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NEW CENTURY MATHS

10

NSW Stages 5.1/5.2

For the
australian curriculum

Klaas Bootsma
David Badger
Sarah Hamper

Robert Yen Series Editor

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Klaas **Bootsma**
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Robert **Yen** Series Editor
NelsonNet print resources by Megan Boltze

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Preface

First published in 1997, *New Century Maths 10 Stages 5.1/5.2* has now been completely rewritten to reflect the spirit of the new Australian curriculum and the NSW syllabus.

For Years 9 and 10, we publish two levels of mathematics books:

- *New Century Maths Stages 5.1/5.2*
- *New Century Maths Advanced Stages 5.2/5.3*

The NSW mathematics syllabus describes Stage 5 (Years 9–10) as a continuum with content classified into three sub-stages, Stage 5.1, Stage 5.2 and Stage 5.3, with each sub-stage including the content of the previous sub-stage. This book has been designed for students and classes progressing along Stages 5.1 or 5.2 of the continuum.

We have retained those familiar features that have made New Century Maths a leading series: clear worked examples, graded exercises, multiple-choice questions, Investigation, Technology, Mental skills, SkillCheck pre-chapter exercise, Power Plus extension exercise, Language of Maths, Topic overview with mind map, glossary, video tutorials and worksheets.

New features include:

- *GeoGebra* activities in our Technology sections
- Chapter outlines showing coverage of the Working Mathematically proficiencies
- *NelsonNet* student and teacher websites of print and multimedia resources
- *NelsonNetBook* web version of this book with interactive features

We always aim to write textbooks to provide flexibility for teachers to plan and teach for a variety of pathways. We thank our families and teaching colleagues for their continued support and patience, and editors Anna Pang and Alan Stewart for transforming our manuscript into this fine text. Finally, we wish all teachers and students using this book every success in embracing the new Australian mathematics curriculum.

About the authors

Klaas Bootsma was head teacher of mathematics at Ambarvale High School in Campbelltown and has taught at Lurnea and Grantham high schools. He was a senior HSC examiner and has worked on the HSC Advice Line and School Certificate marking. Klaas has wide experience teaching all types of students, and his interests are in curriculum and the use of technology (ICT) in learning.

David Badger is principal of Toongabbie Christian School, was deputy principal at Mt Annan Christian College and head teacher of mathematics at Eagle Vale High School in Campbelltown. He has been involved in HSC and School Certificate marking and has worked on the HSC Advice Line. David's passion is to make mathematics interesting, practical and accessible to all students.

Sarah Hamper teaches at Abbotsleigh School in Wahroonga and has taught at Meriden and Tara Anglican schools. She has an interest in gifted and talented (GAT) students and girls education. Sarah's expertise is in using modelling, problem solving and ICT for the effective learning of mathematics, and she has presented workshops for MANSW and nationally.

Series editor **Robert Yen** has taught at Hurlstone Agricultural, Eagle Vale and Ambarvale high schools in southwest Sydney. He co-wrote *New Century Maths Essentials 9–10* and *General Mathematics*, writes and presents for MANSW, and has co-edited its journal, Reflections. He now works for Cengage Learning as a mathematics publisher.

Contributing author

Megan Boltze wrote and edited many of the *NelsonNet* print resources (blackline masters) and is head teacher of mathematics at Ashcroft High School.

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Real numbers	2 Working with numbers 5 Indices 11 Coordinate geometry and graphs	7 Graphs
Money and financial mathematics	2 Working with numbers	1 Interest and depreciation
Patterns and algebra	3 Algebra 5 Indices	4 Algebra 6 Equations and inequalities
Linear and non-linear relationships	7 Equations 11 Coordinate geometry and graphs	2 Coordinate geometry 7 Graphs 6 Equations and inequalities 9 Simultaneous equations
MEASUREMENT AND GEOMETRY		
Using units of measurement	10 Surface area and volume	3 Surface area and volume
Geometric reasoning	6 Geometry 13 Congruent and similar figures	11 Geometry
Pythagoras and trigonometry	1 Pythagoras' theorem 4 Trigonometry	8 Trigonometry
STATISTICS AND PROBABILITY		
Chance	12 Probability	10 Probability
Data representation and interpretation	9 Investigating data	5 Investigating data

Curriculum grid: NSW syllabus

Strand and substrand	New Century Maths 9 Stages 5.1/5.2 chapter	New Century Maths 10 Stages 5.1/5.2 chapter
NUMBER AND ALGEBRA		
Financial mathematics (Stages 5.1, 5.2)	2 Working with numbers 8 Earning money	1 Interest and depreciation
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Linear relationships (Stages 5.1, 5.2)	7 Equations 11 Coordinate geometry and graphs	2 Coordinate geometry
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Ratios and rates (Stage 5.2)	2 Working with numbers 11 Coordinate geometry and graphs	7 Graphs
Algebraic techniques (Stage 5.2)	3 Algebra	4 Algebra
Equations (Stage 5.2)	7 Equations	6 Equations and inequalities 9 Simultaneous equations
MEASUREMENT AND GEOMETRY		
Area and surface area (Stages 5.1, 5.2)	10 Surface area and volume	3 Surface area and volume
Numbers of any magnitude (Stage 5.1)	5 Indices 10 Surface area and volume	
Right-angled triangles (Trigonometry) (Stages 5.1, 5.2)	1 Pythagoras' theorem 4 Trigonometry	8 Trigonometry
Properties of geometrical figures (Stages 5.1, 5.2)	6 Geometry 13 Congruent and similar figures	11 Geometry
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STATISTICS AND PROBABILITY		
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Curriculum grid: Year 10 content descriptions

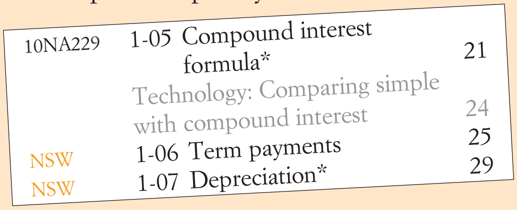
This is an extract from the Australian Curriculum, except for the additional NSW content.

Content description	**New Century Maths 10 Stages 5.1/5.2 chapter**
NUMBER AND ALGEBRA	
Money and financial mathematics	
ACMNA299: Connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies	1 Interest and depreciation
NSW STAGE 5.1: Solve problems involving earning money	1 Interest and depreciation
Patterns and algebra	
ACMNA230: Factorise algebraic expressions by taking out a common algebraic factor	4 Algebra
ACMNA231: Simplify algebraic products and quotients using index laws	4 Algebra
ACMNA232: Apply the four operations to simple algebraic fractions with numerical denominators	4 Algebra
ACMNA233: Expand binomial products and factorise monic quadratic expressions using variety of strategies	4 Algebra
ACMNA234: Substitute values into formulas to determine an unknown	6 Equations and inequalities
Linear and non-linear relationships	
NSW STAGE 5.2: Interpret and graph linear relationships using the gradient-intercept form of the equation of a straight line	2 Coordinate geometry
ACMNA235: Solve problems involving linear equations, including those derived from formulas	6 Equations and inequalities
ACMNA236: Solve linear inequalities and graph their solutions on a number line	6 Equations and inequalities
ACMNA237: Solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology	9 Simultaneous equations
ACMNA238: Solve problems involving parallel and perpendicular lines	2 Coordinate geometry
ACMNA239: Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate	7 Graphs
ACMNA240: Solve linear equations involving simple algebraic fractions	6 Equations and inequalities
ACMNA241: Solve simple quadratic equations using a range of strategies	6 Equations and inequalities

Content description	New Century Maths 10 Stages 5.1/5.2 chapter
MEASUREMENT AND GEOMETRY	
Using units of measurement	
ACMMG242: Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids	3 Surface area and volume
Geometric reasoning	
ACMMG243: Formulate proofs involving congruent triangles and angle properties	11 Geometry
ACMMG244: Apply logical reasoning, including the use of congruence and similarity, to proofs and numerical exercises involving plane shapes	11 Geometry
NSW STAGE 5.2: Apply interior and exterior angle sum results for polygons to find the sizes of unknown angles	11 Geometry
Pythagoras and trigonometry	
ACMMG245: Solve right-angled triangle problems including those involving direction and angles of elevation and depression	8 Trigonometry
STATISTICS AND PROBABILITY	
Chance	
ACMSP246: Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence.	10 Probability
ACMSP247: Use the language of 'if … then', 'given', 'of', 'knowing that' to investigate conditional statements and identify common mistakes in interpreting such language	10 Probability
Data representation and interpretation	
ACMSP248: Determine quartiles and interquartile range	5 Investigating data
ACMSP249: Construct and interpret box plots and use them to compare data sets	5 Investigating data
ACMSP250: Compare shapes of box plots to corresponding histograms and dot plots	5 Investigating data
ACMSP251: Use scatter plots to investigate and comment on relationships between two numerical variables	5 Investigating data
ACMSP252: Investigate and describe bivariate numerical data where the independent variable is time	5 Investigating data
ACMSP253: Evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data	5 Investigating data

Coverage of the Australian curriculum and NSW syllabus

- *New Century Maths 10 Stages 5.1/5.2* covers both the Australian curriculum and the NSW syllabus, as shown by the table of contents and curriculum grids on the previous pages. The previous two pages lists all of the Year 10 content descriptions explicitly.
- This book contains Stages 5.1 and 5.2 content, including NSW syllabus content that is not covered by the Australian curriculum. NSW-only content is highlighted in orange, while Stage 5.2 content is marked with a *.



10NA229	1-05 Compound interest formula*	21
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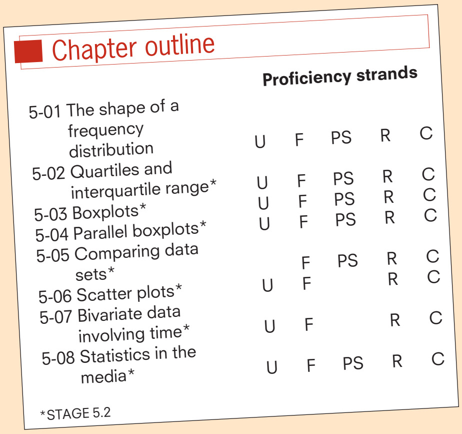
- There are three content strands in Mathematics:
  - **NA** = Number and Algebra
  - **MG** = Measurement and Geometry
  - **SP** = Statistics and Probability
- Each chapter begins with a **chapter outline** that includes the Working Mathematically proficiencies covered in each section:
  - **U** = Understanding
  - **F** = Fluency
  - **PS** = Problem solving
  - **R** = Reasoning
  - **C** = Communicating

Further references to the Working Mathematically proficiencies can be found in the teaching program.



Chapter outline

	Proficiency strands				
5-01 The shape of a frequency distribution	U	F	PS	R	C
5-02 Quartiles and interquartile range*	U	F	PS	R	C
5-03 Boxplots*	U	F	PS	R	C
5-04 Parallel boxplots*	U	F	PS	R	C
5-05 Comparing data sets*		F	PS	R	C
5-06 Scatter plots*	U	F		R	C
5-07 Bivariate data involving time*	U	F		R	C
5-08 Statistics in the media*	U	F	PS	R	C

*STAGE 5.2

Working Mathematically explained

While the three **content strands** are the 'nouns' of the mathematics curriculum, the five **Working Mathematically proficiencies** are the 'verbs': the doing and thinking processes that go hand-in-hand with the content being taught.

- **Understanding** is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.
- **Fluency** is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.
- **Problem solving** is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.
- **Reasoning** is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.
- **Communicating** is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

In each chapter

- **Wordbank** is a sample glossary of 6–8 terms from the chapter
- There is also a full glossary at the back of the book: in this book, all terms printed in red appear in the glossary.
- Curriculum content descriptions are listed at the beginning of each chapter, with the Australian Curriculum descriptions printed with the AC symbol.
- **SkillCheck** and **StartUp assignment** review prerequisite skills and knowledge for the chapter.

- Important facts and formulas are highlighted in a **Summary** box.

Summary

Finding the quartiles of a data set

- sort the scores in order, find the median and call it Q_2
- find the median of the bottom half of the scores and call it Q_1 (or Q_L)
- find the median of the top half of scores and call it Q_3 (or Q_U).

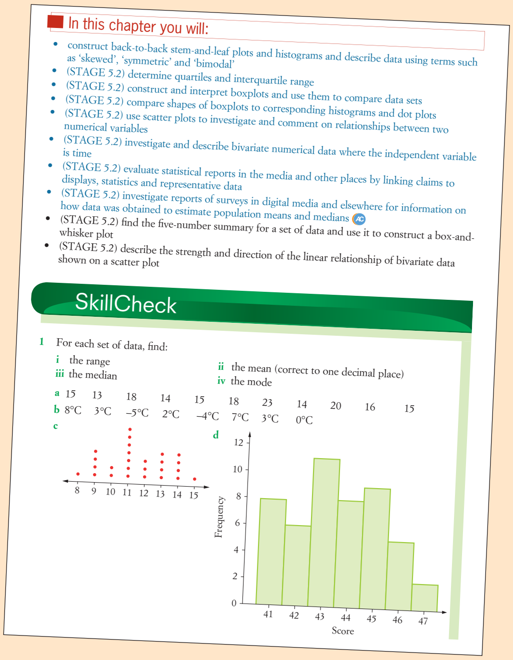
- Pages containing **Stage 5.2 content** are marked by a shaded margin.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.

Investigation: Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas – from population, employment, weekly earnings, weight and obesity in adults, to health of children in Australia.

Visit the ABS website **www.abs.gov.au** to answer the following questions.

1 a What is the current population of Australia?
  b What is the predicted population for:
    i 2020? ii 2030? iii 2040?
  c What is Australia's rate of population increase?
2 Go to 2011 Census Data by Location, and then to Data and analysis.
  a What was the population in NSW and its increase from 2006?
  b Which state had the:
    i largest increase in population?
    ii the smallest increase in population?

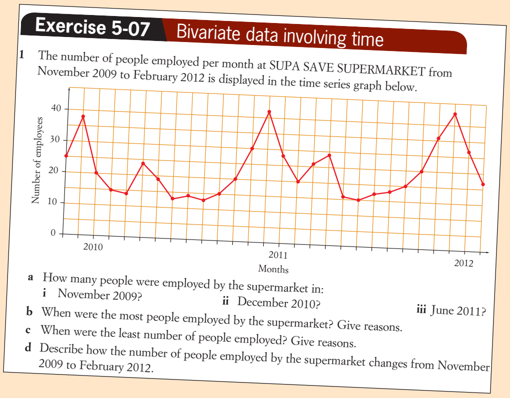


In this chapter you will:

- construct back-to-back stem-and-leaf plots and histograms and describe data using terms such as 'skewed', 'symmetric' and 'bimodal'
- (STAGE 5.2) determine quartiles and interquartile range
- (STAGE 5.2) construct and interpret boxplots and use them to compare data sets
- (STAGE 5.2) compare shapes of boxplots to corresponding histograms and dot plots
- (STAGE 5.2) use scatter plots to investigate and comment on relationships between two numerical variables
- (STAGE 5.2) investigate and describe bivariate numerical data where the independent variable is time
- (STAGE 5.2) evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data
- (STAGE 5.2) investigate reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians
- (STAGE 5.2) find the five-number summary for a set of data and use it to construct a box-and-whisker plot
- (STAGE 5.2) describe the strength and direction of the linear relationship of bivariate data shown on a scatter plot

SkillCheck

1 For each set of data, find:
  i the range
  ii the mean (correct to one decimal place)
  iii the median
  iv the mode
  a 15 13 18 14 15 18 23 14 20 16 15
  b 8°C 3°C –5°C 2°C –4°C 7°C 3°C 0°C
  c
  d



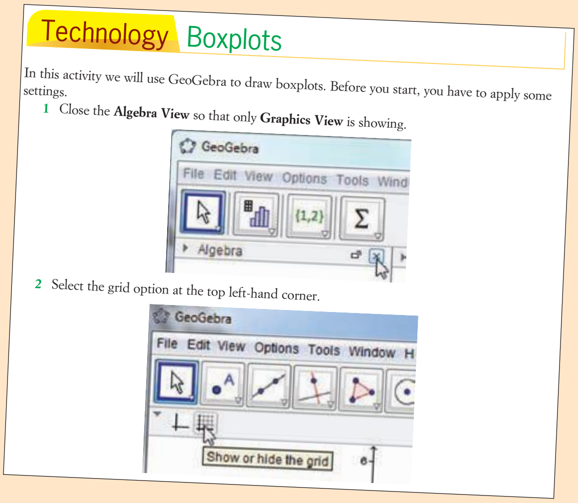
Exercise 5-07 Bivariate data involving time

1 The number of people employed per month at SUPA SAVE SUPERMARKET from November 2009 to February 2012 is displayed in the time series graph below.

a How many people were employed by the supermarket in:
  i November 2009? ii December 2010? iii June 2011?
b When were the most people employed by the supermarket? Give reasons.
c When were the least number of people employed? Give reasons.
d Describe how the number of people employed by the supermarket changes from November 2009 to February 2012.

- **Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.
- **Just for the record** contains interesting facts and applications of the mathematics learned in the chapter

- **Technology** promotes ICT in the classroom, using spreadsheets, *GeoGebra* and the Internet.

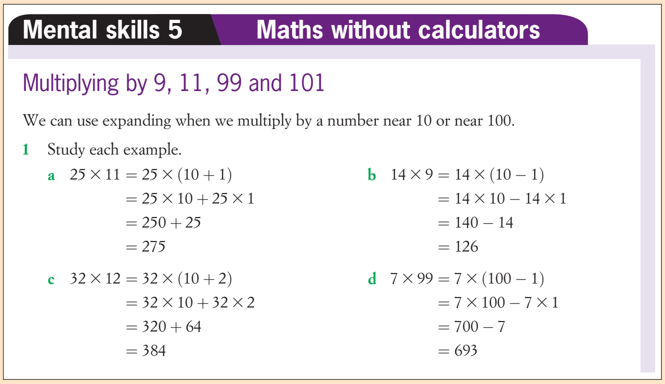


Technology Boxplots

In this activity we will use GeoGebra to draw boxplots. Before you start, you have to apply some settings.

1 Close the **Algebra View** so that only **Graphics View** is showing.

2 Select the grid option at the top left-hand corner.



Mental skills 5 Maths without calculators

Multiplying by 9, 11, 99 and 101

We can use expanding when we multiply by a number near 10 or near 100.

1 Study each example.

a $25 \times 11 = 25 \times (10 + 1)$
$= 25 \times 10 + 25 \times 1$
$= 250 + 25$
$= 275$

b $14 \times 9 = 14 \times (10 - 1)$
$= 14 \times 10 - 14 \times 1$
$= 140 - 14$
$= 126$

c $32 \times 12 = 32 \times (10 + 2)$
$= 32 \times 10 + 32 \times 2$
$= 320 + 64$
$= 384$

d $7 \times 99 = 7 \times (100 - 1)$
$= 7 \times 100 - 7 \times 1$
$= 700 - 7$
$= 693$

- **Mental skills** reinforce mental calculation strategies ('maths without calculators').

NelsonNet resources

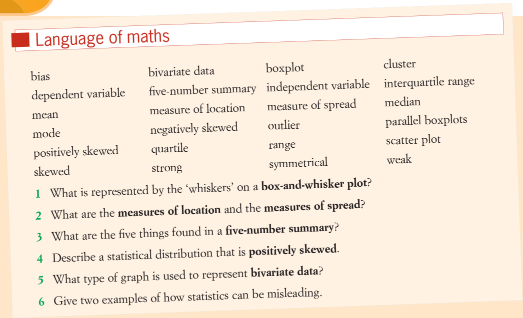
Margin icons link to print (PDF) and multimedia resources found on the NelsonNet website, **www.nelsonnet.com.au**. These include:

Worksheet Conditional probability	**Worksheets**	Video tutorial Interquartile range	**Video tutorials** of worked examples
Puzzle sheet Coordinate geometry crossword	**Puzzle sheets** of matching activities, crosswords and find-a-word puzzles	Technology worksheet Graphing non-linear equations	**Technology worksheets:** additional technology activities
Skillsheet Percentage calculations	**Skillsheets** of examples and exercises of prerequisite skills and knowledge	Quiz Geometry	**ExamView quizzes:** interactive and self-marking

Each resource has a unique identifier code. For example, the video tutorial 'Simplifying surds' has the code **MAT10NAVT10028**, which stands for Mathematics, Year 10, Number and Algebra strand, Video Tutorial 10028.

At the end of each chapter

- **Power plus** is an extension/challenge exercise.
- **Language of maths** has a chapter word list and literacy questions.
- **Topic overview** has reflection questions and an incomplete mind map.
- **Chapter revision** is a review exercise with links to each exercise set of the chapter.
- **Mixed revision** is a review exercise after every 3–4 chapters.



Language of maths

bias
dependent variable
mean
mode
positively skewed
skewed

bivariate data
five-number summary
measure of location
negatively skewed
quartile
strong

boxplot
independent variable
measure of spread
outlier
range
symmetrical

cluster
interquartile range
median
parallel boxplots
scatter plot
weak

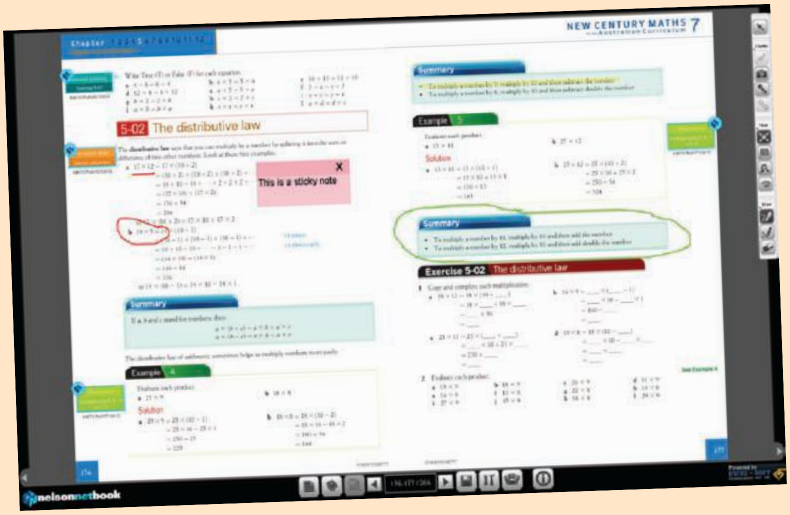
1 What is represented by the 'whiskers' on a **box-and-whisker plot**?
2 What are the **measures of location** and the **measures of spread**?
3 What are the five things found in a **five-number summary**?
4 Describe a statistical distribution that is **positively skewed**.
5 What type of graph is used to represent **bivariate data**?
6 Give two examples of how statistics can be misleading.

At the end of the book

- **General Revision** exercise
- **Instructional and Mathematical glossaries** (in this book, words printed in red also appear in the glossary)
- **Answers** and **index**

NelsonNetBook

- **NelsonNetBook** is the interactive digital version of this textbook found on NelsonNet, containing margin icon links directly to NelsonNet resources.
- You can add notes, voice and sound bites, highlighting, weblinks and bookmarks to each page of the NelsonNetBook.
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students.



Number and Algebra

1

Interest and depreciation

The value of an investment increases over time as a result of interest being added to it, whether it be simple or compound interest. On the other side of the coin, the value of assets and items such as cars and office equipment decreases over time due to age and wear-and-tear. Compound interest and depreciation use formulas that involve repeated percentage increase and decrease respectively.

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Chapter outline

	Proficiency strands				
1-01 Earning an income	U	F	PS		C
1-02 Income tax	U	F			C
1-03 Simple interest	U	F	PS		C
1-04 Compound interest	U	F	PS		C
1-05 Compound interest formula*	U	F	PS	R	C
1-06 Term payments	U	F	PS		C
1-07 Depreciation*	U	F	PS	R	C

*STAGE 5.2

Wordbank

allowable deduction A part of a person's yearly income that is not taxed, such as work-related expenses and donations to charities

compound interest Interest calculated on the principal invested as well as on any accumulated interest

depreciation The decrease in the value of items over time due to ageing

instalment or **repayment** The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan

net pay Pay received after deductions from gross pay; 'take-home' pay

per annum (p.a.) Per year

principal The original amount of money invested or borrowed, for the purpose of earning interest

simple interest Interest calculated on the original principal invested only

In this chapter you will:

- solve problems involving simple interest
- (STAGE 5.2) connect the compound interest formula to repeated applications of simple interest using appropriate digital technologies
- calculate weekly, fortnightly, monthly and yearly incomes
- calculate wages, salaries, overtime, commission, piecework and annual leave loading
- use tables to calculate income tax and PAYG tax
- calculate compound interest for two to three years by repeated percentage increase
- (STAGE 5.2) calculate compound interest using the compound interest formula $A = P(1 + R)^n$
- solve problems involving term payments
- (STAGE 5.2) calculate depreciation using the compound interest formula

SkillCheck

Worksheet
StartUp assignment 1
MAT10NAWK10001

Worksheet
Percentage shortcuts
MAT10NAWK10002

Skillsheet
Mental percentages
MAT10NASS10001

Skillsheet
Percentage calculations
MAT10NASS10002

1 Convert each percentage to a decimal.

a 4% **b** 22% **c** 18.3% **d** 4.7%
e $9\frac{1}{2}\%$ **f** 6.75% **g** $15\frac{1}{4}\%$ **h** 20%

2 Find:

a 6% of $1200 **b** 2.5% of $4650 **c** 12% of $37 450

3 Increase:

a $7000 by 5% **b** $3955 by 2% **c** $8600 by 1.6%

4 How many months are there in:

a 3 years? **b** 2 years? **c** 5 years?

5 Copy and complete:

a One year = __________ weeks **b** One year = __________ fortnights
c One year = __________ days **d** 48 months = __________ years
e 84 days = __________ weeks **f** 100 months = __________ years __________ months

6 If $P = mvt$, find:
a P when $m = 1600$, $v = 0.072$, $t = 10$
b m when $P = 120$, $v = 0.3$, $t = 8$
c v when $P = 18$, $m = 60$, $t = 5$

7 Evaluate, correct to the nearest cent:

a $\$5000 \times (1.045)^4$ **b** $\$28\,000 \times (1.03)^6$
c $\$15\,300 \times (1.065)^3$ **d** $\$32\,400 \times (1.072)^{10}$

1-01 Earning an income

NSW

Wages, salaries and overtime

A **wage** is calculated from the number of hours worked and is usually paid weekly. Wage earners can make more income by working extra hours (overtime).

A **salary** is a fixed annual amount, paid weekly, fortnightly or monthly. Salary earners do not earn overtime pay but can receive benefits such as a computer, company car, expense account, shares in the company or paid medical expenses.


Alamy/Keith Morris

Summary

1 year = 12 months
1 fortnight = 2 weeks
1 year = 52 weeks for wage earners
1 year = 52.18 weeks for salary earners

The two most common rates of overtime pay are:

- **time-and-a-half** = 1.5 × normal hourly rate
- **double time** = 2 × normal hourly rate

Example 1

Thomas earns a salary of \$70 400 p.a.
How much does he earn:

p.a. = per annum = per year'

a each week? **b** each fortnight? **c** each month?

Solution

a Weekly income = \$70 400 ÷ 52.18
= \$1349.1759 . . .
≈ \$1349.18 Rounded to the nearest cent.

b Fortnightly income = 2 × \$1349.18 1 fortnight = 2 weeks
= \$2698.36

c Monthly income = \$70 400 ÷ 12 1 year = 12 months
= \$5866.6666 . . .
= \$5866.67 Rounded to the nearest cent.

Example 2

Noor earns \$22.65 per hour at normal rates. Last week, she worked 38 hours at normal rates, 6 hours at time-and-a-half and 3 hours at double time. Calculate Noor's total earnings for the week.

Solution

$$\begin{aligned}\text{Normal pay} &= \$22.65 \times 38\\ &= \$860.70\end{aligned}$$

$$\begin{aligned}\text{Time-and-a-half pay} &= 6 \times \$22.65 \times 1.5 \\ &= \$203.85\end{aligned}$$

6 hours

$$\begin{aligned}\text{Double time pay} &= 3 \times \$22.65 \times 2 \\ &= \$135.90\end{aligned}$$

3 hours

$$\begin{aligned}\text{Total earnings} &= \$860.70 + \$203.85 + \$135.90\\ &= \$1200.45\end{aligned}$$

Commission, piecework and annual leave loading

Shutterstock.com/Radu Razvan

Commission is earned by salespeople and agents, and is a percentage of the total value of items sold.

Piecework is earned according to the number of items made or tasks completed.

Annual leave loading or **holiday loading** is extra pay given during annual leave (holidays), and is 17.5% of four weeks' normal pay.

Example 3

Georgia is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of $750. How much will Georgia earn if she sells an apartment for $52 500?

A **retainer** is a fixed amount paid regardless of how many items are sold.

Solution

$$\begin{aligned}\text{Commission} &= 2.5\% \text{ of } \$52\,500\\ &= \$1312.50\end{aligned}$$

$$\begin{aligned}\text{Total earnings} &= \text{commission} + \text{retainer}\\ &= \$1312.50 + \$750\\ &= \$2062.50\end{aligned}$$

∴ Georgia earns $2062.50.

Example 4

Emad is a jewellery designer. He makes handmade jewellery and is paid at the following rates:

- $278 per necklace
- $62 per pair of earrings
- $95 per bracelet

This month, Emad made 23 necklaces, 7 pairs of earrings and 19 bracelets. How much did he earn?

Solution

$$\begin{aligned}\text{Monthly earnings} &= 23 \times \$278 + 7 \times \$62 + 19 \times \$95\\ &= \$8633\end{aligned}$$

Example 5

Sanjay's annual salary is $70 590. For his Christmas holidays, he received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. Calculate Sanjay's:

a normal weekly pay
b annual leave loading
c total pay for the Christmas holiday.

Solution

a
$$\begin{aligned}\text{Weekly pay} &= \$70\,590 \div 52.18\\ &= \$1352.8171\ldots\\ &\approx \$1352.82\end{aligned}$$

b Annual leave loading $= 17.5\% \times \$1352.82 \times 4$
$= \$946.974$
$\approx \$946.97$

c Total holiday pay $= (4 \times \$1352.82) + \$946.97$
$= \$6358.25$

Exercise 1-01 Earning an income

Express all answers correct to the nearest cent where necessary.

1 Find the weekly wage for each person.
  - **a** Mary earns $21.85 per hour and works for 40 hours.
  - **b** Connor works 8 hours a day, Monday to Friday, and is paid $23.47 per hour.
  - **c** Yoshe works on Monday and Tuesday from 8:30 a.m. until 4:00 p.m. and Thursday from midday until 9:00 p.m., and earns $15.30 per hour.

2 Greta earns $19.56 an hour and works for 31 hours each week. Chandler earns $21.44 per hour for his 27 hours of work. Who earns more per week and by how much?

See Example 1 **3** Maggie earns a salary of $180 640 p.a. How much does she earn:
a each week? **b** each fortnight? **c** each month?

4 Rakitu considers two jobs, one locally with an annual salary of $57 640 p.a. and the other one in the city with a fortnightly pay of $2320. Calculate the weekly income for each job, determine which one pays more per week, and by how much.

See Example 2 **5** Anan works 38 hours at normal rates, 7 hours at time-and-a-half and 4 hours at double-time. Calculate Anan's total earnings if he earns $19.40 per hour at normal rates.

6 Jacqui works 8.5 hours per day from Tuesday to Friday. She is paid $18.78 per hour. She also works on Saturday for 4.5 hours at a special rate of $21.49 per hour. How much did Jacqui earn for the week?

7 Idra works the following hours in a week at the clothing chain *Shop til U Drop*.

Day	Hours worked
Monday	9:00 a.m. – 5:00 p.m.
Tuesday	9:00 a.m. – 4:00 p.m.
Thursday	11:00 a.m. – 7:30 p.m.
Friday	10:00 a.m. – 5:00 p.m.
Saturday	10:30 a.m. – 5:00 p.m.

She is paid at the following rates.

Day	Rate of pay
Monday to Friday	$19.62 per hour
Saturday Thursday after 4:00 p.m.	$23.15 per hour

What is Idra's total income for the week?

See Example 3 **8** Fatimah is paid a commission of 2.5% on the value of goods she sells. She also receives a weekly retainer of $875. How much will Fatimah earn if she sells goods to the value of $41 600 in one week? Select the correct answer **A**, **B**, **C** or **D**.

A $1915 **B** $1061.88 **C** $2187.50 **D** $1018.13

9 Nathan is a real estate agent whose commission is calculated on the value of the properties he sells:

- 3% paid on first \$300 000
- 1.5% paid on next \$250 000
- 0.75% paid on any value thereafter

How much commission did Nathan earn for selling a house for \$625 000?

10 Brandon designed an app, *iCalc*, that is selling for \$2.49. If he makes 70% profit on the sale price of each app sold, how much would he make from selling 800 units of this app?

11 Matt charges \$60 for each lawn he mows and \$45 for trimming hedges in each yard. In a week, he mows 24 lawns and trims 15 hedges. How much does he earn for the week?

See Example 4

Newspix/Stephen Harman

12 *Clean 2 Swim* charges \$86 to clean backyard pools. If this business earned \$4644 in the first week of summer, how many pools were cleaned?

13 Jade makes homemade eco-friendly soaps, shampoos and cleaning products. A customer purchases 3 homemade soaps, 2 bottles of shampoo and 3 of the cleaning sprays. How much does Jade receive for these purchases?

	Homemade soaps	\$ 5.60
	Eco-friendly shampoo	\$12.70
	Natural cleaning spray	\$ 7.25

14 Calculate the annual leave loading for each person if it is 17.5% of 4 weeks' pay.

a Peter earns \$610 per week

b Jamilla earns \$2000 per fortnight

c Samir earns \$5944 per month

d Ellie earns \$46 630 p.a.

15 For his annual holidays, Jake received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. If Jake's annual salary is \$50 725, find his:

See Example 5

a normal weekly pay

b annual leave loading

c total pay for the four-week holiday.

Investigation: Workers' entitlements

The Australian Government sets the minimum standards for pay and conditions for all Australian workers. Different industries can have different needs from employees in terms of:

- normal and overtime hours worked, breaks allowed
- allowances
- dress codes, such as uniforms
- working conditions

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1 Visit the Fair Work Ombudsman website **www.fairwork.gov.au** and select **Industries**.
2 Select two industries and identify any similarities and differences in the requirements of those industries.
3 Write a summary of your findings.
4 Give a report in class.

NSW

1-02 Income tax

Not all of a person's income is taxed. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable deductions** (or **tax deductions**) and are not taxed. Examples of allowable deductions are tools of trade, uniforms, car-related expenses, subscriptions to professional organisations and journals.

Summary

Income tax is calculated on a person's **taxable income**, which is the **gross income** (total earnings) less all allowable deductions, **rounded down to the nearest dollar**.
Taxable income = gross income − allowable deductions

The more a person earns, the higher the rate of tax to be paid. The table below shows how income tax is calculated.

Tax rates for Australian residents

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

Source: © Australian Taxation Office for the Commonwealth of Australia

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Example 6

Sophia earned $62 348 last financial year and collected bank interest of $440.81. She had allowable deductions of $427.52 in work expenses and $110 in donations to charities.

a Calculate her taxable income.

b Use the tax table to calculate the income tax that Sophie must pay.

Solution

a Taxable income = \$62 348 + \$440.81 − \$427.52 − \$110

= \$62 251.29

≈ \$62 251 Rounded down to the nearest dollar.

b According to the table, a taxable income of \$62 251 is in the \$37 001 – \$80 000 tax bracket.

Income tax = \$3572 + 0.325 × (\$62 251 − \$37 000)

= \$11 778.575

≈ \$11 778.58

'32.5c for each \$1' means 32.5% or 0.325

PAYG tax and net pay

Income tax deducted from your pay by your employer every payday is called **PAYG (Pay As You Go) tax**. The total amount of PAYG tax paid over the year is usually more than the actual income tax payable, so at the end of the financial year you will receive the difference as a **tax refund**. **Gross pay** is the total amount a person earns or receives, but most workers have a variety of deductions taken from their pay before they receive it, including PAYG tax, superannuation contributions, union fees and health fund payments. The amount of income left after the deductions is called **net pay**.

Summary

Net pay = gross pay − tax − other deductions

Example 7

Jay earns a gross pay of \$2290.33 per fortnight. His deductions are for PAYG tax, \$44.10 for private health insurance and \$55.82 for superannuation.

a Use this PAYG tax table to find Jay's PAYG tax per fortnight.

b Calculate Jay's net pay.

c Calculate Jay's total deductions as a percentage of his gross income (correct to one decimal place).

Fortnightly earnings (\$)	PAYG tax withheld (\$)
2274–2279	450
2280–2285	452
2286–2291	454
2292–2297	456
2298–2303	458
2304–2309	460

Solution

a In the table, \$2290.33 falls in the \$2286 – \$2291 range.

Fortnightly PAYG tax = \$454

b
$$\begin{aligned}\text{Net pay} &= \$2290.33 - (\$454 + \$44.10 + \$55.82)\\ &= \$2290.33 - \$553.92\\ &= \$1736.41\end{aligned}$$

Net pay = gross pay − total deductions

c Total deductions = \$553.92

$$\begin{aligned}\text{Deductions percentage} &= \frac{\$553.92}{\$2290.33} \times 100\%\\ &= 24.1851\ldots\%\\ &\approx 24.2\%\end{aligned}$$

$$\frac{\text{Total deductions}}{\text{Gross pay}} \times 100\%$$

Exercise 1-02 Income tax

See Example 6

1 Mrs Shepherd earns \$47 628 in a year and has allowable deductions of \$1930.46.

a Calculate her taxable income, rounded down to the nearest dollar.

b Use the tax table on page 10 to calculate the income tax that Mrs Shepherd must pay.

2 Adam is an environmental engineer who had a gross income of \$118 742 this year and work-related expenses totalling \$4022.80, which are tax-deductible. Calculate:

a Adam's taxable income, rounded down to the nearest dollar

b Adam's payable income tax.

3 Erin is a graphic designer who earns an annual salary of \$90 541 and has collected \$1029.45 in bank interest. She has allowable deductions of \$379 for tools related to her work and \$287 in donations to charity. Calculate:

a Erin's taxable income **b** the amount of tax payable.

4 Riley the builder had a gross income of \$56 922 this year. He is entitled to these tax deductions: tools \$1538, training courses \$445 and outdoor protective clothing \$506. How much should Riley pay in tax? Select the correct answer **A**, **B**, **C** or **D**.

A \$13 046.65 **B** \$10 855.58 **C** \$9237.73 **D** \$6884.27

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5 Michaela is a nurse earning \$87 996 per year. Her allowable deductions are the cost of non-slip footwear –\$225, the cost of laundering uniforms –\$1046, and union fees –\$297.60. How much should Michaela pay in tax?

6 William owns a photography business and earned \$196 000 last year. His allowable deductions were Internet costs for his website –\$968, photographic equipment –\$23 672, and travel to photo locations –\$15 930. Calculate the amount that William should pay in tax.

7 Jackson earns a gross weekly income of \$1075.26. His weekly deductions are \$309.11 PAYG tax, \$44.55 for private health insurance and \$25.18 for superannuation. Calculate Jackson's net weekly pay.

8 Isha earns a gross income of \$788.20 per week. Her deductions are \$132.44 tax and \$32.24 for private health insurance. Calculate Isha's net income.

Use the PAYG table from Example **7** on page 12 to answer questions **9** to **12**.

9 Every fortnight, Mr Singh earns \$2280 and pays \$22.80 in union fees and \$94.10 in superannuation.

See Example 7

a Find how much PAYG tax he pays per fortnight.

b Calculate Mr Singh's fortnightly net pay.

c What percentage (correct to one decimal place) of his gross pay do the deductions make up?

10 Holly earns a gross pay of \$2300 per fortnight. Her deductions are PAYG tax, \$64.35 for superannuation and \$30 for life insurance. Find Holly's:

a PAYG tax

b net pay

c total deductions as a percentage of her gross income (correct to one decimal place).

11 Stefan earns \$1148 per week.

a If he is paid fortnightly, what is his fortnightly gross pay?

b Find the PAYG tax that is taken out of his gross pay.

c Stefan's deductions are \$141.94 for his health fund and \$51.33 for superannuation. Calculate Stefan's net pay.

12 Anne earns a salary of \$59 944 p.a. Each fortnight she has deductions of \$256.20 for family health insurance and \$35 for superannuation taken from her gross income.

a Calculate Anne's fortnightly gross income.

b How much PAYG tax does she pay per fortnight?

c Calculate Anne's fortnightly net income.

13 Copy and complete this pay slip.

<table>
<tr><th colspan="3">Employee: Ziad Chaker</th><th>Hourly pay rate: $19.45</th></tr>
<tr><th colspan="2">Hours worked</th><th colspan="2">Deductions</th></tr>
<tr><td>Normal</td><td>39</td><td>Tax: $200.72</td><td>Other: $168.38</td></tr>
<tr><td>Time-and-a-half</td><td>2</td><td>Gross weekly income</td><td></td></tr>
<tr><td>Double time</td><td>0</td><td>Total deductions</td><td></td></tr>
<tr><td></td><td></td><td>Net weekly income</td><td></td></tr>
</table>

Technology Online income tax calculators

The Australian Taxation Office (ATO) website **www.ato.gov.au** has online calculators for income tax and PAYG tax. Visit the website and search 'Simple Tax Calculator' to find the income tax calculator for individuals.

1. Enter the taxable income $63 000 as '63000' (no spaces).
2. Select the current financial year.
3. Select 'Resident for full year' and click 'Next'.
4. The estimated tax payable will be shown on a new screen.
5. Repeat for at least two more taxable incomes.
6. Find the PAYG tax calculator and use it to find the PAYG tax payable and net pay for each of the following gross pays.
   a $1408 weekly **b** $2870 fortnightly **c** $5610 monthly

1-03 Simple interest

Worksheet
Simple interest
MAT10NAWK10003

Worksheet
Simple interest table
MAT10NAWK00023

Puzzle sheet
Simple interest
MAT10NAPS00027

- When you invest money, you **receive interest** from your investment.
- When you borrow money, you **pay interest** on your loan.
- The original amount of money invested or borrowed is called the **principal**.
- This interest rate is a percentage of the principal, usually written as a rate **per annum** ('per year'), abbreviated 'p.a.'
- **Simple interest** (or **flat rate interest**) is interest calculated simply on the original principal.

Simple interest is calculated using the following formula.

Summary

$I = PRN$, where:

I is the simple interest, P is the principal,
R is the interest rate per time period, expressed as a fraction or decimal, and
N is the number of time periods

Example 8

Find the simple interest on:

a $4000 at 3.5% p.a. for 6 years **b** $13 500 at 5.5% p.a. for 7 months
c $75 640 at 18.3% p.a. for 210 days.

Solution

a $P = \$4000$, $R = 3.5\% = 0.035$, $N = 6$ years

$$\begin{aligned} I &= PRN \\ &= \$4000 \times 0.035 \times 6 \\ &= \$840 \end{aligned}$$

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b $P = \$13\ 500,\ R = 5.5\% = 0.055,\ N = \frac{7}{12}$ years

$$\begin{aligned} I &= PRN \\ &= \$13\,500 \times 0.055 \times \frac{7}{12} \\ &= \$433.125 \\ &\approx \$433.13 \end{aligned}$$

Rounded up to the nearest cent.

c $P = \$75\ 640,\ R = 18.3\% = 0.183,\ N = \frac{210}{365}$ years

$$\begin{aligned} I &= PRN \\ &= \$75\,640 \times 0.183 \times \frac{210}{365} \\ &= \$7963.9594\ldots \\ &\approx \$7963.96 \end{aligned}$$

Rounded up to the nearest cent.

Example 9

Petra invests \$17 400 for 2 years at 3.75% p.a. flat rate interest. To what final value will her investment grow?

Solution

$P = \$17\ 400,\ R = 3.75\% = 0.0375,\ N = 2$ years

$$\begin{aligned} I &= PRN \\ &= \$17\,400 \times 0.0375 \times 2 \\ &= \$1305 \end{aligned}$$

$$\begin{aligned} \text{Value of investment} &= \$17\,400 + \$1305 \\ &= \$18\,705 \end{aligned}$$

Principal + interest

Example 10

After 4 years, an investment of \$13 000 has earned \$2808 in simple interest. What is the annual interest rate?

Solution

$I = \$2808,\ P = \$13\ 000,\ N = 4$ years

$$\begin{aligned} I &= PRN \\ \$2808 &= \$13\,000 \times R \times 4 \\ \$2808 &= \$52\,000R \\ R &= \frac{\$2808}{\$52000} \\ &= 0.054 \\ &= 5.4\% \end{aligned}$$

$\therefore$ Annual interest rate = 5.4%

Example 11

For how many months will \$10 000 need to be invested to earn \$250 in simple interest at 3.25% p.a.?

Solution

$I = \$250$, $P = \$10\,000$, $R = 3.25\% = 0.0325$

$$\begin{aligned} I &= PRN \\ \$250 &= \$10\,000 \times 0.0325 \times N \\ \$250 &= \$325N \\ N &= \frac{\$250}{\$325} \\ &= 0.7692\ldots \text{ years} \\ &= 0.7692\ldots \times 12 \text{ months} \\ &= 9.230\ldots \text{ months} \\ &\approx 10 \text{ months} \end{aligned}$$

Rounded up to the nearest month.

Exercise 1-03 Simple interest

In this exercise, give all money answers correct to the nearest cent.

See Example 8

1 Calculate the simple interest earned on each investment.
- **a** \$35 000 for 4 years at 3.6% p.a.
- **b** \$26 850 at 5.5% p.a. for 2 years
- **c** \$8200 invested for 5 months at 3% p.a.
- **d** \$6590 invested for 17 weeks at 4.5% p.a.
- **e** \$5250 invested for 250 days at 2% p.a.
- **f** \$18 400 invested for 153 days at 5.85% p.a.

2 Calculate the flat rate interest charged on each loan.
- **a** \$1250 for 2 years at 3.5% p.a.
- **b** \$18 900 for $5\frac{1}{2}$ years at 5.7% p.a.
- **c** \$1.15 million at 8.5% p.a. for 4 years.
- **d** \$12 000 for 10 months at 8.2% p.a.
- **e** \$9750 for 16 weeks at 9.3% p.a.
- **f** \$24 720 for 136 days at 7.85% p.a.

3 Harry owed \$783.26 on his credit card. The credit card company charged him one month's simple interest at 18% p.a. How much interest was he charged? Select the correct answer **A**, **B**, **C** or **D**.

A \$11.75 **B** \$14.10 **C** \$27.39 **D** \$43.51

See Example 9

4 Find the final value of each investment using simple interest.
- **a** \$10 000 invested for 3 years at 4% p.a.
- **b** \$1500 invested for 2 years at 2.3% p.a.
- **c** \$8500 invested for 3.5 years at 3% p.a.
- **d** \$9250 invested for 6 years at 3.75% p.a.

5 Liong borrowed \$6000 to go on an overseas holiday, at 12% p.a. flat rate interest for 2 years. Calculate:

a the total interest **b** the total amount Liong must repay.

6 The interest on a loan of \$2500 over 4 years is \$450. Calculate the flat rate of interest p.a. ***See* Example 10**

7 Katy took out a loan for \$12 000 over 3 years. If her total loan repayments amounted to \$18 750, calculate:

a the interest charged **b** the flat rate of interest (p.a.)

8 After 5 years, the interest on a loan of \$8000 amounts to \$3900. Calculate the annual simple interest rate.

9 For how many years will \$4200 need to be invested to earn \$200 interest, if the interest rate is 2.5% p.a.? ***See* Example 11**

10 How many weeks will it take for \$50 000 to earn \$1500 in interest if the rate is 6% p.a.?

11 How many days will it take for \$20 000 to earn \$300 in interest if the rate is 4% p.a.?

12 To save for a holiday, Dariya invested \$3480 for 6 months at a simple interest rate of 5.4% p.a. How much will this investment be worth by the end of the period? Select the correct answer **A**, **B**, **C** or **D**.

A \$32.40 **B** \$93.96 **C** \$3573.96 **D** \$3866.67

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13 For how long will \$20 000 need to be invested to amount to \$22 000, if interest is paid at the rate of 4% p.a.? (Give your answer in years.)

14 What is the flat rate of interest (as a percentage, correct to one decimal place) when \$1650 earns \$85 in interest over 2 years?

15 Toula used a credit card to buy a netbook computer for \$799 and some extra accessories for \$246. She pays off this debt in 30 days. The credit card charges 22% p.a. simple interest daily.

a Calculate the simple interest charged.

b How much will Toula pay after 30 days?

1-04 Compound interest

Most investments earn **compound interest** rather than simple interest. With compound interest, the interest earned after one time period is **added** to the principal so that next time, the interest is calculated on a larger principal. This means that more interest can be earned, because we are also earning interest on the interest we have already earned as well as on the original principal. The word **compound** means 'combined'.

Example 12

A principal of \$23 000 is invested at 4% p.a. interest, compounded yearly for 2 years.

a What is the total value of the investment after 2 years?

b What is the amount of compound interest earned?

Solution

a The interest for each year is calculated separately.

After the first year:

$I = \$23\,000 \times 0.04$

$= \$920$

Investment $= \$23\,000 + \$920$ Principal + interest

$= \$23\,920$

After the second year:

$I = \$23\,920 \times 0.04$

$= \$956.80$

Investment $= \$23\,920 + \$956.80$ New principal + interest

$= \$24\,876.80$

b Compound interest earned = final investment − principal

$= \$24\,876.80 - \$23\,000$

$= \$1876.80$

Compound interest involves repeated percentage increase

In Example **12**, to calculate compound interest on a principal of \$23 000 over two years at 4% p.a., we are actually increasing \$23 000 by 4% twice. Adding 4% to the principal is the same as increasing the principal by 4%, which is the same as multiplying the principal by 104% or 1.04.

Investment after first year = \$23 000 × 1.04 = \$23 920

Investment after second year = \$23 920 × 1.04 = \$24 876.80

We can even combine these two steps into one step by repeated percentage increases on the original principal of \$23 000:

Investment after second year = \$23 000 × 1.04 × 1.04 = \$24 876.80

Using repeated percentage increases can simplify our compound interest calculations.

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Example 13

A principal of \$9000 is invested at 3.7% p.a. compounded yearly over 3 years. What is:

a the value of the investment after 3 years?

b the compound interest earned?

Solution

a Adding 3.7% interest to the principal is the same as multiplying the principal by 1.037.

$$\begin{aligned}\therefore \text{Investment after 3 years} &= \$9000 \times 1.037 \times 1.037 \times 1.037\\ &= \$9000 \times (1.037)^3\\ &= \$10\ 036.4188\ldots\\ &\approx \$10\ 036.42 \qquad \text{Rounded to the nearest cent.}\end{aligned}$$

b

$$\begin{aligned}\text{Compound interest earned} &= \text{final investment} - \text{original principal}\\ &= \$10\ 036.42 - \$9000\\ &= \$1036.42\end{aligned}$$

Exercise 1-04 Compound interest

In this exercise, give all money answers correct to the nearest cent.

1 A principal of \$23 000 is invested at 5% p.a. interest, compounded yearly over 2 years. **See Example 12**

a Copy and complete the following working to calculate the value of the investment after 2 years.

After the first year:

$I = \$23\ 000 \times 0.05 = \$\_\_\_\_\_$ Investment $= \$23\ 000 + \$\_\_\_\_\_$

$= \$\_\_\_\_\_\_$

After the second year:

$I = \$\_\_\_\_\_ \times 0.05 = \$\_\_\_\_\_\_$ Investment $= \$\_\_\_\_\_ + \$\_\_\_\_\_$

$= \$\_\_\_\_\_\_$

b Copy and complete the following working to calculate the amount of compound interest earned.

$$\begin{aligned}\text{Compound interest earned} &= \text{final investment} - \text{principal}\\ &= \$\_\_\_\_\_\_\_\_ - \$23\ 000\\ &= \$\_\_\_\_\_\_\_\_\end{aligned}$$

2 Finn invests \$15 000 at 2.5% p.a. compounded yearly over 3 years. Show all working (as in question **1**) to find:

a the value of the investment after 3 years

b the total amount of compound interest earned.

3 Selina invests \$34 100 at 6.2% p.a. interest compounded yearly over 2 years. Calculate:

a the final value of the investment **b** the compound interest earned.

See Example 13

4 Use repeated percentage increases to calculate the final value of each investment compounded annually, then calculate the compound interest earned.

a \$5000 for 2 years at 4% p.a.
b \$27 800 for 3 years at 2.85% p.a.
c \$9600 for 3 years at 5% p.a.
d \$39 500 for 2 years at 3% p.a.
e \$18 400 for 4 years at 1.25% p.a.

5 For each investment, calculate the compound interest earned.

a \$30 400 at 5% p.a. interest for 3 years.
b \$19 150 at 4.2% p.a. interest for 2 years.
c \$8750 at 1.75% p.a. interest for 2 years.
d \$36 000 at 3.5% p.a. interest for 3 years.
e \$18 960 at 6.35% p.a. interest for 5 years.

Mental skills 1 Maths without calculators

Percentage of a quantity

Learn these commonly-used percentages and their fraction equivalents.

Percentage	50%	25%	12.5%	75%	20%	10%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{3}$

Now we will use them to find a percentage of a quantity.

1 Study each example.

a $$\begin{aligned}20\% \times 25 &= \frac{1}{5} \times 25\\ &= 5\end{aligned}$$

b $$\begin{aligned}50\% \times 120 &= \frac{1}{2} \times 120\\ &= 60\end{aligned}$$

c $$\begin{aligned}12.5\% \times 32 &= \frac{1}{8} \times 32\\ &= 4\end{aligned}$$

d $$\begin{aligned}75\% \times 56 &= \frac{3}{4} \times 60\\ &= (\frac{1}{4} \times 60) \times 3\\ &= 15 \times 3\\ &= 45\end{aligned}$$

e $$\begin{aligned}33\frac{1}{3}\% \times 27 &= \frac{1}{3} \times 27\\ &= 9\end{aligned}$$

f $$\begin{aligned}66\frac{2}{3}\% \times 60 &= \frac{2}{3} \times 60\\ &= (\frac{1}{3} \times 60) \times 2\\ &= 20 \times 2\\ &= 40\end{aligned}$$

2 Now simplify each expression.

a 25% × 44
b $33\frac{1}{3}\% \times 120$
c 20% × 35
d $66\frac{2}{3}\% \times 36$
e 10% × 230
f $12\frac{1}{2}\% \times 48$
g 50% × 86
h 20% × 400
i 75% × 24
j $33\frac{1}{3}\% \times 45$
k 25% × 160
l 10% × 650
m 12.5% × 88
n $66\frac{2}{3}\% \times 21$
o 20% × 60
p 75% × 180

1-05 The compound interest formula

There is a formula for calculating the final amount of an investment earning compound interest. Note the following pattern.

- Final amount of \$23 000 at 4% p.a. interest for 2 years $= \$23\,000 \times (1.04)^2$
- Final amount of \$9000 at 3.7% p.a. interest for 3 years $= \$9000 \times (1.037)^3$
- Final amount of \$18 960 at 6.35% p.a. interest for 5 years $= \$18\,960 \times (1.0635)^5$

Summary

Compound interest

$A = P(1 + R)^n$, where:

A is the total (final) amount of the investment
P is the principal
R is the interest rate per compounding period, expressed as a decimal
n is the number of compounding periods

The compound interest is then calculated using this formula:

$$\text{Compound interest} = \text{total amount} - \text{principal}$$
$$I = A - P$$

Example 14

For each of the following investments, calculate:

i the total amount of the investment
ii the compound interest earned.

a \$26 750 is invested at 4% p.a. for 3 years with interest compounded annually.
b \$52 000 is invested at 3.8% p.a. for 5 years with interest compounded annually.

Solution

a **i** $P = \$26\,750,\ R = 4\% = 0.04,\ n = 3$

$$\begin{aligned} A &= P(1 + R)^n \\ &= \$26\,750(1 + 0.04)^3 \\ &= \$26\,750(1.04)^3 \\ &= \$30\,090.112\ldots \\ &\approx \$30\,090.11 \end{aligned}$$

The total amount of the investment is \$30.090.11.

ii $\text{Compound interest} = \$30\,090.11 - \$26\,750 = \$3340.11$ $\quad I = A - P$

Stage 5.2

Worksheet
Compound interest
MAT10NAWK10004

Technology worksheet
Excel worksheet: Comparing interest rates
MAT10NACT00014

Technology worksheet
Excel spreadsheet: Interesting facts
MAT10NACT00044

Technology worksheet
Excel worksheet: Simple and compound interest calculator
MAT10NACT00015

Technology worksheet
Excel spreadsheet: Simple and compound interest
MAT10NACT00045

Puzzle sheet
Compound interest with annual rests
MAT10NAPS00028

Puzzle sheet
Compound interest with non-annual rests
MAT10NAPS00029

Stage 5.2

b **i** $P = \$52\ 000,\ R = 3.8\% = 0.038,\ n = 5$

$$\begin{aligned} A &= P(1+R)^n \\ &= \$52\,000(1+0.038)^5 \\ &= \$52\,000(1.038)^5 \\ &= \$62\,659.9597\ldots \\ &\approx \$62\,659.96 \end{aligned}$$

ii Compound interest $= \$62\,659.96 - \$52\,000$

$$= \$10\,659.96$$

Video tutorial
Compound interest
MAT10NAVT10021

Example 15

Calculate the compound interest when \$24 500 is invested at 6.3% p.a. for 5 years

a compounded annually

b compounded monthly.

Solution

a $P = \$24\ 500,\ R = 0.063,\ n = 5$

$$\begin{aligned} A &= \$24\,500\,(1+0.063)^5 \\ &= \$24\,500\,(1.063)^5 \\ &= \$33\,253.1205\ldots \\ &\approx \$33\,253.12 \\ I &= \$33\,253.12 - \$24\,500 \\ &= \$8753.12 \end{aligned}$$

b Because interest is compounded monthly, R and n must be expressed in months, not years.

$$P = \$24500,\ R = \frac{0.063}{12} = 0.00525 \text{ per month},\ n = 5 \times 12 = 60 \text{ months}$$

$$\begin{aligned} A &= \$24\,500(1+0.005\,25)^{60} \\ &= \$24\,500(1.005\,25)^5 \\ &= \$33\,543.701\,98\ldots \\ &\approx \$33\,543.70 \\ I &= \$33\,543.70 - \$24\,500 \\ &= \$9043.70 \end{aligned}$$

Note: More interest is earned when it is compounded monthly rather than yearly. Why do you think this is so?

9780170194655

Exercise 1-05 The compound interest formula

Stage 5.2

In this exercise, give all money answers correct to the nearest cent.

See Example 14

1 An amount of $13 000 is invested at 5% p.a. interest, compounded over 2 years. What is the total value of the investment?

2 For each of the following investments, where interest is compounded yearly, calculate:

i the total amount of the investment **ii** the compound interest earned.

a $6500 invested at 7% p.a. for 6 years
b $10 000 invested at 8.5% p.a. for 4 years
c $12 240 invested at 1.6% p.a. for 2 years
d $34 600 invested at 4.9% p.a. for 5 years
e $8000 invested at 1.75% p.a. for 3 years

3 Calculate the amount of interest earned on an investment of $6500 if it is invested at 2.5% p.a. compounded annually for 8 years. Select the correct answer **A**, **B**, **C** or **D**.

A $131.14 **B** $832.81 **C** $1300 **D** $1419.62

4 Find the amount of interest earned on one million dollars invested at 14.9% p.a. for 6 years.

5 Find the amount of interest charged on a loan of $25 000 if it is borrowed over 10 years at 8% p.a. compounded annually. Select the correct answer **A**, **B**, **C** or **D**.

A $31 250 **B** $28 973.12 **C** $28 589.72 **D** $20 000

See Example 15

6 For each investment, calculate:

i the total amount **ii** the interest earned.

a $10 000 for 5 years at 6% p.a., compounded monthly
b $35 500 for 10 years at 4% p.a., compounded twice a year
c $8900 for 2 years at 3% p.a., compounded quarterly
d $42 000 for 5 years at 4.8% p.a., compounded monthly
e $16 500 for 3 years at 5.6% p.a., compounded every 6 months
f $4900 for 1 year at 5.5% p.a., compounded daily

Quarterly means '4 times per year' or 'every three months'.

7 Find the total value of an investment of $4300 over 5 years at 4.6% p.a. interest, compounded every 6 months. Select the correct answer **A**, **B**, **C** or **D**.

A $4817.78 **B** $5384.27 **C** $5397.90 **D** $8506.24

8 **a** Reese invested $6000 for 2 years at a flat rate of 5% p.a. Calculate the amount of interest earned.

b Tegan invested $6000 for 2 years at an interest rate of 5% p.a. compounded annually. Calculate the amount of interest earned.

c Whose investment earned more interest? How much more?

9 Lisa is setting up a trust account for her new grandson Stefan. In 18 years' time, she wants the investment to be worth $30 000, to help with the cost of university fees or the purchase of a car. Suppose the interest rate for the account is 6% p.a. compounding yearly.

a How much should Lisa invest now to achieve the $30 000 target?

b If Lisa opened a trust account that earns 7% p.a. compounding monthly instead, how much less would she need to invest?

Stage 5.2

10 Zoe is 5 years old and about to start school. Her parents want to invest $15 000, for her high school education expenses, in an account that earns 6% p.a. over 7 years.

a Calculate the total interest earned if interest is compounded:

i yearly **ii** every six months **iii** quarterly **iv** monthly

b Which compounding period should Zoe's parents choose? Why?

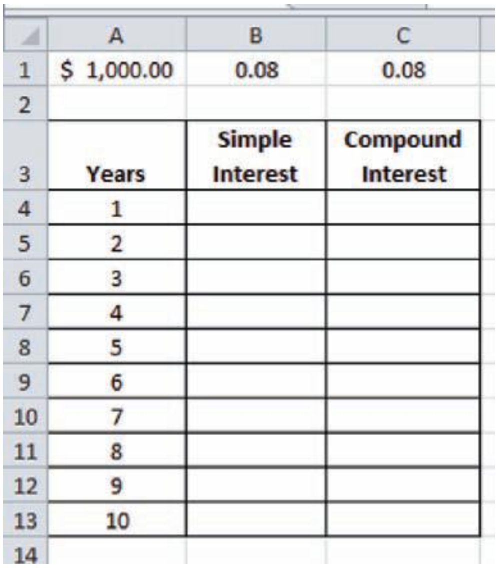
Skillsheet
Spreadsheets
MAT10NASS10003

Technology
GeoGebra: Compound and simple interest investments
MAT10NATC00009

Technology Comparing simple with compound interest

In this activity you will compare the interest earned on an investment of $1000 for 10 years at 8% p.a. simple interest and 8% p.a. compound interest, compounded annually.

1 Create this spreadsheet. The principal (P) is entered in cell A1 and the annual interest rates (in decimal form) in cells B1 and C1.



	A	B	C
1	$ 1,000.00	0.08	0.08
2			
3	Years	Simple Interest	Compound Interest
4	1		
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		
14			

2 To calculate the simple interest in column B, in cell B4 enter the formula =**A1*B1*A4**. Now **Fill Down** from cell B4 to B13.

3 To calculate the compound interest in column C, in cell C4 enter the formula =**A1*(1+C1)^A4-A1**. Now **Fill Down** from cell C4 to C13.

4 Highlight cells A3 to C13. **Insert 'Scatter with Smooth lines and markers'**.

5 When the interest rate is the same, which account pays better interest – simple or compound interest? (Type your answer in cell A15)

6 Now compare the interest earned on an investment of $1000 for 10 years at 9% p.a. simple interest and 7% p.a. compound interest, compounded annually. Change the interest rates in cells B1 (0.09) and C1 (0.07) respectively.

Answer the following questions in the spreadsheet cells indicated in brackets.

7 After how many years did the compound interest rate pay more than the simple interest rate? (A16)

8 How much extra interest did the compound interest rate pay at the end of the 10 years? (A17)

9 Change the interest rate in B1 to 10% (0.1) and C1 to 9% (0.09). How does the change in interest rate affect the amount of interest paid? Include calculations to justify your answer. (A18)

10 Change the interest rate in B1 to 12% (0.12) and C1 to 8.5% (0.085). After how many years did the amount of compound interest earned overtake the amount of simple interest earned? (A19)

11 What is the difference in the amount of compound interest earned for the 10-year period compared to the simple interest investment? Is it a significant amount? Justify your answer. (A20)

1-06 Term payments

NSW

Worksheet

Term payments: Spreadsheet

MAT10NAWK10005

Many customers buy expensive household items **on terms**, which means 'paying off' the item by regular instalments over time, after paying a deposit. A **term payments** plan is also called **hire-purchase** because the purchaser actually hires the item until it is completely paid off. Special offers can include interest-free periods, but there may be other conditions such as establishment fees and extra charges if the regular repayments are not paid on time. Also, if the purchaser fails to keep up with the payments, higher interest may be charged or the item may be **repossessed** (taken back).

Example 16

Sonia purchases a new fridge and dishwasher package valued at \$4925. She pays a 10% deposit and repays the balance in monthly instalments over 3 years. Interest on the balance is charged at a flat rate of 12% p.a.


iStockphoto/gabyjalbert

Find:

a the deposit paid
b the balance owing
c the interest charged on the balance
d the total to be repaid
e the amount of each instalment
f the total price paid for the package.

Solution

a Deposit = 10% × \$4925
= \$492.50

b Balance owing = \$4925 − \$492.50
= \$4432.50

or 90% × \$4925

c Interest charged on the balance is flat or simple interest.

$P = \$4432.50 \times R = 0.12, N = 3$

$$\begin{aligned} I &= PRN \\ &= \$4432.50 \times 0.12 \times 3 \\ &= \$1595.70 \end{aligned}$$

d $$\begin{aligned} \text{Total to be repaid} &= \text{balance} + \text{interest} \\ &= \$4432.50 + \$1595.70 \\ &= \$6028.20 \end{aligned}$$

e $$\begin{aligned} \text{Monthly instalment} &= \$6028.20 \div 36 \\ &= \$167.45 \end{aligned}$$

$$\begin{aligned} 3 \text{ years} &= 3 \times 12 \text{ months} \\ &= 36 \text{ months} \end{aligned}$$

f $$\begin{aligned} \text{Total price paid} &= \text{deposit} + \text{total repayments} \\ &= \$492.50 + \$6028.20 \\ &= \$6520.70 \end{aligned}$$

or cash price + interest
or \$4925 + \$1595.70

Deferred payment plan

With a **deferred payment plan**, the customer does not make any repayments until a later date, such as after three years.

Deferred means ‘delayed’.

Example 17

Ilhea and Robert purchase a \$1600 home theatre system on a deferred payment plan over 2 years. They make no repayments for the first 3 months, then pay \$105 per month.

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a Calculate the total repayments.

b How much interest did they pay?

c Ilhea and Robert were also charged the following fees for the plan.

- One-off establishment fee of \$25
- Account service fee of \$4.95 per month

i How much was paid in fees over the 2-year period?

ii What percentage (correct to one decimal place) of the purchase price was paid in fees?

Solution

a $$\begin{aligned} \text{Total cost} &= \$105 \times 21 \\ &= \$2205 \end{aligned}$$

2 years − 3 months = 21 months

b Interest = total cost − cash price
= \$2205 − \$1600
= \$605

c **i** Total fees paid = \$25 + \$4.95 × 24
= \$143.80

ii $\text{Percentage} = \frac{\$143.80}{\$1600} \times 100\%$
$= 8.9875\%$
$\approx 9.0\%$

Exercise 1-06 Term payments

In this exercise, give all money answers correct to the nearest cent.

1 Gaspard purchases a laundry package of washing machine and dryer valued at \$1755. He pays a 10% deposit and repays the balance in 18 months. Interest on the balance is charged at a flat rate of 13.85% p.a. Find:

See Example 16

- **a** the deposit Gaspard paid
- **b** the balance owing
- **c** the interest charged on the balance
- **d** the total to be repaid
- **e** the amount of each monthly instalment
- **f** the total price paid for the package

2 Maree bought a backyard swimming pool for \$25 500. She paid a 5% deposit and will repay the balance over 5 years, with flat-rate interest charged 8.6% p.a. Calculate:

- **a** the deposit
- **b** the balance owing
- **c** the interest charged
- **d** the total to be repaid
- **e** the monthly instalment
- **f** the total price paid.

3 Sarah wants to buy a new 3-piece lounge suite that she saw advertised at the lounge sale pictured, costing \$6895.

- **a** How much deposit will she have to pay?
- **b** What will be the flat-rate interest on the balance?
- **c** Find the size of each monthly repayment.

4 Peter pays a \$1200 deposit on his holiday and then \$185 per month for a year. The cash price of the holiday is \$2700.

- **a** How much did Peter pay for his holiday?
- **b** Calculate the interest paid.
- **c** Calculate the balance owing after the deposit was paid.
- **d** Calculate, as a percentage correct to one decimal place, the flat interest rate charged p.a.

5 Mrs Allan buys a used car for her daughter by paying a 20% deposit and $275 per month for 4 years. If the car has a cash price of $10 400, find:

- **a** the deposit Mrs Allan paid
- **b** the balance owing after the deposit is paid
- **c** the total amount Mrs Allan repaid in instalments
- **d** the interest charged
- **e** the flat interest rate (p.a.), correct to one decimal place.

6 Ming wanted to buy a new smartphone costing $329.

- **a** How much deposit did she pay?
- **b** How much did she pay altogether for the smartphone?
- **c** What was the total interest paid for the year?
- **d** What was the annual simple interest rate for this purchase? Answer correct to 2 decimal places.

See Example 17

7 Derek bought a laptop computer priced at $800 on a deferred payment plan: nothing to pay for 3 months and then 9 monthly payments of $110. However, a monthly account fee of $6.95 was added to the plan.

- **a** Find the total cost of the laptop computer to Derek.
- **b** How much in excess of the cash price was paid?
- **c** What was the flat rate of interest p.a. (correct to one decimal place) charged?

8 Tahlia bought a new outdoor setting and BBQ for her backyard that retails for $2899. She paid $300 deposit, no payments for 6 months and then fortnightly payments of $63 for 2 years.

- **a** Find the balance owing after Tahlia had paid the deposit.
- **b** Calculate the total cost of the outdoor setting and BBQ.
- **c** How much in excess of the cash price did Tahlia pay under this plan?
- **d** What is the annual flat rate of interest charged, correct to one decimal place?

9 Sophie bought a home cinema system priced at $2100 on interest-free terms for one year with no repayments for the first 4 months.

- **a** If Sophie makes 8 equal monthly repayments, what is the amount of each payment?
- **b** There is a service charge of $12.95 every month for this deferred payment plan. What percentage (correct to one decimal place) of the purchase price was paid in service charges?

1-07 Depreciation

Worksheet
Depreciation
MAT10NAWK10006

Depreciation is the decrease in value of an item over time. When items we buy lose value because of age or frequency of use, they are said to **depreciate**.
The compound interest formula can be adapted to find the depreciated value of an item. While compound interest involves repeated percentage increases, depreciation involves repeated percentage *decreases*, so its formula has a minus sign.

Summary

Depreciation formula

$A = P(1 - R)^n$, where:
A is the final value of the item
P is the original value of the item
R is the rate of depreciation per period, expressed as a decimal
n is the number of periods of depreciation
The amount of depreciation is then calculated using this formula:
Depreciation = original value − final value

Example 18

An accountant's computer and printer depreciate by 15% each year.

a If the computer and printer are currently valued at $2600, what will their value be in 5 years?

b What is the depreciation over this time?

Solution

a $P = \$2600, R = 15\% = 0.15, n = 5$

$$\begin{aligned} A &= P(1-R)^n \\ &= \$2600(1-0.15)^5 \\ &= \$2600(0.85)^5 \\ &= \$1153.633\ldots \\ &\approx \$1153.63 \end{aligned}$$

The value of the items after 5 years is $1153.63.

b $\text{Depreciation} = \$2600 - \$1153.63$ original value − final value

$$= \$1446.37$$

Stage 5.2

Example 19

An industrial oven in a restaurant originally costs \$19 800, then depreciates at a rate of 12% p.a.

a Find the value of the oven after 6 years, correct to the nearest dollar.

b Express the depreciated value as a percentage of the cost price, correct to one decimal place.

Solution

a $P = \$19\,800$, $R = 0.12$, $n = 6$

$$\begin{aligned} A &= P(1 - R)^n \\ &= \$19\,800(1 - 0.12)^6 \\ &= \$19\,800(0.88)^6 \\ &= \$9195.200\ldots \\ &\approx \$9195 \end{aligned}$$

b $$\begin{aligned}\text{Percentage of cost price} &= \frac{\$9195}{\$19800} \times 100\% \\ &= 46.4393\ldots\% \\ &\approx 46.4\%\end{aligned}$$

This means that after 6 years, the oven is worth approximately 46% of its original price (or has lost 54% of its original value).

Exercise 1-07 Depreciation

In this exercise, give all money answers correct to the nearest cent.

See Example 18

1 Find the value of a photocopier after 5 years if its purchase price was \$2850 and the annual depreciation rate is 20%.

2 **a** Find the value of a car after 7 years if it is purchased new for \$49 990 and it depreciates at 12% p.a.

b Find the amount of depreciation over this time.

3 For each item shown in the table, calculate:

i its value after 4 years of depreciation

ii its value after 4 years as a percentage of its original value, correct to one decimal place.

	Item	Original value	Depreciation rate (p.a.)
a	Stove	\$1100	12%
b	Fishing boat	\$5440	18%
c	Library	\$8460	12%
d	Computer	\$1900	20%
e	Furniture	\$27 500	15.5%
f	Bike	\$2300	22%
g	Electrical tools	\$870	17.5%
h	Air conditioner	\$1600	9%

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Stage 5.2

See Example 19

4 A DVD recorder originally valued at $225 depreciates at 10% p.a.

a What percentage (to the nearest whole number) of the original value remains after:

i 1 year? **ii** 3 years? **iii** 6 years? **iv** 7 years?

b Approximately how long would it take the DVD recorder to halve its original value?

5 A security system costs a company $12 500 to buy new. It depreciates at a yearly rate of 20%.

a Find the value of the system after:

i 1 year **ii** 2 years **iii** 5 years

b Find the value of the system after 5 years as a percentage of its original value. Answer correct to one decimal place.

6 Paul pays $15 800 for a new car. The car will depreciate in value by an average of 11% p.a.

a Find the market value of the car in 3 years.

b Calculate the amount of depreciation in the car after 3 years.

7 Asha has spent $12 400 on equipment to set up his painting business. The equipment depreciates at 15% per year.

a Find the value of the equipment after 4 years.

b Find the amount of depreciation in the equipment after 2 years.

c Find, by trial and error, how long it will take for the value to be over $3000. Answer in years and months.

d Find the value of Asha's equipment after 9 years as a percentage of its original value, correct to one decimal place.

8 A salesperson claims that, at 10% p.a. depreciation, a car will lose half its value after 7 years. Is the salesperson correct? Show all working to justify your answer.

9 Office equipment worth $12 000 when new, depreciates at 15% p.a. as shown in the table on the right.

a How much did the office equipment lose in value in the first year?

b After how many years did the office equipment fall below half its original value?

c By how much did the office equipment depreciate between the 5th and 6th years?

d Will the value of the office equipment ever fall below $100?

e Will the value of the office equipment ever be zero?

Year	Depreciated value
0	$12 000
1	$10 200
2	$8670
3	$7369.50
4	$6264.08
5	$5324.46
6	$4525.79
7	$3846.93
8	$3269.89
9	$2779.40
10	$2362.49
11	$2008.12
12	$1706.90
13	$1450.87
14	$1233.24

Power plus

1 How long, in years and days, will it take an investment of $3000 to earn $500 in simple interest at 4% p.a.?

2 What amount should I invest to earn $100 in simple interest if the investment will last for 9 months and the interest rate is 3% p.a.?

3 A principal of $10 000 is invested for 5 years at an interest rate of 5% p.a., with interest compounded weekly. Calculate the final value of the investment.

4 Ali needs $80 000 in 4 years time. What amount should she invest now at an interest rate of 6% p.a., with interest compounded annually, to reach her target?

5 A painting appreciates in value at a rate of 3% p.a. while a computer depreciates in value at a rate of 10% p.a. If I bought the painting for $1200 and the computer for $1500 new, what would be their combined value in 5 years time?

6 A bacteria colony is growing at a rate of 20% per hour. If there are 10 000 bacteria now, use the compound interest formula to calculate how many there will be after 1 day. (Give your answer correct to the nearest 10 000.)

7 **a** You invest $2000 in a bank account at an interest rate of 4% p.a. with interest compounded annually. How long will it take for your investment to double in value?

b If you invested $4000 instead of $2000 at the same interest rate, how long will it take to double in value?

c Does the size of the principal make any difference to the time taken for it to double?

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Chapter 1 review

Language of maths

Puzzle sheet

Interest and depreciation crossword

MAT10NAPS10007

- allowable deductions
- annual leave loading
- compound interest
- deposit
- depreciation
- double time
- flat rate
- fortnightly
- gross pay
- income tax
- interest
- net pay
- overtime
- PAYG tax
- per annum (p.a.)
- principal
- quarterly
- repayment
- salary
- simple interest
- taxable income
- term payments
- time-and-a-half
- wage

1 When investing, why is **compound interest** better than **simple interest**?

2 What do the P and R stand for in the formulas $I = PRN$ and $A = P(1 + R)^n$?

3 What is another name for **flat-rate interest**?

4 What word means a decrease in the value of an item over time?

5 Why is **gross pay** higher than **net pay**?

6 Use a dictionary to find at least two different meanings of **principal**.

Topic overview

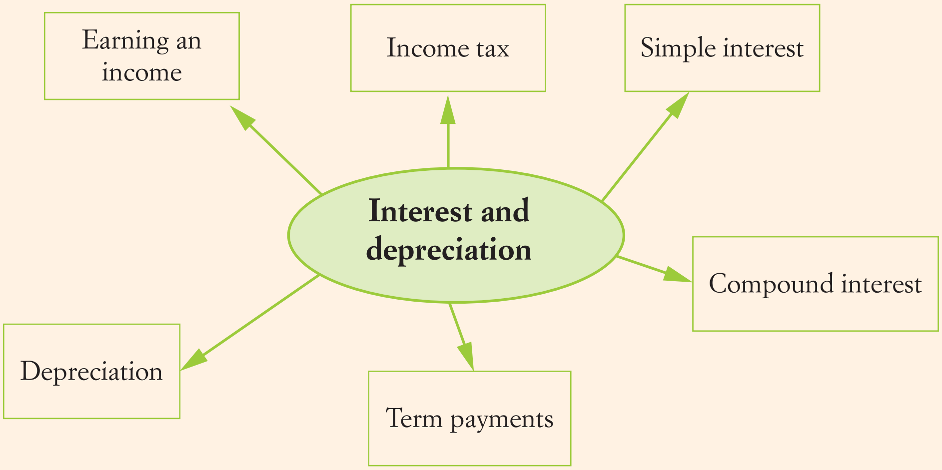
Quiz

Money and finance

MAT10NAQZ00009

- Which parts of this chapter were revision of Year 9 knowledge and skills?
- Which parts of this chapter were new to you?
- Do you know how to use the simple interest and compound interest formulas?
- How is income tax calculated?
- How is the depreciation formula similar to the compound interest formula?

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 1 revision

In this exercise, give all money answers correct to the nearest cent.

See Exercise 1-01 **1** Hayley is paid a commission of 2.5% on the value of properties she sells. She also receives a weekly retainer of $1150. How much will Hayley earn if she sells a house for $475 830?

See Exercise 1-01 **2** Thomas earns a salary of $70 400 p.a. How much is he paid each week?

See Exercise 1-01 **3** A supermarket cashier is employed under the following award.

Normal rate: $19.45 per hour	
Normal rate	For 0 to 38 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 40 hours **b** 46 hours.

See Exercise 1-01 **4** For his Christmas holidays, Jacan received 4 weeks normal pay plus 17.5% annual leave loading for the 4 weeks. If Jacan's annual salary is $54 920, find:

a his normal weekly pay

b his leave loading

c his total pay for the four-week holiday.

See Exercise 1-02 **5** Alia earns a salary of $68 650 p.a. Her allowable deductions are donations to charities of $540 and work-related expenses of $385.

a Calculate Alia's taxable income.

b Use the tax table on page 10 to calculate the income tax Alia should pay.

See Exercise 1-03 **6** Calculate the simple interest earned on each investment.

a $20 000 invested for 3 years at 4% p.a.

b $7850 invested at 2.5% p.a for 2 years

c $4500 invested for 6 months at 3.5% p.a.

d $25 200 invested for 100 days at 9% p.a.

See Exercise 1-04 **7** An amount of $5000 is invested at 6% p.a. interest, compounded over 3 years.

a What is the total value of the investment after 3 years?

b What is the amount of compound interest earned?

Stage 5.2

See Exercise 1-05 **8** Calculate the value of the investment when $34 200 is invested at 3% p.a. for 2 years, with interest compounded annually.

See Exercise 1-05 **9** Find the final value if $11 000 is invested for 4 years at 8.4% p.a., with interest compounded monthly.

See Exercise 1-05 **10** Find the interest earned when $4895 is invested at 5.5% p.a. for 3 years, with interest compounded annually.

See Exercise 1-05 **11** Calculate the interest earned when $46 230 is invested for 9 years at 7.8% p.a., with interest compounded twice a year.

12 Yang purchases a furniture package valued at $4875. She pays a 10% deposit and repays the balance in 36 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find:

See Exercise 1-06

a the deposit Yang paid
b the balance owing
c the interest charged
d the total to be repaid
e the amount of each instalment
f the total price Yang paid for the package.

Stage 5.2

See Exercise 1-07

13 Caroline bought a new car for $14 990, which depreciates by 10% p.a.

a Find the depreciated value of the car after 5 years.
b What is the depreciation over this time?
c Express the depreciated value as a percentage of the original price (correct to one decimal place).

2

Number and Algebra

Coordinate geometry

Straight lines are an important part of our environment. We play sport on courts using parallel and perpendicular lines, and skyscrapers would not be standing without straight lines. We can also use straight lines to model different types of data and predict future outcomes.

Shutterstock.com/Greg Epperson

Chapter outline

*STAGE 5.2

Wordbank

general form Any linear equation expressed as $ax + by + c = 0$, where a, b and c are integers and a is positive

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$

gradient–intercept form Any linear equation expressed as $y = mx + b$, where m is the gradient and b is the y-intercept

linear equation An equation whose graph is a straight line

parallel lines Lines that point in the same direction and have the same gradient

perpendicular lines Lines that cross at right angles (90°) and have gradients whose product is -1

***y*-intercept** The y-value at which a graph cuts the y-axis

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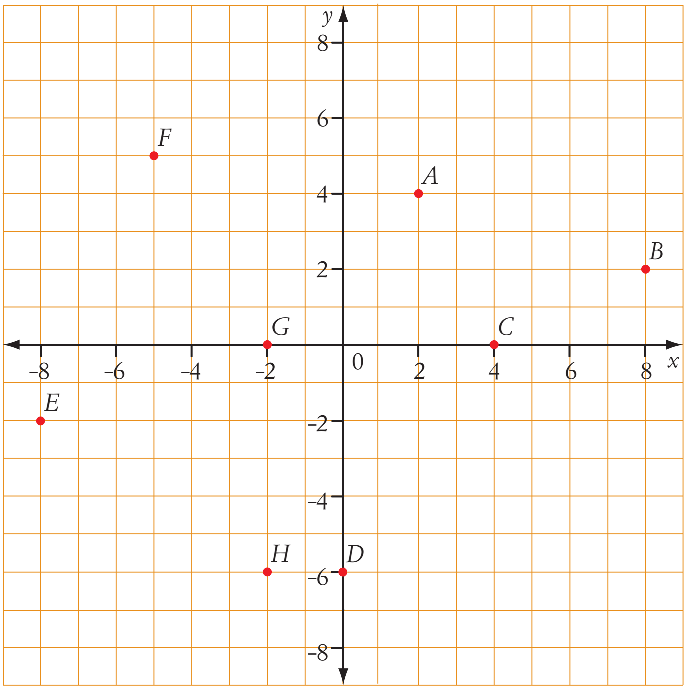
In this chapter you will:

- find the distance between two points located on the Cartesian plane using a range of strategies, including graphing software
- find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software
- sketch linear graphs using the coordinates of two points and solve linear equations
- solve problems involving parallel and perpendicular lines
- test whether a point lies on a line
- (STAGE 5.2) use the gradient-intercept equation of a straight line $y = mx + b$
- (STAGE 5.2) find the equation of a line from its graph
- (STAGE 5.2) recognise the general form of the equation of a straight line and convert it to the gradient-intercept equation
- (STAGE 5.2) find the equation of a line that is parallel or perpendicular to a given line

SkillCheck

Worksheet
StartUp assignment 2
MAT10NAWK10008

Skillsheet
Pythagoras' theorem
MAT10MGSS10004



1 For this number plane, find:
- **a** the midpoint of interval *BC*
- **b** the midpoint of interval *HE*
- **c** the length of interval *GC*
- **d** the length of interval *GH*
- **e** the lengths of *AC* and *BC*, correct to one decimal place
- **f** the type of triangle △*ABC* is
- **g** the gradient of *GE*
- **h** the gradient of *EH*

2 For each linear equation, copy and complete the table of values and graph the equation.

a $y = x - 3$

x	0	1	2	3
y				

b $y = 3x + 2$

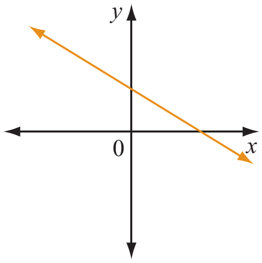
x	-2	-1	0	1
y				

c $y = 1 - 2x$

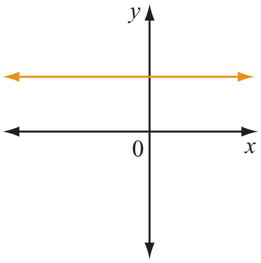
x	-1	0	1	2
y				

3 State whether each line's gradient is positive, negative or neither.

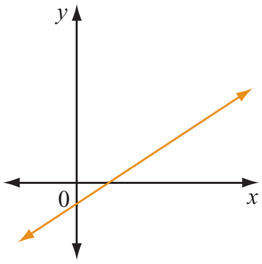
a



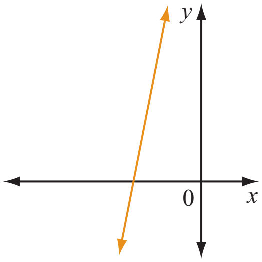
b



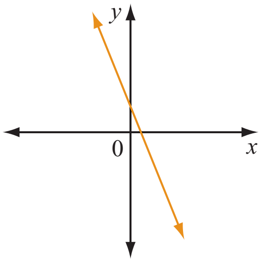
c



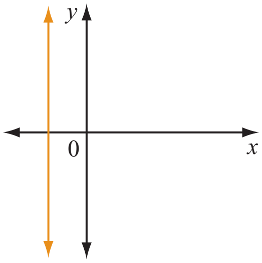
d



e



f



Worksheet
Gradient, midpoint, distance
MAT10NAWK00014

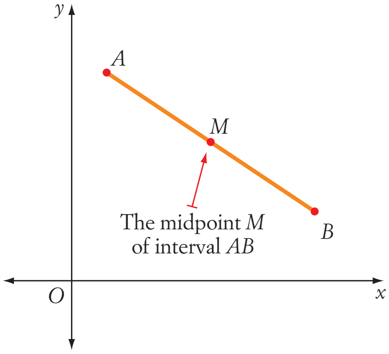
Puzzle sheet
Intervals match-up
MAT10NAPS10009

Technology worksheet
Excel worksheet: Midpoint and distance between two points
MAT10NACT00008

Technology worksheet
Excel spreadsheet: Midpoint and distance
MAT10NACT00038

2-01 Length, midpoint and gradient of an interval

The **length** of an interval AB (or the **distance** between A and B) can be calculated using Pythagoras' theorem if we know the coordinates of A and B.

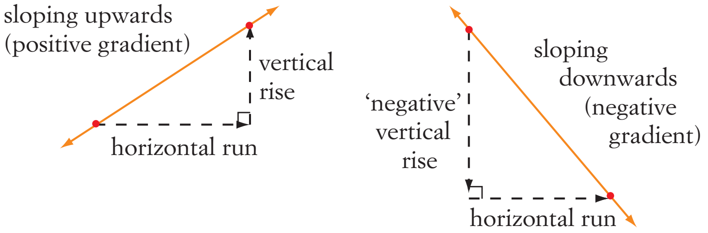


The **midpoint** of an interval *AB* is the point in the middle of *AB* or halfway between *A* and *B*.

- Its x-coordinate is the average of the x-coordinates of *A* and *B*.
- Its y-coordinate is the average of the y-coordinates of *A* and *B*.

The **gradient** of an interval measures its steepness. It is given by the formula:

$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$



- A line **sloping upwards** has a **positive rise** and a **positive gradient**.
- A line **sloping downwards** has a **negative rise** and a **negative gradient**.
- The **run** is always **positive**.

Example 1

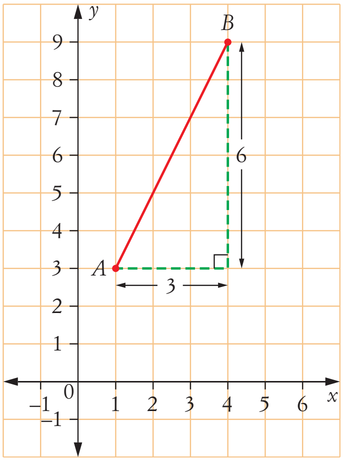
For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place.
ii the midpoint of the interval
iii the gradient of the interval

a $A(1, 3)$ and $B(4, 9)$ **b** $P(-5, 8)$ and $Q(3, 6)$

Solution

a **i** Draw a right-angled triangle on the number plane, with *AB* as the hypotenuse.



Height $= 9 - 3 = 6$ Difference between y-coordinates

Base $= 4 - 1 = 3$ Difference between x-coordinates

$$\begin{aligned} AB^2 &= 6^2 + 3^2 \\ &= 45 \\ AB &= \sqrt{45} \\ &= 6.7082\ldots \\ &\approx 6.7 \text{ units} \end{aligned}$$ by Pythagoras' theorem

ii For $A(1, 3)$ and $B(4, 9)$, the average of the x-coordinates is $\frac{1+4}{2} = 2\frac{1}{2}$.

The average of the y-coordinates is $\frac{3+9}{2} = 6$.

$\therefore$ The midpoint of AB is $(2\frac{1}{2}, 6)$.

From the diagram above, a midpoint at $(2\frac{1}{2}, 6)$ looks reasonable.

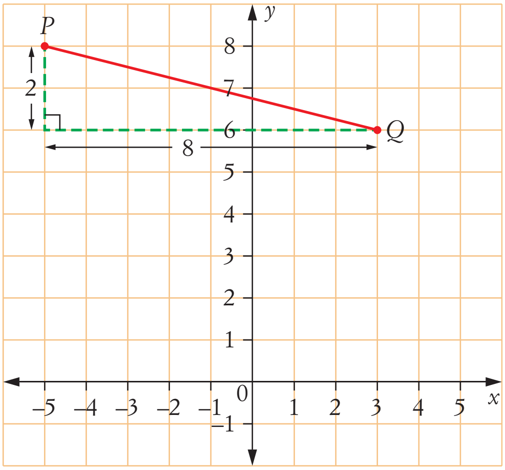
iii The rise is 6 units.

The run is 3 units.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{6}{3} \\ &= 2 \end{aligned}$$

b **i** Draw a right-angled triangle on the number plane with PQ as the hypotenuse.

The height of the triangle is 2 units.
The base of the triangle is 8 units.



$$\begin{aligned} AB^2 &= 2^2 + 8^2 \\ &= 68 \\ AB &= \sqrt{68} \\ &= 8.2462\ldots \\ &\approx 8.2 \text{ units} \end{aligned}$$ by Pythagoras' theorem

ii For $P(-5, 8)$ and $Q(3, 6)$, the average of the x-coordinates is $\frac{-5+3}{2} = -1$.

The average of the y-coordinates is $\frac{8+6}{2} = 7$.

$\therefore$ The midpoint of PQ is $(-1, 7)$.

From the diagram above, a midpoint at $(-1, 7)$ looks reasonable.

iii The rise is −2 units.

The run is 8 units.

Line slopes downwards.

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{8} = -\frac{1}{4}$$

Optional: The distance, midpoint and gradient formulas

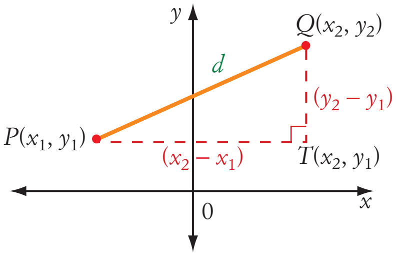
The methods for finding the length, midpoint and gradient of an interval can each be summarised by a formula.

The **distance formula** is used to calculate the distance (d) between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, in other words, the length of the interval PQ.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

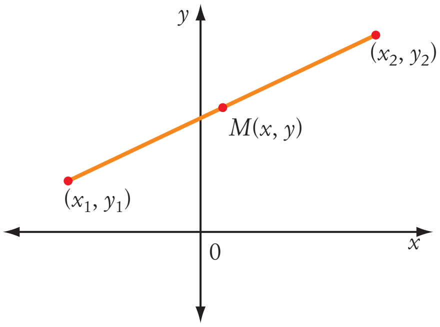
$$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

by Pythagoras' theorem



The **midpoint formula** gives the coordinates of the point M, the midpoint of the interval joining $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



The **gradient formula** gives the gradient of the interval or line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Vertical rise = difference in y-coordinates $= y_2 - y_1$

Horizontal run = difference in x-coordinates $= x_2 - x_1$

$$\text{Gradient, } m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

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Example 2

For the interval joining $P(-5, 8)$ and $Q(3, 6)$ from Example **1b**, use a formula to find:

a the length of the interval, correct to one decimal place

b the midpoint of the interval

c the gradient of the interval.

Video tutorial

Distance, midpoint and gradient formulas

MAT10NAVT10010

Solution

For $P(\underbrace{-5}_{\uparrow},\ \underbrace{8}_{\uparrow})$ and $Q(\underbrace{3}_{\uparrow},\ \underbrace{6}_{\uparrow})$: $x_1 = -5, y_1 = 8, x_2 = 3, y_2 = 6$.

(x_1, y_1) (x_2, y_2)

a
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-5))^2 + (6 - 8)^2} \\ &= \sqrt{68} \\ &= 8.2462\ \ldots \\ &\approx 8.2 \text{ units} \end{aligned}$$

Apply the distance formula.

b
$$\begin{aligned} M(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{-5 + 3}{2}, \frac{8 + 6}{2}\right) \\ &= (-1, 7) \end{aligned}$$

Apply the midpoint formula.

c
$$\begin{aligned} m &= \frac{\text{difference in } y}{\text{difference in } x} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 8}{3 - (-5)} \\ &= \frac{-2}{8} \\ &= -\frac{1}{4} \end{aligned}$$

Apply the gradient formula.

Example 3

a Plot the points $A(0, 6)$, $B(5, 6)$, $C(5, 2)$ and $D(-4, 2)$ on a number plane and join them to make the quadrilateral $ABCD$.

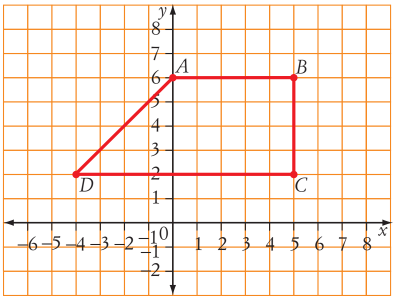
b What type of quadrilateral is $ABCD$?

c Find the exact length of AD.

d Hence find the perimeter of $ABCD$ correct to two decimal places.

Solution

a



Join the points in the correct order.

b Since $AB \parallel CD$, the quadrilateral is a trapezium.

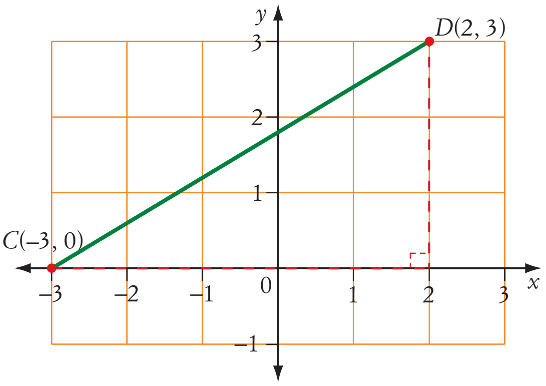
c
$$\begin{aligned} AD^2 &= 4^2 + 4^2 \\ &= 32 \\ AD &= \sqrt{32} \text{ units} \end{aligned}$$

In exact surd form.

d By counting grid squares, $AB = 5$, $BC = 4$, $CD = 9$.
$$\begin{aligned} \text{Perimeter of } ABCD &= 5 + 4 + 9 + \sqrt{32} \\ &= 25.211\ldots \\ &\approx 25.21 \text{ units} \end{aligned}$$

Exercise 2-01 Length, midpoint and gradient of an interval

Questions **1**, **2** and **3** refer to this diagram of interval *CD*.



See Examples 1, 2

1 What is the length of interval *CD*? Select the correct answer **A**, **B**, **C** or **D**.

A 2 units	**B** 5.8 units	**C** 3.2 units	**D** 8 units

2 What is the midpoint of *CD*? Select **A**, **B**, **C** or **D**.

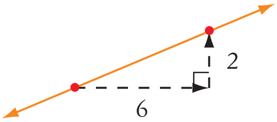
A (−1, 3)	**B** (−5, 3)	**C** (−0.5, 1.5)	**D** (−2.5, 1.5)

3 What is the gradient of *CD*? Select **A**, **B**, **C** or **D**.

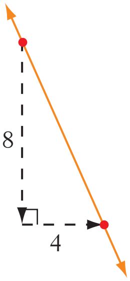
A $\frac{3}{5}$	**B** -3	**C** $-\frac{5}{3}$	**D** 2

4 Calculate the gradient of each line.

a



b



c

5 For the interval joining each pair of points given, find:

i the length of the interval correct to one decimal place

ii the midpoint of the interval

iii the gradient of the interval.

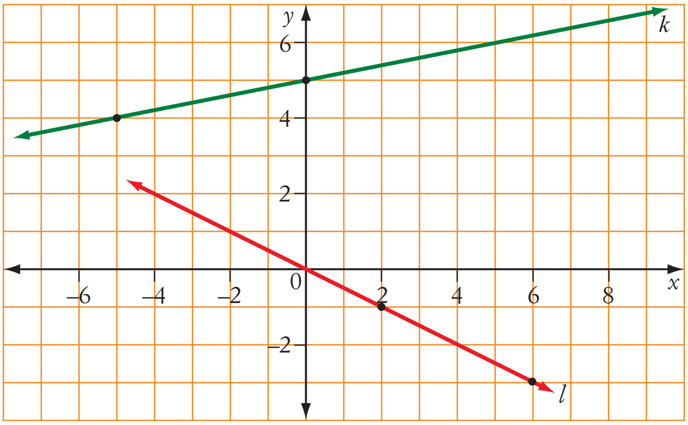
a $A(5, 3)$ and $B(7, 2)$ **b** $J(-1, 0)$ and $K(8, 6)$ **c** $M(0, -3)$ and $N(-5, 2)$

d $R(-3, -6)$ and $S(4, -9)$ **e** $A\ (-7, 2)$ and $B(-5, -8)$ **f** $U(3, -2)$ and $V(7, 2)$

6 Calculate, in exact (surd) form, the distance between each pair of points.

a $(-8, -1)$ and $(0, 4)$ **b** $(12, -6)$ and $(-1, -1)$ **c** $(7, -2)$ and $(-2, -3)$

7 Find the gradient of the lines labelled k and l.



8 Which expression gives the y-coordinate of the midpoint of the interval joining points (3, 8) and (−1, 5)? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{-1+5}{2}$ **B** $\frac{8+5}{2}$ **C** $\frac{8-5}{2}$ **D** $\frac{5-8}{2}$

9 The vertices of triangle ABC are $A(-1, -1)$, $B(1, 3)$ and $C(3, 1)$.

See Example 3

a Draw $\triangle ABC$ on a number plane.

b Find the exact length of each side of the triangle.

c Are any sides of the triangle equal in length?

d What type of triangle is ABC?

e Find the perimeter of $\triangle ABC$, correct to one decimal place.

10 The vertices of quadrilateral *KLMP* are $K(1, 6)$, $L(7, 2)$, $M(3, -4)$ and $P(-3, 0)$.

a Draw the quadrilateral on a number plane.

b What type of quadrilateral is *KLMP*?

c Find the gradients of sides *KL* and *PM*.

d Find the gradients of sides *KP* and *LM*.

e What do you notice about the gradients of opposite sides of this quadrilateral? What does that mean about those sides?

f Find the exact length of each side of *KLMP*.

g Find the perimeter of *KLMP*, correct to one decimal place.

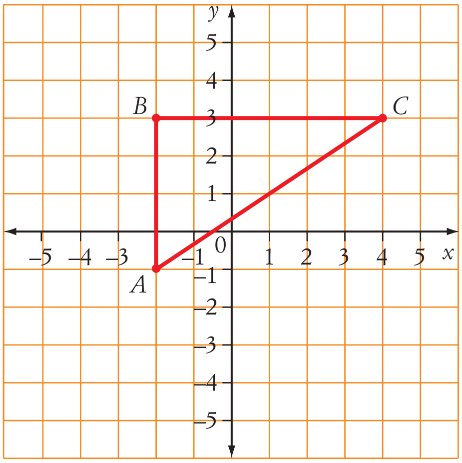
h Find the area of *KLMP*.

11 This diagram shows a right-angled triangle with vertices $A(-2, -1)$, $B(-2, 3)$ and $C(4, 3)$.

a Copy the diagram and find the coordinates of *P* and *Q*, the midpoints of *BA* and *BC* respectively. Mark *P* and *Q* on your diagram.

b Calculate, correct to one decimal place, the lengths of *PQ* and *AC*. What do you notice about your answers?

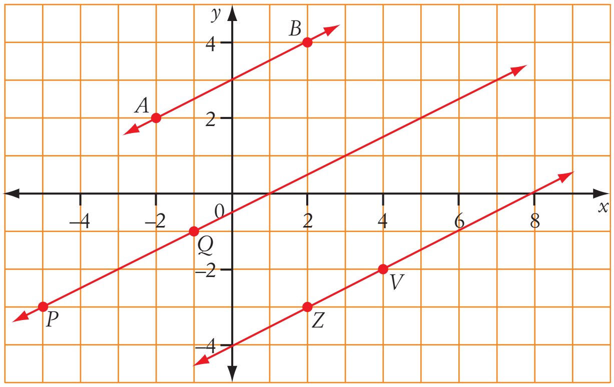
c Find the gradients of *PQ* and *AC*. What do you notice about your answers?



Investigation: Parallel and perpendicular lines

1 These three lines are parallel. Calculate the gradient of:

a *AB* **b** *PQ* **c** *ZV*



2 What can you conclude about the gradients of parallel lines?

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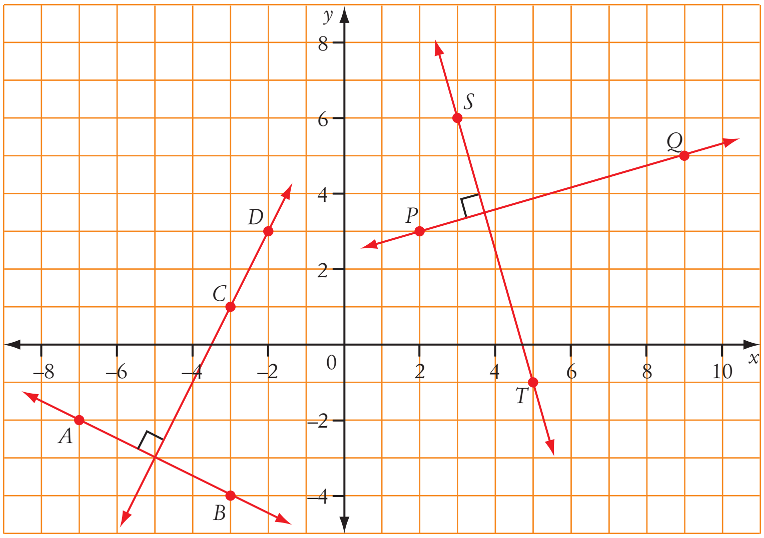
3 This diagram shows two pairs of perpendicular lines. $AB \perp CD$ and $PQ \perp ST$.



Calculate the gradient of:

a AB **b** CD **c** PQ **d** ST

4 Is there a relationship between:

a the gradients of AB and CD? **b** the gradients of PQ and ST?

5 Calculate the product of (multiply):

a the gradients of AB and CD **b** the gradients of PQ and ST

6 What can you conclude about the gradients of perpendicular lines?

2-02 Parallel and perpendicular lines

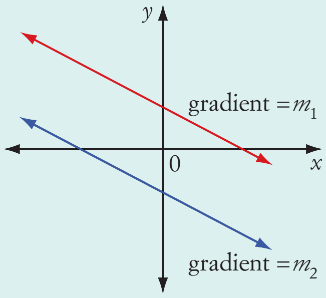
Puzzle sheet
Gradients of parallel and perpendicular lines
MAT10NAPS00012

Technology
GeoGebra: Perpendicular lines
MAT10NATC00005

Parallel lines

Summary

Parallel lines have the same gradient.
If two lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$

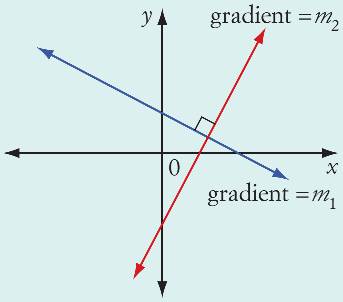


Stage 5.2

Perpendicular lines

Summary

Perpendicular lines have gradients whose product is -1.
If two lines with gradients m_1 and m_2 are perpendicular, then $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.



Note that m_2 is the **negative reciprocal** of m_1.

Example 4

State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{2}$, $m_2 = 2$ **b** $m_1 = 0.4$, $m_2 = \frac{2}{5}$ **c** $m_1 = 1\frac{3}{5}$, $m_2 = -\frac{5}{8}$

Solution

a $m_1 \neq m_2$ so the lines are not parallel.

$$\begin{aligned} m_1 \times m_2 &= \frac{1}{2} \times 2 \\ &= 1 \\ &\neq -1 \end{aligned}$$

so the lines are not perpendicular.
$\therefore$ The lines are neither parallel nor perpendicular.

b $m_2 = \frac{2}{5} = 0.4$

$m_1 = m_2$

$\therefore$ The lines are parallel.

c $m_1 = 1\frac{3}{5} = \frac{8}{5}$

$$\begin{aligned} m_1 \times m_2 &= \frac{8}{5} \times \left(-\frac{5}{8}\right) \\ &= -1 \end{aligned}$$

$\therefore$ The lines are perpendicular.

Example 5

Stage 5.2

Find the gradient of a line that is perpendicular to a line with gradient:

a 2 **b** −3 **c** $\frac{3}{4}$ **d** −0.6

Solution

a $m_1 = 2$

$$m_2 = \frac{-1}{m_1} \text{ for perpendicular lines}$$

$$= \frac{-1}{2}$$

$$= -\frac{1}{2}$$

The negative reciprocal of m_1.

The gradient is $-\frac{1}{2}$.

b $m_1 = -3$

$$m_2 = \frac{-1}{m_1}$$

$$= \frac{-1}{-3}$$

$$= \frac{1}{3}$$

The gradient is $\frac{1}{3}$.

c $m_1 = \frac{3}{4}$

$$m_2 = \frac{-1}{m_1}$$

$$= \frac{-1}{\left(\frac{3}{4}\right)}$$

$$= -\frac{4}{3}$$

The gradient is $-\frac{4}{3}$.

d $m_1 = -0.6 = -\frac{3}{5}$

$$m_2 = \frac{-1}{\left(-\frac{3}{5}\right)}$$

$$= \frac{5}{3}$$

The gradient is $\frac{5}{3}$.

Example 6

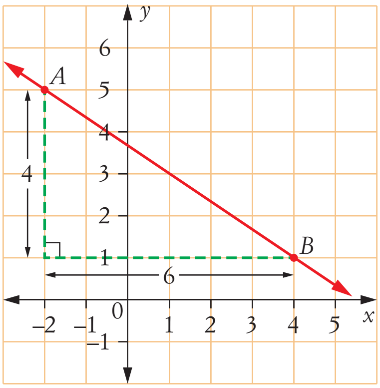
A line passes through the points $A(-2, 5)$ and $B(4, 1)$. What is the gradient of a line:

a parallel to AB? **b** perpendicular to AB?

Solution

Find the gradient of AB by calculating the rise and run.

Sometimes drawing a diagram helps you to understand the problem.



Rise $= 1 - 5 = -4$ Difference between y-coordinates.

Run $= 4 - (-2) = 6$ Difference between x-coordinates.

$$\text{Gradient } AB = \frac{-4}{6} = -\frac{2}{3}$$ $\frac{\text{rise}}{\text{run}}$

a Any line parallel to AB will have the same gradient as AB.

$\therefore m = -\frac{2}{3}$

Stage 5.2

b The gradient of a line perpendicular to AB will be given by:

$$m = \frac{-1}{\left(-\frac{2}{3}\right)} = \frac{3}{2}$$

Exercise 2-02 Parallel and perpendicular lines

See Example 4

1 State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{4}, m_2 = 4$ **b** $m_1 = 3, m_2 = -\frac{1}{3}$ **c** $m_1 = 0.5, m_2 = \frac{1}{2}$

d $m_1 = \frac{2}{7}, m_2 = \frac{7}{2}$ **e** $m_1 = \frac{3}{10}, m_2 = 0.3$ **f** $m_1 = 1\frac{1}{5}, m_2 = -\frac{6}{5}$

2 Find the gradient of a line that is parallel to a line with gradient:

a 4 **b** -2 **c** $\frac{1}{3}$ **d** -0.2

Stage 5.2

See Example 5

3 Find the gradient of a line that is perpendicular to a line with gradient:

a 1 **b** −6 **c** −1.5 **d** $\frac{5}{2}$

4 What is the gradient of a line that is perpendicular to a line with a gradient of 0.8? Select the correct answer **A**, **B**, **C** or **D**.

A 0.2 **B** −0.2 **C** 1.25 **D** −1.25

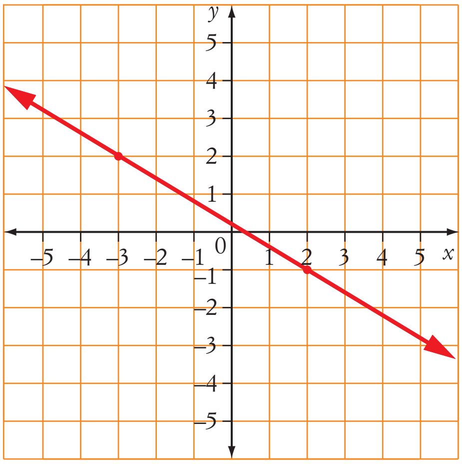
5 What is the gradient of a line that is parallel to a line that goes through $P(0, 3)$ and $Q(5, -2)$? Select **A**, **B**, **C** or **D**.

A 1 **B** −1 **C** $\frac{1}{5}$ **D** $-\frac{1}{5}$

Stage 5.2

See Example 6

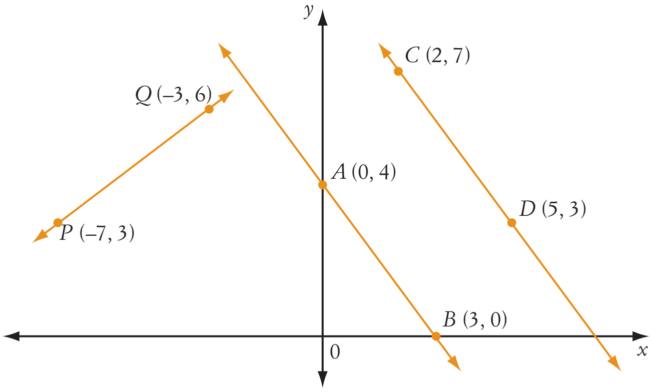
6 What is the gradient of a line perpendicular to the line shown below? Select **A**, **B**, **C** or **D**.



A $\frac{5}{3}$ **B** −5 **C** $\frac{3}{5}$ **D** $\frac{1}{5}$

7 Calculate the gradient of each line shown below and test whether:

a $AB \parallel CD$ **b** $PQ \perp CD$.



8 A line passes through the points $R(-5, 2)$ and $S(1, 4)$. What is the gradient of a line:

a parallel to RS? **b** perpendicular to RS?

Skillsheet
Starting GeoGebra
MAT10MGSS10006

Technology Parallel and perpendicular lines

This activity uses GeoGebra to find out if sets of linear equations are parallel or perpendicular.

Parallel lines

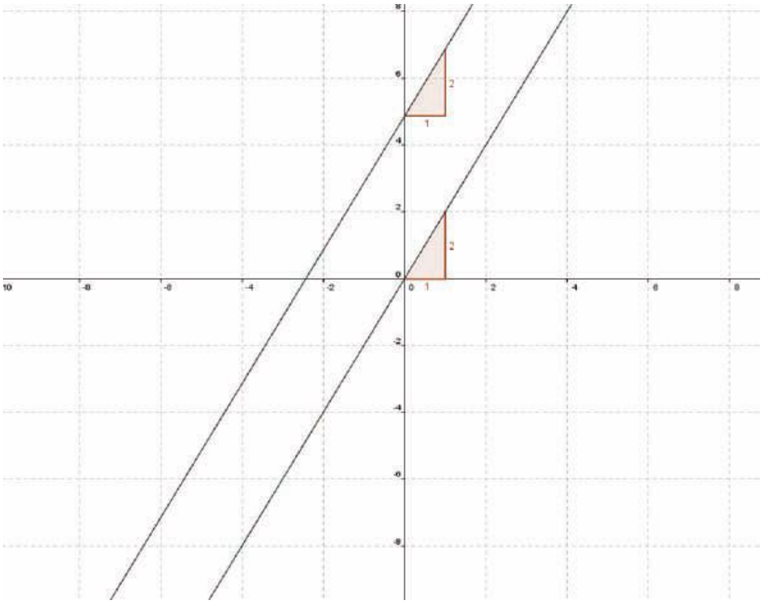
1 Show the **Axes and Grid**.

2 Use the **Input bar** to enter the pair of linear equations $y = 2x + 5$ and $y = 2x$.

3 Use **Move Graphics View** and **Zoom In** to enlarge the axes if required.

4 Find the **Slope** (gradient) of each line.

5 Check if the two lines are parallel, using $m_1 = m_2$



Since $m_1 = m_2 = 2$, this pair of lines is parallel.

6 Repeat steps **1** to **5** for the pairs of equations below. Decide if the lines are parallel or not.

a $5x - 3y = 0$ and $y = \frac{5x}{3}$ **b** $x + y + 4 = 0$ and $x + y - 6 = 0$

c $x - 2y = 0$ and $y = 0.5x$ **d** $y = 5x - 9$ and $5x - y - 1 = 0$

Perpendicular Lines

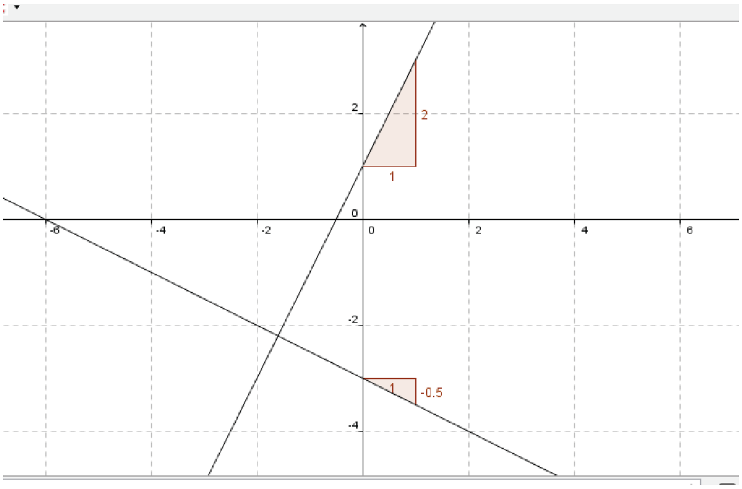
1 Show the **Axes and Grid**.

2 Use the **Input bar** to enter the pair of linear equations $y = 2x + 1$ and $y = -0.5x - 3$.

3 Use **Move Graphics View** and **Zoom In** to enlarge the axes if required.

4 Find the **Slope** (gradient) of each line.

5 Check if the two lines are perpendicular, using $m_1 \times m_2 = -1$



Since $2 \times (-0.5) = -1$, the two lines are perpendicular.

6 Repeat steps **1** to **5** for the pairs of equations below. Decide if the lines are perpendicular or not.

a $y = 0.6x + 2$ and $y = \frac{5}{3}x$

b $x - 4y + 1 = 0$ and $y = -4x - 3$

c $3x - 2y = 0$ and $y = -\frac{2x}{3}$

d $y = 2x + 4$ and $x - 2y - 1 = 0$

2-03 Graphing linear equations

Worksheet
Graphing linear equations
MAT10NAWK10010

Skillsheet
Graphing linear equations
MAT10NASS10005

A relationship between two variables, x and y, whose graph is a straight line is called a **linear relationship**. The expression of that relationship as an algebraic formula, such as $y = 3x + 2$, is called a **linear equation**.

Example 7

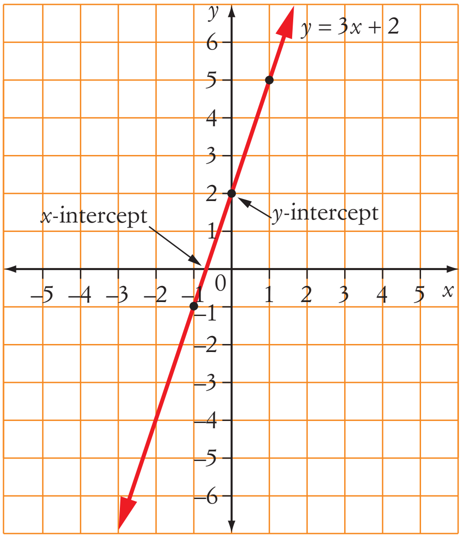
Graph $y = 3x + 2$ on a number plane.

Solution

Complete a table of values. Choose x-values close to 0 for easy calculation and graphing.

x	−1	0	1
y	−1	2	5

Graph (−1, −1), (0, 2) and (1, 5) on a number plane. Rule a straight line through the points, place arrows at each end, and label the line with its equation.



Note:

- the **x-intercept** of the line is $-\frac{2}{3}$: this is the x-value where the line cuts the x-axis
- the **y-intercept** of the line is 2: this is the y-value where the line cuts the y-axis
- every point on the line follows the linear equation $y = 3x + 2$. For example, $(-1, 1)$, $(0, 2)$ and $(1, 5)$ lie on the line and follow the rule $y = 3x + 2$
- there are an infinite number of points that follow the rule. Arrows on both ends of the line indicate that it has infinite length.

Testing if a point lies on a line

Summary

A point lies on a line if its (x, y) coordinates satisfy the equation of the line.

Example 8

Which of the following points lie on the line $x - 2y = 5$?

a $(17, 6)$

b $(8, -4)$

Solution

- Separate the equation into its left-hand side (LHS) and right-hand side (RHS)
- Substitute the coordinates of the point into both sides
- If LHS = RHS, the point satisfies the equation and so lies on the line
- If LHS ≠ RHS, the point does **not** lie on the line.

a Substitute $x = 17$, $y = 6$ into $x - 2y = 5$.

$$\begin{aligned}\text{LHS} &= x - 2y \\ &= 17 - 2 \times 6 \\ &= 5\end{aligned} \qquad \text{RHS} = 5$$

LHS = RHS, so $(17, 6)$ lies on the line.

b Substitute $x = 8$, $y = -4$ into $x - 2y = 5$.

$$\begin{aligned}\text{LHS} &= x - 2y \\ &= 8 - 2 \times (-4) \\ &= 16\end{aligned} \qquad \text{RHS} = 5$$

LHS ≠ RHS, so $(8, -4)$ does not lie on the line.

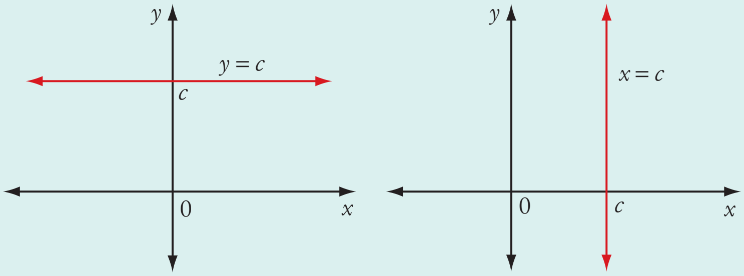
9780170194655

Horizontal and vertical lines

Technology worksheet
Horizontal and vertical lines
MAT10NACT10001

Summary

The **equation of a horizontal line** is of the form $y = c$ (where c is a constant number).
The **equation of a vertical line** is of the form $x = c$ (where c is a constant number).

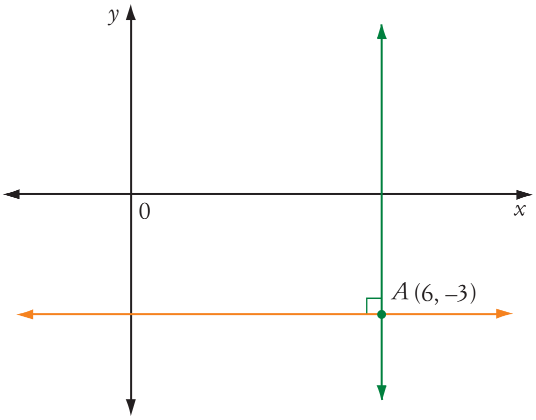


Example 9

For the graph below, find the equation of:

a the vertical line

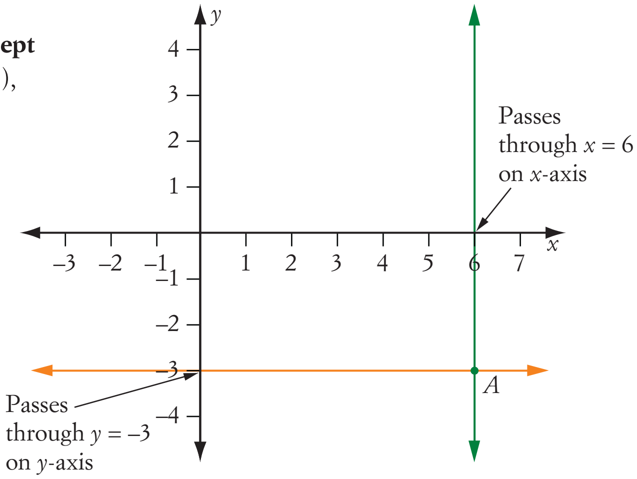
b the horizontal line



Solution

a The vertical line has an ***x*-intercept of 6** and passes though $A(\mathbf{6}, -3)$, so its equation is $x = 6$.

b The horizontal line has a ***y*-intercept of −3** and passes though $A(6, \mathbf{-3})$, so its equation is $y = -3$.



Example 10

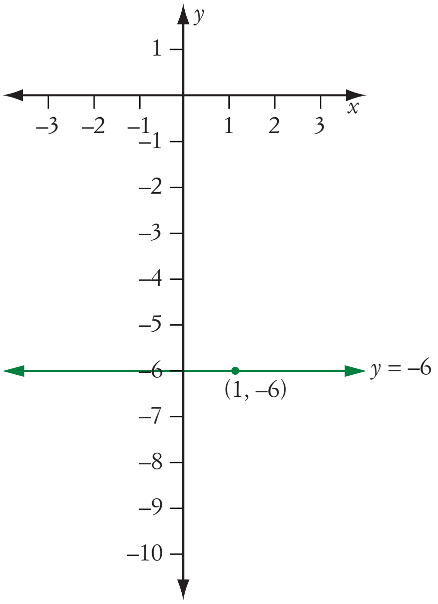
Find the equation of the line that is:

a parallel to the x-axis and passes through the point $(1, -6)$

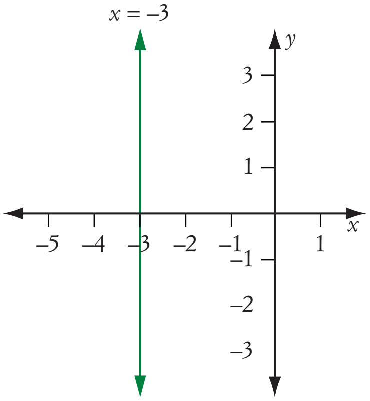
b always 3 units to the left of the y-axis.

Solution

a The equation of the line is $y = -6$.



b The equation of the line is $x = -3$.



Exercise 2-03 Graphing linear equations

1 Graph each linear equation on a number plane, and write: **See Example 7**

i its x-intercept **ii** its y-intercept.

a $y = 3x - 1$ **b** $y = 2x + 5$ **c** $y = -x + 4$

d $y = -2x - 2$ **e** $y = 4x$ **f** $y = \frac{x}{2} + 3$

2 Test whether the point (3, −1) lies on each line. **See Example 8**

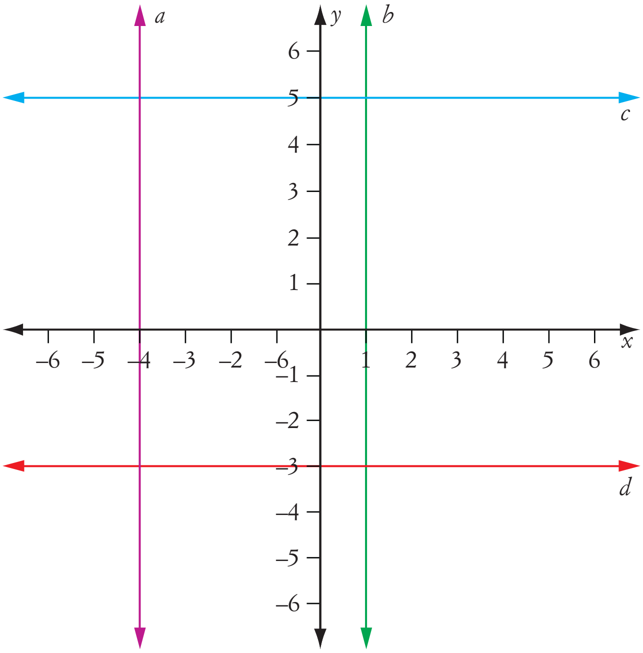
a $y = 2x - 5$ **b** $x - y = 4$ **c** $y + 2x = 5$

d $y = x - 4$ **e** $x + y = 5$ **f** $3x + y + 8 = 0$

3 Which of these points lies on the line $y = 6x - 5$? Select the correct answer **A**, **B**, **C** or **D**.

A (−1, 11) **B** (3, −13) **C** (−2, −17) **D** (−5, 25)

4 Find the equation of each line shown below. **See Example 9**



5 Graph each set of lines on a number plane.

a $x = 2\frac{1}{2}, y = -3, y = 1$ **b** $x = 6, y = -2, x = -\frac{1}{2}$

6 Find the equation the line that is: **See Example 10**

a horizontal and passes through the y-axis at 2

b vertical with an x-intercept of 4

c parallel to the y-axis and passes through the point (−1, 4)

d parallel to the x-axis and passes through the point (0, −2)

e 3 units above the x-axis

f 1 unit to the left of the y-axis

g drawn through the points (−1, 6) and (2, 6)

h drawn through the points (−1, 8) and (−1, 2).

7 Which of these points lies on the line $4x + y = 1$? Select **A**, **B**, **C** or **D**.

A $(-1, 5)$ **B** $(-2, 7)$ **C** $(6, 9)$ **D** $(-\frac{1}{2}, 1)$

8 Which equation represents a line that is horizontal and passes through the point $(8, -2)$? Select **A**, **B**, **C** or **D**.

A $y = 8$ **B** $x = 8$ **C** $y = -2$ **D** $x = -2$

9 **a** What is another name for the line $y = 0$?

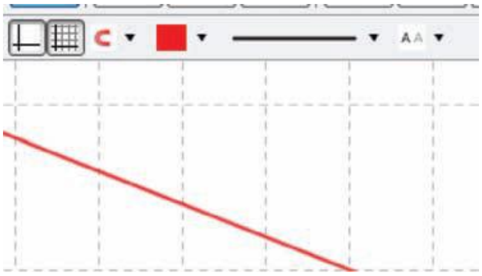
b What is another name for the line $x = 0$?

Technology Graphing $y = mx + b$

1 Show the **Axes and Grid**.

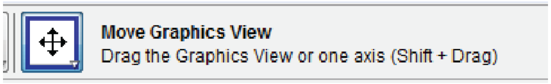
2 Enter the four lines $y = 3x + 2$, $y = 5x + 2$, $y = -2x + 2$, $y = -0.1x + 2$, using **Input** at the bottom of the screen.

3 Each straight line can be a different colour. Right-click on a line and choose a colour.



4 Find the **Slope** of each line.

5 Find the y-intercept of each line. Click on the right drop-down menu and use the mouse to zoom in on the y-intercept. Read off the value on the y-intercept (you may need to use Move Graphics View to locate it).



Move Graphics View
Drag the Graphics View or one axis (Shift + Drag)

6 Save your GeoGebra file.

7 Record your results in a table as shown.

	Equation	Gradient	y-intercept
a			
b			
c			
d			

8 What do you notice about your results?

9 Repeat the steps above for each set of equations.

a $y = -4x$
$4x + y + 1 = 0$
$y = -4x - 10$

b $y = 2x + 3$
$7x + y - 3 = 0$
$0.2x - y + 3 = 0$

c $x + y + 1 = 0$
$y = -x - 1$
$x + y = -1$

10 For each set of lines drawn in question **9**, complete a table as shown in Step **7**.

11 What do you notice about each set of lines? Identify any key features of each set of graphs, such as gradients and *y*-intercepts.

2-04 The gradient–intercept equation $y = mx + b$

Stage 5.2

NSW

Puzzle sheet

Equations in gradient form

MAT10NAPS00011

Technology worksheet

Excel spreadsheet: Drawing linear graphs: gradient and *y*-intercept

MAT10NACT00039

Video tutorial

The gradient–intercept formula

MAT10NAVT10011

Summary

The equation of a straight line is $y = mx + b$, where m is the **gradient** and b is the ***y*-intercept**.
For this reason, $y = mx + b$ is also called the **gradient–intercept form of a linear equation**.

Example 11

Find the gradient and *y*-intercept of the line with equation:

a $y = -4x + 9$ **b** $y = 10 - 6x$ **c** $y = \frac{5x + 4}{2}$ **d** $3x + 2y - 6 = 0$

Solution

a $y = -4x + 9$ is in the form $y = mx + b$.

$\therefore$ Gradient $m = -4$ and y-intercept $b = 9$.

b $y = 10 - 6x$ can be rewritten as $y = -6x + 10$.

$\therefore$ Gradient $m = -6$ and y-intercept $b = 10$.

c For $y = \frac{5x + 4}{2} = \frac{5x}{2} + \frac{4}{2} = \frac{5x}{2} + 2$, gradient $m = \frac{5}{2}$ and y-intercept $b = 2$

d $3x + 2y - 6 = 0$ can be rearranged in the form $y = mx + b$.

$$\begin{aligned} 3x + 2y - 6 &= 0 \\ 3x + 2y - 6 - 3x &= 0 - 3x \\ 2y - 6 &= -3x \\ 2y - 6 + 6 &= -3x + 6 \\ 2y &= -3x + 6 \\ y &= \frac{-3x + 6}{2} \\ y &= \frac{-3x}{2} + 3 \end{aligned}$$

$\therefore 3x + 2y - 6 = 0$ has gradient $m = -\frac{3}{2}$ and y-intercept $b = 3$.

Stage 5.2

Example 12

Write the equation of a line with a gradient of −4 and a *y*-intercept of −6.

Solution

$m = -4$, $b = -6$

$\therefore$ The equation of the line is $y = -4x - 6$.

Example 13

Graph each linear equation by finding the gradient and *y*-intercept first.

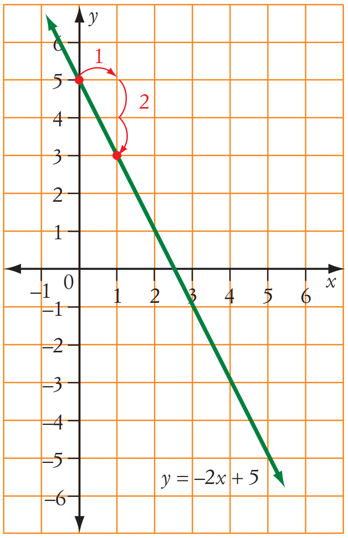
a $y = -2x + 5$ **b** $y = \frac{3}{4}x - 2$

Solution

a $y = -2x + 5$ has a gradient of −2 and a *y*-intercept of 5.

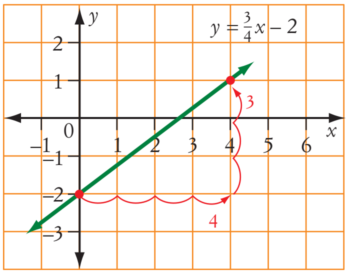
- Plot the *y*-intercept 5 on the *y*-axis.
- Make a gradient of −2 by moving **across** 1 unit (**run**) and **down** 2 units (**'negative' rise**) and marking the point at (1, 3).
- Rule a line through this point and the *y*-intercept.

Don't forget to label the line with its equation '$y = -2x + 5$'



b $y = \frac{3}{4}x - 2$ has a gradient of $\frac{3}{4}$ and a *y*-intercept of −2.

- Plot the *y*-intercept −2 on the *y*-axis.
- Make a gradient of $\frac{3}{4}$ by moving **across** 4 units (**run**) and **up** 3 units (**rise**) and marking the point at (4, 1).
- Rule a line through this point and the *y*-intercept.



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Example 14

Which lines are parallel to $y = -2x + 3$?

A $y = 2x + 3$ **B** $y = -2x + 1$ **C** $y = -2x$ **D** $y = 5x + 3$

Solution

Parallel lines have the **same gradient**. The line $y = -2x + 3$ has the gradient $m = -2$.

- **A** $y = 2x + 3$ has gradient 2.
- **B** $y = -2x + 1$ has gradient -2.
- **C** $y = -2x$ has gradient -2
- **D** $y = 5x + 3$ has gradient 5.

$\therefore$ The lines **B** ($y = -2x + 1$) and **C** ($y = -2x$) are parallel to $y = -2x + 3$.

Exercise 2-04 The gradient–intercept formula $y = mx + b$

1 Find the gradient and y-intercept of each line below. **See Example 11**

a $y = 3x - 2$ **b** $y = -2x + 7$ **c** $y = x + 4$ **d** $y = 9 - x$

e $y = \frac{3x}{4} + 6$ **f** $y = x$ **g** $y = \frac{x}{2} - 11$ **h** $y = \frac{2x + 18}{3}$

i $y = \frac{-24 - x}{3}$ **j** $y = 2(x - 3)$ **k** $11 - 3x = y$ **l** $\frac{2x - 7}{2} = y$

2 Find the equation of a line with: **See Example 12**

a a gradient of 2 and a y-intercept of 1 **b** a gradient of $\frac{3}{4}$ and a y-intercept of 2

c a gradient of -7 and a y-intercept of 5 **d** a gradient of $-\frac{2}{5}$ and a y-intercept of 3

e $m = -2, b = -3$ **f** $m = -3, b = \frac{1}{2}$

3 Graph each linear equation by finding the gradient and y-intercept first. **See Example 13**

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = 2x$ **d** $y = \frac{x}{2} - 1$

e $y = -2x + 3$ **f** $y = -\frac{3x}{4}$ **g** $y = \frac{-5x + 2}{2}$ **h** $y = \frac{3x - 20}{5}$

4 Write the equation of a line with a gradient of 2 that passes through the origin.

5 Select the lines that are parallel to the given line each time. There may be more than one answer. **See Example 14**

a $y = x + 6$

A $y = 6x$ **B** $y = 6 - x$ **C** $y = x + 1$ **D** $y = 2x$

b $y = 3x + 10$

A $y = 10x + 3$ **B** $y = 3x - 1$ **C** $y = 1 - 3x$ **D** $y = 4 + 3x$

c $y = \frac{x}{2} + 5$

A $y = 2x - 1$ **B** $y = \frac{x + 6}{2}$ **C** $y = 1 - \frac{x}{2}$ **D** $y = x + 2$

Stage 5.2

d $y = 6$

A $y = 2x + 6$ **B** $y = 6x$ **C** $y = -1$ **D** $y = 10$

e $y = 4x$

A $y = 4x - 2$ **B** $y = 4x + 3$ **C** $y = 4$ **D** $y = 1 - 4x$

f $x = 10$

A $y = 10$ **B** $y = 10x$ **C** $x = 2y$ **D** $x = -6$

6 For each set of linear equations, find a pair of equations whose graphs are parallel lines.

a $y = 4x + 3$ $y = x + 2$ $y = 4x - 6$ $y = 2x$
b $y = 5x + 1$ $3x - y + 7 = 0$ $y = 3x - 2$ $y = -5x + 2$

Mental skills 2 Maths without calculators

Finding 15%, $2\frac{1}{2}$%, 25% and $12\frac{1}{2}$%

- To find 10% or $\frac{1}{10}$ of a number, divide by 10
- To find 5% of a number, find 10% first, then halve it (since 5% is half of 10%).
- So to find 15% of a number, find 10% and 5% of the number separately, then add the answers together.

1 Study each example.

a $15\% \times 80 = (10\% \times 80) + (5\% \times 80) = 8 + 4 = 12$
b $15\% \times \$170 = (10\% \times \$170) + (5\% \times \$170) = \$17 + \$8.50 = \$25.50$
c $15\% \times 3600 = (10\% \times 3600) + (5\% \times 3600) = 360 + 180 = 540$
d $15\% \times \$28 = (10\% \times \$28) + (5\% \times \$28) = \$2.80 + \$1.40 = \$4.20$

2 Now find 15% of each amount.

a 120 **b** $840 **c** 260 **d** $202
e $50 **f** 72 **g** $180 **h** 400
i $1600 **j** $22 **k** 6000 **l** $350

To find $2\frac{1}{2}$% of a number, first find 5%, then halve it.

3 Study each example.

a $2\frac{1}{2}\% \times 600$

$10\% \times 600 = 60$

$5\% \times 600 = \frac{1}{2} \times 60 = 30$

$2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15$

b $2\frac{1}{2}\% \times \$820$

$10\% \times \$820 = \$82$

$5\% \times \$820 = \frac{1}{2} \times 82 = \$41$

$2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50$

4 Now find $2\frac{1}{2}$% of each amount

a 400 **b** 6640 **c** $2000 **d** $880
e 1500 **f** $232 **g** 5400 **h** $904

To find 25% of a number, halve the number twice as $25\% = \frac{1}{4}$.

5 Study each example.

a $25\% \times 700$

$50\% \times 700 = \frac{1}{2} \times 700 = 350$

$25\% \times 700 = \frac{1}{2} \times 350 = 175$

b $25\% \times \$86$

$50\% \times \$86 = \frac{1}{2} \times \$86 = \$43$

$\therefore 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50$

6 Now find 25% of each amount.

a 2000	**b** \$80	**c** 18	**d** \$25
e \$324	**f** \$140	**g** 66	**h** 298
i \$780	**j** \$1700	**k** \$126	**l** 1160

To find $12\frac{1}{2}\%$ of a number, find 25% first, then halve it. In other words, halve three times because $12\frac{1}{2}\% = \frac{1}{8}$.

7 Study each example.

a $12\frac{1}{2}\% \times 400$

$50\% \times 400 = \frac{1}{2} \times 400 = 200$

$25\% \times 400 = \frac{1}{2} \times 200 = 100$

$12\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$

b $12\frac{1}{2}\% \times \$144$

$50\% \times \$144 = \frac{1}{2} \times \$144 = \$72$

$25\% \times \$144 = \frac{1}{2} \times \$72 = \$36$

$12\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$

8 Now find $12\frac{1}{2}\%$ of each amount.

a 1280	**b** \$12	**c** 60	**d** \$260
e \$540	**f** \$250	**g** 304	**h** 1360

2-05 The general form of a linear equation $ax + by + c = 0$

Stage 5.2

NSW

Puzzle sheet

Linear equations code puzzle

MAT10NAPS10011

Worksheet

Parallel and perpendicular lines

MAT10NAWK00015

A linear equation written in **gradient–intercept form**, such as $y = -\frac{3}{4}x + 2$, can also be written in **general form** ($3x + 4y - 8 = 0$). Note that, for the general form, all of the terms on the left-hand side of the equation are written with no fractions, and only 0 is on the right-hand side. Sometimes the general form is neater and more convenient.

Summary

The **general form of a linear equation** is written as $ax + by + c = 0$, where a, b and c are integers and a is positive.

Stage 5.2

Example 15

Write each linear equation in general form.

a $y = 6x + 2$ **b** $y = -\frac{2}{3}x + 2$ **c** $y = 2x - \frac{3}{5}$

Solution

a

$$\begin{aligned} y &= 6x + 2 \\ 0 &= 6x - y + 2 \\ 6x - y + 2 &= 0 \end{aligned}$$

Subtracting y from both sides.

Swapping sides so that zero appears on the RHS.

b

$$\begin{aligned} y &= -\frac{2}{3}x + 2 \\ 3y &= 3\left(-\frac{2}{3}x + 2\right) \\ &= -2x + 6 \\ 2x + 3y &= 6 \\ 2x + 3y - 6 &= 0 \end{aligned}$$

Multiplying both sides by 3 to remove the fraction.

Adding $2x$ to both sides.

Subtracting 6 from both sides.

c

$$\begin{aligned} y &= 2x - \frac{3}{5} \\ 5y &= 5\left(2x - \frac{3}{5}\right) \\ 5y &= 10x - 3 \\ 0 &= 10x - 5y - 3 \\ 10x - 5y - 3 &= 0 \end{aligned}$$

Multiplying both sides by 5 to remove the fraction.

Subtracting $5y$ from both sides.

Swapping sides so that zero appears on the RHS.

Example 16

Find the gradient and y-intercept of the line whose equation is $5x + 2y - 10 = 0$.

Solution

Rewrite $5x + 2y - 10$ in the form $y = mx + b$.

$$\begin{aligned} 5x + 2y - 10 &= 0 \\ 2y - 10 &= -5x \\ 2y &= -5x + 10 \\ \frac{2y}{2} &= \frac{-5x + 10}{2} \\ y &= \frac{-5x}{2} + 5 \end{aligned}$$

Aim to have y on its own on the LHS of the equation.

Subtracting $5x$ from both sides.

Adding 10 to both sides.

Dividing both sides by 2.

$\therefore$ Gradient: $m = -\frac{5}{2}$

y-intercept: $b = 5$

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Exercise 2-05 The general form of a linear equation $ax + by + c = 0$

Stage 5.2

See Example 15

1 Write each linear equation in general form.

a $y = x + 2$	**b** $y = 3x - 1$	**c** $y = 8 + 5x$
d $x + 2y = 3$	**e** $x - 2y = 6$	**f** $y = 8x + 2$
g $y + 3 = 6x$	**h** $2y = x - 6$	**i** $y = \frac{3}{5}x + 2$

See Example 16

2 Find the gradient and y-intercept of the line with each equation.

a $2x + y = 6$	**b** $8x - 2y = 10$	**c** $3x - 2y + 4 = 0$
d $y + 2x - 1 = 0$	**e** $2x + y + 5 = 0$	**f** $4x + 3y - 12 = 0$

3 Find the gradient, m, and the y-intercept, b, of the line with equation $x - 3y + 5 = 0$. Select the correct answer **A**, **B**, **C** or **D**.

A $m = -1, b = 5$ **B** $m = \frac{1}{3}, b = \frac{5}{3}$ **C** $m = 1, b = -5$ **D** $m = \frac{1}{3}, b = -\frac{5}{3}$

4 Which statement is false about the line whose equation is $3x + y - 6 = 0$? Select **A**, **B**, **C** or **D**.

A The gradient is -3. **B** The y-intercept is -6.
C The x-intercept is 2. **D** It is parallel to the line $y = -3x$.

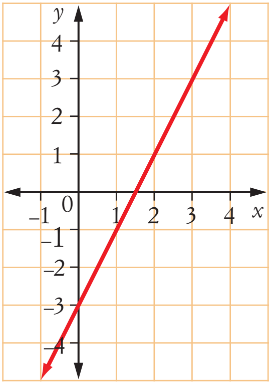
2-06 Finding the equation of a line

NSW

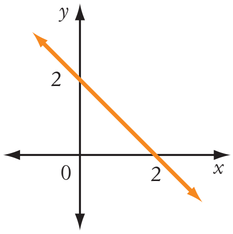
Example 17

Find the equation of each line.

a



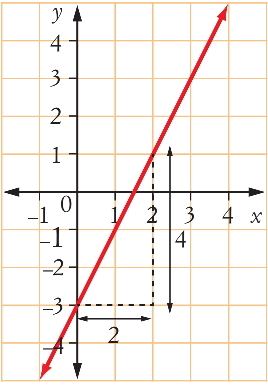
b



Solution

a Select two points on the line to find the gradient, say (0, –3) and (2, 1).

Gradient $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$



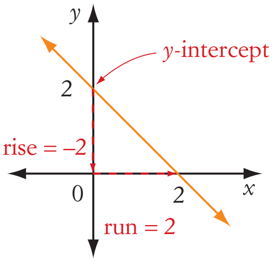
Stage 5.2

y-intercept: $b = -3$	from the graph
$\therefore$ The equation of the line is $y = 2x - 3$.	$y = mx + b$

We can check that this equation is correct for any point on the line, say (3, 3).
When $x = 3$, $y = 2 \times 3 - 3 = 3$.

b Find the gradient of the line passing through (0, 2) and (2, 0).

Gradient $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{-2}{2} = -1$

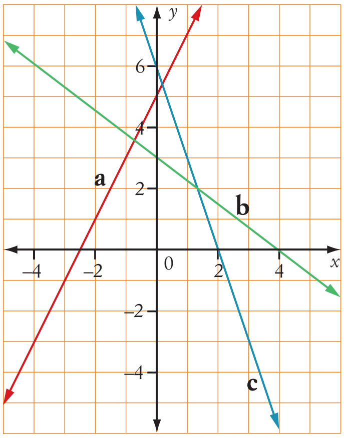


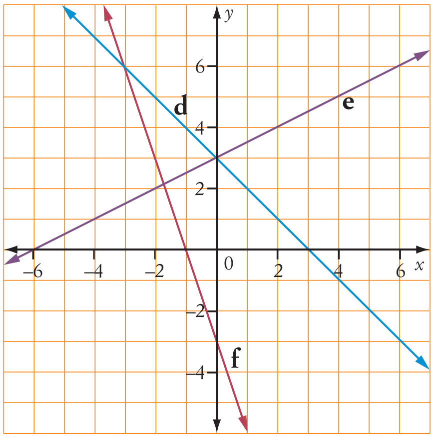
y-intercept: $b = 2$	from the graph
$\therefore$ The equation of line is $y = -x + 2$.	$y = mx + b$

Exercise 2-06 Finding the equation of a line

See Example 17

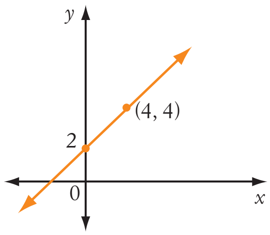
1 Find the equation of each line.



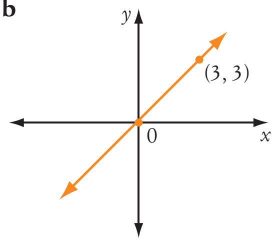


2 Find the equation of each line.

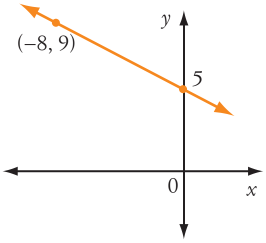
a



b



c



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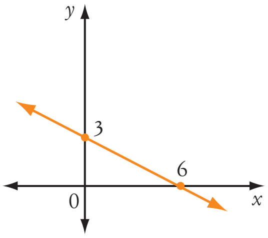
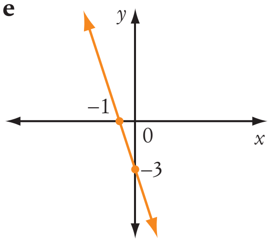
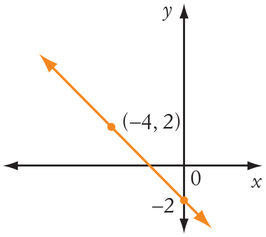


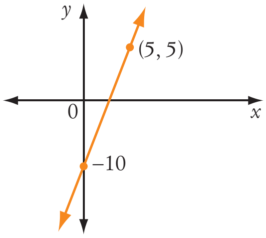
e



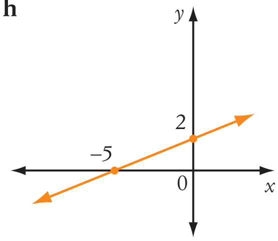
f



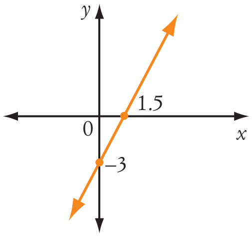
g



h



i



Investigation: Sausage sizzle

A local football club is organising a sausage sizzle on Saturday to raise money to buy new equipment. It costs $25 to hire a gas bottle to run the barbecue and each sandwich costs $0.90 to make.

Fairfax Syndication/Craig Abraham

1 Copy and complete this table to show the cost of making sausage sandwiches. Include the cost of hiring the gas bottle.

No. of sandwiches (x)	0	10	20	30	40	50	60	70	80	90	100
Cost (\$$y$)	25	34									

2 Find the linear equation (formula) for y that represents the cost of making x sausage sandwiches.

3 Use an appropriate scale to construct a graph that shows the cost of making from $x = 0$ to $x = 100$ sandwiches. Label your axes and give your graph an appropriate title.

4 How much will it cost to make 35 sausage sandwiches?

5 How many sandwiches can be made for $98.80?

6 How much would it cost to make 120 sausage sandwiches?

7 **a** If the club sold 75 sausage sandwiches for $3 each, how much money would they take?

b How much **profit** would the club make?

Stage 5.2

Puzzle sheet
Linear equations match-up
MAT10NAPS10012

Worksheet
Writing equations of lines
MAT10NAWK10013

Puzzle sheet
Equations of parallel lines
MAT10NAPS00013

2-07 Equations of parallel and perpendicular lines

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Summary

If two lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.
If two lines with gradients m_1 and m_2 are perpendicular, then $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

Technology
GeoGebra: Perpendicular lines
MAT10NATC00005

Example 18

Find the equation of the line parallel to $y = 8 - 3x$ that passes through the point $(-1, 6)$.

Solution

For $y = 8 - 3x$ (or $y = -3x + 8$), the gradient is $m = -3$.
A line parallel to $y = 8 - 3x$, will also have $m = -3$.
The equation of this line is

$$y = mx + b = -3x + b,$$

where b is a constant.
To find the value of b, substitute the point $(-1, 6)$ into the equation:

$$y = -3x + b$$

$$6 = -3 \times (-1) + b \qquad x = -1, y = 6$$

$$6 = 3 + b$$

$$b = 3$$

∴ The equation is $y = -3x + 3$.

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Example 19

Find the equation of the line perpendicular to $3x - 4y + 6 = 0$, which passes through the point (5, 4).

Solution

To find the gradient of $3x - 4y + 6 = 0$, first convert it to the form $y = mx + b$:

$$\begin{aligned}3x - 4y + 6 &= 0\\ 3x + 6 &= 4y\\ 4y &= 3x + 6\\ y &= \frac{3x+6}{4}\\ y &= \frac{3}{4}x + \frac{3}{2}\end{aligned}$$

$y = mx + b$

$\therefore$ Gradient $= \frac{3}{4}$

$\therefore$ Gradient of perpendicular line $= \frac{-1}{\left(\frac{3}{4}\right)}$

$= -\frac{4}{3}$

The negative reciprocal of $\frac{3}{4}$.

$\therefore$ The equation of this line is $y = -\frac{4x}{3} + b$

To find the value of b, substitute the point (5, 4) into the equation.

$x = 5, y = 4$

$$\begin{aligned}4 &= \left(-\frac{4}{3}\right) \times 5 + b\\ &= -\frac{20}{3} + b\\ 4 + \frac{20}{3} &= b\\ b &= \frac{32}{3}\end{aligned}$$

$\therefore$ The equation is $y = -\frac{4x}{3} + \frac{32}{3}$ or $y = \frac{-4x + 32}{3}$

or, converting to the neater general form:

$3y = -4x + 32$

$4x + 3y - 32 = 0$

Stage 5.2

Exercise 2-07 Equations of parallel and perpendicular lines

See Example 18

1 Find the equation of the line that is parallel to:

- **a** $y = 2x + 9$ and has a y-intercept of 4
- **b** $y = 3x$ and has a x-intercept of -2
- **c** $y = 5 - \frac{x}{2}$ and passes through $(-1, 6)$
- **d** $2x - y = 6$ and passes through $(5, -2)$
- **e** $y = -5x - 8$ and passes through the midpoint of $(3, -10)$ and $(-5, -6)$
- **f** $2y = x - 3$ and passes through $(6, -7)$

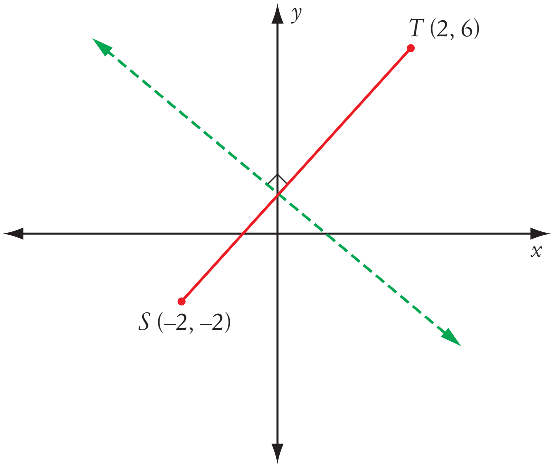
See Example 19

2 Find the equation of a line that is perpendicular to:

- **a** $y = \frac{x}{2}$ and has a y-intercept of -2
- **b** $y = -5x$ and has a x-intercept of 1
- **c** $y = 3x - 1$ and passes through the x-axis at 4
- **d** $y = \frac{x-6}{3}$ and passes through $(1, -6)$
- **e** $x + y - 6 = 0$ and passes through $(-4, 2)$
- **f** $3x - y - 9 = 0$ and passes through $(-10, -7)$

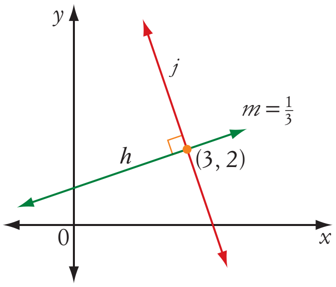
3

- **a** Find the gradient of interval ST in the diagram on the right.
- **b** Find the midpoint of ST.
- **c** The dotted line is perpendicular to ST and passes through its midpoint. What is its gradient?
- **d** Find the equation of the dotted line, in the form $y = mx + b$.

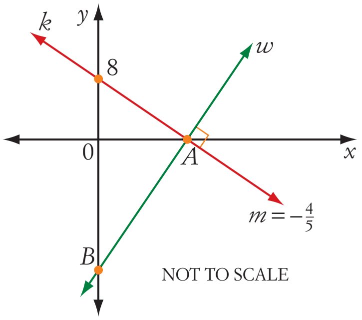


4

- **a** Find the equation of line h in the diagram.
- **b** Find the gradient of line j (which is perpendicular to line h).
- **c** Find the equation of line j.



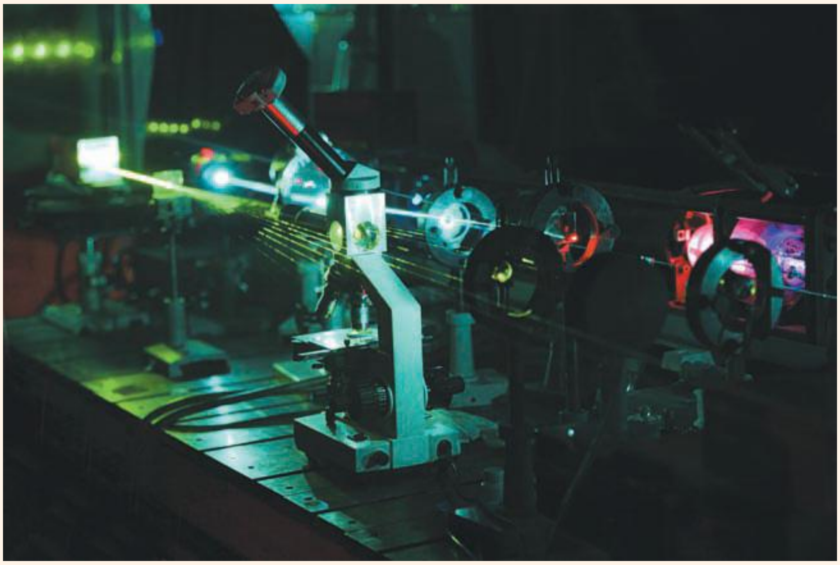
5 **a** Find the equation of line *k*.
b Find the coordinates of point *A*.
c Find the gradient of line *w*.
d Find the equation of line *w*.
e Find the coordinates of point *B*.



Just for the record Constants

Expressions like '*k* is a constant' are often used in mathematics, but constants are also used in areas such as physics, chemistry, biology and astronomy. A constant may be:

- a numerical part of an algebraic expression. For example, in the expression $3x^2 + 5$, the 3 and 5 are constants and 5 is usually called the constant term.
- a quantity that has a fixed value for an expression or calculation. For example, in the equation of a line, $ax + by + c = 0$, the a, b and c are constants (while x and y are variables).
- a number or quantity that does not change in any circumstances. Examples are c (the speed of light) in the formula $E = mc^2$, and π.



Shutterstock.com/Pavel L Photo and Video

Other constants that do not change include Faraday's constant, Planck's constant, Boltzmann's constant, Avogadro's number, 1 astronomical unit, and the gravitational constant.

1 Find the symbol and value of each of the constants listed above.

2 Explain the meaning of the word 'constants' in this statement: 'There are only two constants in life—death and taxes'.

Power plus

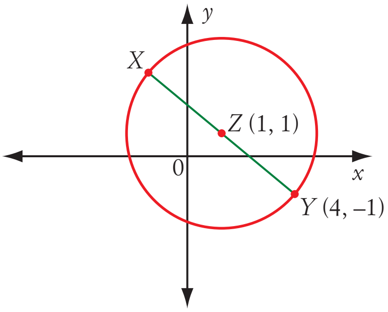
1 A line is drawn through the points $A(0, -2)$ and $B(3, 0)$. The x-coordinate of a point C on AB is 9. Find:

a the gradient of AB **b** the equation of AB **c** the y-coordinate of C.

2 The point $(-1, 6)$ lies on the line $kx + 3y - 13 = 0$, where k is a constant number. Find k.

3 $Z(-1, 3)$ is the midpoint of the interval joining $A(-4, 7)$ and B. Find the coordinates of B.

4 The circle has XY as a diameter and centre Z. What are the coordinates of X?



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Chapter 2 review

Language of maths

Puzzle sheet
Coordinate geometry crossword
MAT10NAPS10014

axes
distance
exact answer
general form
gradient
gradient–intercept form
horizontal
interval
length
linear
linear equation
midpoint
parallel
perpendicular
reciprocal
rise
run
surd
vertical
x-axis
x-intercept
y-axis
y-intercept

1 What is the difference between the ***y*-axis** and the ***y*-intercept**?

2 When finding the length of an interval on a number plane, what is meant by an **exact answer**?

3 What measurement is the fraction given by the vertical rise of a line divided by the horizontal run?

4 What is the everyday meaning of the word **intercept**? Look it up in a dictionary.

5 What is the property of the gradients of perpendicular lines?

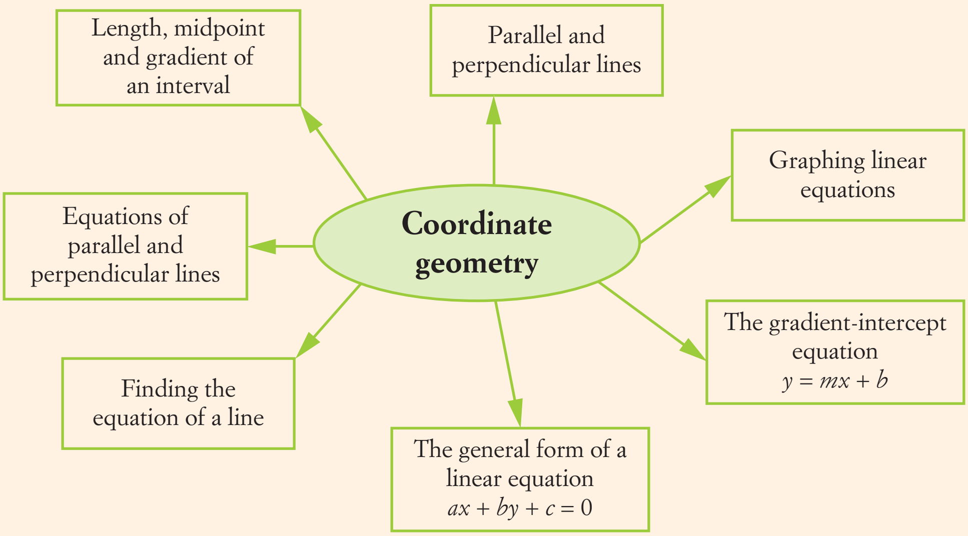
6 What form of the linear equation is $ax + by + c = 0$?

Topic overview

Quiz
Coordinate geometry
MAT10NAQZ00005

- How can you find the gradient of a line?
- What is $y = mx + b$?
- How can you test whether a pair of lines are perpendicular?
- What parts of this topic did you find difficult?

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 2 revision

See Exercise 2-01

1 An interval is formed by joining the points $K(5, 6)$ and $L(-7, 2)$.

a Find, correct to one decimal place, the length of interval KL.

b Find the midpoint of KL.

c Find the gradient of KL.

See Exercise 2-01

2 The vertices of a quadrilateral $HJKL$ are $H(-8, -5)$ $J(-1, -2)$ $K(2, 5)$ $L(-5, 2)$.

a Find the exact length of the sides of the quadrilateral.

b Find the gradient of each side of $HJKL$.

c Find the exact length of the diagonals HK and JL.

d What type of quadrilateral is $HJKL$?

See Exercise 2-02

3 A line passes through the points $V(8, -1)$ and $W(10, -2)$. What is the gradient of a line:

a parallel to VW? **b** perpendicular to VW?

See Exercise 2-03

4 Graph the linear equation $y = -5x - 1$ on a number plane.

See Exercise 2-03

5 Test which of the following points lie on the line of $3x + y = 2$. Select the correct answer **A**, **B**, **C** or **D**.

A $(1, 0)$ **B** $(2, 4)$ **C** $(-1, 5)$ **D** $(-1, -5)$

See Exercise 2-03

6 What is the equation of the line through $(-2, 3)$ and parallel to the x-axis? Select the correct answer **A**, **B**, **C** or **D**.

A $x = -2$ **B** $x = 3$ **C** $y = -2$ **D** $y = 3$

Stage 5.2

See Exercise 2-04

7 Write the gradient, m, and y-intercept, b, for each linear equation.

a $y = 2x - 10$ **b** $y = 4x + 3$ **c** $y = \dfrac{4 - 3x}{8}$

See Exercise 2-05

8 Convert each equation to general form $ax + by + c = 0$.

a $y = 3x + 5$ **b** $y = \dfrac{2x}{5} - 10$ **c** $x = 3y + 6$

See Exercise 2-05

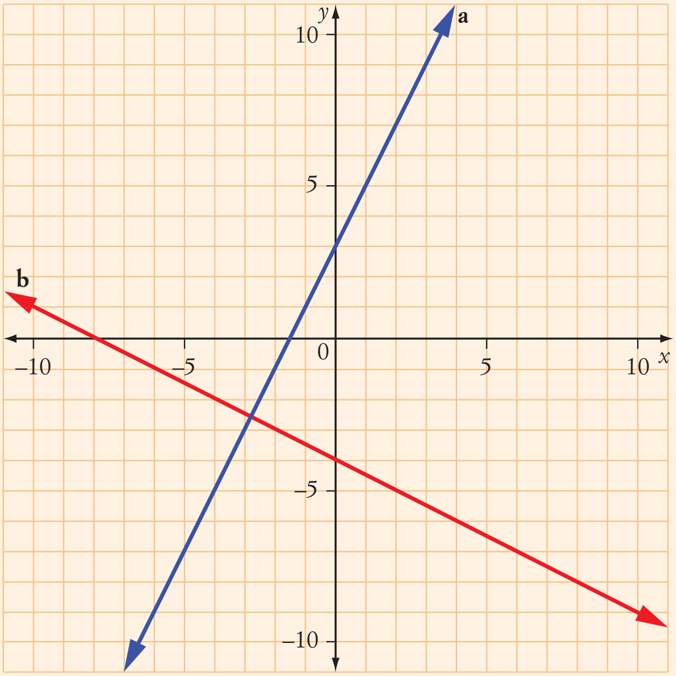
9 Rewrite each equation in the form $y = mx + b$, then state the value of the gradient, m, and the y-intercept, b.

a $x - y + 2 = 0$ **b** $2x - 8y + 8 = 0$ **c** $3x + y - 9 = 0$

Stage 5.2

See Exercise 2-06

10 Find the equation of each line.



See Exercise 2-07

11 Find the equation of a line that is:

a parallel to $y = 3x + 1$ and passes through the x-axis at 2

b perpendicular to $y = \frac{x}{2}$ and passes through the origin.

Mixed revision 1

See Exercise 1-01

1 Cassie earns a salary of $84 750. How much is she paid each week?

See Exercise 2-01

2 An interval is formed by joining the points $M(2, -3)$ and $N(-5, 4)$.

a Find the length of interval MN, correct to one decimal place.

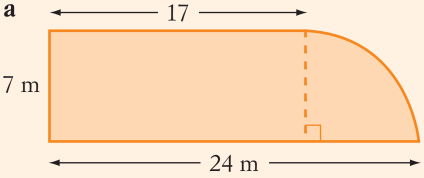
b Find the midpoint of MN.

c Find the gradient of MN.

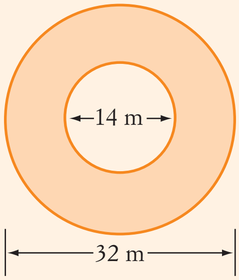
See Exercise 3-01

3 Find, correct to one decimal place, the area of each shape.

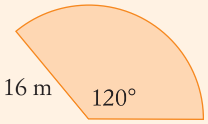
a



b



c



See Exercise 1-01

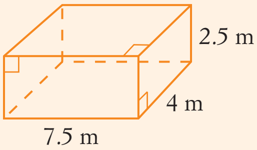
4 Aymin is paid four weeks normal pay plus 17.5% annual leave loading for his four-week holiday. If Aymin's salary is $52 270, find his:

a normal weekly pay **b** leave loading **c** total pay for the four-week holiday.

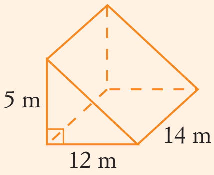
See Exercise 3-02

5 Find the surface area of each prism.

a



b



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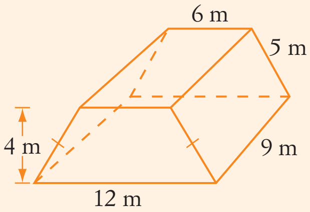


See Exercise 1-01

6 A call centre operator is employed under the following award.

Normal rate: $21.25 per hour	
Normal rate	For 0 to 36 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 20 hours **b** 39 hours **c** 45 hours

See Exercise 2-02

7 A line passes through the points $H(6, -3)$ and $K(8, -6)$. Calculate the gradient of the line:

a parallel to HK

b perpendicular to HK.

Stage 5.2

See Exercise 1-02

8 Crystal earns a weekly wage of $950. She has annual deductions of $623 for a health fund and $1567 for work expenses.

a Calculate Crystal's taxable income.

b Use the tax table on page 10 to calculate the income tax that Crystal should pay.

9 Graph the linear equations $y = 3x - 2$ and $y = -2x + 3$ on a number plane. Where do the lines intersect?

See Exercise 2-03

10 Calculate the simple interest earned on each investment.

a \$25 000 invested for 4 years at 3% p.a.

b \$500 invested at 4.5% p.a for 9 months

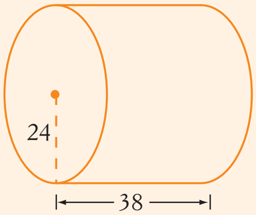
See Exercise 1-03

11 Calculate, correct to one decimal place, the surface area of each solid. All lengths shown are in centimetres.

Stage 5.2

See Exercise 3-03

a



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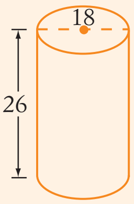

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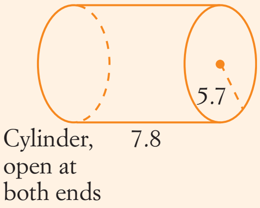


12 Joshua invests \$12 000 at 4% p.a. interest, compounded annually for 2 years.

a What is the total value of his investment after 2 years?

b What is the amount of interest earned after 2 years?

See Exercise 1-04

13 Which of the following points lie on the line of $2x + y = 3$? Select the correct answer **A**, **B**, **C** or **D**.

A (1, 0) **B** (2, −1) **C** (−1, −1) **D** (−1, −5)

See Exercise 2-03

14 Find the equation of the line passing through (−3, 2) that is parallel to the y-axis. Select the correct answer **A**, **B**, **C** or **D**.

A $x = 2$ **B** $x = -3$ **C** $y = 2$ **D** $y = -3$

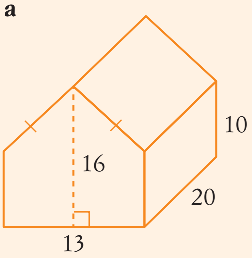
See Exercise 2-03

15 Calculate, correct to nearest square metre, the surface area of each solid. All lengths shown are in metres.

Stage 5.2

See Exercise 3-04

a



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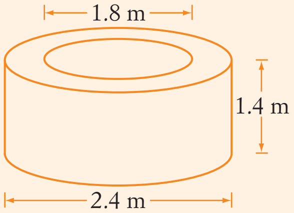

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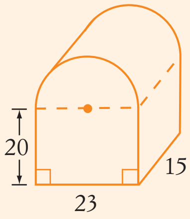


16 Calculate the value of a investment if \$4000 is invested at 6% p.a. for 3 years with interest compounded:

a annually **b** quarterly **c** monthly

See Exercise 1-05

17 Find the compound interest earned when \$50 000 is invested for 10 years at 6.5% p.a., with interest compounded twice a year.

See Exercise 1-05

See Exercise 1-06

18 Jovan purchases a home theatre system valued at \$7680. He pays a 10% deposit and repays the balance in 48 monthly instalments. Interest on the balance is charged at a flat rate of 12.5% p.a. Find:

a the deposit paid
b the balance owing
c the interest charged
d the total amount to be repaid
e the amount of each instalment
f the total price Jovan paid for the system.

Stage 5.2

See Exercise 2-04

19 Write the gradient, m, and y-intercept, b, for each linear equation.

a $y = -3x + 8$ **b** $y = 7 + x$ **c** $y = \frac{3 - 2x}{3}$

See Exercise 2-05

20 Convert each equation to general form $ax + by + c = 0$.

a $y = 2x - 3$ **b** $2x = y + 5$ **c** $y = \frac{3x}{4} + 6$

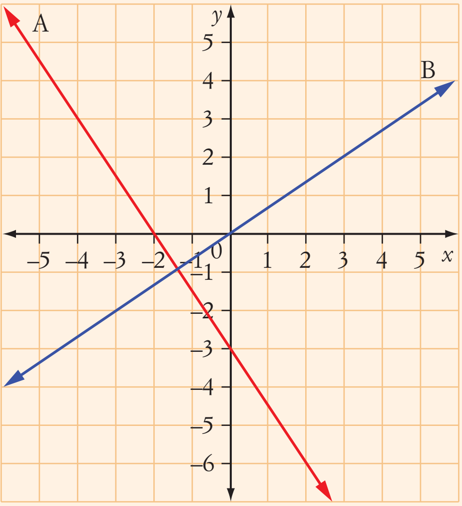
See Exercise 2-05

21 Rewrite each equation in the form $y = mx + b$.

a $5x - y + 3 = 0$ **b** $4x + y - 8 = 0$ **c** $2x - 6y + 8 = 0$

See Exercise 2-06

22 Find the equation of each line.



See Exercise 2-07

23 Find the equation of a line that is:

a parallel to $y = 4x - 3$ and has an x-intercept of -8
b perpendicular to $y = -\frac{x}{5}$ and passes through (0, 0).

See Exercise 1-07

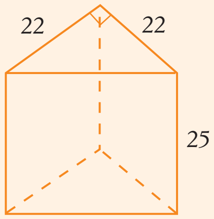
24 Aileen purchases a new car for \$29 990, which depreciates by 10% p.a.

a Find the depreciated value of the car after 4 years.
b What is the depreciation over this time?
c Express the depreciated value as a percentage of the cost price (correct to one decimal place).

25 Calculate, correct to the nearest cubic centimetre, the volume of each solid. All lengths shown are in centimetres.

See Exercise 3-05

a



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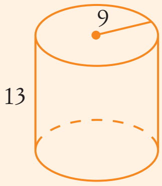

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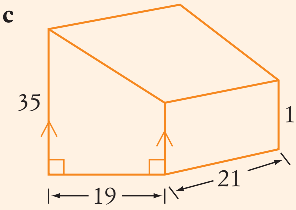


26 A circular swimming pool has a radius of 3.5 m and a depth of 2 m. The pool is to be filled to within 30 cm from the top.

See Exercise 3-05

a Calculate the volume of water in the pool to the nearest m^3.

b If water costs \$1.25 per kilolitre, find the cost of filling the pool.

3

Measurement and Geometry

Surface area and volume

Some theme parks have wave pools, which are big swimming pools that simulate the movement of the water at a beach. A large volume of water is quickly released into one end of the pool which produces a large wave that moves from one end of the pool to the other. The excess water in the pool is recycled so that it can be used to produce more waves.

Shutterstock.com/CJPhoto

Chapter outline

	Proficiency strands				
3-01 Areas of composite shapes	U	F	PS	R	
3-02 Surface area of a prism	U	F	PS	R	C
3-03 Surface area of a cylinder*	U	F	PS	R	
3-04 Surface areas of composite solids*	U	F	PS	R	C
3-05 Volumes of prisms and cylinders	U	F	PS	R	C

*STAGE 5.2

Wordbank

capacity	The amount of fluid (liquid or gas) in a container
composite shape	A shape made up of two or more basic shapes
cross-section	A 'slice' of a solid, taken across the solid rather than along it
curved surface area	The area of the curved surface of a solid such as a cylinder or sphere. The curved surface of a cylinder is a rectangle when flattened.
cylinder	A can-shaped solid with identical cross-sections that are circles
prism	A solid with identical cross-sections that are polygons
sector	A region of a circle cut off by two radii, shaped like a piece of pizza
surface area	The total area of all faces of a solid shape

In this chapter you will:

- solve problems involving the surface areas and volumes of right prisms
- (STAGE 5.2) calculate the surface area and volume of cylinders and solve related problems
- calculate areas of composite shapes, including circular shapes involving sectors
- (STAGE 5.2) calculate the surface areas and volumes of composite solids
- calculate volumes and capacities of right prisms and cylinders

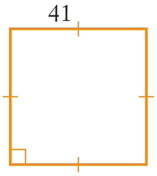
SkillCheck

Worksheet
StartUp assignment 3
MAT10MGWK10015

Skillsheet
Solid shapes
MAT10MGSS10007

1 Calculate the area of each shape. All measurements are in centimetres.

a



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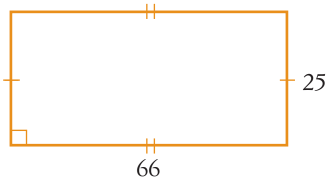

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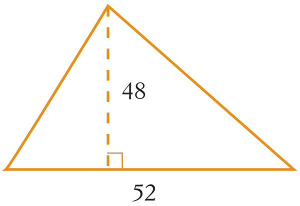

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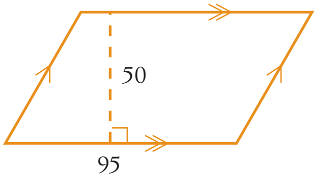

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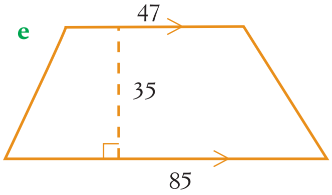

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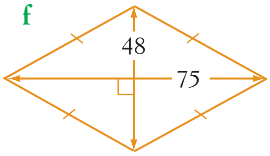

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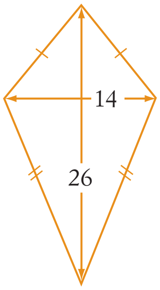

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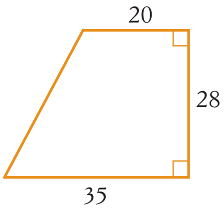

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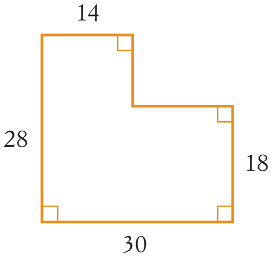
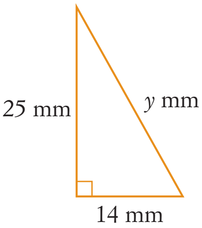


2 Use Pythagoras' theorem to find, correct to one decimal place, the value of each pronumeral.

a



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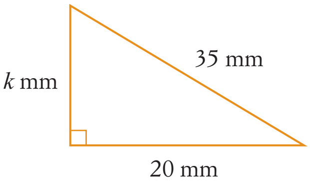

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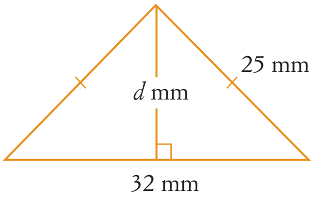
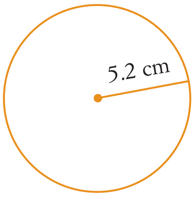


3 For each circle, find correct to two decimal places:

i its circumference **ii** its area.

a



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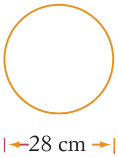

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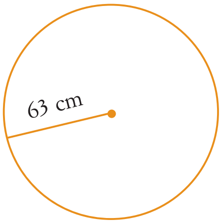

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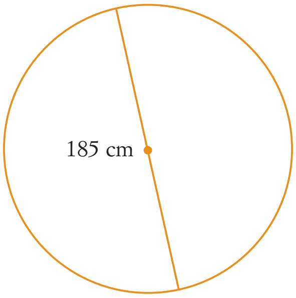
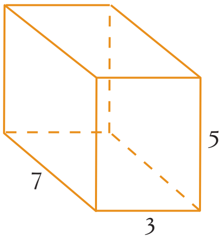


4 Calculate the volume of each solid. All measurements are in metres.

a



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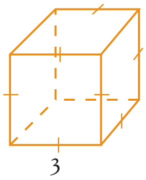

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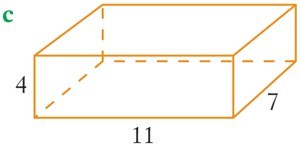

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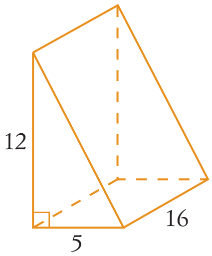

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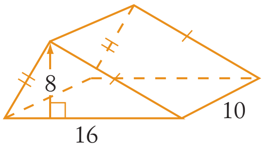

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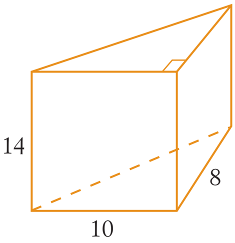


Skillsheet

What is volume?

MAT10MGSS10008

Puzzle sheet

Area

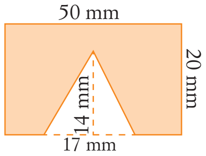
MAT10MGPS00010

3-01 Areas of composite shapes

Example 1

Find the area of each composite shape, correct to one decimal place where appropriate.

a



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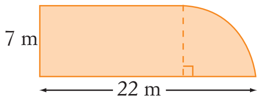

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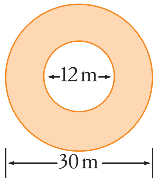


Solution

a $\text{Area} = 50 \times 20 - \frac{1}{2} \times 17 \times 14$

$= 881 \text{ mm}^2$

Area of rectangle − area of triangle

b The shape is made up of a rectangle and a quadrant.

Radius of quadrant = 7 m
Length of rectangle = 22 − 7 = 15 m
Area of shape = area of rectangle + quadrant

$$= 15 \times 7 + \frac{1}{4} \times \pi \times 7^2$$

$$= 143.4845\ldots$$

$$\approx 143.5 \text{ m}^2$$

c This ring shape is an **annulus**, its area is enclosed by two circles with the same centre.

$$\text{Radius of large circle} = \frac{1}{2} \times 30 \text{ m}$$

$$= 15 \text{ m}$$

$$\text{Radius of small circle} = \frac{1}{2} \times 12 \text{ m}$$

$$= 6 \text{ m}$$

$$\text{Area of annulus} = \pi \times 15^2 - \pi \times 6^2$$

$$= 593.7610\ldots$$

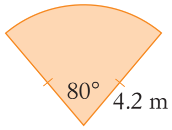
$$\approx 593.8 \text{ m}^2$$

Large circle − small circle

Example 2

Calculate, correct to two decimal places, the area of each sector.

a



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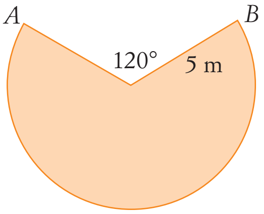


A **sector** is a fraction of a circle 'cut' along two radii, like a pizza slice.

Solution

a Area $= \dfrac{80}{360} \times \pi \times 4.2^2$

$= 12.31504\ldots$

$\approx 12.32 \text{ m}^2$

$\dfrac{80}{360} \times$ area of circle

There are 360° in a circle, but a sector is a fraction of a circle

b Sector angle $= 360° - 120° = 240°$

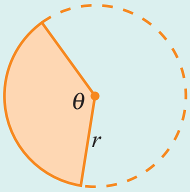
Area of sector $= \dfrac{240}{360} \times \pi \times 5^2$

$= 52.35987\ldots$

$\approx 52.36 \text{ m}^2$

Summary

Area of a sector $= \dfrac{\theta}{360} \times \pi r^2$

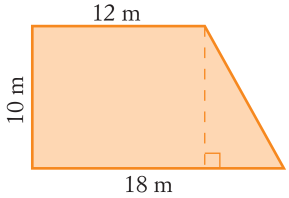


Exercise 3-01 Area of composite shapes

1 Find the area of each composite shape.

See Example 1

a



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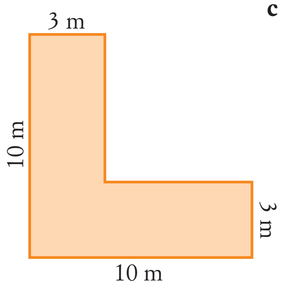

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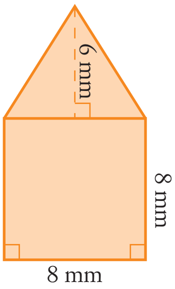

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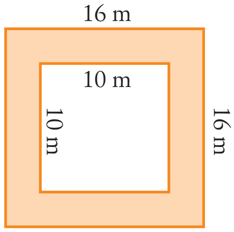

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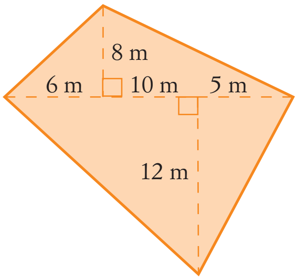

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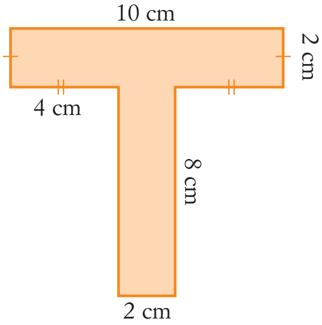

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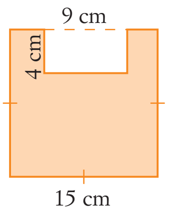

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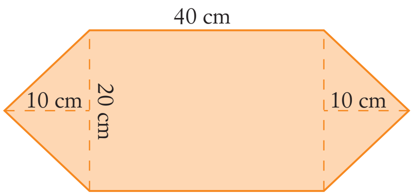

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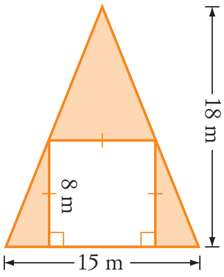

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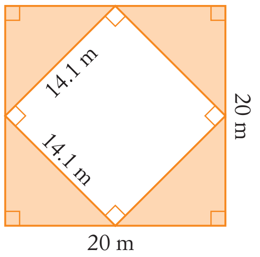

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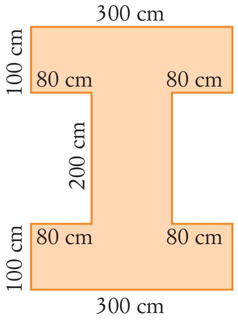
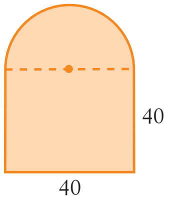


2 Calculate, correct to one decimal place, the area of each shape. All measurements are in metres.

a



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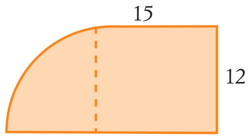

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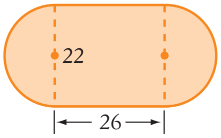

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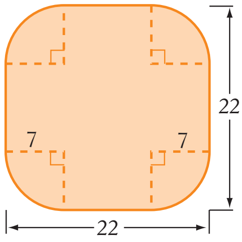

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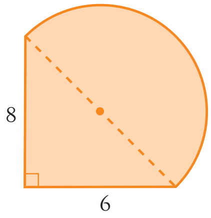

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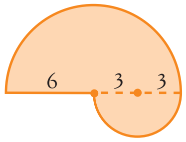

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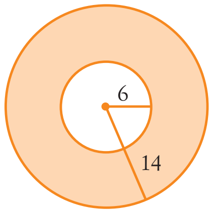

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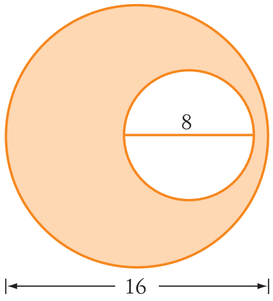

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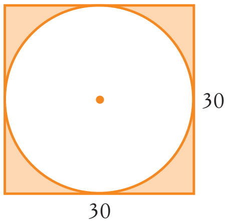

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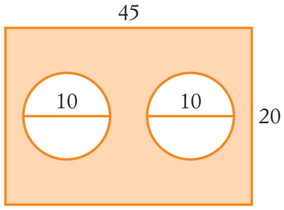

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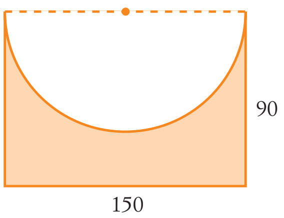
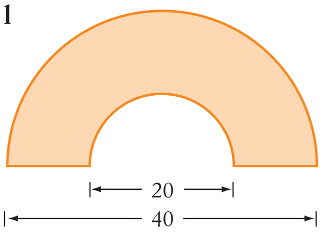


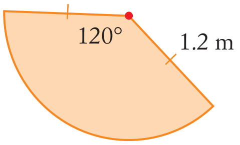
l



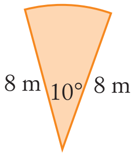
See Example 2

3 Find, correct to two decimal places, the area of each sector.

a



b



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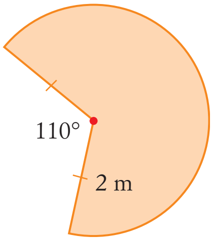


4 A circular playing field has a radius of 80 m. A rectangular cricket pitch measuring 25 m by 2 m is placed in the middle. The field, excluding the pitch, is to be fertilised.

a Calculate the area to be fertilised.

b How much will this cost if the fertiliser is $19.95 per 100 square metres? Give your answer correct to the nearest dollar.

5 A circular plate of diameter 2 m has 250 holes of diameter 10 cm drilled in it. What is the remaining area of the plate? Answer correct to the nearest 0.1 m^2.

6 A circular pond of diameter 10 m is surrounded by a path one metre wide.

a Calculate the area of the path correct to two decimal places.

b If pavers are $75 per square metre laid, what is the cost of the path?

7 A circular sports ground of diameter 120 m has a rectangular soccer pitch measuring 100 m by 50 m inside it. The area outside the soccer pitch is to be painted in the team colour of red.

a Calculate the area that is to be painted red, correct to the nearest m^2.

b If the cost of paint is $29.50 per 10 m^2, calculate the cost of painting this area.

Dreamstime.com/Patrick Allen

8 A new tractor tyre has a diameter of 120 cm, while a worn tyre has a diameter of 115 cm.

a Calculate the difference in circumference between a new and a worn tyre, correct to three decimal places.

b Over 1000 revolutions, how much further (to the nearest metre) will a new tyre travel compared to a worn tyre?

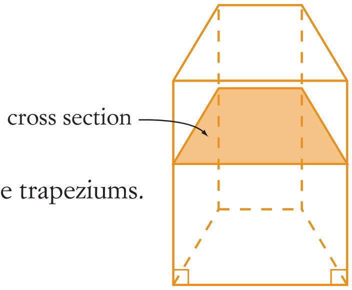
9 A square courtyard measuring 5 m by 5 m has a semi-circular area added to each side.

a Calculate the total area of the semi-circular additions, correct to two decimal places.

b By what percentage (to the nearest whole number) has the area of the courtyard increased? (This can be calculated as $\frac{\text{increase in area}}{\text{original area}} \times 100\%$).

3-02 Surface area of a prism

A **cross-section** of a solid is a 'slice' of the solid cut *across* it, parallel to its end faces, rather than along it. A **prism** has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides).
The trapezoidal prism shown here has cross-sections that are trapeziums.



A right prism

Summary

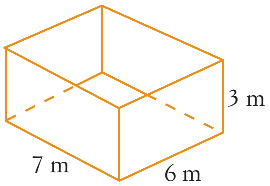
The **surface area** of a solid is the total area of all the faces of the solid. To calculate the surface area of a solid, find the area of each face and add the areas together.

It is often useful to draw the net of a solid when finding its surface area. A net may be used to form an open solid or a closed solid. A sealed cardboard box is an example of a **closed solid**. A cardboard box with the lid removed is an example of an **open solid**.

Example 3

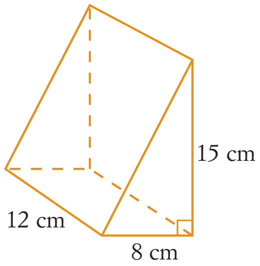
Find the surface area of each prism.

a



Open rectangular prism

b



Closed triangular prism

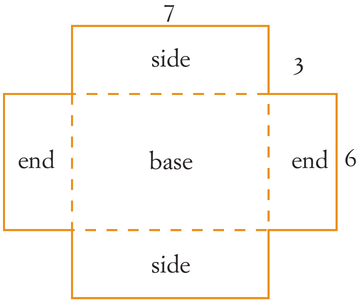
Solution

a This open prism has five faces.

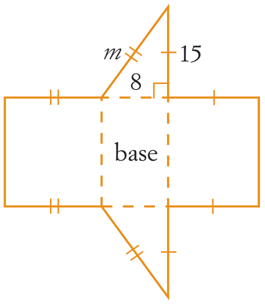
Surface area $= 2$ ends $+ 2$ sides $+$ base

$= 2 \times (3 \times 6) + 2 \times (3 \times 7) + (6 \times 7)$

$= 120\ \text{m}^2$



b This open prism has five faces: two identical triangles (front and back) and three different rectangles.
Using Pythagoras' theorem to find m, the hypotenuse of the triangle:



$$m^2 = 8^2 + 15^2$$
$$= 289$$
$$m = \sqrt{289}$$
$$= 17$$

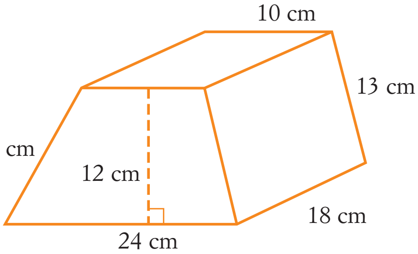
Surface area = 2 triangles + 3 rectangles

$$= 2 \times \left(\frac{1}{2} \times 8 \times 15\right) + (17 \times 12) + (8 \times 12) + (15 \times 12)$$
$$= 600 \text{ cm}^2$$

Example 4

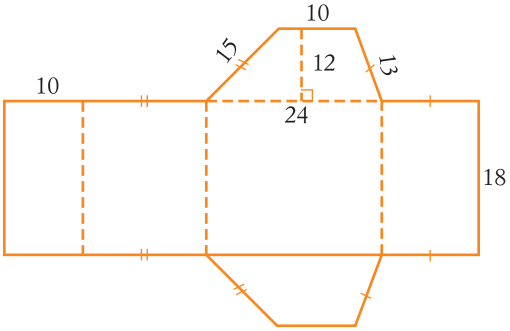
Stage 5.2

Calculate the surface area of this trapezoidal prism.



Solution

This trapezoidal prism has 6 faces: two identical trapeziums (front and back) and four different rectangles.



$$\text{Area of each trapezium} = \frac{1}{2} \times (10 + 24) \times 12$$
$$= 204 \text{ cm}^2$$

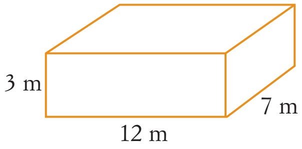
$$\text{Surface area} = (2 \times 204) + (18 \times 10) + (18 \times 15) + (18 \times 24) + (18 \times 13)$$
$$= 1524\text{cm}^2$$

Exercise 3-02 Surface area of a prism

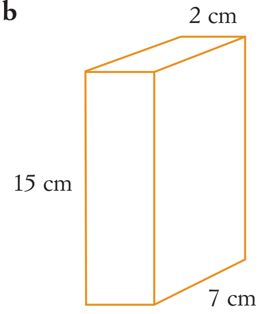
See Example 3

1 Find the surface area of each prism.

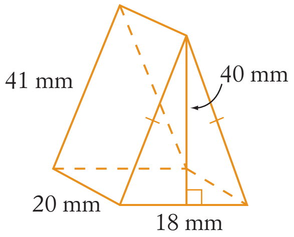
a



b



c



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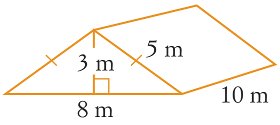

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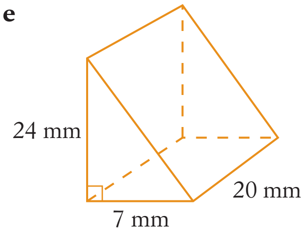

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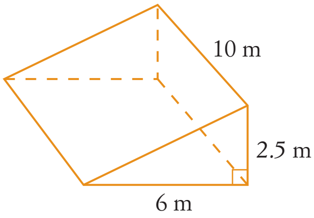
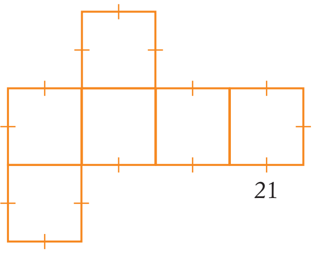
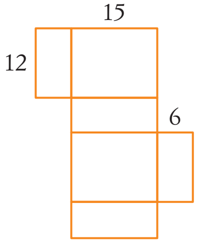


2 Identify the prism that each net represents, then calculate the surface area of the prism. All lengths are in metres.

a



b



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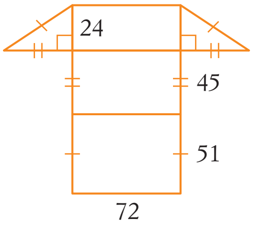
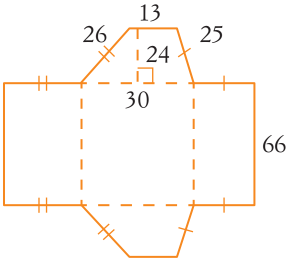


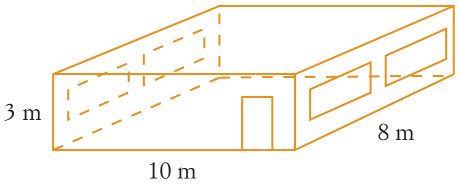
d



3 This classroom is being renovated. Find:

a the area of the floor to be carpeted and the total cost, at \$105 per square metre

b the ceiling and wall area to be painted, if the room contains four windows, each 2.5 m by 1.5 m, and a doorway 2 m by 0.8 m.

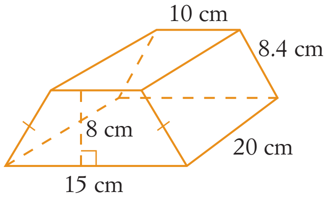


Stage 5.2

***See* Example 4**

4 Calculate the surface area of each prism.

a



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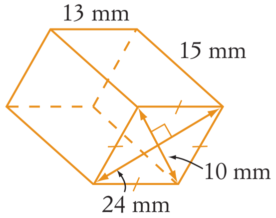

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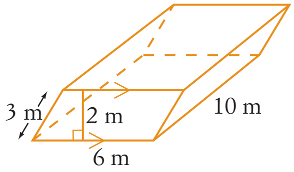

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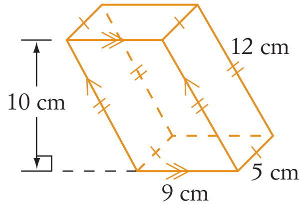

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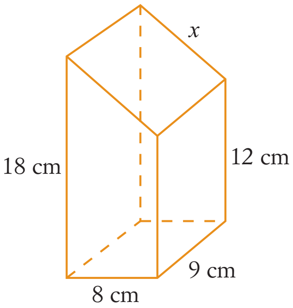

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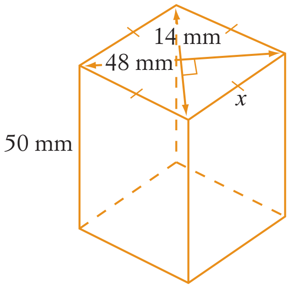
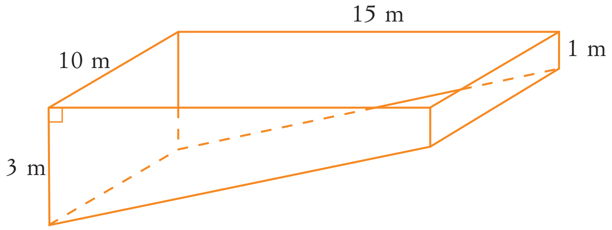


5 This swimming pool is 15 m long and 10 m wide. The depth of the water ranges from 1 m to 3 m. Calculate, correct to two decimal places:

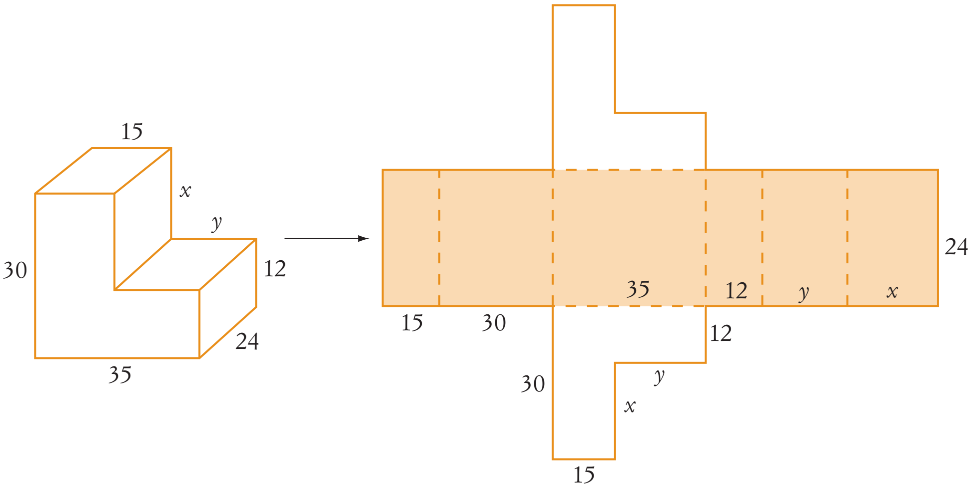
a the area of the floor of the pool

b the total surface area of the pool.



Investigation: A surface area short cut

1 Consider this L-shaped prism and its net. We will find its surface area.



- **a** Find x and y.
- **b** This prism has eight faces: 2 ‘L-shaped’ ends and 6 rectangles. Instead of calculating the areas of the six rectangles separately, we can combine them into one long rectangle, as shaded in the net above. The length of the rectangle is the same as the perimeter of the L-shape. What is the length of this long rectangle?
- **c** What is the area of this long rectangle?
- **d** Copy and complete: Length of shaded rectangle = p ______________ of the L-shape.
- **e** Find the surface area of the prism by copying and completing the following:

Surface area = 2 ‘L-shaped’ ends + shaded rectangle

$= 2 \times (15 \times 30 + 20 \times 12) + \_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_$

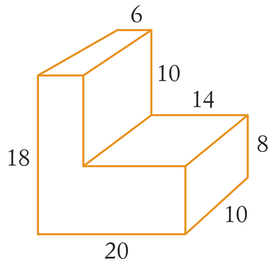
2 From question **1**, it can be seen that the surface area of any prism with end faces of area A and perpendicular height (distance between end faces) h can be calculated using the formula:

$$\text{SA} = 2A + Ph$$

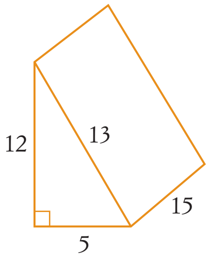
where P = perimeter of end face.

Use this method to calculate the surface area of each prism. All measurements are in centimetres.

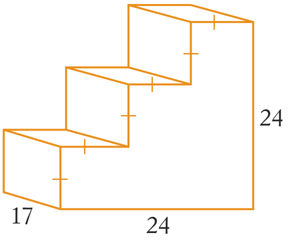
a



b



c



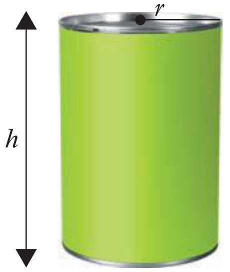
3-03 Surface area of a cylinder

Stage 5.2

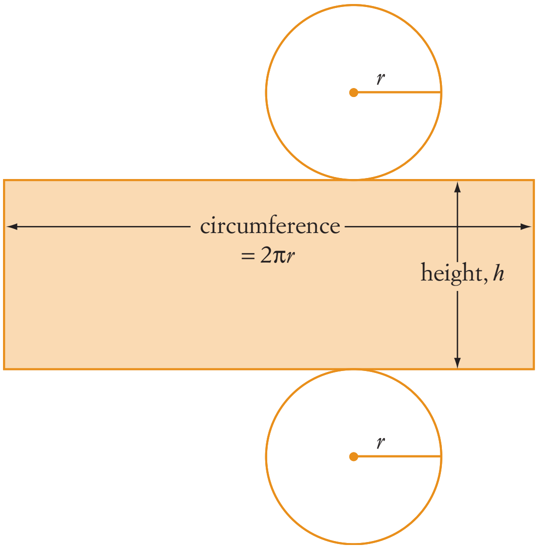
Worksheet
Surface area
MAT10MGWK10016

Puzzle sheet
Surface area
MAT10MGPS00009

A closed cylinder has three faces: two circles (the ends) and a rectangle (the curved surface). The length of the rectangle is the circumference of the circular end, while the width of the rectangle is the height of the cylinder.



Shutterstock.com/Sarah2



$$\begin{aligned}\text{Surface area of a cylinder} &= \text{area of two circles} + \text{area of rectangle}\\ &= 2 \times \pi r^2 + 2\pi r \times h\\ &= 2\pi r^2 + 2\pi rh\end{aligned}$$

Summary

Surface area of a closed cylinder

$$A = 2\pi r^2 + 2\pi rh$$

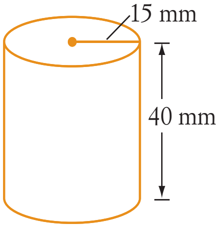
where r = radius of circular base and h = perpendicular height

The area of the *two* circular ends $= 2\pi r^2$ and the area of the curved surface $= 2\pi rh$.

Stage 5.2

Example 5

Find, correct to the nearest mm^2, the surface area of this cylinder.



Solution

$$\begin{aligned}\text{Surface area} &= \text{area of 2 ends} + \text{area of the curved surface}\\ &= 2\pi r^2 + 2\pi rh\\ &= 2\times\pi\times 15^2 + 2\times\pi\times 15\times 40 \qquad r = 15,\ h = 40\\ &= 5183.627\ldots\\ &\approx 5184\ \text{mm}^2\end{aligned}$$

Example 6

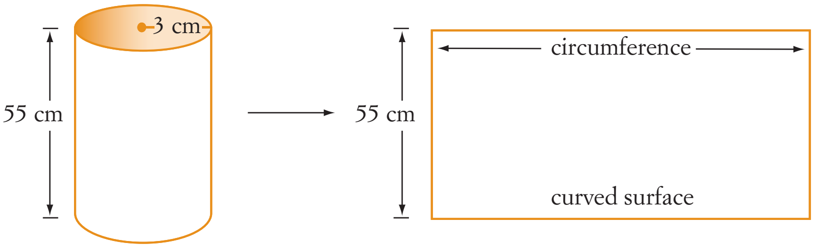
Find, correct to two significant figures, the surface area of:

a a cylindrical tube, open at both ends, with radius 3 cm and length 55 cm

b an open half-cylinder with radius 0.5 m and height 3 m.

Solution

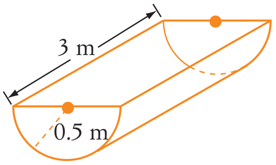
a

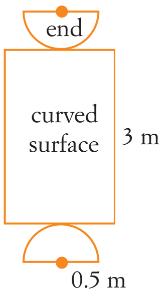


$$\begin{aligned}\text{Surface area} &= \text{curved surface}\\ &= 2\pi rh\\ &= 2\times\pi\times 3\times 55 \qquad r = 3 \text{ and } h = 55\\ &= 1036.725\ldots\\ &\approx 1000\ \text{cm}^2\end{aligned}$$

9780170194655

b





$$\text{Surface area} = 2 \text{ semicircle ends} + \frac{1}{2} \times \text{curved surface}$$
$$= 2 \times \left(\frac{1}{2} \times \pi \times 0.5^2\right) + \frac{1}{2} \times (2 \times \pi \times 0.5 \times 3)$$
$$= 5.49778 \ldots$$
$$\approx 5.5 \text{ cm}^2$$

Exercise 3-03 Surface area of a cylinder

See Example 5

1 Calculate, correct to two decimal places, the surface area of a cylinder with:

a radius 7 m, height 10 m
b diameter 35 mm, height 15 mm
c diameter 6.2 cm, height 7.5 cm
d radius 0.8 m, height 2.35 m

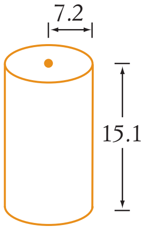
2 Find, correct to the nearest whole number, the curved surface area of a cylinder with:

a radius 1.5 m, height 3.75 m
b diameter 27 cm, height 41 cm

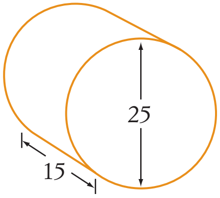
See Example 6

3 Calculate, correct to the nearest square metre, the surface area of each solid. All lengths shown are in metres.

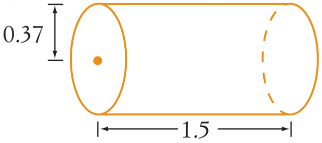
a closed cylinder



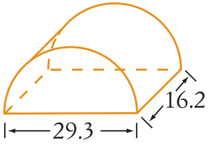
b closed cylinder



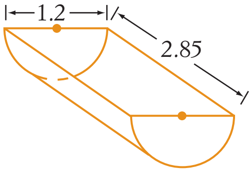
c cylinder with one open end



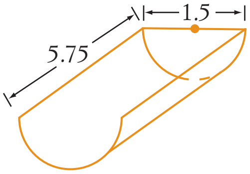
d closed half cylinder



e half cylinder with open top

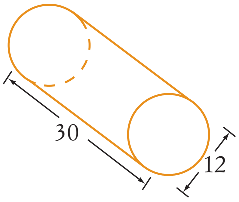


f half cylinder with open top, one end open

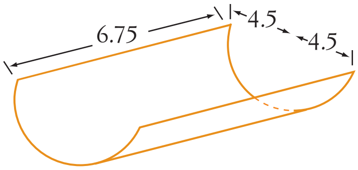


Stage 5.2

g cylinder open both ends

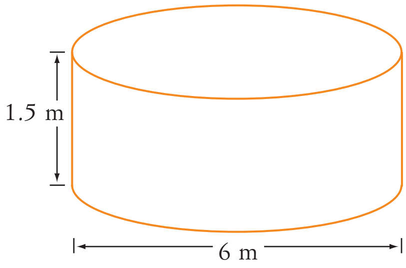


h half cylinder, open both ends



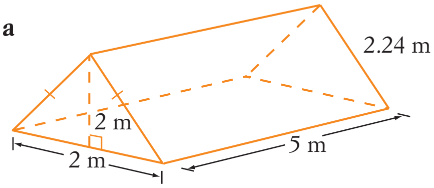
4 A swimming pool is in the shape of a cylinder 1.5 m deep and 6 m in diameter. The inside of the pool is to be repainted, including the floor. Find:

- **a** the area to be repainted, correct to one decimal place
- **b** the number of whole litres of paint needed if coverage is 9 m^2 per litre.

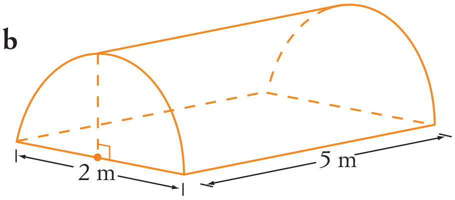


5 Which tent has the greater surface area?

a



b



(*Note*: the floor is included for both tents)

3-04 Surface area of composite solids

Worksheet

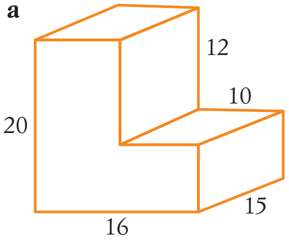
A page of prisms and cylinders

MAT10MGWK10017

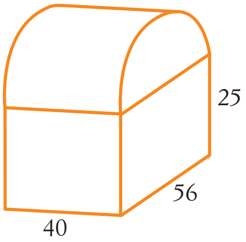
Example 7

Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres.

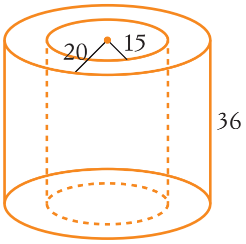
a



b



c

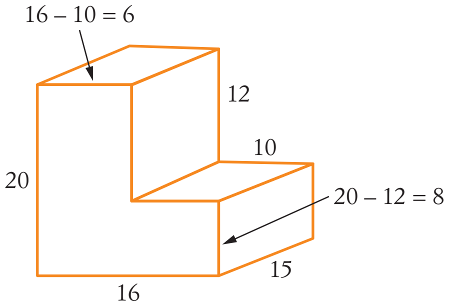


9780170194655

Solution

a This prism has 8 faces: 2 identical L-shapes (front and back) and 6 different rectangles.

$$\begin{aligned}\text{Area of L-shape} &= 16 \times 20 - 10 \times 12\\ &= 200 \text{ cm}^2\end{aligned}$$



$$\begin{aligned}\text{Surface area} &= \text{Front and back L-faces} + \text{1st top} + \text{1st right} + \text{2nd top}\\ &\quad + \text{2nd right} + \text{bottom} + \text{left}\\ &= (2 \times 200) + (6 \times 15) + (12 \times 15)\\ &\quad + (10 \times 15) + (8 \times 15) + (16 \times 15)\\ &\quad + (20 \times 15)\\ &= 1480 \text{ cm}^2\end{aligned}$$

Length of long rectangle
= perimeter of L
= 6 + 12 + 10 + 8 + 16 + 20
= 72

Note that the six rectangles can also be thought of as one long rectangle of width 15 cm:

$$\begin{aligned}\text{Surface area} &= (2 \times 200) + (72 \times 15)\\ &= 1480 \text{ cm}^2\end{aligned}$$

b The solid is made up of a half-cylinder (3 faces) and a rectangular prism (5 faces).

Radius of semi-circle
$= \frac{1}{2} \times 56 = 28$

$$\begin{aligned}\text{Surface area of half-cylinder} &= \text{2 semi-circular ends} + \text{curved surface area}\\ &= 2 \times \frac{1}{2} \times \pi \times 28^2 + \frac{1}{2} \times 2 \times \pi \times 28 \times 40\\ &= 5981.5924\ldots \text{ cm}^2\end{aligned}$$

Do not round this partial answer, the final answer will be inaccurate.

$$\begin{aligned}\text{Surface area of rectangular prism} &= \text{Front and back faces} + \text{2 side faces} + \text{bottom face}\\ &= (2 \times 40 \times 25) + (2 \times 56 \times 25) + (40 \times 56)\\ &= 7040 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total surface area} &= 5981.5924\ldots + 7040\\ &= 13\,021.5924\ldots\\ &= 13\,021.6 \text{ cm}^2\end{aligned}$$

c The hollow cylinder is made up of 2 annulus (ring) faces, an outside curved surface area and an inside curved surface area.

$$\begin{aligned}\text{Surface area of annulus faces} &= 2 \times (\pi \times 20^2 - \pi \times 15^2)\\ &= 1099.5574\ldots \text{ cm}^2\end{aligned}$$

2 × area between two circles

$$\begin{aligned}\text{Outside curved surface area} &= 2 \times \pi \times 20 \times 36\\ &= 4523.8334\ldots \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Inside curved surface area} &= 2 \times \pi \times 15 \times 36\\ &= 3392.9200\ldots \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total surface area} &= 1099.5574\ldots + 4523.8334\ldots + 3392.9200\ldots\\ &= 9016.3108\ldots\\ &= 9016.3 \text{ cm}^2\end{aligned}$$

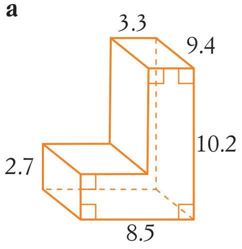
Stage 5.2

Exercise 3-04 Surface areas of composite solids

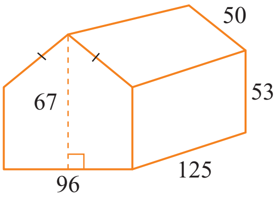
See Example 7

1 Find the surface area of each prism. All measurements are in centimetres.

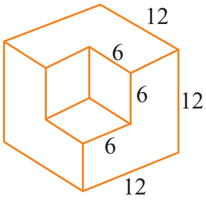
a



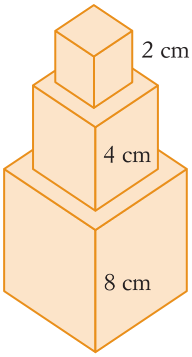
b



c



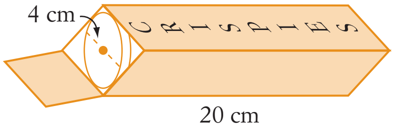
2 Three cubes of length 2 cm, 4 cm and 8 cm are glued on top of each other. Calculate the surface area of the new solid.



3 Circular cracker biscuits of diameter 4 cm are packed in a cardboard box of length 20 cm.

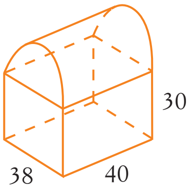
a Calculate the surface area of the box.

b How much cardboard would be saved if the biscuits were packed into a cylindrical box?

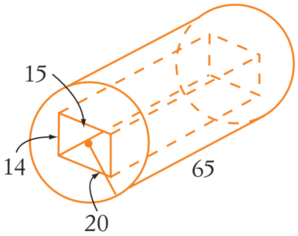


4 Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres.

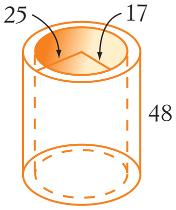
a



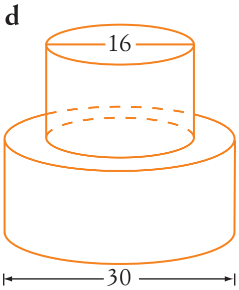
b



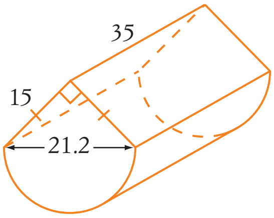
c



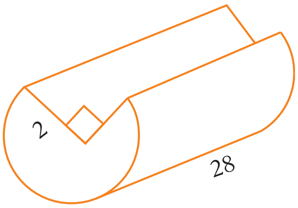
d



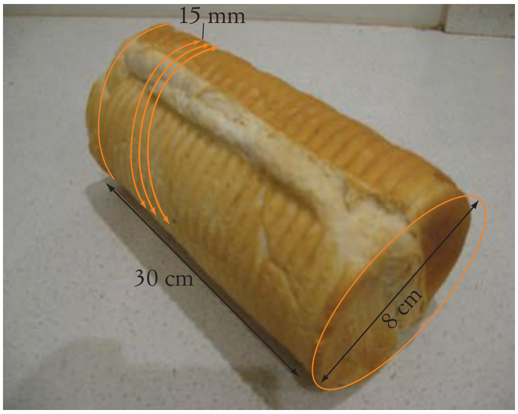
e

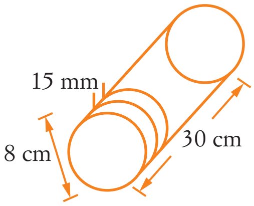


f



5 A cylindrical loaf of bread that is 30 cm long with a diameter of 8 cm is cut into slices 15 mm thick.





a Calculate the surface area of the loaf of bread before it is sliced, correct to two decimal places.

b Find the number of slices in a loaf.

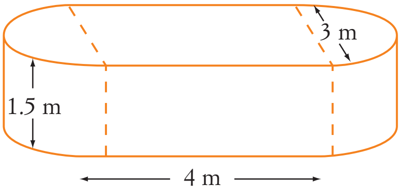
c Calculate the surface area of each slice, correct to the nearest cm^2.

6 A wedding cake with three tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total visible surface area, correct to the nearest cm^2.

Shutterstock.com/John Wollwerth

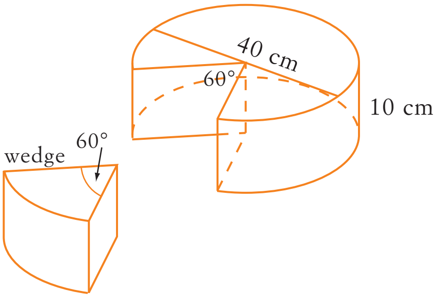
7 **a** Find, correct to two decimal places, the total external area of the wall of this above-ground swimming pool.

b Calculate the area of the water surface, correct to the nearest m^2.



Stage 5.2

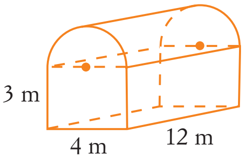
8 A wedge of cheese is cut from a cylindrical block of height 10 cm and diameter 40 cm. Find the total surface area of the wedge, correct to two decimal places.



9 The curved roof of a greenhouse is to be covered in shade cloth.

a Calculate, correct to one decimal place, the area of shade cloth needed if there are no overlaps.

b Shade cloth is sold in 1.5 m wide rolls. How many linear metres of shade cloth are needed to cover the curved roof? Answer to the nearest 0.1 metre.



Mental skills 3 Maths without calculators

Time differences

1 Study each example.

a What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

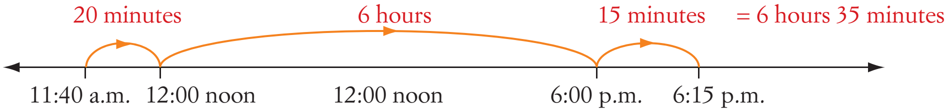
Count: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

5 hours + 20 min + 15 min = 6 hours 35 min

OR:



b What is the time difference between 2030 and 0120?

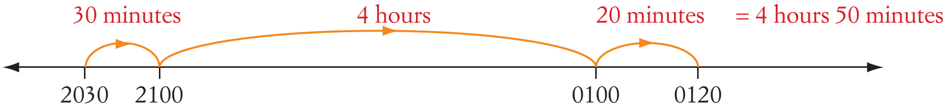
From 2030 to 0030 = 4 hours (24 − 20 = 4)

From 0030 to 0100 = 30 min

From 0100 to 0120 = 20 min

4 hours + 30 minutes + 20 minutes = 4 hours 50 minutes

OR:



2 Now find the time difference between:

a 11:10 a.m. and 7:40 p.m.
b 6:20 pm. and 12:00 midnight
c 4:45 p.m. and 8:10 p.m.
d 2:35 a.m and 10:50 a.m.
e 1:05 p.m. and 12:30 a.m.
f 9:35 a.m. and 11:15 a.m.
g 0425 and 0935
h 1440 and 2025
i 7:55 a.m. and 3:50 p.m.
j 2:40 p.m. and 10:20 p.m.

Technology Surface areas and volumes of solids

Technology worksheet
Excel worksheet: Volume calculator
MAT10MGCT00006

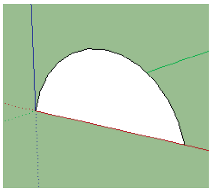
Technology worksheet
Excel spreadsheet: Volume calculator
MAT10MGCT00036

Technology worksheet
Excel worksheet: Volume of a box
MAT10MGCT00007

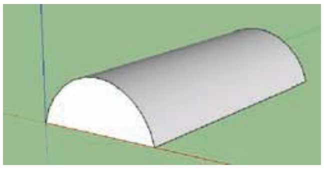
Technology worksheet
Excel spreadsheet: Volume of a box
MAT10MGCT00037

In this activity, we will use Google Sketchup to construct and measure solid shapes.

1 Use the **arc** tool and the **line** tool to create a semicircle.



2 To make a solid, select **Push/Pull**.

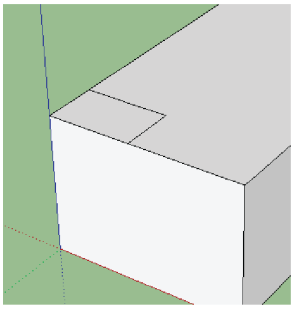


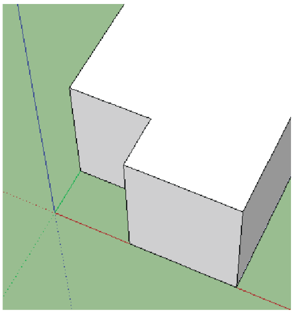
3 Use the **Orbit** tool to reorientate your solid.

4 Use the **Dimension** tool to obtain the dimensions of your half-cylinder. Calculate its surface area and volume.

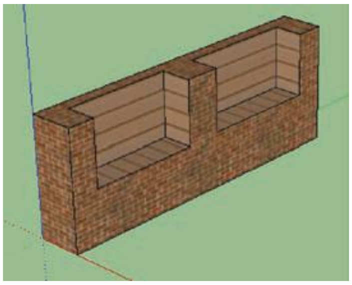
5 Draw a rectangular prism using the **Rectangle** tool and the **Push/Pull** tool.

6 The **Push/Pull** tool can be used to cut away parts of a solid. Use the **Rectangle** tool to create rectangles on the top of the prism. Then use the **Push/Pull** tool to remove it. An example is shown below.



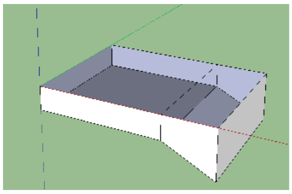


7 Start with a rectangular prism and cut out 2 rectangles to create a seat. Click **Window** and **Materials** to change the appearance of the seat.

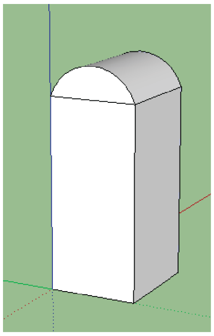


8 Draw each solid shown below and find its surface area and volume.

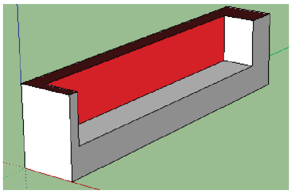
a swimming pool



b a bin



c a bench



3-05 Volumes of prisms and cylinders

The **volume** of a solid is the amount of space it occupies. Volume is measured in **cubic units**, for example, cubic metres (m^3) or cubic centimetres (cm^3).

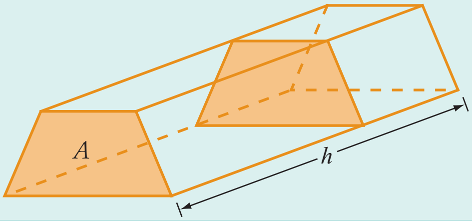
As a **prism** is made up of identical cross-sections, its volume can be calculated by multiplying the **area of its base** by its **perpendicular height** (the length or depth of the prism).

Summary

Volume of a prism

$V = Ah$

where A = area of base and
h = perpendicular height



Worksheet
A page of prisms and cylinders
MAT10MGWK10017

Puzzle sheet
Formula matching game
MAT10MGPS10018

Worksheet
Volumes of solids
MAT10MGWK10020

Worksheet
Back-to-front problems
MAT10MGWK10021

Worksheet
Volume and capacity
MAT10MGWK10022

Animated example
Volumes of shapes
MAT10MGAE00004

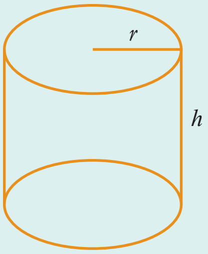
A **cylinder** is like a 'circular prism' because its cross-sections are identical circles. Because of this, we can also use $V = Ah$ to find the volume of a cylinder. But for a circle, $A = \pi r^2$, so:

Summary

Volume of a cylinder

$$V = \pi r^2 h$$

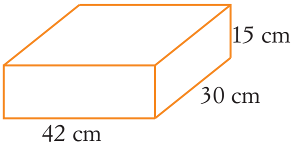
where r = radius of circular base and h = perpendicular height



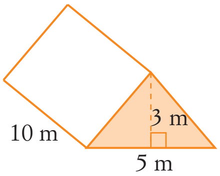
Example 8

Find the volume of each prism.

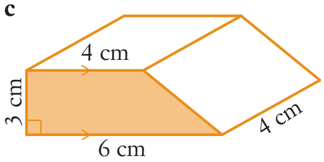
a



b



c



Solution

a $V = 42 \times 30 \times 15$ — For a rectangular prism, volume = length × width × height ($V = lwh$)

$= 18\,900 \text{ cm}^3$

b $A = \frac{1}{2} \times 5 \times 3$ — Area of a triangle

$= 7.5$

$V = 7.5 \times 10$ — $V = Ah$ where height $h = 10$

$= 75 \text{ m}^3$

c $A = \frac{1}{2} \times (4 + 6) \times 3$ — Area of a trapezium

$= 15 \text{ cm}^2$

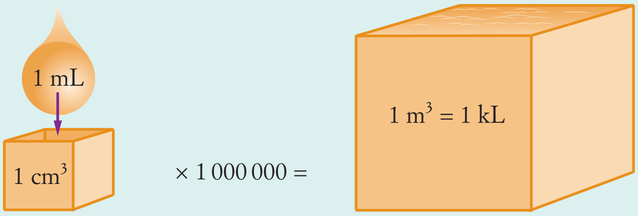
$V = 15 \times 4$ — $V = Ah$ where height $h = 4$

$= 60 \text{ cm}^3$

The **capacity** of a container is the amount of fluid (liquid or gas) it holds, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

Summary

1 cm^3 contains 1 mL.
1 m^3 contains 1000 L or 1 kL

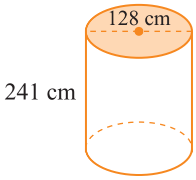


Example 9

For this cylinder, calculate:

a its volume correct to the nearest cm^3

b its capacity in kL, correct to 1 decimal place.



Solution

a Radius $= \frac{1}{2} \times 128$ cm $\qquad \frac{1}{2}$ of diameter

$= 64$ cm

$V = \pi \times 64^2 \times 241 \qquad V = \pi r^2 h$

$= 3\,101\,179.206\ldots$

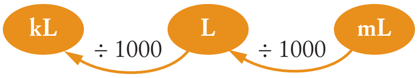
$\approx 3\,101\,179\ \text{cm}^3$

b Capacity $= 3\,101\,179$ mL $\qquad 1\ \text{cm}^3 = 1$ mL

$= (3\,101\,179 \div 1000 \div 1000)$ kL

$= 3.101\,179$ kL

≈ 3.1 kL



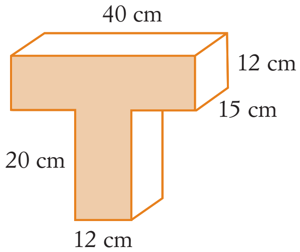
9780170194655

Example 10

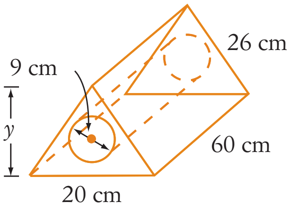
Stage 5.2

Find, correct to the nearest whole number, the volume of each solid.

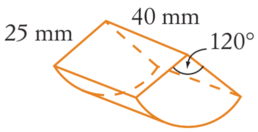
a



b



c



Solution

a $A = 40 \times 12 + 20 \times 12$ Area of T cross-section

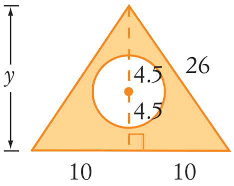
$= 720 \text{ cm}^2$

$V = Ah$

$= 720 \times 15$

$= 10\,800 \text{ cm}^3$

b Cross-section is a triangle minus a circle.



Use Pythagoras' theorem to find *y*.

$26^2 = y^2 + 10^2$

$y^2 = 26^2 - 10^2$

$= 576$

$y = \sqrt{576}$

$= 24 \text{ cm}$

Radius of circle $= \frac{1}{2} \times 9 = 4.5$

$A = \frac{1}{2} \times 20 \times 24 - \pi \times 4.5^2$ Area of triangle − area of circle

$= 176.3827\ldots \text{ cm}^2$ Do not round this partial answer.

$V = Ah$

$= 176.3827\ldots \times 60$

$= 10\,582.9649\ldots$

$\approx 10\,583 \text{ cm}^3$

c $A = \frac{120}{360} \times \pi \times 25^2$ Area of sector

$= 654.498\ldots \text{ mm}^2$ Do not round this partial answer.

$V = Ah$

$= 654.498\ldots \times 40$

$= 26\,179.938\ldots$

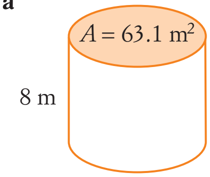
$\approx 26\,180 \text{ mm}^3$

Exercise 3-05 Volumes of prisms and cylinders

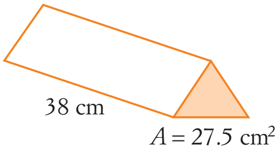
See Example 8

1 Find the volume of each solid, given the shaded area and height.

a



b



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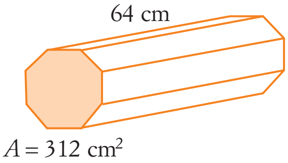
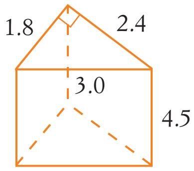


2 Calculate, correct to one decimal place, the volume of each solid. All lengths are in metres.

a



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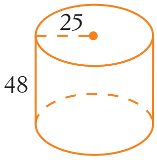

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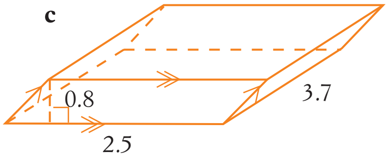

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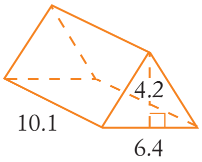

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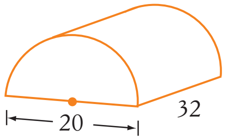

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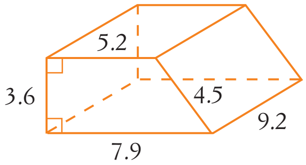

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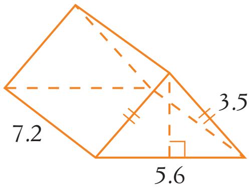

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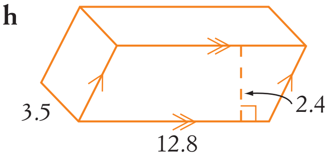
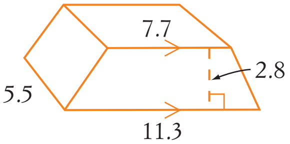


i



See Example 9

3 For each cylinder with the given measurements, calculate:

i its volume, correct to the nearest whole number

ii its capacity

a radius 7 m, height 10 m

b diameter 35 cm, height 15 cm

c diameter 6.2 m, height 7.5 m

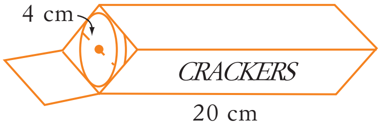
d radius 0.8 cm, height 2.35 cm

4 Rice crackers of diameter 4 cm are packed in a cardboard box of height 20 cm. Calculate, correct to one decimal place:

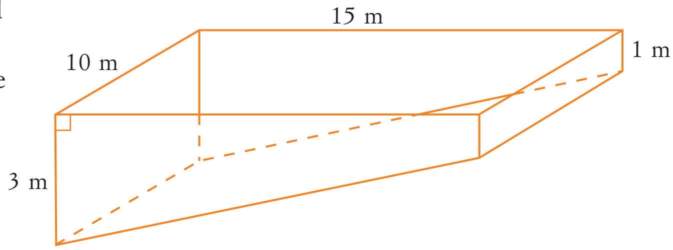
a the volume of the crackers in the box

b the volume of the box

c the percentage of the box that is empty space.



5 This swimming pool is 15 m long and 10 m wide. The depth of the water ranges from 1 m to 3 m. Calculate the capacity of this pool in litres.



6 A wedding cake with three tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total volume of the cake, correct to the nearest cm^3.

Shutterstock.com/John Wollwerth

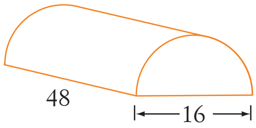
7 A fish tank that is 60 cm long, 30 cm wide and 40 cm high is filled with water to 5 cm below the top. Calculate the volume of the water in litres.

8 Find, correct to two decimal places, the volume of each solid. All lengths shown are in centimetres.

Stage 5.2

See Example 10

a



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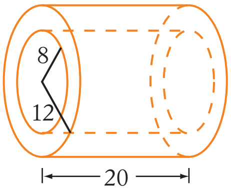

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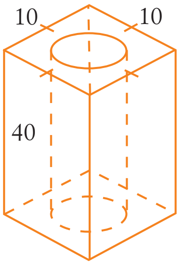
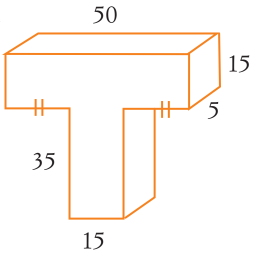


radius of circle = 4 cm

d



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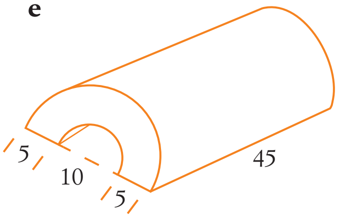

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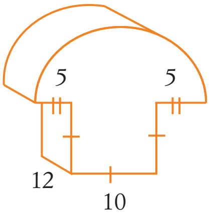

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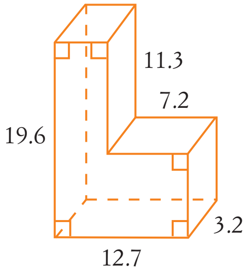

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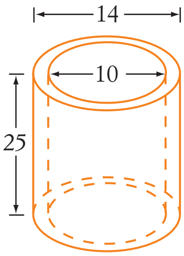

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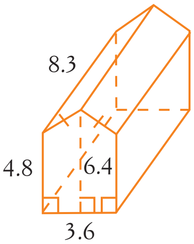
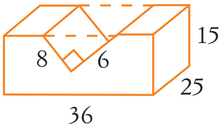


Stage 5.2

j



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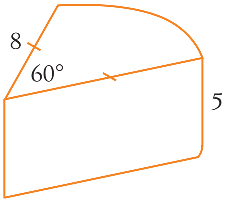

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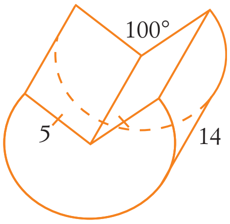
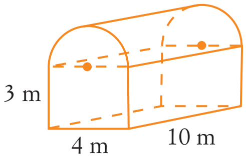


9 **a** Find, correct to two decimal places, the volume of this greenhouse.

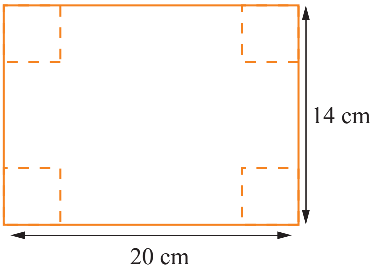
b If this greenhouse costs 0.5c per m^3 per hour to heat, how much is this per day (correct to the nearest cent)?

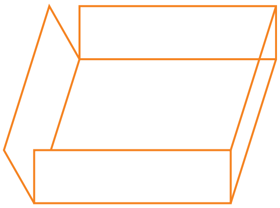


Technology Biggest volume

Worksheet
Biggest volume
MAT10MGWK10019

A rectangular sheet of metal measures 20 cm × 14 cm. Square corners are to be cut from it so that the remaining piece can be folded and welded to form an open tray.





What size must the cut-out squares be for the tray to have the largest possible volume? We will use a spreadsheet to solve this problem.

1 Create this spreadsheet.

	A	B	C	D	E
1	Side of square	Length	Width	Height	Volume
2	0	=20−2*A2	=14−2*A2	=A2	=B2*C2*D2
3					
4					
5					

2 In cell A3, enter the formula =**A2+1**. Use **Fill Down** to copy corresponding formulas into cells A4 to A9.

3 Enter appropriate formulas for cells B3, C3, D3 and E3. Hint: Look at the formulas in row 2.

4 Use **Fill Down** to copy corresponding formulas into rows 4 to 9.

5 The length of the cut-out square cannot be more than 7 cm. Explain why this is so.

6 The spreadsheet suggests that a cut-out square length of 3 cm will give the biggest volume. Test values above and below 3 cm (correct to one decimal place) to see whether you can find a bigger volume.

7 What changes would we need to make to the spreadsheet if the starting dimensions were different?

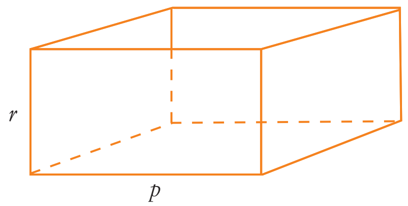
Power plus

1 The total surface area of a cube is 864 cm^2. Find its volume.

2 A cylinder has a volume of 3619.11 cm^3. Its height is 18 cm. Calculate the radius of its base.

3 Find a formula for the surface area, SA, of each solid.
  a A square prism of base length p and height r.
  b A cylinder of diameter and height x.

a



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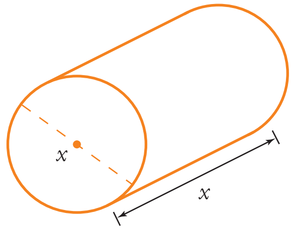
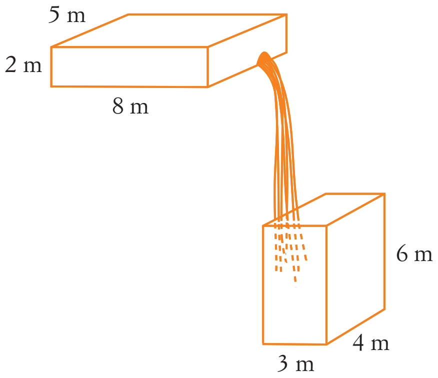
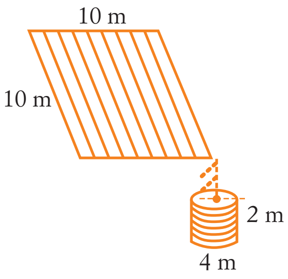


4 The surface area of the curved surface of a can is 27 143.4 mm^2. If its height is 120 mm, find the radius of the can.

5 Water flows from the top tank to the bottom tank at a constant rate. The level of water in the top tank falls at a rate of 15 cm/h. At what rate is the level of water rising in the bottom tank?



6 A 10 m flat square roof drains into a cylindrical rainwater tank with a diameter of 4 m. If 5 mm of rain falls on the roof, by how much (to the nearest millimetre) does the level of the water in the tank rise?



Chapter 3 review

Language of maths

Puzzle sheet

Surface area and volume crossword

MAT10MGPS10023

annulus
area
base
capacity
circle
circumference
cross-section
cubic
curved surface
cylinder
diameter
external
kilolitre
litre
net
open
perpendicular height
prism
quadrant
radius
sector
solid
surface area
volume

1 Which word means a ‘slice’ of a prism or cylinder?

2 What is the formula for the curved surface area of a cylinder?

3 What is the formula $V = \pi r^2 h$ used for?

4 What is the difference between **volume** and **capacity**?

5 What is an **annulus**?

6 What type of measurement has units of **cubic metres**?

Topic overview

Quiz

Area and volume

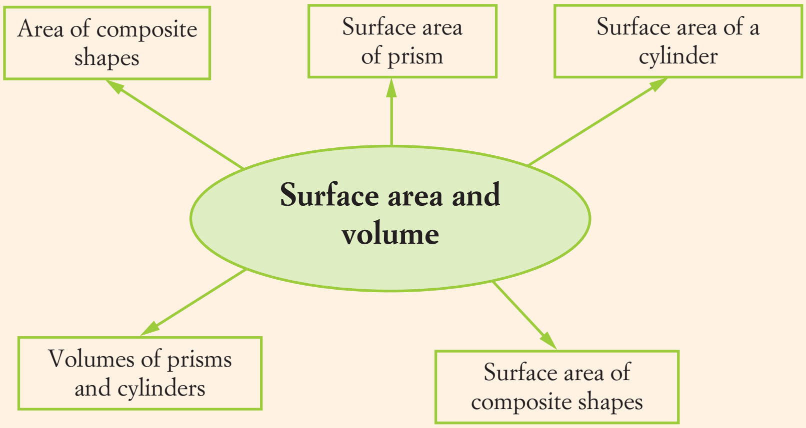
MAT10MGQZ00004

Copy and complete the table below.

🙂	The best part of this chapter was …
☹	The worst part was …
😮	New work …
😕	I need help with …

Chapter 3 review

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

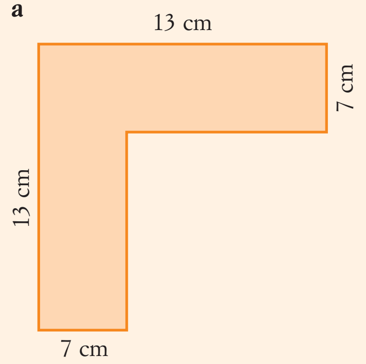


Chapter 3 revision

See Exercise 3-01

1 Find the area of each shape. Give your answers correct to one decimal place where necessary.

a



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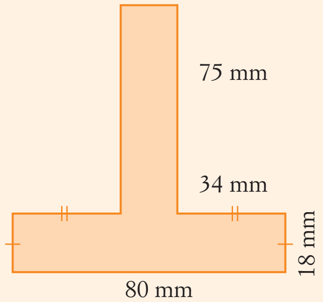

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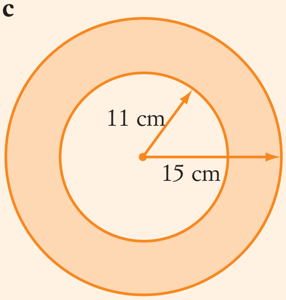

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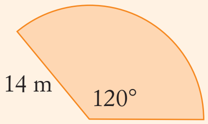

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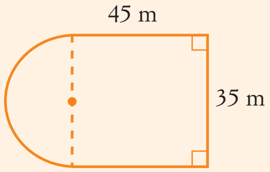

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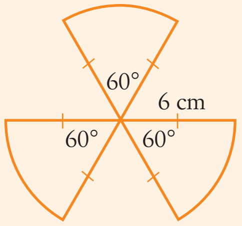
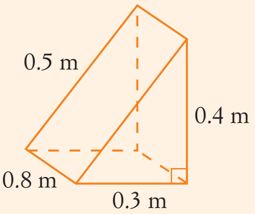


See Exercise 3-02

2 Find the surface area of each prism.

a



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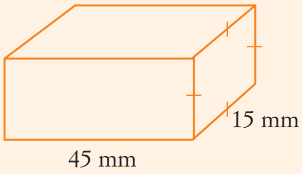

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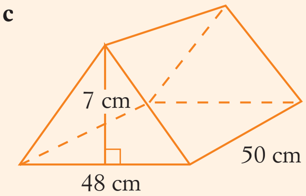

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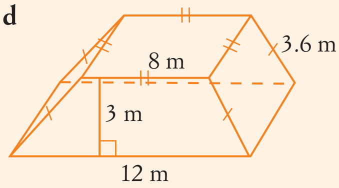

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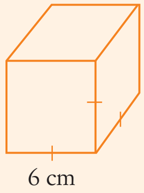

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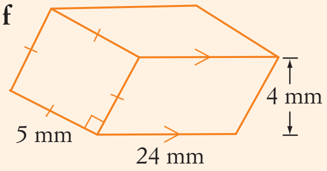
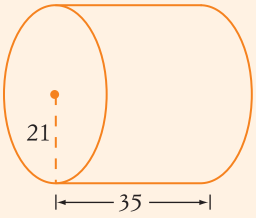


Stage 5.2

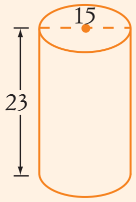
See Exercise 3-03

3 Calculate, correct to one decimal place, the surface area of each solid. All lengths shown are in metres.

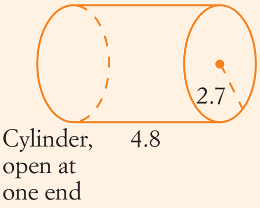
a



b



c

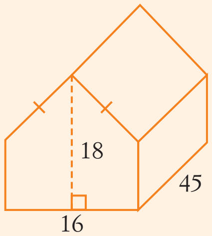


Stage 5.2

See Exercise 3-04

4 Calculate, correct to nearest square centimetre, the surface area of each solid. All lengths shown are in centimetres.

a



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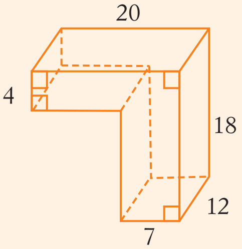

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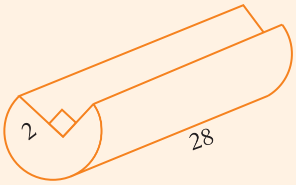

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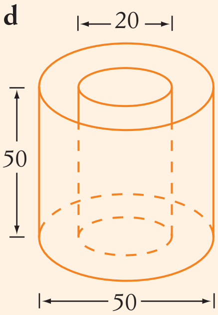

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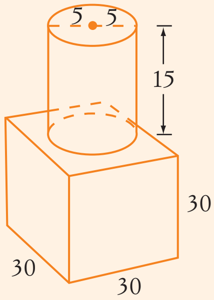

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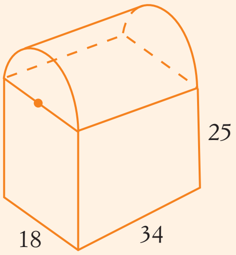
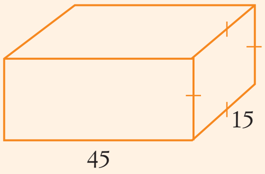


See Exercise 3-05

5 Calculate, correct to nearest cubic metre, the volume of each solid. All lengths shown are in metres.

a



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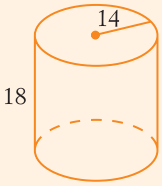

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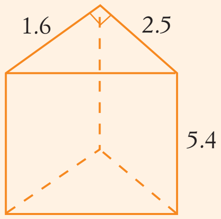
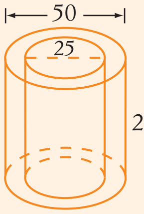


Stage 5.2

d



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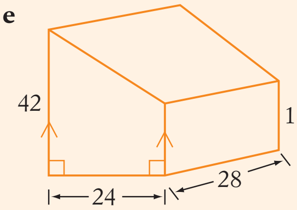
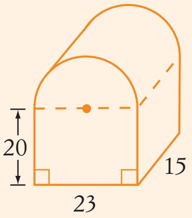


f



6 A rectangular fish tank measures 75 cm long by 55 cm wide by 35 cm deep. Find the capacity of the tank in litres if it is filled to 15 cm from the top.

See Exercise 3-05

7 A cylindrical rain water tank has a radius of 2.8 m and a height of 2.4 m.

See Exercise 3-05

a Calculate, correct to two decimal places, the capacity of the tank in kilolitres.

b If the tank is 60% full, what is the height of the water in the tank? Answer correct to two decimal places.

4

Number and Algebra

Algebra

Rubik's Cube is a puzzle that was invented by Hungarian Professor of Architecture Erno Rubik in 1974. In Hungary, it was originally called the Magic Cube. The cube has 8 vertices and 12 edges, and $2^{10} \times 3^7 \times 8! \times 12! \approx$ 43 252 003 274 489 856 000 different possible arrangements (8! or '8 factorial' means $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$). However, the cube has been solved in as little as 20 moves in 6 seconds!

Shutterstock.com/bin255

Chapter outline

	Proficiency strands			
4-01 The index laws	U	F	R	C
4-02 Adding and subtracting algebraic fractions*	U	F	R	C
4-03 Multiplying and dividing algebraic fractions*	U	F	R	C
4-04 Expanding and factorising expressions	U	F	R	C
4-05 Expanding binomial products*	U	F	R	C
4-06 Factorising quadratic expressions*	U	F	R	C

*STAGE 5.2

Wordbank

binomial An algebraic expression that consists of two terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product Binomials multiplied together, for example, $(x + 9)(3x - 4)$

expand To rewrite an expression such as $5(2k - 6)$ without grouping symbols; for example, $5(2k - 6)$ expands to $10k - 30$

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor. Factorising is the opposite of expanding; for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

index laws Rules for simplifying algebraic expressions involving powers of the same base; for example, $a^m \div a^n = a^{m-n}$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $2x^2 + 5x - 3$ or $x^2 + 2$

In this chapter you will:

- apply index laws to numerical expressions with integer indices
- (STAGE 5.2) apply index laws to algebraic expressions with integer indices
- simplify algebraic products and quotients using index laws
- (STAGE 5.2) expand binomial products and factorise monic quadratic expressions using a variety of strategies
- interpret and use zero and negative indices
- (STAGE 5.2) add, subtract, multiply and divide simple algebraic fractions
- expand and factorise algebraic expressions
- (STAGE 5.2) expand and factorise algebraic expressions involving terms with indices

SkillCheck

Worksheet
StartUp assignment 4
MAT10NAWK10024

1 Simplify each expression.

a $g^4 \times g^5$ **b** $r^8 \div r^2$ **c** $(d^5)^3$ **d** $(-k)^2$
e $h \times h^9$ **f** $m^5 \div m$ **g** a^1 **h** a^0
i $3e^2 \times 2e^5$ **j** $18n^6 \div 6n^2$ **k** $(10w^3)^3$ **l** $25q^0$
m $(vw)^5$ **n** $\left(\frac{v}{w}\right)^3$ **o** y^{-1} **p** y^{-2}

2 Evaluate each expression.

a $\frac{a}{5} + \frac{3a}{4}$ **b** $\frac{7p}{2} - \frac{10p}{3}$ **c** $\frac{8}{t} \times \frac{5t}{24}$ **d** $\frac{x}{14} \div \frac{y}{2}$

3 Expand each expression.

a $6m(3m + 11)$ **b** $-5(3g - 8)$

4 Factorise each expression.

a $4x + 24$ **b** $20 - 15a$ **c** $q^2 + q$
d $18a^2 - 12a$ **e** $-2y - 30$ **f** $-18w + 24$

5 Find two numbers whose:

a product is 18 and sum is 9 **b** product is 8 and sum is −6
c product is −20 and sum is −1 **d** product is −16 and sum is 6

4-01 The index laws

Worksheet
Index laws review
MAT10NAWK10025

Animated example
Index laws
MAT10NAAE00001

Video tutorial
Numbers and powers
MAT10NAVT00001

Index is another name for **power**. The plural of index is **indices** (pronounced 'in-de-sees'). The following rules are called **index laws**.

Summary

When **multiplying terms with the same base**, add the powers

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with the same base**, subtract the powers

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

When **raising a term with a power to another power**, multiply the powers

$$(a^m)^n = a^{m \times n}$$

When **raising a product of terms to a power**, raise each term to that power

$$(ab)^n = a^n b^n$$

When **raising a quotient of terms to a power**, raise each term to that power

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Any number **raised to the power of zero** is equal to 1

$$a^0 = 1$$

A number **raised to a negative power** gives a fraction (with a numerator of 1)

$$a^{-n} = \frac{1}{a^n}$$

A number **raised to a power of −1** gives its reciprocal

$$a^{-1} = \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

Example 1

Simplify each expression.

a $5p^2q^3 \times 4p^3q^4$ **b** $\dfrac{24e^6n^{12}}{8en^4}$ **c** $(3t^5)^2$

d $(4d^4q^2r)^3$ **e** $\left(\dfrac{2c^2}{d}\right)^4$ **f** $31x^0 - (31x)^0$

Solution

a
$$\begin{aligned}5p^2q^3 \times 4p^3q^4 &= (5 \times 4)p^{2+3}q^{3+4}\\ &= 20p^5q^7\end{aligned}$$

b
$$\begin{aligned}\frac{24e^6n^{12}}{8en^4} &= \frac{{}^{3}\cancel{24}e^6n^{12}}{{}_{1}\cancel{8}en^4}\\ &= 3e^{6-1}n^{12-4}\\ &= 3e^5n^8\end{aligned}$$

c
$$\begin{aligned}(3t^5)^2 &= 3^2 \times t^{5\times 2}\\ &= 9t^{10}\end{aligned}$$

d
$$\begin{aligned}(4d^4q^2r)^3 &= 4^3d^{4\times 3}q^{2\times 3}r^3\\ &= 64d^{12}q^6r^3\end{aligned}$$

e
$$\begin{aligned}\left(\frac{2c^2}{d}\right)^4 &= \frac{(2c^2)^4}{d^4}\\ &= \frac{2^4c^{2\times 4}}{d^4}\\ &= \frac{16c^8}{d^4}\end{aligned}$$

f
$$\begin{aligned}31x^0 - (31x)^0 &= 31 \times 1 - 1\\ &= 30\end{aligned}$$

Stage 5.2

Example 2

Simplify each expression.

a $9x^{-1}$ **b** $(4q)^{-3}$ **c** $\left(\dfrac{2}{3}\right)^{-1}$

Solution

a
$$\begin{aligned}9x^{-1} &= 9 \times \frac{1}{x}\\ &= \frac{9}{x}\end{aligned}$$

b
$$\begin{aligned}(4q)^{-3} &= \frac{1}{(4q)^3}\\ &= \frac{1}{64q^3}\end{aligned}$$

c
$$\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$$

Exercise 4-01 The index laws

See Example 1

1 Simplify each expression.

a $3p^2 \times 2p^5$ **b** $20w^9 \div 4w^3$ **c** $(-m^3)^{10}$

d $(3q^3)^2$ **e** $5n^8t \times 6n^8t^4$ **f** $32x^6y^8 \div 8xy^4$

g $5e^4g^3 \times (-e^6g)$ **h** $\dfrac{45a^{15}b^5}{-5a^5}$ **i** $(10y^{10})^2$

j $(-4p)^3$ **k** $\dfrac{36pq^3r^6}{24qr^2}$ **l** $9u^3v \times 6uv^2w^8$

2 Simplify each expression.

a $(l^3m^5)^6$ **b** $\left(\frac{n}{2}\right)^3$ **c** $7x^0$

d $\left(\frac{w^2}{t^3}\right)^5$ **e** $\left(\frac{2}{3}\right)^0$ **f** $(-8ky^5)^2$

g $(16a)^0 - 16a^0$ **h** $\left(\frac{2b}{3d}\right)^4$ **i** $(-5d^3y^5)^3$

j $\left(-\frac{3k^4}{10}\right)^3$ **k** $-9(a^2b^3)^0$ **l** $(3p^2q^3r^4)^4$

3 Evaluate each expression.

a 4^0 **b** $(-4)^0$ **c** 7×2^0 **d** $(7 \times 2)^0$

e $(-2)^3$ **f** $(-3)^2$ **g** $(5^2)^2$ **h** $2^4 \times 2^3$

i $(7^2)^0$ **j** $4^5 \div 4^2$ **k** $4^2 \div 4^5$ **l** $10^3 \div 10^3$

m $5^2 \div 5^0$ **n** $10^{-2} \div 10^2$ **o** $\left(\frac{1}{2}\right)^0$ **p** $10^{-2} \times 10^2$

4 Express each of the terms below as fractions.

a 5^{-2} **b** 2^{-5} **c** 20^{-1} **d** 10^{-3}

Stage 5.2

See Example 2

5 Simplify each expression using a positive index.

a 8^{-7} **b** 3^{-5} **c** y^{-1} **d** x^{-3}

e $(5b)^{-2}$ **f** $5b^{-2}$ **g** $(ab)^{-1}$ **h** ab^{-1}

i $11t^{-3}$ **j** $(11t)^{-3}$ **k** p^3q^{-5} **l** mw^{-3}

m $8u^{-3}v^{-4}$ **n** $-2r^6y^{-5}$ **o** $10e^{-1}f^3$ **p** $\frac{1}{2}k^{-4}n^7$

6 Simplify each expression.

a $10r^{-6}$ **b** $\left(\frac{5}{2}\right)^{-1}$ **c** $\left(\frac{1}{3}\right)^{-1}$ **d** $\left(\frac{1}{x}\right)^{-1}$

e $4h^{-2}$ **f** $\left(\frac{k}{3}\right)^{-1}$ **g** $(4h)^{-2}$ **h** $\left(-\frac{m}{2}\right)^{-1}$

i $\left(\frac{5}{3g^3}\right)^{-1}$ **j** $\left(\frac{2r}{3t}\right)^{-1}$ **k** m^3np^{-2} **l** $\left(\frac{5b}{4a^2}\right)^{-1}$

7 Simplify each expression.

a $(10x^{10}y)^3 \times 5x^{-2}y^3$ **b** $(10x^{10}y \div 5x^{-2}y^3)^3$ **c** $(3q^{-5}r^3 \div 6qr^2)^2$

d $3q^{-5}r^3 \div (6qr^2)^2$ **e** $\left(\frac{4a^5x^6}{a^6x^2}\right)^4$ **f** $\left(\frac{4a^5x^6}{a^6x^2}\right)^{-4}$

g $(4p^{-3}h)^2 \times (-2p^6h^9)$ **h** $(4p^{-3}h)^2 \div (-2p^6h^9)$ **i** $[4p^3h \times (-2p^2h^9)]^2$

Investigation: Videos going viral

Use a spreadsheet to help you with this investigation.

With the invention of social media, we can now communicate with each other instantly. However, what often starts off as a simple message between friends or an online video posting can quickly multiply at an alarming rate and 'go viral' (spread like a virus). Suppose a video is shared with a friend who 10 seconds later shares it with two other friends. This occurs every 10 seconds so that every 10 seconds the number of views doubles.

1 Use a spreadsheet to calculate the total number of views after:

a 30 s **b** 40 s **c** 1 min **d** 1 min 30 s
e 2 min **f** 3 min **g** 4 min **h** 5 min

2 Find how long it will take until the total number of views reaches:

a 64 **b** 256 **c** over 500 **d** over 1000
e over 3000 **f** over 10 000 **g** over 1 million **h** over 4 million

3 Use the spreadsheet's **Graph Option** to graph the viral pattern. Describe the shape of the graph.

9780170194655

4-02 Adding and subtracting algebraic fractions

Stage 5.2

Summary

To **add or subtract fractions**, convert them (if needed) so that they will have the same denominator, then simply add or subtract the numerators.

Example 3

Video tutorial
Algebraic fractions
MAT10NAVT10007

Simplify each expression.

a $\frac{a}{2}+\frac{a}{3}$ **b** $\frac{2x}{5}-\frac{x}{3}$ **c** $\frac{5}{6}+\frac{7b}{15}$

Solution

a
$$\begin{aligned}\frac{a}{2}+\frac{a}{3}&=\frac{3\times a}{3\times 2}+\frac{2\times a}{2\times 3}\\&=\frac{3a}{6}+\frac{2a}{6}\\&=\frac{5a}{6}\end{aligned}$$

Common denominator $= 2 \times 3 = 6$

b
$$\begin{aligned}\frac{2x}{5}-\frac{x}{3}&=\frac{3\times 2x}{3\times 5}-\frac{5\times x}{5\times 3}\\&=\frac{6x}{15}-\frac{5x}{15}\\&=\frac{x}{15}\end{aligned}$$

c
$$\begin{aligned}\frac{5}{6}+\frac{7b}{15}&=\frac{25}{30}+\frac{14a}{30}\\&=\frac{25+14a}{30}\end{aligned}$$

The lowest common denominator is 30.

Stage 5.2

Exercise 4-02 Adding and subtracting algebraic fractions

See Example 3

1 Simplify each expression.

a $\frac{h}{5}+\frac{2h}{5}$ **b** $\frac{7m}{13}-\frac{2m}{13}$ **c** $\frac{4}{x}+\frac{5}{x}$ **d** $\frac{7}{3d}-\frac{2}{3d}$

e $\frac{n}{2}-\frac{n}{7}$ **f** $\frac{3c}{2}+\frac{c}{5}$ **g** $\frac{4r}{7}+\frac{r}{2}$ **h** $\frac{7y}{8}-\frac{5y}{3}$

i $\frac{4t}{3}+\frac{t}{9}$ **j** $\frac{y}{16}-\frac{3y}{8}$ **k** $\frac{11t}{12}-\frac{5t}{9}$ **l** $\frac{3a}{10}+\frac{4a}{15}$

m $\frac{p}{z}+\frac{3}{z}$ **n** $\frac{5u}{8g}-\frac{3u}{8g}$ **o** $\frac{4}{9f}-\frac{1}{9f}$ **p** $\frac{7e}{10}-\frac{3e}{10}$

q $\frac{5a}{6}+\frac{5a}{6}$ **r** $\frac{5}{2d}-\frac{1}{2d}$ **s** $\frac{7}{5k}+\frac{13}{5k}$ **t** $\frac{12}{4a}-\frac{8}{4a}$

2 Simplify each expression.

a $\frac{1}{3}+\frac{x}{4}$ **b** $\frac{s}{2}-\frac{2}{7}$ **c** $\frac{a}{5}+\frac{b}{3}$ **d** $\frac{p}{3}+\frac{q}{2}$

e $\frac{m}{8}-\frac{n}{5}$ **f** $\frac{5t}{4}-\frac{2t}{5}$ **g** $\frac{2a}{15}+\frac{3h}{10}$ **h** $\frac{d}{16}-\frac{r}{24}$

i $\frac{3c}{2}-\frac{c}{5}$ **j** $\frac{2d}{11}-\frac{r}{3}$ **k** $\frac{3h}{5}+\frac{2a}{3}$ **l** $\frac{5}{6}+\frac{4w}{5}$

m $\frac{3}{4}+\frac{2a}{7}$ **n** $\frac{7e}{8}-\frac{2e}{3}$ **o** $\frac{m}{2}-\frac{n}{7}$ **p** $\frac{2k}{5}+\frac{k}{6}$

Worksheet
Algebraic fractions
MAT10NAWK10026

4-03 Multiplying and dividing algebraic fractions

Summary

- To **multiply fractions**, cancel any common factors, then multiply the numerators and denominators separately.
- To **divide by a fraction** $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.

Stage 5.2

Video tutorial

Algebraic fractions

MAT10NAVT10007

Example 4

Simplify each expression.

a $\frac{3x}{4}\times\frac{2x}{9}$ **b** $\frac{4}{k}\times\frac{3k}{16}$ **c** $\frac{2}{v}\div\frac{3}{w}$ **d** $\frac{xy}{5}\div\frac{3x}{25}$

Solution

a $$\begin{aligned}\frac{3x}{4}\times\frac{2x}{9} &= \frac{3x\times 2x}{4\times 9}\\ &= \frac{6x^2}{36}\\ &= \frac{x^2}{6}\end{aligned}$$

or $$\begin{aligned}\frac{3x}{4}\times\frac{2x}{9} &= \frac{{}^{1}\cancel{3}x}{{}_{2}\cancel{4}}\times\frac{{}^{1}\cancel{2}x}{\cancel{9}_{3}}\\ &= \frac{x^2}{6}\end{aligned}$$

b $$\begin{aligned}\frac{4}{k}\times\frac{3k}{16} &= \frac{{}^{1}\cancel{4}}{\cancel{k}}\times\frac{3\cancel{k}}{\cancel{16}_{4}}\\ &= \frac{3}{4}\end{aligned}$$

c $$\begin{aligned}\frac{2}{v}\div\frac{3}{w} &= \frac{2}{v}\times\frac{w}{3}\\ &= \frac{2w}{3v}\end{aligned}$$

d $$\begin{aligned}\frac{xy}{5}\div\frac{3x}{25} &= \frac{\cancel{x}y}{\cancel{5}_{1}}\times\frac{\cancel{25}^{5}}{3\cancel{x}}\\ &= \frac{5y}{3}\end{aligned}$$

Exercise 4-03 Multiplying and dividing algebraic fractions

See Example 4

1 Simplify each product.

a $\frac{w}{3}\times\frac{1}{2}$ **b** $\frac{s}{5}\times\frac{t}{4}$ **c** $\frac{3}{h}\times\frac{5}{k}$ **d** $\frac{4}{m}\times\frac{3}{n}$

e $\frac{d}{3}\times\frac{9}{e}$ **f** $\frac{12}{v}\times\frac{2}{3}$ **g** $\frac{2x}{3}\times\frac{9}{8y}$ **h** $\frac{6}{5b}\times\frac{10b}{18}$

i $\frac{d}{e}\times\frac{e}{g}$ **j** $\frac{4ad}{9}\times\frac{3d}{16a}$ **k** $\frac{5p}{4r}\times\frac{8r}{15p}$ **l** $\frac{20a}{3k}\times\frac{9a}{5k}$

2 Simplify each quotient.

a $\frac{x}{2}\div\frac{y}{5}$ **b** $\frac{t}{18}\div\frac{r}{3}$ **c** $\frac{k}{2}\div\frac{k}{8}$ **d** $\frac{h}{k}\div\frac{k}{h}$

e $\frac{3y}{5}\div\frac{2y}{d}$ **f** $\frac{q}{4}\div\frac{3q}{4}$ **g** $\frac{4t}{9}\div\frac{3t}{4}$ **h** $\frac{5m}{n}\div\frac{2m}{3n}$

i $\frac{8a}{3c}\div\frac{2a}{9c}$ **j** $\frac{3}{2ab}\div\frac{b}{6a}$ **k** $\frac{3p^2}{2}\div\frac{p}{4}$ **l** $\frac{8y}{5}\div\frac{32y^2}{3g}$

Stage 5.2

3 Simplify each expression.

a $\frac{xy}{z} \times \frac{5}{y}$ **b** $\frac{a}{b} \times \frac{c}{b} \times \frac{b}{3a}$ **c** $\frac{5}{2g} \div \frac{2}{g}$ **d** $5 \div \frac{3}{5b}$

e $\frac{4t}{9} \div 3$ **f** $\frac{5mn}{2d} \times \frac{4d}{n} \times \frac{1}{15mn}$ **g** $5p \div \frac{7}{10pt}$ **h** $\frac{s}{3} \div \frac{5s}{2} \times \frac{3s}{7}$

i $\frac{7}{h} \times \frac{1}{42h}$ **j** $\frac{5ty}{k} \times \frac{5ky}{t}$ **k** $\frac{6}{r} \times \frac{5r}{9} \div \frac{15}{yh}$ **l** $\frac{6cf}{q} \div \frac{5}{qf} \div \frac{10c}{a}$

Worksheet
Expanding and factorising
MAT10NAWK10027

Puzzle sheet
The distributive law
MAT10NAPS00008

Skillsheet
Algebra using diagrams
MAT10NASS10009

Skillsheet
HCF by factor trees
MAT10NASS10010

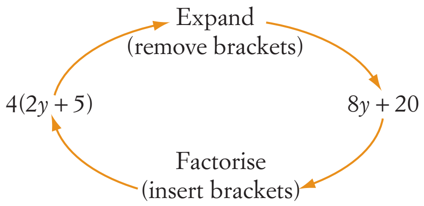
Skillsheet
Factorising using diagrams
MAT10NASS10011

Technology
GeoGebra: Simplifying expressions
MAT10NATC00003

Worksheet
Algebra 4
MAT10NAWK00009

4-04 Expanding and factorising expressions

Expanding and **factorising** are inverse operations.
When $4(2y + 5)$ is **expanded**, the answer is $8y + 20$.
When $8y + 20$ is **factorised**, the answer is $4(2y + 5)$.



Summary

Expanding an expression

Multiply each term inside the brackets by the term outside.

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Example 5

Expand each expression.

a $2d(3d + 10)$ **b** $-3(2t - 7)$

Solution

a $2d(3d + 10) = 2d \cdot 3d + 2d \cdot 10$
$= 6d^2 + 20d$

Multiplying out the brackets.

b $-3(2t - 7) = -3 \cdot 2t - (-3) \cdot 7$
$= -6t - (-21)$
$= -6t + 21$

Multiplying out the brackets.

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Stage 5.2

Example 6

Expand and simplify by collecting like terms.

a $-3r^2(4r + 2) - 5r^3$ **b** $9(m - 3) + m(m - 10)$

Solution

a $-3r^2(4r + 2) - 5r^3 = -3r^2 \times 4r + (-3r^2) \times 2 - 5r^3$ Expanding.

$= -12r^3 + (-6r^2) - 5r^3$

$= -17r^3 - 6r^2$ Collecting like terms to simplify.

b $9(m - 3) + m(m - 10) = 9(m - 3) + m(m - 10)$

$= 9 \times m - 9 \times 3 + m \times m - m \times 10$

$= 9m - 27 + m^2 - 10m$

$= m^2 + 9m - 10m - 27$ It's neater to place m^2 at the front.

$= m^2 - m - 27$ Collecting like terms to simplify.

Summary

Factorising an expression

- Find the HCF of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets
  $ab + ac = a(b + c)$ $ab - ac = a(b - c)$
- To check that the factorised answer is correct, expand it

Example 7

Factorise each expression.

a $25b^2 - 20ab$ **b** $x(4 + y) + 2(4 + y)$ **c** $-b^2 + 8b$

Solution

a The HCF of $25b^2$ and $20b$ is $5b$.

$\therefore 25b^2 - 20b = 5b \times 5b - 5b \times 4$ Rewrite the expression using the HCF $5b$.

$= 5b(5b - 4)$ Write the HCF at the front of the brackets.

b The HCF of $x(4 + y) + 2(4 + y)$ is $(4 + y)$.

$\therefore x(4 + y) + 2(4 + y) = (4 + y) \times x + (4 + y) \times 2$

$= (4 + y)(x + 2)$

c When factorising expressions that begin with a **negative** term, we use the 'negative' HCF.
The highest 'negative' common factor of $-b^2$ and $8b$ is $-b$.

$$\therefore -b^2 + 8b = (-b) \times b + (-b) \times (-8)$$
$$= (-b)[b + (-8)]$$
$$= -b(b - 8)$$

$(-b) \times (-8) = +8b$

Stage 5.2

Example 8

Factorise each expression.

a $8a^3 + 4a^2$ **b** $20h^3k + 25h^4k - 10h^2k$

Solution

a The HCF of $8a^3$ and $4a^2$ is $4a^2$.

$$\therefore 8a^3 + 4a^2 = 4a^2 \times 2a + 4a^2 \times 1$$
$$= 4a^2(2a + 1)$$

Rewrite the expression using the HCF $4a^2$.

b The HCF is $5h^2k$.

$$\therefore 20h^3k + 25h^4k - 10h^2k = 5h^2k \times 4h + 5h^2k \times 5h^2 - 5h^2k \times 2.$$
$$= 5h^2k(4h + 5h^2 - 2)$$

Exercise 4-04 Expanding and factorising expressions

See Example 5

1 Expand each expression.

a $4(h + 6)$ **b** $-3(r + 10)$ **c** $7(x - 9y)$ **d** $-4(a - 5z)$
e $-(2 - t^2)$ **f** $-10e(2e + 3)$ **g** $6y(1 + 7y)$ **h** $4xy(3xy - 1)$
i $8rt(2t - r)$ **j** $3ab(4b - 7a)$ **k** $-6h^2(1 - 3h)$ **l** $-5x(5x^2 + 4y)$

2 Expand $-5u(8 + 2u)$. Select the correct answer **A**, **B**, **C** or **D**.

A $3u - 3u^2$ **B** $-40u + 10u^2$ **C** $-40u - 10u^2$ **D** $-40u - 3u^2$

3 Use the substitution $x = 2$ to test whether each equation is correct or incorrect.

a $4(x + 10) = 4x + 40$ **b** $5(x - 1) = 5x - 6$ **c** $x(3 - x) = 3x - x^2$

Stage 5.2

See Example 6

4 Expand and simplify by collecting like terms.

a $7m(2m + 3) + m^2$ **b** $-3e(1 - 5e) - 6e$ **c** $9w^3 - 3w(5 + 2w^2)$
d $24x^3 - 5x^2(2x^2 - 5x)$ **e** $t(t + 4) + 3(t + 4)$ **f** $4(3 - h) - h(7 + 2h)$
g $3x^2(2x + 5) + 4(2 + 5x^2)$ **h** $-v(2v + 3) + 6(v + 1)$ **i** $3(1 - 2w) - w(2 - w)$
j $3y(3y - 7) - 5(3y - 7)$ **k** $3m(m + 5m^2) - m^2(1 - m)$ **l** $4x(2y + 5) - 6y(10 - 2x)$

See Example 7

5 Factorise each expression.

a $24x + 30$ **b** $36 - 27a$ **c** $x^2 + x$
d $30y - 20y^2$ **e** $36d^2 + 24d$ **f** $16r^2 - 12r$
g $a(a - 3) + 6(a - 3)$ **h** $t(8 + t) - 3(8 + t)$ **i** $b(3b + 5) - 2(3b + 5)$
j $-q^2 - 36q$ **k** $-6t + 10t^2$ **l** $-3y^2 - 6xy$
m $-hn^2 + h^2n$ **n** $-20e^2 - 22e$ **o** $45m^2 - 54m$

9780170194655

6 Factorise $-10kr + 4rn$. Select the correct answer **A**, **B**, **C** or **D**.

A $-2r(5k - 4n)$ **B** $-2r(5k - 2n)$ **C** $-5r(2k - 2n)$ **D** $-2r(5k + 2n)$

7 Factorise each expression.

Stage 5.2

See Example 8

a $12x^2y - 16xy$ **b** $18p^2r + 16pr$ **c** $36m^2n - 108mn^2$
d $36ab^2c - 144bc$ **e** $48v^2w + 64vw^2$ **f** $75g^3h^2 - 125gh$
g $-4p^3 - 8p^2 + p$ **h** $6mn^2 + 3mn + 48m^2n$ **i** $32p^3g + 8pg^2 - 8pg$
j $18a^5 - 12a^2 + 15a^4$

Just for the record CAPTCHA

For security purposes on an Internet website, have you ever been asked to enter letters, words or numbers that have been displayed in a wavy, difficult-to-read format like the one below?

Type the code shown

This process is called **CAPTCHA**, which stands for **Completely Automated Public Turing Test to Tell Computers and Humans Apart**. It is designed to ensure that the person accessing the website is a human and not another computer that could be hacking into the site, sending spam or viruses. CAPTCHA uses optical character recognition (OCR), which is a technology that can convert images of text into editable text.

Mental skills 4 Maths without calculators

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

a
$$\begin{aligned} 55 + 132 - 34 + 17 - 78 &\approx 60 + 130 - 30 + 20 - 80 \\ &= (60 + 20 - 80) + (130 - 30) \\ &= 0 + 100 \\ &= 100 \text{ (Actual answer} = 92) \end{aligned}$$

b
$$\begin{aligned} 78 \times 7 &\approx 80 \times 7 \\ &= 560 \text{ (Actual answer} = 546) \end{aligned}$$

c
$$\begin{aligned} 510 \div 24 &\approx 500 \div 20 \\ &= 50 \div 2 \\ &= 25 \text{ (Actual answer} = 21.25) \end{aligned}$$

2 Now estimate each answer.

a $27 + 11 + 87 + 142 + 64$
b $55 + 34 - 22 - 46 + 136$
c $684 + 903$
d $35 + 81 + 110 + 22 + 7$
e $517 - 96$
f $210 - 38 - 71 + 151 - 49$
g $766 - 353$
h 367×2
i 83×81
j 984×16
k $828 \div 3$
l $507 \div 7$

3 Study each example involving decimals.

a
$$\begin{aligned} 20.91 - 11.3 + 2.5 &\approx 21 - 11 + 3 \\ &= 13 \text{ (Exact answer} = 12.11) \end{aligned}$$

b
$$\begin{aligned} 4.78 \times 19.2 &\approx 5 \times 20 \\ &= 100 \text{ (Exact answer} = 91.776) \end{aligned}$$

c
$$\begin{aligned} \frac{37.6 + 9.3}{41.2 - 12.7} &\approx \frac{38 + 9}{40 - 13} \\ &= \frac{47}{27} \\ &\approx \frac{50}{30} \\ &\approx 1.6 \text{ (Exact answer} = 1.6456\ldots) \end{aligned}$$

4 Now estimate each answer.

a $3.75 + 9.381 + 4.6 + 10.5$
b $14.807 + 6.6 - 7.22$
c 18.47×9.61
d 4.27×97.6
e $\dfrac{11.07 + 18.4}{12.2}$
f $\dfrac{38.18}{17.2 - 9.6}$
g $\dfrac{18.46 \times 4.9}{39.72 - 15.2}$
h $62.13 \div 10.7$
i $(4.89)^2$

Stage 5.2

Worksheet
Mixed expansions
MAT10NAWK10028

Puzzle sheet
Expanding binomials
MAT10NAPS00007

Puzzle sheet
Trinominoes
MAT10NAPS00022

Worksheet
Algebra 2
MAT10NAWK00007

4-05 Expanding binomial products

$(k + 3)$ and $(k - 7)$ are called **binomial expressions** because each expression has exactly two terms (binomial = '2 terms'). $(k + 3)(k - 7)$ is called a **binomial product** because it is a product (multiplication answer) of two binomial expressions.

Example 9

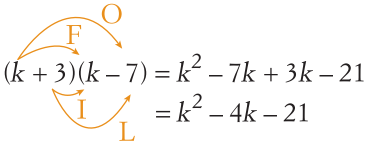
Expand each binomial product.

a $(k + 3)(k - 7)$
b $(3t - 1)(2t - 5)$

Solution

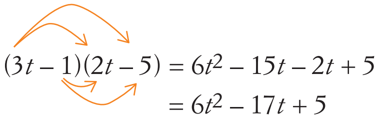
a
$$\begin{aligned} (k + 3)(k - 7) &= k(k - 7) + 3(k - 7) && \text{Each term in } (k + 3) \text{ is multiplied by } (k - 7). \\ &= k^2 - 7k + 3k - 21 && \text{Expanding.} \\ &= k^2 - 4k - 21 && \text{Simplifying.} \end{aligned}$$

One way of remembering which pairs of terms to multiply together in a binomial product is called the FOIL method, as shown below.



$$\begin{aligned}(k+3)(k-7) &= k^2 - 7k + 3k - 21\\ &= k^2 - 4k - 21\end{aligned}$$

- **F** means multiply the **f**irst terms: $k \times k = k^2$
- **O** means multiply the **o**utside terms: $k \times (-7) = -7k$
- **I** means multiply the **i**nside terms: $3 \times k = 3k$
- **L** means multiply the **l**ast terms: $3 \times (-7) = -21$



b $(3t-1)(2t-5) = 6t^2 - 15t - 2t + 5$ Using FOIL.

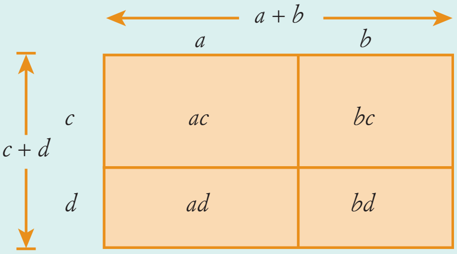
$= 6t^2 - 17t + 5$ Simplifying.

Summary

Expanding a binomial product

Multiply each term in the first binomial by each term in the second binomial.

$$(a+b)(c+d) = ac + ad + bc + bd$$



Exercise 4-05 Expanding binomial products

See Example 9

1 Expand each binomial product.

a $(m+4)(m+3)$ **b** $(w+5)(w+5)$ **c** $(y+12)(y-12)$
d $(a-8)(a+3)$ **e** $(b-2)(9+b)$ **f** $(u-8)(u-7)$
g $(15-k)(k+1)$ **h** $(r-11)(r-7)$ **i** $(6-c)(3-c)$
j $(t-1)(t+2)$ **k** $(x-4)(x+10)$ **l** $(n+11)(9-n)$

2 Expand $(b+7)^2$. Select the correct answer **A**, **B**, **C** or **D**.

A b^2+49 **B** b^2+49b **C** $b^2+7b+49$ **D** $b^2+14b+49$

3 Expand each binomial product.

a $(x+3)(2x+5)$ **b** $(3e+7)(3e+7)$ **c** $(10+3p)(p-1)$
d $(7d-2)(7d-2)$ **e** $(2f-2)(3f+5)$ **f** $(4m-5)(5+3m)$
g $(3-h)(2+5h)$ **h** $(4p-5)(4p-5)$ **i** $(2m-3)(4-5m)$
j $(6t+1)(2t-1)$ **k** $(5y-5)(5y+5)$ **l** $(6-7a)(7a-6)$

Stage 5.2

4 A family room in a house is to be extended. The room is a metres long and b metres wide. The length is to be increased by 3 metres and the width by 1 metre.

- **a** Write down expressions for the new length and width.
- **b** Write down a binomial expression for the new area of the room.
- **c** Expand and simplify your expression for the area.
- **d** By how much has the area of the room increased?

Investigation: Factorising quadratic expressions

1 **a** Show that $(x + 3)(x + 5) = x^2 + 8x + 15$.

- **b** The quadratic expression $x^2 + 8x + 15$ has three terms. The coefficient of x (the number in front of the x) is 8. How are the 3 and 5 in $(x + 3)(x + 5)$ related to the 8?
- **c** The constant term in $x^2 + 8x + 15$ is 15, the number with no x at the end. How are the 3 and 5 related to the 15?

2 **a** Expand $(x + 9)(x + 2)$.

- **b** What is the coefficient of x? How are 9 and 2 related to it?
- **c** What is the constant term? How are 9 and 2 related to it?

3 **a** Expand $(x + 8)(x - 3)$.

- **b** What is the coefficient of x? How are 8 and -3 related to it?
- **c** What is the negative constant term? How are 8 and -3 related to it?

4 **a** Expand $(x - 4)(x - 1)$.

- **b** What is the negative coefficient of x? How are -4 and -1 related to it?
- **c** What is the positive constant term? How are -4 and -1 related to it?

5 In the expansion of any binomial product, how are the coefficient of x and the constant term related to the numbers in the binomials?

6 Copy and complete:

- **a** $(x + \_\_\_)(x + \_\_\_) = x^2 + 5x + 4$
- **b** $(x + \_\_\_)(x + \_\_\_) = x^2 + 8x + 15$
- **c** $(x + \_\_\_)(x + \_\_\_) = x^2 + 7x + 12$
- **d** $(x + \_\_\_)(x - \_\_\_) = x^2 - 4x - 32$
- **e** $(x + \_\_\_)(x - \_\_\_) = x^2 + 2x - 3$
- **f** $(x - \_\_\_)(x - \_\_\_) = x^2 - 9x + 20$

9780170194655

4-06 Factorising quadratic expressions

Stage 5.2

Puzzle sheet
Factorominoes
MAT10NAPS10029

Puzzle sheet
Trinominoes
MAT10NAPS00022

Worksheet
Simplifying algebraic fractions
MAT10NAWK10030

A **quadratic expression** is an algebraic expression in which the highest power of the variable is 2, such as $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$.
The expansion of $(x + 2)(x + 4)$ is $x^2 + 6x + 8$, a quadratic expression.
$\therefore$ The factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

Summary

In the factorisation of a quadratic trinomial such as $x^2 + 6x + 8$:

- each factor must have an x term to give x^2 — $x^2 + 6x + 8 = (x + 2)(x + 4)$
- $2 + 4 = 6$, which is the **coefficient** of x, the number in front of the x — $x^2 + 6x + 8 = (x + 2)(x + 4)$
- $2 \times 4 = 8$, which is the **constant term** with no x — $x^2 + 6x + 8 = (x + 2)(x + 4)$

Video tutorial
Factorising quadratic expressions
MAT10NAVT10016

Example 10

Factorise each quadratic expression.

a $x^2 + 7x + 12$ **b** $x^2 + 9x + 8$

Solution

a Find the two numbers that have a sum of 7 and a product of 12.

It is best to test numbers that have a **product of 12** and then check if their sums equal 7.

Pair of numbers	Product	Sum
6, 2	$6 \times 2 = 12$	$6 + 2 = 8$
3, 4	$3 \times 4 = 12$	$3 + 4 = 7$

The correct numbers are 3 and 4.
$\therefore x^2 + 7x + 12 = (x + 3)(x + 4)$

b Find two numbers with a sum of 9 and a product of 8.
Test numbers that have a **product of 8** and check if their sums equal 9.

Pair of numbers	Product	Sum
4, 2	$4 \times 2 = 8$	$4 + 2 = 6$
1, 8	$1 \times 8 = 8$	$1 + 8 = 9$

The correct numbers are 1 and 8.
$\therefore x^2 + 9x + 8 = (x + 1)(x + 8)$

Stage 5.2

Summary

Factorising quadratic expressions of the form $x^2 + bx + c$

- Find two numbers that have a sum of b and a product of c
- Use these two numbers to write a binomial product of the form $(x$ ___ $)(x$ ___ $)$

Example 11

Video tutorial
Factorising quadratic expressions
MAT10NAVT10016

Factorise each quadratic expression.

a $x^2 + x - 6$ **b** $a^2 - 2a - 15$ **c** $y^2 - 6y + 8$

Solution

a $x^2 + x - 6$

Find two numbers that have a product of -6 and a sum of 1.
Since the product is negative, one of the numbers must be negative.
They are $+3$ and -2.
$\therefore x^2 + x - 6 = (x + 3)(x - 2)$

b $a^2 - 2a - 15$

Product $= -15$, sum $= -2$.
Since the product is negative, one of the numbers must be negative.
They are -5 and $+3$.
$\therefore a^2 - 2a - 15 = (a - 5)(a + 3)$

c $y^2 - 6y + 8$

Product $= 8$, sum $= -6$.
Since the sum is negative, one of the numbers must be negative.
Since the product is positive, both of the numbers must be negative.
They are -4 and -2.
$\therefore y^2 - 6y + 8 = (y - 4)(y - 2)$

Exercise 4-06 Factorising quadratic expressions

Stage 5.2

See Example 10

1 Find two numbers whose:

a product is 6 and sum is −7 **b** product is −12 and sum is 1
c product is −15 and sum is −2 **d** product is 12 and sum is 7
e product is 20 and sum is −9 **f** product is −14 and sum is 5
g product is −10 and sum is 3 **h** product is −25 and sum is 0
i product is −2 and sum is −1 **j** product is −18 and sum is −7

See Example 11

2 Factorise each quadratic expression.

a $x^2 + 7x + 12$ **b** $x^2 + 12x + 35$ **c** $x^2 + 5x + 4$
d $x^2 + 7x + 10$ **e** $x^2 + 9x + 20$ **f** $t^2 + 6t + 5$
g $e^2 + 5e + 6$ **h** $h^2 + 4h + 4$ **i** $n^2 + 11n + 10$
j $a^2 + 11a + 30$ **k** $d^2 + 10d + 24$ **l** $y^2 + 15y + 44$
m $n^2 - 2n - 3$ **n** $r^2 - 5r - 14$ **o** $h^2 - 3h - 4$
p $w^2 - 7w - 18$ **q** $f^2 - 6f - 27$ **r** $a^2 - 4a - 12$

3 Factorise each quadratic expression.

a $x^2 + 3x - 4$ **b** $t^2 + 5t - 24$ **c** $m^2 + 2m - 15$
d $a^2 + a - 2$ **e** $k^2 + 5k - 14$ **f** $w^2 + 4w - 12$
g $m^2 - 5m + 4$ **h** $s^2 - 6s + 8$ **i** $x^2 - 12x + 35$
j $p^2 - 10p + 24$ **k** $n^2 - 3n + 2$ **l** $r^2 - 6r + 9$
m $m^2 + 4m + 4$ **n** $p^2 - 20p + 100$ **o** $c^2 - 10c + 25$

Power plus

1 Simplify each expression using index notation.

a $13^7 \times (13^5 \div 13^{11})$ **b** $(8^{-4})^5 \div (8^5)^{-4}$ **c** $\left(\frac{(7^2)^3}{7^5 \times 7^2}\right)^{-1}$
d $\left(\frac{1}{10}\right)^{-10} \times \left(\frac{1}{10}\right)^{10}$ **e** $\left(-2\frac{1}{2}\right)^3$ **f** $\left(\frac{1}{2}\right)^{-10} - \left(\frac{1}{4}\right)^{-5}$

2 Simplify each expression.

a $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$ **b** $\frac{1}{x} \times \frac{1}{y} \div \frac{1}{z}$ **c** $\frac{1}{x} \times \left(\frac{1}{y} + \frac{1}{z}\right)$

3 Expand each expression.

a $(a - b)(a + b - c)$ **b** $(x - y + 1)^2$ **c** $t^{-1}(3t^2 + 4t - 1)$

4 Factorise each expression.

a $x^2 - 133x + 1000$ **b** $y^2 + 14y - 1800$
c $b^2 + 82b + 1681$ **d** $n^2 - 2500$

Chapter 4 review

Language of maths

Puzzle sheet
Algebra crossword
MAT10NAPS10031

algebraic fraction
base
binomial
brackets
coefficient
constant term
denominator
expand
factorise
highest common factor
index law
indices
numerator
power
quadratic expression
reciprocal
term

1 Dividing by a fraction is the same as multiplying by its what?

2 Any number raised to the power of zero is equal to what?

3 What is the difference between **expand** and **factorise**?

4 What power is associated with the **reciprocal** of a term or number?

5 In the quadratic expression $2x^2 - 3x + 6$, what is:

a the constant term? **b** the coefficient of x?

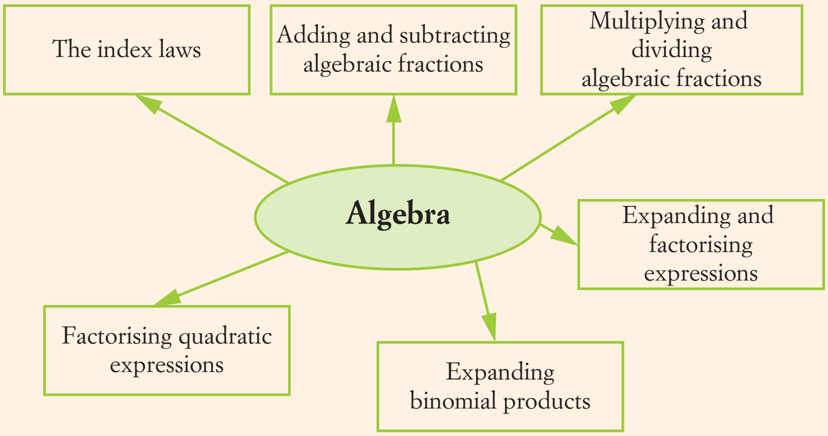
6 Copy and complete: To factorise quadratic expressions of the form $x^2 + bx + c$, first find two numbers that have a ________ of b and a ________ of c.

Topic overview

Quiz
Numbers and powers
MAT10NAQZ00001

- What was this topic about? What was the main theme?
- What content was new and what was revision for you?
- Write three index laws in both words and symbols.
- Write 10 questions (with solutions) that could be used in a test for this chapter. Include some questions that you have found difficult to answer.
- List the sections of work in this chapter that you did not understand. Follow up this work.

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 4 revision

1 Simplify each expression.

See Exercise 4-01

a $3v^4w^2 \times 2vw^5$ **b** $\dfrac{24t^8h^8}{3th^2}$ **c** $(5xy^2)^2$

d $\left(\dfrac{2p}{3}\right)^0$ **e** $(4k)^{-1}$ **f** $\left(\dfrac{2p}{3}\right)^3$

2 Simplify each expression.

Stage 5.2

See Exercise 4-01

a $(4m)^{-2}$ **b** $4m^{-2}$ **c** $\left(\dfrac{5b^8y^6}{b^2y^3}\right)^4$

d $(4t^4u^5)^3 \times 8t^2u$ **e** $45c^6d^8 \div (-3cd^2)^2$ **f** $\left(\dfrac{45ab^4}{54a^2b^3}\right)^{-1}$

3 Simplify each expression.

See Exercise 4-02

a $\dfrac{t}{4} - \dfrac{2t}{5}$ **b** $\dfrac{5g}{3} + \dfrac{3g}{2}$ **c** $\dfrac{5x}{16} - \dfrac{x}{4}$

d $\dfrac{3b}{7} + \dfrac{2}{7}$ **e** $\dfrac{5w}{8} - \dfrac{3}{2}$ **f** $\dfrac{7}{12} - \dfrac{y}{5}$

4 Simplify each expression.

See Exercise 4-03

a $\dfrac{3}{m} \times \dfrac{p}{4}$ **b** $\dfrac{1}{q} \times \dfrac{3}{q}$ **c** $\dfrac{2d}{3} \times \dfrac{15y}{14d}$

d $\dfrac{r}{4} \div \dfrac{r}{5}$ **e** $\dfrac{4}{x} \div \dfrac{12x}{5}$ **f** $\dfrac{12md}{5} \div \dfrac{3m}{10d}$

5 Expand each expression.

See Exercise 4-04

a $9(m - 8)$ **b** $b(10a + b)$ **c** $-3y(4x^2 - 5y)$

d $8tp(7p - 5t)$ **e** $-(3n - 10)$ **f** $-5h^2(3h + 7)$

6 Expand and simplify each expression.

Stage 5.2

See Exercise 4-04

a $4fg(g - 6f) - 6f^2g$ **b** $12(9 - n) - 5(2n + 3)$

c $x^2(6x + x^2) + 2x(3x^3 + x^2)$ **d** $3(7 - 2y) - 5y(7 - 2y)$

7 Factorise each expression.

See Exercise 4-04

a $8t - 72$ **b** $b^2 + 36b$ **c** $-3m - 33$

d $16ar^2 + 24ar$ **e** $-24p + 18q$ **f** $2(5x - 1) - 3x(5x - 1)$

8 Factorise each expression.

Stage 5.2

See Exercise 4-04

a $15xy^2 - 30x^3y^3$ **b** $6pt^2 + 12p^2t - 48p^3$ **c** $32r^2s^4 + 12r^4s^3$

d $50x^4y^3 - 75x^3y^4$ **e** $-8p^3q^3 + 48p^3q^6$ **f** $n(n^2 + 6) - (n^2 + 6)$

9 Expand each binomial product.

See Exercise 4-05

a $(b + 3)(b + 10)$ **b** $(d + 8)(d - 7)$ **c** $(t - 6)(9 - t)$

d $(5x + 7)(4x - 3)$ **e** $(7y - 3)(7y + 3)$ **f** $(5p - 8)(5p - 8)$

10 Factorise each quadratic expression.

See Exercise 4-06

a $y^2 + 10y + 25$ **b** $x^2 - 21x + 20$ **c** $n^2 + 8n - 33$

d $a^2 - 11a + 28$ **e** $m^2 - 5m - 84$ **f** $p^2 + 3p - 54$

5

Statistics and probability

Investigating data

Which capital city in Australia has the highest average temperature? Does Melbourne have higher rainfall than Sydney?

To answer these questions, sets of data need to be collected and then compared by looking at the shape of their displays or by analysing their measures of location and spread.

Shutterstock.com/Gordon Bell

Chapter outline

	Proficiency strands				
5-01 The shape of a frequency distribution	U	F	PS	R	C
5-02 Quartiles and interquartile range*	U	F	PS	R	C
5-03 Boxplots*	U	F	PS	R	C
5-04 Parallel boxplots*	U	F	PS	R	C
5-05 Comparing data sets*		F	PS	R	C
5-06 Scatter plots*	U	F		R	C
5-07 Bivariate data involving time*	U	F		R	C
5-08 Statistics in the media*	U	F	PS	R	C

*STAGE 5.2

Wordbank

bivariate data Data that measures two variables, represented by an ordered pair of values that can be graphed on a scatter plot

boxplot (also called **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest scores; the box contains the middle 50% of scores while the lines or 'whiskers' extend to the two extremes

five-number summary For a set of numerical data, the lowest score, lower quartile, median, upper quartile, highest score

interquartile range (IQR) The difference between the upper quartile and lower quartiles, IQR = $Q_3 - Q_1$, representing the middle 50% of scores

quartile The values Q_1, Q_2, Q_3 that divide a set of data into quarters (four equal parts)

scatter plot A graph consisting of dots on a number plane that represent bivariate data

9780170194655

In this chapter you will:

- construct back-to-back stem-and-leaf plots and histograms and describe data using terms such as ‘skewed’, ‘symmetric’ and ‘bimodal’
- (STAGE 5.2) determine quartiles and interquartile range
- (STAGE 5.2) construct and interpret boxplots and use them to compare data sets
- (STAGE 5.2) compare shapes of boxplots to corresponding histograms and dot plots
- (STAGE 5.2) use scatter plots to investigate and comment on relationships between two numerical variables
- (STAGE 5.2) investigate and describe bivariate numerical data where the independent variable is time
- (STAGE 5.2) evaluate statistical reports in the media and other places by linking claims to displays, statistics and representative data
- (STAGE 5.2) investigate reports of surveys in digital media and elsewhere for information on how data was obtained to estimate population means and medians
- (STAGE 5.2) find the five-number summary for a set of data and use it to construct a box-and-whisker plot
- (STAGE 5.2) describe the strength and direction of the linear relationship of bivariate data shown on a scatter plot

SkillCheck

Worksheet
StartUp assignment 5
MAT10SPWK10032

Skillsheet
Statistical measures
MAT10SPSS10012

Worksheet
Statistical match-up
MAT10SPWK10033

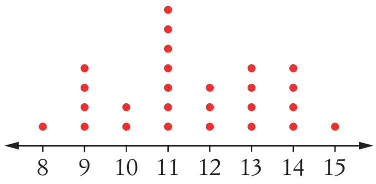
1 For each set of data, find:

i the range　　**ii** the mean (correct to one decimal place)
iii the median　　**iv** the mode

a 15　13　18　14　15　18　23　14　20　16　15

b 8°C　3°C　–5°C　2°C　–4°C　7°C　3°C　0°C

c



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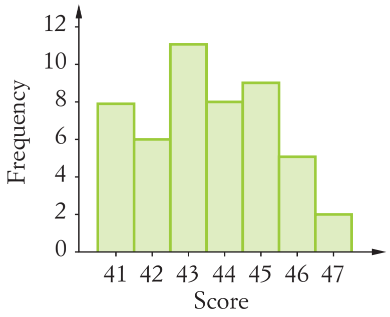


e

Stem	Leaf
1	0 3 6
2	1 4 4 7 8
3	2 3 4 5 5 7 9
4	0 5 7 8
5	2 6 8

f

Score	Frequency
0	2
1	5
2	8
3	4
4	3
5	1

2 A cricketer made the following scores in 10 innings.

34 21 78 30 26 19 41 36 16 32

a Find:

i the median **ii** the mean **iii** the range.

b Which score is the outlier?

c **i** Calculate the median, mean and range if the outlier is not included in the scores.
ii What effect does the outlier have on the mean, median and range?

123rf/Lance Bellers

5-01 The shape of a frequency distribution

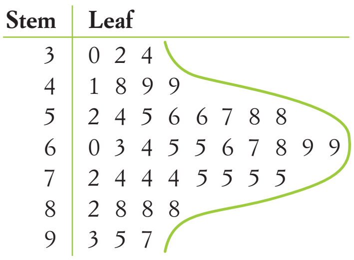
Technology worksheet
Excel worksheet: Skewness
MAT10SPCT00005

Technology worksheet
Excel spreadsheet: Skewness
MAT10SPCT00035

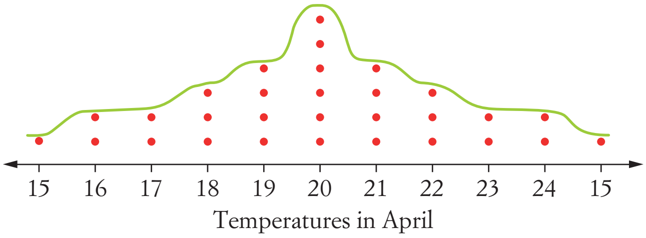
A **statistical distribution** is the way the scores of a data set are arranged, especially when graphed. When looking at histograms, dot plots and stem-and-leaf plots, an overall pattern can be seen from the shape of the display.

The shape of a statistical distribution shows how the data is spread and can be seen by drawing a curve around the graph or display.

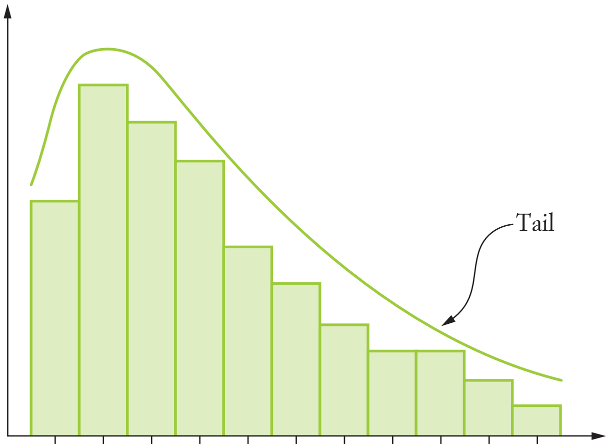
A distribution is **symmetrical** if the data is evenly spread or balanced about the centre.

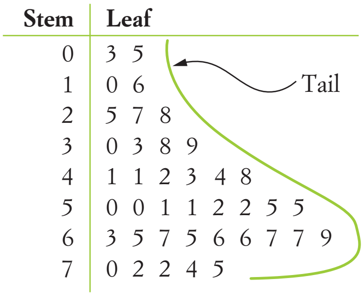


Stem	Leaf
3	0 2 4
4	1 8 9 9
5	2 4 5 6 6 7 8 8
6	0 3 4 5 5 6 7 8 9 9
7	2 4 4 4 5 5 5 5
8	2 8 8 8
9	3 5 7



A distribution is **skewed** if most of the data is bunched or clustered at one ‘end’ of the distribution and the other ‘end’ has a ‘tail’.



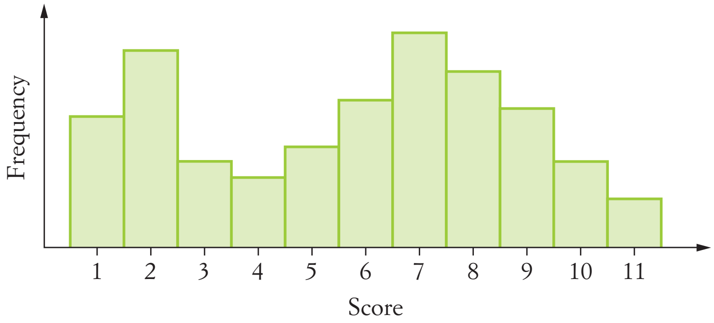


Stem	Leaf
0	3 5
1	0 6
2	5 7 8
3	0 3 8 9
4	1 1 2 3 4 8
5	0 0 1 1 2 2 5 5
6	3 5 7 5 6 6 7 7 9
7	0 2 2 4 5

A distribution is **positively skewed** if its tail points to the right (higher scores).

A distribution is **negatively skewed** if its tail points to the left (lower scores).

A distribution is **bimodal** if it has two peaks. The higher peak is the mode, while the other peak indicates another score that has a high frequency.
For example, this frequency histogram has two peaks at 2 and 7 so it is bimodal. The mode, however, is 7.



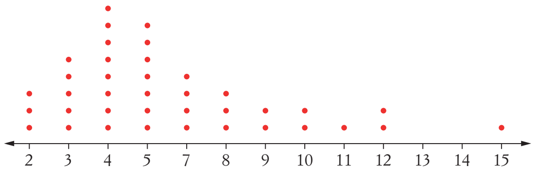
Example 1

For each statistical distribution:

i describe the shape

ii identify any outliers and clusters

a



b

Stem	Leaf
10	4 5
11	3 4 4 9
12	1 2 2 6 8
13	0 1 5 5 7 9 9 9
14	4 5 6 8 8
15	0 0 1 1
16	0 2

Solution

a **i** The shape is positively skewed (tail points towards the higher scores).
ii 15 is an outlier and clustering occurs at 4 and 5.
b **i** The shape is symmetrical (the data is balanced about the stem of 13).
ii There are no outliers but clustering occurs in the 130s.

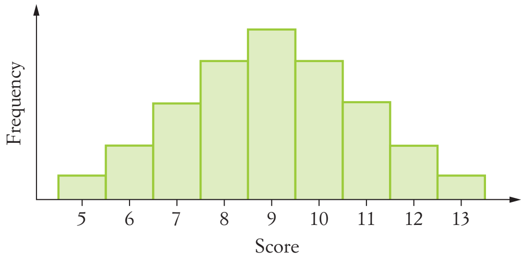
Exercise 5-01 The shape of a distribution

See Example 1

1 For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

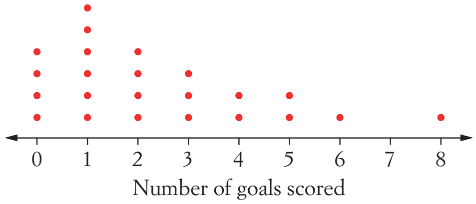
a



b

Stem	Leaf
2	4 5 6 9
3	1 2 3 3 4 5 7 8
4	0 4 4 6 8 9
5	4 5 5 8
6	0 0 2 3 5 6 7 8 9 9
7	3 5 7 8 8 9 9
8	1 1 3 5 6
9	0 3 5 6

c



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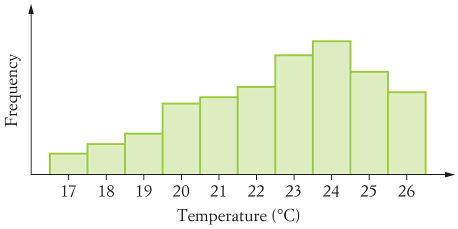
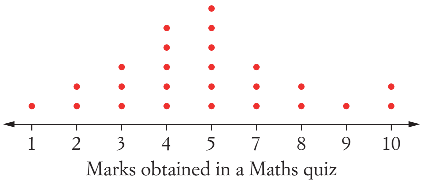


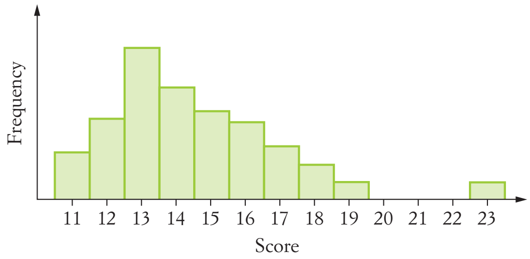
e

Stem	Leaf
12	0 2 4 9
13	2 4 6 7 8 8 8
14	3 3 4 4 5 5 8 9 9 9
15	0 1 1 5 7 8 9 9
16	1 1 5 6 7
17	2 4 5 8
18	0 3 9
19	5 8
20	6 8

f



g



h

Stem	Leaf
5	3 4 4 6 7 8 9
6	0 0 5 9 9
7	2 4 5 6
8	5 7 8
9	3 3 6 7 8
10	2 4 6 8 8 8 8
11	
12	
13	6

2 These are the final round scores for players in a golf tournament.

66	70	67	72	75	72	70	74	75	72
74	72	73	71	71	69	70	71	71	74
72	69	75	73	69	75	73	69	69	67
74	72	72	73	71	73	77	68	72	72

- **a** Arrange the data into a frequency table and construct a frequency histogram.
- **b** Are there any outliers?
- **c** Describe the shape of the distribution.
- **d** Give a possible reason for the shape of the distribution.
- **e** Where does clustering occur?
- **f** Find the mode, the mean and the median and show their position in the histogram.

3 The stem-and-leaf plot shows the number of hours that students spend on their computers during the week.

Stem	Leaf
0	1 1 1 1 1 2 2 2 2 3 3 3 5 6 6 7 7 7 7 9 9
1	0 1 1 2 4 4 5 6 8 8 9
2	0 5 5 5 8 8
3	0 0 0 1 5
4	0 0

- **a** How many students were surveyed?
- **b** Where does the clustering occur?
- **c** Are there any outliers?
- **d** Describe the shape of the distribution.
- **e** Give a possible reason for the shape of the distribution.
- **f** Find the mean, median and mode.

4 The following scores are the heights (in cm) of thirty Year 8 students.

162 155 153 162 182 173 165 165 142 167
164 168 150 155 143 153 123 163 170 169
153 162 161 170 160 162 172 151 160 171

a Arrange the data into an ordered stem-and-leaf plot.

b Describe the shape of the distribution.

c Are there any outliers?

d Where does clustering occur?

e Find the mode, median and mean.

5 The daily maximum temperatures (correct to one decimal place) for July 2013 at the Sydney Observatory are shown in the stem-and-leaf plot.

a Describe the shape of the distribution.

b Are there any outliers?

c What is the mode?

d Find the mean, correct to one decimal place.

e What is the median?

f Find the range.

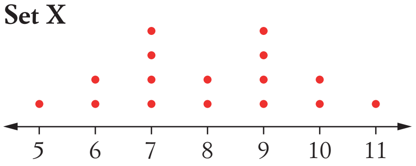
g Is the range a good indicator of the spread of the temperatures? Give reasons.

Stem	Leaf
13	8
14	
15	9
16	3
17	0 2 4 4 7
18	4 4 4 7 8
19	1 2 5 6 8 9
20	1 2 3 4 4
21	5 6
22	0 6
23	4
24	0 3

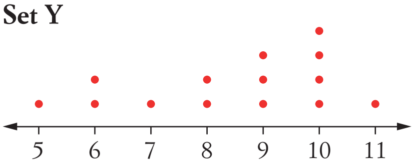
Source: © Bureau of Meteorology

6 Which statement is true about the sets of data below? Select the correct answer **A**, **B**, **C** or **D**.

Set X



Set Y



A X is symmetrical and Y is positively skewed.

B X is bimodal and Y is negatively skewed.

C X has two peaks and the median of Y is 8.

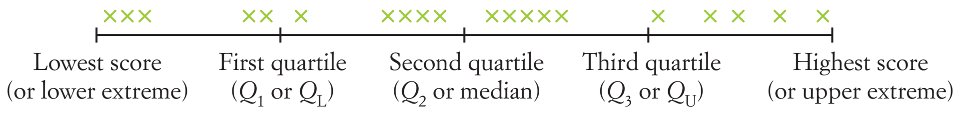
D The median of X and Y is 8.

Stage 5.2

5-02 Quartiles and interquartile range

Quartiles

The median, being the middle score, divides a set of data into two equal parts (halves). **Quartiles** are the values Q_1, Q_2 and Q_3 that divide the set of data into four equal parts (quarters).



Scores (in order)

The **first quartile** Q_1, also called the **lower quartile** Q_L, is the value that divides the lower 25% of scores. $\frac{1}{4}$ of the scores lie below Q_1.

The **second quartile** Q_2 is the value that divides the lower 50% of scores, so it is also the **median**. $\frac{1}{2}$ of the scores lie below Q_2.

The **third quartile** Q_3, also called the **upper quartile** Q_U, is the value that divides the lower 75% of scores from the upper 25% of scores. $\frac{3}{4}$ of the scores lie below Q_3, $\frac{1}{4}$ of the scores lie above it.

Summary

Finding the quartiles of a data set

- sort the scores in order, find the median and call it Q_2
- find the median of the bottom half of the scores and call it Q_1 (or Q_L)
- find the median of the top half of scores and call it Q_3 (or Q_U).

Example 2

Find the quartiles for each set of data.

a 65 84 75 82 97 70 68 76 93 48 79 54 80 79 82 96 63 85 72 70
b 9 3 8 7 6 8 4 6 2 10 9
c 15 18 7 16 23 9 15 20 16 14 13 11 19

Solution

a Arranging the 20 scores in ascending order, we have:

48 54 63 65 68 70 70 72 75 76 79 79 80 82 82 84 85 93 96 97

$$Q_1 = \frac{68 + 70}{2} = 69$$

$$Q_2 \text{ (median)} = \frac{76 + 79}{2} = 77.5$$

$$Q_3 = \frac{82 + 84}{2} = 83$$

Stage 5.2

When finding the quartiles, first find the median, then the lower and upper quartiles.

Q_1 (lower quartile) = 69

Q_2 (median) = 77.5

Q_3 (upper quartile) = 83

b Arranging the 11 scores in ascending order, we have:

2 3 4 6 6 7 8 8 9 9 10

Lower quartile $Q_1 = 4$

Median $Q_2 = 7$

Upper quartile $Q_3 = 9$

c Arranging the 13 scores in ascending order, we have:

7 9 11 13 14 15 15 16 16 18 19 20 23

Lower quartile $Q_1 = \frac{11 + 13}{2} = 12$

Median $Q_2 = 15$

Upper quartile $Q_3 = \frac{18 + 19}{2} = 18.5$

The interquartile range

The **range** is a **measure of spread** because it gives an indication of how widely the scores are spread in a set of data. Another measure of spread is the **interquartile range**.

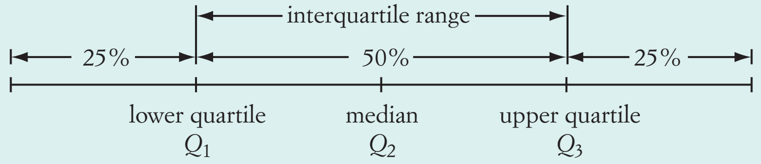
The **interquartile range** is the difference between the upper and lower quartiles and so it is the range of the middle 50% of the data.

Worksheet
Interquartile range
MAT10SPWK10034

Video tutorial
Interquartile range
MAT10SPVT10003

Summary

$$\text{Interquartile range (IQR)} = \text{upper quartile} - \text{lower quartile}$$
$$= Q_3 - Q_1$$



The interquartile range takes into account the middle 50% of scores and ignores very low or very high scores (outliers), so sometimes it is better than the range as a measure of spread.

Stage 5.2

Example 3

Getty Images Sport/Cameron Spencer

The number of points scored by the NSW Waratahs per rugby match during the 2013 season were:

17 31 6 26 30 23 29 25 19 72 21 28 22 28 12

a Find the range.

b Find the interquartile range.

c Which is the better measure of spread of the points scored by the Waratahs – the range or interquartile range?

Solution

First arrange the scores in order:

6 12 17 19 21 22 23 25 26 28 28 29 30 31 72

Lower quartile
$Q_1 = 19$

Median
$Q_2 = 25$

Upper quartile
$Q_3 = 29$

a Range $= 72 - 6$

$= 66$

b Interquartile range $= Q_3 - Q_1$

$= 29 - 19$

$= 10$

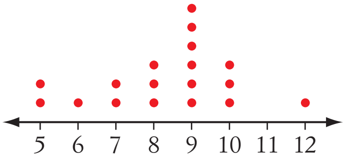
c The interquartile range is the better measure of spread as the outlier of 72 is excluded. The score of 72 has affected the range, making it very big.

Stage 5.2

Example 4

Find the interquartile range of each set of data.

a



b

Stem	Leaf
1	2 7
2	0 3 4 4 5
3	1 2 2 4 6 8 8 9
4	0 1 3 7
5	1 2

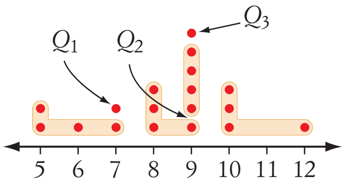
Solution

a There are 18 scores, so the median is 'between' the 9th and 10th scores.

Median, $Q_2 = \frac{9+9}{2} = 9$

Q_1 is the median of the lower half of scores.
$Q_1 = 7$.
Q_3 is the median of the upper half of scores. $Q_3 = 9$.

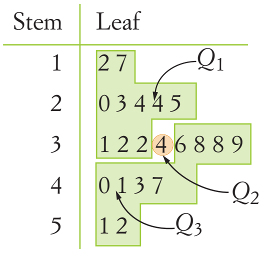
$$\begin{aligned}\therefore \text{IQR} &= Q_3 - Q_1\\ &= 9 - 7\\ &= 2\end{aligned}$$



b There are 21 scores so the median is the 11th score. $Q_2 = 34$
Lower quartile, $Q_1 = \frac{24+24}{2} = 24$

Upper quartile, $Q_3 = \frac{40+41}{2} = 40.5$

$$\begin{aligned}\therefore \text{IQR} &= 40.5 - 24\\ &= 16.5\end{aligned}$$



Exercise 5-02 Quartiles and interquartile range

1 Find the quartiles for each set of data. **See Example 2**

a 3 7 9 5 5 6 2 8 9 7

b 15 19 18 12 20 34 28 18 28 20 23 25

c 34 45 32 38 29 40 37 33 35 30 34 35
38 37 38 31 30 34

2 Calculate the range and the interquartile range of each data set in question **1**. **See Example 3**

Stage 5.2

3 Calculate the interquartile range for each set of data below.

a 5 6 6 7 8 9 9 10 14 14 15 16

b 2 0 3 5 2 1 0 6 4 3 8 4 2

4 The monthly rainfall figures for Ulladulla one year were:
31 174 288 89 15 123 26 5 8 275 38 58

Getty Images/Peter Harrison

For this data, find:

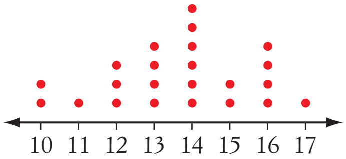
a the range

b the interquartile range

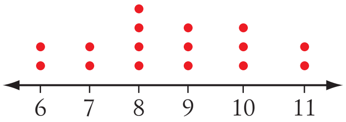
See Example 4

5 Find the interquartile range for each set of data.

a



b



c

Stem	Leaf
3	2 7
4	0 3 3 5
5	2 4 5 6 7 8 8
6	3 4 7
7	2

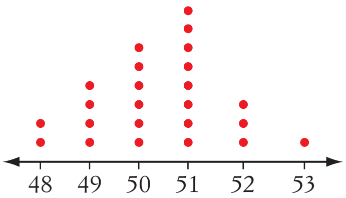
d

Stem	Leaf
1	3 5 8 9
2	0 1 3 3 4 5 6
3	5 8 9 9
4	1 3
5	4

e

Stem	Leaf
10	3 5 5 6 6
11	0 1 2
12	3 4 6 7 8
13	4 7
14	1

f



6 The pulse rates for a group of students are as follows.

82 81 72 58 79 77 62 66 92 78 80 67 91 75 72 68

a Find the range.

b Find the interquartile range.

c **i** List the scores that lie between the lower and upper quartiles.

ii What percentage of scores lie between Q_1 and Q_3?

d What percentage of scores lie above the lower quartile?

7 The number of goals per game scored by the Sydney Swifts netball team during 2013 were:

55 35 49 53 51 55 42 48 63 43 48 48 62

a Find:

i the range **ii** the interquartile range

b Which is the better measure of spread?

c List the scores that lie between Q_1 and Q_3. What percentage of the scores is this?

Just for the record Statistics: Where did it all begin?

In prehistoric times, when the number of people and animals was recorded in pictures and symbols on the walls of caves, a simple form of statistics was being used.

Alamy/Zev Radovan

Before 3000 BCE, ancient Babylonians used clay tablets to record crop yields and trade data, and around 2650 BCE the Egyptians ‘surveyed’ the population and wealth of their country before building the pyramids. Forms of statistics were also used in the Bible in the ‘Book of Numbers’ and the ‘First Book of Chronicles’. Numerical records existed in China before 2000 BCE, and the Greeks (to help collect taxes) held a census in 594 BCE. The Roman Empire was the first government to collect information about the population. In 1086 a census was conducted in England. The information obtained in this census was recorded in the Domesday Book.

Use your library or the Internet to find out more about the Domesday Book. Write a one-page report suitable for a classroom presentation.

5-03 Boxplots

Stage 5.2

Video tutorial
Box-and-whisker plots
MAT10SPVT10004

Video tutorial
Statistics
MAT10SPVT00002

Worksheet
Five number summaries
MAT10SPWK10035

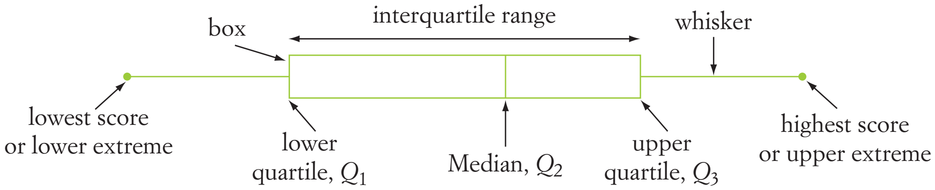
Puzzle sheet
Mode, median and mean
MAT10SPPS00044

Technology
GeoGebra: Boxplot and dot plot
MAT10SPTC00002

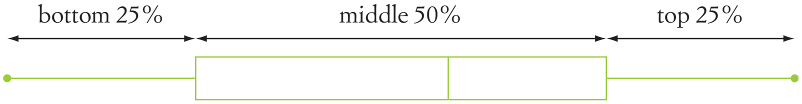
Technology worksheet
Excel worksheet: Five number summary
MAT10SPCT00002

Technology worksheet
Excel spreadsheet: Five number summary
MAT10SPCT00032

A **boxplot** (or box-and-whisker plot) displays the quartiles of a set of data and the lowest and highest scores (lower and upper extremes).



The 'box' represents the middle 50% of scores and the interquartile range, while the 'whiskers' represent the lowest and highest 25% of scores.



Summary

A boxplot gives a **five-number summary** of a data set:

- the lower extreme (or lowest score)
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the upper extreme (or highest score)

Example 5

The number of hours per week that Nick worked at the Big Chicken over summer were:

5 5 4 8 10 3 12 7 7 3 8 8 15

a Find a five-number summary for this data.

b Represent this data on a box-and-whisker plot.

Solution

a First arrange the scores in order.

3 3 4 5 5 7 ⑦ 8 8 8 10 12 15

↑ Q_1 ↑ median Q_2 ↑ Q_3

Lower extreme = 3

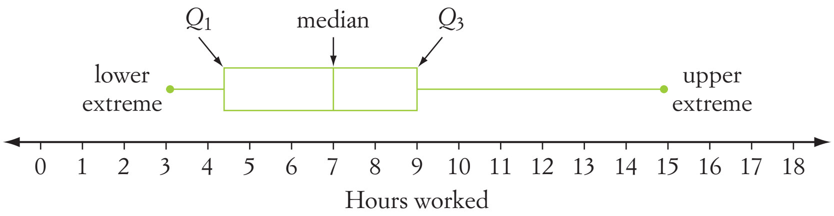
Lower quartile $= \frac{4+5}{2} = 4.5$

Median = 7

Upper quartile $= \frac{8+10}{2} = 9$

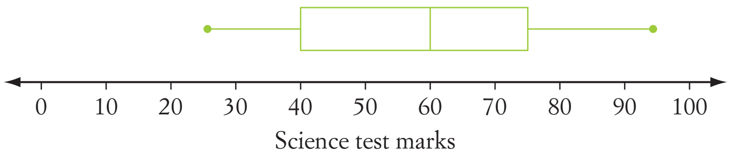
Upper extreme = 15

b



Example 6

The boxplot represents the results of 80 students in a Science test.



a Find the range of the test results.

b Find the median test score.

c What is the interquartile range?

d How many students had a test mark between:

i 25 and 75? **ii** 40 and 60?

e What percentage of students scored more than 75?

Solution

a Range = highest score − lowest score

$= 95 - 25$

$= 70$

b Median = 60

c Interquartile range $= Q_3 - Q_1$

$= 75 - 40$

$= 35$

d i 25 is the lowest score and 75 is Q_3, so 75% × 80 = 60 students had a mark between 25 and 75.

ii 40 is Q_1 and 60 is the median, so 25% × 80 = 20 students had a mark between 40 and 60.

e 75 is the third quartile so 25% × 80 = 20 students scored more than 75.

Stage 5.2

Exercise 5-03 Boxplots

See Example 5

1 The number of orders taken per hour at Bramavale Pizza on a weekend were:

3 5 1 2 4 6 8 10 7 6
12 15 10 3 5 18 5 8 9 10

a Find the five-number summary for this data.

b Represent this data on a box-and-whisker plot.

2 The daily amount of snow (in cm) that fell at Thredbo during one ski season was:

2 5 5 2 5 7 1 2 2 2 2 2 12
20 12 5 40 50 10 40 13 30 5 35 2 6

a On how many days did it snow?

b Find a five-number summary for this data.

c Represent this data on a box-and-whisker plot.

3 The monthly rainfall figures for Penrith in 2012 were:

98 266 149 94 15 65 19 5 24 34 67 28

Source: © Bureau of Meteorology

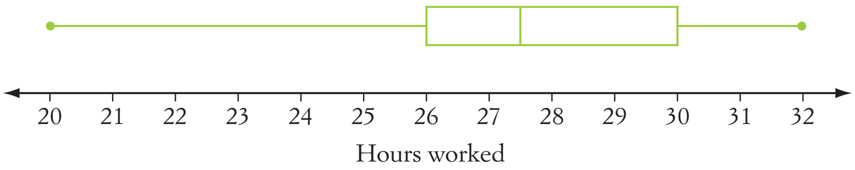
a Find the range.

b Find the five-number summary.

c Represent the data on a boxplot.

See Example 6

4 This boxplot represents the number of hours worked in one week by the staff at a supermarket.



a What is the median number of hours worked?

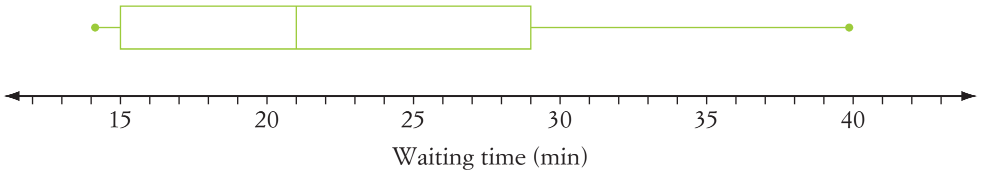
b What is the lower quartile?

c What is the upper quartile?

d Find the interquartile range.

e Estimate the percentage of employees that worked between 26 and 30 hours.

5 The ages of 16 people waiting at a bus stop are displayed by the boxplot below.



a What is the range?

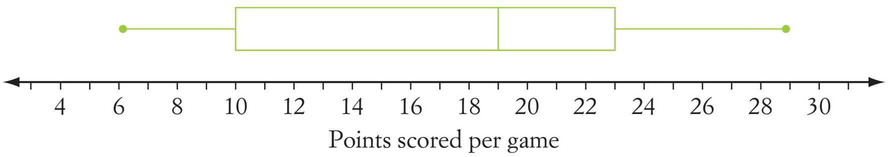
b What is the median age?

c Find the interquartile range.

d What percentage of people were aged from:

i 21 to 29? **ii** 15 to 40?

6 The box-and-whisker plot shows the number of points per game scored by Ben in 28 basketball games during the season.



a What is the five-number summary for the boxplot?

b Find the interquartile range.

c In how many games did Ben score:

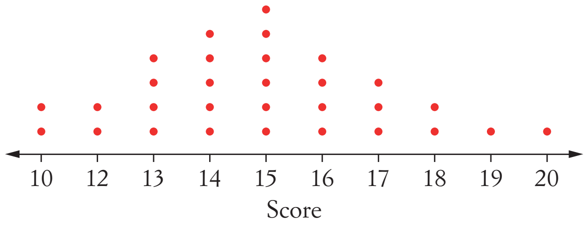
i more than 19 points? **ii** between 19 and 23 points?
iii less than 10 points? **iv** at least 10 points?

7 For each set of data, find the five-number summary and draw a boxplot.

a

Stem	Leaf
2	0 2 3 5
3	3 7
4	4 6 7 8 8 9 9
5	0 1 1 5 6
6	0 3 3 8 8
7	2 5 6
8	5 5 7 8

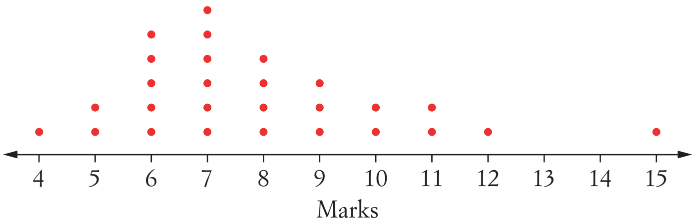
b



c

Stem	Leaf
3	0 7
4	2 6 6
5	1 2 5 9
6	0 4 7 7 9
7	2 3 5 6 8
8	3 4
9	5

8 The results of a general knowledge quiz (out of 15) taken by Year 10 students are displayed by the dot plot.



a Find the five-number summary for the dot plot and then draw a box-and-whisker plot.

b Describe the shape of the dot plot and compare it to the shape of the boxplot.

c What is the outlier?

d Find the five-number summary for the data in the dot plot without the outlier and draw a boxplot.

e Compare the two boxplots. How are they:

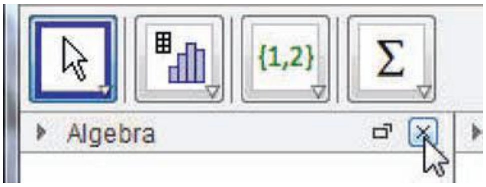
i similar? **ii** different?

Stage 5.2

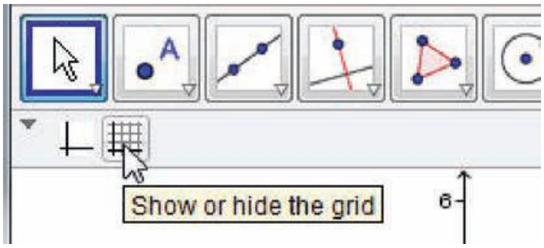
Technology Boxplots

In this activity we will use GeoGebra to draw boxplots.

1 Close the **Algebra View** so that only **Graphics View** is showing.



2 Select the grid option at the top left-hand corner.

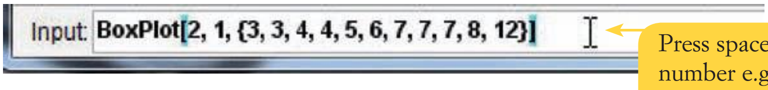


3 Data for boxplots is entered in the format shown below.

Boxplot[y-position, width of box, {data set}]

The **y-position** is where you want the boxplot to sit above the *x*-axis. For multiple plots, you should use different **y-position** values so that they sit on top of each other. In the Input Bar at the bottom, type

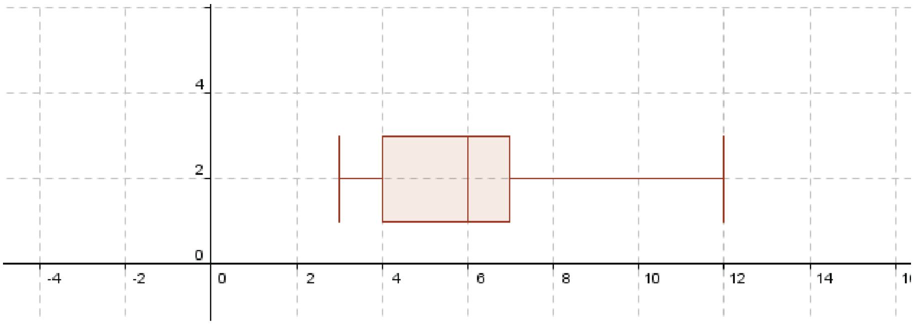
BoxPlot[2, 1, {3, 3, 4, 4, 5, 6, 7, 7, 7, 8, 12}].



Input: BoxPlot[2, 1, {3, 3, 4, 4, 5, 6, 7, 7, 7, 8, 12}]

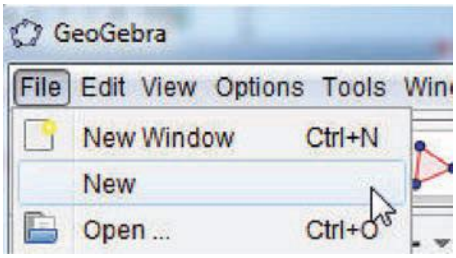
Press spacebar after each number e.g. {3, SPACE 3, SPACE 4, etc.}

4 To move the screen view, use **Move Graphics View** ✥. Your boxplot should look exactly like the one below.



5 Write down the five-number summary for this data set.

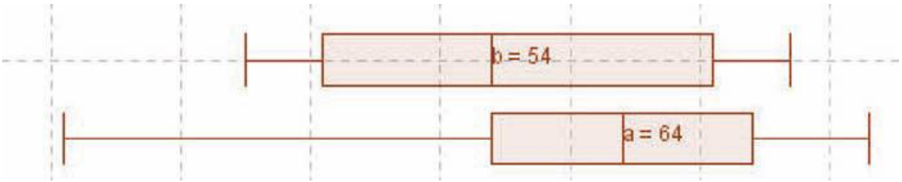
6 We will show the results of an English exam completed by classes 10A and 10B using a boxplot. To start up a new file with the same settings, select **File**, **New**.



In the Input Bar, enter the following formula for the results for 10A.
BoxPlot[4, 2, {21, 81, 33, 58, 67, 76, 64, 74, 56, 60, 54, 74, 49, 83, 66}]

7 Move the screen view as before. To zoom in, use **Zoom In**. To zoom out, use **Zoom Out**. This will allow you to view the boxplot.

8 In the Input Bar, enter the following formula for the results for 10B.
BoxPlot[10, 2, {77, 63, 63, 35, 51, 42, 54, 55, 71, 43, 41, 41, 40, 76, 72}]
You will now have two boxplots to compare.



9 Complete a five-number summary for each class.
10 What is the IQR for each class?
11 Which class had the highest mark?
12 Which class had the lowest mark?
13 Which class performed better? Give reasons for your answer, including explanations using the five-number summaries you found in step **9**.

Stage 5.2

Worksheet
Box-and-whisker plots
MAT10SPWK10036

Worksheet
Data 1
MAT10SPWK00032

Animated example
Analysing data
MAT10SPAE00002

Technology worksheet
Excel worksheet: Parallel boxplots
MAT10SPCT00004

Technology worksheet
Excel spreadsheet: Parallel boxplots
MAT10SPCT00034

5-04 Parallel boxplots

Parallel box-and-whisker plots can be used to compare two or more sets of data. They are drawn on the same scale, but above each other.

Example 7

Two sprinters run the following times (in seconds) over 100 metres.

Sam	10.9	10.5	11.0	9.9	10.7	10.5	10.0	11.2	11.5	10.3
Jesse	11.0	11.4	10.1	9.8	10.8	11.4	10.7	10.3	11.1	11.6

a Find the five-number summary for each sprinter.

b Draw parallel boxplots to display the data for both sprinters.

c Find the interquartile range for each sprinter.

d Find the range for each sprinter.

e Which sprinter is more consistent? Justify your answer.

Alamy/moodboard

Solution

a Sam: 9.9 10.0 10.3 10.5 10.5 10.7 10.9 11.0 11.2 11.5

9.9 ↑ lowest score

10.3 ↑ Q_1

$Q_2 = \frac{10.5 + 10.7}{2}$
$= 10.6$

11.0 ↑ Q_3

11.5 ↑ highest score

Jesse: 9.8 10.1 10.3 10.7 10.8 11.0 11.1 11.4 11.4 11.6

9.8 ↑ lowest score

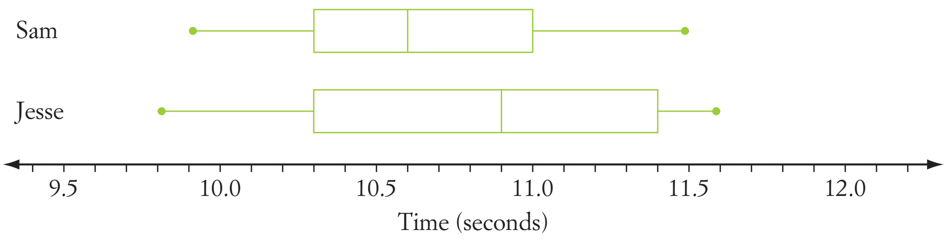
10.3 ↑ Q_1

$Q_2 = \frac{10.8 + 11.0}{2}$
$= 10.9$

11.4 ↑ Q_3

11.6 ↑ highest score

b



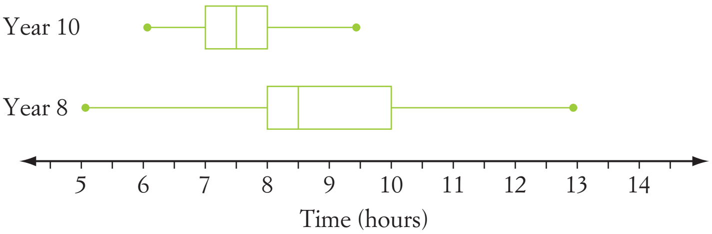
c Interquartile range for Sam = 11.0 − 10.3 = 0.7
Interquartile range for Jesse = 11.4 − 10.3 = 1.1

d Range for Sam = 11.5 − 9.9 = 1.6
Range for Jesse = 11.6 − 9.8 = 1.8

e Sam is the more consistent sprinter, since both the range and interquartile range of his times are lower than those of Jesse.

Exercise 5-04 Parallel boxplots

1 The parallel boxplots show the number of hours of sleep that Year 8 and Year 10 students usually have on a school night.



a For each Year group, find:

i the range **ii** the median **iii** the interquartile range

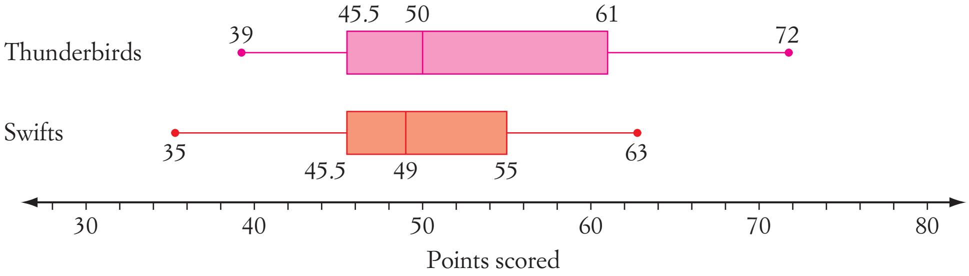
b What percentage of students usually had at most 8 hours of sleep on a school night in:

i Year 8? **ii** Year 10?

c 40 students in both Year 8 and Year 10 were surveyed. How many students usually had at least 10 hours of sleep in:

i Year 8? **ii** Year 10?

2 The number of points scored per match by the Adelaide Thunderbirds and the Sydney Swifts during the 2013 netball season are shown in the parallel box-and-whisker plot.



a Find the range of points scored by:
i the Adelaide Thunderbirds
ii the Sydney Swifts

b What is the median number of points scored for each team?

c Find the interquartile range for each team.

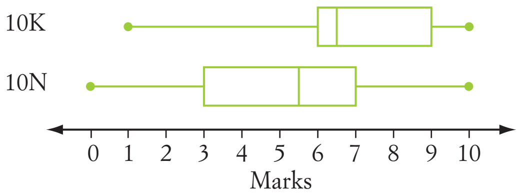
d Which team is more consistent?

e Which team performed better? Give reasons.

AAP/Jenny Evans

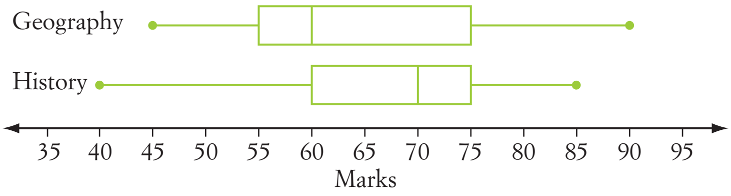
Stage 5.2

3 The boxplots show the test results of students from two different classes.



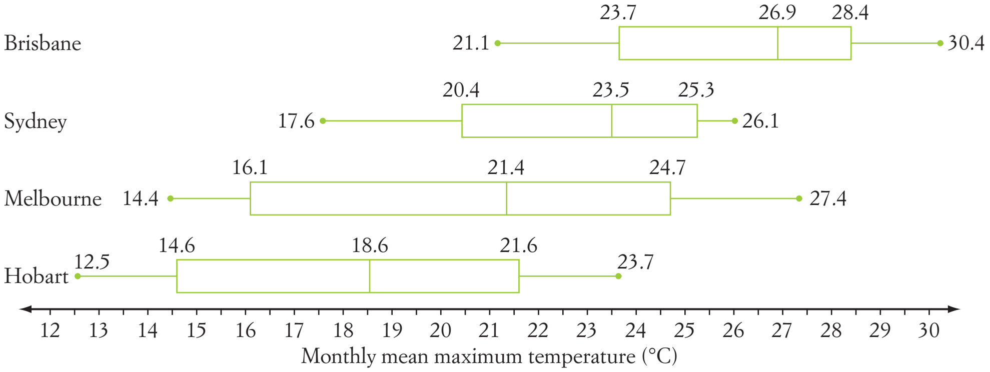
a Find the range of marks for each class.
b Find the median mark for each class.
c Find the interquartile range for each class.
d Which class is more consistent?
e Find the percentage of students who scored 6 or more in 10K.

4 In a Year 10 class of 28 students, the marks for History and Geography tests were displayed on a double boxplot.



Which of the following statements could be true?
A In Geography, more students scored between 60 and 75 than between 55 and 60.
B Fourteen students scored the same or more in History than the median mark in Geography.
C More students scored 60 or more in History than they did in Geography.
D The interquartile range for Geography is 5 less than the interquartile range for History.

5 The monthly mean maximum temperatures for four Australian capital cities are shown in the boxplots below.



a Find the median, range and interquartile range for each city.
b Which capital city had the most spread in temperature?
c Which capital city had the highest mean monthly temperatures? Justify your answer.
d Which city is warmer – Sydney or Melbourne? Give reasons.
e Which city was more consistent - Sydney or Melbourne? Give reasons.

9780170194655

Stage 5.2

See Example 7

6 The number of text messages received by a group of students in one hour are as follows.

Male: 2 0 3 0 1 2 5 6 2 1 3 2 3 7 4
Female: 4 5 6 3 7 5 8 7 4 2 4 5 10 4 3

a Find the five-number summary for each sex.

b Draw parallel box-and-whisker plots to display the data.

c Find the interquartile range for each sex.

d Find the range for each sex.

e Compare the number of text messages that males and females receive. Are there any significant differences between the spread of the two sets of data?

7 Students in a PE class had their heights measured (in cm).

Male: 174 167 164 175 189 145 165 166 165 167 171 169
Female: 163 155 171 162 165 183 172 175 166 163 150 186

a Find the five-number summary for each group and draw a parallel boxplot to display the data.

b Find the range and interquartile range for each group.

c How does the spread of heights of male students compare with the spread of heights of female students?

8 Students at a university were asked whether their frequency of exercise was high or low and then had their pulse taken. The results are as follows.

Low: 90 78 80 84 70 66 92 80 80 77 64 88
High: 96 71 68 56 64 60 50 76 78 49 68 74

a Find a five-number summary for each group and then draw parallel boxplots to show the information.

b Find the range and interquartile range for each group.

c Compare the spread between the two groups. Are there significant differences between them?

d Which group had the lower pulse rates?

9 The average monthly temperatures for Sydney and Brisbane in 2012 are as follows.

Sydney: 26.1 25.8 24.7 23.6 20.9 17.7 17.6 19.9 22.5 23.3 24.1 26.0
Brisbane: 28.7 29.8 28.2 26.5 24.0 21.1 21.4 23.3 25.5 27.3 28.2 30.4

Source: © Bureau of Meteorology

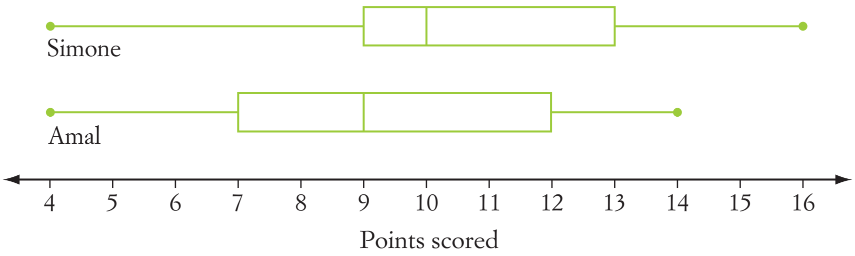
a Find the five-number summary for each city and draw a parallel boxplot.

b Find the range and interquartile range for each city.

c Which city had more consistent average monthly temperatures? Give reasons.

Stage 5.2

10 These box-and-whisker plots show the numbers of points scored by two basketball players during the season.



a Which player has the most points scored for a single game?

b What is the range of the points scored by each player?

c By just looking at the range, which player would seem to be more consistent? Justify your answer.

d Find the median score of each player.

e Find the interquartile range for each player.

f Which player is more consistent?

g Estimate the percentage of games in which Simone scored 9 or 10 points.

h Estimate the percentage of games in which Amal scored more than 12 points.

Mental skills 5 Maths without calculators

Multiplying by 9, 11, 99 and 101

We can use expansion when we multiply by a number near 10 or near 100.

1 Study each example.

a
$$\begin{aligned} 25 \times 11 &= 25 \times (10 + 1) \\ &= 25 \times 10 + 25 \times 1 \\ &= 250 + 25 \\ &= 275 \end{aligned}$$

b
$$\begin{aligned} 14 \times 9 &= 14 \times (10 - 1) \\ &= 14 \times 10 - 14 \times 1 \\ &= 140 - 14 \\ &= 126 \end{aligned}$$

c
$$\begin{aligned} 32 \times 12 &= 32 \times (10 + 2) \\ &= 32 \times 10 + 32 \times 2 \\ &= 320 + 64 \\ &= 384 \end{aligned}$$

d
$$\begin{aligned} 7 \times 99 &= 7 \times (100 - 1) \\ &= 7 \times 100 - 7 \times 1 \\ &= 700 - 7 \\ &= 693 \end{aligned}$$

e
$$\begin{aligned} 27 \times 101 &= 27 \times (100 + 1) \\ &= 27 \times 100 + 27 \times 1 \\ &= 2700 + 27 \\ &= 2727 \end{aligned}$$

f
$$\begin{aligned} 18 \times 8 &= 18 \times (10 - 2) \\ &= 18 \times 10 - 18 \times 2 \\ &= 180 - 36 \\ &= 144 \end{aligned}$$

2 Now evaluate each product.

a 16 × 11 **b** 33 × 11 **c** 29 × 9 **d** 45 × 9
e 62 × 11 **f** 7 × 101 **g** 18 × 101 **h** 36 × 99
i 19 × 8 **j** 45 × 12 **k** 21 × 102 **l** 6 × 98
m 32 × 9 **m** 7 × 99 **o** 39 × 101 **p** 71 × 12

5-05 Comparing data sets

Stage 5.2

Worksheet

Comparing city temperatures

MAT10SPWK10037

Example 8

The back-to-back stem-and-leaf plot shows the results in Year 10 Maths and Science tests.

Maths		Science
5 2	3	6 8
8 6 3 0	4	4 6
8 7 7 4 1	5	1 5 9
8 8 7 6 6 3 2 0	6	0 2 8 9
6 5 4 2 1 1	7	2 3 4 4 5 8 8
6 4 3	8	0 0 2 4 5 6 7 8 9
6 0	9	0 4 4

a Find the mean mark (correct to one decimal place) for each subject.

b Find the median for each subject.

c Find the range and interquartile range for each subject.

d For each subject:

i describe the shape **ii** identify any outliers and clusters.

e In which subject have the students performed better? Justify your answer.

Solution

a Mean for Maths $= \dfrac{1919}{30}$

$= 64.0$

Mean for Science $= \dfrac{2151}{30}$

$= 71.7$

b Median for Maths = 66

Median for Science = 74.5

Average of the 15th and 16th scores.

c Range for Maths = 96 − 32 = 64

Interquartile range = 74 − 54 = 20

Range for Science = 94 − 36 = 58

Interquartile range = 85 − 60 = 25

d i The results for Maths are symmetrical, while the results for Science are negatively skewed.

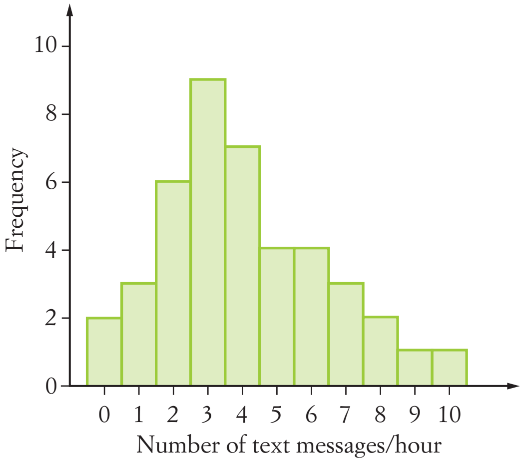
ii There is some clustering for the Maths results in the 60s and in Science the clustering occurs in the 70s and 80s.

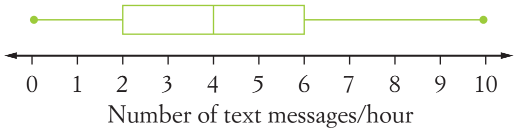
e The students have performed better in Science as the mean and median for it are greater than the mean and median for Maths.

Stage 5.2

Example 9

The number of text messages received per hour by a group of teenagers are displayed in the frequency histogram and the boxplot below.





a How many teenagers received more than 6 text messages per hour?

b Find:

i the mode **ii** the median

iii the range **iv** the interquartile range.

c The shape of the distribution is positively skewed. How is this shown by:

i the frequency histogram? **ii** the boxplot?

d According to the boxplot, what percentage of teenagers received 2 or more text messages?

e What information is better seen on:

i the frequency histogram? **ii** the boxplot?

Solution

a Number of teenagers receiving more than 6 text messages

$= 3 + 2 + 1 + 1$

$= 7$

Using the frequency histogram.

b **i** Mode $= 3$

Using the frequency histogram.

ii Median $= 4$

Using the boxplot.

iii Range $= 10 - 0$

$= 10$

Using the frequency histogram or boxplot.

iv Interquartile range $= 6 - 2$

$= 4$

Using the boxplot.

c **i** The tail of the frequency histogram leans towards the higher scores.

ii The length of the boxplot to the right of the median (Q_2) is greater than its length to the left of the median.

d $Q_1 = 2$, so 75% of teenagers received 2 or more text messages/hour.

e **i** The mode and information regarding the number of text messages received by teenagers can be determined from the frequency histogram.

ii The median, quartiles and interquartile range are easily determined from the boxplot.

Stage 5.2

Exercise 5-05 Comparing data sets

See Example 8

1 The back-to-back stem-and-leaf plot shows the amount of cash (in dollars) carried by a sample of Year 11 students at Mavbalear Senior High.

Boys		**Girls**
5 5 3	0	5 5 6 8 9
8 5 5 2 0	1	0 2 2 5 5 8 8 9
9 6 5 5 5 0 0	2	0 5 6 8 8 8
8 5 5 4 3 2 0 0	3	0 1 4 5 6
5 4 4 2 2 0	4	0 0 5 6
6 6 5 4 3	5	0 3 5
4 2 2	6	5 5 8
5	7	0 4

a Find the mean amount of cash (correct to the nearest cent) carried by each group.

b Find the median amount of cash carried by each group.

c Find the range and interquartile range for each group.

d For each group:

i describe the shape **ii** identify any outliers and clusters.

e Who generally carries more cash – boys or girls? Justify your answer.

2 The back-to-back histogram shows the number of goals scored by two football teams during a season.

a How many games were played by each team?

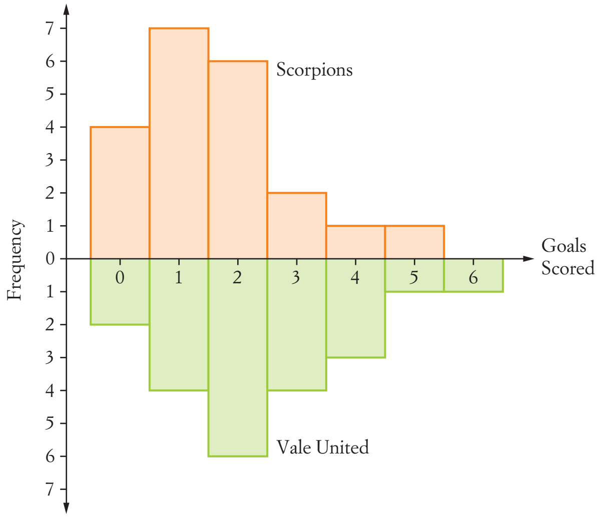
b How many goals were scored by
i Scorpions **ii** Vale United?

c Find the mean number of goals scored by each team.

d What is the range for each team?

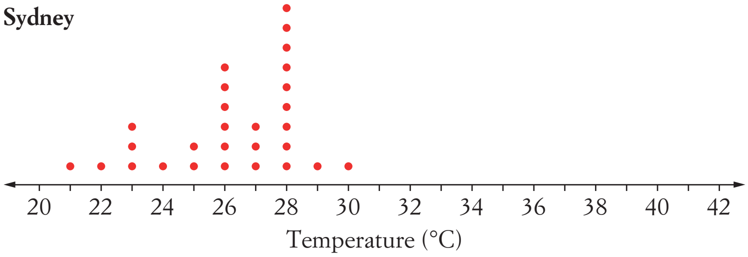
e Describe the shape of each team's results.

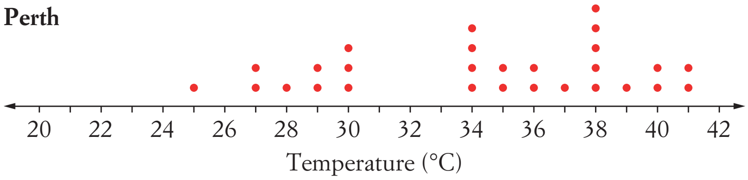
f Which team performed better? Give reasons.



Stage 5.2

3 The daily maximum temperatures for Sydney and Perth in February are shown below.





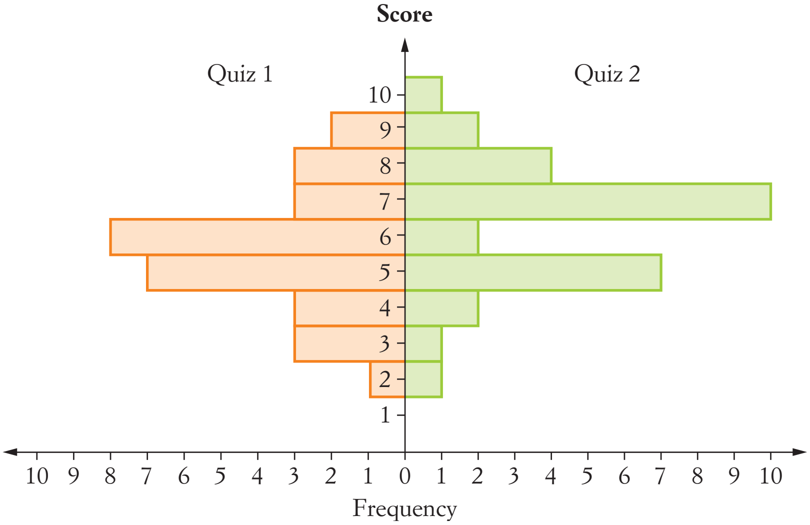
a Find the mean, median and modal temperatures for each city.

b Find the range and interquartile range of temperatures for each city.

c Describe the distribution shape of the temperatures for each city and identify any outliers or clusters.

d Compare the temperatures in Sydney and Perth. Comment on measures of location (the mean, median and mode), and measures of spread (range and interquartile range).

4 The results for two quizzes taken by a Year 10 History class are shown below.



a How many students are in the Year 10 History class?

b Find the mean and mode for each quiz.

c Find the median for each quiz.

d For each quiz, find:
- **i** the range
- **ii** the interquartile range.

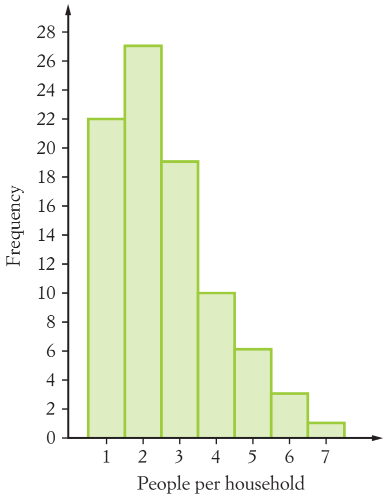
e Describe the distribution for each quiz, identifying any clusters and outliers.

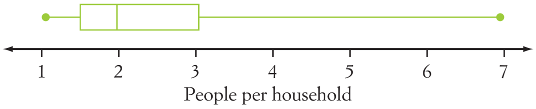
f Are there significant differences between the results of the two quizzes? Justify your answer.

Stage 5.2

***See* Example 9**

5 A survey to determine the number of people per household was conducted in several shopping centres. The results are shown in the frequency histogram and boxplot below.





a How many households had more than 3 or more people?

b Find the:

i mode

ii median

iii range

iv interquartile range.

c Describe the shape of the distribution.

d According to the boxplot, what percentage of households had 2 or more people?

e Clustering occurs at 1 to 3 people per household. How is this shown on the:

i frequency histogram?

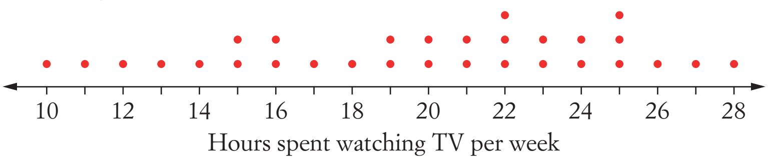
ii boxplot?

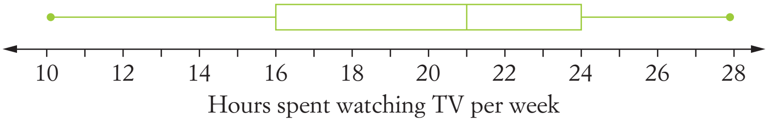
f What information is better seen on:

i the frequency histogram?

ii the boxplot?

6 The dot plot and box-and-whisker plot show the number of hours that Year 10 students spent watching TV during one week.





a How many students watched TV for:

i fewer than 15 hours per week?

ii more than 20 hours per week?

b Find the:

i the mode

ii the range

iii the interquartile range

c What is the shape of the distribution? How is this shown by:

i the dot plot?

ii the boxplot?

d Which display of data, the dot plot or boxplot, is better for finding:

i the mode?

ii the median?

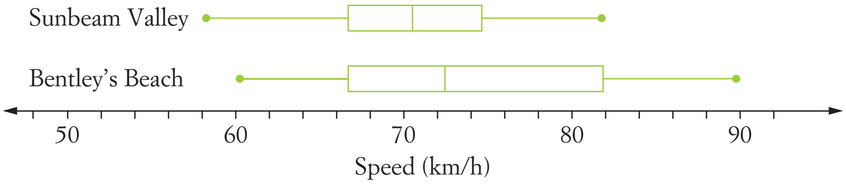
iii the number of students who watched TV for 25 hours?

iv the interquartile range?

Stage 5.2

7 The speeds of cars were monitored along a main road in two different suburbs. The results are shown in the back-to-back stem-and-leaf plot and the parallel boxplots.

Sunbeam Valley		**Bentley's Beach**
8	5	
9 8 8 7 4 3 3 3 2 0	6	0 0 1 2 3 5 5 7 8 9
9 9 6 5 5 4 4 3 3 2 2 1 1 0 0 0	7	0 0 2 2 3 3 5 5 5 6 6
2 0 0	8	0 2 3 4 5 5 5 8
	9	0



a Find the range, median and interquartile range for each suburb.

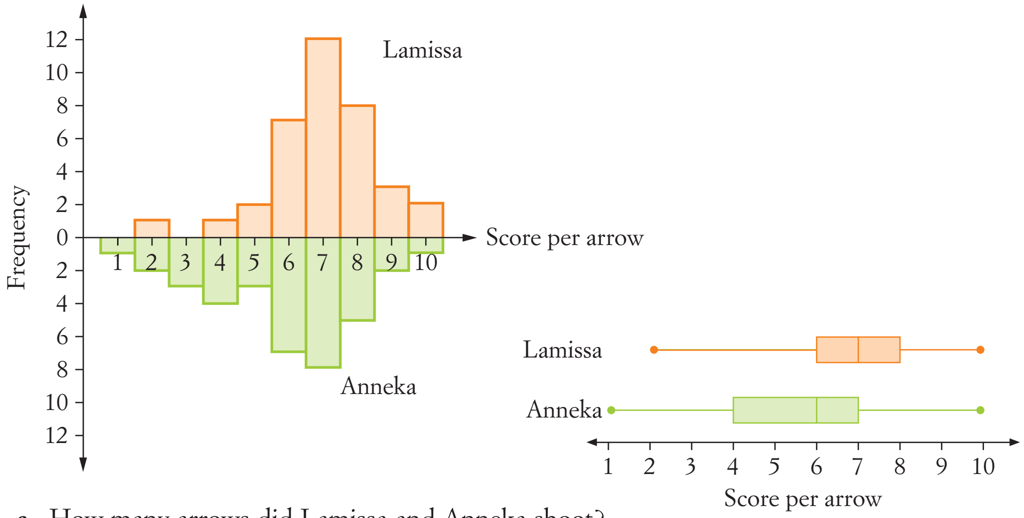
b What is the shape of the distribution for each suburb?

c Are there any clusters or outliers in either suburb?

d According to the boxplot, what percentage of drivers in Bentley's Beach drive faster than all drivers in Sunbeam Valley?

e In which suburb do drivers generally drive faster? Give a possible reason for your answer.

8 Lamissa and Anneka each shot arrows at a target 50 m away during an archery contest. They scored 10 for a bulls-eye down to 1 for the outer ring. Their results are displayed in the back-to-back histogram and the parallel box-and-whisker plots below.



a How many arrows did Lamissa and Anneka shoot?

b Find the mode and median score per arrow for each contestant.

c Find the range and interquartile range for each contestant.

d Describe the shape of the distribution for each contestant.

e According to the boxplots, on what percentage of arrows shot was a score of 6 or less achieved by:
  i Lamissa? **ii** Anneka?

f Who was the better archer during this contest? Justify your answer by referring to the measures of location and spread.

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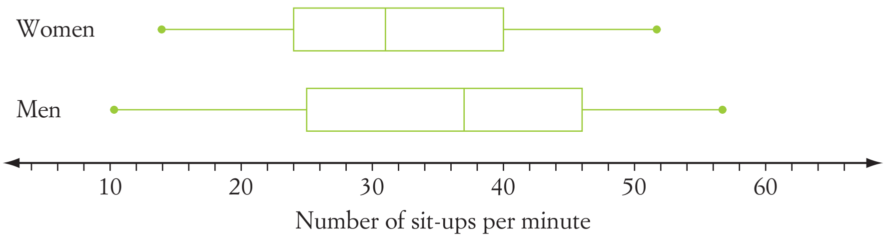
9 The number of sit-ups per minute completed by men and women at the Full On Fitness Centre are displayed in the back-to-back histogram and parallel boxplots.

Women		**Men**
8 7 5 4	1	0 6 7 9 9
9 9 9 8 8 7 4 4 3 3 1 0	2	0 2 3 4 4 5 5 7 7 8
7 6 5 5 5 4 3 2 1 0 0	3	0 2 4 5 6 6 7 7 8 8 8 8 9
7 5 4 3 2 0 0	4	1 3 4 6 6 6 6 7 7 9
2 1 0	5	0 1 3 4 7 7



a Why would a dot plot be an inappropriate display of the data shown above?

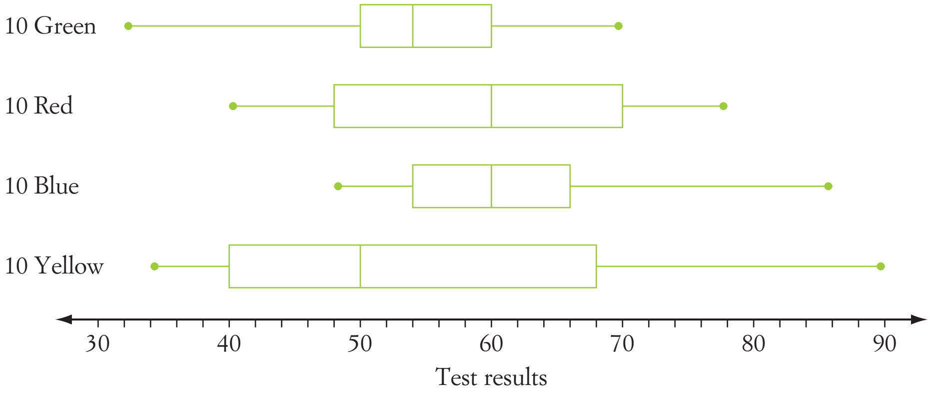
b What is the median number of sit-ups per minute completed by each group?

c Find the range and interquartile range for each group.

d Describe the shape of the distributions for each group.

e Which group has more spread in the number of sit-ups completed per minute? Give reasons for your answer.

10 The results of a Maths test given to four Year 10 classes are shown below.



a What is the range of test results for:
i 10 Yellow? **ii** 10 Blue?

b For which class are the test results:
i positively skewed? **ii** negatively skewed? **iii** symmetrical?

c Which class had:
i the lowest interquartile range? **ii** the highest test score?
iii the highest median?

d Which class had the best test results overall? Give reasons.

Stage 5.2

Puzzle sheet

Scatter plots matching game

MAT10SPPS10038

Worksheet

Scatter plots

MAT10SPWK00002

5-06 Scatter plots

Bivariate data is data that measures two variables, such as a person's height and arm span (distance between outstretched arms). Bivariate data is represented by an ordered pair of values that can be graphed on a **scatter plot** for analysis.

A scatter plot is a graph of points on a number plane. Each point represents the values of the two different variables and the resulting graph may show a pattern that may be linear or non linear. If there is a pattern, then a relationship may exist between the two variables.

Example 10

The heights and arm spans of a group of students are shown in the table.

Height, *H* cm	162	182	153	145	172	163	150	142	183	145	192	171
Arm span, *S* cm	158	185	145	143	174	165	151	141	181	158	191	178

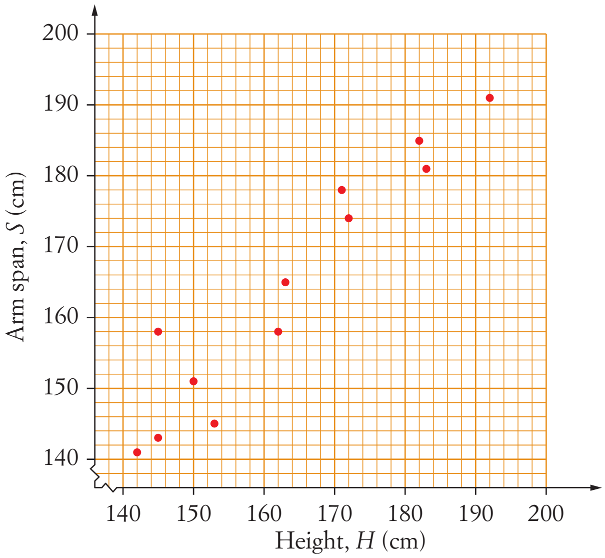
a Plot the data on a scatter plot.

b Describe the pattern of the plotted points.

c Describe the relationship between the students' heights and arm spans.

Solution

a



b The points form a linear pattern.

c As the heights of students increase, their arm spans tend to increase.

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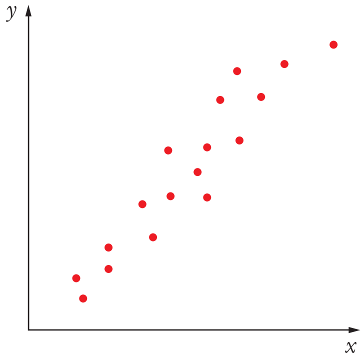
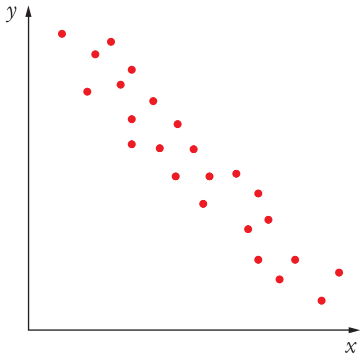
Strength and direction of linear relationships

The type of linear pattern will indicate the **strength** and **direction** of the relationship between the two variables.

Two variables x and y have a **positive relationship** if y increases as x increases.



Two variables x and y have a **negative relationship** if y decreases as x increases.



Summary

The strength of a relationship between two variables can be described as:

- **strong** if the points are close together
- **weak** if the points are more spread out
- **perfect** if all points lie on a straight line

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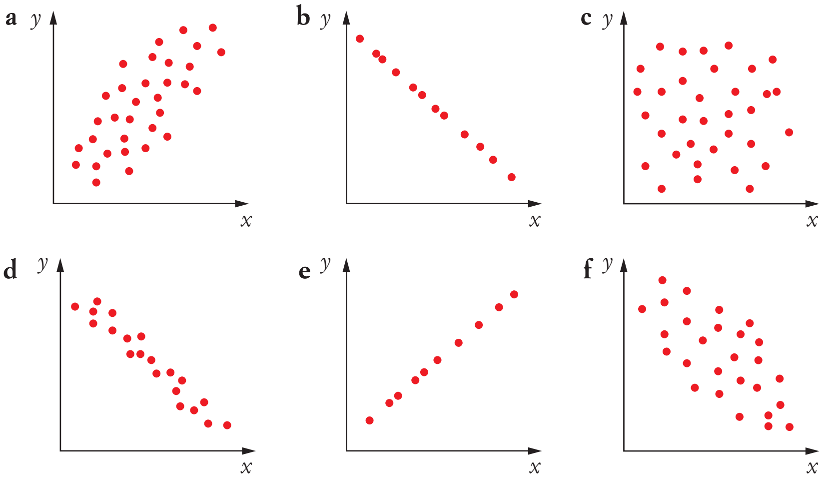
Stage 5.2

Example 11

Describe the strength and direction of the relationship shown in each scatterplot.



Solution

a	weak positive relationship	The points can be seen to form a line but they are spread out.
b	perfect negative relationship	The points lie on a decreasing straight line.
c	no relationship	The points are very spread out with no pattern.
d	strong negative relationship	The points can be seen to form a decreasing line and they are close together.
e	perfect positive relationship	The points lie on an increasing straight line.
f	weak negative relationship	The points can be seen to form a decreasing line but they are spread out.

Dependent and independent variables

If a variable y depends on the value of the variable x, y is called the **dependent variable**, and x is called the **independent variable**. For example, stride length (the length of a person's walking step or pace) depends on the person's height, so stride length is the dependent variable and height is the independent variable. When graphing, the dependent variable is shown on the vertical (y-) axis while the independent variable is shown on the horizontal (x-) axis.

Exercise 5-06 Scatter plots

See Example 10

1 The heights and handspans of a group of students are shown in the table.

Height, H cm	168	175	175	156	160	173	171	180	185	175	182	180
Handspan, S cm	20.0	21.1	17.6	16.5	17.5	19.0	20.8	22.5	25.0	23.0	20.2	21.1

a Plot the data on a scatter plot.

b Describe the pattern of the plotted points.

c Describe the relationship between the students' heights and their hand spans.

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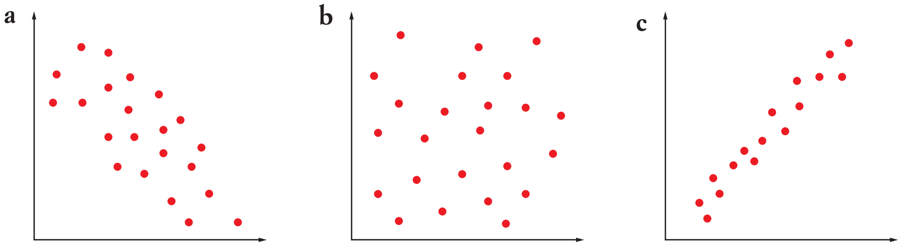
Stage 5.2

See Example 11

2 Describe the strength and direction of the relationship shown in each scatter plot.



3 Describe the strength and direction between the variables height, H and handspan, S in question **1**.

4 The height and stride length measurements of some students are shown in the table below.

Height, *H* cm	174	160	158	180	169	172	171	171	148	190	166	173
Stride length, *L* cm	72.2	64.0	66.4	74.7	70	71.5	70.9	71.2	61.4	78.9	68.0	71.9

a Explain why stride length is the dependent variable.

b Graph this data on a scatter plot.

c Describe the pattern of the plotted points.

d Describe the relationship between the students' heights and stride lengths.

e Describe the strength and direction of the relationship.

f Predict the stride length of a student who is 175 cm tall.

5 The table lists the points scored for and against each NRL team one season.

a Graph this data on a scatter plot.

b Is the pattern of the points linear?

c Describe the strength and direction of the relationship between points scored for and points scored against.

Points scored for, *F*	Points scored against, *A*
568	369
579	361
559	438
497	403
597	445
545	536
445	441
481	447
405	438
506	551
449	477
448	488
462	626
497	609
409	575
431	674

Dreamstime/Vselenka

Stage 5.2

6 Year 10 students were surveyed on the number of hours in a week they spent doing homework and the number of hours they spent on the computer. The results are shown in the table.

Homework, *H*	2	15	12	5	4	2	4	15	14	5	2	5	20	4	2	11
Computer, *C*	25	30	18	35	6	30	20	22	6	40	8	3	20	30	5	8

a Plot the points on a scatter plot.

b Describe the strength and direction of the relationship between hours spent doing homework and hours spent on the computer.

7 A survey was conducted to see whether there was a relationship between height and the age of students in a high school. The results are in the table below.

Age, *A* (years)	14	16	15	13	11	14	17	15	12	11	14	16	13	18
Height, *H* (cm)	162	174	182	162	132	173	187	160	154	145	165	171	151	181

a Graph the points on a scatter plot.

b Which variable could be considered as the dependent variable? Give reasons.

c Describe the strength and direction of the relationship between the age and height of students.

Technology Scatter plot patterns

Investigate one of the following pairs of bivariate data for a group of students or people. You will need instruments (measuring tapes and/or trundle wheels) and stopwatches to help you collect your data.

- Height vs arm span
- Reaction time vs hours of sleep
- Stride length vs 50 m sprint time

Alamy/Kim Karpeles

1 Enter your data into a spreadsheet. Graph it using **Scatter with Smooth Lines and Markers**.

2 Analyse your graph. What type of linear relationship does it show? Positive or negative? Strong or weak?

3 Write a brief summary describing the relationship between the two variables.

5-07 Bivariate data involving time

Bivariate data involving time, or **time series data**, is two-variable data that has time as the independent variable. Examples of time series data are population changes over time, weekly share prices, daily rainfall and patients' heart rates.

Example 12

This table shows the average household size between 1961 and 2011, according to the Census.

Year	1961	1966	1971	1976	1981	1986	1991	1996	2001	2006	2011
Average household size	3.6	3.5	3.3	3.1	3.0	2.9	2.8	2.6	2.6	2.6	2.6

Source: CC © Copyright 2013, Commonwealth of Australia

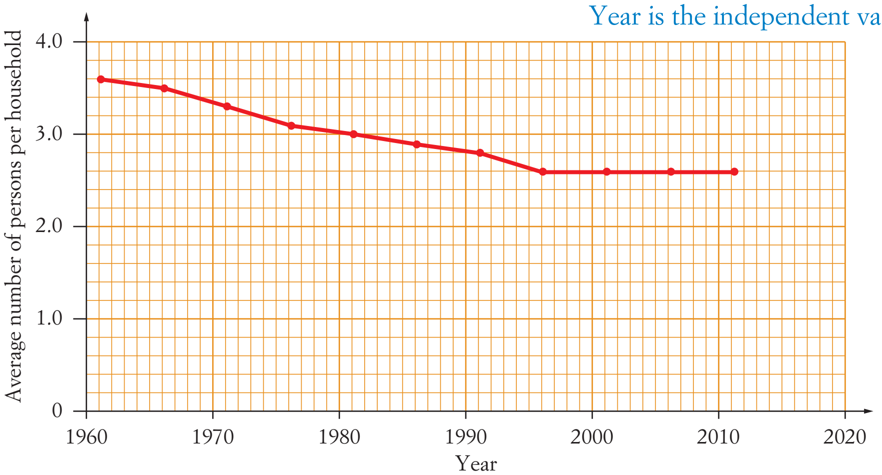
a Graph the data on a scatter plot and join the points.

b Use your graph to describe the change in average household size from 1961 to 2011.

c Based on your time series graph, estimate the household size for 2021.

Solution

a



Year is the independent variable.

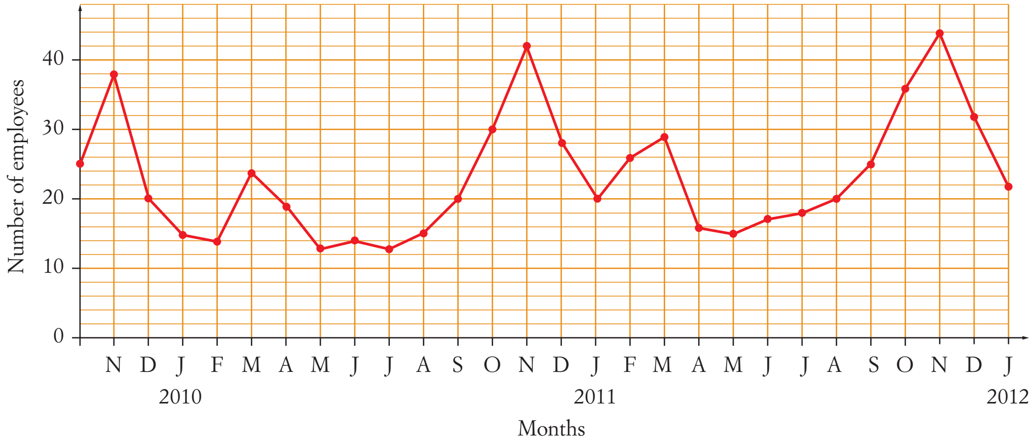
b The average household size decreased from 3.6 in 1961 to 2.6 in 1996 and since then there has been no change.

c 2.4 to 2.6 people per household.

Stage 5.2

Exercise 5-07 Bivariate data involving time

1 The number of people employed per month at SUPA SAVE SUPERMARKET from November 2009 to February 2012 is displayed in the time series graph below.



a How many people were employed by the supermarket in:
- **i** November 2009?
- **ii** December 2010?
- **iii** June 2011?

b In which month of the year were the most people employed by the supermarket? Suggest a reason why.

c In which month of the year were the least number of people employed? Suggest a reason why.

d Describe how the number of people employed by the supermarket changes from November 2009 to February 2012.

2 The population figures for Australia from 1960 to 2010 are given in the table below.

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005	2010
Population (millions)	10.28	11.39	12.51	13.89	14.70	15.76	17.07	18.07	19.15	20.39	22.3

a Graph the data on a scatter plot and join the points.

b Between which years was the greatest population increase?

c Use your graph to describe the change in Australia's population from 1960 to 2010.

d Based on your time series graph, estimate the population for Australia in:
- **i** 2020
- **ii** 2045.

3 The table below shows the fatalities on NSW roads from 1950 to 2010.

Year	1950	1960	1970	1980	1990	2000	2010
Fatalities	634	978	1309	1303	797	603	405

a Draw a time series graph for this data.

b Describe the change in road fatalities from 1950 to 2010.

c Give possible reasons for the reduction in road fatalities from a high of 1309 in 1970 to a low of 405 in 2010.

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Stage 5.2

4 The annual mean maximum temperatures for Sydney from 1990 to 2012 are given in the tables below.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Temperature (°C)	22.3	22.8	21.5	22.3	22.6	21.8	22.1	22.4	22.7	22.1	22.7	23.1

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Temperature (°C)	23.1	22.7	23.4	23.4	23.1	22.7	22.1	22.1	22.6	22.6	22.7

Source: Bureau of Meteorology

a Draw a time series graph for temperatures from:
i 1990 to 2000 **ii** 2001 to 2012.

b Has there been much change in Sydney's temperature from:
i 1990 to 2000? **ii** 2001 to 2012?

Justify your answer.

c Are there differences in temperature between the periods 1990–2000 and 2001–2012? Give reasons.

5 The table below shows the annual emissions of carbon (measured in Megatonnes, Mt) from 2002 to 2012.

Year	2002	2003	2004	2005	2006	2007
Annual emissions (Mt CO_2-e)	509.5	514.5	529.2	530.2	539.8	546.5

Year	2008	2009	2010	2011	2012
Annual emissions (Mt CO_2-e)	554	542.8	551.8	553.2	551.9

a Draw a time series graph for this data.

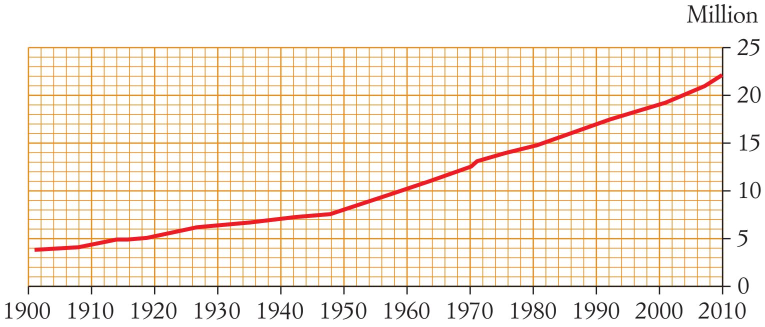
b Describe the change in carbon emissions from 2002 to 2008.

c What happens to the carbon emissions after 2010?

d Give a possible reason for your answer to part **c**.

e What is your estimate of carbon emissions for:
i 2015? **ii** 2025?

6 The graph below shows Australia's population from 1901 to 2010.



Source: Australian Historical Population Statistics (3105.0.65.001); Australian Demographic Statistics (3101.0).

Stage 5.2

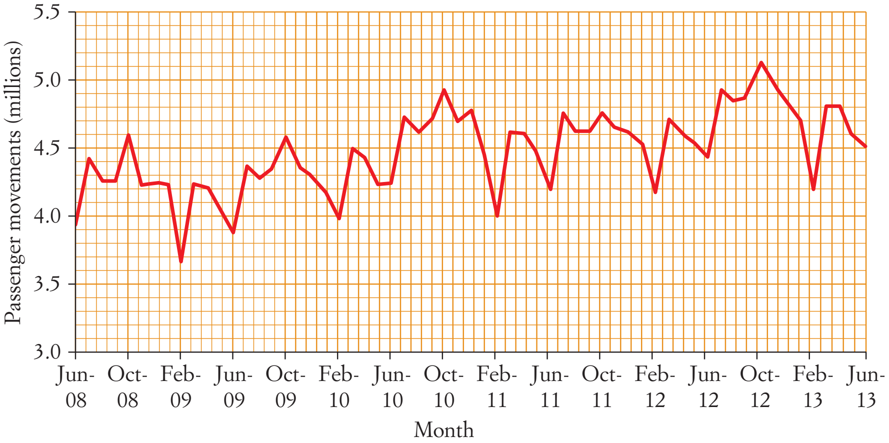
a What was Australia's population in 1901?

b By how much had Australia's population increased between 1901 and 2010?

c What was the average annual rate of increase in population between 2000 and 2010?

d If this trend continues, what is the expected population in 2025?

7 The time series graph below shows the monthly amount of passenger traffic on Australian domestic commercial airlines.



Source: CC © Copyright 2013, Commonwealth of Australia

a Describe the trend in domestic passenger traffic for June 2008–June 2013.

b What was the approximate amount of passenger traffic per month in:
  i June 2008? **ii** June 2010? **iii** June 2011? **iv** June 2013?

c What was the percentage increase in domestic passenger movements from June 2008 to June 2013?

Investigation: Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas – from population, employment, weekly earnings, weight and obesity in adults, to health of children in Australia.

Visit the ABS website **www.abs.gov.au** to answer the following questions.

1 **a** What is the current population of Australia?
  b What is the predicted population for:
    i 2020? **ii** 2030? **iii** 2040?
  c What is Australia's rate of population increase?

2 Go to 2011 Census Data by Location, and then to Data and analysis.
  a What was the population in NSW and its increase from 2006?
  b Which state had the:
    i largest increase in population?
    ii the smallest increase in population?

Stage 5.2

5-08 Statistics in the media

We live in a world of 24-hour news, whether it is from newspapers, TV or the Internet, which often quote results from surveys. When survey data is used in the media we need to consider:

- where the news comes from and what samples the statistics are based on
- who supplied the information
- the number of samples and the sample size used
- the way in which the collected data has been presented

123rf/Oleksiy Mark

Example 13

What concerns could be raised about the following claim?
'The Daily Sun newspaper reports that it has an average issue readership of 1.385 million and that its Travel liftout has a readership of 1.455 million.'

Solution

The newspaper is reporting about its own readership and so it may be biased. It also states that its Travel liftout has a higher readership that its issue readership.

Example 14

The weights (in kg) of a large group of 18–20-year-olds attending University are:

57	58	62	84	64	74	57	55	56	90
68	63	49	66	63	65	60	60	46	70
85	60	70	41	73	75	67	63	70	85
51	49	75	77	87	54	60	75	58	68
55	65	66	57	85	75	56	60	62	75
74	58	51	62	50	55	71	57	58	100
72	58	103	64	52	55	80	96	45	87
81	80	48	54	65	54	59	50	78	60
74	70	64	59	72	78	104	63	102	95

a How many students were in the group?

b Randomly select four groups of 10 and for each sample calculate:

i the mean **ii** the median **iii** the interquartile range.

c Use your results to estimate the mean, median and interquartile range of the population from your four samples.

d Compare your estimates to the mean, median and interquartile range of the population.

Stage 5.2

Solution

a There were 90 students in the group.

b Randomly select four samples of 10 from the 'population'.

Sample 1:	90	63	75	48	74	85	51	96	60	78
Sample 2:	62	75	103	64	65	54	55	54	60	75
Sample 3:	68	70	57	52	78	74	60	63	58	87
Sample 4:	72	54	52	80	45	87	49	77	54	58

The statistics for each group are:

Sample 1: $\bar{x} = 72$ median $= 74.5$ interquartile range $= 25$
Sample 2: $\bar{x} = 66.7$ median $= 63$ interquartile range $= 20$
Sample 3: $\bar{x} = 66.7$ median $= 65.5$ interquartile range $= 16$
Sample 4: $\bar{x} = 62.8$ median $= 56$ interquartile range $= 25$

c Taking averages, population statistics estimates are:

$$\text{Mean, } \bar{x} = \frac{72 + 66.7 + 66.7 + 62.8}{4} = 67.1 \text{ (correct to 1 decimal place)}$$

$$\text{Median} = \frac{74.5 + 63 + 65.5 + 56}{4} = 64.8 \text{ (correct to 1 decimal place)}$$

$$\text{Interquartile range} = \frac{25 + 20 + 16 + 25}{4} = 21.5$$

d The statistics for the population are:

Mean, $\bar{x} = 66.9$ (correct to 1 decimal place)
Median $= 64$
Interquartile range $= 18$

The estimates for the mean, median and interquartile range compare very favourably with the population statistics.

Exercise 5-08 Statistics in the media

See Example 13

1 A TV network surveys 300 people in shopping centres between 9 a.m. and 11 a.m. to get feedback on its new game show.

- **a** How may this survey be biased?
- **b** Suggest a better method for obtaining feedback about its game show.

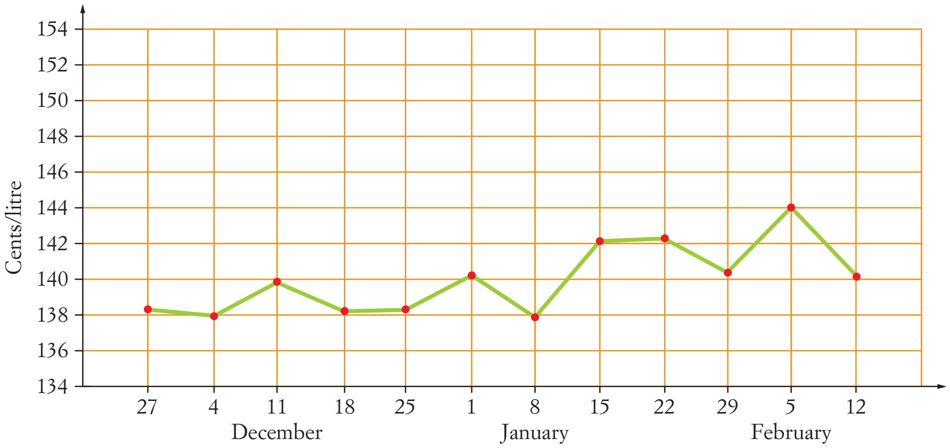
2 A report about hot-water systems recommended a heat pump system. The report stated that people in Queensland who had the heat pump hot-water system saved 30% of their electricity bill per quarter. The company is using this information in their advertising of the product in NSW and Victoria.

Should people in NSW and Victoria install this type of hot-water system? Give reasons.

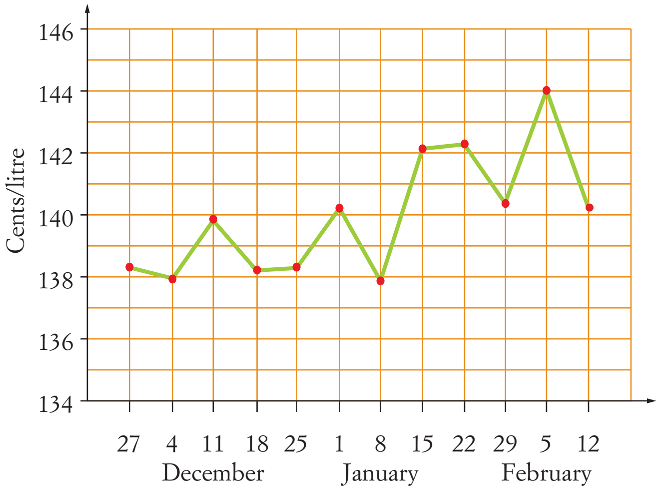
Stage 5.2

3 A report on petrol pricing was conducted by two companies. The following graphs, showing the price of petrol for the same 12-week period, were used to present their findings on the price of petrol.

Petrol pricing: Company A



Petrol pricing: Company B



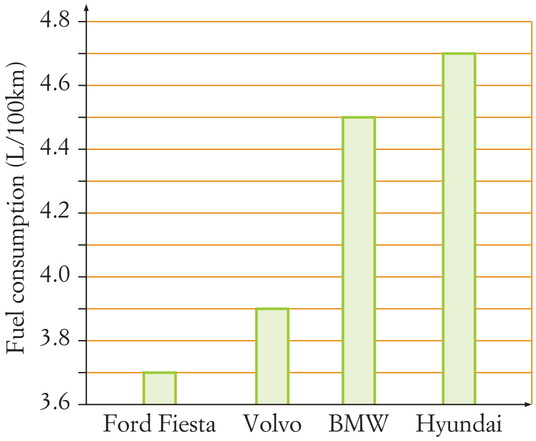
a What is being implied about petrol prices by:

i Company A? **ii** Company B?

b How could both graphs be improved to give a true picture of changing petrol prices?

Stage 5.2

4 A report on the diesel fuel consumption of different cars was published in a motoring magazine.



- **a** What is the magazine report implying about the fuel consumption of the different cars?
- **b** What is the difference in fuel consumption between the:
  - **i** Ford Fiesta and the Volvo?
  - **ii** Ford Fiesta and the Hyundai?
  - **iii** BMW and the Hyundai?
- **c** How should the graph be redrawn so that it is not biased towards the Ford Fiesta and the Volvo?

5 A company manufactures a product. After 3 months, they conduct a survey and customers are asked to rate the product as Excellent, Good or Satisfactory. Is the survey biased? Justify your answer.

6 A market research company working for a car manufacturer needs to determine the most popular car colours.

- **a** Give an example of a biased question for this survey.
- **b** What other information should the market research company use, apart from the survey, to determine the most popular colour car?

See Example 14

7
- **a** Randomly select four samples of 10 weights from the population shown in Example **14**, and for each sample calculate:
  - **i** the mean
  - **ii** the median
  - **iii** the interquartile range.
- **b** Use your results to estimate the mean, median and interquartile range of the population from your four samples.
- **c** How do the statistics of your samples compare to the mean, median and interquartile range of the population?
- **d** How do the estimated statistics compare to the population statistics?

8
- **a** Repeat the process of question **7** by taking two samples: of size:
  - **i** 5
  - **ii** 15
  - **iii** 20
- **b** Do the sample statistics become more accurate and move closer to the population statistics as sample size increases?

Power plus

1 The strength and direction of the relationship between two variables can be measured by the correlation coefficient (r).

- **a** Between which two values does the correlation coefficient lie?
- **b** What is the strength and direction of the relationship if the correlation coefficient is zero?
- **c** Write a possible value for the correlation coefficient for each relationship described.
  - **i** perfect positive
  - **ii** weak negative
  - **iii** strong negative

2 Two variables may have a strong relationship, but this does not mean a change in one variable causes a change in the other.
Which of the following pairs of variables have a causal relationship?

- **a** height and weight of people
- **b** the time it takes to walk to school and the distance from home to school
- **c** the number of children per household and the number of mobile phones per household
- **d** the age of people and their reaction time
- **e** the price of petrol and the amount of petrol sold
- **f** the interest rate of loans and the number of new home loans

3 The following scores are the test results on a History exam for a class of 20 students.

13	14	16	12	14	16	18	13	15	10
9	15	13	14	13	10	8	14	16	14

- **a** Find the mean, median and mode.
- **b** Find the range and interquartile range.
- **c** An error was made in recording the scores and 4 marks need to be added to each score. What effect will this have on the statistics calculated in parts **a** and **b**?

Chapter 5 review

Puzzle sheet
Data crossword
MAT10SPPS10039

Quiz
Statistics
MAT10SPQZ00002

Language of maths

bias
bivariate data
boxplot
cluster
dependent variable
five-number summary
independent variable
interquartile range
mean
measure of location
measure of spread
median
mode
negatively skewed
outlier
parallel boxplots
positively skewed
quartile
range
scatter plot
skewed
strong
symmetrical
weak

1 What is represented by the 'whiskers' on a **box-and-whisker plot**?

2 What are the **measures of location** and the **measures of spread**?

3 What are the five things found in a **five-number summary**?

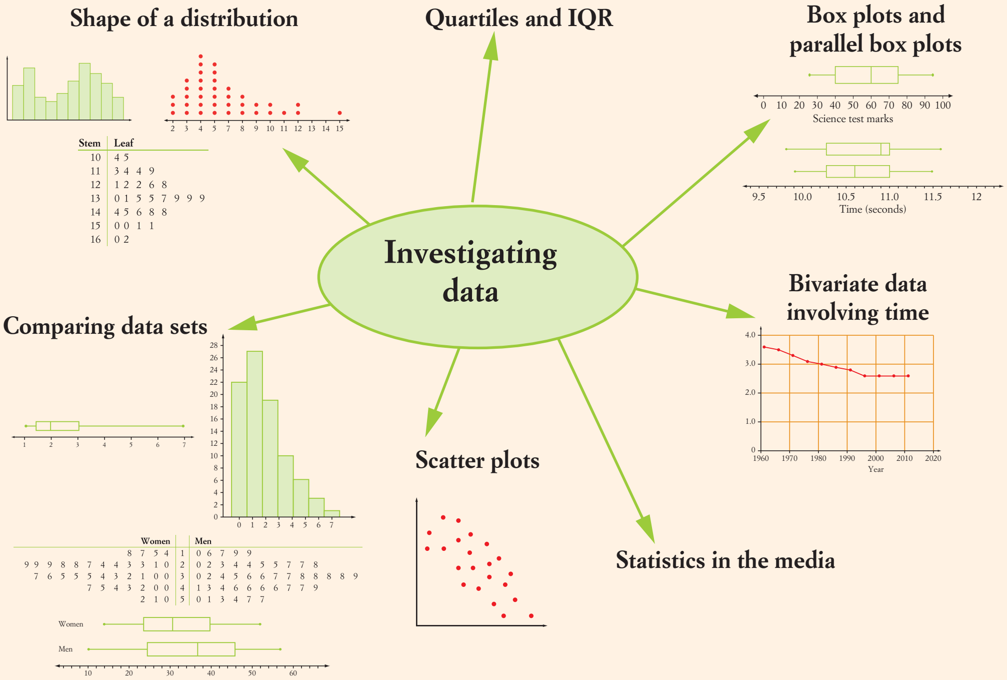
4 Describe a statistical distribution that is **positively skewed**.

5 What type of graph is used to represent **bivariate data**?

6 Give two examples of how statistics can be misleading.

Topic overview

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 5 revision

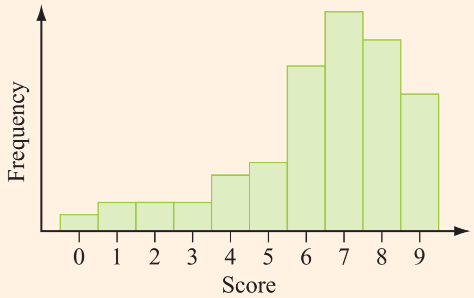
See Exercise 5-01

1 For each statistical distribution:

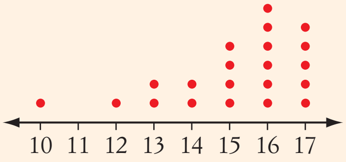
i describe the shape

ii identify any outliers and clusters.

a



b



c

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

Stage 5.2

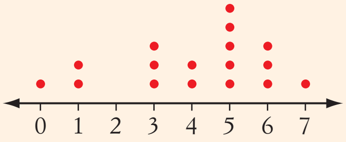
See Exercise 5-02

2 Find the interquartile range of each set of data.

a 5 8 8 10 12 13 14 15 18

b 24 15 23 28 20 20 18 30 21 18

c



d

Stem	Leaf
3	0 1 2
4	3 5 8 8 9 9 9
5	4 5 6 6 8
6	0 1 3 7
7	2

e

Score	Frequency
10	3
11	8
12	15
13	18
14	10
15	5

Stage 5.2

See Exercise 5-03

3 The number of goals scored by the Under-18s Vale soccer team are:

2	0	0	4	2	1	1	2	3
2	3	7	4	3	1	0	4	2

a Find the range and interquartile range of the scores.
b Find the five-number summary for the data
c Draw a box-and-whisker plot for the data.

See Exercise 5-04

4 The pulse rates of students were taken before and after exercising. The results were:

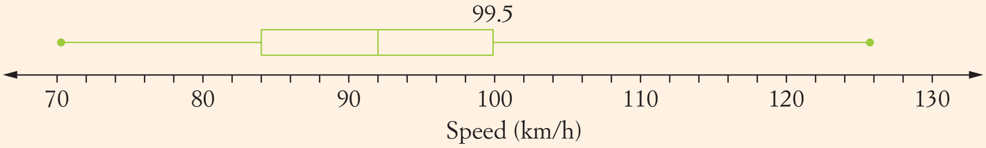
Before exercise:	78	80	66	70	56	64	68	65	50	76	80	70	70	59
After exercise:	141	140	89	95	110	126	84	82	90	88	146	98	96	92

a Find the five-number summary for the pulse rates before and after exercise.
b Construct parallel boxplots for the two sets of data.
c Find the range and interquartile range of the pulse rates:
  i before exercising **ii** after exercising.
d Compare the two sets of pulse rates. Are there significant differences between them? Justify your answer.

See Exercise 5-05

5 The speeds of cars (in km/h) were monitored between 1:00 and 1:30 p.m. on a main road. The results are shown in the stem-and-leaf plot and boxplot below.

Stem	**Leaf**
7	0 3 7 9
8	0 2 2 3 5 6 8 8 9
9	0 0 1 3 5 5 7 8 9 9 9
10	0 0 4 4 6
11	0 1
12	6



a Which display best indicates:
  i skewness? **ii** clustering and outliers?
b Why would a dot plot be unsuitable for displaying the data?
c Find:
  i the median **ii** the interquartile range.
d What percentage of cars were travelling at a speed of at least 92 km/h?

Stage 5.2

See Exercise 5-06

6 Eleven boxes containing 60 oranges each were placed in cold storage for different periods. After storage, the number of good oranges in each box was counted.

Weeks in storage (W)	2	5	12	8	14	6	5	9	10	3	11
Number of good oranges (N)	58	50	33	40	28	50	52	38	35	55	33

a Which is the independent variable? Give reasons.
b Graph this data on a scatter plot.
c Describe the pattern of the plotted points.
d Describe the relationship between the number of weeks in storage and the number of good oranges.
e Describe the strength and direction of the relationship between the variables W and N.

See Exercise 5-07

7 The mean maximum temperatures in Blacktown, NSW for the month of January from 2004 to 2013 are shown in the table.

Year (t)	2004	2005	2006	2007	2008
Temperature (T, °C)	30.6	29.1	29.0	30.1	28.5

Year (t)	2009	2010	2011	2012	2013
Temperature (T, °C)	31.7	30.6	29.9	27.4	30.0

Source: Bureau of Meteorology

a What is the independent variable?
b Graph the data on a scatter plot and join the points.
c Has any change occurred to the temperatures in Blacktown for the month of January over the 10-year period? Give reasons for your answer.

See Exercise 5-08

8 An advertisement in a magazine states that a product is 75% fat-free.
a What impression is the advertisement trying to make about the product?
b What doesn't the advertisement say about the product?

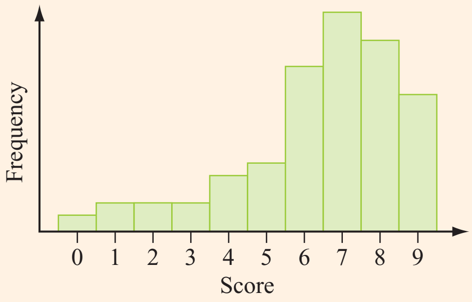
Mixed revision 2

See Exercise 5-01

1 For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

a



b

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

See Exercise 4-01

2 Simplify each expression.

a $(3nm^2)^4$ **b** $5p^4q^2 \times 4qp^5$ **c** $\frac{32a^5b^6}{8ab^3}$

d $(2h)^0$ **e** 3^{-3} **f** $\left(\frac{5}{2}\right)^3$

Stage 5.2

See Exercise 5-02

3 Find the interquartile range of this set of data.

24 15 23 28 20 20 18 30 21 18

See Exercise 6-01

4 Solve each equation.

a $5b + 12 = 37$ **b** $7y + 6 = 2y + 21$

c $3(m - 2) = 27$ **d** $2(2a + 1) = 3(a - 10)$

Stage 5.2

See Exercise 5-03

5 The number of properties sold by a real estate agency per month over a year were:

1 4 6 3 4 6 3 5 7 10 5 5

a Find the range and interquartile range for the scores.

b Find the five-number summary for the data.

c Draw a boxplot for the data.

See Exercise 4-01

6 Simplify each expression.

a $(3y)^{-2}$ **b** $3y^{-2}$ **c** $\left(\frac{4x^6y^6}{x^3y^2}\right)^3$

d $(2h^4k^5)^3 \times 3h^2k$ **e** $48v^5w^8 \div (-4vw^2)^2$ **f** $\left(\frac{36a^4b^2}{48ab^4}\right)^{-1}$

7 Simplify each expression. *See* Exercise 4-02

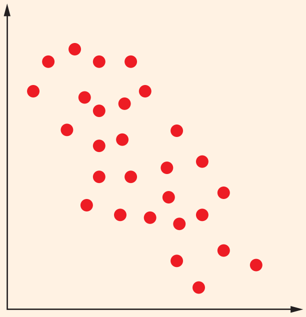
a $\frac{3g}{4} - \frac{2g}{5}$ **b** $\frac{4x}{15} - \frac{x}{3}$ **c** $\frac{3t}{8} + \frac{1}{8}$ **d** $\frac{6}{5} - \frac{r}{2}$

8 Simplify each expression. *See* Exercise 4-03

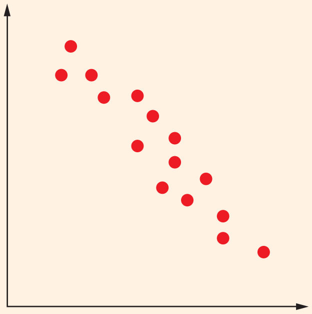
a $\frac{x}{7} \times \frac{6}{y}$ **b** $\frac{m}{8} \div \frac{m}{16}$ **c** $\frac{21e}{10} \times \frac{15d}{7e}$ **d** $\frac{12pv}{5} \div \frac{3p}{10v}$

9 Describe the strength and direction of the relationship shown in each scatter plot. *See* Exercise 5-06

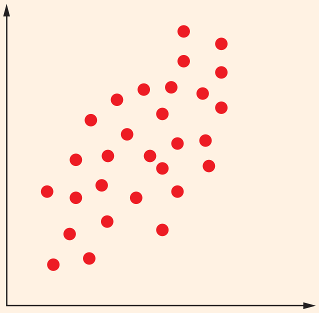
a



b



c



10 Solve each equation.

Stage 5.2

See Exercise 6-02

a $\frac{2a+3}{4} = 5$ **b** $\frac{k}{7} = \frac{5}{4}$ **c** $\frac{3g+2}{4} = \frac{2g-1}{2}$ **d** $\frac{2w}{5} - \frac{w}{10} = 3$

11 Expand each expression. *See* Exercise 4-04

a $7(y - 9)$ **b** $-n(8n + m)$ **c** $-(5w - 6)$ **d** $4f^2(6f + 7g)$

12 The weight of students (in kg) was recorded before and after an exercise program. The results are:

Stage 5.2

See Exercise 5-04

Before program	142	141	90	96	111	127	85	80	91
After program	99	101	77	81	85	96	79	77	70

a Find the five-number summary before and after the exercise program.

b Construct parallel boxplots for the two sets of data.

c Find the range and interquartile range for student weights:

i before the program **ii** after the program.

d Are there significant differences between the two sets of data? Justify your answer.

13 Expand and simplify each expression. *See* Exercise 4-04

a $3ab(b - 6a) - 6a^2b$ **b** $4(8 - 3r) - 6r(8 - 3r)$

See Exercise 4-04

14 Factorise each expression.

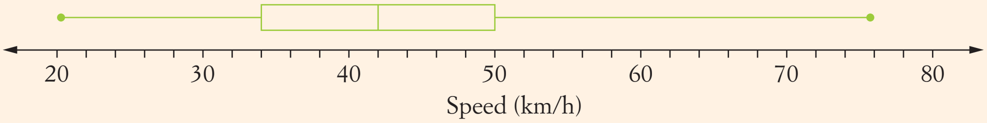
a $7w + 42$ **b** $p^2 - 25p$ **c** $-4n - 44$ **d** $3(7a - 2) - 5a(7a - 2)$

Stage 5.2

See Exercise 5-05

15 The speeds of cars (in km/h) were monitored during 2:30 – 4:00 p.m. in a school zone. The results are shown in the stem-and-leaf plot and boxplot below.

Stem	**Leaf**
2	0 3 7 9
3	0 2 2 3 5 6 8 8 9
4	0 0 1 3 5 5 7 8 9 9 9
5	1 1 4 4 6
6	0 1
7	6



a Which plot gives the better indication of:

i skewness? **ii** clustering and outliers?

b Find the median.

c Find the interquartile range.

See Exercise 6-03

16 Solve each quadratic equation, correct to one decimal place where necessary.

a $p^2 - 144 = 0$ **b** $9y^2 = 36$ **c** $5x^2 - 5 = 0$ **d** $\frac{t^2}{4} = 25$

e $q^2 = 77$ **f** $3x^2 - 70 = 0$ **g** $\frac{3w^2}{5} = 23$

See Exercise 4-05

17 Expand and simplify each expression.

a $(q + 3)(2q + 9)$ **b** $(3f + 8)(f - 7)$ **c** $(4g - 7)(5g - 3)$ **d** $(3x - 7)(3x + 7)$

See Exercise 4-06

18 Factorise each quadratic expression.

a $r^2 + 11r + 24$ **b** $y^2 - 31y + 30$ **c** $x^2 + 9x - 36$ **d** $t^2 - t - 72$

See Exercise 5-07

19 Ten boxes containing 60 apples each were placed in cold storage for different periods. After each period of storage, a box was withdrawn and the number of good apples was counted.

Weeks in storage (W)	2	3	5	6	7	8	9	10	12	14
Number of good apples (N)	56	53	48	48	50	39	36	30	34	26

a Which is the independent variable? Give reasons.

b Graph this data on a scatter plot.

c Describe the relationship between the weeks in storage and the number of good apples.

d Describe the strength and direction of the relationship between the variables W and N.

20 Solve each quadratic equation.

a $x^2 + 8x + 7 = 0$ **b** $h^2 - 8h - 9 = 0$ **c** $u^2 + 4u - 77 = 0$ **d** $w^2 = 9w$

See Exercise 6-03

21 Solve each problem using an equation.

a The sum of three consecutive numbers is 63. Find the three numbers.

b One angle in a triangle is double the smallest angle, and the third angle in the triangle is 60 more than three times the smallest angle. Find the size of each angle.

See Exercise 6-04

Stage 5.2

22 The volume of a pyramid is given by the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

a the volume of a pyramid if the base area is 96 mm^2 and the perpendicular height is 15 mm

b the base area of a pyramid if its volume is 600 cm^3 and its perpendicular height is 12 cm

c the perpendicular height of a pyramid if its volume is 48 m^3 and its base area is 40 m^2.

See Exercise 6-05

23 Graph each inequality on a number line.

a $x \geq 0$ **b** $x < 4$ **c** $x \leq 1$ **d** $x > -6$

See Exercise 6-06

24 Solve each inequality.

a $n - 4 \geq 1$ **b** $5a \leq -15$ **c** $4h + 25 > -7$ **d** $\frac{5x}{2} < 9$

See Exercise 6-07

6

Number and Algebra

Equations and inequalities

Many scientific, natural and social phenomena can be modelled by equations and inequalities.
Historically, algebra dates back to ancient Egypt and Babylon, where linear and quadratic equations were solved. In ancient Babylon, quadratic equations were solved by very similar methods to those still relevant and taught today.

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Chapter outline

	Proficiency strands				
6-01 Equations	U	F		R	
6-02 Equations with algebraic fractions*	U	F		R	
6-03 Quadratic equations $x^2 + bx + c = 0$*	U	F		R	C
6-04 Equation problems	U	F	PS	R	C
6-05 Equations and formulas*	U	F	PS	R	C
6-06 Graphing inequalities on a number line*	U	F			C
6-07 Solving inequalities*	U	F		R	

*STAGE 5.2

Wordbank

$>$ The symbol for ‘is greater than’

$<$ The symbol for ‘is less than’

equation A mathematical statement that two quantities are equal, involving algebraic expressions and an equals sign (=)

formula A rule written as an algebraic equation, using variables

inequality A mathematical statement that two quantities are not equal, involving algebraic expressions and an inequality sign ($>$, $\geq$, $<$, or $\leq$)

quadratic equation An equation involving a variable squared (power of 2), such as $3x^2 - 6 = 69$.

solution The answer to an equation, inequality or problem; the correct value(s) of the variable that makes an equation or inequality true

9780170194655

In this chapter you will:

- (STAGE 5.2) solve linear equations involving simple algebraic fractions
- (STAGE 5.2) solve simple quadratic equations using a range of strategies
- (STAGE 5.2) substitute values into formulas to determine an unknown
- (STAGE 5.2) solve linear inequalities and graph their solutions on a number line AC
- solve linear equations and problems involving equations

SkillCheck

Worksheet
StartUp assignment 6
MAT10NAWK10040

1 Solve each equation.

a $8y = 16$ **b** $10x = 120$ **c** $\frac{m}{5} = 2$
d $w + 6 = 10$ **e** $m - 3 = 12$ **f** $n + 6 = -4$

2 Expand each expression.

a $5(x + 10)$ **b** $4(y - 1)$ **c** $2(5 - 3y)$

3 Solve each equation.

a $2x + 3 = 23$ **b** $3x - 5 = 19$ **c** $4a + 5 = 2a - 19$
d $\frac{3x+2}{5} = 4$ **e** $4(2 - x) = -24$

6-01 Equations

Example 1

Solve each equation.

a $3m - 6 = 12$ **b** $5 - 2a = 3a$
c $9x + 10 = 7x - 6$ **d** $5(p + 6) = 3p + 5$

Solution

a

$$3m - 6 = 12$$

$$3m - 6 + 6 = 12 + 6$$ Adding 6 to both sides.

$$3m = 18$$

$$\frac{3m}{3} = \frac{18}{3}$$ Dividing both sides by 3.

$$m = 6$$

Check:
LHS = $3 \times 6 - 6 = 12$
RHS = 12
LHS = RHS

b

$$\begin{aligned} 5-2a &= 3a \\ 5-2a+2a &= 3a+2a \\ 5 &= 5a \\ 5a &= 5 \\ \frac{5a}{5} &= \frac{5}{5} \\ a &= 1 \end{aligned}$$

Adding $2a$ to both sides.

Dividing both sides by 5.

Check:

LHS $= 5-2\times 1 = 3$

RHS $= 3\times 1 = 3$

LHS = RHS

c

$$\begin{aligned} 9x+10 &= 7x-6 \\ 9x+10-7x &= 7x-6-7x \\ 2x+10 &= -6 \\ 2x+10-10 &= -6-10 \\ 2x &= -16 \\ \frac{2x}{2} &= \frac{-16}{2} \\ x &= -8 \end{aligned}$$

d

$$\begin{aligned} 5(p+6) &= 3p+5 \\ 5p+30 &= 3p+5 \\ 5p+30-3p &= 3p+5-3p \\ 2p+30 &= 5 \\ 2p+30-30 &= 5-30 \\ 2p &= -25 \\ \frac{2p}{2} &= \frac{-25}{2} \\ p &= -12\frac{1}{2} \end{aligned}$$

Example 2

Stage 5.2

Solve $4(y+1)+3(y-5)=8$

Solution

$$\begin{aligned} 4(y+1)+3(y-5) &= 8 \\ 4y+4+3y-15 &= 8 \\ 7y-11 &= 8 \\ 7y-11+11 &= 8+11 \\ 7y &= 19 \\ \frac{7y}{7} &= \frac{19}{7} \\ y &= 2\frac{5}{7} \end{aligned}$$

Check:

$$\begin{aligned} \text{LHS} &= 4(2\tfrac{5}{7}+1)+3(2\tfrac{5}{7}-5) \\ &= 8 \end{aligned}$$

RHS $= 8$

LHS = RHS.

Exercise 6-01 Equations

See Example 1

1 Solve each equation,

a $\frac{k}{6} = 10$ **b** $w + 3 = -6$
c $5y - 1 = 9$ **d** $3a + 10 = 25$
e $2x + 6 = 22$ **f** $15a - 2 = 13$
g $12 - r = 18$ **h** $7w - 10 = 32$
i $9y - 6 = -24$ **j** $11 - 6a = -10$
k $9y = 3y + 32$ **l** $5a = a - 7$

2 What is the solution to $6x - 3 = 27$? Select the correct answer **A**, **B**, **C** or **D**.

A $x = 4$ **B** $x = 5$ **C** $x = 10$ **D** $x = 18$

3 What is the solution to $10 - 2a = 20$? Select **A**, **B**, **C** or **D**.

A $a = -15$ **B** $a = 8$ **C** $a = 32$ **D** $a = -5$

4 Solve each equation.

a $5y + 10 = 3y + 30$ **b** $8a + 20 = 4a + 10$
c $6y - 1 = 3y + 14$ **d** $12a + 30 = 5a + 9$
e $5y + 3 = 8y - 21$ **f** $14x - 20 = 8x - 14$
g $9y + 1 = 3y - 5$ **h** $15x - 15 = 8x - 85$
i $8m - 10 = 5 - 2m$ **j** $18 - 3y = 6 - 2y$
k $1 - 7a = 10 + 2a$ **l** $11 - 5x = 3x + 43$

5 Solve $4y = y - 15$. Select **A**, **B**, **C** or **D**.

A $y = -3$ **B** $y = \frac{7}{4}$ **C** $y = -5$ **D** $y = 11$

6 Solve each equation.

a $3(x - 6) = 30$ **b** $5(m + 10) = 80$
c $2(5y + 3) = 46$ **d** $3(y + 2) = 5y - 10$
e $5(y + 4) = 3y + 6$ **f** $10(x - 3) = 5(x + 5)$
g $2(3m + 6) = 4(m - 1)$ **h** $5(2a + 7) = 5(4 - a)$
i $3(1 - 2y) = 18 - 3y$

7 Solve $2(y - 3) = 5 + 4y$. Select **A**, **B**, **C** or **D**.

A $y = -9$ **B** $y = -5$ **C** $y = -\frac{11}{2}$ **D** $y = -\frac{1}{2}$

Stage 5.2

See Example 2

8 Solve each equation.

a $3(d + 3) + 4(d + 1) = 15$ **b** $3(y - 1) + 5(y + 4) = 10$
c $7(k + 1) + 2(k - 6) = 3$ **d** $5(g - 3) + 2(g - 2) = 4$
e $6(2h + 3) + 5(h - 3) = 9$ **f** $2(1 + p) + 3(4 + p) = 5$

Investigation: Make your own equation

Here are two equations that have the same solution, $x = 6$:

$5x - 1 = 23 + x$ and $\dfrac{3x + 12}{10} = 3$

1 For each solution below, make up two equations that have that solution.

a $x = 4$ **b** $x = \dfrac{1}{2}$ **c** $x = 10$

d $x = 1.5$ **e** $x = 0$ **f** $x = -2$

2 Compare your answers with those of other students. Check that each equation is correct.

6-02 Equations with algebraic fractions

Stage 5.2

Puzzle sheet
Equations code puzzle
MAT10NAPS10041

Puzzle sheet
Equations order activity
MAT10NAPS10042

Puzzle sheet
Solving linear equations 1
MAT10NAPS00035

Puzzle sheet
Solving linear equations 2
MAT10NAPS00036

Example 3

Solve each equation.

a $\dfrac{y + 2}{5} = -1$ **b** $\dfrac{3(p - 3)}{2} + 6 = 1$

Solution

a

$$\begin{aligned}\frac{y+2}{5} \times 5 &= -1 \times 5\\ y + 2 &= -5\\ y + 2 - 2 &= -5 - 2\\ y &= -7\end{aligned}$$

b

$$\begin{aligned}\frac{3(p-3)}{2} + 6 &= 1\\ \frac{3(p-3)}{2} + 6 - 6 &= 1 - 6\\ \frac{3(p-3)}{2} &= -5\\ \frac{3(p-3)}{2} \times 2 &= -5 \times 2\\ 3(p-3) &= -10\\ 3p - 9 &= -10\\ 3p - 9 + 9 &= -10 + 9\\ 3p &= -1\\ \frac{3p}{3} &= \frac{-1}{3}\\ p &= -\frac{1}{3}\end{aligned}$$

Stage 5.2

Video tutorial

Equations with algebraic fractions

MAT10NAVT10026

Example 4

Solve each equation.

a $\dfrac{2a+4}{5}=\dfrac{2}{3}$ **b** $\dfrac{2m}{3}-\dfrac{m}{2}=2$

Solution

a $\dfrac{2a+4}{5}=\dfrac{2}{3}$

For equations where all terms are fractions, multiply both sides by a common multiple of the denominators to remove the fractions.

The lowest common multiple (LCM) of 5 and 3 is 15, so multiply both sides by 15.

$$\begin{aligned}
\frac{2a+4}{\cancel{5}_1}\times\cancel{15}^3 &= \frac{2}{\cancel{3}_1}\times\cancel{15}^5\\
3(2a+4) &= 10\\
6a+12 &= 10\\
6a &= -2\\
a &= \frac{-2}{6}\\
a &= -\frac{1}{3}
\end{aligned}$$

b $\dfrac{2m}{3}-\dfrac{m}{2}=2$

Multiply both sides by 6, the LCM of 3 and 2.

$$\begin{aligned}
6\left(\frac{2m}{3}-\frac{m}{2}\right) &= 6\times 2\\
\cancel{6}^2\times\frac{2m}{\cancel{3}_1}-\cancel{6}^3\times\frac{m}{\cancel{2}_1} &= 12\\
4m-3m &= 12\\
m &= 12
\end{aligned}$$

Exercise 6-02 Equations with algebraic fractions

See Example 3

1 What is the solution to each equation? Select the correct answer **A**, **B**, **C** or **D**.

a $\dfrac{3y}{4}=6$

A $y=9$ **B** $y=8$ **C** $y=7$ **D** $y=6$

b $\dfrac{a+1}{2}=3$

A $a=4$ **B** $a=5$ **C** $a=6$ **D** $a=7$

c $\dfrac{x-1}{4}+2=10$

A $x=49$ **B** $x=37$ **C** $x=33$ **D** $x=3$

Stage 5.2

2 Solve each equation.

a $\frac{3y}{5}=9$ **b** $\frac{2a}{9}=2$ **c** $\frac{m+5}{2}=6$ **d** $\frac{k-2}{5}=11$

e $\frac{n+5}{3}=-10$ **f** $\frac{y-1}{4}=-2$ **g** $\frac{x+1}{4}+2=10$ **h** $\frac{y-1}{5}-6=3$

i $\frac{m+2}{5}-1=3$ **j** $\frac{x-6}{5}+7=0$ **k** $\frac{2(x+1)}{5}=10$ **l** $\frac{3(m-2)}{4}=6$

m $\frac{8(n+1)}{3}+2=4$ **n** $\frac{5(1-n)}{2}-1=3$ **o** $\frac{4(1+d)}{3}+1=7\frac{1}{3}$

3 Solve each equation.

See Example 4

a $\frac{2k}{3}=\frac{5}{4}$ **b** $\frac{3w}{10}=\frac{2}{5}$ **c** $\frac{5x}{2}=-\frac{10}{3}$

d $\frac{x-1}{2}=\frac{x+1}{4}$ **e** $\frac{y+2}{5}=\frac{y-1}{2}$ **f** $\frac{a+5}{3}=\frac{a-1}{8}$

g $\frac{p+2}{5}=\frac{p-5}{2}$ **h** $\frac{2y-1}{5}=\frac{y+1}{4}$ **i** $\frac{3y+2}{3}=\frac{2y+1}{4}$

j $\frac{w}{5}+\frac{w}{2}=7$ **k** $\frac{w}{2}-\frac{w}{5}=15$ **l** $\frac{2w}{3}-\frac{w}{4}=4$

m $\frac{3a}{2}+\frac{a}{3}=1$ **n** $\frac{2y}{5}-\frac{y}{3}=4$ **o** $\frac{a}{3}+\frac{3a}{4}=2$

p $\frac{2m}{5}-\frac{m}{10}=1$ **q** $\frac{4h}{3}+\frac{h}{5}=3$ **r** $\frac{5y-2}{7}=\frac{3y+5}{3}$

4 Solve each equation. Select the correct answer **A**, **B**, **C** or **D**.

a $\frac{4m}{5}-\frac{m}{3}=2$

A $m=10$ **B** $m=12$ **C** $m=\frac{30}{7}$ **D** $m=\frac{4}{3}$

b $\frac{m+1}{2}=\frac{3+2m}{5}$

A $m=1$ **B** $m=5$ **C** $m=\frac{5}{3}$ **D** $m=\frac{2}{3}$

6-03 Quadratic equations $x^2+bx+c=0$

iStockphoto/Lagui

An equation in which the highest power of the variable is 2 is called a **quadratic equation**; for example, $x^2=5$, $3m^2+7=10$, $d^2-d-6=0$ and $4y^2-3y=8$.

Stage 5.2

Worksheet
Equations review
MAT10NAWK10043

Solving $ax^2 = c$

Summary

The quadratic equation $x^2 = c$ (where c is a positive number) has two solutions,

$$x = \pm\sqrt{c} \text{ (which means } x = \sqrt{c} \text{ and } x = -\sqrt{c}\text{)}$$

Video tutorial
Simple quadratic equations
MAT10NAVT10028

Example 5

Solve each quadratic equation.

a $m^2 = 16$ **b** $3x^2 = 75$ **c** $3m^2 - 12 = 0$

Solution

a
$$\begin{aligned} m^2 &= 16 \\ m &= \pm\sqrt{16} \\ &= \pm 4 \end{aligned}$$

Finding the square root of both sides.

b
$$\begin{aligned} 3x^2 &= 75 \\ x^2 &= \frac{75}{3} \\ x^2 &= 25 \\ x &= \pm\sqrt{25} \\ &= \pm 5 \end{aligned}$$

Finding the square root of both sides.

c
$$\begin{aligned} 3m^2 - 12 &= 0 \\ 3m^2 - 12 + 12 &= 0 + 12 \\ 3m^2 &= 12 \\ m^2 &= \frac{12}{3} \\ m^2 &= 4 \\ m &= \pm\sqrt{4} \\ &= \pm 2 \end{aligned}$$

Stage 5.2

Example 6

Solve $7x^2 - 88 = 0$, writing the solution correct to one decimal place.

Solution

$$\begin{aligned}7x^2 - 88 &= 0\\ 7x^2 &= 88\\ x^2 &= \frac{88}{7}\\ x &= \pm\sqrt{\frac{88}{7}}\\ x &= \pm 3.54562\ \ldots\\ &\approx \pm 3.5\end{aligned}$$

Solving $x^2 + bx + c = 0$ by factorising

To solve quadratic equations of the form $x^2 + bx + c = 0$, we need to factorise the quadratic expression on the LHS, which we learnt in Chapter 4, Algebra.

Example 7

Video tutorial
Quadratic equations by factorising
MAT10NAVT10029

Solve $x^2 + 5x + 6 = 0$.

Solution

To factorise $x^2 + 5x + 6$, find two numbers that have a sum of 5 and a product of 6. The correct numbers are 2 and 3.

$$\begin{aligned}x^2 + 5x + 6 &= 0\\ (x + 2)(x + 3) &= 0\end{aligned}$$

The LHS has been factorised into two factors, $(x + 2)$ and $(x + 3)$, whose product is 0. If two numbers have a product of 0, then one of the numbers *must* be 0.

$\therefore x + 2 = 0$ or $x + 3 = 0$

$\therefore x = -2$ or $x = -3$

$\therefore$ The solution to $x^2 + 5x + 6 = 0$ is $\boldsymbol{x = -2}$ or $\boldsymbol{x = -3}$.

Check:

When $x = -2$,
$\text{LHS} = (-2)^2 + 5 \times (-2) + 6 = 0$
$\text{RHS} = 0$
$\text{LHS} = \text{RHS}$
When $x = -3$,
$\text{LHS} = (-3)^2 + 5 \times (-3) + 6 = 0$
$\text{RHS} = 0$
$\text{LHS} = \text{RHS}$.

Stage 5.2

Summary

When **solving quadratic equations by factorising**, the following property is used.
If $pq = 0$, then $\boldsymbol{p} = \mathbf{0}$ or $\boldsymbol{q} = \mathbf{0}$.

Example 8

Solve each quadratic equation.

a $x^2 - x - 2 = 0$

b $u^2 + 3u - 28 = 0$

c $a^2 - 2a = 0$

d $w^2 - 10w + 25 = 0$

Solution

a $x^2 - x - 2 = 0$

Find two numbers that have a sum of −1 and a product of −2.
They are −2 and 1.

$(x - 2)(x + 1) = 0$

$\therefore x - 2 = 0$ or $x + 1 = 0$

$\therefore x = 2$ or $x = -1$

$\therefore$ The solution to $x^2 - x - 2 = 0$ is $x = 2$ or $x = -1$.

Check:

When $x = 2$,
$\text{LHS} = 2^2 - 2 - 2 = 0$
$\text{RHS} = 0$
LHS = RHS.

When $x = -1$,
$\text{LHS} = (-1)^2 - (-1) - 2 = 0$
$\text{RHS} = 0$
LHS = RHS.

b $u^2 + 3u - 28 = 0$

Find two numbers that have a sum of 3 and a product of −28.
They are 7 and −4

$(u + 7)(u - 4) = 0$

$\therefore u + 7 = 0$ or $u - 4 = 0$

$\therefore u = -7$ or $u = 4$

$\therefore$ The solution to $u^2 + 3u - 28 = 0$ is $u = -7$ or $u = 4$.

c $a^2 - 2a = 0$

This requires a simpler factorisation as there are only two terms, both involving a.

$a(a - 2) = 0$

$\therefore a = 0$ or $a - 2 = 0$

$\therefore a = 0$ or $a = 2$

$\therefore$ The solution to $a^2 - 2a = 0$ is $a = 0$ or $a = 2$.

d $w^2 - 10w + 25 = 0$

Find two numbers that have a sum of -10 and a product of 25.

They are -5 and -5.

$(w - 5)(w - 5) = 0$

$\therefore w - 5 = 0$ or $w - 5 = 0$

$\therefore w = 5$ or $w = 5$

$\therefore$ The equation $w^2 - 10w + 25 = 0$ has only one solution, $w = 5$.

Exercise 6-03 Quadratic equations $x^2 + bx + c = 0$

See Example 5

1 Solve each quadratic equation.

a $m^2 = 144$ **b** $x^2 = 400$ **c** $y^2 = 225$ **d** $k^2 - 169 = 0$

e $y^2 - 1 = 0$ **f** $w^2 - 16 = 0$ **g** $x^2 + 10 = 14$ **h** $t^2 - 9 = 7$

i $\frac{a^2}{2} = 8$ **j** $5k^2 = 180$ **k** $3w^2 = 300$ **l** $d^2 + 60 = 204$

m $\frac{k^2}{2} = 0.5$ **n** $\frac{w^2}{10} = 2.5$ **o** $4x^2 = 1$ **p** $\frac{m^2}{4} = 9$

q $5y^2 = 5$ **r** $2p^2 + 3 = 21$ **s** $\frac{3k^2}{4} + 5 = 8$ **t** $\frac{y^2}{5} - 2 = 18$

See Example 6

2 Solve each equation (correct to one decimal place, where necessary).

a $5m^2 - 20 = 0$ **b** $\frac{4a^2}{9} = 36$ **c** $m^2 = 28$ **d** $9m^2 - 2 = 32$

e $9k^2 + 10 = 13$ **f** $\frac{2x^2}{5} = 23$ **g** $\frac{k^2}{16} = 6$ **h** $\frac{3k^2}{10} = 27$

i $6y^2 = 0.726$ **j** $3a^2 + 11 = 267$ **k** $2y^2 - 14 = 63$ **l** $\frac{2w^2}{5} - 1 = 19$

3 Solve each quadratic equation. Select the correct answer **A**, **B**, **C** or **D**.

a $x^2 = 121$

A $x = 12, -12$ **B** $x = 11, -11$ **C** $x = 10, 11$ **D** $x = 12, -11$

b $9m^2 - 1 = 35$

A $m = 3, -3$ **B** $m = 2, -2$ **C** $m = 8, -8$ **D** $m = 9, -9$

See Example 7

4 Solve the following.

a $x^2 + 3x + 2 = 0$ **b** $y^2 + 5y + 4 = 0$ **c** $y^2 + 16y + 48 = 0$

d $x^2 + x - 12 = 0$ **e** $x^2 + 2x - 3 = 0$ **f** $x^2 + 3x - 40 = 0$

See Example 8

5 Solve the following.

a $x^2 - x - 30 = 0$ **b** $x^2 - 8x + 16 = 0$ **c** $x^2 - 5x - 66 = 0$

d $d^2 - 2d = 0$ **e** $x^2 - 3x - 10 = 0$ **f** $n^2 + 4n = 0$

g $k^2 - 7k = 0$ **h** $y^2 = 5y$ **i** $v^2 = 12v$

6 Explain why the quadratic equation $x^2 = -25$ has no solutions.

7 State which of these quadratic equations have no solutions. Give reasons.

a $x^2 = -9$ **b** $2k^2 + 5 = 9$ **c** $3m^2 + 8 = 4$

d $\frac{9w^2}{2} - 1 = 1$ **e** $4 + \frac{d^2}{3} = 8$ **f** $\frac{5a^2}{2} + 3 = 2$

Stage 5.2

8 Solve each quadratic equation. Select the correct answer **A**, **B**, **C** or **D**.

a $x^2 + 4x - 60 = 0$

A $x = -10, 6$ **B** $x = 12, -5$ **C** $x = 10, -6$ **D** $x = -12, 5$

b $q^2 + 3q = 0$

A $q = 3, -3$ **B** $q = 6, -3$ **C** $q = 0, -3$ **D** $q = 0, 3$

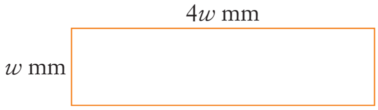
6-04 Equation problems

Example 9

A rectangle is four times as long as it is wide. The perimeter of the rectangle is 180 mm. Find the dimensions of the rectangle.

Solution

Let the width of the rectangle be w mm. Then the length is $4w$ mm.



$$\begin{aligned} \therefore \text{ Perimeter: } w + 4w + w + 4w &= 180 \\ 10w &= 180 \\ w &= \frac{180}{10} \\ &= 18 \end{aligned}$$

$\therefore$ The width of the rectangle is 18 mm and its length is $4 \times 18 = 72$ mm.
Check: Perimeter $= 18 + 72 + 18 + 72 = 180$ mm.

Example 10

Jennifer is 7 years older than Amy. Ten years from now, the sum of their ages will be 43. How old are they now?

Solution

Let $x =$ Amy's age now.
Then Jennifer's age now $= x + 7$.
Break the information into 'Now' and 'In 10 years time'

	Now	**In 10 years time**
Amy	x	$x + 10$
Jennifer	$x + 7$	$x + 7 + 10 = x + 17$

In 10 year's time:

$$\begin{aligned}\text{Sum of ages: } (x+10)+(x+17) &= 43\\ 2x+27 &= 43\\ 2x &= 16\\ x &= 8\end{aligned}$$

Amy is 8 years old now and Jennifer is $8 + 7 = 15$ years old now.
Check: In 10 years time, the sum of their ages will be $18 + 25 = 43$.

Exercise 6-04 Equation problems

For each question, write an equation and solve it to answer the problem.

1 The longer sides of an isosceles triangle are twice as long as the shorter side. The perimeter of the triangle is 90 mm. Find the lengths of the sides of the triangle.

2 The length of a rectangle is three times as long as its width. The perimeter of the rectangle is 152 mm. Find its dimensions.

See Example 9

3 The length of a rectangle is three more than twice its width. Find the dimensions of the rectangle if its perimeter is 84 cm.

4 The sum of three consecutive integers is 186. Find the integers.

5 A father is nine times the age of his son. In 5 years, he will be four times the age of his son. How old are they now?

See Example 10

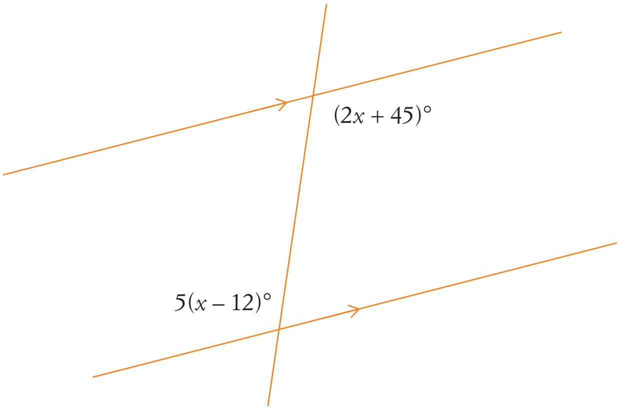
6 When 15 is subtracted from three times a certain number, the answer is 63. What is the number?

7 The product of 2 and a number is the same as 12 minus the number. Find the number.

8 The sum of the present ages of Vatha and Chris is 36. In 4 years time, the sum of their ages will equal twice Vatha's present age. How old are they now?

9 Four consecutive integers have a sum of 858. Find the four integers.

10 Find the size of x in the diagram below.



11 Manori's bag contained 10-cent and 20-cent coins. She had 202 coins, with a total value of \$31.90. How many 20-cent coins did Manori have?

12 If 17 more than a number is 5 more than three times the number, what is the number?

13 The sum of Scott's age and his mother's age is 45. In 5 years time, three times Scott's age less 9 will be the same as his mother's age. Find the present ages of Scott and his mother.

14 One angle in a triangle is double the smallest angle, and the third angle in the triangle is 5 more than four times the smallest angle. Find the size of each angle.

Mental skills 6 Maths without calculators

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a **To multiply by 5**, halve the number, then multiply by 10.

$$\begin{aligned} 18 \times 5 &= 18 \times \frac{1}{2} \times 10 \text{ (or } 9 \times 2 \times 10) \\ &= 9 \times 10 \\ &= 90 \end{aligned}$$

b **To multiply by 50**, halve the number, then multiply by 100.

$$\begin{aligned} 26 \times 50 &= 26 \times \frac{1}{2} \times 100 \text{ (or } 13 \times 2 \times 100) \\ &= 13 \times 100 \\ &= 1300 \end{aligned}$$

c **To multiply by 25**, quarter the number, then multiply by 100.

$$\begin{aligned} 44 \times 25 &= 44 \times \frac{1}{4} \times 100 \text{ (or } 11 \times 4 \times 25) \\ &= 11 \times 100 \\ &= 1100 \end{aligned}$$

d **To multiply by 15**, halve the number, then multiply by 30.

$$\begin{aligned} 8 \times 15 &= 8 \times \frac{1}{2} \times 30 \text{ (or } 4 \times 2 \times 15) \\ &= 4 \times 30 \\ &= 120 \end{aligned}$$

e **To divide by 5**, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$$\begin{aligned} 140 \div 5 &= 140 \div 10 \times 2 \\ &= 14 \times 2 \\ &= 28 \end{aligned}$$

f **To divide by 50**, divide by 100 and double the answer. This is because there are two 50s in every 100.

$$\begin{aligned} 400 \div 50 &= 400 \div 100 \times 2 \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

g **To divide by 25**, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$$\begin{aligned} 600 \div 25 &= 600 \div 100 \times 4 \\ &= 6 \times 4 \\ &= 24 \end{aligned}$$

h **To divide by 15**, divide by 30 and double the answer. This is because there are two 15s in every 30.

$$\begin{aligned} 240 \div 15 &= 240 \div 30 \times 2 \\ &= 8 \times 2 \\ &= 16 \end{aligned}$$

2 Now evaluate each expression.

a 32×5 **b** 14×5 **c** 48×5 **d** 18×50
e 52×50 **f** 36×25 **g** 28×5 **h** 12×25
i 12×15 **j** 22×35 **k** $90 \div 5$ **l** $170 \div 5$
m $230 \div 5$ **n** $1300 \div 50$ **o** $900 \div 50$ **p** $300 \div 25$
q $1000 \div 25$ **r** $360 \div 45$ **s** $210 \div 15$ **t** $360 \div 15$

Stage 5.2

Puzzle sheet

Getting the right formula

MAT10NAPS10044

6-05 Equations and formulas

A **formula** is an equation that describes a relationship between variables. For example, the formula for the perimeter of a rectangle is $P = 2(l + w)$, where P is the perimeter, l is the rectangle's length and w is the width. Because the formula is for the perimeter, P is called the **subject** of the formula and it is the variable on its own on the left side of the '=' sign.

Example 11

The cost, C, in dollars, of hiring a taxi is $C = 5 + 2.4d$, where d is the distance travelled, in kilometres.

iStockphoto/davidf

a Find the cost of a taxi trip if the distance travelled is 15 km.

b Find the distance travelled by the taxi if the cost of the trip was \$78.20.

Solution

a When $d = 15$:

$$\begin{aligned} C &= 5 + 2.4 \times 15 \\ &= 41 \end{aligned}$$

The cost was \$41.

b When $C = 78.20$:

$$\begin{aligned} 78.20 &= 5 + 2.4d \\ 73.20 &= 2.4d \\ d &= \frac{73.20}{2.4} \\ &= 30.5 \end{aligned}$$

The distance travelled was 30.5 km.

Example 12

The surface area of a sphere is given by the formula $A = 4\pi r^2$, where r is the radius.
Find (correct to one decimal place):

a the surface area of a sphere with radius 2.8 cm

b the radius of a sphere with surface area 40 m^2.

9780170194655

Solution

a When $r = 2.8$:

$$\begin{aligned} A &= 4 \times \pi \times 2.8^2 \\ &= 98.520\ldots \\ &\approx 98.5 \text{ cm}^2 \end{aligned}$$

b When $r = 40$:

$$\begin{aligned} 40 &= 4\pi r^2 \\ 4\pi r^2 &= 40 \\ r^2 &= \frac{40}{4\pi} \\ &= 3.183\ldots \\ r &= \sqrt{3.183} \\ &= 1.784\ldots \\ &\approx 1.8 \text{ m} \end{aligned}$$

Since $r > 0$, the radius is positive.

Exercise 6-05 Equations and formulas

See Example 11

1 The formula for the perimeter of a rectangle is $P = 2(l + w)$.

a Find the perimeter of a rectangle with length 10 cm and width 16 cm.

b Find the width of a rectangle whose perimeter is 58 m and length is 12 m.

2 A formula for converting speed expressed in m/s (metres/second) to a speed expressed in km/h is $k = 3.6M$, where M is the speed in m/s. Convert each speed to km/h.

a 10 m/s **b** 24 m/s **c** 50 m/s

3 A car is travelling at a speed of 110 km/h on a freeway. Use the formula from question **2** to calculate how fast this is in m/s.

4 The average of two numbers, m and n, is $A = \frac{m+n}{2}$. If two numbers have an average of 28 and one of the numbers is 13, use the formula to find the other number.

5 The formula for converting temperature recorded in °F to temperature in °C is $C = \frac{5}{9}(F - 32)$. Express each temperature in °C, correct to the nearest degree.

a 80°F **b** 32°F **c** 212°F **d** 102°F

See Example 12

6 Pythagoras' theorem for a right-angled triangle with sides a, b and c (the hypotenuse) is $c^2 = a^2 + b^2$. Find, correct to one decimal place where necessary:

a c, if $a = 5$ and $b = 10$ **b** a, if $c = 41$ and $b = 40$ **c** b, if $c = 20$ and $a = 10$

7 The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius. Find, correct to one decimal place:

a the circumference of a circle with radius 2.4 m

b the radius of a circle whose circumference is 200 cm.

Stage 5.2

8 The body mass index (BMI) of an adult is $B = \frac{M}{h^2}$, where M is the mass in kilograms and h is the height in metres. Find, correct to one decimal place:

- **a** the BMI of Dean who is 1.85 m tall and has a mass of 72 kg
- **b** the mass of a person with a BMI of 24, who is 2.1 m tall.

9 The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. Find, correct to one decimal place:

- **a** the volume of a sphere with radius 3.2 cm
- **b** the radius of a sphere with a volume of 500 m^3.

10 The average speed in km/h of a car is given by the formula $S = \frac{D}{T}$, where D is the distance covered in kilometres and T is the time taken in hours. Find, correct to the nearest whole number:

- **a** the average speed of a car that takes 4.5 hours to travel a distance of 420 km
- **b** the distance travelled, if a car maintains a speed of 87.2 km/h for 5 hours
- **c** the time taken, if a distance of 650 km is covered at a speed of 91 km/h.

11 The cost, C, (in dollars) of hiring a car is $C = 45 + 0.15d$, where d is the number of kilometres travelled. Calculate:

- **a** the cost of hiring a car to travel 350 km
- **b** the distance travelled, if the cost is \$138.

12 The surface area of a closed cylinder is given by the formula $\text{SA} = 2\pi r^2 + 2\pi rh$. Calculate, correct to one decimal place:

- **a** the surface area of a cylinder with radius 2.1 m and height 3.5 m
- **b** the height of a cylinder with surface area 1255.38 cm^2 and radius 9 cm.

6-06 Graphing inequalities on a number line

Worksheet
Graphing inequalities
MAT10NAWK10045

An **inequality** looks like an equation except that the equals sign (=) is replaced by an inequality symbol $>$, $\geq$, $<$ or $\leq$.

$2x - 7 = 15$ is an **equation**. There is only one value of x that makes it true.

$2x - 7 \leq 15$ is an **inequality**. There is a range of values of x that make it true.

Summary

$>$ means 'is greater than'
$<$ means 'is less than'
$\geq$ means 'is greater than or equal to'
$\leq$ means 'is less than or equal to'

The inequality $x \geq 3$ is read 'x is greater than or equal to 3'. It includes 3 and all the numbers above 3, such as 3.01, 4, 10, 20 000, etc.

The inequality $x > 3$ is read 'x is greater than 3' and means all the numbers above 3, but **not** 3.

Inequality	In words	Meaning
$x > 3$	x is greater than 3	Values above 3
$x < 3$	x is less than 3	Values below 3
$x \geq 3$	x is greater than or equal to 3	Values above and including 3
$x \leq 3$	x is less than or equal to 3	Values below and including 3

For convenience, we can represent all the values in an inequality using a number line.

Stage 5.2

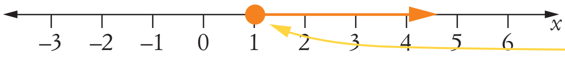
Example 13

Graph each inequality on a number line.

a $x \geq 1$ **b** $x < 5$ **c** $x > -3$

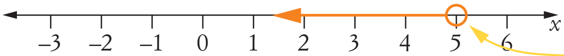
Solution

a $x \geq 1$ means that x can be any number greater than 1 or equal to 1.



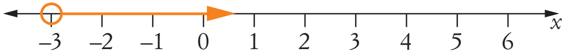
The filled circle at 1 means we include 1.

b $x < 5$ means that x can be any number less than 5, but not including 5.



The open circle on 5 means that 5 is not included

c $x > -3$ means that x can be any number greater than –3, but not including –3.



Exercise 6-06 Graphing inequalities on a number line

See Example 13

1 Graph each inequality on a separate number line.

a $x \geq 2$ **b** $x < -3$ **c** $x \leq 1$ **d** $x > 7$

e $x \leq 4$ **f** $x > 0$ **g** $x \geq -2$ **h** $x < 10$

2 Write the inequality illustrated by each number line.

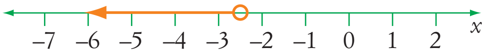
a

b

c

d

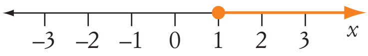
3 Which inequality is graphed below? Select the correct answer **A**, **B**, **C** or **D**.



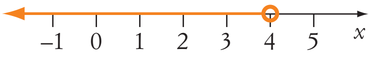
A $x > -2.5$ **B** $x < -2.5$ **C** $x < -3.5$ **D** $x > -3.5$

4 Write the inequality represented on each number line.

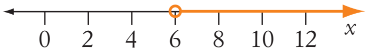
a



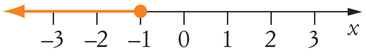
b



c



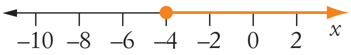
d



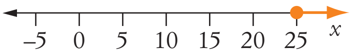
e

f

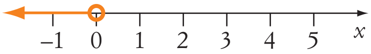
g



h



i



Stage 5.2

Investigation: The language of inequalities

Work in pairs to complete this activity.
Use inequality symbols to write each statement algebraically.

- **a** The minimum height (H) for rides at an amusement park is 1.3 m.
- **b** The speed limit in a school zone is 40 km/h.
- **c** To be eligible to vote, you must be at least 18 years old (A = age).
- **d** The overseas tour is only for people whose age (A) is from 18 to 35.
- **e** The cost (A) of a tennis racquet will be at least \$95 but no more than \$360.
- **f** A new flute (F) costs at least \$475.
- **g** The price of units (U) in a new block start at \$240 000.

Investigation: Solving inequalities

We have solved equations by doing the same thing to both sides (keeping the equation 'balanced'). Will this method work with inequalities, such as $x + 4 > 10$ or $6x < 13$?

1. Start with an inequality that is true, such as $7 > 4$.
2. Add 5 (or any number you choose) to both sides of the inequality; for example $7 > 4$ becomes $12 > 9$. Is the new inequality true or false?
3. Subtract 9 (or any number you choose) from each side of the original inequality; for example $7 > 4$ becomes $-2 > -5$. Is the new inequality true or false?
4. Multiply both sides of the original inequality by 4 (or any positive number you choose); for example $7 > 4$ becomes $28 > 16$. Is the new inequality true or false?
5. Divide both sides of the original inequality by 2 (or any positive number you choose); for example $7 > 4$ becomes $3\frac{1}{2} > 2$. Is the new inequality true or false?
6. Multiply both sides of the original inequality by -3 (or any negative number you choose); for example $7 > 4$ becomes $-21 > -12$. Is the new inequality true or false?
7. Divide both sides of the original inequality by -4 (or any negative number you choose), for example $7 > 4$ becomes $-1\frac{3}{4} > -1$. Is the new inequality true or false?
8. Which of the six operations used in questions **2** to **7** can be used on inequalities to give a true result?
9. Which of the six operations used in questions **2** to **7** cannot be used with inequalities because they give a false result?
10. Copy and complete the following inequality statements.

    a $6 < 8$
    $6 \times 3 < 8 \times$ ___ (multiplying both sides by 3)
    $\therefore 18$ ___ 24

    b $10 > -4$
    $10 \div 2$ ___ $-4 \div$ ___ (dividing both sides by 2)
    $\therefore$ ___________

    Does the inequality sign (< or >) stay the same when multiplying or dividing by a positive number?
11. **a** Is it true that $5 < 8$?
    b Multiply both sides by -2. Is it true that $-10 < -16$?
    c What needs to be reversed to change $-10 < -16$ into a true inequality statement?
    d Copy and complete the following to make a true inequality statement: -10 ______ -16.

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Stage 5.2

12 **a** Is it true that $18 > -6$?
b Divide both sides by -3. Is it true that $-6 > 2$?
c What needs to be reversed to change $-6 > 2$ into a true inequality statement?
d Copy and complete the following to make a true inequality statement: -6 ____ 2.

13 Copy and complete:
When multiplying or d__________ both sides of an inequality by a n__________ number, the inequality sign must be r__________.

6-07 Solving inequalities

Worksheet
Inequalities review
MAT10NAWK10046

Example 14

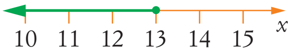
Solve each inequality and graph its solution on a number line.

a $2x - 10 \leq 16$ **b** $2(y - 1) \geq 12$ **c** $\dfrac{w+3}{2} > -1$

Solution

a

$$\begin{aligned} 2x - 10 &\leq 16 \\ 2x - 10 + 10 &\leq 16 \\ 2x &\leq 26 \\ \frac{2x}{2} &\leq \frac{26}{2} \\ x &\leq 13 \end{aligned}$$

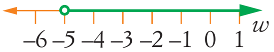


b

$$\begin{aligned} 2(y - 1) &\geq 12 \\ 2y - 2 &\geq 12 \\ 2y - 2 + 2 &\geq 12 + 2 \\ 2y &\geq 14 \\ \frac{2y}{2} &\geq \frac{14}{2} \\ y &\geq 7 \end{aligned}$$

c

$$\begin{aligned} \frac{w+3}{2} &> -1 \\ \frac{w+3}{2} \times 2 &> -1 \times 2 \\ w + 3 &> -2 \\ w + 3 - 3 &> -2 - 3 \\ w &> -5 \end{aligned}$$



Stage 5.2

Summary

Inequalities can be solved algebraically in the same way as equations, by using inverse operations. However, when **multiplying** or **dividing** both sides of an inequality by a **negative** number, you must **reverse the inequality sign**.

Example 15

Solve each inequality.

a $1 - 2x \geq -11$ **b** $4 - r < 7$ **c** $\frac{a+5}{-3} > 4$

Solution

a

$$\begin{aligned} 1 - 2x &\geq -11 \\ 1 - 2x - 1 &\geq -11 - 1 \\ -2x &\geq -12 \\ \frac{-2x}{-2} &\leq \frac{-12}{-2} \\ x &\leq 6 \end{aligned}$$

Dividing both sides by a negative number reverses the inequality sign.

b

$$\begin{aligned} 4 - r &< 7 \\ 4 - r - 4 &< 7 - 4 \\ -r &< 3 \\ \frac{-r}{-1} &> \frac{3}{-1} \\ r &> -3 \end{aligned}$$

Dividing both sides by a negative number reverses the inequality sign.

c

$$\begin{aligned} \frac{a+5}{-3} &> 4 \\ \frac{a+5}{-3} \times (-3) &< 4 \times (-3) \\ a + 5 &< -12 \\ a + 5 - 5 &< -12 - 5 \\ a &< -17 \end{aligned}$$

Multiplying both sides by a negative number reverses the inequality sign.

Exercise 6-07 Solving inequalities

See Example 14

1 Solve each inequality and graph its solution on a number line.

a $x - 1 > 6$ **b** $3y \geq 12$ **c** $m + 4 \leq 2$
d $\frac{x}{5} \geq -20$ **e** $12x < 60$ **f** $5y > -20$
g $4a \geq 2$ **h** $3w \leq -30$ **i** $8a + 5 \geq 45$
j $3a + 1 \leq 10$ **k** $6a + 4 \geq -2$ **l** $3w - 3 < -12$
m $5a + 3 \leq -27$ **n** $5y + 1 \leq 16$ **o** $4a + 5 < 15$

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2 What is the solution to $3a - 3 > -18$? Select the correct answer **A**, **B**, **C** or **D**.

A $a > -5$ **B** $a > -7$ **C** $a > 5$ **D** $a > 9$

3 Solve each inequality.

a $3(x + 2) \geq 9$ **b** $5(m - 4) \leq 10$ **c** $2(y + 5) \leq -6$

d $3(w - 2) > -6$ **e** $5(2w + 3) \leq 15$ **f** $4(2m - 5) \geq 8$

g $\frac{m + 5}{3} \geq 1$ **h** $\frac{x - 1}{2} \leq 2$ **i** $\frac{w - 2}{5} > -1$

j $\frac{2a + 1}{3} < 3$ **k** $\frac{5a + 2}{4} \geq 8$ **l** $\frac{2(m + 1)}{3} \leq 3$

m $\frac{5(m - 1)}{4} > 3$ **n** $\frac{4(m - 2)}{3} \geq -6$ **o** $3 + \frac{x}{5} < 10$

4 What is the solution to $\frac{x - 2}{5} \geq -1$? Select **A**, **B**, **C** or **D**.

A $x \geq -7$ **B** $x \leq -3$ **C** $x \leq 10$ **D** $x \geq -3$

***See* Example 15**

5 Solve each inequality and graph its solution on a number line.

a $5 - x \leq 2$ **b** $15 > 7 - y$ **c** $1 - k < 12$

d $7 - m \geq 7$ **e** $2 - p > 8$ **f** $-t + 6 \geq 10$

6 Solve each inequality.

a $-2x < 6$ **b** $\frac{k}{-3} \geq 4$ **c** $-5t > 12$

d $\frac{-x}{3} \leq -4$ **e** $4 - 3w > 7$ **f** $-4y + 3 \leq 11$

g $3 - 2x \geq -5$ **h** $8 - 5a < 3$ **i** $-2d - 3 > 8$

j $\frac{5 + w}{-3} > 2$ **k** $\frac{x - 4}{-4} \geq 3$ **l** $\frac{-p + 2}{-3} < -2$

Just for the record Film and game classification

In Australia, films, publications and computer games are rated by the Classification Board. Films and videos are rated G, PG, M, MA15+ or R18+, with each category containing a list of guidelines related to the film's use of violence, coarse language, adult themes, sex and nudity. **General** (**G**) means suitable for all ages. Children can watch films classified G without adult supervision.

Parental guidance (**PG**) means that parental guidance is recommended for persons under 15 years of age. These films contain material that may be confusing or upsetting to children, but not harmful or disturbing. Parents should watch the film with their children or preview it to check elements such as language used or inappropriate themes.

Mature (**M**) means recommended for mature audiences, 15 years and over. The film or computer game may contain material that is harmful or disturbing to children, but the impact is not so strong as to require restriction.

Mature accompanied (**MA15+**) means legal restrictions apply to persons under the age of 15. Children are not allowed to see MA films unless accompanied by a parent or guardian, because they contain material that is likely to be harmful or disturbing to them.

Restricted (**R18+**) means legally restricted to adults, 18 years and over. It applies to films that deal with issues and scenes that require an adult perspective, and so are unsuitable for persons under 18 years of age. A person will be asked for proof of age before buying, hiring or viewing films or computer games in this category.

1 Each of the classifications is represented by a logo (as shown) with the letter inside a particular shape. What shape is each logo?
2 Write each classification category as an inequality.

Power plus

1 Solve each equation.
   a $\frac{3(1-y)}{5} = 4 - 2y$ **b** $\frac{50}{2y} = 10$ **c** $\frac{2m+5}{3} - 1 = m + 4$
2 If $y = \frac{ab+cd}{e}$, find d if $y = -12$, $a = -1$, $b = -8$, $c = 7$ and $e = 4$.
3 James is ten years older than Brett. In three years time, James will be twice as old as Brett. How old are James and Brett now?
4 One third of a number added to one-sixth of a number is 18. What is the number?
5 Graph each inequality on a number line.
   a $1 \leq x \leq 4$ **b** $-2 \leq x \leq 3$ **c** $-12 < 4x \leq 4$

Chapter 6 review

Puzzle sheet

Equations and inequalities crossword

MAT10NAPS10047

Language of maths

brackets
check
equation
expand
factorise
formula
fraction
greater than
inequality
inverse operation
LHS
less than
lowest common multiple (LCM)
number line
quadratic equation
RHS
solution
solve
square root
subject
substitute
variable

1 What type of equation has 2 as the highest power of x? Write an example of this type of equation.

2 True or false? $10 \geq 10$.

3 What is the difference between an **equation** and an **inequality**?

4 Why is it possible for a **quadratic equation** to have more than one solution?

5 When checking the solution to an equation, you need to show that 'LHS = RHS'. What does that mean?

6 What does the inequality symbol '$\leq$' mean?

Topic overview

Quiz

Equations

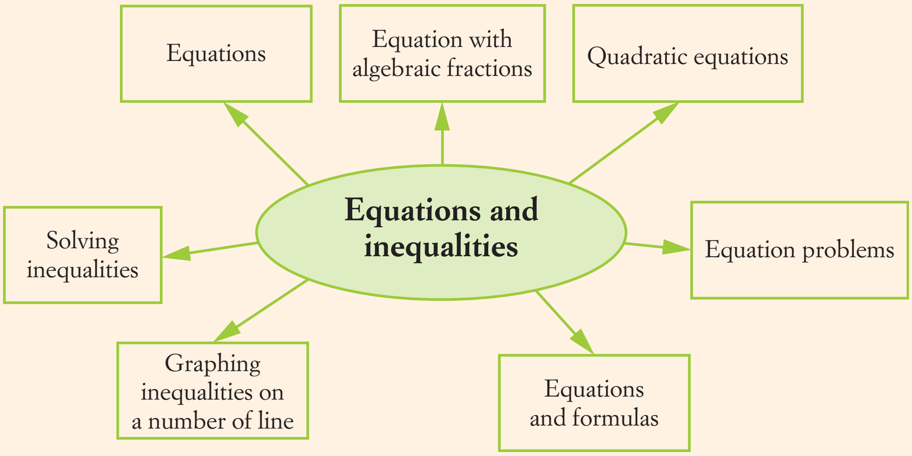
MAT10NAQZ00011

Copy and complete the table below.

🙂	The best part of this chapter was …	
🙁	The worst part was …	
😮	New work consisted of …	
😕	I need help with …	

Chapter 6 review

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 6 revision

1 Solve each equation.

See Exercise 6-01

a $3a + 10 = 43$ **b** $8y + 5 = 2y + 21$ **c** $2a - 12 = 6a$

d $9 - 2y = 5 + 2y$ **e** $3(m - 2) = 27$ **f** $2(2a + 1) = 3(a + 10)$

g $5(h + 1) + 3(h - 2) = 12$ **h** $4(2y + 1) + 3(1 + 4y) = 20$

2 Solve each equation.

Stage 5.2

See Exercise 6-02

a $\frac{3w + 2}{5} = 4$ **b** $\frac{y}{5} = \frac{7}{4}$ **c** $\frac{2a + 1}{2} = \frac{3a - 1}{4}$

d $\frac{3m + 5}{6} = \frac{10 - m}{3}$ **e** $\frac{2s}{3} - \frac{s}{6} = 2$ **f** $\frac{x}{10} + \frac{x}{2} = 1$

3 What is the solution to $\frac{2(p - 1)}{3} = 4$? Select the correct answer **A**, **B**, **C** or **D**.

See Exercise 6-02

A $p = 7$ **B** $p = 11$ **C** $p = 5$ **D** $p = 4$

4 Solve each quadratic equation.

See Exercise 6-03

a $y^2 = 4$ **b** $p^2 - 100 = 0$ **c** $4x^2 = 36$

d $3m^2 - 3 = 0$ **e** $\frac{2w^2}{5} = 10$ **f** $x^2 + 8x + 7 = 0$

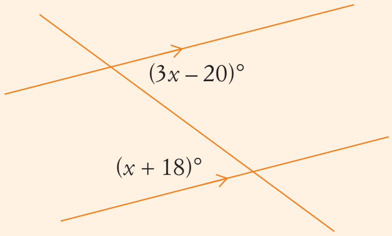
g $h^2 - 8h - 9 = 0$ **h** $u^2 + 4u - 77 = 0$ **i** $k^2 + 5k = 0$

j $m^2 - 2m = 0$ **k** $b^2 + 20b + 100 = 0$ **l** $w^2 = 9w$

5 The sum of four consecutive numbers is 374. Find the four numbers.

See Exercise 6-04

6 Find the value of x.



7 The braking distance (in metres) of a bicycle travelling at a speed of v metres/second is $d = \frac{v(v + 1)}{2}$. Calculate the braking distance when the speed of the bicycle is 15 m/s.

Stage 5.2

See Exercise 6-05

8 The volume of a pyramid is given by the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

a the volume of a pyramid with a base area of 48 mm^2 and a perpendicular height of 10 mm

b the base area of a pyramid with a volume of 500 m^3 and a perpendicular height of 5 m

9 Graph each inequality on a number line.

See Exercise 6-06

a $x \geq 0$ **b** $x < 3$ **c** $x \leq -2$ **d** $x > -5$

10 Solve each inequality.

See Exercise 6-07

a $y - 6 \geq 10$ **b** $2y \leq -15$ **c** $3a + 10 > -5$

d $10 - 6x < 28$ **e** $\frac{a + 2}{-4} > \frac{7}{2}$ **f** $\frac{3 - 5x}{2} \geq 9$

7

Number and Algebra

Graphs

When an object is thrown upwards, its path is a curve called a parabola. The shape and length of the path will depend on the initial speed of the object. Furthermore, car headlights, and satellite dishes all use mirrors or reflectors that have the shape of a parabola.

Shutterstock.com/HSNphotography

Chapter outline

	Proficiency strands				
7-01 Direct proportion*	U	F	PS	R	C
7-02 Inverse proportion*	U	F	PS	R	C
7-03 Conversion graphs*	U	F	PS	R	C
7-04 The parabola	U	F		R	C
7-05 The exponential curve	U	F		R	C
7-06 The circle	U	F		R	C
7-07 Identifying graphs*		F		R	C

*STAGE 5.2

Wordbank

asymptote A line that a curve gets very close to but never touches, for example, the *x*-axis is an asymptote of the exponential curve

direct proportion A relationship between two variables of the form $y = kx$, where k is a constant; for example, if $y = 8.5x$, then y is directly proportional to x

exponential equation An equation involving a variable as a power, such as $y = 3^x$, whose graph is an exponential curve

inverse proportion A relationship between two variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x

parabola A U-shaped curve that is the graph of a quadratic equation

quadratic equation An equation involving a variable squared (power of 2), such as $y = 3x^2 - 6$, whose graph is a curve called a parabola

In this chapter you will:

- (STAGE 5.2) solve problems involving direct proportion and explore the relationship between graphs and equations corresponding to simple rate problems
- explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate AC
- (STAGE 5.2) solve problems involving inverse proportion
- (STAGE 5.2) read and interpret conversion graphs
- graph parabolas of the form $y = ax^2$ and $y = ax^2 + c$
- graph exponential curves of the form $y = a^x$
- graph circles of the form $x^2 + y^2 = r^2$
- match graphs to their equations

SkillCheck

Worksheet
StartUp assignment 7
MAT10NAWK10048

1 If $A = 2x^2 - 3$, find A if:

a $x = 1$ **b** $x = 4$ **c** $x = 0$ **d** $x = -6$

2 If $y = 5^x$, find y if:

a $x = 4$ **b** $x = 5$ **c** $x = 0$ **d** $x = -2$

Stage 5.2

Technology worksheet
Direct proportion
MAT10NACT10004

7-01 Direct proportion

Two variables are **directly proportional** to each other if one variable is a constant multiple of the other; when one variable changes, the other one changes by the same factor.

Summary

If y is **directly proportional** to x, then $y = kx$, where k is a constant (number) called the **constant of proportionality** or **constant of variation**.

- A direct linear relationship exists between x and y
- If x increases (or decreases), y increases (or decreases)
- If x is doubled (or halved), y is doubled (or halved)
- Another way of saying 'y is directly proportional to x' is y **varies directly** with x'
- The graph of direct proportion is a straight line going through (0, 0) with gradient k

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Example 1

The distance (d) in metres travelled by a car is directly proportional to the number of rotations (r) of its tyres. After 540 rotations, a distance of 950 m is travelled.

a What distance (correct to the nearest metre) will be travelled after 800 rotations?

b How many full rotations will be needed to cover 360 km?


Getty Images/John Borthwick

Solution

a d is directly proportional to r

$\therefore d = kr$

To find k, substitute the information given for r and d.

When $r = 540$, $d = 950$:

$$950 = k \times 540$$

$$k = \frac{950}{540}$$

$$= 1.759\ldots$$

Do not round the value of k.

$\therefore d = 1.759\ldots \times r$

When $r = 800$,

$$d = 1.759\ldots \times 800$$

$$= 1407.4074\ldots$$

$$\approx 1407 \text{ m}$$

After 800 rotations, the distance travelled will be 1407 m.

b When $d = 360$ km = 36 000 m,

$$36\ 000 = 1.759\ldots \times r$$

$$r = \frac{36\ 000}{1.759\ldots}$$

$$= 20\ 463.157\ldots$$

Do not round the value of k.

$$\approx 20\ 463 \text{ rotations}$$

Rounding down for full rotations.

For a distance of 360 km, there will be 20 463 rotations.

Stage 5.2

Summary

Solving a direct proportion problem

1. Identify the two variables (say x and y) and form a proportion equation, $y = kx$
2. Substitute values for x and y to find k, the constant of proportionality
3. Rewrite $y = kx$ using the value of k
4. Substitute a value for x or y into $y = kx$ to solve the problem.

Example 2

M varies directly with n. If when $n = 6$, $M = 103.8$, find M when $n = 14.2$.

Solution

Form the proportion equation: $M = kn$

To find k, substitute $n = 6$ and $M = 103.8$

$$\begin{aligned} 103.8 &= k \times 6 \\ 6k &= 103.8 \\ k &= \frac{103.8}{6} \\ &= 17.3 \end{aligned}$$

$\therefore M = 17.3n$

Substitute $n = 14.2$ to find M.

$$\begin{aligned} M &= 17.3 \times 14.2 \\ &= 245.66 \end{aligned}$$

Exercise 7-01 Direct proportion

See Example 1

1 The distance, D, travelled by Craig, a marathon runner, varies directly with time, T.

Time, T (min)	Distance, D (m)
1	190
2	380
3	570

a Write a variation equation for D.

b How far in kilometres will Craig run in:

i 20 minutes? **ii** 45 minutes?

c How long would it take Craig to run 12.35 kilometres? Answer in hours and minutes.

Stage 5.2

2 Mehta's earnings for working a shift at the local nursery are directly proportional to the number of hours she works. Yesterday, she earned \$222.70 for working an 8.5 hour shift.

a If Mehta's earnings are represented by E, and the number of hours worked is represented by h, write an equation for E.

b How much will she earn for working a 7-hour shift?

c How many hours did she work today if she earned \$144.10 for the shift?

3 The amount of interest, I, earned for one year on an investment account varies directly with the size of the deposit, D.

a If Caterina earns \$16 interest on an investment of \$425, find the variation equation for I.

b Hence, how much will she earn on an investment of \$900?

c If Caterina's uncle doubles the size of her investment in **b**, how much will she earn in interest?

***See* Example 2**

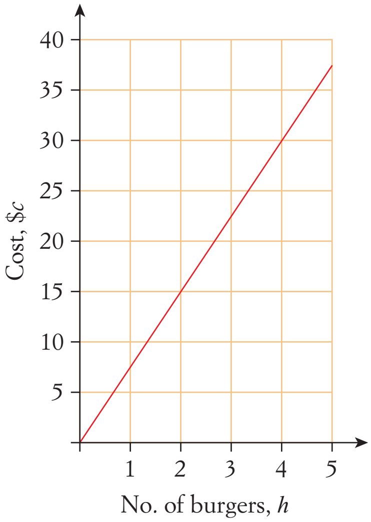
4 S varies directly with t. If when $t = 14$, $S = 106.4$, what is the value of S when $t = 0.3$? Select the correct answer **A**, **B**, **C** or **D**.

A 2.28 **B** 27.72 **C** 36.12 **D** 446.88

5 Find the linear formula for b in terms of a for this table of values.

a	4	8	12	16	20
b	10	20	30	40	50

6 The graph on the right shows that the cost of hamburgers purchased from the local takeaway store depends directly on the number of burgers purchased.



a Copy the table below and use the graph to complete it.

No. of burgers, ***h***	Cost, ***c*** (\$)
1	
2	
3	

b Find the variation equation to represent the relationship between the cost (\$$c$) and the number of burgers (h).

c If Kim buys 6 hamburgers, what is the total cost of the hamburgers?

d The total cost of one order of hamburgers is \$82.50. How many hamburgers were ordered?

e Find the gradient of the line. How is it related to the constant of variation?

7 K varies directly with L. If $L = 9.5$ when $K = 1045$, what is the value of K when $L = 1.65$? Select **A**, **B**, **C** or **D**.

A 0.015 **B** 93.7 **C** 181.5 **D** 1708.575

Stage 5.2

8 A linear relationship exists between the mass of a car (*m* kg) and its fuel consumption rate (*F* L/100 km).

a Find the variation equation for *F* if a 1000 kg car uses fuel at a rate of 6 L/ 100 km.

b Find the fuel consumption of a 2500 kg car.

9 For an object that is cooling, the drop in temperature varies directly with time. If the temperature drops 8°C in 5 minutes, how long would it take to drop 10°C? Select the correct answer **A**, **B**, **C** or **D**.

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

10 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars.

a Calculate how much a 60 kg astronaut weighs on Mars, correct to 1 decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to 1 decimal place.

NSW

7-02 Inverse proportion

Worksheet
Direct and inverse proportion
MAT10NAWK10049

Two variables are **inversely proportional** to each other if, when one variable increases, the other one decreases by the same factor.

The table below shows the different speeds of a car (*s* km/h), and the time it takes to travel 100 km (*t* min). As the speed increases, the time taken decreases.

Speed (*s* km/h)	**Time (*t* min)**
50	120
60	100
80	75
100	60

Summary

If y is **inversely proportional** to x, then $y = \frac{k}{x}$, where k is a constant (number) called the **constant of proportionality** or **constant of variation**.

- If x increases, y decreases ('inverse' means 'opposite')
- If x decreases, y increases
- If x is doubled, y is halved
- If x is halved, y is doubled
- Another way of saying 'y is inversely proportional to x' is 'y **varies inversely** with x'

Example 3

Stage 5.2

The time (t) in minutes taken by a car to travel 100 km is inversely proportional to the speed (s km/h) of the car, as shown in the table on the previous page. At 50 km/h, the time taken is 120 minutes.

a Find the inverse variation equation for t.

b How long did the car take to travel 100 km at:

i 40 km/h? **ii** 110 km/h?

c Find the car's speed if it took 45 minutes to travel 100 km.

Solution

a t is inversely proportional to s.

$$\therefore t = \frac{k}{s}$$

To find k, substitute the information given for s and t.

When $s = 50$, $t = 120$:

$$\begin{aligned}120 &= \frac{k}{50}\\ k &= 120 \times 50\\ &= 6000\end{aligned}$$

$$\therefore t = \frac{6000}{s}$$

b **i** When $s = 40$, $t = \frac{6000}{40} = 150$ min

At 40 km/h, the trip takes 150 min (or 2 h 30 min).

ii When $s = 110$,

$$\begin{aligned}t &= \frac{6000}{110}\\ &= 54.5454\ \ldots\\ &\approx 55 \text{ min}\end{aligned}$$

At 110 km/h, the trip takes 55 min.

c When $t = 45$,

$$\begin{aligned}45 &= \frac{6000}{s}\\ 45s &= 6000\\ s &= \frac{6000}{45}\\ &= 133\frac{1}{3} \text{ km/h}\end{aligned}$$

For a travel time of 45 min, the speed must be $133\frac{1}{3}$ km/h.

Stage 5.2

Summary

Solving an inverse proportion problem

1 Identify the two variables (say x and y) and form a proportion equation, $y = \frac{k}{x}$

2 Substitute values for x and y to find k, the constant of proportionality.

3 Rewrite $y = \frac{k}{x}$ using the value of k.

4 Substitute a value for x or y into $y = \frac{k}{x}$ to solve the problem.

Example 4

The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 600 m above sea level, the temperature is 8°C.

a What is the temperature at 1000 m above sea level?

b Graph the relationship between temperature and height above sea level, for heights between 0 and 5000.

Solution

a T is inversely proportional to h.

$$T = \frac{k}{h}$$

Substitute $h = 600$ and $T = 8$ to find k.

$$8 = \frac{k}{600}$$

$$k = 8 \times 600$$

$$= 4800$$

$$\therefore T = \frac{4800}{h}$$

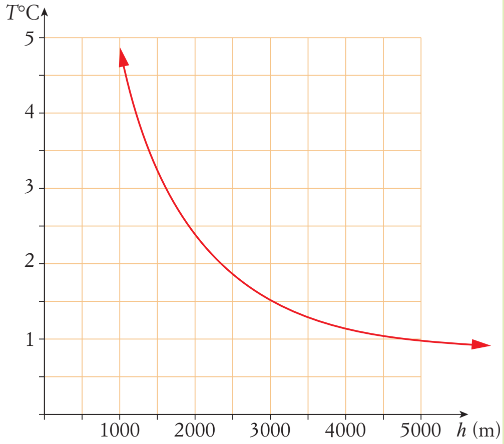
When $h = 1000$, $T = \frac{4800}{1000} = 4.8°\text{C}$

The temperature at a height of 1000 metres above sea level is 4.8°C.

b Draw a table of values for $T = \frac{4800}{h}$

h	1000	2000	3000	4000	5000
T	4.8	2.4	1.6	1.2	0.96

Note that as h increases, T decreases.



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Example 5

P varies inversely with q. If when $q = 4$, $P = 38$, find P when $q = 5$.

Solution

$P = \frac{k}{q}$ — P varies inversely with q.

Substitute $q = 4$, $P = 38$

$38 = \frac{k}{4}$

$k = 38 \times 4 = 152$

$\therefore P = \frac{152}{q}$

Substitute $q = 5$ to find P.

$P = \frac{152}{5} = 30.4$

Exercise 7-02 Inverse proportion

1 The time taken, T hours, to travel from Sydney to Melbourne varies inversely with the speed, s km/h. **See Example 3**

- **a** If it takes 11.5 hours at an average speed of 80 km/h, find the variation equation for T.
- **b** If the average speed is increased to 90 km/h, how long will the journey take? Answer in hours and minutes.
- **c** Find the average speed needed to complete the trip in 10 hours.

2 The rate of vibration of a string varies inversely as its length. A string that is 8 cm long vibrates at 9375 Hz (hertz). What length of string will vibrate at 6250 Hz? Select the correct answer **A**, **B**, **C** or **D**.

A 5 cm **B** 7 cm **C** 12 cm **D** 73 cm

Shutterstock.com/Vereshchagin Dmitry

3 The temperature, T (in degrees Celsius), of the air varies inversely with the height, h (in metres), above sea level. At 150 m above sea level, the temperature is 30°C. **See Example 4**

- **a** What is the temperature at:
  - **i** 300 m above sea level? **ii** 2500 m above sea level?
- **b** What is the height above sea level when the temperature is:
  - **i** 8°C? **ii** 22.5°C?
- **c** Graph the relationship between temperature and height above sea level. Use T on the vertical axis and h on the horizontal axis with $h = 0, 500, 1000, 1500, \ldots, 3000$.

4 The number of people who attend a concert varies inversely with the amount of space allocated to each person. If 80 cm^2 is allowed per person, the ground can hold 3400 people. How many people could attend the concert if only 60 cm^2 was allocated per person?

Stage 5.2

5 Which equation represents the table of values shown below? Select **A**, **B**, **C** or **D**.

x	2	5	8	10
y	2.5	1	0.625	0.5

A $y = \frac{10}{x}$ **B** $y = \frac{5}{x}$ **C** $y = \frac{2.5}{x}$ **D** $y = \frac{1}{x}$

See Example 5

6 K is inversely proportional to L. If $L = 2$ when $K = 7$, find K when $L = 15$.

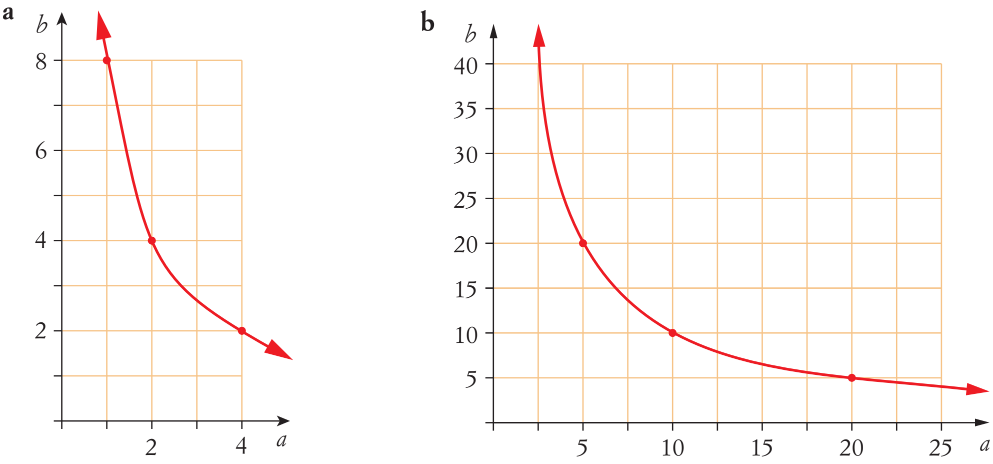
7 Paul believes that at a train station, the number of people waiting on the platform is inversely proportional to the time until the next train arrives. According to his model, when there are 16 people waiting, the train will arrive in 2.5 minutes.

Fairfax Syndication/Janie Barrett

a When will the train arrive if there are 5 people waiting?

b How many people are waiting at the station 10 minutes before the train arrives?

8 Each graph below shows an inverse relationship between a and b. Find each variation equation.



9 The frequency, F beats per second, that a bird beats its wings varies inversely as the length, L cm, of its wings. A bird with wings of length 14 cm beats them at a frequency of 8 beats per second.

a Find the variation equation for F in terms of L.

b Calculate, to the nearest whole number, the wingbeat frequency for wings of length 18 cm.

c A bird beats its wings with a frequency of 4.5 beats per second. What is the length of its wings, correct to the nearest centimetre?

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10 For a certain equation, y varies inversely with x.

- **a** Given $x = 0.2$ when $y = 10$, find y when $x = 32$.
- **b** Find x when $y = 1.6$.

11 The amount of time it takes Sarah to move house is inversely proportional to the number of friends she has to help her. When she has 4 friends helping, the job takes $3\frac{3}{4}$ hours.

- **a** How long will it take if she has 6 friends helping?
- **b** How many friends must she have to help her to move house in 3 hours?

7-03 Conversion graphs

NSW

Worksheet
Currency conversion graph
MAT10NAWK10050

A **conversion graph** is used to convert from one unit to another; for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0).

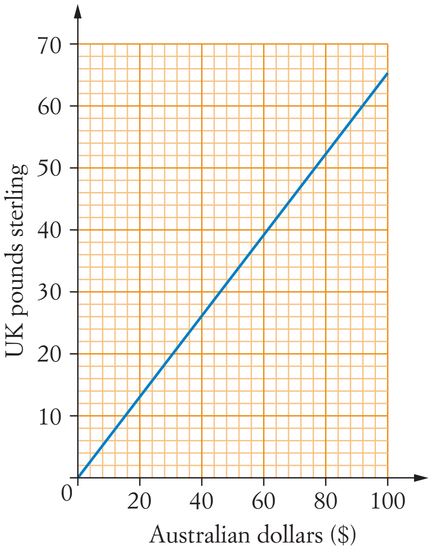
Example 6

Exchange rates change daily but suppose the exchange rate between the Australian dollar and the UK pound sterling is \$A1 = £0.653, then \$A100 = £65.30 sterling. These values are used to draw this conversion graph.

Use the graph to convert:

- **a** \$A50 to pounds
- **b** £10 to Australian dollars.

Australian dollars to UK pounds sterling



Solution

Reading from the graph:

- **a** \$A50 ≈ £33
- **b** £10 ≈ \$A16

Stage 5.2

Exercise 7-03 Conversion graphs

See Example 6

1 Use the graph in Example **6** to answer the following questions.

a Convert to pounds.

i \$A40 **ii** \$A88

b Convert to Australian dollars.

i £18 **ii** £60

c In June 2008, \$A1 = £0.49.

i How much **less** was \$A40 worth in UK pounds sterling in 2008 than it is using this more recent conversion graph?

ii How much **more money** was £60 worth in Australian dollars for visiting tourists, than it is using this more recent conversion graph?

2 The furlong is an imperial measure once used to measure length. This conversion graph shows distances in furlongs converted to kilometres.

a Convert to kilometres.

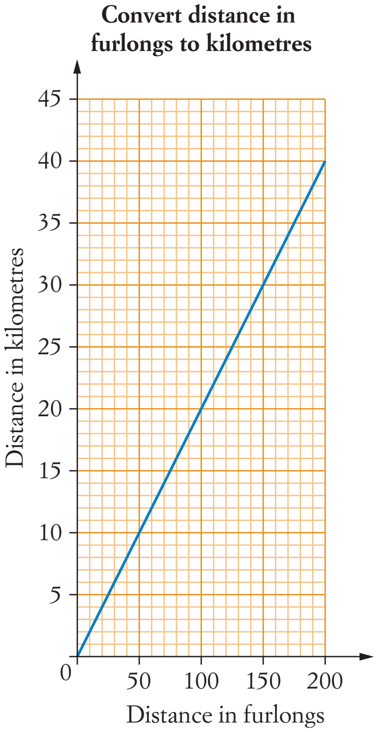
i 10 furlongs
ii 100 furlongs
iii 170 furlongs

b Convert to furlongs.

i 10 km
ii 25 km
iii 36 km

c Use an answer from part **a** to convert 300 furlongs to kilometres.

d Use an answer from part **b** to convert 100 kilometres to furlongs.



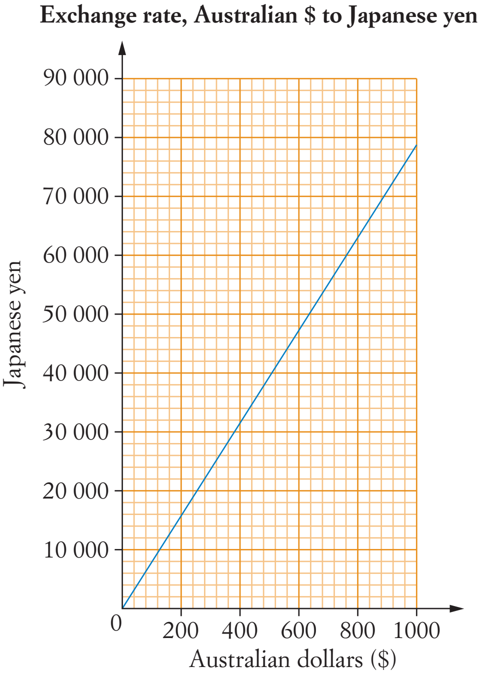
3 The graph below shows the exchange rate to convert Australian dollars to Japanese yen (¥).

a Convert to Japanese yen.

- **i** $A200
- **ii** $A800
- **iii** $A1000

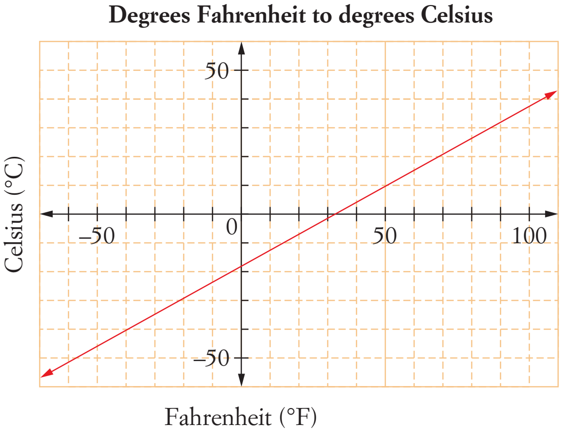
b Convert to Australian dollars.

- **i** ¥20 000
- **ii** ¥60 000
- **iii** ¥72 000



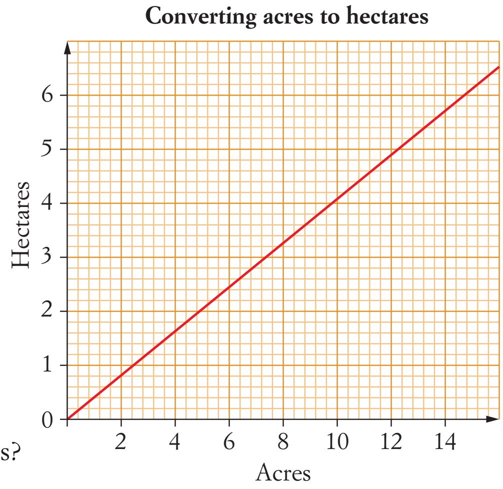
4 The graph below shows the temperature conversion from degrees Fahrenheit to degrees Celsius. Convert:

- **a** 0°F to °C
- **b** 50°F to °C
- **c** 80°F to °C
- **d** 0°C to °F
- **e** −10°C to °F
- **f** 30°C to °F



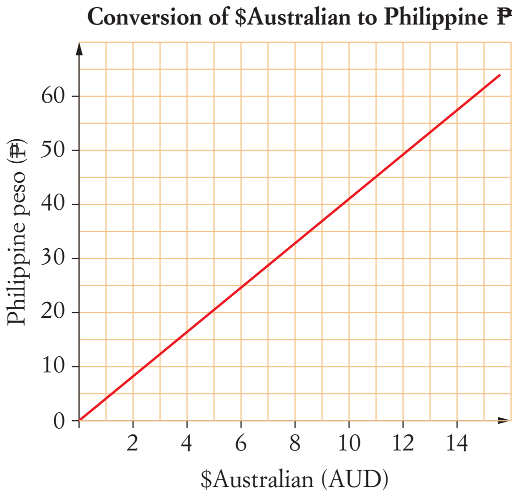
5 This conversion graph is used to convert acres to hectares. The acre is an Imperial measure of land area while the hectare (ha) is the metric measure.

- **a** Use the graph to convert 12 acres to hectares.
- **b** A garden has an area of 5 acres. What is this area in hectares?
- **c** Use the graph to convert 4.4 hectares to acres.
- **d** Mr Ferguson has a property with an area of 5 hectares. How big is this in acres?
- **e** A rectangular playing field measures 250 m by 128 m.
  - **i** What is the area of the field in square metres?
  - **ii** What is the area of the field in hectares?
  - **iii** What is the area of the field in acres?



6 This graph is used to convert Australian dollars (AUD) to Philippine pesos (₱).

- **a** Change into pesos, correct to the nearest whole peso.
  - **i** \$15 **ii** \$50 **iii** \$88
- **b** Change ₱50 to AUD.
- **c** How many Australian dollars would you receive for ₱20?
- **d** Calculate the number of pesos you should get for \$120.



Worksheet
Graphing parabolas
MAT10NAWK10051

Technology worksheet
Graphing non-linear equations
MAT10NACT10005

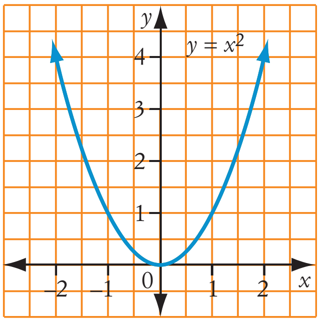
Technology worksheet
Excel worksheet: Investigating parabolas 1
MAT10NACT00010

Technology worksheet
Excel spreadsheet: Investigating parabolas 1
MAT10NACT00040

7-04 The parabola

An equation in which the highest power of the variable is 2 is called a **quadratic equation**; for example, $y = 2x^2 - 5$, $y = x^2 + 7x + 12$ and $y = -5x^2$. The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**.

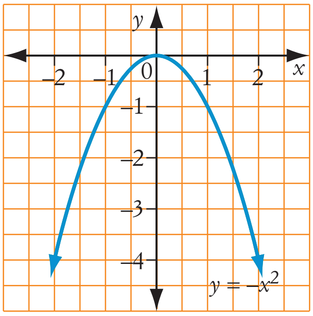
The graphs of $y = x^2$ and $y = -x^2$ are shown below.



$y = x^2$

Concave up (looks like a smile ☺)

Minimum value of the parabola is 0.



$y = -x^2$

Concave down (looks like a frown ☹)

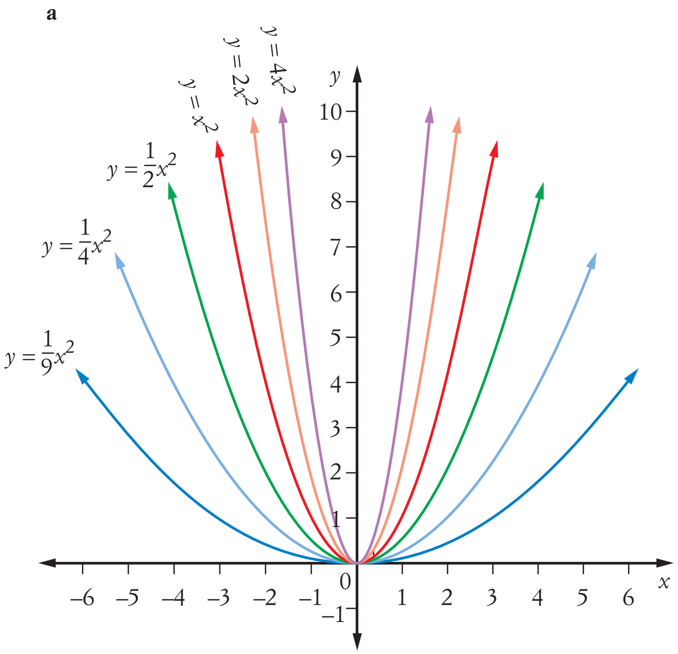
Maximum value of the parabola is 0.

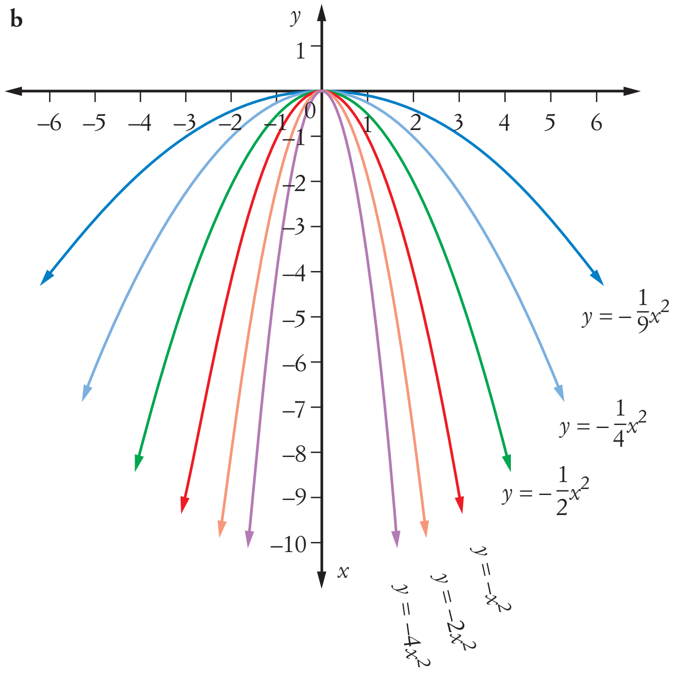
Stage 5.2

The graph of $y = ax^2$

For the graph of a quadratic equation in the form $y = ax^2$, where a is a constant (number), the size of a (the **coefficient** of x^2) affects whether the parabola is 'wide' or 'narrow'.

As the size of a increases, the parabola becomes 'narrower' and as the size of a decreases, the parabola 'widens'. If a is negative, then the parabola is concave down.





- The axis of symmetry, called the **axis of the parabola,** is the y-axis.
- The **vertex** or **turning point** is (0, 0).

The graph of $y = ax^2 + c$

For the graph of a quadratic equation in the form $y = ax^2 + c$, where a and c are constants, the effect of c is to move the parabola up or down from the origin. Also, c is the y-intercept of the parabola.

Stage 5.2

Example 7

Graph each set of quadratic equations, showing the vertex of each parabola.

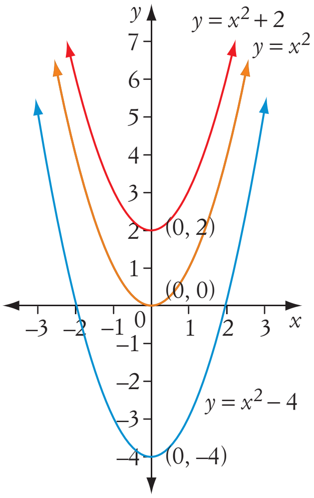
a $y = x^2, y = x^2 - 4, y = x^2 + 2$ **b** $y = -x^2, y = -x^2 - 4, y = -x^2 + 5$

Solution

a First draw the graph of $y = x^2$. Its vertex is at (0, 0).

The graph of $y = x^2 - 4$ is identical to that of $y = x^2$, but it is moved 4 units down. Its vertex is at (0, −4).

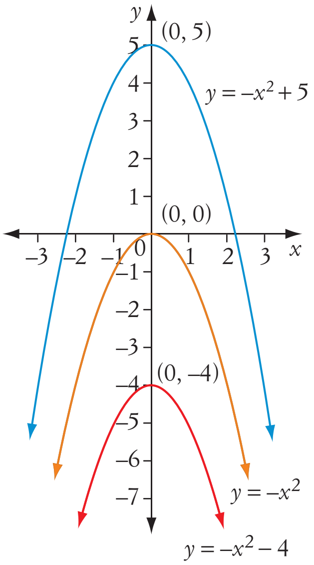
The graph of $y = x^2 + 2$ is identical to that of $y = x^2$, but it is moved 2 units up. Its vertex is at (0, 2).



b The graph of $y = -x^2$ is the graph of $y = x^2$ reflected across the x-axis. Its vertex is at (0, 0) as well.

The graph of $y = -x^2 - 4$ is identical to that of $y = -x^2$, but it is moved 4 units down. Its vertex is at (0, −4).

The graph of $y = -x^2 + 5$ is identical to that of $y = -x^2$, but it is moved 5 units up. Its vertex is at (0, 5).



Note:

- In part **a**, all parabolas are concave up, because of the positive coefficient of x^2
- In part **b**, all parabolas are concave down, because of the negative coefficient of x^2
- For $y = ax^2 + c$, the y-intercept of the parabola is c

Example 8

For the graph of each given quadratic equation, state:

i whether the parabola is wider or narrower than the graph of $y = x^2$
ii whether the parabola has moved up or down when compared to the graph of $y = x^2$
iii the y-intercept.

a $y = 3x^2 - 1$ **b** $y = \frac{1}{3}x^2 + 2$

Solution

a i The coefficient of x^2 is 3, while the coefficient of x^2 in $y = x^2$ is 1.
$\therefore$ The parabola will be narrower than $y = x^2$.

ii The constant term is -1.
$\therefore$ The parabola has moved down.

iii The y-intercept is -1.

b i The coefficient of x^2 is $\frac{1}{3}$.
$\therefore$ The parabola will be wider than $y = x^2$.

ii The constant term is 2.
$\therefore$ The parabola has moved up.

iii The y-intercept is 2.

Example 9

A parabola has the equation $y = 3x^2 - 1$. Find the x-coordinate of the point on the parabola that has a y-coordinate of:

a 11 **b** 191.

Solution

a Substitute $y = 11$ into $y = 3x^2 - 1$.

$$\begin{aligned} 11 &= 3x^2 - 1 \\ 12 &= 3x^2 \\ 3x^2 &= 12 \\ x^2 &= \frac{12}{3} \\ &= 4 \\ x &= \pm\sqrt{4} \\ &= \pm 2 \end{aligned}$$

This means that there are two points on the parabola with a y-coordinate of 11.
They are (2, 11) and (−2, 11).

b Substitute $y = 191$ into $y = 3x^2 - 1$

$$\begin{aligned} 191 &= 3x^2 - 1 \\ 192 &= 3x^2 \\ 3x^2 &= 192 \\ x^2 &= \frac{192}{3} \\ &= 64 \\ x &= \pm\sqrt{64} \\ &= \pm 8 \end{aligned}$$

Exercise 7-04 The parabola

Some of this exercise may also be completed using a graphics calculator or graphing software.

See Example 7

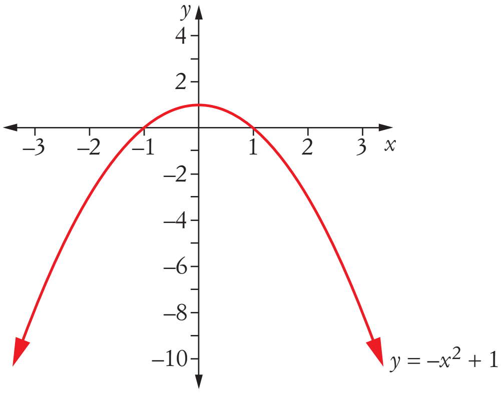
1 **a** Graph each quadratic equation, showing the vertex of each parabola.

$y = x^2$ $y = -x^2$ $y = x^2 + 2$ $y = -2x^2$ $y = x^2 - 1$

b State which graphs you have drawn in part **a**:

i are concave up **ii** are concave down **iii** have a turning point at (0, 0).

2 Which statement is false about this parabola? Select **A**, **B**, **C** or **D**.

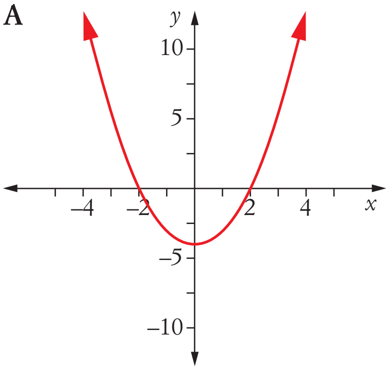


A Its axis of symmetry is the x-axis. **B** It is concave down.

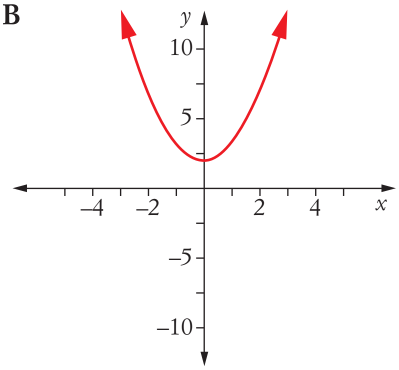
C Its vertex is (0, 1). **D** It has a maximum value.

3 Which diagram shows the graph of $y = x^2 - 2$? Select **A**, **B**, **C** or **D**.

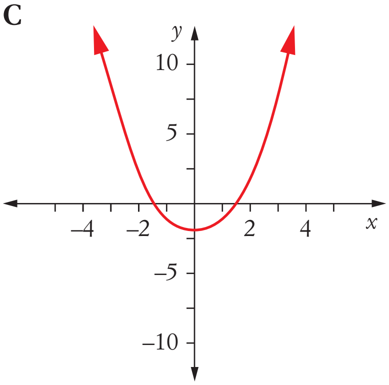
A



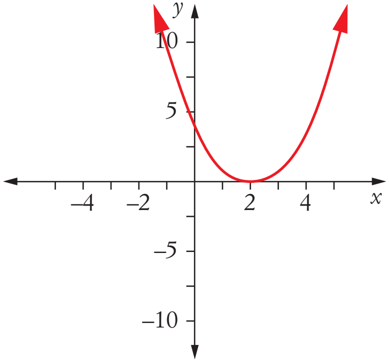
B



C



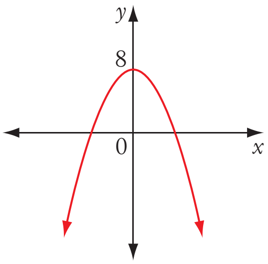
D



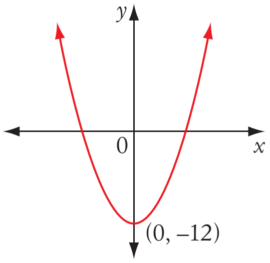
Stage 5.2

4 Match each graph with its correct quadratic equation.

a



b



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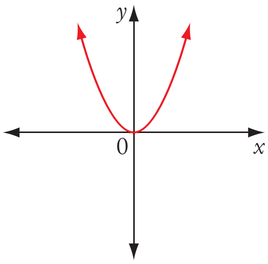

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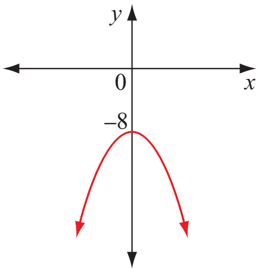
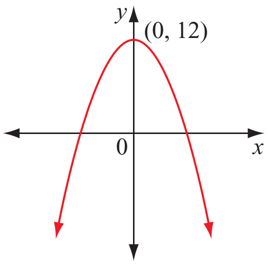
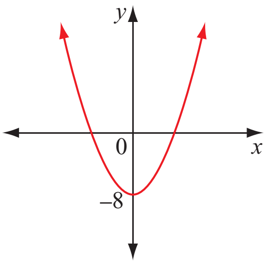


e



f



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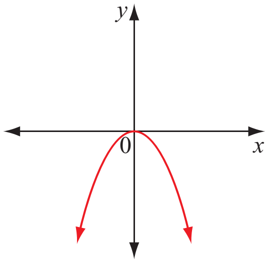

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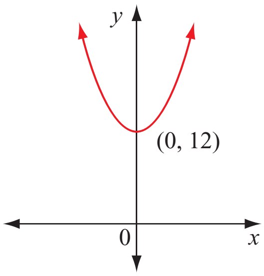

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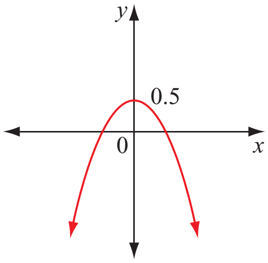

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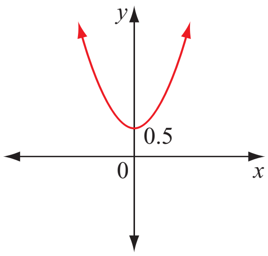

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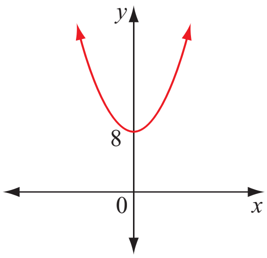

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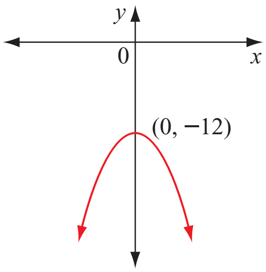


i $y = x^2$

ii $y = -x^2$

iii $y = x^2 - 8$

iv $y = -12 - x^2$

v $y = \frac{1}{2} + x^2$

vi $y = 8 - x^2$

vii $y = 8 + x^2$

viii $y = -x^2 + \frac{1}{2}$

ix $y = x^2 - 12$

x $y = 12 - x^2$

xi $y = -x^2 - 8$

xii $y = x^2 + 12$

Stage 5.2

5 Find the equation of each of the following parabolas in the form $y = x^2 + c$ or $y = -x^2 + c$ (where c is a constant), given:

a vertex (0, 0), concave down

b concave up, turning point (0, 0)

c axis of symmetry $x = 0$, maximum $y = -\frac{1}{4}$

d concave down, maximum $y = -9$

e turning point $\left(0, \frac{1}{2}\right)$, concave down

f axis of symmetry y-axis, minimum $y = 9$

6 **a** Graph $y = 2x^2 + 1$ after copying and completing this table.

x	-2	-1	0	1	2
y					

b State the turning point (vertex).

c Is the parabola concave up or concave down?

d What is its minimum value?

7 **a** Graph $y = -3x^2 + 2$ after copying and completing this table.

x	-2	-1	0	1	2
y					

b Find the vertex.

c Write the equation of its axis of symmetry.

d Find its maximum value.

8 Which statement is false about the graph of $y = 4x^2 - 1$? Select **A**, **B**, **C** or **D**.

A Its axis of symmetry is $y = 0$.

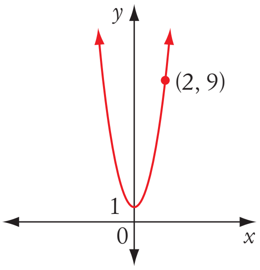
B It is concave up.

C The vertex is $(0, -1)$.

D It has a minimum value of $y = -1$.

9 Match each graph with its correct quadratic equation.

a



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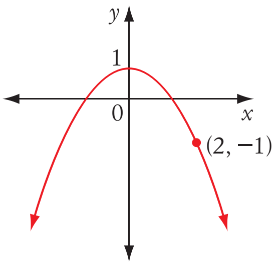

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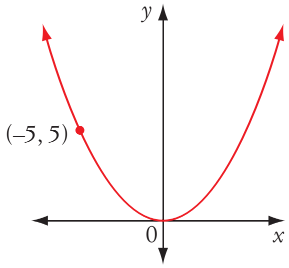

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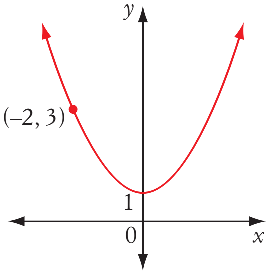

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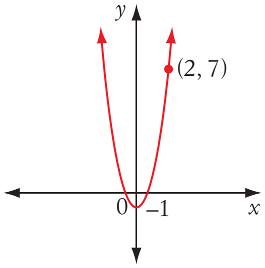

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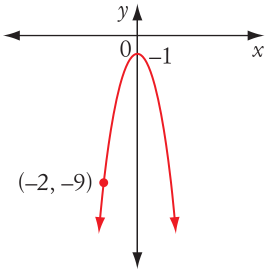

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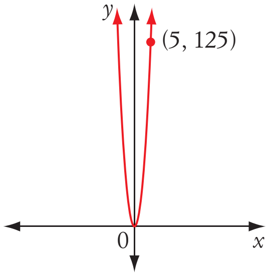

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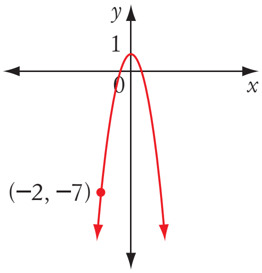

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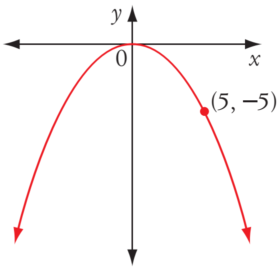

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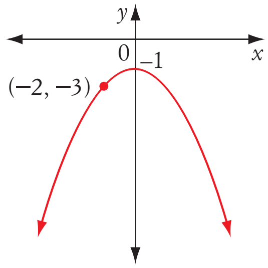

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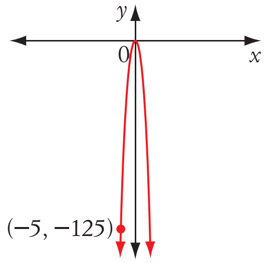

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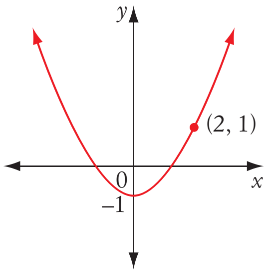


i $y = 5x^2$

ii $y = 2x^2 + 1$

iii $y = \frac{1}{2}x^2 - 1$

iv $y = \frac{1}{5}x^2$

v $y = 2x^2 - 1$

vi $y = -5x^2$

vii $y = -\frac{1}{2}x^2 + 1$

viii $y = -\frac{1}{5}x^2$

ix $y = -2x^2 - 1$

x $y = \frac{1}{2}x^2 + 1$

xi $y = -2x^2 + 1$

xii $y = -\frac{1}{2}x^2 - 1$

10 For the graph of each given quadratic equation, state:

i whether the parabola is wider or narrower than the graph of $y = x^2$

ii whether the parabola has moved up or down when compared to the graph of $y = x^2$

iii the y-intercept.

a $y = 2x^2 + 3$ **b** $y = \frac{1}{2}x^2 + 1$ **c** $y = 6x^2 - 5$ **d** $y = 0.2x^2 - 12$

See Example 8

11 A parabola has the equation $y = x^2 - 5$. Find the x-coordinates for the points on the parabola that have a y-coordinate of:

a 11 **b** 116.

See Example 9

12 A stone is dropped from a cliff and its height (h metres) at any time (t seconds) is given by $h = 80 - 4.8t^2$.

a Draw a graph of the equation for values of t from 0 to 5.

b What is the height of the cliff?

c What is the height of the stone after 3 seconds?

d When will the stone hit the ground?

e How long after it is dropped is the stone 5 metres above the ground? Answer correct to 2 decimal places.

13 A parabola has the equation $y = 2x^2 + 3$. Find the x-coordinates of the point on the parabola that has a y-coordinate of:

a 165 **b** 396.

Just for the record Parabolas in architecture

There are many examples of parabolas in architecture and engineering.
The Notre Dame Cathedral in Paris, France is almost 900 years old and has flying buttresses on the outside that have the shape of parabolas.

Alamy/Stephen Bay

Bridges also often use parabolic curves in their construction. One modern application is the cables used in the suspension of the Golden Gate Bridge in San Francisco.

Shutterstock.com/Mark Schwettmann

Find 2 different uses of parabolas in real-life constructions and create a presentation with pictures.

Mental skills 7 Maths without calculators

Multiplying decimals

1 Study each example.

a $3 \times 8 = 24$, so $\underset{\uparrow}{3} \times \underset{\uparrow}{0.8} = \underset{\uparrow}{2.4}$

0 dp + 1 dp = 1 dp (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation: $3 \times 0.8 \approx 3 \times 1 = 3$ $(2.4 \approx 3)$

b $6 \times 5 = 30$, so $\underset{\uparrow}{0.6} \times \underset{\uparrow}{0.5} = \underset{\uparrow}{0.30} = 0.3$

1 dp + 1 dp = 2 dp

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ $(0.3 \approx 0.25)$

c $7 \times 3 = 21$, so $\underset{\uparrow}{0.07} \times \underset{\uparrow}{0.3} = \underset{\uparrow}{0.021}$

2 dp + 1 dp = 3 dp

By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ $(0.021 \approx 0.02)$

2 Now evaluate each product.

a 0.7×5 **b** 12×0.2 **c** 0.4×0.3 **d** $(0.6)^2$
e 8×0.1 **f** 0.03×0.9 **g** 4×0.05 **h** 1.1×8
i 0.3×0.8 **j** 0.2×0.06 **k** 9×0.2 **l** 0.07×0.4

3 Study each example.
Given that $15 \times 23 = 345$, evaluate each product.

a $\underset{\uparrow}{1.5} \times \underset{\uparrow}{2.3} = \underset{\uparrow}{3.45}$

1 dp + 1 dp = 2 dp (*Estimate*: $1.5 \times 2.3 \approx 2 \times 2 = 4$)

b $150 \times 0.23 = 15 \times 10 \times 0.23 = \underset{\uparrow}{15} \times \underset{\uparrow}{0.23} \times 10 = \underset{\uparrow}{3.45} \times 10 = 34.5$

0 dp + 2 dp = 2 dp

(*Estimate*: $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

c $0.15 \times 2300 = \underset{\uparrow}{0.15} \times \underset{\uparrow}{23} \times 100 = \underset{\uparrow}{3.45} \times 100 = 345$

2 dp + 0 dp = 2 dp

(*Estimate*: $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

4 Now given that $39 \times 17 = 663$, evaluate each product.

a 3.9×17 **b** 39×170 **c** 39×0.17 **d** 0.39×1.7
e 3.9×1.7 **f** 390×1.7 **g** 3.9×0.17 **h** 3.9×170
i 3900×1.7 **j** 39×1.7 **k** 39×0.017 **l** 0.39×0.17

Investigation: Graphing $y = 2^x$

This activity can also be completed using a graphics calculator or graphing software.

1 Copy and complete this table of values for $y = 2^x$:

x	-3	-2	-1	0	1	2	3	4
y								

2 Graph the points from the table and join them with a smooth curve. The equation $y = 2^x$ is called an **exponential equation** and its graph is called an **exponential curve** (exponent means 'power').

3 Graph $y = 2^{-x}$ in a similar way.

4 Compare the graphs of $y = 2^x$ and $y = 2^{-x}$. Describe any similarities and differences.

5 The y-intercept of any graph with equation $y = a^x$ (where a is a positive constant) is always 1. Explain why.

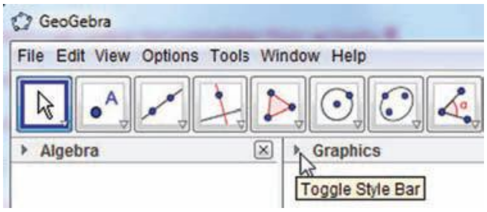
6 The graph of $y = 2^x$ is increasing. Is the graph of $y = 2^{-x}$ increasing or decreasing? Give reasons.

7 Describe what happens to the graph of $y = 2^x$ when:
   a x approaches a large positive number
   b x approaches a large negative number.

Technology Exponential curves

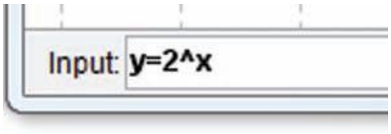
Use **GeoGebra**, **Fx-Graph** or a graphics calculator to complete this activity.
The instructions below are provided using **GeoGebra**.

1 Open up **GeoGebra** and click the little arrow in front of **Graphics**.



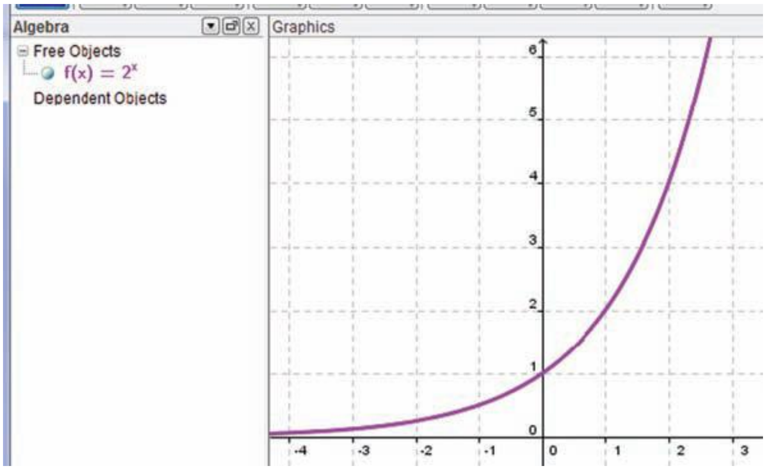
From the new panel that pops up, select the grid option at the top left-hand side.

2 Enter the function $y = 2^x$ into the **Input** bar, using '**^**'.



Input: y=2^x

Press ENTER. The colour of the graph can be changed by right-clicking on the graph and choosing **Object Properties** and **Colour**. The thickness of the curve can also be changed by clicking **Object Properties** and **Style**. The **Algebra View** shows the equation of each graph in the same colour as its graph.



3 Repeat step 2 above, by entering each of the following equations. Change the colours as required.

$y = 2^{-x}$ (enter y = 2^−x), $y = -2^x$ (enter y = −2^−x)
$y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$

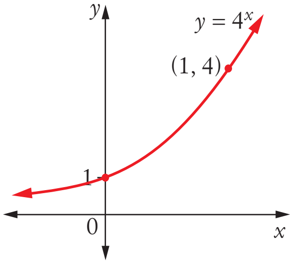
4 **a** Which graphs are similar?
b Identify any features such as y-intercepts
c Which graphs are similar as:
i x becomes larger? **ii** x becomes smaller?

5 Repeat steps **1** to **3** and answer question **4** for the exponential curves below.
$y = 3^x$, $y = 3^{-x}$, $y = -3^x$, $y = -3^{-x}$, $y = 3^x + 1$, $y = 3^x - 1$

7-05 The exponential curve

Worksheet
Graphing exponentials
MAT10NAWK10052

An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, is called an **exponential equation**, for example, $y = 5^x$, $y = 2^x$ and $y = 3^x$. The graph of a quadratic equation is a smooth curve called an **exponential curve**.
The graph of $y = 4^x$ is shown.



- The y-intercept of $y = a^x$ is 1 since $a^0 = 1$.
- As x increases (to the right in the positive direction), a^x becomes very large. Graphically, this means that the graph of $y = a^x$ increases sharply with a steep gradient.
- As x decreases (to the left in the negative direction), a^x approaches zero. This means that the graph of $y = a^x$ flattens out and approaches the x-axis as x approaches a large negative number. The x-axis is an **asymptote** because the curve approaches it but never touches it.
- The exponential curve is always above the x-axis because the value of a^x is always positive.

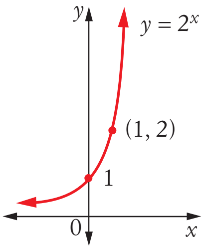
Example 10

Sketch each exponential equation and mark the y-intercept on each curve.

a $y = 2^x$ **b** $y = 3^{-x}$

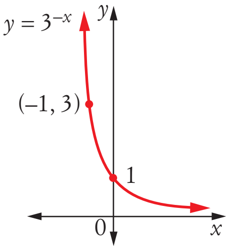
Solution

a
- The y-intercept of $y = 2^x$ is 1
- At $x = 1, y = 2$
- As x increases (to the right in the positive direction), 2^x becomes very large (steep gradient)
- As x decreases (to the left in the negative direction), 2^x approaches zero. The x-axis is an asymptote.



Stage 5.2

b
- The y-intercept of $y = 3^{-x}$ is 1
- At $x = -1, y = 3$
- As x decreases (to the left in the negative direction), 3^{-x} becomes very large (steep gradient)
- As x increases (to the right in the positive direction), 3^{-x} approaches zero. The x-axis is an asymptote.



Note that the graph of $y = 3^{-x}$ (and of $y = a^{-x}$ in general) is decreasing, and is actually a reflection of the graph of $y = 3^x$ across the y-axis.

Exercise 7-05 The exponential curve

Some of this exercise may also be completed using a graphics calculator or graphing software.

See Example 10

1 **a** Graph these exponential equations on the same axes.

i $y = 2^x$ **ii** $y = 3^x$ **iii** $y = 5^x$

b What is the y-intercept of each graph?

c Describe what happens to the graph $y = a^x$ as a increases.

Stage 5.2

2 **a** Graph $y = 4^x$ and $y = 4^{-x}$ on the same axes.

b Copy and complete:

i The reflection of $y = 4^x$ in the y-axis is …

ii The reflection of $y = a^x$ in the y-axis is …

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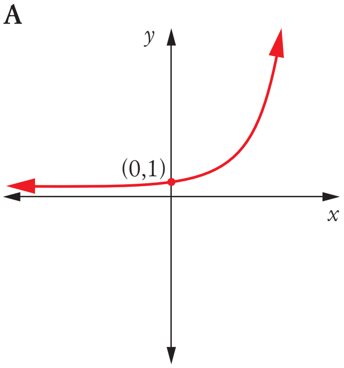
3 Which graph represents $y = 2^{-x}$? Select the correct answer **A**, **B**, **C** or **D**.

A



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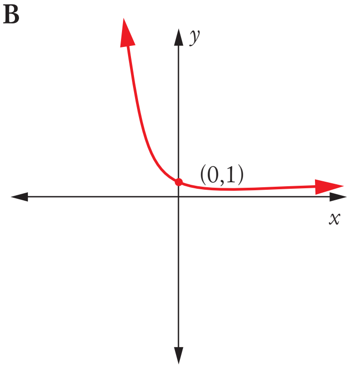

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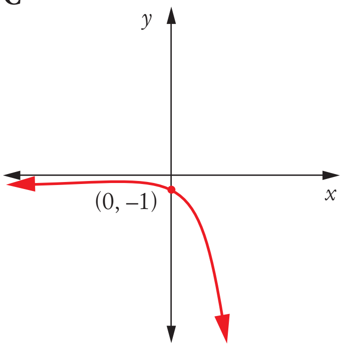

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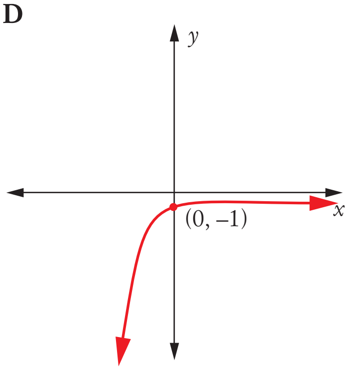


4 **a** Graph $y = 2^x$ and $y = -2^x$ on the same axes.

b How are the two graphs related?

c Copy and complete: The reflection of $y = a^x$ in the x-axis is …

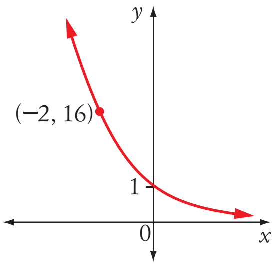
5 Graph $y = 3^x + 1$ and $y = 3^x - 1$ on the same axes and describe how they are related.

6 Sketch each exponential curve, showing the y-intercept.

a $y = 2^x$ **b** $y = 3^{-x}$ **c** $y = -4^x$

d $y = -2^{-x}$ **e** $y = 4^x + 1$ **f** $y = 4^x - 1$

7 Find an exponential equation for this graph.



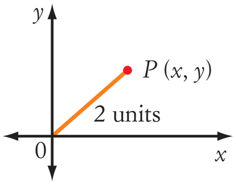
Just for the record Exponential growth

When an increase can be described using an exponential equation, it is called **exponential growth**. Examples include the growth of population (people and bacteria) and monetary investments.

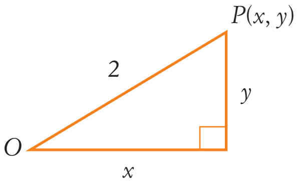
Population growth is monitored in different countries through the fertility (birth) and mortality (death) rates as well as migration. The data collected for these figures can often be modelled as an **exponential function**. By modelling the changes in population, predictions of future changes in population can be simulated and towns and cities can prepare for possible expansion in the numbers of schools, hospitals, housing and other necessary infrastructure.

At what rate is the population of Australia growing? What about the world's population?

7-06 The circle



Let $P(x, y)$ be any point on a number plane so that the distance $OP = 2$ units, where O is the origin. If we plotted every possible position of P, we would have a graph of a circle centred at O with a radius of 2. We can use Pythagoras' theorem to find the equation of this circle by drawing a right-angled triangle where OP is the hypotenuse. Since P has coordinates (x, y), the triangle must have a base length of x and a height of y, so by Pythagoras' theorem:



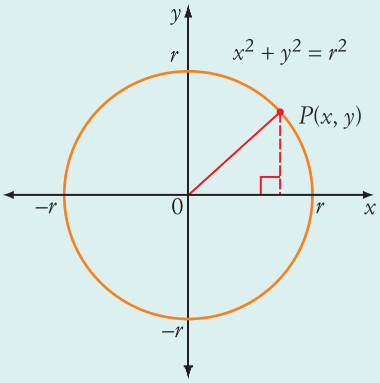
$$x^2 + y^2 = 2^2 = 4$$

∴ The equation of a circle with centre (0, 0) and radius 2 is $x^2 + y^2 = 4$.

This can be generalised for a circle of any radius.

Summary

The **equation of a circle** with centre (0, 0) and radius r units is $\boldsymbol{x^2 + y^2 = r^2}$



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Example 11

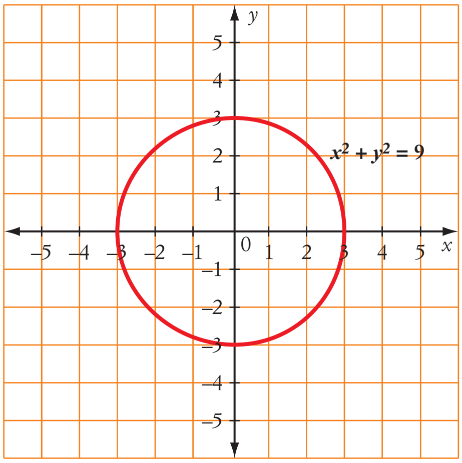
Graph the circle $x^2 + y^2 = 9$.

Solution

The centre is (0, 0). The radius is r, where $r^2 = 9$.

$$r = \sqrt{9} = 3$$

Radius = 3 units



Example 12

Find the equation of a circle with centre (0, 0) and diameter 14 units.

Solution

Radius $= \frac{1}{2} \times 14 = 7$ units.

The equation of the circle is

$x^2 + y^2 = 7^2$

$x^2 + y^2 = 49$

Exercise 7-06 The circle

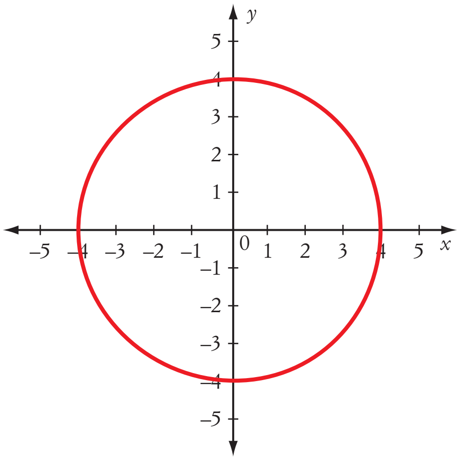
Some of this exercise may also be completed using a graphics calculator or graphing software.

See Example 11

1 Find the centre and radius of the circle given by each equation.

a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 64$
d $x^2 + y^2 = 100$ **e** $x^2 + y^2 = 81$ **f** $2x^2 + 2y^2 = 50$

2 What is the equation of the circle shown below? Select the correct answer **A**, **B**, **C** or **D**.



A $x^2 + y^2 = 2$ **B** $x^2 + y^2 = 4$ **C** $x^2 + y^2 = 8$ **D** $x^2 + y^2 = 16$

3 Which equation represents a circle with centre (0, 0) and radius 3 units? Select the correct answer **A**, **B**, **C** or **D**.

A $x^2 + y^2 = -9$ **B** $x^2 + y^2 = 3$ **C** $x^2 + y^2 = -3$ **D** $x^2 + y^2 = 9$

See Example 12

4 Find the equation of a circle with centre (0, 0) and:

a radius 1 **b** diameter 6 **c** diameter 10 **d** radius $\frac{1}{3}$

5 Graph each circle.

a $x^2 + y^2 = 16$ **b** $x^2 + y^2 = 121$ **c** $x^2 + y^2 = \frac{1}{4}$

6 Which equation represents a circle with centre (0, 0) and radius 10 units? Select **A**, **B**, **C** or **D**.

A $x^2 + y^2 = 10$ **B** $2x^2 + 2y^2 = 20$ **C** $3x^2 + 3y^2 = 300$ **D** $4x^2 + 4y^2 = 14$

7 **a** Show that the point (8, 6) lies on the circle $x^2 + y^2 = 100$.

b Show that the point (5, 9) does not lie on the circle $x^2 + y^2 = 100$.

c Does (5, 9) lie inside or outside this circle?

8 Given the equation of the circle $x^2 + y^2 = 4$, substitute each of the following points into the equation and determine whether the points are inside, on or outside the circle:

a (0, 0) **b** (2, 0) **c** (3, 1) **d** (1, 1) **e** (−5, 2)

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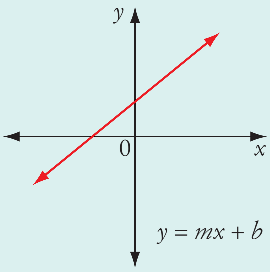
7-07 Identifying graphs

Stage 5.2

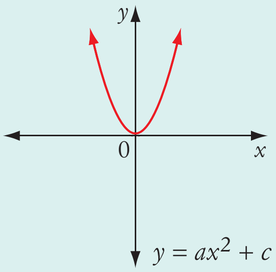
Puzzle sheet
Matching graphs
MAT10NAPS10053

Summary

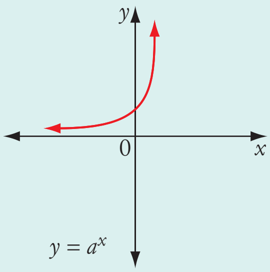
Straight line (linear)



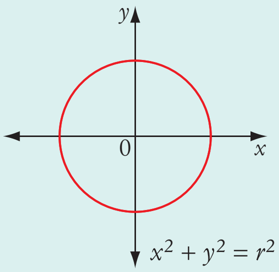
Parabola (quadratic)



Exponential



Circle



When matching graphs with equations, the *x*-value and *y*-value of a point on the graph may need to be substituted into the equation to verify that the equation represents the graph.

Example 13

State whether each equation represents a straight line, a parabola, an exponential curve or a circle.

a $y = 3x^2 - 1$ **b** $y = 5x + 7$ **c** $y = -\frac{1}{2}x^2 - 10$ **d** $x^2 + y^2 = 4$ **e** $y = 3^x$

Solution

a $y = 3x^2 - 1$ is a parabola because it is of the form $y = ax^2 + c$.

b $y = 5x + 7$ is a straight line because it is of the form $y = mx + b$.

c $y = -\frac{1}{2}x^2 - 10$ is a parabola because it is of the form $y = ax^2 + c$.

d $x^2 + y^2 = 4$ is a circle because it is of the form $x^2 + y^2 = r^2$.

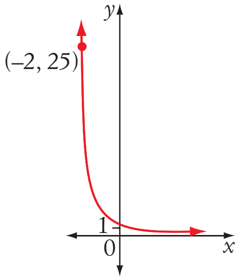
e $y = 3^x$ is an exponential because it is of the form $y = a^x$.

Stage 5.2

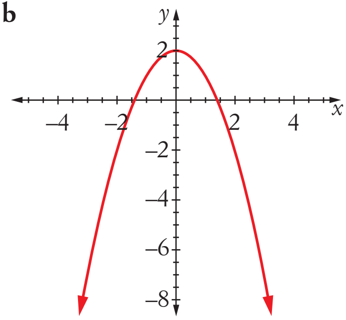
Example 14

Match each graph with its equation.

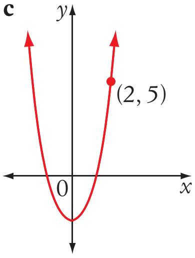
a



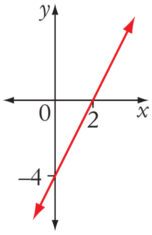
b



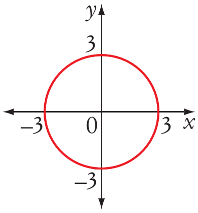
c



d



e



i $y = 2x - 4$ **ii** $x^2 + y^2 = 9$ **iii** $y = 2x^2 - 3$ **iv** $y = 5^{-x}$ **v** $y = -x^2 + 2$

Solution

a This is an exponential curve. The only possible match is **iv**, $y = 5^{-x}$

Test point: (–2, 25)
LHS = 25
RHS $= 5^{-(-2)} = 5^2 = 25 =$ LHS

b This is a concave down parabola with a y-intercept of 2. The only possible match is **v**, $y = -x^2 + 2$

c This is a concave up parabola that matches with **iii**, $y = 2x^2 - 3$

Test point: (2, 5)
LHS = 5
RHS $= 2 \times 2^2 - 3 = 5 =$ LHS

d This is a straight line with a y-intercept of -4 that matches with **i**, $y = 2x - 4$

e This is a circle with centre (0, 0) and radius 3. The only possible match is **ii**, $x^2 + y^2 = 9$

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Exercise 7-07 Identifying graphs

Stage 5.2

See Example 13

1 For each of the following equations, state whether its graph is a straight line (L), a parabola (P), an exponential (E) or a circle (C).

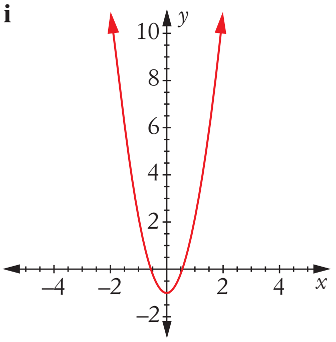
a $y = 9x^2 - 4$ **b** $y = 9x$ **c** $y = 9^x$ **d** $y = 9$
e $x^2 + y^2 = 81$ **f** $y = 3x - 8$ **g** $y = 3x^2 - 8$ **h** $y = 2x + 5$
i $y = -x^2 + 6$ **j** $y = 10^{-x}$ **k** $y = 7x^2 + 2$ **l** $x^2 + y^2 = 36$

See Example 14

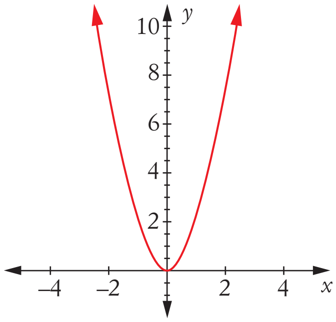
2 Match each equation with its graph.

a $x = 4$ **b** $y = -\frac{1}{2}x + 1$ **c** $y = 1 - x^2$ **d** $y = 5$ **e** $y = 3x^2 - 1$
f $y = 3^x$ **g** $x^2 + y^2 = 9$ **h** $y = 3^{-x}$ **i** $y = 2x^2$ **j** $y = 9x^2 - 4$

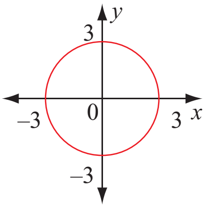
i



ii



iii



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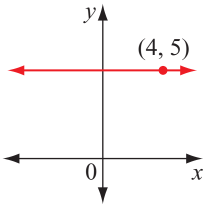

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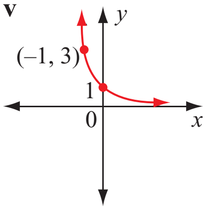

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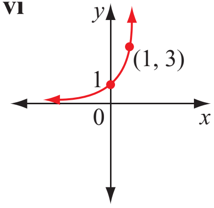

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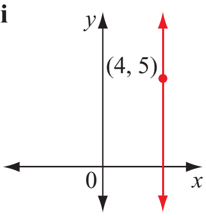
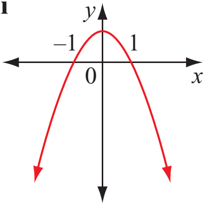


viii



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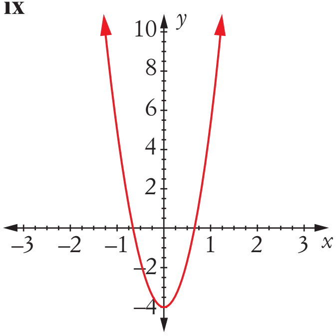
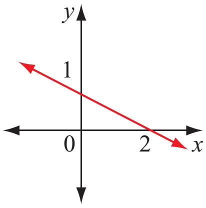


x



3 Sketch each equation. Show a point on the curve each time.

a $y = x^2 - 3$ **b** $y = 5^x$ **c** $y = -x^2 + 4$ **d** $x^2 + y^2 = 49$
e $y = \frac{1}{2}x^2$ **f** $y = -2x + 4$ **g** $x^2 + y^2 = 144$

4 Find the y-intercept of the graph of each equation.

a $y = 3^x$ **b** $y = 2x^2 + 3$ **c** $y = -7x^2 - 6$ **d** $y = 5^{-x}$

Stage 5.2

Technology Identifying graphs

1 Use a graphics calculator, Fx-Graph or GeoGebra to draw the following equations and classify each as either a straight line (L), a parabola (P), an exponential (E) or a circle (C).

a $y = 2x$ **b** $y = x^2$ **c** $y = x^2 + 1$ **d** $y = 2^x$ **e** $y = 2x^2 + 3$
f $y = 4 - 2x$ **g** $y = 3^x$ **h** $y = 2 - x$ **i** $y = 4 - x^2$ **j** $y = 5^{-x}$

2 Without using graphing technology, classify each equation.

a $y = 3x - 2$ **b** $y = x^2 + 3$ **c** $y = 2^x + 1$ **d** $y = 3 - x^2$ **e** $y = 4x^2 - 1$
f $y = 3^x - 2$ **g** $y = 4^x - 1$ **h** $y = 3x^2 - 4$ **i** $y = 10 - 2x^2$ **j** $y = -2x^2$

3 Check your answers to question **2** by drawing each equation using graphing technology.

4 State briefly in words how you distinguish between each type of equation in question **2**.

5 Use TRACE to find the x-intercepts and y-intercepts (if they exist) of the graphs in questions **1** and **3**. Provide approximate answers where necessary.

Power plus

1 On the same set of axes, draw the graphs of these equations.

a $y = x^2$ **b** $y = (x + 1)^2$ **c** $y = (x - 2)^2$

2 On the same set of axes, draw the graphs of these equations.

a $y = -x^2$ **b** $y = -(x - 3)^2$ **c** $y = -(x + 2)^2$

3 For the graph of the parabola $y = (x + a)^2$, describe the effect on the graph of different values of a.

4 Sketch the graph of each equation and find the centre and radius of the semicircle.

a $y = \sqrt{16 - x^2}$ **b** $y = \sqrt{25 - x^2}$ **c** $y = -\sqrt{9 - x^2}$

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Chapter 7 review

Puzzle sheet

Graphs crossword

MAT10NAPS10054

Language of maths

asymptote
axis
centre
circle
coefficient
concave down
concave up
constant
conversion graph
curve
direct proportion
exponential
inverse proportion
parabola
quadratic
radius
table of values
turning point
variable
vertex
x-intercept
y-intercept

1 What is the coefficient of x^2 in the quadratic equation $y = 3x^2 + 10$?

2 What is the graph of a **quadratic equation** called?

3 True or false?: The exponential curve $y = 2^x$ passes through the point (0, 0).

4 In the variation equation $y = \frac{k}{x}$, which is the **constant of proportionality**?

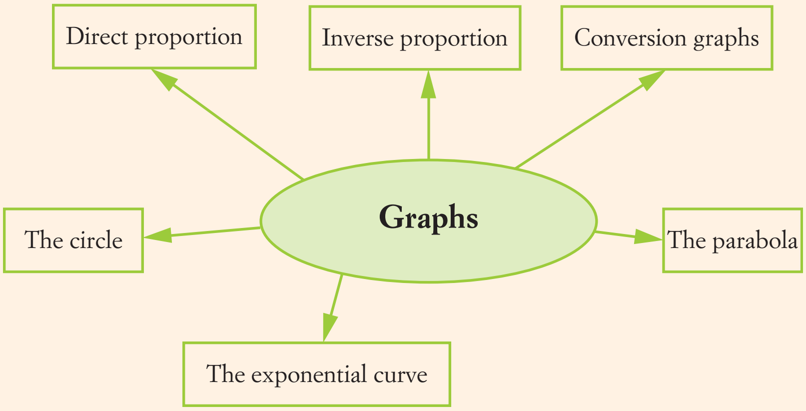
5 Write down the equation of a parabola that is concave down and has a y-intercept of 3.

6 What is the asymptote of the exponential curve $y = a^x$?

Topic overview

- Which parts of this chapter were revision of Year 9 knowledge and skills?
- Which parts of this chapter were new to you?
- What is the difference between direct and inverse proportion?
- Do you know the equations of a parabola, exponential curve and circle, and how to graph them?
- Explain how the graph of $y = 2x^2 + 3$ is different to the graph of $y = -2x^2 + 3$. How are they similar?

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 7 revision

Stage 5.2

See Exercise 7-01

1 H is directly proportional to t. If when $t = 12$, $H = 138$, find H when $t = 27$.

See Exercise 7-02

2 The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 400 m above sea level, the temperature is 15°C. What is the temperature at 600 m above sea level?

See Exercise 7-03

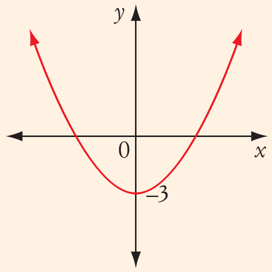
3 The graph in Example **6** on page 229 converts Australian dollars to UK pounds sterling. Use the graph to convert:

a \$A70 to £ **b** £56 to Australian dollars

See Exercise 7-04

4 Match each graph with its correct equation.

a



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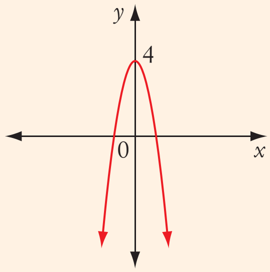

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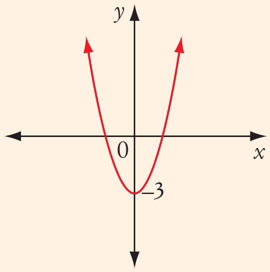

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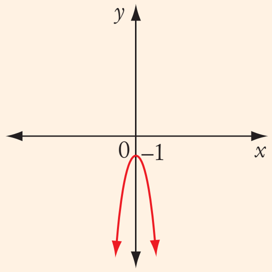

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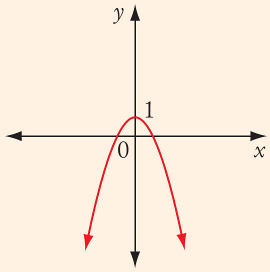

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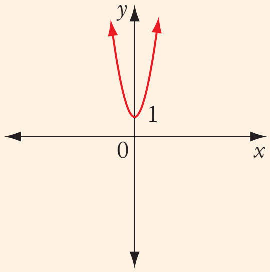


i $y = x^2 - 3$ **ii** $y = 3x^2 + 1$ **iii** $y = \frac{1}{2}x^2 - 3$

iv $y = -x^2 + 1$ **v** $y = -4x^2 - 1$ **vi** $y = 4 - 3x^2$

See Exercise 7-05

5 Sketch each curve described below.

a $y = 4^x$ **b** $y = 4^{-x}$ **c** $y = -4^x$ **d** $y = -4^{-x}$

See Exercise 7-06

6 Find the centre and radius of each circle described below.

a $x^2 + y^2 = 100$ **b** $x^2 + y^2 = 36$ **c** $x^2 + y^2 = 49$

7 What is the equation of the circle with centre (0, 0) and radius 8 units?

See Exercise 7-06

Stage 5.2

See Exercise 7-07

8 Match each equation with its correct graph.

a $y = \frac{1}{4}x^2$

b $y = 3^x$

c $y = -2x^2 - 1$

d $x = -5$

e $y = -3x^2$

f $y = 2x^2 - 1$

g $y = 3^{-x}$

h $x^2 + y^2 = 25$

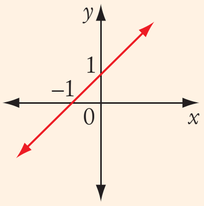
i $y = x^2$

j $y = x + 1$

k $y = -5$

l $y = -2 - 2x$

i



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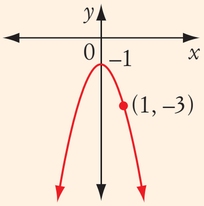

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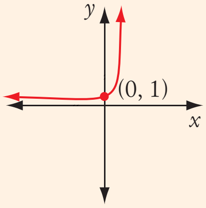

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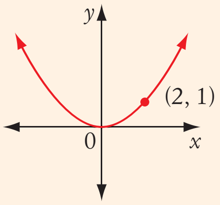

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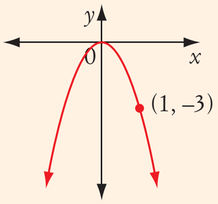

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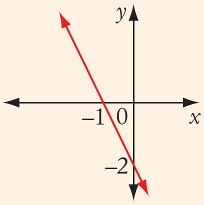

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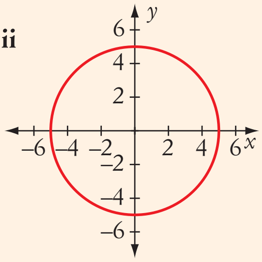

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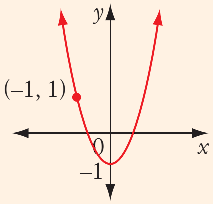
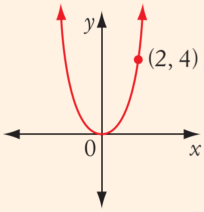
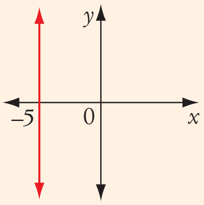


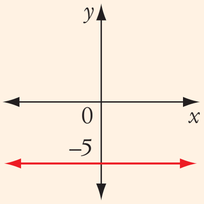
ix



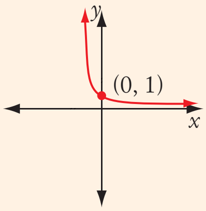
x



xi



xii



8

Measurement and Geometry

Trigonometry

The word **trigonometry** comes from the Greek language: *trigonon*, meaning triangle, and *metron*, meaning measure. Trigonometry uses triangles to find unknown lengths and angles that cannot be measured physically. It has wide applications in engineering, surveying, navigation, astronomy, electronics and construction.

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Chapter outline

	Proficiency strands				
8-01 Pythagoras' theorem	U	F	PS	R	C
8-02 The trigonometric ratios	U	F			C
8-03 Finding an unknown side	U	F	PS		
8-04 Finding an unknown angle	U	F	PS		
8-05 Angles of elevation and depression	U	F	PS	R	C
8-06 Bearings*	U	F		R	C
8-07 Problems involving bearings*		F	PS	R	C

Wordbank

adjacent side	In a right-angled triangle, the side that is next to a given angle and pointing to the right angle
angle of depression	The angle of looking **down**, measured from the horizontal
angle of elevation	The angle of looking **up**, measured from the horizontal
bearing	The angle used to show the direction of one location from a given point
minute ($'$)	A unit for measuring angle size, $\frac{1}{60}$ of a degree
opposite side	In a right-angled triangle, the side that is facing a given angle and not one of its arms
theta (θ)	A letter of the Greek alphabet used as a pronumeral for angles
trigonometric ratio	The ratio of two sides in a right-angled triangle; for example, sine is the ratio of the opposite side to the hypotenuse

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In this chapter you will:

- investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles
- apply trigonometry to solve right-angled triangle problems AC
- find unknown sides and angles in right-angled triangles, where the angle is measured in degrees
- (STAGE 5.2) find unknown sides and angles in right-angled triangles where the angle is measured in degrees and minutes
- apply trigonometry to problems involving angles of elevation and depression
- (STAGE 5.2) apply trigonometry to problems involving bearings

SkillCheck

Worksheet
StartUp assignment 8
MAT10MGWK10055

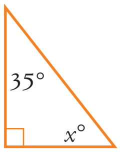
Worksheet
Trigonometric calculations
MAT10MGWK10056

1 Solve each equation.

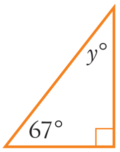
a $\frac{x}{5} = 7$ **b** $\frac{h}{4} = 8.3$ **c** $\frac{45}{y} = 9$

2 Find the value of each pronumeral.

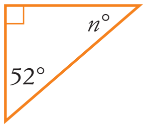
a



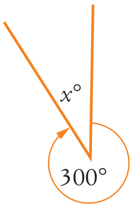
b



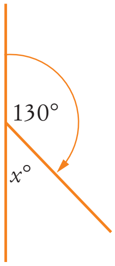
c



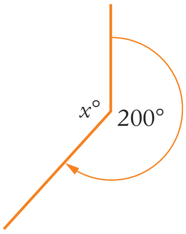
d



e

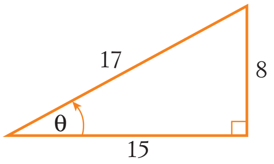


f



3 For each triangle, name the hypotenuse, opposite and adjacent sides for angle θ.

a



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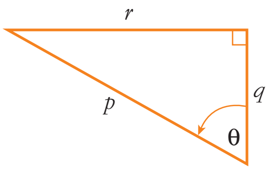

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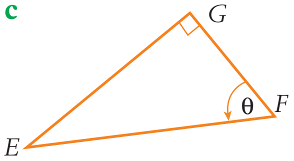


Stage 5.2

4 Round each angle to the nearest degree.

a 64° 27′ **b** 25° 43′ **c** 12° 8′ 50″

5 Round each angle to the nearest minute.

a 50° 19′ 26″ **b** 31° 55′ 55″ **c** 64° 18′ 30″

6 Evaluate each expression correct to four decimal places.

a cos 32° **b** sin 50.9° **c** tan 8° 45′

d 200 tan 18° **e** 14 sin 87° 40′ **f** $\frac{13}{\cos 18°27'}$

7 Convert each angle to degrees and minutes, correct to the nearest minute

a 45.8° **b** 33.175° **c** 5.346°

Stage 5.2

8-01 Pythagoras' theorem

Puzzle sheet
Pythagorean two-step problems
MAT10MGPS00031

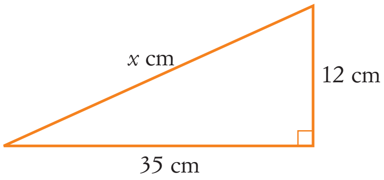
Puzzle sheet
Pythagorean triads
MAT10MGPS00030

Pythagoras' theorem can be used to find the length of an unknown side in a right-angled triangle, or to prove that a triangle is right-angled.

Example 1

Find the value of each pronumeral, correct to one decimal place where necessary.

a



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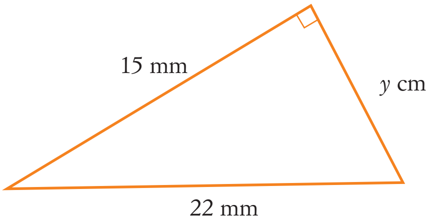


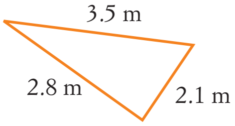
Solution

a
$$\begin{aligned} x^2 &= 12^2 + 35^2 \\ &= 1369 \\ x &= \sqrt{1369} \\ &= 37 \end{aligned}$$

b
$$\begin{aligned} 22^2 &= y^2 + 15^2 \\ 484 &= y^2 + 225 \\ y^2 + 225 &= 484 \\ y^2 &= 484 - 225 \\ &= 259 \\ y &= \sqrt{259} \\ &\approx 16.1 \end{aligned}$$

Example 2

Test whether this triangle is right-angled.



Solution

$$\begin{aligned} 3.5^2 &= 12.25 \\ 2.8^2 + 2.1^2 &= 12.25 \\ \therefore 3.5^2 &= 2.8^2 + 2.1^2 \end{aligned}$$

∴ The triangle is right-angled (the right angle is opposite the 3.5 m side).

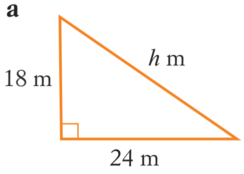
9780170194655

Exercise 8-01 Pythagoras' theorem

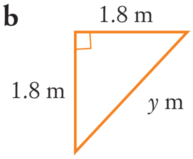
See Example 1

1 Find the value of each pronumeral, correct to one decimal place where necessary.

a



b



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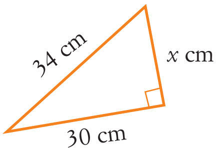

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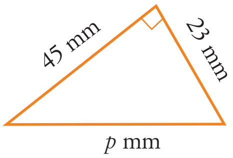
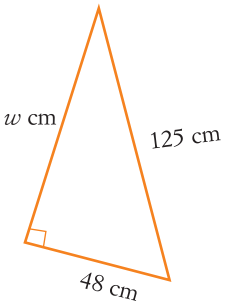


e



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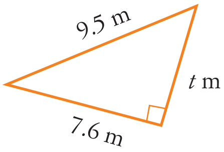
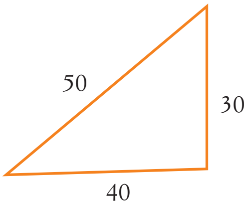


See Example 2

2 Test whether each triangle is right-angled.

a



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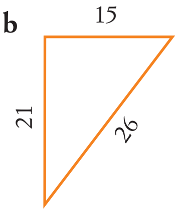

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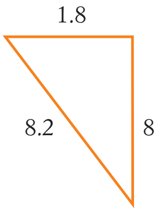
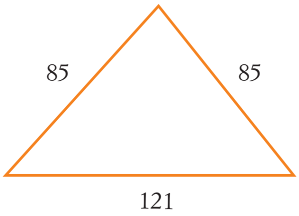


d



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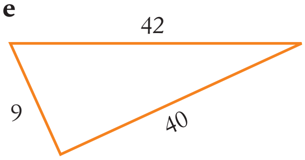

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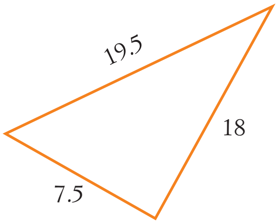
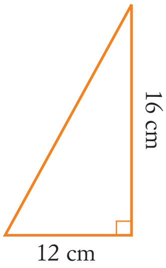


3 Find the perimeter of each shape, correct to one decimal place where necessary.

a



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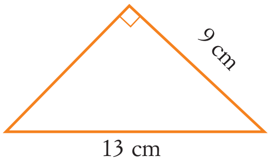

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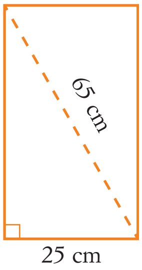

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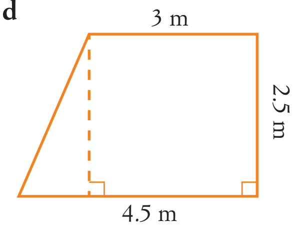

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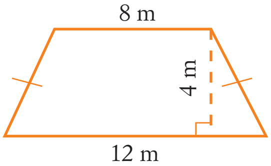

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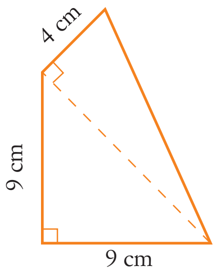
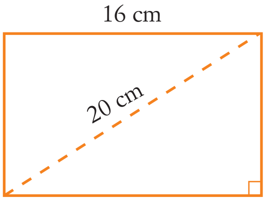


4 Find the area of each shape, correct to one decimal place where necessary.

a



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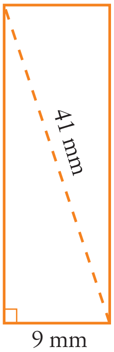

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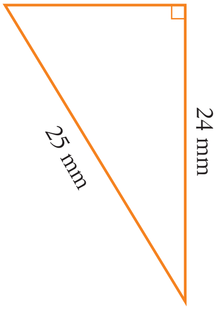

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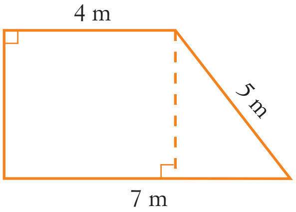

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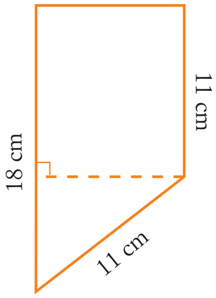
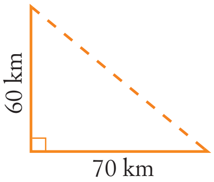
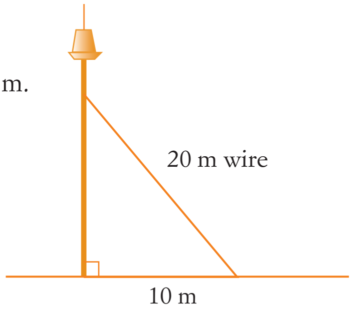


5 A ship sails 60 km south and then 70 km east. How far is it from its starting point, correct to one decimal place?



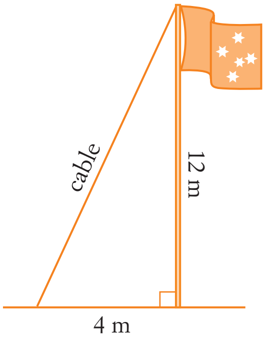
6 A tower is supported by a wire that is 20 m long and attached to the ground 10 m from the base of the tower. How high does the wire reach up the tower? Answer correct to the nearest 0.1 m.



7 A park is in the shape of a rectangle with sides of 96 m and 72 m. Find the shortest distance across the park from one corner to the other.

8 What is the length of the cable used to stabilise a flagpole that is 12 metres high, if the cable is secured to the ground 4 m from the base of the flagpole? Select the correct answer **A**, **B**, **C** or **D**.

A 11.3 m
B 12.6 m
C 16 m
D 80 m



9 A triathlon course consists of three legs forming a right-angled triangle. If the longest leg is 11.5 km and the shortest leg is 6.9 km, find the length of the other leg.

Just for the record The Greek alpha-bet

Here are eight letters (in lower-case and capitals) from the Greek alphabet.

α, A alpha	β, B beta	γ, Γ gamma	δ, Δ delta
θ, Θ theta	ϕ, Φ phi	σ, Σ sigma	ω, Ω omega

The ancient Greeks had a great influence on the development of mathematics. It is traditional to use Greek letters as variables, particularly in geometry and trigonometry.

1 Find out how many letters there are in the Greek alphabet, and name each one.
2 Compare the Greek alphabet with our Roman alphabet.
3 Can you see where the word *alphabet* comes from? Explain how it originated.

8-02 The trigonometric ratios

Worksheet
Investigating trigonometry ratios
MAT10MGWK00027

There are three special fractions called **trigonometric ratios** that relate the lengths of two sides of a right-angled triangle: **sine**, **cosine** and **tangent**.

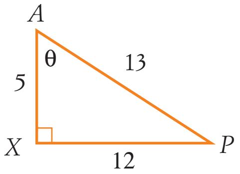
Summary

The trigonometric ratios

Ratio	Abbreviation	Meaning
Sine	sin	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
Cosine	cos	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
Tangent	tan	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

Example 3

In $\triangle AXP$, find $\sin\theta$, $\cos\theta$ and $\tan\theta$.



Solution

For angle θ, opposite = 12, adjacent = 5, hypotenuse = 13.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{5}$$

Mnemonics for sin, cos and tan

A useful mnemonic (memory aid) for remembering the three ratios is to look at the initials of the words in the ratios.

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{O}}{\text{H}} \quad \text{S.O.H.} \qquad \cos = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{A}}{\text{H}} \quad \text{C.A.H.}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{O}}{\text{A}} \quad \text{T.O.A.}$$

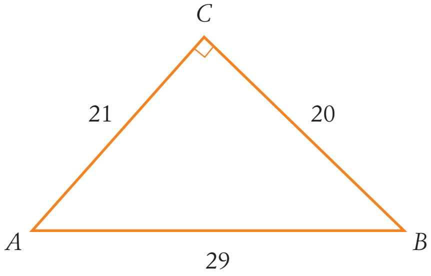
If you remember **SOH-CAH-TOA** (pronounced ‘so-car-toe-ah’), then you can remember the ratios for sin, cos and tan. Some students also learn a phrase where the first letter of each word follows the SOH-CAH-TOA sequence; for example, ‘**S**un **O**ver **H**ead **C**aused **A** **H**uge **T**an **O**n **A**rms’. Find your own mnemonic to remember this.

Example 4

For the triangle below, find:

a $\sin A$ **b** $\cos B$

c $\tan A$ **d** $\sin B$



Solution

For angle A, opposite = 20, adjacent = 21, hypotenuse = 29

For angle B, opposite = 21 adjacent = 20, hypotenuse = 29

a $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{20}{29}$

b $\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{20}{29}$

c $\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{20}{21}$

d $\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{21}{29}$

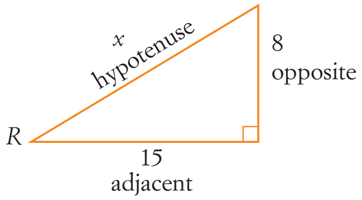
Given one ratio, finding another ratio

Example 5

If $\tan R = \frac{8}{15}$, find the value of $\sin R$ and $\cos R$.

Solution

$\tan R = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$, so draw a right-angled triangle that has an angle R with opposite side 8 and adjacent side 15. Let x be the length of the hypotenuse.
Find x using Pythagoras' theorem.

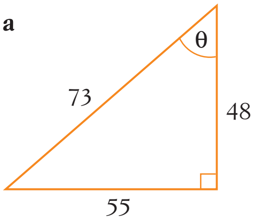


$$
\begin{aligned}
x^2 &= 8^2 + 15^2 \\
&= 289 \\
x &= \sqrt{289} \\
&= 17 \\
\therefore \sin R &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17} \\
\cos R &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17}
\end{aligned}
$$

Exercise 8-02 The trigonometric ratios

See Example 3 **1** For each marked angle, find the sine, cosine and tangent ratios.

a



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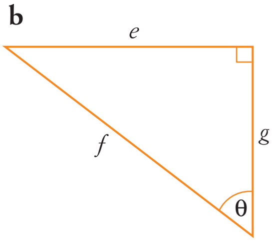

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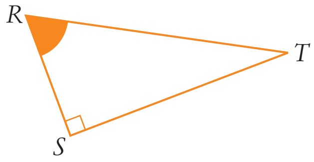

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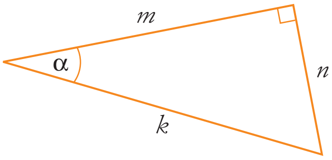

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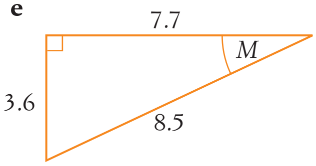

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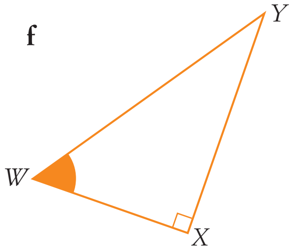
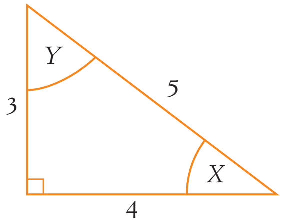
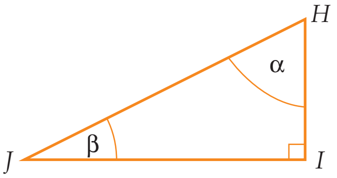


See Example 4 **2** For the triangle below, find:

a $\cos X$ **b** $\tan Y$
c $\sin X$ **d** $\sin Y$



3 Complete each statement below with the correct angle (α or β).



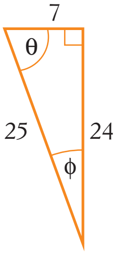
a $\sin \square = \frac{IJ}{HJ}$ **b** $\sin \square = \frac{HI}{HJ}$ **c** $\cos \square = \frac{IJ}{HJ}$

d $\cos \square = \frac{HI}{HJ}$ **e** $\tan \square = \frac{IJ}{IH}$ **f** $\tan \square = \frac{IH}{IJ}$

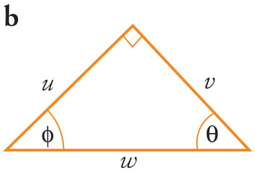
4 For each triangle below, find:

i $\tan \theta$ **ii** $\cos \theta$ **iii** $\cos \phi$ **iv** $\tan \phi$ ← ϕ is the Greek letter 'phi'

a



b



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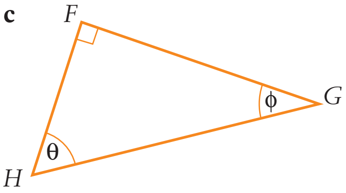
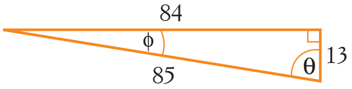
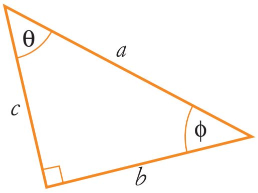


d



e



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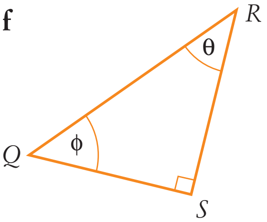
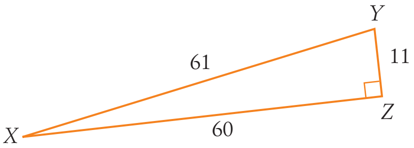


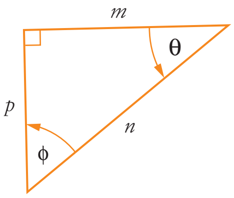
5 For each fraction, write a correct trigonometric ratio involving X or Y in the triangle.



a $\frac{60}{11}$ **b** $\frac{11}{60}$

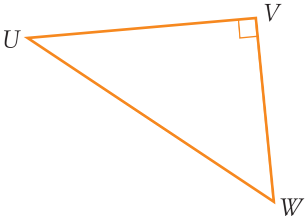
c $\frac{11}{61}$ **d** $\frac{60}{61}$

6 Which ratio is equal to $\frac{p}{m}$? Select the correct answer **A**, **B**, **C** or **D**.



A $\cos \theta$ **B** $\cos \phi$
C $\tan \theta$ **D** $\tan \phi$

7 Which statement is true for this triangle?
Select the correct answer **A**, **B**, **C** or **D**.



A $\sin U = \cos W$ **B** $\tan U = \sin W$
C $\cos U = \tan W$ **D** $\tan U = \tan W$

8 Sketch a right-angled triangle for each trigonometric ratio, then use Pythagoras' theorem to find the length of the unknown side and the other two trigonometric ratios for the same angle.

See Example 5

a $\tan A = \frac{5}{12}$ **b** $\sin B = \frac{3}{5}$ **c** $\cos X = \frac{9}{41}$ **d** $\sin Y = \frac{7}{25}$

8-03 Finding an unknown side

Puzzle sheet
Trigonometry equations
MAT10MGPS00032

We can use a trigonometric ratio to calculate the length of an unknown side in a right-angled triangle if one other side and angle are known. We need to select the correct ratio that links the given angle to the unknown side and known side.

Summary

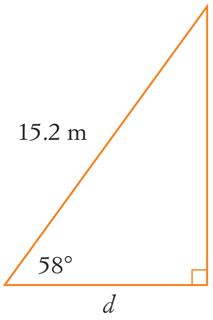
Finding an unknown side in a right-angled triangle

1 Identify the two labelled sides and decide whether to use sin, cos or tan.
2 Write an equation using the ratio, the given angle and the variable.
3 Solve the equation to find the value of the variable.

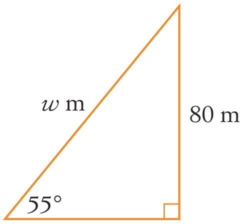
Example 6

Find the value of each pronumeral, correct to two decimal places.

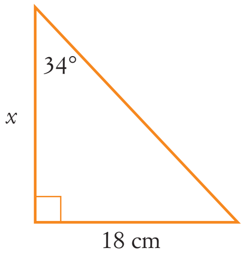
a



b



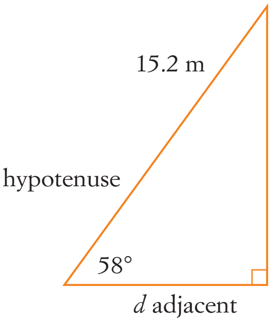
c



Solution

a SOH, CAH or TOA?

The marked sides are the adjacent (A) side and the hypotenuse (H), so use cos.



$$\begin{aligned}\cos 58^\circ &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ &= \frac{d}{15.2}\\ \cos 58^\circ \times 15.2 &= \frac{d}{15.2} \times 15.2 \\ 15.2\ \cos 58^\circ &= d \\ d &= 15.2\ \cos 58^\circ \\ &= 8.05477\ldots \\ &\approx 8.05\end{aligned}$$

Multiplying both sides by 15.2.

Swapping sides.

From the diagram, a length of 8.05 m looks reasonable.

9780170194655

b 80 m is opposite, w m is hypotenuse, so use sin.

$$\sin 55^\circ = \frac{80}{w}$$

Note that the variable w appears in the denominator of the equation.

$$\sin 55^\circ \times w = \frac{80}{w} \times w$$

Multiplying both sides by w.

$$w \ \sin 55^\circ = 80$$

$$\frac{w \ \sin 55^\circ}{\sin 55^\circ} = \frac{80}{\sin 55^\circ}$$

Dividing both sides by sin 55°.

$$\begin{aligned} w &= \frac{80}{\sin 55^\circ} \\ &= 97.66196\ldots \\ &\approx 97.66 \end{aligned}$$

Note that when the unknown appears in the denominator of an equation, it can swap positions with the trigonometric ratio, so that

$\sin 55^\circ = \dfrac{80}{w}$ becomes $w = \dfrac{80}{\sin 55^\circ}$.

c 18 cm is opposite, x is adjacent, so use tan.

$$\tan 34^\circ = \frac{18}{x}$$

x appears in the denominator

$$\begin{aligned} x &= \frac{18}{\tan 34^\circ} \\ &= 26.6860\ldots \\ &\approx 26.69 \end{aligned}$$

Swapping the position of x with tan 34°.

Alternative method

To avoid having x in the denominator, we could use tan with the third angle of the triangle.

$$\begin{aligned} \text{Third angle} &= 180^\circ - 90^\circ - 34^\circ \\ &= 56^\circ \end{aligned}$$

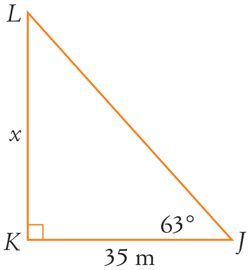
$$\tan 56^\circ = \frac{x}{18}$$

$$\begin{aligned} x &= 18 \tan 56^\circ \\ &= 26.6860\ldots \\ &\approx 26.69 \end{aligned}$$

Example 7

$\triangle JKL$ is right-angled at K, $JK = 35$ m and $\angle J = 63°$. Find the length of LK correct to the nearest metre.

Solution



Draw a diagram.
Let the length of LK be x.
x is opposite, 35 m is adjacent, so use tan.

$$\tan 63^\circ = \frac{x}{35}$$

$$x = 35\ \tan 63^\circ$$

$$= 68.6913\ldots$$

$$\therefore LK \approx 69 \text{ m}$$

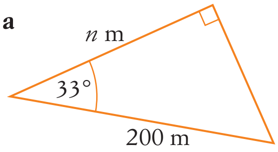
From the diagram, a length of 69 m looks reasonable.

Exercise 8-03 Finding an unknown side

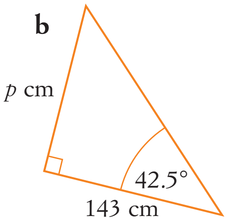
See Example 6

1 Find the value of the pronumeral in each triangle, correct to one decimal place.

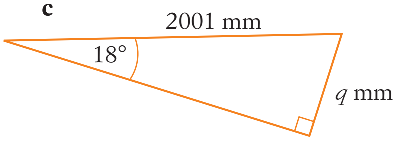
a



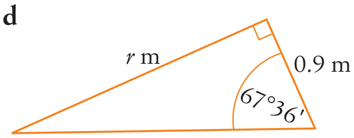
b



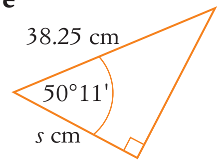
c



d



e



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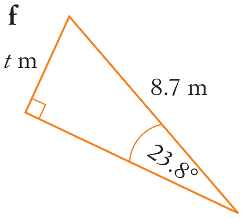
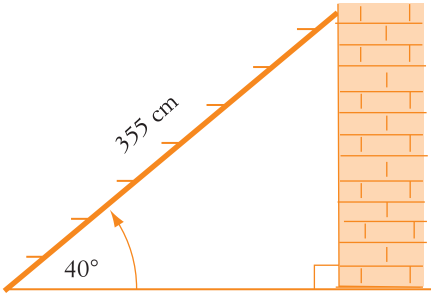
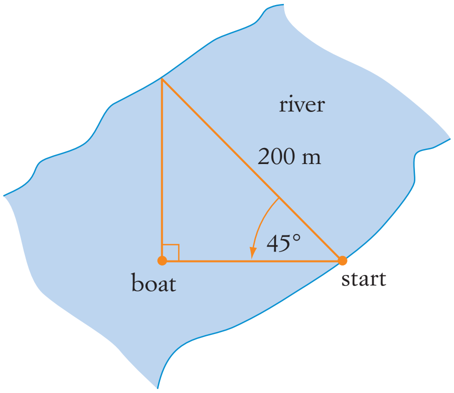


2 Find each length or distance, correct to one decimal place.

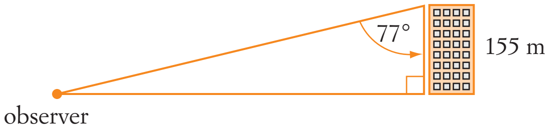
a How high the stairs go up the wall.



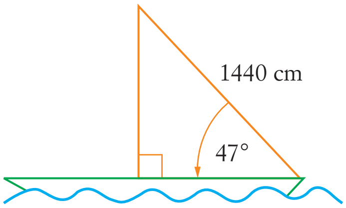
b The distance between the boat and the start.



c The distance from the observer to the base of the building.

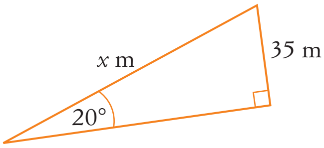


d The height of the boat's mast.

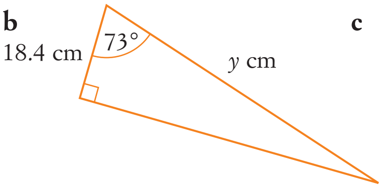


3 Find the value of each pronumeral, correct to one decimal place.

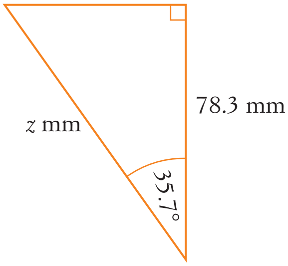
a



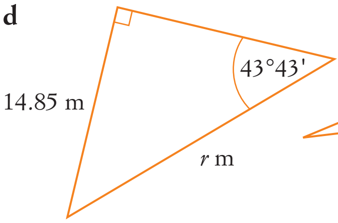
b



c



d



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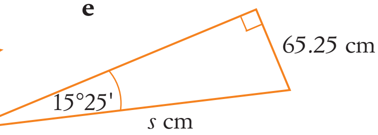

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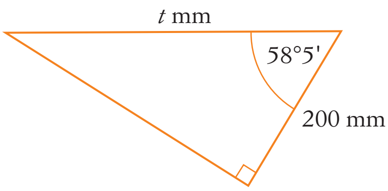
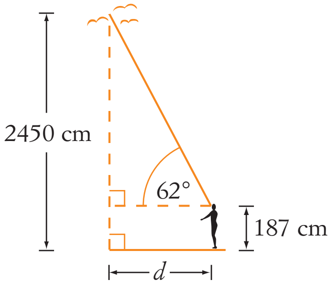
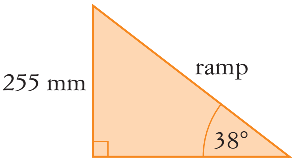


4 Find each length or distance, correct to one decimal place.

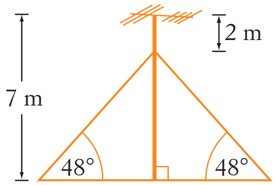
a How far the person is from being directly under the birds.



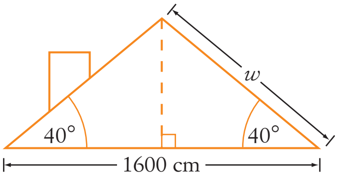
b The length of the ramp.



c The length of the support wire.



d The slant height of the roof.



5 A rectangular gate has a diagonal brace that makes an angle of 60° with the bottom of the gate. The length of the diagonal brace is 1860 mm. Calculate the height of the gate. Select the correct answer **A**, **B**, **C** or **D**.

A 2148 mm **B** 930 mm **C** 1610 mm **D** 3221 mm

6 A ladder rests against a wall. The foot of the ladder is 355 cm from the wall and makes an angle of 63° with the ground. How long (correct to the nearest cm) is the ladder?

7 A glider is flying at an altitude (height) of 1.5 km. To land, it descends at an angle of 18° to the horizontal. How far must the glider travel before landing? (Give your answer correct to the nearest 0.1 km.)

Shutterstock.com/Margo Harrison

8 A supporting wire is attached to the top of a flagpole. The wire meets the ground at an angle of 51° and the flagpole is 15 m high. How far from the base of the flagpole is the wire anchored to the ground? (Give your answer correct to the nearest 0.1 m.)

Alamy/John Simmons

9 A boat is anchored by a rope 5.5 m long. If the anchor rope makes an angle of 23° with the vertical, calculate the depth of the water (correct to one decimal place).

10 A hot air balloon is anchored to the ground by a rope. When it drifts 20 m sideways, it makes an angle of 75° with the ground. How long is the rope (correct to one decimal place)?

iStockphoto/Pinnacle Marketing

11 $\triangle ABC$ is right-angled at B, $AC = 14.8$ m and $\angle C = 56°$. Find the length of side AB, correct to one decimal place.

See Example 7

12 $\triangle XYZ$ is right-angled at Z, $ZY = 230$ mm and $\angle Y = 45°$. Find the length of XY, correct to the nearest millimetre.

13 In $\triangle HMT$, $\angle T = 90°$, $\angle M = 19° 47'$ and side $HT = 18.4$ cm. Find the length of side HM, correct to one decimal place

14 $\triangle MNR$ is right-angled at M, $MN = 19$ cm and $\angle N = 27°$. Find the length of MR, correct to the nearest centimetre.

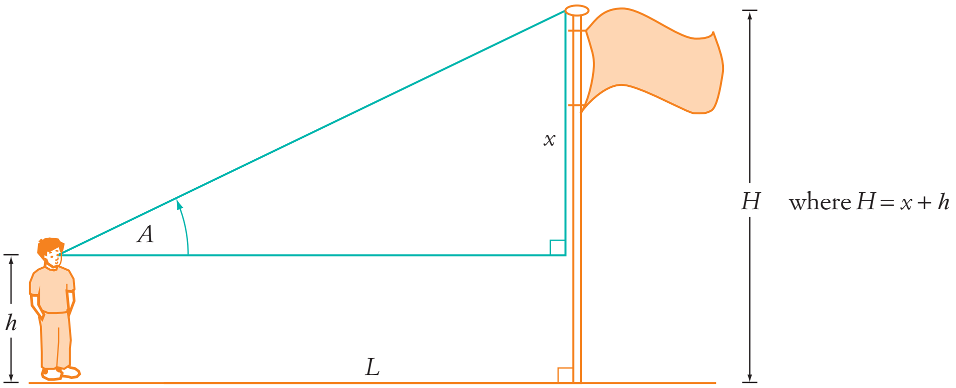
15 In $\triangle KLW$, $\angle L = 90°$, $KL = 12$ m and $\angle W = 75.2°$. Find KW, correct to one decimal place.

16 $\triangle CDE$ is right-angled at D, $\angle E = 36°$ and $CD = 5$ m. Find the length of side DE, correct to two decimal places.

Investigation: Calculating the height of an object

You will need: tape measure or trundle wheel, and a clinometer (or protractor) to measure the angle.

Trigonometry can be used to find the heights of buildings, flagpoles and trees without actually measuring them. This can be done by measuring the distance along the ground from the base of the object to a person who measures the angle to the top of the object. For example, the height of a flagpole can be calculated using the set-up shown in the diagram below.



h is the eye height of the person who measures the angle, A, to the top of the flagpole. L is the distance the person is from the base of the flagpole, x is the height of the flagpole above the person's eye, $H = x + h$ is the height of the flagpole above the ground.

1. Select a tall object outside to measure.
2. Work with a partner to measure (in cm) the distance, L, along the ground, the height, h, of the person, and the angle (in degrees) to the top of the object. Copy this table and record your information in the first row.

Distance, L (cm)	Angle, $A°$	Height of person, h (cm)	Calculated height, x cm	Height of flagpole, H cm

3. Use the tan ratio to calculate the value of x, correct to the nearest whole number.
4. Hence find H, the height of the kite to the nearest centimetre. Write your answers in the table.
5. Repeat the measurements and calculations three more times from different positions, with different people measuring the angle. This will help improve the accuracy of our results and minimise errors. Write your results in the table.
6. Did you find similar values for H? Do they seem reasonable for the height of the object?
7. Calculate the average value for H.

8-04 Finding an unknown angle

Puzzle sheet
Finding an unknown angle
MAT10MGPS00033

Worksheet
Trigonometry review
MAT10MGWK10057

Puzzle sheet
Solving triangles
MAT10MGPS00034

Stage 5.2

A scientific calculator can be used to evaluate a trigonometric ratio such as sin 38°, but it can also be used to find an unknown angle, θ, if the trigonometric ratio of the angle is known, for example, if $\sin \theta = 0.9063$.

An unknown angle can be found using the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ keys on the calculator. These are called the **inverse sin**, **inverse cos** and **inverse tan** functions, found by pressing the SHIFT or 2ndF key before the sin, cos or tan key.

Example 8

a If $\cos \alpha = \frac{4}{7}$, find the value of angle α, correct to the nearest degree.
b If $\tan X = 3.754$, find the value of angle X, correct to the nearest minute.

Solution

a $\cos \alpha = \frac{4}{7}$

$\alpha = 55.1500\ldots°$ On a calculator: SHIFT cos $\frac{4}{7}$ =

$\approx 55°$

b $\tan X = 3.754$

$X = 75.0837\ldots°$ On a calculator: SHIFT tan 3.754 =

$= 75°5'1.62''$ On a calculator: ° ’ ” or DMS

$\approx 75°5'$

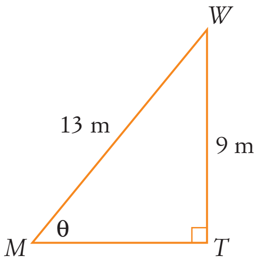
Summary

Finding an unknown angle in a right-angled triangle

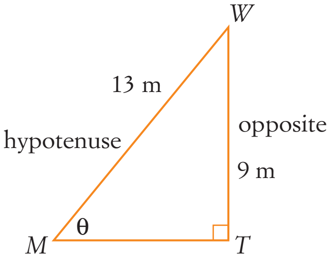
1. Identify the two known sides and decide whether to use the sin, cos or tan ratio.
2. Write an equation using the ratio, the angle variable and the two sides as a fraction.
3. Use the calculator's inverse trigonometric function to find the size of the angle.

Example 9

Find the size of angle θ, correct to the nearest degree.



Solution



SOH, CAH or TOA?
The known sides are the opposite (O) side and the hypotenuse (H), so use sin.

$$\sin \theta = \frac{9}{13}$$

$$\theta = 43.8130\ldots^\circ$$
$$\approx 44^\circ$$

On a calculator: SHIFT sin $\frac{9}{13}$ =

From the diagram, an angle size of 44° looks reasonable.

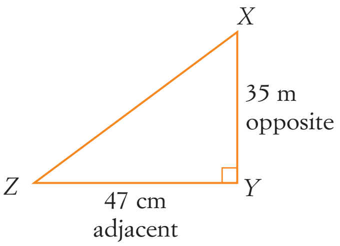
Example 10

$\triangle XYZ$ is right-angled at Y, with $XY = 35$ cm and $YZ = 47$ cm. Find $\angle Z$, correct to the nearest degree.

Solution

Draw a diagram.
SOH, CAH or TOA?
The known sides are the opposite (O) and adjacent (A), so use tan.

$$\tan \theta = \frac{35}{47}$$



$$\theta = 36.6743\ldots^\circ$$
$$\approx 37^\circ$$

On a calculator: SHIFT tan $\frac{35}{47}$ =

From the diagram, an angle size of 37° looks reasonable.

Exercise 8-04 Finding an unknown angle

See Example 8

1 Find the size of angle θ, correct to the nearest degree.

a $\cos\theta = 0.76$ **b** $\tan\theta = 2.0532$ **c** $\sin\theta = \frac{\sqrt{3}}{2}$ **d** $\tan\theta = 6$

e $\sin\theta = \frac{7}{8}$ **f** $\cos\theta = \frac{13}{15}$ **g** $\sin\theta = \frac{1}{10}$ **h** $\cos\theta = \frac{1}{\sqrt{2}}$

Stage 5.2

2 Find the size of angle A, correct to the nearest minute.

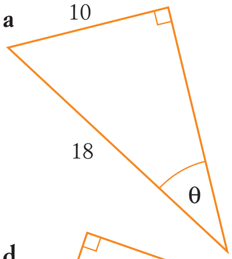
a $\tan A = \frac{15}{7}$ **b** $\sin A = 0.815$ **c** $\cos A = \frac{4}{5}$ **d** $\cos A = 0.9387$

e $\tan A = \frac{19}{20}$ **f** $\cos A = \frac{3}{10}$ **g** $\sin A = \frac{5}{11}$ **h** $\tan A = 0.88$

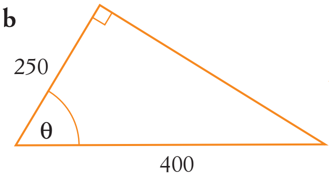
See Example 9

3 Find the value of angle θ, correct to the nearest degree.

a



b



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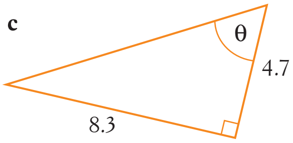
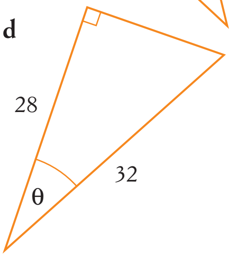
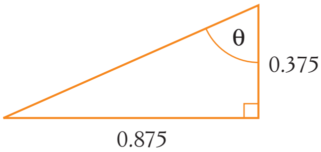


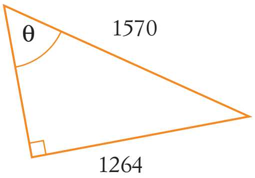
d



e



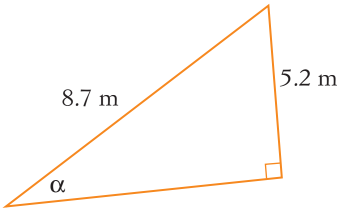
f



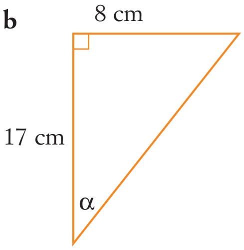
Stage 5.2

4 Find the size of angle α, correct to the nearest minute.

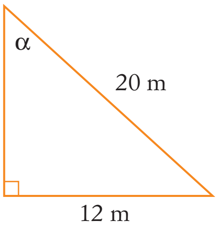
a



b



c



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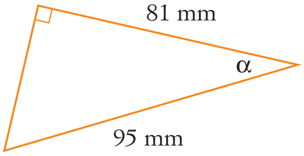

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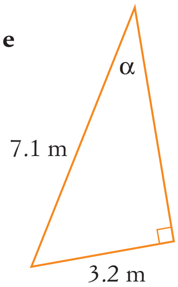

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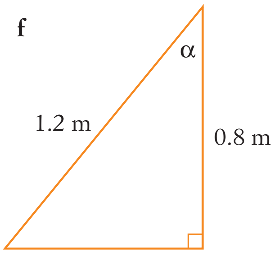
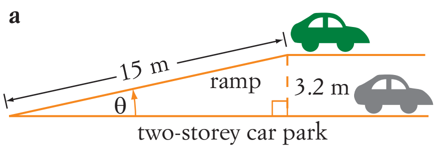
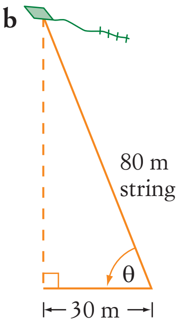
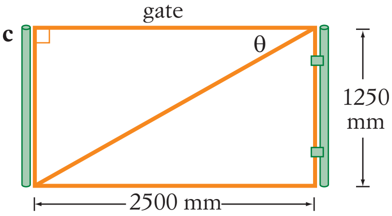


5 Find the size of angle θ, correct to the nearest degree.







For questions **6** to **11**, give your answers correct to the nearest degree.

6 A stretch of freeway rises 55 m for every 300 m travelled along the road. Find the angle at which the road is inclined to the horizontal.

7 A ladder 20 m long is placed against a building. If the ladder reaches 16 m up the building, find the angle of inclination of the ladder to the building.

8 An aircraft is descending in a straight line to an airport. At a height of 1270 m, it is horizontally 1500 m from the airport. Find its angle of descent to the horizontal. Select the correct answer **A**, **B**, **C** or **D**.

A 32° **B** 40° **C** 50° **D** 58°

9 A tree that is 8.5 m high casts a shadow that is 3 m long. What is the angle of the Sun from the ground?

10 At a resort, an artificial beach slopes down at a steady angle to the horizontal. After walking 8.5 m down the slope from the water's edge, the water has a depth of 1.6 m. At what angle is the beach inclined to the horizontal?

11 A pile of wheat is in the shape of a cone that has a diameter of 35 m and measures 27 m up the slope to the apex. Calculate the angle of repose of the wheat (the angle the sloping side makes with the horizontal base).

iStockphoto/guidemark50

See Example 10

12 In $\triangle XYW$, $\angle X = 90°$, $XY = 8$ cm and $XW = 10$ cm. Find $\angle W$, correct to the nearest degree.

Stage 5.2

13 In $\triangle FGH$, $\angle G = 90°$, $GH = 3.7$ m and $FH = 19.5$ m. Find the size of angle F, correct to the nearest minute.

14 $\triangle HTM$ is right-angled at T, $HM = 45$ m and $MT = 35$ m. Find $\angle M$, correct to one decimal place.

15 $\triangle TSV$ is right-angled at S, $TV = 9.5$ cm, and $ST = 8.4$ cm. Find $\angle V$, correct to the nearest degree.

Mental skills 8 Maths without calculators

Divisibility tests

How can you tell if a number is divisible by 2? Look at its last digit. If that digit is 2, 4, 6, 8 or 0, then the number is divisible by 2 (that is, it is even).
How can you tell if a number is divisible by 5? If its last digit is 0 or 5, then the number is divisible by 5.
These are examples of **divisibility tests**—rules for checking whether or not a number is divisible by a certain number. The table below shows some common divisibility tests.

A number is divisible by:	if:
2	its last digit is 2, 4, 6, 8 or 0
3	the sum of its digits is divisible by 3
4	its last two digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Test whether 748 is divisible by 2, 3 or 4.
- Last digit is 8 (even), $\therefore$ 748 is divisible by 2
- Sum of digits $= 7 + 4 + 8 = 19$, which is not divisible by 3, $\therefore$ 748 is not divisible by 3
- 48 is divisible by 4, $\therefore$ 748 is divisible by 4 ($748 \div 4 = 187$)

b Test whether 261 is divisible by 5 or 9.
- Last digit is 1, not 0 or 5, $\therefore$ 261 is not divisible by 5
- $2 + 6 + 1 = 9$, which is divisible by 9, $\therefore$ 261 is divisible by 9. ($261 \div 9 = 29$).

c Test whether 570 is divisible by 4, 6 or 10.
- 70 is not divisible by 4, $\therefore$ 570 is not divisible by 4
- 570 is even and $5 + 7 + 0 = 12$, which is divisible by 3, $\therefore$ 570 is divisible by 6 ($570 \div 6 = 95$)
- Last digit is 0, $\therefore$ 570 is divisible by 10 ($570 \div 10 = 57$)

2 Test whether each number is divisible by 2, 3, 5 or 6.

a 250 **b** 189 **c** 78 **d** 465
e 1024 **f** 840 **g** 715 **h** 627

3 Test whether each number is divisible by 4, 9 or 10.

a 144 **b** 280 **c** 522 **d** 4170
e 936 **f** 726 **g** 342 **h** 5580

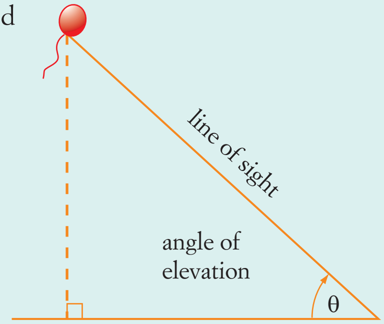
8-05 Angles of elevation and depression

Video tutorial
Trigonometry
MAT10MGVT00010

Summary

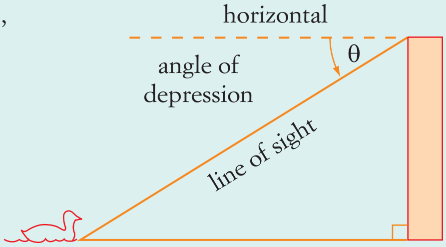
The **angle of elevation** is the angle of looking **up,** measured from the horizontal.

When you feel elevated, things are 'looking up'!



The **angle of depression** is the angle of looking **down**, measured from the horizontal.

When you feel depressed, things are 'looking down'!



Problems involving angles of elevation and depression usually require the tan ratio in their solutions.

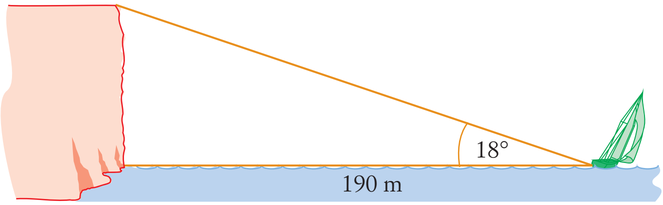
An instrument for measuring an angle of elevation or depression is a **clinometer**. It is like a protractor with a sighting tube attached.

Alamy/Paul Williams

Example 11

Video tutorial
Angles of elevation and depression
MAT10MGVT10023

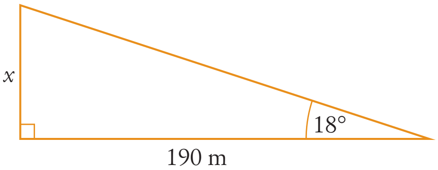
The angle of elevation from a yacht to the top of a cliff is 18°. If the yacht is 190 m from the base of the cliff, find the height of the cliff, correct to one decimal place.



Solution

Let the height be x metres.

$$\begin{aligned} \tan 18^\circ &= \frac{x}{190} \\ x &= 190 \tan 18^\circ \\ &= 61.73474\ldots \\ &\approx 61.7 \text{ m} \end{aligned}$$

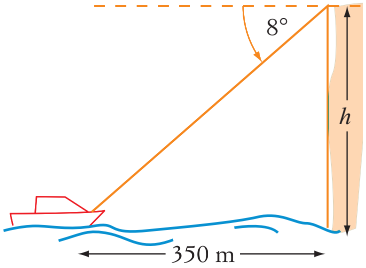


The height of the cliff is 61.7 m.

Example 12

Video tutorial
Angles of elevation and depression
MAT10MGVT10023

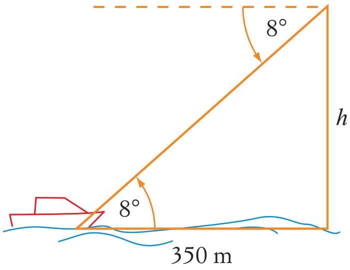
The angle of depression of a boat from the top of a cliff is 8°. If the boat is 350 m from the base of the cliff, calculate the height of the cliff, correct to the nearest metre.



Solution

Using the property of alternate angles, the angle of elevation of the top of the cliff from the boat is also 8°.

$$\begin{aligned} \tan 8^\circ &= \frac{h}{350} \\ h &= 350 \tan 8^\circ \\ &= 49.1892\ldots \\ &\approx 49 \end{aligned}$$



The height of the cliff is 49 m.

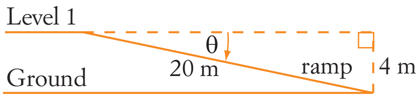
Alternative method
The third angle in the triangle (adjacent to the angle of depression) $= 90^\circ - 8^\circ = 82^\circ$.

$$\begin{aligned} \tan 82^\circ &= \frac{350}{h} \\ h &= \frac{350}{\tan 82^\circ} \\ &= 49.1892\ldots \\ &\approx 49 \end{aligned}$$

The height of the cliff is 49 m.

Example 13

The ramp from one level to the next in a car park is 20 m long and drops 4 m. Find the angle of depression of the ramp, correct to the nearest degree.



Solution

$$\sin\theta = \frac{4}{20}$$

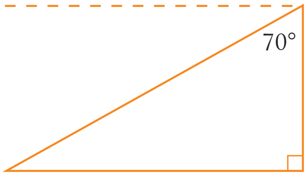
$$\begin{aligned}\theta &= 11.5369\ldots^\circ \\ &\approx 12^\circ\end{aligned}$$

The angle of depression of the ramp is 12°.

Exercise 8-05 Angles of elevation and depression

1 Copy each diagram, mark the angle of elevation θ and find its size.

a



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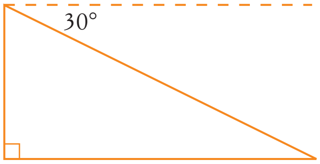

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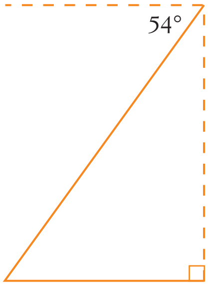
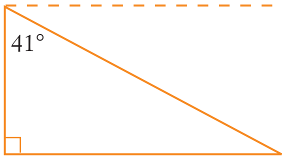


2 Copy each diagram, mark the angle of depression θ and find its size.

a



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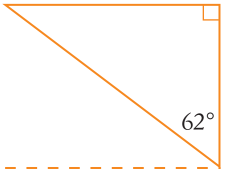

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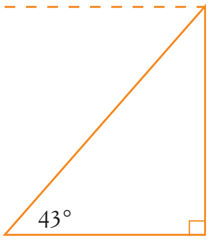
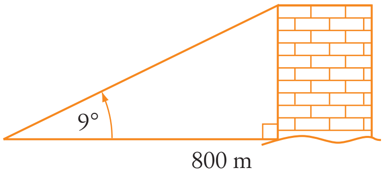
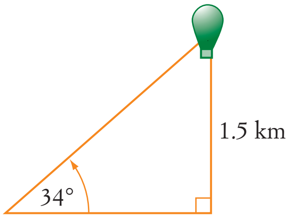


See Example 11

3 Sang Koo stands 800 m from the base of a building. Using a clinometer, he finds that the angle of elevation of the top is 9°. Find the height of the building, correct to the nearest metre.

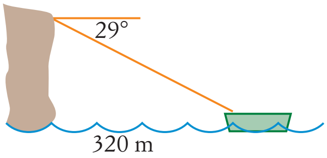


4 The angle of elevation of a weather balloon at a height of 1.5 km is 34°. How far (correct to the nearest metre) is the observer from being directly under the balloon?

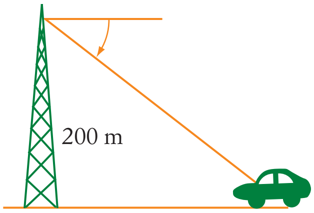


See Example 12

5 A raft is 320 m from the base of a cliff. The angle of depression of the raft from the top of the cliff is 29°. Find the height of the cliff, correct to the nearest metre.



6 From the top of a 200 m tower, the angle of depression of a car is 48°. How far is the car from the foot of the tower? Answer correct to the nearest metre.



See Example 13

7 A 275 m radio mast is 1.7 km from a school. Find, correct to the nearest degree, the angle of elevation of the top of the mast from the school.

8 In a concert hall, Bill is sitting 20 m from the stage by line of sight. He is also 5 m above the level of the stage. At what angle of depression is the stage? Answer correct to the nearest minute.

9 A monument that is 24 m high casts a shadow 20 m long. Calculate, correct to the nearest degree, the angle of elevation of the Sun at this time of day.

10 A plane is 340 m directly above one end of a 1000 m runway. Find, correct to the nearest minute, the angle of depression to the far end of the runway.

Shutterstock.com/whitelook

11 An observer who is 174 cm tall is standing 11.6 m from the base of a flagpole. The angle of elevation to the top of the flagpole is 43°. How high is the flagpole, correct to the nearest cm?

12 A flagpole is mounted on top of a tall building. At a distance of 250 m from the base of the building, the angles of elevation of the bottom and top of the flagpole are 38° and 40° respectively. Calculate the height of the flagpole, correct to one decimal place.

13 A news helicopter hovers at a height of 500 m. The angles of depression of a fire moving in the direction of the helicopter are first 10° and then 15°. How far (correct to the nearest metre) has the fire moved between the two observations?

14 The angle of elevation to the bottom of a transmission tower on a hill from an observer is 5°. The angle of elevation to the top of the tower from the observer is 6.8°. If the distance from the observer to the base of the hill is 1.8 km, find the height of the tower, correct to the nearest metre.

8-06 Bearings

Stage 5.2

Worksheet
A page of bearings
MAT10MGWK10058

Worksheet
Bearings match-up
MAT10MGWK10061

Worksheet
NSW map bearings
MAT10MGWK10060

Bearings are used in navigation. A **bearing** is an angle measurement used to describe precisely the direction of one location from a given reference point.

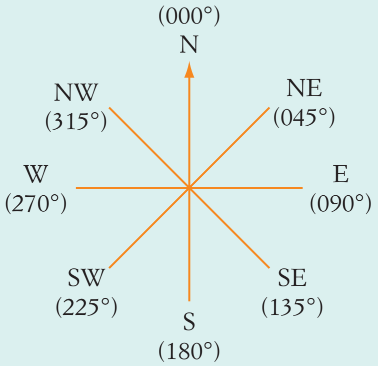
Three-figure bearings

Three-figure bearings, also called **true bearings**, use angles from 000° to 360° to show the amount of turning measured **clockwise from north 000°**. Note that the angles are always written with three digits.

The **compass rose** below shows the three-figure bearings of eight points on the compass. A bearing of due east is 090°, while a compass direction of southwest (SW) is 225°.

Summary

Bearings from 000° to 090° are in the NE quadrant.
Bearings from 090° to 180° are in the SE quadrant.
Bearings from 180° to 270° are in the SW quadrant.
Bearings from 270° to 360° are in the NW quadrant.



Stage 5.2

Worksheet

16 points of the compass

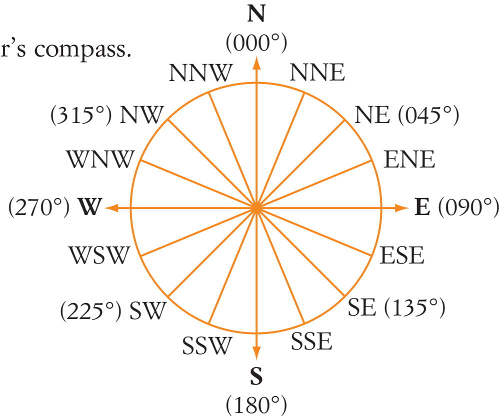
MAT10MGWK10059

Compass bearings

Compass bearings refer to sixteen points of a mariner's compass.

N = north
NE = northeast
E = east
SE = southeast
S = south
SW = southwest
W = west
NW = northwest

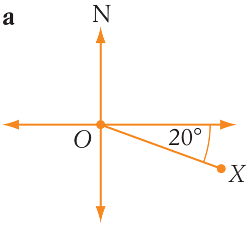
NNE = north-northeast
ENE = east-northeast
ESE = east-southeast
SSE = south-southeast
SSW = south-southwest
WSW = west-southwest
WNW = west-northwest
NNW = north-northwest



Example 14

Write the three-figure bearing of each point from O.

a



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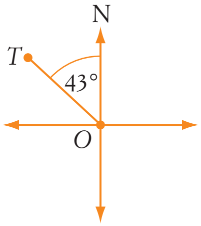

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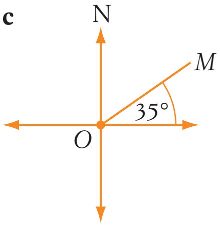


Solution

a The bearing of X from O is $90° + 20° = 110°$.

b The bearing of T from O is $360° - 43° = 317°$.

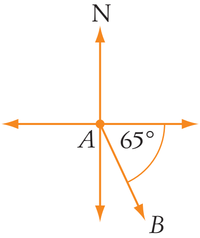
c The bearing of M from O is $90° - 35° = 055°$ Must be written as a three-digit angle.

Example 15

Sketch point B on a compass rose if B has a bearing of 155° from A.

Solution

Draw the compass rose on the point where the bearing is measured from.
155° is between 90° and 180°, so B is in the southeast (SE) quadrant.
$155° - 90° = 65°$, so B is 65° from east (E).



Stage 5.2

Example 16

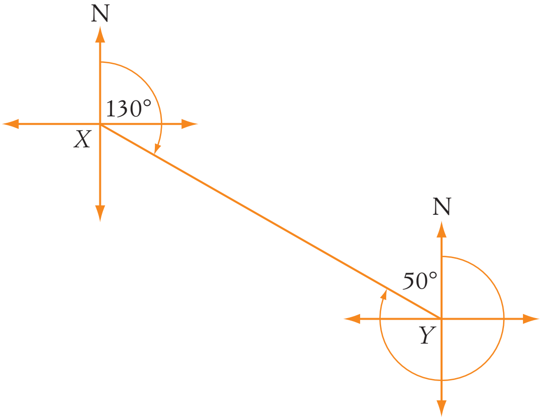
The bearing of Y from X is 130°. What is the bearing of X from Y?

Solution

Sketch the bearing of Y from X.
On the same diagram, draw a compass rose at Y and find $\angle NYX$.

$\angle NYX = 50°$ (co-interior angles, $NX \parallel NY$)

$$\therefore \text{Bearing of } X \text{ from } Y = 360° - 50°$$
$$= 310°$$

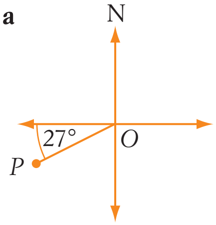


Exercise 8-06 Bearings

See Example 14

1 Write the bearing of each point from O.

a



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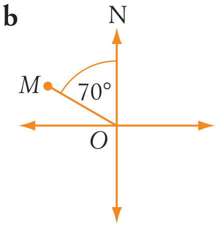
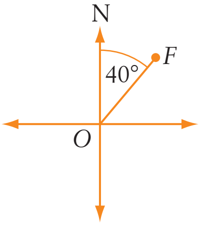


c



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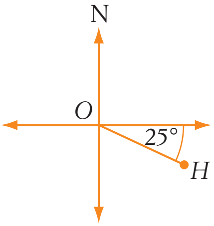
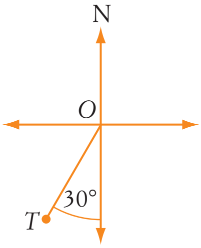
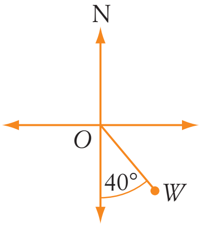


e



f



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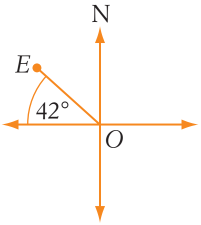

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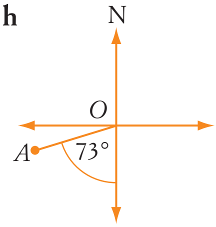
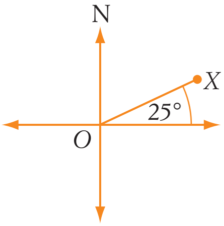
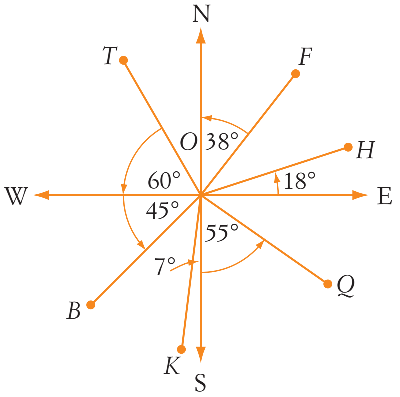


i



2 What is the bearing of each point from *O*?

a N **b** E **c** S **d** W **e** *F*
f *Q* **g** *T* **h** *B* **i** *H* **j** *K*



3 What is the compass direction shown by point *B* in question **2**?

4 Sketch each bearing on a compass rose.

a 220° **b** 060° **c** 260° **d** 125°
e 350° **f** 267° **g** 171° **h** 32°

5 **a** What is the compass direction halfway between northwest and west?

b What is the three-figure bearing of this compass direction?

6 Sketch *P* on a compass rose if *P* has a bearing of:

a 132° from *T* **b** 260° from *M* **c** 335° from *X* **d** 010° from *K*

***See* Example 15**

7 If the bearing of *P* from *A* is 060°, what is the bearing of *A* from *P*?

***See* Example 16**

8 The bearing of *T* from *Y* is 100°. What is the bearing of *Y* from *T*?

9 What is the angle size between:

a S and SW? **b** NE and SE? **c** E and NW?

10 The compass bearing of *H* from *M* is WNW. Find the compass bearing of *M* from *H*.

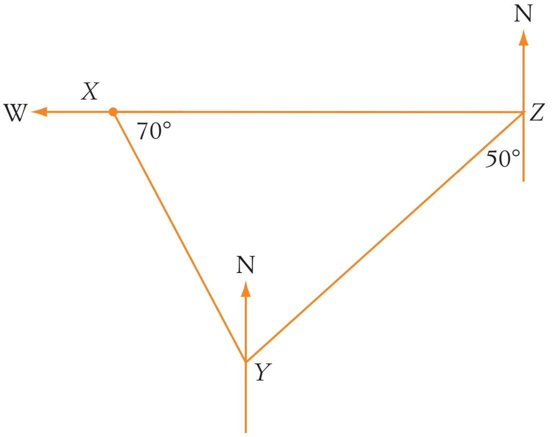
11 Draw a diagram for each situation described.

a A plane flies on a bearing of 280° for 150 km and then another 250 km on a bearing of 080°.

b A cyclist travels 15 km due east and then 20 km on a SW bearing.

12 For this diagram, find the bearing of:

a *Y* from *Z* **b** *X* from *Z*
c *Y* from *X* **d** *X* from *Y*
e *Z* from *Y* **f** *Z* from *X*

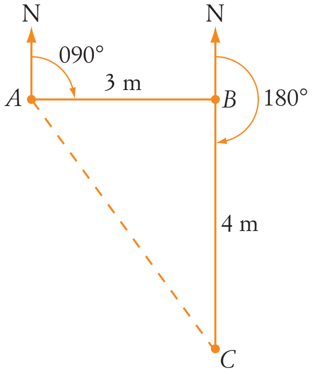


Investigation: Compass walks

You need: a directional compass and a tape measure or trundle wheel.
This activity can also be done in the classroom using scale drawings, graph paper, a ruler and a protractor.

A triangular walk

1. Starting at *A*, walk due east for 3 m to *B*.
2. From *B*, walk due south for 4 m to *C*.
3. How far is *C* from *A*?
4. What is the bearing of:
   - **a** *A* from *C*? **b** *C* from *A*?



A square walk

1. Starting at *P*, walk a bearing of 045° for 8 m to *Q*.
2. From *Q*, walk a bearing of 315° for 8 m to *R*.
3. From *R*, walk a bearing of 225° for 8 m to *S*.
4. How far is *S* from *P*?
5. What is the bearing of:
   - **a** *P* from *S*? **b** *S* from *P*?

A pentagonal walk

1. Starting at *U*, walk a bearing of 130° for 4 m to *V*.
2. From *V*, walk a bearing of 40° for 7 m to *W*.
3. From *W*, walk a bearing of 320° for 4.8 m to *X*.
4. From *X*, walk a bearing of 270° for 4.5 m to *Y*.
5. How far is *Y* from *U*?
6. What is the bearing of:
   - **a** *U* from *Y*? **b** *Y* from *U*?

Worksheet
Elevations and bearings
MAT10MGWK10062

Stage 5.2

Video tutorial
Bearings
MAT10MGVT10022

Video tutorial
Trigonometry
MAT10MGVT00010

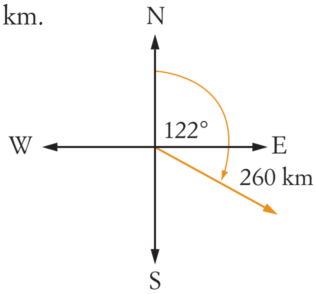
8-07 Problems involving bearings

Example 17

A plane leaves a town and remains on a bearing of 122° for 260 km.

a How far south of the town is the plane, correct to one decimal place?

b What is the bearing of the town from the plane?



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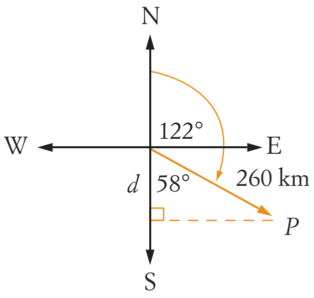
Stage 5.2

Solution

a Let d km = distance south

$$\angle SOP = 180° - 122° \text{ (angles on a straight line)}$$
$$= 58°$$
$$\cos 58° = \frac{d}{260}$$
$$d = 260 \cos 58°$$
$$= 137.7790\ldots$$
$$\approx 137.8$$

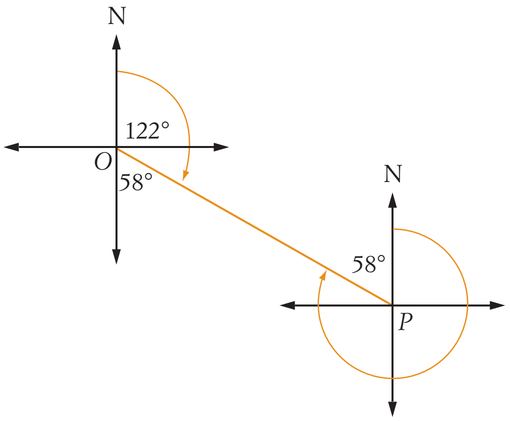
The plane is 137.8 km south of the town.



b Draw a compass rose with the centre at P.
$\angle OPN = 58°$ (alternate angles on parallel lines)

$$\text{Bearing of } O \text{ from } P = 360° - 58°$$
$$= 302°$$

The bearing of the town from the plane is 302°.



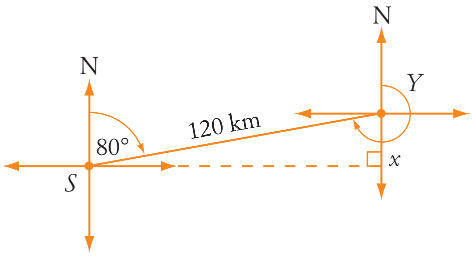
Exercise 8-07 Problems involving bearings

See Example 17

1 A yacht leaves Sydney and sails 120 km on a bearing of 080°.

a How far north of Sydney is the yacht?

b What is the bearing of Sydney from the yacht?

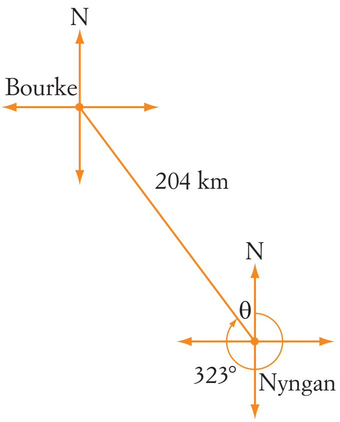


2 Colin leaves Nyngan and drives 204 km to Bourke. The bearing of Bourke from Nyngan is 323°.

a Find the value of θ.

b How far north of Nyngan is Bourke?

c What is the bearing of Nyngan from Bourke?



Stage 5.2

3 The distance 'as the crow flies' from Sydney to Wollongong is 69 km. If the bearing of Wollongong from Sydney is 205°, calculate:

a how far south Wollongong is from Sydney

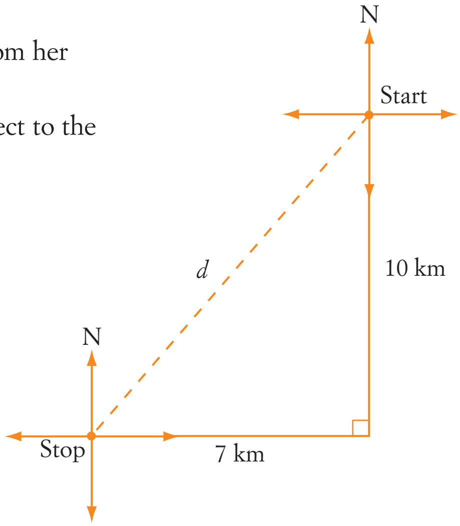
b how far east Sydney is from Wollongong

c the bearing of Sydney from Wollongong.

4 Jana cycles 10 km due south, then 7 km due west.

a How far (correct to one decimal place) is Jana from her starting point?

b What is her bearing from the starting point, correct to the nearest degree?



5 A triathlete cycles 20 km on a SSE bearing to the finish line.

a How far (correct to one decimal place) has the triathlete travelled in a southerly direction?

b What is the compass bearing of the starting point from the finish line?

6 A hiking group walks from Sandy Flats to Black Ridge (a distance of 20.9 km) in the direction 078°. They then turn and hike due south to Rivers End, then due west back to Sandy Flats. How far have they hiked altogether (to the nearest 0.1 km)?

7 A triangular orienteering run starts at Alpha and passes through the checkpoints of Bravo and Charlie before finishing at Alpha. Bravo is 8.5 km due east of Alpha, and Charlie is 10.5 km due south of Bravo.

a Calculate, correct to three decimal places, the distance from Charlie to Alpha.

b Find the bearing of Alpha from Charlie, correct to the nearest degree.

8 A plane takes off from Darwin at 10:15 a.m. and flies on a bearing of 150° at 700 km/h.

a How far (correct to the nearest km) due south of Darwin is the plane at 1:45 p.m.?

b What is the bearing (correct to the nearest degree) of Darwin from the plane?

9 A fishing trawler sails 30 km from port on a bearing of 120° until it reaches a submerged reef. How far (correct to the nearest km) is the port:

a north of the reef? **b** west of the reef?

10 Two racing pigeons are set free at the same time. The first bird flies on a course of 040° while the second bird flies on a course of 130°.

a The first bird flies 200 km until it is due north of the second bird. Find their distance apart, correct to two decimal places.

b How far has the second bird flown?

9780170194655

11 Two horse riders start from the same stable. The rider of the black horse goes due west for 5.5 km and stops. The rider of the chestnut horse travels in a direction of 303° until she is due north of the black horse. How far did the rider of the chestnut horse travel? Answer correct to three decimal places.

Alamy/Mikhail Kondrashov "fotomik"

12 Two ships leave from the same port. One ship travels on a bearing of 157° at 20 knots. The second ship travels on a bearing of 247° at 35 knots. (1 knot is a speed of 1 nautical mile per hour.)

a How far apart are the ships after 8 hours, correct to the nearest nautical mile?

b Calculate the bearing of the second ship from the first, correct to the nearest minute.

Power plus

1 **a** Copy and complete each pair of trigonometric ratios, correct to three decimal places.

i sin 20° = _____, cos 70° = _____ **ii** sin 47° = _____, cos 43° = _____

iii sin 55° = _____, cos 35° = _____ **iv** sin 85° = _____, cos 5° = _____

b What do you notice about each pair of answers in part **a**?

c What do you notice about each pair of angles in part **a**?

d If cos 30° ≈ 0.8660 and sin θ ≈ 0.8660, what is the value of θ?

e Copy and complete each equation.

i sin 75° = cos ____ **ii** ___ 80° = cos 10° **iii** cos ____ = sin 72°

iv sin 30° = ____ 60° **v** cos 65° = sin ____ **vi** sin ____ = cos 58°

f Copy and complete this general rule: sin x = cos (_________).

g Use a right-angled triangle with one angle x and sides a, b and c to prove that the above rule is true.

2 A plane is flying at an angle of 15° inclined to the horizontal.

a How far will the plane have to travel along its line of flight to increase its altitude (height) by 500 m?

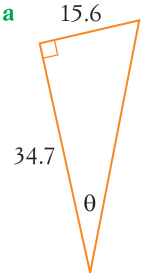
b At what angle must the plane climb to achieve an increase in altitude of 500 m in half the distance needed at an angle of 15°?

3 If $\sin 30° = \frac{1}{2}$, express the following as surds.

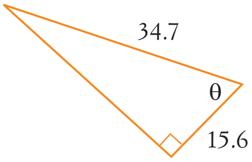
a $\cos 30°$ **b** $\tan 30°$

4 Find the value of angle θ, correct to the nearest second.

a



b

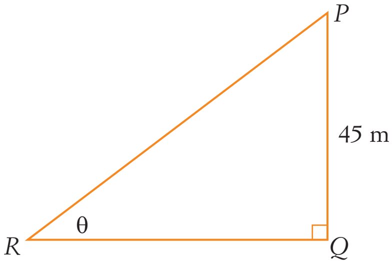


5 By drawing an appropriate triangle, prove that:

a $\tan 45° = 1$ **b** $\sin 45° = \frac{1}{\sqrt{2}}$ **c** $\cos 45° = \frac{1}{\sqrt{2}}$

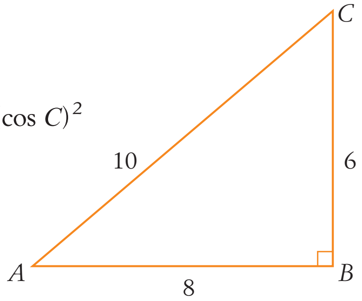
6 In △*PQR*, $PQ = 45$ m and $\sin \theta = \frac{3}{5}$.

What are the lengths of *RP* and *RQ*?



7 For △*ABC*, find the value of:

a $\sin A$
b $\cos A$
c $(\sin A)^2 + (\cos A)^2$
d $\sin C$
e $\cos C$
f $(\sin C)^2 + (\cos C)^2$



8 Evaluate each expression

a $(\sin 20°)^2 + (\cos 20°)^2$ **b** $(\sin 48°)^2 + (\cos 48°)^2$

9 Sketch a right-angled triangle with side lengths *a*, *b* and *c*, where *c* is the hypotenuse. Let θ be one of the acute angles. Find sin θ and cos θ, and hence prove that $(\sin \theta)^2 + (\cos \theta)^2 = 1$.

10 A truck travels at 80 km/h on a straight country road. In the distance, at an angle of 38° on the right, the driver sees a bushfire. Exactly 1.5 minutes later the truck is directly opposite the fire. Calculate how close the truck comes to the fire. (Answer correct to the nearest 0.1 km.)

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Language of maths

Puzzle sheet
Trigonometry crossword
MAT10MGPS10063

- adjacent
- angle of depression
- angle of elevation
- bearing
- clinometer
- compass bearing
- cosine (cos)
- degree (°)
- denominator
- horizontal
- hypotenuse
- inverse ($^{-1}$)
- minute (′)
- opposite
- right-angled
- second (″)
- sine (sin)
- tangent (tan)
- theta (θ)
- three-figure bearing
- trigonometry
- trigonometric ratio
- unknown
- vertical

1 What is an **angle of depression**?

2 What word means ‘next to’?

3 Which side of a right-angled triangle is fixed and does not depend on the position of an angle?

4 Copy and complete: A **bearing** is an __________ used to describe precisely the __________ of one location from a given reference point.

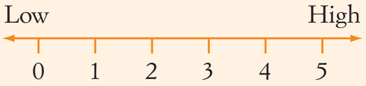
5 The word **minute** has an alternative pronunciation and meaning. What is its alternative meaning?

6 What does **inverse** mean and how is it used in trigonometry?

Topic overview

Quiz
Trigonometry
MAT10MGQZ00010

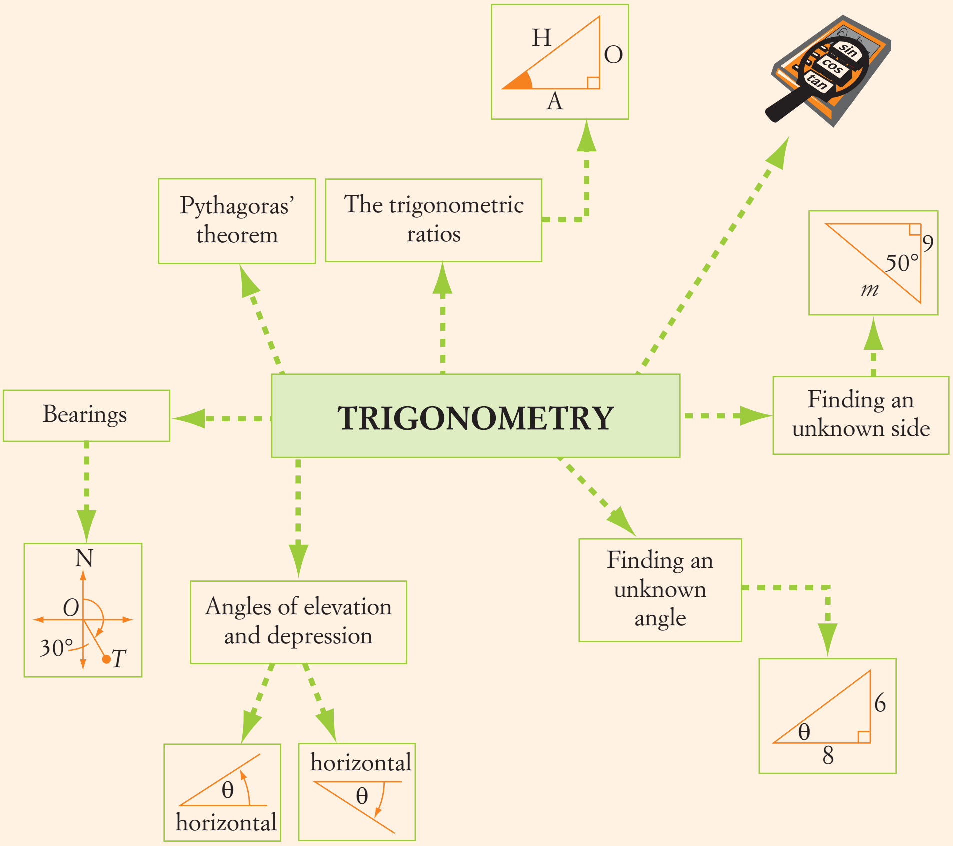
For each statement about the topic, give a rating from 0 to 5 using this scale.



- I can use Pythagoras’ theorem to find an unknown side in a right-angled triangle and to prove that a triangle is right-angled
- I understand the meaning of the sine, cosine and tangent ratios
- I am able to use the trigonometric ratios to find unknown sides
- I am able to use the trigonometric ratios to find unknown angles
- I am able to solve trigonometric problems involving angles of elevation and depression, and bearings

Chapter 8 revision

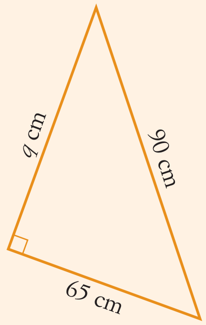
Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 8 revision

1 Find the value of each pronumeral, correct to one decimal place. *See* Exercise 8-01

a



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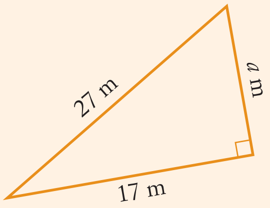

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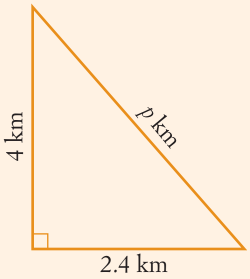
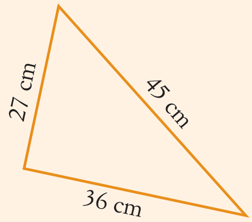


2 Test whether each triangle is right-angled. *See* Exercise 8-01

a



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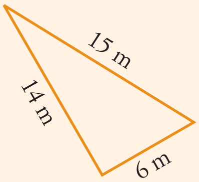
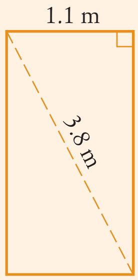


3 Calculate the perimeter of each shape, correct to one decimal place. *See* Exercise 8-01

a



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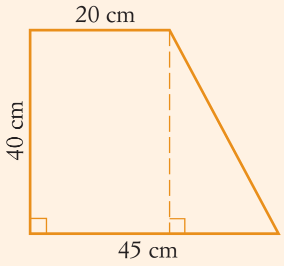
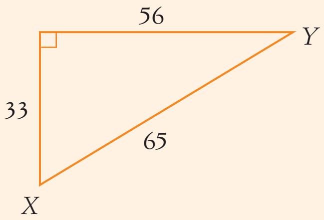


4 For this triangle, write as a fraction: *See* Exercise 8-02

a sin Y **b** tan Y **c** sin X **d** cos X

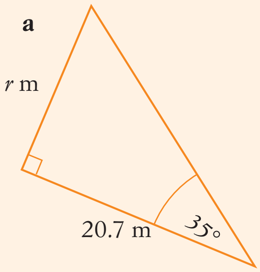


5 If $\sin \alpha = \frac{36}{85}$, express the values of cos α and tan α as fractions. (Draw a diagram.) *See* Exercise 8-02

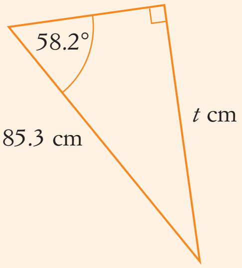
See Exercise 8-03

6 Find the value of each pronumeral, correct to two decimal places.

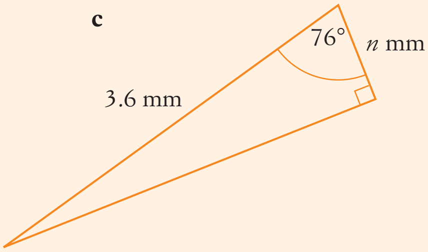
a



b



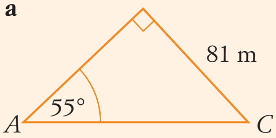
c



See Exercise 8-03

7 For each triangle, find the length of side AC, correct to one decimal place.

a



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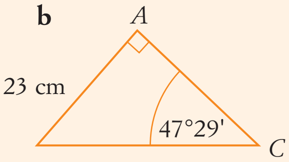

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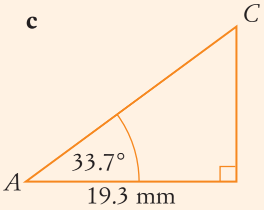


See Exercise 8-04

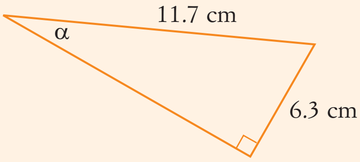
8 Find the size of angle θ, correct to the nearest degree.

a $\tan\theta = 2.57$ **b** $\cos\theta = \frac{4}{7}$ **c** $\sin\theta = \frac{1.5}{1.6}$

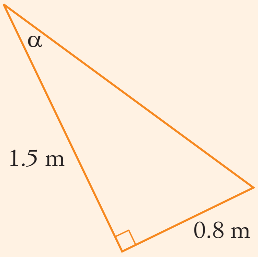
See Exercise 8-04

9 Find the size of angle α, correct to the nearest degree.

a



b



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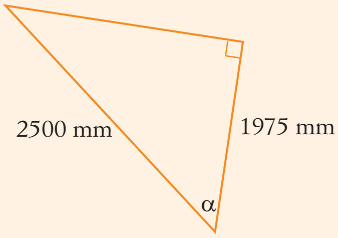


Stage 5.2

See Exercise 8-04

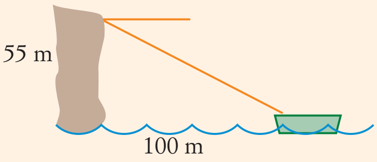
10 In $\triangle AEC$, $\angle C = 90°$, $CE = 3.9$ m and $AE = 4.2$ m. Find $\angle A$, correct to the nearest minute.

See Exercise 8-05

11 The angle of elevation of a tower roof is 26° at a point 400 m from its base. Find the height of the tower, correct to the nearest metre.

See Exercise 8-05

12 Find the angle of depression (correct to the nearest degree) of a boat that is 100 m from the base of a 55 m cliff.

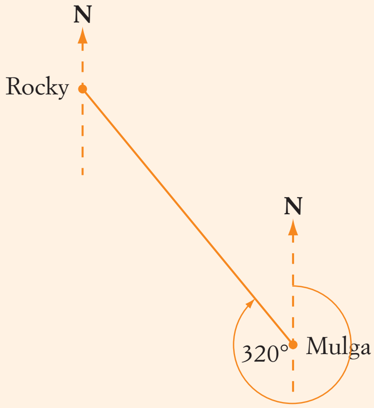


Stage 5.2

See Exercise 8-06

13 What is the bearing of:

a Rocky from Mulga? **b** Mulga from Rocky?



See Exercise 8-06

14 Sketch point *B* on a compass rose so that *B* has a bearing from *A* of

a 220° **b** 020° **c** 120°

See Exercise 8-07

15 Two planes leave an airport at the same time. The first travels on a bearing of 063° at 500 km/h. The second travels on a bearing of 153° at 400 km/h.

a How far apart are the planes after 2 hours (correct to the nearest km)?

b Calculate, correct to the nearest degree, the bearing of the first plane from the second plane.

Mixed revision 3

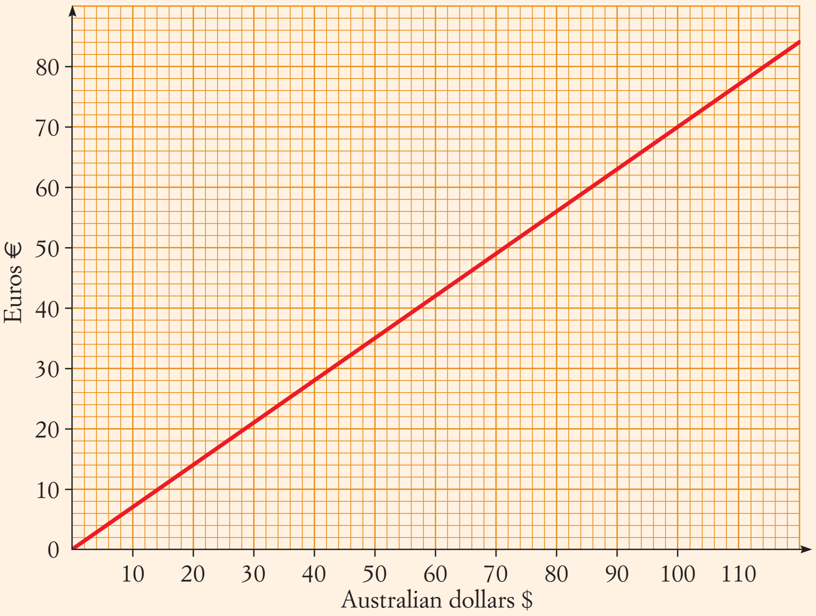
Stage 5.2

1 Solve each pair of simultaneous equations graphically. *See* Exercise 9-01

a $y = 2x - 3$ and $x + y = 6$ **b** $2x + y = 1$ and $y = 3x - 4$

2 This conversion graph shows the exchange rate between the Australian dollar and the Euro. *See* Exercise 7-03

Australian dollars to Euros



Use the graph to convert:

a AU\$25 to euros **b** €60 to Australian dollars **c** AU\$140 to euros.

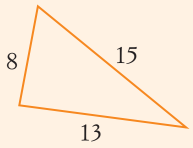
3 T varies directly with h. If $T = 48$ when $h = 5$, find T when $h = 16.5$. *See* Exercise 7-01

4 Which of the following points lies on the circle with equation $x^2 + y^2 = 9$? *See* Exercise 7-06

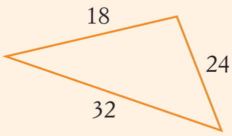
A (0, 3) **B** (2, 2) **C** (4, 1) **D** (1.5, 2)

5 Which of the following is a right-angled triangle? Select **A**, **B**, **C** or **D**. *See* Exercise 8-01

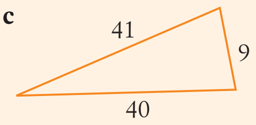
a



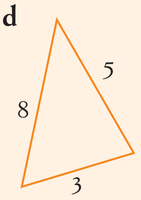
b



c



d



6 **a** Graph each quadratic equation, showing the vertex of each parabola. *See* Exercise 7-04

i $y = x^2$ **ii** $y = 4 - x^2$ **iii** $y = 3x^2 + 1$ **iv** $y = 3 - 2x^2$

b State which graphs you have drawn in **a**:

i are concave up **ii** are concave down

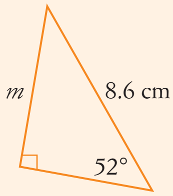
iii have a turning point at (0, 1).

Mixed revision 3

See Exercise 8-03

7 Find the value of each pronumeral, correct to one decimal place.

a



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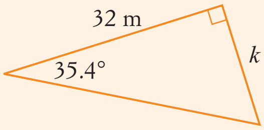

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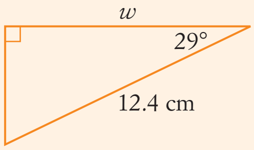


Stage 5.2

See Exercise 7-02

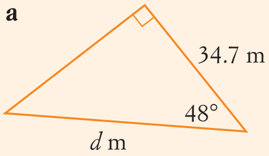
8 The time taken, t seconds, to complete a race is inversely proportional to the speed, S m/s, of the sprinter.

- **a** If it takes 9.8 seconds to complete a race at a speed of 10.2 m/s, find the variation equation for t.
- **b** If the time taken is 10.5 seconds, what is the speed of the sprinter?
- **c** If the sprinter's average speed for the race is 10.3 m/s, how long will it take to complete the race?

See Exercise 8-03

9 Calculate the value of each pronumeral, correct to one decimal place.

a



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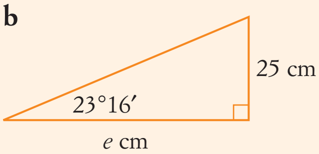

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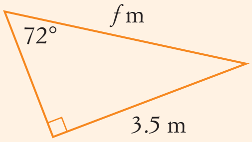


See Exercise 9-02

10 Solve each pair of simultaneous equations using the elimination method.

a $3g - 2w = 8$
$g - 2w = 4$

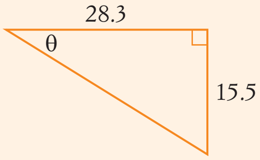
b $2y + 3f = 15$
$5y - 2f = 9$

c $3a - 4c = 5$
$5a - 3c = 1$

See Exercise 8-04

11 Find θ, correct to the nearest degree.

a



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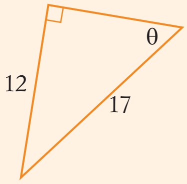

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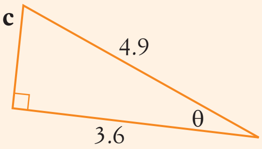


See Exercise 7-06

12 Graph each equation.

a $x^2 + y^2 = 6.25$ **b** $x^2 + y^2 = 1$ **c** $x^2 + y^2 = 49$

See Exercise 8-03

13 A kite is flying at the end of a string that is 85 m long. The string makes an angle of 57° with the ground. At what height is the kite flying? Answer correct to the nearest metre.

Stage 5.2

See Exercise 7-02

14 The amount of time (in hours) it takes to paint a house varies inversely with the number of painters used to paint the house. It takes 5 painters 28 hours to paint the house.

- **a** How long will it take 8 painters to paint the house?
- **b** How many painters are required to paint the house in 24 hours?

Stage 5.2

See Exercise 9-04

15 Tickets to the school play cost \$20 for adults and \$15 for children. Altogether, 395 people attended and ticket sales totalled \$6700. Let A stand for the number of adults and C for the number of children that attended the school play.

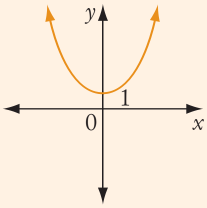
a Write a pair of simultaneous equations to represent this situation.

b Solve the simultaneous equations to find the number of children that attended the play.

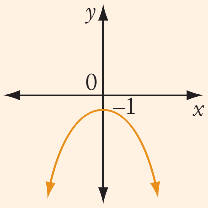
See Exercise 7-04

16 Which of the following could be a graph of $y = 1 - 2x^2$? Select **A**, **B**, **C** or **D**.

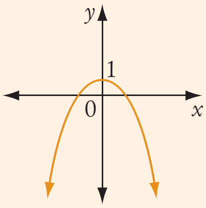
A



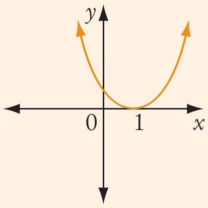
B



C

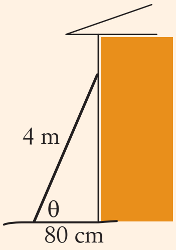


D



See Exercise 8-04

17 A 4 m ladder is placed against the side of a house. The foot of the ladder is 80 cm from the base of the house. Find the angle between the ladder and the ground, correct to the nearest degree.



Stage 5.2

See Exercise 9-03

18 Solve each pair of simultaneous equations using the substitution method.

a $y = x + 3$
$y = 5x - 7$

b $2w + p = 5$
$p = 2w - 3$

c $3k - 2g = 8$
$k = 4g + 1$

See Exercise 7-06

19 Which equation represents a circle with centre (0, 0) and radius 4 units? Select **A**, **B**, **C** or **D**.

A $x^2 + y^2 = 4$ **B** $2x^2 + 2y^2 = 32$ **C** $x^2 + y^2 = 8$ **D** $4x^2 + 4y^2 = 32$

Stage 5.2

See Exercise 7-04

See Exercise 7-05

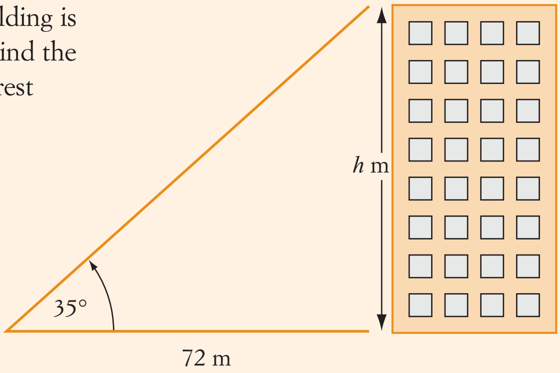
20 A parabola has the equation $y = 4x^2 - 3$. Find the x-coordinates of the points on the parabola that have a y-coordinate of 13.

21 Sketch each exponential curve, showing the y-intercept.

a $y = 10^x$ **b** $y = 2^x - 3$ **c** $y = 4^{-x}$ **d** $y = 5^x + 2$

See Exercise 8-05

22 The angle of elevation of the top of a building is 35° at a distance of 72 m from its base. Find the height of the building, correct to the nearest metre.



Stage 5.2

See Exercise 7-07

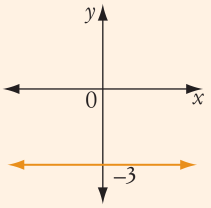
23 Match each equation to its graph below.

a $y = 2x^2 - 2$ **b** $x = -3$ **c** $y = 2^{-x}$ **d** $x + y = 1$

e $y = 2 - x^2$ **f** $x^2 + y^2 = 1$ **g** $y = 2x^2$ **h** $y = 3^x$

i $3x^2 + 3y^2 = 75$ **j** $y = x$ **k** $y = -3$ **l** $y = 4^x - 1$

A



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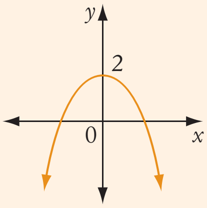

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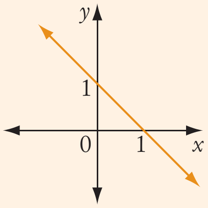

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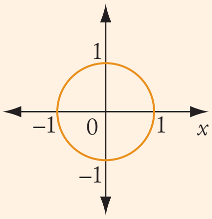

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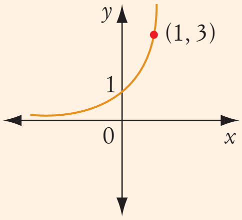
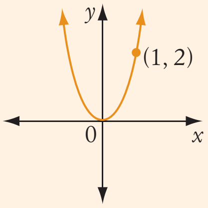
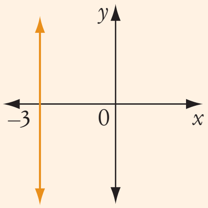


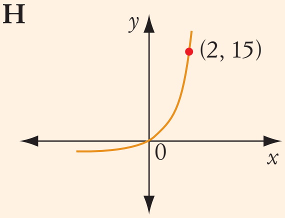
F



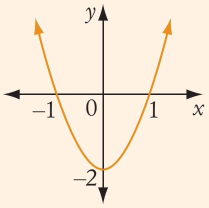
G



H



I



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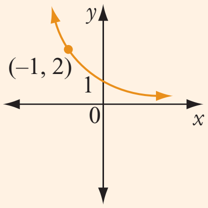

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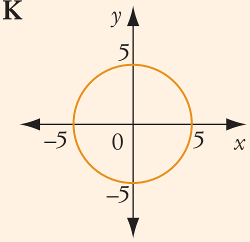
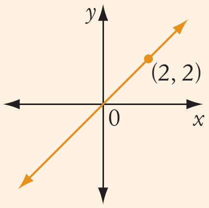


L



Stage 5.2

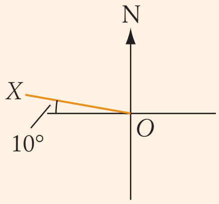
24 In $\triangle ABC$, $\angle C = 90°$, $AC = 56$ mm and $AB = 72$ mm. Find the size of $\angle B$, correct to the nearest minute.

See Exercise 8-04

25 Write the bearing of point X from O in each diagram.

See Exercise 8-06

a



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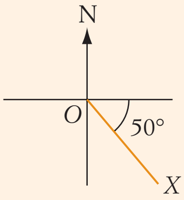

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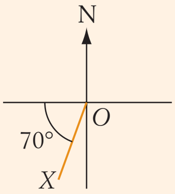
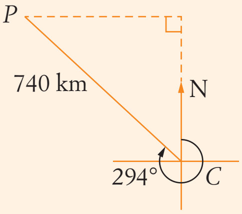


26 A plane flies on a bearing of 294° from Canberra for a distance of 740 km.

a How far north has the plane travelled (correct to the nearest kilometre)?

b What is the bearing of Canberra from the plane's position?

See Exercise 8-07



9

Number and Algebra

Simultaneous equations

Many scientific, natural, economic and social phenomena can be modelled by equations. Often these models consist of more than one equation. For example, when manufacturing milk, equations can be written that describe relationships between quantity, cost and income. These equations can then be solved simultaneously to obtain information on pricing and the quantities that need to be produced and sold to make a profit.

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Chapter outline

*STAGE 5.2

Wordbank

coefficient The numerical part of an algebraic term. For example, in $3x^2 + 7x - 1$ the coefficient of x is 7.

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables

graphical method A method of solving simultaneous equations that involves graphing them on a number plane and identifying the point(s) of intersection

simultaneous equations Two (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1, y = 3$.

substitution method A method of solving simultaneous equations that involves substituting one equation into another equation

In this chapter you will:

- (STAGE 5.2) solve linear simultaneous equations, using algebraic and graphical techniques including using digital technology AC
- solve linear simultaneous equations by graphing them on a number plane and finding the point of intersection of the lines
- solve linear simultaneous equations algebraically using the elimination and substitution methods
- solve problems using linear simultaneous equations

SkillCheck

Worksheet
StartUp assignment 9
MAT10NAWK10064

1 Given the equation $y = 2x + 5$, find y when:

a $x = 0$ **b** $x = 4$ **c** $x = \frac{1}{2}$ **d** $x = -3$

2 Given the equation $y = 4 - 3x$, find y when:

a $x = 5$ **b** $x = 1$ **c** $x = -1$ **d** $x = -\frac{1}{2}$

3 By completing a table of values, graph each equation.

a $y = x + 1$ **b** $y = 3x$ **c** $y = \frac{x}{2} - 1$
d $y = 3 - x$ **e** $x + y = 4$ **f** $2x - y = 5$

4 Test whether the point $(-2, 3)$ lies on the line represented by each equation.

a $y = 1 - x$ **b** $x + y = 3$ **c** $2x - y = 7$
d $\frac{1}{2}x + y = 2$ **e** $y = 3x + 7$ **f** $2y = 3x$

5 **a** Show that the point $(2, 5)$ lies on both the lines $y = 2x + 1$ and $x + y = 7$.
b At what point do these two lines intersect?

6 Use the y-intercept and the gradient to graph each equation.

a $y = -2x + 3$ **b** $y = \frac{5}{2}x - 2$ **c** $y = -\frac{4}{3}x + 5$

Investigation: When two lines meet

1 Copy and complete the table of values for each equation.

a $x + 2y = 0$

x	−2	−1	0	1	2
y					

b $y = x + 4$

x	−2	−1	0	1	2
y					

2 Which coordinates satisfy both equations?

3 On the same set of axes draw the graphs of $x + 2y = 0$ and $y = x + 4$.

4 **a** Do the lines you drew in question **3** intersect?
b What are the coordinates of the point of intersection?

9780170194655

5 Repeat questions **1** to **4** for these pairs of equations.

a $x - y = 5$
$2x + y = 1$

b $3x + y = 8$
$x + 2y = 1$

6 Copy and complete.

a The coordinates of the p_____ of intersection between two lines satisfy both equations.

b The values of x and y that satisfy both equations are the coordinates of the __________.

9-01 Solving simultaneous equations graphically

Stage 5.2

Worksheet
Testing simultaneous equations
MAT10NAWK10065

Worksheet
Intersection of lines
MAT10NAWK10066

Technology worksheet
Excel worksheet: Solving simultaneous equations
MAT10NACT10017

Technology worksheet
Excel spreadsheet: Simultaneous equations solver
MAT10NACT10047

A linear equation in one variable such as $3x + 5 = 11$ has only one solution for x ($x = 2$). However, a linear equation in **two** variables, such as $x + 3y = 5$, has more than one solution (for example, $x = 2$, $y = 1$, or $x = 5$, $y = 0$, and so on). The equation actually has an **infinite number** of solutions.

We will now look at solving two equations simultaneously to see if there is a solution that satisfies both equations.

Simultaneous equations can be solved **graphically** or **algebraically**.

Summary

- Linear simultaneous equations can be graphed as lines on the same number plane.
- If two (non-parallel) lines are drawn, the lines will intersect.
- At the point of intersection, the x-coordinate and y-coordinate represent the solution to the simultaneous equations.

Example 1

On the same set of axes, graph $3x + y = 4$ and $x + y = -2$, then solve the equations simultaneously.

Solution

Step 1

Construct tables of values.

$3x + y = 4$

x	0	1	2
y	4	1	−2

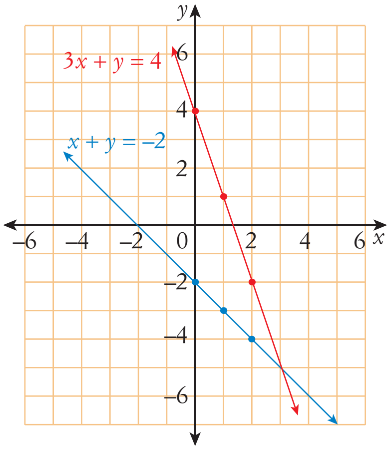
$x + y = -2$

x	0	1	2
y	−2	−3	−4

Stage 5.2

Step 2
Graph the equations.
The lines intersect at (3, –5).
∴ The solution of the simultaneous equations $3x + y = 4$ and $x + y = -2$ is $x = 3$, $y = -5$.

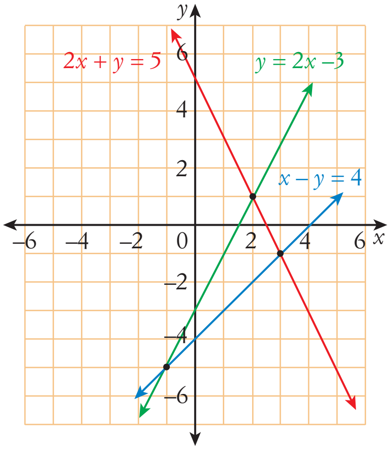
Check that $x = 3$, $y = -5$ satisfies both equations.



Exercise 9-01 Solving simultaneous equations graphically

1 Use the graph to write the solution to each pair of simultaneous equations.

a $x - y = 4$ and $2x + y = 5$
b $2x + y = 5$ and $y = 2x - 3$
c $x - y = 4$ and $y = 2x - 3$



See Example 1

2 Graph each pair of equations on the same set of axes. Then find the solution to the pair of simultaneous equations.

a $y = 2x$ and $y = 3 - x$
b $y = 2x + 1$ and $y = x - 4$
c $x + y = 3$ and $4x + y = 6$
d $y = -x + 2$ and $y = 3x + 4$
e $y = 2x - 5$ and $y = 5x + 1$
f $2x + y = 6$ and $y = 1 - x$
g $y = 7 - x$ and $y = 3x + 5$
h $x + 2y = 7$ and $2x - y = 4$
i $3x - 2y = 12$ and $x + 2y = 8$
j $y = x + 3$ and $2x - y = 2$
k $5x - y = 5$ and $x + y = 4$
l $5x + 3y = 20$ and $y = x - 4$

3 **a** On the same set of axes, draw the graphs of $y = 1 - 2x$ and $2x + y = 4$.
b Why isn't there a solution to the simultaneous equations $y = 1 - 2x$ and $2x + y = 4$?

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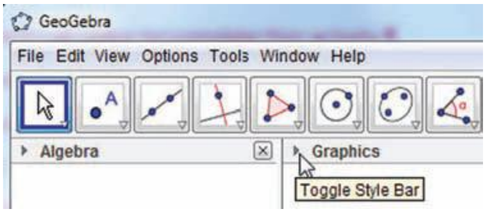
Technology Solving simultaneous equations graphically

You can use GeoGebra or other graphing software to solve simultaneous equations graphically. Write each answer as coordinates in the form (*x*, *y*) representing the **point of intersection**.

Using GeoGebra

Before you start, apply these settings:

1 Open up **GeoGebra** and click the little arrow in front of **Graphics**.



From the new panel that pops up, select the grid option at the top left-hand side.

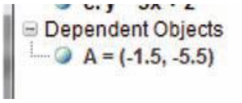
2 Enter these equations in the **Input bar**.

$y = -x + 1$

$y = x + 3$

3 If the points are difficult to read on screen, select Intersect Two Objects from the second icon drop-down menu.

4 In the **Algebra View** (left) you will see the exact coordinates of the point of intersection. They are listed as **Dependent Objects**.



Dependent Objects
A = (-1.5, -5.5)

5 Enter each pair of equations using step **2** above.

a $y = 2$
$x = 3$

b $y = -2x + 4$
$y = x - 5$

c $y = 5x + 2$
$y = 3x - 1$

d $y = -1$
$x = 0$

e $y = -x - 8$
$y = -3x + 4$

f $y = 2x + 6$
$y = x + 9$

g $y = -x$
$x = -4$

h $y = x + 4$
$y = x + 6$

i $y = -2x + 2$
$y = -2x$

HINT: Click Move Graphics View to locate the point of intersection

What do you notice about these equations? Do they intersect?

6 Enter each set of equations using step **2** above. Find the point of intersection.

a $y = 3x, y = -x + 2$ and $x = 0.5$

b $y = -4x + 1, y = -5x$ and $y = x + 6$

Stage 5.2

Technology

GeoGebra: Simultaneous equations

MAT10NATC00011

Video tutorial

Simultaneous equations

MAT10NAVT10027

9-02 The elimination method

Using graphs to solve simultaneous equations can be time-consuming and inaccurate. **Algebraic methods** provide a better way of solving things. There are two algebraic methods: the **elimination method** and the **substitution method**.
In the elimination method, equations are added or subtracted to eliminate one of the variables.

Example 2

Solve the simultaneous equations $x + 3y = 7$ and $4x - 3y = 13$.

Solution

Label each equation.

$$\begin{aligned} x + 3y &= 7 \quad [1] \\ 4x - 3y &= 13 \quad [2] \end{aligned}$$

Since there is the same number of ys in each equation, and since they are **opposite** in sign ($3y$ and $-3y$), **add** equations [1] and [2] to eliminate the variable y.

$$5x = 20 \quad [1] + [2]$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$\therefore x = 4$$

Substitute $x = 4$ into equation [1] to find the y-value.

$$\begin{aligned} x + 3y &= 7 \\ 4 + 3y &= 7 \\ 4 + 3y - 4 &= 7 - 4 \\ 3y &= 3 \\ \therefore y &= 1 \end{aligned}$$

$\therefore$ The solution is $x = 4$, $y = 1$.

Example 3

Solve $2k + 3m = 9$ and $2k - 5m = 1$.

Solution

Label each equation.

$$\begin{aligned} 2k + 3m &= 9 \quad [1] \\ 2k - 5m &= 1 \quad [2] \end{aligned}$$

Since there is the same number of ks in each equation, and because they have the **same** sign ($2k$ and $2k$), **subtract** equation [2] from [1] to eliminate k.

$$8m = 8 \quad [1] - [2]$$
$$\frac{8m}{8} = \frac{8}{8}$$
$$\therefore m = 1$$

Substitute $m = 1$ into equation [1] to find the value of k.

$$2k + 3m = 9$$
$$2k + 3 \times 1 = 9$$
$$2k + 3 = 9$$
$$2k + 3 - 3 = 9 - 3$$
$$2k = 6$$
$$\frac{2k}{2} = \frac{6}{2}$$
$$\therefore k = 3$$

$\therefore$ The solution is $m = 1$, $k = 3$.

Example 4

Solve $3a + 4c = 8$ and $2a - 3c = 11$.

Solution

Label each equation.

$$3a + 4c = 8 \quad [1]$$
$$2a - 3c = 11 \quad [2]$$

In this case, neither adding nor subtracting equations [1] and [2] will eliminate a variable. Let's choose to eliminate c. We need to make the **coefficient** of c the same in both equations ($12c$).

The **coefficient** of c is the number in front of the c in the equation

$$9a + 12c = 24 \quad [3]$$ Multiplying both sides of [1] by 3.
$$8a - 12c = 44 \quad [4]$$ Multiplying both sides of [2] by 4.
$$17a = 68 \quad [3] + [4]$$
$$\therefore a = 4$$

Substitute $a = 4$ in [1] to find c.

$$3a + 4c = 8$$
$$3 \times 4 + 4c = 8$$
$$12 + 4c = 8$$
$$4c = -4$$
$$c = -1$$

$\therefore$ The solution is $a = 4$, $c = -1$.

Stage 5.2

Exercise 9-02 The elimination method

See Example 2

1 For each pair of simultaneous equations, eliminate one variable by adding the equations, then solve the equations.

a $4k + d = 5$
$2k - d = 7$

b $2x - w = 6$
$x + w = 9$

c $3g + 5h = 4$
$2g - 5h = 6$

d $7p - 4n = -20$
$3p + 4n = 10$

e $4q + 3r = 8$
$-q - 3r = 7$

f $-5k - 3x = 8$
$5k + 4x = -3$

g $-4c - 6e = -12$
$4c - 10e = -4$

h $-3y + 5k = 21$
$3y + k = -3$

i $a + 3f = 8$
$-a + 4f = 6$

See Example 3

2 For each pair of simultaneous equations, eliminate one variable by subtracting the equations, then solve the equations.

a $5k + d = 16$
$3k + d = 4$

b $4a + 3c = 7$
$a + 3c = 4$

c $4h + 3y = 24$
$4h - y = 8$

d $3x + 5e = 16$
$3x - 2e = -5$

e $4q - 2w = -1$
$7q - 2w = 8$

f $6p + c = 16$
$4p + c = 10$

g $5y + 3m = 18$
$2y + 3m = 6$

h $3a + 2r = 8$
$a + 2r = 10$

i $-x + 5w = 8$
$-x + 3w = 4$

See Example 4

3 Solve each pair of simultaneous equations.

a $3w + q = 6$
$2w - 3q = 15$

b $2x + m = 5$
$3x + 2m = 3$

c $2d + 3h = 25$
$d + 4h = -5$

d $-3g + 2n = 9$
$g + 5n = 14$

e $5m - h = 10$
$m - 3h = 2$

f $2y + 3e = -6$
$5y - 2e = 23$

g $3q - 2w = 11$
$2q - 5w = 22$

h $5a + 3d = 4$
$4a + 2d = 3$

i $-2p + 3k = 19$
$7p + 4k = 6$

j $5a + 2f = -14$
$2a - 3f = 2$

k $5r - 3c = 2$
$-3r + 2c = -14$

l $5y - 4x = 1$
$2y - 3x = 6$

9-03 The substitution method

Stage 5.2

Puzzle sheet
Simultaneous equations order activity
MAT10NAPS10067

Puzzle sheet
Simultaneous equations by substitution
MAT10NAPS00039

With the substitution method, substitute the x or y variables from one equation into the other equation.

Example 5

Solve the simultaneous equations $y = x + 4$ and $y = 3x - 2$.

Solution

Label each equation.

$$\begin{aligned} y &= x + 4 \quad [1] \\ y &= 3x - 2 \quad [2] \end{aligned}$$

Use equation [1] to substitute for y in equation [2] and solve for x.

$$\begin{aligned} x + 4 &= 3x - 2 \\ x + 4 - 3x &= 3x - 2 - 3x \\ -2x + 4 &= -2 \\ -2x + 4 - 4 &= -2 - 4 \\ -2x &= -6 \\ \frac{-2x}{-2} &= \frac{-6}{-2} \\ x &= 3 \end{aligned}$$

Now substitute $x = 3$ in equation [1] to find y.

$$\begin{aligned} y &= x + 4 \\ y &= 3 + 4 \\ &= 7 \end{aligned}$$

$\therefore$ The solution is $x = 3$ and $y = 7$.

Example 6

Video tutorial
Simultaneous equations
MAT10NAVT10027

Solve the simultaneous equations $5x + 3y = 9$ and $y = 7 - 3x$.

Solution

Label each equation.

$$\begin{aligned} 5x + 3y &= 9 \quad [1] \\ y &= 7 - 3x \quad [2] \end{aligned}$$

Since y is the subject in [2], substitute equation [2] into equation [1] to give an equation using x only.

Stage 5.2

$$\begin{aligned} 5x + 3(7 - 3x) &= 9 \\ 5x + 21 - 9x &= 9 \\ -4x &= -12 \\ \frac{-4x}{-4} &= \frac{-12}{-4} \\ x &= 3 \end{aligned}$$

Now substitute $x = 3$ into equation [2] to find y.

$$\begin{aligned} y &= 7 - 3x \\ y &= 7 - 3 \times 3 \\ &= -2 \end{aligned}$$

∴ The solution is $x = 3$ and $y = -2$.

Exercise 9-03 The substitution method

See Example 5

1 Use the substitution method to solve each pair of simultaneous equations.

a $y = 2x + 1$ and $y = x + 3$ **b** $y = 5 - 2x$ and $y = 3x + 2$
c $x = 3 + 2y$ and $x = 9 - y$ **d** $y = -x$ and $y = 3x - 8$
e $x = 1 - 4y$ and $x = 2y + 7$ **f** $x = 2y$ and $x = 6 - y$

See Example 6

2 Solve each pair of simultaneous equations.

a $y = 2x + 3$ and $3x - y = 6$ **b** $y = x - 2$ and $3x + y = 18$
c $y = 1 - 4x$ and $4x + 2y = 3$ **d** $x = 2y - 5$ and $4x - y = -13$
e $x = 3y - 4$ and $5x - 4y = 2$ **f** $x = 5 - 3y$ and $4y - x = 23$
g $2x - 5y = -1$ and $y = 10 - x$ **h** $6y - 2x = 9$ and $y = \frac{x+2}{2}$
i $x = \frac{9-y}{3}$ and $3x + 2y = 10$ **j** $y = 3x + 5$ and $4x - 3y = 1$

Investigation: Elimination or substitution method?

With two algebraic methods of solving simultaneous equations, often it is more efficient to use one method than another.

1 Consider these pairs of simultaneous equations.

a $x - 2y = 9$ [1] **b** $4a + 3c = 18$ [1] **c** $3a - 2y = -5$ [1]
$3x + 2y = 11$ [2] $4a - 3c = -6$ [2] $2a + 5y = 3$ [2]

i Why might the elimination method be the more appropriate method to use with these equations?
ii What feature in the pairs of equations do you look for to decide if the elimination method is the best one to use?
iii Solve the three pairs of simultaneous equations using the elimination method.

9780170194655

2 Consider these pairs of simultaneous equations.

a $m = 2p$ [1]
$m + p = 15$ [2]

b $m = 4 - p$ [1]
$4m - 3p = -6$ [2]

c $p = 2m - 5$ [1]
$5m - 3p = 11$ [2]

i Why might the substitution method be the more appropriate method to use with these equations?

ii What feature in the pairs of equations do you look for to decide if the substitution method is the best one to use?

iii Solve the three pairs of simultaneous equations using the substitution method.

3 Using whichever method is more efficient, solve each of these pairs of simultaneous.

a $7c + 2y = 13$ [1]
$3c + 2y = 1$ [2]

b $m = 5 - k$ [1]
$2m - k = 4$ [2]

c $3x + 8y = 10$ [1]
$x = 3 - 2y$ [2]

d $4h - 3w = 8$ [1]
$4h + 7w = 12$ [2]

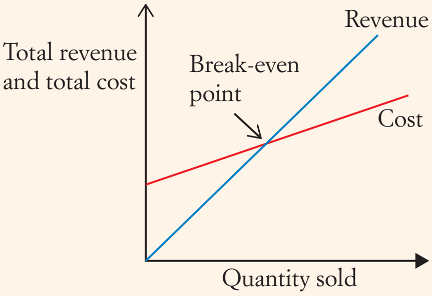
e $3d = q$ [1]
$q + 4d = 14$ [2]

f $3h + 5r = 7$ [1]
$2h - 3r = -8$ [2]

Just for the record

Break-even point

Manufacturers use simultaneous equations to make decisions about how many products they should make and sell. Linear equations can be formed to determine total revenue (the amount made from selling products) and total costs (the cost of making the products).
Total revenue = cost per item × number of items made, while total costs includes rent and production costs.



The equations can be graphed as shown. The point where the two lines intersect is called the **break-even point** and occurs when total revenue is equal to total cost.
A publisher receives \$35 per book sold. There are fixed costs of \$110 000 and production costs per book are \$8.50.

a Determine the equations for total revenue and total costs.

b Graph the equations to find the break-even point.

c How many books must be sold before the publisher makes a profit?

Stage 5.2

Worksheet
Simultaneous equations problems
MAT10NAWK10068

Animated example
Simultaneous equations
MAT10NAAE00011

9-04 Problems involving simultaneous equations

Sometimes, worded problems can be solved using simultaneous equations.

- Read the problem carefully
- Identify the variables to be used
- Use the variables to write simultaneous equations from the information given in the problem
- Solve the equations
- Solve the problem by answering in words

Example 7

At an art show there were 520 guests. If there were 46 more women than men, how many women attended the show?

Solution

Let the number of women attending be w.

Let the number of men attending be m.

$$\therefore w + m = 520 \quad [1]$$

520 people altogether.

$$\text{and } w = m + 46 \quad [2]$$

46 more women than men.

Use equation [2] to substitute for w in equation [1].

$$\begin{aligned} m + 46 + m &= 520 \\ 2m + 46 &= 520 \\ 2m + 46 - 46 &= 520 - 46 \\ 2m &= 474 \\ m &= 237 \end{aligned}$$

Substitute $m = 237$ into equation [2] to find w.

$$\begin{aligned} w &= 237 + 46 \\ &= 283 \end{aligned}$$

∴ There were 283 women who attended the art show.

Example 8

Anita and Ben spent \$931 on shrubs and trees for their new home. Altogether they bought 70 plants. The shrubs cost \$11 each while the trees cost \$18 each.
How many of each plant did they buy?

123rf/Shariff Che'Lab

Solution

Let x be the number of shrubs.

Let y be the number of trees.

$\therefore x + y = 70$ [1]

and $11x + 18y = 931$ [2]

Neither adding nor subtracting equations [1] and [2] will eliminate a variable. Let's choose to eliminate x. We will need to make the coefficient of x the same in both equations ($11x$).

$11x + 18y = 931$ [2]

$11x + 11y = 770$ [3] Multiplying both sides of equation [1] by 11.

$7y = 161$ [2] − [3]

$y = 23$

Substitute $y = 23$ in [1] to find the value of x.

$x + y = 70$

$x + 23 = 70$

$\therefore x = 47$

So Anita and Ben bought 47 shrubs and 23 trees.

Exercise 9-04 Problems involving simultaneous equations

1 At a school concert there were 640 guests. There were 70 more women than men. How many of the audience were men? *See Example 7*

2 At a circus, there were twice as many children as there were adults in attendance. Altogether, 1020 attended the circus. How many were children?

3 Tickets to a concert cost \$5 for children and \$14 for adults. Altogether, 650 people attended the concert and ticket sales totalled \$5824. Let a stand for the number of adults and c stand for the number of children. *See Example 8*

a Explain why the equations $a + c = 650$ and $14a + 5c = 5824$ correctly match the information.

b Solve the equations simultaneously to find the number of children that attended the concert.

4 Tracey bought a total of 17 DVDs and CDs. Each DVD cost her \$25 and each CD cost \$18. Altogether, Tracey spent \$390. How many DVDs did she buy?

5 Aaron is three times as old as Sejuti. The sum of their ages is 48. How old are Aaron and Sejuti?

6 The sum of the ages of Mrs Bui and her daughter Hayley is 70. The difference between their ages is 38 years. How old is Hayley?

Stage 5.2

7 A business bought a total of 60 ink cartridges. Some of them were black, costing \$42 each. The others were colour, each costing \$35. How many of each type did the business buy if the total cost of the ink cartridges was \$2352?

8 Five pies and two sausage rolls cost a total of \$23.40, while two pies and 3 sausage rolls cost \$15.30. Find the cost of a pie and the cost of a sausage roll.

9 Pete's Pizzas sells Supreme pizzas for \$15.90 each and Vegetarian pizzas for \$13.50 each. If 45 pizzas were sold at lunchtime, totalling \$684.30, how many of each pizza were sold?

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10 Nasser bought 3 punnets of strawberries and 5 punnets of blueberries for \$35.45 and Sarah bought 7 punnets of strawberries and 2 punnets of blueberries for \$34.48. What was the cost of each punnet of strawberries and blueberries?

11 A money box contains only 20-cent coins and 50-cent coins. Altogether, there are 245 coins in the money box and they amount to \$76.30. Let x be the number of 20-cent coins and y be the number of 50-cent coins.

a Explain why the equations $x + y = 245$ and $20x + 50y = 7630$ correctly match the information.

b Solve the equations to determine the number of 20-cent and 50-cent coins in the money box.

12 The initial cost for producing bottles of fresh orange juice is \$135 plus \$1.20 for each bottle. The bottles of juice are sold for \$3 each. C is the cost in dollars, R is the total sales in dollars and n is the number of bottles produced and/or sold.

a Explain why the equations $C = 135 + 1.2n$ and $R = 3n$ correctly match the information.

b Copy and complete the tables of values below for both equations.

$C = 135 + 1.2n$

n	0	50	100
C			

$R = 3n$

n	0	50	100
R			

c Draw the graphs of both equations on the same axes for values of 0 to 100 for n on the horizontal axis and values of \$0 to \$300 on the vertical axis.

d For what value of n is total sales equal to total cost (the break-even point)?

Mental skills 9 Maths without calculators

Simplifying fractions and ratios

When simplifying a fraction or a ratio, look for a common factor to divide into both the numerator and the denominator, preferably the highest common factor (HCF).

1 Study each example.

a Simplify $\frac{27}{45}$

$\frac{\cancel{27}^{9}}{\cancel{45}_{15}} = \frac{9}{15}$ Dividing numerator and denominator by 3.

$\frac{\cancel{9}^{3}}{\cancel{15}_{5}} = \frac{3}{5}$ Dividing numerator and denominator by 3 again.

Note: This fraction could be simplified in one step if you divided by 9, the highest common factor (HCF) of 27 and 45.

b Simplify $\frac{160}{400}$.

$\frac{16\cancel{0}}{40\cancel{0}} = \frac{16}{40}$ Dividing numerator and denominator by 10.

$\frac{\cancel{16}^{2}}{\cancel{40}_{5}} = \frac{2}{5}$ Dividing numerator and denominator by 8.

Note: This fraction could be simplified in one step if you divided by 80, the HCF of 160 and 400.

c Simplify 24 : 36.

$24 : 36 = \cancel{24}^{4} : \cancel{36}^{6} = 4 : 6$ Dividing both terms by 6.

$\cancel{4}^{2} : \cancel{6}^{3} = 2 : 3$ Dividing both terms by 2.

Note: This fraction could be simplified in one step if you divided by 12, the HCF of 24 and 36.

d Simplify 135 : 90.

$135 : 90 = \cancel{135}^{27} : \cancel{90}^{18} = 27 : 18$ Dividing both terms by 5.

$\cancel{27}^{3} : \cancel{18}^{2} = 3 : 2$ Dividing both terms by 9.

e Calculate $\frac{3}{8} \times \frac{2}{15}$ in simplest form.

$\frac{3}{8} \times \frac{2}{15} = \frac{3}{{}_{4}\cancel{8}} \times \frac{\cancel{2}^{1}}{15}$ Dividing 2 and 8 by 2.

$= \frac{{}^{1}\cancel{3}}{{}_{4}\cancel{8}} \times \frac{\cancel{2}^{1}}{\cancel{15}_{5}}$ Dividing 3 and 15 by 3.

$= \frac{1}{20}$

f What fraction is 36 minutes of 1 hour?

$\frac{36}{1 \text{ h}} = \frac{36 \text{ min}}{60 \text{ min}} = \frac{3}{5}$

2 Now simplify each fraction or ratio.

a $\frac{10}{15}$ **b** $\frac{16}{20}$ **c** $\frac{30}{42}$ **d** $\frac{8}{16}$

e $\frac{20}{80}$ **f** $\frac{6}{36}$ **g** $\frac{20}{24}$ **h** $\frac{12}{30}$

i 20 : 36 **j** 25 : 45 **k** 18 : 40 **l** 28 : 35

m 27 : 21 **n** 16 : 12 **o** $\frac{5}{6} \times \frac{18}{25}$ **p** $\frac{12}{50} \times \frac{10}{21}$

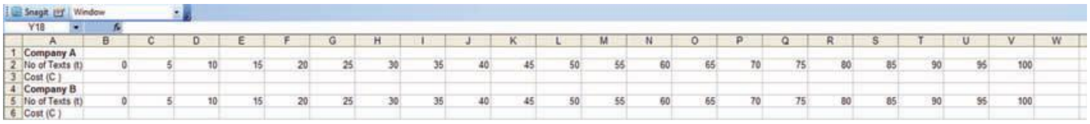
3 Express each as a simplified fraction.

a 425 g of 1 kg **b** 8 months of 1 year **c** 64 cm of 1 m

d 750 mL of 3 L **e** 10 hours of 2 days **f** 80c of $10

Technology SMS plans

Use a spreadsheet to solve the problem below.
Madhu wants to change her mobile phone provider. She is looking for a better deal on text messages as this is her preferred method of contacting her friends and family. After Madhu has researched various company rates, she decides on a comparison of 2 providers.
Company A: Costs $49 per month plus 25 cents per SMS, per recipient.
Company B: Monthly fee is $35 but charges 42c per SMS, per recipient.
To calculate and graph the cost of each mobile phone text messaging plan, create a spreadsheet for both companies for $t = 0, 5, 10, 15, 20, \ldots, 95, 100$ text messages.



	A	B	C	D	E	F	G	H	I	J	K
1	Company A										
2	No of Texts (t)	0	5	10	15	20	25	30	35	40	45
3	Cost (C)										
4	Company B										
5	No of Texts (t)	0	5	10	15	20	25	30	35	40	45
6	Cost (C)										

	A	L	M	N	O	P	Q	R	S	T	U	V	W
1	Company A												
2	No of Texts (t)	50	55	60	65	70	75	80	85	90	95	100	
3	Cost (C)												
4	Company B												
5	No of Texts (t)	50	55	60	65	70	75	80	85	90	95	100	
6	Cost (C)												

1 In cell B3, enter the formula for *Company A*. Formula: =**49+0.25*B2**
Fill Right to continue the formula from cell B3 to V3.

2 Highlight B3 to V3 and B6 to V6. Click **Format Cells** to convert to currency (to 2 decimal places).

3 In cell B6, create a formula for *Company B*. [Hint: a formula must start with =]
Fill Right to continue the formula from cell B6 to V6.

4 Create an **XY Scatter** graph of the data for rows 2, 3 and 6 only.
Give your graph an appropriate title and label both axes.

5 Answer the following questions in the specified cells on your spreadsheet.
[Hint: some questions might need a formula]

a Cell A8: For small numbers of text messages, which company charges more?

b Cell A9: How much more did Company A charge if 30 text messages were sent per month? [Write a formula]

c Cell A10: How much cheaper was Company B if 45 text messages were sent in the month? [Write a formula]

d Cell A11: Between which 2 values for text messages sent per month did the graphs intersect? (e.g., between 45 and 50). Explain the significance of the point of intersection between the two graphs.

e If Madhu's mother sets a limit of \$60 per month for text messaging:

i A12: What is the most number of text messages that Madhu could possibly send? [Answer correct to nearest whole number]

ii A13: How many **more** texts will Madhu be able to send with one company than the other?

f A14: Extend rows 2, 3, 5 and 6 to 120 text messages sent per month.

g **i** A15: If Madhu's mother sets a limit of 60 text messages per month, which is the better plan? Give a reason for your answer.

ii A16: If Madhu starts a part time job and contributes \$20 extra per month to the limit her mother has set, how many **more** texts will Madhu be able to send? [Answer correct to nearest whole number]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Power plus

1 With simultaneous equations in two variables, we have two equations to solve. With simultaneous equations in **three** variables, we have **three** equations to solve.

Step 1: Take two of the equations and eliminate one of the variables.
Step 2: Take another two of the equations and eliminate the same variable.
Step 3: Solve the two new simultaneous equations from Steps 1 and 2.
Step 4: Use substitution to find the values of the other two variables.

Use the above steps to solve the following sets of simultaneous equations.

a $2x + y - 3w = -16$
$x - y + 4w = 25$
$3x - y + 2w = 19$

b $3a - 2c + d = 5$
$5a + 2c + d = 25$
$4a + 3c - d = 10$

c $2m + 3n - p = 9$
$3m - 2n + 5p = 27$
$4m + 3n + 2p = 13$

2 **a** Show that the solutions to the simultaneous equations $ax + by = c$ and $dx + ey = f$ are $x = \dfrac{ce - bf}{ae - bd}$ and $y = \dfrac{af - cd}{ae - bd}$.

b The above solutions do not work when $ae = bd$. Explain why.

c Solve the equations $3x - 2y = 11$ and $5x + y = 14$ by either the substitution or elimination method. Check that the results in part **a** give the same answer.

d Set up a spreadsheet to solve simultaneous equations of the form $ax + by = c$ and $dx + ey = f$ using the solutions $x = \dfrac{ce - bf}{ae - bd}$ and $y = \dfrac{af - cd}{ae - bd}$.

Use your spreadsheet to solve the simultaneous equations below.

i $3x + y = 4$
$2x - y = 6$

ii $3x - 5y = 4$
$2x - 3y = 8$

iii $15x + 6y = 17$
$2x + 3y = 8$

Chapter 9 review

Language of maths

Puzzle sheet

Simultaneous equations crossword

MAT10NAPS10069

algebraic
axes
coefficient
elimination method
graphical
linear
point of intersection
satisfy
simultaneous equations
solution
substitution method
variable

1 How do you think **simultaneous equations** got their name?

2 What are the two algebraic methods for solving simultaneous equations?

3 Which algebraic method involves cancelling one of the variables?

4 What word means the answer to an equation or problem?

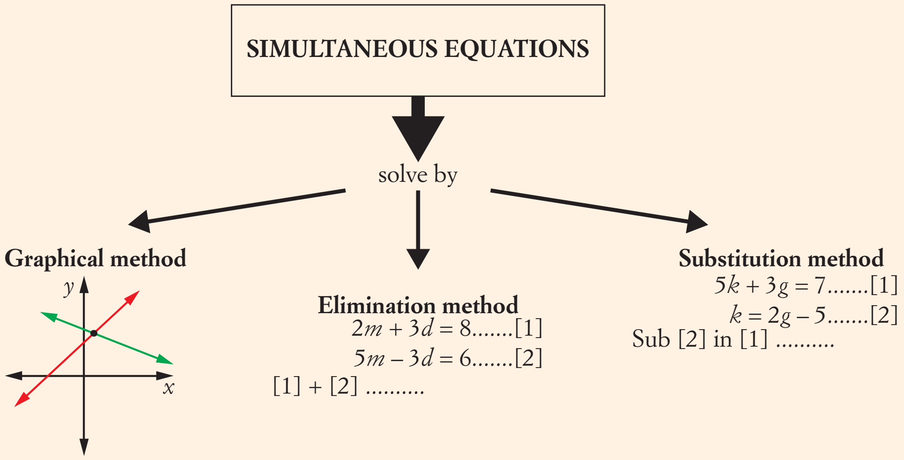
5 What does '**linear**' mean?

6 Which method of solving simultaneous equations involves the point of intersection of lines on a number plane?

Topic overview

- In your own words, write down the new things you have learnt about simultaneous equations.
- What parts of this topic did you like?
- What parts of the topic did you find difficult or not understand?
- Copy and complete the following topic overview, and refer to the **Language of maths** word list for keywords you might like to include.

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Chapter 9 revision

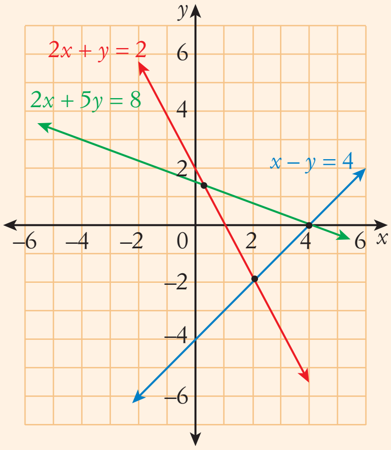
Stage 5.2

1 Use the graph to write the solution to each pair of simultaneous equations.

See Exercise 9-01

a $x - y = 4$ and $2x + y = 2$

b $2x + 5y = 8$ and $x - y = 4$



2 Graph each pair of simultaneous equations on the same set of axes. By finding their point of intersection, write the solution to each pair of equations.

See Exercise 9-01

a $y = x + 2$
$y = 6 + 2x$

b $y = 3 - \frac{x}{2}$
$y = 2x - 7$

c $y = 4 - 3x$
$y = x$

d $y = 2x + 3$
$y = 9 - x$

e $x + y = 7$
$y = 2x + 1$

f $y = 5 - 2x$
$y = -1 - x$

3 Use the elimination method to solve each pair of simultaneous equations.

See Exercise 9-02

a $5m + 2c = 6$
$3m + 2c = -4$

b $2x + 3y = 5$
$5x - 3y = 9$

c $3a + 4d = 7$
$3a + d = 4$

d $4x - y = 9$
$x - y = -9$

e $x - 4y = 3$
$x + 2y = -9$

f $3d - 2w = 11$
$2d - 5w = 44$

4 Use the substitution method to solve each pair of simultaneous equations.

See Exercise 9-03

a $y = 7x - 3$
$y = x + 9$

b $m = 4 - p$
$m = -2 + p$

c $h = 3t - 2$
$h = t + 6$

d $a = 4 - 2c$
$a = 6c$

e $x + 2y = 3$
$y = 2 - x$

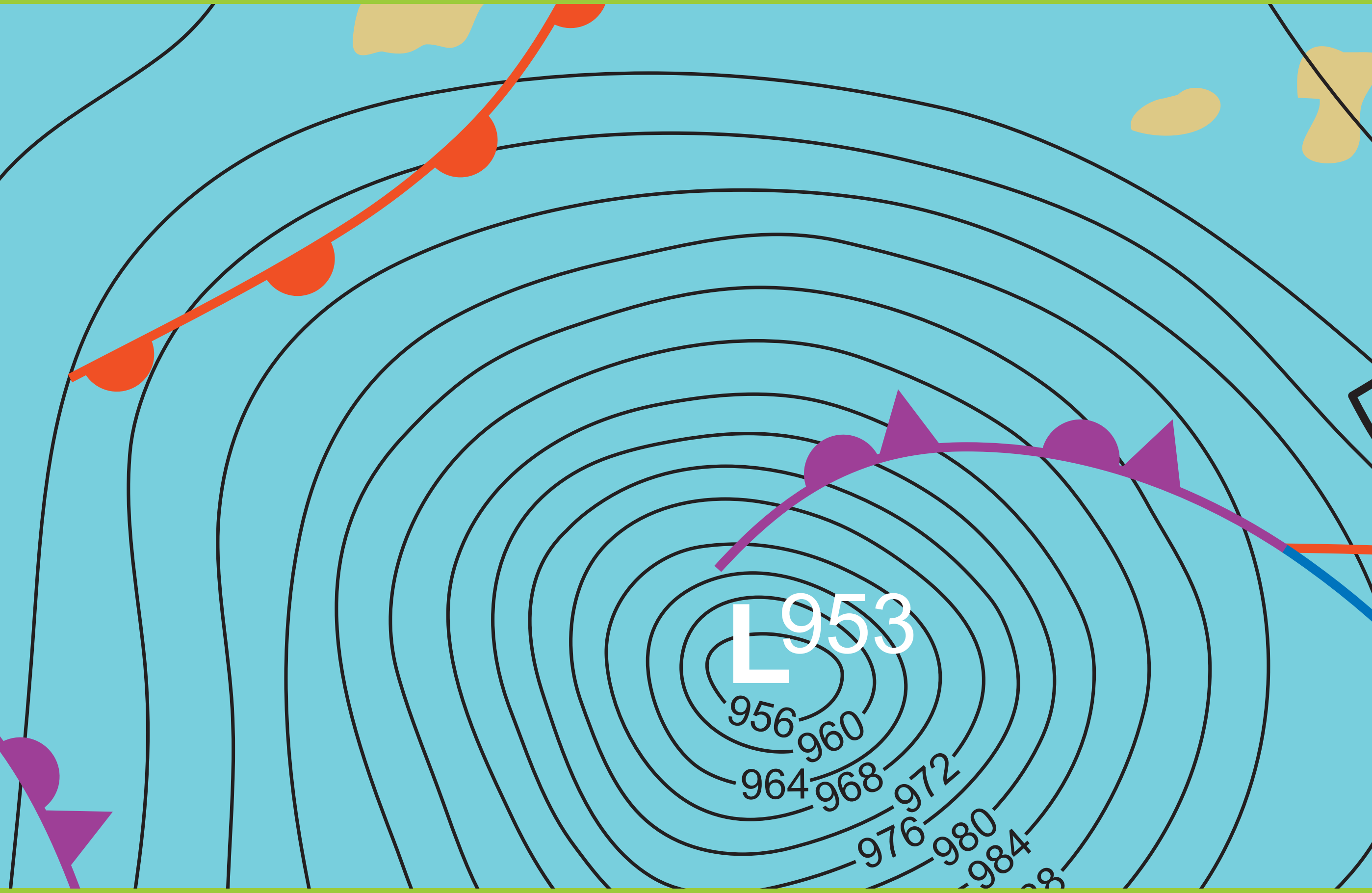
f $p = 4 - 2q$
$p = 3q + 24$

Stage 5.2

See Exercise 9-04

5 Solve each problem using simultaneous equations.

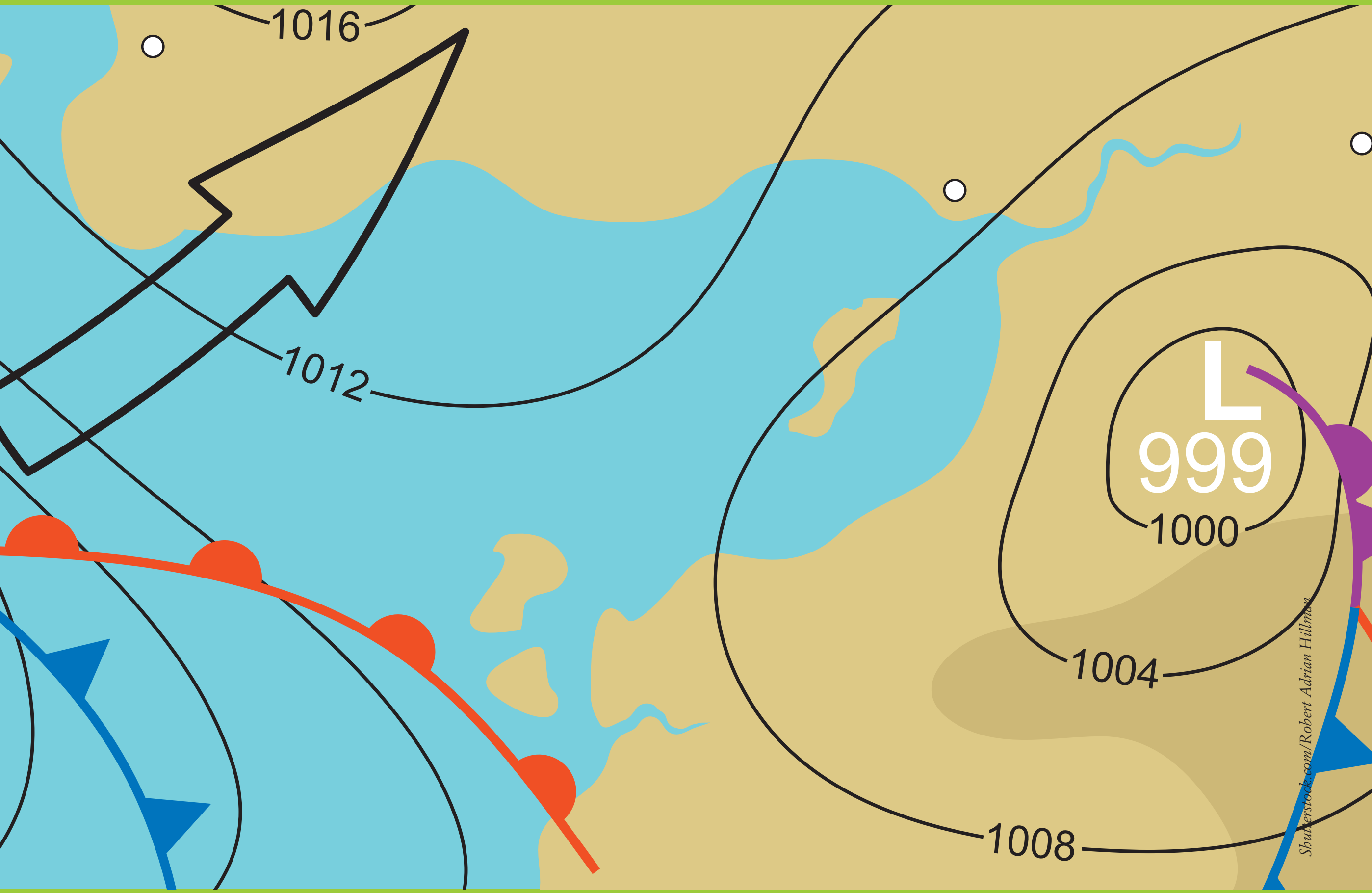
- **a** In an audience of 2500, there were 700 more adults than children. Find the number of adults and the number of children that were in the audience.
- **b** Robyn bought 30 CDs and DVDs for a total cost of $696. Each CD cost $25 and each DVD cost $22. How many of each did she buy?
- **c** It costs 2 adults and 5 children $191 to go to a football game, while the cost of 3 adults and 2 children is $160. Find the cost of an adult ticket.
- **d** At the cake stall, the SRC sell two types of cakes – cheesecakes for $4 each and mudcakes for $3 each. In total, they sold 75 cakes for a total of $253. How many of each cake did they sell?
- **e** In Year 10, there are 213 students. There are 27 more boys than girls. Find the number of boys and girls in Year 10.



Statistics and Probability

10 Probability

Probability theory, the study of chance, began in the 17th century when two great mathematicians, Blaise Pascal and Pierre de Fermat, corresponded over problems arising from games of chance. Since then, probability has become an essential branch of mathematics that is used widely in fields such as weather forecasting, finance, insurance, politics and risk assessment of new medical treatments.



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Chapter outline

	Proficiency strands				
10-01 Relative frequency	U	F	PS	R	C
10-02 Venn diagrams	U	F	PS	R	C
10-03 Two-way tables	U	F	PS	R	C
10-04 Two- and three-step experiments*	U	F	PS	R	C
10-05 Selecting with and without replacement*	U	F	PS	R	C
10-06 Dependent and independent events*	U	F	PS	R	C
10-07 Conditional probability*	U	F	PS	R	C

*STAGE 5.2

Wordbank

relative frequency The frequency of an event over repeated trials as a fraction of the total number of trials

tree diagram A diagram of branches for listing all the possible outcomes of a multi-step chance experiment

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-step experiment A chance experiment with two steps or stages, such as rolling a pair of dice

two-way table A way of grouping items into two overlapping categories, such as gender and the ability to drive a car

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories

In this chapter you will:

- calculate relative frequencies from given and collected data to estimate probabilities of events involving 'and' or 'or'
- describe events using language of probability of 'at least', 'A or B but not both', 'A or B or both' and 'and'
- (STAGE 5.2) list all outcomes for two- and three-step chance experiments, with and without replacement, using tree diagrams or arrays, and assign probabilities to outcomes and determine the probabilities for events
- represent events in Venn diagrams and two-way tables and solve related problems
- (STAGE 5.2) understand the difference between dependent and independent events, and the product rule for independent events
- (STAGE 5.2) solve problems involving compound events and conditional probability

SkillCheck

Worksheet
StartUp assignment 10
MAT10SPWK10070

1 If a die is rolled, which of the following is more likely? Select the correct answer **A**, **B**, **C** or **D**.

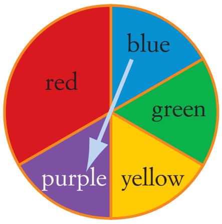
A a number less than 3 **B** an even number
C a number that is 3 or more **D** a number that is a prime number

2 A bag contains five 10-cent coins, four 20-cent coins and three 50-cent coins. A coin is drawn at random from the bag.

a How many outcomes are in the sample space?
b Are the outcomes equally likely?

3 For the spinner shown, the red sector is twice as large as each of the other sectors. Find the probability that when the spinner is spun, the arrow lands on:

a red **b** purple or blue **c** not green



4 A normal die is rolled. What is the probability of rolling:

a a 7? **b** a number less than 7?

5 The probability of Danica hitting a bullseye when playing darts is 0.6. What is the probability of Danica not hitting the bullseye?

6 Aditi bought 10 tickets in a raffle in which 400 tickets were sold and there is only one prize. What is the probability that Aditi will win the prize? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{400}$ **B** 2.5% **C** 0.0025 **D** 0.975

7 The probability of cloudy skies tomorrow is 0.85. What is the probability of clear skies?

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10-01 Relative frequency

Technology worksheet
Excel worksheet: Long-run proportion
MAT10SPCT00023

Technology worksheet
Excel spreadsheet: Long-run proportion
MAT10SPCT00053

Puzzle sheet
Dice probability
MAT10SPPS00017

Probability that is calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is called the **theoretical probability** (or **calculated probability**).
We can also determine probability based on the results of a trial that has been repeated many times, such as testing the effectiveness of 100 light globes, or rely on past statistics, such as the number of babies born last year that were girls. This type of probability is called **experimental probability**, which is based on a **relative frequency**–the number of times an event occurs compared to the total frequency of outcomes.

Summary

Experimental probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

Expected frequency = theoretical probability × number of trials.

Example 1

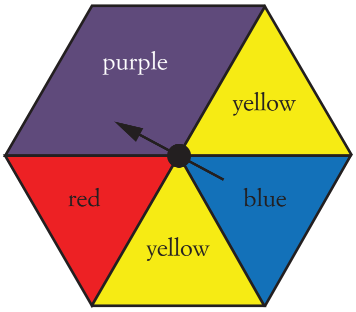
This spinner was spun 160 times and the results are shown in the table.

a Calculate, as a decimal:
  i the experimental probability that the arrow stops on blue
  ii the theoretical probability that the arrow stops on blue.

b Are the experimental and theoretical probabilities similar?

c If the spinner is spun 500 times, calculate the expected frequency of stopping on purple based on the theoretical probability.

Outcome	Frequency
Red	30
Blue	32
Yellow	50
Purple	48



Solution

a **i** Experimental probability:

$$P(\text{Blue}) = \frac{32}{160} = 0.2$$

ii Theoretical probability:

$$P(\text{Blue}) = \frac{1}{6} = 0.1666\ldots \approx 0.17$$

b By comparing the decimals for the two answers, we see that the experimental and the theoretical probabilities are similar.

c Expected frequency of purple $= \frac{1}{3} \times 500 = 166.6666\ldots \approx 167$

Probability × number of trials

Example 2

James rolled a die 100 times and recorded the results in a table.

Outcome	Frequency
1	23
2	19
3	11
4	12
5	18
6	17

a Find the experimental probability of rolling:
  i an even number
  ii an even number or a number greater than 4
  iii an even number less than or equal to 4.

b Calculate the probability of rolling a 2 or 3
  i as an experimental probability
  ii as a theoretical probability.

c If the die is rolled 100 times, what is the expected number of times of obtaining a 2 or a 3? How does this compare with James' observed number of times?

Solution

a **i** Rolls of even numbers $= 19 + 12 + 17$
$= 48$

Frequencies of 2, 4, 6

$$\text{Experimental probability } P(\text{even}) = \frac{48}{100} = \frac{12}{25}$$

ii Rolls of even numbers or numbers greater than 4
$= 19 + 12 + 17 + 18$
$= 66$

Frequencies of 2, 4, 6, 5

$$\text{Experimental probability } P(\text{even or} > 4) = \frac{66}{100} = \frac{33}{50}$$

iii Rolls of even numbers less than or equal to 4 $= 19 + 12$
$= 31$

Frequencies of 2 and 4

Experimental probability

$$P(\text{even and} \leq 4) = \frac{31}{100}$$

b **i** Rolls of 2 or 3 $= 19 + 11$
$= 30$

Frequencies of 2 and 3

$$\text{Experimental probability } P(2 \text{ or } 3) = \frac{30}{100} = \frac{3}{10}$$

ii Theoretical probability

$$P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$$

c Expected number of rolls of 2 or 3

$$= \frac{1}{3} \times 100 = 33.333\ldots \approx 33$$

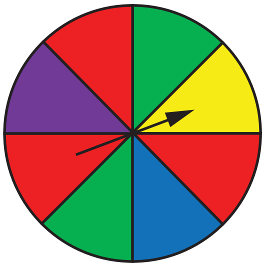
Probability × number of trials

From the table, the observed number of heads $= 19 + 11 = 30$, which is close to 33.

Exercise 10-01 Relative frequency

See Example 1

1 Aashima spun this spinner 200 times and recorded the results in the table.



Event	Frequency
Red	85
Green	42
Blue	28
Yellow	15
Purple	30

a Calculate, as a decimal, the experimental probability (relative frequency) that the arrow points to:

i red **ii** blue **iii** green

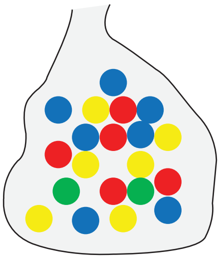
b Calculate, as a decimal, the theoretical probability that the arrow points to:

i red **ii** blue **iii** green

c Are the experimental and theoretical probabilities similar?

d For 200 spins, what is the expected frequency of red or purple based on the theoretical probability? How does this compare with the observed frequency?

2 A bag contains 7 blue, 6 yellow, 5 red and 2 green marbles. Lamisa selects a marble at random, records its colour and then returns it to the bag. Lamisa repeats this process 100 times and the results are shown in the table.



Outcome	Frequency
Red	20
Blue	38
Yellow	33
Green	9

a Find the relative frequency of selecting a marble that is:

i red **ii** blue **iii** yellow **iv** green

b What is the theoretical probability of selecting a marble that is:

i red? **ii** blue? **iii** yellow? **iv** green?

c Are the experimental and theoretical probabilities similar?

d If the process is repeated 100 times, what is the expected frequency of a selecting a yellow or green marble? How does this compare with the observed frequency?

3 A coin is tossed.

Outcome	Frequency
Head	
Tail	

a What is the expected number of heads if the coin is tossed 100 times?

b Toss a coin 100 times. Copy this table and record your results in it.

c Calculate, as a decimal:

i the experimental probability of tossing a head

ii the theoretical probability of tossing a tail.

d Are the experimental and theoretical probabilities similar?

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See Example 2

4 A die was repeatedly rolled and the results are shown in the table.

Outcome	Frequency
1	95
2	119
3	108
4	87
5	78
6	113

a How many times was the die rolled?

b Find the experimental probability (as a decimal) of rolling:

i an odd number
ii a number less than 4
iii a 2 or a 3
iv a number less than 4 or an even number.

c Find the theoretical probability (as a decimal) of rolling:

i an odd number
ii a number less than 4
iii a 2 or a 3
iv a number less than 4 or an even number.

d Compare the experimental probabilities to the theoretical probabilities.

5 Place 5 blue counters, 2 red counters and 3 yellow counters in a bag. Select a counter at random from the bag, note its colour and return the counter to the bag. Repeat this 60 times. Copy this table and record your results in it.

Event	Tally	Frequency
Blue		
Red		
Yellow		

a What is the experimental probability of selecting:

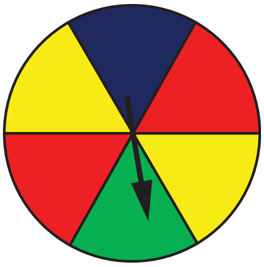
i a blue counter?
ii a red counter?
iii a yellow counter?
iv a red or blue counter?

b What is the theoretical probability of selecting:

i a blue counter?
ii a red counter?
iii a yellow counter?
iv a red or blue counter?

c Are the experimental probabilities similar to the theoretical probabilities?

6 Denise spun this spinner 50 times and the results are shown in the table.



Event	Frequency
Red	15
Blue	6
Yellow	24
Green	5

a What is the experimental probability (as a decimal) of the arrow stopping on

i red?
ii blue?
iii yellow?
iv green?

b What is the calculated probability (as a decimal) of the arrow stopping on

i red?
ii blue?
iii yellow?
iv green?

c Are the experimental and calculated probabilities similar?

d What is the expected number of times of the arrow stopping on a colour that is not yellow? How does this compare with Denise's observed number of times?

7 A die is rolled 100 times.

a What is the calculated probability of rolling a 6? (Express your answer as a fraction and as a decimal.)

b How many times would you expect a 6 to appear if the die was rolled 100 times?

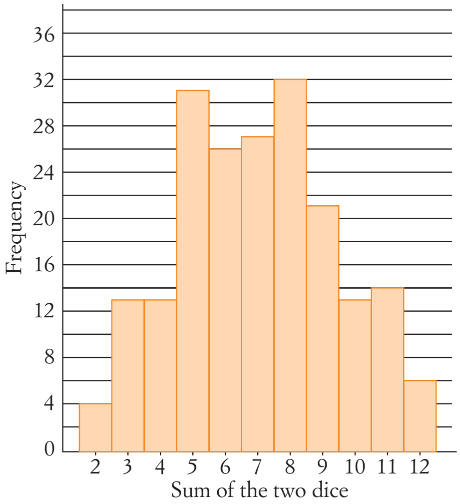
c Roll a die 100 times and record your results in a table similar to the one shown.

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

d What is the experimental probability or relative frequency of rolling a 6? (Express your answer as a fraction and as a decimal.)

e How does the calculated probability of rolling a 6 compare with the experimental probability?

8 Two dice are rolled and the sum of the numbers appearing uppermost was recorded in the frequency histogram.



a How many times were the two dice rolled?

b Based on these results, what is the experimental probability (or relative frequency) of obtaining:

- **i** a sum of 2?
- **ii** a sum of 7?
- **iii** a sum of 10?
- **iv** a sum greater than 7?
- **v** a sum less than 7?
- **vi** a sum of 7 or 8?
- **vii** a sum that is even and greater than 6?

9 School-aged children at several shopping centres were asked how they travelled to school.

Mode of transport	Frequency
Walk	27
Bus	80
Car	62
Train	21
Bicycle	5
Skateboard	1
Other	4

a How many students were surveyed?

b Based on these results, find the probability that a student chosen at random will:

- **i** walk to school
- **ii** be driven to school
- **iii** catch a bus to school
- **iv** catch a train to school
- **v** ride a skateboard to school

c What mode of transport could '**Other**' include?

d Survey 100 students at your school and make up a table showing the results. How do the results from your school compare with the results from the survey?

10-02 Venn diagrams

Video tutorial
Venn diagrams
MAT10SPVT10014

Puzzle sheet
Venn diagrams matching activity
MAT10SPPS10071

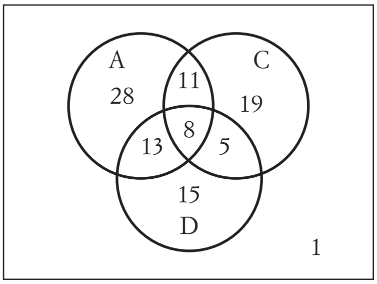
Puzzle sheet
And/or problems
MAT10SPPS00016

A **Venn diagram** is a diagram of circles (usually overlapping) that is used to group items into categories. A rectangle represents the whole group while the circles represent categories. Items common to two or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).

Pictures from History

Example 3

The Venn diagram shows the results of a survey on what type of movies – action (A), comedy (C) or drama (D) – that students prefer to watch.



a How many students were surveyed?

b How many students preferred to watch two types of movies only?

c Calculate, as a decimal, the probability of selecting a student who prefers to watch:

- **i** action movies only
- **ii** action or comedy movies but not dramas
- **iii** action and drama movies
- **iv** all types.

d A student is chosen from those who like action and comedy movies. What is the probability that they also like to watch drama movies?

e What is the probability of selecting a student who does not like watching any of the three types?

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Solution

a Number of students $= 28 + 11 + 8 + 13 + 5 + 19 + 15 + 1$
$= 100$

b 29 students only preferred two types of movies.

$11 + 13 + 5 = 29$

c **i** Students preferring action movies only $= 28$

$$P(\text{action only}) = \frac{28}{100} = 0.28$$

The region of A that doesn't overlap C or D.

ii Students preferring action or comedy only $= 28 + 19 + 11 = 58$

$$P(\text{action or comedy only}) = \frac{58}{100} = 0.58$$

The regions of A and C that don't overlap with D.

iii Students preferring action and drama $= 13 + 8 = 21$

$$P(\text{action and drama}) = \frac{21}{100} = 0.21$$

The regions where A and D intersect.

iv Students preferring all types $= 8$

$$P(\text{all types}) = \frac{8}{100} = 0.08$$

The region where the three circles intersect.

d Students preferring action and comedy $= 11 + 8 = 19$

Students preferring action and comedy and drama $= 8$

$$P(\text{drama if preferring action and comedy}) = \frac{8}{19} = 0.42 \text{ (to 2 decimal places)}$$

e There is one student who doesn't prefer action, comedy or drama.

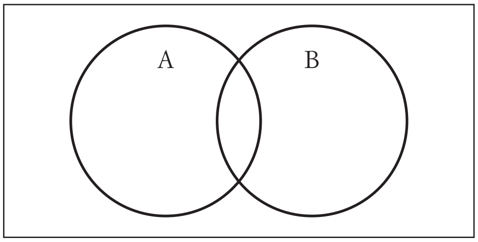
$$P(\text{not action, comedy or drama}) = \frac{1}{100} = 0.01$$

'And' vs 'or'

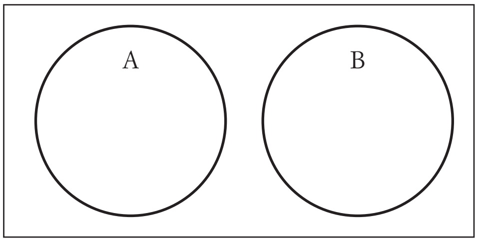
For two categories or events A and B, the phrase **'A and B'** means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a bus' means to do both things.

If A and B are **overlapping**, the phrase **'A or B'** means to have A or B or both. For example, 'to drive a car' **or** 'to ride a bus' means to drive a car only, or to ride a bus only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.

If A and B are **mutually exclusive**, this means that they are **not overlapping** and on a Venn diagram they appear as two separate circles. For mutually exclusive categories or events, the phrase **'A or B'** means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.



Overlapping events: 'A or B' means A or B or both



Mutually exclusive events: 'A or B' means A or B but not both

Example 4

A survey of 110 students at Lambavare High showed that 34 students take Art, 65 students study Computing, and 23 students do Computer Studies and Art.

Corbis/Purestock

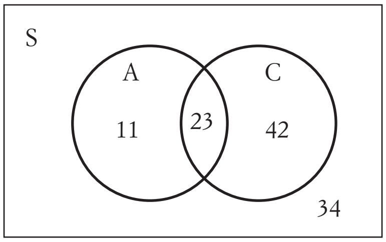
a Represent this information on a Venn diagram.

b How many students do Art and Computer Studies but not both?

c What is the probability of randomly selecting a student from this group who does:

- **i** Computer Studies?
- **ii** Art and Computing?
- **iii** Art or Computer Studies?
- **iv** neither Art nor Computer Studies?

Solution



a S = students at Lambavare High
A = students doing Art
C = students doing Computer Studies

There are 23 students who take both Art and Computer Studies.

∴ Students doing Art only = 34 − 23
= 11

∴ Students doing Computer Studies only = 65 − 23
= 42

∴ Students who take neither Art nor Computer Studies = 110 − 11 − 42 − 23
= 34

b Number of students doing Art and Computer Studies only = 11 + 42
= 53

c **i** 65 students take Computer Studies.

$P(\text{Computer Studies}) = \frac{65}{110}$

ii $P(\text{Art and Computing Studies}) = \frac{23}{110}$

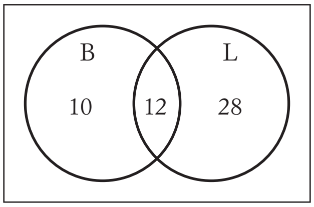
iii Number of students who do Art or Computer Studies = 11 + 23 + 42
= 76

$$P(\text{Art or Computer Studies}) = \frac{76}{110} = \frac{38}{55}$$

iv $$P(\text{neither Art nor Computer Studies}) = \frac{34}{110} = \frac{17}{55}$$

Exercise 10-02 Venn diagrams

See Example 3



1 Fifty people were asked as to whether they have breakfast (B) or lunch (L) on a weekday. The results are shown in the Venn diagram.

a What is the probability of selecting a person who has:
- **i** breakfast?
- **ii** lunch?
- **iii** breakfast but not lunch?
- **iv** breakfast and lunch?
- **v** breakfast or lunch only?

b Of the people who have lunch, find the probability that a person also had breakfast.

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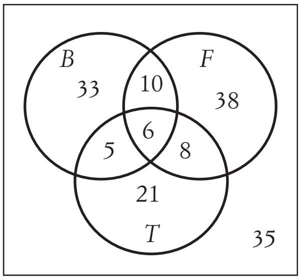
See Example 4

2 The Venn diagram shows the number of Year 10 students who play basketball (*B*), touch football (*F*) or tennis (*T*).



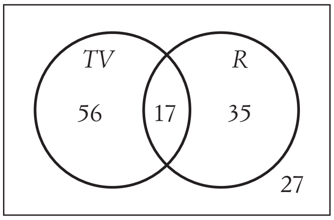
a How many students are in Year 10?

b Find the probability of selecting a student who plays:

i basketball only
ii tennis only
iii touch football and tennis
iv touch football or tennis
v basketball but not touch football
vi all three sports.

c Of the students that play touch football, find the probability of selecting a student who also plays tennis.

3 The Venn diagram shows the results of a survey asking people how they relax at home: watching television (*TV*) or reading (*R*).



a How many people were surveyed?

b Find the probability of selecting a person who only watches TV to relax.

c What is the probability of selecting a person who doesn't watch TV or read to relax?

d Of the people who read, find the probability that they also watch TV to relax.

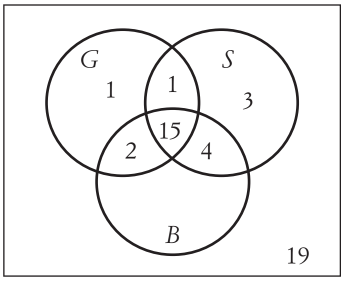
4 Of the 54 Year 10 Music students, 23 students sing (*S*), 43 students play a musical instrument (*P*) and 12 students sing and play a musical instrument.

a Show this information on a Venn diagram.

b Find the probability of selecting a Music student who:

i sings or plays an instrument
ii sings only
iii plays a musical instrument only
iv sings or plays an instrument, but not both.

5 The Venn diagram shows the number of countries that won medals at the XXI Winter Olympic Games in Vancouver, 2010.



a How many countries competed at these winter Olympics?

b What is the probability of selecting a country at random that:

i won only gold medals?

ii won gold, silver and bronze medals?

iii won gold or silver medals, but not bronze?

iv did not win a gold or silver medal?

c Of the countries that won medals, what is the probability of selecting a country that

i won gold medals?

ii won bronze, but no gold or silver?

6 At Riverside College, Year 10 students are asked what language they are studying. 64 students take Indonesian (*I*), 47 students take Japanese (*J*), 15 students take both Indonesian and Japanese, and 27 do not study a language.

a How many students are in Year 10?

b Show the information on a Venn diagram.

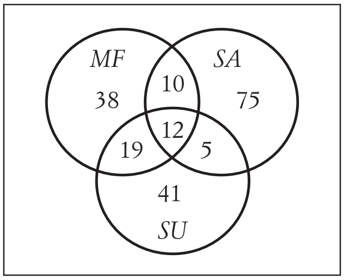
c How many students studied only one language?

d Find the probability of selecting a Year 10 student at random who studies:

i Indonesian but not Japanese **ii** Japanese but not Indonesian

iii no languages **iv** only one language.

7 People at several shopping centres were asked on which day they preferred to shop – Monday to Friday (*MF*), Saturday (*SA*) or Sunday (*SU*). The results are shown in the Venn diagram.



a How many people were surveyed?

b What is the probability of selecting a person who prefers to shop on:

i Monday to Friday? **ii** Saturday?

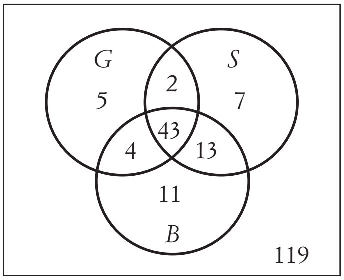
iii Sunday? **iv** on the weekend only?

v on Saturday or Sunday? **vi** any day (has no preference)?

c Find the probability of selecting a person who only prefers Saturday or Sunday but not both.

d Is it necessary to include the rectangle in this Venn diagram? Give reasons.

8 The Venn diagram shows the number of countries that won gold, silver, bronze or no medals at the 2012 Summer Olympic Games in London.



a Find the total number of countries that competed at the 2012 London games.

b What is the probability of randomly selecting a country that:

i won a silver medal only? **ii** won one medal only?

iii won at least two medals? **iv** won at most one medal only?

c Out of the countries that won gold medals, find the probability of selecting a country that:

i won gold and silver but not bronze **ii** won gold, silver and bronze.

Worksheet
Two-way tables
MAT10SPWK10072

Puzzle sheet
Combined events: Two-way tables
MAT10SPPS00014

10-03 Two-way tables

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot easily be represented by a Venn diagram.

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Example 5

Year 11 students at Southbank College were surveyed on whether they had part-time jobs.

	Part-time work	No part-time work
Male	43	27
Female	35	31

a How many students are in Year 11 at Southbank College?

b How many students had part-time work?

c How many male students were in Year 11?

d What is the probability of selecting a student at random that:

  i works part-time? **ii** is female and works part-time?

  iii is male and doesn't work? **iv** doesn't work?

e What is the probability of selecting a student working part-time given that:

  i the student is male? **ii** the student is female?

Solution

a Number of Year 11 students $= 43 + 27 + 35 + 31$

$= 136$

b Students with part-time work $= 43 + 35$

$= 78$

c Male students in Year 11 $= 43 + 27$

$= 70$

d i $P(\text{student works part-time}) = \frac{78}{136}$

$= \frac{39}{68}$

ii There are 35 female students who work part-time.

$P(\text{female and part-time work}) = \frac{35}{136}$

iii There are 27 males who don't work.

$P(\text{male and not working}) = \frac{27}{136}$

iv Number of students not working $= 27 + 31 = 58$

$P(\text{not working}) = \frac{58}{136} = \frac{29}{68}$

e i There are 70 male students and 43 of them work part-time.

$P(\text{working part-time given that student is male}) = \frac{43}{70}$

ii There are 66 female students and 35 of them work part-time.

$P(\text{working part-time given that student is female}) = \frac{35}{66}$

Exercise 10-03 Two-way tables

See Example 5

1 People attending the Staying Alive Fitness Centre early on a Saturday morning either went swimming or did a workout in the gym. The numbers are shown in the table on the right.

	Swimming	Gym
Male	32	53
Female	24	41

a How many people went to the fitness centre?

b Find the probability that a person selected at random:

i was female and went swimming

ii was male and did a workout in the gym

iii went swimming.

c Find the percentage (to the nearest whole number) of females who did a workout in the gym.

2 Year 10 students at Baramvale High were given a choice of two activities, bowling or indoor soccer on a wet sports afternoon. Their choices are shown in the table.

	Bowling	Indoor soccer
Boys	25	48
Girls	43	12

a How many students were in Year 10?

b How many students:

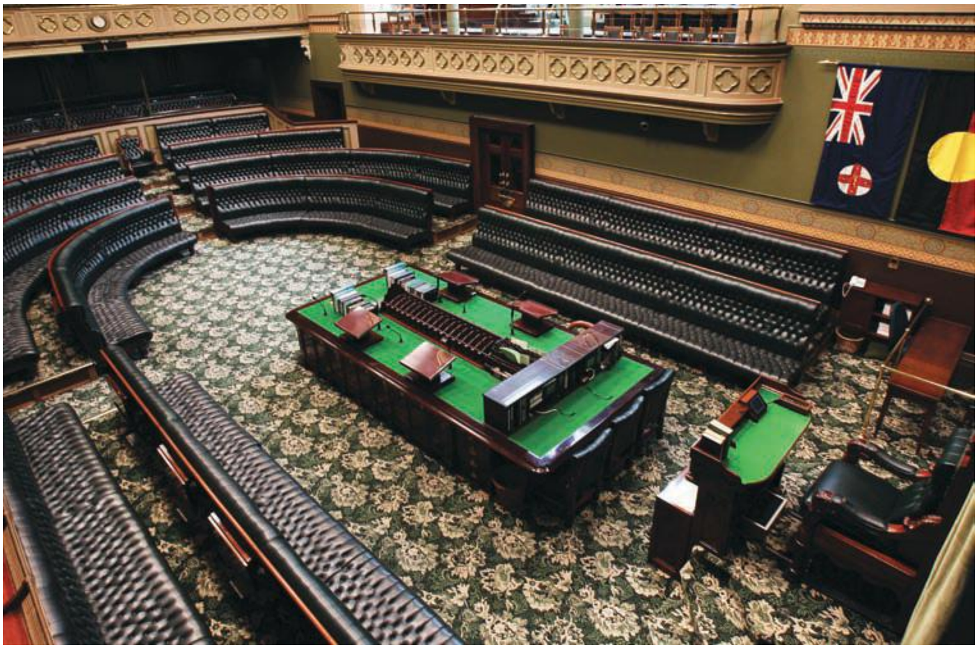
i went bowling?

ii played indoor soccer?

c What is the probability of randomly selecting a student who went bowling?

d What is the probability of randomly selecting a girl who played indoor soccer?

3 The composition of the Legislative Assembly (the lower house) in the NSW State Parliament (in 2012) is shown in the table on the next page.



Newspix/Craig Greenhill

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	Liberal/Nationals	Labor	Independents/Greens
Male	59	11	3
Female	11	9	0

a How many members of parliament (MPs) are there in the Legislative Assembly?

b Find the percentage probability of randomly selecting an MP who is:

i female **ii** a male and in the Opposition **iii** an independent or Green

c What percentage probability of:

i Government MPs are female? **ii** Opposition MPs are female?

d Compare your answers to part **c** and comment on the difference between the two results.

4 People were asked to name their favourite takeaway food. The results are shown in the table on the right.

	Pizza	Hamburger	Fish and Chips
Men	22	35	18
Women	43	26	6

a How many people were surveyed?

b Find the probability (as a decimal) that a person selected at random:

i is male **ii** is female and likes fish and chips

iii likes pizza **iv** is male and likes hamburgers.

c If a male is selected at random, what is the probability that his favourite takeaway food is pizza?

5 Year 7 students were asked about their favourite drink. The results are in the table on the right.

	Boys	Girls
Water	21	35
Milk	11	12
Juice	15	17
Soft drink	31	18

a How many students were in Year 7?

b What is the probability of randomly selecting a student that:

i prefers water?

ii is a boy and likes milk?

iii is a girl and likes soft drinks?

c What is the probability that if a girl is randomly selected, she prefers water?

6 A survey looked at whether people ate breakfast and whether they exercised regularly. The results are shown in the two-way table.

	Exercise	No exercise
Ate breakfast	72	27
Did not eat breakfast	38	63

a How many people were surveyed?

b What percentage of people exercised?

c Find the percentage probability of picking a person at random who:

i eats breakfast

ii does not exercise regularly

iii eats breakfast and exercises regularly

iv does not eat breakfast and doesn't exercise.

d Of the people who exercise regularly, what is the probability of picking someone who eats breakfast?

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7 Students at Granthorp High School were asked to indicate their preference for dark or milk chocolate in a survey.

		Milk chocolate	
		Like	**Dislike**
Dark chocolate	**Like**	545	134
	Dislike	157	42

a How many students attended Granthorp High?

b What is the probability (correct to three decimal places) of selecting a student at random who:

- **i** likes dark chocolate?
- **ii** likes both milk chocolate and dark chocolate?
- **iii** likes dark chocolate but dislikes milk chocolate?
- **iv** dislikes both dark and milk chocolate?

Stage 5.2

10-04 Two- and three-step experiments

A **two-** or **three-step experiment** is a chance experiment that has two or three parts or stages, for example:

- rolling two or three dice
- drawing two or three prizes in a raffle
- observing the weather each day over a weekend or a long weekend
- throwing two or three coins together

The sample space for two-step experiments can be displayed using lists, tables or tree diagrams but the sample space of three-step experiments is best displayed using a tree diagram.

Example 6

A coin is tossed and a die is rolled.

a Use a table to display the sample space.

b Find the probability of obtaining:

i a tail and a 3 **ii** a head and an even number.

Solution

a The sample space of a coin is a head (H) and a tail (T).

The sample space for a die is 1, 2, 3, 4, 5 and 6.
The sample space of tossing a coin and rolling a die is shown in the table below.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**H**	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

Using a table ensures that all outcomes are counted

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b **i** There are 12 outcomes in the sample space.

$\therefore P(\text{a tail and a 3}) = P(\text{T3}) = \frac{1}{12}$

ii There are three outcomes that make up the event a head and an even number: H2, H4, H6

$$\therefore P(\text{a head and an even number}) = \frac{3}{12} = \frac{1}{4}$$

Tree diagrams

A **tree diagram** lists all the possible outcomes of each stage. Branches stretch out to show the possible pathways of outcomes at each step or stage. An outcomes column at the end of the diagram lists the sample space.

Example 7

Two coins are tossed.

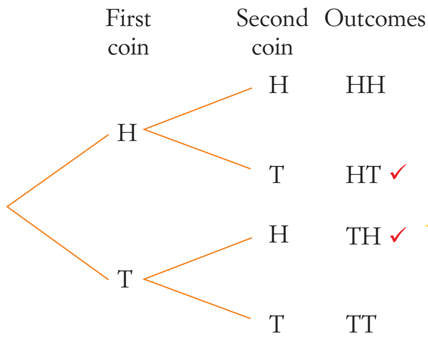
a Use a tree diagram to list the sample space.

b Find the probability of tossing:

i two heads **ii** a head and a tail (in any order).

Solution

a There are 2 outcomes for the first coin, followed by 2 outcomes for the second coin. There are $2 \times 2 = 4$ possible outcomes.



Using a tree diagram ensures that all outcomes are counted

b **i** There is one outcome out of a possible 4 for two heads.

$\therefore P(\text{2 heads}) = \frac{1}{4}$

ii There are 2 outcomes for a head and a tail (ticked on the tree diagram).

$$\therefore P(\text{a head and a tail}) = \frac{2}{4} = \frac{1}{2}$$

Stage 5.2

Video tutorial
Tree diagrams
MAT10SPVT10013

Worksheet
Tree diagrams
MAT10SPWK10073

Puzzle sheet
Combined events: Tree diagrams
MAT10SPPS00015

Stage 5.2

Exercise 10-04 Two- and three-step experiments

See Example 6

1 The positions of captain and vice-captain of a netball team are to be selected from Cassandra, Ashleigh, Gemma, Jodie, Erin and Robyn.

a List the possible pairings of captain and vice-captain.

b What is the probability of Ashleigh being captain or vice-captain?

c What is the probability of Erin becoming vice-captain?

2 Two dice are rolled.

a Copy and complete the table below to list the sample space.

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	1, 1	1, 2				
	2						
	3						
	4			4, 3			
	5						
	6						

b How many possible outcomes are there?

c Find the probability of:

i rolling doubles **ii** rolling two even numbers

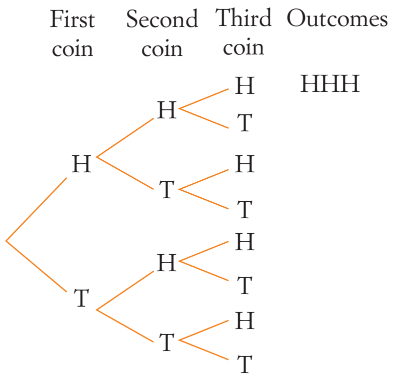
iii rolling at least one 2 **iv** rolling two numbers greater than 3

v rolling one odd and one even number

vi rolling two numbers so that the first number is greater than the second number.

See Example 7

3 Three coins are tossed.



a Copy and complete the tree diagram to list the sample space.

b How many outcomes are there in the sample space?

c Use the tree diagram to find these probabilities.

i $P(3\text{ heads})$

ii $P(2\text{ heads})$

iii $P(3\text{ tails})$

iv $P(\text{head, then tail and then head})$

v $P(2\text{ heads or }3\text{ heads})$

d Find the probability of:

i tossing at least 1 tail **ii** tossing at most 2 tails.

e If three coins are tossed 200 times, find the expected number of:

i tossing two heads **ii** tossing no tails.

4 Use a tree diagram to display all possible outcomes when a coin is tossed and a die is rolled.

5 A tetrahedral die (numbered 1, 2, 3 and 4) and a normal six-sided die are rolled.

a Construct a table to list the outcomes in the sample space.

b How many outcomes are in the sample space?

c Find the probability of rolling:

i doubles
ii two even numbers
iii one even and one odd number
iv a pair of numbers that are both less than 4
v a pair of numbers that are both greater than 4.

6 Two normal dice are rolled and the sum of the two numbers is calculated.

a Copy and complete this table to show all possible sums.

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2					
	2						
	3				7		
	4						
	5						
	6					11	

b Find the of probability of rolling a sum:

i of 5
ii of 12
iii of 7
iv that is even
v less than 2
vi more than 7
vii at least 7
viii between 4 and 8.

7 Four coins are tossed.

a Use a tree diagram to list the sample space.

b Find the probability of tossing:

i 4 heads
ii 1 head
iii 2 tails
iv at least 1 tail
v 2 heads and then 2 tails
vi not more than 1 tail

c If four coins are tossed 1000 times, find the expected number of:

i 4 heads
ii 2 heads and 2 tails
iii at least one tail.

Mental skills 10 Maths without calculators

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	1%	5%	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

1 Study each example.

a Increase 360 by 25%.

$$\begin{aligned}25\% \text{ of } 360 &= \frac{1}{4} \times 360\\ &= 360 \div 4\\ &= 90\end{aligned}$$

$360 + 90 = 450.$

b Increase \$80 by 5%.

$$\begin{aligned}5\% \text{ of } \$80 &= \frac{1}{20} \times \$80\\ &= \$80 \div 20\\ &= \$4\end{aligned}$$

or

$$\begin{aligned}10\% \text{ of } \$80 &= \$8\\ \therefore 5\% \text{ of } \$80 &= \$8 \div 2\\ &= \$4\end{aligned}$$

$\$80 + \$4 = \$84.$

2 Now increase:

a \$340 by 20%
b 66 by 50%
c 150 by $33\frac{1}{3}\%$
d \$400 by 1%
e 640 by 5%
f \$72 by $12\frac{1}{2}\%$
g \$470 by 10%
h 180 by 25%
i 420 by $33\frac{1}{3}\%$
j \$80 by 5%
k \$280 by 25%
l 70 by 20%

3 Study each example.

a Decrease \$225 by $33\frac{1}{3}\%$.

$$\begin{aligned}33\frac{1}{3}\% \text{ of } \$225 &= \frac{1}{3} \times \$225\\ &= \$225 \div 3\\ &= \$75\end{aligned}$$

$\$225 - \$75 = \$150.$

b Decrease \$70 by 15%

$$\begin{aligned}10\% \text{ of } \$70 &= \frac{1}{10} \times \$70 = \$7\\ \therefore 5\% \text{ of } \$70 &= \frac{1}{2} \times \$7 = \$3.50\\ \therefore 15\% \text{ of } \$70 &= (10\% \times \$70) + (5\% \times \$70)\\ &= \$7 + \$3.50 = \$10.50\end{aligned}$$

4 Now decrease:

a 440 by 25%
b \$300 by 20%
c 2400 by $33\frac{1}{3}\%$
d \$500 by 15%
e \$250 by 10%
f \$120 by 50%
g \$72 by $12\frac{1}{2}\%$
h 80 by 5%
i \$85 by 20%
j \$3800 by 1%
k \$440 by 15%
l \$150 by $33\frac{1}{3}\%$

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Investigation: The birth month paradox

A **paradox** is a statement or proposition that seems impossible but is actually true.

1 Copy this table.

Group	Outcome (Y or N)
1	
2	
3	
4	
5	

2 Randomly select a group of 5 people and ask them what month they were born in. If two or more people have the same birth month, record a Y in the table for Group 1, otherwise write N.

3 Repeat this process 4 more times, recording your results in the table.

4 Combine your results with those of six other students so that you have the outcomes for 30 groups.

5 What fraction of the groups had repeated birth months?

6 Obtain the results of another group of six students. What fraction of the groups had a repeated birth month?

7 The birth month paradox is that in any randomly selected group of 5 people, the probability that at least two people have the same birth month is greater than 0.5. Have your results shown this to be true?

8 Can you show the following?

- **a** For every 23 people selected at random, the probability that at least 2 people will share the same birthday is 50%.
- **b** If 30 people are selected at random, this probability is 70%.
- **c** If 50 people are selected at random, this probability is 97%

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Stage 5.2

Worksheet
Multi-step experiments
MAT10SPWK10074

10-05 Selecting with and without replacement

In two- and three-step experiments where an item is selected repeatedly, the outcome of the second or third step may or may not be affected by the outcome of the previous step. This depends upon whether each selected item is returned to the set of items before the next item is selected. If it is, then this is called **selecting** '**with replacement**'. If it isn't, then it is called **selecting** '**without replacement**'.

Example 8

Two cards are drawn from a set of cards numbered 1 to 5, to form a two-digit number.

a Make a list of all possible outcomes if the cards are drawn:

i with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that the number formed is:

i even **ii** greater than 30 **iii** divisible by 5.

c If the first card is not replaced, find the probability that the number formed is:

i even **ii** odd **iii** less than 20

Solution

a i The possible outcomes, **with replacement**, are:

11	21	31	41	51
12	22	32	42	52
13	23	33	43	53
14	24	34	44	54
15	25	35	45	55

There are $5 \times 5 = 25$ different outcomes possible.

ii The possible outcomes, **without replacement**, are:

12	21	31	41	51
13	23	32	42	52
14	24	34	43	53
15	25	35	45	54

There are $5 \times 4 = 20$ different outcomes possible.

b i There are 10 even numbers.

$$P(\text{even number}) = \frac{10}{25} = \frac{2}{5}$$

ii There are 15 numbers greater than 30.

$$P(\text{number} > 30) = \frac{15}{25} = \frac{3}{5}$$

iii There are 5 numbers divisible by 5.

15, 25, 35, 45 and 55

$$P(\text{number divisible by } 5) = \frac{5}{25} = \frac{1}{5}$$

c **i** There are 8 even numbers without replacement.

$P(\text{even}) = \frac{8}{20} = \frac{2}{5}$

ii There are 12 odd numbers.

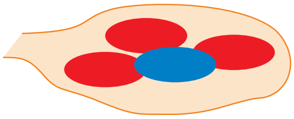
$P(\text{odd}) = \frac{12}{20} = \frac{3}{5}$

iii There are 4 numbers less than 20.

$P(\text{number} < 20) = \frac{4}{20} = \frac{1}{5}$

Example 9

A bag contains 3 red counters and a blue counter.
Three counters are drawn at random without replacement.



a Use a tree diagram to display all possible outcomes.

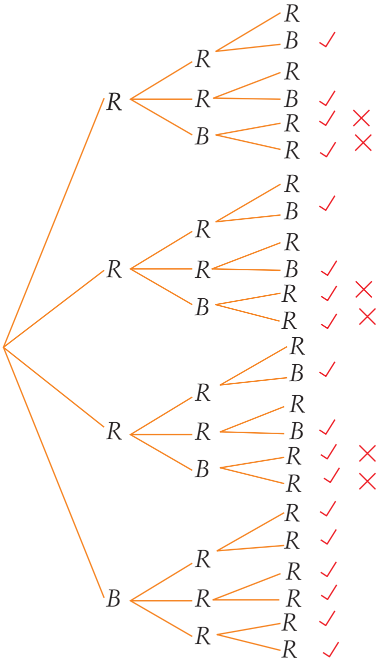
b Find the probability of drawing:

- **i** two red counters
- **ii** a red, blue, and red in that order
- **iii** at least one red counter.

Solution

a The tree diagram will have 4 branches for the first step or stage, followed by 3 branches for the second step, followed by 2 branches for the third step.

So there are $4 \times 3 \times 2 = 24$ outcomes in the sample space.



b **i** There are 18 outcomes with two red counters (ticked on the tree diagram)

$\therefore P(\text{two red counters}) = \frac{18}{24} = \frac{3}{4}$

ii Red, blue, red occurs 6 times (crossed × on the tree diagram).

$\therefore P(\text{red, blue, red}) = \frac{6}{24} = \frac{1}{4}$

iii All outcomes contain at least one red counter.

$\therefore P(\text{at least one red counter}) = \frac{24}{24} = 1$

Stage 5.2

Exercise 10-05 Selecting with and without replacement

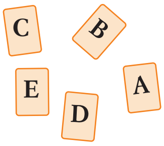
See Example 8

1 A boy and a girl are to be chosen from a group of 4 boys and 6 girls: Ben, Christian, Ewan, William, Becky, Cassandra, Emily, Marianne, Roz and Siana.

a List all the possible pairs of a boy and a girl.

b Find the probability of selecting:

- **i** Christian and Roz
- **ii** a boy and a girl whose names begin with a B or a C
- **iii** a pair that includes Emily.



2 Two cards are drawn from a set of cards labelled A, B, C, D and E.

a Make a list of all possible outcomes if the cards are drawn:

- **i** with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that:

- **i** both letters are the same **ii** both letters are vowels
- **iii** one letter is a vowel and the other is a consonant.

c If the first card is not replaced, find the probability that:

- **i** both letters are vowels **ii** one letter is a vowel and the other is a consonant
- **iii** the first letter is a B or a D **iv** the last letter is not A.

3 Two coins are tossed together.

a Copy and complete the table to find all the outcomes in the sample space.

b What is the probability of tossing:

- **i** two tails?
- **ii** a head and a tail?
- **iii** at least one head?

		2nd coin	
		H	**T**
1st coin	**H**		
	T		

4 When staying at a hotel, David and Sarah can select one item from each course of a breakfast menu.

1st course	**2nd course**
Cereal (C)	Bacon and eggs (B)
Fruit (F)	Ham and cheese croissants (H)
Yoghurt (Y)	Pancakes (P)
	Sausages and tomatoes (S)
	Toast and jam (T)

Stage 5.2

a Copy and complete the table to list all the different 2-course breakfasts available.

		2nd course				
		B	**H**	**P**	**S**	**T**
1st course	**C**					
	F					
	Y					

b If one of the combinations of breakfasts is chosen at random, what is the probability that it includes:

i fruit? **ii** cereal but not bacon and eggs? **iii** fruit and croissants?

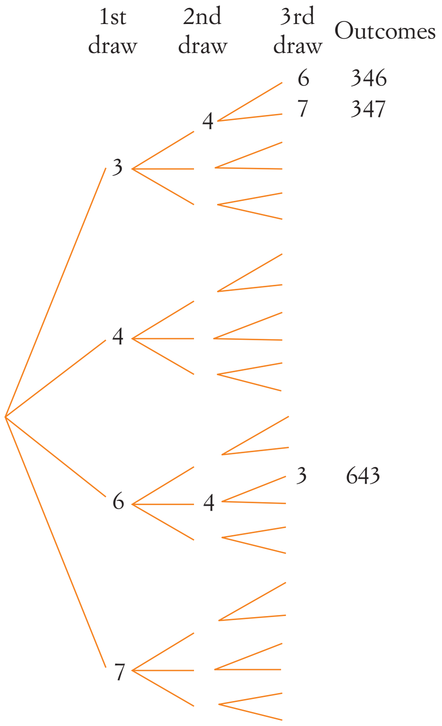
See Example 9

5 The numbers 3, 4, 6 and 7 are written on separate cards and placed in a bag. Three cards are drawn at random without replacement to form a three-digit number.

a Copy and complete the tree diagram to list the sample space.

b Find the probability of forming:

i an even number
ii a number greater than 400
iii a number between 400 and 700
iv an even number greater than 400.



6 The cards in question **5** are to be now drawn *with replacement*, that is, the same card can be used twice.

a Use a tree diagram to show all possible outcomes.

b Find the probability that the three-digit number formed is:

i a number with all digits the same **ii** an odd number
iii greater than 500 **iv** begins and ends with a 4.

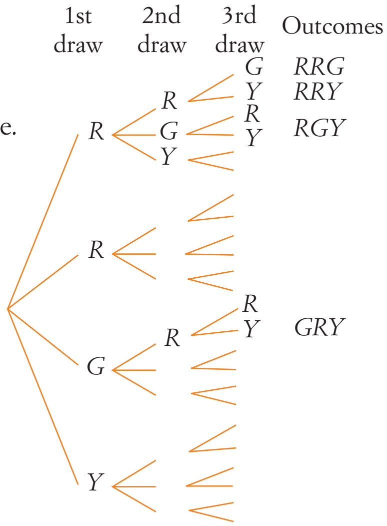
Stage 5.2

7 A bag contains 2 red marbles, 1 green marble and 1 yellow marble. Three marbles are drawn from the bag at random without replacement.

a Copy and complete the tree diagram to list the sample space.

b Find the probability of drawing:

- **i** two red counters
- **ii** a red, green, and red in that order
- **iii** at least one red counter.



8 A family has three children.

a Use a tree diagram to list all possible outcomes in the sample space.

b What is the probability that the family consists of:

- **i** 3 boys?
- **ii** 3 girls?
- **iii** 2 girls and a boy?
- **iv** a girl and then two boys?

9 The weather on a long weekend will either be fine or rain each day, with each outcome being equally likely.

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that:

- **i** it rains on all three days?
- **ii** it is fine on two of the three days?
- **iii** it is fine on Saturday and Sunday, but rains on Monday?
- **iv** it rains on at least one day of the long weekend?

10 Two dice are rolled.

a How many outcomes are possible?

b What is the probability of rolling two 6s?

c What is the probability of rolling at least one 6?

11 Three counters are drawn from a bag containing 3 yellow counters, 1 blue counter and 1 white counter.

a Draw a tree diagram to show the sample space if the counters are drawn:

- **i** with replacement
- **ii** without replacement.

b If the counters are drawn with replacement, find the probability of obtaining:

- **i** three counters of the same colour
- **ii** three counters of different colours
- **iii** no blue counters
- **iv** at least one yellow counter.

c If the counters are drawn without replacement, find the probability of drawing:

- **i** three counters of the same colour
- **ii** three of different colours
- **iii** 2 blue counters and a white counter
- **iv** no white counter.

Investigation: Dependent or independent?

Work in pairs.

You will need: a coin, 3 blue counters and 2 red counters.

1 **a** **i** Toss a coin and record the outcome.
  ii What is the probability of obtaining your outcome?
 b **i** Toss the coin a second time and record the outcome.
  ii What is the probability of obtaining the second outcome?
 c Is the outcome of the second toss affected by the outcome of the first toss? Is the probability of the second outcome independent or dependent on the first outcome? Justify your answer.

2 **a** Copy this table.

With replacement	1st draw	2nd draw
Blue		
Red		
	40	40

 b Put 3 blue counters and 2 red counters in a bag. Randomly draw a counter from the bag and record the outcome.
 c Put back the counter you drew in part **a** and shake the bag. Again draw a counter and record your outcome.
 d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.
 e Use your results to find:
  i P(blue counter drawn first) **ii** P(blue counter drawn second)
 f **i** Are your two results for part **e** the same?
  ii Would you expect the results to be the same? Give reasons.
 g Is the outcome of the second draw dependent on the outcome of the first draw?

3 **a** Copy this table.

Without replacement	1st draw	2nd draw
Blue		
Red		
	40	40

 b Again, place 3 blue counters and 2 red counters in a bag. Randomly draw a counter from the bag and record your outcome.
 c Without replacing the counter you drew in part **a**, shake the bag and draw a second counter, recording your outcome.
 d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.
 e Use your results to find:
  i P(blue counter drawn first) **ii** P(blue counter drawn second)
 f **i** Are your two results for part **e** the same?
  ii Would you expect your results to be the same? Give reasons.
 g Is the outcome of the second draw dependent on the outcome of the first draw? Compare your results with those of other students in your class.

Stage 5.2

10-06 Dependent and independent events

Two events are **independent** if the outcome of one event **does not affect** the outcome of the other event and so the probability of one event is not affected by the probability of the other event occurring. For example, if a coin is tossed and a die is rolled, the two events are independent as the outcome on the coin does not affect the outcome on the die.

Two events are **dependent** if the outcome of one event **affects** the outcome of the other event and so the probability of one event is affected by the probability of the other event occurring. For example, when drawing two marbles from a bag without replacement of the first marble, the two events are dependent because the outcome of the second draw is affected by the outcome of the first draw.

Example 10

A coin is tossed and a die is rolled.

a List the outcomes in the sample space.

b Find:

- **i** P(tossing a head) **ii** P(rolling an even number)
- **iii** P(tossing a head and rolling an even number)

c Is P(tossing a head and rolling an even number) $=$ P(tossing a head) $\times$ P(rolling an even number)?

d Are the two events dependent or independent?

Solution

a The outcomes are
H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5 and T6.

b i $P(\text{H}) = \frac{1}{2}$

ii $P(\text{even}) = \frac{3}{6}$

$= \frac{1}{2}$

iii $P(\text{H and even}) = \frac{3}{12}$ H2, H4 and H6

$= \frac{1}{4}$

c Yes, since $P(\text{H and even}) = \frac{1}{4}$ and

$P(\text{H}) \times P(\text{even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

d The two events are independent since the outcome when tossing a coin does not affect the outcome when rolling a die.

Summary

Two events are **independent** if the outcome of one event does not affect the outcome of the other event.

If A and B are two independent events, then $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$.

Example 11

A bag contains 3 blue marbles and 1 yellow marble. Two marbles are drawn from the bag, without replacing the marble from the first draw.

a Find the probability of:

  i selecting a blue marble with the first draw

  ii selecting a blue marble with the second draw if the first marble was blue.

b Are the two events dependent or independent?

Solution

a **i** $P(\text{blue on the first draw}) = \frac{3}{4}$

  ii After drawing a blue marble, there are 3 marbles left, of which 2 are blue.

  $\therefore P(\text{blue on the second draw}) = \frac{2}{3}$

b The bag contains 2 blue marbles and 1 yellow marble for the second draw, so $P(\text{blue})$ changes from $\frac{3}{4}$ to $\frac{2}{3}$.

The second event is dependent on the first event.

Exercise 10-06 Dependent and independent events

1 State whether the following pairs of events are dependent or independent.

  a rolling a die to obtain a 4 and rolling a second die to obtain an even number

  b rolling a die and obtaining a 6 and tossing a coin to obtain a tail

  c a person who trains hard and that person succeeding in sport

  d drawing a red ball from a bag containing red and blue balls, replacing it and then drawing a blue ball from the bag

  e electing a captain from a group of players and then electing a vice-captain from the same group

  f tossing two coins and obtaining a head on the first coin and a head on the second coin.

  g drawing a King from a normal pack of cards, not replacing it and then drawing an Ace.

2 In Lotto, 6 balls and 2 supplementary balls are drawn without replacement. Are the events of drawing each of the balls dependent or independent events? Give reasons.

3 A coin is tossed 3 times and the result is heads each time.

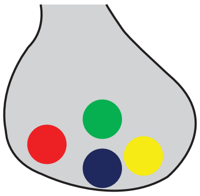
  a Are each of the three coin tosses dependent or independent events?

  b The coin is tossed a fourth time. What is the probability of obtaining a head on the fourth toss?

Stage 5.2

See Example 10

4 A normal die is rolled and a marble is drawn from a bag containing 1 yellow marble, 1 green marble, 1 blue marble and 1 red marble.



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a Find the probability of:

i rolling a number less than 3 with the die

ii drawing a green marble from the bag.

b List the outcomes for rolling the die and drawing a marble from the bag.

c What is the probability of rolling a number less than 3 and drawing a green marble?

d Is P(rolling a number less than 3) × P(drawing a green marble) = P(a number less than 3 and a green marble)?

e Are the events of rolling a number less than 3 and drawing a green marble dependent or independent?

5 A red die and a blue die are rolled.

a Find the following probabilities.

i P(5 on the red die) **ii** P(an even number on the blue die)

b Are the events '5 on the red die' and 'an even number on the blue die' dependent or independent?

See Example 11

6 A bag contains 5 red balls and 4 yellow balls. Two balls are drawn at random without replacement.

a What is the probability of drawing a red ball first?

b What is the probability of obtaining a red ball on the second draw if the first ball was red?

c Are the two events dependent or independent? Give reasons.

7 A bag contains 5 yellow counters and 3 red counters. Two draws are made with no replacement. Find the probability of drawing:

a **i** a yellow counter on the first draw

ii a yellow counter on the second draw after a yellow counter was drawn with the first draw

b **i** a red counter on the first draw

ii a yellow counter on the second draw after a red counter was drawn on the first draw

c **i** a yellow counter on the first draw

ii a red counter on the second draw after a yellow counter was drawn on the first draw

d **i** a red counter on the first draw

ii a red counter on the second draw after a red counter was drawn on the first draw.

8 Three children in a family are all girls. What is the probability that the next child in this family will be a girl?

Just for the record Lotteries and Lotto

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A lottery is a game of chance in which numbered tickets are drawn from tickets which have been sold. Lotteries were introduced by the State Government to raise money for hospitals. The first lottery was drawn on 20 August 1931 with a first prize of £5000.

Lotteries have been used to celebrate special events and to help finance special projects. The Opera House Lottery, which commenced selling on 25 November 1957, was used to finance the construction of the Sydney Opera House.

Other games of chance have been introduced, including Lotto (1979), Instant Scratchies (1982), OzLotto (1994) and Powerball (1996).

Research the probability of winning Lotto, OZ lotto and Powerball.

10-07 Conditional probability

Stage 5.2

Worksheet
Conditional probability
MAT10SPWK10075

Puzzle sheet
Conditional probability: Two-way tables
MAT10SPPS00018

In many practical situations, events are not independent. For example, the probability of a student arriving to school on time if catching a bus may be dependent on the amount of traffic. Conditional probability is used to calculate probabilities for dependent events.

Summary

The **conditional probability** of an event B given event A, is the probability that event B occurs, given that event A has already occurred.

Example 12

A bag contains 3 red marbles and 2 yellow marbles. Two marbles are drawn at random from the bag without replacement. What is the probability that the second marble is yellow, given that the first marble was also yellow?

Solution

If the first marble is yellow, there are 3 red marbles and 1 yellow marble left in the bag.

$\therefore P(\text{second marble yellow, given the first marble is yellow}) = \frac{1}{4}$

Stage 5.2

Example 13

Two dice are rolled and their total is calculated.

a Use a table to show all possible totals.

b Given that the total is 7, what is the probability that one of the dice shows a 3?

c Given that one of the dice shows a 4, what is the probability that the total is 10?

d Given that the total is 6, what is the probability of a double?

e Given that a double is rolled, what is the probability of:

i a total of 12? **ii** a total less than 10?

Solution

a

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b There are 6 outcomes that give a total of 7. (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 2)

If one of the dice shows a three, the possible outcomes are (3, 4) and (4, 3).

$$
\begin{aligned}
P(\text{one die shows 3, given total} = 7) &= \frac{2}{6} \\
&= \frac{1}{3}
\end{aligned}
$$

c There are 11 outcomes that have 4 showing on one of the dice. (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)

Of these outcomes, only two have a total of 10. (6, 4) and (4, 6)

$$P(\text{total} = 10, \text{ given one die shows 4}) = \frac{2}{11}$$

d There are 5 outcomes that give a total of 6. (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)

There is only 1 double. (3, 3)

$$P(\text{double, given total} = 6) = \frac{1}{5}$$

e There are 6 doubles that can be rolled. (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

i (6, 6) is the only double with a total of 12.

$$P(\text{total} = 12, \text{ given double rolled}) = \frac{1}{6}$$

ii (1, 1), (2, 2), (3, 3) and (4, 4) are the doubles with a total less than 10.

$$P(\text{total} < 10, \text{ given double rolled}) = \frac{4}{6} = \frac{2}{3}$$

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Exercise 10-07 Conditional probability

Stage 5.2

See Example 12

1 A bag contains 4 yellow and 3 red marbles. Two marbles are drawn from the bag without replacement.
  a Given that the first marble was red, what is the probability that the second marble is also red?
  b Given that the first marble was yellow, what is the probability that the second marble is red?

2 A shop takes delivery of 10 radio controlled cars, 3 of which are defective. Two cars are randomly selected and tested. What is the probability that the second car tested is defective, given that the first car tested was defective and was not replaced?

Alamy/Oleksiy Maksymenko Photography

3 In a chemistry class there are 5 boys and 7 girls. Two students are asked to come and help with an experiment. If the first student was a boy, what is the probability that:
  a both students chosen were boys?
  b a boy and a girl were chosen?

4 In Aiden's pencil case, there are 3 red pens, 4 blue pens and 5 black pens. Aiden takes out two pens at random. What is the probability that:
  a if one pen is red, the other one is also red?
  b if one pen is red, the other pen is blue?
  c if one pen is black, the other pen is black?
  d if one pen is blue, the other pen is black?

5 A die is rolled and a number less than 4 is the result. What is the probability that the number is even?

6 A coin is tossed and a die is rolled at the same time. Knowing that an even number has been rolled, what is the probabililty of the result being a head and a 4?

See Example 13

7 Two dice are rolled and the sum of the numbers is calculated.
  a Make up a table to show all possible sums.
  b Given that the sum is 9, find the probability that:
    i one of the dice shows a 4
    ii one of the dice shows an even number.
  c Knowing that one of the dice shows a 6, find the probability that the sum is 11.
  d Given that one of the dice shows an even number, find the probability that:
    i the sum is even
    ii the sum is 7.
  e If the dice show a double, what is the probability of a sum of 4?

Stage 5.2

8 A drawer contains 5 different pairs of coloured socks – black, blue, red, brown and white.

a Haylee randomly takes two socks from the drawer. If one of the socks is blue, what is the probability that Haylee has:

i a matching pair? **ii** two different coloured socks?

b Haylee selects two socks at random from the drawer. Given that she selected a red sock and a blue sock, what is the probability that the third sock she selects will form a matching pair?

c What is the maximum number of socks that Haylee will need to take from the drawer before she has at least one matching pair of socks?

9 Lotto in NSW is a game of chance in which 6 balls are selected at random from a barrel containing balls numbered 1 to 45. What is the chance of Cassie winning Lotto with the sixth ball, given that she has the first five numbers?

10 Three cards are chosen at random from a normal deck of 52 cards. Given that the three cards are hearts, what is the probability that the fourth card is also a heart?

11 A card is drawn at random from a deck of 52 cards. What is the probability of drawing a queen, given that the card is a heart?

12 Two dice are rolled. What is the probability of rolling a 6 on the second die, given that the first die is a 6?

13 Two dice are rolled and the difference between the numbers is calculated.

a Copy and complete the table to show all possible outcomes.

		1st die					
		1	**2**	**3**	**4**	**5**	**6**
2nd die	**1**	0					
	2					3	
	3						
	4			1			
	5						
	6						

6 − 1 = 5

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The difference between these numbers is 5.

b What is the probability of obtaining a:

i difference of 0? **ii** difference of 5? **iii** difference greater than 3?

c Knowing that the difference is 4, what is the probability that:

i one of the dice shows a 1? **ii** one of the dice shows a 3?

d If one of the dice shows a 4, find the probability that:

i the difference is 0 **ii** the difference is 1.

e Given that the difference is odd, find the probability that:

i one of the dice shows a 3 **ii** one of the dice shows an even number.

Power plus

1 Students at Arunel High School are surveyed about which sport they like to watch and what type of movies they like to see. The results are shown in the table below.

	Horror/Drama	Fantasy	Comedy	Action
Football	23	34	30	48
Cricket	25	12	45	34
Tennis	8	12	32	17

a How many students were surveyed?

b If a student is selected at random, what is the probability that the student likes to watch:

- **i** horror/drama movies?
- **ii** comedy and football?
- **iii** tennis, but not fantasy?
- **iv** action, but not cricket or tennis?

c Given that a student likes to watch football, find the probability that the student also likes to watch action movies.

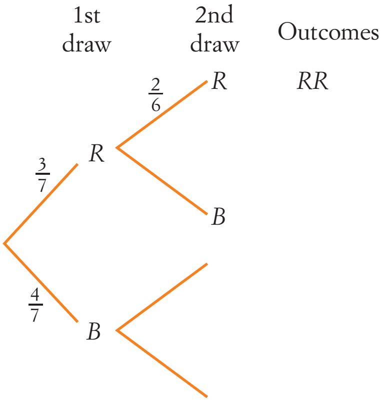
d Of the students who like comedy, what is the probability that they also like to watch cricket?

2 A bag contains 3 red and 4 blue marbles. Two marbles are taken out of the bag without replacement.

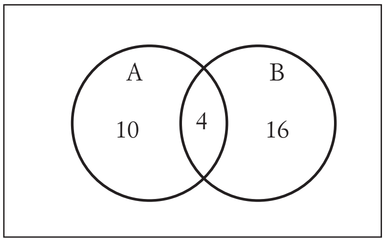
a A **probability tree diagram** is a tree diagram that has the probability of each step or stage listed on the branches.
Copy and complete the probability tree diagram shown on the right to show the possible outcomes.

b Use the probability tree diagram to find the probability of drawing:

- **i** 2 red marbles
- **ii** 2 blue marbles
- **iii** a blue and a red marble
- **iv** at least one blue marble.



3 The conditional probability of event A occurring, given that event B has occured can be written as $P(\text{A}|\text{B})$. Answer the following for the information shown in the Venn diagram.



a Find the probabilities:

- **i** $P(\text{A})$
- **ii** $P(\text{B})$
- **iii** $P(\text{A and B})$
- **iv** $P(\text{A}|\text{B})$
- **v** $P(\text{B}|\text{A})$

b **i** Find the value of $\frac{P(\text{A and B})}{P(\text{B})}$

ii Is $P(\text{A}|\text{B}) = \frac{P(\text{A and B})}{P(\text{B})}$?

c Show, by calculation, that $P(\text{B}|\text{A}) = \frac{P(\text{A and B})}{P(\text{A})}$.

Chapter 10 review

Language of maths

Puzzle sheet

Probability crossword

MAT10SPPS10076

at least
compound event
conditional probability
dependent event
die/dice
event
expected frequency
experimental probability
independent event
mutually exclusive
overlapping
random
relative frequency
sample space
table
three-step experiment
theoretical probability
tree diagram
trial
two-step experiment
two-way table
Venn diagram
with replacement
without replacement

1 What is the meaning of **expected frequency**?

2 What term from the above list is another name for **experimental probability**?

3 On a **Venn diagram**, what does the rectangle represent?

4 Give an example of **dependent events**.

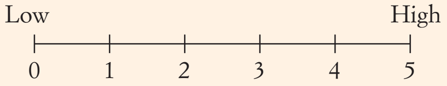
5 When are **tree diagrams** used in probability?

6 For two events A and B, what is the difference between '**A or B**' and '**A and B**'?

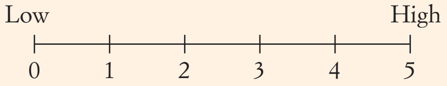
Topic overview

Rate your understanding of and your ability to do the work in this chapter by copying and completing the following scales. Circle your rating on each scale

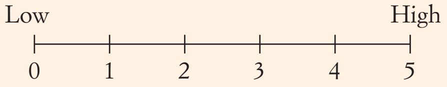
a Calculate relative frequencies to estimate probabilities of events.



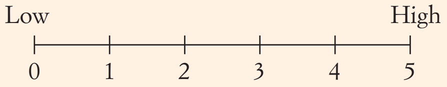
b Represent events in Venn diagrams and two-way tables and solve related problems involving probabilities.



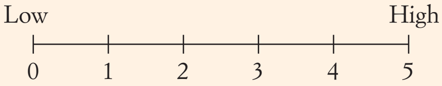
c List all outcomes for two- and three-step chance experiments, with and without replacement, using tree diagrams or tables, and determine the probabilities of events.



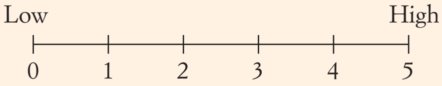
d Calculate the probabilities of compound events.



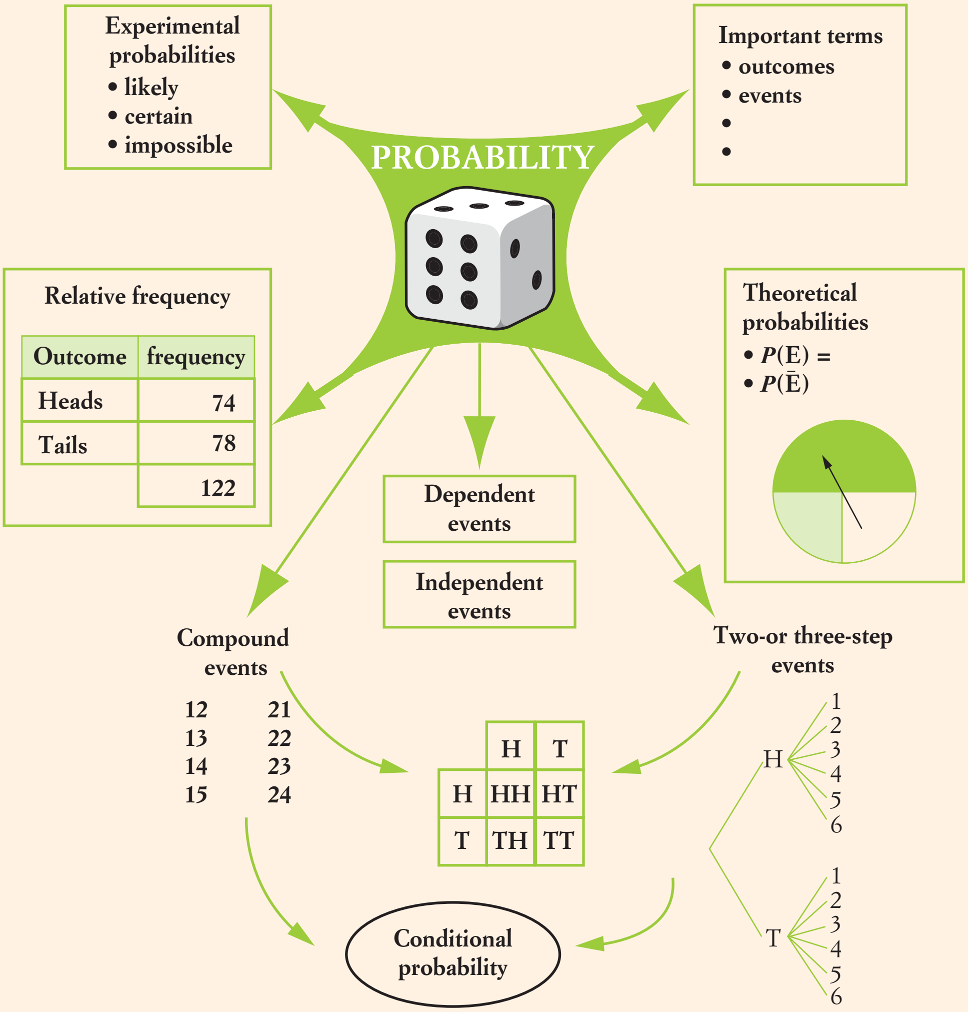
e Understand the difference between dependent and independent events.



f Calculate conditional probabilities.



Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Outcome	frequency
Heads	74
Tails	78
	122

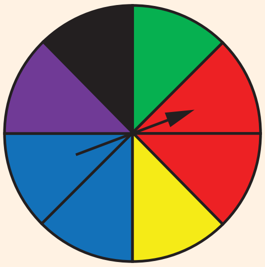
12	21
13	22
14	23
15	24

	H	T
H	HH	HT
T	TH	TT

See Exercise 10-01

1 For the spinner shown, the red and blue sectors are twice as large as the other sectors. Rafiya spun the spinner 100 times and her results are shown in the table.

Outcome	Frequency
Red	22
Blue	29
Yellow	10
Purple	12
Black	13
Green	14



a What is the experimental probability (as a decimal) of the arrow stopping on:
i red? **ii** blue?
iii yellow? **iv** green?

b What is the calculated probability (as a decimal) of the arrow stopping on:
i red? **ii** blue?
iii yellow? **iv** green?

c Are the experimental and calculated probabilities similar?

d What is the experimental probability of the arrow stopping at purple or black?

e What is the expected number of times of the arrow stopping on a colour that is not purple or black? How does this compare with Rafiya's observed number of times?

See Exercise 10-01

2 Three coins are tossed 150 times and the number of heads at each trial is recorded in the table.

Number of heads	Frequency
0	20
1	53
2	64
3	13

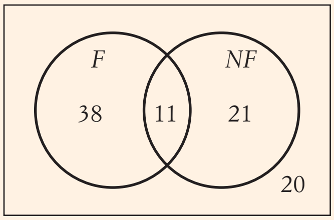
a Find the relative frequency (experimental probability) of tossing:
i one head **ii** two heads
iii three heads **iv** at least two heads.

b Find the experimental probability of:
i at least one head **ii** three tails.

c Are the answers in part **b** the same or different? Explain why.

See Exercise 10-02

3 People at a train station were asked whether they prefer to read fiction (F) or non-fiction (NF) books. The results are shown in the Venn diagram.



a How many people were surveyed?

b Find the probability of selecting a person who only reads fiction books.

c What is the probability of selecting a person who doesn't read fiction or non-fiction books?

d Of the people who read fiction books, find the probability that they read non-fiction books.

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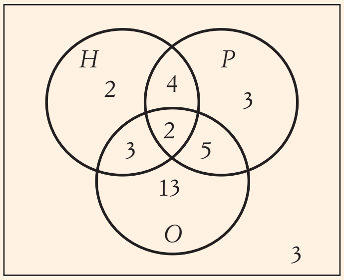
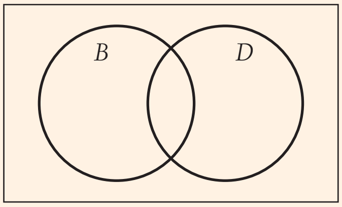
4 The Venn diagram shows the results of a survey on the types of music that school students listen to – Hip Hop/Rap (*H*), Pop (*P*) and Other(*O*), which includes R&B, Dance, Metal, Rock and Alternative.



See Exercise 10-02

- **a** How many students were surveyed?
- **b** Find the probability of selecting a student who likes to listen to all types of music.
- **c** What is the probability of selecting a student who listens to
  - **i** Hip Hop/Rap and Pop?
  - **ii** Hip Hop/Rap or Pop?
  - **iii** Pop music only?
- **d** Why are three students in the rectangle but not in the circles?

5 Of 20 people in a lift, 6 have blue eyes (*B*), 8 have dark hair (*D*) and 3 have blue eyes and dark hair.



See Exercise 10-02

- **a** Copy and complete the Venn diagram to show the given information.
- **b** What is the probability of selecting a person at random from the lift who has:
  - **i** blue eyes only?
  - **ii** dark hair?
  - **iii** blue eyes and dark hair?
  - **iv** hair that is not dark?
- **c** What is the probability of selecting a person at random who has neither blue eyes nor dark hair?

6 Students were asked what type of activities they would like to do on a camp. The results are shown in the table.

See Exercise 10-03

	Hiking	**Rock climbing**	**Kayaking**
Boys	25	38	47
Girls	45	23	22

- **a** How many students were surveyed?
- **b** Find the probability (as a decimal) that a student selected at random:
  - **i** likes rock climbing
  - **ii** likes kayaking and is a girl
  - **iii** is a girl who likes hiking
  - **iv** is a boy who likes rock climbing or kayaking
- **c** If a boy is selected at random, what is the probability that his favourite activity is hiking?
- **d** If a student that likes kayaking is chosen, what is the probability that the student is:
  - **i** a boy?
  - **ii** a girl?

Stage 5.2

See Exercise 10-04

7 Two 4-sided dice (numbered 1, 2, 3 and 4) are rolled.

a Copy and complete the table below to list the sample space.

		2nd die			
		1	**2**	**3**	**4**
1st die	**1**				
	2		2, 2		
	3				
	4			4, 3	

b How many possible outcomes are there?

c Find the probability of:

i rolling one odd and one even number

ii rolling two even numbers

iii rolling at least one 3

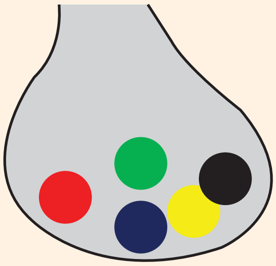
iv rolling two numbers less than 3

v rolling a double

vi rolling two numbers so that the first number is odd.

See Exercise 10-05

8 Two marbles are drawn from a bag containing a red, a blue, a green, a yellow and a black marble.



a Make up a list to show all the possible outcomes if the marbles are taken:

i with replacement

ii without replacement.

b If the first marble is replaced before the second marble is drawn, find the probability of drawing:

i two red marbles

ii two marbles of the same colour

iii a yellow and a black marble

iv at least one green marble.

c If the first marble is not replaced, find the probability of drawing:

i a green marble and a yellow marble

ii no red marbles.

See Exercise 10-05

9 The numbers 2, 4, and 7 are written on separate cards and placed in a bag. Three cards are drawn at random to form a three-digit number.

a Make up a tree diagram to list the sample space if the cards are drawn:

i with replacement

ii without replacement.

b If the cards are drawn with replacement, find the probability of forming:

i an even number

ii a number less than 400

iii the numbers 222, 444, or 777

iv an odd number greater than 400.

c If the cards are drawn without replacement, find the probability of forming:

i an odd number

ii a number greater than 400

iii a number beginning with 7

iv a number divisible by 4.

Stage 5.2

See Exercise 10-06

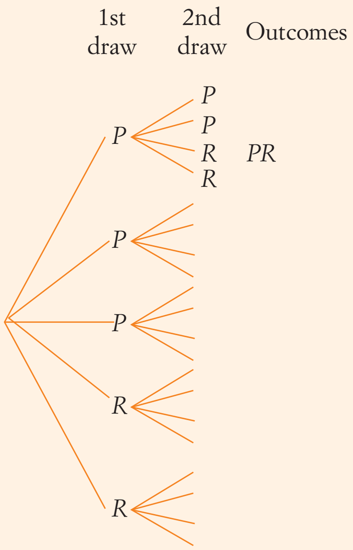
10 State whether the following pairs of events are dependent or independent.

- **a** obtaining a tail from tossing a coin and then tossing the coin again and obtaining a head
- **b** drawing a ticket in a raffle and winning a first prize and then drawing a second ticket and winning a second prize.
- **c** electing a president for a cricket club and then electing the vice-president of the cricket club
- **d** a family's first three children are girls and then the fourth child is also a girl
- **e** rolling a die and obtaining an even number and then rolling the die again and obtaining an even number again

See Exercise 10-07

11 Two marbles are drawn without replacement from a bag containing 3 purple and 2 red marbles.

- **a** Copy and complete the tree diagram to show the sample space.



- **b** Given that the first marble drawn is purple, what is the probability that the second marble will be:
  - **i** red?
  - **ii** purple?
- **c** Given that the first marble drawn is red, find the probability that the second marble is:
  - **i** red
  - **ii** purple.
- **d** Given that the drawn marbles are the same colour, what is the probability that they are:
  - **i** both red?
  - **ii** both purple?

See Exercise 10-07

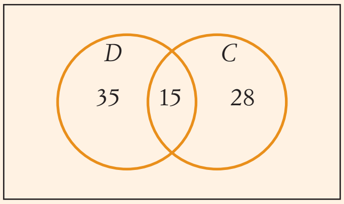
12 Two 4-sided dice (with numbers 1, 2, 3 and 4) are rolled and the sum of the numbers is calculated.

- **a** Draw up a table to show all possible sums.
- **b** Given that the sum is 5, find the probability that:
  - **i** one of the dice shows a 2
  - **ii** one of the dice shows an odd number.
- **c** Knowing that one of the dice shows a 4, find the probability that the sum is 7.
- **d** Given that one of the dice shows an even number, find the probability that:
  - **i** the sum is even
  - **ii** the sum is 5.
- **e** If the dice show a double, what is the probability of a sum of 2?

Mixed revision 4

See Exercise 10-02

1 The Venn diagram shows the results of a survey on whether people prefer to watch movies by legal download (D) or at the cinema (C).



a How many people were surveyed?

b Find the probability of selecting a person who prefers to watch movies:

i by download **ii** only at the cinema

c What is the probability of selecting a person who only likes to download movies?

Stage 5.2

See Exercise 10-05

2 Two dice are rolled.

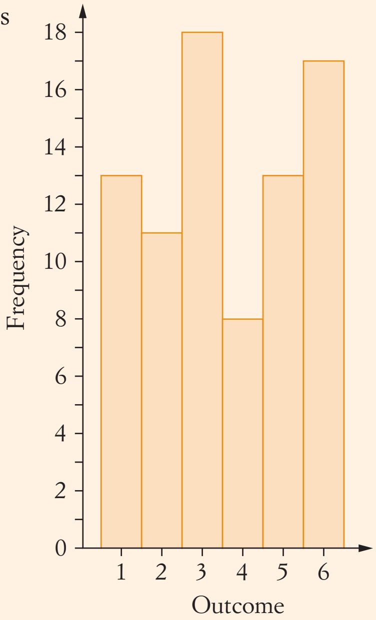
a How many outcomes are possible?

b What is the probability of rolling:

i two 1s? **ii** two of the same number? **iii** two numbers both less than 4?

See Exercise 10-01

3 A die was rolled and the number appearing uppermost was recorded in the frequency histogram.



a How many times was the die rolled?

b Use these results to find the experimental probability of rolling:

i a 1

ii an even number

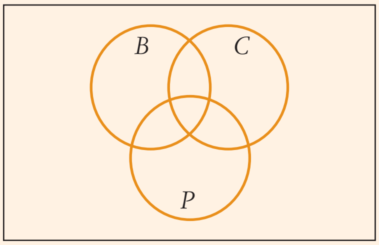
iii a number less than 4

iv at least a 3.

c What is the theoretical probability of obtaining a 6? How does this compare with the experimental probability of obtaining a 6?

See Exercise 10-02

4 Of 160 Year 11 students at Westvale High, 54 do Biology (B), 75 take Chemistry (C) and 68 study Physics (P). 55 students take both Chemistry and Physics, 20 do Biology and Chemistry and 10 students take all three.



a Copy and complete the Venn diagram to show this information.

b Find the probability of selecting a student who:

i only takes Physics

ii does not do any Science subject

iii does Chemistry and Physics but not Biology

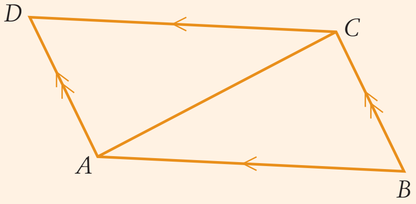
iv studies Chemistry or Biology

v only does one Science subject

c From the students that do Biology, what is the probability that the student also takes Physics?

9780170194655

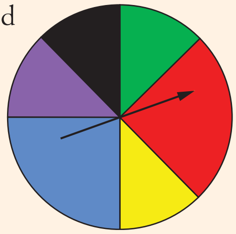
5 Prove that $\triangle ABC \equiv \triangle CDA$



Stage 5.2

See Exercise 11-03

6 Olivia spun the spinner twice, obtaining red on both spins. Is the second spin dependent on or independent of the outcome of the first spin?

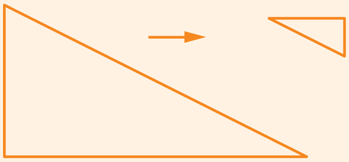


See Exercise 10-06

7 By measurement, calculate the scale factor for each pair of similar figures.

See Exercise 11-05

a



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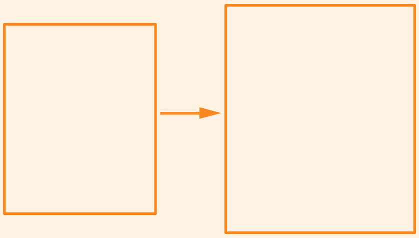


8 A bag contains 4 red, 5 black and 3 green marbles. Sadiya selects a marble at random, records its colour and then returns the marble to the bag. Sadiya repeats this process 150 times. The results are shown in the table.

See Exercise 10-01

Outcome	Frequency
red	58
black	65
green	27

a Find, correct to two decimal places, the relative frequency of selecting a marble that is:

i red **ii** black **iii** green.

b Find, correct to two decimal places, the theoretical probability of selecting a marble that is

i red **ii** black **iii** green.

c For which outcome are the experimental and theoretical probabilities most similar?

d What is the expected frequency of selecting a red or green marble? How does this compare with the observed frequency?

9 The angle sum of a regular polygon is 6120°.

Stage 5.2

See Exercise 11-01

a How many sides does the polygon have?

b Find the size of each angle.

Stage 5.2

See Exercise 10-04

10 A four-sided die (numbered 1, 2, 3 and 4) and a normal six-sided die are rolled and the product of the two numbers is calculated.

	1	**2**	**3**	**4**	**5**	**6**
1		2				
2						
3			9			
4					20	

a Copy and complete this table to show all possible products.

b Find the probability of rolling a product:

- **i** of 20
- **ii** of 6
- **iii** that is odd
- **iv** of at least 12
- **v** less than 10
- **vi** between 10 and 24.

See Exercise 10-03

11 Shoppers at a mall were asked whether they had a pet dog or cat. The results are shown in the two-way table.

	Cat	**No cat**
Dog	28	35
No dog	32	40

a How many shoppers were surveyed?

b Find the probability of selecting a shopper from the survey who does not have a cat or dog.

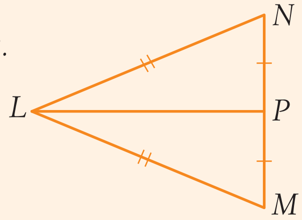
c What is the probability of a shopper from the survey:

- **i** having only a dog or cat (not both)?
- **ii** having a cat as a pet?
- **iii** not having a dog?

Stage 5.2

See Exercise 11-03

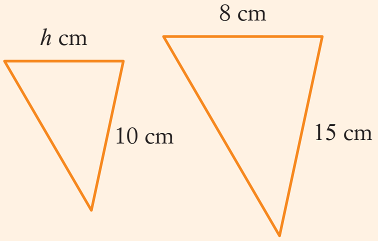
12 $LM = LN$ and P is the midpoint of MN. Prove that $\triangle LMP \equiv \triangle LNP$ and hence that $\angle LPM = \angle LPN = 90°$.



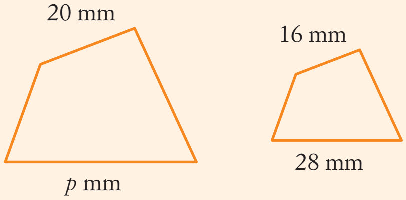
See Exercise 11-06

13 Find the value of the pronumeral in each pair of similar figures.

a



b



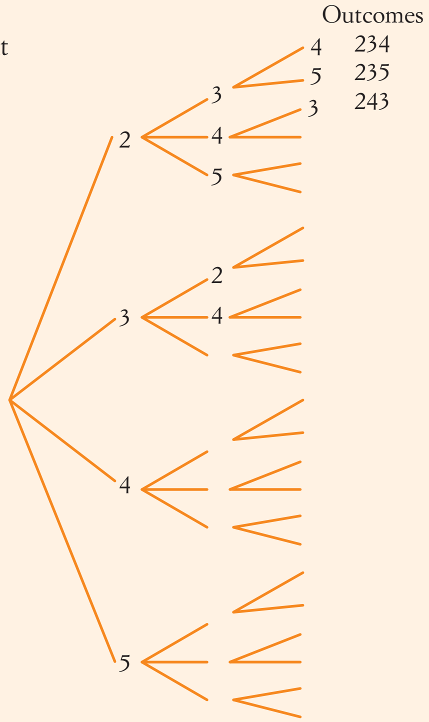
14 Three cards are drawn from a set of cards numbered 2, 3, 4 and 5, without replacement, to form a three digit number.

See Exercise 10-05

a Copy and complete the tree diagram to list all possible outcomes.

b Find the probability of forming:

- **i** an even number
- **ii** a number ending in 3
- **iii** a number greater than 400
- **iv** a number between 200 and 500
- **v** a number divisible by 5.



15 Find the angle sum of a polygon that has:

See Exercise 11-01

a 15 sides **b** 20 sides **c** 8 sides **d** 48 sides.

16 Find the number of sides in a regular polygon if each of its exterior angles is 10°.

See Exercise 11-02

17 A bag contains 3 yellow and 2 red marbles. Two marbles are drawn from the bag, without replacing the marble from the first draw.

See Exercise 10-06

a Find the probability of:

- **i** selecting a red marble with the first draw
- **ii** selecting a red marble with the second draw if the first marble was yellow.

b Are the two draws dependent or independent? Justify your answer.

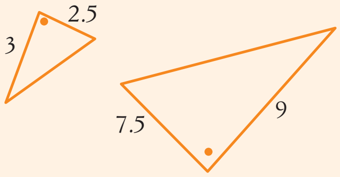
c If the two draws are made with replacement of the marble after the first draw, are the draws dependent or independent? Justify your answer.

18 Which test can be used to prove that each pair of triangles are similar?

Stage 5.2

See Exercise 11-07

a



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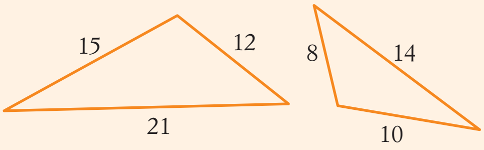

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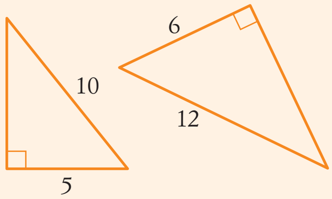

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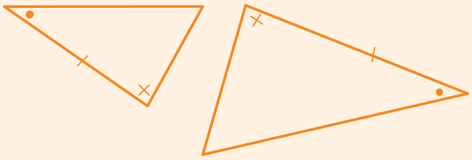
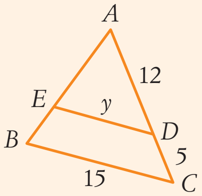


See Exercise 11-06

19 If $\triangle ABC \,|||\, \triangle AED$, find the value of y (correct to one decimal place).



See Exercise 10-07

20 Two dice are rolled and the product of the two numbers is calculated.

a Copy and complete the table to show all possible outcomes.

		1st die					
		1	**2**	**3**	**4**	**5**	**6**
2nd die	**1**	1					
	2					10	
	3						
	4			12			
	5						
	6						

b Given that one of the numbers was 3, what is the probability of obtaining a product that is odd?

c Given that one of the numbers is 4, what is the probability of obtaining a product:

i that is even? **ii** that is odd?

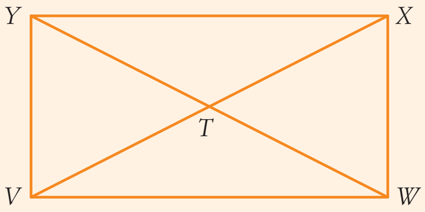
d Given that the first number is odd, find the probability of obtaining an even product.

e Given that an even number and an odd number are obtained, find the probability of obtaining an even product.

Stage 5.2

See Exercise 11-04

21 $VWXY$ is a rectangle with diagonals intersecting at T.



a Since $YX \parallel VW$, list pairs of equal alternate angles.

b Why is $YX = VW$?

c Which congruence test can now be used to prove that $\triangle YXT \equiv \triangle WVT$?

d Explain why $YT = WT$ and $XT = VT$.

e Which property of a rectangle has been proved?

General revision

See Chapter 1

1 Kim is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of $1250. How much will Kim earn if she sells an apartment for $578 000?

Stage 5.2

See Chapter 2

2 Find the gradient, m, and y-intercept, b, of each linear equation.

a $y = 3x - 4$ **b** $y = 5 - 2x$ **c** $4x + 3y - 9 = 0$

See Chapter 4

3 Factorise each expression.

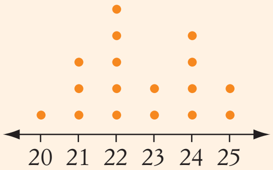
a $8x^2 + 16x$ **b** $y^2 - 25$ **c** $a^2 + 6a + 8$ **d** $5p + 3 - 2p^2$

See Chapter 5

4 Find the interquartile range for each set of data.

a 4 7 8 12 5 8 10 7 13 6 9 2

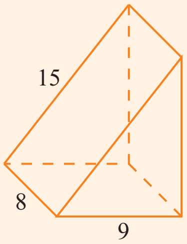
b



See Chapter 3

5 Find the surface area of each solid correct to the nearest mm^2. All measurements shown are in millimetres.

a



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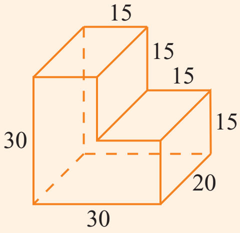
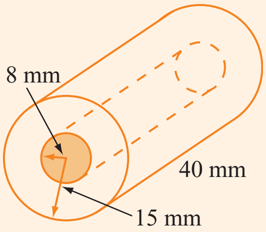


c



See Chapter 10

6 Students in Year 10 at a particular high school were asked if they had studied French at high school.

	Male	Female
French	35	87
No French	67	21

a How many students are in Year 10 at the school?

b What is the probability of selecting a Year 10 student at random who is:

i male and has studied French at high school?

ii female or studied French at high school?

c Find the probability, expressed as a percentage to the nearest whole number, of randomly selecting a male in Year 10 who has not studied French at high school.

d Given that a student is a Year 10 female, what is the probability that she has studied French at high school?

Stage 5.2

See Chapter 11

7 **a** If a regular polygon has 9 sides, calculate the size of each interior angle.

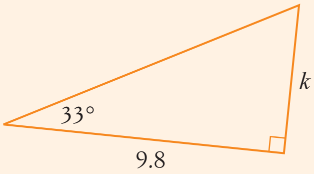
b Find the size of each exterior angle.

General revision

See Chapter 8

8 Find the value of the pronumeral in each diagram, correct to one decimal place.

a



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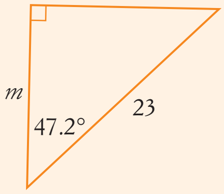

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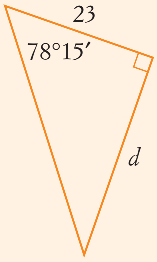

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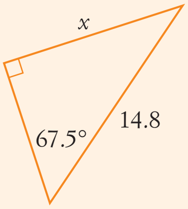

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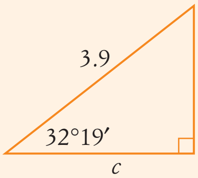

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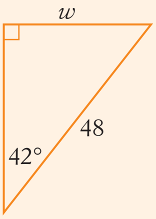


See Chapter 4

9 Simplify each expression, writing your answer with a positive index where necessary.

a $7x^5 \times 8x^7$ **b** $4x^2 \div 16x^{-3}$ **c** $(3y)^{-2}$ **d** $m^{-6}n^3 \times mn^{-1} \div m^2n$

See Chapter 6

10 Solve each equation.

a $2k - 5 = 8$ **b** $3(m + 7) = 12$ **c** $4(x - 3) - 2(x - 1) = 5$

See Chapter 8

11 The angle of elevation of the top of a tree is 65° at a distance of 12 m from its base. Find the height of the tree, to the nearest metre.

Stage 5.2

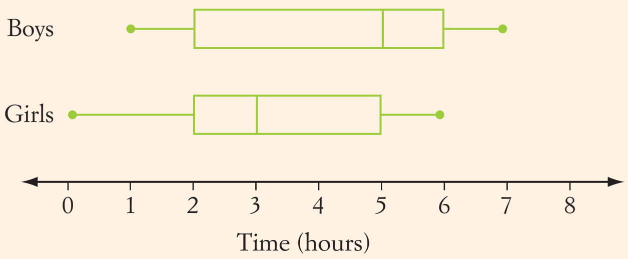
See Chapter 6

12 Solve each equation.

a $\frac{a - 2}{3} = 7$ **b** $\frac{4y}{5} - 2 = 3$ **c** $\frac{3m + 8}{2} = -1$ **d** $\frac{5k}{3} - \frac{k}{2} = 4$

13 In a school, students were surveyed about the amount of time they typically spend on the Internet over a weekend. The results are displayed separately for boys and girls in the boxplots below.

See Chapter 5

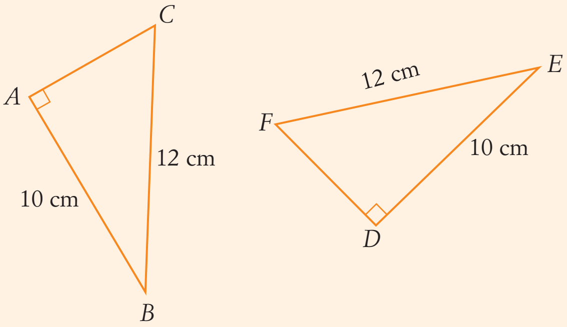


a Calculate the interquartile range for the boys.
b What is the median amount of time spent on the Internet by the girls surveyed?
c What percentage of girls usually spend fewer than 5 hours on the Internet over the weekend?

Stage 5.2

See Chapter 11

14 Prove that $\triangle ABC \equiv \triangle DEF$.



15 Solve each inequality and graph each solution on a number line.

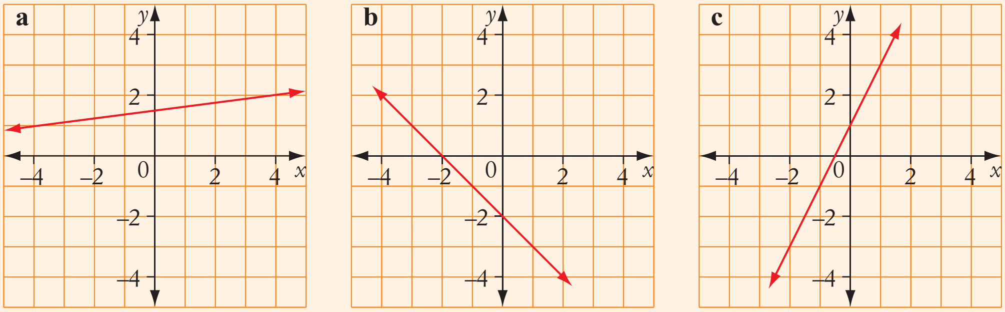
See Chapter 6

a $5y + 3 \geq -2$ **b** $\dfrac{2x+5}{2} < 4$ **c** $5 - 4x > 17$

16 For each line, find:

See Chapter 2

i the gradient **ii** the y-intercept **iii** the equation of the line.



Stage 5.2
See Chapter 9

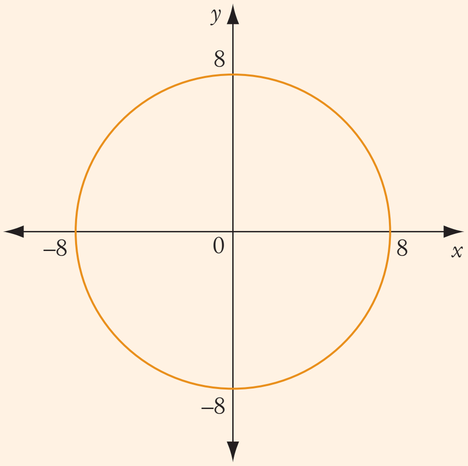
17 Solve each pair of simultaneous equations.

a $3x - y = 4$
$2x + y = 6$

b $2m - 3p = 5$
$5m - 2p = 7$

See Chapter 7

18 Find the equation of this circle.



See Chapter 1

19 Calculate the simple interest on each investment.

a \$500 invested at 4% p.a. for 2 years
b \$280 invested at 2.5% p.a. for 7 months

See Chapter 8

20 Find θ, correct to the nearest degree, if:

a $\sin\theta = \frac{3}{7}$ **b** $\tan\theta = 6$ **c** $\cos\theta = 0.816$

Stage 5.2
See Chapter 1

21 Calculate the final amount for each investment.

a \$800 invested at 4% p.a. for 4 years, compounded annually
b \$1260 invested at 8% p.a. for 3 years, compounded quarterly

See Chapter 2

22 For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place
ii the midpoint of the interval
iii the gradient of the interval

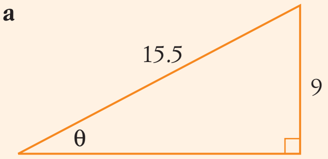
a $C(2, 1)$ and $D(6, 9)$ **b** $X(-7, -2)$ and $Y(5, 4)$

See Chapter 8
Stage 5.2

23 Find θ:

i correct to the nearest degree
ii correct to the nearest minute

a



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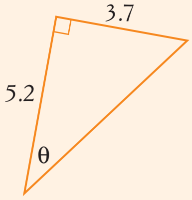

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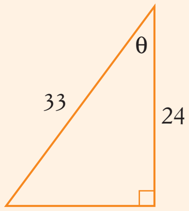


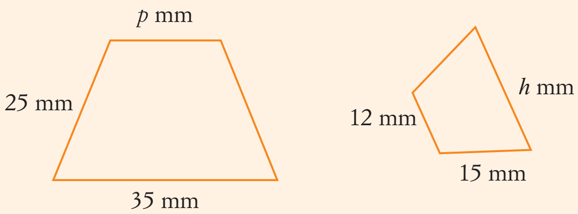
24 Graph each equation on a number plane.

See Chapter 7

a $y = x^2$ **b** $y = -x^2 + 3$ **c** $y = 3^x$

25 Find the value of h and p in this pair of similar figures.

See Chapter 11

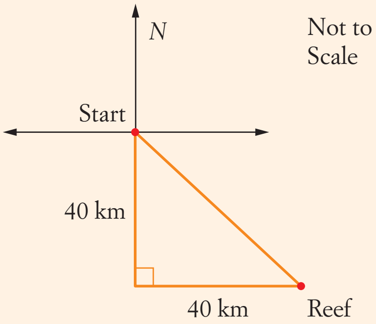


Stage 5.2

See Chapter 8

26 Imogen sailed due south for 40 km. Then she sailed due east for 40 km to a reef. What is the bearing of the reef from Imogen's starting point? Select the correct answer **A**, **B**, **C** or **D**.

A 225° **B** 045° **C** 090° **D** 135°



Stage 5.2

See Chapter 10

27 The weather on a long weekend will either be fine or rainy each day, with each outcome being equally likely.

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that it is fine:

i on all three days? **ii** on exactly two of the days?

iii on at least one of the days?

See Chapter 7

28 For an object that is cooling, the drop in temperature is directly proportional to the time. If the temperature drops 5°C in 12 minutes, how long would it take to drop 8°C?

11

Measurement and geometry

Geometry

The word ‘geometry’ comes from the Greek word *geometria*, which means ‘land measuring’. The principles and ideas of geometry are evident in many aspects of our lives. For example, geometry can be seen in the design of buildings, bridges, roads and transport networks.

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Chapter outline

	Proficiency strands				
11-01 Angle sum of a polygon*	U	F		R	C
11-02 Exterior angle sum of a polygon*	U	F		R	C
11-03 Congruent triangle proofs*	U	F	PS	R	C
11-04 Proving properties of triangles and quadrilaterals*	U	F	PS	R	C
11-05 Similar figures	U	F		R	C
11-06 Finding unknown sides in similar figures	U	F		R	C
11-07 Tests for similar triangles*	U	F	PS	R	C

*STAGE 5.2

Wordbank

congruence test	One of four tests for proving that triangles are congruent: SSS, SAS, AAS and RHS
congruent	Identical, exactly the same (symbol: ≡)
enlargement	An increase in the size of a shape
included angle	The angle between two given sides of a shape
convex polygon	A polygon whose vertices all point outwards
regular polygon	A polygon with all angles equal and all sides equal, such as an equilateral triangle or a square
scale factor	The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$
similar	To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: |||)

In this chapter you will:

- (STAGE 5.2) formulate proofs involving congruent triangles and angle properties
- solve problems using ratio and scale factors in similar figures AC
- (STAGE 5.2) solve problems involving the angle sum of a polygon and the exterior angle sum of a convex polygon
- (STAGE 5.2) write formal proofs for congruent triangles
- (STAGE 5.2) prove properties of triangles and quadrilaterals using congruent triangles
- explain similarity and investigate the properties of similar figures
- (STAGE 5.2) identify and use the four tests for similar triangles

SkillCheck

Worksheet
StartUp assignment 11
MAT10MGWK10077

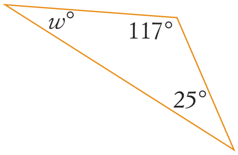
Puzzle sheet
Finding angles
MAT10MGPS00026

Video tutorial
Geometry
MAT10MGVT00008

Skillsheet
Starting Geometer's SketchPad
MAT10MGSS10013

1 Find the value of each pronumeral.

a



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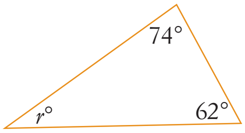

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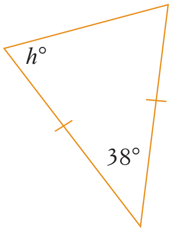

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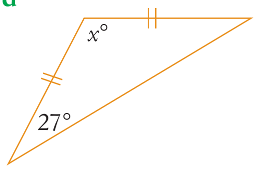

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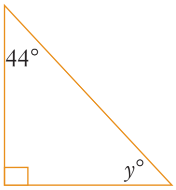

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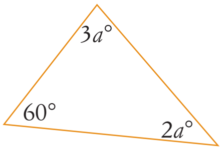
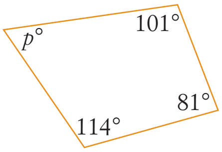


2 Find the value of *p* in each diagram.

a



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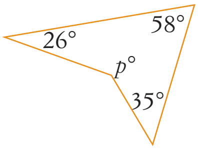

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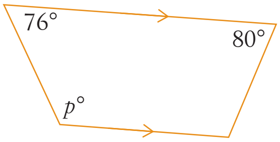

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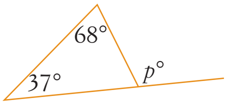

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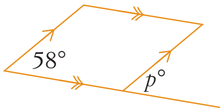

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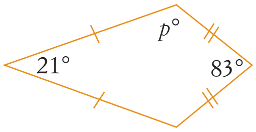
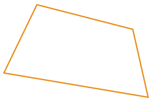


3 Match shapes that are congruent.

a



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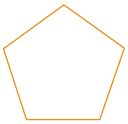

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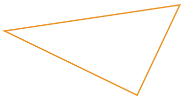
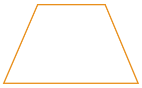


e



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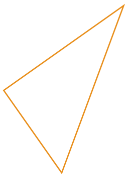

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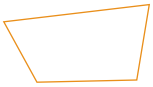

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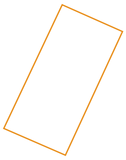

i

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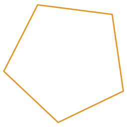

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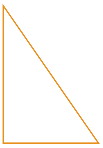

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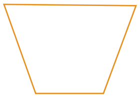
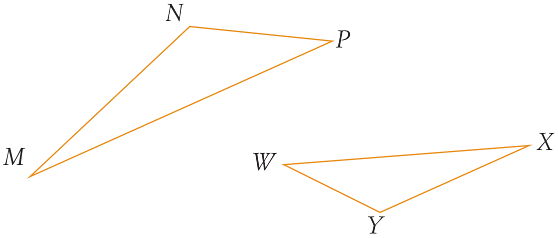


4 Triangles *MNP* and *WXY* are similar.

a List all pairs of matching angles.

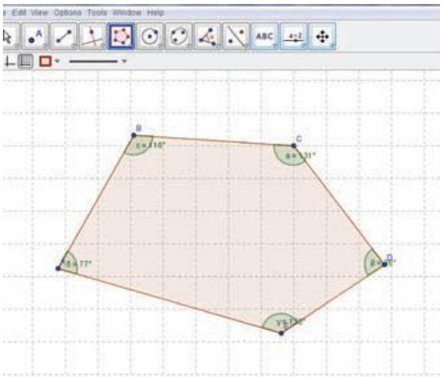
b List all pairs of matching sides.



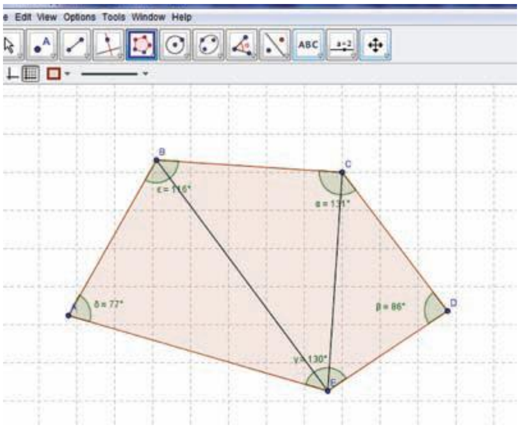
Technology Angle sum of a polygon

In this activity, you will use *GeoGebra* to find a rule for the angle sum of a polygon.

1. Before you start, remove **Axes** and click **Grid** on. Also, click **Options Rounding 0 decimal places**.
2. Click **polygon** and construct any pentagon. Click **Angle** and in a *clockwise direction*, measure the size of each of the five angles in the pentagon (these are called the **interior angles**). Find the total sum of the five interior angles. Write your answer in your book.



3 Click **Interval between Two Points** and from only one vertex, draw as many triangles as possible in your pentagon (as shown below).



4 Copy and complete this table for the above pentagon.

No. of sides in polygon	Angle sum of each triangle	Angle sum of all triangles in polygon	$\frac{\text{Angle sum of polygon}}{\text{Angle sum of triangle}}$

5 Repeat Steps **1** and **2** for the following shapes:

- regular hexagon (use **Regular Polygon** to construct it)
- regular pentagon
- octagon (use **Polygon** to construct it)
- nonagon (9 sides)

6 Continue the table from Step **4** for each shape. Can you see the relationship between the number of sides in a polygon (n) and the angle sum of a triangle?

7 Complete the rule for the angle sum of any polygon.

$$\text{Angle sum of a polygon with } n \text{ sides} = 180 \times (n - \_\_\_\_)$$

Stage 5.2

NSW

Technology

GeoGebra: Naming polygons

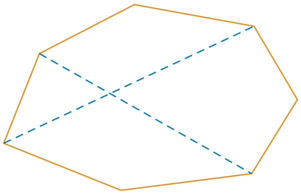
MAT10MGTC00008

11-01 Angle sum of a polygon

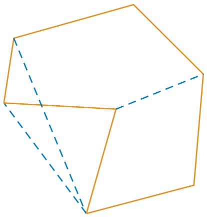
Alamy/Raymond Warren

Stage 5.2

A **polygon** is any shape with straight sides. A polygon may be either **convex** or **non-convex** (**concave**).



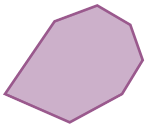
Convex polygon



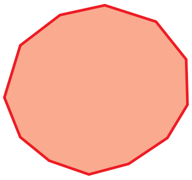
Non-convex polygon

In a convex polygon, all vertices point outwards, all diagonals lie within the shape and all angles are less than 180°. In a non-convex polygon, some vertices point inwards, some diagonals lie outside the shape and some angles are more than 180° (reflex angles).
A polygon's name is determined by the number of sides that it has.

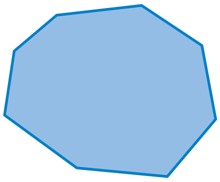
Name	Number of sides
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10
Hendecagon	11
Dodecagon	12



heptagon



dodecagon



octagon

Summary

The **angle sum of a polygon** with n sides is given by the formula $A = 180(n - 2)°$.
This formula applies to both convex and non-convex polygons.

Example 1

Find the angle sum of a 15-sided polygon.

Solution

$$\begin{aligned}\text{Angle sum} &= 180(15 - 2)^\circ \qquad n = 15\\ &= (180 \times 13)^\circ\\ &= 2340^\circ\end{aligned}$$

Stage 5.2

Example 2

Find the number of sides in a polygon that has an angle sum of 1080°.

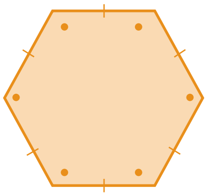
Solution

$$\begin{aligned} 180(n-2) &= 1080 \\ 180n - 360 &= 1080 \\ 180n &= 1440 \\ n &= \frac{1440}{180} \\ &= 8 \end{aligned}$$

∴ The polygon has 8 sides (octagon).

Regular polygons

A **regular polygon** has all its angles and sides equal.
For example, a regular hexagon has 6 equal angles and 6 equal sides.
A square is a regular polygon but a rhombus is not.



Summary

The size of each angle in a **regular polygon** with n sides $= \dfrac{\text{Angle sum}}{\text{Number of sides}}$

$$= \frac{180(n-2)^\circ}{n}$$

Example 3

Find the size of one angle in a regular pentagon.

Solution

A pentagon has 5 sides ($n = 5$).

$$\begin{aligned} \text{Size of one angle} &= \frac{180(5-2)^\circ}{5} \\ &= \frac{(180 \times 3)^\circ}{5} \\ &= 108^\circ \end{aligned}$$

Each angle in a regular pentagon is 108°.

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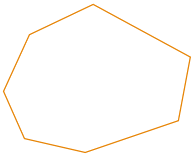
Exercise 11-01 Angle sum of a polygon

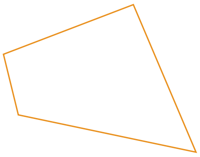
Stage 5.2

1 Name each polygon.

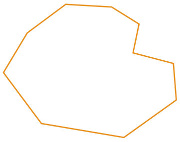
a



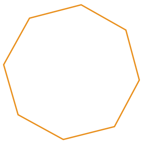
b



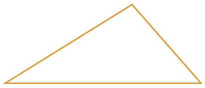
c



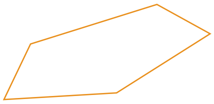
d



e



f



2 Which polygons from question **1** are:

a convex? **b** regular?

3 What is the more common name for a regular triangle? Select the correct answer **A**, **B**, **C** or **D**.

A isosceles **B** scalene **C** equilateral **D** acute

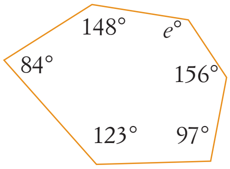
4 Find the angle sum of a polygon with:

See Example 1

a 12 sides **b** 10 sides **c** 9 sides **d** 20 sides **e** 15 sides.

5 Find the value of each pronumeral.

a



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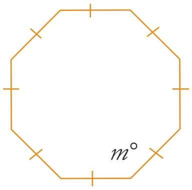

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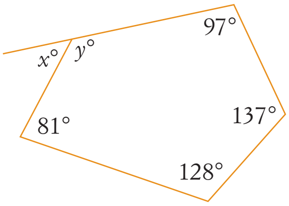

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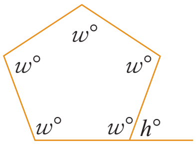

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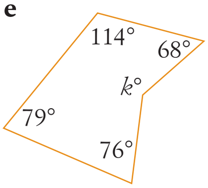

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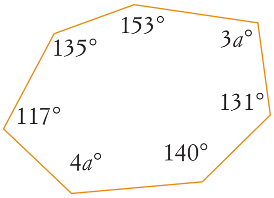


6 Find the number of sides in a polygon that has an angle sum of:

See Example 2

a 720° **b** 3420° **c** 1980° **d** 5040° **e** 1260°.

7 The angle sum of a regular polygon is 2520°.

a How many sides does the polygon have?

b Find the size of each angle.

8 Find the size of one angle in a regular

See Example 3

a decagon **b** octagon **c** hexagon **d** dodecagon.

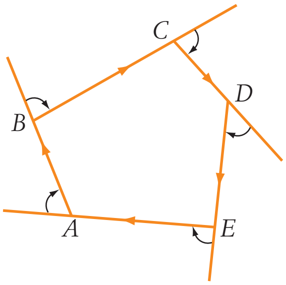
9 How many sides does a regular polygon have if each of its angles is:

a 168°? **b** 156°? **c** 172°? **d** 165.6°?

Stage 5.2

Investigation: Exterior angle sum of a convex polygon

1 Draw any convex polygon and extend the sides as shown. Label the vertices A, B, C, etc.



2 Use a protractor to measure all of the exterior angles of the polygon.
3 What is the sum of the exterior angles of the polygon?
4 Start at A and move around the polygon, turning in the direction indicated at each vertex, until you return to A, facing the same direction that you started from.
5 What must be the sum of the turns in any round trip of a convex polygon?
6 Test whether this rule works for a non-convex polygon.

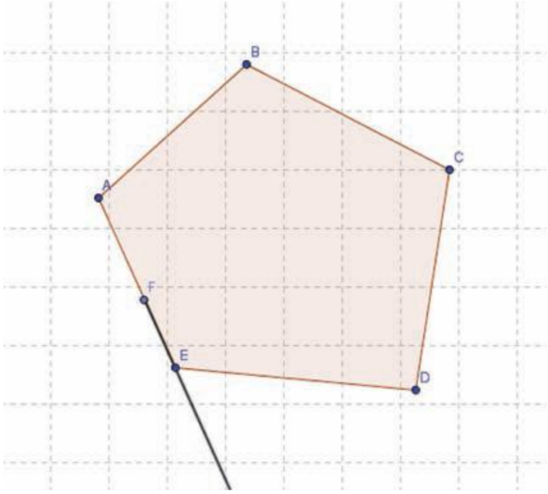
Technology Exterior angle sum of a polygon

In this activity, you will use *GeoGebra* find a rule for the exterior angle sum of a polygon.

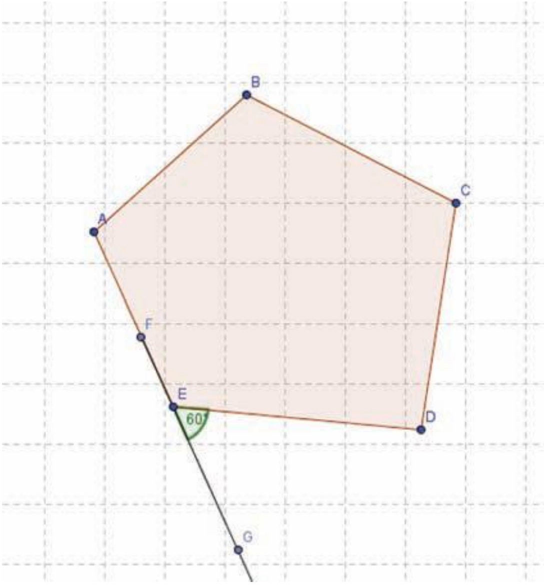
1 Before you start, remove **Axes** and click **Grid on**. Also, click **Options Rounding 0 decimal places**.
2 Click **Polygon** and construct any pentagon.
3 Click

Ray through Two Points
Select starting point, then point on ray

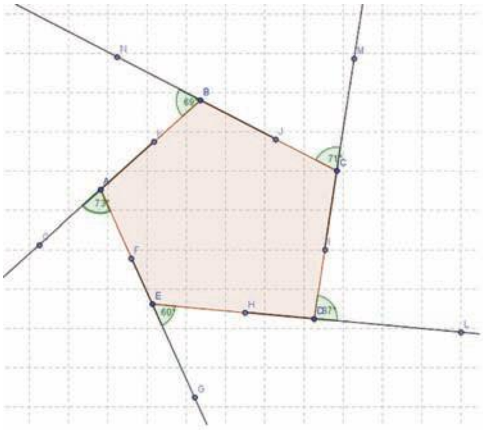
To produce the side AE, click a point on AE and then point E.



4 Insert **New Point** on the produced side AE (in diagram shown as point G). Click **Angle** and in a *clockwise direction*, measure the size of each of the angles outside the pentagon (these are called the **exterior angles**). This is shown below as ∠GED = 60°.



5 Repeat step **3** to produce the remaining 4 sides of the pentagon. Use **Angle** to find the size of each exterior angle. When completed, your diagram should look like the one shown below, with 5 exterior angles.



6 Calculate the total sum of the five exterior angles. Write the answer in your book.

7 Copy and complete this table for the above pentagon.

No. of sides in polygon	Angle sum of all exterior angles of polygon

Stage 5.2

8 Repeat steps 1 to 6 for the following shapes.
- octagon (use **Polygon** to construct it)
- regular hexagon (use **Regular Polygon** to construct it)
- regular heptagon (use **Regular Polygon** to construct it)

9 Continue the table from step **7** for each shape. What is the exterior angle sum of any polygon?

10 Complete the rule for the exterior angle sum of any polygon:

Exterior angle sum of a polygon = _______

11 Using the results of the regular polygons from step **8**, complete the rule for the exterior angle of any regular polygon:

Exterior angle of a regular polygon with n sides = _______

NSW

11-02 Exterior angle sum of a convex polygon

Worksheet
Angles in polygons
MAT10MGWK10078

Summary

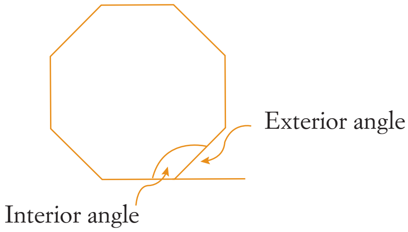
The **sum of the exterior angles** of a convex polygon is 360°.

Example 4

For a regular octagon, find the size of:

a each exterior angle

b each (interior) angle.



Solution

a Sum of exterior angles = 360°

One exterior angle = 360° ÷ 8

= 45°

b Each angle = 180° – 45° (angles on a straight line)

= 135°

OR : Each angle $= \dfrac{180(8-2)^\circ}{8}$

= 135°

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Example 5

Stage 5.2

Find the number of sides in a regular polygon if:

a each exterior angle is 24° **b** each (interior) angle is 140°.

Solution

a $$\begin{aligned}\text{Number of exterior angles} &= 360 \div 24\\ &= 15\end{aligned}$$

$\therefore$ The regular polygon has 15 sides.

b $$\begin{aligned}\text{Exterior angle} &= 180^\circ - 140^\circ \quad \text{(angles on a straight line)}\\ &= 40^\circ\end{aligned}$$

$$\text{Sum of exterior angles} = 360^\circ$$

$$\begin{aligned}\text{Number of exterior angles} &= 360 \div 40\\ &= 9\end{aligned}$$

$\therefore$ The regular polygon has 9 sides.

OR: $$\begin{aligned}\frac{180(n-2)}{n} &= 140\\ 180(n-2) &= 140n\\ 180n - 360 &= 140n\\ 40n - 360 &= 0\\ 40n &= 360\\ n &= \frac{360}{40}\\ &= 9\end{aligned}$$

$\therefore$ The regular polygon has 9 sides.

Exercise 11-02 Exterior angle sum of a polygon

1 Find the size of each exterior angle in a regular: See Example 4

a pentagon **b** dodecagon **c** 18-sided polygon **d** hexagon.

2 Find the size of each angle in a regular:

a nonagon **b** 20-sided polygon **c** decagon **d** 30-sided polygon.

3 Find the number of sides in a regular polygon if each exterior angle is: See Example 5

a 15° **b** 72° **c** 20° **d** 40° **e** 5° **f** 12°.

4 Find the number of sides in a regular polygon if each angle is:

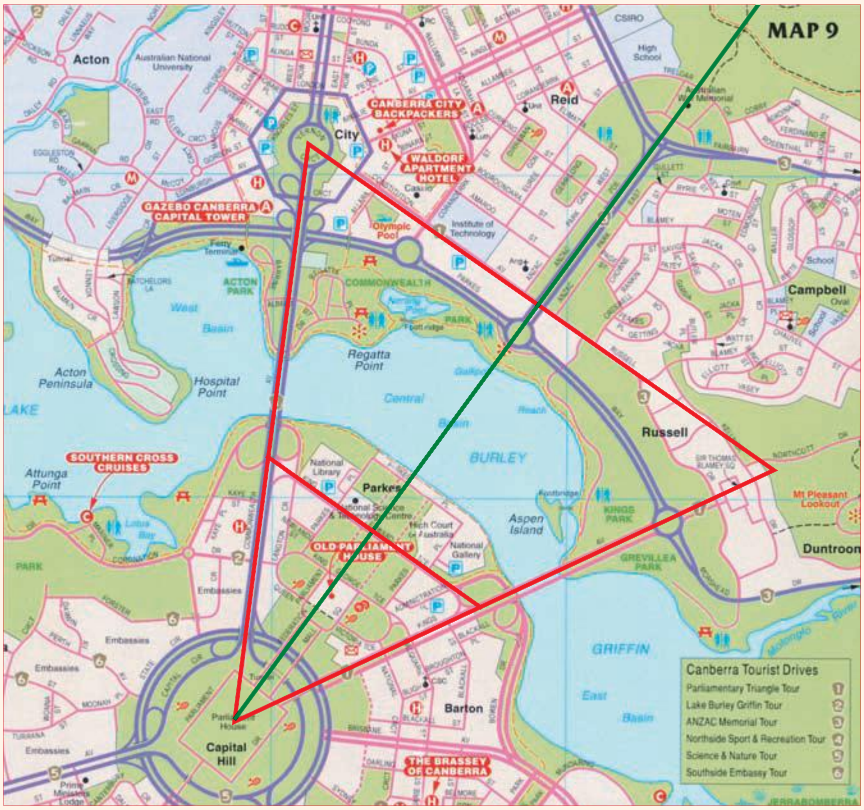
a 135° **b** 144° **c** 156° **d** 178° **e** 165° **f** 150°.

Stage 5.2

Just for the record The geometry of Canberra

Canberra is located 300 km south-west of Sydney and was designed by the American architect Walter Burley Griffin. Construction of Australia's capital city began in 1913. The 'centre' of Canberra is based on an equilateral triangle, bounded by the 'sides' Commonwealth Avenue, Kings Avenue and Constitution Avenue. The smaller 'Parliamentary triangle' is bounded by Commonwealth Avenue, Kings Avenue and King Edward Terrace. The axis of symmetry of the triangle runs from Parliament House, across Lake Burley Griffin, directly along Anzac Parade to the Australian War Memorial.

What other geometrical features can you see in Canberra's design?



Worksheet
Congruent triangles proofs
MAT10MGWK10079

Worksheet
Congruent triangles
MAT10MGWK00022

Video tutorial
Congruent triangles proofs
MAT10MGVT10019

Worksheet
Congruent and similar triangle proofs
MAT10MGWK10083

11-03 Congruent triangle proofs

Two figures are **congruent** if they are identical in shape and size. For congruent figures:

- **matching sides** are equal
- **matching angles** are equal

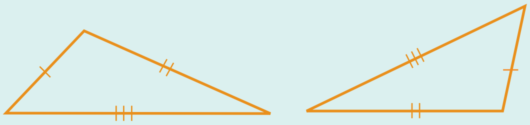
There are four sets of conditions that can be used to determine if **two triangles** are congruent. These are called the **tests for congruent triangles** or **congruence tests**.

9780170194655

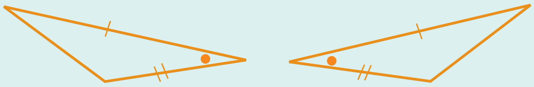
Summary

There are four tests for congruent triangles: **SSS**, **SAS**, **AAS** or **RHS**.
Two triangles are congruent if:

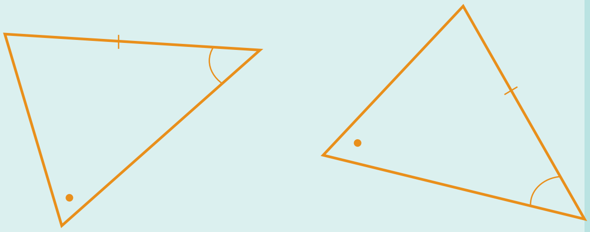
- the three sides of one triangle are respectively equal to the three sides of the other triangle (**SSS** rule)



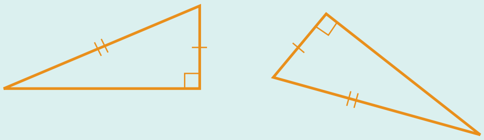
- two sides and the **included angle** of one triangle are respectively equal to two sides and the **included angle** of the other triangle (**SAS** rule)



- two angles and one side of one triangle are respectively equal to two angles and the matching side of the other triangle (**AAS** rule)



- they are right-angled and the hypotenuse and another side of one triangle are respectively equal to the hypotenuse and another side of the other triangle (**RHS** rule).



The congruence symbol ≡

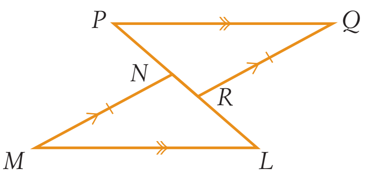
The symbol for 'is congruent to' is a special equals sign, written as '≡' (which also means 'is identical to'). The two triangles below are congruent, so we can write $\triangle \boldsymbol{ABC} \equiv \triangle \boldsymbol{XYZ}$, which is read as 'triangle ABC is congruent to triangle XYZ'.

When using this notation, we must make sure that the vertices (angles) of the congruent figures are written in matching order: $\triangle ABC \equiv \triangle XYZ$ means $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$.
To formally prove that two triangles are congruent, we need to use one of the four tests for congruence SSS, SAS, AAS or RHS.

Stage 5.2

Example 6

In the diagram, $PQ \parallel LM$, $QR \parallel MN$ and $QR = MN$.
Prove that $\triangle PQR \equiv \triangle LMN$.



Solution

In $\triangle PQR$ and $\triangle LMN$:

Identifying the triangles in matching order of vertices.

$QR = MN$ (given)

Stating each part of the congruence test, giving reasons.

$\angle P = \angle L$ (alternate angles, $PQ \parallel LM$)

$\angle QRP = \angle MNL$ (alternate angles, $QR \parallel MN$)

$\therefore \triangle PQR \equiv \triangle LMN$ (AAS)

Concluding the congruence proof, stating the test used.

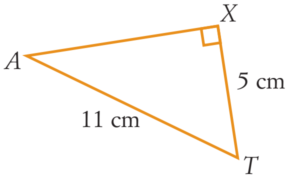
Exercise 11-03 Congruent triangle proofs

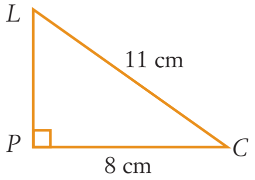
1 For each set of triangles:

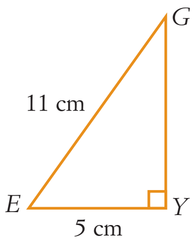
i decide which two are congruent and state the congruence test used

ii use the correct notation to write a congruency statement relating those two triangles.

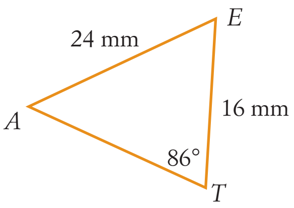
a

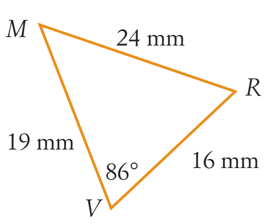


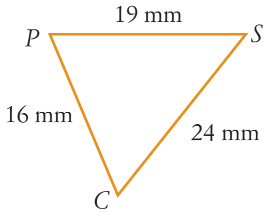




b

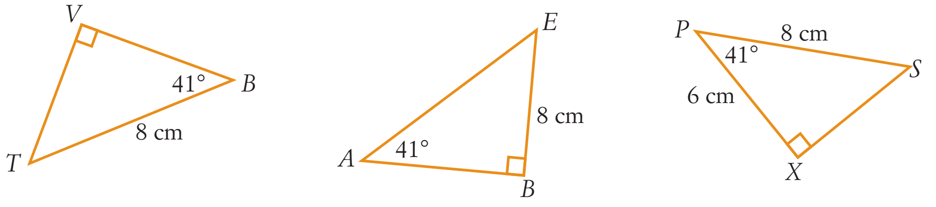




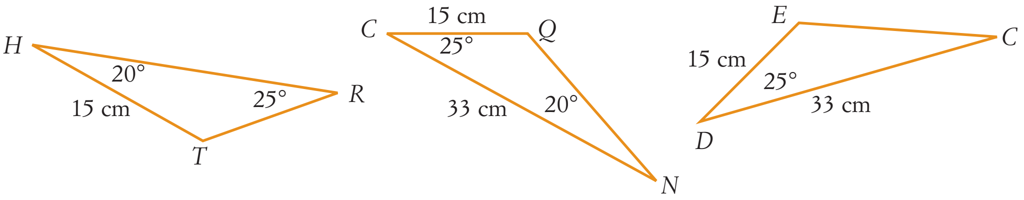


Stage 5.2

c

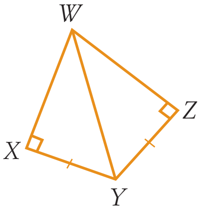


d

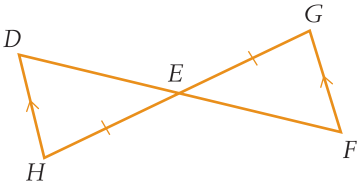


2 Prove that $\triangle WXY \equiv \triangle WZY$.

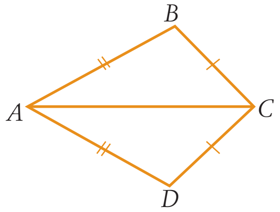
***See* Example 6**



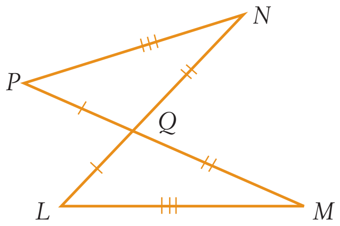
3 In the diagram, $EG = EH$ and $DH \parallel FG$.
Show that $\triangle DEH \equiv \triangle FEG$.



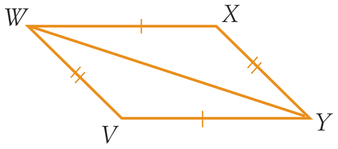
4 For this kite $ABCD$, prove that $\triangle ABC \equiv \triangle ADC$.



5 If $PQ = LQ$ and $NQ = MQ$, prove that $\triangle PQN \equiv \triangle LQM$.

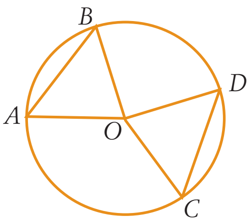


6 Prove that $\triangle WXY \equiv \triangle YVW$.

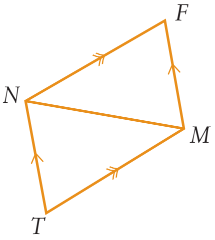


Stage 5.2

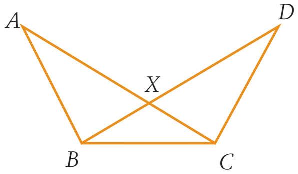
7 O is the centre of the circle and $AB = CD$.
Prove that $\triangle AOB \equiv \triangle COD$.



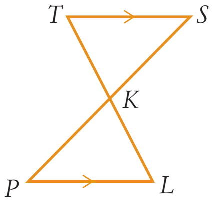
8 Prove that $\triangle FNM \equiv \triangle TMN$.



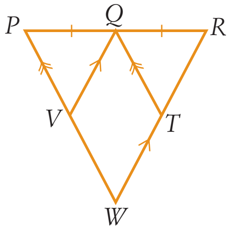
9 If $\angle ABC = \angle DCB$ and $AB = DC$ in the diagram, prove that $\triangle ABC \equiv \triangle DCB$.



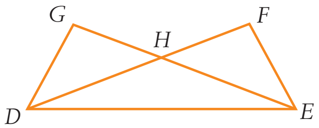
10 $TS \parallel PL$ and K is the midpoint of TL. Prove that $\triangle TSK \equiv \triangle LPK$.



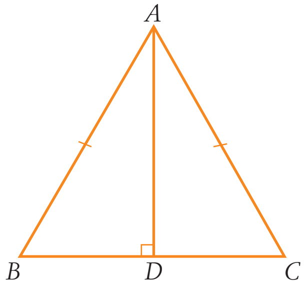
11 In the diagram, $PW \parallel QT$, $RW \parallel QV$ and $PQ = QR$.
Prove that $\triangle PVQ \equiv \triangle QTR$.



12 If $\angle DEG = \angle EDF$ and $GE = FD$,
prove that $\triangle DEG \equiv \triangle EDF$.



13 In $\triangle ABC$, $AB = AC$ and $AD \perp BC$. Prove that $\triangle ABD \equiv \triangle ACD$ and hence AD bisects $\angle BAC$.



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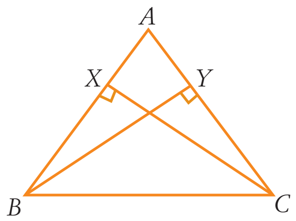
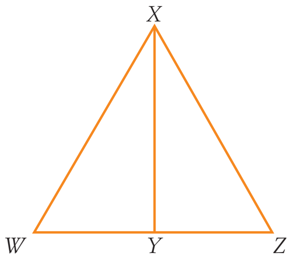
14 If $CX \perp AB$, $BY \perp AC$ and $XC = YB$, prove that $\triangle BCX \equiv \triangle CBY$.



15 $XW = XZ$ in the isosceles triangle and Y is the midpoint of WZ. Prove that $\triangle WYX \equiv \triangle ZYX$.



Stage 5.2

Worksheet
Quadrilaterals: True or false?
MAT10MGWK00020

Technology
GeoGebra: Making quadrilaterals
MAT10MGTC00012

Worksheet
Proving properties of quadrilaterals
MAT10MGWK10080

Animated example
Geometric problems and proofs
MAT10MGAE00008

Puzzle sheet
Geometrical proofs order activity
MAT10MGPS10081

Worksheet
Congruent and similar triangle proofs
MAT10MGWK10083

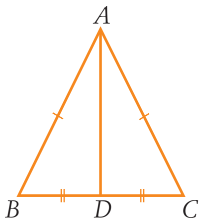
11-04 Proving properties of triangles and quadrilaterals

Properties of triangles and quadrilaterals can be proved using the congruence tests.

Example 7

$\triangle ABC$ is an isosceles triangle with $AB = AC$. D is the midpoint of BC.

a Which congruence test can be used to prove that $\triangle ABD \equiv \triangle ACD$?
b Explain why $\angle B = \angle C$.
c What geometrical result about isosceles triangles does this prove?



Solution

a For $\triangle ABD$ and $\triangle ACD$:

$AB = AC$ (given)
AD is common.
$BD = CD$ (D is the midpoint of BC)
$\therefore$ The congruence test is SSS.

b $\angle B = \angle C$ because they are matching angles of congruent triangles.

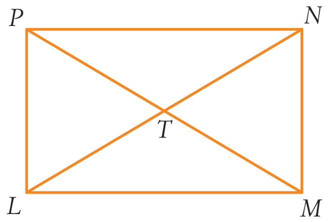
c The angles opposite the equal sides of an isosceles triangle are equal.

Stage 5.2

Example 8

a If *LMNP* is a rectangle, prove that $\triangle PNT \equiv \triangle MLT$.

b Prove that the diagonals of a rectangle bisect each other.



Solution

a In $\triangle PNT$ and $\triangle MLT$:

$PN = ML$ (opposite sides of a rectangle)
$\angle PNT = \angle MLT$ (alternate angles, $PN \parallel ML$)
$\angle PTN = \angle MTL$ (vertically opposite angles)
$\therefore \triangle PNT = \triangle MLT$ (AAS)

b $\therefore PT = MT$ and $NT = LT$ (matching sides of congruent triangles)
$\therefore$ *T* is the midpoint of the diagonals *LN* and *MP*.
$\therefore$ The diagonals of a rectangle bisect each other.

Exercise 11-04 Proving properties of triangles and quadrilaterals

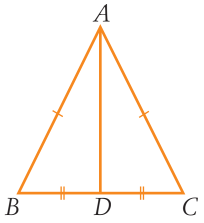
See Example 7

1 $\triangle ABC$ is an isosceles triangle, with $AB = AC$. *D* is the midpoint of *BC*.

a Which congruence test can be used to prove that $\triangle ABD \equiv \triangle ACD$?

b Explain why $\angle ADB = \angle ADC$.

c Hence prove that $AD \perp BC$.

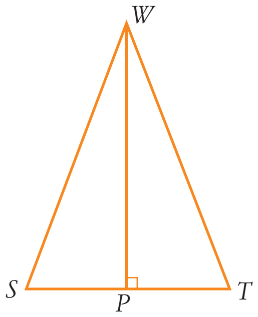


2 In the diagram, $\angle S = \angle T$ and $WP \perp ST$.

a Which congruence test can be used to prove that $\triangle SPW \equiv \triangle TPW$?

b Explain why $WS = WT$.

c What geometrical result about triangles does this prove?



3 $\triangle ABC$ is an equilateral triangle ($AB = BC = AC$). X is the midpoint of BC.

a Which congruence test can be used to prove that $\triangle ABX \equiv \triangle ACX$?

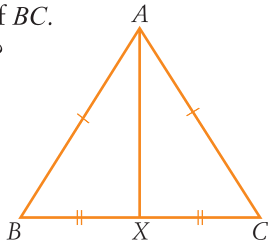
b Explain why $\angle B = \angle C$.

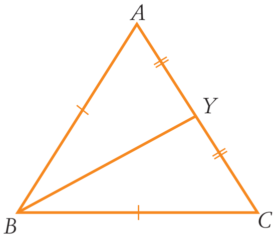
c In the second diagram, $\triangle ABC$ is redrawn so that Y is the midpoint of AC. Which congruence test can be used to prove that $\triangle BAY \equiv \triangle BCY$?

d Is $\angle A = \angle C$? Why?

e Calculate the sizes of the three angles of $\triangle ABC$.

f What geometrical result about equilateral triangles does this prove?





4 In $\triangle PMN$, $\angle M = \angle N$ and YP bisects $\angle MPN$.

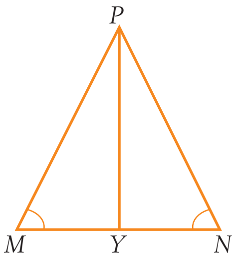
a Explain why $\angle MPY = \angle NPY$.

b Which congruence test can be used to prove that $\triangle PMY \equiv \triangle PNY$?

c Explain why $MY = NY$.

d Is $\angle PYM = \angle PYN$? Why?

e Prove that $PY \perp MN$.



See Example 8

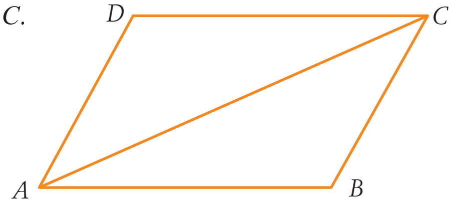
5 $ABCD$ is a quadrilateral whose opposite sides are equal.

a Prove that $\triangle ABC \equiv \triangle CDA$.

b Explain why $\angle BAC = \angle DCA$ and $\angle BCA = \angle DAC$.

c Hence state why $AB \parallel CD$ and $AD \parallel CB$.

d What type of quadrilateral is $ABCD$?



6 $WXYZ$ is a parallelogram whose opposite sides are parallel.

a Copy the diagram into your book.

b On your diagram, show two pairs of equal alternate angles.

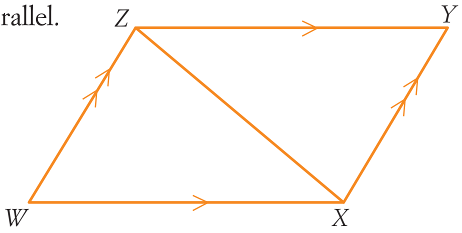
c Prove that $\triangle WXZ \equiv \triangle YZX$.

d Explain why $\angle W = \angle Y$.

e Draw the other diagonal WY and prove that $\triangle WXY \equiv \triangle YZW$.

f Explain why $\angle WXY = \angle YZW$.

g What angle property of a parallelogram does this prove?

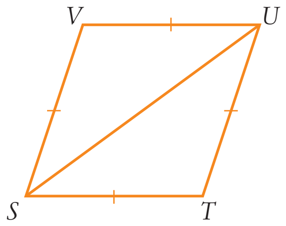


Stage 5.2

7 *STUV* is a rhombus, so all sides are equal.

a Prove that $\triangle VUS \equiv \triangle TUS$.

b Prove that the diagonal *US* bisects $\angle VUT$ and $\angle VST$.

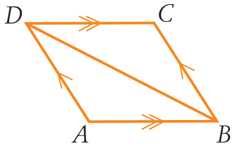


8 *ABCD* is a parallelogram with opposite sides parallel.

a Prove that $\triangle ABD \equiv \triangle CDB$.

b Explain why $AB = CD$ and $AD = CB$.

c What property of a parallelogram does this prove?



9 *BEGH* is a rhombus (a parallelogram with equal sides) whose diagonals *BG* and *EH* intersect at *L*.

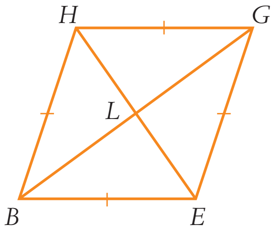
a Prove that $\triangle BEL = \triangle GHL$.

b Prove that the diagonals of a rhombus bisect each other.

c $\triangle BEH$ is isosceles, so which two angles are equal?

d Hence prove that $\triangle BEL \equiv \triangle BHL$.

e Hence prove that the diagonals of a rhombus cross at right angles.



10 *ABCD* is a kite, so adjacent sides are equal.

a Prove that $\triangle ABD \equiv \triangle CBD$.

b Prove that $\angle A = \angle C$.

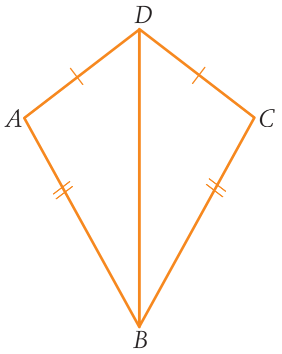
c Prove that diagonal *DB* bisects $\angle ADC$ and $\angle ABC$.

d Copy the diagram and draw the other diagonal *AC*, intersecting *DB* at point *X*.

e Prove that $\triangle DAX \equiv \triangle DCX$.

f Prove that diagonal *DB* bisects diagonal *AC*.

g Prove that $DB \perp AC$.



Mental skills 11 Maths without calculators

Dividing a quantity in a given ratio

1 Study this example.

Divide \$5600 between Alice and Peter in the ratio 5 : 3.
Total number of parts = 5 + 3 = 8.
1 part = \$5600 ÷ 8 = \$700
Alice's share = 5 × \$700 = \$3500
Peter's share = 3 × \$700 = \$2100
Check: \$3500 + \$2100 = \$5600 (original amount)

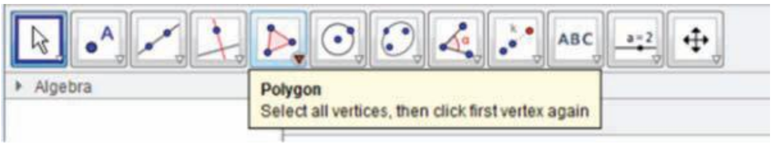
2 Now divide each of these quantities in the given ratio.

- **a** Divide $150 between Mark and Jenni in the ratio 2 : 1.
- **b** Divide $2100 between Simon and Sunil in the ratio 4 : 3.
- **c** Divide $720 between Lisa and Bree in the ratio 2 : 7.
- **d** Divide $2000 between William and Adriana in the ratio 1 : 3.
- **e** Divide $4500 between Ed and Pete in the ratio 3 : 2.
- **f** Divide $3000 between Sharanya and Asam in the ratio 3 : 7.
- **g** Divide $3600 between Cindy and Carmen in the ratio 5 : 1.
- **h** Divide $1600 between Nancy and John in the ratio 3 : 5.
- **i** Divide $990 between Carol and Louis in the ratio 5 : 4.
- **j** Divide $4000 between Yvette and Andre in the ratio 1 : 4.
- **k** Divide $4900 between Arden and Ivan in the ratio 3 : 4.
- **l** Divide $3200 between Tan and Mai in the ratio 5 : 3.

Technology Properties of similar figures

We will use GeoGebra to look at the properties of similar figures.

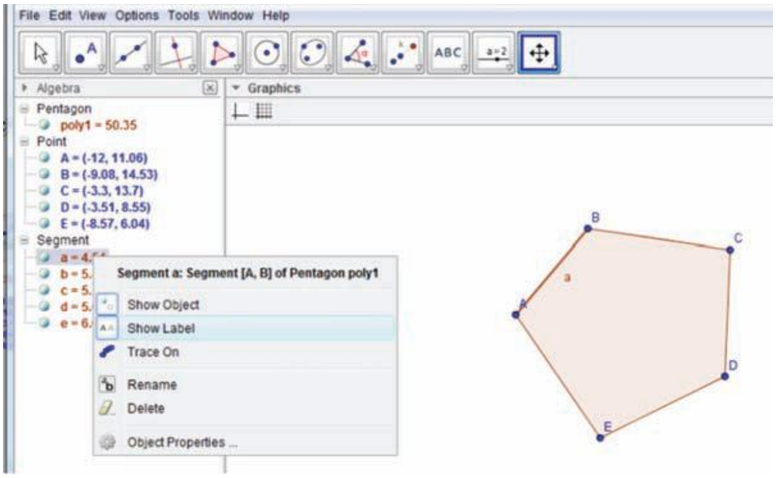
1 Go to Graphics and hide the axes and grid.



Polygon
Select all vertices, then click first vertex again

2 Construct a 5-sided polygon as shown.

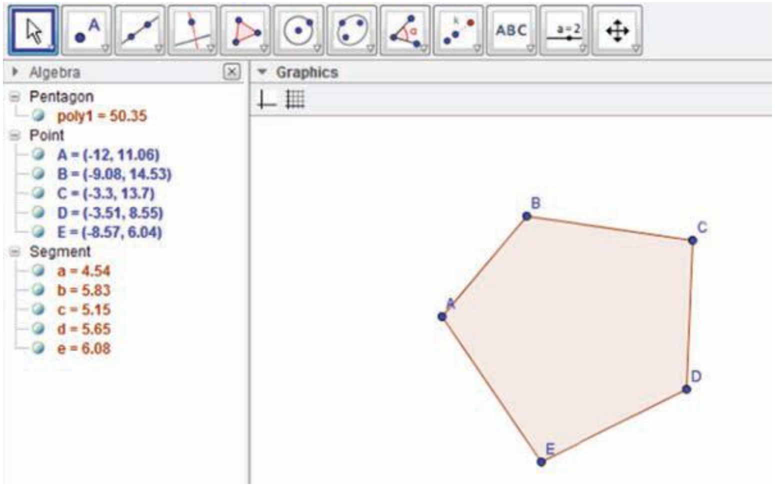
3 To label the sides of the polygon, select **Segment a**, then select **Show Label**. Label the other sides of the polygon.



```
Pentagon
  poly1 = 50.35
Point
  A = (-12, 11.06)
  B = (-9.08, 14.53)
  C = (-3.3, 13.7)
  D = (-3.51, 8.55)
  E = (-8.57, 6.04)
Segment
  a = 4.
  b = 5.
  c = 5.
  d = 5.
  e = 6.
```

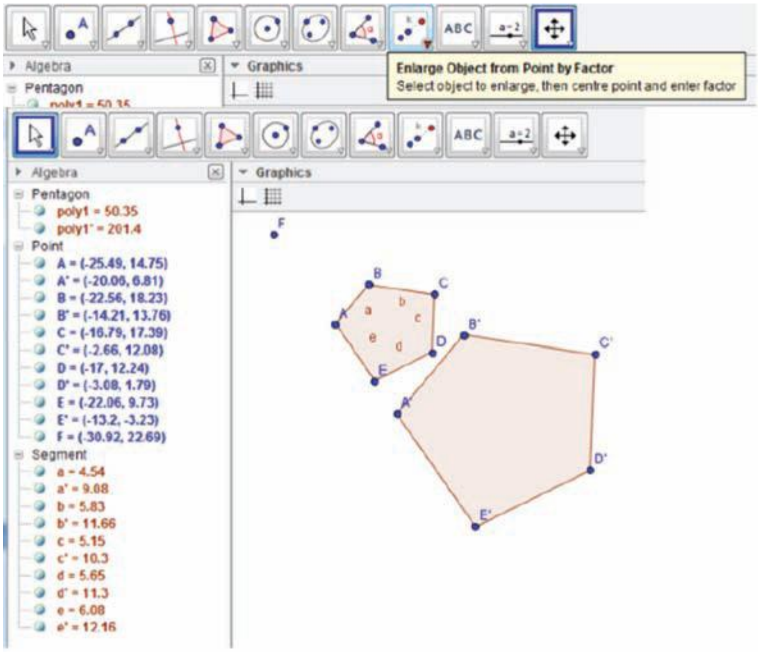
Segment a: Segment [A, B] of Pentagon poly1

4 The lengths of the sides of the polygon are shown in the Algebra View, where segment a, with length 4.54, is the side AB.



Pentagon
poly1 = 50.35
Point
A = (-12, 11.06)
B = (-9.08, 14.53)
C = (-3.3, 13.7)
D = (-3.51, 8.55)
E = (-8.57, 6.04)
Segment
a = 4.54
b = 5.83
c = 5.15
d = 5.65
e = 6.08

5 Enlarge the polygon by a scale factor of 2 to obtain the image A′B′C′D′E′ as shown.

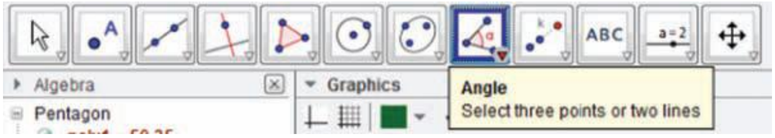


Enlarge Object from Point by Factor
Select object to enlarge, then centre point and enter factor

Pentagon
poly1 = 50.35
poly1' = 201.4
Point
A = (-25.49, 14.75)
A' = (-20.06, 6.81)
B = (-22.56, 18.23)
B' = (-14.21, 13.76)
C = (-16.79, 17.39)
C' = (-2.66, 12.08)
D = (-17, 12.24)
D' = (-3.08, 1.79)
E = (-22.06, 9.73)
E' = (-13.2, -3.23)
F = (-30.92, 22.69)
Segment
a = 4.54
a' = 9.08
b = 5.83
b' = 11.66
c = 5.15
c' = 10.3
d = 5.65
d' = 11.3
e = 6.08
e' = 12.16

The lengths of the sides of the image are shown in the Algebra View. Is the ratio of matching sides the same for all sides?

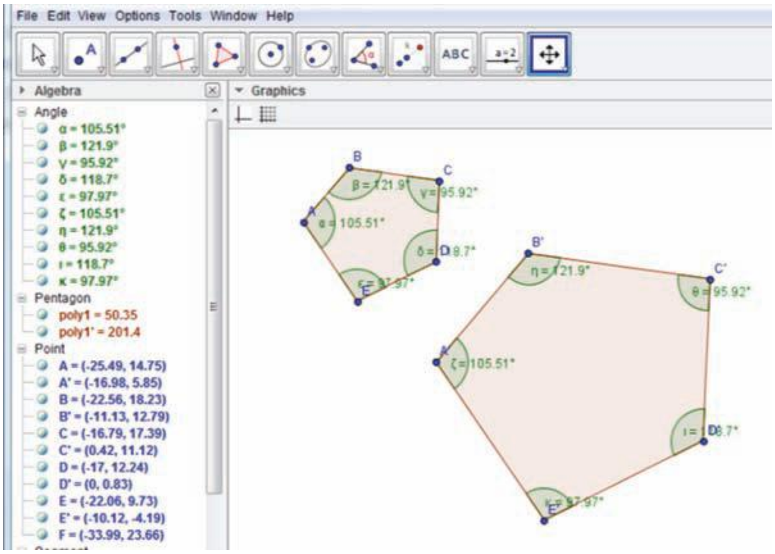
6 Measure the angles of the polygon and its image. (It may be necessary to delete the labels on the sides of the polygon.)



Angle
Select three points or two lines

9780170194655

The angles of the polygon and its image are shown on the diagrams and in the Algebra view. Are the matching angles equal?



```
Angle
  α = 105.51°
  β = 121.9°
  γ = 95.92°
  δ = 118.7°
  ε = 97.97°
  ζ = 105.51°
  η = 121.9°
  θ = 95.92°
  ι = 118.7°
  κ = 97.97°
Pentagon
  poly1 = 50.35
  poly1' = 201.4
Point
  A = (-25.49, 14.75)
  A' = (-16.98, 5.85)
  B = (-22.56, 18.23)
  B' = (-11.13, 12.79)
  C = (-16.79, 17.39)
  C' = (0.42, 11.12)
  D = (-17, 12.24)
  D' = (0, 0.83)
  E = (-22.06, 9.73)
  E' = (-10.12, -4.19)
  F = (-33.99, 23.66)
```

7 Repeat the above process for:

a a triangle **b** a quadrilateral.

8 For figures that are similar:

a are matching angles equal? **b** are matching sides in the same ratio?

11-05 Similar figures

Similar figures have the same shape but are not necessarily the same size.
When a figure is enlarged or reduced, a **similar figure** is created. The original figure is called the **original**, while the enlarged or reduced figure is called the **image**.
The scale factor describes by how much a figure has been enlarged or reduced.

Summary

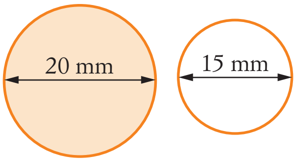
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

- If the scale factor is greater than 1, then the image is an enlargement.
- If the scale factor is between 0 and 1, then the image is a reduction.

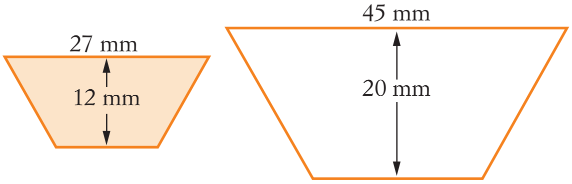
Example 9

Find the scale factor for each pair of similar figures.

a



b



Solution

a Scale factor $= \frac{15}{20}$ $\quad\frac{\text{Image length}}{\text{Original length}}$

$= \frac{3}{4}$

b Scale factor $= \frac{45}{27}$ $\left(\text{or } \frac{20}{12}\right)$ $\quad\frac{\text{Image length}}{\text{Original length}}$

$= \frac{5}{3}$

The similarity symbol |||

The symbol for 'is similar to' is '|||'. As with congruence notation, we must make sure that the vertices (angles) of similar figures are written in matching order.

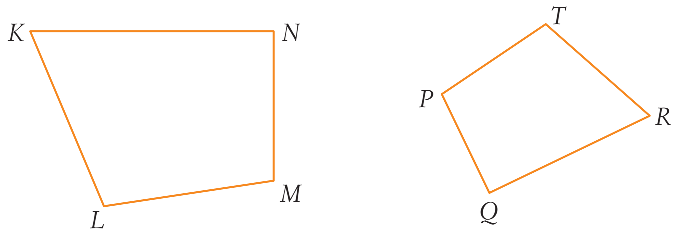
Summary

If two figures are similar, then:

- the matching angles are equal
- the matching sides are in the same ratio

Example 10

The two quadrilaterals *KLMN* and *PQRT* are similar.



a List all pairs of matching sides and matching angles.

b Use the correct notation to write a similarity statement relating these two quadrilaterals.

Solution

a By rotating the figure *KLMN*, its shape can be matched with *PQRT*.

The pairs of matching sides are:	The pairs of matching angles are:
KN and *QR*	$\angle K$ and $\angle R$
MN and *PQ*	$\angle N$ and $\angle Q$
ML and *PT*	$\angle M$ and $\angle P$
LK and *TR*.	$\angle L$ and $\angle T$.

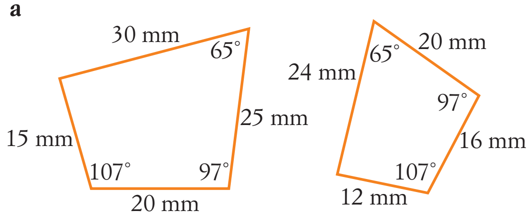
b $\angle K$ matches with $\angle R$, $\angle L$ matches with $\angle T$, $\angle M$ matches with $\angle P$, $\angle N$ matches with $\angle Q$.

$\therefore$ *KLMN* ||| *RTPQ* Matching order of vertices.

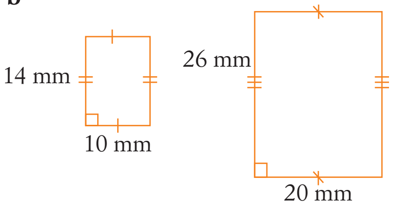
Example 11

Test whether each pair of figures are similar.

a



b



Solution

a For the two quadrilaterals, matching angles are equal and the ratios of matching sides are equal.
$\therefore$ The quadrilaterals are similar.

$$\frac{20}{16} = \frac{25}{20} = \frac{30}{24} = \frac{15}{12} = \frac{5}{4}$$

b For the two rectangles, matching angles are equal (90°) but the ratios of matching sides are not equal.
$\therefore$ The rectangles are not similar.

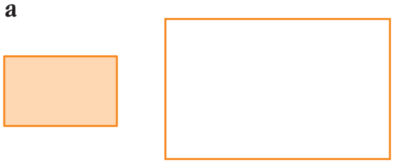
$$\frac{10}{20} = \frac{1}{2} \text{ but } \frac{14}{26} = \frac{7}{13}$$

Exercise 11-05 Similar figures

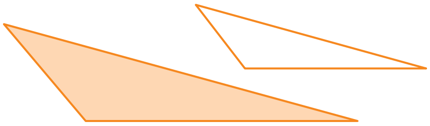
See Example 9

1 By measurement, find the scale factor for each pair of similar figures.

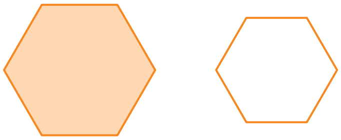
a



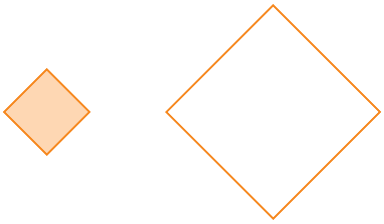
b



c

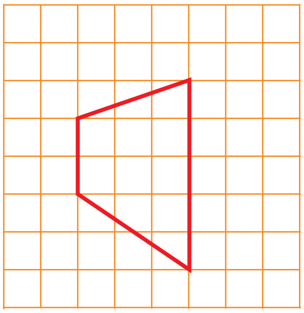


d

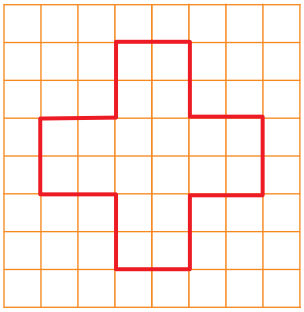


2 Copy each figure onto graph paper and draw its image using the given scale factor.

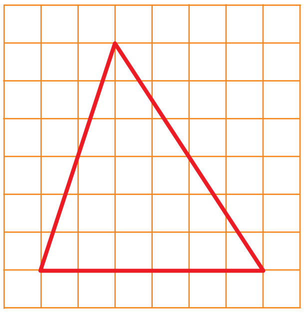
a Scale factor = 2



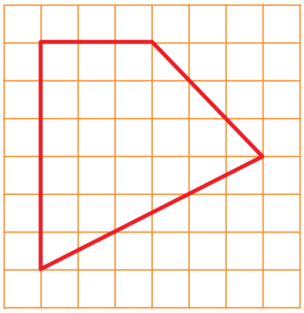
b Scale factor = 2.5



c Scale factor = $\frac{1}{2}$



d Scale factor = $\frac{2}{3}$



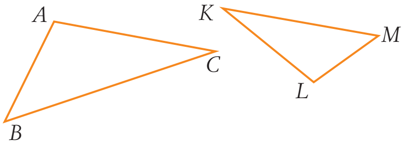
3 For each pair of similar figures: **See Example 10**

i list all pairs of matching angles

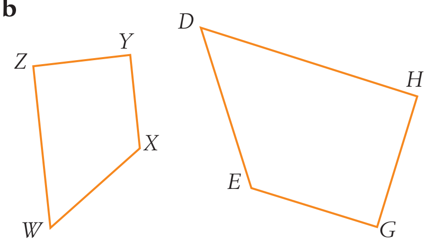
ii list all pairs of matching sides

iii use the correct notation to write a similarity statement relating them.

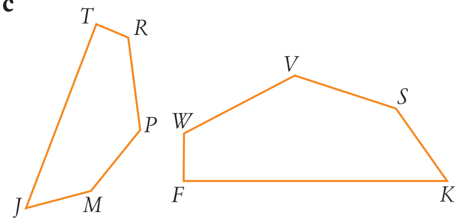
a



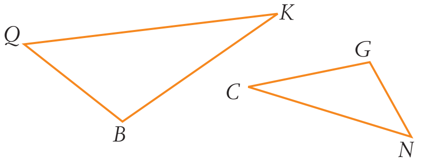
b



c

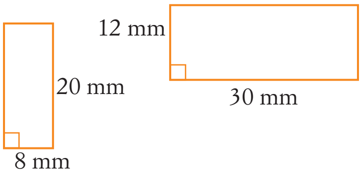


d

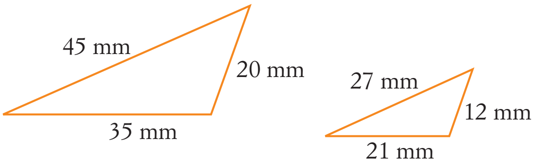


4 Test whether each pair of figures are similar. **See Example 11**

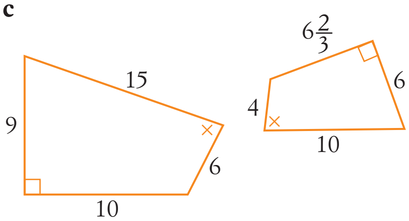
a



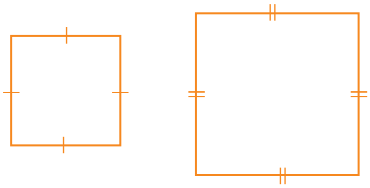
b



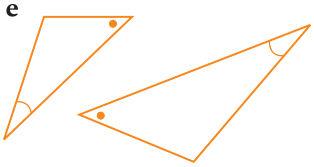
c



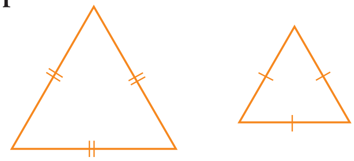
d



e



f



Skillsheet
Finding sides in similar triangles
MAT10MGSS10014

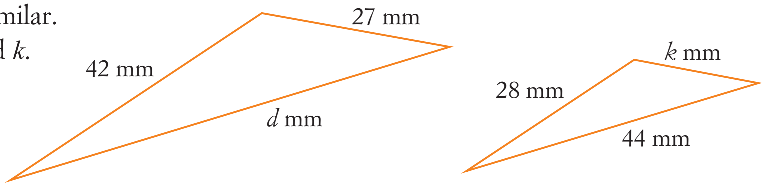
Worksheet
Finding sides in similar figures
MAT10MGWK10082

Puzzle sheet
Similar triangles
MAT10MGPS00025

11-06 Finding unknown lengths in similar figures

Example 12

The two triangles are similar.
Find the values of d and k.



Solution

Since the triangles are similar, the ratios of matching sides are equal.

$$\begin{aligned}\frac{d}{44} &= \frac{42}{28}\\ d &= \frac{42}{28} \times 44\\ &= 66\end{aligned}$$

$$\begin{aligned}\frac{k}{27} &= \frac{28}{42}\\ k &= \frac{28}{42} \times 27\\ &= 18\end{aligned}$$

OR

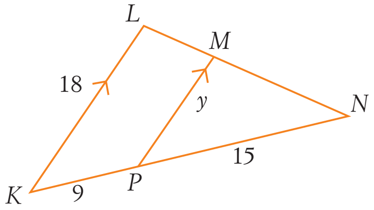
$$\text{Scale factor} = \frac{28}{42} = \frac{2}{3}$$

$$\begin{aligned}d &= 44 \div \frac{2}{3}\\ &= 66\end{aligned}$$

$$\begin{aligned}k &= 27 \times \frac{2}{3}\\ &= 18\end{aligned}$$

Example 13

$\triangle KLN \,|||\, \triangle PMN$. Find the value of y.



Solution

$$\frac{MP}{LK} = \frac{PN}{KN}$$

Ratios of matching sides are equal.

$$\frac{y}{18} = \frac{15}{24}$$

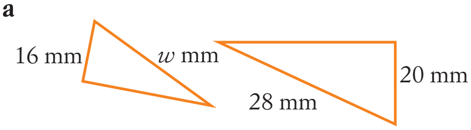
$KN = 9 + 15 = 24$

$$\begin{aligned}y &= \frac{15}{24} \times 18\\ &= 11\frac{1}{4}\end{aligned}$$

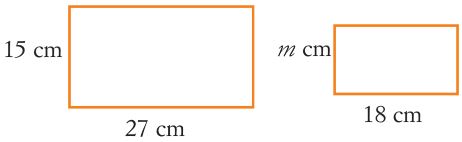
Exercise 11-06 Finding unknown lengths in similar figures

1 Find the value of every pronumeral in each pair of similar figures. **See Example 12**

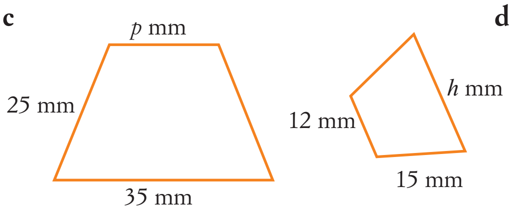
a



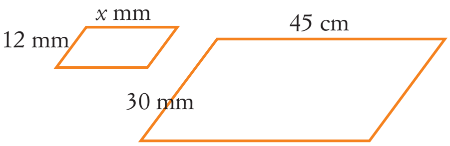
b



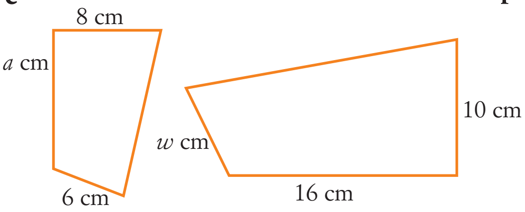
c



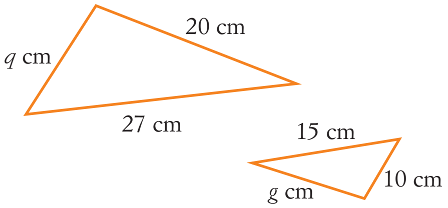
d



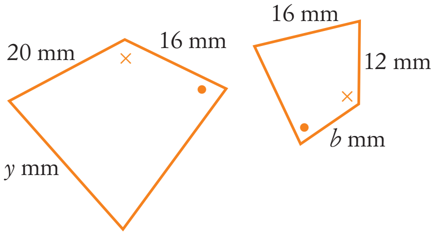
e



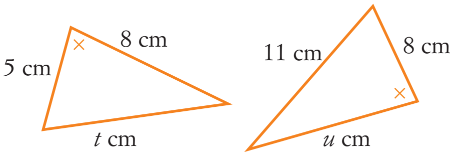
f



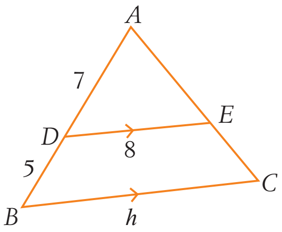
g



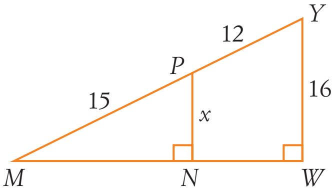
h



2 $\triangle ABC \parallel\!| \triangle ADE$. Find the value of h. **See Example 13**



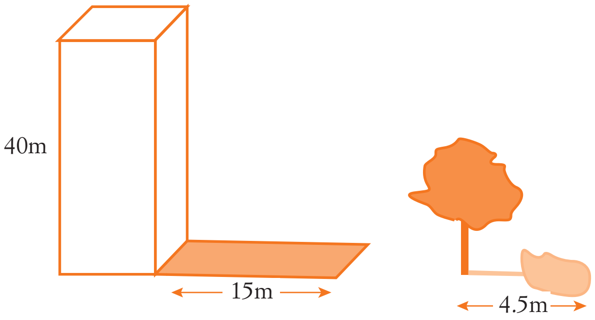
3 $\triangle MNP \parallel\!| \triangle MWY$. Find the value of x.



4 This photograph of the Sydney Harbour Bridge has been enlarged so that its length is 24 cm. If the dimensions of the original photo were 15 cm × 10 cm, what is the width of the enlargement?

Shutterstock.com/clearviewstock

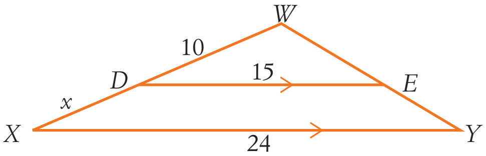
5 A building that is 40 m high casts a shadow 15 m long. At the same time, the shadow of a tree is 4.5 m long. What is the height of the tree?



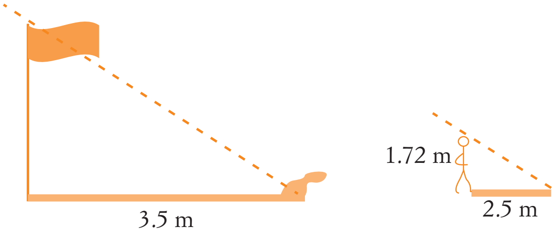
6 $\triangle WXY \,|||\, \triangle WDE$. What is the value of x? Select the correct answer **A**, **B**, **C** or **D**.

A 4 **B** 6

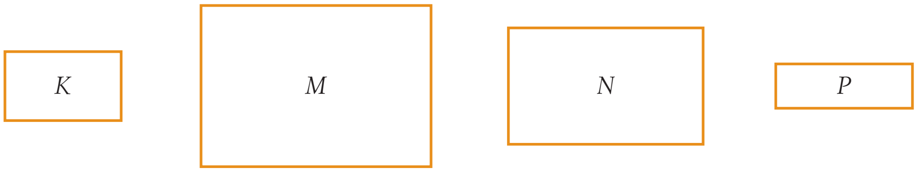
C 8 **D** 10



7 Katrina is 1.72 m tall and casts a shadow that is 2.5 m long. At the same time, a flagpole casts a shadow that is 3.5 m long. How long is the flagpole?



8 Which two rectangles are similar? Select the correct answer **A**, **B**, **C** or **D**.



A M and N **B** K and P **C** M and P **D** K and N

9 A 2 m high fence casts a shadow of 1.4 m. How long is the shadow cast by a pole that is 3.2 m high at the same time?

11-07 Tests for similar triangles

Stage 5.2

Worksheet

Congruent and similar triangle proofs

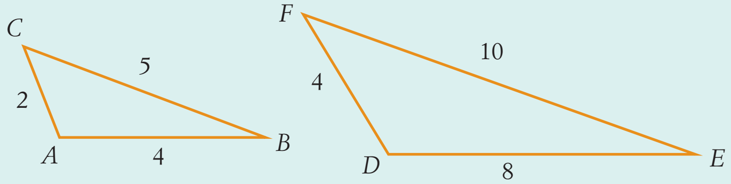
MAT10SPWK10083

There are four sets of conditions that can be used to determine if two triangles are similar. These are called the tests for similar triangles or **similarity tests**.

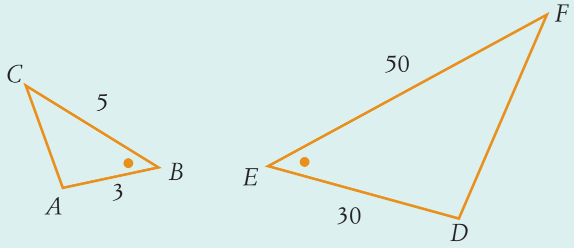
Summary

There are four tests for similar triangles.
Two triangles are similar if:

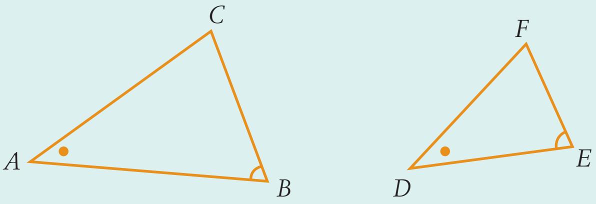
- the three sides of one triangle are proportional to the three sides of the other triangle (**'SSS'**)



- two sides of one triangle are proportional to two sides of the other triangle, and the **included angles** are equal (**'SAS'**)

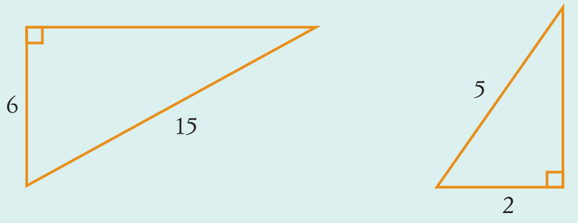


- two angles of one triangle are equal to two angles of the other triangle (**'AA'** or **'equiangular'**)



equiangular means 'equal angles'

- they are right-angled and the hypotenuse and a second side of one triangle are proportional to the hypotenuse and a second side of the other triangle (**'RHS'**).

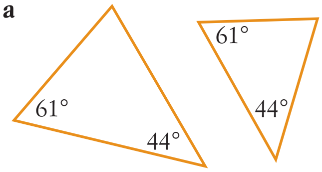


Stage 5.2

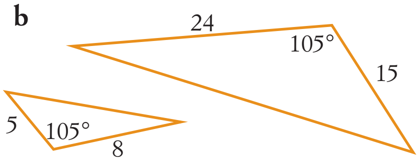
Example 14

Which test can be used to prove that each pair of triangles are similar?

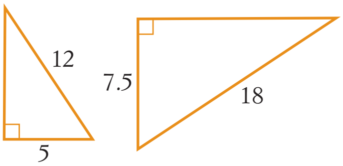
a



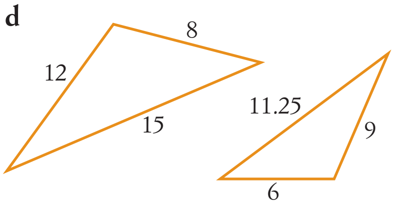
b



c



d



Solution

a Two pairs of angles are equal, or equiangular ('AA').

b Two pairs of matching sides are in the same ratio and the included angles in both triangles are equal ('SAS'). $\frac{15}{5} = \frac{24}{8} = 3$

c Both have right angles, and the pairs of hypotenuses and second sides are in the same ratio ('RHS'). $\frac{7.5}{5} = \frac{18}{12} = \frac{3}{2}$

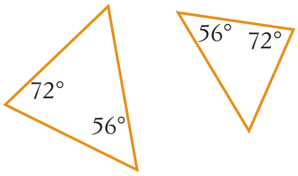
d All three pairs of matching sides are in the same ratio ('SSS'). $\frac{11.25}{15} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$

Exercise 11-07 Tests for similar triangles

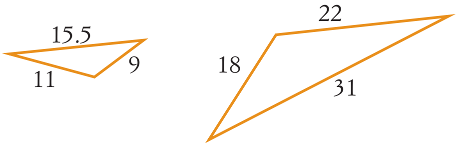
See Example 14

1 Which test can be used to prove that each pair of triangles are similar?

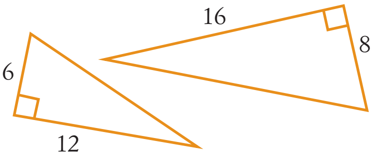
a



b



c



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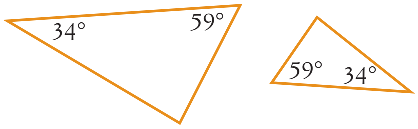
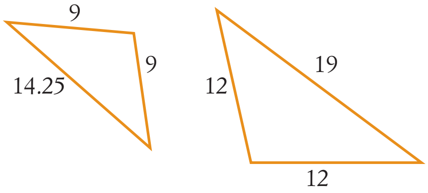
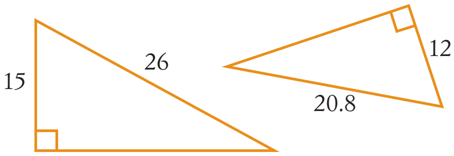


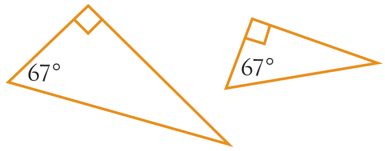
e



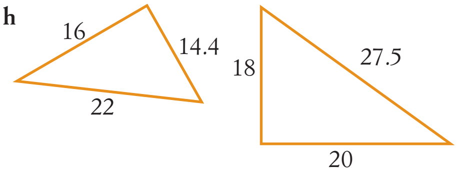
f



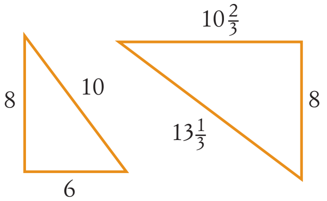
g



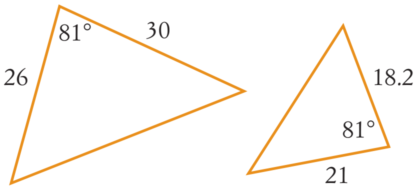
h



i

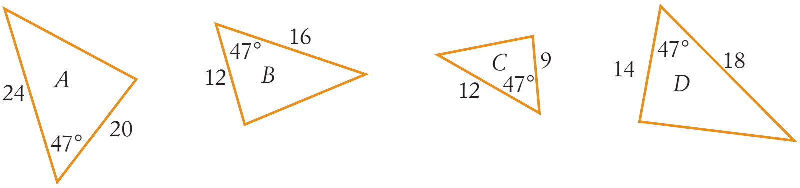


j



2 For each set of triangles, find the pair of similar triangles.

a



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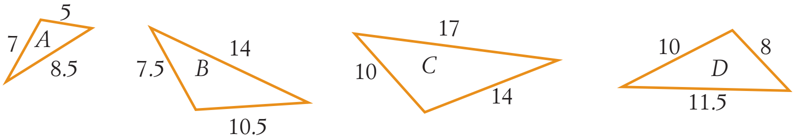

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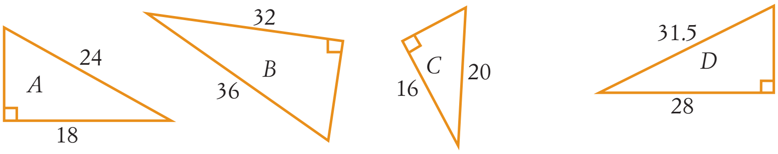
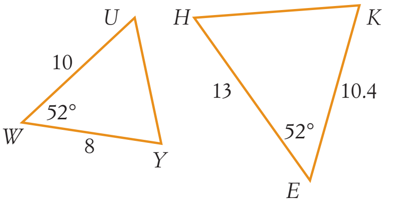


3 Use the correct notation to write a similarity statement relating each pair of similar triangles.

a



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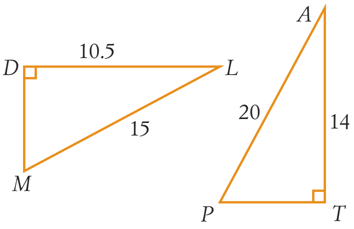
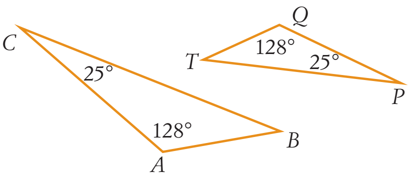
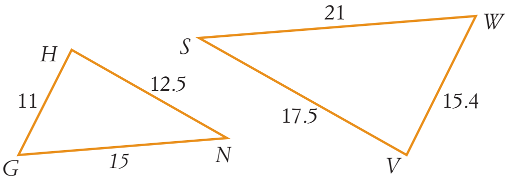


c

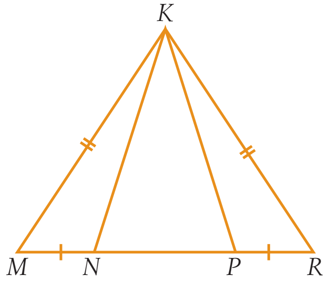


d

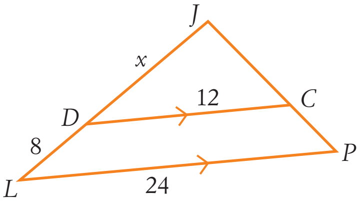


Power plus

1 **a** Explain why $\angle KMN = \angle KRP$.
b Prove that $\triangle KMN \equiv \triangle KRP$.
c Hence prove that $KN = KP$ and that $\triangle KNP$ is an isosceles triangle.



2 $\triangle JDC \mathrel{|||} \triangle JLP$. Find the value of x.

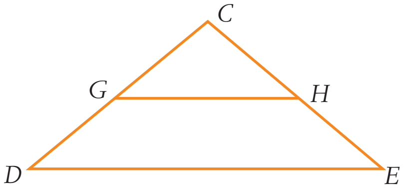


3 G and H are the midpoints of CD and CE respectively. Prove that:

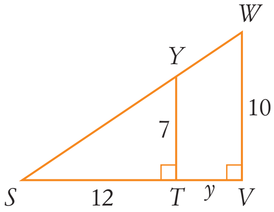
a $\triangle CGH \mathrel{|||} \triangle CDE$

b $GH \parallel DE$

c $GH = \frac{1}{2}DE$



4 **a** Which similarity test proves that $\triangle STY \mathrel{|||} \triangle SVW$?
b Find the value of y.



Chapter 11 review

Language of maths

Puzzle sheet
Geometry crossword
MAT10MGPS10084

AAS
angle sum
congruence test
congruent (≡)
convex polygon
enlargement
equiangular
exterior angle
hypotenuse
image
included angle
matching
original
polygon
proof
proportional
reduction
regular polygon
RHS
SAS
scale factor
similar (|||)
similarity test
SSS

1 What is a **convex polygon**?

2 Explain the difference between the **interior** and **exterior angles** of a polygon.

3 What is the symbol and meaning of 'is similar to'?

4 What happens to a figure that is changed by a scale factor of $\frac{1}{2}$?

5 What are the four tests for similar triangles?

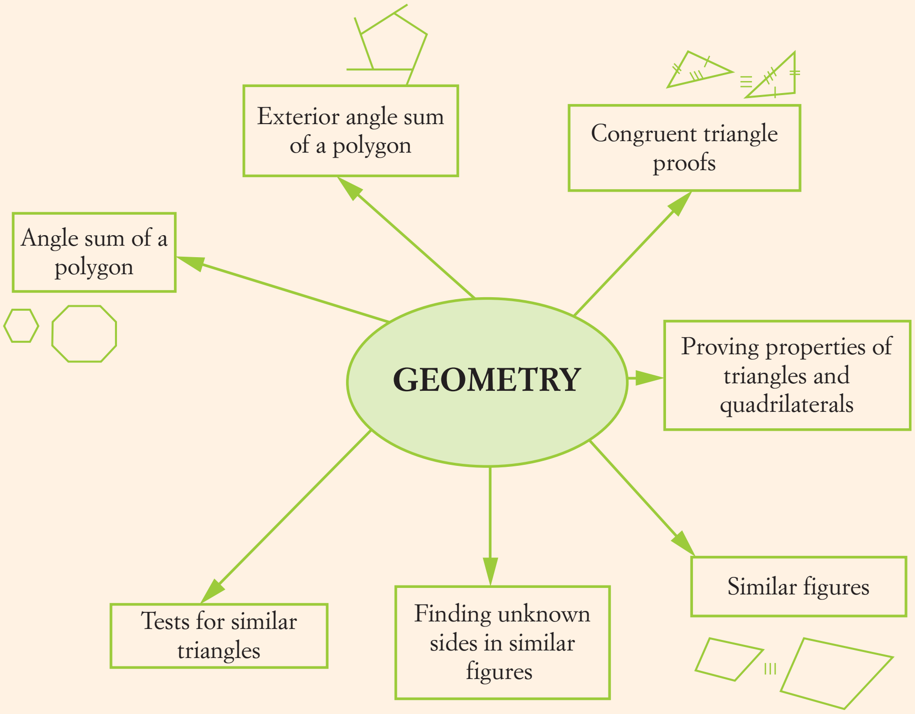
6 What is the meaning of the 'A' in the SAS test for congruent triangles?

7 What does **equiangular** mean in the similarity tests?

Topic overview

Quiz
Geometry
MAT10MGQZ00008

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask you teacher to check your work.



Chapter 11 revision

Stage 5.2

See Exercise 11-01

1 Find the angle sum of a polygon with:

a 15 sides **b** 24 sides **c** 12 sides **d** 48 sides

See Exercise 11-01

2 Find the size of one angle in a regular 15-sided polygon.

See Exercise 11-01

3 The angle sum of a polygon is 6120°. How many sides does the polygon have?

See Exercise 11-02

4 Find the number of sides in a regular polygon if each exterior angle is:

a 10° **b** 24° **c** 45° **d** 15°

See Exercise 11-02

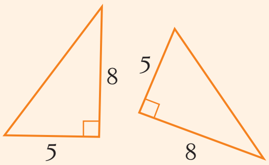
5 Each angle of a regular polygon is 162°. How many sides does the polygon have? Select the correct answer **A**, **B**, **C** or **D**.

A 18 **B** 20 **C** 22 **D** 24

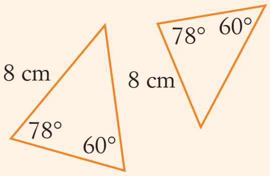
See Exercise 11-03

6 Which congruence test (SSS, SAS, AAS or RHS) can be used to prove that each pair of triangles are congruent?

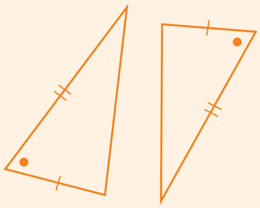
a



b

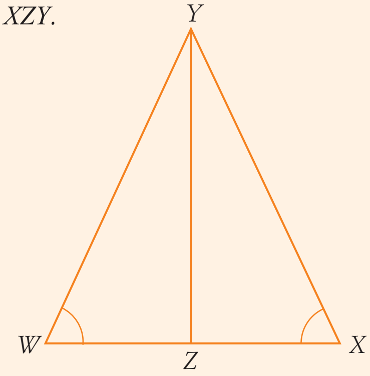


c



See Exercise 11-03

7 In $\triangle WXY$, $\angle W = \angle X$ and $YZ \perp WX$. Prove that $\triangle WZY \equiv \triangle XZY$.



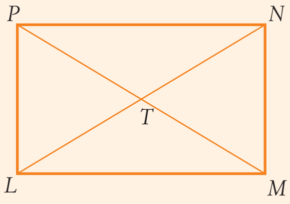
See Exercise 11-04

8 *PNML* is a rectangle.

a Which congruence test can be used to prove that $\triangle PML \equiv \triangle NLM$?

b Hence explain why $PM = NL$.

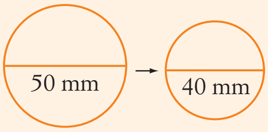
c What geometrical result about rectangles does this prove?



See Exercise 11-05

9 Calculate the scale factor between each pair of similar figures.

a



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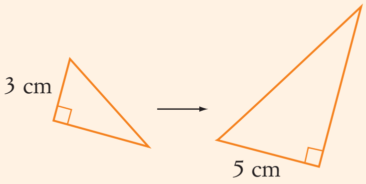
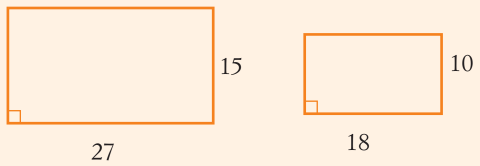


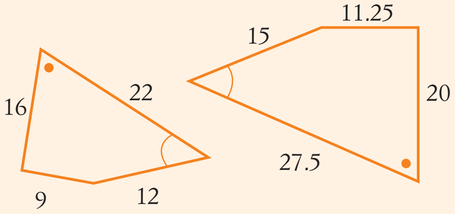
10 Test whether each pair of figures are similar.

See Exercise 11-05

a



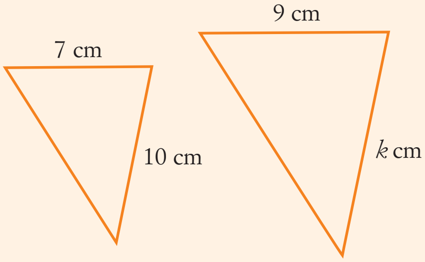
b



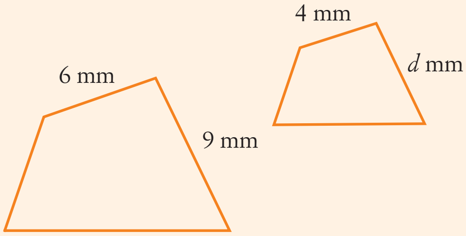
11 Find the value of the pronumeral in each pair of similar figures.

See Exercise 11-06

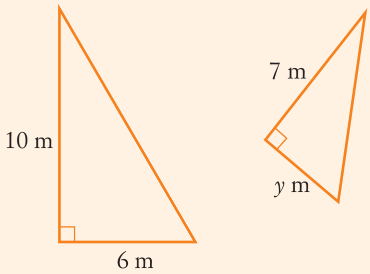
a



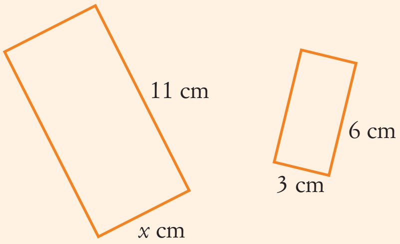
b



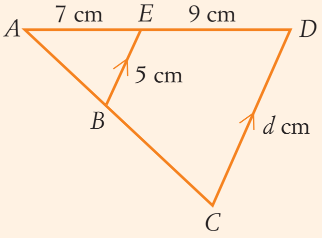
c



d



12 If $\triangle ABE \mid\mid\mid \triangle ACD$, find the value of d.



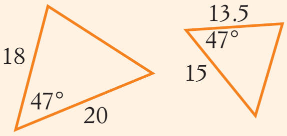
See Exercise 11-06

13 Which test can be used to prove that each pair of triangles are similar?

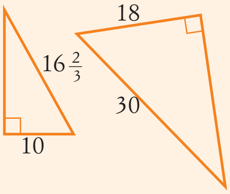
Stage 5.2

See Exercise 11-07

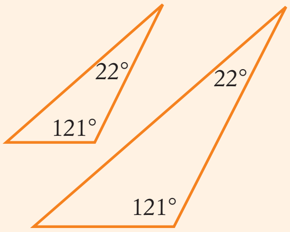
a



b



c



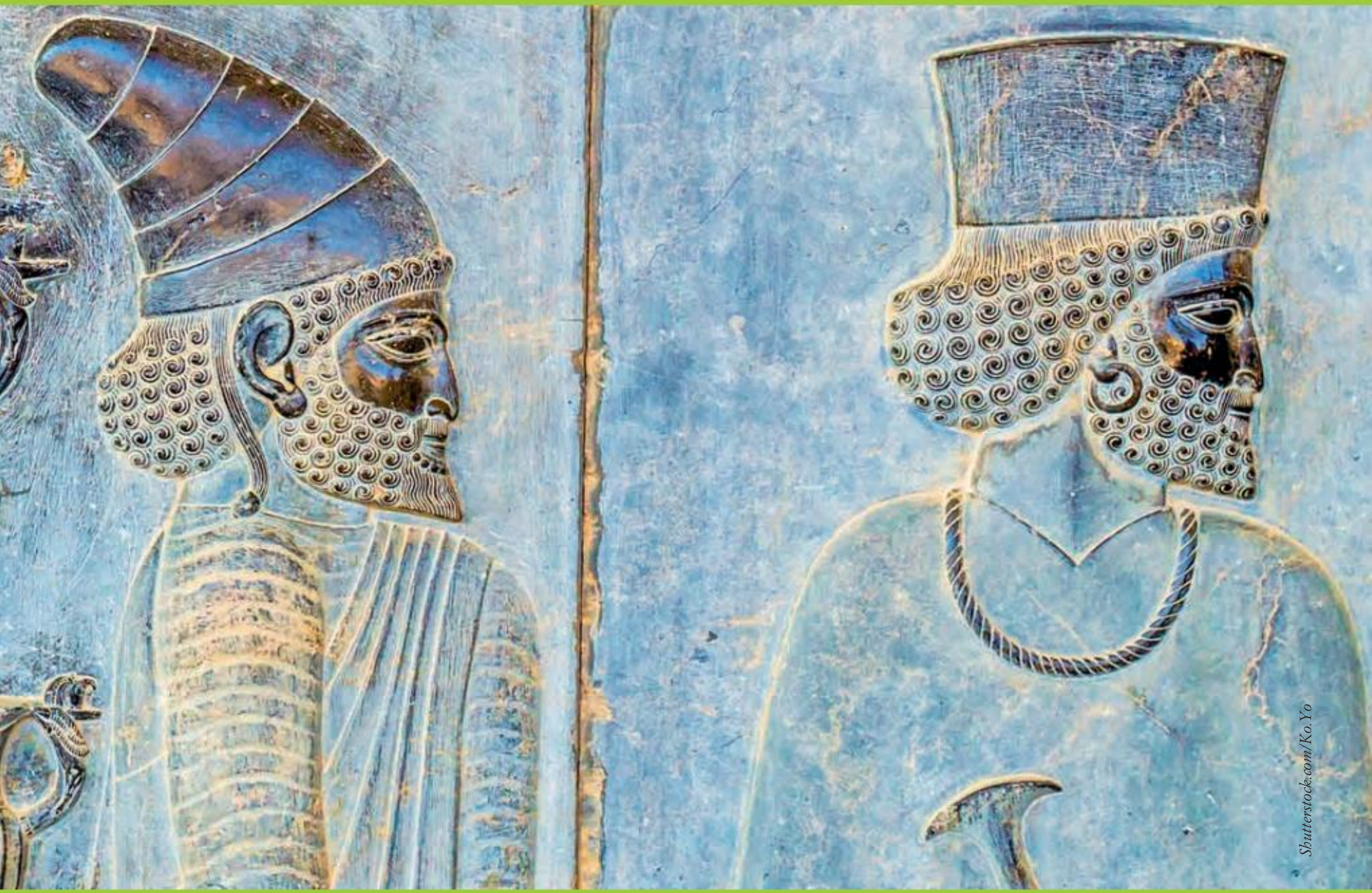
12

Number and Algebra

Products and factors

Optional Stage 5.3 topic

In 825 CE, the Persian mathematician al-Khwarizmi used the Arabic word ‘al-jabr’ to describe the process of adding equal quantities to both sides of an equation. When al-Khwarizmi’s book was translated into Latin and introduced to Europe, ‘al-jabr’ became ‘algebra’ and the word was adopted as the name for the branch of mathematics that uses formulas to describe patterns and relationships in our world.



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Chapter outline

	Proficiency strands			
12-01 Perfect squares	U	F	R	C
12-02 Difference of two squares	U	F	R	C
12-03 Mixed expansions		F	R	C
12-04 Factorising special binomial products	U	F	R	C
12-05 Factorising quadratic expressions of the form $ax^2 + bx + c$	U	F	R	C
12-06 Mixed factorisations		F	R	C
12-07 Factorising algebraic fractions	U	F	R	C

Wordbank

binomial	An algebraic expression that consists of two terms; for example, $4a + 9$, $3 - y$, $x^2 - 4x$
binomial product	Binomials multiplied together; for example, $(x + 9)(3x - 4)$.
factorise	To rewrite an expression with grouping symbols by taking out the highest common factor; factorising is the opposite of expanding; for example, $9r^2 + 36r$ factorised is $9r(r + 4)$
highest common factor (HCF)	The largest term that is a factor of two or more terms; for example, the HCF of $9r^2 + 36r$ is $9r$
perfect square	A square number or an algebraic expression that represents one; for example, 64, $(x + 9)^2$
quadratic expression	An algebraic expression in which the highest power of the variable is 2; for example, $2x^2 + 5x - 3$ or $x^2 + 2$

In this chapter you will:

- (STAGE 5.3) expand special binomial products such as $(a+b)^2$, $(a-b)^2$, $(a+b)(a-b)$
- (STAGE 5.3) factorise algebraic expressions using grouping in pairs, perfect squares, the difference of two squares and quadratic expressions of the form ax^2+bx+c
- (STAGE 5.3) factorise and simplify complex algebraic expressions involving algebraic fractions

SkillCheck

Worksheet
StartUp assignment 12
MAT10NAWK10085

1 Expand each binomial product.

a $(x+1)(x+7)$ **b** $(d+3)(d-4)$ **c** $(t-5)(t+5)$
d $(y+2)(3y-6)$ **e** $(7y-3)(7y-3)$ **f** $(5p+5)(2p-7)$

2 Factorise each quadratic expression.

a $y^2+10y+25$ **b** $x^2-21x+20$ **c** $n^2+8n-33$
d $a^2-11a+28$ **e** $m^2-5m-84$ **f** $p^2+3p-54$

Stage 5.3

12-01 Perfect squares

Worksheet
Special products
MAT10NAWK10086

In Chapter 4, we learnt how to expand **binomial products** such as $(k+3)(k-7)$. Now we will examine the special binomial product where a binomial is multiplied by itself, for example, $(x+4)(x+4)=(x+4)^2$ or $(2a-9)(2a-9)=(2a-9)^2$.
16, 49, v^2 and $(y+5)^2$ are called **perfect squares** because they are square numbers. The rule for expanding the perfect square of a binomial is:

Summary

$$(a+b)^2=a^2+2ab+b^2$$
$$(a-b)^2=a^2-2ab+b^2$$

Proof

$$\begin{aligned}(a+b)^2&=(a+b)(a+b)\\&=a(a+b)+b(a+b)\\&=a^2+ab+ba+b^2\\&=a^2+2ab+b^2\end{aligned}$$

$$\begin{aligned}(a-b)^2&=(a-b)(a-b)\\&=a(a-b)-b(a-b)\\&=a^2-ab-ba+b^2\\&=a^2-2ab+b^2\end{aligned}$$

9780170194655

Example 1

Copy and complete the expansion of each perfect square.

a $(x+7)^2 = x^2 + \_\_\_\_ + 49$

b $(y-6)^2 = y^2 - 12y + \_\_\_\_$

c $(5g+9)^2 = \_\_\_\_ + 90g + 81$

d $(3d-5)^2 = 9d^2 - \_\_\_\_ + 25$

Solution

a In the expansion,

$$\begin{aligned}\_\_\_\_ &= 2 \times x \times 7\\ &= 14x\end{aligned}$$

Doubling the product of the two terms.

$$\therefore (x+7)^2 = x^2 + 14x + 49$$

b In the expansion,

$$\begin{aligned}\_\_\_\_ &= 6^2\\ &= 36\end{aligned}$$

The second term squared.

$$\therefore (y-6)^2 = y^2 - 12y + 36$$

c

$$\begin{aligned}\_\_\_\_ &= (5g)^2\\ &= 25g^2\end{aligned}$$

The first term squared.

$$\therefore (5g+9)^2 = 25g^2 + 90g + 81$$

d

$$\begin{aligned}\_\_\_\_ &= 2 \times 3d \times 5\\ &= 30d\end{aligned}$$

Doubling the product of the two terms.

$$\therefore (3d+5)^2 = 9d^2 - 30d + 25$$

Example 2

Video tutorial
Special binomial products
MAT10NAVT10006

Expand each perfect square.

a $(n-5)^2$

b $(k+4)^2$

c $(3y-8)^2$

Solution

a

$$\begin{aligned}(n-5)^2 &= n^2 - 2 \times n \times 5 + 5^2\\ &= n^2 - 10n + 25\end{aligned}$$

1st term squared − double product + 2nd term squared

b

$$\begin{aligned}(k+4)^2 &= k^2 + 2 \times k \times 4 + 4^2\\ &= k^2 + 8k + 16\end{aligned}$$

c

$$\begin{aligned}(3y-8)^2 &= (3y)^2 - 2 \times 3y \times 8 + 8^2\\ &= 9y^2 - 48y + 64\end{aligned}$$

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Stage 5.3

Exercise 12-01 Perfect squares

See Example 1

1 Copy and complete the expansion of each perfect square.

a $(x + 10)^2 = x^2 + \underline{\qquad} + 100$

b $(m - 8)^2 = \underline{\qquad} - 16m + 64$

c $(p - t)^2 = p^2 - 2pt + \underline{\qquad}$

d $(h + 4)^2 = h^2 \underline{\qquad} + 16$

e $(k - 9)^2 = k^2 \underline{\qquad} + 81$

f $(8 + 5f)^2 = 64 \underline{\qquad} + 25f^2$

g $(2d + 3)^2 = \underline{\qquad} + \underline{\qquad} + 9$

h $(6a + 1)^2 = \underline{\qquad} + 12a + \underline{\qquad}$

See Example 2

2 Expand each perfect square.

a $(m + 9)^2$

b $(u + 3)^2$

c $(y - 6)^2$

d $(8 + k)^2$

e $(5 - h)^2$

f $(7 + k)^2$

g $(f + 20)^2$

h $(q - 11)^2$

i $(10 + t)^2$

j $(x - w)^2$

k $(a + g)^2$

l $(2m - 3)^2$

m $(5x - 6)^2$

n $(9a + 2)^2$

o $(3e - 4)^2$

p $(5 + 7b)^2$

q $(4 - 5p)^2$

r $(11 - 2c)^2$

s $(10g + 3)^2$

t $(3k + 11)^2$

u $(5 + 2v)^2$

3 Expand each perfect square.

a $(7h + 2k)^2$

b $(8a - 3y)^2$

c $(xy + z)^2$

d $\left(1 + \frac{1}{y}\right)^2$

e $\left(t - \frac{1}{t}\right)^2$

f $\left(\frac{3}{w} + w\right)^2$

4 Use expansion to evaluate each square number without using a calculator.

a $21^2 = (20 + 1)^2$

b $45^2 = (40 + 5)^2$

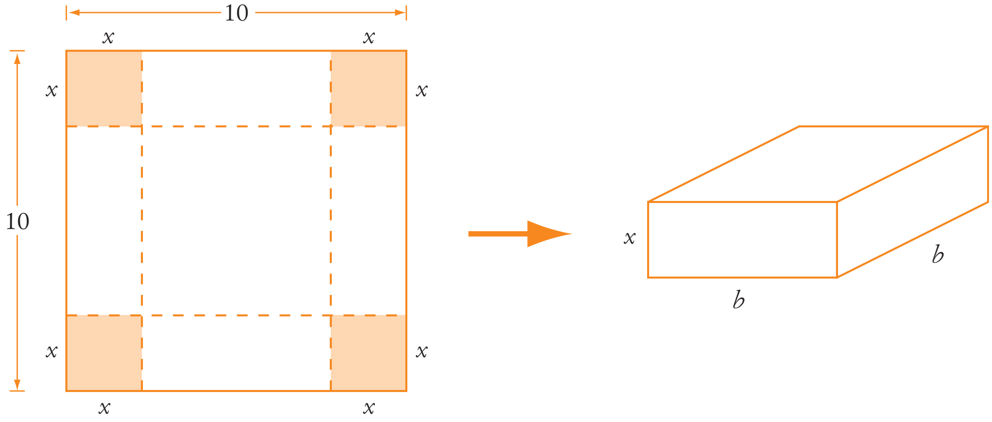
c $29^2 = (30 - 1)^2$

d $59^2 = (60 - 1)^2$

e $102^2 = (100 + 2)^2$

f $98^2 = (100 - 2)^2$

5 A 10 cm square sheet of cardboard has a square of length x cm cut from each corner. It is then folded to form a square-based prism.



a Why is the length of the square base (b) equal to $(10 - 2x)$ cm?

b Find the area of the square base.

c Find the area of one side face of the prism.

d Hence show that the surface area of the prism is $(100 - 4x^2)$ cm^2.

9780170194655

12-02 Difference of two squares

Stage 5.3

Worksheet
Special products
MAT10NAWK10086

The rule for expanding $(a + b)(a - b)$ is:

Summary

$(a + b)(a - b) = a^2 - b^2$

The answer is called the **difference of two squares**.

Proof

$$\begin{aligned}(a - b)(a + b) &= a(a + b) - b(a + b)\\ &= a^2 + ab - ba - b^2\\ &= a^2 - b^2\end{aligned}$$

When the sum of two terms is multiplied by their difference, the answer is the square of the first term minus the square of the second term (the difference of two squares).

Example 3

Video tutorial
Special binomial products
MAT10NAVT10006

Expand each expression.

a $(d + 3)(d - 3)$ **b** $(2 + r)(2 - r)$ **c** $(7x + 2)(7x - 2)$ **d** $(4k - 5p)(4k + 5p)$

Solution

a $$\begin{aligned}(d+3)(d-3) &= d^2 - 3^2\\ &= d^2 - 9\end{aligned}$$

b $$\begin{aligned}(2+r)(2-r) &= 2^2 - r^2\\ &= 4 - r^2\end{aligned}$$

c $$\begin{aligned}(7x+2)(7x-2) &= (7x)^2 - 2^2\\ &= 49x^2 - 4\end{aligned}$$

d $$\begin{aligned}(4k-5p)(4k+5p) &= (4k)^2 - (5p)^2\\ &= 16k^2 - 25p^2\end{aligned}$$

Exercise 12-02 Difference of two squares

1 Expand each expression.

See Example 3

a $(m + 5)(m - 5)$ **b** $(c - 10)(c + 10)$ **c** $(a + 12)(a - 12)$
d $(6 - y)(6 + y)$ **e** $(8 - m)(8 + m)$ **f** $(p + 1)(p - 1)$
g $(5 + e)(5 - e)$ **h** $(v + 11)(v - 11)$ **i** $(w - 3)(w + 3)$
j $(x - 10)(x + 10)$ **k** $(q + 7)(q - 7)$ **l** $(9 - g)(9 + g)$
m $(b - 2)(b + 2)$ **n** $(15 - r)(15 + r)$ **o** $(d + 13)(d - 13)$

Stage 5.3

2 Expand each each expression.

a $(2h - 3)(2h + 3)$
b $(5r + 4)(5r - 4)$
c $(5b + 8)(5b - 8)$
d $(4p - 7)(4p + 7)$
e $(3 - 8k)(3 + 8k)$
f $(7x - 5)(7x + 5)$
g $(2 + 9m)(2 - 9m)$
h $(9k - 4r)(9k + 4r)$
i $(7n + 8m)(7n - 8m)$
j $(4g - 5h)(4g + 5h)$
k $(7u + 3w)(7u - 3w)$
l $(11a + 3b)(11a - 3b)$
m $\left(t + \frac{1}{t}\right)\left(t - \frac{1}{t}\right)$
n $\left(\frac{w}{3} - 2\right)\left(\frac{w}{3} + 2\right)$
o $\left(1 - \frac{1}{r}\right)\left(1 + \frac{1}{r}\right)$

3 Susan's age is p years.

a What was Susan's age last year?
b What will Susan's age be next year?
c Write an expression for (Susan's age last year) × (Susan's age next year).
d If (Susan's age last year) × (Susan's age next year) is equal to 48, what is Susan's age?

4 By expressing 31×29 as $(30 + 1)(30 - 1)$, use the difference of two squares to find the value of 31×29.

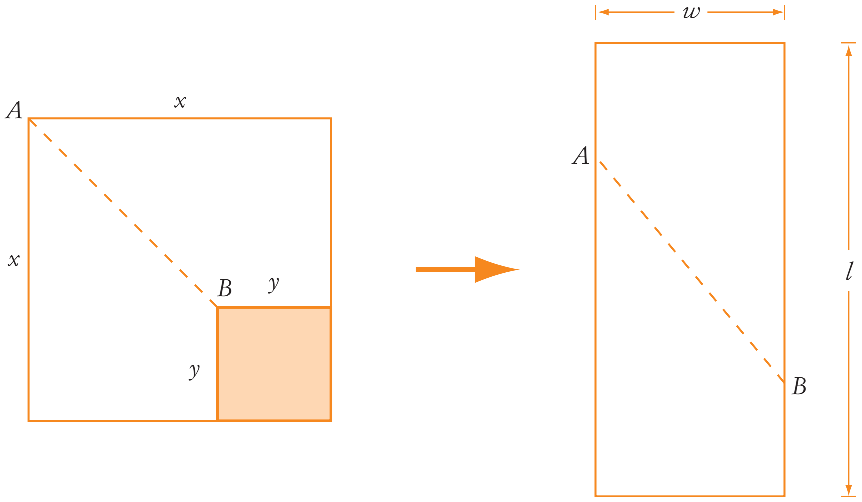
5 Use the method in question **4** to evaluate each expression.

a 21×19
b 51×49
c 89×91
d 78×82

6 Use 'sum by difference' to evaluate each expression.

a $15^2 - 14^2$
b $24^2 - 23^2$
c $65^2 - 35^2$
d $101^2 - 99^2$
e $23^2 - 17^2$
f $50^2 - 48^2$

7 A square sheet of paper of length x cm has a smaller square of length y cm cut from one corner. It is then cut on the diagonal AB and rearranged to form a rectangle as shown.



a Find an expression for:

i the length (l) of the rectangle
ii the width (w) of the rectangle
iii the area of the rectangle.

b The area of the rectangle should be the same as the area of the square on the left minus the area of the smaller square. Find an expression for this area. What does this prove?

12-03 Mixed expansions

Stage 5.3

Puzzle sheet
Products and factors squaresaw
MAT10NAPS10087

Worksheet
Mixed expansions
MAT10NAWK10089

Example 4

Expand and simplify each expression.

a $(4r+5)(1-2r)$ **b** $(7+9x)^2$ **c** $(2d-10)(2d+10)$

d $(a+6)(a-6)+(a+12)(a+3)$ **e** $(m-2)^2-(m-2)(m+2)$

Solution

a $(4r+5)(1-2r) = 4r(1-2r)+5(1-2r)$ Expanding

$= 4r - 8r^2 + 5 - 10r$

$= -8r^2 - 6r + 5$ Rearrange and put the higher order terms first.

b $(7+9x)^2 = 7^2 + 2\times 7\times 9x + (9x)^2$ Expand the perfect square.

$= 49 + 126x + 81x^2$

$= 81x^2 + 126x + 49$ Rearrange and put the higher order terms first.

c $(2d-10)(2d+10) = (2d)^2 - 10^2$ Expand into the difference of two squares.

$= 4d^2 - 100$

d $(a+6)(a-6)+(a+12)(a+3) = a^2 - 36 + a^2 + 3a + 12a + 36$

$= 2a^2 + 15a$

e $(m-2)^2 - (m-2)(m+2) = m^2 - 4m + 4 - (m^2 - 4)$

$= m^2 - 4m + 4 - m^2 + 4$

$= -4m + 8$

Exercise 12-03 Mixed expansions

See Example 4

1 Expand and simplify each expression.

a $(2m-1)(2m+1)$ **b** $(y+4)(5y-3)$ **c** $(2k-7)^2$

d $(d+9)^2$ **e** $(2e-1)(e+1)$ **f** $(5a+4)(5a-4)$

g $(2-p)(p-2)$ **h** $(10-6y)(10+6y)$ **i** $(h-3z)^2$

j $(2x-3)(y+3)$ **k** $(11a-4b)(11a+4b)$ **l** $\left(u-\frac{1}{u}\right)^2$

2 Expand and simplify each expression.

a $(m-5)(m+5)+25$ **b** $6y+(y-3)^2+9$

c $(3x+1)(2-x)+2x+4$ **d** $(d+4)^2-8d+5$

e $16+(4k-8)(4k+8)$ **f** $(x-y)^2-(x+y)^2$

g $20t-(t-2)(t-5)+t^2$ **h** $2(f-2)(f+2)$

i $(2h+3)^2-(2h-3)(2h+3)$ **j** $7xy-(2x-3)(y+3)$

Stage 5.3

3 Expand and simplify each expression.

a $(8a - 1)(8a + 1) - 4a^2 + 1$
b $(n + 1)^2 + 2n + 3$
c $3(4 - t)(4 + t) + (t - 12)(t + 4)$
d $(2m - n)^2 + (2m + n)^2$
e $(x - 2)(x + 3) - (x - 2)(x + 2)$
f $2(b - 1)^2 - (2b - 1)^2$
g $(y + 1)^2 + (y + 2)^2 + (y + 3)^2$
h $(x - 3)(x + 3) + (x + 3)^2 + (x - 3)^2$
i $(5n + 3)(5n - 3) + (3n - 5)(3n + 5)$
j $2(a - b)(a + b) - (a + b)^2 - (a - b)^2$

12-04 Factorising special binomial products

Puzzle sheet
Grouping
MAT10NAPS00021

Factorising by grouping in pairs

An **algebraic expression with four terms** can often be factorised in pairs, that is, two terms at a time, to make a binomial product.

Example 5

Factorise each expression.

a $3ac + 2bd + 2bc + 3ad$ **b** $4km + 6mn - 6kp - 9np$ **c** $10xw - 6yw - 10xt + 6yt$

Solution

a $3ac + 2bd + 2bc + 3ad = 3ac + 3ad + 2bd + 2bc$ Grouping into pairs for factorising.
$= 3a(c + d) + 2b(d + c)$ Factorising each pair.
$= (c + d)(3a + 2b)$ Factorising again.

b $4km + 6mn - 6kp - 9np = 2m(2k + 3n) - 3p(2k + 3n)$ Factorising each pair.
$= (2k + 3n)(2m - 3p)$ Factorising again.

c $10xw - 6yw - 10xt + 6yt = 2(5xw - 3yw - 5xt + 3yt)$ Factorising all terms first.
$= 2[w(5x - 3y) - t(5x - 3y)]$ Factorising each pair.
$= 2(5x - 3y)(w - t)$ Factorising again.

Factorising the difference of two squares

Puzzle sheet
Difference of two perfect squares
MAT10NAPS00019

Puzzle sheet
Mixed factorisations
MAT10NAPS00023

You should recall the product $(a + b)(a - b) = a^2 - b^2$.
If we use this rule in reverse, then the factors of $a^2 - b^2$ are $(a - b)$ and $(a + b)$.

Summary

$a^2 - b^2 = (a + b)(a - b)$

Stage 5.3

Example 6

Factorise each expression.

a $x^2 - 4$ **b** $9 - 16b^2$ **c** $20d^2 - 5a^2$ **d** $y^3 - y$

Solution

a $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

b $9 - 16b^2 = 3^2 - (4b)^2$
$= (3 + 4b)(3 - 4b)$

c $20d^2 - 5a^2 = 5(4d^2 - a^2)$
$= 5[(2d)^2 - a^2]$
$= 5(2d + a)(2d - a)$

d $y^3 - y = y(y^2 - 1)$
$= y(y + 1)(y - 1)$

Exercise 12-04 Factorising special binomial products

See Example 5

1 Factorise each expression.

a $4ab + 5bc + 4ad + 5cd$
b $2xy - 5wy + 2xt - 5wt$
c $9ac + 6bc + 12ad + 8bd$
d $10x^2 + 30 + x^3 + 3x$
e $3a^2 + 3ab + 3ac + 3bc$
f $6rt - 18wt + 6rp - 18wp$
g $14e - 21 + 2de - 3d$
h $hk - h^2 - 2k + 2h$
i $3mn - 6m + pn - 2p$
j $9p^2 - 27 + qp^2 - 3q$
k $fg - fh - 10g + 10h$
l $9kl - 12ml + 9kn - 12mn$
m $2p - 2c - p^2 + pc$
n $l^3 + lm^2 - 3l^2 - 3m^2$
o $a(x + 1) + y(x + 1) - ka - ky$
p $p(a - b) - 2q(a - b) + 3qp - 6q^2$

See Example 6

2 Factorise each expression.

a $w^2 - 9$
b $y^2 - 36$
c $k^2 - 1$
d $m^2 - 121$
e $p^2 - 64$
f $t^2 - 100$
g $4e^2 - f^2$
h $a^2 - 9b^2$
i $16y^2 - 1$
j $4 - b^2$
k $25 - e^2$
l $1 - 16x^2$
m $y^2 - z^2$
n $49 - 16m^2$
o $b^2 - 121d^2$
p $36c^2 - 25k^2$
q $16 - 81h^2$
r $25a^2 - 64m^2$
s $100 - 49n^2$
t $121p^2 - 144q^2$
u $\frac{1}{4} - 25c^2$
v $4t^2 - \frac{1}{9}$
w $25h^2 - 2\frac{1}{4}$
x $1 - m^2n^2$

3 Factorise each expression.

a $2a^2 - 2b^2$
b $7k^2 - 28$
c $3 - 75u^2$
d $x^3 - 49x$
e $k - 16k^3$
f $50q^2 - 2$
g $3d^2 - 12v^2$
h $5t^5 - 125t^3$
i $2a^2b^2 - 2$
j $x^2y^2 - x^2w^2$
k $192f^2 - 108g^2$
l $45d^2 - \frac{5}{4}$
m $2x^2 - 8a^2$
n $100 - 25w^2$
o $1\frac{1}{4} - 80e^2$
p $9c^2 - 6\frac{1}{4}$

4 Factorise each expression.

a $\frac{p^2}{16} - \frac{x^2}{25}$
b $x^2 - \frac{1}{9}$
c $\frac{v^2}{49} - \frac{u^2}{81}$
d $\frac{2y^2}{9} - \frac{2m^2}{121}$
e $\frac{16a^2}{49} - \frac{25b^2}{4}$
f $t^4 - 81$
g $100 - n^4$
h $(x + y)^2 - x^2$
i $(p - 2q)^2 - (2p + q)^2$
j $\frac{x^2}{4} - \frac{y^2}{36}$
k $(a + b)^2 - (a - b)^2$

12-05 Factorising quadratic expressions of the form $ax^2 + bx + c$

Stage 5.3

Puzzle sheet
Products and factors squaresaw
MAT10NAPS10087

Puzzle sheet
Factorising puzzle
MAT10NAPS10090

Technology worksheet
Excel worksheet: Factorising trinomials
MAT10NACT00011

Technology worksheet
Excel spreadsheet: Factorising trinomials
MAT10NACT00041

Puzzle sheet
Perfect squares
MAT10NAPS00020

Puzzle sheet
Trinominoes
MAT10NAPS00022

Video tutorial
Factorising quadratic expressions (Advanced)
MAT10NAVT10017

Video tutorial
Advanced algebra and functions
MAT10NAVT00007

In Chapter 4, Algebra, we factorised **quadratic expressions** of the type $x^2 + bx + c$. For example, the factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.
We will now factorise quadratic expressions of the type $ax^2 + bx + c$, such as $6x^2 + 19x + 15$, where x^2 has a **coefficient**.

Example 7

Factorise each quadratic expression.

a $3g^2 + 12g - 36$ **b** $48 - 8p - p^2$

Solution

a $3g^2 + 12g - 36 = 3(g^2 + 4g - 12)$ Taking out the HCF of 3 first.
$= 3(g - 2)(g + 6)$ Product $= -12$, sum $= 4$

b $48 - 8p - p^2 = -p^2 - 8p + 48$ Rearranging the terms to make the p^2 term first.
$= -1(p^2 + 8p - 48)$ Taking out a common factor of -1.
$= -(p + 12)(p - 4)$ Product $= -48$, sum $= 8$

Example 8

Factorise $3x^2 + 8x + 4$.

Solution

There is no HCF, so we need to split up the middle term $8x$.
Find two numbers that have a product of 12 and a sum of 8.

$$3x^2 + 8x + 4 = \underbrace{3x^2 + \overset{\text{sum of 8}}{8x} + 4}_{\text{product of 12 } (3 \times 4)}$$

The two numbers are $+6$ and $+2$, so we will split $8x$ into $6x$ and $2x$.

$\therefore 3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$
$= 3x(x + 2) + 2(x + 2)$ Factorising by grouping in pairs.
$= (x + 2)(3x + 2)$ Factorising again.

Summary

Factorising quadratic trinomials of the form $ax^2 + bx + c$ (with no HCF)

- find two numbers that have a sum of b and a product of ac
- use these two numbers to split the middle term bx into two terms
- factorise by grouping in pairs

Example 9

Factorise each quadratic expression.

a $3x^2 - 11x + 10$ **b** $4x^2 - 3x - 7$ **c** $6t^2 + t - 12$

Solution

a $3x^2 - 11x + 10$

$3 \times 10 = 30$. Find two numbers that have a product of 30 and a sum of -11.
Since the sum is negative, one of the numbers must be negative.
Since the product is positive, both of the numbers must be negative.
They are -6 and -5. Split $-11x$ into $-6x$ and $-5x$.

$$\begin{aligned} 3x^2 - 11x + 10 &= 3x^2 - 6x - 5x + 10 \\ &= 3x(x - 2) - 5(x - 2) \qquad \text{Factorising by grouping in pairs.} \\ &= (x - 2)(3x - 5) \end{aligned}$$

b $4x^2 - 3x - 7$

$4 \times (-7) = -28$. Find two numbers with a product of -28 and a sum of -3.
Since the product is negative, one of the numbers must be negative.
They are -7 and 4.

$$\begin{aligned} 4x^2 - 3x - 7 &= 4x^2 - 7x + 4x - 7 \\ &= x(4x - 7) + 1(4x - 7) \\ &= (4x - 7)(x + 1) \end{aligned}$$

c $6t^2 + t - 12$

$6 \times (-12) = -72$. Find two numbers with a product of -72 and a sum of 1.
They are -8 and $+9$.

$$\begin{aligned} 6t^2 + t - 12 &= 6t^2 - 8t + 9t - 12 \\ &= 2t(3t - 4) + 3(3t - 4) \\ &= (3t - 4)(2t + 3) \end{aligned}$$

Stage 5.3

Example 10

Factorise each quadratic expression.

a $18a^2 - 18a - 8$ **b** $10 - 7x - 12x^2$

Solution

a
$$\begin{aligned} 18a^2 - 18a - 8 &= 2(9a^2 - 9a - 4) && \text{Taking out the HCF of 2 first.} \\ &= 2(9a^2 - 12a + 3a - 4) && \text{Product} = -36\text{, sum} = -9 \\ &= 2[3a(3a - 4) + 1(3a - 4)] \\ &= 2(3a - 4)(3a + 1) \end{aligned}$$

b
$$\begin{aligned} 10 - 7x - 12x^2 &= -12x^2 - 7x + 10 && \text{Rearranging the terms to make the } x^2 \text{ term first.} \\ &= -(12x^2 + 7x - 10) && \text{Taking out a common factor of } -1. \\ &= -(12x^2 + 15x - 8x - 10) && \text{Product} = -120\text{, sum} = 7 \\ &= -[3x(4x + 5) - 2(4x + 5)] \\ &= -(4x + 5)(3x - 2) \end{aligned}$$

Exercise 12-05 Factorising quadratic expressions of the form $ax^2 + bx + c$

See Example 7

1 Factorise each quadratic expression. Look for the highest common factor first.

a $3m^2 + 9m + 6$ **b** $2y^2 + 2y - 4$ **c** $5t^2 - 10t - 400$
d $5e^4 + 25e^3 - 120e^2$ **e** $x^3 - x^2 - 110x$ **f** $4b^2 - 4b - 168$
g $4w^2 + 4w - 48$ **h** $3a^3 - 9a^2 - 12a$ **i** $2e^2 + 18e + 40$
j $24 - 5t - t^2$ **k** $42 + u - u^2$ **l** $28 + 3x - x^2$
m $12 - b - b^2$ **n** $7k - 12 - k^2$ **o** $12x - 35 - x^2$

See Example 8

2 Factorise each quadratic expression.

a $2x^2 + 11x + 5$ **b** $4x^2 + 13x + 3$ **c** $5x^2 + 17x + 6$
d $6x^2 + 19x + 10$ **e** $2w^2 + 31w + 15$ **f** $4e^2 + 15e + 9$
g $8f^2 + 14f + 3$ **h** $3d^2 + 5d + 2$ **i** $2b^2 + 9b + 7$
j $5y^2 + 16y + 11$ **k** $8g^2 + 26g + 15$ **l** $6a^2 + 23a + 21$

See Example 9

3 Factorise each quadratic expression.

a $2y^2 - 11y + 12$ **b** $10k^2 - 19k + 6$ **c** $6e^2 - 13e + 6$
d $4b^2 - 13b + 3$ **e** $6w^2 - 23w + 15$ **f** $8t^2 + 26t + 15$
g $9x^2 - 12x + 4$ **h** $12f^2 - 25f + 12$ **i** $4h^2 - 36h + 81$
j $5y^2 - 6y - 11$ **k** $4d^2 - d - 5$ **l** $2m^2 - 3m - 9$
m $3t^2 - t - 30$ **n** $6h^2 - h - 7$ **o** $2y^2 - 5y - 12$
p $8a^2 - 2a - 3$ **q** $15u^2 - 7u - 4$ **r** $9c^2 - 12c - 5$

4 Factorise each quadratic expression.

a $5m^2 + 2m - 7$ **b** $6g^2 + g - 12$ **c** $3p^2 + 4p - 4$
d $7w^2 + 6w - 1$ **e** $5y^2 + 14y - 3$ **f** $3n^2 + 10n - 8$
g $4b^2 + 9b - 9$ **h** $8m^2 + 10m - 3$ **i** $3x^2 + 2x - 16$

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Stage 5.3

5 Factorise each of the perfect squares below.

a $81w^2 - 180w + 100$
b $4y^2 + 8y + 4$
c $25h^2 - 40h + 16$

6 Factorise each quadratic expression by first taking out a common factor.

See Example 10

a $6t^2 + 10t - 4$
b $6g^2 + 15g - 36$
c $24e^2 - 28e - 12$
d $8a^2 - 10a - 12$
e $12t^2 + 20t - 8$
f $-25q^2 - 5q + 6$
g $-12m^2 + 14m - 4$
h $20 - h - 12h^2$
i $18 + 48c + 24c^2$
j $15 + 9z - 6z^2$
k $12d^2 + 2d - 30$
l $22x - 12 - 6x^2$

7 Factorise each quadratic expression.

a $2a^2 + 5a + 3$
b $12m^2 - 32m + 5$
c $4x^2 + 11x - 3$
d $7w^2 - 8w + 1$
e $4h^2 - 7h - 15$
f $8x^2 - 2x - 3$
g $5r^2 + 26r + 5$
h $2d^2 - 15d + 7$
i $6n^2 - 7n - 3$
j $8 - 6m - 9m^2$
k $3 - 2c - 5c^2$
l $15g^2 + 19g + 6$
m $15 + 14q - 8q^2$
n $3x^2 - 13x + 14$
o $16 - 8d - 3d^2$

12-06 Mixed factorisations

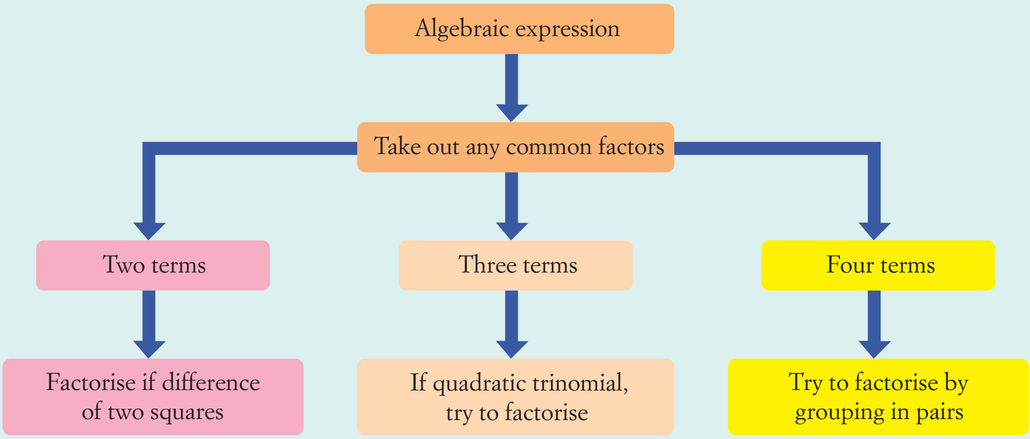
Puzzle sheet
Mixed factorisations
MAT10NAPS00023

Puzzle sheet
Factorominoes
MAT10NAPS10029

Summary

Factorisation strategies

- Look for any common factors and factorise first.
- If there are two terms, try factorising using the difference of two squares.
- If there are three terms, try factorising as a quadratic trinomial.
- If there are four terms, try factorising by grouping in pairs.



Stage 5.3

Example 11

Factorise each quadratic expression.

a $3a^2 - 27$ **b** $5a^2 + 100$ **c** $20b^2 - 52b + 24$ **d** $d^3 - d^2 - d + 1$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$ Taking out the HCF of 3 first.

$= 3(a + 3)(a - 3)$ Difference of two squares.

b $5a^2 + 100 = 5(a^2 + 20)$ Two terms, but not a difference of two squares.

c $20b^2 - 52b + 24 = 4(5b^2 - 13b + 6)$

$= 4(5b^2 - 10b - 3b + 6)$ Product = 30, sum = −13

$= 4[5b(b - 2) - 3(b - 2)]$

$= 4(b - 2)(5b - 3)$

d $d^3 - d^2 - d + 1 = d^2(d - 1) - 1(d - 1)$ Factorising by grouping in pairs.

$= (d - 1)(d^2 - 1)$

$= (d - 1)(d + 1)(d - 1)$ Difference of two squares.

$= (d - 1)^2(d + 1)$

Exercise 12-06 Mixed factorisations

See Example 11

1 Factorise each expression.

a $m^2 - 16m + 64$
b $3d^2 - 3d$
c $3d^2 - 4d - 15$
d $3k - 15 - 5h + hk$
e $25y^2 - 64$
f $100f^2 - 64$
g $q^2 + 3q - 3pq$
h $3 + 2g - g^2$
i $24b^2 + 44b - 40$
j $25r^2 - 1$
k $b^3 + b^2 + b + 1$
l $4x^2 - 20x + 25$
m $4 - d - 5d^2$
n $b^3 - b^2 - b + 1$
o $8 - 2v^2$
p $mn^2 + mnp + 3mn + 3mp$
q $2w^2 - 24w + 72$
r $36h^2 + 12h + 1$
s $15r^2 - 31rt - 24t^2$
t $4d^2 + 4d + 1$
u $9g^2 - 36k^2$

2 Factorise each expression.

a $e^3 - 3e^2 - 10e$
b $5(p + q)^2 - 125(p - q)^2$
c $28x^2 - 7$
d $a^2 - b^2 + 4a - 4b$
e $c^3 - 2c^2 - 4c + 8$
f $6a^2 + 13a - 5$
g $t^2 - 3t + 5t - 35$
h $18p^2 + 24p + 8$
i $1 - 2a - 24a^2$
j $9x^2 - 27x + 18x - 54$
k $2a^2b - 6ab - 3a + 9$
l $2a^2 + 12a + 18$
m $25u^2 - 10u + 1$
n $4k^2 - 5k - 21$
o $48 - 3w^2$
p $3 - 27s^2$
q $k^3 + 4k^2 - 16k - 64$
r $5y^3 - 10y^2 + 15y$
s $m^3n - 4mn$
t $8 - 2a^2$
u $32c^2 - 40c - 12$

12-07 Factorising algebraic fractions

Stage 5.3

NSW

Example 12

Simplify each expression.

a $\dfrac{10a+25b}{5}$ **b** $\dfrac{9y^2-16}{6y+8}$ **c** $\dfrac{x^2+x}{-4x-4}$ **d** $\dfrac{t^2-3t+2}{3t^2-5t-2}$

Solution

a
$$\begin{aligned}\frac{10a+25b}{5}&=\frac{5(2a+5b)}{5}\\&=2a+5b\end{aligned}$$

b
$$\begin{aligned}\frac{9y^2-16}{6y+8}&=\frac{(3y+4)(3y-4)}{2(3y+4)}\\&=\frac{\cancel{(3y+4)}(3y-4)}{2\cancel{(3y+4)}}\\&=\frac{3y-4}{2}\end{aligned}$$

c
$$\begin{aligned}\frac{x^2+x}{-4x-4}&=\frac{x\cancel{(x+1)}}{-4\cancel{(x+1)}}\\&=\frac{x(x+1)}{-4(x+1)}\\&=-\frac{x}{4}\end{aligned}$$

d
$$\begin{aligned}\frac{t^2-3t+2}{3t^2-5t-2}&=\frac{(t-2)(t-1)}{(t-2)(3t+1)}\\&=\frac{\cancel{(t-2)}(t-1)}{\cancel{(t-2)}(3t+1)}\\&=\frac{t-1}{3t+1}\end{aligned}$$

Example 13

Simplify each expression.

a $\dfrac{4}{x^2+x}-\dfrac{2}{x^2-1}$ **b** $\dfrac{3m-6}{4}\times\dfrac{8m}{m^2-2m}$ **c** $\dfrac{d^2+3d+2}{d^2-9}\div\dfrac{d^2+d}{3d+9}$

Solution

a
$$\begin{aligned}\frac{4}{x^2+x}-\frac{2}{x^2-1}&=\frac{4}{x(x+1)}-\frac{2}{(x+1)(x-1)}\\&=\frac{4(x-1)}{x(x+1)(x-1)}-\frac{2x}{x(x+1)(x-1)}\\&=\frac{4x-4-2x}{x(x+1)(x-1)}\\&=\frac{2x-4}{x(x+1)(x-1)}\\&=\frac{2(x-2)}{x(x+1)(x-1)}\end{aligned}$$

Factorising denominators.

Using common denominators.

Stage 5.3

b $$\frac{3m-6}{4} \times \frac{8m}{m^2-2m} = \frac{3(m-2)}{4} \times \frac{8m}{m(m-2)}$$

$$= \frac{3\cancel{(m-2)}}{\cancel{4}_1} \times \frac{{}^2\cancel{8}\cancel{m}}{\cancel{m}\cancel{(m-2)}}$$

$$= 6$$

c $$\frac{d^2+3d+2}{d^2-9} \div \frac{d^2+d}{3d+9} = \frac{d^2+3d+2}{d^2-9} \times \frac{3d+9}{d^2+d}$$

$$= \frac{(d+2)(d+1)}{(d+3)(d-3)} \times \frac{3(d+3)}{d(d+1)}$$

$$= \frac{(d+2)\cancel{(d+1)}}{\cancel{(d+3)}(d-3)} \times \frac{3\cancel{(d+3)}}{d\cancel{(d+1)}}$$

$$= \frac{3(d+2)}{d(d-3)}$$

Exercise 12-07 Factorising algebraic fractions

See Example 12

1 Simplify each expression.

a $\frac{3x+3y}{3}$ **b** $\frac{5}{10t-10r}$ **c** $\frac{ab-ac}{a^2}$

d $\frac{y-1}{1-y}$ **e** $\frac{w^2-16}{w+4}$ **f** $\frac{5d-5t}{d^2-t^2}$

g $\frac{(k+5)^2}{k^2-25}$ **h** $\frac{6c^2-6}{2c+2}$ **i** $\frac{am-an+m-n}{m^2-n^2}$

j $\frac{y^2+9y+20}{2y+10}$ **k** $\frac{k^2-3k-4}{k^2-16}$ **l** $\frac{16a^2-25c^2}{4a^2-9ac+5c^2}$

m $\frac{s^2+4s+4}{s^2-s-6}$ **n** $\frac{1-c-2c^2}{3c^2+2c-1}$ **o** $\frac{ap+4p-2a-8}{2p^2-8}$

See Example 13

2 Simplify each expression.

a $\frac{5}{m(m+1)} + \frac{2}{(m+1)(m+2)}$ **b** $\frac{6}{(w+5)(w+3)} - \frac{4}{w(w+3)}$

c $\frac{3}{(b+2)(b-1)} + \frac{1}{(b-1)(b-3)}$ **d** $\frac{2}{k^2+k} - \frac{3}{k^2-1}$

e $\frac{5}{4h+4} + \frac{3}{h^2+h}$ **f** $\frac{3}{d^2+3d+2} - \frac{4}{d+2}$

g $\frac{3}{r^2-36} - \frac{5}{4r+24}$ **h** $\frac{3}{d^2+2d} + \frac{d}{d^2-4}$

i $\frac{5}{k^2-3k-4} - \frac{k}{k^2-1}$ **j** $\frac{2}{q^2-1} + \frac{3}{q+1}$

Stage 5.3

Quiz

Advanced algebra and equations

MAT10NAQZ00007

3 Simplify each expression.

a $\dfrac{3m+9}{2} \times \dfrac{4m}{m+3}$

b $\dfrac{5d-10}{3d-9} \times \dfrac{5d-15}{8d-16}$

c $\dfrac{4}{e+2} \times \dfrac{e^2+2e}{8e}$

d $\dfrac{3k+6}{5} \times \dfrac{10k}{k+2}$

e $\dfrac{5h}{3h+9} \times \dfrac{6h+18}{h^2+h}$

f $\dfrac{4}{a^2-b^2} \times \dfrac{3a+3b}{8}$

g $\dfrac{r+t}{t^2-r^2} \times \dfrac{r^2-rt}{5r+5t}$

h $\dfrac{20m+16}{7m-7} \times \dfrac{7m}{5m+4}$

i $\dfrac{p^2+2p+1}{p^2-1} \times \dfrac{4p-4}{p^2+p}$

j $\dfrac{y+2}{5y} \div \dfrac{7y+14}{15y}$

k $\dfrac{5}{x^2-4} \div \dfrac{15}{2x+4}$

l $\dfrac{4n+8}{n+5} \div \dfrac{6n+12}{5n+25}$

m $\dfrac{d^2+d}{d+3} \div \dfrac{6d}{d^2-9}$

n $\dfrac{1}{f^2-6f+9} \div \dfrac{4}{f^2-9}$

o $\dfrac{3f+6}{f^2+f-6} \div \dfrac{f^2-2f-8}{f^2-f-12}$

13

Number and Algebra

Surds

Optional Stage 5.3 topic

When applying Pythagoras' theorem, we have found lengths that cannot be expressed as an exact rational number. Pythagoras encountered this when calculating the diagonal of a square of side length 1 unit. A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found.

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Chapter outline

	Proficiency strands			
13-01 Surds and irrational numbers	U	F	R	C
13-02 Simplifying surds	U	F	R	
13-03 Adding and subtracting surds	U	F	R	
13-04 Multiplying and dividing surds	U	F	R	
13-05 Binomial products involving surds	U	F	R	C
13-06 Rationalising the denominator	U	F	R	C

Wordbank

irrational number	A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction $\frac{a}{b}$
rational number	Any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$
rationalise the denominator	To simplify a fraction involving a surd by making its denominator rational (that is, not a surd)
real number	A number that is either rational or irrational and whose value can be graphed on a number line
simplify a surd	To write a surd $\sqrt{x}$ in its simplest form so that x has no factors that are perfect squares
surd	A square root (or other root) whose exact value cannot be found

In this chapter you will:

- (STAGE 5.3) define rational and irrational numbers and perform operations with surds
- (STAGE 5.3) describe real, rational and irrational numbers and surds
- (STAGE 5.3) add, subtract, multiply and divide surds
- (STAGE 5.3) expand and simplify binomial products involving surds
- (STAGE 5.3) rationalise the denominator of expressions of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$

SkillCheck

Worksheet
StartUp assignment 13
MAT10NAWK10091

1 Simplify each expression.

a $(5y)^2$ **b** $(4m)^3$ **c** $(-3x)^2$

2 Expand each expression.

a $5(x + 2)$ **b** $4(y - 3)$ **c** $3(1 + 2w)$
d $2(5 - y)$ **e** $-5(2a + 3)$ **f** $k(1 + 2k)$

3 Select the square numbers from the following list of numbers.

44 81 25 100 75
72 16 50 64 32

4 Expand and simplify each expression.

a $(m + 3)(m + 7)$ **b** $(y + 1)(y - 4)$ **c** $(n - 2)(n - 3)$
d $(2d + 3)(1 + 3d)$ **e** $(1 - 5p)(4 + 3p)$ **f** $(3a + 2f)(a + 5f)$
g $(x + 4)^2$ **h** $(y - 3)^2$ **i** $(2k + 1)^2$
j $(a - 5)(a + 5)$ **k** $(t + 7)(t - 7)$ **l** $(3m + 4)(3m - 4)$

Stage 5.3

13-01 Surds and irrational numbers

A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fractional value cannot be found. As a decimal, its digits run endlessly *without repeating* (like π), so they are neither terminating nor recurring decimals.

$\sqrt{7}$ is read as 'the square root of 7' or simply 'root 7'.

Rational numbers such as fractions, terminating or recurring decimals, and percentages, can be expressed in the form $\frac{a}{b}$, where a and b are integers (and $b \neq 0$). Surds are **irrational numbers** because they cannot be expressed in this form.

Rational numbers

can be expressed in the form $\frac{a}{b}$

Integers

$\frac{4}{1} = 4$, $\frac{26}{1} = 26$, $\frac{-3}{1} = -3$

Recurring decimals

$\frac{2}{3} = 0.666 \ldots$

$\frac{5}{6} = 0.833 \ldots$

$\frac{4}{11} = 0.3636 \ldots$

Terminating decimals

0.5, $7\frac{1}{8} = 7.125$,

$16\% = 0.16$, 1.32

Irrational numbers

cannot be expressed in the form $\frac{a}{b}$

Surds

$\sqrt{5}$, $-\sqrt{2}$, $\frac{\sqrt{11}}{3}$, $8\sqrt{6}$

Transcendental numbers

Have no pattern and are non-recurring

e.g. $\pi = 3.14159\ldots$, $1.257308\ldots$, $0.0097542\ldots$

Example 1

Select the surds from this list of square roots: $\sqrt{72}$ $\sqrt{121}$ $\sqrt{64}$ $\sqrt{90}$ $\sqrt{28}$

Solution

$\sqrt{72} = 8.4852\ldots$

$\sqrt{121} = 11$

$\sqrt{64} = 8$

$\sqrt{90} = 9.4868\ldots$

$\sqrt{28} = 5.2915\ldots$

So the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

Example 2

Is each number rational or irrational?

a $4\frac{2}{5}$ **b** $\sqrt[3]{-8}$ **c** $-\sqrt{7}$ **d** $0.\dot{6}$ **e** 5π

Solution

a $4\frac{2}{5} = \frac{22}{5}$ which is in the form of a fraction $\frac{a}{b}$

$\therefore 4\frac{2}{5}$ is a rational number.

Stage 5.3

b $\sqrt[3]{-8} = -2$ which can be written as $\frac{-2}{1}$

$\therefore \sqrt[3]{-8}$ is a rational number.

c $-\sqrt{7} = -2.645\,751\,311\ \ldots$ The digits run endlessly without repeating.

$\therefore -\sqrt{7}$ is an irrational number.

d $0.\dot{6} = 0.666\ldots$ which is a recurring decimal

$= \frac{2}{3}$ which is a fraction

$\therefore 0.\dot{6}$ is a rational number.

e $5\pi = 15.707\ 963\ 27\ldots$ The digits run endlessly without repeating.

$\therefore 5\pi$ is an irrational number.

Square roots

The symbol $\sqrt{\ }$ stands for the positive square root of a number. For example, $\sqrt{4} = 2$ (not -2). Furthermore, it is not possible to find the square root of a negative number. It is only possible to find the square root of a positive number or zero, because the square of any real number is positive or zero.

Summary

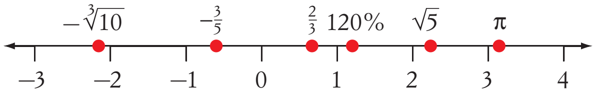
For $x > 0$, $\sqrt{x}$ is the positive square root of x.
For $x = 0$, $\sqrt{x}$ is 0.
For $x < 0$, $\sqrt{x}$ is undefined.

Your calculator will tell you that there is a mathematical error if you enter, for example, $\sqrt{-5}$.

Worksheet
Surds on the number line
MAT10NAWK10092

Surds on a number line

The rational and irrational numbers together make up the **real numbers**. Any real number can be represented by a point on the number line.



$-\sqrt[3]{10} \approx -2.1544\ldots$	irrational (surd)
$-\frac{3}{5} = -0.6$	rational (fraction)
$\frac{2}{3} \approx 0.6666\ldots$	rational (fraction)
$120\% = 1.2$	rational (percentage)
$\sqrt{5} \approx 2.2360\ldots$	irrational (surd)
$\pi \approx 3.1415\ldots$	irrational (pi)

9780170194655

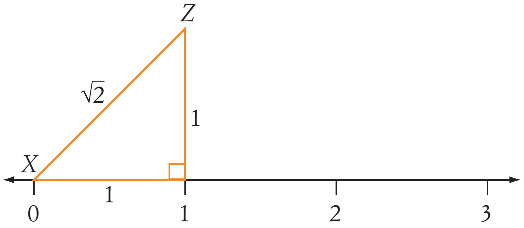
Example 3

Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

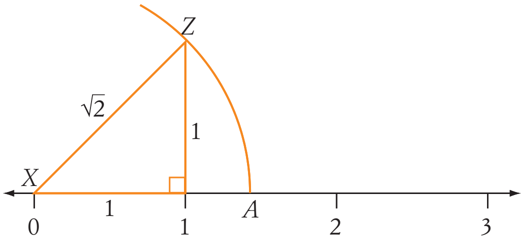
Solution

Step 1
Using a scale of 1 unit to 2 cm, draw a number line as shown.

Step 2
Construct a right-angled triangle on the number line with base length and height 1 unit as shown. By Pythagoras' theorem, show that $XZ = \sqrt{2}$ units.



Step 3
With 0 as the centre, use compasses with radius $XZ\left(\sqrt{2}\right)$ to draw an arc to meet the number line at A as shown. The point A represents the value of $\sqrt{2}$ and should be approximately 1.4142…



Exercise 13-01 Surds and irrational numbers

See Example 1

1 Which one of the following is a surd? Select the correct answer **A**, **B**, **C** or **D**.

A $\sqrt{9}$ **B** $\sqrt{225}$ **C** $\sqrt{160}$ **D** $\sqrt{81}$

2 Which one of the following is NOT a surd? Select the correct answer **A**, **B**, **C** or **D**.

A $\sqrt{77}$ **B** $\sqrt{144}$ **C** $\sqrt{18}$ **D** $\sqrt{200}$

3 Select the surds from the following list of square roots.

$\sqrt{32}$ $\sqrt{33}$ $\sqrt{289}$ $\sqrt{81}$ $\sqrt{4.9}$

$\sqrt{52}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{196}$ $\sqrt{200}$

See Example 2

4 Is each number rational (R) or irrational (I)?

a $5.\dot{6}$ **b** $\sqrt{8}$ **c** $\sqrt{4}$ **d** $3\frac{1}{7}$

e $\sqrt[3]{27}$ **f** $1.3\dot{5}$ **g** $\sqrt[3]{-64}$ **h** $27\frac{1}{2}\%$

i $\sqrt{5 \times 10^3}$ **j** $\frac{3}{11}$ **k** $\frac{\sqrt{50}}{3}$ **l** $\sqrt{\sqrt{4}}$

5 Arrange each set of numbers in descending order.

a $1\frac{4}{7}, \sqrt{2}, \frac{\pi}{2}$ **b** $\sqrt[3]{20}, 2.\dot{6}, 2\frac{7}{9}$

Stage 5.3

6 Express each real number correct to one decimal place and graph them on a number line.

a $-1\frac{4}{5}$ **b** 74% **c** $\frac{4}{11}$ **d** $-\sqrt{12}$

e $-\sqrt[3]{15}$ **f** $2\frac{5}{9}$ **g** $\frac{\pi}{2}$ **h** 187%

See Example 3

7 Use the method from Example **3** to estimate the value of $\sqrt{2}$ on a number line.

8 **a** Use the method from Example **3** to estimate the value of $\sqrt{5}$ on a number line by constructing a right-angled triangle with base length 2 units and height 1 unit.

b Use a similar method to estimate the following surds on a number line.

i $\sqrt{10}$ **ii** $\sqrt{17}$

Puzzle sheet
Simplifying surds
MAT10NAPS10093

Technology worksheet
Excel worksheet: Simplifying surds quiz
MAT10NACT00019

Technology worksheet
Excel spreadsheet: Simplifying surds
MAT10NACT00049

13-02 Simplifying surds

Summary

For $x > 0$ (positive):

$$(\sqrt{x})^2 = \sqrt{x} \times \sqrt{x} = x$$
$$\sqrt{x^2} = x$$

Example 4

Simplify each expression.

a $(\sqrt{7})^2$ **b** $(3\sqrt{5})^2$ **c** $(-2\sqrt{3})^2$

Solution

a $(\sqrt{7})^2 = 7$

b $$\begin{aligned}(3\sqrt{5})^2 &= 3\sqrt{5} \times 3\sqrt{5}\\ &= 3^2 \times (\sqrt{5})^2\\ &= 9 \times 5\\ &= 45\end{aligned}$$

$3\sqrt{5}$ means $3 \times \sqrt{5}$

c $$\begin{aligned}(-2\sqrt{3})^2 &= (-2)^2 \times (\sqrt{3})^2\\ &= 4 \times 3\\ &= 12\end{aligned}$$

Stage 5.3

Summary

The square root of a product

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

A surd $\sqrt{n}$ can be simplified if n can be divided into two factors, where one of them is a square number such as 4, 9, 16, 25, 36, 49, …

Example 5

Video tutorial

Simplifying surds

MAT10NAVT10002

Simplify each surd.

a $\sqrt{8}$ **b** $\sqrt{108}$ **c** $4\sqrt{45}$ **d** $\dfrac{\sqrt{288}}{3}$

Solution

a $\sqrt{8} = \sqrt{4} \times \sqrt{2}$ 4 is a square number.

$= 2 \times \sqrt{2}$

$= 2\sqrt{2}$

b Method 1

$\sqrt{108} = \sqrt{36} \times \sqrt{3}$

$= 6 \times \sqrt{3}$

$= 6\sqrt{3}$

Method 2

$\sqrt{108} = \sqrt{4} \times \sqrt{27}$

$= 2 \times \sqrt{9} \times \sqrt{3}$

$= 2 \times 3 \times \sqrt{3}$

$= 6\sqrt{3}$

Method 2 involves simplifying surds *twice* ($\sqrt{108}$ and $\sqrt{27}$). Method 1 shows that when simplifying surds, you should look for the highest square factor possible.

c $4\sqrt{45} = 4 \times \sqrt{9} \times \sqrt{5}$

$= 4 \times 3 \times \sqrt{5}$

$= 12\sqrt{5}$

d $\dfrac{\sqrt{288}}{3} = \dfrac{\sqrt{144} \times \sqrt{2}}{3}$

$= \dfrac{12\sqrt{2}}{3}$

$= \dfrac{\cancel{12}^{4}\sqrt{2}}{\cancel{3}^{1}}$

$= 4\sqrt{2}$

Exercise 13-02 Simplifying surds

See Example 4

1 Simplify each expression.

a $(\sqrt{2})^2$ **b** $(\sqrt{5})^2$ **c** $(3\sqrt{3})^2$ **d** $(5\sqrt{10})^2$

e $(\sqrt{0.09})^2$ **f** $(-2\sqrt{7})^2$ **g** $(-3\sqrt{5})^2$ **h** $(-5\sqrt{2})^2$

Stage 5.3
See Example 5

2 Simplify each surd.

a $\sqrt{50}$ **b** $\sqrt{27}$ **c** $\sqrt{24}$ **d** $\sqrt{54}$
e $\sqrt{243}$ **f** $\sqrt{45}$ **g** $\sqrt{48}$ **h** $\sqrt{200}$
i $\sqrt{96}$ **j** $\sqrt{63}$ **k** $\sqrt{288}$ **l** $\sqrt{108}$
m $\sqrt{75}$ **n** $\sqrt{147}$ **o** $\sqrt{32}$ **p** $\sqrt{242}$
q $\sqrt{162}$ **r** $\sqrt{245}$ **s** $\sqrt{125}$ **t** $\sqrt{512}$

3 Simplify each expression.

a $5\sqrt{50}$ **b** $3\sqrt{8}$ **c** $4\sqrt{27}$ **d** $8\sqrt{98}$
e $\frac{\sqrt{40}}{2}$ **f** $\frac{\sqrt{243}}{9}$ **g** $\frac{\sqrt{28}}{6}$ **h** $3\sqrt{24}$
i $9\sqrt{68}$ **j** $\frac{\sqrt{3125}}{10}$ **k** $\frac{1}{2}\sqrt{72}$ **l** $\frac{3}{4}\sqrt{48}$
m $10\sqrt{160}$ **n** $3\sqrt{75}$ **o** $7\sqrt{68}$ **p** $\frac{\sqrt{52}}{6}$

4 Which one of the following is equivalent to $4\sqrt{50}$? Select **A**, **B**, **C** or **D**.

A $8\sqrt{5}$ **B** $20\sqrt{2}$ **C** $8\sqrt{2}$ **D** $20\sqrt{5}$

5 Which one of the following is equivalent to $\frac{\sqrt{250}}{10}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{5}}{10}$ **B** $\frac{\sqrt{10}}{2}$ **C** $2\sqrt{10}$ **D** $5\sqrt{10}$

6 Decide whether each statement is true (T) or false (F).

a $3\sqrt{7} = \sqrt{21}$ **b** $\sqrt{12} = 6$ **c** $(\sqrt{9.4})^2 = 9.4$ **d** $\sqrt{75} = 5\sqrt{3}$
e $\sqrt{3} \approx 1.7$ **f** The exact value of $\sqrt{10}$ is 3.162 277 8

13-03 Adding and subtracting surds

Puzzle sheet
Surds code puzzle
MAT10NAPS10094

Just as you can only add or subtract 'like terms' in algebra, you can only add or subtract 'like surds'. You may first need to express all the surds in their simplest forms.

Example 6

Simplify each expression.

a $4\sqrt{2} + 5\sqrt{2}$ **b** $7\sqrt{3} - 2\sqrt{3}$ **c** $5\sqrt{2} - 3\sqrt{3} + \sqrt{2}$
d $\sqrt{50} + \sqrt{32}$ **e** $\sqrt{8} - \sqrt{27} + \sqrt{18}$ **f** $5\sqrt{20} - 3\sqrt{125}$

Solution

a $4\sqrt{2} + 5\sqrt{2} = 9\sqrt{2}$

b $7\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

c $5\sqrt{2} - 3\sqrt{3} + \sqrt{2} = 6\sqrt{2} - 3\sqrt{3}$

d $\sqrt{50} + \sqrt{32} = \sqrt{25}\sqrt{2} + \sqrt{16}\sqrt{2}$ Simplifying each surd.
$= 5\sqrt{2} + 4\sqrt{2}$
$= 9\sqrt{2}$

Stage 5.3

e $$\begin{aligned}\sqrt{8}-\sqrt{27}+\sqrt{18}&=\sqrt{4}\sqrt{2}-\sqrt{9}\sqrt{3}+\sqrt{9}\sqrt{2}\\&=2\sqrt{2}-3\sqrt{3}+3\sqrt{2}\\&=5\sqrt{2}-3\sqrt{3}\end{aligned}$$

f $$\begin{aligned}5\sqrt{20}-3\sqrt{125}&=5\sqrt{4}\sqrt{5}-3\sqrt{25}\sqrt{5}\\&=5\times2\sqrt{5}-3\times5\sqrt{5}\\&=10\sqrt{5}-15\sqrt{5}\\&=-5\sqrt{5}\end{aligned}$$

Exercise 13-03 Adding and subtracting surds

See Example 6

1 Simplify each expression.

a $5\sqrt{7}+2\sqrt{7}$ **b** $3\sqrt{2}-8\sqrt{2}$ **c** $7\sqrt{5}-\sqrt{5}$
d $\sqrt{5}+3\sqrt{5}$ **e** $5\sqrt{17}-5\sqrt{17}$ **f** $3\sqrt{10}-2\sqrt{10}$
g $4\sqrt{15}-3\sqrt{15}+7\sqrt{15}$ **h** $5\sqrt{6}-2\sqrt{6}-4\sqrt{6}$ **i** $3\sqrt{3}+4\sqrt{3}-5\sqrt{3}$
j $4\sqrt{5}+7\sqrt{5}-\sqrt{5}$ **k** $8\sqrt{10}-5\sqrt{10}+3\sqrt{10}$ **l** $10\sqrt{3}-3\sqrt{3}-12\sqrt{3}$

2 Simplify each expression.

a $3\sqrt{5}-8+2\sqrt{5}$ **b** $11\sqrt{10}+3\sqrt{2}+2\sqrt{10}$
c $-4\sqrt{3}+5\sqrt{2}-5\sqrt{3}$ **d** $3\sqrt{15}+3\sqrt{2}+4\sqrt{15}+5\sqrt{2}$
e $\sqrt{7}-3\sqrt{5}-4\sqrt{7}+\sqrt{5}$ **f** $4\sqrt{6}-3\sqrt{3}-2\sqrt{6}-5\sqrt{3}$
g $10\sqrt{11}-5\sqrt{3}+3\sqrt{11}+4\sqrt{3}$ **h** $\sqrt{13}+8\sqrt{7}-7\sqrt{13}+3\sqrt{7}$
i $2\sqrt{5}-3\sqrt{7}-2\sqrt{5}-3\sqrt{7}$ **j** $4\sqrt{10}-3\sqrt{5}-4\sqrt{10}$

3 For each expression, select the correct simplified answer **A**, **B**, **C** or **D**.

a $\sqrt{3}+\sqrt{12}$

A $5\sqrt{3}$ **B** $\sqrt{15}$ **C** $2\sqrt{6}$ **D** $3\sqrt{3}$

b $4\sqrt{5}-2\sqrt{125}$

A $-6\sqrt{5}$ **B** $\sqrt{5}$ **C** $-\sqrt{45}$ **D** $-46\sqrt{5}$

4 Simplify each expression.

a $\sqrt{8}+\sqrt{32}$ **b** $\sqrt{108}-\sqrt{27}$ **c** $\sqrt{20}-\sqrt{80}$
d $\sqrt{28}-\sqrt{63}$ **e** $3\sqrt{6}+\sqrt{24}$ **f** $2\sqrt{5}+\sqrt{125}$
g $\sqrt{40}-\sqrt{90}$ **h** $5\sqrt{11}+\sqrt{99}$ **i** $3\sqrt{2}+\sqrt{18}$
j $\sqrt{27}+5\sqrt{3}$ **k** $\sqrt{200}-7\sqrt{2}$ **l** $\sqrt{50}+\sqrt{32}$
m $5\sqrt{3}+2\sqrt{27}$ **n** $3\sqrt{20}-\sqrt{245}$ **o** $7\sqrt{12}-5\sqrt{48}$
p $4\sqrt{27}+2\sqrt{243}$ **q** $3\sqrt{63}-2\sqrt{28}$ **r** $2\sqrt{98}+3\sqrt{162}$
s $-5\sqrt{6}+2\sqrt{150}$ **t** $4\sqrt{50}+3\sqrt{18}$ **u** $5\sqrt{27}-6\sqrt{75}$
v $3\sqrt{112}-2\sqrt{252}$ **w** $\sqrt{32}+\sqrt{8}+\sqrt{12}$ **x** $\sqrt{27}+\sqrt{54}+\sqrt{243}$
y $\sqrt{98}-3\sqrt{20}-2\sqrt{8}$ **z** $3\sqrt{96}-2\sqrt{150}+\sqrt{24}$

Stage 5.3

Worksheet
Multiplying and dividing surds
MAT10NAWK10095

Puzzle sheet
Surds
MAT10NAPS00043

Technology worksheet
Excel worksheet: Simplifying surds quiz
MAT10NACT00019

13-04 Multiplying and dividing surds

Summary

The square root of products and quotients

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$
$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

Example 7

Simplify each expression.

a $\sqrt{3} \times \sqrt{5}$ **b** $\sqrt{10} \times \sqrt{6}$ **c** $3\sqrt{7} \times 5\sqrt{7}$

d $5\sqrt{27} \times 3\sqrt{6}$ **e** $\sqrt{54} \div (-\sqrt{2})$ **f** $\frac{15\sqrt{32}}{5\sqrt{8}}$

Solution

a $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

b
$$\begin{aligned}\sqrt{10} \times \sqrt{6} &= \sqrt{60}\\ &= \sqrt{4} \times \sqrt{15}\\ &= 2\sqrt{15}\end{aligned}$$

c
$$\begin{aligned}3\sqrt{7} \times 5\sqrt{7} &= 3 \times 5 \times \sqrt{7} \times \sqrt{7}\\ &= 15 \times 7\\ &= 105\end{aligned}$$

d
$$\begin{aligned}5\sqrt{27} \times 3\sqrt{6} &= 5 \times 3 \times \sqrt{27} \times \sqrt{6}\\ &= 15\sqrt{162}\\ &= 15 \times \sqrt{81} \times \sqrt{2}\\ &= 15 \times 9\sqrt{2}\\ &= 135\sqrt{2}\end{aligned}$$

e
$$\begin{aligned}\sqrt{54} \div (-\sqrt{2}) &= -\frac{\sqrt{54}}{\sqrt{2}}\\ &= -\sqrt{27}\\ &= -\sqrt{9} \times \sqrt{3}\\ &= -3\sqrt{3}\end{aligned}$$

f
$$\begin{aligned}\frac{15\sqrt{32}}{5\sqrt{8}} &= 3\sqrt{4}\\ &= 3 \times 2\\ &= 6\end{aligned}$$

Stage 5.3

Example 8

Simplify $\dfrac{5\sqrt{2}\times 4\sqrt{12}}{10\sqrt{8}}$.

Solution

$$\begin{aligned}\frac{5\sqrt{2}\times 4\sqrt{12}}{10\sqrt{8}} &= \frac{20\sqrt{24}}{10\sqrt{8}}\\ &= 2\sqrt{3}\end{aligned}$$

Exercise 13-04 Multiplying and dividing surds

See Example 7

1 Simplify each expression.

a $\sqrt{7}\times\sqrt{2}$
b $-\sqrt{5}\times\sqrt{7}$
c $\sqrt{6}\times\sqrt{8}$
d $\sqrt{12}\times\sqrt{3}$
e $\sqrt{10}\times(-\sqrt{5})$
f $3\sqrt{3}\times 5\sqrt{3}$
g $5\sqrt{10}\times 3\sqrt{3}$
h $-2\sqrt{7}\times 5\sqrt{3}$
i $7\sqrt{5}\times 4\sqrt{5}$
j $2\sqrt{3}\times(-5\sqrt{6})$
k $4\sqrt{3}\times\sqrt{27}$
l $-3\sqrt{5}\times 4\sqrt{10}$
m $-7\sqrt{2}\times 4\sqrt{8}$
n $\sqrt{18}\times 8\sqrt{3}$
o $10\sqrt{2}\times 2\sqrt{8}$
p $3\sqrt{18}\times 5\sqrt{12}$
q $3\sqrt{44}\times(-2\sqrt{99})$
r $5\sqrt{8}\times 4\sqrt{40}$
s $8\sqrt{3}\times 3\sqrt{54}$
t $-8\sqrt{32}\times\sqrt{27}$
u $\sqrt{90}\times\sqrt{72}$
v $-5\sqrt{20}\times 3\sqrt{8}$
w $7\sqrt{18}\times 3\sqrt{24}$
x $3\sqrt{48}\times 2\sqrt{12}$

2 Simplify each expression.

a $\sqrt{15}\div\sqrt{3}$
b $\sqrt{18}\div(-\sqrt{6})$
c $\dfrac{6\sqrt{48}}{2\sqrt{8}}$
d $10\sqrt{54}\div 5\sqrt{27}$
e $-3\sqrt{98}\div 6\sqrt{14}$
f $\dfrac{7\sqrt{18}}{\sqrt{2}}$
g $2\sqrt{24}\div 4\sqrt{6}$
h $\dfrac{\sqrt{128}}{\sqrt{2}}$
i $15\sqrt{18}\div 3\sqrt{6}$
j $\dfrac{-20\sqrt{10}}{-4\sqrt{5}}$
k $36\sqrt{24}\div 9\sqrt{8}$
l $16\sqrt{30}\div 8\sqrt{5}$
m $12\sqrt{14}\div 6$
n $\dfrac{3\sqrt{2}}{-12}$
o $\sqrt{80}\div 4\sqrt{5}$
p $5\sqrt{60}\div\sqrt{15}$
q $6\sqrt{8}\div 3\sqrt{2}$
r $\dfrac{-42\sqrt{54}}{6\sqrt{3}}$
s $12\sqrt{63}\div 3\sqrt{7}$
t $\dfrac{8\sqrt{50}}{2\sqrt{200}}$
u $6\sqrt{3}\div\sqrt{243}$

3 Simplify the expressions below.

a $\sqrt{6}\times\sqrt{6}$
b $\sqrt{7}\times\sqrt{7}$
c $2\sqrt{3}\times\sqrt{3}$
d $5\sqrt{y}\times 3\sqrt{y}$
e $\sqrt{x}\times\sqrt{x}$
f $\sqrt{a^2}\times\sqrt{a}$

4 Simplify $3\sqrt{2}\times\sqrt{6}$. Select the correct answer **A**, **B**, **C** or **D**.

A 6
B $6\sqrt{2}$
C $6\sqrt{3}$
D $12\sqrt{2}$

Stage 5.3

5 Simplify $20\sqrt{10} \div 5\sqrt{2}$. Select **A**, **B**, **C** or **D**.

A $4\sqrt{5}$ **B** $15\sqrt{5}$ **C** 10 **D** 20

See Example 8

6 Simplify each expression.

a $\dfrac{3\sqrt{5} \times 4\sqrt{2}}{3\sqrt{40}}$ **b** $\dfrac{3\sqrt{12} \times 8\sqrt{6}}{4\sqrt{27}}$ **c** $\dfrac{5\sqrt{8} \times 2\sqrt{90}}{10\sqrt{24}}$

d $\dfrac{4\sqrt{5}}{2\sqrt{15} \times 5\sqrt{27}}$ **e** $\dfrac{10\sqrt{686} \times 3\sqrt{12}}{5\sqrt{28} \times \sqrt{18}}$ **f** $\dfrac{8\sqrt{80} \times 3\sqrt{2}}{4\sqrt{5} \times 6\sqrt{8}}$

13-05 Binomial products involving surds

Surd expressions involving brackets can be expanded in the same way as algebraic expressions, as we did in Chapter 4, Algebra.

Example 9

Expand and simplify each expression.

a $\sqrt{3}(\sqrt{5} + \sqrt{7})$ **b** $2\sqrt{11}(3\sqrt{11} - 5\sqrt{2})$

Solution

a
$$\begin{aligned}&\sqrt{3}(\sqrt{5} + \sqrt{7})\\&= \sqrt{3} \times \sqrt{5} + \sqrt{3} \times \sqrt{7}\\&= \sqrt{15} + \sqrt{21}\end{aligned}$$

b
$$\begin{aligned}&2\sqrt{11}(3\sqrt{11} - 5\sqrt{2})\\&= 2\sqrt{11} \times 3\sqrt{11} - 2\sqrt{11} \times 5\sqrt{2}\\&= 6 \times 11 - 10 \times \sqrt{22}\\&= 66 - 10\sqrt{22}\end{aligned}$$

Example 10

Expand and simplify each expression.

a $(\sqrt{7} + \sqrt{5})(3\sqrt{2} - \sqrt{3})$ **b** $(3 - 2\sqrt{10})(\sqrt{5} - 3\sqrt{2})$

Solution

a
$$\begin{aligned}(\sqrt{7} + \sqrt{5})(3\sqrt{2} - \sqrt{3}) &= \sqrt{7}(3\sqrt{2} - \sqrt{3}) + \sqrt{5}(3\sqrt{2} - \sqrt{3})\\&= \sqrt{7} \times 3\sqrt{2} - \sqrt{7} \times \sqrt{3} + \sqrt{5} \times 3\sqrt{2} - \sqrt{5} \times \sqrt{3}\\&= 3\sqrt{14} - \sqrt{21} + 3\sqrt{10} - \sqrt{15}\end{aligned}$$

b
$$\begin{aligned}(3 - 2\sqrt{10})(\sqrt{5} - 3\sqrt{2}) &= 3(\sqrt{5} - 3\sqrt{2}) - 2\sqrt{10}(\sqrt{5} - 3\sqrt{2})\\&= 3 \times \sqrt{5} - 3 \times 3\sqrt{2} - 2\sqrt{10} \times \sqrt{5} + 2\sqrt{10} \times 3\sqrt{2}\\&= 3\sqrt{5} - 9\sqrt{2} - 2\sqrt{50} + 6\sqrt{20}\\&= 3\sqrt{5} - 9\sqrt{2} - 2(5\sqrt{2}) + 6(2\sqrt{5})\\&= 3\sqrt{5} - 9\sqrt{2} - 10\sqrt{2} + 12\sqrt{5}\\&= 15\sqrt{5} - 19\sqrt{2}\end{aligned}$$

Summary

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

Example 11

Expand and simplify each expression.

a $\left(\sqrt{7}-\sqrt{5}\right)^2$ **b** $\left(2\sqrt{3}+3\sqrt{5}\right)^2$
c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$ **d** $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)$

Solution

a
$$\begin{aligned}\left(\sqrt{7}-\sqrt{5}\right)^2 &= \left(\sqrt{7}\right)^2 - 2\times\sqrt{7}\times\sqrt{5}+\left(\sqrt{5}\right)^2\\ &= 7-2\sqrt{35}+5\\ &= 12-2\sqrt{35}\end{aligned}$$

Using $(a-b)^2 = a^2 - 2ab + b^2$

b
$$\begin{aligned}\left(2\sqrt{3}+3\sqrt{5}\right)^2 &= \left(2\sqrt{3}\right)^2 + 2\times2\sqrt{3}\times3\sqrt{5}+\left(3\sqrt{5}\right)^2\\ &= (4\times3)+12\sqrt{15}+(9\times5)\\ &= 12+12\sqrt{15}+45\\ &= 57+12\sqrt{15}\end{aligned}$$

Using $(a+b)^2 = a^2 + 2ab + b^2$

c
$$\begin{aligned}\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) &= \left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2\\ &= 5-2\\ &= 3\end{aligned}$$

Using $(a+b)(a-b) = a^2 - b^2$

Note that because of the 'difference of two squares', the answer is not a surd but a rational number.

d
$$\begin{aligned}\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right) &= \left(3\sqrt{11}\right)^2-4^2\\ &= (9\times11)-16\\ &= 83\end{aligned}$$

Using $(a+b)(a-b) = a^2 - b^2$

Exercise 13-05 Binomial products involving surds

1 Expand and simplify each expression.

See Example 9

a $\sqrt{5}\left(\sqrt{3}+\sqrt{2}\right)$ **b** $\sqrt{6}\left(\sqrt{2}-1\right)$ **c** $\sqrt{2}\left(\sqrt{3}+\sqrt{7}\right)$
d $\sqrt{5}\left(3\sqrt{2}-\sqrt{5}\right)$ **e** $3\sqrt{2}\left(\sqrt{2}+2\sqrt{3}\right)$ **f** $-\sqrt{11}\left(4-\sqrt{5}\right)$
g $2\sqrt{7}\left(3\sqrt{7}-4\right)$ **h** $5\sqrt{5}\left(1+3\sqrt{5}\right)$ **i** $3\sqrt{2}\left(4\sqrt{2}+\sqrt{3}\right)$

Stage 5.3

See Example 10

2 Which expression is equivalent to $\left(\sqrt{3}+2\sqrt{5}\right)\left(5\sqrt{2}+\sqrt{3}\right)$? Select the correct answer **A**, **B**, **C** or **D**.

A $20\sqrt{10}$ **B** $2\sqrt{15}+5\sqrt{6}$

C $5\sqrt{6}+3+10\sqrt{10}+2\sqrt{15}$ **D** $5\sqrt{5}+\sqrt{3}+7\sqrt{7}+4\sqrt{2}$

3 Expand and simplify each expression.

a $\left(\sqrt{5}-3\right)\left(2\sqrt{5}+\sqrt{2}\right)$ **b** $\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+2\right)$

c $\left(7\sqrt{3}+2\right)\left(4\sqrt{2}+\sqrt{3}\right)$ **d** $\left(3\sqrt{2}-\sqrt{5}\right)\left(5\sqrt{2}+2\sqrt{5}\right)$

e $\left(\sqrt{7}+2\sqrt{11}\right)\left(3\sqrt{7}+4\sqrt{11}\right)$ **f** $\left(5\sqrt{3}-2\sqrt{2}\right)\left(4\sqrt{3}-3\sqrt{2}\right)$

g $\left(6+2\sqrt{10}\right)\left(3\sqrt{10}-1\right)$ **h** $\left(\sqrt{7}-2\sqrt{5}\right)\left(3\sqrt{5}+2\sqrt{7}\right)$

See Example 11

4 Which expression is equivalent to $\left(5+\sqrt{7}\right)^2$? Select **A**, **B**, **C** or **D**.

A 12 **B** 32 **C** $32+10\sqrt{7}$ **D** $32+5\sqrt{7}$

5 Expand and simplify each expression.

a $\left(\sqrt{5}-\sqrt{3}\right)^2$ **b** $\left(\sqrt{7}+\sqrt{2}\right)^2$ **c** $\left(\sqrt{5}-2\right)^2$ **d** $\left(3+\sqrt{10}\right)^2$

e $\left(5\sqrt{2}+3\sqrt{3}\right)^2$ **f** $\left(5\sqrt{7}-2\right)^2$ **g** $\left(3\sqrt{2}+2\sqrt{5}\right)^2$ **h** $\left(2\sqrt{5}+\sqrt{3}\right)^2$

6 Expand and simplify each expression.

a $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$ **b** $\left(5+\sqrt{3}\right)\left(5-\sqrt{3}\right)$

c $\left(6+2\sqrt{7}\right)\left(6-2\sqrt{7}\right)$ **d** $\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)$

e $\left(\sqrt{11}-\sqrt{10}\right)\left(\sqrt{11}+\sqrt{10}\right)$ **f** $\left(5\sqrt{7}+3\right)\left(5\sqrt{7}-3\right)$

g $\left(3\sqrt{2}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{5}\right)$ **h** $\left(4\sqrt{2}-5\sqrt{3}\right)\left(4\sqrt{2}+5\sqrt{3}\right)$

7 Which expression is equivalent to $\left(5\sqrt{2}-4\sqrt{3}\right)\left(5\sqrt{2}+4\sqrt{3}\right)$? Select **A**, **B**, **C** or **D**.

A $25\sqrt{2}-16\sqrt{3}$ **B** $10\sqrt{2}+10\sqrt{6}$ **C** 2 **D** 26

8 Expand and simplify each expression.

a $\left(3\sqrt{7}-5\right)^2$ **b** $\left(5\sqrt{2}-4\right)\left(\sqrt{2}+5\right)$

c $\left(2\sqrt{7}+3\sqrt{5}\right)\left(\sqrt{5}+\sqrt{7}\right)$ **d** $\left(4\sqrt{3}+5\right)^2$

e $\left(4\sqrt{2}+\sqrt{3}\right)\left(4\sqrt{2}-\sqrt{3}\right)$ **f** $\left(3\sqrt{10}-\sqrt{2}\right)^2$

13-06 Rationalising the denominator

If $\sqrt{2}\approx 1.4142$, what is the value of $\frac{3}{\sqrt{2}}$? Fractions containing surds in the denominator are difficult to work with. When approximating the value of $\frac{3}{\sqrt{2}}$, it is difficult to mentally divide by 1.4142. We can overcome this by making the denominator **rational** (that is, **not** a surd).

Surds of the form $\frac{1}{\sqrt{5}}, \frac{3}{2\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{2}}, \frac{5\sqrt{7}}{\sqrt{3}}, \ldots$ have denominators that are irrational. These expressions may be rewritten with a rational denominator by multiplying both the numerator and denominator by the surd that appears in the denominator. This method is called **rationalising the denominator**.

Example 12

Rationalise the denominator of each surd.

a $\frac{3}{\sqrt{2}}$ **b** $\frac{5}{4\sqrt{3}}$ **c** $\frac{8\sqrt{2}}{3\sqrt{5}}$

Solution

a $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ because $\frac{\sqrt{2}}{\sqrt{2}} = 1$

$= \frac{3\sqrt{2}}{2}$

Because $\frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$, it is easier to approximate $\frac{3}{\sqrt{2}}$ by mentally multiplying $\frac{3}{2}$ by 1.4142 than by dividing 3 by 1.4142.

b $\frac{5}{4\sqrt{3}} = \frac{5}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{3}}{4 \times 3}$

$= \frac{5\sqrt{3}}{12}$

c $\frac{8\sqrt{2}}{3\sqrt{5}} = \frac{8\sqrt{2}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{8\sqrt{10}}{3 \times 5}$

$= \frac{8\sqrt{10}}{15}$

Exercise 13-06 Rationalising the denominator

See Example 12

1 By rationalising the denominator, which surd is equivalent to $\frac{2}{\sqrt{6}}$? Select the correct answer **A**, **B**, **C** or **D**.

A $2\sqrt{6}$ **B** $\frac{\sqrt{6}}{3}$ **C** $\frac{\sqrt{6}}{6}$ **D** $\frac{2\sqrt{6}}{3}$

2 Rationalise the denominator of each surd.

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{7}}$ **c** $\frac{1}{\sqrt{3}}$ **d** $\frac{3}{\sqrt{2}}$

e $\frac{2}{\sqrt{7}}$ **f** $\frac{1}{3\sqrt{2}}$ **g** $\frac{1}{2\sqrt{3}}$ **h** $\frac{1}{4\sqrt{7}}$

i $\frac{7}{3\sqrt{5}}$ **j** $\frac{\sqrt{2}}{3\sqrt{5}}$ **k** $\frac{3\sqrt{2}}{2\sqrt{6}}$ **l** $\frac{5\sqrt{3}}{4\sqrt{5}}$

3 Which surd is equivalent to $\frac{\sqrt{3}}{2\sqrt{5}}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{15}}{10}$ **B** $2\sqrt{15}$ **C** $\frac{\sqrt{15}}{3}$ **D** $\sqrt{5}$

4 Which surd is equivalent to $\frac{\sqrt{27}}{3\sqrt{18}}$? Select **A**, **B**, **C** or **D**.

A $\frac{1}{2}$ **B** $\frac{\sqrt{2}}{2}$ **C** $\frac{\sqrt{5}}{6}$ **D** $\frac{\sqrt{6}}{6}$

14

Number and Algebra

Quadratic equations and the parabola

Optional Stage 5.3 topic

A farmer wants to make two rectangular paddocks with 5000 metres of fencing. The length has to be twice the width and he wants the greatest area for his paddocks. What dimensions should the paddocks have? Problems like this can be solved using quadratic equations.

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Chapter outline

		Proficiency strands				
14-01	Quadratic equations	U	F		R	C
14-02	Completing the square	U	F		R	C
14-03	The quadratic formula	U	F		R	C
14-04	Higher-order quadratic equations	U	F		R	C
14-05	Quadratic equation problems	U	F	PS		C
14-06	The parabola $y = ax^2 + bx + c$	U	F		R	C
14-07	The axis of symmetry and vertex of a parabola	U	F		R	C
14-08	Point of intersection of a line and a curve	U	F		R	C

Wordbank

coefficient The number in front of a variable; for example, in $y = 3x^2 + 4x - 6$, the coefficient of x^2 is 3

quadratic equation An equation in which the highest power of the variable is 2; for example, $2x^2 - 12x + 10 = 0$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $3x^2 + 4x - 6$

quadratic formula A formula for solving quadratic equations of the form $ax^2 + bx + c = 0$

solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true

***x*-intercept** The x-value(s) at which a graph intersects the x-axis

In this chapter you will:

- (STAGE 5.3) factorise monic and non-monic quadratic expressions and solve a wide range of quadratic equations derived from a variety of contexts
- (STAGE 5.3) solve quadratic equations of the form $ax^2 + bx + c = 0$ by factorising, completing the square and using the quadratic formula
- (STAGE 5.3) simplify and solve higher-order quadratic equations such as $x^4 - 2x^2 - 8 = 0$
- (STAGE 5.3) solve problems involving quadratic equations
- (STAGE 5.3) graph parabolas of the form $y = ax^2 + bx + c$ and identify their features
- (STAGE 5.3) find the axis of symmetry and the vertex of a parabola
- (STAGE 5.3) find the point of intersection of a line with a parabola, circle or hyperbola by solving linear and non-linear simultaneous equations

SkillCheck

Worksheet
StartUp assignment 14
MAT10NAWK10096

1 Solve each quadratic equation.

a $4x^2 = 100$ **b** $3m^2 - 12 = 0$ **c** $x^2 - x - 2 = 0$
d $u^2 + 3u - 28 = 0$ **e** $k^2 + 3k = 0$ **f** $w^2 - 10w + 25 = 0$

2 Factorise each expression.

a $16 - m^2$ **b** $d^2 - 121$ **c** $14y - 2y^2$
d $10p^2 + 25p$ **e** $5x^2 - 320$ **f** $18w^2 - 50$
g $k^2 + 5k + 4$ **h** $y^2 - 10y + 16$ **i** $m^2 - m - 56$
j $u^2 + 8u - 65$ **k** $w^2 - 10w + 21$ **l** $x^2 - 2x - 24$

3 If $y = x^2 + 4x - 7$, find the value of y if:

a $x = 1$ **b** $x = -1$ **c** $x = 2$ **d** $x = -3$

4 Factorise each expression.

a $3a^2 + 10a + 3$ **b** $5x^2 - 13x - 6$ **c** $6y^2 + y - 40$
d $15t^2 + 7t - 4$ **e** $5v^2 - 32v - 21$ **f** $8y^2 + 34y + 35$
g $15h^2 - 23h + 4$ **h** $12p^2 + 11p - 15$ **i** $16d^2 + 40d + 25$

Stage 5.3

Puzzle sheet
Factorising quadratic equations
MAT10NAPS0037

14-01 Quadratic equations

In Chapter 6, Equations and inequalities, we learnt that in a **quadratic equation**, the highest power of the variable is 2; for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4y^2 - 3y = 8$.

The general form of a quadratic equation is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b and c are constants (numbers).

Solving a quadratic equation means finding those values of the variable that satisfy the equation (or make the statement true). When checking solutions, substitute the possible solutions into the equation and show that the left-hand side (LHS) of the equation is equal to the right-hand side (RHS) of the equation.

Stage 5.3

Solving $ax^2 + bx + c = 0$ by factorising

To solve quadratic equations of the form $ax^2 + bx + c = 0$, we need to factorise the quadratic expression on the LHS, which we learnt in Chapter 12, Products and factors.

Summary

When solving quadratic equations by factorising, the following property is used.
If $pq = 0$, then $\boldsymbol{p = 0}$ or $\boldsymbol{q = 0}$.

Example 1

Solve each quadratic equation.

a $4w(3w + 2) = 0$ **b** $(5m + 2)(2m - 7) = 0$

Solution

a $4w(3w + 2) = 0$

$$\begin{array}{rcr} \therefore 4w = 0 & \text{or} & 3w + 2 = 0 \\ w = 0 & \text{or} & 3w = -2 \\ w = 0 & \text{or} & w = -\dfrac{2}{3} \end{array}$$

b $(5m + 2)(2m - 7) = 0$

$$\begin{array}{rcr} 5m + 2 = 0 & \text{or} & 2m - 7 = 0 \\ 5m = -2 & \text{or} & 2m = 7 \\ m = -\dfrac{2}{5} & \text{or} & m = \dfrac{7}{2} \\ m = -\dfrac{2}{5} & \text{or} & m = 3\dfrac{1}{2} \end{array}$$

Example 2

Video tutorial
Quadratic equations by factorising
MAT10NAVT10029

Solve each quadratic equation. Use substitution to check your solutions.

a $6p^2 - 9p = 0$ **b** $2x^2 - x - 15 = 0$ **c** $6y^2 = -7y + 5$

Solution

a $6p^2 - 9p = 0$

$3p(2p - 3) = 0$ Factorising

$$\begin{array}{rcr} 3p = 0 & \text{or} & 2p - 3 = 0 \\ p = 0 & \text{or} & 2p = 3 \\ p = 0 & \text{or} & p = \dfrac{3}{2} \\ p = 0 & \text{or} & p = 1\dfrac{1}{2} \end{array}$$

Check: When $p = 0$, LHS $= 6 \times 0^2 - 9 \times 0 = 0 =$ RHS

When $p = 1\frac{1}{2}$, LHS $= 6 \times \left(1\frac{1}{2}\right)^2 - 9 \times 1\frac{1}{2} = 0 =$ RHS

b

$$\begin{aligned} 2x^2 - x - 15 &= 0 \\ 2x^2 - 6x + 5x - 15 &= 0 \\ 2x(x-3) + 5(x-3) &= 0 \\ (x-3)(2x+5) &= 0 \end{aligned}$$

$$\begin{array}{lcl} x - 3 = 0 & \text{or} & 2x + 5 = 0 \\ x = 3 & \text{or} & 2x = -5 \\ x = 3 & \text{or} & x = -\frac{5}{2} \\ x = 3 & \text{or} & x = -2\frac{1}{2} \end{array}$$

Check: When $x = 3$, LHS $= 2 \times 3^2 - 3 - 15 = 0 =$ RHS

When $x = -2\frac{1}{2}$, LHS $= 2 \times \left(-2\frac{1}{2}\right)^2 - \left(-2\frac{1}{2}\right) - 15 = 0 =$ RHS

c

$$\begin{aligned} 6y^2 &= -7y + 5 \\ 6y^2 + 7y - 5 &= 0 \\ 6y^2 - 3y + 10y - 5 &= 0 \\ 3y(2y-1) + 5(2y-1) &= 0 \\ (2y-1)(3y+5) &= 0 \end{aligned}$$

$$\begin{array}{lcl} 2y - 1 = 0 & \text{or} & 3y + 5 = 0 \\ 2y = 1 & \text{or} & 3y = -5 \\ y = \frac{1}{2} & \text{or} & y = -\frac{5}{3} \\ y = \frac{1}{2} & \text{or} & y = -1\frac{2}{3} \end{array}$$

Check: When $y = \frac{1}{2}$:

LHS $= 6 \times \left(\frac{1}{2}\right)^2 = 1\frac{1}{2}$

RHS $= -7 \times \left(\frac{1}{2}\right) + 5 = 1\frac{1}{2}$

LHS = RHS

When $y = -1\frac{2}{3}$:

LHS $= 6 \times \left(-1\frac{2}{3}\right)^2 = 16\frac{2}{3}$

RHS $= -7 \times \left(-1\frac{2}{3}\right) + 5 = 16\frac{2}{3}$

LHS = RHS

Exercise 14-01 Quadratic equations

Stage 5.3

See Example 1

1 Solve each quadratic equation.

a $(m + 7)(m + 3) = 0$
b $(d - 3)(d - 7) = 0$
c $(y - 3)(y + 5) = 0$
d $k(k - 3) = 0$
e $t(t + 7) = 0$
f $2p(p - 3) = 0$
g $w(3w - 2) = 0$
h $(2n + 1)(n - 3) = 0$
i $(5a - 3)(2a - 1) = 0$
j $(3x + 1)(2x + 3) = 0$
k $(2c - 5)^2 = 0$
l $(2f - 1)^2 = 0$
m $(3c + 1)(4c + 1) = 0$
n $(1 - 2h)(h + 1) = 0$
o $(5 - 7e)(1 - e) = 0$

See Example 2

2 Solve each quadratic equation.

a $2y^2 + 7y + 6 = 0$
b $2g^2 + 5g + 3 = 0$
c $3d^2 + 5d + 2 = 0$
d $5t^2 + 16t + 11 = 0$
e $2p^2 - 11p + 12 = 0$
f $10x^2 - 19x + 6 = 0$
g $8y^2 - 2y - 3 = 0$
h $6a^2 - 5a - 4 = 0$
i $4w^2 - 7w - 15 = 0$
j $5c^2 + 2c - 7 = 0$
k $8e^2 + 10e - 3 = 0$
l $3q^2 + 4q - 15 = 0$
m $4g^2 - 20g + 25 = 0$
n $18m^2 - 3m - 10 = 0$
o $16 - 8w - 3w^2 = 0$
p $36 + 3y - 3y^2 = 0$
q $2f^2 - 24f + 72 = 0$
r $12h^2 + 3h - 9 = 0$

3 Express each quadratic equation in the form $ax^2 + bx + c = 0$ and solve.

a $2x^2 = x + 15$
b $4t(t + 2) = 5$
c $41u = -8u^2 - 5$
d $7m^2 = 8m - 1$
e $p(p - 3) = 28$
f $(e - 2)^2 = 9$
g $t(2t - 13) = -15$
h $7 = 6d^2 + 11d$
i $5h^2 = 125$
j $8f^2 = 4f$
k $6w^2 + 3 = 19w$
l $4a(3a + 5) - 8 = 0$

4 A certain positive number, plus its square, minus 72, equals zero. Find the number.

14-02 Completing the square

Puzzle sheet

Complete the square order activity

MAT10NAPS10097

Quadratic equations can be solved by using a method called **completing the square**. We try to make the LHS of the equation a perfect square.
The method of **completing the square** is based on the following results for perfect squares.

$$(x + a)^2 = x^2 + 2ax + a^2$$
$$(x - a)^2 = x^2 - 2ax + a^2$$

We note that the last term, a^2, is the square of 'half the coefficient of x'.

Example 3

Find the numbers that complete the square in each equation.

a $x^2 + 10x + \ldots = (x + \ldots)^2$

b $x^2 - 14x + \ldots = (x - \ldots)^2$

Solution

a The coefficient of x is 10.
Half of 10 is 5, and $5^2 = 25$.
The perfect square is
$x^2 + 10x + 25 = (x + 5)^2$

b The coefficient of x is -14.
Half of -14 is -7, and $(-7)^2 = 49$.
The perfect square is
$x^2 - 14x + 49 = (x - 7)^2$

Stage 5.3

Video tutorial

Simple quadratic equations

MAT10NAVT10028

Example 4

Solve $(k+3)^2 = 7$.

Solution

$(k+3)^2 = 7$

$k+3 = \pm\sqrt{7}$ Taking the square root of both sides.

$k+3-3 = \sqrt{7}-3$ **or** $k+3-3 = -\sqrt{7}-3$

This solution is usually written as:

$k = -3+\sqrt{7}$ **or** $k = -3-\sqrt{7}$

Notice that the two solutions are surds (not rational), of the form $a+\sqrt{b}$ and $a-\sqrt{b}$.

Example 5

Solve $x^2+6x+7=0$ by completing the square.

Solution

Step 1

Move the constant term to the RHS.

$x^2+6x = -7$

Step 2

Halve the coefficient of x, square it and then add the square to both sides.

$x^2+6x+3^2 = -7+3^2$

$x^2+6x+9 = 2$

Step 3

Express the LHS as a perfect square.

$(x+3)^2 = 2$

Step 4

Solve the resulting equation.

$x+3 = \pm\sqrt{2}$ Taking the square root of both sides.

$x = -3\pm\sqrt{2}$

$x = -3+\sqrt{2}$ or $x = -3-\sqrt{2}$

This answer in surd form is called the '**exact answer**'.

Stage 5.3

Example 6

Solve $2x^2 - 3x - 4 = 0$, writing the solution correct to two decimal places.

Solution

$$2x^2 - 3x - 4 = 0$$

$$2x^2 - 3x = 4$$

Moving the constant term to the RHS.

$$x^2 - \frac{3}{2}x = 2$$

Dividing both sides by 2, the coefficient of x^2.

$$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = 2 + \left(-\frac{3}{4}\right)^2$$

Completing the square.

$$\left(x - \frac{3}{4}\right)^2 = \frac{41}{16}$$

Expressing the LHS as a perfect square and leaving the RHS as an improper fraction.

$$x - \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$$

Taking the square root of both sides.

$$= \pm\frac{\sqrt{41}}{4}$$

Simplifying the RHS.

$$x = \frac{3}{4} \pm \frac{\sqrt{41}}{4}$$

This is the exact answer in surd form.

$$= \frac{3 \pm \sqrt{41}}{4}$$

$$x = \frac{3 + \sqrt{41}}{4} \quad \text{or} \quad x = \frac{3 - \sqrt{41}}{4}$$

$$x = 2.35078\ldots \quad \text{or} \quad x = -0.85078\ldots$$

$$x \approx 2.35 \quad \text{or} \quad x \approx -0.85$$

This is an approximate answer in decimal form.

Exercise 14-02 Completing the square

See Example 3

1 Find the numbers that ‘complete the square’ in each equation.

a $x^2 + 2x + \_\_\_ = (x + \_\_\_)^2$

b $p^2 - 6p + \_\_\_ = (p - \_\_\_)^2$

c $m^2 - 8m + \_\_\_ = (m - \_\_\_)^2$

d $k^2 + 4k + \_\_\_ = (k + \_\_\_)^2$

e $y^2 - 7y + \_\_\_ = (y - \_\_\_)^2$

f $w^2 - 3w + \_\_\_ = (w - \_\_\_)$

g $x^2 + x + \_\_\_ = (x + \_\_\_)^2$

h $h^2 - 5h + \_\_\_ = (h - \_\_\_)^2$

i $a^2 + \frac{7}{2}a + \_\_\_ = (a + \_\_\_)^2$

j $v^2 + \frac{5}{3}v + \_\_\_ = (v + \_\_\_)^2$

Stage 5.3

See Example 4

2 Solve each equation, writing the solution in surd form.

a $(d+3)^2 = 7$ **b** $(x-5)^2 = 5$ **c** $(p+1)^2 = 10$

d $(y-1)^2 = 2$ **e** $\left(m-\frac{1}{2}\right)^2 = 5$ **f** $\left(t+\frac{2}{3}\right)^2 = 3$

g $(c+1)^2 = \frac{21}{2}$ **h** $(w-3)^2 = \frac{41}{2}$ **i** $\left(n+\frac{2}{3}\right)^2 = \frac{7}{9}$

j $\left(e-\frac{3}{2}\right)^2 = \frac{71}{4}$ **k** $(d-2)^2 = 5$ **l** $\left(x-\frac{3}{4}\right)^2 = 2$

See Example 5

3 Solve each equation by completing the square. Leave your answers in exact form.

a $h^2 + 2h - 5 = 0$ **b** $r^2 - 2r - 1 = 0$ **c** $m^2 + 6m + 2 = 0$

d $w^2 - 4w - 60 = 0$ **e** $a^2 - 10a - 5 = 0$ **f** $y^2 + 8y - 3 = 0$

g $p^2 + 12p - 5 = 0$ **h** $x^2 - 4x + 2 = 0$ **i** $u^2 + 9u + 14 = 0$

j $d^2 + d - 7 = 0$ **k** $c^2 - 9c + 2 = 0$ **l** $e^2 + 5e + 2 = 0$

m $y^2 - 3y - 8 = 0$ **n** $b^2 - b - 5 = 0$ **o** $q^2 - 3q + 1 = 0$

p $2g^2 + 7g - 3 = 0$ **q** $2x^2 + 5x - 7 = 0$ **r** $3f^2 + 4f - 6 = 0$

See Example 6

4 Solve each of these by completing the square. Give your answers correct to two decimal places.

a $x^2 + 12x + 9 = 0$ **b** $m^2 - 16m - 7 = 0$ **c** $g^2 + 4g - 3 = 0$

d $2h^2 + 3h - 7 = 0$ **e** $5w^2 - 4w - 3 = 0$ **f** $3y^2 + y - 5 = 0$

g $3p^2 + 2p - 8 = 0$ **h** $4e^2 - e - 4 = 0$ **i** $2n^2 + 3n - 5 = 0$

Worksheet
Investigating quadratic equations
MAT10NAWK10098

Puzzle sheet
Quadratic equations puzzle
MAT10NAPS10099

Technology worksheet
Excel worksheet: The quadratic formula
MAT10NACT00018

Technology worksheet
Excel spreadsheet: The quadratic formula
MAT10NACT00048

Puzzle sheet
The quadratic formula
MAT10NAPS0038

14-03 The quadratic formula

There is a formula for solving a quadratic equation of the form $ax^2 + bx + c = 0$ that involves the coefficients a, b and c.

Alamy/Robert Harding World Imagery

Stage 5.3

Summary

The **quadratic formula** for the solution to the quadratic equation $ax^2 + bx + c = 0$ is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 7

Solve each quadratic equation using the quadratic formula.

a $x^2 - 3x + 2 = 0$ **b** $6x^2 + x - 2 = 0$ **c** $3x^2 + 11x + 2 = 0$

Video tutorial
The quadratic formula
MAT10NAVT10030

Solution

a For $x^2 - 3x + 2 = 0$:

$a = 1$, $b = -3$, and $c = 2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 1 \times 2}}{2 \times 1} \\ &= \frac{3 \pm \sqrt{1}}{2} \\ &= \frac{3 \pm 1}{2} \end{aligned}$$

$x = \frac{3+1}{2}$ or $x = \frac{3-1}{2}$

$x = 2$ or $x = 1$

b For $6x^2 + x - 2 = 0$:

$a = 6$, $b = 1$, and $c = -2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4 \times 6 \times (-2)}}{2 \times 6} \\ &= \frac{-1 \pm \sqrt{49}}{12} \\ &= \frac{-1 \pm 7}{12} \end{aligned}$$

$x = \frac{-1+7}{12}$ or $x = \frac{-1-7}{12}$

$x = \frac{6}{12}$ or $x = \frac{-8}{12}$

$x = \frac{1}{2}$ or $x = -\frac{2}{3}$

c For $3x^2 + 11x + 2 = 0$:

$a = 3$, $b = 11$ and $c = 2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times 2}}{2 \times 3} \\ &= \frac{-11 \pm \sqrt{97}}{6} \end{aligned}$$

$x = \frac{-11 + \sqrt{97}}{6}$ or $x = \frac{-11 - \sqrt{97}}{6}$ In exact surd form.

$x = 0.1918 \ldots$ or $x = -3.4748\ldots$

$x \approx 0.19$ or $x \approx -3.47$ Rounded correct to two decimal places.

Stage 5.3

Example 8

Solve $2x^2 - 6 = 9x$ using the quadratic formula, expressing the answer correct to two decimal places.

Solution

$$2x^2 - 6 = 9x$$

$$2x^2 - 9x - 6 = 0$$

Writing the quadratic equation in the form $ax^2 + bx + c = 0$

Substitute $a = 2$, $b = -9$, $c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times (-6)}}{2 \times 2}$$

$$= \frac{9 \pm \sqrt{129}}{4}$$

$$x = \frac{9 + \sqrt{129}}{4} \quad \text{or} \quad x = \frac{9 - \sqrt{129}}{4}$$

$$x = 5.0894\ldots \quad \text{or} \quad x = -0.5894\ldots$$

$$x \approx 5.09 \quad \text{or} \quad x \approx -0.59$$

Exercise 14-03 The quadratic formula

See Example 7

1 Solve each quadratic equation using the quadratic formula. Write each solution in exact form.

a $x^2 + 6x + 2 = 0$
b $m^2 - 5m - 3 = 0$
c $w^2 - 8w + 3 = 0$
d $k^2 + 3k - 5 = 0$
e $y^2 - 4y - 1 = 0$
f $p^2 + p - 5 = 0$
g $u^2 - 7u - 3 = 0$
h $2a^2 + 3a - 7 = 0$
i $5q^2 - 6q + 1 = 0$
j $3c^2 + 2c - 2 = 0$
k $4e^2 - 5e - 2 = 0$
l $3x^2 + 8x + 2 = 0$
m $2d^2 - 4d - 5 = 0$
n $3a^2 - 10a - 2 = 0$
o $2t^2 + 3t - 5 = 0$
p $3y^2 + 8y + 4 = 0$
q $6k^2 - 11k + 5 = 0$
r $2n^2 - 5n - 11 = 0$

See Example 8

2 Solve each quadratic equation using the quadratic formula, expressing the solution as a surd.

a $5y^2 - 9y = 3$
b $3m^2 = 7 - 2m$
c $4x^2 = 3x + 2$
d $1 - 4k - k^2 = 0$
e $3m^2 - 1 - 3m = 0$
f $1 - 2g - 5g^2 = 0$
g $8 = 9h - 2h^2$
h $2w + 2 = 3w^2$
i $4p - 3p^2 = -1$
j $2 - 4u - 5u^2 = 0$
k $6a^2 = 9 - 4a$
l $10 = 3y + 2y^2$

3 Solve each quadratic equation, writing the solutions correct to two decimal places.

a $k^2 - 9k + 1 = 0$
b $c^2 - 2 = 0$
c $m^2 - 5 = 2$
d $2n^2 + 2 = 7n$
e $2p^2 + 3p - 4 = 0$
f $6w^2 + 5w - 2 = 0$
g $3x^2 + 2 - 8x = 0$
h $h^2 = 7 + 2h$
i $1 + x - x^2 = 0$
j $36 = 13a - a^2$
k $5v^2 - 11 = 0$
l $5c^2 + 8 = 15c$
m $t^2 = 5(t + 5)$
n $(x - 6)^2 = 3$
o $12 = 2d^2 - 3d$

14-04 Higher-order quadratic equations

Higher-order equations are equations where the highest power of the variable is greater than 2. Equations such as $x^4 - 6x^2 + 8 = 0$, $2y^6 + 15y^3 - 8 = 0$ are examples of higher order 'quadratic' equations. Substitution can be used so that the equation is 'simplified' or reduced to a quadratic equation.

Example 9

Solve each quadratic equation.

a $x^4 - 11x^2 + 18 = 0$

b $y^4 + 3y^2 - 4 = 0$

Solution

a This is a quadratic equation in x^2.
Let $u = x^2$.

$\therefore x^4 - 11x^2 + 18 = 0$ becomes

$$u^2 - 11u + 18 = 0$$
$$(u - 9)(u - 2) = 0$$

$$u - 9 = 0 \quad \text{or} \quad u - 2 = 0$$
$$u = 9 \quad \text{or} \quad u = 2$$

But $u = x^2$

$$\therefore x^2 = 9 \quad \text{or} \quad x^2 = 2$$
$$x = \pm 3 \quad \text{or} \quad x = \pm\sqrt{2}$$

The solutions of the equation are:

That is $x = \sqrt{2}, -\sqrt{2}, 3$ or -3.

$$x = \pm\sqrt{2} \quad \text{or} \quad x = \pm 3$$

This quadratic equation has 4 solutions

b $y^4 + 3y^2 - 4 = 0$ is a quadratic equation in y^2.

Let $u = y^2$

$\therefore y^4 + 3y^2 - 4 = 0$ becomes

$$u^2 + 3u - 4 = 0$$
$$(u + 4)(u - 1) = 0$$

$$\therefore u + 4 = 0 \quad \text{or} \quad u - 1 = 0$$
$$u = -4 \quad \text{or} \quad u = 1$$

But $u = y^2$

$$y^2 = -4 \quad \text{or} \quad y^2 = 1$$

which has no solution or $y = \pm 1$

$\therefore$ The solutions of the equation are $y = \pm 1$.

This quadratic equation has 2 solutions

Stage 5.3

Exercise 14-04 Higher-order quadratic equations

See Example 9

1 Use the substitution given in the brackets to solve each quadratic equation.

a $y^4 - 12y^2 + 32 = 0$ [let $u = y^2$]
b $m^4 - 13m^2 - 48 = 0$ [let $u = m^2$]
c $w^4 - 8w^2 + 12 = 0$ [let $u = w^2$]
d $m^6 - 9m^3 + 8 = 0$ [let $u = m^3$]
e $k^6 - 26k^3 - 27 = 0$ [let $u = k^3$]
f $w^6 - 4w^3 - 32 = 0$ [let $u = w^3$]
g $(3x + 5)^2 - 8(3x + 5) + 12 = 0$ [let $u = 3x + 5$]
h $(5y - 1)^2 - (5y - 1) - 30 = 0$ [let $u = 5y - 1$]
i $(2a + 7)^2 - 10(2a + 7) + 24 = 0$ [let $u = 2a + 7$]
j $p^6 + 6p^3 + 5 = 0$ [let $u = p^3$]
k $8g^6 - 65g^3 + 8 = 0$ [let $u = g^3$]
l $125c^6 + 999c^3 - 8 = 0$ [let $u = c^3$]

2 Solve each equation, and give your solutions correct to one decimal place.

a $2m^4 - 19m^2 + 24 = 0$ [let $u = m^2$]
b $5x^4 - 14x^2 + 8 = 0$ [let $u = x^2$]
c $7a^6 - 9a^3 - 10 = 0$ [let $u = a^3$]
d $2y^6 + y^3 - 10 = 0$ [let $u = y^3$]
e $3w^6 - 16w^3 - 35 = 0$ [let $u = w^3$]
f $12e^4 + 5e^2 - 2 = 0$ [let $u = e^2$]

3 Solve each equation, correct to one decimal place where necessary.

a $w^4 - 21w^2 - 100 = 0$ **b** $x^6 + 6x^3 - 16 = 0$ **c** $k^6 - 64 = 0$
d $m^4 - 4m^2 + 4 = 0$ **e** $2y^4 + 7y^2 - 15 = 0$ **f** $v^6 + 5v^3 - 24 = 0$

Worksheet
Problems involving quadratic equations
MAT10NAWK00029

14-05 Quadratic equation problems

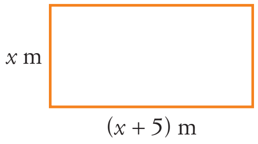
Example 10

A rectangular garden is 5 m longer than it is wide. The area of the rectangle is 84 m^2. Find the dimensions of the garden.

Solution

Let the width of the rectangular garden be x cm.
$\therefore$ The length of the garden is $(x + 5)$ cm.

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= x(x + 5)\ \text{m}^2 \end{aligned}$$



$$\begin{aligned}\therefore x(x+5) &= 84\\ x^2+5x &= 84\\ x^2+5x-84 &= 0\\ (x+12)(x-7) &= 0\end{aligned}$$

$$x+12=0 \quad \text{or} \quad x-7=0$$

$$\therefore x=-12 \quad \text{or} \quad x=7$$

Since x represents a measurement of width, $x \neq -12$.

$\therefore$ The width is 7 m and the length is $7 + 5 = 12$ m

Check: Area = 7 m × 12 m = 84 m^2

Example 11

A ball is thrown upwards and its height, h metres, after t seconds is given by the formula $h = 30t - 5t^2$. At what times did the ball reach a height of 24 m? Express the answer correct to two decimal places.

Science Photo Library/Dr Gary Settles

Solution

The equation is $h = 30t - 5t^2$

When $h = 24$,

$$24 = 30t - 5t^2$$

Rearranging the equation,

$$5t^2 - 30t + 24 = 0$$

Using the quadratic formula,

$$\begin{aligned}t &= \frac{-b \pm \sqrt{b^2-4ac}}{2a}\\ &= \frac{-(-30) \pm \sqrt{(-30)^2 - 4\times 5\times 24}}{2\times 5}\\ &= \frac{30 \pm \sqrt{420}}{10}\end{aligned}$$

$$t = \frac{30+\sqrt{420}}{10} \quad \text{or} \quad t = \frac{30-\sqrt{420}}{10}$$

$$t = 5.049\ldots \quad \text{or} \quad t = 0.950\ldots$$

$$t \approx 5.05 \quad \text{or} \quad t \approx 0.95$$

Why are there two answers to this problem? Why is the ball at a height of 24 m at two different times?

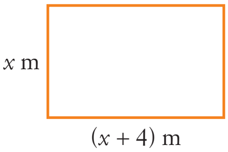
The ball reaches a height of 24 m after 0.95 s and after 5.05 s.

Stage 5.3

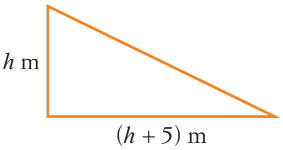
Exercise 14-05 Quadratic equation problems

See Example 10

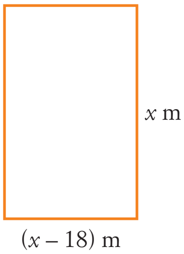
1 A garden is in the shape of a rectangle and its length is 4 m longer than its width. If the area of the garden is 96 m^2, find the dimensions of the garden.



2 A park is in the shape of a right-angled triangle, with its base 5 m longer than its perpendicular height. If the area of the park is 700 m^2, find the dimensions of the park.



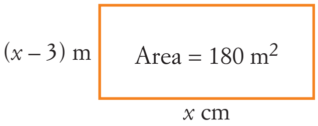
3 A rectangular block of land has its width 18 m shorter than its length. If the area of the block is 1008 m^2, find the dimensions of the block of land.



4 A block of land is in the shape of a rectangle with its width 20 m shorter than its length. If the area of the block is 2204 m^2, find the length and width of the block of land.

5 A rectangular garden bed is twice as long as it is wide. Its area is 84.5 m^2. Find the length of the garden bed.

6 Find the dimensions of this rectangle.



See Example 11

7 A golf ball is thrown upwards and its height, h metres, after t seconds is given by the formula $h = 18t - 5t^2$. At what times did the ball reach a height of 8 m? Answer correct to two decimal places.

8 After jumping from a plane, the height (in metres) of a skydiver above the ground is given by $h = 4000 - 5t^2$, where t is the time (in seconds) after jumping.

a How high was the plane at the moment the skydiver jumped?

b What was the skydiver's height after 20 seconds?

c The skydiver opened his parachute at 1000 m. How long did it take the skydiver to reach this height? (Give your answer correct to one decimal place.)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Stage 5.3

9 A ball is thrown from a balcony and its height (in metres) after t seconds is given by the formula $h = 30 + 12t - 5t^2$.

a What is the height of the ball after 2 seconds?

b When does the ball hit the ground? Answer correct to one decimal place.

c At what time (correct to one decimal place) is the ball at a height of:

i 35 m? **ii** 10 m?

10 The sum of a number and its square is 72. What is the number?

11 The product of two consecutive integers is 600. Find the integers.

12 When a number is subtracted from its square, the result is 1190. What is the number?

13 The difference between two positive integers is 12 and their product is 405. Find the integers.

14 The production costs, $\$C$, of a factory producing n toy boats each week is given by:

$C = 0.05n^2 - 12n + 2700$

Find:

a the cost of producing 500 toy boats

b the profit made on 500 toy boats if each one sells for \$29.50

c the (whole) number of toy boats that can be produced at a production cost of \$8150.

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Worksheet
Graphing parabolas 2
MAT10NAWK10100

Technology worksheet
Investigating parabolas 1
MAT10NACT00010

Technology worksheet
Excel spreadsheet: Investigating parabolas 1
MAT10NACT00040

Technology worksheet
GeoGebra: Parabola
MAT10NATC00007

14-06 The parabola $y = ax^2 + bx + c$

The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**. We have already graphed simple parabolas of the form $y = ax^2 + c$ in Chapter 7, Graphs. Now we will graph parabolas of the form $y = ax^2 + bx + c$.

Example 12

For each quadratic equation:

i complete a table of values and draw the graph

ii find the x-intercepts

iii find the y-intercept

iv solve $y = 0$ and compare the solutions to the x-intercepts.

a $y = x^2 + 6x + 5$ **b** $y = 2x^2 - x - 10$

Solution

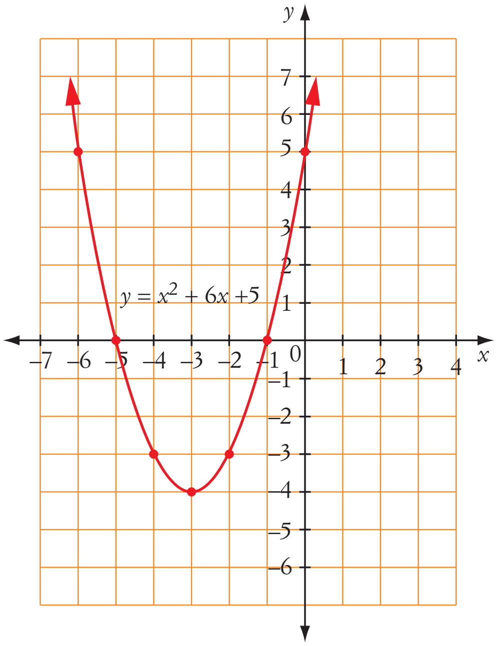
a **i** $y = x^2 + 6x + 5$

ii The x-intercepts are -5 and -1.

iii The y-intercept is 5.

Note that this is the constant term, $c = 5$, in $y = x^2 + 6x + 5$.

x	-6	-5	-4	-3	-2	-1	0	1
y	5	0	-3	-4	-3	0	5	12



iv $x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

$x + 5 = 0$ or $x + 1 = 0$

$x = -5$ or $x = -1$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = x^2 + 6x + 5$.

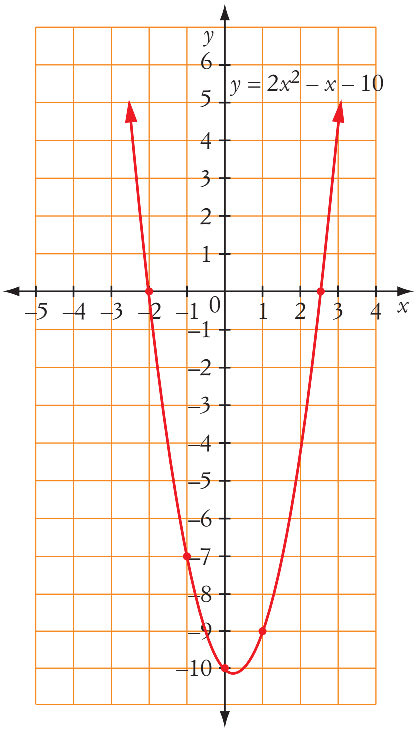
b **i** $y = 2x^2 - x - 10$

ii The x-intercepts are -2 and $2\frac{1}{2}$.

iii The y-intercept is -10.

Note that this is the constant term, $c = -10$, in $y = 2x^2 - x - 10$

x	-3	-2	-1	0	1	2	3
y	11	0	-7	-10	-7	0	5



iv $2x^2 - x - 10 = 0$

$(2x - 5)(x + 2) = 0$

$2x - 5 = 0$ or $x + 2 = 0$

$x = 2\frac{1}{2}$ or $x = -2$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = 2x^2 - x - 10$.

Stage 5.3

Summary

For the **graph of the parabola $y = ax^2 + bx + c$**:

- if $a > 0$ (positive), the parabola is **concave up**
- if $a < 0$ (negative), the parabola is **concave down**
- the **y-intercept** of the parabola is c
- the **x-intercepts** of the parabola are the solutions to the quadratic equation $ax^2 + bx + c = 0$

Example 13

Graph each quadratic equation, showing its x- and y-intercepts.

a $y = 2x(x - 5)$ **b** $y = 2x^2 + x - 6$ **c** $y = -2x^2 - 3x + 9$

Solution

a $y = x(2x - 5)$

$= 2x^2 - 5x$

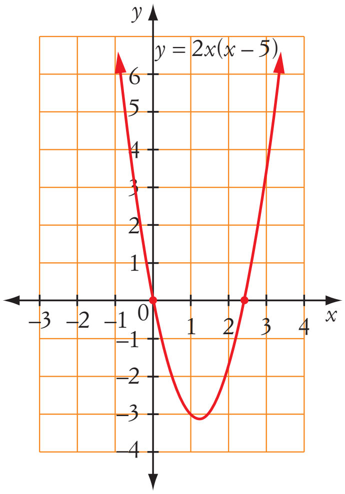
$a = 2 > 0$, so the parabola is concave up

x-intercepts: $x(2x - 5) = 0$

$x = 0$ and $2x - 5 = 0$

$x = 0$ and $x = 2\frac{1}{2}$

y-intercept: $c = 0$



b $y = 2x^2 + x - 6$

$a = 2 > 0$, so the parabola is concave up

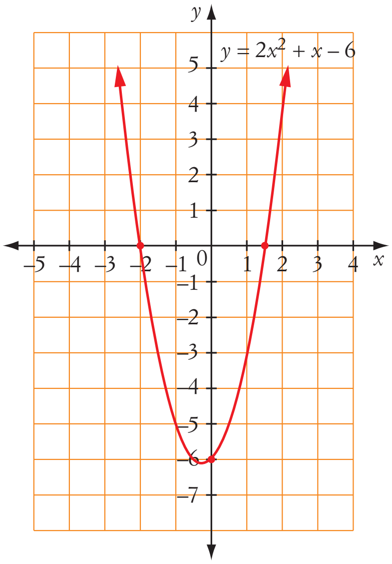
x-intercepts: $2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ and $x + 2 = 0$

$x = 1\frac{1}{2}$ and $x = -2$

y-intercept: $c = -6$



Stage 5.3

c $y = -2x^2 - 3x + 9$

$a = -2 < 0$, so the parabola is concave down

x-intercepts: $-2x^2 - 3x + 9 = 0$

$$2x^2 + 3x - 9 = 0$$

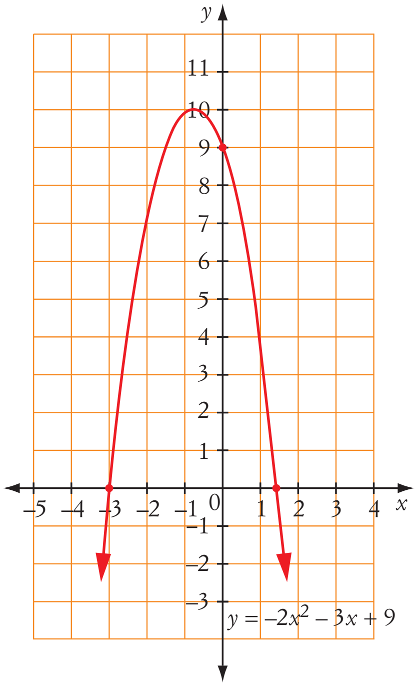
Dividing by –1

$$(2x - 3)(x + 3) = 0$$

$2x - 3 = 0$ and $x + 3 = 0$

$x = 1\frac{1}{2}$ and $x = -3$

y-intercept: $c = 9$



Exercise 14-06 The parabola $y = ax^2 + bx + c$

See Example 12

1 For each quadratic equation:

i complete a table of values and draw the graph

ii find the x-intercepts

iii find the y-intercept

iv solve $y = 0$ and compare the solutions to the x-intercepts

a $y = x^2 + 4x + 3$ **b** $y = 2x^2 + 7x + 3$

c $y = 2x^2 - 3x - 9$ **d** $y = x^2 - 2x$

2 Find the y-intercept of the parabolas with equation:

a $y = 3x^2 - 2x - 5$ **b** $y = 2x^2 + 6x + 3$ **c** $y = 5x^2 - 10x$

See Example 13

3 Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(x - 4)$ **b** $y = 3x(x + 2)$ **c** $y = (x + 3)(x - 5)$

d $y = (x - 2)(2x - 5)$ **e** $y = -(x + 1)(3x + 2)$ **f** $y = x^2 + 6x + 8$

g $y = -x^2 + 4x + 5$ **h** $y = 3x^2 + 11x - 20$ **i** $y = -2x^2 + 5x - 2$

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Stage 5.3

Investigation: The axis of symmetry and vertex of a parabola

1 **a** The graph of the parabola $y = x^2 - 6x + 8$ and its axis of symmetry have been drawn.

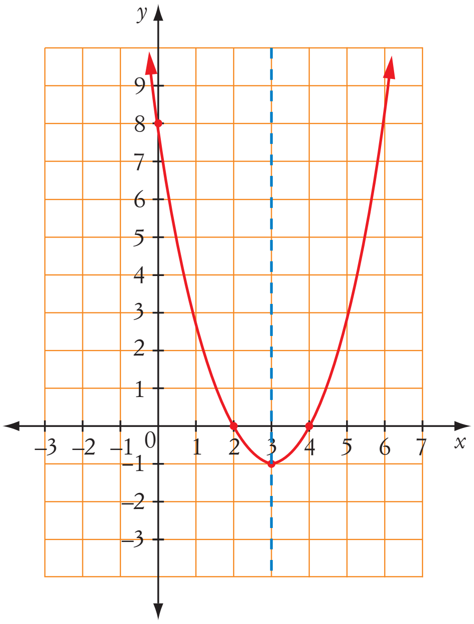
b Explain why the equation of the axis of symmetry is $x = 3$.

c **i** What are the x-intercepts of the parabola $y = x^2 - 6x + 8$?

ii What is the midpoint of the interval joining the x-intercepts?

d How can the x-intercepts be used to find the equation of the axis of symmetry? (Use your results from parts **b** and **c**.)

e Copy and complete. The equation of the axis of symmetry can be found by finding the ________ of the x-intercepts.



2 **a** For $y = x^2 - 6x + 8$, what is the value of:

i a (the coefficient of x^2)? **ii** b (the coefficient of x)?

b Find the value of $-\frac{b}{2a}$.

c How is the value of $-\frac{b}{2a}$ related to the equation of the axis of symmetry of the parabola $y = x^2 - 6x + 8$?

d Copy and complete. $x = -\frac{b}{2a}$ is the ____________ of the axis of ____________ of the parabola $y = ax^2 + bx + c$.

3 The vertex of a parabola is where the axis of symmetry intersects with the parabola.

a The equation of the axis of symmetry of $y = x^2 - 6x + 8$ is $x = 3$. The coordinates of the vertex are $(3, -1)$. Explain why the y-coordinate of the vertex is -1.

b The equation of the axis of symmetry of $y = -x^2 + 4x - 7$ is $x = 2$. Show that the coordinates of the vertex are $(2, -3)$.

Stage 5.3

Worksheet
Features of a parabola
MAT10NAWK10102

Worksheet
A page of parabolas
MAT10NAWK10103

Worksheet
Parabolas
MAT10NAWK00019

14-07 The axis of symmetry and vertex of a parabola

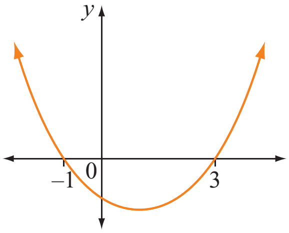
Summary

For the **graph of the parabola $y = ax^2 + bx + c$**:

- the equation of its **axis of symmetry** is $x = -\dfrac{b}{2a}$
- the **axis of symmetry** passes through the point halfway between the two x-intercepts of the parabola
- the **vertex** of the parabola lies on the axis of symmetry, so its **x-coordinate** is $x = -\dfrac{b}{2a}$ and its **y-coordinate** can be found by substituting the x-coordinate into the equation of the parabola

Example 14

Find the equation of the axis of symmetry for the parabola shown.



Solution

The x-intercepts are -1 and 3.
The equation of the axis of symmetry is:

$$x = \frac{-1+3}{2}$$
$$= 1$$

Finding the average of the x-intercepts.

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Example 15

For the parabola with equation $y = 2x^2 - 4x + 3$, find:

a the equation of its axis of symmetry

b the coordinates of the vertex of the parabola.

Solution

a For $y = 2x^2 - 4x + 3$, $a = 2$, $b = -4$ and $c = 3$.

The axis of symmetry is:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{-4}{2 \times 2} \\ &= 1 \end{aligned}$$

$x = 1$ is the equation of the axis of symmetry.

b The vertex lies on the axis of symmetry.

Substitute $x = 1$ in $y = 2x^2 - 4x + 3$

$$\begin{aligned} y &= 2 \times 1^2 - 4 \times 1 + 3 \\ &= 1 \end{aligned}$$

The vertex is (1, 1).

Example 16

Sketch the parabola $y = 2x^2 + 5x + 1$.

Solution

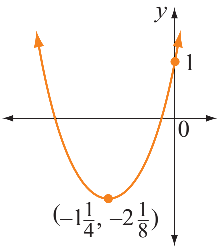
$a = 2 > 0$, so the parabola is concave up.

y-intercept $= 1$.

$y = 2x^2 + 5x + 1$ cannot be factorised, so we cannot find the x-intercepts precisely.

For the vertex:

$$\begin{aligned} x &= -\frac{b}{2a} \qquad \text{where } a = 2, \text{ and } b = 5 \\ &= -\frac{5}{2 \times 2} \\ &= -1\frac{1}{4} \end{aligned}$$



Substitute $x = -1\frac{1}{4}$ into $y = 2x^2 + 5x + 1$

$$\begin{aligned} y &= 2 \times \left(-1\frac{1}{4}\right)^2 + 5 \times \left(-1\frac{1}{4}\right) + 1 \\ &= -2\frac{1}{8} \end{aligned}$$

$\therefore$ The vertex has coordinates $\left(-1\frac{1}{4}, -2\frac{1}{8}\right)$.

Stage 5.3

Summary

To graph $y = ax^2 + bx + c$:

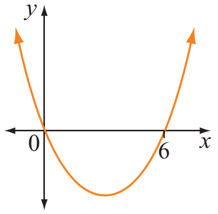
- use the sign of a (the coefficient of x^2) to determine whether the parabola is **concave up** or **down**
- find the **y-intercept**
- find the **x-intercepts** where possible by solving a quadratic equation
- use $x = -\dfrac{b}{2a}$ to find the **axis of symmetry** and the **vertex**

Exercise 14-07 The axis of symmetry and vertex of a parabola

See Example 14

1 Write the equation for the axis of symmetry in each parabola.

a



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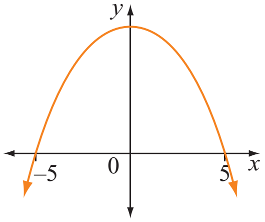
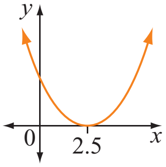
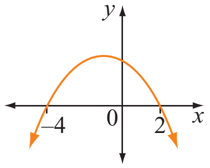


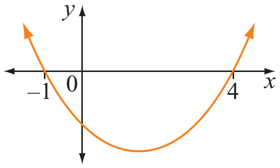
c



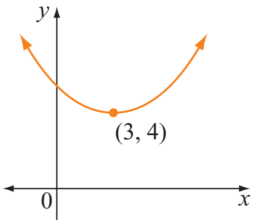
d



e



f



See Example 15

2 For each parabola whose equation is given below:

i find the equation for its axis of symmetry **ii** find the coordinates of its vertex

a $y = x^2 - 6x + 8$ **b** $y = -x^2 + 10x - 9$ **c** $y = x^2 - 2x + 10$

d $y = -x^2 + 8x + 9$ **e** $y = -x^2 + x - 25$ **f** $y = 5x^2 + 40x$

g $y = 24x - 3x^2$ **h** $y = 4x^2 + 2x - 1$ **i** $y = 1 - 3x - 9x^2$

See Example 16

3 Graph the parabola with each given equation, showing:

i the x-intercepts (correct to one decimal place where necessary)

ii the y-intercept

iii the equation of the axis of symmetry

iv the coordinates of the vertex

v the concavity

a $y = x^2 - 6x - 40$ **b** $y = x^2 - 3x$ **c** $y = 2x^2 + 3x + 4$

d $y = -x^2 + 6x + 5$ **e** $y = -4x^2 - 12x + 21$ **f** $y = x^2 - 8x + 3$

g $y = 5x^2 + 7x + 4$ **h** $y = 8x - 2x^2$ **i** $y = -2x^2 + 7x - 6$

14-08 Point of intersection of a line and a curve

The points of intersection of a line with a parabola, circle or hyperbola may be found either graphically or algebraically. The algebraic method involves solving simultaneous equations using the substitution method.

Example 17

Find the points of intersection of:

a the line $y = 2x - 3$ and the parabola $y = x^2 - 3x + 1$

b the line $y = x + 2$ and the circle $x^2 + y^2 = 4$

c the line $y = x + 5$ and the hyperbola $y = \frac{6}{x}$

Solution

a
$$y = 2x - 3 \quad [1]$$
$$y = x^2 - 3x + 1 \quad [2]$$

Use [2] to substitute for y in [1].

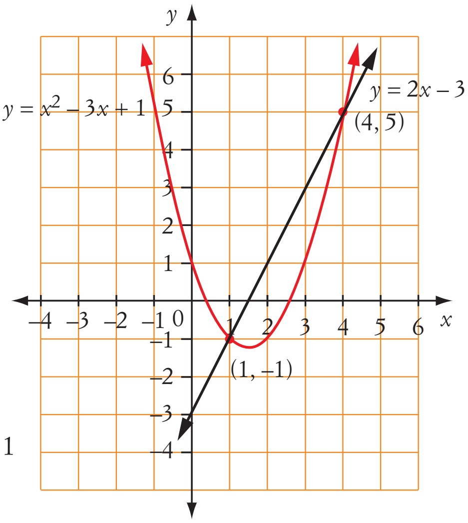
$$x^2 - 3x + 1 = 2x - 3$$
$$x^2 - 5x + 4 = 0$$
$$(x - 4)(x - 1) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 4 \quad \text{or} \quad x = 1$$

Substitute $x = 4$ and $x = 1$ into [1] to find y.

$$y = 2 \times 4 - 3 \quad \text{and} \quad y = 2 \times 1 - 3$$
$$= 5 \qquad\qquad = -1$$

The solutions are $x = 4$, $y = 5$ and $x = 1$, $y = -1$

The points of intersection are (4, 5) and (1, −1).



b
$$y = x + 2 \quad [1]$$
$$x^2 + y^2 = 4 \quad [2]$$

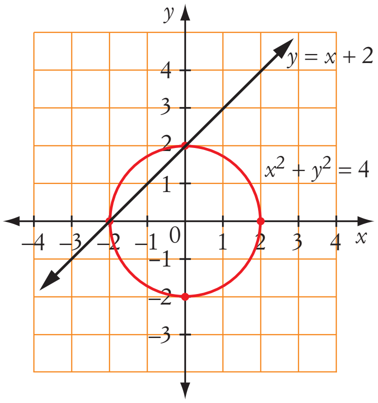
Use [1] to substitute for y in [2].

$$x^2 + (x + 2)^2 = 4$$
$$x^2 + x^2 + 4x + 4 = 4$$
$$2x^2 + 4x = 0$$
$$2x(x + 2) = 0$$
$$2x = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 0 \quad \text{or} \quad x = -2$$

Substitute $x = 0$ and $x = -2$ into [1] to find y.

$$y = 0 + 2 \quad \text{and} \quad y = -2 + 2$$
$$= 2 \qquad\qquad = 0$$

The points of intersection are (0, 2) and (−2, 0).



Stage 5.3

c $y = x + 5$ [1]

$y = \frac{6}{x}$ [2]

Use [1] to substitute for y in [2].

$$x + 5 = \frac{6}{x}$$

$$x(x + 5) = x \times \frac{6}{x}$$

$$x^2 + 5x = 6$$

$$x^2 + 5x - 6 = 0$$

$$(x + 6)(x - 1) = 0$$

$x + 6 = 0$ or $x - 1 = 0$

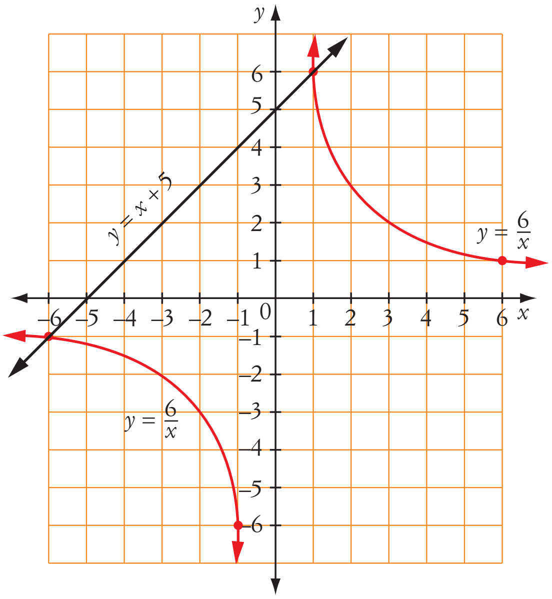
$x = -6$ or $x = 1$

Substitute $x = -6$ and $x = 1$ into [1] to find y.

$y = -6 + 5$ and $y = 1 + 5$

$= -1$ $= 6$

The points of intersection are $(-6, -1)$ and $(1, 6)$.



Exercise 14-08 Point of intersection of a line and a curve

See Example 17

1 Find the points of intersection of the line and the parabola with equations:

a $y = x^2$ and $y = 5x - 6$
b $y = 2x^2$ and $y = 8x$
c $y = x^2 + 10$ and $y = 2x + 18$
d $y = 5x^2$ and $y = x + 6$
e $y = 3x^2 + 2x + 10$ and $y = 12 - 3x$
f $y = 9x - x^2$ and $y = 6x - 10$
g $y = 6x - 3$ and $y = x^2 - 8x + 46$
h $x + y = 2$ and $y = x^2 - 2x$

2 Find the points of intersection of:

a the line $y = x - 3$ and the hyperbola $y = -\frac{2}{x}$

b the line $y = 1 - x$ and the circle $x^2 + y^2 = 1$

c the line $x + y = 3$ and the circle $x^2 + y^2 = 9$

d the line $y = x + 5$ and the circle $x^2 + y^2 = 25$

e the line $y = 5x + 2$ and the hyperbola $y = \frac{7}{x}$

f the line $2x - y = 1$ and the hyperbola $y = \frac{15}{x}$

g the line $y = 2x + 5$ and the hyperbola $y = -\frac{3}{x}$

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Notes

Appendix

Glossaries

Instructional glossary

Mathematical 'doing' words

analyse To study something in great detail by breaking it into its parts.

bisect To cut in half.

calculate To find the value of a numerical expression. See also **evaluate**.

classify To sort into categories or types.

complete To fill in detail that makes a statement or diagram correct or finished.

construct To draw a geometrical figure accurately.

convert To change from one form to another. For example, convert a fraction to a decimal, or convert dollars to cents.

decrease To make smaller by subtracting or dividing.

estimate To make an educated guess for a number or answer; to find roughly or approximately.

evaluate To find the value of an expression. For example, evaluate 3×8^2, or evaluate $4x + 1$ when $x = 5$.

expand To remove the brackets in an algebraic expression; the opposite of **factorise**. Expanding $3(2y + 1)$ gives $6y + 3$.

factorise To take out the highest common factor (HCF) of an expression and insert brackets; the opposite of **expand**. Factorising $5x – 20$ gives $5(x – 4)$.

give reasons When solving a problem, to show the mathematical rules or thinking used.

graph To display on a number line, number plane or statistical graph.

hence find/prove To find an answer or prove a result using previous answers or information supplied.

increase To make larger by adding or multiplying.

measure To find the size of something using an instrument. For example, to find a length using a ruler.

prove/show that In questions where the answer is given, to use mathematical reasoning to prove that the answer is true.

reduce (a fraction) to its lowest terms. See **simplify (a fraction).**

round (a number) To find the nearest approximation for a number. For example, round 4.3 to the nearest whole number (4), \$12.9598 to the nearest cent (\$12.96), 0.166 66 to three decimal places (0.167).

show working To show the steps you used to find an answer.

simplify To give the answer in its simplest, shortest, neatest form.

simplify (a fraction) To reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF). When the numerator and denominator are as small as possible, the fraction has been simplified or reduced to its lowest terms.

simplify (a ratio or rate) To reduce the terms or units of a ratio or rate by dividing by a common factor, similar to simplifying a fraction.

simplify (a surd) To write a surd $\sqrt{x}$ in its simplest form so that x has no factors that are perfect squares.

sketch To draw a rough diagram. Less accurate than to **construct**.

Glossaries

solve To find the value of the unknown variable in an equation or inequality.

substitute To replace a variable with a number, to evaluate an expression. For example, substituting $t = 5$ in the expression $t^2 + 6$ gives 31.

write correct to See **round (a number)**.

write/state To write an answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

Appendix

Glossaries

Mathematical glossary

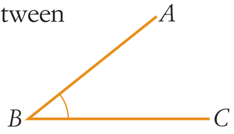
Some common symbols

$\%$	per cent
$(\ )$	parentheses, round brackets
$[]$	(square) brackets
$\{\}$	braces
$=$	is equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\therefore$	therefore
-3	negative 3
$\sqrt{\ }$	the square root of
$\sqrt[3]{\ }$	the cube root of
± 3	positive or negative 3
$0.\dot{6}$	the recurring decimal 0.666 666 . . .

π	pi, the decimal 3.14159 . . .
$3x$	3 multiplied by x, $3 \times x$
$\frac{x}{2}$	x divided by 2, $x \div 2$
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\overline{x}$	the mean (average)
Σ	the sum of
$42°17'54''$	42 degrees, 17 minutes, 54 seconds
$\angle ABC$	angle ABC
$\triangle ABC$	triangle ABC
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\equiv$	is congruent (identical) to
$\vert\vert\vert$	is similar to
$P(E)$	The probability of an event, E
$P(\overline{E})$	The probability of the complementary event, $\overline{E}$

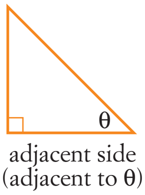
A

acute angle An angle between 0° and 90°.



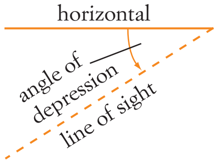
acute-angled triangle A triangle with all three angles acute.

adjacent side In a right-angled triangle, the side ‘next to’ the given angle, leading to the right angle.

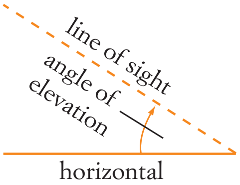


allowable (tax) deduction A part of a person’s yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**.

angle of depression The angle of looking **down**, measured from the horizontal.



angle of elevation The angle of looking **up**, measured from the horizontal.

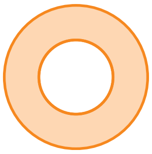


angle sum The total of the sizes of the angles in a shape. The angle sum of a triangle is 180°.

annual leave loading (or **holiday loading**) Extra payment to a worker during annual leave based on17.5% of four weeks’ pay.

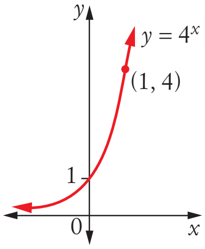
Glossaries

annulus A ring shape between two different-sized circles with the same centre.



ascending order Going up, increasing, from smallest to largest (1-2-3). The opposite of **descending order**.

asymptote A line that a curve gets very close to but never touches, for example, the x-axis is an asymptote of the exponential curve.



average See **measure of location**.

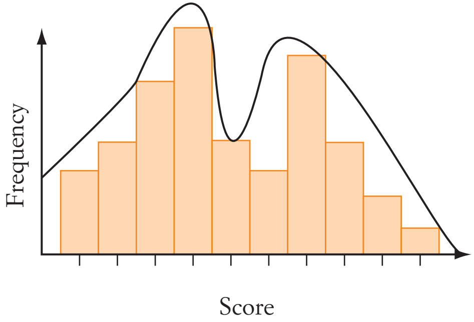
B

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5, the 3 is called the base.

bias In statistics, something that causes a sample to not truly represent the population.

bisect To cut in half.

bimodal distribution A statistical distribution that has two peaks.

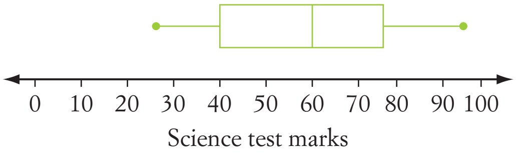


binomial expression An algebraic expression with two terms, for example, $x + 9$, $2y – 12$.

bivariate data Data that measures two variables, such as a person's height and arm span, represented by an ordered pair of values that can be graphed on a **scatter plot** for analysis.

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline.

boxplot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest scores; the 'box' contains the middle 50% of scores while the 'whiskers' extend to the two extremes.



C

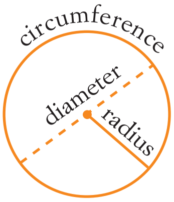
capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). See also **volume**.

categorical data Data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not **numerical**.

census A survey of the entire **population** of people or items, not just a survey of a **sample**.

chance experiment An activity or process that involves chance; for example, rolling a die or tossing a coin.

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius.



cluster A group of data scores that are bunched or close together.

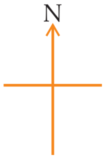
Glossaries

coefficient The number in front of a variable in an algebraic term. For example, the coefficient of x in $2x - 5$ is 2.

commission Pay earned by salespeople and agents, calculated as a percentage of the value of items sold or income made.

compass bearing A bearing that refers to one of the sixteen points of a mariner's compass; for example, north-northwest (NNW). See also **bearing** and **three-figure bearing**.

compass rose A cross-shaped diagram that shows the direction of north. See also **compass bearing**.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

composite shape A shape made up of two or more basic shapes.

compound interest Interest that is calculated as a percentage of the original principal and the accumulated interest. See also **simple interest**.

congruent Identical, exactly the same. The symbol '≡' means 'is congruent to' or 'is identical to'.

congruent figures Identical figures, having the same shape and size.

congruence test One of four tests for proving that two triangles are congruent: SSS, SAS, AAS and RHS.

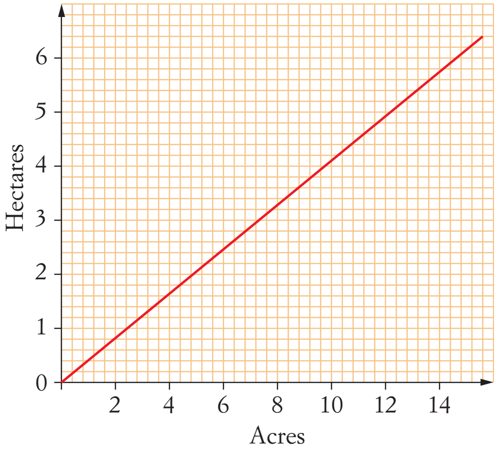
consecutive numbers Any series of integers that follow each other in order; for example, 8, 9 and 10.

constant term The term in an algebraic expression that is a number only, with no variable. For example, the constant term in $x^2 - 4x + 6$ is 6.

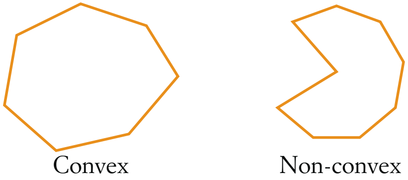
continuous data Numerical data that can be measured on a smooth scale without any gaps, and can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**.

conversion graph A line graph for converting between different units or currencies; for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0).

Converting acres to hectares



convex polygon A polygon whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180°.

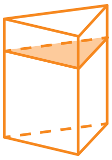


cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to }\theta}{\text{hypotenuse}}$$

where θ is an angle. See also **sine** and **tangent**.

cross-section A 'slice' of a solid cut across it rather than along it.

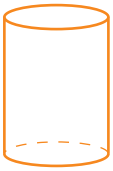


cube (of a number) The number raised to the power of 3. For example, 7 cubed $= 7^3 = 7 \times 7 \times 7 = 343$.

cube root (of a number) The value which, if cubed, gives the number. For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$.

cumulative frequency A progressive or running total of frequencies, the sum of frequencies of a particular score and all scores below it.

cylinder A can-shaped solid with ends that are circles.



D

data Information, a collection of facts.

denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3.

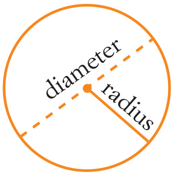
dependent event An event whose outcome (and probability) depends upon the outcome of another event; for example, the colour of the second marble drawn from a bag depends on the colour of the first marble drawn.

dependent variable A variable that depends on another variable for its value. For example, if y depends on x, then the dependent variable is y and the **independent variable** is x.

depreciation The decrease in the value of items over time due to ageing or use.

descending order Going down, decreasing, from largest to smallest (3-2-1). The opposite of **ascending order**.

diameter An interval joining two points on the circumference and passing through the centre of a circle, or the length of that interval. The diameter is double the **radius**.

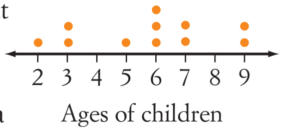


difference of two squares An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$.

discrete data Numerical data that are counted or measured, only taking on distinct, separate values, such as the number of children in a family (0, 1, 2, . . .). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**.

direct proportion (or **direct variation**) A relationship between two variables of the form $y = kx$, where k is a constant; for example, if $y = 8.5x$, then y is directly proportional to x.

dot plot A graph that uses dots above a number line to show the frequencies of data scores.

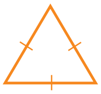


double time Overtime pay that is calculated at 2 times the normal pay rate.

E

equation A mathematical statement that two quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$.

equiangular All angles equal.



equilateral triangle A triangle with all three sides equal (and all angles 60°).

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables.

expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials.

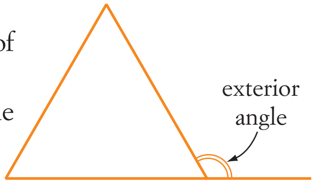
experimental probability An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula

$$P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

exponential curve The graph of an exponential equation $y = a^x$. See **asymptote** for a diagram.

exponential equation An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, for example, $y = 4^x$.

exterior angle An 'outside' angle of a shape created by extending one of the sides of the shape.



event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the three outcomes {2, 4, 6}.

F

factor (of a number) A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. Also called **divisor**.

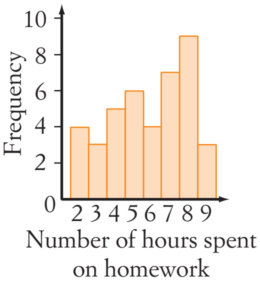
five-number summary For a set of numerical data, the lowest score, lower quartile, median, upper quartile, highest score; used to draw a **boxplot**.

formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$.

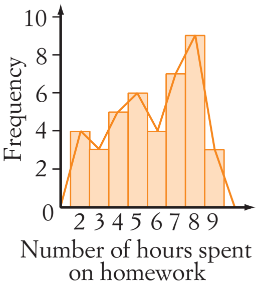
fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data.

frequency histogram A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings.



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the tops of the columns of a histogram. The graph looks like a mountain.

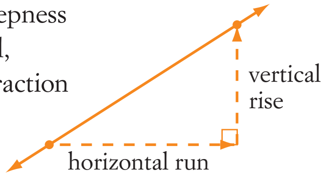


frequency (distribution) table A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx.

G

general form of a linear equation The equation of a straight line $ax + by + c = 0$, where a, b and c are integers and a is positive.

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$



gradient–intercept form of a linear equation The equation of a straight line $y = mx + b$, where m is the **gradient** and b is the y-intercept.

greatest common divisor (GCD) See **Highest common factor**.

gross pay Pay received before tax and other deductions are taken out.

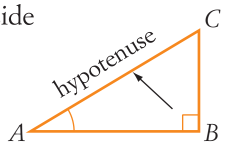
H

highest common factor (HCF) Also called **greatest common divisor (GCD)**. The largest factor shared by two or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$.

hire-purchase See **term payments**.

horizontal Going across, sideways, flat.

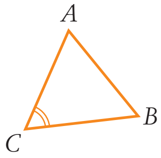
hypotenuse The longest side of a right-angled triangle, opposite the right angle.



I

image A transformed shape after it has been enlarged or reduced.

included angle The angle between two given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$.



income tax A tax paid to the government based on the size of a person's income.

independent event An event whose outcome (and probability) does not depend upon the outcome of another event; for example, the number rolled on the second die does not depend on the number rolled on the first die.

independent variable A variable whose value does not depend on another variable. For example, if y depends on x, then the **dependent variable** is y and the independent variable is x.

inequality A mathematical statement that two quantities are not equal, involving algebraic expressions and an inequality sign ($>$, $\geq$, $<$, or $\leq$), for example, $-3 > -10$ or $2x - 7 \leq 15$.

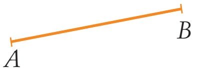
index (Plural: **indices**, pronounced 'in-da-sees') See **power**.

index law An algebraic rule for simplifying expressions involving powers of the same base; for example, $a^m \times a^n = a^{m+n}$.

instalment (or **repayment**) The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan.

interquartile range (IQR) The difference between the upper quartile and lower quartiles, $\text{IQR} = Q_3 - Q_1$, representing the middle 50% of scores.

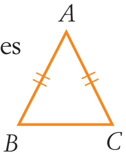
interval A section of a line with a definite length, such as AB on the right.



inverse proportion (or **inverse variation**) A relationship between two variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x.

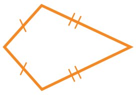
irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction (rational number). In decimal form, its digits run endlessly without repeating. See also **rational number** and **real number**.

isosceles triangle A triangle with two equal sides (and two equal angles opposite those sides).



K

kite A quadrilateral with two pairs of equal adjacent sides.



L

LHS The left-hand side (of an equation).

like terms Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms.

linear equation A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$.

M

mean The average of a set of data, represented by $\overline{x}$, calculated by dividing the sum of the scores by the number of scores.

measure of location An average, middle or typical value of a set of data. The three measures of location are the **mean**, **median** and **mode**.

measure of spread A statistical value that describes how the scores in a data set are spread; for example, **range** or **interquartile range**.

median The middle score when the scores are arranged in order. If the number of values is even, then the median is the average of the two middle values.

midpoint The point in the middle of an interval or halfway between two given points.

minute (symbol ′) A measure of angle size. $\frac{1}{60}$ of a degree. $1° = 60'$.

mode The most common or frequent score(s) in a set of data.

mutually exclusive events Events or categories that have no items in common.

N

negatively skewed See **skewed distribution**.

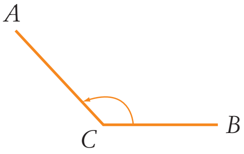
net pay Pay received after deductions from gross pay; ‘take-home’ pay.

numerator The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2.

numerical data Data that can be measured or counted, such as a person’s height or the number of goals scored. Data that is not **categorical**.

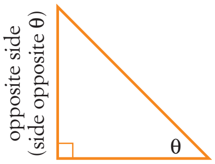
O

obtuse angle An angle greater than 90° but less than 180°.



obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°).

opposite side In a right-angled triangle, the side directly facing the given angle.



outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4.

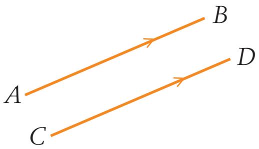
outlier An extreme data value that is very different from the other values in a set.

overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay.

P

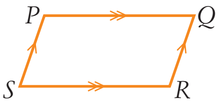
parabola A U-shaped curve that is the graph of a quadratic equation such as $y = x^2$.

parallel lines Lines that point in the same direction and do not intersect.



$AB \parallel CD$ means 'AB is parallel to CD'.

parallelogram A quadrilateral in which the opposite sides are parallel.



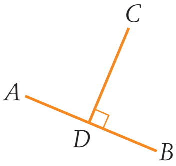
PAYG (Pay As You Go) tax Income tax deducted from your pay each payday by your employer.

perfect square A square number or an algebraic expression that represents one; for example, 64, $(x + 9)^2$, $(a - b)^2$.

perimeter The distance around the outside of a shape. The sum of the lengths of its sides.

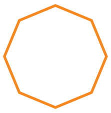
per annum (p.a) Per year.

perpendicular lines Lines that intersect to form a right angle. $AB \perp CD$ means 'AB is perpendicular to CD'.



piecework Earnings based on the number of items processed, made or delivered, paid at a rate per item rather than on the number of hours worked.

polygon Any flat shape made up of straight sides.



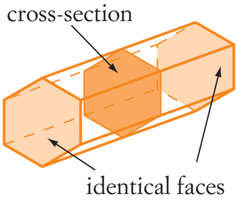
population In statistics, all of the items being studied, the entire group.

positively skewed See **skewed distribution**.

power (or **index**) The number of times a base is multiplied by itself. In 2^5, the power is 5. Also called the *exponent*.

principal An amount of money invested or borrowed, on which interest is calculated.

prism A solid shape with identical cross-sections and straight sides.

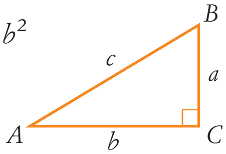


probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1.

product The result of a multiplication. The product of 7 and 3 is 21.

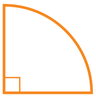
pronumeral Another name for **variable**.

Pythagoras' theorem The relationship $c^2 = a^2 + b^2$ for a right-angled triangle, where c is the length of the hypotenuse and a and b are the lengths of the other two shorter sides.



Q

quadrant (of a circle) A sector that is a quarter of a circle, containing a right angle.

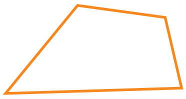


quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$.

quadratic equation An equation in which the highest power of the variable is 2, that is, a variable squared; for example, $3x^2 - 6 = 69$ or an equation such as $y = 3x^2 - 6$, whose graph is a **parabola**.

quadratic formula The formula for solving quadratic equations of the form $ax^2 + bx + c = 0$, which is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

quadrilateral Any polygon with four sides.

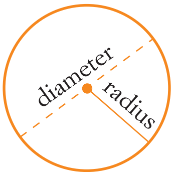


quarterly Occurring regularly four times a year, that is, every three months.

quartile The values Q_1, Q_2, Q_3 that divide a set of data into four equal parts. The 1st quartile Q_1 is the lower quartile, the 2nd quartile Q_2 is the **median**, the 3rd quartile Q_3 is the upper quartile.

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**.



random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely.

random sampling In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population.

range In a set of data, the difference between the highest and lowest scores.

rate A relationship between two quantities measured in different units. For example, a speed of 107 km/h compares distance travelled (in kilometres) with time (in hours).

ratio A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3 : 40 (read '3 to 40').

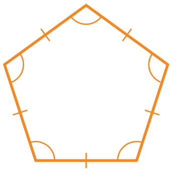
rational number A number that can be written as a fraction in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. See also **irrational number** and **real number**.

real number A **rational** or **irrational number**, that can be ordered on a number line.

reciprocal The product of any number and its reciprocal is 1. The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

rectangle A quadrilateral with four right angles.

regular polygon A polygon that has all sides equal and all angles equal. For example, this regular pentagon has five equal sides and five equal angles.

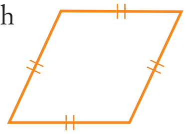


relative frequency The number of times an event or score occurred, written as a fraction of the total number of events or scores. See also **experimental probability**.

repayment See **instalment**.

retainer A fixed amount paid to a salesperson before **commission** is added.

rhombus A quadrilateral with four equal sides.



RHS The right-hand side (of an equation).

right-angled triangle A triangle with one 90° angle.

rise Short for 'vertical rise', this is the change in vertical position between two points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. See **gradient**.

run Short for 'horizontal run', this is the change in horizontal position between two points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. See **gradient**.

S

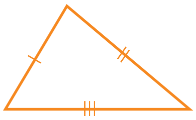
salary A fixed yearly amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked.

sample In statistics, a group of people or items selected from a population for study.

sample space In a probability situation, the set of all possible outcomes.

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

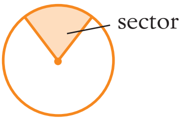
scalene triangle A triangle with no equal sides.



scatter plot A graph of points on a number plane. Each point represents the values of the two different variables and the resulting graph may show a pattern.

second (″) A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$.

sector A region of a circle cut off by two radii, shaped like a slice of pizza.



shape of a distribution The way the data in a frequency distribution is spread, can be **symmetrical**, positively **skewed** or negatively **skewed**.

significant figures The meaningful digits in a number that show its level of accuracy, the first non-zero digits; for example, 31 487 000 has 5 significant figures.

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: |||).

similarity test One of four tests for proving that two triangles are similar.

simple interest Interest that is calculated as a percentage of the original principal. See also **compound interest**.

simultaneous equations Two (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1, y = 3$.

sine A ratio in a right-angled triangle:

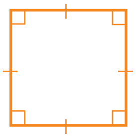
$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. See also **cosine** and **tangent**.

skewed distribution A distribution in which most of the scores are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. See also **symmetrical distribution**.

solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true.

square A quadrilateral with four equal sides and four right angles.



stem-and-leaf plot A ‘number graph’ that lists all the data scores, in groups. Each score is split into a ‘stem’ and a ‘leaf’. This stem-and-leaf plot shows 12 test scores, from 42 to 82.

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key: 5|8 stands for 58

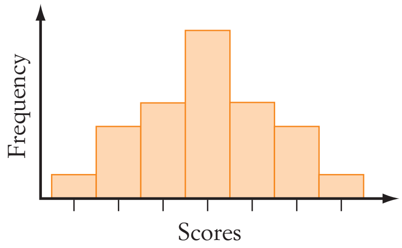
subject of a formula The variable for which a formula is written, the variable on the left-hand side of a formula. The subject of the formula $A = \frac{1}{2}bh$ is A.

substitution method A method of solving **simultaneous equations** that involves substituting one equation into another equation.

surd A square root (or other root) whose exact value cannot be found because it is **irrational**, such as $\sqrt{10}$ or $\sqrt[3]{7}$.

surface area The total area of all the faces of a solid shape.

symmetrical distribution A distribution in which all scores are distributed equally on both sides of the centre, its shape having line symmetry. See also **Skewed distribution**.



T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

where θ is an angle. See also **sine** and **cosine**.

tax deduction See **allowable deduction**.

taxable income The part of a person’s income that is taxed, equal to yearly income minus allowable deductions.

term payments Paying for an expensive item through a loan in which regular instalments are made over time. Also called **hire-purchase**.

term (of an expression) A part of an algebraic expression. For example, $b^2 + 6b - 9$ has three terms: b^2, $6b$ and -9.

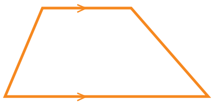
theoretical probability (or **calculated probability**) Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

three-figure bearing (or **true bearing**) A bearing that uses three-digit angles from 000° to 360° to show the amount of turning measured clockwise from north. See also **bearing** or **compass bearing**.

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate.

trapezium A quadrilateral with one pair of opposite sides parallel.



trial One go or run of a repeated probability experiment; for example, one roll of a die.

tree diagram A diagram of branches for listing all of the possible outcomes of a multi-step chance experiment.

two-step experiment (or **two-stage experiment**) A chance experiment with two steps or stages, such as rolling a pair of dice.

two-way table A table that shows the number of items belonging to overlapping categories.

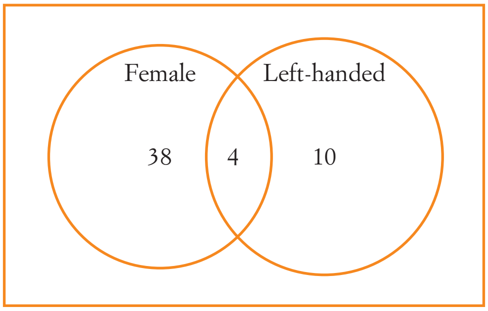
	Can swim	Cannot swim
Boys	13	2
Girls	9	3

V

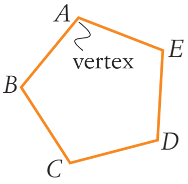
variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**.

variation See **direct proportion**.

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories.



vertex (plural: **vertices)** A corner of a shape, angle or curve.



vertical Going up and down, at a right angle to the **horizontal**.

volume The amount of space occupied by a solid object, measured in cubic units.

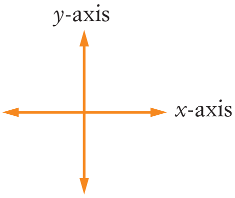
W

wage An amount of money paid to people for work, calculated on the number of hours worked.

X

***x*-axis** The horizontal axis of a number plane (running across).

***x*-intercept** The *x*-value at which a graph cuts the *x*-axis



Y

***y*-axis** The vertical axis of a number plane (running up and down).

***y*-intercept** The *y*-value at which a line cuts the *y*-axis.

Answers

Chapter 1

SkillCheck

1 **a** 0.04 **b** 0.22 **c** 0.183 **d** 0.047
e 0.095 **f** 0.0675 **g** 0.1525 **h** 0.2
2 **a** $72 **b** $116.25 **c** $4494
3 **a** $7350 **b** $4034.10 **c** $8737.60
4 **a** 36 **b** 24 **c** 60
5 **a** 52 **b** 26 **c** 365
d 4 **e** 12 **f** 8 years and 4 months
6 **a** 1152 **b** 50 **c** 0.06
7 **a** $5962.59 **b** $33 433.46
c $18 481.63 **d** $64937.10

Exercise 1-01

1 **a** $874 **b** $938.80 **c** $367.20
2 Greta earns more per week by $27.48.
3 **a** $3461.86 **b** $6923.73 **c** $150 053.33
4 Job1: $1104.64; Job2: $1160; Job2 by $55.36
5 $1096.10 **6** $735.23 **7** $761.24
8 A **9** $13 312.50 **10** $1394.40
11 $2115 **12** 54 **13** $63.95
14 **a** $427 **b** $700 **c** $956.87 **d** $625.55
15 **a** $972.12 **b** $680.48 **c** $4568.96

Exercise 1-02

1 **a** $45 697 **b** $6398.53
2 **a** $114 719 **b** $30 393.03
3 **a** $90 904 **b** $21 581.48
4 C **5** $19 924.99
6 $45 456.10 **7** $696.42 **8** $623.52
9 **a** $452 **b** $1711.10 **c** 25.0%
10 **a** $458 **b** $1747.65 **c** 24.0%
11 **a** $2296 **b** $456 **c** $1646.73
12 **a** $2297.59 **b** $456 **c** $1550.39
13 Gross weekly income = $816.90; Total deductions = $369.10; Net income = $447.80

Exercise 1-03

1 **a** $5040 **b** $2953.50 **c** $102.50
d $96.95 **e** $71.92 **f** $451.20
2 **a** $87.50 **b** $5925.15 **c** $391 000
d $820 **e** $279 **f** $723.04
3 A
4 **a** $11 200 **b** $1569
c $9392.50 **d** $11 331.25
5 **a** $1440 **b** $7440
6 4.5% **7** **a** $6750 **b** 18.75%
8 9.75% p.a. **9** 2 years **10** 26 weeks
11 137 days **12** C **13** 2.5 years
14 2.6% p.a. **15** **a** $18.90 **b** $1063.90

Exercise 1-04

1 **a** Check with your teacher. Investment after 1st yr = $24 150; Investment after 2nd yr = $25 357.50
b Compound interest = $2357.50
2 **a** $16 153.36 **b** $1153.36
3 **a** $38 459.48 **b** $4359.48
4 **a** $5408, $408 **b** $30 245.29, $2445.29
c $11 113.20, $1513.20 **d** $41 905.55, $2405.55
e $19 337.39, $937.39
5 **a** $4791.80 **b** $1642.38
c $308.93 **d** $3913.84
e $6834.42

Mental skills 1

2 **a** 11 **b** 40 **c** 7 **d** 24
e 23 **f** 6 **g** 43 **h** 80
i 18 **j** 15 **k** 40 **l** 65
m 11 **n** 14 **o** 12 **p** 135

Exercise 1-05

1 $14 332.50
2 **a** **i** $9754.75 **ii** $3254.75
b **i** $13 858.59 **ii** $3858.59
c **i** $12 634.81 **ii** $394.81
d **i** $43 949.46 **ii** $9349.46
e **i** $8427.39 **ii** $427.39
3 D **4** $1 301 018.83 **5** B
6 **a** **i** $13 488.50 **ii** $3488.50
b **i** $52 751.13 **ii** $17 251.13
c **i** $9448.23 **ii** $548.23
d **i** 53 366.91 **ii** $11 366.91
e **i** $19 473.44 **ii** $2973.44
f **i** $5177.03 **ii** $277.03
7 C
8 **a** $600 **b** $615 **c** Tegan by $15.
9 **a** $10 510.31 **b** $1969.48 less
10 **a** **i** $7554.45 **ii** $7688.85
iii $7758.33 **iv** $7805.54
b Monthly, because it earns the most interest.

Exercise 1-06

1 **a** $175.50 **b** $1579.50 **c** $328.14
d $1907.64 **e** $105.98 **f** $2083.14
2 **a** $1275 **b** $24 225 **c** $10 416.75
d $34 641.753 **e** $577.36 **f** $35 916.75
3 **a** $1379 **b** $2316.72 **c** $217.58
4 **a** $3420 **b** $720 **c** $1500 **d** 48%
5 **a** $2080 **b** $8320 **c** $13 200
d $4880 **e** 14.7%

6 a $32.90 b $437.42 c $108.42 d 36.62%
7 a $1073.40 b $273.40 c 34.2%
8 a $2599 b $3576 c $677 d 10.4%
9 a $262.50 b 7.4%

Exercise 1-07

1 $933.89
2 a $20 429.69 b $29 560.31
3 a i $659.66 ii 60%
  b i $2459.54 ii 45.2%
  c i $5073.42 ii 60%
  d i $778.24 ii 41%
  e i $14 020.37 ii 51%
  f i $851.35 ii 37%
  g i $403.03 ii 46.3%
  h i $1097.20 ii 68.6%
4 a i 90% ii 73% iii 53% iv 48%
  b By trial and error, in approx 6.6 years.
5 a i $10 000 ii $8000 iii $4096
  b 32.8%
6 a $11 138.51 b $4661.49
7 a $6472.88 b $3441
  c Approx 8 years and 9 months.
  d 23.2%
8 Yes, it will lose approximately 52% after 7 years.
9 a $1800 b 5 years c $798.67
  d Yes, in the 30th year. e No

Power plus

1 4 years and 61 days 2 $4444.44
3 $12 838.71 4 $63 367.50
5 $2276.87 6 790 000
7 a 18 years. b 18 years.
  c No. The size of the interest rate and the number of compounding periods determine how quickly the principal takes to double in value.

Chapter 1 revision

1 $13 045.75 2 $1349.18
3 a $797.45 b $1011.40
4 a $1052.51 b $736.76 c $4946.80
5 a $67 725 b $13 557.63
6 a $2400 b $392.50 c $78.75 d $621.37
7 a $5955.08 b $955.08
8 $36 282.78 9 $15 374.72 10 $852.91
11 $45 815.75
12 a $487.50 b $4387.50 c $1908.56
  d $6296.06 e $174.89 f $6783.56
13 a $8851.45 b $6138.55 c 59%

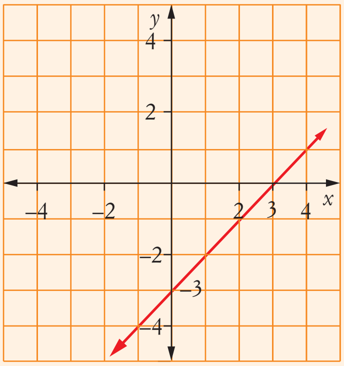
Chapter 2

SkillCheck

1 a (6, 1) b (−5, −4) c 6 d 6
  e $AC = BC = 4.5$ f isosceles
  g $\frac{1}{3}$ h $-\frac{2}{3}$

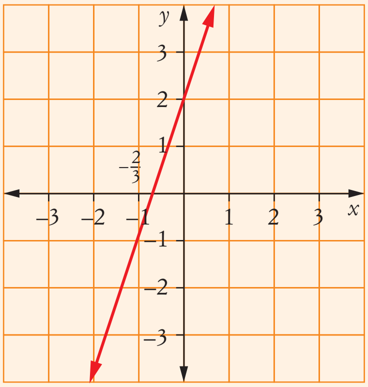
2 a

x	0	1	2	3
y	−3	−2	−1	0



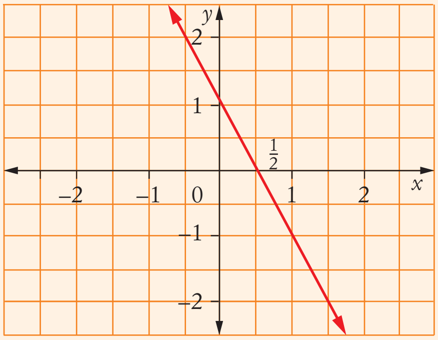
b

x	−2	−1	0	1
y	−4	−1	2	5



c

x	−1	0	1	2
y	3	1	−1	−3

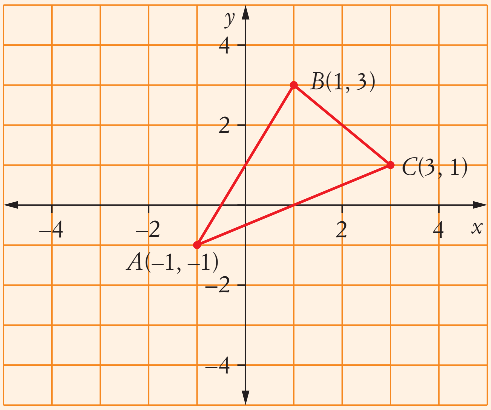


Answers

3 **a** negative **b** neither **c** positive
d positive **e** negative **f** neither

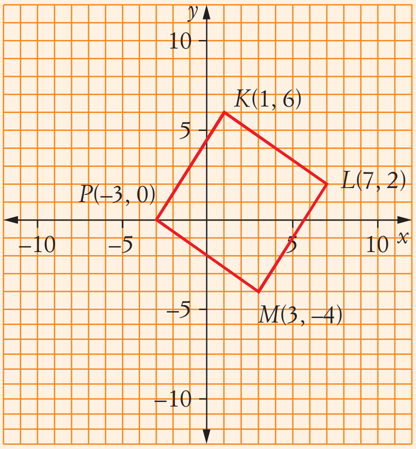
Exercise 2-01

1 B **2** C **3** A
4 **a** $\frac{1}{3}$ **b** -2 **c** $\frac{7}{3}$
5 **a** **i** 2.2 **ii** (6, 2.5) **iii** $-\frac{1}{2}$
b **i** 10.8 **ii** (3.5, 3) **iii** $\frac{2}{3}$
c **i** 7.1 **ii** (−2.5, −0.5) **iii** −1
d **i** 7.6 **ii** (0.5, −7.5) **iii** $-\frac{3}{7}$
e **i** 10.2 **ii** (−6, −3) **iii** −5
f **i** 5.7 **ii** (5, 0) **iii** 1
6 **a** $\sqrt{89}$ **b** $\sqrt{194}$ **c** $\sqrt{82}$
7 k: $m = \frac{1}{5}$; l: $m = -\frac{1}{2}$ **8** B
9 **a**



b $AB = AC = \sqrt{20}$ or $2\sqrt{5}$, $BC = \sqrt{8}$ or $2\sqrt{2}$
c $AB = AC = \sqrt{20}$ or $2\sqrt{5}$
d isosceles
e 11.8

10 **a**



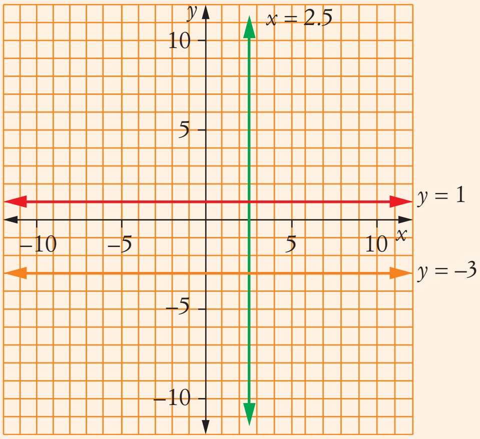
b square **c** $m_{KL} = -\frac{2}{3}$, $m_{PM} = -\frac{2}{3}$
d $m_{KP} = \frac{3}{2}$, $m_{LM} = \frac{3}{2}$
e the gradients are equal, they are parallel
f $KL = LM = PM = KP = \sqrt{52}$ or $2\sqrt{13}$
g 28.8 **h** 52 sq. units
11 **a** $P(-2, 1)$, $Q(1, 3)$
b $PQ = 3.6$, $AC = 7.2$, $AC = 2 \times PQ$
c $m_{PQ} = \frac{2}{3}$, $m_{AC} = \frac{2}{3}$; the gradients are equal.

Exercise 2-02

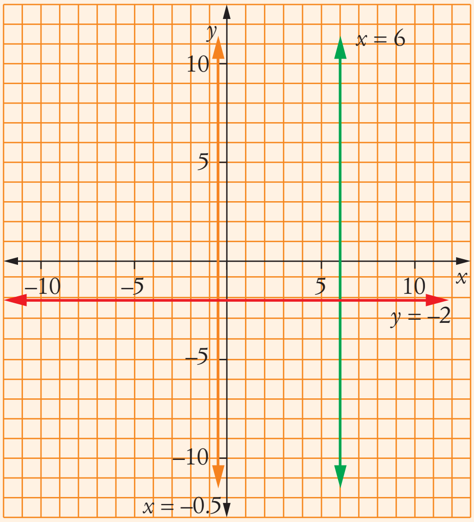
1 **a** neither **b** perpendicular **c** parallel
d neither **e** parallel **f** neither
2 **a** 4 **b** −2 **c** $\frac{1}{3}$ **d** −0.2
3 **a** −1 **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $-\frac{2}{5}$
4 D **5** B **6** A
7 **a** $m_{AB} = -\frac{4}{3}$, $m_{CD} = -\frac{4}{3}$; $\therefore AB \parallel CD$
b $m_{PQ} = \frac{3}{4}$, $m_{CD} = -\frac{4}{3}$; $\therefore PQ \perp CD$
8 **a** $\frac{1}{3}$ **b** −3

Exercise 2-03

1 **a** **i** $\frac{1}{3}$ **ii** −1
b **i** −2.5 **ii** 5
c **i** 4 **ii** 4
d **i** −1 **ii** −2
e **i** 0 **ii** 0
f **i** −6 **ii** 3
2 **a** no **b** yes **c** yes
d yes **e** no **f** no
3 C
4 **a** $x = -4$ **b** $x = 1$ **c** $y = 5$ **d** $y = -3$
5 **a**



b



6 **a** $y = 2$ **b** $x = 4$ **c** $x = -1$ **d** $y = -2$
e $y = 3$ **f** $x = -1$ **g** $y = 6$ **h** $x = -1$

7 A **8** C

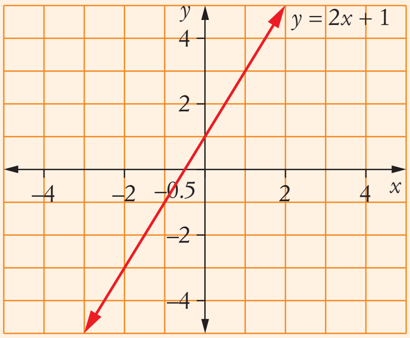
9 **a** x-axis **b** y-axis

Exercise 2-04

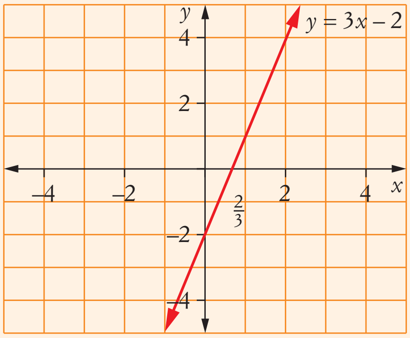
1 **a** $m = 3, b = -2$ **b** $m = -2, b = 7$
c $m = 1, b = 4$ **d** $m = -1, b = 9$
e $m = \frac{3}{4}, b = 6$ **f** $m = 1, b = 0$
g $m = \frac{1}{2}, b = -11$ **h** $m = \frac{2}{3}, b = 6$
i $m = -\frac{1}{3}, b = -8$ **j** $m = 2, b = -6$
k $m = -3, b = 11$ **l** $m = 1, b = -\frac{7}{2}$

2 **a** $y = 2x + 1$ **b** $y = \frac{3}{4}x + 2$
c $y = -7x + 5$ **d** $y = -\frac{2}{5}x + 3$
e $y = -2x - 3$ **f** $y = -3x + \frac{1}{2}$

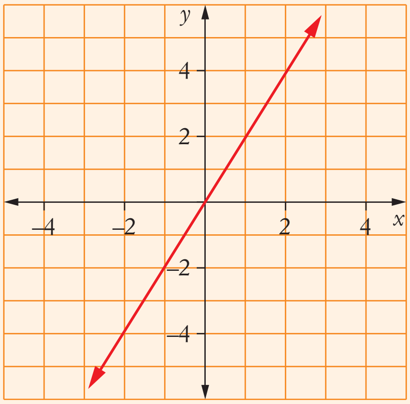
3 **a** $m = 2, b = 1$



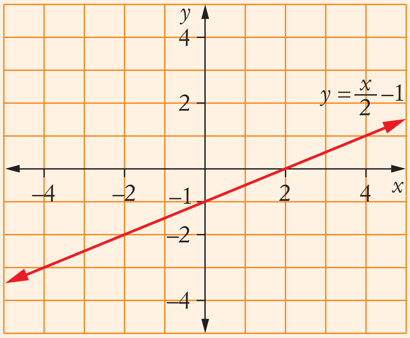
b $m = 3, b = -2$



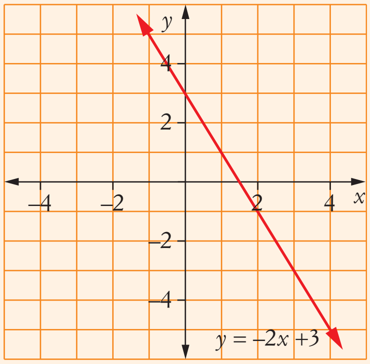
c $m = 2, b = 0$



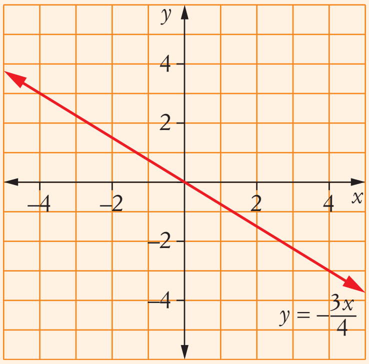
d $m = \frac{1}{2}, b = -1$



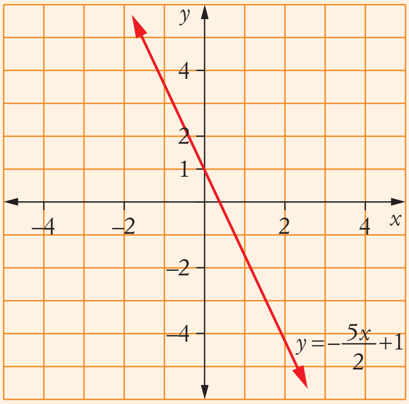
e $m = -2, b = 3$



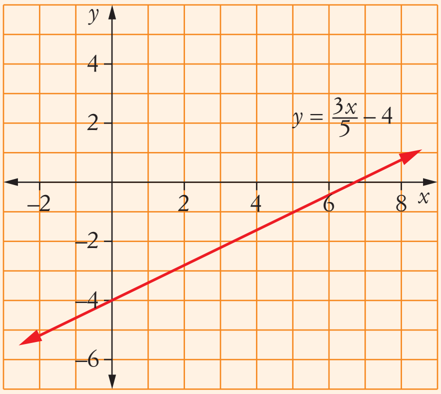
f $m = -\frac{3}{4}, b = 0$



g $m = -\frac{5}{2}, b = 1$



h $m = \frac{3}{5}, b = -4$



4 $y = 2x$

5 a C **b** B, D **c** B
d C, D **e** A, B **f** D

6 a $y = 4x + 3, y = 4x - 6$ **b** $3x - y + 7 = 0, y = 3x - 2$

Mental skills 2

2 a 18 **b** $126 **c** 39 **d** $30.30
e $7.50 **f** 10.8 **g** $27 **h** 60
i $240 **j** $3.30 **k** 900 **l** $52.50
4 a 10 **b** 166 **c** $50 **d** $22
e 37.5 **f** $5.80 **g** 135 **h** $22.60
6 a 500 **b** $20 **c** 4.5 **d** $6.25
e 81 **f** $35 **g** 16.5 **h** 74.5
i $195 **j** $425 **k** $31.50 **l** 290
8 a 160 **b** $1.50 **c** 7.5 **d** $32.50
e $67.50 **f** $31.25 **g** 38 **h** 170

Exercise 2-05

1 a $x - y + 2 = 0$ **b** $3x - y - 1 = 0$
c $5x - y + 8 = 0$ **d** $x + 2y - 3 = 0$
e $x - 2y - 6 = 0$ **f** $8x - y + 2 = 0$
g $6x - y - 3 = 0$ **h** $x - 2y - 6 = 0$
i $3x - 5y + 10 = 0$
2 a $m = -2, b = 6$ **b** $m = 4, b = -5$
c $m = \frac{3}{2}, b = 2$ **d** $m = -2, b = 1$
e $m = -2, b = -5$ **f** $m = -\frac{4}{3}, b = 4$
3 B **4** B

Exercise 2-06

1 a $y = 2x + 5$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -3x + 6$
d $y = -x + 3$ **e** $y = \frac{1}{2}x + 3$ **f** $y = -3x - 3$
2 a $y = \frac{1}{2}x + 2$ **b** $y = x$ **c** $y = -\frac{1}{2}x + 5$
d $y = -\frac{1}{2}x + 3$ **e** $y = -3x - 3$ **f** $y = -x - 2$
g $y = 3x - 10$ **h** $y = \frac{2}{5}x + 2$ **i** $y = 2x - 3$

Exercise 2-07

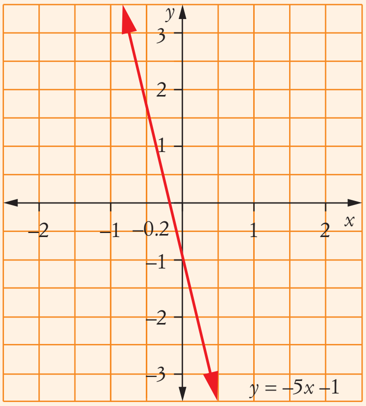
1 a $y = 2x + 4$ **b** $y = 3x + 6$ **c** $y = -\frac{1}{2}x + \frac{11}{2}$
d $y = 2x - 12$ **e** $y = -5x - 13$ **f** $y = \frac{1}{2}x - 10$
2 a $y = -2x - 2$ **b** $y = \frac{1}{5}x - \frac{1}{5}$ **c** $y = -\frac{1}{3}x + \frac{4}{3}$
d $y = -3x - 3$ **e** $y = x + 6$ **f** $y = -\frac{1}{3}x - \frac{31}{3}$
3 a $m = 2$ **b** $M(0, 2)$ **c** $-\frac{1}{2}$ **d** $y = -\frac{1}{2}x + 2$
4 a $y = \frac{1}{3}x + 1$ **b** -3 **c** $y = -3x + 11$
5 a $y = -\frac{4}{5}x + 8$ **b** $A(10, 0)$ **c** $\frac{5}{4}$
d $y = \frac{5}{4}x - \frac{25}{2}$ **e** $(0, -12.5)$

Power plus

1 a $\frac{2}{3}$ **b** $y = \frac{2}{3}x - 2$ **c** $y = 4$
2 $k = 5$ **3** $B(2, -1)$ **4** $X(-2, 3)$

Chapter 2 revision

1 a 12.6 **b** $M(-1, 4)$ **c** $\frac{1}{3}$
2 a $HJ = JK = KL = HL = \sqrt{58}$
b $m_{HJ} = \frac{3}{7}, m_{JK} = \frac{7}{3}, m_{KL} = \frac{3}{7}, m_{HL} = \frac{7}{3}$
c $HK = \sqrt{200}$ or $10\sqrt{2}$, $JL = \sqrt{32}$ or $4\sqrt{2}$
d rhombus
3 a $-\frac{1}{2}$ **b** 2
4



5 C **6** D
7 a $m = 2, b = -10$ **b** $m = 4, b = 3$
c $m = -\frac{3}{8}, b = \frac{1}{2}$
8 a $3x - y + 5 = 0$ **b** $2x - 5y - 50 = 0$
c $x - 3y - 6 = 0$
9 a $m = 1, b = 2$ **b** $m = \frac{1}{4}, b = 1$
c $m = -3, b = 9$
10 a $y = \frac{5x}{2} + 3$ **b** $y = -\frac{1}{2}x - 4$
11 a $y = 3x - 6$ **b** $y = -2x$

Chapter 3

SkillCheck

1 a 1681 cm^2 **b** 1650 cm^2 **c** 1248 cm^2
d 4750 cm^2 **e** 2310 cm^2 **f** 1800 cm^2
g 182 cm^2 **h** 770 cm^2 **i** 680 cm^2
2 a 28.7 mm **b** 28.7 mm **c** 19.2 mm
3 a i 32.67 cm **ii** 84.95 cm^2
b i 87.96 cm **ii** 615.75 cm^2
c i 395.84 cm **ii** 12 468.98 cm^2
d i 581.19 cm **ii** 26 880.25 cm^2
4 a 105 m^3 **b** 27 m^3 **c** 308 m^3
d 480 m^3 **e** 640 m^3 **f** 560 m^3

Exercise 3-01

1 a 150 m^2 **b** 51 m^2 **c** 88 mm^2
d 156 m^2 **e** 210 m^2 **f** 36 cm^2
g 189 cm^2 **h** 1000 cm^2 **i** 71 m^2
j 201.19 m^2 **k** 88 000 cm^2
2 a 2228.3 m^2 **b** 293.1 m^2 **c** 952.1 m^2
d 441.9 m^2 **e** 63.3 m^2 **f** 70.7 m^2
g 502.7 m^2 **h** 150.8 m^2 **i** 193.1 m^2
j 742.9 m^2 **k** 4664.3 m^2 **l** 471.2 m^2
3 a 1.51 m^2 **b** 5.59 m^2 **c** 8.73 m^2
4 a 20 056 m^2 **b** \$4001
5 1.2 m^2
6 a 34.56 m^2 **b** \$2592
7 a 6310 m^2 **b** \$18 614.50
8 a 15.708 cm **b** 157 m
9 a 39.27 m^2 **b** 157% increase

Exercise 3-02

1 a 282 m^2 **b** 298 cm^2 **c** 2720 mm^2
d 204 m^2 **e** 1288 mm^2 **f** 165 m^2
2 a cube, 2646 m^2 **b** rectangular prism, 684 m^2
c triangular prism, 9720 m^2 **d** trapezoidal prism, 6378 m^2
3 a 80 m^2, \$8400 **b** 171.4 m^2
4 a 1036 cm^2 **b** 1020 mm^2 **c** 204 m^2
d 390 cm^2 **e** 672 cm^2 **f** 5672 mm^2
5 a 151.33 m^2 **b** 251.33 m^2

Exercise 3-03

1 a 747.70 m^2 **b** 3573.56 mm^2
c 206.47 cm^2 **d** 15.83 m^2
2 a 35 m^2 **b** 3478 cm^2
3 a 1009 m^2 **b** 2160 m^2 **c** 4 m^2 **d** 1895 m^2
e 7 m^2 **f** 14 m^2 **g** 1131 m^2 **h** 95 m^2
4 a 56.5 m^2 **b** 7 L
5 The triangular prism tent by 7.6 m^2.

Answers

Exercise 3-04

1 **a** 446.96 cm^2 **b** 49 270 cm^2 **c** 864 cm^2
2 464 cm^2
3 **a** 352 cm^2 **b** 76 cm^2
4 **a** 9721.7 cm^2 **b** 14 031.4 cm^2 **c** 14 778.1 cm^2
d 2858.8 cm^2 **e** 2793.5 cm^2 **f** 394.7 cm^2
5 **a** 854.51 cm^2 **b** 20 slices **c** 138 cm^2
6 2953 cm^2
7 **a** 26.14 m^2 **b** 19 m^2
8 1028.32 cm^2
9 **a** 75.4 m^2 **b** 50.3 m

Mental skills 3

2 **a** 8 hours 30 mins **b** 5 hours 40 mins
c 3 hours 25 mins **d** 8 hours 15 mins
e 11 hours 25 mins **f** 1 hour 40 mins
g 5 hours 10 mins **h** 5 hours 45 mins
i 7 hours 55 mins **j** 7 hours 40 mins

Exercise 3-05

1 **a** 504.8 m^3 **b** 1045 cm^3 **c** 19 968 cm^3
2 **a** 9.7 m^3 **b** 94 247.8 m^3 **c** 7.4 m^3
d 137.7 m^3 **e** 5026.5 m^3 **f** 216.9 m^3
g 42.3 m^3 **h** 107.5 m^3 **i** 146.3 m^3
3 **a** **i** 1539 m^3 **ii** 1539 kL
b **i** 14 432 cm^3 **ii** 14 432 mL
c **i** 226 m^3 **ii** 226 kL
d **i** 5 cm^3 **ii** 5 mL
4 **a** 251.3 cm^3 **b** 320 cm^3 **c** 21.5%
5 300 kL **6** 13 666cm^3 **7** 63 L
8 **a** 4825.49 cm^3 **b** 5026.55 cm^3 **c** 1989.38 cm^3
d 6375.00 cm^3 **e** 5301.44 cm^3 **f** 3084.96 cm^3
g 536.19 cm^3 **h** 1884.96 cm^3 **i** 167.33 cm^3
j 12 900 cm^3 **k** 167.55 cm^3 **l** 794.12 cm^3
9 **a** 182.83 m^3 **b** $21.94 per day

Power plus

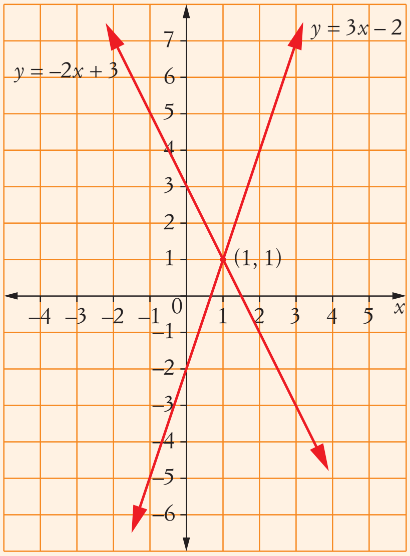
1 1728 cm^3 **2** 8 m (to the nearest m)
3 **a** $p^2 + 4pr$ **b** $\frac{3\pi x^2}{2}$
4 36 mm (to the nearest mm)
5 0.5 m/h or 500 cm/h **6** 40 mm

Chapter 3 revision

1 **a** 133 m^2 **b** 2340 mm^2 **c** 326.7 cm^2
d 205.3 m^2 **e** 2056.1 m^2 **f** 56.5 cm^2
2 **a** 1.08 m^2 **b** 3150 mm^2 **c** 5236 cm^2
d 277.6 m^2 **e** 216 cm^2 **f** 434 mm^2
3 **a** 7389.0 m^2 **b** 1437.3 m^2 **c** 104.3 m^2
4 **a** 3180 cm^2 **b** 1268 cm^2 **c** 395 cm^2
d 14 294 cm^2 **e** 5871 cm^2 **f** 4428 cm^2
5 **a** 10 125 m^3 **b** 11 084 m^3 **c** 11 m^3
d 36 816 m^3 **e** 20 160 m^3 **f** 10 016 m^3
6 82.5 L
7 **a** 59.11 kL **b** 1.44 m

Mixed revision 1

1 $1624.19
2 **a** 9.9 **b** $\left(\frac{-3}{2}, \frac{1}{2}\right)$ **c** -1
3 **a** 157.5 m^2 **b** 650.3 m^2 **c** 268.1 m^2
4 **a** $1001.72 **b** $701.20 **c** $4708.08
5 **a** 117.5 m^2 **b** 480 m^2 **c** 324 m^2
6 **a** $425 **b** $860.63 **c** $1105
7 **a** $-\frac{3}{2}$ **b** $\frac{2}{3}$
8 **a** $47 210 **b** $6890.25
9



10 **a** $3000 **b** $16.88
11 **a** 9349.4 cm^2 **b** 1979.2 cm^2 **c** 279.4 cm^2
12 **a** $12 979.20 **b** $979.20
13 B **14** B
15 **a** 1352 m^2 **b** 22 m^2 **c** 2822 m^2
16 **a** $4764.06 **b** $4782.47 **c** $4786.73
17 $44 791.90
18 **a** $768 **b** $6912 **c** $3456
d $10 368 **e** $216 **f** $11 136
19 **a** $m = -3, b = 8$ **b** $m = 1, b = 7$ **c** $m = -\frac{2}{3}, b = 1$
20 **a** $2x - y - 3 = 0$ **b** $2x - y - 5 = 0$ **c** $3x - 4y + 24 = 0$
21 **a** $y = 5x + 3$ **b** $y = -4x + 8$ **c** $y = \frac{1}{3}x + \frac{4}{3}$
22 line A: $y = -\frac{3x}{2} - 3$; line B: $y = \frac{2x}{3}$
23 **a** $y = 4x + 32$ **b** $y = 5x$
24 **a** $19 676.44 **b** $10 313.56 **c** 65.6%
25 **a** 6050 cm^3 **b** 3308 cm^3 **c** 10 175 cm^3
26 **a** 65 **b** $81.25

Chapter 4

SkillCheck

1 **a** g^9 **b** r^6 **c** d^{15} **d** k^2
e h^{10} **f** m^4 **g** a **h** 1
i $6e^7$ **j** $3n^4$ **k** $1000w^9$ **l** 25
m v^5w^5 **n** $\frac{v^3}{w^3}$ **o** $\frac{1}{y}$ **p** $\frac{1}{y^2}$

2 **a** $\frac{19a}{20}$ **b** $\frac{p}{6}$ **c** $\frac{5}{3}$ **d** $\frac{x}{7y}$

3 **a** $18m^2 + 66m$ **b** $-15g + 40$

4 **a** $4(x + 6)$ **b** $5(4 - 3a)$ **c** $q(q + 1)$
d $6a(3a - 2)$ **e** $-2(y + 15)$ **f** $-6(3w - 4)$

5 **a** 3 and 6 **b** −2 and −4
c 4 and −5 **d** 8 and −2

Exercise 4-01

1 **a** $6p^7$ **b** $5w^6$ **c** m^{30}
d $9q^6$ **e** $30n^{16}t^5$ **f** $4x^5y^4$
g $-5e^{10}g^4$ **h** $-9a^{10}b^5$ **i** $100y^{20}$
j $-64p^3$ **k** $\frac{3pq^2r^4}{2}$ **l** $54u^4v^3w^8$

2 **a** $l^{18}m^{50}$ **b** $\frac{n^3}{8}$ **c** 7
d $\frac{w^{10}}{t^{15}}$ **e** 1 **f** $64k^2y^{10}$
g −15 **h** $\frac{16b^4}{81d^4}$ **i** $-125d^9y^{15}$
j $\frac{-27k^{12}}{1000}$ **k** −9 **l** $81p^8q^{12}r^{16}$

3 **a** 1 **b** 1 **c** 7 **d** 1
e −8 **f** 9 **g** 625 **h** 128
i 1 **j** 64 **k** $\frac{1}{64}$ **l** 1
m 25 **n** $\frac{1}{10\ 000}$ **o** 1 **p** 1

4 **a** $\frac{1}{25}$ **b** $\frac{1}{32}$ **c** $\frac{1}{20}$ **d** $\frac{1}{1000}$

5 **a** $\frac{1}{8^7}$ **b** $\frac{1}{3^5}$ **c** $\frac{1}{y}$ **d** $\frac{1}{x^3}$
e $\frac{1}{25b^2}$ **f** $\frac{5}{b^2}$ **g** $\frac{1}{ab}$ **h** $\frac{a}{b}$
i $\frac{11}{t^3}$ **j** $\frac{1}{1331t^3}$ **k** $\frac{p^3}{q^5}$ **l** $\frac{m}{w^3}$
m $\frac{8}{u^3v^4}$ **n** $\frac{-2r^6}{y^5}$ **o** $\frac{10f^3}{e}$ **p** $\frac{n^7}{2k^4}$

6 **a** $\frac{10}{r^6}$ **b** $\frac{2}{5}$ **c** 3 **d** x
e $\frac{4}{h^2}$ **f** $\frac{3}{k}$ **g** $\frac{1}{16h^2}$ **h** $\frac{-2}{m}$
i $\frac{3g^3}{5}$ **j** $\frac{3t}{2r}$ **k** $\frac{m^3n}{p^2}$ **l** $\frac{4a^2}{5b}$

7 **a** $5000x^{28}y^6$ **b** $\frac{8x^{36}}{y^6}$ **c** $\frac{r^2}{4q^{12}}$
d $\frac{1}{12q^7r}$ **e** $\frac{256x^{16}}{a^4}$ **f** $\frac{a^4}{256x^{16}}$
g $-32h^{11}$ **h** $\frac{-8}{p^{12}h^7}$ **i** $64p^{10}h^{20}$

Exercise 4-02

1 **a** $\frac{3h}{5}$ **b** $\frac{5m}{13}$ **c** $\frac{9}{x}$ **d** $\frac{5}{3d}$
e $\frac{5n}{14}$ **f** $\frac{17c}{10}$ **g** $\frac{15r}{14}$ **h** $\frac{-19y}{24}$
i $\frac{13t}{9}$ **j** $\frac{-5y}{16}$ **k** $\frac{13t}{36}$ **l** $\frac{17a}{30}$
m $\frac{p+3}{z}$ **n** $\frac{u}{4g}$ **o** $\frac{1}{3f}$ **p** $\frac{2e}{5}$
q $\frac{5a}{3}$ **r** $\frac{2}{d}$ **s** $\frac{4}{k}$ **t** $\frac{1}{a}$

2 **a** $\frac{4+3x}{12}$ **b** $\frac{7s-4}{14}$ **c** $\frac{3a+5b}{15}$ **d** $\frac{2p+3q}{6}$
e $\frac{5m-8n}{40}$ **f** $\frac{17t}{20}$ **g** $\frac{4a+9h}{30}$ **h** $\frac{3d-2r}{48}$
i $\frac{13c}{10}$ **j** $\frac{6d-11r}{33}$ **k** $\frac{9h+10a}{15}$ **l** $\frac{25+24w}{30}$
m $\frac{21+8a}{28}$ **n** $\frac{5e}{24}$ **o** $\frac{7m-2n}{14}$ **p** $\frac{17k}{30}$

Exercise 4-03

1 **a** $\frac{w}{6}$ **b** $\frac{st}{20}$ **c** $\frac{15}{hk}$ **d** $\frac{12}{mn}$
e $\frac{3d}{e}$ **f** $\frac{8}{v}$ **g** $\frac{3x}{4y}$ **h** $\frac{2}{3}$
i $\frac{d}{g}$ **j** $\frac{d^2}{12}$ **k** $\frac{2}{3}$ **l** $\frac{12a^2}{k^2}$

2 **a** $\frac{5x}{2y}$ **b** $\frac{t}{6r}$ **c** 4 **d** $\frac{h^2}{k^2}$
e $\frac{3d}{10}$ **f** $\frac{1}{3}$ **g** $\frac{16}{27}$ **h** $\frac{15}{2}$
i 12 **j** $\frac{9}{b^2}$ **k** $6p$ **l** $\frac{3g}{20y}$

3 **a** $\frac{5x}{z}$ **b** $\frac{c}{3b}$ **c** $\frac{5}{4}$ **d** $\frac{25b}{3}$
e $\frac{4t}{27}$ **f** $\frac{2}{3n}$ **g** $\frac{50p^2t}{7}$ **h** $\frac{2s}{35}$
i $\frac{1}{6h^2}$ **j** $25y^2$ **k** $\frac{2yh}{9}$ **l** $\frac{3af^2}{25}$

Exercise 4-04

1 **a** $4h + 24$ **b** $-3r - 30$ **c** $7x - 63y$
d $-4a + 20z$ **e** $-2 + t^2$ **f** $-20e^2 - 30e$
g $6y + 42y^2$ **h** $12x^2y^2 - 4xy$ **i** $16rt^2 - 8r^2t$
j $12ab^2 - 21a^2b$ **k** $-6h^2 + 18h^3$ **l** $-25x^3 - 20xy$

2 C

3 **a** Yes **b** No **c** Yes

4 **a** $15m^2 + 21m$ **b** $15e^2 - 9e$
c $3w^3 - 15w$ **d** $49x^3 - 10x^4$
e $t^2 + 7t + 12$ **f** $12 - 11h - 2h^2$
g $6x^3 + 35x^2 + 8$ **h** $6 + 3v - 2v^2$
i $w^2 - 8w + 3$ **j** $9y^2 - 36y + 35$
k $16m^3 + 2m^2$ **l** $20x + 20xy - 60y$

5 **a** $6(4x + 5)$ **b** $9(4 - 3a)$
c $x(x + 1)$ **d** $10y(3 - 2y)$
e $12d(3d + 2)$ **f** $4r(4r - 3)$
g $(a - 3)(a + 6)$ **h** $(8 + t)(t - 3)$
i $(3b + 5)(b - 2)$ **j** $-q(q + 36)$
k $-2t(3 - 5t)$ **l** $-3y(y + 2x)$
m $-hn(n - h)$ **n** $-2e(10e + 11)$
o $9m(5m - 6)$

6 B

7 **a** $4xy(3x - 4)$ **b** $2pr(9p + 8)$
c $36mn(m - 3n)$ **d** $36bc(ab - 4)$
e $16vw(3v + 4w)$ **f** $25gh(3g^2h - 5)$
g $p(1 - 8p - 4p^2)$ **h** $3mn(2n + 1 + 16m)$
i $8pg(4p^2 + g - 1)$ **j** $3a^2(6a^3 - 4 + 5a^2)$

Mental skills 4

Exact answers shown

2 **a** 331 **b** 157 **c** 1587 **d** 255
e 421 **f** 203 **g** 413 **h** 734
i 6723 **j** 15 744 **k** 276 **l** $72\frac{3}{7}$

4 **a** 28.231 **b** 14.187 **c** 177.4967 **d** 416.752
e 2.4156 **f** 5.0237 **g** 3.6890 **h** 5.8065
i 23.9121

Exercise 4-05

1 **a** $m^2 + 7m + 12$ **b** $w^2 + 10w + 25$
c $y^2 - 144$ **d** $a^2 - 5a - 24$
e $b^2 + 7b - 18$ **f** $u^2 - 15u + 56$
g $15 + 14k - k^2$ **h** $r^2 - 18r + 77$
i $c^2 - 9c + 18$ **j** $t^2 + t - 2$
k $x^2 + 6x - 40$ **l** $99 - 2n - n^2$

2 D

3 **a** $2x^2 + 11x + 15$ **b** $9e^2 + 42e + 49$
c $3p^2 + 7p - 10$ **d** $49d^2 - 28d + 4$
e $6f^2 + 4f - 10$ **f** $12m^2 + 5m - 25$
g $6 + 13h - 5h^2$ **h** $16p^2 - 40p + 25$
i $-10m^2 + 23m - 12$ **j** $12t^2 - 4t - 1$
k $25y^2 - 25$ **l** $-49a^2 + 84a - 36$

4 **a** length = $(a + 3)$ m; width = $(b + 1)$ m
b Area = $(a + 3)(b + 1)$
c Area = $ab + a + 3b + 3$
d Increase in area = $a + 3b + 3$

Exercise 4-06

1 **a** −1, −6 **b** 4, −3 **c** 3, −5 **d** 3, 4
e −4, −5 **f** −2, 7 **g** −2, 5 **h** −5, 5
i −2, 1 **j** −9, 2

2 **a** $(x + 3)(x + 4)$ **b** $(x + 5)(x + 7)$
c $(x + 4)(x + 1)$ **d** $(x + 5)(x + 2)$
e $(x + 4)(x + 5)$ **f** $(t + 1)(t + 5)$
g $(e + 3)(e + 2)$ **h** $(h + 2)(h + 2)$
i $(n + 1)(n + 10)$ **j** $(a + 6)(a + 5)$
k $(d + 4)(d + 6)$ **l** $(y + 4)(y + 11)$
m $(n - 3)(n + 1)$ **n** $(r - 7)(r + 2)$
o $(h - 4)(h + 1)$ **p** $(w - 9)(w + 2)$
q $(f - 9)(f + 3)$ **r** $(a - 6)(a + 2)$

3 **a** $(x + 4)(x - 1)$ **b** $(t + 8)(t - 3)$
c $(m + 5)(m - 3)$ **d** $(a + 2)(a - 1)$
e $(k + 7)(k - 2)$ **f** $(w + 6)(w - 2)$
g $(m - 4)(m - 1)$ **h** $(s - 4)(s - 2)$
i $(x - 7)(x - 5)$ **j** $(p - 6)(p - 4)$
k $(n - 2)(n - 1)$ **l** $(r - 3)(r - 3)$
m $(m + 2)(m + 2)$ **n** $(p - 10)(p - 10)$
o $(c - 5)(c - 5)$

Power plus

1 **a** 13 **b** 1 **c** 7 **d** 1
e $-\frac{125}{8}$ **f** 0

2 **a** $\frac{yz+xz-xy}{xyz}$ **b** $\frac{z}{xy}$ **c** $\frac{y+z}{xyz}$

3 **a** $a^2 - b^2 - ac + bc$ **b** $x^2 + y^2 - 2xy + 2x - 2y + 1$
c $3t + 4 - \frac{1}{t}$

4 **a** $(x - 8)(x - 125)$ **b** $(y + 50)(y - 36)$
c $(b + 41)(b + 41) = (b + 41)^2$ **d** $(n - 50)(n + 50)$

Chapter 4 revision

1 **a** $6v^5w^7$ **b** $8t^7h^6$ **c** $25x^2y^4$
d 1 **e** $\frac{1}{4k}$ **f** $\frac{8p^3}{27}$

2 **a** $\frac{1}{16m^2}$ **b** $\frac{4}{m^2}$ **c** $625b^{24}y^{12}$
d $512t^{14}u^{16}$ **e** $5c^4d^4$ **f** $\frac{6a}{5b}$

3 **a** $\frac{-3t}{20}$ **b** $\frac{19g}{6}$ **c** $\frac{x}{16}$
d $\frac{3b+2}{7}$ **e** $\frac{5w-12}{8}$ **f** $\frac{35-12y}{60}$

4 **a** $\frac{3p}{4m}$ **b** $\frac{3}{q^2}$ **c** $\frac{5y}{7}$
d $\frac{5}{4}$ **e** $\frac{5}{3x^2}$ **f** $8d^2$

5 **a** $9m - 72$ **b** $10ab + b^2$ **c** $-12x^2y + 15y^2$
d $56tp^2 - 40t^2p$ **e** $-3n + 10$ **f** $-15h^3 - 35h^2$

6 **a** $4fg^2 - 30f^2g$ **b** $93 - 22n$
c $8x^3 + 7x^4$ **d** $10y^2 - 41y + 21$

7 **a** $8(t - 9)$ **b** $b(b + 36)$
c $-3(m + 11)$ **d** $8ar(2r + 3)$
e $-6(4p - 3q)$ **f** $(5x - 1)(2 - 3x)$

8 **a** $15xy^2(1 - 2x^2y)$ **b** $6p(t^2 + 2pt - 8p^2)$
c $4r^2s^3(8s + 3r^2)$ **d** $25x^3y^3(2x - 3y)$
e $-8p^3q^3(1 - 6q^3)$ **f** $(n^2 + 6)(n - 1)$

9 **a** $b^2 + 13b + 30$ **b** $d^2 + d - 56$
c $15t - 54 - t^2$ **d** $20x^2 + 13x - 21$
e $49y^2 - 9$ **f** $25p^2 - 80p + 64$

10 **a** $(y + 5)(y + 5)$ **b** $(x - 20)(x - 1)$
c $(n + 11)(n - 3)$ **d** $(a - 7)(a - 4)$
e $(m - 12)(m + 7)$ **f** $(p + 9)(p - 6)$

Chapter 5

SkillCheck

1 **a** **i** 10 **ii** 16.5 **iii** 15 **iv** 15
b **i** 13 **ii** 1.8 **iii** 2.5 **iv** 3
c **i** 7 **ii** 11.5 **iii** 11 **iv** 11
d **i** 6 **ii** 43.6 **iii** 43 **iv** 43
e **i** 48 **ii** 34.3 **iii** 34.5 **iv** 24, 35
f **i** 5 **ii** 2.2 **iii** 2 **iv** 2

2 **a** **i** 31 **ii** 33.3 **iii** 62
b 78
c **i** Median = 30, mean = 28.3, range = 25.
ii The outlier has increased the median (by 1), the mean (by 5), and the range (by 37).

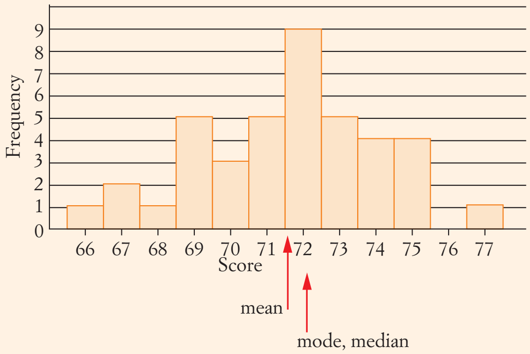
Exercise 5-01

1 **a** **i** symmetrical
  ii clustering at 9, no outliers
b **i** not symmetrical, not skewed
  ii clustering in the 30s and 60s, no outliers
c **i** positively skewed
  ii clustering at 1, 8 is an outlier
d **i** negatively skewed
  ii clustering at 23–24, no outliers
e **i** positively skewed
  ii clustering at 130–150, no outliers
f **i** symmetrical
  ii clustering at 5, no outliers
g **i** positively skewed
  ii clustering at 13, 23 is an outlier
h **i** symmetrical
  ii clustering at 50s and 100s, 13 is an outlier

2 **a**

Score	Frequency
66	1
67	2
68	1
69	5
70	3
71	5
72	9
73	5
74	4
75	4
76	0
77	1

b



c negatively skewed, no outliers
d The lower the score below par, the fewer the golfers that achieve that score.
e clustering at 72
f mode = 72, $\bar{x} = 71.6$, median = 72

3 **a** 45 **b** 1–9 hours (stem of 0)
c no outliers **d** positively skewed
e Most students spend limited time on their computers, and have other commitments and do other activities such as sport. Only a few students spend many hours on the computer during the week.
f Mode = 1, $\bar{x} = 14$, median = 11

4 **a**

Stem	Leaf
12	3
13	
14	2 3
15	0 1 3 3 3 5 5
16	0 0 1 2 2 2 2 3 4 5 5 7 8 9
17	0 0 1 2 3
18	2

b Symmetrical
c 123 is an outlier
d Clustering occurs in the 160s
e mode = 162, median = 162, $\bar{x} = 160.2$

5 **a** slight positive skew **b** 13.8 is an outlier
c 18.4 **d** 19.5
e 19.5 **f** 10.5
g No, the range has been affected by the outlier 13.8.

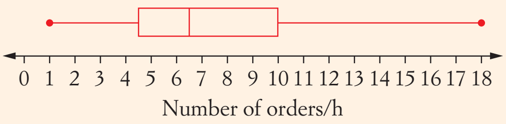
6 B

Exercise 5-02

1 **a** 5, 6.5, 8 **b** 18, 20, 26.5 **c** 32, 34.5, 38
2 **a** range = 7, IQR = 3 **b** range = 22, IQR = 8.5
c range = 16, IQR = 6
3 **a** 7.5 **b** 3
4 **a** 283 mm **b** 128 mm
5 **a** 3 **b** 2.5 **c** 17.5
d 19 **e** 21.5 **f** 1.5
6 **a** 34 **b** 13
c **i** 68, 72, 72, 75, 77, 78, 79, 80 **ii** 50%
d 75%
7 **a** **i** 28 **ii** 9.5
b The interquartile range, as it is not affected by the score of 35.
c 48, 48, 48, 49, 51, 53, 55; 54%

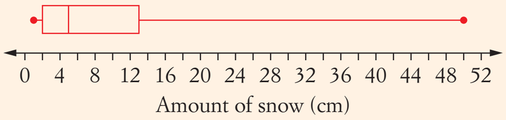
Exercise 5-03

1 **a** 1, 4.5, 6.5, 10, 18
b



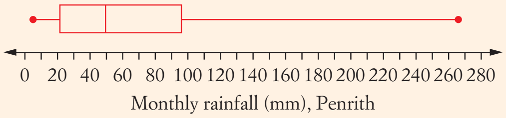
2 a 26 **b** 1, 2, 5, 13, 50

c



3 a 261 **b** 5, 21.5, 49.5, 96, 266

c



4 a 27.5 h **b** 26 h **c** 30 h **d** 4 h **e** 50%

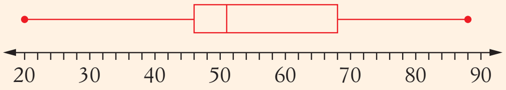
5 a 26 **b** 21 **c** 14

d i 25% **ii** 75%

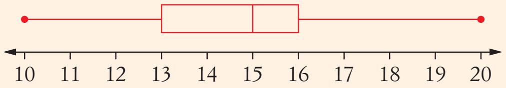
6 a 6, 10, 19, 23, 29 **b** 13

c i 14 **ii** 7 **iii** 7 **iv** 21

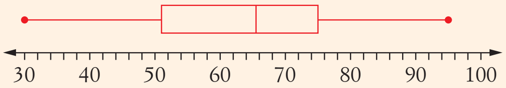
7 a 20, 46, 51, 68, 88



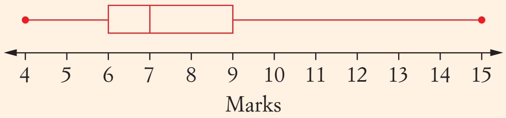
b 10, 13, 15, 16, 20



c 30, 51, 65.5, 75, 95



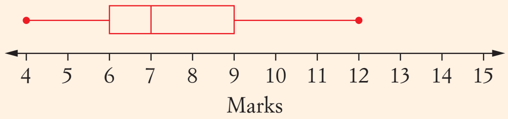
8 a 4, 6, 7, 9, 15



b Dot plot is positively skewed. The length of the boxplot from the median to the highest score is greater than the length from the median to the lowest score.

c 15

d 4, 6, 7, 9, 12



e i The boxplots are the same up to Q_3.

ii The whisker from Q_3 is reduced without the outlier.

Exercise 5-04

1 a i Year 10: 3.5; Year 8: 8

ii Year 10: 7.5; Year 8: 8.5

iii Year 10: 1; Year 8: 2

b i 25% **ii** 75%

c i 10 **ii** 0

2 a i 33 **ii** 28

b Thunderbirds: 50; Swifts: 49

c Thunderbirds: 15.5; Swifts: 9.5

d The range for both teams is similar but the IQR of the Swifts is less than the IQR of the Thunderbirds, indicating that the Swifts are more consistent in their performance.

e The position of the Thunderbirds' boxplot shows that the Thunderbirds scored more points in games than the Swifts and so performed better in the season.

3 a 10K: 9; 10N: 10 **b** 10K: 6.5; 10N: 5.5

c 10K: 3; 10N: 4 **d** 10K – lower range and IQR.

e 75%

4 C

5 a Brisbane: 26.9, 9.3, 4.7
Sydney: 23.5, 8.5, 4.9
Melbourne: 21.4, 13, 8.6
Hobart: 18.6, 11.2, 7

b Melbourne – it has the highest range and IQR.

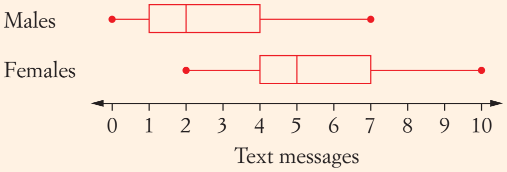
c Brisbane, more than half of the mean monthly temperatures are higher than most of the mean monthly temperatures of the other cities.

d Sydney's median temperature is significantly higher than Melbourne's, so Sydney is the warmer city.

e Sydney has the smaller range and IQR of mean monthly temperatures so it is more consistent.

6 a Male: 0, 1, 2, 4, 7; Female: 2, 4, 5, 7, 10

b

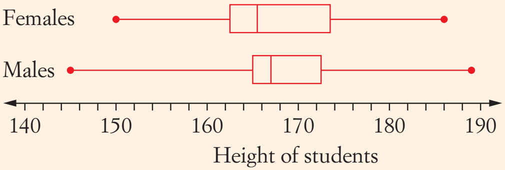


c Male: 3; Females: 3

d Males: 7; Females: 8

e Both are positively skewed, the interquartile range is the same, and the range of females is one more than that of the males. Females do receive more text messages, as the boxplot shows that 75% of females receive more messages than 75% of males.

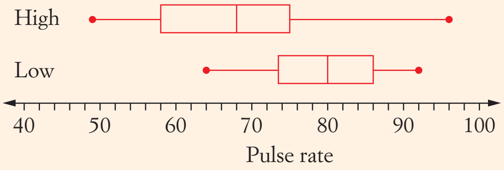
7 a Male: 145, 165, 167, 172.5, 189; Female: 150, 162.5, 165.5, 173.5, 186



b Male: range = 44 IQR = 7.5
Female: range = 36 IQR = 11

c Male students have a greater range (44 compared to 36), but a smaller interquartile range (7.5 compared to 12).

8 **a** Low: 64, 73.5, 80, 86, 92; High: 49, 58, 68, 75, 96

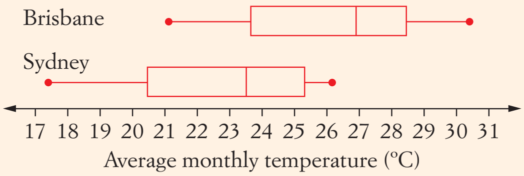


b **i** 28, 12.5 **ii** 47, 17

c The range and interquartile range of the High Frequency group are both greater than the Low Frequency group.

d The High Frequency group.

9 **a** Sydney: 17.6, 20.4, 23.45, 25.25, 26.1; Brisbane: 21.1, 23.65, 26.9, 28.45, 30.4



b Sydney: 8.5°, 4.85° Brisbane: 9.3°, 4.8°

c Sydney average monthly temperatures were slightly more consistent than Brisbane, since its range was 0.8° less than Brisbane's, while Brisbane's IQR was only 0.05° less than Sydney's.

10 **a** Simone

b Simone: 12 Amal: 10

c Amal, smaller range. **d** Simone: 10; Amal: 9

e Simone: 4; Amal: 5

f Not enough information given to make a valid decision. The interquartile range and range only differ by 1.

g 25% **h** 25%

Mental skills 5

2	**a** 176		**b** 363		**c** 261		**d** 405	
	e 682		**f** 707		**g** 1818		**h** 3564	
	i 152		**j** 540		**k** 2142		**l** 588	
	m 288		**n** 693		**o** 3939		**p** 852	

Exercise 5-05

1 **a** Boys: \$34.58 Girls: \$31.78

b Boys: \$33.50 Girls: \$28

c Boys: Range = 72, IQR = 25 Girls: Range = 69, IQR = 30.5

d **i** Boys are positively skewed slightly, girls are positively skewed.

ii There are no outliers, clustering occurs for the boys in the 20–30s and for the girls in the 10–20s.

e Boys generally carry more cash – they have a higher mean than the girls and the shape of the data for girls is more positively skewed.

2 **a** 21 games

b **i** 34 **ii** 51

c Scorpions: $\bar{x} = 1.6$ goals; Vale United: $\bar{x} = 2.4$

d Scorpions 5, Vale United 6

e The shape of both teams' results is positively skewed. Clustering for Scorpions occurs at 1 and 2 and for Vale United it occurs at 2.

f Vale United performed better as its mean was 2.4 goals/game compared to Scorpions 1.6 goals/game.

3 **a** Sydney: $\bar{x} = 26.2$, median = 26.5, mode = 28
Perth: $\bar{x} = 34.25$, median = 35, mode = 38

b Sydney: Range = 9°, IQR = 3
Perth: Range = 16°, IQR = 8

c The temperatures for Sydney and Perth are both negatively skewed, there are no outliers. Sydney's temperatures are clustered in the high 20s, while Perth's are clustered from 34–38.

d Sydney's temperatures are lower than Perth's, as evidenced by the significantly lower mean, median and mode. The range and interquartile range for Perth are greater than the range and interquartile range for Sydney, indicating greater spread.

4 **a** 30

b Quiz 1: $\bar{x} = 5.6$, mode = 6; Quiz 2: $\bar{x} = 6.3$, mode = 7

c Quiz 1: 6; Quiz 2: 7

d Quiz 1: **i** Range = 7 Quiz 2: **i** Range = 8
ii IQR = 2 **ii** IQR = 2

e Quiz 1: Results are symmetrical with clustering 5–6, no outliers.
Quiz 2: Results show negative skewness with clustering at 5 and 7–8, no outliers.

f Scores for Quiz 2 are just better than Quiz 1, as the mean of Quiz 2 is higher than the mean of Quiz 1.The spread for both quizzes are similar as there is only a difference of 1 between the ranges and the IQRs are equal.

5 **a** 39

b **i** mode = 2 **ii** median = 2
iii range = 6 **iv** IQR = 1.5

c positively skewed, no outliers

d 50%

e **i** By the highest columns.

ii By the short length of the box when compared to the whole length of the boxplot.

f **i** The shape of the distribution, the frequency for each household size and the mode. The mean can also be calculated from the histogram.

ii The shape of the distribution, the median and the quartiles Q_1 and Q_2.

6 **a** **i** 5 **ii** 16

b **i** mode = 22 **ii** range = 18
iii IQR = 24 − 16 = 8

c Negatively skewed.

i The tail of the dot plot goes to the left.

ii The length of the boxplot from the lowest score to the median is longer than from the median to the highest score.

d **i** dot plot **ii** boxplot
iii dot plot **iv** boxplot

7 **a** Sunbeam Valley: range = 24, median = 71, IQR = 8
Bentley's Beach: range = 30, median = 73, IQR = 15

b Sunbeam Valley: negatively skewed (slight)
Bentley's Beach: positively skewed

c Sunbeam Valley's speeds are clustered in the 70s.

d 25%

e Bentley's Beach – higher median, positively skewed. 25% of drivers drive faster than all drivers in Sunbeam Valley. This may be due to more main roads with higher speed limits.

8 **a** 36

b Lamissa: mode 7, median = 7
Anneka: mode = 7, median = 6

c Lamissa: range = 8, IQR = 8 − 6 = 2
Anneka: range = 9, IQR = 7 − 4 = 3

d Lamissa's distribution of scores is negatively skewed with clustering at 7. Anneke's distribution is negatively skewed with clustering at 6 and 7.

e **i** 25% **ii** 50%

f Lamissa is the better archer. Her median score is higher than Anneke's, 30.5% of scores are less than 6 compared to Anneke's 55.6%. Also, from the boxplot, 50% of Lamissa's scores are equal to or better than 75% of Anneke's.

9 **a** The range (47) is too large.

b Women: 31 Men: 37

c Women: Range = 38, IQR = 40 − 24 = 16
Men: Range = 47, IQR = 46 − 25 = 21

d Distribution for women is positively skewed with clustering in the 20s. Distribution for men is symmetrical with clustering in the 30s.

e Men have the greater spread in the number of sit-ups completed, as the range and IQR are both greater than those for women.

10 **a** **i** 56 **ii** 38

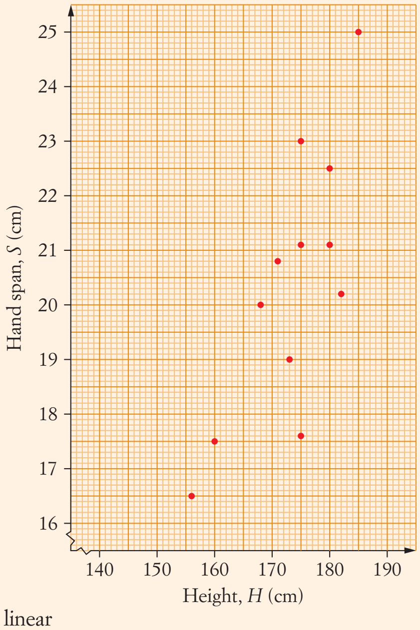
b **i** 10 Blue, 10 Yellow **ii** 10 Green
iii 10 Red

c **i** 10 Green **ii** 10 Yellow
iii 10 Red, 10 Blue

d 10 Blue. It shares the highest median with 10 Red but its lowest score is still higher than 25% of 10 Red's scores.

Exercise 5-06

1 **a**



b linear

c As the heights of students increase, their handspans tend to increase.

2 **a** weak negative relationship

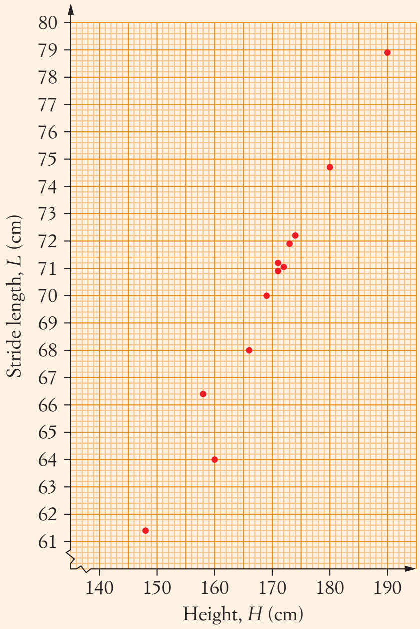
b no relationship

c strong positive relationship

3 Weak positive.

4 **a** Stride length depends on a person's height; the taller the person, is, the longer their legs are.

b



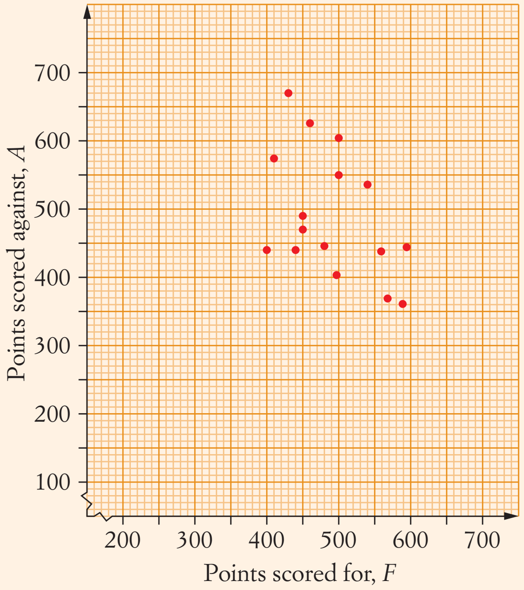
c linear

d Students' stride length increases with height.

e strong positive relationship

f Near 72.5–73 cm

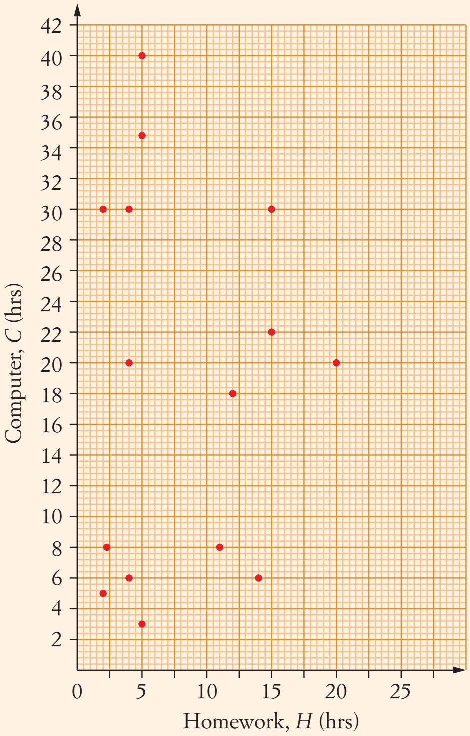
5 a



b Yes

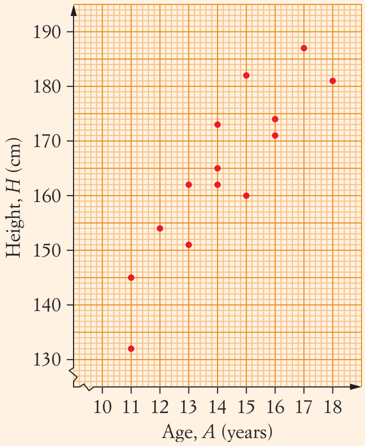
c Weak negative relationship

6 a



b no relationship

7 a



b Age, because as a young person ages, he usually grows in height.

c weak positive relationship

Exercise 5-07

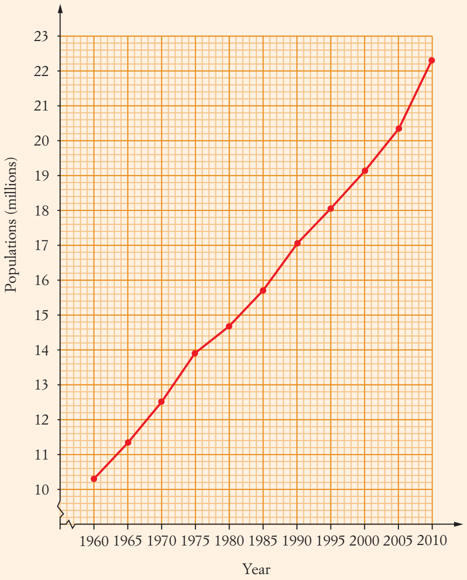
1 a **i** 25 **ii** 42 **iii** 15

b December, more customers due to summer and Christmas holiday season.

c June, fewer customers due to winter, busy end-of-financial year season.

d Number of people employed peaks in December, then falls, only to increase in March, April (the Easter holiday period). It then falls again to a low in June, July and then slowly the number of people employed rises to a peak in December. From 2010 to 2012, the number of people employed is showing a slow increase.

2 a



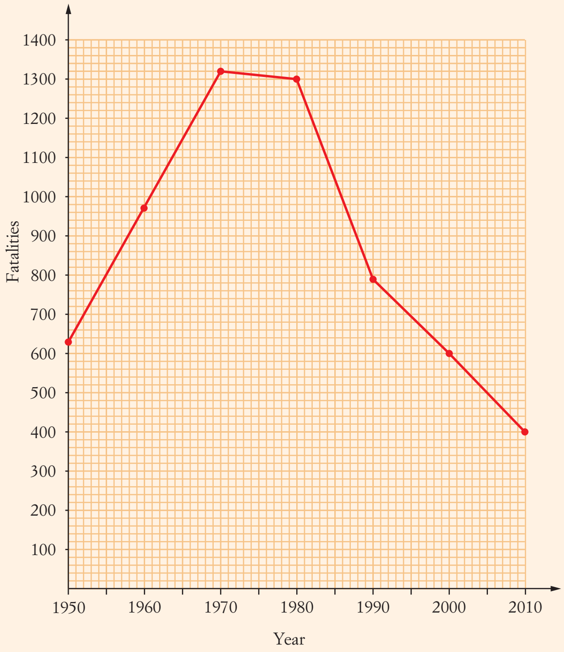
b 2005–2010

c Australia's population increased at 1.1–1.2 million every 5 years up to 1975. The population growth then slowed down for 5 years. From 1980, the population grew at a steady rate of just over a million people every 5 years, but in 2005–2010, the rate increased to 1.9 million for the 5-year period.

d **i** 26–27 million

ii 32–34 million, teacher to check.

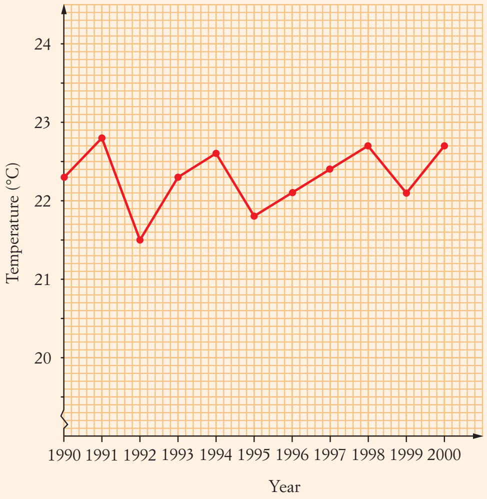
3 **a**



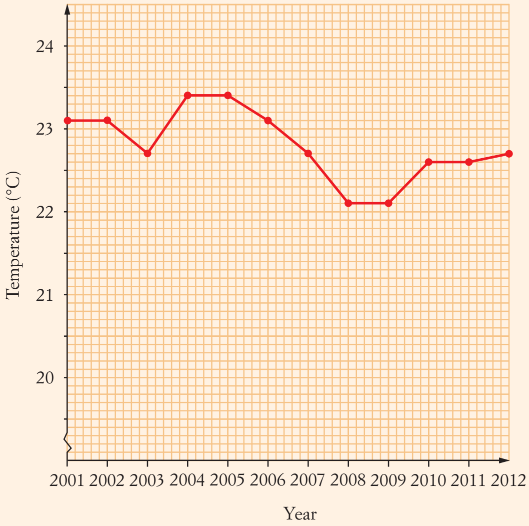
b From 1960, road fatalities rose at a steady rate, reaching a peak of approximately 1300 in 1970–1980. Road fatalities then fell at a rapid rate between 1980–1990 and continued to fall by about 200/5 years until 2010.

c Improved safety in cars, with seat belts being compulsory, then drink driving laws introduced.

4 **a** **i**



ii

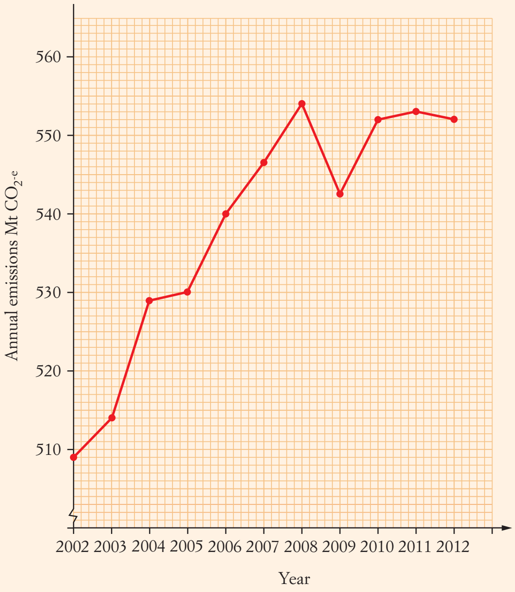


b **i** Starting at 22.3° in 1990, the temperature has seen a series of increases of less than 1°, followed by falls of less than 1°. The increase from 1990 to 2000 is 0.4°.

ii Starting at 23.1° in 2001, the temperature falls to 22.7° and then increases to a high of 23.4° for two years before falling to 22.1°. From 2009, the temperature rose, then remained steady before a slight increase to 22.7° in 2012.

c The temperature from 1990–2000 continually increased and decreased by less than 1°. The temperature in 2001–2012 started at 23.1, rose to a high of 23.4° in 2004, before falling for three years. This was followed by a slight increase. The range of annual temperatures for both periods is 1.3°, but the minimum and maximum temperatures for 2001–2012 are 0.6° higher than for 1990–2000.

5 **a**



b Carbon emissions increased by 55 Mt.
c Carbon emissions stablised.
d More environmentally-friendly policies and practices in Australia.
e **i, ii** Teacher to check and discuss.

6 a Approximately 4 million. **b** 18 million
c 300 000 persons per year **d** 26.5 million

7 a Gradual increase in passenger movements with peaks in October and troughs in February.
b **i** 3.9–4.0 million **ii** 4.25 million
iii 4.2 million **iv** 4.5 million
c 15%

Exercise 5-08

1 a Just surveying 300 people between 9 a.m. and 11 p.m. in shopping centres only targets a narrow group of people in certain areas.
b The sample needs to be more random and over a large area, not just in shopping centres. A telephone survey should produce more accurate feedback.

2 The report does not say what conditions are needed for the hot water system to work effectively. The temperature in Queensland is much warmer than in NSW and Victoria. Consequently, with the cooler climate in NSW and Victoria, especially in winter, the heat pump system may not provide the savings that people in Queensland obtain.

3 a **i** The price of petrol has shown little increase from December to February.
ii The price of petrol has shown marked rises and falls over the period from December to February.
b Both graphs could be improved by starting the vertical scale at 0 cents/litre.

4 a That there is a marked difference between the fuel consumption of the different cars.
b **i** 0.2 L/100 km **ii** 1 L/100 km **iii** 0.2 L/100 km
c Begin the scale on the vertical axis with 0 and use a scale of 1 cm = 0.5 L/100 km instead of 1 cm = 0.2 L/100 km.

5 Yes, as there is no option for a customer to rate the product as unsatisfactory or poor.

6 a An example of a biased question could be: Which of these colours do you prefer – red, black, silver, blue?
b Apart from surveying people, they need to look at the sales figures of all cars. This will give information about the most popular car colour.

7 Teacher to check. **8** Teacher to check.

Power plus

1 a –1 and 1
b There is no relationship between the variables.
c **i** 1 **ii** – 0.2 **iii** – 0.8

2 b, d, f

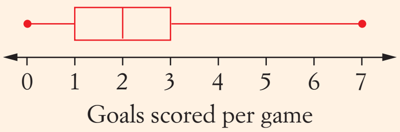
3 a $\bar{x} = 13.35$, median $= 14$, mode $= 14$
b Range $= 10$, IQR $= 15 - 12.5 = 2.5$
c The mean, median, and mode will increase by 4, the range and the interquartile range remain unchanged.

Chapter 5 revision

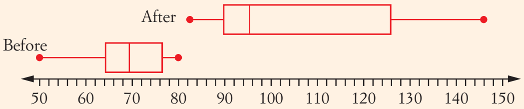
1 a **i** negatively skewed
ii clustering at 7 and 8, no outliers
b **i** negatively skewed
ii clustering at 16 and 17, 10 is an outlier
c **i** positively skewed
ii clustering at 40s and 50s, no outliers

2 a 6.5 **b** 6 **c** 2.5 **d** 12.5 **e** 2

3 a Range $= 7$, IQR $= 3 - 1 = 2$
b 0, 1, 2, 3, 7
c



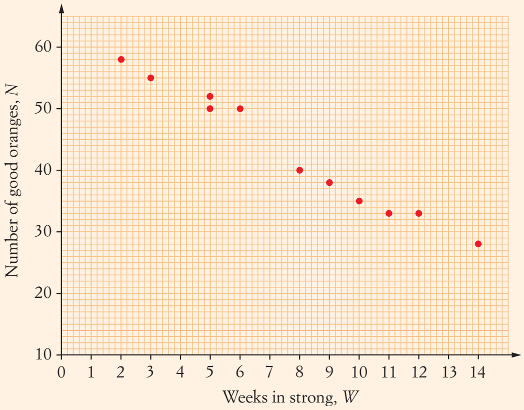
4 a Before: 50, 64, 69, 76, 80; After: 82, 89, 95.5, 126, 146
b



c **i** Range $= 30$, IQR $= 12$ **ii** Range $= 64$, IQR $= 37$
d The pulse rates for after exercise are significantly higher. In fact, all the rates for after exercise are above all the rates for before exercise. The median pulse for after exercise is 95.5 compared to the median pulse of 69 for before exercise. The range and interquartile range are also greater for the after exercise pulse rate.

5 a **i** Both **ii** Stem-and-leaf plot
b The range ($126 - 70 = 56$) is too large.
c **i** median $= 92$ **ii** IQR $= 99.5 - 84 = 15.5$
d 50%

6 a Weeks in storage – this determines how many oranges stay good.
b



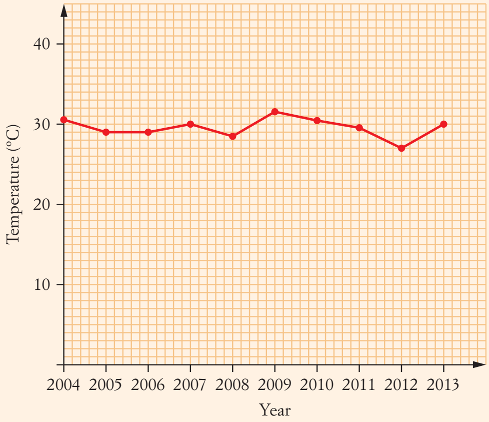
c linear

d The longer the oranges remain in storage, the fewer good oranges there are in the box.

e strong negative correlation

7 a Year

b



c For the last 10 years, the mean maximum temperatures at Blacktown, after staying at 30.6°, have ranged from a low of 27.4° to a high of 31.7°, finishing at a temperature of 30.0° in 2013. This shows there has been little change in temperature for the month of January over the last 10 years.

8 a That the product is healthy.

b There is no data given on the actual fat content in the product. This should also be stated in terms of the daily percentage requirement of fat or in mg of fat.

Chapter 6

SkillCheck

1 a $y = 2$ **b** $x = 12$ **c** $m = 10$
d $w = 4$ **e** $m = 15$ **f** $n = -10$
2 a $5x + 50$ **b** $4y - 4$ **c** $10 - 6y$
3 a $x = 10$ **b** $x = 8$ **c** $a = -12$
d $x = 6$ **e** $x = 8$

Exercise 6-01

1 a $k = 60$ **b** $w = -9$ **c** $y = 2$
d $a = 5$ **e** $x = 8$ **f** $a = 1$
g $r = -6$ **h** $w = 6$ **i** $y = -2$
j $a = 3\frac{1}{2}$ **k** $y = 5\frac{1}{3}$ **l** $a = -1\frac{3}{4}$
2 B **3** D
4 a $y = 10$ **b** $a = -2\frac{1}{2}$ **c** $y = 5$
d $a = -3$ **e** $y = 8$ **f** $x = 1$
g $y = -1$ **h** $x = -10$ **i** $m = 1\frac{1}{2}$
j $y = 12$ **k** $a = -1$ **l** $x = -4$
5 C
6 a $x = 16$ **b** $m = 6$ **c** $y = 4$
d $y = 8$ **e** $y = -7$ **f** $x = 11$
g $m = -8$ **h** $a = -1$ **i** $y = -5$
7 C

8 a $d = \frac{2}{7}$ **b** $y = -\frac{7}{8}$ **c** $k = \frac{8}{9}$
d $y = 3\frac{2}{7}$ **e** $h = \frac{6}{17}$ **f** $p = -1\frac{4}{5}$

Exercise 6-02

1 a B **b** B **c** C
2 a $y = 15$ **b** $a = 9$ **c** $m = 7$
d $k = 57$ **e** $n = -35$ **f** $y = -7$
g $x = 31$ **h** $y = 46$ **i** $m = 18$
j $x = -29$ **k** $x = 24$ **l** $m = 10$
m $n = -\frac{1}{4}$ **n** $n = -\frac{3}{5}$ **o** $d = 3\frac{3}{4}$
3 a $k = 1\frac{7}{8}$ **b** $w = 1\frac{1}{3}$ **c** $x = -1\frac{1}{3}$
d $x = 3$ **e** $y = 3$ **f** $a = -8\frac{3}{5}$
g $p = 9\frac{2}{3}$ **h** $y = 3$ **i** $y = -\frac{5}{6}$
j $w = 10$ **k** $w = 50$ **l** $w = 9\frac{3}{5}$
m $a = \frac{6}{11}$ **n** $y = 60$ **o** $a = 1\frac{11}{13}$
p $m = 3\frac{1}{3}$ **q** $h = 1\frac{22}{23}$ **r** $y = -6\frac{5}{6}$
4 a C **b** A

Exercise 6-03

1 a $m = \pm 12$ **b** $x = \pm 20$ **c** $y = \pm 15$
d $k = \pm 13$ **e** $y = \pm 1$ **f** $w = \pm 4$
g $x = \pm 2$ **h** $t = \pm 4$ **i** $a = \pm 4$
j $k = \pm 6$ **k** $w = \pm 10$ **l** $d = \pm 12$
m $k = \pm 1$ **n** $w = \pm 5$ **o** $x = \pm\frac{1}{2}$
p $m = \pm 6$ **q** $y = \pm 1$ **r** $p = \pm 3$
s $k = \pm 2$ **t** $y = \pm 10$
2 a $m = \pm 2$ **b** $a = \pm 9$ **c** $m \approx \pm 5.3$
d $m \approx \pm 1.9$ **e** $k \approx \pm 0.6$ **f** $x \approx \pm 7.6$
g $k \approx \pm\, 9.8$ **h** $k \approx \pm 9.5$ **i** $y \approx \pm 0.3$
j $a \approx \pm 9.2$ **k** $y \approx \pm 6.2$ **l** $w \approx \pm 7.1$
3 a B **b** B
4 a $x = -2, -1$ **b** $y = -4, -1$ **c** $y = -4, -12$
d $x = -4, 3$ **e** $x = -3, 1$ **f** $x = -8, 5$
5 a $x = 6, -5$ **b** $x = 4$ **c** $x = 11, -6$
d $d = 0, 2$ **e** $x = 5, -2$ **f** $n = 0, -4$
g $k = 0, 7$ **h** $y = 0, 5$ **i** $v = 0, 12$
6 You cannot take the square root of a negative number.
7 a, c, f: cannot find the square root of a negative number.
8 a A **b** C

Exercise 6-04

1 18 mm, 36 mm, 36 mm **2** 57 mm, 19 mm
3 13 cm, 29 cm **4** 61, 62, 63
5 Son is 3, Father is 27 **6** 26 **7** 4
8 Vatha is 22, Chris is 14 **9** 213, 214, 215, 216
10 $x = 35$ **11** 117
12 6 **13** Scott is 11, Mother is 34 **14** 25°, 50°, 105°

Mental skills 6

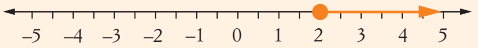
2 **a** 160 **b** 70 **c** 240 **d** 900
e 2600 **f** 900 **g** 140 **h** 300
i 180 **j** 770 **k** 18 **l** 34
m 46 **n** 26 **o** 18 **p** 12
q 40 **r** 8 **s** 14 **t** 24

Exercise 6-05

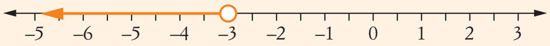
1 **a** 52 **b** 17
2 **a** 36 km/h **b** 86.4 km/h **c** 180 km/h
3 $30.\dot{5}$ km/h
4 43
5 **a** 27°C **b** 0°C **c** 100°C **d** 39°C
6 **a** 11.2 **b** 9 **c** 17.3
7 **a** 15.1 m **b** 31.8 cm
8 **a** 21.0 **b** 105.8 kg
9 **a** 137.3 cm^3 **b** 4.9 m
10 **a** 93 km/h **b** 436 km **c** 7 h
11 **a** $97.50 **b** 620 km
12 **a** 73.9 cm^2 **b** $h = 13.2$ cm

Exercise 6-06

1 **a**

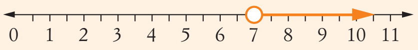


b



c

d



e

f

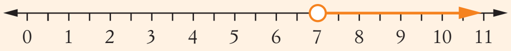
g

h

2 **a** $x < 4$ **b** $x \geq 2$ **c** $x > -6$ **d** $x \leq 1$
3 B
4 **a** $x \geq 1$ **b** $x < 4$ **c** $x > 6$
d $x \leq -1$ **e** $x > -6$ **f** $x < 2$
g $x \geq -4$ **h** $x \geq 25$ **i** $x < 0$

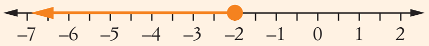
Exercise 6-07

1 **a** $x > 7$



b $y \geq 4$

c $m \leq -2$

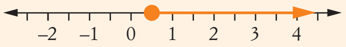


d $x \geq -100$

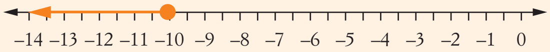
e $x < 5$

f $y > -4$

g $a \geq \frac{1}{2}$

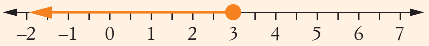


h $w \leq -10$



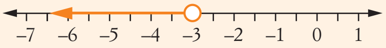
i $a \geq 5$

j $a \leq 3$

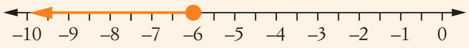


k $a \geq -1$

l $w < -3$

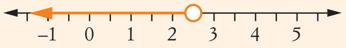


m $a \leq -6$



n $y \leq 3$

o $a < 2\frac{1}{2}$

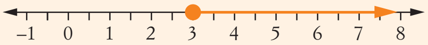


2 A

3 **a** $x \geq 1$ **b** $m \leq 6$ **c** $y \leq -8$
d $w > 0$ **e** $w \leq 0$ **f** $m \geq 3\frac{1}{2}$
g $m \geq -2$ **h** $x \leq 5$ **i** $w > -3$
j $a < 4$ **k** $a \geq 6$ **l** $m \leq 3\frac{1}{2}$
m $m > 3\frac{2}{5}$ **n** $m \geq -2\frac{1}{2}$ **o** $x < 35$

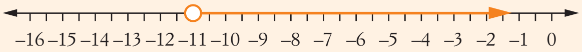
4 D

5 **a** $x \geq 3$



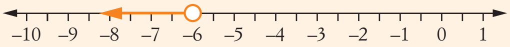
b $y > -8$

c $k > -11$



d $m \leq 0$

e $p < -6$



f $t \leq -4$

6 **a** $x > -3$ **b** $k \leq -12$ **c** $t < -2\frac{2}{5}$
d $x \geq 12$ **e** $w < -1$ **f** $y \geq -2$
g $x \leq 4$ **h** $a > 1$ **i** $d < -5\frac{1}{2}$
j $w < -11$ **k** $x \leq -8$ **l** $p < -4$

Power plus

1 **a** $y = 2\frac{3}{7}$ **b** $y = 2\frac{1}{2}$ **c** $m = -10$

2 $d = -8$ **3** Brett is 7, James is 17.

4 The number is 36.

5 **a**

b

c

Chapter 6 revision

1 **a** $a = 11$ **b** $y = 2\frac{2}{3}$ **c** $a = -3$
d $y = 1$ **e** $m = 11$ **f** $a = 28$
g $h = 1\frac{5}{8}$ **h** $y = \frac{13}{20}$

2 **a** $w = 6$ **b** $y = \frac{35}{4}$ **c** $a = -3$
d $m = 3$ **e** $s = 4$ **f** $x = 1\frac{2}{3}$

3 A

4 **a** $y = \pm 2$ **b** $p = \pm 10$ **c** $x = \pm 3$
d $m = \pm 1$ **e** $w = \pm 5$ **f** $x = -7, -1$
g $h = 9, -1$ **h** $u = 7, -11$ **i** $k = 0, -5$
j $m = 0, 2$ **k** $b = -10$ **l** $w = 9, 0$

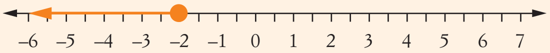
5 92, 93, 94, 95 **6** $x = 19$ **7** 120 m

8 **a** 160 mm^3 **b** 300 m^2

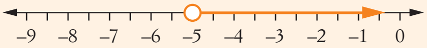
9 **a** $x \geq 0$

b $x < 3$

c $x \leq -2$



d $x > -5$



10 **a** $y \geq 16$ **b** $y \leq -7\frac{1}{2}$ **c** $a > -5$
d $x > -3$ **e** $a < -16$ **f** $x \leq -3$

Mixed revision 2

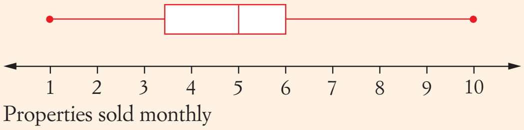
1 **a** **i** negatively skewed
ii no outliers, clusters at 7 and 8
b **i** positively skewed
ii outlier 98, clustering in 40s and 50s

2 **a** $81n^4m^8$ **b** $20p^9q^3$ **c** $4a^4b^3$
d 1 **e** $\frac{1}{27}$ **f** $\frac{125}{8}$

3 6

4 **a** $b = 5$ **b** $y = 3$ **c** $m = 11$ **d** $a = -32$

5 **a** range = 9, interquartile range = 2.5
b 1, 3.5, 5, 6, 10
c



6 **a** $\frac{1}{9y^2}$ **b** $\frac{3}{y^2}$ **c** $64x^9y^{12}$
d $24h^{14}k^{16}$ **e** $-3v^3w^4$ **f** $\frac{4b^2}{3a^2}$

7 **a** $\frac{7g}{20}$ **b** $\frac{-x}{15}$ **c** $\frac{3t+1}{8}$ **d** $\frac{12-5r}{10}$

8 **a** $\frac{6x}{7y}$ **b** 2 **c** $\frac{9d}{2}$ **d** $8v^2$

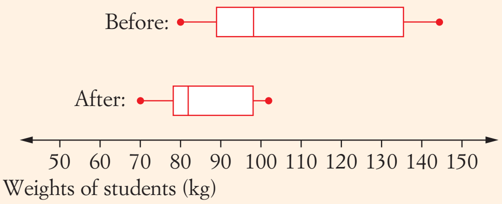
9 **a** weak negative **b** strong negative
c weak positive

10 **a** $a = 8.5$ **b** $k = 8\frac{3}{4}$ **c** $g = 4$ **d** $w = 10$

11 **a** $7y - 63$ **b** $-8n^2 - nm$
c $-5w + 6$ **d** $24f^3 + 28f^2g$

12 **a** Before: 80, 87.5, 96, 134, 142 After: 70, 77, 81, $97\frac{1}{2}$, 101

b



c Before: range = 62, interquartile range = 46.5
After: range = 31, interquartile range = 20.5

d Yes, there is a significant difference as both the range and the interquartile have decreased markedly meaning the students have reached more consistent weights.

13 **a** $3ab^2 - 24a^2b$ **b** $18r^2 - 60r + 32$

14 **a** $7(w + 6)$ **b** $p(p - 25)$
c $-4(n + 11)$ **d** $(7a - 2)(3 - 5a)$

15 **a** **i** stem-and-leaf plot
ii stem-and-leaf plot
b median = 42 **c** interquartile range = 16

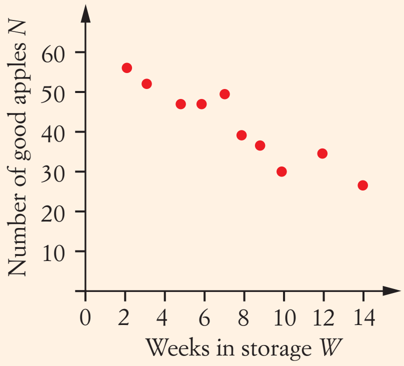
16 **a** $p = \pm 12$ **b** $y = \pm 2$ **c** $x = \pm 1$ **d** $t = \pm 10$
e $q = \pm 8.8$ **f** $x = \pm 4.8$ **g** $w = \pm 6.2$

17 **a** $2q^2 + 15q + 27$ **b** $3f^2 - 13f - 56$
c $20g^2 - 47g + 21$ **d** $9x^2 - 49$

18 **a** $(r + 8)(r + 3)$ **b** $(y - 30)(y - 1)$
c $(x + 12)(x - 3)$ **d** $(t - 9)(t + 8)$

19 **a** The independent variable is W, the weeks in storage. The number of weeks in storage is set first, after which time the number of good apples is counted.

b



c The number of good apples decreases the longer the apples are in storage.

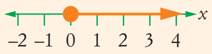
d There is a strong and negative relationship between the variables W and N.

20 **a** $x = -7, x = -1$ **b** $h = 9, h = -1$
c $u = -11, u = 7$ **d** $w = 0, w = 9$

21 **a** 20, 21, 22 **b** 20, 40, 120

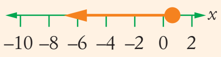
22 **a** 480 mm^3 **b** 150 cm^2 **c** 3.6 m

23 **a**

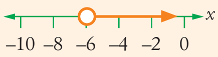


b

c



d



24 **a** $n \geq 5$ **b** $a \leq -3$ **c** $h > -8$ **d** $x < 3\frac{3}{5}$

Chapter 7

SkillCheck

1 **a** -1 **b** 29 **c** -3 **d** 69
2 **a** 625 **b** 3125 **c** 1 **d** $\frac{1}{25}$

Exercise 7-01

1 **a** $D = 190T$
b **i** 3.8 km **ii** 8.55 km
c 1 h 5 min

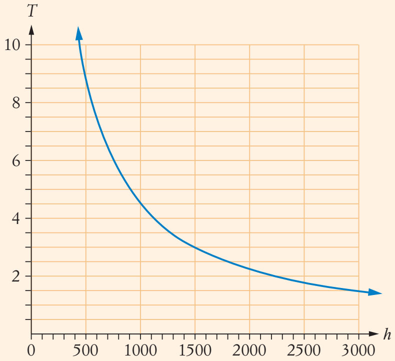
2 **a** $E = 26.2$ h **b** \$183.40 **c** 5.5 h
3 **a** $I = \frac{16D}{425}$ **b** \$33.88 **c** \$67.76
4 A **5** $b = 2.5a$
6 **a**

h	*c*
1	\$ 7.50
2	\$15
3	\$22.50

b $c = 7.5$ h **c** \$45 **d** 11
e 7.5. It is the same.
7 C
8 **a** $F = 0.006m$ **b** 15 L/100 km
9 A
10 **a** 22.8 kg **b** 84.1 kg

Exercise 7-02

1 **a** $T = \frac{920}{s}$ **b** 10 h 13 min **c** 92 km/h
2 C
3 **a** **i** 15°C **ii** 1.8°C
b **i** 562.5 m **ii** 200 m
c



4 4533 people
5 B **6** $\frac{14}{15}$
7 **a** 8 min **b** 4 people
8 **a** $b = \frac{8}{a}$ **b** $b = \frac{100}{a}$
9 **a** $F = \frac{112}{L}$ **b** 6 beats/sec **c** 25 cm
10 **a** $y = \frac{1}{16}$ **b** $x = 1\frac{1}{4}$
11 **a** 2.5 h **b** 5 friends

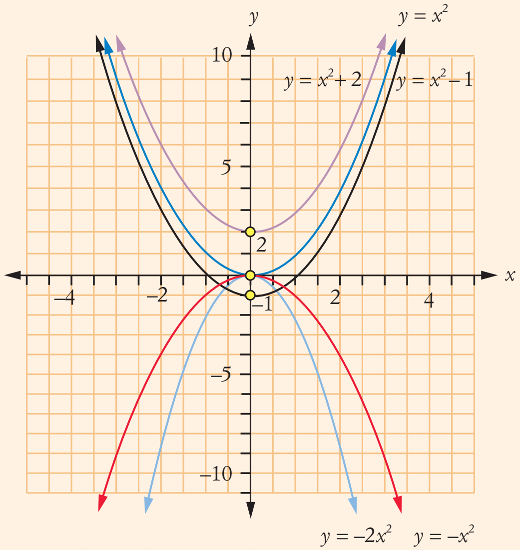
Answers

Exercise 7-03

1 a i £26 **ii** £58
b i $A28 **ii** $A92
c i approx. £7 **ii** approx. £30
2 a i 2 km **ii** 20 km **iii** 34 km
b i 50 furlongs **ii** 122 furlongs **iii** 180 furlongs
c 60 km **d** 500 furlongs
3 a i ¥16 000 **ii** ¥63 000 **iii** ¥78 000
b i $A250 **ii** $A760 **iii** $A920
4 a −18°C **b** 10°C **c** 26°C
d 32°F **e** 14°F **f** 86°F
5 a 4.9 ha **b** 2 ha **c** 10.8 acres **d** 12.2 acres
e i 32 000 m^2 **ii** 3.2 ha **iii** 7.8 acres
6 a i ₱62 **ii** ₱210 **iii** ₱360
b $A12 **c** $A5 **d** ₱490

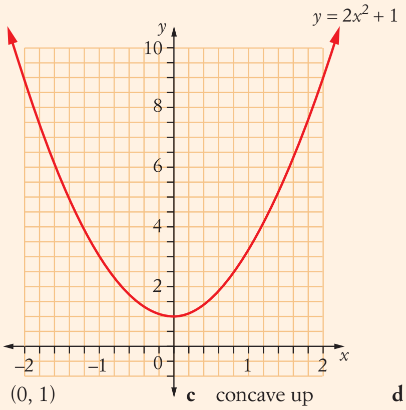
Exercise 7-04

1 a



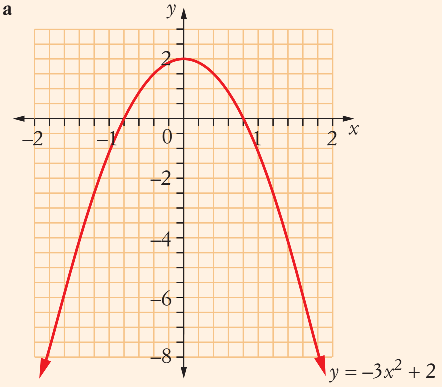
b i $y = x^2, y = x^2 + 2, y = x^2 - 1$
ii $y = -x^2, y = -2x^2$
iii $y = x^2, y = -x^2, y = -2x^2$
2 A **3** C
4 a vi **b** ix **c** i **d** xi
e x **f** iii **g** ii **h** xii
i viii **j** v **k** vii **l** iv
5 a $y = -x^2$ **b** $y = x^2$ **c** $y = -x^2 - \frac{1}{4}$
d $y = -x^2 - 9$ **e** $y = \frac{1}{2} - x^2$ **f** $y = x^2 + 9$

6 a



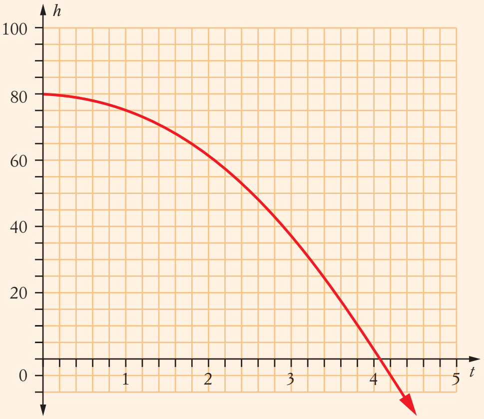
b (0, 1) **c** concave up **d** $y = 1$

7 a



b (0, 2) **c** $x = 0$ **d** $y = 2$
8 A
9 a ii **b** vii **c** iv **d** x
e v **f** ix **g** i **h** xi
i viii **j** xii **k** vi **l** iii
10 a i narrower **ii** up **iii** (0, 3)
b i wider **ii** up **iii** (0, 1)
c i narrower **ii** down **iii** (0, −5)
d i wider **ii** down **iii** (0, −12)
11 a $x = \pm 4$ **b** $x = \pm 11$

12 a



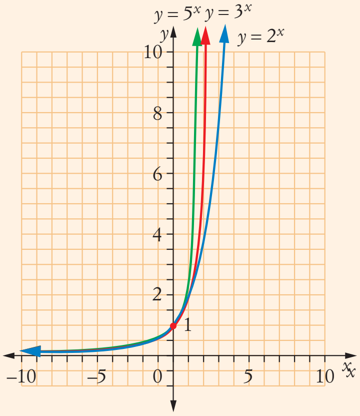
b 80 m **c** 36.8 m **d** 4.1 s **e** 3.95 s
13 a $x = \pm 9$ **b** $x = \pm 14.02$

Mental skills 7

2 **a** 3.5 **b** 2.4 **c** 0.12 **d** 0.36
e 0.8 **f** 0.027 **g** 0.2 **h** 8.8
i 0.24 **j** 0.012 **k** 1.8 **l** 0.028
4 **a** 66.3 **b** 6630 **c** 6.63 **d** 0.663
e 6.63 **f** 663 **g** 0.663 **h** 663
i 6630 **j** 66.3 **k** 0.663 **l** 0.0663

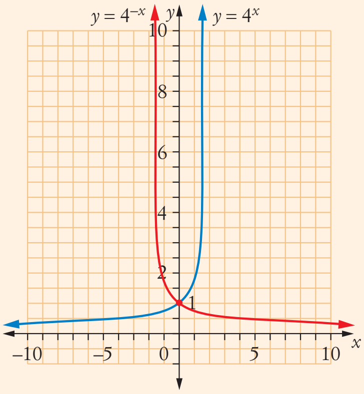
Exercise 7-05

1 **a**



b 1 **c** becomes steeper, in 1st quadrant

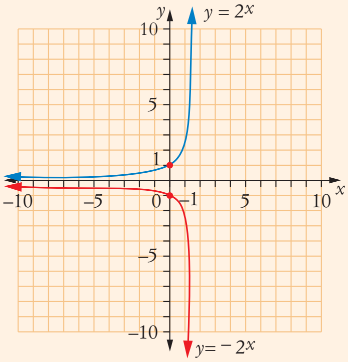
2 **a**



b **i** $y = 4^{-x}$ **ii** $y = a^{-x}$

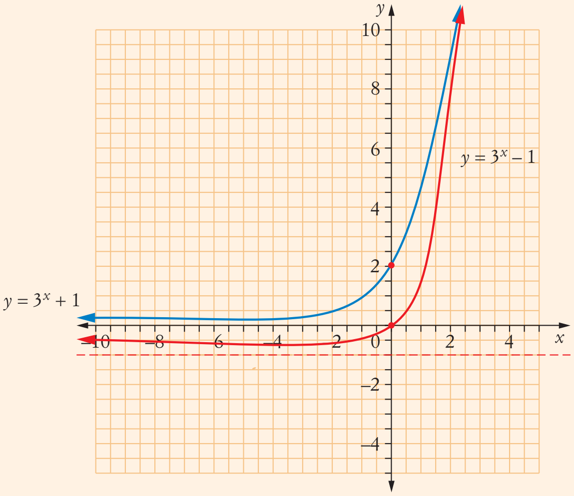
3 B

4 **a**

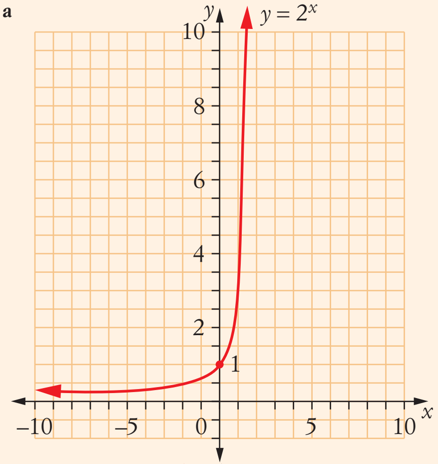


b one is the reflection of the other in the *x*-axis **c** $y = -a^x$

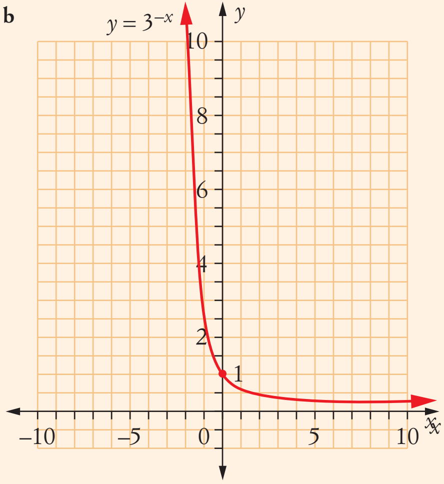
5 Same shape, shifted down 2 units.



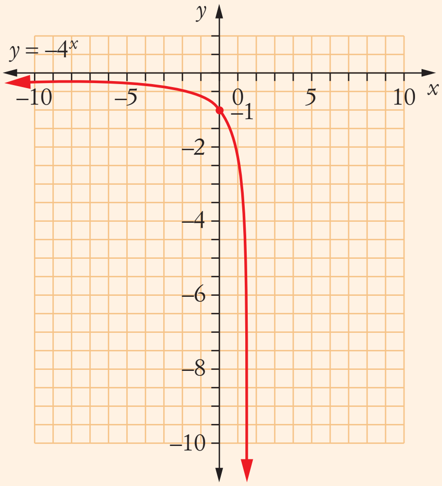
6 **a**



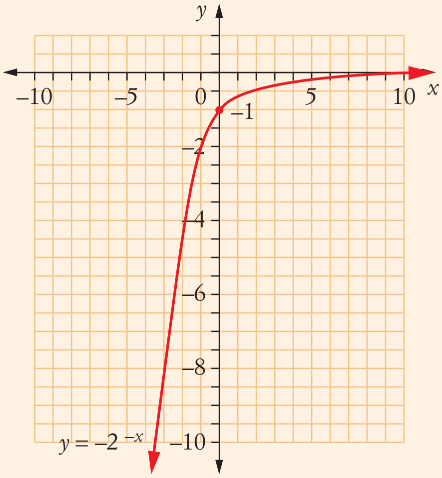
b



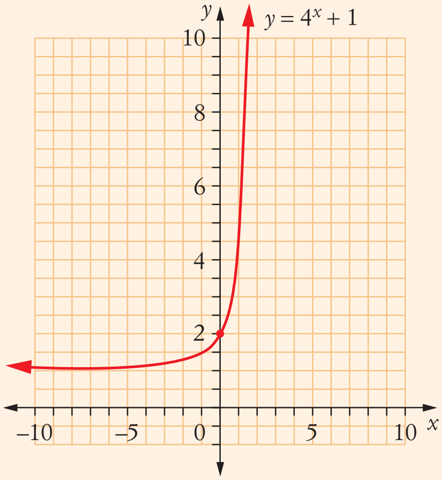
c



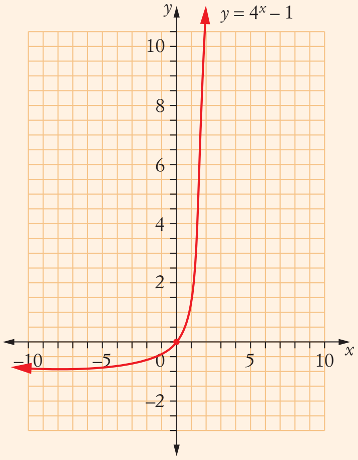
d



e



f



7 $y = 4^{-x}$

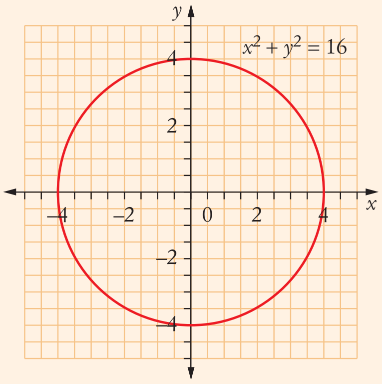
Exercise 7-06

1 **a** centre (0, 0), $r = 2$ **b** centre (0, 0), $r = 6$
c centre (0, 0), $r = 8$ **d** centre (0, 0), $r = 10$
e centre (0, 0), $r = 9$ **f** centre (0, 0), $r = 5$

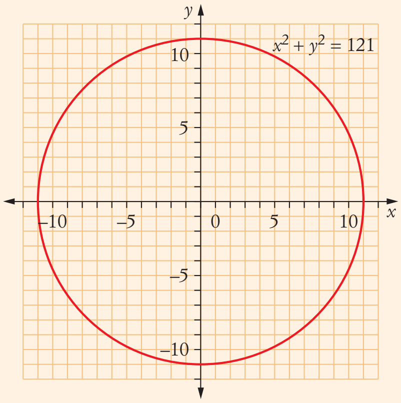
2 D **3** D

4 **a** $x^2 + y^2 = 1$ **b** $x^2 + y^2 = 9$
c $x^2 + y^2 = 25$ **d** $x^2 + y^2 = \frac{1}{9}$

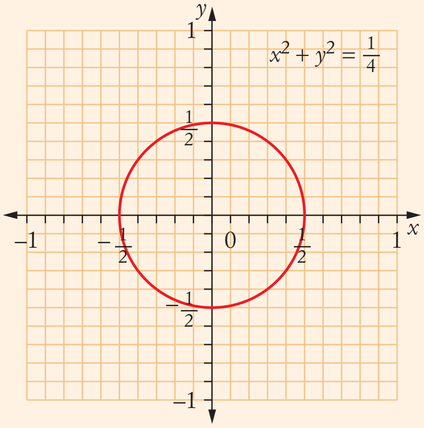
5 **a**



b



c



6 C

7 **a** $\text{LHS} = 8^2 + 6^2$

$= 64 + 36$

$= 100$

$= \text{RHS}$

$\therefore$ (8, 6) lies on $x^2 + y^2 = 100$

b $\text{LHS} = 5^2 + 9^2$

$= 25 + 81$

$= 106$

$\neq \text{RHS}$

$\therefore$ (5, 9) does not lie on $x^2 + y^2 = 100$

c outside

8 **a** inside **b** on **c** outside

d inside **e** outside

Exercise 7-07

1 **a** P **b** L **c** E **d** L

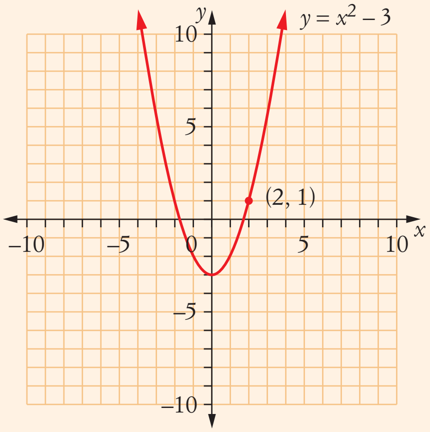
e C **f** L **g** P **h** L

i P **j** E **k** P **l** C

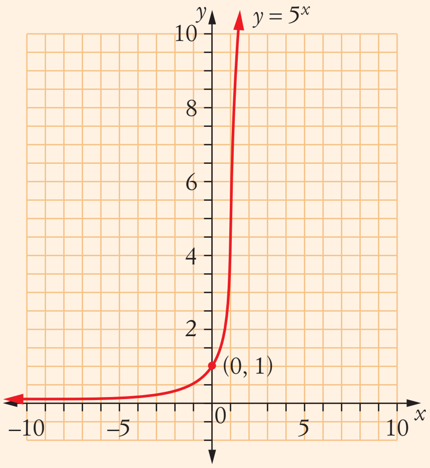
2 **a** vii **b** x **c** viii **d** iv **e** i

f vi **g** iii **h** v **i** ii **j** ix

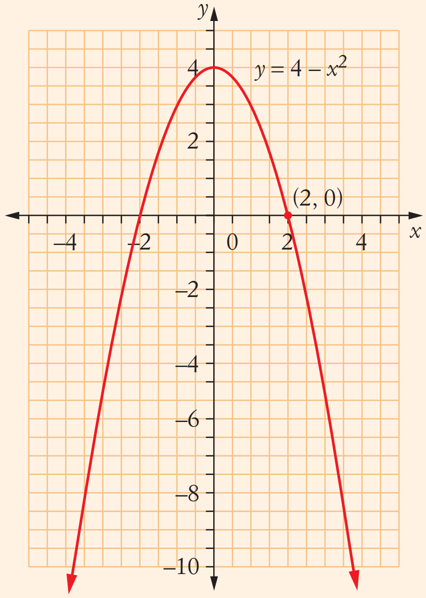
3 **a**



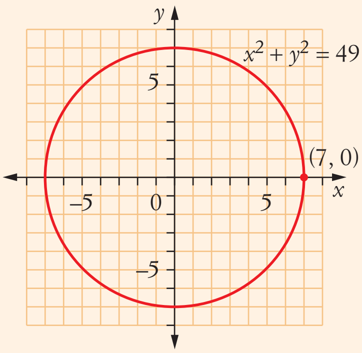
b



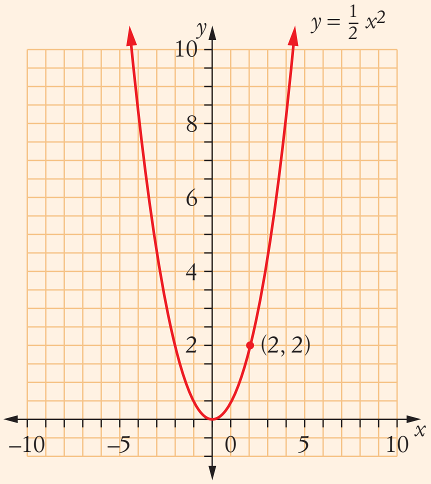
c



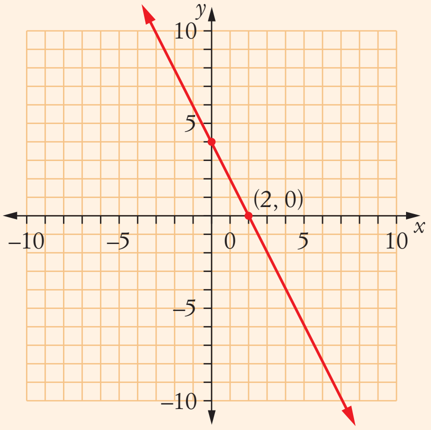
d



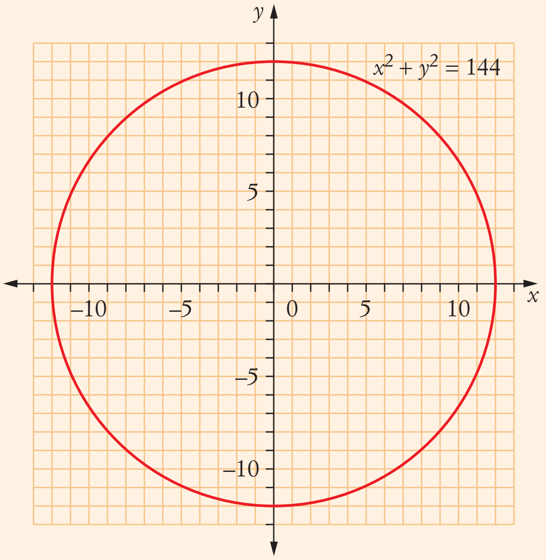
e



f



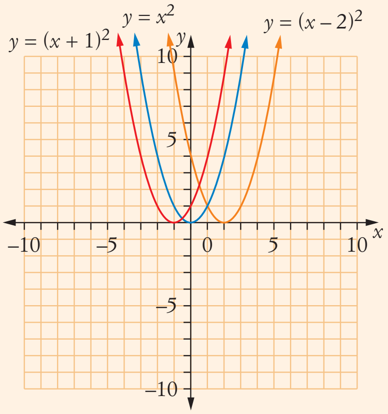
g



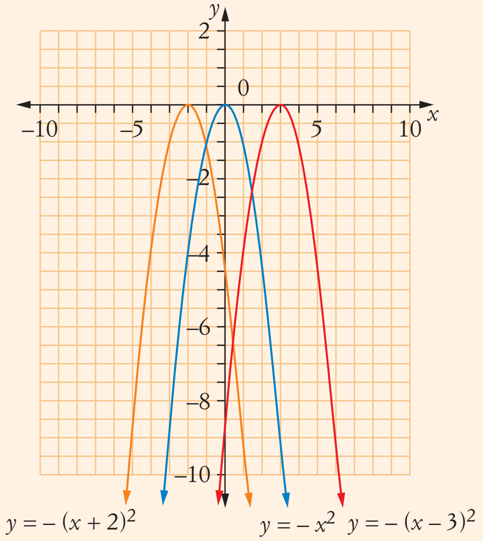
4 **a** 1 **b** 3 **c** −6 **d** 1

Power plus

1 **a** **b** **c**

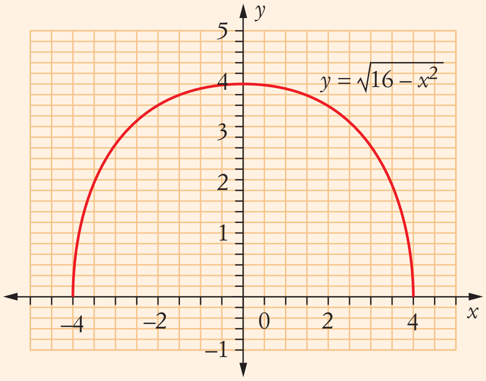


2 **a** **b** **c**

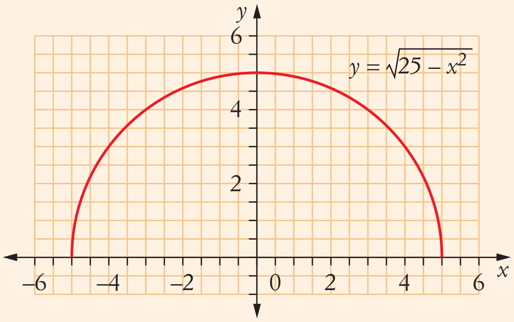


3 If a is positive, the parabola $y = x^2$ is shifted left a units. If a is negative, the parabola $y = x^2$ is shifted right a units.

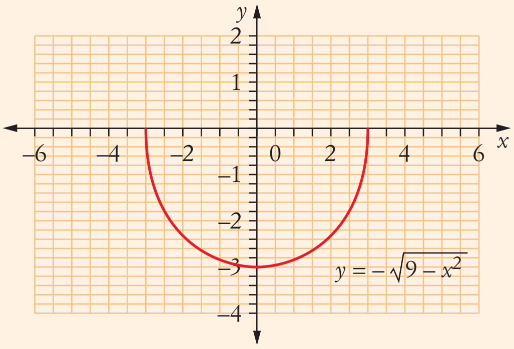
4 **a** centre (0, 0) and $r = 4$



b centre (0, 0) and $r = 5$



c centre (0, 0) and $r = 3$



Chapter 7 revision

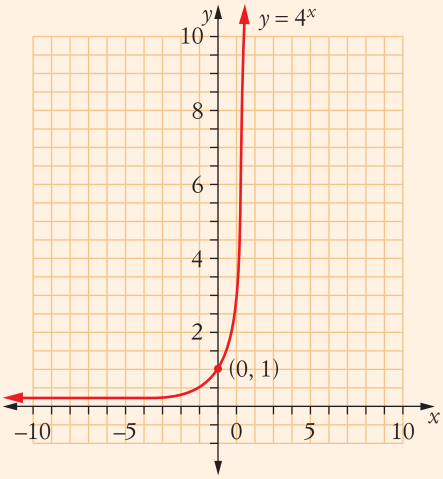
1 $H = 310.5$ **2** 10°C

3 a £46 **b** $A85

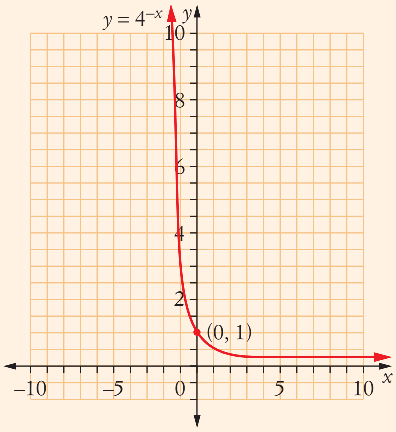
4 a iii **b** vi **c** i

d v **e** iv **f** ii

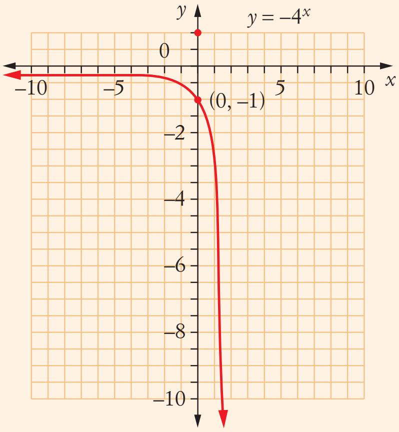
5 a



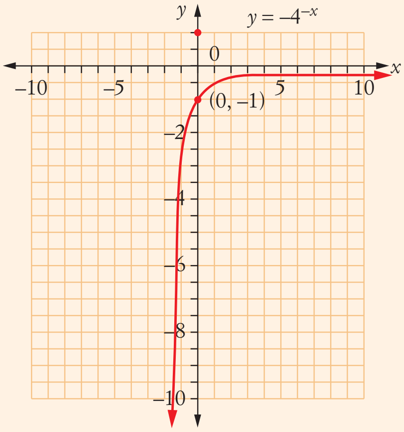
b



c



d



6 a centre (0, 0), $r = 10$ **b** centre (0, 0), $r = 6$

c centre (0, 0), $r = 7$

7 $x^2 + y^2 = 64$

8 a iv **b** iii **c** ii **d** x

e v **f** viii **g** xii **h** vii

i ix **j** i **k** xi **l** vi

Chapter 8

SkillCheck

1 a 35 **b** 33.2 **c** 5

2 a 55 **b** 23 **c** 38

d 60 **e** 50 **f** 160

3 a h: 17, o: 8, a: 15 **b** h: p, o: r, a: q

c h: EF, o: EG, a: GF

4 a 64° **b** 26° **c** 12°

5 a 50°19′ **b** 31°56′ **c** 64°19′

6 a 0.8480 **b** 0.7760 **c** 0.1539

d 64.9839 **e** 13.9884 **f** 13.7044

7 a 45°48′ **b** 33°11′ **c** 5°21′

Exercise 8-01

1 a 30 m **b** 2.5 m **c** 16 m

d 50.5 m **e** 115.4 m **f** 5.7 m

2 a Yes **b** No **c** Yes

d No **e** No **f** Yes

3 a 48 cm **b** 31.4 cm **c** 170 cm

d 12.9 m **e** 28.9 m **f** 35.3 cm

4 a 192 cm^2 **b** 360 mm^2 **c** 84 mm^2

d 22 m^2 **e** 123.0 cm^2

5 92.2 km **6** 17.3 m **7** 120 m **8** B **9** 9.2 km

Exercise 8-02

1 a $\sin\theta = \frac{55}{73}$ $\cos\theta = \frac{48}{73}$ $\tan\theta = \frac{55}{48}$

b $\sin\theta = \frac{e}{f}$ $\cos\theta = \frac{g}{f}$ $\tan\theta = \frac{e}{g}$

c $\sin R = \frac{ST}{RT}$ $\cos R = \frac{RS}{RT}$ $\tan R = \frac{ST}{RS}$

d $\sin\alpha = \frac{n}{k}$ $\cos\alpha = \frac{m}{k}$ $\tan\alpha = \frac{n}{m}$

e $\sin M = \frac{3.6}{8.5}$ $\cos M = \frac{7.7}{8.5}$ $\tan M = \frac{3.6}{7.7}$

f $\sin W = \frac{XY}{WY}$ $\cos W = \frac{WX}{WY}$ $\tan W = \frac{XY}{WX}$

2 a $\frac{4}{5}$ **b** $\frac{4}{3}$ **c** $\frac{3}{5}$ **d** $\frac{4}{5}$

3 a α **b** β **c** β

d α **e** α **f** β

4 a i $\frac{24}{7}$ **ii** $\frac{7}{25}$ **iii** $\frac{24}{25}$ **iv** $\frac{7}{24}$

b i $\frac{u}{v}$ **ii** $\frac{v}{w}$ **iii** $\frac{u}{w}$ **iv** $\frac{v}{u}$

c i $\frac{FG}{FH}$ **ii** $\frac{FH}{HG}$ **iii** $\frac{FG}{HG}$ **iv** $\frac{FH}{FG}$

d i $\frac{84}{13}$ **ii** $\frac{13}{85}$ **iii** $\frac{84}{85}$ **iv** $\frac{13}{84}$

e i $\frac{b}{c}$ **ii** $\frac{c}{a}$ **iii** $\frac{b}{a}$ **iv** $\frac{c}{b}$

f i $\frac{QS}{RS}$ **ii** $\frac{RS}{QR}$ **iii** $\frac{QS}{QR}$ **iv** $\frac{RS}{QS}$

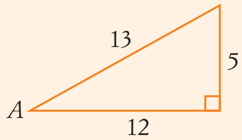
5 a $\tan Y = \frac{60}{11}$ **b** $\tan X = \frac{11}{60}$

c $\sin X = \frac{11}{61}$, $\cos Y = \frac{11}{61}$ **d** $\cos X = \frac{60}{61}$, $\sin Y = \frac{60}{61}$

6 C

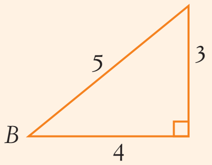
7 A

8 a



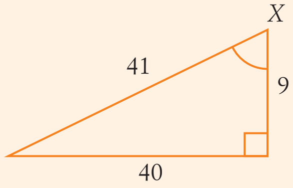
$\sin A = \frac{5}{13}$ $\cos A = \frac{12}{13}$

b



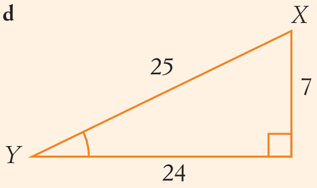
$\cos B = \frac{4}{5}$ $\tan B = \frac{3}{4}$

c



$\sin X = \frac{40}{41}$ $\tan X = \frac{40}{9}$

d



$\cos Y = \frac{24}{25}$ $\tan Y = \frac{7}{24}$

Exercise 8-03

1 a 167.7 **b** 131.0 **c** 618.3

d 2.2 **e** 24.5 **f** 3.5

2 a 228.2 cm **b** 141.4 m

c 671.4 m **d** 1053.1 cm

3 a 102.3 **b** 62.9 **c** 96.4

d 21.5 **e** 245.5 **f** 378.3

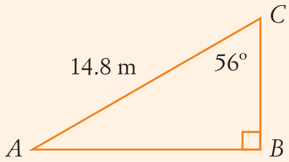
4 a 1203.3 cm **b** 414.2 mm

c 6.7 m **d** 1044.3 cm

5 C **6** 782 cm **7** 4.9 km

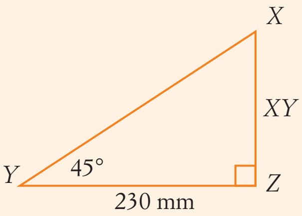
8 12.1 m **9** 5.1 m **10** 77.3 m

11



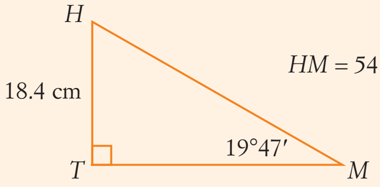
$AB = 12.3$ m

12



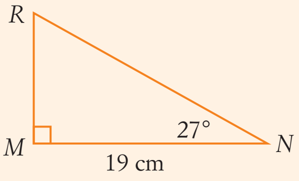
XY = 325 mm

13



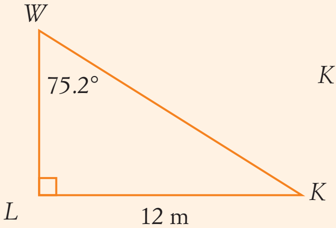
HM = 54.4 cm

14



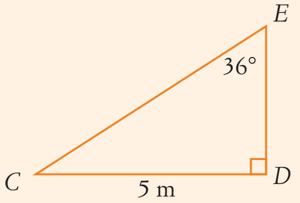
MR = 10 cm

15



KW = 12.4 m

16

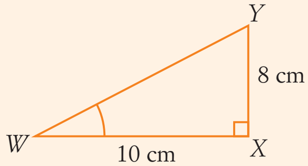


DE = 6.88 m

Exercise 8-04

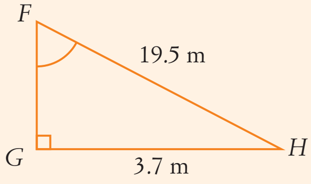
1	**a**	41°	**b**	64°	**c**	60°	**d**	81°
	e	61°	**f**	30°	**g**	6°	**h**	45°
2	**a**	64°59′	**b**	54°35′	**c**	36°52′		
	d	20°10′	**e**	43°32′	**f**	72°33′		
	g	27°2′	**h**	41°21′				
3	**a**	34°	**b**	51°	**c**	60°		
	d	29°	**e**	67°	**f**	54°		
4	**a**	36°42′	**b**	25°12′	**c**	36°52′		
	d	31°30′	**e**	26°47′	**f**	48°11′		
5	**a**	12°	**b**	68°	**c**	27°		
6		10°	**7**	37°	**8**	B		
9		71°	**10**	10°	**11**	50°		

12



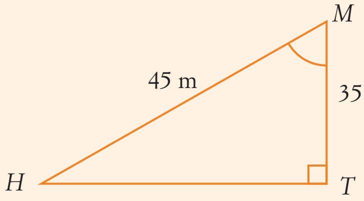
∠*W* = 39°

13



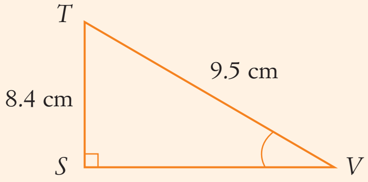
∠*F* = 10° 56'

14



∠*M* = 38.9°

15



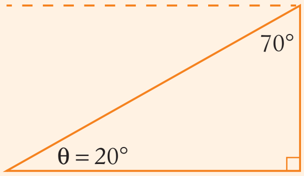
∠*V* = 62°

Mental skills 8

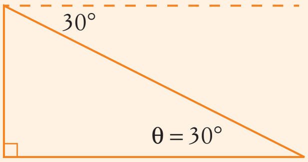
2	**a**	2, 5	**b**	3	**c**	2, 3, 6
	d	3, 5	**e**	2	**f**	2, 3, 5, 6
	g	5	**h**	3		
3	**a**	4, 9	**b**	4, 10	**c**	9
	d	10	**e**	4, 9	**f**	none of these
	g	9	**h**	4, 9, 10		

Exercise 8-05

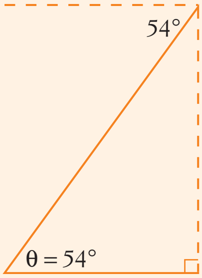
1 a



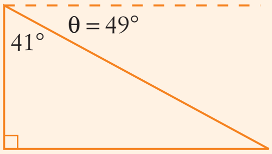
b



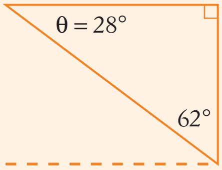
c



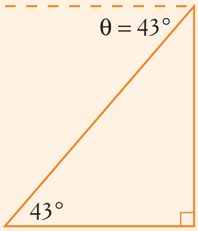
2 a



b



c

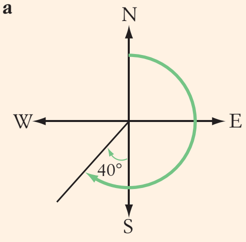


3 127 m **4** 2224 m or 2.224 km
5 177 m **6** 180 m **7** 9°
8 14°29′ **9** 50° **10** 18°47′
11 12.56 m or 1256 cm **12** 14.5 m **13** 970 m
14 57 m

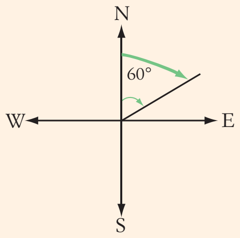
Exercise 8-06

1 a 243° **b** 290° **c** 040°
d 115° **e** 210° **f** 140°
g 312° **h** 253° **i** 065°
2 a 000° **b** 090° **c** 180° **d** 270°
e 038° **f** 125° **g** 330° **h** 225°
i 072° **j** 187°
3 SW

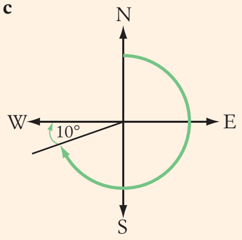
4 a



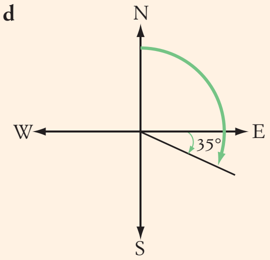
b



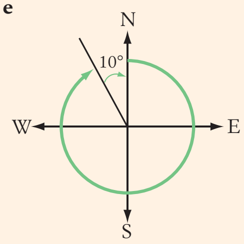
c



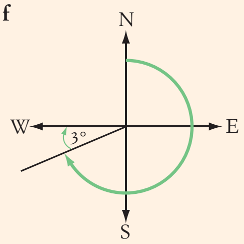
d



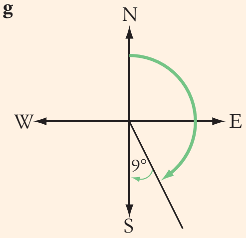
e



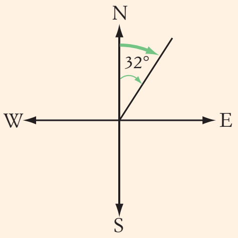
f



g

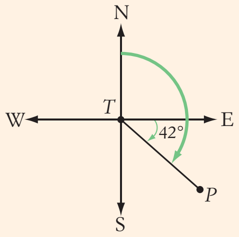


h

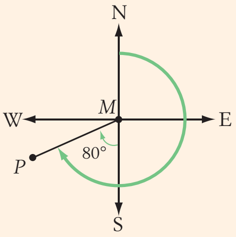


5 a NNW **b** 337.5°

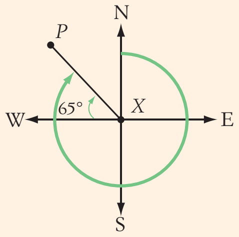
6 a



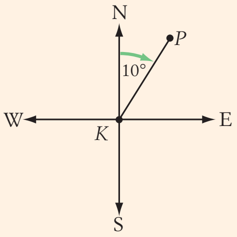
b



c



d

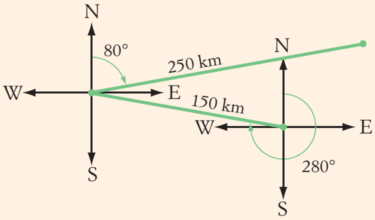


7 240° **8** 280°

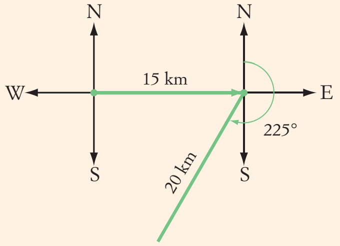
9 a 45° **b** 90° **c** 135° or 225°

10 ESE

11 a



b



12 a 230° **b** 270° **c** 160°

d 340° **e** 050° **f** 090°

Exercise 8-07

1 a 21 km **b** 260°

2 a 37° **b** 163 km **c** 143°

3 a 63 km **b** 29 km **c** 025°

4 a 12.2 km **b** 035°

5 a 18.5 km **b** NNW

6 45.7 km

7 a 13.509 km **b** 321°

8 a 2122 km **b** 330°

9 a 15 km **b** 26 km

10 a 261.08 km **b** 167.82 km

11 6.558 km

12 a 322 nautical miles **b** 276°45′

Power plus

1 a i 0.342, 0.342 **ii** 0.731, 0.731

iii 0.819, 0.819 **iv** 0.996, 0.996

b Each pair of trigonometric ratios has the same value.

c The pairs of angles are complementary (add to 90°).

d 60°

e i 15° **ii** sin **iii** 18°

iv cos **v** 25° **vi** 32°

f $\sin x = \cos(90° - x)$ **g** Teacher to check.

2 a 1932 m **b** 31°

3 a $\cos 30° = \frac{\sqrt{3}}{2}$ **b** $\tan 30° = \frac{1}{\sqrt{3}}$

4 a 24°12′26″ **b** 63°17′3″

5 Teacher to check.

6 $RP = 75$ m $RQ = 60$ m

7 a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** 1

d $\frac{4}{5}$ **e** $\frac{3}{5}$ **f** 1

8 a 1 **b** 1

9 Teacher to check. **10** 1.6 km

Chapter 8 revision

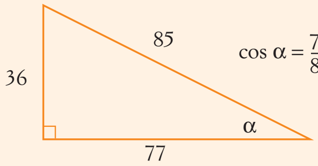
1 a 62.2 cm **b** 21.0 m **c** 4.7 km

2 a Yes **b** No

3 a 9.5 m **b** 152.2 cm

4 a $\frac{33}{65}$ **b** $\frac{33}{56}$ **c** $\frac{56}{65}$ **d** $\frac{33}{65}$

5 $\cos \alpha = \frac{77}{85}$ $\tan \alpha = \frac{36}{77}$



$\cos \alpha = \frac{77}{85}$ $\tan \alpha = \frac{36}{77}$

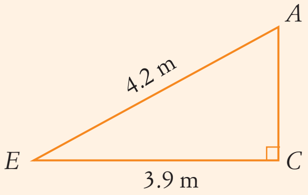
6 a 14.49 **b** 72.50 **c** 0.87

7 a 98.9 **b** 21.1 **c** 23.2

8 a 69° **b** 55° **c** 70°

9 a 33° **b** 28° **c** 38°

10

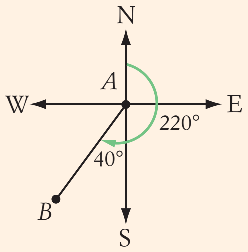


∠A = 68°13′

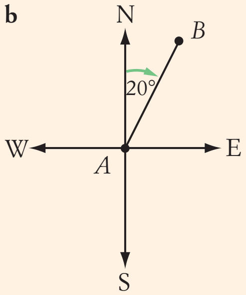
11 195 m **12** 29°

13 a 320° **b** 140°

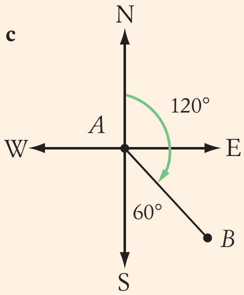
14 a



b



c



15 a 1281 km **b** 024°

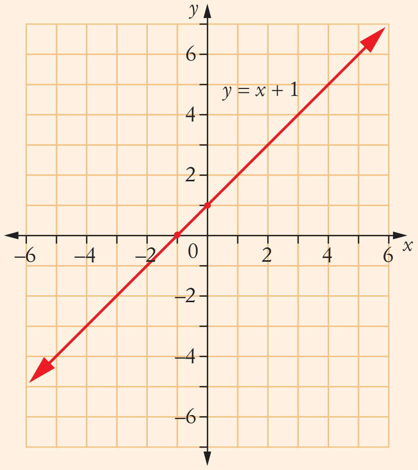
Chapter 9

SkillCheck

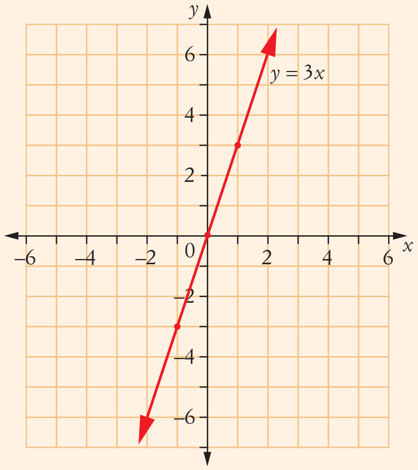
1 **a** 5 **b** 13 **c** 6 **d** −1

2 **a** −11 **b** 1 **c** 7 **d** $5\frac{1}{2}$

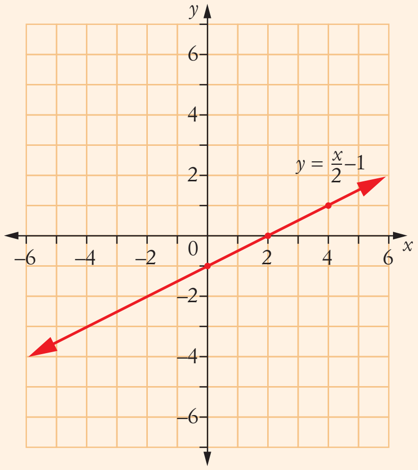
3 **a**



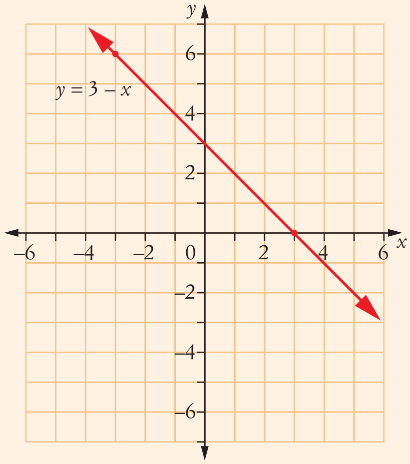
b



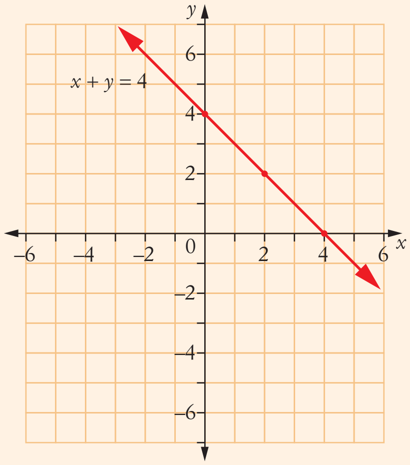
c



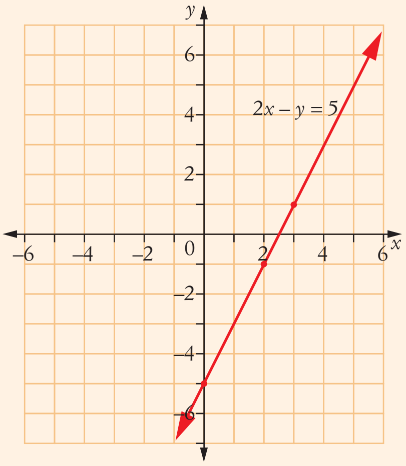
d



e



f



4 $(-2, 3)$ lies on a, d

5 **a** For $y = 2x + 1$, when $x = 2$, $y = 2 \times 2 + 1 = 5$

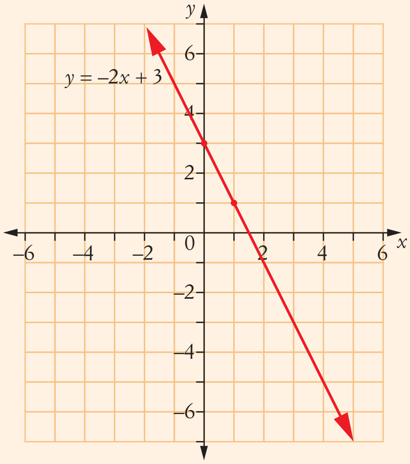
$\therefore (2, 5)$ lies on $y = 2x + 1$

For $x + y = 7$, when $x = 2$, $y = 5$, $2 + 5 = 7$

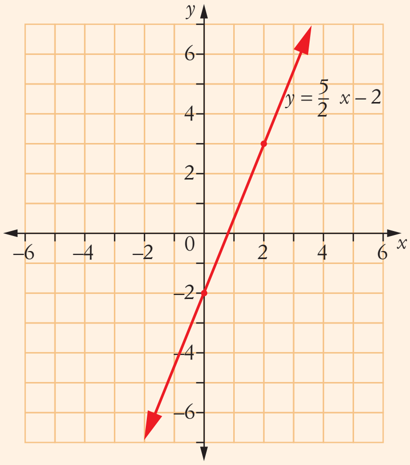
$\therefore (2, 5)$ lies on $x + y = 7$

b $(2, 5)$

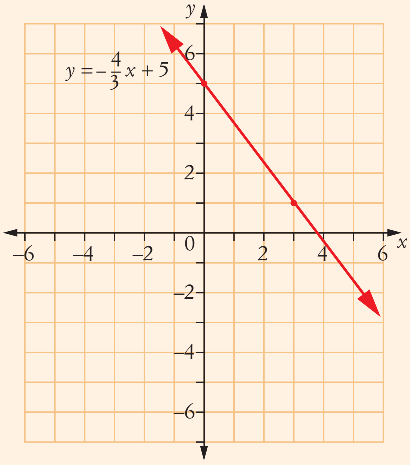
6 a $m = -2, b = 3$



b $m = \frac{5}{2}, b = -2$



c $m = -\frac{4}{3}, b = 5$



Exercise 9-01

1 a $x = 3, y = -1$ **b** $x = 2, y = 1$

c $x = -1, y = -5$

2 a $x = 1, y = 2$ **b** $x = -5, y = -9$

c $x = 1, y = 2$ **d** $x = -\frac{1}{2}, y = 2\frac{1}{2}$

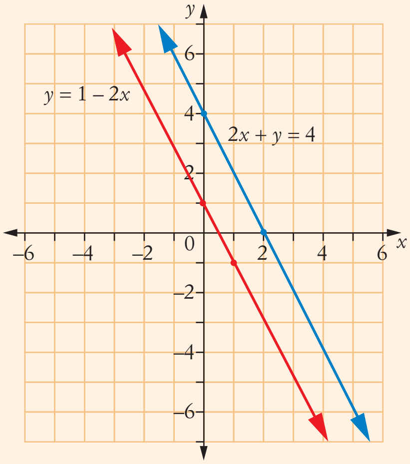
e $x = -2, y = -9$ **f** $x = 5, y = -4$

g $x = \frac{1}{2}, y = 6\frac{1}{2}$ **h** $x = 3, y = 2$

i $x = 5, y = 1\frac{1}{2}$ **j** $x = 5, y = 8$

k $x = 1\frac{1}{2}, y = 2\frac{1}{2}$ **l** $x = 4, y = 0$

3 a



b The lines are parallel.

Exercise 9-02

1 a $d = -3, k = 2$ **b** $x = 5, w = 4$

c $g = 2, h = -\frac{2}{5}$ **d** $n = 3.25, p = -1$

e $q = 5, r = -4$ **f** $k = -4\frac{3}{5}, x = 5$

g $c = 1\frac{1}{2}, e = 1$ **h** $k = 3, y = -2$

i $a = 2, f = 2$

2 a $d = -14, k = 6$ **b** $a = 1, c = 1$

c $h = 3, y = 4$ **d** $e = 3, x = \frac{1}{3}$

e $q = 3, w = 6\frac{1}{2}$ **f** $c = -2, p = 3$

g $m = -\frac{2}{3}, y = 4$ **h** $a = -1, r = 5\frac{1}{2}$

i $x = 2, w = 2$

3 a $q = -3, w = 3$ **b** $m = -9, x = 7$

c $d = 23, h = -7$ **d** $g = -1, n = 3$

e $h = 0, m = 2$ **f** $e = -4, y = 3$

g $q = 1, w = -4$ **h** $a = \frac{1}{2}, d = \frac{1}{2}$

i $k = 5, p = -2$ **j** $a = -2, f = -2$

k $c = -64, r = -38$ **l** $x = -4, y = -3$

Exercise 9-03

1 a $x = 2, y = 5$ **b** $x = \frac{3}{5}, y = 3\frac{4}{5}$

c $x = 7, y = 2$ **d** $x = 2, y = -2$

e $x = 5, y = -1$ **f** $x = 4, y = 2$

2 **a** $x = 9, y = 21$ **b** $x = 5, y = 3$
c $x = -\frac{1}{4}, y = 2$ **d** $x = -3, y = 1$
e $x = 2, y = 2$ **f** $x = -7, y = 4$
g $x = 7, y = 3$ **h** $x = 3, y = 2\frac{1}{2}$
i $x = 2\frac{2}{3}, y = 1$ **j** $x = -3\frac{1}{5}, y = -4\frac{3}{5}$

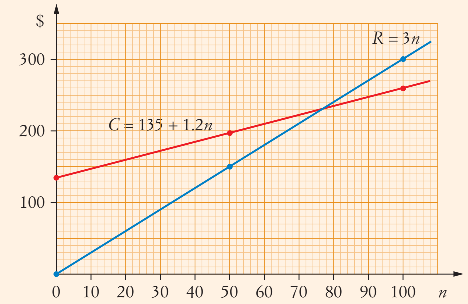
Exercise 9-04

1 285 **2** 680
3 **a** Teacher to check **b** 364
4 12 **5** Aaron 36, Sejuti 12 **6** 16
7 black 36, colour 24 **8** Pie: \$3.60, Sausage roll: \$2.70
9 Supreme 32, Vegetarian 13
10 Strawberries \$3.50; Blueberries \$4.99
11 **a** Teacher to check.
b 20-cent coins: 154, 50-cent coins: 91
12 **a** Teacher to check.
b $C = 135 + 1.2n$ $R = 3n$

n	0	50	100
C	135	195	255

n	0	50	100
R	0	150	300

c



d $n = 75$

Mental skills 9

2 **a** $\frac{2}{3}$ **b** $\frac{4}{5}$ **c** $\frac{5}{7}$ **d** $\frac{1}{2}$
e $\frac{1}{4}$ **f** $\frac{1}{6}$ **g** $\frac{5}{6}$ **h** $\frac{2}{5}$
i 5 : 9 **j** 5 : 9 **k** 9 : 20 **l** 4 : 5
m 9 : 7 **n** 4 : 3 **o** $\frac{3}{5}$ **p** $\frac{4}{35}$
3 **a** $\frac{17}{40}$ **b** $\frac{2}{3}$ **c** $\frac{16}{25}$
d $\frac{1}{4}$ **e** $\frac{5}{24}$ **f** $\frac{2}{25}$

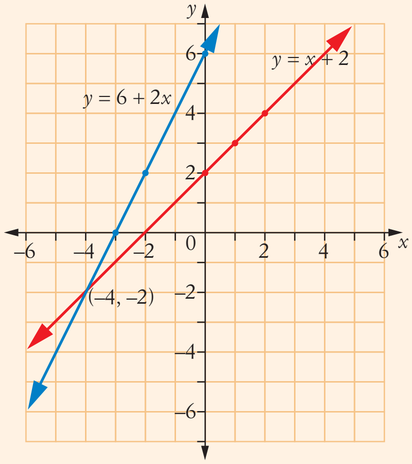
Power plus

1 **a** $x = 1\frac{1}{2}, y = -5\frac{1}{2}, w = 4\frac{1}{2}$
b $a = 1\frac{7}{13}, c = 4\frac{3}{13}, d = 8\frac{11}{13}$
c $p = -11\frac{3}{13}, m = 18\frac{11}{13}, n = -13\frac{4}{13}$

2 **a** Teacher to check.
b $ae - bd = 0$ and a fraction cannot have denominator 0.
c $x = 3, y = -1$
d Teacher to check.
i $x = 2, y = -2$ **ii** $x = 28, y = 16$
iii $x = \frac{1}{11}, y = 2\frac{20}{33}$

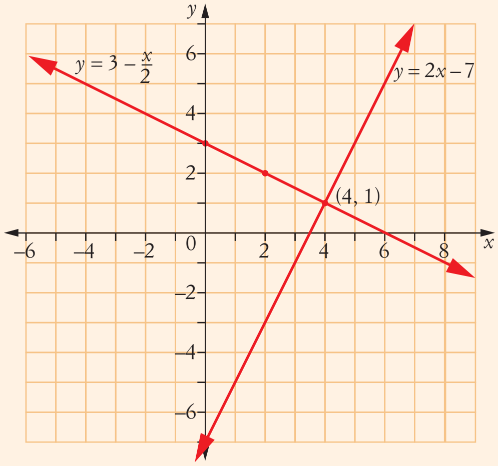
Chapter 9 revision

1 **a** $x = 2, y = -2$ **b** $x = 4, y = 0$
2 **a**



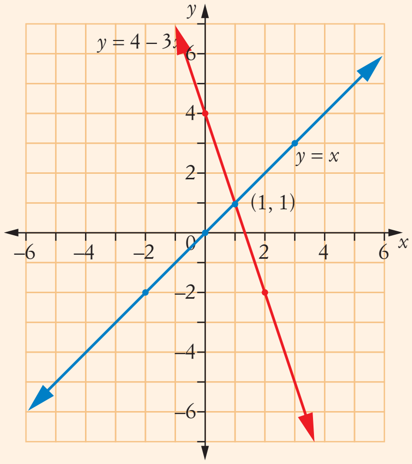
$x = -4, y = -2$

b



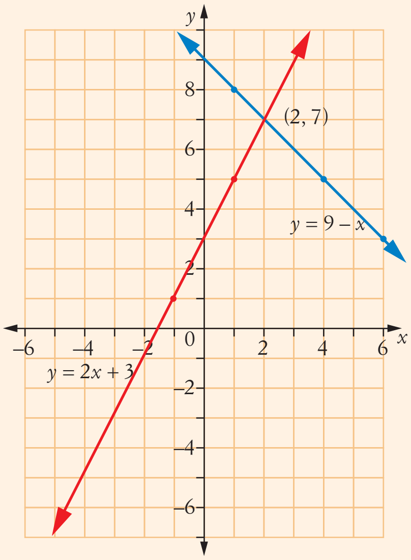
$x = 4, y = 1$

c



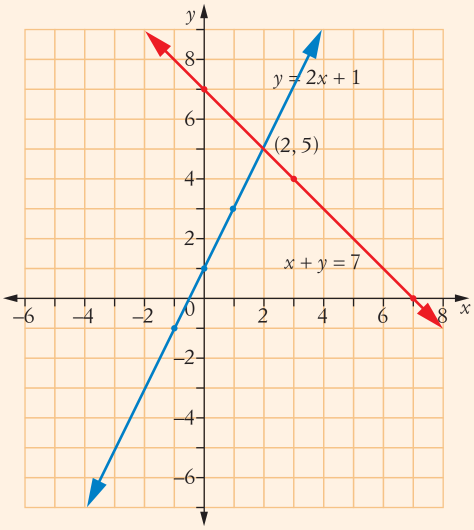
$x = 1, y = 1$

d



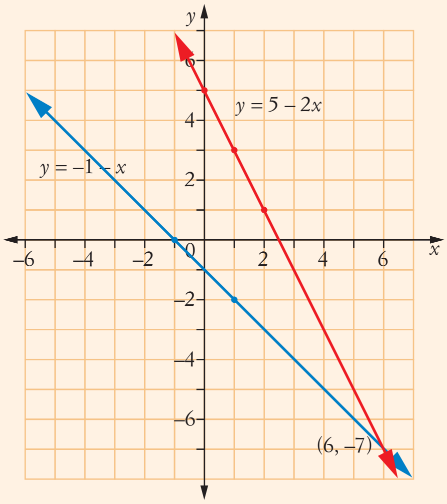
$x = 2, y = 7$

e



$x = 2, y = 5$

f



$x = 6, y = -7$

3 a $m = 5, y = -9\frac{1}{2}$ **b** $x = 2, y = \frac{1}{3}$

c $a = 1, d = 1$ **d** $x = 6, y = 15$

e $x = -5, y = -2$ **f** $d = -3, w = -10$

4 a $x = 2, y = 11$ **b** $m = 1, p = 3$

c $h = 10, t = 4$ **d** $a = 3, c = \frac{1}{2}$

e $x = 1, y = 1$ **f** $p = 12, q = -4$

5 a 1600 adults, 900 children

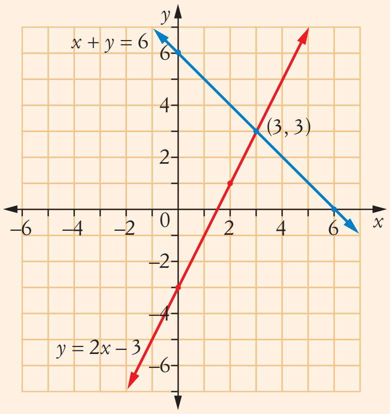
b 18 DVDs, 12 CDs **c** $38

d 28 cheesecakes, 47 mudcakes

e 120 boys, 93 girls

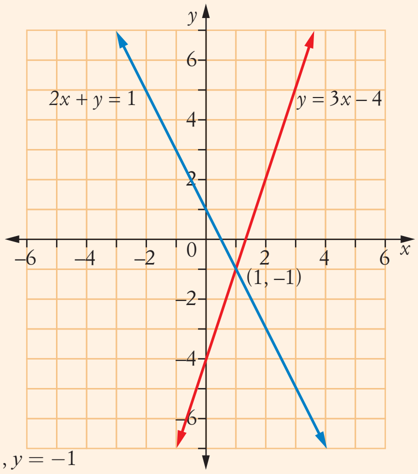
Mixed revision 3

1 a



$x = 3, y = 3$

b

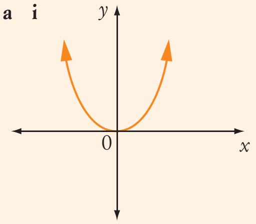


$x = 1, y = -1$

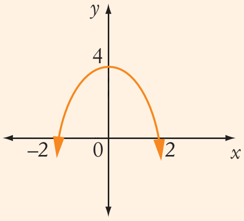
2 **a** € 17.5 **b** AU$86 **c** € 98

3 158.4 **4** A **5** C

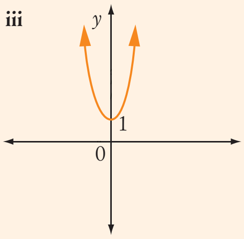
6 **a** **i**



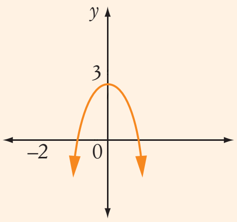
ii



iii



iv



b **i** $y = x^2, y = 3x^2 + 1$ **ii** $y = 4 - x^2, y = 3 - 2x^2$

iii $y = 3x^2 + 1$

7 **a** 6.8 cm **b** 22.7 m **c** 10.8 cm

8 **a** $t = \frac{99.96}{s}$ **b** 9.52 m/s **c** 9.70 s

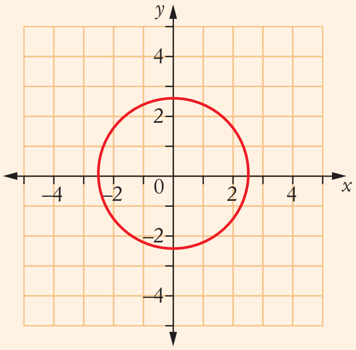
9 **a** $d = 51.9$ **b** $e = 58.1$ **c** $f = 3.7$

10 **a** $g = 2, w = -1$ **b** $f = 3, y = 3$

c $a = -1, c = -2$

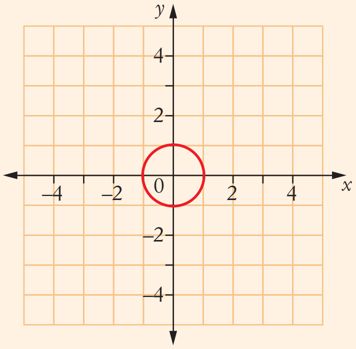
11 **a** 29° **b** 45° **c** 43°

12 **a**



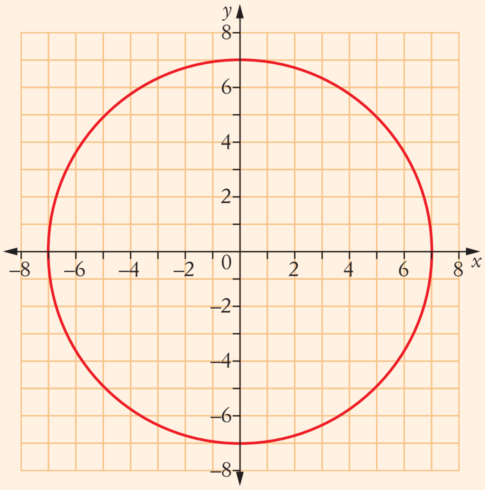
centre (0, 0), radius = 2.5

b



centre (0, 0), radius = 1

c



centre (0, 0), radius = 7

13 71 m

14 **a** 17.5 hours **b** 6 painters

15 **a** $A + C = 395, \quad 20A + 15C = 6700$

b 240 children

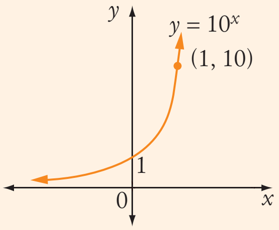
16 C **17** 78°

18 **a** $x = 2\frac{1}{2}, y = 5\frac{1}{2}$ **b** $w = 2, p = 1$

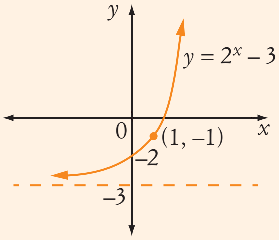
c $g = \frac{1}{2}, k = 3$

19 B **20** $x = \pm 2$

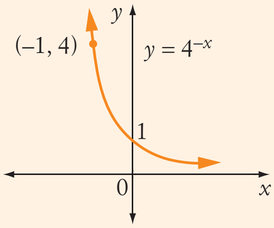
21 **a**



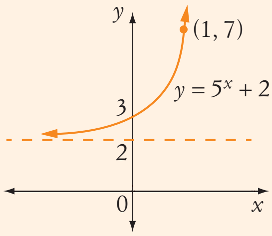
b



c



d



22 50 m

23 **a** I **b** G **c** J **d** C

e B **f** D **g** F **h** E

i K **j** L **k** A **l** H

24 51°3′

25 **a** 280° **b** 140° **c** 200°

26 **a** 301 km **b** 114°

Chapter 10

SkillCheck

1 C

2 **a** 3

b No, $P(\text{10c coin}) = \frac{5}{12}$, $P(\text{20c coin}) = \frac{1}{3}$, $P(\text{50c coin}) = \frac{1}{4}$

3 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{5}{6}$

4 **a** 0 **b** 1

5 0.4 **6** B **7** 0.15

Exercise 10-01

1 **a** **i** 0.425 **ii** 0.14 **iii** 0.21

b **i** 0.375 **ii** 0.125 **iii** 0.25

c Yes

d Expected frequency = 100. The observed frequency of red or purple is 115, which is more than the expected frequency.

2 **a** **i** $\frac{1}{5} = 0.2$ **ii** $\frac{19}{50} = 0.38$

iii $\frac{33}{100} = 0.33$ **iv** $\frac{9}{100} = 0.09$

b **i** $\frac{1}{4} = 0.25$ **ii** $\frac{7}{20} = 0.35$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{1}{10} = 0.1$

c Yes

d Expected frequency = 40. The expected frequency compares very favourably with the observed frequency of 42.

3 **a** 50 **b** Teacher to check.

c **i** Teacher to check. **ii** $\frac{1}{2}$

d Teacher to check.

4 **a** 600

b **i** $\frac{281}{600} = 0.468$ **ii** $\frac{322}{600} = 0.537$

iii $\frac{227}{600} = 0.378$ **iv** $\frac{522}{600} = 0.87$

c **i** 0.5 **ii** 0.5 **iii** 0.33 **iv** 0.83

d The probabilities are similar.

5 **a** Teacher to check.

b **i** $\frac{1}{2} = 0.5$ **ii** $\frac{1}{5} = 0.2$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{7}{10} = 0.7$

c Teacher to check.

6 **a** **i** $\frac{3}{10} = 0.3$ **ii** $\frac{3}{25} = 0.12$

iii $\frac{12}{25} = 0.48$ **iv** $\frac{1}{10} = 0.1$

b **i** 0.33 **ii** 0.17 **iii** 0.33 **iv** 0.17

c Yes

d Expected frequency of yellow is 33. This is more than the observed frequency of 26.

7 **a** $\frac{1}{6} \approx 0.17$

b 16 or 17 times

c, d, e Teacher to check.

8 **a** 200

b **i** $\frac{4}{200} = 0.02$ **ii** $\frac{27}{200} = 0.135$

iii $\frac{13}{200} = 0.065$ **iv** $\frac{86}{200} = 0.43$

v $\frac{87}{200} = 0.435$ **vi** $\frac{59}{200} = 0.295$

vii $\frac{51}{200} = 0.255$

9 **a** 200

b **i** $\frac{27}{200} = 0.135$ **ii** $\frac{62}{200} = 0.31$

iii $\frac{80}{200} = 0.4$ **iv** $\frac{21}{200} = 0.105$

v $\frac{1}{200} = 0.005$

c Ferry, light rail (tram) **d** Teacher to check.

Answers

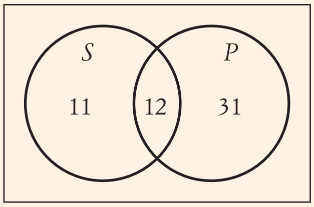
Exercise 10-02

1 **a** **i** $\frac{11}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{1}{5}$ **iv** $\frac{6}{25}$ **v** $\frac{19}{25}$
b $\frac{3}{10}$

2 **a** 156
b **i** $\frac{11}{52}$ **ii** $\frac{7}{52}$ **iii** $\frac{7}{78}$
iv $\frac{22}{39}$ **v** $\frac{19}{78}$ **vi** $\frac{1}{26}$
c $\frac{7}{31}$

3 **a** 135 **b** $\frac{56}{135}$ **c** $\frac{1}{5}$ **d** $\frac{17}{52}$

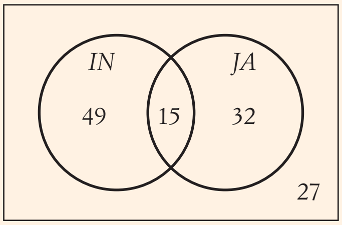
4 **a**



b **i** 1 **ii** $\frac{11}{54}$ **iii** $\frac{31}{54}$ **iv** $\frac{7}{9}$

5 **a** 45
b **i** $\frac{1}{45}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{9}$ **iv** $\frac{19}{45}$
c **i** $\frac{19}{26}$ **ii** 0

6 **a** 123
b



c 81
d **i** $\frac{49}{123}$ **ii** $\frac{32}{123}$ **iii** $\frac{27}{123}$ **iv** $\frac{81}{123}$

7 **a** 200
b **i** $\frac{79}{200}$ **ii** $\frac{51}{100}$ **iii** $\frac{77}{200}$
iv $\frac{121}{200}$ **v** $\frac{81}{100}$ **vi** $\frac{3}{50}$
c $\frac{29}{40}$
d No, because all the people surveyed indicated a day on which they preferred to shop.

8 **a** 204
b **i** $\frac{7}{204}$ **ii** $\frac{23}{204}$ **iii** $\frac{31}{102}$ **iv** $\frac{71}{102}$
c **i** $\frac{2}{54}$ **ii** $\frac{43}{54}$

Exercise 10-03

1 **a** 150
b **i** $\frac{4}{25}$ **ii** $\frac{53}{150}$ **iii** $\frac{28}{75}$
c 63%

2 **a** 128
b **i** 68 **ii** 60
c $\frac{17}{32}$ **d** $\frac{3}{32}$

3 **a** 93
b **i** 21.5% **ii** 11.8% **iii** 3.2%
c **i** 15.7% **ii** 45%
d The percentage composition of women in the opposition is three times that of the percentage composition of women in the government.

4 **a** 150
b **i** 0.5 **ii** 0.04 **iii** 0.43 **iv** 0.23
c $\frac{22}{75} = 0.293$

5 **a** 160
b **i** $\frac{7}{20} = 0.35$ **ii** $\frac{11}{160} = 0.069$ **iii** $\frac{9}{80} = 0.113$
c $\frac{35}{82} = 0.43$

6 **a** 200 **b** 55%
c **i** 49.5% **ii** 45%
iii 36% **iv** 31.5%
d 65.5%

7 **a** 878
b **i** $\frac{679}{878} = 0.773$ **ii** $\frac{545}{878} = 0.621$
iii $\frac{67}{439} = 0.153$ **iv** $\frac{21}{439} = 0.048$

Exercise 10-04

1 **a**

CA	CG	CJ	CE	CR	AC	AG	AJ	AE	AR
GC	GA	GJ	GE	GR	JC	JA	JG	JE	JR
EC	EA	EG	EJ	ER	RC	RA	RG	RJ	RE

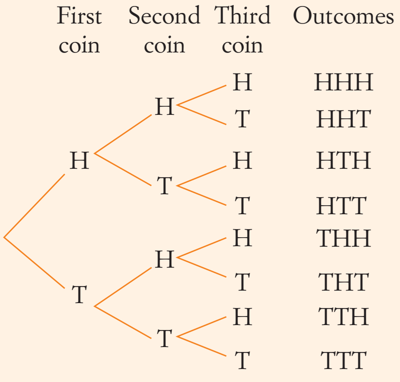
b $\frac{1}{3}$ **c** $\frac{1}{6}$

2 **a**

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

b 36
c **i** $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{11}{36}$
iv $\frac{1}{4}$ **v** $\frac{1}{2}$ **vi** $\frac{5}{12}$

3 **a**



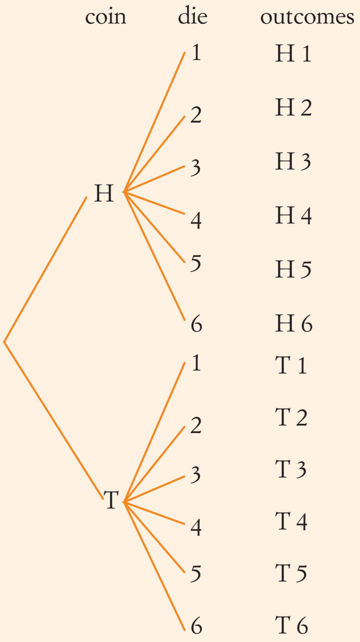
b 8

c **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{8}$ **v** $\frac{1}{2}$

d **i** $\frac{7}{8}$ **ii** $\frac{7}{8}$

e **i** 75 **ii** 25

4



5 **a**

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6

b 24

c **i** $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$

iv $\frac{3}{8}$ **v** 0

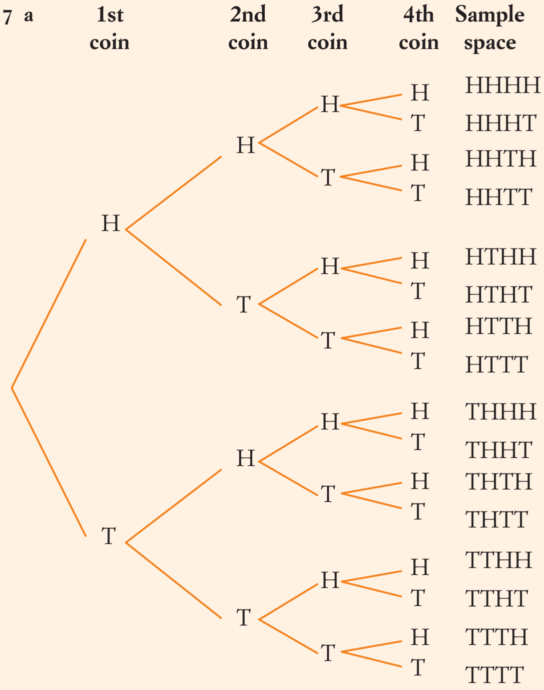
6 **a**

		2nd die					
		1	**2**	**3**	**4**	**5**	**6**
1st die	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b **i** $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

v 0 **vi** $\frac{5}{12}$ **vii** $\frac{7}{12}$ **viii** $\frac{5}{12}$

7 **a**



b **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{8}$

iv $\frac{15}{16}$ **v** $\frac{1}{16}$ **vi** $\frac{5}{16}$

c **i** 63 **ii** 250 **iii** 937

Mental skills 10

2 **a** \$408 **b** 99 **c** 200 **d** \$404

e 672 **f** \$81 **g** \$517 **h** 225

i 560 **j** \$84 **k** \$350 **l** 84

4 **a** 330 **b** \$240 **c** 1600 **d** \$425

e \$225 **f** \$60 **g** \$63 **h** 76

i \$68 **j** \$3762 **k** \$374 **l** \$100

Exercise 10-05

1 **a**

		Girls					
		Be	**Ca**	**Em**	**M**	**R**	**S**
Boys	**Ben**	Ben, Be	Ben, Ca	Ben, Em	Ben, M	Ben, R	Ben, S
	C	C, Be	C, Ca	C, Em	C, M	C, R	C, S
	Ew	Ew, Be	Ew, Ca	Ew, Em	Ew, M	Ew, R	Ew, S
	W	W, Be	W, Ca	W, Em	W, M	W, R	W, S

b **i** $\frac{1}{24}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$

2 **a** **i**

	A	**B**	**C**	**D**	**E**
A	AA	AB	AC	AD	AE
B	BA	BB	BC	BD	BE
C	CA	CB	CC	CD	CE
D	DA	DB	DC	DD	DE
E	EA	EB	EC	ED	EE

ii

	A	B	C	D	E
A		AB	AC	AD	AE
B	BA		BC	BD	BE
C	CA	CB		CD	CE
D	DA	DB	DC		DE
E	EA	EB	EC	ED	

b **i** $\frac{1}{5}$ **ii** $\frac{4}{25}$ **iii** $\frac{12}{25}$

c **i** $\frac{1}{10}$ **ii** $\frac{3}{5}$ **iii** $\frac{2}{5}$ **iv** $\frac{4}{5}$

3 **a**

	H	T
H	HH	HT
T	TH	TT

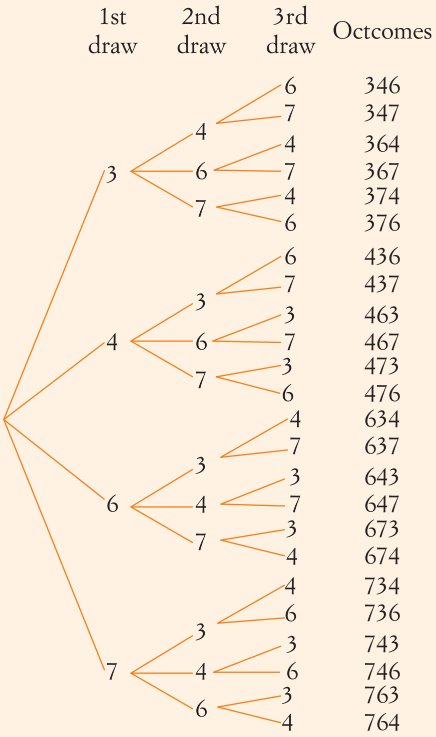
b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

4 **a**

		2nd course				
		B	**H**	**P**	**S**	**T**
1st course	**C**	CB	CH	CP	CS	CT
	F	FB	FH	FP	FS	FT
	Y	YB	YH	YP	YS	YT

b **i** $\frac{1}{3}$ **ii** $\frac{4}{15}$ **iii** $\frac{1}{15}$

5 **a**

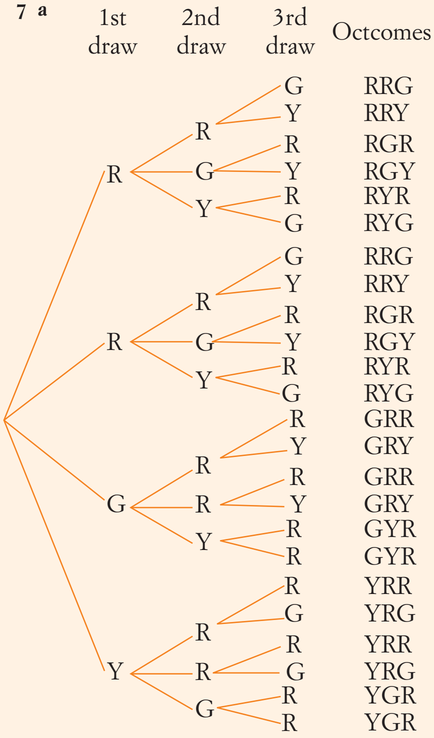


b **i** $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{3}$

6 **a** Teacher to check. 64 outcomes, beginning with 333, 334, 336, 337, 343, 344, 346, 347, …, 776, 777.

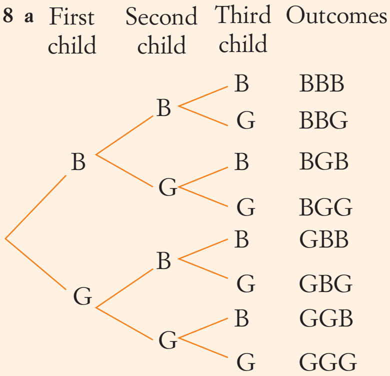
b **i** $\frac{1}{16}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{16}$

7 **a**



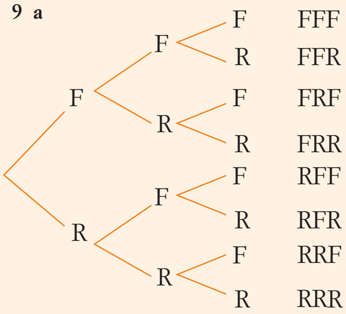
b **i** $\frac{1}{2}$ **ii** $\frac{1}{12}$ **iii** 1

8 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{8}$

9 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

10 **a** 36 **b** $\frac{1}{36}$ **c** $\frac{11}{36}$

11 a i 125 outcomes. Teacher to check.
ii 60 outcomes. Teacher to check.
b i $\frac{29}{125}$ **ii** $\frac{18}{125}$ **iii** $\frac{64}{125}$ **iv** $\frac{117}{125}$
c i $\frac{1}{10}$ **ii** $\frac{3}{10}$ **iii** 0 **iv** $\frac{2}{5}$

Exercise 10-06

1 a independent **b** independent **c** dependent
d independent **e** dependent **f** independent
g dependent
2 Dependent, as the balls are not replaced when drawn.
3 a independent **b** $\frac{1}{2}$
4 a i $\frac{1}{3}$ **ii** $\frac{1}{4}$
b 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 1G, 2G,3G, 4G, 5G, 6G, 1B, 2B, 3B, 4B, 5B, 6B, 1R, 2R, 3R, 4R, 5R, 6R,
c $\frac{1}{12}$ **d** Yes, $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$
e independent
5 a i $\frac{1}{6}$ **ii** $\frac{1}{2}$
b independent
6 a $\frac{5}{9}$ **b** $\frac{4}{8} = \frac{1}{2}$
c Dependent, as the first draw changes the contents of the bag.
7 a i $\frac{5}{8}$ **ii** $\frac{4}{7}$
b i $\frac{3}{8}$ **ii** $\frac{5}{7}$
c i $\frac{5}{8}$ **ii** $\frac{3}{7}$
d i $\frac{3}{8}$ **ii** $\frac{2}{7}$
8 $\frac{1}{2}$

Exercise 10-07

1 a $\frac{2}{6} = \frac{1}{3}$ **b** $\frac{3}{6} = \frac{1}{2}$
2 $\frac{2}{9}$
3 a $\frac{4}{11}$ **b** $\frac{7}{11}$
4 a $\frac{2}{11}$ **b** $\frac{4}{11}$ **c** $\frac{4}{11}$ **d** $\frac{5}{11}$
5 $\frac{1}{3}$ **6** $\frac{1}{6}$
7 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b i $\frac{1}{2}$ **ii** 1
c $\frac{2}{11}$
d i $\frac{9}{27} = \frac{1}{3}$ **ii** $\frac{6}{27} = \frac{2}{9}$
e $\frac{1}{6}$

8 a i $\frac{1}{9}$ **ii** $\frac{8}{9}$ **b** $\frac{2}{8} = \frac{1}{4}$ **c** 6
9 $\frac{1}{40}$ **10** $\frac{10}{49}$ **11** $\frac{1}{13}$ **12** $\frac{1}{6}$

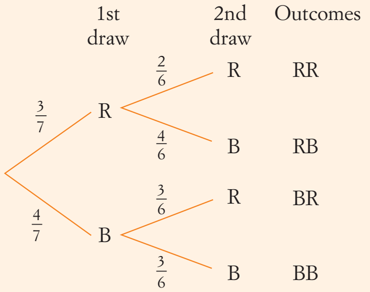
13 a

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b i $\frac{1}{6}$ **ii** $\frac{1}{18}$ **iii** $\frac{1}{6}$
c i $\frac{1}{2}$ **ii** 0
d i $\frac{1}{11}$ **ii** $\frac{4}{11}$
e i $\frac{6}{18} = \frac{1}{3}$ **ii** 1

Power plus

1 a 320
b i $\frac{7}{40} = 0.175$ **ii** $\frac{3}{32} = 0.094$
iii $\frac{57}{320} = 0.178$ **iv** $\frac{3}{20} = 0.15$
c $\frac{16}{45}$ **d** $\frac{45}{107}$
2 a



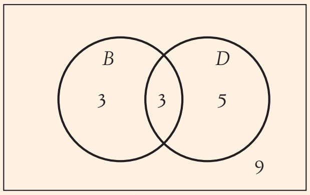
b i $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{4}{7}$ **iv** $\frac{6}{7}$
3 a i $\frac{14}{30} = \frac{7}{15}$ **ii** $\frac{20}{30} = \frac{2}{3}$ **iii** $\frac{4}{30} = \frac{2}{15}$
iv $\frac{4}{20} = \frac{1}{5}$ **v** $\frac{4}{14} = \frac{2}{7}$
b i $\frac{\frac{2}{15}}{\frac{2}{3}} = \frac{1}{5}$ **ii** Yes
c $P(B|A) = \frac{2}{7}, \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{2}{15}}{\frac{7}{15}} = \frac{2}{7}$

Chapter 10 revision

1 a i 0.22 **ii** 0.29 **iii** 0.1 **iv** 0.14
b ii 0.25 **ii** 0.25 **iii** 0.125 **iv** 0.125
c Yes **d** 0.25
e The expected number of times the arrow stops at a colour not purple or black is 75, which is the same as the observed number of times.
2 a i 0.353 **ii** 0.427 **iii** 0.087 **iv** 0.513
b i $\frac{13}{15} = 0.867$ **ii** $\frac{2}{15} = 0.133$
c Different – at least one head occurring excludes zero heads occurring, which is the same as three tails occurring. The events are complementary.

3 a 90 **b** $\frac{19}{45}$ **c** $\frac{2}{9}$ **d** $\frac{11}{49}$
4 a 35 **b** $\frac{2}{35}$
c i $\frac{6}{35}$ **ii** $\frac{19}{35}$ **iii** $\frac{3}{35}$
d They don't like the types of music mentioned in the survey.

5 **a**



b **i** $\frac{3}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{20}$ **iv** $\frac{3}{5}$

c $\frac{9}{20}$

6 **a** 200

b **i** 0.305 **ii** 0.11 **iii** 0.225 **iv** 0.425

c $\frac{25}{110} = 0.227$

d **i** $\frac{47}{69} = 0.681$ **ii** $\frac{22}{69} = 0.319$

7 **a**

	1	**2**	**3**	**4**
1	1, 1	1, 2	1, 3	1, 4
2	2, 1	2, 2	2, 3	2, 4
3	3, 1	3, 2	3, 3	3, 4
4	4, 1	4, 2	4, 3	4, 4

b 16

c **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{16}$

iv $\frac{1}{4}$ **v** $\frac{1}{4}$ **vi** $\frac{1}{2}$

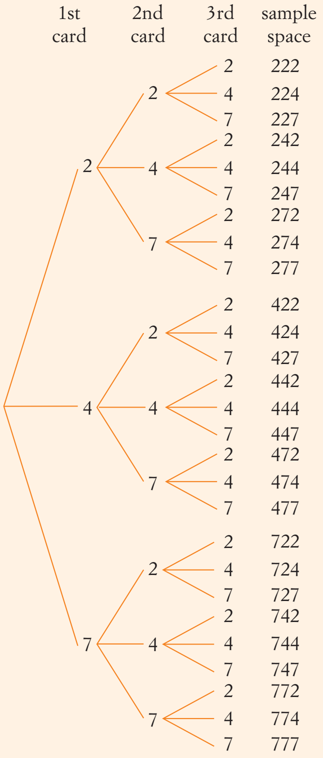
8 **a** **i** RR, RB, RG, RY, RBla, BR, BB, BG, BY, BBla, GR, GB, GG, GY, GBla, YR, YB, YG, YY, YBla, BlaR, BlaB, BlaG, BlaY, BlaBla

ii RB, RG, RY, RBla, BR, BG, BY, BBla, GR, GB, GY, GBla, YR, YB, YG, YBla, BlaR, BlaB, BlaG, BlaY

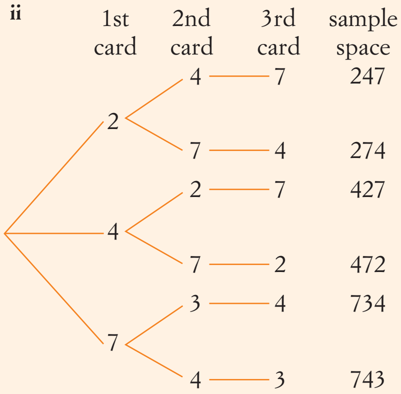
b **i** $\frac{1}{25}$ **ii** $\frac{1}{5}$ **iii** $\frac{2}{25}$ **iv** $\frac{9}{25}$

c **i** $\frac{1}{10}$ **ii** $\frac{3}{5}$

9 **a** **i**



ii



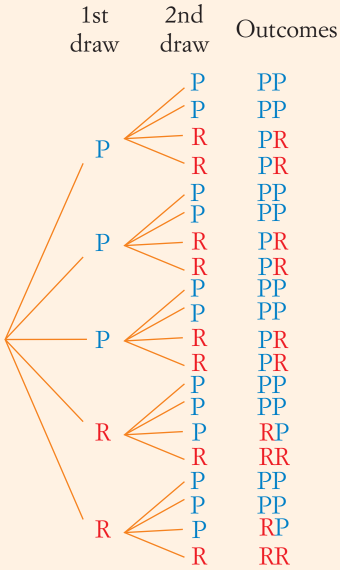
b **i** $\frac{2}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{9}$ **iv** $\frac{2}{9}$

c **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{6}$

10 **a** independent **b** dependent **c** dependent

d independent **e** independent

11 **a**



b **i** $\frac{1}{2}$ **ii** $\frac{1}{2}$

c **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

d **i** $\frac{1}{6}$ **ii** $\frac{5}{6}$

12 **a**

	1	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b **i** $\frac{1}{2}$ **ii** 1

c $\frac{2}{7}$

d **i** $\frac{1}{3}$ **ii** $\frac{1}{3}$

e $\frac{1}{4}$

Chapter 11

SkillCheck

1 a $w = 38$ **b** $r = 44$ **c** $h = 71$
d $x = 126$ **e** $y = 46$ **f** $a = 24$
2 a $p = 64$ **b** $p = 241$ **c** $p = 104$
d $p = 105$ **e** $p = 58$ **f** $p = 128$
3 *a* and *g*, *c* and *j*, *d* and *f*, *h* and *i*
4 a $\angle M$ and $\angle X$, $\angle N$ and $\angle Y$, $\angle P$ and $\angle W$
b *MN* and *XY*, *NP* and *YW*, *MP* and *XW*

Exercise 11-01

1 a heptagon **b** quadrilateral **c** decagon
d octagon **e** triangle **f** pentagon
2 a **a**, **b**, **d**, **e**, **f** **b** **d**
3 C
4 a 1800° **b** 1440° **c** 1260°
d 3240° **e** 2340°
5 a $e = 112$ **b** $m = 135$ **c** $x = 83, y = 97$
d $w = 108, h = 72$ **e** $k = 203$ **f** $a = 32$
6 a 6 **b** 21 **c** 13 **d** 30 **e** 9
7 a 16 **b** 157.5°
8 a 144° **b** 135° **c** 120° **d** 150°
9 a 30 **b** 15 **c** 45 **d** 25

Exercise 11-02

1 a 72° **b** 30° **c** 20° **d** 60°
2 a 140° **b** 162° **c** 144° **d** 168°
3 a 24 **b** 5 **c** 18 **d** 9 **e** 72 **f** 30
4 a 8 **b** 10 **c** 15
d 180 **e** 24 **f** 12

Exercise 11-03

1 a $\triangle AXT \equiv \triangle GYE$ (RHS) **b** $\triangle MVR \equiv \triangle SPC$ (SSS)
c $\triangle BTV \equiv \triangle PSX$ (AAS) **d** $\triangle CNQ \equiv \triangle DCE$ (SAS)

2 In $\triangle WXY$ and $\triangle WZY$
$\angle X = \angle Z = 90°$ (given)
$XY = ZY$ (given)
WY is common.
$\therefore \triangle WXY \equiv \triangle WZY$ (RHS)

3 In $\triangle DEH$ and $\triangle FEG$
$\angle D = \angle F$ (alternate angles, $HD \parallel GF$)
$\angle H = \angle G$ (alternate angles, $HD \parallel GF$)
$EH = EG$ (given)
$\therefore \triangle DEH \equiv \triangle FEG$ (AAS)

4 In $\triangle ABC$ and $\triangle ADC$
$BC = DC$ (given)
$AB = AD$ (given)
AC is common.
$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

5 In $\triangle PQN$ and $\triangle LQM$
$\angle PQN = \angle LQM$ (vertically opposite angles)
$PQ = LQ$ (given)
$NQ = MQ$ (given)
$\therefore \triangle PQN \equiv \triangle LQM$ (SAS)

6 In $\triangle WXY$ and $\triangle YVW$
$XY = VW$ (given)
$WX = YV$ (given)
WY is common.
$\therefore \triangle WXY \equiv \triangle YVW$ (SSS)

7 In $\triangle AOB$ and $\triangle COD$
$OA = OC$ (equal radii)
$OB = OD$ (equal radii)
$AB = CD$ (given)
$\therefore \triangle AOB \equiv \triangle COD$ (SSS)

8 In $\triangle FNM$ and $\triangle TMN$
$\angle FNM = \angle TMN$ (alternate angles, $FN \parallel TM$)
$\angle FMN = \angle TNM$ (alternate angles, $TN \parallel FM$)
NM is common.
$\therefore \triangle FNM \equiv \triangle TMN$ (AAS)

9 In $\triangle ABC$ and $\triangle DCB$
$\angle ABC = \angle DCB$ (given)
$AB = DC$ (given)
BC is common.
$\therefore \triangle ABC \equiv \triangle DCB$ (SAS)

10 In $\triangle TSK$ and $\triangle LPK$
$\angle T = \angle L$ (alternate angles, $TS \parallel PL$)
$\angle S = \angle P$ (alternate angles, $TS \parallel PL$)
$TK = LK$ (*K* is the midpoint of *TL*)
$\therefore \triangle TSK \equiv \triangle LPK$ (AAS)

11 In $\triangle PVQ$ and $\triangle QTR$
$\angle VPQ = \angle TQR$ (corresponding angles, $PW \parallel QT$)
$\angle VQP = \angle TRQ$ (corresponding angles, $RW \parallel VQ$)
$PQ = QR$ (given)
$\therefore \triangle PVQ \equiv \triangle QTR$ (AAS)

12 In $\triangle DEG$ and $\triangle EDF$
$\angle DEG = \angle EDF$ (given)
$GE = FD$ (given)
ED is common.
$\therefore \triangle DEG \equiv \triangle EDF$ (SAS)

13 In $\triangle ABD$ and $\triangle ACD$
$AB = AC$ (given)
AD is common.
$\angle ADB = \angle ADC = 90°$ ($AD \perp BC$)
$\therefore \triangle ABD \equiv \triangle ACD$ (RHS)
$\therefore \angle BAD = \angle CAD$ (matching angles of congruent triangles)
$\therefore$ *AD* bisects $\angle BAC$

14 In $\triangle BCX$ and $\triangle CBY$
$\angle BXC = \angle CYB = 90°$ ($CX \perp AB$, $BY \perp AC$)
$XC = YB$ (given)
BC is common.
$\therefore \triangle BCX \equiv \triangle CBY$ (RHS)

15 In $\triangle WYX$ and $\triangle ZYX$
$XW = XZ$ (given)
$WY = ZY$ (Y is midpoint of WZ)
XY is common.
$\therefore \triangle WYX \equiv \triangle ZYX$ (SSS)

Exercise 11-04

1 **a** SSS
b $\angle ADB = \angle ADC$ (matching angles of congruent triangles)
c $\angle ADB + \angle ADC = 180°$ (angles on a line)
$\therefore \angle ADB = \angle ADC = 90°$
$\therefore AD \perp BC$

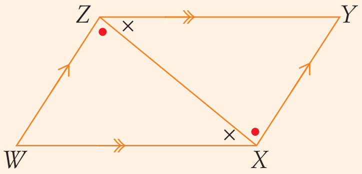
2 **a** AAS
b $WS = WT$ (matching sides of congruent triangles)
c The sides opposite the equal angles are equal.

3 **a** $\triangle ABX \equiv \triangle ACX$ (SSS)
b $\angle B = \angle C$ (matching angles of congruent triangles)
c $\triangle BAY \equiv \triangle BCY$ (SSS)
d $\angle A = \angle C$ (matching angles of congruent triangles)
e Since $\angle B = \angle C$, $\angle A = \angle C$,
$\angle A = \angle B = \angle C$
$\angle A + \angle B + \angle C = 180°$ (angle sum of a triangle)
$\therefore \angle A = \angle B = \angle C = 60°$
f Each angle in an equilateral triangle is 60°.

4 **a** $\angle MPY = \angle NPY$ (PY bisects $\angle MPN$)
b $\triangle PMY \equiv \triangle PNY$ (AAS)
c $MY = NY$ (matching sides of congruent triangles)
d $\angle PYM = \angle PYN$ (matching angles of congruent triangles)
e $\angle PYM + \angle PYN = 180°$ (angles on a line)
$\therefore \angle PYM = \angle PYN = 90°$
$\therefore PY \perp MN$

5 **a** In $\triangle ABC$ and $\triangle CDA$
$CB = AD$ (given)
$AB = CD$ (given)
AC is common.
$\therefore \triangle ABC \equiv \triangle CDA$ (SSS)
b $\angle BAC = \angle DCA$ (matching angles of congruent triangles)
$\angle BCA = \angle DAC$ (matching angles of congruent triangles)
c $\therefore AB \parallel CD$ and $AD \parallel BC$ (alternate angles are equal)
d $ABCD$ is a parallelogram since opposite sides are parallel.

6 **b**



c In $\triangle WXZ$ and $\triangle YZX$
$\angle WXZ = \angle YZX$ (alternate angles, $WX \parallel YZ$)
$\angle WZX = \angle YXZ$ (alternate angles, $WZ \parallel YX$)
XZ is common.
$\therefore \triangle WXZ \equiv \triangle YZX$ (AAS)
d $\angle W = \angle Y$ (matching angles of congruent triangles)
e In $\triangle WXY$ and $\triangle YZW$
$\angle XYW = \angle ZWY$ (alternate angles, $XY \parallel ZW$)
$\angle XWY = \angle ZYW$ (alternate angles, $WX \parallel YZ$)
WY is common.
$\therefore \triangle WXY \equiv \triangle YZW$ (AAS)
f $\angle WXY = \angle YZW$ (matching angles of congruent triangles)
g Opposite angles of a parallelogram are equal.

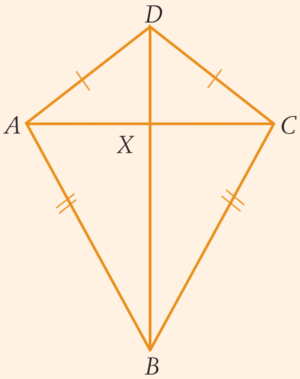
7 **a** In $\triangle VUS$ and $\triangle TUS$
$VU = TU$ (given)
$VS = TS$ (given)
SU is common.
$\therefore \triangle VUS \equiv \triangle TUS$ (SSS)
b $\angle VUS = \angle TUS$ (matching angles of congruent triangles)
$\angle VSU = \angle TSU$ (matching angles of congruent triangles)
$\therefore VS$ bisects $\angle VUT$ and $\angle VST$.

8 **a** In $\triangle ABD$ and $\triangle CDB$
$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel CD$)
$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel CB$)
BD is common.
$\therefore \triangle ABD \equiv \triangle CDB$ (AAS)
b $AD = CB$ (matching sides of congruent triangles)
$AB = CD$ (matching sides of congruent triangles)
c $\therefore$ opposite sides of a parallelogram are equal.

9 **a** In $\triangle BEL$ and $\triangle GHL$
$\angle BEL = \angle GHL$ (alternate angles, $BE \parallel GH$)
$\angle BLE = \angle GLH$ (vertically opposite angles are equal)
$BE = GH$ (given)
$\therefore \triangle BEL \equiv \triangle GHL$ (AAS)
b $BL = GL$ (matching sides of congruent triangles)
$EL = HL$ (matching sides of congruent triangles)
$\therefore$ diagonals of a rhombus bisect each other.
c $\angle BEH = \angle BHE$ (angles opposite equal sides in an isosceles triangle)
$\therefore \angle BEL = \angle BHL$
d In $\triangle BEL$ and $\triangle BHL$
$\angle BEL = \angle BHL$ (proven in **c**)
$BE = BH$ (given)
$EL = HL$ (proved in **b**)
$\therefore \triangle BEL \equiv \triangle BHL$ (SAS)
e $\angle BLE = \angle BLH$ (matching angles of congruent triangles)
and $\angle BLE + \angle BLH = 180°$ (angles on a line)
$\therefore \angle BLE = \angle BLH = 90°$
$\therefore$ The diagonals of a rhombus are at right angles.

10 **a** In $\triangle ABD$ and $\triangle CBD$
$AB = CB$ (given)
$AD = CD$ (given)
BD is common.
$\therefore \triangle ABD \equiv \triangle CBD$ (SSS)
b $\angle A = \angle C$ (matching angles of congruent triangles)
c $\angle ADB = \angle CDB$ (matching angles of congruent triangles)
$\angle ABD = \angle CBD$ (matching angles of congruent triangles)
$\therefore DB$ bisects $\angle ADB$ and $\angle ABC$.

d



e In $\triangle DAX$ and $\triangle DCX$
$AD = CD$ (given)
DX is common.
$\angle ADX = \angle CDX$ (matching angles of congruent triangles proved in **a**)
$\therefore \triangle ADX \equiv \triangle CDX$ (SAS)

f $AX = CX$ (matching sides of congruent triangles)
$\therefore$ X is the midpoint of AC so diagonal DB bisects diagonal AC.

g $\angle AXD = \angle CXD$ (matching angles of congruent triangles)
and $\angle AXD + \angle CXD = 180°$ (angles on a line)
$\therefore \angle AXD = \angle CXD = 90°$
$\therefore DX \perp AC$
$\therefore BD \perp AC$

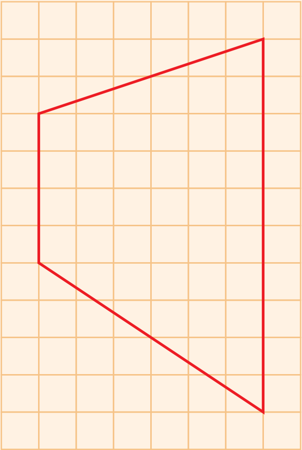
Mental skills 11

2 **a** \$100, \$50
b \$1200, \$900
c \$160, \$560
d \$500, \$1500
e \$2700, \$1800
f \$900, \$2100
g \$3000, \$600
h \$600, \$1000
i \$550, \$440
j \$800, \$3200
k \$2100, \$2800
l \$2000, \$1200

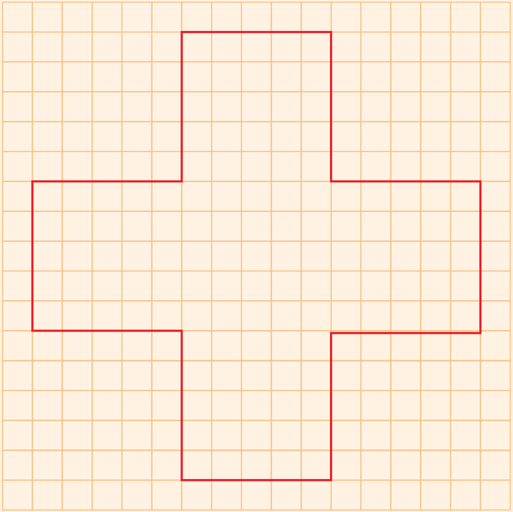
Exercise 11-05

1 **a** 2 **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $2\frac{1}{2}$

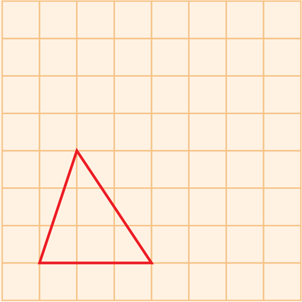
2 **a**



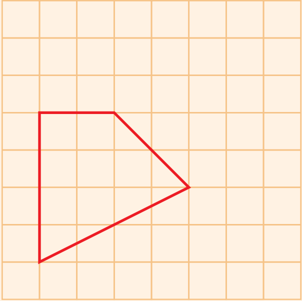
b



c



d



3 **a** **i** $\angle A$ and $\angle L$, $\angle B$ and $\angle M$, $\angle C$ and $\angle K$.
ii AC and LK, AB and LM, BC and MK.
iii $\triangle ABC \,|||\, \triangle LMK$

b **i** $\angle W$ and $\angle D$, $\angle X$ and $\angle E$, $\angle Y$ and $\angle G$, $\angle Z$ and $\angle H$.
ii WX and DE, XY and EG, YZ and GH, WZ and DH.
iii $WXYZ \,|||\, DEGH$

c **i** $\angle T$ and $\angle F$, $\angle R$ and $\angle W$, $\angle P$ and $\angle V$, $\angle M$ and $\angle S$, $\angle J$ and $\angle K$.
ii TR and FW, RP and WV, PM and VS, MJ and SK, TJ and FK.
iii $TRPMJ \,|||\, FWVSK$

d **i** $\angle Q$ and $\angle N$, $\angle K$ and $\angle C$, $\angle B$ and $\angle G$.
ii QB and NG, BK and GC, KQ and CN.
iii $\triangle QKB \,|||\, \triangle NCG$

Answers

4 a Yes ($\frac{8}{12} = \frac{20}{30} = \frac{2}{3}$)

b Yes ($\frac{12}{20} = \frac{27}{45} = \frac{21}{35} = \frac{3}{5}$)

c Yes ($\frac{4}{6} = \frac{6}{9} = \frac{6\frac{2}{3}}{10} = \frac{10}{15} = \frac{2}{3}$)

d Yes, all squares are similar.

e Yes, matching angles are equal.

f Yes, all equilateral triangle are similar.

Exercise 11-06

1 a $w = 22.4$ **b** $m = 10$

c $p = 20, h = 21$ **d** $x = 18$

e $a = 12.8, w = 7.5$ **f** $g = 11\frac{1}{9}, q = 18$

g $y = 26\frac{2}{3}, b = 9\frac{3}{5}$ or 9.6 **h** $u = 12\frac{4}{5}$ or 12.8, $t = 6\frac{7}{8}$ or 6.875

2 $h = 13\frac{5}{7}$ **3** $x = 8\frac{8}{9}$ **4** $w = 16$ cm

5 12 m **6** B **7** $h = 2.408$ m

8 D **9** 2.24 m

Exercise 11-07

1 a Two pairs of angles are equal (AA).

b All three pairs of matching sides are in the same ratio, $\frac{9}{18} = \frac{11}{22} = \frac{15.5}{31} = \frac{1}{2}$ (SSS).

c Two pairs of matching sides are in the same ratio $\frac{6}{8} = \frac{12}{16} = \frac{3}{4}$ and the included angles are equal (SAS).

d Two pairs of angles are equal (AA).

e All three pairs of matching sides are in the same ratio $\frac{9}{12} = \frac{9}{12} = \frac{14.25}{19} = \frac{3}{4}$ (SSS).

f In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{12}{15} = \frac{20.8}{26} = \frac{4}{5}$ (RHS).

g Two pairs of angles are equal (AA).

h All three pairs of matching sides are in the same ratio $\frac{18}{14.4} = \frac{27.5}{22} = \frac{20}{16} = \frac{5}{4}$ (SSS).

i All three pairs of matching sides are in the same ratio $\frac{6}{8} = \frac{8}{10\frac{2}{3}} = \frac{10}{13\frac{1}{3}} = \frac{3}{4}$ (SSS).

j Two pairs of matching sides are in the same ratio $\frac{26}{18.2} = \frac{30}{21} = \frac{10}{7}$, and the included angles are equal (SAS).

2 a B and C (SAS) **b** A and C (SSS)

c B and D (RHS)

3 a $\triangle UWY\,|||\,\triangle HEK$ (SAS) **b** $\triangle DML\,|||\,\triangle TPA$ (RHS)

c $\triangle ABC\,|||\,\triangle QTP$ (AA) **d** $\triangle GHN\,|||\,\triangle WVS$ (SSS)

Power plus

1 a Angles opposite equal sides of isosceles $\triangle KMR$ are equal.

b In $\triangle KMN$ and $\triangle KRP$

$KM = KR$ (given)

$MN = PR$ (given)

$\angle KMN = \angle KRP$ (angles opposite equal sides of isosceles triangle KMR)

$\therefore \triangle KMN \equiv \triangle KRP$ (SAS)

c $KN = KP$ (matching sides of congruent triangles)

$\therefore \triangle LNP$ is isosceles (two sides equal)

2 $x = 8$

3 a In $\triangle CGH$ and $\triangle CDE$

$\frac{CG}{CD} = \frac{1}{2}$ (G is the midpoint of CD)

$\frac{CH}{CE} = \frac{1}{2}$ (H is the midpoint of CE)

$\angle C$ is common.

$\therefore \triangle CGH\,|||\,\triangle CDE$ (SAS)

b Since $\triangle CGH\,|||\,\triangle CDE$

$\angle CGH = \angle CDE$ (matching angles of similar triangles)

$\therefore GH \parallel DE$ (corresponding angles are equal)

c $\frac{CG}{CD} = \frac{GH}{DE}$ (pairs of matching sides of similar triangles in the same ratio)

But $\frac{CG}{CD} = \frac{1}{2}$

$\therefore \frac{GH}{DE} = \frac{1}{2}$

$\therefore GH = \frac{1}{2}DE$

4 a Two pairs of angles are equal (AA). **b** $y = 5\frac{1}{7}$

Chapter 11 revision

1 a 2340° **b** 3960° **c** 1800° **d** 8280°

2 156° **3** 36

4 a 36 **b** 15 **c** 8 **d** 24

5 B

6 a SAS **b** AAS **c** SAS

7 In $\triangle WYZ$ and $\triangle XYZ$

$\angle W = \angle X$ (given)

$\angle WZY = \angle XZY = 90°$ ($YZ \perp WX$)

YZ is common.

$\therefore \triangle WYZ \equiv \triangle XYZ$ (AAS)

8 a $\triangle PML \equiv \triangle NLM$ (SAS)

b $PM = NL$ (matching sides of congruent triangles)

c The diagonals of a rectangle are equal.

9 a $\frac{4}{5}$ **b** $\frac{5}{3}$

10 a Yes, ($\frac{18}{27} = \frac{10}{15} = \frac{2}{3}$) **b** Yes ($\frac{9}{11.25} = \frac{12}{15} = \frac{22}{27.5} = \frac{16}{20} = \frac{4}{5}$)

11 a $k = 12\frac{6}{7}$ **b** $d = 6$

c $y = 4\frac{1}{5}$ or 4.2 **d** $x = 5\frac{1}{2}$ or 5.5

12 $d = 11\frac{3}{7}$

13 a Two pairs of matching sides are in the same ratio $\frac{18}{13.5} = \frac{20}{15} = \frac{4}{3}$ and the included angles are equal (SAS).

b In both right-angled triangles, the pairs of hypotenuses and second sides are in the same ratio $\frac{16\frac{2}{3}}{30} = \frac{10}{18} = \frac{5}{9}$ (RHS)

c Two pairs of angles are equal (AA).

Mixed revision 4

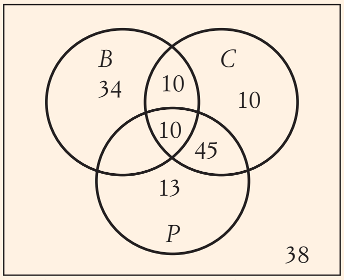
1 a 78 **b i** $\frac{25}{39}$ **ii** $\frac{14}{39}$ **c** $\frac{35}{78}$

2 a 36 **b i** $\frac{1}{36}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{4}$

3 a 80 **b i** $\frac{13}{80}$ **ii** $\frac{36}{80} = \frac{9}{20}$ **iii** $\frac{42}{80} = \frac{21}{40}$ **iv** $\frac{56}{80} = \frac{7}{10}$

c $\frac{1}{6} = 0.16$, which is lower than the experimental probability of $\frac{17}{80} = 0.2125$

4 **a**



b **i** $\frac{13}{160}$ **ii** $\frac{19}{80}$ **iii** $\frac{9}{32}$ **iv** $\frac{109}{160}$ **v** $\frac{57}{160}$

c $\frac{5}{27}$

5 In $\triangle ABC$ and $\triangle CDA$,

$\angle CAB = \angle ACD$ (alternate angles, $AB \| CD$)

$\angle BCA = \angle DAC$ (alternate angles, $AD \| CB$)

AC is common.

$\therefore \triangle ABC \equiv \triangle CDA$ (AAS)

6 independent

7 **a** $\frac{1}{4}$ or 0.25 **b** $\frac{5}{4}$ or 1.25

8 **a** **i** 0.39 **ii** 0.43 **iii** 0.18

b **i** 0.33 **ii** 0.42 **iii** 0.25

c drawing a black marble

d 88 which is close to the observed frequency of 85

9 **a** 36 **b** 170°

10 **a**

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24

b **i** $\frac{1}{24}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$

iv $\frac{1}{3}$ **v** $\frac{5}{8}$ **vi** $\frac{7}{24}$

11 **a** 135 **b** $\frac{40}{135} = \frac{8}{27}$

c **i** $\frac{67}{135}$ **ii** $\frac{60}{135} = \frac{4}{9}$ **iii** $\frac{72}{135} = \frac{8}{15}$

12 In $\triangle LMP$ and $\triangle LNP$,

$LM = LN$ (given)

$MP = NP$ (P is the midpoint of MN)

LP is common

$\therefore \triangle LMP \equiv \triangle LNP$ (SSS)

$\therefore \angle LPM = \angle LPN$ (matching angles of congruent triangles)

But $\angle LPM + \angle LPN = 180°$ (angles on a straight line)

$\therefore \angle LPM = \angle LPN = 90°$

13 **a** $h = 5\frac{1}{3}$ **b** $p = 35$

14 **a** Teacher to check.

b **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$ **v** $\frac{1}{4}$

15 **a** 2340° **b** 3240° **c** 1080° **d** 8280°

16 36

17 **a** **i** $\frac{2}{5}$ **ii** $\frac{1}{2}$

b Dependent as the sample space has reduced from 5 to 4.

c Independent as the number of marbles in the bag remains the same.

18 **a** SAS **b** SSS **c** RHS **d** AA

19 10.6

20 **a**

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b $\frac{5}{36}$ **c** **i** 1 **ii** 0 **d** $\frac{1}{2}$ **e** 1

21 **a** $\angle YXV = \angle WVX$, $\angle XYW = \angle VWY$

b Opposite sides of a rectangle are equal.

c AAS

d Matching sides of congruent triangles are equal.

e The diagonals of a rectangle bisect each other.

General Revision

1 \$15 700

2 **a** $m = 3, b = -4$ **b** $m = -2, b = 5$ **c** $m = -\frac{4}{3}, b = 3$

3 **a** $8x(x + 2)$ **b** $(y + 5)(y - 5)$

c $(a + 4)(a + 2)$ **d** $-(2p + 1)(p - 3)$

4 **a** 4 **b** 2.5

5 **a** 396 mm^2 **b** 3750 mm^2 **c** 6792 mm^2

6 **a** 210 **b** **i** $\frac{1}{6}$ **ii** $\frac{143}{210}$

c 32% **d** $\frac{29}{36}$

7 **a** 140° **b** 40°

8 **a** 6.4 **b** 15.6 **c** 110.6 **d** 13.7

e 3.3 **f** 32.1

9 **a** $56x^{12}$ **b** $\frac{x^5}{4}$ **c** $\frac{1}{9y^2}$ **d** $\frac{n}{m^7}$

10 **a** $k = 6\frac{1}{2}$ **b** $m = -3$ **c** $x = 7\frac{1}{2}$

11 26 m

12 **a** $a = 23$ **b** $y = 6\frac{1}{4}$ **c** $m = -3\frac{1}{3}$ **d** $k = 3\frac{3}{7}$

13 **a** 4 **b** 3 h **c** 75%

14 In $\triangle ABC \equiv \triangle DEF$:

$AB = DE = 10$ cm (given)

$CB = FE = 12$ cm (given)

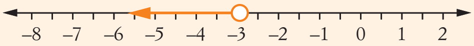
$\angle A = \angle D = 90°$ (given)

$\therefore \triangle ABC \equiv \triangle DEF$ (RHS)

15 **a** $y \geq -1$

b $x < \frac{3}{2}$

c $x < -3$



16 **a** **i** $\frac{1}{8}$ **ii** $\frac{3}{2}$ **iii** $y = \frac{x}{8} + \frac{3}{2}$

b **i** -1 **ii** -2 **iii** $y = -x - 2$

c **i** 2 **ii** 1 **iii** $y = 2x + 1$

17 **a** $x = 2, y = 2$ **b** $m = 1, p = -1$

18 $x^2 + y^2 = 64$

19 **a** \$40 **b** \$4.08

20 **a** 25° **b** 81° **c** 35°

21 **a** \$935.89

b \$1597.98

22 **a** **i** 8.9 **ii** (4, 5) **iii** 2

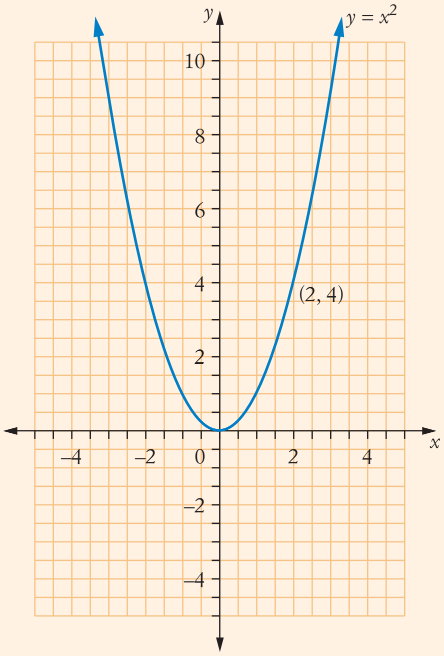
b **i** 13.4 **ii** (−1, 1) **iii** $\frac{1}{2}$

23 **a** **i** 35° **ii** 35°30′

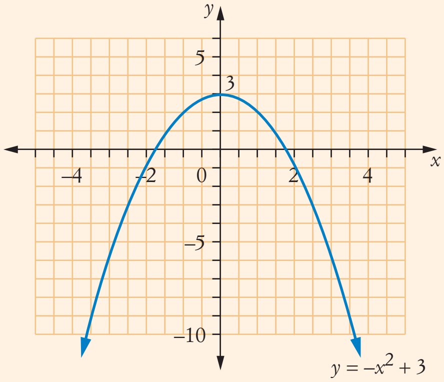
b **i** 35° **ii** 35°26′

c **i** 43° **ii** 43°21′

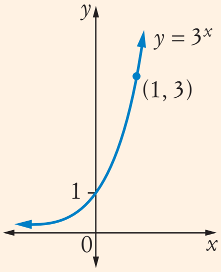
24 **a**



b



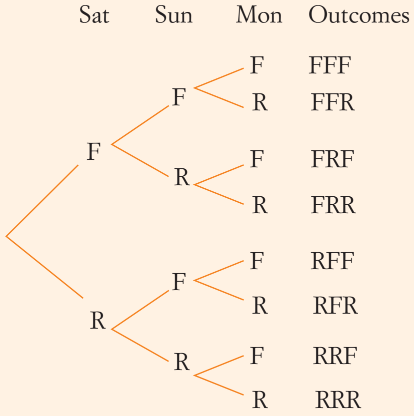
c



25 $h = 21, p = 20$

26 D

27 **a** F = fine, R = rain



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

28 19.2 min

Chapter 12

SkillCheck

1 **a** $x^2 + 8x + 7$ **b** $d^2 - d - 12$ **c** $t^2 - 25$

d $3y^2 - 12$ **e** $49y^2 - 9$ **f** $10p^2 - 25p - 35$

2 **a** $(y + 5)^2$ **b** $(x - 20)(x - 1)$

c $(n + 11)(n - 3)$ **d** $(a - 7)(a - 4)$

e $(m - 12)(m + 7)$ **f** $(p + 9)(p - 6)$

Exercise 12-01

1 **a** $20x$ **b** m^2

c t^2 **d** $+ 8h$

e $- 18k$ **f** $+ 80f$

g $4d^2 + 12d$ **h** $36a^2$, 1

2 **a** $m^2 + 18m + 81$ **b** $u^2 + 6u + 9$

c $y^2 - 12y + 36$ **d** $64 + 16k + k^2$

e $25 - 10h + h^2$ **f** $49 + 14k + k^2$

g $f^2 + 40f + 400$ **h** $q^2 - 22q + 121$

i $100 + 20t + t^2$ **j** $x^2 - 2xw + w^2$

k $a^2 + 2ag + g^2$ **l** $4m^2 - 12m + 9$

m $25x^2 - 60x + 36$ **n** $81a^2 + 36a + 4$

o $9e^2 - 24e + 16$ **p** $25 + 70b + 49b^2$

q $16 - 40p + 25p^2$ **r** $121 - 44c + 4c^2$

s $100g^2 + 60g + 9$ **t** $9k^2 + 66k + 121$

u $25 + 20v + 4v^2$

3 **a** $49h^2 + 28hk + 4k^2$ **b** $64a^2 - 48ay + 9y^2$

c $x^2y^2 + 2xyz + z^2$ **d** $1 + \frac{2}{y} + \frac{1}{y^2}$

e $t^2 - 2 + \frac{1}{t^2}$ **f** $\frac{9}{w^2} + 6 + w^2$

4 **a** 441 **b** 2025 **c** 841

d 3481 **e** 10 404 **f** 9604

5 **a** Two squares of length x cm have been cut from the ends, so the length of the base is $(10 - 2x)$ cm.

b $100 - 40x + 4x^2$ **c** $10x - 2x^2$

d $100 - 40x + 4x^2 + 4(10x - 2x^2) = 100 - 4x^2$

Answers

Exercise 12-02

1 a $m^2 - 25$ **b** $c^2 - 100$ **c** $a^2 - 144$
d $36 - y^2$ **e** $64 - m^2$ **f** $p^2 - 1$
g $25 - e^2$ **h** $v^2 - 121$ **i** $w^2 - 9$
j $x^2 - 100$ **k** $q^2 - 49$ **l** $81 - g^2$
m $b^2 - 4$ **n** $225 - r^2$ **o** $d^2 - 169$
2 a $4h^2 - 9$ **b** $25r^2 - 16$ **c** $25b^2 - 64$
d $16p^2 - 49$ **e** $9 - 64k^2$ **f** $49x^2 - 25$
g $4 - 81m^2$ **h** $81k^2 - 16r^2$ **i** $49n^2 - 64m^2$
j $16g^2 - 25h^2$ **k** $49u^2 - 9w^2$ **l** $121a^2 - 9b^2$
m $t^2 - \frac{1}{t^2}$ **n** $\frac{w^2}{9} - 4$ **o** $1 - \frac{1}{r^2}$
3 a $p - 1$ **b** $p + 1$ **c** $p^2 - 1$ **d** 7
4 899
5 a 399 **b** 2499 **c** 8099 **d** 6396
6 a 29 **b** 47 **c** 3000
d 400 **e** 240 **f** 196
7 a i $l = x + y$ **ii** $w = x - y$ **iii** $x^2 - y^2$
b $(x + y)(x - y) = x^2 - y^2$

Exercise 12-03

1 a $4m^2 - 1$ **b** $5y^2 + 17y - 12$
c $4k^2 - 28k + 49$ **d** $d^2 + 18d + 81$
e $2e^2 + e - 1$ **f** $25a^2 - 16$
g $-p^2 + 4p - 4$ **h** $100 - 36y^2$
i $h^2 - 6hz + 9z^2$ **j** $2xy + 6x - 3y - 9$
k $121a^2 - 16b^2$ **l** $u^2 - 2 + \frac{1}{u^2}$
2 a m^2 **b** $y^2 + 18$
c $-3x^2 + 7x + 6$ **d** $d^2 + 21$
e $16k^2 - 48$ **f** $-4xy$
g $27t - 10$ **h** $2f^2 - 8$
i $12h + 18$ **j** $5xy - 6x + 3y + 9$
3 a $60a^2$ **b** $n^2 + 4n + 4$
c $-2t^2 - 8t$ **d** $8m^2 + 2n^2$
e $x - 2$ **f** $1 - 2b^2$
g $3y^2 + 12y + 14$ **h** $3x^2 + 9$
i $34n^2 - 34$ **j** $-4b^2$

Exercise 12-04

1 a $(b + d)(4a + 5c)$ **b** $(y + t)(2x - 5w)$
c $(3c + 4d)(3a + 2b)$ **d** $(10 + x)(x^2 + 3)$
e $3(a + b)(a + c)$ **f** $6(t + p)(r - 3w)$
g $(7 + d)(2e - 3)$ **h** $(h - 2)(k - h)$
i $(3m + p)(n - 2)$ **j** $(9 + q)(p^2 - 3)$
k $(f - 10)(g - h)$ **l** $3(l + n)(3k - 4m)$
m $(2 - p)(p - c)$ **n** $(l - 3)(l^2 + m^2)$
o $(a + y)(x + 1 - k)$ **p** $(a - b + 3q)(p - 2q)$
2 a $(w + 3)(w - 3)$ **b** $(y + 6)(y - 6)$
c $(k + 1)(k - 1)$ **d** $(m + 11)(m - 11)$
e $(p + 8)(p - 8)$ **f** $(t + 10)(t - 10)$
g $(2e + f)(2e - f)$ **h** $(a + 3b)(a - 3b)$
i $(4y + 1)(4y - 1)$ **j** $(2 + b)(2 - b)$
k $(5 + e)(5 - e)$ **l** $(1 + 4x)(1 - 4x)$
m $(y + z)(y - z)$ **n** $(7 + 4m)(7 - 4m)$
o $(b + 11d)(b - 11d)$ **p** $(6c + 5k)(6c - 5k)$
q $(4 + 9h)(4 - 9h)$ **r** $(5a + 8m)(5a - 8m)$
s $(10 + 7n)(10 - 7n)$ **t** $(11p + 12q)(11p - 12q)$
u $\left(\frac{1}{2} + 5c\right)\left(\frac{1}{2} - 5c\right)$ **v** $\left(2t + \frac{1}{3}\right)\left(2t - \frac{1}{3}\right)$
w $\left(5h + \frac{3}{2}\right)\left(5h - \frac{3}{2}\right)$ **x** $(1 + mn)(1 - mn)$
3 a $2(a + b)(a - b)$ **b** $7(k + 2)(k - 2)$
c $3(1 + 5u)(1 - 5u)$ **d** $x(x + 7)(x - 7)$
e $k(1 + 4k)(1 - 4k)$ **f** $2(5q + 1)(5q - 1)$
g $3(\mathrm{d} + 2v)(d - 2v)$ **h** $5t^3(t + 5)(t - 5)$
i $2(ab + 1)(ab - 1)$ **j** $x^2(y + w)(y - w)$
k $12(4f + 3g)(4f - 3g)$
l $5\left(3d + \frac{1}{2}\right)\left(3d - \frac{1}{2}\right)$ or $\frac{5}{4}(6d + 1)(6d - 1)$
m $2(x + 2a)(x - 2a)$ **n** $25(2 + w)(2 - w)$
o $5\left(\frac{1}{2} + 4e\right)\left(\frac{1}{2} - 4e\right)$ or $\frac{5}{4}(1 + 8e)(1 - 8e)$
p $\left(3c + 2\frac{1}{2}\right)\left(3c - 2\frac{1}{2}\right)$ or $\frac{1}{4}(6c + 5)(6c - 5)$
4 a $\left(\frac{p}{4} + \frac{x}{5}\right)\left(\frac{p}{4} - \frac{x}{5}\right)$ **b** $\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right)$
c $\left(\frac{v}{7} + \frac{u}{9}\right)\left(\frac{v}{7} - \frac{u}{9}\right)$ **d** $2\left(\frac{y}{3} - \frac{m}{11}\right)\left(\frac{y}{3} + \frac{m}{11}\right)$
e $\left(\frac{4a}{7} + \frac{5b}{2}\right)\left(\frac{4a}{7} - \frac{5b}{2}\right)$ **f** $(t + 3)(t - 3)(t^2 + 9)$
g $(10 + n^2)(10 - n^2)$ **h** $y(2x + y)$
i $-(p + 3q)(3p - q)$ **j** $\left(\frac{x}{2} + \frac{y}{6}\right)\left(\frac{x}{2} - \frac{y}{6}\right)$
k $4ab$

Exercise 12-05

1 a $3(m + 1)(m + 2)$ **b** $2(y + 2)(y - 1)$
c $5(t - 10)(t + 8)$ **d** $5e^2(e + 8)(e - 3)$
e $x(x - 11)(x + 10)$ **f** $4(b - 7)(b + 6)$
g $4(w + 4)(w - 3)$ **h** $3a(a - 4)(a + 1)$
i $2(e + 5)(e + 4)$ **j** $-(t + 8)(t - 3)$
k $-(u - 7)(u + 6)$ **l** $-(x - 7)(x + 4)$
m $-(b + 4)(b - 3)$ **n** $-(k - 3)(k- 4)$
o $-(x - 5)(x - 7)$
2 a $(x + 5)(2x + 1)$ **b** $(x + 3)(4x + 1)$
c $(x + 3)(5x + 2)$ **d** $(2x + 5)(3x + 2)$
e $(w + 15)(2w + 1)$ **f** $(e + 3)(4e + 3)$
g $(2f + 3)(4f + 1)$ **h** $(d + 1)(3d + 2)$
i $(b + 1)(2b + 7)$ **j** $(y + 1)(5y + 11)$
k $(4g + 3)(2g + 5)$ **l** $(3a + 7)(2a + 3)$
3 a $(y - 4)(2y - 3)$ **b** $(2k - 3)(5k - 2)$
c $(2e - 3)(3e - 2)$ **d** $(b - 3)(4b - 1)$
e $(w - 3)(6w - 5)$ **f** $(2t + 5)(4t + 3)$
g $(3x - 2)^2$ **h** $(3f - 4)(4f - 3)$
i $(2h - 9)^2$ **j** $(y + 1)(5y - 11)$
k $(4d - 5)(d + 1)$ **l** $(2m + 3)(m - 3)$
m $(3t - 10)(t + 3)$ **n** $(6h - 7)(h + 1)$
o $(y - 4)(2y + 3)$ **p** $(2a + 1)(4a - 3)$
q $(5u - 4)(3u + 1)$ **r** $(3c + 1)(3c - 5)$
4 a $(5m + 7)(m - 1)$ **b** $(3g - 4)(2g + 3)$
c $(3p - 2)(p + 2)$ **d** $(7w - 1)(w + 1)$
e $(5y - 1)(y + 3)$ **f** $(3n - 2)(n + 4)$
g $(4b - 3)\ (b + 3)$ **h** $(4m - 1)(2m + 3)$
i $(3x + 8)(x - 2)$
5 a $(9w - 10)^2$ **b** $4(y + 1)^2$ **c** $(5h - 4)^2$

Answers

6 **a** $2(t + 2)(3t - 1)$ **b** $3(g + 4)(2g - 3)$
c $4(2e - 3)(3e + 1)$ **d** $2(a - 2)(4a + 3)$
e $4(t + 2)(3t - 1)$ **f** $-(5q + 3)(5q - 2)$
g $-2(2m - 1)(3m - 2)$ **h** $-(3h + 4)(4h - 5)$
i $6(2c + 3)(2c + 1)$ **j** $-3(z + 1)(2z - 5)$
k $2(2d - 3)(3d + 5)$ **l** $-2(x - 3)(3x - 2)$
7 **a** $(a + 1)(2a + 3)$ **b** $(2m - 5)(6m - 1)$
c $(4x - 1)(x + 3)$ **d** $(w - 1)(7w - 1)$
e $(h - 3)(4h + 5)$ **f** $(4x - 3)(2x + 1)$
g $(r + 5)(5r + 1)$ **h** $(d - 7)(2d - 1)$
i $(3n + 1)(2n - 3)$ **j** $-(3m - 2)(3m + 4)$
k $-(5c - 3)(c + 1)$ **l** $(3g + 2)(5g + 3)$
m $-(4q + 3)(2q - 5)$ **n** $(x - 2)(3x - 7)$
o $-(3d - 4)(d + 4)$

Exercise 12-06

1 **a** $(m - 8)^2$ **b** $3d(d - 1)$
c $(d - 3)(3d + 5)$ **d** $(3 + h)(k - 5)$
e $(5y + 8)(5y - 8)$ **f** $4(5f + 4)(5f - 4)$
g $q(q + 3 - 3p)$ **h** $-(g - 3)(g + 1)$
i $4(2b + 5)(3b - 2)$ **j** $(5r + 1)(5r - 1)$
k $(b^2 + 1)(b + 1)$ **l** $(2x - 5)^2$
m $-(5d - 4)(d + 1)$ **n** $(b - 1)^2(b + 1)$
o $2(2 + v)(2 - v)$ **p** $m(n + 3)(n + p)$
q $2(w - 6)^2$ **r** $(6h + 1)^2$
s $(3r - 8t)(5r + 3t)$ **t** $(2d + 1)^2$
u $9(g + 2k)(g - 2k)$
2 **a** $e(e - 5)(e + 2)$ **b** $-20(2p - 3q)(3p - 2q)$
c $7(2x + 1)(2x - 1)$ **d** $(a - b)(a + b + 4)$
e $(c - 2)^2(c + 2)$ **f** $(3a - 1)(2a + 5)$
g $(t + 7)(t - 5)$ **h** $2(3p + 2)^2$
i $-(6a - 1)(4a + 1)$ **j** $9(x + 2)(x - 3)$
k $(a - 3)(2ab - 3)$ **l** $2(a + 3)^2$
m $(5u - 1)^2$ **n** $(k - 3)(4k + 7)$
o $3(4 + w)(4 - w)$ **p** $3(1 + 3s)(1 - 3s)$
q $(k + 4)^2(k - 4)$ **r** $5y(y^2 - 2y + 3)$
s $mn(m + 2)(m - 2)$ **t** $-2(a + 2)(a - 2)$
u $4(2c - 3)(4c + 1)$

Exercise 12-07

1 **a** $x + y$ **b** $\frac{1}{2(t-r)}$ **c** $\frac{b-c}{a}$
d -1 **e** $w - 4$ **f** $\frac{5}{d+t}$
g $\frac{k+5}{k-5}$ **h** $3(c - 1)$ **i** $\frac{a+1}{m+n}$
j $\frac{y+4}{2}$ **k** $\frac{k+1}{k+4}$ **l** $\frac{4a+5c}{a-c}$
m $\frac{s+2}{s-3}$ **n** $\frac{-(2c-1)}{3c-1}$ **o** $\frac{a+4}{2(p+2)}$
2 **a** $\frac{7m+10}{m(m+1)(m+2)}$ **b** $\frac{2w-20}{w(w+3)(w+5)}$
c $\frac{4b-7}{(b-1)(b+2)(b-3)}$ **d** $\frac{-k-2}{k(k+1)(k-1)}$
e $\frac{5h+12}{4h(h+1)}$ **f** $\frac{-4d-1}{(d+2)(d+1)}$
g $\frac{42-5r}{4(r+6)(r-6)}$ **h** $\frac{d^2+3d-6}{d(d+2)(d-2)}$
i $\frac{-k^2+9k-5}{(k+1)(k-1)(k-4)}$ **j** $\frac{3q-1}{(q+1)(q-1)}$

3 **a** $6m$ **b** $1\frac{1}{24}$ **c** $\frac{1}{2}$
d $6k$ **e** $\frac{10}{h+1}$ **f** $\frac{3}{2(a-b)}$
g $\frac{-r}{5(r+t)}$ **h** $\frac{4m}{m-1}$ **i** $\frac{4}{p}$
j $\frac{3}{7}$ **k** $\frac{2}{3(x-2)}$ **l** $3\frac{1}{3}$
m $\frac{(d+1)(d-3)}{6}$ **n** $\frac{f+3}{4(f-3)}$ **o** $\frac{3}{f-2}$

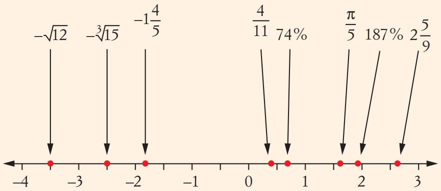
Chapter 13

SkillCheck

1 **a** $25y^2$ **b** $64m^3$ **c** $9x^2$
2 **a** $5x + 10$ **b** $4y - 12$ **c** $3 + 6w$
d $10 - 2y$ **e** $-10a - 15$ **f** $k + 2k^2$
3 81, 25, 100, 16, 64
4 **a** $m^2 + 10m + 21$ **b** $y^2 - 3y - 4$
c $n^2 - 5n + 6$ **d** $6d^2 + 11d + 3$
e $4 - 17p - 15p^2$ **f** $3a^2 + 17af + 10f^2$
g $x^2 + 8m + 16$ **h** $y^2 - 6y + 9$
i $4k^2 + 4k + 1$ **j** $a^2 - 25$
k $t^2 - 49$ **l** $9m^2 - 16$

Exercise 13-01

1 C **2** B
3 $\sqrt{32}$, $\sqrt{33}$, $\sqrt{4.9}$, $\sqrt{52}$, $\sqrt{200}$
4 **a** R **b** I **c** R **d** R
e R **f** R **g** R **h** R
i I **j** R **k** I **l** I
5 **a** $1\frac{4}{7}$, $\frac{\pi}{2}$, $\sqrt{2}$ **b** $2\frac{7}{9}$, $\sqrt[3]{20}$, $2.\dot{6}$
6 **a** -1.8 **b** 0.7 **c** 0.4 **d** -3.5
e -2.5 **f** 2.6 **g** 1.6 **h** 1.9



7 1.4
8 Teacher to check, $\sqrt{5} \approx 2.24$, $\sqrt{10} \approx 3.16$, $\sqrt{17} \approx 4.12$

Exercise 13-02

1 **a** 2 **b** 5 **c** 27 **d** 250
e 0.09 **f** 28 **g** 45 **h** 50
2 **a** $5\sqrt{2}$ **b** $3\sqrt{3}$ **c** $2\sqrt{6}$ **d** $3\sqrt{6}$
e $9\sqrt{3}$ **f** $3\sqrt{5}$ **g** $4\sqrt{3}$ **h** $10\sqrt{2}$
i $4\sqrt{6}$ **j** $3\sqrt{7}$ **k** $12\sqrt{2}$ **l** $6\sqrt{3}$
m $5\sqrt{3}$ **n** $7\sqrt{3}$ **o** $4\sqrt{2}$ **p** $11\sqrt{2}$
q $9\sqrt{2}$ **r** $7\sqrt{5}$ **s** $5\sqrt{5}$ **t** $16\sqrt{2}$
3 **a** $25\sqrt{2}$ **b** $6\sqrt{2}$ **c** $12\sqrt{3}$ **d** $56\sqrt{2}$
e $\sqrt{10}$ **f** $\sqrt{3}$ **g** $\frac{\sqrt{7}}{3}$ **h** $6\sqrt{6}$
i $18\sqrt{17}$ **j** $\frac{5\sqrt{5}}{2}$ **k** $3\sqrt{2}$ **l** $3\sqrt{3}$
m $40\sqrt{10}$ **n** $15\sqrt{3}$ **o** $14\sqrt{17}$ **p** $\frac{\sqrt{13}}{3}$

4 B **5** B

6 **a** false **b** false **c** true

d true **e** true **f** false

Exercise 13-03

1 **a** $7\sqrt{7}$ **b** $-5\sqrt{2}$ **c** $6\sqrt{5}$ **d** $4\sqrt{5}$

e 0 **f** $\sqrt{10}$ **g** $8\sqrt{15}$ **h** $-\sqrt{6}$

i $2\sqrt{3}$ **j** $10\sqrt{5}$ **k** $6\sqrt{10}$ **l** $-5\sqrt{3}$

2 **a** $5\sqrt{5}-8$ **b** $13\sqrt{10}+3\sqrt{2}$ **c** $5\sqrt{2}-9\sqrt{3}$

d $7\sqrt{15}+8\sqrt{2}$ **e** $-2\sqrt{5}-3\sqrt{7}$ **f** $2\sqrt{6}-8\sqrt{3}$

g $13\sqrt{11}-\sqrt{3}$ **h** $11\sqrt{7}-6\sqrt{13}$ **i** $-6\sqrt{7}$

j $-3\sqrt{5}$

3 **a** D **b** A

4 **a** $6\sqrt{2}$ **b** $3\sqrt{3}$ **c** $-2\sqrt{5}$

d $-\sqrt{7}$ **e** $5\sqrt{6}$ **f** $7\sqrt{5}$

g $-\sqrt{10}$ **h** $8\sqrt{11}$ **i** $6\sqrt{2}$

j $8\sqrt{3}$ **k** $3\sqrt{2}$ **l** $9\sqrt{2}$

m $11\sqrt{3}$ **n** $-\sqrt{5}$ **o** $-6\sqrt{3}$

p $30\sqrt{3}$ **q** $5\sqrt{7}$ **r** $41\sqrt{2}$

s $5\sqrt{6}$ **t** $29\sqrt{2}$ **u** $-15\sqrt{3}$

v 0 **w** $6\sqrt{2}+2\sqrt{3}$ **x** $12\sqrt{3}+3\sqrt{6}$

y $3\sqrt{2}-6\sqrt{5}$ **z** $4\sqrt{6}$

Exercise 13-04

1 **a** $\sqrt{14}$ **b** $-\sqrt{35}$ **c** $4\sqrt{3}$ **d** 6

e $-5\sqrt{2}$ **f** 45 **g** $15\sqrt{30}$ **h** $-10\sqrt{21}$

i 140 **j** $-30\sqrt{2}$ **k** 36 **l** $-60\sqrt{2}$

m -112 **n** $24\sqrt{6}$ **o** 80 **p** $90\sqrt{6}$

q -396 **r** $160\sqrt{5}$ **s** $216\sqrt{2}$ **t** $-96\sqrt{6}$

u $36\sqrt{5}$ **v** $-60\sqrt{10}$ **w** $252\sqrt{3}$ **x** 144

2 **a** $\sqrt{5}$ **b** $-\sqrt{3}$ **c** $3\sqrt{6}$ **d** $2\sqrt{2}$

e $-\frac{\sqrt{7}}{2}$ **f** 21 **g** 1 **h** 8

i $5\sqrt{3}$ **j** $5\sqrt{2}$ **k** $4\sqrt{3}$ **l** $2\sqrt{6}$

m $2\sqrt{14}$ **n** $-\frac{\sqrt{2}}{4}$ **o** 1 **p** 10

q 4 **r** $-21\sqrt{2}$ **s** 12 **t** 2

u $\frac{2}{3}$

3 **a** 6 **b** 7 **c** 6

d $15y$ **e** x **f** $a\sqrt{a}$

4 C **5** A

6 **a** 2 **b** $4\sqrt{6}$ **c** $\sqrt{30}$

d $\frac{2}{45}$ **e** $14\sqrt{3}$ **f** 2

Exercise 13-05

1 **a** $\sqrt{15}+\sqrt{10}$ **b** $2\sqrt{3}-\sqrt{6}$ **c** $\sqrt{6}+\sqrt{14}$

d $3\sqrt{10}-5$ **e** $6+6\sqrt{6}$ **f** $\sqrt{55}-4\sqrt{11}$

g $42-8\sqrt{7}$ **h** $5\sqrt{5}+75$ **i** $24+3\sqrt{6}$

2 C

3 **a** $10+\sqrt{10}-6\sqrt{5}-3\sqrt{2}$ **b** $7+2\sqrt{7}-\sqrt{21}-2\sqrt{3}$

c $28\sqrt{6}+21+8\sqrt{2}+2\sqrt{3}$ **d** $20+\sqrt{10}$

e $109+10\sqrt{77}$ **f** $72-23\sqrt{6}$

g $16\sqrt{10}+54$ **h** $-16-\sqrt{35}$

4 C

5 **a** $8-2\sqrt{15}$ **b** $9+2\sqrt{14}$ **c** $9-4\sqrt{5}$

d $19+6\sqrt{10}$ **e** $77+30\sqrt{6}$ **f** $179-20\sqrt{7}$

g $38+12\sqrt{10}$ **h** $23+4\sqrt{15}$

6 **a** 1 **b** 22 **c** 8 **d** 2

e 1 **f** 166 **g** 13 **h** -43

7 C

8 **a** $88-30\sqrt{7}$ **b** $21\sqrt{2}-10$ **c** $5\sqrt{35}+29$

d $73+40\sqrt{3}$ **e** 29 **f** $92-12\sqrt{5}$

Exercise 13-06

1 B

2 **a** $\frac{\sqrt{2}}{2}$ **b** $\frac{\sqrt{7}}{7}$ **c** $\frac{\sqrt{3}}{3}$ **d** $\frac{3\sqrt{2}}{2}$

e $\frac{2\sqrt{7}}{7}$ **f** $\frac{\sqrt{2}}{6}$ **g** $\frac{\sqrt{3}}{6}$ **h** $\frac{\sqrt{7}}{28}$

i $\frac{7\sqrt{5}}{15}$ **j** $\frac{\sqrt{10}}{15}$ **k** $\frac{\sqrt{3}}{2}$ **l** $\frac{\sqrt{15}}{4}$

3 A **4** D

Chapter 14

SkillCheck

1 **a** $x=\pm 5$ **b** $m=\pm 2$ **c** $x=2$ or -1

d $u=-7$ or 4 **e** $k=0$ or -3 **f** $w=5$

2 **a** $(4-m)(4+m)$ **b** $(d-11)(d+11)$

c $2y(7-y)$ **d** $5p(2p+5)$

e $5(x-8)(x+8)$ **f** $2(3w-5)(3w+5)$

g $(k+1)(k+4)$ **h** $(y-8)(y-2)$

i $(m-8)(m+7)$ **j** $(u+13)(u-5)$

k $(w-3)(w-7)$ **l** $(x-6)(x+4)$

3 **a** $y=-2$ **b** $y=-10$

c $y=5$ **d** $y=-10$

4 **a** $(3a+1)(a+3)$ **b** $(5x+2)(x-3)$

c $(2y-5)(3y+8)$ **d** $(3t-1)(5t+4)$

e $(5v+3)(v-7)$ **f** $(2y+5)(4y+7)$

g $(3h-4)(5h-1)$ **h** $(4p-3)(3p+5)$

i $(4d+5)^2$

Exercise 14-01

1 **a** $m=-7$ or -3 **b** $d=3$ or 7

c $y=-5$ or 3 **d** $k=0$ or 3

e $t=-7$ or 0 **f** $p=0$ or 3

g $w=0$ or $\frac{2}{3}$ **h** $n=-\frac{1}{2}$ or 3

i $a=\frac{1}{2}$ or $\frac{3}{5}$ **j** $x=-\frac{1}{3}$ or $-1\frac{1}{2}$

k $c=\frac{5}{2}$ **l** $f=\frac{1}{2}$

m $c=-\frac{1}{3}$ or $-\frac{1}{4}$ **n** $h=-1$ or $\frac{1}{2}$

o $e=\frac{5}{7}$ or 1

2 **a** $y=-2$ or $-1\frac{1}{2}$ **b** $g=-1$ or $-1\frac{1}{2}$

c $d=-1$ or $-\frac{2}{3}$ **d** $t=-2\frac{1}{5}$ or -1

e $p=1\frac{1}{2}$ or 4 **f** $x=\frac{2}{5}$ or $1\frac{1}{2}$

g $y=\frac{3}{4}$ or $-\frac{1}{2}$ **h** $a=-\frac{1}{2}$ or $1\frac{1}{3}$

i $w=-1\frac{1}{4}$ or 3 **j** $c=1$ or $-1\frac{2}{5}$

k $e=\frac{1}{4}$ or $-1\frac{1}{2}$ **l** $q=-3$ or $1\frac{2}{3}$

m $g=2\frac{1}{2}$ **n** $m=-\frac{2}{3}$ or $\frac{5}{6}$

o $w=-4$ or $1\frac{1}{3}$ **p** $y=-3$ or 4

q $f=6$ **r** $h=-1$ or $\frac{3}{4}$

Answers

3 **a** $x = -\,2\frac{1}{2}$ or 3
b $t = -\,2\frac{1}{2}$ or $\frac{1}{2}$
c $u = -\frac{1}{8}$ or -5
d $m = \frac{1}{7}$ or 1
e $p = -4$ or 7
f $e = -1$ or 5
g $t = \frac{3}{2}$ or 5
h $d = -\frac{7}{3}$ or $\frac{1}{2}$
i $h = \pm 5$
j $f = 0$ or $\frac{1}{2}$
k $w = \frac{1}{6}$ or 3
l $a = -2$ or $\frac{1}{3}$

4 8

Exercise 14-02

1 **a** $x^2 + 2x + 1 = (x+1)^2$
b $p^2 - 6p + 9 = (p-3)^2$
c $m^2 - 8m + 16 = (m-4)^2$
d $k^2 + 4k + 4 = (k+2)^2$
e $y^2 - 7y + \frac{49}{4} = \left(y - \frac{7}{2}\right)^2$
f $w^2 - 3w + \frac{9}{4} = \left(w - \frac{3}{2}\right)^2$
g $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$
h $h^2 - 5h + \frac{25}{4} = \left(h - \frac{5}{2}\right)^2$
i $a^2 + \frac{7}{2}a + \frac{49}{16} = \left(a + \frac{7}{4}\right)^2$
j $v^2 + \frac{5}{3}v + \frac{25}{36} = \left(v + \frac{5}{6}\right)^2$

2 **a** $d = -3 + \sqrt{7}, -3 - \sqrt{7}$
b $x = 5 + \sqrt{5}, 5 - \sqrt{5}$
c $p = -1 + \sqrt{10}, -1 - \sqrt{10}$
d $y = 1 + \sqrt{2}, 1 - \sqrt{2}$
e $m = \frac{1+2\sqrt{5}}{2}, \frac{1-2\sqrt{5}}{2}$
f $t = \frac{-2+3\sqrt{3}}{3}, \frac{-2-3\sqrt{3}}{3}$
g $c = \frac{-2+\sqrt{42}}{2}, \frac{-2-\sqrt{42}}{2}$
h $w = \frac{6+\sqrt{82}}{2}, \frac{6-\sqrt{82}}{2}$
i $n = \frac{-2+\sqrt{7}}{3}, \frac{-2-\sqrt{7}}{3}$
j $e = \frac{3+\sqrt{71}}{2}, \frac{3-\sqrt{71}}{2}$
k $d = 2 + \sqrt{5}, 2 - \sqrt{5}$
l $x = \frac{3+4\sqrt{2}}{4}, \frac{3-4\sqrt{2}}{4}$

3 **a** $h = -1 \pm \sqrt{6}$
b $r = 1 \pm \sqrt{2}$
c $m = -3 \pm \sqrt{7}$
d $w = -6, 10$
e $a = 5 \pm \sqrt{30}$
f $y = -4 \pm \sqrt{19}$
g $p = -6 \pm \sqrt{41}$
h $h = 2 \pm \sqrt{2}$
i $u = -7, -2$
j $d = \frac{-1\pm\sqrt{29}}{2}$
k $c = \frac{9\pm\sqrt{73}}{2}$
l $e = \frac{-5\pm\sqrt{17}}{2}$
m $y = \frac{3\pm\sqrt{41}}{2}$
n $b = \frac{1\pm\sqrt{21}}{2}$
o $q = \frac{3\pm\sqrt{5}}{2}$
p $g = \frac{-7\pm\sqrt{73}}{4}$
q $x = -3\frac{1}{2},\ 1$
r $u = \frac{-2\pm\sqrt{22}}{3}$

4 **a** $x = -11.20$ or -0.80
b $m = -0.43$ or 16.43
c $g = -4.65$ or 0.65
d $h = 1.27$ or -2.77
e $w = 1.27$ or -0.47
f $y = 1.14$ or -1.47
g $p = -2$ or 1.33
h $e = 1.13$ or -0.88
i $n = 1$ or -2.5

Exercise 14-03

1 **a** $x = -3 \pm \sqrt{7}$
b $m = \frac{5\pm\sqrt{37}}{2}$
c $w = 4 \pm \sqrt{13}$
d $k = \frac{-3\pm\sqrt{29}}{2}$
e $y = 2 \pm \sqrt{5}$
f $p = \frac{-1\pm\sqrt{21}}{2}$
g $u = \frac{7\pm\sqrt{61}}{2}$
h $a = \frac{-3\pm\sqrt{65}}{4}$
i $q = \frac{1}{5}, 1$
j $c = \frac{-1\pm\sqrt{7}}{3}$
k $e = \frac{5\pm\sqrt{57}}{8}$
l $x = \frac{-4\pm\sqrt{10}}{3}$
m $d = \frac{2\pm\sqrt{14}}{2}$
n $a = \frac{5\pm\sqrt{31}}{3}$
o $t = -2\frac{1}{2}, 1$
p $y = -\frac{2}{3}, -2$
q $k = \frac{5}{6}, 1$
r $n = \frac{5\pm\sqrt{113}}{4}$

2 **a** $y = \frac{9\pm\sqrt{141}}{10}$
b $m = \frac{-1\pm\sqrt{22}}{3}$
c $x = \frac{3\pm\sqrt{41}}{8}$
d $k = -2 \pm \sqrt{5}$
e $m = \frac{3\pm\sqrt{21}}{6}$
f $g = \frac{-1\pm\sqrt{6}}{5}$
g $h = \frac{9\pm\sqrt{17}}{4}$
h $w = \frac{1\pm\sqrt{7}}{3}$
i $p = \frac{2\pm\sqrt{7}}{3}$
j $u = \frac{-2\pm\sqrt{14}}{5}$
k $a = \frac{-2\pm\sqrt{58}}{6}$
l $y = \frac{-3\pm\sqrt{89}}{4}$

3 **a** $k = 8.89, 0.11$
b $c = 1.41, -1.41$
c $m = 2.65, -2.65$
d $n = 3.19, 0.31$
e $p = 0.85, -2.35$
f $w = 0.30, -1.13$
g $x = 2.39, 0.28$
h $h = 3.83, -1.83$
i $x = 1.62, -0.62$
j $a = 4, 9$
k $v = 1.48, -1.48$
l $c = 2.31, 0.69$
m $t = 8.09, -3.09$
n $x = 4.27, 7.73$
o $d = 3.31, -1.81$

Exercise 14-04

1 **a** $y = \pm 2,\ \pm 2\sqrt{2}$
b $m = \pm 4$
c $w = \pm\sqrt{2},\ \pm\sqrt{6}$
d $m = 1, 2$
e $k = -1, 3$
f $w = 2,\ \sqrt[3]{-4}$
g $x = -1,\ \frac{1}{3}$
h $y = -\frac{4}{5},\ 1\frac{2}{5}$
i $a = -1\frac{1}{2}, -\frac{1}{2}$
j $p = -\sqrt[3]{5},\ -1$
k $g = \frac{1}{2}, 2$
l $c = -2, \frac{1}{5}$

2 **a** $m \approx \pm 1.2, \pm 2.8$
b $x \approx \pm 0.9, \pm 1.4$
c $a \approx -0.9, 1.3$
d $y \approx -1.4, 1.3$
e $w \approx -1.2, 1.9$
f $e \approx \pm 0.5$

3 **a** $w = \pm 5$
b $x = -2, 1.3$
c $k = \pm 2$
d $m = \pm 1.4$
e $y = \pm 1.2$
f $v = -2, 1.4$

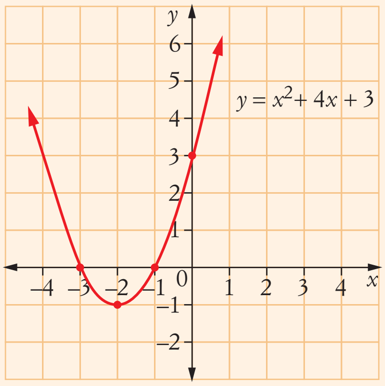
Exercise 14-05

1 12 m by 8 m
2 40 m by 35 m
3 42 m by 24 m
4 Length 58 m, width 38 m
5 Length 13 m
6 15 m by 12 m
7 0.52 s, 3.08 s
8 **a** 4000 m **b** 2000 m **c** 24.5 s
9 **a** 34 m **b** 3.9 s
c **i** 0.5 s and 1.9 s **ii** 3.5 s
10 8 or -9
11 24, 25 or -24, -25
12 35 or -34
13 27 and 15
14 **a** \$9200 **b** \$5550 **c** 471

Exercise 14-06

1 **a** **i**

x	-3	-2	-1	0	1	2	3
y	0	-1	0	3	8	15	24

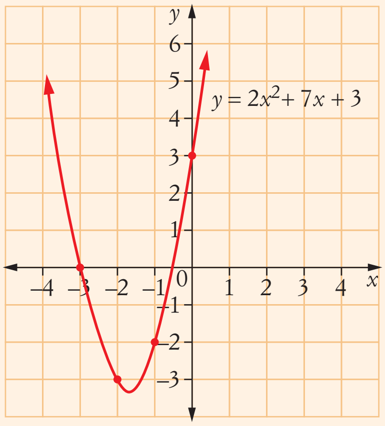


ii -3 and -1 **iii** 3

iv $x = -3$ and -1, same as x-intercepts

b i

x	-3	-2	-1	0	1	2	3
y	0	-3	-2	3	12	25	42

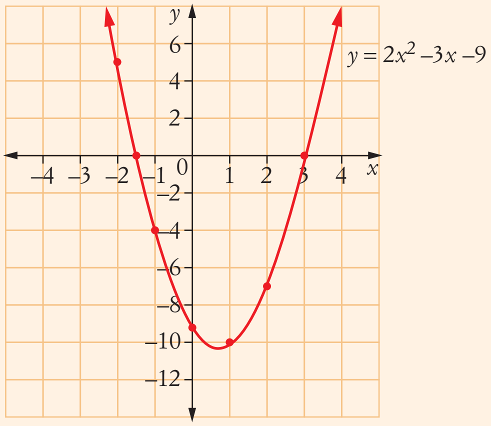


ii -3 and $-\frac{1}{2}$ **iii** 3

iv $x = -3$ and $-\frac{1}{2}$, same as x-intercepts

c i

x	-3	-2	-1	0	1	2	3
y	18	5	-4	-9	-10	-7	0

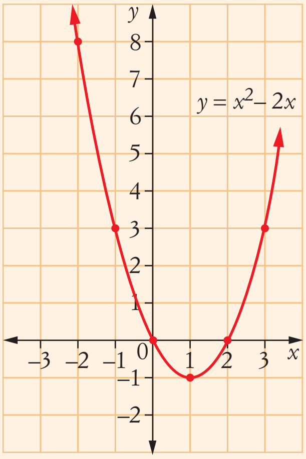


ii $-1\frac{1}{2}$ and 3 **iii** -9

iv $x = -1\frac{1}{2}$ and 3, same as x-intercepts

d i

x	-3	-2	-1	0	1	2	3
y	15	8	3	0	-1	0	3

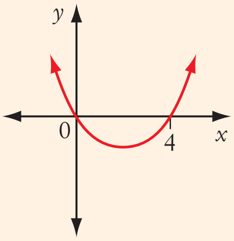


ii 0 and 2 **iii** 0

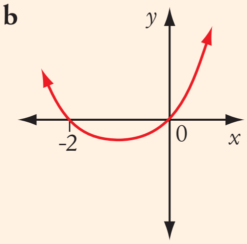
iv $x = 0$ and 2, same as x-intercepts

2 a -5 **b** 3 **c** 0

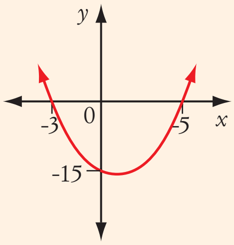
3 a



b



c



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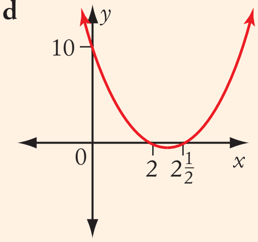

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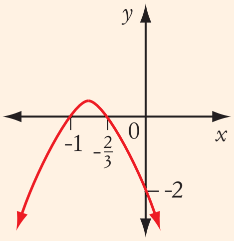
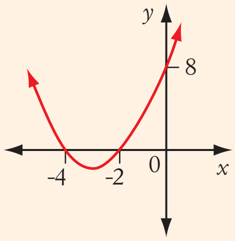
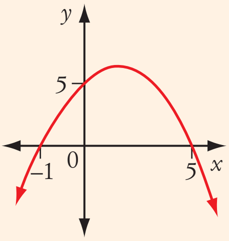


f



g



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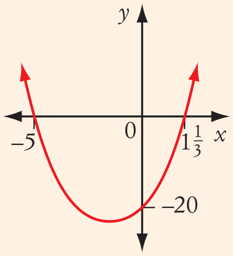
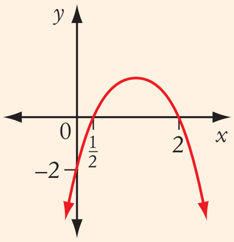


i



Exercise 14-07

1 **a** $x = 3$ **b** $x = 0$ **c** $x = 2\frac{1}{2}$

d $x = -1$ **e** $x = 1\frac{1}{2}$ **f** $x = 3$

2 **a** **i** $x = 3$ **ii** $(3, -1)$

b **i** $x = 5$ **ii** $(5, 16)$

c **i** $x = 1$ **ii** $(1, 9)$

d **i** $x = 4$ **ii** $(4, 25)$

e **i** $x = \frac{1}{2}$ **ii** $(\frac{1}{2}, -24\frac{3}{4})$

f **i** $x = -4$ **ii** $(-4, -80)$

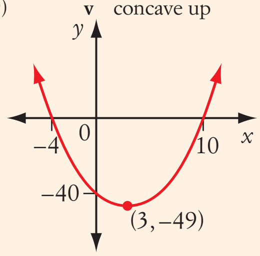
g **i** $x = 4$ **ii** $(4, 48)$

h **i** $x = -\frac{1}{4}$ **ii** $(-\frac{1}{4}, -1\frac{1}{4})$

i **i** $x = -\frac{1}{6}$ **ii** $(-\frac{1}{6}, 1\frac{1}{4})$

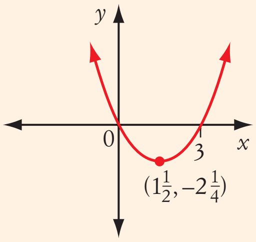
3 **a** **i** $-4, 10$ **ii** -40 **iii** $x = 3$

iv $(3, -49)$ **v** concave up



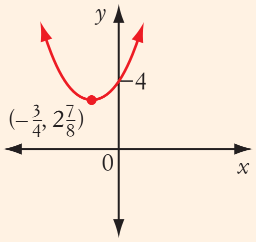
b **i** $0, 3$ **ii** 0 **iii** $x = 1\frac{1}{2}$

iv $(1\frac{1}{2}, -2\frac{1}{4})$ **v** concave up



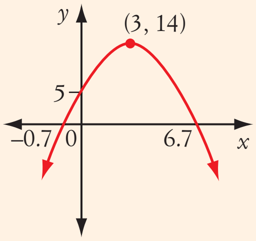
c **i** No *x*-intercepts. **ii** 4 **iii** $x = -\frac{3}{4}$

iv $(-\frac{3}{4}, 2\frac{7}{8})$ **v** concave up



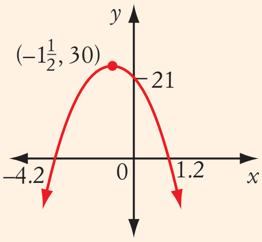
d **i** $-0.7, 6.7$ **ii** 5 **iii** $x = 3$

iv $(3, 14)$ **v** concave down



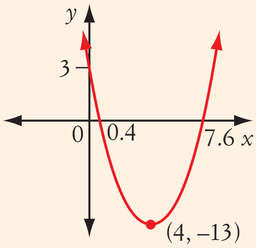
e **i** $-4.2, 1.2$ **ii** 21 **iii** $x = -1\frac{1}{2}$

iv $(-1\frac{1}{2}, 30)$ **v** concave down

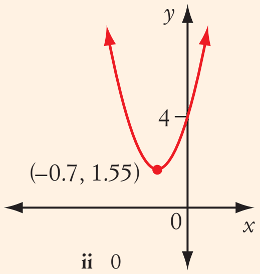


f **i** $0.4, 7.6$ **ii** 3 **iii** $x = 4$

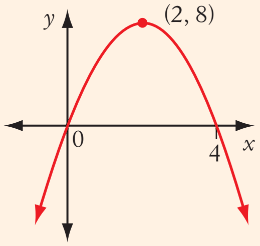
iv $(4, -13)$ **v** concave up



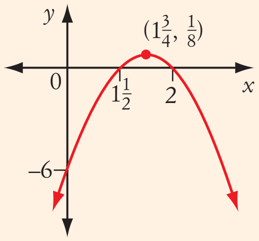
g **i** No x-intercepts. **ii** 4 **iii** $x = -0.7$
iv $(-0.7, 1.55)$ **v** concave up



h **i** 0, 4 **ii** 0 **iii** $x = 2$
iv $(2, 8)$ **v** concave down



i **i** $1\frac{1}{2}, 2$ **ii** -6 **iii** $x = 1\frac{3}{4}$
iv $\left(1\frac{3}{4}, \frac{1}{8}\right)$ **v** concave down



Exercise 14-08

1 **a** $(3, 9), (2, 4)$
b $(0, 0), (4, 32)$
c $(4, 26), (-2, 14)$
d $(-1, 5), \left(1\frac{1}{5}, 7\frac{1}{5}\right)$
e $(-2, 18), (\frac{1}{3}, 11)$
f $(-2, -22), (5, 20)$
g $(7, 39)$
h $(2, 0), (-1, 3)$

2 **a** $(1, -2), (2, -1)$
b $(0, 1), (1, 0)$
c $(0, 3), (3, 0)$
d $(0, 5), (-5, 0)$
e $(1, 7), \left(-1\frac{2}{5}, -5\right)$
f $(3, 5), \left(-2\frac{1}{2}, -6\right)$
g $(-1, 3), \left(-1\frac{1}{2}, 2\right)$

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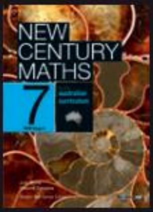

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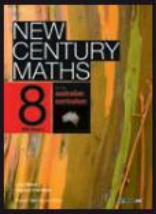

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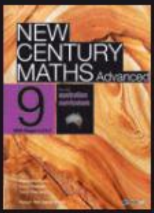

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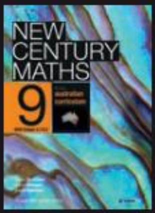

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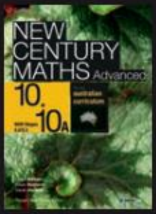

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