

**NEW
FORMAT**

AJ Sadler

**Mathematics
Applications**

2

Student Book

Unit 2





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PREFACE

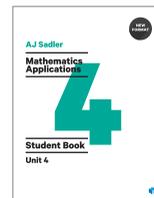
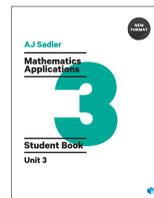
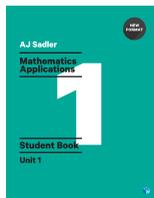
This text targets Unit Two of the West Australian course *Mathematics Applications*, a course that is organised into four units altogether, the first two for year eleven and the last two for year twelve.

The West Australian course, *Mathematics Applications*, is based on the Australian Curriculum Senior Secondary course *General Mathematics*. The main difference between Unit Two of these two courses is the inclusion of some work on the Normal Distribution in the West Australian course, covered in the final chapter of this text. Hence, by excluding the final chapter this text is also suitable for anyone following Unit Two of the Australian Curriculum course, *General Mathematics*.

The book contains text, examples and exercises containing many carefully graded questions. A student who studies the appropriate text and relevant examples should make good progress with the exercise that follows.

The book commences with a section entitled **Preliminary work**. This section briefly outlines work of particular relevance to this unit that students should either already have some familiarity with from the mathematics studied in earlier years, or for which the brief outline included in the section may be sufficient to bring the understanding of the concept up to the necessary level.

As students progress through the book they will encounter questions involving this preliminary work in the **Miscellaneous Exercises** that feature at the end of each chapter. These miscellaneous exercises also include questions involving work from preceding chapters to encourage the continual revision needed throughout the unit.



Some chapters commence with a **'Situation'** or two for students to consider, either individually or as a group. In this way students are encouraged to think and discuss a situation, which they are able to tackle using their existing knowledge, but which acts as a forerunner and stimulus for the ideas that follow. Students should be encouraged to discuss their solutions and answers to these situations and perhaps to present their method of solution to others. For this reason answers to these situations are generally not included in the book.

At times in this series of books I have found it appropriate to go a little outside the confines of the syllabus for the unit involved. In this regard readers will find in this text I have included some consideration of pie charts as a method of data display. Similarly, with Linear Relationships, whilst the syllabus concentrates on determining the slope and intercept from the equation or plot, I also include tables of values and include determining the equation knowing the gradient and one other point on the line or knowing two points on the line. When using the sine rule to determine the size of an unknown angle in non right triangles, I do not confine consideration to acute angles. However, when an obtuse angle is involved, the reader is told this fact so that ambiguous situations are still avoided.

Alan Sadler

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IMPORTANT NOTE

This series of texts has been written based on my interpretation of the appropriate *General Mathematics* syllabus documents (Australian Curriculum) and *Mathematics Applications* syllabus documents (Western Australia) as they stand at the time of writing.

It is likely that as time progresses some points of interpretation will become clarified and perhaps even some changes could be made to the original syllabus. I urge teachers of these courses, and students following the courses, to check with the appropriate curriculum authority to make themselves aware of the latest version of the syllabus current at the time they are studying the course.

Acknowledgements

As with all of my previous books I am again indebted to my wife, Rosemary, for her assistance, encouragement and help at every stage.

To my three beautiful daughters, Rosalyn, Jennifer and Donelle, thank you for the continued understanding you show when I am ‘still doing sums’ and for the love and belief you show in me.

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I wish to thank the following companies and organisations for allowing me to use some of their data as the basis for some of the questions, and occasionally in the theory sections, of this text:

Australian Bureau of Statistics

Bureau of Meteorology

National Heart Foundation of Australia

The New Internationalist Magazine

The Western Australian Police

West Australian Department of Education
and Training

Alan Sadler

PRELIMINARY WORK

This book assumes that you are already familiar with a number of mathematical ideas from your mathematical studies in earlier years.

This section outlines the ideas which are of particular relevance to Unit Two of the *Mathematics Applications* course and for which some familiarity will be assumed, or for which the brief explanation given here may be sufficient to bring your understanding of the concept up to the necessary level.

Read this ‘Preliminary work’ section and if anything is not familiar to you, and you don’t understand the brief explanation given here, you may need to do some further reading to bring your understanding of those concepts up to an appropriate level for this unit. (If you do understand the work but feel somewhat ‘rusty’ with regards to applying the ideas some of the chapters afford further opportunities for revision, as do some of the questions in the miscellaneous exercises at the end of chapters.)

- Chapters in this book will continue some of the topics from this preliminary work by building on the assumed familiarity with the work.
- The miscellaneous exercises that feature at the end of each chapter may include questions requiring an understanding of the topics briefly explained here.

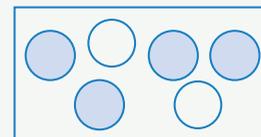
Use of number

The understanding and appropriate use of the rule of order, fractions, decimals, percentages, rounding, truncation, square roots and cube roots, numbers expressed with positive integer powers, e.g. 2^3 , 5^2 , 2^5 etc, expressing numbers in standard form, e.g. 2.3×10^4 (= 23 000), 5.43×10^{-7} (= 0.000 000 543), also called scientific notation, and familiarity with the symbols $>$, \geq , $<$, and \leq is assumed.

Ratios

The idea of comparing two or more quantities as a *ratio* should be something you are familiar with.

For example, for the diagram on the right the ratio of unshaded circles to shaded circles is 2 : 4 which simplifies to 1 : 2.



Suppose the ratio of males to females in a school is 17 : 21.

If we know that there are 231 females in the school we can determine the number of males.

$$\begin{aligned} \text{Males : females} &= 17 : 21 \\ &= ? : 231 \end{aligned} \quad \begin{array}{l} \nearrow \\ \searrow \end{array} \times 11$$

The number of males = 17×11 , i.e. 187.

$$231 / 21$$

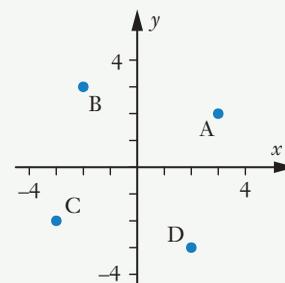
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Coordinates

It is assumed that you are familiar with the idea that points on a graph can be located by stating the coordinates of the point.

For example

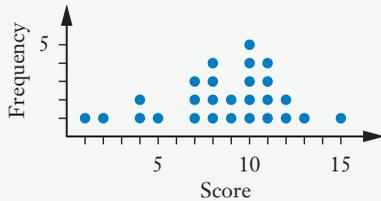
point A	has coordinates	(3, 2),
point B	has coordinates	(-2, 3),
point C	has coordinates	(-3, -2),
point D	has coordinates	(2, -3).



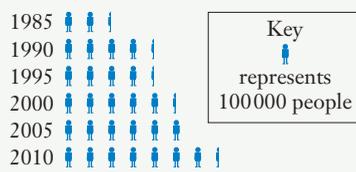
Data display

It is anticipated that you have already encountered the idea of counting or *tallying* data and organising it into tables, frequency tables and two way classification tables. The following forms of data presentation are also assumed to be familiar.

Dot frequency

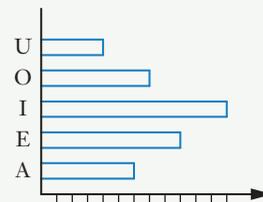


Pictogram



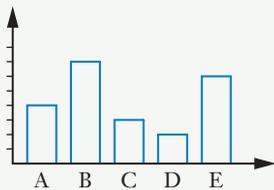
Bar graph

(horizontal bars)

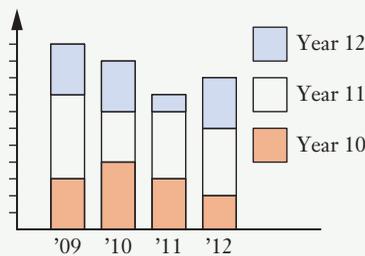


Bar graph (see below)

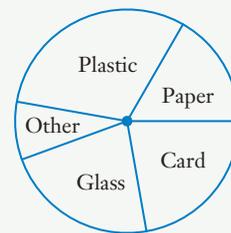
(vertical bars)



Proportional bar graph

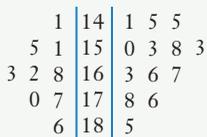


Circle graph

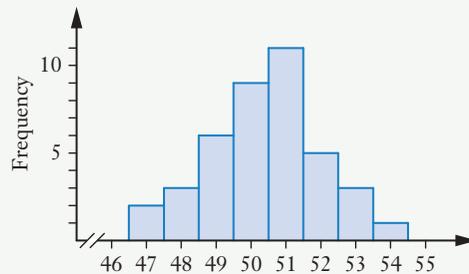


Some texts call a bar graph with vertical bars a column graph and restrict the name 'bar graph' to those with horizontal bars. In this text we will not make that distinction and will refer to both as bar graphs.

Stem and leaf plot



Frequency histogram



Data analysis

The **mean**, the **median** and the **mode** are all measures used to summarise a set of scores. They are all ways of giving an *average* or *typical* score for the set.

The **mean** is found by summing the scores and then dividing by the number of scores there are. The mean is the arithmetic average of the scores.

We use the symbol \bar{x} to represent the mean of a set of scores.

For example, for the ten numbers: 19, 25, 29, 28, 23, 15, 27, 22, 24, 21

$$\bar{x} = \frac{19 + 25 + 29 + 28 + 23 + 15 + 27 + 22 + 24 + 21}{10}$$

The mean of the ten numbers is 23.3.

The mean is a very useful measure of central tendency and is frequently used when analysing data. One disadvantage of the mean is that it can be greatly influenced by extreme scores, called **outliers**. For example if we add an eleventh score of 97 to the previous list the mean jumps from 23.3 to 30, i.e. the mean now exceeds all of the original ten scores. Clearly the score of 97 is a long way from the other scores. It is an extreme value and alters the mean significantly.

The **median** is the middle score in an ordered set of scores. If there are an even number of scores we say that the median is the mean of the ‘middle two’ in the ordered set.

For example, to determine the median of the seven numbers

29, 13, 27, 18, 33, 16, 29

we write them in order and choose the middle one:

13, 16, 18, 27, 29, 29, 33

The median of the seven scores is 27.

To determine the median of the eight numbers

29, 13, 27, 18, 33, 16, 29, 13

we write them in order and find the mean of the middle two:

13, 13, 16, 18, 27, 29, 29, 33

The median of the eight scores is 22.5.

The **mode**, or modal score, in a set of scores is the one that appears most frequently. There is of course no guarantee that the mode represents a score that is anywhere near the middle of the set of scores. It can be a useful and informative measure but is not necessarily ‘central’. The mode is used when we want the ‘most popular’ value.

If there are two scores that are equally ‘most popular’ we say the set of scores is **bimodal**, because it has two modes. We do **not** find the mean of the two modes. (Sets of scores with more than two modes could be referred to as multimodal.)

For the ten numbers: 7, 8, 5, 9, 9, 11, 9, 11, 8, 5 the mode is 9.

The set of ten numbers: 5, 8, 5, 9, 9, 11, 9, 11, 8, 5 is bimodal.

The modes are 5 and 9.

Make sure you agree with the given mean, median and mode for the following set of numbers:

3, 3, 1, 4, 3, 0, 5, 5, 5, 3, 1, 4, 5, 4, 2.

The mean of the set of numbers is 3.2, the median is 3 and the set is bimodal with modes of 3 and 5.

As well as wanting to summarise the data using a mean or median or mode we may also be interested to know how widely spread the data is.

One way of indicating this is by stating the **range**, which is simply the difference between the highest score and the lowest score.

For example the set of numbers 7, 8, 5, 9, 9, 11, 9, 11, 8, 5 have a range of 6, obtained by working out $(11 - 5)$.

Whilst the range is easy to calculate it is determined using just two of the scores and does not take any of the other scores into account. For this reason it is of limited use.

Formulae

From previous work, probably from *Unit One* of the *Mathematics Applications* course, you should be familiar with the idea of using a formula to determine the value of a variable, or pronumeral, that appears in the formula by itself and on one side of the equals sign, given the values of the variables, or pronumerals appearing on the other side.

For example:

Given $A = P + I$ we could determine A , given P and I :

If $P = 200$ and $I = 15$ it follows that

$$\begin{aligned}A &= 200 + 15 \\ &= 215\end{aligned}$$

Given $C = 2\pi r$ we could determine C , knowing π and given r :

If $r = 4$, it follows that

$$\begin{aligned}C &= 2 \times \pi \times 4 \\ &= 25.133 \text{ (rounded to three decimal places)}\end{aligned}$$

Given $s = ut + \frac{1}{2}at^2$ we could determine s , given u , a and t :

If $u = 4$, $a = 10$ and $t = 6$ it follows that

$$\begin{aligned}s &= 4 \times 6 + \frac{1}{2} \times 10 \times 6^2 \\ &= 204\end{aligned}$$

Algebra

You should also be familiar with evaluating **expressions** such as $2x + 3$, $5x - 2$, $5y + 4$, $2(x + 3)$, $3xy + 2z$ etc, given the values of x , y , and z .

If $x = 2$, $y = 3$ and $z = -5$ then $2x + 3 = 7$

$$5x - 2 = 8$$

$$5y + 4 = 19$$

$$2(x + 3) = 10$$

$$3xy + 2z = 8$$

The idea of **expanding brackets** should also be familiar to you:

The expression $3(x + 2)$ means 'three lots of $(x + 2)$ '. Think of the bracket as a parcel containing an x and a 2.

If we open the three parcels we have three x s and three 2s, i.e. $3x + 6$. We call this *expanding* the brackets.

Thus $5(x + 4)$ expands to $5x + 20$

$7(2x + 5)$ expands to $14x + 35$

$-2(3x - 4)$ expands to $-6x + 8$

If we are expanding several brackets we may be able to simplify our answer.

For example $3(2x + 1) + 5(x + 3) = 6x + 3 + 5x + 15$
 $= 11x + 18$

For example $4(x + 3) - 3(x + 2) = 4x + 12 - 3x - 6$
 $= 1x + 6$

Usually written: $x + 6$.



Substitution code puzzle



Collecting like terms



Multiplying and dividing



Expanding algebra



Similar triangles

To know whether two triangles are similar we can:

- see if the 3 angles of one triangle are equal to the 3 angles of the other triangle.
- OR
- see if the lengths of corresponding sides are in the same ratio.
- OR
- see if the lengths of two pairs of corresponding sides are in the same ratio and the angles between the sides are equal.

Once we know that two triangles are similar then we know that corresponding sides are in the same ratio.

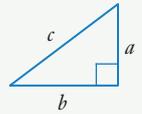
The Pythagorean theorem

From unit one of this course you should be familiar with the following fact:

The square of the length of the hypotenuse of a right angled triangle is equal to the sum of the squares of the lengths of the other two sides.

Thus, for the triangle shown on the right, $c^2 = a^2 + b^2$

The theorem of Pythagoras allows us to determine the length of one side of a right triangle, knowing the lengths of the other two sides.



Probability

The work of chapter 13 assumes a basic understanding of the idea that the probability of something happening is a measure of the likelihood of it happening and this measure is given as a number between zero (no chance of happening) to 1 (certain to happen).

If we roll a normal die once the probability of getting a 3 is one sixth.

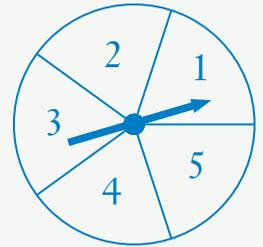
We write this as: $P(3) = \frac{1}{6}$.

For each spin of the spinner shown on the right

$$P(1) = \frac{1}{5}, \quad P(2) = \frac{1}{5},$$

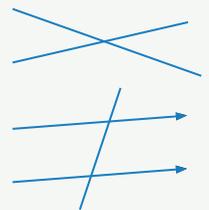
$$P(\text{odd number}) = \frac{3}{5}, \quad P(\text{even number}) = \frac{2}{5},$$

$$P(\text{number} > 5) = 0, \quad P(\text{number} < 6) = 1.$$



Geometry

It is assumed that you are familiar with the fact that when two straight lines intersect, the vertically opposite angles are equal, and with angle facts relating to alternate angles, corresponding angles and co-interior angles with parallel lines.

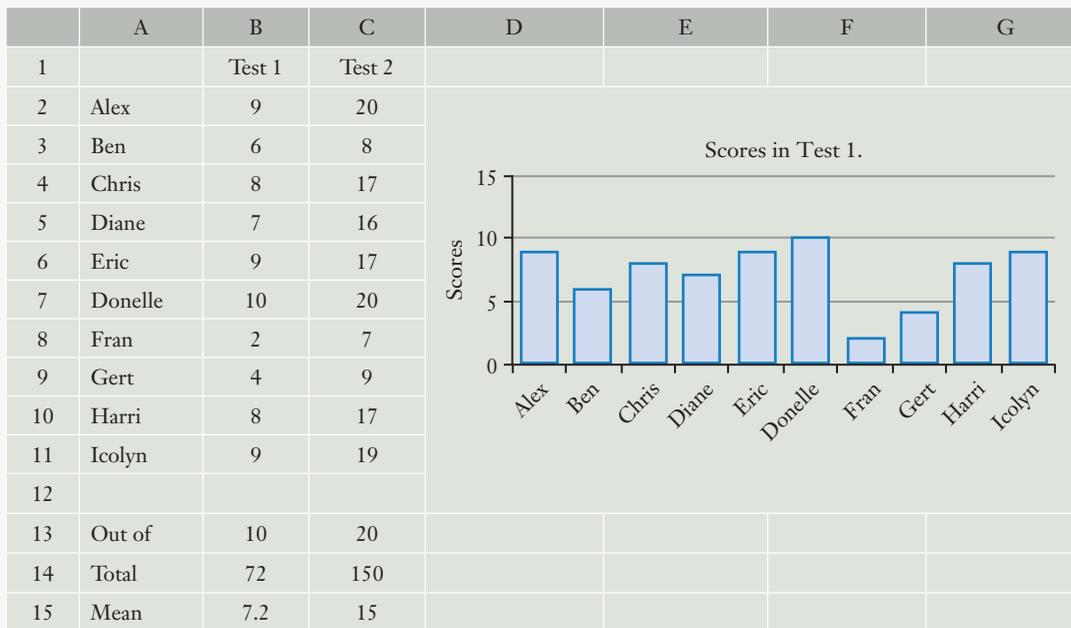
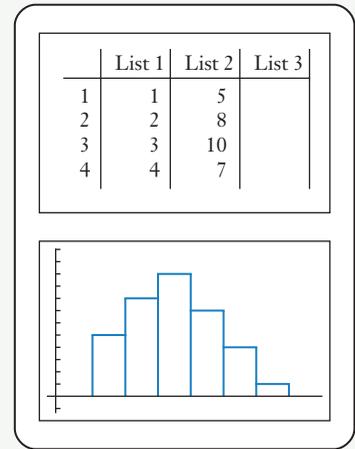


Use of technology to process and display data

Students following this course may come to this unit with very varied abilities in the use of technology such as graphic calculators and computers. Whatever your initial ability you are encouraged to make use of such technology whenever appropriate.

As the course progresses, do try to:

- a** gain familiarity with entering data into the columns of a calculator with statistical capabilities and showing various statistical information and displays about that data.
- b** become familiar with entering data into the columns of a spreadsheet on a computer or calculator and of carrying out straightforward operations on those entries such as adding a list of numbers, finding their average and presenting the data as a graph, as shown below.



1.

Univariate data: Classify, organise and display

- Types of data
- Variables
- Categorical variables
- Displaying categorical data
- Numerical variables
- Displaying numerical data
- Histograms and bar charts
- Miscellaneous exercise one

Situation: Cardiovascular disease

A doctor is to give a lecture on cardiovascular disease, i.e. diseases of the heart or blood vessels, including heart attack and stroke. Using statistics for the previous year she wants to start her lecture by saying:

‘Did you know that of the _____ deaths in Australia last year, from all causes, _____% of these were due to cardiovascular disease. This means that last year, on average, one Australian died from cardiovascular disease every _____ minutes.’

Copy and complete the above introduction to her talk using the information given in the table below to fill in the blanks. (Give the percentage to the nearest percent and the time to the nearest minute.)

TOTAL AUSTRALIAN DEATHS FOR YEAR PRIOR TO TALK: All ages

CAUSE OF DEATH	MALE	FEMALE	PERSONS
CARDIOVASCULAR DISEASE			
Coronary heart disease	12 433	11 137	23 570
Stroke	4 668	6 845	11 513
Other cardiovascular	4 856	6 195	11 051
(Sub-total)	21 957	24 177	46 134
CANCERS	22 039	17 183	39 222
TRANSPORT ACCIDENTS	1 224	414	1 638
ALL OTHER	22 021	21 699	43 720
ALL CAUSES	67 241	63 473	130 714

[Source of data: National Heart Foundation of Australia and The Australian Bureau of Statistics.]

The situation above involved you in making sense of information, or **data**, that was presented as a table, extracting relevant information from that table, and summarising that data in terms of percentages and times.

The data involved in the table would have been collected from government data bases where the cause of death for each deceased person, as stated on the death certificate, would be stored.

Data collection is often carried out to investigate some aspect of our lives. The methods by which we collect that data can vary as can the types of data we collect. The initial chapters of this text consider

- the various types of data,
- organising and displaying data,
- describing and interpreting data,
- comparing sets of data.



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Types of data

For some investigations we collect the data ourselves by asking questions, by measuring, by experiment etc. This is called **primary data** – data we have collected ourselves.

Sometimes it is appropriate to use the data already collected by others, as in the situation on the previous page. For us this would be **secondary data** – data collected by others.

Note: Internet access provides us with a ready supply of information regarding all sorts of subjects. Whilst much of this may well be valid we need to be cautious before assuming certain things about the information. We might consider such things as:

- Was the method of data collection appropriate?
- Is the information fairly presented?
- Is the data collected from everyone in a particular situation or was sampling involved?
- Are any summary statements correct?
- Are any conclusions reasonable?

One way to have confidence in the validity of the information is to use data from a reputable source. *The Australian Bureau of Statistics* (ABS), for example, would be one such credible source, as would other Government departments. Such sources are likely to have correctly considered collection methods, suitable presentation of data, appropriate conclusions etc.

Variables

If we ask someone what their favourite colour is or how tall they are the answers we get will vary from one person to another. Not everyone has blue as their favourite colour, not everyone is 163 cm tall etc. The responses will vary. Favourite colour and height are examples of **variables**.

If we are considering *one* variable, such as favourite colour, any data we collect will be **univariate**. However if we wanted to investigate how favourite colour may change with the age of a person we would be considering two variables – favourite colour and age. In such cases we would be considering **bivariate** data. This unit will only consider univariate data. Unit three of the *Mathematics Applications* course considers bivariate data.

Categorical variables

Consider the following questions that might be asked in some data collection activities:

- How do you get to school (or work) – walk, cycle, car, bus, train, other?
- Are you male or female?
- What is your favourite colour?

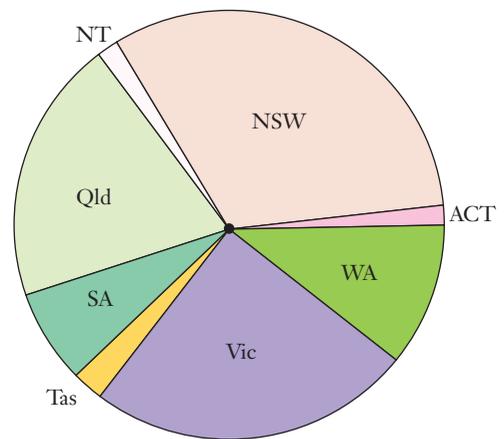
Displaying categorical data

We tend to use tables, pie charts and bar graphs to organise and display categorical data, as shown below.

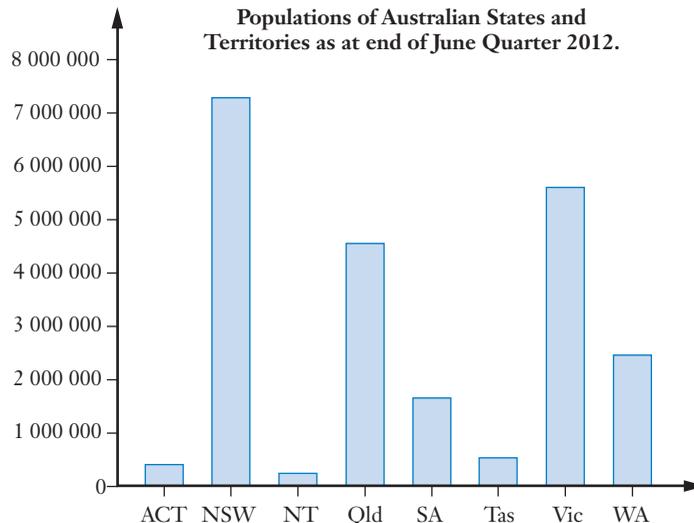
State or Territory	Population at end of June quarter 2012 (nearest hundred)
Australian Capital Territory (ACT)	374 700
New South Wales (NSW)	7 290 300
Northern Territory (NT)	234 800
Queensland (Qld)	4 560 100
South Australia (SA)	1 654 800
Tasmania (Tas)	512 000
Victoria (Vic)	5 623 500
Western Australia (WA)	2 430 300
Total	22 680 500

Source: Australian Bureau of Statistics.

Pie Chart of Australian State and Territory populations as at end of June Quarter 2012. (Total population 22 680 500.)



Populations of Australian States and Territories as at end of June Quarter 2012.



Notice that we may choose some particular order in which to display this nominal categorical data, for example in alphabetical order (as used above), or perhaps in order of population size or land area, or perhaps in order of state or territory formation date, or in order of number of letters in their non-abbreviated titles (!), etc, but on the basis of the categories themselves they have no natural order about them.



Statistical data
match-up

Numerical variables

Consider the following questions that might be asked in some data collection activities:

- How many people live in your house?
- How many people in your Mathematics class?
- How many pets do you have?
- How many of your teeth have been filled in some way?
- How tall are you?
- How far have you walked today?
- What is your blood pressure reading?
- What weight are you?

The responses to each of these questions will be **numerical** and the number will not just be a label, it will indicate a size or an amount. Some form of counting or measuring will be required to be done, or to have been done, for the response to be given. In each case the variable concerned, be it the number of people in a house, how many pets you have, your height, your blood pressure reading etc, are all examples of **numerical variables**. Data associated with a numerical variable is called **numerical data**.

Notice that the first four questions each involve responses that can only take specific values, in this case integer values. We cannot have 2.3 people living in a house, we cannot have 28.6 people in a Mathematics class. Numerical variables that can only take integer values are called **discrete variables**. Data associated with discrete variables is **discrete data**.

However the last four questions each involve responses that can take any value (usually within some realistic range). We can be 164.5 cm tall, we can walk 5.278 km in a day. Responses no longer have to be integer values and in practice the limit on the values taken are those of reasonableness (eg we cannot have 18.24 metres for the height of someone) and the accuracy of the measurement instrument used. Numerical variables that can take any value in an interval are called **continuous variables**. Data associated with a continuous variable is **continuous data**.

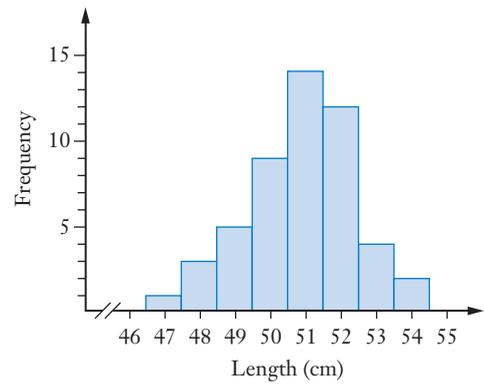
Generally, if counting is involved we have a discrete variable, if measurement is involved we have a continuous variable.

Exercise 1B

For 1 to 12 classify each of the numerical variables given as either *discrete* or *continuous*.

- 1 The number of rooms in a house.
- 2 The area of the block of land a house stands on.
- 3 The number of brothers and sisters a person has.
- 4 The length of a person's handspan.
- 5 The time it takes to get to school.
- 6 The lifetime of a rechargeable battery before it needs recharging.

There was 1 baby recorded as 47 cm, when measured to the nearest centimetre. Thus for the 47 cm class the lower class boundary is 46.5 cm and the upper class boundary is 47.5 cm. This gives the first column in the histogram shown.

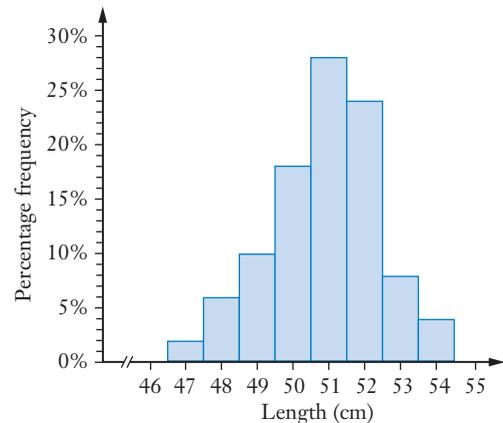
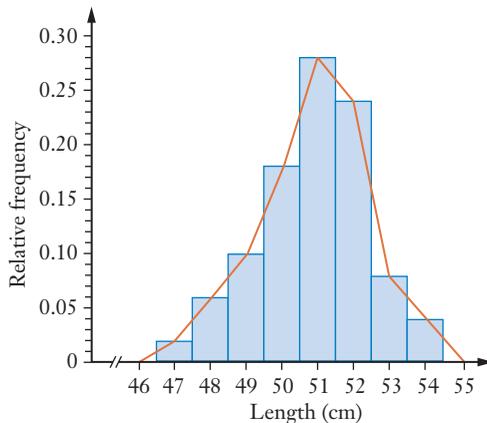


Note

- A histogram does not have gaps between the columns because, with continuous data, each class begins where the previous one leaves off. (Except by the apparent 'gap' between columns if a column has a frequency of zero.)
- We would usually have about 6 to 10 intervals. Too many and the table can be unmanageable, too few causes unnecessary bunching.
- It is usual to have class intervals of equal width. Intervals of different width could be misleading (and will be avoided in this text).
- The vertical axis on a histogram is always frequency.
- The horizontal axis on a histogram is a number line.
- Sometimes the frequency in each class may be shown as a fraction of the whole (*relative frequency*) or as a percentage of the whole (*percentage frequency*).

Note that one of the histograms below also shows a **frequency polygon**, formed by connecting the middle of the top of each bar to the next, thus making the overall shape and continuity more evident.

Length (cm)	47	48	49	50	51	52	53	54
Frequency	1	3	5	9	14	12	4	2
Relative frequency	0.02	0.06	0.10	0.18	0.28	0.24	0.08	0.04
Percentage frequency	2%	6%	10%	18%	28%	24%	8%	4%



EXAMPLE 1

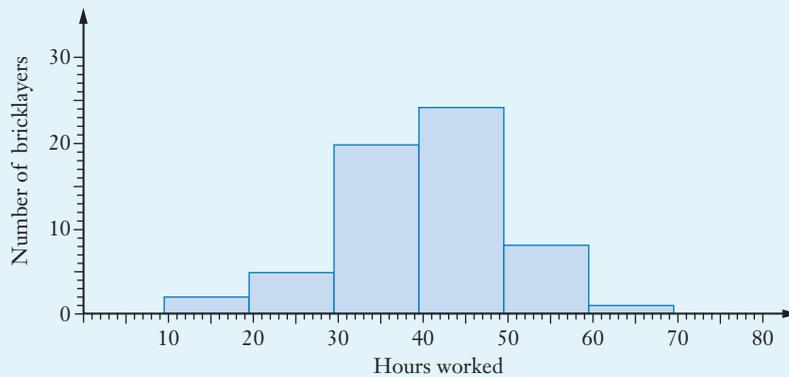
Sixty self-employed bricklayers were surveyed regarding the number of hours they worked during one particular week (to the nearest hour). The results of the survey are shown.

Display the data as a histogram.

Solution

The first column of the histogram will extend from 9.5 to 19.5, the second column from 19.5 to 29.5 and so on. The completed histogram is shown below.

Number of hours	Frequency (i.e. No. of bricklayers)
10 → 19	2
20 → 29	5
30 → 39	20
40 → 49	24
50 → 59	8
60 → 69	1



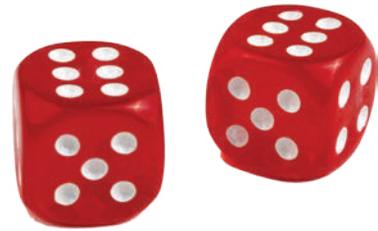
EXAMPLE 2

The road accident statistics for a country for one year showed that for motorcyclists (drivers not passengers) in the age range fifteen to fifty-nine, 186 had died in road accidents with the distribution of the ages of these riders as shown on the right.

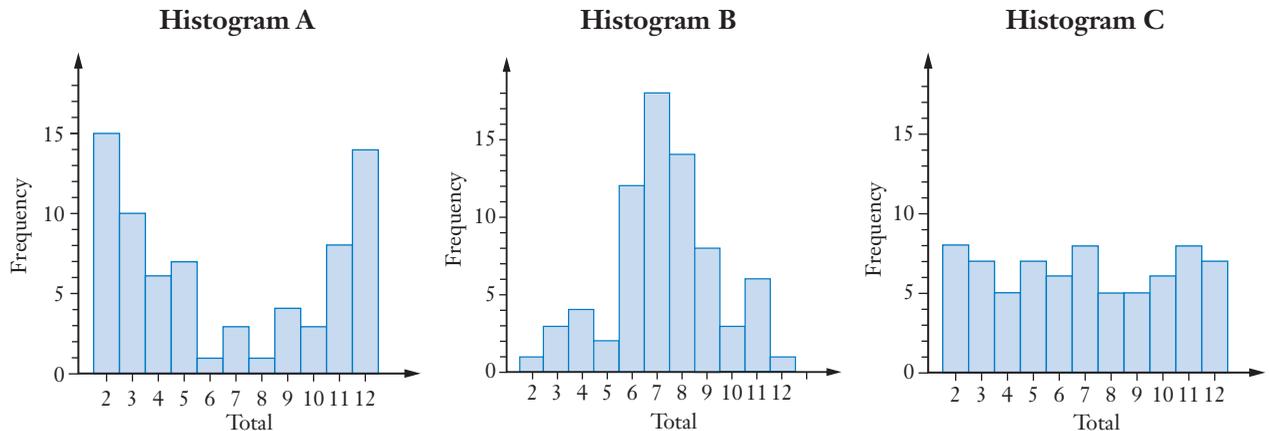
Display this information as a frequency histogram.

Age (x yrs)	Drivers killed
$15 \leq x < 20$	40
$20 \leq x < 25$	59
$25 \leq x < 30$	29
$30 \leq x < 35$	19
$35 \leq x < 40$	16
$40 \leq x < 45$	11
$45 \leq x < 50$	8
$50 \leq x < 55$	2
$55 \leq x < 60$	2

- 3** Two normal fair six-sided dice are rolled and the numbers displayed on the two uppermost faces are added together and the total noted. This process is carried out 72 times. Given that one of the three frequency histograms shown below displays the results obtained for this activity which of the three is it most likely to be?



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- 4** What shape will the histogram be?

Sketch what you consider to be a reasonable histogram for each of the following. Your sketch should show what you consider to be a reasonable shape for the histogram to have. There is no need to include any numbers on the axes of your sketch graphs except for parts **a** and **b** which should have numbers on the horizontal axis.

- a** Rolling a fair 8-sided die approximately one hundred times.
- b** The number of children in approximately one hundred randomly selected families, each of which have at least one child.
- c** The heights of a large number of adult males.
- d** The straight line distance from home to school for students at your school.
- e** The lengths of new-born babies.

For at least some of Questions 5 to 10 generate the required histogram using a computer or calculator.

5 HORTICULTURE

Some seeds were planted and, some weeks later, the heights of the seedlings were measured and recorded, to the nearest centimetre. The results were as follows:

Length (cm)	6	7	8	9	10	11	12	13	14	15
Frequency	2	4	10	15	14	11	9	7	3	1

Display this information as a frequency histogram.

9 WEIGHT

200 males and 200 females, all aged between 30 and 40, took part in a survey which recorded (amongst other things) the weight of each person, recorded to the nearest kg. The results were as follows:

MALES	
Weight (kg)	Frequency
30 – 39	0
40 – 49	2
50 – 59	10
60 – 69	35
70 – 79	68
80 – 89	53
90 – 99	21
100 – 109	9
110 – 119	2
Total	200

FEMALES	
Weight (kg)	Frequency
30 – 39	5
40 – 49	12
50 – 59	73
60 – 69	66
70 – 79	27
80 – 89	8
90 – 99	5
100 – 109	3
110 – 119	1
Total	200

Display these results as two separate frequency histograms and include the frequency polygon on each one.

10 HEIGHT

50 males and 50 females, all aged between 20 and 30, took part in a survey which recorded (amongst other things) the height of each person, recorded to the nearest cm. The results were as follows:

MALES	
Height (cm)	Frequency
140 – 149	0
150 – 159	1
160 – 169	6
170 – 179	26
180 – 189	14
190 – 199	2
200 – 209	1

FEMALES	
Height (cm)	Frequency
140 – 149	1
150 – 159	11
160 – 169	28
170 – 179	8
180 – 189	2
190 – 199	0
200 – 209	0

Display these results as two separate **percentage** frequency histograms.

8 Expand each of the following and simplify where possible.

a $3(5x - 2)$

b $4(7 - 2x)$

c $-3(2x + 7)$

d $8(1 - 2x)$

e $5(2p - 7)$

f $3(2h - 5)$

g $5 + 2(1 + 3x)$

h $4(2x + 1) - 5(3 + 2x)$

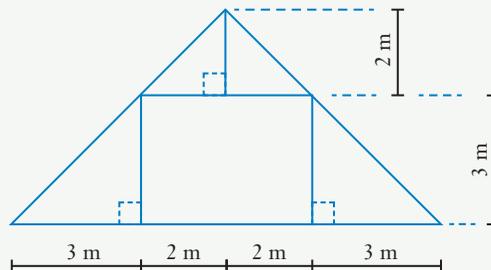
i $2x + 3(5 - 2x)$

j $2(5 + q) - 3(1 - 2q)$

k $6(2w + 3) - 5w + 4$

l $2(p + 6) - 4(3 - p)$

9 The framework shown is to be made of steel. Find the total length of steel required for the framework giving your answer rounded up to the next whole metre.



10 HOLIDAY ACTIVITIES

A sports club offers holiday activities for young people aged at least 5 but under 15. The enrolment forms required participants to give, amongst other things, their age in years and months. To determine which ages their programme of activities suited, club officials considered the ages of participants. These are shown below.

5y 3m	14y 5m	7y 2m	6y 2m	10y 5m	8y 10m	8y 3m	7y 5m
12y 2m	6y 1m	13y 3m	9y 9m	6y 1m	6y 3m	11y 5m	8y 5m
9y 11m	13y 0m	7y 10m	12y 2m	8y 0m	14y 7m	6y 9m	9y 11m
8y 7m	7y 1m	8y 0m	7y 5m	7y 0m	5y 3m	10y 0m	6y 7m
14y 10m	11y 2m	5y 8m	12y 1m	11y 7m	7y 3m	12y 3m	6y 11m
6y 1m	7y 9m	10y 2m	11y 2m	5y 2m	7y 3m	7y 4m	9y 2m
13y 2m	12y 4m	7y 2m	6y 3m	9y 4m	8y 0m	6y 9m	7y 7m
11y 11m	6y 1m	13y 9m	13y 7m	6y 9m	12y 7m	11y 2m	6y 7m
5y 1m	9y 0m	7y 7m	14y 0m	13y 7m	6y 9m	9y 7m	12y 11m
8y 7m	14y 1m	13y 9m	5y 11m	8y 11m	12y 7m	5y 7m	8y 1m
8y 6m	7y 11m	6y 4m	13y 10m	7y 6m	6y 5m	6y 2m	6y 6m

- a Arrange these ages into a grouped frequency table as shown (tally shows first column entered):
- b Display the grouped data as a frequency histogram.

Age (x years)	Tally	Frequency
$5 \leq x < 6$		
$6 \leq x < 7$		
$7 \leq x < 8$		
$8 \leq x < 9$		
$9 \leq x < 10$		
$10 \leq x < 11$		
$11 \leq x < 12$		
$12 \leq x < 13$		
$13 \leq x < 14$		
$14 \leq x < 15$		

2.

Summarising data and describing distributions

- Combining groups
- Use of statistical functions on a calculator
- Grouped data
- Describing a distribution of scores
- Miscellaneous exercise two

Situation: Company salaries

A particular company employs thirteen people:

1 managing director,
2 managers,
1 accountant,
3 chargehands,
6 machine operators.

The salary structure in the company is shown below.

Managing director		\$115 000
Managers		\$90 000 each
Accountant		\$78 000
Chargehands		\$62 500 each
Machine operators		\$55 000 each

- Find the company's mean salary, median salary and modal salary.
- If a group were arguing that higher salaries should be awarded to the people working for this company which of the previous three answers would they claim to be the average to best suit their argument?
- If the managing director wished to quote an average salary for this company, in support of her claim that the average salary was already high, which average best suits her argument?
- If this company took over another company, retaining all 7 employees of the other company on their existing salaries, which had a mean of \$66 000, what would be the mean salary of the new '20 employee' company?



iStock.com/George Pchemyan

In the situation on the previous page you had to summarise information by determining the mean, median and mode, concepts that you were reminded of in the *Preliminary work* section at the beginning of this book and that some of the questions in *Miscellaneous exercise one* required you to find. The mean, median and mode summarise the location of a set of scores. They are *summary statistics* and are *measures of location*. The mean and the median give an indication of *central tendency*.

The next three examples remind you how to determine these quantities from a frequency table (example 1), a dot frequency graph (example 2) and a stem and leaf plot (example 3).



Mode, median and mean



Measures of central tendency



Mean, median, mode and range



Frequency distribution table

EXAMPLE 1

The fifty scores shown below

14 17 15 17 12 16 19 16 17 16
 16 10 15 17 18 17 14 16 17 16
 18 19 20 14 15 18 18 18 15 17
 15 16 17 18 16 16 13 15 18 15
 17 17 15 16 19 15 17 18 13 14

can be neatly displayed in the form of a frequency table, as shown below:

Score	10	11	12	13	14	15	16	17	18	19	20
Frequency	1	0	1	2	4	9	10	11	8	3	1

Determine the mode, the mean and the median of the fifty scores.

Solution

The mode is readily determined from this table by seeing which score has the greatest frequency. The mode of the set of scores is 17.

The table tells us that we have one 10, zero 11s, one 12, two 13s etc. Hence, to determine the total of the fifty scores we calculate:

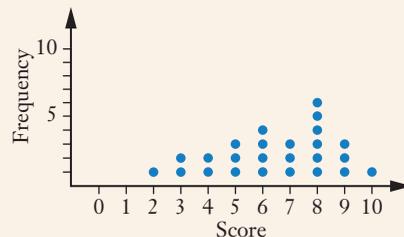
$$\begin{aligned} & 1 \times 10 + 1 \times 12 + 2 \times 13 + 4 \times 14 + 9 \times 15 + 10 \times 16 + 11 \times 17 + 8 \times 18 + 3 \times 19 + 1 \times 20 \\ = & 10 + 12 + 26 + 56 + 135 + 160 + 187 + 144 + 57 + 20 \\ = & 807 \end{aligned}$$

Hence the mean will be given by $\frac{807}{50}$ i.e. 16.14.

The median of fifty scores will be the mean of the 25th and 26th scores once the scores have been written in order. Summing the frequencies in the above table from the left end we see that the 25th and 26th scores will both be 16. Thus the median score is 16.

EXAMPLE 2

Determine the mode, mean, median and range for the set of scores shown in the dot frequency graph.



Combining groups

Knowing the number of scores and their sum we can calculate the mean of the scores. For example if 15 scores have a sum of 108 the scores have a mean of

$$\frac{108}{15} = 7.2$$

It also follows that if we know the number of scores and their mean we can find the sum of the scores.

For example, knowing that 15 scores have a mean of 7.2 the scores must have a sum of

$$15 \times 7.2 = 108$$

Indeed you probably used this idea if you managed the last dot point of the *Situation* at the beginning of the chapter. In that situation we were told that the 7 employees of a company had a mean salary of \$66 000 from which we could calculate the total amount paid out for these salaries as

$$7 \times \$66\,000 = \$462\,000$$

This idea of determining the total of a set of scores, knowing the mean and the number of scores can be useful when solving some problems as examples 4, 5 and 6 show.

EXAMPLE 4

The mean of six scores is 23.5. If five of the scores were 17, 20, 19, 25 and 30 find the sixth score.

Solution

If six scores have a mean of 23.5 then these six scores have a sum of $23.5 \times 6 = 141$.

The five given scores have a sum of $17 + 20 + 19 + 25 + 30 = 111$

Thus the sixth score must be $141 - 111 = 30$

The sixth score is 30.

EXAMPLE 5

To pass a particular course a student needs to gain a mean of at least 55% in the five tests that form the course assessment. In the first four tests the student achieves marks of 46%, 57%, 54% and 57%. What percentage mark must the student gain in test five if they are to pass the course?

Solution

To gain a 55% average in 5 tests the total marks in the 5 tests must be $55 \times 5 = 275$

The first 4 tests have a sum of $46 + 57 + 54 + 57 = 214$

Thus in the fifth test the student needs $275 - 214 = 61$

The student needs to score at least 61% in the fifth test in order to pass the course.

EXAMPLE 6

In a test the 15 girls in a class score a mean mark of 21.2 and the ten boys score a mean mark of 22.4. Calculate the mean for the whole group of 25 students.

Solution

The 15 girls achieved a mean of 21.2 thus they gained a total mark of $15 \times 21.2 = 318$.

The 10 boys achieved a mean of 22.4 thus they gained a total mark of $10 \times 22.4 = 224$.

Thus the 25 students gained a total mark of $318 + 224 = 542$

The 25 students achieved a mean mark of $\frac{542}{25} = 21.68$

Exercise 2A

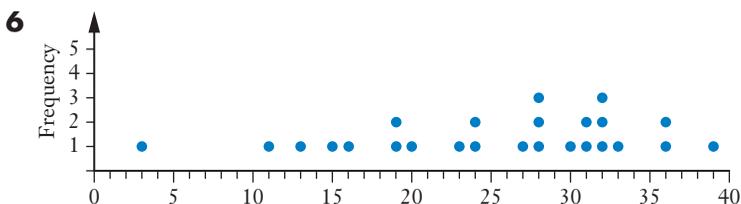
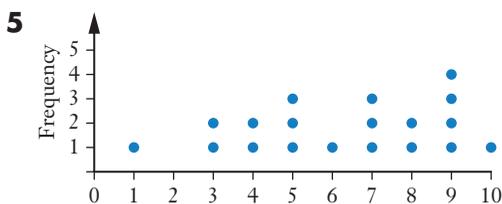
Find the mean, median, mode and range of each of the distributions shown in questions 1 to 6 (correct to one decimal place if necessary).

1	Score	0	1	2	3	4	5
	Frequency	1	2	2	3	4	7

2	Score	0	1	2	3	4	5	6	7	8	9	10
	Frequency	1	0	2	3	2	4	7	6	2	3	1

3	Score	15	16	17	18	19	20
	Frequency	1	2	7	8	3	1

4	Score	98	100	101	103	104	105
	Frequency	1	1	3	3	1	1



- 7** The stem and leaf diagram on the right can accommodate numbers from 110 to 169, the first two digits being the stem and the last digit the leaf. The diagram shows the scores recorded by 25 competitors in a shooting competition. For the scores recorded determine:

16	4	4
15	3	1 8 7
14	0	1 7 9 5 3
13	0	9 7 4 2 5 6 2
12	0	6 5
11	9	8

- a** the lowest score **b** the highest score
c the median score **d** the mean score

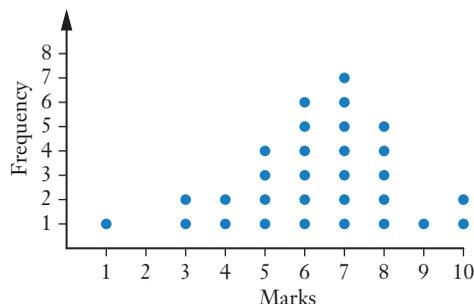
- 8** The stem and leaf diagram on the right shows scores achieved by 24 students in an exam that was marked out of 120. The stem is the tens digit and the leaf is the units digit. Determine

9	4	0	2
8	2	5	8 1 2 5 4 3
7	5	7	4 4 7
6	1	7	9 8 4 8 6
5			
4			8

- a** the lowest score
b the highest score
c the median score
d the mean score (correct to 1 decimal place)

- 9** Estimate the mean number of thumbs per Australian adult.

- 10** The dot frequency diagram on the right shows the marks obtained by 30 year 8 students in a mental arithmetic test. Determine the mean, median and mode for this distribution of marks.



- 11** A class of school students calculated their mean age as 17.2 years, correct to one decimal place. This average did not include the teacher. If the teacher were to be included would the mean be increased or decreased?

- 12** The mean of eight scores is 54.25. If seven of the scores were 60, 50, 37, 60, 55, 32 and 65 find the eighth score.

- 13** Three mathematics classes sat the same exam. The mean marks for the classes were 55%, 62% and 56% and the number of students in each class were 24, 15 and 21 respectively. Find the mean for the three groups put together.



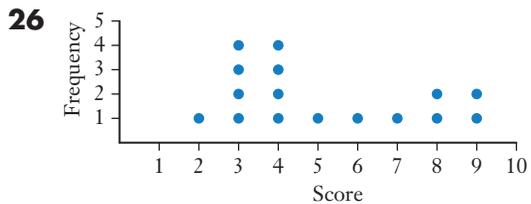
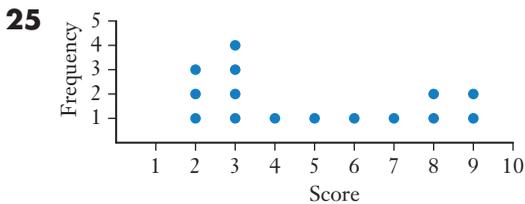
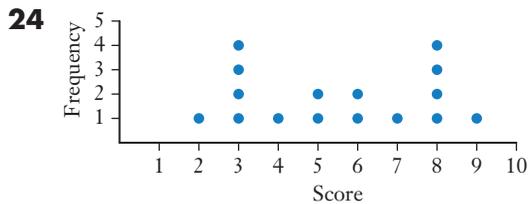
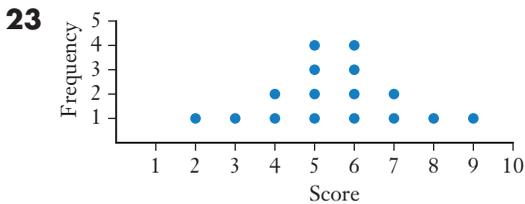
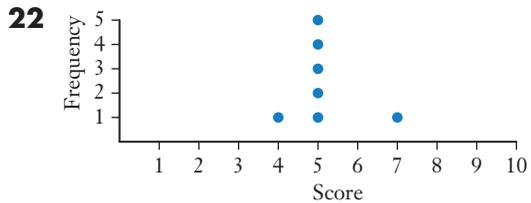
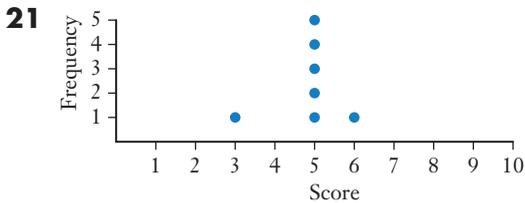
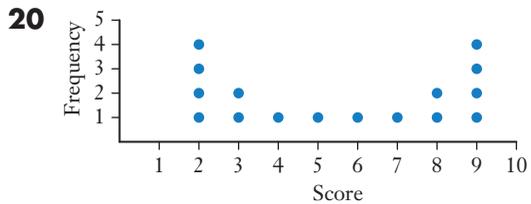
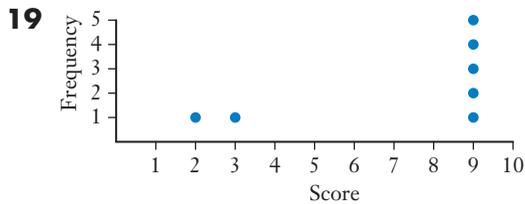
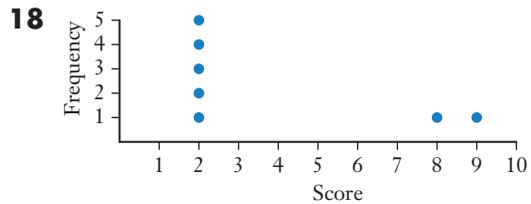
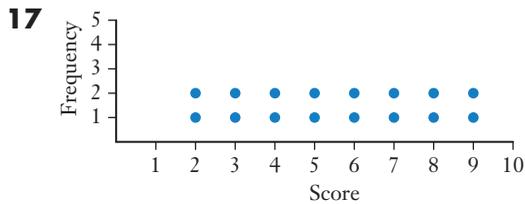
- 14** To pass a particular course a student has to achieve a mean of at least 50% in the ten pieces of work that form the assessment items. In the first nine of these pieces of work the student achieves a mean of 46%. What percentage mark must the student achieve in the tenth item if he is to pass the course?

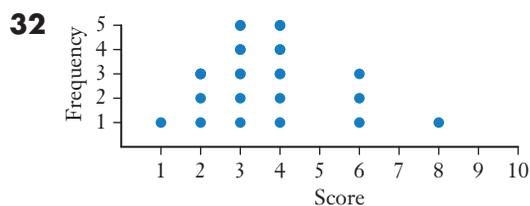
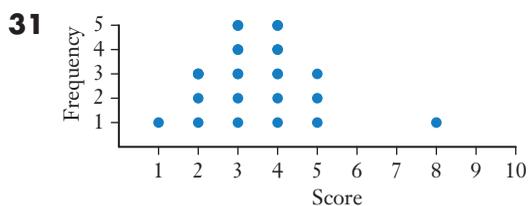
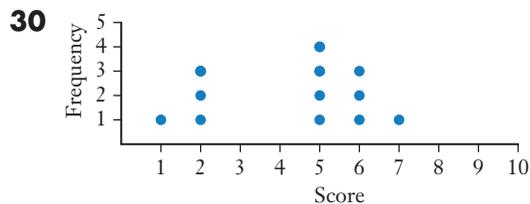
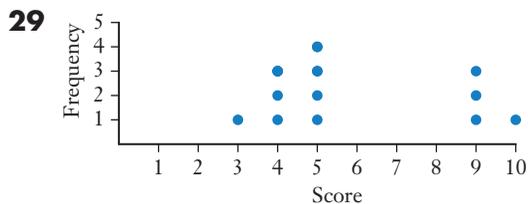
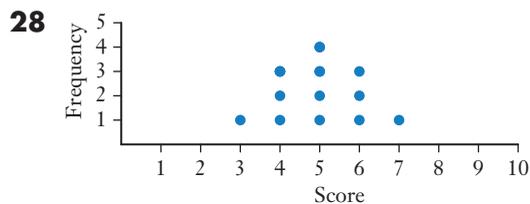
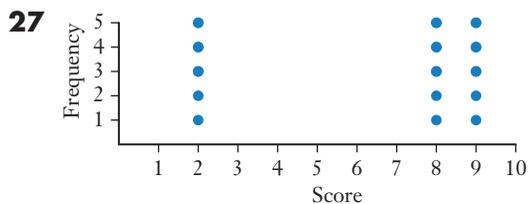
- 15** The mean of 25 scores is 54. If 20 of the scores had a mean of 55 find the mean of the other five scores.

- 16** The 13 boys in a class gained a mean mark of 57% in a test in which the class mean was 59%. If the class consisted of 20 students altogether find the mean achieved by the girls in the class. (Give your answer correct to one decimal place.)

For each of the dot frequency graphs shown in questions 17 to 32, without actually calculating the mean and the median, state which one of the following statements apply:

- The mean is the same as the median.
- The mean is greater than the median.
- The mean is less than the median.



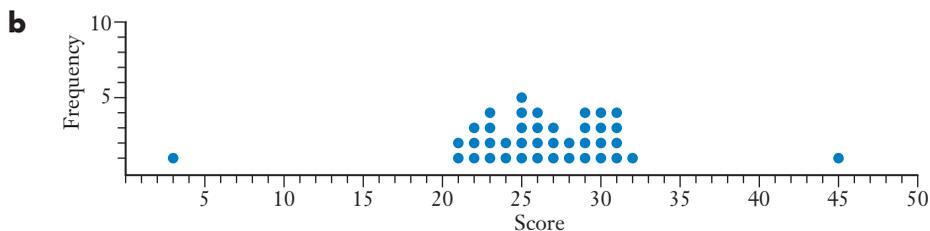
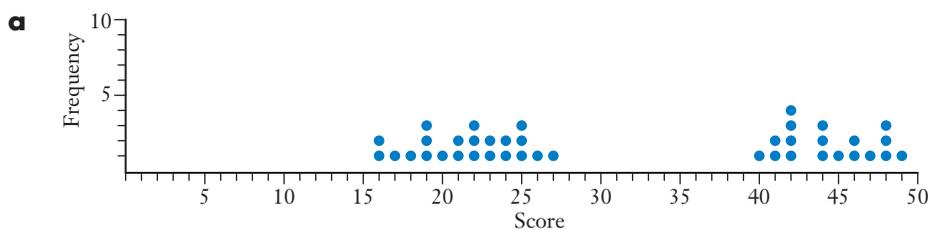


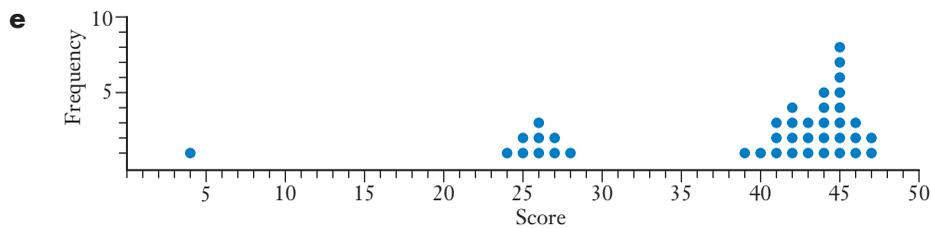
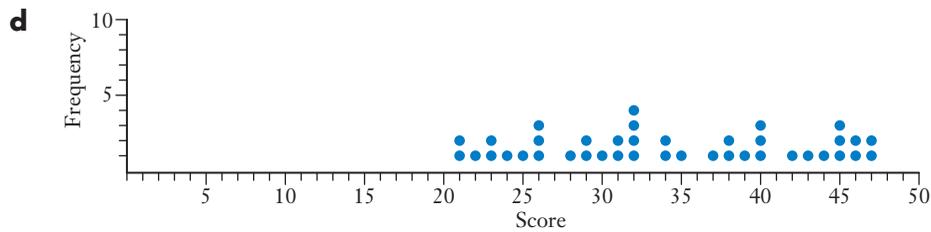
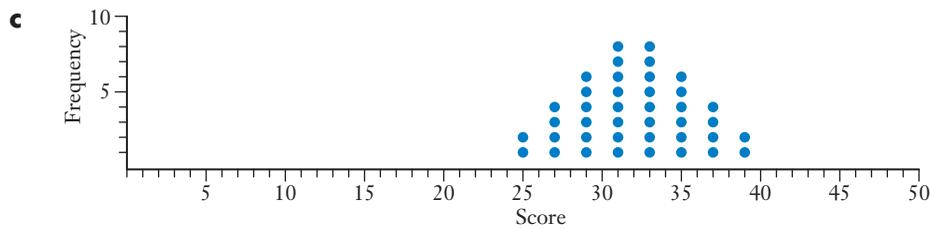
33 Whilst each of the dot frequency diagrams shown in this question feature 40 data points they show quite different distributions of the 40 scores.

Write a few sentences describing each distribution of 40 scores.

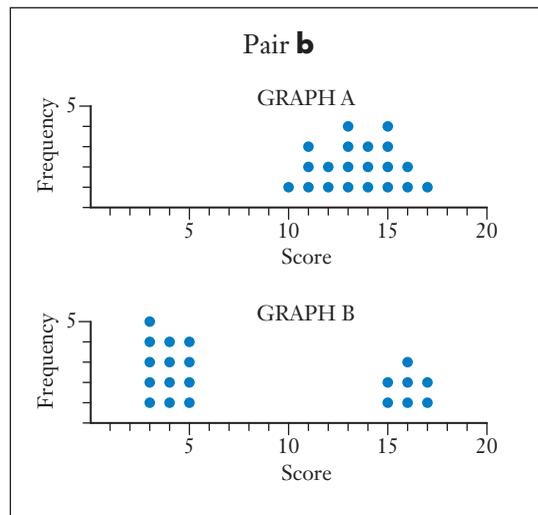
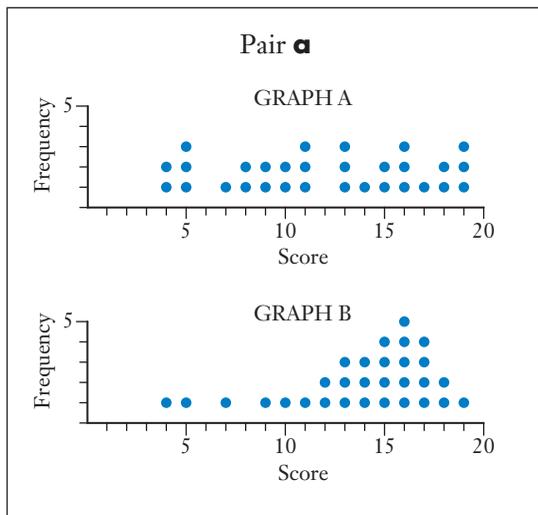
Some useful words and phrases that you might consider using in your descriptions could include:

- | | | | |
|--------------|---------------|----------------|------------|
| lowest score | highest score | tightly packed | spread out |
| clusters | gaps | outliers | uniform |
| groups | dense regions | symmetrical | |





34 In each part of this question two dot frequency graphs are given, Graph A and Graph B. In each pair the two graphs involve sets of data with the same number of data points. However, in each case the two graphs are quite different. Write a few sentences comparing graphs A and B in each case. Whilst you do not need to determine the mean and the median exactly for any of the graphs your answers should include comparisons of means, medians and spread.



Use of statistical functions on a calculator

When asked to determine the mean of a set of scores are you adding up all of the scores and then dividing by the number of scores or are you putting the scores into a calculator and using its ability to determine the mean?

Many calculators have an inbuilt ability to output various summary statistics for data that is put into the calculator.

This usually involves the following steps:

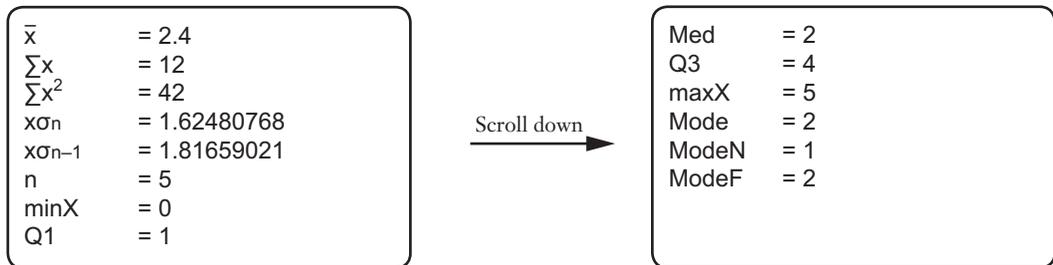
- 1 Setting or selecting the statistics facility of your calculator.
- 2 Clearing any statistical data already in your calculator.
- 3 Inputting the data.
- 4 Outputting the required data.

Your calculator is able to output statistical information for the set of data you put in. The calculator may display more statistical information than you are familiar with at present but somewhere in the list it may well display the number of scores in the set, often listed as ' n ', the mean of the scores, often listed as \bar{x} , the median, the mode and various other things, some of which you will encounter in later mathematical studies.

For example, below left shows such a display for the data set:

3, 5, 2, 0, 2

Scrolling down such a display shows more information as shown below right.



Can you locate the mean,
the number of scores,
the median,
the mode?

-
- Get to know **your** calculator with regard to inputting data and getting it to display summary statistics for the data.
-
-

If the data is presented as a frequency table the information can be put into many calculators in this frequency form, i.e. we do not need to input all of the scores individually. Usually one column is used for the scores and another for the frequencies. We then set the calculator to read the information in each column appropriately.

For example, the following table

Score	10	11	12	13	14	15	16	17	18	19	20
Frequency	1	0	1	2	4	9	10	11	8	3	1

could be put into two columns, see display below left, and the summary statistics displayed, see below right (more statistics being available if we were to scroll down).

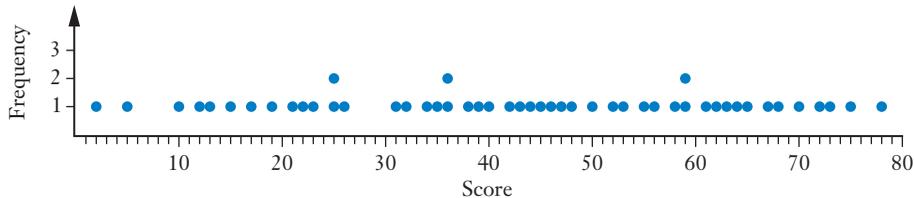
	List 1	List 2	List 3	List 4
1	10	1		
2	11	0		
3	12	1		
4	13	2		
5	14	4		
6	15	9		

\bar{x}	= 16.14
$\sum X$	= 807
$\sum X^2$	= 13205
$x\sigma_n$	= 1.897472
$x\sigma_{n-1}$	= 1.91673617
n	= 50
minX	= 10
Q1	= 15

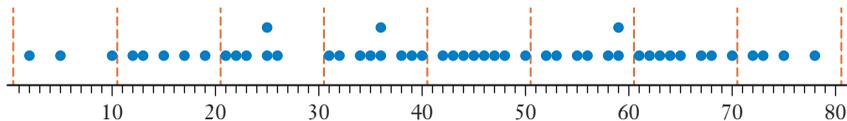
Once again, get to know how to put information from a frequency table into your calculator and how to output the mean etc.

Grouped data

As was mentioned in chapter one, continuous numerical data is often grouped because of rounding and sometimes discrete numerical data is grouped for convenience. For example suppose we are given a set of fifty numbers with very few repeats. If we were to display the scores as a dot frequency graph, we would have almost as many 'columns' to our graph as we have scores as most of the scores occur just once:



The data may be better presented in groups or class intervals:



giving the grouped frequency table:

Score	1 → 10	11 → 20	21 → 30	31 → 40	41 → 50	51 → 60	61 → 70	71 → 80
Frequency	3	5	6	9	8	7	8	4

If we only have the grouped data and do not know the original scores we talk of the **modal group** or **modal class**, rather than referring to a mode. For the frequency table just encountered, and shown again below, the modal class is the 31 → 40 class.

Score	1 → 10	11 → 20	21 → 30	31 → 40	41 → 50	51 → 60	61 → 70	71 → 80
Frequency	3	5	6	9	8	7	8	4

Similarly we find the class in which the median lies, called the **median group** or **median class**, in this case the 41 → 50 class.

To determine a mean we assume all the scores in an interval are at the **midpoint** of the interval. Clearly this is unlikely to be the case but it will give a reasonable estimate for the mean when a large number of scores are involved. Thus for the above table we calculate the mean based on three scores of 5.5, five scores of 15.5, six scores of 25.5 etc.

Check that you agree that applying this idea to the above table of grouped data gives an estimated mean of 42.9.

Exercise 2B (Use the statistical capability of your calculator.)

Find the mean, median, mode and range of each of the sets of data given in questions 1 to 7 (correct to one decimal place if necessary).

1 125, 137, 137, 143, 153, 162, 165.

2 85, 85, 78, 72, 83, 78, 90, 89, 78.

3 8, 34, 19, 14, 25, 15, 40, 26, 17, 30.

4 55, 42, 36, 63, 45, 35, 76, 50, 50, 58,
40, 72, 35, 80, 75, 66, 48, 35, 62, 35,
66, 40, 56, 52, 38.

5

Score	0	1	2	3	4	5	6	7
Frequency	8	12	18	20	9	3	0	1

6

Score	5	6	7	8	9	10	11	12
Frequency	24	35	17	28	33	31	27	19

7

Score	15	16	17	18	19	20
Frequency	1	3	5	13	16	22

In the next four questions use the midpoint of each class interval to determine the mean of each of the following distributions, correct to one decimal place.

8

Score	Frequency
1 → 5	15
6 → 10	28
11 → 15	7
16 → 20	3
21 → 25	1
26 → 30	1

9

Score	Frequency
1 → 4	3
5 → 8	8
9 → 12	15
13 → 16	8
17 → 20	3

10

Score	Frequency
20 → 24	6
25 → 29	10
30 → 34	17
35 → 39	7
40 → 44	5
45 → 49	4
50 → 54	1

11

Score (x)	Frequency
$0 \leq x < 20$	5
$20 \leq x < 40$	13
$40 \leq x < 60$	21
$60 \leq x < 80$	72
$80 \leq x < 100$	54

- 12** As part of the process of assessing the value of a property a real estate agent considers the prices of other properties recently sold in the same area. The selling prices of ten such properties were as follows:

\$437 000 \$425 000 \$456 000 \$421 000 \$442 000
 \$445 000 \$441 000 \$437 000 \$432 000 \$540 000

Find the mean and median of these prices.

How many of the ten prices are lower than the mean?

How many of the ten prices are lower than the median?



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- 13** A real estate survey investigated the number of bedrooms in each of 100 houses in a particular area. The results were as shown below:

Number of bedrooms	1	2	3	4	5	6	7
Frequency	2	3	37	49	5	3	1

Find the mean number of bedrooms per house for these houses.

- 14** A company employs 25 people and has seven salary levels. The number of employees on each level are shown in the following table.

Salary	\$62 000	\$68 000	\$71 000	\$78 000	\$85 000	\$100 000	\$110 000
No. of employees	4	10	5	1	2	2	1

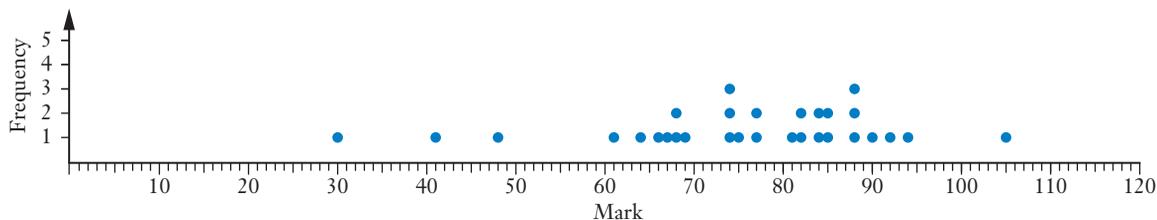
- Calculate
- a** the modal salary,
 - b** the median salary,
 - c** the mean salary.

- 15** In one particular year, the number of hours of sunlight recorded each day in December at a particular weather recording location were as follows.

			1 Dec	2 Dec	3 Dec	4 Dec
			11.2 h	9.4 h	8.5 h	12.2 h
5 Dec	6 Dec	7 Dec	8 Dec	9 Dec	10 Dec	11 Dec
19.2 h	11.2 h	11.8 h	10.4 h	8.7 h	10.3 h	9.1 h
12 Dec	13 Dec	14 Dec	15 Dec	16 Dec	17 Dec	18 Dec
11.1 h	10.0 h	12.0 h	11.2 h	12.9 h	13.1 h	12.0 h
19 Dec	20 Dec	21 Dec	22 Dec	23 Dec	24 Dec	25 Dec
11.7 h	9.3 h	9.0 h	11.1 h	12.8 h	10.3 h	13.2 h
26 Dec	27 Dec	28 Dec	29 Dec	30 Dec	31 Dec	
12.1 h	7.3 h	5.2 h	9.9 h	10.1 h	11.3 h	

Calculate the mean and the median number of hours of sunlight per day for the December of this particular year at this location.

- 16** The scores obtained by 30 students in an exam are shown in the dot frequency diagram below. The exam was marked out of 120.



- Calculate
- the mean of these scores (correct to one decimal place),
 - the median of these scores.
 - the number of students scoring less than 60%.
 - the percentage of students scoring greater than 75%.

- 17** The heights of a group of 29 students, measured to the nearest centimetre, were as shown in the stem and leaf diagram below.

Girls			Boys	
		18	6	
	5	17	9	3 0 5 2
8 3 5 0		16	9	6 3 2 8 5 9
7 9 5 8 5		15	9	5 9
9 8		14	9	

- How many girls were there in the group?
- How tall was the shortest girl?
- Find the mean height for the girls (to nearest cm).
- Find the mean height for the boys (to nearest cm).
- Find the mean height for the group of 29 students (to nearest cm).
- Display the heights of the 29 students as a dot frequency diagram, making some distinction on your diagram between boys and girls.

18 a Calculate the mean of the 50 scores shown below.

23 42 40 47 31 42 39 31 43 26
 33 50 23 40 49 30 42 40 29 37
 44 48 43 26 36 43 50 32 31 44
 47 31 45 37 48 32 42 41 43 23
 28 47 36 26 36 45 23 49 29 48

b Rearrange the above 50 scores as grouped data using the class intervals:

21 – 25, 26 – 30, 31 – 35, 36 – 40, 41 – 45 and 46 – 50.

Use the midpoints of the intervals to determine the mean for this grouped data.

19 One hundred students were asked to note the number of hours they study in the ‘study week’ they are given prior to an examination period. The frequency table on the right shows the results of this survey.

Number of hours (h)	Number of students
$0 \leq h < 10$	3
$10 \leq h < 20$	4
$20 \leq h < 30$	10
$30 \leq h < 40$	20
$40 \leq h < 50$	29
$50 \leq h < 60$	18
$60 \leq h < 70$	9
$70 \leq h < 80$	4
$80 \leq h < 90$	2
$90 \leq h < 100$	1

a Find the modal class for the number of hours spent studying in the week.

b Use the class midpoints to determine the mean for the distribution.

20 As part of the process of assessing the value of a block of land a real estate agent considers other blocks recently sold in the area. The agent is able to access such information from data held in his computer. For 68 recent sales the information was as follows:

Price (\$ C)	Midpoint of interval	Frequency
$200000 \leq C < 210000$	205000	2
$210000 \leq C < 220000$	215000	13
$220000 \leq C < 230000$	225000	10
$230000 \leq C < 240000$	235000	15
$240000 \leq C < 250000$	245000	7
$250000 \leq C < 260000$	255000	7
$260000 \leq C < 270000$	265000	4
$270000 \leq C < 280000$	275000	6
$280000 \leq C < 290000$	285000	3
$290000 \leq C < 300000$	295000	1

a In which class interval does the median price of the 68 blocks lie?

b Use the interval midpoints to calculate the mean price (to nearest \$1000).

Describing a distribution of scores

The last two questions of the first exercise in this chapter, **Exercise 2A**, required you to write some sentences describing some distributions. Indeed one of those questions suggested the following useful words that you could consider using:

lowest score	highest score	tightly packed	spread out
clusters	gaps	outliers	uniform
groups	dense regions	symmetrical	

Let us now formalise what aspects we should consider when asked to describe a data distribution.

If asked to write a description of a set of scores we should comment on such things as

- the location of the scores
- how spread out they are
- the 'shape' of the distribution

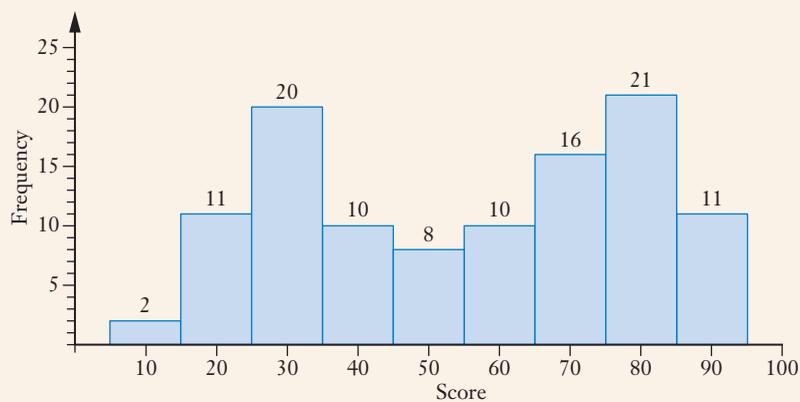
and

- anything else of relevance.
- **Location:** The mean and the median of the scores give information about location.
- **Spread:** The range of the scores gives information about their spread.
- **Shape:** Symmetry, gaps, clustering, more dense/less dense regions, outliers, modality (does the data have one modal class or is it perhaps bimodal with two modal classes) all convey information about the shape of a distribution.

Note: From work of earlier years, some readers may also be familiar with *standard deviation* as a measure of spread, and with the idea that a distribution can be *skewed*. These terms are not included in the descriptions of distributions given in this section because later chapters cover these concepts. (Standard deviation is covered in chapter 3 and skewness in chapter 4.)

EXAMPLE 7

The histogram below shows the distribution of scores achieved by the students of a school in a mathematics exam.



Describe the distribution of marks in this exam.

Organising, describing and interpreting data

Lizards

A survey of a particular species of lizard involved capturing about one hundred different lizards of the species, measuring their length, marking them with an identifiable tag and then releasing them back into the wild. (The tagging was to ensure that the same lizard did not feature more than once in the measuring process and also to allow for follow up studies to be carried out later.)

The lizard lengths, recorded in millimetres, are shown below (the shortest and longest lengths in the list are shown in bold):



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114	140	161	110	88	153	112	163	107	151
54	113	39	115	116	106	69	165	103	104
100	110	173	93	158	109	160	60	175	112
160	155	117	162	33	168	152	106	156	159
90	116	104	154	161	101	164	111	107	103
167	110	158	109	115	154	42	163	158	106
105	104	47	113	163	77	158	167	197	119
162	156	147	160	105	163	109	108	162	111
156	63	104	104	170	131	108	115	111	153
107	122	164	116	102	162	57	160	103	115
158	105								

- Organise the data into a grouped frequency table involving what you consider to be an appropriate number of equal width intervals.
- Display the data graphically and write some sentences describing the distribution of lengths using suitable statistical vocabulary, e.g. range, outliers, clusters, modal class, mean length, etc.
- If the distribution of lengths shows any particularly notable features suggest some possible reasons why this might be the case. (The scientific team can then consider exploring such suggestions in further surveys.)

- 6 List six prices for secondhand cars with a mean of \$28 000 but for which this mean value is not a particularly good choice to represent a price that is centrally located with regard to the six prices.

7 HEALTH SURVEY

One hundred men and one hundred women took part in a health survey. Fifty of the men and fifty of the women were aged between 20 and 25 and the remaining fifty men and fifty women were aged between 65 and 70.

One of the variables measured was the diastolic blood pressure of these two hundred people. The results of such measurements are shown below with the blood pressures given to the nearest millimetre of mercury (mmHg).

Males (aged 20 to 25)	
Blood pressure	Frequency
50 – 59	1
60 – 69	10
70 – 79	21
80 – 89	14
90 – 99	3
100 – 109	1
110 – 119	0
Total	50

Males (aged 65 to 70)	
Blood pressure	Frequency
50 – 59	0
60 – 69	3
70 – 79	12
80 – 89	17
90 – 99	13
100 – 109	4
110 – 119	1
Total	50

Females (aged 20 to 25)	
Blood pressure	Frequency
50 – 59	4
60 – 69	22
70 – 79	19
80 – 89	5
90 – 99	0
100 – 109	0
110 – 119	0
Total	50

Females (aged 65 to 70)	
Blood pressure	Frequency
50 – 59	1
60 – 69	3
70 – 79	13
80 – 89	20
90 – 99	10
100 – 109	2
110 – 119	1
Total	50

- a Use the centre of each class interval to determine a mean blood pressure for each of these four groups.
- b Draw frequency histograms for the two male groups.
- c Draw frequency histograms for the two female groups.
- d Comment on any trends suggested from parts a, b and c.



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3.

Measures of dispersion or spread

- Standard deviation
- Use of statistical functions on a calculator
- Frequency tables
- Outliers
- Grouped data
- Central tendency and spread – An investigation
- Miscellaneous exercise three

This reliance on just the lowest and highest scores makes the range of limited use and so we need to consider other measures of spread.

We will now consider two other ways of quantifying spread, namely

the **mean deviation**,
and the **standard deviation**.

Each of these considers how much each score deviates from the mean score. In this way we can obtain measures that will tell us how concentrated the scores are about the mean value and that use each and every score in the data set in their determination.

Consider again the heights of team A from the previous page and listed again below:

Team A (mean 184 cm)		
Player	Height	Deviation from the mean
1	211 cm	211 cm – 184 cm = +27 cm
2	184 cm	184 cm – 184 cm = 0 cm
3	184 cm	184 cm – 184 cm = 0 cm
4	172 cm	172 cm – 184 cm = –12 cm
5	169 cm	169 cm – 184 cm = –15 cm

If we sum the deviations from the mean the answer is zero (as you may have expected from your understanding of the mean). Thus we cannot find the average of these deviations as they are. To avoid this problem we could:

- ignore the negative signs and find the average of the absolute values of the deviations. This technique gives the **mean deviation** of the heights.

$$\begin{aligned} \text{For team A, mean deviation of heights} &= \frac{27+0+0+12+15}{5} \\ &= 10.8 \end{aligned}$$

Alternatively we could:

- square the deviations, find the average of these square deviations (this is called the **variance** of the scores) and then square root this variance. (This final step of finding the square root is to give a measure that has the same units as the original deviations that had been squared.)

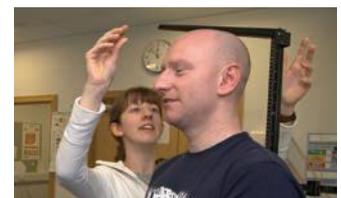
This technique gives the **standard deviation** of the scores.

$$\begin{aligned} \text{For team A, variance of heights} &= \frac{(27)^2 + (0)^2 + (0)^2 + (-12)^2 + (-15)^2}{5} \\ &= 219.6 \end{aligned}$$

$$\begin{aligned} \text{Thus the standard deviation} &= \sqrt{219.6} \\ &= 14.8, \text{ correct to 1 decimal place.} \end{aligned}$$

Note: Finding the standard deviation may seem a more complicated process than that of finding the mean deviation. However it is the standard deviation that is the more commonly used measure of dispersion in data analysis.

For this reason we will focus our attention on the standard deviation and will not pursue the concept of a mean deviation.



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Standard deviation

EXAMPLE 1

Find the mean, the variance and the standard deviation of the set of scores:

4, 7, 10, 13, 21.

Solution

$$\begin{aligned}\text{Mean} &= \frac{4+7+10+13+21}{5} \\ &= 11\end{aligned}$$

$$\begin{aligned}\text{Variance} &= \frac{(4-11)^2 + (7-11)^2 + (10-11)^2 + (13-11)^2 + (21-11)^2}{5} \\ &= 34\end{aligned}$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{34} \\ &= 5.83, \text{ correct to two decimal places.}\end{aligned}$$

The scores have a mean of 11, a variance of 34 and a standard deviation of 5.83 (correct to 2 decimal places).

Exercise 3A

Just as the various measures of central tendency can be obtained using the statistical capabilities of many calculators so too can some of the measures of dispersion. You will be encouraged to use your calculator to obtain such measures soon but for this exercise obtain the variance and standard deviation 'the long way', as in the above example, to gain understanding of the concepts.

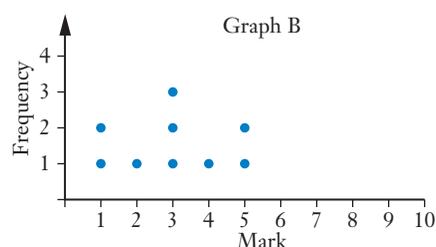
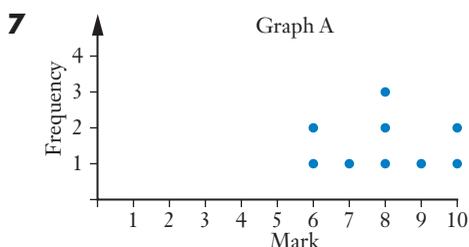
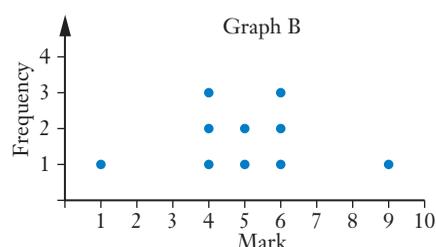
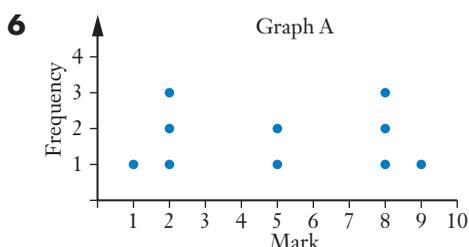
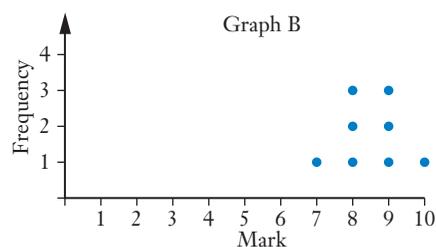
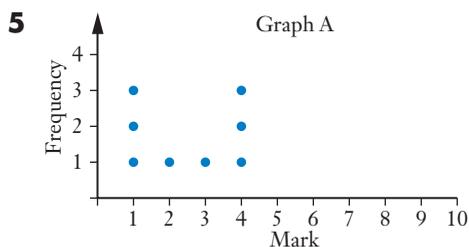
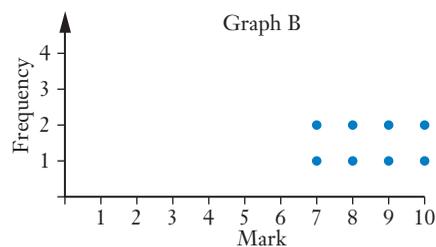
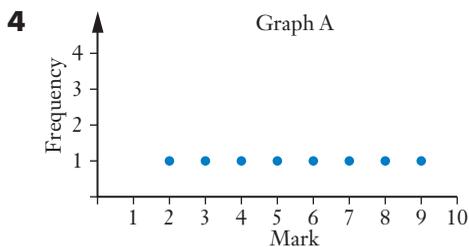
- Find the range of each of the following sets of scores.
 - 5, 7, 11, 12, 17, 19, 21, 36.
 - 104, 115, 117, 117, 118, 121, 122, 125, 125, 146.
 - 121 000, 109 000, 128 000, 90 000, 110 000, 95 000, 112 000, 107 000.
- Find the variance of each of the following sets of scores.
 - 5, 5, 6, 8, 10, 12, 12, 13, 14, 15.
 - 7, 9, 10, 12, 15, 18, 18, 19.
- Find the standard deviation of each of the following sets of scores.
(Give answers correct to 2 decimal places when rounding is necessary.)
 - 15, 15, 15, 15, 15, 15, 15.
 - 5, 7, 10, 11, 13, 13, 16, 21.
 - 104, 115, 117, 117, 118, 121, 122, 125, 125, 146.
 - 65, 67, 72, 83, 84.

For questions 4 to 7 state which of the two diagrams shows the set of scores with

a the greater mean

b the greater standard deviation.

(You should **not** need to calculate the means and standard deviations.)



8 For each of parts **a** to **d** given below, two sets of scores are given. For each part state which set, I or II, has the greater standard deviation (you should **not** need to calculate the standard deviations).

a Set I: 6, 6, 6, 6, 6, 6, 6 mean = 6

Set II: 4, 5, 6, 6, 6, 7, 8 mean = 6

b Set I: 6, 8, 9, 9, 10, 11, 11, 12, 14 mean = 10

Set II: 6, 6, 6, 9, 10, 11, 14, 14, 14 mean = 10

c Set I: 20, 21, 21, 22, 23, 24, 25, 25, 26 mean = 23

Set II: 13, 15, 21, 22, 23, 24, 25, 31, 33 mean = 23

d Set I: 1, 1, 2, 2, 10, 18, 18, 19, 19 mean = 10

Set II: 1, 1, 9, 9, 10, 11, 11, 19, 19 mean = 10

- 9 Find the range, the mean and the standard deviation of the following set of scores.
(Give answers correct to two decimal places if rounding is necessary.)

8, 8, 9, 11, 11, 15, 16, 19, 19, 20, 20, 21, 21, 25, 32.

Use of statistical functions on a calculator

As was stated at the beginning of the previous exercise, just as the various measures of central tendency can be obtained using the statistical capabilities of many calculators so too can some of the measures of dispersion. Also, you should already know how to put data into a calculator and output statistical measures such as the mean and the median.

The standard deviation of a set of scores can similarly be obtained using your calculator.

We tend to use either s or σ as symbols for standard deviation. σ is a letter from the Greek alphabet and is pronounced sigma. It is a 'lower case' sigma, capital sigma is written Σ .

The diagram below shows a typical graphic calculator display for the data set:

3 5 2 0 2

1-Variable		
\bar{x}	= 2.4	← The mean of the scores
$\sum x$	= 12	← The sum of the scores
$\sum x^2$	= 42	← The sum of the squares of the scores
$x\sigma_n$	= 1.62480768	← The standard deviation of the scores
$x\sigma_{n-1}$	= 1.81659021	← The different standard deviation – see note next page
n	= 5	← The number of scores

Scrolling down such a display would allow further statistical information for this set of scores to be viewed.

For this set of scores: Mean = 2.4
Standard deviation = 1.625 (correct to 3 decimal places)

Hint

Make sure that **you** can obtain these two values from **your** calculator.

- Note
- The standard deviation is a measure of spread. For most distributions very few, if any, of the scores would be more than three standard deviations from the mean, i.e. the vast majority of the scores (and probably all of them) would lie between

$$\bar{x} - 3\sigma \text{ and } \bar{x} + 3\sigma$$

Indeed we would frequently find that about two thirds of the scores would lie within 1 standard deviation of the mean, i.e. between $\bar{x} - \sigma$ and $\bar{x} + \sigma$.

Note • The calculator display shown has two different standard deviations,

$$\sigma_n \text{ and } \sigma_{n-1}$$

σ_n is the standard deviation of the five scores.

σ_{n-1} , sometimes shown as s_x , gives an answer a little bigger than σ_n by dividing the sum of the squared deviations by $(n - 1)$ rather than n . This would be used if the five scores were a sample taken from a larger population and we wanted to use the deviation of the sample to estimate the standard deviation of the whole population, as is often the case in real life. Division by $(n - 1)$ rather than n compensates for the fact that there is usually less variation in a *small* sample than there is in the entire population. (This is known as Bessel's correction.) If we make the sample large then n will be large and there will be little difference between σ_n and σ_{n-1} . This book makes the distinction between σ_n and σ_{n-1} and if a question asks for the standard deviation of a set of scores to be determined σ_n is given, unless the question *specifically* states that the task is to use the small sample data to estimate the standard deviation of the population of which the sample is a part. However in some states of Australia this distinction may not be part of the course and instead you might simply be expected to give σ_{n-1} , or s_x , whenever standard deviation is requested. Hence it is important that you make sure you know which standard deviation, σ_n or σ_{n-1} , you are expected to give in examinations when a question simply asks you to determine a standard deviation. To assist readers in states that only require σ_{n-1} to be determined answers in the back of this text will tend to give both values for those cases where, to the given accuracy, the values differ.



EXAMPLE 2

Find the mean and standard deviation for the following set of scores.

$$9, 10, 11, 13, 19, 20, 21, 21, 22, 24, 25, 31, 31, 32, 44.$$

a How many of the scores are such that

$$\bar{x} - 1 \text{ standard deviation} < \text{score} < \bar{x} + 1 \text{ standard deviation?}$$

b How many of the scores are such that

$$\bar{x} - 2 \text{ standard deviations} < \text{score} < \bar{x} + 2 \text{ standard deviations?}$$

Solution

Using a calculator $\bar{x} = 22.2$, standard deviation (σ_n) = 9.27 (2 decimal places)

a $\bar{x} - \text{standard deviation} \approx 12.93$ and $\bar{x} + \text{standard deviation} \approx 31.47$.

10 of the 15 scores lie between $\bar{x} - \sigma$ and $\bar{x} + \sigma$.

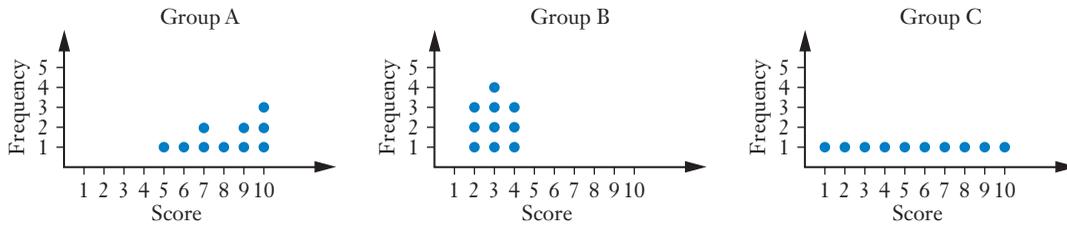
b $\bar{x} - 2 \text{ standard deviations} \approx 3.66$ and $\bar{x} + 2 \text{ standard deviations} \approx 40.74$.

14 of the 15 scores lie between $\bar{x} - 2\sigma$ and $\bar{x} + 2\sigma$.

Exercise 3B

Find the mean and the standard deviation for each of the sets of scores given in numbers 1 to 5.
(Give answers correct to one decimal place when rounding unless stated otherwise.)

- 1 10, 11, 12, 13, 14, 15.
- 2 15, 26, 47, 16, 33, 49, 8, 11, 41, 26, 19, 14.
- 3 31, 29, 33, 32, 34, 29, 30, 30.
- 4 6.6, 6.2, 7.3, 8.1, 6.8, 7.0, 6.9, 7.1, 6.9, 7.0. (Answers correct to 2 decimal places)
- 5 30, 29, 34, 27, 26, 25, 26, 38, 38, 23, 39, 35, 26, 27, 29, 32, 29, 31, 32, 30, 31, 27, 29, 32, 30, 32, 31, 28, 32, 30, 29, 30.
- 6 Three groups of 10 students do a spelling test marked out of 10. The scores achieved by each group are shown in the dot frequency graphs below.



- a Without calculating values but just by looking at the graphs state which of the three groups have scores with
 - i the greatest standard deviation
 - ii the smallest standard deviation
 - iii the greatest mean
 - iv the smallest mean.
 - b Calculate the mean and standard deviation for each group.
- 7 In a particular sporting competition contestants are awarded a score by each of eight judges. The eight scores for one competitor were:
- 5.9 5.9 5.7 6.0 5.1 5.8 5.8 5.9
- a Find the mean and standard deviation of these scores. (To 2 decimal places.)
 - b If the highest score and the lowest score are discarded find the mean and standard deviation of the remaining scores. (To 2 decimal places.)
- 8 An engineering company makes a particular component that theoretically is to be of length 63 cm. Quality control imposes certain restrictions that any randomly selected sample of ten of these components must satisfy. One such random sample has lengths (in cm):
- 63.0, 62.9, 63.0, 63.2, 63.0, 63.1, 63.0, 62.9, 63.1, 63.1.
- a Does this sample satisfy the restriction: $62.95 \text{ cm} < \text{mean} < 63.05 \text{ cm}$?
 - b Does this sample satisfy the restriction: $\text{standard deviation} < 0.1 \text{ cm}$?

- 9** The 25 students in a year 8 class have the following heights (nearest cm).
 181 145 162 158 165 150 164 155 173 160 164 154 161
 152 167 169 148 163 175 153 166 153 166 147 160
- a** Find the mean and standard deviation of these 25 heights (to 2 decimal places).
b These 25 heights are to be used to estimate the standard deviation of the 231 year 8 students in this school. What would this estimated standard deviation be (to 2 decimal places)?

- 10** An entomologist catches an adult moth that he is sure is one belonging to a particular species but he is not sure whether it is type A of the species or type B. Some of the smaller examples of type B moths can easily be mistaken for a type A moth and positive examination then requires analysis of body tissue. The entomologist refers to a reference book which states that in an extensive survey involving thousands of these adult moths it was found that the body length of the two types were such that:



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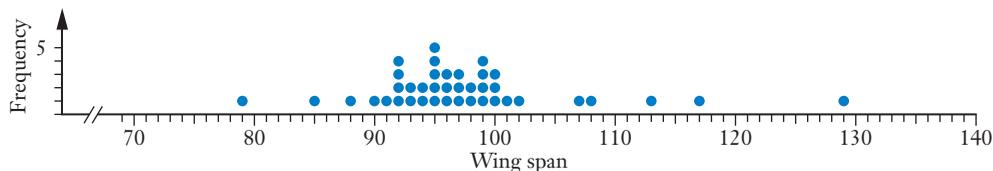
For the type A moths surveyed: Mean 15 mm, Standard deviation 1mm.

For the type B moths surveyed: Mean 22 mm, Standard deviation 3mm.

The entomologist measures the body length of 'his' moth as 18 mm.

Decide which it is more likely to be, a large type A or a small type B, and explain your choice.

- 11** A scientist collects 40 butterflies of a particular species and measures the lengths of the wing span of each one. The lengths, to the nearest millimetre, are shown in the dot frequency diagram below.



- a** Calculate the mean and standard deviation (st. devⁿ) for this set of lengths, giving the standard deviation correct to 3 decimal places.
b What percentage of the 40 lengths lie within one standard deviation of the mean?
 i.e. What percentage of the lengths are such that:
 $(\text{mean} - 1 \text{ st. dev}^n) < \text{length} < (\text{mean} + 1 \text{ st. dev}^n)$
c What percentage of the 40 lengths lie within two st. dev^{ns} of the mean?
d What percentage of the 40 lengths lie within three st. dev^{ns} of the mean?
e If the scientist wanted to use these 40 lengths to estimate the standard deviation of the entire butterfly population of this species what would this estimated standard deviation be (to 2 decimal places)?
- 12** A scientific experiment involved students determining the temperature at which a particular chemical reaction took place. Ten groups carried out the experiment and obtained the following answers: 156°C, 163°C, 154°C, 158°C, 159°C, 161°C, 121°C, 163°C, 159°C, 159°C.
 One of the groups discovered that they had made a number of errors in carrying out the experiment and in calculating the answer.
 Determine the mean and standard deviation of the results if
- a** the answer likely to be from the group making errors **is** included,
b the answer likely to be from the group making errors is **not** included.

15 The marks achieved in an exam sat by 100 candidates are shown below.

53	58	45	61	89	55	60	49	62	26	65	92	51	59	40
56	21	61	65	56	80	40	69	54	83	59	83	47	77	36
46	58	62	52	69	97	66	64	75	14	77	62	56	58	81
64	36	69	64	66	72	47	80	50	70	56	43	68	52	28
40	69	52	78	32	78	67	56	83	62	43	67	64	56	85
62	49	59	89	62	66	42	62	53	47	85	74	86	79	75
70	75	53	72	23	70	77	74	80	71					

- a** Calculate the mean and standard deviation for this set of marks (correct to 1 decimal place).
- b** Display the data as a dot frequency graph.
- c** Show on your graph the ‘grade borderlines’ and state the number of candidates awarded each grade given that grades were awarded as follows:
- A: exam mark \geq (mean + 1.25 \times st. devⁿ.)
- B: (mean + 0.5 \times st. devⁿ.) \leq exam mark $<$ (mean + 1.25 \times st. devⁿ.)
- C: (mean - 0.5 \times st. devⁿ.) \leq exam mark $<$ (mean + 0.5 \times st. devⁿ.)
- D: (mean - 1.5 \times st. devⁿ.) \leq exam mark $<$ (mean - 0.5 \times st. devⁿ.)
- F: exam mark $<$ (mean - 1.5 \times st. devⁿ.)

16 A company manufactures components for aircraft engines. The quality control for one particular component involves 25 of the components being randomly selected and measured from every batch of 500. If **any** one of the conditions stated below is found to apply to the sample then production is halted, each of the other 475 components in the batch is checked and the machine is reset.

- Sample reject, condition 1: mean $<$ 64.8 cm
- Sample reject, condition 2: mean $>$ 65.2 cm
- Sample reject, condition 3: standard deviation $>$ 0.15 cm
- Sample reject, condition 4: any one component $>$ 65.3 cm
- Sample reject, condition 5: any one component $<$ 64.7 cm

For each of the following samples of 25, determine whether the sample passes these checks and, for any sample that does not pass, state the reason for it not passing. (All measurements are in centimetres.)

Sample A				
65.0	65.0	64.7	64.9	65.1
65.1	65.0	65.0	65.1	65.0
65.0	64.8	64.9	65.0	64.8
64.8	65.0	65.0	65.1	65.0
65.0	64.9	65.1	65.0	65.2

Sample B				
65.0	65.0	65.0	65.0	65.0
65.1	65.1	65.1	64.9	65.0
65.0	65.0	64.8	64.6	64.9
64.9	65.1	65.0	64.9	65.0
65.0	65.0	64.8	65.0	65.0

Sample C				
64.9	64.9	64.8	65.1	65.0
64.9	65.0	65.1	65.0	65.0
65.0	65.1	65.0	65.1	65.0
65.1	65.1	64.9	65.0	64.8
65.1	65.0	65.1	65.0	65.0

Sample D				
64.8	65.2	65.0	65.0	65.2
65.0	64.8	65.1	65.2	64.8
65.0	65.1	65.1	64.8	65.2
64.7	64.8	64.8	65.2	65.0
64.9	65.1	65.0	65.2	64.8

Grouped data

Just as we use the midpoint of each interval to estimate the mean for grouped data, we do the same thing to obtain an estimate for the standard deviation.

Score	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Frequency	3	11	18	28	24	10	6

$$n = 100, \quad \bar{x} = 55.8, \quad \sigma_n = 14.3 \text{ (1 decimal place).}$$

L1	L2	L3
24.5	3	
34.5	11	
44.5	18	
54.5	28	
64.5	24	
74.5	10	
84.5	6	

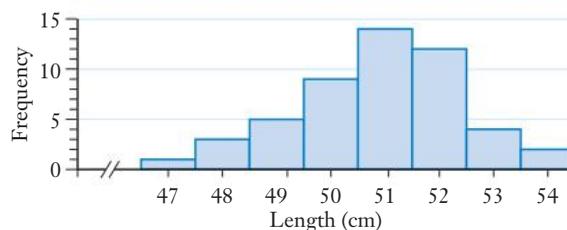


1-Var Stats	
\bar{x}	= 55.8
$\sum x$	= 5580
$\sum x^2$	= 331895
Sx	= 14.4008277
σx	= 14.32864264
n	= 100

Remember: Many measured quantities are naturally grouped by rounding and the data is often displayed as a histogram.

Confirm that for the histogram shown on the right:

$$\begin{aligned} \text{Mean} &= 50.88, \\ \text{Standard deviation} &= 1.5 \text{ (1 decimal place).} \end{aligned}$$



Exercise 3C

Find the mean and standard deviation of each of the distributions shown in questions 1 to 5. (Give answers correct to one decimal place.)

1	Score	0	1	2	3	4	5
	Frequency	3	7	15	24	19	12

2	Score	15	20	25	30	35	40	45	50
	Frequency	1	3	4	8	8	4	3	1

3	Score	15	20	25	30	35	40	45	50
	Frequency	8	4	3	1	1	3	4	8

4	Score	1	2	3	4	5	6	7	8	9	10
	Frequency	1	0	4	6	10	4	2	1	1	1

5	Score	10	20	30	40	50	60	70	80	90	100
	Frequency	1	2	4	3	5	7	15	19	12	7

Use the midpoint of each class interval to determine the mean and standard deviation of the following distributions shown in questions 6 to 10. (Give answers correct to one decimal place if rounding is necessary.)

6

Score	Frequency
20 → 24	7
25 → 29	12
30 → 34	18
35 → 39	20
40 → 44	24
45 → 49	13
50 → 54	6

7

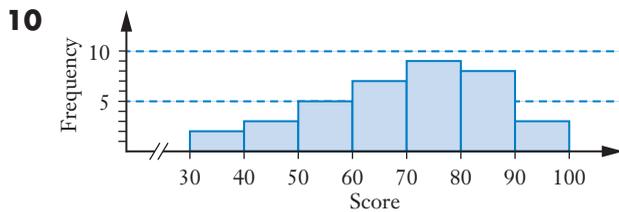
Score (x)	Frequency
$0 \leq x < 10$	17
$10 \leq x < 20$	13
$20 \leq x < 30$	9
$30 \leq x < 40$	7
$40 \leq x < 50$	4

8

Score	Frequency
0 → 9	3
10 → 19	8
20 → 29	15
30 → 39	24
40 → 49	34
50 → 59	16

9

Score (x)	Frequency
$0 \leq x < 20$	1
$20 \leq x < 40$	3
$40 \leq x < 60$	10
$60 \leq x < 80$	9
$80 \leq x < 100$	7



11 A golf club organises a ‘club members championship’ each year for the top 35 ranked players in the club. In a particular year the scores achieved by these players in the championship round were as shown below.

Score	67	69	71	72	73	74	75	76	78	82	85	91
Number of players	1	1	2	4	7	5	3	5	3	2	1	1

- a** How many standard deviations from the mean was the best (i.e. lowest) score?
(Answer correct to one decimal place.)
- b** How many standard deviations from the mean was the worst (i.e. highest) score?
(Answer correct to one decimal place.)

- 12** A company wishes to test a coating it is developing for seeds. The coating is designed to provide the seeds with essential nutrients and to stimulate germination and growth. The company arranges 200 trays each containing identical potting mix. In each of 100 of these trays 50 coated seeds are planted and in each of the other 100 trays 50 uncoated seeds are planted, all the seeds being of the same quality and type. After a certain number of weeks the company counts the number of successful germinations in each tray – a ‘success’ being a healthy seedling at least 6 cm in height. The results were as follows:

Uncoated Seeds	
No. of successes in tray.	No. of trays.
21 – 25	7
26 – 30	21
31 – 35	28
36 – 40	23
41 – 45	15
46 – 50	6

Coated Seeds	
No. of successes in tray.	No. of trays.
21 – 25	1
26 – 30	2
31 – 35	10
36 – 40	17
41 – 45	42
46 – 50	28

Calculate the mean and standard deviation for each set of 100 trays and comment on your results.

- 13** The time that thirty patients had to wait beyond their allotted appointment time at a particular health centre was noted. The results are shown tabulated below.

Time (t mins)	$0 \leq t < 10$	$10 \leq t < 20$	$20 \leq t < 30$	$30 \leq t < 40$	$40 \leq t < 50$	$50 \leq t < 60$
No. of patients	8	15	4	2	0	1

Find the mean and standard deviation of this distribution of waiting times both with and without the outlier included. (Round to one decimal place if necessary.)

- 14** One hundred primary schools are surveyed regarding the number of students on the roll of each school. The information collected is shown tabulated below:

No. of students	1 to 50	51 to 100	101 to 150	151 to 200	201 to 250
No. of schools	5	5	10	9	18

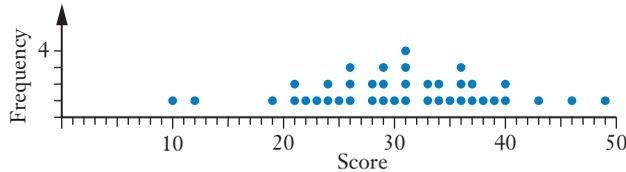
No. of students	251 to 300	301 to 350	351 to 400	401 to 450	451 to 500
No. of schools	22	15	6	7	2

No. of students	501 to 550	551 to 600	601 to 650	651 to 700	701 to 750
No. of schools	0	0	0	0	1

- a** By taking the centre of each interval as the number of students in each school in that interval, determine estimates for the mean number of students per school and the standard deviation of the distribution.
- b** Determine the mean and standard deviation of the data once the outlier in the data is removed.

Central tendency and spread – An investigation

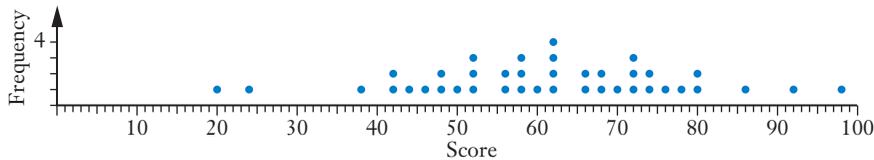
Forty students sat a maths test marked out of 50. The scores they achieved are shown in the dot frequency graph below.



Find the mean (to 2 decimal places) and standard deviation (to 1 decimal place) of the forty scores.

The teacher wanted these scores as percentages so she multiplied each score by 2.

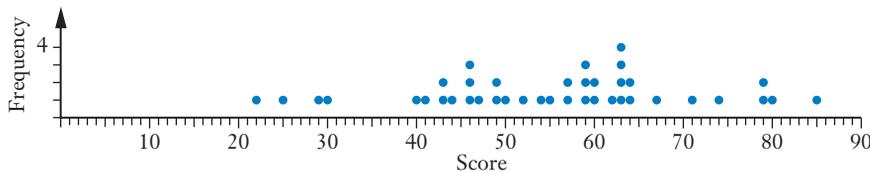
The dot frequency graph of the percentage scores is given below.



Find the mean and standard deviation of these forty percentage scores.

A second test, given to the same forty students, was in two parts. Part A consisted of 10 mental questions and was marked out of 10. Part B was more involved and was marked out of 90.

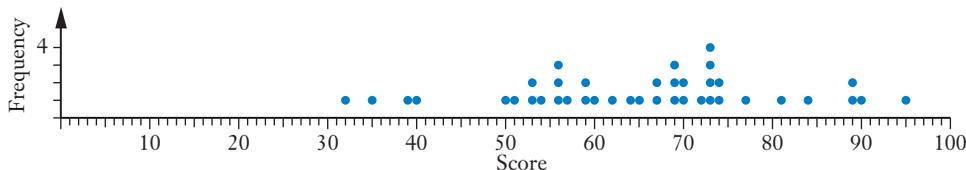
The dot frequency for the scores achieved in part B is shown below.



Find the mean and standard deviation of these forty scores.

Part A proved to be very straightforward and all of the students scored 10 out of 10!

The dot frequency for the combined score out of 100 is given below.



Find the mean and standard deviation of these forty scores.

Suggest how the mean and the standard deviation of a set of scores are affected

- if
- the scores have the same number added to each of them (or subtracted from each them),
 - and/or
 - each of the scores is multiplied by (or divided by) the same number.

Test your suggestions (conjectures) by putting a set of scores into your graphic calculator, creating a new set of scores by adding a number to each of the scores in the first set, and/or by multiplying each set of scores in the first set by a number, and then comparing the mean and standard deviation of the two sets of scores.

Miscellaneous exercise three

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1 Write the following numbers in order of size, smallest first.

0.201 0.12 0.102 0.21 0.1 0.021 0.012 0.2

- 2 Without the assistance of a calculator, write the following numbers in order of size, smallest first.

$\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{10}$ $\frac{1}{100}$ $\frac{3}{5}$

- 3 Evaluate each of the following.

a $3 + 2 \times 4$

b $5 \times 4 + 6$

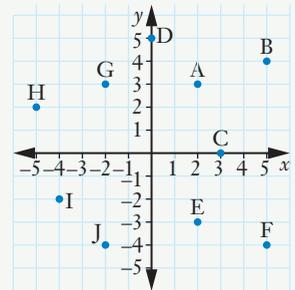
c $3 + 5^2$

d $32 \div 4 \div 2$

e $32 \div (4 \div 2)$

f $(5 + 3)^2 - 8 \times 3$

- 4 State the coordinates of each of the points A to J shown on the right.
(All coordinates involve integer values only.)



- 5 The statistics regarding commercial orchard fruit in Western Australia for a particular year are shown in the following table:

Fruit	Number of trees (1000s)	Production (tonnes)	Gross value of production (\$1000s)
Apples	778	37 418	19 497
Pears	175	8 399	4 886
Lemons & Limes	18	1 125	738
Mandarins	55	1 315	1 830
Oranges	183	5 304	1 830
Nectarines	147	2 333	2 333
Peaches	126	2 507	4 070
Plums & Prunes	190	3 494	4 392

[Source of data: Australian Bureau of Statistics.]

- a** What type of variable does the first column of the table involve?

For this particular year:

- b** how many commercial apple trees were in Western Australia?

- c** how many tonnes of peaches were produced commercially?

- d** find the gross value per tonne for **i** oranges? **ii** nectarines?

- e** on average, how many kilograms of apples did each commercial apple tree yield?

11 (Pythagoras)

A vertical mast is 80 metres tall and is to be held in place by a number of wires. These wires are each one of three different lengths and are classified as short, medium or long.

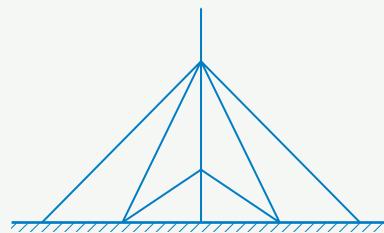
Each short wire is to have one end attached to a point one quarter of the way up the mast and the other to a point on the ground, level with the base of the mast and thirty metres from it.

Each medium wire is to have one end attached to a point twenty metres from the top of the mast and the other to a point on the ground, level with the base of the mast and thirty metres from it.

Each long wire is to have one end attached to a point twenty metres from the top of the mast and the other to a point on the ground, level with the base of the mast and sixty metres from it.

The wires are to be made a little longer than is required, brought to the site, and then suitably adjusted. Each wire is to be made to 'the accurate length plus 50 cm then round up to the next 10 cm'.

Find the length that each classification of wire should be made to.



- 12** The assessment of her college course involves Suzanne sitting five exams, one in each of the units A, B, C, D and E. The twenty-five students following this course all took the five exams and their results in the exam for unit A were as follows:

55 50 54 49 14 53* 50 37 37 48
40 71 20 57 61 55 9 46 30 44
50 43 48 34 43

*Suzanne's result.

- a** Determine the mean and standard deviation for these scores.

The mean and standard deviation for the marks obtained by these students in the other four exams are shown below, together with Suzanne's score.

Unit B:	Mean	67	Standard deviation	12	Suzanne's score	61
Unit C:	Mean	37	Standard deviation	8	Suzanne's score	49
Unit D:	Mean	83	Standard deviation	5	Suzanne's score	80
Unit E:	Mean	72	Standard deviation	10	Suzanne's score	79

- b** In order to compare her marks in the five exams Suzanne decides to standardise the marks by expressing each mark in terms of the number of standard deviations the mark is above or below the mean. For example in a course having a mean of 55 and a standard deviation of 12 then a score of 67 ($= 55 + 1(12)$) becomes 1, a score of 79 ($= 55 + 2(12)$) becomes 2, a score of 43 ($= 55 - 1(12)$) becomes -1 etc.

List the units in order from the one that Suzanne achieved her highest standardised score to the one with her lowest standardised score and state the standardised score for each unit.

4.

Boxplots, histograms and more about describing distributions

- Box and whisker diagrams (boxplots)
- Boxplot or histogram?
- More about the shape of a distribution – skewness
- Miscellaneous exercise four

Box and whisker diagrams (boxplots)

A simple diagram that shows the way a set of scores is distributed is a **box and whisker diagram** or **box plot**. This type of diagram does *not* show all the individual scores (unlike a dot frequency diagram which does show all of the scores) but instead it concentrates our attention on specific features of the data.

Just as the median divides the distribution into two halves then so the **quartiles** divide the distribution into four quarters. Box and whisker diagrams show the locations of:

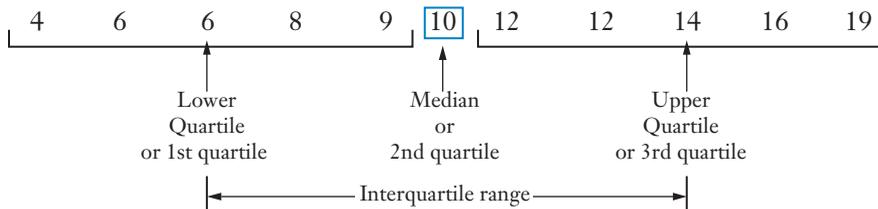
- the **lowest score**,
- the **highest score**,
- the **median**,
- the **lower quartile**
- and
- the **upper quartile**.

Using this **five-number summary**, boxplots give a visual impression of the location of the data and how widely spread it is. The range (highest score minus lowest score) and the **interquartile range** (upper quartile minus lower quartile) can be determined.

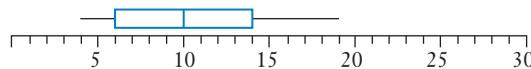
For example, consider the set of scores

12, 6, 10, 19, 9, 12, 4, 14, 8, 16, 6.

Listing the scores in order allows the quartiles and interquartile range to be determined:



A box plot can then be drawn with the 'box' extending from the lower quartile to the upper quartile with a line in the box indicating the median. 'Whiskers' then extend from the lower quartile to the lowest score and from the upper quartile to the highest score.



- WS
Box-and-whisker plots
- WS
Interquartile range
- WS
Boxplots 1
- WS
Boxplots 2
- WS
Five-number summaries
- WS
Statistical calculations
- WS
Statistics review

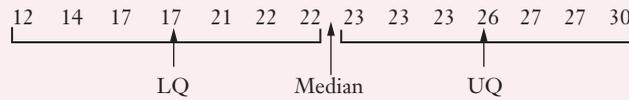
EXAMPLE 1

Draw box and whisker plots for each of the following sets of scores.

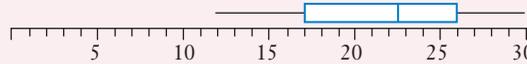
- a** 12, 22, 22, 23, 27, 14, 27, 23, 21, 30, 26, 17, 23, 17.
b 7, 11, 11, 11, 8, 17, 10, 12, 10, 14, 9, 15, 9.
c 21, 18, 28, 30, 23, 17, 30, 27, 28, 19, 29, 20.

Solution

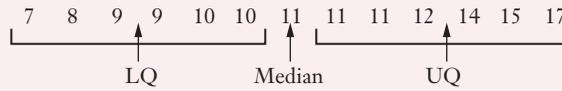
- a** Order the scores and find the median, lower quartile and upper quartile:



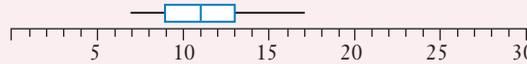
Hence draw the box plot:



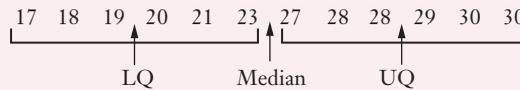
- b** Order the scores and find the median, lower quartile and upper quartile:



Hence draw the box plot:



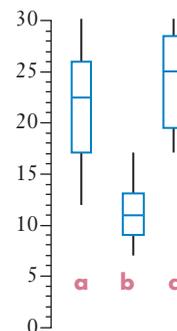
- c** Order the scores and find the median, lower quartile and upper quartile:



Hence draw the box plot:

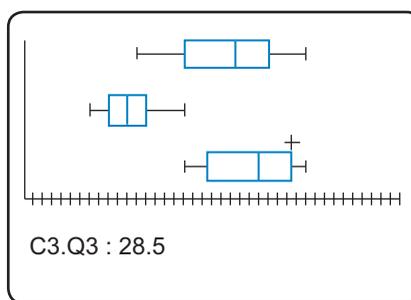


- Note
- The box and whisker diagrams shown in the previous example have all been drawn horizontally but they may also be drawn vertically, as shown on the right.

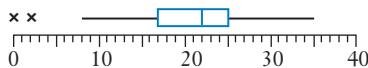


- Some graphic calculators can display data as box plots. The display below left shows the data of example 1 (remaining data could be viewed by scrolling down) and the display below right shows the box plots.

n	C1	C2	C3	C4
1	12	7	21	
2	22	11	18	
3	22	11	28	
4	23	11	30	
5	27	8	23	
6	14	17	17	
				12



- A possible refinement of this type of diagram is to indicate scores that may be considered as unusually high or unusually low compared to the others, as separate points on the diagram (i.e. show *outliers* as separate points.) The whiskers are drawn to include all scores that are within 1.5 times the interquartile range of the nearest quartile. Any scores outside that are considered outliers and are marked separately. The diagram below shows an example of this. The interquartile range is 8. The whiskers only extend to include marks that are no more than 12 marks ($= 1.5 \times 8$) from the nearest quartile. Any scores beyond this are marked with a \times .



Most of the questions in this book will involve the simpler boxplots where the whiskers are drawn to the lowest and highest scores. However the idea of using 'more than $1.5 \times$ interquartile range beyond the upper and lower quartiles' as the criteria for identifying possible outliers should be remembered.

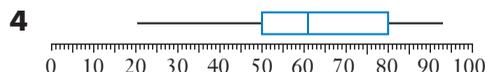
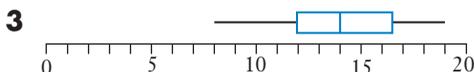
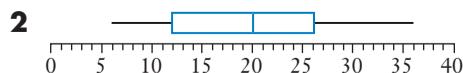
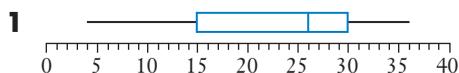


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Exercise 4A

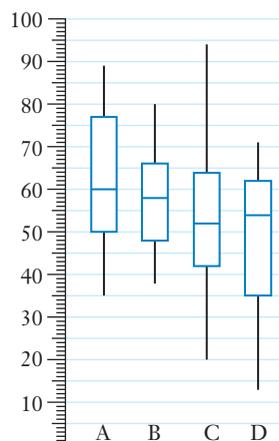
For each of the box and whisker diagrams shown in numbers 1 to 4 state:

- a** the median **b** the lower quartile **c** the upper quartile
d the lowest score **e** the highest score **f** the interquartile range



- 5** Four year 10 maths classes, A, B, C and D, take the same test, marked out of 100. The diagram on the right shows box plots for the results.

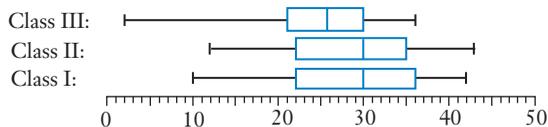
- a** In which class is the student who scored the highest mark?
b In which class is the student who scored the lowest mark?
c Which class had the highest median?
d Which class had the lowest median?
e Which class had the smallest interquartile range?
f Which class had the greatest range of marks?
g Which class had the smallest range of marks?



Draw box plots for each of the following data sets.

- 6** 5, 6, 11, 12, 12, 15, 16, 18, 18, 19, 20, 22, 25, 29, 31.
7 11, 14, 7, 16, 16, 5, 14, 14, 24, 7, 12, 15, 14, 9.
8 7, 10, 17, 23, 9, 12, 20, 2, 15, 5, 10, 12, 1.
9 1, 14, 11, 25, 16, 14, 1, 1, 7, 18, 20, 5.

- 10** The box plots on the right are for scores achieved by three classes, in the same test.
 Comment on each of the following statements.



- a** Class III had more scores below the median than above it.
b The class I marks were more spread out than the class II marks.
c The class III marks were more spread out than the class I marks.
d The class I marks and the class II marks were similarly distributed.
e Based on this test the top student in class III would be the twenty fifth student if they moved to class I.
f Class III had lots of students who scored a lower mark than the lowest mark from the other two classes.

Boxplot or histogram?

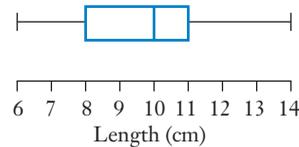
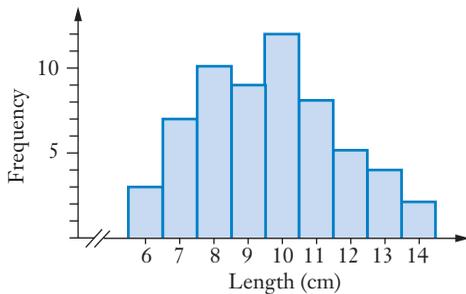
The lengths of the beaks of sixty male birds of a particular species were measured and the lengths, recorded to the nearest centimetre, were as follows:



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8	7	10	13	11	9	9	7	10	12	8	11	7	11	8
11	13	9	14	10	7	6	10	8	11	10	8	9	10	7
9	12	11	8	6	8	10	12	9	7	10	8	12	6	9
7	10	14	9	8	10	13	8	10	12	11	11	13	9	10

Below left shows the data displayed as a histogram and below right it is displayed as a boxplot.



Question: Which is the better form of display?

Answer: Well, they are each useful in their own way and each allows us to visualise how the data is distributed.

Box plots can be drawn quickly, allow a five number summary consisting of lowest score, lower quartile, median, upper quartile and highest score to be readily obtained and the range and the interquartile range to be determined. Their compact nature and ease of production allows several boxplots to be drawn in close proximity thus allowing distributions to be compared easily.

Histograms convey the overall 'shape' of a distribution allowing aspects such as symmetry, grouping, gaps, modes etc to be noticed. They allow the mean and the standard deviation of the data to be determined, or at least estimated if grouped data is involved. However certain features can be hidden if we choose too few or too many class intervals.

Hence which is 'better' depends upon how much information we are wanting to show, whether we want a quickly produced visual summary of the data or a more detailed picture.

Thus box plots and histograms are both useful methods of data display, each enabling us, in their own way, to build up a picture of how a set of scores are distributed. They complement each other. Sometimes both forms of display may be given for the same set of data, as was the case above. Each form of display provides information about three key aspects of a data set:

- its **location**, (Where is it?)
- its **dispersion**, (How spread out is it?)
- its **shape**. (What does it look like?)

The three aspects of **location**, **spread** (dispersion) and **shape**, together with any other information we may notice about a distribution, were the aspects of a distribution that we were encouraged to consider when describing a distribution in chapter two. The work of chapter three now allows us to include mention of standard deviation when considering spread and the following section covers **skewness**, an aspect we can consider when describing the shape of a distribution.

More about the shape of a distribution – skewness

Consider the histogram shown on the right.

Using the centre of each class interval to determine the mean and standard deviation of the distribution gives:

$$\text{Mean} = 115$$

(As we would expect considering the symmetry of the histogram.)

$$\text{St. dev.}^n.(\sigma_n) = 16.04 \text{ (2 decimal places)}$$

Suppose we now take the scores to the left of the central column, 40 scores in this case, and spread them further out on this left side, as shown on the second histogram.

The median score would still lie in the central column, and would therefore be unchanged, but consider what will have happened to the mean and the standard deviation?

We now have more extreme scores to the left of the centre score and these scores will drag the mean left, and increase the standard deviation.

$$\text{Mean} = 112.71$$

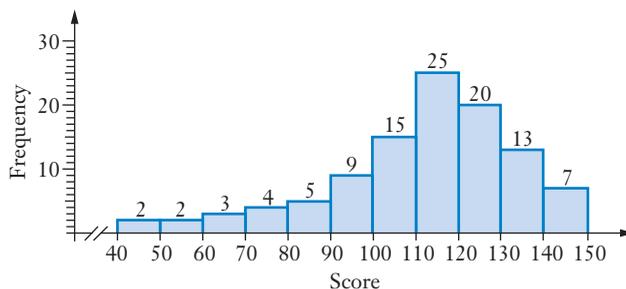
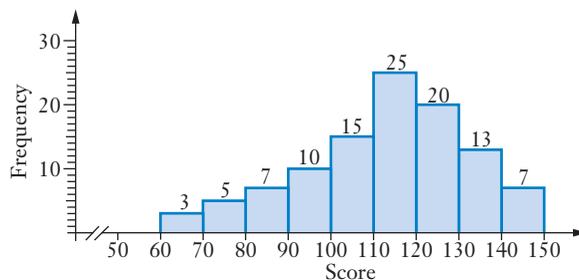
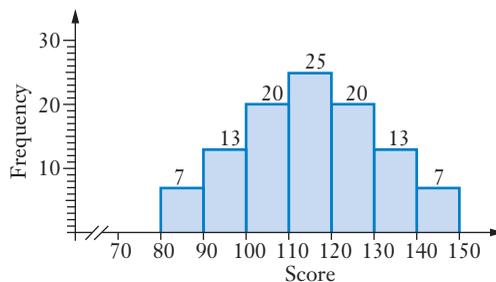
$$\text{St. dev.}^n.(\sigma_n) = 19.58 \text{ (2 decimal places)}$$

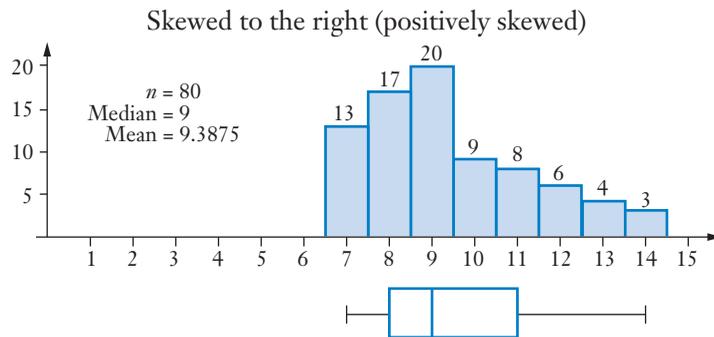
Spreading the left half of the distribution further left drags the mean further left and increases the standard deviation yet more.

$$\text{Mean} = 111.38$$

$$\text{St. dev.}^n.(\sigma_n) = 22.39 \text{ (2 decimal places)}$$

Whilst the first histogram was symmetrical we say that the second and third histograms are **skewed to the left**, also referred to as being **negatively skewed**.



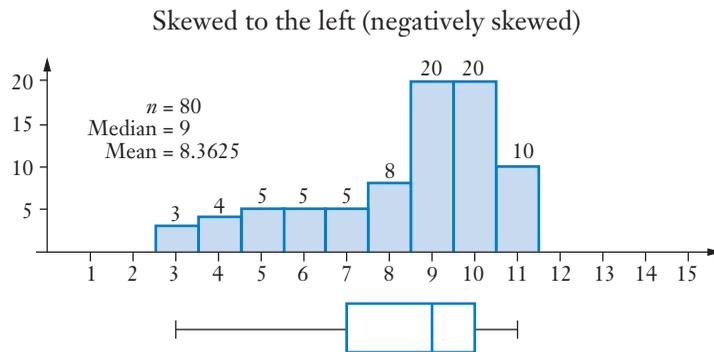


If a distribution is skewed to the right, i.e. positively skewed, the longer ‘tail’ will be in the positive direction. The mean will usually be to the right of the median, i.e. for most positively skewed distributions we would expect

$$\text{mean} > \text{median}$$

because the ‘tail’ of high scores to the right will tend to drag the mean right.

The box plot will tend to be longer to the right of the median than it is to the left.



If a distribution is skewed to the left, i.e. negatively skewed, the longer ‘tail’ will be in the negative direction. The mean will usually be to the left of the median, i.e. for most negatively skewed distributions we would expect

$$\text{mean} < \text{median}$$

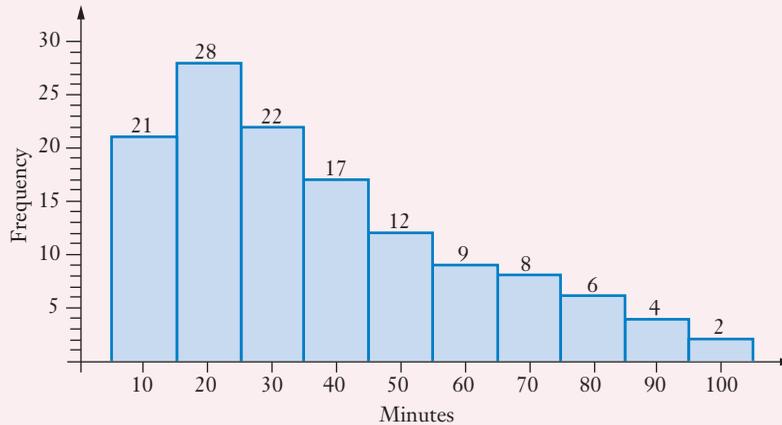
because the ‘tail’ of low scores will tend to drag the mean to the left.

The box plot will tend to be longer to the left of the median than it is to the right.

Note: The explanation of skewness given here is somewhat simplistic and does not, for example, consider what skewness might mean for a multimodal distribution, nor does it attempt to ‘quantify’ skewness. However the explanation given here is sufficient for a basic understanding of the idea.

EXAMPLE 2

Following a traffic warning that due to a number of accidents and roadworks long delays were likely to occur for people making their way home, the workers of one company decided that they would each record how long it took them to get home that evening. The histogram below shows the distribution of recorded times.



Describe the distribution of times.

Solution

129 times were recorded.

An estimate for the mean time is 37.6 minutes.
The median time lies in the 25 to 35 minute class.

The times were spread out from about 5 minutes to 105 minutes, i.e. the range was about 100 minutes.
An estimate for the standard deviation (σ_n) is 23.4 minutes

15 – 25 minutes is the modal class (which is just the second of the ten classes).
The long tail to the right indicates that the distribution is positively skewed.

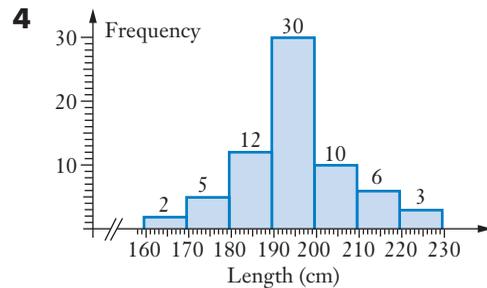
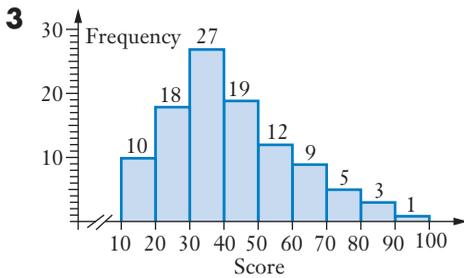
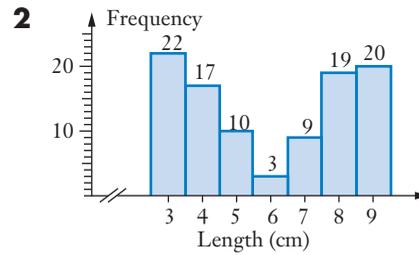
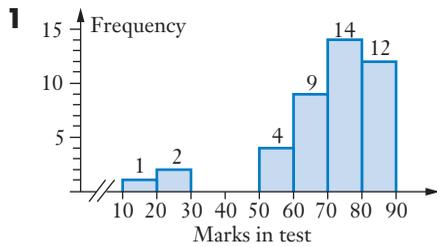
Whilst the times ranged from approx 5 minutes to 105 minutes over half (55%) were between 5 minutes and 35 minutes.

Descriptions could also include mention of: gaps,
clusters,
more dense/less dense regions,
extreme values or outliers.

- ← Any relevant information perhaps not covered in location, spread and shape?
- ← Comment about location. Mean, median (whichever can be determined).
- ← Comment about spread. Range, interquartile range, standard deviation (whichever can be determined)
- ← Comment on shape. Symmetry, modality, skewness, as appropriate.
- ← Anything else you notice of relevance.

Exercise 4B

Describe each of the distributions shown in questions 1 to 6.



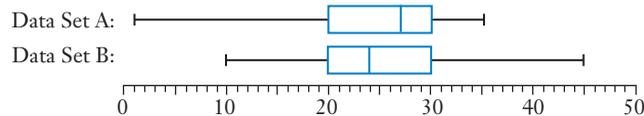
5

Score	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55	56 – 60	61 – 65
Frequency	15	12	16	15	13	14	15

6

Score (x)	$0 \leq x < 10$	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$
Frequency	46	29	13	6	3	2	1

7 Compare the distributions shown in the box plots below.

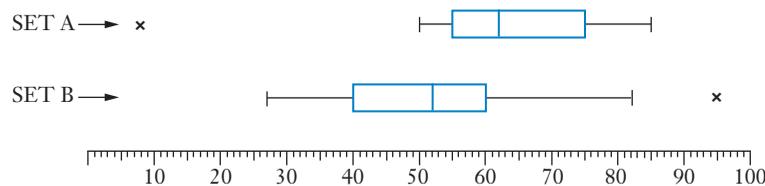


8 Compare the distributions shown in the box plots below which were formed using the results of two maths classes, set A and set B, taking the same test, with set A being the top set and expected to do better than set B which was the second set.

Each boxplot shows any outliers that are more than

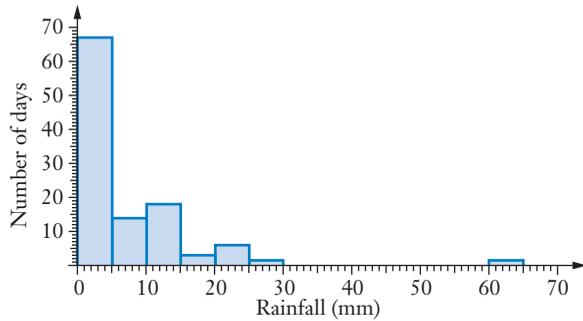
$1.5 \times$ the interquartile range from the nearest quartile

as separate crosses.



- 9 The rainfall figures recorded at a Regional Meteorology Station for each day that some rain fell at the location, from 1 January to 31 December of a particular year, are shown in the table and graph below:

Rainfall (x mm)	$0 < x < 5$	$5 \leq x < 10$	$10 \leq x < 15$	$15 \leq x < 20$	$20 \leq x < 25$	$25 \leq x < 30$	$60 \leq x < 65$
Number of days	67	14	18	3	6	1	1



Shutterstock.com/Zorabc

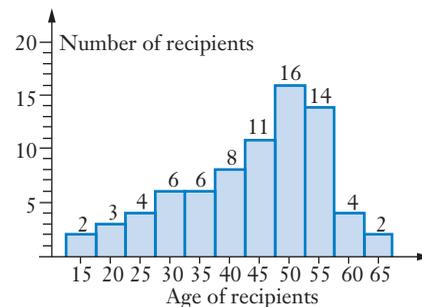
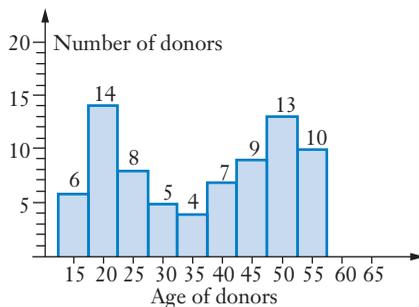
Write a report describing the rainfall for this region in the year for which the data applies.

- 10 The table below shows the scores obtained by the 196 students sitting a particular examination.

Score	21	31	41	51	61	71	81	91	101	111
	to 30	to 40	to 50	to 60	to 70	to 80	to 90	to 100	to 110	to 120
Number of students	2	3	11	12	17	32	37	41	24	17

Write a summary describing the performance of the students in this examination.

- 11 A survey of the age of the donors and the recipients of a particular organ transplant procedure led to the following histograms:



Write a report describing and comparing the data.

Miscellaneous exercise four

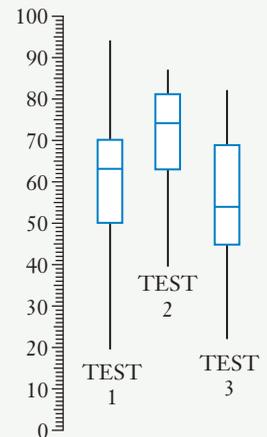
This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1 Find the mean and the standard deviation (correct to two decimal places) of the following set of scores both with and without the outlier included.

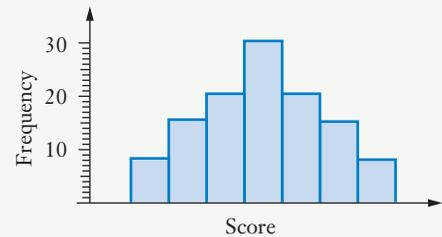
4, 5, 6, 6, 8, 9, 9, 9, 34.

- 2 The diagram on the right shows boxplots for percentage scores in three tests taken by a maths class.

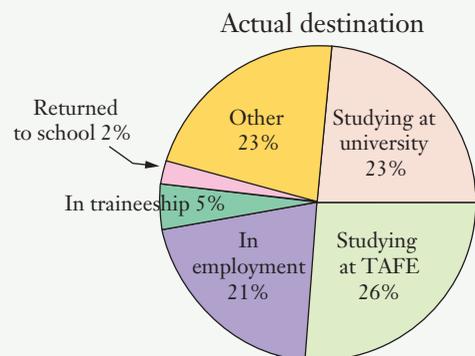
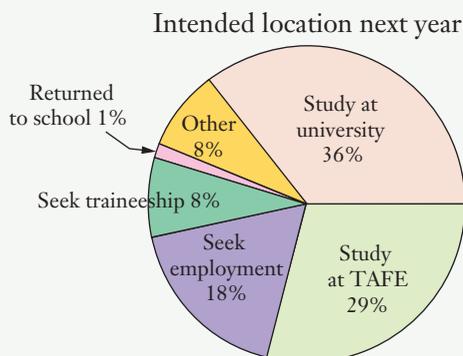
- In which test was the highest mark scored?
- In which test was the lowest mark scored?
- Which test had the highest median?
- Which test had the greatest interquartile range?
- Which test had the greatest range of marks?
- Which test had the smallest range of marks?
- What percentage of the students scored 50% or above in test one?
- In which test did at least three quarters of the students achieve a mark of more than 60%?



- 3 If five of the scores from the central column of the histogram shown on the right were removed from the data, would the standard deviation increase or decrease? Justify your answer with appropriate reasoning.



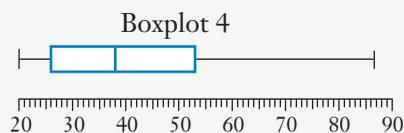
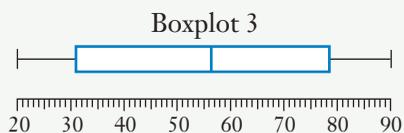
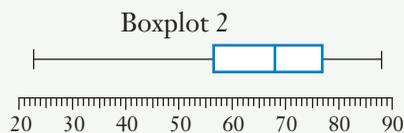
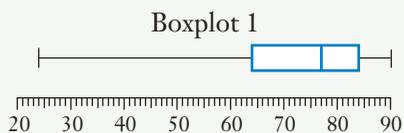
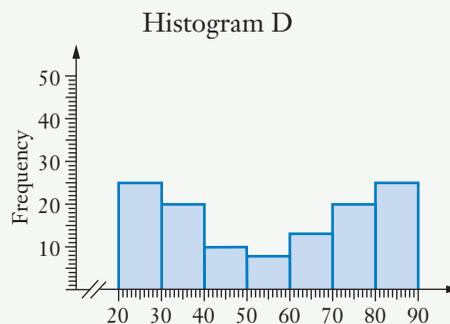
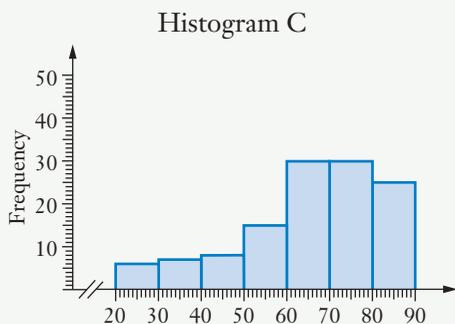
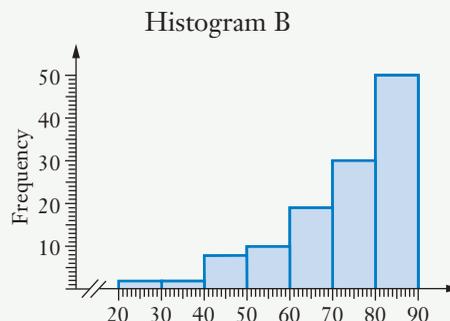
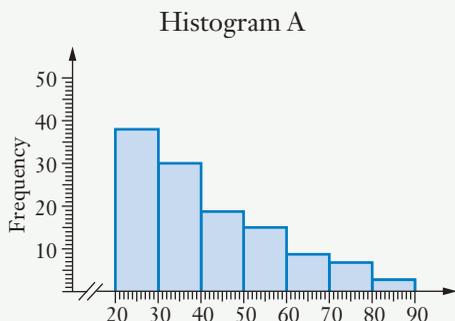
- 4 The pie chart below left shows (for a particular year) the intended destinations for the following year for the year 12 students in Western Australia. The pie chart below right shows where these same students actually were the following year.



[Source of data: Western Australian Department of Education and Training.]

Imagine you are a newspaper reporter asked to write a short article using some of the information contained in these pie charts. Concentrate on just one or two sectors, e.g. university study or TAFE study, and write the article with an appropriate headline included. (There were approximately 18 000 year 12 students in WA that year.)

- 5 Given that the four sets of data that were used to create the histograms below were also used to create the four boxplots shown, match each histogram with its corresponding boxplot.

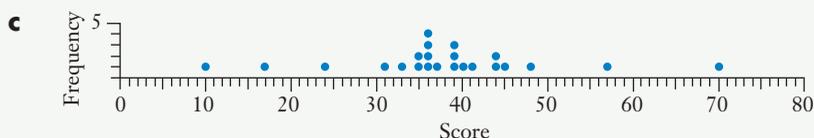


6 Classifying an outlier as:

*any score that is more than $1.5 \times$ the interquartile range
from either quartile 1 or quartile 3, whichever is the nearer*

which scores in the following distributions would be classified as outliers?

- a** A distribution with a lower quartile of 28 and upper quartile of 40.
b A distribution with a median of 35, which is 5 above the lower quartile and 13 below the upper quartile.



d

73	63	79	91	87	75	89	77	79	57
87	82	71	85	81	98	62	85	83	87
65	93	80	89	88	42	91	68	80	75
88	83	68	82	74	80	79	78	50	88
80	49	87	77	83	78	86	62	76	80

7 A test involved ten questions and was sat by 120 students. Copy and complete the following table showing the marks obtained. (Each question was either correct, 1 mark, or incorrect, 0 marks.)

- Whilst you may not be familiar with the term *cumulative frequency* used in the third row of the table, with thought you should be able to determine what it means.

Mark	0	1	2	3	4	5	6	7	8	9	10
Frequency	3	0	5				15	21	18		
Cumulative frequency	3	3	8	15	28	42				113	120

- a** How many students scored a mark of 9?
b How many students scored a mark less than 10?
c What percentage of students scored a mark greater than 7?
d What fraction of students scored 3 or less?
e Display the data as a boxplot.
f Display the data as a frequency histogram.
g Describe the distribution.
- 8** Find, correct to two decimal places, the standard deviation of the five numbers:

$$(a - 4), \quad (a - 2), \quad (a + 1), \quad (a + 3), \quad (a + 7).$$

5.

The statistical investigation process

- Implementing the statistical investigation process
- Miscellaneous exercise five

Implementing the statistical investigation process

Collect data to investigate one of the questions posed below (or negotiate with your teacher to collect data that will involve collecting and comparing data across two or more groups to investigate an alternative question of your own).

You need to think about what data you will collect,
how you will collect it,
how you will record it.

Present your findings as a report and include in your report:

- the question you decided to investigate,
- what data you collected, how you collected it and how you recorded it,
- the collected data, tabulated and/or graphed as appropriate,
- your analysis of that data,
- your conclusions.

Are year 11s the fittest year group in the school?

Do females have quicker reaction times to males?

Are female year 8s better at mental arithmetic than male year 8s?

How does your *Mathematics Applications* class compare with other *Mathematics Applications* classes in recent tests?

Do males have bigger heads than females?

Are left-handers generally taller than right-handers?



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Les Gibbon / Alamy Stock Photo

Miscellaneous exercise five

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1 The times taken to perform a particular task were recorded for 174 people and the results, in seconds, are shown below:

20.41	30.13	60.34	19.72	81.31	27.14	54.14	70.08	68.73	17.24
51.34	20.32	26.83	84.72	25.12	41.13	24.23	53.24	31.62	67.81
26.14	57.24	29.81	70.02	76.24	55.82	58.21	24.55	24.62	44.14
60.82	26.08	63.54	20.80	10.73	40.21	27.23	64.18	75.64	53.80
29.24	48.24	85.34	57.89	38.13	24.32	39.23	75.00	35.88	23.14
74.18	80.79	11.23	33.15	77.34	68.81	28.14	24.21	16.81	31.54
79.31	38.72	88.23	70.73	69.14	78.85	69.86	70.13	76.88	75.21
18.74	70.54	33.65	38.94	37.24	29.53	48.87	30.82	23.22	45.61
76.31	38.24	56.18	66.84	60.98	62.41	15.71	49.00	64.92	31.70
38.16	48.94	47.32	18.31	39.41	28.32	77.41	39.91	34.73	67.34
74.23	60.93	85.23	74.89	34.13	75.32	61.24	75.87	74.98	23.91
33.24	29.11	36.40	65.11	20.81	51.24	13.75	43.24	29.18	35.24
47.41	71.34	89.10	71.23	76.71	28.71	71.92	67.14	58.90	53.71
79.58	42.09	61.82	85.67	65.01	75.93	61.34	58.32	31.23	47.24
21.32	77.23	88.24	14.63	76.91	42.90	34.89	23.62	53.28	72.81
57.31	18.13	65.23	76.52	48.21	73.24	78.51	23.51	42.64	15.23
42.71	79.22	42.83	36.02	62.13	22.04	30.24	51.89	31.24	64.99
77.14	29.72	21.41	81.25						

The histogram for these results, using the intervals

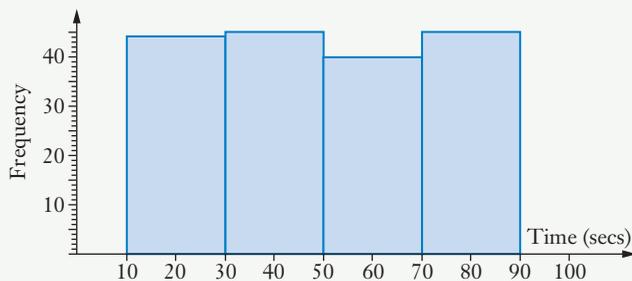
$$10 \leq \text{time} < 30$$

$$30 \leq \text{time} < 50$$

$$50 \leq \text{time} < 70$$

$$70 \leq \text{time} < 90$$

is shown below.



This histogram suggests that the times are evenly, or *uniformly*, distributed from a low of about 10 seconds to a high of about 90 seconds.

However, is this 'uniform distribution' really the case?

Draw the histogram for this data using the intervals

$$10 \leq \text{time} < 20$$

$$20 \leq \text{time} < 30$$

$$30 \leq \text{time} < 40$$

etc.,

and comment on your findings.

2 Expand and simplify each of the following:

a $2(x + 4) + 5(x + 3)$

c $12(x + 1) - 5(x + 2)$

e $3(2x + 5) - 1(x - 3)$

b $5(2x + 3) + 2(3x + 1)$

d $3(x - 4) - 2(x - 1)$

f $2(3x + 2) - (5x + 2)$

3 The box plot and the histogram for a set of data are shown on the right.

For this set of data, determine

a the lowest score,

b the median,

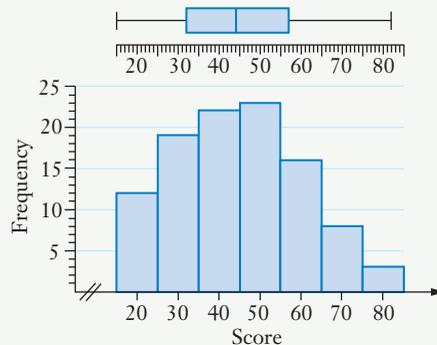
c the highest score,

d the interquartile range,

and use your calculator to determine

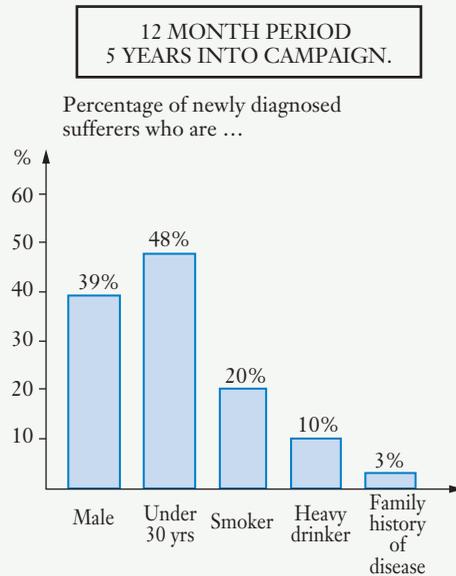
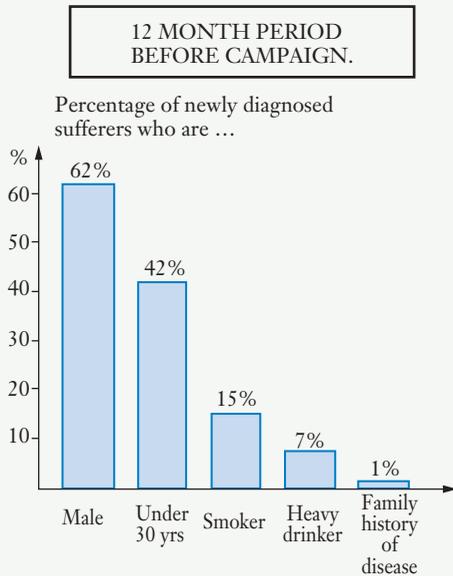
e an estimate for the mean,

f an estimate for the standard deviation.



4 Concerned about the spread of a certain disease, the health department launches a long-term campaign aimed at increasing the population's awareness of the risks. To monitor the success of the campaign amongst various sections of the community, all newly diagnosed sufferers are noted according to a number of characteristics.

The graphs below show the percentage of newly diagnosed sufferers in each category, before the campaign started and 5 years into the campaign.



a The percentages for each graph add up to more than 100%! Explain.

b Would it be correct to say that the graphs indicate that the number of newly diagnosed sufferers who are male has decreased but the number who are under 30 has increased? Justify your answer.

6.

Solving equations

- Solving equations
- Equations with brackets or fractions
- Linear equations
- Miscellaneous exercise six

The *Preliminary work* section at the beginning of this book reminded us of work encountered previously involving

using a formula to determine the value of a variable, or pronumeral, that appears in the formula by itself and on one side of the equals sign, given the values of the variables, or pronumerals appearing on the other side.

In the examples given, there we used $A = P + I$ to determine A , given P and I ,

$C = 2\pi r$ to determine C knowing π and given r ,

$s = ut + \frac{1}{2}at^2$ to determine s , given u , a and t .

Suppose instead, in this last formula, we were asked to determine u given s , t and a .

Substituting the given values into the formula gives us a statement of equality, called an **equation**, which will have to be **solved** to determine the unknown quantity.

For example, if $s = 63$, $t = 3$ and $a = 10$, $s = ut + \frac{1}{2}at^2$ becomes

$$63 = u(3) + \frac{1}{2}(10)(3)^2$$

i.e. $63 = 3u + 45$

Solving this **equation** means finding the value of u for which $3u + 45$ does equal 63.

What is the required value of u for which

$$3u + 45 = 63?$$

Note: The word *variable* was used in chapter one when considering data. In that case there were *various* responses that could be given to a question like ‘What is your favourite colour?’ Hence the use of the word variable there.

In the formula $A = P + I$, A can take *various* values, dependent upon the values of P and I . Hence the use of the word variable in this situation. Given specific values of P and I , the particular value of A can be determined.



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One- and two-step equations

Solving equations

To solve $3u + 45 = 63$, did you proceed mentally, or did you perhaps use the ability of some calculators to solve equations or did you carry out a step-by-step process to isolate u ?

These three approaches are shown below and in the examples that follow.

Solving $3u + 45 = 63$ mentally:

We know that *eighteen* plus forty-five equals 63.

Thus $3u = 18$.

But three multiplied by *six* is eighteen.

Hence $u = 6$.



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Using the solve facility on a calculator.

solve ($63=3\cdot u+45$, u)
{ $u=6$ }

Using a **step-by-step process** of doing something to both sides of the equation in order to isolate the unknown whilst retaining the correctness of the equality statement.

Starting with the given equation: $63 = 3u + 45$

We subtract 45 from each side to isolate $3u$: $18 = 3u$

We divide each side by 3 to isolate u : $6 = u$

Thus $u = 6$.

EXAMPLE 1

Solve the following equations

a $x + 9 = 21$

b $5x - 7 = 23$

c $15 - 2x = 4$

Solution

a $x + 9 = 21$

Mentally:

We know that *twelve* add nine equals twenty-one.

Thus $x = 12$.

Using the solve facility:

solve ($x+9=21$, x)
{ $x=12$ }

Step by step approach to isolate x :

We are given the equation: $x + 9 = 21$
 We subtract 9 from each side to isolate x : $x = 21 - 9$
 $= 12$
 Thus $x = 12$

b $5x - 7 = 23$

Mentally:

We know that *thirty* take seven equals twenty-three.

Thus $5x = 30$.

But five times *six* equals 30 and so $x = 6$.

Using the solve facility:

solve ($5x-7=23, x$)
{ $x=6$ }

Step by step approach to isolate x :

We are given the equation: $5x - 7 = 23$
 We add 7 to both sides to isolate $5x$: $5x = 23 + 7$
 $= 30$
 Now we divide both sides by 5 to isolate x : $x = 30 \div 5$
 $= 6$
 Thus $x = 6$

c $15 - 2x = 4$

Mentally:

We know that fifteen take *eleven* equals four.

Thus $2x = 11$ and so $x = 5.5$.

Using the solve facility:

solve ($15-2x=4, x$)
{ $x=5.5$ }

Step by step approach to isolate x :

We are given the equation: $15 - 2x = 4$
 We add $2x$ to both sides to make the x term positive: $15 = 4 + 2x$
 We subtract 4 from both sides to isolate $2x$: $15 - 4 = 2x$
 $\therefore 11 = 2x$
 Now we divide both sides by 2 to isolate x : $5.5 = x$
 $\therefore x = 5.5$



More complex
equations

Equations with brackets or fractions

Some equations may involve brackets. For example, $3(2x + 1) - 5 = 40$.

Some equations may involve fractions. For example, $\frac{2x + 3}{5} = 4$.

EXAMPLE 2

Solve the following equations

a $3(x - 1) = 21$

b $3(2x + 1) - 5 = 40$

c $\frac{2x + 3}{5} = 4$

Solution

a $3(x - 1) = 21$

Mentally:

We know that three times *seven* equals twenty-one.

Thus $x - 1 = 7$.

But *eight* take one equals 7 and so $x = 8$.

Using the solve facility:

solve $(3(x-1)=21, x)$
 $\{x=8\}$

Step by step approach to isolate x :

We are given the equation:

$$3(x - 1) = 21$$

Expand to remove bracket:

$$3x - 3 = 21$$

Add 3 to both sides to isolate $3x$:

$$3x = 24$$

Now divide both sides by 3 to isolate x :

$$x = 8$$

b $3(2x + 1) - 5 = 40$

Mentally:

We know that three *fifteens* take away five is equal to 40.

Thus $2x + 1 = 15$.

But *fourteen* add one is equal to fifteen.

Hence $2x = 14$ and so $x = 7$.

Using the solve facility:

$$\text{solve } (3(2x+1)-5=40, x) \\ \{x=7\}$$

Step by step approach to isolate x :

We are given the equation: $3(2x + 1) - 5 = 40$

Expand to remove bracket: $6x + 3 - 5 = 40$

Hence $6x - 2 = 40$

Add 2 to both sides to isolate $6x$: $6x = 42$

Now divide both sides by 6 to isolate x : $x = 7$

c $\frac{2x + 3}{5} = 4$

Mentally:

We know that *twenty* divided by five is equal to four.

Thus $2x + 3$ must equal 20.

But *seventeen* plus three is equal to twenty.

Hence $2x = 17$ and so $x = 8.5$.

Using the solve facility:

$$\text{solve } \left(\frac{2 \cdot x + 3}{5} = 4, x \right) \\ \{x=8.5\}$$

Step by step approach to isolate x :

We are given the equation: $\frac{2x + 3}{5} = 4$

Multiply both sides by 5 to remove fractions: $2x + 3 = 20$

Subtract 3 from both sides to isolate $2x$: $2x = 17$

Now we divide both sides by 2 to isolate x : $x = 8.5$



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If the equations are more involved the mental approach can be too difficult but we can still use either

- the solve facility on a graphic calculator, or
- the step by step process to isolate the unknown,

as shown below.

EXAMPLE 3

Solve the following equations:

a $2(x + 3) - 3(2x + 1) = -5$

b $\frac{x}{7} = \frac{3}{10} + \frac{2x}{21}$

Solution

a Using the solve facility:

solve $(2(x+3)-3(2x+1))=-5, x)$
{x=2}

Step by step approach to isolate x :

We are given the equation:

Expand to remove brackets:

Collect like terms:

We add $4x$ to both sides to make the x term positive:

Add 5 to both sides to isolate $4x$:

Now we divide both sides by 4 to isolate x :

$$\begin{aligned}2(x + 3) - 3(2x + 1) &= -5 \\2x + 6 - 6x - 3 &= -5 \\-4x + 3 &= -5 \\3 &= -5 + 4x \\8 &= 4x \\2 &= x \\\therefore x &= 2\end{aligned}$$

b Using the solve facility:

solve $\left(\frac{x}{7} = \frac{3}{10} + \frac{2x}{21}, x\right)$
{x=6.3}

Step by step approach to isolate x :

We are given the equation:

$$\frac{x}{7} = \frac{3}{10} + \frac{2x}{21}$$

Multiply both sides by 210 to remove fractions: $210 \times \frac{x}{7} = 210 \times \frac{3}{10} + 210 \times \frac{2x}{21}$

i.e.:

$$30x = 63 + 20x$$

Subtract $20x$ from each side:

$$10x = 63$$

Now we divide both sides by 10 to isolate x :

$$x = 6.3$$

EXAMPLE 6

Formula: $S = 2\pi r^2 + 2\pi rh$

Find h given that $S = 545$ and $r = 5$ giving your answer correct to 1 decimal place.

Solution

Either

Determine the value of h for the given values of S and r using the ability of some calculators to determine unknown values in a formula given sufficient information.

Equation:

$$S = 2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot h$$

$S = 545$

$r = 5$

$h = 12.3478887970166$

Lower = $-9E+999$

Upper = $9E+999$

Or

Substitute $S = 545$ and $r = 5$ into the given formula and use the step by step approach to determine h , as shown below.

We are given the formula:

$$S = 2\pi r^2 + 2\pi rh$$

Substitute $S = 545$ and $r = 5$:

$$545 = 2\pi(5)^2 + 2\pi(5)h$$

$$\therefore 545 = 50\pi + 10\pi h$$

Subtract 50π from both sides to isolate $10\pi h$:

$$545 - 50\pi = 10\pi h$$

Now we divide both sides by 10π to isolate h :

$$\frac{545 - 50\pi}{10\pi} = h$$

$$\therefore h = 12.3 \text{ (to one decimal place)}$$

Thus when $S = 545$ and $r = 5$, $h = 12.3$ (to one decimal place).



Equations code puzzle

Linear equations

The six equations shown below are all examples of **linear equations in one variable**.

$$2x + 17 = 5$$

$$5p - 7 = 32$$

$$15 - 3z = 6$$

$$\frac{w}{2} = 5$$

$$2(3q - 5) + 1 = 15$$

$$\frac{2n + 3}{5} = 4$$

Each equation, after expansion of any brackets and separation of fractions, only involves terms that are either just a number, or the variable multiplied or divided by a number. Linear equations do *not* involve the variable squared (x^2), cubed (x^3), square rooted (\sqrt{x}), in the denominator of a fraction $\left(\frac{3}{x-1}\right)$, as a power (2^3) etc.

Each of the above equations can, with a bit of work, be written in the form $ax + b = 0$, the basic form of a linear equation in one variable:

$$2x + 12 = 0$$

$$5p - 39 = 0$$

$$-3z + 9 = 0$$

$$w - 10 = 0$$

$$6q - 24 = 0$$

$$2n - 17 = 0$$

(As we will see later in this text, and as you may already be familiar with, equations of the form $y = ax + b$ give straight line, or **linear**, graphs.)

The emphasis in this text will be on solving **linear** equations.

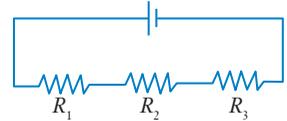
$$6 \quad s = \frac{(u+v)}{2}t$$

- a Find t given that $s = 56$, $u = 3$ and $v = 5$.
- b Find u given that $s = 92$, $v = 14$ and $t = 8$.
- c Find v given that $s = 22.5$, $u = -6$ and $t = 5$.

- 7 If three resistors, R_1 , R_2 and R_3 are placed in an electrical circuit as shown in the diagram they are said to be in **series**. The total resistance, R , is then given by:

$$R = R_1 + R_2 + R_3$$

- a Find R if $R_1 = 2$, $R_2 = 5$, $R_3 = 4$.
- b Find R if $R_1 = 10$, $R_2 = 15$, $R_3 = 20$.
- c Find R_1 if $R = 32$, $R_2 = 8$, $R_3 = 14$.
- d Find R_3 if $R = 80$, $R_1 = 25$, $R_2 = 28$.



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$$8 \quad S = 2\pi r^2 + 2\pi r h$$

- a Find h given $S = 250$ and $r = 4$. (Answer correct to two decimal places.)
- b Find h given $S = 1000$ and $r = 4$. (Answer correct to two decimal places.)

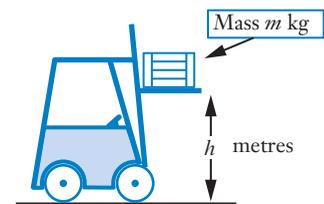
- 9 A certain mass of gas is held under pressure at a constant temperature. V , the volume of the gas, is related to the pressure according to the rule:

$$V \times P = 150.$$

- a Find V given that $P = 6$,
- b Find P given that $V = 15$,
- c Find V given that $P = 7.5$,
- d Find P given that $V = 60$.

- 10 When a forklift truck lifts an object of mass m kg through a height h metres it gives the object potential energy equal to P Joules where $P = 9.8 mh$.

- a Find P when $m = 50$ and $h = 1.5$.
- b Find P when $m = 400$ and $h = 2$.
- c Find m when $P = 2940$ and $h = 2$.
- d Find h when $P = 13\,230$ and $m = 600$.



- 11 A car rental firm charges $\$C$ for renting a particular vehicle where C depends on d , the number of days hired, and k , the number of kilometres travelled. C is calculated according to the rule:
 $C = 30 + 70d + 0.1k$.

- a Find the cost of renting the vehicle for 7 days and travelling 500 km.
- b Find the cost of renting the vehicle for 3 days and travelling 1200 km.
- c A person wishes to rent the vehicle for six days but does not want the hire costs to exceed $\$700$. How far could the person travel in the car in the six days?

- 12** When an agent of a particular real estate company sells a house for A thousand dollars the commission the agent receives is $\$P$ where

$$P = 600 + 4A$$

- a** Find the amount the agent receives in commission for the sale of each of the following houses:



- b** An agent sells a house and receives a commission of $\$2380$. How much was the house sold for?
c An agent sells a house and receives a commission of $\$4000$. How much was the house sold for?
- 13** If, in an archaeological dig, human bones are found, or if in a macabre murder case parts of a body are discovered, these bones can be used to estimate the height the person was when alive. In particular the bone in the upper arm from elbow to shoulder, called the humerus, is a good indicator of height.

If the humerus is of length h cm then a reasonable estimate for the height of a male with this length humerus is:

$$(2.9h + 71) \text{ cm,}$$

and a reasonable estimate for the height of a female with this length humerus is:

$$(2.75h + 71) \text{ cm.}$$

- a** In an archaeological dig the remains of a male are uncovered and the humerus is found to be 34 cm long. Estimate the height of the male.
b What would be the expected humerus length of a 1.81 metre tall female?
- 14** A company has five thousand calendars printed. If it sells x of these calendars, where x is from a low of zero to a high of 5000, the profit produced will be $\$P$ where P is given by:

$$P = 12.7x - 29750$$

- a** What will be the profit if the firm sells 2500 of the calendars?
b What will be the profit if the firm sells 3500 of the calendars?
c What will be the profit if the firm sells all but 800 of the calendars?
d What is the least number of the calendars the firm needs to sell to make a profit that exceeds $\$9000$?
e If the firm are left with 3200 of the calendars unsold determine whether they have made a profit or a loss and state how much?
f What is the greatest profit the firm can make from this venture?
g If the firm sold none of the calendars how much would they lose?
h What is the least number of calendars the firm must sell to avoid making a loss?

Miscellaneous exercise six

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

1 Solve the following equations.

a $3x - 1 = 20$

b $5x + 6 = 24 - 4x$

c $3x + 7 = 21 - x$

d $6(2x + 1) - 5 = 31$

e $\frac{x-2}{3} - 3 = 1$

f $\frac{2x}{9} = \frac{5}{3}$

2 Formula: $A = 4bh + b^2$

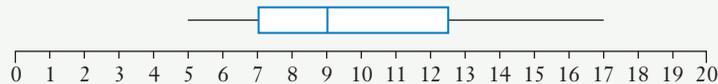
a Find A given that $b = 3$, $h = 5$.

b Find h given that $A = 119$, $b = 7$.

3 The following nine scores are listed in ascending order, from left to right.

$a + 2$, $a + 3$, b , $c - 1$, $c - 1$, $c + 1$, d , $e - 1$, $a + e$.

The box and whisker diagram for these nine scores is as follows:



Determine a , b , c , d and e .

4 To pass a particular course a student needs to gain a mean of at least 60% in the six tests that form the course assessment. In the first five tests the student achieves marks of 65%, 58%, 71%, 60% and 59%. What percentage mark does the student require in test six in order to pass the course?

5 The box plots on the right are for four sets of data, A, B, C and D. Which of the data sets:

a seems to involve the greatest variability?

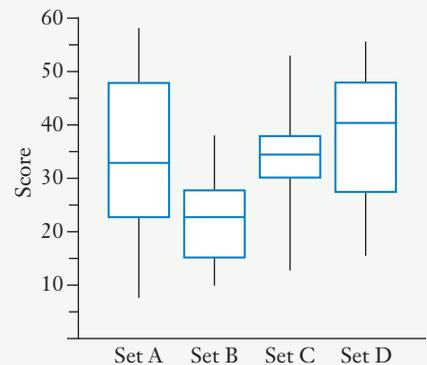
b has the smallest interquartile range?

c has the smallest range?

d contains the lowest of all the scores?

e could the following apply to?

More than half of the scores in set ___ exceed all of the scores in set ___.



- 6 The marks obtained by 47 students in an examination marked out of 140 are shown below:

25 32 50 54 59 66 67 69 71 73 75 75
 76 76 78 80 81 82 83 83 84 85 88 88
 89 89 89 89 90 92 94 95 96 99 100 104
 106 108 109 109 111 111 113 114 115 115 117

With the mean of these marks being \bar{x} and the standard deviation σ , grades are awarded to these 47 students as follows

		exam mark	\geq	$\bar{x} + 1.25\sigma$	Grade A
$\bar{x} + 0.5\sigma$	\leq	exam mark	$<$	$\bar{x} + 1.25\sigma$	Grade B
$\bar{x} - 0.5\sigma$	\leq	exam mark	$<$	$\bar{x} + 0.5\sigma$	Grade C
$\bar{x} - 2\sigma$	\leq	exam mark	$<$	$\bar{x} - 0.5\sigma$	Grade D
		exam mark	$<$	$\bar{x} - 2\sigma$	Fail grade

Determine the number of students obtaining each grade.

- 7 A real estate agent in a particular region wants to publish the average price of houses sold in the town each month. Past figures indicate that each month somewhere between 10 and 50 houses are sold each month. Most houses in the area are of a similar nature except for a small number of beachside luxury properties. These do not come up for sale very often but when they do they are priced very much above most others in the region.

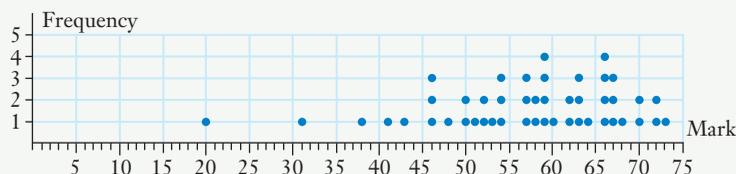
Which average (mean, median or mode) would you advise the agent to use for the average monthly price and why?

- 8 The salaries of the 187 full time employees of a large manufacturing company were distributed as follows:

Category	Salary \$\$	Number of Employees
A	$40\,000 \leq S < 50\,000$	23
B	$50\,000 \leq S < 60\,000$	64
C	$60\,000 \leq S < 70\,000$	43
D	$70\,000 \leq S < 80\,000$	25
E	$80\,000 \leq S < 90\,000$	14
F	$90\,000 \leq S < 100\,000$	9
G	$100\,000 \leq S < 110\,000$	5
H	$110\,000 \leq S < 120\,000$	3
I	$120\,000 \leq S < 130\,000$	1

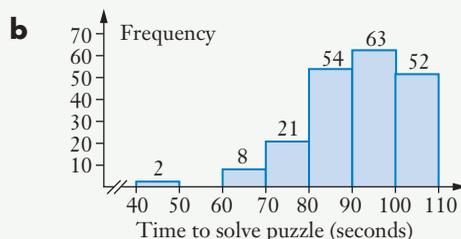
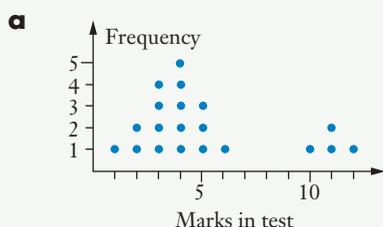
- a Calculate the mean and standard deviation of this distribution.
- b With increased automation in the manufacturing processes the company no longer requires such a large workforce. Through voluntary redundancy and non-replacement of retirees the company reduced its workforce to 173 by losing 5 employees from category A, 4 from category B, 3 from category C and 2 from category F. Calculate the mean and standard deviation of the salaries of this workforce of 173.

- 9 One hundred and twenty-seven people applied for a particular job vacancy and the company involved decided to invite what they considered to be the best 50 applicants to take an aptitude test. On test day 47 of the invited 50 turned up and the marks obtained (out of 75) were as shown in the following diagram:

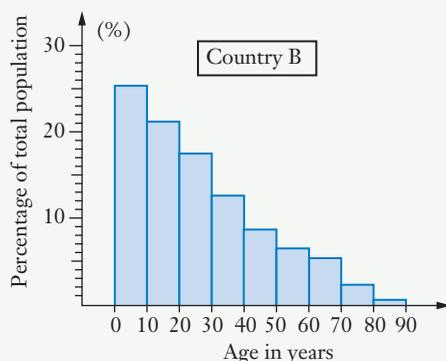
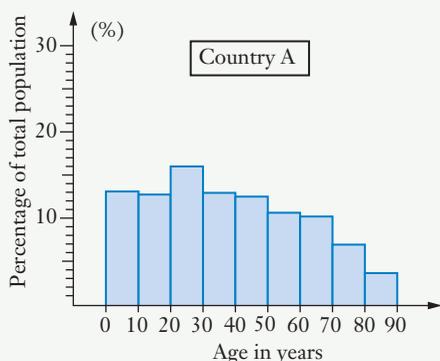


The company decides to invite for interview any of the applicants who achieved a mark in the test that is more than 1 standard deviation above the mean. How many of the applicants do they invite for interview?

- 10 Describe each of the following distributions.



- 11 The graphs below show the age distribution of the human population of two countries, A and B.



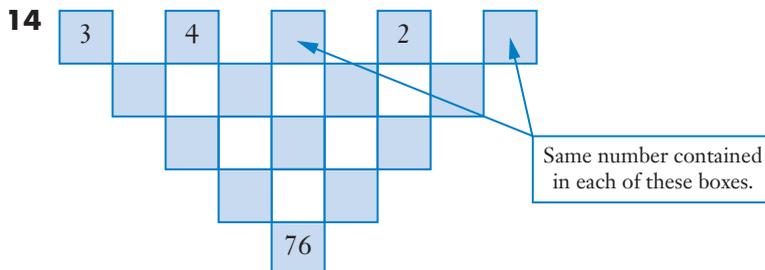
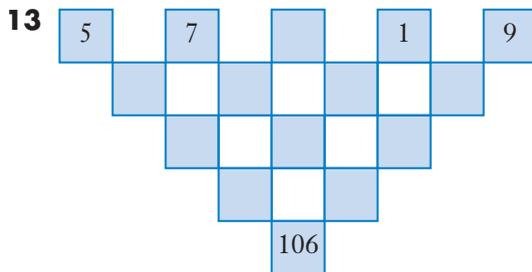
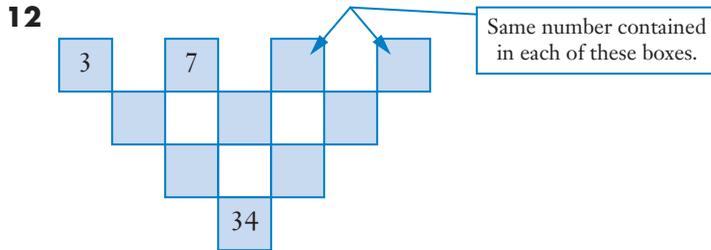
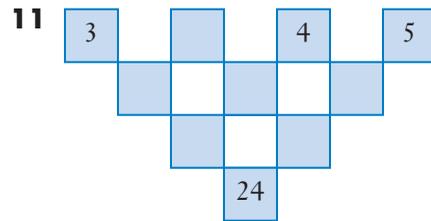
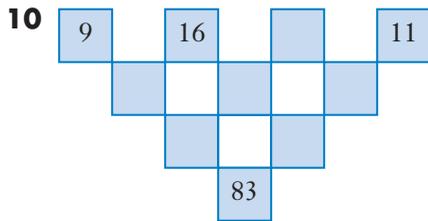
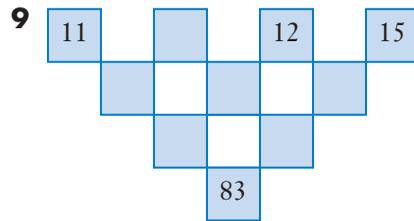
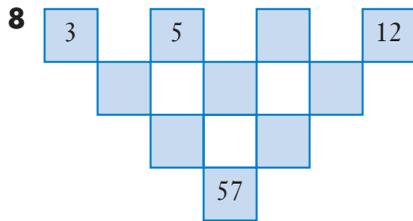
- Which of the two countries has the greater population?
- If country A has a total population of approximately eighty two million people, approximately how many of these are 70 or over?
- Produce a report describing the population distributions of each country, commenting on any similarities and differences between the distributions of the two countries and comment on possible implications for future government policy given these population distributions.

7.

Using equations to solve problems

- Pyramids
- Number puzzles
- Solving problems
- Equations from simple interest formula
- Equations from ratios
- Miscellaneous exercise seven

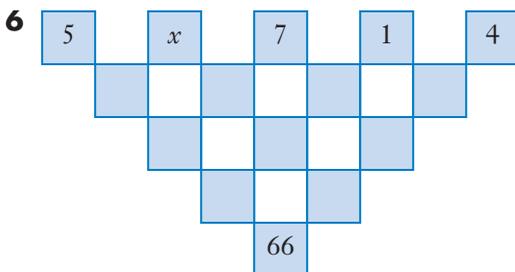
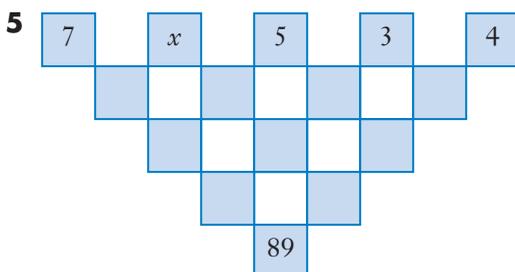
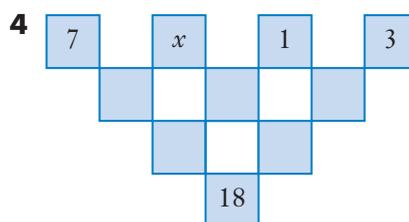
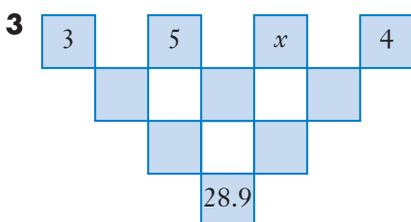
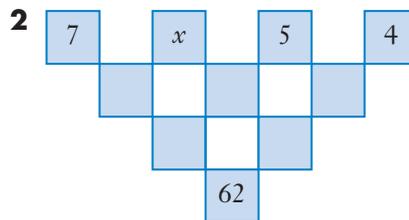
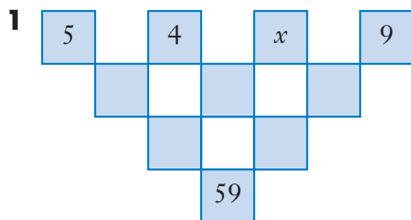
Pyramids 1 to 7 should not have caused you too much trouble but now try pyramids 8 to 14 below. They are not so easy and may involve fractions.



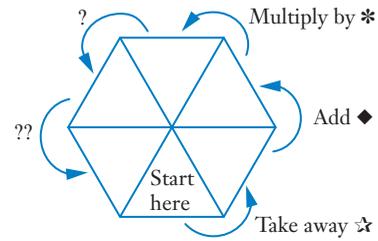
This technique of introducing a symbol, frequently x , is a useful mathematical technique for solving questions in which we seem to have sufficient information but we can't seem to 'get started'.

Exercise 7A

Find the value of x in each of the following pyramids.



7 The 'hexapatterns' shown below **all** follow the pattern shown on the right. The symbols ☆, ◆ and * all represent numbers.



- a What number does ☆ represent?
- b What number does ◆ represent?
- c What number does * represent?
- d What instruction should go in place of '?' ?
- e What instruction should go in place of '??' ?
- f Copy and complete the 'hexapatterns' E to L shown below.

A		B		C		D	
E		F		G		H	
I		J		K		L	

Number puzzles

So far in this chapter we have seen an x introduced to enable us to 'get started' on certain problems. Once started we then obtained an equation which allowed x to be determined. This technique of introducing an x in order to get started can be used to solve other types of puzzles and problems.



Word problems with equations



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EXAMPLE 1

I think of a number, multiply it by three, add seven to the answer and then add the number I first thought of. If my answer is 51 what number did I think of?

Solution

Question We need to multiply the number by three and then add seven etc. but how can we get started if we do not know what the number is?

Answer To allow us to get started let x be the number first thought of.

Let x be the number first thought of:	x
Multiply by 3:	$3x$
Add seven to the answer:	$3x + 7$
Add the number first thought of:	$3x + 7 + x$
The result is 51:	$\therefore 3x + 7 + x = 51$
Collect like terms:	$4x + 7 = 51$
Solving:	Step-by-step OR
	$4x + 7 = 51$
	$4x = 51 - 7$
	$4x = 44$
	$x = 11$

The number I first thought of was 11.

By calculator

solve($4x+7=51$, x)
 $\{x=11\}$

Note: In the last example we did not leave the answer as $x = 11$. The initial problem asked us to find the number thought of and had no mention of x which we introduced to help solve the question. Our final sentence should clearly answer this question and not simply say ' $x = 11$ ' unless of course the question itself had introduced, and asked us to find, x .

EXAMPLE 2

I think of a number, subtract it from twelve, multiply the result by five, add the number I first thought of and end up with 24. Find the number first thought of.

Solution

Let x be the number first thought of:	x
Subtract it from 12:	$12 - x$
Multiply by 5:	$5(12 - x)$
Add the number first thought of:	$5(12 - x) + x$
The result is 24:	$\therefore 5(12 - x) + x = 24$
Solving:	
	$5(12 - x) + x = 24$
	$60 - 5x + x = 24$
	$60 - 4x = 24$
	$60 = 24 + 4x$
	$36 = 4x$
	$\therefore x = 9$

The number I first thought of was 9.

solve($5(12-x)+x=24$, x)
 $\{x=9\}$

Note: Trial and adjustment is a perfectly acceptable alternative method for solving this type of question. For example 2:

Try 3: Subtract 3 from 12: 9
 Multiply by 5: 45
 Add number first thought of: 48
 Thus 3 was not correct as we want to end up with 24.

Try 5: Subtract 5 from 12: 7
 Multiply by 5: 35
 Add number first thought of: 40
 Thus 5 was not correct but was better than 3.

Continuing in this way will eventually give us the correct answer of 9. Trial and adjustment is a very useful technique but it can be a lengthy process, particularly if the answer is not an integer. In the examples that follow the calculator approach or the step by step method will tend to be shown.

EXAMPLE 3

I think of a number, add five, multiply the result by four, and end up with an answer that is 29 more than the number I first thought of. Find the number first thought of.

Solution

Let x be the number first thought of.

Thus $4(x + 5) = x + 29$

Solving:

$$\text{solve}(4(x+5)=x+29, x) \\ \{x=3\}$$

Check carefully that you understand how this equation has been arrived at.

The number I first thought of was 3.

EXAMPLE 4

I think of a number, add fifteen, divide the result by two, and end up with four more than the number I first thought of. Find the number first thought of.

Solution

Let x be the number first thought of.

Thus $\frac{x + 15}{2} = x + 4$

Solving:

Multiply by two: $x + 15 = 2x + 8$

Subtract x : $15 = x + 8$

Subtract 8: $7 = x$

$\therefore x = 7$

The number I first thought of was 7.

Check carefully that you understand how this equation has been arrived at.

Exercise 7B

- 1 If x represents 'the number' write each of the following statements in terms of x .

Example: Three times the number then add one.

Answer: $3x + 1$

- a Multiply the number by five then add six.
- b Take the number away from fourteen.
- c Add six to the number and then multiply by five.
- d Double the number then take away seven.
- e Take seven from the number and then double your answer.
- f Double the number, add five and then multiply your answer by three.

- 2 If x is used to represent 'the number I think of' in each of the statements A to H below, select the equation from the box on the right that matches each statement and then solve the equation.

A: I think of a number, double it and add one and my answer is 10.

B: If I take one from the number I think of and double the result my answer is ten.

C: I think of a number and subtract it from ten and my answer is one.

D: I think of a number, add one and then double the result and my answer is ten.

E: I think of a number, divide it by two and then subtract one and end up with 10.

F: I think of a number, take ten away and my answer is one.

G: I think of a number, subtract one and then divide by two and my answer is ten.

H: Twice the number I thought of exceeds ten by one.

- 3 I think of a number, multiply by three, subtract eleven from the answer and then add the number I first thought of. If my answer is 25 what was the number I first thought of?
- 4 I think of a number, add seven, multiply the result by two and end up with an answer that is 17 more than the number I first thought of. Find the number I first thought of.
- 5 I think of a number, double it, add seven and end up with an answer that is 17 more than the number I first thought of. Find the number I first thought of.
- 6 I think of a number, subtract four, multiply the result by three and end up with an answer that is two less than the number I first thought of. Find the number I first thought of.
- 7 I think of a number, subtract five, divide the answer by two and end up with seven less than the number I first thought of. Find the number I first thought of.
- 8 I think of a number, add two, multiply the answer by three and then subtract eleven. This gives an answer equal to twice the number first thought of. Find the number I first thought of.

$2x + 1 = 10$
 $2x - 10 = 1$
 $x - 10 = 1$
 $1 - \frac{x}{2} = 10$
 $10 - x = 1$
 $\frac{x}{2} - 1 = 10$
 $2(x - 1) = 10$
 $\frac{x-1}{2} = 10$
 $\frac{1-x}{2} = 10$
 $2(x + 1) = 10$

EXAMPLE 6

Rosalyn is 8 years older than Jennifer. In six years' time their ages will be such that Rosalyn will be twice as old as Jennifer. How old is Jennifer now?

Solution

Let Jennifer's age now be	x years
Thus Rosalyn's age now is	$(x + 8)$ years
In 6 years' time Jennifer will be	$(x + 6)$ years
In 6 years' time Rosalyn will be	$(x + 8) + 6$ years
Thus	$(x + 8) + 6 = 2(x + 6)$
This equation can be solved to give	$x = 2$
Jennifer is 2 years old now.	

Note: Alternatively 'trial and adjustment' can be used or the answer can be 'reasoned through' as follows: If Rosalyn is 8 years older than Jennifer now, she will always be 8 years older. Thus, in 6 years' time, when Rosalyn is twice as old as Jennifer, the difference in their ages will be both 8 years and one lot of Jennifer's age. Thus in 6 years' time Jennifer will be 8 and Rosalyn will be 16. Thus Jennifer is 2 years old now.

EXAMPLE 7

An amateur drama group hire a theatre for their production. They expect to sell all of the 1200 tickets, some at \$10 and the rest at \$7. The group require the ticket sales to cover their \$4150 production costs, to allow a donation of \$4000 to be made to charity and to provide a profit of \$1000 to aid future productions. If they are to exactly achieve this target and their expectations regarding ticket sales are correct how many of the total 1200 tickets should they charge \$10 for and how many should they charge \$7 for?



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Solution

Let the number of \$7 tickets be	x
These will give an income of	$7x$ dollars
The number of \$10 tickets will then be	$(1200 - x)$
These will give an income of	$10(1200 - x)$ dollars
Thus	$7x + 10(1200 - x) = 4150 + 4000 + 1000$
which can be simplified to	$12\,000 - 3x = 9150$
Solving gives	$x = 950$
The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.	

- 8** An amateur drama group hire a theatre for their production. They expect to sell all 850 tickets, some at \$12 and the rest at \$8. The group require the ticket sales to cover their \$3760 production costs and to make a profit of \$4000. If they are to exactly achieve this target and their expectations regarding ticket sales are correct how many of the 850 tickets should they charge \$12 for and how many should they charge \$8 for?
- 9** A farmer has a certain number of acres that she wishes to use to grow wheat, barley and lupins. Whatever acreage she decides to use for the lupins she likes to have 2000 acres more than this for barley. She also likes to use twice as many acres for wheat as she does for barley.
- a** If she uses x acres for lupins, find
- an expression in terms of x for the number of acres she uses for barley,
 - an expression in terms of x for the number of acres she uses for wheat.
- b** If she decides to use a total of 18 000 acres for the three products, determine how many acres she uses for each.
- 10** A firm making fertiliser produces a new fertiliser *QuickGrow*. Each 50 kg bag of *QuickGrow* contains x kg of compound X and $(50 - x)$ kg of compound Y . Each kilogram of X contains 150 g of a particular element and each kilogram of Y contains 80 g of this element. The company wants each 50 kg of *QuickGrow* to contain 6.24 kg of this element.
- a** Find
- an expression in terms of x for the amount of the particular element contained in x kg of compound X and state the units,
 - an expression in terms of x for the amount of the particular element contained in $(50 - x)$ kg of compound Y and state the units.
- b** How much of each compound, X and Y , should each 50 kg of *QuickGrow* contain to give the desired total amount of the particular element?
- 11** A book shop owner orders some hardback and some softback versions of a book. The hardback version costs the shop owner \$20 each and the softback \$12 each. The total order was for 300 books.
- When the order arrives there were only 200 books! The shopkeeper wishes to query the order but cannot find his copy of the original order. However his records do tell him that it was going to cost him \$5080. How many of each type of the book was his original order for?
- 12** An investor has \$5000 to invest and decides to invest some of it with company A and the rest with company B. After one year each \$1 invested with company A has grown to \$1.20, each dollar invested with company B has grown to \$1.05 and the \$5000 has grown to \$5670. How much of the original \$5000 was invested with each company?



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Equations from ratios

The *Preliminary work* at the beginning of this text reminded us of the idea of a ratio. In particular the following example was given:

Suppose the ratio of males to females in a school is 17 : 21.

If we know that there are 231 females in the school we can determine the number of males

$$\begin{aligned} \text{males : females} &= 17 : 21 && \times 11 \\ &= ? : 231 \end{aligned}$$

The number of males = 17×11 , i.e. 187.

231 / 21

11

Alternatively, if we let the number of males be m , we could set up, and solve, an equation, as follows:

$$\begin{aligned} \text{males : females} &= 17 : 21 \\ \therefore m : 231 &= 17 : 21 \end{aligned}$$

$$\text{Hence} \quad \frac{m}{231} = \frac{17}{21}$$

$$\times \text{ by } 231 \text{ to eliminate fractions:} \quad 231 \times \frac{m}{231} = 231 \times \frac{17}{21}$$

$$\begin{aligned} m &= 231 \times \frac{17}{21} \\ &= 187 \end{aligned}$$

The number of males is 187, as before.

Suppose the previous situation had instead given us the ratio as:

$$\text{females : males} = 21 : 17.$$

We would then have proceeded as follows:

$$\therefore 231 : m = 21 : 17$$

$$\text{Hence} \quad \frac{231}{m} = \frac{21}{17}$$

Now with the variable in the denominator this is not a linear equation but this need not trouble us. We could solve the equation by:

- first multiplying by $17m$ to eliminate the fractions, as shown below left, or
- using the fact that if $\frac{a}{b} = \frac{c}{d}$, then it follows that $\frac{b}{a} = \frac{d}{c}$, as shown below right.

$$\text{Given} \quad 231 : m = 21 : 17$$

$$\frac{231}{m} = \frac{21}{17}$$

$$\times \text{ by } 17m: \quad 17m \times \frac{231}{m} = \frac{21}{17} \times 17m$$

$$\text{Hence} \quad 17 \times 231 = 21m$$

$$\text{Solving gives} \quad m = 187$$

$$\text{OR} \quad \text{Given} \quad 231 : m = 21 : 17$$

$$\frac{231}{m} = \frac{21}{17}$$

$$\text{Hence} \quad \frac{m}{231} = \frac{17}{21}$$

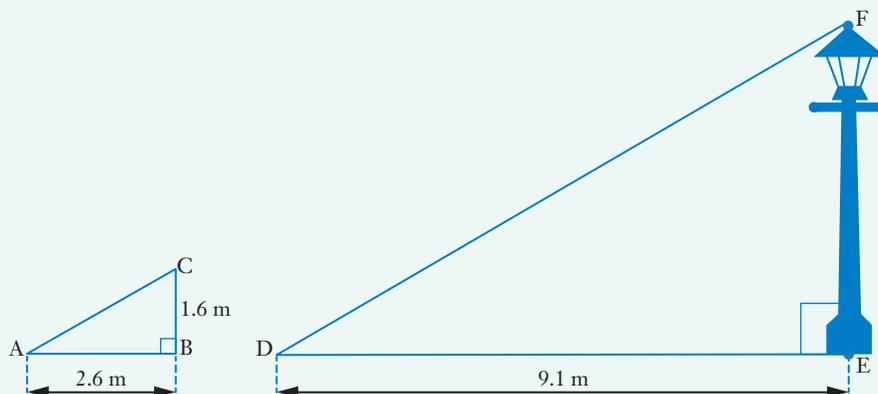
$$\times \text{ by } 231: \quad 231 \times \frac{m}{231} = \frac{17}{21} \times 231$$

$$\text{Hence} \quad m = 187$$

EXAMPLE 12

So how tall is the street light?

On a sunny day, at the same time that a street light standing on horizontal ground casts a shadow of length 9.1 metres, a 1.6 metre stick held vertically on the same ground casts a shadow of length 2.6 metres.



What does this information suggest that the height of the street light is?

Solution

In triangles ABC and DEF: $\angle CAB = \angle FDE$ (angle Sun's rays make with the ground)
 $\angle CBA = \angle FED (= 90^\circ)$

Hence the third angles will be equal and so $\triangle ABC \sim \triangle DEF$, corresponding angles equal.

Hence $AB : DE = BC : EF$

Let the height of the street light be h metres.

$$2.6 : 9.1 = 1.6 : h$$

i.e.
$$\frac{2.6}{9.1} = \frac{1.6}{h}$$

Hence
$$2.6h = 9.1 \times 1.6$$

and so
$$h = 5.6$$

The information suggests that the height of the street light is 5.6 metres.

Exercise 7D

1 Find the values of $a, b, c, \dots i$ in the following.

a $a : 9 = 2 : 3$

b $b : 10 = 2 : 5$

c $4 : 3 = 8 : c$

d $d : 3 = 5 : 2$

e $2e : 9 = 4 : 5$

f $f : 2 = 7 : 5$

g $6 : g = 4 : 1$

h $17 : 2h = 5 : 1$

i $5 : 8 = 3 : i$

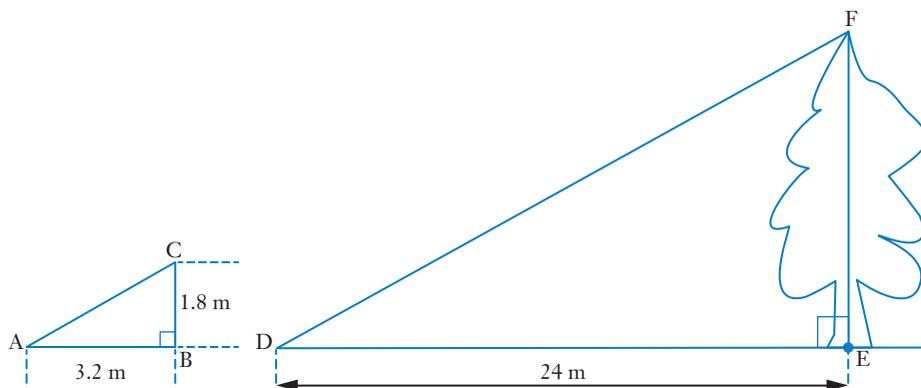
- 2 Determine how much money needs to be invested for 3 years at 8% per annum simple interest to accrue interest of \$1008.
- 3 What annual rate of simple interest would cause an investment of \$6400 to grow to \$7360 in two years?
- 4 Sally invests a sum of money into an account paying 8.2% simple interest. After three years the account has earned interest of \$209.10. What was the initial sum invested?
- 5 For how long must an investment of \$10 000 be invested in an account paying 8.6% per annum simple interest for it to become \$13 870?
- 6 How many days must a sum of \$8650 be left to accrue interest at a rate of 5% per annum simple interest to become \$8823?
- 7 What annual rate of simple interest is needed to see an initial investment of \$6720 become \$7011.20 in 8 months?
- 8 \$65 000 is invested in an account paying simple interest at a rate of $R\%$ per annum. Nine months later the account is closed and the total balance of principal plus interest is then \$68 997.50. Find R .
- 9 What annual rate of simple interest is required to see an investment of eight and a half thousand dollars grow to eight thousand eight hundred and six dollars in 219 days?
- 10 The ratio of males to females in the audience for a particular event was 5 : 7. If there were 1045 males in the audience, how many females were in the audience?
- 11 Suppose the ratio of male students to female students in a school is 15 : 17. If there are 345 male students in the school, how many female students are there in the school?

12 So how tall is the tree?

At the same time as a tree, stood on horizontal ground, casts a shadow of length 24 metres, a 1.8 metre stick held vertically on the same ground casts a shadow of length 3.2 metres.

What does this information suggest that the height of the tree is?

(Your working should include justification that triangles assumed similar are indeed similar.)

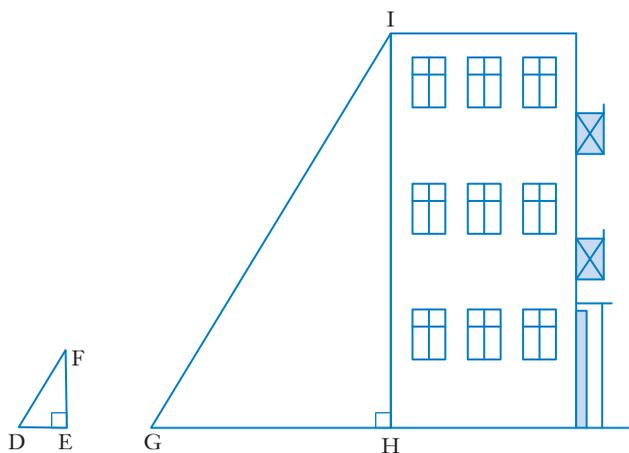


13 So how tall is the building?

When a vertical signpost that is known to have a height of 3 metres casts a 1.8 metre shadow a nearby building casts a shadow of length 15 metres.

How tall is the building?

(Your working should include justification that triangles assumed similar are indeed similar.)



14 So how tall is the flag pole? (I)

A stick of length 2.1 metres held vertically on horizontal ground casts a shadow of length 0.6 metres.

At the same time, a nearby flagpole casts a shadow of length x metres.

If the height of the flagpole is h metres find an expression for h in terms of x .

If $x = 1.5$ find h .

(Your working should include justification that triangles assumed similar are indeed similar.)



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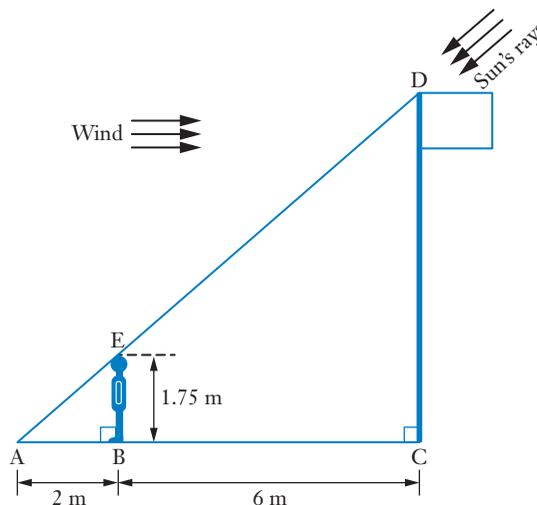
15 So how tall is the flag pole? (II)

Standing on level ground, and with the sun shining, Panji starts at the base of a flagpole and walks along the flagpole's shadow until the tip of his own shadow is at the same point on the ground as the tip of the shadow cast by the flagpole.

Panji, who's own height is 1.75 metres, is then 2 metres from the tip of his shadow and 6 metres from the base of the flagpole.

How tall is the flagpole?

(Your working should include justification that triangles assumed similar are indeed similar.)



THINK OF A NUMBER

Think of a whole number between one and ten (and remember it).

Double it

Add 3

Double your answer

Add on the number you first thought of

Add four

Divide by 5

Add 1

Take away the number you first thought of.

Your answer is 3 (or at least it should be if you have followed the instructions correctly!)

Why does the above puzzle always work no matter what whole number between one and ten is chosen as the starting number? Write an explanation of why the puzzle works.

Does it work for numbers other than whole numbers between one and ten? Explain.

A TRICK INVOLVING FIFTEEN COINS

Ranii placed fifteen coins on the table and said to her sister Jenna,

'While I shut my eyes you put some of the coins in your left hand, the rest in your right hand, and then put your hands behind your back so that I cannot see them.'

Jenna followed the instructions and then Ranii said,

'Now multiply the number of coins you have in your left hand by two, add on the number of coins you have in your right hand and tell me the answer.'

Again Jenna followed the instructions and announced,

'That makes 22.'

Performing some simple arithmetic Ranii was quickly able to announce,

'You have 7 in your left hand and 8 in your right.'

'That's right,' exclaimed Jenna, *'How did you know that?'*

How did she know it? **Explain what she did and why it works.**

Miscellaneous exercise seven

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1 Suppose the ratio of male students to female students in a school is 21 : 19. If there are 720 students in the school altogether how many female students are there in the school?
- 2 Find the value of x in each of the following.
 - a $\frac{x}{3} = \frac{7}{5}$
 - b $\frac{x}{10} = \frac{3}{4}$
 - c $\frac{5}{x} = \frac{4}{7}$
 - d $\frac{2}{x} = \frac{5}{7}$
- 3 Find m if $5(m + 3) + 2(3 - 2m) = 36$.
- 4 The mean of ten numbers is 10.8. Seven of these numbers have a mean of 12 and the other three numbers are a , $(a + d)$ and $(a + 2d)$. Determine $(a + d)$.
- 5 A number of adults, some male and some female, were asked ten questions about a particular issue. The number answered correctly by the adults are shown in the frequency distribution table below and, below that is the frequency table for the females in the group.

	Frequency distribution for entire group										
Number correct	0	1	2	3	4	5	6	7	8	9	10
Frequency	3	1	4	12	20	25	25	32	16	14	13

	Frequency table for the females in the group										
Number correct	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	10	17	10	8	9	7	12	13

- a How many females were asked the ten questions?
- b How many males were asked the ten questions?
- c Determine the range of the female scores.
- d Determine the range of the male scores.
- e Naomi claims that:

The range of the male scores is bigger than the range of the female scores and therefore the male scores are more spread out than the female scores.

Comment on her claim.

- 6 I think of a number, multiply it by two, add one and then double the result. To this answer I add on half of the number I first thought of and end up with 83. Find the number I first thought of.
- 7 To the nearest \$50, how much needs to be invested into an account paying 6% simple interest for the account to be worth at least \$19 000 in 5 years?

- 8** A company invests \$80 000, some into an account paying 6.3% per annum simple interest and the remainder in an account paying 5.4% per annum simple interest. After 2 years the \$80 000 had grown to \$89 612.

How much went into each account?

- 9** A maths exam was sat by 2145 candidates. The exam was marked out of 120 and the marks gained were distributed as follows.

Class interval	1 – 20	21 – 40	41 – 60	61 – 80	81 – 100	101 – 120
No. of candidates	113	263	340	720	478	231

- a** What is the midpoint of the 61 – 80 class?
b Use the midpoint of each interval to determine a mean and standard deviation for this grouped distribution.

- 10** The scores achieved by two classes in a maths test are given below:

Class One

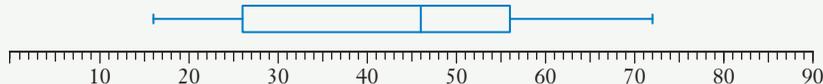
39 33 35 44 5 37 40 24 41 30 42
 12 46 52 16 58 35 22 44 37 26 28
 40 50 31

Class Two

41 45 48 40 24 47 42 37 44 43 39
 45 49 41 51 50 43 48 45 32 36 46

Draw a box and whisker diagram for each set of results and write a brief report comparing the distributions.

- 11** The scores obtained by the fifty students who sat a mathematics test are shown on this boxplot:



Construct a possible histogram for this set of fifty scores using the class intervals:

$$10 \leq \text{score} < 20, \quad 20 \leq \text{score} < 30, \quad 30 \leq \text{score} < 40, \text{ etc.}$$

- 12** Two loads, load A and load B, of a particular valuable metal are for sale.

The weight of load A is greater than that of load B but load B is of a higher quality and therefore has a greater price per kilogram. The ratio of the weights of the loads is as follows:

$$\text{weight of load A} : \text{weight of load B} = 15 : 8.$$

The ratio of the per kilogram price of the two loads is as follows:

$$\text{price of each kg of load A} : \text{price of each kg of load B} = 3 : 5.$$

If the price of load A is \$72 000 and the price of load B is \$ x , find x .

8.

Linear relationships

- Straight line graphs
- The gradient of a straight line
- Table of values
- What's my rule?
- Table – rule – graph
- Lines parallel to the axes
- Use of a calculator with a graphing facility
- More about $y = mx + c$, the equation of a straight line
- It may not look like $y = mx + c$ but it may still be linear
- Determining the equation of a straight line
- A useful rule
- Calculator routines
- Linear relationships in practical situations
- Miscellaneous exercise eight

Situation One

Suppose that each copy of a particular book weighs 1.5 kg.

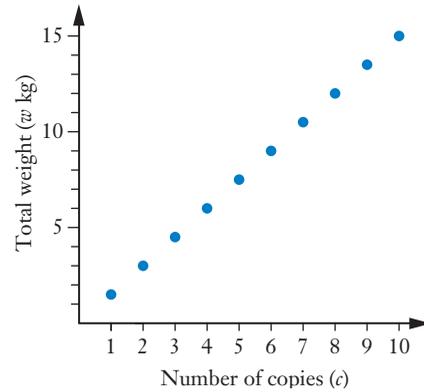
If we place copies of this book on a set of scales, each time we add one more book so the weight shown will increase by 1.5 kg, as shown in the graph on the right.

Copy and complete the following table for the situation:

Number of copies, c	1	2	3	4	5	6	7	8	9	10
Number of kg, w										

Which of the following rules agree with the figures in your table?

$$w = 2c \quad c = w \quad w = 1.5c \quad c = 1.5w \quad c + w = 1.5$$



Situation Two

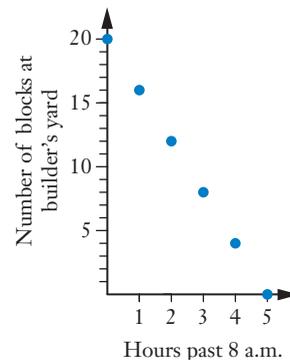
At 8 a.m. one morning there are 20 large concrete blocks in a builder's yard that need to be delivered to a worksite. A truck from a transport company is due to arrive at the builder's yard in one hour to pick up four blocks, take them to the worksite and then return for four more one hour later, repeating the process until all twenty have been removed from the builder's yard. Check that you agree that the graph on the right is consistent with this information.

Copy and complete the following table for this situation:

Number of hours past 8 a.m., h	0	1	2	3	4	5
Number of blocks at builder's yard, n						

Which of the following rules agree with the figures in your table?

$$n = 20h \quad n = -4h + 20 \quad n = 4h - 20 \quad n = 24h \quad n = 20 - h$$



Situation Three

The graph on the right shows the amount charged by a plumber working at your house for up to 2 hours.

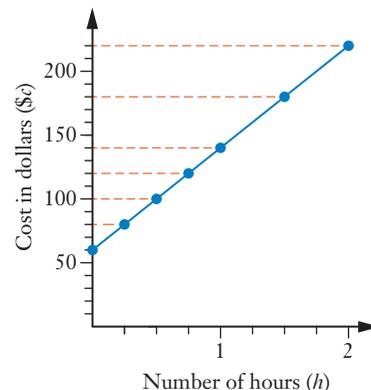
Copy and complete the following table for this situation:

Number of hours, h	0	0.25	0.5	0.75	1	1.5	2
Cost in dollars, c							

Which of the following rules agree with your table?

$$c = 80 + 60h \quad c = 80h + 60 \quad c = 60 - 80h$$

Why are the dots joined in this graph and not for situations one and two?

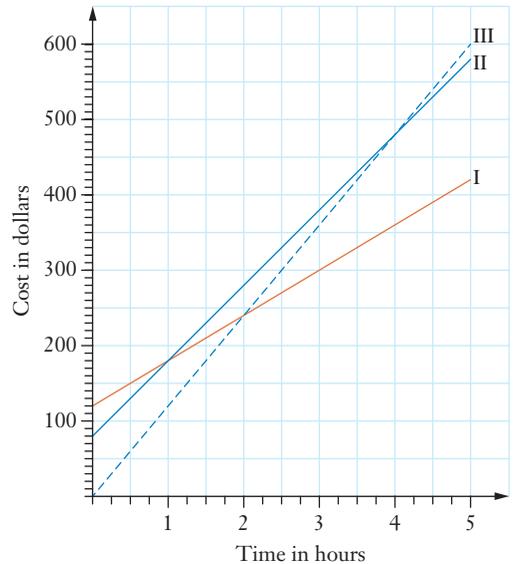


Situation Four

Three electricians, Sparky, Flash and Voltman, have different ways of calculating a customer's bill.

- Sparky charges a standard rate per hour and has no other charges.
- Flash has a fixed, or 'standing' charge and then charges a certain amount per hour on top of that.
- Voltman has a higher standing charge than Flash but then charges less per hour.

These three methods are shown graphed below:



- Which line, I, II or III, corresponds to **a** Sparky, **b** Flash, **c** Voltman?
- Ignoring the standing charges who charges most per hour?
What feature of the graph shows this?
- With the charge, or cost, being $\$C$ and the time being t hours the equation of line I is

$$C = 60t + 120.$$

Determine the equations of lines II and III.

- If you were considering using one of the three electricians for a job and wanted to keep the cost to a minimum which of the three could you dismiss from your considerations?

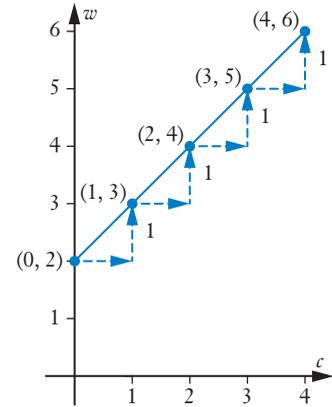


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Table of values

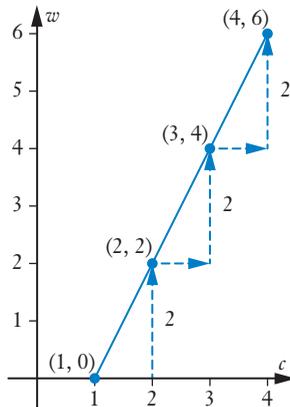
Using points with whole number horizontal coordinates to create a table of values for the three graphs just encountered, with c as the horizontal coordinate and w the vertical coordinate, the table for the first graph (shown again on the right) is:

		1	1	1	1
		↘	↘	↘	↘
c	0	1	2	3	4
w	2	3	4	5	6
		↗	↗	↗	↗
		1	1	1	1

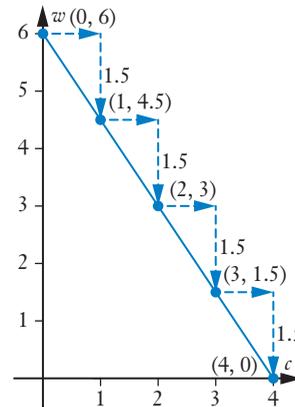


Notice how the gradient of the line, i.e. the constant increase in the w -values for each unit increase in the c -values, is evident from the table.

For the other two graphs the tables (and graphs) are shown below:



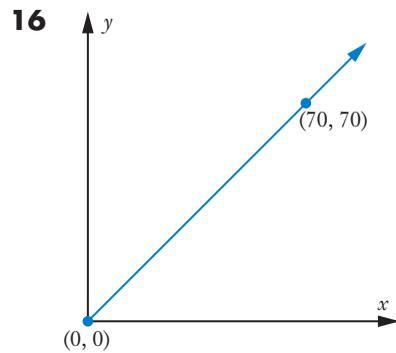
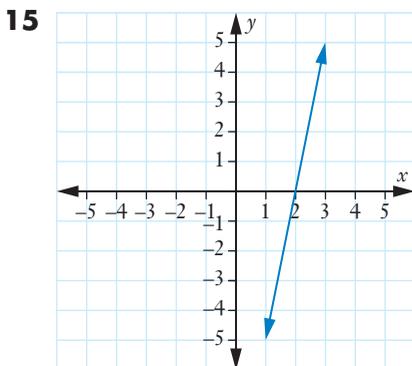
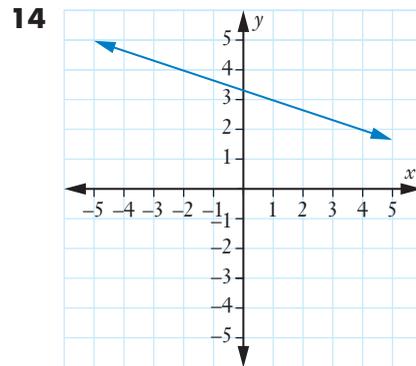
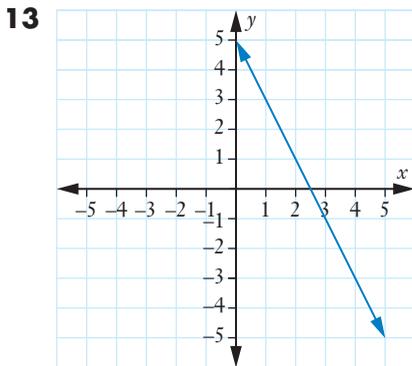
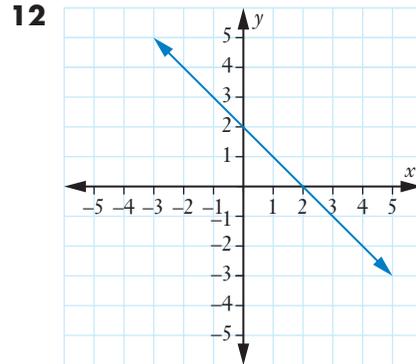
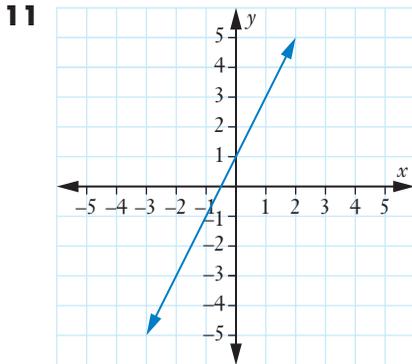
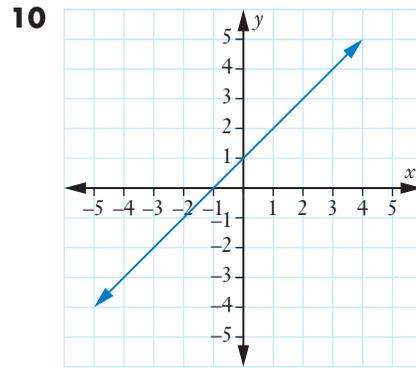
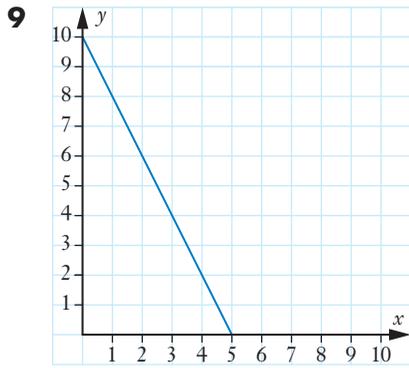
		1	1	1
		↘	↘	↘
c	1	2	3	4
w	0	2	4	6
		↗	↗	↗
		2	2	2



		1	1	1	1
		↘	↘	↘	↘
c	0	1	2	3	4
w	6	4.5	3	1.5	0
		↗	↗	↗	↗
		-1.5	-1.5	-1.5	-1.5

Notice again how the gradient of 2 and of -1.5 are evident from these tables.

- The gradient of a straight line graph is the vertical rise in the graph for each unit moved to the right. (A fall in the graph for each unit moved right indicates that the gradient is negative.) This gradient, or *slope*, is sometimes described as ‘*the rise divided by the run*’.
- With the horizontal coordinate increasing by 1 unit each time, the table of values for a straight line graph will show a constant difference pattern in the vertical coordinate equal to the gradient of the line. (If, for a constant increase in the values of the horizontal coordinate, the values of the vertical coordinate do not show a constant difference pattern the table of values is not for a straight line graph.)





Gradient and
y-intercept



Finding the equation
of a line

What's my rule?

Notice that the straight line graph on the right passes through the following points (as well as others):

$$\begin{array}{cccc} (-3, -5), & (-2, -3), & (-1, -1), & (0, 1), \\ (1, 3), & (2, 5), & (3, 7). & \end{array}$$

For every one of these points the x and y coordinates fit the rule

$$y = 2x + 1.$$

Indeed every point lying on the given straight line will have coordinates that fit the rule

$$y = 2x + 1$$

and all points not on the line will not fit the rule.

For example: The point $(1.5, 4)$ lies on the line and

$$4 = 2 \times 1.5 + 1$$

The point $(-2.5, -4)$ lies on the line and

$$-4 = 2 \times (-2.5) + 1$$

The point $(2, 3)$ does not lie on the line and

$$3 \neq 2 \times 2 + 1$$

The point $(-3, 0)$ does not lie on the line and

$$0 \neq 2 \times (-3) + 1$$

We say that the rule for the straight line shown is $y = 2x + 1$.

Note that the line with a rule of $y = 2x + 1$ has

- a gradient of 2
- and • cuts the vertical axis at $(0, 1)$.

Similarly: $y = 1x + 4$ has

- a gradient of 1
- and • cuts the y -axis at $(0, 4)$.

$y = 0.5x - 2$ has

- a gradient of 0.5
- and • cuts the y -axis at $(0, -2)$.

$y = -2x + 1$ has

- a gradient of -2
- and • cuts the y -axis at $(0, 1)$.

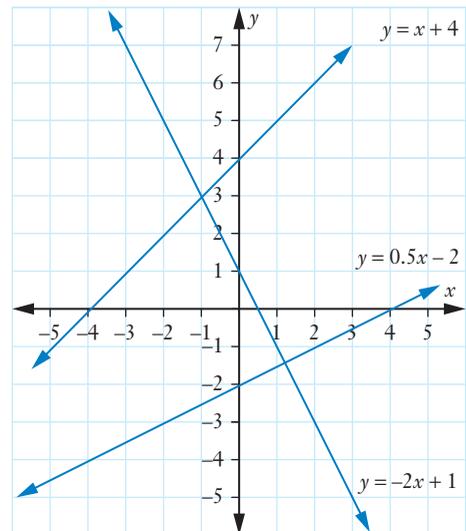
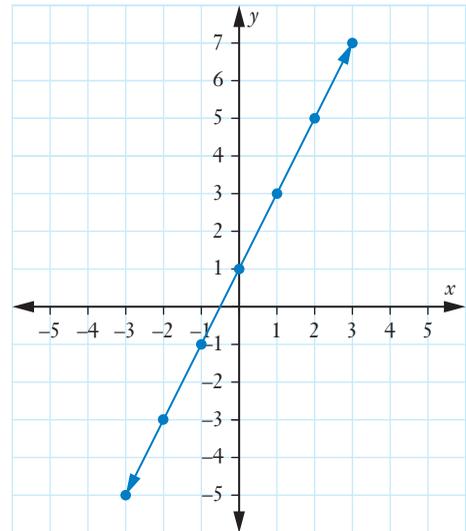
To generalise:

The straight line with the rule $y = mx + c$
has a gradient of m
and cuts the y -axis at the point $(0, c)$.

And:

If a straight line has gradient m and cuts the y -axis at the point $(0, c)$, it has the rule:

$$y = mx + c$$



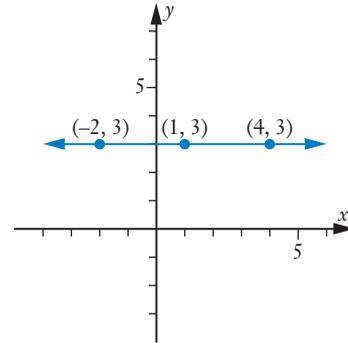
Lines parallel to the axes

I Lines parallel to the x -axis.

Consider the line parallel to the x -axis and passing through $(-2, 3)$, $(1, 3)$ and $(4, 3)$ as shown on the right.

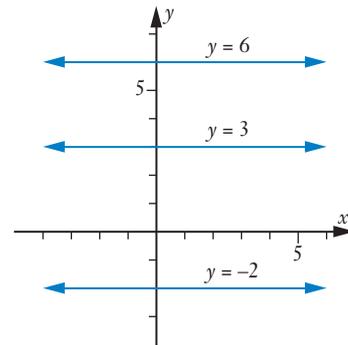
The rule for this line is $y = 3$ because the y coordinates of all points lying on this line will equal 3.

Note that this is consistent with the $y = mx + c$ idea because the line has a gradient of zero and a y intercept of 3.



The graph shown on the right shows the horizontal lines:

$$\begin{aligned}y &= 6, \\y &= 3, \\ \text{and } y &= -2.\end{aligned}$$



II Lines parallel to the y -axis.

The diagram on the right shows some lines drawn parallel to the y -axis.

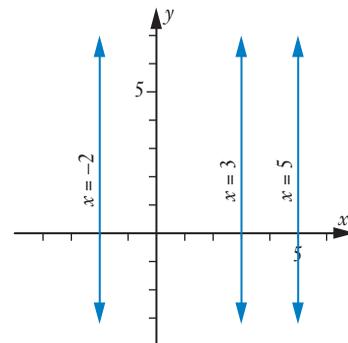
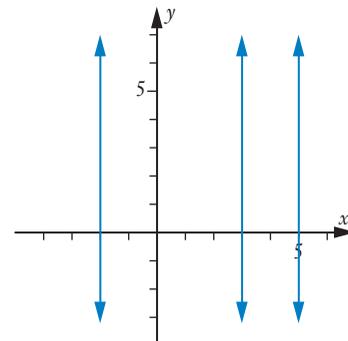
The gradient of each of these vertical lines is undefined because we cannot find the vertical rise in the line for each horizontal unit increased because the line rises vertically for zero increase horizontally! Hence we should not expect the rules for vertical lines to be of the form $y = mx + c$ because the gradient, m , is undefined. Indeed straight lines parallel to the y -axis are the only straight lines having rules that are *not* of the form $y = mx + c$.

Lines parallel to the y -axis have rules of the form $x = c$.

The graph on the right shows the vertical lines:

$$\begin{aligned}x &= -2 \\x &= 3, \\ \text{and } x &= 5.\end{aligned}$$

Even though these vertical lines have rules of a different form, points lying on each line must still 'obey' the rule. For example, for a point to lie on $x = 3$ the point must have an x -coordinate equal to 3.

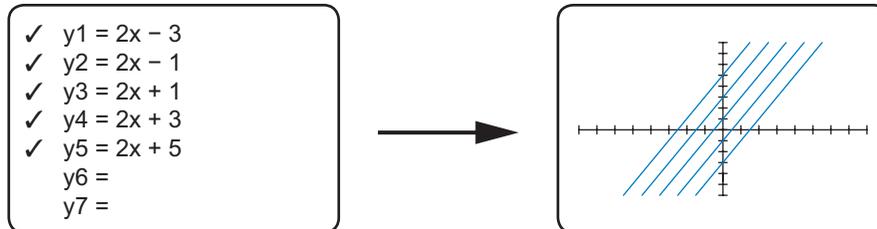


Use of a calculator with a graphing facility

Calculators with a graphing facility can display graphs of lines given the rules for the lines. For example, entering the following rules into such a calculator

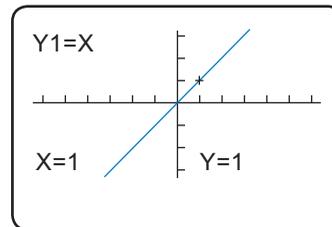
$$y = 2x - 3, \quad y = 2x - 1, \quad y = 2x + 1, \quad y = 2x + 3 \quad \text{and} \quad y = 2x + 5$$

allows the graphs of these lines to be displayed.

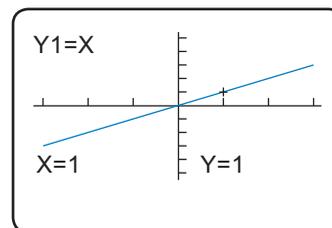
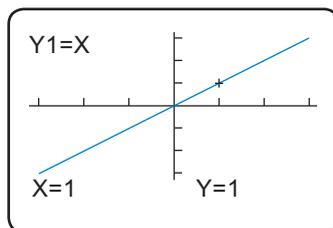


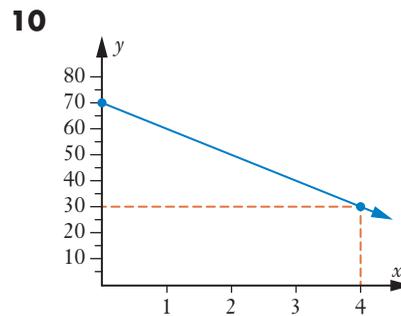
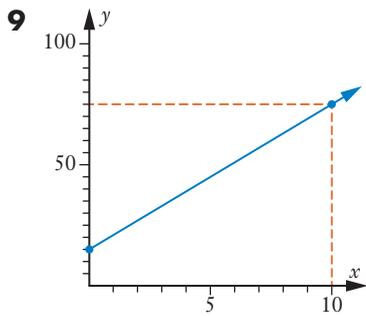
What feature of the five rules allows us to anticipate that the lines would be parallel to each other?

Note: The display on the right shows the line $y = x$. As we would expect this line has a gradient of 1 and cuts the y -axis at the point $(0, 0)$, i.e. the origin. The line passes through all those points for which the x -coordinate equals the y -coordinate, for example $(0, 0)$, $(1, 1)$, $(2, 2)$ etc.

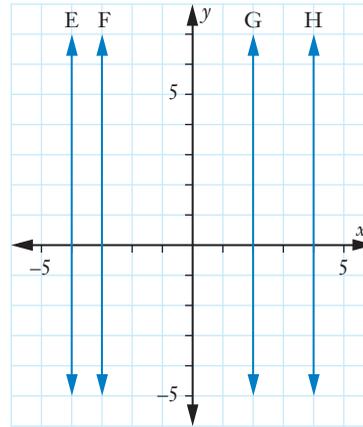
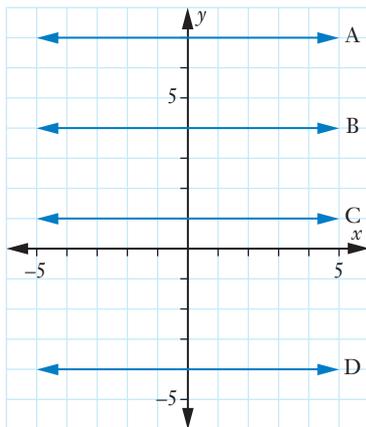


Because the same scale is used on each axis the line $y = x$ makes an angle of 45° with each axis. However, do not expect this 45° property of the line $y = x$ to be evident if different scales are used on each axis. Both of the displays shown below show the line $y = x$ but with different scales used on each axis, and with the two displays using different scales, the two displays appear different and neither shows the 45° nature of the line.





11 Determine the rules for each of the lines A to H shown below.



For each of the tables shown in questions 12 to 25, determine whether the relationship between x and y is linear and, for any for which it is, determine the rule for the relationship.

12

x	1	2	3	4	5	6
y	4	7	10	13	16	19

13

x	1	2	3	4	5	6
y	21	17	13	9	5	1

14

x	0	1	2	3	4	5
y	-3	2	7	12	17	22

15

x	0	1	2	3	4	5
y	25	16	9	4	1	0

16

x	1	2	3	4	5	6
y	2	3.5	5	6.5	8	9.5

17

x	5	6	7	8	9	10
y	5	10	20	40	80	160

18

x	-2	-1	0	1	2	3
y	0	1	2	3	4	5

19

x	0	2	4	6	8	10
y	1	3	6	10	15	21

20

x	1	3	5	7	9	11
y	12	10	8	6	4	2

21

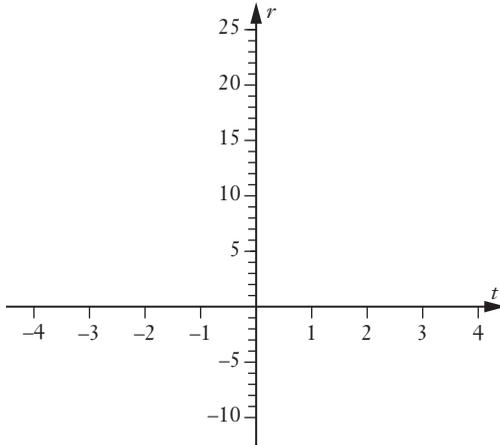
x	0	5	10	15	20	25
y	21	31	41	51	61	71

30 Table:

t	-4	-3	-2	-1	0	1	2	3	4
r	-2	0	2	4	6	8	10	12	14

Rule: $r =$

Graph:

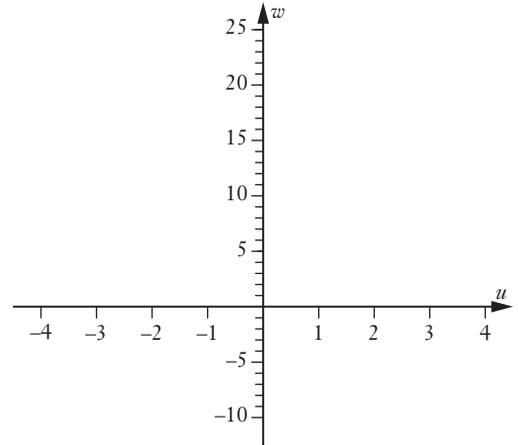


31 Table:

u	-4	-3	-2	-1	0	1	2	3	4
w	-9	-6	-3	0	3	6	9	12	15

Rule: $w =$

Graph:

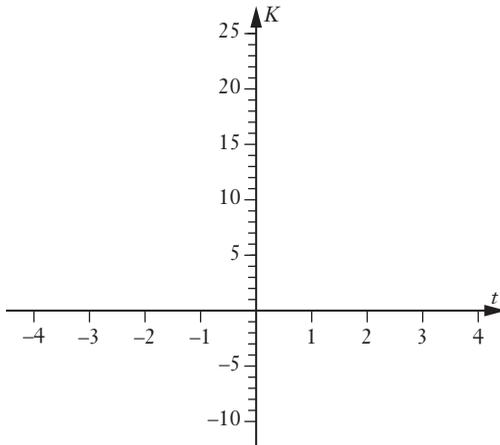


32 Table:

t	-4	-3	-2	-1	0	1	2	3	4
K									

Rule: $K = 4t + 9$

Graph:

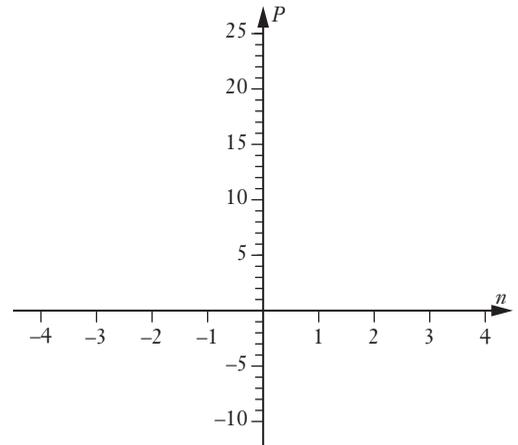


33 Table:

n	-4	-3	-2	-1	0	1	2	3	4
P									

Rule: $P = -2n + 7$

Graph:





More about $y = mx + c$, the equation of a straight line

From earlier work we know that straight lines have equations that are of the form:

$$y = mx + c$$

1 Why should equations of the form $y = mx + c$ produce straight line graphs?

A straight line must clearly have a constant gradient. Thus wherever we are on the line, if we journey one further unit horizontally, then the line will always rise a constant vertical amount.

Consider the equation $y = mx + c$.

- If x changes from 2 to 3, y changes from $(2m + c)$ to $(3m + c)$, a rise of m .
- If x changes from 3 to 4, y changes from $(3m + c)$ to $(4m + c)$, a rise of m .
- If x changes from 98 to 99, y changes from $(98m + c)$ to $(99m + c)$, a rise of m .

Each unit increase in the x -coordinate produces an increase of m units in the y -coordinate. i.e. a straight line of gradient m .

If an equation cannot be expressed in the form $y = mx + c$ then the y -coordinate will not rise by a constant amount for each unit increase in the x -coordinate.

For example, consider $y = x^2$.

- If x changes from 2 to 3, y changes from 4 to 9, i.e. a rise of 5.
- If x changes from 3 to 4, y changes from 9 to 16, i.e. a rise of 7.
- If x changes from 98 to 99, y changes from 9604 to 9801, i.e. a rise of 197.

Each increase of one unit in the x -coordinate does not produce the same change in the y -coordinate. i.e. not a straight line graph.

2 The equation of a line is like the membership ticket for the line.

As was mentioned earlier, if a point lies on a particular line then the coordinates of that point will 'fit' the equation of the line, and if it does not lie on the line the coordinates will not 'fit' the equation. The equation of a line is the rule which all points lying on the line must 'obey'. In this way the equation is like the membership ticket for the line. If a point does not fit the equation it cannot lie on the line.

EXAMPLE 2

State whether each of the points A to C lie on the line $y = 2x + 7$.

$$A(1, 8) \quad B(3, 13) \quad C(-5, -3)$$

Solution

If A (1, 8) lies on the line then $x = 1$ and $y = 8$ must 'fit' the equation.

Substituting $x = 1$ into $y = 2x + 7$ gives $y = 9$.

Thus point A (1, 8) *does not* lie on the line $y = 2x + 7$.

If B (3, 13) lies on the line then $x = 3$ and $y = 13$ must 'fit' the equation.

Substituting $x = 3$ into $y = 2x + 7$ gives $y = 13$.

Thus point B (3, 13) *does* lie on the line $y = 2x + 7$.

If C (-5, -3) lies on the line then $x = -5$ and $y = -3$ must 'fit' the equation.

Substituting $x = -5$ into $y = 2x + 7$ gives $y = -3$.

Thus point C (-5, -3) *does* lie on the line $y = 2x + 7$.

EXAMPLE 5

(Given two points that lie on the line)

Find the equation of the straight line through the points $(-2, 8)$ and $(4, -1)$.

Solution

Starting at the point with the lower x -coordinate $(-2, 8)$, and moving to the point $(4, -1)$, we travel across 6 units and down 9 units. Thus in moving across 1 unit we travel *down* $\frac{9}{6}$ units, i.e. $\frac{3}{2}$ units.

The gradient of the line is $-\frac{3}{2}$.

Thus the given line will have an equation of the form $y = -1.5x + c$.

The line passes through the point $(4, -1)$.

Thus the values $x = 4$ and $y = -1$ must 'fit' the equation, i.e. $-1 = -1.5(4) + c$
giving $c = 5$.

Thus the given line has equation $y = -1.5x + 5$.

(The reader should confirm that using the point $(-2, 8)$ and saying that the values $x = -2$ and $y = 8$ must 'fit' the equation also gives $c = 5$.)



Finding the gradient
between two points
on a line

A useful rule

A useful rule to remember when determining the gradient of the line through two points, A and B is:

$$\text{Gradient} = \frac{\text{the change in the } y\text{-coordinate in going from A to B}}{\text{the change in the } x\text{-coordinate in going from A to B}}$$

Thus if A has coordinates (x_1, y_1) and B has coordinates (x_2, y_2)
then the gradient of the line through A and B = $\frac{y_2 - y_1}{x_2 - x_1}$.

Note: In this formula $\frac{y_1 - y_2}{x_1 - x_2}$ would also give the correct answer but $\frac{y_1 - y_2}{x_2 - x_1}$ and $\frac{y_2 - y_1}{x_1 - x_2}$ would not.

Hence make sure that 'whichever point you get the first y -coordinate from is also where you get the first x -coordinate from'.

Calculator routines

Your calculator may have programmed routines that allow the equation of a line to be determined simply by inputting the coordinates of two points on the line, or inputting the gradient and the coordinates of just one point on the line. Such routines can be useful but make sure that you understand the underlying ideas and can apply them without the assistance of calculator programs if required.



Linear relationships in practical situations

Situation One at the start of this chapter involved copies of a particular book. Each copy weighed 1.5 kg so one book would weigh 1.5 kg, two books 3 kg, three books 4.5 kg, four books 6 kg and so on.

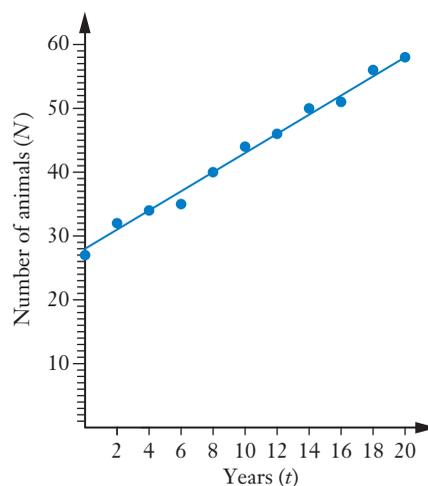
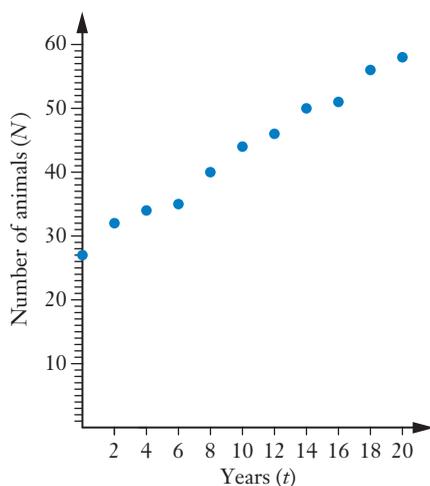
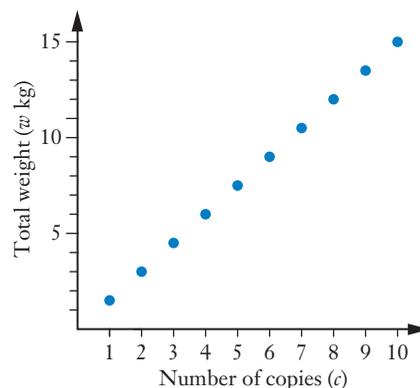
In this example the relationship between the total weight, w kg, and the number of copies, c , is exactly linear with all points exactly lying on the straight line

$$w = 1.5c$$

This is an example of a linear relationship in real life.

Some other situations may involve data that does not exactly fit a straight line but for which a straight line may be a reasonable *model* to use to summarise what is going on, and that might allow reasonable predictions and statements to be made.

Suppose, for example, that in the process of monitoring the survival of a particular endangered species of animal the numbers of these animals held in zoos around the world is recorded every two years over a period of twenty years. Suppose the numbers were as shown in the graph below left. Whilst the values do not exactly lie in a straight line, the rule $N = 28 + 1.5t$ could be a reasonable linear model to use for this data, as shown below right.



In this linear model ($N = 1.5t + 28$):

- the vertical intercept is 28, indicating that the number of these animals kept in zoos at the beginning of the 20 year period was approximately 28,
- the gradient is 1.5, indicating that on average each 1 year increase saw an increase of 1.5 in the number of these animals kept in zoos (i.e. an increase of approximately 3 every two years).

Exercise 8D

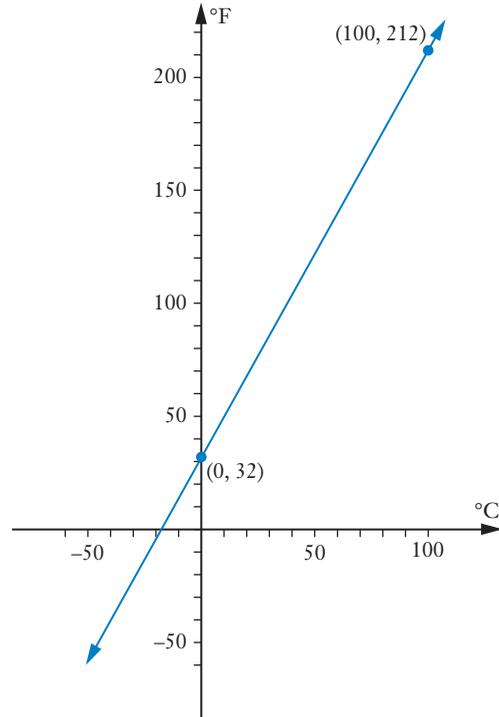
1 If we plot degrees Celsius, ($^{\circ}\text{C}$), on the x -axis and degrees Fahrenheit, ($^{\circ}\text{F}$), on the y -axis, the graph for converting from one scale to the other is a straight line.

- a Given that 100°C is the same as 212°F and 0°C is the same as 32°F find the equation of the line in the form

$$F = mC + b,$$

where m and b are constants.

- b What does the value of m tell us about $^{\circ}\text{C}$ and $^{\circ}\text{F}$ temperatures?
 c Convert 55°C to $^{\circ}\text{F}$.
 d Convert -10°C to $^{\circ}\text{F}$.
 e Convert 59°F to $^{\circ}\text{C}$.
 f Convert -4°F to $^{\circ}\text{C}$.
 g Is there a temperature for which the number of degrees Celsius is the same as the number of degrees Fahrenheit, and if so what is that temperature?

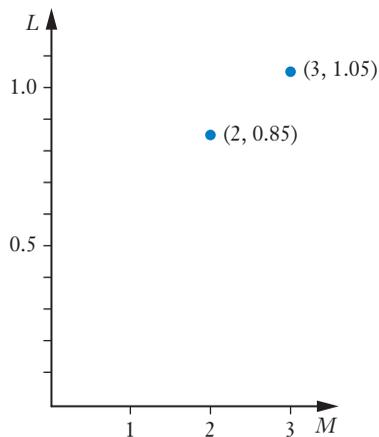


2 When a particular spring has a mass of M kg suspended from one end the total length of the spring is L metres where

$$L = kM + L_0 \text{ where } k \text{ and } L_0 \text{ are constants.}$$

- a What will the value of k tell us about this situation?
 b What will the value of L_0 tell us about this situation?
 c A graph of M plotted on the x -axis and L on the y -axis passes through the points $(2, 0.85)$ and $(3, 1.05)$.

Calculate k and L_0 and hence determine how much the spring is extended **beyond its natural length** when a mass of 250 g is suspended from it.

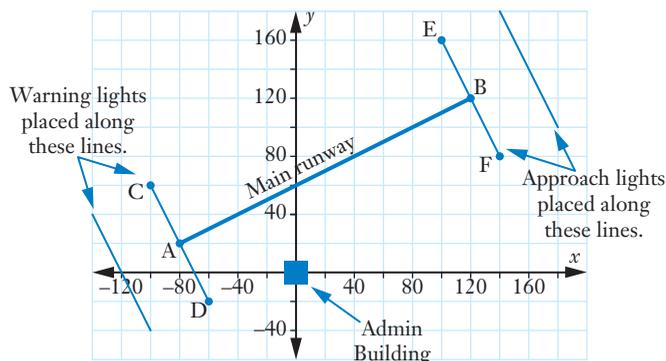
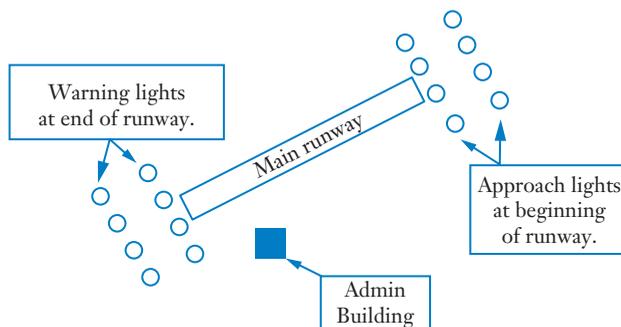


- 3** The diagram on the right shows the proposed layout of a small airfield. The diagram shows the main runway, the approach lights, the warning lights and the administration building.

The second diagram shows the proposal as a graph with lengths in metres and the admin building as the origin.

Find:

- the coordinates of the points A, B, C, D, E and F, (all divisible by 20)
- the equation of the straight line through A and B
- the equation of the straight line through C and D
- the equation of the straight line through E and F.



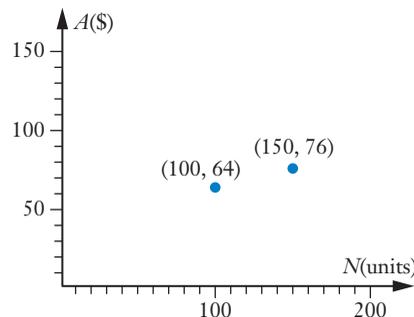
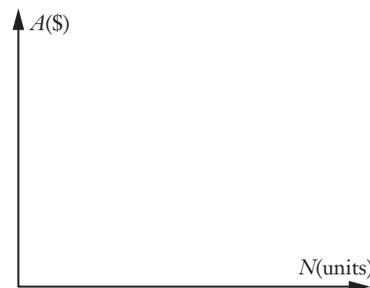
- 4** If we plot the 'Number of metered units', N , on the x -axis and the 'Amount to be paid', $\$A$, on the y -axis then the graph for calculating a telephone bill from one particular company is a straight line with equation

$$A = mN + c,$$

where m and c are constants.

In the context of this question:

- what does the value of m tell us about A and N ?
- what does the value of c tell us?
- If the bill for 100 units is \$64 and for 150 units is \$76, determine the equation of the line.
- What would be the bill for 200 units?
- If the bill was for \$82 how many units were used?



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- 7** A taxi company charges a fixed start fee (called the ‘flag fall’ in the taxi industry) of \$4.90 followed by a charge of \$1.85 per kilometre.

If the cost for a journey of x kilometres is $\$C$, write a rule in the form

$$C = c + mx.$$

- 8** A water tank initially contains 1000 litres of water.

The tank develops a leak at its base, which causes water to leak out at a constant rate of 200 millilitres every minute.

If V litres is the volume of water in the tank t minutes after the leak commenced, write a rule relating V and t .

- 9** It costs a small business \$800 to run the business each week.

The company imports just one type of product from overseas and sells each one for \$75 more than it buys each one for.

Find an expression for $\$P$, the weekly profit the company makes, if it sells n of these products in that week.

- 10** A company sells copies of a book that it prints on a book by book basis, as orders arrive. Each book costs the company \$12 to produce in this way.

The company sells each book for \$22, which includes 10% goods and services tax.

The company forwards the goods and services tax amount to the government and the company keeps the rest.

If the company makes a profit of $\$P$ when it sells x copies of this book write a rule for P in the form $P = mx$.

- 11** To cook a joint of meat a recipe book advises preheating the oven to 180°C and then, when the oven temperature has reached this temperature, place the meat in the oven for ‘20 minutes per kilogram + 20 minutes over’.

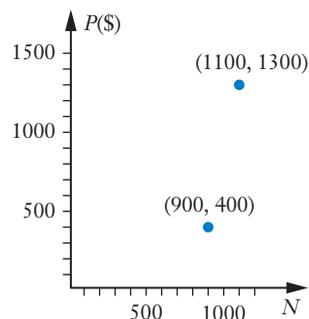
Express as a rule the time, t hours, a joint of meat weighing k kilograms should be placed in the hot oven for it to cook according to these instructions.

- 12** A linear relationship exists between the profit, $\$P$, that the organisers of a concert make, and N , the number of tickets they sell. With P plotted on the vertical, y , axis and N on the horizontal, x , axis the line of this relationship passes through the points (900, 400) and (1100, 1300).

Find the equation of this line in the form

$$P = mN + c, \text{ where } m \text{ and } c \text{ are constants.}$$

- What does the value of m tell us in this context?
- What does the value of c tell us in this context?
- What will be the profit when 1500 tickets are sold?
- If the concert hall has a maximum capacity of 2500 what profit will the organisers make if they give away 150 complimentary tickets and sell all the rest?
- What is the least number of tickets the organisers could sell and still not make a loss?



- 13** The owner of a computer shop calculates that his weekly profit from computer sales is linearly related to the number of computers sold that week.

If he sells 10 computers in a week his total profit is \$560.

If he only sells 5 computers in the week he makes a profit of \$10.

The rule relating his total profit for the week, \$ P , to the number of computers sold, x , is given by:

$$\text{Total profit in dollars } (P) = mx - c,$$

\$ c being the fixed weekly cost of running the shop.

- a** Calculate m and c .
- b** What is his weekly profit from computer sales in a week that he sells 20 computers?
- 14** The membership secretary of a club monitors the growth in membership over a 5 year period. Plotting the 5 years (1, 2, 3, 4 and 5) on the horizontal ' t ' axis and the membership numbers on the vertical, ' N ', axis the secretary finds there is an almost perfect linear relationship between t and N . Express the relationship in the form $N = mt + c$ given that when $t = 1$, $N = 250$ and when $t = 5$, $N = 410$.
- What do the values of the gradient, m , and the vertical intercept, c , tell us in the context of this question?
- Use your equation to predict the value of N when $t = 10$, assuming the linear relationship continues.

- 15** The table below shows the profit, \$ P , that a company makes from the sale of x copies of a particular book it has had printed.

x	0	100	200	300	400	500	600	700	800
P	-3750	-2250	-750	750	2250	3750	5250	6750	8250

- a** Determine the rule for the relationship between P and x .
- b** How many copies of the book must the company sell to achieve a profit of more than \$10 000?
- 16** The monitoring of the numbers of a particular endangered species of animal found that over a number of years, from the time the monitoring started, the numbers thought to be in existence in the wild showed a steady decline. Indeed with N representing the number of these animals thought to be in existence in the wild, t years into the monitoring program, N and t were approximately following the rule:

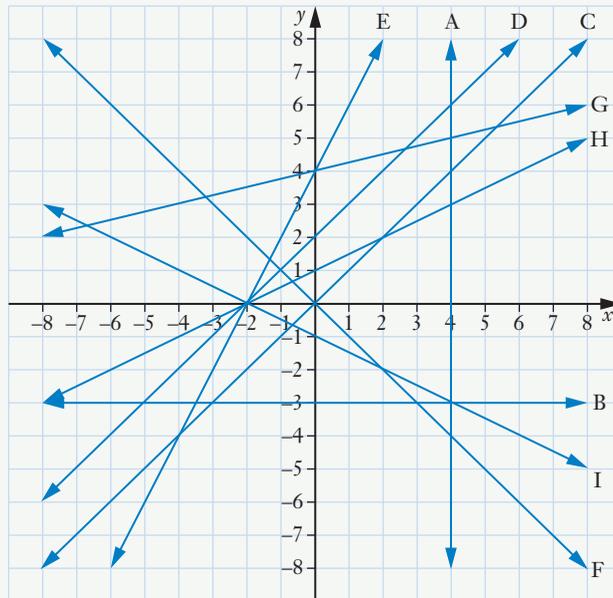
$$N = 5740 - 350t.$$

- a** Interpret what the numbers 5740 and 350 mean in the context of this situation.
- b** Graph the rule $N = 5740 - 350t$ with N plotted on the vertical axis and t on the horizontal axis.
- c** State the coordinates of the point where the line $N = 5740 - 350t$ cuts the horizontal axis and explain the significance of this point in the context of this question.

Miscellaneous exercise eight

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1 Determine the equation of each of the straight lines A to I shown below.



- 2 A particular straight line with a gradient of b and cutting the y -axis at the point with coordinates $(0, a)$ has equation $y = a + bx$. The line passes through the point $(5, -2)$.

Which of the following equations must be true?

Equation 1
 $5 = a - 2b$

Equation 2
 $y = 5 - 2x$

Equation 3
 $a + 5b + 2 = 0$

- 3 Find the values of a, b, c, \dots, h in the following.

a $a : 2 = 6 : 5$

b $6 : 7 = 3 : b$

c $5 : 2 = 15 : c$

d $d : 2 = 19 : 10$

e $e : 3 = 9 : 2$

f $2 : 3 = 9 : f$

g $4 : g = 8 : 9$

h $4 : h = 9 : 8$

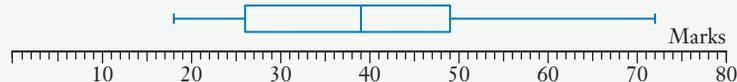
- 4 The ratio of females to males in a particular workforce is $7 : 9$.

There are 252 males in this workforce.

How many females are there in this workforce?

- 5 In the first three units of a course a student achieves a mean of 64%. In the next two units the student achieves a mean of 51%. What is the least mark the student needs to achieve in the sixth and final unit to gain an overall mean of at least 60%?

- 7 Forty one students sat a test. The boxplot of their results is shown below.

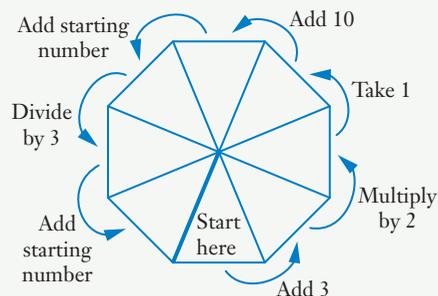


With the student who gained the top mark ranked 1st, the students ranked 19th and 21st in the test scored identical marks.

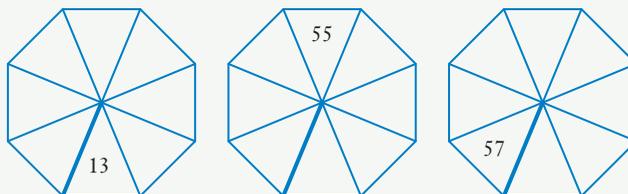
The top twelve students scored twelve different marks.

The student ranked 11th in the test scored 47.

- What was the top mark achieved in the test?
 - What score did the student ranked 20th achieve?
 - What score did the student ranked 10th achieve?
- 8 The three 'octapatterns' below all follow the pattern shown on the right.



Copy and complete each of the octapatterns shown below.



- 9 Certain types of cricket chirp more frequently as the temperature rises. For a particular species the number of chirps per minute (N) is found to be related to the Celsius temperature (C°) according to the rule:

$$N \approx 7C - 16$$

- If we were to graph $N = 7C - 16$, with C on the horizontal axis and N on the vertical axis it would give a straight line with gradient 7 and intersecting the vertical axis at -16 . Interpret these two numbers in the context of this 'chirping crickets' situation.
- What does the rule suggest as the temperature below which we would not expect a cricket to chirp?
- Roughly how many chirps will a cricket of this species make per minute if the temperature is
 - 14°C ?
 - 28°C ?
- Estimate the temperature if a cricket of this species is chirping
 - 200 times per minute
 - 50 times per 20 seconds.

9.

Piecewise defined relationships

- Piecewise defined relationships
- Miscellaneous exercise nine

Piecewise defined relationships

The three situations on the previous page (and five of the six graphs) involved circumstances in which the relationship between two variables involved a number of different linear relationships. Which relationship applied depended on where on the horizontal axis we were. In this way different rules applied for different *pieces* of the horizontal axis. Such relationships are said to be **piecewise defined**.

One of the six graphs on the previous page is not a piecewise defined linear graph – which one?

Two of the six graphs on the previous page are also called **step graphs** – which two?

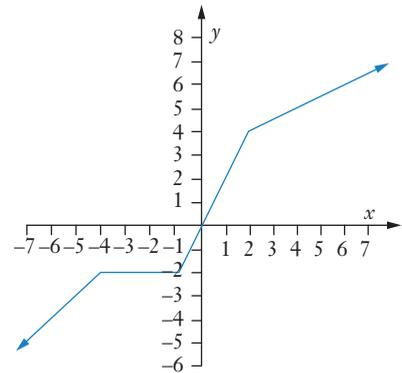
Consider, for example, the graph on the right.

In this case, for x less than -4 one rule applies, for x from -4 to -1 another rule applies, etc.

We write this as:

For $x < -4$,	$y = x + 2$
for $-4 \leq x < -1$,	$y = -2$
for $-1 \leq x < 2$,	$y = 2x$
for $x \geq 2$,	$y = 0.5x + 3$

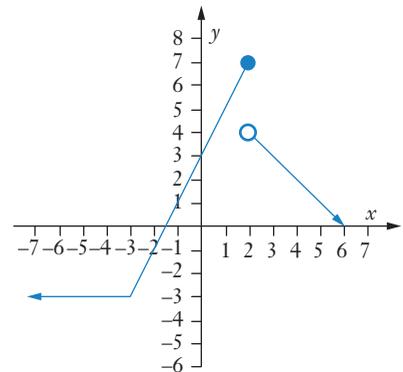
(Note that whilst in the above listing we have attached the $x = -4$ value to the $y = -2$ rule it could equally well have been attached to the $y = x + 2$ rule. Similarly the $x = -1$ and $x = 2$ values could be differently ‘attached’.)



In the piecewise defined relationship shown on the right the filled circle shows where the value for $x = 2$ **is**, and the open circle shows where the value for $x = 2$ **is not**. (Did you notice this aspect in one of the situations on the previous page?)

In this case

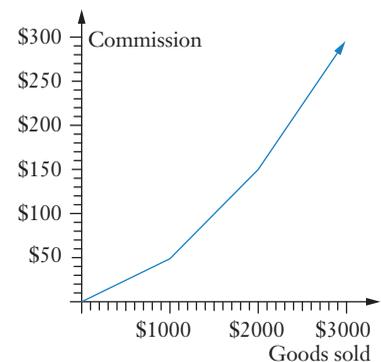
For $x < -3$,	$y = -3$
for $-3 \leq x \leq 2$,	$y = 2x + 3$
for $x > 2$,	$y = -x + 6$



The above examples involved abstract lines without any real context attached but, as the first three situations demonstrated, piecewise defined relationships do occur in real life.

For example consider the situation of a company paying commission for sales achieved by its sales people according to the following rules:

Sales	Commission
\$0 → \$1000	5% of sales
\$1000 → \$2000	\$50 + 10% of each \$1 over \$1000
\$2000 and over	\$150 + 15% of each \$1 over \$2000



The graph of this situation is shown on the right.

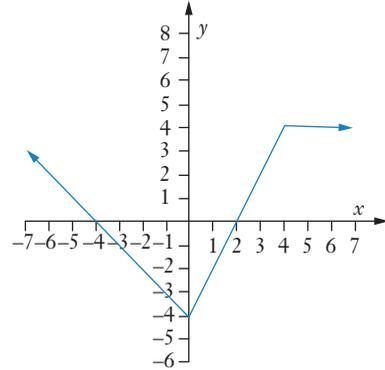
Exercise 9A

- 1 Copy and complete the following statements for the piecewise defined function shown on the right.

For $x < 0$, $y =$

for $0 \leq x < 4$, $y =$

for $x \geq 4$, $y =$



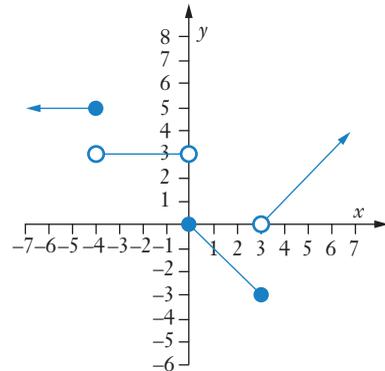
- 2 Copy and complete the following statements for the piecewise defined function shown on the right.

For $x \leq -4$ $y =$

for $-4 < x \leq 3$ $y =$

for $3 < x < 4$ $y =$

for $x \geq 4$ $y =$



- 3 Draw the following piecewise defined function:

For $x \leq -5$, $y = -x - 5$

for $-5 < x \leq 3$, $y = x + 5$

for $x > 3$, $y = 8$.

- 4 Draw the following piecewise defined function:

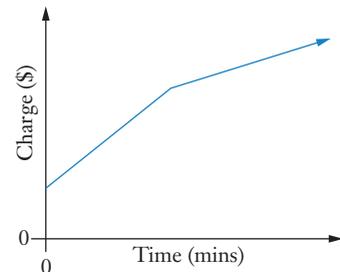
For $x \leq 0$, $y = -3$

for $0 < x < 4$, $y = 2x$

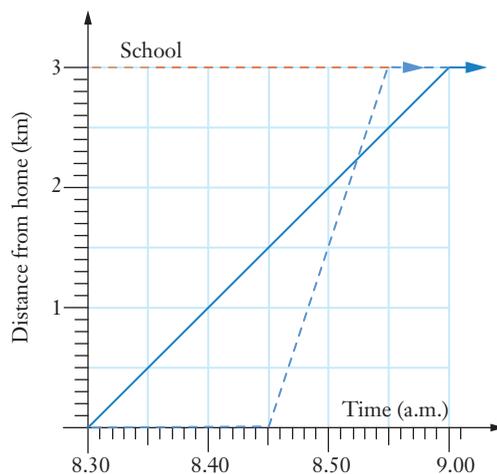
for $x = 4$, $y = 10$

for $x > 4$, $y = -x + 7$.

- 5 The fare charged by a taxi company depended upon the number of minutes the journey lasted. The graph shows the charge in dollars graphed against the time of the journey in minutes. Write a few sentences describing your interpretation of the situation based on information given by the graph.

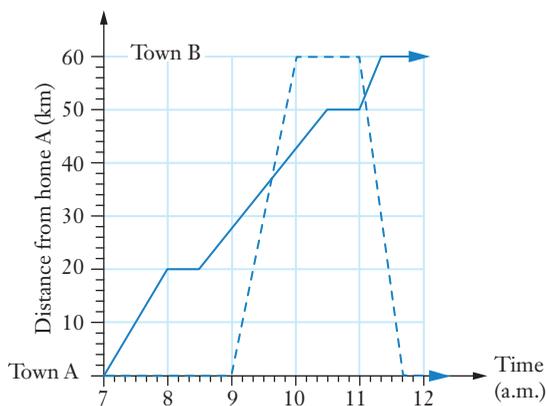


- 6** The distance time graph on the right is for two brothers journeying from home to the same school. One brother leaves early and walks to school and the other leaves later and cycles.
- Does the blue broken line '----' represent the journey of the walker or the cyclist?
 - Estimate the time when the cyclist passes the walker.
 - How many minutes did the walker take to walk to school?
 - What was the steady speed maintained by the walker during his walk?
 - How many minutes did the cyclist take to ride to school?
 - What was the steady speed maintained by the cyclist during his ride?



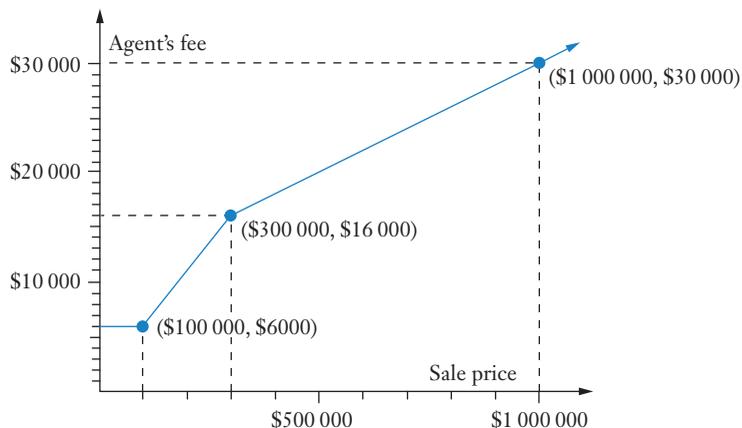
- 7** The distance time graph shown on the right is for the motion of a cyclist travelling from town A to town B, 60 km away, and a delivery truck making the round trip from A to B and back to A again.

- When did the cyclist leave town A?
- When did the cyclist reach town B?
- The cyclist stopped twice for a rest. How long was each stop?
- What speed did the cyclist maintain
 - prior to the first stop?
 - between the two stops?
 - after the second stop?
- What speed did the delivery truck maintain
 - from town A to B?
 - from town B back to A?
- Estimate the time and distance from A of the place where the delivery truck passed the cyclist when they were both travelling towards B.
- Estimate the time and distance from A of the place where the delivery truck passed the cyclist when the truck was returning to A.



- 11** When a real estate agent arranges for the sale of a house the owner of the house pays the agent a fee, often based on the amount the house sells for.

Let us suppose that one agent's fee structure is as shown in the graph below.



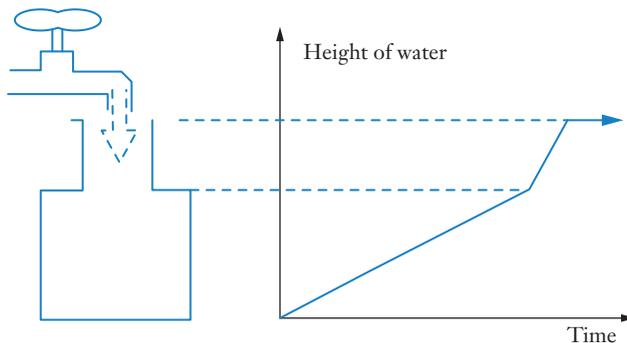
Express this piecewise defined relationship as follows by copying and completing the sentences.

For the first \$_____ of the sale price the agent's fee is a fixed \$_____.

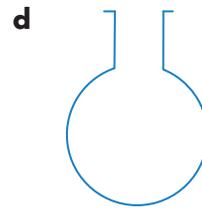
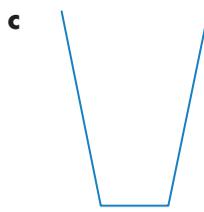
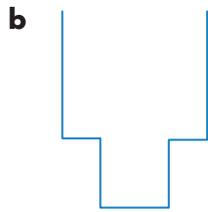
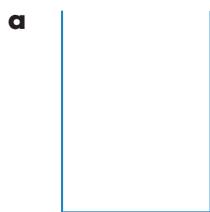
From \$_____ to \$_____ the fee is \$_____ plus _____% of the amount over \$_____.

From \$_____ and over the fee is \$_____ plus _____% of the amount over \$_____.

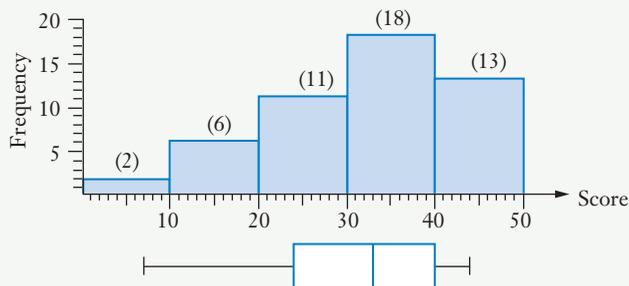
- 12** Suppose water is flowing at a constant rate into a container. For a container shaped as shown on the right the graph of the height of the water level plotted against time would be as shown.



Sketch the graph of the height of the water level against time for each of the following containers.



- 8** A set of numbers consists of 2 fours, 8 fives, 11 sixes, 9 sevens and a number of eights. If the mean of the set is 6.2 determine the number of eights in the set.
- 9** The histogram and the box plot for a data set are shown below.

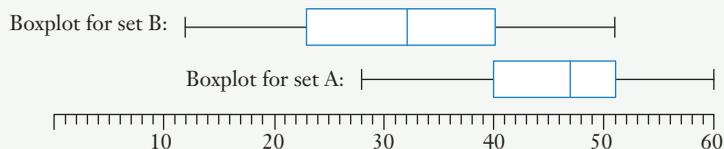


Determine

- a** the median,
- b** the range,
- c** the interquartile range,
- d** the modal class.

Use your calculator to determine an estimate for the mean.

- 10** Two sets of students sat the same test and the boxplots of their marks are shown below.



- a** What percentage of students in set A exceeded the highest mark obtained by students in set B?
- b** Which two features of the boxplots suggest that the marks in set B were more variable (i.e. more spread out) than those in set A?

If the top 25% of the students in set B, as defined by the test results, were all moved to set A how would:

- c** the median mark of those left in set B compare with that of set B before the move?
- d** the range of the scores of those left in set B compare with that of set B before the move?
- e** the range of scores of the new set A compare with the range of the scores of set A before the move?
- f** the interquartile range of the new set A compare with the interquartile range of the scores of set A before the move?

10.

Trigonometry for right triangles

- Right angled triangles
- Trigonometry
- Hypotenuse, opposite and adjacent
- Notes regarding calculator usage
- Applications
- Accuracy and trigonometry questions
- Bearings
- Elevation and depression
- More vocabulary
- Miscellaneous exercise ten

How did you get on with the situations on the previous page?

Did you think of drawing scale diagrams to determine the required lengths?

Perhaps instead you have encountered some *trigonometry* work in earlier years and remembered how to apply that to determine lengths of sides in right triangles.

Perhaps you used your calculator to determine lengths of unknown sides in right triangles.

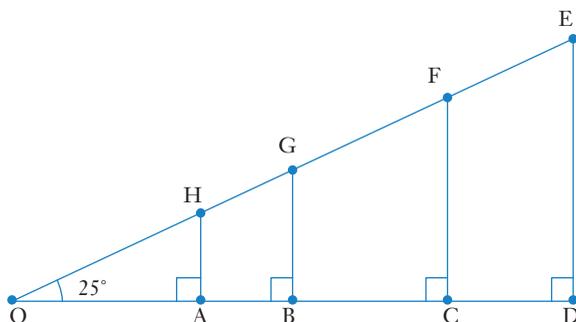
In this chapter we will consider the use of *trigonometry* to determine sides and angles in right triangles.

Right angled triangles

The four triangles OAH, OBG, OCF and ODE shown below all have angles of 25° , 65° and 90° .

As you know from unit one of this course, the four triangles are **similar**.

Each triangle is an enlargement, or reduction, of the others.



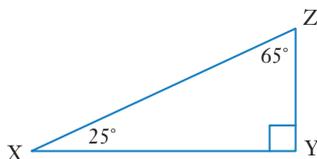
In the above diagram measure the length of ED and the length of OD and use your calculator to

determine: $\frac{\text{length of ED}}{\text{length of OD}}$.

Did you get an answer of approximately 0.47?

On a piece of A4 paper accurately draw a large triangle with angles of 25° , 65° and 90° .

Label your triangle XYZ as shown in the diagram below.

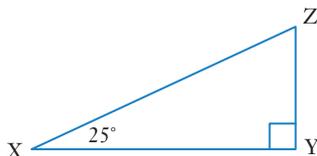


Measure the lengths of XY and YZ and determine: $\frac{\text{length of YZ}}{\text{length of XY}}$

Trigonometry

The **tangent** of an angle is one of three ratios commonly used in the branch of mathematics called **trigonometry**. The three ratios are

- the **tangent** ratio (or **tan**)
- the **sine** ratio (or **sin**) and
- the **cosine** ratio (or **cos**).



$$\tan 25^\circ = \frac{ZY}{XY}$$

$$\sin 25^\circ = \frac{ZY}{XZ}$$

$$\cos 25^\circ = \frac{XY}{XZ}$$

The values of these ratios can be obtained from a scientific, or graphic, calculator.

Correct to two decimal places:

$$\tan 25^\circ = 0.47$$

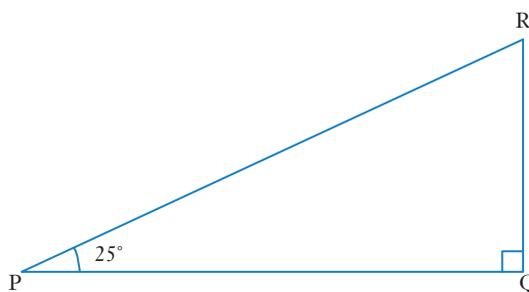
$$\sin 25^\circ = 0.42$$

$$\cos 25^\circ = 0.91$$

tan 25	0.4663076582
sin 25	0.4226182617
cos 25	0.906307787

Copy and complete the lines in the box below for $\triangle PQR$ shown.

Measure the lengths as accurately as possible and perform the division using your calculator. Then see how the answers you obtain compare with the values given above.



Side PQ is of length _____ cm

Side RQ is of length _____ cm

Side PR is of length _____ cm

$$\tan 25^\circ = \frac{RQ}{PQ} = \underline{\hspace{2cm}}$$

$$\sin 25^\circ = \frac{RQ}{PR} = \underline{\hspace{2cm}}$$

$$\cos 25^\circ = \frac{PQ}{PR} = \underline{\hspace{2cm}}$$

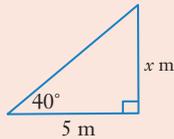


The following examples show how these ratios can be used to determine unknown sides and angles in right angled triangles.

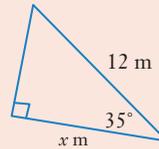
EXAMPLE 1

Find the value of x in each of the following, correct to one decimal place.

a



b



Solution

- a** We require the side **opposite** the 40° angle and we know the **adjacent**. Thus we use the tangent ratio because it involves these two sides.

$$\tan 40^\circ = \frac{\text{Opp}}{\text{Adj}}$$

$$\therefore \tan 40^\circ = \frac{x}{5}$$

Multiply both sides by 5 to eliminate fractions.

$$5 \times \tan 40^\circ = x$$

we write this as

$$5 \tan 40^\circ = x$$

Thus $x = 4.2$
(correct to 1 decimal place)

- b** We require the side **adjacent** to the 35° angle and we know the **hypotenuse**. Thus we use the cosine ratio because it involves these two sides.

$$\cos 35^\circ = \frac{\text{Adj}}{\text{Hyp}}$$

$$\therefore \cos 35^\circ = \frac{x}{12}$$

Multiply both sides by 12 to eliminate fractions.

$$12 \times \cos 35^\circ = x$$

we write this as

$$12 \cos 35^\circ = x$$

Thus $x = 9.8$
(correct to 1 decimal place)

or, using the solve facility on a calculator:

$$\text{solve} \left\{ \tan(40) = \frac{x}{5}, x \right\}$$

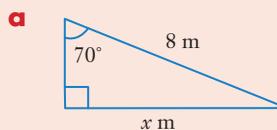
$$\{x=4.195498156\}$$

$$\text{solve} \left\{ \cos(35) = \frac{x}{12}, x \right\}$$

$$\{x=9.829824531\}$$

EXAMPLE 2

Find the value of x in each of the following, correct to one decimal place.



Solution

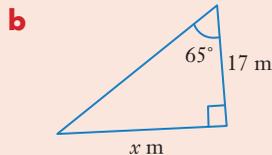
- a** We require the side **opposite** the 70° angle and we know the **hypotenuse**. Thus we use the sine ratio.

$$\sin 70^\circ = \frac{\text{Opp}}{\text{Hyp}}$$

$$\therefore \sin 70^\circ = \frac{x}{8}$$

$$\text{i.e. } 8 \sin 70^\circ = x$$

$$\text{Thus } x = 7.5 \text{ (1 decimal place)}$$



- b** We require the side **opposite** the 65° angle and we know the **adjacent**. Thus we use the tangent ratio.

$$\tan 65^\circ = \frac{\text{Opp}}{\text{Adj}}$$

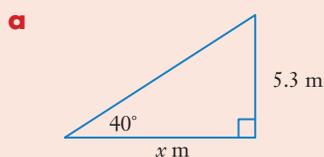
$$\therefore \tan 65^\circ = \frac{x}{17}$$

$$\text{i.e. } 17 \tan 65^\circ = x$$

$$\text{Thus } x = 36.5 \text{ (1 decimal place)}$$

EXAMPLE 3

Find the value of x in each of the following, correct to one decimal place.



Solution

- a** We require the side **adjacent** to the 40° angle and we know the **opposite**. Thus we use the tangent ratio.

$$\tan 40^\circ = \frac{\text{Opp}}{\text{Adj}}$$

$$\therefore \tan 40^\circ = \frac{5.3}{x}$$

Multiply both sides by x

$$x \tan 40^\circ = 5.3$$

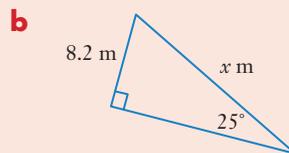
Divide both sides by $\tan 40^\circ$.

$$x = \frac{5.3}{\tan 40^\circ}$$

Thus $x = 6.3$ (1 decimal place)

(Or alternatively use the solve facility

on a calculator to solve $\tan 40^\circ = \frac{5.3}{x}$)



- b** We require the **hypotenuse** and we know the side **opposite** the 25° angle. Thus we use the sine ratio.

$$\sin 25^\circ = \frac{\text{Opp}}{\text{Hyp}}$$

$$\therefore \sin 25^\circ = \frac{8.2}{x}$$

Multiply both sides by x

$$x \sin 25^\circ = 8.2$$

Divide both sides by $\sin 25^\circ$.

$$x = \frac{8.2}{\sin 25^\circ}$$

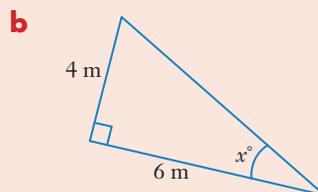
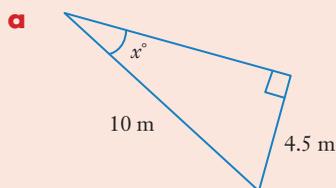
Thus $x = 19.4$ (1 decimal place)

(Or alternatively use the solve facility

on a calculator to solve $\sin 25^\circ = \frac{8.2}{x}$)

EXAMPLE 4

Find the value of x in each of the following, to the nearest integer.



Solution

- a** We know the side **opposite** the required angle and we know the **hypotenuse**. Thus we use the sine ratio.

$$\begin{aligned}\sin x^\circ &= \frac{4.5}{10} \\ &= 0.45\end{aligned}$$

We require the angle whose sine is equal to 0.45.

Using 'inverse sine' or 'arc sine' on a calculator, often shown as \sin^{-1} , arcsin or perhaps ASIN we obtain

$$x = 27 \text{ (nearest integer)}$$

$$\begin{array}{l} \sin^{-1}(4.5 \div 10) \\ 26.74368395 \end{array}$$

- b** We know the side **opposite** the required angle and the side **adjacent** to the required angle. Thus we use the tangent ratio.

$$\begin{aligned}\tan x^\circ &= \frac{4}{6} \\ &= 0.\bar{6}\end{aligned}$$

We require the angle whose tangent is equal to $0.\bar{6}$.

Using 'inverse tan' or 'arc tan' on a calculator, often shown as \tan^{-1} , arctan or perhaps ATAN we obtain

$$x = 34 \text{ (nearest integer)}$$

$$\begin{array}{l} \tan^{-1}(4 \div 6) \\ 33.69006753 \end{array}$$

Notes regarding calculator usage

- 1 We can use the solve facility on some graphic calculators to solve equations such as $\sin x^\circ = \frac{4.5}{10}$ and $\tan x^\circ = \frac{4}{6}$. However, this needs care. In this chapter we are using sine, cosine and tangent in situations involving right triangles. We therefore know that when solving an equation like $\sin x = 0.45$ our answer must be between 0° and 90° . However, in more advanced mathematics (some of which we will see in the next chapter), meaning can be given to the sine, cosine and tangent of angles that are bigger than 90° and even to the sine, cosine and tangent of negative angles! Whilst there is only one value of x between 0° and 90° for which $\sin x = 0.45$ there are many angles outside this interval which have a sine equal to 0.45. The solve facility on a graphic calculator may give us one of these other answers instead. (See displays at the top of the next page.)

Remember: When determining the lengths of sides in right triangles, the **Pythagorean theorem** can also be of use in situations where we are given the lengths of two sides and need to find the length of the third, as was mentioned in the *Preliminary work* section at the beginning of this book.

Applications

The previous examples all involved abstract triangles in which we had to determine an unknown side length or angle size. Some questions will be more applied and will refer to a particular situation in which a right angled triangle is involved. A simple, neat, clear diagram will then need to be drawn.

EXAMPLE 5

A ladder of length 5 metres leans against a vertical wall and just reaches the top of the wall. If the wall is 4.4 metres high calculate the angle the ladder makes with the horizontal ground (to the nearest degree) and the distance from the foot of the ladder to the wall (in metres correct to one decimal place).

Solution

First draw a diagram:

Knowing the hypotenuse and the side opposite the required angle we choose the sine ratio.

$$\sin x^\circ = \frac{4.4}{5}$$

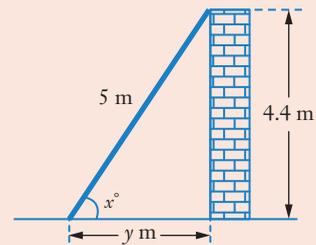
Solving gives $x \approx 62$

Using Pythagoras' theorem.

$$5^2 = 4.4^2 + y^2$$

Solving gives $y \approx 2.375$

The ladder makes an angle of 62° with the horizontal ground and the foot of the ladder is 2.4 metres from the wall.



Notice

- The final answers are not given as $x = 62$ and $y = 2.4$. The letters x and y were not part of the original question, we introduced them to help us obtain a solution. The final answer is given as a sentence that gives what was asked for.
- Having determined x , we could alternatively have then used 'tan' or 'cos' to determine the value of y :

$$\tan x^\circ = \frac{4.4}{y}$$

Multiply both sides by y

$$y \tan x^\circ = 4.4$$

Being sure to use the accurate value of x , not the rounded value of 62 , solving gives:

$$y \approx 2.375, \text{ as before.}$$

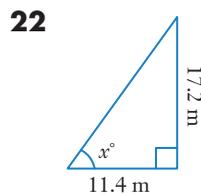
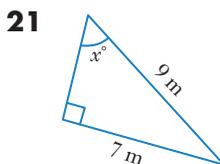
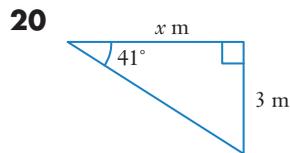
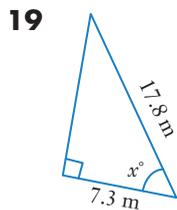
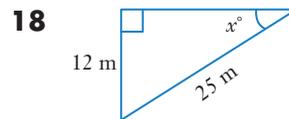
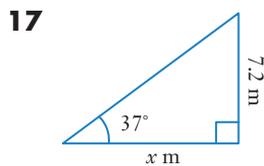
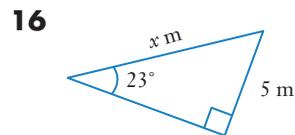
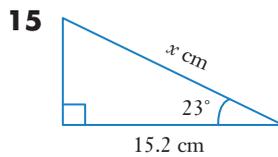
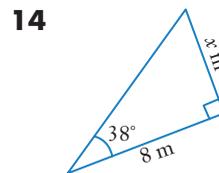
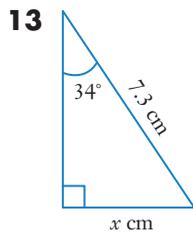
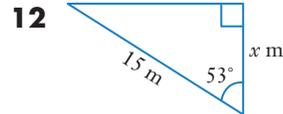
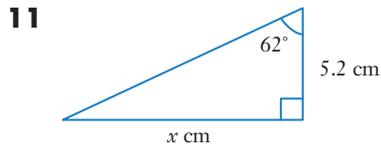
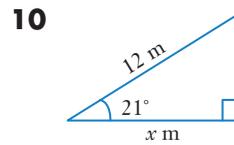
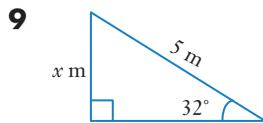
$$\cos x^\circ = \frac{y}{5}$$

Multiply both sides by 5

$$5 \cos x^\circ = y$$

$$y \approx 2.375, \text{ as before.}$$

Find the value of x (and y if applicable) in each of numbers 9 to 30 clearly showing your use of trigonometry or Pythagoras in each one. (Give answers correct to one decimal place if rounding is necessary.)



- 34** The outdoor light that illuminates the driveway of a two-storey house has a light globe that needs replacing. A ladder of length 5 metres is placed with its foot on the horizontal ground and 2 metres from the vertical wall of the house. In this position the ladder just reaches the light. Find



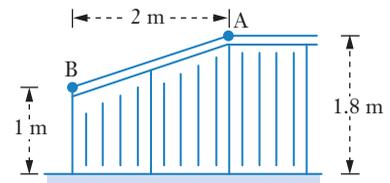
iStock.com/karamysh

- the angle the ladder makes with the ground, to the nearest degree,
- the height of the light above the ground, in metres and correct to one decimal place.

- 35** A person flying a kite holds the line 1 metre above level ground and has 45 metres of line out. If the line is straight and makes 62° with the horizontal what is the height of the kite above ground level (to the nearest metre)?

- 36** To reduce the force acting on the end of a garden fence due to the wind the fence can be 'raked down'.

The diagram on the right shows a fence raked down from a height of 1.8 metres to 1 metre in a horizontal distance of 2 m.



Find

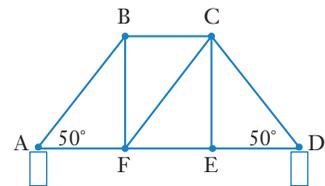
- the acute angle AB makes with the horizontal, to the nearest degree.
- the length of AB, to the nearest centimetre.

- 37** The diagram on the right shows a simple bridge design.

AD and BC are horizontal. FB and EC are vertical.

$\angle BAF = \angle CDE = 50^\circ$. AD = 24 m and AF = FE = ED.

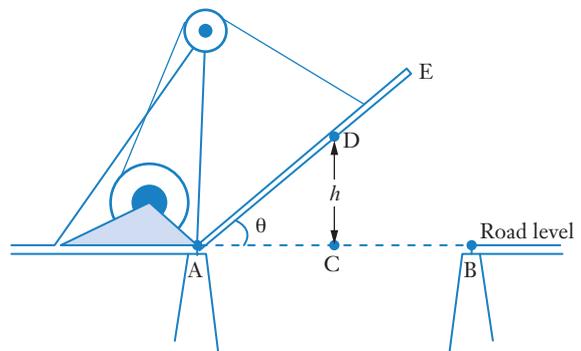
Calculate the lengths of AB, BF and FC giving all answers to the nearest centimetre.



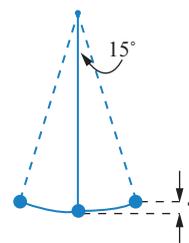
- 38** The diagram shows a road bridge that can be opened to allow tall ships to pass underneath.

$AB = AE = 20$ metres and h is the distance from C, the mid-point of AB, to D, the point on the bridge vertically above C.

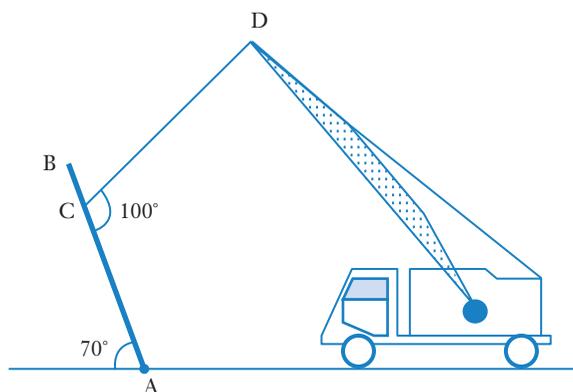
If h needs to be 8 m find θ in degrees correct to one decimal place. For this value of θ find the length of AD as a percentage of AE (to the nearest percent).



- 43** A pendulum of length 80 cm swings 15° either side of the vertical.
What is the vertical rise the bob of the pendulum makes above its lowest position, to the nearest millimetre?

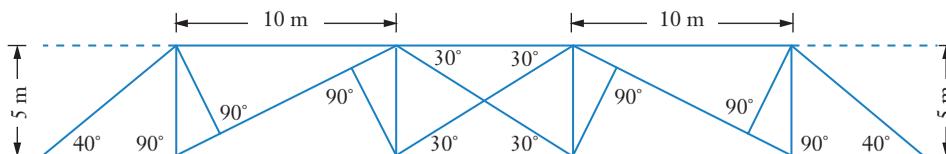


- 44** The diagram shows a mobile crane lifting an 8 metre pole into the vertical position. The cable from the crane is attached to a point C where AC is four fifths of AB. At the instant shown in the diagram $CD = 8$ metres, how high is point D above the horizontal ground (to the nearest metre)?



- 45** (Challenge.)

The framework shown below is to be made out of lengths of steel.

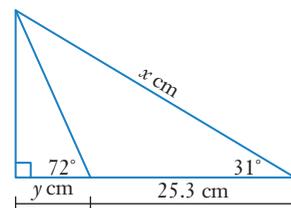


The framework consists of a right angled triangle on each end with three rectangles in the middle. The company contracted to make it needs to know the total length of steel required. Find the length of steel required, to the nearest whole metre.

- 46** (Challenge? Maybe, but hint makes it okay.)

Given the diagram shown on the right determine the values of x and y giving each answer correct to one decimal place.

Hint: Obtain the height in terms of y from one triangle, and then find the height in terms of y from another triangle and then ...





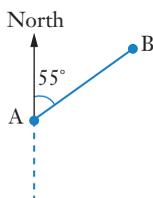
Identifying bearings



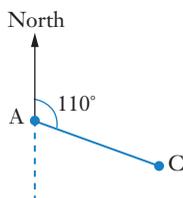
Bearings

Bearings

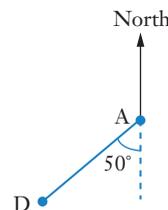
From one location, the direction we would need to travel to reach a second location can be given as a bearing. These are angles, expressed as three figures, and are measured from North, clockwise, as shown below.



Bearing of B from A is 055°



Bearing of C from A is 110°



Bearing of D from A is 230°

Alternatively, bearings can be expressed as a certain number of degrees (between 0 and 90) to the East or West of North or South. Thus the above *three figure bearings* could be expressed as the following *compass bearings*:

Bearing of B from A is
N 55° E

Bearing of C from A is
S 70° E

Bearing of D from A is
S 50° W

EXAMPLE 6

From town A, town B lies 7.2 km away on a bearing of 070° .

From town B, town C lies 8.4 km away on a bearing of 160° .

Find the distance and bearing of C from A.

Solution

First make a sketch of the situation:

Note that with the given bearings $\angle ABC = 90^\circ$.

In $\triangle ABC$, by Pythagoras' theorem, $AC^2 = 7.2^2 + 8.4^2$

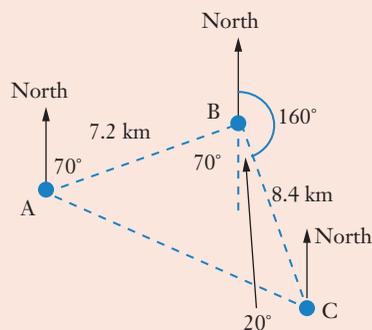
Thus $AC \approx 11.06$ km

Also $\tan \angle BAC = \frac{8.4}{7.2}$

giving $\angle BAC \approx 49.4^\circ$

Thus the bearing of C from A is approximately 119° , i.e. $(49^\circ + 70^\circ)$.

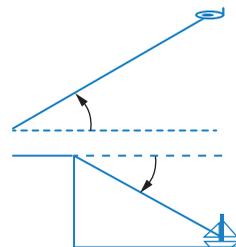
Town C is 11.1 km from A, on a bearing of 119° .



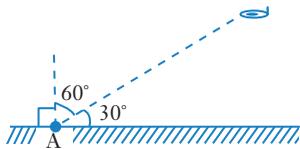
Elevation and bearings

Elevation and depression

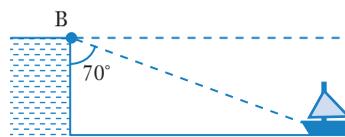
- **Angles of elevation** are measured from the horizontal, up.
- **Angles of depression** are measured from the horizontal, down.



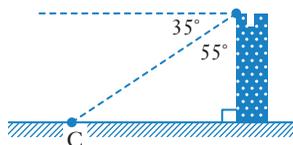
- 2 a** What is the angle of elevation of the aeroplane from A?



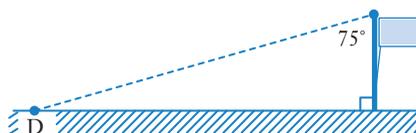
- b** What is the angle of depression of the ship from B?



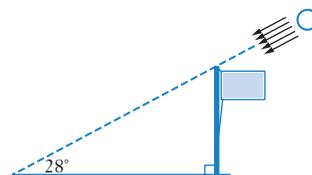
- c** What is the angle of depression of point C from the top of the tower?



- d** What is the angle of elevation of the top of the flagpole from D?



- 3** When the angle of elevation of the sun is 28° a vertical flag pole casts a shadow of length 22.4 metres on horizontal ground. Calculate the height of the flagpole.



- 4** Find the angle of elevation of the sun if a 2.0 metre pole held vertically on horizontal ground casts a shadow of length 4.1 metres. Give your answer correct to the nearest degree.
- 5** A flagpole stands vertically on level ground. When the sun's elevation is 24° the flagpole casts a shadow of length 22.5 metres. Find the height of the flagpole.
- 6** A and B are two points on horizontal ground. A mast of length 540 cm is to stand vertically with its base at B. From A, the top of the mast will have an angle of elevation of 17° . A straight wire is to run from the top of the mast to the point A. How far is this, rounded up to the next metre?
- 7** At 9 a.m. one morning two ships leave a harbour and head out to sea. One ship travels at a steady 4 km/h on a bearing 110° and the other ship maintains 5 km/h on a bearing 200° . To the nearest kilometre how far apart are the ships one and a half hours later?



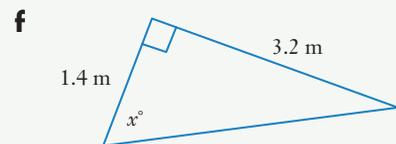
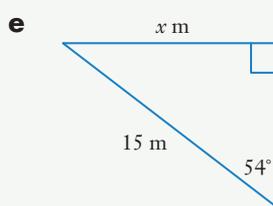
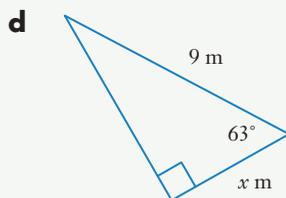
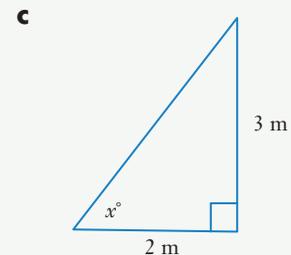
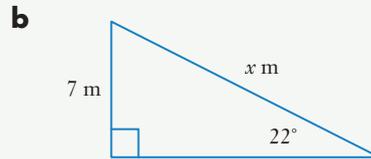
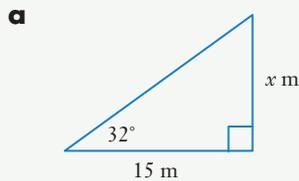
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- 8** The three points A, B and C lie on horizontal ground and form a straight line with B between A and C. A vertical tower of height 40 metres stands at C. The angle of elevation of the top of the tower is 18° from A and 35° from B. How far is B from A (to the nearest metre)?

Miscellaneous exercise ten

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1 Determine the value of x in each of the following, giving your answers correct to one decimal place.



- 2 Find the mean and the median of the following six amounts:

\$13 600 \$5700 \$23 400 \$2100 \$14 600 \$98 700

- 3 The 35 students in a class sat a test that was marked out of 40. The 20 boys had a mean score of 24.35 and the class mean was 23. What was the mean score of the girls in the class?

- 4 Solve the following equations.

a $7x - 15 = 132$

b $3(2x - 1) + 2x = 17$

c $\frac{2x+1}{3} = 5$

d $\frac{5}{x} = 8$

- 5 Formula: $v^2 = u^2 + 2as$

a Find s given that $v = 13$, $u = 5$ and $a = 24$.

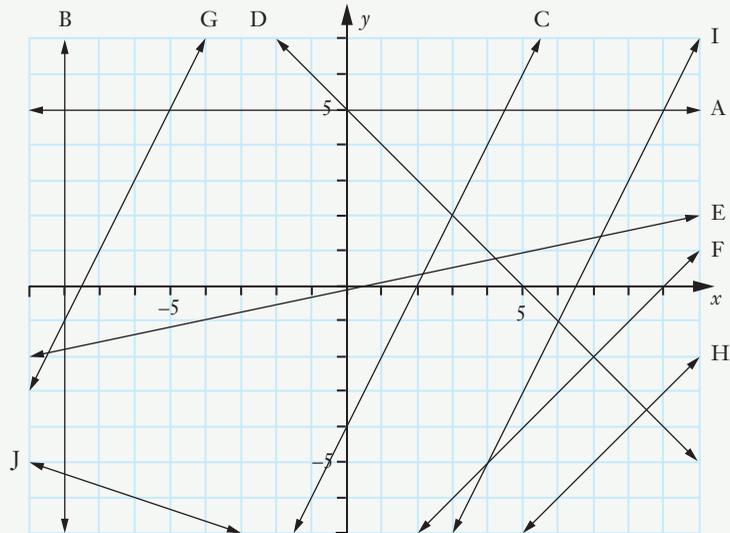
b Find a given that $v = 21$, $u = 17$ and $s = 19$.

- 6 I think of a number, multiply it by three, add seven and then divide the answer by two. At the end of all this the number I end up with is eleven more than the number I first thought of. Find the number first thought of.

- 7 John takes out a loan which involves simple interest charged at the rate of 7.5% per annum. After 4 years John repays \$11 180 which clears the loan and interest.

How much did John borrow in the first place?

12 Find the equations of the lines A to J shown below.

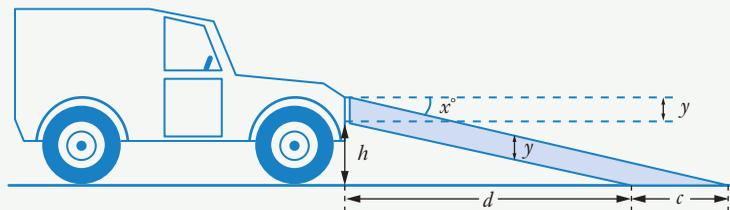


13 Scientists investigating levels of pollution in a particular stretch of river estimate that N , the number of fish that part of the river can support, depends on P , the number of tonnes of pollutant in that part of the river, according to the rule:

$$N \approx 60\,000 - 2100P$$

- How many fish does this rule suggest this stretch of the river can support if $P = 5$.
- If the level of pollution reaches 18 tonnes what number of fish could this stretch of the river support according to the above rule.
- If fish numbers are not to drop below 45 000 what does the formula suggest the pollution level must not exceed?

14 The diagram below shows a headlamp beam adjusted down to avoid dazzling oncoming drivers.



The light beam is angled at x° below the horizontal and the distances h , d , c and y are as shown in the diagram. Assuming that the beam does not spread out at all and that the ground is horizontal, find d and c given that:

$$x = 4, h = 80 \text{ cm and } y = 20 \text{ cm.}$$

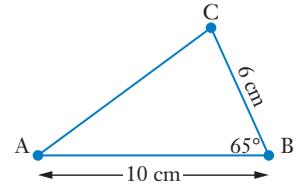
11.

Trigonometry for triangles that are not right angled

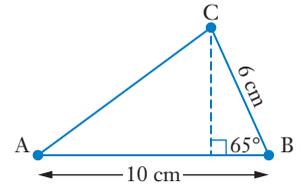
- Area of a triangle
- Triangles that are not right angled
- Area of a triangle given two sides and the included angle
- The sine rule
- The cosine rule
- Miscellaneous exercise eleven

Triangles that are not right angled

On the previous page we were asked, amongst other things, to determine the area of $\triangle ABC$, given the lengths of BA and BC and the size of $\angle ABC$. (i.e. given two sides and the angle between them.)



By drawing the perpendicular from C to AB we obtain right triangles. This allows trigonometry to be used to determine the height, and hence the area, of $\triangle ABC$.



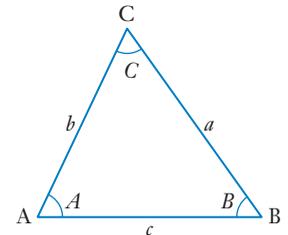
(Alternatively $\triangle ABC$ could be drawn accurately from the given information and the perpendicular height could be measured.)

This approach of drawing the perpendicular from one vertex to the opposite side allows trigonometry to be used for a triangle that is not right angled. We will use this approach in this chapter to obtain three formulae that are useful when dealing with triangles that are not right angled.

We will consider:

- a formula for the area of a triangle,
- the sine rule formula,
- the cosine rule formula.

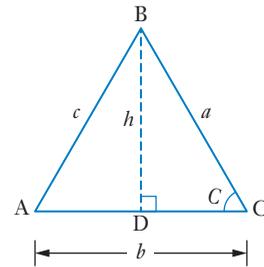
Note: In obtaining these formulae we will use the usual convention for naming the sides and angles of a triangle. i.e. in triangle ABC, the three angles are labelled A , B and C according to their vertex and the sides opposite these angles are labelled a , b and c respectively.



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Area of a triangle given two sides and the included angle

Consider $\triangle ABC$ with the perpendicular from B to AC meeting AC at D (see diagram).



In $\triangle BDC$,
$$\sin C = \frac{h}{a}$$

Multiplying by a to isolate h ,
$$h = a \sin C$$

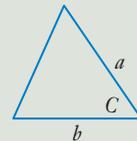
Using
$$\text{Area of triangle} = \frac{1}{2} \text{ base} \times \text{height}$$

we have
$$\text{Area of triangle} = \frac{1}{2} \times b \times a \sin C$$

Thus:

$$\text{Area of a triangle} = \frac{ab \sin C}{2}$$

i.e. the area of a triangle is half the product of two sides multiplied by the sine of the angle between them.

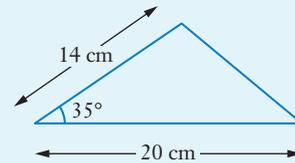


EXAMPLE 1

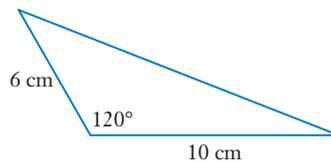
Find the area of the triangle shown.

Solution

$$\begin{aligned} \text{Area} &= \frac{20 \times 14 \times \sin 35^\circ}{2} \\ &\approx 80.3 \text{ cm}^2 \end{aligned}$$



Now that we are dealing with any triangle, not just right triangles, we could have an obtuse angle between the two sides of known length, as shown below.



Applying our formula will involve the sine of an obtuse angle:

$$\text{Area} = \frac{10 \times 6 \times \sin 120^\circ}{2}$$

How does your calculator respond when asked for $\sin 120^\circ$ or $\sin 130^\circ$ or $\sin 140^\circ \dots$?

Did you notice that $\sin 120^\circ = \sin 60^\circ$
 $\sin 130^\circ = \sin 50^\circ$
 $\sin 140^\circ = \sin 40^\circ$

sin 120	0.8660254038
sin 60	0.8660254038
sin 130	0.7660444431
sin 50	0.7660444431

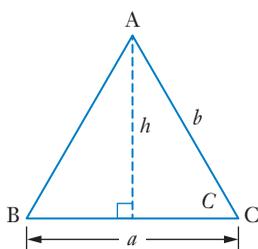
in fact, to generalise: $\sin(180^\circ - C) = \sin C$?

This fact means that we can use our area formula,

$$A = \frac{1}{2} ab \sin C$$

for all triangles, even those for which angle C is obtuse.

Consider the acute angled $\triangle ABC$ below left and the obtuse angled $\triangle ABC$ below right.

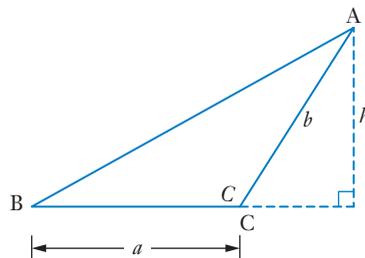


$$\sin C = \frac{h}{b}$$

$$\therefore h = b \sin C$$

$$\text{Area} = \frac{1}{2} a \times h$$

$$= \frac{1}{2} ab \sin C$$



$$\sin (180^\circ - C) = \frac{h}{b}$$

$$\therefore h = b \sin (180^\circ - C)$$

$$\text{Area} = \frac{1}{2} a \times h$$

$$= \frac{1}{2} ab \sin (180^\circ - C)$$

$$= \frac{1}{2} ab \sin C$$

Thus for all triangles our area formula $A = \frac{1}{2} ab \sin C$ applies.

However, the fact that $\sin(180^\circ - C) = \sin C$ does present a difficulty if a question gives the area of a triangle and the lengths of two sides of the triangle and asks for the size of the angle between the two sides of known length. Which answer do we give – the acute angle or the obtuse angle?

For example, suppose $\triangle ABC$ has an area of 3 cm^2 and is such that $a = 4 \text{ cm}$ and $b = 3 \text{ cm}$.

Using $\text{Area} = \frac{1}{2} ab \sin C$

$$3 = \frac{1}{2} \times 4 \times 3 \sin C$$

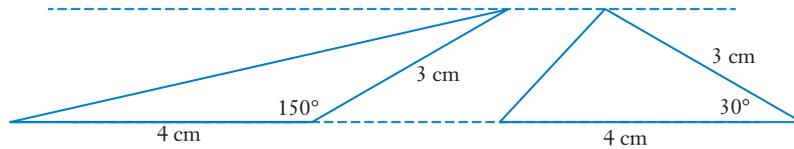
$$3 = 6 \sin C$$

and so $\sin C = 0.5$

Now comes our dilemma:

Does $C = 30^\circ$ or does $C = 150^\circ$?

The dilemma is genuine because for the information we are given there *are* two possible triangles that 'fit the facts':



The two triangles each have a base of 4 cm and are the same height as each other. Hence their areas will indeed be equal, and in this case each equal to 3 cm^2 .

The information we have been given about triangle ABC is said to be *ambiguous*. (The word ambiguous meaning open to more than one interpretation.) However, there is no need to panic. In this unit, we will be given sufficient information for such ambiguity to be avoided, as in the next example.

EXAMPLE 2

Triangle DEF is such that $e = 16 \text{ cm}$, $d = 12 \text{ cm}$, the area of the triangle is 78 cm^2 and $\angle DFE$ is an acute angle. Find the size of $\angle DFE$ giving your answer to the nearest 0.1° .

Solution

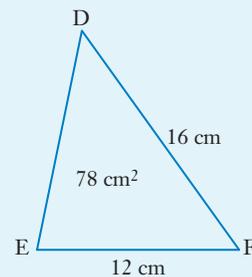
Using $\text{Area} = \frac{1}{2} ab \sin C$

$$78 = \frac{1}{2} \times 12 \times 16 \sin \angle DFE$$

$\therefore \sin \angle DFE = 0.8125$

and so, given that $\angle DFE$ is an acute angle, using a calculator:

$$\angle DFE = 54.3^\circ \text{ (to nearest } 0.1^\circ\text{)}$$



(If instead we had been told that $\angle DFE$ was an obtuse angle our final answer would have been 125.7° , i.e. $180^\circ - 54.3^\circ$. Had we not been told anything about $\angle DFE$ then two possible triangles would exist that each satisfied the given facts.)

$\sin 150$	0.5
$\sin 30$	0.5

EXAMPLE 3

Given that the triangle sketched on the right has an area of 7 cm^2 find x correct to one decimal place.

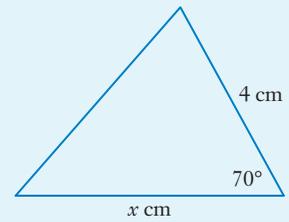
Solution

$$\text{Area} = \frac{1}{2}(x) 4 \sin 70^\circ$$

$$\therefore 7 = \frac{1}{2}(x) 4 \sin 70^\circ$$

$$\text{i.e. } 7 = 2x \sin 70^\circ$$

Solving this equation gives $x = 3.7$, correct to 1 decimal place



Note: It is also possible to determine the area of a triangle, given the lengths of the three sides of the triangle, using a result known as **Heron's 's' formula**:

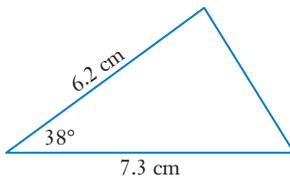
$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{a+b+c}{2}.$$

One of the questions of a later exercise in this chapter, and one of the questions in a later Miscellaneous Exercise, reminds you of this formula and requires you to use it.

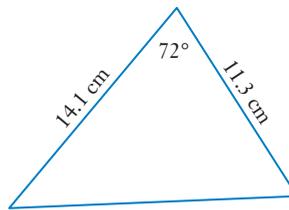
Exercise 11B

Find the area of each triangle in questions 1 to 11, giving your answers in square centimetres and correct to one decimal place. (Diagrams not necessarily to scale).

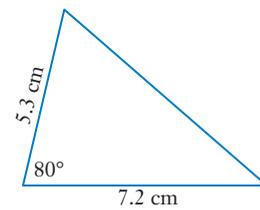
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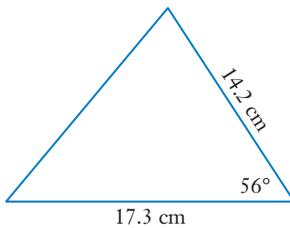
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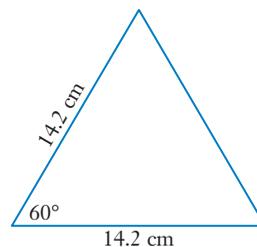
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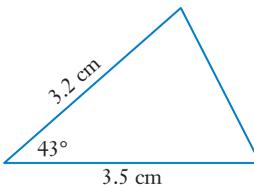
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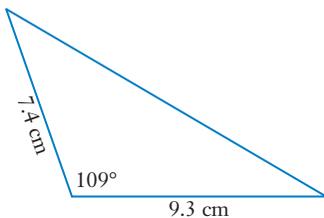
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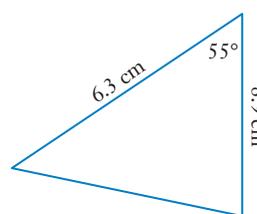
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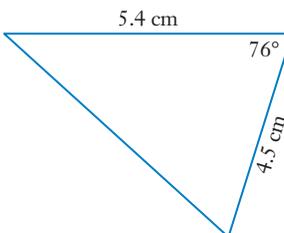
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8



9

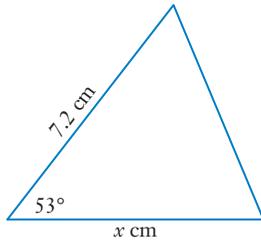


10 $\triangle ABC$ given that $AB = 8 \text{ cm}$, $BC = 7 \text{ cm}$, $AC = 5 \text{ cm}$ and $\angle BAC = 60^\circ$.

11 $\triangle PQR$ given that $PQ = 7 \text{ cm}$, $PR = 8 \text{ cm}$, $RQ = 3 \text{ cm}$ and $\angle PRQ = 60^\circ$.

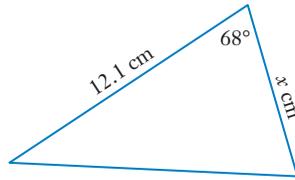
Find the value of x in each of the following, correct to one decimal place, given that the area of each triangle is as stated. (Diagrams not necessarily drawn to scale.)

12



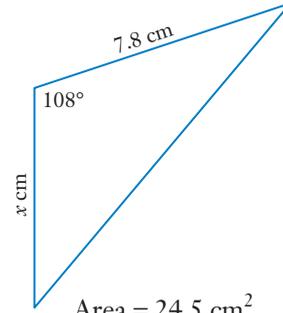
Area = 19.6 cm^2

13



Area = 40.9 cm^2

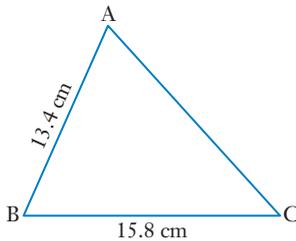
14



Area = 24.5 cm^2

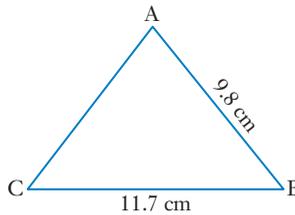
Find the size of $\angle ABC$ in each of the following, correct to the nearest degree, given that each triangle is acute angled and the area of each triangle is as stated. (The diagrams are not necessarily drawn to scale.)

15



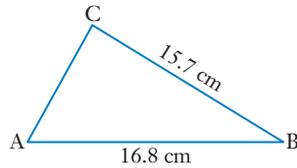
Area = 97.4 cm^2

16



Area = 45.2 cm^2

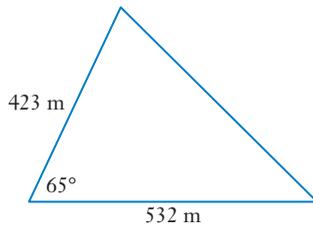
17



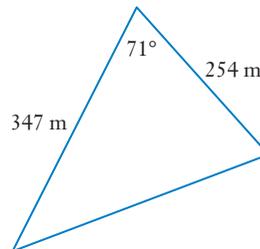
Area = 69.9 cm^2

18 If farming land in a particular region costs \$12 300 per hectare find the cost of each of the following areas, to the nearest \$1000. (1 hectare = $10\,000 \text{ m}^2$.)

a



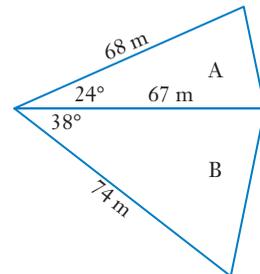
b



19 A triangular block of land has two sides of lengths 45 m and 30 m and the angle included between them is 70° . A second triangular block has two sides of lengths 48 m and 35 m and the angle included between them is 50° .

Which block has the greater area and by how much (to the nearest square metre)?

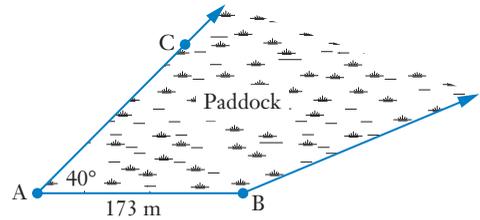
20 The owners of two neighbouring triangular blocks of land, shown as A and B in the diagram on the right, are offered a total of \$1 250 000 by a property developer for the two blocks together. If they were to accept this offer and divide the money between them in the ratio of the land areas of the blocks how much would each owner receive?



21 (Note: A square of side 100 metres has an area of 1 hectare.)

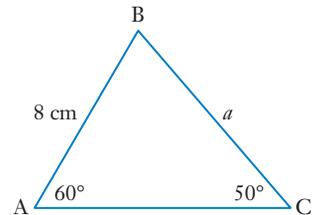
A farmer wishes to lease one hectare of his land to an investor who wishes to use it to grow Tasmanian Blue Gum trees. The investor intends harvesting these fast growing trees and selling the wood to a paper making company as woodchip. The farmer, for his part, simply has to fence off suitable land for the investor to use.

Rather than having to use new fencing around the whole area the farmer chooses a triangular site that allows existing fencing to be used on two sides (AB and AC in the diagram). The farmer measures the distance AB as 173 metres and measures $\angle CAB$ as 40° . He wishes to locate point C so that $\triangle ABC$ will have an area of one hectare. He asks you to calculate the length AC for him. Calculate this length, rounding your answer *up* to the next whole metre.



The sine rule

If we were given the triangle on the right and were asked to find a we could accurately draw the triangle and measure the required length. However, great accuracy is not easy to achieve and drawing can be time consuming. Once again we could proceed by drawing the perpendicular from B to AC so that our right triangle trigonometry work can be used, as shown below.



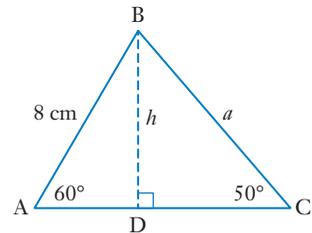
Draw the perpendicular from B to meet AC at D (see diagram).

$$\begin{aligned} \text{In } \triangle ABD, \quad \sin 60^\circ &= \frac{h}{8} \\ \therefore h &= 8 \sin 60^\circ \end{aligned}$$

$$\text{In } \triangle BCD, \quad \sin 50^\circ = \frac{h}{a}$$

$$\therefore \sin 50^\circ = \frac{8 \sin 60^\circ}{a}$$

$$\text{Solving gives} \quad a \approx 9.04 \text{ cm}$$

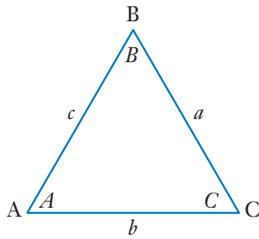


$$\begin{aligned} \text{solve } \left(\sin(50) = \frac{8 \cdot \sin(60)}{x}, x \right) \\ \{x = 9.044126999\} \end{aligned}$$

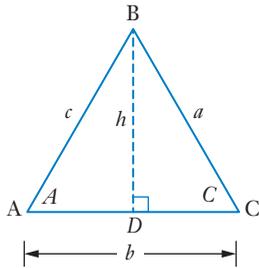
As we will see on the next page, if we apply this technique to a general triangle ABC we obtain the **sine rule**:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Consider a triangle ABC as shown below left for an acute angled triangle and below right for an obtuse angled triangle.



Drawing the perpendicular from B to meet AC at D:



From $\triangle ABD$: $\sin A = \frac{h}{c}$
 $\therefore h = c \sin A$ [1]

From $\triangle CBD$: $\sin C = \frac{h}{a}$
 $\therefore h = a \sin C$ [2]

Thus for both the acute triangle and the obtuse triangle:

From [1] and [2] $c \sin A = a \sin C$

Thus $\frac{c}{\sin C} = \frac{a}{\sin A}$ [3]

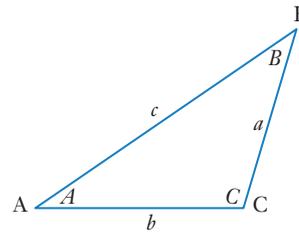
If instead we draw the perpendicular from A to BC we obtain

$$\frac{b}{\sin B} = \frac{c}{\sin C} \quad [4]$$

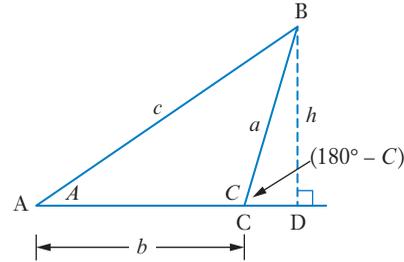
From [3] and [4] it follows that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This is the **sine rule**.



Drawing the perpendicular from B to meet AC produced at D:



From $\triangle ABD$: $\sin A = \frac{h}{c}$
 $\therefore h = c \sin A$ [1]

From $\triangle CBD$: $\sin (180^\circ - C) = \frac{h}{a}$
 $\therefore h = a \sin C$ [2]

Hint

Rather than learning this formula, notice the pattern:

Any side on the sine of the opposite angle is equal to any other side on the sine of its opposite angle.

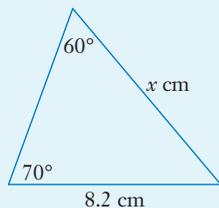


The sine rule—finding lengths of sides

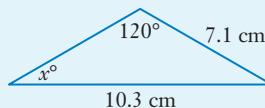
EXAMPLE 4

Find the value of x in the following, giving answers correct to one decimal place.

a



b



Solution

a By the sine rule

$$\frac{x}{\sin 70^\circ} = \frac{8.2}{\sin 60^\circ}$$

Multiply by $\sin 70^\circ$ to isolate x

$$\begin{aligned} x &= \frac{8.2 \sin 70^\circ}{\sin 60^\circ} \\ &= 8.9 \text{ (to 1 decimal place)} \end{aligned}$$

b By the sine rule

$$\frac{10.3}{\sin 120^\circ} = \frac{7.1}{\sin x^\circ}$$

Multiply by $(\sin x^\circ)(\sin 120^\circ)$

$$10.3 \sin x^\circ = 7.1 \sin 120^\circ$$

$$\therefore \sin x^\circ \approx 0.5970$$

$$x = 36.7 \text{ (to 1 decimal place)}$$

Or, using the 'solve' ability of some calculators:

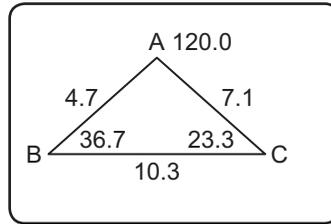
$$\text{solve} \left(\frac{x}{\sin(70)} = \frac{8.2}{\sin(60)}, x \right) \\ \{x = 8.897521316\}$$

$$\text{solve} \left(\frac{10.3}{\sin(120)} = \frac{7.1}{\sin(x)}, x \right) | 0 \leq x \leq 180 \\ \{x = 143.346877, x = 36.65312298\}$$

Note:

- In part **b** we went from $\sin x^\circ \approx 0.5970$ to $x = 36.7$ (to 1 decimal place) despite there being another value of x between 0 and 180 for which $\sin x^\circ = 0.5970$, and that is $(180 - 36.7)$, i.e. 143.3, as the calculator shows when asked for solutions in the interval $0 \leq x \leq 180$. However, in the given triangle x cannot be 143.3 because the triangle already has one obtuse angle and cannot have another. However, whilst this may not always be the case, and an *ambiguous* situation could occur when both answers are possible, this unit will not include such situations and sufficient information will be given to be able to dismiss one of the solutions.

As was mentioned in the previous chapter on right triangles, some calculator programs allow the user to put in the known sides and angles of a triangle and, provided the information put in is sufficient, the program will determine the remaining sides and angles.



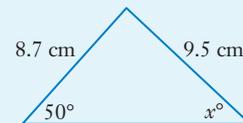
Also some calculators allow us to create a scale drawing of the triangle and find lengths and angles that way.

These programs can be useful but make sure that you understand the underlying idea of the sine rule (and the cosine rule which we will see later in this chapter) and can demonstrate the appropriate use of these rules when required to do so.

EXAMPLE 5

Find the value of x in the triangle shown on the right.

(Give the answer correct to one decimal place.)



The sine rule—
finding angles

Note

Note first that x° , being opposite a side of length 8.7 cm, must be less than the 50° which is opposite a side of length 9.5 cm. (For any two sides of a triangle, the larger of the two sides has the larger opposite angle.)

Solution

By the sine rule:

$$\frac{9.5}{\sin 50^\circ} = \frac{8.7}{\sin x^\circ}$$

Multiply by $(\sin 50^\circ)(\sin x^\circ)$:

$$9.5 \sin x^\circ = 8.7 \sin 50^\circ$$

\therefore

$$\sin x^\circ = \frac{8.7 \sin 50^\circ}{9.5}$$

Thus

$$x \approx 44.6 \text{ (correct to 1 decimal place)}$$

Or, using the 'solve' ability of some calculators:

We then dismiss the obtuse angle because x had to be smaller than 50 . (Or, had we not noticed this from the side lengths, we would reject the obtuse angle as the angle sum of the triangle would exceed 180°)

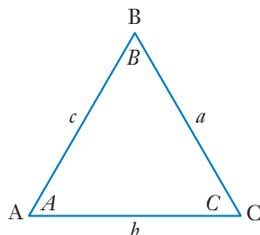
Thus, as before, $x = 44.6$ (correct to 1 decimal place).

$$\text{solve} \left\{ \frac{9.5}{\sin(50)} = \frac{8.7}{\sin(x)}, x \mid 0 \leq x \leq 180 \right. \\ \left. \{x = 135.4496775, x = 44.55032253\} \right.$$

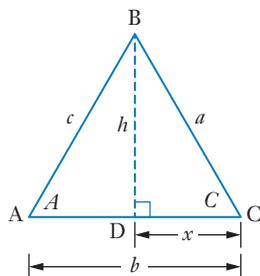
The cosine rule

Again, consider a triangle ABC as shown below left for an acute angled triangle and below right for an obtuse angled triangle. However, in this case, we use the fact that for cosines:

$$\cos(180^\circ - C) = -\cos C$$



Again we draw the perpendicular from B to meet AC at D:



From $\triangle CBD$: $a^2 = h^2 + x^2$ [1]

From $\triangle ABD$: $c^2 = h^2 + (b-x)^2$
 $= h^2 + (b-x)(b-x)$

i.e. $c^2 = h^2 + b^2 + x^2 - 2bx$

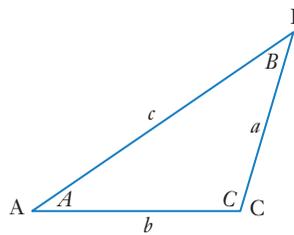
Using [1]: $c^2 = a^2 + b^2 - 2bx$ [2]

From $\triangle CBD$: $\cos C = \frac{x}{a}$

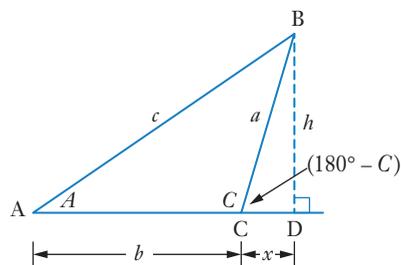
$\therefore x = a \cos C$ [3]

Using [2] and [3]:

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Again we draw the perpendicular from B to meet AC produced at D:



From $\triangle CBD$: $a^2 = h^2 + x^2$ [1]

From $\triangle ABD$: $c^2 = h^2 + (b+x)^2$
 $= h^2 + (b+x)(b+x)$

i.e. $c^2 = h^2 + b^2 + x^2 + 2bx$

Using [1]: $c^2 = a^2 + b^2 + 2bx$ [4]

From $\triangle CBD$: $\cos(180^\circ - C) = \frac{x}{a}$

$\therefore x = -a \cos C$ [5]

Using [4] and [5]:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Thus for both the acute triangle and the obtuse triangle $c^2 = a^2 + b^2 - 2ab \cos C$.

This is the **cosine rule**:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Similarly $a^2 = b^2 + c^2 - 2bc \cos A$ and $b^2 = a^2 + c^2 - 2ac \cos B$

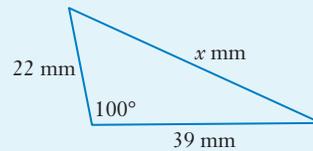
Hint

As was said with the sine rule, rather than learning the rule as a formula instead notice the pattern of what it is telling you:

The square of any side of a triangle is equal to the sum of the squares of the other two sides take away twice the product of the other two sides multiplied by the cosine of the angle between them.

EXAMPLE 6

Find the value of x for the triangle shown.



Solution

By the cosine rule:

$$\begin{aligned}x^2 &= 22^2 + 39^2 - 2(22)(39)\cos 100^\circ \\ &\approx 2302.98 \\ x &= 48 \text{ to the nearest integer.}\end{aligned}$$

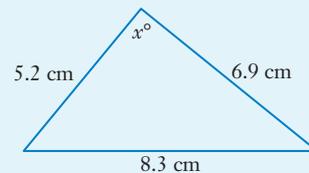
$$\begin{aligned}22^2 + 39^2 - 2 \times 22 \times 39 \times \cos(100) \\ 2302.980273 \\ \sqrt{\text{ans}} \\ 47.98937667\end{aligned}$$



The cosine rule—
angles and sides

EXAMPLE 7

Find the value of x for the triangle shown.



Solution

By the cosine rule:

$$\begin{aligned}8.3^2 &= 5.2^2 + 6.9^2 - 2(5.2)(6.9)\cos x^\circ \\ \cos x^\circ &= \frac{5.2^2 + 6.9^2 - 8.3^2}{2(5.2)(6.9)} \\ &\approx 0.08027 \\ x &= 85 \text{ to the nearest integer.}\end{aligned}$$

$$\begin{aligned}\frac{5.2^2 + 6.9^2 - 8.3^2}{2 \times 5.2 \times 6.9} \\ 0.08026755853 \\ \cos^{-1}(\text{ans}) \\ 85.39605483\end{aligned}$$

Note:

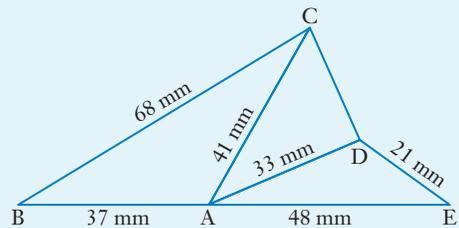
- If you prefer to use the solve facility on your calculator make sure you can obtain the same answers as those shown.
- With the cosine rule, when solving equations of the form $\cos x = c$ we do not have to worry about there being two solutions in the range 0° to 180° . The cosine of an acute angle is positive whilst the cosine for an obtuse angle is negative. Thus an equation of the form $\cos x = c$ does *not* have two solutions for x in the range 0° to 180° . If c is positive the one solution will be an acute angle and if c is negative it will be an obtuse angle.

EXAMPLE 8

The sketch on the right shows a system of three triangles with lengths and angles as indicated.

BAE is a straight line.

Find the length of CD.



Thoughts

CD is one side of $\triangle ACD$. In this triangle we know the lengths of AC and AD so if we knew the size of $\angle CAD$ we could apply the cosine rule to find the length of CD. We can find the size of $\angle CAD$ if we first find the size of $\angle CAB$ and the size of $\angle DAE$.

Solution

For $\triangle ABC$, applying the cosine rule:

$$68^2 = 41^2 + 37^2 - 2 \times 41 \times 37 \cos \angle BAC$$

$$\cos \angle BAC = \frac{41^2 + 37^2 - 68^2}{2 \times 41 \times 37}$$

$$\therefore \angle BAC \approx 121.3^\circ$$

For $\triangle DAE$, applying the cosine rule:

$$21^2 = 33^2 + 48^2 - 2 \times 33 \times 48 \cos \angle DAE$$

$$\cos \angle DAE = \frac{33^2 + 48^2 - 21^2}{2 \times 33 \times 48}$$

$$\therefore \angle DAE \approx 21.3^\circ$$

$(41^2 + 37^2 - 68^2) \div (2 \times 41 \times 37)$	-0.5187870798
\cos^{-1} Ans	121.2509263
Ans \rightarrow A	121.2509263

$(33^2 + 48^2 - 21^2) \div (2 \times 33 \times 48)$	0.9318181818
\cos^{-1} Ans	21.27996647
Ans \rightarrow B	21.27996647

For $\triangle CAD$, applying the cosine rule:

$$CD^2 = 33^2 + 41^2 - 2 \times 33 \times 41 \cos \angle CAD$$

$$\approx 622.3$$

$$\therefore CD \approx 24.9$$

CD is of length 25 mm, to the nearest millimetre.

$33^2 + 41^2 - 2 \times 33 \times 41 \cos(180 - A - B)$	622.297976
$\sqrt{\text{Ans}}$	24.94590099

Notice from the calculator displays that the more accurate values for $\angle BAC$ and $\angle DAE$ were stored and later recalled for use, thus avoiding the risk of introducing unnecessary rounding errors.

Exercise 11C

The sine rule

Given that each of the following equations are formed by applying the sine rule to an acute angled triangle solve for x , giving your answer correct to one decimal place in each case.

1 $\frac{x}{\sin 50^\circ} = \frac{7.3}{\sin 75^\circ}$

2 $\frac{x}{\sin 32^\circ} = \frac{12.1}{\sin 78^\circ}$

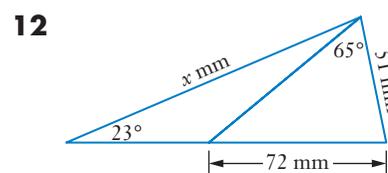
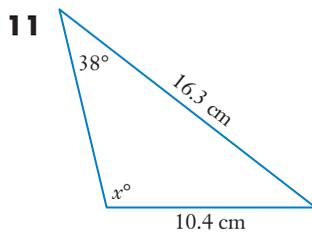
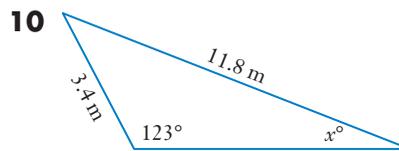
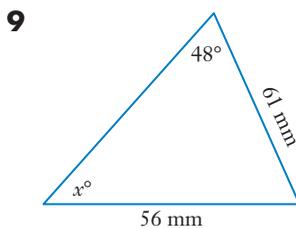
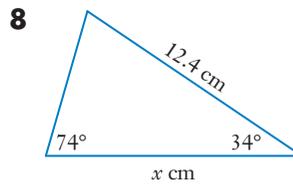
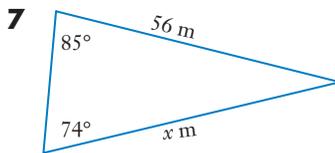
3 $\frac{12.3}{\sin 60^\circ} = \frac{x}{\sin 65^\circ}$

4 $\frac{8.2}{\sin x^\circ} = \frac{10}{\sin 85^\circ}$

5 $\frac{7.8}{\sin x^\circ} = \frac{8.3}{\sin 50^\circ}$

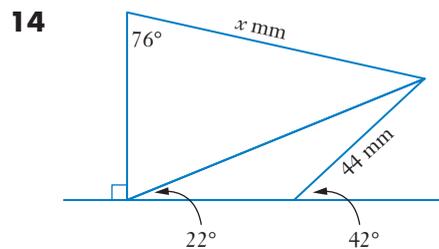
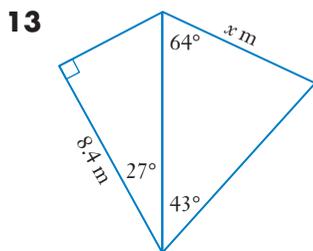
6 $\frac{6.8}{\sin 50^\circ} = \frac{7.2}{\sin x^\circ}$

Find the value of x in each of the following.



x is between 0 and 90.

x is between 90 and 180.

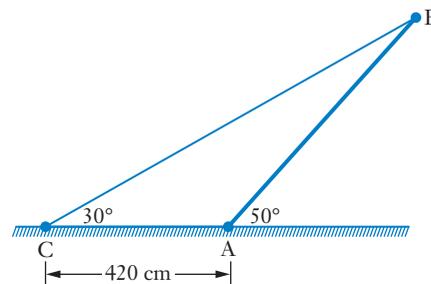


- 15** The diagram shows a pole AB with end A fixed on horizontal ground and the pole supported by a wire attached to end B and to a point C on the ground with $AC = 420$ centimetres.

The pole makes an angle of 50° with the ground and the wire makes an angle of 30° with the ground, as shown in the diagram.

Points A, B and C all lie in the same vertical plane.

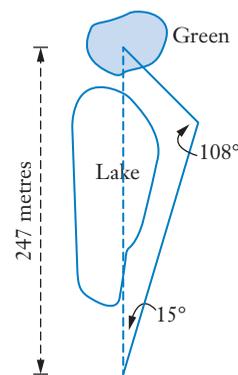
Find the length of the pole giving your answer to the nearest centimetre.



- 16** Rather than risking the direct shot over a lake a golfer prefers to take two shots to get to the green as shown in the diagram on the right. How much further is this two shot route than the direct route?



Shutterstock.com/ameinc 181



The cosine rule

Given that each of the following equations are formed by applying the cosine rule to an acute angled triangle solve for x in each case, giving your answers correct to one decimal place.

17 $x^2 = 7^2 + 8^2 - 2(7)(8) \cos 56^\circ$

18 $x^2 = 3^2 + 2^2 - 2(3)(2) \cos 32^\circ$

19 $3^2 = 5^2 + 7^2 - 2(5)(7) \cos x^\circ$

20 $12^2 = 9^2 + 11^2 - 2(9)(11) \cos x^\circ$



Finding an unknown side

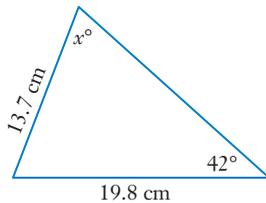


Finding an unknown angle

Miscellaneous

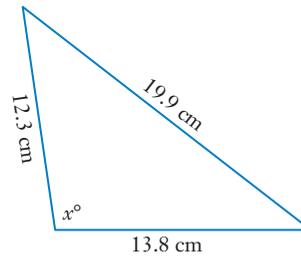
Find the value of x in each of the following.

32

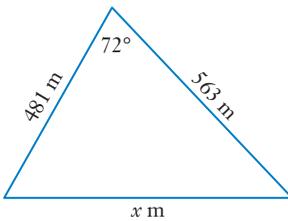


x is between 0 and 90.

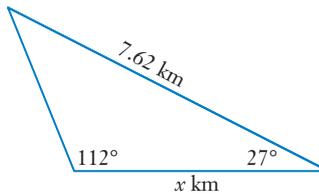
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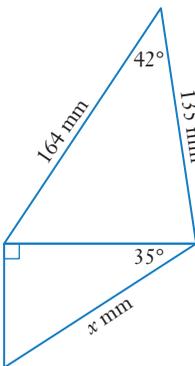
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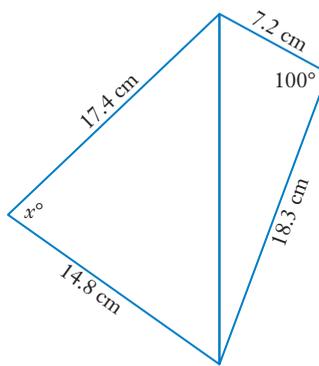
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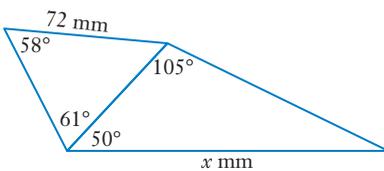
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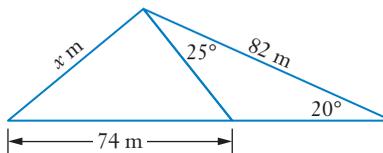
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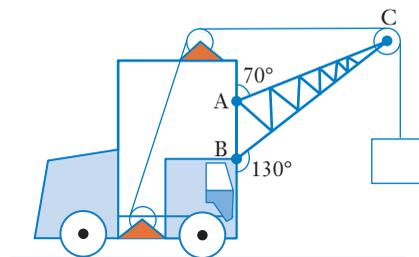
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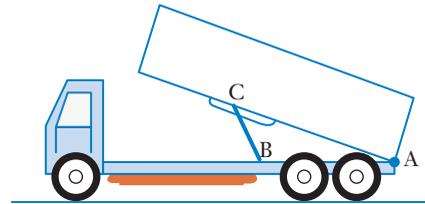


- 40 The diagram on the right shows a mobile crane used to lift containers from ships and transfer them to waiting container trucks. If AB is of length 300 centimetres find the lengths of AC and BC .

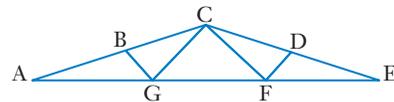


- 41** A triangle has sides of length 12.7 cm, 11.9 cm and 17.8 cm. Find the size of the smallest angle of the triangle, giving your answer to the nearest degree.
- 42** A parallelogram has sides of length 3.7 cm and 6.8 cm and the acute angle between the sides is 48° . Find the lengths of the diagonals of the parallelogram.
- 43** The diagonals AC and BD, of parallelogram ABCD, intersect at E.
If $\angle AED = 63^\circ$ and the diagonals are of length 10.4 cm and 14.8 cm use the fact that the diagonals of a parallelogram bisect each other to determine the lengths of the sides of the parallelogram.

- 44** The tray of the tip truck shown on the right is tipped by the motor driving rod BC clockwise about B. As the tray tips, end C moves along the guide towards A.
If $AB = 2$ metres and $BC = 1$ metre find the size of $\angle CAB$ when AC is
- a** 2.6 metres **b** 2.1 metres.

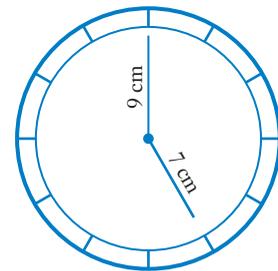


- 45** The 'W-type roof truss' shown on the right is to be constructed with $AE = 900$ cm and $AG = GF = FE$.
 $\angle DEF = 20^\circ$, $ED = DC$ and the truss is symmetrical with the vertical line through C as the line of symmetry.
Calculate the following lengths, correct to the nearest cm



- a** CE **b** ED **c** DF **d** CF

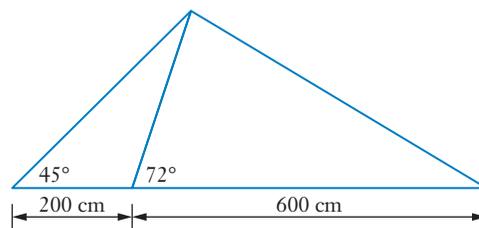
- 46** Find, to the nearest millimetre, the distance between the tip of the 70 mm hour hand and the tip of the 90 mm minute hand of a clock at
- a** 5 o'clock
b 10 minutes past 5.



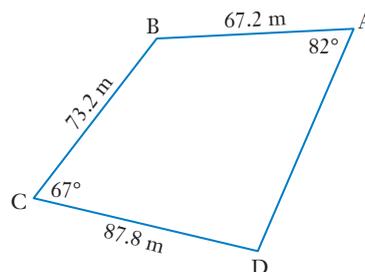
- 47** A coastal observation position is known to be 2.50 km from a lighthouse. The coastguard in the observation position is in radio and visual contact with a ship in distress at sea. If the coastguard looks towards the lighthouse and then towards the ship these two directions make an angle of 40° with each other. If the captain on the ship looks towards the observation position and then towards the lighthouse these two directions make an angle of 115° with each other. (The ship, the lighthouse and the observation position may all be assumed to be on the same horizontal level.)
How far is the ship from
- a** the lighthouse?
b the coastal observation position?
- 48** From a lighthouse, ship P is 7.3 km away on a bearing 070° .
A second ship Q is on a bearing 150° from P and 130° from the lighthouse.
- a** How far is Q from P?
b How far is Q from the lighthouse?

- 49** From a lighthouse, ship A is 15.2 km away on a bearing 030° and ship B is 12.1 km away on a bearing 100° . How far, and on what bearing, is B from A?

- 50** Ignoring any wastage needed for cutting, joining etc. what total length of steel would be needed to make twelve of the steel frameworks shown sketched on the right, rounding your answer up to the next ten metres.

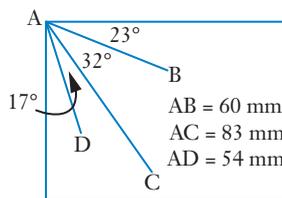
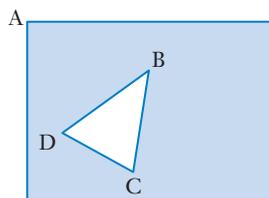


- 51** The diagram on the right shows the sketch made by a surveyor after taking measurements for a block of land ABCD. Find the area and the perimeter of the block.



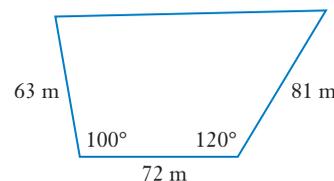
- 52** An engineering component consists of a rectangular metal plate with a triangular piece removed, as in the diagram below left.

The removed piece is cut away by a computer controlled machine that is programmed to cut a triangle with vertices at the distances and angles shown on the diagram below right.



Find the area and the perimeter of the triangular piece that is removed.

- 53** The diagram on the right shows the sketch made by a surveyor after taking measurements for a block of land. Find the area of the block.

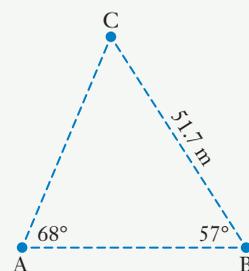


- 54** Make use of the cosine rule, and the rule for the area of a triangle given two sides and the included angle, to determine the area of a triangular block of land with sides of length 63 m, 22 m and 55 m and then check that your answer agrees with the following statement of the rule known as **Heron's rule**.

Area of a triangle with sides of length a , b and c is given by:

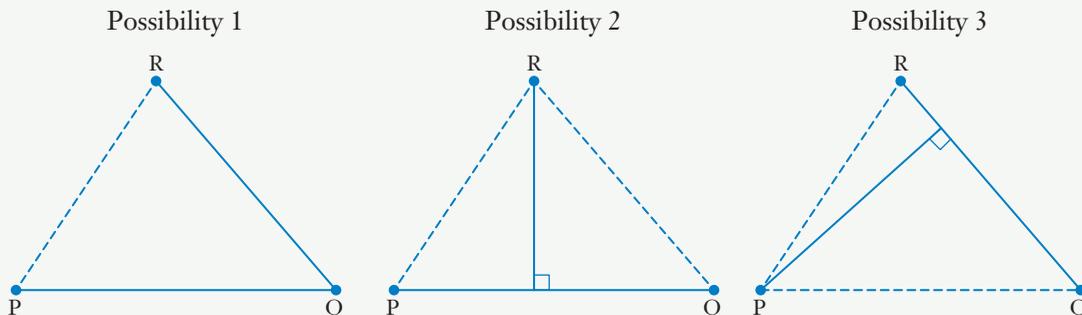
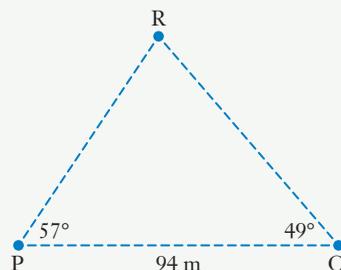
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}.$$

- 12** For the situation shown on the right how much shorter is the direct journey from A to C than the journey from A to C via B.



- 13** Electrical cabling is to be installed to connect three locations, P, Q and R whose relative positions are as shown in the diagram on the right.

Direct connection from P to R is not feasible so three possibilities are considered as shown below:



Find the total length of each of these giving each answer to the nearest metre.

- 14** To provide cover for nurses absent due to sickness and other reasons, a health authority maintains a 'pool' of nurses who are not attached to any particular ward or hospital but who can be directed to any particular area that is suffering a shortage. To assess how many nurses they need 'in pool' they collect information regarding how many nurses are absent on each day for one year. The information is shown in the following table:

No. of nurses absent	0 to 4	5 to 9	10 to 14	15 to 19	20 to 24	25 to 29	30 to 34
No. of days	56	137	97	46	18	9	2

Use the interval midpoints to calculate the mean number of nurses absent per day over this 365 day period and determine the standard deviation of the distribution.

If the authority decides to have n nurses in pool where n is given by:

$$n = \text{the next integer after } (\text{mean} + 0.5 \times \text{standard deviation}),$$

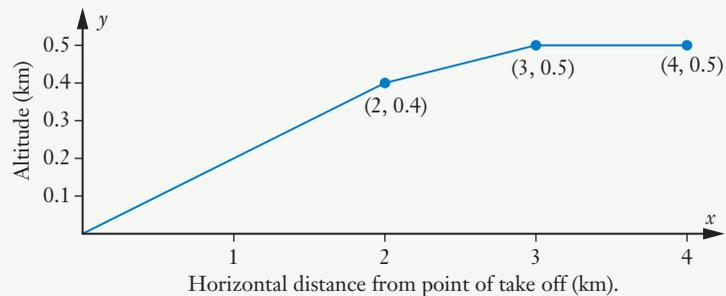
how many do they decide to have in pool?

15 The graphs below show the ‘population pyramids’ of 2 countries, A and B.

AGE DISTRIBUTION COUNTRY A					AGE DISTRIBUTION COUNTRY B				
Age	% of Popl	MALE	FEMALE	% of Popl	% of Popl	MALE	FEMALE	% of Popl	Age
65+	4.1			5.0	2.5			1.8	65+
55-64	4.3			4.6	2.6			2.3	55-64
45-54	5.2			5.2	3.0			3.2	45-54
35-44	7.1			7.0	4.3			5.0	35-44
25-34	8.3			8.3	6.2			6.5	25-34
15-24	8.2			8.3	8.2			8.6	15-24
5-14	8.7			8.6	15.0			14.3	5-14
0-4	3.6			3.5	8.2			8.3	0-4

- In each pyramid the shaded bar at the bottom is smaller than the one above it? Suggest a reason why this might be.
- For which of the age ranges does A have more males than females?
- For which of the age ranges does B have more females than males?
- Country B has a population of 52 000 000. How many people aged 55 and over does the country have?
- Which of the two countries do you think is the more highly developed? Give reasons for your choice.

16 The diagram below shows the approximate path of an aircraft from take off to a point 4 km horizontally from take off drawn as three straight line sections.



Find the equation of each of the three lines giving your answer in the form:

For x from 0 to 2: $y = \dots$

For x from 2 to ...: $y = \dots$

For $x \dots$: $y = \dots$



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12.

Simultaneous linear equations

- Introducing two variables
- Solving word problems
- Miscellaneous exercise twelve

The following example is a repeat of the example on the previous page but the solution given below uses *two* variables and demonstrates the three methods of solution mentioned on the previous page.

EXAMPLE 2

Example 1 revisited

An amateur drama group hire a theatre for their production. They expect to sell all of the 1200 tickets, some at \$10 and the rest at \$7. The group require the ticket sales to cover their \$4150 production costs, to allow a donation of \$4000 to be made to charity and to provide a profit of \$1000 to aid future productions. If they are to exactly achieve this target, and their expectations regarding ticket sales are correct, how many of the total 1200 tickets should they charge \$10 for and how many should they charge \$7 for?

Solution

Let the number of \$7 tickets be

x

and the number of \$10 be

y

They expect to sell 1200 tickets $\therefore x + y = 1200$ [1]

The total revenue must be \$9150 $\therefore 7x + 10y = 9150$ [2]

1 Solve using the equation solving capability of some calculators.

The display on the right shows the two equations put into a calculator and the values

$$x = 950$$

and $y = 250$ displayed.

These are the values that 'fit' both the equation

$$x + y = 1200 \text{ and } 7x + 10y = 9150.$$

$$\text{Check: } 1(950) + 1(250) = 1200 \quad \checkmark$$

$$\text{and } 7(950) + 10(250) = 9150 \quad \checkmark$$

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.

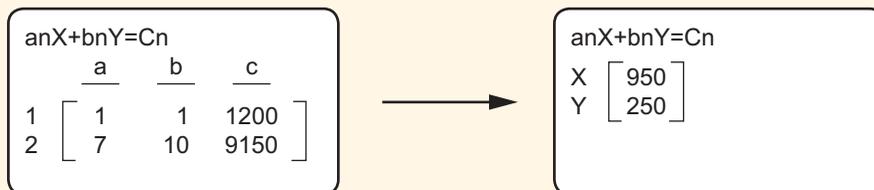
Some calculators require you to input the equations in a different form.

The display below left, for example, shows the equations put into the calculator

by entering: $1 \quad 1 \quad 1200$ for $1x + 1y = 1200$

and $7 \quad 10 \quad 9150$ for $7x + 10y = 9150$

Below right shows the pair of values that satisfy these two equations.



As before, the values that fit both equations are $x = 950$ and $y = 250$.

Note: In the equation $7x + 10y = 9150$, 7 and 10 are the **coefficients** of x and y .

**3 Solve using a graphical approach.**

The equation $x + y = 1200$ has many possible solutions, some of which are shown below:

x	0	1	10	15	33	98	117	900	950	1116
y	1200	1199	1190	1185	1167	1102	1083	300	250	84
$x + y$	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200

The equation $7x + 10y = 9150$ has many possible solutions, some of which are shown below:

x	0	10	20	50	100	210	250	900	950	1120
y	915	908	901	880	845	768	740	285	250	131
$7x + 10y$	9150	9150	9150	9150	9150	9150	9150	9150	9150	9150

Notice that the above tables both include $x = 950$ and $y = 250$, the pair of values that fit both of the equations. Thus $x = 950$ and $y = 250$ are the solutions to the equations. However we could not be sure of being this lucky when we randomly list possible pairs for each of two equations. However we can use this idea to solve the equations graphically.

The graph of the equation $x + y = 1200$ will pass through *all* of the points whose x and y coordinates fit the equation $x + y = 1200$.

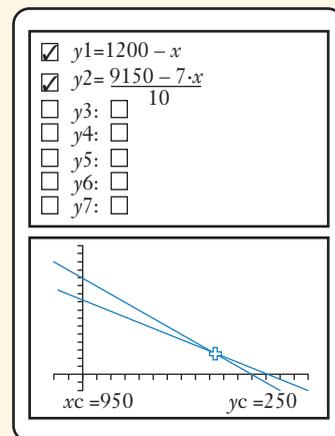
Similarly the graph of $7x + 10y = 9150$ will pass through *all* of the points whose x and y coordinates fit the equation $7x + 10y = 9150$.

Using a graphic calculator to plot both lines, the x and y coordinates of the point where the lines intersect will fit both equations (and of course with both equations being for straight lines there will be only one such point of intersection). The display on the right shows the equations, their graphs and the coordinates of the point of intersection.

$$\text{Thus solving the equations } \begin{cases} x + y = 1200 \\ 7x + 10y = 9150 \end{cases}$$

gives $x = 950$ and $y = 250$.

The group should sell 950 tickets at \$7 each and 250 tickets at \$10 each.



- Some calculators require the equations to be input in the form ' $y =$ ', as shown.

Thus $x + y = 1200$ was entered as $y = 1200 - x$

$$7x + 10y = 9150 \text{ was entered as } y = \frac{9150 - 7x}{10}.$$

- You may need to adjust the viewing window of your calculator so that the point of intersection can be seen (though some calculators will state the coordinates of the point of intersection even if it is off screen).

Question: Which of these methods for solving simultaneous equations should you use?

Answer: Unless your teacher, or a particular question, requires you to use a particular method, use whichever method appeals to you most for that question, and with which you would expect to make least mistakes. **However, do make sure you know which methods you might be required to demonstrate in this unit and practise those.**

The method(s) you are likely to use need practice. The next two examples further demonstrate the technique of using two equations in two unknowns to give one equation in one unknown. This is not to suggest that this should be your chosen method for these questions but rather to further demonstrate its application and allow those readers who intend using other methods to check that they can obtain the same answers 'their way'.

Example 3 shows the substitution approach and **Example 4** the elimination approach.

EXAMPLE 3

a Solve
$$\begin{cases} 3x = 5y - 10 \\ x + y = 34 \end{cases}$$

Solution

a

From [2]:
Substitute into [1]:
Expand:
Add $5x$ to both sides:
i.e.
Divide both sides by 8
But $y = 34 - x$, thus

Hence $x = 20$ and $y = 14$.

b Solve
$$\begin{cases} 0.3A + 0.1P = 161 \\ 5A - 3P = 1050 \end{cases}$$

b

\times [1] by 10 to avoid decimals:
Thus
Substitute into [2]:
Expand:
i.e.
Add 4830 to both sides:
Divide both sides by 14:
But $P = 1610 - 3A$ and so
and so
Thus $A = 420$ and $P = 350$.

$$\begin{aligned} 3x &= 5y - 10 & [1] \\ x + y &= 34 & [2] \\ y &= 34 - x \\ 3x &= 5(34 - x) - 10 \\ 3x &= 170 - 5x - 10 \\ 8x &= 170 - 10 \\ 8x &= 160 \\ x &= 20 \\ y &= 34 - 20 \\ &= 14 \end{aligned}$$

$$\begin{aligned} 0.3A + 0.1P &= 161 & [1] \\ 5A - 3P &= 1050 & [2] \\ \times [1] \text{ by } 10 & \text{ to avoid decimals:} \\ 3A + P &= 1610 \\ \text{Thus} & P = 1610 - 3A \\ \text{Substitute into [2]:} & 5A - 3(1610 - 3A) = 1050 \\ \text{Expand:} & 5A - 4830 + 9A = 1050 \\ \text{i.e.} & 14A - 4830 = 1050 \\ \text{Add } 4830 \text{ to both sides:} & 14A = 5880 \\ \text{Divide both sides by } 14: & A = 420 \\ \text{But } P = 1610 - 3A \text{ and so} & P = 1610 - 3 \times 420 \\ \text{and so} & P = 350 \end{aligned}$$



Solving simultaneous equations



Simultaneous equations order activity

EXAMPLE 4

a Solve $\begin{cases} 3x + 2y = 11 \\ x + 2y = 1 \end{cases}$

b Solve $\begin{cases} 5x - 2y = 6 \\ 3x + 2y = 26 \end{cases}$

c Solve $\begin{cases} 2x + 3y = 12 \\ x + 4y = 11 \end{cases}$

Solution

a Notice that both equations feature '+ 2y'. Taking one equation from the other will take '2y' from itself and thus eliminate one variable.

$$\begin{array}{r} [1] - [2]: \\ \text{i.e.} \\ \text{Thus} \\ \text{Substitute } x = 5 \text{ into } [2]: \\ \text{Take 5 from both sides:} \\ \text{Hence:} \\ \text{Thus } x = 5 \text{ and } y = -2. \end{array} \quad \begin{array}{r} 3x + 2y = 11 \quad [1] \\ \underline{x + 2y = 1} \quad [2] \\ 2x + 0y = 10 \\ 2x = 10 \\ x = 5 \\ 5 + 2y = 1 \\ 2y = -4 \\ y = -2 \end{array}$$

b Notice that one equation features '- 2y' and the other features '+ 2y'. Adding the equations together will allow these to eliminate each other.

$$\begin{array}{r} [1] + [2]: \\ \text{i.e.} \\ \text{Thus} \\ \text{Substitute } x = 4 \text{ into } [2]: \\ \text{Take 12 from both sides:} \\ \text{Hence:} \\ \text{Thus } x = 4 \text{ and } y = 7. \end{array} \quad \begin{array}{r} 5x - 2y = 6 \quad [1] \\ \underline{3x + 2y = 26} \quad [2] \\ 8x + 0y = 32 \\ 8x = 32 \\ x = 4 \\ 12 + 2y = 26 \\ 2y = 14 \\ y = 7 \end{array}$$

c If we leave the first equation unchanged but multiply the second equation by 2 we will have two equations each featuring '2x'. Taking one equation from the other will then eliminate one variable.

$$\begin{array}{r} 2x + 3y = 12 \quad [1] \quad \rightarrow \\ x + 4y = 11 \quad [2] \quad \times 2 \rightarrow \\ \text{Subtracting:} \\ \text{Giving} \\ \text{From } [2]: \\ \text{Thus } x = 3 \text{ and } y = 2. \end{array} \quad \begin{array}{r} 2x + 3y = 12 \\ \underline{2x + 8y = 22} \\ -5y = -10 \\ y = 2 \\ x + 4(2) = 11 \\ x = 3 \end{array}$$

Note: It is *not* the intention here to claim that these questions can only be solved by introducing two letters, building up two equations and solving them simultaneously. Questions like the previous example can be solved by introducing just one variable, as we saw at the start of this chapter.

Alternatively the solution could be ‘reasoned through’ as follows:

Selling all 4000 at \$19 would have raised $4000 \times \$19 = \$76\,000$

However the ticket sales raised \$83 200, i.e. \$7200 ‘extra’. This extra must come from the extra \$9 charged on some tickets.

\$7200 is 800 lots of \$9 so 800 seats were priced at \$28 and 3200 at \$19.

Yet another method would be to *guess* the number of \$28 tickets there should be, *check* whether our guess works and then *improve* our guess, i.e. *guess, check and improve*.

Introducing two letters and solving the resulting pair of equations simultaneously can be very useful but other methods can be just as effective. In general, for each question you should choose the method you consider most appropriate for you to use to solve that question. However do use the next exercise to practise the techniques shown in this chapter.



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Exercise 12A

Use the graph shown on the right to solve the following pairs of equations simultaneously.

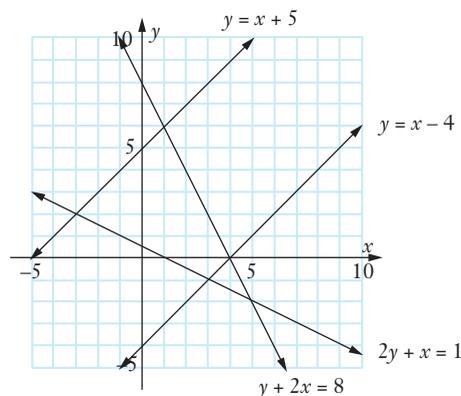
1 $y = x + 5$ and $y + 2x = 8$

2 $y = x - 4$ and $2y + x = 1$

3 $y = x - 4$ and $y + 2x = 8$

4 $y + 2x = 8$ and $2y + x = 1$

5 $y = x + 5$ and $2y + x = 1$



Solve the following pairs of equations.

6
$$\begin{cases} 2x + y = 19 \\ x - y = 2 \end{cases}$$

7
$$\begin{cases} 3x + 2y = 17 \\ x + 2y = 11 \end{cases}$$

8
$$\begin{cases} 5x + 3y = 9 \\ 5x - y = 17 \end{cases}$$

9
$$\begin{cases} 3x + 2y = -1 \\ -x + 2y = 27 \end{cases}$$

10
$$\begin{cases} 3x - y = 16 \\ 2x + y = 29 \end{cases}$$

11
$$\begin{cases} 2x - 3y = 16 \\ x - 3y = 11 \end{cases}$$

12
$$\begin{cases} 3x + 5y = 47 \\ 3x + 2y = 26 \end{cases}$$

13
$$\begin{cases} -x + 7y = 3 \\ x - 3y = 1 \end{cases}$$

14
$$\begin{cases} 2x + 3y = 12 \\ 2x - y = -12 \end{cases}$$

$$15 \begin{cases} y = x - 3 \\ 2x + 3y = 11 \end{cases}$$

$$16 \begin{cases} 2x + y = 7 \\ 3x - 2y = 14 \end{cases}$$

$$17 \begin{cases} 3x - 2y = 6 \\ 2x - 3y = -1 \end{cases}$$

$$18 \begin{cases} y = 11 - 2x \\ 2x + 3y = 21 \end{cases}$$

$$19 \begin{cases} 3x - 5y = 6 \\ x = 2y + 1 \end{cases}$$

$$20 \begin{cases} 3A + 2B = 11 \\ 3A - 2B = 19 \end{cases}$$

$$21 \begin{cases} 2p - 3q = 2 \\ 4p + 2q + 1 = 29 \end{cases}$$

$$22 \begin{cases} 0.5x + 0.2y = 7 \\ 2x - 3y = -29 \end{cases}$$

$$23 \begin{cases} 2(x + 5) = 3y \\ x + 2y = 30 \end{cases}$$

24 One day a baker bakes x white loaves and y wholemeal loaves.

- a** The number of white loaves baked that day together with the number of wholemeal loaves baked that day totalled 600.

Which of the following equations correctly expresses this information?

Equation 1: $x - y = 600$	Equation 2: $y - x = 600$	Equation 3: $x + y = 600$
------------------------------	------------------------------	------------------------------

- b** The number of white loaves baked that day exceeded the number of wholemeal loaves baked that day by 140. Which of the following equations correctly expresses this information?

Equation 4: $x - y = 140$	Equation 5: $y - x = 140$	Equation 6: $x + y = 140$
------------------------------	------------------------------	------------------------------

- c** Given that both of the statements from **a** and **b** are correct, solve your equations from parts **a** and **b** to determine the number of each type of loaf the baker baked that day.

25 At a dog show there were x people (each with two legs) and y dogs (each with four legs).

- a** The total number of legs at the show that were either human legs or dog legs equalled 1758.

Which of the following equations correctly expresses this information?

Equation 1: $2x + 4y = 1758$	Equation 2: $4x + 2y = 1758$	Equation 3: $x + y = 1758$
---------------------------------	---------------------------------	-------------------------------

- b** If the number of dogs at the show is multiplied by 5 and the answer subtracted from the number of people at the show the number obtained is 403.

Which of the following equations correctly expresses this information?

Equation 4: $5x - y = 403$	Equation 5: $x - 5y = 403$	Equation 6: $5y - x = 403$
-------------------------------	-------------------------------	-------------------------------

- c** Given that both of the statements from **a** and **b** are correct solve your equations from parts **a** and **b** to determine the number obtained by adding the number of people at the show to the number of dogs at the show.

26 The smaller of two numbers is x and the larger is y .

a The larger of the two numbers is 5 more than the smaller.

Which of the following equations correctly expresses this information?

Equation 1:

$$x - y = 5$$

Equation 2:

$$y - x = 5$$

Equation 3:

$$x + y = 5$$

b Twice the smaller added to three times the larger equals 70.

Which of the following equations correctly expresses this information?

Equation 4:

$$2x + 3y = 70$$

Equation 5:

$$3x + 2y = 70$$

Equation 6:

$$x + y = 70$$

c Given that both of the statements from **a** and **b** are correct solve your equations from parts **a** and **b** to determine the two numbers.

27 Sally saves \$1 and 50 cent coins by putting them into a piggy bank. When Sally opens the piggy bank she finds that it contains x \$1 coins and y 50 cent coins.



a The piggy bank contained 46 coins altogether, all either \$1 coins or 50 cent coins. Which of the following correctly expresses this information?

Equation 1:

$$x + y = 46$$

Equation 2:

$$xy = 46$$

Equation 3:

$$x + 0.5y = 46$$

b The total value of the coins in the piggy bank came to \$32.

Which of the following correctly expresses this information?

Equation 4:

$$2x + y = 32$$

Equation 5:

$$x + 0.5y = 32$$

Equation 6:

$$0.5x + y = 32$$

c Solve your equations from parts **a** and **b** to determine how many of each type of coin the piggy bank contained.

28 A seamstress buys:

x metres of material A, costing \$28 per metre,
and y metres of material B, costing \$35 per metre.

a The seamstress buys a total of 23 metres of these two materials.

Which of the following correctly expresses this information?

Equation 1:

$$x + y = 28$$

Equation 2:

$$x + y = 35$$

Equation 3:

$$x + y = 23$$

b The two quantities cost the seamstress a total of \$700 altogether.

Which of the following correctly expresses this information?

Equation 4:

$$35x + 28y = 700$$

Equation 5:

$$xy + 28 + 35 = 700$$

Equation 6:

$$28x + 35y = 700$$

c Solve your equations from parts **a** and **b** to determine what length of each material the seamstress bought.



29 An investor invests \$ x in company X and \$ y in company Y.

- a** The investor invests a total of \$25 000 in these two companies.

Which of the following correctly expresses this information?

Equation 1: $X + Y = 25\ 000$	Equation 2: $x + y = \$25\ 000$	Equation 3: $x + y = 25\ 000$
----------------------------------	------------------------------------	----------------------------------

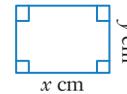
- b** The investment in company X achieves a 4% loss in value and the investment in company Y achieves a 12% increase in value to make the total investment now worth \$25 120.

Which of the following correctly expresses this information?

Equation 4: $-4x + 12y = 25\ 120$	Equation 5: $0.04x + 0.12y = 25\ 120$	Equation 6: $0.96x + 1.12y = 25\ 120$
--------------------------------------	--	--

- c** Solve your equations from parts **a** and **b** to determine the amount the investor put into each company.

30 A rectangle has a length of x cm and a height of y cm.



- a** The perimeter of the rectangle is 70 cm.

Which of the following correctly expresses this information?

Equation 1: $x + y = 35$	Equation 2: $xy = 70$	Equation 3: $x + y = 70$
-----------------------------	--------------------------	-----------------------------

- b** Three heights exceeds two lengths by 15 cm.

Which of the following correctly expresses this information?

Equation 4: $3x - 2y = 15$	Equation 5: $3y - 2x = 15$	Equation 6: $3x + 2y = 15$
-------------------------------	-------------------------------	-------------------------------

- c** Solve your equations from parts **a** and **b** to determine x and y and hence determine the area of the rectangle.

31 Entry into a particular event costs \$ x for each adult and \$ y for each child.

- a** For 16 adults and 7 children the total cost is \$256.

Which of the following equations correctly expresses this information?

Equation 1: $16x + 7y = 265$	Equation 2: $16y + 7x = 256$	Equation 3: $16x + 7y = 256$
---------------------------------	---------------------------------	---------------------------------

- b** For 20 adults and 11 children the total cost is \$338.

Which of the following equations correctly expresses this information?

Equation 4: $20y + 11x = 338$	Equation 5: $20x + 11y = \$338$	Equation 6: $20x + 11y = 338$
----------------------------------	------------------------------------	----------------------------------

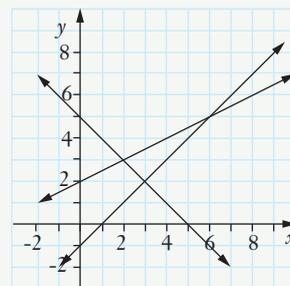
- c** Solve your equations from parts **a** and **b** to determine the total cost of entry for five adults and three children.

32 Two numbers have a sum of 41 whilst three times the larger added to twice the smaller totals 106. By letting the smaller number be x and the larger be y express the given information as two equations and hence determine the two numbers.

- 33** Two numbers have a difference of eleven whilst five times the smaller exceeds twice the larger by seventeen. Find the two numbers.
- 34** A chemist is asked to make 100 mL of a particular medicine. This 100 mL should contain 20 g of a certain compound. The chemist has available two bottles of this medicine but neither is to the desired concentration. The solution in bottle A has 0.15 g per mL and the solution in bottle B has 0.4 g per mL. How many mL should the chemist use from each bottle to make the 100 mL of the desired concentration?
(Hint: Let the required mix consist of x mL from A and y mL from B.)
- 35** A person has \$12 000 to invest in two companies, Acorp and Bcorp. The person invests \$ x with Acorp and \$ y with Bcorp. After one year each \$1 invested with Acorp has grown to \$1.12 and each \$1 invested with Bcorp has grown to \$1.05. The person's \$12 000 has grown to \$13 195.
- Write two equations involving x and y .
 - Solve these equations to determine x and y .
- 36** The total amount received from the sale of 1500 tickets for a play is \$13 800. Some of the tickets were sold for \$12 each and the rest for \$8 each. How many tickets were sold for \$12 and how many for \$8?
- 37** A coach hire company has 25 coaches. Some of these can carry 56 passengers each and the rest 35 passengers each. With all 25 coaches full 1211 passengers can be carried. How many of each size coach does the company have?
- 38** Two numbers are such that seven times the smaller exceeds three times the larger by one whilst twice the larger exceeds four times the smaller by four. Find the two numbers.
- 39** A stall at a school fete sold jars of jam, for \$2.50 per jar, and jars of relish, for \$2 per jar. In all they sold 78 jars of these two commodities, receiving a total of \$179. How many jars of each commodity did they sell?
- 40** To start a new company a person borrows \$120 000 from a bank. Under the terms of the loan the company will pay interest on this loan calculated at 14% per annum on part of the loan and 17% per annum on the remainder and does not have to repay any of the \$120 000 capital until the second year. If the first year interest bill totalled \$18 150 how much of the \$120 000 was borrowed at 14% and how much at 17%?
- 41** A mathematics multiple choice test consisted of 25 questions. Candidates were awarded 4 marks for each correct answer, they lost 3 marks for each incorrect answer but there was no penalty for any questions that were left unattempted. David attempted 23 of the questions and scored 64 marks altogether. How many did he answer correctly?
Suppose instead that whilst 4 marks were still awarded for each correct answer and 3 were lost for each incorrect answer, 3 marks were also lost for each question not attempted. How many questions must be correctly answered for a mark of at least 50?



- 3** Using only the graph shown on the right, determine the solutions to each of the following pairs of simultaneous equations.



a
$$\begin{cases} y = x - 1 \\ y = 0.5x + 2 \end{cases}$$

b
$$\begin{cases} y = x - 1 \\ y = -x + 5 \end{cases}$$

c
$$\begin{cases} y = 0.5x + 2 \\ y = -x + 5 \end{cases}$$

- 4** Solve the following equations.

a $5x - 7 = 11$

b $3(2x - 5) + 6 = 40 - x$

c $\frac{x+3}{2} - 8 = 1$

d
$$\begin{cases} 5x - 3y = 46 \\ x + 2y = 17 \end{cases}$$

e $\frac{x}{7} = \sin 30^\circ$

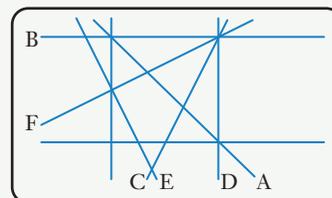
f $\frac{7}{x} = \sin 30^\circ$

- 5** The display on the right shows the lines

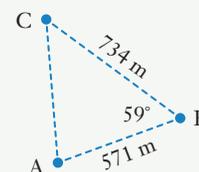
$$\begin{array}{ll} x = 60 & y = 60 \\ y = 2x - 60 & y = 0.5x + 30 \\ y = -x + 60 & y = -2x + 30 \end{array}$$

labelled A to F.

Allocate the correct rule to each line.

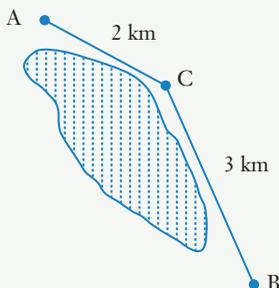


- 6** For the situation shown on the right how much shorter is the direct journey from A to C than the journey from A to C via B.



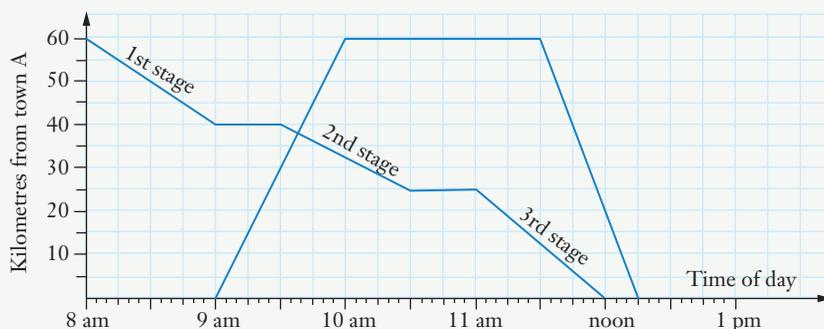
- 7** A particular ‘family’ of straight lines are related by the fact that they all have equations of the form $y = 3x + c$, each member having a different value for c .
What feature do the graphs of all members of this family have in common?
- 8** A particular ‘family’ of straight lines are related by the fact that they all have equations of the form $y = mx - 7$, each member having a different value for m .
What feature do the graphs of all members of this family have in common?
- 9** A particular ‘family’ of straight lines are related by the fact that they all have equations of the form $x + 2y = c$, each member having a different value for c .
What feature do the graphs of all members of this family have in common?

- 10** From Lookout No. 1 a fire is spotted on a bearing 050° . From Lookout No. 2 the fire is seen on a bearing 020° . Lookout No. 2 is 10 km from Lookout No. 1 on a bearing 120° . Assuming that the fire and the two lookouts are all on the same horizontal level find how far the fire is from each lookout.
- 11** The road from A to B consists of two straight sections AC, length 2 km, and CB, length 3 km (see diagram). The bearing of C from A is 108° and the bearing of B from A is 132° .



How much further is the road route from A to B than the straight line distance AB?

- 12** Two towns A and B are 60 km apart and are separated by a long road that can be assumed straight. A cyclist sets off from town B at 8 a.m. one morning and travels to town A in three stages, maintaining an approximately constant speed over each stage and resting for half an hour between stages. A delivery van sets off from town A, travels to town B at an approximately constant speed, stays in B for unloading etc, and then returns to A, again maintaining an approximately constant speed. The distance – time graph of this situation is shown below.

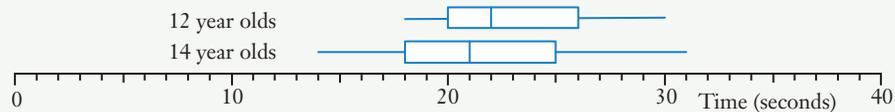


- Without doing any calculations it is possible to tell from the graph on which of the three stages the cyclist maintained the greatest average speed. Explain.
- What was the cyclist's average speed over each of the three stages?
- What was the delivery van's average speed from town A to town B?
- What was the delivery van's average speed from town B to town A?
- Without leaving town B earlier than it did what average speed would the delivery van have needed to travel back to A at if it was to arrive back at town A before the cyclist reached there?

- 13** An investor has \$50 000 to invest for one year. She decides to put some of it in a secure deposit account and the rest in a more risky investment. At the end of the year the deposit account pays interest of 8%, the more risky investment pays 18% and the investor receives a total of \$5600 in interest from these two sources. How much of the \$50 000 had the investor put in the secure deposit account and how much in the more risky investment?
- 14** The investor of question 13 decides to re-invest her money for a second year. She adds the interest to her original \$50 000 so she now has \$55 600 to invest. She still opts to keep all of this money invested with either the secure deposit account or the more risky investment but she changes the balance of her portfolio. At the end of this second year her total interest is \$4360 with the deposit account paying 10% and the more risky investment paying 7%. How much of the \$55 600 had the investor put in the deposit account and how much in the more risky investment?
- 15** Triangle ABC has $a = 5$ cm, $b = 7$ cm and $c = 6$ cm.
- Use the cosine rule to determine the size of $\angle C$.
 - Use $\frac{1}{2} ab \sin C$ to determine the area of $\triangle ABC$.
 - When given the lengths of the sides of a triangle an alternative way of determining the area is to use **Heron's 's' formula**:

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where} \quad s = \frac{a+b+c}{2}.$$
 Calculate the area of $\triangle ABC$ using this formula.

- 16** The times taken for some 12 year old students and some 14 year old students to run a particular distance were noted. Box plots of the times are shown below.



Each of the following statements are either incorrect or their correctness cannot be concluded from the boxplot.

- Write a few sentences about each of the following statements.
 - For the 12 year old results the box plot extends further to the right of the median than to the left. This shows there are more results involved to the right of the median than there are to the left.
 - More 14 year olds were involved than 12 year olds.
 - The interquartile range for the 14 year olds was much bigger than the interquartile range for the 12 year olds.
- Write at least five statements that in some way compare the times of the 12 year olds to those of the 14 year olds, with the first two of your five statements being completed versions of the following:
 - The range of times for the 14 year olds (____ seconds) exceeded the range of times for the 12 year olds (____ seconds).
 - The fastest 25% of the 14 year olds were ____ than the fastest of the 12 year olds.

13.

Standard scores and the normal distribution

- Standard scores
- Normal distribution
- Using a calculator
- In the old days: Using a book of tables
- Notation
- Quantiles
- Miscellaneous exercise thirteen

Situation

Test 1:
27

Kym sits a Mathematics test and achieves a mark of 27.
In the next test she scores 30. Has she improved?

Test 2:
30

*Before answering this question we might first ask:
What was each test out of?*

$\frac{27}{40}$

Suppose that test 1 was out of 40 and test 2 was out of 50. Can we now decide whether she has improved?

$\frac{30}{50}$

Before answering we may want to know if the tests were of similar difficulty. What was the mean mark in each test?

Mean:
23

Suppose the mean in test 1 was 23 and in test 2 was 25. Now can we judge whether her test 2 mark shows an improvement?

Mean:
25

What if we also knew the standard deviation for each test as well?

St dev:
5

Suppose the standard deviation in test 1 was 5 marks and in test 2 was 10 marks

St dev:
10

Now can you suggest whether or not Kym's mark in test 2 was an improvement on her mark in test 1?



Standard scores

In the *Situation* on the previous page, did you consider expressing Kym's test scores in terms of the number of standard deviations each was from the mean? (An idea also encountered in one question of **Miscellaneous exercise three** earlier in this book.)

Expressing a score as a number of standard deviations above or below the mean is called **standardising** the score. We obtain the **standard score**.

$$\text{Standardised score} = \frac{\text{Raw score} - \text{mean}}{\text{standard deviation}}$$

EXAMPLE 1

Jennifer scores 23, 35 and 17 in tests A, B and C respectively. If the mean and standard deviation in each of these tests are as given below express each of Jennifer's test scores as standardised scores.

Test A:	mean	30	standard deviation	5
Test B:	mean	32	standard deviation	6
Test C:	mean	15	standard deviation	2.5

Solution

In Test A, Jennifer's standardised score is $\frac{23-30}{5}$ i.e. -1.4 .

In Test B, Jennifer's standardised score is $\frac{35-32}{6}$ i.e. 0.5 .

In Test C, Jennifer's standardised score is $\frac{17-15}{2.5}$ i.e. 0.8 .

Exercise 13A

- Express each of the following as a standard score.
 - A score of 65 in a test that had a mean of 60 and a standard deviation of 5.
 - A score of 72 in a test that had a mean of 55 and a standard deviation of 10.
 - A score of 50 in a test that had a mean of 58 and a standard deviation of 4.
 - A score of 60 in a test that had a mean of 58 and a standard deviation of 4.
 - A score of 58 in a test that had a mean of 64 and a standard deviation of 8.
- SuMin scores 30, 50, 7 and 26 in tests A, B, C and D respectively. If the mean and standard deviation in each of these tests are as given below express each of SuMin's test scores as standardised scores.

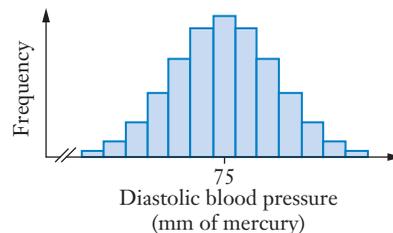
Test A:	mean	20	standard deviation	4
Test B:	mean	60	standard deviation	10
Test C:	mean	6	standard deviation	0.8
Test D:	mean	25	standard deviation	5



The normal curve

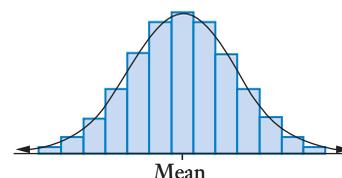
Normal distribution

Suppose the diastolic blood pressure of a large number of adults was measured and the mean value was found to be 75 mm of mercury (mm of mercury being the units blood pressure is measured in). The data collected, if presented as a histogram, could well have a shape similar to the diagram shown on the right, i.e. a symmetrical distribution with many values close to the mean and the number of values decreasing as we move further from the mean.



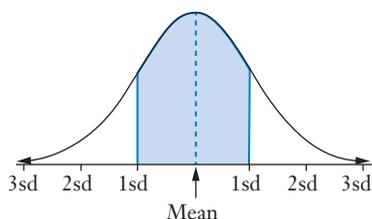
Fitting a smooth curve to the midpoints of the columns we obtain a 'bell shaped curve' as shown on the right.

If we make many measurements of something that occurs naturally, for example the heights of many adult females, the weights of many domestic cats, the foot lengths of many adult males, etc., the histogram of the data often follows this sort of shape.

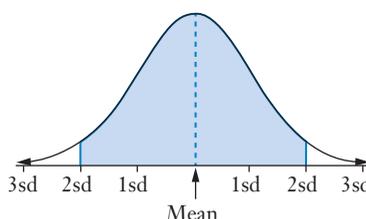


Data of this kind is said to be **normally distributed**. In **normal distributions**, approximately two thirds of the population lie within one standard deviation of the mean, 95% would lie within two standard deviations of the mean and almost all would lie within three standard deviations of the mean.

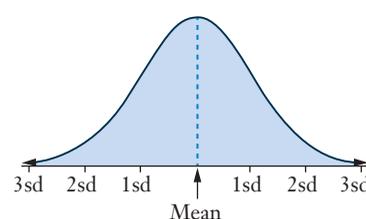
This is the 68%, 95%, 99.7% rule.



68.3% of area
under curve shaded



95.4% of area
under curve shaded



99.7% of area
under curve shaded

In terms of probabilities, we could say that the probability of a randomly selected individual from a normally distributed population being within

- one standard deviation of the mean is 0.683,
- two standard deviations of the mean is 0.954,
- three standard deviations of the mean is 0.997.

Note

The normal distribution is also referred to as the Gaussian distribution, after the German mathematician Carl Gauss.

Using a calculator

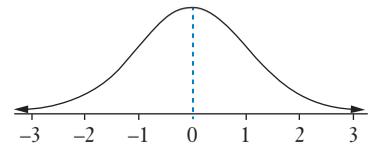
The various calculators have different capabilities and routines with regard to displaying probabilities for normally distributed sets of data.

You will gain familiarity with the ability of *your* calculator in this regard in the next exercise.

In the old days: Using a book of tables

Prior to the ready availability of calculators with built in statistical routines for determining probabilities associated with normal distributions, these probabilities were determined using books of statistical tables.

These books give probabilities for just one normal distribution, the **standard normal distribution**. For this the random variable has a mean of 0 and a standard deviation of 1, as shown on the right.



Normal distributions having means and standard deviations not equal to these standard values needed to be standardised.

We encountered this idea of standardising data by expressing it as a number of standard deviations above or below the mean at the beginning of this chapter. Calling the original score an 'x score' and the standardised score a 'z score' we have:

$$z \text{ score} = \frac{x \text{ score} - \text{mean of } x \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

Thus before the ready availability of sophisticated calculators, to answer the previous example which required us to determine the probability that from a normally distributed set of data, X , with mean 512 and standard deviation 8, a randomly selected item would have a value less than 500 we would have changed the 500 to a standard score:

$$\begin{aligned} \text{standard score} &= \frac{500 - 512}{8} \\ &= -1.5 \end{aligned}$$

(i.e. a score of 500 is 1.5 standard deviations below the mean)

and then used the table of probabilities for the standard normal distribution to determine the required probability.

z	0.00	0.01	0.02	0.03
-1.9	0.0287	0.0281	0.0274	0.0268
-1.8	0.0359	0.0351	0.0344	0.0336
-1.7	0.0446	0.0436	0.0427	0.0418
-1.6	0.0548	0.0537	0.0526	0.0516
-1.5	0.0668	0.0655	0.0643	0.0630
-1.4	0.0808	0.0793	0.0778	0.0764
-1.3	0.0968	0.0951	0.0934	0.0918

$$\begin{aligned} P(X < 500) &= P(Z < -1.5) \\ &= 0.0668 \end{aligned}$$

Thus, as before, the probability that a randomly chosen box of the cereal contains less than 500 grams is 0.0668.

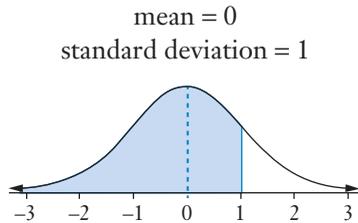
Exercise 13B

Get to know the capabilities of your calculator with regard to normal probability distributions.

The questions of this exercise refer to data sets involving normally distributed scores, X .

Using your calculator make sure that you can obtain each of the probabilities given in questions 1 to 8 below (correct to 4 decimal places), and each value of k in questions 9 to 17.

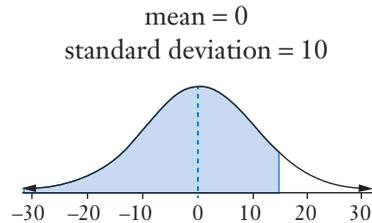
1



$$P(X < 1) = 0.8413$$

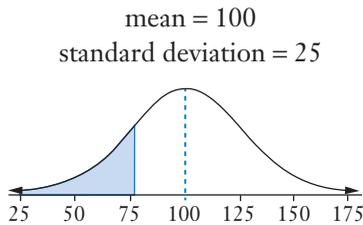
Can you also get 0.84 using the 68%, 95%, 99.7% rule?

2



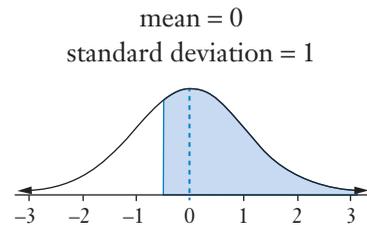
$$P(X < 15) = 0.9332$$

3



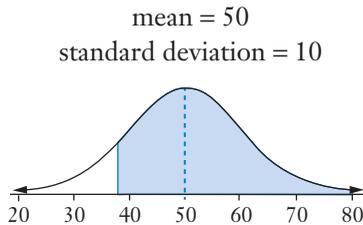
$$P(X < 78) = 0.1894$$

4



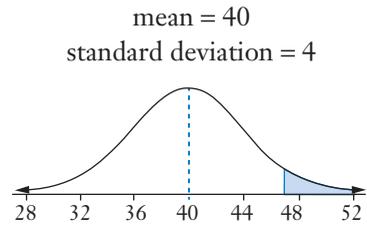
$$P(X > -0.5) = 0.6915$$

5



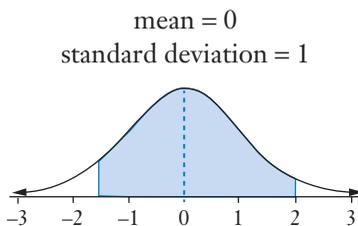
$$P(X > 38) = 0.8849$$

6



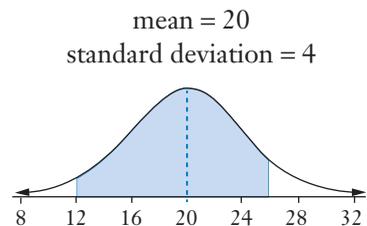
$$P(X > 47) = 0.0401$$

7



$$P(-1.5 < X < 2) = 0.9104$$

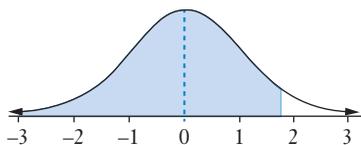
8



$$P(12 < X < 26) = 0.9104$$

9

mean = 0
standard deviation = 1

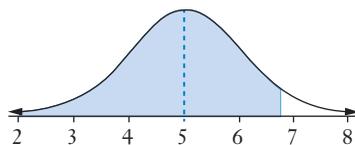


$$P(X < k) = 0.9573$$

$$\therefore k = 1.72 \text{ (2 dp)}$$

10

mean = 5
standard deviation = 1

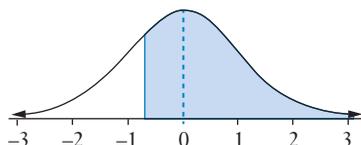


$$P(X < k) = 0.9671$$

$$\therefore k = 6.84 \text{ (2 dp)}$$

11

mean = 0
standard deviation = 1

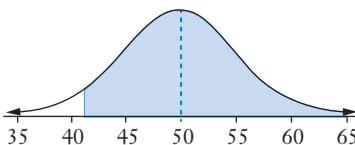


$$P(X > k) = 0.7517$$

$$\therefore k = -0.68 \text{ (2 dp)}$$

12

mean = 50
standard deviation = 5

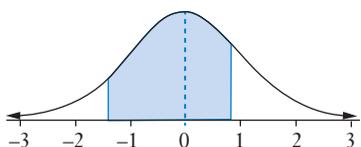


$$P(X > k) = 0.9656$$

$$\therefore k = 40.9 \text{ (1 dp)}$$

13

mean = 0
standard deviation = 1

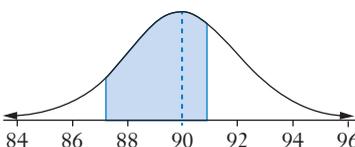


$$P(-1.4 < X < k) = 0.7215$$

$$\therefore k = 0.85 \text{ (2 dp)}$$

14

mean = 90
standard deviation = 2



$$P(87.2 < X < k) = 0.5964$$

$$\therefore k = 90.92 \text{ (2 dp)}$$

15

mean = 40,
standard deviation = 15.
 $P(X < k) = 0.9850$

$$\therefore k = 72.55 \text{ (2 dp)}$$

16

mean = 10,
standard deviation = 0.5.
 $P(X > k) = 0.0721$

$$\therefore k = 10.73 \text{ (2 dp)}$$

17

mean = 0.1,
standard deviation = 0.01.
 $P(0.08 < X < k) = 0.3036$

$$\therefore k = 0.0955 \text{ (4 dp)}$$

Notation

If we use X to represent the possible values of a normally distributed set of measurements having a mean μ and standard deviation σ (and hence variance σ^2) this is sometimes written:

$$X \sim N(\mu, \sigma^2).$$

' μ ' is a Greek letter, mu, (pronounced myew) and σ is sigma so this is read as:

X is normally distributed with mean myew and standard deviation sigma.

EXAMPLE 4

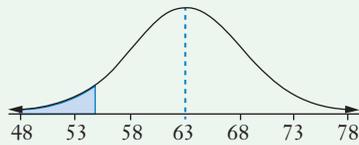
If $X \sim N(63, 25)$ determine $P(X < 55)$.

Solution

X is normally distributed with a mean of 63 and a standard deviation of 5.

Using a calculator:

$$P(X < 55) = 0.0548$$



Using a tables book:

$$\begin{aligned} P(X < 55) &= P(Z < -1.6) \\ &= 0.0548 \end{aligned}$$

(Shown for interest only.)

EXAMPLE 5

Eight thousand two hundred and forty students were given an IQ test. The scores were normally distributed with a mean of 100 and a standard deviation of 16.

- Determine how many of the students, to the nearest ten, achieved a score in excess of 128.
- What were the minimum and maximum scores of the middle 60% of students on this test?

Solution

- Let X be the scores obtained in the test.
Thus $X \sim N(100, 16^2)$.

We require $P(X > 128)$.

Using a calculator, $P(X > 128) = 0.0401$.

Number scoring more than 128:

$$0.0401 \times 8240 \approx 330$$

Approximately 330 students achieved a score in excess of 128.

- If p is the lowest score achieved by the middle 60% then

$$P(X < p) = 0.2$$

$$\text{i.e. } p = 86.53$$

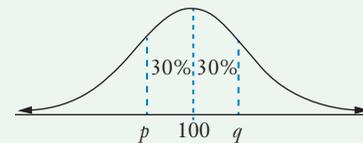
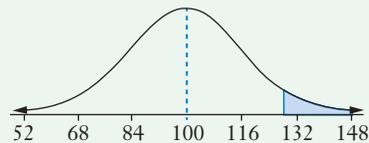
and if q is the highest score achieved by the middle 60% then

$$P(X < q) = 0.8$$

$$\text{i.e. } q = 113.47$$

(Some calculators can determine p and q more directly for this symmetrical situation.)

The lowest and highest scores achieved by the middle 60% are 86.5 and 113.5 respectively (to the nearest half mark).



EXAMPLE 6

If $X \sim N(40, 10^2)$ determine each of the following probabilities using the 68%, 95%, 99.7% rule, and not the statistical capability of your calculator.

- a** $P(30 < X < 50)$ **b** $P(20 < X < 60)$ **c** $P(40 < X < 60)$ **d** $P(X \leq 50)$

Solution

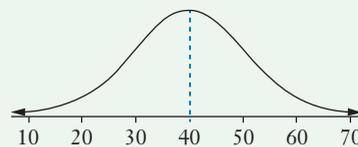
- a** 30 is one standard deviation below the mean and
50 is one standard deviation above the mean.

$$\text{Thus } P(30 < X < 50) = 0.68$$

- b** $P(20 < X < 60) = 0.95$

- c**
$$P(40 < X < 60) = \frac{0.95}{2}$$
$$= 0.48 \text{ (correct to 2 decimal places)}$$

- d**
$$P(X \leq 50) = 0.5 + \frac{0.68}{2}$$
$$= 0.84$$



Note that we make no distinction between $P(X \leq 50)$ and $P(X < 50)$. Including the line or not makes no difference to the area of the region.

EXAMPLE 7

Let us suppose that the time from Simon getting out of bed until his arrival at school is normally distributed with a mean of 55 minutes and a standard deviation of 5 minutes. Simon's arrival at school is classified as being late if it occurs after 9.10 am.

- a** One day Simon gets out of bed at 8.08 am. What is the probability of him arriving late?
b For a period of time Simon always gets out of bed at the same time but finds that he arrives late approximately 85% of the time! What time is he getting out of bed (to the nearest minute)?

Solution

- a** Let T minutes be the time from getting out of bed until arrival at school.

$$\text{Thus } T \sim N(55, 5^2).$$

Simon has 62 minutes to get to school before he is late.

We require: $P(T > 62)$

Calculator gives: $P(T > 62) = 0.0808$.

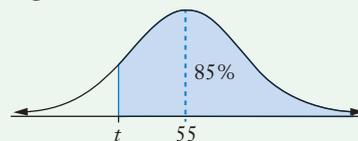
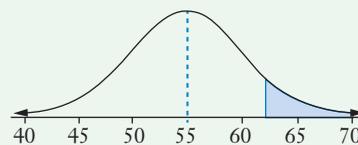
If Simon gets out of bed at 8.08 am the probability of him arriving late is 0.0808.

- b** The time that Simon is allowing himself to get to school is causing him to be late approximately 85% of the time.

We require t for which $P(T > t) = 0.85$.

Calculator gives: $t \approx 49.8$

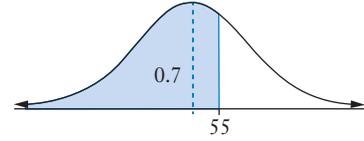
Thus Simon is allowing approximately 50 minutes to get to school and for 85% of the days the journey takes longer than this, causing him to be late 85% of the time. Simon is getting out of bed at 8.20 am.



Quantiles

Quantiles are the values which a particular proportion of the distribution falls below.

Thus if 0.7 (70%) of the distribution is below 55, then 55 is the 0.7 quantile.



Alternatively we can refer to 55 as being the 70th **percentile**.

Note

- We are already accustomed to referring to the 0.25 quantile as the first, or lower, **quartile** and the 0.75 quantile as the third, or upper, quartile.
- If the quartiles divide a distribution into four equal parts and the percentiles divide the distribution into 100 equal parts what might deciles and quintiles do?

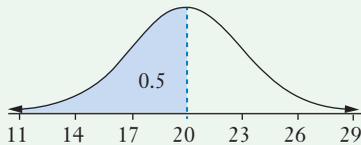
EXAMPLE 8

If $X \sim N(20, 3^2)$ determine

- a** the 0.5 quantile
- c** the 24th percentile
- b** the 0.82 quantile
- d** the 62nd percentile

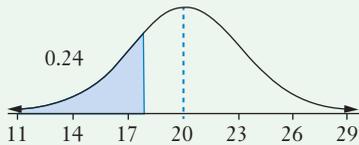
Solution

a



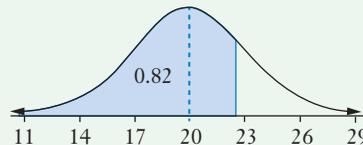
By inspection:
The 0.5 quantile is 20.

c



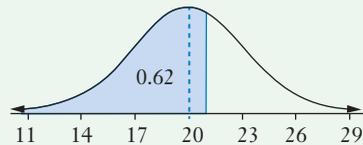
Using a calculator:
The 24th percentile is 17.9.

b



Using a calculator:
The 0.82 quantile is 22.7.

d



Using a calculator:
The 62nd percentile is 20.9.

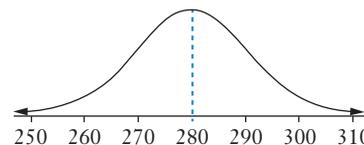
Exercise 13C

- 1 The random variable, X , is normally distributed with a mean of 12 and a standard deviation of 2, i.e. $X \sim N(12, 2^2)$. Determine $P(X \geq 13.5)$.
- 2 The random variable, X , is normally distributed with a mean of 240 and a variance of 400, i.e. $X \sim N(240, 20^2)$. Determine $P(218 < X < 255)$.

- 3** $X \sim N(62, 64)$, i.e. X , is normally distributed with a mean of 62 and a standard deviation of 8. Given that $P(X > k) = 0.8238$ determine k .
- 4** If $X \sim N(0, 1)$ determine
- | | |
|-------------------------------|-------------------------------|
| a the 0.72 quantile, | b the 0.26 quantile, |
| c the 89th percentile, | d the 23rd percentile. |
- 5** If $X \sim N(20, 3^2)$ determine
- | | |
|-------------------------------|-------------------------------|
| a the 0.44 quantile, | b the 0.74 quantile, |
| c the 33rd percentile, | d the 85th percentile. |
- 6** Using the 68%, 95%, 99.7% rule, and *not* the statistical capability of your calculator, determine the following probabilities.
- | | |
|---|---|
| a $P(-1 < X < 1), X \sim N(0, 1^2)$. | b $P(-2 < X < 2), X \sim N(0, 1^2)$. |
| c $P(-3 < X < 3), X \sim N(0, 1^2)$. | d $P(8 < X < 32), X \sim N(20, 6^2)$. |
| e $P(4 < X < 16), X \sim N(10, 2^2)$. | f $P(0 < X < 1), X \sim N(0, 1^2)$. |
| g $P(X < 1), X \sim N(0, 1^2)$. | h $P(X > 1), X \sim N(0, 1^2)$. |
| i $P(X < 5), X \sim N(0, 5^2)$. | j $P(X > 70), X \sim N(60, 10^2)$. |

- 7** Let us suppose that the duration of pregnancy, for a naturally delivered human baby, is a normally distributed variable with a mean of 280 days and a standard deviation of 10 days.

Using the 68%, 95%, 99.7% rule, and *not* the statistical capability of your calculator, determine estimates for the percentage of human pregnancies, for naturally delivered babies, that:



- | |
|---|
| a are between 250 days and 310 days, |
| b exceed 290 days, |
| c are between 260 days and 270 days. |
- 8** A machine produces components whose weights are normally distributed with a mean of 500 g and standard deviation of 5 g. According to the 68%, 95%, 99.7% rule, what percentage of the components will have:
- | |
|---------------------------------------|
| a a weight of less than 495 g? |
| b a weight of less than 490 g? |
- 9** A box of breakfast cereal has 'contains 300 grams of breakfast cereal' printed on it. Suppose that in fact the weight of breakfast cereal contained in these boxes is normally distributed with a mean of 310 grams and a standard deviation of 4 grams. Determine the probability that a randomly chosen box of this cereal contains:
- | |
|---|
| a more than 312 grams of breakfast cereal, |
| b less than 300 grams of breakfast cereal. |

- 16** The marks achieved in a particular exam are normally distributed with a mean of 64 and a standard deviation of 12.

Grades are to be awarded as follows:

- Top 12% of candidates: Grade A
Next 25% of candidates: Grade B
Next 40% of candidates: Grade C
Next 15% of candidates: Grade D
Remainder of candidates: Grade F

Determine the marks that form the A/B, B/C, C/D, and D/F grade boundaries, giving your answers correct to the nearest whole number.

- 17** Let us suppose that the time, in minutes, from Monica leaving home until she arrives at work is a normally distributed random variable with a mean of 45 and a standard deviation of 5. Monica's arrival at work is classified as late if it occurs after 8.30 am.
- a** One day Monica leaves home at 7.40 am. What is the probability of her arriving late?
 - b** For a period of time Monica leaves home at the same time each day. During this period she finds that she arrives late approximately 8% of the time. What time is she leaving home (to the nearest minute)?
 - c** What is the latest time (involving whole minutes) that Monica should leave home each day if she wishes to cut her late arrivals to less than 1%?

- 18** The annual rainfall in an area in the south west of Western Australia is normally distributed with a mean of 1200 mm and a standard deviation of 200 mm.

According to this model, and assuming the situation does not change, in every one hundred years how many years would you expect the annual rainfall to be

- a** less than 800 mm?
- b** more than 1500 mm?
- c** between 800 mm and 1500 mm?

(Challenge.) Given that a year has an annual rainfall of more than 1300 mm what is the probability that the rainfall for the year is less than 1500 mm?

- 19** The weight of each apple harvested from a particular orchard determines where the apple will be sent:

- If weight of apple \geq 250 g send to premium outlet
150 g < weight of apple < 250 g send to normal market
weight of apple \leq 150 g send for juicing.

The weights of the apples are normally distributed with mean 180 g and standard deviation 40 g.

- a** In a random sample of 1000 apples how many would you expect to go to the premium outlet?
- b** (Challenge.) Given that an apple does not go to the premium outlet what is the probability that it is sent for juicing?



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Miscellaneous exercise thirteen

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the Preliminary work section at the beginning of the book.

- 1** A particular straight line with a gradient of m and cutting the y -axis at the point with coordinates $(0, c)$ has equation $y = mx + c$.

- a** The line passes through the point $(3, 4)$.

Which of the following equations must be true?

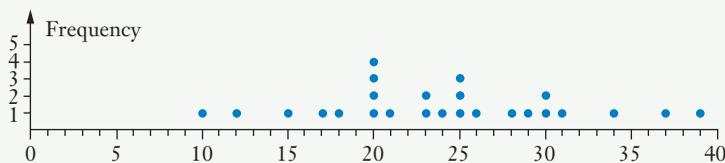
Equation 1: $y = 3x + 4$	Equation 2: $3 = 4m + c$	Equation 3: $4 = 3m + c$
-----------------------------	-----------------------------	-----------------------------

- b** The line also passes through the point $(8, 19)$.

Which of the following equations must be true?

Equation 4: $8 = 19m + c$	Equation 5: $19 = 8m + c$	Equation 6: $y = 8x + 19$
------------------------------	------------------------------	------------------------------

- c** Solve your equations from parts **a** and **b** to determine the equation of the straight line.
- 2** Find the mean, median, mode and range for the set of scores shown in the dot frequency graph.



- 3** One hot dog and three burgers cost \$13.30. Four hot dogs and four burgers cost \$25.20. What would be the cost of four hot dogs and six burgers? (Assume that the individual cost of a hot dog and of a burger remain unchanged throughout.)
- 4** A rectangle is of length x cm and width y cm.
Four times the width exceeds one length by 4 cm.
The perimeter of the rectangle is 42 cm.
Find the values of x and y and hence determine the area of the rectangle.
- 5** In support of a primary school fete a parent makes lots of high quality home made chocolates and lollies. The organisers decide to sell bags containing 20 chocolates for \$6.00 and bags containing 20 lollies for \$4.00. They also wish to sell a mixed bag of 20 chocolates and lollies for \$4.80. How many chocolates and how many lollies should each of these bags hold for the price of \$4.80 to be consistent with the prices of the other bags?

- 6** A road construction company charges 125 million dollars for constructing a 20 km stretch of highway and 245 million dollars for constructing 40 km of similar highway. Based on these costs and assuming a linear relationship exists between the total cost and the length of road constructed, determine the cost of constructing

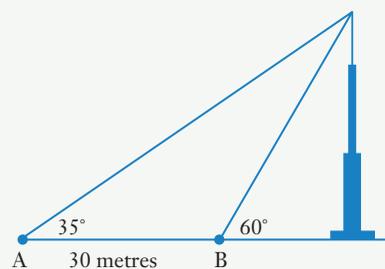


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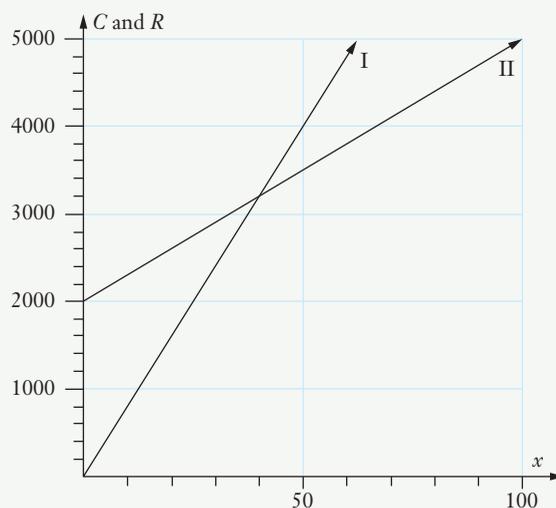
- a** 25 km of similar highway
b 52 km of similar highway.

- 7 a** Three consecutive integers have a sum of 504. Find the integers.
b Three consecutive even integers have a sum of 504. Find the integers.
- 8** Jack walks 2.4 km on a bearing 060° followed by 4.4 km on a bearing 190° . On what bearing and for what distance should he now walk to return directly to his starting point?

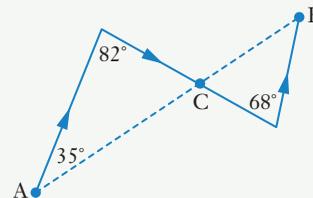
- 9** From a point A, level with the base of a monument, the angle of elevation of the topmost point of the monument is 35° . From point B, also at ground level but 30 metres closer to the monument, the same point has an angle of elevation of 60° . Find how high the topmost point is above ground level. (Give your answer correct to the nearest metre.)



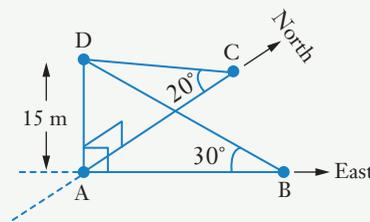
- 10** When a company sells x units of a particular product the revenue raised, $\$R$, is given by one of the lines on the graph shown and the other line shows $\$C$, the cost of producing these x units.
- a** Which of the two lines, I or II, is likely to be the revenue line and which the cost line? (Explain your answer.)
b What does the graph suggest is the value of x for 'break even'? (i.e. revenue raised = cost of production.)
c Suggest equations for each of the two lines shown.



- 11** A yacht travels from location A to location B by tacking, as shown in the diagram. The direct distance from A to B is 580 metres. The path the yacht takes causes it to cross the direct line from A to B at a point C where the distance from A to C is 65% of the distance from A to B. How far did the yacht travel altogether in its journey from A to B?



- 16** Points A, B and C all lie on horizontal ground. A vertical tower DA has its base at A and is of height 15 metres. C lies due North of A and B is due East of A. The angle of elevation of D is 20° from C and 30° from B (see diagram).



- Calculate
- how far C is from A
 - how far B is from A
 - how far C is from B
 - the bearing of C from B.

- 17** The tank in a water irrigation system holds 80 000 litres of water.

The tank is initially three quarters full and each day, from 6am to noon, water flows from the tank at the rate of 1000 litres per hour.

This occurs for 6 days with no water entering the tank.

On day 7 rain is forecast so the system is switched off for days 7 and 8. This rain means that not only is no water taken off during these two days but instead, on day 7, from 6 am to 6 pm, 24000 litres flows in. Unfortunately no water flows in on day 8.

The usual 6 am to noon daily outflow then recommences for days 9, 10, 11 and 12 and during these four days no more rain falls.

Sketch a piecewise graph showing the amount of water in the tank for these 12 days.

- 18** The 5347 candidates who sat a national mathematics test scored marks that had a mean of 127 and a standard deviation of 17.

- Distinction certificates are to be awarded to the top 15% of students.
- Participation certificates are to be awarded to students scoring less than 115. Candidates scoring between the above categories receive other certificates.
- The top 1% of students were awarded prizes.

Modelling the results as a normal distribution with mean 127 and standard deviation 17, determine each of the following:

- The lowest mark, rounded to the next whole mark *down*, that would achieve a distinction certificate.
- The number of students, to the nearest 10 students, who would receive a participation certificate.
- The lowest mark, rounded to the next whole number *up*, that would achieve a prize.
- The lowest and highest marks, each rounded to one decimal place, achieved by the middle 20% of students.

- 19** (Challenge) A river runs East to West. A tree stands on the edge of one bank and from a point C, on the opposite bank and due South of the tree, the angle of elevation of the top of the tree is 28° . From point D, 65 m due East of C the angle of elevation of the top of the tree is 20° .

- Calculate
- the height of the tree,
 - the width of the river at C.

ANSWERS

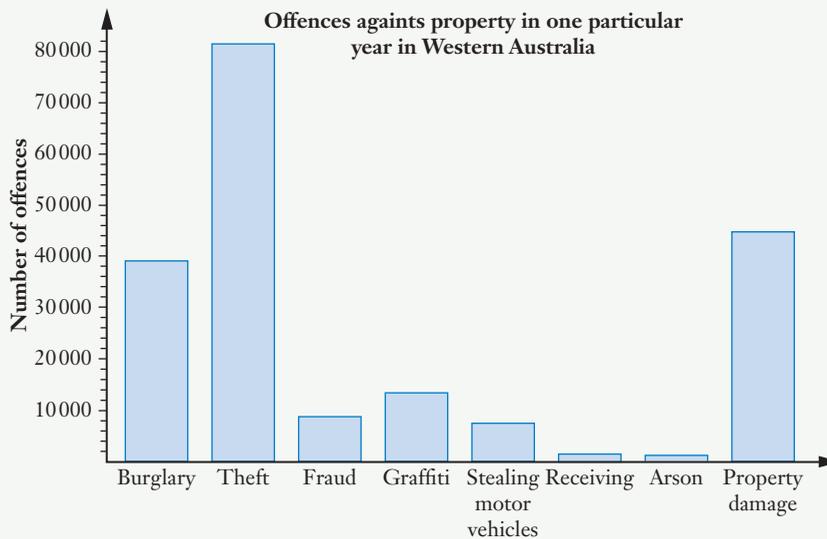
Note:

- For questions that do not stipulate a specific level of rounding the answers given here have been rounded to a level considered appropriate for the question.
- If a question asks for an answer to be given 'to the nearest centimetre' it does not necessarily have to be given 'in centimetres'. In such a situation an answer of 174.9256 metres could be written as 174.93 m or as 17493 cm, both answers being to the nearest centimetre.

Exercise 1A PAGE 7

- | | |
|-------------------|-------------------|
| 1 Nominal | 2 Nominal |
| 3 Ordinal | 4 Ordinal |
| 5 Nominal | 6 Ordinal |
| 7 Ordinal | 8 Nominal |
| 9 Nominal | 10 Ordinal |
| 11 Nominal | 12 Nominal |

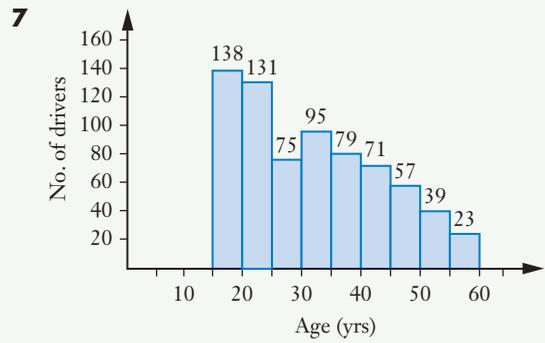
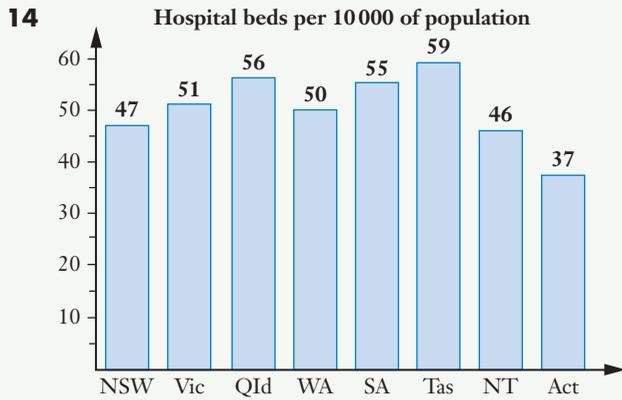
13



The advantage of the pie chart is that it shows how the total number of offences are divided up – i.e. the numbers of each type of offence as a proportion of the whole is shown. Whilst the pie chart also allows the numbers of each offence to be compared, if two categories are close in number it could be difficult to determine which has the greater number.

The bar chart on the other hand allows for very good comparison between categories but the proportion that each category is of the whole is not so evident.

Both the pie chart and the bar graph shown fail to show the accurate numbers given in the question but do allow overall comparisons to be made. However this weakness could be overcome if it was felt to be significant by including the accurate figures, with each category title in the pie graph, or at the top of each bar in the bar graph.

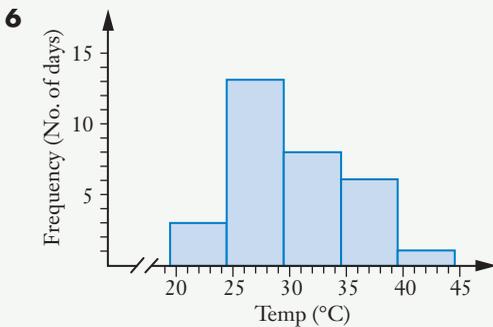
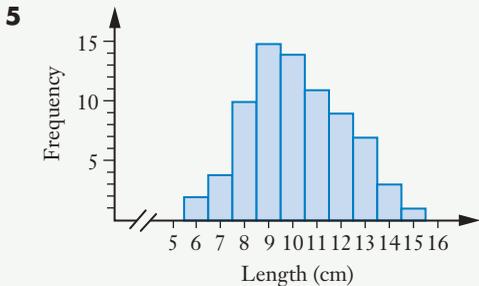


Exercise 1B PAGE 8

- | | |
|---------------------|----------------------|
| 1 Discrete | 7 Discrete |
| 2 Continuous | 8 Continuous |
| 3 Discrete | 9 Discrete |
| 4 Continuous | 10 Continuous |
| 5 Continuous | 11 Continuous |
| 6 Continuous | 12 Continuous |

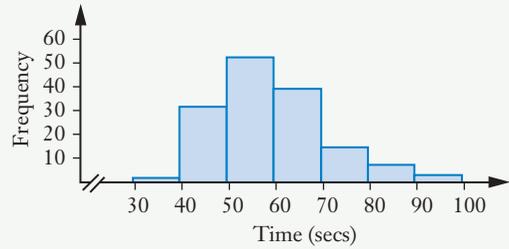
Exercise 1C PAGE 13

- 1, 2, 3** Compare your choice with those of others in your class and discuss with your teacher.
- 4** Compare and discuss your sketches with those of others in your class. Discuss the reasonableness of your sketches and their sketches.

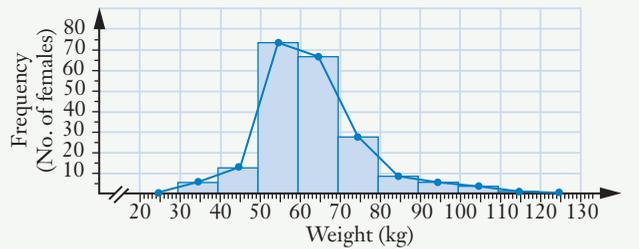
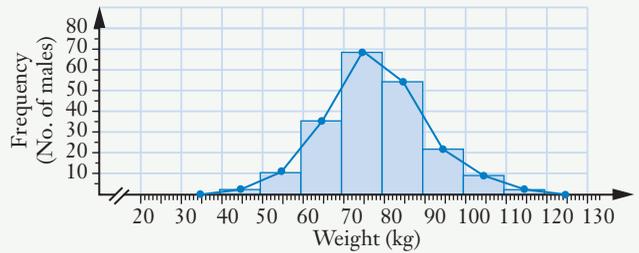


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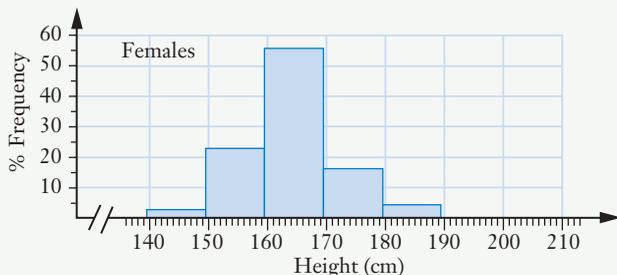
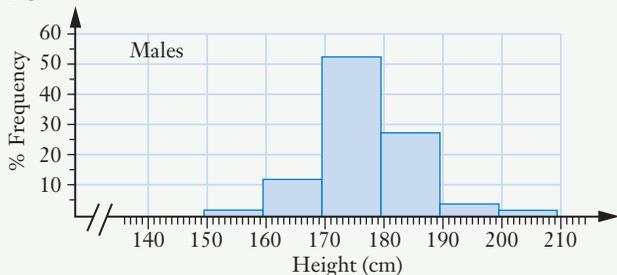
Time (secs)	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	32	53	39	14	7	3



9



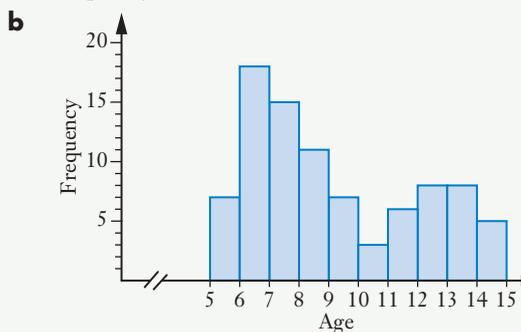
10



Miscellaneous exercise one PAGE 17

- 1 a False b False c False
 d False e False f False
- 2 a Nominal categorical
 b Continuous numerical
 c Discrete numerical
 d Nominal categorical
 e Ordinal categorical
 f Nominal categorical
 g Continuous numerical
 h Discrete numerical
- 3 a 130 b 2.1 c 29.3
 4 a 22 b 21 c 13.5
 5 a 3 b 13 and 17 c 22
- 6 7.5 litres of water should be added to 25 millilitres of concentrate.
- 7 a mean 37.6, median 38, mode 40, range 7
 b mean 102, median 107, mode 131, range 63
 c mean 17.2, median 17.5, mode 18, range 4
- 8 a $15x - 6$ b $28 - 8x$
 c $-6x - 21$ d $8 - 16x$
 e $10p - 35$ f $6h - 15$
 g $7 + 6x$ h $-2x - 11$
 i $-4x + 15$ j $7 + 8q$
 k $7w + 22$ l $6p$
- 9 The framework requires 37 metres of steel (rounded up to the next whole metre).

10 a Frequency: 7, 18, 15, 11, 7, 3, 6, 8, 8, 5.



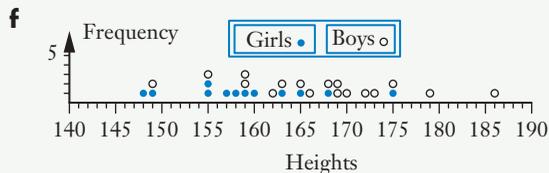
Exercise 2A PAGE 25

- 1 mean 3.5 (1dp), median 4, mode 5, range 5.
 2 mean 5.7 (1dp), median 6, mode 6, range 10.
 3 mean 17.6 (1dp), median 18, mode 18, range 5.
 4 mean 101.9, median 102, mode 101 and 103, range 7.
 5 mean 6.3 (1dp), median 7, mode 9, range 9.
 6 mean 25.2, median 28, mode 28 and 32, range 36.
 7 a 118 b 164 c 139 d 139.8
 8 a 48 b 94 c 77 d 76.4
 9 A little less than two.
 10 mean 6.3, median 6.5, mode 7.
 11 Assuming the teacher is older than 17.2 years, a reasonable assumption given that a school class is involved, the mean would be increased.
 12 The eighth score is 75.
 13 The mean for the three groups is 57.1%.
 14 The student must achieve 86% or more in the tenth item.
 15 The mean of the other five scores is 50.
 16 The girls achieved a mean of 62.7%.
 17 The mean is the same as the median.
 18 The mean is greater than the median.
 19 The mean is less than the median.
 20 The mean is the same as the median.
 21 The mean is less than the median.
 22 The mean is greater than the median.
 23 The mean is the same as the median.
 24 The mean is the same as the median.
 25 The mean is greater than the median.
 26 The mean is greater than the median.
 27 The mean is less than the median.
 28 The mean is the same as the median.
 29 The mean is greater than the median.
 30 The mean is less than the median.

- 31** The mean is greater than the median.
32 The mean is greater than the median.
33 and **34** Answers not given here. Compare your answers with those of someone else in your class and discuss the merits of each.

Exercise 2B PAGE 32

- 1** Mean 146, median 143, mode 137, range 40
2 Mean 82, median 83, mode 78, range 18
3 Mean 22.8, median 22, no mode, range 32
4 Mean 52.4, median 50, mode 35, range 45
5 Mean 2.3 (1 dp), median 2, mode 3, range 7
6 Mean 8.4 (1 dp), median 9, mode 6, range 7
7 Mean 18.8 (1 dp), median 19, mode 20, range 5
8 8.5 **9** 10.5 **10** 33.1 **11** 69.0
12 \$447 600, \$439 000. 8 are lower than the mean. 5 are lower than the median.
13 The mean number of bedrooms per house is 3.65.
14 **a** The modal salary is \$68 000.
b The median salary is \$68 000
c The mean salary is \$73 640
15 The mean number of sunlight hours is 10.9 (1 dp) and the median is 11.1.
16 **a** The mean is 75.4 (1 dp).
b The median is 77.
c 10 students scored less than 60%.
d 10% (3 of the 30) scored greater than 75%.
17 **a** There were 12 girls in the group.
b The shortest girl was 148 cm tall.
c The mean height for the girls is 159 cm (nearest cm).
d The mean height for the boys is 167 cm.
e The mean height for the 29 students is 164 cm (nearest cm).



18

Score	21 - 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50
Freq.	4	7	7	9	13	10

- a** 37.8 **b** 38
19 **a** $40 \leq h < 50$ is the modal class
b Mean = 44.7 hours

- 20** **a** Median lies in $230\,000 \leq C < 240\,000$
b Mean price = \$240 000

Exercise 2C PAGE 37

(Answers are given to suggest typical comments that could be made.)

- 1** The displayed data involves 64 test marks altogether. Using the mid-point of each class interval gives an estimated mean of 61.25, the median lies in the 60 to 70 interval and the modal class is the 70 to 80 interval. The marks are spread from about 20 to about 100, a range of approximately 80, but almost all of the marks (61 out of the 64) are actually spread from about 40 to 80.
The 61 marks from 40 to 80 are reasonably evenly spread amongst the four intervals 40 to 50 (15 marks), 50 to 60 (13 marks), 60 to 70 (16 marks) and 70 to 80 (17 marks).
There were no marks between 30 and 40 and none between 80 and 90 so with 1 mark between 20 to 30 and the 2 marks between 90 and 100 the histogram features two 'gaps'.
- 2** The displayed data involves 82 lengths altogether. With length involved it is likely that each class interval shows lengths rounded to the nearest whole centimetre. Using these whole centimetre values gives a mean length of 4.3 cm (to one decimal place). The median length is in the 3.5 cm to 4.5 cm interval and this is also the modal class.
Whilst the lengths are spread from about 2.5 cm to about 9.5 cm, a range of approximately 7 cm, almost all (77 out of the 82, approx 94%) are actually between 2.5 cm and 5.5 cm.
The 77 lengths from 2.5 cm to 5.5 cm are reasonably evenly spread amongst the three intervals centred around 3 cm (24 lengths), 4 cm (27 lengths) and 5 cm (26 lengths).
The histogram features a 'gap' with no lengths between 5.5 cm and 7.5 cm.
5 of the recorded lengths are unusually long compared to the other 77 lengths.
- 3** The displayed data involves 85 lengths altogether. With length involved it is likely that each class interval shows lengths rounded to the nearest whole centimetre. Using these whole centimetre values gives a mean length of 6.0 cm (to one decimal place). The median length is in the 5.5 cm to 6.5 cm interval and this is also the modal class.
The lengths are spread from about 2.5 cm to 9.5 cm, a range of approximately 7 cm.

The distribution of scores is reasonably uniform with each 1 cm class interval containing roughly the same number of lengths (from a low of 10 to a high of 14).

The histogram has no gaps.

- 4** The displayed data involves 100 lengths altogether.

Using the mid-point of each class interval gives an estimated mean of 37.1 cm.

The median length lies in the 34.5 cm to 39.5 cm interval and this is also the modal class.

The distribution of the lengths is very symmetrical in nature with a tall central column and the frequencies falling away on either side of this centre.

Whilst the lengths are spread from a low of 19.5 cm to a high of 54.5 cm, a range of 35 cm, seventy three of the 100 lengths are between 29.5 cm and 44.5 cm, a range of just 15 cm, i.e. most lengths are close to the central mean and median values.

- 5** The tabulated data involves 96 scores altogether.

Using the centre of each class interval gives an estimated mean of 30.3 (to one decimal place). The median lies in the 26–30 interval and the modal class is the 21–25 interval.

Although the scores range from about 21 to about 55, a range of 34, the first interval (scores of 21–25) has the highest frequency with approximately 36% of the

scores, and the frequencies then decrease as the scores increase.

Almost 60% of the scores are from 20.5 to 30.5 and just less than 5% are from 45.5 to 55.5.

- 6** The displayed data involves 106 scores altogether.

Using the centre of each class interval gives an estimated mean for the scores of 34.7, to one decimal place. The median lies in the 30 to 40 interval.

The scores are spread from about 0 to about 70, i.e. a range of approximately 70.

The scores are reasonably symmetrically spread about a mid-point of about 35. Frequencies rise on either side of this centre to give a bi-modal appearance peaking at the first interval, 28 scores for which $0 \leq \text{score} < 10$, and again at the last interval, 25 scores for which $60 \leq \text{score} < 70$. Half of the scores are either in this first interval or in the last interval.

Miscellaneous exercise two PAGE 39

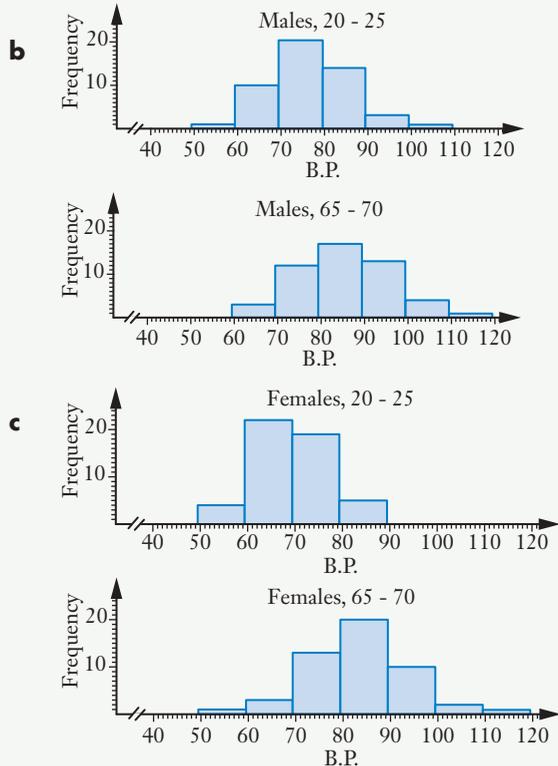
- 1** **a** $6x + 15$ **b** $35x - 15$ **c** $-2 + 10x$
d $22x - 9$ **e** $-2x + 11$ **f** $4x + 9$
g $x + 5$ **h** $14x - 1$

- 2** Nominal categorical.

(With just two categories, Yes and No, it could also be referred to as binary, or binomial data.)

3	Advantages	Disadvantages
Mean	Commonly understood as what we mean by the average. Output from statistical calculators. Every score in the set is used when determining the mean. May not be the central score but likely to be reasonably central if no outliers.	Can be greatly influenced by extreme scores (outliers). May not be one of the scores itself. (For example the mean number of children per married couple may not itself be a whole number of children.)
Median	As many scores below the median as above so it is central. Not affected by extreme scores. Output from statistical calculators. Easily calculated for small amounts of data. Can simplify measuring tasks - for example to find a median height we need only place the items in order and measure the middle one whereas for the mean we would need to measure all. To determine the median time for a team of 5 cyclists we can stop the clock when the third one crosses the line, we don't need to time them all.	May not be one of the scores itself. Can be tedious to rearrange a lot of scores in order if doing it manually.
Mode	It will be one of the scores. Not affected by an isolated outlier. Easy to work out for small data sets. Output from statistical calculators. Gives the most common score.	Not necessarily at all central. There may not be a mode.

- 4 The student needs at least 102% in test eight, i.e. cannot pass the course.
- 5 Answers not given here. Compare your answers with those of someone else in your class and discuss the merits of each.
- 6 Many possibilities, for example: \$14 000, \$15 000, \$16 000, \$17 000, \$18 000, \$88 000.
- 7 a Males 20 – 25: 76.7. Males 65 – 70: 85.7.
Females 20 – 25: 69.5. Females 65 – 70: 83.5.



- d For both sexes the 65 to 70 year olds generally have higher diastolic blood pressures than the 20 to 25 year olds suggesting diastolic blood pressure may increase with age. In both age groups the mean male diastolic blood pressure is higher than the mean for the corresponding female group.

Exercise 3A PAGE 45

- 1 a 31 b 42 c 38 000
- 2 a 12.8 b 18.75
- 3 a 0 b 4.72 (2 d.p.)
c 10.12 (2 d.p.) d 7.93 (2 d.p.)
- 4 a B b A
- 5 a B b A
- 6 a Neither mean is greater, both = 5.
b A

- 7 a A
b Neither standard deviation is greater, they are both the same.
- 8 a II b II c II d I
- 9 Range = 24, mean = 17, standard deviation = 6.58 (2 d.p.).

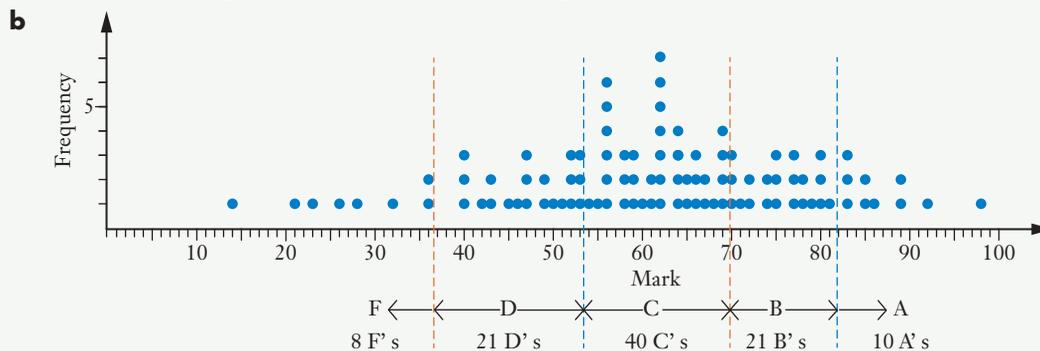
Exercise 3B PAGE 49

Note: The answers below give σ_n when the standard deviation of a given set of scores is asked for. However it is recognised that in some states σ_{n-1} may be expected to be given whenever the standard deviation is requested. For this reason, if to the given accuracy the two values differ the σ_{n-1} value is shown in brackets.

- 1 Mean 12.5, Standard deviation 1.7 (1.9)
- 2 Mean 25.4, Standard deviation 13.5 (14.2)
- 3 Mean 31, Standard deviation 1.7 (1.9)
- 4 Mean 6.99, Standard deviation 0.47 (0.49)
- 5 Mean 30.2, Standard deviation 3.7
- 6 a i C ii B
iii A iv B
- b A: Mean = 8.1, Standard deviation = 1.7 (1.8)
B: Mean = 3, Standard deviation = 0.8 (1 dp)
C: Mean = 5.5, Standard deviation = 2.9 (1 dp) (3.0)
- 7 a 5.76, 0.26 (0.28) b 5.83, 0.07 (0.08)
- 8 a Yes b Yes
- 9 a Mean = 160.44 cm,
Standard deviation = 8.87 cm (9.06 cm)
b Now use σ_{n-1} because sample is used to predict standard deviation of population.
Estimated standard deviation of population = 9.06 cm.
- 10 More likely to be a small type B. 18 mm is a little more than 1 standard deviation from the type B mean which is more likely than being 3 standard deviations from the type A mean.
- 11 a Mean = 97.45 cm,
Standard deviation = 8.346 cm (3 dp) (8.452 cm)
b 80% c 92.5%
d 97.5% e 8.45 cm
- 12 a Mean = 155.3°C,
Standard deviation = 11.74°C (2 dp) (12.37°C)
b Mean = 159.1°C (1 dp)
Standard deviation = 2.81°C (2 dp) (2.98°C)
- 13 31.5

- 14 a** Mean = 30.4°C (1 dp),
Range = 19.3°C,
Standard deviation = 4.9°C (1 dp). (5.0°C)
- b** The particular year involved has a mean maximum daily temperature for December that is approximately 3°C higher than long term mean.
- c** Mean = 15.8°C (1 dp),
Range = 14.4°C,
Standard deviation = 3.3°C (1 dp).
- d** The particular year involved has a mean minimum daily temperature for December that is approximately 0.5°C lower than long term mean.

- 15 a** Mean = 61.4 (1 dp), Standard deviation = 16.3 (1 dp) (16.4)



- 16** A passes. B rejected, condition 5. C passes. D rejected, condition 3.

Exercise 3C PAGE 54

Note: The answers below give σ_n when the standard deviation of a given set of scores is asked for. However it is recognised that in some states σ_{n-1} may be expected to be given whenever the standard deviation is requested. For this reason, if to the given accuracy the two values differ the σ_{n-1} value is shown in brackets.

- 1** Mean 3.1 Standard deviation 1.3
- 2** Mean 32.5 Standard deviation 8.1 (8.2)
- 3** Mean 32.5 Standard deviation 14.3 (14.5)
- 4** Mean 5.1 Standard deviation 1.8 (1.9)
- 5** Mean 70.8 Standard deviation 21.1 (21.2)
- 6** Mean 37.25 Standard deviation 8.0 (8.1)
- 7** Mean 18.6 Standard deviation 12.9 (13.1)
- 8** Mean 37.1 Standard deviation 12.9
- 9** Mean 62 Standard deviation 21.0 (21.4)
- 10** Mean 69.6 Standard deviation 16.0 (16.3)
- 11 a** 1.8 standard deviations from mean
- b** 3.6 standard deviations from mean
- 12** Uncoated seeds:
Mean number of successes per tray = 34.8
Standard deviation = 6.6 (correct to 1 decimal place).

Coated seeds:

Mean number of successes per tray = 42.05
Standard deviation = 5.4 (correct to 1 decimal place).
The trays with coated seeds tend to produce more successes per tray with less variability in the number of successes.

- 13** With outlier: Mean 16.3 minutes
Standard deviation 10.9 minutes (11.1 minutes)
Without outlier: Mean 15 minutes
Standard deviation 8.3 minutes (8.5 minutes)
- 14 a** Mean number of students per school 253,
standard deviation 116 (117) (nearest integers).
- b** Mean number of students per school 248,
standard deviation 107 (nearest integers).

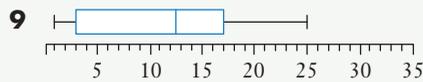
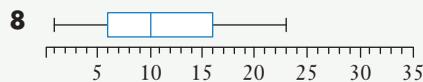
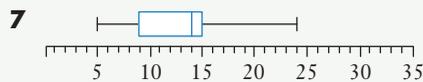
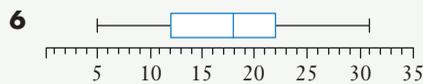
Miscellaneous exercise three PAGE 58

- 1** 0.012, 0.021, 0.1, 0.102, 0.12, 0.2, 0.201, 0.21.
- 2** $\frac{1}{100}, \frac{1}{5}, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{7}{10}, \frac{3}{4}$.
- 3 a** 11 **b** 26 **c** 28
- d** 4 **e** 16 **f** 40
- 4** A (2, 3), B (5, 4), C (3, 0), D (0, 5), E (2, -3),
F (5, -4), G (-2, 3), H (-5, 2), I (-4, -2), J (-2, -4).

- 5 a** Nominal categorical.
b For the year in question there were approximately 778 000 apple trees in W.A.
c 2507 tonnes of peaches were produced commercially.
d i The gross value per tonne for oranges was \$345
ii The gross value per tonne for nectarines was \$1000
e On average each tree yielded 48 kilograms of apples.
- 6** Advantage: Easily determined.
 Disadvantage: Only two scores involved so variation amongst other scores not taken into account.
- 7** Mean 6.68, median 7, mode 9, standard deviation 2.32 (2.38)
- 8** The new mean is 83.6.
- 9** The mean birth weight is 3.00 kg (to 2 dp)
- 10 a** The survey involved people who were trading in an old vehicle and so all those surveyed had owned at least one vehicle.
b For those surveyed the mean number of vehicles owned prior to the latest purchase is indeed 4.4, to one decimal place, but the two outlying values significantly influence this value. Quoting the median value of 3 vehicles owned prior to the latest purchase may be more representative. Also the median has the advantage of being an integer value and it may be preferable to quote the average as a whole numbers of cars.
- 11** Each short wire is to be made of length 36.6 metres
 Each medium wire is to be made of length 67.6 metres.
 Each long wire is to be made of length 85.4 metres.
- 12 a** Mean 43.92, Standard deviation 13.99 (14.28)
b Unit C: 1.5, Unit E: 0.7,
 Unit A: 0.649 (0.636), Unit B: -0.5,
 Unit D: -0.6

Exercise 4A PAGE 66

- | | | | |
|---------------|--------------|---------------|-------------|
| 1 a 26 | b 15 | c 30 | d 4 |
| e 36 | f 15 | | |
| 2 a 20 | b 12 | c 26 | d 6 |
| e 36 | f 14 | | |
| 3 a 14 | b 12 | c 16.5 | d 8 |
| e 19 | f 4.5 | | |
| 4 a 61 | b 50 | c 80 | d 20 |
| e 93 | f 30 | | |
| 5 a C | b D | c A | d C |
| e B | f C | g B | |



- 10 a** The statement is incorrect. There will be the same number of scores below the median as there are above it.
- b** If we rank spread on the basis of range (32 for class I and 31 for class II) or interquartile range (14 for class I and 13 for class II) then we could say that the class I marks are more spread out than the class II marks. However, even if we only consider these measures there is not much in it. Furthermore box plots do not show us all of the individual marks and the spread summaries of range and interquartile range are each determined using only two scores. Using other criteria to judge spread, for example standard deviation which uses all of the scores in its determination, we could well end up disagreeing with the given statement.
- c** Considering only the range of scores, 34 for class III and 32 for class I, might tempt us to suggest that class III marks are more spread out but the interquartile range, 9 for class III and 14 for class I, suggests the opposite conclusion. The long lower whisker in the class III boxplot could be caused by just one outlying score. Thus whilst the statement could conceivably be correct under some suitably chosen definition of spread it would be unwise to claim the statement true under a more general understanding of 'more spread out'.
- d** Based on the 'five point' nature of the information a box plot provides (i.e. lowest, Q1, median, Q3, highest) the statement seems correct and justified. Were we to know the individual scores we might find the distribution of scores within each quarter differs markedly between the two classes but without this information the statement is a reasonable statement to make based on the boxplot data.
- e** Based on this test the top student in class III would be about 25% of the way down the rank positions of the students in class I, not necessarily the 25th student. Hence the statement is not one that can be concluded from the given information. Box plots do not tell us the number of data points involved.

- f** Class II certainly had at least one student who scored a lower mark than the lowest mark from the other two classes but we cannot conclude that there were ‘lots’ of students for whom this could be said.

Exercise 4B PAGE 72

(Answers are given to most questions to suggest some typical comments that could be made. For the questions for which comments are not given compare and discuss your descriptions with those of others in your class.)

Note: The answers below give σ_n when the standard deviation of a given set of scores is asked for. However it is recognised that in some states σ_{n-1} may be expected to be given whenever the standard deviation is requested. For this reason, if to the given accuracy the two values differ the σ_{n-1} value is shown in brackets.

- 1** The displayed data involves 42 test marks altogether.
The distribution of marks give a mean mark of 70, the median lies in the 70 to 80 interval and the modal class is also the 70 to 80 interval.
The marks are spread from about 10 to about 90, a range of approximately 80, and the standard deviation is 16.4. (16.5)
The distribution of marks is skewed to the left with 26 of the 42 marks (almost 62%) being 70 or more and 35 of the 42 (approximately 83%) being 60 or more. The distribution features a ‘gap’ with no marks between 30 and 50 but 3 marks were between 10 and 30.
- 2** The displayed data involves 100 lengths altogether.
The mean length is 5.97 cm and the median length (nearest centimetre) is 6 cm.
Lengths are spread from a low of 3 cm to a high of 9 cm, a range of 6 cm. (However the measurements probably involve rounding so the low could be 2.5 cm and the high could be 9.5 cm to give a range of 7 cm.)
The standard deviation of the lengths is 2.3 cm.
The distribution of the lengths is approximately symmetrical in nature about the central value of 6 cm. It is bimodal with peaks at 3 cm and 9 cm. Seventy eight of the 100 recorded lengths are either between 2.5 cm and 4.5 cm or between 7.5 cm and 9.5 cm. The remaining 22 lengths fall in the 4.5 cm to 7.5 cm interval. Hence most of the lengths are situated away from the central mean and median values.
- 3** The displayed data involves 104 scores altogether.
The distribution of scores give a mean score of 42.0, the median lies in the 30 to 40 interval and the modal class is also the 30 to 40 interval.
The scores are spread from about 10 to about 100, a range of approximately 90, and the standard deviation is 18.2. (18.3)
The distribution of scores is skewed to the right with 74 of the 104 scores being between 10 and 50 and the remaining 30 being between 50 and 100. Just 9 of the 104 scores were between 70 and 100.
- 4** Comments not given here.
Compare and discuss your descriptions with those of others in your class.
- 5** The tabulated data involves 100 scores altogether.
The distribution of scores give a mean of 48.05, the median lies in the 46–50 interval and the modal class is the 41–45 class.
The scores are spread from about 31 to about 65, a range of approximately 34, and the standard deviation is 10.04. (10.09)
The distribution of scores is uniform with each class interval containing roughly the same number of scores (from a low of 13 to a high of 16).
- 6** The tabulated data involves 100 scores altogether.
The distribution gives a mean score of 15.1, the median lies in the $10 \leq x < 20$ interval and is probably much nearer to 10 than it is to 20. The modal class is the $0 \leq x < 10$ interval.
The scores are spread from about 0 to about 70, a range of approximately 70, and the standard deviation is 12.9. (13.0)
With almost half of the scores in the $0 \leq x < 10$ interval and the frequencies decreasing as we move right, the distribution of scores is skewed to the right. Eighty eight of the 100 scores are such that $0 \leq \text{score} < 30$ whilst just 6 are such that $40 \leq \text{score} < 70$.
- 7** The median of data set A (27) is higher than the median of data set B (24).
With the lower quartile at 20 and the upper quartile at 30 in each set the middle 50% of the data points in each set are spread over the same scores and each data set has an interquartile range of 10. The range of the two data sets is very similar, 34 for set A and 35 for set B. However for data set A the left whisker is the longer whisker, and there is more of the box to the left of the median than the right, indicating that data set A is skewed to the left. Conversely data set B has the right whisker noticeably longer than the left, and more of the box to the right of the median indicating that data set B is skewed to the right.
- 8** Comments not given here.
Compare and discuss your comments with those of others in your class.

- 9 Assuming the location of the Regional Meteorology Station to be typical for the region as a whole the region experienced rain on 110 days of the year, i.e. approximately 30% of the days in the year had some rain.

On more than half of the rainy days the rainfall was less than 5 mm. The total rainfall for the year was approximately 775 mm with approximately 8% of this total falling on just one day. If we include the days on which no rain fell the average daily rainfall for the year was approximately 2.1 mm per day. Considering only days on which rain fell the average daily rainfall was approximately 7 mm per rainy day. If we discount the one day of unusually high rainfall these averages become 1.95 mm and 6.5 respectively. For the days that rain fell the rainfall figures have a standard deviation of 8.1 mm but if we discount the one day of unusually high rainfall this falls to 6.1 mm.

The overall distribution is skewed to the right with one extreme value in the 60 to 65 mm interval and all other rainy days recording less than 30 mm.

- 10 The distribution of scores of the 196 students give a mean score of 83.8. The median score is in the 81 to 90 interval and the modal class is the 91 to 100 interval with approximately 21% of the scores in this interval. 151 of the 196 students (77% of them) achieved a score of over 70 in the exam (remember though that this 70 is a raw score out of at least 120 and is not a percentage score) and 119 of them achieved a raw score over 80.

The students achieved scores from a low of about 21 to a high of about 120, hence the range of scores was approximately 99. The scores had a standard deviation of 20.4.

With the distribution showing a long tail to the left the scores were skewed to the left.

- 11 The 76 donors involved had a mean age of 36 (nearest year), standard deviation 14.

Their ages ranged from around 15 to almost 60 with a median age of approximately 40.

The 76 recipients involved had a mean age of 44 (nearest year), standard deviation 12.

Their ages ranged from around 15 to mid sixties with a median age of approximately 45.

Thus whilst the ages of the donors and the recipients were spread across similar age ranges, with just six of the recipients older than the oldest of the donors, the donors tended on average to be younger than the recipients.

The age distribution of the donors is bimodal, peaking around 20 and again around 50, and is roughly symmetrical rising on either side of a low frequency central age of approximately 35. In contrast the age

distribution of recipients is negatively skewed with frequencies tailing off to the left of the modal age of about 50. Over half of the recipients were between 42 and 58.

Approx 40% of donors but only 15% of the recipients (approx) were aged under 30.

Miscellaneous exercise four PAGE 74

- 1 With the outlier: Mean = 10
Standard deviation = 8.67 (9.19).

Without the outlier: Mean = 7
Standard deviation = 1.87 (2).

- 2 a 1 b 1 c 2 d 3
e 1 f 2 g 75% h 2

- 3 The standard deviation will increase.

If we remove scores that are close to the mean, as those in the central column are, the average distance from the mean will increase. Hence the standard deviation will increase.

- 4 Ask others in your class to read and comment on your article and you do the same for theirs.

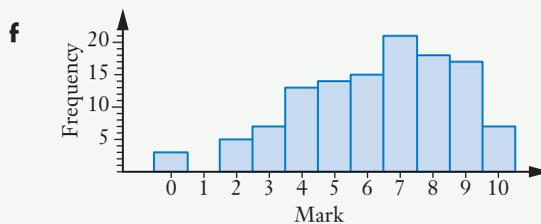
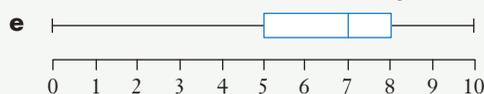
- 5 A with boxplot 4. B with boxplot 1.
C with boxplot 2. D with boxplot 3.

- 6 a Any scores less than 10 or greater than 58.
b Any scores less than 3 or greater than 75.
c The scores of 10, 17 and 70 are outliers under the given definition.
d The scores of 42, 49 and 50 are outliers under the given definition.

7

Mark	0	1	2	3	4	5	6	7	8	9	10
Frequency	3	0	5	7	13	14	15	21	18	17	7
Cumulative frequency	3	3	8	15	28	42	57	78	96	113	120

- a 17 b 113
c 35% d One eighth



- g** The marks have a mean of 6.31 and a median of 7. The modal score is also 7.

With the lowest score of zero (3 students) and highest of 10 (7 students) the range of the scores is 10. No student scored a total of 1 but all other possible totals featured.

With a lower quartile of 5 and an upper quartile of 8 the interquartile range is 3.

The standard deviation is 2.35 (2.36) and the distribution is skewed to the left as can be seen from the histogram, as the longer box and whisker to the left of the median than the right suggests, and the fact that mean (6.31) is less than median (7) further reinforces.

- 8** 3.85 (4.30)

Miscellaneous exercise five PAGE 81

- 1** Histogram with new class intervals not shown here.

Histogram shows that the apparent uniform distribution of times is not the case. The new histogram indicates a distribution that dips on either side of two peaks - i.e. a more bi-modal shape to the distribution. Distribution is reasonably symmetrical about the time of 50 seconds. The apparent uniformity of the original distribution misses the bi-modal feature because the small number of class intervals cause some detail to be lost.

- 2** **a** $7x + 23$ **b** $16x + 17$ **c** $7x + 2$
d $x - 10$ **e** $5x + 18$ **f** $x + 2$
- 3** **a** 15 **b** 44 **c** 82
d 25 **e** 44.7 **f** 15.6 (15.7)
- 4** **a** Some of the newly diagnosed sufferers will be counted in more than one category, for example a male smoker under 30.
b No. Statement not necessarily true. The graphs show *percentage* of newly diagnosed sufferers in each category, the overall *number* of male sufferers could have increased.

Exercise 6A PAGE 93

- 1** **a** $x = 6$ **b** $x = -26$ **c** $x = 28$
d $x = 6$ **e** $x = 4.5$ **f** $x = 3$
g $x = 4$ **h** $x = 8$ **i** $x = 1$
j $x = 13$ **k** $x = 5$ **l** $x = 1.4$
m $x = 15$ **n** $x = 42$ **o** $x = 5$
p $x = 1.4$ **q** $x = 84$ **r** $x = 7$
s $x = 2$ **t** $x = -1$ **u** $x = 12$
v $x = 2$ **w** $x = -0.5$ **x** $x = 18$
y $x = 13$

- 2** **a** $P = 650$ **b** $A = 1335$ **c** $I = 55$
3 **a** $v = 20$ **b** $u = 18$ **c** $a = 2$
d $a = -3$ **e** $t = 3$ **f** $t = 11$
4 **a** $r = 3.98$ **b** $r = 15.12$ **c** $C = 50.27$
d $r = 64$
5 **a** $A = 25.13$ **b** $r = 2.84$ **c** $h = 5.31$
6 **a** $t = 14$ **b** $u = 9$ **c** $v = 15$
7 **a** $R = 11$ **b** $R = 45$
c $R_1 = 10$ **d** $R_3 = 27$
8 **a** $h = 5.95$ **b** $h = 35.79$
9 **a** $V = 25$ **b** $P = 10$
c $V = 20$ **d** $P = 2.5$
10 **a** $P = 735$ **b** $P = 7840$
c $m = 150$ **d** $h = 2.25$
11 **a** The cost is \$570.
b The cost is \$360.
c The person could travel 2500 km, or less.
12 **a** The commissions received are \$3100, \$3400 and \$2100 respectively.
b The house was sold for \$445 000.
c The house was sold for \$850 000.
13 **a** The male has an estimated height of 1.7 m.
b The expected humerus length is 40 cm.
14 **a** The profit will be \$2000.
b The profit will be \$14700.
c The profit will be \$23 590.
d The least number is 3052.
e The firm would make a loss of \$6890.
f The greatest profit is \$33 750.
g The firm would lose \$29 750.
h The firm must sell at least 2343 calendars to avoid making a loss.

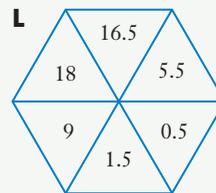
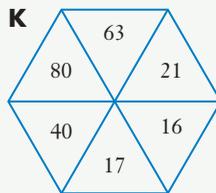
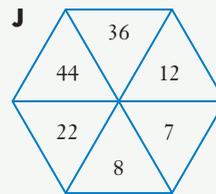
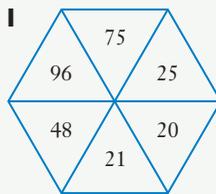
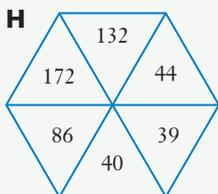
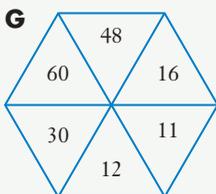
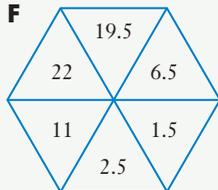
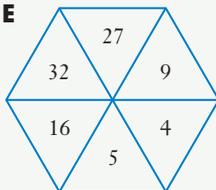
Miscellaneous exercise six PAGE 96

- 1** **a** $x = 7$ **b** $x = 2$ **c** $x = 3.5$
d $x = 2.5$ **e** $x = 14$ **f** $x = 7.5$
2 **a** $A = 69$ **b** $h = 2.5$
3 $a = 3, b = 8, c = 10, d = 12, e = 14.$
4 The student requires a mark of 47% or more in test six.
5 **a** A **b** C **c** B **d** A
e D and B (in that order)
6 5 A, 9 B, 21 C, 10 D, 2 Fail.

- 7** The median would be the fairest to use as it will not be unduly influenced by the occasional sale of one of the luxury properties. It might suit the agent to be able to quote a higher average by using the mean, with one or more luxury properties included in the calculation, but for a steadier monthly value that allows a general trend to be observed, free from the occasional monthly 'spike' when a luxury property is sold, the median would be more suitable. The mode might occasionally be central but not necessarily, so that too would not be as good as the median.
- 8 a** Mean \$65 588 Standard deviation \$16 488 (\$16 532)
- b** Mean \$66 098 Standard deviation \$16 393 (\$16 441)
- 9** Six applicants are invited for interview.
- 10** Discuss your descriptions with those of others in your class and with your teacher.
- 11 a** Question cannot be answered from the given information. The graphs indicate percentage of the population in the various age intervals, not population numbers.
- b** Approximately nine million of country A's population are aged 70 or over.
- c** Let others read and comment on your report and you read and comment on their report.

Exercise 7A PAGE 104

- 1** 11 **2** 12 **3** 2.3 **4** $1\frac{2}{3}$
- 5** 9 **6** 2.75
- 7 a** 1 **b** 5 **c** 3
- d** add starting number **e** divide by two



Exercise 7B PAGE 108

- 1 a** $5x + 6$ **b** $14 - x$ **c** $5(x + 6)$
- d** $2x - 7$ **e** $2(x - 7)$ **f** $3(2x + 5)$
- 2 A:** $2x + 1 = 10, x = 4.5$ **B:** $2(x - 1) = 10, x = 6$
- C:** $10 - x = 1, x = 9$ **D:** $2(x + 1) = 10, x = 4$
- E:** $\frac{x}{2} - 1 = 10, x = 22$ **F:** $x - 10 = 1, x = 11$
- G:** $\frac{x-1}{2} = 10, x = 21$ **H:** $2x - 10 = 1, x = 5.5$
- 3** The number first thought of was 9.
- 4** The number first thought of was 3.
- 5** The number first thought of was 10.
- 6** The number first thought of was 5.
- 7** The number first thought of was 9.
- 8** The number first thought of was 5.
- 9** The number first thought of was 7.
- 10** The number first thought of was 13.
- 11** The number first thought of was 9.5.
- 12** The number first thought of was 12.

Exercise 7C PAGE 111

- 1** Bob contributes \$6500, Tony contributes \$12 000.
- 2** Sue should receive \$12 000, Lyn \$18 000 and Paul \$17 000.
- 3** Bill is 43 years old now.
- 4 a** $\$(5000 + 22x)$ **b** 278
- 5 a** $3x$ hours
- b** $4(120 - x)$ hours
- c** 75 standard and 45 deluxe.
- 6 a** The width should be 85 m.
- b** The length should be 95 m.
- c** The area should be 8075 m^2 .

- 7 $(5x - 2)$ years. Heidi is 9 years old now.
- 8 They need 240 tickets at \$12 each and 610 at \$8 each.
- 9 **a i** $x + 2000$
ii $2(x + 2000)$
- b** 3000 acres of lupins, 5000 acres of barley and 10000 acres of wheat.
- 10 **a i** 150x grams
ii $80(50 - x)$ grams
- b** Each 50 kg of *QuickGrow* should contain 32 kg of X and 18 kg of Y.
- 11 The original order was for 185 hardback and 115 softback.
- 12 \$2800 was invested with company A and \$2200 was invested with company B.

Exercise 7D PAGE 116

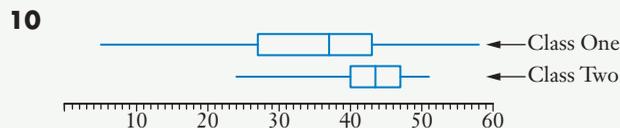
- 1 **a** $a = 6$ **b** $b = 4$ **c** $c = 6$
d $d = 7.5$ **e** $e = 3.6$ **f** $f = 2.8$
g $g = 1.5$ **h** $h = 1.7$ **i** $i = 4.8$
- 2 \$4200 needs to be invested.
- 3 7.5%
- 4 The initial investment was \$850.
- 5 Four and a half years.
- 6 146 days
- 7 Annual rate of 6.5% required.
- 8 $R = 8.2$
- 9 6% is the required annual rate of simple interest.
- 10 There were 1463 females in the audience.
- 11 There are 391 female students in the school.
- 12 The height of the tree is approximately 13.5 metres.
 Note: Using ratios an 'exact' answer of 13.5 metres is obtained. However the word approximately is used in the answer because the nature of the situation means that this answer would have to be regarded as being something of an approximation. Will the highest point of the tree be directly above the centre of the base? Will the end of the shadow be somewhat blurry and difficult to locate accurately? (This applies to some later questions in this exercise too.)
- 13 The building is approximately 25 metres tall.
- 14 $h = 3.5x$. If $x = 1.5$, $h = 5.25$.
- 15 The flagpole is approximately 7 metres tall.
- 16 The river is approximately 54 metres wide.
- 17 The pylon is approximately 8.6 metres tall.
- 18 Mary borrowed \$1200 in the first place.
- 19 Mai borrowed \$6800 in the first place.

Miscellaneous exercise seven PAGE 121

- 1 There are 342 female students in the school.
- 2 **a** $x = 4.2$ **b** $x = 7.5$
c $x = 8.75$ **d** $x = 2.8$
- 3 $m = 15$
- 4 8
- 5 **a** 86 **b** 79 **c** 7 **d** 9
- e** Naomi is correct in her statement that the range of the male scores is bigger than the range of the females. ($\text{Range}_{\text{Male}} = 9$, $\text{Range}_{\text{Female}} = 7$).

However the range depends only on the lowest score and the highest score so it is unwise to claim that the male scores as a whole are more spread out than the female scores based solely on this fact. Indeed other measures of spread namely the interquartile range (5 for female and 2 for male) and the standard deviation (2.4 for female and 2.0 for male) disagree with Naomi's statement and suggest that the female scores are more spread out than the male scores.

- 6 The number first thought of was 18.
- 7 At least \$14650 needs to be invested for the account to be at least \$19000 in 5 years.
- 8 \$54000 into the account paying 6.3% and \$26000 into the account paying 5.4%.
- 9 **a** 70.5
b Mean 68 Standard deviation 26.5



Overall class two performed better even though top mark was in class one:

The median for class one (37) is lower than the median for class two (43.5).

Having the individual scores available we can also determine the means: 34.7 for class one and 42.5 for class two.

The upper quartile in class one is lower than the median in class two.

The marks of class one are more spread out than those of class two:

The range of class one (53) greatly exceeds the range of class two (27) as does the interquartile range (16 for class one compared to 7 for class two).

Having the individual scores available we can also determine the standard deviations: 12.2 (12.5) for class one and 6.2 (6.3) for class two.

(Answer continued over page.)

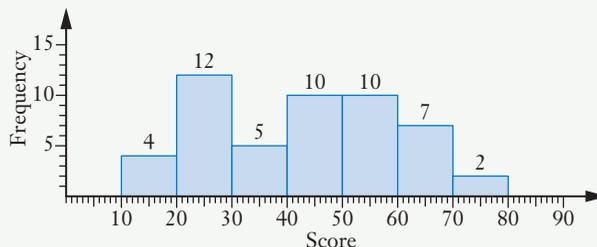
The box plot for class one suggests a reasonably symmetrical distribution perhaps skewed left a little as suggested by the longer left whisker and the greater part of the box being to the left of the median. The class two boxplot also suggests a skew to the left because of the longer left whisker though within the box the median is centrally placed. (Such skewness is further suggested by the fact that mean < median in each class.)

The two distributions were based on a very similar number of data points, 25 for class one and 22 for class two.

11 Many possible answers but all must have:

- a total of 50 scores,
- at least one score in the 10 → 20 interval (as lowest score was 16),
- the 13th score (counting from the low end) in the 20 → 30 interval (as 1st quartile = 26),
- the mean of the 25th and 26th score in the 40 → 50 interval (as median was 46),
- the 13th score (counting from the top end) in the 50 → 60 interval (as 3rd quartile = 56),
- at least one score in the 70 → 80 interval (as highest score was 72).

One possibility is shown below:



12 $x = 64000$.

Exercise 8A PAGE 129

- | | | | |
|---------------|--------------------------|---------------|---------------|
| 1 2 | 2 0.5 | 3 1 | 4 -1 |
| 5 -0.5 | 6 2 | 7 -0.5 | 8 3 |
| 9 -2 | 10 1 | 11 2 | 12 -1 |
| 13 -2 | 14 $-\frac{1}{3}$ | 15 5 | 16 1 |
| 17 3 | 18 0.2 | 19 5 | 20 -20 |
- 21** Points would lie in a straight line. Gradient = 2.
22 Points would lie in a straight line. Gradient = -2.
23 Points would not lie in a straight line.
24 Points would not lie in a straight line.
25 Points would lie in a straight line. Gradient = 5.
26 Points would not lie in a straight line.

- 27** Points would lie in a straight line. Gradient = -5.
28 Points would not lie in a straight line.
29 Points would lie in a straight line. Gradient = -3.
30 Points would lie in a straight line. Gradient = 3.
31 Points would lie in a straight line. Gradient = 2.
32 Points would lie in a straight line. Gradient = 3.
33 Points would not lie in a straight line.
34 Points would lie in a straight line. Gradient = 2.5.

Exercise 8B PAGE 137

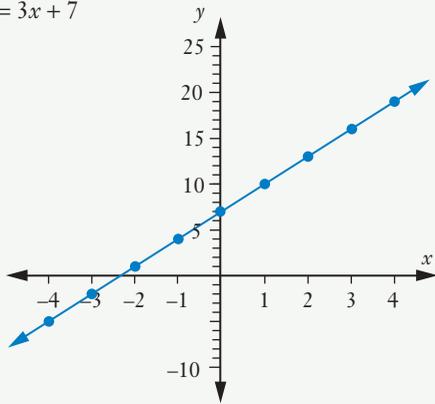
- | | | |
|-----------------|------------------|--------------------------|
| 1 a 1 | b (0, 2) | c $y = x + 2$ |
| 2 a 2 | b (0, -3) | c $y = 2x - 3$ |
| 3 a -1 | b (0, 3) | c $y = -x + 3$ |
| 4 a 0.5 | b (0, 3) | c $y = 0.5x + 3$ |
| 5 a -3 | b (0, 2) | c $y = -3x + 2$ |
| 6 a -2 | b (0, -3) | c $y = -2x - 3$ |
| 7 a 40 | b (0, 10) | c $y = 40x + 10$ |
| 8 a 5 | b (0, 4) | c $y = 5x + 4$ |
| 9 a 6 | b (0, 15) | c $y = 6x + 15$ |
| 10 a -10 | b (0, 70) | c $y = -10x + 70$ |
- 11** A: $y = 7$, B: $y = 4$, C: $y = 1$, D: $y = -4$,
E: $x = -4$, F: $x = -3$, G: $x = 2$, H: $x = 4$.
- 12** Relationship is linear. $y = 3x + 1$
13 Relationship is linear. $y = -4x + 25$
14 Relationship is linear. $y = 5x - 3$
15 The relationship is not linear.
16 Relationship is linear. $y = 1.5x + 0.5$
17 The relationship is not linear.
18 Relationship is linear. $y = x + 2$
19 The relationship is not linear.
20 Relationship is linear. $y = -x + 13$
21 Relationship is linear. $y = 2x + 21$
22 Relationship is linear. $y = 3x + 4$
23 Relationship is linear. $y = 5x - 12$
24 Relationship is linear. $y = 2x - 2$
25 Relationship is linear. $y = 3x + 1$
26 a 5 **b** (0, -10) **c** $y = 5x - 10$
27 a -6.25 **b** (0, 37.5) **c** $y = -6.25x + 37.5$

28 Table:

x	-4	-3	-2	-1	0	1	2	3	4
y	-5	-2	1	4	7	10	13	16	19

Rule: $y = 3x + 7$

Graph:

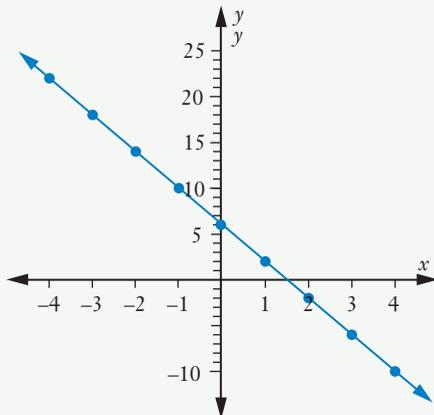


29 Table:

x	-4	-3	-2	-1	0	1	2	3	4
y	22	18	14	10	6	2	-2	-6	-10

Rule: $y = -4x + 6$

Graph:

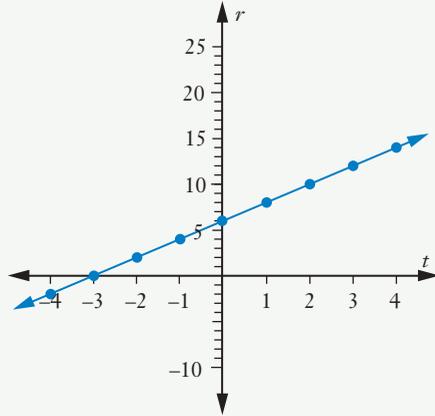


30 Table:

t	-4	-3	-2	-1	0	1	2	3	4
r	-2	0	2	4	6	8	10	12	14

Rule: $r = 2t + 6$

Graph:

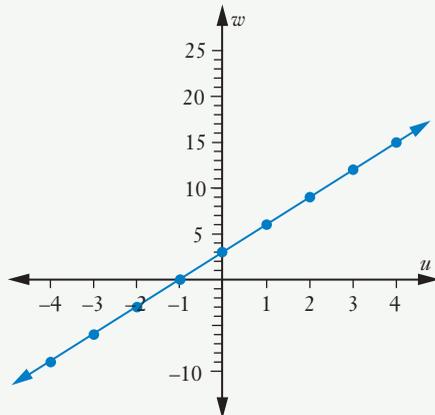


31 Table:

u	-4	-3	-2	-1	0	1	2	3	4
w	-9	-6	-3	0	3	6	9	12	15

Rule: $w = 3u + 3$

Graph:

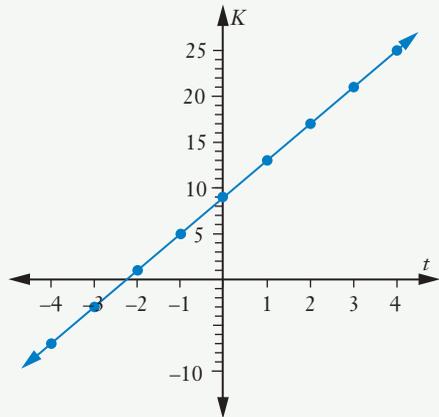


32 Table:

t	-4	-3	-2	-1	0	1	2	3	4
K	-7	-3	1	5	9	13	17	21	25

Rule: $K = 4t + 9$

Graph:

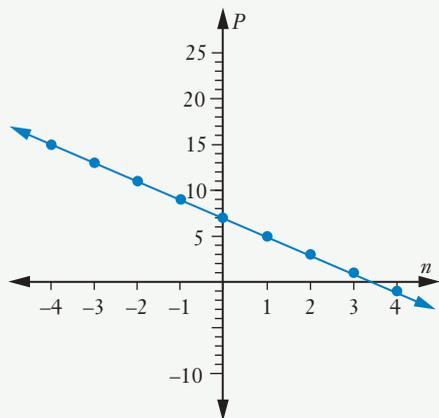


33 Table:

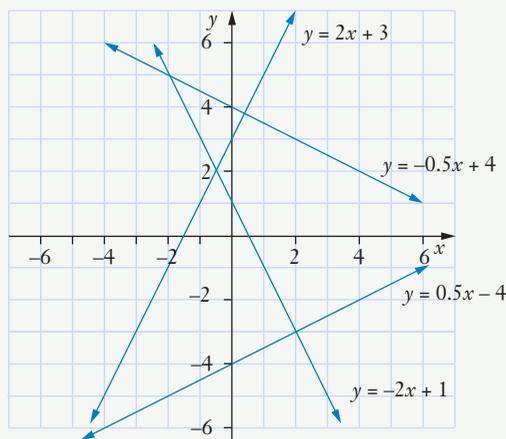
n	-4	-3	-2	-1	0	1	2	3	4
P	15	13	11	9	7	5	3	1	-1

Rule: $P = -2n + 7$

Graph:



34



35 A: $y = -\frac{2}{3}x + 2$

B: $y = -1.5x + 3$

C: $y = 2.5x + 4$

D: $y = 4x$

E: $y = 3x$

F: $y = 3x - 2$

36 A: $y = -2$

B: $y = 0.5x + 2$

C: $y = -x + 2$

D: $y = 2x - 2$

E: $y = 0.5x - 2$

37 A: $y = 10x + 60$

B: $y = 30$

C: $y = -10x + 30$

D: $y = 30x$

E: $y = 30x - 90$

38 The graphs appear different because the scales used on the axes are not the same in the two graphs. Hence the intercepts with the y -axes appear different but in both cases will be at $(0, 3)$ and the intercepts with the x -axes appear different but in both cases will be at $(-1.5, 0)$.

Exercise 8C PAGE 145

1 a 2 b 4 c 2 d 0.5

e 0.25 f 1 g -2 h 2.5

i 0.5 j -4 k 2.5 l 0.5

2 a Gradient 3 Cuts y -axis at $(0, -17)$

b Gradient -2 Cuts y -axis at $(0, 13)$

c Gradient -7 Cuts y -axis at $(0, 5)$

d Gradient $-\frac{2}{3}$ Cuts y -axis at $(0, 8)$

e Gradient -0.4 Cuts y -axis at $(0, 1.6)$

f Gradient $\frac{2}{3}$ Cuts y -axis at $(0, 3)$

g Gradient -0.5 Cuts y -axis at $(0, 11)$

h Gradient -2.5 Cuts y -axis at $(0, 15)$

i Gradient -1.2 Cuts y -axis at $(0, 12)$

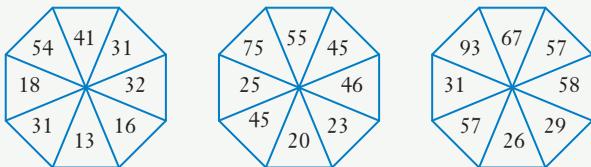
3 $y = 0$

- 2 Equation 3 must be true.
- 3 **a** $a = 2.4$ **b** $b = 3.5$ **c** $c = 6$
d $d = 3.8$ **e** $e = 13.5$ **f** $f = 13.5$
g $g = 4.5$ **h** $h = 3\frac{5}{9}$

- 4 There are 196 females in this workforce.
- 5 The student needs at least 66% in the final unit.
- 6 Dot frequency 1 with Boxplot C
Dot frequency 2 with Boxplot A
Dot frequency 3 with Boxplot D
Dot frequency 4 with Boxplot B

- 7 **a** 72 **b** 39 **c** 51

8

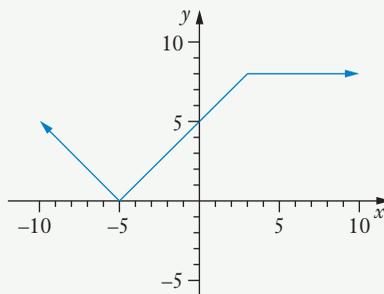


- 9 **a** The gradient of 7 means that for each degree rise in temperature the number of chirps per minute goes up by 7.
In theory the -16 suggests that at 0°C the cricket would make -16 chirps! However 'negative chirps' is a rather meaningless concept. The equation is really only valid for $N \geq 0$.
- b** For $N \geq 0$ we need the temperature to be greater than 2.3°C . Indeed, according to the rule, for at least one chirp per minute the temperature needs to be approximately 2.5°C .
- c** **i** The cricket makes roughly 82 chirps per minute.
ii The cricket makes roughly 180 chirps per minute.
- d** **i** The temperature is approximately 31°C .
ii The temperature is approximately 24°C .

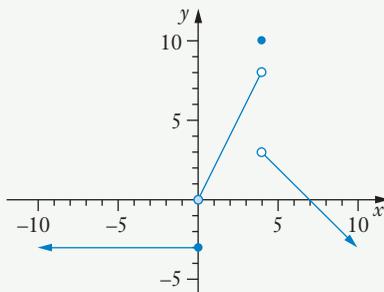
Exercise 9A PAGE 159

- 1 For $x < 0$ $y = -x - 4$
for $0 \leq x < 4$ $y = 2x - 4$
for $x \geq 4$ $y = 4$
- 2 For $x \leq -4$ $y = 5$
for $-4 < x < 0$ $y = 3$
for $0 \leq x \leq 3$ $y = -x$
for $x > 3$ $y = x - 3$

3



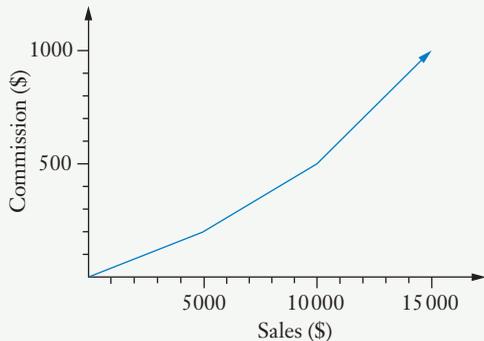
4



- 5 Ask someone in your class to read and constructively comment on your answer and you do the same for theirs.
- 6 **a** The blue broken line represents the journey of the cyclist.
b The cyclist passes the walker between 8.52 a.m. and 8.53 a.m.
c The walker took 30 minutes to walk to school.
d The walker maintained a steady speed of 6 km/h.
e The cyclist took 10 minutes to ride to school.
f The cyclist maintained a steady speed of 18 km/h.
- 7 **a** The cyclist left town A at 7 a.m.
b The cyclist reached town B at 11.20 a.m.
c For the cyclist each stop was for 30 minutes.
d **i** Prior to the first stop the cyclist maintained 20 km/h.
ii Between the two stops the cyclist maintained 15 km/h.
iii After the second stop the cyclist maintained 30 km/h.
e **i** From town A to town B the delivery truck maintained 60 km/h.
ii From town B back to town A the delivery truck maintained 90 km/h.
f When they were both travelling towards B the delivery truck passes the cyclist at about 9.35 a.m., and about 36 km from A.
g When returning to A the truck passed the cyclist at about 11.05 a.m., and about 52 km from A.

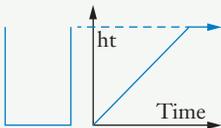
- 8** (Graph not shown here.)
- The car reaches C at 11.54 a.m. and truck reaches town C at 12.15 p.m.
 - From 8.30 a.m. to 9.30 a.m. the truck maintained a steady speed of 100 km/h.
 - The average speed of the truck from A to B was 87 km/h (to the nearest km/h).
 - The car passes the truck at 10.30 a.m. in town B, just as the truck is about to leave B.
- 9**
- Someone with a taxable income of \$30 000 would pay \$7000 in tax.
 - Someone with a taxable income of \$40 000 would pay \$11 000 in tax.
 - Someone with a taxable income of \$48 000 would pay \$15 000 in tax.
 - Someone with a taxable income of \$3000 would pay no tax at all.
Someone paying tax of \$20 000 would have a taxable income of \$58 000.

10

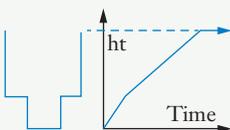


- 11** For the first \$100 000 of the sale price the agent's fee is a fixed \$6000.
From \$100 000 to \$300 000 the fee is \$6000 plus 5% of the amount over \$100 000.
From \$300 000 and over the fee is \$16 000 plus 2% of the amount over \$300 000.

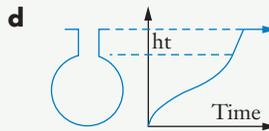
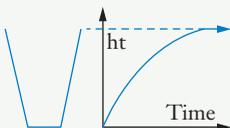
12 a



b



c



Miscellaneous exercise nine PAGE 163

- $P = 3t - 5$
- $a = -1, b = 4, c = 9, d = 19, e = 29, f = 11, g = 99$
- | | |
|--------------------|-----------------|
| A: $y = -3$ | B: $y = 1$ |
| C: $y = -0.5x + 5$ | D: $x = 5$ |
| E: $y = x + 3$ | F: $y = 9$ |
| G: $x = -3$ | H: $y = 3x + 2$ |
| I: $x = 7$ | J: $y = x$ |
- The 5200 and the 16 tell us that the fixed cost of production is \$5200, even when no radios are produced, and then each radio produced adds \$16 to this cost of production.
 - When 100 radios are produced the mean cost per radio is \$68.
 - When 500 radios are produced the mean cost per radio is \$26.40.
 - When 1000 radios are produced the mean cost per radio is \$21.20.
- There are 210 year eight students in the school.
- 4
- The number first thought of was 6.
- There were 5 eights in the set.
- | | | |
|------------------|-------------|-------------|
| a 33 | b 37 | c 16 |
| d 30 - 40 | | |

Mean = 31.8
- 25%
 - The range of the set B marks (39) exceed the range of the set A marks (32).
The interquartile range for set B (17) exceeds the interquartile range of set A (11).
 - The median of those left in set B would be lower than that of set B before the move.
 - The range of those left in set B would be less than that of set B before the move.
 - The range of the set A scores would be unchanged.
 - The interquartile range would be reduced.

Exercise 10A PAGE 177

- | | | | |
|-----------------|---------------|---------------|---------------|
| 1 a 0.34 | b 0.98 | c 0.36 | d 0.84 |
| e 3.08 | f 0.60 | g 0.77 | h 0.77 |

- 3 a** 11.5 **b** 66.4 **c** 52.4 **d** 17.5
e 75.5 **f** 53.1 **g** 63.4 **h** 25.8
- 4 a** 1.3 **b** 3.4 **c** 2.9 **d** 20.5
e 14.0 **f** 12.6
- 5 a** 23.6 **b** 44.4 **c** 54.5
- 6 a** $\frac{a}{b}$ **b** $\frac{c}{b}$ **c** $\frac{a}{c}$ **d** $\frac{a}{b}$
e $\frac{c}{b}$ **f** $\frac{c}{a}$
- 7** $p^2 = q^2 + r^2$
- 8 a i** 5.3 **ii** 5.27
b i 31 **ii** 31.0
- 9** 2.6 **10** 11.2 **11** 9.8 **12** 9.0
- 13** 4.1 **14** 6.3 **15** 16.5 **16** 12.8
- 17** 9.6 **18** 28.7 **19** 65.8 **20** 3.5
- 21** 51.1 **22** 56.5 **23** 6.5 **24** 10.8
- 25** 9.2 **26** 3.7 **27** 40.8
- 28** $x = 16.8, y = 11.8$ **29** $x = 16.7, y = 33.7$
- 30** 53.3
- 31 a** 14.9 cm **b** 9.3 cm
- 32 a** 36° **b** 8.6 cm
- 33 a** The ladder reaches 7.06 m up the wall, to the nearest centimetre.
b The horizontal distance from the foot of the ladder to the wall is 3.76 m, to nearest cm.
- 34 a** The ladder makes an angle of 66° with the ground, to the nearest degree.
b The light is 4.6 metres above ground, correct to one decimal place.
- 35** The kite is 41 m above ground level, to the nearest metre.
- 36 a** AB makes an angle of 22° with the horizontal, to the nearest degree.
b AB is of length 2.15 m, to the nearest centimetre.
- 37** To the nearest centimetre, AB = 12.45 m, BF = 9.53 m and FC = 12.45 m.
- 38** $\theta = 38.7^\circ$ correct to one decimal place.
The length of AD is 64% of the length of AE, to the nearest percent.
- 39 a** Each short wire is of length 7.81 m (nearest cm) and makes an angle of 40° with the ground (nearest degree).
b Each long wire is of length 11.66 m (nearest cm) and makes an angle of 59° with the ground (nearest degree).
- 40** To the nearest centimetre, AC is of length 5.22 m, CG is of length 3.36 m, BH is of length 1.68 m, HC is of length 3.91 m.

- 41** To the nearest degree, each wire makes an angle of 62° with the horizontal.
- 42 a** The largest possible length of the support wire EB is, to the nearest cm, 15.24 m.
b The shortest possible length of the support wire EB is, to the nearest cm, 12.56 m.
- 43** The bob rises 27 mm above its lowest position (to the nearest millimetre).
- 44** To the nearest metre point D is 10 metres above the horizontal ground.
- 45** The total length of steel required is 165 metres (to the nearest whole metre).
- 46** $x = 36.7, y = 6.1$

Exercise 10B PAGE 185

- 1 a** 035° (or N35°E) **b** 080° (or N80°E)
c 110° (or S70°E) **d** 145° (or S35°E)
e 200° (or S20°W) **f** 300° (or N60°W)
g 215° (or S35°W) **h** 260° (or S80°W)
i 290° (or N70°W) **j** 325° (or N35°W)
k 020° (or N20°E) **l** 120° (or S60°E)
- 2 a** 30° **b** 20°
c 35° **d** 15°
- 3** The height of the flagpole is 11.9 m, correct to one decimal place.
- 4** The angle of elevation of the sun is 26° (nearest degree).
- 5** The height of the flagpole is 10.0 m, correct to one decimal place.
- 6** Rounded up to the next metre the length is 19 metres.
- 7** To the nearest kilometre the ships are 10 km apart.
- 8** B is 66 m from A (to the nearest metre).
- 9** Ship C is approximately 26.9 km from ship A.
- 10** C is 216 m from A (to the nearest metre).
- 11** B is 91 m from A (to the nearest metre).
- 12** The height of the second tower is 65 metres, to the nearest metre.
- 13** The tree is approximately 14.8 metres tall.
- 14** The smoke is approximately 11.4 km from the first observation tower.
- 15** The ships are approximately 270 metres apart.
- 16** To the nearest metre the flagpole is 17 metres long.
- 17** The height of the flagpole is 15.5 metres, correct to one decimal place.
- 18** The required angle of elevation is 32° (nearest degree).

Miscellaneous exercise ten PAGE 188

- 1 **a** $x = 9.4$ **b** $x = 18.7$ **c** $x = 56.3$
d $x = 4.1$ **e** $x = 12.1$ **f** $x = 66.4$
- 2 The mean of the six amounts is \$26 350 and the median is \$14 100.
- 3 The mean for the girls was 21.2.
- 4 **a** $x = 21$ **b** $x = 2.5$ **c** $x = 7$
d $x = 0.625$
- 5 **a** $s = 3$ **b** $a = 4$
- 6 The number first thought of was 15.
- 7 John borrowed \$8600.
- 8 **a**

24	12
29	5

b

29	14.5
36.5	7.5
- c**

184	92
269	85

d

34	17
44	10
- e**

36	18
47	11

f

14	7
14	0
- g**

44	22
59	15

h

60	30
83	23
- 9 **a** Approximately 27%.
b 37.9% of the income of the country is earned by the richest 10% of the population.
- 10 $y = 0.5x + 2.5$, $f = 7$, $g = -2$, $h = 13$, $i = -2$, $j = 4.4$.
- 11 **a** 59% **b** More girls than boys.
- 12 **A:** $y = 5$ **B:** $x = -8$
C: $y = 2x - 4$ **D:** $y = -x + 5$
E: $y = 0.2x$ **F:** $y = x - 9$
G: $y = 2x + 15$ **H:** $y = x - 12$
I: $y = 2x - 13$ **J:** $y = -\frac{1}{3}x - 8$
- 13 **a** 49 500 **b** 22 200
c 7.14 tonnes (to 2 d.p.)
- 14 $d = 11.44$ m, $c = 2.86$ m

Exercise 11A PAGE 193

- 1 5.4, 27.2 cm² 2 45 cm²
3 27 cm² 4 20 cm²
5 9.6 cm² 6 211 m²
7 8.8 cm² 8 15.6 cm²
9 18.1 cm² 10 1730 mm²

Exercise 11B PAGE 198

- 1 13.9 cm² 2 75.8 cm² 3 18.8 cm²
4 101.8 cm² 5 87.3 cm² 6 3.8 cm²
7 32.5 cm² 8 23.0 cm² 9 11.8 cm²
10 17.3 cm² 11 10.4 cm² 12 6.8
13 7.3 14 6.6 15 67°
16 52° 17 32°
18 **a** \$125 000 **b** \$51 000
- 19 The second block has the greater area, by 9 m², to the nearest square metre.
- 20 The owner of block A receives \$472 193.55 and the owner of block B receives \$777 806.45.
- 21 AC needs to be 180 metres long, rounded up to the next whole metre.

Exercise 11C PAGE 207

- 1 5.8 2 6.6 3 12.9
4 54.8 5 46.0 6 54.2
7 58 (nearest integer) 8 12.3 (1 d.p.)
9 54 (nearest integer) 10 14 (nearest integer)
11 105 (nearest integer) 12 126 (nearest integer)
13 6.7 (1 d.p.) 14 75 (nearest integer)
15 The pole is of length 614 cm, to the nearest centimetre.
16 The two shot journey is 38 metres further than the direct route, to the nearest metre.
17 7.1 18 1.7
19 21.8 20 73.0
21 59 (nearest integer) 22 14.4 (1 d.p.)
23 43 (nearest integer) 24 111 (nearest integer)
25 44 (nearest integer) 26 62 (nearest integer)
27 11.9 (1 d.p.) 28 146 (nearest integer)
29 The boat is then 13.4 km from its initial position, correct to one decimal place.
30 After eight seconds Jim and Toni are 10.7 metres apart, correct to one decimal place.
31 B is 13.1 km from C, correct to one decimal place.
32 75 (nearest integer) 33 99 (nearest integer)
34 617 (nearest integer) 35 5.39 (2 d.p.)
36 135 (nearest integer) 37 80 (nearest integer)
38 160 (nearest integer) 39 54 (nearest integer)
40 The lengths of AC and BC are 672 cm and 824 cm respectively, each answer given to the nearest cm.
41 The smallest angle of the triangle is of size 42°, to the nearest degree.

- 42** The parallelogram has diagonals of length 5.1 cm and 9.7 cm, correct to one decimal place.
- 43** The parallelogram has sides of length 6.8 cm and 10.8 cm, correct to one decimal place.
- 44** **a** When AC is 2.6 metres $\angle CAB = 20^\circ$, to the nearest degree.
b When AC is 2.1 metres $\angle CAB = 28^\circ$, to the nearest degree.
- 45** **a** 479 cm **b** 239 cm
c 111 cm **d** 222 cm
- 46** **a** At 5 o'clock the distance between the tip of the hour hand and the tip of the minute hand is 155 mm, to the nearest mm.
b At 10 minutes past 5 the distance between the tip of the hour hand and the tip of the minute hand is 119 mm, to the nearest mm.
- 47** **a** The ship is 1.77 km from the lighthouse, correct to 2 decimal places.
b The ship is 1.17 km from the coastal observation position, correct to 2 decimal places.
- 48** **a** Q is 18.5 km from P, correct to one decimal place.
b Q is 21.0 km from the lighthouse, correct to one decimal place.
- 49** Ship B is approximately 15.9 km from ship A, on a bearing 164° .
- 50** Twelve of the steel frameworks would require a total of 260 metres of steel (to the next 10 metres).
- 51** The block has an area of 5270 m^2 and a perimeter of 298 metres, both answers given to the nearest integer.
- 52** The triangular piece that has been removed has an area of 752 mm^2 and a perimeter of 128 mm, both answers given to the nearest whole number.
- 53** The block has an area of 6399 m^2 , to the nearest square metre.

Miscellaneous exercise eleven PAGE 213

- 1** $y = 2x - 1$, B and E.
- 2** **a** $x = 17$ **b** $x = 7$ **c** $x = 2$
d $x = -2$ **e** $x = 35$ **f** $x = 20$
- 3** The estimated mean = 13.9. The median lies in the 11 \rightarrow 15 interval.
- 4** The mean for the whole class of students is 23.8.
- 5** The number I first thought of was 17.
- 6** Four and a half years.
- 7** The flagpole is of height 13.7 metres, correct to one decimal place.

- 8** $a = 6, b = 5, c = 19, d = 7, e = 3$.
5, 7, 11, 13, 15, 21, 21, 25, 30, 31.
- 10** The boats are 7.4 km apart ninety minutes after leaving the harbour, to the nearest 100 m.
- 11** **a** 4 m **b** 3.46 m
c 1 m **d** 1 m
- 12** The direct journey from A to C is 50.6 m shorter than the journey via B (correct to one dp).
- 13** Possibility 1 is of length 176 m, possibility 2 is of length 156 m and possibility 3 is of length 153 m (all to the nearest metre).
- 14** A mean number of nurses absent per day is 10.2, standard deviation 6.1.
They decide to have 14 in pool.
- 15** **a** The bottom bar covers fewer years (0 – 4) than the bar above it (5 – 14).
b Country A has more males than females for the age ranges 0 – 4, 5 – 14 and 35 – 44.
c Country B has more females than males for the age ranges 0 – 4, 15 – 24, 25 – 34, 35 – 44, and 45 – 54.
d Country B has approximately 4 800 000 people aged 55 and over.
e Discuss answer and reasons with your teacher.
- 16** For x from 0 to 2: $y = 0.2x$.
For x from 2 to 3: $y = 0.1x + 0.2$.
For x from 3 to 4: $y = 0.5$.

Exercise 12A PAGE 226

- 1** $x = 1, y = 6$ **2** $x = 3, y = -1$
3 $x = 4, y = 0$ **4** $x = 5, y = -2$
5 $x = -3, y = 2$ **6** $x = 7, y = 5$
7 $x = 3, y = 4$ **8** $x = 3, y = -2$
9 $x = -7, y = 10$ **10** $x = 9, y = 11$
11 $x = 5, y = -2$ **12** $x = 4, y = 7$
13 $x = 4, y = 1$ **14** $x = -3, y = 6$
15 $x = 4, y = 1$ **16** $x = 4, y = -1$
17 $x = 4, y = 3$ **18** $x = 3, y = 5$
19 $x = 7, y = 3$ **20** $A = 5, B = -2$
21 $p = 5.5, q = 3$ **22** $x = 8, y = 15$
23 $x = 10, y = 10$
- 24** **a** Equation 3: $x + y = 600$
b Equation 4: $x - y = 140$
c The baker baked 370 white loaves and 230 wholemeal loaves that day.

- 25 a** Equation 1: $2x + 4y = 1758$
b Equation 5: $x - 5y = 403$
c Adding the number of people at the show to the number of dogs at the show gives total of 811.
- 26 a** Equation 2: $y - x = 5$
b Equation 4: $2x + 3y = 70$
c The two numbers are 11 and 16.
- 27 a** Equation 1: $x + y = 46$
b Equation 5: $x + 0.5y = 32$
c The piggy bank contains 18 coins that are \$1 coins and 28 that are 50 cent coins.
- 28 a** Equation 3: $x + y = 23$
b Equation 6: $28x + 35y = 700$
c The seamstress bought 15 metres of material A and 8 metres of material B.
- 29 a** Equation 3: $x + y = 25\,000$
b Equation 6: $0.96x + 1.12y = 25\,120$
c The investor put \$18 000 into company X and \$7 000 into company Y.
- 30 a** Equation 1: $x + y = 35$
b Equation 5: $3y - 2x = 15$
c $x = 18, y = 17$. The area of the rectangle is 306 cm^2 .
- 31 a** Equation 3: $16x + 7y = 256$
b Equation 6: $20x + 11y = 338$
c For five adults and three children the cost would be \$86.50.
- 32** The equations are $x + y = 41$ and $3y + 2x = 106$. The two numbers are 17 and 24.
- 33** The two numbers are 13 and 24.
- 34** The chemist should use 80 mL from bottle A and 20 mL from bottle B.
- 35 a** $x + y = 12\,000, 1.12x + 1.05y = 13\,195$
b $x = 8500, y = 3500$
- 36** 450 tickets were sold for \$12 each and 1050 were sold for \$8 each.
- 37** The company has 16 of the 56 seaters and 9 of the 35 seaters.
- 38** The two numbers are 7 and 16.
- 39** They sold 46 jars of jam and 32 jars of relish.
- 40** \$75 000 was borrowed at 14% and \$45 000 was borrowed at 17%.
- 41** David answered 19 correctly.
 At least 18 questions must be correctly answered for a mark of at least 50.

- 1 a**
- $74 - 70 \cos x$ does not equal $4 \cos x$. Rule of order not being followed.
 - If $16 = 4 \cos x$, $\cos x = 4$ not 0.25.
 - Inappropriate degree of accuracy in final answer for accuracy of given data.
- b**
- Not correct to use $A = 0.5ab \sin C$ for given information as the angle of known size is not between the two sides of known length.
 - $\frac{76 \times 72 \times \sin 64^\circ}{2}$ does not equal $38 \times 36 \times \sin 32^\circ$.
 Could write it as $38 \times 72 \times \sin 64^\circ$ or as $76 \times 36 \times \sin 64^\circ$ but not what is written in given 'solution'.
 - Final answer claims to be an area but has units of length.
- c**
- Given working involves the sine rule not the cosine rule as claimed.
 - Rearrangement should give $\sin \angle BCA = \frac{6.8 \sin 65^\circ}{7.1}$, not what is claimed.
 - Obtuse angle should not be claimed as being a solution because $\angle BCA$ is not opposite longest side and so cannot be obtuse.
- d**
- Should be taking away $2 \times 5 \times 7 \cos 130^\circ$ not adding it on.
 - $74 + 70 \cos 130^\circ$ does not equal $144 \cos 130^\circ$. Rule of order not being followed.
 - $144 \cos 130^\circ$ gives a negative value for x^2 , not a positive value. A negative value would then mean that x could not be determined.
 - Given value for x includes units but x is a number not a length.
- 2 a** Nominal categorical
b Continuous numerical
c Ordinal categorical
d Discrete numerical
e Continuous numerical
f Continuous numerical
g Discrete numerical
h Nominal categorical
i Nominal categorical
j Discrete numerical
- 3 a** $x = 6, y = 5$ **b** $x = 3, y = 2$ **c** $x = 2, y = 3$
- 4 a** $x = 3.6$ **b** $x = 7$ **c** $x = 15$
d $x = 11, y = 3$ **e** $x = 3.5$ **f** $x = 14$

- 5 A: $y = -x + 60$ B: $y = 60$
 C: $y = 2x - 60$ D: $x = 60$
 E: $y = -2x + 30$ F: $y = 0.5x + 30$
- 6 The direct journey from A to C is 647 m shorter than the journey via B, to the nearest metre.
- 7 The graphs of all members of the family will have a gradient of 3.
- 8 The graphs of all members of the family will pass through the point (0, -7).
- 9 The graphs of all members of the family will have a gradient of -0.5.
- 10 The fire is approximately 19.7 km from lookout No.1 and 18.8 km from lookout No.2.
- 11 0.285 km, i.e. approximately 300 metres.
- 12 a The steeper the line the greater the speed. Hence we can see from the graph that the third stage was the one with the greater average speed.
 b 1st stage: 20 km/h. 2nd stage: 15 km/h. 3rd stage: 25 km/h.
 c From town A to town B the delivery van averaged 60 km/h.
 d From town B to town A the delivery van averaged 80 km/h.
 e The delivery van would need to average more than 120 km/h to arrive back at A before the cyclist got there!
- 13 \$34 000 secure, \$16 000 risky
- 14 \$15 600 secure, \$40 000 risky
- 15 Answers not given here. Check that your part **b** and **c** answers are the same.
- 16 a i The statement is incorrect. There are the same number of results to the right of the median as there are to the left of the median.
 ii The box plots do not tell us how many students were involved. Hence we cannot conclude that there were more 14 year olds than 12 year olds.
 iii The word 'much' is open to interpretation. Better to quantify rather than use words like 'much'. The reader can then decide if they wish to interpret the difference as 'much'. Instead could say, for example: The interquartile range for the 14 year olds was 7 seconds which exceeded that of the 12 year olds which was 6 seconds.
- b Discuss the reasonableness of your statements with others in your class.

Exercise 13A PAGE 238

- 1 a 1 b 1.7 c -2
 d 0.5 e -0.75
- 2 Test A: 2.5, Test B: -1, Test C: 1.25, Test D: 0.2
- 3 Computing (1.216), Chemistry (0.278), Mathematics (-0.385), Electronics (-0.616)
- 4 English, Mathematics, Science, Social Studies.
- 5 Jill: 'Well I got 1.'
 Jill: 'The mean was zero.'
 Jill: 'Oh he got -0.25.'

Exercise 13C PAGE 247

- 1 0.2266 2 0.6377 3 54.56
- 4 a 0.5828 b -0.6433 c 1.2265
 d -0.7388
- 5 a 19.5 b 21.9 c 18.7
 d 23.1
- 6 a 0.68 b 0.95 c 0.997
 d 0.95 e 0.997 f 0.34
 g 0.84 h 0.16 i 0.84
 j 0.16
- 7 a 99.7% b 16% c 13.5%
- 8 a 16% b 2.5%
- 9 a 0.3085 b 0.0062
- 10 a 0.5 b 0.34
- 11 a 0.3085 b 0.2902 c 0.0228
- 12 a approx 11 b approx 11 c approx 39
- 13 0.0548
- 14 a 415 b 217
- 15 To nearest 0.5 cm: 158.5 cm, 191.5 cm
- 16 A/B: 78, B/C: 68, C/D: 55, D/F: 47
- 17 a 0.1587 b 7.38 am c 7.33 am
- 18 a 2 yrs b 7 yrs
 c 91 yrs 0.783
- 19 a approx 40 b 0.236

Miscellaneous exercise thirteen PAGE 251

- 1 a Equation 3: $4 = 3m + c$
 b Equation 5: $19 = 8m + c$
 c The straight line equation is $y = 3x - 5$.
- 2 mean 24.08, median 24, mode 20, range 29.
- 3 Four hot dogs and six burgers would cost a total of \$32.20.

- 4 $x = 16, y = 5$. The rectangle has an area of 80 cm^2 .
- 5 Each mixed bag should contain 8 chocolates and 12 lollies.
- 6 **a** The cost of constructing 25 km of similar highway would be 155 million dollars.
b The cost of constructing 52 km of similar highway would be 317 million dollars.
- 7 **a** 167, 168, 169 **b** 166, 168, 170
- 8 $337^\circ, 3.4 \text{ km}$
- 9 The topmost point is 35 metres above ground (to the nearest metre).
- 10 **a** Revenue is likely to be zero for zero units sold. Thus line I is likely to be revenue line.
 Costs are likely to have some fixed costs for 'set up' then an amount per unit made. Thus line II is likely to be cost line.
b Break even when $x = 40$ (then Cost = Revenue = \$3200).
c I: $R = 80x$ II: $C = 2000 + 30x$
- 11 The yacht travelled 918 metres, to the nearest metre
- 12 If the agent attempts to calculate a mean he will have to decide what values to assume for the one property '\$400 000 or less' and the six that are 'over \$1 000 000'. Hence determining the mean presents a problem. Using the modal class will not be an indicator of *central* tendency. The median is probably the most suitable to work with. The median will be the 15th sale price and will lie in the \$700 001 \rightarrow \$800 000

interval. The 15th will be the 'last' value in this class so he could suggest a value in this interval but towards the \$800 000 end. Hence a reasonable average for the given data could be about \$780 000.

In reality a lot may depend on what he wants the average for. If a customer is asking it may be more informative to show the customer the whole table as this shows the quite considerable range of prices. Just knowing an average may not be that helpful. If the average is really needed perhaps the agent should attempt to find out the actual sale prices of the 29 properties. Just how recent the sales were made could also be an important consideration.

- 13 **a** The total length of aluminium is 775 cm, to the nearest centimetre.
b The area of glass is $19\,580 \text{ cm}^2$, to the nearest 10 square centimetres.
- 14 The distance between the hands is 471 mm, to the nearest millimetre.
- 15

Initials	PA	CB	JB	CC	JD	KD	LF	LJ	MJ	EK	IM	PN
Grade	C	D	A	D	B	C	F	C	B	C	B	C
Initials	RN	PP	AR	TR	VR	AS	PS	TS	BV	PV	IW	RZ
Grade	D	B	D	C	B	C	C	C	D	B	F	C

- 16 **a** 41 m (nearest metre) **b** 26 m (nearest metre)
c 49 m (nearest metre) **d** 328° (nearest degree)

- 17 Graph of amount of water in tank over 12 day period.



- 18 **a** 144 **b** 1280
c 167 **d** 122.7, 131.3
- 19 **a** $\sim 32.5 \text{ m}$ **b** $\sim 61 \text{ m}$

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