Essential Mathematics Units 3 and 4

Donna Buckley - Paula McMahon - Michelle Ostberg

The Mathematical Association of Western Australia Inc.

MAΣWA

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The Mathematical Association of Western Australia

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Essential Mathematics

Senior Secondary Course

Units 3 and 4

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The Mathematical Association of Western Australia

About this book

Sufficient material has been included in this text to provide students with the opportunity to develop their understanding of every item of content in Units 3 and 4 of Essential Mathematics as published by the Australian Curriculum Assessment and Reporting Authority and the content of Mathematics: Essential as published by the School Curriculum and Standards Authority. There are likely to be concepts, however, of which students have already developed a sound understanding. As such, it is expected that teachers will modify the requirement on students to complete every topic as appropriate to their learning needs.

The majority of the content is dealt with through contexts that are relevant and/or interesting to students in this age range. The four units over Years 11 and 12 are explored through contexts that build from a focus on the individual in their immediate environment through their engagement with their community and country to culminate in global contexts and applications of the mathematics.

While the writers have endeavoured to make every topic approximately an hour in length, they have also ensured that the topics are completed at a natural learning end-point. As such, depending on the teacher's choice of activities from those suggested, topics may either extend beyond this time period or require additional material to be added.

The proficiencies as outlined within the Australian Curriculum materials have been fully integrated within the lessons provided. Opportunities are provided for students to develop their *understanding* of the concepts in the course and to build *fluency* in using the mathematics they have learned. In order for students to build their *fluency* further in recognising what mathematics to use for a variety of contexts, the lessons have been interspersed with miscellaneous exercises that build cumulatively so that the final miscellaneous questions cover content from throughout the two units. Students also engage in *reasoning* through questions that require critical analysis and interpretation of information and results, as well as through justification of choices of what mathematics and processes to use. The final proficiency of *problem solving* is an integrated element of the materials as each topic culminates in a range of applications which require students to use their mathematics in either a discrete or integrated manner in order to solve problems.

A Teacher's Guide is also available that contains supplementary materials such as a sample program, sample assessment items, solutions to warm-up questions and blank proformas of relevant tables and activities to help students to complete the tasks in this student text. The sample programs and assessment items are constructed to meet the requirements of the School Curriculum and Standards Authority of Western Australia.

The Mathematics: Essential course for Western Australia also requires that students apply mathematical thinking processes to real-world problems through:

- Interpreting the task and gathering key information;
- Identifying the mathematics which could help to complete the task;
- Analysing information and data from a variety of sources;
- Applying their existing mathematical knowledge and strategies to obtain a solution;
- Verifying the reasonableness of their solutions; and
- Communicating their findings in a systematic and concise manner.

These processes are woven through the text and, where relevant and appropriate, students are given opportunities to perform them, in line with the ACARA proficiencies. Some of these processes, however, are more suitable for the teacher to incorporate as part of lessons and in student interactions, rather than through formal text questions.

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Unit 3

Unit 3 syllabus coverage WA curriculum

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3.1.1	✓																								
3.1.2		✓	✓		✓	✓	✓																		
3.1.3			✓																						
3.1.4					✓	✓																			
3.1.5							✓																		
3.1.6									✓		✓														
3.1.7											✓														
Topic 3.2: Scales, plans and models																									
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Topic 3.3: Graphs in practical situations																									
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3.4.5	✓	✓	✓	✓	✓	
3.4.6	✓	✓	✓	✓	✓	
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3.4.8			✓	✓	✓	
3.4.9			✓	✓	✓	
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3.4.12						✓
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Unit 3 syllabus coverage ACARA curriculum

	Chapter																				
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ACMEM091	✓																				
ACMEM092		✓	✓	✓																	
ACMEM093		✓	✓	✓																	
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ACMEM104											✓										
Topic 2: Scales, plans and models																					
ACMEM105												✓	✓	✓							
ACMEM106														✓							
ACMEM107													✓								
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ACMEM114													✓	✓							
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ACMEM116																	✓				
ACMEM117																		✓	✓		✓
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ACMEM120																					✓

	Chapter									
Content	22	23	24	25	26	27	28	29	30	31
Topic 3: Graphs										
ACMEM121	✓	✓	✓							
ACMEM122		✓	✓							
ACMEM123		✓	✓							
ACMEM124				✓						
ACMEM125		✓	✓	✓						
ACMEM126		✓	✓	✓						
Topic 4: Data Collection										
ACMEM127					✓					
ACMEM128					✓					
ACMEM129					✓	✓	✓	✓	✓	
ACMEM130					✓	✓	✓	✓	✓	
ACMEM131					✓	✓	✓	✓	✓	
ACMEM132					✓	✓	✓	✓	✓	
ACMEM133					✓	✓	✓	✓	✓	
ACMEM134							✓	✓	✓	
ACMEM135							✓	✓	✓	
ACMEM136							✓	✓	✓	
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ACMEM143										✓
ACMEM144										✓
ACMEM145										✓
ACMEM146										✓
ACMEM147										✓

1 Perimeter

Materials required: Calculator

Warm-up

1.	6cm = ______ mm	2.	13cm = _______ mm
3.	2.7cm = ______ mm	4.	140mm = ______ cm
5.	257mm = ______ cm	6.	1.5m = ______ cm
7.	3.86m = ______ cm	8.	840cm = ______ m
9.	459cm = ______ m	10.	1m = ______ mm

Activity

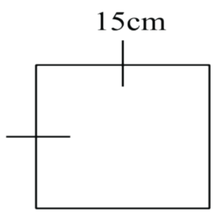
Brainstorm situations that would require you to determine the perimeter of two dimensional shapes. Remember to think of situations from the home, garden or workplace.

Make a list of various measuring devices and units that could be used when determining the perimeter.

Exercises

1. Find the perimeter of the following shapes. Remember to include the units with your answer.

a)



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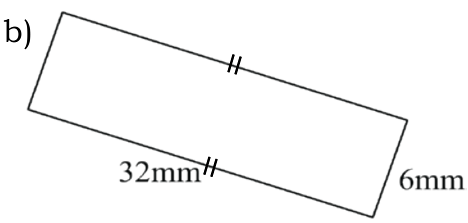

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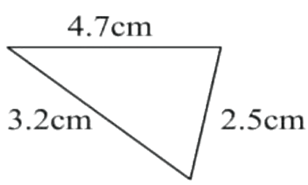

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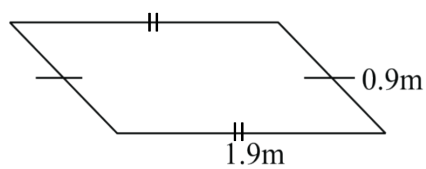

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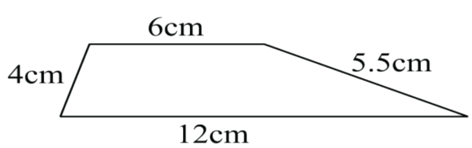

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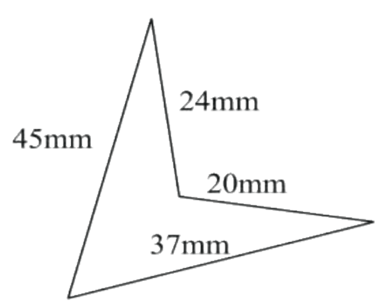
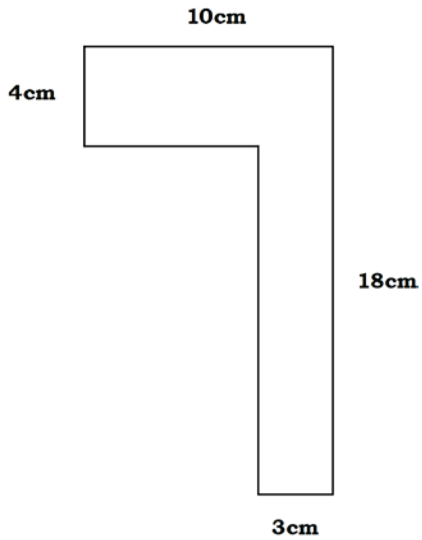


2. Find the perimeter of the following shapes. Give your answers correct to one decimal place, if appropriate.

a)



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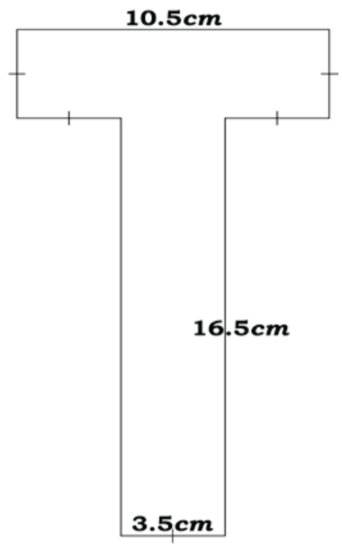

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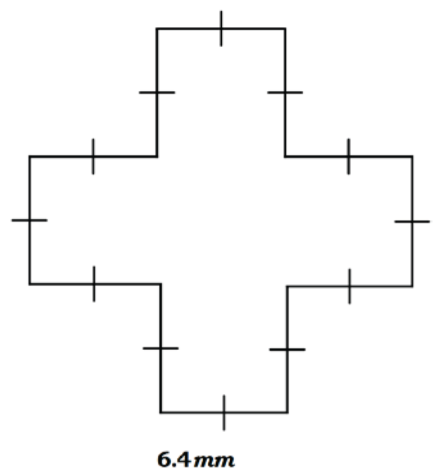

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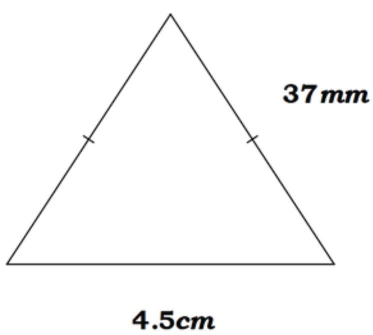

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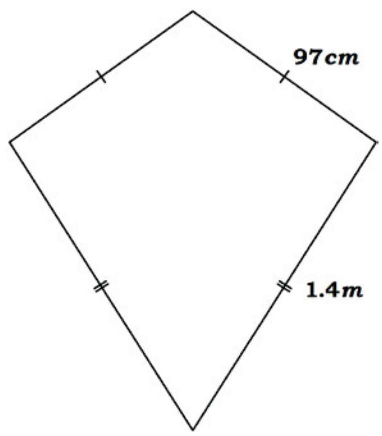


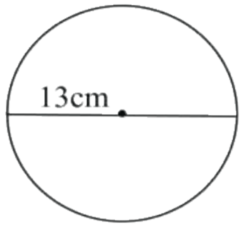
3. State the diameters of the circles with each of the following radii.

a) 40cm   b) 58cm   c) 87cm

d) 22.6mm   e) 1.4m   f) 0.96m

4. Find the circumference (perimeter) of the following circles. Give your answer correct to one decimal place.

a)



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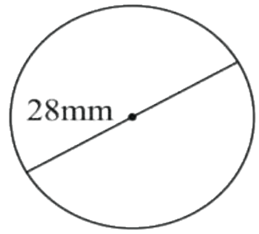

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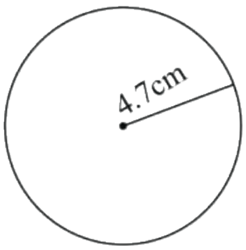

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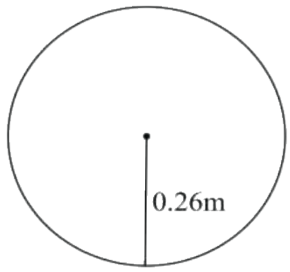


5. The centre circle on a netball court has a diameter of 0.9m. Determine the circumference of the circle.

6. Peter has a cylindrical present for his wife. The radius of the cylinder is 15cm. What length of ribbon will he need to buy to wrap three times around the cylinder?

7. Mr Smith has a rectangular paddock that is 157m by 359m. He wants to fence the paddock, leaving 2.8 metres for a gate. How many metres of fencing will he need to purchase?

8. A netball court is 15.25m by 30.6m. It is divided into thirds along the longest side. Determine the perimeter of the whole court and the centre third.

Extended Problem Solving

Mr Harris, the Physical Education teacher, needs to have all the lines re-marked on the school basketball court. He is able to purchase a suitable tape in rolls of 50 metres from Gilmore's Tape Company. The representative from Gilmore's has told Mr Harris that he should add an extra 5 metres to the length needed to allow for errors, off-cuts and corners.

Determine how many rolls of tape Mr Harris will need to order to complete the task, remembering to include the extra 5 metres.

2 Area I

Materials required: Calculators

Warm-up

1.	$1cm^2$ = ______mm^2	2.	$1m^2$ = ______cm^2
3.	$1km^2$ = ______m^2	4.	$2\,000mm^2$ = ______cm^2
5.	$8\,000mm^2$ = ______cm^2	6.	$4\,700mm^2$ = ______cm^2
7.	$9\,000cm^2$ = ____m^2	8.	$3.5m^2$ = _____cm^2
9.	$2.75m^2$ = _____cm^2	10.	$2.1km^2$ = _____m^2

Discussion

The area of a two dimensional shape is the number of squares that the shape covers. The area is measured in square units, that is, square millimetres mm^2, square centimetres cm^2, square metres m^2 or square kilometres km^2, depending on the size of the squares used to measure the area. One hectare is equivalent to 10 000 m^2, or the area of a square that is 100m long, and the abbreviation for hectares is *ha*.

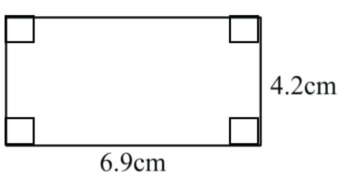
For common two-dimensional shapes (square, rectangle, parallelogram, trapezium, circle and triangle) there are a variety of different formulae that can be used to accurately find the area.

Brainstorm the formulae that you know, include a diagram for each one to assist you. Ask your teacher to provide you with any formulae or diagrams that you are unsure of.

Exercises

1. Determine the areas of the following shapes. Give your answers correct to two decimal places.

a)



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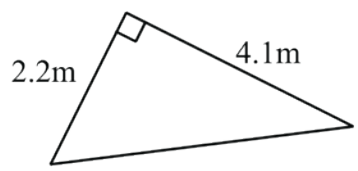

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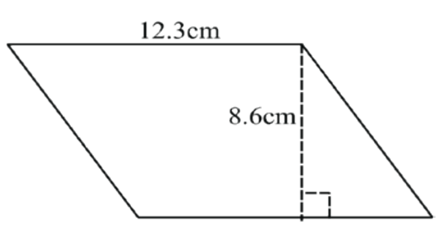

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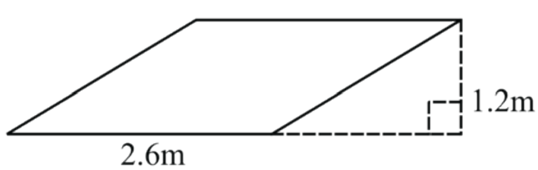

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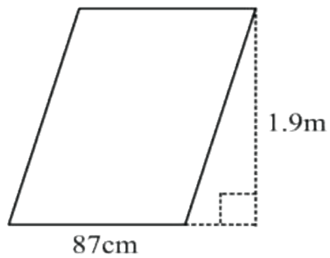

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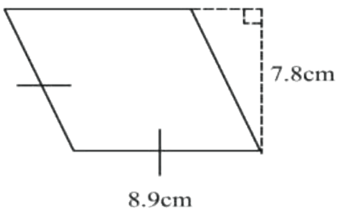
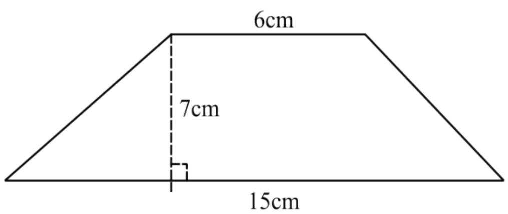


2. Find the areas of the following trapeziums. Give your answers correct to 1 decimal place. It is good to remember that if a shape has one pair of parallel lines then it is a trapezium.

a)



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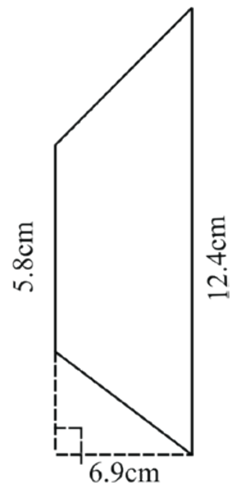

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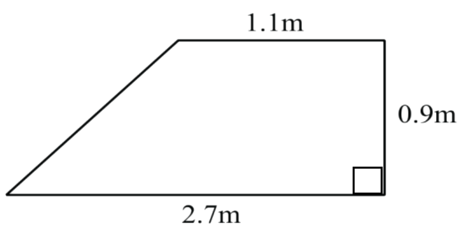

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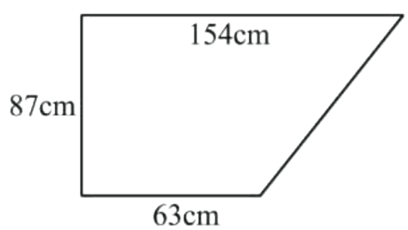


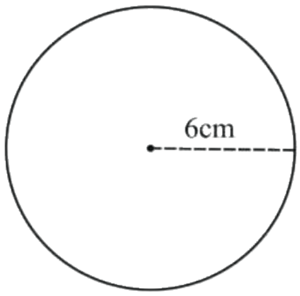
3. State the radii of each of the circles with the following diameters.

a) 50cm b) 97cm c) 41.5cm

d) 132.3mm e) 0.97m f) 1.1m

4. Find the areas of the following circles and semicircles. Remember to use the π button on your calculator. Round your answers to one decimal place.

a)



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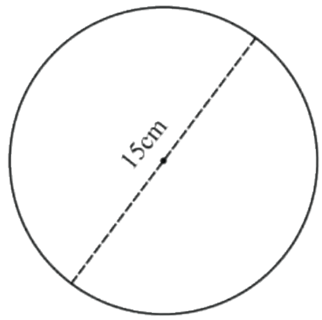

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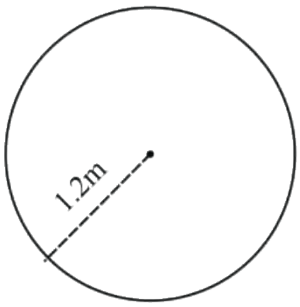

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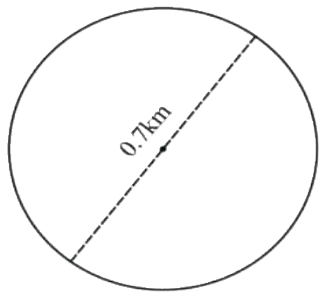

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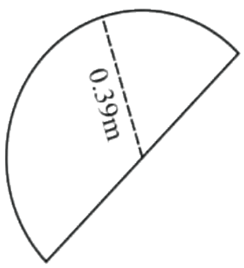

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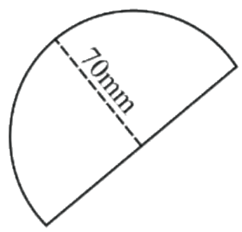

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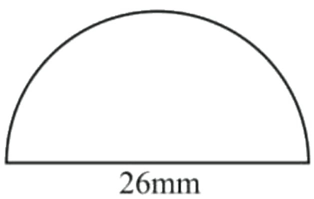

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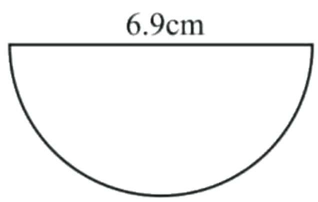


Extended Problem Solving

Determine the area of the two keys on the basketball court at your school. Remember to include the thickness of the lines.

3 Area II

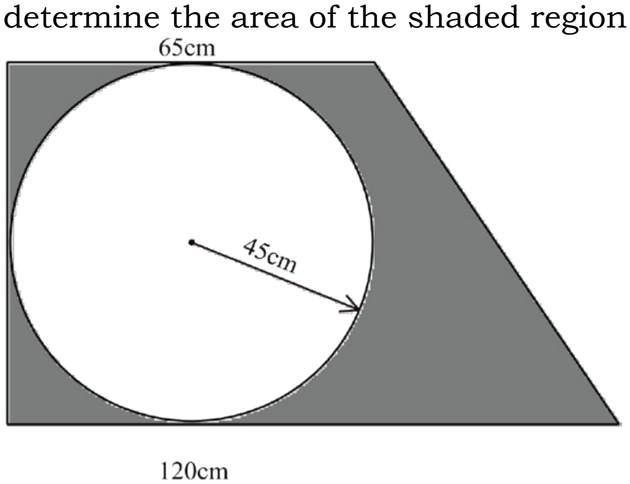
Materials required: Calculators

Warm-up

1.	Estimate 987 - 395	2.	Estimate 378 + 429 -215
3.	Estimate 27 × 34	4.	Estimate 53 × 78
5.	Estimate 7 × 43 × 4	6.	Estimate 32 × 4 × 9
7.	Estimate 9.6^2	8.	Estimate 2.6^2
9.	Estimate 4.1^3	10.	Estimate 5.4^3

Introductory problem

Use the information from the previous lesson on area to assist you to determine the area of the shaded region in the diagram below.

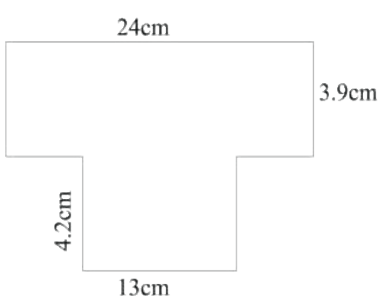


Being able to find the area of composite shapes by either adding or subtracting the areas of two or more smaller shapes is an important skill for people who quote, sell and/or install stone bench tops, swimming pool covers or floor tiles.

Exercises

1. Find the areas of the following shapes.

a)



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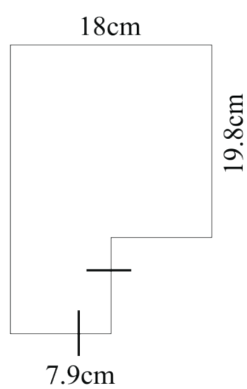

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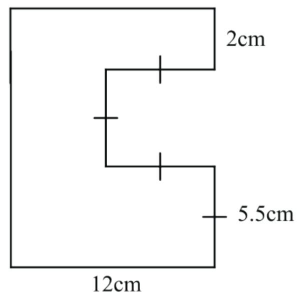

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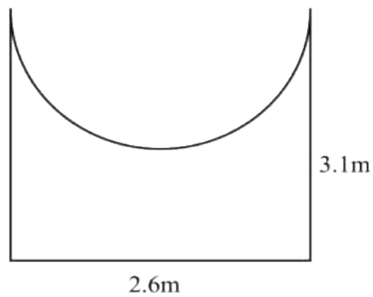

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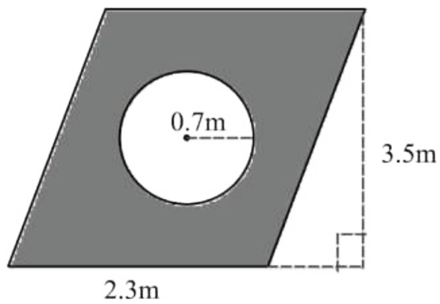

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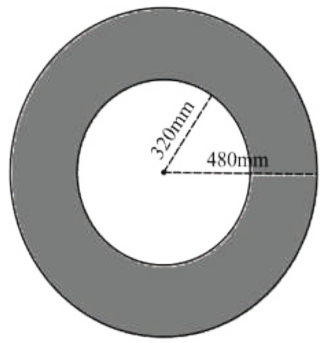
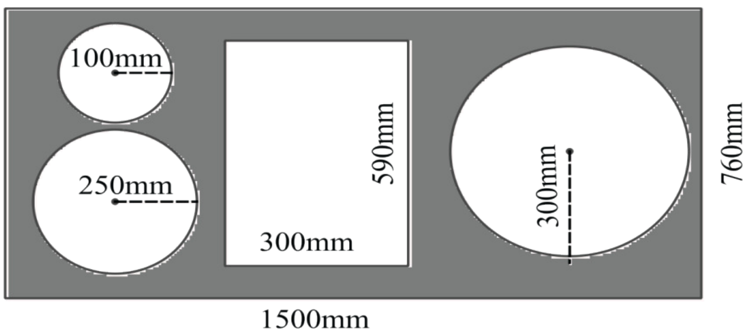


4. Adele has had a new induction cooktop installed in her kitchen. The specifications are shown in the diagram below. Determine the total area of the 4 elements (three circular and one rectangular) shown. Give your answer in cm^2 and m^2.



Extended Problem Solving

1. Determine the area of the walls in your classroom.
2. How many bricks would be needed to build the walls of your classroom?
3. How many bricks would the bricklayer need to order if he needs 10% more? Why would the bricklayer need 10% more than you calculated?
4. Determine the area of the bricks in your classroom (internal or external) that are exposed. Remember that the bricks behind whiteboards, pin up boards, signs and/or posters are not exposed.

4 Area of Sectors

Materials required: Calculators, A4 paper, rulers and protractors

Warm-up

Express as a simplified fraction

1.	90 ÷ 360	2.	180 ÷ 360
3.	45 ÷ 360	4.	135 ÷ 360
5.	270 ÷ 360	6.	300 ÷ 360
7.	80 ÷ 360	8.	30 ÷ 360
9.	330 ÷ 360	10.	60 ÷ 360

Introductory Activity

Many children used to make paper fans on hot days from a piece of A4 paper. Search the internet to find instructions to make a simple origami fan or take a sheet of A4 paper and place in the portrait direction. Fold the bottom 2cm up. Turn the paper over and again fold the bottom 2cm up. Turn the paper over and repeat the folding process until all of the paper is folded. Turn the end 4cm over to form a 'handle' and hold the fan with one hand. Spread the top of your fan out.

What is the area of your fan?

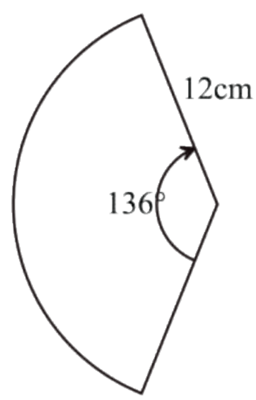
Measure the angle at the top of the 'handle' of your fan. This will not be a very accurate measurement. What fraction is the fan of a circle? For example if the angle measures 120°, then the fan is $\frac{1}{3}$ of a circle. Now measure the radius of your fan, not including the handle.

A sector is part of a circle. The area of the sector is found by calculating what fraction of a full circle it is. The formula for the area of a sector is: $A = \frac{\phi}{360}\pi r^2$, where ϕ is the number of degrees (angle) between the two radii. In the example below ϕ is 136°.

Determine the area of the fan you have made. Make sure your calculator is set to use degrees as the units.

For the example on the right

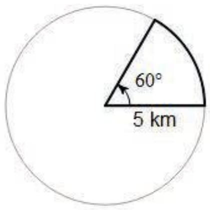
$A = \frac{136}{360}\pi(12)^2$, that is $170.9cm^2$



Exercises

1. Find the area of the following sectors.

a)



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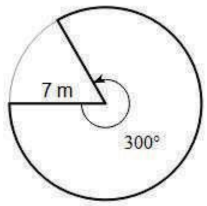

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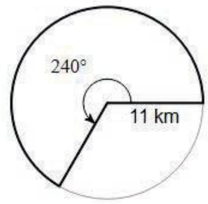

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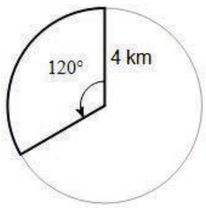

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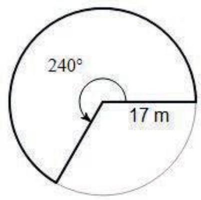

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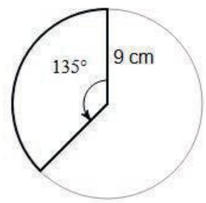

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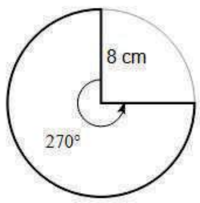

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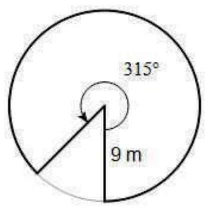

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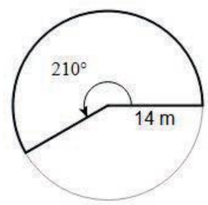


2. Mr Douglas is curious about the area that each of the wipers on his windscreen covers. Calculate the areas given:

   a) the front wiper is 60 centimetres long and the angle it moves through is 95°.
   b) the rear wiper is 45 centimetres long and the angle it moves through is 150°.

3. Ms Le has brought some Japanese fans into her Japanese class. Mia decides to find the area of the paper section of each fan.

   a) Fan A has a radius of 18.5cm and the angle is 120°.
   b) Fan B has a radius of 10cm and the angle is 100°.

Extended Problem Solving

Investigate – What is:

a) the effect on the area of a sector when you double the radius?

b) the effect on the area of a sector when you double the angle?

5 Surface Area I

Materials required: Calculator, assorted boxes and ruler.

Warm-up

Use $x=6, y=6$ and $z=4$

1.	Evaluate $x-x+y-z$	2.	Evaluate $z-z+x=$
3.	Evaluate $x-yz=$	4.	Evaluate $xyz=$
5.	Evaluate $y(x+1)=$	6.	Evaluate $3+xz=$
7.	Evaluate $yz-y=$	8.	Evaluate $8(x+z)-y=$
9.	Evaluate $\frac{48}{z}=$	10.	Evaluate $\sqrt{z}(x+y)\div 3=$

Activity

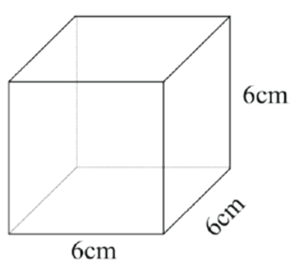
Take a cereal box or similar and carefully cut it to create a net.

1. Measure the dimensions (length and width) of each face.
2. Calculate the area of each face.
3. Add the area of all faces to determine the total surface area.
4. Discuss a method for efficiently finding the surface area of rectangular prisms.
5. Discuss how this method could be altered to find the surface area of cubes.

Exercises

1. Find the surface area of the following objects.

a)



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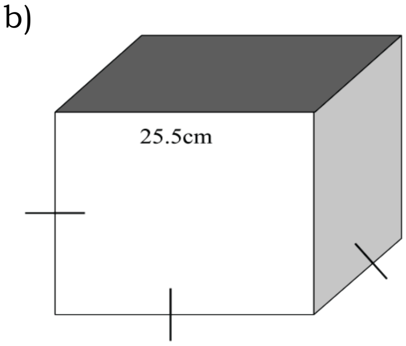

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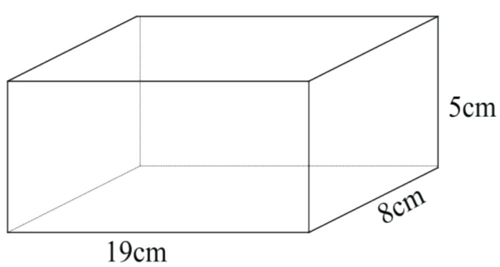

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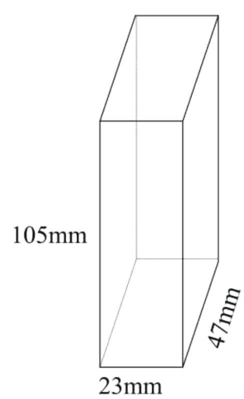

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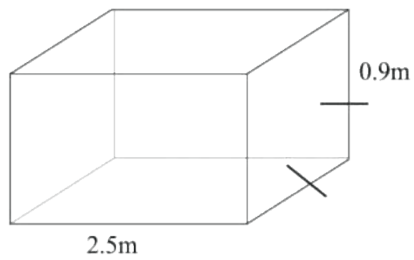

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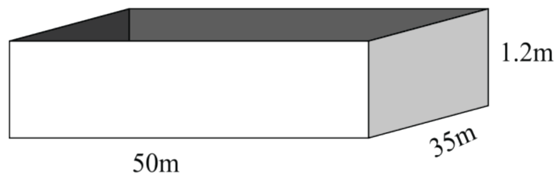


2. Mrs Pearson wants to completely paint the outside of her rectangular wardrobe. She has measured the dimensions. The height is 1.7 metres, the width is 1.2 metres and the depth is 0.75 metres. The paint tin says it will cover a surface area of 6 square metres. Determine how many tins Mrs Pearson will need to buy to give the wardrobe two coats of paint.

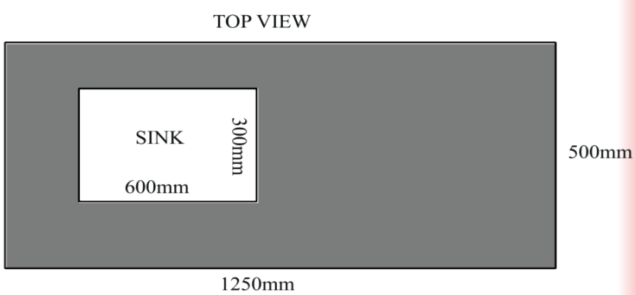
3. Magic Pool Restoration Company will recoat the inside surface of fiberglass pools. To provide a quote they assume that all pools are rectangular prisms. My pool is 7.4 metres long, 2.9 metres wide and on average a depth of 1 metre.

   a) Determine the internal surface area of my pool.

   b) If it costs $125 per square metre to recoat a pool, how much will it cost me?

4. Cubic Furniture make cubic shaped office stools. Each side of the stool is 60cm. How much leather is required to cover one stool? Give the answer in square metres.

5. Peter wishes to replace the laminex on his bathroom cabinet top. The top view is provided. The height of the cabinet top is 70mm. Determine the amount of laminex required, giving your answer in square metres.



Extended Problem Solving

Given that the surface area of a cube is $300m^2$, determine the dimensions of this cube. If we double the surface area to $600m^2$, will this double the dimensions of the cube? If the dimensions do not double, state the ratio of the original dimensions to the new ones.

6 Surface Area II

Materials required: Calculator and assorted Toblerone boxes.

Warm-up

1.	Simplify $20:10$	2.	Simplify $45:15$
3.	Simplify $3:6$	4.	Simplify $12:60$
5.	Simplify $150:275$	6.	Simplify $12:6:2$
7.	Simplify $25:100:75$	8.	Simplify $70:50:35$
9.	Simplify $2.5:3.5:3.0$	10.	Simplify $5.2:10.4:20.8$

Introductory problem

What is the surface area of an orange, apple or banana? What is the surface area of your own skin?

Doctors, pharmacologists and other health workers mostly use body surface area, BSA, if they need to determine an accurate dosage for medicines. There are five common formulae that will give an approximation of a human's surface area.

Mosteller's Formula is $BSA=\dfrac{\sqrt{W}\times\sqrt{H}}{60}$, where W is weight in kilograms and H is height in centimetres.

Du Bois' Formula is $BSA=\dfrac{W^{0.425}\times H^{0.725}}{139.2}$, where W is weight in kilograms and H is height in centimetres.

For a person who weighs 68.3kg and height 162cm, using Mosteller's formula the surface area of the skin is $BSA=\dfrac{\sqrt{68.3}\times\sqrt{162}}{60}$ that is $1.75m^2$, while using Du Bois' formula the answer is $BSA=\dfrac{68.3^{0.425}\times162^{0.725}}{139.2}$ that is $1.72m^2$.

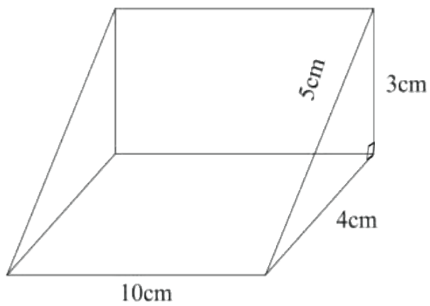
Find the height and weight of the average 17 year old and determine the associated body surface area.

Find the height and weight of the tallest and shortest people recorded in the Guinness Book of Records and determine their bodies' surface areas.

Exercises

1. Find the surface areas of the following triangular prisms.

a)



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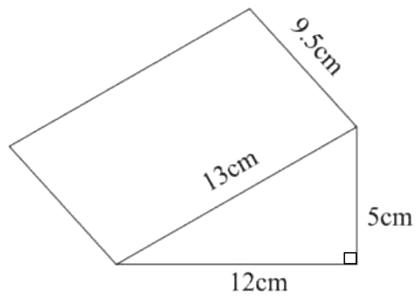

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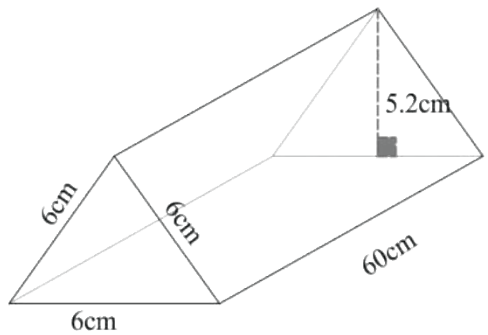

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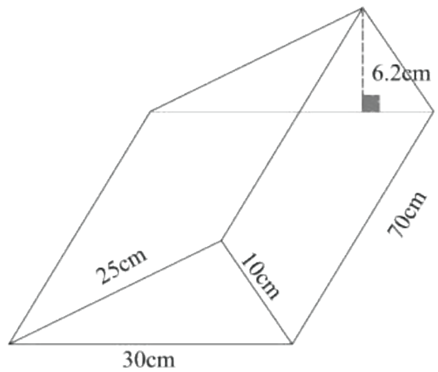
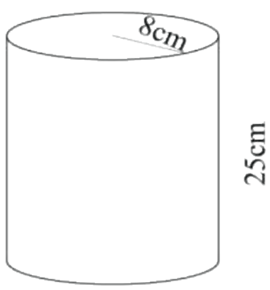


2. Find the surface areas of the following cylinders and spheres. Round your answers to one decimal place.

a)



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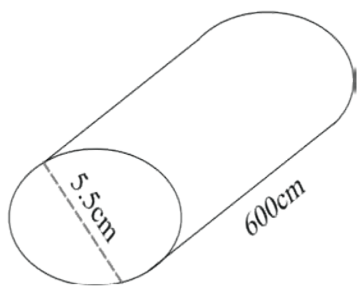

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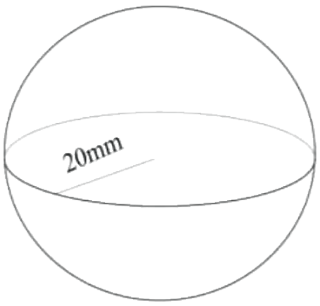

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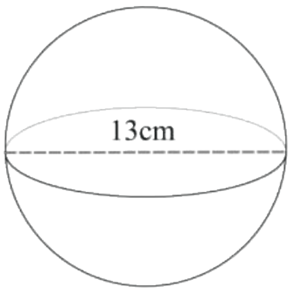


3. Australia Post has a variety of cylindrical containers that can be used to safely pack posters. The containers come in the following sizes:

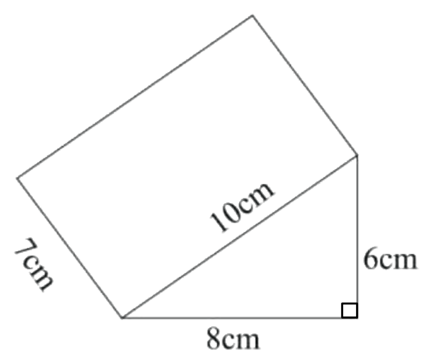
   a) radius 4cm and height 1.5 metres
   b) radius 5cm and height 2 metres
   c) radius 3.5cm and height 1 metre

   Determine the surface area of each container.

4. The longest piece of spaghetti ever made was 3.3 millimetres in diameter and 548.7 metres long. What was the outside surface area of this piece of spaghetti? Keep in mind that spaghetti is an open-ended tube.

5. Chad Fell, from the United States of America, blew a bubblegum bubble with a diameter of 50.8 cm without using his hands at the Double Springs High School, on 24 April 2004. What was the surface area of the bubble?

6. Does the surface area of a triangular prism double if the dimensions double? Use the following example to assist you in investigating.



Extended Problem Solving

Find the surface areas of the various Toblerone® boxes.

What is the surface area of one piece of the chocolate when it is broken off the block? Is it as simple as dividing the surface area of the box by the number of pieces of chocolate?

7 Surface Area III

Materials required: Calculator, ruler and connective cubes

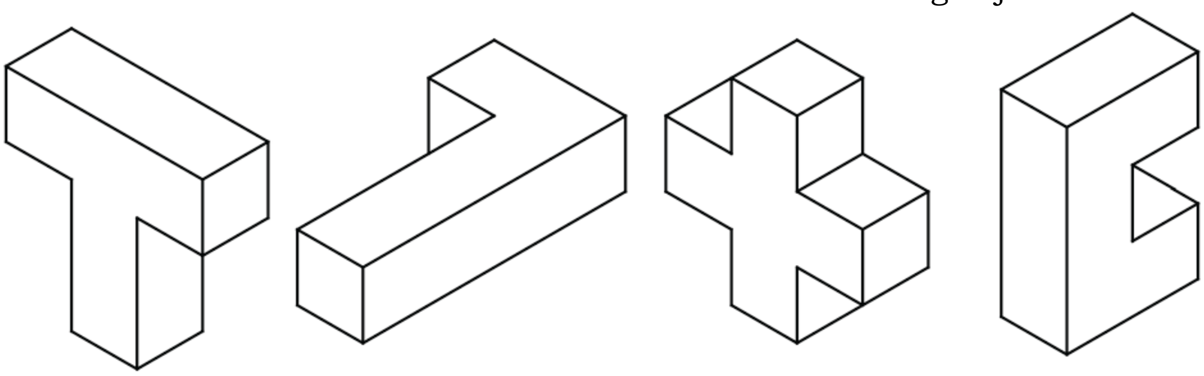
Warm-up

Use $x=9, y=4$ and $z=-2$

1.	Evaluate $x+z-y=$	2.	Evaluate $xy=$
3.	Evaluate $\sqrt{xy}=$	4.	Evaluate $z\sqrt{xy}=$
5.	Evaluate $x^2=$	6.	Evaluate $x^2y=$
7.	Evaluate $\sqrt{x}\times\sqrt{y}\div z=$	8.	Evaluate $(\sqrt{y}\,z)^2=$
9.	Evaluate $\frac{xy}{z}=$	10.	Evaluate $2^3\times yz=$

Activity

Use five connective cubes to construct each of the following objects.

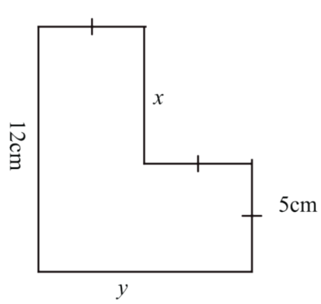


Find the total surface area of each by counting the exposed faces. Discuss, with a partner, a suitable method for determining the total surface area of complex objects. Test your method with the objects above.

Exercises

1. Find the missing side lengths.

a)



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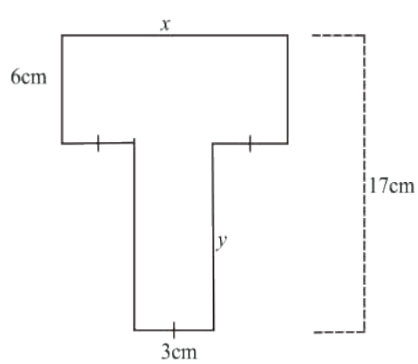
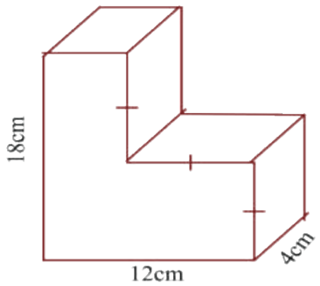


2. Find the total surface area of each of the following objects.

a)



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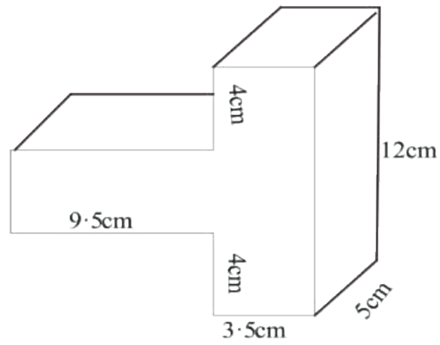

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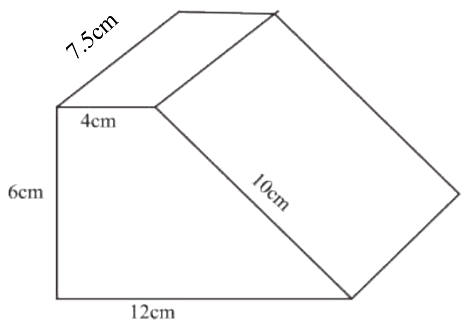

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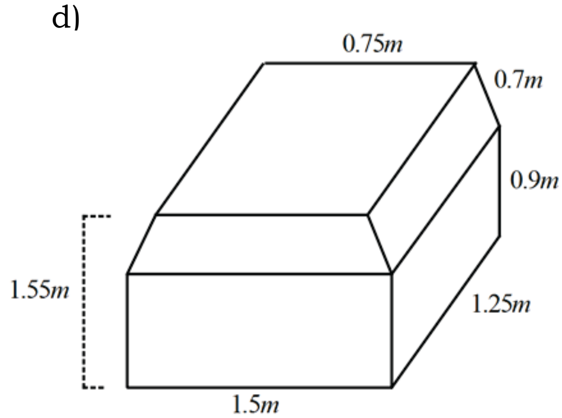


Extended Problem Solving

The principal of your school wishes to paint your mathematics classroom.

Find the surface area of your classroom to assist with determining how many litres of paint are needed to apply two coats of paint.

Approximately 200 millilitres are required for one square metre of coverage.

8 Units of Mass

Materials required: Calculator

Warm-up

1.	6000g = _________ kg	2.	4250g = _________ kg
3.	9823g = _________ kg	4.	450g = __________ kg
5.	856g = ___________ kg	6.	2.4kg = __________ g
7.	4.98kg = ____________ g	8.	0.9kg = __________ g
9.	10.45kg = ___________ g	10.	63.2kg = _________ g

Discussion

Often we hear people say 'I weigh 63kg'. Is this scientifically correct? What is the difference between mass and weight?

Mass is a measure of how much matter an object has. Weight is a measure of how strongly gravity pulls on that matter.

So on Earth I have a mass of 63 kilograms and this will not change, no matter which planet I travel to in our solar system. My weight will change because of the effect of gravity. My weight on Earth is 63kg but on Venus it is 57kg, on Mercury it is 24kg, on Neptune it is 72.6kg and on Jupiter my weight is a whopping 171kg. I think I will start saying 'My mass is 63kg' or not head into space!

Use the internet to explore the weight of everyday items on other planets.

Exercises

We are all familiar with kilograms and grams but there are other measures of mass. These include tonnes, milligrams and micrograms.

Unit of Mass	Abbreviation	Conversion
Tonne	t	1 tonne = 1000kg
Kilogram	kg	1kg = 1000g
Gram	g	1g = 1000mg
Milligram	mg	1mg = 1000mcg
Microgram	mcg	

1. Use the table above to convert the following:

a) 5g to mg b) 9.45g to mg c) 12.45g to mg

d) 6000mg to g e) 2340mg to g f) 600mg to g

2. Use the table to convert the following:

   a) 0.3kg to g
   b) 0.65kg to g
   c) 0.16kg to g
   d) 0.3kg to mg
   e) 0.65kg to mg
   f) 0.16kg to mg

Manufacturers of medicines, chemicals and processed foods use milligrams. The masses of ingredients in medicines, chemicals or processed foods are labelled as measured in milligrams, but the ingredients in bulk containers are measured in kilograms.

For example, in a jar of salsa the mass of sodium (salt) in a serving is 322mg, but there would have been kilograms of sodium added to the vat that the salsa was cooked in.

3. Max takes Antigout medication. There are 60 tablets in the bottle. The total weight of the tablets is 108grams.

   a) What is the mass of one tablet in grams?

   b) What is the mass of one tablet in milligrams?

   c) Each tablet contains 300mg of chemical X. What is the combined mass of the other chemicals?

4. Mr Silva needs to take calcium tablets. Each bottle contains 120 tablets. The active ingredient is calcium carbonate. There is 220mg in each tablet.

   a) What is the total mass of the calcium carbonate in the bottle in milligrams and grams?

   b) The chemist purchases 3 boxes of tablets. Each box contains 24 bottles. What is the total mass of the calcium carbonate in grams and kilograms?

   c) The manufacturer mixes the ingredients in a very large vat. From one vat they are able to produce 10 million tablets. How many kilograms of calcium carbonate were added to the vat?

Extended Problem Solving

Collect ten processed food items and read the nutrition information. If you had one serve of each of these items in a day how much sodium would you have consumed? Is this above the recommended level for one day?

9 Volume and Capacity

Materials required: calculator, cubic centimetre and cubic metre

Warm-up

1.	7 + 8 ÷ 2 =	2.	7 × (10 – 7) + 3 =
3.	9 ÷ 3 + 3 × 5 =	4.	15 ÷5 × 3 =
5.	27 – 11 × 2 =	6.	8 ÷ 4 × 2 =
7.	10 ×5 + 9 × (10 – 6) =	8.	10 – 2 + 7 =
9.	20 – 4 ÷ 2 × 7 =	10.	18 – 7 × 2 + 13 – 4 ÷ 2 =

Discussion

A can of soft drink says it contains 375mL. Is this a measure of volume or capacity?

A new fridge has $2m^3$ written on the carton. Is this a measure of volume or capacity?

What is the difference between volume and capacity?

Most people are familiar with the conversion of millilitres to litres and litres to kilolitres. Have you ever heard of megalitres or gigalitres?

Use the internet to create a conversion chart to assist you to convert between millilitres, litres, kilolitres, megalitres and gigalitres.

Exercises

1. State which unit of capacity (millilitres, litres, kilolitres, megalitres or gigalitres) you would use when measuring the capacity of:

a)	a car petrol tank	b)	a small water bottle
c)	a backyard swimming pool	d)	Mundaring Weir dam
e)	an eyedropper	f)	an Olympic diving pool
g)	a soft drink can	h)	a medicine bottle
i)	Sydney Harbour	j)	a standard bucket
k)	a kettle	l)	Lake Argyle

2. Convert the following measurements to litres.

a)	4000mL	b)	0.5kL	c)	870mL
d)	50mL	e)	2.8kL	f)	0.96kL

3. Convert the following measurements to millilitres.

a)	2L	b)	9.5L	c)	0.65L
d)	1kL	e)	0.25kL	f)	0.1ML

4. Convert the following measures to megalitres and then gigalitres.

a)	7 000 000L	b)	12 750 000L
c)	100 000 000L	d)	50 000kL
e)	800 000kL	f)	100 000 000 000mL

5. Sumi is planning a party for himself and 45 friends. He believes that each person will consume 2.4 litres of soft drink.

   a) How many litres should Sumi buy?

   b) He wants to buy twenty 2-litre bottles and the rest in 375mL cans. How many cans should he buy?

6. Mrs Pearson has a very large fishpond in her back yard. The automatic tap adds 800mL of water every hour. How much water, in litres, will be added in:

a)	6 hours?	b)	one day?
c)	45 minutes?	d)	1 minute?

7. A backyard swimming pool has a capacity of 24 000 litres. If the owner wants to fill the pool in 15 hours how many litres must be added each hour?

Often we wish to know if a container will hold a certain amount of liquid. If we find the volume of a container the measurements are often in cubic millimetres, cubic centimetres or cubic metres. The capacity of the liquid, though, is measured in millilitres or litres.

It is important that we can convert between these two types of measurements.

We need to know that $1m^3$ has a capacity of 1 kilolitre and $1cm^3$ contains 1mL. Ask your teacher to show you the actual size of a cubic centimetre and a cubic metre.

8. Use the above information to calculate the capacities of the following volumes in the units indicated.

a) $4.5\,m^3$ to kL and L

b) $7.1\,m^3$ to kL and L

c) $10.25\,m^3$ to kL and L

d) $70.5\,cm^3$ to mL

e) $4600\,cm^3$ to mL

f) $750\ 000\,cm^3$ to mL

Extended Problem Solving

Fried's, Clark's and Young's rules are three different methods for working out the amount of medicine to give to a child when only the adult dosage, or amount, is known.

Fried's Formula is: $\dfrac{\textit{adult dosage} \times \textit{child's age (in months)}}{150}$

Clark's Rule is: $\dfrac{\textit{weight of child (in pounds)}}{150} \times \textit{adult dose}$

Young's Rule is: $\textit{adult dose} \times (\textit{age} \div (\textit{age} + 12))$

Note: age is in years.

Given the following information and using each of the rules determine the amount of each child's dosage.

1. Kenzie is 2 years old, weighs 32 pounds and the adult dosage is 45mL.

2. Byron is 7½ years old, weighs 132 pounds and the adult dosage is 60mL.

Comment on how close the three answers are to each other. Which rule would you use? Justify your answer.

10 Volume of Regular Objects

Materials required: Calculators

Warm-up

Draw a neat sketch of each of these 3D objects

1.	Rectangular prism	2.	Triangular prism
3.	Cylinder	4.	Cone
5.	Hexagonal Prism	6.	Cube
7.	Square-based pyramid	8.	Triangular-based pyramid
9.	Sphere	10.	Hemisphere

Activity & Discussion

Use eight one-centimetre cubes to build a larger cube. Draw a sketch of this object including its dimensions.

What is the volume of the large cube?

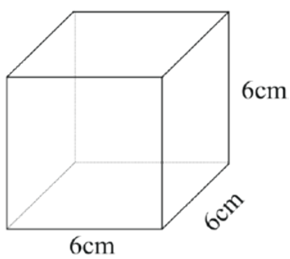
If you double all of the dimensions what is the effect on the volume?

What is the effect on the volume if you triple the dimensions?

Exercises

1. Determine the volumes of the following cubes and rectangular prisms.

a)



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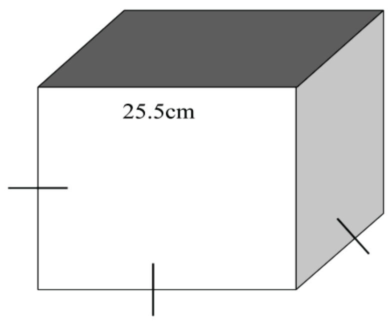

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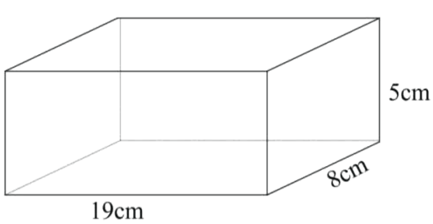

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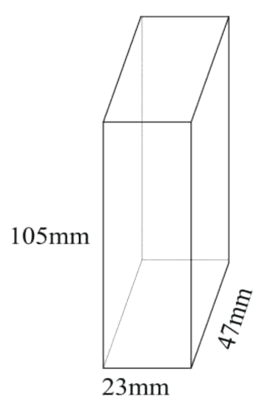

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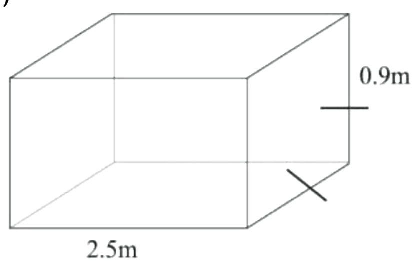

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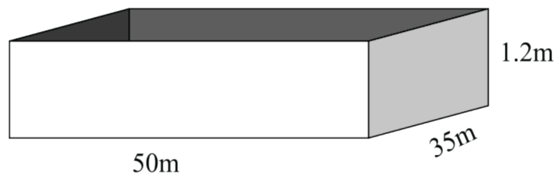
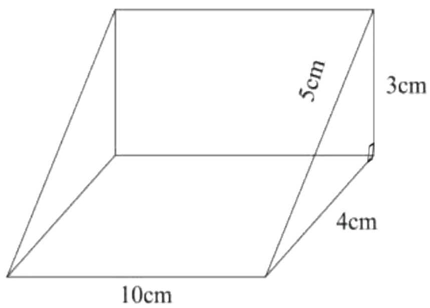
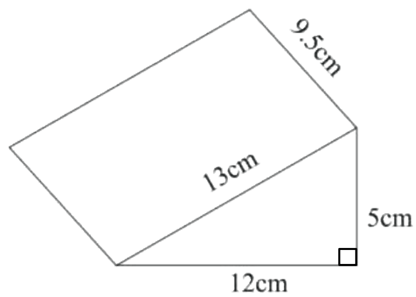


2. Determine the volumes of the following triangular prisms.

a)



b)



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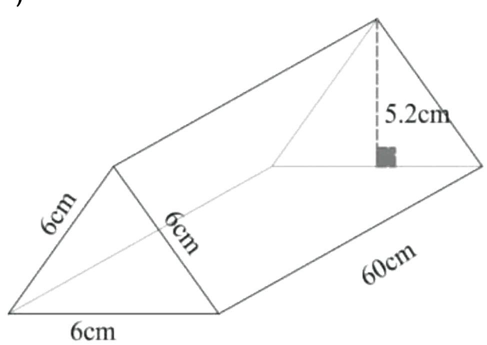

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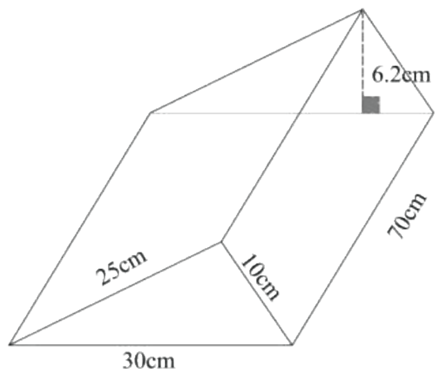


3. Mr Swamp has brought a rectangular fish tank to school. The dimensions are 1.2m by 45cm by 55 cm.

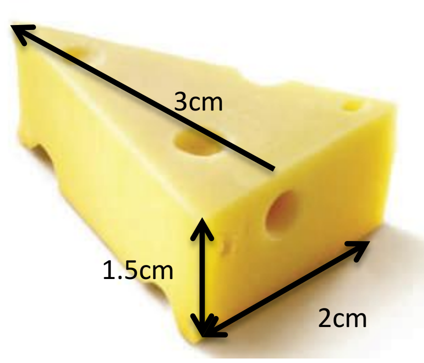
   a) What is the volume of the fish tank in cubic centimetres?

   b) Ideally each medium fish should have approximately 10 000 cubic centimetres of space. What is the maximum number of medium fish that Mr Swamp should add to the tank, if it is full of water?

   c) Given $0.1m^3$ = 100 litres, how many litres of water are needed to fill the tank to a height of 50 centimetres?

4. a) Determine the volume of the wedge of cheese.

   b) If 16 wedges make up the cheese block, what is the volume of the block?



5. A marble cake is 30cm by 12cm by 4cm. How many 6cm by 3cm by 4cm pieces can it be cut into?

Extended Problem Solving

1. If you use exactly 100 cubes to form the tallest rectangular block (rectangular prism) possible,

   (i) what are the dimensions of the prism?
   (ii) sketch the block including its dimensions.

2. Sketch the flattest rectangular prism that can be made using the 100 cubes, what is the length width and height of this prism?

3. Liam forms a rectangular prism with a length of 10 cubes, a width of 5 and a height of 2. Did Liam use all of the 100 cubes to build this rectangular prism? Justify your answer.

4. Eryn forms a rectangular prism with a length of 4 cubes and a width of 3 cubes.

   (i) What should the height be, so that she will use as many of the 100 cubes as possible?
   (ii) How many cubes are left unused?

5. How many different rectangular prisms can be formed that use all 100 cubes? Make an organised list to help you identify all possibilities.

11 Volume

Materials required: Calculator

Warm-up

1.	$20+2\times3=$	2.	$15\div3\times10=$
3.	$18\div2\times7=$	4.	$16\div(6-4)=$
5.	$(12-4)\times(3+2)=$	6.	$4^2+20\div5=$
7.	$10-5+3\times7=$	8.	$\sqrt{100}-(6-5)=$
9.	$27-\sqrt{144}\times2=$	10.	$64\div8\times2-\sqrt{4}=$

Introductory problem

Khufu was the first pharaoh to build a pyramid in Giza. He built the largest one of a group of three, known as the Great Pyramid. They are close to the Great Sphinx on the Giza Plateau. Interesting fact: the Great Pyramid is located in Giza, which is 20 kilometres from the centre of Cairo. The Giza Pyramids are basically in the suburbs of Cairo (in fact there is a well-known fast food store next to them)!

It took approximately 20 years to build Khufu's pyramid and it used 2 300 000 building blocks that weighed on average 2.5 tonnes (2500 kilograms) each. The base of Khufu's pyramid is a square of length of 230.4 metres and a perpendicular height of 147 metres. What is the volume of this pyramid?
(Hint: Volume of a pyramid = $\frac{1}{3}$ × volume of prism with same base and perpendicular height)

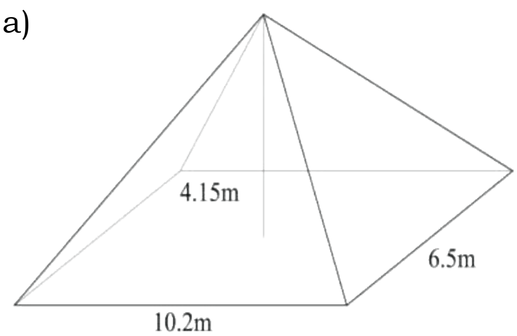
How does it compare with the volume of your classroom? Your bedroom?

Would the pyramid fit on your school oval or the entire site that your school is built on?

Exercises

1. Find the volumes of the following rectangular or square based pyramids, rounded to two decimal places.

a)



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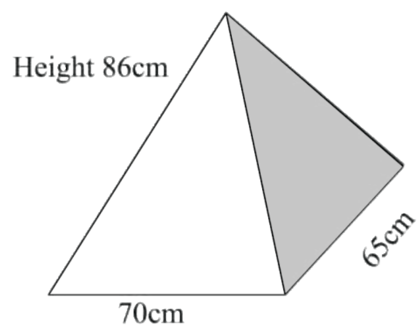

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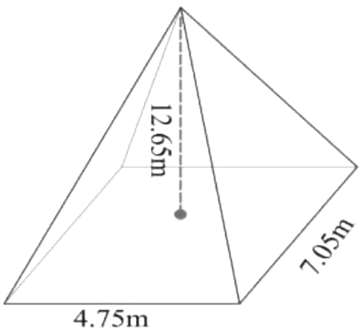

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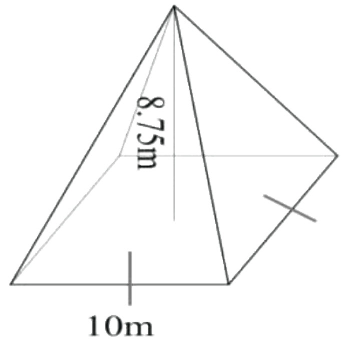
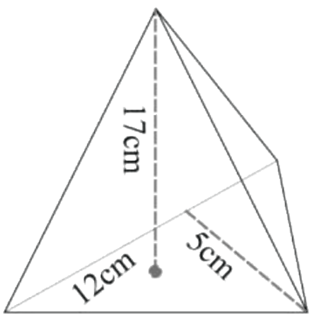


2. Find the volumes of the following pyramids and cones. Round your answers to two decimal places.

a)



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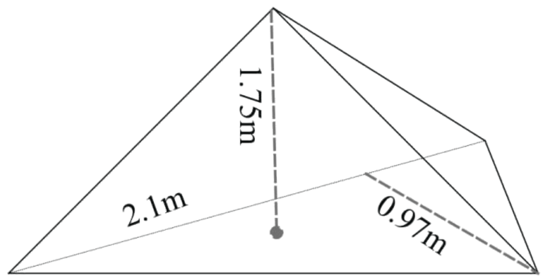

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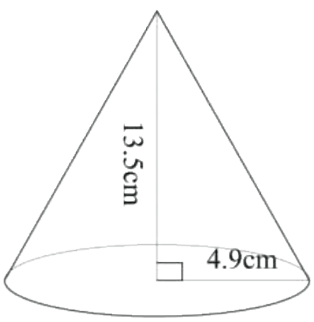

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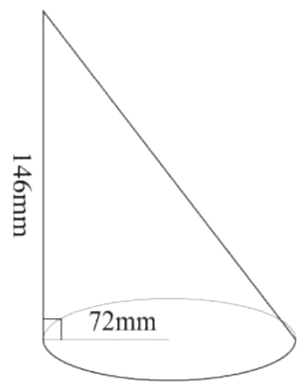

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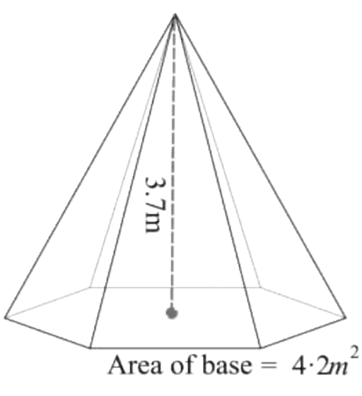

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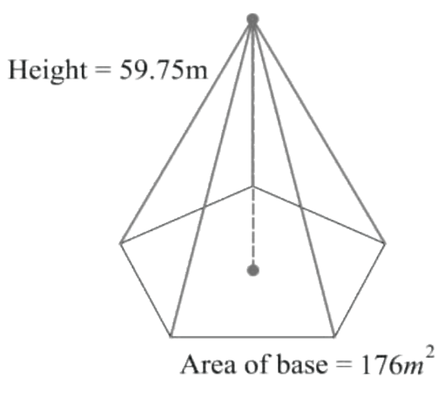
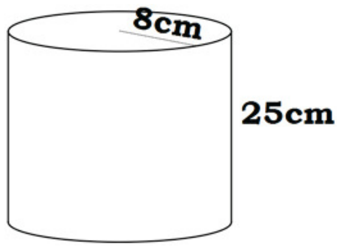
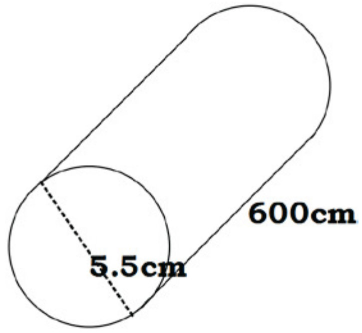


3. Find the volumes of the following cylinders to 2 decimal places.

a)



b)



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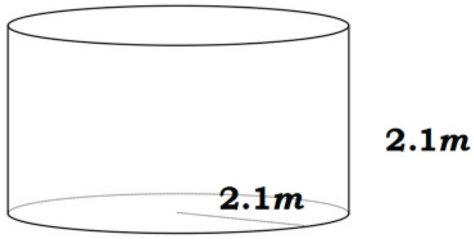

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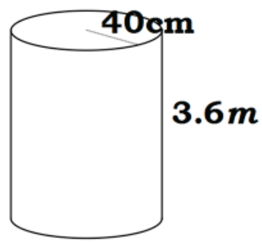


4. Peta lives in a rural area and wants to buy a new rain water tank for the property. From her water bills, she estimates that she will need a tank with a capacity of at least 100kL to get through the dry spells in summer. She has the following tanks to choose from:

Model	Diameter	Height
Residential 1	5.49m	2.31m
Residential 2	7.06m	2.31m
Residential 3	7.84m	2.31m
Low profile 1	8.63m	1.82m
Slimline 1	5.49m	5.11m

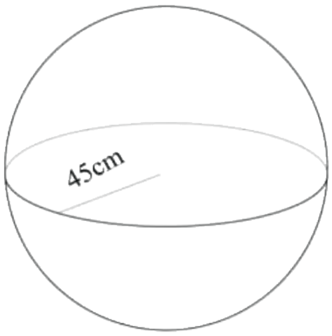
a) Find the capacity of each of the tanks in litres and kilolitres.

b) Which tank or tanks will meet Peta's needs?

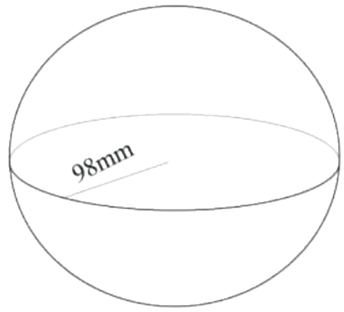
c) Given the cost is calculated from the sum of the diameter and the height, which of the suitable tanks would be cheapest?

5. Use the internet to find out how to find the volume of a sphere. Find the volumes of the following spheres. Give your answers correct to one decimal place. Remember to use the radius.

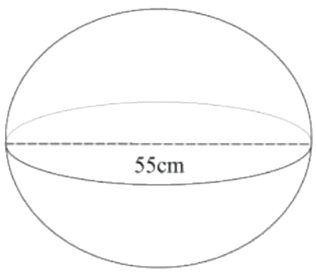
a)



b)



c)



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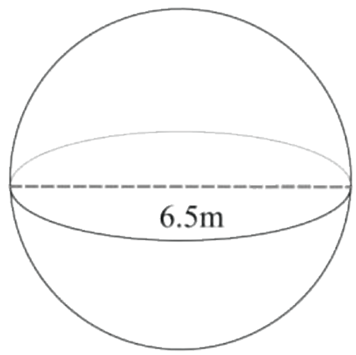
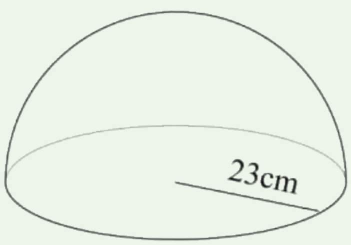


Extended Problem Solving

1. Find the volume of the following hemisphere.



2. Find the volume of the following everyday spherical objects. You will need to determine a method to find the radius of the objects. They are a basketball, volleyball, tennis ball, exercise ball and a beach ball.

12 Two-Dimensional Shapes

Materials required: Matching activity (from Teacher's Guide), scissors and glue

Warm-up

Spelling practice – write each word three times

1.	quadrilateral	2.	isosceles
3.	scalene	4.	equilateral
5.	perpendicular	6.	parallelogram
7.	congruent	8.	septagon
9.	rhombus	10.	dimensions

Definitions and Activity

Two-dimensional straight-sided shapes are all polygons, 'poly' meaning many and 'gon' meaning angle. Polygons are categorised according to the number and type of sides and angles. There are six names for triangles, six special types of quadrilaterals and pentagons, hexagons, septagons etc. Did you know that a 10-sided shape is a decagon and a 13-sided shape is a tridecagon (*tri-* meaning three and *deca-* meaning ten)?

To increase the accuracy of the definition of a polygon you can include the following characteristics.

Concave – A polygon that has "dents" or indentations in it. The polygon has at least one internal angle that is greater than 180°. You can remember it by con-"cave" (it has a cave in it or it has caved in).
Convex – A polygon that has no "dents" or indentations in it. The polygon has all internal angles smaller than 180°.
Regular – A polygon that has all sides equal and all interior angles equal.
Irregular – A polygon whose sides are not all the same length or whose interior angles are not all the same size.

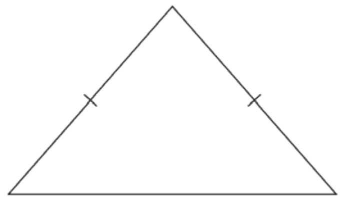
Using the matching activity, provided by your teacher, match the names, diagrams and definitions. Using the internet may be of assistance.

Exercises

1. Provide a full description of each of the following polygons. A clear, detailed description using correct geometric language is essential.

   a) rectangle
   b) trapezium
   c) equilateral triangle
   d) kite

e)



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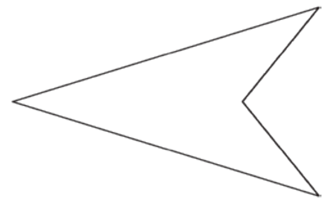

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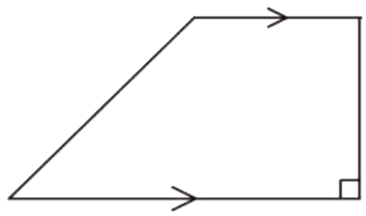

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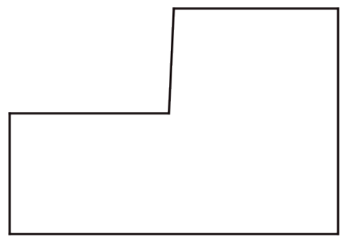


2. **Match** the following descriptions with at least one of the following polygons.

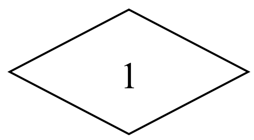
   (a) A concave polygon ....

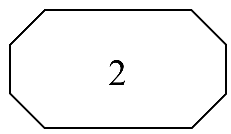
   (b) A regular polygon ....

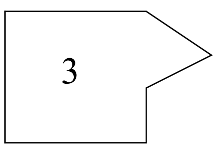
   (c) A polygon with congruent angles ....

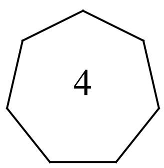
   (d) A convex polygon with three pairs of parallel sides ....

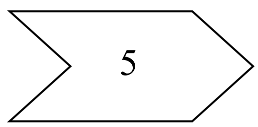
   (e) A polygon with right angles ....





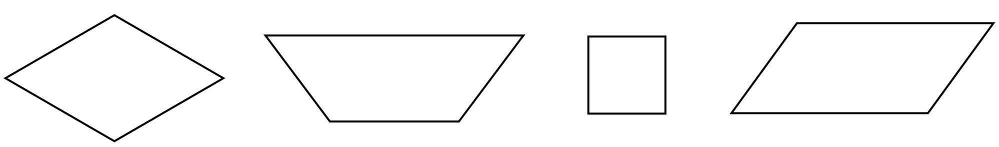




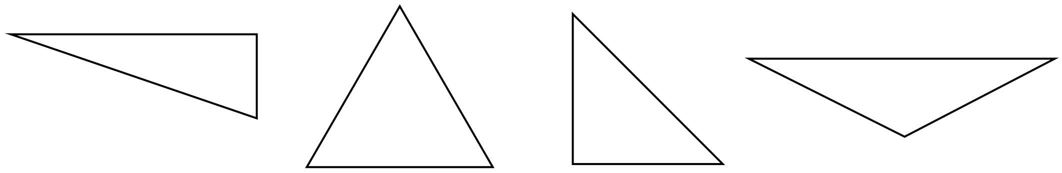


3. In each of the following sets of polygons, there is one that does not belong. Identify the odd one out. Explain how your chosen shape is different from the other three shapes.

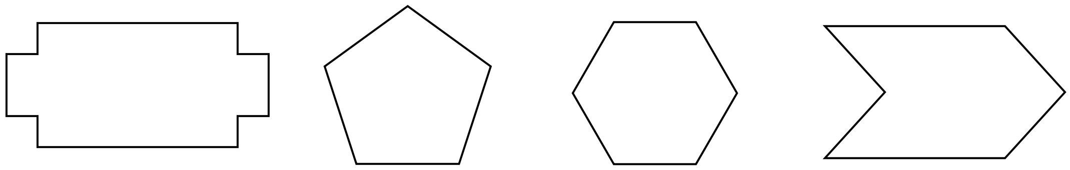
   Set A



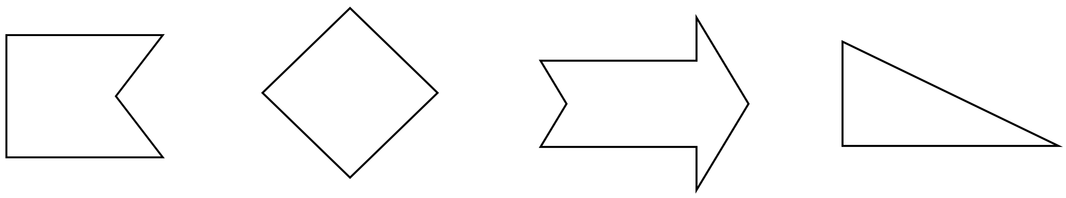
Set B



Set C



Set D



4. Draw neat diagrams that match each of the following descriptions.

   a) A quadrilateral with exactly three acute angles and exactly two congruent sides.

   b) A hexagon with five right angles and three pairs of congruent sides.

   c) A convex hexagon with one pair of congruent, parallel sides and one pair of unequal, parallel sides. It also has exactly two right angles.

   d) A concave pentagon with no right angles and no parallel sides.

5. Draw all polygons that have exactly two right angles.

Extended Problem Solving

In Australia many road signs are polygons. The STOP sign is the only octagonal shaped sign.

Use the internet to research the road signs, collect pictures of each and give each sign its mathematically correct name.

13 Three-Dimensional Objects

Warm-up
Use a dictionary to write a short mathematical definition of the following:

1.	Net	2.	Edge
3.	Face	4.	Vertex
5.	Parallel	6.	Perpendicular
7.	Point	8.	Line
9.	Ray	10.	Angle

Activity
3D objects can be classified into three large groups – prisms, pyramids and others. For each of the objects below colour the base/s and use this information to determine its mathematical name and then the classification of the object as a prism, pyramid or other.

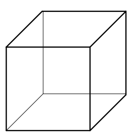
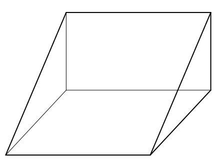
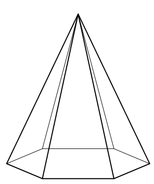
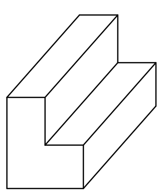
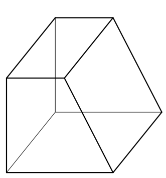
Diagram	***Prism, Pyramid or Other***	***Mathematical Name***
a. 		
b. 		
c. 		
d. 		
e. 		

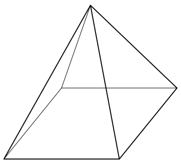
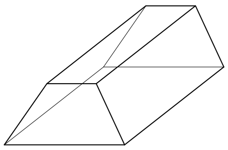
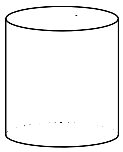
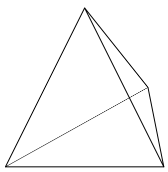
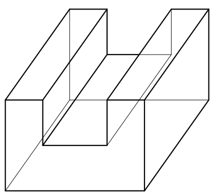
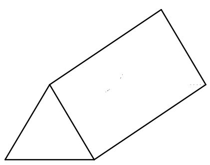
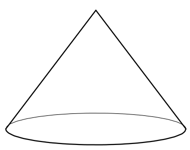
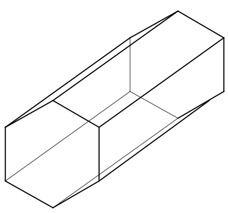
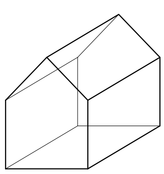
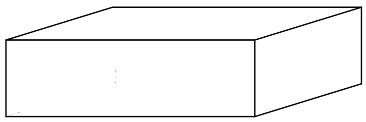
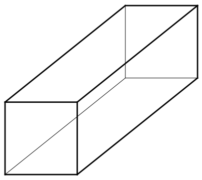
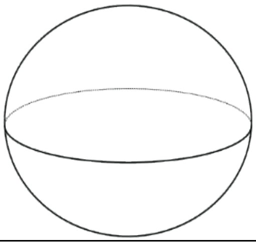
Diagram	***Prism, Pyramid or Other***	***Mathematical Name***
f. 		
g. 		
h. 		
i. 		
j. 		
k. 		
l. 		
m. 		
n. 		

Diagram	*Prism, Pyramid or Other*	*Mathematical Name*
o. 		
p. 		
q. 		

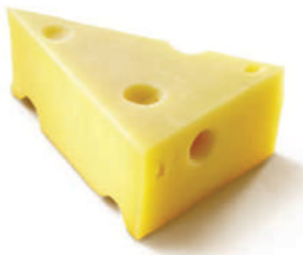
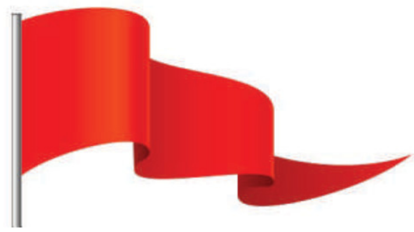
In your own words write a definition that clearly shows the difference between a prism and a pyramid.

Exercises

1. For each of the prisms and pyramids from the introductory activity, determine the number of edges, vertices and faces. Copy and extend the table below to record your findings.

Mathematical Name of Object	**Number of Faces**	**Number of Edges**	**Number of Vertices**
a) Cube			
b) Triangular Prism			
c) Hexagonal Pyramid			
d) Hexagonal Prism			

2. Give the mathematical three-dimensional name for the object that is represented by these everyday items.

a) 	b) 	c) 
d) 	e) 	f) 
g) 	h) 	i) 
j) 	k) 	l) planter box only 

3. Write an accurate mathematical description for each of the following objects.

a)

b)

c)

d)

Extended Problem Solving

Leonhard Euler, born in Basel Switzerland in1707, was a mathematician who made contributions to the fields of geometry, calculus, mechanics and number theory. Euler's number is e, which is approximately 2.71828.

He is also credited with a law that connects the number of faces, vertices and edges in three-dimensional objects.

Use your answers to question 1 to explore and determine Euler's Polyhedron Law. Does it apply to all three dimensional objects or just prisms and pyramids?

14 Nets and Perspective

Materials required: Pencil, centimetre graph paper, ruler, scissors

Warm-up

1.	Find 50% of $400	2.	Find 25% of $700
3.	Find 75% of $600	4.	Find 10% of $500
5.	Find 1% of $65	6.	Find 15% of $30
7.	Find 12% of $200	8.	Find 12.5% of $200
9.	Find 7% of $90	10.	Find 7.5% of $90

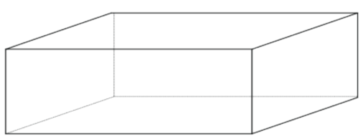
Activity

A cube is a six-sided object with all faces being square and congruent, that is, all the same size and shape. There are eleven different nets of a cube. Using centimetre graph paper, find all eleven nets.

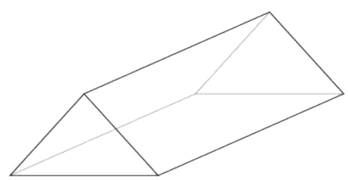
Exercises

1. Draw a net for each of the following solids.

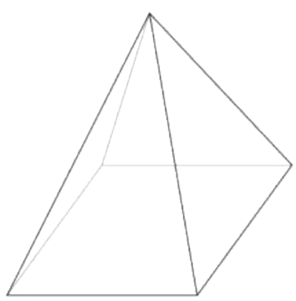
a)



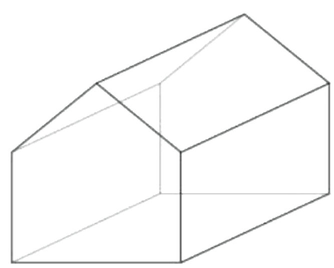
b)



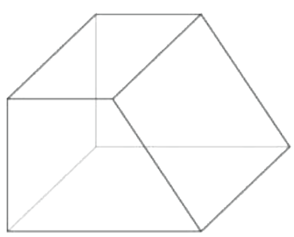
c)



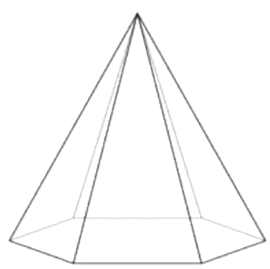
d)



e)

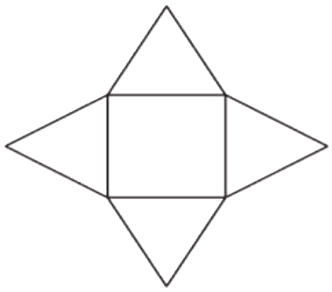


f)

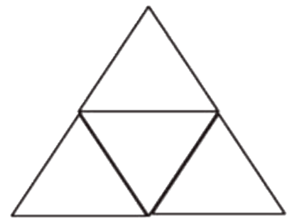


2. Name the solid that is represented by each of the following nets.

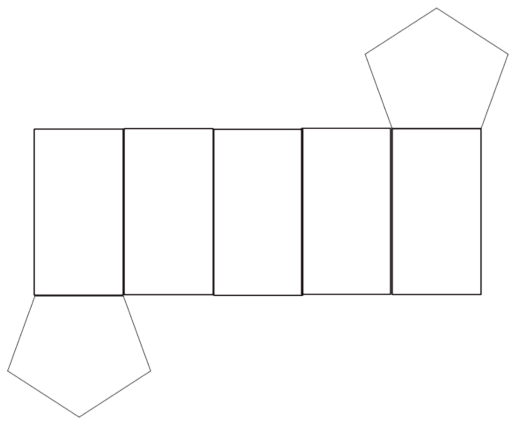
a)



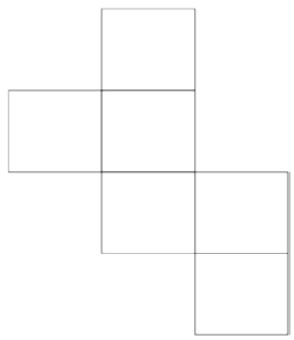
b)



c)

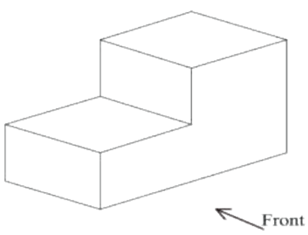


d)

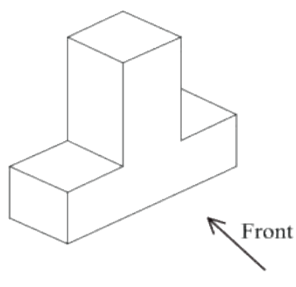


3. For each of the following objects draw a front, top and left view. Using graph paper will be of assistance.

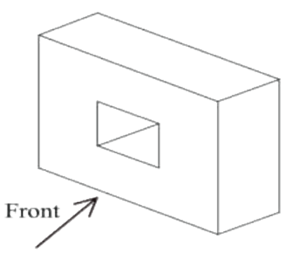
a)



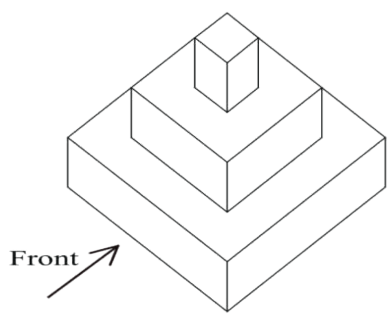
b)



c)



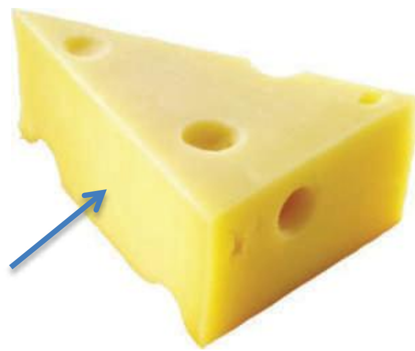
d)



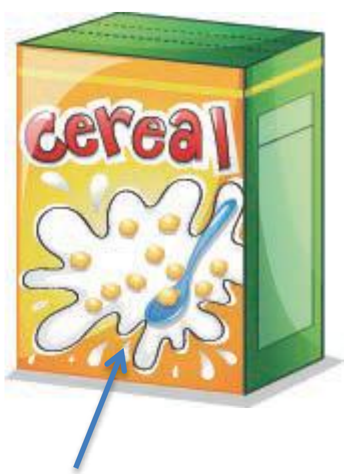
4. Draw the top view, front and right elevations of the following items. The arrow indicates the front elevation.

a)

b)



c)



d)

Extended Problem Solving

Platonic solids are three-dimensional objects. There are only five platonic solids. We are very familiar with one of these, the cube.

Use the internet to write a definition of a platonic solid and find the other four.

Use the internet to assist you in finding a net for each of the platonic solids, then using straws or other construction materials, construct the nets of this special group of solids.

Why was 6 afraid of 7?

Because 7 8 9

What do you get if you add 3 apples and 2 apples?

A maths problem

15 Scale Drawings

Materials required: Rulers, 1cm grid paper, protractors, and calculators.

Warm-up

1.	Convert 3000mm to metres	2.	Convert 5000mm to metres
3.	Convert 2000mm to metres	4.	Convert 9000mm to metres
5.	Convert 1500mm to metres	6.	Convert 7500mm to metres
7.	Convert 4500mm to metres	8.	Convert 6250mm to metres
9.	Convert 9450mm to metres	10.	Convert 300mm to metres

Introductory problem

The Hubble Space Telescope was launched into space by the space shuttle Discovery in April 1990. This telescope observes far into space and transmits pictures back to Earth. These images have been amazing and our knowledge and understanding of astronomy has changed with Hubble's discoveries.

This telescope is an example of an instrument that produces a product that is a scale drawing or photograph of a real object. A scale drawing is a reduced or enlarged version of a real object. In the case of the Hubble Space Telescope the real object is faraway galaxies and star systems and the image is much smaller than the real object

Below is an image on the left of the Horsehead Nebula and the Flaming Tree Nebula in the constellation Orion. Next to it is a close-up of the Horsehead Nebula taken by the Hubble telescope. The Horsehead Nebula can be seen with an ordinary telescope.

"Horsehead-Hubble" by NASA, NOAO, ESA and The Hubble Heritage Team STScI/AURA - http://www.spacetelescope.org/images/html/heic0105a.html. Licensed under Public Domain via Wikimedia Commons - https://commons.wikimedia.org/wiki/File:Horsehead-Hubble.jpg#/media/File:Horsehead-Hubble.jpg

Brainstorm other devices or examples of where scale is used to produce images of objects. The example of the Sydney Harbour Bridge below was taken with a device that produces a scale drawing or representation.

Scales are always written as a ratio. For example the ratio used in the UBD Perth Street Directory is 1:19 000. This shows the ratio between the image to the actual object. In this case 1cm on the map is 19 000cm (that is 190 metres) in real life.

Another way to write a scale is as a statement: 1cm represents 10m.

Exercises

1. For each scale diagram:
   (i) find the diagram's length indicated, in millimetres
   (ii) use the ratio to find the actual length in millimetres
   (iii) convert the actual length to an appropriate unit

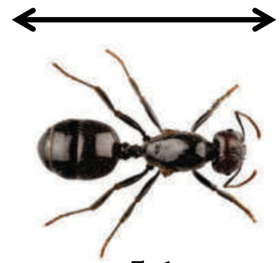
   a) 1:800

   b) 2:1

   c) 1:720

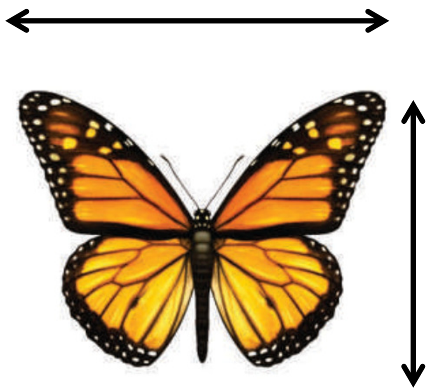
d) 2:5

e)

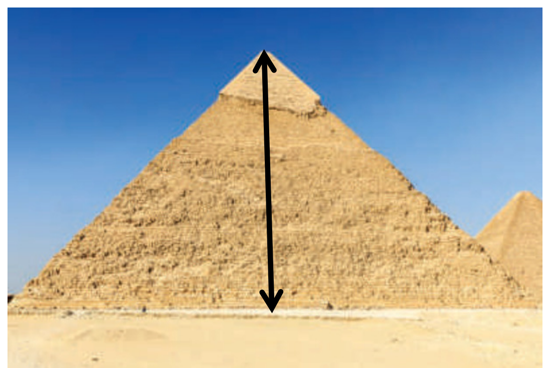


5:1

f) 1:2



g) 1:3900



2. Andrew has a keen interest in building model cars. Determine the length of the actual car, in metres, given the length of the model and its scale.

   a) Car A 241mm scale 1:18

   b) Car B 178mm scale 1:24

   c) Car C 127mm scale 1:32

   d) Car D 76.2mm scale 1:64

3. Using the scale 1:64, what would be the length of a model school bus given that the actual school bus measures 14.6 metres? If we used a scale of 1:32 for a second model, show that the new model would be twice as big as the previous model.

4. Agnes and George have a map of Western Australia and the scale is 1cm represents 50km. Determine the:

   a) distance between Perth and Albany, given it measures 8.3cm on the map

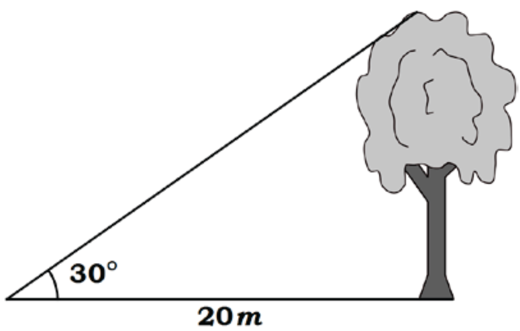
   b) distance between Bunbury and Geraldton given it measures 11.9cm on the map

   c) distance on the map between Perth and Broome given the actual distance between them is 2240km

   d) distance on the map between Perth and Kalgoorlie given the actual distance between them is 594km.

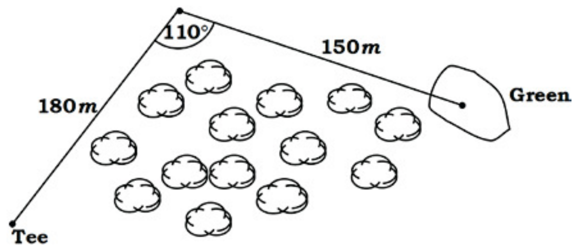
5. Use the information provided to draw a scale diagram and then answer the question.

   a) Use scale 1cm = 2m



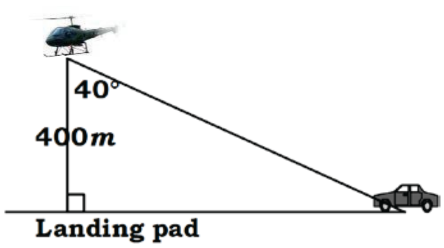
How tall is the tree?

   b) Use scale 1cm = 20m



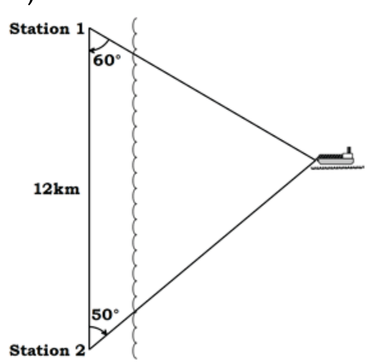
What is the length of a direct hit from the tee to the green?

   c) Use scale 1cm = 50m



How far is the car from the landing pad?

   d) Use scale 1cm = 2km



How far is the boat from each coastal watch station?

6. Use the scale 1:50 to draw a top view of your classroom. Remember to include the positions of the desks and any cupboards or filing cabinets.

Extended Problem Solving

The distances between each planet in our solar system will vary depending on each orbit. For example, the distance between Mercury, the planet closest to the sun, and Earth can range from 77 million kilometres to 222 million kilometres. Astronomers look at the average distance between planets rather than a specific distance at a given time.

Astronomers use a unit called an astronomical unit (AU) where 1AU = the distance between Sun and Earth = 149,598,000 kilometres.

Determine the distance, in kilometres, of the following planets from Earth:

a) Mercury 0.61AU

b) Venus 0.28AU

c) Mars 0.52AU

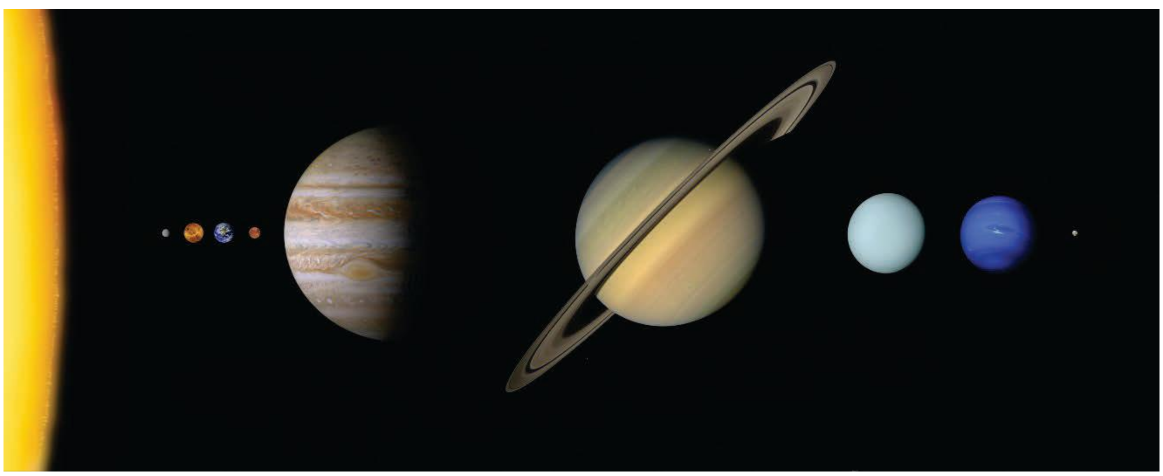
d) Jupiter 4.2AU

e) Saturn 8.54AU

f) Uranus 18.14AU

g) Neptune 29.06AU

The circumference of the Earth is 40075.16km at the equator. Use this measurement to determine how many times you would need to travel around Earth's equator to equal the distance to Mercury.



16 House Plans

Materials required: House plan (provided by teacher), calculator

Warm-up

1.	Convert 1450mm to metres	2.	Convert 5990mm to metres
3.	Convert 4290mm to metres	4.	Convert 13430mm to metres
5.	Convert 1570mm to metres	6.	Convert 2520mm to metres
7.	Convert 3930mm to metres	8.	Convert 1430mm to metres
9.	Convert 1830mm to metres	10.	Convert 600mm to metres

Introductory problem

Architects and drafters prepare house plans for either individual 'one-off' designs or house building companies. Estimators, interior designers, plumbers, electricians, painters and carpenters use house plans. It is important that house plans are drawn accurately and, with the advancements in technology, they are now created using computer software rather than hand drawn. Computer software also allows 3D representations to be shown to clients.

Architects and drafters produce scale drawings that contain many symbols and abbreviations. Most of the abbreviations and symbols use common sense and as such a legend or key is rarely provided.

Use the house plan below to answer the following questions.

a) How many bedrooms does this house have?

b) What is the area of the kitchen, in square metres?

c) How many windows are in the garage?

d) What do the following symbols represent:

(i) 

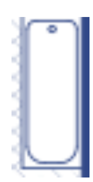
(ii) 

(iii) 

(iv) 

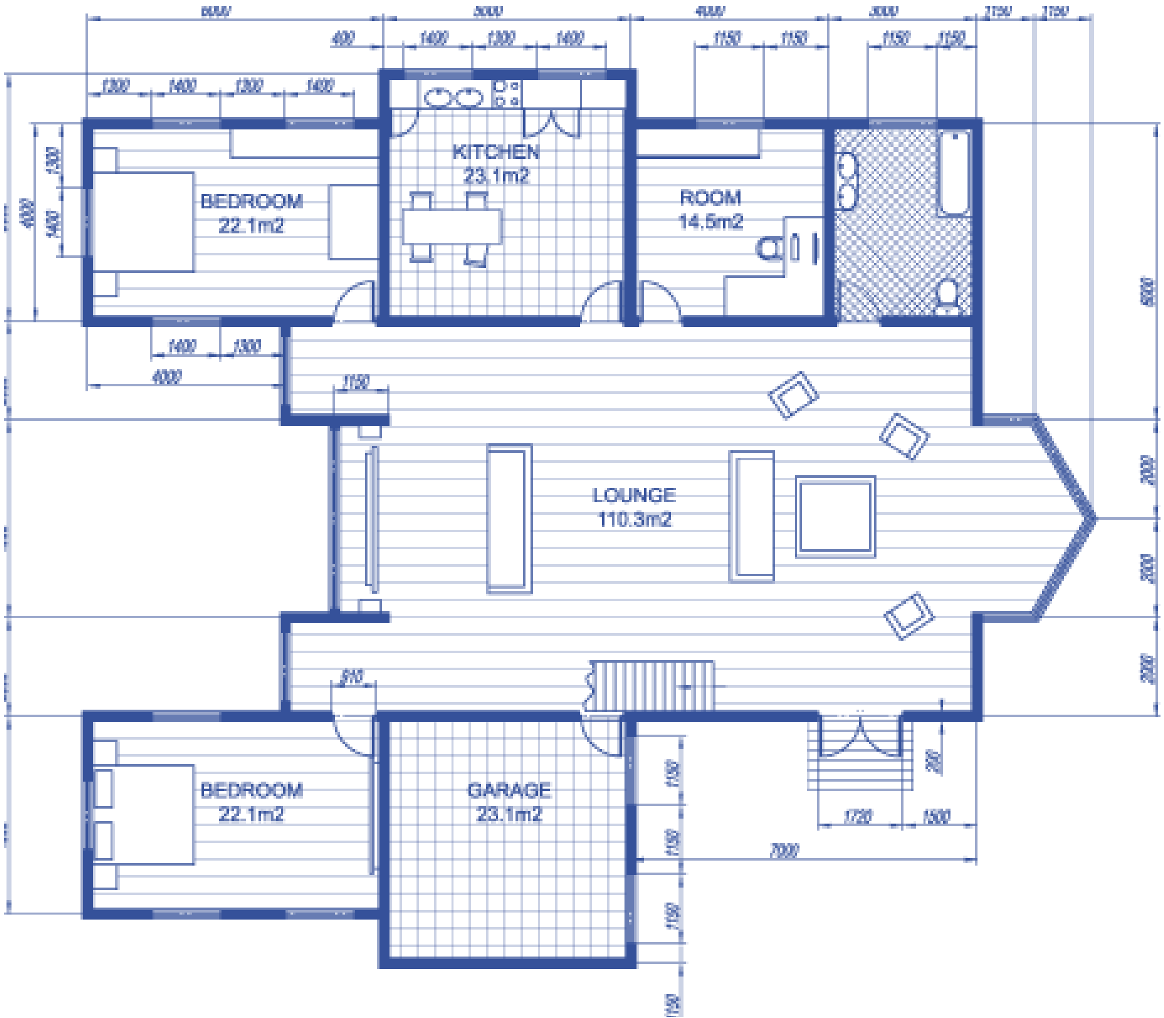
(v) 

(vi) 

(vii) 

(ix)

(x) 



Exercises

Use the house plan provided by your teacher to complete the following questions. You may need to quickly look back at the lessons on perimeter, area, volume, surface area and scale drawings.

1. What do the following abbreviations represent:

a)	ENS	b)	L'DRY	c)	WIP
d)	S&R	e)	LIN	f)	25c CEILING
g)	WC	h)	OBS (toilet and bathrooms)		

2. Determine the volume of concrete needed to lay the slab, given that the slab is 100mm thick. The alfresco area is not included. A cement truck holds $10m^3$ of cement. How many truckloads are needed for the house slab?

3. Determine the number of square metres of tiling required for all wet areas, that is bathrooms, toilets and laundry. Be careful not to include the wall thickness.

4. Determine the amount of carpet needed to carpet the four bedrooms, including built in robes, and the theatre room. Allow an extra 10% for wastage.

5. Determine the amount of floating floorboards required for all rooms not mentioned in questions 3 and 4. If it costs $150 per square metre for supply and installation, how much will it cost to have the floorboards completed?

6. The height of the external walls in this house is 25 courses. Each course is 9 centimetres, including mortar.
   (i) What is a course?
   (ii) What is the height of the garage wall, in metres?
   (iii) Determine the number of external bricks needed to build the garage, if there are 50 standard bricks to the square metre.

7. You wish to paint all of the wet areas with two coats of paint. If 4 litres of paint covers approximately $34.5m^2$, how many litres of paint will you need to buy? If it costs $79.90 for a 4-litre tin, how much will you spend on paint?

8. You wish to air condition your house with a reverse cycle system. Heffer Air Con Company has a choice of units. Unit A can be used in a house with a volume of less than $250m^3$, Unit B is used for houses with a volume of between $250m^3$ and $350m^3$ and Unit C for houses with a volume of greater than $350m^3$. If you worked for Heffer Air Con which unit would you recommend and why? Show all calculations to support your answer.

9. You have decided that you wish to put a wallpaper border in bedroom 4. Determine the length of border required. The border will be placed under the window. Allow 20% extra for pattern matching.

10. Gilmore Smart Wiring has been employed to put in all data cables in your house. As a guide they recommend two and a half times the perimeter of your house in cabling.
    a) Determine how much wiring will need to be ordered.
    b) Suggest locations in the house that will need data points (used for the internet), TV antenna points and landline phone points.

11. You wish to lay liquid limestone in the alfresco area. The limestone needs to be 150 millimetres thick.
    a) How much liquid limestone will you need to order?
    b) How much sand will you need to remove from the alfresco area, given that it is flat and level with the top of the house slab?

Extended Problem Solving

Rainwater tanks are becoming more common in urban households. Designers have ensured that the tanks are attractive and add to the value of the house.

Household rainwater tanks can be above ground and underground. The above ground tanks are cylindrical or rectangular prisms. Underground tanks are rectangular prisms or spheres.

Given that $1m^3$ holds 1000 litres, calculate the capacity of each of the following rainwater tanks.

Above Ground
Cylindrical tanks:

- (i) Diameter of 855mm and a height of 1025mm.
- (ii) Diameter of 700mm and a height of 2025mm.

Rectangular slimline tanks:

- (i) 395mm × 1795mm × 1890mm
- (ii) 290mm × 1935mm × 2400mm

Underground
Spherical tank:

- (i) Diameter of 2450mm

Ask your parents how much water is consumed in your house. This is found on the accounts sent from your water provider. The accounts are sent every 8 weeks.

Use this information to determine how many of each of the cylindrical tanks would be needed to meet your family's water consumption needs.

17 Pythagoras

Materials required: Calculators and rulers

Warm-up

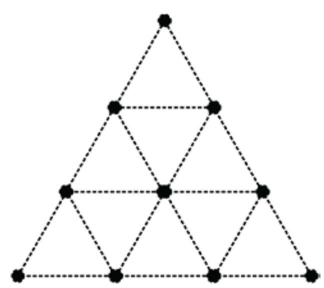
1.	3^2	2.	8^2
3.	2^2	4.	11^2
5.	9^2	6.	$\sqrt{16}$
7.	$\sqrt{100}$	8.	$\sqrt{1}$
9.	$\sqrt{2500}$	10.	$\sqrt{4900}$

Discussion

Pythagoras of Samos was an Ionian Greek philosopher, mathematician and founder of the religious movement called Pythagoreanism. He was born on the island of Samos, and it is believed he travelled widely in his youth, visiting Egypt and other places seeking knowledge. He is said to have died in Metapontum.

He is credited with three major discoveries in mathematics, though some believe that he never discovered them himself but that his followers may have made the discoveries. First is Pythagoras' Theorem of the relationship between the side lengths in a right-angled triangle. Second, that musical notes can be translated into mathematical equations and he may be responsible for discovering the properties of string length.

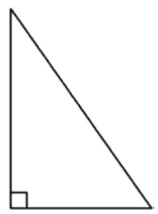
Lastly, Pythagoras was also credited with devising the tetractys, the triangular figure of four rows which add up to the perfect number, ten. As a mystical symbol, it was very important to the worship of the Pythagoreans who would swear oaths by it.

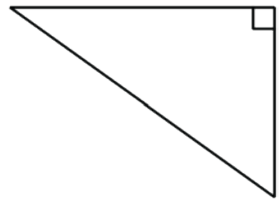


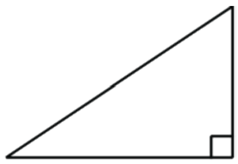
Exploration: See what you can find out about Pythagoras of Samos, the stories of his religion and followers (what they believed and what they did) and anything else interesting about him, them or their actions. Hint: see if you can find out what they thought of beans, or how Pythagoras died.

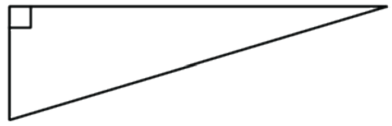
Exercises

1. Label the hypotenuse in each of the following triangles.

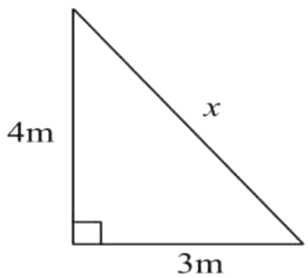
a) 

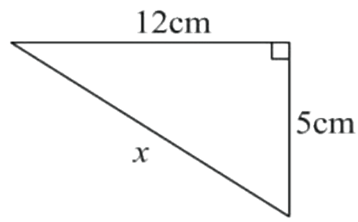
b) 

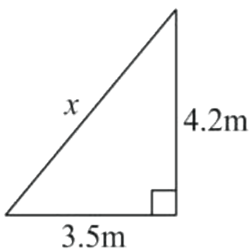
c) 

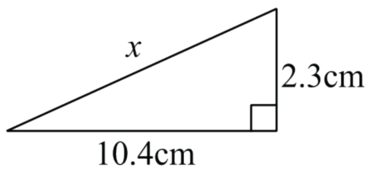
d) 

2. Find the length of the hypotenuse to one decimal place, if appropriate.

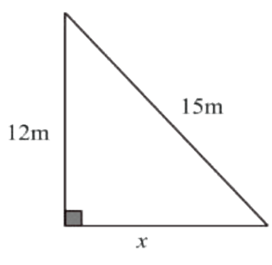
a) 

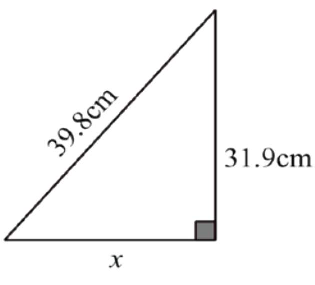
b) 

c) 

d) 

3. Find the length of the shorter side giving answers correct to two decimal places.

a) 

b) 

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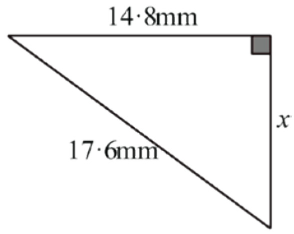
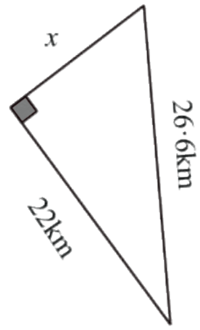
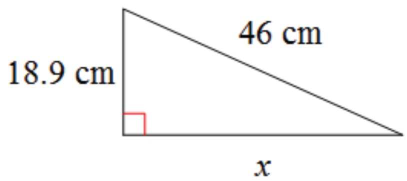


d)

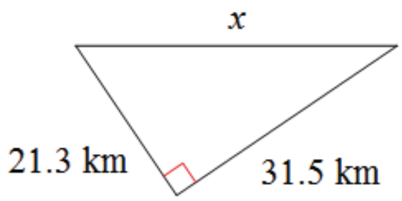


4. Find the length of the unknown side, giving your answers correct to one decimal place.

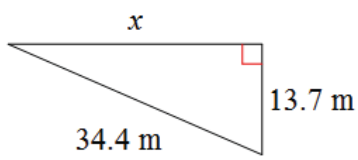
a)



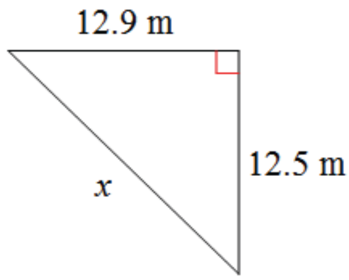
b)



c)

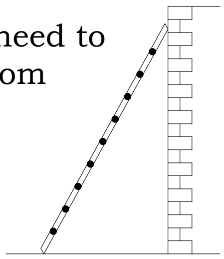


d)



5. A rectangular gate that is 1.8m high and 2.2m wide needs to be repaired using a single brace, a straight support piece fixed across opposite corners of the gate. Determine how long the brace will be for this gate. Hint: draw a sketch to assist.

6. How far from the base of the wall does a 3.2m ladder need to be placed so that the ladder can reach a point 2.9m from the ground up the wall?



Extended Problem Solving

Many trades people use Pythagoras' Theorem to determine if walls, paving, door frames, window frames or brickwork are square, that is at right angles.

One method is to use a version of the 3m, 4m, 5m right angled triangle. Find a suitable corner within your classroom, such as a door frame or window frame. Measure 30cm along one edge and 40cm along the other edge. If the diagonal that joins these two points is 50cm, then the item is square.

Check at least three other items in your classroom.

"Mathematics expresses values that reflect the cosmos, including orderliness, balance, harmony, logic, and abstract beauty."

— Deepak Chopra

"With me, everything turns into mathematics."

— René Descartes

18 Tangent Ratio

Materials required: Calculators, 1cm graph paper, rulers and protractors.

Warm-up

1.	180° – 120° =	2.	180° – 95° =
3.	180° – 137° =	4.	180° – 168° =
5.	180° – 45° – 90° =	6.	180° – 37° – 90° =
7.	180° – 64° – 90° =	8.	180° – 36° – 87° =
9.	180° – 27° – 105° =	10.	180° – 15° – 132° =

Activity

On a blank sheet of paper accurately construct three different size triangles with internal angles of 30°, 60°, and 90°. Inside the triangle label the right angle and the 30° angle. Carefully measure the length of the sides opposite and adjacent to the 30° angle.

Record your findings in a table similar to the one below:

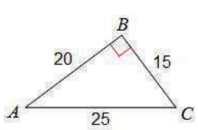
Triangle Number	Length of opposite side (in mm)	Length of adjacent side (in mm)	Ratio of sides $\frac{\textit{length of opposite side}}{\textit{length of adjacent side}}$
1			
2			
3			

Record the information from three other people. Comment on your findings in the 'ratio of sides' column, remembering that the accuracy of the findings depends heavily on the accuracy of the drawings and measuring. This ratio is called the 'tan' ratio of the 30° angle and it can be found for either of the smaller angles in any right-angled triangle.

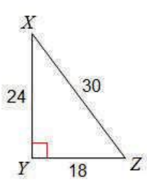
Exercises

1. State the tangent ratio, as indicated in each of the following examples.

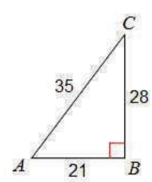
a) $\tan C$



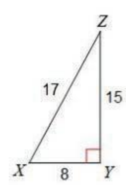
b) $\tan X$



c) $\tan A$

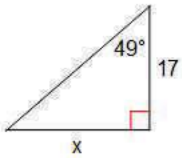


d) $\tan Z$

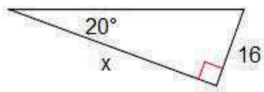


2. Use the tangent ratio to find the length of the unknown side correct to 2 decimal places. All lengths are in centimetres.

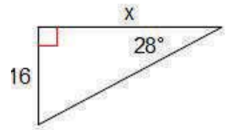
a)



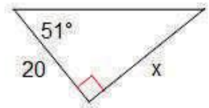
b)



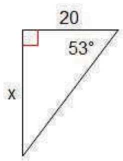
c)



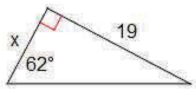
d)



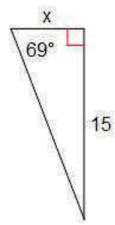
e)



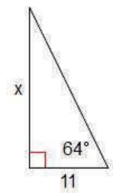
f)



g)



h)



3. Use a scientific calculator to determine the size of the angles, correct to the nearest whole degree.

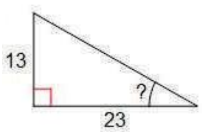
a) $tan^{-1}0.4$ b) $tan^{-1}0.56$ c) $tan^{-1}0.85$

d) $tan^{-1}1$ e) $tan^{-1}\frac{3}{4}$ f) $tan^{-1}\frac{17.2}{24}$

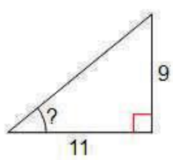
g) $tan^{-1}\frac{27.3}{26.1}$ h) $tan^{-1}\frac{0.9}{0.2}$ i) $tan^{-1}\frac{8}{3}$

4. Use the tangent ratio to find the size of the marked angle, to the nearest whole number.

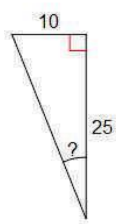
a)



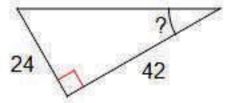
b)



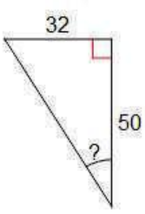
c)



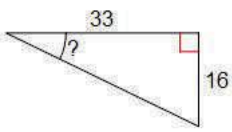
d)



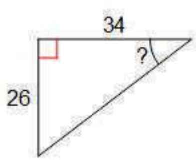
e)



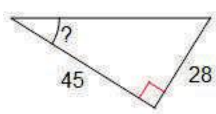
f)



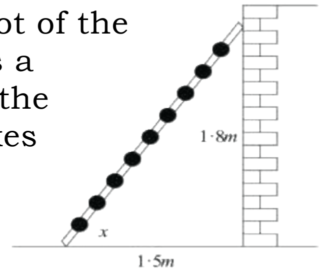
g)



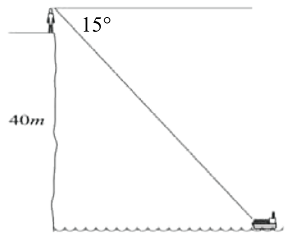
h)



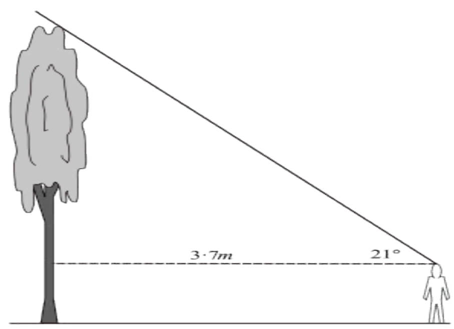
5. Quinny placed a ladder against a wall. The foot of the ladder is 1.5 metres from the wall and reaches a height of 1.8 metres. Determine the angle, to the nearest degree, that the foot of the ladder makes with the ground.



6. Michaela was standing 40 metres above sea level on the edge of a cliff. She looks directly out to the horizon and then lowers her eyes through 15° to look at a boat. How far out to sea is the boat?



7. Mr Ackers has asked his students to work out the height of a tree in the school grounds. Mario is 1.7 metres tall and he stands 3.7 metres from the tree. Using a clinometer, the angle from his eye level to the top of the tree is 21°. Determine the height of the tree, to the nearest metre.



8. The Golden Gate Bridge is a famous landmark in San Francisco. The main span, which is between the two large pylons, is 1 280 metres long. The pylon is 230 metres above road level. The suspension wire is fixed at the top of the pylon and half way along the main span. Determine the size of the angle between the wire and the pylon and between the wire and the main span, assuming the wire is stretched taut (ie a straight line).

Extended Problem Solving

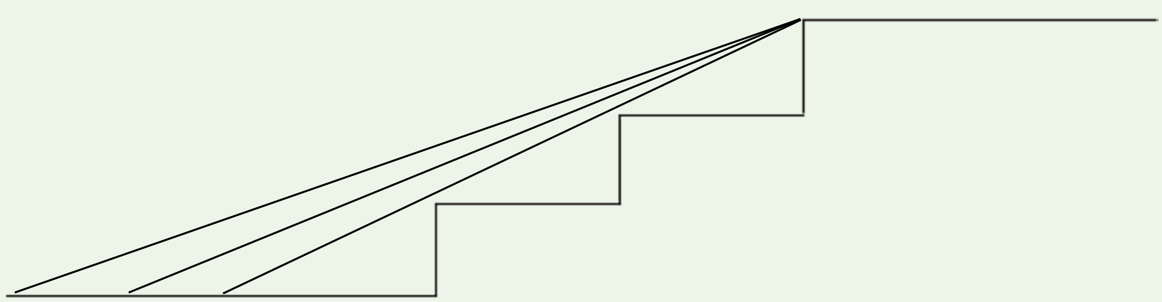
There are five steps leading up to the front door of the building. A ramp is to be put on part of the stairs to make the building wheelchair accessible.

Each step is 1.1 metres wide. The riser, the vertical measurement of each step, is 25 centimetres high and the tread, the horizontal section of each stair that is the part you step on, is 60 centimetres deep.

The following information is provided to assist you:
Slope is given as height:depth
Slope 1:20 Accessible pathway
Slope <1:10 Assistance required
Slope >1:10 Hazardous

For wheelchair accessibility, the optimum ratio is 1:12.

You have been given the task of preparing a report to determine the most suitable angle/s of a ramp for wheelchair use. Clearly show all of your calculations to support your chosen angle/s of the ramp.



19 Angles of Elevation and Depression

Materials required: Calculators, rulers, pencils, tape measures, and clinometers

Warm-up

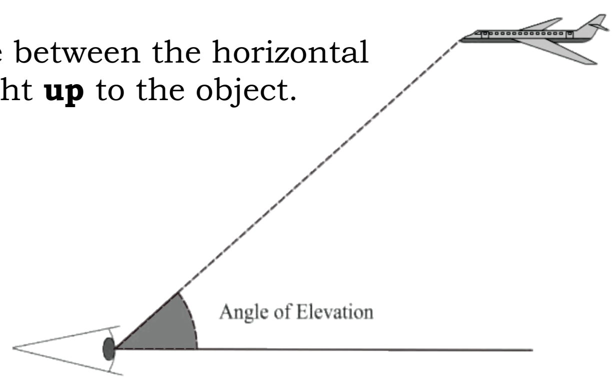
1.	Round 7.84 to 1 decimal place	2.	Round 3.79 to 1 decimal place
3.	Round 2.07 to 1 decimal place	4.	Round 3.55 to 1 decimal place
5.	Round 2.96 to 1 decimal place	6.	Round 13.04 to 1 decimal place
7.	Round 7.346 to 2 decimal places	8.	Round 0.473 to 2 decimal places
9.	Round 9.955 to 2 decimal places	10.	Round 9.95 to 1 decimal place

Discussion

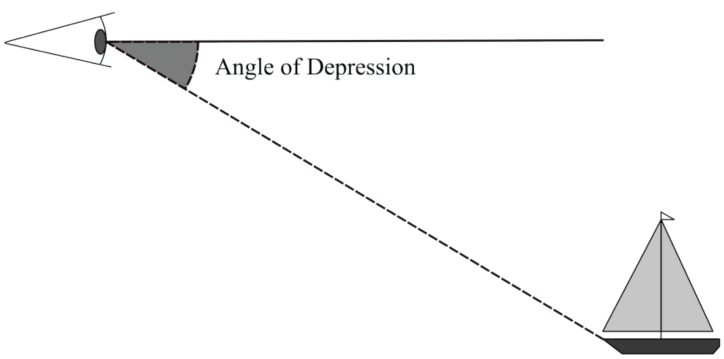
Angles of elevation or depression are used in many aspects of mathematics. They are especially useful in navigation, surveying and architectural designing.

Angles of elevation and depression are measured from the horizontal.

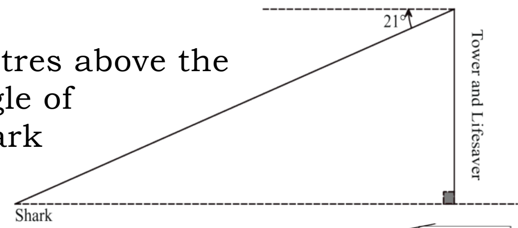
The angle of elevation is the angle between the horizontal line from the observer's line of sight **up** to the object.



The angle of depression is the angle between the horizontal line from the observer's line of sight **down** to the object.

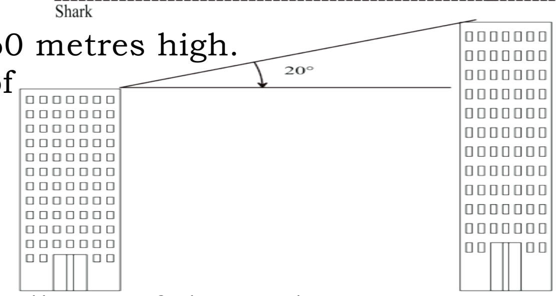


Exercises



1. A surf lifesaver, in her tower, is 3 metres above the ground. She spots a shark at an angle of depression of 21°. How far is the shark from the lifesaver's tower?

2. Two buildings are 75 metres and 60 metres high. The angle of elevation from the roof of the shorter to the taller is 20°. Find the distance between the two walls.



3. James observes a yacht from the walkway of the Bathurst Lighthouse on Rottnest Island. He is standing 30 metres above sea level and observes the yacht at an angle of depression of 17°. How far is the yacht from the base of the lighthouse?

4. Brookfield Place, in Perth, is where you will find the BHP building. The building is 244 metres tall. An observer measures the angle of elevation to be 41°. Approximately how far is the observer from the base of the building?

5. A bird is sitting on top of a lamppost. The angle of depression from the bird to the feet of an observer is 35°. The distance from the bird to the observer is 3.1 metres. How tall is the lamppost?

6. A pilot is flying a jet at a height of 3.23 kilometres above the ground. The distance, measured along the ground, from a point directly below the jet to the airport is 8.04 kilometres. What is the angle of depression from the pilot's eye-line to the airport?

7. A man, who is 1.8 metres tall, stands on the ground 30 metres from a tree. He measures the angle of elevation, with a clinometer, to the top of the tree as 27°. Estimate the height of the tree. Comment on why this is an estimation rather than an exact distance.

Extended Problem Solving

Native American Indians would estimate the height of a tree by bending over and looking through their legs.
Search the internet for this method and use it to estimate the height of trees, flagpoles or buildings.

Then use a clinometer, measuring tape and trigonometry to check the accuracy of the Native American Indian method.

20 Sine and Cosine Ratios

Materials required: rulers, protractors, scientific calculators

Warm-up

1. Solve $7 = \frac{x}{3}$
2. Solve $5 = \frac{x}{4}$
3. Solve $4 = \frac{x}{7}$
4. Solve $8 = \frac{x}{3}$
5. Solve $12 = \frac{x}{5}$
6. Solve $6 = \frac{24}{x}$
7. Solve $5 = \frac{70}{x}$
8. Solve $7 = \frac{63}{x}$
9. Solve $4 = \frac{100}{x}$
10. Solve $9 = \frac{99}{x}$

Activity

On a blank sheet of paper accurately construct three different size triangles with internal angles of 30°, 60° and 90°. Inside the triangles, label the right angle and the 30° angle. Carefully measure the length of the sides opposite and adjacent to the 30° angle and the hypotenuse.

Record your findings in a table similar to the one below:

Triangle Number	Length of opposite side (in mm)	Length of adjacent side (in mm)	Length of hypotenuse (in mm)	Ratio of sides $\frac{\text{length of opposite side}}{\text{length of hypotenuse}}$	Ratio of sides $\frac{\text{length of adjacent side}}{\text{length of hypotenuse}}$
1					
2					
3					

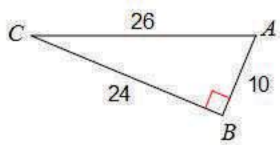
Record the information from three other people. Comment on your findings about the ratio of the opposite side and hypotenuse. This ratio is called the sine ratio for the angle of 30°. Also comment on the ratio of the adjacent side and the hypotenuse. This ratio is called the cosine ratio for the angle of 30°. Remember that the accuracy of the findings depends heavily on the accuracy of the drawings and measuring. These two ratios can be found for either of the smaller angles in any right-angled triangle.

Exercises

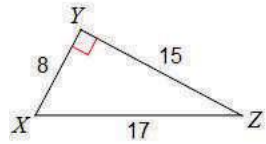
1. Use a scientific calculator to determine the following.
   a) sin 56° b) sin 12° c) sin 45°
   d) cos 75° e) cos 63° f) cos 90°

2. State the ratio, as indicated in each of the following examples.

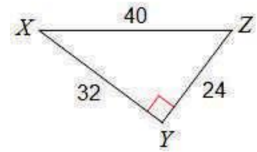
a) cos C



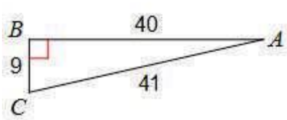
b) sin Z



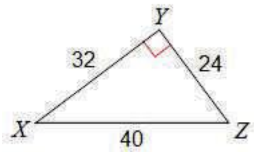
c) sin X



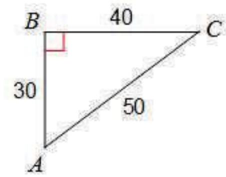
d) cos C



e) sin Z

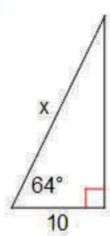


f) cos A

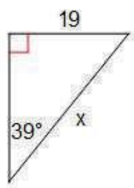


3. Find the length of the side marked with x, giving your answer correct to one decimal place.

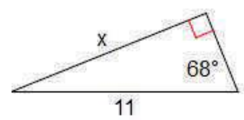
a)



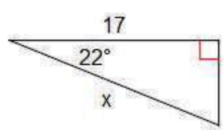
b)



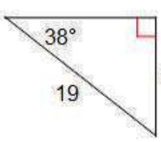
c)



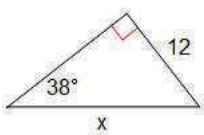
d)



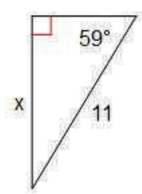
e)



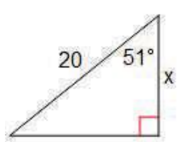
f)



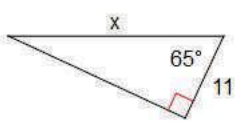
g)



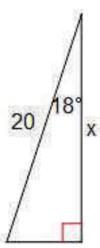
h)



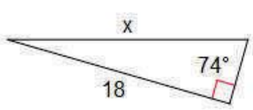
i)



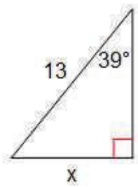
j)



k)

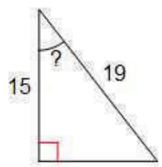


l)

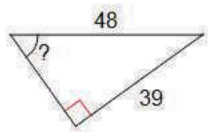


4. Determine the size of the marked angle, giving your answer correct to the nearest whole degree.

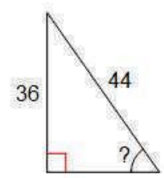
a)



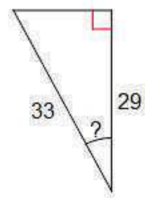
b)



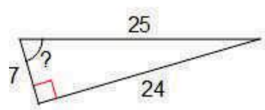
c)



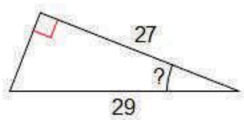
d)



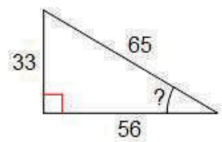
e)



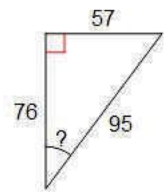
f)



g)



h)



5. Mario is standing on the ground near the Eiffel Tower. The angle of elevation to the top of the tower is 12°. Given that the tower is 300.65 metres tall, determine the length a wire would need to be to reach from Mario to the top of the tower.

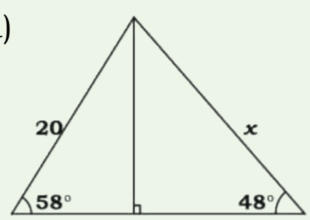
6. The rope that connects the top of a tent to the ground is 4.7 metres long. The tent is 1.8 metres tall. What is the angle, to the nearest degree, between the ground and the rope?

Extended Problem Solving

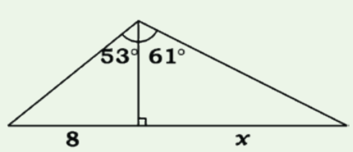
Many roof carpenters use trigonometry to determine the pitch of the roof and the length of the timbers needed to construct the roof.

Determine the lengths of the sides marked x for each of the following cross sections of various roofs. Determine the length of the timber needed for that side. Assume all lengths are in metres.

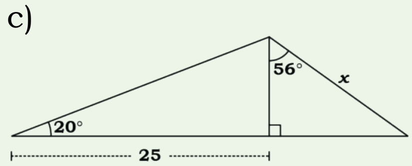
a)



b)



c)



21 Trigonometry and Bearings

Materials required: Calculator, ruler and pencil

Warm-up

1.	$360° - 65° =$	2.	$360° - 170° =$
3.	$360° - 230° =$	4.	$180° - 38° =$
5.	$180° - 96° =$	6.	$60° + 35° =$
7.	$30° + 85° =$	8.	$270° - 15° =$
9.	$270° + 35° =$	10.	$135° + 75° =$

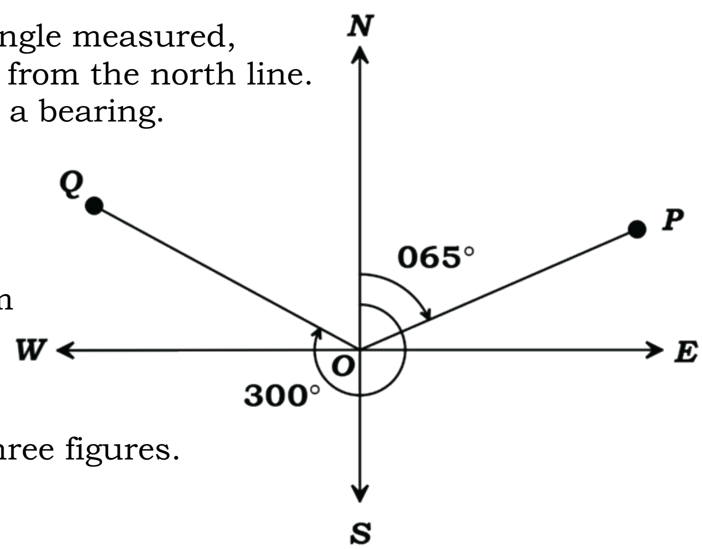
Introductory information

North (N), south (S), east (E) and west (W) are the four main directions on a compass, they are known as the cardinal points. North-east (NE), north-west (NW), south-east (SE) and south-west (SW) are sometimes called the half-cardinal points.

The true bearing to a point is the angle measured, in degrees, in a clockwise direction from the north line. We refer to true bearings simply as a bearing.

In this example point P is on a bearing of 065° from point O.

Point Q is on a bearing of 300° from point O.



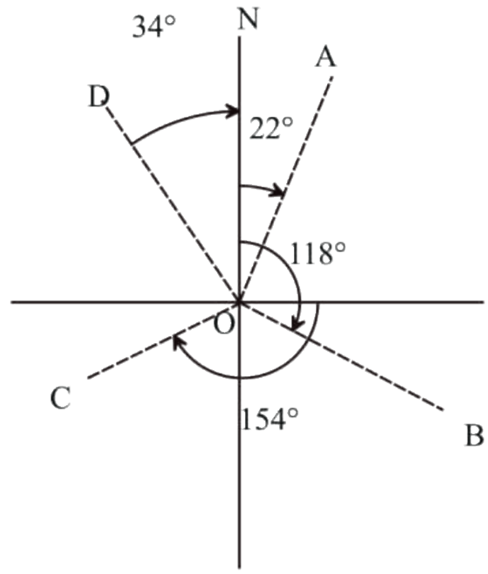
Bearings are always written with three figures.

E.g. 17° = 017°
2° = 002°
190° = 190°

It is important to understand the language of bearings. The bearing **of** P **from** O is 065°, while the bearing **of** O **from** P is 245°.

Exercises

1. Determine the bearing of each of the following points from O.



   a) A b) B

   c) C d) D

2. Draw diagrams to represent the following bearings.

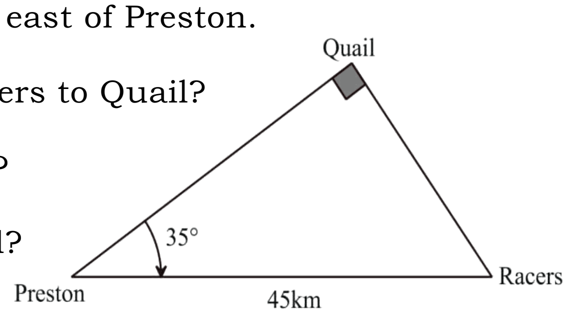
   a) Point X is on a bearing of 070° from point O.

   b) Point Y is on a bearing of 245° from point O.

   c) Point Z is on a bearing of 190° from point O.

   d) Point W is on a bearing of 145° from point O.

3. Three towns Preston, Quail and Racers are situated as shown in the diagram. Racers is 45km due east of Preston.



   a) What is the bearing from Racers to Quail?

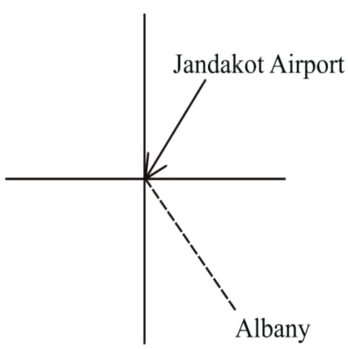
   b) How far is Quail from Racers?

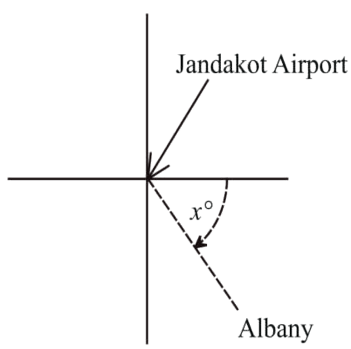
   c) How far is Preston from Quail?

4. A plane flew 405km in a straight line from Jandakot Airport to Albany on a bearing of 119°.

   a) Calculate the size of *x*.

   b) How far is Albany east of Jandakot Airport?





5. Brian and James left home and rode a distance of 5km and 7km respectively. Brian travelled due north of home, while James travelled due west.

   a) Draw a diagram to represent this situation.

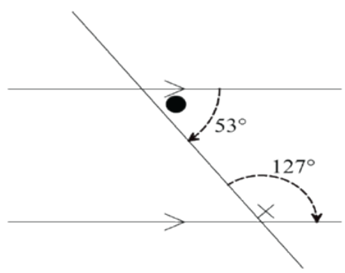
   b) What is the distance between the boys, rounded to one decimal place?

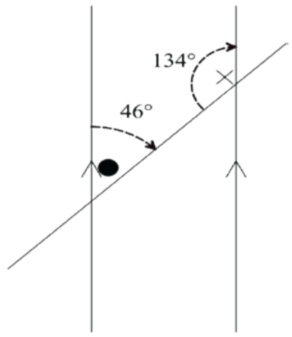
   c) What is the bearing from James to Brian?

   d) What is the bearing from Brian to James?

6. Steve and his mate, Barry, decide to go diving off Rottnest. Steve leaves Rottnest and travels 4.7km on a bearing of 297°. Barry goes on a bearing of 027°. After checking their GPS's they discover that they are 7.9km apart. How far is Barry from Rottnest?

To assist with finding bearings it may be necessary to recall that co-interior angles between parallel lines add to 180°. See the diagrams below.





7. The bearing from a lighthouse (L) of two boats A and B are 034° and 124° respectively. Boat A is 230m from the lighthouse. The bearing from boat A to boat B is 154°.

   a) Draw a diagram to represent this situation.

   b) Determine the size of angle LAB.

   c) How far is boat B from the lighthouse?

   d) How far is boat B from boat A?

22 The Cartesian Plane

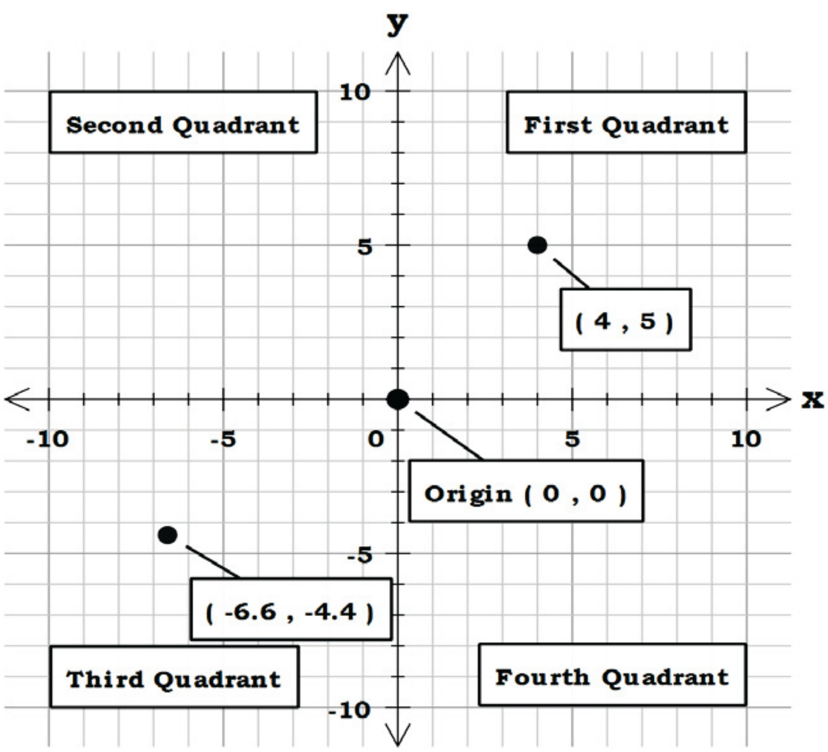
Materials required: 1cm graph paper, rulers, pencils, erasers, template for Battleship in Teacher's Guide

Warm-up

1.	-5 < -7 True or False?	2.	-3.5 > -3.4 True or False
3.	½ > 0.12 True or False?	4.	3 × 4 + 3 × 3 =
5.	3 × (4+3) =	6.	Convert ¾ to a percentage
7.	A six sided polygon is called a __________	8.	$A = \pi r^2$ is the formula for _________
9.	1 m^3 = ________cm^3	10.	1mL = _____cm^3

The Cartesian Plane

Invented in the 17th Century by Rene Descartes the Cartesian plane is used to locate an object's position using a coordinate (x,y) on a two dimensional plane. You may be familiar with the Cartesian plane from using maps or from previous studies of functions in algebra. The Cartesian plane with four quadrants is shown below.



IMPORTANT

When giving coordinates we always run before we jump (like hurdles) - the x-coordinate and then y-coordinate.

Activity: Battleship

The Australian Defence Forces use the Cartesian plane to locate the positions of important landmarks, targets, fighters and other forces when defending the country. Invented by Clifford Von Winkle, the game of Battleship was based on the warring principles of World War I. Battleship is a popular board game that began as a pen and pencil game in the early 1900's. The first board game was released in 1943. In this activity we will use a Cartesian plane to play a game of Battleship. The objective of the game is to use strategies to predict where your opponent's ships are located and by calling out coordinates, sink the enemy.

Equipment: Partner, grid paper (template in TG), ruler, pencil

Rules:

1. Each player gets a navy like this:
   1 aircraft carrier - 6 points (3 × 2) on the grid
   1 battleship - 5 points (5 × 1) on the grid
   1 destroyer - 4 points (4 × 1) on the grid
   2 submarines - 3 points (3 × 1) on the grid
   2 patrol boats - 2 points (2 × 1) on the grid
2. Set up your navy on a Cartesian plane $-5 < x < 5$ and $-5 < y < 5$. Each craft can be placed in a horizontal, vertical or diagonal position.
3. Take turns to predict the opposition's position by using coordinates.
4. Mark an O for a miss and an X for a hit. If you get a hit you can have another shot.
5. The winner is the first player to sink all of their opponent's navy.

Exercises

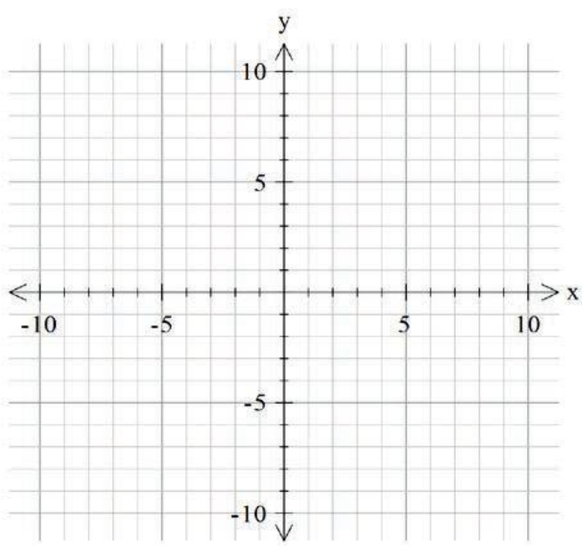
1. Using the grid, describe the shape drawn when connecting the given coordinates.

   a) (5,7),(3,1),(7,1),(5,7)

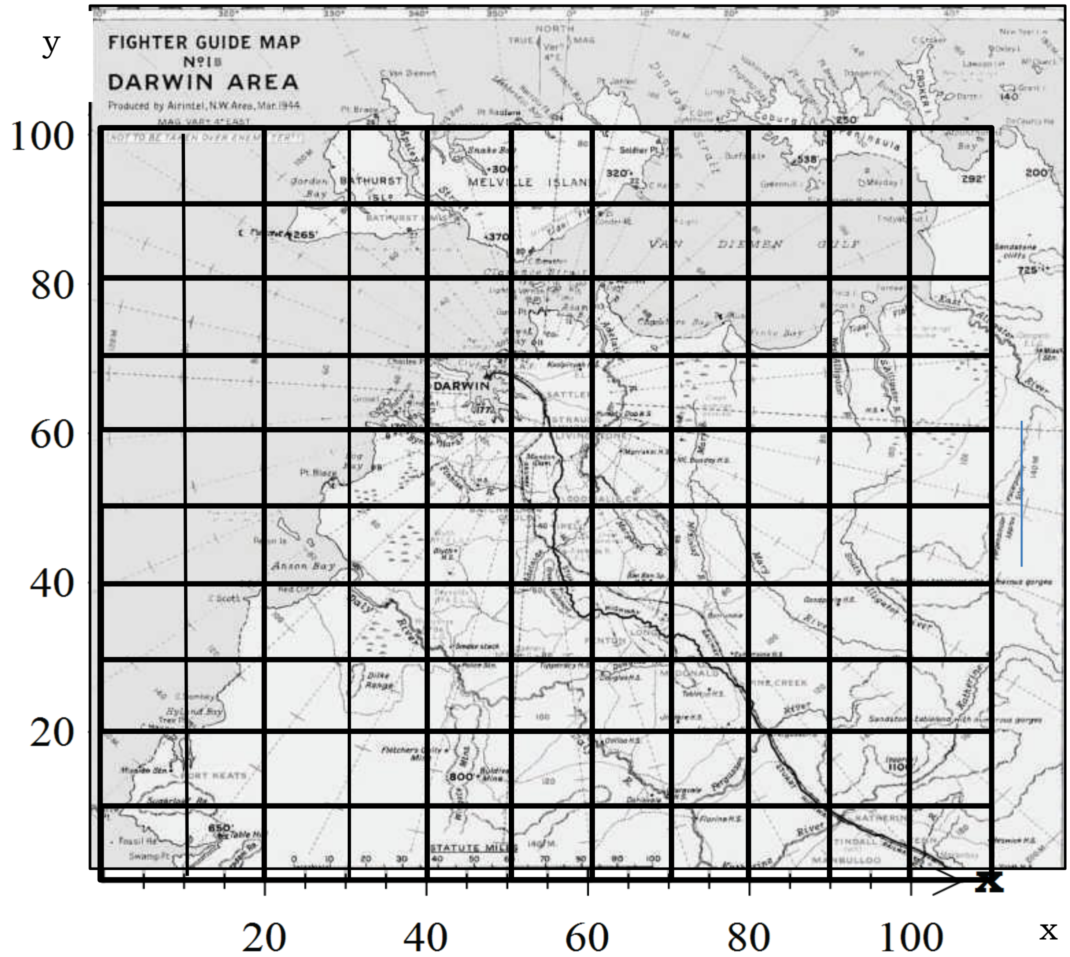
   b) (-7,-2),(-4,-2),(-3,-4),
   (-6,-4),(-7,-2)

   c) (-2,2),(1,2),(3,-2),
   (-2,-2),(-2,2)

   d) (-7,8),(-2,8),(-7,2),(-7,8)



2. The bombing of Darwin by the Japanese in 1942 was the first of several attacks on Australia and resulted in the deaths of 235 Australians. A map produced by the RAAF is shown below that includes many of the air fields targeted by the Japanese aircraft.



http://upload.wikimedia.org/wikipedia/commons/4/40/Darwin_air_defence_map_1944.jpg

Use the grid provided to state the coordinates of:

a) Darwin town centre
b) Police station
c) Soldier Point, Melville Island

3. The coordinates of some special quadrilaterals with at least one line of symmetry are given below, but in each case one coordinate is missing. The coordinates are given going round each quadrilateral in an anti-clockwise direction. Find the missing coordinate and name the quadrilateral.

a) (3,8),(1,6),(3,4),(?,?)
b) (2,1),(2,-2),(7,-2),(?,?)
c) (-4,1),(-7,1),(-9,-2),(?,?)
d) (0,3),(-2,0),(0,-3),(?,?)

4. Using a Cartesian plane of the scale given, join the points stated below to create the mystery pictures.

   a) AXES: -13 ≤ x ≤ 13 and -8 ≤ y ≤ 8
   START (-13,6),(-11,5),(-10,3),(-9.1),(-7,2)(-4,3),(1,3),(11,-1),
   (6½,-1),(6,-2),(10,-2),(2,-4),(-1,-4),(-9,-1),(-10,-3),(-4,-12),
   (-11,-1),(-11,1),(-13,6) STOP
   (-2,3),(-3,5),(-2,5),(-3,6),(-3,8),(-1,6),(0,4) STOP
   (5,0),(6,0),(6,1),(5,1),(5,0) STOP
   (-4,-3),(-6,-5),(-5,-5),(-3,-3) STOP
   (6½ ,-1),(7,-2),(7½,-1),(8,-2),(8½,-1),(9,-2),(9½,-1),(10.-2) STOP
   (-1,-1),(-1,-2),(-2,-5),(-3,-6),(-2,-6).(0,-5),(1,-2),(2,-1) STOP

   b) AXES: -9 ≤ x ≤ 9 and -5 ≤ y ≤ 5
   START (-3,3),(3,3),(4,1),(9,0),(9,-1½),(-6,-3),(2,-3),(2,-2½),
   (-4,-2½),(-4,-3),(-6,-3),(-8,-1),(-8,2),(-7,1),(-6,1),(-3,3) STOP
   (-4,-2½),(-4,2),(0,2),(0,0),(-4,0) STOP
   (-2,0),(-2,-2½) STOP
   (4,1),(1,0,(0,2),(3,3) STOP
   (1,0),(6,-1),(9,0) STOP
   (2,-2½), (2,-1),(4,-1),(4,-3) STOP
   (4,-2½), (-6,-2½),(-9,-1) STOP
   (-6,-2),(-6,-1),(-4,-1) STOP
   (-5½,-1½),(-4½,-1½),(-4½,-2½),(-5½,-2½)(-5½,-1½) STOP
   (2½,-1½),(3½,-1½),(3½,-2½),(2½,-2½),(2½,-1½) STOP
   (-6,-1), (-6,-2½) STOP

   c) AXES: -8 ≤ x ≤ 6 and -6 ≤ y ≤ 11
   START (-8,-6),(5,-6),(5,0),(4,1),(3,0),(2,1),(1,0),(3,1),(½,5),(0,5),
   (-1½,11),(-3,5),(-3½,5),(-4,3),(-4,0),(-5,1),(-6,0),(-7,1),(-8,0),
   (-8,-6) STOP
   (-7,-6),(-7,-3),(-6,-2),(-5,-3),(-5,-6) STOP
   (-3,-6),(-3,-1),(-2,1),(-1,1),(0,-1),(0,-6) STOP
   (2,-6),(2,-3), (3,-2),(4,-3),(4,-6) STOP
   (-1½,2),(-2,4),(-3,5),(-2,5),(-1½,7),(-1,5),(0,5),(-1,4),(-1½,-2)
   STOP

Extended Problem Solving – Mystery Picture

Create your own mystery picture like those in question 4 to give to a partner to solve

23 Linear Graphs

Materials required: cards for silent shuffle in Teacher's Guide, graph paper, rulers, lead pencil

Warm-up

1.	150 ÷ 30 =	2.	15 ÷ 30 =
3.	Complete the pattern 12 ,7, 2, -3, ___, ___	4.	Complete the pattern 785, 887, 989, ____, ____
5.	0.03 × 5 =	6.	12 – 3 × 2 =
7.	A square has an area of 64cm^2. Calculate the perimeter.	8.	A square has a perimeter of 4cm. Calculate the area.
9.	$\frac{1}{3} \times \frac{1}{2} =$	10.	$\frac{1}{3} + \frac{1}{2} =$

Activity – Silent Card Shuffle

- Silent Classification - in groups of four, distribute the cards provided to you by your teacher, evenly between your group. With *no talking* to your group members, classify the cards into each category provided in the cards with **bold** print.
- Challenge, Justify and Refine – once directed by your teacher, you can discuss and challenge the classification. With whole group approval make any card classification changes.
- Circle and Observe - once directed by your teacher, leave a person in your group as a group representative, the other members then walk around and observe the other students' efforts.
- Return and Refine – return to your group and make any changes.
- Solutions – your teacher will give you the solutions and mark your results. Which group achieved the highest score?

A *linear relationship* has a *constant rate of change* between its variables and because of this it forms a straight line when graphed. The constant rate of change is also known as the *slope/gradient* in a graph or *constant first difference* pattern when presented as a table.

Linear relationships are one of the most common and simplest type of patterns observed in the natural world. Pay rates, costs of materials and food prices are some examples of real life situations that can be modelled by a direct linear rule.

Example

'Bright Spark' Electrics charges for electricians to go out and work at an industrial site. They charge $120 for the electrician to go to the site and then charge $30 for every fifteen minutes they work at the site.

a) Create a table showing the charges.
b) Use the table to help you identify the constant rate of change, and state what this represents in the context of the situation.
c) Graph the relationship between the variables.
d) Identify the vertical intercept as a coordinate and comment on its significance.

a)

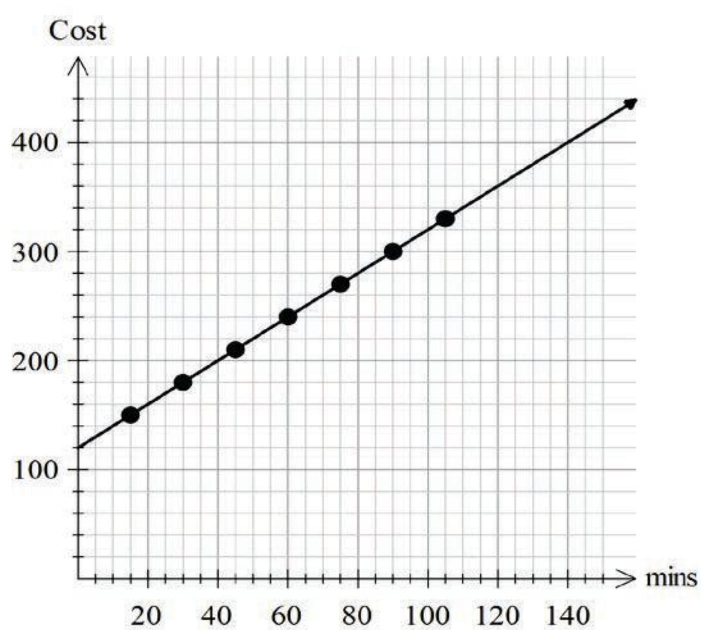
Number of mins	15	30	45	60	75	90	105
Cost in $	150	180	210	240	270	300	330

b) The constant rate of change represents the charge rate per minute, this is calculated by: $30 is charged every 15 mins

then the rate per minute = 30 ÷ 15

= $2 per minute

c) Each point on the table corresponds to a coordinate on the graph (15,150); (30,180) etc. Use this information to choose an appropriate scale for your graph and plot the points.



d) The vertical intercept is (0,120) and represents the initial call out fee.

Exercises

1. For each of the relationships represented in the tables below
   i) State whether or not the relationship is linear.
   ii) For each that is linear, find the constant difference.

a)

Number of bags	0	1	2	3	4
Cost	0	3.5	7	10.5	14

b)

Number of tiles	1	2	5	9	14
Perimeter	4	6	8	10	12

c)

Number of guests	0	1	2	5	7
Cost ($)	40.00	45.50	51.00	67.50	78.50

d)

Tadpole age (days)	0	10	20	30	40
Length of tail (mm)	20	15	10	5	0

2. Blue's taxi cab company charge a flag-fall of $6.25 and $2.40 for every km travelled.
   a) Use this information to complete the table below

Km	0	1	2	3	4	5	10
Cost							

   b) Use graph paper to plot the relationship between the *cost (C)* and *kilometres travelled (n).*
   c) Identify the vertical intercept and comment on its significance in the context of this situation.
   d) Green's taxi cab company charge a flat rate of $3.50 per km travelled. Plot another line on your axes to represent Green's taxi cab company's charges. Use the information in your graph to comment on the advantages and disadvantages of using each company.

3. A young group of actors are producing their first show in a local theatre. The table below shows the linear relationship between the *profit* they will make and the *number of tickets* sold.

Number of tickets (n)	0	25	50	75	100
Profit (P)	-300	-50	200	450	700

a) Use graph paper to represent this information in a graph.
b) Calculate the cost of a single ticket. *Hint: the constant rate of change.*
c) Identify the vertical intercept and comment on its significance in this situation
d) At what point will the students break even (no loss or profit is made)?

4. Janet and Brad have a small landscaping business and have won a contract to landscape Mr Sharman's villa. They have budgeted $180 to buy some agapanthas and gardenias, costing $6.00 each, to plant in the garden beds.

a) Copy and complete the table

Number of agapanthas (x)	0	5	10	15	20	25	30
Number of gardenias (y)	30			15			0

b) Graph the information from the table on a Cartesian plane.
c) Identify the vertical intercept.
d) The constant rate of change for this relationship is -1, comment on the significance of the negative value.
e) Justify whether it is reasonable to join the points of your graph with straight line segments.

You want to know how to rhyme, then learn how to add. It's mathematics.

\- Mos Def

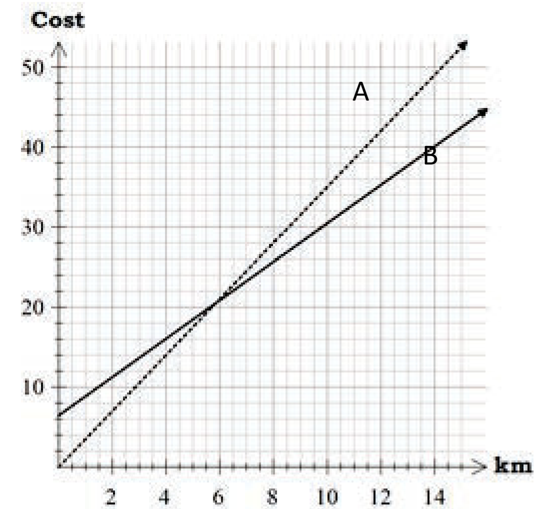
24 Linear Relationships

Materials required: graph paper

Warm-up

1.	For the set {1, 3, 5, 7, 12, 14} calculate the *mean*	2.	For the set {1, 3, 5, 7, 12, 14} calculate the *mode*
3.	For the set {1, 3, 5, 5, 12, 14} calculate the *median*	4.	For the set {1, 3, 5, 5, 12, 14} calculate the *range*
5.	120km/hour = ______m/min	6.	$\frac{2}{3}$ hour = _______minutes
7.	If 1 bag costs $4.25, 8 bags cost?	8.	If 12 boxes cost $120, then one box costs?
9.	15% as a decimal	10.	$\frac{15}{50}$ as a percentage

Opening Problem



Green's and Blue's Taxi companies have two different ways of charging their customers.

Green's charge $3.50 per km travelled.

Blue's charge $2.40 per km travelled and a $6.25 flagfall.

a) The graph on the right represents the two relationships, determine which relationship each line represents.
b) Determine the co-ordinates of the intersection of the two graphs and comment on its significance in the context of this situation.
c) Calculate the constant rate of change and vertical intercept and hence find the relationship between the variables for each situation.

Discussion
How do we use the constant rate of change and the initial value to determine the relationship between two variables?
Can you predict what a linear graph will look like if you know the relationship between the variables?
Given the graph of a function can you determine the relationship between the variables?
What information does the intersection(s) of linear graphs tell us?

Finding the relationship between variables using the vertical intercept and the constant rate of change

In a linear relationship the constant rate of change determines the gradient/slope of a graph and the initial value determines the vertical intercept.

If we know the values of the vertical intercept and gradient we can determine the rule of the relationship by using the general rule

$$\boldsymbol{y} = \mathbf{a}\boldsymbol{x} + \mathbf{b}.$$

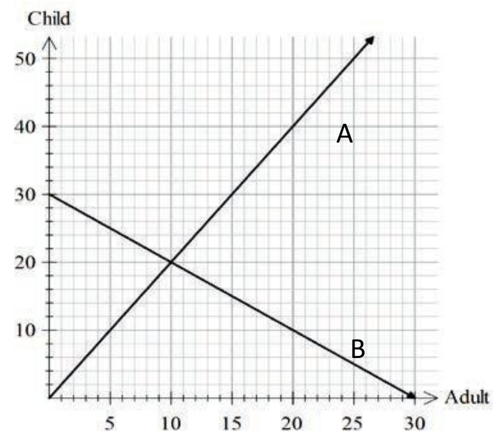
Where **a** represents the gradient/constant rate of change
and **b** is the y value of the vertical intercept.

The *intersection* of linear graphs represents the values of the variables that fit both of the relationships. For example, in the opening problem it tells us the number of kilometres travelled where both companies charge the same price.

Exercises

1. Use the constant rate of change 'a' and the vertical intercept/initial value 'b' to help find the relationship between the variables for each of the following situations from the previous exercise.
   a) Bright Spark Electrics have an initial call out of $120 and a constant rate of change of $2 per minute. Find the relationship between the *number of minutes (n)* on site and the *cost (C)*.
   b) Potting mix bags cost $3.50 each, find the relationship between the *number of bags (n)* of potting mix and the *cost (C)*.
   c) A dinner party will initially cost $40 and has a constant rate of change of $5.50 increase per guest attending. Find the relationship between the *Total Cost (C)* and the *number of guests (n)*.
   d) Tadpole tail length reduces at a constant rate of 0.5mm/day from an initial length of 20mm long. Find the relationship between *tail length (y)* and *time in days (t)*.
   e) Write a relationship between *profit (P)* and the *number of tickets (n)* sold if the vertical intercept is (0,-300) and the constant rate of change of ticket price is $10 per ticket.
   f) Write down the relationship between the *number of agapanthas (x)* and the *number of gardenias (y)* if the vertical intercept is (0,30) and the constant rate of change is -1.

2. A small puppet show is being held in a toy shop to promote a sale on Pinocchio puppets. Each free ticket allows entry for one adult and two children, and 30 people need to attend for the show to begin. The graphs below represent these two relationships between *child (c)* and *adult (a)* numbers.



a) Determine what relationship each line on the graph represents if
   i) $c = 2a$
   ii) $a + c = 30$

b) Determine the co-ordinates of the intersection of the two graphs and comment on its significance in the context of this situation.

3. A hot air balloon **A** is descending from a height of 3km at a constant rate of 5m/sec. A second balloon **B** is ascending at a constant rate of 4m/sec from the ground.

a) Convert the ascent and descent rate of each balloon to m/min.

b) Determine the variables that are used for this linear relationship.

c) Draw a table of values for each of the balloons.

d) Plot the graphs of both linear relationships on the same set of axes.

e) Use your graph to estimate how long it will take for the two balloons to be at the same height. How high will this be? Does this mean they will collide?

f) Determine how long it takes the descending balloon to touch the ground.

Extended Problem Solving – graphing y = ax + b

Use technology such as a graphing calculator or software package to help you complete this task.

1 a) On the one set of axes graph these relationships:

(i) y = x
(ii) y = x + 3
(iii) y = x + 7
(iv) y = x - 5

b) Analyse the graphs and summarise any patterns that you can see by changing the value of 'b'.

2 a) Now on another set of axes graph these relationships:

(i) y = 2x
(ii) y = 4x
(iii) y = 0.5x
(iv) y = -2x
(v) y = -4x

b) Analyse the graphs and summarise any pattern that you can see by changing the value of 'a'.

3 Now on another set of axes graph these relationships.

(i) y = 2x + 3
(ii) y = 4x + 3
(iii) y = 0.5x + 3
(iv) y = -2x + 3
(v) y = -4x + 3

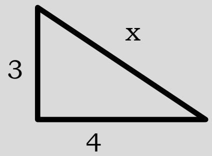
4 What can you generalise about graphs of the form $y = ax + b$?

What do you call a boiling kettle on a mountain?

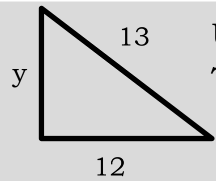
A high-pot-in-use

25 Graphs in Practical Situations

Warm-up

1. 

Use Pythagoras' Theorem to find x

2. 

Use Pythagoras' Theorem to find y

3. A closed triangular prism has ___ faces

4. A cube of length 5cm has a volume of ______cm^3

5. The origin on a Cartesian plane is represented by the point (___,___)

6. 183 cm^3 contains _____ mL

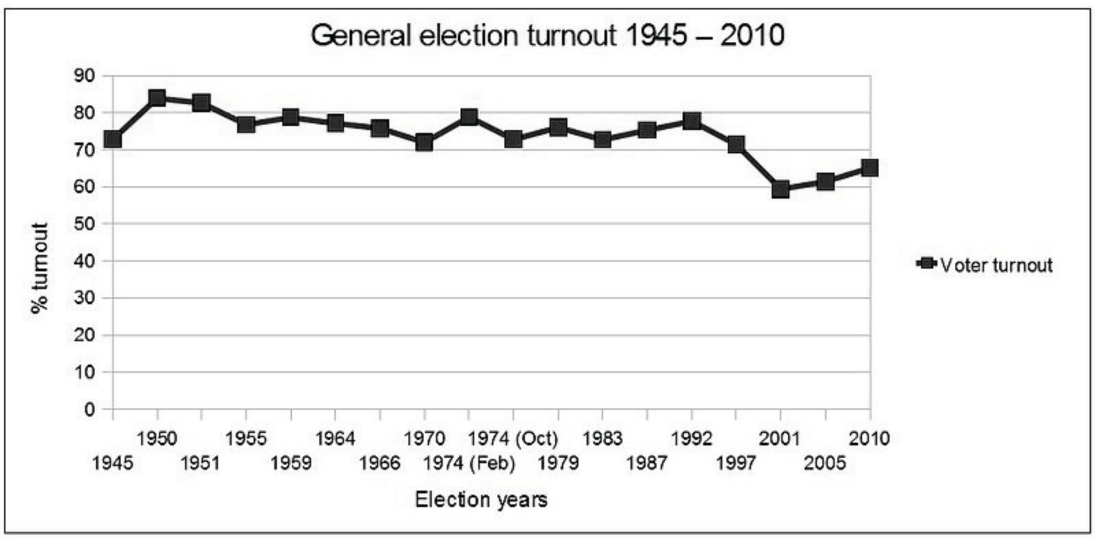
7. 6 – 0.07 =

8. -1^4 =

9. Convert 8% to a decimal

10. Increase 50 by 10%

Introductory problem: Voter turnout



https://en.wikipedia.org/wiki/File:Graph_1_UK_election_voter_turnout_1945-2010.jpg#filelinks

Use the graph above to answer the following:

a) Approximate the percentage of voters who turned up to the 2010 United Kingdom election.

b) In what year were two elections held?

c) Is it appropriate for each data point to be joined by a line? Why?

d) Locate the distortion/error in the graph and discuss reasons why the author may have chosen to distort the graph.

e) Predict how a graph of Australia's election turn-out would compare to the percentage of voters in the UK each year.

Graphs in Practice

A graph is an efficient way to display information visually. It enables the reader to interpret the *trends* in a data set and helps him/her to *summarise* the results. You should be familiar with some types of graphs such as bar charts, histograms and stem and leaf plots from your studies in Unit Two. Always be careful to read the information in graphs carefully as it may be in the author's interest to distort the information to present a particular viewpoint.

Line Graphs

A *line* graph can be used to represent the relationship between two variables in a practical situation that involves *continuous* data. Some examples of where line graphs are used in practice are given below.

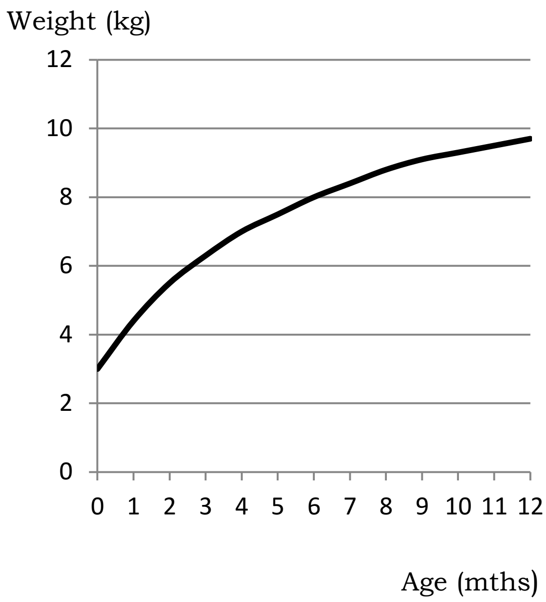
Travel graphs - time and distance
Conversion Graphs - currency conversions, e.g. $AU to $US
measurement conversions, e.g. Fahrenheit to Celsius
Time Series - change in temperature over time
monitoring heart rate

Exercises

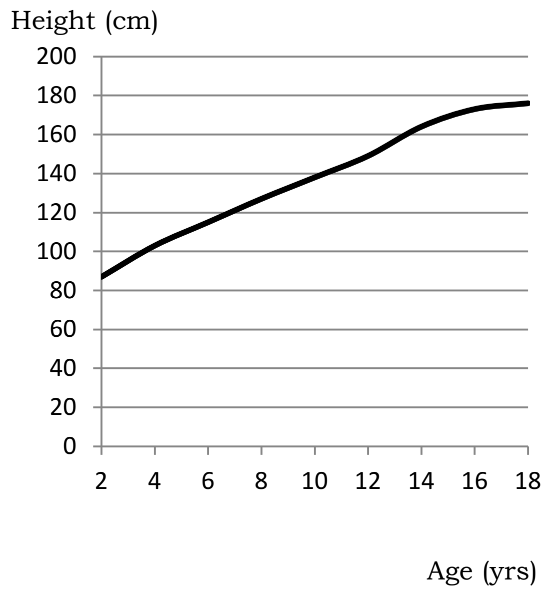
1. For each of the following graphs:
   a) identify the variables.
   b) describe in your own words the *relationship* between them.

Boys' Growth Charts

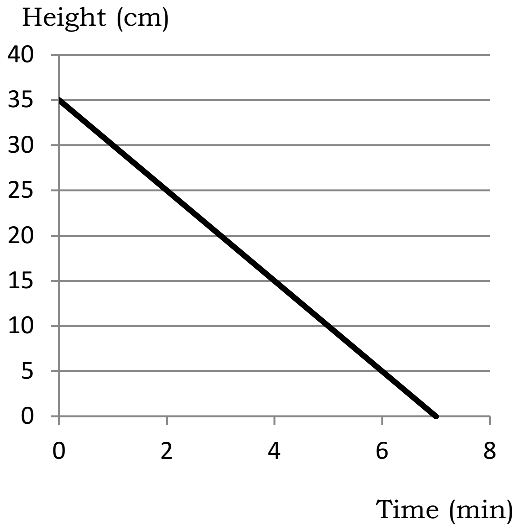
A.



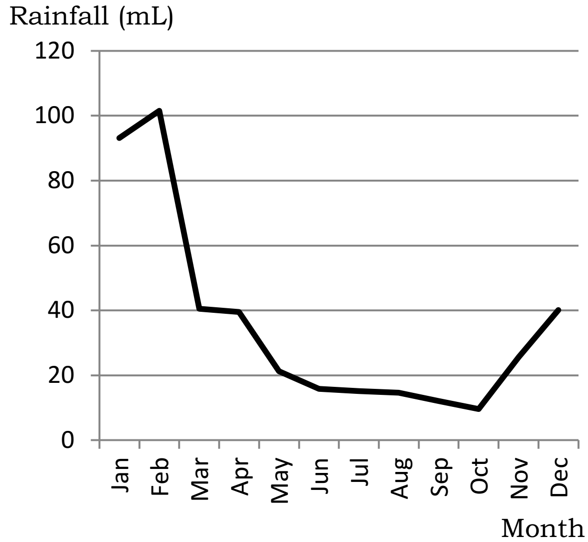
B.



C. Height of water in a jar



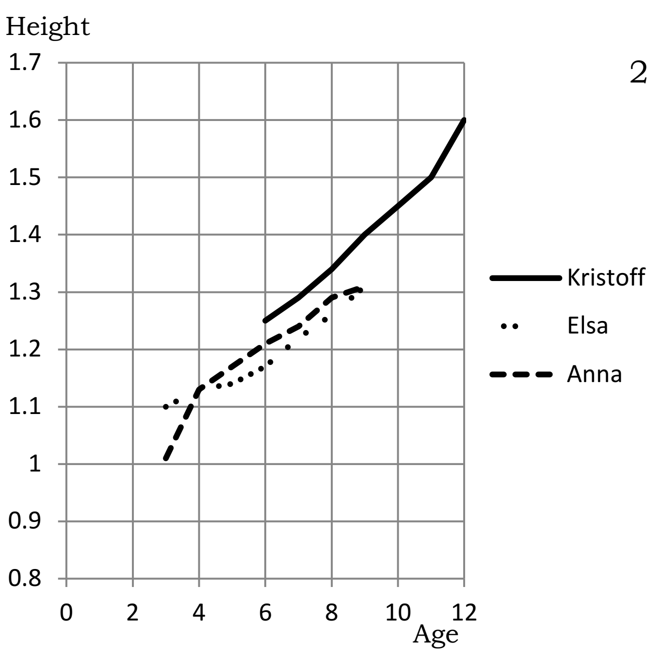
D. Monthly rainfall in Port Hedland



The steepness of a line is called the *slope* or *gradient* and gives information about the *rate of change* between the two variables. In a growth chart the graph is steeper when the height changes a lot in a given amount of time. Slope can be described as:

a) *positive* when both variables are *increasing* (**A,B**).
b) *negative* when as one variable increases the other decreases (**C**).
c) *zero* when there has been no change in the *dependent* variable.

The *independent* variable is, as the name says, independent of the other variable, for example time, and is placed along the x-axis. The *dependent* variable is the variable that is being measured, is dependent on the other and is placed on the y-axis.

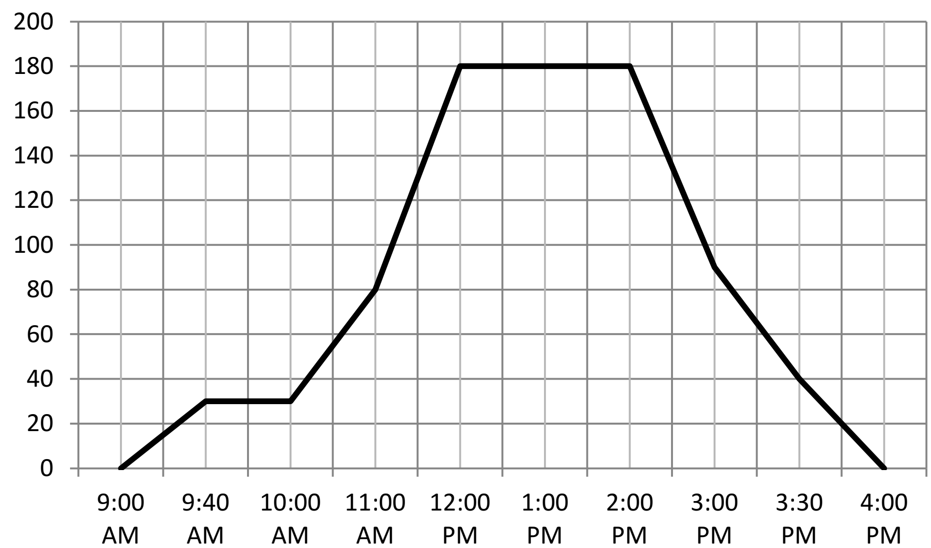


2. The growth charts of three children Kristoff, Elsa and Anna are shown left.
   a) When did Kristoff start his biggest growth spurt?
   b) At what age(s) were Anna and Elsa the same height?
   c) When did Anna have her greatest change in height?
   d) Is there any period where any of the children did not grow? Justify.

3. Match the different graphs with the occupation that uses each:

A	Party Planner	**I**	A line graph of a person's blood pressure over a period of time
B	Personal Trainer	**II**	A spanner conversion chart converting inches to millimetres
C	Nurse	**III**	A network showing the connecting information systems
D	Auto Mechanic	**IV**	A chart showing a person's age and recommended BMI
E	Computer Technician	**V**	A graph showing the number of guests and cost in $

4. Cheryl travelled from her home in Bunbury to visit some friends in Perth. A graph of her journey is shown on the distance time graph below.



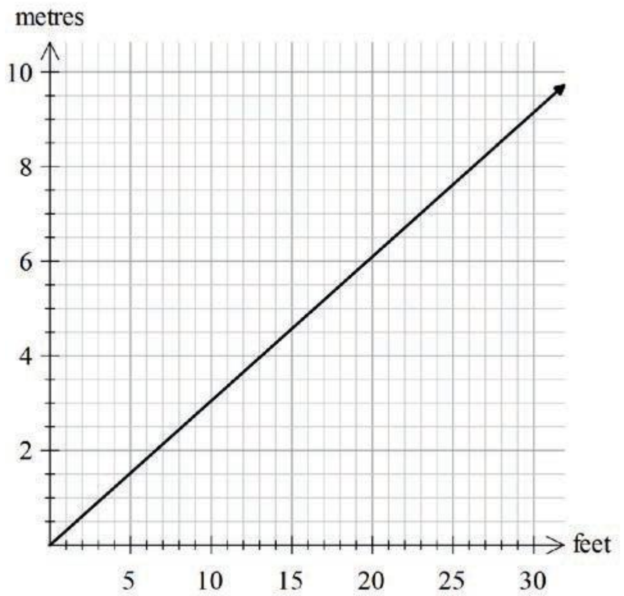
a) Identify any issues you can see with how the graph is constructed.

b) Cheryl stopped twice on her journey to fill up her petrol tank and visit friends. For which time periods did she stop?

c) By identifying the period she was travelling fastest, calculate her maximum speed during the journey.

d) Calculate the total time taken for her trip and the total distance travelled and hence calculate her average speed.

5.



Use the graph to answer the following questions.

a) In a long-jump competition, Jesse jumps 4 metres. Approximate how many feet this is.

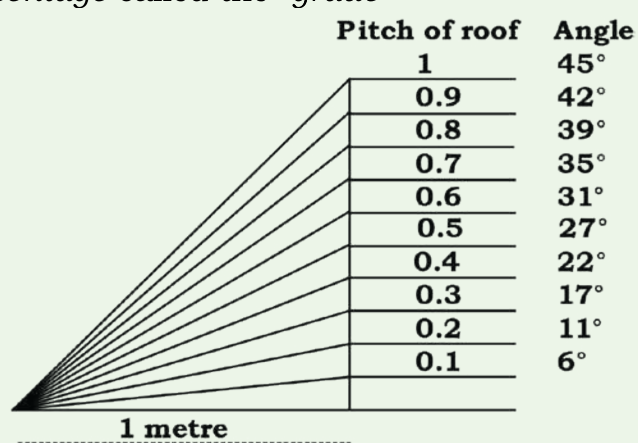
b) Which is longer, 20 feet or 6.5 metres?

c) Penelope says that 8 metres is less than 28 feet. Is she right?

Extended Problem Solving: Roof Pitch

Bicyclists, motorists, carpenters, roofers and others need to calculate slope or at least have some understanding of it. Slope, pitch or inclination can be expressed in three ways:

- As a *ratio* of the rise to the run (for example 1 to 20)
- As an *angle* (almost always in degrees)
- As a *percentage* called the "*grade*"



A slope (to the nearest degree) conversion graph used by roof carpenters is shown above.

a) Identify and verify the mathematics that was used to create this graph. Hint: use trigonometry.

b) Create a new graph that could be used by a roof carpenter to calculate the length of the rafters (hypotenuse) for a given pitch.

c) Generalise your results by communicating a formula that could be used to calculate i) slope and ii) rafter length to one decimal place.

26 Sampling Methods

Materials required: scientific calculators, complete list of Year 12 students

Warm-up

1.	$\sqrt{49}$ =	2.	$\sqrt[3]{27}$ =
3.	- 4 – 16 =	4.	½ × ¼ =
5.	10% of 1350 =	6.	20% of 1350 =
7.	3: 5 = 18: _____	8.	1800mm = ______m
9.	$16m^2$ = _________cm^2	10.	Write $\frac{6}{5}$ as a decimal

Discussion – Census

August 2016 is the date of Australia's 17th national Census. This is the first fully electronic Census managed by the Australian Bureau of Statistics.

Discuss with your class what the national Census is and why do we have one every five years. Who benefits from the information collected and how does a census differ from other data collection techniques?

Information about Census and the information collected about Australia's population can be found on the ABS website's Census Quick Stats.

http://www.abs.gov.au/websitedbs/censushome.nsf/home/data?opendocument&navpos=200

Companies, political parties, scientists and government departments need to collect information about certain issues. One method of determining this information is by conducting a *survey*.

A census is a special survey that surveys the whole *population* involved. The population is the whole group of organisms or objects to be surveyed. In most studies the population that someone is interested in is far too large to measure so a *sample* representing the population is chosen instead.

A larger manageable sample should be chosen as the results will be more representative of the population.

Exercises

1. **Random sampling** is the equivalent of putting the entire population's names in a hat and drawing out the names. One way to do this is to assign a number to each item, then use a number generator to determine which item will be selected.

   a) Number each person on your Year 12 list.
   b) Use the random number generator to select 20% or 20 students (whichever is more appropriate).

   The advantage of random sampling is that each element of the population has an equal chance of being selected. Every element of the population needs to be identified, which is not always possible.

2. **Systematic sampling** also needs the whole population to be identified but it is divided into blocks and every n^{th} member is selected. Use your Year 12 list to choose 20 students using systematic sampling.

3. **Incidental sampling** uses the most accessible and available sample by taking the most convenient set of subjects. It is the most unreliable type of sampling. List two reasons why this type of sampling method should not be used.

4. **Stratified sampling** selects a specific characteristic in the sample that is important in the research. The sample is divided into non-overlapping groups called strata, such as age groups, gender, geographical areas or political affiliation. The advantage of this sampling technique is that it ensures that a group is not missed out. Other sampling procedures are combined with this.

   Sunset Cove High School has an enrolment of 1200 students in Year 7 to Year 12. The school has 8 classes of 25 students in each year. In Year 7 there are 110 girls and 90 boys, in Year 8 there are 120 girls and 80 boys, in Year 9 there are 100 boys and 100 girls, and in Year 10 there are 110 boys and 90 girls. Year 11 and 12 have 100 boys and 100 girls each.

   Design a stratified sample that chooses 10% of the school population.

5. **Cluster sampling** may be used to reduce costs. This technique involves dividing the population into groups or *clusters*. A number of these clusters are randomly selected to represent the population. *All* units within these selected clusters are then surveyed. This is different to stratified sampling. (Why?) Examples of clusters are factories, schools and geographic areas.

   Cluster sampling might be used, for example, to determine what foods 17-year-old students in Australia are eating for lunch. It would be too expensive and take too long to survey every student, or even some students from every school. Instead 100 schools might be selected from all over Australia. These schools would be the clusters. Every 17-year-old student in those 100 schools would be surveyed.

   What are the advantages and disadvantages of cluster sampling?

Extended Problem Solving – Political Polls

In 1916 in the US the Literary Digest conducted a national survey and correctly predicted Woodrow Wilson's election. They would mail out millions of postcards and count the returns. The magazine correctly predicted a subsequent four elections using this method.

In 1936, the Literary Digest's 2.3 million "voters" was considered a huge sample, however they were generally more affluent Americans. The survey found that Alf Landon was far more popular than Roosevelt.

At the same time, George Gallop conducted a far smaller sample of only 5000 but demographically representative. Gallop successfully predicted Roosevelt's landslide victory (60% to Landon's 37%) and election polls took off globally. The Literary Digest lost credibility and subsequently merged with Time magazine.

Political opinion polls make headlines and can claim political scalps. The Australian opinion polls are dominated by six polling companies – Galaxy, Nielsen, Newspoll, Morgan, Essential Research and ReachTEL. Opinion polls are generally done over the telephone and little information about the sample sizes and dates of the survey are recorded in the media.

Your task is to investigate and compare the sampling techniques of each of the six polling companies.

See more at: http://www.thecitizen.org.au/media/opinion-polls-often-matter-opinion-say-pundits#sthash.HLT1NxNg.dpuf.

27 Surveys

Warm-up

1.	Draw a set of parallel lines	2.	Draw a set of perpendicular lines
3.	10% of 650 =	4.	1% of 650 =
5.	12% of 650 =	6.	Increase 650 by 12%
7.	Decrease 650 by 12%	8.	1200 ÷ 6000 =
9.	$6 = 2^3$ True or False	10.	Simplify the ratio 90: 60: 120

The Statistical Investigation Process

When undertaking any kind of survey, there are steps that are vital to the success of any project.

1. Clarify the purpose of the survey and clearly state the aims and objectives. Pose one or more questions that can be answered with data – what are you trying to find out?
2. Design and plan an appropriate type of survey for the study purpose, and identify the target population. Consider full enumeration (census) or a suitable form of sampling.
3. Design the questions. How carefully the questions are worded and factors such as if the responses will be anonymous, will affect the reliability of your results.
4. Collect and analyse the data by applying appropriate graphical or numerical techniques.
5. Interpret the results of this analysis and relate the interpretation to the original question.
6. Communicate your findings in a systematic and concise manner.

A Traffic Problem

Sunset Cove High School is situated on a main road. Students at the school want a pedestrian traffic crossing installed. One class decided to gather data on the traffic passing the school. They thought that they might be able to convince the city councillors that the crossing was needed. A student suggested "One of us should count the cars going by in one day." Other students made different suggestions. They decided to survey the traffic following the statistical investigation process described above.

1. Clarify the aims and objectives of this study.
2. Identify the target population.

When the class discussed what would be the most appropriate survey there were many suggestions:

More than one person should count.
The speeds should be checked.
We should count on more than one day.
We should count during school hours.
We should count on Saturday and Sunday.
Let's count people crossing the street.
Let's count people in the cars.
We don't need to count small cars or motorbikes.
We should start counting at 8:00am.
We should count traffic from only one direction.
We should count for one month.
We should count on sunny days and rainy days.
We should count trucks and bicycles separately.
We should have a trial run.
A special form should be made for recording the data.

3. Which of the suggestions are useful and appropriate? Why?

   There is no reason to collect more data than is necessary. Data collection takes time and usually people must be paid to collect it, so it becomes expensive as well. Even if the data collectors are volunteers, they do not want their free time wasted in pointless or purposeless tasks. If data collection is extended over a longer period, then there are delays in making decisions that are dependent on the results.

   On the other hand, it is important that enough data is collected. A survey is useless if at the analysis stage you discover that you must know certain things that were not measured or asked.

4. Choose the suggestions from the list in question 2 that should be used for the survey. Add any extras that you believe are necessary.

5. Design a survey form for recording the traffic data. Compare your survey form with others in the class. Decide which is the best form and why.

28 Questionnaire Design

Warm-up

1.	A 3cm cube has a volume of ___________	2.	A 3cm cube has a capacity of ___________
3.	Calculate the length of a cube with a volume of $8000cm^3$	4.	Calculate the length of a cube with a capacity of 1000mL
5.	10% of $3050	6.	Decrease $3050 by 20%
7.	A six sided polygon is called a ___________	8.	Write 65% as its simplest fraction
9.	0.2^3	10.	$\frac{1}{5}$ as a percentage

Discussion

Have you ever participated in a survey? What sort of questionnaire did the survey use? Were you offered an incentive to participate? How did you complete the survey? How long did it take? Were there questions that were difficult to answer?

After clarifying the objective of a survey and identifying the target population, it is then necessary to consider the *questionnaire design*. How carefully questions are worded will affect the reliability of the results. The time taken to process responses should also be considered. The aim of a survey should be to collect accurate data. To do this the questionnaire should enable respondents to complete it accurately within a reasonable time, use readily understood language, and be easily processed.

Exercises

Vague words should be avoided in questionnaire design. Vague words do not have definite meanings. Because people will interpret vague words and vague questions differently, results of surveys containing these words and questions will not necessarily be valid.

1. Sort the following words into two lists, a precise list and a vague list.

easy	never	many	good	several
twice	exactly	seldom	poor	sometimes
none	fair	few	two	half
maybe	triple	usually	much	

Good questions for a survey do not show bias by leading the person to a particular answer. The questions in a survey should use neutral language. Descriptive words that could exaggerate the scenario should be avoided as it could also lead to a particular answer.

A good question should be straightforward and avoid double meanings. Closed questions are preferred over open ended questions as they simplify the processing of the data and vague answers are avoided.

2. Choose the better question in each pair:

a) Do you do much homework?
How many hours per week do you spend on homework?

b) Do you agree with most people that girls are less athletic than boys?
Do you think girls are less athletic than boys?

c) Should kids be allowed to ride skateboards on footpaths?
Should kids be allowed to endanger others by riding skateboards on footpaths?

d) Do you like the fantastic music of Taylor Swift?
Do you enjoy the music of Taylor Swift?

e) Because of the increase in cyber bullying, do you think children spend too much time on social media?
Do you think children spend too much time on social media?

f) Should our water supply be fluoridated?
Should our water supply be contaminated with chemicals like fluoride?

g) Should we permit the destruction of the beautiful and irreplaceable old growth forest by the selfish logging industry?
Should we allow ignorant and tree hugging “greenies” to destroy the jobs of those rightfully employed in the logging industry?

h) Should inexperienced teenagers of 17 be allowed to vote?
Should the voting age be lowered to 17?

Surveys provide a snapshot of the demographics, characteristics, attitudes and opinions of those participants who have completed the questionnaire. When designing a survey it is important that you know when you are asking a question of *fact* or a question of *opinion*.

3. Read each question below taken from the *CensusAtSchools* 2014 questionnaire and decide if the answer is a fact(F) or opinion(O).

   a) When were you born?
   b) Are you of Aboriginal or Torres Strait Islander origin?
   c) What is your arm span?
   d) What is your favourite type of take-away food?
   e) What did you eat for breakfast this morning?
   f) In what sport or activity do you most enjoy participating?
   g) What is your favourite type of music?
   h) How do you usually spend your time on the internet?
   i) What actions do you take in your home to conserve the environment in Australia?
   j) Use the sliding scale to decide how important reducing pollution is to you.

4. Looking over your answers to Question 3, is there any way of knowing whether the question is asking for a fact or opinion?

You must be careful to consider issues of privacy and ethics when designing survey questions so that you do not infringe on the rights of others. You must be very cautious about asking questions about illegal activity, for example shop lifting or using illegal drugs. Questions that could be embarrassing or offensive such as income or health details need to be worded carefully so that accurate data is collected. Do not ask questions that are prejudiced against a particular group, either in their wording or content.

5. Criticize and, where possible, rewrite each of the following questions so they would be suitable for a survey.

   a) Are you popular at school?
   b) Have you ever cheated on a test?
   c) Do you exercise to lose weight?
   d) Do you wear glasses or contact lenses?
   e) Are you trustworthy?
   f) Is it ever okay to lie?
   g) How often do you eat take-away meals?
   h) Is your family wealthy?

The sequencing (order) of questions in a survey is a very important factor to consider when designing a questionnaire. Questions ordered in a particular way can create bias in itself. Questions should be grouped together in logical similar sections.

6. Reorder the following questions in a meaningful and appropriate sequence for a questionnaire.
   A. On a typical day, are you likely to use a social networking site?
   B. On a typical day, how many hours of homework are you likely to do?
   C. On a typical day, how many hours do you spend on a social networking site?
   D. On a typical day, are you likely to do homework?
   E. What is your date of birth?
   F. Are you a member of Facebook?
   G. Are you male or female?
   H. Are you able to access the internet from home?
   I. If you could use only one social networking site, which one would you use?

Extended Problem Solving

The opinions of Australians on issues such as climate change, health care and education are regularly reported and debated in the media. The questions in surveys can be presented in a way that encourages a response that supports the agenda of a specific group. Advertising surveys are carefully designed to make the company product look good while making the competition look bad.

For example, two politicians want to survey voters about their attitudes to a school funding bill:

Politician A
"Should we invest more money in our children's future by passing the school funding bill?"

Politician B
"Should we raise taxes to fund more and bigger government bureaucracy by passing the school funding bill?"

Politician A wants voters to support the bill while Politician B wants the bill to be rejected.

Create a pair of questions for two alternative groups on a particular issue. Write the questions in a way that they contain bias by leading the respondents to answer in a particular way.

For example
- Australian Cat Association and Australian Wildlife Conservancy
- A school's student council and P&C association
- Microsoft Corporation and Apple Inc.

29 Survey Response Design

Warm-up

1.	Find the mean of {1, 3, 4, 4, 8}	2.	10% of 65 =
3.	1% of 65 =	4.	12% of 65 =
5.	114 – 3 × 8 =	6.	12 × (0.1 + 0.3) =
7.	3^3=	8.	$\sqrt{121}$=
9.	$\frac{1}{7}$ of 280	10.	$\frac{4}{7}$ of 280

Electronic Surveys

Copy and complete the SWOT Analysis chart to consider if *electronic surveys through the web and email are superior to traditional mail and telephone surveys.*

Strengths	**Weaknesses**
• Reduces postage and telephone costs • •	• People not comfortable with technology will not respond • •
Opportunities	**Threats**
• Can target a specific group through company emails • •	• Privacy issues from emails and by tracing IP address • •

Once it has been decided on the type of survey, an appropriate response format needs to be considered for the questionnaire. As data is often summarized using spread sheeting software, the processing of data needs to be taken into consideration in response design. Possible styles of responses are:

Open ended questions allow the respondent to answer the question in their own words. The advantages are they allow many possible answers, however they are more demanding to process. For example, *What is your attitude to the new development?*

Closed questions provide the respondent with a range of the most likely answers to choose from:

- **binary**, where there are only two possible answers for example *yes/no* or *agree/disagree*
- **scaled**, where the answers are placed on a number line for example *"I enjoy playing computer games"*

1	2	3	4	5
Strongly disagree	Disagree	Undecided	Agree	Strongly agree

- **categorical**, where the answer is chosen from a fixed set, for example:
  What is the total income of your household per year?
  a 0 - $19 999
  b $20 000 - $39 999
  c $40 000 - $59 999
  d $60 000 or more

- **checklist**, where more than one response is valid, for example:
  Which of the following modes of transport have you used in the last week?
  A train
  B bus
  C bike

Exercises

1. Comment on the errors in these categorical questions.

   a) *What age group do you fall into?*

   A. Under 20 **B**. 20 – 30 **C.** 30 – 40 **D.** 40 – 50 **E.** 50+

   b) *How many hours a day do you spend doing homework?*

   [ ] 0 to 1 hour
   [ ] 120 to 180 minutes
   [ ] 4 to 5 hours
   [ ] more than 5 hours

2. The Sunset Cove High School student council wishes to survey parents. Using a simple random sampling method, these questions were emailed to the randomly selected parents.

> 1. Write down three things that you like about Sunset Cove High School.
>   ____
>   ____
>   ____
>
> 2. What are three things you would like to see improved?
>   ____
>   ____
>   ____
>
> 3. Choose the most important item from question 2 and explain how you would make the improvement you want.
>   ____
>   ____
>   ____

a) What type of questions are these?
b) What do you believe is the purpose of the first question?
c) What are the advantages and disadvantages of this type of survey?
d) How could the results be summarised for the school council?
e) The response may be biased to some extent. What part of the survey has introduced this potential bias?
f) Question 3 may be difficult for parents to answer. In what way?

3. Participants in the Census at Schools project were asked to respond to the importance of the following issues on a scale of 1 – 1000 (1000 being most important)

A	Reducing pollution	B	Recycling our rubbish
C	Conserving water	D	Conserving old growth forests
E	Reducing energy usage (electricity, gas, oil for heating, lighting, car travel)	F	Protecting coastal/marine environments
G	Having healthy eating habits	H	Reducing Bullying in schools
I	Owning a computer	J	Access to the internet

a) What style of response does this question have?
b) How important are each of the above issues to you? Use a scale of 1 – 1000.
c) Collate the opinion rating of all of the students in your class and hence calculate the mean importance of each issue.
d) Comment on the scale used for rating the opinions of respondents. Is a smaller or larger scale preferred?
e) How could a scaled response be designed so that the respondent has to give an opinion and cannot sit in the middle of the scale?

4. An auto mechanic repair chain encourages feedback from its customers regarding its outlets and product. They use a survey form displayed on the counter that can be posted to head office (no stamp required) or customers complete an online survey.

> **Tell us about your experience with Repair Auto!**
>
> Thank you for taking the time to give us your feedback. Whether it be good or bad news, we welcome your comments. Your comments and observations about how well we're performing in our workshops are very important to us. Your feedback enables us to improve and refine our high quality standard.
>
> ***As an incentive to provide us with your customer feedback, one lucky respondent per month will win $500 cash.***

a) Does this style of introduction appeal to you? Explain.
b) Would such an introduction be used in a face to face interview?
c) In what other situations could it be used?
d) If you saw this questionnaire with this introduction, would you feel you have been given enough reasons for conducting this survey?
e) Is the opening statement clear, concise and easily understood?
f) Having read this introduction, would you complete the survey? Explain. Would your attitude be the same without the prize?
g) Remembering that the survey forms are on the counter of the workshops for customers to help themselves, can you explain how the sample may become biased?

Extended Problem Solving

In Q3 you gave your opinion on the importance of various issues. Compare the mean results of your class with other target population samples around Australia by generating a sample through Random Sampler on the Census at Schools website.

http://www.cas.abs.gov.au/cgi-local/cassampler.pl

30 Sources of Bias

Materials required: tape measures, graph paper, rulers

Warm-up

1.	Volume of a sphere =	2.	Volume of a prism =
3.	Area of a circle =	4.	Volume of a pyramid =
5.	Surface Area of a sphere =	6.	Surface Area of a cube =
7.	A cube has a side length of 3m. What is the volume?	8.	A cube has a side length of 3m. What's the surface area?
9.	A cube has a volume of $8m^3$. What is the side length?	10.	A cube has a surface area of $24m^2$. What's the side length?

Discussion

Think about this question: What are the possible sources of bias or error in a survey? You might like to make some notes so you don't forget any of your ideas.
Now share your thoughts with the person next to you – share your ideas with each other and generate a list that you both agree with.
Now share your list with the class and together come to an agreed list of possible sources of bias.

Bias is found in the structure, process or analysis of surveys and can lead to a false conclusion being drawn. Sometimes it is deliberate (eg some media polls), but generally it is unintentional. How many of the following did your class identify?

Common sources of bias include:

- Researcher bias – the person conducting the research designs it on the basis of his/her own opinions and often these influence how the survey is designed and the results it generates. Even the best survey designers sometimes have their pre-conceived ideas influence the choices they make in wording or survey methodology.

- Sample bias – the target population of the survey does not reflect the full population.

  Some survey methods, such as phone polls or voluntary internet surveys, capture respondents that aren't representative of the full population. Generally only people who feel strongly about a topic will respond to a phone poll. An internet survey (particularly a single question survey) attracts people on the basis of the websites on which it is advertised. Groups with an interest in the topic may promote it to like-minded people. This voluntary involvement often leads to skewing of the sample, but many surveys assume that those who respond represent the full population.

There are several types of sampling bias. One is called the 'caveman effect' – the remnants of the earliest civilisations are found in caves. This is because paintings, fire pits or burial sites in caves are preserved much better than those in the open or on trees or animal skins. As a result some researchers concluded that early man lived predominantly in caves. It is much more likely, however, that they lived in a range of places, including in the open, but that the evidence of these other habitats has been destroyed by exposure to the elements.

See if you can find out more about other types of sampling bias.

- Response bias – where the method of surveying means that the sampled population may no longer represent the full population.

  This is similar to the population/sample bias, but a completely random process of selection for survey participation may still lead to skewing due to who chooses to respond and who doesn't. The lower the response rate, the less likely it is that your sample will represent the intended population. For this reason, many researchers will request demographic data and weight responses accordingly or only continue to source participants who meet certain criteria once the initial data is collected.

- Respondent bias – where the respondent is unwilling or unable to provide an accurate answer to the question, eg questions about illegal behaviour or of a personal nature if confidentiality is not assured or if they feel the person conducting the survey will judge them on the basis of their responses.

- Measurement error – where the measurement of a value is not the same as the true value. There are two types of measurement error, random error and systematic error.

  Random error is natural – each time the same measurement is taken, there will be slight variation in the value. This is due to accuracy of measuring devices, human differences and naturally occurring variation. Random errors can be both above and below the true value and if multiple measurements are taken, will generally average to the true value.

  Systematic errors will always have the same effect as they affect all measurements taken. This type of error might be caused by the incorrect calibration of a measuring instrument. For example, if you use a scale to measure the mass of some objects, but don't 'zero' the scale first, then all of the measurements taken will be out by the same amount because of the way the scales are set up. It can also be caused by common interference, eg if there is heavy construction work occurring outside the room in which a test is being conducted, the decreased focus of the people taking part is likely to lead to lower performance.

These types of bias or error can be demonstrated as:

- Background information that is provided within the survey that promotes a particular point of view.
- Question wording – use of emotive language or asking leading questions.
- Question type and design – for example, if your rating scale contains more positive than negative options for a question or sequencing questions that lead respondents to a particular conclusion by framing their thinking.
- Identification of a skewed sample population for surveying.
- Low response rates **may** indicate a skewed population.
- Reduction in the time allowed or number of responses sought – if early results support the conclusion being sought or expected then the researcher may prematurely end the process.
- Some sources of measurement error may not be easy to identify and prevent, but will cause the results to no longer accurately reflect the true situation.

Exercises

For the scenarios described below, identify any obvious sources of bias, describe the effect it will have on the results and where possible, suggest how the bias can be removed.

1. A television station runs a phone poll during their 11am news show and asks the question "Are unemployed people all bludgers who are living off hard working tax payers' money?"
2. A television station runs a phone poll during their evening news show as the stock market report is given, and asks the question "Are unemployed people all bludgers who are living off hard working tax payers' money?"
3. A university PhD student creates a drug for treating the common cold. She runs a trial involving 100 people, 50 who are treated with the drug and 50 with a placebo. The results of the first 20 people show that those treated with the drug had an average length of symptoms of 2 days, whereas the placebo group had an average of 5 days. The student ends the trial at this stage, confident that the drug is effective.
4. ABC Classic FM radio station conducts a SMS survey of its listeners and asks "Would you rather listen to Mozart or Taylor Swift? Text M for Mozart or T for Taylor Swift to …"

Extended Problem Solving

With a partner, use the internet to find 3 different surveys that have been conducted. For each one identify any aspects of how it was conducted that may have introduced bias and describe what effect these may have had on the results.

31 Scatter Graphs

Materials required: tape measures, graph paper, rulers

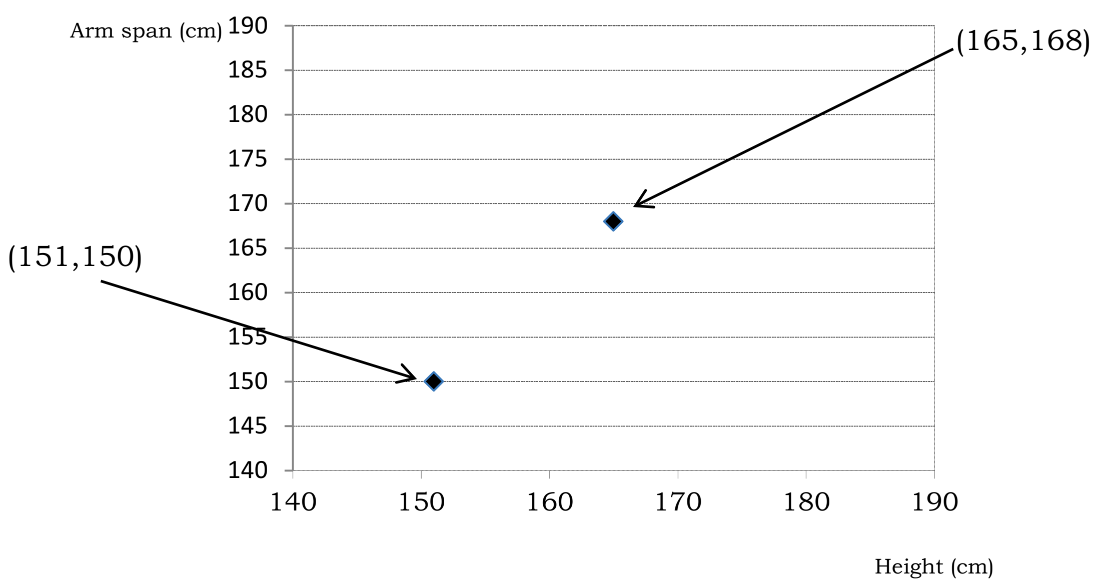
Warm-up

1.	Calculate the perimeter of a square of length 0.12cm	2.	Calculate the area of a square of length 0.12cm
3.	27, 17, 7, ____, ____, ____	4.	0.1^3 =
5.	$\sqrt[3]{81}$ =	6.	A right angle = ____°
7.	A sample of every nth person is called a ___________ sample	8.	Putting all the names in a hat is called a _________ sample
9.	27, 17, 7 …. Find the difference pattern	10.	1.3, 1.7, 2.1, …. Find the difference pattern

Activity: Body Ratios

A *scattergraph* uses Cartesian coordinates to display values for two variables in a data set. In this activity you will create a scattergraph to see if you can predict the length of a person's arm span from their height.

1. With a partner measure your height and your arm span from fingertip to fingertip. Record your results.
2. With the help of your teacher collect the measurements of all the students in your class.
3. Based on the data collected, choose an appropriate scale to use for a scatter graph.
4. Use graph paper to record each student's measurements as a point on the Cartesian plane to create a scattergraph. An example is shown below.



5. Use your scattergraph to comment on the *association* (relationship) between a person's height and the length of their arm span.
6. Draw a *trend* line through your graph. This is a straight line through the data points that approximately distributes the points equally above and below the line and reflects the relationship between the variables.
7. Use your trend line to predict the arm span of a person of height 172cm.
8. Comment on the original question posed: *can you predict the length of a person's arm span from their height?*

Bivariate Data

Often in a statistical investigation, more than one variable is needed to be collected. For example, in large health studies of populations it is common to obtain variables such as age, sex, height, weight, blood pressure, and total cholesterol about each individual. Economic studies may be interested in, among other things, personal income and years of education. Data for two variables is called bivariate data.

Association

The association between two variables is the relationship that can be described between them. The association can be classified by:

1. **form** – whether it is linear or non-linear
2. **direction** – whether it is positive or negative
3. **strength** – whether it is strong, moderate or weak

Trend line

A straight line can be fitted to a scattergraph that indicates the direction of the relationship. The line aims to go through the middle of the points such that there are an equal number of points above and below the line, and the points are as close to the line as possible. The trend line is used to make predictions.

Using the trend line to predict the value of a variable from within the given range of values, the process is called: **interpolation.**

Using the trend line to predict the value of a variable outside the given range of values, the process is called: **extrapolation**

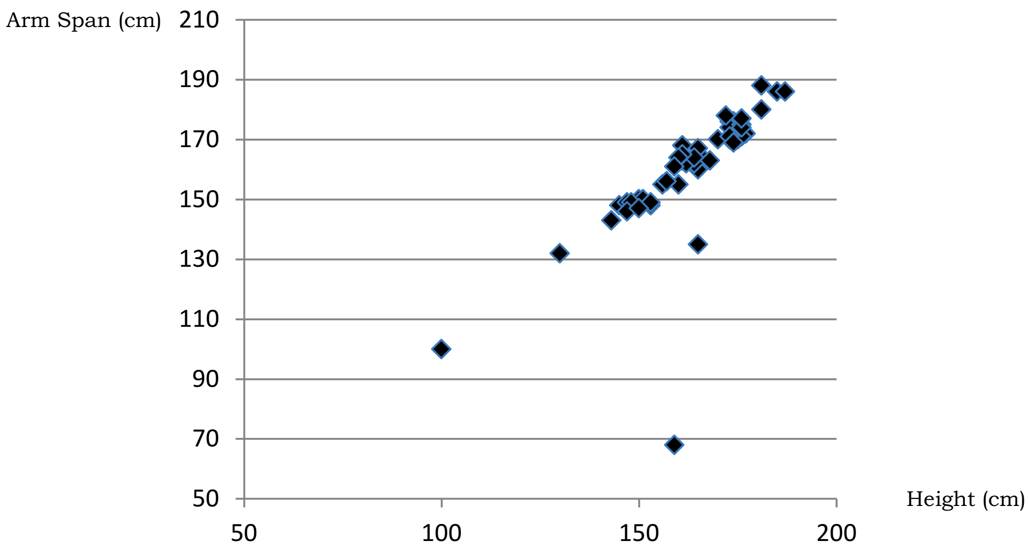
Factors affecting reliability of predictions

- The stronger the association the more accurate the prediction.
- Interpolation is generally more accurate than extrapolation.
- The number of observations (that is the number of points in the scatter plot) – generally the more observations (data points), the more reliable the prediction.

It is important to remember that association does not necessarily mean causation. Just because two variables show a relationship to each other, this does not mean that one causes the other, eg as the number of peas eaten by a person in his/her lifetime increases, so does the total number of hours they have slept. This does not mean that peas make you sleepy, nor that sleeping gives you a craving for peas. In this case, both are linked by a common factor of age.

Exercises

1. This scattergraph below shows the relationship between the arm span and height of 50 students.



   a) Using the three classifications of association, describe the relationship between the two variables.
   b) One of the data points seem unlikely, locate this point and comment on factors that could have caused this result.

2. The table below shows the hours spent doing homework and playing video games per week of 15 Year 12 students.

Hours of homework	10	1	6	2	4	8	5	8	2	5	7	10	15
Hours on video games	0	9	4	6	13	3	7	1	10	1	1	6	3

a) Choose a suitable scale and plot the data in a scatter graph.
b) Comment on the association between the variables.
c) Plot a trend line through your points.

3. The hours of paid work and the amount of income in a week for 18 students is displayed in the table.

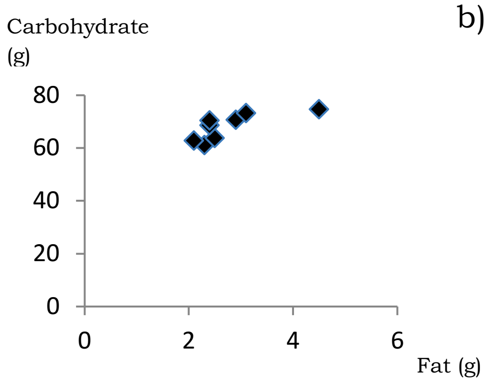
Work	8	7	12	8	4	15	14	13	10
Income	100	150	100	60	45	200	200	120	165

Work	20	9	4	10	16	15	14	11	5
Income	389	150	44	184	190	180	130	120	90

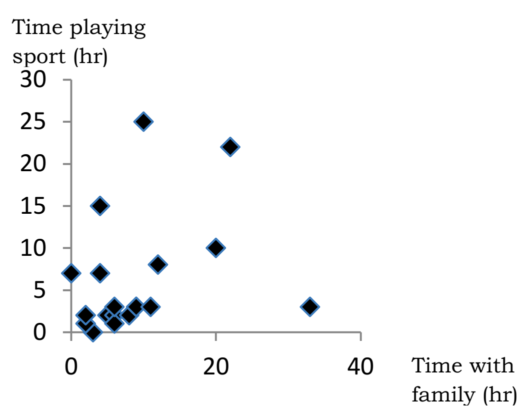
a) Use the data to draw a scatter graph of this information.
b) Comment on the association between the variables.
c) Draw a trend line through your graph.
d) Use your trend line to predict the income of a student who does 7 hours of paid work per week.
e) In full time employment, a person works approximately 38 hours per week. Use your trend line to predict the weekly pay of a person working full time.
f) Dave earned $130 from his part time job. Use your trend line to predict the hours he worked.
g) Comment on the reliability of each of your predictions.
h) Explain why the data collected from this survey could have contained bias.

4. Using the three categories of association describe the relationship between the variables in each of the scatter graphs.

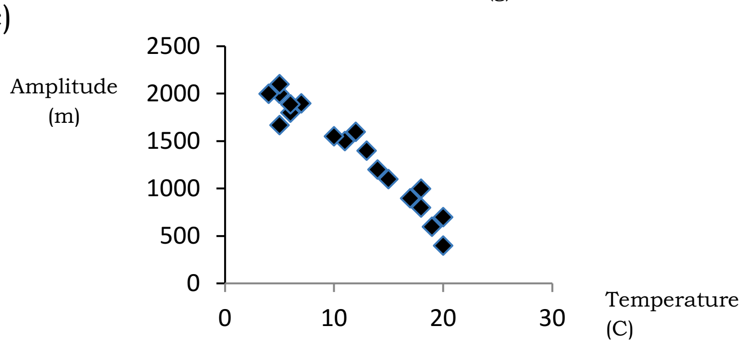
a)



b)



c)



Extended Problem Solving – Gapminder

www.gapminder.org

Gapminder is a non-profit venture promoting sustainable global development and achievement of the United Nations Millennium Development Goals by increased use and understanding of statistics and other information about social, economic and environmental development at local, national and global levels.

This website contains a link to the interactive bubble graph software that compares countries on two different social dimensions in a scatterplot. These graphs also show how the relationship between the variables changes over time, as well as showing population growth. Some example graphs are demonstrated on the site and you can change the variables to create your own graph.

Your task is to use the statistical investigation process to analyse and compare the social, economic or environmental development of a country using the Gapminder website.

1. Explore the website and pose one or more questions that could be answered with the available data.

2. Plan appropriate variables that could be collected to answer your questions.

3. Use a bubble scatter graph to analyse the association between the variables over time.

4. Interpret the results of the analysis and relate the interpretation to the original question, for example describe the trend as increasing or decreasing.

5. Communicate your findings in a systematic and concise manner, for example a URL could be created to show a short clip of your bubble scatter graph and you could record a voice over of your findings.

Unit 4

Unit 4 syllabus coverage WA curriculum

	Chapter													
Content	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Topic 4.1: Probability and relative frequencies														
4.1.1	✓	✓	✓	✓	✓	✓								
4.1.2	✓	✓	✓	✓	✓	✓								
4.1.3		✓												
4.1.4			✓	✓	✓	✓								
4.1.5			✓											
4.1.6			✓											
4.1.7				✓		✓								
4.1.8				✓		✓								
4.1.9				✓		✓								
4.1.10					✓									
4.1.11					✓	✓								
Topic 4.2: Earth geometry and time zones														
4.2.1							✓	✓	✓					
4.2.2								✓	✓					
4.2.3								✓	✓					
4.2.4										✓	✓			
4.2.5										✓	✓			
4.2.6										✓	✓			
4.2.7										✓	✓			
4.2.8										✓	✓			
4.2.9										✓	✓			
Topic 4.3: Loans and compound interest														
4.3.1												✓		
4.3.2												✓	✓	
4.3.3													✓	
4.3.4													✓	
4.3.5												✓	✓	
4.3.6													✓	
4.3.7														✓
4.3.8														✓

Unit 4 syllabus coverage ACARA curriculum

	Chapter													
Content	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Topic 1: Probability and relative frequencies														
ACMEM148	✓	✓	✓	✓	✓	✓								
ACMEM149	✓	✓	✓	✓	✓	✓								
ACMEM150		✓												
ACMEM151			✓	✓	✓	✓								
ACMEM152			✓											
ACMEM153			✓											
ACMEM154				✓		✓								
ACMEM155				✓		✓								
ACMEM156				✓		✓								
ACMEM157					✓									
ACMEM158					✓	✓								
Topic 2: Earth geometry and time zones														
ACMEM159							✓	✓	✓					
ACMEM160								✓	✓					
ACMEM161								✓	✓					
ACMEM162										✓	✓			
ACMEM163										✓	✓			
ACMEM164										✓	✓			
ACMEM165										✓	✓			
ACMEM166										✓	✓			
ACMEM167										✓	✓			
Topic 3: Loans and compound interest														
ACMEM168												✓		
ACMEM169												✓	✓	
ACMEM170													✓	
ACMEM171													✓	
ACMEM172												✓	✓	
ACMEM173													✓	
ACMEM174														✓
ACMEM175														✓

32 Probability Expressions

Materials required: Probability scale (Teacher's Guide), scissors, glue

Warm-up

1.	Express $^1/_5$ as a decimal	2.	Express 25% as a fraction
3.	$1 - \frac{3}{4} =$	4.	Express 0.05 as a percentage
5.	$\frac{1}{4} \times \frac{1}{4} =$	6.	$\frac{1}{2} \times \frac{1}{2} =$
7.	$\frac{3}{4} \times \frac{1}{2} =$	8.	$(0.5)^2 =$
9.	$(0.05)^2 =$	10.	$(0.01)^2 =$

Activity: Using a probability scale

Use the probability scale and events provided by your teacher:
a) Cut out the events from group A, and place these on the probability scale.
b) Cut out the events from group B, and place these on the probability scale.
c) Draw your own probability scale and create six events to place on the scale.

Discussion

The probability scale used in the activity went from 0 to 1. Is it possible to have a probability greater than one? Is it possible to have a probability less than 0? What type of numbers do we use to express the probability of an event?
Discuss with your class members some situations where you may have used probability to make a decision.

Probability can be expressed in a mathematical way. The numerical value that represents the likelihood of an event is called its probability.

$$\text{The probability of an event} = \frac{\text{number of ways the event can occur}}{\text{number of all possible events}}$$

where all the events are equally likely, such as rolling a fair dice.

The probability of an event cannot be less than 0 or more than 1. The value can be written as a fraction, as a decimal or as a percentage.

When the probability of an event = 0, then it is impossible.
When the probability of an event = 1, then it is certain to happen.

Exercises

1. We use words which relate to chance every day, words like: doubtful, perhaps, possibly, maybe, unlikely, inevitable. For each expression listed, place them in order from least likely to most likely.

Buckley's chance	Pigs might fly	Without a doubt
It's in the bag	Once in a blue moon	Fat chance
Maybe yes, maybe no	In your dreams	

2. A manufacturer tested 100 mousetraps. 95 traps passed the test and 5 traps did not pass. Based on these results, the probability that the next trap tested will pass the test is:

A 0.95 **B** 95% **C** $\frac{95}{100}$ **D** A, B and C

3. A deck of 52 playing cards is made up of four suits of 13 cards each: hearts, diamonds, spades and clubs. Calculate the probability, as a simplified fraction, that a card being chosen at random is:
   a) the 4 of spades
   b) a red card
   c) a heart
   d) an ace

4. A standard six-sided die is thrown. What is the probability, as a simplified fraction, that the die shows:
   a) 1?
   b) either 3 or 4?
   c) a number between 5 and 8?
   d) an odd number?

5. Tickets in a raffle are numbered from 1 to 10 000. What is the probability, as a decimal, that the winning number:
   a) is 2 658?
   b) is 7?
   c) is an odd number?
   d) is less than 1 001?
   e) is greater than 15 000?
   f) has two digits?
   g) has three digits?
   h) has five digits?

6. Lauren has three red and five blue marbles in a bag. Stefi has a bag of 10 marbles, including six red marbles. They are arguing over who has the better chance of drawing a red marble without looking, Lauren or Stefi.
   a) Calculate the probability of Lauren drawing a red marble.
   b) Calculate the probability of Stefi drawing a red marble.
   c) Choose an appropriate mathematical strategy to compare the probabilities and hence evaluate which girl has the better chance of drawing a red marble.

7. Cradle Mountain is 1 545 metres above sea level and is situated north-west of the centre of Tasmania. It is surrounded by natural rainforest and has Tasmania's highest mountain and Australia's deepest natural freshwater lake nearby. A special feature of the area and a 'must' for visitors is to walk along many of the tracks in the area. These walks can take from ½ hour to one week.

   The following statistics should be considered before trekking:
   - It rains seven out of every ten days
   - It is cloudy eight out of every ten days
   - The sun shines all day, one in every ten days
   - It snows 54 days each year

   a) Determine the probabilities of the above conditions occurring, as:
      i) a fraction
      ii) a decimal
      iii) a percentage

   b) Rank the conditions in order, from most likely to least likely.

Extended Problem Solving

Brodie really wants to win a car in a raffle. Kalem suggests that Brodie buys every ticket in the raffle so that it is certain he will win the car. Brodie really wants the car but he doesn't want to waste money.

Use the mathematical thinking process to decide if it is a good idea for Brodie to buy every ticket in the raffle to try and win the car.

If 10% of car thieves are left handed, and all polar bears are left handed, does that mean there is a 10% chance that your car will be stolen by a polar bear?

33 Simulations

Materials required: Activity worksheet (Teacher's Guide), dice, calculators, coins

Warm-up

1.	卌III is read as the number __	2.	Write 12 in tally form
3.	The probability of flipping a head on a coin is _____	4.	The probability of rolling a 4 on a normal die is ______
5.	An impossible event has a probability of _____	6.	A certain event has a probability of _____
7.	$1 - \frac{2}{3} =$	8.	$1 -$ ____ $= \frac{3}{5}$
9.	Is a probability of $\frac{1}{20}$ more or less likely than a chance of 20%?	10.	Is a probability of $\frac{3}{4}$ more or less likely than a chance of 0.36?

Class Activity I: Infection simulation

A *simulation* is a representation or model of a real world situation over time. Technology can be used to simulate situations that require large numbers of trials. Simple simulations can be done with dice, counters or a random number generator on your calculator.

In this activity you will *simulate* the spread of a virus. You will require an activity sheet and a pen to record your results.

1. Each student in your class will be allocated a number – don't tell anyone else what your number is. One person in the class has been infected by the virus, but no-one knows who.
2. When instructed, you will walk around the class mingling with the other students. When instructed, you will stop and multiply your number by the number of the person closest to you. For example, if you have a 2 and the person next to you has a 3 then 2 × 3 =6. 6 will now be the new number for both people.
3. Repeat the activity until instructed by your teacher to stop.

Your teacher will tell you who is the 'carrier' and how to identify whether or not you have been infected.

Discuss the rate of infection at each time interval. What would you expect to happen if we had continued the simulation for 100 people? 1 000 people?

What other situations could be simulated by an infection simulation?

This simulation assumed that no person was immune to the virus. Determine other factors that may need to be considered in a real world pandemic simulation.

Class Activity II: Footy Tipping Simulation

You might participate in a footy tipping competition or know someone who does. In this task you will use a simulation to predict the results of each round.

A list of the 18 AFL teams is shown below:

Adelaide	Hawthorn
Brisbane Lions	Melbourne
Carlton	North Melbourne
Collingwood	Port Adelaide
Essendon	Richmond
Fremantle	St Kilda
Geelong	Sydney Swans
Gold Coast	West Coast Eagles

Look up the most recent AFL ladder to rank the 18 AFL teams and use this to simulate the number of scoring shots each team will have.

Rank 1 – 4	Receive 20 scoring shots per game
Rank 5 – 9	Receive 18 scoring shots per game
Rank 10 – 14	Receive 16 scoring shots per game
Rank 15 – 18	Receive 14 scoring shots per game

On average each scoring shot has:

i) a 50% probability of getting a goal (6 points)
ii) a 40% probability of getting a behind (1 point)
iii) a 10% probability of getting no score (0 points)

Each student in your class needs to choose a team to represent.

Look up this week's AFL fixtures to see what teams are playing each other. Find the person who will be your opponent.

Each of you will run a simulation using a random number generator based on the statistics shown above to determine the number of goals, behinds and misses for a game. (Hint: generate random numbers between 0 and 9, 0-4 is a goal, 5-8 is a behind and 9 is a miss; if ranked 1-4, generate 20 numbers and work out the score.)

Continue these simulations to predict what teams will make it to the September finals. Make sure you adjust your simulation to the new rankings for each round.

Note: Current statistics can be found at http://www.afl.com.au/stats

Exercises

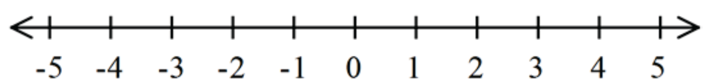
1. Josh's soccer team has lost two out of their last six soccer games. To predict the number of wins for the next ten soccer games, Josh chooses to use a dice simulation where:

   Rolling a 1, 2 = lose   *Rolling a 3, 4, 5, 6 = win*

   a) Roll a die ten times and record the number of wins and losses.
   b) To qualify for the finals, they will need to have won a total of 10 games out of 16 games. Based on your simulation, do you predict that Josh's team will make the finals?

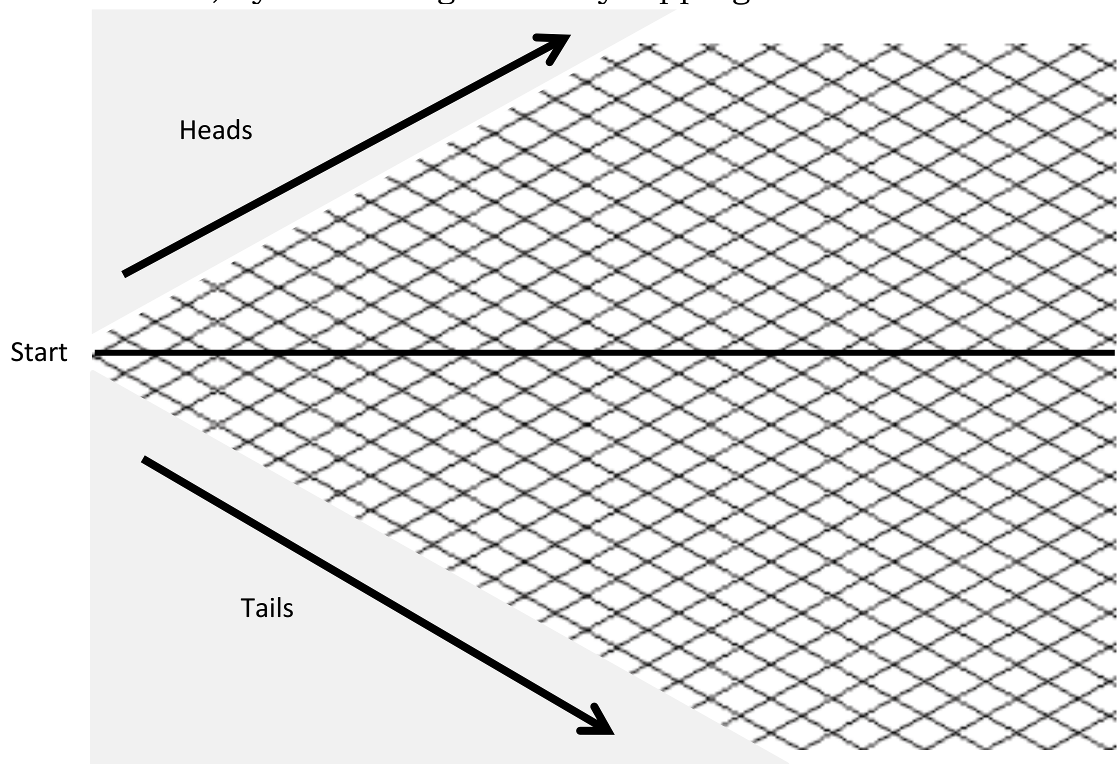
2. The Ebola virus was discovered in 1976 and became a pandemic in 2014 in West Africa.
   a) In 2014, 70% of Ebola virus cases were fatal in West Africa. Show how you could use a random number generator to simulate the number of fatalities.
   b) A community in Africa has 45 people infected with Ebola, predict the number of fatal cases based on a 70% fatality rate.
   c) A simulation takes into consideration the randomness of the situation. Use the random number generator available on your calculator to simulate the number of fatalities predicted among the infected people.
   d) Interpret your results and compare them to your partner's and to your answer to b). Comment on their validity.
   e) Is this simulation model still relevant today? What factors may cause this simulation to no longer model the Ebola fatality rate for West Africa?

3. A random walk is a simulation of a path that consists of a sequence of random steps. They can be used to simulate the path taken by a molecule, a foraging animal or the financial status of a gambler. In this simulation you will use a number line and a coin to produce a random walk.



- Flip a coin. A head is a score of 1 and you move one step in a positive direction, a tail is a score of -1 and moves one step in the negative direction. If you start at 0, where do you land?
- Repeat this process 5 times and record the final number you land on.
- Compare your results with a friend.

4. A random walk in two dimensions can be used to simulate a character moving around in a computer game or the path of a foraging animal. Repeat the activity from Q3 on the diamond grid below, by simulating moves by flipping a coin as follows.



Toss the coin.
Heads means one move up to the right, tails means one move down to the right.
Complete 50 tosses of the coin following a path through the grid.
(You can use isometric paper to draw the path)

a) How many times did you return to the middle line?
b) Of the 50 tosses, how many heads and how many tails did you get?
c) What was the longest run of the same side being tossed?

Extended Problem Solving

Computer generated random walk simulations can be found on the internet, for example a 2D random walk for 25 000 steps can be found on Wikipedia at http://upload.wikimedia.org/wikipedia/commons/c/cb/Random_walk_25000.svg

Research other computer generated random walks on the internet. How can these simulations be used to model a real life context?

34 Experimental Probability

Materials required per student: paper clip, two twenty cent pieces, four counters (3 red, 1 blue)

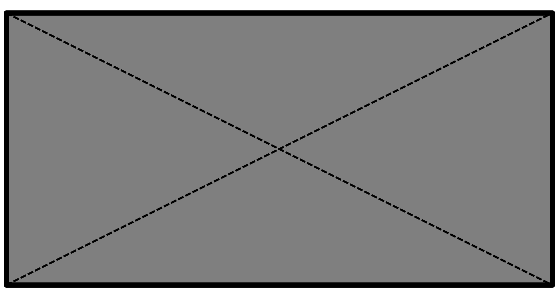
Warm-up

1.	Simplify the fraction $\frac{25}{75}$	2.	Simplify the fraction $\frac{25}{35}$
3.	20% × 1000 =	4.	0.05 × 1000 =
5.	An even chance has a probability of ______	6.	$0.75 > \frac{4}{5}$ true or false
7.	5% as a decimal	8.	2.5% as a decimal
9.	Write $\frac{4}{20}$ as a percentage	10.	Write 2.5% as a simplified fraction

Activity: Design a spinner

Not all experiments involve equally likely outcomes, for example in some games the player must throw a six on a die before beginning. It is less likely than if you throw a number from one to five, though we know it is equally likely that you will throw any *specified* number.

In this activity, you will first determine some probabilities, then design some probabilities and then design a spinner that corresponds to the same probabilities.



- Hold a paperclip about 20cm above the centre of the rectangle above. Drop the paperclip and record where it lands: completely in the rectangle, over the edge of the rectangle, or completely outside the rectangle.
- Repeat dropping the paperclip until you have 50 results. Drop the paperclip the same way each time.
- Use the recording sheet on the following page.

	In rectangle	On the edge	Outside rectangle	Total
Tally				50
Frequency				50
Percentage				100%

Use the percentages in your table to represent the probabilities that the dropped paperclip will land in, on the edge or outside the rectangle

Design a spinner that has the same probabilities for three results as your paperclip experiment.

Experimental Probability is the probability of an event occurring as demonstrated by an experiment.

$$\textit{Experimental Probability} = \frac{\textit{Number of times event occurs}}{\textit{Total number of trials}}$$

For example, if a dice is rolled 1 000 times and the number 2 occurs 176 times, the experimental probability of rolling a 2 is $\frac{176}{1000}$ = 0.176.

Exercises

1. Two red and one blue counters are placed in a bag. Without looking, two counters are taken out together. Is it more likely that the two counters are both red or that one is red and one blue?
   a) Write down which result you think will be more likely. How much more likely?
      - Put the counters behind your back and mix them up in your hands.
      - Without looking, place two counters in one hand and one counter in the other.
      - Check the colours in the hand with two counters.
      - Record S for the same colour, D for different colours.
      - Do the experiment 20 times.

   b) Complete these sentences:
   The counters were the same _______% of the time.
   The counters were different _______% of the time.

   c) The *'law of large numbers'* states that the more experiments/trials we do, the closer the results will be to the true probability. Combine your results with the rest of the class and adjust your percentages accordingly. How do these compare to your prediction in a)?

2. Three red and one blue counters are placed in a bag. Without looking, two counters are taken out together. Is it more likely that the two counters are red or one red and one blue?
   a) Write down which result you think will be more likely. How much more likely?

   b) Repeat the experiment in Q1, except using the four counters. Mix them up behind your back and put two counters in each hand. Check the colours of the ones in your right hand. Do the experiment 20 times and record the percentages.

   c) Combine the results with the rest of the class, adjust your percentages. Which result is more likely? How does this compare to your prediction in part a)?

   d) Compare your results with Q1, explain the differences.

3. Two-up is a traditional Australian game that requires two pennies (20c pieces) to be thrown into the air by a spinner. Players bet on the coins landing on heads or tails. Two-up can be played legally for money at casinos or once a year on Anzac Day.
   a) There are three possible results that can occur in a game of two up: flipping two heads, odds (a tail and a head) or two tails. Predict the probability of a single toss landing on:
      (i) two heads.
      (ii) odds (a head and a tail).
      (iii) two tails.

   b) Using two coins or a computer generated coin simulator such as https://www.random.org/coins record the results of 50 coin tosses of two-up in the table.

Result	Tally	Frequency
H, H		
odds		
T, T		
	Total	50

   c) Use your results in b) to calculate the experimental probability a single two-up toss will result in:
      i) two heads.
      ii) odds (a head and a tail).
      iii) two tails.

   d) Compare the results of your experiment to your prediction in a).

4. What is the probability that I will choose a four letter word from the dictionary (or any other book), if I choose a word at random?

   a) Choose a page from a dictionary or book and count the number of four letter words and non-four letter words. Copy and record your results in the table.

	Tally	**Frequency**
Four Letter Words		
Non four letter words		
	Total words on page	

   b) Use your results to calculate the probability that a word, chosen at random from your book, has four letters.

   c) How could this experiment be improved to make the probability as accurate as possible?

My feet are in the oven and my head is in the freezer, so on average I feel fine.

35 Probability in Games

Warm-up

1.	Write 30% as a fraction	2.	30% × 100 =
3.	$\frac{6}{9} > 0.69$ true or false	4.	$\frac{3}{75} = 0.375$ true or false
5.	$\frac{1}{10} \times 173$	6.	$\frac{\_\_}{21} = \frac{10}{14}$
7.	$1 - \frac{4}{7} =$	8.	1 – 0.23 = ______
9.	0.55 + ____ = 1	10.	Write 12.5% as a simplified fraction.

Activity: Rock Paper Scissors

Rock, paper, scissors is a game that you may have played before. Each player counts to three and then chooses a hand symbol shown right. The winning hand is declared the winner. If both players have the same move then the game is declared a tie.

Get two people to demonstrate the game to the class.

Do you think this game is fair? Explain.

We will use experimental probability to test whether this game is fair. In a fair game what results would you *expect*?

In a group of three, two of you will play 45 games. The other group member will record the results in a frequency table. You might want to share the workload and each record 15 results.

Outcome	Tally	Frequency
Player A wins		
Player B wins		
Tie		

From your experiment do you think the game is fair? Explain.

Theoretical Probability

In theoretical probability, we list all of the possible outcomes to calculate the probability of an event occurring. The set of all possible outcomes in a chance event is called the *sample space*.

The sample space can be represented as a *list* of all the possible outcomes.

For example, the sample space of

i) Rolling a six sided die is {1, 2, 3, 4, 5, 6}

ii) Flipping a coin is {H,T}

The sample space of a two-step random experiment can be represented in a *table*. The sample space of the game rock, paper, scissors is shown in the table below.

		Player A		
Player B		Rock	Paper	Scissors
	Rock	T	B	A
	Paper	A	T	B
	Scissors	B	A	T

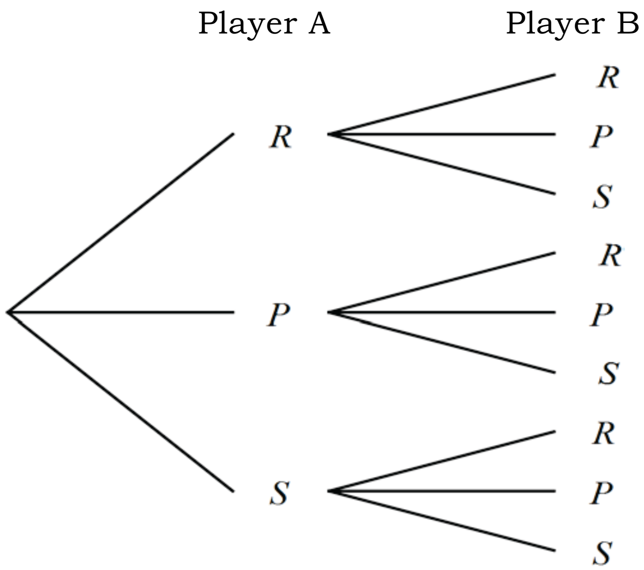
Note: A = A wins, B = B wins, T = Tie

Using the sample space shown in the table, we can evaluate the fairness of the game of rock, paper, scissors.

The probability of each outcome is:

$$\text{Probability (A wins)} = \frac{3}{9} = \frac{1}{3}$$
$$\text{Probability (B wins)} = \frac{3}{9} = \frac{1}{3}$$
$$\text{Probability (tie)} = \frac{3}{9} = \frac{1}{3}$$

Another possible way of representing the sample space is in a *tree diagram.* The tree diagram below shows the sample space of the rock paper scissors game.



R, R – Tie
R, P – B wins
R, S – A wins
P, R – A wins
P, P – Tie
P, S – B wins
S, R – B wins
S, P – A wins
S, S – Tie

Exercises

1. In the previous lesson you calculated the experimental probability of each of the outcomes in a game of two-up. The possible outcomes in the game are tossing two heads, two tails or an 'odds' (a head and a tail). The sample space of two-up has been represented in the table below.

		COIN 1	
		Heads	Tails
COIN 2	Heads	H, H	T, H
	Tails	T, H	T, T

a) Use the table to calculate the probability of
   i) Tossing two heads
   ii) Tossing two tails
   iii) Tossing odds.

b) Comment on how your theoretical probability values compare to the experimental probability in the previous lesson.

c) The sample space of the possible outcomes in a two-up game could also be represented in a tree diagram. Draw a tree diagram to show the sample space.

2. Board games such as Monopoly require players to roll two dice to move around a playing board.

   a) Copy and complete the table below to show the sample space of the sum of two dice.

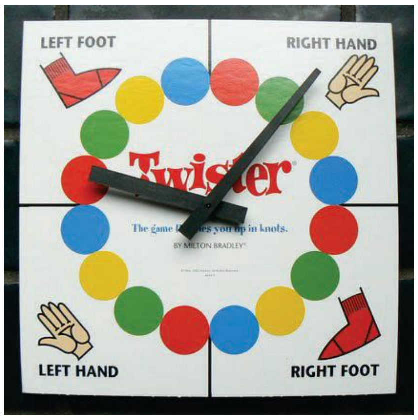
		Die 1					
	+	1	2	3	4	5	6
Die 2	1						
	2						
	3						
	4						
	5						
	6						

   b) Use the sample space to calculate the probability that in a game of Monopoly:
      i) you will roll a total of ten.
      ii) you will roll a total of one.
      iii) you will roll a double.
      iv) you will roll a total of an even number.
      v) you will roll a total greater than seven.

3. Have you ever played Twister? The spinner from a game of Twister is shown. To play, you spin both arms on the spinner. The short arm indicates the colour (red, green, yellow or blue). The long arm indicates the body part that must be placed in a circle of that colour on the playing mat.

   a) Use a table to represent the sample space of the spins in a game of Twister

   b) Calculate the probability that a spin will land on
      i) right hand on red.
      ii) a foot on blue.
      iii) red.
      iv) blue or green.



http://commons.wikimedia.org/wiki/File:Twister_dial.jpg

36 Probability Applications

Materials required: Pot Luck simulation sheet (Teacher's Guide), dice

Warm-up

1.	The probability that a person is born on a Sunday	2.	The probability that a person is born on a day starting with T
3.	¼ = _____%	4.	108% as a decimal
5.	$\frac{2}{5} \times 200 =$	6.	$\frac{1}{8} \times 24\,000 =$
7.	$12 - 3 \times 10 =$	8.	0.5^2
9.	$0.01 \times 5000 =$	10.	0.05×5000

Simulation: A Crayfishing Business

Weather is an essential factor to consider for many Australian businesses. Tourism and agriculture are two industries that rely on accurate predictions of weather for the success of their business. Meteorologists predict the weather based on experimental probability. The patterns of the weather in previous years enables temperatures, rainfall and storm patterns to be predicted.

In this activity you will simulate how much money could be made in a crayfish business that is reliant on weather conditions. A dice will be used to simulate the weather conditions and a copy of the recording sheet is required for your calculations.

- Your business will start with 10 craypots and two boats. Each boat carries a maximum of 10 craypots.
- Pots set close **IN** to shore catch small crayfish and sell for less money.
- By taking a risk and putting the pots **OUT** from shore the crayfish caught are larger and will make you more money. If the weather is bad, though, you lose your pots.
- The roll of the dice simulates the weather:

  1, 2, 3 = good weather
  4 = same as day before
  5, 6 = bad weather
- The money you earn will be determined by the weather and where you placed your pots. The table can be used to determine your income.

	Good	**Bad**
In	\$2	\$6
Out	\$4	Lose pots

- As you make money you can buy new pots for \$5 each and new boats for \$100.

How much money can you make? Can you make a million dollars?

Real Life Applications of Probability

Beside the weather there are many other applications of probability that we use in daily life. A few of these have been listed below:

Sports – be it basketball or football, the probability of a team winning is determined based on past performances. The Melbourne Cup, held on the first Tuesday in November, is known as the race that stops a nation. Large amounts of money are on offer and everyday Australians gamble money based on odds set by bookies or betting companies.

Board Games – as seen in the previous lesson, many games rely on probability. The mathematical methods of probability arose in the letters written between the mathematicians Blaise Pascal and Pierre de Fermat on questions about dice games of chance in 1654.

Medical Decisions – When a patient is advised to undergo surgery, they often want to know the success rate of the operation, which is given as a probability. Based on this information, the patient makes a decision whether or not to go ahead with the operation. The Therapeutic Goods Administration considers probabilities when considering whether the benefits of a medication are higher than the risk of side effects, when considering approving them for public use.

Social Trends and Issues – based on data collected from the same groups of people, the likelihood of events can be calculated. Financial advisors, community groups and governments use these figures to make decisions about future population needs.

We use mathematical probabilities in everyday life. Subconsciously we use it in every step that we take, when we decide what we will wear for the day or the route we will travel. The use of real life probability is an integral part of our world.

Exercises

1. List three real life situations where you use probability.

2. The different types of pizzas available at a restaurant can be represented in a tree diagram. The base can be thin or deep pan, it can have tomato or bbq sauce and the topping can be supreme, vegetarian or meat. Use a tree diagram to find how many different styles of pizza can be made.

3. A sociologist, studying the characteristics of families, wants to calculate the likelihood of different gender distributions.

   a) Assuming that it is equally likely that a child born in a family could be a girl or a boy, draw a tree diagram to represent the sample space of three children being born.

   b) Hence calculate the likelihood (probability) that in a family with three children:
      i) there are three boys
      ii) the oldest child is a boy
      iii) at least two children are girls
      iv) the youngest child is a girl

   c) *The Jones family had three boys in a row. The fourth child is more likely to be a girl.* Comment on the accuracy of this statement.

4. A motorist has to travel through two sets of traffic lights to get to work each day. Assuming it is equally likely that each traffic light is red, amber or green:

   a) Draw a tree diagram to represent this situation.

   b) Hence calculate the probability that on the way to work tomorrow, there will be:
      i) a green light.
      ii) one red light and one green.
      iii) an amber light <u>then</u> a red.

   c) Is this simulation model realistic? Comment on the original assumption and how this could influence the results.

5. When the students challenge the staff to a game of volleyball, the staff generally win two out of every three games.

   a) Show the possible outcomes for the next game on a tree diagram. Make sure the branches reflect the probabilities.
   b) They decide to play two games over the next two weeks. Extend your tree diagram to show the results for two games.
   c) Calculate the probability that the students will win at least one game.
   d) Could you use a table to show the same sample space including the probabilities? How?

When punters bet on a horserace, they need to understand how to interpret the odds. The odds are given as a ratio

chance against : *chance for*

Hence a horse with the odds 2:1 has a probability of winning 1 race for every two races that he loses.

So the probability of a win = $\frac{1}{3}$ and the probability of a loss = $\frac{2}{3}$.

If the horse competed in three races he is likely to win once and lose two times.

6. The Melbourne Cup odds are affected by each horse's performance in races such as the Caulfield Cup, Cox Plate and Victoria Derby earlier in the Melbourne spring racing carnival. Because so many horses come from overseas, their runs in other countries will also affect their Melbourne Cup odds. The odds of the horses who have won the Melbourne Cup at Flemington Raceway from 1999 – 2013 are listed below.

Year	**Horse**	**Odds**	**Probability (win)**	**Rank**
2013	Fiorente	6:1		
2012	Green Moon	19:1		
2011	Dunaden	17:2		
2010	Americain	12:1		
2009	Shocking	9:1		
2008	Viewed	40:1		
2007	Efficient	16:1		
2006	Delta Blues	17:1		
2005	Makybe Diva	17:5		
2004	Makybe Diva	13:5		
2003	Makybe Diva	7:1		
2002	Media Puzzle	11:2		
2001	Ethereal	9:1		
2000	Brew	14:1		
1999	Rogan Josh	5:1		

a) Convert the odds to a probability of a win, to 3 decimal places, for each of the winning horses.
b) Hence, rank the horses as the most likely (1) to least likely (15) to win the Cup based on their pre-race odds.

If the odds for a horse are 3:1, you can work out how much needs be wagered to win $60 if the horse wins:

Note that 3:1 means the probability of winning is $\frac{1}{4}$.

At 3:1, each $1 bet results in $3 being won if the horse wins. To win $60 you must risk losing $20.

7. How much did I win, if I bet $100 on a horse that won at

   a) 4:1?

   b) 10:1?

Extended Problem Solving – A traffic light spinner

Equipment: a pin and a spinner

At a road intersection, a minor road meets a major highway. Many cars use both roads, but in peak times, the traffic is much heavier on the major highway. The traffic lights are adjusted to recognise the greater use of the major highway by allowing traffic to flow for 1½ minutes on the highway and only 20 seconds on the minor road. The traffic lights allow five seconds for the change-over.

This means that for the cars on the major highway, the traffic lights show green for 1½ minutes, amber for five seconds and then red for 25 seconds. This cycle is repeated every two minutes.

For each car on the highway, the probability that it will have a green, amber, or red light at the intersection, is proportional to the times in the cycle.

1. Design a spinner that has the same probabilities of getting green, amber or red as a car travelling along the major highway. You do not have to make it exactly correct, but be as accurate as you can.
2. Connect your spinner using the pin and check that it spins freely. Adjust if necessary.
3. Spin the spinner 60 times and record your results for each spin.

	Green	**Amber**	**Red**	**Total**
Tally				
Frequency				
Percentage				

4. Compare your results with the students near you.
5. Calculate the theoretical probabilities for each colour light. How close are your results to the theoretical results?

37 Expected Values

Warm-up

1.	Simplify $\frac{35}{77}$	2.	Simplify $\frac{8}{24}$
3.	10% of 4 000	4.	1% of 4 000
5.	12% of 4 000	6.	Increase 4 000 by 12%
7.	12% as a decimal	8.	112% as a decimal
9.	$(-1)^3$	10.	$(0.4)^3$

Discussion

"In theoretical probability, the probability of rolling a six on a die is 1 out of 6. So if I throw the die six times I should expect to get one six."

Discuss this statement…

An **expectation** is a number or quantity that can be expected as an outcome. To find the expectation, multiply the number of trials by the probability of the event.

- Find the number of trials.
- Find the probability.
- Multiply the number of trials by the probability.

Example 1:

In a game of 120 two-up spins, how many times would you *expect* the spinner to win by tossing two heads?

Number trials = 120 P (H,H) = $\frac{1}{4}$

Expected value $= \frac{1}{4} \times 120$

$= 30$

You would expect to spin 30 double-heads out of 120 tosses.

Example 2:

To raise money and gain public awareness of their activities, a *Jane Goodall Roots and Shoots* club holds a raffle. In previous years they find that 1 out of 20 ticket buyers become new members of their mailing list. How many additions to the mailing list could the club expect from the raffle?

Number trials = 1000 tickets sold, $P = \frac{1}{20}$

Expected value $= \frac{1}{20} \times 1000$

$= 50$

The Roots and Shoots club can expect 50 additions to their mailing list.

Exercises

1. A cycle touring club has 230 members. 30% of their members join the country tour ride each year. How many members can the club expect at this year's ride?

2. Mrs Simpson knows that 1 in every 7 scratch and win tickets has a prize. How many prizes can she expect if she buys 30 tickets?

3. If 32 families, each with three children, are interviewed, how many would you expect to have:
   a) 3 daughters?
   b) 2 daughters and a son?
   c) 2 daughters and the youngest a son?

4. Keisha knows that in a multiple choice test she usually gets about one-third of her answers wrong. The Biology test has 100 multiple choice questions. How many questions can Keisha expect to get right?

5. The *Roots and Shoots* club must sell at least 200 tickets to break even (cover the cost of prizes). The members find that they sell 1 ticket for every 12 people they ask. In order to break even, how many people must be asked to buy a ticket?

6. In a school with 1200 students, how many students would you expect to have a birthday
   a) on a weekend?
   b) in July?
   c) on 25th December?

7. On average, 12% of the 24 million people in Australia will catch influenza each year.
   a) How many people would you expect to catch influenza in a single year?
   b) 42% of Australians get the flu vaccine each year, and for these people, the chance of getting influenza is 0.1. How many people do you expect to have caught influenza, even though they had been given the flu vaccine?

8. On average, 20% of children suffer from asthma. In a particular town there is a concern that there is an unusually high level of asthma due to high pollution levels. A survey of 50 randomly selected children from the town found that 19 of them had asthma.
   a) Is the asthma rate in the town higher than expected? If so, by how many?
   b) What recommendations would you give the town council based on these results?

9. A drug trial to treat depression involves 1000 people. 50% are given a placebo and 50% have the new drug. The probability a person improves, who is taking the placebo, is 0.35. The probability that a person improves, who is taking the drug, is 0.6. How many people, in total, do you expect to have improved depression in the trial?

Extended Problem Solving

The prizes from Saturday lotto on the 6th June 2015 are shown below, as well as the probabilities of winning each division.

Div	**Prize**	**What you need to win**	**P (winning)**
1	$722 023	6 Winning Numbers	1 in 8,145,060
2	$14 665	5 Winning Numbers + 1 Sup	1 in 678,755
3	$1 441	5 Winning Numbers	1 in 36,690
4	$38	4 Winning Numbers	1 in 733
5	$25	3 Winning Numbers + 1 Sup	1 in 298
6	$12	1 or 2 Winning Numbers + 2 Sup	1 in 144

To calculate the expected value of a ticket, you multiply each probability by the prize amount and then total the results for each division. Verify that the expected value of the 6th June 2015 draw was $0.37.

Investigate the expected value of a ticket in another raffle or lottery. Communicate your findings using a systematic method.

38 Latitude and Longitude

Materials required: Atlas, internet access, a globe

Warm-up

1.	$^1/_3$ as a decimal	2.	0.25 as a simplified fraction
3.	$2\pi r$ is the formula for _____	4.	πr^2 is the formula for ______
5.	A right angle has _______°	6.	A revolution has ______°
7.	Simplify $\frac{45}{360}$	8.	Simplify the ratio 4:98
9.	What is the probability of a win, given the odds are 5:1 against?	10.	Express 10.26056 with 3 significant figures

Discussion: Missing Malaysian Flight 370

At 12:40am local time on Saturday, March 8, 2014, Malaysia Airlines Flight 370 left Kuala Lumpur (2.7° N, 101.7° E) en route to Beijing (40.1° N, 116.6° E).

At 1:21am, Flight 370 was observed on a radar as it passed position (6.9°N 103.6°E) in the Gulf of Thailand. Five seconds later the aircraft disappeared from radar screens. The final data said MH 370 was flying at its assigned cruise altitude of flight level 350 and was travelling at 872 km/h.

Kuala Lumpur ACC contacted Malaysia Airlines' operations centre at 2:34am, inquiring about the communication status with Flight 370, and were informed that Flight 370 was in a normal condition based on a signal download and that it was located at (14.9°N 109.3°E) (Northern Vietnam).

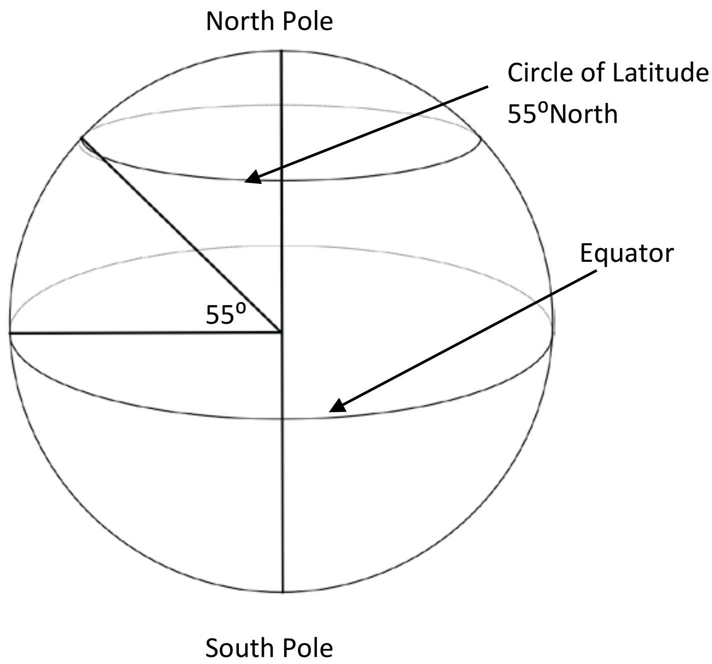
At 3:30am, Malaysia Airlines' operations centre informed Kuala Lumpur ACC that the locations it had provided earlier were "based on flight projection and not reliable for aircraft positioning".

At the writing of this book, the aircraft has not been located and the search is ongoing in the Indian Ocean, off the west coast of Perth, Australia, although debris believed to be from the plane has recently been found on Reunion Island.

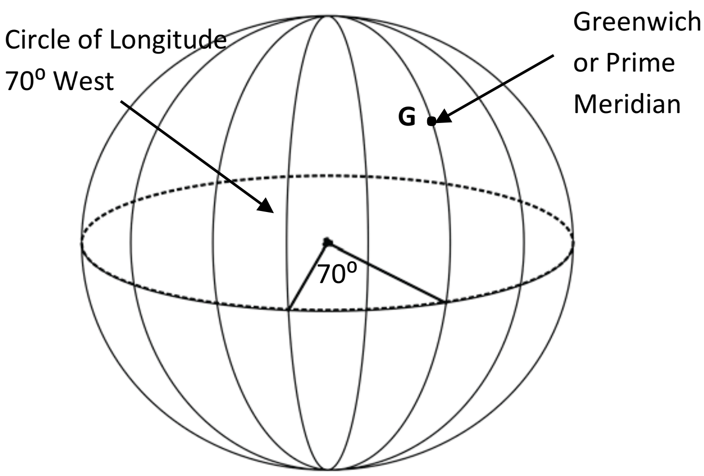
1. *Use a globe or Atlas to locate the planned and recorded flight path of MH370 before it went missing.*
2. *Discuss how the numbers next to each location can be used to find a more 'pinpointed' location of the aircraft's position.*

Latitude and Longitude

We can find any place on Earth as accurately as possible by using its *latitude* and *longitude*. We can consider lines of latitude as horizontal circles that are parallel to the equator. Lines of longitude can be considered as vertical lines running through the North and South Poles.

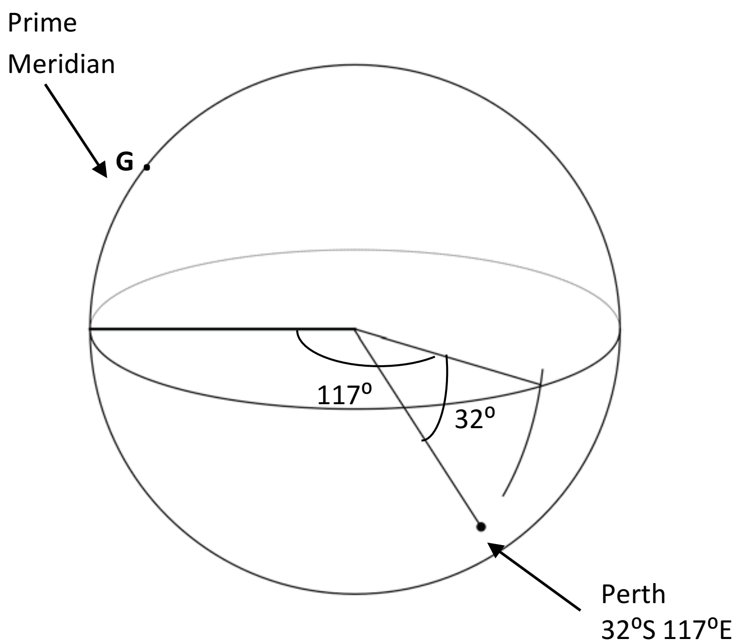


Each circle representing a *line of latitude* is given as a certain number of degrees South or North of the Equator (0°).



Each circle on a *line of longitude* is given as a certain number of degrees East or West of the *prime meridian*, which passes through Greenwich, a place near London, England.

Discuss: What are the largest and smallest values that can be given to a circle of latitude? What about a circle of longitude?



Any point on the earth's surface can be located by stating its latitude and longitude.

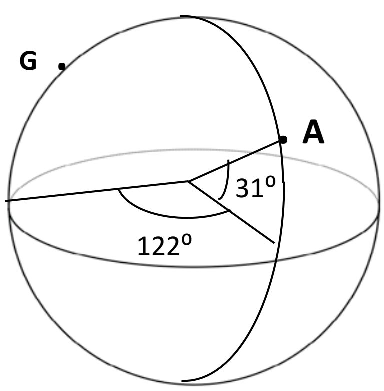
The location of the city of Perth, Australia, shown as the point on the globe has a unique location of a latitude of 32° South and a longitude of 117° East.

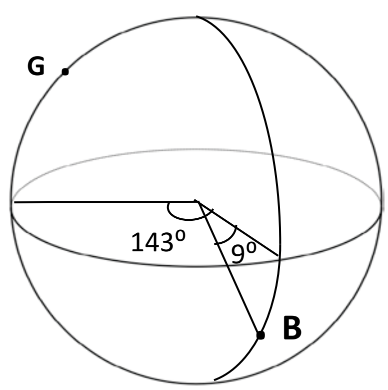
Exercises

1. Use an Atlas or Globe to locate the latitude and longitude position of the "Seven Wonders" listed below:

   a) Great Pyramid of Giza, Egypt
   b) Harbour of Rio de Janeiro, Brazil
   c) Grand Canyon, United States
   d) Great Barrier Reef, Australia
   e) Mount Everest, Tibet
   f) Machu Picchu, Peru
   g) Victoria Falls, Zambia

2. State the latitude and longitude of points A and B shown.





3. Use a global positioning system such as that used on the website www.latlong.net to calculate the latitude and longitude of the following top mathematicians' university locations.

Name	**Nationality**	**University**	**Claim to Fame**	**Latitude and Longitude**
Peter Hall	Australian	University of Melbourne	The Bootstrap method	
Manjul Bhargava	Canadian	Princeton University	Gauss Composition Laws	
Stanislav Smirnov	Russian	University of Geneva	Percolation Theory	
Ingrid Daubechies	Belgian	Duke University	Wavelets	
Robert Langlands	Canadian	Yale University	Langlands program	
Joseph B. Keller	American	New York University	Einstein-Brillouin-Keller method	
Brian D. Ripley	British	St Peter's College	Smith's Prize	
Frank Kelly	British	University of Cambridge	Loss networks	
Mikhail Gromov	Russian	Institut des Hautes Etudes Scientfiques	Geometry	
Endre Szemeredi	Hungarian	Rutgers University	Szemeredi's theorem	
Bernard Silverman	British	University of Oxford	Functional data analysis	
Wendelin Werner	French	ETH Zurich	Random walks	
Elon Lindenstrauss	Israeli	Hebrew University of Jerusalem	Ergodic theory	
Yurij Manin	Russian	Max-Plank-Institut für Mathematik	Algebraic geometry	
Andrew Wiles	British	Princeton University	Proving Fermat's Last Theorem	
Terence Tao	Australian	University of California	Gren-Tao Theorem	

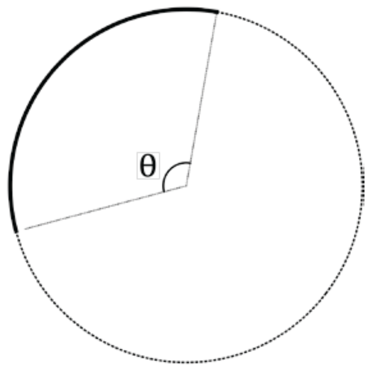
39 Arc Length

Materials required: calculators

Warm up

1.	Simplify 36:360	2.	18 – 2 × 4 + 7 =
3.	Round 34.78965 to 2 decimal places	4.	Estimate 1203 × 3900 by using leading digits
5.	0.04 × 500 =	6.	10% of $345
7.	1% of $345	8.	8% of $345
9.	A cube of length 5cm has a volume of ______	10.	A cube of length 5cm has a surface area of ________

An arc is a section of the circumference of a circle. The length of an arc can be found by calculating the ratio of the arc to the circumference of the circle.



The ratio of the *arc angle* θ to a *full angle (360°)* is the same as the ratio of the arc length, *a*, to the circle's circumference.

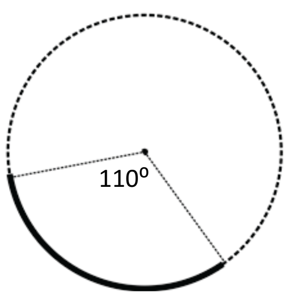
The circumference of a circle can be found using the formula $C = 2\pi r$.

The arc length can be determined by using the formula:

$$a = \frac{\theta}{360^\circ} \times 2\pi r$$

where a = arc length, θ = arc angle, r = radius of a circle

Example



Radius = 5cm

θ = 110°, r=5 — Determine θ and r

$$a = \frac{\theta}{360} \cdot 2\pi r$$

Substitute θ and r into the arc length formula

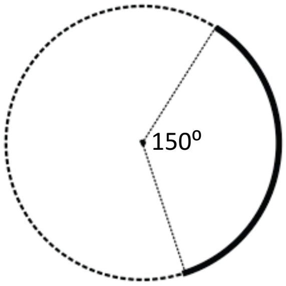
$$= \frac{110}{360} \cdot 2\pi(5)$$

$$= 9.6\text{cm}$$

Use your calculator to determine the length of the arc

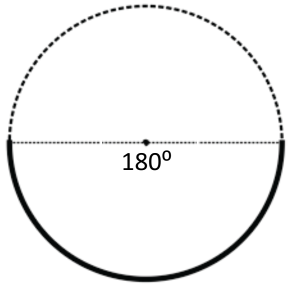
Exercises

1.



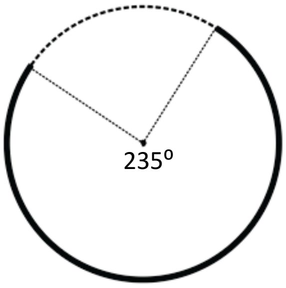
Radius = 10cm

2.



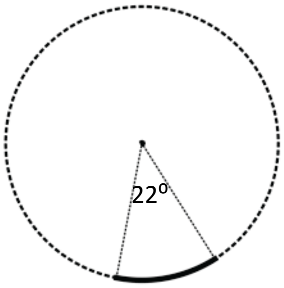
Radius = 25mm

3.



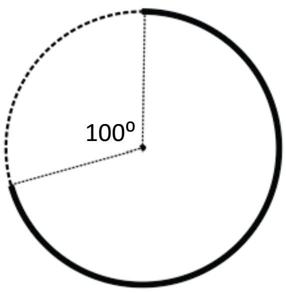
Radius = 8cm

4.



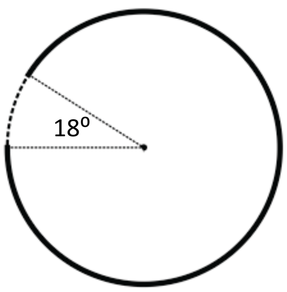
Radius = 14mm

5.



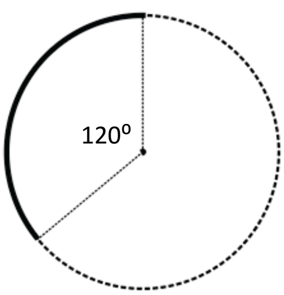
Radius = 30cm

6.



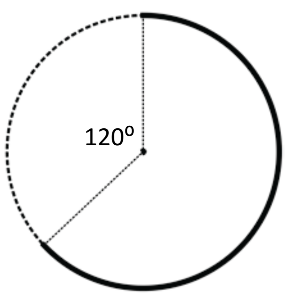
Radius = 12mm

7.



Radius = 11cm

8.



Radius = 11cm

40 Determining Distances

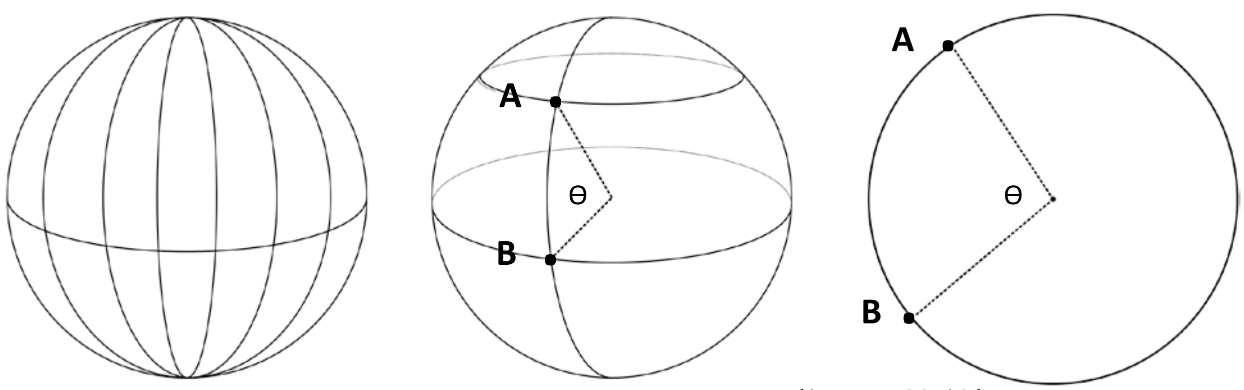
Materials required: calculators

Warm-up

1.	100^3	2.	$(0.1)^3$
3.	14cm = ____mm	4.	$14cm^2$ = _____mm^2
5.	$14cm^3$ = ____mm^3	6.	$2\pi \approx$ _____
7.	If P(X) = 20% How many X events would you expect in 3000 trials	8.	If P(Y) = 0.15 How many Y events would you expect in 60 trials
9.	100 ÷ ½ =	10.	1000 ÷ 0.25 =

Determining distances using the arc length formula

The distance between two places on Earth on the *same* longitude can be determined by using the arc length formula. The radius of the Earth is approximately 6350km and each line of longitude is a *great circle* of radius 6350km. (Note: All lines of longitude are on circles that are the same size as each other, but circles of latitude are different sizes and get smaller the further from the equator they are.)



radius = 6350km

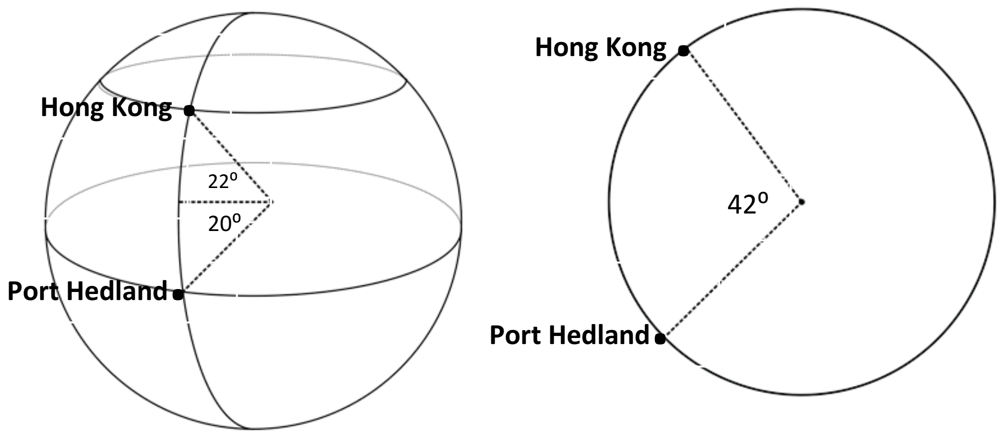
The distance between A and B can be found by:

$$distance = \frac{\theta}{360°} \times 2\pi r$$

Where r = 6350km and θ can be determined by using the latitude of the two points.

Example

Determine the distance between Hong Kong, China (22^{o}N, 115^{o}E) and Port Hedland, Australia (20^{o}S, 115^{o}E)



As one city is south and one is north we add the angles to find the angle, θ, between them.

$$\text{Distance} = \frac{\theta}{360} \cdot 2 \cdot \pi \cdot 6350$$

$$\text{Distance} = \frac{20 + 22}{360} \cdot 2 \cdot \pi \cdot 6350$$

$$= \frac{42}{360} \cdot 2 \cdot \pi \cdot 6350$$

$$= 4655 \text{ km}$$

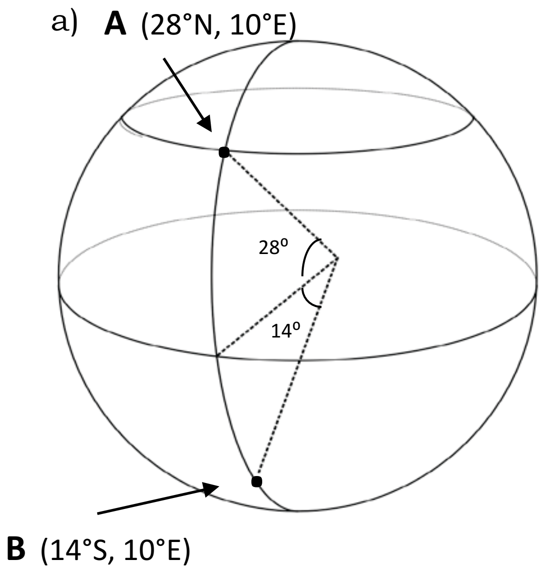
$\therefore$ distance between Hong Kong and Port Hedland is 4655 km, along the line of longitude 115^{o}E.

Note: if both locations are on the same side of the equator (both N or both S), to find θ you need to find the difference between the latitudes.

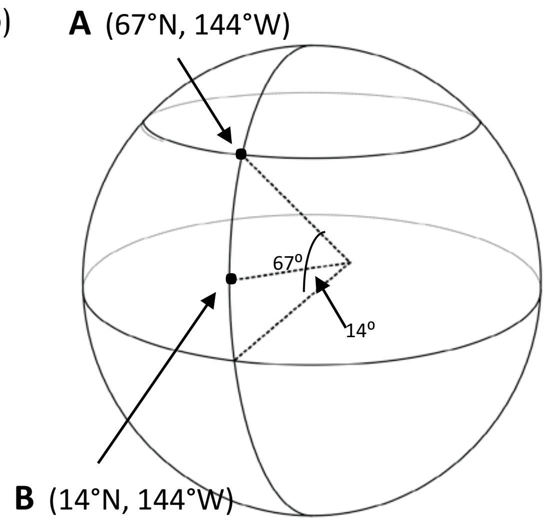
Exercises

1. Calculate the angle between points A and B that lie on the same line of longitude.

a)



b)



2. Find the distance between each of the following pairs of locations along their common circle of longitude:

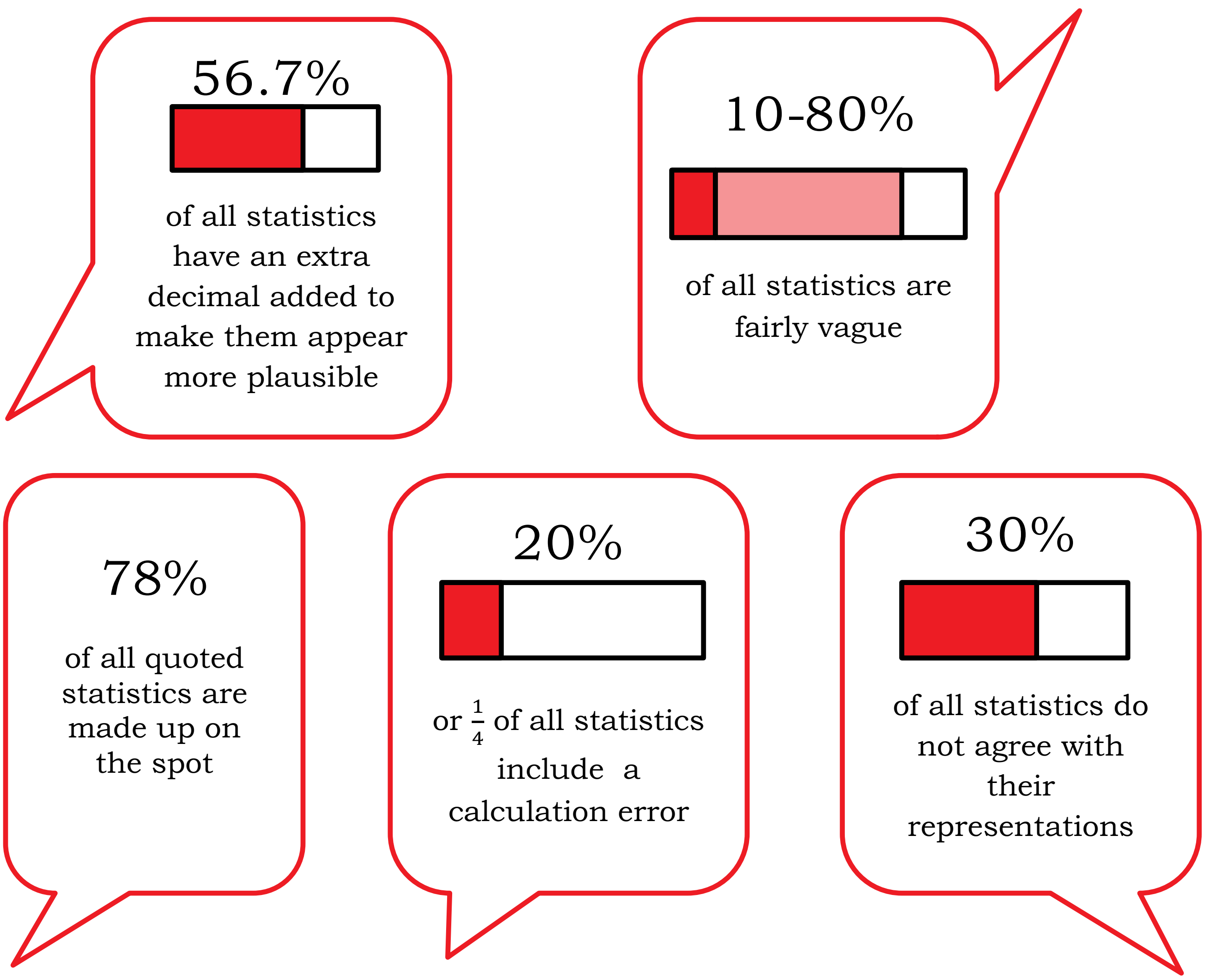
   a) Darwin, Australia (13°S,132°E) to Hiroshima, Japan (34°N,132°E)
   b) New York City, United States (40°N,74°W) and Iquitos, Peru (3°S, 74°W)
   c) Melbourne, Australia (37°S,145°E) to Cairns, Australia (16°S, 145°E)
   d) Hobart, Tasmania (42°S, 147°E) to ACT, Canberra (35°S, 147°E)
   e) Seoul South Korea (37°N, 125°E) to Manila, Philippines (14°N, 125°E)
   f) Honolulu, Hawaii (22°N, 158°W) to Barrow, Alaska (71°N, 158°W)

3. The distances between two places on Earth can also be found using technology.
   a) What sort of technology could you use to determine the distance between two points on the earth?
   b) Check your solutions to Question 2 by using technology.
   c) Compare the level of accuracy of each method used to find distances.

Extended Problem Solving –
The most remote city on earth

Perth, Western Australia is often considered the most remote city in the world. Others, however, say that in fact Auckland, New Zealand, is more remote than Perth.

1. Identify what information and factors would need to be considered to identify the most remote city in the world.
2. Choose an effective method to determine the remoteness of Perth and Auckland.
3. Determine and justify which city you consider to be the most remote.
4. Are there any other cities in the world that could also be considered the most remote in the world? Justify.



41 Time Zones 1

Materials required: A3 paper, rulers and pencils

Warm-up

1.	Convert 1am to 24 hour time	2.	Convert 7am to 24 hour time
3.	Convert 11:30am to 24 hour time	4.	Convert 9:45am to 24 hour time
5.	Convert 1pm to 24 hour time	6.	Convert 6pm to 24 hour time
7.	Convert 2:30pm to 24 hour time	8.	Convert 8:45pm to 24 hour time
9.	Convert 12pm to 24 hour time	10.	Convert 12am to 24 hour time

Introductory information and problem

A line of longitude is an imaginary great circle on the surface of the Earth passing through the North and South poles at right angles to the equator. Lines of longitude measure how far something is east or west of the Meridian Line, also referred to as the zero longitude.

The Meridian Line, also known as the Prime Meridian, is an imaginary line that runs from the North Pole to the South Pole. By international convention it runs through the Royal Observatory in Greenwich. The International Date Line is the meridian that is 180° from the Prime Meridian (the other half of its great circle). About 15 degrees of longitude is equivalent to one hour of time; there are 24 standard time zones in the world. Adelaide is one of the few non-standard time zones. Interestingly, when representing time zones, most lines of longitude weave around countries or islands rather than directly join the North and South Poles.

As more sophisticated timepieces, such as the atomic clocks, became available there was a need for a new international time standard. With this ability to measure time more precisely, UTC (Coordinated Universal Time) was born and is now used along with Greenwich Mean Time (GMT)

In Western Australia we are Greenwich Mean Time plus 8 hours (GMT+8). Use the internet or an atlas to find other countries or cities that are in the same time zone as WA.

How many time zones are there in Australia? Which states are in which time zones?

What do the abbreviations AWST, ACST, AEST, ACDT, AEDT and CXT stand for?

Why is Adelaide considered to be in a non-standard time zone?

What is daylight saving and how does it affect time zones?

Exercises

1. Given the Western Standard Times (WST) below, write the Central Standard Time (CST) and Eastern Standard Time (EST)
   a) 9am b) 1pm c) 8pm
   d) 12:00am e) 12:00pm f) 5:30pm
   g) 3:30am h) 7:30pm i) 10:30am
   j) 2:45pm k) 11:45am l) 11:30pm

2. Given the GMT+hours, approximate how many degrees the city is to the east of Greenwich.
   a) Beijing GMT+8 hours b) Auckland GMT+12 hours
   c) Tokyo GMT+9 hours d) Jakarta GMT+7 hours
   e) New Delhi GMT+$5\frac{1}{2}$ hours f) Abu Dhabi GMT+4 hours

3. Given the GMT-hours, determine how many degrees the city is to the west of Greenwich.
   a) New York GMT-5 hours b) Sao Paulo GMT-3 hours
   c) San Francisco GMT-7 hours d) Toronto GMT-8 hours
   e) Cook Island GMT-10 hours f) Reykjavik GMT-0 hours

4. Use your knowledge of Australian time zones, ignoring daylight savings, to answer the following:
   a) It is 10:30am in Perth, what is the time in Melbourne?
   b) An AFL game starts at 3:30pm in Sydney, what time is it in Adelaide?
   c) It is 10:45pm in Adelaide, what time is it in Perth?
   d) New Year's Eve fireworks start at midnight in Brisbane, what time is it in:
      (i) Hobart?
      (ii) Darwin?
      (iii) Perth?
      (iv) Auckland?

This table shows the times in various cities in relation to GMT, when daylight savings does not apply.

City	Hours from GMT	City	Hours from GMT
Amsterdam	+1	London	+0
Auckland	+12	Los Angeles	-8
Bali	+8	Mumbai	+5.30
Bangkok	+7	New York	-5
Beijing	+8	Melbourne	+10
Boston	-5	Perth	+8
Cairo	+2	Portugal	+0
Darwin	+9.30	Seoul	+9
Doha	+3	Singapore	+8
Dubai	+4	Suva	+12
Hanoi	+7	Sydney	+10
Hawaii	-11	Tokyo	+9
Hong Kong	+8	Toronto	-4
Johannesburg	+2	Vancouver	-7
Kuala Lumpur	+8	Zurich	+1

5. If it is 8am on Wednesday in Perth, state the time in the following cities:

   a) Dubai b) New York c) Portugal

   d) Hawaii e) Hanoi f) Tokyo

   g) Toronto h) Cairo i) Mumbai

6. Why can you leave Perth at 10am, fly for 3 hours and arrive in Bali at 1pm?

During the summer months many countries convert to daylight savings time. This means that the clocks are advanced an hour at the beginning of daylight savings and rewound the hour at the end of the daylight savings period.

7. Use the internet to determine:
   a) When the daylight savings period starts and finishes in Australia.
   b) Which states and/or territories of Australia use daylight savings?

8. Using daylight savings times, determine the time in each of the following places if it is November 12 at 11:20am in Adelaide.

   a) Perth b) Brisbane c) Hobart
   d) Darwin e) Melbourne f) Sydney

Extended Problem Solving

Use the internet to research time zones in Western Australia and South Australia. Research when and where daylight savings starts. Use the information from the internet and the following holiday information to provide a timeline to travel from Perth to Adelaide. You may wish to include pictures of places of interest or information about main towns.

Leave Perth by car on Wednesday 30 September 2015 at 9am and travel to Kalgoorlie. Travel time is 8 hours, including a 1hour stop in Merredin.

Spend the next day exploring the sights of Kalgoorlie, including the Superpit, Royal Flying Doctors Service Visitor Centre and Hannan's North Tourist Mine.

Leave Kalgoorlie the following day at 9am and travel 2½ hours to Norseman. Stop for 45 minutes for a late morning tea. Continue your drive to Balladonia, taking 3 hours. Stop for 15 minutes to refuel and get a take away lunch. Complete your day's travel at Caiguna at 6pm or is it 6:45pm? Congratulations you have just travelled 146.6 kilometres of straight road; it is one of the longest in the world.

You spend the night at the roadhouse caravan park. At 10am the next morning you drive 4½ hours to Eucla and spend the rest of the day exploring the telegraph station and the Great Australian Bight.

On Sunday 4 October, you leave Eucla at 9am and drive 15 minutes to the WA/SA border. You then drive a further 5 hours and arrive in Ceduna. You stay overnight but missed the whale-watching season.

At 10am the next morning you leave Ceduna and drive 135 km to Poochera, which takes 2½ hours. There is not much to see here as the town is surrounded by agriculture and is a strategic grain exchange point. Poochera is also home to one of only two colonies of the Dinosaur Ant! You do not find this particularly interesting so at 1pm you start the journey to Port Augusta, but 2 hours and 45 minutes later you stop in Kimba to see the Giant Galah. The Galah stands at the point that is half way between the west and east coasts. You arrive in Port Augusta at 6pm and wish to ring your parents in Perth. What time will it be in Perth?

You stay in Port Augusta for the following day and explore Wadlata Outback Centre and the famous pink lakes.

On Wednesday you leave Port Augusta at 11am and travel 306km to Adelaide. You arrive in Adelaide at 2:30pm ACDT.

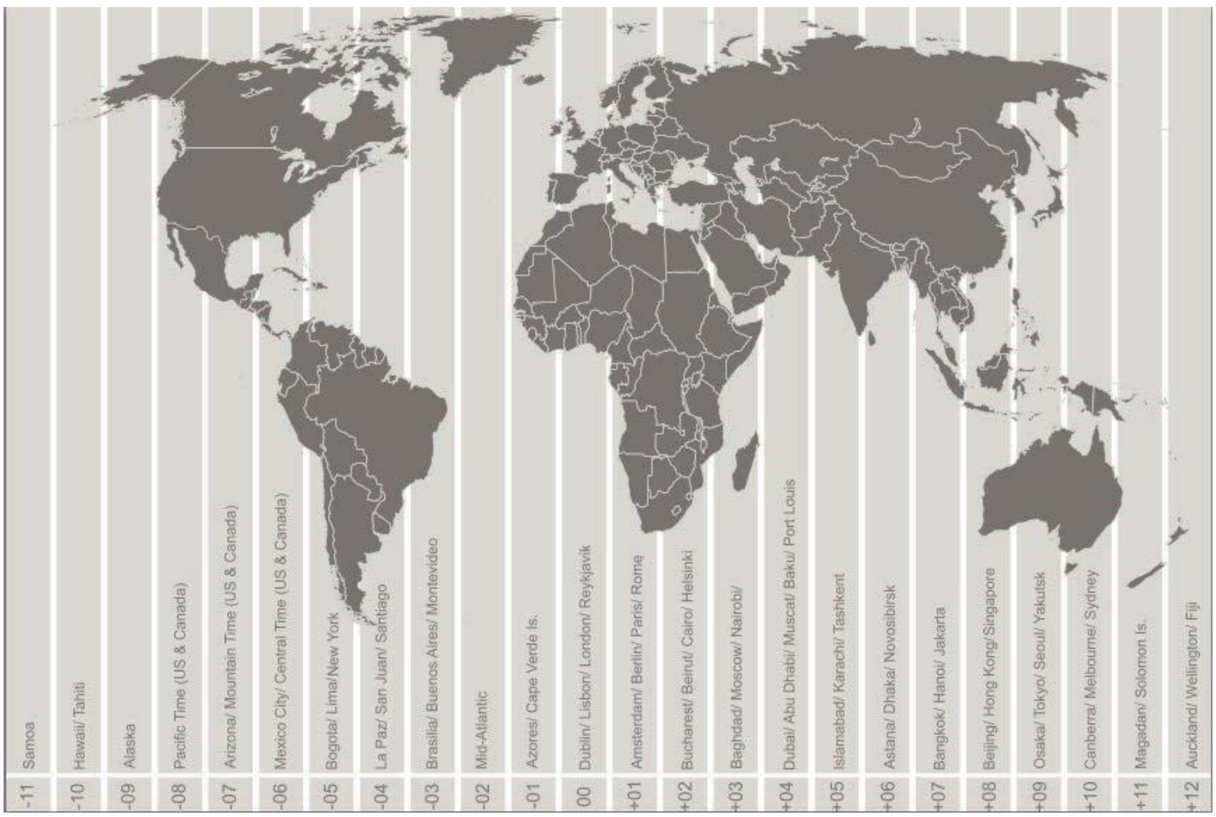
42 Time Zones 2

Materials required: calculators, clock may be helpful

Warm-up

1.	Convert 1600h to 12 hour time	2.	Convert 0800h to 12 hour time
3.	Convert 1100h to 12 hour time	4.	Convert 2000h to 12 hour time
5.	Convert 0330h to 12 hour time	6.	Convert 1730h to 12 hour time
7.	Convert 1335h to 12 hour time	8.	Convert 1950h to 12 hour time
9.	Convert 1200h to 12 hour time	10.	Convert 0000h to 12 hour time

Discussion



The world map above is a representation of the time zones of the world. There are 24 standard time zones. Use the internet to find the countries or states that use a non-standard time zone. Interestingly, we have two in Australia.

Discuss why countries, like China, chose to be in one time zone. What would be the effect if Australia operated with only one time zone?

Exercises

This table shows the times in various cities in relation to GMT.

City	Hours from GMT	City	Hours from GMT
Amsterdam	+1	London	+0
Auckland	+12	Los Angeles	-8
Bali	+8	Mumbai	+5.30
Bangkok	+7	New York	-5
Beijing	+8	Melbourne	+10
Boston	-5	Perth	+8
Cairo	+2	Porto	+0
Darwin	+9.30	Seoul	+9
Doha	+3	Singapore	+8
Dubai	+4	Suva	+12
Hanoi	+7	Sydney	+10
Hawaii	-11	Tokyo	+9
Hong Kong	+8	Toronto	-4
Johannesburg	+2	Vancouver	-7
Kuala Lumpur	+8	Zurich	+1

1. Neil lives in Perth and is a Manchester United fan and wishes to watch as many games as he can during the season. The games are televised live from England with a 12:30pm, 3:00pm or 5:30pm kick-off on Saturdays. What day and time will kick-off be in Perth?

2. Daniel loves to watch the Formula One Grand Prix. He lives in Melbourne and wants to watch the races in Australia (Melbourne), Malaysia (Kuala Lumpur), China (Beijing), Canada (Vancouver), Portugal (Porto) and London. Each race starts at 2:00pm local time. What will be the time when Daniel starts watching each race?

3. Mustafa is currently in London, his mother is in Johannesburg and his sister is in New York. He Skypes them every Monday at 7pm London time. What day and time is it in Johannesburg and New York?

4. Carly is living in Dubai and she wishes to conference call her offices in Boston and Sydney. What time(s), in Dubai, could she make the call so that it is different days in Boston and Sydney?

5. The 2018 FIFA World Cup will be held in Russia. Most of the games will be played in Moscow (GMT+3hours). Kick-off is at 11:00am, 2:00pm or 5:00pm local time. Channel Sports will televise the games live in London, Sydney and Los Angeles. At what time will kick-off be in each city?

6. Angela and Martin are planning a holiday in Italy and Germany. Their flight leaves Adelaide airport at 4:30am ACST on Saturday. The flight stops at Dubai, 13 hours and 24 minutes later.

   a) What is the local time in Adelaide when they arrive in Dubai?
   b) What is their arrival time in Dubai?

   Angela and Martin leave Dubai at 4pm local time and fly to Rome, which is in the same time zone as Zurich. The flight to Rome takes 5 hours and 16 minutes.

   c) What time is it in Rome when their plane leaves Dubai?
   d) What is their arrival time in Rome?
   e) On what day will they arrive in Rome?

Extended Problem Solving

Grant's son, Jamie, is going to attend a college in Toronto to play ice hockey. Jamie will leave Perth at 2:00pm on Saturday 10 August. Below are his flight details:

Perth to Sydney	Flight time: 5 hours and 5 minutes Stopover time 3 hours
Sydney to Los Angeles	Flight time: 14 hours and 43 minutes Stopover time 2 hours
Los Angeles to New York	Flight time: 6 hours and 6 minutes Stopover time 1 hour
New York to Toronto	Flight time: 1 hour and 27 minutes

You need to prepare Jamie's itinerary, which is to include the day, date and local time for each flight's departure and arrival.

43 Simple Interest

Materials required: Calculators

Warm-up

1.	Convert 36 months to years	2.	Convert 54 months to years
3.	Convert 18 months to years	4.	Convert 3 months to years
5.	Convert 6 months to years	6.	Convert 2 months to years
7.	Convert 8 months to years	8.	Convert 15 months to years
9.	Convert 27 months to years	10.	Convert 7.5 months to years

Introductory problem

Simple interest is also known as flat rate interest. Simple interest is calculated at the same rate every year regardless of any changes to the original amount invested or borrowed.

Simple Interest = Principal × Rate × Time

Kasey wishes to invest $5000 at 4% pa for 5 years. She has the choice to invest it at simple interest for the 5 years or invest the $5000 for one year at 4% pa and then add the interest to the principal and reinvest it for another year and so on for a total of five years. Which investment option is best and by how much?

Exercises

1. Calculate the simple interest for each of the following:

   a) $2500 at 6% pa for 3 years   b) $5000 at 2.5% pa for 5 years.

   c) $1250 at 3.5% pa for 4 years   d) $750 at 8.3% pa for 6 years.

2. Calculate the simple interest for each of the following; remember to change the time into years.

   a) $3000 at 4% pa for 18 months

   b) $1450 at 6.5% pa for 64 months

   c) $10 000 at 2% pa for 100 months

   d) $6000 for 125 months at 5.2% pa.

   e) $100 for 9 months at 3% pa.

3. Felicity took out a loan for $20 000 to start up a beauty shop. She takes this loan for 9 months at 6.35% pa.

   a) How much interest does she owe at the end of the nine months?
   b) What is the total amount Felicity needs to repay?

4. Oliver is a local plumber. He completed a job for Speedy Couriers, that cost $750. He charges a 10% pa rate for accounts that are not paid on the day of service. Speedy Couriers did not pay their account for 3 months.

   a) How much interest did Oliver charge?

   b) What was the amount that Speedy Couriers have to pay?

5. The McMahon Bank pays different interest rates for large investments.

Balance	**Interest Rate**
$1 - $9 999	3.5%
$10 000 - $49 999	4.25%
$50 000 - $99 999	5%
Over $100 000	6.05%

   a) Sara invested $8000 for 2 years. How much interest did she earn?

   b) Adele invested her inheritance of $75 000 for five and a half years. How much did her investment grow to?

   c) Jaymz invested one million dollars for 4 months. How much interest did he earn in that time?

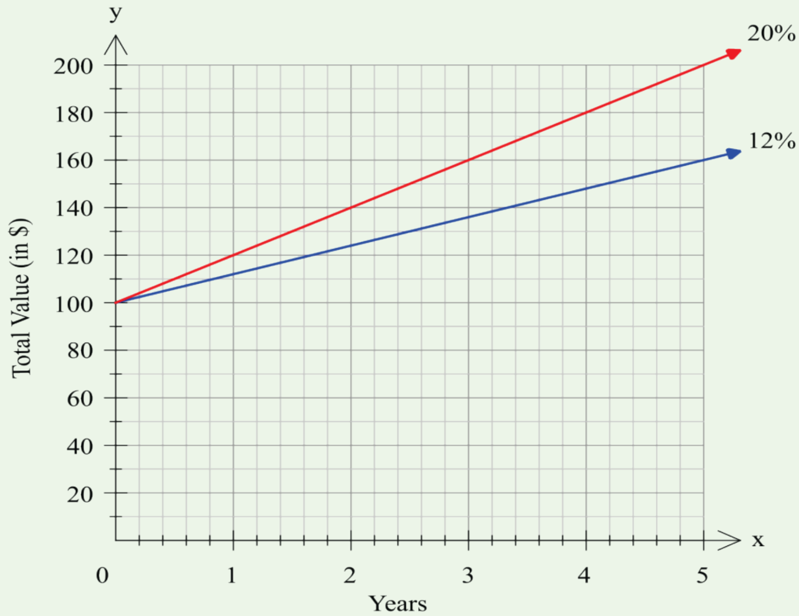
6. Mrs Gates has decided to invest $10 000 at 5% pa for 5 years. She has two options for her investment. Your task is to advise her as to which is best, showing all calculations.

   OPTION 1 Simple interest investment for the 5 years.

   OPTION 2 Simple interest investment, on a yearly arrangement, where the interest earned after each year is added to the principal and re-invested.

This second option is called compound interest and is for a situation where your interest is effectively re-invested and earns further interest (ie the interest compounds).

Extended Problem Solving



This graph shows the total value of $100 invested using a simple interest rate of 12% and 20%.

a) How much is the investment worth after 3 years at 20% pa?

b) How much simple interest is earned after 4 years at 12% pa?

c) How much more is the investment worth after 5 years at 20% compared to 12%?

d) Both of the lines start at 100. Explain why.

> "I guess a sock is also a geometric shape – technically – but I don't know what you'd call it.
> A socktagon?"
>
> — Stephen King

44 Compound Interest

Materials required: Calculator and computer

Warm-up

1.	Half of 5% =	2.	Half of 4.8% =
3.	Half of 6.7% =	4.	Half of 9.9% =
5.	Quarter of 8% =	6.	Quarter of 6%
7.	Quarter of 12.8%	8.	Quarter of 9.2%
9.	18% divided by 12	10.	15% divided by 12

Discussion

Your teacher will provide you with three graphs that show the growth of the Australian population. In groups, discuss the information in each of the graphs. Prepare a written summary, about one paragraph, of the information shown in each graph.

Using the combined information in the three graphs, write a short summary of trends in the Australian population between 1970 and 2010.

Compound interest is best calculated using a spread sheet, but a formula can be used instead to find the amount of money an investment will yield (original investment plus interest):

When compounded once per year, $Amount = P\,(1 + r)^t$

When compounded n times per year, $Amount = P\,(1 + \frac{r}{n})^{nt}$

P = Principal (amount invested), r = annual interest rate as a decimal, t = number of years of the investment.

Exercises

1. Sarah wishes to invest $2500 for 3 years at 4.25% pa.

   a) How much interest did she earn in the first year?
   b) How much interest did she earn in the second year?
   c) Calculate the value of Sarah's investment after 3 years.
   d) How much interest did Sarah earn after the 3 years?
   e) If Sarah invested the money for one more year, how much extra interest would she earn?

2. Find the accumulated amount in each of the following investments. Truncate your answers to 2 decimal places.

   a) $15 000 invested for 10 years at 9% per annum, with interest calculated yearly.
   b) $8 000 invested at 5% pa, with interest compounded yearly, for 3 years.
   c) An investment of $750 for one and a half years with an annual interest rate of 3%, compounded annually.
   d) A loan of $5 000, with an annual interest rate of 9.9%, compounded annually for 36 months.

Interest rates are quoted as per annum but interest can be compounded yearly, every 6 months, quarterly (4 times a year), monthly or even daily.

3. Determine the interest rates for the given time periods.

   a) 3.4% pa paid every six months
   b) 7.5% pa paid every six months
   c) 6% pa paid quarterly
   d) 6.7% pa paid quarterly
   e) 18% pa paid monthly
   f) 9% pa paid monthly
   g) 10% pa paid daily (round answer to 4 decimal places)
   h) 6% pa paid daily (round answer to 4 decimal places)

4. Find the accumulated amount for each of the following investments. Truncate your answers to 2 decimal places.

   a) $6 000 invested for 6 years with interest of 7.2% per annum, calculated 6 monthly
   b) $5 000 invested for 9 years at 6.4% pa, compounded every six months
   c) $10 000 invested for 3 years at 12% pa, compounded quarterly
   d) $3 000 invested for 4 years at 10% pa, compounded monthly
   e) $6 000 invested for 2 years at 15% pa, compounded monthly.

5. For each of the examples in Question 4, determine the amount of interest earned.

Spread sheets are an excellent tool for comparing investments or loans that use compound interest. Over the page is a screen shot of an Excel spread sheet that can be used for compound interest.

This example shows a $1000 investment at 2.25% compounded quarterly for 10 years. Cell E2 contains the formula for compound interest and cell F2 contains a formula as well.

	A	B	C	D	E	F
1	Amount Invested	Annual Percentage Rate (as a decimal)	Number of Times Compounded Annually	Number of Years	Future Value	Interest
2	$1,000.00	0.0225	4	10	1251.53344	$251.53
3						

Create a spread sheet that will allow you to quickly answer the following questions.

6. a) Investigate how the returns on an investment of $50 000 for 10 years changes when the interest rate increases by 0.5% pa from 2% pa to 5% pa, compounded quarterly.

   b) Investigate how the returns on an investment of $100 000 for 5 years changes when the compounding period changes from yearly, to 6 monthly, quarterly, monthly and daily, if it is invested at an interest rate of 4.6% pa.

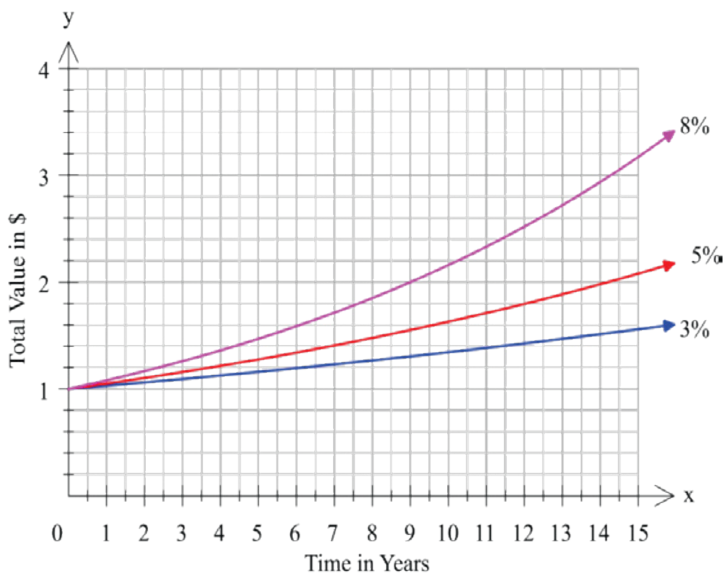
7. The graph below shows the total value of an investment of $1, compounded annually, at different interest rates. Use the graph to answer the following questions.

   a) Describe why the graphs are not straight lines.

   b) Approximately how long, at each interest rate, for your investment to increase to:

      (i) $1.50

      (ii) $2.00



Extended Problem Solving

Shenhav has $10 000 to invest. She can get the following interest rates: 2.5% simple interest, 3.71% per annum compounded yearly or 4.06% compounded monthly. Shenhav would like her investment to grow to $12 000.

How long will she need to invest her money to achieve her goal, for each of these options?

45 Reducible Interest

Materials required: Calculator, dictionary and computer

Warm-up

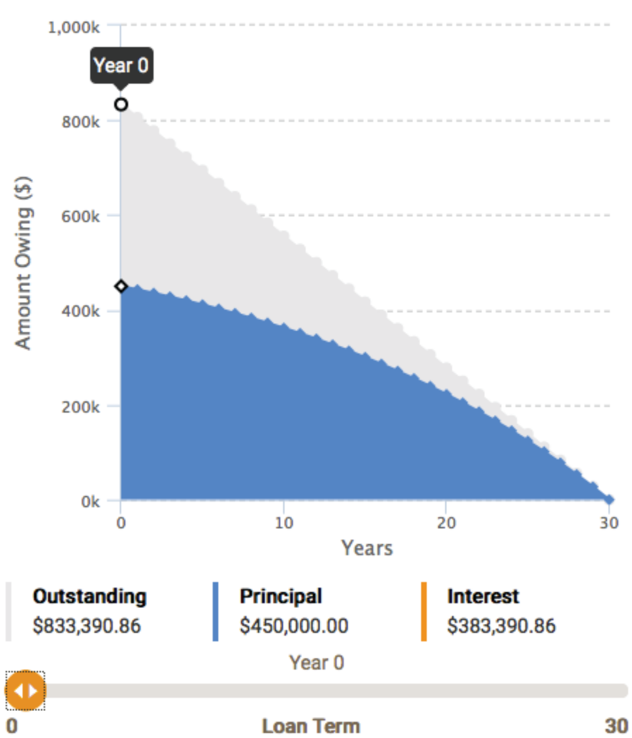
Write a brief mathematical definition of the following financial terms

1.	Principal	2.	Term of loan
3.	Interest rate	4.	Accumulated amount
5.	Mortgage	6.	Interest
7.	Repayments	8.	Annum
9.	Compound interest	10.	Reducible interest

Discussion

Home loans are called mortgages. Most banks, credit unions and lending organisations have a mortgage calculator on their website. You will need to enter the value of the home, the amount of the mortgage and the length of the mortgage. After entering all of this information, you can view a graph of how much of your repayment goes in interest and how much reduces the principal.

Below is an example of a $450 000 loan at 4.63% pa for 30 years with monthly repayments.



Use the Internet to explore the effects of increasing the principal, increasing and decreasing the interest rate and the frequency of repayments.

Mortgages are an example of reducible interest as repayments are made, but the interest is compounded.

Exercises

1. a) Complete the following table to determine the amount owing on a $5000 loan, at an interest rate of 3.5% pa, with a yearly payment of $700 after:

   (i) one year.
   (ii) two years.
   (iii) five years.

Time	Principal	Interest	Repayment	Amount Owing
End of 1st Year	$5000	$175	$700	
End of 2nd Year	$4475	$156.63	$700	
End of 3rd Year		$137.61		$4093.74
End of 4th Year		$117.92	$700	
End of 5th Year			$700	

   b) How much were the total repayments for five years?
   c) How much interest was paid in the first five years?

2. A personal loan of $3000 is borrowed at 12.2% pa reducible yearly. If repayments of $500 are paid each year, how much is owing after 4 years.

3. Jevin has decided to borrow $10 000 to help start up his IT company. The bank charges 7.25% pa reducible every 6 months. Jevin makes repayments of $1500 twice a year.

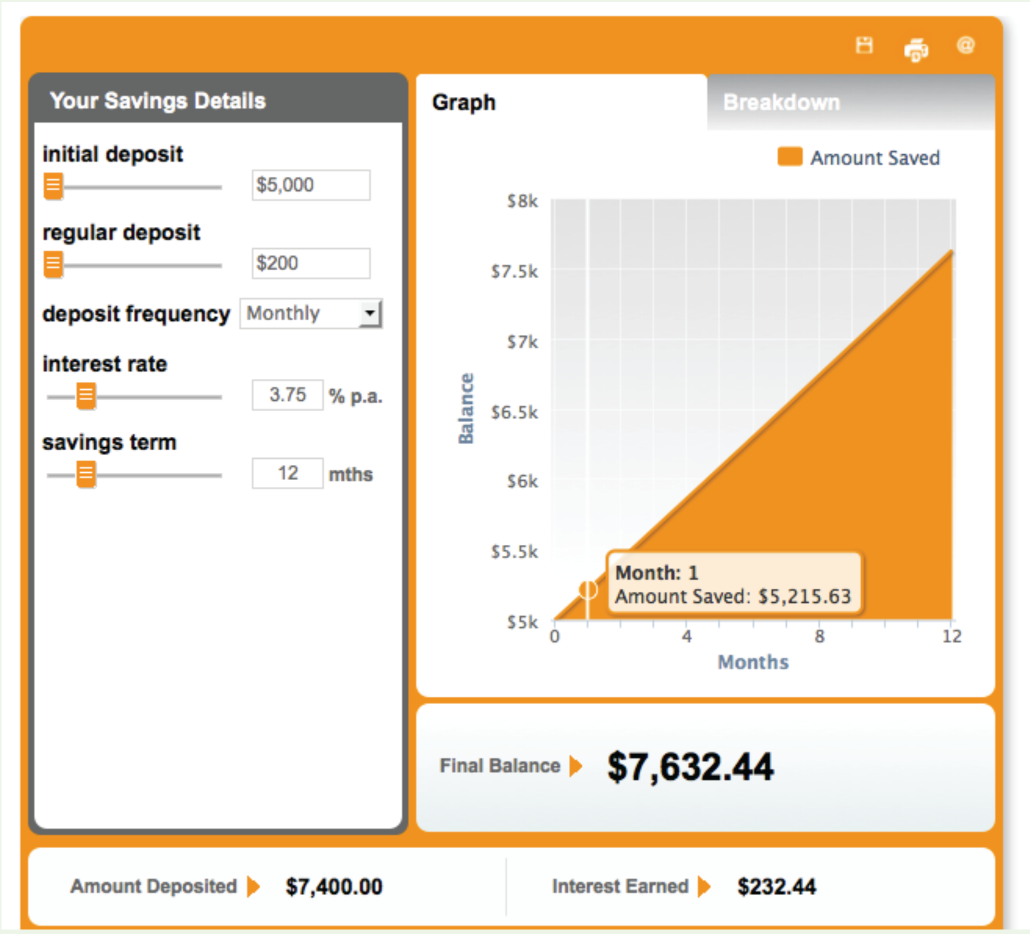
   a) What is the six monthly interest rate?
   b) How much will he owe after 3 years?

4. Ping and Zhou wish to purchase a house for $220 000. They have an $80 000 deposit and the bank charges 2.6% per month, reducible monthly. They have agreed to repay $3500 per month.

   a) How much did Ping and Zhou borrow from the bank?
   b) How much do they owe after 6 months?
   c) Why do Ping and Zhou owe more money than they borrowed, after 6 months?

5. Carter wishes to invest $500 at a rate of 4.5% pa, paid every 6 months. He adds $75 to his investment every 6 months. How much will his investment be worth after 3 years?

Extended Problem Solving



Many lending institutions have online calculators that let you investigate how long it will take to reach your savings goals. Use the internet to find a savings calculator to explore the following situation.

Stanley has $5000 and wishes to make regular monthly deposits of $350.

a) What interest rate will he need, to achieve a final balance of $10 000 in one year?

b) If the interest rate is 1.9%, how long will it take for him to achieve a balance of $10 000?

Answers

"A sum can be put right: but only by going back till you find the error and working it afresh from that point, never by simply going on."

— C.S. Lewis

"Failure is instructive. The person who really thinks learns quite as much from his failures as from his successes."

— John Dewey

Unit 3

1 Perimeter

1.	a)	60cm	b)	76mm	c)	10.4cm		
	d)	5.6m	e)	27.5m	f)	126mm		
2.	a)	56cm	b)	46cm	c)	61cm		
	d)	76.8mm	e)	119mm or 11.9cm				
	f)	474cm or 4.7m to 1 d.p.						
3.	a)	80cm	b)	116cm	c)	174cm		
	d)	45.2mm	e)	2.8m	f)	1.92m		
4.	a)	40.8cm	b)	88.0mm	c)	29.5cm	d)	1.6m
5.	2.83m		6.	2.83m	7.	1 029.2m		
8.	91.7m (whole court), 50.9m (centre third)							

2 Area I

1.	a)	$28.98cm^2$	b)	$4.51m^2$	c)	$105.78cm^2$
	d)	$3.12m^2$	e)	$1.653m^2$ or $16530cm^2$		
	f)	$69.42cm^2$				
2.	a)	$73.5cm^2$	b)	$62.8cm^2$	c)	$1.7m^2$
	d)	$9439.5cm^2$				
3.	a)	25cm	b)	48.5cm	c)	20.75cm
	d)	66.15mm	e)	0.485m	f)	0.55m
4.	a)	$113.1cm^2$	b)	$176.7cm^2$	c)	$4.5m^2$
	d)	$0.4km^2$	e)	$0.2m^2$	f)	$7\ 696.9mm^2$
	g)	$265.5mm^2$	h)	$18.7cm^2$		

3 Area II

1.	a)	$148.2cm^2$	b)	$418.81cm^2$	c)	$125.75cm^2$
	d)	$5.41m^2$	e)	$6.51m^2$	f)	$402\ 123.86mm^2$
2.	Total of the 4 elements is $6\ 875.09cm^2$ that is $0.69m^2$					

4 Area of Sectors

1.	a)	$13.1km^2$	b)	$128.3m^2$	c)	$253.4km^2$
	d)	$16.8km^2$	e)	$605.3m^2$	f)	$95.4cm^2$
	g)	$150.8cm^2$	h)	$222.7m^2$	i)	$359.2m^2$
2.	a)	$2984.5cm^2$	b)	$2650.7cm^2$		
3.	a)	$358.4cm^2$	b)	$87.3cm^2$		

5 Surface Area I

1.	a)	$216cm^2$	b)	$3\ 901.5cm^2$	c)	$574cm^2$
	d)	$16\ 862mm^2$	e)	$10.62m^2$	f)	$3\ 704m^2$
2.	$8.43m^2$ for one coat. Therefore three tins of paint.					
3.	a)	$42.06m^2$	b)	$5 257.50		
4.	$2.16m^2$		5.	$0.69m^2$ (top and sides, not base)		

6 Surface Area II

1. a) $132cm^2$ b) $345cm^2$ c) $1\ 111.2cm^2$
   d) $4\ 736cm^2$
2. a) $1\ 658.8cm^2$ b) $10\ 391.0cm^2$ c) $5\ 026.5mm^2$
   d) $530.9cm^2$
3. a) $3\ 870.4cm^2$ b) $6\ 440.3cm^2$ c) $2\ 276.1cm^2$
4. $5.69m^2$ 5. $8\ 107.3cm^2$ 6. SA = $216cm^2$, SA = $864cm^2$.

Therefore surface area is not doubled, it is four times more.

7 Surface Area III

1. a) x = 7cm, y = 10cm b) x = 9cm, y = 11cm
2. a) $510cm^2$ b) $410cm^2$ c) $336cm^2$
   d) $10.08m^2$

8 Units of Mass

1. a) 5000 b) 9450 c) 12 450
   d) 6 e) 2.34 f) 0.6
2. a) 300 b) 650 c) 160
   d) 300 000 e) 650 000 f) 160 000
3. a) 1.8g b) 1800mg c) 1500mg
4. a) 26 400mg, 26.4g
   b) 1900.8g, 1.9008kg
   c) 2200kg

9 Volume and Capacity

1. a) litres b) millilitres c) litres
   d) megalitres e) millimetres f) kilolitres
   g) millilitres h) millilitres i) gigalitres
   j) litres k) litres l) gigalitres
2. a) 4L b) 500L c) 0.87L
   d) 0.05L e) 2800L f) 960L
3. a) 2000mL b) 9500mL c) 650mL
   d) 1 000 000mL e) 250 000mL f) 100 000 000mL
4. a) 7ML (0.007gL) b) 12.75ML (0.01275gL)
   c) 100ML (0.1gL) d) 50ML (0.05gL)
   e) 800ML (0.8gL) f) 100 000ML (100gL)
5. a) 110.4L b) 188 cans
6. a) 4.8L b) 19.2L c) 0.6L
   d) 0.01L
7. 1600L/hour
8. a) 4.5kL (4 500L) b) 7.1kL (7 100L)
   c) 10.25kL (10 250L) d) 70.5mL
   e) 4 600ml f) 750 000mL

10 Volume of Regular Objects

1. a) $216cm^3$ b) $16\ 581.375cm^3$
   c) $760cm^3$ d) $113\ 505\ mm^3$
   e) $2.025m^3$ f) $2\ 100m^3$
2. a) $60cm^3$ b) $285cm^3$
   c) $936cm^3$ d) $6510cm^3$
3. a) $297\ 000cm^3$ b) approximately 29 or 30 fish
   c) 270L
4. a) $4.5cm^3$ b) $72cm^3$
5. 20 pieces

11 Volume

1. a) $91.72m^3$ b) $130\ 433.33cm^3$
   c) $141.21m^3$ d) $291.67m^3$
2. a) $170cm^3$ b) $0.59m^3$
   c) $339.43m^3$ d) $792\ 586.13mm^3$
   e) $5.18m^3$ f) $3\ 505.33m^3$
3. a) $5\ 026.55cm^3$ b) $14\ 254.98cm^3$
   c) $29.09m^3$ d) $1\ 809\ 557.37cm^3$ or $1.81m^3$
4. a)

Model	Capacity (L)	Capacity (kL)
Residential 1	54 682.3	54.7
Residential 2	90 429.7	90.4
Residential 3	111 515.2	111.5
Low profile 1	1 064.6	1.1
Slimline 1	120 963.8	121.0

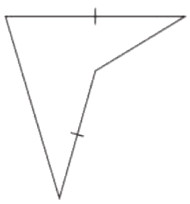
   b) Residential 3 and Slimline 1
   c) Residential 3
5. a) $381\ 703.5cm^3$ b) $3\ 942\ 455.8mm^3$
   c) $87\ 113.7cm^3$ d) $143.8m^3$

12 Two-Dimensional Shapes

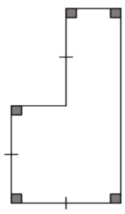
1. a) A quadrilateral with two pairs of parallel sides and four 90° angles.
   b) A quadrilateral with exactly one pair of parallel sides.
   c) A triangle with all sides equal length and all angles are 60°.
   d) A quadrilateral with two pairs of adjacent congruent sides and one pair of opposite equal angles.
   e) A triangle with two congruent sides and two congruent angles.
   f) An irregular concave quadrilateral with no sides the same length and one angle greater than 180°.
   g) A quadrilateral with exactly one pair of parallel sides and one 90° angle.
   h) An irregular concave hexagon with one angle of greater than 180°.
2. a) 3 & 5 b) 4 c) 1, 2, 3, 4 & 5
   d) 2 e) 3

3. Other answers may be possible:
    a) Trapezium – only has one pair of parallel sides.
    b) Obtuse triangle – has an angle greater than 90°.
    c) Pentagon – has an odd number of sides.
    d) Octagon (arrow) – no 90° angle.
4. Other possible answers may exist.

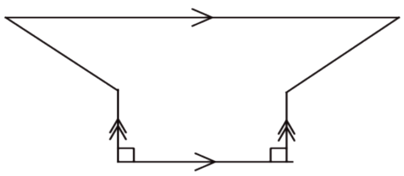
a)



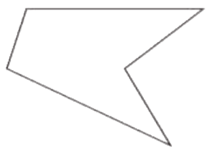
b)



c)



d)



5. It is possible to draw every type of polygon with exactly two right angles, except a triangle. They may be convex or concave.

13 Three-Dimensional Objects

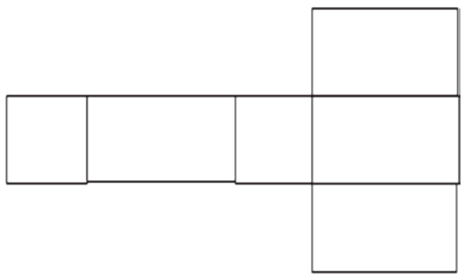
1.

Mathematical Name of Object	Number of Faces	Number of Edges	Number of Vertices
a) Cube	6	12	8
b) Triangular Prism	5	9	6
c) Hexagonal Pyramid	7	12	7
d) Hexagonal Prism	8	18	12
e) Trapezoid prism	6	12	8
f) Rectangular pyramid	5	8	5
g) Trapezoid prism	6	12	8
h) Cylinder or circular prism	3	2	0
i) Triangular pyramid or tetrahedron	4	6	4
j) Octagonal prism	10	24	16
k) Triangular prism	5	9	6
l) Cone or circular pyramid	2	1	1
m) Hexagonal prism	8	18	12
n) Pentagonal prism	7	15	10
o) Rectangular prism	6	12	8
p) Square based prism	6	12	8
q) Sphere	1	0	0

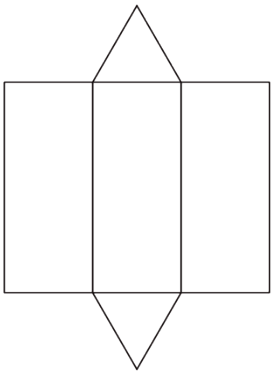
2. a) pentagonal prism
   b) triangular prism
   c) sphere
   d) dodecagonal prism
   e) cube
   f) triangular prism
   g) cylinder
   h) rectangular prism
   i) septagonal (or heptagonal) prism
   j) cone
   k) hexagonal prism
   l) trapezoid prism
3. a) square based pyramid or rectangular based pyramid
   b) triangular prism
   c) octagonal prism
   d) cylinder

14 Nets and Perspective

1. a)



b)



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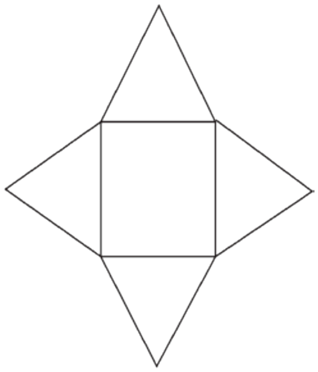

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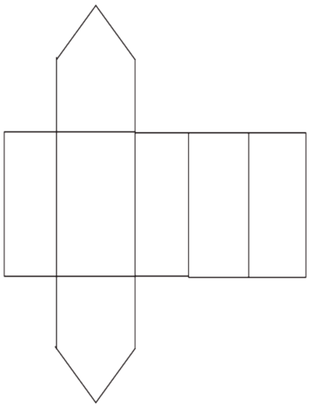
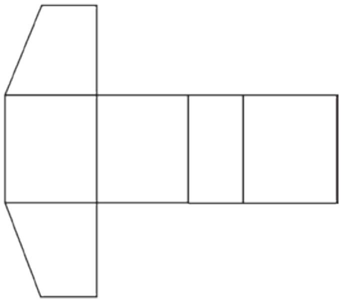
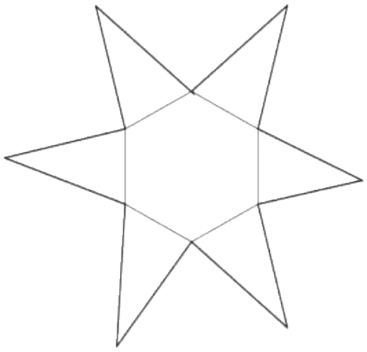


e)

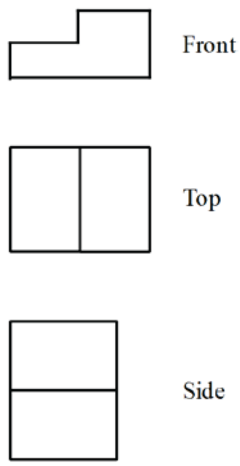


f)



2. a) rectangular based pyramid
   b) triangular pyramid
   c) pentagonal prism
   d) cube

3. a)



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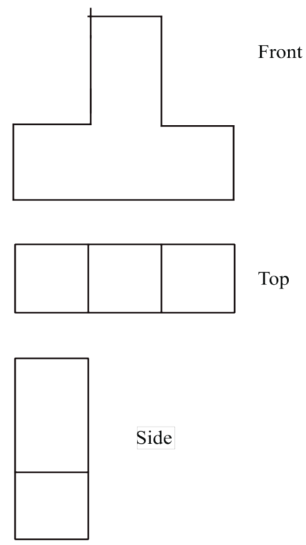

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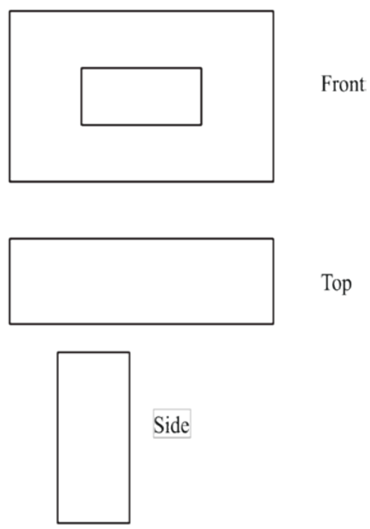

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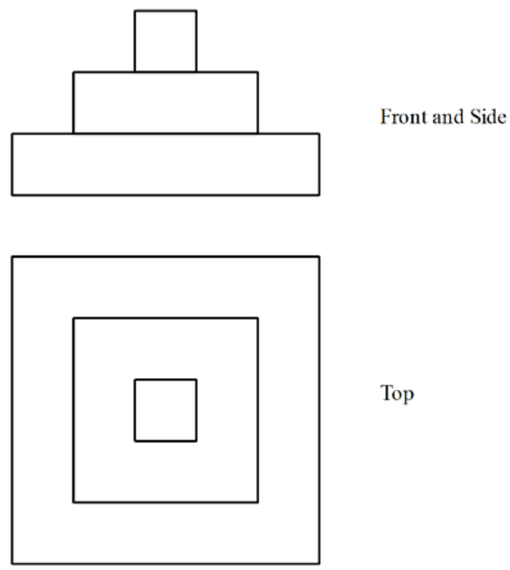
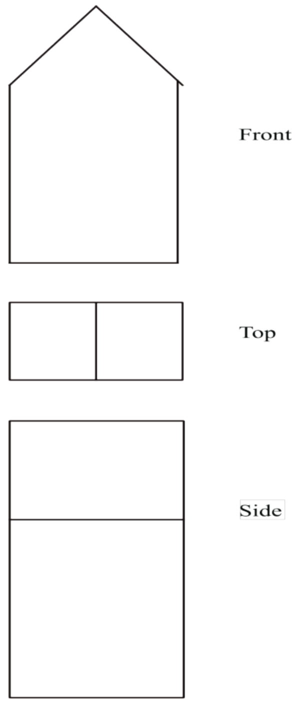
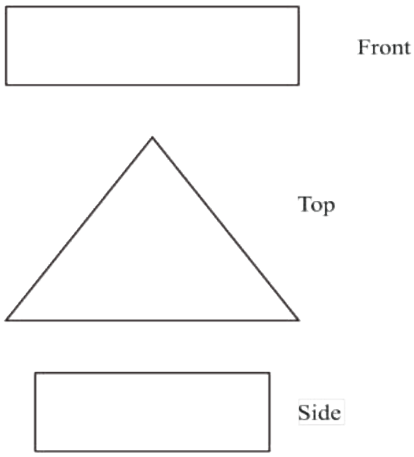


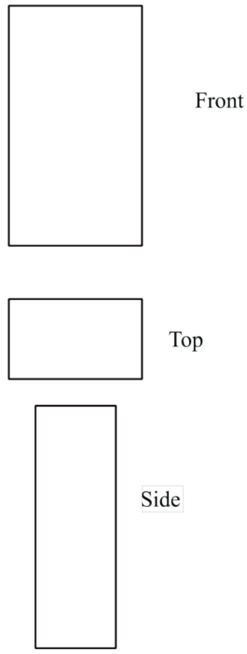
4. a)



b)



c)



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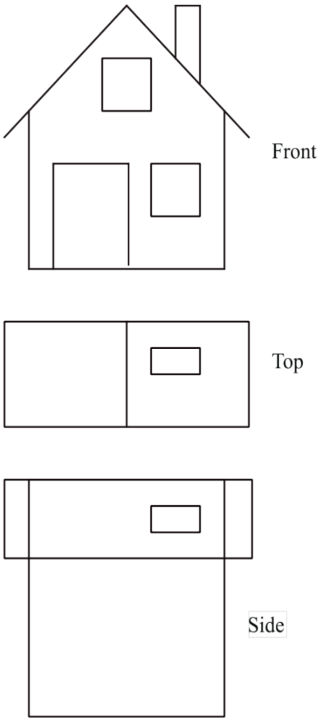


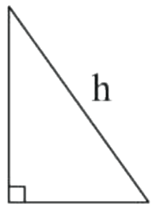
15 Scale Drawings

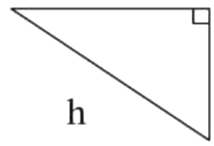
1. a) (i) 76mm (ii) 60 800mm (iii) 60.8m
   b) (i) 32mm (ii) 16mm (iii) 1.6cm
   c) (i) 64mm (ii) 46 080mm (iii) 46.08m
   d) (i) 72mm (ii) 180mm (iii) 18cm
   d) (i) 35mm (ii) 87.5mm (iii) 8.75cm
   e) (i) 35mm (ii) 7mm (iii) 7mm
   f) (i) 51mm (ii) 102mm (iii) 10.2cm
   f) (i) 38mm (ii) 76mm (iii) 7.6cm
   g) (i) 36mm (ii) 140 400mm (iii) 140.4m
2. a) 4.338m b) 4.272m c) 4.064m
   d) 4.8768m
3. 1:64 l = 228.125mm 1:32 l = 456.25mm
   228.125 × 2 = 456.25. Therefore model is twice as big using the scale 1:32
4. a) 415km b) 595km c) 44.8cm
   d) 11.88cm
5. a) 11.5m b) 270m c) 335m
   d) Station 1 is 9.6km and station 2 is 11km
6. Answers will need to be determined by the teacher, as the size of classrooms will be different.

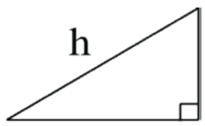
16 House Plans

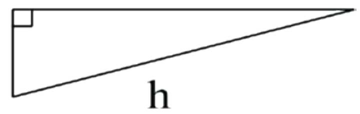
1. a) ensuite b) laundry c) walk in pantry
   d) shelf and robe e) linen cupboard
   f) 25 bricks high or 25 brick to ceiling height
   g) water closet (toilet)
   h) obscured glass
2. 9.954m^3 one truck load of cement
3. Bathroom (1810x1730), laundry (1570x2520), WC (1000x1830) and ensuite (3930x1680). Total of 15.53m^2
4. Bed 1 (3690x3150), Dressing (1790x2170), Bed 2 (2810x3810), Bed 3 (3810x3270), Bed 4 (3270x2850) and theatre (3810x3510). Total carpet including 10% extra is 67.51m^2
5. 113.72m^2 Total cost is $17 058.00
6. (i) a row of bricks and mortar
   (ii) 2.16m (iii) Total area is 37.044m^2, therefore 1853 bricks
7. Measurements do not include windows and assume floor to ceiling tiles in ensuite shower. Area of all walls is 53.3088m^2. Two coats is 107m^2. Therefore will need 4 tins and spend $319.60 on paint.
8. Volume is 411.696m^3. Your will need Unit C.
9. Perimeter of walls is 7.37m. Will need to purchase 8.9 metres (accept 9 metres)
10. a) 172.7 metres b) data points – activity & theatre, TV points – family, theatre and possibly each bedroom, landline phone points – activity and family.
11. a) 3.732m^3 b) 3.732m^3

17 Pythagoras

1. a)


b)


c)


d)


2. a) 5m b) 13cm c) 5.5m d) 10.7cm
3. a) 9m b) 23.80cm c) 9.52mm d) 14.95km
4. a) 41.9cm b) 38.0km c) 31.6m d) 18.0m
5. 2.84m (maths classroom answer) 2.85m (real world answer)
6. 1.35m

18 Tangent Ratio

1. a) $\tan C = \frac{20}{15}$ b) $\tan X = \frac{18}{24}$
   c) $\tan A = \frac{28}{21}$ d) $\tan Z = \frac{8}{15}$
2. a) 19.56cm b) 43.96cm c) 30.09cm
   d) 24.70cm e) 26.54cm f) 10.10cm
   g) 5.76cm h) 22.55cm
3. a) 22° b) 29° c) 40°
   d) 45° e) 37° f) 36°
   g) 46° h) 77° i) 69°
4. a) 29° b) 39° c) 22°
   d) 30° e) 33° f) 26°
   g) 37° h) 32°
5. 50° 6. 149.3m 7. 3.1m
8. (i) 70.23° (ii) 19.77°

19 Angles of Elevation and Depression

1. 7.8m 2. 41.2m 3. 98m 4. 208.7m
5. 2.2m 6. 22° 7. 17m this is an estimate as errors could have been made with the measurement of the height of the man and whilst using the clinometer to measure the angle of elevation.

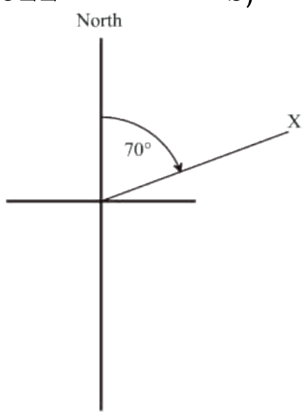
20 Sine and Cosine Ratios

1. a) 0.83 b) 0.21 c) 0.71
   d) 0.26 e) 0.45 f) 0
2. a) $\cos C = \frac{24}{26}$ b) $\sin Z = \frac{8}{17}$ c) $\sin X = \frac{24}{40}$
   d) $\cos C = \frac{9}{41}$ e) $\sin X = \frac{24}{40}$ f) $\cos A = \frac{30}{50}$

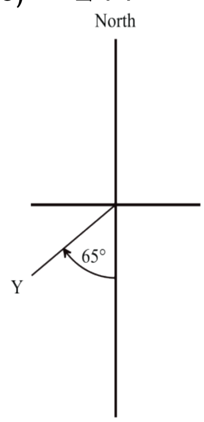
3. a) 22.8 units b) 30.2 units c) 10.2 units
   d) 18.3 units e) 11.7 units f) 19.5 units
   g) 9.4 units h) 12.6 units i) 26.0 units
   j) 19.0 units k) 18.7 units l) 8.2 units
4. a) 38° b) 54° c) 55°
   d) 29° e) 74° f) 21°
   g) 31° h) 37°
5. 1446.05m 6. 23°

21 Trigonometry and Bearings

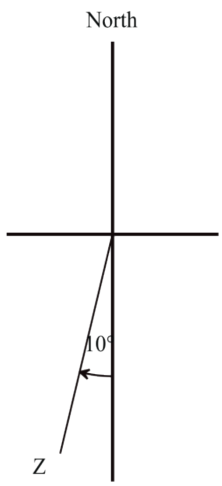
1. a) 022° b) 118° c) 244° d) 326°
2. a)



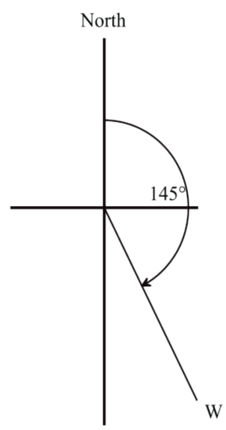
b)



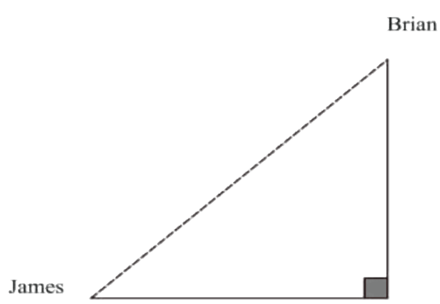
c)



d)

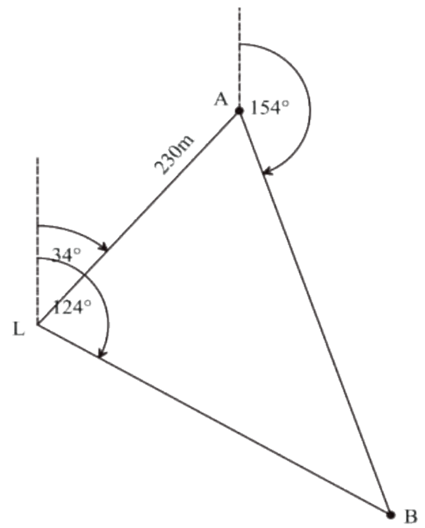


3. a) 325° b) 25.8km c) 36.9km
4. a) 29° b) 354.2km
5. a)



   b) 8.6km c) 054° d) 234°
6. 6.3km

7. a)



b) 60° c) 398.4m d) 460m

22 The Cartesian Plane

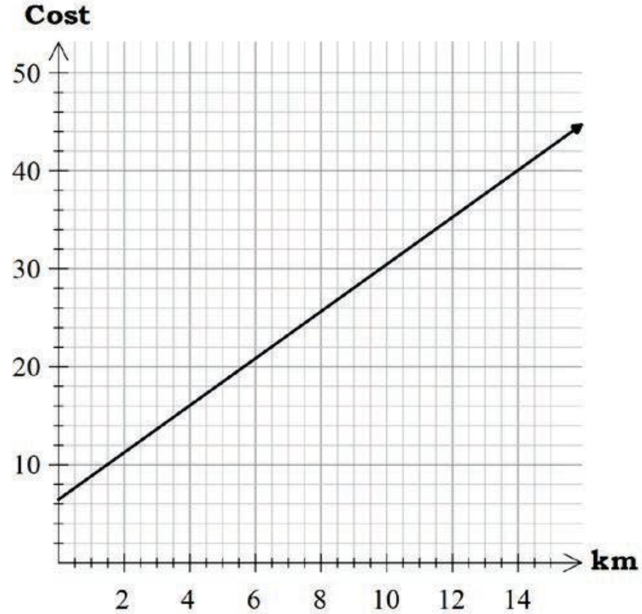
1 a) isosceles triangle b) parallelogram c) trapezium
d) right angled triangle
2 a) (45,63) b) (42,26) c) (68,96)
3 a) (5,6) square b) (7,1) rectangle c) (-2, -2) trapezium
d) (2,0) rhombus
4 a) shark b) car c) castle

23 Linear Graphs

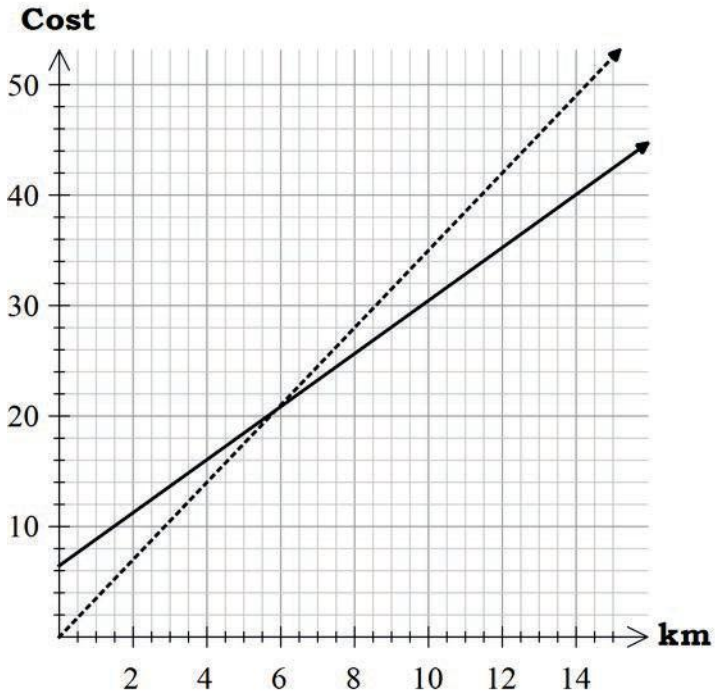
1 a) i) linear ii) 3.5 b) i) not linear
c) i) linear ii) $5.50 d) i) linear ii) – 0.5
2 a)

Km	0	1	2	3	4	5	10
Cost	6.25	8.65	11.05	13.45	15.85	18.25	30.25

b)

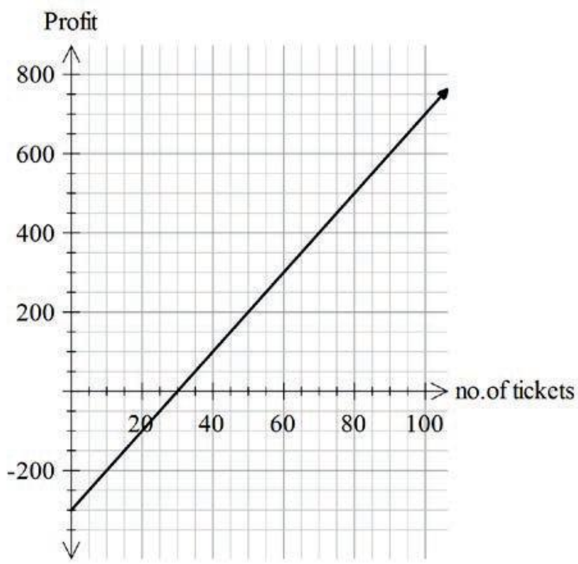


c) The vertical intercept is (0, 6.25), it corresponds to the flag fall charge



d) Green's is cheaper for shorter trips of less than 6km but more expensive than Blues taxis for trips that are longer than 6km

3. a)



b) 250 ÷ 25 = 10, tickets sell for $10 each

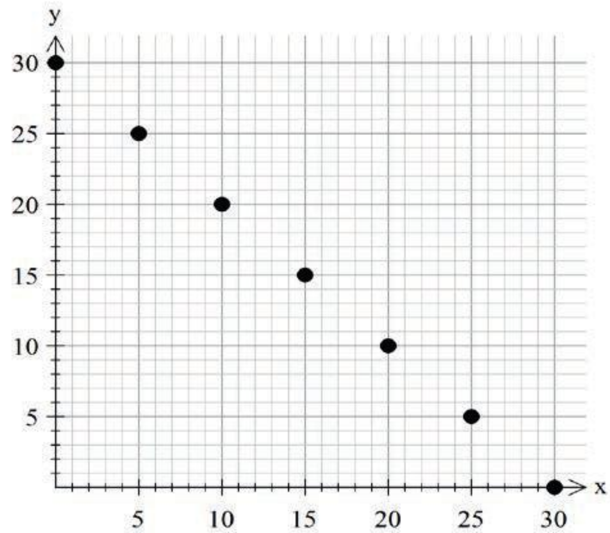
c) (0, -300) the vertical intercept represents the cost incurred for the show

d) When they sell 30 tickets

4. a)

Number of agapanthas (x)	0	5	10	15	20	25	30
Number of gardenias (y)	30	25	20	15	10	5	0

b)



c) (0,30) d) One variable is decreasing as the other increases, so the graph has a negative slope

e) No it is not reasonable to join the points as the values are discrete as we can only have whole numbers of plants

24 Linear Relationships

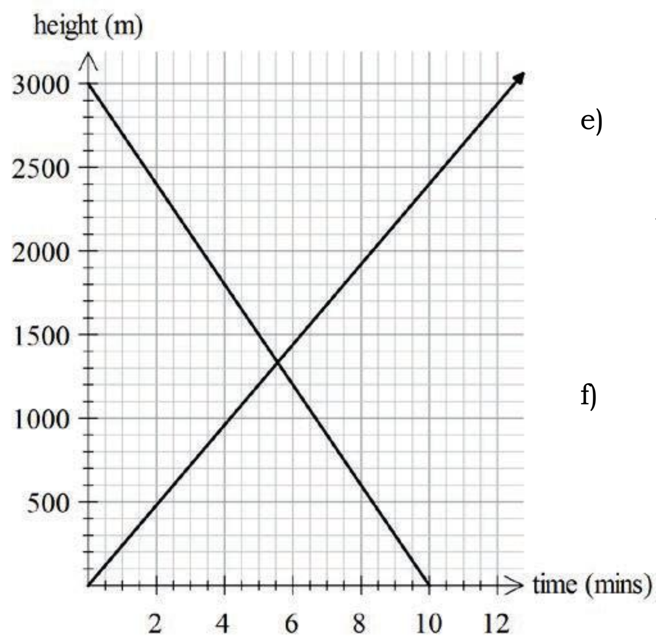
1. a) C = 2n + 120 b) C = 3.5n c) C = 5.5n + 40
   d) y = 20 – 0.5t e) P = 10n – 300 f) y = 30 – x
2. a) i) A ii) B
   b) (10,20) – the point at which both requirements are met, ie there are two children and one adult per ticket and a total of 30 people for the show to begin.
3. a) **A** 300m/min, **B** 240m/min b) height and time
   c) Balloon A

Time (mins)	0	2	4	6	8	10
Height (metres)	3000	2400	1800	1200	600	0

Balloon B

Time (mins)	0	2	4	6	8	10
Height (metres)	0	480	960	1440	1920	2400

d)



e) 1320m at approximately 5.5 mins, they probably won't crash – are at the same height, but not necessarily the same position

f) 10 minutes

25 Graphs in Practical Situations

1 A Weight and Age, as age increases the weight is also increasing at a decreasing rate
  B Height and Age, height increases at a steady rate over time, but the rate slows from 14 years
  C height and time, as time passes the height is decreasing at a constant rate
  D month and rainfall, the average rainfall starts at about 90mL in Jan increasing to 100mL in Feb, the rainfall then drops quickly to 40mL for March and April after which it then slowly decreases until Oct when rainfall begins to increase again

2. a) 11 years b) 4 years and 9 years
   c) from 3 to 4 years d) No, as the slope is always positive

3. a) V b) IV c) I d) II e) III

4. a) The time scale is not equally distributed so will distort the slope of the line
   b) 9.40am – 10.00am and 12.00pm – 2.00pm
   c) From 11 – 12 she travelled 100km hence 100km/hr
   d) 360km in 7 hours so average speed is 51.4 km/hr

5. a) 13 feet b) 6.5m c) yes

26 Sampling Methods

Answers will vary

27 Surveys

Answers will vary

28 Questionnaire Design

1. *Vague*: easy, maybe, many, seldom, usually, few, sometimes, several, poor, usually, fair, much
   Precise: never, twice, exactly, none, half, triple, two

2. a) How many hours per week do you spend on homework?
   b) Do you think girls are less athletic than boys?
   c) Should kids be allowed to ride skateboards on footpaths?
   d) Do you enjoy the music of Taylor Swift?
   e) Do you think children spend too much time on social media?
   f) Should our water supply be fluoridated?
   g) neither
   h) Should the voting age be lowered to 17?

3. a) F b) F c) F d) O e) F
   f) O g) O h) F i) F j) O

4. Those with a measurable or observable answer that can be independently verified are facts; opinions are personal choices/preferences.

5. a) Embarrassing question with vague meaning. *"On a scale of 1 -10 rate how positive you feel about the relationship you have with your peers"*
   b) Response required about immoral behaviour, if questionnaire is confidential then respondents are more likely to answer honestly
   c) Embarrassing question. *"Do you exercise? Why?"*
   d) Privacy issues, if questionnaire is confidential then respondents are more likely to answer honestly
   e) Respondents will most likely answer yes, vague question with moral judgements
   f) Vague judgement question. *"Describe a situation that you believe it is OK to misrepresent the truth."* Never can be given as an option
   g) Response required about potentially embarrassing behaviour, if questionnaire is confidential then respondents are more likely to answer honestly
   h) Wealthy is a vague definition, asking for household income within income brackets within a confidential questionnaire will encourage honest responses
6. G, E, D, B, H, A, F, C, I (other answers are possible, check with your teacher)

29 Survey Response Design

1 a) Overlapping interval boundaries
  b) Inconsistent units used and missing categories
2. a) Open ended b) To get positive feedback
   c) Allows answer to be in own words, difficult to process
   d) Categorise answers into subgroups
   e) Target audience is the school population and the sample is not representative as it is only parents
   f) Not familiar with strategies for the improvement
3. a) Scaled
   d) Larger scale encourages a greater variety of responses
   e) The scale should have an even number of possible responses
4. Various answers, discuss with a partner and teacher

30 Sources of Bias

1. During the 11am news, more likely to have unemployed people watching than employed viewers, so skewed population – sample bias. Phone poll requires people to have a phone and care enough to call in (only people with strongly held views will respond) – sample bias. Language is biased (bludgers, hard working tax payers) and probably reflects the view-point of the person who wrote it – researcher bias.
2. During the stock market report, so more likely to have wealthy employed people with investments watching than unemployed viewers, so skewed population – sample bias. Phone poll requires people to have a phone and care enough to call in (only people

with strongly held views will respond) – sample bias. Language is biased (bludgers, hard working tax payers) and probably reflects the view-point of the person who wrote it – researcher bias.

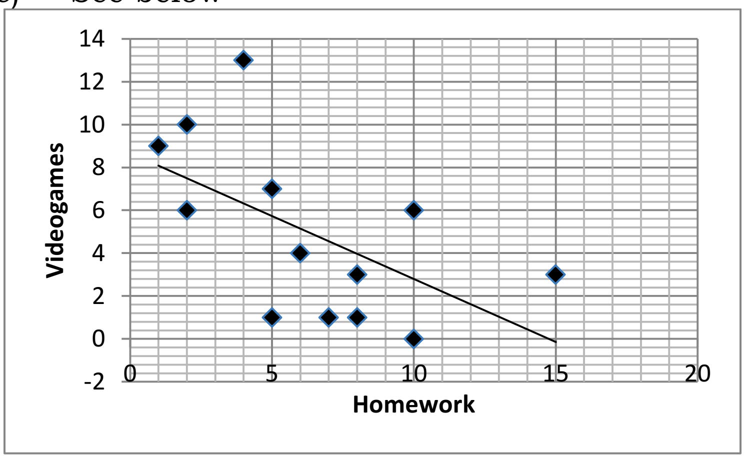
3. Early end of trial when initial results are positive reflects researcher bias. Early finish leads to sample/response bias. Small sample likely to lead to measurement bias where the small sample size means the results are less likely to reflect the full population.

4. On ABC Classic FM radio, so more likely to have classical music fans listening than Taylor Swift fans, so skewed population – sample bias. Phone poll requires people to have a phone, know how to send text messages (challenging for some older listeners) and care enough to call in (only people with strongly held views will respond) – sample bias. Likely to be researcher bias as the person who is asking the question works for a classical music radio station.

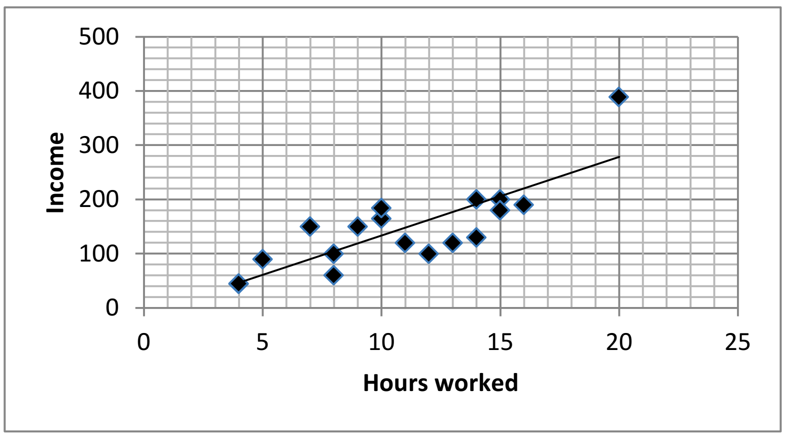
31 Scattergraphs

1 a) Positive strong linear relationship
  b) (160, 66), data input errors

2. a) See below b) Negative moderate linear association
  c) See below



3. a) see below b) Moderate, positive, linear association
  c) see below d) approx. $90
  e) approx. $520 f) 10 hours
  g) Prediction d) and f) are more reliable because they involve interpolation, e) is extrapolated data.



4. a) Positive non-linear strong association
   b) no association
   c) strong negative linear

Unit 4

32 Probability Expressions

1. Pigs might fly; Once in a blue moon; Buckley's chance; in your dreams; fat chance; maybe yes, maybe no; it's in the bag; without a doubt
2. D
3. a) $\frac{1}{52}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{13}$
4. a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$
5. a) 0.0001 b) 0.0001 c) 0.5 d) 0.1
   e) 0 f) 0.009 g) 0.09 h) 0.0001
6. a) $\frac{3}{8}$ b) $\frac{3}{5}$ c) $\frac{3}{5} > \frac{3}{8}$ hence Stefi has the better chance of drawing a red marble
7. a) i) $\frac{7}{10}, \frac{8}{10}, \frac{1}{10}, \frac{54}{365}$ ii) 0.7, 0.8, 0.1, 0.15
   iii) 70%, 80%, 10%, 15%
   b) cloudy, rain, snow, sun shines all day

33 Simulations

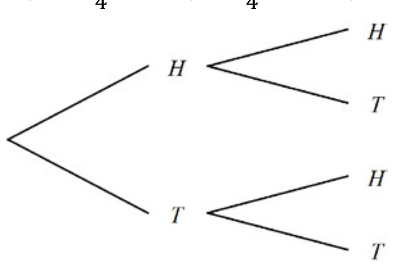
1. Various solutions, check with your teacher
2. a) {0, 1, 2, 3, 4, 5, 6 – fatal case} {7, 8, 9 – survival}
   b) 31 fatal cases c) various solutions
   d) various solutions
   e) better treatments of infected people may have decreased the fatality rate increasing the chance of survival
3. Various solutions
4. Various solutions

34 Experimental Probability

Results will vary as the repetition of chance events is likely to produce different results. Compare your results with other students in your class. Do they seem reasonable? The larger the number of trials in an experiment the closer the results will be to the theoretical probability.

35 Probability in Games

1. a) i) $\frac{1}{4}$ ii) $\frac{1}{4}$ iii) $\frac{1}{2}$ b) check with your teacher
   c)



2. a)

	Die 1						
Die 2	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b) i) $\frac{3}{36} = \frac{1}{12}$ ii) 0 iii) $\frac{6}{36} = \frac{1}{6}$ iv) $\frac{23}{36} = \frac{1}{2}$ v) $\frac{15}{36}$

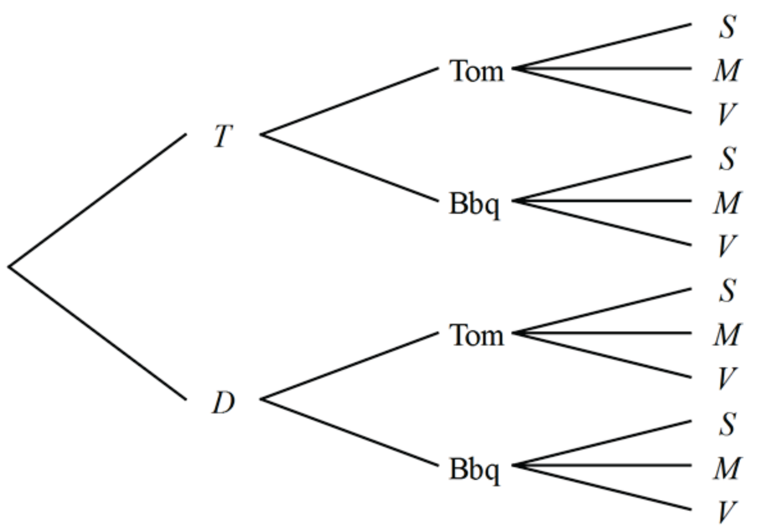
3. a)

	Red	Blue	Green	Yellow
Left Foot	RLF	BLF	GLF	YLF
Right Foot	RRF	BRF	GRF	YRF
Left Hand	RLH	BLH	GLH	YLH
Right Hand	RRH	BRH	GRH	YRH

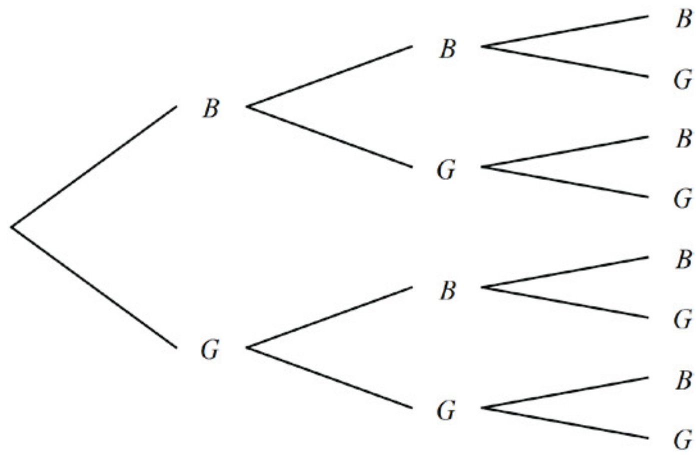
b) i) $\frac{1}{16}$ ii) $\frac{2}{16} = \frac{1}{8}$ iii) $\frac{4}{16} = \frac{1}{4}$ iv) $\frac{1}{2}$

36 Probability Applications

1. Choosing what you do for the day based on weather, sporting predictions, course selections at school, risks associated with certain activities (sky diving)
2. 12:



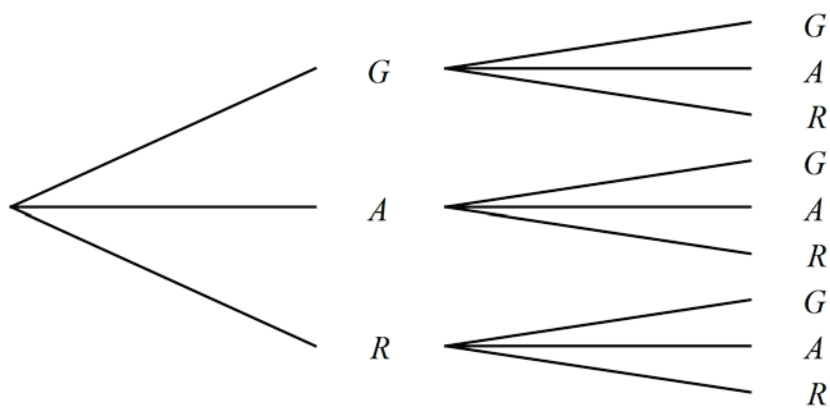
3.



b) i) $\frac{1}{8}$ ii) $\frac{1}{2}$ iii) $\frac{4}{8} = \frac{1}{2}$ iv) $\frac{4}{8} = \frac{1}{2}$

c) In a family, the sex of each new child is not affected by the sex of the previous siblings. The fourth child is equally likely to be a boy or a girl.

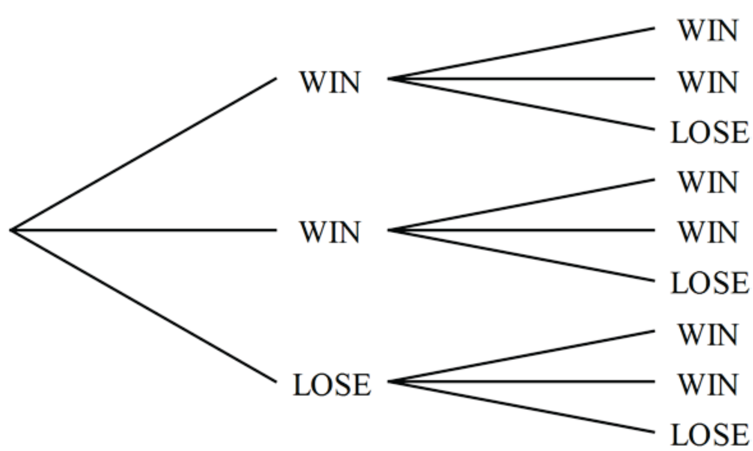
4. a)



b) i) $\frac{5}{9}$ ii) $\frac{2}{9}$ iii) $\frac{1}{9}$

c) This is not a realistic model as the assumption that the colours of the lights are equally likely is not true. For example, an amber light is on for less time than a red or green. See extended problem solving for further examples.

5. a), b)



c) $\frac{3}{9} = \frac{1}{3}$ for students to win, the staff must lose

		Game One		
		Win	**Win**	**Lose**
Game Two	**Win**	WW	WW	LW
	Win	WW	WW	LW
	Lose	WL	WL	LL

6.

Year	**Horse**	**Odds**	**Probability (win)**	**Rank**
2013	Fiorente	6:1	0.143	5
2012	Green Moon	19:1	0.05	13
2011	Dunaden	17:2	0.105	7
2010	Americain	12:1	0.077	9
2009	Shocking	9:1	0.1	8
2008	Viewed	40:1	0.024	14
2007	Efficient	16:1	0.059	11
2006	Delta Blues	17:1	0.056	12
2005	Makybe Diva	17:5	0.227	2
2004	Makybe Diva	13:5	0.278	1
2003	Makybe Diva	7:1	0.125	6
2002	Media Puzzle	11:2	0.154	4
2001	Ethereal	9:1	0.1	8
2000	Brew	14:1	0.067	10
1999	Rogan Josh	5:1	0.167	3

7. a) Return of $500, $400 is winnings
   b) Return of $1 100, $1 000 is winnings

37 Expected Values

1. 69 2. 4 tickets 3. a) 4 b) 12 c) 4
4. 67 5. 2400 6. a) 343 b) 100 c) 3
7. a) 2 880 000 b) 1 008 000
8. a) It is 9 more than expected
   b) Further investigation into factors that could be increasing the asthma rates need to be considered
9. 475

38 Latitude and Longitude

1. a) 30°N, 31°E b) 22°S, 43°W c) 36°N, 112°W
   d) 16°S, 146°E e) 28°N, 87°E f) 14°S, 72°W
   g) 18°S, 26°E
2. A = 31°N 122°E B = 9°S 143°E

3.

Name	**Nationality**	**University**	**Claim to Fame**	**Latitude and Longitude**
Peter Hall	Australian	University of Melbourne	The Bootstrap method	37.8°S 145.0°E
Manjul Bhargava	Canadian	Princeton University	Gauss Composition Laws	40.3°N 74.7°W
Stanislav Smirnov	Russian	University of Geneva	Percolation Theory	46.2°N 6.1°E
Ingrid Daubechies	Belgian	Duke University	Wavelets	41.9°N 91.6°W
Robert Langlands	Canadian	Yale University	Langlands program	41.3°N 72.9°W
Joseph B. Keller	American	New York University	Einstein-Brillouin-Keller method	40.7°N 74.0°W
Brian D. Ripley	British	St Peter's College	Smith's Prize	40.7°N 74.1°W
Frank Kelly	British	University of Cambridge	Loss networks	52.2°N 0.11°E
Mikhail Gromov	Russian	Institut des Hautes	Geometry	48.6°N 7.8°E
Endre Szemeredi	Hungarian	Rutgers University	Szemeredi's theorem	40.5°N 74.4°W
Bernard Silverman	British	University of Oxford	Functional data analysis	51.8°N 1.3°W
Wendelin Werner	French	ETH Zurich	Random walks	47.4°N 8.5°E
Elon Lindenstra uss	Israeli	Hebrew University of Jerusalem	Ergodic theory	12.9°N 80.1°E
Yurij Manin	Russian	Max-Plank-Institut für Mathematik	Algebraic geometry	50.7°N 7.1°E
Andrew Wiles	British	Princeton University	Proving Fermat's Last Theorem	40.3°N 74.7°W
Terence Tao	Australian	University of California	Gren-Tao Theorem	37.0°N 122.1°W

39 Arc Length

1.	26.2cm	2.	78.5mm	3.	32.8cm	4.	5.4mm
5.	136.1cm	6.	71.6mm	7.	23cm	8.	46.1cm

40 Determining Distances

1. a) 42° b) 53°
2. a) 5209km b) 4766km c) 2327km
   d) 446km e) 5431km f) 5431km
3. Check with your teacher

41 Time Zones I

1. a) 10:30am/11:00am b) 2:30pm/3:00pm
   c) 9:30pm/10:00pm d) 1:30am/2:00am
   e) 1:30pm/2:00pm f) 7:00pm/7:30pm
   g) 5:00am/5:30am h) 9:00pm/9:30pm
   i) 12:00pm/12:30pm j) 4:15pm/4:45pm
   k) 1:15pm/1:45pm l) 1:00am/1:30am
2. a) 120°E b) 180°E c) 135°E
   d) 105°E e) 82.5°E f) 60°E
3. a) 75°W b) 45°W c) 105°W
   d) 120°W e) 150°W f) 0°W
4. a) 12:30pm b) 3:00pm c) 9:15pm
   d) (i) 12am (ii) 11:30pm (iii) 10pm
   (iv) 2:00am
5. a) 4am Wednesday b) 7pm Tuesday
   c) 12am Wednesday d) 1pm Tuesday
   e) 7am Wednesday f) 9am Wednesday
   g) 8pm Tuesday h) 2am Wednesday
   i) 5:30am Wednesday
6. Perth and Bali are in the same time zone.
7. a) Daylight savings usually starts the first weekend in October and ends the first weekend in April.
   b) New South Wales, ACT, Victoria, Tasmania and South Australia are the states that have daylight savings.
8. a) 8:50am b) 10:50am c) 11:50am
   d) 10:20am e) 11:50am f) 11:50am

42 Time Zones II

1. Saturday 8:30pm and 11:00pm and Sunday 1:30am
2. Australia 2pm, Malaysia 12:00pm, China 12:00pm, Canada 7am (the next day), Portugal 12:00am (the next day) and London 12:00am (the next day)
3. Johannesburg 9pm Monday and New York 2pm Monday
4. Anytime between 7:01pm and 8:59am
5. 11am kick off London 8:00am, Sydney 6:00pm, Los Angeles 12:00am (the same day)
   2:00pm kick off London 11:00am, Sydney 9:00pm, Los Angeles 3:00am
   5:00pm kick off London 2:00pm, Sydney 12:00am (the next day), Los Angeles 6:00am
6. a) 5:54pm b) 12:24pm c) 1pm
   d) 6:16pm e) Saturday

43 Simple Interest

1. a) $450 b) $625 c) $175
   d) $373.50
2. a) $180 b) $502.67 c) $1 666.67
   d) $3250 e) $2.25
3. a) $952.50 b) $20 952.50
4. a) $18.75 b) $768.75
5. a) $560 b) $95 625 c) $20 166.67
6. Option 1: $2500 Option 2: $2 762.82
   She should select option 2.

44 Compound Interest

1. a) $106.25 b) $110.76 c) $2 832.49
   d) $332.49 e) $120.38
2. a) $35 510.45 b) $9 261.00 c) $784.00
   d) $6 636.86
3. a) 1.7% b) 3.75% c) 1.5%
   d) 1.675% e) 1.5% f) 0.75%
   g) 0.0274% h) 0.0164%
4. a) $9 172.09 b) $8 814.63 c) $14 257.60
   d) $4 468.06 e) $8 084.10
5. a) $3 172.09 b) $3 814.63 c) $4 257.60
   d) $1 468.06 e) $2 084.10
6. a)

Amount Invested	Annual Percentage Rate (as a decimal)	Number of times compounded annually	Number of Years	Future Value	Interest
$50 000.00	0.02	4	10	$61 039.71	$11 039.71
$50 000.00	0.025	4	10	$64 151.34	$14 151.34
$50 000.00	0.03	4	10	$67 417.43	$17 417.43
$50 000.00	0.035	4	10	$70 845.44	$20 845.44
$50 000.00	0.04	4	10	$74 443.19	$24 443.19
$50 000.00	0.045	4	10	$78 218.84	$28 218.84
$50 000.00	0.05	4	10	$82 180.97	$32 180.97

   b)

Amount Invested	Annual Percentage Rate (as a decimal)	Number of times compounded annually	Number of Years	Future Value	Interest
$100 000.00	0.046	1	5	$125 215.60	$25 215.60
$100 000.00	0.046	2	5	$125 532.55	$25 532.55
$100 000.00	0.046	4	5	$125 694.92	$25 694.92
$100 000.00	0.046	12	5	$125 804.67	$25 804.67
$100 000.00	0.046	365	5	$125 858.18	$25 858.18

7. a) Each year the principal increases thus total value increases at an ever increasing rate.
   b) (i) 3% - 14 years   5% - 8.5 years   8% - 5.5 years
   (ii) 3% ~ 24 years (not shown on graph so estimate)
   5% - 14 years   8% - 9 years

45 Reducible Interest

1. a) Missing values in bold

Time	Principal	Interest	Repayment	Amount Owing
End of 1st Year	$5000	$175	$700	**$4475**
End of 2nd Year	$4475	$156.63	$700	**$3931.63**
End of 3rd Year	**$3931.63**	$137.61	**$700**	$4093.74
End of 4th Year	**$4093.74**	$117.92	$700	**$2787.16**
End of 5th Year	**$2787.16**	**$97.55**	$700	**$2184.71**

   (i) $4475.00
   (ii) $3931.63
   (iii) $2184.71
   b) $3500
   c) $684.71
2. $2357.69
3. a) 3.625%   b) $2 525.77
4. a) $140 000   b) $140 896.53
   c) The interest charged was more than the repayment.
5. $1047.50

Why are mathematics books always sad?

They have lots of problems

