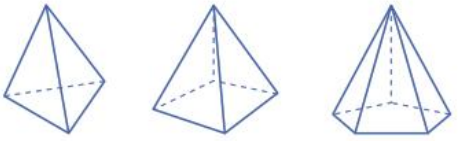

## Prisms




## Non-polyhedra

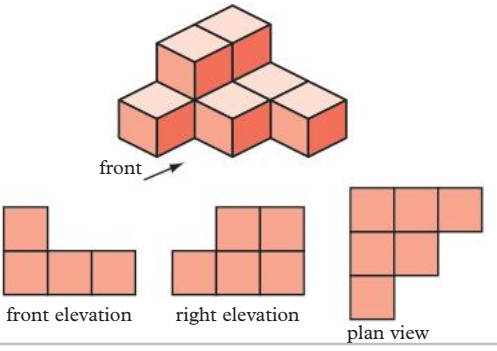
sphere cone cylinder

## Pyramids

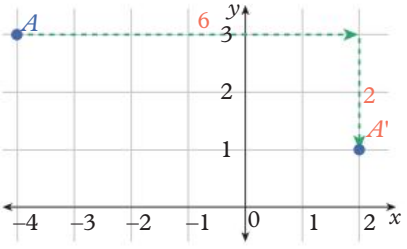


triangular-based pyramid square-based pyramid pentagonal-based pyramid

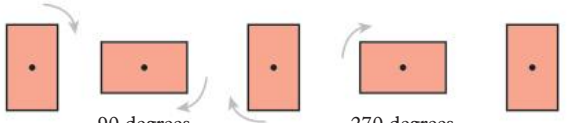
## Plans and elevations




## Translations

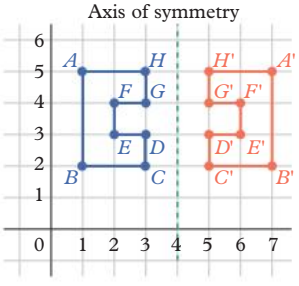



## Rotational symmetry

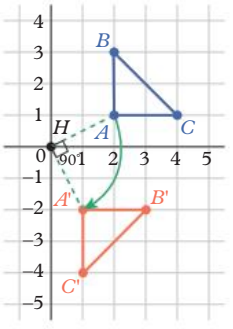


Original shape 90 degrees Original = 180 degrees 270 degrees Original = 360 degrees

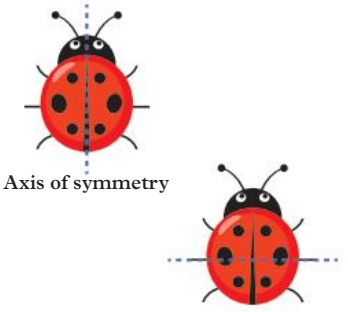
## Reflections




## Rotations




## Line symmetry




Not an axis of symmetry

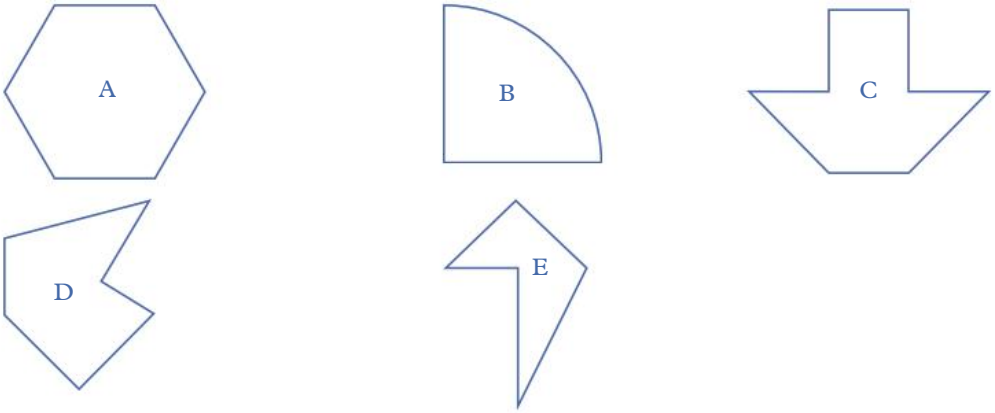
# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

Use Figures A–E to answer questions **1** and **2**.



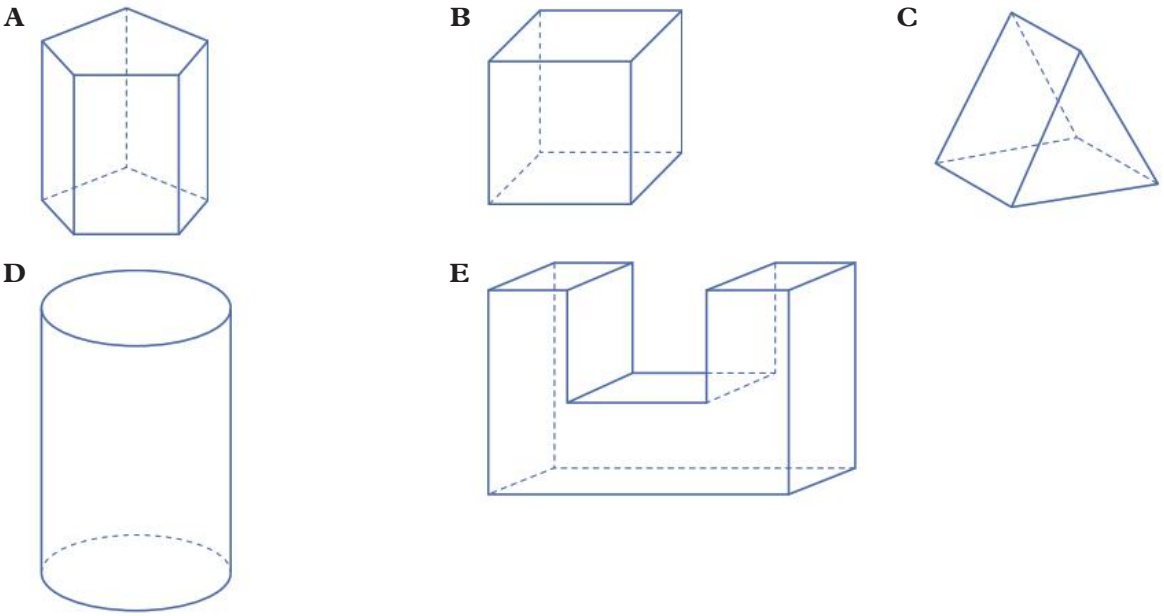

8A **1** Which shape is not a polygon?

**A** A **B** B **C** C **D** D **E** E

8A **2** Which shape is a pentagon?

**A** A **B** B **C** C **D** D **E** E

Use Figures A–E to answer questions **3** and **4**.



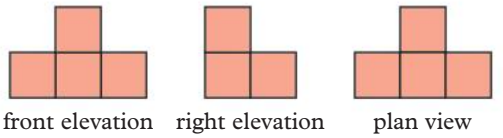

8B **3** Which of the 3D objects above is not a prism?

**A** **B** **C** **D** **E**

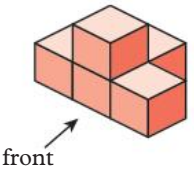
8B **4** Which of the 3D objects above is not a polyhedron?

**A** **B** **C** **D** **E**

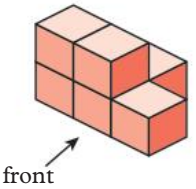
**8C** **5** Which 3D object could this set of a plan and elevations represent?



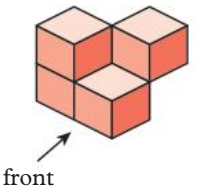

**A**



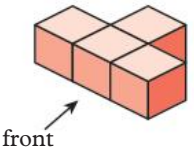

**B**



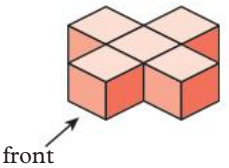

**C**



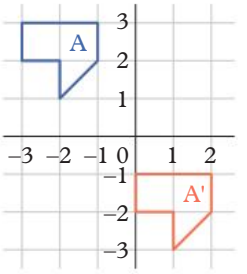

**D**




**E**

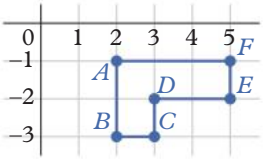



**8D** **6** What is the translation shown in this diagram?



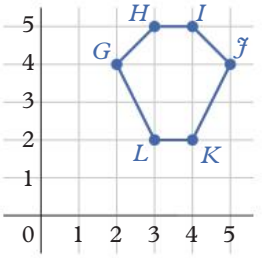

**A** 4 units down and 5 units right
**B** 5 units down and 6 units right
**C** 6 units down and 5 units right
**D** 3 units down and 4 units right
**E** 4 units down and 3 units right

**8E** **7** What will be the coordinates of point $A'$ if this shape is rotated 90° clockwise around the origin?



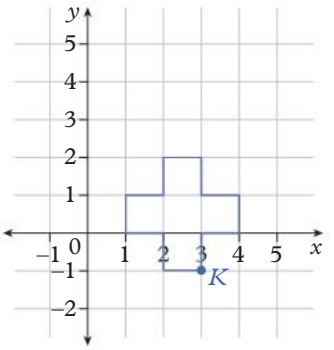

**A** (1, 2)
**B** (−1, 2)
**C** (−1, −2)
**D** (1, −2)
**E** (−2, −1)

**8E** **8** What will be the coordinates of point $J'$ if this shape is reflected across the $y$-axis?




**A** (−5, 4)
**B** (4, −5)
**C** (5, −4)
**D** (−5, −4)
**E** (−4, 5)

**8F** **9** This shape is translated 3 units to the left and 4 units down, and then the translated image is reflected across the $x$-axis. What are the coordinates of vertex $K''$ in the final image?




**A** (5, 0)
**B** (0, 5)
**C** (0, 3)
**D** (0, −5)
**E** (−1, 4)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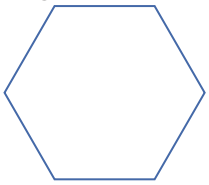
8D **10** What is the order of line symmetry for this hexagon?



**A** 1 **B** 2 **C** 4 **D** 6 **E** infinite number

8G **11** What is the order of rotational symmetry for the regular hexagon in question **10**?

**A** 1 **B** 2 **C** 4 **D** 6 **E** infinite number

## Short answer

Grids to complete the chapter review transformations can be found in the Chapter Review BLM.

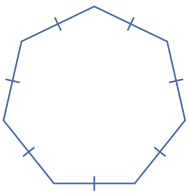
8A **1** Consider each of the following polygons.

**i** State whether it is regular or irregular.

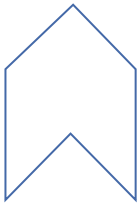
**ii** Write the number of sides it has.

**iii** Does it have an internal reflex angle?

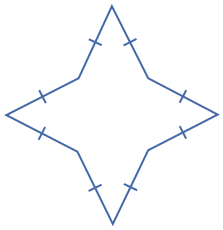
**a**



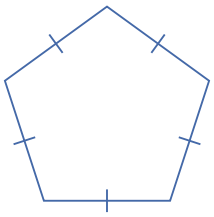
**b**



**c**



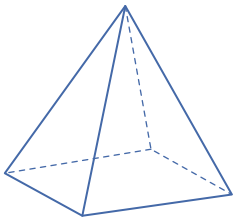
**d**



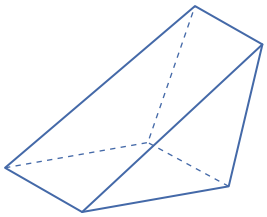
8A **2** Give each of the polygons in question **1** a name using your answer to parts **i**, **ii** and **iii**.

8B **3** Are these 3D objects prisms?

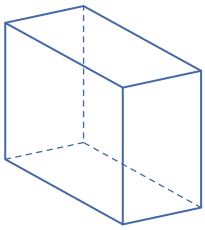
**a**



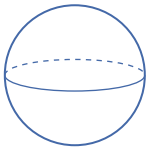
**b**



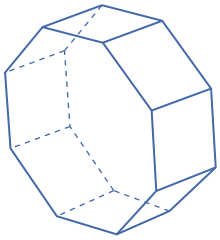
**c**



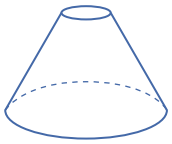
**d**



**e**

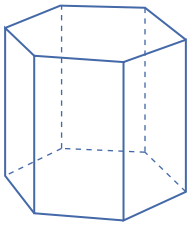


**f**

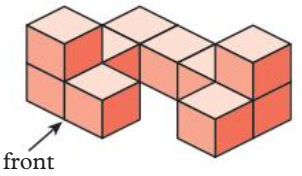


8B **4** Name each of the prisms from question **3**.

8C **5** Draw a set of a plan and elevations for the following prism.

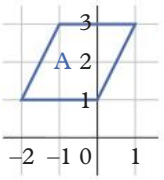


**8C** **6** Draw a set of a plan and elevations for this 3D object.

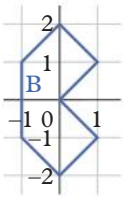



**8D** **7** Translate each of the shapes by the given amounts.

**a** 4 units down and 1 unit left

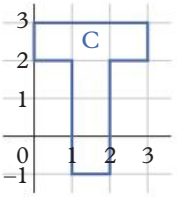



**b** 3 units up and 2 units left

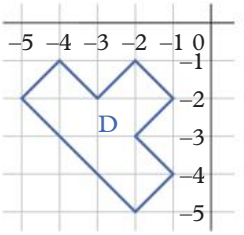



BLM
Chapter 8 review

**c** 1 unit up and 3 units right




**d** 2 units up and 5 units right

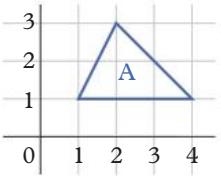



**8D** **8** Describe the translation required to move the first point to the second point.

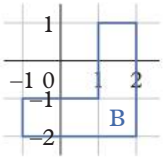
**a** $A(7, 3)$ to $A'(6, 1)$
**b** $B(1, -2)$ to $B'(1, 5)$
**c** $C(-3, -2)$ to $C'(2, -1)$
**d** $D(0, -1)$ to $D'(4, 5)$
**e** $E(3, 1)$ to $E'(-2, 4)$
**f** $F(8, -4)$ to $F'(-1, -5)$

**8E** **9** Rotate each of these shapes 90° clockwise around the origin.

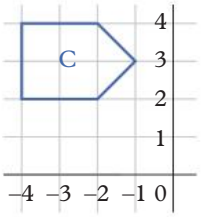
**a**



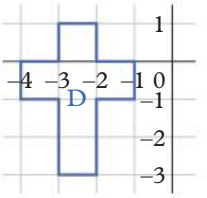

**b**




**c**



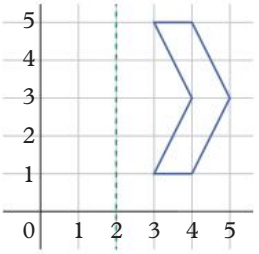

**d**



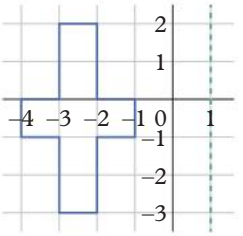

**8E** **10** Rotate each of the shapes in question **9** 180° anticlockwise around the origin.

**8E** **11** Reflect each of these shapes across the given axis of symmetry.

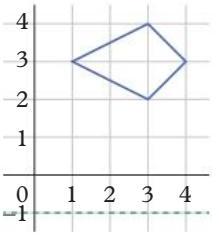
**a**



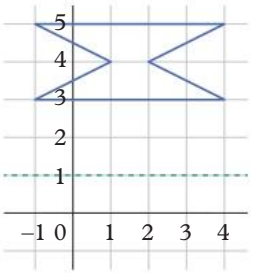
**b**



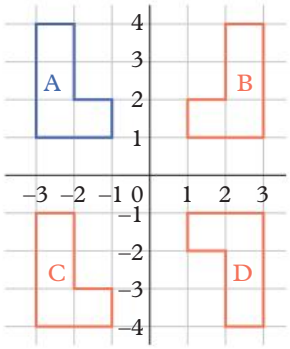
**c**



**d**



8F **12** Consider the shapes shown on this grid. Describe the single transformation that produces each of the shapes B to D if the original shape is A.

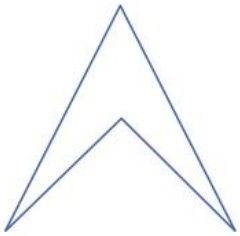



8G **13** How many axes of symmetry do each of these shapes have?

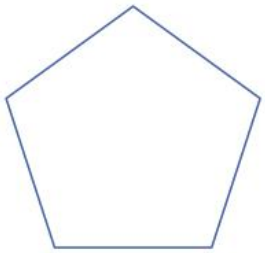
**a**

**b**

**c**



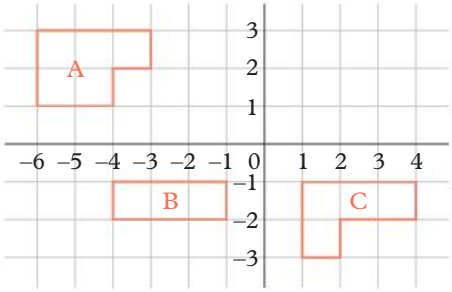
**d**



8G **14** What is the order of rotational symmetry of each of the shapes from question **13**?

## Analysis

**a** Transform each of the following shapes as instructed.




A: reflect in the *y*-axis

B: translate 5 units up and 7 units right

C: rotate 90° anticlockwise around the origin

**b** Together the images from the transformations form a polygon. State the vertices of the polygon that has been formed.

**c** **i** Is the polygon regular or irregular?

**ii** Is the polygon convex or concave?

**d** Use isometric dot paper to draw a prism, with the shape formed in part **a** being the base of the prism.

**e** **i** How many faces does your prism have?

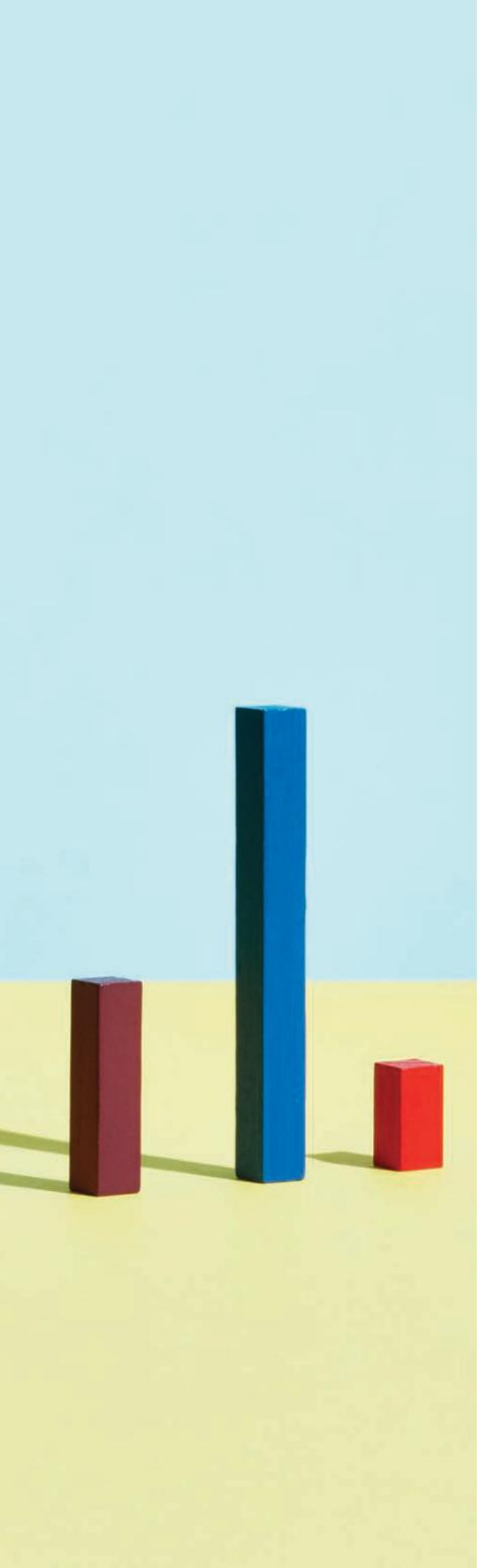
**ii** How many edges does your prism have?

**iii** How many vertices does your prism have?

**f** Draw a set of plan and elevations for your prism.

# 9

# Length, area and volume



## Index


**9A** Length and perimeter
**9B** Area of a rectangle
**9C** Area of a parallelogram
**9D** Area of a triangle
**9E** Surface area
**9F** Volume and capacity


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-text, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Perimeter using a square grid
- ✔ Area using a square grid
- ✔ Volume using cubes
- ✔ Identifying shapes within composite shapes
- ✔ Multiplying and dividing by powers of 10

## Curriculum links

- ✔ Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (VCMMG258)
- ✔ Calculate volumes of rectangular prisms (VCMMG259)

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## Materials

- ✔ Calculator
- ✔ Grid paper

# 9A Length and perimeter

## Learning intentions

✔ I can convert between metric units of length.
✔ I can calculate the perimeter of 2D shapes.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Length and perimeter |
| Year 8 | 8A Length and perimeter |
| Year 9 | 6A Area of composite shapes |

## Metric units

- The metric system is based on place value. Metric prefixes are added to units to describe different values.

| **Place value** | thousands | hundreds | tens | units | tenths | hundredths | thousandths |
|---|---|---|---|---|---|---|---|
| **Value** | 1000 | 100 | 10 | 1 | 0.1 | 0.01 | 0.001 |
| **Prefix** | kilo- | hecto- | deca- | N/A | deci- | centi- | milli- |
| **Abbreviation** | k- | h- | da- | N/A | d- | c- | m- |

## Length

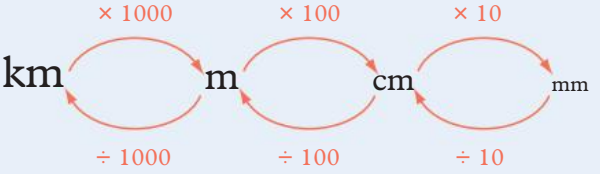
- Length is a measure of distance. To specify a length, you need a numerical value as well as a unit of length, for example, 3 kilometres.

| **Metric units of length** | |
|---|---|
| **Name** | **Abbreviation** |
| millimetre | mm |
| centimetre | cm |
| metre | m |
| kilometre | km |

- To convert between units of length, multiply or divide by the appropriate **conversion factor**.

  Conversion factors:

  1 cm = 10 mm
  1 m = 100 cm
  1 km = 1000 m

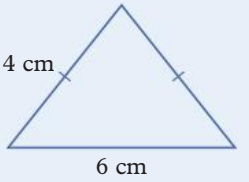



  For example,

  5 km = 5 × 1000
  = 5000 m

## Perimeter

- The **perimeter** of a two-dimensional (2D) shape is the length of the shape's boundary.
- To calculate the perimeter of a shape, add the lengths of each side.
  For example, the perimeter of this triangle is calculated as: 4 + 4 + 6 = 14 cm.
  → All length measurements must be in the same unit before being added.
  → Sides of equal length are marked using one or more hatch marks.





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## **Example 9A.1** Converting between units of length

Complete these length conversions.

**a** 5.4 km = _____ m **b** 900 mm = _____ cm

**THINK**

**a** Identify the appropriate conversion factor. To convert from a larger to a smaller unit, *multiply* the length by the conversion factor.

**b** Identify the appropriate conversion factor. To convert from a smaller to a larger unit, *divide* the length by the conversion factor.

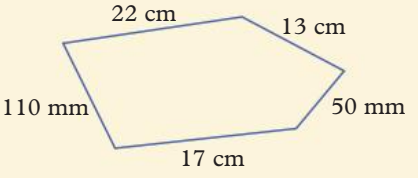
**WRITE**

**a** 1 km = 1000 m
5.4 km = 5.4 × 1000
= 5400 m

**b** 1 cm = 10 mm
900 mm = 900 ÷ 10
= 90 cm

## **Example 9A.2** Calculating perimeter

Calculate the perimeter of this shape in centimetres.




**THINK**

1 Convert all measurements into the unit specified by the question. In this case it is centimetres.

2 Write out the sum of the side lengths. There are five sides to the shape so there should be five lengths in the sum.

3 Add all the lengths together. Include the appropriate unit in the answer.

**WRITE**

110 mm = 110 ÷ 10
= 11 cm
50 mm = 50 ÷ 10
= 5 cm
perimeter = 22 + 13 + 5 + 17 + 11
= 68 cm

**Helpful hints**

✔ To specify a length, you must include a unit of length as well as the numerical value. Make sure that your answers contain the correct units of length.

✔ Once the units are the same, we can leave them out of calculations. However, you must remember to include them in your final answer.

ANS p599

# Exercise 9A Length and perimeter

1(1st column), 2(a, b, e), 3(1st, 2nd columns), 4(a, c, f), 5, 6(a, c, e), 7(a, c, e), 8(a, b), 11, 12(a–c), 16(a–c), 17

1(g–l), 2(b, e, f), 3(2nd column), 4(d–f), 5(c), 6(d–f), 7(e, f), 8(c, d), 9, 11, 12(c, g), 13, 18, 19, 22, 23

2(d, f), 3(2nd column), 4(f), 6(d–f), 7(f), 8(d), 10, 11, 14, 15, 20–24

UNDERSTANDING AND FLUENCY

9A.1 **1** Complete these length conversions.

**a** 8 km = _____ m
**b** 3 m = _____ cm
**c** 700 mm = _____ cm
**d** 500 cm = _____ m
**e** 16 cm = _____ mm
**f** 2000 m = _____ km
**g** 3.5 m = _____ cm
**h** 7.9 mm = _____ cm
**i** 0.06 km = _____ m
**j** 0.47 cm = _____ mm
**k** 4310 mm = _____ km
**l** 51.2 cm = _____ m

**2** Complete these length conversions.

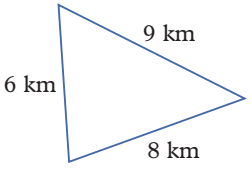
**a** 4 km = _____ m = _____ cm
**b** 12 000 mm = _____ cm = _____ m
**c** 73 m = _____ cm = _____ mm
**d** 8400 cm = _____ m = _____ km
**e** _____ mm = 250 cm = _____ m
**f** _____ cm = 362 m = _____ km

**3** Convert these lengths to the units shown in brackets.

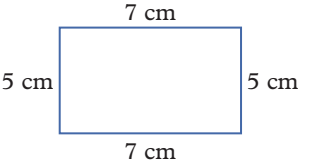
**a** 55 cm (mm)
**b** 27 000 m (km)
**c** 41 mm (cm)
**d** 6.2 m (cm)
**e** 3.6 km (m)
**f** 89 cm (m)
**g** 1.7 km (cm)
**h** 500 mm (m)
**i** 7000 cm (km)
**j** 3.65 m (mm)
**k** 0.94 km (mm)
**l** 1800 mm (km)

**4** Calculate the perimeter of each shape.

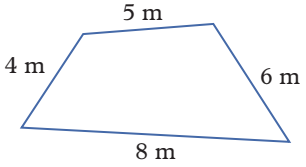
**a**



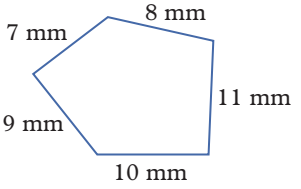

**b**



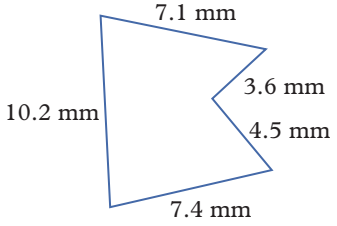

**c**



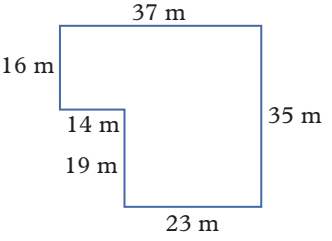

**d**




**e**




**f**





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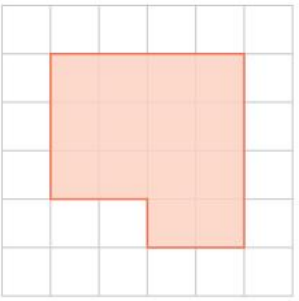
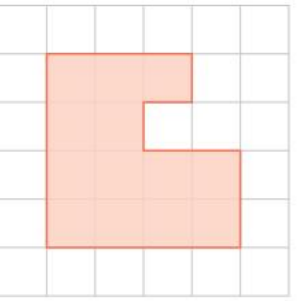
**5** Calculate the perimeter of these shapes. Assume each square in the grid has a length of 1 cm.

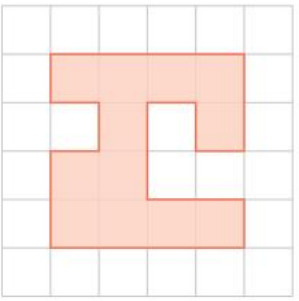
**a**



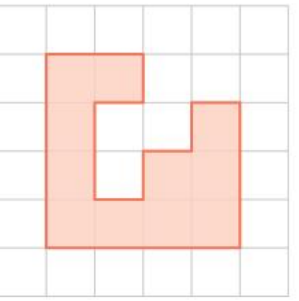
**b**



**c**

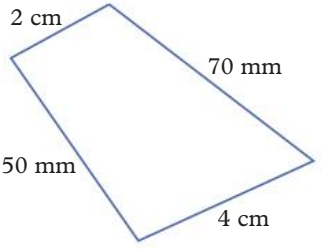


**d**

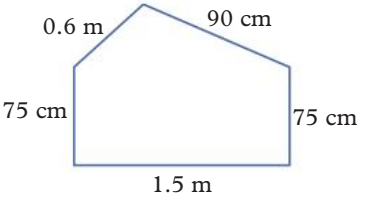


9A.2 **6** Calculate the perimeter of each shape in centimetres.

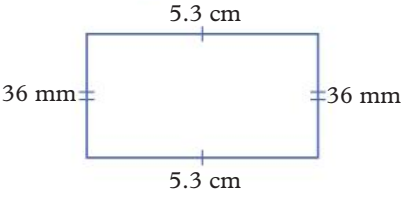
**a**



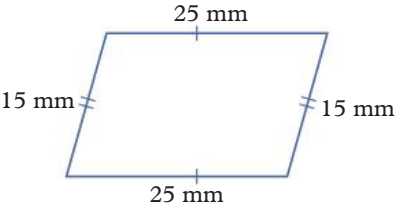

**b**



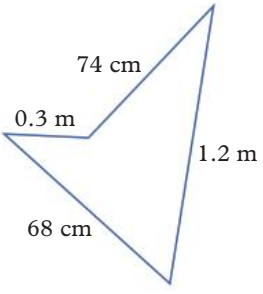

**c**



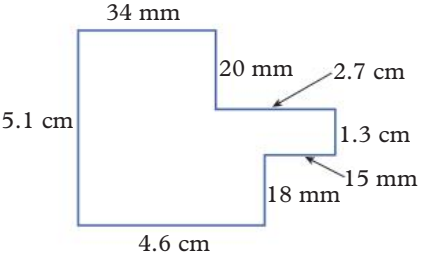

**d**




**e**




**f**




**7** List the following length measurements in ascending order, from smallest to largest.

**a** 5 cm, 15 mm, 60 mm

**b** 2 m, 190 cm, 210 mm

**c** 0.7 km, 701 m, 7001 cm

**d** 82 mm, 8 cm, 0.8 m

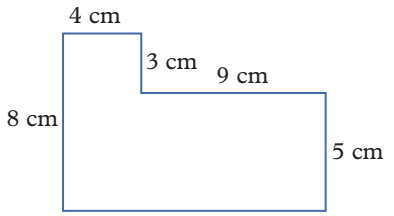
**e** 3 km, 3000 mm, 300 m, 30 cm

**f** 5500 mm, 0.005 km, 5.7 m, 560 cm

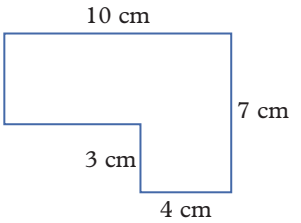
UNDERSTANDING AND FLUENCY

**8** Calculate the perimeter of each of the following shapes.

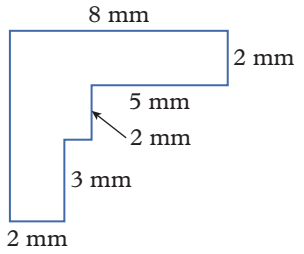
**a**



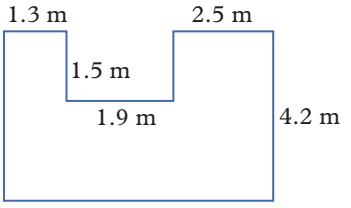

**b**




**c**




**d**




PROBLEM SOLVING AND REASONING

**9** A volleyball court is 18 m long and 9 m wide.

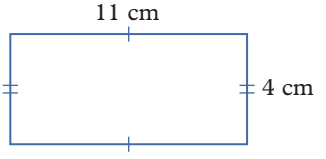
**a** Draw a diagram of the court showing its measurements and the net.

**b** Calculate its perimeter.

**c** To warm up, each team jogs five laps around their half of the court. That is, the players jog along the outside lines and along the net on their side of the court. How far does a player jog during the warm-up?

**10** A gardener plans to dig a rectangular herb garden with a border 36 m long. If the length of the garden is twice as long as the width, what are the dimensions?

**11** A rectangle has length 11 cm and width 4 cm.




**a** Calculate the perimeter of this rectangle.

**b** If $l$ is used to represent the length of a rectangle and $w$ is used to represent the width of a rectangle, write a formula for finding the perimeter, $P$, of a rectangle.

**12** Use the formula from question **11** to find the perimeter of these rectangles.

**a** length 5 cm, width 3 cm

**b** length 12 m, width 10 m

**c** length 47 mm, width 35 mm

**d** length 100 cm, width 24 cm

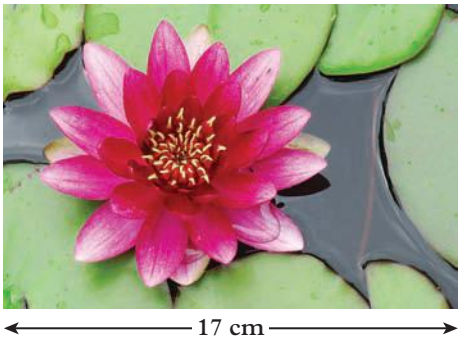
**e** length 6 m, width 4.5 m

**f** length 3.2 cm, width 1.9 cm

**g** length 8.62 m, width 425 cm

**h** length 15 mm, width 1.2 cm

**13** The photo shown has a perimeter of 52 cm. What is the width of the photo?



**14** If $P$ is used to represent the perimeter of a rectangle and $w$ is used to represent the width of a rectangle, write a formula for finding the length, $l$, of a rectangle.

**15** A rectangular tin is 9.8 cm wide and has a perimeter of 50.8 cm. What is the length of the tin?

**16** Some students made errors when they were converting length measurements. Explain the error in each case and give the correct answer.

**a** 55 cm = 5.5 mm

**b** 24 km = 2400 m

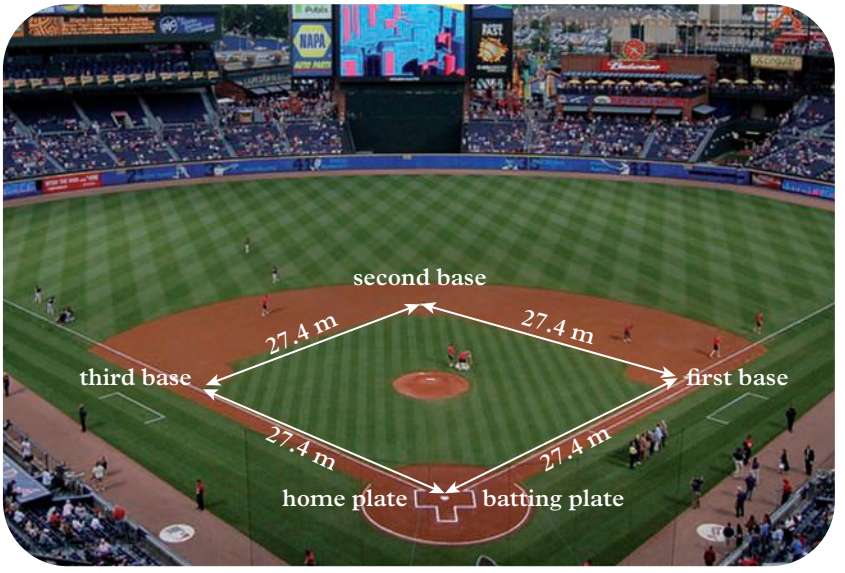
**c** 0.66 m = 60.6 cm

**d** 1.73 mm = 0.0173 cm

**e** 420 cm = 42 m

**f** 800 m = 0.08 km

**17** In baseball, a batter must run from the batting plate to first base, then second base, third base and finally home. Use the measurements shown to answer the following.




**a** How far does a batter run to reach first base after hitting the ball?

**b** How far does a batter run to reach third base after hitting the ball?

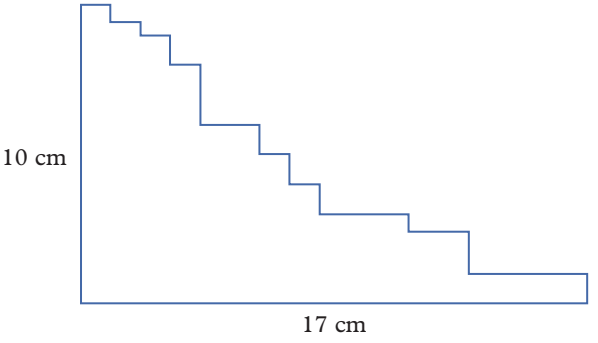
**c** How far does a batter run to reach the home plate when scoring a home run?

**d** Which one out of parts **a**, **b** and **c** is the same as the perimeter of the baseball diamond?

**18** A square drink coaster has a perimeter of 342 mm. What is the length of the drink coaster?

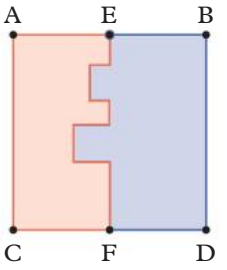
**19** **a** Explain how you can calculate the perimeter of the shape shown below when only knowing two side lengths. Note: all corners are right angles.

**b** What is the perimeter of the shape shown below?

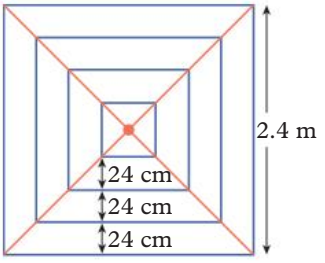


PROBLEM SOLVING AND REASONING

**20** In the square shown below, E is at the midpoint of AB and F is at the midpoint of CD. Does the orange or blue shape have a larger perimeter or are their perimeters the same? Explain your answer.



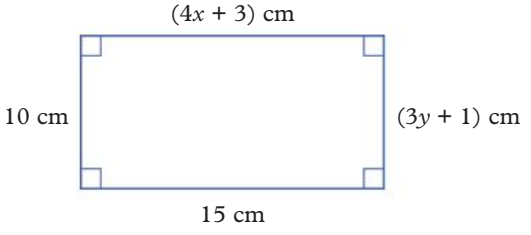

**21** A Hills Hoist clothesline has four lines arranged in squares. Use the measurements shown on the diagram, which shows the clothesline from above, to calculate the total length of clothesline used.



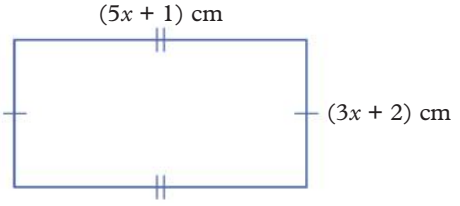
**22** A rectangular swimming pool of length 7.5 m and width 4.6 m is to have a safety fence built around it. If the fence is to be built 2 m from the edge of the pool, calculate the total length of fencing needed, including the gate.

CHALLENGE

**23** Find the values of $x$ and $y$.




**24** If the perimeter of this shape is 38 cm, find the value of $x$.




Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Converting between units of length

**Investigation** Metric and imperial units of length

**Worksheet** Converting units of length

**Topic quiz** 9A

**Interactive skillsheet** Perimeter

# 9B Area of a rectangle

## Learning intentions

✔ I can calculate the area of a rectangle using length and width.

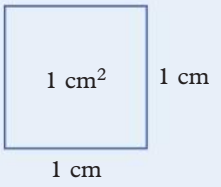
**Inter-year links**

| | |
|---|---|
| Years 5/6 | Understanding area |
| Year 8 | 8C Area of rectangles and triangles |
| Year 9 | 6A Area of composite shapes |

## Area

- The **area** of a 2D shape is the amount of space enclosed by that shape.

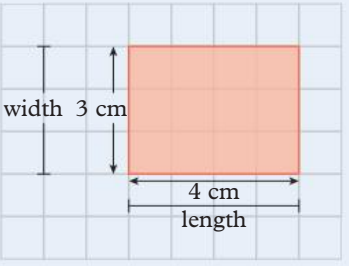
| Metric units of area | |
|---|---|
| **Name** | **Abbreviation** |
| square millimetre | $mm^2$ |
| square centimetre | $cm^2$ |
| square metre | $m^2$ |
| square kilometre | $km^2$ |




## Area of a rectangle

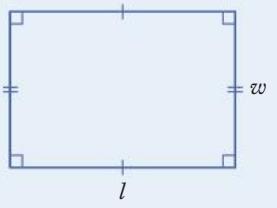
- Rectangles have two dimensions:
  - → The **length** of a rectangle is the length of its longest side.
  - → The **width** of a rectangle is the length of its shortest side.
- A rectangle can be divided into squares to calculate its area. The area of a rectangle is equal to the number of squares that make up its length multiplied by the number of squares that make up its width, or:

Area of a rectangle = length × width.




→ Expressing the rule using pronumerals, where $A$ = area of a rectangle, $l$ = length and $w$ = width, the formula for the area of a rectangle is:

$$A = l \times w \text{ or } A = lw$$



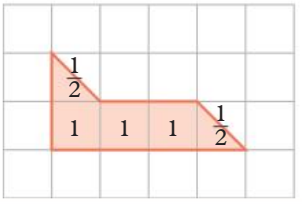

→ The measurements for $l$ and $w$ must be in the same unit before they are substituted into the formula.

## **Example 9B.1** Finding area by counting squares

Find the exact area of this shape. Assume each square in the grid has an area of 1 $cm^2$.

**THINK**

Calculate the area of the shape by adding up the number of squares that make up the shape. As each square in the grid has an area of 1 $cm^2$, 4 squares = 4 $cm^2$.



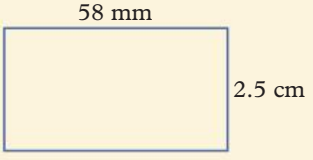

**WRITE**

3 squares + 2 half squares = 3 squares + 1 square
= 4 squares

Area = 4 $cm^2$

## **Example 9B.2** Calculating the area of a rectangle using the formula

Calculate the area of this rectangle.




**THINK**

1. Identify the measurements for $l$ and $w$. Convert all measurements into the same unit.
2. Substitute the values for $l$ and $w$ into the formula for the area of a rectangle.
3. Calculate the area of the rectangle. Remember to include the appropriate unit in the answer. As the length and width are in centimetres, the area will be in square centimetres, $cm^2$.

**WRITE**

$l = 58$ mm
$= 5.8$ cm
$w = 2.5$ cm

$A = l \times w$
$= 5.8 \times 2.5$
$= 14.5$ $cm^2$

Alternatively, if $w$ was converted into millimetres, $A = 1450$ $mm^2$.

**Helpful hints**

- ✔ The perimeter and area of 2D shapes are not the same thing! The perimeter is a measure of the length around the boundary of a shape, whereas the area is a measurement of the amount of space that the shape covers.
- ✔ When working with areas, the units will be square units. Remember to include a superscript 2 to indicate this, for example, $cm^2$.

ANS p599

# Exercise 9B Area of a rectangle

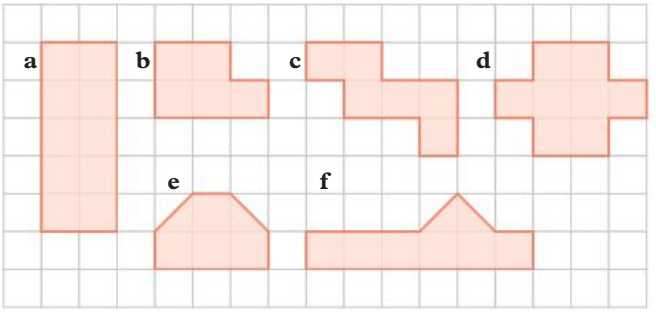
1, 2, 3(a–c), 4(a, e, f), 5(a–c), 6(1st column), 7, 8, 9(a, e, f), 10, 11(a, b), 12, 15(a, b)

1(d–f), 3(c, e, f), 4(e, f), 5(d–f), 6(2nd column), 7(c, d), 8, 9(e–h), 11, 13, 14, 15(c, d), 17, 18, 21

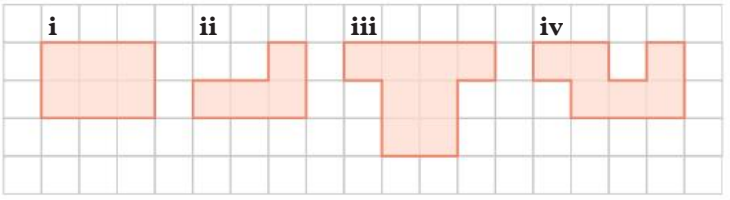
1(e, f), 4(b, e, f), 5(d–f), 6(d, f), 7(c, d), 8, 9(f, h), 11(c, d), 13, 14, 16, 19–22

UNDERSTANDING AND FLUENCY

**9B.1** **1** Find the exact area of these shapes. Assume each square in the grid has an area of 1 cm$^2$.




**2** These shapes are shown on 1-cm grid paper.



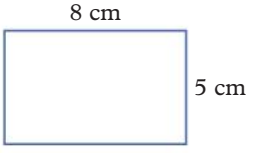

**a** Find the perimeter of each shape.

**b** Find the area of each shape.

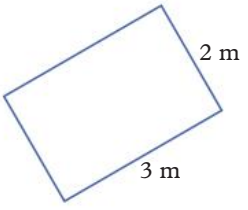
**c** Explain the difference between finding the perimeter and finding the area of each shape.

**9B.2** **3** Calculate the area of each rectangle.

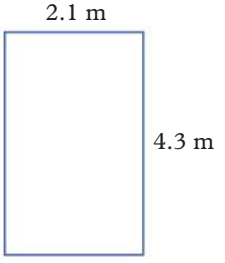
**a**



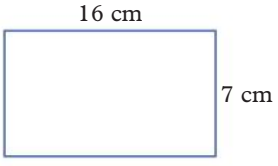

**b**



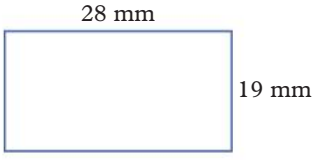

**c**



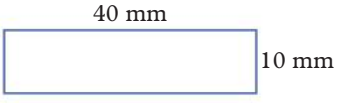

**d**




**e**




**f**




**4** Find the area of the rectangles with these dimensions.

**a** length 9 cm, width 7 cm

**b** length 13 mm, width 4 mm

**c** length 6 m, width 1 m

**d** length 10.4 cm, width 5 cm

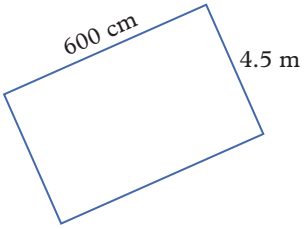
**e** length 14.2 mm, width 9.7 mm

**f** length $17\frac{1}{2}$ m, width $5\frac{1}{4}$ m

**5** Calculate the area of each rectangle.

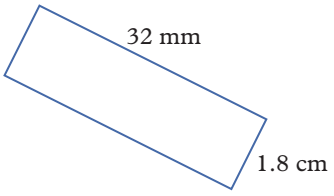
**a**

**b**



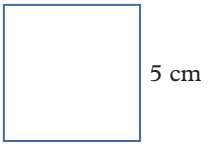

**c**

**d**




**e**

**f**

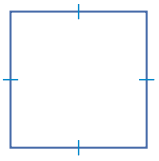



**6** Find the area of each of the following rectangles.

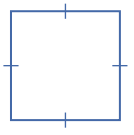
**a** length 3 m, width 100 cm
**b** length 8 cm, width 50 mm
**c** length 4 km, width 280 m
**d** length 600 mm, width 30 cm
**e** length 3.1 cm, width 20 mm
**f** length 8.4 cm, width 0.05 m

**7** Calculate the area of each square.

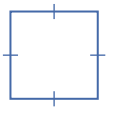
**a**



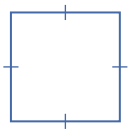

**b**




**c**




**d**




**8** Use your working from question **7** to determine the formula for the area ($A$) of a square of length $l$.

**9** Find the area of the squares with these side lengths.

**a** 7 m
**b** 15 cm
**c** 11 km
**d** 60 mm
**e** 6.4 cm
**f** 180 m
**g** 0.7 mm
**h** 0.53 km

**10** The shape shown in Figure A can be divided into two rectangles as shown in Figure B.

**Figure A**

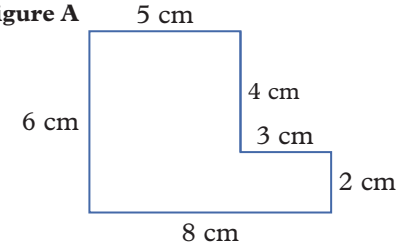
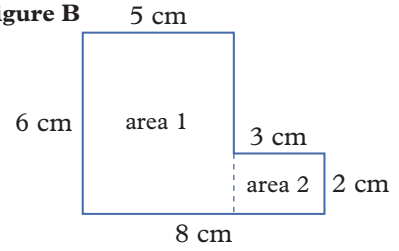



**Figure B**




**a** Calculate the area of the rectangle labelled as area 1 in Figure B.

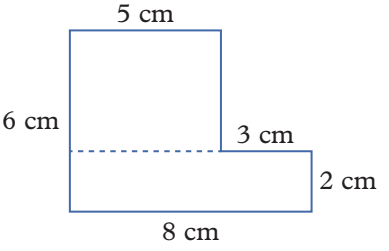
**b** Calculate the area of the rectangle labelled as area 2 in Figure B.

**c** What is the total area of the shape?

Figure A can also be divided into two other rectangles, as shown in Figure C.

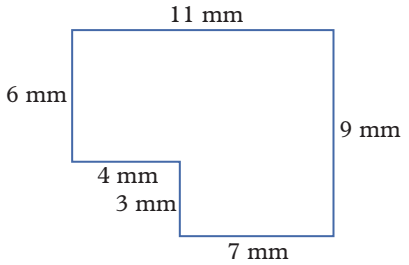
**d** Calculate the area of the shape using Figure C.

**Figure C**

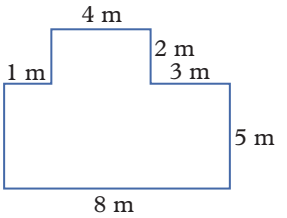


**11** Calculate the area of these shapes by first dividing them into rectangles.

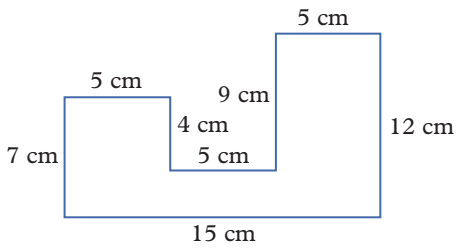
**a**



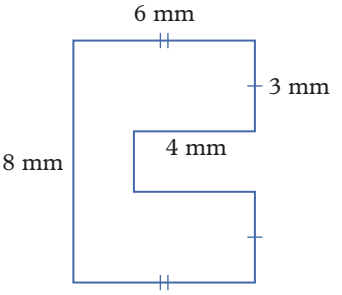

**b**



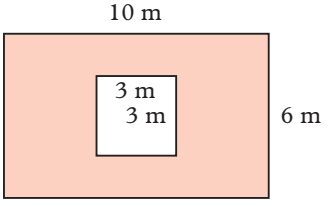

**c**




**d**




**12** In questions **10** and **11**, the shapes are divided into separate rectangles and the areas of each rectangle are added together to find the total area of the shape.

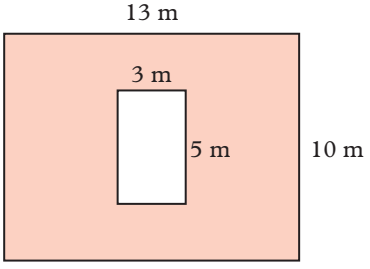



**a** Explain how you could calculate the area of the shaded region in this shape using subtraction instead of addition. Both shapes are rectangles.

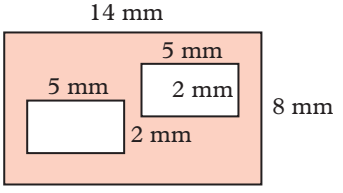
**b** What is its area?

**13** Find the area of the shaded region in each of the following diagrams. All shapes are rectangles.

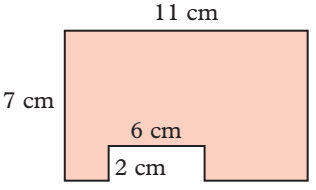
**a**



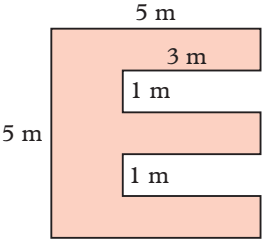

**b**




**c**




**d**




**14** Use the images of these two phones to determine the following.

**a** If the area of the screen on the phone in Figure B is 3000 mm$^2$, what is its length?

**b** What is the width of the screen on the phone in Figure A, if its area is 3750 mm$^2$?

**Figure A**

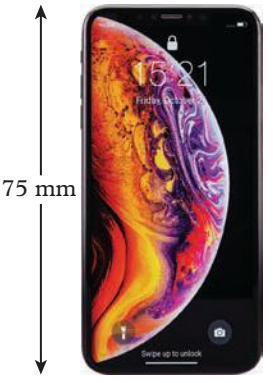
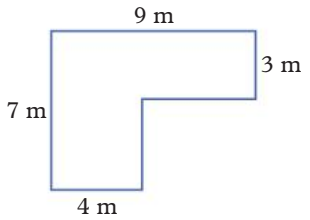



**Figure B**

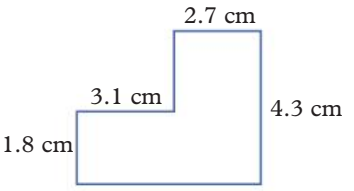
PROBLEM SOLVING AND REASONING

**15** Find **i** the area and **ii** the perimeter of each shape. Note: all corners are right angles.

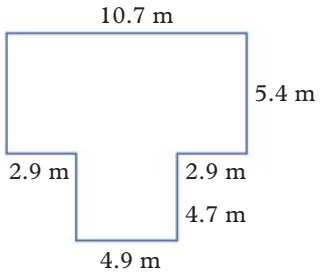
**a**



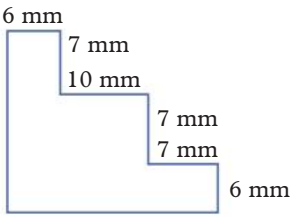

**b**




**c**




**d**



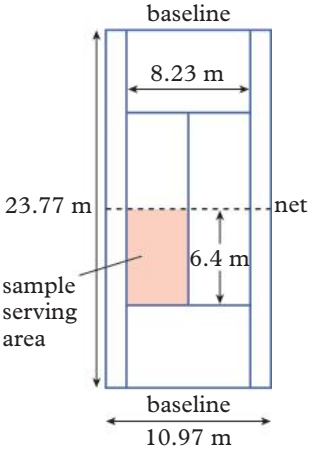

**16** The perimeter of a rectangle with a length of 14 cm is 46 cm. What is its area?

**17** The perimeter of a square is 80 m. What is its area?

**18** The area of a square is 16 $mm^2$. What is its perimeter?

**19** Explain why two rectangles with the same area do not always have the same perimeter. Give an example in your answer.

**20** A tennis court has a playing area that is divided by lines drawn on the ground.




**a** Use the dimensions labelled in the diagram to calculate:

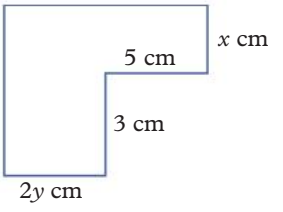
- **i** the distance from the baseline to the net
- **ii** the width of the doubles playing area (that is, the width of the full court)
- **iii** the width of the singles playing area, given the outer sidelines are not used in singles matches
- **iv** the playing area used in a doubles match
- **v** the playing area used in a singles match
- **vi** the area of the serving area.

**b** When it starts to rain, a waterproof cover is placed over the playing surface. What area of waterproof material would be required for the cover?

CHALLENGE

**21** Is it possible for a rectangle's perimeter to have the same value in cm as its area in $cm^2$? Explain your answer.

**22** Write four equivalent expressions for the area of the shape to the right.




- **a** One that contains products added together.
- **b** Two that contain a sum multiplied by a term.
- **c** One that contains a subtraction.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Area of a rectangle

**Investigation** Floor plan of a house

**Topic quiz** 9B

# 9C Area of a parallelogram

## Learning intentions

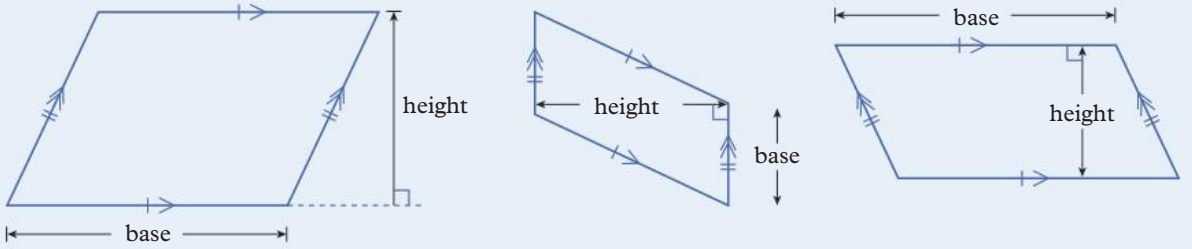
✔ I can calculate the area of a parallelogram using the formula.

**Inter-year links**

| | |
|---|---|
| Year 5/6 | Understanding area |
| Year 8 | 8D Area of quadrilaterals |
| Year 9 | 6A Area of composite shapes |

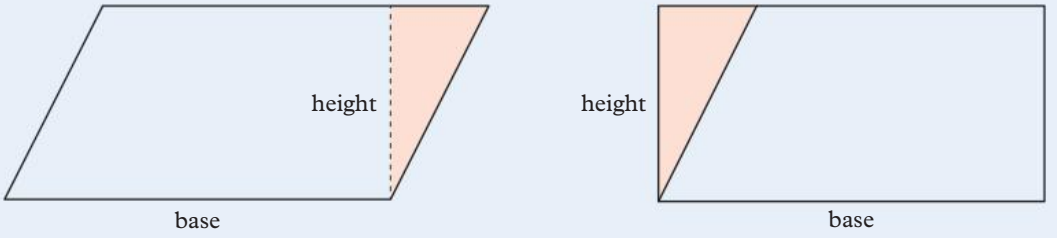
## Area of a parallelogram

- Parallelograms have two dimensions:
  - → The **base** is the length of one side of the parallelogram.
  - → The **height** is the perpendicular distance from the base to the opposite side of the parallelogram.



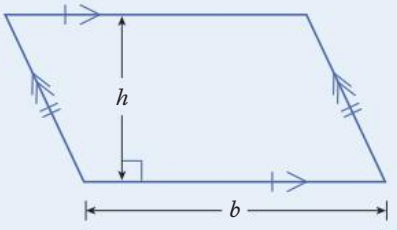

- Parallel sides are identified using one or more arrow heads.
- Every parallelogram can be rearranged into a rectangle with the same base length and height. Therefore, the area of a parallelogram can be calculated using the rule:

Area of a parallelogram = base × height



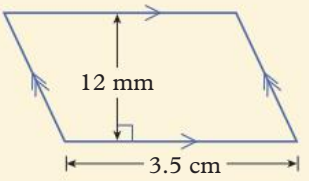

- Expressing the rule using pronumerals, where $A$ = area of a parallelogram, $b$ = base and $h$ = height, the formula for the area of a parallelogram is:

  $A = b \times h$ or $A = bh$
  - → The base and height must be perpendicular to each other.
  - → The measurements for $b$ and $h$ must be in the same unit before they are substituted into the formula.



## Example 9C.1 Calculating the area of a parallelogram using the formula

Calculate the area of this parallelogram in square centimetres, $cm^2$.




| THINK | WRITE |
|---|---|
| **1** Identify the measurements for $b$ and $h$. All measurements must be in the same unit before they are substituted into the formula. Convert the height into the units specified by the question. | $b = 3.5$ cm<br>$h = 12$ mm<br>$= 1.2$ cm |
| **2** Substitute the values for $b$ and $h$ into the formula. | $A = b \times h$<br>$= 3.5 \times 1.2$ |
| **3** Calculate the area of the parallelogram. Remember to include the appropriate unit in the answer. | $= 4.2$ $cm^2$ |

**Helpful hints**

- ✔ The base can be any side of a parallelogram; it doesn't need to be at the bottom of a diagram.
- ✔ The height of a parallelogram is the perpendicular length between two parallel sides; it is not the length of the sloping edge! The height is used to calculate the area, and the sloping edges are used to calculate the perimeter.

ANS p600

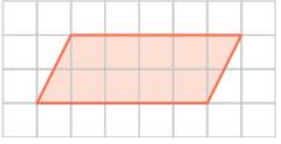
# Exercise 9C Area of a parallelogram

△ 1–6, 7(a–c), 8(a–c), 9, 11, 12 ▢ 2(c, d), 3–8, 10, 11, 13, 14, 15(a) ⬠ 3(d–f), 5–8, 11, 13, 15, 16

UNDERSTANDING AND FLUENCY

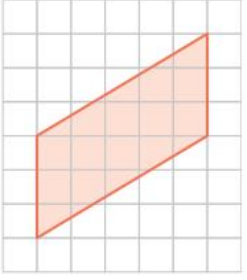
**1** Find the area of these parallelograms shown on 1-cm grid paper.

**a**

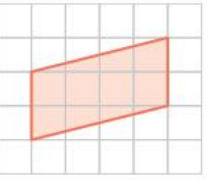


**b**

**c**

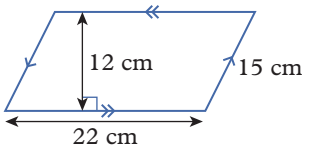


**d**

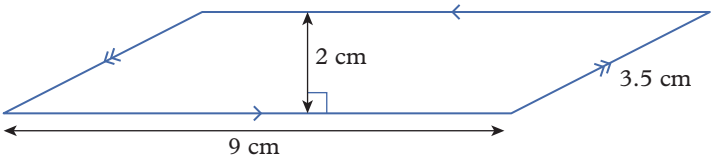


**2** State the base length and the height of each of these parallelograms.

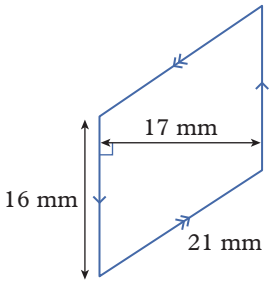
**a**



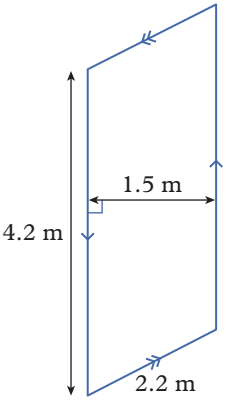

**b**




**c**

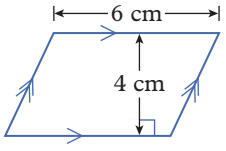



**d**

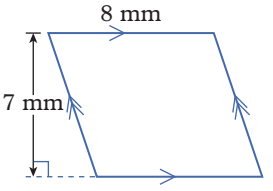



**3** Calculate the area of these parallelograms.

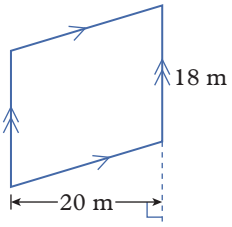
**a**



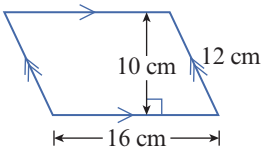

**b**



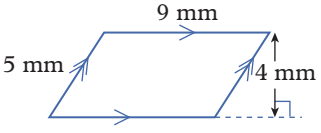

**c**



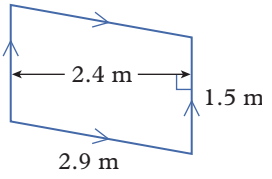

**d**




**e**




**f**




**4** Find the area of the parallelograms with these dimensions.

**a** base length 5 cm, height 3 cm

**b** base length 10 mm, height 17 mm

**c** base length 4 m, height 1.5 m

**d** base length 2.5 cm, height 4.1 cm

**5** Find the area of each parallelogram with these dimensions in the units specified in brackets.

**a** base length 30 mm, height 5 cm ($cm^2$)

**b** base length 58 m, height 200 cm ($m^2$)

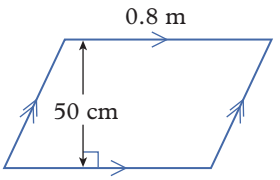
**c** base length 0.06 m, height 40 mm ($mm^2$)

**d** base length 27.4 cm, height 27.4 mm ($cm^2$)

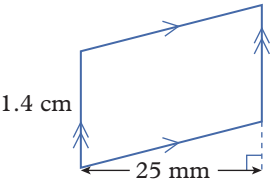
UNDERSTANDING AND FLUENCY

**9C.1** **6** Calculate the area of each parallelogram in square centimetres, $cm^2$.

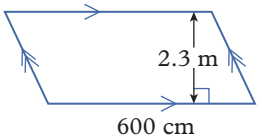
**a**



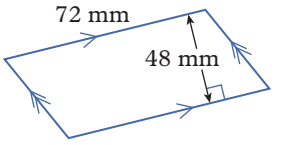

**b**



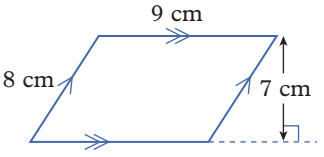

**c**



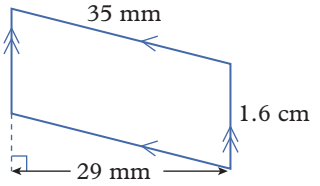

**d**




**e**



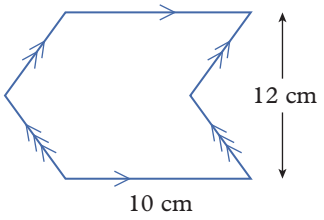

**f**



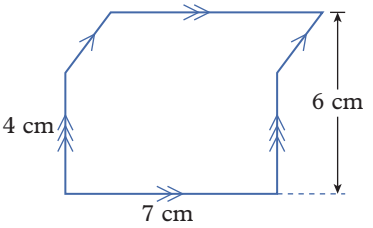

PROBLEM SOLVING AND REASONING

**7** Divide these shapes into two or more rectangles and/or parallelograms to calculate their area.

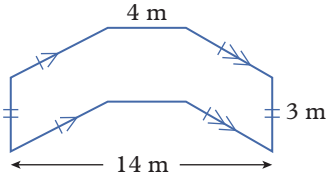
**a**



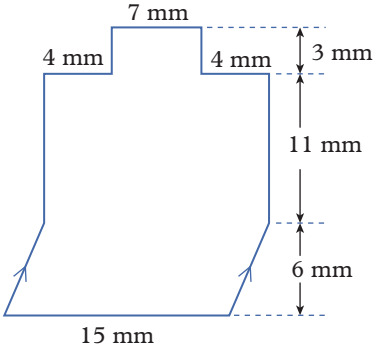

**b**




**c**

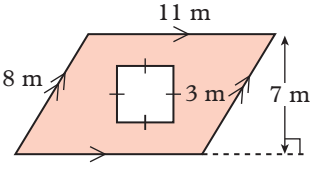



**d**

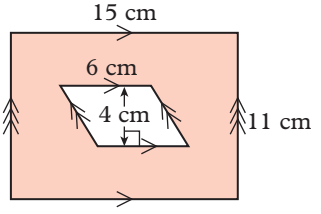



**8** Find the area of the shaded region in each of these. Consider subtracting areas as well as adding areas to obtain your answer.

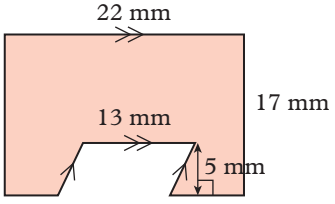
**a**



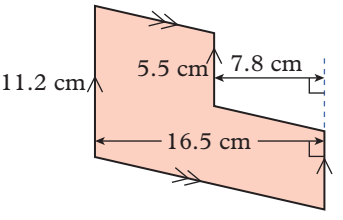

**b**



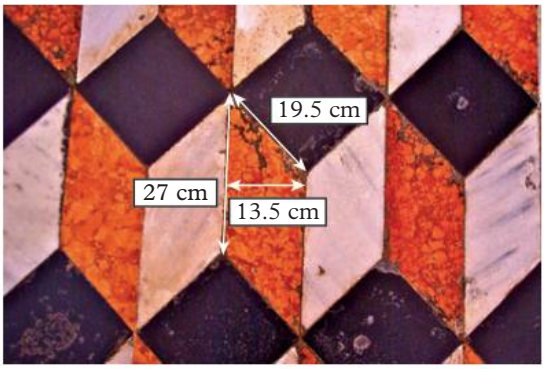

**c**



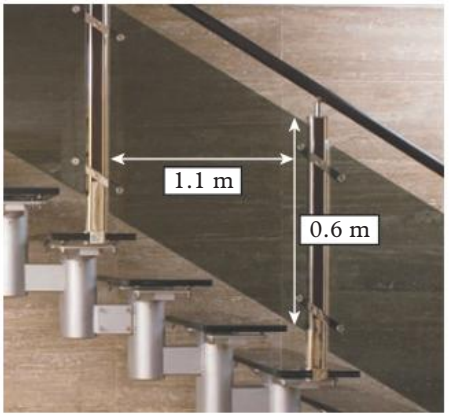

**d**



**9** Use the measurements shown on this photo of floor tiles to calculate the area of a red tile.



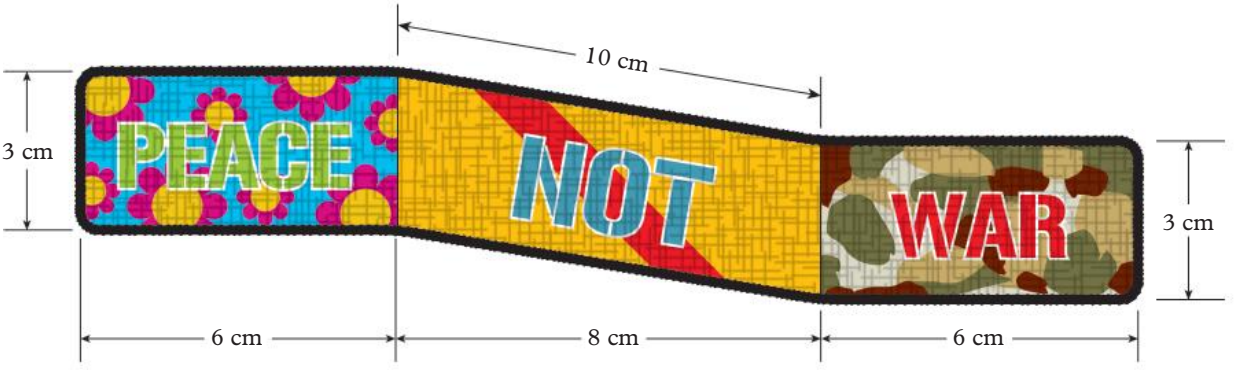

**10** A stair rail is supported by five identical glass panels.




**a** Use the dimensions shown in the photo above to calculate the area of glass needed for all the glass panelling.

**b** If each glass panel were to be 10 cm taller, what area of glass would be needed?

**11** A badge for a jacket is made by sewing three pieces of fabric as shown below. The centre piece is in the shape of a parallelogram and the other two pieces of fabric are rectangles.




**a** What area of fabric is needed for the parallelogram?

**b** What area of fabric is needed for each rectangle?

**c** Calculate the total area of fabric needed for this badge.

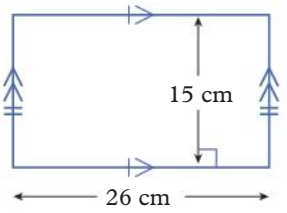
**d** Black cord is sewn around the outer edge of the badge to make it stand out on the jacket. Calculate how much black cord is needed.

**12** The area of a parallelogram is 20 $cm^2$. If its height is 4 cm, how long is the base of the parallelogram?

PROBLEM SOLVING AND REASONING

**13** The area of a parallelogram is 36 cm$^2$. If its base is 45 mm long, what is the height of the parallelogram?

**14** Consider the rectangle shown.

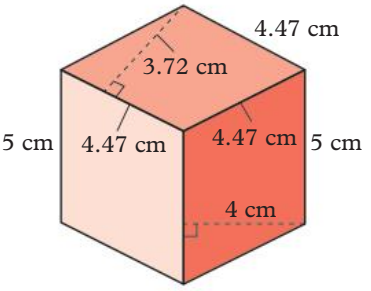



**a** Calculate its area using the formula for the area of a rectangle, $A = l \times w$.

**b** Calculate its area using the formula for the area of a parallelogram, $A = b \times h$.

**c** Explain how you obtain the same answer in each case.

**15** The image of a rectangular prism is drawn using two identical parallelograms representing the two visible faces joined by a straight edge of a prism, and a rhombus representing one of the matching ends of the prism.



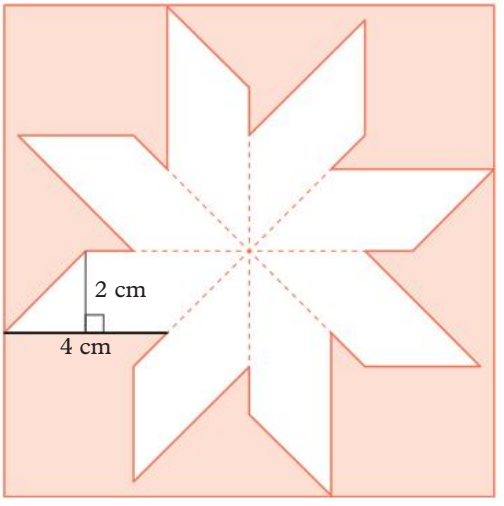

**a** Determine the total area of the two parallelograms and the rhombus.

**b** If a rectangular prism has rectangular faces with the same lengths as in the image, determine the total area of the three visible faces.

**c** Determine the percentage that the area of the image is of the total area of the three visible faces. Give your answer to two decimal places.

CHALLENGE

**16** Eight identical parallelograms have been cut out of a square piece of paper as shown. Find the area of the remaining paper.




Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Area of a parallelogram

**Investigation** The secret of the parallelogram

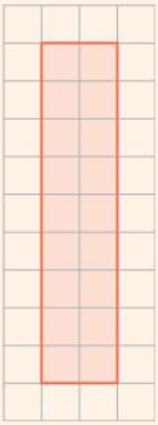
**Topic quiz** 9C

# Checkpoint

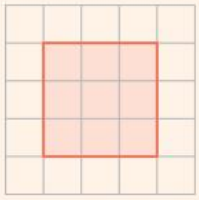
**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

9A **1** Calculate the perimeter of the following shapes shown on 1-cm grid paper.

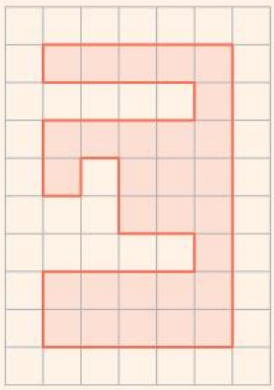
**a**



**b**



**c**

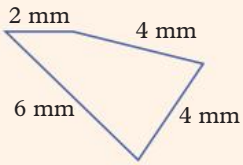


9A **2** Convert the following lengths to the unit in brackets.

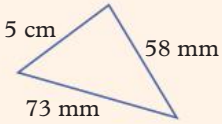
**a** 15 mm (cm) **b** 25 912 km (m) **c** 23 cm (m) **d** 0.3 m (mm)

9A **3** Calculate the perimeter of the following shapes.

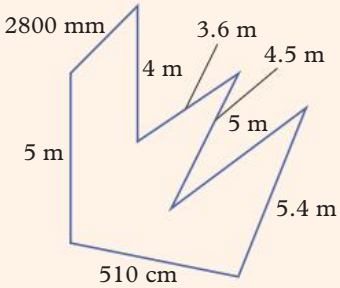
**a**




**b**



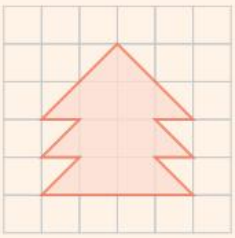

**c**



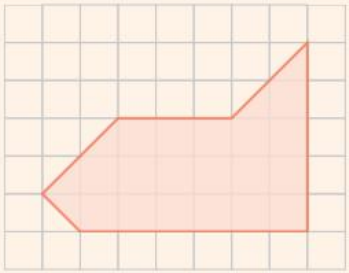

9B **4** Calculate the area of the shapes shown in question **1**.

9B **5** Calculate the area of the following shapes shown on 1-cm grid paper.

**a**

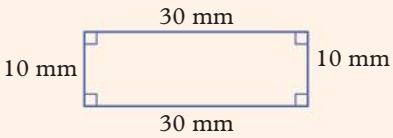


**b**

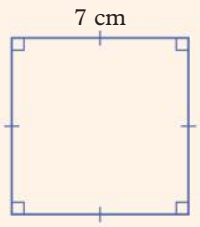


9B **6** Calculate the area of the following shapes.

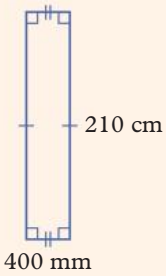
**a**



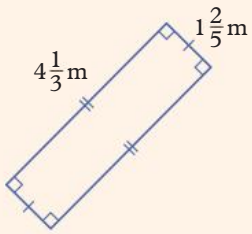

**b**




**c**

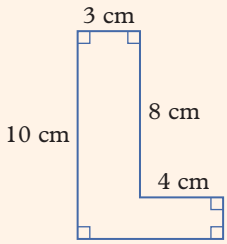



**d**

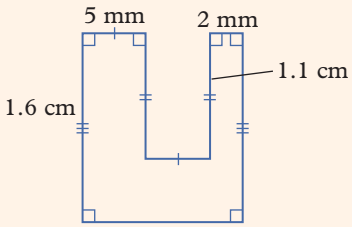


9B **7** Calculate the perimeter and area of the following shapes.

**a**

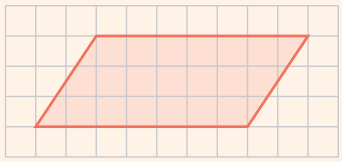



**b**

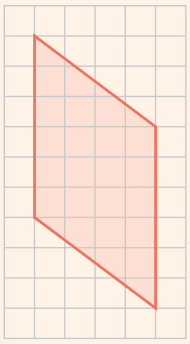



9C **8** Calculate the area of the following shapes shown on 1-cm grid paper.

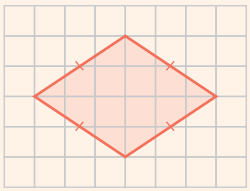
**a**



**b**

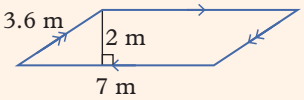


**c**

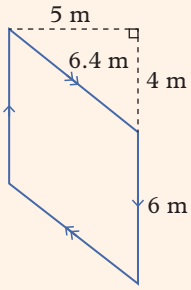


9C **9** Calculate the area of the following shapes.

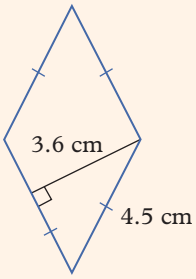
**a**



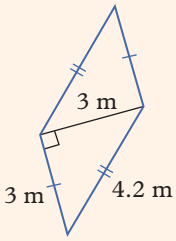

**b**




**c**

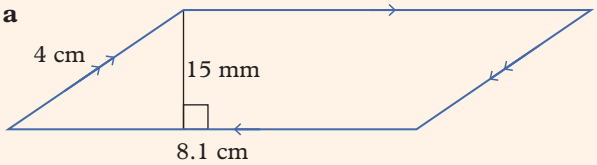



**d**



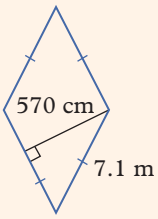

9C **10** Calculate the area of the following shapes in the units specified in brackets.

**a**




($mm^2$)

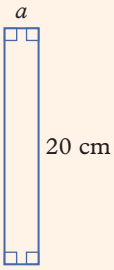
**b**



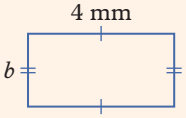

($m^2$)

9C **11** Find the value of the pronumeral in each of the following shapes.

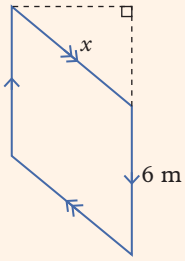
**a** $A = 20\ cm^2$



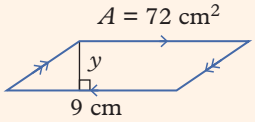

**b** $P = 12$ mm




**c** $P = 36$ m




**d** $A = 72\ cm^2$



# 9D Area of a triangle

## Learning intentions

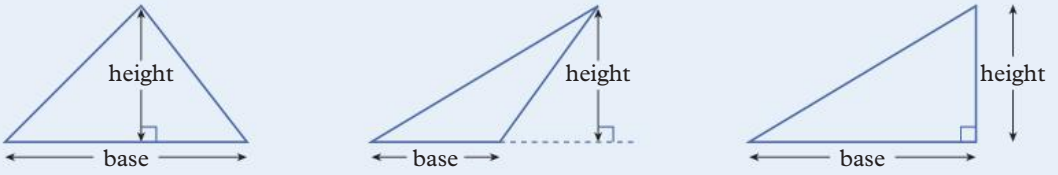
✔ I can calculate the area of a triangle using the base and height.

**Inter-year links**

| | |
|---|---|
| Year 5/6 | Understanding area |
| Year 8 | 8C Area of rectangles and triangles |
| Year 9 | 6A Area of composite shapes |

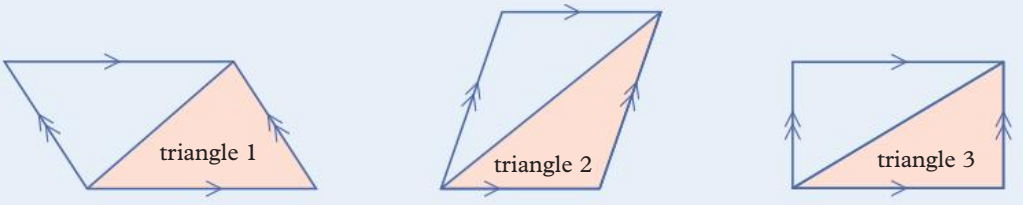
## Area of a triangle

- Triangles have two dimensions:
  - → The base is the length of the base of the triangle.
  - → The height is the perpendicular distance from the base to the opposite point of the triangle.



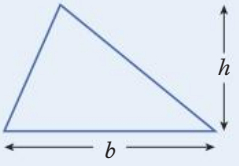

- Every triangle can be enclosed by a corresponding parallelogram or rectangle which has the same base length and height as the triangle.
- The corresponding parallelogram or rectangle is made up of exactly two of the triangles that it encloses, therefore the area of a triangle can be calculated using the rule:

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$




- Expressing the rule using pronumerals, where $A$ = area of a triangle, $b$ = base and $h$ = height, the formula for the area of a triangle is:

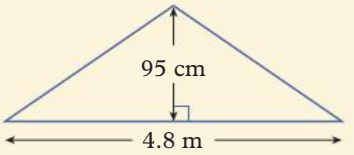
$$A = \frac{1}{2}b \times h \text{ or } A = \frac{1}{2}bh$$




  - → The base and height must be perpendicular to each other.
  - → The measurements for $b$ and $h$ must be in the same unit before they are substituted into the formula.

## **Example 9D.1** Calculating the area of a triangle using the formula

Calculate the area of this triangle in $m^2$.




| THINK | WRITE |
|---|---|
| **1** Identify the measurements for $b$ and $h$. Convert the height into the units specified by the question. | $b = 4.8$ m<br>$h = 95$ cm<br>$= 0.95$ m<br>$A = \frac{1}{2}b \times h$ |
| **2** Substitute the values for $b$ and $h$ into the formula. | $A = \frac{1}{2} \times 4.8 \times 0.95$ |
| **3** Calculate the area of the triangle. Remember to include the appropriate unit in the answer. | $= 2.28$ $m^2$ |

**Helpful hints**

The base can be any side of a triangle; it does not always have to be at the bottom! Every triangle will have three potential bases, along with three corresponding perpendicular heights.

ANS p601

# **Exercise 9D** Area of a triangle

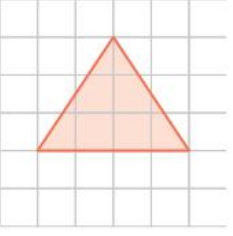
1, 2(a, c, e), 3–5, 6(a, c, e), 7, 9, 13(a–c) | 3(b, d, f), 4, 5, 6(b, d, f), 7, 8, 10, 12–14 | 4, 6(d–f), 7, 11–16

UNDERSTANDING AND FLUENCY

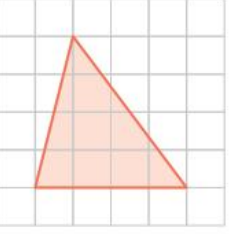
**1** Find the area of these triangles shown on 1-cm grid paper.

**a**

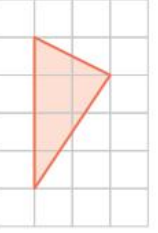
**b**



**c**



**d**



**2** Find the area of the triangles with these dimensions.

**a** base 12 cm, height 9 cm

**b** base 24 mm, height 4 mm

**c** height 3 m, base 5 m

**d** base 8.6 cm, height 7 cm

**e** base 14.2 mm, height 9.7 mm

**f** height 15.4 m, base 17.03 m



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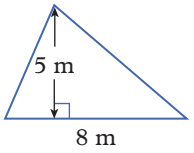
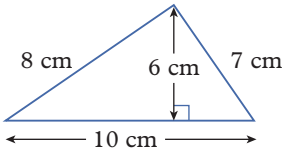
**3** State the base length and the height of each of these triangles.

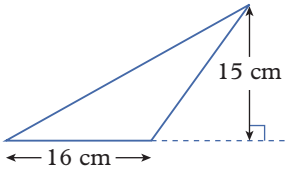
**a**



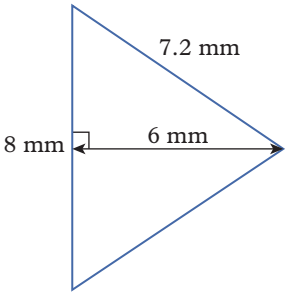

**b**



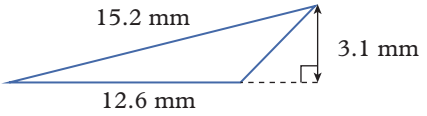

**c**



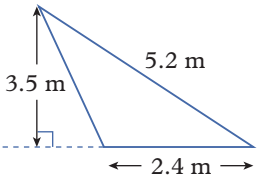

**d**




**e**

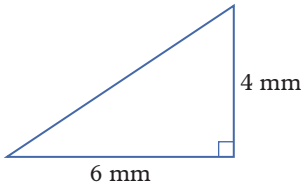



**f**

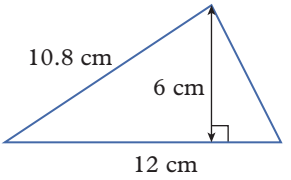



**4** Calculate the area of each triangle.

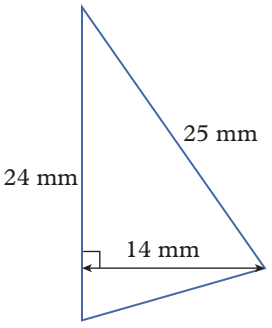
**a**



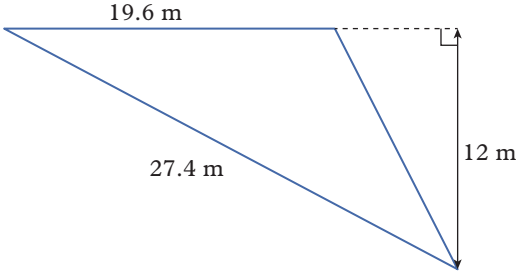

**b**



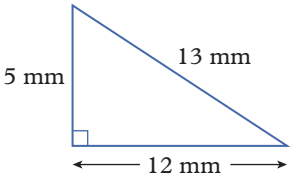

**c**



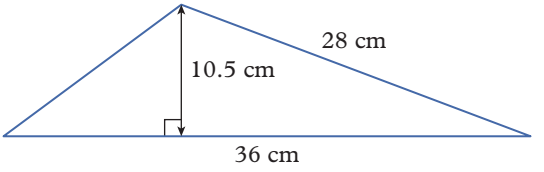

**d**




**e**




**f**

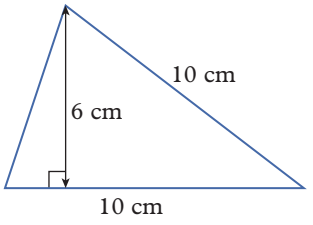



**5** Complete the following for each of these triangles.

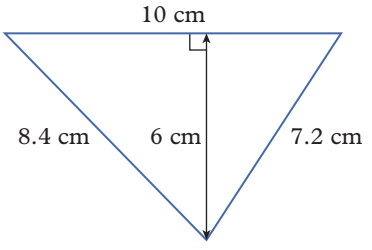
**i** Calculate the area.

**ii** Comment on your answers to part **i**.

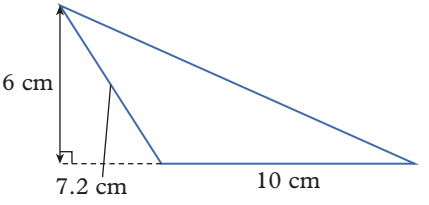
**a**




**b**




**c**



UNDERSTANDING AND FLUENCY

**6** Find the area of each triangle with these dimensions.

**a** base 8 cm, height 30 mm

**b** height 3 cm, base 16 mm

**c** base 4 m, height 50 cm

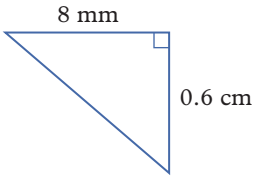
**d** base 310 cm, height 10 m

**e** height 20 mm, base 0.7 cm

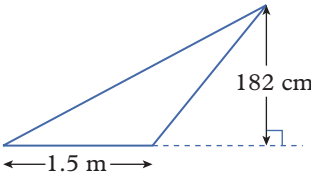
**f** height 97 cm, base 2.8 m

9D.1 **7** Calculate the area of each triangle.

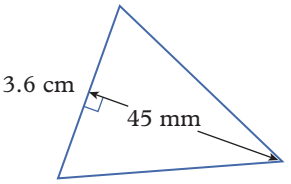
**a**



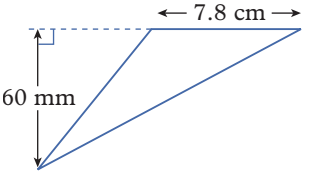

**b**



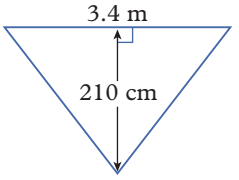

**c**



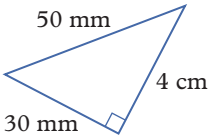

**d**




**e**



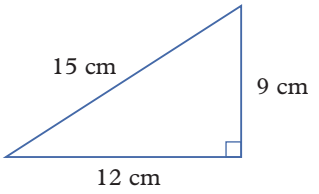

**f**



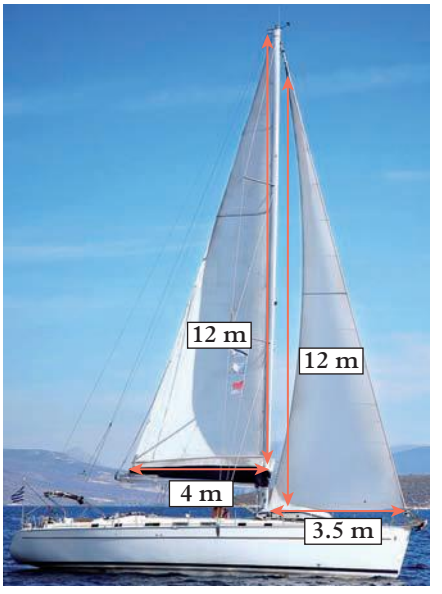

PROBLEM SOLVING AND REASONING

**8** **a** A student has calculated the area of the triangle shown at right to be 90 cm². Explain the mistake that the student has made and find the correct area of the triangle.

**b** Calculate the perimeter of this triangle.




**9** Calculate the amount of sailcloth needed for the two triangular sails on the boat on the right.



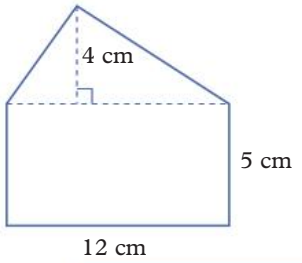

**10** The area of a triangle is 20 cm². If its base is 10 cm long, what is the height of the triangle?

**11** The area of a triangle is 400 mm². If its height is 2 cm, what is the base length in mm?

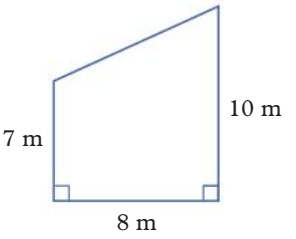
**12** Use your understanding of how to find the area of a triangle to calculate the area of metal needed for this sign.

**13** A composite shape is made from two or more basic shapes. Calculate the area of each of these composite shapes.

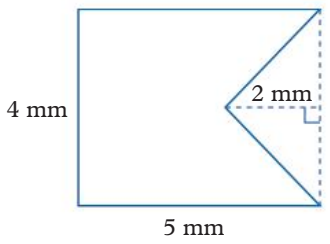
**a**



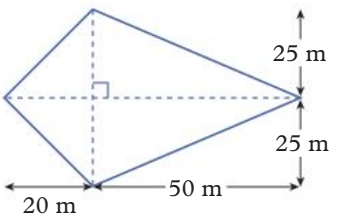

**b**



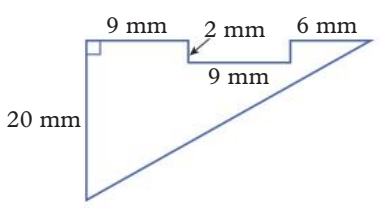

**c**



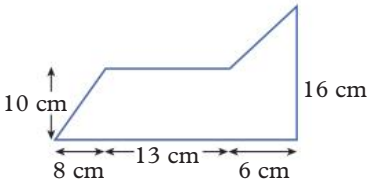

**d**




**e**

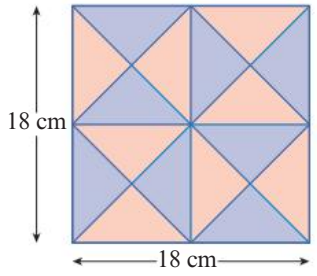



**f**

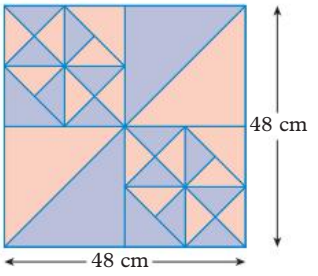



**14** Calculate the total area that is shaded orange in each of the graphics below.

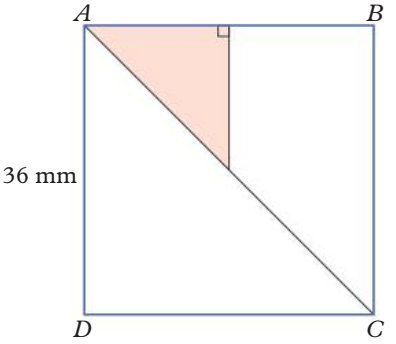
**a**



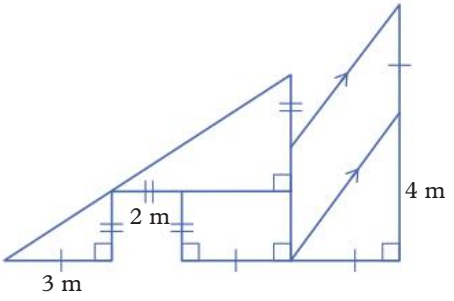

**b**




**15** This square has a side length of 36 mm. A vertical line has been drawn from the midpoint of side *AB* to meet the diagonal. What is the area of the shaded region?




**16** Find the shaded area of this shape. Hint: recreate the shape on square paper.






Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Area of a triangle

**Investigation** Mighty mathematician, the superhero

**Topic quiz** 9D

# 9E Surface area

## Learning intentions

- ✔ I can calculate the surface area of a cube.
- ✔ I can calculate the surface area of a rectangular prism.
- ✔ I can calculate the surface area of a triangular prism.

Inter-year links

| | |
|---|---|
| Year 8 | 8C Area of rectangles and triangles |
| Year 9 | 6B Surface area |

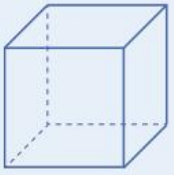
## Surface area

- The **surface area** of a three-dimensional (3D) object is the total area of the outer surface of that object.
- To calculate the total surface area of a 3D object:
  1. determine the number of faces
  2. calculate the area of each face
  3. add the areas of all the faces together.

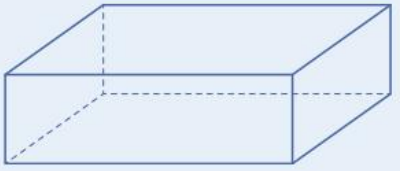
## 3D prisms

- A **cube** has six faces that are all identical squares.
- A **rectangular prism** has six faces. There are three pairs of identical rectangular faces.
- A **triangular prism** has five faces. The two ends of a triangular prism are identical triangular faces.

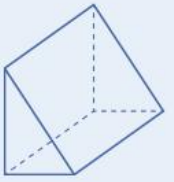
Cube (6 faces)



Rectangular prism (6 faces)



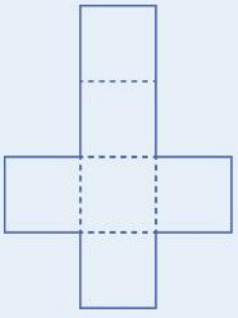
Triangular prism (5 faces)



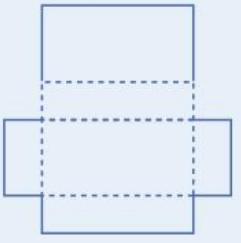
## Nets

- A **net** is a two-dimensional (2D) composite shape that can be folded to make a 3D prism.
- Nets are useful diagrams for visualising the total surface area of a 3D prism.

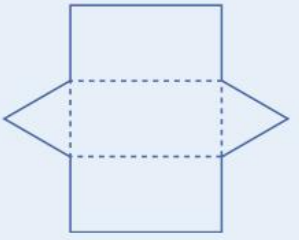
Net of a cube



Net of a rectangular prism



Net of a triangular prism





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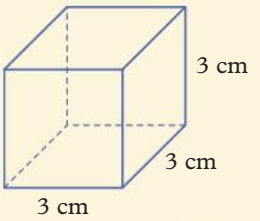
## **Example 9E.1** Calculating the surface area of a cube

Calculate the surface area of this cube.




**THINK**

1. Determine the number of faces. A cube has six identical square faces.
2. Calculate the area of each face.
3. Calculate the total surface area of the cube. As all the faces of a cube are identical, the total surface area is six times the area of one face.

**WRITE**

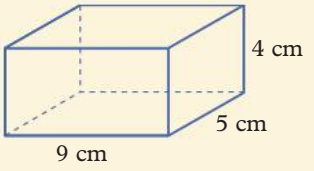
There are 6 faces.

Area of one face $= 3 \times 3$
$= 9 \text{ cm}^2$

Total surface area $= 6 \times$ area of one face
$= 6 \times 9$
$= 54 \text{ cm}^2$

## **Example 9E.2** Calculating the surface area of a rectangular prism

Calculate the surface area of this rectangular prism.




**THINK**

1. Determine the number of faces. A rectangular prism has six rectangular faces; three pairs of identical faces.
2. Calculate the area of each face.
3. Add the areas of all the faces together. Check that all six faces have been included in the sum.

**WRITE**

Area of front and back faces $= 9 \times 4$
$= 36 \text{ cm}^2$

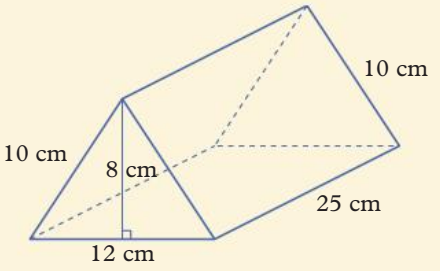
Area of side faces $= 5 \times 4$
$= 20 \text{ cm}^2$

Area of top and bottom faces $= 9 \times 5$
$= 45 \text{ cm}^2$

Total surface area $= 2 \times 36 + 2 \times 20 + 2 \times 45$
$= 202 \text{ cm}^2$

## **Example 9E.3** Calculating the surface area of a triangular prism

Calculate the surface area of this triangular prism.




**THINK**

1. Determine the number of faces. This triangular prism has two identical triangular faces, two identical rectangular sides and a rectangular base.
2. Calculate the area of each face.
3. Add the area of each face together. Check that all five faces have been included in the sum.

**WRITE**

Area of front and back faces $= \frac{1}{2} \times 12 \times 8$
$= 48 \text{ cm}^2$

Area of rectangular sides $= 10 \times 25$
$= 250 \text{ cm}^2$

Area of base $= 12 \times 25$
$= 300 \text{ cm}^2$

Total surface area $= 2 \times 48 + 2 \times 250 + 300$
$= 896 \text{ cm}^2$

**Helpful hints**

All outside surfaces of 3D objects must be included in your total surface area calculations, including those which you can't see directly – don't forget to include the base unless otherwise instructed!

ANS p601

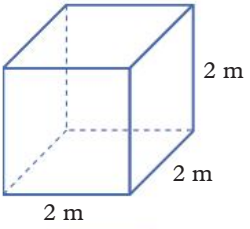
# **Exercise 9E** Surface area

1(a, b, c, e), 2, 3(a, b, d, e), 4, 5(a, b, c, e), 6(a, c, d), 7, 8, 10, 12(a) | 1(d–f), 3, 5, 6, 8, 9, 11, 12, 14(b) | 1(e, f), 3(d–f), 5(d–f), 6, 8, 11, 13–15

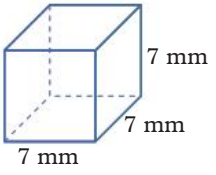
UNDERSTANDING AND FLUENCY

**9E.1** **1** Calculate the surface area of each cube.

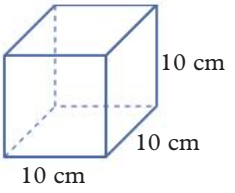
**a**



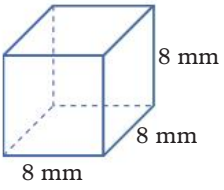

**b**



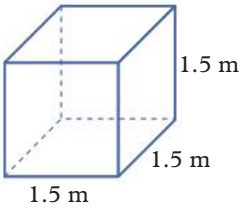

**c**



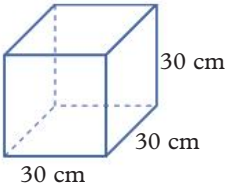

**d**




**e**




**f**



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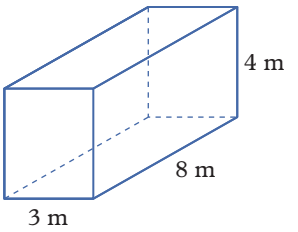
**2** **a** How many faces does this rectangular prism have?

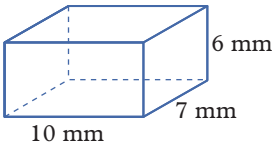
**b** Calculate the area of each pair of identical faces of the prism.

**c** Use your answer from part **b** to calculate the total surface area of the prism.

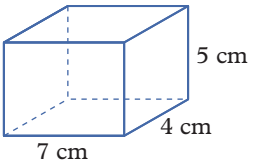



**9E.2** **3** Calculate the surface area of these rectangular prisms.

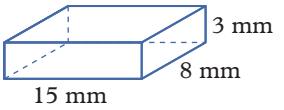
**a**



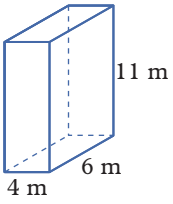

**b**



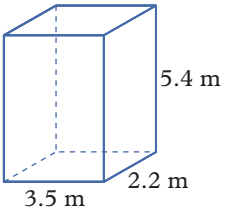

**c**



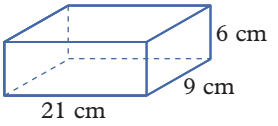

**d**




**e**



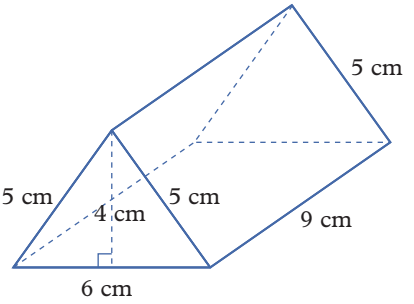

**f**




**4** **a** How many faces does this triangular prism have?

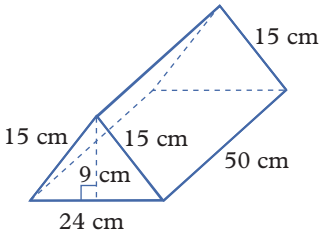
**b** Calculate the area of each face of the triangular prism.

**c** Use your answer from part **b** to calculate the surface area of the triangular prism.

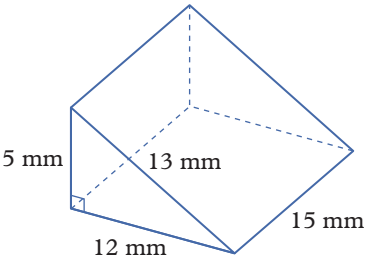



**9E.3** **5** Calculate the surface area of each triangular prism.

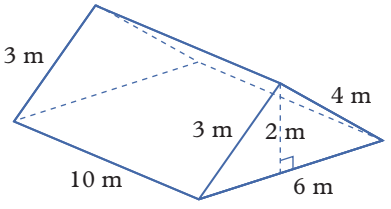
**a**



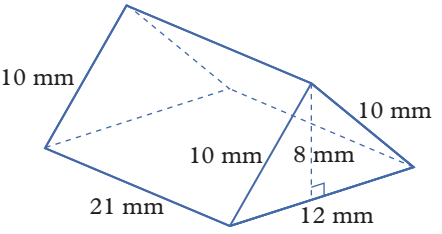

**b**



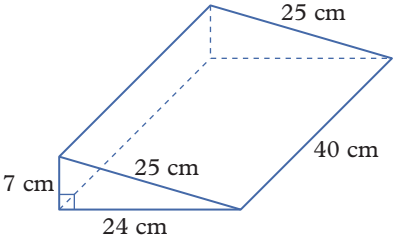

**c**



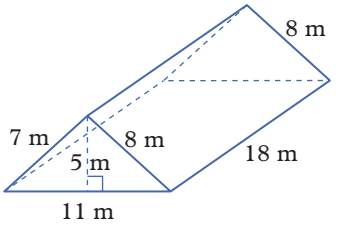

**d**




**e**




**f**

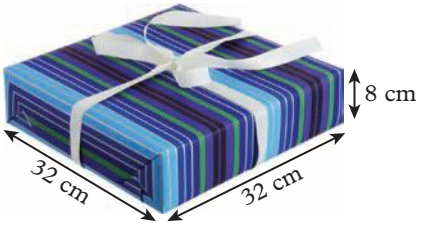


UNDERSTANDING AND FLUENCY

**6** Calculate the surface area of each of these objects.

**a**

**b**




**c**

**d**

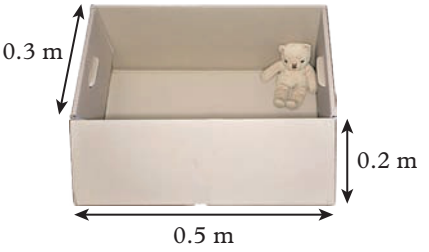
PROBLEM SOLVING AND REASONING

**7** This toy box is to be covered with coloured contact paper.

**a** What is the external surface area of the box?

**b** What is the internal surface area of the box?

**c** What area of paper will be needed to cover both the inner and outer surfaces of the box?

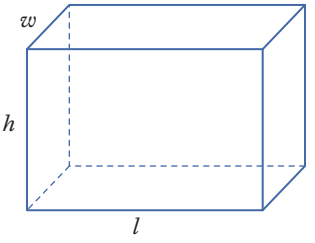



**8** Write a formula that can be used to calculate the total surface area, $T$, of a cube given the side length, $s$, of the cube.

**9** A cube has a surface area of 150 $cm^2$. What is the side length of the cube?

**10** A sculpture shaped as a cube has a surface area of 24 $m^2$. What are the dimensions of the sculpture?

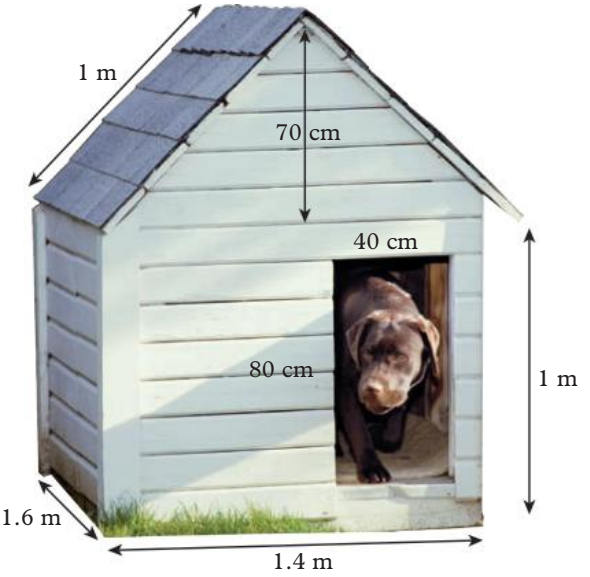
**11** Write a formula that can be used to calculate the total surface area, $T$, of a rectangular prism given the height, $h$, width, $w$, and length, $l$, of the prism.



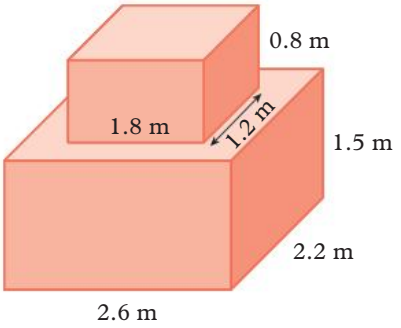
**12** The outside walls and the roof of the dog kennel are to be painted.

**a** Calculate the area of the kennel to be painted in $m^2$.

**b** If a small can of paint covers 8 $m^2$, how many cans of paint will be needed to paint the kennel?

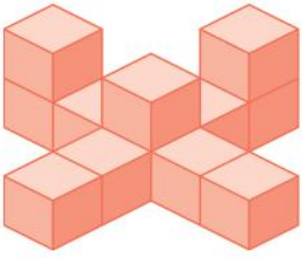



**13** A sculpture is constructed in the shape of a rectangular box with a smaller box placed on top. The dimensions of the boxes are shown in the diagram. To finish the sculpture, the artist must work out the surface area to be painted. Calculate the surface area of the sculpture excluding the base, which will not be visible to the public. Note: the overlapping parts of the boxes will also not be painted.

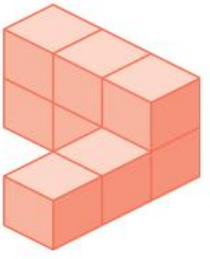



**14** These objects are made of 1-cm blocks. Work out their surface area (including the base).

**a**

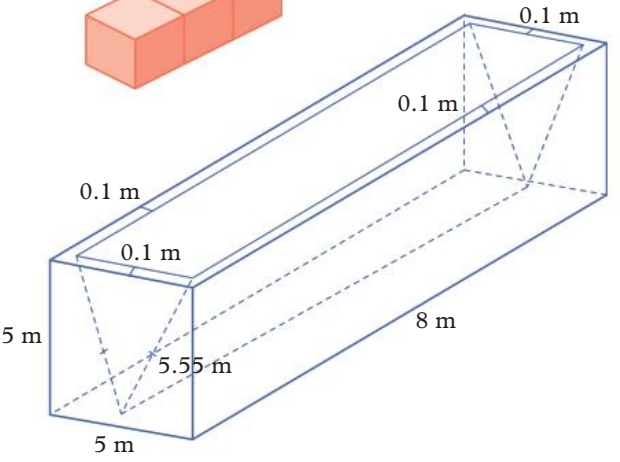


**b**





**15** Find the surface area of this trough. Include the exterior and interior faces.




Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Surface area of a rectangular prisms

**Investigation** Wedding cake

**Worksheet** Surface area of rectangular prisms

**Topic quiz** 9E

# 9F Volume and capacity

## Learning intentions

✔ I can calculate the volume of rectangular prisms using the formula $V = lwh$.

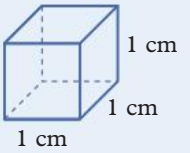
**Inter-year links**

| | |
|---|---|
| Years 5/6 | Volume and capacity |
| Year 8 | 8F Volume of prisms |
| Year 9 | 6C Volume |

## Volume

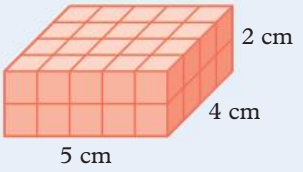
- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.
- The metric units of volume include cubic millimetres ($mm^3$), cubic centimetres ($cm^3$) and cubic metres ($m^3$).

| Metric units of volume | |
|---|---|
| **Name** | **Abbreviation** |
| cubic millimetre | $mm^3$ |
| cubic centimetre | $cm^3$ |
| cubic metre | $m^3$ |




- The volume of a rectangular prism can be determined by multiplying the base area by the height of the prism:

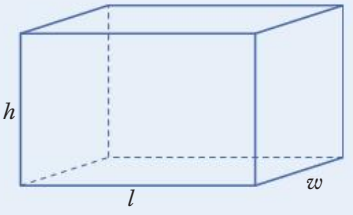
Volume of a rectangular prism = base area × height




- As the area of a rectangle is equal to its length multiplied by its width, the volume of a rectangular prism is equal to its length, multiplied by its width, multiplied by its height, or:

  Volume of a rectangular prism = length × width × height
- Expressing the rule using pronumerals, where $V$ = volume of a rectangular prism, $l$ = length, $w$ = width and $h$ = height, the formula for the volume of a rectangular prism is:

$$V = l \times w \times h \text{ or } V = lwh$$




→ The base and height must be perpendicular to each other.

→ The measurements for $l$, $w$ and $h$ must be in the same unit before they are substituted into the formula.



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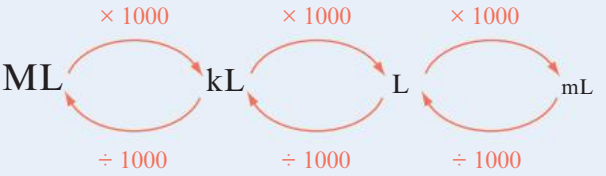
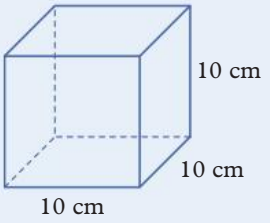
# Capacity

- The **capacity** of a 3D object is a measure of how much that object can hold.

| Metric units of capacity | |
|---|---|
| **Name** | **Abbreviation** |
| millilitres | mL |
| litres | L |
| kilolitres | kL |
| megalitres | ML |



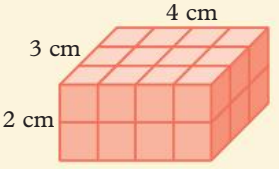

→ A container with an inside volume of 1 $cm^3$ holds 1 mL of liquid.
→ A container with an inside volume of 1000 $cm^3$ holds 1 L of liquid.
→ A container with an inside volume of 1 $m^3$ holds 1 kL of liquid.




Volume = 1000 $cm^3$
Capacity = 1 L

## Example 9F.1 Finding volume by counting cubes

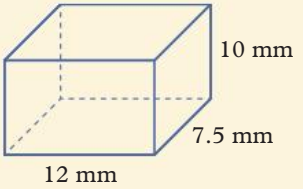
Find the volume of this rectangular prism.




| THINK | WRITE |
|---|---|
| **1** Work out the number of cubes in one layer of the prism. | $4 \times 3 = 12$ cubes in one layer of the prism |
| **2** Calculate the volume of the rectangular prism by multiplying the volume of one layer by the number of layers in the prism. As each cube has a side length of 1 cm, the volume of one cube is 1 $cm^3$. | $12 \times 2 = 24$ cubes in the prism<br>Volume of the rectangular prism = 24 $cm^3$ |

## Example 9F.2 Calculating the volume of a rectangular prism using the formula

Calculate the volume of this rectangular prism.




**THINK**

1. Identify the measurements for $l$, $w$ and $h$. Check that each measurement is in the same unit.
2. Substitute the values for $l$, $w$ and $h$ into the formula.
3. Calculate the volume of the rectangular prism. Remember to include the appropriate unit in the answer.

**WRITE**

$l = 12$ mm

$w = 7.5$ mm

$h = 10$ mm

$V = l \times w \times h$

$= 12 \times 7.5 \times 10$

$= 900 \text{ mm}^3$

**Helpful hints**

Volume and capacity are not the same! Volume is a measure of how much space an object occupies (expressed in $\text{mm}^3$, $\text{cm}^3$ or $\text{m}^3$), whereas capacity is a measure of how much an object can hold (expressed in mL, L, kL or ML).

ANS p601

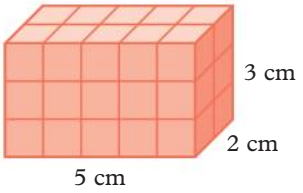
# Exercise 9F Volume and capacity

△ 1(a–c), 2–8, 11, 13, 14, 17(a) | □ 1(e, f), 2(b–e), 3(b, d), 4–9, 12, 15, 17, 18(a, b), 19 | ⬠ 2(d–f), 3(d), 4, 5, 6(c, d), 7(c, d), 9, 10, 12, 16, 17(c, d), 18(b, c), 19, 20

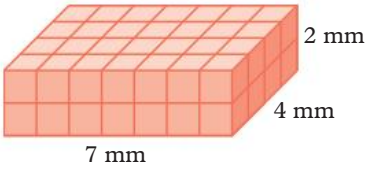
UNDERSTANDING AND FLUENCY

9F.1 **1** Find the volume of each rectangular prism.

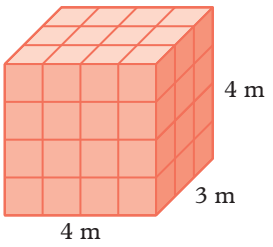
**a**



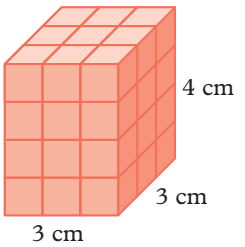

**b**



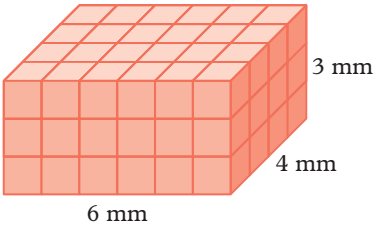
**c**



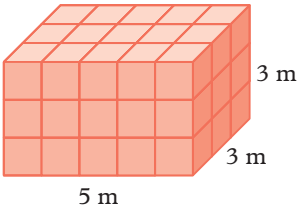

**d**




**e**

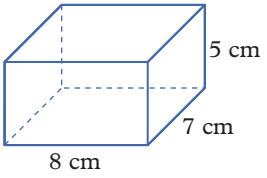



**f**

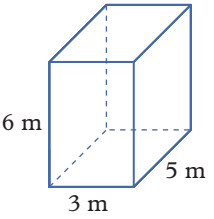



9F.2 **2** Calculate the volume of each rectangular prism.

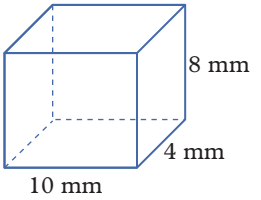
**a**



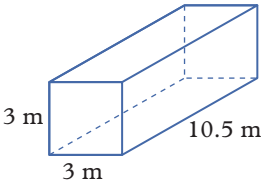

**b**



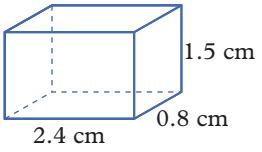

**c**



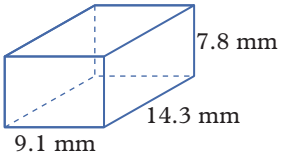

**d**




**e**

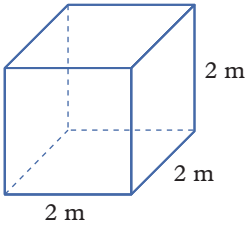



**f**

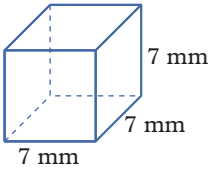



**3** Calculate the volume of each cube.

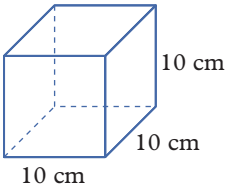
**a**



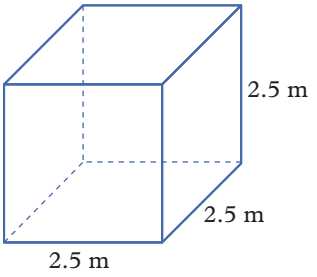

**b**




**c**




**d**




**4** Use the conversion factors on page 421 to complete these statements.

**a** 8 L = _____ mL

**b** 15 kL = _____ L

**c** 70 mL = _____ L

**d** 6.4 ML = _____ kL

**e** 2500 L = _____ kL

**f** 3 kL = _____ mL

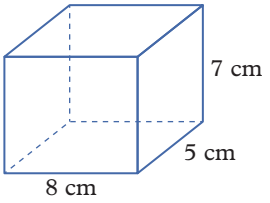
UNDERSTANDING AND FLUENCY

**5** Complete the table by working out the missing value for each container.

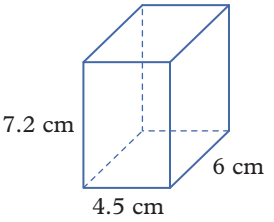
| | Inside volume of container | Capacity of container |
|---|---|---|
| **a** | 14 $cm^3$ | _____ mL |
| **b** | _____ $cm^3$ | 3 L |
| **c** | 5.8 $m^3$ | _____ kL |
| **d** | 750 $cm^3$ | _____ L |
| **e** | _____ $m^3$ | 1000 kL |
| **f** | _____ $m^3$ | 6.4 ML |

**6** Calculate the volume of these 3D objects in $cm^3$.

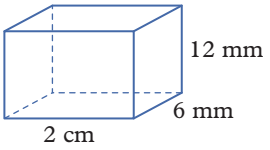
**a**



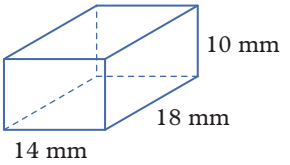

**b**




**c**

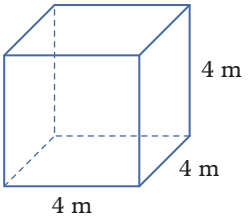



**d**

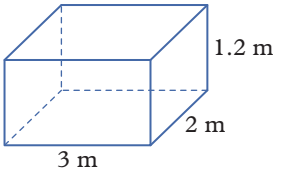



**7** Calculate the volume of these 3D objects in $m^3$.

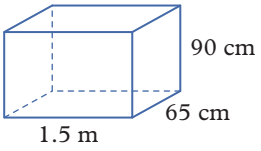
**a**



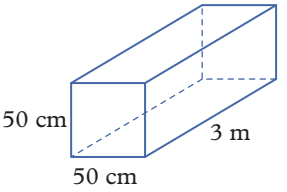

**b**




**c**




**d**



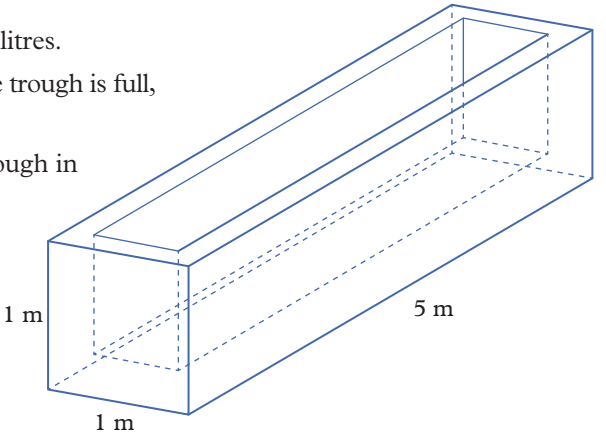

PROBLEM SOLVING AND REASONING

**8** A concrete water trough is designed as follows. A 1 m × 5 m × 1 m concrete block is formed around a 80 cm × 480 cm × 90 cm mould to create a cavity to fill with water.

**a** Determine the maximum capacity of the trough in millilitres.

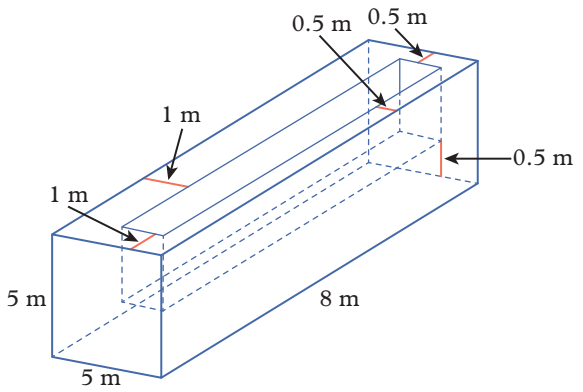
**b** Determine the total volume of trough and water when the trough is full, in cubic centimetres.

**c** Determine the volume of concrete used to create the trough in cubic centimetres.

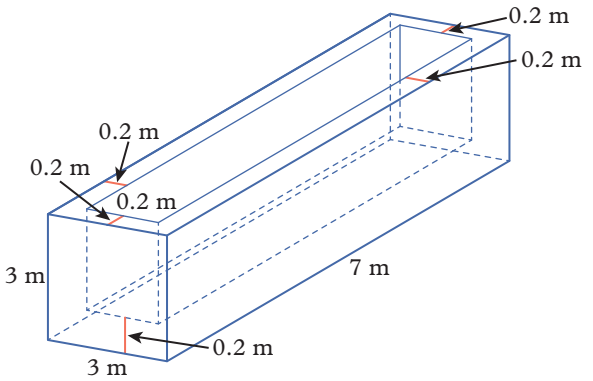


**9** Calculate the capacity of each of the following troughs in litres.

**a**




**b**

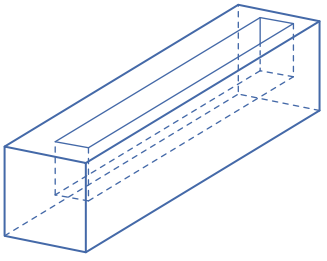



**10** Consider the following three troughs made from the same sized block.

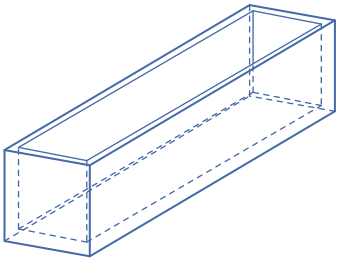
**a** Put these troughs in ascending order according to their capacity.

**b** Put these troughs in ascending order according to their volume.

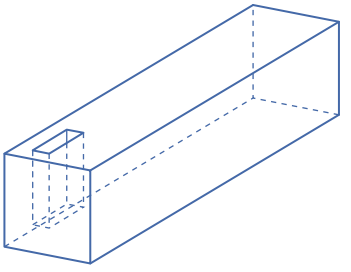
**A**



**B**



**C**



**11** Soil has been dug from a garden to form a rectangular hole of length 5 m, width 2.5 m and depth 50 cm. How many cubic metres of soil have been removed from the garden?

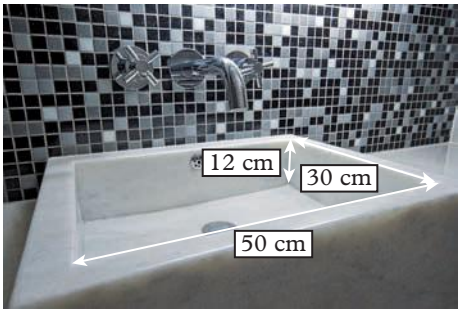
**12** A delivery of 3 $m^3$ of concrete is to be spread to form a driveway that is 7.5 m long and 3.2 m wide. How thick will the layer of concrete be?

**13** A sugar cube has the dimensions 1 cm by 1 cm by 1 cm.

**a** What is the volume of one sugar cube?

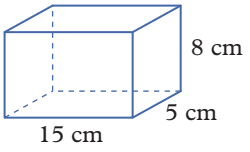
**b** How many sugar cubes would fit into a box with dimensions 12 cm by 10 cm by 7 cm?

**14** Find how many litres of water this rectangular sink will hold if filled to the top.

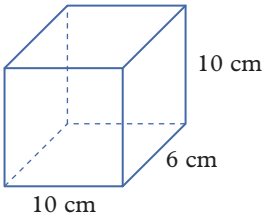



**15** Which of these 600 $cm^3$ boxes has the greatest surface area?

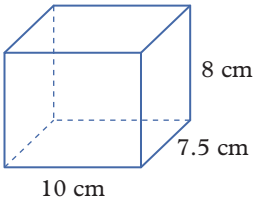
**A**



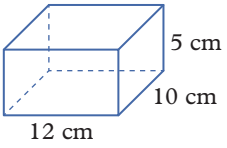

**B**



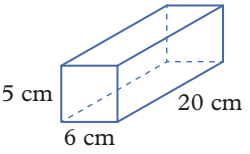

**C**




**D**



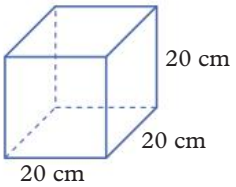

**E**



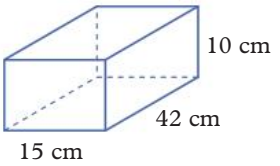
PROBLEM SOLVING AND REASONING

**16** By calculating the surface area and volume of both of these boxes, demonstrate that boxes with the same surface area do not always have the same volume.

**a**

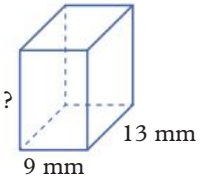



**b**



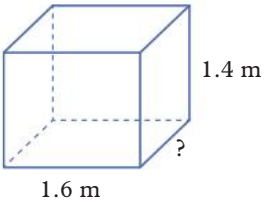

**17** Find the missing length in these rectangular prisms.

**a**



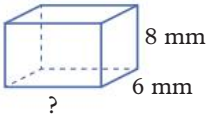

Volume = 1989 mm³

**b**



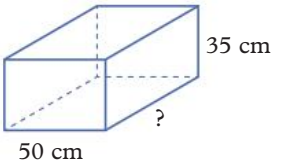

Volume = 2.912 m³

**c**




Volume = 672 mm³

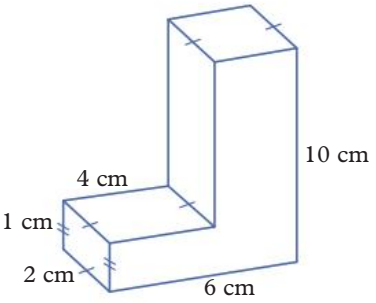
**d**



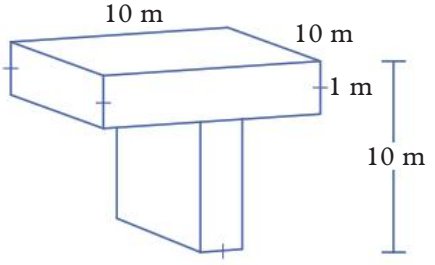

Volume = 0.14 m³

**18** Find the volume of these composite solids made up from rectangular prisms.

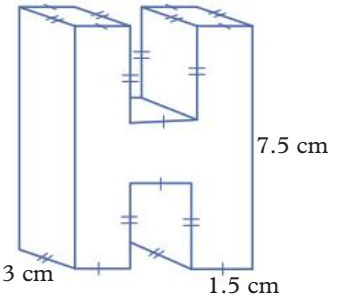
**a**




**b**




**c**




CHALLENGE

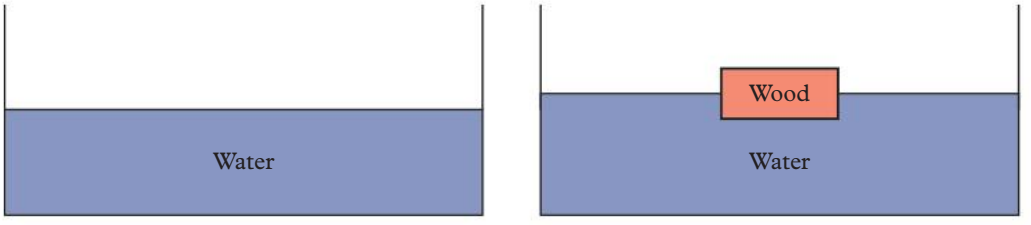
**19** How many whole number sets of dimensions (in cm) are there for a rectangular prism with a volume of 20 cm³? List them all.

**20** Archimedes knows that when an object is submerged under water, the water level will rise as the water becomes *displaced* by the object. He also knows that the volume of the submerged object is equal to the volume of the displaced water.

Archimedes has a rectangular glass container with base dimensions 30 cm x 40 cm. He fills it with 12 L of water and finds that the water level is exactly halfway up the total height of the glass container.

**a** Determine the height of Archimedes' container.

Archimedes then floats a rectangular wooden block on the surface of the water. The wooden block has dimensions 10 cm x 8 cm x 5 cm. With a ruler, he measures that the water level is now at a height of 202 mm.




**b** Determine the difference in the water level in cm.

**c** Determine the volume of displaced water.

**d** Determine what fraction of the block's volume is submerged under water.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Volume of prisms

**Interactive skillsheet** Capacity

**Investigation** Building a vegetable garden

**Topic quiz** 9F

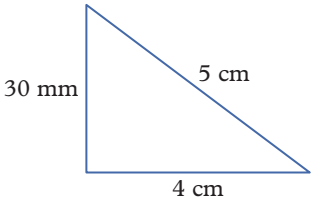
# Chapter summary

### Metric units of length

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

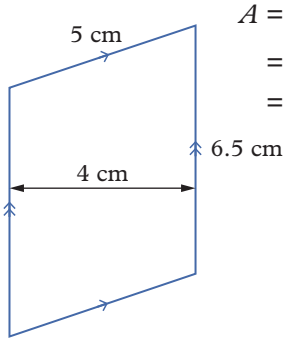
5 km = 5 × 1000
= 5000 m

### Perimeter



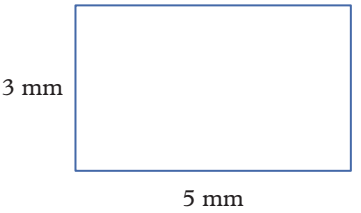

$P$ = 30 mm + 4 cm + 5 cm
= 3 cm + 4 cm + 5 cm
= 12 cm

### Area of a parallelogram



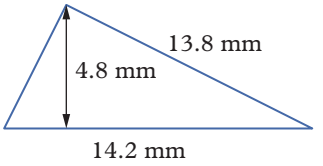

$A = bh$
$= 6.5 \times 4$
$= 26\ \text{cm}^2$

### Area of a rectangle



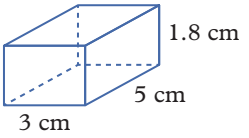

$A = lw$
$= 3 \times 5$
$= 15\ \text{mm}^2$

### Area of a triangle



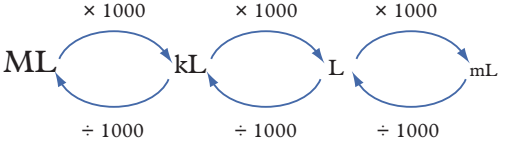

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 14.2 \times 4.8$
$= 34.08\ \text{mm}^2$

### Volume of a rectangular prism



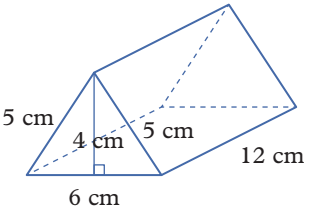

$V = lwh$
$= 5 \times 3 \times 1.8$
$= 27\ \text{cm}^3$

### Capacity




### Surface area

**1** Determine the number of faces.
**2** Calculate the area of each face.
**3** Add the area of the faces together.




Area of triangular faces $= \frac{1}{2} \times 4 \times 6$
$= 12\ \text{cm}^2$

Area of rectangular sides $= 5 \times 12$
$= 60\ \text{cm}^2$

Area of base $= 6 \times 12$
$= 72\ \text{cm}^2$

TSA $= 2 \times 12 + 2 \times 60 + 72$
$= 216\ \text{cm}^2$

# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

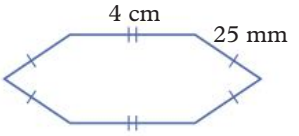
**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

9A **1** Which length measurement is equivalent to 123 cm?

**A** 1230 mm **B** 12.3 m **C** 0.123 km **D** 12 300 m **E** 1.23 km

9A **2** The perimeter of this shape is:



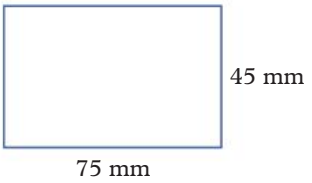

**A** 6.5 cm **B** 108 mm **C** 18 cm
**D** 29 mm **E** 10 cm

9B **3** The area of a rectangle with length 9 cm and width 3 cm is:

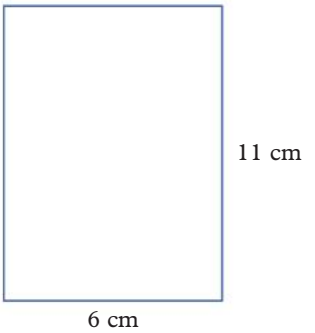
**A** 6 $cm^2$ **B** 12 $cm^2$ **C** 13.5 $cm^2$ **D** 24 $cm^2$ **E** 27 $cm^2$

9B **4** Which of these rectangles has the largest area?

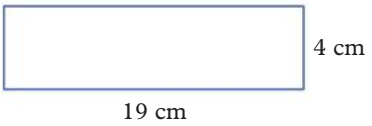
**A**



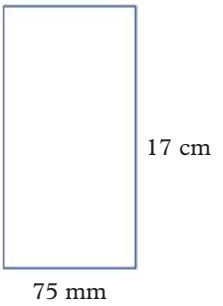

**B**



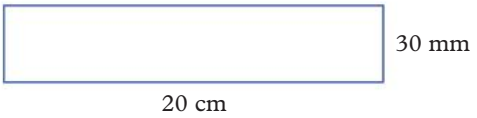

**C**



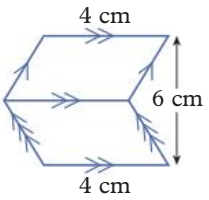

**D**




**E**

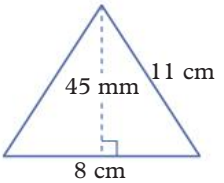



9C **5** The area of this shape is:




**A** 14 $cm^2$ **B** 24 $cm^2$ **C** 48 $cm^2$ **D** 49 $cm^2$ **E** 144 $cm^2$

9D **6** The base length and height of this triangle respectively are:




**A** 8 cm, 4.5 cm
**B** 8 cm, 11 cm
**C** 11 cm, 4.5 cm
**D** 4.5 cm, 8 cm
**E** 11 cm, 8 cm

9D **7** What is the height of a triangle with a base length of 22 mm and an area of 6.16 $cm^2$?

**A** 5.6 mm **B** 5.6 cm **C** 27.104 cm **D** 56 cm **E** 271.04 cm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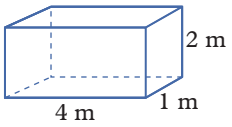
9E **8** The surface area of this rectangular prism is:

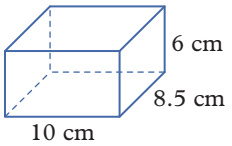



**A** 8 $m^2$ **B** 14 $m^2$ **C** 16 $m^2$ **D** 28 $m^2$ **E** 56 $m^2$

9F **9** Which pair of volume and capacity measurements are equivalent?

**A** 40 mL and 4 $cm^3$ **B** 1 $m^3$ and 1 L **C** 250 mL and 2.5 L
**D** 500 $cm^3$ and 0.5 L **E** 150 mL and 1.5 $mm^2$

9F **10** What is the volume of this rectangular prism?




**A** 60 $cm^3$ **B** 85 $cm^3$ **C** 196 $cm^3$ **D** 392 $cm^3$ **E** 510 $cm^3$

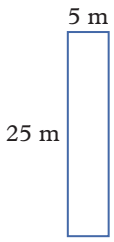
## Short answer

9A **1** Convert these units.

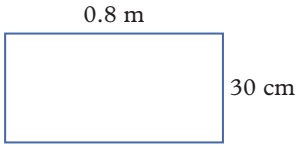
**a** 34 cm into mm **b** 170 cm into m
**c** 45 mm into cm **d** 8.7 km into m

9A **2** Calculate the perimeter of each rectangle.

**a**

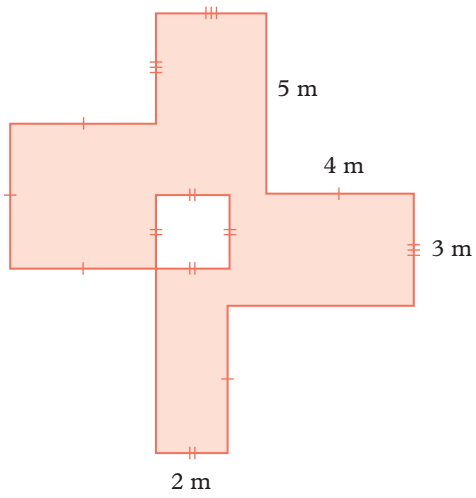



**b**



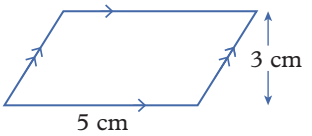

9B **3** Calculate the area of each rectangle shown in question **2**.

9B **4** Calculate the area of this composite shape.

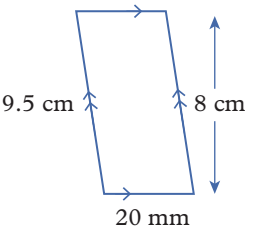


9C **5** Calculate the area of each parallelogram.

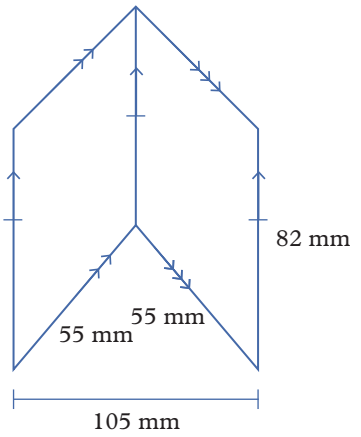
**a**




**b**

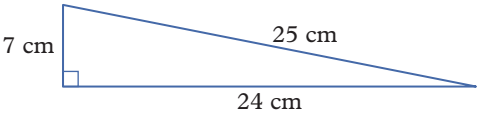



9C **6** Calculate the area of this composite shape.

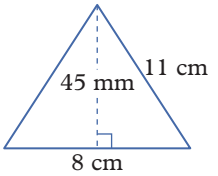



9D **7** Calculate the area of each triangle.

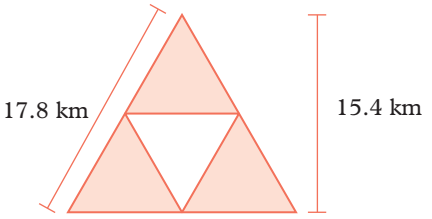
**a**




**b**

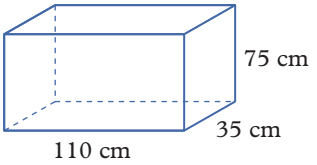



9D **8** Calculate the area of this composite shape consisting of equilateral triangles.

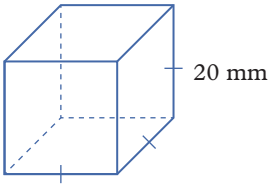



9E **9** Calculate the surface area of each object.

**a**



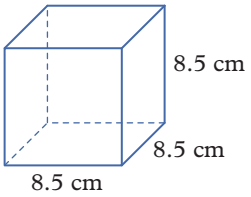

**b**



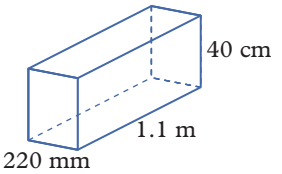

9F **10** Calculate the volume of each object shown in question **9**.

9F **11** Calculate the volume of each object.

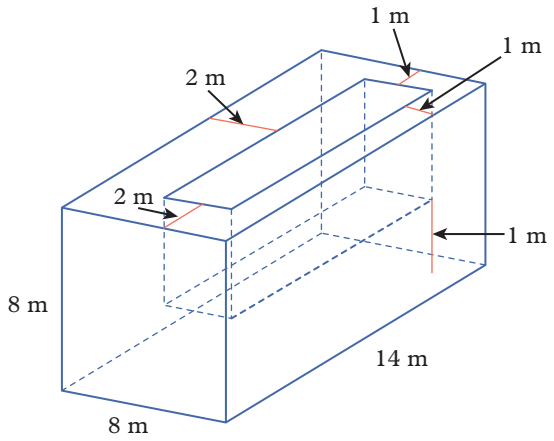
**a**




**b**

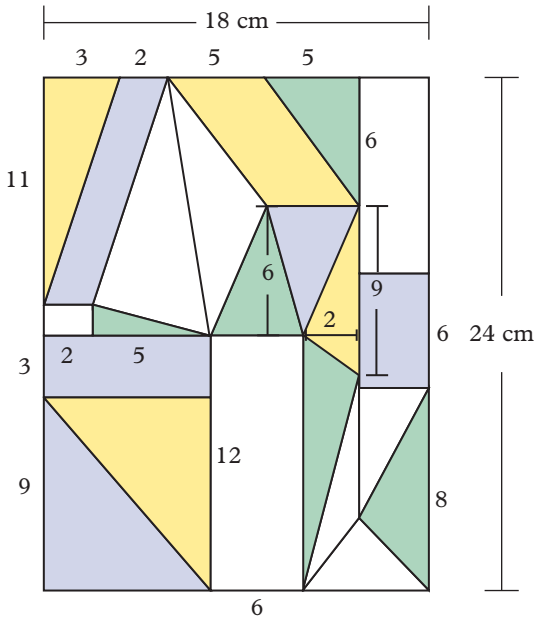


9F **12** Calculate the capacity of this trough.



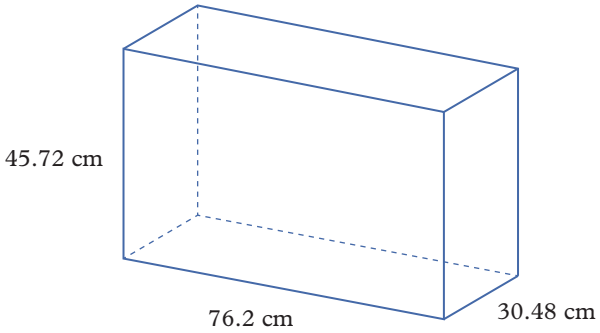

## Analysis

**1** Gertrude is crafting a geometric painting using yellow and blue paints, which she mixes to make a green paint. She plans to use an 18 cm × 24 cm canvas. Her design is shown on the right. Note: all lengths are in cm.

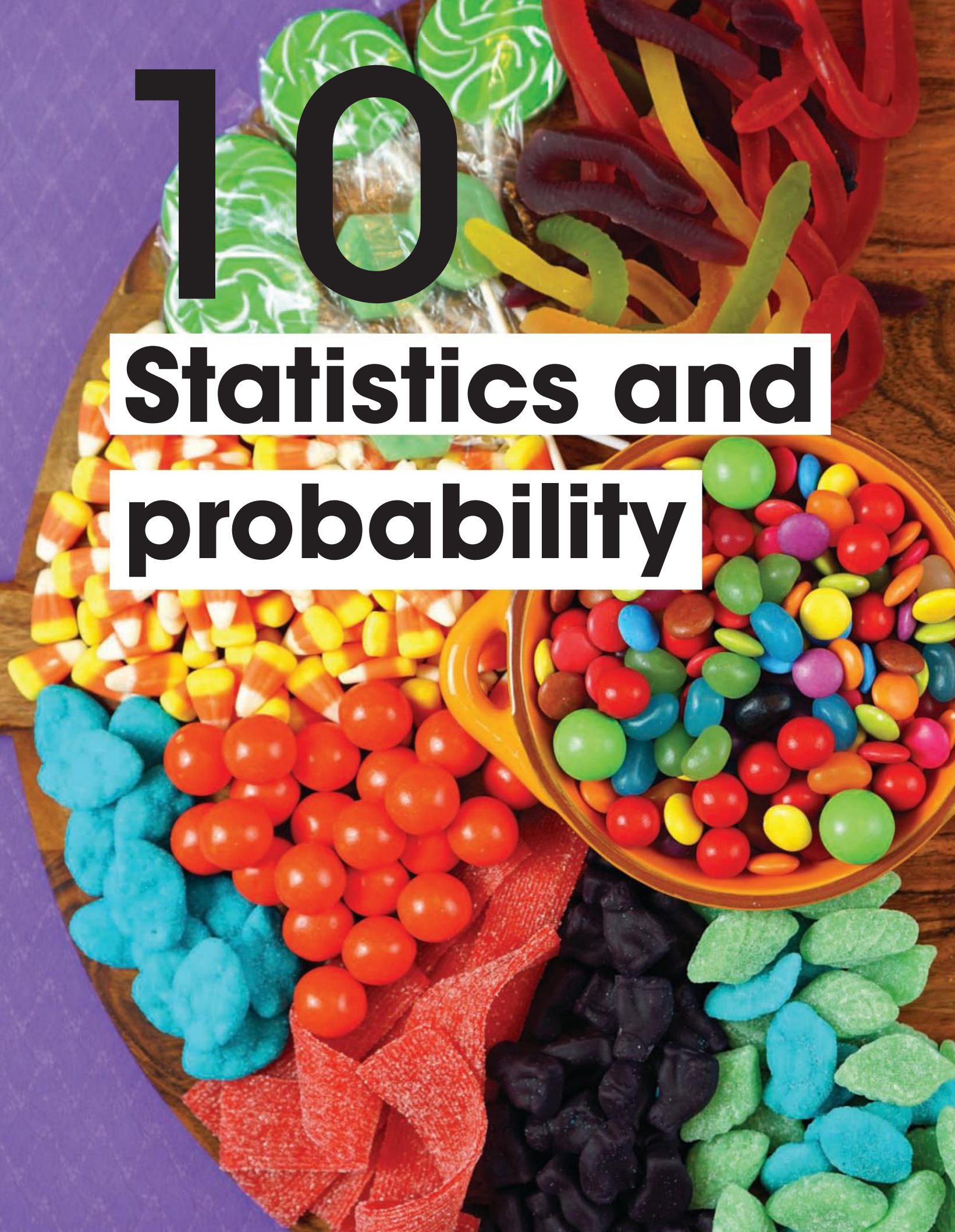



- **a** Calculate the total yellow area.
- **b** Calculate the total blue area.
- **c** Calculate the total green area.
- **d** Calculate the total unpainted (white) area.
- **e** Gertrude bought 0.25 litres of yellow paint and half a litre of blue paint. How much paint does she have in cubic centimetres?
- **f** If the yellow and blue paint is mixed in the ratio 1 : 2 to make green paint, how much yellow and blue paint does she mix to make 0.45 L of green paint?
- **g** Gertrude gets her painting framed with a 2 cm wide frame. What is the perimeter of the framed painting?

**2** Alicia buys a new fish tank. The tank has glass sides and base but has a plastic lid with UV lights built in.




- **a** Determine the exterior surface area of the glass on the fish tank to the nearest square centimetre.
- **b** If the glass is 14 mm thick, determine the capacity of the open tank to the nearest litre.
- **c** Alicia reads a recommendation that for each 1.5 L of water you can have up to 1 cm of fish length. What is the maximum amount of fish length Alicia's new tank can hold?



# 10 Statistics and probability

## Index


**10A** Collecting and classifying data
**10B** Summary statistics
**10C** Constructing and interpreting graphs
**10D** Stem-and-leaf plots
**10E** Describing probability
**10F** Theoretical probability
**10G** Experimental probability


## Prerequisite skills

**Diagnostic pre-test**
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

**Interactive skillsheets**
After completing the diagnostic pre-text, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Interpreting graphs
- ✔ Fractions, decimals and percentages
- ✔ Calculating percentages
- ✔ Multiplying and dividing fractions

## Curriculum links

- Construct sample spaces for single-step experiments with equally likely outcomes (VCMSP266)
- Assign probabilities to the outcomes of events and determine probabilities for events (VCMSP267)
- Identify and investigate issues involving numerical data collected from primary and secondary sources (VCMSP268)
- Construct and compare a range of data displays including stem-and-leaf plots and dot plots (VCMSP269)
- Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (VCMSP270)
- Describe and interpret data displays using median, mean and range (VCMSP271)

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## Materials

- ✔ Ruler
- ✔ Protractor
- ✔ Calculator

# 10A Collecting and classifying data

## Learning intentions

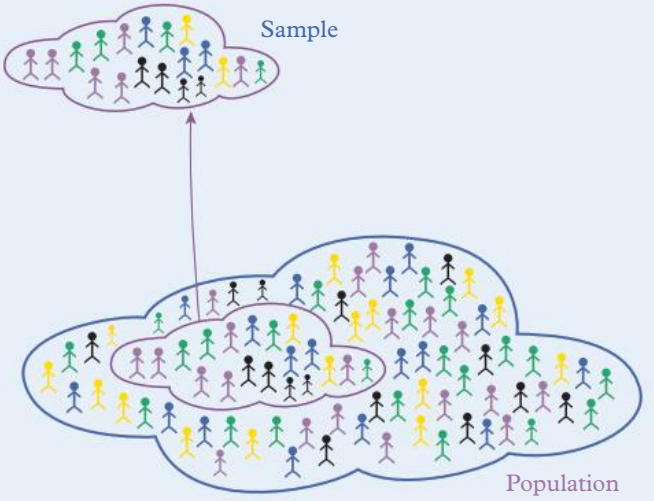
- ✔ I can classify data.
- ✔ I can record data in a frequency table.
- ✔ I can differentiate between the population and a sample.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Collecting data |
| Year 8 | 9A Collecting data and sampling methods |
| Year 9 | 8A Classifying and displaying data |

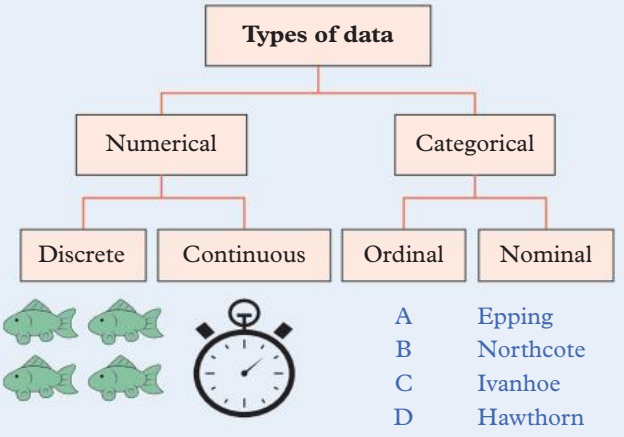
## Collecting data

- When collecting **data**, the **population** refers to all potential pieces of data under consideration.
- A **census** is a survey of an entire population.
- A **sample** is a selection of data that is part of the population.
  - → Samples are often used instead of censuses because it is quicker, cheaper and more practical to take samples.
- When you collect your own data, it is called **primary data**. If you use somebody else's data, it is called **secondary data**.




## Classifying data

- **Numerical data** can be counted or measured. There are two types of numerical data: discrete and continuous.
  - → **Discrete data** can be counted in regular intervals; for example, the number of fish in an aquarium.
  - → **Continuous data** can be measured; for example, the time it takes to listen to a song.
- **Categorical data** can be put into categories. There are two types of categorical data: ordinal and nominal.
  - → **Ordinal data** can be put in order; for example, student exam results ordered by letter grades.
  - → **Nominal data** can be put into unordered categories; for example, the suburbs Year 7 students live in.





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- The **frequency** of a data value is a measure of how often a piece of data occurs in a sample.
- A **frequency table** can be used to tally frequencies in a data set.
  → For example, this frequency table shows a group of 18 people divided by age.

| Age | Tally | Frequency |
|---|---|---|
| 11 | 𝍸 \| | 6 |
| 12 | \|\|\| | 3 |
| 13 | \|\| | 2 |
| 14 | 𝍸 \|\| | 7 |
| **Total** | | **18** |

## Example 10A.1 Identifying primary and secondary data

Determine whether the following situations involve primary or secondary data.

**a** Using data from the Australian Bureau of Statistics (ABS) website to write a report about house prices in Australia.

**b** Conducting a survey to find out which football teams students support.

**THINK**

Consider whether the data used is collected first-hand, or whether someone else's data is being used.

**WRITE**

**a** The ABS collected the data, so this is an example of secondary data.

**b** The data is collected first-hand; this is an example of primary data.

## Example 10A.2 Classifying data

Classify the following data.

**a** The heights of a class of 26 Year 7 students in cm.

**b** Opinions on a local issue, ranging from strongly disagree to strongly agree.

**THINK**

**a** Consider whether the data can be put into categories, or if it can be counted or measured. Heights in cm are measured, so this is numerical, continuous data.

**b** **1** Consider whether the data can be put into categories, or if it can be counted or measured. Opinions can be put into categories, so this is categorical data.

**2** Consider whether these categories can be ordered or if they are unordered. These opinions can be ordered, so this is ordinal data.

**WRITE**

**a** Heights of students are numerical, continuous data.

**b** These opinions are categorical, ordinal data.

## **Example 10A.3** Recording data in a frequency table

Alexis asked her friends what their favourite chocolate bar was out of Time Out, Crunchie, Picnic and Mars Bar. She collected the following data.

Time Out, Crunchie, Picnic, Mars Bar, Crunchie, Mars Bar, Mars Bar, Time Out, Mars Bar, Picnic, Picnic, Picnic, Mars Bar, Time Out, Mars Bar

**a** Record the data in a frequency table.

**b** How many people were surveyed?

**c** What was the most popular chocolate bar?

**THINK**

**a** **1** Create a table with three columns headed: Category, Tally and Frequency.

**2** Enter the types of chocolate into the category column.

**3** Go through the list and add a tally mark in the appropriate row of the tally column for each piece of data. Every fifth tally mark should cross over the previous four marks to form groups of five.

**4** Write the final number for each chocolate bar in the frequency column.

**b** The total of the frequency column is the number of people who were surveyed.

**c** The category with the highest frequency represents the most popular chocolate bar.

**WRITE**

**a**

| Category | Tally | Frequency |
|---|---|---|
| Time Out | \|\|\| | 3 |
| Crunchie | \|\| | 2 |
| Picnic | \|\|\|\| | 4 |
| Mars Bar | ~~\|\|\|\|~~ \| | 6 |
| **Total** | | **15** |

**b** 15 people were surveyed.

**c** Mars Bar was the most popular chocolate bar.

**Helpful hints**

✔ Be careful, as categorical data may also involve numbers. For example, Australian postcodes consist of numbers, but these numbers represent a region, or category, as opposed to a numerical value. If it doesn't make sense to use it in a calculation, it is not numerical data!

✔ Always check your frequency table entries by ensuring the sum of the frequency column equals the total number of data points.

ANS p602

# Exercise 10A Collecting and classifying data

△ 1, 2, 3(a, b, c, g), 4–6, 7(a, b), 8(a–c), 9(a, b), 10(a, c), 11

□ 1, 2a(v–viii), b, c, 3, 4, 6, 7(c, d), 8, 9(c–e), 10(c, d), 12, 13, 14(a, b)

⬠ 1(d, e), 3, 4, 7, 8, 9(d, e), 10(c, d), 12, 14, 15

UNDERSTANDING AND FLUENCY

10A.1 **1** Determine whether the following situations involve primary or secondary data.

**a** Surveying pedestrian traffic by counting how many people pass an intersection when the lights change.

**b** Measuring the lung capacity of students across different year levels.

**c** Using the internet to research the weights of various animals.

**d** Using data from an environmental report to analyse how our climate is changing.

**e** Creating an online poll to research people's favourite hobbies.

**2** **a** A student surveys their classmates to collect the following data. Classify each set of data as either numerical or categorical.

**i** Weight

**ii** Favourite animal

**iii** Grandparents' nationality

**iv** Time spent on the internet per week

**v** Hair colour

**vi** Number of pens in your pencil case

**vii** How much people like dogs on a scale of 1–10

**viii** Hand span

**b** Classify each example of numerical data as either discrete or continuous.

**c** Classify each example of categorical data as either nominal or ordinal.

10A.2 **3** Classify the following data as numerical or categorical, and as discrete, continuous, ordinal or nominal.

**a** Screen size of a mobile phone

**b** Number of people at a birthday party

**c** Favourite sports

**d** Amount of screen-time per week

**e** How much people like exercise on a scale of 1–10

**f** Number of runs scored in a game of cricket

**g** Size of a takeaway coffee

**h** Number of spots on a leopard

**4** Give two of your own examples for each of these data categories.

**a** Discrete, numerical data

**b** Continuous, numerical data

**c** Ordinal, categorical data

**d** Nominal, categorical data

UNDERSTANDING AND FLUENCY

**5** Consider the frequency table below.

**a** What type of data is movie genres?

**b** Which movie genre is:

**i** most popular **ii** least popular?

| Movie genre | Tally | Frequency |
|---|---|---|
| Action | 𝍸 \|\| | 7 |
| Comedy | 𝍸 𝍸 𝍸 | 15 |
| Horror | \|\| | 2 |
| Animated | 𝍸 𝍸 \|\| | 12 |

10A.3 **6** Harry conducts a survey to find out how many siblings each member of his class has and collects the following data.

1 2 1 0 2 2 4 1 2 1 1 3 1 2 3 0 1 2 1 1 2 2 8 1 2 1

**a** Record the data in a frequency table.

**b** How many people were surveyed?

**c** What was the most common number of siblings?

**7** Jasmine completes four different surveys. In each instance determine:

**i** the type of data collected

**ii** whether Jasmine is taking a sample or carrying out a census.

**a** Researching birthday months in her class by asking all students in her class

**b** Researching favourite sports in Year 7 statewide by asking all students in her class

**c** Researching height in her class by asking four of her classmates

**d** Researching the number of languages spoken at school by asking each person in the school

PROBLEM SOLVING AND REASONING

**8** Ryan grabs a handful of Fruit Loops from a box.

**a** Create a frequency table showing the frequency of the different colours in the handful.

**b** What is the most common colour?

**c** What is the least common colour?

**d** There are four pink Fruit Loops in this sample of 24. How many might you expect to find in the box if it contains 3000 Fruit Loops?

**e** Describe one advantage and one disadvantage of taking a sample of 24 Fruit Loops rather than completing a census of the entire box of 3000 Fruit Loops.

**9** Decide whether it would be better to use a primary or secondary source of data to research the following. Explain the reasoning behind your decision.

**a** Shoe sizes of Year 7 students in your school

**b** Shoe sizes of 13-year-olds across Australia

**c** Attendances at AFL games in 2021

**d** Most popular fast food outlet of students in your class

**e** Average income of workers in Victoria

**10** Give a reason why it would be more practical to take a sample rather than completing a census in the following situations.

**a** Surveying fish populations in the Pacific Ocean

**b** Identifying Victorian Year 7 students' favourite YouTube personality

**c** Investigating the length of pieces of grass in a field

**d** Estimating the most frequent number of words for questions in this textbook

**11** Jaime measures the heights of his Year 7 class at the start of the year. Rosa measures the heights of her Year 7 class at the end of the year. Jaime and Rosa found that Rosa's class was generally taller than Jaime's class. What bias has affected their conclusion?

**12** Kate surveys members of her soccer team regarding their favourite sports and presents the data to her class as a sample of the favourite sports of Year 7 students. Explain why Kate's choice of sample may not be representative of the population.

**13** The AFL ladder shown below displays the position of the top eight football teams at the end of a season. Each team's position on the ladder is indicated by the 1st, 2nd, 3rd, etc. labels – these are called ordinal numbers.

| **AFL 2019 Final Ladder** | |
|---|---|
| 1st | Geelong |
| 2nd | Brisbane Lions |
| 3rd | Richmond |
| 4th | Collingwood |
| 5th | West Coast |
| 6th | Greater Western Sydney |
| 7th | Western Bulldogs |
| 8th | Essendon |

**a** Why do you think they are called ordinal numbers?

**b** Explain why team ranking is an example of categorical data and not numerical data, even though it involves numbers.

**c** Provide another example of categorical data that involves numbers.

**14** Classify the following data.

**a** Home postcodes

**b** Shoe sizes

**c** Percentage on a test above 50% or below 50%

**d** Number of days in a month

**e** Favourite number

**15** For each of the following write:

**i** a possible unit of measure or counting if the data is numerical

**ii** possible categories if the data is categorical.

**a** Size of a suburb

**b** Amount of traffic at an intersection

**c** Number of bees in a beekeeper's hives

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Classifying data

**Investigation** Sales analysis

**Worksheet** Presenting data in a frequency table

**Topic quiz** 10A

# 10B Summary statistics

## Learning intentions

- ✔ I can calculate the mean, median, mode and range of a numerical data set.
- ✔ I can interpret summary statistics in the context of the data.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Understanding data |
| Year 8 | 9B Summary statistics |
| Year 9 | 8C Summary statistics from tables and displays |

## Summary statistics

- **Summary statistics** provide an overview of numerical data and can be used to make quick comparisons between data sets.
- The **mean** is a numerical average of the values in a data set. To find the mean, add all the numbers together, then divide by how many numbers are in the set.

  For example, for the data set 1, 2, 3, 4, 6:

$$\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$
$$= \frac{1 + 2 + 3 + 4 + 6}{5}$$
$$= 3.2$$

- The **median** is the middle of a data set when the numbers are listed in order. The median divides a data set in half.
  - → If there is an even number of values in a data set, the median will be the average of the two middle values.

  For example,

$$2, 3, 4, 5, 6 \qquad 1, 2, 3, 4, 5, 6$$
$$\text{Median} = 4 \qquad \text{Median} = \frac{3 + 4}{2} = 3.5$$

- The **mode** is the most common value in a numerical data set. There can be no mode or more than one mode, and if there are no repeated values, there is no mode.

  For example,

| | | |
|---|---|---|
| 1, 2, 3, 4, 4, 5 | 1, 2, 2, 3, 4, 4, 5 | 1, 2, 3, 4, 5, 6 |
| Mode = 4 | Mode = 2, 4 | No mode |

- The **range** is the distance between the largest (maximum) and smallest (minimum) values in a data set.

  Range = maximum value – minimum value
- An **outlier** is an extreme data point that is much higher or lower than the other values in a data set. For example, 100 is an outlier in the data set 1, 2, 3, 3, 100.


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## **Example 10B.1** Calculating summary statistics

The lengths in cm of a school of fish are listed below.

6, 7, 13, 7, 12, 7, 8, 10, 11

**a** Find the mean.

**b** Find the mode.

**c** Find the median.

**d** Find the range.

**e** A fish of length 15 cm is added to the data set. What is the new median?

**THINK**

**a** Add the numbers together and divide by the total number of values.

**b** List the numbers in the data set in ascending order and identify the number which occurs most often.

**c** List the numbers in the data set in ascending order and find the middle value.

**d** Identify the minimum and maximum values of the data set. Subtract the minimum value from the maximum value.

**e** List the numbers in the data set in ascending order and find the middle value. As there are two middle values, add them together and divide the total by 2.

**WRITE**

**a** $\text{Mean} = \dfrac{\text{sum of all data points}}{\text{total number of data points}}$

$= \dfrac{6 + 7 + 13 + 7 + 12 + 7 + 8 + 10 + 11}{9}$

$= \dfrac{81}{9}$

$= 9$

**b** 6, 7, 7, 7, 8, 10, 11, 12, 13

Mode = 7

**c** 6, 7, 7, 7, 8, 10, 11, 12, 13

Median = 8

**d** Range = maximum value − minimum value

$= 13 - 6$

$= 7$

**e** 6, 7, 7, 7, 8, 10, 11, 12, 13, 15

$\text{Median} = \dfrac{8 + 10}{2}$

$= 9$

**Helpful hints**

- ✔ Order your data sets first before calculating the median and mode. This will decrease the likelihood of errors!
- ✔ When using a calculator to find the mean find the value of the total first before dividing. You can use brackets around the addition to ensure the sum is divided, not just the last number.
- ✔ A good method for making sure all data points are included when you re-order a data set is to put a dot above or below the values as you list them, and to count the number of data points at the end.

  For example,

  $\dot{4}, 5, \dot{2}, 7, \dot{1}, 5$

  1, 2, 4, _, _, _

ANS p603

# Exercise 10B Summary statistics

△ 1, 2, 3(a, c, e, g), 4, 5, 7, 8, 11, 13 ▢ 2, 3(b, d, g, h), 4, 6, 9, 10, 12, 15, 18(a) ⬠ 3(d, g, h), 4(c, d), 5, 9, 14–17, 18(b), 19

UNDERSTANDING AND FLUENCY

**10B.1** **1** A zoologist collected the following data on the number of young in the litters of several fat-tailed dunnarts.

7 8 6 7 6 7 9 8 15 7 8

**a** Find the mean.

**b** Find the mode.

**c** Find the median.

**d** Find the range.

**e** A new litter with 10 young is added to the data set. What is the new median?

**2** Information was collected on the number of online games students play in a normal week. The results are below.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 4 | 2 | 1 | 1 | 3 | 2 | 1 | 0 | 0 | 2 | 1 | 0 |
| 1 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 1 | 1 | 1 | 0 | 0 |

**a** Find the mean correct to two decimal places.

**b** Find the median.

**c** Find the mode.

**d** Find the range.

**3** Find the **i** mean, **ii** median, **iii** mode and **iv** range for each data set. Round your answers to two decimal places where appropriate.

**a** 4, 8, 2, 5, 3, 7, 5, 7, 7, 7, 1, 2, 6

**b** 1, 1, 1, 2, 5, 6, 8, 4, 5, 8, 6, 9, 6, 4

**c** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

**d** 4, 7, 5, 8, 3, 6, 9, 9, 1, 0, 3, 5, 4, 7, 2

**e** 4, 8, 3, 7, 11, 15, 4, 9, 5

**f** 17, 13, 11, 7, 5, 3, 2

**g** 11, 17, 13, 9, 13, 5, 18, 12

**h** 8, 5, 8, 6, 8, 1, 93, 5, 5, 5, 8

**4** What will be the position of the median in an ordered set of data containing:

**a** 7 data points? **b** 8 data points?

**c** 13 data points? **d** 22 data points?

**5** Match each summary statistic to the correct data set.

**a** Mean = 9 **A** 15, 8, 4, 7, 3, 10, 1

**b** Mode = 4 **B** 11, 4, 6, 8, 3

**c** Median = 7 **C** 3, 17, 8, 6, 14, 8, 7

**d** Range = 8 **D** 1, 7, 4, 5, 9, 4, 2, 3, 3, 4, 10

**6** A bag of 8 tangerines weighs 720 g.

**a** What is the mean weight of the tangerines in the bag?

**b** A tangerine is added to the bag. State whether the mean increases, decreases, or stays the same if the tangerine weighs:

**i** 120 g **ii** 90 g **iii** 60 g **iv** 91 g

**7** Dorothy measured the heights of all her classmates and calculated the mean, median, mode and range of the heights. To write up her report, she needs to interpret the statistics by writing them in a sentence that describes what the statistics mean. State the statistic each statement is interpreting.

**a** The heights of my classmates were spread out over a 50 cm interval.

**b** The most common height was 150 cm.

**c** Half of the class had heights greater than 155 cm.

**d** If everyone was the same height, but the total height stayed the same, then everyone would be 157 cm.

**8** In a science experiment, different model carts are built to see how far they will move after the same force is applied to them. The following statistics were obtained.

| Mean | Median | Mode | Range |
|---|---|---|---|
| 6.4 m | 6.2 m | 4.0 m | 10.1 m |

Write a sentence to interpret each of the statistics.

**9** A group of Year 7 students were asked how much money they spend on their mobile phones every month. The results, in dollars, are:

20, 50, 30, 10, 30, 20, 20, 15, 20, 30, 20, 20, 25.

**a** Find the mean, median, mode and range of the data set.

**b** Use your answers from part **a** to write a three-sentence description of the data.

**10** After finishing the survey in question **9**, students were then asked how much pocket money they earned per week. The results, in dollars, were:

5, 5, 10, 20, 5, 5, 0, 5, 0, 5, 10, 5, 100.

**a** Find the mean, median and mode of the data set.

**b** What is the difference in value between the mean and median?

**c** Explain why the mean is not a good indicator of the centre of this data set.

**d** Would any of these students be able to afford to pay for their phone bill from question **9** using just their pocket money? Explain.

**11** A company claims that the average salary of one of their employees is $80 000. One of the employees, who earns much lower than this, seems suspicious of the average and decides to ask some of her colleagues what they earn. She gathers the following salaries, including her own.

$42 000, $51 000, $39 000, $40 000, $45 000, $60 000, $25 000, $33 000

**a** Calculate the mean and median salary of this sample of employees.

**b** Based on your answer from part **a**, do you think the claim that the average salary is $80 000 is reasonable? Why or why not?

**c** It turns out that the chief executive officer and chief financial officer earn much more than the rest of the company.

**i** Which average was probably used to calculate the $80 000 figure? Why?

**ii** Which summary statistic would be a better measure of the average salary? Why?

**12** At the end of the year, two students received their exam scores for each of their subjects.

Yolanda's scores: 40%, 41%, 43%, 44%, 42%, 95%

Zoe's scores: 99%, 97%, 98%, 96%, 94%, 42%

Their parents asked how they went on average this year.

**a** To one decimal place, calculate the mean and median score:

**i** Yolanda received **ii** Zoe received.

**b** To make themselves look better, which average (mean or median) should:

**i** Yolanda use? **ii** Zoe use?

**c** To give a more representative statistic, which average (mean or median) should:

**i** Yolanda use? **ii** Zoe use?

**13** Two friends were comparing the delays on their train lines. Dylan thought that his train was the worst because it once ran 20 minutes late. Costa argued that his train was always late but could only remember it being at worst 10 minutes late. They decided to record how many minutes late their trains were for two weeks. The results are below.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dylan: | 5 | 3 | 1 | 4 | 2 | 2 | 5 | 15 | 20 | 3 | 2 | 4 | 2 | 3 |
| Costa: | 4 | 6 | 8 | 5 | 6 | 7 | 7 | 9 | 8 | 8 | 7 | 9 | 9 | 9 |

**a** Whose train line had the largest range of delays? What was the range of this line's delays?

**b** Calculate the mean delay for each train correct to two decimal places.

**c** Whose train do you think is the most reliable? Justify your decision using the results from parts **a** and **b**.

**14** Three zoologists examined 15 cheetahs to determine how many spots each cheetah had on its coat. The results of their survey are listed below. One zoologist claimed that the average number of spots is 466, another calculated the average of the data set to be 456, and the third said that it is 472. Use your knowledge of statistics to discuss why all three are correct.

450 495 413 456 517 472 436 485 476 499 486 444 456 424 481

**15** Two local footballers were competing to be the leading goalkicker in the Central Suburbs Football League.
The number of goals each player scored in the 15 games of the season is listed below.

Player 1: 3, 5, 3, 2, 6, 4, 3, 1, 4, 4, 5, 3, 3, 2, 3

Player 2: 1, 3, 2, 4, 1, 4, 3, 1, 13, 4, 3, 5, 1, 2, 5

**a** Which player won the leading goalkicker award by kicking the most goals in the season?

**b** Find the mean, median and mode number of goals kicked by each player. Give your answers to two decimal places where necessary.

**c** Which player was the more consistent goalkicker? Explain your answer.

**d** There is an outlier in the number of goals Player 2 kicked. Identify the outlier and explain why this value is an outlier.

**e** Recalculate the mean and median number of goals kicked for Player 2 with the outlier removed from the data set. Give your answers to two decimal places where necessary.

**f** Considering your answers to parts **b** and **e**, discuss how the outlier affected the mean and median values.

**16** A recent census found there is an average of 1.97 children in every Australian family.

**a** Which summary statistic do you think they used to get this number? Why?

**b** Braeden is confused as to how a family can have 1.97 children and how it is possible that every family has the same number of children. Explain the meaning of this average to Braeden.

**17** There are five values in a data set. They are all different positive integers. The median and the mean are both 4.

**a** Find all potential data sets that meet these criteria.

**b** Explain how you can be sure that you have found all potential data sets.

**18** Find the **i** mean, **ii** median, **iii** mode, and **iv** range for each data set. Round your answers to two decimal places where appropriate.

**a** 7.4, 6.8, 9.2, 11.3, 8.1, 6.8, 4.6, 13.3, 10.5, 5.3

**b** 22.4, 28.0, 33.3, 19.7, 38.5, 42.9, 29.5, 21.3, 20.8, 16.4, 45.2, 51.1

**19** Find the **i** mean, **ii** median, **iii** mode, and **iv** range for each data set. Round your answers to two decimal places where appropriate.

**a**

| **Score** | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| **Frequency** | 2 | 3 | 3 | 4 | 6 |

**b**

| **Score** | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|
| **Frequency** | 1 | 3 | 2 | 5 | 3 | 5 |

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** The mean

**Interactive skillsheet** The median

**Interactive skillsheet** The mode

**Interactive skillsheet** The range

**Investigation** Travel calculations

**Topic quiz** 10B

# 10C Constructing and interpreting graphs

## Learning intentions

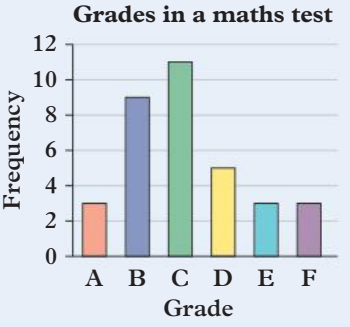
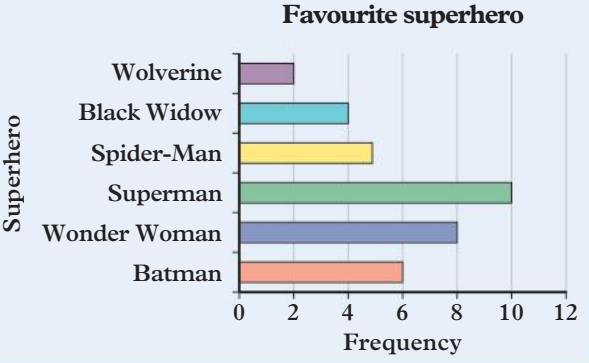
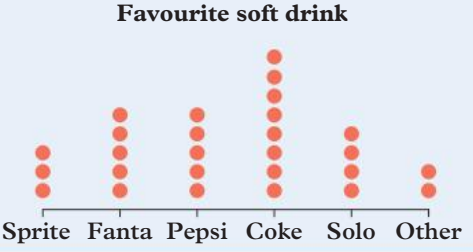
- ✔ I can construct column graphs, bar charts and dot plots.
- ✔ I can choose the most appropriate graph to represent data.
- ✔ I can locate the mean, median, mode and range on graphs.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Data displays |
| Year 8 | 9C Presenting data |
| Year 9 | 8D Grouped data and histograms |

## Types of graphs

- **Column graphs**, **bar charts** and **dot plots** can all be used to display either discrete numerical or categorical data.

| Graph type | Features | Example |
|---|---|---|
| Column graph | • Categories or values are represented using vertical columns<br>• The height of each column indicates the frequency of each category or value |  |
| Bar charts | • Horizontal representation of a column graph |  |
| Dot plots | • The frequency of a category or value is indicated using dots |  |

# Constructing graphs

- Column graphs and bar charts should always include a title and clearly labelled axes, including any units.
  - → For the frequency axis, an **interval** should be chosen so that the frequency of each category is clear. Intervals of 2, 5 or 10 are common.
  - → The scale of the axes should be consistent and set to accommodate the minimum and maximum frequencies.
- For dot plots:
  - → The dots should be equally sized.
  - → The dots should be vertically aligned in columns and equally spaced vertically.
  - → The spacing on the horizontal axis should be consistent.

# Choosing which graph type to use

- Dot plots should only be used for relatively small data sets.
- Categories with long names are better suited to bar charts rather than column graphs.

# Interpreting graphs

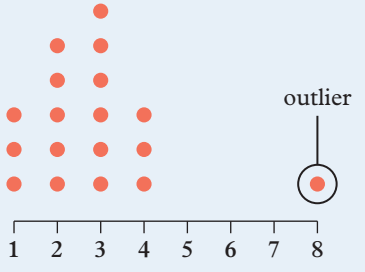
- A graph is an effective way to display data in an easy-to-read, visual format.
- Graphs can be used to help make conclusions about data sets.

  For example, the column graph in the table shows that B or C grades were a lot more common than the other grades.

# Summary statistics in graphs

- The mean can be calculated by identifying the individual data values from a graph, and then using the formula.
- The mode is the most common piece of data. It is the tallest column in a column graph.
- The median can be found by identifying the individual data pieces on a graph, and then finding the middle piece.
- The range is found by subtracting the maximum data value from the minimum data value.
- Outliers are easier to identify when numerical data is presented in a graph.

**Questions answered in five minutes**



## Example 10C.1 Reading a column graph

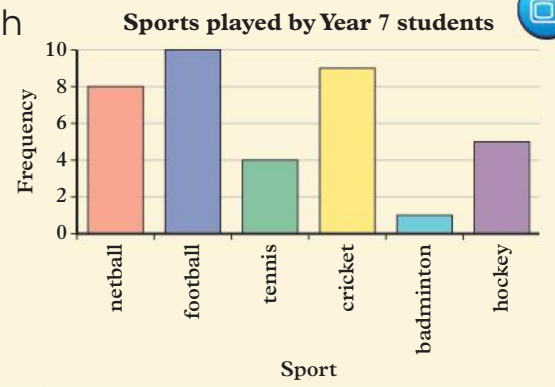
This column graph shows the results of a survey of Year 7 students and the sports they play.

**a** What is the most common sport played and how many students play it?

**b** What is the least common sport played and how many students play it?

**c** How many sports are in the survey?

**d** How many students were surveyed?




**THINK**

**a** The most common sport has the tallest column. Read the height of the column off the vertical axis.

**b** The least common sport has the shortest column. Read the height of the column off the vertical axis.

**c** Count the number of categories listed along the horizontal axis.

**d** Read the frequency of each column and add them together.

**WRITE**

**a** Football is the most common sport. 10 students play football.

**b** Badminton is the least common sport. One student plays badminton.

**c** There are six types of sports.

**d** Total number of students surveyed
= 8 + 10 + 4 + 9 + 1 + 5
= 37

## Example 10C.2 Drawing a column graph

Sanam conducted a survey to find out which mobile phone network students in her school used and recorded the following results in a frequency table.

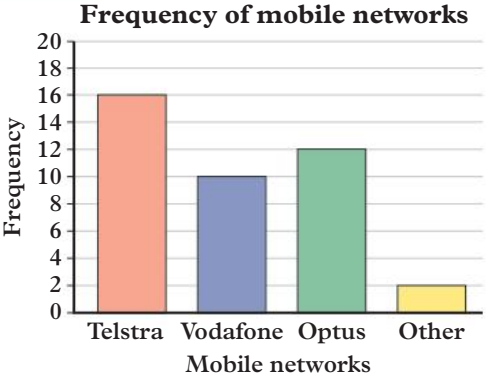
Create a column graph from the data.

| Mobile network | Frequency |
|---|---|
| Telstra | 16 |
| Vodafone | 10 |
| Optus | 12 |
| Other | 2 |

**THINK**

1. Identify the categories with the smallest and largest frequencies.
2. Draw the vertical axes. The top of the vertical scale should be a number that is at least equal to the largest frequency.
3. Choose an interval so that the frequency of each category is clear.
4. Draw the horizontal scale, providing enough space to fit the categories and allowing for space between each column.
5. Use a ruler to draw each column accurately.
6. Include a title and axis labels.

**WRITE**



## **Example 10C.3** Drawing a dot plot

A group of students were asked what their favourite colour is. Present the data below in a dot plot.
red, pink, green, red, pink, purple, blue, red, blue, green, purple, blue, blue, blue, green, blue, red, purple

**THINK**

1 Create a frequency table to record the data.

| Colour | Tally | Frequency |
|---|---|---|
| Red | \|\|\|\| | 4 |
| Pink | \|\| | 2 |
| Green | \|\|\| | 3 |
| Blue | ~~\|\|\|\|~~ \| | 6 |
| Purple | \|\|\| | 3 |

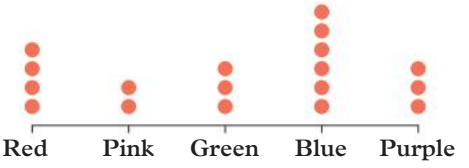
2 Draw the horizontal axis. The axis should be long enough to fit all the categories, and there should be enough space above the axis to place the dots.

3 Place the dots in columns above the appropriate category. The dots should be aligned with the categories and evenly spaced.

4 Include a title.

**WRITE**

**Favourite colour of students**




## **Example 10C.4** Identifying summary statistics from a dot plot

The following dot plot shows the number of attempts it took a group of learner drivers to pass their practical driving test.

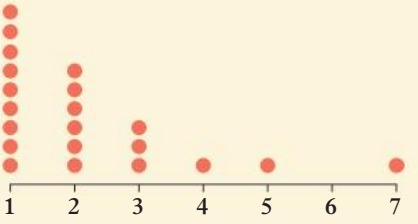
**a** How many learners were surveyed?

**b** What is the range of number of attempts?

**c** What is the mode number of attempts?

**d** What is the median number of attempts?

**Number of attempts taken to pass the practical driving test**




**THINK**

**a** Count the total number of dots on the plot.

**b** Subtract the lowest number of attempts from the highest number of attempts.

**c** Identify the number of attempts that has the highest frequency or number of dots.

**d** Count the dots to the median value of the data set. There are 21 values, so the median is the 11th value.

**WRITE**

**a** 21 learners were surveyed.

**b** Range = 7 − 1
= 6
The range of the number of attempts is 6.

**c** The mode number of attempts is 1.

**d** The median number of attempts is 2.

**Helpful hints**

- ✔ Watch out for graphs that are misleading! Graphs with frequency scales that do not start from zero and graphs with inconsistent scales can be particularly misleading.
- ✔ In a dot plot, the number of categories or values are shown on the horizontal axis and should not be confused with the frequency, which is represented by the heights of the columns of dots.
- ✔ When drawing an axis, make sure your scale covers the minimum and maximum frequencies. All values on your scale should also be easy to read.

ANS p604

# Exercise 10C Constructing and interpreting graphs

1–4, 7, 8, 10, 14, 15, 18 | 3, 5–9, 11, 12, 13, 15, 16, 19 | 5, 7–9, 11, 13, 14, 17, 20–22

UNDERSTANDING AND FLUENCY

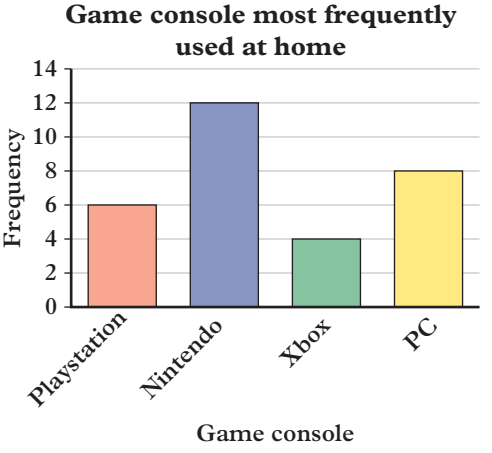
10C.1 **1** A class of Year 7 students were surveyed about what game consoles they use at home.

**a** What is the most common console and how many students use it?

**b** What is the least common console and how many students use it?

**c** How many different game consoles are in the survey?

**d** How many students were surveyed?




10C.2 **2** Sanjeev conducted a survey on people's favourite clothing store and recorded the following results in a frequency table.

| Clothing store | Frequency |
|---|---|
| Rebel Sport | 18 |
| Ghanda | 6 |
| Osmosis | 27 |
| Sportsgirl | 9 |
| Cotton On | 12 |

**a** How many categories are there?

**b** What is the minimum value?

**c** What is the maximum value?

**d** Create a column graph from the data.

10C.3 **3** A group of music students were asked what instrument they played. Use this data to construct a dot plot.

piano, saxophone, violin, guitar, guitar, saxophone, trombone, piano, clarinet, bass, drums, clarinet, saxophone, guitar, cello, violin, flute, trombone, flute, saxophone, guitar, violin, piano, guitar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**4** Present the data in this table in a bar chart.

| State/Territory | Number of people on tour |
|---|---|
| Australian Capital Territory | 4 |
| New South Wales | 11 |
| Northern Territory | 9 |
| Queensland | 4 |
| South Australia | 8 |
| Tasmania | 13 |
| Victoria | 10 |
| Western Australia | 7 |

**5** The results of a survey on the make of the family car are shown below.

| | | | | |
|---|---|---|---|---|
| Holden | Toyota | Ford | Audi | Ford |
| Ford | Volvo | Holden | Holden | Hyundai |
| BMW | Holden | Toyota | Audi | Ford |
| Ferrari | Toyota | Ford | Ford | BMW |
| Hyundai | Volvo | Volvo | Toyota | Holden |
| Ford | Hyundai | Ford | Holden | Hyundai |

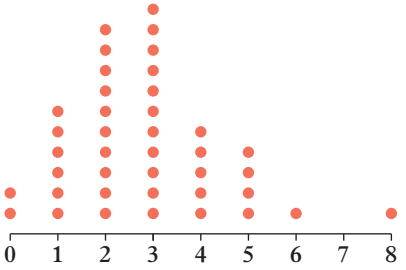
**a** Create a bar chart to show the data.

**b** What is the most common type of car?

**c** What is the least common type of car?

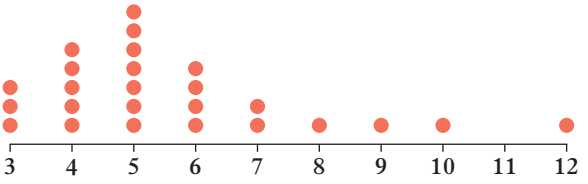
**6** Sarah conducted a survey on the number of televisions in homes and produced a dot plot. Change the dot plot into a column graph.

**Number of TVs in homes**




10C.4 **7** The following dot plot shows the number of pairs of shoes owned by members of a class of Year 7 students.

**Number of pairs of shoes owned by Year 7 students**



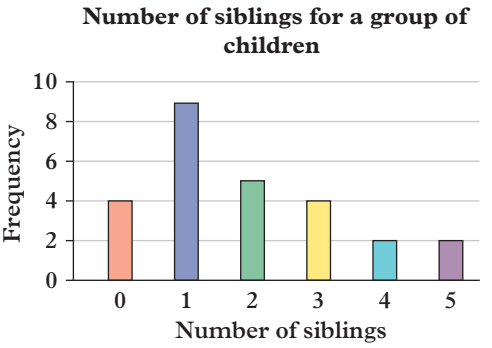

**a** How many students are in the class?

**b** What is the range of pairs of shoes owned?

**c** What is the mode number of pairs of shoes owned?

**d** What is the median number of pairs of shoes owned?

**8** The column graph on the right shows the number of siblings for a group of children.




**a** How many children are in the group?

**b** What is the range of the number of siblings?

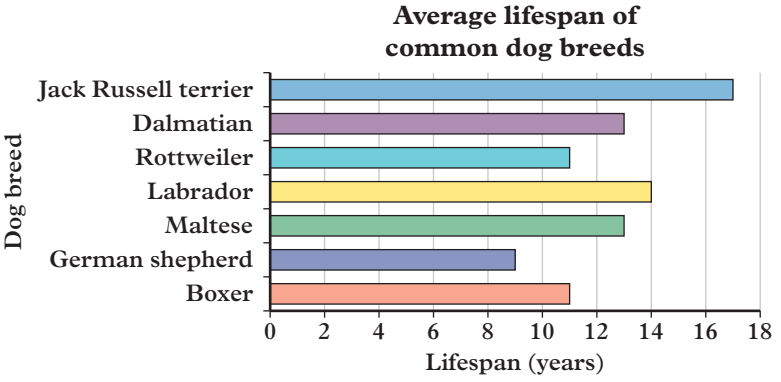
**c** What it the mode number of siblings?

**d** What is the median number of siblings?

**e** What is the mean number of siblings, correct to two decimal places?

**f** Use the summary statistics to write two sentences about the number of siblings for this group of children.

**9** Consider the bar chart shown below.



**a** What is topic of the survey?

**b** Write two sentences to describe what the graph shows.



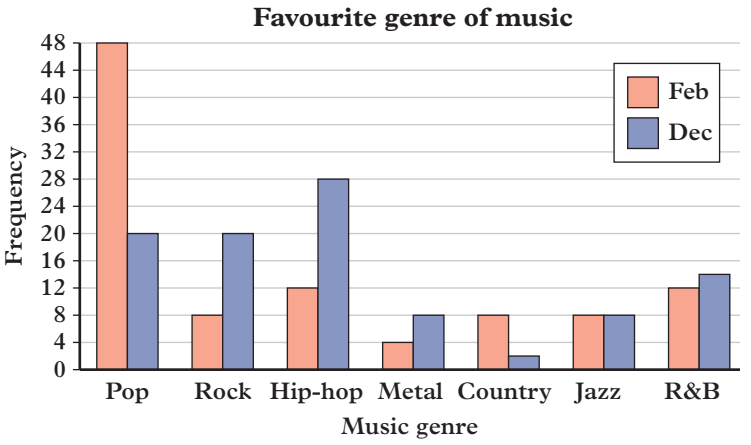
**10** Present the data in the table using an appropriate graph. Explain your choice of graph.

| Hair colour | Frequency |
|---|---|
| Black | 13 |
| Blonde | 8 |
| Brown | 15 |
| Red | 4 |
| Dyed | 7 |

**11** Present the data in the table below using an appropriate graph. Explain your choice of graph.

| Birth month | Frequency |
|---|---|
| January | 17 |
| February | 31 |
| March | 28 |
| April | 20 |
| May | 16 |
| June | 25 |
| July | 18 |
| August | 33 |
| September | 54 |
| October | 42 |
| November | 21 |
| December | 26 |

**12** This column graph shows the favourite music genre for Year 7 students at the start and end of the year.




**a** What type of data is this?

**b** What was the most popular genre of music at the beginning of the year? How many students chose it?

**c** What was the most popular genre of music at the end of the year? How many students chose it?

**d** Which genre of music showed the greatest increase in popularity over the year?

**e** What was the least popular genre of music throughout the year?

**f** If you pick two different genres of music to play at a whole school function in the middle of the year, which two genres would you pick to get the most positive response from the audience? Why?

**13** Eleonora asked her classmates their favourite genre of film. She made a tally in the table below.

| Genre | Tally | Frequency | Percentage frequency |
|---|---|---|---|
| Action | 𝍸 𝍸 \|\| | | |
| Drama | \|\|\|\| | | |
| Romance | \| | | |
| Comedy | 𝍸 \| | | |
| Other | \|\| | | |

**a** Copy the table and complete the frequency column.

**b** Complete the percentage frequency column by writing each frequency as a percentage of the total frequency.

**c** Construct a column graph for the data using percentage frequency on the vertical scale.

**d** State the most common category. What percentage of Eleonora's classmates chose this category?

**e** State the least common category. What percentage of Eleonora's classmates chose this category?

**14** Rachael has been keeping track of her scores out of 10 on her weekly retrieval quizzes. She decides to turn her list of scores into a dot plot which she can easily add new scores to. Her scores from her retrieval quizzes are:

8, 9, 8, 8, 6, 8, 7, 9, 8, 9.

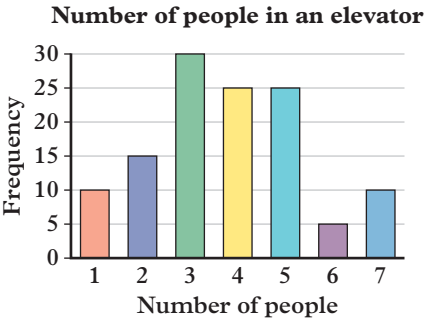
**a** Create Rachael's dot plot.

**b** Calculate Rachael's mean retrieval score.

**c** On the next retrieval quiz, Rachael scores 10 out of 10. Add this to her dot plot.

**d** Correct to two decimal places, how much has Rachael's mean retrieval score increased?

**15** Consider the column graph on the right.

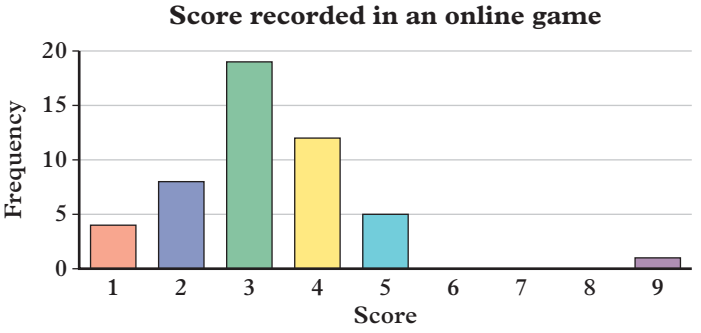



**a** What is the mode of the data?

**b** What is the range of the number of people in an elevator at any one time?

**c** Write the data from this graph as a list of numbers and use the list to find the mean and median of the data.

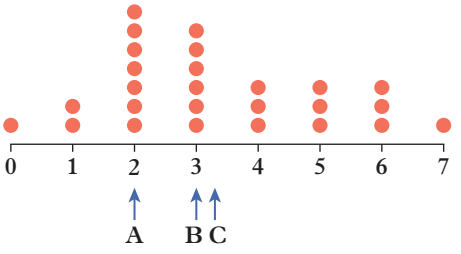
**16** Salwa produced this column graph showing the scores people recorded in an online game.



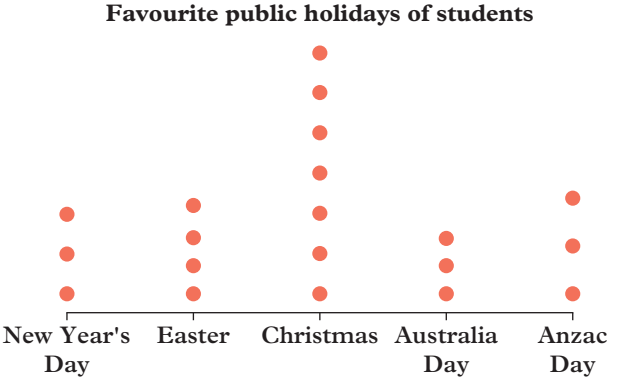

**a** What is the value of the outlier?

**b** Salwa realises she misread the score of the outlier when plotting the graph. The score of the outlier is actually 3. If you were to redraw the column graph based on this information, what change(s) would you need to make to the graph?

**17** The three arrows on the following dot plot represent the values of the mean, median and mode. Without performing any calculations, identify which arrow belongs to which summary statistic. Give reasons for your answer.

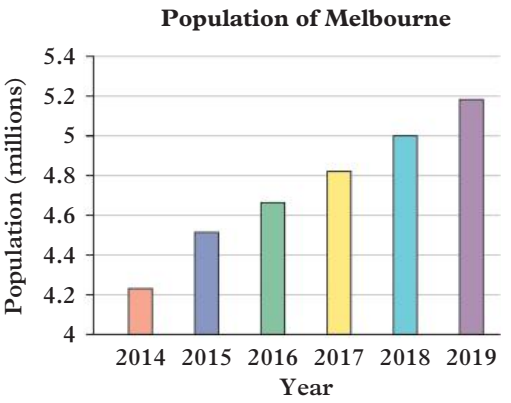



**18** Gabriel produced the following dot plot, which shows the favourite public holidays of students in his class.

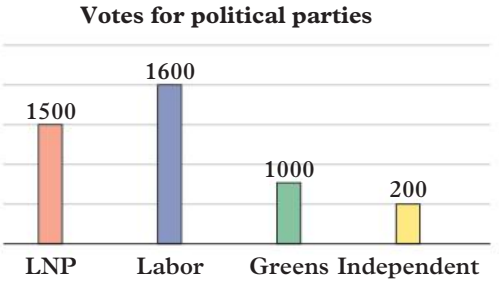



By considering only the vertical spacing, explain why this graph could be misleading.

**19** The following graph shows the population of Melbourne for each year from 2014 to 2019. By considering only the vertical scale, explain why this graph could be misleading.




**20** In the lead-up to the next election, a sample of voters were asked which party they planned to vote for. The surveyors created this graph to show how the sample planned to vote.



**a** Describe why this graph is misleading.

**b** Redraw the graph accurately.

**21** The bottom three rows of dots on a dot plot are removed and every column of dots still has dots remaining. Which statistics (mean, median, mode and range) will *not* change? Why?

**22** Construct a dot plot with the following statistics:

- number of data points: 11
- mean: 3
- median: 3
- mode: 4
- range: 7

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Column graphs and dot plots

**Investigation** TV audiences

**Topic quiz** 10C

PROBLEM SOLVING AND REASONING

CHALLENGE

# 10D Stem-and-leaf plots

## Learning intentions

✔ I can interpret stem-and-leaf plots.
✔ I can construct stem-and-leaf plots.

Inter-year links
Year 8 9C Presenting data
Year 9 8E Comparing data

## Stem-and-leaf plots

- **Stem-and-leaf plots** or stem plots can be used to display numerical data. In stem-and-leaf plots:
  - → Each piece of data is split into two parts: a *stem* (the first digit or digits) and a *leaf* (the last digit).
  - → All stems should have the same place value.
  - → A key is necessary to indicate the value of the data. For example, 1 | 5 = 15 indicates a stem of 1 and a leaf of 5, which represents the data point 15.
  - → The value of the leaves increases as they move away from the stem.

7, 11, 15, 16, 20, 22, 29, 29, 43, 47

| Stem | Leaf |
|---|---|
| 0 | 7 |
| 1 | 1 5 6 |
| 2 | 0 2 9 9 |
| 3 | |
| 4 | 3 7 |

Key: 1 | 5 = 15

### Example 10D.1 Identifying summary statistics from a stem-and-leaf plot

Consider the stem-and-leaf plot on the right.

**a** Find the range of this data set.
**b** Find the mode of this data set.
**c** Find the median of this data set.
**d** Calculate the mean of this data set.

| Stem | Leaf |
|---|---|
| 0 | 2 5 8 |
| 1 | 0 7 7 9 |
| 2 | 2 6 |

Key: 1 | 9 = 19

**THINK**

**a** Use the key to interpret the plot. Subtract the minimum value from the maximum value to find the range.
**b** Identify the value which occurs most often.
**c** Count to the median value of the ordered list of numbers. There are 9 values, so the median is the 5th value.
**d** Add all the numbers together and divide by the total number of values.

**WRITE**

**a** Range = 26 – 2
= 24
The range of the data is 24.
**b** The mode of this data set is 17.
**c** 2, 5, 8, 10, 17, 17, 19, 22, 26
The median of this data set is 17.
**d** Mean $= \frac{2 + 5 + 8 + 10 + 17 + 17 + 19 + 22 + 26}{9}$
$= \frac{126}{9}$
= 14
The mean of this data set is 14.

## **Example 10D.2** Constructing a stem-and-leaf plot

Arrange this set of data into a stem-and-leaf plot.

12, 17, 23, 46, 8, 34, 33, 15, 28, 22, 42, 10, 4, 35, 39, 50

**THINK**

1. Identify the minimum and maximum stem values. The minimum value is 4 and the maximum value is 50. 4 has a stem of '0' and 50 has a stem of '5'.
2. Place all stems from 0 to 5 in a single vertical column.
3. Construct the leaves by entering the data points from the list into the appropriate row of the plot.
4. Rewrite the stem-and-leaf plot, putting the data points in ascending order as you move away from the stem. Make sure the numbers are evenly spaced.
5. Include a key.

**WRITE**

Unordered stem plot

| Stem | Leaf |
|---|---|
| 0 | 8 4 |
| 1 | 2 7 5 0 |
| 2 | 3 8 2 |
| 3 | 4 3 5 9 |
| 4 | 6 2 |
| 5 | 0 |

Ordered stem plot

| Stem | Leaf |
|---|---|
| 0 | 4 8 |
| 1 | 0 2 5 7 |
| 2 | 2 3 8 |
| 3 | 3 4 5 9 |
| 4 | 2 6 |
| 5 | 0 |

Key: 1|0 = 10

**Helpful hints**

- ✔ The minimum and maximum stems will not always have the same number of digits.
  For example, if a data set has a minimum value of 55 and a maximum value of 120, the minimum stem will be 5 and the maximum stem will be 12.
- ✔ If a stem has no leaf, there are no data points corresponding to that stem.
- ✔ Data points ending in zero are represented using a '0' leaf.
  For example, 1|0 = 10.
- ✔ When calculating summary statistics from a stem-and-leaf plot remember to use the full data values, not just the values of the leaves.
- ✔ Make sure you use the key to decode the data values.
  For example, 2|7 may represent 2.7, 27, 270 or 2700.

ANS p606

# Exercise 10D Stem-and-leaf plots

1–3, 4(a, b), 5, 6, 9, 10(a–c), 11 | 2, 3, 4(c, d), 5, 7, 8, 10, 11, 12 | 4(c, d), 6, 7, 8, 10–13

UNDERSTANDING AND FLUENCY

**1** The number of people at a local skate park over the course of a month is displayed in this stem-and-leaf plot.

| Stem | Leaf |
|---|---|
| 1 | 4 8 |
| 2 | 2 5 5 6 9 |
| 3 | 1 1 2 4 5 6 6 7 7 7 |
| 4 | 4 4 8 9 |
| 5 | 1 2 3 |
| 6 | 0 0 1 6 |
| 7 | 4 8 |
| 8 | |
| 9 | 8 |

**a** Add an appropriate key to the plot.

**b** What was the smallest number of people at the park during the month?

**c** What was the largest number of people?

**d** What is the range of the number of people in the park for this month?

**e** What is the most common number of people in the park for this month?

**f** How many days were there in this month?

10D.1 **2** Consider the stem-and-leaf plot below.

| Stem | Leaf |
|---|---|
| 0 | 1 5 6 |
| 1 | 2 8 9 9 |
| 2 | 1 4 8 |
| 3 | 0 1 4 |
| 4 | 3 |

Key: 2|1 = 21

**a** Find the range of the data.

**b** Find the mode of the data.

**c** Find the median of the data.

**d** Calculate the mean of the data correct to two decimal places.

10D.2 **3** Arrange each set of data into a stem-and-leaf plot.
Remember to include a key with each plot.

**a** 11, 14, 18, 19, 23, 23, 26, 31, 38, 42, 42, 43

**b** 13, 18, 22, 46, 54, 59, 31, 36, 42, 19, 4, 28

**c** 54, 34, 33, 65, 39, 27, 35, 51, 42, 49, 37, 53

**d** 41, 37, 53, 29, 31, 59, 67, 23, 41, 59, 29, 41



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**4** Consider the stem-and-leaf plot below.

| Stem | Leaf |
|---|---|
| 5 | 1 2 9 8 |
| 6 | 4 6 7 8 2 |
| 7 | 1 1 5 8 6 |
| 8 | 3 3 4 9 2 2 1 3 |
| 9 | |
| 10 | 8 |

Key: 5|1 = 51

**a** Redraw the stem-and-leaf plot so that the data points are in ascending order as you move away from the stem.

**b** What is the smallest number in the data set?

**c** What is the largest number in the data set?

**d** What is the mode?

**e** Add the number 112 to the plot. (Hint: add another stem.)

**5** The ages of customers sitting in a sushi restaurant at lunch time were recorded and are displayed below.

15 23 29 45 19 65 18 29 31 34 22
24 19 33 25 32 54 39 28 19 26

**a** How old was the youngest and oldest customer in the restaurant?

**b** Draw a stem-and-leaf plot to represent this data. Remember to include a key.

**c** What was the most common customer age?

**d** What was the most common age bracket (10–19, 20–29, 30–39, etc.)? (Hint: consider which stem this would be.)

**6** The number of points scored by a junior basketball team in each match over a season are shown in the stem-and-leaf plot below.

| Stem | Leaf |
|---|---|
| 1 | 1 5 6 |
| 2 | 0 0 1 2 3 4 4 5 |
| 3 | 0 1 2 3 6 7 7 |
| 4 | 1 1 1 3 |
| 5 | 2 5 |
| 6 | 7 |

Key: 1|5 = 15

**a** What is the range of the scores?

**b** What is the mode score?

**c** What is the median score?

**d** What is the mean score correct to two decimal places?

**7** The ages of a sample of people that use Instagram on a daily basis are given below.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 18 | 20 | 21 | 29 | 34 | 25 | 15 | 17 | 19 | 13 | 14 | 31 |
| 14 | 14 | 18 | 23 | 27 | 36 | 54 | 45 | 46 | 76 | 53 | 41 | 22 |
| 46 | 29 | 30 | 18 | 21 | 24 | 41 | 33 | 15 | 16 | 25 | 12 | 10 |

**a** Create a stem-and-leaf plot to display this data.

**b** What is the age of the youngest Instagram user in this sample group?

**c** What is the age of the oldest Instagram user in this sample group?

**d** What is the most common age bracket to use Instagram in this sample group?

**8** A sample of feathertail gliders (tiny possums native to Australia) were weighed to monitor their health. The results are shown in this stem-and-leaf plot.

| Stem | Leaf |
|---|---|
| 10 | 1 4 8 |
| 11 | 2 3 6 6 |
| 12 | 1 2 4 5 5 6 9 |
| 13 | 1 2 4 4 4 6 8 8 8 8 |
| 14 | 0 1 |
| 15 | |
| 16 | 1 |

Key: 10|1 = 10.1 g

**a** What is the mass of the lightest glider?

**b** What is the mass of the heaviest glider?

**c** Will said that the most common mass recorded was 138 g. Identify and explain his mistake, and then state the most common recorded mass.

**d** Identify the outlier in this data set and state the mass of the heaviest glider, excluding the outlier.

**e** Another five feathertail gliders were weighed and their masses (in grams) are recorded below. Redraw the stem-and-leaf plot, including these masses in the plot.

14.3 12.9 13.1 13.6 11.0

**9** The figures below indicate the approximate number of people lined up at a sample of cinemas an hour before the premiere of the latest Spider-Man movie.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 120 | 250 | 360 | 80 | 150 | 430 | 570 | 220 | 160 | 130 | 170 | 240 | 220 |
| 60 | 110 | 140 | 200 | 130 | 120 | 100 | 180 | 220 | 260 | 130 | 150 | 100 |

A stem-and-leaf plot has been started below.

| Stem | Leaf |
|---|---|
| 0 | 6 8 |
| 1 | |
| 2 | |

Key: 1|2 = 120

**a** How is the key different from those you have seen before?

**b** Complete the stem-and-leaf plot.

**c** How many people were in the smallest line?

**d** How many people were in the largest line?

**e** Calculate the mean number of people lining up an hour before the premier. Give your answer to the nearest whole number.

**10** The schedule on the right lists the times a tram arrives at a certain stop in Melbourne on a Sunday.

**a** How is the schedule different from a normal stem-and-leaf plot?

**b** What is the earliest time the tram leaves the stop?

**c** At what time does the last tram leave the stop?

**d** How many trams run each hour from midday to 7 pm?

**e** Between which times do the shortest waiting periods occur?

**f** Between which times do the longest waiting periods occur?

| Sunday | |
|---|---|
| AM | |
| 6 | |
| 7 | 16 46 |
| 8 | 16 46 |
| 9 | 16 46 |
| 10 | 12 30 43 56 |
| 11 | 08 18 30 42 54 |
| PM | |
| 12 | 06 18 30 42 54 |
| 1 | 06 18 30 42 54 |
| 2 | 06 18 30 42 54 |
| 3 | 06 18 30 42 54 |
| 4 | 06 18 30 42 54 |
| 5 | 06 18 30 42 54 |
| 6 | 06 18 30 42 54 |
| 7 | 04 19 34 |
| 8 | 04 35 |
| 9 | 05 35 |
| 10 | 04 31 |
| 11 | 01 31 59 |

**11** Consider the stem-and-leaf plot below. Identify and describe three things that need to be fixed.

| Stem | Leaf |
|---|---|
| 2 | 1 4 9 2 |
| 3 | 3 9 8 3 5 |
| 4 | 1 1 6 7 5 3 1 |
| 5 | 0 6 3 |
| 8 | 7 8 |
| 9 | 9 4 2 2 |

**12** A set of numerical data contains no repeating values. Which graph (column graph, bar chart, dot plot, or stem-and-leaf plot) would be most appropriate to display the data? Explain your answer.

**13** While working on an ordered stem-and-leaf plot worksheet, Sal spills his drink. The bottom section of the plot is smudged, obscuring three leaves, the final stem and the key. He already had the following information:

- The range was 0.026
- The mean was 0.14065

**a** Determine the missing maximum value.

**b** Determine all possible pairs of numbers for the two remaining illegible digits.

| Stem | Leaf |
|---|---|
| 12 | 9 |
| 13 | 3 4 4 8 8 8 9 9 9 |
| 14 | 0 0 1 1 4 5 6 |

Key: 12 | 9 = 0.129

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Stem-and-leaf plots

**Investigation** Study plan results

**Topic quiz** 10D

PROBLEM SOLVING AND REASONING

CHALLENGE

# Checkpoint

**Checkpoint quiz**
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

10A **1** State whether the following data:

**i** is a sample or a census
**ii** involves primary or secondary data.

**a** Researching the top 10 highest grossing movies ever made by looking up the sales figures for all movies premiered in the last 5 years.
**b** Researching the average number of video games a person with a console owns by looking up how many copies of all video games and consoles have been sold.
**c** Researching the most common reason why people visit the doctor by standing at the door of a medical centre and asking patients why they are going to the doctor as they arrive.

10A **2** Classify the following data.

**a** Number of pair of shoes a person owns
**b** Movies on the highest grossing film list
**c** Amount of sugar required for a dessert recipe
**d** Reason patients visit the doctor

10B **3** Match the summary statistics for numerical data to how it is calculated.

| | |
|---|---|
| **a** Mean | **A** Find the number with the highest frequency |
| **b** Median | **B** Split the data into two equal sized groups, find the value in between. |
| **c** Mode | **C** maximum value – minimum value |
| **d** Range | **D** $\dfrac{\text{sum of all data points}}{\text{total number of data points}}$ |

10B **4** Calculate the following for the data sets (to two decimal places where needed).

**i** mean **ii** median **iii** mode **iv** range.

**a** 6, 7, 7, 8, 9, 10, 12, 15
**b** $2.5, 8.3, 9\frac{1}{7}, 5\frac{2}{3}, 2.5, 2.5, 1\frac{3}{14}$

**5** State the statistic each statement is interpreting.

**a** Half of the students scored under 65 on the science test.
**b** The most common weight was 1.2 kg.
**c** The difference in height between the shortest and tallest basketballer is 46 cm.
**d** If all the prizes were shared equally, everyone would receive $75.

10C **6** A group of 13 teenagers at a skatepark were comparing how many bones they had broken while skateboarding. The number of bones broken were 3, 1, 1, 2, 4, 2, 0, 1, 1, 7, 2, 1, 1. Display this data on a dot plot.

10C **7** A class of 30 students wrote down their birth month. The data was collected and put into the frequency table below.

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 4 | 2 | 2 | 3 | 5 | 0 | 2 | 4 | 0 | 2 | 2 |

Construct a bar chart for this data.


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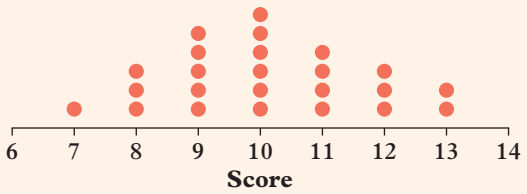
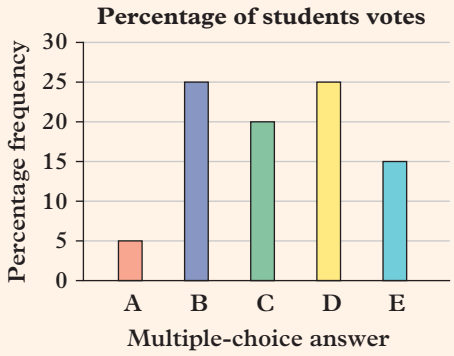
10C **8** Consider the dot plot below.




**a** Create a frequency table for the data.
**b** Calculate the mean correct to two decimal places.
**c** Calculate the median.
**d** Calculate the mode.
**e** Calculate the range.

10C **9** A number of students answered a multiple-choice question and the column graph shows the percentage of students that voted for each response.




**a** What was (were) the most common response(s)?
**b** What was (were) the least common response(s)?
**c** If there were 25 people that answered the question, how many chose option C?
**d** If 8 people chose option B, how many people answered the question?

10D **10** Construct a stem-and-leaf plot for the following data.

15, 42, 60, 27, 28, 12, 45, 37, 19, 28, 39, 31, 52, 35, 30, 40, 41, 47, 32, 23

10D **11** Consider the following stem-and-leaf plot.

| Stem | Leaf |
|---|---|
| 9 | 8 |
| 10 | 0 1 2 4 9 |
| 11 | 1 6 8 8 |
| 12 | 1 2 |
| 13 | |

Key: 9|8 = 9.8

**a** What is the value of the 4 leaf on the 10 stem?
**b** Calculate the mean.
**c** Calculate the median.
**d** Calculate the mode.
**e** Calculate the range.

# 10E Describing probability

## Learning intentions

- ✔ I can use probability terminology.
- ✔ I can list sample spaces of experiments.
- ✔ I can distinguish between equally likely outcomes and outcomes that are not equally likely.

**Inter-year links**

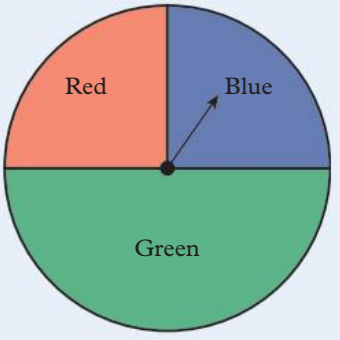
| | |
|---|---|
| Years 5/6 | Understanding probability |
| Year 8 | 9E Theoretical probability |

## Probability terminology

- An **experiment** is a repeatable procedure that has a clearly defined set of possible results.
- A **trial** is a single performance within an experiment.
  For example, a roll of a standard die.
- An **outcome** is a possible result of a trial.
  For example, rolling a 3.
- The **sample space** of an experiment is a set of all the possible outcomes of that experiment.
  For example, the sample space for the experiment of rolling one die is: $\{1, 2, 3, 4, 5, 6\}$.
- An **event** refers to an outcome or group of outcomes in a sample space.
  For example, a single outcome is rolling a 2; a group of outcomes is rolling a 2 or rolling a 4.

## Describing probability

- **Probability** describes the chance that a particular event will occur. It is expressed on a scale from impossible to certain.
  - → Events that are less likely to occur than to not occur can be described as *unlikely*.
  - → Events that are equally likely to occur or not occur can be described as *even chance*.
  - → Events that are more likely to occur than to not occur can be described as *likely*.
- An event that is certain to occur is twice as likely as an event that has an even chance of occurring.
- **Equally likely outcomes** have the same probability of occurring.
  For example, it is equally likely that the spinner will land on blue or red, but not equally likely that the spinner will land on blue compared to green.





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## **Example 10E.1** Listing the sample space

List the sample space for each of the following experiments.

**a** Flipping a coin

**b** Drawing a card from a standard deck of cards and recording the suit

| THINK | WRITE |
|---|---|
| **a** There are two possible outcomes. Place these within curly brackets separated by commas. | **a** {head, tail} |
| **b** Although there are 52 cards in a deck, there are only four different suits. Place these within curly brackets separated by commas. | **b** {hearts, diamonds, clubs, spades} |

## **Example 10E.2** Describing probability

Describe the probability of the following events using one of the following five terms: impossible, unlikely, even chance, likely, certain.

**a** Rolling a six-sided die and obtaining a six

**b** Flipping a coin and obtaining a tail

**c** May following April

**d** Selecting a non-picture card from a deck of cards

| THINK | WRITE |
|---|---|
| **a** Rolling a six is unlikely, but not impossible. | **a** Unlikely |
| **b** The chance of obtaining a tail is equal to the chance of obtaining a head. As there are two equally likely outcomes, this is an even chance. | **b** Even chance |
| **c** May always follows April. | **c** Certain |
| **d** Each suit is made up of 10 cards that do not have a picture, and 3 picture cards, so this is a likely event. | **d** Likely |

**Helpful hints**

✔ If there are two outcomes in a sample space, this does not automatically mean they are equally likely to occur.

For example, given the sample space {will win the lottery, will not win the lottery} there is not an even chance you'll win!

✔ Events should not be described as impossible unless they will never happen, and events should not be described as certain unless they will definitely happen.

ANS p608

# Exercise 10E Describing probability

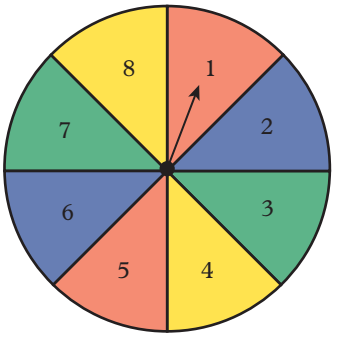
1(a–d), 2–6, 7(a–c), 10(a–d), 12(a, b), 13

1(a, b, e, f), 2(d–f), 3, 5–7, 8, 10(d–f), 11, 12(a–c), 13

1(e, f), 2(d, f), 5, 7, 9, 10(e, f), 11, 12(c, d), 14, 15

UNDERSTANDING AND FLUENCY

10E.1 **1** List the sample space for each of the following experiments and surveys.




**a** The spinner on the right is spun and the colour is recorded.

**b** The spinner on the right is spun and the number is recorded.

**c** Justine asks her friends what their favourite season is.

**d** Lisa conducts a survey on the month in which her classmates were born.

**e** Selecting one coin at random from a wallet containing at least one of every Australian coin.

**f** Rishi conducts a survey about how many days a week people work.

**2** State whether all of the outcomes of the following experiments are equally likely or not equally likely.

**a** The spinner labelled Figure A is spun and the colour recorded.

**b** A card is drawn at random from a standard deck after someone has slipped a second ace of spades into the deck.

**c** A fair 20-sided die labelled 1–20 is rolled.

**d** The spinner labelled Figure B is spun and the colour is recorded.

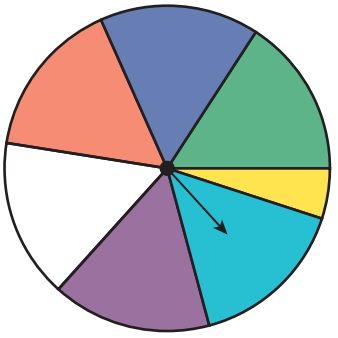


**Figure A**

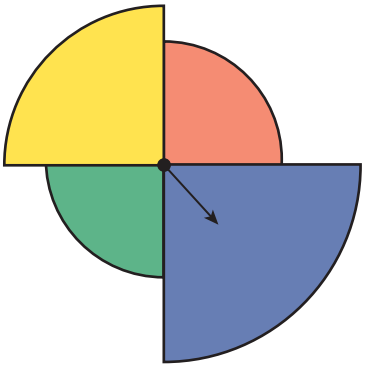


**Figure B**

**e** A biased six-sided die labelled 1–6 is rolled such that a 6 is more likely to be rolled and a 1 is less likely to be rolled.

**f** Without looking, a token is taken from a bag containing 15 small red tokens and 15 large blue tokens.

10F.2 **3** Describe the probability of each event using one of the following five terms: impossible, unlikely, even chance, likely, certain.

**a** Rolling a 1, 2, 3, 4 or 5 on a standard six-sided die

**b** A woman giving birth to triplets

**c** Selecting a red card from a standard deck of cards

**d** Picking the winning option out of three equally likely choices

**e** Having at least one hot day in summer in Australia

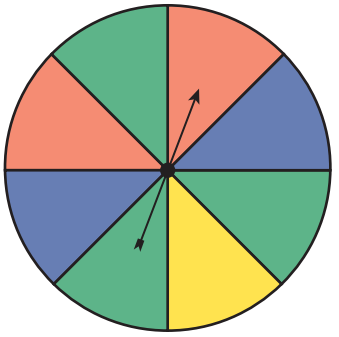
**4** Match each event to the most appropriate probability.

| | |
|---|---|
| **a** Winning a coin toss | **A** Likely |
| **b** Rolling more than 2 on a standard six-sided die | **B** Certain |
| **c** Living to 100 years old | **C** Even chance |
| **d** Tuesday following Monday | **D** Unlikely |

**5** This spinner below is spun once. Provide an example of an outcome that is:

**a** Unlikely **b** Likely **c** Impossible

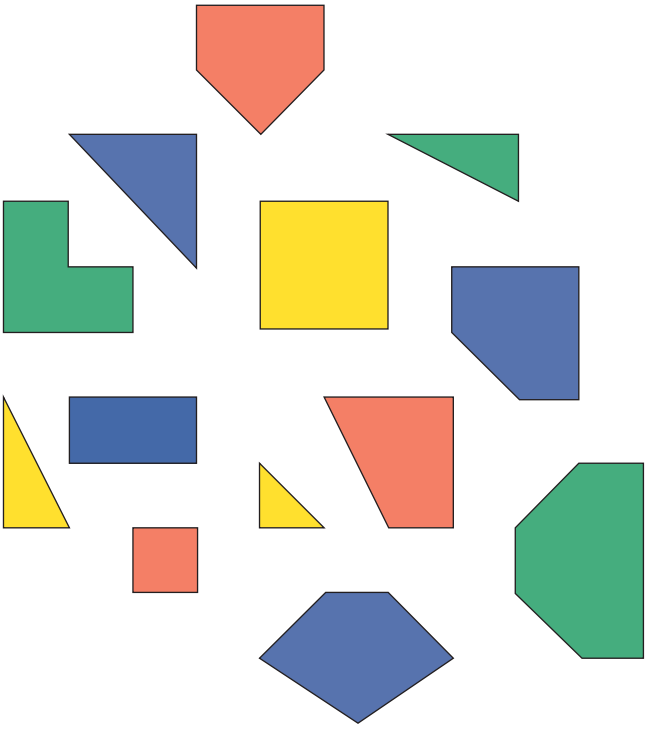
**d** Even chance **e** Certain



**6** Look at the picture of polygons. List the sample space if:

**a** the polygons are grouped by colour

**b** the polygons are grouped by number of sides.



**7** The number of elements in a sample space is equal to the number of possible outcomes. For example, the sample space for flipping a coin consists of two elements, {heads, tails}, because there are two possible outcomes: heads or tails. Find the number of elements in the sample space of the following experiments.

**a** Randomly choosing a letter from all the vowels in the alphabet

**b** Randomly choosing a day of the week

**c** Randomly drawing a letter from the word FIRST

**d** Randomly drawing a letter from the word TEXT

**e** Randomly selecting one counter out of a bag containing 8 red, 7 green and 10 blue coloured counters

**8** A fair 10-sided die labelled 1, 2, 2, 3, 4, 4, 5, 6, 6, 7 is rolled.

**a** If each individual face is considered a different outcome, are the outcomes equally likely?

**b** If each number is considered a different outcome, are the outcomes equally likely?

**9** A board game lets you choose from a collection of differently labelled six-sided dice which are rolled to determine the number of spaces you move. For each of the following dice, determine:

**i** if the die has equally likely outcomes based on the numbers

**ii** the mean die roll if each face is equally likely to appear.

**a** $\{0, 0, 0, 0, 10, 10\}$

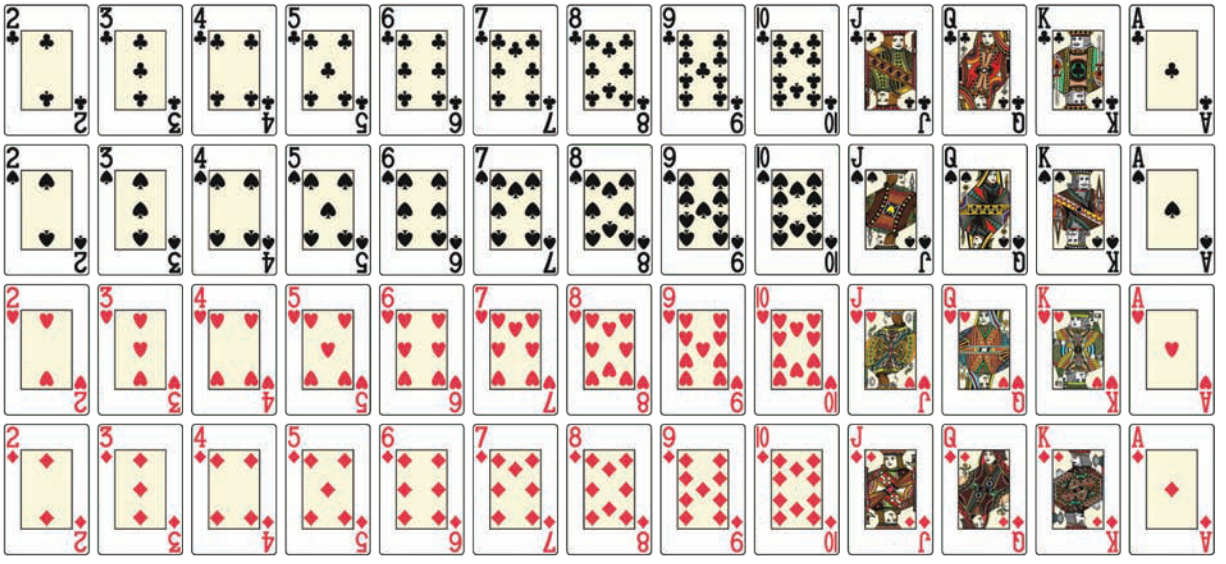
**b** $\{0, 0, 0, 7, 7, 7\}$

**c** $\{1, 3, 3, 3, 4, 4\}$

**d** $\{-3, -2, -1, 1, 2, 3\}$

**e** $\{0, 1, 3, 5, 7, 9\}$

**10** A standard deck of playing cards has four suits: clubs, spades, hearts and diamonds. There are 13 cards in each suit – Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King – making a total of 52 cards.



20 cards are drawn at random from a standard deck of cards. State whether the following statements are true or false.

**a** There will always be an equal number of red and black cards.

**b** All of the picture cards (Jack, Queen, King) can be drawn.

**c** Five sets of four-of-a-kind can be drawn.

**d** There will always be red and black cards drawn.

**e** There will always be a card from each of the four suits.

**f** If A = 1, J = 11, Q = 12 and K = 13, then the total of the 20 cards will never be less than 60.

**11** Not all experiments have outcomes that are equally likely. For example, you are equally likely to select a 'B' from the word 'BALLOON' as you are to select an 'A', but you are more likely to select a 'B' from the word 'BABY' than you are to select an 'A'.

Determine whether or not the outcomes to the experiments in question **7** are equally likely.

**12** The sample space of an experiment can continue indefinitely. For example, the number of goals scored in a soccer match is $\{0, 1, 2, 3, 4, \ldots\}$ or the temperature outside to the nearest integer is $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The ellipses '…' means to continue the pattern of the previous numbers in the set. Write the sample spaces of the following experiments which continue indefinitely.

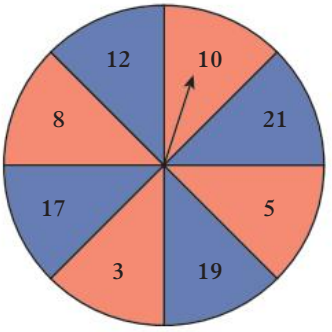
**a** A random number generator is programmed to print a positive multiple of 3.

**b** The score at the end of a game of Pac-Man where 10 points are scored for every pellet he eats.

**c** A person's elevation (distance above or below sea level) at a given point in time to the nearest tenth of a metre.

**d** The number of bacteria in a petri dish after some time has passed, where there is initially one bacterium and the number of bacteria doubles every hour.

**13** This spinner with equally sized segments is spun once and the number it lands on is recorded. By considering the number of equally likely outcomes and the number of outcomes in the event, state whether each of these events are impossible, unlikely, even chance, likely or certain.




**a** Landing on an even number

**b** Landing on a number greater than 10

**c** Landing on a square number

**d** Landing on a multiple of 3

**e** Landing on a composite number

**14** The events $A$, $B$, $C$, $D$ and $E$ all have different probabilities of occurring.

- $E$ is less likely to occur than $B$ but more likely to occur than $A$.
- $A$ is more likely to occur than $C$.
- $D$ is certain to occur.

**a** Use the clues provided to place events $A$ to $E$ in order of likelihood to occur (from least likely to most likely).

**b** Which is the only event that might be impossible?

**15** The events $A$, $B$ and $C$ are either impossible, unlikely, even chance, likely, or certain, with no two events being in the same category. One, and only one, of the following statements about the events is false. Determine which statement is false and correct the statement.

**1** Event $C$ is likely to happen.

**2** Event $C$ is twice as likely as $B$.

**3** Event $A$ is twice as likely as $C$.

**4** Event $B$ is unlikely to happen.

**5** Event $A$ is certain to happen.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet** Describing probability

**Investigation** Probability snap

**Topic quiz** 10E

# 10F Theoretical probability

## Learning intentions

✔ I can calculate theoretical probabilities.

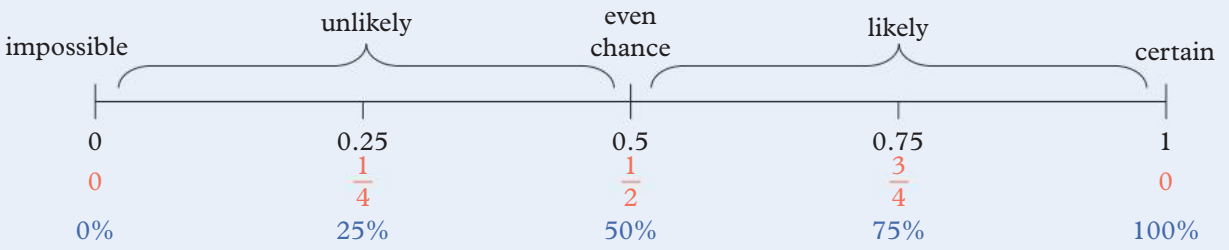
✔ I can express probabilities as fractions, percentages and decimals.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Understanding probability |
| Year 8 | 9E Theoretical probability |
| Year 9 | 9A Two-step chance experiments |

## Expressing probabilities

- All events have a probability between 0 (impossible) and 1 (certain) of occurring.
- Probabilities can be expressed as decimals, fractions and percentages.




## Theoretical probability

- **Theoretical probability** is probability that is determined based on mathematical reasoning, not experiments.
- Events are made up of one or more outcomes. A **favourable outcome** is a result that you are interested in.

  For example, the event of 'rolling an even number' with a standard die is made up of the favourable outcomes 2, 4 and 6.
- Pr(event) means the probability of the stated event occuring.
- The theoretical probability of an event is calculated by dividing the number of favourable outcomes by the total number of equally likely outcomes:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$$

→ To calculate the theoretical probability of an event:

1. Count all the possible equally likely outcomes.
2. Count all the favourable outcomes that make up the event.
3. Divide the number of favourable outcomes by the total number of equally likely outcomes.

- If an event has a higher probability of occuring than a second event, it is more likely to occur in a random trial of an experiment.

## **Example 10F.1** Calculating theoretical probabilities

A standard six-sided die is rolled.

**a** List the sample space.

**b** What is the probability of rolling a 4?

**c** What is the probability of rolling a number greater than 4?

**THINK**

**a** List the possible outcomes in curly brackets.

**b** **1** Count the number of equally likely outcomes. There are six equally likely outcomes.

**2** Count the number of favourable outcomes. There is one favourable outcome.

**3** Use the probability formula to calculate the required probability.

**c** **1** Count the number of favourable outcomes. There are two favourable outcomes.

**2** Use the probability formula to calculate the required probability.

**3** Simplify the fraction.

**WRITE**

**a** $\{1, 2, 3, 4, 5, 6\}$

**b** $\Pr(\text{rolling a } 4) = \dfrac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$

$= \dfrac{1}{6}$

**c** $\Pr(\text{rolling number} > 4) = \dfrac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$

$= \dfrac{2}{6}$

$= \dfrac{1}{3}$

**Helpful hints**

- ✔ Always simplify fractional probabilities where possible.
- ✔ List the sample space to avoid making errors when determining the number of favourable outcomes and the total number of equally likely outcomes.

ANS p609

# **Exercise 10F** Theoretical probability

△ 1, 3, 4, 5(a, b), 6, 7, 9, 12–14, 16, 18(b), 19

□ 2, 5, 8, 10, 11, 13(d–f), 15, 16, 18(a, b), 20, 21, 22(a, b)

⬠ 5, 8, 10(d, e), 11, 13(d–f), 17, 18, 20, 21(c, d), 22(c, d), 23

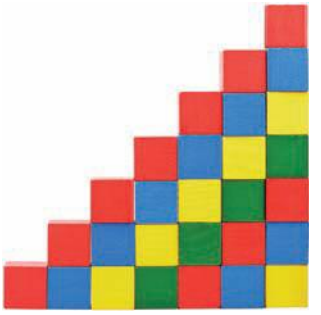
**1** Theo selects a block at random from the group of blocks on the right.

**a** What is the total number of equally likely outcomes?

**b** If Theo is looking to find the probability of selecting a red block, what is the number of favourable outcomes?

**c** What is the probability that Theo selects a red block?

2 Consider selecting a block at random from the staircase of blocks on the right.

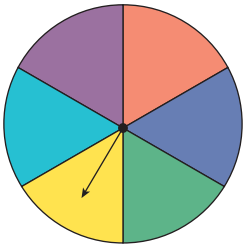


**a** What is the total number of outcomes?

**b** What is the probability of selecting a red block?

**c** What is the probability of selecting a green block?

10F.1 3 The spinner with equally sized segments on the right is spun once and the result is recorded.



**a** List the sample space.

**b** What is the probability of landing on the yellow section?

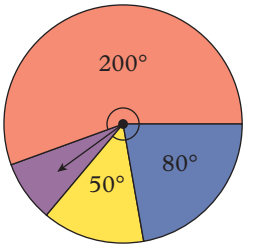
**c** What is the probability of landing on a green or purple section?

4 A fair coin is flipped.

**a** What is the probability of getting a head? Give your answer as a percentage.

**b** What is the probability of getting a tail? Give your answer as a percentage.

5 To determine the theoretical probability of landing on a sector in a spinner, divide the angles at the centre by the number of degrees in a full turn.




**a** Determine the angle of the purple sector.

**b** Determine the theoretical probability of landing on the:

**i** purple sector **ii** yellow sector

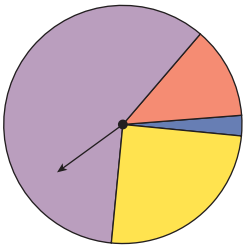
**iii** blue sector **iv** red sector.

When the angles at the centre of a spinner are not known, we can measure the angles using a protractor.

**c** Measure the angles at the centre of the:

**i** purple sector **ii** yellow sector

**iii** blue sector **iv** red sector.



**d** Determine the theoretical probability of this spinner landing on the:

**i** purple sector **ii** yellow sector

**iii** blue sector **iv** red sector.

6 A standard six-sided die is rolled.

**a** What is the probability of rolling a 2?

**b** What is the probability of rolling an odd number?

**c** What is the probability of rolling a number less than 5?

**d** What is the probability of rolling a multiple of 3 or a 4?

7 A card is selected at random from a standard deck and the suit is recorded.

**a** What is the probability of selecting a club?

**b** What is the probability of selecting a red card?

**c** What is the probability of selecting a green card?

**8** A letter is selected at random from the word STATISTICS.

**a** What is the probability, given as a decimal, of selecting a T?

**b** What is the probability, given as a decimal, of selecting a C?

**c** What is the probability, given as a decimal, of selecting a vowel?

**9** A box of Favourites contains six Crunchies, six Cherry Ripes, four Caramels and four Flakes. A Favourite is picked at random from the bag.

**a** How many Favourites are there in total?

**b** What is the probability, given as a decimal, of:

**i** picking a Crunchie? **ii** picking a Caramel?

**10** A card is randomly drawn from a standard deck.

**a** What is the probability of selecting a picture card (Jack, Queen or King)?

**b** What is the probability of selecting an Ace?

**c** What is the probability of selecting the Queen of spades?

**d** What is the probability of selecting an even number from the deck?

**e** What is the probability of selecting a prime number from the deck?

**11** The spinner from the game Twister is shown.

What is the probability, given as a percentage, of:

**a** spinning a yellow?

**b** spinning a left foot?

**c** spinning a blue right hand?

**d** spinning a green or red?

**e** spinning a right foot or blue (or both)?

**12** A deck of UNO cards has four suits: red, yellow, green and blue. Each suit contains the cards 0–9, Reverse, Skip and Draw Two. There are two copies of all cards except the 0s. The deck also includes four standard Wild Cards and four Draw Four Wild Cards. This gives a total of 108 cards. A card is randomly selected from the deck.

**a** What is the probability of selecting a 0?

**b** What is the probability of selecting a 6?

**c** What is the probability of selecting a Reverse?

**d** What is the probability of selecting a Wild Card (standard) or Draw Four?

**e** What is the probability that a card selected will make another player draw cards?

**13** Express the following probabilities using decimals.

**a** Muhammed has a 70% chance of kicking this goal.

**b** There is a $\frac{1}{4}$ chance it will rain tomorrow.

**c** Sylvia is 95% likely to pass her English exam.

**d** There is a $\frac{2}{5}$ chance that Hawthorn will win the Premiership.

**e** The likelihood that Priya will get the lead role is 65%.

**f** There is a 100% chance that Tuesday will follow Monday.

**14** Tess takes a handful of jellybeans from a packet and says that because there are four different colours of jellybeans, the probability of selecting a green jellybean is $\frac{1}{4}$. Describe Tess's mistake and calculate the correct probability of selecting a green jellybean from the handful shown on the right.

**15** Can an outcome have a 200% chance of occurring? Explain your answer.

**16** Explain why, when calculating theoretical probability using the formula, all the outcomes must be equally likely.

**17** In 1889 Nintendo was founded to sell handmade hanafuda cards (hanafuda translates to 'flower cards'). The hanafuda deck contains 48 cards divided into 12 suits representing the months of the year. The cards are categorised into different types: 24 plains, 10 ribbons (3 with poetry, 3 blue, 4 plain red), 9 animals and 5 brights.

**a** Which month is most likely to be a plain card? What is the chance of drawing that month's plain card(s)?

**b** Which month is most likely to not be a plain card? What is the chance of drawing that month's plain card(s)?

**c** You've drawn a ribbon card, what is the chance that it has poetry on it?

**d** You've already drawn three plain cards, what is the chance you draw another one?

**18** An event with a probability of 0.5 is five times more likely to occur than an event with a probability of 0.1. For each of these pairs of events, determine how many times more likely the second event is to occur than the first.

**a** Drawing a random card from a standard deck and checking if it is a spade or if it is red.

**b** Rolling a fair, standard, six-sided die and getting a 5 or getting an odd number.

**c** A bag contains 18 red marbles, 6 blue marbles and 3 pink marbles. A marble is picked at random and checked if:

**i** it is red or if it is blue **ii** if it is pink or it is blue **iii** if it is red or it is pink.

**19** A drawer contains 5 wood, 5 stone, 5 marble and 5 glass coasters. A coaster is pulled out of the drawer at random and its type {wood, stone, marble, glass} is noted.

**a** Are the events equally likely?

**b** The coaster is returned to the drawer after it is pulled out before the next one is pulled out. Are the events equally likely?

**c** The coaster is not returned to the drawer after it is pulled out before the next one is pulled out. Are the events equally likely?

**d** A different type of coaster is returned to the coasters after one is pulled out before the next one is pulled out. Are the events equally likely?

**e** Ten coasters are removed from the drawer at random before the next one is pulled out. Are the events equally likely?

**20** Determine if the following statements are true or false.

**a** A fraction always gives the exact probability.

**b** Percentages allow for comparisons more easily than fractions.

**c** A decimal must always be rounded to give the probability.

**d** Decimals are more useful than percentages when the probabilities are very close to 0 or 1 (e.g. 0.0001 and 0.99999).

**e** Fractions always make the probability easy to interpret.

**21** Determine the probability of each outcome of the experiments listed below. Give your answers in the forms given in brackets, then add all the probabilities of the outcomes of each experiment together.

**a** A fair, standard, six-sided die is rolled and the number is recorded. (Fraction)

**b** A fair coin is tossed and the result is recorded. (Decimal)

**c** A counter is chosen from bag containing 9 red and 16 blue counters and the colour is recorded. (Fraction)

**d** A sticker is chosen from a shuffled pile containing 31 'Good Work' stickers, 52 'Awesome' stickers and 17 'Fabulous' stickers, and the type is recorded. (Percentage)

**22** Determine the probability of the following outcomes.

**a** A biased die is rolled. The probability of rolling a 1 is 0.2, rolling a 2 is 0.1, rolling a 3 is 0.2, rolling a 4 is 0.1, rolling a 5 is 0.2. What is the probability of rolling a 6?

**b** A rigged coin is flipped. The probability of the coin landing on heads is 100%. What is the probability of the coin landing on tails?

**c** The probability of correctly choosing the 6 winning numbers from 1 to 49 in the lottery is 1 in 13 983 816. What is the probability of not choosing all 6 winning numbers?

**d** A marble is picked out of a bag containing orange, pink and aqua marbles. The probability of picking an orange marble is $\frac{1}{3}$ and the probability of selecting a pink marble is $\frac{1}{4}$. What is the probability of selecting an aqua marble?

**23** A fair 10-sided die with the numbers 0–9 is rolled.

**a** If the die is rolled once, what is the probability of:

**i** rolling an odd number? **ii** rolling an even number? **iii** rolling a prime number?

**b** Draw a table to show all possible outcomes when rolling the die twice.

**c** Use your answer to part **b** to help you find the probability of throwing a total of 15 when rolling the die twice.

Check your Student obook pro for these digital resources and more:

**Interactive skillsheet**
Calculating theoretical probability

**Topic quiz**
10F

PROBLEM SOLVING AND REASONING

CHALLENGE

# 10G Experimental probability

## Learning intentions

✔ I can calculate the expected results of an experiment.
✔ I can calculate experimental probabilities.

**Inter-year links**

| | |
|---|---|
| Years 5/6 | Chance experiments |
| Year 8 | 9F Two-way tables |
| Year 9 | 9D Relative frequency |

## Experimental probability

- A **successful trial** is a trial where the desired event occurs.
  For example, an experiment performed to determine the probability of rolling a 6 on a standard die has a successful trial when a 6 is rolled.
- The **relative frequency** (or **experimental probability**) of an event occurring is determined by an experiment involving a number of trials.

$$\text{Relative frequency} = \frac{\text{number of sucessful trials}}{\text{total number of trials}}$$

  → For a small number of trials, the relative frequency may be significantly different from the theoretical probability.
  → As the number of trials increases, the relative frequency is likely to approach (get close in value to) the theoretical probability.
- The theoretical probability (or relative frequency) of an event can be used to determine the expected number of successful trials:

$$\text{Expected number of successful trials} = \Pr(\text{event}) \times \text{total number of trials}$$

  For example, if the expected outcome of rolling a die 1000 times is required, this formula could be used instead of rolling a die 1000 times.
- **Simulation** involves using digital technology or simple devices like coins or dice to simulate real experiments.

### Example 10G.1 Calculating expected results and experimental probabilities

A fair coin is flipped 50 times.

**a** Calculate the theoretical probability of obtaining a head.

**b** How many times would you expect to obtain a head based on the theoretical probability?

**c** When the experiment was carried out, 27 heads were obtained. Calculate the relative frequency of obtaining a head.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**THINK**

**a** Use the theoretical probability formula to calculate the probability of obtaining a head. There is one head and two possible outcomes: heads or tails.

**b** Calculate the expected number of successful trials by multiplying the theoretical probability by the number of trials in the experiment.

**c** Calculate the relative frequency by dividing the number of heads flipped by the total number of trials.

**WRITE**

**a** $\Pr(\text{head}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of all possible outcomes}}$

$= \dfrac{1}{2}$

**b** Expected number of heads = Pr(head) × total number of trials

$= \dfrac{1}{\cancel{2}^{1}} \times \cancel{50}^{25}$

$= 25$

**c** Relative frequency $= \dfrac{\text{number of heads}}{\text{total number of trials}}$

$= \dfrac{27}{50}$

**Helpful hints**

- ✔ If you repeat an experiment more than once, you are likely to get different results! For example, if you flip a coin 10 times you won't get 5 heads every time you repeat the experiment.
- ✔ Remember to simplify fraction answers where possible.
- ✔ The expected number of successful trials for an outcome may not be a whole number. Instead it is the mean result if the experiment was repeated indefinitely.

ANS p610

## Exercise 10G Experimental probability

△ 1–5, 8, 9, 12(a–d) ▢ 3–7, 9, 10, 12, 13(a–d) ⬠ 4, 6, 9–14

10G.1 **1** A fair coin is flipped 20 times.

**a** Calculate the theoretical probability of obtaining a tail.

**b** How many times would you expect to obtain a tail based on the theoretical probability?

**c** When the experiment was carried out, 12 tails were obtained. Calculate the relative frequency of obtaining a tail.

UNDERSTANDING AND FLUENCY

**2** A standard six-sided die was rolled 60 times.

**a** Calculate the theoretical probability of obtaining a 1.

**b** How many times would you expect to obtain a 1 based on the theoretical probability?

**c** When the experiment was carried out, eight 1s were obtained. Calculate the relative frequency of obtaining a 1.

**3** A fair coin is flipped 20 times and 13 tails are obtained.

**a** Calculate the relative frequency of obtaining a tail.

**b** Compare this relative frequency to the theoretical probability of obtaining a tail.

**c** If the coin was flipped 2000 times, would you expect the relative frequency of obtaining a tail to increase, decrease or stay the same? Explain.

**4** A standard six-sided die is rolled 60 times and six 1s are obtained.

**a** Calculate the relative frequency of obtaining a 1.

**b** Compare this relative frequency to the theoretical probability of obtaining a 1.

**c** If the die was rolled 6000 times, would you expect the relative frequency of obtaining a 1 to increase, decrease or stay the same? Explain.

**5** To determine the shopping habits of people at a shopping centre, 100 people were surveyed as they left the shopping centre. They were asked: 'What was the main type of shopping you did from this list?' {groceries, clothing, electrical and entertainment, medical, window shopping (browsing)}. The data was tabulated as shown.

| **Type of shopping** | **Frequency** |
|---|---|
| Groceries | 61 |
| Clothing | 18 |
| Electrical and entertainment | 12 |
| Medical | 4 |
| Window shopping | 5 |

**a** What is the probability that the next person surveyed will have mainly done grocery shopping based on the relative frequencies? Give your answer as a decimal.

**b** The relative frequency of a shopper having mainly done clothes shopping is 18%. Why is the percentage equal to the frequency?

**c** If 11 000 people were at the shopping centre on Tuesday, and the survey is representative of the population, then how many people would be expected to have been window shopping?

**6** David was rolling a four-sided die and keeping track of the number of 1s, 2s and 3s he had rolled. He decided to determine the relative frequency of each number after 50 rolls but realised that he hadn't been tracking the number of 4s rolled. His table looks like this:

| **Number** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Relative frequency** | 0.24 | 0.16 | 0.34 | |

**a** Determine the relative frequency of the number 4 appearing.

**b** Determine the amount of 4s he must have rolled.

**7** Jade flips a coin 50 times, recording 27 heads and 23 tails. April also records 27 heads and 23 tails when she flips a coin 50 times. Would you expect their friend, Peter, to record the same results if he flips a coin 50 times? Explain why or why not.

**8** Eric rolled a 6-sided die 36 times and recorded his results in the following table.

| Number | Expected frequency | Actual frequency | Theoretical probability | Relative frequency |
|---|---|---|---|---|
| 1 | 6 | 3 | $\frac{6}{36}$ | $\frac{3}{36}$ |
| 2 | 6 | 5 | $\frac{6}{36}$ | $\frac{5}{36}$ |
| 3 | 6 | 7 | $\frac{\quad}{36}$ | $\frac{\quad}{36}$ |
| 4 | 6 | 5 | | |
| 5 | 6 | 5 | | |
| 6 | 6 | 11 | | |

**a** Complete the table by calculating the theoretical probability and the relative frequency of throwing each number.

**b** Is the relative frequency of rolling a 6 larger or smaller than the theoretical probability?

A small number of trials can lead to misrepresentative results. However, results can also be intentionally skewed, or distorted, by tampering with the equipment used in an experiment. To find out if this die is unfair, another 3600 trials were conducted.

The results of the additional trials are given in the table below.

**c** Would you say that this is a fair die? Why or why not?

| Number | Expected frequency | Actual frequency |
|---|---|---|
| 1 | 600 | 285 |
| 2 | 600 | 576 |
| 3 | 600 | 745 |
| 4 | 600 | 467 |
| 5 | 600 | 512 |
| 6 | 600 | 1015 |

**9** In a monster-catching game, a rare monster is claimed to appear 5% of the time. You want to test the validity of that claim, so you ask your friend to join you in an experiment where you will encounter 50 monsters each and write down how many times the rare monster appears. You encountered the rare monster two times and your friend encountered it seven times.

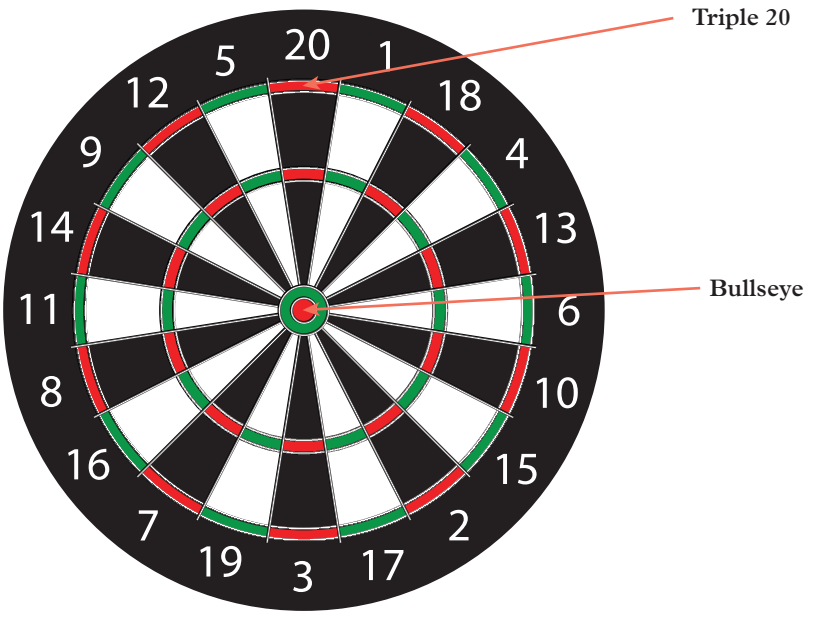
**a** Calculate the relative frequency of your sample and your friend's sample as a percentage.

**b** Do you believe the claim that the monster appears 5% of the time? Why or why not?

**c** A special form of the rare monster has a chance of appearing of 1 in 4096. Write this as a percentage to four decimal places.

**d** If you encounter the rare monster 4095 times, without seeing the special form, are you guaranteed to encounter the special form on your next encounter? Why or why not?

**10** Pritesh plays darts. He wants to determine how likely it is to score a bullseye and a triple 20, provided the dart hits the board. He takes a picture of his dart board and writes a computer program to define each pixel on the board as a point the dart can hit. He then tells it to throw 1000 darts at the board randomly. Of the 1000 darts, 1 dart hits the bullseye and 5 darts hit the triple 20.




**a** Calculate, as a simplest fraction, the relative frequency of getting:

**i** a bullseye **ii** a triple 20.

Pritesh gets the computer to count the number of pixels on the board for the bullseye and for the triple 20. The program prints the following number of pixels:

Total: 916 088, Bullseye: 1283, Triple 20: 4214.

**b** Calculate, as a percentage to two decimal places, the probability of getting:

**i** a bullseye **ii** a triple 20.

**11** A game show has a large tub containing 10 000 coloured marbles. The aim of the game is to get the closest guess of the number of red marbles in the tub. You're allowed to take three samples of the marbles and let a machine count the number of red marbles and total marbles in the sample.

The first contestant takes her three samples and gets 49 reds out of 87, 34 reds out of 57, and 43 reds out of 74.

**a** Calculate the relative frequency of each of the three samples as a percentage correct to the nearest per cent.

**b** She decides to use her median relative frequency to estimate the number of red marbles. Calculate her estimate.

The second contestant takes his three samples and gets 54 reds out of 95, 19 reds out of 32, and 31 reds out of 53.

**c** Calculate the relative frequency of each of the three samples as a percentage correct to the nearest per cent.

**d** He decides to calculate the mean relative frequency to estimate the number of red marbles. Calculate his estimate.

**e** Still not confident in his estimate, the second contestant decides to combine his three samples and calculate the relative frequency to the nearest hundredth of a per cent. He uses this figure to re-estimate the number of red marbles. Calculate his second estimate.

**f** The second contestant locks in his second estimate. If the number of red marbles in the tub was 5821,

**i** which contestant won?

**ii** did the second contestant choose his best estimate?

**12** An experiment involves rolling two dice and recording the sum of the values shown on each die.

| | | Die 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** |
| **Die 1** | **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| | **2** | 3 | 4 | | | | |
| | **3** | | | | | | |
| | **4** | | | | | | |
| | **5** | | | | | | |
| | **6** | | | | | | |

**a** Complete the table.

**b** How many chances are there of rolling a total of 6?

**c** What is the theoretical probability of rolling a total of 6?

**d** How many times would you expect to roll a total of 6 if you rolled the dice 36 times?

**e** Perform the experiment and tally the results.

**f** Compare the results of your experiment to the theoretical probability of rolling a total of 6. Comment on the reasons for the similarities or differences between the two figures.

**13** The chocolate company Mars often run a promotion in which you can win a free Mars Bar. One out of every six wrappers is printed with a token for a free bar.

**a** What is the theoretical probability of winning a Mars Bar?

**b** What is the theoretical probability of *not* winning a Mars Bar?

**c** Using your theoretical probabilities, calculate how many Mars Bars you could expect to buy in order to win five free Mars Bars.

**d** Describe a probability experiment that has six possible outcomes and can be used to simulate the chance of winning a free Mars Bar.

**e** Conduct your experiment 30 times. How many free Mars Bars did you win?

**f** For better results, simulations should always be repeated. Repeat this experiment another four times and record the number of free Mars Bars you win each time.

**g** What are some of the advantages and disadvantages of running multiple simulations when conducting an experiment?

**14** A six-sided die was rolled a number of times. The relative frequency of each number appearing is shown below. Determine the smallest number of total die rolls that could have been rolled to give these probabilities.

| **Number** | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **Relative frequency** | $\frac{1}{10}$ | $\frac{1}{3}$ | $\frac{1}{7}$ | $\frac{1}{5}$ | $\frac{1}{6}$ | $\frac{2}{35}$ |

Check your Student obook pro for these digital resources and more:

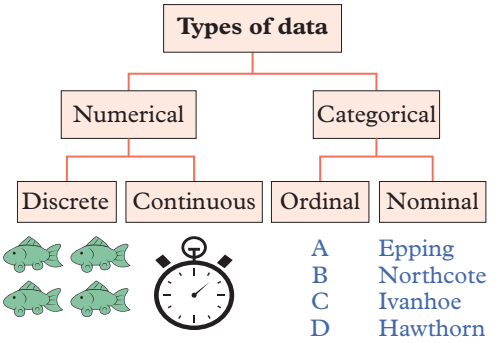
**Interactive skillsheet** Relative frequency

**Investigation** Rolling a 10-sided die

**Topic quiz** 10G

# Chapter summary

## Classifying data




## Summary statistics

Data: 2, 3, 4, 4, 6, 7, 10

Mean (numerical average) $= \dfrac{2+3+4+4+6+7+10}{7}$

$= 5$

Median (middle): 2, 3, 4, 4, 6, 7, 10

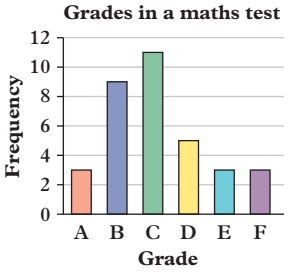
Mode (most common): = 2, 3, 4, 4, 6, 7, 10

Range = 10 – 2

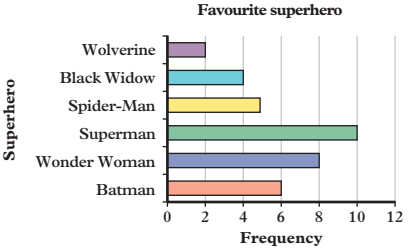
= 8

Outlier: 1, 2, 3, 3, 100

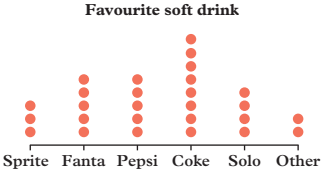
## Column graph




## Bar chart




## Dot plot



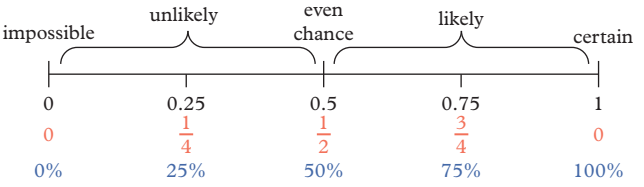

## Stem-and-leaf plot

7, 11, 15, 16, 20, 22, 29, 29, 43, 47

| Stem | Leaf |
|---|---|
| 0 | 7 |
| 1 | 1 5 6 |
| 2 | 0 2 9 9 |
| 3 | |
| 4 | 3 7 |

Key: 1 | 5 = 15

## Theoretical probability




For a standard 6-sided die,
Sample space = {1, 2, 3, 4, 5, 6}

$\Pr(\text{rolling a 3}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$

$= \dfrac{1}{6}$

## Experimental probability

Relative frequency $= \dfrac{\text{number of successful trials}}{\text{total number of trials}}$

Expected number = Pr(event) × number of trials



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# Chapter review

**Chapter review quiz**
Take the chapter review quiz to assess your knowledge of this chapter.

**Quizlet**
Test your knowledge of this topic by working individually or in teams.

## Multiple-choice

10A **1** Which of the following is an example of discrete numerical data?

**A** Your height
**B** Your finishing position in a race
**C** The number of pets that you have
**D** The colour of your hair
**E** The time taken to answer this question

10B **2** What is the median of the data set: 5, 3, 4, 7, 9, 16, 5, 6, 8?

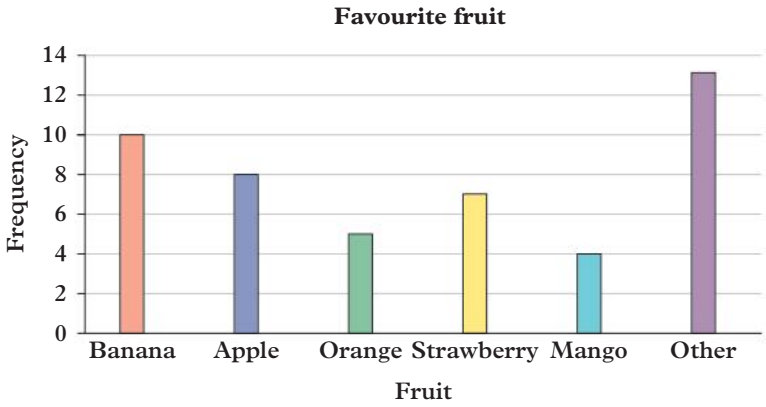
**A** 5 **B** 6 **C** 7 **D** 9 **E** 16

10B **3** What is the mean of the data set: 6, 9, 11, 7, 3, 15, 7, 14?

**A** 7 **B** 8 **C** 9 **D** 12 **E** 15

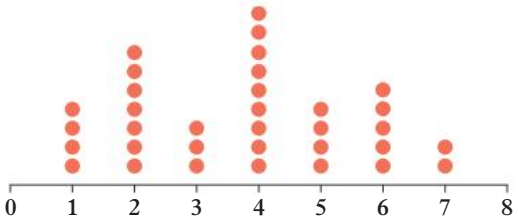
10C **4** How many people were surveyed to create the following column graph?

**A** 4 **B** 6 **C** 10 **D** 13 **E** 47




10C **5** What is the mode of the data presented in the dot plot below?

**A** 3 **B** 4 **C** 7 **D** 8 **E** 6




10D **6** The key to a stem-and-leaf plot is 1|2 = 12. What is the leaf of the data point 48?

**A** 4 **B** 8 **C** 12 **D** 48 **E** 40

10E **7** How could the possibility of Friday following on from Thursday be described?

**A** Impossible **B** Even chance **C** Unlikely **D** Certain **E** Likely

10F **8** What is the probability of selecting a heart from a standard deck of cards?

**A** $\frac{1}{13}$ **B** $\frac{1}{52}$ **C** $\frac{1}{2}$ **D** $\frac{1}{4}$ **E** $\frac{1}{3}$

10G **9** If a standard die is rolled 120 times, how many times would you expect to obtain a four?

**A** 4 **B** 20 **C** 30 **D** 120 **E** 15

## Short answer

10A **1** Diana surveys her class to see who can roll their tongues. The results of the survey are below. Y represents 'yes' and N represents 'no'.

Y N N Y Y N N Y Y Y N N Y

**a** Classify the data.

**b** Record the data in a frequency table.

**c** How many people were surveyed?

**d** Which was the most common attribute?

10B **2** **a** Calculate the **i** mean (to one decimal place), **ii** median, **iii** mode and **iv** range for this group of data.

7, 1, 8, 12, 71, 6, 14, 10, 2, 2, 3, 5, 9

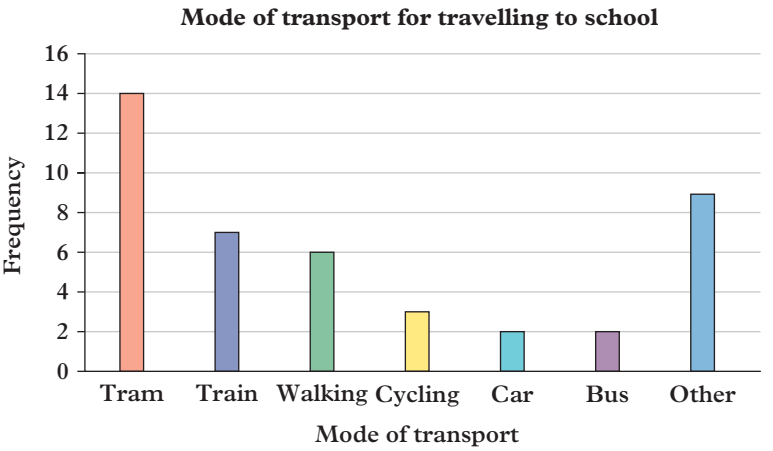
**b** Explain why the mean is not the most appropriate summary statistic to represent this group of data.

10C **3** This bar chart shows the mode of transport used by a group of students to travel to school. Determine whether the following statements are true or false.

**a** The tram is the most common mode of transport.

**b** More than half of the students take a tram or train.

**c** A larger number of students take the train rather than the bus.




10C **4** Present the data in the frequency table below in a:

**a** dot plot **b** bar chart.

| Favourite holiday activity | Frequency |
|---|---|
| Swimming at the beach | 11 |
| Camping | 4 |
| Watching TV | 7 |
| Reading a book | 2 |
| Hiking | 5 |
| Snow-skiing | 4 |
| Other | 3 |

10D **5** This stem-and-leaf plot shows the lengths of 23 worms in a worm farm.

| Stem | Leaf |
|---|---|
| 8 | 1 5 9 |
| 9 | 2 2 3 5 6 8 |
| 10 | 1 1 1 2 2 4 6 7 8 |
| 11 | 0 1 2 2 |
| 12 | 7 |

Key 8 | 1 = 8.1 cm

**a** Write down the values represented by the leaves 4 and 9.

**b** Which stem has the highest frequency?

**c** What was the mean worm length correct to the nearest tenth of a centimetre?

**d** What was the median worm length?

**e** What was the modal worm length?

**f** What was the range of worm lengths?

**g** Write two sentences that describe the lengths of the worms in the worm farm.

10D **6** Arrange the data below into a stem-and-leaf plot.

13 16 45 32 62 17 22 28 55 9 12 38 42

51 59 31 16 13 12 25 30 41 20 5 23 33

10E **7** For each of the following experiments:

**i** list the sample space

**ii** categorise the outcome in brackets as: impossible, unlikely, even chance, likely, certain.

**a** flipping a fair coin (head)

**b** rolling a standard, six-sided die (odd number)

**c** selecting a card from a standard 52-card deck and recording the suit (diamond)

**d** randomly selecting a counter from a bag containing 12 red, 7 blue and 2 white counters (red counter).

10F **8** Calculate the theoretical probability of:

**a** rolling a standard six-sided die and obtaining a 3

**b** flipping a fair coin and landing on tails

**c** selecting a consonant from the word REVISION

10G **9** A card is drawn from a standard deck of 52 cards. The suit is noted, then the card is returned to the deck and the deck is shuffled. The first 10 recorded results are:

hearts, clubs, hearts, clubs, diamonds, hearts, diamonds, spades, hearts, clubs

**a** Calculate the relative frequency, as a decimal, of obtaining:

**i** a heart **ii** a spade **iii** a club

**iv** a diamond **v** a red card (hearts or diamonds).

**b** Which relative frequency (or relative frequencies) from part **a** is (or are) closest to the theoretical probability of the same event?

## Analysis

**1** A survey was conducted on the number of movies a group of Year 7 students had watched in the past month. The results are below.

1 5 3 2 7 1 4 2 3 6 2 5 7 13 2 2 1 0 4 2 5 2 1 0 5 1

**a** What type of data is this?

**b** How many people were surveyed?

**c** Calculate the **i** mean (to one decimal place), **ii** median, **iii** mode and **iv** range for this data set.

**d** Record the results in a frequency table.

**e** Present the data in a graph and explain why you think this is the best graph to present this particular data set.

**2** A bag contains 20 marbles. 10 are bronze, 5 are silver, 3 are gold, and the rest are iron. A marble will be pulled out of the bag at random and then replaced.

**a** State the sample space of the experiment.

**b** Calculate the theoretical probability of selecting an iron marble.

**c** A marble is picked from the bag and then replaced. This is repeated 12 times with the following outcomes:

silver, iron, silver, bronze, bronze, bronze, bronze, bronze, silver, gold, silver, bronze.

Calculate the relative frequency of selecting an iron marble.

**d** Another 12 trials are repeated with the following outcomes:

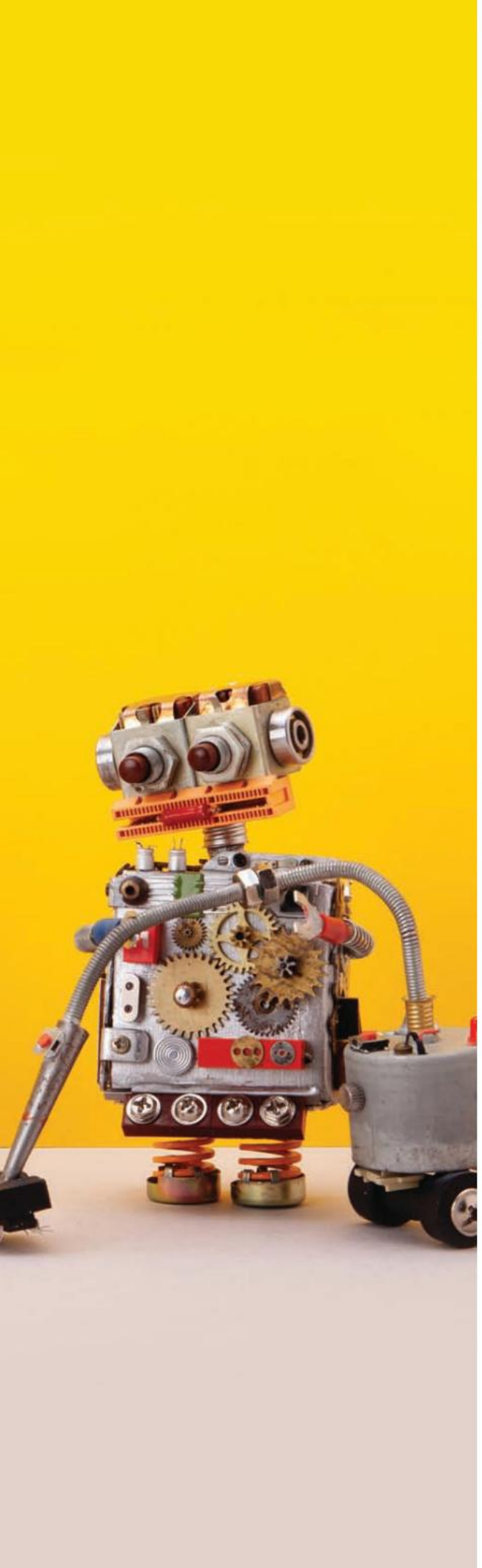
bronze, iron, bronze, bronze, bronze, bronze, bronze, silver, iron, bronze, gold, bronze.

Calculate the relative frequency of selecting an iron marble.

**e** Give a reason why the relative frequencies are not equal to each other or to the theoretical probability in part **b**.

# 11

# Computational thinking



## Index


**11A** Printing information to the computer screen
**11B** Variables in coding
**11C** Asking a user for input
**11D** Conditions in code


**Document**
Why Python?

**Document**
Writing and executing code in Python

**Inter-year links**

| | |
|---|---|
| Year 8 | 10 Computational thinking |
| Year 9 | 10 Computational thinking |
| Year 10 | 12 Computational thinking |

## Curriculum links

- Design and implement mathematical algorithms using a simple general purpose programming language (VCMNA254)

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## Materials

- Computer
- Python (online or downloaded)

  Note: Python is used as the coding language in this chapter. Instructions will vary for other coding languages.

# 11A Printing information to the computer screen

## Learning intentions

✔ I can use the terms string and characters to describe text in Python.

✔ I can provide instructions in Python to print information on the computer screen.

## Coding

- An **algorithm** is a clear set of instructions for solving a problem. For computer algorithms to be clear, they need to follow the rules of the computer language that is being used (in this text, Python).
- **Coding** is the process of designing algorithms and writing them in a computer language such as Python.

## Printing

- When we **print** information in coding, we are displaying information on the screen.
- A **string** in Python is a sequence of characters. Strings are commonly used to display words on the screen.
- Quotation marks are used at the beginning and end of each string to let the computer know that it should expect a string.
- **Characters** include all letters, symbols and spaces between the quotation marks.

**Quotation marks**

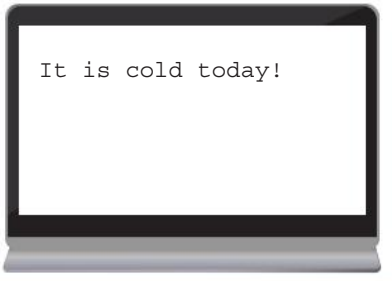
`"It is cold today!"`

Letters, spaces and symbols are all **characters.**

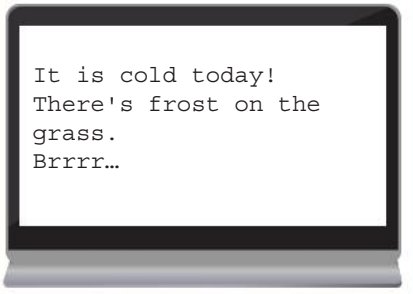
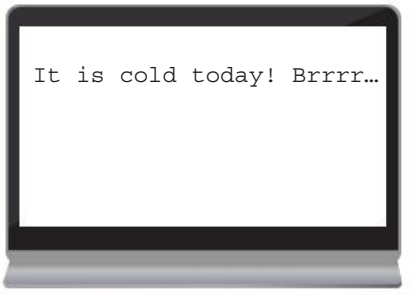
This **string** has **17 characters.**

## Printing text to the screen

- When coding in Python, the command `print( )` is used to print information to the screen. Any information you want to see on the screen should be placed inside the brackets.
- Note that the quotation marks `(" ")` do not appear on the screen.

| What we type | What we see on the screen |
|---|---|
| `print("It is cold today!")` |   |

- To print more than one piece of information, we can give the computer multiple `print( )` commands. This will print each of the pieces of information on a new line.
- To print multiple statements on the same line, we can use commas to separate each statement inside one `print( )` command.

| What we type | What we see on the screen |
|---|---|
| `print("It is cold today!")`<br>`print("There's frost on the grass.")`<br>`print("Brrrr…")` |  |
| `print("It is cold today!", "Brrrr…")` |  |

## **Example 11A.1** Printing a string

Write down the code used to print the following string on the screen:

`I can code in Python`

**THINK**

To display information on the screen, used the `print( )` command and put quotation marks around the words you want to print.

**WRITE**

```
print("I can code in Python")
```

**Helpful hints**

✔ It is very common to forget the brackets after the print or to forget the quotation marks around the string. This will cause an error and often frustration!

✔ When printing multiple strings, the location of the comma matters. Remember that if a comma is inside the quotation marks, it will be printed. If it is between quotation marks it will be used to tell the computer that there are multiple strings.

| Correct | Incorrect |
|---|---|
| `print("Hello!","Good bye!")` | `print("Hello!," "Good bye!")` |

ANS p612

# Exercise 11A Printing information to the computer screen

1 Write down the number of characters in each of the following strings.

**a** `"Hello world!"`

**b** `"Now is better than never."`

**c** `"$5 is the smallest note"`

**d** `"I don't speak Python"`

2 Each of the following `print( )` commands has a minor error that will prevent the computer from understanding it. Describe the error.

**a** `print(Hello world!)`

**b** `print "Hello world!"`

**c** `print("Hello world"!)`

3 A student would like to print the names of her three favourite subjects on the screen. What are two different ways she can do this?

11A.1 4 Write down the code used to print the following strings on the screen.

**a** `Too hot!`

**b**
```
Too hot!
Too cold!
Just right :)
```

**c** `Too hot! Too cold! Just right :)`

5 The print function has a default where it places a space between each string of characters. For example, the code `print ("Hello", "world")` will print `Hello world`. Notice the space separating the two words.

We can use the parameter `sep`, short for separator, to change the space default to any other character or string. For example, the code `print ("Hello", "world", sep = "-")` will print `Hello-world`. You need to add an equals sign and include inverted commas around the characters or string you want to use as your separator.

Write down the code used to print the following strings on the screen including the separating parameter. (Hint: The code `\n` means new line.)

**a** `Helloworld`

**b**
```
Hello
world
```

**c** `I/can/code`

**d**
```
I can code well.
Very well indeed.
```

**e** `chicken -> egg -> chicken -> egg`

# 11B Variables in coding

## Learning intentions

- ✔ I can define variables and call them in lines of code.
- ✔ I can understand how variables in coding are similar to variables in algebra.
- ✔ I can choose names for variables based on the information they are going to store.

## Defining variables

- Defining a variable in Python always involves the following structure:

An equals sign:
This tells the computer to **store** the information on the right-hand side and the variable on the left

The **name** of the variable:
A letter or letters on the left-hand side

```
a = 5
```

The **value** of the variable:
The information we want to store in the variable

### Maths Curriculum link

- ✔ Introduce the concept of variables as a way of representing numbers using letters (VCMNA251)

## Calling a variable

- When a line of code refers to a variable that has been defined earlier in the code, this is known as **calling a variable**. This is identical to substituting in a variable in algebra.

  For example, line 3 below is calling the variables x and y, adding them together and then displaying the result.

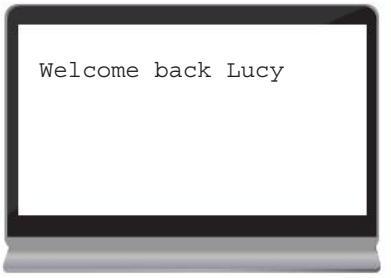
```
1 x = 8
2 y = -2
3 print(x+y)
```

| Algebra | Coding |
|---|---|
| Evaluate $4x - y$ by substituting $x = 3$ and $y = 5$ into the expression. | Describe what the code below displays on a screen.<br>`1 x = 3`<br>`2 y = 5`<br>`3 print(4*x-y)` |
| **THINK:**<br>**1** Replace the $x$ with 3 and the $y$ with 5.<br>**2** Perform the multiplication, then the subtraction. | **THINK:**<br>**1** Consider what has been defined. Line 1 defines the value of x as 3.<br>Line 2 defines the values of y as 5.<br>**2** In line 3, the expression inside the brackets calls the variables x and y, replacing the letters with their values (x = 3 and y = 5).<br>**3** The * symbol in line 3 represents a multiplication symbol in Python.<br>**4** The item that is printed is the result of the expression $4 \times 3 - 5$. |
| **WRITE:**<br>$4x - y = 4 \times 3 - 5$<br>$= 12 - 5$<br>$= 7$ | **WRITE:**<br>7 will be displayed on the screen. |

# Storing strings as variables

- As well as storing numbers – as in algebra – variables in coding can store strings.
- Numbers will be recognised immediately by Python; however, to indicate a string, quotation marks are required at the start and end.

  For example, below x is defined to have a value of "Lucy".

| What we type | What we see on the screen |
|---|---|
| `x="Lucy"`<br>`print("Welcome back", x)`<br><br>The variable x is **called** inside the print command, and so `"Lucy"` is displayed on the screen after `"Welcome back"` rather than x |  |

→ The first line of the code above stores the text `"Lucy"` in the variable x and the second line prints two pieces of information, separated by a comma.

→ There are no quotation marks around x, indicating to the computer that x is a variable, and not a string.

- The names of variables can be words rather than just letters. Variables that have two words for a name need to be separated by an underscore, such as blue_car.

  For example,

```
name = "Lucy"
print("Welcome back to your computer",name)
```

→ Using the variable name instead of x makes the code easier to read, as it is clear what information the variable is expected to contain. This is good practice.

## Example 11B.1 Printing mathematical calculations

Consider the code below. What will it display on the screen?

```
1 x = 4
2 y = 1
3 print(4*x-3*y)
```

**THINK**

1. The first two lines of the code define the variables x and y.
2. Line 3 calls both x and y. The calculation is performed inside the print command before it is displayed.

**WRITE**

$$\begin{aligned} 4x - 3y &= 4(4) - 3(1) \\ &= 16 - 3 \\ &= 13 \end{aligned}$$

The code will print: `13`

## **Example 11B.2** Printing variables and strings

Consider the code below:

```
1 x = "Federation Square"
2 y = "Flinders Street"
3 z = "Melbourne"
4 print(x, "on",y, "is a well-known landmark in",z)
```

**a** What is the above code displaying on the screen?

**b** Rewrite the code with new variable names so that the names describe the information they contain.

**THINK**

**a** **1** Line 1, line 2 and line 3 are defining the variables `x`, `y` and `z` respectively.

**2** Line 4 is printing five pieces of information. Pieces 1, 3 and 5 correspond to variables `x`, `y` and `z`. Pieces 2 and 4 are text because they start and end with quotation marks `(" ")`.

**b** Based on the information stored in the variables, it would be better to rename variable `x` to `landmark`, `y` to `street` and `z` to `city`. This way, when someone reads the code, it is clear what each variable is supposed to be storing.

**WRITE**

**a**
```
Federation Square on Flinders Street is a well-known
landmark in Melbourne
```

**b**
```
landmark="Federation Square"
street="Flinders Street"
city="Melbourne"
print(landmark, "on",street, "is a well-known landmark in",city)
```

## **Example 11B.3** Identifying errors in code

Why will the following code produce an error?

```
1 x = 2
2 print(3*y+2*x)
```

**THINK**

**1** Line 1 defines a variable named `x` and gives it a value of 2.

**2** Line 2 attempts to evaluate the expression $3y + 2x$ by calling `y` and `x` and displaying it. The variable `y` was not defined before being called in line 2, so the computer has nothing to multiply by 3.

**WRITE**

When the computer attempts to call the variable `y` in line 2, it will not be able to do so as `y` has not been defined.

**Helpful hints**

✔ Do not forget to define variables before trying to call them. Consider the following example:

→ If you have a dog, and someone asks you "What colour is your dog?" that's an easy question to answer. But, if you don't have a dog, it is very confusing!

✔ If you forget to define a variable, the computer will report an error. The error will end with the line

```
NameError: name __ is not defined.
```

# ANS p612 Exercise 11B Variables in coding

11B.1 **1** For each for the following programs, write down what will be displayed on the screen.

**a**
```
x = 7
y = 5
print(x-y)
```

**b**
```
x = 12
y = 4
print(x+y)
```

**c**
```
x = -2
y = 5
print(x*y)
```

**d**
```
x = 3
y = 1
print(2*x-5*y)
```

**e**
```
x = 6
y = 3
z = 2
print(2*x-3*y+5*z)
```

**2** For each of the following, write down whether Python will read it as a string or as a variable.

**a** `v` **b** `"text"` **c** `"variable"` **d** `text` **e** `variable`

**3** For each of the following programs, write down what will be displayed on the screen.

**a**
```
favourite_subject = "Maths"
print("My favourite subject is", favourite_subject)
```

**b**
```
sunrise = "6.45am"
sunset = "7.15pm"
print("The sun rose at",sunrise,"today and set at",sunset)
```

**c**
```
piggybank = 37
goal = "bike"
print("I have",piggybank,"dollars in my piggybank and I am saving for a",goal)
```

11B.2 **4** For each of the following programs, rewrite the code using different names for the variables (`x` and `y`) so that the code is easier to read.

**a**
```
x = 14
y = "wheatbix"
print("I once ate",x,y,"in one sitting")
```

**b**
```
x = "Africa"
y = "The Nile"
z = 6650
print(y,"is the longest river in",x,"and runs for" ,z,"km")
```

**c**
```
a = "Ms. Tiong"
b = "Grade 5"
c = "helpful"
print(a, "was my favourite teacher in",b,"because she was", c)
```

11B.3 **5** Each of the blocks of code below contains one error. Proofread the code, run it in Python to see what error it generates and write down the correct code.

**a**
```
y = 7
z = 2
print(x+y)
```

**b**
```
y = 7
z = 2
print(4y-2z)
```

**c**
```
x = Australia
y = 44
print("The hottest day of summer in",x,"was",y,"degrees last year")
```

**d**
```
name = "Kelvin"
print "Welcome back to your computer,", name
```

# 11C Asking a user for input

## Learning intentions

✔ I can request input from a user of my code and store it in a variable.

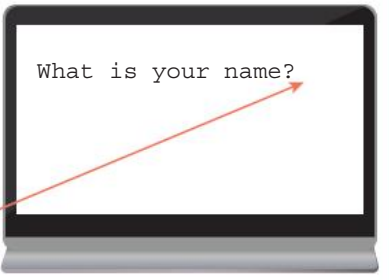
✔ I can specify the type of input I want from a user.

## Requesting user input

- The `input()` command is used to request the user of the computer to type something into the program.
- The default expectation for the `input()` command is a string.

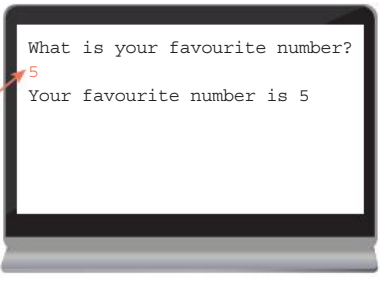
**Maths Curriculum link**

✔ Calculate volumes of rectangular prisms [VCMMG259]

| What we type | What we see on the screen |
|---|---|
| The input( ) command tells the computer to expect the user to type something.<br>`name = input ("What is your name?")`<br>The input the user types will be stored in the variable `name`<br>The text or **string** inside the brackets is what the user will see.<br>Type your response here |  |

## Working with whole numbers

- When the user's answer is expected to be a number rather than text, there is one extra step that is required to store the number.
- The command `int()`, short for integer, tells Python that the number is an integer and to store it as such. For example,

| What we type | What we see on the screen |
|---|---|
| `num = input ("What is your favourite number?")`<br>`num = int(num)`<br>`print("Your favourite number is",num)`<br>Type your response here |  |

## Application

- In the topic of measurement, when finding areas or volumes, the same formulas are used over and over. Instead, we can write a bit of code to do the work for us!

## **Example 11C.1** Using inputs

Consider the line of code below:

```
colour = input("What is your favourite colour?:")
```

**a** What is displayed on the computer screen?

**b** Suppose the user typed green and pressed Enter. What is stored in the variable colour?

**THINK**

**a** **1** Determine what is being defined. The line of code has an equal to sign and a word `colour` to the left of the equals sign. Therefore, `colour` is being defined as a variable.

**2** The `input()` command is present, suggesting that the user is being asked to type something into their computer.

**3** The text inside the brackets of the `input()` command is what is printed to the screen.

**b** Once the user types green and presses Enter, `green` will be stored in the variable `colour`.

**WRITE**

**a** `What is your favourite colour?:`

**b** `green`

## **Example 11C.2** Writing code requiring user input

Write code that asks a user for a whole number, and then prints double that number.

**THINK**

**1** To ask a user to type some input, the `input()` command will need to be used.

**2** Inside the brackets of the `input()` command, include a string that the user will see when using the program.

**3** For Python to know that `num` is an integer, we use the `int()` command and store the result back into the variable `num`.

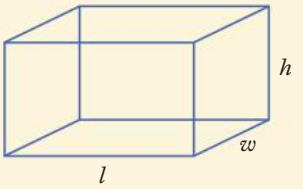
**4** Finally, recall that to multiply in Python the `*` symbol is used.

**WRITE**

```
num = input("What number would you like to double?")
num = int(num)
print(2*num)
```

## **Example 11C.3** Writing code to apply a formula

Write code that asks the user for the length, width and height of a rectangular prism, and prints the volume of that prism.




**THINK**

1 From Section 9F, recall that the formula for the volume of a rectangular prism is:

volume = length × width × height

2 Given that the three variables that are being defined will be used in a calculation, we would like to flag them as integers.

3 Finally, perform the calculation using the formula for volume, and print the answer.

**WRITE**

```
length = input("What is the length?")
width = input("What is the width?")
height = input("What is the height?")
length = int(length)
width = int(width)
height = int(height)
volume = length*width*height
print(volume)
```

**Helpful hints**

- ✔ Remember to convert variables that contain user input into integers before using them in calculations.
- ✔ Do not forget to store the user input in a variable.
  For example, writing

  ```
  input("Height?")
  ```

  will ask the user to type in the height, but this information is not being stored anywhere and so will be lost when the user presses Enter.
- ✔ Remember to close your quotation marks within the brackets. If you don't, you will encounter an error ending in:

  ```
  SyntaxError: invalid syntax.
  ```
- ✔ The ^ symbol above the error usually points to where in the code an error has occurred. This is very helpful when you're trying to find it!

ANS p612

# Exercise 11C Asking a user for input

11C.1 **1** Consider the line of code below:

```
1 x = input("What did you have for breakfast?")
2 print(x)
```

**a** What will line 1 display on the screen?

**b** What command in the code above is requesting that the user types something?

**c** What will line 2 display on the screen?

11C.2 **2** Write code that asks a user for two integers and prints the sum of the two integers.

**3** Write code that asks a user for two integers and prints the product of the two integers.

**4** Write code that asks a user for an integer and prints the next two consecutive integers (for example, if the user types 17, print 18 and 19).

**5** Write code that asks a user to enter their favourite subject and then incorporates that subject in a sentence. For example, if the user enters `"coding"`, it prints `Hey! I like coding too!`

11C.3 **6** Write code that asks a user for the following:

**a** The side lengths of a rectangle and prints its area

**b** The base and height of a triangle and prints its area

**c** The side length of a cube and prints its volume

**d** The base and height of a parallelogram and prints its area

**7** Write code that asks a user for the dimensions of a rectangular prism and prints its surface area.

**8** The function, `int()`, can be applied to the input function as the user is being asked for a value.

For example, in Example 11C.3, the first line of code could instead be `length = int(input("What is the length?")`. This removes the need for the line of code that reads `length = int(length)`.

Write the missing lines of code in each of the following.

**a**
```
first_number = int(____________________________)
second_number = int(input("Enter second number"))
____________________________________
print("The addition of the two numbers is: ", sum1)
```

**b**
```
x = int(input____________________
y = ______("Enter the value for y: "))
y = 2*x+1
print("The correct y value for your x value is: ",______)
```



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# 11D Conditions in code

## Learning intentions

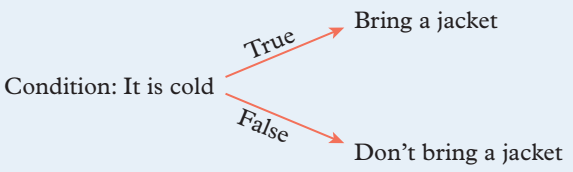
- ✔ I can write code that only works if a certain condition is met.
- ✔ I can explain the purpose of indentation in code and use indentation to structure my code.
- ✔ I can use else statements to capture all conditions that are not given in my if statement.

## Conditions in code

- **Conditions** are statements that are either true or false.
- In writing code, we can develop a result based on:
  - → checking whether a condition is true
  - → providing different actions for the code to perform depending on whether that condition is true.

**Maths Curriculum link**

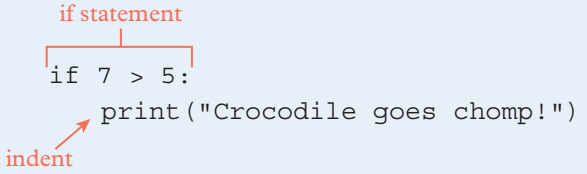
- ✔ Classify triangles according to their side and angle properties [VCMMG262]




## Checking conditions in Python

- The word `if` is used in Python to check whether a certain condition is true.
  - → An if statement is a line of code which has the word if, followed by some condition, and finishing with a colon.
- An **indent** is a gap used in Python to indicate a logical structure.

  For example, the indent below indicates to the computer that it should only perform the `print()` command if the if statement is true.




  - → A single indent in Python is denoted by exactly 3 spaces or 1 tab.
- Coding conditions are represented by different symbols from mathematical relationships. See the table for comparison.

| | Maths | Python |
|---|---|---|
| **Equals** | $a = b$ | `a == b` |
| **Not equals** | $a \neq b$ | `a != b` |
| **Less than** | $a < b$ | `a < b` |
| **Less than or equal to** | $a \leq b$ | `a <= b` |
| **Greater than** | $a > b$ | `a > b` |
| **Greater than or equal to** | $a \geq b$ | `a >= b` |

# Checking multiple conditions in Python

- Sometimes, it is necessary to check multiple conditions.
- Use `or` in your if statement to ensure that only one of the conditions is required for the statement to be true.
- Use `and` in your if statement to ensure that both of the conditions are required for the statement to be true.

  For example, when deciding what to wear, you might wear a jacket if it is cold *or* if it is raining. The word *or* here makes it clear that we only need at least one of the conditions to be true.

  On the other hand, when deciding whether to see a movie, you might want to check if the movie has good reviews *and* if friends also want to see it. The word *and* here makes it clear that we need both conditions to be true.

# Using if / else pairs

- In many situations, it is important to do something different if the condition is false.
- An `else` statement indicates what should be done `if` the if statement is false.

```
if colour is "green":
  print("Green is a great colour!")
else:
  print("I prefer green")
```

→ The `else` statement is similar to how people use the word 'otherwise' in speech when describing options.

# Using an if / elif / else structure

- In some situations you may need to perform more than one check of a condition before getting to the else statement. We can introduce more conditions with the `elif` statement, which is short for 'else if'. You can have many `elif` statements, but you need to use the following structure.

  → Consider the problem of determining whether a number is positive, negative or zero.

| | **Code** | **Explanation** |
|---|---|---|
| | `num = input("Enter a number:")`<br>`num = int(num)` | A user is asked for a number, that number is stored in the variable num and flagged as an integer. |
| **If statement** | `if num > 0:`<br>` print("Positive")` | If the number is positive, `Positive` will be printed. |
| **Else if statement** | `elif num == 0:`<br>` print("Zero")` | Or if the number is zero, `Zero` will be printed. |
| **Else statement** | `else:`<br>` print("Negative number")` | Or if neither of the above statements are true, `Negative` will be printed. |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## Example 11D.1 If statements and indentation

For each of the following if statements, write down what will be printed on the screen.

**a**
```
if 7 > 5:
    print("Crocodile goes chomp!")
```
**b**
```
if 3 > 5:
    print("Crocodile goes chomp!")
```
**c**
```
if 3 > 5:
    print ("Crocodile goes chomp!")
print ("This line has no indent")
```

**THINK**

**a** 7 > 5 is a true statement, so the computer will perform any command that is indented underneath the corresponding if statement.

**b** 3 > 5 is a false statement, so the computer will ignore any command that is indented underneath the corresponding if statement.

**c** 3 > 5 is a false statement, so the computer will ignore any command that is indented underneath the corresponding if statement. The code that is not indented does not depend on the if statement and will always be carried out by the computer.

**WRITE**

**a** `Crocodile goes chomp!`

**b** Nothing is printed on the screen.

**c** `This line has no indent.`

## Example 11D.2 If and else statements on user input

Write code that asks a user to pick a whole number between 1 and 10. If the number is 7, print "That is my favourite number!". Otherwise, print "My favourite number is 7."

**THINK**

1. Use the `input()` command to ask the user to pick a number between 1 and 10. Use the `int()` command to save the number as an integer.
2. Check if the number is 7 using the if statements. Include the `if`, a double equals sign for the condition and end the statement with a colon.
3. Indent your next line and print the required statement from the question.
4. Use the command `else` followed by a colon to cover all possibilities where the number is not 7. Indent the next line and print the required statement.

**WRITE**

```
num = input("Pick a number between 1 and 10:")
num = int(num)

if num == 7:
  print("That is my favourite number!")
else:
  print("My favourite number is 7.")
```

## **Example 11D.3** Checking multiple conditions

Write code that asks a user to pick two numbers, and print "You picked a negative number!" if at least one of the user's numbers is less than zero.

**THINK**

1. Use the `input()` command twice to ask the user to pick two numbers, storing the numbers in different variables. Use the `int()` command to save the numbers as integers.
2. Use `or` in your if statement to ensure that only one of the conditions is required for the statement to be true.
3. Indent your next line and print the required statement from the question.

**WRITE**

```
num1 = input("Pick any integer: ")
num2 = input("Pick another integer: ")
num1 = int(num1)
num2 = int(num2)

if num1<0 or num2<0:
   print ("You picked a negative number!")
```

## **Example 11D.4** If, elif and else statements to classify triangles

Write code that requires a user to input the integer lengths of three sides of a triangle, and classifies the triangle as one of equilateral, isosceles or scalene.

**THINK**

1. Ask the user to input three integer side lengths using the `input()` command.
2. To check if the triangle is an equilateral triangle, write an if statement to check if all the side lengths are equal. Remember to use the double equal to sign.
3. To check if the triangle is an isosceles triangle, write an if statement to check if two side lengths are equal. Consider all three combinations.
4. Since a triangle that is neither equilateral nor isosceles is necessarily scalene, there is no need to explicitly state the condition for a scalene triangle. Use the `else:` command.

**WRITE**

```
side1 = input("Length of side 1:")
side2 = input("Length of side 2:")
side3 = input("Length of side 3:")
side1 = int(side1)
side2 = int(side2)
side3 = int(side3)

if side1 == side2 == side3:
  print("Equilateral triangle")
elif side1 == side2 or side1 == side3 or side2 == side3:
  print("Isosceles triangle")
else:
  print("Scalene triangle")
```

**Helpful hints**

✔ Indentation is very tricky if you are new to coding. Practice makes perfect, but it may help to think of an indent as a gate: if the condition is true, the gate below the condition opens. Otherwise, the computer moves on past the gate to the next line.

```
if 1 + 2 == 4:
    print("This will never be printed.")

if 1 + 2 == 3:
    print("You're good at Maths.")
```

ANS p613

# Exercise 11D Conditions in code

**11D.1** **1** For each of the codes below, write down what is printed on the screen.

**a**
```
if 10 == 3:
  print("True!")
```

**b**
```
if 10>3:
  print("True!")
```

**c**
```
if 10<3:
  print("True!")
  print("This line is indented")
print("This line has no indent")
```

**d**
```
if 10>3:
  print("True!")
  print("This line is indented")
print("This line has no indent")
```

**11D.2** **2** Write code that will ask a user to select from chicken, fish or tofu. If chicken is selected, print "We're out of chicken!". Otherwise, print "Coming right up!"

**11D.3** **3** Write code that asks a user to pick two numbers, and print "Both your numbers are negative!" if both of the user's numbers are less than zero.

**11D.4** **4** Write code that requires a user to input the integer values of two angles of a triangle, and classifies the triangle as one of equilateral, isosceles or scalene.

**5** A pair of jeans has a regular retail price (RRP) of $70. Write a program that asks the user how much they would be willing to pay for those jeans. If the user states a price that is more than 20% off the RRP, print "They will never be that cheap!". Otherwise, if the user states a price that is between 5% and 20% off, print "Better wait for a sale…". Otherwise, print "What are you waiting for? Go and buy them". (Hint: You may wish to review Exercise 4F before attempting this question.)

**6** In Example 11D.4, it is assumed the user will always provide the dimensions of a valid triangle. In reality, however, the user may – accidentally or on purpose – input 3 side lengths that cannot form a triangle. Consider:

side1 = 2
side2 = 5
side3 = 11

Describe why 2, 5 and 11 cannot be the side lengths of a triangle, and write code that would print "That is not a triangle!" if a user entered any three side lengths that did not form a valid triangle. (Hint: Draw a scale diagram to help understand why a triangle with these side lengths is not possible.)

# Semester 2 review

## Short answer

**1** Decide whether the following statements about $3x + \frac{1}{2}y - 4xy + 0.1$ are true or false. For the statements that are false, provide a reason for your answer.

**a** It is an expression.

**b** There are 3 terms.

**c** The coefficient of the $y$ term is $\frac{1}{2}$.

**d** The coefficient of the $xy$ term is 4.

**e** The constant term is 0.1.

**2** **a** Use the rule $y = 2x - 1$ to complete the table of values below.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** List the values as a set of coordinates with the form $(x, y)$.

**c** Plot the coordinates on the Cartesian plane.

**3** Simplify the following expressions by collecting the like terms where possible.

**a** $4a + 10a - 3a - a$

**b** $5x + y - 10x + 5$

**c** $fg - \frac{1}{2}gf + fgh - \frac{3}{4}gfh$

**d** $5x^2 + 3x - 7 - 2x^2 + 3$

**4** Solve the following equations to find the value of $x$.

**a** $2x = 10$

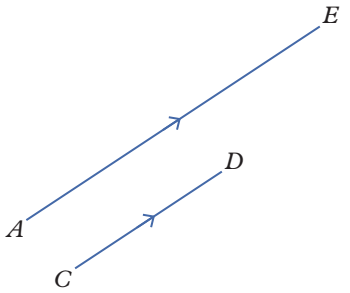
**b** $x - 9 = 15$

**c** $3x + 2 = 8$

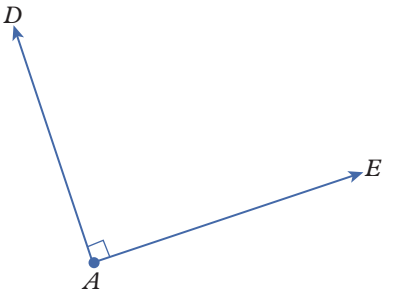
**d** $\frac{x - 6}{4} = 9$

**5** Describe the following diagrams using standard naming conventions and correct mathematical terminology.

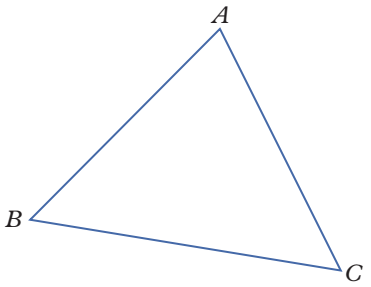
**a**




**b**




**6** **a** Measure $\angle ABC$ to the nearest degree.




**b** If $\angle ACB = 54°$, what type of triangle is $\Delta ABC$?

**7** For each of the following, state if the chance of it occurring is impossible, unlikely, even chance, likely or certain.

**a** You buy one ticket for the lottery and win first prize.

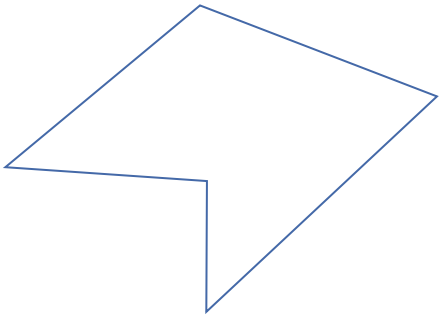
**b** A number is chosen at random from 1 to 20 and the result is even.

**c** A randomly generated polygon has straight edges.

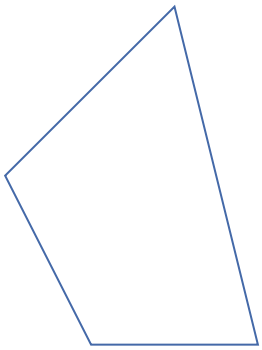
**d** A randomly selected chair at a furniture shop has four legs.

**8** State whether each of the following shapes are polygons or not.

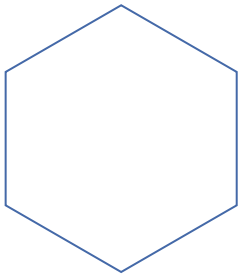
**a**



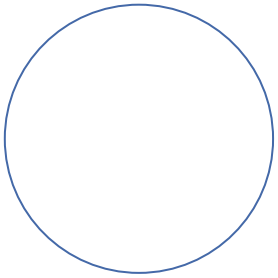
**b**



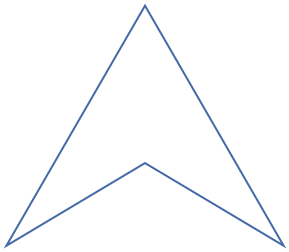
**c**



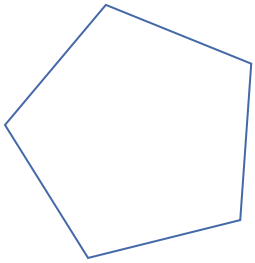
**d**



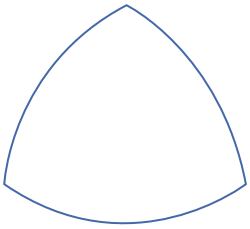
**e**



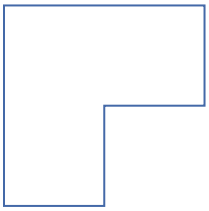
**f**



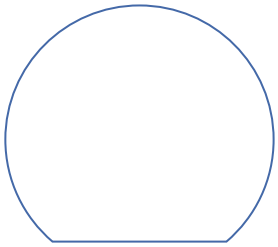
**g**



**h**



**i**



**9** Complete the following for each of the polygons in question **7**.

- **i** State whether it is concave or convex.
- **ii** State whether it is regular or irregular.
- **iii** Give the mathematical name.

**10** Draw two parallel lines cut by a transversal to form an acute angle of 35°. Show that your lines are parallel by measuring and labelling the alternate, corresponding and co-interior angle to your 35° angle.

**11** Determine the size of the angles described below.

- **a** Complement of 19°
- **b** Supplement of 19°
- **c** Angle that is vertically opposite to an angle of 19°
- **d** Angle that is co-interior to an angle of 19° between parallel lines
- **e** Angle that is alternate to an angle of 19° between parallel lines.

**12** Write an expression, equation or formula for each of the following statements.

- **a** Five more than three lots of $b$
- **b** The product of five-sevenths and three less than $m$ is equal to $n$.
- **c** Seven more than a number divided by 8 is equal to three-fifths. Let $u$ be the number.
- **d** The sum of two numbers divided by 2 is equal to the product of the two numbers. Let $p$ and $q$ be the two numbers.
- **e** Nine lots of the difference between $t$ and 8, where $t$ is greater than 8, is equal to 41.

**13** Evaluate the expression $a + bc + d$ by substituting the following values.

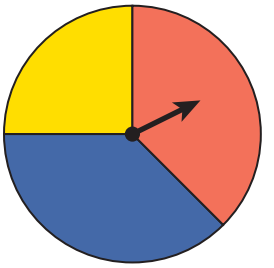
**a** $a = 2,\ b = 3,\ c = 9,\ d = 12$

**b** $a = -2,\ b = 3,\ c = 9,\ d = -12$

**c** $a = \frac{1}{2},\ b = \frac{3}{4},\ c = \frac{1}{9},\ d = \frac{5}{12}$

**d** $a = 2.5,\ b = 3.7,\ c = 0.9,\ d = 1.24$

**14** State whether the chance of each outcome is equally likely.

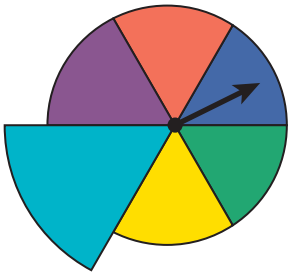


**a** The pointer on the spinner on the right with three segments is spun and the colour that the pointer lands on is recorded.

**b** A fair coin is flipped, and it is noted if the coin landed on heads or tails.

**c** A deck contains 20 brown cards, 10 blue cards and 10 purple cards. A card is drawn from the deck and the colour is noted.

**d** The faces of a fair, 12-sided die are numbered 1, 2, 3, 4, 4, 6, 7, 8, 9, 10, 11, 12. The die is rolled and the number is noted.



**e** The pointer on the spinner on the right with 6 segments is spun and the colour that the pointer lands on is recorded.

**15** For each of the following state:

**i** if the data collected is primary or secondary

**ii** if the data collected is from a census or a sample

**iii** the type of data collected

**iv** an appropriate graph or display that could be used.

**a** A student used data from the Australian Bureau of Statistics to investigate the average number of children in Australian families.

**b** A researcher used data from an existing study on 1000 high school students' study habits and their exam results to determine what was the most and least common study technique.

**c** A class measured the height of all students in Year 7 at their school to approximate the average height of all Year 7 students in Victoria.

**d** Top Games used its yearly sales data to determine the 10 best-selling games sold by Top Games that year.

**16** Use the table below to solve the equation $3x - 9 = 5(x - 2)$.

| **Guess** $x$ | **LHS** $3x - 9$ | **RHS** $5(x - 2)$ | **LHS = RHS?** |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| $\frac{1}{2}$ | | | |
| 0.4 | | | |

**17** Solve the following equations for the unknown.

**a** $0.8x - 8.9 = 3.5$

**b** $\frac{5}{12}y + \frac{5}{3} = \frac{17}{6}$

**c** $8 : b = 12 : 11$

**d** $\frac{15}{m} = \frac{50}{21}$

**18** A rectangle has a perimeter of 24 cm.

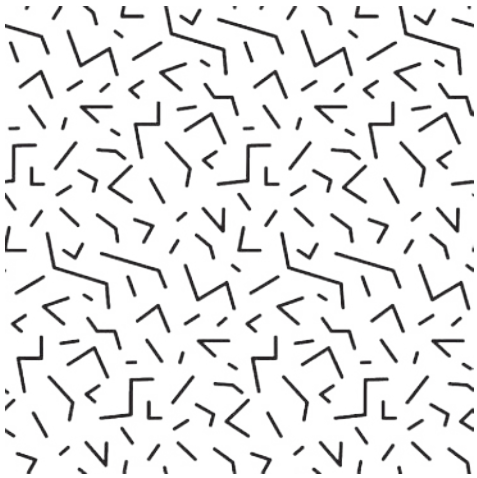
**a** If the length and width of the rectangle are whole numbers, state all possible pairs of length and width.

**b** Which pair of lengths creates the rectangle with the greatest area? What is this area?

A different rectangle has a perimeter of 48 cm.

**c** If the length is one-and-a-half times as long as the width, calculate the dimensions of the rectangle. Write your answer using decimals.

**19** A dressmaker measures a random sample of 12 angles from the print shown on the right. The angles are 19°, 214°, 321°, 191°, 239°, 225°, 142°, 147°, 271°, 124°, 315°, 26°.



**a** Calculate the following values for the sample, correct to two decimal places:

**i** mean

**ii** median

**iii** mode.

**b** Determine the relative frequency of selecting:

**i** an acute angle

**ii** an obtuse angle

**iii** a reflex angle.

**20** A rectangle has an area of 48 $cm^2$.

**a** If the length and width of the rectangle are whole numbers, state all possible pairs of length and width.

**b** Calculate the perimeter of each pair of length and width.

**c** One of these rectangles is randomly selected. Calculate the probability that the selected rectangle:

**i** has a side length that is a multiple of 4

**ii** has a side length that is odd

**iii** has a perimeter greater than 30 cm.

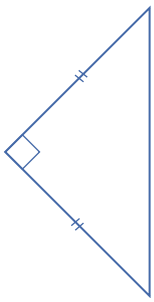
**d** If a square has an area of 48 $cm^2$, the side length is $\sqrt{48}$ cm. Between what two whole numbers is this side length?

**e** Consider all the possible whole number side lengths. Calculate the mean and median side length.

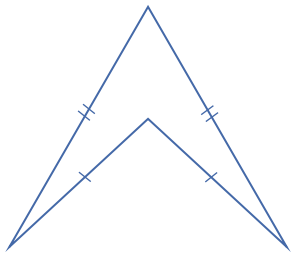
**f** Calculate how many times larger the length is than the width of each possible pair.

**21** State all the possible names of the triangles and quadrilaterals shown below.

**a**

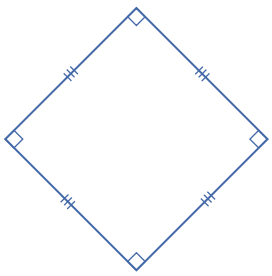


**b**

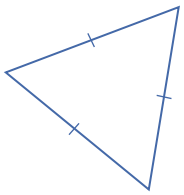


**c**

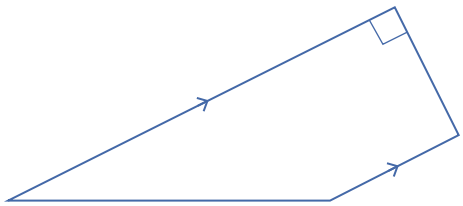
**d**



**e**



**f**



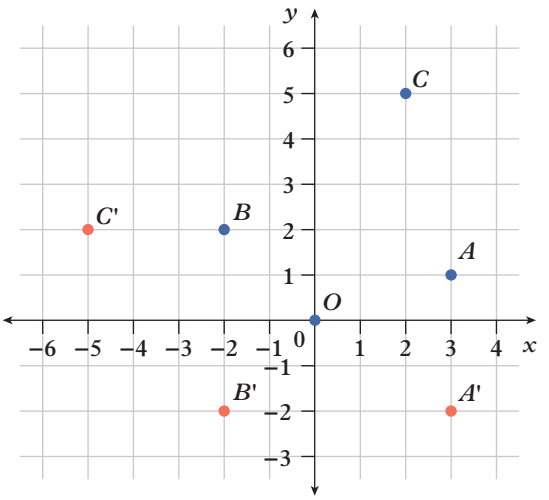
**22** In the diagram to the right, each of the points $A$, $B$ and $C$ have been transformed using a different one of the following transformations to create their images $A'$, $B'$ and $C'$:

→ A rotation of 90° about $O$ either clockwise or anticlockwise

→ A reflection in the $x$-axis or in the $y$-axis

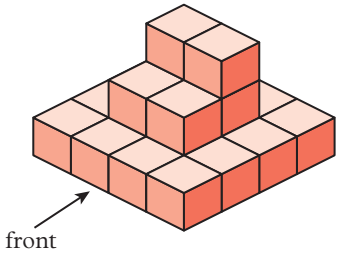
→ A horizontal translation or a vertical translation, but not both

Determine which transformation has been applied to points $A$, $B$ and $C$ to create their images.

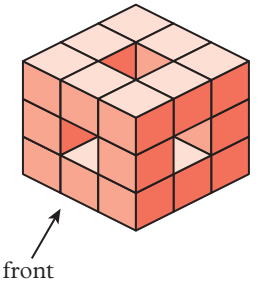



**23** Draw the front elevation, side elevation, and plan view of the following 3D objects. For part **b**, assume that every face has the middle cube missing.

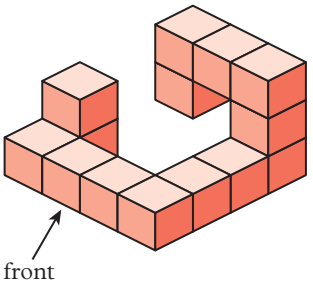
**a**



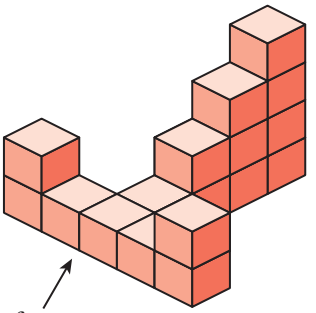

**b**




**c**




**d**




**24** For each of the following:

**i** define the unknown variable

**ii** write an equation to represent the problem

**iii** solve the equation.

Hint: draw a labelled diagram to represent each problem.

**a** One of the co-interior angles between parallel lines is 50°. Determine the size of the other co-interior angle.

**b** Three of the interior angles in a quadrilateral are 81.5°, 52.3° and 95.1°. Determine the size of the fourth interior angle.

**c** Each side of a hexagon is 2.1 cm longer than the previous side. Determine the length of the shortest side if the perimeter is 51.9 cm.

**d** The area of a square is 81 $cm^2$. Determine the side length of the square.

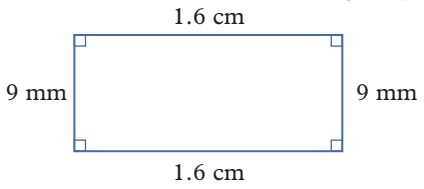
**e** The area of a triangle with a side length of 4 cm is 40 $cm^2$. Determine the length of the perpendicular height.

**f** The volume of a cube is 64 $cm^3$. Determine the side length of the cube.

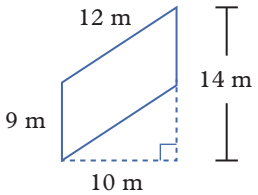
**g** The mean of 3, 5, 8, 12, 4 and one other number is 7. Determine the value of the missing number.

**25** Calculate the area of the following shapes.

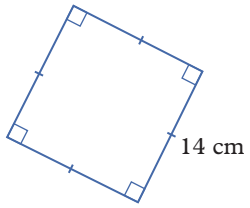
**a**



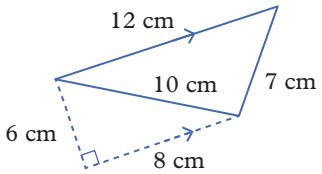

**b**



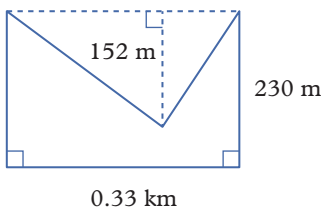

**c**



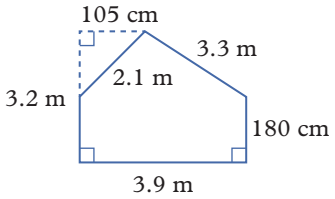

**d**




**e**




**f**



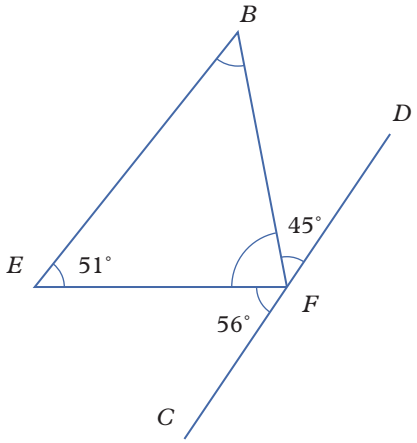

**26** Consider the diagram shown.

**a** Use three letters to name the angle that is 56° in size.

**b** Are $\overline{EB}$ and $\overline{CF}$ parallel? Use the angles in the diagram to justify your answer.

**c** If $\angle CFD$ is a straight angle, determine the size of angle $\angle EFB$.

**d** Determine the size of angle $\angle FBE$. Is $\Delta BFE$ an isosceles triangle? Why or why not?

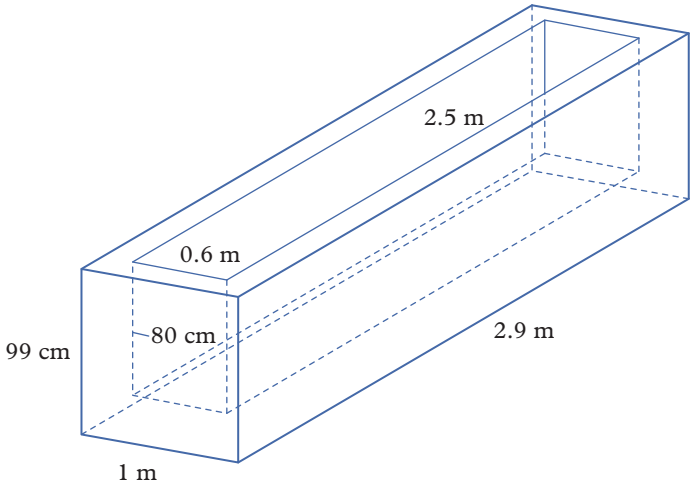



**27** A rectangular prism has side lengths 1 m, 2.9 m and 99 cm.

**a** Calculate the volume of the rectangular prism.

**b** A rectangular prism with side lengths 0.6 m, 2.5 m and 80 cm is carved out of the original rectangular prism. Calculate the capacity of the carved out prism.

**c** Calculate the new volume of the original object.



**28** A supermarket chain launched a 'collectathon' promotion, where customers were given limited edition stickers every time they shopped and encouraged to collect all 50. After three months, the supermarket chain surveyed customers at their Lilydale store about the number of unique stickers they had collected. The data they collected is:

15, 2, 32, 43, 39, 7, 37, 44, 12, 45, 49, 26, 20, 34, 21, 26, 33, 22, 48, 2

**a** Create a stem-and-leaf plot for this data.

**b** Calculate the:

**i** mean

**ii** median

**iii** mode(s)

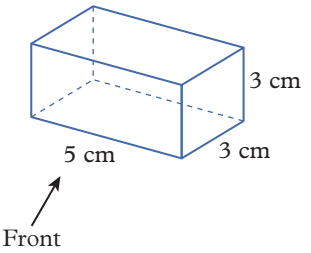
**iv** range.

**c** Write two sentences that describe the number of stickers that surveyed customers have.

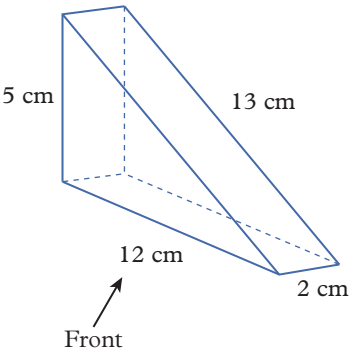
**29** What must be added to the expression $4x + 3y + 9x + 8 + 2x - 8x - 5y - 15$ so that it is equal to 0?

**30** Tyler is making two 3D models.

**Object 1:**




**Object 2:**




**a** Give the mathematical name of the two objects Tyler is making.

**b** Determine the number of **i** faces, **ii** edges and **iii** vertices for each object.

**c** Draw the front, side and top plans for both objects.

**d** Calculate the surface area of both objects.

**31** Xavier and Yvette are writing equations that have a solution of 2. Xavier writes his equations using the pronumeral $x$ and Yvette writes her equations using the pronumeral $y$. They currently have the following equations:

| | | | |
|---|---|---|---|
| $4x = 8$ | $x - 4 = -2$ | $\frac{y}{9} = 18$ | $y + 51 = 53$ |
| $\frac{x+1}{3} = 1$ | $9(y - 2) = 36$ | $\frac{x}{3} + \frac{5}{6} = \frac{3}{2}$ | $7y - 30 = -16$ |

Some of the equations do not have the correct solution.

**a** Determine which equations do not have a solution of 2.

**b** Yvette argues that $y = 2$ is an equation with the solution $y = 2$. Explain why she is correct.

**c** Xavier argues that $x = 2$ is the solution to the equation $x + x + x + x + x + x + x + x = x + x + x + x$. Explain why he is incorrect.

## Analysis

**1** Matt uses the table below to record the height of his plant over the first few days of growth.

| **Days** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Height (cm)** | 2 | 4 | 6 | | |

**a** Write a formula for the height, $h$, of Matt's plant after $d$ days.

**b** Use the formula to complete the table of values.

**c** Determine how many days it will take for the plant to reach a height of 22 cm.

Matt gives a cutting of his plant to his friend, Niamh. When Niamh plants the cutting, it is 1 cm tall, and it grows 3 cm each day.

**d** Write a formula for the height, $h$, of Niamh's plant after $d$ days.

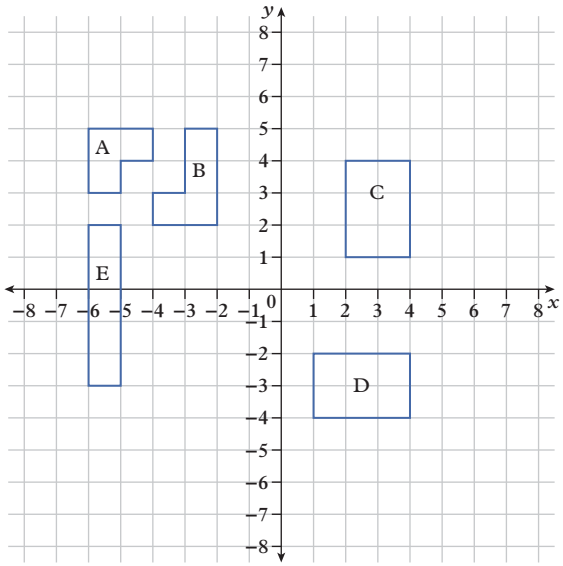
**e** Determine how many days it will take for Niamh's plant to reach a height of 22 cm.

To make her plant grow faster, Niamh feeds her plant Supergrowth plant food. The instructions on the packet state that the plant should be fed two-thirds of 4 less than the height of the plant each day in grams.

**f** Write a formula for the number of grams, $g$, of Supergrowth that should be fed to the plant when it is $h$ cm tall.

**g** How much Supergrowth should Niamh feed the plant when it is 22 cm tall?

**2** Adriana and Beethal are designing an artwork. Here is their current plan.




**a** Complete their design by following their instructions:

- Reflect shape A in the $y$-axis.
- Reflect shape B in the $x$-axis.
- Rotate shape C 90° anti-clockwise about the point $(2, 0)$.
- Rotate shape D 90° clockwise about the point $(0, -2)$.
- Translate shape E 11 units right and 1 unit up.
- Reflect the image of A (A') in the $x$-axis.

**b** What single transformation can be applied to shape A to get the image shape A".

**c** For each shape, A, B and C, state the order of:

**i** rotational symmetry

**ii** reflection symmetry.

**d** State which of shapes A, B, C, D or E have:

**i** the greatest area.

**ii** the greatest perimeter.

**e** State the geometric name of shapes A and E.

**3** A class of 30 students in a Year 7 game-design class have a competition to see who can clear the most levels of Pac-Man. The class recorded the highest level each player reached.

3, 7, 5, 2, 2, 7, 9, 8, 7, 10, 10, 1, 3, 5, 3, 9, 10, 5, 7, 2, 6, 2, 3, 4, 6, 2, 6, 9, 1, 5

**a** State whether the data collected is:

**i** primary or secondary

**ii** ordinal, nominal, discrete or continuous.

**b** Create a frequency table and a dot plot for the data.

**c** Calculate the:

**i** mean (to one decimal place)

**ii** median

**iii** mode

**iv** range.

**d** Write two sentences that describe the highest levels the students reached.

A student in the class wants to use their class' data to describe all players of Pac-Man.

**e** If this is the case, is the data a sample or a census?

**f** Would the data collected be representative of the population of Pac-Man players?

# EXPLORATIONS 2

## 1 Substitutes

What happens when you substitute the expression $2x + 1$ into itself? That is, think about the expression '$2 \times \square + 1$' and replace the box with another copy of '$2x + 1$'. Expanding brackets and simplifying, we get

$$\begin{aligned} 2(2x + 1) + 1 &= 4x + 2 + 1 \\ &= 4x + 3. \end{aligned}$$

**a** Starting with the answer $4x + 3$, show that substituting $2x + 1$ for $x$ results in the expression $8x + 7$.

**b** What is the next answer if you substitute $2x + 1$ for $x$ again?

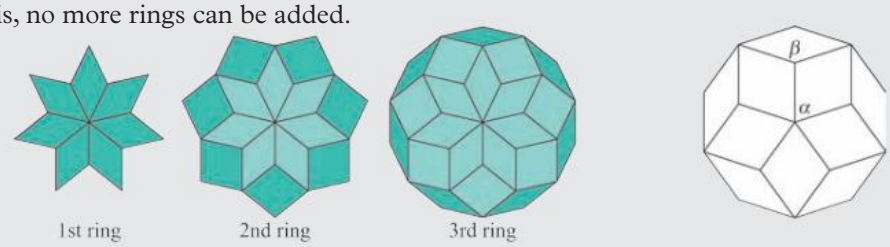
**c** Without doing the calculation, can you predict the next two answers when you repeat this substitution?

**d** Write down a general rule for the final expression when you repeat the substitution $n$ times. Can you explain why this works?

**e** Investigate what expressions you get when you substitute $x + y$ into itself as follows: first substitute $x + y$ for $x$, then substitute $x + y$ for $y$, then substitute $x + y$ for $x$, and so on, alternating between $x$ and $y$ at each stage.

## 2 Rhombus Rings

The first diagram shows a ring of 7 identical rhombuses all sharing a common centre point. In the second diagram, another ring of 7 rhombuses is added around the first. Likewise, a third ring is added in the third diagram. After this, no more rings can be added.




**a** The fourth diagram shows a pattern with 5 rhombuses in each ring. Find the size of the angles $\alpha$ and $\beta$.

**b** A pattern has 8 rhombuses in each ring. What are the angles in each rhombus in the last ring?

**c** In another pattern, each rhombus in the last ring has an angle of 20° and each rhombus in the second last ring has an angle of 60°. What are the angles in each rhombus in the third last ring? How many rings are there in total and how many rhombuses are in each ring?

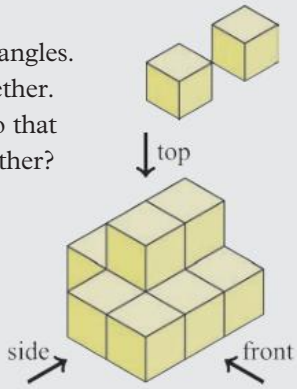
**d** Let $\alpha$ be the size of the angles at the centre of a rhombus ring pattern. Show that rhombuses in the second ring have an angle of $2\alpha$ and rhombuses in the third ring have an angle of $3\alpha$. Does this pattern continue?

**e** The third ring of a rhombus ring pattern consists of squares. How many squares are there?

**f** A rhombus ring pattern has 7 rings. How many rhombuses are there at the centre? Find all possible answers.

## 3 Ambiguous Stacks

Isometric view is a convenient way to draw three-dimensional diagrams on a grid of equilateral triangles. But sometimes the diagrams are ambiguous, that is, you cannot tell exactly how the shapes fit together. Are these two cubes on the same level and one cube-width apart, is one floating above the other so that they touch at a corner, or is one floating further away so that their left faces are in line with each other?




Nine identical cubes are stacked on a table, as shown on the right. Cubes cannot float above the table and adjacent cubes must share an entire face.

**a** Draw the side view, as seen by looking in the direction of the arrow marked 'side'.

**b** Draw one possibility for the front view. Also draw the corresponding top view and the isometric view as seen from above but from the opposite corner.

**c** Draw a different possibility for the front view, the corresponding top view and the opposite isometric view. (Hint: some cubes may only *look* like they are touching!)

**d** Using eight cubes, design two different stacks that have the same isometric view, one which is three cubes high and the other only two cubes high. Draw the common isometric view and the side, front and top views of each.

## 4 Laptagons

A *laptagon* is a right-angled shape with integer side lengths, in centimetres, which is formed from two overlapping rectangles. The overlap must have non-zero area but it cannot completely cover either of the two rectangles. So the first three diagrams show legitimate laptagons but the final two diagrams do not.

**a** Show how a $6 \times 6$ square and a $7 \times 7$ square can overlap to form one laptagon with perimeter 48 cm and another with perimeter 30 cm. Which other perimeters can be achieved using these squares?

**b** A laptagon with perimeter 32 cm is formed from two squares with 1 $cm^2$ of overlap. Find all possible sizes of the two squares.

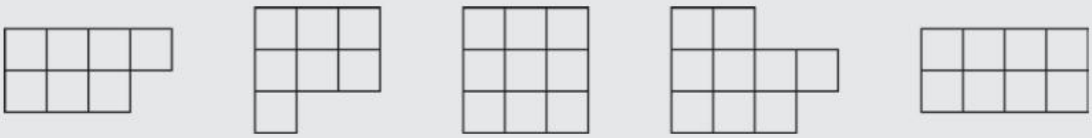
**c** A laptagon with perimeter 30 cm is formed from two squares with 12 $cm^2$ of overlap. Find all possible sizes of the two squares.

**d** A laptagon with perimeter 30 cm is formed from two rectangles (not necessarily squares) with 12 $cm^2$ of overlap. What is the largest possible area of the laptagon?

## 5 Magical Averages

A grid consisting of $n$ cells is called *magical* if it can be filled with the numbers 1 to $n$ in such a way that the average of the numbers in every row and column is the same. Note that some rows or columns may only have one cell.

**a** Show that the following grids are all magical:



**b** Explain why the following grids are not magical:

**c** Investigate rectangular grids with two equal length rows. Can you come up with a conjecture (guess) about which are magical and which are not?

## 6 Plus Printer

Pseudo-code is an informal way to describe a computer program without worrying about the precise symbols and commands that are needed to correctly write it up in a specific programming language. An example of pseudo-code is shown on the right.

```
enter a number n
loop
  if n is 0
    quit
  else if n is odd
    print "+"
    reduce n by 1
  else if n is even
    divide n by 2
```

**a** How many plus symbols are printed if $n$ is 26?

**b** How many plus symbols are printed if $n$ is 89?

**c** Find two consecutive values of $n$: one that prints 6 plus symbols, and the other that prints only one.

**d** Investigate whether it is possible for three consecutive values of $n$ to print the same number of plus symbols.

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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# NAPLAN PRACTICE

## CHAPTER 1 Whole numbers

### Non-calculator

**1** Jane's total electricity bill this year is \$2386. Last year it was \$1857. What is the difference between the two yearly amounts?

\$ ☐

Questions **2** and **3** refer to this road sign.

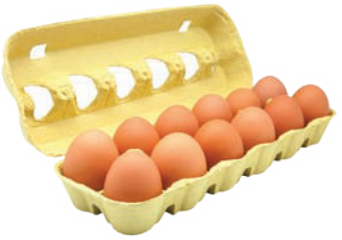
**2** What is the distance between Miles and Surat?

☐ km

**3** If a person drives from this intersection to Miles, then to Injune and then to Mitchell, how far have they travelled in total?

☐ km

**4** This carton contains eggs of about the same size. The total mass inside the carton is 660 g.



What is the mass of one egg?

- ☐ 55 g
- ☐ 66 g
- ☐ 648 g
- ☐ 672 g

**5** What is the value of $\sqrt{100}$?

☐

**6** If $17^2 = 289$, what is the value of $\sqrt{289}$?

| 2 | 17 | 289 | 578 |
|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ |

**7** Which calculation produces the largest result?

- ☐ $2^4 \times 5^2$
- ☐ $5^2 \times 3^2$
- ☐ $4^2 \times 6^2$
- ☐ $3^2 \times 2^5$

**8** Which calculation has the same value as $12 \times 4$?

- ☐ $3 + 9 \times 5$
- ☐ $6 \times 9 - 5$
- ☐ $6 \times 3 + 5$
- ☐ $(3 + 6) \times 5$

**9** What is the value of $8 \times (3 + 6^2 \div 9) - 2$?

☐

### Calculator

**10** Which number is six thousand and forty-eight?

- ☐ 6480
- ☐ 6048
- ☐ 6408
- ☐ 6084

**11** Which group of numbers is listed in ascending order?

- ☐ 236, 245, 254, 263, 236
- ☐ 7503, 7053, 7350, 7530
- ☐ 4965, 4695, 4659, 4569
- ☐ 603, 623, 630, 632, 662

**12** Hayden kept a record of how many laps of the pool he swam each day.

| Day | Number of laps |
|---|---|
| Monday | 18 |
| Tuesday | 25 |
| Wednesday | 19 |
| Thursday | 32 |
| Friday | 28 |
| Saturday | 15 |
| Sunday | 24 |

What is the total number of laps he swam in a week?

☐

**13** A set of DVDs contains 14 episodes of a television show. Each episode runs for 86 minutes. What is the total running time?

☐ minutes

**14** Alexis bought five books, three photo frames, seven magazines and two board games. If the books cost \$11 each, photo frames \$14 each, magazines \$6 each and board games \$21 each, how much did she spend?

\$ ☐

**15** To estimate the value of $751 \times 329$, each number is first rounded to the nearest hundred.

Which calculation will be performed?

- ☐ $700 \times 300$
- ☐ $700 \times 400$
- ☐ $800 \times 300$
- ☐ $800 \times 400$



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**16** Two families paid $10 each to buy one lottery ticket. Their ticket was one of five winners in a $1 000 000 prize. If each family has four members, how much did each person receive?

- ◯ $250 000
- ◯ $125 000
- ◯ $50 000
- ◯ $25 000

**17** Zahra goes for a bike ride and after 17 minutes she has ridden 4250 metres. How many kilometres will she cover in an hour?

15 ◯ 17 ◯ 72 ◯ 255 ◯

**18** What is another way of writing $4^3$?

- ◯ $3 \times 3 \times 3 \times 3$
- ◯ $4 \times 3$
- ◯ $4 \times 4 \times 4$
- ◯ $4 + 4 + 4$

## CHAPTER 2 Factors and primes

### Non-calculator

**1** Christos walks around an athletic track in 6 minutes while Lisa jogs around the same track in 4 minutes. They both begin their laps of the oval at the same time and from the same starting position. When will they next pass the starting position at the same time?

[ ] minutes after the start

**2** How is 72 written as a product of its prime factors?

- ◯ $8 \times 9$
- ◯ $2 \times 2 \times 2 \times 3 \times 3$
- ◯ $3 \times 4 \times 6$
- ◯ $2 \times 3 \times 3 \times 4$

**3** What is the lowest common multiple of 6 and 9?

3 ◯ 6 ◯ 9 ◯ 18 ◯

**4** What is the highest common factor of 6 and 9?

3 ◯ 6 ◯ 9 ◯ 18 ◯

**5** Which of these numbers is *not* a factor of 24?

3 ◯ 6 ◯ 8 ◯ 9 ◯

**6** Which of these numbers is *not* a multiple of 6?

3 ◯ 6 ◯ 12 ◯ 18 ◯

**7** Which of these numbers is divisible by both 4 and 5?

100 ◯ 104 ◯ 105 ◯ 110 ◯

### Calculator

**8** Which number comes next in this sequence?
48, 72, 96, 120, 144, …

148 ◯ 160 ◯ 168 ◯ 192 ◯

**9** Which of these is *not* a prime number?

2 ◯ 43 ◯ 51 ◯ 67 ◯

**10** How many prime factors does 30 have?

1 ◯ 2 ◯ 3 ◯ 8 ◯

**11** What is the sixth term in the sequence 4, 7, 10, …?

13 ◯ 15 ◯ 16 ◯ 19 ◯

**12** What is the missing term in the sequence
6, 12, _____, 48, 96 …?

15 ◯ 16 ◯ 19 ◯ 24 ◯

**13** The first number in a pattern is 36.
Each number in the pattern is formed by subtracting 3 from the previous number.
What is the third number in this pattern?

33 ◯ 30 ◯ 27 ◯ 39 ◯

**14** Two buses leave the bus stop at the same time.
The first bus makes a stop every 20 minutes and the second bus makes a stop every 30 minutes.
After they leave the bus stop, when will they next stop at the same time?

- ◯ 20 minutes
- ◯ 30 minutes
- ◯ 40 minutes
- ◯ 60 minutes

**15** There are 50 houses in a street.
The houses are numbered from 1 to 50. All the houses that have a number which is a multiple of 7 have yellow letterboxes. All the houses that have a number which is a multiple of 2 have red doors.
How many houses have both a yellow letterbox and a red door?

1 ◯ 2 ◯ 3 ◯ 4 ◯

# CHAPTER 3 Fractions and ratios

## Non-calculator

**1** What is $3\frac{4}{7}$ as an improper fraction?

**2** Angela needs $\frac{33}{5}$ m of material. How many whole metres should she buy?

33 ◯ 7 ◯ 6 ◯ 5 ◯

**3** Which fraction is equivalent to $\frac{5}{12}$?

$\frac{15}{25}$ ◯ $\frac{7}{14}$ ◯ $\frac{10}{24}$ ◯ $\frac{10}{25}$ ◯

**4** Write $\frac{24}{16}$ in its simplest form.

**5** Hayley has 18 red and 6 orange icy-poles. What fraction are orange?

$\frac{1}{2}$ ◯ $\frac{1}{3}$ ◯ $\frac{1}{4}$ ◯ $\frac{1}{6}$ ◯

**6** What is $3\frac{1}{5} - 2\frac{4}{5}$?

**7** What is $\frac{4}{5} \times \frac{2}{3}$?

**8** Angelina has a recipe that uses 3 cups of flour. She decides to make a $\frac{3}{4}$ batch. How much flour will Angelina need?

◯ 3 cups ◯ $\frac{1}{2}$ a cup

◯ 2 cups ◯ $2\frac{1}{4}$ cups

**9** What is $\frac{8}{5} \div \frac{4}{9}$?

**10** What is $\frac{7}{8} \div 3\frac{1}{2}$?

$\frac{7}{48}$ ◯ $\frac{49}{16}$ ◯ $4\frac{3}{8}$ ◯ $\frac{1}{4}$ ◯

**11** If $4\frac{1}{2}$ L of juice is divided evenly between five friends, how much does each person get?

4 L ◯ $\frac{9}{10}$ L ◯ $1\frac{1}{8}$ L ◯ $\frac{4}{5}$ L ◯

**12** Write the ratio 36 : 54 in its simplest form.

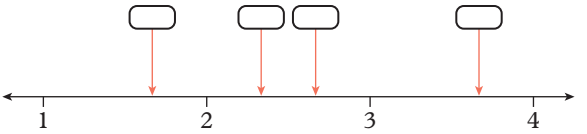
**13** The ratio of girls to boys at camp is 7 : 6. If there are 54 boys, how many girls are there?

## Calculator

**14** What is $\frac{24}{5}$ as a mixed number?

$2\frac{4}{5}$ ◯ $4\frac{4}{5}$ ◯ $\frac{5}{24}$ ◯ $4\frac{2}{5}$ ◯

**15** Which arrow is pointing closest to $2\frac{2}{3}$?




**16** Which list is ordered smallest to largest?

◯ $\frac{3}{4}, \frac{7}{2}, 2\frac{1}{4}, \frac{5}{8}, \frac{1}{2}$

◯ $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$

◯ $1\frac{1}{3}, \frac{12}{5}, 2\frac{4}{5}, \frac{13}{3}, \frac{26}{15}$

◯ $\frac{2}{5}, 1\frac{3}{4}, \frac{9}{4}, 2\frac{7}{10}, \frac{16}{5}$

**17** What is $\frac{64}{5} - \frac{16}{3}$?

◯ $7\frac{3}{5}$ ◯ $7\frac{7}{15}$

◯ $3\frac{7}{15}$ ◯ $7\frac{1}{3}$

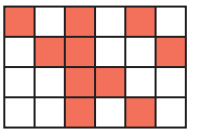
**18** What fraction is halfway between $\frac{7}{11}$ and $\frac{8}{11}$?

**19** What is $\left(\frac{3}{7}\right)^2$?

**20** What is $\left(2\frac{1}{4}\right)^2$?

$\frac{1}{4}$ ◯ $\frac{81}{16}$ ◯ $4\frac{1}{16}$ ◯ $\frac{16}{81}$ ◯

Questions **21** and **22** refer to this diagram.



**21** In its simplest form, what fraction is *not* shaded?

$\frac{14}{24}$ ◯ $\frac{7}{12}$ ◯ $\frac{10}{24}$ ◯ $\frac{5}{12}$ ◯

**22** What is the ratio of shaded segments to unshaded segments?

**23** Monique is 1.21 m tall. Lachlan is 98 cm tall. What is the ratio of Lachlan's height to Monique's?

- ◯ 98 : 1.21
- ◯ 98 : 121
- ◯ 1.21 : 98
- ◯ 121 : 98

**24** Alex has 16 jelly beans: 7 pink, 5 white and the rest are blue. What is the ratio of blue to pink jelly beans?

**25** Which ratio is *not* equivalent to the ratio 3 : 7?

- ◯ 9 : 21
- ◯ 12 : 28
- ◯ 6 : 10
- ◯ 30 : 70

**26** A netball game consists of four quarters and three rest breaks. The ratio of game time to rest time is 15 : 4. If total rest time is 16 minutes, what is the game's duration?

- ◯ 16 minutes
- ◯ 44 minutes
- ◯ 60 minutes
- ◯ 76 minutes

## **CHAPTER 4** Decimals and percentages

### Non-calculator

**1** What is 8.605 – 2.077?

**2** What is 1.26 × 0.55?

- ◯ 0.693
- ◯ 0.0693
- ◯ 1.2655
- ◯ 1.81

**3** What is the cost of 3 m of material if the price is $5.99 per metre?

**4** If eight DVDs cost $47.92, how much do 10 DVDs cost?

**5** What is $\frac{11}{20}$ as a decimal number?

**6** What is 0.875 as a fraction in its simplest form?

- ◯ $\frac{875}{100}$
- ◯ $\frac{825}{1000}$
- ◯ $\frac{175}{200}$
- ◯ $\frac{7}{8}$

**7** An 18-carat gold necklace is made of 75% gold, 12.5% silver and the rest is copper.
What percentage of the necklace is copper?

%

**8** Melinda notices that three out of every 10 passing cars are red. What percentage of cars that pass are *not* red?

%

**9** What is 59.23% written as a decimal number?

**10** Which list correctly shows the numbers 45%, $\frac{1}{8}$, 0.76, $\frac{4}{5}$, 0.12 and 9% in descending order?

- ◯ $\frac{4}{5}$, 0.76, 45%, $\frac{1}{8}$, 0.12, 9%
- ◯ 0.76, $\frac{4}{5}$, 45%, 0.12, $\frac{1}{8}$, 9%
- ◯ 9%, 0.12, $\frac{1}{8}$, 45%, 0.76, $\frac{4}{5}$
- ◯ 45%, 0.76, $\frac{4}{5}$, 0.12, $\frac{1}{8}$, 9%

**11** What is 35% of $620?

### Calculator

Questions **12** and **13** refer to 164.506 483 2.

**12** What is the place value of the digit 0?

- ◯ Hundreds
- ◯ Tens
- ◯ Tenths
- ◯ Hundredths

**13** Round the number to two decimal places.

**14** Which list is written in ascending order?

- ◯ 1.26, 1.623, 1.86, 1.88
- ◯ 2.1, 2.89, 2.506, 2.4896
- ◯ 8.0175, 8.454, 9.9, 8.64
- ◯ 3.84, 3.521, 3.1133, 3.005

**15** Which is the correct setting out for the sum of 1.3, 4.06 and 0.008?

- ◯ $\begin{array}{r} 1.3 \\ 4.6 \\ +\ 0.8 \\ \hline \end{array}$
- ◯ $\begin{array}{r} 1.3 \\ 4.06 \\ +\ 0.008 \\ \hline \end{array}$
- ◯ $\begin{array}{r} 1.03 \\ 4.06 \\ +\ 0.008 \\ \hline \end{array}$
- ◯ $\begin{array}{r} 1.300 \\ 4.060 \\ +\ 0.008 \\ \hline \end{array}$

**16** Andrew, James and Mark want to buy a new kettle. If Andrew has $16.55, James has $13.20 and Mark has $12.60, how much more money do they need to buy this kettle for $49?

$

**17** What is 0.548 ÷ 0.2?

- ◯ 0.1096
- ◯ 0.274
- ◯ 1.096
- ◯ 2.74

**18** Which option to buy tea bags is better value?

- ◯ 10 for $1.77
- ◯ 25 for $2.98
- ◯ 50 for $4.08
- ◯ 100 for $7.48

**19** Which option is the best buy for milk?

- ◯ 0.6 L for $1.98
- ◯ 1 L for $2.98
- ◯ 2 L for $5.92
- ◯ 3 L for $8.90

Questions **20–22** refer to this information.

Chelsea wants to buy some tights. One store is selling three pairs for $9.99 or $3.99 each. Chelsea has $35.20 and buys as many pairs as she can.

**20** How many pairs of tights can she buy?

**21** How much money does she have left afterwards?

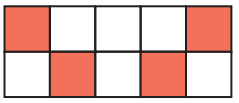
$1.24 ◯ $5.23 ◯ $29.97 ◯ $33.96 ◯

**22** On average, how much did each pair cost?

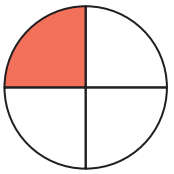
$3.00 ◯ $3.33 ◯ $3.40 ◯ $8.49 ◯

**23** Which diagram is 40% unshaded?

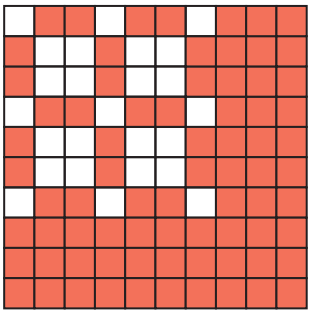
◯ A



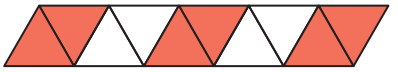
◯ B



◯ C



◯ D



**24** Enrique scored a mark of $\frac{52}{80}$ on his maths test. What is this as a percentage?

**25** What is the sale price of an item that costs $49.99 before it is discounted by 25%?

- ◯ $12.50
- ◯ $24.99
- ◯ $37.49
- ◯ $1249.75

Questions **26** and **27** refer to this information.

Michelle plans to buy 25 cupcakes. Her local bakery sells cupcakes for $2.10 each. The closest supermarket sells 10 cupcakes for $14.99.

**26** If the bakery has a 30% off everything sale, how much would it cost to buy 25 cupcakes?

- ◯ $52.50
- ◯ $36.75
- ◯ $31.48
- ◯ $15.75

**27** If the supermarket has 20% off bakery items, how much would it cost per cupcake if Michelle bought three packets?

- ◯ $35.98
- ◯ $1.50
- ◯ $1.20
- ◯ $1.05

## CHAPTER 5 Integers and the Cartesian plane

### Non-calculator

**1** When Andrew wakes up it is −3°C. By midday, it has warmed by 7°C. What is the temperature at midday?

Questions **2** and **3** refer to the following information.

Miners are working at Level 5 below the surface of the ground. Ground level is Level 0.

**2** At the end of a shift, workers use a lift that travels six levels up the vertical mine shaft. At what level do the miners leave the lift?

- ◯ Level 1 above the surface
- ◯ Level 1 below the surface
- ◯ Level 11 above the surface
- ◯ Level 11 below the surface

**3** On the way, the lift stops after travelling two levels up the mine shaft. At which level do the lift doors open?

**4** Complete the following number statement.

−5 + ☐ = −11

**5** What is the result of −4 − (−3)?

−7 ◯ −1 ◯ 1 ◯ 7 ◯

**6** A cool room is kept at 4°C. Somebody turns the temperature down by 10°C. What is the new temperature of the cool room?

**7** A scuba diver jumps from the side of a boat and travels a vertical distance of 16 m to touch the bottom of the seabed. If the boat's deck is 2 m above sea level, how deep is the water?

m

**8** Complete the following number statement.

$-4 -$ ____ $= 12$

**9** Transactions for Peter's bank account are shown below.

| Date | Reference | Transaction | Balance |
|---|---|---|---|
| 1 June | – | – | +$86 |
| 5 June | Deposit | +$25 | |
| 7 June | Cinemas | −$15 | |
| 11 June | Woolworths | −$33 | |
| 21 June | JB Hifi | −$243 | |
| 30 June | Deposit | +$200 | |

How much is in Peter's bank account at the end of the month?

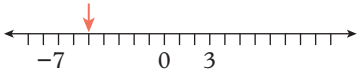
$

**10** Tess catches an elevator from floor 6 and gets off at floor −2. How many floors did Tess travel?

## Calculator

**11** The arrow is pointing to a position on the number line below.

What number belongs at this position?



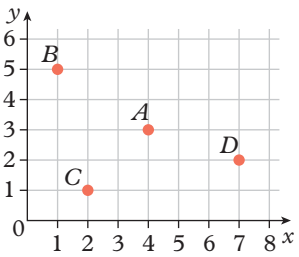

**12** Which number is the largest?

1 −5 2 −9

**13** Which number is the smallest?

0 −20 −10 5

Questions **14–16** refer to the following figure. The points *A*, *B*, *C* and *D* are plotted on this Cartesian plane.




**14** What are the coordinates of the point *A*?

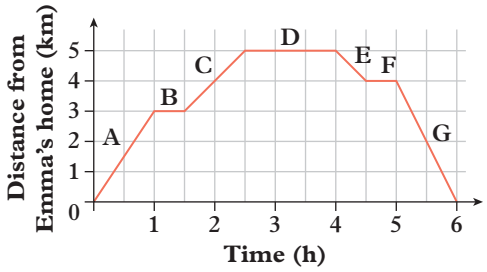
(1, 5) (3, 4) (1, 2) (4, 3)

**15** Which point has the coordinates (1, 5)?

*A* *B* *C* *D*

**16** The *x*-coordinate of point *C* is

Questions **17–19** refer to this distance–time graph.




Emma walks to her friend's house and together they walk to the cinema. After the film, they go to a cafe before returning home.

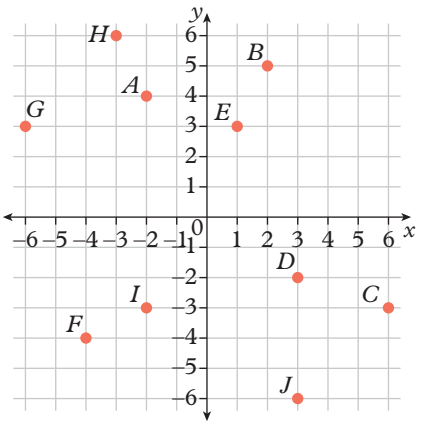
**17** Which section of the graph represents their time at the cinema?

B C D E

**18** Answer true or false. Emma's home is closer to the cafe than it is to her friend's house.

**19** In which section did Emma walk the fastest?

Questions **20–22** refer to the following figure.
The points $A$ to $J$ are plotted on this Cartesian plane.




**20** Write the coordinates of point $C$.

**21** Which point has the coordinates $(-3, 6)$?

$A$ ☐ $C$ ☐ $F$ ☐ $H$ ☐

**22** Which point has the same $x$-coordinate as point $I$?

## CHAPTER 6 Algebra

### Non-calculator

**1** The cost of buying slurpees is recorded in this table.

| **Number of slurpees** | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| **Cost ($)** | 15 | 20 | 25 | 30 | 35 |

What is the cost of nine slurpees?

$

Questions **2** and **3** refer to this information.
An artist builds sculptures out of old tin cans. The relationship between the number of cans used ($n$) and the number of sculptures produced ($s$) is shown in the table.

| ***n*** | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| ***s*** | 1 | 2 | | 4 | 5 |

**2** What is the missing value in the table?

**3** Which formula best describes the relationship?

☐ $s = n + 5$ ☐ $s = 5n$
☐ $s = \frac{n}{5}$ ☐ $s = n - 5$

**4** The cost of lunch (in dollars) at a bakery is described by the expression $5s + 3d + 2p$, where $s$ is number of sandwiches, $d$ is number of drinks and $p$ is number of pastries. A group of people buy six sandwiches, eight drinks and four pastries. What is the total cost?

☐ \$28 ☐ \$62
☐ \$118 ☐ \$180

Questions **5** and **6** refer to this information.
Entry to an animal farm is \$4 per adult and \$2 per child.

**5** Which expression describes the total entry cost (in dollars) for $a$ adults and $c$ children?

☐ $4 + 2$ ☐ $6n$
☐ $6ac$ ☐ $4a + 2c$

**6** A family of five (two adults and three children) visits the farm. What is their total entry cost?

**7** James has a number of lollies. He eats four of them, then buys more to double what was left. His mother then gives him three more lollies.

If James ends up with seven lollies, how many did he originally have?

**8** Which of these equations has the solution $x = 6$?

☐ $3(x + 6) = 24$ ☐ $3x - 6 = 24$
☐ $3(x - 2) = 24$ ☐ $3(x + 2) = 24$

**9** Sava has a plant that grows according to the formula $H = 5t + 40$.
$H$ is the height of the plant in centimetres and $t$ is the number of weeks the plant has been growing.
What is the height of the plant after 6 weeks?

☐ 30 cm ☐ 45 cm
☐ 70 cm ☐ 46 cm

### Calculator

**10** Mario has five more chocolates than Bill. If Mario has $m$ chocolates and Bill has $b$ chocolates, which of these statements best shows the relationship between $m$ and $b$?

☐ $m = b + 5$ ☐ $m = 5b$
☐ $b = 5 + m$ ☐ $5bm$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Questions **11** and **12** refer to this information.

Erica buys five apples and then gives \$3 to a busker. In total, she has spent \$13.

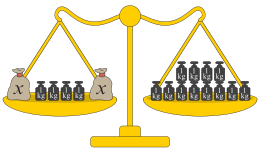
**11** Which equation matches this situation?

- ☐ $5a + 3 = 10$
- ☐ $8a = 13$
- ☐ $5a = 16$
- ☐ $5a + 3 = 13$

**12** What is the cost of a single apple?

\$ ☐

Questions **13** and **14** refer to this diagram of a set of scales that shows two identical mystery items and a number of 1 kg weights. The scales are balanced.




**13** Which equation matches this situation?

- ☐ $x + 4 = 12$
- ☐ $2x = 12$
- ☐ $2x + 4 = 12$
- ☐ $2x - 4 = 12$

**14** What is the mass of a single mystery item?

☐ kg

**15** I think of a number, add 2 then divide by 5, giving the result of 3. Which equation matches these operations?

- ☐ $\frac{x+2}{5} = 3$
- ☐ $\frac{5+x}{2} = 3$
- ☐ $5(x + 2) = 3$
- ☐ $3(x + 2) = 5$

**16** I think of a number, subtract 6 then multiply by 4, giving the result of 16. Which equation matches these operations?

- ☐ $\frac{x-6}{4} = 16$
- ☐ $\frac{6-x}{4} = 16$
- ☐ $4(x - 6) = 16$
- ☐ $4(x + 6) = 16$

**17** Ben and Tiana buy dumplings. Ben buys 2 times the number Tiana does less 3 dumplings.

Let $d$ be the number of dumplings Tiana buys.

Which expression shows the number of dumplings Ben buys?

- ☐ $\frac{d+3}{2}$
- ☐ $2d - 3$
- ☐ $2(d - 3)$
- ☐ $\frac{2d}{3}$

**18** Vlad has an ant farm. Unfortunately, each day some ants escape. He records the number of ants in the farm each day.

| **Day, *d*** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Number of ants, *n*** | 100 | 95 | 90 | 85 | 80 | 75 |

The relationship between the number of ants that have escaped, $n$, and the number of days that have passed is

- ☐ $n = 100 - 5d$
- ☐ $n = 100 - d$
- ☐ $n = 80 + d$
- ☐ $n = 100 + 5d$

**19** When the expression $4x + 5xy + 10x - 2xy$ is fully simplified, what is the result?

- ☐ $9x + 8xy$
- ☐ $14x + 7xy$
- ☐ $14x - 3xy$
- ☐ $14x + 3xy$

**20** Roshni hires a bicycle for 5 hours at a total cost of \$45. What is the formula for the cost, $C$, of hiring a bicycle for $t$ hours?

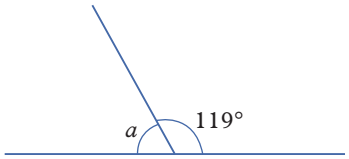
- ☐ $C = 5t$
- ☐ $C = 9t$
- ☐ $C = t + 9$
- ☐ $C = t + 5$

# CHAPTER 7 Angles

## Non-calculator

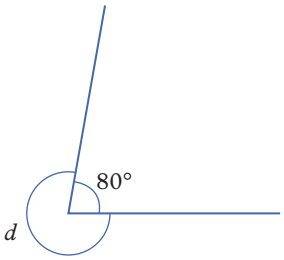
**1** What is the size of angle $a$ in this diagram?

☐ °

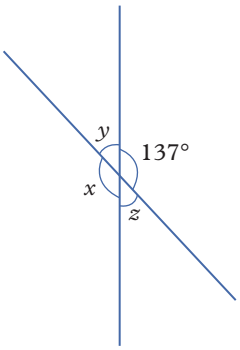



**2** What is the size of angle $d$ in this diagram?

☐ °




Questions **3–5** refer to this diagram.




**3** What is the size of angle $x$?

- ☐ 137°
- ☐ 360°
- ☐ 43°
- ☐ 223°

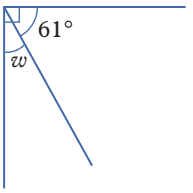
**4** What is the size of angle $y$?

- ☐ 137°
- ☐ 360°
- ☐ 43°
- ☐ 223°

**5** What is the size of angle $z$?

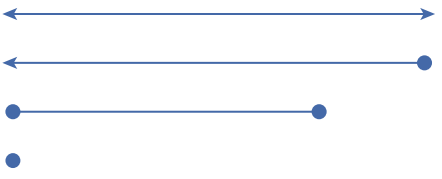
137° ☐ 360° ☐ 43° ☐ 223° ☐

**6** What is the size of angle $w$ in this diagram?




## Calculator

**7** Which of the following is a ray?



**8** Which of these statements is true?

- ◯ Parallel lines meet at a 90° angle.
- ◯ A ray has a definite start and finish.
- ◯ A segment starts from a point and continues infinitely.
- ◯ A pair of lines that meet at right angles are called perpendicular.

**9** Line A is perpendicular to line B. Ray C is perpendicular to ray D. If line B is parallel to ray D, which of these statements is false?

- ◯ Line B extends in both directions forever.
- ◯ Ray C is parallel to line A.
- ◯ Ray D starts from a point.
- ◯ Line A is parallel to ray D.

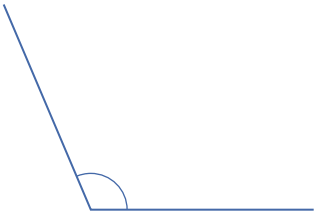
**10** Supplementary angles add to:

[ ] °

**11** Complementary angles add to:

[ ] °

Questions **12** and **13** refer to this angle.



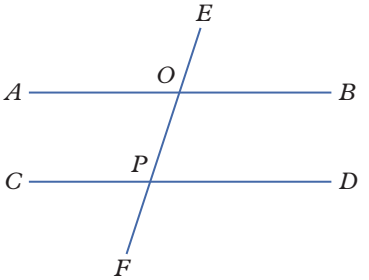
**12** What is the best description of the angle above?

- ◯ Obtuse
- ◯ Acute
- ◯ Reflex
- ◯ Revolution

**13** What is the size of the angle?

113° ◯ 67° ◯ 247° ◯ 90° ◯

Questions **14–20** refer to this diagram.




**14** Which pair of angles is corresponding?

- ◯ $\angle CPE$ and $\angle FPD$
- ◯ $\angle AOF$ and $\angle EPD$
- ◯ $\angle AOF$ and $\angle CPE$
- ◯ $\angle AOE$ and $\angle CPE$

**15** Which pair of angles is alternate?

- ◯ $\angle CPE$ and $\angle FPD$
- ◯ $\angle AOF$ and $\angle EPD$
- ◯ $\angle AOF$ and $\angle CPE$
- ◯ $\angle AOE$ and $\angle CPE$

**16** Which pair of angles is co-interior?

- ◯ $\angle CPE$ and $\angle FPD$
- ◯ $\angle AOF$ and $\angle EPD$
- ◯ $\angle AOF$ and $\angle CPE$
- ◯ $\angle AOE$ and $\angle CPE$

If $\angle AOE = 125°$

**17** What is the size of angle $\angle FOB$?

[ ] °

**18** What is the size of angle $\angle EPD$?

[ ] °

**19** What is the size of $\angle AOF$?

[ ] °

**20** What is the size of angle $\angle CPF$?

[ ] °

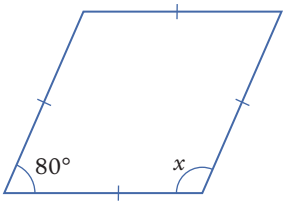
**21** What type of triangle has exactly two equal sides?

- ◯ Right-angled triangle
- ◯ Equilateral triangle
- ◯ Scalene triangle
- ◯ Isosceles triangle

**22** What is the size of angle $x$ in this diagram?

[ ] °

Questions **23** and **24** refer to this figure.



**23** What is the best description of this shape?

- ◯ Quadrilateral
- ◯ Parallelogram
- ◯ Rhombus
- ◯ Kite

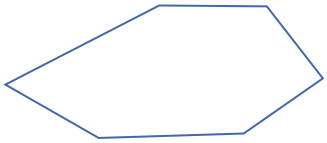
**24** What is the size of angle $x$?

[ ] °

## CHAPTER 8 Shapes and objects

### Non-calculator

**1** What is the best description of this shape?



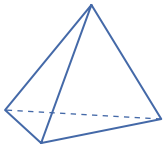
- ◯ Irregular heptagon
- ◯ Irregular hexagon
- ◯ Irregular kite
- ◯ Irregular concave hexagon

**2** How many sides does a decagon have?

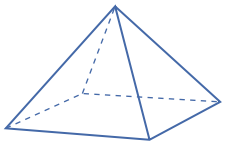
[ ]

**3** Which of these is a triangular prism?

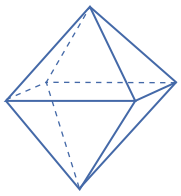
◯ A



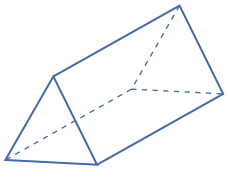
◯ B



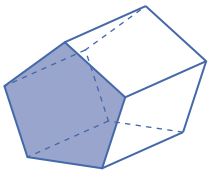
◯ C



◯ D



Questions **4** and **5** refer to this figure.

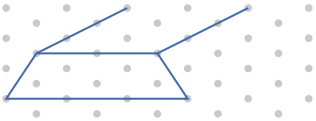


**4** What 2D shape forms the matching ends of this object?

**5** What is the best name for this object?

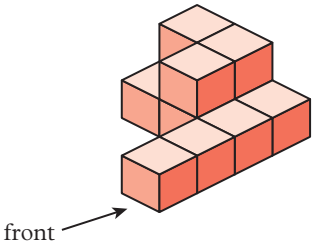
- ◯ Pentagonal prism
- ◯ Pentagonal pyramid
- ◯ Triangular prism
- ◯ Rectangular pyramid

**6** When completed, what will this object be?

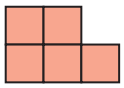


- ◯ Parallelogram-based prism
- ◯ Rectangular pyramid
- ◯ Trapezoidal prism
- ◯ Rectangular prism

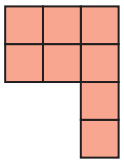
**7** Which figure best represents the top view of this object?



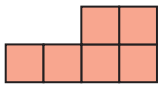

◯ A



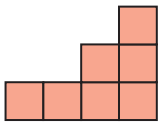
◯ B



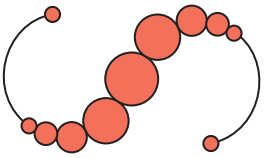
◯ C



◯ D



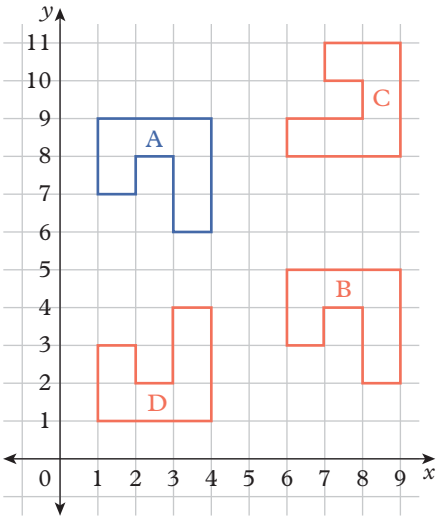
Questions **8** and **9** refer to this figure.



**8** What is the order of line symmetry for the object?

**9** What is the order of rotational symmetry for the object?

Questions **10–13** refer to this figure. Shapes B, C and D are all transformations of shape A.




**10** How has shape A been translated to produce shape B?

- 2 units right and 1 unit down
- 2 units right and 4 units down
- 5 units right and 4 units down
- 5 units left and 4 units up

**11** To produce shape C, shape A has been rotated in a clockwise direction by

°

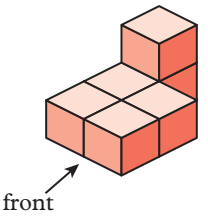
**12** What has been done to shape A to produce shape D?

- Reflected in a vertical mirror line
- Translated 2 units down
- Rotated 180° anti-clockwise
- Reflected in a horizontal mirror line

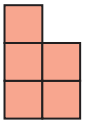
**13** A fifth shape (shape E) was drawn onto the grid. It was produced by translating shape A 4 units right and 3 units down, then 2 units right and 1 unit up, and finally 3 units left and 2 units down. The total translation can be described as follows.

- 3 units left and 2 units down
- 9 units left and 6 units down
- 9 units right and 6 units up
- 3 units right and 4 units down

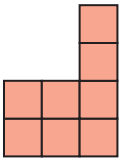
**14** Which figure best represents the plan view of this object?



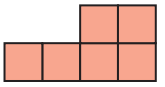

A



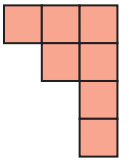
B



C



D



**15** In preparation to take a shot, Jaime rotates the racquet to the right 90 degrees clockwise. Which of these shows the new position of the racquet and ball?

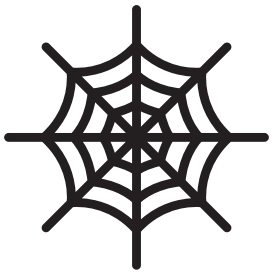
A

B

C

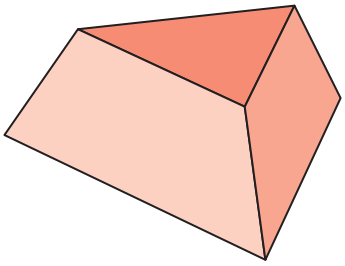
D

**16** How many lines of symmetry does this cobweb have?



4 ☐ 6 ☐ 8 ☐ 1 ☐

**17** The shape below is a truncated triangular pyramid. How many edges does a truncated triangular pyramid have?



6 ☐ 7 ☐ 8 ☐ 9 ☐

## CHAPTER 9 Length, area and volume

### Non-calculator

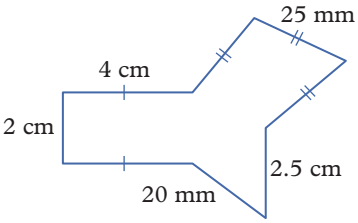
**1** Kyle knows that it is exactly 798 m from his house to school. Convert this distance to kilometres.

☐ km

**2** Which length measurement is the largest?

☐ 6500 mm ☐ 5600 cm
☐ 7.5 m ☐ 0.006 km

**3** What is the perimeter of this shape?




☐ 53.5 mm ☐ 13 cm
☐ 22 cm ☐ 107.5 mm

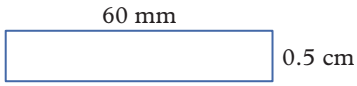
**4** What is the perimeter of a square of length 8 mm?

☐ mm

**5** What is the area of a square of length 8 mm?

☐ $mm^2$

**6** What is the area of this rectangle?




☐ 30 $mm^2$ ☐ 3 $cm^2$
☐ 30 $cm^2$ ☐ 121 $mm^2$

**7** A rectangle has a width of 30 mm and an area of 24 $cm^2$. What is the length of the rectangle in centimetres?

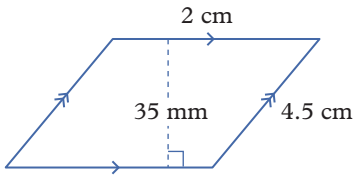
☐ cm

**8** Which shape has the largest area?

☐ Rectangle with length 70 mm and width 3 cm
☐ Triangle with base length 10 cm and height 40 mm
☐ Square with length 5 cm
☐ Parallelogram with base length 8 cm and height 30 mm

### Calculator

Questions **9** and **10** refer to this shape.



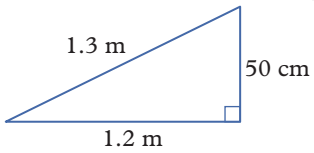

**9** What is the name of the shape?

☐ Rectangle ☐ Triangle
☐ Parallelogram ☐ Square

**10** What is the area of the shape?

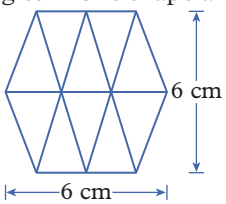
☐ 130 $mm^2$ ☐ 7 $cm^2$
☐ 9 $cm^2$ ☐ 1575 $mm^2$

**11** What is the area of this shape?




☐ 30 $m^2$ ☐ 60 $m^2$
☐ 300 $cm^2$ ☐ 3000 $cm^2$

**12** All of the triangles in this shape are identical.

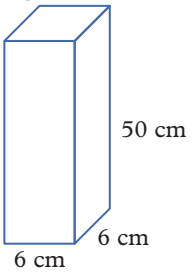



What is the area of the entire shape?

☐ 3 $cm^2$ ☐ 30 $cm^2$
☐ 45 $cm^2$ ☐ 60 $cm^2$

Questions **13–15** refer to this object.

Ella bought a vase for her new house.




**13** What is the surface area of the outside of the vase?

- ☐ 62 $cm^2$
- ☐ 300 $cm^2$
- ☐ 1236 $cm^2$
- ☐ 1800 $cm^2$

**14** What is the volume of the vase?

- ☐ 50 $cm^3$
- ☐ 62 $cm^3$
- ☐ 300 $cm^3$
- ☐ 1800 $cm^3$

**15** What is the amount of water that this vase could hold?

- ☐ 1.8 L
- ☐ 18 L
- ☐ 180 L
- ☐ 1800 L

Questions **16** and **17** refer to a new storage room of length 6 m, width 4 m and height 3 m.

The storage room needs to be painted. There is a door to the room with dimensions 200 cm by 150 cm that has already been painted.

**16** What is the area of the walls, floor and ceiling that would need to be painted?

☐ $m^2$

**17** What volume of goods could be stored in the room?

☐ $m^3$

## CHAPTER 10 Statistics and probability

**1** This frequency table is missing a value. If 50 students were surveyed, how many students like green eyes?

| Favourite eye colour | Frequency |
|---|---|
| Brown | 13 |
| Hazel | 9 |
| Green | |
| Blue | 17 |
| Total | 50 |

Questions **2–4** refer to a box containing Lindt chocolate balls. There are four milk chocolate balls, three white chocolate, four hazelnut and three dark chocolate.

**2** What is the sample space for the box of chocolates?

- ☐ {4, 3, 4, 3}
- ☐ {12}
- ☐ {milk, hazelnut, white, dark}
- ☐ {chocolate, box}

**3** What is the probability (written as a fraction) of selecting a white chocolate from the box?

☐

**4** What is the probability of selecting a chocolate that is not dark?

- ☐ $\frac{3}{14}$
- ☐ $\frac{1}{2}$
- ☐ $\frac{1}{4}$
- ☐ $\frac{11}{14}$

Questions **5** and **6** refer to this experiment. A fair die was rolled 100 times and a 6 was obtained 20 times.

**5** What is the experimental probability of obtaining a 6? Write your answer as a decimal number.

☐

**6** What would you expect to happen to the experimental probability if the experiment was repeated 1000 times?

- ☐ Increase
- ☐ Decrease
- ☐ Stay the same
- ☐ Become 1

### Calculator

**7** What type of data is hair colour?

- ☐ Categorical, ordinal
- ☐ Numerical, discrete
- ☐ Numerical, continuous
- ☐ Categorical, nominal

Questions **8** and **9** refer to this data set:

4, 7, 5, 9, 1, 3, 4, 11, 9, 4, 4, 8, 9

**8** The mean, median and mode for this set are:

- ☐ 4, 5 and 6
- ☐ 6, 5 and 4
- ☐ 5, 4 and 6
- ☐ 6, 4 and 5

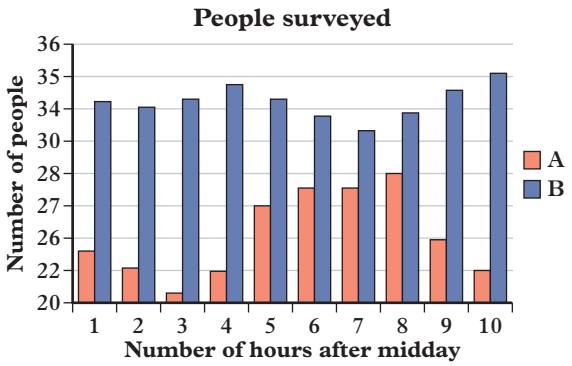
**9** What is the range of the data set?

☐

**10** Which data set has a mode of 8?

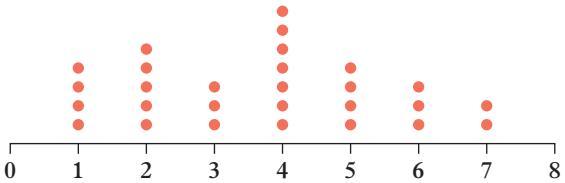
- ☐ 8, 3, 8, 5, 7, 3
- ☐ 3, 6, 4, 8, 5, 6, 8, 7, 8, 1, 2, 9
- ☐ 2, 4, 6, 8, 10, 12, 14
- ☐ 7, 3, 6, 8, 5, 2, 6, 9

**11** Which of the following characteristics of the graph below does *not* need fixing?



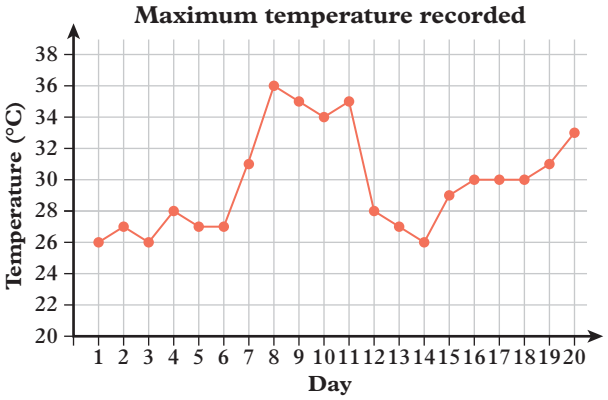

- ◯ The horizontal scale
- ◯ The vertical scale
- ◯ The legend
- ◯ The title

**12** How many people were surveyed for this dot plot?




[ ]

**13** According to the line graph below, what was the maximum temperature on day eight?




[ ] °C

**14** What is the largest number shown in this stem-and-leaf plot?

| Stem | Leaf |
|---|---|
| 1 | 1 4 5 7 |
| 2 | 2 3 4 5 9 9 |
| 3 | 1 9 |
| 4 | 0 3 3 3 5 6 |
| 5 | 0 0 4 5 8 8 |

Key: 1 | 1 = 11

8 ◯ 9 ◯ 5 ◯ 58 ◯

**15** Which of these situations would be best described as 'likely'?

- ◯ The sun rising tomorrow
- ◯ Rolling a die and getting an even number
- ◯ Selecting a non-picture card from a deck of cards
- ◯ Winning the lottery

Questions **16** and **17** refer to the following stem and leaf plot.

This stem and leaf plot shows the ages of employees at a local supermarket.

| Stem | Leaf |
|---|---|
| 1 | 7 7 8 9 9 |
| 2 | 0 0 1 4 5 |
| 3 | 3 6 8 |
| 4 | 5 9 |

Key 3 | 4 = 34

**16** What was the median age of the employees?

◯ 19 ◯ 20
◯ 21 ◯ 24

**17** How many employees were over 30 years of age?

◯ 0 ◯ 10
◯ 15 ◯ 5

[STEAM project 1]

# How can we reduce contaminants in local waterways so that biodiversity in the area is protected?

**Your task**

Design a device that will reduce the contaminants that are entering your local waterway, in order to protect biodiversity in the area.

**Figure 1** Algal blooms occur when an oversupply of nutrients in the water allows algae populations to quickly increase, covering the water's surface. Contaminants such as industrial fertiliser running into waterways can cause algal blooms. Algal blooms are often toxic to other aquatic life.

Australia is one of the driest continents in the world. But according to an Australian Government report, Australians consume more water per person than any other country, using an average of 100 000 L per person every year.

Water is an important resource in Australia, so it is critical to manage our waterways carefully.

Australia has many waterways, including rivers, groundwater systems, wetland environments and other human-made passages for water. Waterways are vital to our existence and are valuable economic assets.

Waterways play an important role in supporting biodiversity in our local areas, by providing habitats for wildlife (such as fish and turtles) and plants.

Humans rely on local waterways for drinking water, irrigation of crops, industrial processes and recreational activities. But sometimes these human activities can impact waterways, endangering the biodiversity of a local area.

Waterways also hold spiritual significance for many people. Due to the importance of local waterways to Aboriginal and Torres Strait Islander peoples, waterways are part of our cultural heritage.

We must, therefore, manage our waterways to maintain their complex ecosystems.

## Water quality and contaminants

A contaminant is a substance that pollutes or poisons something. Contaminants can occur naturally, or be caused by humans – such as microplastics, pesticides and litter. Litter is an example of a physical contaminant, while pesticides are examples of chemical contaminants. There are many types of contaminants, which are often more heavily concentrated in industrial, urbanised or agricultural areas.

Human land-use and major weather events, such as floods and bushfires, can introduce contaminants into local waterways and affect the water quality.

When water becomes contaminated, it can affect the health of an entire ecosystem, leading to serious environmental issues, such as acidic soil or algal blooms. So it is important for all waterways to be managed to protect the organisms that rely on them to survive.

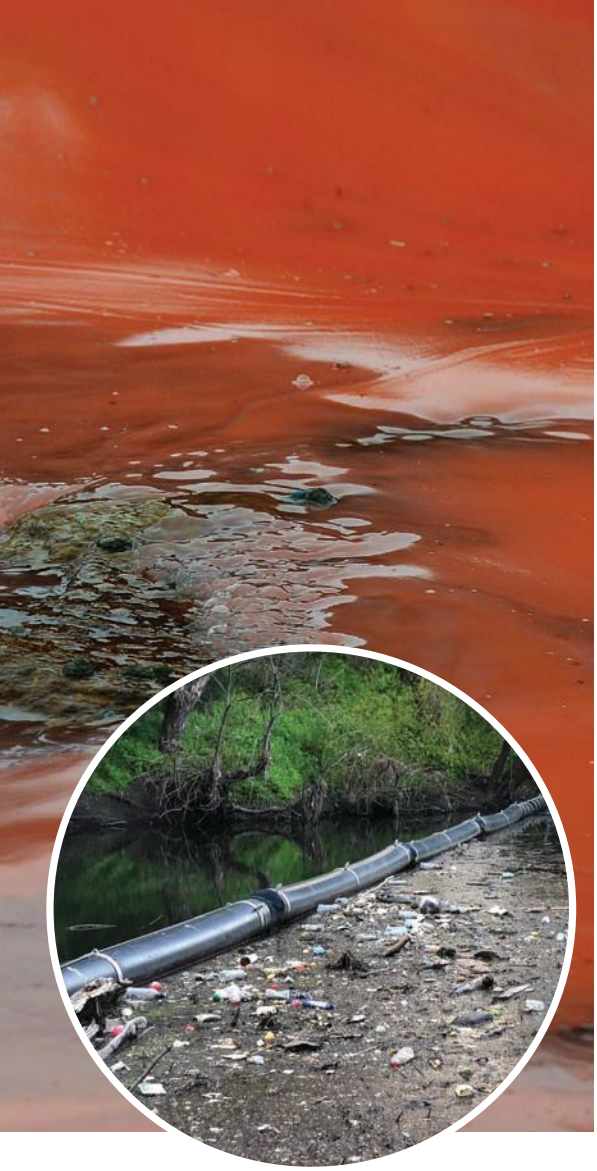


**Figure 2** Litter is a contaminant in local waterways, such as the River Torrens in Adelaide.

**Figure 3** The Yarra River is known to local Wurundjeri people as Birrarung, and has great cultural significance.

## HUMANITIES

In Geography this year, you will learn about water as a resource and how it connects places as it moves through the environment. You will also study the variability and scarcity of water in Australia.

In History, you will investigate the importance of water in sustaining ancient civilisations.

To complete this task successfully, you will need to investigate the health of a local waterway and the nearby land uses that may be introducing contaminants to the water. You will then need to research strategies that will help to reduce these contaminants.

You will find more information on this in Chapter 2 'Water as a resource' and Chapter 3 'Valuing and managing water' of *Oxford Humanities 7 Victorian Curriculum*.

## MATHS

In Maths this year, you will consolidate your understanding of volume and capacity and different units for measuring them. You will learn to perform calculations involving fractions, decimals and percentages – both with and without digital technology.

To complete this task successfully, you will need to combine these mathematical skills with your understanding of chemical and physical changes. You can then determine the scale of the problem and design your prototype in detail. You may need to perform calculations that relate the concentrations of contaminants, the dimensions of your prototype, the volume of water that can and needs to be processed, and the quantity of contaminants that need to be disposed of.

You will find help for applying these maths skills in Chapter 3 'Fractions and ratios', Chapter 4 'Decimals and percentages', and Chapter 9 'Length, area and volume' of *Oxford Maths 7 Victorian Curriculum*.

## SCIENCE

In Science this year, you will learn about how particles move in solids, liquids and gases. Each of these states of matter occurs during the water cycle, which is influenced by both nature and humans. When water is combined with other particles, it forms different types of solutions and mixtures. The unique properties of each particle (including the water molecule) can allow it to be isolated and purified once more.

To complete this task successfully, you will need to consider the properties of each contaminant, and how these properties can be used to separate the contaminants from the water. You will also need to be familiar with the scientific method, and understand how to conduct a fair test.

You will find more information on this in Chapter 2 'Particle model' and Chapter 3 'Mixtures' of *Oxford Science 7 Victorian Curriculum*.

# The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

## Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems?
- What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

## Define

Before you start to design your device, you need to define the criteria that you will use to test that the problem is solved.

### Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and the reason it is important to solve it. Use the following question as a guide:

'How can we help (the group) to solve (the problem) so that (the reason)?'

### Determine the criteria

1. Define each contaminant that is present in the waterway. Describe the properties of each contaminant.
2. Describe how you could test whether the contaminant was present in the water.
3. Describe how the contaminant would affect the biodiversity of the area if it were not removed from the waterway.

## Ideate

Once you know who you're designing for, and what the criteria are, it's time to get creative!

- Outline the criteria or requirements your device must fulfil (for example, the weight and height of your design).
- Brainstorm at least one idea per person that fulfils the criteria.
- Consider whether your idea will prevent contamination from occurring or solve the problem after it has already occurred.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

## Build

Draw each individual design for your device. Label each part of the design. Include the materials that will be used for its construction.

Include in the individual designs:

**a** the method you will use to isolate each contaminant

**b** the location of the device in the waterway.

If there is more than one separation method used in your design, identify the order in which you will carry out each method.

Present your design to your group. Use the criteria or requirements that you identified to decide which design your group will build.

### Build the prototype

Build and test the prototype of your group's chosen device.

Use the following questions as a guideline for your prototype:

- What materials will you need to build your prototype?
- How will you test whether each step of the design is successful? What will the outcome of each step look like?
- How will you record the steps you use when testing your device?
- How will you record the details of each extracted contaminant?
- How will you dispose of the extracted contaminants after your project is completed?

## Test

Use the scientific method to design and experiment with each separation method to ensure its success. You will need to control your variables between each test.

What criteria will you use to determine the success of your prototype?

Conduct your tests and record your results in an appropriate table.

## Communicate

Present your design to the class as though you are trying to get your peers to invest in your device.

In your presentation, you will need to:

- explain why removing the contaminant is important for the local wildlife
- describe the key features of your design and how it will reduce the amount of contaminant in the waterways
- construct a labelled diagram of your prototype in the natural environment
- explain the principles that support your design – the importance of water in the local environment and in sustaining civilisations, how some waterways become contaminated and how these contaminants can be reduced
- estimate the number of devices needed to reduce contaminants in the waterways in your local area
- calculate the cost of implementing your design.

**Check your Student obook pro for the following digital resources to help you with this STEAM project:** pro

**Student guidebook**
This helpful booklet will guide you step-by-step through the project.

**What is the design cycle?**
This video will help you to better understand each phase in the design cycle.

**How to manage your project**
This 'how-to' video will help you to manage your time throughout the design cycle.

**How to pitch your idea**
This 'how-to' video will help you with the 'Communicate' phase of your project.

**Check your Teacher obook pro for these digital resources and more:** pro

**Implementation advice**
Find curriculum links and advice for this project.

**Assessment resources**
Find information about assessment for this project.

[STEAM project 2]

# How can we reduce waste so that we don't exploit resources?

In 2019, the United Nations estimated that every year 90 billion tonnes of resources (including fossil fuels, precious metals and non-metals) are extracted from the earth and turned into usable products. When these products are no longer used or wanted, only 9 per cent are recycled.

If we want to have enough resources left for future generations, humans cannot continue to extract materials from the earth in this way. It is unsustainable.

## E-waste

In Australia, millions of electronic devices are thrown away every year. This is known as e-waste. E-waste can be difficult to recycle, but it often contains valuable metals.

The average mobile phone contains 0.034 g of gold, 16 g of copper and 0.35 g of silver. When a phone is thrown into landfill (along with all the other e-waste that once used electricity or batteries) the materials can take many decades to break down (sometimes releasing toxins into the ground).

Recycling these materials reduces the need to mine new resources.

## Fast fashion

Fast fashion is the term used for affordable clothing produced rapidly for the general public. It is designed and produced quickly to reflect current fashion trends. When fashion becomes outdated, the clothes or products that reflect that fashion are often thrown away. The Australian Bureau of Statistics (2020) identified that, on average, each person sends 23 kg of clothing to landfill each year. Over 60 per cent of this clothing is made of synthetic fibres (made from fossil fuels) that can take many years to biodegrade.

**Your task**

Develop a way to encourage residents or businesses to be part of the circular economy by reducing the amount of waste that is sent to the local landfill site.

**Figure 1** In Australia, 6000 kg of textiles and clothing are dumped in landfill every 10 minutes.

## What is a circular economy?

One potential solution to our unsustainable use of resources is a 'circular economy'. In a circular economy, resources are used and reused as much as possible. This benefits businesses because the longer a resource can be used, the more value it has. A circular economy is based on three key principles:

1. eliminate waste or pollution in the production of a product
2. keep products and materials in use
3. regenerate natural systems.

Some companies are already using the principles of a circular economy in the products they sell in Australia.

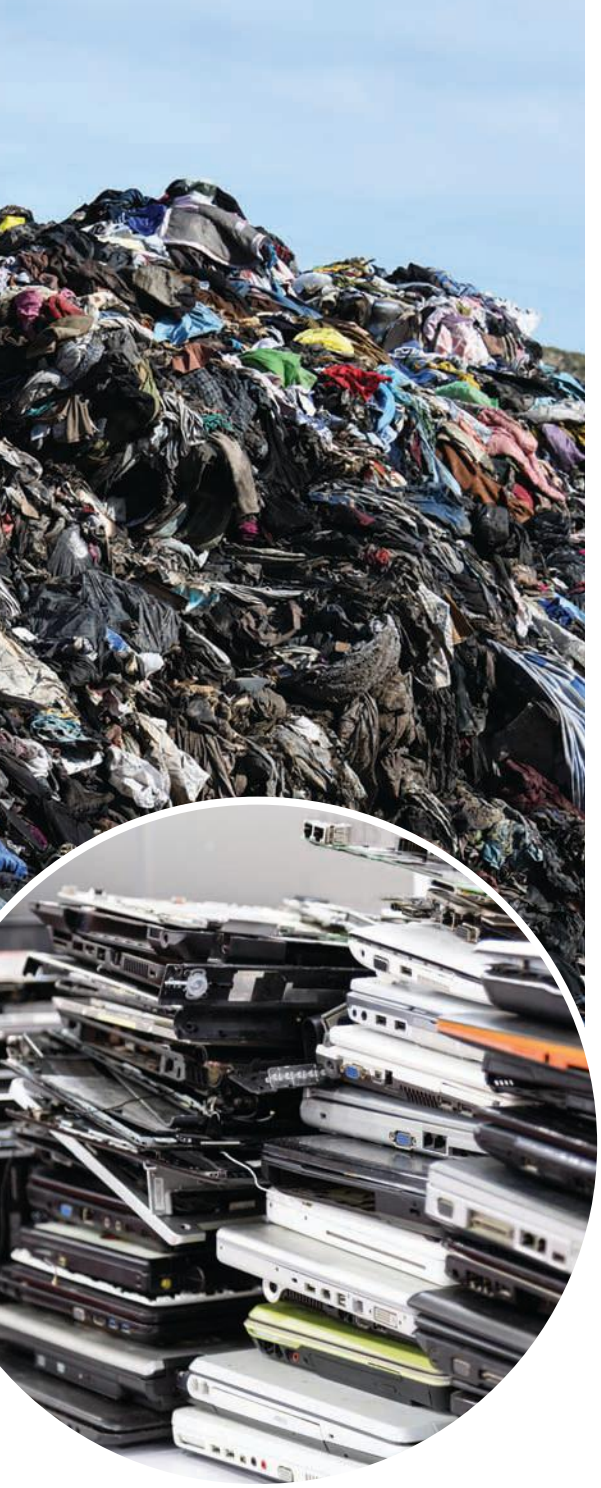


**Figure 2** Electronic waste does not belong in landfill. Laptops that are no longer working or wanted are examples of e-waste.

**Figure 3** The average mobile phone contains gold, copper and silver.

## HUMANITIES

In Economics and Business this year, you will learn how consumers and producers respond to and influence each other in the market, particularly through price mechanisms. You will consider how resources are distributed, allocated or used in the production of food, clothing and electronics, and the growing consumer demand for sustainable products.

In Geography, you will investigate how the environment can affect the liveability of a place. You may survey the local area to understand the role of services and facilities provided to minimise, reduce and prevent waste, and strategies used to enhance liveability.

To complete this task successfully, you will need to consider the products you buy and the decisions businesses make when deciding what to produce, and the consequences when a product reaches its end-of-life. You will also need to gain an understanding of people's perceptions in your local area towards waste management, recycling and the importance of the environment in measuring liveability.

You will find more information on this in Chapter 5 'Liveable cities', and Chapter 17 'Economic choices' of *Oxford Humanities 7 Victorian Curriculum*.

## MATHS

In Maths this year, you will use fractions, percentages and decimals to represent numbers, and ratios between quantities. You will consolidate your knowledge of volume, learning about cubic units and how to convert between different units of volume. You will perform calculations with and without digital technology.

To complete this task successfully, you will need to quantify the problem, which will include using ratios or fractions to scale between individual, local, national and global situations. You will need to cost your solution, accounting for any costs saved by recycling valuable materials.

You will find help for applying these maths skills in Chapter 3 'Fractions and ratios', Chapter 4 'Decimals and percentages', and Chapter 9 'Length, area and volume' in *Oxford Maths 7 Victorian Curriculum*.

## SCIENCE

In Science this year, you will learn how different resources can take different amounts of time to be regenerated. You will need to use your knowledge to consider the role of renewable and 'non-renewable' resources in different products, and how constantly sourcing new materials will affect the surrounding ecosystem.

To complete this task successfully, you may need to consider how the change in state of matter can aid a circular economy. You will also need to be familiar with the scientific method and understand how to conduct a fair test.

You will find more information on resources in Chapter 5 'Resources' in *Oxford Science 7 Victorian Curriculum*.

# The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

## Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users. This stage involves thinking about the problem (not possible solutions).

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- Who is producing the waste? Why is the waste being produced?
- What does it feel like to face these problems?
- What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

## Define

Before you start to design your solution, you need to define the criteria that you will use to test the success of your solution.

### Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and the reason it is important to solve it. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

### Determine the criteria

1. Describe the product that is being used. How much product is needed for normal functions?
2. Describe the waste that is being produced. In what units could you measure the amount of waste? How could you estimate how much waste is currently being produced?
3. Describe the different things that currently happen to this waste. To what fraction of the waste does this happen?
4. Describe the criteria that you will use to measure the success of your design.

## Ideate

Once you know who you're designing for, and what the criteria are, it's time to get creative!

As a group, brainstorm ways to solve the problem. Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Once you have many possible solutions, select three to five ideas and research whether these ideas have already been produced by someone else. If the prototype idea is already on the market, can you make a better version? If it's not, what will be needed to make it?

## Build

Draw your top two ideas. Label each part of the designs. Include the materials or skills required for their construction.

Include in the designs:

a a description of how the users will interact with the prototype idea

b a description of how the amount of waste will be decreased, and by roughly how much

c a description of how the design will contribute to the circular economy

d at least one advantage and disadvantage of each design.

Select one of the designs to take to the building and testing stage.

## Build the prototype

You will need to build at least three versions of your prototype idea. The first version will be tested for usefulness; the second will be used to test or survey the group you are helping; the third will be used for the presentation.

Use the following questions as a guideline for your prototype idea.

- What skills will you need?
- How will you produce a physical version of your prototype idea?
- How will you collect data on the effectiveness of your idea?

# Test

## Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first prototype. You will test the prototype more than once so that you can compare, but you will need to control your variables between tests.

What criteria will you use to determine the success of your solution?

Conduct your tests and record your results in an appropriate table.

## Prototype 2

If your prototype will be used to reduce waste, then you will need to generate a survey to test whether the prototype is appropriate for the user. (How would they use it? Would it make their work easier or harder? How likely do you think they are to buy it, and why? How will the prototype affect normal behaviours? How will the production of the prototype affect the environment?)

## Prototype 3

Use the information you have obtained from testing the first two versions to adapt your last prototype to be more effective and usable for the group you are helping. You may want to use the first two prototypes to demonstrate how the design has been improved over time.

# Communicate

Present your solution to the class as though your peers are going to invest their money. How will you convince them it is a good idea?

In the presentation, you will need to:

- explain why we need to reduce the amount of waste going to the local landfill
- describe the key features of your design and how they will reduce the amount of waste in the landfill, using calculations to justify a quantitative estimate of that reduction
- construct a labelled diagram of your prototype in the natural environment
- describe how the ecosystem will be impacted by your prototype idea
- explain the principles that support your design – such as the circular economy
- use calculations to estimate the cost of implementing your design.

**Check your Student obook pro for the following digital resources to help you with this STEAM project:**

**Student guidebook**
This helpful booklet will guide you step-by-step through the project.

**What is the design cycle?**
This video will help you to better understand each phase in the design cycle.

**How to manage your project**
This 'how-to' video will help you to manage your time throughout the design cycle.

**How to define a problem**
This 'how-to' video will help you to narrow your ideas down and define a specific problem.

**Check your Teacher obook pro for these digital resources and more:**

**Implementation advice**
Find curriculum links and advice for this project.

**Assessment resources**
Find information about assessment for this project.

# Answers

## CHAPTER 1 Whole numbers

### EX p8 1A Place value

**1** **a** 50 **b** 5 **c** 50 000
**d** 500 **e** 5000 **f** 500 000

**2** **a** 90 **b** 900 **c** 9
**d** 9000 **e** 90 **f** 900 000

**3** **a** $4 \times 10 + 6 \times 1$
**b** $5 \times 100 + 6 \times 10 + 8 \times 1$
**c** $1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1$
**d** $7 \times 1000 + 6 \times 100 + 9 \times 10 + 2 \times 1$
**e** $8 \times 10\,000 + 9 \times 1000 + 4 \times 100 + 5 \times 10 + 7 \times 1$
**f** $4 \times 100\,000 + 5 \times 10\,000 + 7 \times 1000 + 8 \times 10 + 2 \times 1$

**4** **a** 800 **b** 500 **c** 100
**d** 6500 **e** 38 400 **f** 6000
**g** 8500 **h** 23 000

**5** **a** 5000 **b** 9000 **c** 1000
**d** 8000 **e** 37 000 **f** 51 000
**g** 90 000 **h** 100 000

**6** **a** 90 **b** 8
**c** **i** 10 **ii** 500 **iii** 1000

**7** **a** 62 000 **b** 978 **c** 304
**d** 250 112 **e** 12 543 **f** 9026

**8** **a** Three hundred and sixty-two
$3 \times 100 + 6 \times 10 + 2 \times 1$
**b** Seven thousand, two hundred and fifteen
$7 \times 1000 + 2 \times 100 + 1 \times 10 + 5 \times 1$
**c** Forty-five thousand, seven hundred and thirty-three
$4 \times 10\,000 + 5 \times 1000 + 7 \times 100 + 3 \times 10 + 3 \times 1$
**d** Two hundred and thirty-four thousand, six hundred and one
$2 \times 100\,000 + 3 \times 10\,000 + 4 \times 1000 + 6 \times 100 + 1 \times 1$
**e** Six million, four hundred and twenty thousand and fifty-eight
$6 \times 1\,000\,000 + 4 \times 100\,000 + 2 \times 10\,000 + 5 \times 10 + 8 \times 1$
**f** Fifty-five thousand, five hundred and fifty-five
$5 \times 10\,000 + 5 \times 1000 + 5 \times 100 + 5 \times 10 + 5 \times 1$

**9** **a** $8530 > 7503$ **b** $46\,249 < 64\,249$
**c** $317\,294 > 37\,294$ **d** $709 < 4503$
**e** $5678 < 5876$ **f** $10\,462 > 10\,248$

**10** **a** 9, 15, 28, 40, 58, 72, 88
**b** 805, 806, 846, 856, 890
**c** 625, 6105, 9472, 9902, 10 417
**d** 234, 2347, 2374, 23 074, 23 704

**11** **a** 8000, 1800, 870, 807, 87
**b** 3999, 3909, 3099, 399, 309
**c** 75 561, 75 516, 75 126, 72 156
**d** 2 567 291, 1 967 219, 256 291

**12** 2, 5, 11, 16, 20, 28

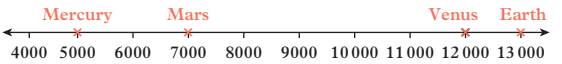
**13** 108, 132, 150, 173, 185, 200

**14** 2070, 2100, 2250, 2500, 2850, 2920

**15** **a** 4503 **b** 30 372 **c** 68 539
**d** 203 004 **e** 890 403 **f** 674 326

**16** **a** **i** 200 **ii** 300 **iii** 300
**iv** 200 **v** 300
**b** **i** 6000 **ii** 5000 **iii** 6000
**iv** 6000 **v** 5000
**c** If the number is 250 or greater, it is closer to 300.
If the number is 5500 or greater, it is closer to 6000.

**17** **a** Mercury: 5000 km, Venus: 12 000 km,
Earth: 13 000 km, Mars: 7000 km
**b**




**c** Sample answer: Mars is slightly larger than Mercury, whereas Earth and Venus are nearly the same size and approximately twice as large as the smaller two planets.

**18** **a** **i** 3000 km **ii** 300 000 km
**b** The distance from the Earth to the Moon is approximately 100 times greater than the diameter of the Moon. The diameter of the Moon is approximately 3000 km. If you multiply this by 100 you get 300 000 km, which is approximately the distance from the Earth to the Moon.

**19** **a** Two different two-digit numbers; 35, 53
**b** Four different two-digit numbers; 55, 53, 35, 33

**20** **a** Six different two-digit numbers; 74, 72, 47, 42, 27, 24
**b** Nine different two-digit numbers; 22, 24, 27, 42, 44, 47, 72, 74, 77

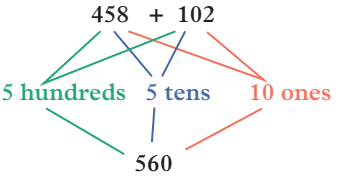
**21** **a** 9631 **b** 1111 **c** 9316
**d** 3691, 3916, 3961, 6139, 6193, 6319

**22** **a** 1 200 000 **b** 0, 1, 2, 3 or 4 **c** 2 000 000
**d** 480 000

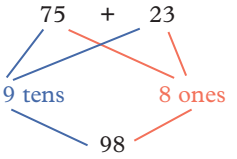
**23** 23 570, 98 500, 79 500, 86 000

### EX p14 1B Adding whole numbers

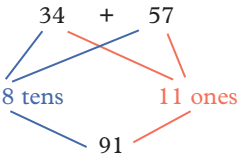
**1** **a** 560



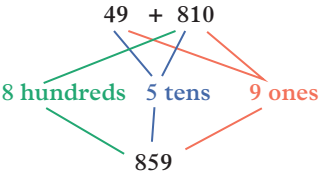

**b** 98



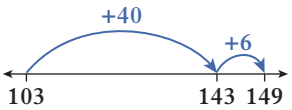

**c** 91



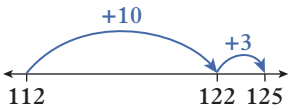

**d** 859



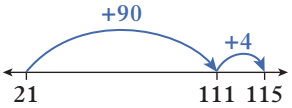
**2** **a** 149



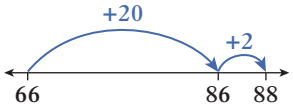

**b** 125



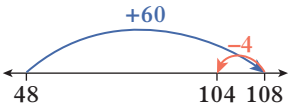

**c** 115



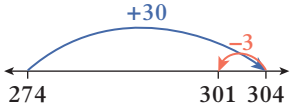

**d** 88



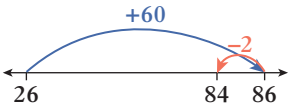

**3** **a** 104



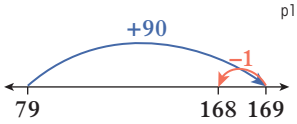

**b** 301




**c** 84




**d** 168




**4** **a** 80 **b** 180 **c** 380
**d** 38 **e** 77 **f** 90
**g** 100 **h** 468

**5** **a** 70 **b** 900 **c** 80
**d** 100 **e** 90 **f** 99
**g** 400 **h** 609

**6** **a** 958 **b** 692 **c** 1219
**d** 1502

**7** **a** 1119 **b** 453 **c** 7598
**d** 5251 **e** 2660 **f** 5839
**g** 17 189 **h** 43 723

**8** **a** 1569 **b** 1804 **c** 1597
**d** 960 **e** 11 508 **f** 7889

**9** **a** 138 849 **b** 92 203 **c** 349 908
**d** 686 461

**10** **a** 11 374 **b** 102 409

**12** **a** **i** False **ii** True **iii** True
**b** Sample answer:
32 + 45 + 59 = (30 + 40 + 50) + (2 + 5 + 9)
**c** 136 km

**13** **a** Sample answer: 42 + 11 = 53
**b** **i** Sample answer: 233 + 153 = 386
**ii** Sample answer: 214 + 172 = 386
**c** Sample answer: 367 + 1289 + 3551 = 5207

**14** **a** 75 mins **b** 6:30 pm

**15** **a** **i** 750 km **ii** 1591 km **iii** 626 km
**iv** 3430 km

**16** **a** 39 000 **b** 39 174
**c** Depends on the commentator. Some like to round the nearest ten thousand, some like to tell you the actual number.

**17** **a** 55
**b**
(1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6)
= 11 + 11 + 11 + 11 + 11
= 55

**18** 210

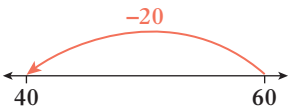
**19** **a** **i** 45 **ii** 190
**b** There will be one number that is not part of a pair and will need to be added separately.
**c** **i** 35 **ii** 150 **iii** 315

**20** **a** 26 and 27 **b** 37, 38 and 39
**c** 46, 47, 48 and 49
**d** Answers will vary.

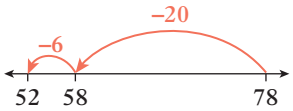
## EX 1C Subtracting whole numbers

p19

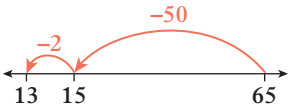
**1** **a** 40



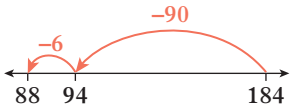

**b** 52



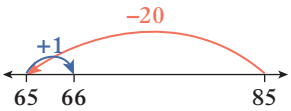

**c** 13



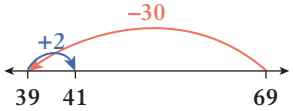

**d** 88



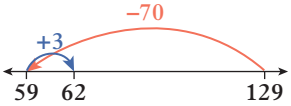

**2** **a** 66



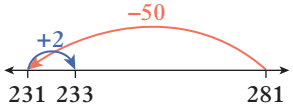

**b** 41




**c** 62




**d** 233




**3** **a** 20 **b** 100 **c** 140
**d** 210 **e** 690 **f** 120
**g** 235 **h** 112

**4** **a** 60 **b** 500 **c** 10
**d** 109 **e** 515 **f** 32
**g** 42 **h** 435

**5** **a** 33 **b** 41 **c** 265
**d** 272

**6** **a** 42 **b** 45 **c** 222
**d** 287 **e** 439 **f** 2122
**g** 1877 **h** 1656

**7** **a** 6112 **b** 5315 **c** 1747
**d** 6928 **e** 3563 **f** 4571

**8** **a** 8181 **b** 370 595 **c** 175 128
**d** 73 390

**9** **a** 12 620 **b** 52 514

**11 a** **i** 30 **ii** 32
**b** **i** 600 **ii** 579
**c** **i** 5100 **ii** 4871
**d** **i** 18 000 **ii** 19 880
**e** **i** 120 **ii** 122
**f** **i** 1260 **ii** 1271

**12 a** **i** False **ii** True **iii** False **iv** False
**b** Sample answer:
6650 − 2740 = 6700 − 2700 − 50 − 40
**c** 3910 km

**13 a** Sample answer: 95 − 52 = 43
**b** **i** Sample answer: 259 − 91 = 168
**ii** Sample answer: 578 − 410 = 168
**c** **i** Sample answer: 486 − 200 = 286
**ii** 429 − 143 = 286

**14 a** 16 742 km **b** 16 112 km
**c** The Singapore stopover is the shortest by 630 km.

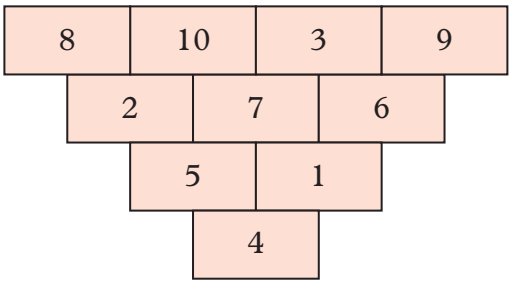
**15 a** Washington Monument: 659 m, Eiffel Tower: 528 m, Empire State Building: 447 m, Sydney Tower: 519 m, Petronas Towers: 376 m, Taipei 101: 319 m, Q1: 505 m, Eureka Tower: 531 m
**b** Taipei 101 and the Q1 give a total height of 832 m.
**c** Washington Monument, Empire State Building and Eureka Tower give a total height of 847 m.

**16 a** Mount Everest is 6620 m higher than Mount Kosciuszko.
**b** 4148 m higher, 2152 m lower, 12 323 m lower
**c** 16 471 m **d** 8772 m

**17** 193

**18 a** **i** 645 200 000 km
**ii** 1 365 691 000 km
**iii** 50 291 000 km
**b** 929 392 000 km
**c** **i** 720 491 000 km
**ii** 1 315 400 000 km
**iii** 2 203 392 000 km
**d** 78 340 000 km

**19 a** If the number 10 is on a lower row then the two numbers diagonally above it would need to have a difference of 10, however this is not possible since 10 is the largest number in the set.
**b**




## EX 1D Multiplying whole numbers

p25

**1 a** 28 **b** 18 **c** 132 **d** 42
**2 a** 45 **b** 112 **c** 95 **d** 112
**3 a** 54 **b** 84 **c** 288 **d** 165
**4 a** 592 **b** 186 **c** 135 **d** 182
**e** 960 **f** 7236 **g** 3192 **h** 1124
**5 a** 50 **b** 320 **c** 6390 **d** 4800
**e** 700 **f** 10 300 **g** 516 230 **h** 2 784 000
**6 a** 120 **b** 420 **c** 180
**d** 6000 **e** 40 000 **f** 123 000
**g** 712 000 **h** 500 000
**7 a** 480 **b** 120 **c** 840
**d** 3720 **e** 10 850 **f** 126 900
**g** 41 600 **h** 128 400

**8 a, b**

| First number × second number | Product | Number of zeros in the first number | Number of zeros in the second number | Number of zeros in the product |
|---|---|---|---|---|
| 10 × 10 | 100 | 1 | 1 | 2 |
| 100 × 10 | 1000 | 2 | 1 | 3 |
| 1000 × 10 | 10 000 | 3 | 1 | 4 |
| 10 000 × 10 | 100 000 | 4 | 1 | 5 |

**c** The total number of zeros in the product is the sum of the number of zeros in the original calculation.
**d** **i** 1000 **ii** 10 000 **iii** 100 000 **iv** 1 000 000

**9 a** **i** 100 000 **ii** 400 000
**b** **i** 3000 **ii** 70 000 **iii** 600 000
**iv** 2 000 000 **v** 5000 **vi** 400 000
**vii** 80 000 000 **viii** 900 000

**10 a** **i** 6 **ii** 1000 **iii** 6000
**b** **i** 8000 **ii** 90 000 **iii** 120 000
**iv** 4 500 000 **v** 56 000 **vi** 2 400 000
**vii** 210 000 000 **viii** 16 000 000

**11 a** 80 000 **b** 162 000 000
**c** 5 600 000 **d** 120 000 000

**12 a** **i** True **ii** True **iii** True **iv** False **v** True
**b** Sample answer: 37 × 28 = 37 × 20 + 37 × 8
**c** $1036

**13 a** 1150 m **b** 1074 m
**c** Divide 1000 m by the full stroke length of 5 m. Then multiply by 3. The result is 600.

**14 a** 528 **b** 996 **c** 1524
**d** 1778

**15 a** 1290 **b** 5160 minutes
**c** If it takes 4 minutes to install 1 light bulb, it takes 60 minutes to install 15 light bulbs as 4 × 15 = 60.

**16** Sample answer:

$$24 \times 6 = 6\begin{array}{|c|c|}\hline 120 & 24 \\ \hline\end{array} = 144 \quad (20\ \ 4)$$

$$24 \times 6 = 6\begin{array}{|c|c|c|}\hline 60 & 60 & 24 \\ \hline\end{array} = 144 \quad (10\ \ 10\ \ 4)$$

$$24 \times 6 = \begin{array}{r} 3 \\ 3 \end{array}\begin{array}{|c|c|}\hline 60 & 12 \\ \hline 60 & 12 \\ \hline\end{array} = 144 \quad (20\ \ 4)$$

$$24 \times 6 = \begin{array}{r} 5 \\ 1 \end{array}\begin{array}{|c|c|c|}\hline 50 & 50 & 20 \\ \hline 10 & 10 & 4 \\ \hline\end{array} = 144 \quad (10\ \ 10\ \ 4)$$

$$24 \times 6 = \begin{array}{r} 2 \\ 2 \\ 2 \end{array}\begin{array}{|c|c|}\hline 40 & 8 \\ \hline 40 & 8 \\ \hline 40 & 8 \\ \hline\end{array} = 144 \quad (20\ \ 4)$$

## 1 Checkpoint

**1** **a** Eight hundred and thirty-six
$8 \times 100 + 3 \times 10 + 6 \times 1$

**b** Nine thousand, six hundred and fifty-four
$9 \times 1000 + 6 \times 100 + 5 \times 10 + 4 \times 1$

**c** Six hundred and twenty-nine thousand
$6 \times 100\,000 + 2 \times 10\,000 + 9 \times 1000$

**2** **a** 419 **b** 96 200 **c** 325 656

**3** **a** 9000 **b** 26 000 **c** 136 000

**4** **a** 114 **b** 149 **c** 600

**5** **a** 1041 **b** 15 173 **c** 35 128

**6** 920 568

**7** **a** 290 **b** 540 **c** 144

**8** **a** 232 **b** 4295 **c** 3519

**9** **a** 3407 **b** 543

**10** **a** 91 **b** 108 **c** 184

**11** **a** 4824 **b** 17 480 **c** 472 500

**12** **a** 144

**b** 18 cups milk, 12 eggs, 24 teaspoons vanilla extract, 24 cups self-raising flour, 6 cups sugar, 300 g butter

## EX p30 1E Long multiplication

**1** **a** 2538 **b** 143 **c** 450 **d** 814

**2** **a** 570 **b** 1488 **c** 1472 **d** 6205 **e** 3120 **f** 4536 **g** 1782 **h** 2208

**4** **a** 2527 **b** 6970 **c** 8232 **d** 17 984

**5** **a** 9545 **b** 15 282 **c** 4551 **d** 12 936 **e** 38 318 **f** 40 986 **g** 19 062 **h** 111 468

**7** **a** 43 250 **b** 300 155 **c** 157 872 **d** 585 625 **e** 66 712 **f** 91 333 **g** 256 452 **h** 197 262

**8** See answers from question 7.

**9** **a** **i** 48 000 **ii** 180 000 **iii** 2 400 000 **iv** 360 000 000

**b** **i** 48 462 **ii** 202 275 **iii** 2 388 224 **iv** 355 868 293

**10** **a** 4320 **b** 103 680

**11** **a** 800 **b** 733

**12** 2576 menus

**13** **a** 1040 m **b** 18 000 m

**c** **i** 1848 months
**ii** 56 210 days
**iii** 1 349 040 hours
**iv** 80 942 400 minutes
**v** 4 856 544 000 seconds

**14** Sample answer: If on your next birthday, you turn 12 years old:

**a** **i** 144 months
**ii** 4380 days
**iii** 105 120 hours
**iv** 6 307 200 minutes
**v** 378 432 000 seconds

**15** 1 200 000 000 km

**16** **a** 1 568 000 hectares

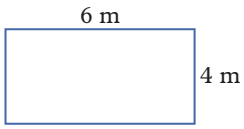
**b** 180 million animals

**c** 2 744 000 hectares

**17** Sample answer:

$$\begin{array}{r} 243 \\ \times\ \ 15 \\ \hline 1215 \\ +2430 \\ \hline =3645 \end{array}$$

**18** **a**




**b** 24 $m^2$

**c** 24 cm by 16 cm

**d** 384 $cm^2$

**e** 400 matchsticks

## EX p35 1F Dividing whole numbers

**1** **a** **i** 9 **ii** 2 **iii** 4 **iv** 1

**b** **i** 17 **ii** 7 **iii** 2 **iv** 3

**c** **i** 30 **ii** 5 **iii** 6 **iv** 0

**2** **a** 6 remainder 1 **b** 4 remainder 3
**c** 5 remainder 2 **d** 2 remainder 5
**e** 4 remainder 6 **f** 6 remainder 2
**g** 7 remainder 3 **h** 8 (or 8 remainder 0)

**3** **a** 134 remainder 2 **b** 126
**c** 57 remainder 1 **d** 245 remainder 4
**e** 175 remainder 2 **f** 69 remainder 6
**g** 83 remainder 4 **h** 308 remainder 2

**4** **a** 7 **b** 245
**c** 94 **d** 417

**5** **a** 25 remainder 17 **b** 41 remainder 1
**c** 34 **d** 66 remainder 6
**e** 21 remainder 3 **f** 21 remainder 20
**g** 213 remainder 7 **h** 22 remainder 2

**6** **a** 38 remainder 24 **b** 100 remainder 9
**c** 1214 remainder 7 **d** 150 remainder 23

**7** **a** 16 **b** 134
**c** 1473 **d** 694

**8** **a, b**

| **First number ÷ Second number** | 10 ÷ 10 | 100 ÷ 10 | 1000 ÷ 10 | 10 000 ÷ 10 | 100 ÷ 100 | 1000 ÷ 100 | 10 000 ÷ 100 |
|---|---|---|---|---|---|---|---|
| **First number / Second number** | 10/10 | 100/10 | 1000/10 | 10 000/10 | 100/100 | 1000/100 | 10 000/100 |
| **Quotient** | 1 | 10 | 100 | 1000 | 1 | 10 | 100 |
| **Number of zeros in the first number** | 1 | 2 | 3 | 4 | 2 | 3 | 4 |
| **Number of zeros in the second number** | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| **Number of zeros in the quotient** | 0 | 1 | 2 | 3 | 0 | 1 | 2 |

**c** The total number of zeros in the quotient is the difference in the number of zeros in the original calculation.

**d** **i** 1000 **ii** 1000 **iii** 1
**iv** 10

**9** **a** **i** 10 **ii** 50
**b** **i** 20 **ii** 6 **iii** 90
**iv** 4 **v** 80 **vi** 30
**vii** 50 **viii** 600

**10** **a** **i** 3 **ii** 10 **iii** 30
**b** **i** 30 **ii** 400 **iii** 20
**iv** 50 **v** 20 **vi** 20
**vii** 100 **viii** 20

**11** **a** **i** 20 **ii** 300 **iii** 20
**iv** 20
**b** **i** 19 **ii** 330 **iii** 18
**iv** 22

**12** **a** A
**b** One possible answer is: How many times does 15 go into 165?
**c** 11

**13** **a** 4 minutes **b** 4 × 68 = 272

**14** 30 laps

**15** **a** 54 **b** 95

**16** **a** 16 **b** 6
**c** 1350 food parcels if each contains one bag of rice. The next largest is 675 food parcels if each one contains 2 bags of rice.

**17** **a** 10 **b** 11

**18** 110: 10, 111: 3, 112: 8, 113: 1, 114: 6, 115: 5, 116: 4, 117: 9, 118: 2, 119: 7, 120: 12, 121: 11

**19** Sample answer:
4 + 5 = 9
9 × 6 = 54
7 × 3 = 21
21 + 9 = 30
2 × 8 = 16
30 + 16 = 46
46 × 1 = 46
46 + 54 = 100

**20** Sample answer:

```
      14 654
18)263 772
   -18
     83
    -72
     117
    -108
       97
      -90
        72
       -72
         0
```

## EX 1G Indices and square roots

p41

**1** **a** $5^9$ **b** $4^7$ **c** $11^3$
**d** $20^5$ **e** $9^8$ **f** $300^4$

**2** **a** 5 to the power of 9
**b** 4 to the power of 7
**c** 11 to the power of 3
**d** 20 to the power of 5
**e** 9 to the power of 8
**f** 300 to the power of 4

**3** **a** $7^3 \times 9^4$ **b** $4^6 \times 2^3$
**c** $3^2 \times 5^6$ **d** $8^4 \times 13^2$
**e** $2^4 \times 6^2 \times 7^2$ **f** $19^2 \times 23^4 \times 31$

**4** **a** 3 × 3
**b** 2 × 2 × 2 × 2 × 2 = 32
**c** 7 × 7 × 7 = 343
**d** 5 × 5 = 25
**e** 9 × 9 × 9 × 9 = 6561
**f** 4 × 4 × 4 = 64
**g** 1 × 1 × 1 × 1 × 1 = 1
**h** 6 × 6 × 6 = 216

**5** **a** 16 **b** 81 **c** 49
**d** 125 **e** 1331 **f** 144
**g** 1000 **h** 256

**6** **a** 2 × 2 × 2 × 4 × 4 = 128
**b** 3 × 3 × 3 × 3 × 5 × 5 = 2025
**c** 8 × 8 × 10 × 10 × 10 = 64 000

**d** $3 \times 3 \times 3 \times 3 \times 3 \times 1 \times 1 \times 1 \times 1 = 243$

**e** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 3136$

**f** $10 \times 10 \times 3 \times 3 \times 3 = 2700$

**g** $9 \times 9 \times 9 \times 9 \times 11 \times 11 = 793\,881$

**h** $6 \times 6 \times 3 \times 3 = 324$

**7** **a** 3 **b** 6 **c** 8
**d** 1 **e** 7 **f** 11
**g** 12 **h** 20

**8** **a** 11 **b** 35 **c** $\sqrt{3721}$
**d** $\sqrt{88\,804}$

**9** **a** 89 **b** 125 **c** 0
**d** 63 **e** 4 **f** 48
**g** 59 **h** 84 **i** 3600
**j** 160 **k** 31 752 **l** 209 952

**10** $3^5$

**11** **a** $2^3, 3^2, 5^4, 4^5$ **b** $1^{50}, 7^6, 50^3, 6^7$
**c** $10^3, 9^4, 6^5, 3^{10}$ **d** $12^2, 8^3, 4^6, 3^8$

**12** **a** False **b** False **c** True
**d** True

**13** **a** True **b** True **c** False
**d** False

**14** **a** $3 < \sqrt{15} < 4$ **b** $5 < \sqrt{33} < 6$
**c** $6 < \sqrt{40} < 7$ **d** $9 < \sqrt{99} < 10$
**e** $10 < \sqrt{120} < 11$ **f** $14 < \sqrt{214} < 15$

**15** **a**

| **Index form** | $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ |
|---|---|---|---|---|---|
| **Expanded form** | 10 | 10 × 10 | 10 × 10 × 10 | 10 × 10 × 10 × 10 | 10 × 10 × 10 × 10 × 10 |
| **Basic numeral** | 10 | 100 | 1000 | 10 000 | 100 000 |
| **Number of zeros in the basic numeral** | 1 | 2 | 3 | 4 | 5 |

**b** The index is the same as the number of zeros in the basic numeral.

**c** **i** 100 000 000 **ii** 1 000 000
**iii** 1 000 000 000 000 000
**iv** 10 000 000 000

**d** 1

**e** **i** $10^2$ **ii** $10^4$ **iii** $10^0$
**iv** $10^7$

**16** Finding the square root of a number is the opposite operation to finding the square of a number.

**17** **a** 2 **b** $2^1, 2^2$ **c** $2^3$
**d** $2^5$ or 32 times bigger

**18** **a**

| **Number of days** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Expanded form** | 3 | 3 × 3 | 3 × 3 × 3 | 3 × 3 × 3 × 3 | 3 × 3 × 3 × 3 × 3 |
| **Index form** | $3^1$ | $3^2$ | $3^3$ | $3^4$ | $3^5$ |
| **Number of bacteria (in millions)** | 3 | 9 | 27 | 81 | 243 |

**b** **i** 729 **ii** 2187 **iii** 4 782 969

**c** **i** 3 days **ii** 5 days **iii** 8 days

**19** **a** 84 **b** $4 + 4^2 + 4^3$

**20** The boss should pay Taylor \$100 per week, making a total of \$400. If the boss used the other method, the boss would have to pay Taylor
\$5 + \$25 + \$125 + \$625 = \$780.

**21** $1 + 8 = 9, 2 + 7 = 9, 3 + 6 = 9, 4 + 5 = 9,$
$9 + 16 = 25, 10 + 15 = 25, 11 + 14 = 25, 12 + 13 = 25$

**22** **a**

| **Consecutive square numbers** | **Difference** |
|---|---|
| 1 and 4 | 4 − 1 = 3 |
| 4 and 9 | 9 − 4 = 5 |
| 9 and 16 | 16 − 9 = 7 |
| 16 and 25 | 25 − 16 = 9 |
| 25 and 36 | 36 − 25 = 11 |
| 36 and 49 | 49 − 36 = 13 |
| 49 and 64 | 64 − 49 = 15 |
| 64 and 81 | 81 − 64 = 17 |
| 81 and 100 | 100 − 81 = 19 |
| 100 and 121 | 121 − 100 = 21 |

**b** They are consecutive odd numbers.

**c** $(n + 1)^2 - (n)^2$

**d** 2 multiplied by an even number is always an even number. Adding 1 means that the result will always be an odd number. This equation provides the solution to each difference when $n$ is the number of pairs.

**23** **a** Person A weighs just below 10 kg, Person B weighs just below 12 kg, Person C weighs between 7 and 8 kg, Person D weighs just above 9 kg.

**b** Person A weighs just above 3 kg, Person B weighs between 3 and 4 kg, Person C weighs between 2 and 3 kg, Person D weighs just above 3 kg.

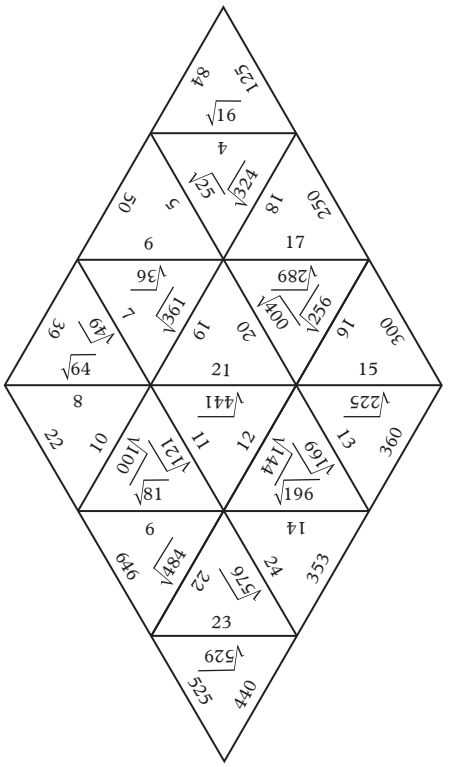
**c** Person A weighs just below 2 kg, Person B weighs just below 2 kg, Person C weighs between 1 and 2 kg, Person D weighs between 1 and 2 kg.

**d** Each person will weigh approximately 1 kg.

**e** No person will ever weigh 0 kg. Their weight will continue to get closer and closer to 1 kg for each hour for the rest of their life.

**f** Sample answer: Subtract one thirteenth of the initial weight every hour.

**24** Sample answer:




## EX 1H Order of operations

p46

**1** **a** 11 **b** 1 **c** 21
**d** 59 **e** 19 **f** 8
**g** 12 **h** 0 **i** 24
**j** 17 **k** 4 **l** 9

**2** **a** 15 **b** 60 **c** 4
**d** 37 **e** 67 **f** 39
**g** 0 **h** 4 **i** 52
**j** 49 **k** 55 **l** 1

**3** **a** 16 **b** 6 **c** 24
**d** 15 **e** 40 **f** 86
**g** 2 **h** 9 **i** 8
**j** 24 **k** 115 **l** 2880

**4** **a** **i** 860 **ii** 921
**b** **i** 42 **ii** 43
**c** **i** 90 **ii** 83
**d** **i** 2404 **ii** 2266
**e** **i** 610 **ii** 638
**f** **i** 6800 **ii** 7578
**g** **i** 118 **ii** 106
**h** **i** 3395 **ii** 3191

**5** **a** **i** 2700 **ii** 2589
**b** **i** 20 **ii** 14
**c** **i** 8060 **ii** 9831
**d** **i** 340 **ii** 332
**e** **i** 168 **ii** 172
**f** **i** 100 **ii** 108

**6** **a** **i** $11 \times 4 - (10 \div 2)$ **ii** 39
**b** **i** $(12 \div 4) \times 2^3$ **ii** 24
**c** **i** $(9^2 - 3^3) \times 2$ **ii** 108
**d** **i** $((17 - 9) \times \sqrt{4}) \div 2$ **ii** 8
**e** **i** $(\sqrt{16} \times 6) \div 2^3$ **ii** 3
**f** **i** $14 - 7 - 3$ **ii** 4

**7** **a** 11
**b** **i** 19 **ii** 7 **iii** 317
**iv** 10 **v** 53 **vi** 40

**8** 48

**9** **a** 2 **b** 4 **c** \$13

**10** 36

**11** 88

**12** 22

**13** **a** **i** Sharks: $10 \times 6 + 15 \times 1$
**ii** Visitors: $9 \times 6 + 23 \times 1$
**b** Sharks: 75, Visitors: 77
**c** Visitors by 2 points

**14** **a** $8 \times 6 + 4 \times 10 + 7 \times 5$ **b** \$123

**15** **a** $2 \times 2^2$ cannot be simplified to $4^2$. You'd need to apply the index before multiplication.
$35 - 16 - 1$ is 18 not 15.
**b** $(3 + 4)^2$ simplifies to $(7)^2$ not $3 + 4^2$. Brackets should be applied first.
$3 + 16 - 4 \times 3$ simplifies to $3 + 16 - 12$. Multiplication is applied before addition and subtraction.

**16** 5050

**17** Sample answer:

| | |
|---|---|
| **1** | $4^2 - 3 \times 5$ |
| **2** | $3 + 4 - 5$ |
| **3** | $(5 + 4) \div 3$ |
| **4** | $5 + 3 - 4$ |
| **5** | $(4^2 + 3^2) \div 5$ |
| **6** | $5 + (4 - 3)$ |
| **7** | $3 \times 4 - 5$ |
| **8** | $4^2 \div (5 - 3)$ |
| **9** | $3^2 \div (5 - 4)$ |
| **10** | $3^2 + 5 - 4$ |
| **11** | $3 \times 5 - 4$ |
| **12** | $5 + 4 + 3$ |
| **13** | $5^2 - 3 \times 4$ |
| **14** | $4^2 - (5 - 3)$ |
| **15** | $3 \times 5 \times 4^0$ |
| **16** | $(5 + 3)^2 \div 4$ |
| **17** | $(4 \times 5) - 3$ |
| **18** | $5^2 - 4^4 + 3^3$ |
| **19** | $3 \times 5 + 4$ |
| **20** | $3^2 + 4^2 - 5$ |

**18** **a** 20 **b** 5

# CHAPTER 1 review

## Multiple-choice

**1** B **2** E **3** C
**4** D **5** B 6 A
**7** C **8** D **9** B
**10** A

## Short answer

**1** **a** $4 \times 100 + 6 \times 10 + 7 \times 1$
**b** $7 \times 1000 + 8 \times 100 + 5 \times 10 + 6 \times 1$
**c** $9 \times 1000 + 3 \times 10$
**d** $1 \times 10\,000 + 5 \times 1000 + 4 \times 100 + 5 \times 10 + 2 \times 1$

**2** **a** 57, 547, 5347, 53 047
**b** 87 056, 87 506, 87 605

**3** **a** 400 **b** 9500 **c** 600
**d** 12 900

**4** **a** 124 **b** 115 **c** 785
**d** 118

**5** **a** 9267 **b** 402 253

**6** **a** Sample answer: 355 + 123 = 478
**b** Sample answer: 146 + 332 = 478

**7** **a** 4183 **b** 333 578 **c** 7843
**d** 41 333

**8** **a** 2945 **b** 253

**9** **a** 418 **b** 861 **c** 280
**d** 1008 **e** 384 **f** 910

**10** **a** 476 **b** 546 **c** 1638
**d** 6669 **e** 19 664 **f** 71 412

**11** **a** 38 900 **b** 2 472 000 **c** 680 000
**d** 6 000 000 **e** 4800 **f** 139 650

**12** **a** 20 054 **b** 35 568 **c** 177 915
**d** 108 416 **e** 4800 **f** 26 230

**13** **a** 141 remainder 4
**b** 17 111 remainder 1
**c** 2507
**d** 81 029 remainder 1

**14** **a** 4180 remainder 7
**b** 1151 remainder 7
**c** 245 remainder 13
**d** 1745

**15** **a** 700 **b** 40 **c** 500
**d** 300

**16** **a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
**b** $5 \times 5 \times 5 \times 3 \times 3 = 1125$
**c** $2 \times 2 \times 2 \times 7 \times 7 = 392$
**d** $6 \times 6 \times 4 \times 4 \times 4 = 2304$

**17** $10^4$

**18** **a** $7^2 = 49$ **b** $6^2 = 36$ **c** $19^2 = 361$
**d** $8^2 = 64$

**19** **a** 5 **b** 25 **c** 4
**d** 0

**20** **a** 28 **b** 60 **c** 31
**d** 11

**21** **a** 36 **b** 75 **c** 4
**d** 24

## Analysis

**1** **a** Saina's dad performed the calculations from left to right instead of using the order of operations.
**b** 32

**2** **a**

| Item | Quantity | Price per item | Total cost per item |
|---|---|---|---|
| Party pie 24 pack | 4 | $4 | $16 |
| Allen's mixed lollies | 3 | $3 | $9 |
| Chips 100 g | 4 | $2 | $8 |
| M&Ms 250 g | 2 | $4 | $8 |
| Freddo frogs 20 pack | 2 | $5 | $10 |
| Cocktail sausages 2 kg pack | 1 | $6 | $6 |
| Frozen pizza | 1 | $10 | $10 |
| Mudcake | 3 | $4 | $12 |

**b** $79 **c** $21 **d** chips
**e** Cocktail sausages. This is different from part **d** as he only bought one of these, whereas he bought four packets of chips.
**f** 120 lollies
**g** 11 friends (12 people in total)
**h** Three Freddo frogs each with four left over
**i** Four groups of 3, three groups of 4, two groups of 6

**3** **a** 5 km
**b** The third day
**c** 1216 turns
**d** Kate's total time: 36 mins 44 seconds
Toms' total time: 39 mins 54 seconds
**e** Kate's total time on Day 2: 37 mins 31 seconds
Kate was faster on Day 1 by 47 seconds.
**f** Tom should pit stop again; he will be quicker by 9 seconds.

# CHAPTER 2 Factors and primes

## EX 2A Number sequences

p57

**1** **a** Add two **b** Add nine **c** Add five
**d** Subtract four **e** Divide by three
**f** Multiply by four

**2** **a** 8, 24, 72 **b** 500, 100, 20 **c** 23, 16, 9
**d** 1000, 1200, 1400
**e** 25, 30, 40 **f** 18, 12, 9

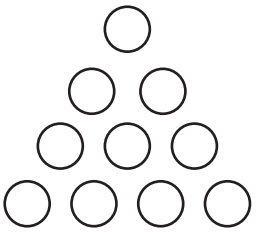
**3** **a** 30, 36 **b** 35, 42 **c** 54, 48
**d** 88, 44 **e** 39, 26 **f** 1024, 4096

**4** **a** 24 **b** 31 **c** 69
**d** 1 **e** 160 **f** 4

**5** **a** 19, 33 **b** 29, 5 **c** 6, 60
**d** 275, 175

**6** **a** 16, 256 **b** 2000, 250 **c** 5, 1215
**d** 81, 27

**7** **a**

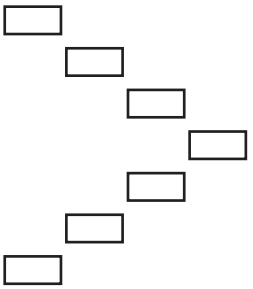


**b** Add a new row to the bottom containing one more shape than the previous bottom row.

**8** **a**

**b** Add a new row to the bottom containing three shapes.

**9** **a**



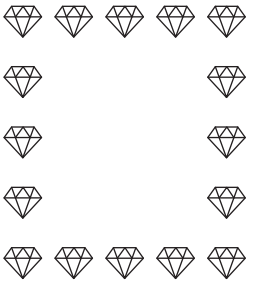
**b** Add one more shape to each end of the lines in the sequence.

**10** **a**

**b** Decrease the side length of the square by 1 shape.

**11** **a** 15 **b** 150 **c** 6
**d** 6561

**12** **a**



**b** 4, 8, 12, …
**c** Starting at 4, add 4 to get the next term
**d** 40
**e** 84

**13** 156 cm

**14** **a** 16 384 **b** 21 845

**15** Sample answer: Saina is correct. There are many different sequences that begin 4, 16, … For example, 'starting at 4, add 12 to get the next term' and 'starting at 4, multiply by 4 to get the next term'. Note: Different sequences are still possible if you starting with three terms.

**16** Sample answers:
**a** 53, 51, 49, 47 **b** 4
**c** 53 **d** 47
**e** Yes, patterns with the following starting numbers work: 101, 53, 44, 32, 29

## EX 2B Divisibility rules

p62

**1** **a** Divisible by 2 as the last digit (6) is even.
**b** Divisible by 3 as the sum of the digits (15) is divisible by 3.
**c** Not divisible by 4 as the number formed from the last two digits (46) is not divisible by 4.
**d** Not divisible by 5 as last digit is not 0 or 5.
**e** Divisible by 6 as it is divisible by 2 and 3.
**f** Not divisible by 8. Prove by division, or because it is not divisible by 4.
**g** Not divisible by 9 as sum of the digits is not divisible by 9.
**h** Not divisible by 10 as the last digit is not 0.

**2** **a** Divisible by 2 as the last digit (0) is even.
**b** Divisible by 3 as the sum of the digits (18) is divisible by 3.
**c** Divisible by 4 as 40 is divisible by 4.
**d** Divisible by 5 as the last digit is 0.
**e** Divisible by 6 as it is divisible by 2 and 3.
**f** Divisible by 8 as 640 is divisible by 8.
**g** Divisible by 9 as the sum of the digits is divisible by 9.
**h** Divisible by 10 as the last digit is 0.

**3** **a** Divisible by 2.
**b** Divisible by 3.
**c** Not divisible by 4.
**d** Divisible by 5.
**e** Divisible by 6.
**f** Not divisible by 8.
**g** Not divisible by 9.
**h** Divisible by 10.

**4** **a** Divisible by 2.
**b** Not divisible by 3.
**c** Divisible by 4.
**d** Not divisible by 5.
**e** Not divisible by 6.

**f** Divisible by 8.
**g** Not divisible by 9.
**h** Not divisible by 10.

**5** **a** No **b** Yes **c** Yes
**d** No **e** Yes **f** No

**6** **a** True **b** True **c** False
**d** False

**7** **a** 2 and 3 **b** 3

**8** Octopuses

**9** 9

**10** 6

**11** Sample answer: If 14 805 is divisible by 3 and 5, then it will also be divisible by 15.

**12** Sample answer: Scott is not correct. There are some numbers which are divisible by 4 and 6 but are not divisible by 24, for example 36.

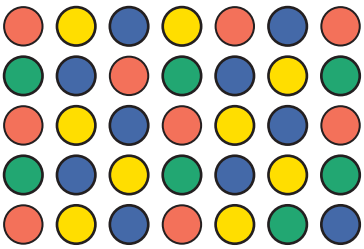
**13** Sample answer: If the sum of the all the digits in the number is divisible by 3 and the number made by the last two digits is divisible by 4, then the number will be divisible by 12. This is because any number which is divisible by both 3 and 4 will also be divisible by 12.

**14** **a** Yes
**b** Sample answer: If a number is divisible by 3, 4 and 5, then it will also be divisible by 60 (the number of minutes in an hour).

**15** **a** No **b** Yes

**16** 50

**17**



**18** The students that say the number is divisible by 16 and by 17.

## EX p66 2C Multiples and the lowest common multiple

**1** **a** 10, 20, 30, 40 **b** 6, 12, 18, 24
**c** 9, 18, 27, 36 **d** 5, 10, 15, 20
**e** 11, 22, 33, 44 **f** 30, 60, 90, 120
**g** 25, 50, 75, 100 **h** 13, 26, 39, 52
**i** 40, 80, 120, 160 **j** 22, 44, 66, 88
**k** 14, 28, 42, 56 **l** 31, 62, 93, 124

**2** 24, 32, 40, 48, 56, 64

**3** 56, 63, 70, 77, 84, 91, 98

**4** 20, 40, 60

**5** 12, 24, 36, 48, 60

**6** **a** Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60
Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
**b** 30, 60 **c** 30

**7** **a** 10 **b** 28 **c** 18
**d** 24 **e** 40 **f** 70
**g** 60 **h** 24

**8** **a** 12 **b** 60 **c** 36
**d** 24 **e** 30 **f** 90
**g** 168 **h** 60

**9** **a** 24 **b** 28 **c** 80
**d** 132 **e** 144 **f** 24
**g** 80 **h** 350

**10** **a** 88 **b** 112
**c** 216 and 224 **d** 296 and 304

**11** **a** 56 **b** 126 **c** 210
**d** 336

**12** **a** 135 **b** 225 **c** 315
**d** 450

**13** 99

**14** August 20

**15** **a** 21 **b** 11

**16** **a** 5:12 am, 5:24 am, 5:36 am, 5:48 am, 6:00 am, 6:12 am
**b** **i** 5:36 am **ii** 6:12 am
**c** 17th train. Sample answer: I noticed that 1 train arrives every 12 minutes, so 5 trains arrive every hour.

**17** 13 packs of hats and 8 packs of water bottles

**18** **a** 105 blocks **b** 15 **c** 35
**d** She runs 55 blocks and flies 50 blocks.

**19** Sample answer: No, as the list of common multiples can extend indefinitely.

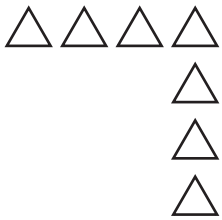
**20** **a** Sample answer: 63, 64, 65. The first set is 3, 4 and 5. To find the next, add the LCM (60) giving 63, 64, 65, then 123, 124, 125 etc.
**b** Sample answer: 3030, 3031, 3032, 3033. As before, the first set is 6, 7, 8 and 9. To find the next, add the LCM (504) giving 510, 511, 512, 513, then 1014, 1015, 1016, 1017, etc.

## 2 Checkpoint

**1** **a** Starting at 5, add 2 to get the next term
**b** Starting at 12, add 12 to get the next term
**c** Starting at 3, multiply by 2 to get the next term

**2** **a** 9, 36, 144 **b** 280, 260, 240
**c** 36, 48, 60 **d** 2000, 400, 80

**3** **a**



**b** Starting with one triangle, add two triangles (one on each arm) to get the next term.
**c** 9

**4** 25

5 a Yes b Yes c Yes
d No e Yes f Yes
g Yes h No

6 a No b No c Yes

7 a True b False c True

8 The boxes that hold 9 books

9 a 7, 14, 21, 28 b 12, 24, 36, 48
c 15, 30, 45, 60 d 24, 48, 72, 96

10 a 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
b 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
c 18, 36, 54, 72, 90
d 18

11 a 15 b 24 c 84

12 a 20 b Sunday

## EX p71 2D Factors and the highest common factor

1 a $20 = 1 \times 20$ or $4 \times 5$
b $24 = 1 \times 24$ or $2 \times 12$ or $3 \times 8$
c $32 = 1 \times 32$ or $2 \times 16$ or $4 \times 8$
d $28 = 1 \times 28$ or $2 \times 14$ or $4 \times 7$
e $49 = 1 \times 49$ or $7 \times 7$
f $50 = 1 \times 50$ or $2 \times 25$ or $5 \times 10$

2 a 1, 3, 5, 15 b 1, 2, 3, 6, 9, 18
c 1, 3, 9 d 1, 2, 3, 4, 6, 12
e 1, 2, 5, 10 f 1, 2, 7, 14

3 a 1, 2, 4, 5, 10, 20
b 1, 2, 3, 4, 6, 8, 12, 24
c 1, 2, 4, 8, 16, 32
d 1, 2, 4, 7, 14, 28
e 1, 7, 49
f 1, 2, 5, 10, 25, 50
g 1, 2, 3, 6
h 1, 2, 3, 6, 11, 22, 33, 66
i 1, 2, 4, 7, 8, 14, 28, 56
j 1, 3, 9, 27, 81
k 1, 3, 9, 27
l 1, 2, 4, 5, 10, 20, 25, 50, 100

4 a 1, 2, 4, 11, 22, 44
b 1, 2, 3, 6, 9, 18, 27, 54
c 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
d 1, 2, 4, 8, 16, 32, 64
e 1, 3, 5, 15, 25, 75
f 1, 3, 5, 7, 15, 21, 35, 105
g 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
h 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132
i 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
j 1, 2, 7, 14, 49, 98
k 1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168
l 1, 3, 7, 11, 21, 33, 77, 231

5 a 1, 2 b 1, 3, 9, 27 c 1, 2, 5, 10, 25, 50
d 1, 2, 4 e 1, 3 f 1, 2, 4, 8
g 1, 3 h 1, 2, 4 i 1, 7

6 a 13, 26 b 11 c 14, 21
d 21 e 14 f 12, 18

7 a Factors of 18: 1, 2, 3, 6, 9, 18
b Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
c 1, 2, 3, 6, 9, 18
d 18

8 a Factors of 21: 1, 3, 7, 21
b Factors of 45: 1, 3, 5, 9, 15, 45
c 1, 3
d 3

9 a Factors of 12: 1, 2, 3, 4, 6, 12
b Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
c 1, 2, 3, 6
d 6

10 a 8 b 3 c 6
d 5 e 4 f 15
g 7 h 50

11 a 4 b 5 c 8
d 3 e 6 f 10
g 2 h 3

12 a LCM = 60, HCF = 6
b LCM = 70, HCF = 5
c LCM = 48, HCF = 4
d LCM = 18, HCF = 1

13 3

14 a One possible answer is: three rows of six pieces.
b Six ways are possible: one row of 18 pieces, two rows of 9 pieces, three rows of 6 pieces, six rows of 3 pieces, nine rows of 2 pieces, 18 rows of 1 piece.

15 6

16 Sample answers include: four bags of 30 cookies, twelve bags of 10 cookies, six bags of 20 cookies.

17 Sample answer: A multiple of a number is the answer obtained when that number is multiplied by another whole number. A factor of a number is a number that divides exactly into the given number.

18 a 9 b 1
c Sample answer: Every lowest common factor is 1 since 1 is a factor of every number.

19 a 8
b 7 purple balloons, 8 green balloons, 13 blue balloons
c 28 balloons

20 a 1 b 2 c 4
d 6 e 16 f 12

21 80 and 16

22 12, 18, 20, 24, 30, 36, 40, 42, 48, 54, 56 and 60

## EX p76 2E Prime and composite numbers

**1 a**

| Number | Factors |
|---|---|
| 2 | 1, 2 |
| 3 | 1, 3 |
| 5 | 1, 5 |
| 7 | 1, 7 |
| 11 | 1, 11 |
| 13 | 1, 13 |
| 17 | 1, 17 |
| 19 | 1, 19 |

**b** 2

**2 a, b**

| Number | Factors | Number of factors |
|---|---|---|
| 4 | 1, 2, 4 | 3 |
| 6 | 1, 2, 3, 6 | 4 |
| 8 | 1, 2, 4, 8 | 4 |
| 9 | 1, 3, 9 | 3 |
| 10 | 1, 2, 5, 10 | 4 |
| 12 | 1, 2, 3, 4, 6, 12 | 6 |
| 14 | 1, 2, 7, 14 | 4 |
| 15 | 1, 3, 5, 15 | 4 |
| 16 | 1, 2, 4, 8, 16 | 5 |
| 18 | 1, 2, 3, 6, 9, 18 | 6 |
| 20 | 1, 2, 4, 5, 10, 20 | 6 |

**c** More than prime numbers

**3** **a** 1, 2, 3, 4, 6, 8, 12, 24 **b** 1, 7, 49
**c** 1, 73 **d** 1, 5, 25, 125
**e** 1, 37 **f** 1, 2, 41, 82

**4** 73 and 37 are prime numbers. The others are composite numbers.

**5** **a** Composite; even number
**b** Composite; even number
**c** Composite; divisible by 3
**d** Prime; only two factors
**e** Composite; even number
**f** Prime; only two factors
**g** Composite; even number
**h** Prime; only two factors
**i** Composite; divisible by 7
**j** Prime; only two factors
**k** Prime; only two factors
**l** Composite; divisible by 3

**6** **a** 2 **b** 2, 5 **c** 2, 3
**d** 5, 7 **e** 2 **f** 2, 5
**g** 3 **h** 2, 5 **i** 2, 3, 7
**j** 2, 7 **k** 3, 11 **l** 2

**7** Sample answer: Prime numbers have only two factors: 1 and the number itself. Composite numbers have more than two factors.

**8** Sample answers:
**a** 7 + 17 = 24 **b** 13 + 23 = 36
**c** 71 + 11 = 82 **d** 61 + 83 = 144

**9** **a** 3 × 7 = 21 **b** 5 × 11 = 55
**c** 2 × 13 = 26 **d** 5 × 23 = 115

**10** C and E

**11** **a** 11, 13, 17, 19, 23, 29
**b** They are all odd numbers.
**c** All prime numbers are odd except for 2.

**12** Sample answer: The number 1 has only one factor so it is not one of the prime numbers (which have two factors) and it is not one of the composite numbers (which have more than two factors).

**13** **a** False, as 4 is an even number and it is not prime.
**b** False, as 9 is odd but is a composite number (not prime).
**c** True, as each number has only two factors.
**d** False, as 9 is a composite number but is odd.
**e** False, as 3 × 5 = 15 is composite and odd.
**f** True, as a composite number has more than two factors and therefore if two prime numbers are multiplied, the result is a composite number.
**g** False, being even is not the reason 0 is neither prime nor composite.
**h** True, as there are fewer prime numbers than composites.

**14** **a** 1 by 30, 2 by 15, 3 by 10, and 5 by 6.
**b** Sample answer: They are prime numbers so can only be arranged into one long row to be a rectangle. The dimensions would be 1 by 29 and 1 by 31.
**c** 1 by 1 by 36, 1 by 2 by 18, 1 by 3 by 12, 1 by 4 by 9, 1 by 6 by 6, 2 by 2 by 9, 2 by 3 by 6, 3 by 3 by 4.

**15** **a** 5 and 7, 11 and 13, 17 and 19
**b** Four: 29 and 31, 41 and 43, 59 and 61, 71 and 73

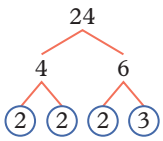
**16** 997

**17** Sample answer: None, as per the divisibility rules 1 + 3 + 5 = 9, so any arrangement of the number will be divisible by 3 and 9.

**18** 14, these are the perfect squares.

## EX p81 2F Prime factorisation

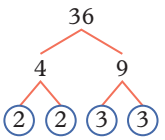
**1 a**




Prime factors: 2, 3

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

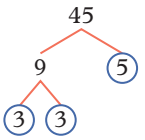
**b**




Prime factors: 2, 3

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

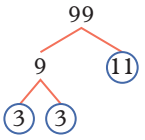
**c**




Prime factors: 3, 5

$45 = 3 \times 3 \times 5 = 3^2 \times 5$

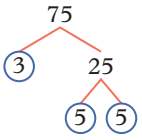
**d**




Prime factors: 3, 11

$99 = 3 \times 3 \times 11 = 3^2 \times 11$

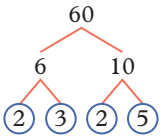
**e**




Prime factors: 3, 5

$75 = 3 \times 5 \times 5 = 3 \times 5^2$

**f**



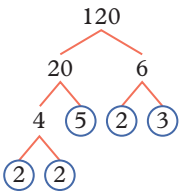

Prime factors: 2, 3, 5

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

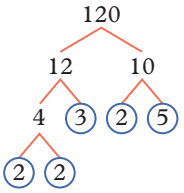
**2** **a** $2 \times 13$ **b** $2^5$ **c** $2 \times 5^2$
**d** $3^2 \times 7$ **e** $2 \times 3 \times 13$ **f** $2 \times 29$
**g** $2^2 \times 3 \times 7$ **h** $2^2 \times 23$ **i** $2^4 \times 3^2$

**3** **a** $2^4 \times 3$ **b** $2^5 \times 3$ **c** $3^3 \times 5$
**d** $2 \times 7 \times 13$ **e** $2^3 \times 3 \times 7$ **f** $3 \times 5 \times 19$
**g** $2^2 \times 61$ **h** $2^5 \times 13$ **i** $2 \times 3^2 \times 5^2$

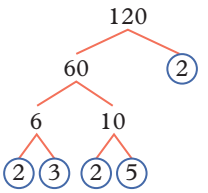
**4** **a** **i** Sample answer:




**ii**




**iii**




**b** $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$. The initial dividing factors do not change the prime factorisation; it will always be the same.

**5** **a**

| | | or | | |
|---|---|---|---|---|
| 2 | 80 | | 2 | 80 |
| 2 | 40 | | 2 | 40 |
| 2 | 20 | | 2 | 20 |
| 2 | 10 | | 5 | 10 |
| 5 | 5 | | 2 | 2 |
| | 1 | | | 1 |

Prime factors: 2, 5

$80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$

**b**

| | | or | | |
|---|---|---|---|---|
| 5 | 225 | | 5 | 225 |
| 5 | 45 | | 3 | 45 |
| 3 | 9 | | 3 | 15 |
| 3 | 3 | | 5 | 5 |
| | 1 | | | 1 |

Prime factors: 3, 5

$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$

**c**

| | | or | | |
|---|---|---|---|---|
| 2 | 570 | | 2 | 570 |
| 5 | 285 | | 3 | 285 |
| 3 | 57 | | 5 | 95 |
| 19 | 19 | | 19 | 19 |
| | 1 | | | 1 |

Prime factors: 2, 3, 5, 19

$570 = 2 \times 3 \times 5 \times 19$

**6** **a** $2^2 \times 7$ **b** $2^3 \times 7$ **c** $2^2 \times 11$
**d** $5 \times 17$ e $2^2 \times 3 \times 11$ **f** $2^2 \times 13$
**g** $2 \times 5^3$ **h** $2 \times 3^2 \times 5$ **i** $2^3 \times 3^2$
**j** $2^2 \times 5 \times 11$ **k** $2^2 \times 5^2$ **l** $2^4 \times 5^2$

**7** **a** $2^2 \times 3 \times 19$ **b** $2^4 \times 5 \times 7$ **c** $2^2 \times 3^4$
**d** $5 \times 13^2$ **e** $7^3$ **f** $2^9$
**g** $2 \times 7 \times 11$ **h** $2^6 \times 3 \times 5$ **i** $2^2 \times 5 \times 37$
**j** $2^2 \times 3 \times 5 \times 19$ **k** $2 \times 3 \times 7 \times 19$
**l** $2^2 \times 3 \times 11 \times 13$

**8** **a** Yes, it is in the expanded form.
**b** No, as there is only one prime factorisation for each number. The first option is the correct factorisation for 36.
**c** No, as $2 \times 17$ doesn't equal 19. 19 is a prime number, so it doesn't have a factorisation.
**d** No, as 4 is not a prime factor.

**9** **a** $2 \times 3 \times 5 \times 7 = 210$

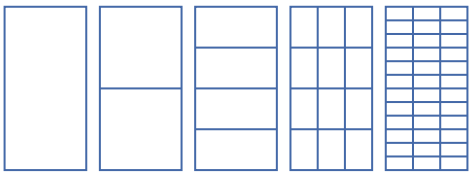
**b** $2^5 = 32$

**c** $2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30\,030$

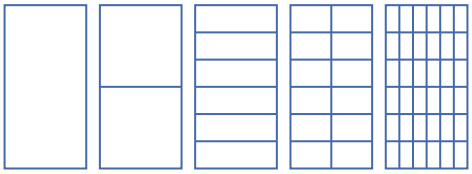
**10** The HCF = 30, so the tickets should be 30 cm by 30 cm.

**11** **a** $36 = 2 \times 2 \times 3 \times 3$. Sample answer: This means the cake can be cut in half to produce two pieces, then each piece is cut in half again, then each of these pieces is cut into equal thirds and then each of these pieces into equal thirds again.

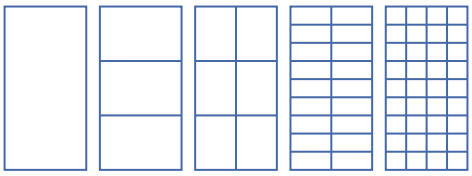
**b** Sample answer:



**c** Sample answer:



or



**12** **a** $100 = 2^2 \times 5^2$

**b** $180 = 2^2 \times 3^2 \times 5$

**c** $1000 = 2^3 \times 5^3$

**13** **a** $315 = 3^2 \times 5 \times 7$

**b** $441 = 3^2 \times 7^2$

**c** $1575 = 3^2 \times 5^2 \times 7$

**14** **a** 2 **b** 5 **c** 8 **d** 12

**15** **a** 15 **b** 7

**16** 26

**17** **a** 420 **b** 315 **c** 4800 **d** 672

**18** 714

**19** 2:30 pm

**20**

| | | | |
|---|---|---|---|
| | 1 | 5 | 7 |
| × | | 2 | 8 |
| 4 | 3 | 9 | 6 |

| | | | |
|---|---|---|---|
| | 2 | 9 | 7 |
| × | | 1 | 8 |
| 5 | 3 | 4 | 6 |

| | | | |
|---|---|---|---|
| | 1 | 3 | 8 |
| × | | 4 | 2 |
| 5 | 7 | 9 | 6 |

| | | | |
|---|---|---|---|
| | 1 | 9 | 8 |
| × | | 2 | 7 |
| 5 | 3 | 4 | 6 |

| | | | |
|---|---|---|---|
| | 4 | 8 | 3 |
| × | | 1 | 2 |
| 5 | 7 | 9 | 6 |

# CHAPTER 2 review

## Multiple-choice

| | | |
|---|---|---|
| **1** A | **2** D | **3** E |
| **4** A | **5** E | **6** B |
| **7** A | **8** B | **9** B |
| **10** B | **11** C | **12** C |

## Short answer

**1** **a** 22, 30 **b** 70 **c** 5, 160

**d** 22, 11

**2** 640

**3** **a** Yes **b** Yes **c** Yes

**d** Yes

**4** **a** 3, 5 and 7 **b** 7

**5** 24, 28, 32, 36, 40, 44

**6** 24, 48, 72, 96

**7** **a** 36 **b** 90

**8** 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

**9** Sample answer: 2 rows of 12, 3 rows of 8, 4 rows of 6

**10** **a** 6 **b** 3

**11** **a** 7, 14, 21 **b** 6, 8, 10 **c** 4, 9, 16

**12** **a** 3 and 11 **b** 2 and 19

**c** 3 and 23, 7 and 19

**d** 7 and 23, 11 and 19, 13 and 17

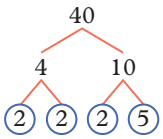
**13** **a** 3 and 5 **b** 3 and 11

**c** 2 and 17 **d** 2 and 29

**14** **a** Composite, as it has more than two factors

**b** Its prime factors are 2 and 5.

Sample answer




**c** $40 = 2^3 \times 5$

**15** **a** 12 **b** 12 **c** 7

## Analysis

**a** 448 **b** 8 **c** 56

**d** 5 **e** 3 **f** Page 30

**g** No pages

# CHAPTER 3 Fractions and ratios

## EX 3A Fractions

p92

**1** **a** $\frac{1}{3}$ **b** $\frac{4}{7}$ **c** $\frac{1}{4}$

**d** $\frac{1}{2}$ **e** $\frac{1}{3}$ **f** $\frac{3}{8}$

**g** $\frac{5}{8}$ **h** $\frac{3}{5}$ **i** $\frac{9}{16}$

**j** $\frac{9}{16}$ **k** $\frac{2}{4} = \frac{1}{2}$ **l** $\frac{7}{17}$

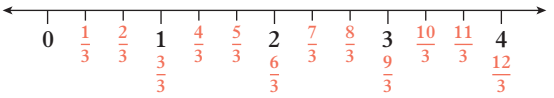
**2** **a** $\frac{2}{3}$ **b** $\frac{3}{7}$ **c** $\frac{3}{4}$
**d** $\frac{1}{2}$ **e** $\frac{2}{3}$ **f** $\frac{5}{8}$
**g** $\frac{3}{8}$ **h** $\frac{2}{5}$ **i** $\frac{7}{16}$
**j** $\frac{7}{16}$ **k** $\frac{2}{4} = \frac{1}{2}$ **l** $\frac{10}{17}$

**3** B. The rectangle is divided into three equal sections and one of the sections is shaded. The other two rectangles are not divided into equal sections.

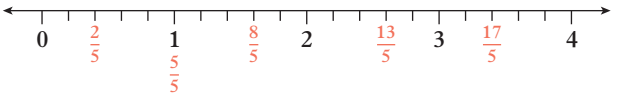
**4** **a** $\frac{7}{8}, \frac{16}{31}, \frac{13}{22}$ **b** $\frac{5}{2}, \frac{17}{2}, \frac{29}{15}$ **c** $2\frac{3}{5}, 1\frac{5}{9}, 12\frac{1}{7}$

**5** **a** **i** $\frac{5}{4}$ **ii** $1\frac{1}{4}$
**b** **i** $\frac{11}{4}$ **ii** $2\frac{3}{4}$
**c** **i** $\frac{4}{3}$ **ii** $1\frac{1}{3}$
**d** **i** $\frac{12}{5}$ **ii** $2\frac{2}{5}$
**e** **i** $\frac{11}{2}$ **ii** $5\frac{1}{2}$
**f** **i** $\frac{12}{4}$ **ii** 3
**g** **i** $\frac{19}{10}$ **ii** $1\frac{9}{10}$
**h** **i** $\frac{12}{3}$ **ii** 4

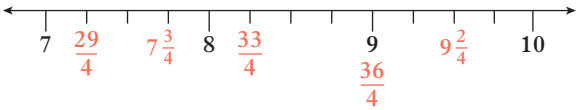
**6** **a** $\frac{1}{3}$
**b**




**7**



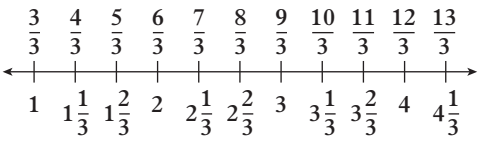

**8**




**9** **a** $6\frac{3}{7}$ or $\frac{45}{7}$ **b** $7\frac{6}{7}$ or $\frac{55}{7}$ **c** $8\frac{4}{7}$ or $\frac{60}{7}$
**d** $9\frac{2}{7}$ or $\frac{65}{7}$

**10** **a** $\frac{12}{23}$ **b** $\frac{7}{12}$ **c** $\frac{23}{50}$
**d** $\frac{8}{41}$ **e** $\frac{17}{22}$ **f** $\frac{21}{8}$

**11** **a**




**b** **i** 2 **ii** $\frac{10}{3}$ **iii** $3\frac{1}{3}$

**12** **a** Blue: $\frac{1}{7}$, red: $\frac{6}{7}$
**b** Blue: $\frac{4}{10} = \frac{2}{5}$, red: $\frac{6}{10} = \frac{3}{5}$
**c** Blue: $\frac{3}{12} = \frac{1}{4}$, yellow: $\frac{3}{12} = \frac{1}{4}$, red: $\frac{6}{12} = \frac{1}{2}$
**d** Blue: $\frac{4}{9}$, red: $\frac{2}{9}$, yellow: $\frac{3}{9} = \frac{1}{3}$

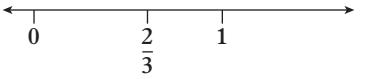
**13** Sample answer: The numerator in a proper fraction is less than the denominator and the fraction is less than 1. The numerator in an improper fraction is greater than or equal to the denominator and the fraction is greater than or equal to 1.

**14** Sample answer: Improper fractions are a fraction only. Mixed numbers are a combination of a whole number and a fraction.

**15** **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{17}{60}$
**d** $\frac{31}{60}$ **e** $\frac{119}{60}$ **f** $5\frac{11}{60}$

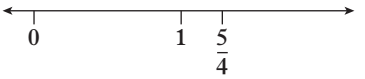
**16** Sample answer: There are 8 rows and 8 columns, so there are 64 boxes. If every second box is shaded, we know that $\frac{1}{2}$ are shaded because there is an even number of boxes.

**17** **a**



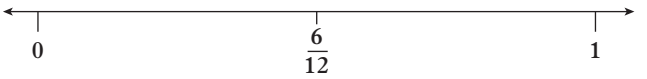

The space between $\frac{2}{3}$ and 1 should be half the size of the space between 0 and $\frac{2}{3}$.

**b**

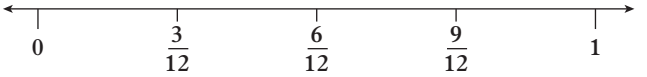



The space between 0 and 1 should be four times the size of the space between 1 and $\frac{5}{4}$.

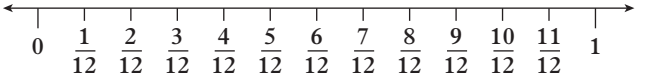
**18** **a**



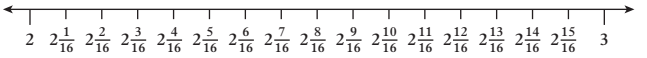

**b**




**c**




**d**




**e** Sample answer: Divide a number line between 0 and 1 into thirds, then divide each third into five equal parts each representing one fifteenth.

**19** $\frac{8}{9}$

**20** **a** $\frac{1}{16}$ **b** $\frac{2}{16}$ or $\frac{1}{8}$ **c** $\frac{4}{16}$ or $\frac{1}{4}$

## EX 3B Equivalent fractions

p99

**1** **a** $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ **b** $\frac{6}{8}, \frac{9}{12}$ **c** $\frac{1}{4}, \frac{2}{8}$
**d** $\frac{4}{10}$ **e** $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}$
**f** $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{8}{8}, \frac{9}{9}, \frac{10}{10}, \frac{11}{11}, \frac{12}{12}$

**2** **a** $\frac{4}{9} = \frac{16}{36}$ **b** $\frac{24}{8} = \frac{12}{4}$ **c** $2\frac{5}{11} = 2\frac{15}{33}$
**d** $\frac{42}{70} = \frac{6}{10}$ **e** $\frac{144}{48} = \frac{36}{12}$ **f** $3\frac{2}{8} = 3\frac{18}{72}$
**g** $\frac{2}{3} = \frac{6}{9} = \frac{60}{90}$ **h** $\frac{5}{6} = \frac{15}{18} = \frac{20}{24}$ **i** $\frac{2}{5} = \frac{10}{25} = \frac{6}{15}$

**3** **a** $5\frac{1}{13} = 5\frac{3}{39} = 5\frac{5}{65}$

**b** $\frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{8}{80}$

**c** $\frac{8}{7} = \frac{24}{21} = \frac{40}{35} = \frac{88}{77}$

**d** $3\frac{3}{5} = 3\frac{18}{30} = 3\frac{9}{15} = 3\frac{42}{70}$

**e** $\frac{1}{12} = \frac{6}{72} = \frac{8}{96} = \frac{12}{144} = \frac{15}{180} = \frac{20}{240}$

**f** $\frac{2}{3} = \frac{4}{6} = \frac{10}{15} = \frac{20}{30} = \frac{50}{75} = \frac{400}{600}$

**4** **a** $\frac{5}{8}$ **b** $\frac{7}{9}$ **c** $\frac{9}{8}$

**d** $2\frac{2}{5}$ **e** $5\frac{11}{12}$ **f** $6\frac{1}{5}$

**5** **a** $\frac{1}{3}$ **b** $\frac{5}{8}$ **c** $\frac{58}{47}$

**d** $7\frac{4}{9}$ **e** $4\frac{7}{8}$ **f** $3\frac{8}{27}$

**6** $\frac{1}{3}, \frac{6}{18}, \frac{16}{48}, \frac{18}{54}, \frac{20}{60}, \frac{52}{156}$

**7** **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{1}{4}$

**d** $\frac{2}{5}$

**8** **a** **i** $\frac{5}{2}$ **ii** $2\frac{1}{2}$

**b** **i** $\frac{11}{3}$ **ii** $3\frac{2}{3}$

**c** **i** $\frac{7}{4}$ **ii** $1\frac{3}{4}$

**9** $\frac{17}{30}$

**10** $\frac{11}{17}$

**11** $\frac{1}{6}$

**12** **a** 75 **b** 135 **c** 169

**13** $\frac{13}{15}$

**14** **a** $\frac{3}{4}$ **b** 157

**15** **a** 21 **b** 18 **c** 756

**16** **a** $\frac{81}{8}$ or $10\frac{1}{8}$ **b** $\frac{3}{5000}$

**c** $\frac{2}{21}$ **d** $\frac{14\,641}{210}$ or $69\frac{151}{210}$

**17** **a** $\frac{8}{3} = 2\frac{2}{3}$ **b** $\frac{4}{3} = 1\frac{1}{3}$ **c** $\frac{2}{3}$

**d** 8 **e** 4 **f** 2

## EX 3C Ordering fractions

p105

**1** **a** $\frac{2}{3}$ **b** $\frac{9}{12}$ **c** $\frac{1}{5}$

**d** $\frac{7}{8}$

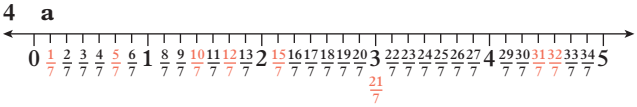
**2** **a** $\frac{5}{6} > \frac{3}{6}$ **b** $\frac{9}{5} > \frac{7}{5}$ **c** $\frac{93}{15} > \frac{32}{15}$

**d** $1\frac{7}{8} > 1\frac{3}{8}$ **e** $3\frac{11}{20} > 3\frac{9}{20}$ **f** $\frac{180}{4} > \frac{108}{4}$

**3** **a** $\frac{2}{3} > \frac{2}{4}$ **b** $\frac{16}{44} > \frac{16}{45}$ **c** $\frac{77}{12} > \frac{77}{100}$

**d** $2\frac{5}{6} > 2\frac{5}{7}$ **e** $4\frac{13}{15} > 4\frac{13}{20}$ **f** $\frac{55}{17} > \frac{55}{30}$

**4** **a**




**b** $\frac{1}{7} < \frac{5}{7} < \frac{10}{7} < \frac{12}{7} < \frac{15}{7} < \frac{21}{7} < \frac{31}{7} < \frac{32}{7}$

The numerators are in ascending order.

**5** **a** $\frac{2}{11} < \frac{5}{11} < \frac{8}{11} < \frac{15}{11}$

**b** $\frac{6}{7} < \frac{14}{7} < \frac{15}{7} < \frac{16}{7}$

**c** $\frac{10}{15} < \frac{10}{13} < \frac{10}{5} < \frac{10}{3}$

**d** $\frac{77}{101} < \frac{77}{89} < \frac{77}{70} < \frac{77}{59}$

**6** **a** $3\frac{2}{5}, 2\frac{1}{5}, 1\frac{4}{5}, 1\frac{2}{5}, \frac{4}{5}, \frac{1}{5}$

**b** $\frac{6}{2}, \frac{6}{3}, \frac{6}{4}, \frac{6}{5}, \frac{6}{6}, \frac{6}{8}$

**c** $\frac{100}{67}, \frac{76}{67}, \frac{67}{67}, \frac{33}{67}, \frac{33}{67}, \frac{1}{67}$

**d** $\frac{38}{2}, \frac{38}{19}, \frac{38}{37}, \frac{38}{40}, \frac{38}{59}, \frac{38}{60}$

**7** **a** $\frac{3}{6} > \frac{5}{12}$ **b** $\frac{7}{10} < \frac{30}{40}$ **c** $\frac{93}{15} < \frac{32}{5}$

**d** $\frac{43}{48} > \frac{7}{8}$ **e** $1\frac{9}{20} < 1\frac{55}{100}$ **f** $\frac{23}{2} < \frac{180}{14}$

**8** **a** $\frac{5}{6} < \frac{8}{9}$ **b** $\frac{10}{20} < \frac{11}{14}$ **c** $2\frac{15}{24} < 2\frac{14}{16}$

**d** $\frac{12}{7} < \frac{13}{5}$ **e** $\frac{6}{17} < \frac{5}{9}$ **f** $\frac{17}{16} > \frac{18}{17}$

**9** **a** $\frac{1}{3}, \frac{2}{5}, \frac{4}{3}, \frac{7}{5}, \frac{8}{3}, \frac{16}{5}$

**b** $\frac{2}{6}, \frac{5}{8}, \frac{9}{8}, \frac{7}{6}, \frac{5}{4}, \frac{11}{4}$

**c** $\frac{2}{9}, \frac{5}{6}, \frac{11}{9}, \frac{4}{3}, \frac{9}{6}, \frac{7}{3}$

**d** $\frac{9}{20}, \frac{8}{15}, \frac{11}{10}, \frac{5}{4}, \frac{10}{6}, \frac{6}{3}$

**10** **a** $\frac{5}{2}, \frac{12}{5}, \frac{10}{6}, \frac{8}{5}, \frac{3}{2}, \frac{7}{6}$

**b** $\frac{11}{12}, \frac{5}{6}, \frac{7}{12}, \frac{3}{8}, \frac{2}{6}, \frac{1}{8}$

**c** $\frac{3}{2}, \frac{10}{7}, \frac{9}{8}, \frac{7}{8}, \frac{6}{7}, \frac{3}{4}$

**d** $\frac{11}{7}, \frac{13}{9}, \frac{4}{3}, \frac{5}{7}, \frac{1}{3}, \frac{2}{9}$

**11** **a** $\frac{1}{3} > \frac{1}{4}$ **b** $\frac{2}{5} < \frac{5}{2}$ **c** $4\frac{5}{6} < \frac{23}{3}$

**d** $\frac{42}{5} > 7\frac{6}{7}$ **e** $\frac{7}{9} < \frac{6}{7}$ **f** $8\frac{5}{6} < \frac{49}{5}$

**12** **a** $\frac{12}{13} < \frac{13}{14}$ **b** $\frac{4}{5} > \frac{3}{4}$ **c** $\frac{1}{6} < \frac{2}{7}$

**d** $\frac{3}{9} > \frac{3}{10}$ **e** $\frac{4}{19} < \frac{4}{17}$ **f** $\frac{5}{16} > \frac{5}{26}$

**13** The second test

**14** Melanie; $\frac{24}{40}$ compared to $\frac{20}{35}$

**15** **a** $\frac{5}{6} < \frac{8}{9}$ **b** $\frac{17}{16} > \frac{18}{17}$ c $\frac{100}{101} > \frac{99}{100}$

**d** $\frac{48}{47} < \frac{47}{46}$

**16** **a** A larger denominator with common numerators indicates a smaller fraction as the five pieces are smaller. $\frac{5}{8} < \frac{5}{7}$

**b** The numerators cannot be directly compared if the denominators are not the same. $\frac{70}{34} > \frac{9}{10}$

**c** This is correct as 17 parts of size $\frac{1}{5}$ is less than 23 parts of size $\frac{1}{5}$.

**17** Tilly is correct as $\frac{25}{30}$ is greater than $\frac{29}{40}$.

**18** $\frac{17}{8}, \frac{10}{3}, \frac{15}{4}, \frac{25}{6}, \frac{21}{5}, \frac{11}{2}, \frac{60}{9}$

**19** $\frac{98}{99}, \frac{997}{999}, \frac{9996}{9999}, \frac{99\,995}{99\,999}, \frac{999\,994}{999\,999}$

## EX p109 3D Mixed numbers and improper fractions

**1** **a** $1\frac{2}{5}$ **b** $8\frac{2}{3}$ **c** $2\frac{7}{9}$
**d** $4\frac{3}{10}$ **e** $9\frac{3}{8}$ **f** $6\frac{10}{13}$
**g** $3\frac{1}{7}$ **h** $4\frac{1}{4}$ **i** $8\frac{1}{6}$
**j** $4\frac{6}{11}$ **k** $13\frac{2}{5}$ **l** $55\frac{1}{2}$

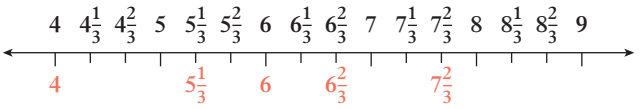
**2** **a** $\frac{11}{9}$ **b** $\frac{19}{6}$ **c** $\frac{53}{9}$
**d** $\frac{28}{11}$ **e** $\frac{54}{5}$ **f** $\frac{35}{4}$
**g** $\frac{17}{7}$ **h** $\frac{37}{3}$ **i** $\frac{37}{8}$
**j** $\frac{53}{10}$ **k** $\frac{85}{12}$ **l** $\frac{34}{15}$

**3** **a** True **b** False **c** False
**d** False **e** True

**4** **a** $\frac{5}{3} > \frac{3}{2}$ or $1\frac{2}{3} > 1\frac{1}{2}$
**b** $\frac{7}{3} < \frac{10}{4}$ or $2\frac{1}{3} < 2\frac{1}{2}$
**c** $\frac{10}{7} < \frac{15}{10}$ or $1\frac{3}{7} < 1\frac{1}{2}$
**d** $\frac{12}{5} > \frac{19}{8}$ or $2\frac{2}{5} > 2\frac{3}{8}$

**5** **a** $\frac{18}{7} > \frac{17}{7}$ **b** $\frac{26}{3} < \frac{28}{3}$ c $\frac{38}{11} > \frac{28}{11}$
**d** $\frac{49}{5} < \frac{51}{5}$

**6** **a, b**




**c** $\frac{12}{3}, 5\frac{2}{6}, \frac{24}{4}, \frac{20}{3}, 7\frac{4}{6} = \frac{23}{3}$

**7** **a** $9\frac{1}{3} > 3\frac{2}{5}$ **b** $8\frac{1}{6} < 9\frac{2}{4}$ **c** $6\frac{1}{2} > 2\frac{8}{9}$
**d** $5\frac{3}{5} < 6\frac{1}{10}$ **e** $6 < 7$ **f** $4\frac{3}{6} > 3\frac{3}{4}$
**g** $6\frac{2}{5} < 7\frac{7}{8}$ **h** $10\frac{1}{2} > 10\frac{1}{4}$

**8** **a** $1\frac{3}{5}$ **b** 32

**9** **a** $1\frac{1}{2}$ **b** 15

**10** **a** Added the whole number to the numerator without first multiplying it by the denominator.
$3\frac{1}{5} = \frac{3 \times 5 + 1}{5} = \frac{16}{5}$
**b** Miscalculated the division $14 \div 5$ as 3 remainder 1 (14 is one less than $3 \times 5 = 15$ not one more).
$\frac{14}{5} = 2\frac{4}{5}$
**c** Added the whole number multiplied by the numerator to the denominator. $6\frac{2}{3} = \frac{6 \times 3 + 2}{3} = \frac{20}{3}$
**d** Assumed that fractions always have a smaller numerator than denominator. $3\frac{4}{7} = \frac{25}{7}$

**11** 64 pieces

**12** **a** **i** $\frac{17}{5}$ **ii** $\frac{23}{17}$ **iii** $\frac{19}{2}$
**iv** $\frac{97}{10}$
**b** The calculation for the numerator in the worked example is the same as in the alternative method, only it is completed with more steps.
**c** **i** $2\frac{2}{3}$ **ii** $8\frac{1}{2}$ **iii** $1\frac{5}{6}$
**iv** $4\frac{7}{10}$
**d** The alternative method takes out the largest multiple of the denominator and considers the remainder as the numerator of the mixed fraction. This is the same as dividing by the denominator in the worked example method.

**13** **a** **i** $\frac{44}{8}$ **ii** $5\frac{4}{8}\left(\text{or } 5\frac{1}{2}\right)$
**b** No, because there will only be 40 slices.
**c** 9 slices **d** $\frac{1}{9}$ **e** 37 slices

**14** $2\frac{9}{38}$

**15** Sample answer: $\frac{11}{10} = 1\frac{1}{10}$

## 3 Checkpoint

**1** **a** $\frac{1}{4}$ **b** $\frac{15}{22}$ **c** $1\frac{5}{7}$ or $\frac{12}{7}$

**2** **a** $\frac{3}{4}$ and $2\frac{1}{4}$ **b** $11\frac{4}{5}$ and $13\frac{2}{5}$

**3** **a** $\frac{17}{20}$ **b** $\frac{51}{100}$ **c** $\frac{15}{16}$

**4** **a** $\frac{12}{5} = \frac{36}{15}$ **b** $\frac{20}{35} = \frac{4}{7}$ **c** $\frac{18}{27} = \frac{2}{3} = \frac{4}{6}$

**5** **a** $\frac{1}{2}$ **b** $\frac{8}{5}$ or $1\frac{3}{5}$ **c** $7\frac{5}{6}$

**6** **a** $\frac{1}{2}$ **b** $2\frac{3}{4}$ or $\frac{11}{4}$ **c** $1\frac{2}{3}$ or $\frac{5}{3}$

**7** **a** $\frac{87}{121} > \frac{78}{121}$ **b** $\frac{55}{8} > \frac{55}{9}$ **c** $\frac{3}{4} < \frac{5}{6}$

**8** **a** $\frac{1}{17}, \frac{9}{17}, \frac{20}{17}, 1\frac{5}{17}$ **b** $\frac{1}{6}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}$
**c** $5\frac{1}{8}, 5\frac{1}{6}, 5\frac{1}{4}, 5\frac{1}{3}$ **d** $\frac{19}{12}, \frac{15}{9}, \frac{17}{6}, \frac{13}{4}$

**9** **a** $\frac{13}{5}$ **b** $\frac{47}{7}$ **c** $\frac{103}{8}$

**10** **a** $6\frac{1}{3}$ **b** $8\frac{3}{8}$ **c** $8\frac{4}{7}$

## EX p116 3E Adding and subtracting fractions

**1** **a** $\frac{3}{5}$ **b** $\frac{6}{7}$ **c** $\frac{2}{3}$
**d** $\frac{11}{18}$ **e** $\frac{19}{37}$ **f** $1\frac{2}{23}$
**g** $2\frac{3}{11}$ **h** $\frac{11}{13}$ **i** $\frac{11}{25}$

**2** **a** $\frac{6}{7}$ **b** $\frac{17}{21}$ **c** $1\frac{1}{13}$
**d** $\frac{2}{3}$ **e** $\frac{1}{27}$ **f** $\frac{16}{33}$
**g** $1\frac{8}{17}$ **h** $\frac{1}{3}$ **i** $\frac{1}{2}$

**3** **a** $\frac{5}{6}$ **b** 1 **c** $\frac{1}{8}$
**d** $\frac{9}{16}$ **e** $\frac{14}{15}$ **f** $\frac{5}{6}$
**g** $\frac{25}{48}$ **h** $\frac{5}{26}$ **i** $\frac{50}{69}$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**4** **a** $1\frac{1}{6}$ **b** $\frac{7}{20}$ **c** $1\frac{5}{24}$

**d** $\frac{1}{18}$ **e** $1\frac{5}{12}$ **f** $\frac{2}{15}$

**g** $1\frac{1}{14}$ **h** $\frac{7}{30}$ **i** $\frac{11}{21}$

**5** **a** $2\frac{1}{3}$ **b** $2\frac{1}{20}$ **c** $2\frac{1}{8}$

**d** $\frac{17}{24}$ **e** $5\frac{17}{30}$ **f** $\frac{19}{40}$

**6** **a** **i** & **C**, **iii** & **A**, **iv** & **B**, **ii** and **D** do not match

**b** **i** $\frac{3}{20}$ **ii** $\frac{17}{20}$ **iii** $\frac{13}{20}$

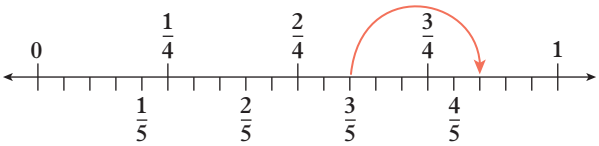
**iv** $\frac{7}{20}$

**c** $\frac{3}{4} - \frac{1}{5} = \frac{11}{20}$

**d**

| $\frac{1}{5}$ | | | | $\frac{1}{5}$ | | | | $\frac{1}{5}$ | | | | $\frac{1}{4}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ | $\frac{1}{20}$ |

or




**7** **a** $7\frac{3}{5}$ **b** $3\frac{2}{5}$ **c** $4\frac{2}{11}$

**d** $3\frac{2}{3}$ **e** $1\frac{2}{3}$ **f** $9\frac{2}{5}$

**g** $\frac{7}{12}$ **h** $3\frac{7}{8}$

**8** **a** $4\frac{1}{6}$ **b** $2\frac{19}{21}$ **c** $5\frac{13}{20}$

**d** $1\frac{13}{15}$ **e** $2\frac{8}{15}$ **f** $5\frac{5}{21}$

**g** $7\frac{1}{3}$ **h** $1\frac{7}{8}$

**9** **a** $8\frac{19}{24}$ **b** $\frac{2}{7}$ **c** $2\frac{67}{70}$

**d** $4\frac{1}{4}$

**10** **a** Dean **b** $\frac{1}{28}$ of an hour

**11** $10\frac{7}{15}$ minutes

**12** **a** 4 **b** $\frac{5}{8}$

**13** **a** $\frac{103}{120}$ **b** $\frac{17}{120}$ will go towards savings.

This is the difference between Laila's pay (1) and her combined expenses.

**14** $14\frac{7}{10}$ cm

**15** **a** $5\frac{4}{15}$ km **b** $1\frac{1}{15}$ km

**16** **a** $\frac{11}{7}, \frac{14}{7}$ (or 2), $\frac{17}{7}, \frac{20}{7}$; increases by $\frac{3}{7}$ each time.

**b** $14\frac{1}{5}, 15\frac{3}{5}, 17, 18\frac{2}{5}$; increases by $1\frac{2}{5}$ each time.

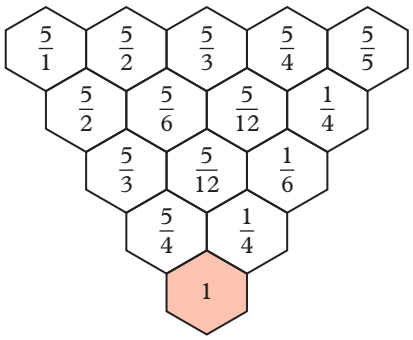
**c** $\frac{10}{3}, \frac{7}{3}, \frac{4}{3}, \frac{1}{3}$; decreases by $\frac{3}{3}$ (or 1) each time.

**d** $6\frac{2}{15}, 3\frac{13}{15}, 1\frac{9}{15}\left(\text{or } 1\frac{3}{5}\right)$; decreases by $2\frac{4}{15}$ each time.

**17** **a** $\frac{1}{2} - \left(\frac{5}{8} - \frac{2}{5}\right) = \frac{11}{40}$

**b** $\frac{19}{28} - \left(\frac{3}{7} + \frac{3}{14}\right) = \frac{1}{28}$

**18**




The number at the bottom will also be 1.

## EX 3F Multiplying fractions

p122

**1** **a** $\frac{6}{5}$ or $1\frac{1}{5}$ **b** $\frac{8}{9}$ **c** 1

**d** $\frac{22}{3}$ or $7\frac{1}{3}$

**2** **a** $\frac{3}{10}$ **b** $\frac{2}{21}$ **c** $\frac{22}{15}$

**d** $\frac{4}{27}$ **e** $\frac{21}{16}$ **f** $\frac{66}{91}$

**g** $\frac{7}{45}$ **h** $\frac{27}{70}$ **i** $\frac{81}{10}$

**3** **a** $\frac{2}{9}$ **b** $\frac{4}{11}$ **c** $\frac{6}{13}$

**d** $\frac{1}{34}$ **e** $\frac{4}{15}$ **f** $\frac{4}{9}$

**g** $\frac{9}{16}$ **h** $\frac{12}{49}$ **i** $\frac{4}{13}$

**4** **a** $\frac{9}{11}$ **b** $4\frac{5}{7}$ **c** $18\frac{1}{5}$

**d** 14 **e** $2\frac{8}{9}$ **f** $13\frac{3}{5}$

**g** $1\frac{21}{25}$ **h** $13\frac{3}{4}$ **i** $7\frac{19}{24}$

**5** **a** 9 **b** 30 **c** $3\frac{1}{5}$

**d** $3\frac{1}{5}$ **e** $\frac{3}{8}$ **f** $\frac{5}{6}$

**g** 40 **h** $\frac{4}{9}$

**6** **a** $\frac{1}{20}$ **b** $\frac{9}{28}$ **c** $\frac{14}{99}$

**d** $1\frac{1}{44}$ **e** 66 **f** $18\frac{1}{3}$

**7** **a** 45 minutes **b** 8 months **c** \$64

**d** 24 weeks **e** 48 seconds **f** 75 L

**g** 6 days **h** \$450 **i** $4\frac{1}{3}$ hours

**j** $15\frac{3}{4}$ km **k** 100 months **l** 160 cents

**8** **a** True **b** False

**9** **a** $\frac{6}{17}$ **b** $\frac{5}{26}$ **c** $35\frac{1}{5}$

**d** $10\frac{5}{6}$ **e** $36\frac{4}{5}$ **f** $63\frac{63}{80}$

**g** $58\frac{1}{2}$ **h** 45

**10** **a** 1 **b** $\frac{16}{15}$ or $1\frac{1}{15}$ **c** $\frac{1}{12}$

**d** $\frac{11}{6}$ or $1\frac{5}{6}$

**11** 100

**12 a** **i** $2\frac{1}{2}$ hours **ii** $\frac{5}{2}$ hours

**b** $12\frac{1}{2}$ hours

**13 a** $8\frac{1}{4}$

**b** $6\frac{12}{72}$ is incorrect; the whole numbers and fractions were multiplied separately rather than converting the mixed numbers to improper fractions.

$6\frac{12}{17}$ is incorrect; as well as multiplying the whole numbers, the denominators of the fractions were added.

**14** \$3792

**15** $2\frac{1}{16}$ km

**16 a** **i** $33 \div 3$ **ii** $40 \div 8$ **iii** $55 \div 101$

**iv** $\frac{4}{9} \div 7$

**b** **i** $\frac{1}{7} \times 56$ **ii** $\frac{1}{10} \times 90$ **iii** $\frac{1}{35} \times 92$

**iv** $\frac{1}{5} \times \frac{8}{11}$

**c** **i** $3 \times (16 \div 8) = 6$

**ii** $8 \times (35 \div 5) = 56$

**iii** $12 \times (63 \div 7) = 108$

**iv** $7 \times \left(\frac{18}{5} \div 6\right) = \frac{21}{5}$

**17 a** **i** $\frac{8}{15}$, less than both

**ii** $\frac{3}{7}$, between both

**iii** $\frac{33}{4}$, greater than both

**iv** $2\frac{13}{36}$, between both

**v** 2, between both

**vi** $\frac{1}{63}$, less than both

**vii** $\frac{100}{9}$ or $11\frac{1}{9}$, greater than both

**viii** $\frac{20}{3}$ or $6\frac{2}{3}$, greater than both

**b** When both are proper fractions (value between 0 and 1)

**c** When both are mixed numbers, whole numbers or improper fractions (value greater than 1)

**d** When one fraction is a proper fraction and the other is a mixed number or improper fraction (one value between 0 and 1, and the other value greater than 1)

**e** 1

**18** She use cancelling to remove most of the calculations.

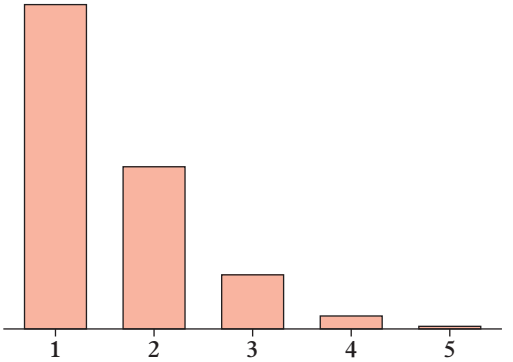
See: $\frac{1}{\cancel{2}^1} \times \frac{\cancel{2}^1}{\cancel{3}^1} \times \frac{\cancel{3}^1}{\cancel{4}^1} \times \frac{\cancel{4}^1}{\cancel{5}^1} \times \frac{\cancel{5}^1}{\cancel{6}^1} \times \frac{\cancel{6}^1}{\cancel{7}^1} \times \frac{\cancel{7}^1}{8} = \frac{1}{8}$

**19 a** $\frac{1}{120}$

**b**

| **Column** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Value** | 1 | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{24}$ | $\frac{1}{120}$ |

**c**




**d** Smaller

**e** Zero

## EX 3G Dividing fractions

p128

**1 a** How many $\frac{1}{8}$s are there in 2?

**b** How many $\frac{1}{3}$s are there in 4?

**c** How many $\frac{1}{12}$s are there in $\frac{1}{2}$?

**d** How many $\frac{1}{9}$s are there in $\frac{1}{3}$?

**2 a** $\frac{2}{3} \div \frac{1}{9} = 6$ **b** $\frac{3}{10} \div \frac{1}{5} = 1\frac{1}{2}$

**c** $\frac{9}{10} \div \frac{2}{5} = 2\frac{1}{4}$ **d** $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$

**e** $1 \div \frac{3}{8} = 2\frac{2}{3}$ **f** $\frac{4}{6} \div \frac{2}{9} = 3$

**3 a** $\frac{31}{65}$ **b** $\frac{3}{11}$ **c** $\frac{5}{2}$ or $2\frac{1}{2}$

**d** $\frac{4}{27}$ **e** 100 **f** $\frac{1}{8}$

**g** $\frac{1}{34}$ **h** $\frac{31}{65}$ **i** $\frac{7}{30}$

**j** $\frac{8}{73}$

**4 a** $\frac{4}{7} \times \frac{9}{5}$ **b** $\frac{8}{3} \times \frac{9}{1}$ **c** $\frac{3}{5} \times \frac{1}{4}$

**d** $\frac{6}{1} \times \frac{10}{3}$ **e** $9\frac{3}{8} \times \frac{2}{3}$ **f** $\frac{8}{3} \times \frac{10}{67}$

**g** $5\frac{1}{5} \times \frac{9}{67}$ or $\frac{26}{5} \times \frac{9}{67}$ **h** $\frac{3}{1} \times \frac{8}{19}$

**i** $4\frac{5}{8} \times \frac{1}{10}$ or $\frac{37}{8} \times \frac{1}{10}$ **j** $21 \times \frac{1}{32}$

**5 a** $\frac{15}{16}$ **b** $\frac{11}{12}$ **c** $2\frac{1}{2}$

**d** $\frac{17}{36}$ **e** $\frac{5}{8}$ **f** $2\frac{1}{4}$

**g** $\frac{4}{9}$ **h** $7\frac{1}{3}$ **i** $\frac{3}{65}$

**j** $1\frac{2}{33}$

**6 a** $\frac{1}{27}$ **b** $\frac{1}{12}$ **c** $\frac{1}{4}$

**d** $\frac{4}{9}$ **e** $2\frac{6}{7}$ **f** 42

**g** 12 **h** $13\frac{1}{2}$ **i** $6\frac{1}{2}$

**j** $\frac{30}{7}$

**7 a** $4\frac{3}{8}$ **b** $\frac{22}{35}$ **c** $5\frac{1}{15}$

**d** $\frac{54}{55}$ **e** $1\frac{4}{5}$ **f** $12\frac{1}{2}$

**g** $1\frac{1}{4}$ **h** 2 **i** $3\frac{1}{3}$

**j** $\frac{7}{15}$ **k** $\frac{10}{11}$ **l** $\frac{2}{5}$

**8** 6

**9** 8

**10** The reciprocal of the reciprocal of a number is the original number.

**11** $\frac{3}{8}$

**12** 72

**13** **a** 11 bags **b** $\frac{1}{3}$ kg

**14** **a** $\frac{4}{3}$ **b** 27 customers

**c** 45 customers **d** $\frac{4}{5}$

**15** **a** 40 T-shirts **b** 11 backpacks

**16** **a** False **b** False **c** True

**d** False

## EX 3H Ratios

p132

**1** **a** **i** 4 : 5 **ii** 4 : 9

**b** **i** 3 : 5 **ii** 3 : 8

**c** **i** 7 : 8 **ii** 7 : 15

**d** **i** 5 : 3 **ii** 5 : 8

**2** **a** 9 : 7 **b** 2 : 57 **c** 2 : 14 : 1

**d** 19 : 6

**3** **a** 44 : 69 **b** 16 : 35 **c** 18 : 11

**d** 85 : 121

**4** **a** 5 : 1 **b** 3 : 4 : 9 **c** 28 : 23

**d** 1 : 4

**5** **a** 7 : 2 **b** 2 : 3 **c** 13 : 16

**d** 3 : 8 **e** 11 : 6 **f** 8 : 9

**g** 65 : 13 **h** 1 : 6

**6** **a** 3 : 1 **b** 1 : 1 **c** 1 : 3 : 3

**7** **a** 3 : 2, 6 : 4, 15 : 10 **b** 4 : 5, 20 : 25, 16 : 20 **c** 30 : 210, 10 : 70, 2 : 14

**d** 144 : 54, 24 : 9, 8 : 3 **e** 165 : 121, 150 : 110, 15 : 11 **f** 147 : 189, 49 : 63, 7 : 9

**8** **a** 3 : 5 **b** 5 : 1 **c** 1 : 6

**d** 7 : 3 **e** 5 : 36 **f** 67 : 120

**g** 11 : 42 **h** 31 : 9

**9** **a** 15, 18 **b** 21, 14 **c** 20, 50

**d** 88, 44 **e** 63, 56 **f** 26, 169

**g** 2048, 256 **h** 555, 999

**10** **a** 87 : 100 **b** 13 : 100 **c** 87 : 13

**11** **a** 3 : 2 **b** 2 : 5 **c** 3 : 5

**d** Parts **b** $\left(\frac{2}{5}\right)$ and **c** $\left(\frac{3}{5}\right)$ can be written as fractions as they are expressed as part of the whole.

**e** Part **a** cannot be written as a fraction as it is comparing shaded to non-shaded sections.

**f** True

**12** **a** 12 **b** 4 : 3

**13** Jesse: \$16 million, James: \$10 million

**14** **a** 2 : 1 : 3 **b** $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$

**15** **a** 4 : 9 **b** 9 L **c** $\frac{4}{9}$

**d** $4\frac{1}{2}$ L

**16** 16 : 9

**17** **a** 1945

**b** **i** 54 : 61 **ii** 3 : 4

**c** 389 : 440

**18** **a** Yes, since the ratios can be expressed in the same simplest form.

**b** Yes, since the ratios can be expressed in the same simplest form.

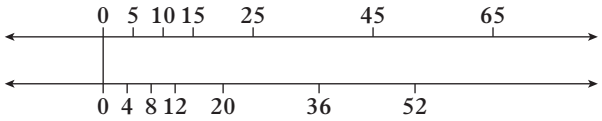
**c** No, since the ratios cannot be expressed in the same simplest form.

**d** No, since the ratios cannot be expressed in the same simplest form.

**19** $3\frac{1}{8}$ cups of self-raising flour and $2\frac{1}{2}$ cups of Greek yoghurt

**20** 450 mL

**21** **a**




| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 5 | 10 | 15 | 25 | 45 | 65 |
| 0 | 4 | 8 | 12 | 20 | 36 | 52 |

**b** Sample answer: We are adding multiples of the number to itself which is the same as multiplying by a whole number. They are the same ratio because we are adding the same number of multiples to each number in the ratio.

**22** 260 marbles

# CHAPTER 3 review

## Multiple-choice

**1** D **2** E **3** B

**4** B **5** A **6** D

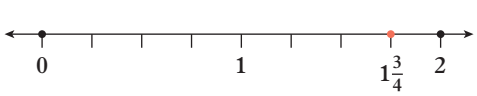
**7** D **8** A **9** C

**10** A **11** D **12** B

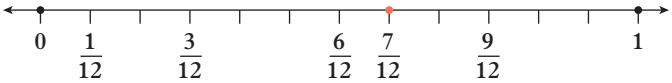
**13** C **14** C **15** D

## Short answer

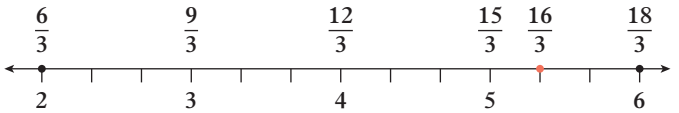
**1** **a**




**b**



**c**




**2** **a** $\frac{5}{3}, \frac{9}{3}$ **b** $\frac{34}{5}, \frac{41}{5}$ **c** $\frac{404}{4}, \frac{409}{4}$

**3** **a** $\frac{10}{25}$ **b** $\frac{27}{24}$ **c** $\frac{21}{49}$ **d** $\frac{10}{24}$

**4** **a** $\frac{4}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{2}$ **d** $\frac{3}{7}$

**5** $\frac{1}{4}, \frac{2}{7}, \frac{11}{20}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}$

**6** **a** $\frac{19}{8}$ **b** $5\frac{1}{2}$ **c** $1\frac{8}{9}$ **d** $\frac{16}{3}$

**7** **a** $\frac{5}{9}$ **b** $\frac{2}{11}$ **c** $\frac{1}{6}$ **d** $\frac{7}{10}$

**8** **a** $5\frac{6}{7}$ **b** $\frac{4}{5}$ **c** $1\frac{1}{8}$ **d** $4\frac{7}{15}$

**9** **a** $\frac{8}{35}$ **b** $\frac{1}{6}$ **c** $\frac{20}{33}$ **d** $1\frac{1}{15}$

**10** **a** $1\frac{1}{5}$ **b** $\frac{17}{35}$ **c** $8\frac{1}{4}$ **d** $5\frac{23}{24}$

**11** **a** $1\frac{1}{8}$ **b** $1\frac{1}{4}$ **c** $4\frac{2}{3}$ **d** $\frac{3}{4}$

**12** **a** $1\frac{2}{3}$ **b** $\frac{2}{3}$ **c** 9 **d** $1\frac{5}{8}$

**13** **a** 4 : 3 **b** 3 : 2 **c** 1 : 14 **d** 5 : 2

**14** **a** 2 : 3 **b** 3 : 4 **c** 5 : 8 **d** 14 : 1

**15** 56

### Analysis

**1** **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** 3 : 2

**d** 25 **e** $\frac{1}{12}$, 25 families

**f** Correct about one pet, because 5 : 10 = 1 : 2; but not about two pets, because $\frac{3}{10} \neq \frac{1}{3}$.

**2** **a** $\frac{3}{2}$ or $1\frac{1}{2}$ **b** $\frac{7}{10}$ **c** $\frac{4}{5}$

**d** $\frac{8}{15}$ **e** $\frac{3}{5}$

**f** Camila and Madelaine

# CHAPTER 4 Decimals and percentages

EX p145

## 4A Decimals

**1** **a** Eight ones or 8

**b** Eight thousandths or $\frac{8}{1000}$

**c** Eight tenths or $\frac{8}{10}$

**d** Eight ten-thousandths or $\frac{8}{10\,000}$

**e** Eight tens or 80

**f** Eight hundredths or $\frac{8}{100}$

**g** Eight hundreds or 800

**h** Eight hundredths or $\frac{8}{100}$

**i** Eight ten-thousandths or $\frac{8}{10\,000}$

**j** Eight tenths or $\frac{8}{10}$

**k** Eight thousands or 8000

**l** Eight tens or 80

**2** **a** 5785.195 **b** 496.283 **c** 0.006 **d** 5.703 **e** 36.07 **f** 40.1007

**3** **a** 3 **b** 5 **c** 2 **d** 6 **e** 0 **f** 1 **g** 8 **h** 1

**4** **a** 0.1 **b** 0.5 **c** 0.73 **d** 1 **e** 1.16 **f** 0.07 **g** 1.87

**5** **a** 0.07 **b** 0.1 **c** 0.5 **d** 0.73 **e** 1 **f** 1.16 **g** 1.87

**6** **a** 0.7 **b** 0.9 **c** 0.33 **d** 0.531 **e** 0.27 **f** 0.11 **g** 0.2978 **h** 0.804

**7** **a** 0.03 **b** 0.17 **c** 0.9 **d** 0.2010 **e** 0.0375 **f** 0.0156 **g** 0.07 **h** 0.921 **i** 0.042 **j** 0.0003 **k** 0.019 81 **l** 0.0086

**8** **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{7}{20}$ **d** $\frac{22}{25}$ **e** $\frac{23}{100}$ **f** $\frac{2}{25}$ **g** $\frac{3}{25}$ **h** $\frac{19}{20}$

**9** **a** $\frac{1}{5}$ **b** $\frac{7}{10}$ **c** $\frac{13}{100}$ **d** $\frac{13}{50}$ **e** $\frac{9}{1000}$ **f** $\frac{213}{500}$ **g** $\frac{64}{125}$ **h** $\frac{18}{25}$ **i** $\frac{1}{2500}$ **j** $1\frac{14}{25}$ **k** $2\frac{3}{8}$ **l** $3\frac{7}{25}$

**10** **a** $5\frac{83}{100}$ **b** $3\frac{451}{500}$ **c** $6\frac{9}{20}$ **d** $8\frac{7}{20}$ **e** $2\frac{1}{1250}$ **f** $3\frac{9}{10}$ **g** $12\frac{22}{25}$ **h** $1\frac{353}{500}$

**11** **a** 0.875 **b** 0.75 **c** 0.8 **d** 1.125 **e** 14.25 **f** 1.875 **g** 6.6 **h** 7.25 **i** 0.15 **j** 0.04 **k** 0.14 **l** 0.225

**12** **a** **i** $\frac{3}{10}$ **ii** 0.3

**b** **i** $\frac{5}{10}$ or $\frac{1}{2}$ **ii** 0.5

**c** **i** $\frac{8}{10}$ or $\frac{4}{5}$ **ii** 0.8

**13** **a** $\frac{1}{5} = \frac{2}{10} = 0.2$

**b** $\frac{3}{4} = \frac{75}{100} = 0.75$

**c** $\frac{7}{20} = \frac{35}{100} = 0.35$

**d** $\frac{4}{5} = \frac{8}{10} = 0.8$



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**e** $\frac{11}{50} = \frac{22}{100} = 0.22$

**f** $\frac{2}{4} = \frac{50}{100} = 0.50$ or $0.5$

**g** $\frac{13}{20} = \frac{65}{100} = 0.65$

**h** $\frac{1}{2} = \frac{5}{10} = 0.5$

**i** $\frac{19}{50} = \frac{38}{100} = 0.38$

**14 a** 0.3

**b** There are 100 small squares; 30 are shaded.

**c** $\frac{30}{100}$ or $\frac{3}{10}$

**d** 0.3 and 0.30 are the same as both are describing 3 tenths and 0 hundredths or 30 hundredths. Zeros on the end of the decimal to the right don't change the value of the number.

**15 a** 10 **b** $\frac{2}{10}$ or $\frac{1}{5}$ **c** 0.2

**d** Apple: $\frac{2}{10}$ or $\frac{1}{5} = 0.2$

Cola: $\frac{1}{10} = 0.1$

Watermelon: $\frac{2}{10}$ or $\frac{1}{5} = 0.2$

Strawberries and cream: $\frac{3}{10} = 0.3$

**16** 0.3 is greater than $\frac{1}{4}$ (which is equivalent to 0.25).

**17** Sample answer: He has failed to put a zero in the tenths place: 42 thousandths means 4 hundredths and 2 thousandths.

**18 a** $4\frac{1}{4}$ m and $1\frac{4}{5}$ m (or 4.25 m and 1.8 m)

**b** $6\frac{1}{20}$ m (or 6.05 m) **c** Yes

**19 a** 0.01 **b** 0.001 **c** 0.00001

**20 a** 3471.49 **b** 23.5437 **c** 39.307

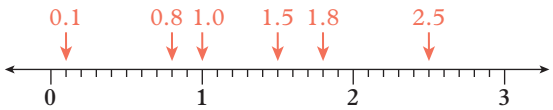
**d** 500 **e** 9999.099 **f** 219.99

**21** $3\frac{162}{200} = 3\frac{81}{100}$

## EX 4B Ordering and rounding decimals

p150

**1**




**2 a** > **b** < **c** >

**d** < **e** > **f** >

**g** > **h** > **i** >

**3 a** 1.90, 1.91, 9.01, 9.10

**b** 0.55, 5.05, 5.5, 5.55

**c** 4.0567, 4.5067, 4.567, 45.67

**d** 0.001, 0.100001, 0.101, 1

**4 a** 0.146, 0.238, 0.328, 0.461, 0.641, 0.823

**b** 1.234, 2.143, 2.341, 2.431, 3.412, 4.132

**c** 32.861, 36.182, 36.218, 36.812, 38.162, 38.261

**d** 18.1524, 18.2145, 18.2154, 18.4125, 18.4152, 18.4251

**5 a** 0.00040, 0.0003, $\frac{21}{100\,000}$, $\frac{2}{10\,000}$, 0.00004, 0.000030

**b** 0.8, $\frac{80\,001}{100\,000}$, 0.801, 0.81, 8.0, $\frac{81}{10}$

**c** 0.0510, 0.15, $\frac{1}{5}$, $\frac{1}{2}$, 0.51, $\frac{501}{100}$

**d** 3.75, $3 + \frac{6}{10} + \frac{2}{100}$, $3 + \frac{2}{10} + \frac{6}{100}$, $3 + \frac{25}{100}$, 3.026, $3 - \frac{25}{100}$

**6 a** 3.547 **b** 3.5465 **c** 3.55

**7 a** 82.63 **b** 59.112 **c** 0.2634

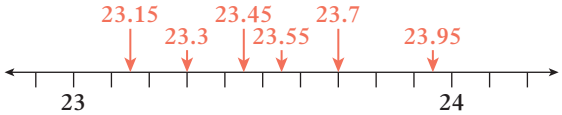
**d** 0.27 **e** 15.000 **f** 2.175499

**g** 0.219 **h** 165.5455 **i** 43.80625

**8 a** 200 **b** 170 **c** 168

**d** 168.2 **e** 168.25 **f** 168.247

**9 a**




**b** 23 **c** 24

**d** If the tenths digit is 0, 1, 2, 3 or 4, round to the whole number part of the number. If the tenths digit is 5, 6, 7, 8 or 9, add 1 to the whole number and discard the remaining digits (the decimal place digits).

**10 a** 21.89 m **b** 22.81 m

**c** The largest value in the highest place value represents the largest distance.

**11 a** $12.021 = 1 \times 10 + 2 \times 1 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100} + 1 \times \frac{1}{1000}$

$12.201 = 1 \times 10 + 2 \times 1 + 2 \times \frac{1}{10} + 0 \times \frac{1}{100} + 1 \times \frac{1}{1000}$

$12.210 = 1 \times 10 + 2 \times 1 + 2 \times \frac{1}{10} + 1 \times \frac{1}{100} + 0 \times \frac{1}{1000}$

**b** 0 tenths, 0 hundredths, 0 thousandths

**c** A different value would result. The zero holds the position of the digits in the correct place value. The result would be greater because zero is replaced by a bigger number in that place value.

**d** Zeros are needed so that the correct decimal number can be represented with all digits in the right positions according to place value.

**e** Trailing zeros can be left out as they do not change the value of the decimal.

**f** 12.021, 12.201, 12.210

**12** The digit in the next place value is a 5, meaning the digit in the third place value must have 1 added to it.

**13 a** **i** 5.4 and 5.41

**ii** 3.0 and 3.001

**iii** 2.99 and 2.9

**iv** 0.00 and 0.00312

**b** **i** 5.4 and 5.42
**ii** 3.0 and 3.001
**iii** 3.00 and 3.0
**iv** 0.00 and 0.003 12

**c** When the number rounds down (the following digit is 0–4).

**d** When the number rounds up (the following digit is 5–9).

**14** **a** 2 decimal places **b** 2 decimal places
**c** 4 decimal places **d** 3 decimal places

**15** **a** Despite 2 > 1, the place value of the 2 (ten thousandths) is much smaller than the place value of the 1 (tenths). 0.0002 < 0.1

**b** Despite 1 < 100, the place value of the 1 is the same in both (thousandth), all other digits are 0. 0.001 = 0.001 00

**c** Despite 5 = 5, the place value of the 5 is not the same in both (hundredth and tenth). 0.05 < 0.5

**d** Despite 1234 < 5678, the first non-zero digits do not have the same place value (1 ten and 5 ones). 12.34 > 5.678

**16** **a** Inequality is incorrect. Reasoning is incorrect – Tyler incorrectly converted the fractions to decimals $\frac{1}{8} = 0.125$ and $\frac{1}{5} = 0.2$.

**b** Inequality is correct. Reasoning is incorrect – Tyler did not compare using the correct place values 3400 < 6789.

**c** Inequality is correct. Reasoning is correct.

**17** **a** To 2 decimal places to $42.22 as we do not pay amounts less than one cent on a card.

**b** To the nearest 5 cents to $40.00 as we do not pay amounts less than five cents using cash.

**c** To the nearest degree to 32° as the hiker only has to travel 20 m so will not be far off course.

**d** To 4 decimal places to 32.2851° as the rocket has to travel a very long distance and it is very expensive if it veers off course and crashes or does not have enough fuel to correct the error.

**e** To 2 decimal places to 1.592 m as we generally can't tell much difference between millimetre differences of height.

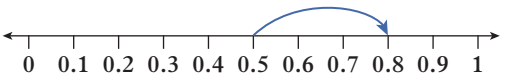
**f** To 3–4 decimal places to 1.592 cm or 1.5924 cm as it is likely the bug is being measured for a report and more accuracy for the comparison would be useful.

**18** **a** 58.3 cm **b** 98.212 L
**c** 214.193 km **d** 214.192 75 km

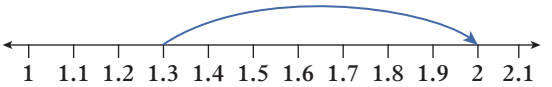
**19** **a** 1 decimal place **b** 3 decimal places
**c** 3 decimal places **d** 5 decimal places

## EX p154 4C Adding and subtracting decimals

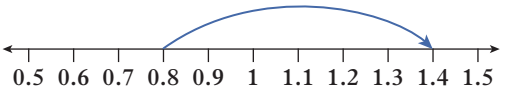
**1** **a** **i** **ii** 0.8



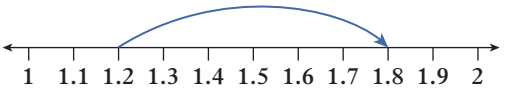

**b** **i** **ii** 2



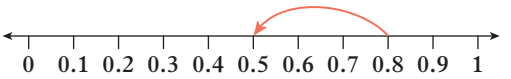

**c** **i** **ii** 1.4



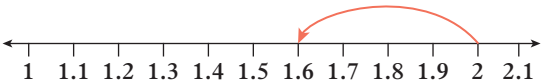

**d** **i** **ii** 1.8



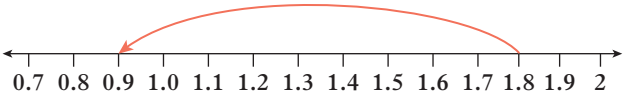

**2** **a** **i** **ii** 0.5



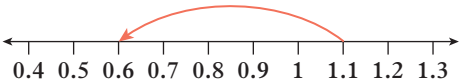

**b** **i** **ii** 1.6




**c** **i** **ii** 0.9




**d** **i** **ii** 0.6




**3** **a** 90 **b** 310 **c** 200
**d** 290 **e** 150 **f** 1210
**g** 940 **h** 8100

**4** **a** 88.21 **b** 316.47 **c** 192.273
**d** 292.065 **e** 149.601 **f** 1208.2163
**g** 944.911 **h** 8102.391

**5** **a** 36.012 **b** 1519.797 **c** 66.017
**d** 3855.1036 **e** 177.9932 **f** 15.971

**6** **a** 60 **b** 30 **c** 400
**d** 20 **e** 1470 **f** 110
**g** 10 **h** 1230

**7** **a** 56.23 **b** 34.72 **c** 394.9
**d** 23.22 **e** 1476.68 **f** 113.27
**g** 15.588 **h** 1233.6118

**8** **a** 123.926 **b** 7.487 **c** 15.47
**d** 25.14 **e** 6.675 **f** 10.338

**9** See answers in questions 5 and 8.

**10** **a** 283.15 **b** 62.369 **c** 15.413
**d** 617.58 **e** 0.23 **f** 158.7

**11** 66.68 m

**12** **a** $43 **b** $7
**c** $42.51, change is $7.49
**d** Only 49 cents away from the actual amount

**13** **a** 42.6 **b** 42.1
**c** Natasha received the higher score by 0.5.

**14** Jonathan 0.09 m, Kamaro 0.13 m, Tan 0.14 m

**15 a** The place values are not lined up, so the incorrect digits are added.

**b** The 7 in the second row has not been subtracted as the additional zero was not added above it.

**c** The 0.25 has been subtracted 1 has been subtracted from the 6 instead of 2.

**d** The student could not perform 2 – 7 without negatives so instead performed 7 – 2 rather than taking 10 tenths from the 5 ones.

**16 a i** 10 + 2

**ii** 12, higher than the actual answer

**iii** 11.25

**iv** Sample answer: May forget to carry across.

**b i** 15 – 9

**ii** 6, higher than the actual answer

**iii** 5.84

**iv** Sample answer: May reverse the .02 and the .18 as it is easier to calculate.

**c i** 14 – 3

**ii** 11, lower than the actual answer

**iii** 11.15

**iv** Sample answer: May forget to subtract the decimals and just write down 12.85.

**d i** 25 – 3 + 6

**ii** 28, lower than the actual answer

**iii** 27.856

**iv** Sample answer: May perform the addition first then subtract a larger number than intended.

**17 a** 12.49 L **b** Three tins

**18** Total monthly rainfall of 64.68 mm is 1.02 mm below September average, so it is very close to the average.

**19 a** Day 1: 187.2 km; Day 2: 148.6 km; Day 3: 249.8 km; Day 4: 144.8 km; Day 5: 251.5 km

**b** 28 071.5 **c** Day 4, 144.8 km

**d** Day 5, 251.5 km **e** 981.9 km

**20 a** 22.09 kg **b** More than 2.09 kg

**21 a i** $453.98, $394.99, $329.99, $79.99, $51.46, $53.24, $48.24

**ii** $407.52 **iii** $127.20 **iv** $48.24

**b** Elisa has $48.24 in her account at the end of the month; not enough to purchase a drone.

**22** 86.945

**23 a i** 462.2 **ii** 1.279 **iii** 0.00 **iv** 12.64

**b i** 28 **ii** 26

**c i** 27.643 **ii** 27.642

**d** the rounded estimate

**e** when all numbers round down

**f** The rounding estimate will always be greater than or equal to the truncating estimate, because when a value is rounded up it increases and when a value is rounded down it is equal to the truncated value.

**g** 0.3 **h** 0.15

## EX p160 4D Multiplying decimals

**1 a** 5 **b** 7 **c** 6

**d** 6 **e** 6 **f** 6

**2 a** 0.12 **b** 0.24 **c** 0.42

**d** 0.072 **e** 0.0036 **f** 0.49

**g** 0.96 **h** 0.000 24 **i** 0.000 000 7

**3 a** 26.6 **b** 39.2 **c** 3.15

**d** 9.12 **e** 36.0072 **f** 19.68

**g** 8.277 **h** 359.8 **i** 79.44

**4 a** 69.16 **b** 456.48 **c** 1258.32

**d** 28.644 **e** 130.974 **f** 964.73

**g** 47.7776 **h** 24.311 92 **i** 2251.1775

**5 a** 32.88 **b** 86.76 **c** 198.15

**d** 622.08 **e** 0.185 68 **f** 7.4684

**g** 12.4 **h** 3.702 **i** 1041.56

**6** See answers for questions 4 and 5.

**7 a** 6000 **b** 4200 **c** 1.284

**d** 1572.1 **e** 0.4 **f** 7.02

**g** 61 **h** 150 **i** 310

**j** 6481.3 **k** 480 000 **l** 128 468 000

**8 a** 9086 **b** 297 600 **c** 104 688

**d** 33 984 **e** 97 500 **f** 12 150

**9 a** 12.0807 **b** 36.62

**c** 12.958 **d** 0.481 74

**10 a** All the calculations involve the same digits in the same order.

**b** The digits represent different place values in the different numbers.

**c, d**

| Original calculation | Estimated answer | Answer |
|---|---|---|
| 38 × 21 | 40 × 20 = 800 | 798 |
| 3.8 × 21 | 4 × 20 = 80 | 79.8 |
| 38 × 2.1 | 40 × 2 = 80 | 79.8 |
| 3.8 × 2.1 | 4 × 2 = 8 | 7.98 |
| 380 × 210 | 400 × 200 = 80 000 | 79 800 |

**e** The number of decimal places in the answer equals the total number of decimal places in the original calculation.

**11 a** $60 **b** $77.48

**c** The estimate is $17.48 lower.

**12 a** $9.74 **b** $6.08 **c** $9.73

**d** $8.28 **e** $8.97 **f** $10.09

**13 a i** 54 **ii** 56.062 245 85 **iii** 2.062 245 85

**b i** 2 400 000 **ii** 1 942 774.728 **iii** 457 225.272

**c i** 4 **ii** 5.345 665 957 **iii** 1.345 665 957

**d i** 0.0054 **ii** 0.005 304 312 **iii** 0.000 095 688

**14 a** 0.000 21 **b** 1.0015

**c** 10.8054 **d** 22.0779

**15 a** $713.40 **b** Option 1 **c** $68.40

**16 a i** 0.04 **ii** 0.16 **iii** 0.0625

**iv** 0.000 064 **v** 1.44

**b** The number of decimal places in the answer is twice the number of decimal places in the number to be squared.

**17** 8 mm

**18** **a** \$258 **b** \$147.60

**19** **a** 3.654 **b** 10.549 85 **c** 60.069 67

**20** Sample answer: $14.679 \times 0.5382 = 7.900\,237\,8$

## 4 Checkpoint

**1** **a** Five thousandths or $\frac{5}{1000}$

**b** Five tenths or $\frac{5}{10}$

**c** Five ten-thousandths or $\frac{5}{10\,000}$

**d** Five hundredths or $\frac{5}{100}$

**2** **a** $\frac{17}{100}$ **b** $5\frac{3}{10} = \frac{53}{10}$

**c** $\frac{3}{20}$ **d** $9\frac{1}{8} = \frac{73}{8}$

**3** **a** 0.71 **b** 2.4 **c** 0.625

**d** 11.35

**4** **a** $1.3 > 0.6$

**b** $0.1 = 0.10$

**c** $0.07 < 0.5$

**d** $31.2729 < 31.8$

**5** **a** 1.087, 1.708, 7.108, 7.180, 17.80

**b** 15.986, 15.689, 1.5698, 1.5689, 1.056 89

**c** 0.08, $\frac{18}{100}$, 0.20, 0.7, $\frac{9}{10}$

**d** $\frac{11}{5}$, 1.80, $\frac{34}{20}$, $\frac{3}{2}$, 1.4

**6** **a** 6.47 **b** 32.183 **c** 0.003 **d** 100.0

**7** **a** 5.9 **b** 2.2 **c** 13.27 **d** 6.58

**8** **a** 86.18 **b** 290.89 **c** 62.11 **d** 382.08

**9** **a** 59.1 **b** 0.412 **c** 124 **d** 5.282

**10** **a** 0.000 54 **b** 1.84 **c** 8.685 **d** 137.28

**11** **a** $3 - 1 + 9 = 11$

**b** $9 + 10 \times 1 = 19$

**c** $37 \times (15 + 5) = 740$

**d** $(18 - 9) \times (0 + 0) = 0$

**12** **a** 11.39 **b** 15.38 **c** 742

**d** 2.024

## EX p166 4E Dividing a decimal by a whole number

**1** **a** 6.23 **b** 13.721 **c** 9.102

**d** 23.14 **e** 11.115 **f** 1.6398

**g** 2.9217 **h** 4.15 **i** 3.51

**j** 0.0256 **k** 0.1208 **l** 0.728 04

**2** **a** 2.5 **b** 6.75 **c** 12.6

**d** 4.35 **e** 2.05 **f** 0.4062

**g** 0.603 75 **h** 39.05 **i** 14.0575

**j** 1.193 75 **k** 1.145 **l** 3.2375

**3** **a** \$1.28 **b** \$217.50 **c** \$1.28

**d** \$0.75 **e** \$11.80 **f** \$7.25

**g** \$1.30 **h** \$1.80

**4** **a** \$3.88 **b** \$4.98 **c** \$3.98

**d** \$11.97 **e** \$6.98 **f** \$7.98

**g** \$1.70 **h** \$4.27

**5** **a** 0.678 543 **b** 9.53 **c** 28.894

**d** 0.157 21 **e** 0.2934 **f** 0.702

**g** 68.9421 **h** 0.150 95 **i** 0.0436

**6** **a** 0.562 78 **b** 1155.6 **c** 559.68

**d** 2.4978 **e** 61.4304 **f** 40.9625

**g** 93.78 **h** 9.36705

**7** \$186.25

**8** \$135.90

**9** \$24.65

**10** **a** \$2.49

**b** 2 L container, as it is cheaper per litre.

**11** \$18.90 ÷ 6 = \$3.15. Whole pizza is better value by \$0.35 per slice.

**12** **a** \$2.75

**b** If using it five days per week, weekly option is cheaper by 20 cents per day.

**c** If planning to travel less than five days per week, buying daily tickets will cost less than a weekly ticket.

**13** **a** \$8.99 ÷ 12 = \$0.75. The carton of eggs is better value.

**b** \$0.15

**c** Some people want to buy less than 12 eggs.

**14** **a** \$3.73 **b** \$1.56 **c** \$0.53

**d** \$1.50

**15** **a** Simon \$0.80, Julia \$0.66 **b** Julia

**16** **a** Firenze \$0.93, Milano \$0.82; Milano is better value

**b** Two, Firenze (\$7.46 compared to \$8.18)

**17** **a** \$357 122.89

**b** Divide the money by 18, then round to two decimal places.

**18** Less than 1 L, as divisor (180) is larger than dividend (100); that is, dividing the smaller amount by the larger amount.

**19** Packing boxes at \$8.13 an hour

**20** **a** \$2.10 **b** \$21.00

**c** The cost of one mango is found by dividing \$6.30 by 3. Multiply by 10 for cost of 10 mangos.

**21** **a** \$7.00 **b** \$6.00

**c** \$6.50 **d** \$8.50

**22** **a** \$12.40 **b** \$25.44

**c** \$3.73 **d** \$0.68

**23** **a** Crinkle cut chips \$2.54, small Surfwave chips \$2.85, large Surfwave chips \$2.81, Blue Stone chips \$2.25

**b** Blue Stone chips

**c** Sample answer: taste, texture

**24** **a** Ava **b** 6.08 km/h

## EX p170 4F Dividing a decimal by a decimal

**1** **a** $0.5 \div 0.1$ **b** $0.8 \div 0.4$

**c** $0.3 \div 0.06$ **d** $0.6 \div 0.04$

**2** **a** 5 **b** 2 **c** 5
**d** 15

**3** **a** 623.56 ÷ 8 **b** 9840 ÷ 8 **c** 1956.74 ÷ 6
**d** 4.62 ÷ 3 **e** 420 ÷ 4 **f** 0.5 ÷ 2
**g** 4 620 850 ÷ 9 **h** 0.56 ÷ 2 **i** 2108 ÷ 5

**4** **a** 9 **b** 6 **c** 7
**d** 200 **e** 3 **f** 800
**g** 90 **h** 30 **i** 7100

**5** **a** 19.3 **b** 281 **c** 84.7
**d** 119.7 **e** 162.8 **f** 185.3
**g** 34 150 **h** 47.1 **i** 364.8

**6** **a** 11.905 **b** 43.375 **c** 5412.5
**d** 0.82 **e** 3187.895 **f** 642.175
**g** 188.25 **h** 2282.45 **i** 14.325

**7** **a** 15 960 **b** 22 **c** 2096.96
**d** 4 **e** 500 **f** 5.674295

**8** **a** 8.40 ÷ 0.60 **b** 84 ÷ 6 **c** 14

**9** 86

**10** **a** **i** 12.00 ÷ 0.50
**ii** 120 ÷ 5 **iii** 24
**b** **i** 12.00 ÷ 0.20
**ii** 120 ÷ 2 **iii** 60
**c** **i** 12.00 ÷ 0.80
**ii** 120 ÷ 8 **iii** 15
**d** **i** 12.00 ÷ 1.20
**ii** 120 ÷ 12 **iii** 10

**11** **a** **i** 5 **ii** 50 **iii** 500
**iv** 5000
**b** The answer increases by a power of 10.
**c** The answer is a value larger than the dividend.
**d** True

**12** **a** A, B, E, G
**b** The dividend and the divisor in each problem is multiplied (or divided) by the same factor.

**13** **a** **i** 1243 **ii** 0.1243
**b** **i** 94 257 **ii** 942.57
**c** **i** 19 831 **ii** 0.001 983 1
**d** **i** 36 492 725 **ii** 0.000 364 927 25

**14** **a** \$2.40 **b** \$2.75 **c** The 1.2 kg bag

**15** **a** 144 pieces **b** Yes, 0.1 m remaining

**16** 45.5 L

**17** Divide cost by volume to find the cost of 1 L of each. Orange mineral water is the better buy (\$1.68 per L compared to \$1.80 per L for cola).

**18** 32.4 ÷ 4.05 = 8

**19** Sample answer: 124.98 ÷ 5.0367 = 24.81387…

**20** **a** $\frac{1052}{315}$ or $3\frac{107}{315}$
**b** 3.34
**c** The calculation alternates between adding and subtracting each term. The denominators increase in odd numbers and the numerator is always 4.
**d** $-\frac{4}{11}+\frac{4}{13}$
**e** 3.28
**f** 7.64
**g** 26.24
**h** The circumference divided by 3.28 is approximately equal to the diameter. The diameter multiplied by 3.28 is approximately equal to the circumference.
**i** The relationships will produce more accurate approximations when more terms are added to the original calculation.

## EX 4G Percentages

p175

**1** **a** 50% **b** 52% **c** 36%
**d** 60% **e** 55% **f** 45%

**2** **a** 50% **b** 48% **c** 64%
**d** 40% **e** 45% **f** 55%

**3** **a** 23% **b** 78% **c** 35%
**d** 99% **e** 13% **f** 7%
**g** 82% **h** 4%

**4** **a** $\frac{7}{100}$ **b** $\frac{19}{100}$ **c** $\frac{43}{100}$
**d** $\frac{57}{100}$ **e** $\frac{3}{100}$ **f** $\frac{63}{100}$
**g** $\frac{11}{100}$ **h** $\frac{91}{100}$ **i** $\frac{79}{100}$
**j** $\frac{21}{100}$ **k** $\frac{99}{100}$ **l** $\frac{33}{100}$

**5** **a** 23% **b** 80% **c** 13%
**d** 67% **e** 60% **f** 170%
**g** 209% **h** 314% **i** 79%
**j** 191% **k** 4% **l** 55%

**6** **a** $\frac{16}{25}$ **b** $\frac{9}{50}$ **c** $\frac{27}{100}$
**d** $\frac{4}{5}$ **e** $\frac{1}{20}$ **f** $\frac{19}{20}$
**g** $\frac{1}{10}$ **h** $\frac{9}{20}$ **i** $\frac{3}{5}$
**j** $\frac{3}{4}$ **k** $\frac{7}{20}$ **l** $\frac{22}{25}$

**7** **a** 45% **b** 22% **c** 175%
**d** 68% **e** 160% **f** 20%
**g** 60% **h** 92% **i** 120%
**j** 225% **k** 80% **l** 20%

**8** **a** **i** 14 sections shaded out of a total of 20 sections
**ii** $\frac{14}{20}$ **iii** $\frac{70}{100}$ **iv** 70%
**v** 0.7
**b** **i** 5 sections shaded out of a total of 10 sections
**ii** $\frac{5}{10}$ **iii** $\frac{50}{100}$ **iv** 50%
**v** 0.5
**c** **i** 1 section shaded out of a total of 4 sections
**ii** $\frac{1}{4}$ **iii** $\frac{25}{100}$ **iv** 25%
**v** 0.25

**9** **a** **i** 76% **ii** 0.76
**b** **i** 40% **ii** 0.4
**c** **i** 75% **ii** 0.75
**d** **i** 30% **ii** 0.3
**e** **i** 35% **ii** 0.35
**f** **i** 6% **ii** 0.06

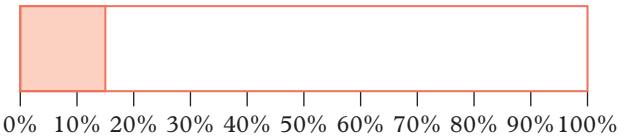
**10** **a** C **b** A **c** D **d** B

**11** **a** 40% **b** 90%
**c** 4% **d** 20%

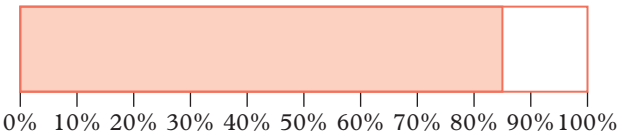
**12** **a** **i** 20% **ii** 10% **iii** 30%
**b** **i** 80% **ii** 90% **iii** 70%
**c** Sample answer: Divide the number of pieces eaten by the total number of pieces and multiply by 100.

**13** **a** 3 : 2 **b** 3 : 1 **c** 9 : 11
**d** 9 : 41

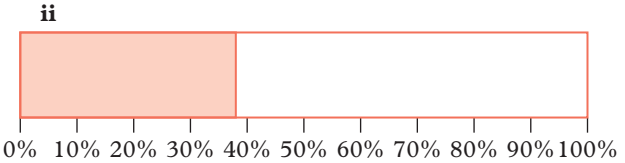
**14** **a** See part **c**
**b** **i** 10 is factor of 100 so a denominator of 10 can easily be expressed as a denominator of 100.
**ii** Each cm will represent 10%.
**iii** Each mm will represent 1%.
**c** 15% = $\frac{15}{100}$ so 15 mm out of 100 mm (or 1.5 cm out of 10 cm) needs to be shaded.



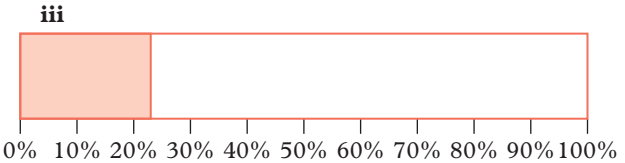

**d** **i**



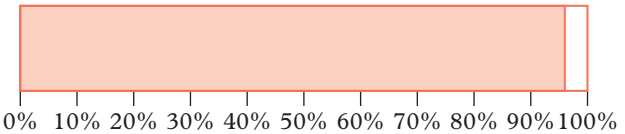

**ii**




**iii**




**iv**




**15** **a** 40% **b** 30% **c** 65% **d** 75%

**16** Sample answer: Percentage is calculated by finding volume of liquid in each container as a fraction of the total volume that container holds. However, 50% of a large container is still going to be more that 50% of a small container.

**17** **a** Three times the whole amount
**b** **i** 2 **ii** 3
**iii** 4 **iv** 1.5

**18** **a** 4.5 **b** 525%

**19** Sample answer: 20% of the full glass (100%) is more than 20% of the partially emptied glass (80%), so it will not fill back to the brim.

**20** **a** The values that can appear here are between 0.4 and 0.65.
**b** The values that can appear here are fractions with a denominator of 15 and are greater than $\frac{6}{15}$.
**c** The values that can appear here are $\frac{9}{15}$, $\frac{8}{15}$, $\frac{7}{15}$, $\frac{6}{15}$, $\frac{5}{15}$, $\frac{4}{15}$, $\frac{3}{15}$, $\frac{2}{15}$, and $\frac{1}{15}$.
**d** The value in this region are $\frac{9}{15}$, $\frac{8}{15}$, and $\frac{7}{15}$.

## EX 4H Fractions, decimals and percentages

p182

**1**

| Percentage | Fraction | Decimal |
|---|---|---|
| 50% | $\frac{1}{2}$ | 0.5 |
| 25% | $\frac{1}{4}$ | 0.25 |
| 75% | $\frac{3}{4}$ | 0.75 |
| 30% | $\frac{3}{10}$ | 0.3 |
| 12.5% | $\frac{1}{8}$ | 0.125 |
| 62.5% | $\frac{5}{8}$ | 0.625 |
| 20% | $\frac{1}{5}$ | 0.2 |
| 40% | $\frac{2}{5}$ | 0.4 |
| 60% | $\frac{3}{5}$ | 0.6 |

**2** **a** $\frac{23}{50}$, 0.46 **b** $\frac{13}{100}$, 0.13 **c** $\frac{99}{100}$, 0.99
**d** $\frac{1}{4}$, 0.25 **e** $\frac{1}{5}$, 0.2 **f** $\frac{1}{2}$, 0.5
**g** $\frac{1}{20}$, 0.05 **h** $\frac{2}{25}$, 0.08 **i** $\frac{1}{100}$, 0.01

**3** **a** 28%, $\frac{7}{25}$ **b** 88%, $\frac{22}{25}$ **c** 15%, $\frac{3}{20}$
**d** 62%, $\frac{31}{50}$ **e** 45%, $\frac{9}{20}$ **f** 72%, $\frac{18}{25}$
**g** 9%, $\frac{9}{100}$ **h** 4%, $\frac{1}{25}$ **i** 30%, $\frac{3}{10}$

**4** **a** 102%, 1.02 **b** 85%, 0.85 **c** 280%, 2.8
**d** 225%, 2.25 **e** 52%, 0.52 **f** 1050%, 10.5
**g** 40.5%, 0.405 **h** 415%, 4.15 **i** 7121%, 71.21

**5** **a** 23.84 = 0.2384 = $\frac{149}{625}$
**b** 19.65% = 0.1965 = $\frac{393}{2000}$
**c** 46.7% = 0.467 = $\frac{467}{1000}$

**d** $3.09\% = 0.0309 = \frac{309}{10\,000}$

**e** $567.4\% = 5.674 = 5\frac{337}{500}$

**f** $0.467\% = 0.00467 = \frac{467}{100\,000}$

**g** $12.895\% = 0.12895 = \frac{2579}{20\,000}$

**h** $73.28\% = 0.7328 = \frac{458}{625}$

**i** $200.5\% = 2.005 = 2\frac{1}{200}$

**6** **a** $51.8\%, \frac{259}{500}$ **b** $902\%, 9\frac{1}{50}$ **c** $10.5\%, \frac{21}{200}$

**d** $70\%, \frac{7}{10}$ **e** $2150\%, 21\frac{1}{2}$ **f** $841\%, 8\frac{41}{100}$

**g** $710\%, 7\frac{1}{10}$ **h** $0.9\%, \frac{9}{1000}$ **i** $17.65\%, \frac{353}{2000}$

**7** **a** 37.5%, 0.375

**b** 7.5%, 0.075

**c** 3.75%, 0.0375

**d** 9.375%, 0.093 75

**e** 212.5%, 2.125

**f** 227.5%, 2.275

**g** 946.25%, 9.4625

**h** 1190.625%, 11.906 25

**i** 11.25%, 0.1125

**8** **a** $\frac{2}{5}$

**b** **i** 40% **ii** 60%

**c** **i** 0.4 **ii** 0.6

**9** **a** 68% **b** 210% **c** 215%

**d** 87.5%

**10** **a** $1\frac{9}{10} = \frac{38}{20} = 1.9 = 190\%; \frac{38}{200} = 19\%;$

$\frac{19}{95} = 20\% = 0.2$

$\frac{19}{5} = 3.8; 1.09 = 109\%; 0.38 = 38\%$

**b** $\frac{38}{200} = 19\% = 0.19; \frac{19}{5} = 3.8 = 380\%$

$1.09 = 109\% = 1\frac{9}{100} = \frac{109}{100}; 0.38 = 38\% = \frac{19}{50}$

**c** 3.8, 1.9, 1.09, 0.38, 0.2, 0.19

**11** **a** $\frac{71}{100}$, 0.71

**b** **i** 0.09% **ii** 0.0009 **iii** $\frac{9}{10\,000}$

**c** 99.853%; values may be rounded and there may be other substances in the Sun.

**12** 20%

**13** Both methods of working are correct.

**14** 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80

**15** One possible solution: $\frac{37}{40} = 92.5\%$

## EX p186 4I Calculating percentages

**1** **a** 75% **b** 125% **c** 60%

**d** 50% **e** 200% **f** 562.5%

**g** 92.5% **h** 87.5%

**2** **a** 195 **b** 12 **c** 104

**d** 40 **e** 7.5 **f** 18

**g** 39 **h** 12

**3** **a** \$4.80 **b** \$76.50 **c** \$55

**d** \$198.90 **e** \$700 **f** \$168.30

**g** \$1485 **h** \$8.95

**4** **a** 4.5 **b** 3.85 **c** 90.96

**d** 30.6 **e** 27.82 **f** 288.75

**g** 50.88 **h** 25.2 **i** 80.94

**j** 21.12 **k** 12.375 **l** 7.956

**5** **a** 44 **b** 73.5 **c** 45.6

**d** 400 **e** 10.68 **f** 450.8

**g** 148.5 **h** 308

**6** **a** Each percentage is greater than 100%.

**b** The final amount is greater than the original amount.

**7** **a** halve **b** triple **c** quadruple

**8** **a** 5980 **b** 520

**9** **a** 72% **b** 76.7%

**c** The second test, as his percentage is higher

**10** **a** \$659.70 **b** \$1539.30 **c** No

**11** **a** \$114 **b** \$456

**12** **a** Ripper Reptiles: \$108, Enclosed: \$162.50

**b** Ripper Reptiles: \$492, Enclosed: \$487.50

**c** Enclosed

**13** **a** 62.5% **b** 160%

**14** **a** 0.2

**b** Wally: 253, Artemis: 115, Megan: 92

**c** **i** 11 : 9 **ii** 11 : 5 **iii** 5 : 4

**d** The percentages of lollies for the three people are:

| | |
|---|---|
| Wally | 55% |
| Artemis | 25% |
| Megan | 20% |

Wally would give you the closest to a 1 : 1 ratio as he has the closest to 50%, and 1 : 1 is the same as 50 : 50.

**15** **a** 72 **b** 180 **c** 60 **d** 900

**16** **a** Water: 29.25 kg, protein: 8.1 kg, fat: 4.5 kg, minerals and vitamins: 2.7 kg, carbohydrates: 0.45 kg

**b** Water: 40.95 kg, protein: 11.34 kg, fat: 6.3 kg, minerals and vitamins: 3.78 kg, carbohydrates: 0.63 kg

**c** Water: 63.7 kg, protein: 17.64 kg, fat: 9.8 kg, minerals and vitamins: 5.88 kg, carbohydrates: 0.98 kg

**17** **a** \$3.75 **b** \$1.30 **c** \$7.35

**d** \$9.20 **e** \$1.75 **f** \$1.20

**18** **a** **i** \$119.90 **ii** \$1318.90

**b** **i** \$20.90 **ii** \$229.90

**c** **i** \$49.90 **ii** \$548.90

**d** **i** \$0.30 **ii** \$3.29

**e** **i** \$3.00 **ii** \$32.99

**f** **i** \$5.40 **ii** \$59.40

**19** Sample answer: Move the decimal point one place to the left of the original price value to obtain the GST amount.

**20** **a** 171 **b** 213.75%

**21** 32%

**22** **a** 205.01 m **b** 6 days **c** More time

**d** Under the new claim, the building will never reach 0 m. After 20 days, the building will be less than 6 m tall.

**e** 15 days

## CHAPTER 4 review

### Multiple-choice

| | | |
|---|---|---|
| **1** A | **2** E | **3** E |
| **4** C | **5** E | **6** E |
| **7** B | **8** A | **9** C |
| **10** A | **11** D | **12** D |
| **13** C | **14** D | **15** A |
| **16** E | **17** D | |

### Short answer

**1** **a** Hundredths **b** Hundredths

**c** Millionths **d** Tenths

**2** **a** 0.87 **b** 0.117

**c** 0.3 **d** 0.07

**3** **a** 0.64 **b** 0.38

**c** 0.75 **d** 0.375

**4** **a** **i** Five **ii** 1589.23

**b** **i** Five **ii** 12.71

**c** **i** Four **ii** 0.93

**d** **i** Seven **ii** 152.88

**5** 18.0958 12.8905 12.5089 12.0985 10.9852

**6** **a** 120.46 **b** 9.532 **c** 1.693

**d** 4.2885

**7** **a** 1263 **b** 153.298 **c** 210

**d** 609.4

**8** **a** 0.21 **b** 0.036 **c** 0.088

**d** 0.0032

**9** **a** 2.58 **b** 0.468 **c** 46.8

**d** 8.037

**10** **a** 7.894 **b** 12.563 59 **c** 12.456

**d** 189.7656

**11** **a** 9.327 **b** 15.76 **c** 29.726

**d** 40.931 25

**12** **a** \$1.56, \$1.80

**b** 500 g pack, cost per 100 g is cheaper

**13** **a** 12.4 **b** 3.75 **c** 21.55

**d** 0.162

**14** **a** **i** $\frac{49}{100}$ **ii** $\frac{49}{100}$

**b** **i** $\frac{22}{100}$ **ii** $\frac{11}{50}$

**c** **i** $\frac{75}{100}$ **ii** $\frac{3}{4}$

**d** **i** $\frac{5}{100}$ **ii** $\frac{1}{20}$

**15** 46%

**16** **a** $\frac{3}{4}$ **b** $\frac{9}{50}$ **c** $\frac{489}{1000}$

**d** $\frac{3}{1000}$

**17** **a** 56% **b** 8% **c** 179%

**d** 27.48%

**18** **a** 0.78 **b** 0.465 **c** 0.09

**d** 0.106 79

**19** **a** 60% **b** 87.5% **c** 100%

**d** 275%

**20** **a** 45 **b** 150 **c** 135

**d** 552.5 **e** 89.94 **f** 142.27

**21** **a** 1496 km **b** 330.125 g **c** \$6.96

**d** 72.9 L

### Analysis

**a** \$39.30

**b** 13, \$0.45 (\$0.43 left over, round to nearest 5c)

**c** \$35.88 **d** \$26.91

**e** 0.375 **f** \$53.82

**g** \$20.1825 **h** Four, \$20.18

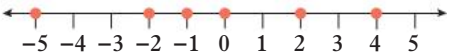
**i** \$33.65, \$5.65 left over **j** \$1.41

## CHAPTER 5 Integers and the Cartesian plane

### EX 5A Negative numbers

p198

**1** **a**




**b** **i** 4 **ii** 0 **iii** 4

**iv** −2 **v** 4 **vi** −1

**c** **i** 1 **ii** 0 **iii** −1

**iv** −3 **v** −1 **vi** −4

**2** **a** $-4 < 6$ **b** $0 > -3$ **c** $-5 > -9$

**d** $-7 < -\frac{1}{7}$ **e** $-2 > -2\frac{2}{3}$ **f** $-\frac{5}{4} > -2$

**g** $-4\frac{2}{5} < -\frac{20}{5}$ **h** $-\frac{7}{2} > -3\frac{2}{3}$ **i** $-\frac{13}{3} < -\frac{21}{5}$

**3** **a** $-40 < 30$ **b** $0 > -22$ **c** $-65 < -55$

**d** $78 > -\frac{100}{2}$ **e** $-\frac{127}{10} < -\frac{328}{100}$

**f** $-150\frac{79}{190} < 0$ **g** $248\frac{23}{25} > -300\frac{67}{70}$

**h** $-\frac{120}{7} < 15\frac{5}{7}$ **i** $-\frac{143}{9} < -\frac{95}{6}$

**4** **a** −6, −2, 0, 4, 6

**b** $-13, -3, -\frac{5}{2}, 4, 10$

**c** $-12, -1, -\frac{2}{3}, 1, \frac{10}{3}$

**d** $-5\frac{2}{3}, -5, -\frac{9}{4}, 5, 10, 16$

**e** $-20\frac{4}{5}, -20.75, -20, 25, 25\frac{17}{20}, 25\frac{35}{40}$

**f** $-1.25, -\frac{12}{10}, -1, -\frac{12}{13}, 0.6, 1\frac{6}{5}$

5 **a** 8, 4.8, 2.1, −5, −6

**b** 80.8, 43, −12, −35, −46.2

**c** 8.9, 6.5, −2.3, −4.7, −12.5

**d** 15, 12.5, 7.6, $-4\frac{1}{3}$, −4.5, $-\frac{11}{2}$

**e** 48.9, $\frac{12}{5}$, $-36\frac{25}{45}$, $-45\frac{4}{5}$, −48.9, −50.3

**f** −0.11, $-\frac{110}{12}$, $-\frac{120}{11}$, −11, −11.25, $-11\frac{5}{6}$

6 −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6

7 Sample answer: $-\frac{5}{6}, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{5}, \frac{1}{6}, \frac{3}{5}$

8 Sample answer: $2\frac{3}{5}, 2\frac{2}{7}, -1\frac{8}{9}, -2\frac{1}{6}, -2\frac{3}{4}, -4\frac{1}{4}$

9 **a** −3 **b** 360 **c** −2

**d** −15 **e** 2230 **f** −28

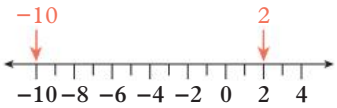
10 **a** −10°C

**b** **i** Higher **ii** Higher

**iii** Lower **iv** Higher

**c** A larger number is higher on the scale.

11 **a**




**b** −10°C

**c** −10°C, −1°C, 2°C

**d** Cooler

12 **a** Level 5 **b** Level −3

**c** Nine levels **d** 0

**e** You go up three levels.

**f** You go down two levels.

**g** You go up four levels.

**h** Levels that are below ground level.

13 **a** 100 **b** $20 **c** −20

14 **a** Paulo: −85.50; Kamilla: 98.05; Amad: 135.98; Jessica: −172.20

**b** Amad **c** Jessica

**d** −172.20, −85.50, 98.05, 135.98. Numbers increase in value going left to right on the number line.

15 **a** Raj's school is at −2 and the hospital is at 9, or Raj's school is at 2 and the hospital is at −9.

**b** 11 km

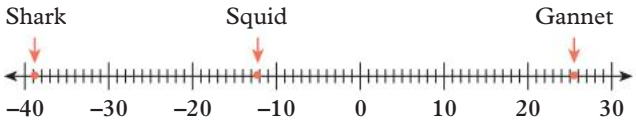
16 **a** 5 **b** −30 **c** 35 m

**d** −3800

17 **a** $25\frac{1}{2}$ **b** $-12\frac{1}{3}$

**c** $-38\frac{7}{30}$ m. The shark is swimming $25\frac{9}{10}$ lower than the squid, so the shark is at a depth of $-12\frac{1}{3} - 25\frac{9}{10} = -38\frac{7}{10}$ m.

**d** The shark is closer to the squid.




18 **a** **i** +5 **ii** −4

**iii** 0 **iv** +5

**b** Diagrams **i** and **iv** have the same electric charge because the 3 negative charges in diagram **iv** cancel out 3 of the 8 positive charges, leaving it with a charge of +5.

19 **a** Sample answer: −1 and 2. The positive integer is larger in magnitude than the negative integer.

**b** Sample answer: −1 and 10. As long as you subtract the positive integer from the negative integer, the result will be negative as subtracting value from a negative integer makes the negative integer more negative.

20 **a** No. The temperature cannot be negative Kelvin because 0 K is the coldest temperature that is physically possible.

**b c**

**d** **i** 0 K **ii** 263 K **iii** 300 K

## EX 5B Adding and subtracting positive numbers

p204

1 **a** 5 + 4 = 9 **b** 7 − 3 = 4

**c** 5 + 3 = 8 **d** 2 − 6 = −4

**e** −8 + 3 = −5 **f** −1 + 4 = 3

**g** −5 + 5 = 0 **h** −1 − 5 = −6

2 **a** 7 **b** 1 **c** −2

**d** 5 **e** −10 **f** 1

**g** −4 **h** 0 **i** −3

3 **a** −5 **b** −13 **c** −5

**d** −80 **e** −59 **f** −27

**g** −10 **h** 2 **i** −132

4 **a** 2 **b** −5 **c** 0

**d** −9 **e** −1 **f** −2

5 **a** −1.7 **b** −7.5 **c** 6.79

**d** −12.78 **e** 8.243 **f** −11.465

6 **a** $\frac{3}{11}$ **b** $-\frac{3}{5}$ **c** $-\frac{25}{7}$ or $-3\frac{4}{7}$

**d** $-\frac{23}{15}$ or $-1\frac{8}{15}$ **e** $-\frac{92}{9}$ or $-10\frac{2}{9}$ **f** $-\frac{17}{60}$

7 **a** 4 **b** −7 **c** −1

**d** 10

8 **a** Negative integer **b** Negative integer

**c** Positive integer **d** Zero

**e** Negative integer **f** Positive integer

9 If the positive number is further from zero than the negative number, the sum will be positive. If the negative number is further from zero than the positive number, the sum will be negative. If the positive number and the negative number are the same distance from zero (opposite numbers), the sum will be zero.

10 **a** −18 + 98 **b** 80°C

11 1 m

12 **a** −8.25 + 5.3 **b** −2.95°C

**13** **a** −50 **b** −50 + 35 **c** \$15

**14** **a** −26.95 **b** −26.95 + 40 **c** \$13.05

**15** **a** 4205 m **b** 4643 m **c** 1362 m

**16** −3, −1, 1 and 3. Half of the highest result gives the highest number, 6 ÷ 2 = 3 and half the smallest result gives the smallest number, −6 ÷ 2 = −3. The second largest number is 4, which would be made up of the largest number, 3, as well as another dice number, in this case 1. Likewise, the next smallest number, −4, would be made up of the smallest number, −3, plus another dice number, in this case −1. Using a table to confirm all the results:

| + | −3 | −1 | 1 | 3 |
|---|---|---|---|---|
| −3 | −6 | −4 | −2 | 0 |
| −1 | −4 | −2 | 0 | 2 |
| 1 | 2 | 0 | 2 | 4 |
| 3 | 0 | 2 | 4 | 6 |

## EX p208 5C Adding and subtracting negative numbers

**1** **a** −3 + 5 **b** 6 − 5
**c** −2 − 4 **d** 7 + 1

**2** **a** −3 + (−6) = −3 − 6
**b** 1 − (−3) = 1 + 3
**c** 8 − (−9) = 8 + 9
**d** −5 + (−7) = −5 − 7

**3** **a** 7 + (−4) = 3
**b** −6 − (−4) = −2
**c** 2 − (−8) = 10
**d** 8 + (−3) = 5
**e** −3 − (−4) = 1
**f** −6 + (−1) = −7
**g** −4 − (−4) = 0
**h** −2 + (−7) = −9

**4** **a** −7 **b** 1 **c** 6
**d** 1 **e** −2 **f** 0
**g** 5 **h** 10 **i** 3

**5** **a** 13 **b** −11 **c** −29
**d** −23 **e** 65 **f** −68
**g** −22 **h** −71 **i** 300

**6** **a** 1.1 **b** 20.1 **c** −3.79
**d** −32.86 **e** 10.56 **f** −2.308

**7** **a** 1 **b** $-1\frac{5}{7}$ **c** −3
**d** $-9\frac{2}{3}$ **e** $-\frac{7}{20}$ **f** $2\frac{61}{84}$

**8** **a** Positive integer **b** Positive integer
**c** Negative integer **d** Negative integer
**e** Zero **f** Negative integer

**9** **a** −5 **b** 2
**c** −8 **d** 7

**10** **a** 4 − (−15) **b** 19°C

**11** **a** Sun: 6°C, Mon: 5°C, Tue: 9°C, Wed: 5°C, Thu: 4°C, Fri: 6°C, Sat: 5°C
**b** Tuesday

**12** **a** 74 **b** −31 − 74 **c** \$105

**13** **a** Turquoise Cave: −56.83 m, Amethyst Cave: −75.29 m
**b** −18.46 m
**c** 18.46 m
**d** Sample answer: If the negative number being subtracted is further from zero than the other negative number, the result will be positive. If the negative number being subtracted is closer to zero than the other negative number, the result will be negative.
**e** Sample answer: The positive number represents the difference in depth as negative differences have no meaning.

**14** **a** Deposit **b** Withdrawal
**c** Ayesha has money in her account.
**d** Ayesha owes money to the bank.
**e** +\$27.83 **f** Withdrawal of \$8.78

**15** **a** −8 **b** 9 **c** 4
**d** 14 **e** −15 **f** −5
**g** −11 **h** 3

**16** **a** **i** −6 **ii** −6 **iii** −6
**b** **i** −6 **ii** −6 **iii** −6
**c** −6, −6 **d** −6

**17** **a**

| | | |
|---|---|---|
| 4 | −3 | 2 |
| −1 | 1 | 3 |
| 0 | 5 | −2 |

**b**

| | | |
|---|---|---|
| 1 | 0 | −4 |
| −6 | −1 | 4 |
| 2 | −2 | −3 |

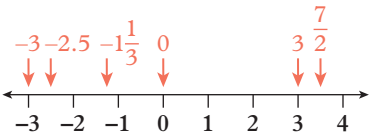
**c**

| | | |
|---|---|---|
| 6 | −6 | −9 |
| −18 | −3 | 12 |
| 3 | 0 | −12 |

**18** **a** $\frac{7}{12}+\frac{3}{4}=1\frac{1}{3}$ **b** $\frac{5}{8}-\left(-\frac{3}{5}\right)=\frac{49}{40}$
**c** $\frac{8}{5}+\left(-\frac{6}{15}\right)=1\frac{1}{5}$

## 5 Checkpoint

**1** **a**




**b** **i** 3 > −3 **ii** −3 < −2.5 **iii** $-1\frac{1}{3}<0$
**iv** $\frac{7}{2}>-1\frac{1}{3}$ **v** $-2.5<\frac{7}{2}$ **vi** 0 > −3

**2** **a** $-9, -4, -1, 0, 7$

**b** $-6, -\frac{9}{2}, -2, 6, 10.5$

**c** $-15, -1.5, -\frac{3}{4}, 2, \frac{20}{3}$

**3** **a** $-6$ **b** $-16$ **c** $-9$
**d** $-80$ **e** $-30$ **f** $-22$

**4** **a** $4$ **b** $-4$ **c** $9$
**d** $-20$ **e** $-5$ **f** $-6$

**5** **a** $-1.2$ **b** $-15.4$ **c** $12.47$
**d** $-11.38$

**6** **a** $-27$ m **b** $-27 + 12$
**c** 15 m below sea level

**7** **a** $-5$ **b** $-2$ **c** $13$
**d** $6$ **e** $-5$ **f** $0$

**8** **a** $1.8$ **b** $15.7$ **c** $-11.5$
**d** $-45.36$ **e** $13.77$ **f** $-19.6$

**9** **a** $\frac{5}{7}$ **b** $1\frac{2}{3}$ **c** $-1\frac{7}{10}$ or $-\frac{17}{10}$
**d** $-1\frac{1}{4}$ or $-\frac{5}{4}$ **e** $6\frac{1}{10}$ or $\frac{61}{10}$ **f** $4\frac{4}{15}$ or $\frac{64}{15}$

**10** **a** $-12.2 + (-3.7) + 5.4$
**b** 10.5 m below sea level

## EX 5D The Cartesian plane

p216

**1** **a** $D$ **b** $A$ **c** $F$
**d** $E$ **e** $C$ **f** $B$

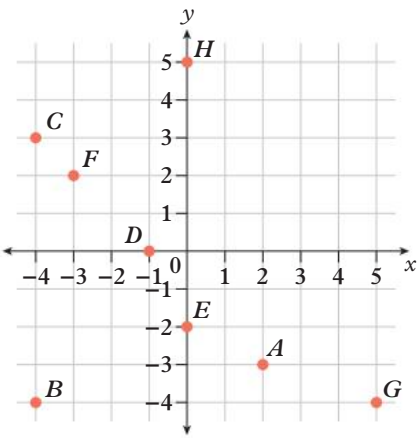
**2** **a** B **b** C

**3** $A(2, 5)$, $B(3, -2)$, $C(4, 0)$, $D(-5, -4)$, $E(-3, 3)$, $F(0, 2)$, $G(2, -4)$, $H(0, -3)$

**4** **a** $F$ and $H$; $A$ and $G$ **b** $D$ and $G$

**5** $(0, 0)$

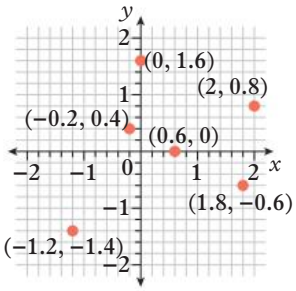
**6**



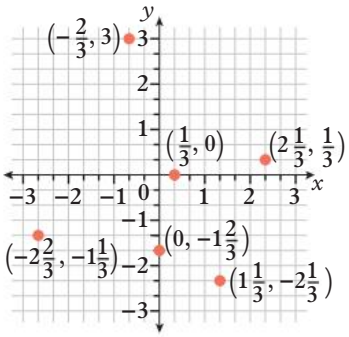

**7** **a** $A(1\frac{1}{2}, 2)$, $B(-2, -3\frac{1}{2})$, $C(-3\frac{1}{2}, -1\frac{1}{2})$, $D(-4\frac{1}{2}, 2\frac{1}{2})$, $E(-2\frac{1}{2}, 3\frac{1}{2})$, $F(4\frac{1}{2}, 3\frac{1}{2})$, $G(\frac{1}{2}, 0)$, $H(0, 4\frac{1}{2})$, $I(3\frac{1}{2}, -2)$, $J(4\frac{1}{2}, -\frac{1}{2})$

**b** $A(-0.4, 0.6)$, $B(0.4, 0.4)$, $C(-0.8, -0.6)$, $D(0.8, 0)$, $E(0.5, -0.7)$, $F(0, -0.5)$, $G(-0.9, 0.3)$, $H(0.2, -0.2)$, $I(0.3, -0.9)$, $J(-0.5, -0.3)$

**8** **a** **i, ii**

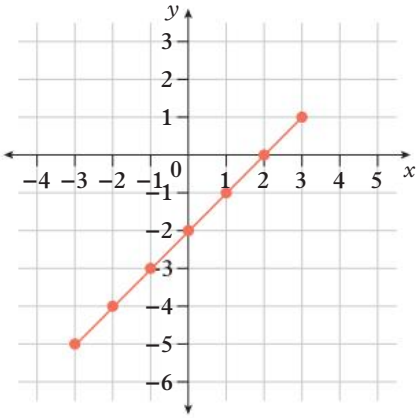



**b** **i, ii**




**9** **a** **i** $(-3, -5)$, $(-2, -4)$, $(-1, -3)$, $(0, -2)$, $(1, -1)$, $(2, 0)$, $(3, 1)$

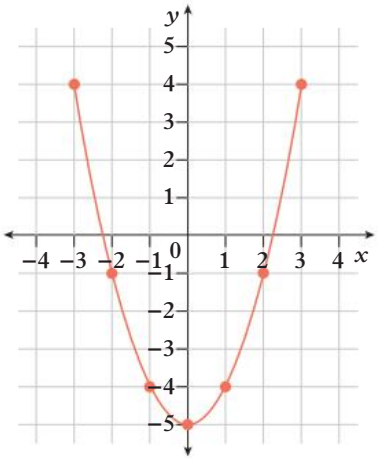
**ii**




**iii** The points lie on a straight line

**b** **i** $(-3, 4)$, $(-2, -1)$, $(-1, -4)$, $(0, -5)$, $(1, -4)$, $(2, -1)$, $(3, 4)$

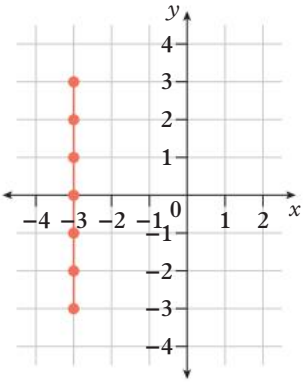
**ii**




**iii** The points do not lie on a straight line

**c** **i** (–3, –3), (–3, –2), (–3, –1), (–3, 0), (–3, 1), (–3, 2), (–3, 3)

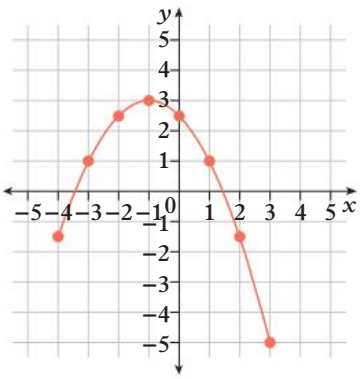
**ii**



**iii** The points lie on a straight line.

**d** **i** (–4, $-1\frac{1}{2}$), (–3, 1), (–2, $2\frac{1}{2}$), (–1, 3), (0, $2\frac{1}{2}$), (1, 1), (2, $-1\frac{1}{2}$), (3, –5)

**ii**



**iii** The points do not lie on a straight line.

**10** **a** On the *y*-axis (vertical axis)

**b** On the *x*-axis (horizontal axis)

**c** At the intersection of the *x*-axis and the *y*-axis (the origin)

**11** **a** **i** (–5, –5) **ii** (3, –3) **iii** (–7, 2) **iv** (7, 3)

**b** Butterfly

**c** Fly

**d** Ladybird and fly

**e** Dragonfly and fly

**12** **a** **i** Alex

**ii** Alex

**b** **i** 3 km

**ii** 2 km

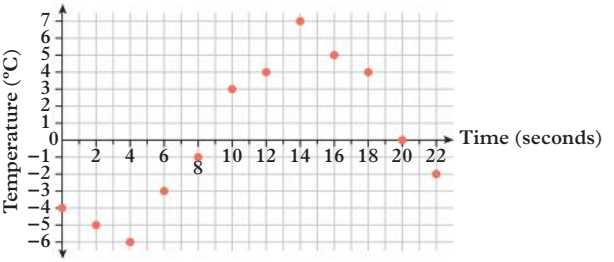
**c** (–3, –5)

**13** **b** Scale should be from 0 to 22 on the horizontal axis. Negative numbers not needed because negative time has no meaning.

**c** Scale should go from –6 to 7 on the vertical axis.

**d** (0, –4), (2, –5), (4, –6), (6, –3), (8, –1), (10, 3), (12, 4), (14, 7), (16, 5), (18, 4), (20, 0), (22, –2)

**a, e**

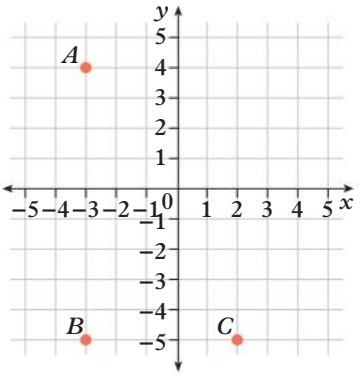



**f** –6°C at 4 am

**g** 7°C at 2 pm

**14** **b** 9 units

**a, c**




**d** 5 units

**e** *D*(2, 4)

**15** **a** 6 units

**b** Sample answer: (–2, –3) and (4, –3)

**c** Other possible answer: (–2, 9) and (4, 9).

**d** 12 units

## EX 5E Interpreting graphs

p222

**1** **a** 2 km **b** 4 h

**c** **i** 1 hour **ii** 2 hours

**d** 4 km

**2** **a** **i** 0 m **ii** 120 m **iii** 360 m

**b** Levi was stationary.

**c** **i** At a local shop

**ii** At his friend's house **iii** At home

**d** **i** 240 m **ii** 360 m

**e** **i** 2 minutes **ii** 4 minutes **iii** 2 minutes

**3** **a** 2 km/h **b** 0 km/h **c** 1 km/h

**4** **a** 3 km **b** 2 pm **c** 1 km

**d** 1 hour **e** 4 pm

**f** From dental clinic to home; 4 km/h

**5** **a** 9:00 pm

**b** 11 μg/m³

**c** Possible answers: 9:00 am and 3:00 pm, 10:00 am and 11:00 am, 12:00 pm and 12:00 am, 6:00 pm and 7:00 pm, 2:00 am and 4:00 am, 6:00 am and 7:00 am

**d** 9:00–10:00 pm

**6** **a** **i** False

**ii** The distance from home is increasing during this section.

**b** **i** True

**ii** The distance from home stays the same during this section.

**c** **i** False

**ii** The slope is steeper for section C than for section A.

**d** **i** True

**ii** The slope is steeper for section D than for section C.

**7** **a** Tom **b** Sue **c** Molly

**8** No; you cannot be in different places at the same time.

**9** **a** –10°C **b** –5°C

**c** After 3 min **d** 35°C

**e** From 3 to $3\frac{1}{2}$ min and from 5 to $5\frac{1}{2}$ min

**f** The rate at which the temperature is changing

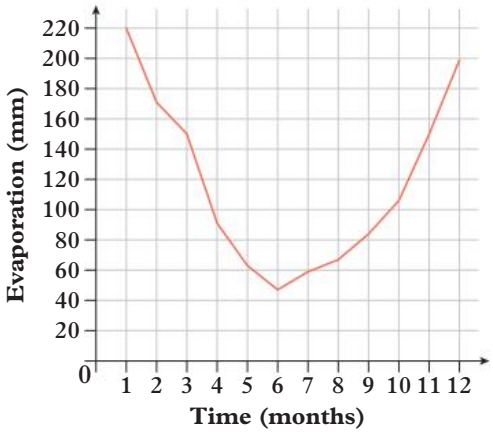
**g** From $3\frac{1}{2}$ to 5 min and from $5\frac{1}{2}$ to 6 min

**h** 1 min

**10** **a** 220 mm **b** 150 mm

**c** The rate at which the evaporation level is changing

**d, e**




**f** January, 220 mm

**g** June, around 50 mm

**h** Sample answer: temperature, humidity and wind affect the amount of evaporation.

**11** **a** 0 to 45

**b** –20 to 1.5

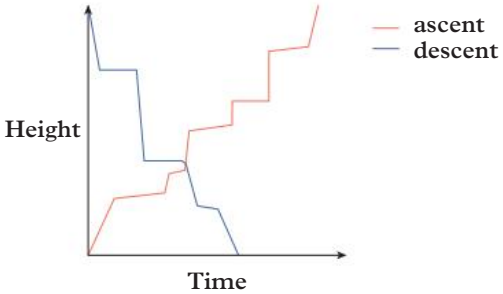
**c** Yes, the vertical axis requires negative values as it measures the depth of the dive above and below the surface of the water.

**d** 30 minutes

**e** To dive to the bottom of the lake. The descent took 30 minutes, whereas the ascent took 15 minutes.

**f** 5 m below the water, 10 minutes

**12** Yes, there will be exactly one point as no matter the speed, the line graphs will intersect at one point.




## CHAPTER 5 review

### Multiple-choice

| | | |
|---|---|---|
| **1** C | **2** A | **3** A |
| **4** D | **5** B | **6** A |
| **7** E | **8** E | **9** D |
| **10** A | **11** B | |

### Short answer

**1** **a** False **b** True

**c** True **d** False

**2** **a** –10, –5, 0, 2, 12, 25

**b** –12, –6, –3, 0, 6, 9

**3** **a** $2\frac{2}{5}, \frac{9}{5}, 1, 0, -\frac{9}{10}, -2$

**b** $5.5, 5\frac{1}{4}, \frac{7}{2}, -2\frac{3}{4}, -\frac{15}{4}, -4.75$

**4** **a** –86 **b** 615

**c** 8848 **d** –35

**5** **a** –8 **b** 1

**c** –14 **d** –1

**6** 3.5 m below sea level

**7** **a** –9 **b** 1

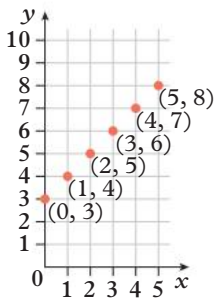
**c** –2 **d** 8

**8** –$50

**9** **a**

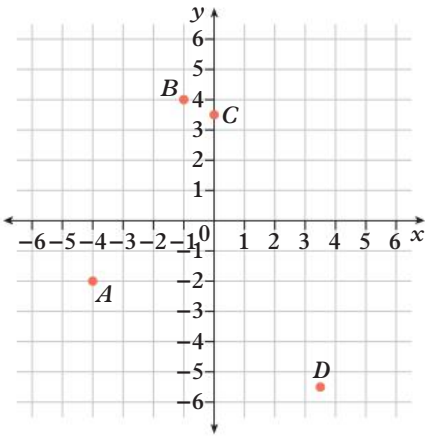
| ***x*-coordinate** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| ***y*-coordinate** | 3 | 4 | 5 | 6 | 7 | 8 |
| **coordinates** | (0, 3) | (1, 4) | (2, 5) | (3, 6) | (4, 7) | (5, 8) |

**b**




**10** A(3, 1), B(–3, 3), C(4, –2), D(1, 0), E(–2, –3), F(0, –2)

**11**




**12** **a** 4 km **b** 30 minutes

**c** **i** 1 hour **ii** 30 minutes **iii** 1 hour

**d** 4 km/h

## Analysis

**a** **i** 10 m **ii** –2 m **iii** 0

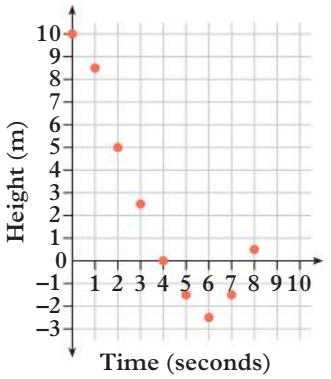
**iv** –8 m

**b** **i** 12 m **ii** 6 m

**c** –6 m

**d** (0, 10), (1, 8.5), (2, 5), (3, 2.5), (4, 0), (5, –1.5), (6, –2.5), (7, –1.5), (8, 0.5)

**e**




**f** A curve

**g** The rate of change of the height of the seagull

**h** The seagull dived to the water, plunged under the water and then came back to the surface.

**i** No

EX p230

# SEMESTER 1 review

## Short answer

**1** **a** $4 \times 1000 + 8 \times 100 + 10 + 7$

**b** $10\,000 + 9 \times 1000 + 3 \times 10$

**c** $6 \times 6 \times 6 \times 6$

**d** $51 \times 51 \times 51$

**2** **a** 6 **b** 23 760 **c** 1043

**d** 81 **e** 752 **f** 462

**3** **a** **i** 135 680 **ii** 140 000 **iii** 100 000

**b** **i** 60 **ii** 57.7 **iii** 57.658

**iv** 57.66 **v** 58

**4** **a** 1331 **b** $\frac{13}{8}$ **c** 56.11

**d** –23 **e** $-\frac{67}{18}$ **f** 0.24

**5** **a** 387 **b** $\frac{7}{8}$ **c** 51.29

**d** –7 **e** $-\frac{95}{18}$ **f** 1.1

**6** **a** 27 375 **b** $\frac{3}{2}$ **c** 1.107

**d** $47\frac{2}{3}$

**7** **a** 62.3 **b** $\frac{5}{2}$ **c** 85

**d** 2

**8** **a** 10 **b** $\frac{3}{4}$ **c** 54

**d** 140% **e** 25 **f** 11 : 3

**9** **a** HCF = 16, LCM = 160

**b** HCF = 8, LCM = 120

**c** HCF = 7, LCM = 294

**d** HCF = 42, LCM = 2310

**10** **a** Start with a cross made up of five squares and form each new shape by adding a square to each arm of the previous shape.

**b** Start with 7 and form each subsequent term, subtracting six from the previous term. Subtract six

**c** Start with 3 and form each subsequent term, doubling the previous term.

**d** Start with $\frac{150}{7}$ and form each subsequent term, by dividing the previous term by 5.

**11** **a** Composite **b** Prime

**c** Prime **d** Composite

**12** **a** True **b** False **c** False

**d** True

**13** **a** 821 **b** 29 **c** 16 191

**d** 342

**14** **a** 33 **b** 10 **c** $\frac{3}{7}$

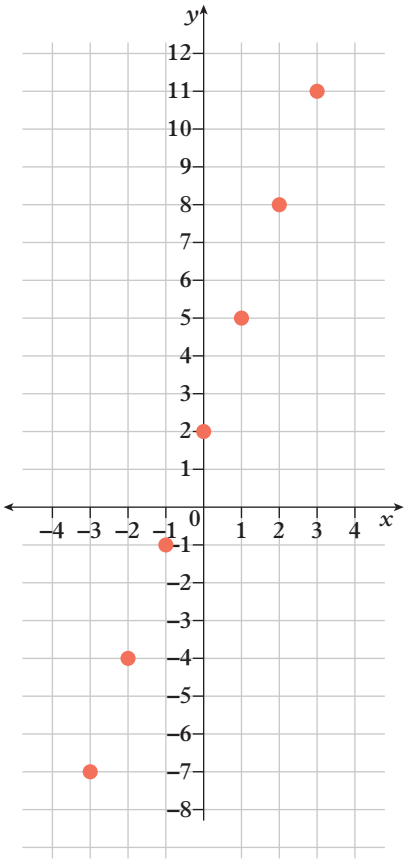
**d** 2.14 **e** –3 **f** 5.73

**15** **a** $8\frac{1}{4}$ **b** $\frac{5}{2}$ or $2\frac{1}{2}$ **c** 11:15

**d** 13:5

**16** **a** (–3, –7), (–2, –4), (–1, –1), (0, 2), (1, 5), (2, 8), (3, 11)

**b**




**c** The points form a straight line.

**17** **a** 19°C **b** 22°C

**18**

| Percentage | Fraction | Decimal |
|---|---|---|
| 35% | $\frac{7}{20}$ | 0.35 |
| 775% | $\frac{31}{4}$ | 7.75 |
| 4.5% | $\frac{9}{200}$ | 0.05 |
| 971.43% | $\frac{68}{7}$ | 9.71 |
| 56% | $\frac{14}{25}$ | 0.56 |
| 642% | $\frac{321}{50}$ | 6.42 |
| 260% | $\frac{13}{5}$ | 2.6 |
| 893.1% | $\frac{8931}{1000}$ | 8.93 |

**19** **a** $\frac{5}{6}, \frac{9}{8}, \frac{8}{6}, \frac{9}{5}$

**b** 0.05, 0.5, 5.0, 5.5

**c** $-4, -3, 1, 5$

**d** $-\frac{11}{25}, -\frac{3}{7}, -0.41, -\frac{2}{5}$

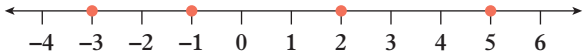
**20** **a** $2^3 \times 7$ **b** $3^2 \times 5$ **c** $2^3 \times 3^3$

**d** $2 \times 5^2 \times 7$

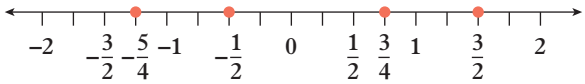
**21** **a** 343 **b** 256 **c** 81

**d** 289

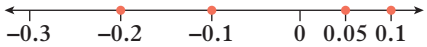
**22** **a**




**b**




**c**




**23** **a** $3\frac{1}{4}$ **b** $\frac{18}{7}$ **c** 47%

**d** 182%

**24** **a** 4 **b** 6

**25** 11

**26** **a** 2520 **b** 14

## Analysis

**1** **a** p, w, r, w, p, y, p, w, r, w, p, y

**b** Plant every 6 m, starting at 6 m.

**c** Plant the first 1m, then 4m, then 2m, then 4m, then 2 m, alternating 4 m and 2 m apart.

**d** White **e** 8

**2** **a** 15

**b** 2 cannolis, 3 eclairs, 7 macarons

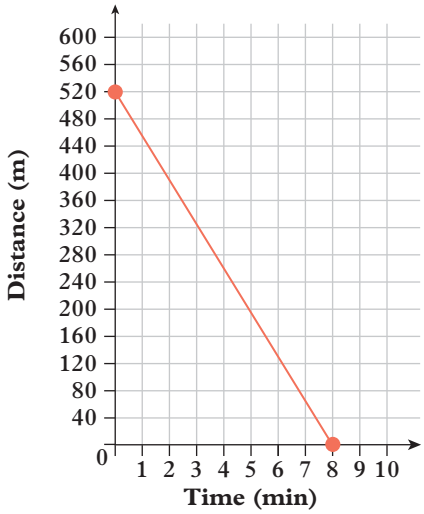
**c** $\frac{7}{12}$ **d** \$140.25 **e** \$520.50

**f** \$34.70

**3** **a** 240 m **b** 520 m **c** 280 m

**d** 12 min **e** 4 min

**f**




**g** 65 m/min

## EX EXPLORATIONS 1

p234

**1** **a** 12, 53 and 54

**b** 75, 76 and 145

**c** 39, 40, 57 and 58

**d** 128

**e** 158

**2** **a** 28 minutes

**b** 60 minutes (HOTSHOTS only takes 4 minutes)

**c** All word-lengths must be factors of 132 and the factors 3, 4 and 11 must each occur in at least one word-length, e.g. THIS WAS COMPLICATED or I DO LIKE COMPLICATED THINGS

**d** 2 minutes using 1- and 2- letter words, e.g. IF IT IS TO BE IT IS UP TO ME (or shorter if non-sensical sentences are allowed)

**e** 420 minutes using 1-, 3-, 4-, 5- and 7-letter words, e.g. I CAN SOLVE THIS PROBLEM

**f** **a** 140 minutes

**b** unchanged

**c** answers will vary

**d** 6 minutes, e.g. IT IS TRICKY TO DO TRICKS (or shorter if non-sensical sentences are allowed)

**e** 840 minutes, e.g. I TRY A TRICKIER PROBLEM

**3** **a** 60%; yes, from 120% then 50%, or vice versa

**b** Approximately 195.3% from 125% three times

**c** 50%-80%-120%-125% or 50%-80%-125%-150%

**d** No: if there are only 3, they must be 50%, 80% and 125%, but then 150% cannot be achieved.

**e** One possibility is 50%-70%-80%-150%, but there are others.

**4** **a** $2, 3, -\frac{1}{3}, 3, 4, -\frac{1}{4}, \frac{3}{4}, -\frac{4}{3}, -\frac{1}{3}, 3$

**b** R T T

**c** R T R T T R T T

**d** Apply R T, then repeatedly apply R T T as many times as necessary.

**e** Apply T R, then repeatedly apply R T R T R as many times as necessary.

**f** Yes: via part **d**, part **e** or repeatedly applying T, any number can be turned into 0 and vice versa.

**5** **a** $(4, -3)$

**b** $(-1, 2)$, $(1, -4)$ and $(-3, -2)$

**c** $P(-3, 0)$ and $Q(0, 1)$

# CHAPTER 6 Algebra

## 6A Variables

EX p240

**1** **a** A runner's heart rate, the speed at which they run

**b** Wage, number of hours spent working

**c** Mass of chicken, time to roast

**d** $x, y$

**e** Time, temperature

**f** Area, length

**g** $x, y$

**h** Hours after 6:00 am number of passengers

**i** $V, n$

**2**

| **Number of bananas** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Total cost ($)** | 2 | 4 | 6 | 8 | 10 |

**3**

| **Time (h)** | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Volume of water in the tank (L)** | 10 | 20 | 30 | 40 | 50 |

**4**

| **Distance travelled (km)** | 0 | 100 | 200 | 300 | 400 | 500 |
|---|---|---|---|---|---|---|
| **Volume of petrol in the tank (L)** | 50 | 40.8 | 31.6 | 22.4 | 13.2 | 4 |

**5**

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $c$ ($) | 15 | 30 | 45 | 60 | 75 |

**6**

| $t$ | 0 | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|---|
| $p$ | 150 | 110 | 70 | 30 | 0 | 0 |

**7** **a**

| $x$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $y$ | 5 | 7 | 9 | 11 | 13 |

**b**

| $x$ | 7 | 12 | 17 | 22 | 27 |
|---|---|---|---|---|---|
| $y$ | 0 | 5 | 10 | 15 | 20 |

**c**

| $a$ | 10 | 11 | 13 | 16 | 20 |
|---|---|---|---|---|---|
| $b$ | 20 | 22 | 26 | 32 | 40 |

**d**

| $m$ | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| $n$ | 1 | 2 | 3 | 4 | 5 |

**8** **a** C **b** A **c** D **d** B

**9** **a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 4 | 5 | 6 | 7 | 8 |

**b**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 | 4 |

**c**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 4 | 8 | 12 | 16 |

**d**

| $x$ | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| $y$ | 12 | 6 | 4 | 3 | 2 |

**e**

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 1 | 3 | 5 | 7 | 9 |

**f**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 1 | 4 | 9 | 16 |

**10** Answers will vary based on the selected values of $x$. Sample answers:

**a**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 10 | 20 | 30 | 40 | 50 |

**b**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | $\frac{1}{3}$ | $\frac{2}{3}$ | 1 | $\frac{4}{3}$ | $\frac{5}{3}$ |

**c**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | −7 | −6 | −5 | −4 | −3 |

**d**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 10 | 13 | 16 | 19 | 22 |

**11 a**

| Number of customers | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Amount of tips earned ($) | 0.50 | 1.00 | 1.50 | 2.00 | 2.50 |

**b** Number of customers, amount of tips earned

**c i** \$2.50 **ii** \$3.00 **iii** \$6.00

**d i** 4 customers

**ii** 20 customers

**iii** 200 customers

**12 a** B **b** C **c** A

**13 a** The length of dough that comes out of the machine is three times the length of dough that goes into the machine.

**b**

| Length of dough in (cm) | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| Length of dough out (cm) | 30 | 60 | 90 | 120 | 150 | 180 |

**c i** 300 cm **ii** 80 cm

**14 a** No. $x$ is not a variable because the value of $x$ is fixed. For the calculation to be true, $x = 20$.

**b i** specific unknown number

**ii** specific unknown number

**iii** variable

**15 a** $x$ represents the number of diamonds in a given track.

**b** $y$ represents the number of concrete slabs required to make up a given track.

**c** The largest track that can be made using 22 concrete slabs would consist of 5 diamonds, joined end-to-end, with one horizontal track at the entrance of the first diamond and the exit of the last.

**d** Sample answer: The number in front of the $x$ should be adjusted to match the number of sides of the chosen shape. For example, a hexagonal design can be represented by the formula $y = 6x + 2$.

## EX p245 6B Writing formulas

**1 a** Add 4 each time

**b** Add 11 each time

**c** Multiply by 10 each time

**d** Divide by 2 each time

**2 a** 40, 47, 54 **b** 62, 56, 50

**c** 36, 49, 64 **d** 720, 5040, 40 320

**3 a** 12 **b** 4

**4 a** 7 **b** 4

**c** 5 **d** 3

**5 a** $y = x + 2$ **b** $t = 3p$ **c** $d = c - 6$

**d** $n = 7k$ **e** $y = \frac{x}{2}$ **f** $h = m + 4$

**6** $n = p - 3$

**7** $t = x + 20$

**8** $n = 2p$

**9 a** $y$ = Ying Ying's age, $s$ = her little sister's age

**b** $y = s + 5$

**10** If $p$ = number of people, $d$ = number of dogs, $d = 2p$

**11 a**

| Number of triangles | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of toothpicks | 3 | 6 | 9 | 12 | 15 | 18 |

**b** $t$ = number of triangles, $p$ = number of toothpicks

$p = 3t$

**c i** 360 toothpicks **ii** 79 triangles

**12 a**

| Time (min) | 6 | 12 | 18 | 24 | 30 |
|---|---|---|---|---|---|
| Distance (km) | 1 | 2 | 3 | 4 | 5 |

**b** $d = \frac{t}{6}$ **c** 10 km

**13 a** \$5 **b** \$50

**14 a** $d$ = number of days **b** \$110

**c** \$25 **d** \$285

**15 a** $y = 2x + 3$ **b** $y = 5x - 2$ **c** $y = 3x + 4$

**16 a** The numbers above the table show the difference between the $x$ values. The numbers below the table show the difference between the $y$ values.

**b** The numbers below the $y$ values are all identical and the same as the number by which $x$ is being multiplied in each formula.

**c** The difference between the $y$ values gives the number by which $x$ is being multiplied and acts as a starting point for more difficult formulas.

**17 a** $y = 3x + 2$ **b** $y = 4x - 3$ **c** $y = 2x + 6$

**d** $y = 7x + 1$ **e** $y = 3x - 8$ **f** $y = 5x - 4$

**18 a i**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5 | 10 | 15 | 20 | 25 |

**ii** $y = 5x$

**b i**

| $x$ | 3 | 5 | 7 | 9 | 11 | 13 |
|---|---|---|---|---|---|---|
| $y$ | 18 | 30 | 42 | 54 | 66 | 78 |

**ii** $y = 6x$

**c i**

| $x$ | 10 | 15 | 20 | 25 | 30 | 35 |
|---|---|---|---|---|---|---|
| $y$ | 2 | 3 | 4 | 5 | 6 | 7 |

**ii** $y = \frac{x}{5}$

**d i**

| $x$ | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|
| $y$ | 21 | 17 | 13 | 9 | 5 |

**ii** $y = 4x + 1$

**e i**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 1 | 3 | 5 | 7 | 9 |

**ii** $y = 2x - 1$

**f** **i**

| $x$ | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | −6 | −2 | 2 | 6 | 10 | 14 |

**ii** $y = 2x - 10$

**19** No. There is more than one possible relationship between 3 and 18 and without another pair of $x$ and $y$ values, we cannot verify which relationship is correct.

**20** **a**

| Time (min) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ants | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |

**b** $t$ = time, $a$ = number of ants
$a = 2t - 1$

**c** 119 ants

## EX 6C Terms, expressions and equations

p251

**1** **a** **i** 2 **ii** $x$
**iii** 5 **iv** 6
**b** **i** 3 **ii** $a, b$
**iii** 2, 4 **iv** 3
**c** **i** 2 **ii** $m$
**iii** 1 **iv** 4
**d** **i** 2 **ii** $k$
**iii** 11 **iv** 2
**e** **i** 2 **ii** $c, d$
**iii** 4, 7 **iv** none
**f** **i** 1 **ii** $y$
**iii** 13 **iv** none
**g** **i** 2 **ii** $f$
**iii** 1 **iv** 5
**h** **i** 4 **ii** $x, w, y$
**iii** 6, 1, 3 **iv** 1
**i** **i** 3 **ii** $p, m$
**iii** 8, 1 **iv** 10

**2** **a** False, it is an equation as it has an equals sign.
**b** True **c** True
**d** False, constant is 3.
**e** True **f** True
**g** False, it is an equation as it has an equals sign.
**h** True **i** True
**j** False, it is an expression as there is no equals sign.
**k** True **l** True

**3** **a** Equation **b** Expression
**c** Equation **d** Equation
**e** Expression **f** Equation
**g** Equation **h** Expression

**4** **d**, **f** and **g**

**5** **a** $a + 7$ **b** $\frac{b}{2}$ **c** $c - 5$
**d** $3d$ **e** $8 - e$ **f** $2f$
**g** $\frac{4}{g}$ **h** $h + 9$ **i** $k + m + n$

**6** **a** 2 **b** 1 **c** 2
**d** 1 **e** 2 **f** 1
**g** 1 **h** 2 **i** 3

**7** **a** $x + 8$ **b** $x - 15$
**c** $3x$ **d** $\frac{x}{7}$

**8** **a** $4n + 5$ **b** $\frac{n}{3} + 7$
**c** $2n - 9$ **d** $6n - 1$

**9** **a** $x + 5 = 11$ **b** $x - 3 = 4$ **c** $6x = 18$
**d** $\frac{x}{2} = 5$ **e** $2x = 24$ **f** $10 - x = 8$
**g** $3x + 1 = 7$ **h** $\frac{2}{5}x - 8 = 0$

**10** **a** $y = \frac{x}{6}$ **b** $x + 11 = 20$ **c** $y = x - 9$
**d** $b = a + 130$ **e** $q = 10p$ **f** $c = \frac{1}{4}d$
**g** $m = 2n$ **h** $f = \frac{3}{10}g$

**11** Similarities: both contain equals signs. Differences: formulas contain two or more variables, equations can contain no variables, one variable, or two or more variables.

**12** **a** $n + 10$ **b** $3n$
**c** $\frac{n}{2}$ **d** $n - 4$

**13** **a** **i** $2p$ **ii** $52p$
**b** $p + w$

**14** **a** $c$ = the cost of hiring a canoe, $n$ = the number of hours
**b** 2
**c** The booking fee

**15** **a** The cost of apples, the cost of bananas and the cost of oranges
**b** 3. The total cost is made up of the cost of 3 different fruits.
**c** One example is: let $a$ represent cost of an apple, let $b$ represent cost of a banana, let $c$ represent cost of an orange.
**d** **i** $10a$ **ii** $7b$ **iii** $6c$
**e** $10a + 7b + 6c$
**f** $w = 10a + 7b + 6c$

**16** If $c$ = the cost of chicken per gram, $d$ = the cost of cheese per gram, $w$ = the total weekly cost of deli goods, $w = 500c + 300d$.

**17** If $n$ = the number of Year 7 students, $p$ = the number of students who want to attend the planetarium, $t$ = the total cost of admissions, $t = 10n + 5p$.

**18** **a** Sample answer: the number of apples in a bowl after 4 apples are added to the bowl
**b** Sample answer: the number of students remaining out a class of 20 when $b$ students are away
**c** Sample answer: the cost of buying $c$ drinks at \$6 each
**d** Sample answer: the number of biscuits each person receives when $d$ biscuits are divided up between 5 people

**19** $y^2 + 3$

**20** **a** $x + 5$ **b** $x + 1$ **c** $x - 4$
**d** $x$ **e** $2x + 7$ **f** $3x + 1$

**21** $x < \frac{1}{2}$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EX p257

## 6D Order of operations in algebra

1 a $7(x+6)$ b $2(x-3)$
c $5(x+1)$ d $8(x-2)$
e $\frac{x+4}{7}$ f $\frac{x-5}{11}$
g $\frac{9+x}{2}$ h $\frac{1-x}{6}$

2 a $10(a+1)$ b $\frac{a-9}{5}$
c $\frac{a}{3}+6$ d $\frac{a+21}{7}$
e $(7a)^2$ f $a^2-12$
g $\sqrt{a}-4$ h $\sqrt{10a}$

3 7 less than $a$ all divided by 2

4 a $y$ is 10 more than 7 times $x$
b The sum of $x$ and 12 all divided by 9 is $y$
c sqrt($x$) less than 6
d $q$ is 8 less than $p$ divided by 3
e The square root of all of 2 times $s$ minus 1
f $s$ is 1 less than the square of $t$
g Half of 1 less than the square of $a$ is $b$
h 16 less than the square of 7 more than $m$ is $n$

5 a Multiply by 3 b Subtract 5 c Add 6
d Multiply by 3 e Divide by 8 f Square
g Square h Multiply by 8 i Square

6 a Number of passengers from the terminal, number of passengers on the bus
b $p-4$ c $t=2(p-4)$

7 a Dollars received on birthday, remaining money
b $m-15$ c $r=\frac{m-15}{2}$

8 $s=4(n+3)$

9 $y=\frac{x+30}{2}+7$

10 Sample answer: $n=\frac{3x-5}{x}$

11 Sample answer: $y=\frac{1}{3}\left(\frac{9}{10}x-75\right)$

12 Sample answer: $y=\frac{450+50x}{x}$

13 Sample answer: $n=2r^2$

14 Sample answer: $d=\frac{50(t-2)}{2}$

15 a Sample answer: $n=\frac{l}{3}+1$
b Sample answer: $n=\frac{4l}{3}$
c Sample answer: If the number of light poles placed along one side of the ground was multiplied by 4, the light poles at each corner would be counted twice.

16 Sample answer: $8n+16(n+8)+11(3n)$, where $n$ is the number of children's tickets sold

## 6 Checkpoint

1 a Number of rooms in hotel, number of guests in hotel
b Time spent driving, speed of the car

2 a

| Number of cups of coffee | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Profit ($) | 2 | 4 | 6 | 8 | 10 |

b Number of cups of coffee, profit
c i \$10 ii \$30
d i 4 ii 8

3 a $y=3x$ b $n=m-6$

4 $g=\frac{15}{n}$

5 a i 2 ii $x$
iii 10 iv 2
b i 1 ii $f$
iii $\frac{1}{2}$ iv none
c i 3 ii $a, b$
iii 1, 5 iv −12
d i 2 ii $x, y$
iii 1, −1 iv none

6 a $t+6$ b $k-11$
c $6b$ d $\frac{h}{9}$

7 a $y=\frac{x}{2}$ b $c=b+9$
c $n=m-20$ d $q=6p$

8 a $7\sqrt{a+6}$ b $\frac{8-b}{3}$
c $\frac{c}{9}+6$ d $(d+4)^2$

9 $4(n+2)$

10 $n=\frac{p-1}{3}$

EX p262

## 6E Substitution

1 a 11 b 4 c 18
d 3 e 38 f 15

2 a 17 b 4 c 40
d 1 e 13 f 0

3 a 8 b 14 c 17
d 26 e 5 f 6

4 a 3 b 15 c 75
d −1 e −5 f −4

5 a 9 b −6 c −8
d −1 e 14 f 20

6 a $\frac{2}{9}$ b 2 c 1
d $\frac{5}{12}$ e $-2\frac{7}{12}$ f $1\frac{1}{4}$

7 a 6 b 9 c 30
d 0 e 17 f 2
g 48 h 1 i 12
j 30 k 5 l 10

8 a $2\times a\times b$ b $a\times a$ c $a\times a\times b$

9 a 9 b 250 c 14
d 17 e 36 f 56
g 135 h 150 i 3
j 4

**10** **a** 7 **b** 1 **c** 13
**d** 2 **e** −2 **f** 1
**g** 10 **h** −6 **i** −8
**j** 6

**11** **a** 4 **b** 52 **c** −14
**d** 36.4 **e** 29.8 **f** 2.8
**g** −5 **h** 2

**12** **a** $5r + 2d$ **b** \$25 **c** \$54

**13** **a** $m = 12y$
**b** **i** 60 **ii** 120 **iii** 1200
**iv** 960

**14** **a** $l = 8n$
**b** **i** 16 **ii** 40
**iii** 64 **iv** 88
**c** 1848

**15** **a** **i** 5 **ii** 4
**iii** 2.1 **iv** 5.5
**b** 20 m/s

**16** **a** **i** 15°C **ii** 30°C
**iii** 0°C **iv** 35°C
**b** **i** 68°F **ii** 212°F
**iii** 77°F **iv** 140°F
**c** 5°C
**d** 50°F = 10°C, so 15°C is higher

**17** **a** **i** $p = 4s$ **ii** 8 cm
**b** **i** $p = 5s$ **ii** 10 cm
**c** $p = ns$

**18** **a** $m = 15x + 18y$ where $x$ is the number of hours Alex works during the week, $y$ is the number of hours Alex works on Sunday and $m$ is the amount of money Alex earns in a week
**b** \$654
**c** 38 hours
**d** \$24

**19** **a** $y = x(x + 1)$ where $x$ is the lower number and $y$ is the product of the consecutive numbers
**b** 506 or 552
**c** The other number could either be 22 or 24.

**20** **a** Possible answers are 19, 29, 39, 49, 59, 69, 79, 89 and 99.
**b** Sample answer: If the digits of the number are $a$ and $b$ respectively then this situation can be modelled by the equation $10a + b = a + b + ab$. The solution to this equation is $b = 9$, so any two-digit number ending in 9 satisfy the question.

## EX p267 6F Simplifying

**1** **a** $3a$, $10a$ and $9a$
**b** $x$, $2x$, $-2x$ and $0.5x$
$2y$ and $0.5y$
**c** $7pq$, $\frac{1}{3}pq$ and $qp$
$18q$ and $-\frac{1}{3}q$
**d** $8m^2$, $m^2$ and $5.56m^2$
$9m$ and $-m$
**e** $11b$ and $\frac{3}{2}b$
$3ab$, $-ab$, $6ba$ and $9ba$
**f** $9xyz$ and $zyx$
$3xy$, $13xy$ and $-yx$
**g** no like terms
**h** $g^2h$ and $0.01g^2h$
$4gh$ and $hg$
$\frac{1}{2}gh^2$ and $10gh^2$

**2** **a** True **b** True
**c** False, the pronumeral components of the terms are different ($a$ and $d$)
**d** True
**e** False, the pronumeral components of the terms are different ($a$ and $ac$)
**f** True
**g** False, the pronumeral components of the terms are different ($p$ and $p^2$)
**h** True

**3** Answers will vary. Sample answer given.
**a** $2y$, $6y$, $\frac{1}{2}y$ **b** $3a$, $-2a$, $9a$
**c** $5n$, $6n$, $11n$ **d** $-ab$, $3ab$
**e** $gh$, $\frac{3}{2}gh$, $5gh$ **f** $kmn$, $4kmn$, $-\frac{1}{2}kmn$
**g** $a^2b$, $0.4a^2b$, $-10a^2b$
**h** $3t^5$, $-\frac{7}{9}t^5$, $35t^5$

**4** **a** $18a$ **b** $11a$ **c** $b$
**d** $2m$ **e** $16ab$ **f** $20fg$
**g** $a^2$ **h** $41x^3$

**5** **a** $-5y$ **b** $5t$
**c** $-13p$ **d** $-ab$

**6** **a** $\frac{1}{3}b$ **b** $q$
**c** $\frac{1}{20}sp$ **d** $\frac{19}{18}r = 1\frac{1}{18}r$

**7** **a** $36a$ **b** $73x^2$
**c** $160abc$ **d** $6000x^3$

**8** **a** $85m$ **b** $3y^2$
**c** $120mn^2$ **d** $210abc$

**9** **a** $12p$ **b** $6a + 2ab$
**c** $9f + 4$ **d** $17m + 3 - 3m^2$
**e** $4x + 6y$ **f** $15i - j + 7$
**g** $15k^2 - 12 + 6k$
**h** $15 + 4g^2 + g^3$

**10** **a** $13a^2 + 9ab^2$
**b** $wx^2 - 7 - 2w^2 + 5w$
**c** $3a^2 - 5a^2b + 6b^2 + 9$
**d** $2xy + 1 + 7x^2y - 5xy^2$

**11** **a** 63 **b** $14m + 5k$ **c** 63
**d** Sample answer: The simplified expression, as less substitution is required.

**12** **a** $13ab + 5a^2b$ **b** 2322

**13** **a** $4x + 8x + 15x$ or $27x$ **b** 540

**14** **a** **i** $m = 6(2x + 4y + 6z)$

**ii** $n = 2(2x + 4y + 6z)$

**b** 140

**15** **a** **i** $s = 6n$ **ii** $t = \frac{59}{12}n = 4\frac{11}{12}n$

**b** 39

**16** Let $x$ be the number. Following the operations gives the expression $\frac{2(x+3)+4}{2} - x$ which equals 5.

**17** **a** **i** The student has subtracted instead of adding the $3x$ term.

**ii** 0

**b** **i** The student has added the indices instead of adding the coefficients of $x^2$.

**ii** $2x^2$

**c** **i** The student has not correctly identified the like terms before simplifying.

**ii** $12x^2y - 3xy^2$

**18** **a** True by the commutative law

**b** True by the commutative law

**c** True as any number multiplied by 1 stays the same

**d** False as $a + 0 = a$

**e** False using the distributive law $2(a + b) = 2a + 2b$

**f** True by the associative law

**g** False, using the associative law $2(3 \times y) = 6y$

**h** True by the distributive law

**19** No, $(a - b) - c = a - b - c$, whereas $a - (b - c) = a - b + c$. Subtracting a negative value gives a positive value.

**20** $-5\frac{7}{10}x - 49\frac{24}{25}y$

## EX p273 6G Solving equations by inspection

**1** **a** True **b** False **c** False

**d** True **e** True **f** False

**2** **a** Yes **b** No **c** No

**d** Yes **e** Yes **f** No

**g** Yes **h** No **i** Yes

**3** **a** $a = 5$ **b** $b = 6$ **c** $c = 8$

**d** $d = 18$ **e** $e = 10$ **f** $f = 16$

**g** $g = 9$ **h** $h = 7$ **i** $i = 4$

**j** $j = 8$ **k** $k = 10$ **l** $l = 11$

**m** $m = 4$ **n** $x = 54$ **o** $x = 0$

**p** $p = 11$ **q** $q = 11$ **r** $t = 12$

**4**

| Guess $x$ | LHS $4x + 11$ | RHS | LHS = RHS? |
|---|---|---|---|
| 3 | 23 | 35 | no |
| 9 | 47 | 35 | no |
| 5 | 31 | 35 | no |
| 7 | 39 | 35 | no |
| 6 | 35 | 35 | yes |

So, $x = 6$

**5** **a** $x = 11$ **b** $x = 8$

**c** $x = 13$ **d** $x = 16$

**6** B

**7** 3

**8** **a** $x = 2$ **b** $x = 4$

**c** $x = 3$ **d** $x = 1$

**9** Sample answers:

**a** $x + 7 = 13$ **b** $4x = 24$

**c** $\frac{x-2}{4} = 1$ **d** $2(x + 3) - 10 = 8$

**10** **a** $y = 8$ **b** $y = 41$

**c** $x = 2$ **d** $x = 5$

**11** **a** $b = 10$ **b** $a = 3$

**c** $b = 0$ **d** $a = 7$

**12** **a** $x + 6 = 13, x = 7$

**b** $9x = 54, x = 6$

**c** $x - 7 = 5, x = 12$

**d** $2x = 15, x = \frac{15}{2}$

**e** $2x + 5 = 23, x = 9$

**f** $3(x - 8) = 24, x = 16$

**g** $\frac{1}{4}x + 2 = 20, x = 72$

**h** $\frac{1}{2}(x + 3) = 13, x = 23$

**13** **a** $15n$ **b** $15n = 60$ **c** $n = 4$

**d** Four T-shirts

**14** **a** $5 - s$ **b** $5 - s = 3$ **c** $s = 2$

**d** \$2

**15** **a** Let $p$ = the number of occupied seats

**b** $p + 17 = 48$ **c** $p = 31$ **d** 31 people

**16** **a** Let $h$ = the number of hours James works in January

**b** 12h = 336 **c** 28 hours

**17** **a** $2g = p$ **b** 4 giraffes

**18** **a** $x + y = 15$ **b** $4x + 2y = 42$

**c** 6 cars and 9 motorbikes

**19** **a** $x = y + 4$ **b** $x + y = 12$

**c** Eleanor played 8 games and Jing played 4 games

**20** **a** Sample answer: Solving by inspection as there is only one operation in the equation.

**b** Sample answer: Guess, check and improve as there is more than one operation in the equation.

**21** **a** Sample answer: If the solution can be easily identified it is quicker to solve equations by inspection. A disadvantage is that it is only easy to use on simple equations.

**b** Sample answer: The guess, check and improve method can be used on any equation. A disadvantage is that on simple equations it can take longer than solving by inspection.

**22** **a** $x = 2$ **b** $x = 12$

**c** $x = 7$ **d** $x = 8$

**23** **a, b** $x = 2$ or $x = 5$

**c** The power of 2 on the $x$ term.

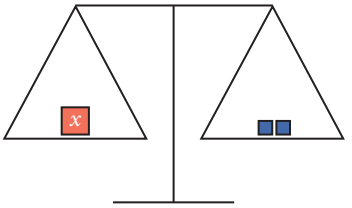
EX p279

## 6H Solving equations using inverse operations

**1** **a** Remove five blue blocks

**b** Remove five blue blocks

**c**

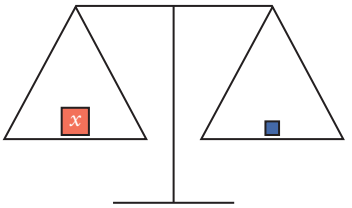



**d** 2 grams

**2** **a** Remove two blue blocks, then divide the number of blocks by three

**b** Remove two blue blocks, then divide the number of blocks by three

**c**




**d** 1 g

**3** **a** $-5$ **b** $+3$ **c** $\div 8$ **d** $\times 2$

**4** **a** $a = 7$ **b** $b = 11$ **c** $c = 4$ **d** $d = 15$

**5** **a** $x = 5$ **b** $a = 10$ **c** $p = 6$ **d** $s = 30$ **e** $y = 23$ **f** $f = 90$ **g** $d = -17$ **h** $x = 3$

**6** **a** $x = 5$ **b** $x = 6$ **c** $x = 8$ **d** $x = 30$ **e** $x = 8$ **f** $x = 17$ **g** $x = 12$ **h** $x = 7$ **i** $x = \frac{1}{3}$ **j** $x = -11$ **k** $x = -180$ **l** $x = -27$

**7** **a** $x = 6$ **b** $x = 10$ **c** $x = 3$ **d** $x = 18$ **e** $x = 28$ **f** $x = \frac{4}{7}$ **g** $x = -6$ **h** $x = -90$ **i** $x = 0$

**8** **a** $x = 5$ **b** $x = 19$ **c** $x = 11$ **d** $x = 11$ **e** $x = 7$ **f** $x = 16$ **g** $x = 4$ **h** $x = -27$ **i** $x = -12$

**9** **a** $x = 1$ **b** $a = 14$ **c** $x = 6$ **d** $x = 11$ **e** $p = 42$ **f** $d = 9$ **g** $f = 8$ **h** $z = 29$ **i** $x = 4$ **j** $t = 30$ **k** $q = -2$ **l** $y = \frac{21}{2}$ **m** $x = 6$ **n** $c = 12$ **o** $n = 2$

**10** **a** $y = 4$ **b** $y = 3\frac{1}{2}$ **c** $x = 4$ **d** $x = -6$

**11** **a** $x = 4$ **b** $x = 3$ **c** $n = 80$ **d** $y = 0$ **e** $b = 3$ **f** $n = -5$ **g** $x = -6$ **h** $p = -4\frac{1}{3}$

**12** **a** $2x - 3 = 15$ **b** 9

**13** **a** $\frac{n+3}{2} + 5$ **b** 8

**14** Let $x$ represent each variable.

**a** $4x = 64$, $x = 16$, 16 tables

**b** $15x + 6 = 81$, $x = 5$, 5 hours

**c** $2x + 12 = 20$, $x = 4$, 4 extra toppings

**d** $24x + 105 = 369$, $x = 11$, 11 weeks

**15** **a** Let $x$ = the original amount of money in your wallet, $2(x + 5) = 136$

**b** $x = 63$, \$63

**16** **a** $c = 1.9m + 6.2$

**b** \$67 **c** 39 minutes

**17** **a** $t = 14.5m + 18$

**b** 90.5°C **c** 6 minutes

**18** 6

**19** \$500

**20** **a** $l = 3f + 12$

**b** Since plural "friends" is used, we assume Liam has at least 2 friends. So the minimum is $l = 18$.

**c** If $f = 3$ then $l = 21$; if $f = 4$ then $l = 24$, etc.

**d** The possible solutions go up in increments of 3.

**21** **a** $x = 3$ **b** $x = 4$ **c** $x = 6$ **d** $x = 15$

**22** $b = \frac{5}{2}$

# CHAPTER 6 review

## Multiple choice

**1** D **2** A **3** E
**4** A **5** D **6** B
**7** A **8** B **9** D
**10** D

## Short answer

**1** **a** Time, total number of worms collected

**b**

| Time | 1:00 pm | 1:30 pm | 2:00 pm | 2:30 pm | 3:00 pm |
|---|---|---|---|---|---|
| Total number of worms collected | 0 | 5 | 10 | 15 | 20 |

**2** **a** $y = x - 4$ **b** $n = m + 1$

**3** **a** **i** 2 **ii** $x, y$ **iii** 7, 3 **iv** none

**b** **i** 2 **ii** $a$ **iii** 1 **iv** 3

**c** **i** 3 **ii** $d, c$ **iii** 5, 2 **iv** 9

**d** **i** 4 **ii** $e, t, s$ **iii** 1, 2, 6 **iv** 4

**4** **a** $7x = 21$ **b** $\frac{x}{3} = 6$ **c** $x + 7 = 10$ **d** $x - 2 = 17$

**5** **a** $3(x + 7)$ **b** $\frac{p+5}{13}$ **c** $\left(\frac{c}{2}\right)^2$ **d** $v^2 - 3$

**6** **a** 7 **b** 6
**c** 6 **d** 2

**7** **a** 14 **b** 5
**c** 2 **d** 162

**8** **a** $x$ **b** $6p + 3p^2$
**c** $4st + 4t$ **d** $18mn + 4m^2 + 3m$

**9** **a** $a = 7$ **b** $b = 31$
**c** $c = 3$ **d** $d = 10$

**10** $x = 20$

**11** **a** $x = 12$ **b** $x = 48$ **c** $x = 4$
**d** $x = 13$ **e** $x = -4$ **f** $x = -55$

### Analysis

**a** Number of hours, money earned

**b** Let $c$ = the total amount that Santo charges for a given session, $r$ = the number of hours in a specific recording session

**c** $c = 250r + 300$

**d** \$1550

**e** $f = 325h$, where $f$ = the total amount that Santo charges and $h$ = the number of hours

**f** 6 hours

**g** The recording session

**h** 4 hours

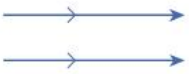
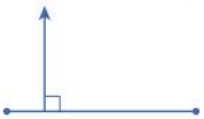
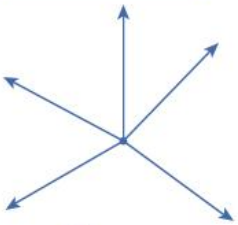
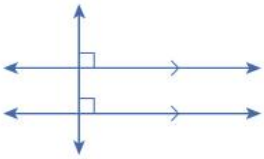
## CHAPTER 7 Angles

EX p292

### 7A Classifying angles and lines

**1** **a** $AB$ **b** $\angle AOB$ or $\angle BOA$
**c** $\overleftrightarrow{CD}$ **d** $\angle XYZ$ or $\angle ZYX$
**e** $\overrightarrow{ST}$ **f** $\angle JOK$ or $\angle KOJ$
**g** $\angle POQ$ or $\angle QOP$
**h** $\angle LMN$ or $\angle NML$

**2** **a** Two parallel line segments
**b** A ray perpendicular to a line segment
**c** Two parallel lines perpendicular to another two parallel lines

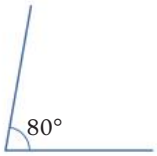
**3** **a**

**b**

**c**

**d**


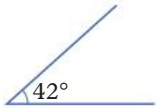
**4** **a** d, e, f, a, c, b
**b** **i** Between 90° and 180°
**ii** Between 0° and 90°
**iii** 90° **iv** 360°
**v** Between 180° and 360° **vi** 180°
**c** **i** Obtuse **ii** Acute
**iii** Right angled **iv** Revolution
**v** Reflex **vi** Straight

**5** Measurements within 2–3° of the answers below are acceptable.
**a** 84°, acute **b** 40°, acute **c** 135°, obtuse
**d** 15°, acute **e** 150°, obtuse **f** 120°, obtuse

**6** **a** $\angle AOB$ or $\angle BOA$, acute
**b** $\angle XOY$ or $\angle YOX$, obtuse
**c** $\angle COD$ or $\angle DOC$, right angled
**d** $\angle SOT$ or $\angle TOS$, acute

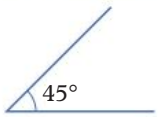
**7** **a** 15° **b** 56°
**c** $\angle COF$ is an acute angle
**d** $\angle AOB$, $\angle AOC$, $\angle BOC$, $\angle BOD$, $\angle COD$, $\angle COE$, $\angle COF$, $\angle DOE$, $\angle DOF$, $\angle EOF$
**e** $\angle AOD$, $\angle AOE$, $\angle AOF$, $\angle BOF$

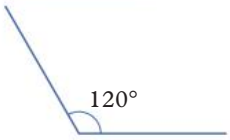
**8** **a**


**b**


**c**

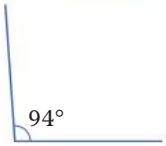
**d**

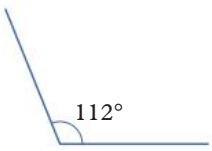
**9** **a**


**b**


**c**

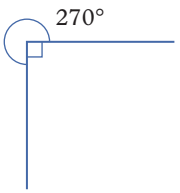
**d**

**e**


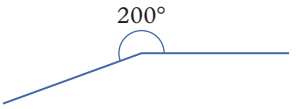
**f**


**10** **a** $a = 270°$ **b** $b = 332°$
**c** $c = 265°$ **d** $d = 186°$

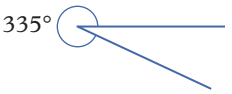
**11** **a**




**b**




**c**




**d**

**12** **a** 2 **b** 4 **c** 2

**13** **a** Straight angle **b** Revolution
**c** Reflex angle **d** Reflex angle

**14** The protractor was read from the right-hand zero line. $\angle ABC = 106°$.

**15** $\angle XYZ = 60°$

**16** 80°

**17** **a** Top right pocket
**b** Top left pocket

## EX 7B Angles at a point

p300

**1** **a** $a = 130°$ **b** $b = 67°$ **c** $c = 99°$
**d** $d = 19°$ **e** $e = 66°$ **f** $f = 84°$

**2** **a** $\angle YOZ = 30°$
**b** $\angle COD = 21°$
**c** $\angle QOR = 132°$
**d** $\angle KOL = 37°$

**3** **a** $\angle WOX$ and $\angle XOY$; $\angle XOY$ and $\angle YOZ$
**b** $\angle AOB$ and $\angle BOC$; $\angle BOC$ and $\angle COD$
**c** $\angle POQ$ and $\angle QOR$; $\angle QOR$ and $\angle ROS$
**d** $\angle JOK$ and $\angle KOL$; $\angle KOL$ and $\angle LOM$

**4** **a** **i** 67° **ii** 13° **iii** 2°
**iv** 81°
**b** **i** 22° **ii** 159° **iii** 80°
**iv** 83°

**5** **a** **i** 44° **ii** 134°
**b** **i** 78° **ii** 168°
**c** **i** 5° **ii** 95°
**d** **i** 63° **ii** 153°

**6** **a** $a = 100°$ **b** $b = 27°$ **c** $c = 38°$
**d** $d = 133°$ **e** $a = 215°$ **f** $b = 300°$
**g** $c = 270°$ **h** $e = 20°$

**7** Yes, these pairs of angles are adjacent as they share a common side and a common vertex.

**8** **a** $r = 147°$ **b** $d = 20°$; $e = 160°$; $f = 20°$
**c** $x = 136°$; $y = 44°$; $z = 136°$ **d** $c = 90°$
**e** $a = 50°$; $b = 50°$; $c = 50°$; $d = 80°$
**f** $g = 60°$; $h = 60°$

**9** **a** $\angle YOZ = 30°$
**b** $\angle POQ = 142°$, $\angle QOR = 135°$
**c** $\angle EOF = 127°$
**d** $\angle JOK = 137°$, $\angle JOM = 43°$, $\angle LOM = 137°$
**e** $\angle TOU = 22°$, $\angle POU = 88°$, $\angle POQ = 70°$, $\angle ROS = 88°$

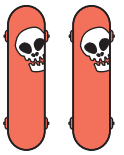
**10** **a** **i** 34° **i** Complementary
**b** **i** 93° **ii** Supplementary
**c** **i** 214° **ii** Full revolution
**d** **i** 30° **ii** Complementary

**11** **a** 8 **b** 45° **c** 10
**d** 72°

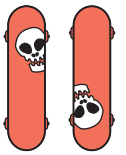
**12** **a** The two angles do not share a common side.
**b** The two angles do not share a common vertex.
**c** The two angles overlap.

**13** 125°

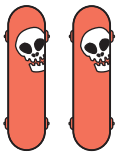
**14** **a**




**b**




**c**




**15** **a** True, the angles around a point must sum to 360°.
$360° - 138° - 35° - 42° - 55° = 90°$
**b** True, two complementary angles add to make 90°.
$55° + 35° = 90°$
**c** True, two supplementary angles add to make 180°.
$42° + 138° = 180°$
**d** True, $55° + 90° + 138° + 35° + 42° = 360°$.

**16** No, the largest acute angle is less than 90°, therefore the sum of two acute angles must always be less than 180°.

**17** Sample answer:
$\angle AOC$ and $\angle BOD$ are both supplementary to $\angle BOC$
hence: $180° - \angle BOC = \angle AOC$
$180° - \angle BOC = \angle BOD$
So, $\angle AOC = \angle BOD$

## EX 7C Angles and parallel lines

p308

**1** **a** Alternate
**b** Co-interior
**c** Corresponding
**d** Corresponding
**e** Alternate
**f** Co-interior
**g** Corresponding
**h** Corresponding
**i** Alternate

**2** The pairs of angles in **a**, **c**, **d**, **e**, **g**, **h** and **i** are equal in size. The pairs of angles in **b** and **f** are supplementary.

**3** **a** $\angle EPD$ **b** $\angle FOB$ **c** $\angle CPE$
**d** $\angle CPE$ **e** $\angle FOB$ **f** $\angle CPE$

**4** **a** $\angle PYT$ and $\angle UXS$, $\angle QYT$ and $\angle RXU$
**b** $\angle RXU$ and $\angle PYT$, $\angle QYT$ and $\angle UXS$
**c** $\angle PYU$ and $\angle RXU$, $\angle PYT$ and $\angle RXT$, $\angle SXT$ and $\angle QYT$, $\angle SXU$ and $\angle QYU$

**5** **a** $\angle EOB$ **b** $\angle DPE$ or $\angle CPF$ or $\angle AOF$ or $\angle BOE$
**c** $\angle CPF$ **d** $\angle DPE$, $\angle CPF$ or $\angle AOF$ or $\angle BOE$

**6** **a** **i** Alternate **ii** 125°
**b** **i** Co-interior **ii** 110°
**c** **i** Corresponding **ii** 131°
**d** **i** Corresponding **ii** 60°
**e** **i** Alternate **ii** 150°
**f** **i** Co-interior **ii** 138°
**g** **i** Alternate **ii** 75°
**h** **i** Co-interior **ii** 90°
**i** **i** Corresponding **ii** 84°

**7** **a** Yes, the alternate angles are equal.
**b** No, the corresponding angles are not equal.
**c** No, the co-interior angles are not supplementary.
**d** Yes, the co-interior angles are supplementary.
**e** No, the corresponding angles are not equal.
**f** No, the alternate angles are not equal.
**g** Yes, the angles vertically opposite the labelled angles are alternate angles and equal.
**h** Yes, the angles supplementary to the labelled angles are co-interior and supplementary.

**8** **a** **i** Supplementary **ii** 125°
**b** **i** Supplementary **ii** 55°
**c** **i** Equal **ii** 71°
**d** **i** Equal **ii** 71°
**e** **i** Equal **ii** 67°
**f** **i** Equal **ii** 123°

**9** **a** $a = 71°$, alternate angles
**b** $b = 122°$, co-interior angles
**c** $c = 65°$, alternate angles
**d** $d = 65°$, co-interior angles
**e** $e = 35°$, alternate angles
**f** $f = 121°$, corresponding angles
**g** $g = 133°$, corresponding angles
**h** $h = 160°$, alternate angles

**10** 35°

**11** 133°

**12** **a** $a = b = c = 64°$
**b** No, if the paintings were parallel then $c$ would be 64°.
**c** Amal would have to move the left-hand side of the bottom painting down to make $c = 64°$.

**13** **a** $\angle AOF$ is corresponding to $\angle CPF$, and $\angle CPF$ and $\angle FPD$ are angles on a straight line (supplementary). Therefore, as $81°+99°= 180°$, the two blue lines are parallel.
**b** Sample answer: $\angle BOF = \angle EPC$, $\angle CPE = \angle AOE$

**14** **a** $\angle AOF$ and $\angle CPE$ are co-interior angles.
$\angle AOF = \angle CPF$ since corresponding angles on parallel lines are equal.
$\angle CPF + \angle CPE = 180°$ since they are angles that form a straight angle.
$\angle AOF + \angle CPE = 180°$ since $\angle AOP = \angle CPF$
Therefore, $\angle AOF$ and $\angle CPE$ are supplementary since they add to 180°.
**b** **i** Let $\angle AOE$ be an acute angle.
The corresponding angle $\angle CPE = \angle AOE$ since corresponding angles on parallel lines are equal, therefore it is also acute.
The alternate angle to $\angle CPE$ is $\angle FOB$, since alternate angles on parallel lines are equal. Therefore, $\angle FOB = \angle CPE = \angle AOE$ and they are all acute.
The corresponding angle to $\angle FOB$ is $\angle FPD$, since corresponding angles on parallel lines are equal. Therefore, $\angle FPD = \angle FOB = \angle CPE = \angle AOE$ and they all all acute.
The remaining four angles each form a straight angle with an acute angle, so must be the supplement of the acute angle. The supplement of an acute angle is always an obtuse angle.
**ii** Let $\angle AOE$ be an acute angle. We know that $\angle FPD = \angle FOB = \angle CPE = \angle AOE$ so they are all acute. The four remaining obtuse angles, $\angle BOE$, $\angle AOP$, $\angle DPO$ and $\angle CPF$ each form a straight angle with an acute angle, so must be the supplement of the acute angle. The supplement of an acute angle is an obtuse angle. Since all the acute angles are equal, each obtuse angle is the supplement of the same angle, and so they must also all be equal.
**iii** All angles on parallel lines are equal or supplementary. If any angle is 90°, then all equal angles must also be 90°, and all supplementary angles are also 90°. Therefore, all angles would equal 90°.

## 7 Checkpoint

**1** **a** C **b** F **c** B
**d** E **e** D **f** A

**2** **a** Point, $P$ **b** Ray, $\overrightarrow{JH}$
**c** Line, $\overleftrightarrow{BY}$ **d** Angle, $\angle SWK$ or $\angle KWS$

**3** **a** 85° **b** 123°

**4** **a** B **b** C **c** D
**d** A

5 a Complement = 71°, supplement = 161°
b Completement = 53°, supplement = 143°
c Complement = 1°, supplement = 91°

6 a $\angle AHJ = 143°$
b $\angle IJC = 106°$
c $\angle FIB = 70°$

7 a True b True c True
d False e False

8 a Co-interior b Alternate
c Vertically opposite
d Corresponding

9 a $h = 87°$ b $u = 146°$ c $n = 99°$

10 a Not parallel. The alternate angles are not equal.
b Not parallel. The co-interior angles are not supplementary.
c Parallel. The angles vertically opposite the labelled angles are alternate angles and equal.
d Not parallel. The angles vertically opposite to the labelled angles are co-interior and are not supplementary.

## EX 7D Classifying triangles

p316

1 a Equilateral triangle b Scalene triangle
c Isosceles triangle d Scalene triangle
e Right-angled triangle f Scalene triangle
g Equilateral triangle h Isosceles triangle
i Isosceles, right-angled triangle

2 a $a = 80°$ b $b = 82°$ c $c = 91°$
d $d = 50°$ e $e = 67°$ f $f = 33°$

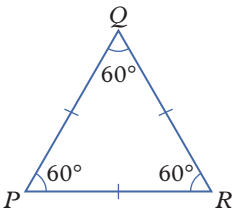
3 a $a = 29°$, scalene triangle
b $b = 48°$, isosceles triangle
c $c = 60°$, equilateral triangle
d $d = 70°$, isosceles triangle
e $e = 85°$, scalene triangle
f $f = 90°$, right-angled triangle

4 a $a = 53°$ b $a = b = c = 60°$
c $a = b = 73°$ d $a = b = 45°$
e $a = 25°$; $b = 130°$
f $a = b = 60°$

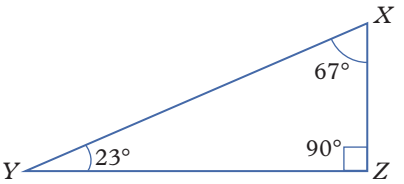
5 a $x = 1.5$ cm b $a = 10$ cm
c $q = 23$ cm d $b = 3$ cm

6 a

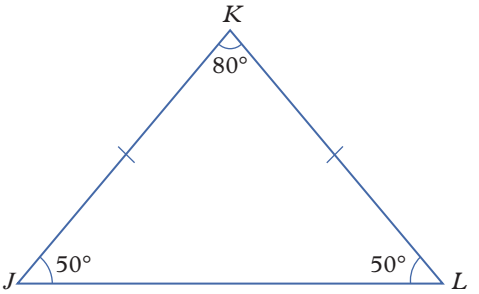
b




c




d




7 a i Possible ii Not possible iii Possible
iv Possible v Not possible vi Possible
b i Right-angled triangle
iii Equilateral triangle
iv Scalene triangle
vi Isosceles triangle

8 Yes, a right-angled triangle with two 45° angles is also an isosceles triangle.

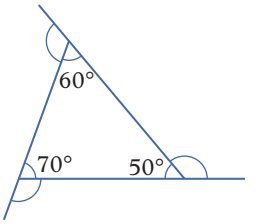
9 Either a scalene or an isosceles triangle can have an obtuse triangle.
In a right-angled triangle the sum of the two other angles must be 90°, so they must both be acute angles.
In an equilateral triangle all angles are 60°, so they are all acute angles.

10 a

| Type of triangle | Number of sides of equal length | Number of internal angles that are equal in size |
|---|---|---|
| Equilateral | 3 | 3 |
| Isosceles | 2 | 2 |
| Scalene | 0 | 0 |

b In any triangle, the number of sides of equal length is the same as the number of angles that are equal in size.

11 a It is outside the triangle.
b 130°
c 130°. They add to the size of the exterior angle.
d



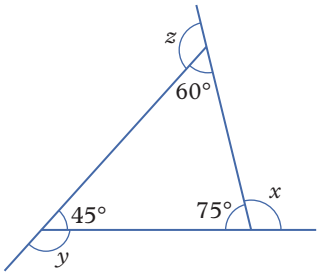

e 110° and 120°
f 110° and 120°. They add to the size of the exterior angle.

**g** An exterior angle is equal to the sum of the two interior angles opposite that exterior angle. This is because an exterior angle is supplementary to the adjacent interior angle and the sum of interior angles in a triangle is 180°, so the two opposite interior angles are also supplementary to the interior angle adjacent to the exterior angle.

**12 a** $x = 105°$ **b** $x = 120°$ **c** $x = 115°$
**d** $x = 40°$ **e** $x = 95°$ **f** $x = 25°$

**13 a** 360°

**b** Sample answer:




**i** $x = 105°$; $y = 135°$; $z = 120°$

**ii** $x + y + z = 360°$

**c** The exterior angles of a triangle add to 360°.

**14 a** $x = 30°$ **b** $x = 10°$ **c** $x = 10°$
**d** $x = 10°$ **e** $x = 12°$ **f** $x = 20°$

**15 a i** Alternate **ii** Alternate

**b** $\angle PBA$, $\angle ABC$ and $\angle QBC$ are angles on a straight line, therefore $\angle PBA + \angle ABC + \angle QBC = 180°$.

## EX p322 7E Classifying quadrilaterals

**1 a** Parallelogram **b** Kite **c** Square
**d** Trapezium **e** Rhombus **f** Trapezium
**g** Irregular quadrilateral
**h** Rectangle **i** Kite

**2 a** Rhombus **b** Trapezium
**c** Rectangle **d** Parallelogram
**e** Square **f** Kite

**3 a** True **b** False **c** True
**d** False

**4 a** $a = 112°$ **b** $b = 135°$ **c** $c = 142°$
**d** $d = 253°$ **e** $e = 59°$ **f** $f = 79°$

**5 a** Irregular quadrilateral; $a = 70°$
**b** Parallelogram; $b = 53°$
**c** Trapezium; $c = 35°$
**d** Irregular quadrilateral; $d = 145°$
**e** Irregular quadrilateral; $e = 65°$
**f** Rhombus; $f = 105°$

**6 a** $d = 60°$; $e = 130°$ **b** $c = 80°$
**c** $x = 150°$; $y = 30°$; $z = 150°$ **d** $k = 45°$
**e** $m = n = 90°$ **f** $f = 135°$
**g** $x = w = 111°$ **h** $e = f = g = 90°$
**i** $s = t = 118°$

**7** Trapezium, rectangle, rhombus, parallelogram, irregular quadrilateral

**8 a** Rhombus **b** Square

**9** Barbecue: trapezium
Dining area: square
Jungle gym: irregular quadrilateral
Lawn: rectangle
Rose garden: rectangle
Sand pit: irregular quadrilateral
Spa: trapezium
Veranda: trapezium
Washing line: parallelogram

**10 a** Angles are incorrectly labelled. As opposite angles are equal in a rhombus, the labelled angles must sum to 180°, otherwise interior angle sum is not 360°.

**b** Angles are incorrectly labelled. The interior angle sum is 380°, not 360°.

**c** The diagram is a trapezium not an irregular quadrilateral. The angles make supplementary pairs so the longer sides must be parallel.

**11 a** Rectangle or square
**b** Rhombus or square
**c** Rectangle, parallelogram, rhombus, or square
**d** Trapezium, parallelogram, rhombus, rectangle, or square
**e** Parallelogram, rectangle, rhombus or square

**12 a** Yes, all four angles are 90°
**b** No, all sides are not equal in length
**c** Yes, there are two pairs of parallel sides and the pairs of opposite angles are equal in size
**d** Yes, all four sides are of equal length

**13 a** Two **b** 180° **c** $2 \times 180° = 360°$
**d** Not all of the triangles' internal angles align with the internal angles of quadrilateral. Each triangle has one internal angle located at the centre of the quadrilateral.
**e** Subtract 360°
**f** Revolution **g** 360°; yes

**14 a** $x = 35°$ **b** $x = 80°$ **c** $x = 15°$
**d** $x = 37°$ **e** $x = 17°$ **f** $x = 12°$

**15 a** 70°, 110°, 70°, 110°
**b** 360°. The exterior angles on a triangle and a quadrilateral both add to 360°.
**c** Sample answer: The exterior angles of all quadrilaterals add to 360°.

# CHAPTER 7 review

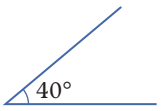
## Multiple choice

| | | |
|---|---|---|
| **1** B | **2** E | **3** C |
| **4** D | **5** D | **6** B |
| **7** E | **8** C | **9** D |
| **10** C | **11** B | **12** B |

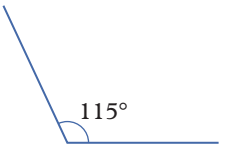
## Short answer

1 **a** 20°, acute **b** 135°, obtuse
**c** 90°, right angle **d** 197°, reflex

2 **a**




**b**




**c**

3 **a** Ray: $\overrightarrow{DJ}$ Line: $\overleftrightarrow{GD}$
**b** Triangle: $\triangle ABC$

4 **a** $a = 320°$ **b** $b = 145°$ **c** $c = 25°$
**d** $d = 57°$, $e = 123°$, $f = 123°$
**e** $g = 26°$, $h = 66°$
**f** $i = 62°$, $j = 270°$

5 **a** **i** 67° **ii** 11°
**b** **i** 171° **ii** 53°

6 **a** $g$ **b** $d$ **c** $d$
**d** **i** 77° **ii** 103° **iii** 103°

7 **a** Yes, the corresponding angles are equal.
**b** No, the corresponding angle to 117° is not supplementary to 73°.

8 **a** Scalene, $x = 30°$
**b** Isosceles, $x = 25°$
**c** Equilateral, $x = 60°$
**d** Right-angled isosceles, $x = 45°$

9 **a** $x = 136°$ **b** $d = 49°$
**c** $m = 22°$ **d** $a = 101°$, $b = 45°$

10 **a** Irregular quadrilateral, $x = 86°$
**b** Trapezium, $x = 113°$
**c** Kite, $x = 111°$
**d** Parallelogram, $x = 59°$

11 **a** **i** 106° **ii** 90° **iii** 35°
**iv** 90°
**b** Kite

## Analysis

1 $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{EF}$
$\overleftrightarrow{IJ}$ is parallel to $\overleftrightarrow{LK}$ as the alternate angles are equal (102°) and the cointerior angles are supplementary, 102°+78°= 180°

2 $a = 140°$, $d = 50°$, $g = 37°$, $k = 130°$, $n = 127°$, $q = 103°$, $t = 40°$, $w = 64°$, $z = 116°$

# CHAPTER 8 Shapes and objects

EX p338

## 8A Polygons

1 **a** Polygon **b** Polygon
**c** Not a polygon **d** Polygon
**e** Polygon **f** Not a polygon
**g** Polygon **h** Polygon
**i** Not a polygon

2 **a** **i** Regular **ii** Convex
**b** **i** Irregular **ii** Convex
**c** **i** Irregular **ii** Concave
**d** **i** Regular **ii** Convex
**e** **i** Irregular **ii** Concave
**f** **i** Irregular **ii** Convex

3 **a** Regular, convex octagon
**b** Irregular, convex pentagon
**c** Irregular, concave pentagon
**d** Regular, convex nonagon
**e** Regular, convex hexagon
**f** Irregular, concave decagon

4 Shapes **a**, **b**, **d**, **e**, **g** and **h** from question **1** are polygons.
**a** **i** Regular **ii** Ten sides
**iii** No **iv** Regular, convex decagon
**b** **i** Irregular **ii** Six sides
**iii** No **iv** Irregular, convex hexagon
**d** **i** Regular **ii** Three sides
**iii** No
**iv** Regular, convex triangle or equilateral triangle
**e** **i** Regular **ii** Five sides
**iii** No **iv** Regular, convex pentagon
**g** **i** Irregular **ii** Four sides
**iii** No **iv** Irregular, convex quadrilateral
**h** **i** Irregular **ii** Eight sides
**iii** Yes **iv** Irregular, concave octagon

5 **a** Irregular, convex hexagon
**b** Irregular, concave decagon
**c** Irregular, concave octagon
**d** Irregular, convex pentagon
**e** Irregular, concave decagon
**f** Irregular, concave heptagon

6 B

7 D

8 **a** One name describes the number of convex points it has and the other name describes how many sides it has.
**b** **i** Four-pointed star or concave octagon
**ii** Six-pointed star or concave dodecagon
**iii** Three-pointed star or concave hexagon
**c** The number of points of the star is half the number of sides.

9 **a** Angles in a triangle add to 180°.
**b** 10 sides **c** Eight **d** 1440°

10 **a** **i** Three **ii** 540°
**b** **i** Four **ii** 720°
**c** **i** Five **ii** 900°
**d** **i** Seven **ii** 1260°

11 Yes; triangles and quadrilaterals have straight sides.

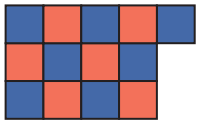
**12** **a** Regular hexagons in the honeycomb and triangles, quadrilaterals, pentagons, hexagons, heptagons and octagons in the Water Cube.

**b** Same: Both photos show shapes filling a space without overlaps or gaps, a tessellation. Different: Only a hexagon is needed to fill the honeycomb, but more than one shape is needed to fill the Water Cube wall.

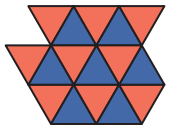
**13** **a** Regular hexagonal, irregular

**b** Sample answers:

**i**



**ii**



**14** Squares and triangles

**15** **a** 108° **b** 120° **c** 128.57° **d** 140°

**16** **a** $a = 120°$ **b** $b = 41°$ **c** $c = 146°$

EX p346

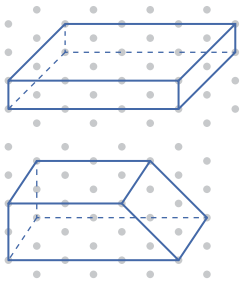
## 8B 3D objects

**1** **a** Five faces **b** Eight edges **c** Five vertices

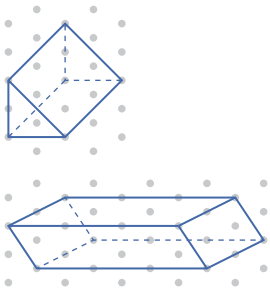
**2** **a** Six faces **b** Ten edges **c** Six vertices

**3** Sample answers:

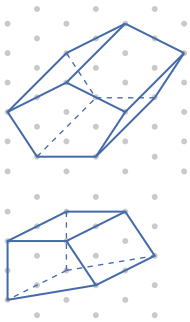
**a** **b**



**c** **d**



**e** **f**



**4** **a** Polyhedra **b** Polyhedra **c** Polyhedra **d** Polyhedra **e** Polyhedra **f** Not polyhedra **g** Polyhedra **h** Not polyhedra **i** Not polyhedra

**5** **a** Heptagon **b** Triangle **c** Square **d** Triangle **e** Decagon or 5-pointed star **g** Rectangle

**6** **a** Heptagonal prism

**b** Triangular pyramid (or tetrahedron)

**c** Square prism (or cube)

**d** Triangular prism

**e** Decagonal pyramid

**g** Rectangular pyramid

**7** **a** Pentagonal prism

**b** Hexagonal pyramid

**c** Heptagonal pyramid

**d** Decagonal prism

**e** Nonagonal prism

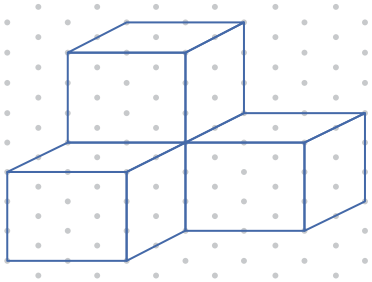
**f** Octagonal pyramid

**8** **a** Rectangular prism

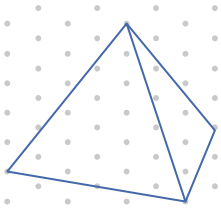
**b** Triangular pyramid (or tetrahedron)

**c** Sphere

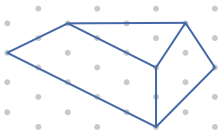
**9** Sample answer:



**10** Sample answer:



**11** Sample answer:



**12** **a** Yes; it has flat faces, straight edges and vertices

**b** No; it has some curved sides

**c** Yes; it has flat faces, straight edges and vertices

**13** **a** **i** Sphere **ii** Cone **iii** Cylinder

**b** They have some curved sides.

**14** Sample answer: no, the faces are slightly curved because of the pressure in the ball.

**15**

| Shape | F | E | V |
|---|---|---|---|
| heptagonal prism | 9 | 21 | 14 |
| triangular pyramid | 4 | 6 | 4 |
| square-based prism | 6 | 12 | 8 |
| triangular prism | 5 | 9 | 6 |
| decagonal pyramid | 11 | 20 | 11 |
| rectangular pyramid | 5 | 8 | 5 |

**16 a–c**

| | Tetrahedron | Cube (hexahedron) |
|---|---|---|
| ***F*** | 4 | 6 |
| **polygon** | triangle | square |
| ***V*** | 4 | 8 |
| ***E*** | 6 | 12 |
| ***F + V − E*** | 2 | 2 |

| | Octahedron | Dodecahedron |
|---|---|---|
| ***F*** | 8 | 12 |
| **polygon** | triangle | pentagon |
| ***V*** | 6 | 20 |
| ***E*** | 12 | 30 |
| ***F + V − E*** | 2 | 2 |

| | Icosahedron |
|---|---|
| ***F*** | 20 |
| **polygon** | triangle |
| ***V*** | 12 |
| ***E*** | 30 |
| ***F + V − E*** | 2 |

**d** $F + V - E = 2$

**17 a** Yes **b** The rule stays the same.

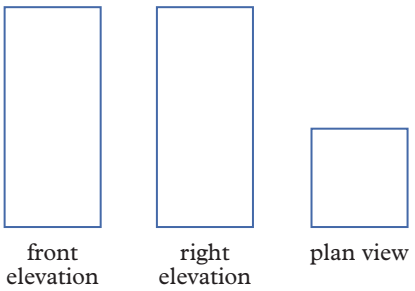
**18 a** 8 **b** 5 **c** 14 **d** 12

**19** The smiley face with hearts for eyes

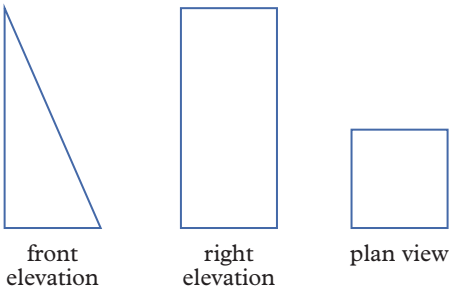
EX p351

## 8C Plans and elevations

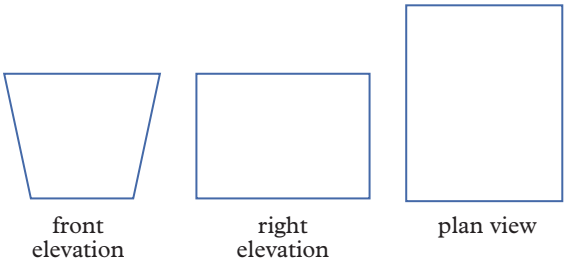
**1 a**



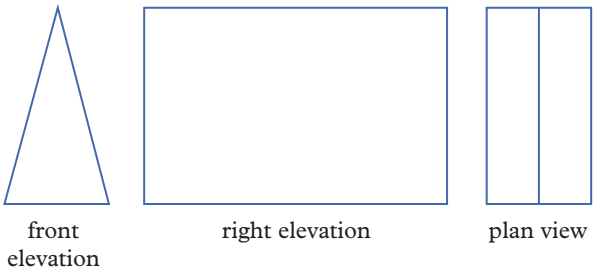

**b**



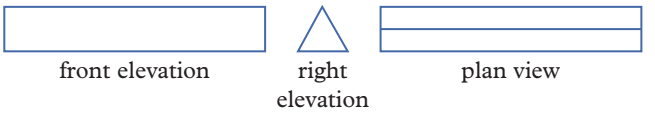

**c**



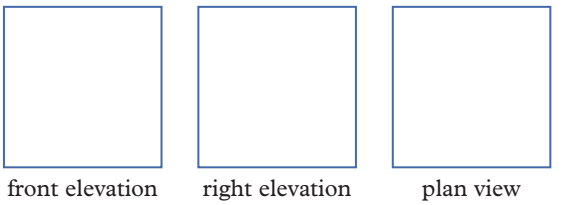

**d**



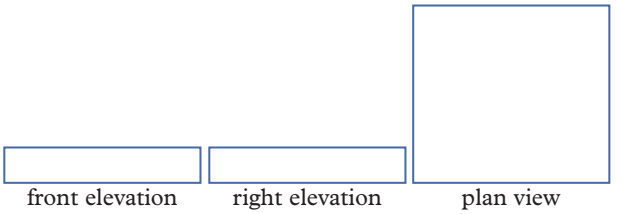

**2 a**



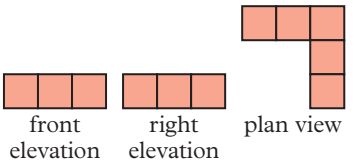

**b**



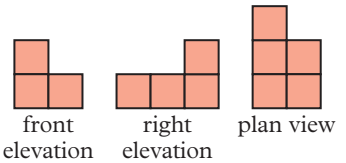

**c**



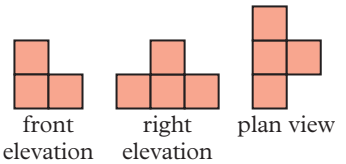

**3 a**



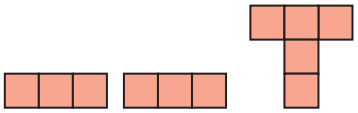

**b**




**c**

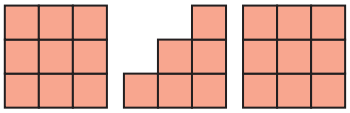


**d**



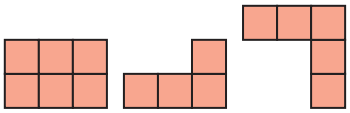
front elevation | right elevation | plan view

**e**



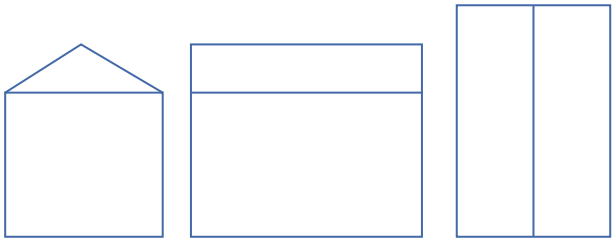
front elevation | right elevation | plan view

**f**



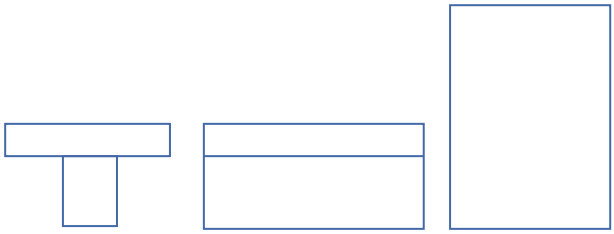
front elevation | right elevation | plan view

**4 a**



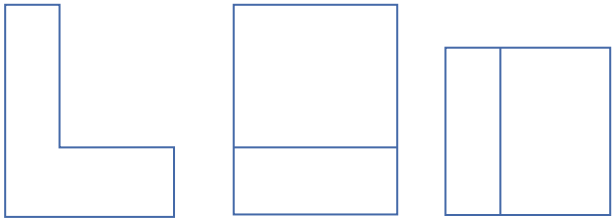
front elevation | right elevation | plan view

**b**



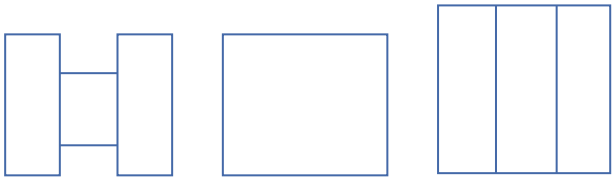
front elevation | right elevation | plan view

**c**



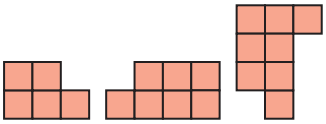
front elevation | right elevation | plan view

**d**



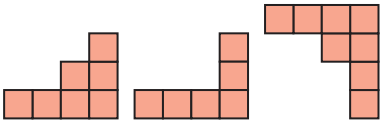
front elevation | right elevation | plan view

**5 a**



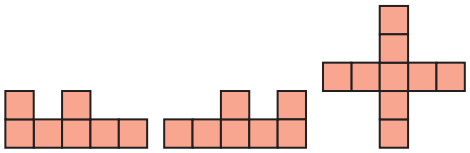
front elevation | right elevation | plan view

**b**



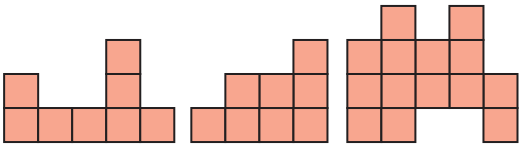
front elevation | right elevation | plan view

**c**



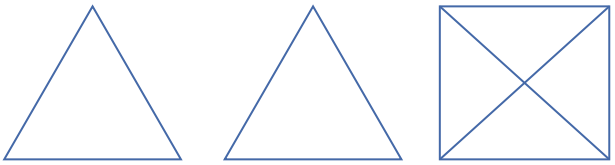
front elevation | right elevation | plan view

**d**



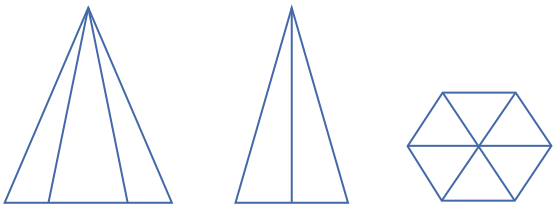
front elevation | right elevation | plan view

**6 a**



front elevation | right elevation | plan view

**b**

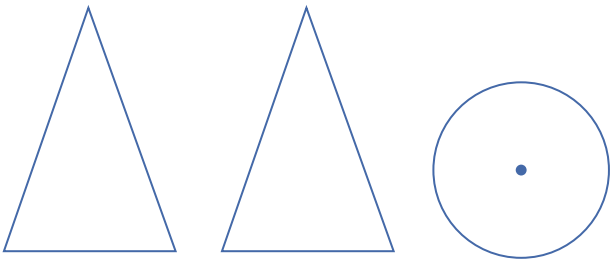


front elevation | right elevation | plan view

**7** When viewed from the side you don't see the curved edge and only see a rectangular shape.

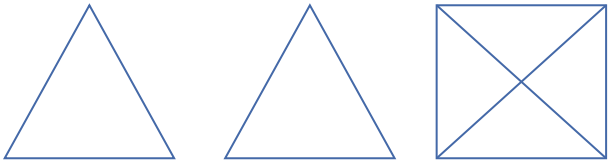
**8** They are all circles.

**9**



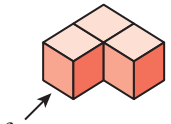
front elevation | right elevation | plan view

**10**

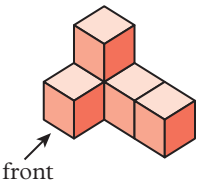


front elevation right elevation plan view

**11 a b**

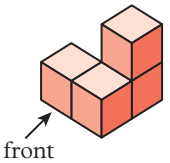


front

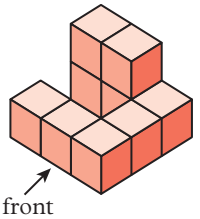


front

**c d**

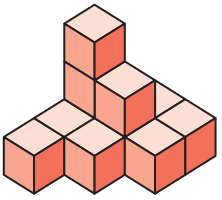


front



front

**12**

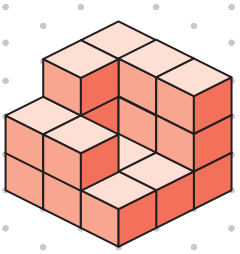


front

**13 a**

front elevation right elevation

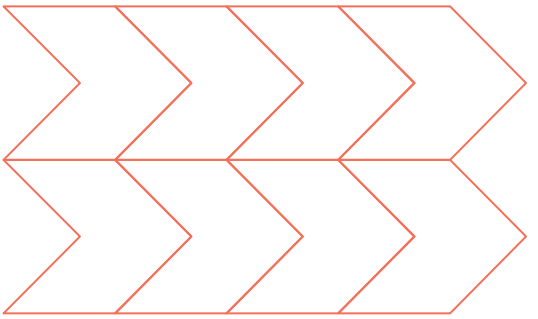
**b**



## 8 Checkpoint

**1 a i** Irregular **ii** Concave
**iii** Irregular, concave pentagon
**b i** Irregular **ii** Concave
**iii** Irregular, concave hexagon
**c i** Regular **ii** Convex
**iii** Regular, convex octagon
**d i** Regular **ii** Convex
**iii** Regular, convex hexagon

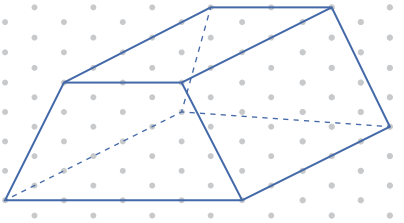
**2** Sample answer:



**3** Rhombus

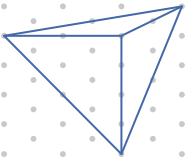
**4 a** 5 **b** 9 **c** 6

**5**

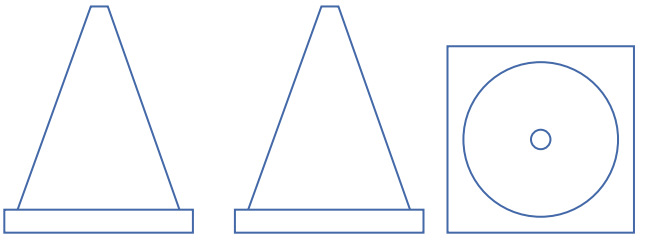


**6 a** Triangular pyramid (or tetrahedron)
**b** Cylinder **c** Sphere

**7**

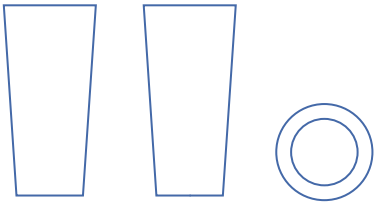


**8 a**



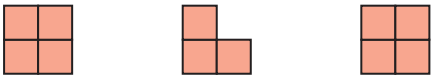
front elevation right elevation plan view

**b**



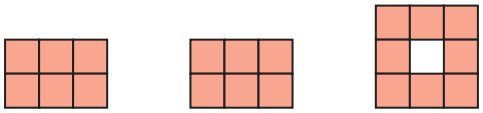
front elevation right elevation plan view

**9** **a**



front elevation right elevation plan view

**b**

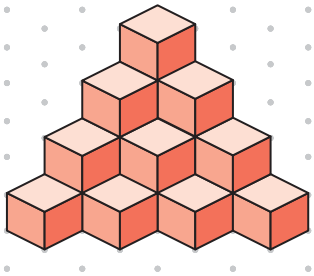


front elevation right elevation plan view

**c**

front elevation right elevation plan view

**10** Sample answer:

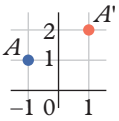


EX p359

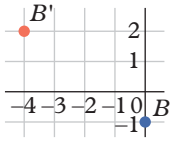
## 8D Translations

**1** **a** 4 units right and 2 units down
**b** 1 unit left and 3 units down
**c** 3 units left and 1 unit up
**d** 4 units right
**e** 4 units left and 4 units down
**f** 4 units right and 1 unit up

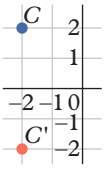
**2** **a**



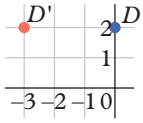

**b**



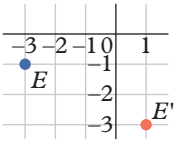

**c**



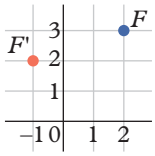

**d**




**e**




**f**




**3** **a** (−5, −3) **b** (−3, 5) **c** (−10, 0)
**d** (2, −2) **e** (−11, −10) **f** (5, 9)

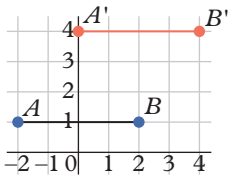
**4** **a** 3 units to the right and 4 units up
**b** 3 units to the right and 1 unit up
**c** 4 units to the left and 3 units down
**d** 4 units to the right and 2 units down
**e** 2 units to the right and 2 units down
**f** 5 units right and 6 units up
**g** 6 units left and 6 units up
**h** 3 units left and 10 units down

**5** **a** 5 units right and 3 units up
**b** 1 unit right and 5 units down
**c** 3 units left and 10 units down
**d** 13 units right and 6 units up
**e** 4 units right
**f** 8 units left and 9 units up

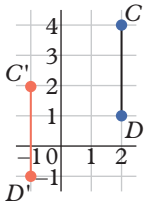
**6** **a** F
**b** E
**c** **i** 2 units left and 3 units up
**ii** 3 units left and 3 units down
**iii** 6 units right and 3 units up
**iv** 1 unit right and 5 units down
**v** 3 units right and 1 unit up
**vi** 2 units left and 1 unit up

**7** **a** 2 units right and 1 unit up
**b** 3 units left and 1 unit up
**c** 1 unit left and 2 units down
**d** 3 units right and 1 unit down

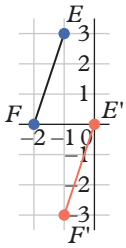
**8** **a**




**b**



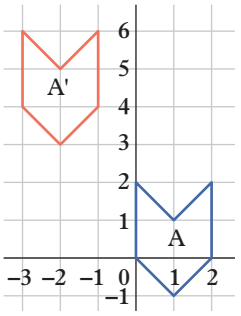

**c**



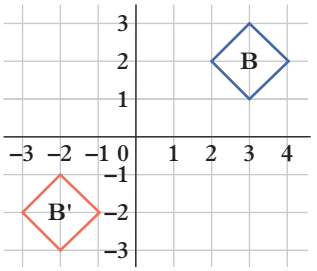

**9** D

**10** **a** 4 units left and 1 unit down
**b** 4 units down
**c** 2 units right and 2 units down
**d** 1 unit left and 3 units down
**e** 4 units left and 2 units up
**f** 3 units right and 2 units up

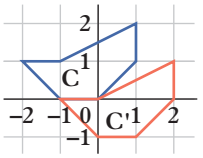
**11 a**



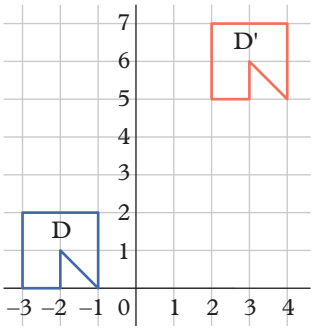

**b**



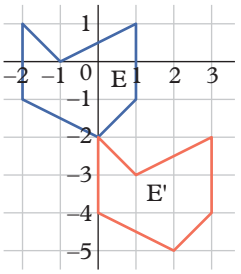

**c**



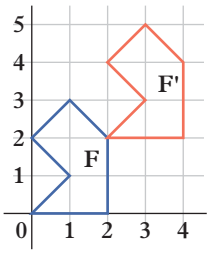

**d**




**e**




**f**




**12 a** Shape C

**b** Sample answer: Jordan has calculated the distance between the closest points on the two shapes; whereas, they should have counted the distance between matching vertices on the two shapes.

**c i** 5 units down and 2 units right

**ii** 10 units down and 8 units right

**iii** 3 units down and 6 units left

**iv** 10 units up and 8 units left

**v** 2 units up and 3 units left

**vi** 9 units right

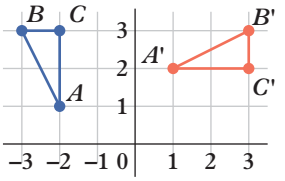
**13** Original figure = shape D, final image = shape A

**14** There are 8 possibilities: translate A and D or A and B 2 units to the right; or translate C and B or C and D 2 units to the left; or translate B and A or B and C 2 units up; or translate D and A or D and C 2 units down.

## EX 8E Rotations and reflections

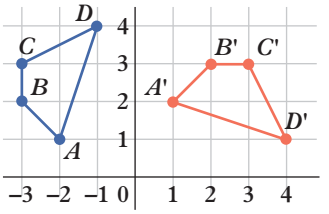
p366

**1 a**



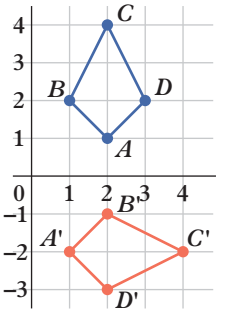

The coordinates of the image are $A'(1, 2)$, $B'(3, 3)$, $C'(3, 2)$.

**b**



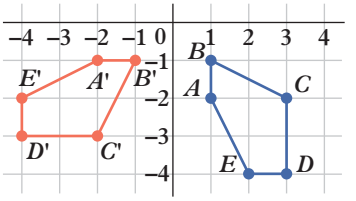

The coordinates of the image are $A'(1, 2)$, $B'(2, 3)$, $C'(3, 3)$, $D'(4, 1)$.

**c**



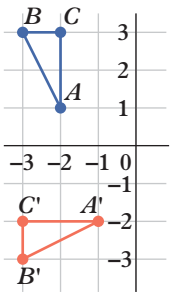

The coordinates of the image are $A'(1, -2)$, $B'(2, -1)$, $C'(4, -2)$, $D'(2, -3)$.

**d**



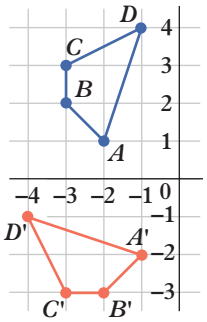

The coordinates of the image are $A'(-2, -1)$, $B'(-1, -1)$, $C'(-2, -3)$, $D'(-4, -3)$, $E'(-4, -2)$.

**2 a**



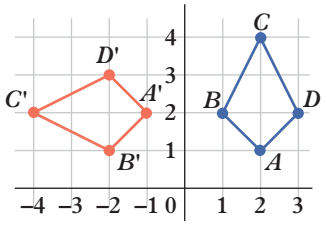

The coordinates of the image are $A'(-1, -2)$, $B'(-3, -3)$, $C'(-3, -2)$.

**b**



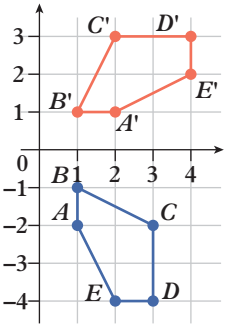

The coordinates of the image are $A'(-1, -2)$, $B'(-2, -3)$, $C'(-3, -3)$, $D'(-4, -1)$.

**c**



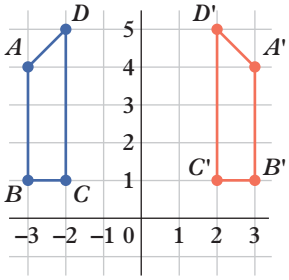

The coordinates of the image are $A'(-1, 2)$, $B'(-2, 1)$, $C'(-4, 2)$, $D'(-2, 3)$.

**d**



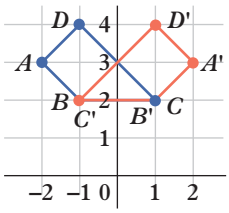

The coordinates of the image are $A'(2, 1)$, $B'(1, 1)$, $C'(2, 3)$, $D'(4, 3)$, $E'(4, 2)$.

**3 a**



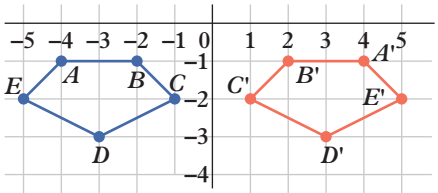

The coordinates of the image are $A'(3, 4)$, $B'(3, 1)$, $C'(2, 1)$, $D'(2, 5)$.

**b**



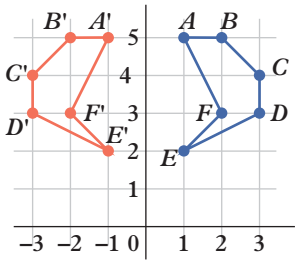

The coordinates of the image are $A'(2, 3)$, $B'(1, 2)$, $C'(-1, 2)$, $D'(1, 4)$.

**c**



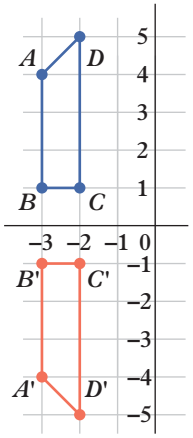

The coordinates of the image are $A'(4, -1)$, $B'(2, -1)$, $C'(1, -2)$, $D'(3, -3)$, $E'(5, -2)$.

**d**



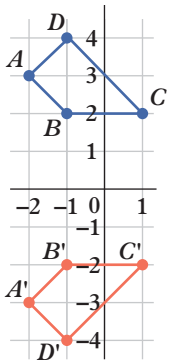

The coordinates of the image are $A'(-1, 5)$, $B'(-2, 5)$, $C'(-3, 4)$, $D'(-3, 3)$, $E'(-1, 2)$, $F'(-2, 3)$.

**4 a**



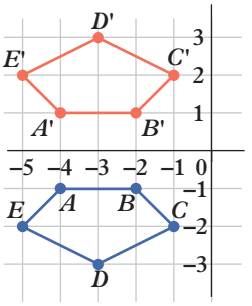

The coordinates of the image are $A'(-3, -4)$, $B'(-3, -1)$, $C'(-2, -1)$, $D'(-2, -5)$.

**b**



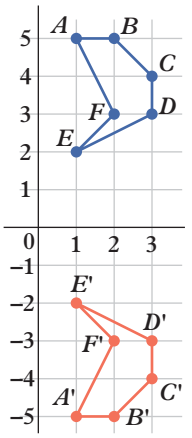

The coordinates of the image are $A'(-2, -3)$, $B'(-1, -2)$, $C'(1, -2)$, $D'(-1, -4)$.

**c**



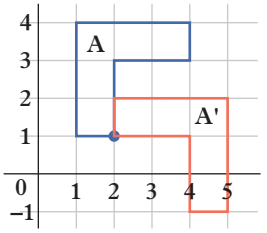

The coordinates of the image are $A'(-4, 1)$, $B'(-2, 1)$, $C'(-1, 2)$, $D'(-3, 3)$, $E'(-5, 2)$.

**d**

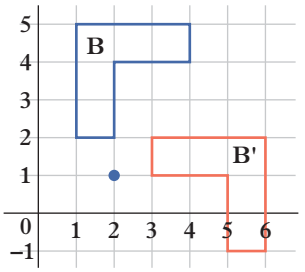



The coordinates of the image are $A'(1, -5)$, $B'(2, -5)$, $C'(3, -4)$, $D'(3, -3)$, $E'(1, -2)$, $F'(2, -3)$.

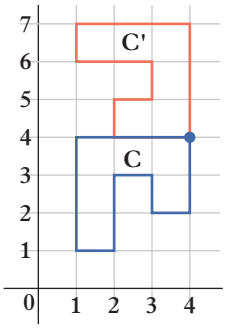
**5 a**



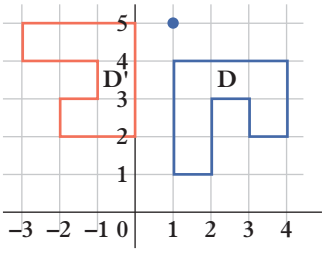

**b**



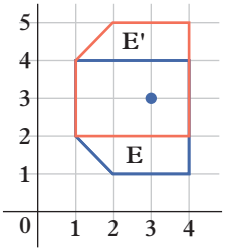

**c**



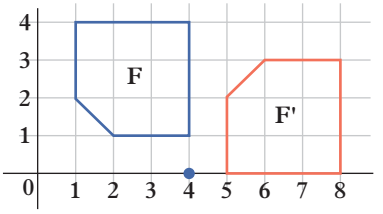

**d**



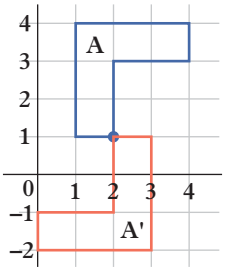

**e**



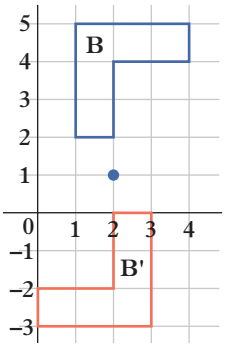

**f**



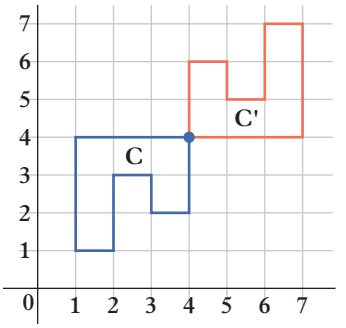

**6 a**




**b**




**c**



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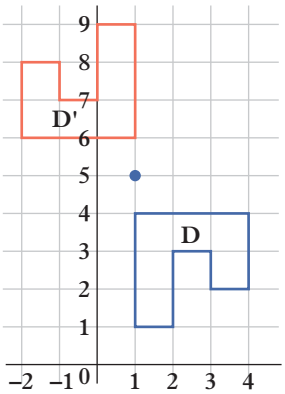


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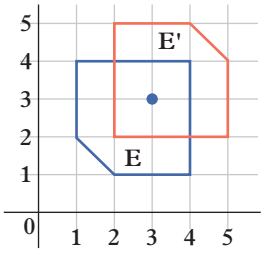


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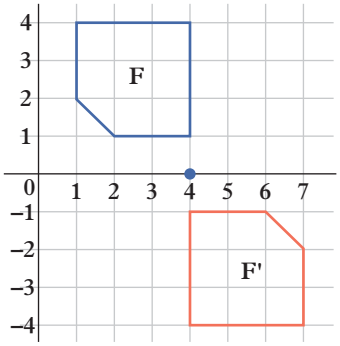
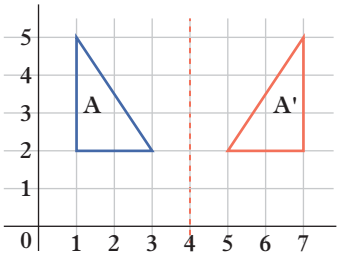



**7 a**

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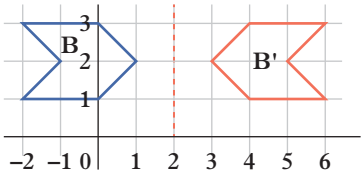


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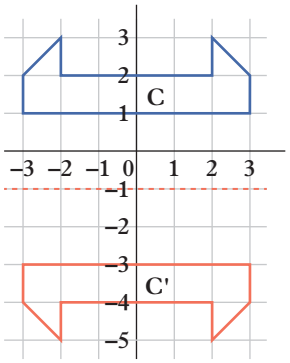


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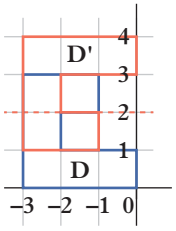



**8 a** 180° **b** 90° **c** 270°

**9** D

**10** D

**11** B: rotation of 90° clockwise (or 270° anticlockwise) about point (2, 2)

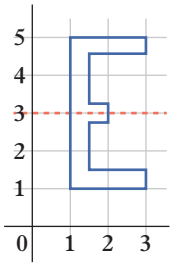
C: rotation of 180° clockwise (or 180° anticlockwise) about point (2, 2)

D: rotation of 90° anticlockwise (or 270° clockwise) about point (2, 2)

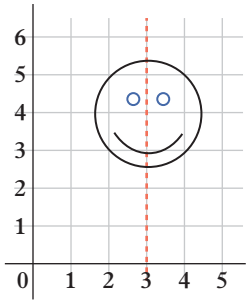
**12 a** (8, 5)

**b** (4, 1)

**13 a**

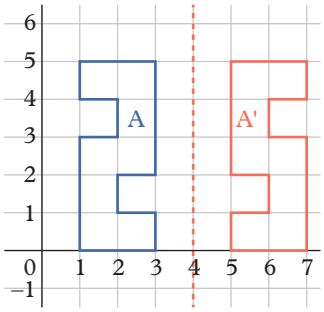


b

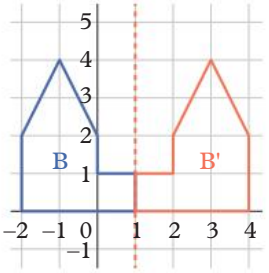


**14** Sample answer: Turning 90°clockwise results in the same position as turning 270° anticlockwise because there are 360° in a full circle and 270° = 360° − 90°.

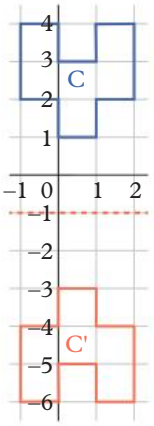
**15 a**



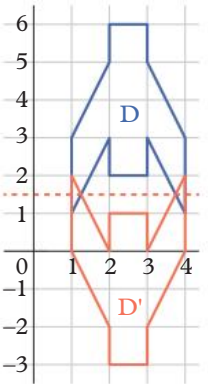
**b**



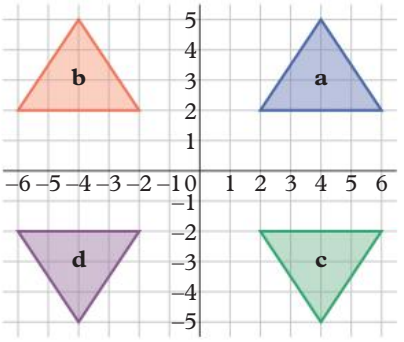

**c**




**d**




**16 a**

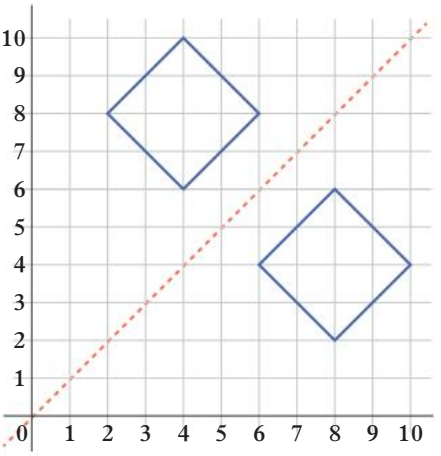



**b** (−2, 2), (−6, 2), (−4, 5)

**c** (2, −2), (6, −2), (4, −5)

**d** (−2, −2), (−6, −2), (−4, −5)

**17 a, b**

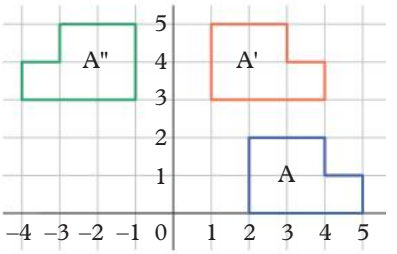


**c** The *x*- and *y*-coordinates have swapped places.

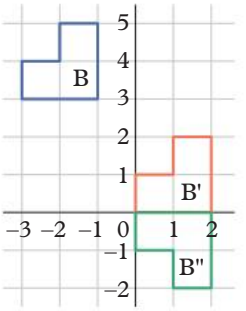
## EX 8F Combined transformations

p371

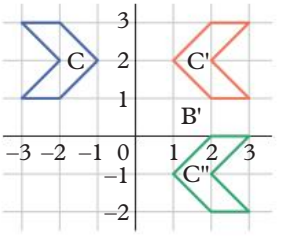
**1 a**



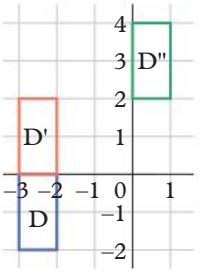

**b**



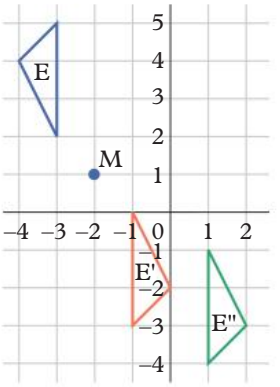

**c**



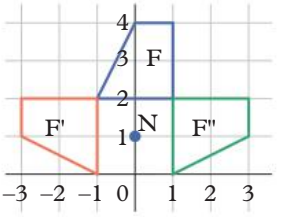

**d**




**e**



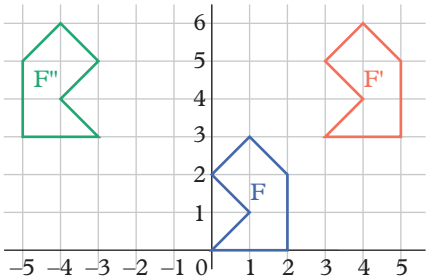

**f**



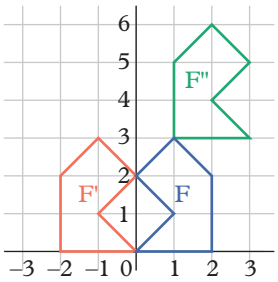
2 (0, 5)

3 A

4 a




b




5 a Reflection across the *x*-axis

b Rotation 90° clockwise (or 270° anticlockwise) about (1, 1)

c Translation 2 units down and 3 units right

d Reflection across the *y*-axis

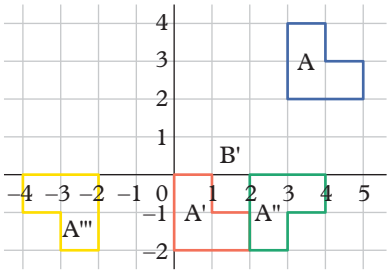
e Rotation 90° anticlockwise (or 270° clockwise) about (2, 6)

f Translation 3 units up and 4 units right

6 Rotation of 180° around the origin

7 Reflection across the *y*-axis or translation 4 units left

8

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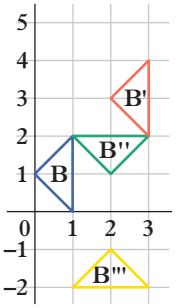



10 Sample answer: two reflections across different vertical mirror lines will move a shape in a horizontal direction and result in the original orientation of the shape.

11 a rotation 90° clockwise around the origin followed by a translation 1 unit down

b rotation 270° clockwise around the origin followed by a translation 7 units right

## EX 8G Line and rotational symmetry

p376

1 a Two b One c Zero
d Zero e Four f Six
g One h Zero i Two

2 a

Note: O has an infinite number of lines of symmetry.

b F, G, J, L, N, P, Q, R, S, Z

c H, I, O, X

3 a One b None c Two
d Four e Five

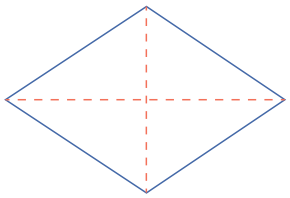
4 a Five b Two c Two
d Five e Four f Two
g One h Two i Two

5 a Three b Two c Five

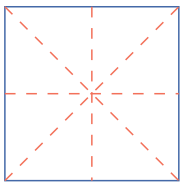
6 H: 2, I: 2, N: 2, O: infinite, S: 2, X: 2, Z: 2

7 Sample answers:

a

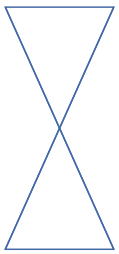


b

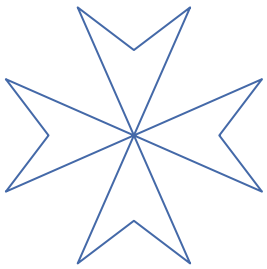


8 Sample answers:

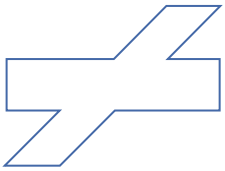
a



b



9 Sample answer: Line symmetry is the number of axes of symmetry a shape has; whereas rotational symmetry is the number of times an object fits exactly onto itself when being rotated 360°. They are not always identical; for example, the following shape has no axes of symmetry but will fit exactly onto itself when rotated 180°.



**10** B: horizontal line through 8 on the *y*-axis; C: vertical line through 8 on the *x*-axis, horizontal line through 5.5 on the *y*-axis; D: vertical line through 13 on the *x*-axis, horizontal line through 5 on the *y*-axis.

**11** **a** Horizontal axis of symmetry

**b** One example is BED.

**12** **a** Vertical symmetry through the letter U

**b** No

**c** One example is 'dob'.

**13** Rotational symmetry. One example is MOW

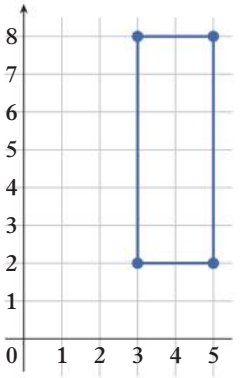
**14** **a** Infinite; between any two given lines of symmetry you can add another line of symmetry.

**b** Infinite; however small you rotate the circle it will still fit onto itself.

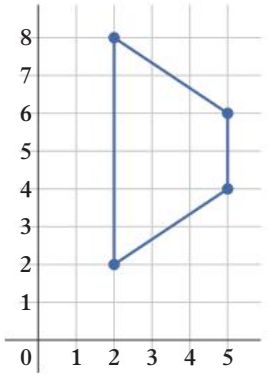
**15** **a** 3 **b** 6

**c** The number of lines of symmetry remains at 6 as the snowflake gets more sides; this is because the changes happen within each one of the six sections of the snowflake.

**16** **a**, **b** The new pair of coordinates are at (3, 2).

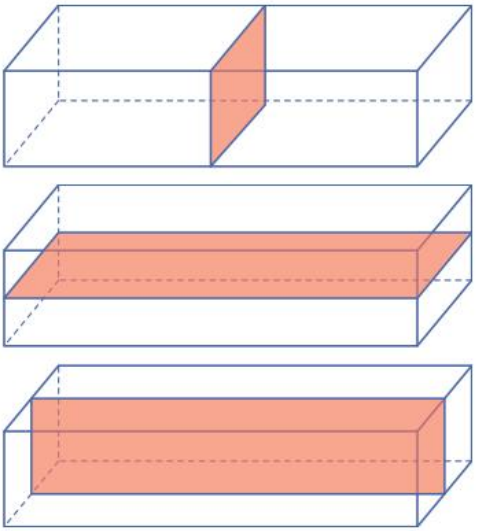



**17** **a**, **b** The new pair of coordinates are at (5, 6).



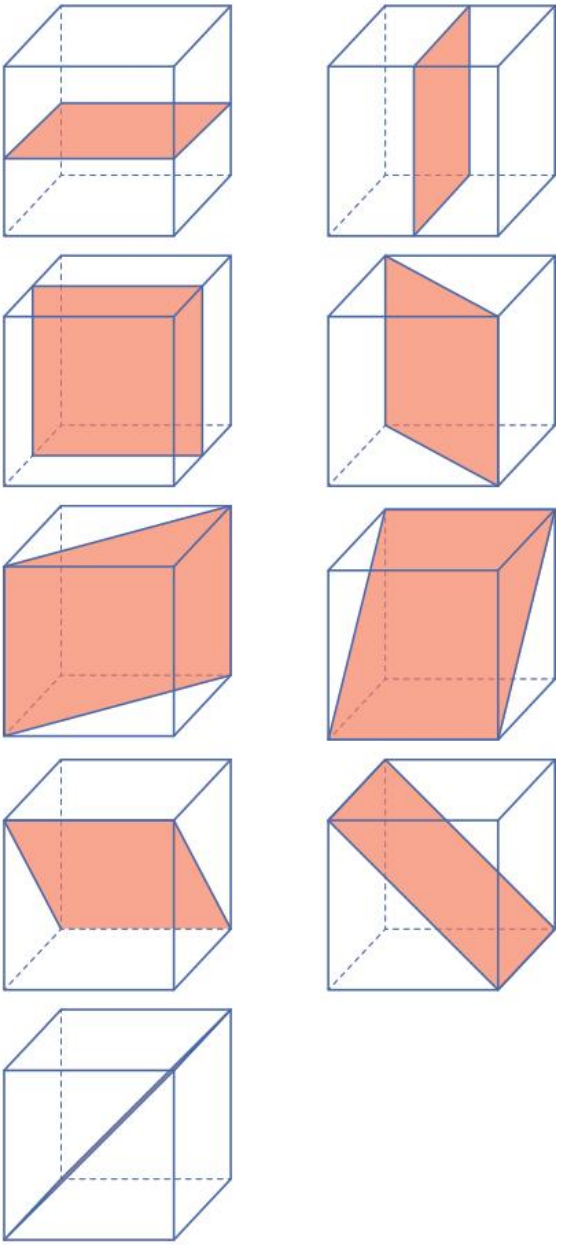

**18** **a** 2

**b**



**19** **a** 9

**b**



# CHAPTER 8 review

## Multiple-choice

| | | | | | |
|---|---|---|---|---|---|
| **1** | B | **2** | E | **3** | D |
| **4** | D | **5** | A | **6** | E |
| **7** | C | **8** | A | **9** | B |
| **10** | D | **11** | D | | |

## Short answer

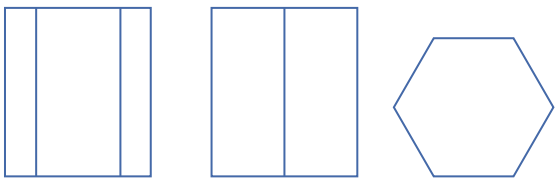
**1** **a** **i** Regular **ii** 7 **iii** No
**b** **i** Irregular **ii** 6 **iii** Yes
**c** **i** Irregular **ii** 8 **iii** Yes
**d** **i** Regular **ii** 5 **iii** No

**2** **a** Regular, convex heptagon
**b** Irregular, concave hexagon
**c** Irregular, concave octagon
**d** Regular, convex pentagon

**3** **a** No **b** Yes **c** Yes
**d** No **e** Yes **f** No

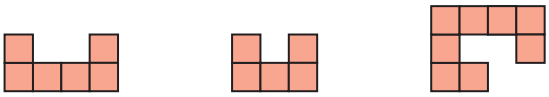
**4** **b** Triangular prism
**c** Rectangular prism
**e** Octagonal prism

**5**



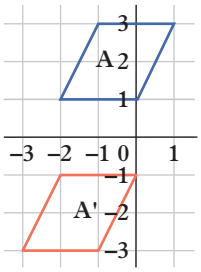
front elevation right elevation plan view

**6**

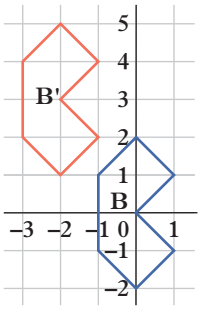


front elevation right elevation plan view

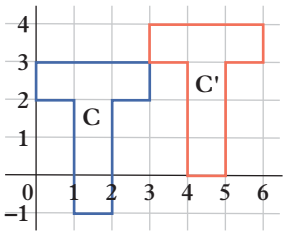
**7** **a**



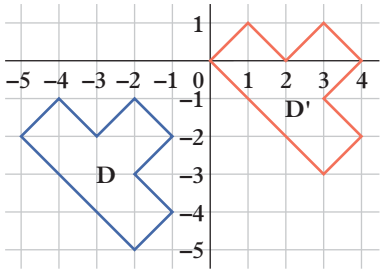

**b**




**c**

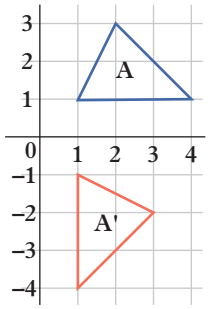



**d**

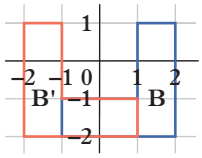



**8** **a** 2 units down and 1 unit left
**b** 7 units up
**c** 1 unit up and 5 units right
**d** 6 units up and 4 units right
**e** 3 units up and 5 units left
**f** 1 unit down and 9 units left

**9** **a**




**b**

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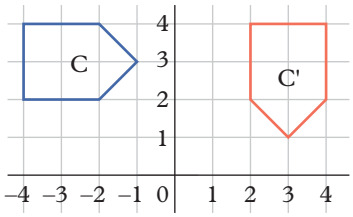
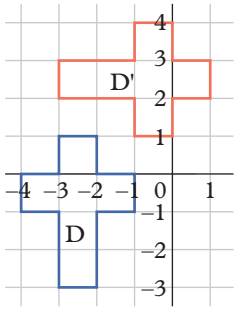
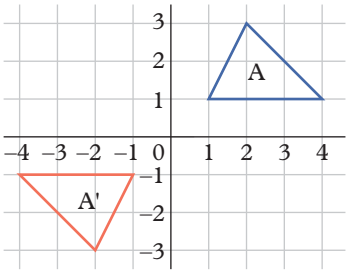


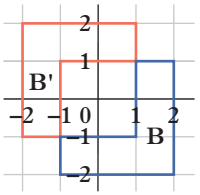

**d**



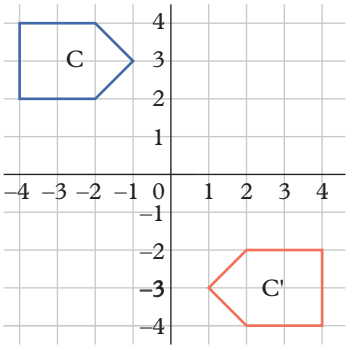
**10 a**



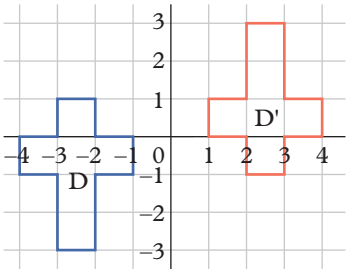

**b**



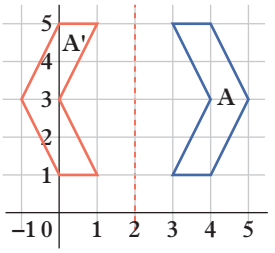

**c**



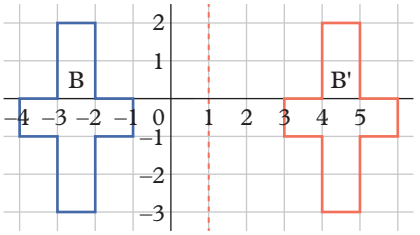

**d**



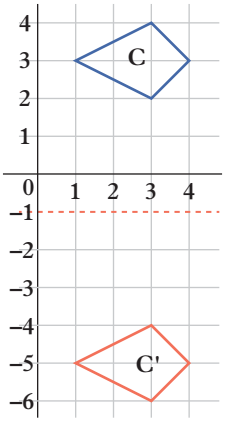

**11 a**



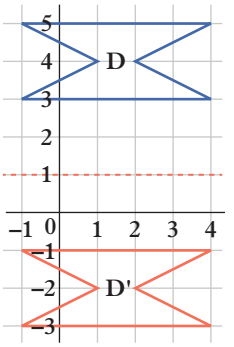

**b**




**c**




**d**




**12** B: reflection in the *y*-axis

C: translation 5 units down

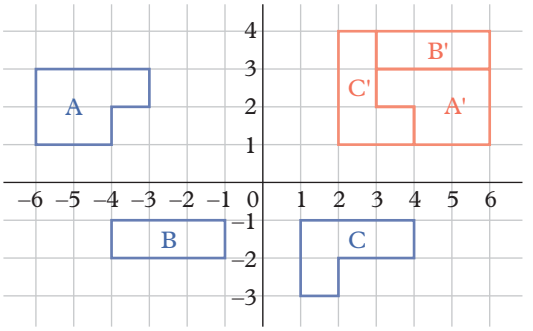
D: rotation 180° around the origin

**13 a** 2 **b** 4 **c** 1 **d** 5

**14 a** 2 **b** 4 **c** 1 **d** 5

## Analysis

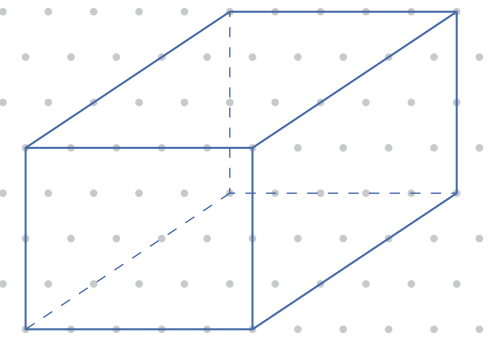
**a**




**b** (2, 1), (2, 4), (6, 4) and (6, 1)

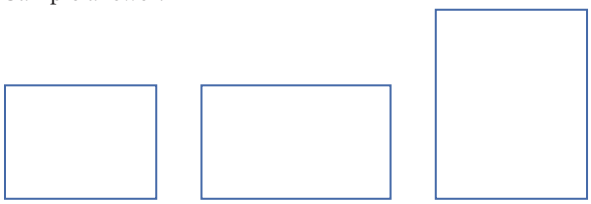
**c i** irregular **ii** convex

**d** Sample answer:



**e** **i** 6 **ii** 12 **iii** 8

**f** Sample answer:




# CHAPTER 9 Length, area and volume

## EX p390 9A Length and perimeter

**1** **a** 8 km = 8000 m
**b** 3 m = 300 cm
**c** 700 mm = 70 cm
**d** 500 cm = 5 m
**e** 16 cm = 160 mm
**f** 2000 m = 2 km
**g** 3.5 m = 350 cm
**h** 7.9 mm = 0.79 cm
**i** 0.06 km = 60 m
**j** 0.47 cm = 4.7 mm
**k** 4310 mm = 0.00431 km
**l** 51.2 cm = 0.512 m

**2** **a** 4 km = 4000 m = 400 000 cm
**b** 12 000 mm = 1200 cm = 12 m
**c** 73 m = 7300 cm = 73 000 mm
**d** 8400 cm = 84 m = 0.084 km
**e** 2500 mm = 250 cm = 2.5 m
**f** 36 200 cm = 362 m = 0.362 km

**3** **a** 550 mm **b** 27 km **c** 4.1 cm
**d** 620 cm **e** 3600 m **f** 0.89 m
**g** 170 000 cm **h** 0.5 m **i** 0.07 km
**j** 3650 mm **k** 940 000 mm **l** 0.0018 km

**4** **a** 23 km **b** 24 cm **c** 23 m
**d** 45 mm **e** 32.8 mm **f** 144 m

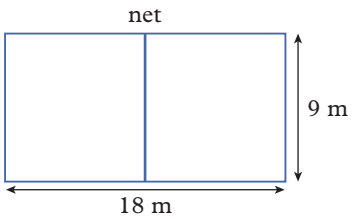
**5** **a** 16 cm **b** 18 cm **c** 24 cm
**d** 22 cm

**6** **a** 18 cm **b** 450 cm **c** 17.8 cm
**d** 8 cm **e** 292 cm **f** 22.4 cm

**7** **a** 15 mm, 5 cm, 60 mm
**b** 210 mm, 190 cm, 2 m
**c** 7001 cm, 0.7 km, 701 m
**d** 8 cm, 82 mm, 0.8 m
**e** 30 cm, 3000 mm, 300 m, 3 km
**f** 0.005 km, 5500 mm, 560 cm, 5.7 m

**8** **a** 42 cm **b** 34 cm **c** 30 mm
**d** 22.8 m

**9** **a**




**b** 54 m **c** 180 m

**10** 12 m by 6 m

**11** **a** 30 cm **b** $P = 2l + 2w$ or $P = 2(l + w)$

**12** **a** 16 cm **b** 44 m **c** 164 mm
**d** 248 cm **e** 21 m **f** 10.2 cm
**g** 25.74 m or 2574 cm
**h** 54 mm or 5.4 cm

**13** 9 cm

**14** $l = \dfrac{P - 2w}{2}$ or $l = \dfrac{P}{2} - w$

**15** 15.6 cm

**16** **a** Divided by 10 instead of multiplying by 10 to convert from cm to mm. Correct answer is 55 cm = 550 mm.
**b** Multiplied by 100 instead of multiplying by 1000 to convert from km to m. Correct answer is 24 km = 24 000 m.
**c** Inserted a zero before multiplying by 100 to convert from m to cm. Correct answer is 0.66 m = 66 cm.
**d** Divided by 100 instead of dividing by 10 to convert from mm to cm. Correct answer is 1.73 mm = 0.173 cm.
**e** Divided by 10 instead of dividing by 100 to convert from cm to m. Correct answer is 420 cm = 4.2 m.
**f** Divided by 10 000 instead of dividing by 1000 to convert from m to km. Correct answer is 800 m = 0.8 km.

**17** **a** 27.4 m **b** 82.2 m
**c** 109.6 m **d** Part **c**

**18** 85.5 mm

**19** **a** Sample answer: The sum of all the horizontal lengths of the staircase is equal to the length at the bottom and the sum of all of the vertical lengths of the staircase is equal to the height on the left.
**b** 54 cm

**20** Sample answer: The shapes have the same perimeter. AE = BE, AC = BD and CF = DF. The remaining side lengths are shared by both shapes.

**21** 26.88 m

**22** 40.2 m

**23** $x = 3, y = 3$

**24** $x = 2$

## EX p397 9B Area of a rectangle

**1** **a** 10 cm$^2$ **b** 5 cm$^2$ **c** 6 cm$^2$
**d** 8 cm$^2$ **e** 5 cm$^2$ **f** 7 cm$^2$

2 **a** **i** 10 cm **ii** 10 cm **iii** 14 cm
**iv** 14 cm
**b** **i** 6 $cm^2$ **ii** 4 $cm^2$ **iii** 8 $cm^2$
**iv** 6 $cm^2$
**c** Sample answer: Area is the space inside a shape; perimeter is the total distance around the edge of a shape.

3 **a** 40 $cm^2$ **b** 6 $m^2$ **c** 9.03 $m^2$
**d** 112 $cm^2$ **e** 532 $mm^2$ **f** 400 $mm^2$

4 **a** 63 $cm^2$ **b** 52 $mm^2$ **c** 6 $m^2$
**d** 52 $cm^2$ **e** 137.74 $mm^2$ **f** $91\frac{7}{8}$ $m^2$

5 **a** 10 $cm^2$ or 1000 $mm^2$
**b** 27 $m^2$ or 270 000 $cm^2$
**c** 0.084 $km^2$ or 84 000 $m^2$
**d** 5.76 $cm^2$ or 576 $mm^2$
**e** 0.035 $m^2$ or 35 000 $mm^2$
**f** 25 $cm^2$ or 2500 $mm^2$

6 **a** 3 $m^2$ or 30 000 $cm^2$
**b** 40 $cm^2$ or 4000 $mm^2$
**c** 1.12 $km^2$ or 1 120 000 $m^2$
**d** 180 000 $mm^2$ or 1800 $cm^2$
**e** 6.2 $cm^2$ or 620 $mm^2$
**f** 42 $cm^2$ or 0.0042 $m^2$

7 **a** 16 $m^2$ **b** 25 $cm^2$
**c** 289 $mm^2$ **d** 12.96 $cm^2$

8 $A = l \times l$ or $A = l^2$

9 **a** 49 $m^2$ **b** 225 $cm^2$ **c** 121 $km^2$
**d** 3600 $mm^2$ **e** 40.96 $cm^2$ **f** 32 400 $m^2$
**g** 0.49 $mm^2$ **h** 0.2809 $km^2$

10 **a** 30 $cm^2$ **b** 6 $cm^2$ **c** 36 $cm^2$
**d** 20 $cm^2$ + 16 $cm^2$ = 36 $cm^2$

11 **a** 87 $mm^2$ **b** 48 $m^2$ **c** 110 $cm^2$
**d** 40 $mm^2$

12 **a** Find the area of the larger rectangle and subtract the area of the smaller rectangle.
**b** 51 $m^2$

13 **a** 115 $m^2$ **b** 92 $mm^2$ **c** 65 $cm^2$
**d** 19 $m^2$

14 **a** 60 mm or 6 cm **b** 50 mm

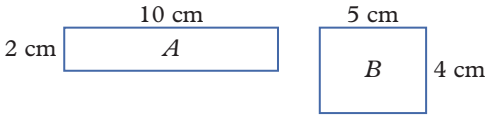
15 **a** **i** 43 $m^2$ **ii** 32 m
**b** **i** 17.19 $cm^2$ **ii** 20.2 cm
**c** **i** 80.81 $m^2$ **ii** 41.6 m
**d** **i** 292 $mm^2$ **ii** 86 mm

16 126 $cm^2$

17 400 $m^2$

18 16 mm

19 Sample answer: Area and perimeter are two different measures. Long, thin rectangles will have a larger perimeter than wider rectangles with the same area. For example, both of the rectangles below have an area of 20 $cm^2$, but rectangle A has a perimeter of 24 cm and rectangle B has a perimeter of 18 cm.




20 **a** **i** 11.885 m **ii** 10.97 m
**iii** 8.23 m **iv** 260.7569 $m^2$
**v** 195.6271 $m^2$
**vi** 26.336 $m^2$
**b** 260.7569 $m^2$ (more material would be required to cover the court surface past the outer lines, particularly past the baselines where players may run during rallies.)

21 Sample answer: Yes; one possible rectangle has length 6 cm and width 3 cm. The perimeter is 18 cm and the area is 18 $cm^2$.

22 **a** $5x + 6y + 2xy$
**b** $x(2y + 5) + 6y$ and $2y(x + 3) + 5x$
**c** $(2y + 5)(x + 3) - 15$

## EX 9C Area of a parallelogram

p402

1 **a** 10 $cm^2$ **b** 5 $cm^2$ **c** 15 $cm^2$
**d** 8 $cm^2$

2 **a** Base length = 22 cm, height = 12 cm
**b** Base length = 9 cm, height = 2 cm
**c** Base length = 16 mm, height = 17 mm
**d** Base length = 4.2 m, height = 1.5 m

3 **a** 24 $cm^2$ **b** 56 $mm^2$ **c** 360 $m^2$
**d** 160 $cm^2$ **e** 36 $mm^2$ **f** 3.6 $m^2$

4 **a** 15 $cm^2$ **b** 170 $mm^2$ **c** 6 $m^2$
**d** 10.25 $cm^2$

5 **a** 15 $cm^2$ **b** 116 $m^2$
**c** 2400 $mm^2$ **d** 75.076 $cm^2$

6 **a** 4000 $cm^2$ **b** 3.5 $cm^2$ **c** 138 000 $cm^2$
**d** 34.56 $cm^2$ **e** 63 $cm^2$ **f** 4.64 $cm^2$

7 **a** 120 $cm^2$ **b** 42 $cm^2$ **c** 42 $m^2$
**d** 276 $mm^2$

8 **a** 68 $m^2$ **b** 141 $cm^2$ **c** 309 $mm^2$
**d** 141.9 $cm^2$

9 364.5 $cm^2$

10 **a** 3.3 $m^2$ **b** 3.85 $m^2$

11 **a** 24 $cm^2$ **b** 18 $cm^2$ **c** 60 $cm^2$
**d** 50 cm

12 5 cm

13 8 cm

14 **a** 390 $cm^2$ **b** 390 $cm^2$
**c** Sample answer: A rectangle is a special parallelogram where the height of the parallelogram equals the width of the rectangle and base of parallelogram is equal to length of rectangle. In both cases, the two dimensions (length and width in a rectangle, and base and height in a parallelogram) are at right angles to each other.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**15** **a** 56.6284 $cm^2$ **b** 64.6809 $cm^2$ **c** 87.55%

**16** 80 $cm^2$

## 9 Checkpoint

**1** **a** 22 cm **b** 12 cm **c** 44 cm

**2** **a** 1.5 cm **b** 25 912 000 m
**c** 0.23 m **d** 300 mm

**3** **a** 16 mm
**b** 18.1 cm = 181 mm
**c** 35.4 m

**4** **a** 18 $cm^2$ **b** 9 $cm^2$ **c** 29 $cm^2$

**5** **a** 10 $cm^2$ **b** 20.5 $cm^2$

**6** **a** 300 $mm^2$ **b** 49 $cm^2$
**c** 8400 $cm^2$ **d** $6\frac{1}{15}$ $m^2$

**7** **a** Perimeter = 34 cm, area = 38 $cm^2$
**b** Perimeter = 78 mm or 7.8 cm, area = 137 $mm^2$ or 1.37 $cm^2$

**8** **a** 21 $cm^2$ **b** 24 $cm^2$ **c** 12 $cm^2$

**9** **a** 14 $m^2$ **b** 30 $cm^2$ **c** 16.2 $cm^2$
**d** 9 $m^2$

**10** **a** 1215 $mm^2$ **b** 40.47 $m^2$

**11** **a** $a = 1$ cm **b** $b = 2$ mm **c** $x = 12$ m
**d** $y = 8$ cm

## EX p410 9D Area of a triangle

**1** **a** 4 $cm^2$ **b** 6 $cm^2$ **c** 8 $cm^2$
**d** 4 $cm^2$

**2** **a** 54 $cm^2$ **b** 48 $mm^2$ **c** 7.5 $m^2$
**d** 30.1 $cm^2$ **e** 68.87 $mm^2$ **f** 131.131 $m^2$

**3** **a** Base length = 8 m, height = 5 m
**b** Base length = 10 cm, height = 6 cm
**c** Base length = 16 cm, height = 15 cm
**d** Base length = 8 mm, height = 6 mm
**e** Base length = 12.6 mm, height = 3.1 mm
**f** Base length = 2.4 m, height = 3.5 m

**4** **a** 12 $mm^2$ **b** 36 $cm^2$ **c** 168 $mm^2$
**d** 117.6 $m^2$ **e** 30 $mm^2$ **f** 189 $cm^2$

**5** **a** **i** 30 $cm^2$ **ii** 30 $cm^2$ **iii** 30 $cm^2$
**b** Sample answer: all these triangles have the same base length and height, so their areas are also the same.

**6** **a** 12 $cm^2$ or 1200 $mm^2$
**b** 240 $mm^2$ or 2.4 $cm^2$
**c** 1 $m^2$ or 10 000 $cm^2$
**d** 155 000 $cm^2$ or 15.5 $m^2$
**e** 70 $mm^2$ or 0.7 $cm^2$
**f** 1.358 $m^2$ or 13 580 $cm^2$

**7** **a** 24 $mm^2$ or 0.24 $cm^2$
**b** 13 650 $cm^2$ or 1.365 $m^2$
**c** 8.1 $cm^2$ or 810 $mm^2$
**d** 23.4 $cm^2$ or 2340 $mm^2$
**e** 3.57 $m^2$ or 35 700 $cm^2$
**f** 6 $cm^2$ or 600 $mm^2$

**8** **a** The student used 15 cm as the height of the triangle instead of 9 cm. The correct area of the triangle is 54 $cm^2$.
**b** 36 cm

**9** 45 $m^2$

**10** 4 cm

**11** 40 mm

**12** 2880 $cm^2$

**13** **a** 84 $cm^2$ **b** 68 $m^2$ **c** 16 $mm^2$
**d** 1750 $m^2$ **e** 222 $mm^2$ **f** 248 $cm^2$

**14** **a** 162 $cm^2$ **b** 1152 $cm^2$

**15** 162 $mm^2$

**16** 31.5 $m^2$

## EX p416 9E Surface area

**1** **a** 24 $m^2$ **b** 294 $mm^2$ **c** 600 $cm^2$
**d** 384 $mm^2$ **e** 13.5 $m^2$ **f** 5400 $cm^2$

**2** **a** Six
**b** 32 $m^2$, 24 $m^2$ and 12 $m^2$
**c** 136 $m^2$

**3** **a** 344 $mm^2$ **b** 166 $cm^2$ **c** 378 $mm^2$
**d** 268 $m^2$ **e** 76.96 $m^2$ **f** 738 $cm^2$

**4** **a** Five
**b** 12 $cm^2$, 12 $cm^2$, 45 $cm^2$, 45 $cm^2$, 54 $cm^2$
**c** 168 $cm^2$

**5** **a** 2916 $cm^2$ **b** 510 $mm^2$ **c** 142 $m^2$
**d** 768 $mm^2$ **e** 2408 $cm^2$ **f** 523 $m^2$

**6** **a** 1678 $cm^2$ **b** 3072 $cm^2$ **c** 580.2 $cm^2$
**d** 13.5 $cm^2$

**7** **a** 0.47 $m^2$ **b** 0.47 $m^2$ **c** 0.94 $m^2$

**8** $T = 6s^2$

**9** 5 cm

**10** Length = 2 m, width = 2 m, height = 2 m

**11** $T = 2(hw + hl + lw)$ or $T = 2hw + 2hl + 2lw$

**12** **a** 9.86 $m^2$ **b** Two cans

**13** 24.92 $m^2$

**14** **a** 50 $cm^2$ **b** 30 $cm^2$

**15** 283.14 $m^2$

## EX p422 9F Volume and capacity

**1** **a** 30 $cm^3$ **b** 56 $mm^3$ **c** 48 $m^3$
**d** 36 $cm^3$ **e** 72 $mm^3$ **f** 45 $m^3$

**2** **a** 280 $cm^3$ **b** 90 $m^3$ **c** 320 $mm^3$
**d** 94.5 $m^3$ **e** 2.88 $cm^3$ **f** 1015.014 $mm^3$

**3** **a** 8 $m^3$ **b** 343 $mm^3$ **c** 1000 $cm^3$
**d** 15.625 $m^3$

**4** **a** 8000 mL **b** 15 000 L **c** 0.07 L
**d** 6400 kL **e** 2.5 kL **f** 3 000 000 mL

5 a 14 mL b 3000 $cm^3$ c 5.8 kL
d 0.75 L e 1000 $m^3$ f 6400 $m^3$
6 a 280 $cm^3$ b 194.4 $cm^3$ c 1.44 $cm^3$
d 2.52 $cm^3$
7 a 64 $m^3$ b 7.2 $m^3$
c 0.8775 $m^3$ d 0.75 $m^3$
8 a 3 456 000 mL
b 5 000 000 $cm^3$
c 1 544 000 $cm^3$
9 a 102 375 L b 48 048 L
10 a C, A, B b B, A, C
11 6.25 $m^3$
12 0.125 m or 12.5 cm
13 a 1 $cm^3$ b 840
14 18 L
15 E
16 a Surface area = 2400 $cm^2$, volume = 8000 $cm^3$
b Surface area = 2400 $cm^2$, volume = 6300 $cm^3$
17 a 17 mm b 1.3 m
c 14 mm d 80 cm or 0.8 $m^3$
18 a 48 $cm^3$ b 190 $m^3$
c 74.25 $cm^3$
19 Four sets: 20 cm by 1 cm by 1 cm, 10 cm by 2 cm by 1 cm, 5 cm by 4 cm by 1 cm, 5 cm by 2 cm by 2 cm
20 a 20 cm b 0.2 cm
c 240 mL d $\frac{3}{5}$

## CHAPTER 9 review

### Multiple-choice

1 A 2 C 3 E
4 D 5 B 6 A
7 B 8 D 9 D
10 E

### Short answer

1 a 340 mm b 1.7 m c 4.5 cm
d 8700 m
2 a 60 m b 220 cm or 2.2 m
3 a 125 $m^2$ b 2400 $cm^2$ or 0.24 $m^2$
4 56 $m^2$
5 a 15 $cm^2$ b 16 $cm^2$ or 1600 $mm^2$
6 8610 $mm^2$
7 a 84 $cm^2$ b 18 $cm^2$ or 1800 $mm^2$
8 102.795 $km^2$
9 a 29 450 $cm^2$ b 2400 $mm^2$
10 a 288 750 $cm^3$ b 8000 $mm^3$
11 a 614.125 $cm^3$
b 96 800 $cm^3$ or 9.68 $m^2$ or 9 680 000 $mm^2$
12 a 140 L b 385 000 L

### Analysis

1 a 87 $cm^2$ b 107.5 $cm^2$ c 59.5 $cm^2$
d 178 $cm^2$ e 750 $cm^3$
f 0.15 L of yellow and 0.3 L of blue
g 100 cm
2 a 12 077 $cm^2$ b 90 L c 60 cm

## CHAPTER 10 Statistics and probability

### EX 10A Collecting and classifying data

p437

1 a Primary data b Primary data
c Secondary data d Secondary data
e Primary data
2 a i Numerical ii Categorical
iii Categorical iv Numerical
v Categorical vi Numerical
vii Categorical viii Numerical
b i Continuous iv Continuous
vi Discrete viii Continuous
c ii Nominal iii Nominal
v Nominal vii Ordinal
3 a Numerical, continuous
b Numerical, discrete
c Categorical, nominal
d Numerical, continuous
e Categorical, ordinal
f Numerical, discrete
g Categorical, ordinal (small, medium or large)
h Numerical, discrete
4 Sample answers:
a number of chocolates in a box
b height of students in a class
c Quality of service, ranging from very good to very bad
d shape of Lego bricks
5 a Categorical, nominal
b i Comedy ii Horror
6 a

| Number of siblings | Tally | Frequency |
|---|---|---|
| 0 | \|\| | 2 |
| 1 | 卌 卌 \| | 11 |
| 2 | 卌 \|\|\|\| | 9 |
| 3 | \|\| | 2 |
| 4 | \| | 1 |
| 5 | | 0 |
| 6 | | 0 |
| 7 | | 0 |
| 8 | \| | 1 |

b 26 c 1
7 a i Categorical, nominal ii Census
b i Categorical, nominal ii Sample
c i Numerical, continuous ii Sample
d i Numerical, discrete ii Census

**8** **a**

| Colour | Tally | Frequency |
|---|---|---|
| Pink | \|\|\|\| | 4 |
| Blue | \|\| | 2 |
| Green | \|\|\| | 3 |
| Purple | ~~\|\|\|\|~~ | 5 |
| Yellow | \|\|\|\| | 4 |
| Orange | ~~\|\|\|\|~~ \| | 6 |

**b** Orange

**c** Blue

**d** 500

**e** Sample answer: it is quicker and easier to take a sample, but you might not get an accurate view of the population.

**9** **a** Primary **b** Secondary **c** Secondary **d** Primary **e** Secondary

**10** Sample answers:

**a** There are millions of fish in the Pacific Ocean and they are regularly moving.

**b** There are thousands of Year 7 students in Victoria.

**c** A field usually contains millions of pieces of grass.

**d** There are many questions in this textbook.

**11** Sample answer: children can grow quite a lot within the space of a year, so measuring Rosa's class at the end of the year gave the students in her class more time to grow taller than Jaime's class.

**12** Sample answer: members of Kate's soccer team are probably more likely to pick soccer as their favourite sport than a Year 7 student selected at random.

**13** Sample answers:

**a** They indicate an order.

**b** You cannot count or measure the data, and you can't find the average team ranking.

**c** Sample answer: Australian post codes

**14** **a** Categorical, nominal

**b** Numerical, discrete

**c** Categorical, ordinal

**d** Numerical, discrete

**e** Categorical, nominal

**15** Sample answers:

**a** **i** Square kilometres

**ii** Small, medium, large

**b** **i** Number of cars passing through the intersection

**ii** Low, medium, high

**c** **i** Number of bees

**ii** Less than 100 bees, 100 bees or more

## EX p442 10B Summary statistics

**1** **a** 8 **b** 7 **c** 7 **d** 9 **e** 7.5

**2** **a** 1.04 **b** 1 **c** 1 **d** 4

**3** **a** **i** 4.92 **ii** 5 **iii** 7 **iv** 7

**b** **i** 4.71 **ii** 5 **iii** 1, 6 **iv** 8

**c** **i** 5.5 **ii** 5.5 **iii** No mode **iv** 9

**d** **i** 4.87 **ii** 5 **iii** 3, 4, 5, 7, 9 **iv** 9

**e** **i** 7.33 **ii** 7 **iii** 4 **iv** 12

**f** **i** 8.29 **ii** 7 **iii** No mode **iv** 15

**g** **i** 12.25 **ii** 12.5 **iii** 13 **iv** 13

**h** **i** 13.82 **ii** 6 **iii** 5, 8 **iv** 92

**4** **a** The 4th value

**b** Between the 4th and 5th value

**c** The 7th value

**d** Between the 11th and 12th value

**5** **a** C **b** D **c** A **d** B

**6** **a** 90 g

**b** **i** Increases **ii** Stays the same **iii** Decreases **iv** Increases

**7** **a** Range **b** Mode **c** Median **d** Mean

**8** Sample answer:

Mean: If all carts travelled the same distance, but the total distance travelled remained the same, then each cart would have travelled 6.4 m.

Median: Half the carts travelled more than 6.2 m.

Mode: The most common distance travelled was 4.0 m.

Range: The distances travelled by the carts were spread out over a 10.1 m interval.

**9** **a** Mean = \$23.85, median = \$20, mode = \$20, range = \$40

**b** Sample answer: More students spent \$20 on their mobile phone than any other amount. The difference between the least and most amount spent on mobile phones each month was \$40. The average spend was \$23.85.

**10** **a** Mean = \$13.46, median = \$5, mode = \$5

**b** \$8.46

**c** Sample answer: Only two students earn more than the mean. The student who gets \$100 has had a big effect on the results. Most students earn much less than the mean.

**d** Sample answer: Most would be able to, but most would have no money left over for anything else; all except two students get \$5 a week or more. \$5 a week is about \$20 a month, which is what most students spend on their mobile phone per month.

**11** **a** Mean = \$41 875, median = \$41 000

**b** Sample answer: Both the mean and median are significantly less than \$80 000, so this claim is hard to justify. However, this is only a sample of the employees, and this may be a sample that is not representative of the population.

**c** **i** The mean, as the two employees skewed the average much higher than it should have been.

**ii** The median, as 50% of employees will earn more than it and 50% of employees will earn less than it.

**12** **a** **i** Mean = 50.8%, Median = 42.5%

**ii** Mean = 87.7%, Median = 96.5%

**b** **i** Mean **ii** Median

**c** **i** Median **ii** Median

**13** **a** Dylan, 19 minutes

**b** Dylan: 5.07 minutes, Costa: 7.29 minutes

**c** One possible answer is that Costa's train is worse because on average it runs later than Dylan's, even though Dylan's train has had two very late departures.

**14** Sample answer:
mean = 466, median = 472, mode = 456.
All three summary statistics can be described as the average.

**15** **a** Player 2

**b** Player 1: mean = 3.4, median = 3, mode = 3
Player 2: mean = 3.47, median = 3, mode = 1

**c** Player 1, whose mean, median and mode are all similar values and who has a much smaller range (5 vs. 12).

**d** 13 is the outlier. This value is significantly different from all the other values in this data set.

**e** Mean = 2.79, median = 3

**f** The outlier made a significant difference to the mean, increasing the value from 2.79 to 3.47, but did not affect the median.

**16** **a** The mean, because of the number of decimal places. The mode will be a whole number, and the median will be either a whole number or a whole number and a half.

**b** The mean is a numerical average that does not have to match any of the values in a data set. The mean is the value given if all the values in a data set are evenly distributed.

**17** **a** 1, 2, 4, 5, 8 1, 2, 4, 6, 7 1, 3, 4, 5, 7
2, 3, 4, 5, 6

**b** The middle value must be 4, and the sum of the values must be 20; therefore, the remaining four values must sum to 16. There are only four potential data sets that meet these criteria.

**18** **a** **i** 8.33 **ii** 7.75
**iii** 6.8 **iv** 8.7

**b** **i** 30.76 **ii** 28.75
**iii** no mode **iv** 34.7

**19** **a** **i** 6.5 **ii** 7
**iii** 8 **iv** 4

**b** **i** 13.11 **ii** 13
**iii** 13 and 15 **iv** 5

## EX 10C Constructing and interpreting graphs

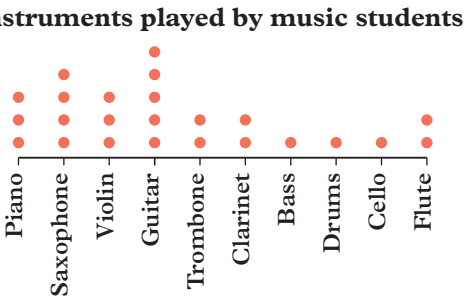
p450

**1** **a** Nintendo, 12 **b** Xbox, 4
**c** 4 **d** 30

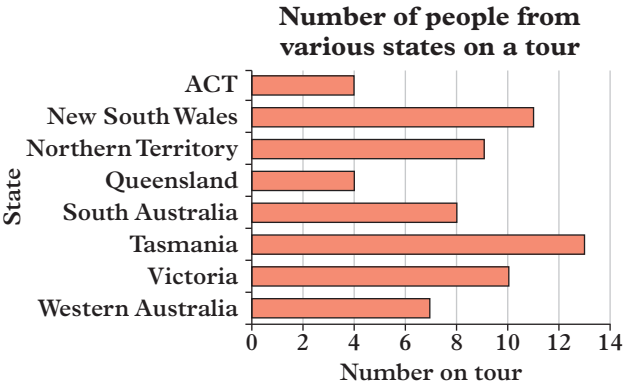
**2** **a** 5 **b** 6 **c** 27

**d**

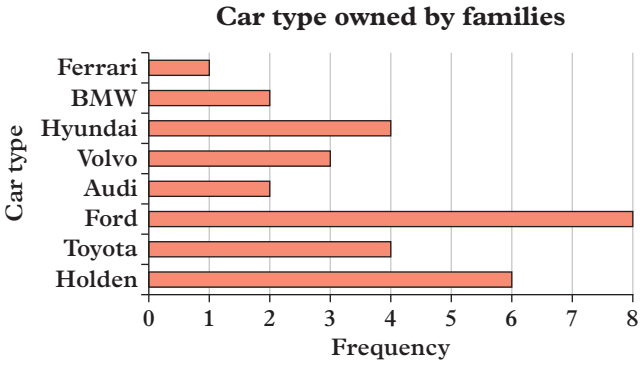
**3**




**4**

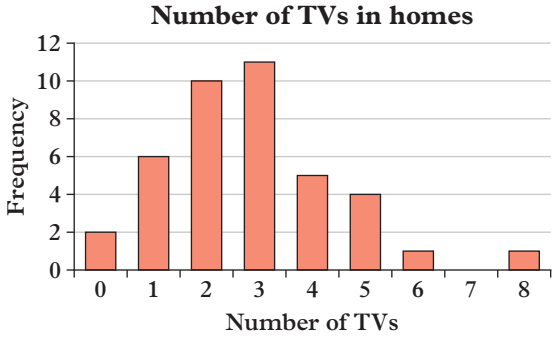


**5** **a**




**b** Ford **c** Ferrari

**6**

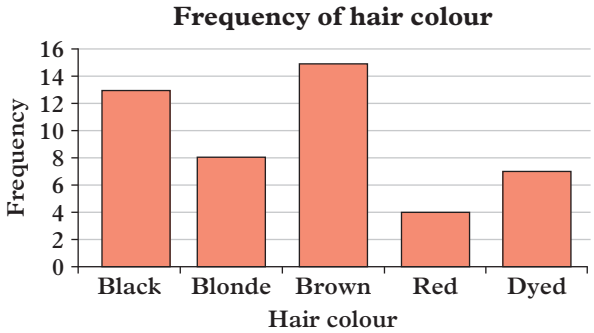



**7** **a** 25 **b** 9 **c** 5
**d** 5

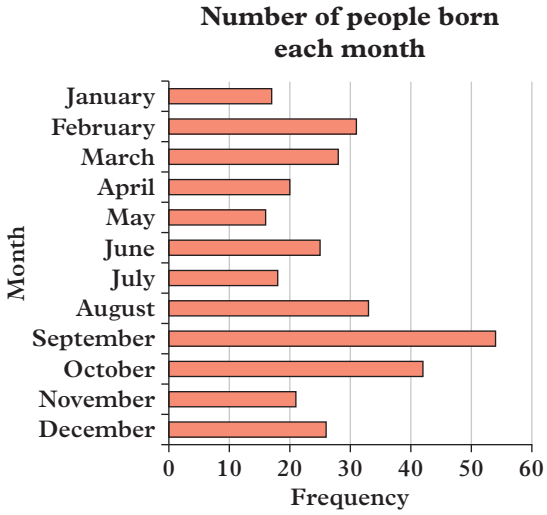
**8** **a** 26 **b** 5 **c** 1
**d** 1.5 **e** 1.88
**f** Sample answer: The number of siblings for this group of children is spread out over 5 siblings (from 0 to 5). The average number of siblings for this group of children is 1–2 (mode: 1, median: 1.5, mean 1.88).

**9** **a** Average lifespan of common dog breeds
**b** Sample answer: the average lifespan of these common dog breeds ranges from 9 years for a German shepherd to 17 years for a Jack Russell terrier. The 'average' lifespan seems to be about 11–13 years old.

**10** Sample answer: a column graph is appropriate because it makes the data easy to interpret. A bar graph could also be used, but there are too many data points for a dot plot.




**11** Sample answer: a bar graph is appropriate as all the categories can be easily read.

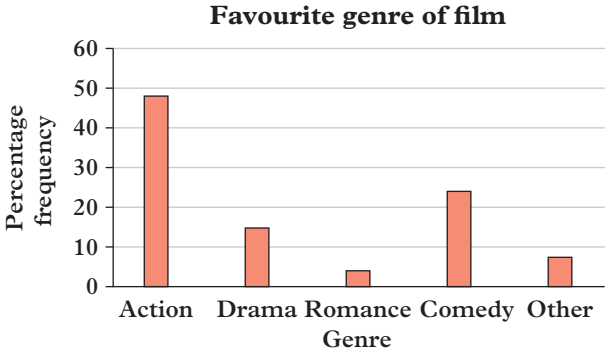



**12** **a** Categorical, nominal
**b** Pop; 48 **c** Hip-hop; 28
**d** Hip-hop **e** Country music
**f** Sample answer: hip-hop and pop, because across the year they were the most liked.

**13** **a, b**

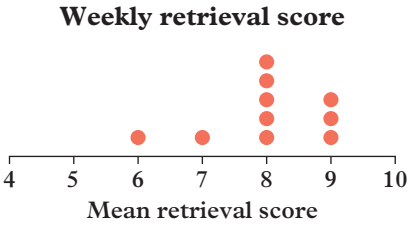
| Genre | Tally | Frequency | Percentage frequency |
|---|---|---|---|
| Action | 𝍸 𝍸 \|\| | 12 | 48% |
| Drama | \|\|\|\| | 4 | 16% |
| Romance | \| | 1 | 4% |
| Comedy | 𝍸 \| | 6 | 24% |
| Other | \|\| | 2 | 8% |

**c**



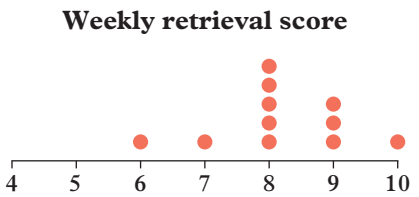

**d** Action, 48%
**e** Romance, 4%

**14** **a**




**b** 8

**c**




**d** 8.18

**15** **a** 3 **b** 6

**c** Mean = 3.79, Median = 4

**16** **a** 9

**b** Sample answer: Delete the column for the score 9, remove the scores above 6 from the horizontal axis, and add 1 to the height of the 3 column.

**17** A = mode, B = median, C = mean

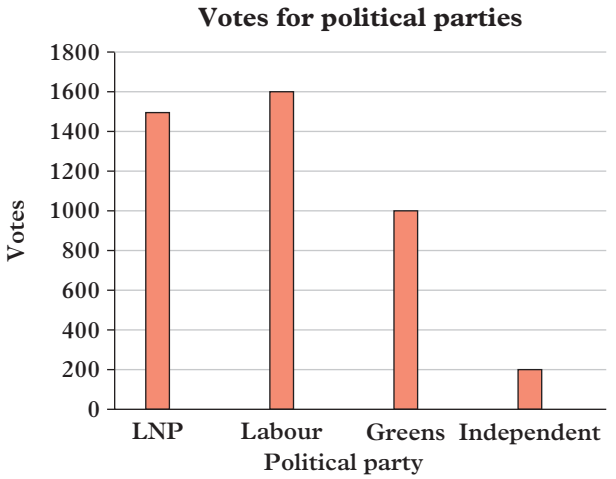
Sample answer: The mode is the highest column (A), the median must be either a whole number or whole number and a half (B) and the mean is the other value (C).

**18** The values are not evenly spaced out vertically, so it appears as though Easter and Australia Day are less popular than they are, and Anzac day is more popular than it is..

**19** The vertical scale does not start at 0 so, for example, it looks like the population has doubled between 2014 and 2015 when in reality it did not.

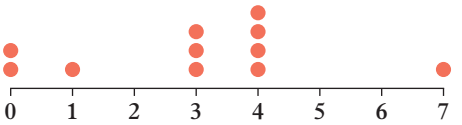
**20** **a** Sample answer: The difference in the heights of the bars is not consistent. There is a 100 vote difference between LNP and Labor; which is one represented by a one line jump; whereas, there is a 800 vote difference between the Greens and Independent, which is represented by a half line jump.

**b**




**21** Sample answer: The range and the mode. The range will not change as the highest and lowest values still have dots. The mode will not change as the tallest column will stay the tallest if the same amount is removed from each column. Since the total number of dots will change, the median and mean are likely to change (although they may not).

**22** Sample answer:




Note: other possible answers exist.

## EX 10D Stem-and-leaf plots

p458

**1** **a** Sample answer: 1 | 4 = 14

**b** 14 **c** 98 **d** 84

**e** 37 **f** 31

**2** **a** 42 **b** 19 **c** 20

**d** 20.79

**3** **a**

| Stem | Leaf |
|---|---|
| 1 | 1 4 8 9 |
| 2 | 3 3 6 |
| 3 | 1 8 |
| 4 | 2 2 3 |

Key: 1 | 1 = 11

**b**

| Stem | Leaf |
|---|---|
| 0 | 4 |
| 1 | 3 8 9 |
| 2 | 2 8 |
| 3 | 1 6 |
| 4 | 2 6 |
| 5 | 4 9 |

Key: 1 | 3 = 13

**c**

| Stem | Leaf |
|---|---|
| 2 | 7 |
| 3 | 3 4 5 7 9 |
| 4 | 2 9 |
| 5 | 1 3 4 |
| 6 | 5 |

Key: 2 | 7 = 27

**d**

| Stem | Leaf |
|---|---|
| 2 | 3 9 9 |
| 3 | 1 7 |
| 4 | 1 1 1 |
| 5 | 3 9 9 |
| 6 | 7 |

Key: 2 | 3 = 23

**4** **a**

| Stem | Leaf |
|---|---|
| 5 | 1 2 8 9 |
| 6 | 2 4 6 7 8 |
| 7 | 1 1 5 6 8 |
| 8 | 1 2 2 3 3 3 4 9 |
| 9 | |
| 10 | 8 |

Key: 5 | 1 = 51

**b** 51 **c** 108 **d** 83

**e**

| Stem | Leaf |
|---|---|
| 5 | 1 2 8 9 |
| 6 | 2 4 6 7 8 |
| 7 | 1 1 5 6 8 |
| 8 | 1 2 2 3 3 3 4 9 |
| 9 | |
| 10 | 8 |
| 11 | 2 |

Key: 5 | 1 = 51

**5** **a** 15 and 65

**b**

| Stem | Leaf |
|---|---|
| 1 | 5 8 9 9 9 |
| 2 | 2 3 4 5 6 8 9 9 |
| 3 | 1 2 3 4 9 |
| 4 | 5 |
| 5 | 4 |
| 6 | 5 |

Key: 1 | 5 = 15

**c** 19 **d** 20–29

**6** **a** 56 **b** 41 **c** 31

**d** 31.88

**7** **a**

| Stem | Leaf |
|---|---|
| 1 | 0 2 3 4 4 4 5 5 6 6 7 8 8 8 9 |
| 2 | 0 1 1 2 3 4 5 5 7 9 9 |
| 3 | 0 1 3 4 6 |
| 4 | 1 1 5 6 6 |
| 5 | 3 4 |
| 6 | |
| 7 | 6 |

Key: 1 | 5 = 15

**b** 10 **c** 76 **d** 10–19

**8** **a** 10.1 g **b** 16.1 g

**c** Sample answer: Will did not read the key properly. It is 13.8 g.

**d** The outlier weighs 16.1 g. 14.1 g is the heaviest glider excluding the outlier.

**e**

| Stem | Leaf |
|---|---|
| 10 | 1 4 8 |
| 11 | 0 2 3 6 6 |
| 12 | 1 2 4 5 5 6 9 9 |
| 13 | 1 1 2 4 4 4 6 6 8 8 8 8 |
| 14 | 0 1 3 |
| 15 | |
| 16 | 1 |

Key: 10 | 1 = 10.1 g

**9** **a** Each stem represents hundreds and each leaf represents tens instead of tens and ones (units).

**b**

| Stem | Leaf |
|---|---|
| 0 | 6 8 |
| 1 | 0 0 1 2 2 3 3 3 4 5 5 6 7 8 |
| 2 | 0 2 2 2 4 5 6 |
| 3 | 6 |
| 4 | 3 |
| 5 | 7 |

Key: 1 | 2 = 120

**c** 60 **d** 570 **e** 192

**10** **a** The leaves have two digits instead of one.

**b** 7:16 am **c** 11:59 pm **d** 5

**e** 6:54 pm and 7:04 pm (10 minutes)

**f** 8:04 pm and 8:35 pm (31 minutes)

**11** The plot is missing a key, the stem should not skip numbers and the leaves should be in order.

**12** A stem-and-leaf plot; the other graphs need to display all of the values on the axes, so if there is a large number of values the graph would be difficult to read. Also, the frequency of every value would be the same (1).

**13** **a** 0.155

**b** 0.146 and 0.154; 0.147 and 0.153; 0.148 and 0.152; 0.149 and 0.151

## 10 Checkpoint

**1** **a** **i** Sample **ii** Secondary

**b** **i** Census **ii** Secondary

**c** **i** Sample **ii** Primary

**2** **a** Numerical, discrete

**b** Categorical, ordinal

**c** Numerical, continuous

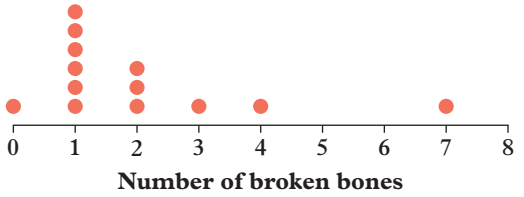
**d** Categorical, nominal

3 **a** D **b** B **c** A
**d** C

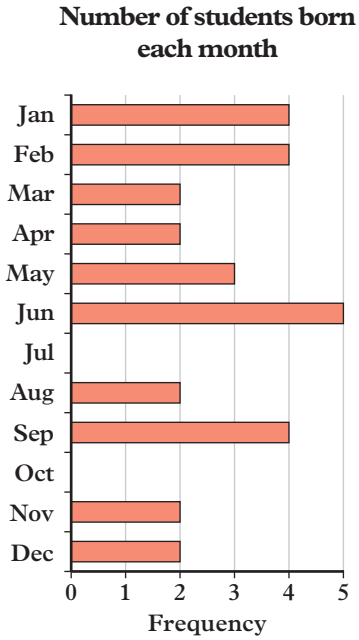
4 **a** **i** 9.25 **ii** 8.5 **iii** 7
**iv** 9
**b** **i** 4.55 **ii** 2.5 **iii** 2.5
**iv** $7\frac{13}{14}$

5 **a** median **b** mode **c** range
**d** mean

6




7




8 **a**

| Score | Frequency |
|---|---|
| 7 | 1 |
| 8 | 3 |
| 9 | 5 |
| 10 | 6 |
| 11 | 4 |
| 12 | 3 |
| 13 | 2 |

**b** 10.08 **c** 10 **d** 10
**e** 6

9 **a** B and D **b** A
**c** 5 students **d** 32 students

10

| Stem | Leaf |
|---|---|
| 1 | 2 5 9 |
| 2 | 3 7 8 8 |
| 3 | 0 1 2 5 7 9 |
| 4 | 0 1 2 5 7 |
| 5 | 2 |
| 6 | 0 |

Key: 1 | 2 = 12

11 **a** 10.4 **b** 11 **c** 11
**d** 11.8 **e** 2.4

## EX 10E Describing probability

p466

1 **a** {red, blue, green, yellow}
**b** {1, 2, 3, 4, 5, 6, 7, 8}
**c** {spring, summer, autumn, winter}
**d** {January, February, March, April, May, June, July, August, September, October, November, December}
**e** {5c, 10c, 20c, 50c, $1, $2}
**f** {0, 1, 2, 3, 4, 5, 6, 7}

2 **a** Not equally likely
**b** Not equally likely
**c** Equally likely
**d** Equally likely
**e** Not equally likely
**f** Not equally likely

3 **a** Likely **b** Unlikely
**c** Even chance
**d** Unlikely **e** Certain

4 **a** C **b** A **c** D
**d** B

5 Sample answers:
**a** Landing on a yellow segment
**b** Landing on an red or green segment
**c** Landing on a purple segment
**d** Landing on a yellow or green segment
**e** Landing on a yellow, red, green or blue segment

6 **a** {red, blue, green, yellow}
**b** {3, 4, 5, 6}

7 **a** 5 **b** 7 **c** 5
**d** 3 (T, E and X are the different letters to choose from)
**e** 3

8 **a** Yes **b** No

9 **a** **i** Not equally likely **ii** $3\frac{1}{3}$
**b** **i** Equally likely **ii** 3.5
**c** **i** Not equally likely **ii** 3
**d** **i** Equally likely **ii** 0
**e** **i** Equally likely **ii** $4\frac{1}{6}$

10 **a** False **b** True **c** True
**d** False **e** False **f** True

11 **a** Equally likely **b** Equally likely **c** Equally likely

**d** Not equally likely

**e** Not equally likely

**12** Sample answers:

**a** $\{3, 6, 9, ...\}$

**b** $\{0, 10, 20, 30, ...\}$

**c** $\{..., -0.2, -0.1, 0, 0.1, 0.2, ...\}$

**d** $\{1, 2, 4, 8, 16, ...\}$

**13** **a** unlikely **b** even chance **c** impossible

**d** unlikely **e** likely

**14** **a** *C*, *A*, *E*, *B*, *D* **b** *C*

**15** Statement 1 is false. Event *C* has an even chance of occurring.

## EX 10F Theoretical probability

p471

**1** **a** 25 **b** 5 **c** $\frac{5}{25} = \frac{1}{5}$

**2** **a** 28 **b** $\frac{10}{28} = \frac{5}{14}$ **c** $\frac{4}{28} = \frac{1}{7}$

**3** **a** {red, dark blue, green, yellow, light blue, purple}

**b** $\frac{1}{6}$ **c** $\frac{2}{6} = \frac{1}{3}$

**4** **a** 50% **b** 50%

**5** **a** 30°

**b** **i** $\frac{1}{12}$ **ii** $\frac{5}{36}$ **iii** $\frac{2}{9}$

**iv** $\frac{5}{9}$

**c** **i** 215° **ii** 90° **iii** 10°

**iv** 45°

**d** **i** $\frac{43}{72}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{36}$

**iv** $\frac{1}{8}$

**6** **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{4}{6} = \frac{2}{3}$

**d** $\frac{3}{6} = \frac{1}{2}$

**7** **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** 0

**8** **a** 0.3 **b** 0.1 **c** 0.3

**9** **a** 20

**b** **i** $\frac{6}{20} = \frac{3}{10}$ **ii** $\frac{4}{20} = \frac{1}{5}$

**10** **a** $\frac{12}{52} = \frac{3}{13}$ **b** $\frac{4}{52} = \frac{1}{13}$ **c** $\frac{1}{52}$

**d** $\frac{20}{52} = \frac{5}{13}$ (2, 4, 6, 8, 10)

**e** $\frac{16}{52} = \frac{4}{13}$ (2, 3, 5, 7)

**11** **a** 25% **b** 25% **c** 6.25%

**d** 50% **e** 43.75%

**12** **a** $\frac{4}{108} = \frac{1}{27}$ **b** $\frac{8}{108} = \frac{2}{27}$ **c** $\frac{8}{108} = \frac{2}{27}$

**d** $\frac{8}{108} = \frac{2}{27}$ **e** $\frac{12}{108} = \frac{1}{9}$

**13** **a** 0.7 **b** 0.25 **c** 0.95

**d** 0.4 **e** 0.65 **f** 1

**14** Sample answer: although there are four different outcomes, each outcome is not equally likely. There are different numbers of each coloured jellybean. There are two green jellybeans out of a total of ten jellybeans, so the correct probability of selecting a green jellybean is $\frac{2}{10}$, which simplifies to $\frac{1}{5}$.

**15** Sample answer: no, the maximum the maximum probability an outcome can have of occurring is 1, which is 100%.

**16** Sample answer: if the outcomes are not all equally likely, then some would be more likely to occur than their theoretical probability and others would be less likely to occur than their theoretical probability. In the formula all outcomes are added equally to each other, showing that they have equal probability of occurring.

**17** **a** December, $\frac{1}{16}$

**b** November, $\frac{1}{48}$

**c** $\frac{3}{10}$ **d** $\frac{7}{15}$

**18** **a** 2 times **b** 3 times

**c** **i** $\frac{1}{3}$ times **ii** 2 times **iii** $\frac{1}{6}$ times

**19** **a** Yes **b** Yes **c** No

**d** No **e** No

**20** **a** True **b** True **c** False

**d** False **e** False

**21** **a** $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

**b** $0.5 + 0.5 = 1$

**c** $\frac{9}{25} + \frac{16}{25} = 1$

**d** 31% + 52% + 17% = 100%

**22** **a** 0.2 **b** 0%

**c** 13 983 815 in 13 983 816 **d** $\frac{5}{12}$

**23** **a** **i** $\frac{5}{10} = \frac{1}{2}$ **ii** $\frac{5}{10} = \frac{1}{2}$ **iii** $\frac{4}{10} = \frac{2}{5}$

**b**

| | | Second roll of die | | | | |
|---|---|---|---|---|---|---|
| | | **0** | **1** | **2** | **3** | **4** |
| First roll of die | **0** | 0, 0 | 0, 1 | 0, 2 | 0, 3 | 0, 4 |
| | **1** | 1, 0 | 1, 1 | 1, 2 | 1, 3 | 1, 4 |
| | **2** | 2, 0 | 2, 1 | 2, 2 | 2, 3 | 2, 4 |
| | **3** | 3, 0 | 3, 1 | 3, 2 | 3, 3 | 3, 4 |
| | **4** | 4, 0 | 4, 1 | 4, 2 | 4, 3 | 4, 4 |
| | **5** | 5, 0 | 5, 1 | 5, 2 | 5, 3 | 5, 4 |
| | **6** | 6, 0 | 6, 1 | 6, 2 | 6, 3 | 6, 4 |
| | **7** | 7, 0 | 7, 1 | 7, 2 | 7, 3 | 7, 4 |
| | **8** | 8, 0 | 8, 1 | 8, 2 | 8, 3 | 8, 4 |
| | **9** | 9, 0 | 9, 1 | 9, 2 | 9, 3 | 9, 4 |

| | | Second roll of die | | | | |
|---|---|---|---|---|---|---|
| | | **5** | **6** | **7** | **8** | **9** |
| First roll of die | **0** | 0, 5 | 0, 6 | 0, 7 | 0, 8 | 0, 9 |
| | **1** | 1, 5 | 1, 6 | 1, 7 | 1, 8 | 1, 9 |
| | **2** | 2, 5 | 2, 6 | 2, 7 | 2, 8 | 2, 9 |
| | **3** | 3, 5 | 3, 6 | 3, 7 | 3, 8 | 3, 9 |
| | **4** | 4, 5 | 4, 6 | 4, 7 | 4, 8 | 4, 9 |
| | **5** | 5, 5 | 5, 6 | 5, 7 | 5, 8 | 5, 9 |
| | **6** | 6, 5 | 6, 6 | 6, 7 | 6, 8 | 6, 9 |
| | **7** | 7, 5 | 7, 6 | 7, 7 | 7, 8 | 7, 9 |
| | **8** | 8, 5 | 8, 6 | 8, 7 | 8, 8 | 8, 9 |
| | **9** | 9, 5 | 9, 6 | 9, 7 | 9, 8 | 9, 9 |

**c** $\frac{3}{100}$

## EX 10G Experimental probability

p477

**1** **a** $\frac{1}{2}$ **b** 10 **c** $\frac{12}{20} = \frac{3}{5}$

**2** **a** $\frac{1}{6}$ **b** 10 **c** $\frac{8}{60} = \frac{2}{15}$

**3** **a** $\frac{13}{20}$

**b** The relative frequency is higher than the theoretical probability.

**c** Decrease; you would expect it to get closer to the theoretical probability.

**4** **a** $\frac{6}{60} = \frac{1}{10}$

**b** The relative frequency is lower than the theoretical probability.

**c** Increase; you would expect it to get closer to the theoretical probability.

**5** **a** 0.61

**b** The number of people surveyed was 100, so the frequency is also per 100.

**c** 550 people

**6** **a** 0.26 **b** 13

**7** Sample answer: It is possible that Peter could record the same results, although it is very unlikely. The results that Jade and April recorded will not have an effect on the results Peter records.

**8** **a**

| Number | Expected frequency | Actual frequency | Theoretical probability | Relative frequency |
|---|---|---|---|---|
| 1 | 6 | 3 | $\frac{6}{36}$ | $\frac{6}{36}$ |
| 2 | 6 | 5 | $\frac{6}{36}$ | $\frac{5}{36}$ |
| 3 | 6 | 7 | $\frac{6}{36}$ | $\frac{7}{36}$ |
| 4 | 6 | 5 | $\frac{6}{36}$ | $\frac{5}{36}$ |
| 5 | 6 | 5 | $\frac{6}{36}$ | $\frac{5}{36}$ |
| 6 | 6 | 11 | $\frac{6}{36}$ | $\frac{11}{36}$ |

**b** Larger

**c** Sample answer: The die is probably not fair; the 6 is rolled too many times.

**9** **a** Your sample: 4%, your friend's sample: 14%

**b** Sample answer: Yes, 5% does not mean exactly five in every hundred. It means that each encounter has a 5% chance of the monster appearing. The sample percentages of 4% and 14% are both possible.

**c** 0.0244%

**d** Sample answer: No. 1 in 4096 does not mean exactly 1 in every 4096. It means that each encounter has a 1 in 4096 chance of encountering the special form. You may encounter the special form more or less frequently than once every 4096 encounters.

**10** **a** **i** $\frac{1}{1000}$ **ii** $\frac{1}{200}$

**b** **i** 0.14% **ii** 0.46%

**11** **a** 56%, 60%, 58%

**b** 5800 red marbles

**c** 57%, 59%, 58%

**d** 5800 red marbles

**e** 5778 red marbles

**f** **i** The first contestant

**ii** No, his first estimate was closer to the actual number.

**12** **a**

| | | Die 2 | | | | | |
|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** |
| Die 1 | **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| | **2** | 3 | 4 | 5 | 6 | 7 | 8 |
| | **3** | 4 | 5 | 6 | 7 | 8 | 9 |
| | **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| | **5** | 6 | 7 | 8 | 9 | 10 | 11 |
| | **6** | 7 | 8 | 9 | 10 | 11 | 12 |

**b** 5 **c** $\frac{5}{36}$ **d** 5

**13** **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** 30

**d** Rolling a die

**e–f** Answers will vary.

**g** Sample answer: The main advantage is that the more simulations you do, the closer the experimental probability is likely to be to the actual probability. The main disadvantage is that running a lot of simulations can take a lot of time, which in some circumstances can also mean a lot of money.

**14** 210

## CHAPTER 10 review

### Multiple-choice

| | | | | | |
|---|---|---|---|---|---|
| **1** | C | **2** | B | **3** | C |
| **4** | E | **5** | B | **6** | B |
| **7** | D | **8** | D | **9** | B |

### Short answer

**1** **a** Nominal categorical

**b**

| Answer | Frequency |
|---|---|
| Y | 7 |
| N | 6 |

**c** 13 **d** Y: tongue rolling

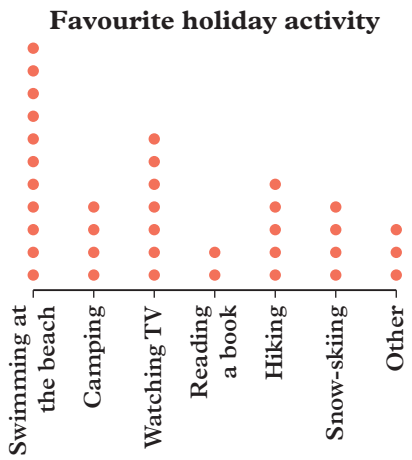
**2** **a** **i** 11.5 **ii** 7 **iii** 2

**iv** 70

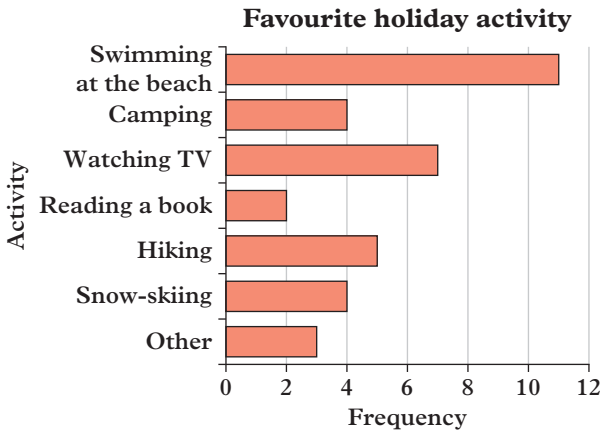
**b** The mean is skewed by a single large data point.

**3** **a** True **b** False **c** True

**4** **a**




**b**




**5** **a** 10 | 4 = 10.4 cm, 8 | 9 = 8.9 cm

**b** The 10 stem **c** 10.1 cm **d** 10.1 cm

**e** 10.1 cm **f** 4.6 cm

**g** The lengths of the worms are spread out over 4.6 cm (from 8.1 cm to 12.7 cm). The average length of a worm is 10.1 cm (mode: 10.1 cm, median: 10.1 cm, mean: 10.1 cm).

**6**

| Stem | Leaf |
|---|---|
| 0 | 5 9 |
| 1 | 2 2 3 3 6 6 7 |
| 2 | 0 2 3 5 8 |
| 3 | 0 1 2 3 8 |
| 4 | 1 2 5 |
| 5 | 1 5 9 |
| 6 | 2 |

Key: 1 | 3 = 13

**7** **a** **i** {head, tail} **ii** Even chance

**b** **i** {1, 2, 3, 4, 5, 6} **ii** Even chance

**c** **i** {club, diamond, heart, spade}

**ii** Unlikely

**d** **i** {red, blue, white}

**ii** Likely

**8** **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

**9** **a** **i** 0.4 **ii** 0.1 **iii** 0.3

**iv** 0.2 **v** 0.6

**b** **iii** and **iv** are equally close

### Analysis

**1** **a** Discrete numerical **b** 26

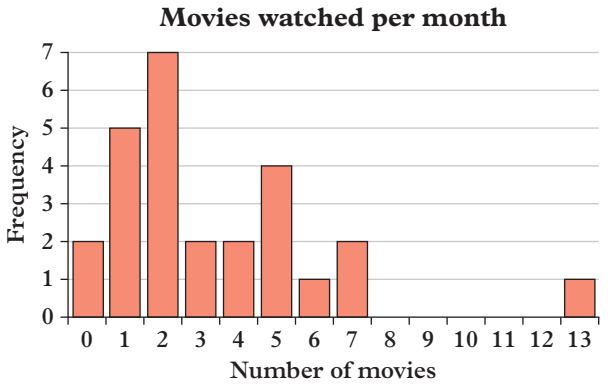
**c** **i** 3.3 **ii** 2 **iii** 2

**iv** 13

**d**

| Number | Frequency |
|---|---|
| 0 | 2 |
| 1 | 5 |
| 2 | 7 |
| 3 | 2 |
| 4 | 2 |
| 5 | 4 |
| 6 | 1 |
| 7 | 2 |
| 13 | 1 |

**e** Sample answer: a column graph allows you to see the range and spread of data accurately. A bar graph or a dot plot would also be appropriate.




**2** **a** {bronze, silver, gold, iron}

**b** $\frac{1}{10}$ **c** $\frac{1}{12}$ **d** $\frac{1}{6}$

**e** Sample answer: The theoretical probability does not guarantee that 1 in every 10 trials will be an iron marble; there will be variation in the number of iron marbles that get picked. The number of trials is not equal to a multiple of 10 and cannot be simplified to a tenth, so the relatively frequency cannot be equal to the theoretical probability.

# CHAPTER 11 Computational thinking

## EX p490 11A Printing information to the computer screen

**1** **a** 12 **b** 25

**c** 23 **d** 20

**2** **a** There are no quotation marks around Hello world! to indicate this is a string.

**b** There are no brackets around the string that is being printed.

**c** The exclamation mark is outside the quotation marks.

**3** Sample answers:

Option 1

```
print("Maths", "English", "Geography")
```

Option 2

```
print("Maths")
print("English")
print("Geography")
```

**4** **a**
```
print("Too hot!")
```

**b**
```
print("Too hot!")
print("Too cold!")
print("Just right :)")
```

**c**
```
print("Too hot! Too cold! Just right :)")
```

**5** **a**
```
print("Hello","world", sep = "")
```

**b**
```
print("Hello","world", sep = "\n")
```

**c**
```
print("I","can","code", sep = "/")
```

**d**
```
print("I can code well.","Very well indeed.", sep = "\n")
```

**e**
```
print("chicken","egg","chicken","egg", sep = " -> ")
```

## EX p494 11B Variables in coding

**1** **a** 2 **b** 16 **c** −10

**d** 1 **e** 13

**2** **a** Variable **b** String **c** String

**d** Variable **e** Variable

**3** **a** My favourite subject is Maths

**b** The sun rose at 6.45am today and set at 7.15pm

**c** I have 37 dollars in my piggybank and I am saving for a bike

**4** Sample answers:

**a**
```
number = 14
food = "wheatbix"
print("I once ate", number, food, "in one sitting")
```

**b**
```
place = "Africa"
river = "The Nile"
length = 6650
print(river, "is the longest river in", place, "and runs for", length, "km")
```

**c**
```
name = "Ms. Tiong"
grade = "Grade 5"
attribute = "Helpful"
print(name, "was my favourite teacher in", grade, "because they were", attribute)
```

**5** **a** The error is `NameError: The name 'x' is not defined` (trying to use the variable x but have only defined y and z). The third line should be: `print(z+y)` or the second line should be: `x = 2`

**b** The error is `SyntaxError: invalid syntax`

A syntax error means Python cannot interpret or make sense of what you've typed. It is often you're missing some symbol. In this case, we are trying to use the Maths shorthand of writing $4 \times y$ as $4y$, but this is not allowed in Python. The last line should be: `print(4*y-2*z)`.

**c** Error is `NameError: name 'Australia' is not defined` (the first line of the code is missing quotation marks around Australia, so Python assumes it is a variable). First line should be: `x = "Australia"`

**d** Error is `SyntaxError: Missing parentheses in call to 'print'` (missing the brackets around what is being printed). The second line should be: `print("Welcome back to your computer,", name)`

## EX p498 11C Asking a user for input

**1** **a** What did you have for breakfast?

**b** input( )

**c** The input the user typed in response to line 1

2
```
number1 = input("Choose a number: ")
number2 = input("Choose another
number: ")
number1 = int(number1)
number2 = int(number2)
print("The sum is", number1+number2)
```

3
```
number1 = input("Choose a number: ")
number2 = input("Choose another
number: ")
number1 = int(number1)
number2 = int(number2)
print("The product is", number1*number2)
```

4
```
number1 = input("Choose a number: ")
number1 = int(number1)
print("The next 2 numbers are",
number1+1, "and", number1+2)
```

5
```
subject = input("What is your favourite
subject? ")
print("Hey! I like", subject, "too!")
```

6 **a**
```
side1 = input("What is the length of
the longer side? ")
side2 = input("What is the length of
the shorter side? ")
side1 = int(side1)
side2 = int(side2)
print("The area is",side1*side2)
```

**b**
```
base = input("What is the length of
the base? ")
height = input("What is the height? ")
base = int(base)
height = int(height)
print("The area of the triangle
is",base*height*0.5)
```

**c**
```
side = input("What is the side length
of the cube? ")
side = int(side)
print("The volume of the cube
is",side*side*side)
```

**d**
```
base = input("What is the length of
the base? ")
height = input("What is the height? ")
base = int(base)
height = int(height)
print("The area of the parallelogram
is",base*height)
```

7
```
l = input("What is the length? ")
w = input("What is the width? ")
h = input("What is the height? ")
l = int(l)
w = int(w)
h = int(h)
surface_area = 2*l*w+2*l*h+2*w*h
print("The surface area is",surface_area)
```

8 **a**
```
first_number = int(input("Enter first
number"))
second_number = int(input("Enter
second number"))
sum1 = first_number + second_number
print("The addition of the two numbers
is: ", sum1)
```

**b**
```
x = int(input("Enter the value
for x: "))
y = int(input("Enter the value
for y: "))
y = 2*x+1
print("The correct y value for your x
value is: ", y)
```

## EX 11D Conditions in code

p503

1 **a** Nothing printed **b** True!
**c** This line has no indent
**d** True!
This line is indented
This line has no indent

2
```
food_choice = input("chicken, fish or
tofu? ")
if food_choice == "chicken":
   print("We're out of chicken!")
else:
   print("Coming right up!")
```

3
```
num1 = input("Choose a number: ")
num2 = input("Choose another number: ")
num1 = int(num1)
num2 = int(num2)
if num1 < 0 and num2 < 0:
   print("Both your numbers are
   negative!")
```

4
```
a1 = input("What is the size of the first
angle? ")
a2 = input("What is the size of the
second angle? ")
a1 = int(a1)
a2 = int(a2)
a3 = 180 - a1 - a2
if a1 == a2 == a3:
   print("Equilateral triangle")
elif a1 == a2 or a1 == a3 or a2 == a3:
   print("Isosceles triangle")
else:
   print("Scalene triangle")
```

5
```
price = input("How much would you be
willing to pay? ")
price = int(price)
if price < 70*0.8:
   print("They will never be that cheap!")
```

```
elif price < 70*0.95:
    print("Better wait for a sale…")
else:
    print("What are you waiting for?
    Go and buy them")
```

**6**
```
s1 = input("What is the length of
side 1? ")
s2 = input("What is the length of
side 2? ")
s3 = input("What is the length of
side 3? ")
s1 = int(s1)
s2 = int(s2)
s3 = int(s3)
if s1 < 0 or s2 < 0 or s3 < 0:
    print("That is not a triangle!")
elif s1 > s2 + s3:
    print("That is not a triangle!")
elif s2 > s1 + s3:
    print("That is not a triangle!")
elif s3 > s1 + s2:
    print("That is not a triangle!")
```

EX p504

## SEMESTER 2 review

### Short answer

**1** **a** True

**b** False. There are 4 terms. Each term is something that is added or subtracted.

**c** True

**d** False. The coefficient is −4.

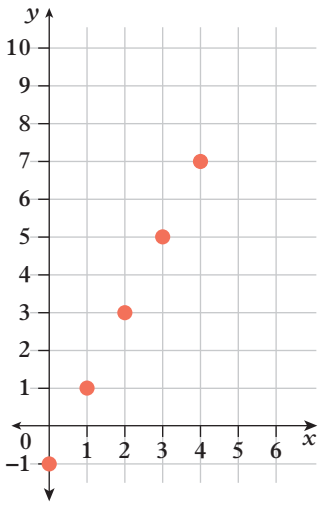
**e** True

**2** **a**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | −1 | 1 | 3 | 5 | 7 |

**b** (0, −1), (1, 1), (2, 3), (3, 5), (4, 7)

**c**




**3** **a** $10a$ **b** $-5x + y + 5$

**c** $\frac{1}{2}fg + \frac{1}{4}fgh$ **d** $3x^2 + 3x - 4$

**4** **a** $x = 5$ **b** $x = 24$ **c** $x = 2$

**d** $x = 42$

**5** **a** $\overline{AE}$ and $\overline{CD}$ are parallel line segments

**b** $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are perpendicular rays

**6** **a** 55° **b** Isosceles triangle

**7** **a** Unlikely **b** Even chance

**c** Certain **d** Likely

**8** **a** Polygon **b** Polygon **c** Polygon

**d** Not a polygon **e** Polygon **f** Polygon

**g** Not a polygon **h** Polygon **i** Not a polygon

**9** **a** **i** Concave **ii** Irregular

**iii** Irregular, concave pentagon

**b** **i** Convex **ii** Irregular

**iii** Irregular, convex quadrilateral

**c** **i** Convex **ii** Regular

**iii** Regular, convex hexagon

**e** **i** Concave **ii** Irregular

**iii** Irregular, concave quadrilateral

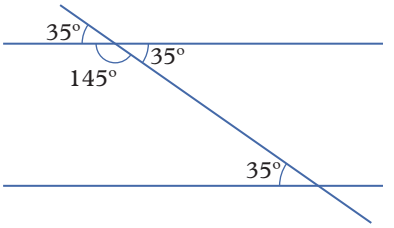
**f** **i** Convex **ii** Regular

**iii** Regular, convex pentagon

**h** **i** Concave **ii** Irregular

**iii** Irregular, concave hexagon

**10** Sample answer: other orientations and labelled angles possible.




**11** **a** 71° **b** 161° **c** 19°

**d** 161° **e** 19°

**12** **a** $3b + 5$

**b** $\frac{5}{7}(m - 3) = n$

**c** $\frac{u}{8} + 7 = \frac{3}{5}$

**d** $\frac{p + q}{2} = pq$

**e** $9(t - 8) = 41$

**13** **a** 41 **b** 13 **c** 1

**d** 7.07

**14** **a** Not equally likely **b** Equally likely

**c** Not equally likely **d** Not equally likely

**e** Equally likely

**15** **a** **i** Secondary

**ii** Census

**iii** Discrete numerical

**iv** Column graph or bar chart

**b** **i** Secondary

**ii** Sample

**iii** Nominal categorical

**iv** Column graph or bar chart

**c** **i** Primary
**ii** Sample
**iii** Continuous numerical
**iv** Stem-and-leaf plot

**d** **i** Primary
**ii** Census
**iii** Discrete numerical
**iv** Column graph or bar chart

**16**

| Guess | LHS | RHS | LHS = RHS? |
|---|---|---|---|
| $x = 1$ | $-6$ | $-5$ | No |
| $x = 2$ | $-3$ | $0$ | No |
| $x = \frac{1}{2}$ | $-\frac{15}{2} = -7.5$ | $-\frac{15}{2} = -7.5$ | Yes |
| $x = 0.4$ | $-7.8$ | $-8$ | No |

**17** **a** $x = 15.5$ **b** $y = \frac{14}{5}$
**c** $b = 7\frac{1}{3}$ **d** $m = \frac{63}{10}$

**18** **a** 1 cm × 11 cm, 2 cm × 10 cm, 3 cm × 9 cm, 4 cm × 8 cm, 5 cm × 7 cm, 6 cm × 6 cm
**b** 6 cm × 6 cm = 36 cm$^2$
**c** 4.8 cm × 7.2 cm

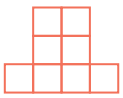
**19** **a** **i** 186.17° **ii** 202.5° **iii** no mode
**b** **i** $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{12}$

**20** **a** 1 cm × 48 cm, 2 cm × 24 cm, 3 cm × 16 cm, 4 cm × 12 cm, 6 cm × 8 cm
**b** 98 cm, 52 cm, 38 cm, 32 cm, 28 cm
**c** **i** 1 **ii** $\frac{2}{5}$ **iii** $\frac{4}{5}$
**d** 6 and 7
**e** Mean = 12.4 cm, median = 7 cm
**f** 48, 12, $\frac{16}{3} = 5\frac{1}{3}$, 3, $\frac{4}{3} = 1\frac{1}{3}$

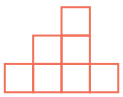
**21** **a** Right-angled triangle, isosceles triangle
**b** Kite
**c** Scalene triangle
**d** Square, rhombus, rectangle, parallelogram, kite
**e** Equilateral triangle, isosceles triangle
**f** Trapezium

**22** *A*: a vertical translation
*B*: a reflection in the *x*-axis
*C*: a rotation of 90° about *O* anticlockwise

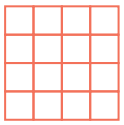
**23** **a** Front elevation:



Side elevation:



Plan view:

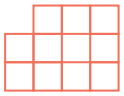


**b** Front elevation:

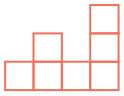
Side elevation:

Plan view:

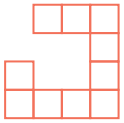
**c** Front elevation:



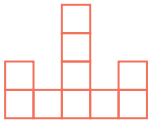
Side elevation:



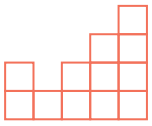
Plan view:



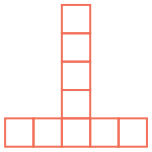
**d** Front elevation:



Side elevation:



Plan view:



**24** Pronumeral choices may vary.
**a** **i** Let $x$ be the size of the other co-interior angle.
**ii** $x + 50° = 180°$
**iii** $x = 130°$
**b** **i** Let $x$ be the size of the fourth interior angle.
**ii** $x + 81.5° + 52.3° + 95.1° = 360°$
**iii** $x = 131.1°$

**c** **i** Let $x$ be the length of the shortest side of the hexagon.

**ii** $x + (x + 2.1) + (x + 4.2) + (x + 6.3) + (x + 8.4) + (x + 10.5) = 51.9$

**iii** $x = 3.4$ cm

**d** **i** Let $x$ be the side length of the square.

**ii** $x^2 = 81$ cm$^2$

**iii** $x = 9$ cm

**e** **i** Let $x$ be the length of the perpendicular height.

**ii** $\frac{1}{2} \times 4 \text{ cm} \times x = 40 \text{ cm}^2$

**iii** $x = 20$ cm

**f** **i** Let $x$ be the side length of the cube.

**ii** $x^3 = 64$ cm$^3$

**iii** $x = 4$ cm

**g** **i** Let $x$ be the value of the missing number.

**ii** $\frac{x + 3 + 5 + 8 + 12 + 4}{6} = 7$

**iii** $x = 10$

**25** **a** 1.44 cm$^2$ or 144 mm$^2$

**b** 90 m$^2$

**c** 196 cm$^2$

**d** 36 cm$^2$

**e** 50 820 m$^2$ or 0.050 82 km$^2$

**f** 12.48 m$^2$ or 124 800 cm$^2$

**26** **a** $\angle CFE$ or $\angle EFC$

**b** No, as the alternate angles $\angle CFE$ and $\angle FEB$ are not equal.

**c** $\angle EFB = 79°$

**d** $\angle FBE = 50°$, not isosceles because there are not two equal angles.

**27** **a** 2.871 m$^3$ or 2 871 000 cm$^3$

**b** 1 200 000 mL or 1200 L

**c** 1.671 m$^3$ or 1 671 000 cm$^3$

**28** **a**

| Stem | Leaf |
|---|---|
| 0 | 2 2 7 |
| 1 | 2 5 |
| 2 | 0 1 2 6 6 |
| 3 | 2 3 4 7 9 |
| 4 | 3 4 5 8 9 |

Key: 1 | 2 = 12

**b** **i** 27.85 **ii** 29 **iii** 2, 26

**iv** 47

**c** Sample answer: The number of stickers is spread out over 47 stickers (from 2 stickers to 49 stickers). The average number of stickers is around 27 (modes: 2, 26, median: 29, mean 27.85).

**29** $-7x + 2y + 7$

**30** **a** Object 1: Rectangular prism

Object 2: Triangular prism

**b** **i** Object 1: 6, Object 2: 5

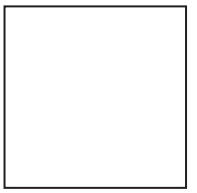
**ii** Object 1: 12, Object 2: 9

**iii** Object 1: 8, Object 2: 6

**c** Object 1:

Front elevation:

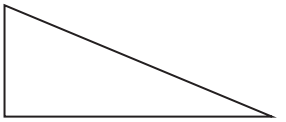
Side elevation:



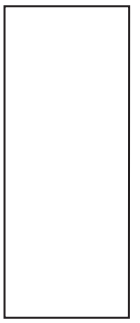
Plan view:

Object 2:

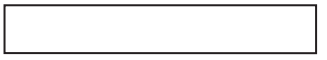
Front elevation:



Side elevation:



Plan view:



**d** Object 1: 78 cm$^2$, Object 2: 120 cm$^2$

**31** **a** $\frac{y}{9} = 18$ and $9(y - 2) = 36$

**b** Sample answer: $y = 2$ is an equation as it has two expressions written equal to each other. $y = 2$ is the solution of the equation as substituting $y = 2$ shows that the left-hand side is equal to the right-hand side.

**c** Sample answer: Substituting $x = 2$ reveals that the left-hand side is equal to 16 and the right-hand side is equal to 8.

## Analysis

**1** **a** $h = 2d$

**b**

| Days | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Height (cm) | 2 | 4 | 6 | 8 | 10 |

**c** 11 days

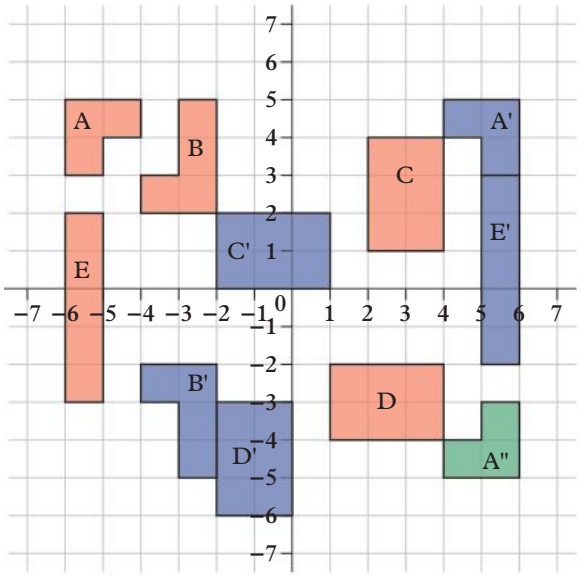
**d** $h = 3d + 1$

**e** 7 days

**f** $g = \frac{2}{3}(h - 4)$

**g** 12 g

**2** **a**




**b** Rotation of 180° about the point (0,0)

**c** **i** $A$: 1, $B$: 1, $C$: 2 **ii** $A$: 1, $B$: 0, $C$: 2

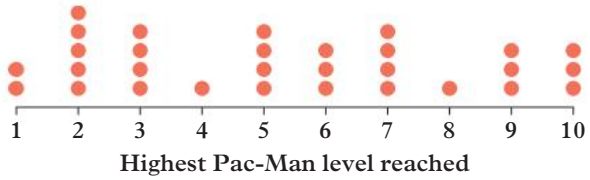
**d** **i** $C$ and $D$ **ii** $E$

**e** $A$ is an irregular, concave hexagon. $E$ is a rectangle.

**3** **a** **i** Primary **ii** Discrete

**b**

| Highest Pac-Man level reached | Frequency |
|---|---|
| 1 | 2 |
| 2 | 5 |
| 3 | 4 |
| 4 | 1 |
| 5 | 4 |
| 6 | 3 |
| 7 | 4 |
| 8 | 1 |
| 9 | 3 |
| 10 | 3 |




**c** **i** 5.3 **ii** 5 **iii** 2 **iv** 9

**d** Sample answer: The highest Pac-Man levels reached are spread out over 9 levels (from level 1 to level 10). The average highest Pac-Man level reached was about 5 (mode: 2, median 5, mean 5.3).

**e** Sample

**f** Sample answer: No, as Year 7 players are not representative of all Pac-Man players who range in factors such as age and experience playing Pac-Man.

## EX EXPLORATIONS 2

p512

**1** **a** Simplify $4(2x + 1) + 3$

**b** $16x + 15$

**c** $32x + 31$ and $64x + 63$

**d** $(2^{n+1})\,x + (2n^{n+1} - 1)$

**e** $x + 2y$, $3x + 2y$, $3x + 5y$, $8x + 5y$, … ; the coefficients are the Fibonacci numbers

**2** **a** $\alpha = 72°$ and $\beta = 144°$

**b** 45° and 135°

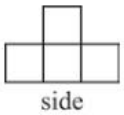
**c** 80° and 100°; 4 rings with 9 rhombuses each

**d** Yes, the pattern continues (for as long as the resulting angle is less than 180°)

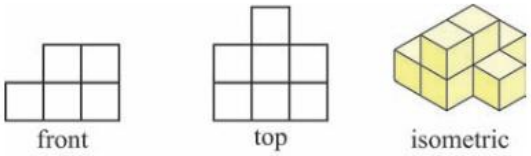
**e** 12

**f** 15 or 16

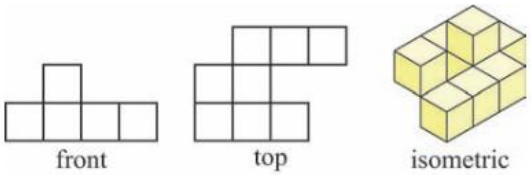
**3** **a**



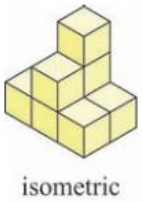

**b**



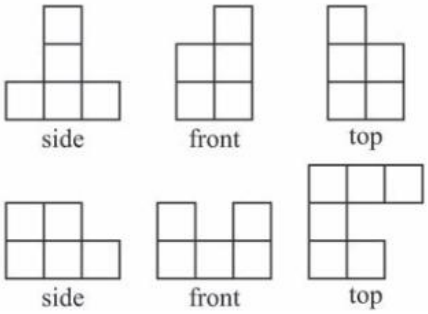

**c**



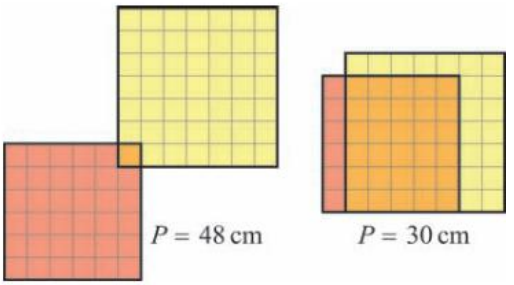

**d**




A reflection of this solution is also valid.



**4** **a**




The only possible perimeters, in centimetres, are the even numbers between 30 and 48, inclusive.

**b** Side lengths 2 cm and 7 cm, 3 cm and 6 cm, or 4 cm and 5 cm

**c** Side lengths 4 cm and 7 cm, or 5 cm and 6 cm

**d** 55 $cm^2$

**5** **a** Here is one possible solution for each grid. There are many others.

| | | | |
|---|---|---|---|
| 1 | 6 | 5 | 4 |
| 7 | 2 | 3 | |

| | | |
|---|---|---|
| 1 | 6 | 5 |
| 7 | 2 | 3 |
| 4 | | |

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

| | | | |
|---|---|---|---|
| 1 | 9 | | |
| 6 | 2 | 7 | 5 |
| 8 | 4 | 3 | |

| | | | |
|---|---|---|---|
| 1 | 7 | 6 | 4 |
| 8 | 2 | 3 | 5 |

**b** The first and last column cannot contain the same number; the bottom-right cell cannot have the same number above it and to its left; the total of 55 cannot be shared equally among the two rows; the average of 5.5 cannot be placed in the bottom-right cell.

**c** The number of columns is even and greater than 2.

**6** **a** 3

**b** 4

**c** 63 and 64

**d** No. The number of plus symbols is the same as the number of 1's in the binary representation of $n$.

EX p514

## NAPLAN practice

### CHAPTER 1 Whole numbers

**1** \$529

**2** 63 km

**3** 375 km

**4** 55 g

**5** 10

**6** 17

**7** $4^2 \times 6^2$

**8** $3 + 9 \times 5$

**9** 54

**10** 6048

**11** 603, 623, 630, 632, 662

**12** 161

**13** 1204 minutes

**14** \$181

**15** $800 \times 300$

**16** \$25 000

**17** 15

**18** $4 \times 4 \times 4$

### CHAPTER 2 Factors and primes

**1** 12 minutes after the start

**2** $2 \times 2 \times 2 \times 3 \times 3$

**3** 18

**4** 3

**5** 9

**6** 3

**7** 100

**8** 168

**9** 51

**10** 3

**11** 19

**12** 24

**13** 30

**14** 60 minutes

**15** 3

### CHAPTER 3 Fractions and ratios

**1** $\frac{25}{7}$

**2** 7

**3** $\frac{10}{24}$

**4** $\frac{3}{2}$ or $1\frac{1}{2}$

**5** $\frac{1}{4}$

**6** $\frac{2}{5}$

**7** $\frac{8}{15}$

**8** $2\frac{1}{4}$ cups

**9** $\frac{18}{5}$ or $3\frac{3}{5}$

**10** $\frac{1}{4}$

**11** $\frac{9}{10}$L

**12** 2 : 3

**13** 63

**14** $4\frac{4}{5}$

**15** The third arrow from the left

**16** $\frac{2}{5}, 1\frac{3}{4}, \frac{9}{4}, 2\frac{7}{10}, \frac{16}{5}$

**17** $7\frac{7}{15}$

**18** $\frac{15}{22}$

**19** $\frac{9}{49}$

**20** $\frac{81}{16}$

**21** $\frac{7}{12}$

**22** 5 : 7

**23** 98 : 121

**24** 4 : 7

**25** 6 : 10

**26** 76 minutes

## **CHAPTER 4** Decimals and percentages

**1** 6.528

**2** 0.693

**3** $17.97

**4** $59.90

**5** 0.55

**6** $\frac{7}{8}$

**7** 12.5%

**8** 70%

**9** 0.5923

**10** $\frac{4}{5}$, 0.76, 45%, $\frac{1}{8}$, 0.12, 9%

**11** $217

**12** Hundredths

**13** 164.51

**14** 1.26, 1.623, 1.86, 1.88

**15**
$$\begin{array}{r} 1.300 \\ 4.060 \\ +\ 0.008 \\ \hline \end{array}$$

**16** $6.65

**17** 2.74

**18** 100 for $7.48

**19** 2 L for $5.92

**20** 10

**21** $1.24

**22** $3.40

**23** D

**24** 65%

**25** $37.49

**26** $36.75

**27** $1.20

## **CHAPTER 5** Integers and the Cartesian plane

**1** 4°C

**2** Level 1 above the surface

**3** Level 3 below the surface

**4** −6

**5** −1

**6** −6°C

**7** 14 m

**8** (−16)

**9** $20

**10** 8

**11** −5

**12** 2

**13** −20

**14** (4, 3)

**15** *B*

**16** 2

**17** D

**18** False

**19** G

**20** (6, −3)

**21** *H*

**22** *A*

## **CHAPTER 6** Algebra

**1** $45

**2** 3

**3** $s = \frac{n}{5}$

**4** $62

**5** $4a + 2c$

**6** $14

**7** 6

**8** $3(x + 2) = 24$

**9** 70 cm

**10** $m = b + 5$

**11** $5a + 3 = 13$

**12** $2

**13** $2x + 4 = 12$

**14** 4 kg

**15** $\frac{x + 2}{5} = 3$

**16** $4(x - 6) = 16$

**17** $2d - 3$

**18** $n = 100 - 5d$

**19** $14x + 3xy$

**20** $C = 9t$

## **CHAPTER 7** Angles

**1** 61°

**2** 280°

**3** 137°

**4** 43°

**5** 43°

**6** 29°

**7** The second answer

**8** A pair of lines that meet at right angles are called perpendicular.

**9** Line A is parallel to ray D.

**10** 180°

**11** 90°
**12** Obtuse
**13** 113°
**14** $\angle AOE$ and $\angle CPE$
**15** $\angle AOF$ and $\angle EPD$
**16** $\angle AOF$ and $\angle CPE$
**17** 125°
**18** 55°
**19** 55°
**20** 55°
**21** Isosceles triangle
**22** 40°
**23** Rhombus
**24** 100°

## CHAPTER 8 Shapes and objects

**1** Irregular hexagon
**2** 10
**3** D
**4** Pentagon
**5** Pentagonal prism
**6** Trapezoidal prism
**7** B
**8** None
**9** Two
**10** 5 units right and 4 units down
**11** 90°
**12** Reflected in a horizontal mirror line
**13** 3 units right and 4 units down
**14** A
**15** B
**16** 8
**17** 9

## CHAPTER 9 Length, area and volume

**1** 0.789 km
**2** 5600 cm
**3** 22 cm
**4** 32 mm
**5** 64 $mm^2$
**6** 3 $cm^2$
**7** 8 cm
**8** Square with length 5 cm
**9** Parallelogram
**10** 7 $cm^2$
**11** 3000 $cm^2$
**12** 30 $cm^2$
**13** 1236 $cm^2$
**14** 1800 $cm^3$
**15** 1.8 L
**16** 105 $m^2$
**17** 72 $m^3$

## CHAPTER 10 Statistics and probability

**1** 11
**2** {milk, hazelnut, white, dark}
**3** $\frac{3}{14}$
**4** $\frac{11}{14}$
**5** 0.2
**6** Decrease
**7** Categorical, nominal
**8** 6, 5 and 4
**9** 10
**10** 3, 6, 4, 8, 5, 6, 8, 7, 8, 1, 2, 9
**11** The horizontal scale
**12** 28
**13** 36°C
**14** 58
**15** Selecting a non-picture card from a deck of cards
**16** 21
**17** 5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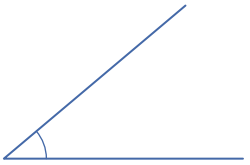
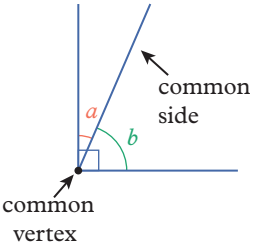
# Glossary

### accuracy
the closeness of a measurement to its exact value

### acute angle
angle greater than 0° but less than 90°

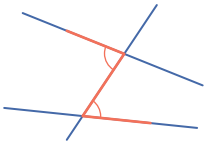


### adjacent angles
angles that share a common side and a common vertex

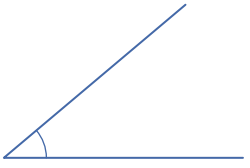



### algorithm
clear set of instructions for solving a problem

### alternate angles
a pair of angles not adjacent to (next to) each other on opposite sides of the transversal between two lines, sometimes called 'Z angles'



### angle
formed when two lines or rays meet at a point



### arc
a portion of a circle; angles are usually marked using an arc

### area
amount of space enclosed by a two-dimensional (2D) shape

### arms
the two lines or rays of an angle

### ascending order
arranging numbers from smallest to largest

### associative law
law that states regardless of how three or more numbers are grouped, when added or multiplied, the result is the same; for example,
$(4 + 5) + 6 = 9 + 6 = 15$ and
$4 + (5 + 6) = 4 + 11 = 15$;
$(4 \times 5) \times 6 = 20 \times 6 = 120$ and
$4 \times (5 \times 6) = 4 \times 30 = 120$

### axis of symmetry
a line drawn on a shape or object to cut it into two symmetrical halves (each half is a reflection of the other), also called a line of symmetry

### bar chart
a graph where the frequency of categorical data is presented in horizontal bars

### base
1. for a value expressed in index form, the base is the number which is repeatedly multiplied; for example, $2^4$ has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$
2. in a 2D shape, it is a nominated side of the shape, where the base and height are perpendicular to each other. *See* height for diagram
3. in a 3D object, it is a nominated face of the object, where the base and height are perpendicular to each other

### BIDMAS
the order of operation of a mathematical expresssion: Brackets, Indices, Division, Multiplication, Addition, Subtraction

### by-parts method
addition method involving adding the digits in each place-value separately

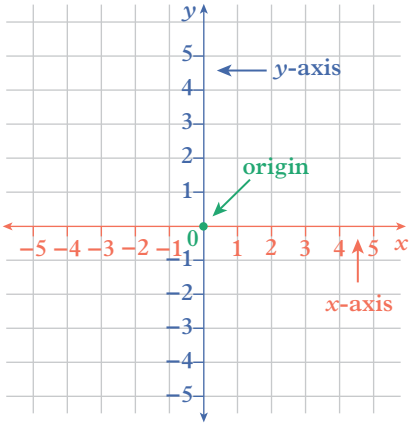
### calling a variable
when a line of code refers to a variable that has been defined earlier in the code

### cancelling (simplifying)
dividing the numerator and the denominator of a fraction by the same factor (usually the highest common factor) to produce an equivalent fraction

### capacity
amount of liquid that a three-dimensional object (container) can hold

### Cartesian coordinates
pair of numbers that describe the position of a point on the Cartesian plane; for example, (3, 5) indicates an $x$-coordinate of 3 and a $y$-coordinate of 5

### Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described




### categorical data
data that can be put into categories. *See* ordinal data, nominal data

### census
a survey of an entire population

### characters
all letters, symbols and spaces between quotation marks in a code

### coding
process of designing algorithms and writing them in a computer language such as Python

### coefficient
the number multiplied by a pronumeral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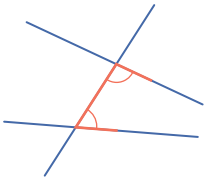
### co-interior angles
a pair of angles on the same side of the transversal between two lines, sometimes called 'C angles'



### column graph
graph where the frequency of categorical data is presented in vertical columns

### common denominator
a common multiple of the denominators of two or more fractions

### common factors
factors that divide exactly into two or more whole numbers; for example, 12 and 9 have a common factor of 3

### common multiples
multiples that appear in the multiples list for two or more different whole numbers. For example: multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and multiples of 4 are 4, 8, 12, 16, 20, 24, …, so the first three common multiples of 3 and 4 are 12, 24 and 36

### commutative law
law that states the order in which two numbers are added or multiplied is not important; for example, $3 + 4 = 7$ and $4 + 3 = 7$; $3 \times 4 = 12$ and $4 \times 3 = 12$

### compensation method
involves rounding one number to make the calculations easier. The amount by which the number was originally rounded is then added or subtracted.

### complementary angles
angles that add to make a right angle (90°)

### composite number
a positive whole number that is exactly divisible by at least one whole number other than itself and 1; for example, 8 has factors of 1, 2, 4 and 8, so is composite

### concave polygon
polygon with at least one interior angle that is greater than 180°

### conditions
statements in coding that are either true or false

### cone
3D object that tapers from a circular base to a point

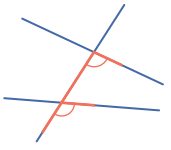
### constant term
a term without any pronumeral component; for example, the constant in the expression $3b - 4c + 10$ is 10

### continuous data
type of numerical data that can be measured; for example, height of a plant

### conversion factor
a multiplier for converting a quantity expressed in one set of units into another quantity expressed in a different set of units; for example, multiply by 10 to convert from centimetres to millimetres

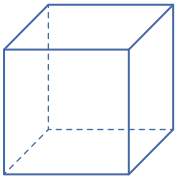
### convex polygon
polygon with interior angles that are all less than 180°

### corresponding angles
a pair of angles on the same side of the transversal and in corresponding positions on the two lines (both above a line or both below a line), sometimes called 'F angles'



### cross-section
the 2D shape obtained when cutting a 3D object

### cube
3D object with six faces that are all identical squares



### cylinder
3D object with a uniform circular cross-section

### data
individual facts or pieces of information that can be collected. Data can be numerical or categorical.

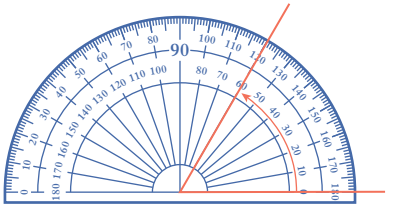
### decimal fraction
*See* decimal number

### decimal number (decimal fraction)
numbers that have a whole number component, which is to the left of the decimal point, and a decimal (or decimal fraction) component, which is to the right of the decimal point; for example, 3.456

### decimal place
position of a digit after the decimal point, where each decimal place represents a different place value

### decimal point
dot that separates the whole number component and the decimal component of a decimal number; for example, 3.456

### degrees
the unit of measure for angles; for example, 60 degrees is written as 60°, as shown on the protractor



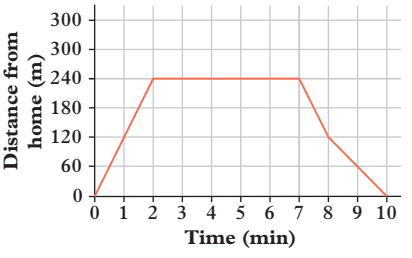
### denominator
bottom number of a fraction; for example, the denominator of $\frac{2}{3}$ is 3

### descending order
arranging numbers from largest to smallest

### digit
a single symbol used to make numerals; for example, the 10 digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

### discrete data
numerical data that can be counted; for example, number of siblings

### distance–time graph
shows the relationship between distance covered over time



### distributive law
states that the same result is obtained if you add the numbers inside a pair of brackets first before multiplying by the common factor, or multiply each number inside the brackets by the number outside the brackets before adding: $a(b + c) = ab + ac$

### dividend
number that is being divided by another number (divisor); for example, 6.84 is the dividend, 2 is the divisor and 3.42 is the quotient in $6.84 \div 2 = 3.42$

### divisor
number that divides into another number (dividend); for example, in $15 \div 3 = 5$, 3 is the divisor

### dot plot
graph that presents every piece of data in the data set as a dot above a matching number or category on a horizontal scale

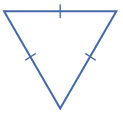
### edge
line segment that join two faces of a polyhedron. *See* polyhedron

### elevation
*See* plan

### equally likely outcomes
have the same probability of occurring

### equation
collection of two or more algebraic terms separated by mathematical operation symbols and an equal to sign; for example, $2d + 5 = 10$

### equilateral triangle
triangle with all three sides equal in length and three identical internal angles of 60°



### equivalent
equal in value or amount

### equivalent equations
equations that have the same solution

### equivalent fractions
different fractions that have the same numerical value; for example, $\frac{2}{3} = \frac{4}{6}$

### estimate
an approximate value of a calculation, usually involving rounding

### evaluate
calculate the numerical value of a term or expression after substitution; for example, evaluating $3x + 4$ when $x = 2$ gives 10

### event
refers to an outcome or group of outcomes in a sample space; for example, a single outcome of rolling a die is a 2; a group of outcomes is rolling a 2 or a 4

### expanded form
where a number is written as the sum of its place value components; for example, $3479 = 3000 + 400 + 70 + 9$

### experiment
repeatable procedure that has a clearly defined set of possible results

### experimental probability
probability of an event based upon the results of a probability experiment. *See* relative frequency

### exponent
*See* index

### expression
either a single term or the sum or difference of two or more terms

### face
flat surface of a polyhedron, where each face is a polygon. *See* polyhedron

### factor pairs
two numbers that when multiplied give a certain product; for example, the factor pairs that give a product of 12 are 1 and 12, 2 and 6, 3 and 4

### factor
whole number that divides exactly into a given whole number; for example, factors of 8 are 1, 2, 4 and 8

### factor tree
tree diagrams where composite numbers are broken down into factor pairs until prime numbers are found

### favourable outcome
outcome within the sample space that we are interested in

### formula
relationship or rule between two or more variables, usually represented using pronumerals; for example, $s = \frac{d}{t}$ is the formula for speed equal distance divided by time

### fraction
a part or a portion of a whole, or multiple wholes

### fraction wall
a visual representation to help compare and identify fractions, set out in the form of a wall

### frequency
number of times that a particular value occurs in a data set

### frequency table
table organised to show data by recording the frequency of each value in the data set

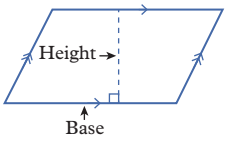
### generalisation
involves writing a rule or formula that describes a pattern or relationship between two or more variables

### gradient
indicates how quickly one quantity is changing compared to and another; the steepness and direction of a line

### grouping symbols
brackets, radicals and fraction lines that show where a group starts and ends, and help to show the order used to apply mathematical operations

### guess, check and improve
a trial and error strategy to solve simple equations; first guess a solution, then check if the number is the solution, and then improve your guess until the solution is found

### height
length measurement from the base to the top or end of a shape or object and is perpendicular to the base



### highest common factor (HCF)
greatest factor that is common to two or more given numbers; for example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so HCF of 8 and 12 is 4

### image
result of completing a transformation on a shape or object

### improper fraction
fraction where the numerator is larger than (or equal to) the denominator; for example, $\frac{9}{7}$ or $\frac{6}{6}$

### indent
a gap/space used in Python to indicate a logical structure

### index (exponent) (plural indices)
for a value expressed in index form, the index indicates the number of times the base is written as a repeated multiplication; for example, $2^4$ has an index of 4 and can be written as $2 \times 2 \times 2 \times 2$

### index form (index notation)
shorter form of writing a repeated multiplication, where a number is written with a base and an index; for example, $2^4$ is written in index form

### integers
whole numbers that are positive, negative or zero

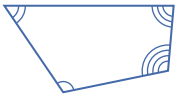
### interior angle (internal angle)
angle inside a two-dimensional shape

### interval
distance between marks on a line which form part of a scale

### inverse operation
operation that reverses the effect of a previous operation; for example, multiplication and division are inverse operations and addition and subtraction are inverse operations

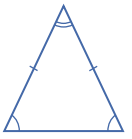
### irregular polygon
polygon that does not have all sides equal in length or all angles equal in size

### irregular quadrilateral
four-sided shape that has four different internal angles



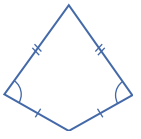
### isometric transformation
type of transformation (such as The translation, rotation or reflection) applied to a shape or object that does not change its shape or size

### isosceles triangle
triangle with two sides equal in length, and the angles opposite the equal sides are also equal



### jump method
involves breaking down one of the given numbers, then adding or subtracting each part of the number in stages

### kite
quadrilateral with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides



### leading digit
first non-zero digit of a number. For example, the leading digit of 328 is 3.

### length
length measurement of the longest side of a shape or object

### like fractions
fractions with same denominators; for example, $\frac{1}{6}$ and $\frac{5}{6}$. *See* unlike fractions

### like terms
terms which contain the same pronumerals; for example, $x$, $5x$ and $\frac{1}{2}x$ are like terms but $5x^2$ is an unlike term

### line
a line that continues to infinity in both directions, which is named using a double-headed arrow; for example, $\overleftrightarrow{AB}$

### line segment
a line joining two points that is named using a bar and the two end points labels; for example, $\overrightarrow{AB}$

### line symmetry
where an object can be divided into two halves that are mirror images of each other

### linear equation
an equation that contains two variables, both of which have a power of 1; for example: $y = x + 3$

### lowest common denominator (LCD)
lowest common multiple of the denominators in two or more fractions; for example, $\frac{1}{3}$ and $\frac{1}{2}$ have a LCD of 6

### lowest common multiple (LCM)
lowest (first) multiple that is common in the multiples list of two or more numbers, excluding zero (0), for example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and multiples of 4 are 4, 8, 12, 16, 20, 24, … so the LCM of 3 and 4 is 12

### magnitude
size of a mathematical object; the magnitude of a number refers to its size independent of its sign; for example, −10 has a greater magnitude than 3

### mean
a measure of the centre of a data set, also known as average, calculated by adding all of the data values together and dividing by the number of values

### median
a measure of the centre of a data set, determined by placing the data in ascending order (smallest to largest) to find the middle number of the data set if there is an odd number of values, and the average of the two middle numbers if there is an even number of values

### mixed number
made up of a whole number component and a proper fraction component; for example, $3\frac{4}{5}$

### mode
the most common value in a numerical data set

### multiple
result of multiplying a given number by a whole number; for example, multiples of 4 are 0, 4, 8, 12, 16, … ($4 \times 0$, $4 \times 1$, $4 \times 2$, $4 \times 3$, $4 \times 4$, …)

### negative number
a number that is less than zero; for example, −25

### net
two-dimensional plan that can be folded to form a 3D object

### nominal data
type of categorical data where the categories are unrelated and cannot be ordered; for example, eye colour

### numerator
top number of a fraction; for example, the numerator of $\frac{2}{3}$ is 2

### numerical data
data that can be counted or measured. *See* continuous data, discrete data

### obtuse angle
angle greater than 90° but less than 180°

### order of line symmetry
the number of axes of symmetry for a shape

### order of rotational symmetry
the number of times an object fits exactly onto itself when being rotated 360°

### ordinal data
type of categorical data that can be placed in categories in a specific order; for example, a rating system from 1–5

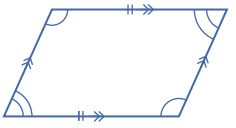
### origin
point where the $x$- and $y$-axes intersect on a Cartesian plane, with coordinates (0, 0)

### outcome
possible result in an experiment or trial; for example, when rolling a die, there are six possible outcomes: 1, 2, 3, 4, 5 and 6

### outlier
extreme piece of data that is much higher or lower than the rest of the data in the data set

### parallel lines
lines (or rays or segments) that are always the same distance apart so that they never meet. Parallel lines are labelled with matching arrowheads.

### parallelogram
quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size



### per cent
for every one hundred (or per hundred); for example, 20 per cent means 20 for every 100

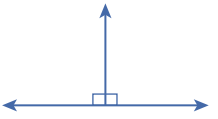
### percentage
a portion of a whole divided into one hundred parts, shown using the symbol %

### perfect square
number that can be expressed as the product of two equal whole numbers; for example, $3 \times 3 = 9$, so 9 is a perfect square

4 9

### perimeter
length of the boundary of a two-dimensional (2D) shape

### perpendicular lines
two lines (or rays or segments) that meet to form a right angle. Perpendicular lines are labelled with a small box at the right angle.



### place value
value of a digit in a number depending on its position; for example, the digit 3 in 2316 represents 3 hundreds or 300 and in 15.34 it represents 3 tenths or $\frac{3}{10}$

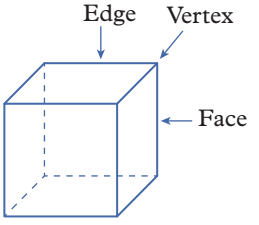
### plan
a drawing of an object from a single view directly above the object

### point
an exact position or location in space that has no length or width, which is represented by a dot labelled with a capital letter or a coordinate

### point of focus
a point chosen for a rotation to concentrate around when rotating a shape

### polygon
a closed, 2D shape with straight sides

### polyhedron (plural polyhedra)
3D object where each face is a polygon




### population
refers to all potential pieces of data under consideration

### primary data
refers to data you have collected yourself

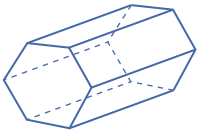
### prime factorisation
the product of the primes that make up a whole number

### prime factors
factors of a composite number that are prime numbers

### prime number
a positive whole number greater than 1 that has exactly two factors, itself and 1; for example, the only factors of 7 are 1 and 7, so 7 is a prime number

### print
to display text, variables or other information on a screen

### prism
object with two ends that are identical polygons and joined by straight edges

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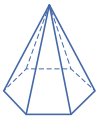
**probability**
describes the chance that a particular event will occur; expressed on a scale from impossible to certain

**product**
result of a multiplication; for example, $10 \times 3 = 30$ where 30 is the product

**pronumeral**
letter or symbol that takes the place of a number

**proper fraction**
fraction where the numerator is smaller than the denominator; for example, $\frac{4}{5}$

**pyramid**
a polyhedron with a base and triangular faces joining the base which all meet at a single point



**quadrilateral**
two-dimensional shape with four straight sides

**quotient**
result of a division; for example, $28 \div 4 = 7$ where 7 is the quotient

**range**
a measure of spread determined by finding the difference between the highest and lowest value in a data set

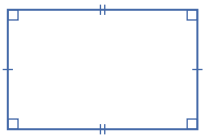
**rate**
a comparison between two or more different quantities

**ratio**
comparison of two or more quantities of the same kind; for example, when comparing the number of red jellybeans to blue jellybeans, the ratio is 2 : 3

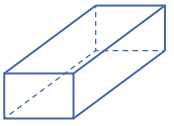
**ray**
a line that starts at a point and continues to infinity that is named using an arrow, which indicates the direction of the ray; for example, $\overrightarrow{AB}$

**reciprocal**
found by first writing a number as a proper or improper fraction and then swapping the numerator and the denominator; for example, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

**rectangle**
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°



**rectangular prism**
prism with three pairs of identical faces (total of six faces)



**reflection**
transformation where a shape or object is reflected (flipped) in an axis of symmetry to produce its exact mirror image

**reflex angle**
angle that is greater than 180° but less than 360°

**regular polygon**
polygon with all sides equal in length and all angles equal in size

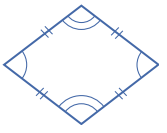
**relative frequency**
the number of times a favourable outcome occurs divided by the total number of outcomes

$$\text{Relative frequency} = \frac{\text{number of occurences}}{\text{total number of outcomes}}$$

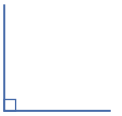
**remainder**
the part of the dividend that is left over when the number is not exactly divisible by the divisor; for example, 22 divided by 7 is 3 remainder 1

**revolution**
angle that is exactly 360°

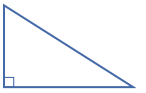
**rhombus**
quadrilateral with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size



**right angle**
angle that is exactly 90°



**right-angled triangle**
triangle with two perpendicular sides, or one right angle, 90°



**right prism**
a prism with right angles between the matching ends and the other faces

**rotation**
transformation of a shape or object that is rotated (turned) around a fixed point, where the movement is described as an angle of rotation in either a clockwise or anti-clockwise direction

**rotational symmetry**
property of a shape or object where it looks the same after a given amount of rotation

**rounding**
replacing the number with an approximation that is simpler and easier to use in calculations, but which will also make any calculations less accurate; for example, $123\,343 \approx 120\,000$

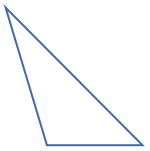
**rule**
1. a description of the relationship between each term and the next term in a sequence
2. a description of the relationship between two or more variables

**sample**
a selection of data that is part of the population

**sample space**
list of all the different outcomes possible for a probability experiment; for example, sample space when rolling a die is {1, 2, 3, 4, 5, 6}

**secondary data**
refers to data that another person or group has collected

### scalene triangle
triangle with three sides of different lengths and with three different internal angles



### sequence
an ordered list of numbers with a set rule; for example, 2, 4, 6, 8, 10, …

### set
a collection of distinct objects or numbers

### simplest form (fractions)
fractions with a highest common factor of 1 between the numerator and the denominator

### simplify (algebraic expressions)
the process of 'collecting like terms' or adding and subtracting the coefficients of like terms; for example, $4a - a = 3a$

### simulation
using technology or simple devices like coins or dice to imitate real-life events

### solution
value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal

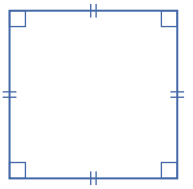
### solve
find the value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal

### solving by inspection
mental strategy to solve simple equations by inspection or observation to identify a number that will make an equation true

### speed
how fast something is travelling, measured as distance per unit of time

### sphere
3D object shaped like a ball

### square
1 quadrilateral with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°
2 to raise to the power of 2; for example, $3^2 = 9$



### square root
a number that when multiplied by itself produces a result that can be represented in a square pattern; for example, the square root of 9 (or $\sqrt{9}$) is 3

### standard form
the way we normally write numbers using digits; for example, thirty-two written in standard form is 32

### stationary
has a speed of zero; it is not moving

### stem-and-leaf plot (stem plot)
a display of data where each piece of data is split into two parts: the last digit becoming the leaf and the other digits becoming the stem

### straight angle
angle that is exactly 180°

### string
a sequence of characters in Python

### substitution
replacement of a pronumeral with a given number; for example, substituting $x = 2$ in $3x + 4$ gives $3 \times 2 + 4$

### successful trial
probability trial where a favourable outcome is obtained

### summary statistics
information such as the mean, median, mode and range that provides a summary of the data set

### supplementary angles
angles that add to make a straight angle (180°)

### surface area
total area of the surface of an object, where the areas of each face or curved surface are added together

### term
1 a single pronumeral, single number, or the product of a number and one or more pronumerals; for example, the expression $3b - 4c + 10$ has three terms
2 an individual number in a sequence

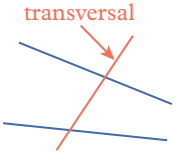
### tessellation
pattern of shapes that has no overlaps or gaps

### theoretical probability
probability determined based on mathematical reasoning, not experiments

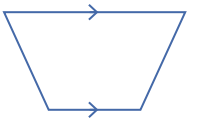
### trailing zero
any zero to the right of the last non-zero digit after the decimal point in a decimal number; for example, 3.900 has two trailing zeros

### translation
transformation of a shape or object that is translated (moved) without turning or changing size, where the movement is often described as the number of units up or down and left or right

### transversal
line that crosses (intersects) a two or more lines




### trapezium
quadrilateral with exactly one pair of opposite sides that are parallel

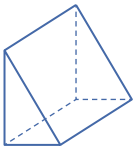


### travel graph
*See* distance–time graph

### trial
performed to obtain results in a probability experiment; for example, rolling a die 50 times in a probability experiment is completing 50 trials

### triangle
two-dimensional shape with three straight sides

### triangular prism
3D object with five faces, two identical triangular faces whose respective vertices are joined by parallel lines



### unit fraction
any fraction that has a numerator of 1; for example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

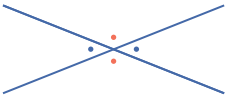
### unknown
a number that is represented using a pronumeral; for example, the x in

unlike fractions
fractions with different denominators; for example, $\frac{3}{2}$ and $\frac{3}{4}$

### unlike fractions
fractions with different denominators; for example $\frac{2}{3}$ and $\frac{5}{4}$. See like fractions

### variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

### vertically opposite angles
a pair of equal angles not adjacent to (next to) each other, formed where two lines intersect; sometimes called 'X angles'



### vertex (plural vertices)
1. corner point where three or more edges of a polyhedron meet. *See* polyhedron
2. the point at which the two lines meet

### volume
amount of space that a 3D object occupies ($cm^3$, $m^3$)

### width
length measurement of the shortest side of a shape or object

### *x*-axis
horizontal number line that forms part of the Cartesian plane

### *x*-coordinate
first of a pair of numbers (coordinates) that describes the point's horizontal distance from the origin on the Cartesian plane

$(-4, 1)$

*x*-coordinate – horizontal distance from the origin

### *y*-axis
vertical number line that forms part of the Cartesian plane

### *y*-coordinate
second of a pair of numbers (coordinates) that describes the point's vertical distance from the origin on the Cartesian plane

$(-4, 1)$

*y*-coordinate – vertical distance from the origin

# Index


## A

Aboriginal and Torres Strait Islander people, water knowledge 528
accuracy 148
acute angle 289, 291
adding
  decimals 153–4, 190
  fractions 113–16, 136
  mental strategies 11, 12, 50
  mixed numbers 114, 115, 136
  negative integers 206–8
  negative numbers 206
  positive integers 202–3, 226
  whole numbers 11–13
addition algorithm 12, 50
  to add three numbers 13
  to add two numbers 13
addition problems, written 203, 207
addition properties 11, 266
adjacent angles 297
algal blooms 528–9
algebra
  order of operations 25–57
  solving equations by inspection 271–4
  solving equations using inverse operations 277–83
  substitution 261–2
  terms, expressions and equations 249–51, 278
  variables 238–40, 284
algebraic expressions *see* expressions
algebraic formulas 244–5
algebraic notation 266, 284
algorithm 488
alternate angles 304, 307
  finding unknown 305, 306
  identifying 305
  to determine if lines are parallel 307
*and* statement (Python) 500
angles 288
  at a point 297, 298–9, 328
  classifying 288–9, 290
  drawing using a protractor 291
  identifying 305
  measuring using a protractor 289, 290, 328
  naming 290, 322
  pairs of 297, 298
  and parallel lines 304–7, 328
  terminology 289–90, 328
arc 288
area, metric units 395, 396
area of a parallelogram 401, 427
  using the formula 401–2
area of a rectangle 395–6, 427
  by counting squares 395, 396
  using the formula 395, 396
area of a triangle 409, 427
  using the formula 409–10
arms 288
ascending order 5
associative law
  of addition 11
  of algebra 266
  of multiplication 23
average 47
axis of symmetry 364, 374, 375, 380

## B

balance model 277
  solving equations using 277, 284
bar charts 446, 447, 483
base (2D shapes) 401, 409
base (indices) 39
BIDMAS 44, 45, 255, 278
biodiversity 528
by inspection, solving equations 271–4, 284
by-parts method 11, 50

## C

calling a variable 491
cancelling 119, 120
capacity 421, 422, 427, 529
  metric units 421
Cartesian coordinates 212–15, 226
Cartesian plane 212–16, 226
  graphs on 219–21
  plotting points from a table of values 215
  plotting points on 214
  rotations and reflections 364–6, 380
  translations 357–59, 380
categorical data 434, 436
census 436
characters (Python) 488
circular economy 532
circumference 172
classifying
  angles and lines 288–292
  data 434–5, 482
  quadrilaterals 320–1
  triangles 314–15
coding (Python) 488
  conditions in code 499–503
  identifying errors in code 493
  printing text to the screen 488–90
  requesting user input 495–7
  to apply a formula 497
  variables 491–3
  working with whole numbers 495
coefficients 249, 250
co-interior angles 207, 304
  finding unknown 305, 306
  identifying 305
  to determine if lines are parallel 307
collecting data 434
column graphs 446, 447, 482
  drawing 448
  reading 448
combined transformations 370
common denominator 103
common factor 69
common multiples 64
commutative law
  of addition 11, 266
  of algebra 266
  of multiplication 23, 266
comparing fractions 103, 104
  methods 102
  using equivalent fractions 103, 105
  using lowest common denominator 103, 105
  with different denominators 104
  with the same denominator 104
  with the same numerator 104
compensation method 11, 17, 50
complementary angles 297, 298
composite numbers 74–5
concave polygons 336
conditions in code 499–3
  checking 499
  checking multiple conditions 500
cones 347, 380
constant terms (constants) 249, 250
contaminant reduction in local waterways (STEAM project) 528–31
continuous data 434
conversion factors 388
convex polygons 336
coordinate points 212–15
corresponding angles 304, 307
  finding unknown 305, 306
  identifying 305
  to determine if lines are parallel 307
cross-section 343, 380
cube 343, 380
  net 414
  surface area 415
cylinders 343, 380

## D

data 434
  classifying 434–5, 482
  collecting 434
  recording in a frequency table 436
  types of 434

decimal fraction 142
decimal number 142
decimal place 148
decimal point 142
decimals 142–4
adding and subtracting 153–4, 190
converting to/from fractions 142, 143, 144, 180–1, 190
converting to/from percentages 174, 180–1, 190
dividing 164–5, 169–70, 190
fractions and percentages 179–82, 190, 529, 533
multiplying 158–9, 190
on a number line 148, 197
ordering and rounding 148–50, 190
place value of digits 142, 143
degrees 289
denominator 90
descending order 4
design cycle 530–1, 534–5
diameter 172
digits 7
discrete data 434
distance–time graphs 219–21, 226
calculating speed from 221
interpreting 220–1
distributive law 23
dividend 33
dividing
mixed numbers 126, 127
using long division 33, 34, 50
using short division 33, 34, 144
whole numbers 33–5
dividing decimals 164–5
by a decimal 169–70, 190
by powers of 10 164, 165, 190
by a whole number 164–5, 190
dividing fractions 125–7, 136
by fractions 125, 126
by whole numbers 126
divisibility of numbers 60, 61, 85
divisibility rules 60–1
division 33–5, 50
divisor 33
dot plots 446, 447, 482
drawing 449
identifying summary statistics 449
drawing
angles 291, 292
plans and elevations of 3D objects 351
plans and elevations of prisms 350
prisms 344

## E

e-waste 532
edges 343, 380
elevations *see* plans and elevations
*elif* statement (Python) 500
else statement (Python) 500
environment, reducing contaminants in waterways 528–9
equally likely outcomes 464, 470
equations 249, 284
checking a solution 271
operations in 249
solving by inspection 270, 271, 284
solving using balance model 277, 284
solving using 'guess, check and improve' method 271, 272, 284
solving using inverse operations 278–80, 284
solving worded problems 273
writing 250
equilateral triangle 314
equivalent equations 277, 278
equivalent expressions 255
equivalent fractions 96–9, 103, 105, 136
estimation 5
evaluating algebraic expressions 261
containing more than one pronumeral 261–2
containing one pronumeral 261
event 464
expanded form 4, 6, 50
expected number 476, 482
experiment 464
experimental probability 476–7, 482
explorations 234–5, 512–13
expressing probabilities 470
expressing a quantity as a percentage of another 185
expressions 243, 284
evaluating 261–2
simplifying 266–8, 284
writing 253, 254

## F

faces 343, 380, 414
factor pairs 69, 70
factor trees 78, 79, 85
factors 69, 70
fast fashion 532
favourable outcomes 470, 482
finding the rule for a sequence 56
formulas 244, 246, 255
substituting into 262
writing 260–1, 256, 263
fraction wall 96, 136
fractions 90–2
adding and subtracting 113–16, 136
converting to/from decimals 142, 143, 180–1, 190
converting to/from percentages 174, 180–1, 190
decimals and percentages 179–82, 190, 529, 533
dividing 125–7, 136
equivalent 96–9, 103, 105, 136
identifying from a diagram 91
improper 90, 108–9, 136
multiplying 119–21, 136
negative 197, 226
on a number line 90, 91
ordering and comparing 102–5, 136
proper 90, 136
reciprocals 125, 126
of a set 90, 92
simplifying 97, 98
writing as a decimal using short division 144
*see also* mixed numbers
frequency 435
frequency table 435, 436
front elevation 350, 380
Fundamental Theorem of Arithmetic 78, 85

## G

generalisations 250
glossary 622–29
gradient 219
distance–time graphs 225
graphs 446–50
choosing which type to use 447
constructing 447
distance–time 219–21, 226
gradient (slope) of 219
interpreting 219–21, 447, 448
summary statistics in 447, 449
types of 446, 482
'greater than' 5, 102
grouping symbols 255–57, 278, 284
'guess, check and improve method' to solve equations 271, 273, 284

## H

habitats, human activity effects 528–9
height (2D shapes) 401, 409
hexagonal prism 343, 380
highest common factor (HCF) 69, 71, 85, 97, 98, 131
Humanities (subject), in STEAM projects 529, 533

## I

*if* / *elif* / *else* structure 500
to classify triangles 502
*if* / *else* pairs 500, 501
*if* statement (Python) 499
and indentation 501
image 356
improper fractions 90
and mixed numbers 108–9, 136
index 39

index form 39, 40
index notation 39
indices 39, 40, 41, 50, 256
*input* ( ) command 495–7
*int* ( ) command 495
integers 196–7, 224
  *see also* negative integers; positive integers
interior angles (internal angles)
  of a quadrilateral 320, 328
  of a triangle 314, 328
interpreting graphs 219–21, 226, 446–51
inverse operations, solving equations using 278–80, 284
irregular polygons 336
irregular quadrilaterals 320
isometric transformations 370
isosceles triangle 314

## J

jump method 11, 17, 50

## K

kite 320

## L

Laws of Arithmetic 266
leading digit 5
length
  converting units of length 388, 389
  metric units 388, 427
  of a rectangle 395
'less than' 5, 102
like fractions, adding and subtracting 113, 114, 136
like terms 266, 267, 284
line 288
line segment 288
line symmetry 374, 375, 376, 380
linear equations 271
linear relationships 215, 226
lines
  classifying 288
  naming 290, 328
litter 528–9
long division 33, 34, 50
long multiplication 29–30, 50, 159
lowest common denominator (LCD) 103, 105
lowest common multiple (LCM) 64, 65, 85

## M

magnitude (of a number) 196
mean 440, 441, 447, 456, 482
measuring angles using a protractor 289, 290, 328
median 440, 441, 447, 449, 456, 482
mental addition strategies 11, 12, 50
mental multiplication strategies 24, 50
mental subtraction methods 17, 18, 50
metric units 388
  of area 395
  of capacity 421
  of length 388, 427
  of volume 420, 533
mixed numbers 90, 108–9, 136
  adding and subtracting 114, 115
  dividing 126, 127
  and improper fractions 108–9, 136
  multiplying 121
mode 440, 441, 447, 449, 456, 482
multiples 64, 65
multiplication algorithm 29–30, 50
multiplication properties 23, 266
multiplying
  by a one-digit number 23, 24
  by powers of 10 24, 25, 50
  by whole numbers 119
  long multiplication 29–30
  mental strategies 24
  mixed numbers 121
  short multiplication 23
  whole numbers 23–5, 29–30
multiplying decimals 158–9, 190
  by powers of 10 158, 159, 190
multiplying fractions 119–21, 136
  with cancelling 119, 120
  with simplifying 120
  with unit conversions 121
  using repeated addition 120

## N

NAPLAN practice 514–27
negative fractions 197, 226
negative integers 197, 226
  adding and subtracting 206–8
negative numbers 196–8
  adding and subtracting 206
negative sign notation 206, 226
nets 414
nominal data 434
non-polygons 336, 380
non-polyhedra 343, 380
notation 39
number lines
  decimals on 148, 197
  fractions on 90, 91, 197
  integers on 196–7
number sequences 56–7
  patterns in 244
numbers
  divisibility rules 60–1
  expanded form 4, 6, 50
  ordering 5, 50
  rounding 5, 7, 50
  standard form 4
  worded form 4, 50
numerator 90
numerical data 434

## O

obtuse angle 289, 291
one-step equations, solving using inverse operations 278
operations in equations 249
*or* statement (Python) 500
order of line symmetry 374, 375
order of operations 45–6, 50
  in algebra 255–57
order of rotational symmetry 374, 375
ordering
  decimals 148–50, 190
  numbers 5, 50
  positive and negative numbers 198
ordering fractions 102–5, 136
  by finding the lowest common denominator 105
ordinal data 434
origin 212, 226
outcome 464
outlier 440, 447, 482

## P

pairs of angles 297, 298
parallel lines 288
  and angles 304–7, 328
  and transversals 304
parallelogram 321
  area 401–2, 427
patterns in sequences 244
pentagonal-based pyramids 344, 380
per cent 173
percentages 173–5
  converting to/from decimals 174, 180–1, 190
  converting to/from fractions 174, 180–1, 190
  expressing a quantity as a percentage of another 185
  fractions and decimals 179–82, 190, 529, 533
  of a quantity 185, 186
perfect square numbers 29, 50
perimeter 388, 389, 427
perpendicular lines 288
place value 4–7, 142, 143
  identifying place value of a digit in a number 6
place value charts/tables 4, 6, 142, 143, 190
plan view 350, 380
plans and elevations 350–1, 380
  of 3D objects 351
  of prisms 350

plotting points
on the Cartesian plane 214
from a table of values and recognising linear relationships 215
point 288
point of focus 364
polygons 336–7, 380
describing 337
naming 338
number of sides and shape 336
polyhedra 343–4, 345
population 434
positive integers 197, 226
adding and subtracting 202–3, 226
positive numbers 196, 197, 198
primary data 434, 435
prime factorisation 78–80
using factor trees 78, 79
using repeated division 79, 80
prime factors 74, 75
prime numbers 74, 75, 85
*print* ( ) command 488, 489
printing 488
mathematical calculations 492
a string 489
text to the screen 488
variables and strings 493
prisms 343, 380
drawing 344
drawing plans and elevations of 350
faces 414
probability 464–5
describing 464, 465
experimental 476–7, 482
expressing 470
terminology 464
theoretical 470–1, 482
pronumerals 244, 246, 255, 256, 288
proper fractions 90, 136
protractor 289, 290–2, 328
pyramids 344, 380
Python
asking a user for input 495–7
checking conditions 499
checking multiple conditions 500, 502
conditions in code 499–503
printing information to the screen 488–9
variables 491–3
whole numbers 495

## Q

quadrilaterals 320, 336
classifying 320–1, 328
finding the unknown angle 322
interior (internal) angles 320, 328
quotation marks 488
quotient 33

## R

range 440, 441, 447, 449, 456, 482
rates 165
ratios 130–2, 533
determining from a diagram 131
dividing a quantity into a 132
simplifying 131
writing with a change of units 130
ray 288
reciprocal of a fraction 125, 126, 136
recording data in a frequency table 436
rectangle 321
area 395–6, 427
rectangular prisms 344, 414
nets 414
surface area 415
volume 420, 421–2, 427
recycling 532–3
reflections 364, 365, 380
reflex angle 191, 289
regrouping 12
regular polygons 336
relative frequency 476, 482
remainder 33
repeated addition 120
repeated division 79, 80, 85, 98
resources, waste reduction to we don't exploit resources (STEAM project) 532–5
revolution 289, 297
rhombus 321
right angle 289, 297
right elevation 350, 380
right prisms 343
right-angled triangle 315
rotational symmetry 374, 375, 380
rotations 364, 365, 380
rounding 5, 7, 50
decimals 148, 149–50, 190
rule
and formula 244
in a number sequence 56
relationship between variables 244, 245

## S

sample 434
sample space 464, 465, 482
scalene triangle 314
Science (subject), in STEAM projects 529, 533
secondary data 434, 435
Semester review 230–3, 504–11
sequences 56–7, 85
generating terms in 57
identifying the rule 56
patterns in 244
short division 33, 34, 144
short multiplication 23, 24
simplest form 97
simplifying
expressions 264–6, 282
fractions 97, 98, 120
ratios 130, 131, 136
simulation 476
solving equations 269
by inspection 269, 269, 283
using balance model 275, 282
using guess, check and improve method 263, 265, 276
using inverse operations 275–8, 282
worded problems 271
speed 219, 226
calculating from distance–time graphs 221
spheres 347, 380
square 323
square a number 39
square-based pyramids 344, 380
square roots 39, 40
standard form 4, 50
stationary 219
STEAM projects
reducing contaminants in local waterways 528–31
reducing waste so we don't exploit resources 532–5
stem-and-leaf plots 456–7, 482
constructing 457
summary statistics from 456
straight angle 287, 297
straight line 215
strings (Python) 488
storing as variables 492
substitution 261, 284
into expressions 261–2
into formulas 262
subtracting
decimals 153–4, 190
fractions 113–16, 136
mental strategies 17, 18, 50
mixed numbers 114, 115, 136
negative integers 206–8, 226
negative numbers 206
positive integers 202–3
whole numbers 17–19
subtraction algorithm 17, 50
to subtract numbers with different numbers of digits 18
to subtract two numbers 18
subtraction problems, written 203, 207
subtraction properties 17
successful trial 476
summary statistics 440–2, 482
from graphs 447, 449
from stem-and-leaf plots 456
supplementary angles 297, 298
surface area 414–16, 427

## T


table of values
  plotting points from 215
  writing formulas from 244
terms (algebra) 249, 250, 284
terms (sequence) 56
tessellations 337
theoretical probability 470–1, 482
3D objects 343–4, 380
  identifying and naming 347
  surface area 414–16
  *see also* polyhedra; prisms
trailing zeros 140
transformations
  combined 370
  *see also* reflections; rotations; translations
transformed object 356
translations 357–59, 380
  finding coordinates after 359
  identifying 358
  performing 358
transversals 34
  and parallel lines 304
trapezium 320
travel graphs 219–21
trial 464
triangle(s) 314, 336
  area 409, 427
  classifying 314–15, 328
  finding an unknown angle in 315
  interior (internal) angles 314, 328
triangular-based pyramids 344, 380
triangular prisms 343, 380, 414
  nets 414
  surface area 416
two-step equations
  involving grouping symbols, solving using inverse operations 279–80
  solving using inverse operations 279


## U


unit fractions 102
unknowns 238, 240
unlike fractions, adding and subtracting 113, 115, 136


## V


value of decimals 142
variables (algebra) 238–40, 278
variables (Python)
  calling 491
  defining 491
  storing strings as 492
vertex 288, 343, 380
vertically opposite angles 297
volume 420, 422, 529, 533
  by counting cubes 421
  metric units 420, 533
  rectangular prisms 420, 421–2, 427
  using the formula 422


## W


waste reduction so we don't exploit resources (STEAM project) 532–5
water, quality and contaminants 528–9
water knowledge, Aboriginal and Torres Strait Islander peoples 528
waterways, contaminant reduction (STEAM project) 528–31
whole numbers 4
  adding 11–13
  coding 495
  dividing 33–5
  dividing decimals by 164–5
  dividing fractions by 126
  multiplying 23–5
  multiplying fractions by 119
  subtracting 17–19
  written as fractions 90
width of a rectangle 395
worded form 4, 50
worded problems
  involving addition and subtraction 203
  involving addition and subtraction of negative integers 207
  using equations to solve 272
  writing formulas from 245
writing equations, with one unknown 250
writing equivalent fractions 97
writing expressions 250
  using grouping symbols 256
writing formulas 244–5, 251
  from a table of values 244
  from worded problems 245
  using grouping symbols 256
writing numbers 4, 50


## X


*x*-axis 212, 226
*x*-coordinate 212–15


## Y


*y*-axis 212, 226
*y*-coordinate 212–15

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**Hexagons are one of only three regular polygons that can be tiled perfectly without any gaps or overlapping. Maybe bees knew this when they first started storing honey ...**

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