PEARSON

Mathematics

STUDENT BOOK | 3RD EDITION

9

AUSTRALIAN CURRICULUM







Pearson

Pearson Mathematics 9 3rd Edition

Student Book

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We pay our respects to Elders, past and present.

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How to use Pearson Mathematics

The components of Pearson Mathematics

This *Student Book* is a complementary resource that offers a print medium for corresponding lessons in *Pearson Secondary Teaching Hub* and the *Student Companion*. It is designed on the principles of current best practice and provides students with an offline version to access the theory, worked examples and practice questions from the digital lessons in the *Hub*, to give the perfect blend of online and offline learning.

The *Student Book* (also available as an eBook) gives you an offline version of the theory, worked examples and independent practice questions. The *Student Book* has been supplied to minimise digital distraction and screen fatigue, supporting teachers and students to find the perfect balance of online and offline learning.

The *Hub* is a one-of-a-kind digital product designed to simplify teaching and energise learning, with high-quality content created by experienced Australian educators who know how to engage students.



The *Student Companion* is a write-in workbook that has been designed to support the guided practice phase of the lesson, giving students scaffolded instruction as an introduction to the skill. The *Student Companion* also supports the creation of a bound reference or portfolio of learning over the year.

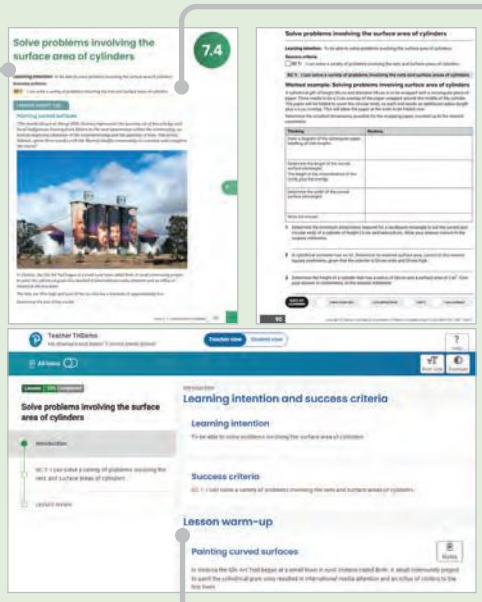
Chunking with learning intentions and success criteria

The structure and relationship between each of the three components in the Pearson Mathematics series (*Hub*, *Student Book* and *Student Companion*) has been designed to manage the cognitive load of our students. Chunking has been used to break complex concepts and strategies into smaller, more manageable sections, defined by the success criteria.

In the *Hub* and the *Student Companion*, the chunking of content directly supports success criteria, while the *Student Book* is chunked by learning intention, to give both teachers and students autonomy in delivery.

Learning intentions are statements aligned to the relevant curriculum. Each of the lessons in the *Hub* have been developed around a single purpose for learning. The learning intentions are communicated using student-friendly language and share the learning goals or objectives from the corresponding digital lesson in *Hub* and pages of the *Student Companion*. They describe what learners should know, understand or be able to do by the end of the lesson.

Every lesson starts with a **Lesson warm-up**, a 3- to 5-minute activity designed to activate prior knowledge and prime students for learning. In the *Hub*, teachers are able to access teaching notes that contain sample solutions, different ways to deliver the activity, and enabling and extending prompts to ensure all learners have an entry point and can be extended.



Success criteria clarify expectations and describe what success looks like. They are used to determine how well a student has met the learning intention. The Pearson Mathematics lesson design is based on cognitive load principles and, as such, each lesson has between one and four success criteria. The success criteria are specific, concrete and measurable so learners can actively engage with and reflect on their evidence of learning within each lesson. The success criteria form the basis of all three components — the *Hub*, the *Student Book* (and eBook) and the *Student Companion* — to ensure they are referred to throughout the lesson and provide the basis for feedback.

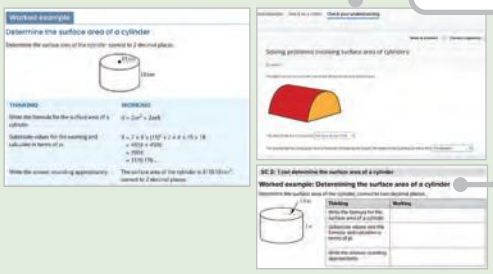
Reducing cognitive load with worked examples

Worked examples provide students with a step-by-step solution to a problem.

The worked examples in the *Student Book* correspond to those in the digital lesson and are provided for each skill to:

- scaffold learning
- support skill acquisition
- reduce the cognitive load.

In the *Hub*, each worked example is presented as a video walkthrough, and has a 'Check your understanding' section with one to three autocorrecting questions. This gives students and teachers immediate feedback on student understanding of the required skills to succeed before moving on to the independent practice phase of the lesson.



The *Student Companion* supports the gradual release of responsibility, supporting the guided practice phase of the lesson by giving students access to a similar problem presented as a 'try yourself' worked example and one to three additional scaffolded practice questions. The *Student Companion* also provides students with the opportunity to develop a learning portfolio and can be used in a flipped environment, in classroom learning, as homework or in preparation to create a bound reference or summary.

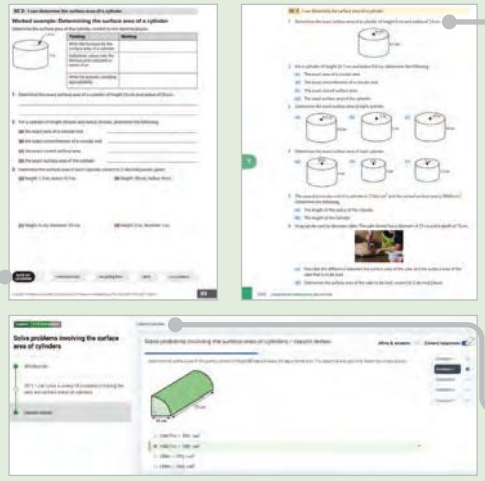
Gradually releasing responsibility with print and digital

I do: Students and teachers are supported by new information being demonstrated and modelled with worked examples and the worked example videos. The 'Check your understanding' questions provide immediate feedback to both students and teachers on the student's level of understanding of the new concept.

We do: The *Student Companion* offers a 'try yourself' version of the worked example and a small set of scaffolded questions designed to facilitate the guided practice phase of the lesson.

You do: The success criteria each have a set of 'Practice' questions available to access in both the *Hub* and the *Student Book*, designed to be used in the independent practice phase of the lesson.

Each lesson in the *Student Companion* contains a space for students to reflect on their understanding. The simple and intuitive design of the **lesson reflection tool** allows students to scale their confidence, reflect on their learning and identify areas in which they need additional support.



The **Practice** section in the *Student Book* contains the independent practice questions for the lesson, still broken down by success criteria. These questions are provided in both *Hub* and the *Student Book* to give teachers and students the choice of digital or print for the independent practice phase of the lesson. These questions are designed to ensure learners build confidence and demonstrate proficiency.

Each lesson in the *Hub* contains an additional six to eight auto-correcting questions. These questions provide a summative assessment of the lesson and utilise the skills and knowledge obtained throughout the entire lesson. These assessments give students and teachers immediate feedback on student readiness to move forward in the topic sequence.

Pearson Mathematics 3E

Simplified teaching, energised learning for the new Australian Curriculum Mathematics Year levels 7–10

Powered by the Pearson Secondary Teaching Hub

How have we simplified teaching?

In consultation with Australian teachers and students, we've built a simple-to-use hub of lesson content for the new Australian Curriculum Mathematics.

How have we energised learning?

Engaging videos, worksheets, auto-correcting questions, worked examples and more, with the perfect mix of online and offline resources

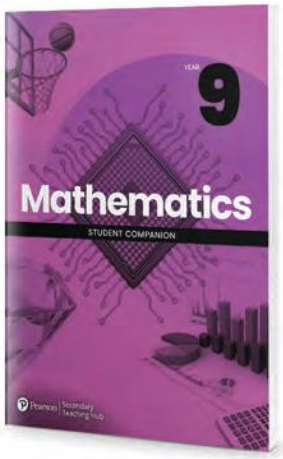
Student Books





Teaching Hub with eBooks

Student Companions



Student Books link to the Hub for flexible teaching

Giving you the autonomy to teach online or offline to meet the needs of your students

- Developed using the latest teaching and learning research
- Access to all 7-10 content in the Hub for students that need revision, or those that need extension
- eBook versions available in the Hub

Student Companions provide write-in practice books

Accompanying the Teaching Hub, with try yourself worked examples and exercise questions to support guided practice

- 1000s of practice questions across the series
- Use as a homework activity program, guided practice in class, or to support multiple exposures

Discover Pearson Mathematics at **pearson.com.au/schools/maths-ac**

TOPIC
1

Real numbers

Recall 2

1.1 Identify and define irrational numbers 3

1.2 Apply irrational numbers 9

1.3 Plot and locate irrational numbers on a number line by estimation and by construction 15

Topic review 23

Why learn this?

Grasping real number concepts, including the ability to identify and define irrational numbers, helps individuals:

- understand the complete set of numbers used in everyday mathematics, which includes both rational and irrational numbers
- accurately represent quantities that cannot be expressed as simple fractions, which is vital for precise scientific calculations, technical measurements and financial computations
- develop a foundational understanding for algebra and calculus, which are key in higher education and numerous scientific and engineering careers
- solve problems more efficiently by approximating and applying irrational numbers in various contexts, from computing square roots to exploring the properties of circles
- interpret and work with real-world data where irrational numbers often arise, such as measuring natural phenomena or analysing statistical information.

RECALL

ANSWERS Page 607

I can identify prime and composite numbers

1 From the set of numbers
21, 19, 2, 12, 3, 9, 25, 37, 4, 41, 33, 28, 7, 6, 16, 14
identify all the:

(a) even numbers

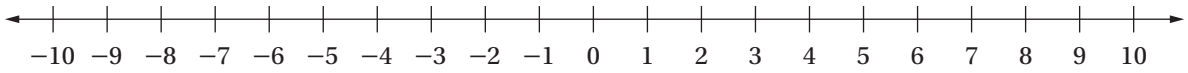
(b) prime numbers

(c) composite numbers

(d) perfect squares.

I can locate and plot integers on a number line

1 Plot each of the following integers on a number line.

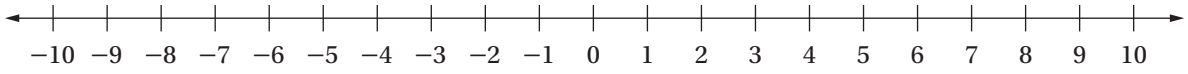


(a) 3 **(b)** −7 **(c)** 8

(d) −1 **(e)** 0 **(f)** −5

I can locate and plot mixed numbers on a number line

1 Plot each of the following fractions onto a number line.



(a) $2\frac{1}{2}$ **(b)** $-3\frac{1}{2}$ **(c)** $\frac{8}{16}$

(d) $\frac{11}{2}$ **(e)** $-\frac{26}{4}$ **(f)** $1\frac{2}{3}$

I can round decimals to a required number of decimal places

1 Round each of the following numbers to 2 decimal places.

(a) 9.2483 **(b)** 6.1529 **(c)** 0.0382

2 Round each of the following numbers to 3 decimal places.

(a) 0.1572 **(b)** 1.2068 **(c)** 13.2369

3 Round each of the following numbers to the nearest whole number.

(a) 12.745 **(b)** 1.259 **(c)** −3.641

Identify and define irrational numbers

1.1

Learning intention: To be able to identify and define irrational numbers

Success criteria:

SC 1 I can define a rational number and determine whether a number is rational or irrational.

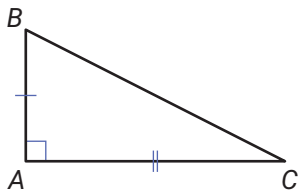
SC 2 I can use a number line to indicate the solution interval for inequalities.

Lesson warm-up

Golden ratio

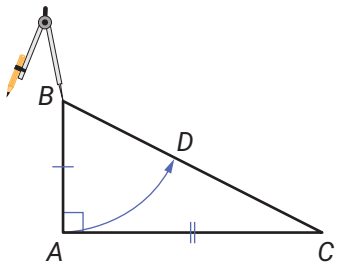
Time for some strange magic involving triangles.

Take a look at this special right-angled triangle ABC.

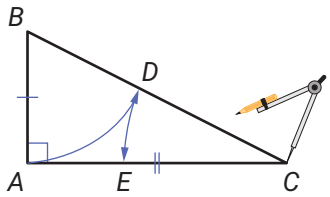


The length of side AC is exactly double the length of side AB.

Using a compass with the centre placed at vertex B, and a radius of AB, an arc can be drawn to the hypotenuse BC. Label this point D.



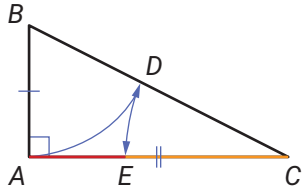
Now place the centre point at vertex C, and with a radius of CD, draw an arc from D on the hypotenuse to the side AC. Label this point E.



The ratio of the lengths AE to EC will always be ~ 1 : 1.618...

This ratio is a special irrational number known as the golden ratio and is denoted by the Greek letter phi (φ).

Neatly draw a right-angled triangle of any reasonable size, ensuring one of the shorter sides is double the length of the other. Now carefully and neatly, follow the steps outlined above and compare the lengths AE to EC for your triangle.



Did you get close to the golden ratio?

SC 1 I can define a rational number and determine whether a number is rational or irrational

A rational number is any number that can be written in the form $\frac{a}{b}$, where $b \neq 0$ and a and b are integers. This set of numbers includes all the integers as well as positive and negative fractions. An irrational number is a number that cannot be written in the form $\frac{a}{b}, b \neq 0$. This set of numbers contains surds, such as $\sqrt{3}$, non-terminating, non-recurring decimals and some special numbers, called transcendental numbers, the most common of which is π. There is another transcendental number used in secondary mathematics courses and that is Euler's number e, which has an approximate value of 2.718 281 8...

Together, the set of rational numbers and the set of irrational numbers make the set of real numbers.

Worked example

Identifying rational numbers in a list

Label each of the following numbers as either rational or irrational.

$\sqrt{16}$, -6π, 1.83, $1.\dot{6}$, $\sqrt{5}$ and $\sqrt[3]{8}$

THINKING	WORKING
Recall the definition of a rational number.	A number that is rational can be written in the form $\frac{a}{b}$.
Examine the square roots.	The square root of a square number is rational. $\sqrt{16} = \sqrt{4^2}$ $= 4$ $= \frac{4}{1}$
	If the number is not a perfect square, the number is irrational. $\sqrt{5}$
Examine the cube root.	The cube root of a perfect cube is rational. $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2}$ $= 2$ $= \frac{2}{1}$

Identify any recurring or terminating decimals. Recurring and terminating decimals are rational.	$1.83 = \frac{183}{100}$ $1.\dot{6} = 1.666\,66\ldots$ $= 1\frac{2}{3}$ $= \frac{5}{3}$
Identify any terms that contain transcendental numbers (π or e), which are irrational.	-6π is irrational because π is transcendental.
Write the answer.	The rational numbers are $\sqrt{16}$, 1.83, $1.\dot{6}$ and $\sqrt[3]{8}$. The irrational numbers are $\sqrt{5}$ and -6π.

SC 2 I can use a number line to indicate the solution interval for inequalities

An equation has a finite number of solutions. For example,

$2x - 5 = 3$ has the solution $x = 4$

and

$2x^2 = 8$ has solutions $x = -2$ and $x = 2$.

An inequality has an infinite number of solutions because the solution will include an inequality sign. For example,

$3x - 4 > 5$

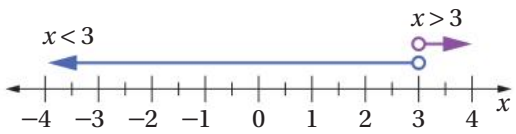
$3x > 9$

$x > 3$

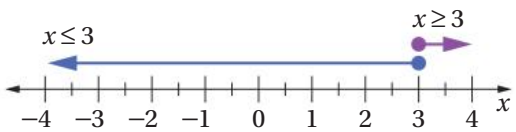
A number line is a useful way of representing the solution for an inequality.

There are some mathematical conventions regarding showing an inequality on a number line.

An open circle, ○, indicates that the end value is not included in the solution, such as in $x > 3$ or $x < 3$.



A closed circle, ●, indicates that the end value is included in the solution, such as in $x \leq 3$ or $x \geq 3$.

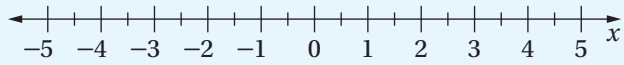
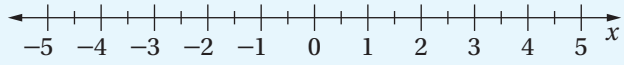
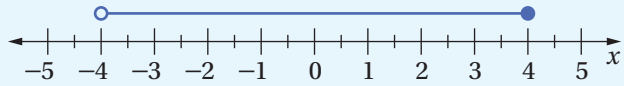


Worked example

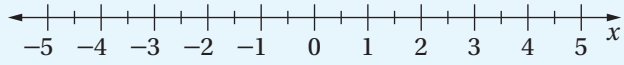
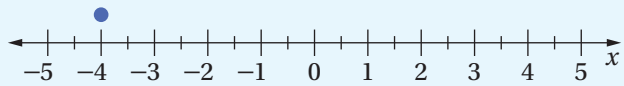
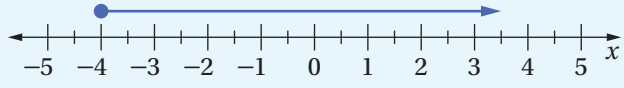
Showing a solution interval on a number line

Show the following intervals on a number line.

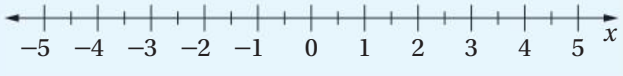
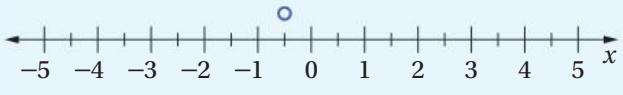
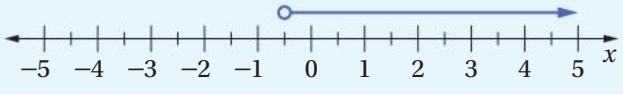
(a) $-4 < x \leq 4$

THINKING	WORKING
Draw a number line big enough to show all the values required. (Here, –5 to 5 will be used.)	
Plot the endpoint/s on the number line using a closed circle for an included value and an open circle if it is not included.	
Complete the number line.	

(b) $x + 6 \geq 2$

THINKING	WORKING
Solve the inequality to determine the value of the unknown.	$x + 6 \geq 2$ $x + 6 - 6 \geq 2 - 6$ $x \geq -4$
Draw a number line big enough to show all the values required.	
Plot the endpoint/s on the number line using a closed circle for an included value and an open circle if it is not included.	The endpoint is at –4. 
Complete the number line.	

(c) $-2x < 1$

THINKING	WORKING
Solve the inequality to determine the value of the unknown. Note that when multiplying or dividing by a negative number, the inequality sign is reversed.	$-2x < 1$ $-x < \frac{1}{2}$ $x > -\frac{1}{2}$
Draw a number line big enough to show all the values required.	
Plot the endpoint/s on the number line using a closed circle for an included value and an open circle if it is not included.	The endpoint is at $-\frac{1}{2}$. 
Complete the number line.	

Practice

ANSWERS Page 607

SC 1 I can define a rational number and determine whether a number is rational or irrational

1 Label each of the following numbers as rational or irrational. For the rational numbers, show the value expressed in the form $\frac{a}{b}$, where a and b are integers.

(a) $\sqrt[3]{125}$ **(b)** $6e$ **(c)** $1\frac{15}{22}$

(d) $9.\dot{3}$ **(e)** $\sqrt{48}$ **(f)** $-\sqrt{81}$

2 Express each of the following numbers in the form $\frac{a}{b}$, thus confirming they are rational. In each case, express the fraction in simplest form.

(a) 1.9696... **(b)** $1.3\dot{5}$ **(c)** $2.\overline{04}$ **(d)** $0.\dot{1}2\dot{3}$

3 Demonstrate why 0 is a rational number by expressing it in the form $\frac{a}{b}$.

4 Explain why $\sqrt[3]{-8}$ is rational, but $\sqrt{-4}$ is not even a real number.

5 Complete the following sentences.

(a) Given that $\sqrt{n}$ is rational, then n is a ____.

(b) Assuming that $\sqrt[3]{n}$ is rational, then n is a ____. The value of n can be ____ or ____.

(c) Every repeating fraction is ____.

6 Label each of the following numbers as rational or irrational. For the rational numbers, show the value expressed in the form $\frac{a}{b}$, where a and b are integers. For the irrational numbers, state a brief justification.

(a) $\sqrt{32}$ **(b)** $\sqrt{\pi^4}$ **(c)** $(1.\dot{2})^3$ **(d)** $\sqrt[3]{250}$

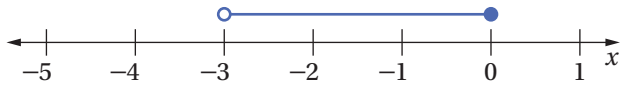
SC 2 I can use a number line to indicate the solution interval for inequalities

1 Draw separate number lines to show the following solution intervals.

(a) $-2 < x < 3$ **(b)** $x < -2$ **(c)** $x \geq 2$ **(d)** $-1 \leq x < 4$

2 State the solution interval shown in each of the following diagrams.

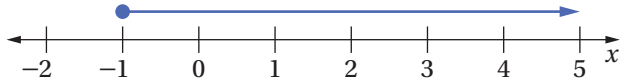
(a)



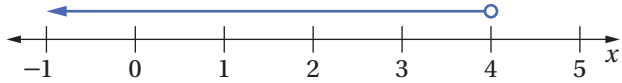
(b)



(c)



(d)



3 State the solution interval shown in each of the following diagrams.

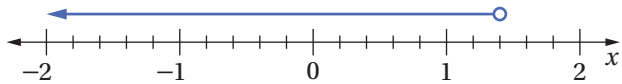
(a)



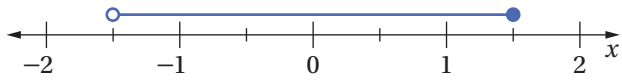
(b)



(c)



(d)



4 Solve each of the following inequalities and show the solution interval on a number line.

(a) $3x + 4 < 7$ **(b)** $2x - 1 \geq 4$ **(c)** $6x + 2 \leq -1$ **(d)** $4x - 3 > -2$

5 Solve each of the following inequalities and show the solution interval on a number line.

(a) $2 - 3x > 1$ **(b)** $2(x - 1) \leq -4$ **(c)** $5 - 2x \geq -1$ **(d)** $-3(x + 2) < 1$

6 Complete the following statements.

(a) The solution interval $x \geq a$ does/does not include the value a.

(b) When solving an inequality, the direction of the inequality changes only if _____ .

(c) The value $0.\dot{3}$ is/is not part of the solution for the inequality $18x > 6$.

(d) The following statement is true/false. The endpoints of solution intervals must be rational numbers.

Apply irrational numbers

1.2

Learning intention: To be able to apply irrational numbers

Success criteria:

SC 1 I can write approximate values for irrational numbers.

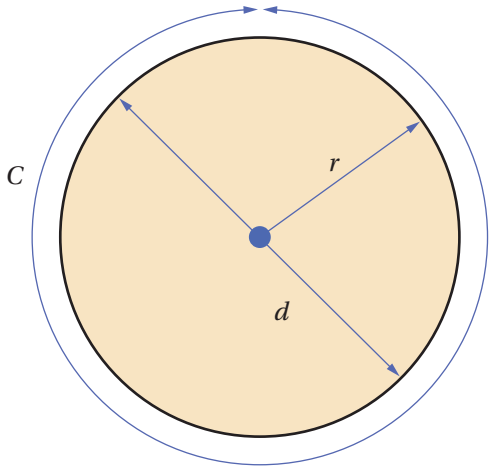
SC 2 I can apply irrational numbers in the solution of problems.

Lesson warm-up

Approximating pi

From calculations involving the circumference of circles, the value of pi (π) was introduced as a special ratio of the circumference to the diameter (or two times the radius).





$C = \pi d$
$= 2\pi r$

Pi is an irrational number and in some texts is approximated using $\frac{22}{7}$.

An even better approximation of pi is $\frac{355}{113}$.

1 Write down and compare the decimal values of π, $\frac{22}{7}$ and $\frac{355}{113}$ accurate to 6 decimal places.

2 Explain the apparent contradiction that approximating pi with $\frac{355}{113}$ represents.

3 Explain why it is called an approximation, even if it is identical to the real value for pi up to 6 decimal places.

4 What does pi being irrational tell you about its decimal values?

SC 1 I can write approximate values for irrational numbers

An irrational number may not have an exact decimal value, but an approximation to a set number of decimal places can be easily obtained using a calculator.

For example, a calculator may give the value of π as 3.141 592 654. Although there is no indication given on the calculator that this is a non-repeating, non-terminating decimal number, it is expected that you know this. The following approximations would be acceptable for π.

2 decimal places: 3.14

3 decimal places: 3.142

6 decimal places: 3.141 593

The approximation to 2 decimal places is often used in measurement applications related to circles.

The first few digits of the irrational number e might suggest it is a repeating decimal: 2.718 281 82...

However, looking further down, the list of digits shows that this is not the case:
2.718 281 828 459 045 235 3...

Worked example

Writing a decimal approximation for an irrational number

Write the value of $\sqrt{7}$ correct to:

(a) 2 decimal places

THINKING	WORKING
Use your calculator to obtain a value for the irrational number.	$\sqrt{7} = 2.645\,75131\ldots$
Round this value to the appropriate number of decimal places.	$\sqrt{7} = 2.64\underline{5}\,75131$ $= 2.65\,(2\text{ d.p.})$

(b) 5 decimal places

THINKING	WORKING
Round the calculator value to the appropriate number of decimal places.	$\sqrt{7} = 2.64575\underline{1}31$ $= 2.64575\,(5\text{ d.p.})$

SC 2 I can apply irrational numbers in the solution of problems

Circumference of a circle formula

Many formulas in mathematics include irrational values represented by their special character/symbol.

The circumference (C) of a circle can be calculated using the formula $C = \pi d$, where d is the diameter of the circle.

Pi (π) represents an irrational number with a value of 3.141 59 accurate to 5 decimal places.

For a diameter of a circle that is a rational, real measure such as 10 cm, the perimeter can be written in two ways, following substitution into the formula:

$C = 10\pi$ cm or $C = 31.4159$ cm (accurate to 4 decimal places)

The first way, $C = 10\pi$ cm, is referred to as the exact answer because the symbol for pi represents the entire irrational number, not just a rounded approximation.

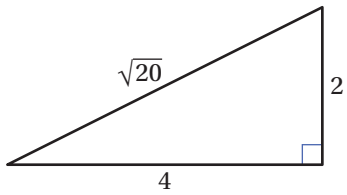
Using the symbol, and not actually calculating an approximate value, this form represents an exact real number representation of the circumference.

The second way, $C = 31.1459$ cm, is an approximate decimal answer since the infinite decimal places for pi are not included.

Rounding the answer after calculating (or rounding the value of pi before calculating) can only result in an approximate decimal value of the circumference.

Pythagoras' theorem

Calculations resulting in, or involving, irrational square roots can also be represented as exact or approximate answers.



From Pythagoras' theorem, the hypotenuse of the triangle is calculated using $c^2 = a^2 + b^2$.

$$\begin{aligned} c^2 &= 4^2 + 2^2 \\ &= 16 + 4 \\ &= 20 \end{aligned}$$

$\therefore c = \sqrt{20}$ units

$c = 4.472$ units (3 d.p.)

$\sqrt{20}$ represents the exact value of the hypotenuse, as this form includes all the decimal places.

4.472 represents the approximate decimal value.

It is always best to leave the exact form throughout all calculations and only round in the final calculation.

Worked example

Determining the area of a circle

The radius (r) of a circle is 5 cm. Substitute this into the formula for the area of a circle ($A = \pi r^2$) to determine:

(a) the exact value of the area

THINKING	WORKING
Recall the formula for the area of a circle.	$A = \pi r^2$
Substitute the known values and simplify.	$A = \pi \times 5^2$ $= 25\pi \text{ cm}^2$
Write the answer.	The exact area of a circle with a radius of 5 cm is $25\pi \text{ cm}^2$.

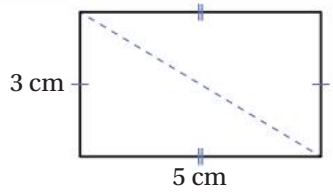
(b) the area of the circle accurate to 2 decimal places.

THINKING	WORKING
Recall the formula for the area of a circle.	$A = \pi r^2$
Substitute the known values and calculate the value using a calculator. Round the answer as required.	$A = \pi \times 5^2$ $= 78.539816\ldots$ $= 78.54 \text{ (2 d.p.)}$
Write the answer.	The area of a circle with a radius of 5 cm is 78.54 cm^2 accurate to 2 decimal places.

Worked example

Calculating with Pythagoras' theorem

Calculate the exact length of the diagonal for this rectangle.



THINKING	WORKING
Recall Pythagoras' theorem for right-angled triangles.	$c^2 = a^2 + b^2$
Substitute the shorter side values of the right-angled triangle formed by the diagonal values and simplify, leaving the answer as a square root.	$c^2 = 3^2 + 5^2$ $= 9 + 25$ $= 34$ $\therefore c = \sqrt{34} \text{ cm}$
Write the answer.	The diagonal of the rectangle has a length of $\sqrt{34}$ cm.

Practice

ANSWERS Page 608

SC 1 I can write approximate values for irrational numbers

1 Write each of the following irrational numbers as a decimal, correct to:

(i) 2 decimal places

(ii) 5 decimal places.

(a) $\sqrt{11}$ **(b)** $\sqrt{22}$ **(c)** $\sqrt[3]{4}$ **(d)** $\sqrt[3]{20}$

2 Write each of the following irrational numbers as a decimal, correct to:

(i) 2 decimal places

(ii) 5 decimal places.

(a) $\frac{3\pi}{4}$ **(b)** $2\pi - 2$ **(c)** $3e$ **(d)** $\frac{2e+1}{3}$

3 Complete the following regarding the value of $\sqrt{5}$.

(a) The nearest perfect square lower than 5 is ___.

(b) The square root of this number is ___.

(c) The nearest perfect square greater than 5 is ___.

(d) The square root of this number is ___.

(e) Hence $\sqrt{5}$ lies between ___ and ___ and will be closer to ___.

4 Between which two whole numbers is it certain that the following irrational numbers will be found?

(a) $\sqrt{30}$ **(b)** $\sqrt{47}$ **(c)** $\sqrt{106}$ **(d)** $\sqrt{211}$

5 Between which two whole numbers is it certain that the following irrational numbers lie?

(a) $\sqrt[3]{20}$ **(b)** $\sqrt[3]{76}$ **(c)** $\sqrt[4]{15}$ **(d)** $\sqrt[4]{90}$

SC 2 I can apply irrational numbers in the solution of problems

1 Calculate the exact circumference of each circle with the following diameters.

(a) $d = 6$ mm **(b)** $d = 17$ mm **(c)** $d = \frac{1}{2}$ cm **(d)** $d = 2.2$ cm

2 Use the rule $C = 2\pi r$ to calculate the circumference of each circle with the following radii, leaving your answers in:

(i) exact form, and

(ii) accurate to 2 decimal places.

(a) $r = 9$ cm **(b)** $r = 13$ m **(c)** $r = 3.3$ mm **(d)** $r = 1.75$ cm

3 Use the rule $A = \pi r^2$ to calculate the area of each circle with the following radii, leaving your answers in:

(i) exact form, and

(ii) accurate to 2 decimal places.

(a) $r = 6$ cm **(b)** $r = 17$ mm **(c)** $r = \frac{1}{2}$ cm **(d)** $r = 2.2$ m

4 Use the rule $V = \pi r^2 h$ to calculate the volume of the cylinders with the following dimensions, leaving your answers in:

(i) exact form, and

(ii) accurate to 2 decimal places.

(a) base radius $r = 3$ m and height $h = 5$ m

(b) base radius $r = 4$ mm and height $h = 20.5$ mm

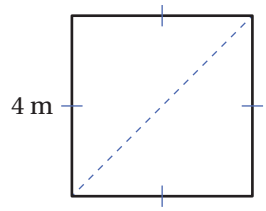
(c) base radius $r = 5$ cm and height $h = 4.2$ cm.

5 Which of the following representations for the surface area (S) of a cylinder is the most accurate?

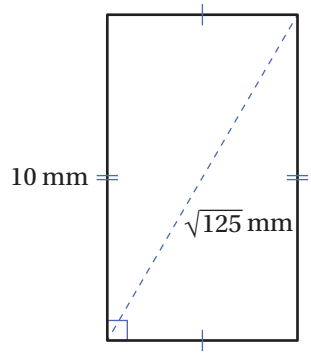
$S = 32\pi$ cm^2 or $S = 100.530\,964\,9$ cm^2

Give reasons for your answer.

6 Calculate the exact length of the diagonal for this square.



7 What is the value of the unknown shorter side of this rectangle?



8 Write the integer dimensions of the rectangles with the following diagonal lengths.

(a) $\sqrt{52}$ cm **(b)** $\sqrt{117}$ m

Plot and locate irrational numbers on a number line by estimation and by construction

1.3

Learning intention: To be able to plot and locate irrational numbers on a number line by estimation and by construction

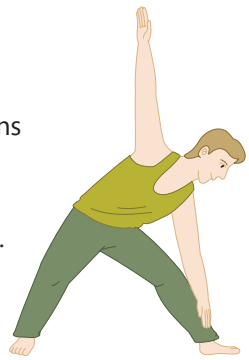
Success criteria:

SC 1 I can estimate the location of irrational numbers on a number line.

SC 2 I can use construction techniques to accurately locate irrational numbers on a number line.

Lesson warm-up

Pythagoras lock puzzle



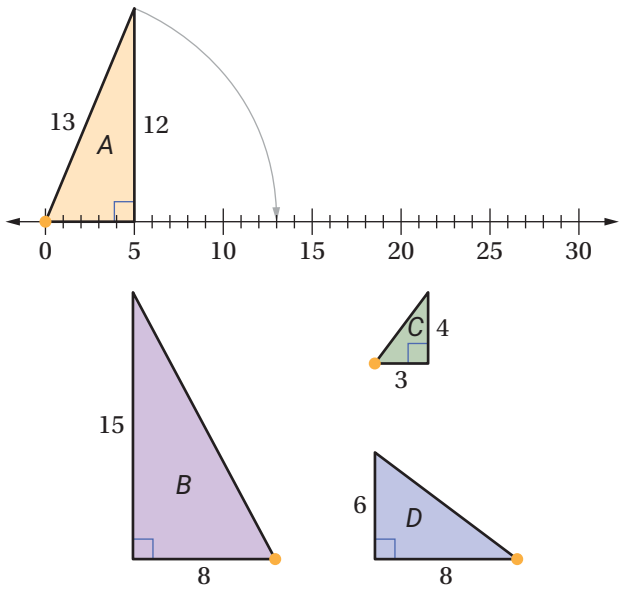
A strange and ancient Pythagorean lock has been uncovered!

To solve it, all the right-angled triangles must be placed in the correct positions on the number line. When in the correct positions, all arcs, centred at the yellow dots with a radius equal to that triangle's hypotenuse, will intersect at 13 on the number line. The orientation of the triangles cannot be changed. The first triangle has been placed in the correct position to get you started.

Can you solve the lock?

Use Pythagoras' theorem to first calculate the length of each hypotenuse.

Write down the triangle letter and value on the number line where each vertex with the dot must be placed.



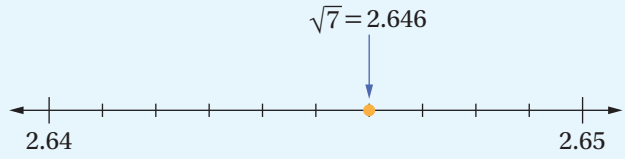
SC 1 I can estimate the location of irrational numbers on a number line

A calculator can be used to estimate the value of an irrational number to whatever degree of accuracy is required. This estimated value can then be located on a number line. With such number lines, it is usually enough to just show the part of the number line immediately surrounding the value.

Worked example

Plotting an approximate value for an irrational number on a number line

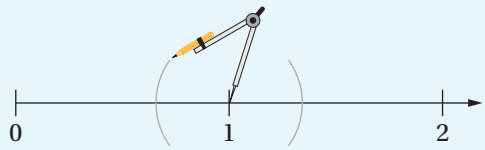
Write the value of $\sqrt{7}$ correct to 3 decimal places and plot its position on a number line.

THINKING	WORKING
Write the approximate value of the irrational number.	$\sqrt{7} = 2.64575131\ldots$ Correct to 3 decimal places, this is 2.646.
Identify the two nearest decimals that the value lies between.	This lies between 2.64 and 2.65.
Draw a number line showing the approximate value.	

SC 2 I can use construction techniques to accurately locate irrational numbers on a number line

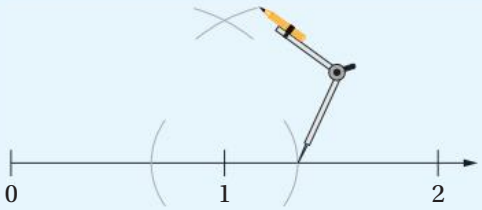
Constructing perpendicular lines at a point on a number line using a compass

A compass, or a pair of dividers, can be used to locate the position of irrational numbers accurately on a number line. This section shows how an irrational number can be represented in a physical sense, then transferred to a number line.

INSTRUCTION	EXAMPLE
Step 1 Before representing irrational numbers on a number line, it is necessary to know how to draw a line perpendicular to a point on a line using a compass. Place the compass point at 1 and draw two small arcs of equal radius, crossing the number line on either side.	

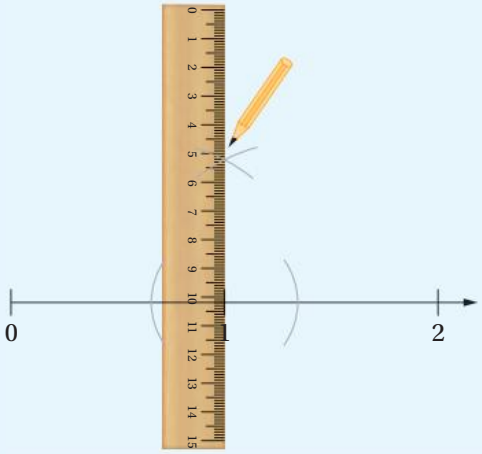
Step 2

Increase the radius of the compass, then place the point where each previous arc crossed the number line and draw two arcs with the same radius such that they form a small cross above the point (1).



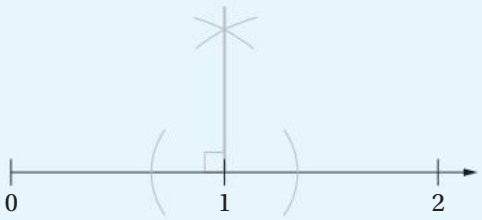
Step 3

Draw a straight line from the point (1) on the number line, through the intersection of the small cross.



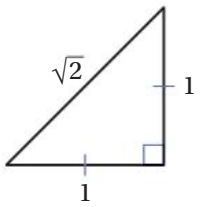
Step 4

Mark this new line as perpendicular to the number line.



Displaying an irrational number on a number line using construction

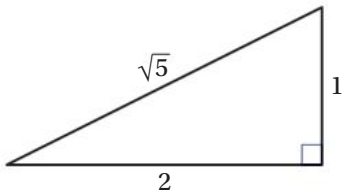
Consider the following triangle.



Applying Pythagoras' theorem to this triangle will show that the length of the hypotenuse of a right-angled isosceles triangle with equal shorter side lengths of 1 unit will be $\sqrt{2}$ units.

Using this triangle, the position of $\sqrt{2}$ on a number line can be located.

Consider the following triangle:



Applying Pythagoras' theorem to this triangle will show that the length of the hypotenuse of a right-angled triangle with shorter side lengths of 1 unit and 2 units will be $\sqrt{5}$ units.

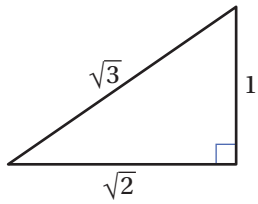
Using this triangle, the position of $\sqrt{5}$ on a number line can be located.

Using an irrational number to locate another irrational number

For each subsequent integer square root, the location of the previous integer root on the number line can be used to locate the next.

Keeping one shorter side length of a right-angled triangle equal to 1, the other shorter side length is determined using Pythagoras' theorem.

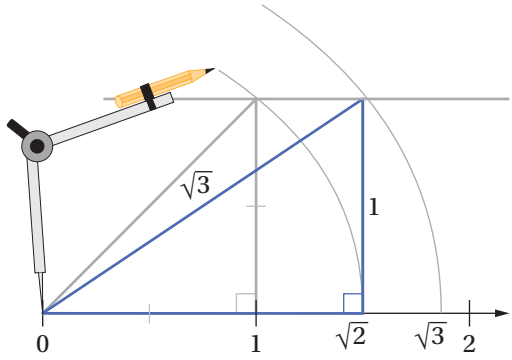
Therefore the right-angled triangle would look like this, because $\left(\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + 1^2$.



The previous worked example showed how to locate $\sqrt{2}$ on the number line.

Using that location and the same method as before, drawing a perpendicular line at $\sqrt{2}$, and forming a right-angled triangle, the position of $\sqrt{3}$ can be found.

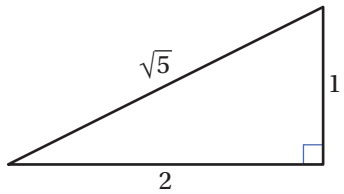
Because the length of both shorter sides is 1, construct a perpendicular line at 1.



This pattern can be continued for higher irrational square roots of integers.

Use a parallel line 1 unit above the number line to help locate each subsequent vertex for the next right-angled triangle.

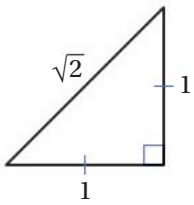
Note, $\sqrt{4}$ is next, which is not irrational because it has a value of 2. So, the right-angled triangle for $\sqrt{5}$ is located on the number line using:

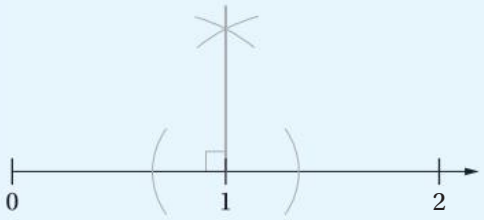
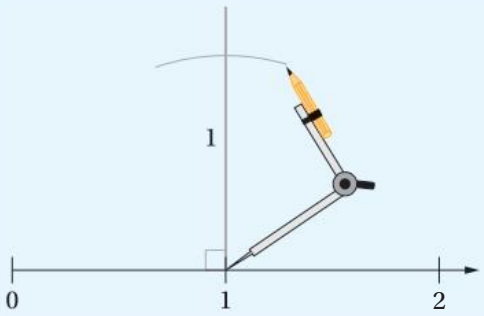
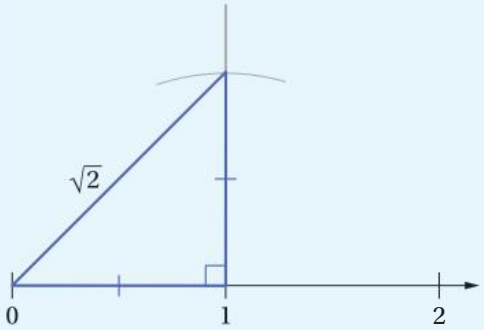
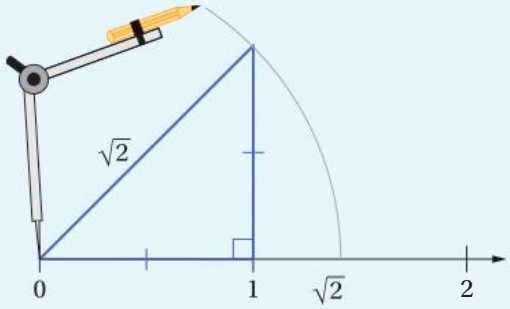


Worked example

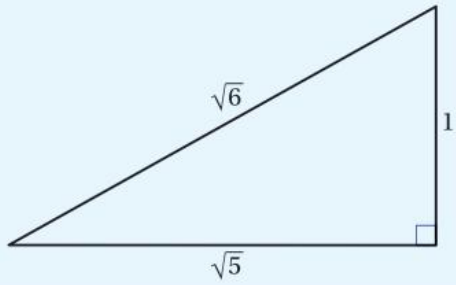
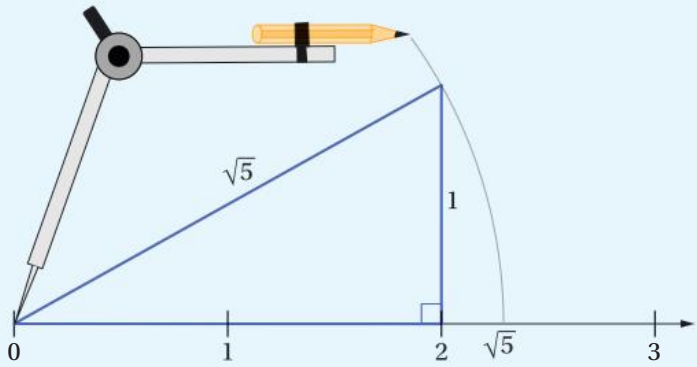
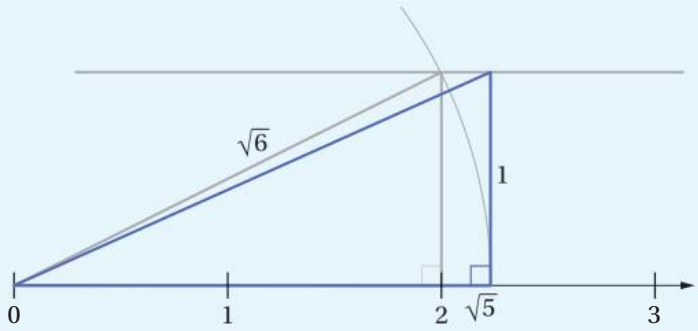
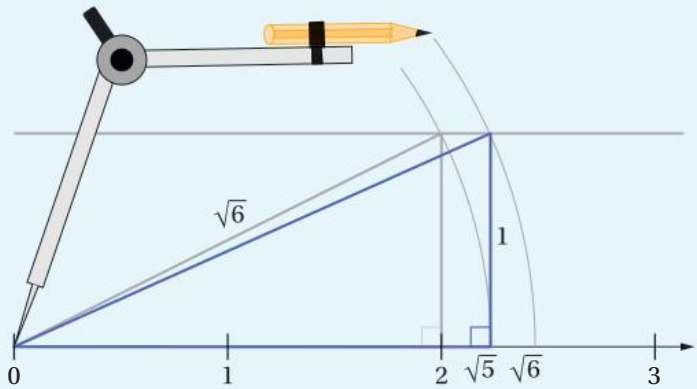
Using construction to show the location of an irrational number on a number line

(a) Construct the right-angled triangle on a number line and use it to plot the position of $\sqrt{2}$.



THINKING	WORKING
Because the length of both shorter sides is 1, construct a perpendicular line at 1.	
Set the compass radius to a distance of 1 unit using the number line. Then draw an arc of that radius across the perpendicular line.	
Draw a line from 0 to the intersection of the arc and perpendicular line. Form a right-angled triangle with equal shorter-side lengths of 1 unit and a hypotenuse of $\sqrt{2}$ units.	
Set the compass radius to the length of the hypotenuse ($\sqrt{2}$ units). With the compass point at 0, draw an arc from the vertex to the number line. Mark the point at which the arc intersects the number line as the location of $\sqrt{2}$.	

(b) Use constructions to locate the position of $\sqrt{6}$ on a number line.

THINKING	WORKING
Identify the right-angled triangle needed with a shorter side 1 unit long.	$\left(\sqrt{6}\right)^2 = \left(\sqrt{5}\right)^2 + 1^2$ 
Locate $\sqrt{5}$ on the number line.	$\left(\sqrt{5}\right)^2 = 2^2 + 1^2$ 
Draw a vertical line from $\sqrt{5}$ and a horizontal line 1 unit above the number line. Use this to form a new right-angled triangle with a hypotenuse of $\sqrt{6}$.	
Set the compass radius to the length of the hypotenuse ($\sqrt{6}$ units). With the compass point at 0, draw an arc from the vertex to the number line. Mark the point at which the arc intersects the number line as the location of $\sqrt{6}$.	

Practice

ANSWERS Page 608

SC 1 I can estimate the location of irrational numbers on a number line

1 Plot each of the following irrational numbers on a number line to show their position, as a decimal, correct to 3 decimal places.

(a) $\sqrt{27}$ **(b)** $\sqrt{63}$ **(c)** $\frac{\pi}{6}$ **(d)** $\frac{e}{2}$

2 Complete the sentences to state the correct decimal values between which each of the following irrational values will lie, when written correct to 4 decimal places.

(a) π lies between ___ and ___.

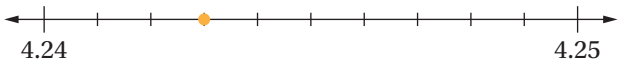
(b) e lies between ___ and ___.

(c) $3\pi - 2e$ lies between ___ and ___.

(d) $\frac{2e - 3\pi}{4}$ lies between ___ and ___.

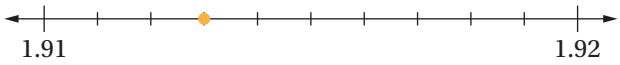
3 Choose the best option for the irrational number shown on each of the following number lines.

(a)



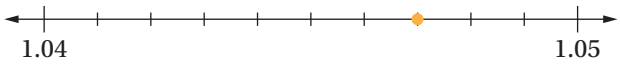
A $\sqrt{17}$ **B** $3\sqrt{2}$ **C** $\sqrt{19}$ **D** $\sqrt{20}$

(b)



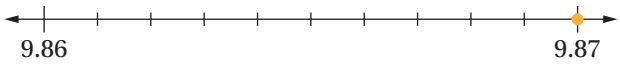
A $\sqrt{3.7}$ **B** $\sqrt{3.8}$ **C** $\sqrt[3]{7}$ **D** $\sqrt[3]{9}$

(c)



A $\frac{3\sqrt{2}}{4}$ **B** $\frac{3\pi}{10}$ **C** $\sqrt[3]{2}$ **D** $\frac{\pi}{3}$

(d)



A $\sqrt{97}$ **B** $\sqrt{98}$ **C** π^2 **D** e^4

4 List the following irrational numbers in order, from smallest to largest.

$\frac{\pi}{5}$, $\sqrt{3}$, $\sqrt[3]{5}$, $\frac{2e}{3}$, $\frac{2\pi - e}{4}$

5 List the following irrational numbers in order, from smallest to largest.

$2e - 3\pi$, $\sqrt[3]{-11}$, $\frac{\sqrt[3]{-20}}{2}$, $-\frac{3\pi}{4}$, $\sqrt{15} - \sqrt{8}$

SC 2 I can use construction techniques to accurately locate irrational numbers on a number line

1 Use a compass to construct a perpendicular line at the value 3 on a number line. Show all construction lines.

2 Draw a right-angled triangle with shorter side 1 unit long that could be used to locate these irrational numbers on a number line.

(a) $\sqrt{10}$ **(b)** $\sqrt{17}$ **(c)** $\sqrt{37}$

3 Draw a right-angled triangle with shorter side 1 unit long that could be used to locate these irrational numbers on a number line.

(a) $\sqrt{10}$ **(b)** $\sqrt{26}$

4 For each given Pythagorean calculation, draw a right-angled triangle with integer values for the shorter sides that could be used to locate each irrational number on a number line.

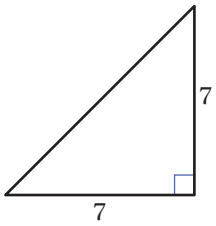
(a) $\left(\sqrt{13}\right)^2 = 9 + 4$ **(b)** $\left(\sqrt{34}\right)^2 = 25 + 9$ **(c)** $\left(\sqrt{41}\right)^2 = 25 + 16$

5 Draw a right-angled triangle with shorter side 2 units long that could be used to locate these irrational numbers on a number line.

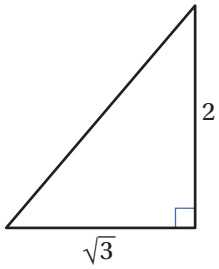
(a) $\sqrt{13}$ **(b)** $\sqrt{29}$

6 Identify the irrational number that could be accurately located on a number line, given the diagrams.

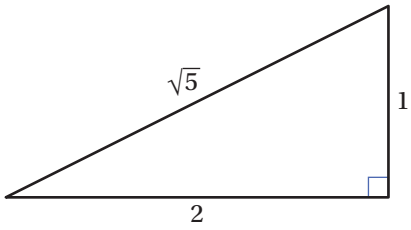
(a)



(b)



7 Given $\sqrt{20} = \sqrt{4} \times \sqrt{5}$, describe how the value $\sqrt{20}$ could be located on a number line using the triangle below.



TOPIC REVIEW

Multiple choice

ANSWERS Page 660

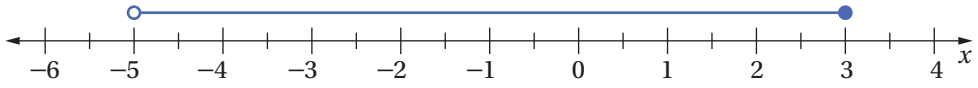
1 Which of the following values is an irrational number?

A $\sqrt[3]{2.744}$ **B** $18\pi^2$ **C** $5.30\dot{6}$ **D** $3\frac{6}{29}$

2 Which of the following values is *not* an irrational number?

A $-2e$ **B** $\sqrt[3]{-15}$ **C** $4 - 7\pi$ **D** $2\sqrt{-7}$

3 What does the number line show?



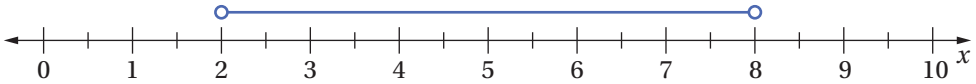
A $-5 < x \leq 3$ **B** $-5 < x < 3$ **C** $-5 \leq x \leq 3$ **D** $-5 \leq x < 3$

4 Which number line represents $2 \leq x < 8$?

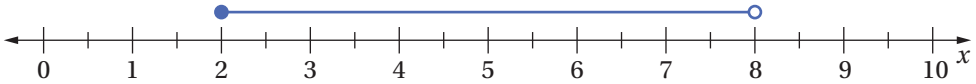
A



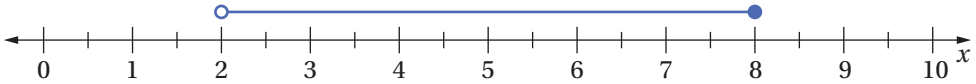
B



C



D



5 The number 8.15 is an approximation of which of the following, correct to 2 decimal places?

A $3e$ **B** 2.6π **C** $2\sqrt{14} + 0.65$ **D** $\sqrt[3]{600} - 0.3$

6 $\sqrt{15} + \sqrt[3]{12}$ is closest to:

A 6.13 **B** 6.14 **C** 6.15 **D** 6.16

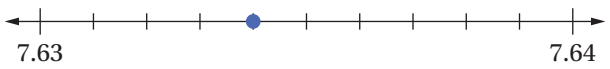
7 The exact circumference of a circle of radius 15 cm is:

A 15π cm **B** 30π cm **C** 225π cm **D** 450π cm

8 The exact length of the diagonal of a rectangle of sides 4 m and 7 m is:

A $\sqrt{11}$ m **B** $\sqrt{28}$ m **C** $\sqrt{65}$ m **D** $\sqrt{130}$ m

9 What does the number line show?



A $\sqrt{56}$ **B** $\sqrt{57}$ **C** e^2 **D** 2.43π

10 Which pair of values could *not* be used to construct a diagonal of a rectangle of $\sqrt{41}$ units?

A 2 units and 6 units **B** 4 units and 5 units

C $\sqrt{40}$ units and 1 unit **D** 3 units and $4\sqrt{2}$ units

Short answer

1 Express each of the following numbers in the simplest form of $\frac{a}{b}$, where $b \neq 0$ and a and b are integers.

(a) 2.0202… **(b)** $15.1\dot{6}$ **(c)** $12.\overline{45}$ **(d)** $30.\dot{6}1\dot{8}$

2 Simplify the following expressions to classify each value as not real, irrational or rational.

(a) $\sqrt[3]{-27}$ **(b)** $\sqrt{-25}$ **(c)** $\sqrt[3]{40}$ **(d)** $\sqrt{4320}$

3 Solve each of the following inequalities, graphing each solution on a number line.

(a) $x-2>-3$ **(b)** $3x+2\leq 7$ **(c)** $5(x-1)<20$

4 Solve each of the following inequalities, graphing each solution on a number line.

(a) $3\geq x$ **(b)** $6-x<2$ **(c)** $4\leq 5x+2$

5 Write each of the following irrational numbers correct to 3 decimal places.

(a) $\sqrt{19}$ **(b)** $\sqrt[3]{20}$ **(c)** $\frac{7\pi}{8}$ **(d)** $\frac{5e-1}{6}$

6 For each of these irrational numbers, write two consecutive whole numbers such that the first is less than the irrational number and the second is greater.

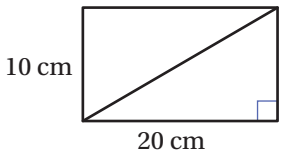
(a) $\sqrt{13}$ **(b)** $\sqrt[3]{45}$ **(c)** $\sqrt[4]{200}$

7 The volume of a hemisphere can be calculated using the formula $V=\frac{2}{3}\pi r^3$. Determine the exact volume of a hemisphere of radius:

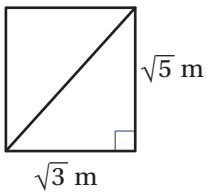
(a) 8 cm **(b)** 6 m.

8 Calculate the exact length of the diagonal in each of the following rectangles.

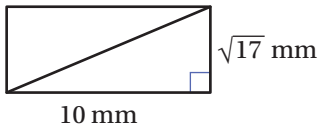
(a)



(b)



(c)



9 Locate each of the following sets of values on a single number line and use the display to write the values from smallest to largest.

(a) $\frac{3\pi}{5}$, $\sqrt{6}$, $\sqrt[3]{10}$, $\frac{4e-1}{3}$ **(b)** $\sqrt{6}-\sqrt{12}$, $e-\pi$, $\sqrt[3]{-5}$, $-\frac{6}{e}$

10 Determine a pair of whole-number lengths for the sides of a right-angled triangle that would result in each of the following lengths for the hypotenuse.

(a) $\sqrt{37}$ units **(b)** $\sqrt{65}$ units **(c)** $\sqrt{58}$ units

Extended response

1 Determine all possible solutions for the integer n, given:

(a) $\sqrt{n}$ is a rational number and

(i) $0 < n < 20$ **(ii)** $50 < n < 90$

(b) $\sqrt[3]{n}$ is a rational number and

(i) $200 < n < 400$ **(ii)** $-200 < n < -100$.

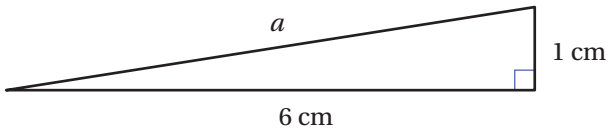
2 Consider each of the sets of three irrational numbers.

(i) For each number, write a pair of decimal numbers, each with 1 decimal place and that are 0.1 apart, such that the first number is less than the irrational number and the second is greater.

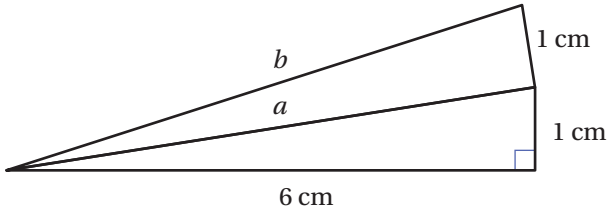
(ii) Plot the set of three irrational numbers on the same number line.

(a) $\sqrt{20}$, $\sqrt[3]{20}$, $\sqrt[4]{20}$ **(b)** $\sqrt{75}$, $\sqrt[3]{75}$, $\sqrt[4]{75}$

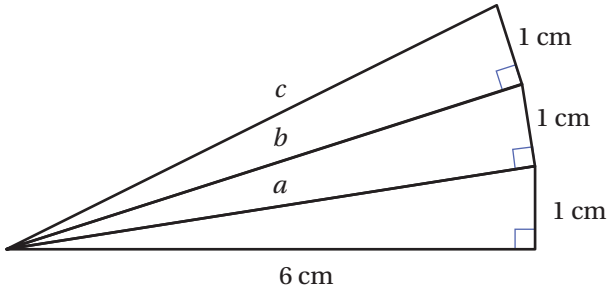
3 (a) For the right-angled triangle shown, determine the exact length of a.



(b) For the compound diagram, determine the exact length of b.



(c) For the further stage shown, determine the exact length of c.



(d) Continue the pattern for two more stages and determine the exact lengths of d and e.

TOPIC

2

Exponents

Recall 27

2.1 Establish and apply the exponent law for multiplication 28

2.2 Establish and apply the exponent law for division 31

2.3 Establish and apply the exponent law for raising a power to a power 35

2.4 Establish and apply the exponent law for raising to the power of 0 38

2.5 Apply exponent laws to numerical expressions with negative integer exponents 42

2.6 Apply exponent laws to simplify numerical expressions 45

2.7 Choose and apply efficient strategies to exponent laws of numerical expressions 51

2.8 Apply exponent laws to simplify algebraic expressions 55

Topic review 61

Why learn this?

Learning about exponents provides a foundation for understanding more complex mathematical concepts, and it helps to simplify numerical and algebraic expressions, making calculations more manageable. Understanding exponents can streamline problem-solving in various fields, such as physics, engineering, computer science and finance. It can also assist in everyday tasks, such as calculating interest or interpreting data presented in exponent form.

RECALL

ANSWERS Page 610

I can write numbers raised to a power in expanded form

1 Write each of these numbers in expanded form.

(a) 5^2 **(b)** 6^4 **(c)** 2^8 **(d)** $(-3)^4$ **(e)** $\left(\frac{1}{2}\right)^3$

I can write repeated multiplications in exponent form

1 Write each of these multiplications as a single term using exponent notation.

(a) $3 \times 3 \times 3 \times 3$

(b) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

(c) $11 \times 11 \times 11 \times 11 \times 11 \times 11$

(d) $(-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2)$

(e) $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

2 Simplify these products to multiplications written using exponent notation.

(a) $2 \times 2 \times 2 \times 3 \times 3$

(b) $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

(c) $10 \times 10 \times 10 \times 10 \times 9 \times 9$

(d) $2 \times 3 \times 3 \times 5 \times 3 \times 2$

(e) $3 \times 4 \times 5 \times 4 \times 5 \times 4 \times 5 \times 5$

I can calculate squares and cubes of numbers

1 Calculate each of the following.

(a) 4^2 **(b)** 15^2 **(c)** $(-3)^2$ **(d)** 2^3 **(e)** $(-4)^3$

I understand the effect of raising a number to a power

1 Arrange the digits from 1 to 6 in the squares below to make the sum:

(a) as large as possible **(b)** as small as possible.

$\square^{\square} + \square^{\square} + \square^{\square}$

2.1 Establish and apply the exponent law for multiplication

Learning intention: To be able to establish and apply the exponent law for multiplication

Success criteria:

SC 1 I can write the expanded form of a multiplication and multiply numbers using exponent notation.

SC 2 I can multiply variables using exponent notation.

Lesson warm-up

Rice on the double!

A person with a chessboard (a square board with a total of 64 small squares) was asked:

How many times would you need to double the number of grains of rice in each square, starting from 1 rice grain in the first square, to reach a total greater than 1000 grains of rice in the next square?



Predict the answer before trying to calculate!

How would you express this calculation mathematically?

SC 1 I can write the expanded form of a multiplication and multiply numbers using exponent notation

When a number is multiplied by itself any number of times, exponent notation is used to simplify the expression. The number being multiplied is called the base and the number of times this factor occurs in the multiplication is written as the exponent.

For example, $2 \times 2 \times 2 \times 2 \times 2 = 2^5$, where 2 is the base and 5 is the exponent, since there are 5 factors of 2.

2^5 is the fifth power of 2.

A single factor can be written in exponent form with an exponent of 1, so $2 = 2^1$.

In general, $a \times a \times a \times \ldots$ to m factors $= a^m$ and $a = a^1$.

When two powers with the same base are multiplied together, adding the exponents gives the total number of factors of the base, so the exponent law for multiplication is: $a^m \times a^n = a^{m+n}$. This law is also known as the product law.

Worked example

Applying the exponent law for multiplication to numbers

Write the following in simplest exponent form.

(a) $5^3 \times 5$

THINKING	WORKING
Write the product in expanded form. Simplify into exponent form.	$5^3 \times 5 = (5 \times 5 \times 5) \times 5$ $= 5^4$
Confirm using exponent laws. Recall the exponent law for multiplication.	$a^m \times a^n = a^{m+n}$
Apply the exponent law by adding exponents of the same base.	Remember: $5 = 5^1$ $5^3 \times 5^1 = 5^{(3+1)} = 5^4$

(b) $5^2 \times 5^5 \times 7^4 \times 7^3$

THINKING	WORKING
Write the product in expanded form. Simplify into exponent form.	$5^2 \times 5^5 \times 7^4 \times 7^3$ $= (5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5) \times (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7)$ $= 5^7 \times 7^7$
Confirm using exponent laws.	$5^2 \times 5^5 \times 7^4 \times 7^3 = 5^{(2+5)} \times 7^{(4+3)}$ $= 5^7 \times 7^7$

SC 2 I can multiply variables using exponent notation

Exponent notation is used in the same way for algebraic factors as for numerical factors: $a \times a \times a \times \ldots$ to m factors $= a^m$ and $a = a^1$.

The exponent law for multiplication remains as: $a^m \times a^n = a^{m+n}$.

Note that for algebraic bases, as with algebraic factors generally, the multiplication symbol is unnecessary, so is removed from expressions in simplest form.

$3 \times a^2 \times b^4$ would be written as $3a^2b^4$.

Worked example

Applying the exponent law for multiplication to variables

Write the following in simplest exponent form.

(a) $x^5 \times x$

THINKING	WORKING
Write the product in expanded form. Simplify into exponent form.	$x^5 \times x = (x \times x \times x \times x \times x) \times x$ $= x^6$
Confirm using exponent laws. Recall the exponent law for multiplication.	$a^m \times a^n = a^{m+n}$
Apply the exponent law for multiplication by adding the exponents of the same base.	Remember: $a = a^1$ $x^5 \times x^1 = x^{(5+1)} = x^6$

(b) $5 \times 5^2 \times x^3 \times x \times y^4 \times y^2$

THINKING	WORKING
Write the product in expanded form. Simplify into exponent form. Leave out multiplication symbols next to powers of variables.	$5 \times 5^2 \times x^3 \times x \times y^4 \times y^2$ $= 5 \times (5 \times 5) \times (x \times x \times x) \times x \times (y \times y \times y \times y) \times (y \times y)$ $= 5^3x^4y^6$
Confirm using exponent laws. Apply the exponent law for multiplication by adding the exponents of the same base.	$5 \times 5^2 \times x^3 \times x \times y^4 \times y^2$ $= 5^{(1+2)} \times x^{(3+1)} \times y^{(4+2)}$ $= 5^3x^4y^6$

Practice

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SC 1 I can write the expanded form of a multiplication and multiply numbers using exponent notation

1 Write the following expressions in expanded notation to demonstrate the exponent law for multiplication, then write them in simplest exponent form.

(a) $2^5 \times 2$ **(b)** $2^4 \times 2^2$ **(c)** $2^2 \times 2^3$ **(d)** $2^2 \times 2 \times 2^4$

2 Write the following in simplest exponent form.

(a) $5 \times 7 \times 7^3$ **(b)** $7^4 \times 7^2 \times 7^3$ **(c)** $5^4 \times 5^2 \times 7^2$ **(d)** $5^3 \times 5^4 \times 5$

3 Express each number in terms of powers of prime numbers.

(a) 256 **(b)** 625 **(c)** 1000

(d) 36 **(e)** 90 000 **(f)** 825

4 Simplify, expressing your answer in terms of powers of prime numbers.

(a) $3 \times 125 \times 27$ **(b)** $10^3 \times 15^2$ **(c)** $36^3 \times 30^4$ **(d)** $33^2 \times 300^3$

SC 2 I can multiply variables using exponent notation

1 Write the following expressions in expanded notation to demonstrate the exponent law for multiplication, then write them in simplest exponent form.

(a) $x^4 \times x$ **(b)** $x^5 \times x^2$ **(c)** $x \times x^4 \times x^3$ **(d)** $x^2 \times x^2 \times x^3$

2 Write the following in simplest exponent form.

(a) $x \times x^3 \times y^4$ **(b)** $3 \times x \times x^3 \times y^2$

(c) $3^5 \times 3^3 \times x^2 \times y \times y^3$ **(d)** $3^4 \times 3 \times x^5 \times y^3 \times x^3$

3 Write the following with numbers and variables in simplest exponent form.

(a) $11^2a^2 \times 11^4a^3$ **(b)** $7b \times 7^2b^4$ **(c)** $3c^3 \times 9c$ **(d)** $5d^3 \times 5d^2$

4 Write the following with numbers as a single coefficient and variables in simplest exponent form.

(a) $2x^4 \times 3x^2y \times y^5$ **(b)** $xy^3 \times 5xy \times x^2y^3$ **(c)** $xy \times yz \times xyz$ **(d)** $y^3 \times 3x^2 \times x^2y$

Establish and apply the exponent law for division

2.2

Learning intention: To be able to establish and apply the exponent law for division

Success criteria:

SC 1 I can write the expanded form of a division and divide numbers using exponent notation.

SC 2 I can divide powers of variables using exponent notation.

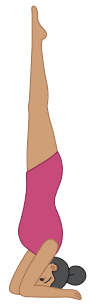
Lesson warm-up

Simplifying fractions with big numbers

How would you go about simplifying the fraction $\frac{40\,960}{1024}$?

You know that both the numerator and denominator are even, however, dividing by 2 just gives you the slightly smaller fraction $\frac{20\,480}{512}$ with still a lot more work to do.

How could you keep track of how many times you need to divide by 2 to simplify the fraction in a systematic and easy-to-communicate way?



SC 1 I can write the expanded form of a division and divide numbers using exponent notation

When two powers with the same base are divided, some of the factors cancel out. When the number of factors in the numerator is larger than the number of factors in the denominator, the difference between the exponents gives the number of factors remaining in the numerator.

The exponent law for division is:

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

This law is also known as the quotient law.

Worked example

Applying the exponent law for division to powers of numbers

Write the following in simplest exponent form.

(a) $\frac{3^5}{3}$

THINKING	WORKING
Write the quotient in expanded form. Simplify into exponent form.	$\frac{3^5}{3} = \frac{3\times3\times3\times3\times\cancel{3}}{\cancel{3}}$ $= \frac{3\times3\times3\times3}{1}$ $= 3^4$
Recall the exponent law for division.	$a^m \div a^n = a^{m-n}$
Apply the exponent law by subtracting exponents of the same base.	$\frac{3^5}{3^1} = 3^{(5-1)} = 3^4$

(b) $\frac{5^5 \times 7^3}{5^4 \times 7}$

THINKING	WORKING
Write the quotient in expanded form. Simplify into exponent form.	$\frac{5^5\times7^3}{5^4\times7} = \frac{5\times\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}\times7\times7\times\cancel{7}}{\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{5}\times\cancel{7}}$ $= 5\times7\times7$ $= 5\times7^2$
Apply the exponent law for division by subtracting exponents of the same base.	$\frac{5^5\times7^3}{5^4\times7} = 5^{(5-4)}\times7^{(3-1)}$ $= 5^1\times7^2$ $= 5\times7^2$

SC 2 I can divide powers of variables using exponent notation

Division of powers works in the same way for algebraic factors as for numerical factors.

It is best to show factors cancelling down using fractions.

The exponent law for division remains as: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

Worked example

Applying the exponent law for division to variables

Write the following in simplest exponent form.

(a) $\frac{a^6}{a}$

THINKING	WORKING
Write the quotient in expanded form. Simplify into exponent form.	$\frac{a^6}{a} = \frac{a \times a \times a \times a \times a \times \cancel{a}}{\cancel{a}}$ $= \frac{a^5}{1}$ $= a^5$
Recall the exponent law for division.	$a^m \div a^n = a^{m-n}$
Apply the exponent law by subtracting exponents of the same base.	$\frac{a^6}{a^1} = a^{(6-1)} = a^5$

(b) $\frac{a^5b^4}{a^2b^3}$

THINKING	WORKING
Write the quotient in expanded form. Simplify into exponent form.	$\frac{a^5b^4}{a^2b^3} = \frac{a \times a \times a \times \cancel{a} \times \cancel{a} \times b \times \cancel{b} \times \cancel{b} \times \cancel{b}}{\cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{b}}$ $= a \times a \times a \times b$ $= a^3b$
Apply the exponent law for division by subtracting exponents of the same base.	$\frac{a^5b^4}{a^2b^3} = a^{(5-2)} \times b^{(4-3)}$ $= a^3 \times b^1$ $= a^3b$

Practice

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SC 1 I can write the expanded form of a division and divide numbers using exponent notation

1 Write the following expressions in expanded notation to demonstrate the exponent law for division, then write them in simplest exponent form.

(a) $\frac{3^3}{3}$ **(b)** $\frac{3^8}{3^7}$ **(c)** $\frac{3^5 \times 3^4}{3^6}$ **(d)** $\frac{3^3 \times 3^5 \times 3^2}{3^2 \times 3^4}$

2 Write the following in simplest exponent form.

(a) $\frac{13^4 \times 11^6}{11 \times 13^2}$ **(b)** $\frac{2^5 \times 7^4}{2^2 \times 7^3}$ **(c)** $\frac{5^6 \times 3^7}{3^2 \times 5^3}$ **(d)** $\frac{2^{10} \times 3^5}{3^2 \times 2^5}$

3 Simplify, by first expressing your answer in terms of powers of prime numbers, then as a single number.

(a) $\frac{243}{27}$ **(b)** $\frac{8 \times 32}{16}$ **(c)** $\frac{25^3 \times 125^2}{5^8}$ **(d)** $\frac{11^3 \times 11 \times 11^5}{121^3}$

4 Simplify, expressing your answer in terms of powers of prime numbers.

(a) $\frac{50 \times 363 \times 8}{20 \times 15}$ **(b)** $\frac{81 \times 14 \times 63 \times 35}{18 \times 49}$ **(c)** $\frac{60 \times 33 \times 75}{45 \times 4}$ **(d)** $\frac{220 \times 18^2 \times 45^2}{12 \times 9^3}$

SC 2 I can divide powers of variables using exponent notation

1 Write the following expressions in expanded notation to demonstrate the exponent law for division, then write them in simplest exponent form.

(a) $\frac{a^5}{a}$ **(b)** $\frac{a^6}{a^2}$ **(c)** $\frac{a^8}{a^6}$ **(d)** $\frac{a^4 \times a^4}{a^3}$

2 Write the following in simplest exponent form.

(a) $\frac{a^2b^5}{ab^3}$ **(b)** $\frac{c^5d^8}{c^3d^6}$ **(c)** $\frac{e^7f^7}{e^2f^5}$ **(d)** $\frac{g^6h^{12}}{g^4h^6}$

3 Write the following with numbers and variables in simplest exponent form.

(a) $\frac{2^4a^9}{2^2a^5}$ **(b)** $\frac{3^5b^5}{3^4b^3}$ **(c)** $\frac{5^8c^4}{5^6c^2}$ **(d)** $\frac{3^3d^4 \times 3^2d^2}{3d^6}$

4 Write the following with numbers and variables in simplest exponent form.

(a) $\frac{5^6x^2y^4}{5xy}$ **(b)** $\frac{3^4x^5y^6}{3^2x^3y}$ **(c)** $\frac{2^8x^3y^5}{2^4xy^2}$ **(d)** $\frac{7^5x^3y^6}{7^4x^3y^4}$

Establish and apply the exponent law for raising a power to a power

2.3

Learning intention: To be able to establish and apply the exponent law for raising a power to a power

Success criteria:

SC 1 I can write the expanded form for powers of numbers raised to a power.

SC 2 I can raise a power to a power using a variable base.

Lesson warm-up

The Googolplex



In 1920, nine-year-old Milton Sirotta developed the terms 'googol', which is 10^{100}, and 'googolplex', which was the number one, followed by as many zeroes as you could write before getting tired.

A googolplex was later formalised to be 10^{googol} or $\left(10^{10}\right)^{100}$.

How big is this number really?

Could you write the whole number before getting tired?

SC 1 I can write the expanded form for powers of numbers raised to a power

When a power is raised to a power, multiplying the exponents gives the total number of factors of the base, so the exponent law for the power of a power is:

$(a^m)^n = a^{m \times n}$ or a^{mn}

The law can be demonstrated by expanding each of the powers.

Worked example

Applying the exponent law for a power raised to a power to numbers

Simplify $(3^2)^4$. Leave your answer in exponent form.

THINKING	WORKING
Write the power of a power in expanded form. Simplify into exponent form.	$(3^2)^4 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$ $= 3^8$
Recall the exponent law for raising a power to a power.	$(a^m)^n = a^{m \times n}$
Apply the exponent law by multiplying exponents of the same base.	$(3^2)^4 = 3^{2 \times 4} = 3^8$

SC 2 I can raise a power to a power using a variable base

Exponent notation is used in the same way for algebraic factors as for numerical factors.

Hence, the exponent law for raising a power to a power remains as: $(a^m)^n = a^{m \times n} = a^{mn}$.

Worked example

Applying the exponent law for a power raised to a power to variables

Simplify $(x^3)^6$. Leave your answer in exponent form.

30 THINKING	WORKING
Write the power of a power in expanded form. Simplify into exponent form.	$(x^3)^6 = (x \times x \times x) \times (x \times x \times x) \times (x \times x \times x) \times (x \times x \times x) \times (x \times x \times x) \times (x \times x \times x)$ $= x^{18}$
Recall the exponent law for raising a power to a power.	$(a^m)^n = a^{m \times n}$
Apply the exponent law by multiplying exponents of the same base.	$(x^3)^6 = x^{3 \times 6} = x^{18}$

Practice

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SC 1 I can write the expanded form for powers of numbers raised to a power

1 Simplify the following by expanding, then applying the exponent law to raise a power to a power. Leave your answer in exponent form.

(a) $(5^2)^2$ **(b)** $(5^3)^4$ **(c)** $(5^2)^5$

(d) $(2^3)^5$ **(e)** $(2^4)^2$ **(f)** $(2^5)^2$

2 Copy and complete the following table.

Expression	Expansion	Simplified form	Multiplying the exponents
$(7^2)^3$	$7^2 \times 7^2 \times 7^2$	7^6	$2 \times 3 = 6$
$(5^4)^2$			
$(11^3)^3$			
$(2^4)^3$			

3 Explain why $(7^6)^4 = (7^4)^6$.

4 Simplify the following. Leave your answer in exponent form.

(a) $(5^3)^2 \times (5^5)^4$ **(b)** $5^3 \times (5^2)^5$ **(c)** $(5^4)^6 \times 5$

(d) $2^4 \times (2^3)^2$ **(e)** $(2^3)^3 \times (2^3)^2$ **(f)** $(2^2)^2 \times 2$

5 Simplify the following. Leave your answer in exponent form.

(a) $\frac{(3^4)^2}{3^2}$ **(b)** $\frac{(3^3)^4}{3^8}$ **(c)** $\frac{(3^2)^5}{3^3}$

(d) $\frac{(3^3)^4}{(3^2)^3}$ **(e)** $\frac{(3^6)^2}{(3^4)^3}$ **(f)** $\frac{3^5}{(3^2)^2}$

6 Simplify the following. Leave your answer as powers of prime numbers.

(a) $8^3 \times 16^2$ **(b)** $27^4 \times 9^3$ **(c)** $125^2 \times 25^4$

(d) $\frac{49^3}{7^4}$ **(e)** $\frac{32^2}{4^3}$ **(f)** $\frac{27^5}{81^2}$

SC 2 I can raise a power to a power using a variable base

1 Simplify the following by expanding, then applying the exponent law to raise a power to a power. Leave your answer in exponent form.

(a) $(a^7)^2$ **(b)** $(b^3)^3$ **(c)** $(c^2)^5$

(d) $(d^4)^2$ **(e)** $(e^5)^3$ **(f)** $(f^6)^2$

2 Copy and complete the following table.

Expression	Expansion	Simplified form	Multiplying the exponents
$(x^5)^2$	$x^5 \times x^5$	x^{10}	$5 \times 2 = 10$
$(y^4)^3$			
$(p^3)^7$			
$(q^2)^8$			

3 Write the following in simplest exponent form.

(a) $a^3 \times (a^4)^2$ **(b)** $(b^5)^3 \times b^2$ **(c)** $(c^2)^8 \times (c^3)^3$ **(d)** $(d^5)^5 \times (d^3)^3$

4 Write the following in simplest exponent form.

(a) $(x^5)^3 \times (y^2)^5$ **(b)** $x^6 \times (y^2)^3$ **(c)** $(x^8)^3 \times y^7$ **(d)** $(x^5)^3 \times (y^2)^4$

5 Write the following in simplest exponent form.

(a) $\frac{(x^3)^2}{x^4}$ **(b)** $\frac{(x^4)^3}{x^6}$ **(c)** $\frac{(x^4)^4}{(x^2)^5}$ **(d)** $\frac{x^9}{(x^2)^3}$

6 Write the following in simplest exponent form.

(a) $\frac{(x^4)^2 \times (y^3)^3}{x^6 y^5}$ **(b)** $\frac{(x^3)^5 \times y^6}{x^3}$ **(c)** $\frac{(x^2)^5 \times (y^4)^5}{y^{10}}$ **(d)** $\frac{x^6 \times (y^5)^2}{xy}$

2.4 Establish and apply the exponent law for raising to the power of 0

Learning intention: To be able to establish and apply the exponent law for raising to the power of 0

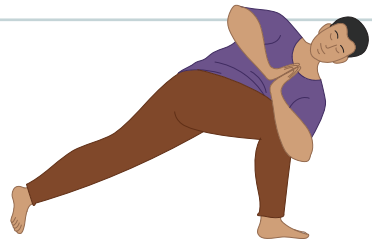
Success criteria:

SC 1 I can demonstrate that any natural number raised to the power of 0 is equal to 1.

SC 2 I can solve problems raising variables to the power of 0.

Lesson warm-up

The zero effect



Operating with the number zero has many different outcomes.

- Adding zero has no effect.
- Subtracting zero has no effect.
- Multiplying by zero makes the result zero.
- Dividing by zero is impossible and is undefined.

Raising a number or variable to the power zero is a curious concept – what do you think it does? Can you justify why?

SC 1 I can demonstrate that any natural number raised to the power of 0 is equal to 1

To explain the zero exponent for all natural numbers, we draw on the patterns seen in the division of powers with the same base.

When two powers with the same base are divided, some of the factors cancel out. When the number of factors in the numerator is the same as the number of factors in the denominator, all of the factors cancel out.

Since $\frac{a^m}{a^m} = 1$ and $\frac{a^m}{a^m} = a^{m-m} = a^0$, the conclusion for the zero exponent is that: $a^0 = 1$, where $a \neq 0$.

The exponent law for raising to the power of zero is also known as the zero power law or zero exponent law.

Worked example

Using the exponent law for division to explain the zero exponent

(a) Calculate the value of $\frac{6^4}{6^4}$ by writing the numerator and denominator in expanded form.

THINKING	WORKING
Write the numerator and denominator in expanded form. Cancel common factors. Write the answer.	$\frac{6^4}{6^4} = \frac{\cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6}}{\cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6}}$ $= \frac{1}{1}$ $= 1$

(b) Simplify $\frac{6^4}{6^4}$ using the exponent law for division.

THINKING	WORKING
Recall the exponent law for division.	$a^m \div a^n = a^{m-n}$
Use this law to simplify the division of terms with the same base. Write the answer.	$\frac{6^4}{6^4} = 6^{4-4}$ $= 6^0$

(c) What do you conclude from your answers to parts **(a)** and **(b)**?

THINKING	WORKING
Compare the results from parts **(a)** and **(b)**.	$\frac{6^4}{6^4} = 1$ and $\frac{6^4}{6^4} = 6^0$
Write the answer.	Hence $6^0 = 1$

SC 2 I can solve problems raising variables to the power of 0

Division of powers works in the same way for algebraic factors as for numerical factors.

Hence: $a^0 = 1$, where $a \neq 0$.

Worked example

Applying the zero exponent law to variables

Assuming $p \neq 0$, evaluate the following.

(a) p^0

THINKING	WORKING
Recall the exponent law for raising to the power of 0.	$a^0 = 1$
Apply the exponent law to numbers and variables raised to the power of 0.	$p^0 = 1$

(b) $4p^0$

THINKING	WORKING
Apply the exponent law to numbers and variables raised to the power of 0, then simplify.	$4p^0 = 4 \times 1$ $= 4$

(c) $(p^3)^0$

THINKING	WORKING
Apply the exponent law to numbers and variables raised to the power of 0, then simplify.	$(p^3)^0 = 1$ Alternatively: $(p^3)^0 = p^{3 \times 0}$ $= p^0$ $= 1$

Practice

ANSWERS Page 611

SC 1 I can demonstrate that any natural number raised to the power of 0 is equal to 1

1 Copy and complete the following table.

Question	Calculated value	Exponent form	Simplest exponent form answer
$5 \div 5 =$	1		5^0
$3^4 \div 3^4 =$		3^{4-4}	
$4^6 \div 4^6 =$			
$12^3 \div 12^3 =$			
$6^8 \div 6^8 =$			

2 **(a)** Copy and complete this table.

2^5	2^4	2^3	2^2	2^1	2^0
32					

(b) Describe the pattern in the exponent values.

(c) Describe the pattern in the value of the numbers.

(d) What does this tell you about the value of 2^0?

3 Solve for x in each of the following equations.

(a) $\frac{2^x \times 2^3}{2^4 \times 2^7} = 1$ **(b)** $\frac{3^2 \times 3^3 \times 3^5}{3^x \times 3^4} = 1$ **(c)** $\frac{\left(5^2\right)^3}{5^x} = 1$ **(d)** $\frac{7^3 \times 7^x}{\left(7^4\right)^3} = 1$

4 Determine the solutions for x and y in each of the following equations.

(a) $\frac{3^2 \times 3 \times 5^y}{3^x \times 5^6} = 1$ **(b)** $\frac{2^3 \times 2^x \times \left(6^4\right)^y}{2^7 \times 6^{12}} = 1$

(c) $\frac{\left(11^4\right)^3 \times 11^x \times 13^5}{11^{14} \times 13^y} = 1$ **(d)** $\frac{6^{14} \times 5^2 \times 5^y}{\left(6^2\right)^x \times 5^5} = 1$

SC 2 I can solve problems raising variables to the power of 0

1 Evaluate the following.

(a) m^0 **(b)** $2m^0$ **(c)** $-5m^0$ **(d)** $(4m)^0$ **(e)** $-(3m)^0$

2 Simplify the following.

(a) $(x^4)^0$ **(b)** $(p^0)^5$ **(c)** $(3m^0)^4$

(d) $(5a^4)^0$ **(e)** $(7n^5)^0$ **(f)** $(4b^0)^3$

3 **(a)** Copy and complete this table.

x^5	x^4	x^3			

(b) Describe the pattern in the exponent values.

(c) Describe the operation pattern to get the value of the next expression.

(d) What does this tell you about the value of x^0?

4 Solve for x in each of the following equations.

(a) $\frac{a^x \times a^5}{a^3 \times a^4} = 1$ **(b)** $\frac{2^3 \times b^6 \times b^2}{b^x \times b^5} = 8$ **(c)** $\frac{6\left(c^4\right)^5}{2c^x} = 3$ **(d)** $\frac{5^2 \times d^{12}}{\left(d^3\right)^x} = 25$

5 Determine the solutions for x and y in each of the following equations.

(a) $\frac{q^5 \times 4r^y}{q^x \times r^7} = 4$ **(b)** $\frac{3^{3g^x}\left(h^y\right)^5}{g^7 h^{15}} = 27$

(c) $\frac{\left(u^2\right)^5 \times 7^x \times v^8}{u^{10} \times v^y} = 49$ **(d)** $\frac{6^{11} \times e^{21} \times d^y}{\left(e^2\right)^x \times 6^{10} d^2} = 6e$

2.5 Apply exponent laws to numerical expressions with negative integer exponents

Learning intention: To be able to apply exponent laws to numerical expressions with negative integer exponents

Success criteria:

SC 1 I can write any natural number raised to a power with a negative integer exponent as a power with a positive exponent.

Lesson warm-up

Does it make sense to raise a base to a negative power?

We have already seen that it is possible to raise a base to the power of zero.

What would happen if we continued this pattern of reducing powers by one beyond zero and into negative integer powers?

$2^2 = 4, 2^1 = 2, 2^0 = 1, 2^{-1} =$ ____, $2^{-2} =$ ____

Explore what happens as the powers become larger negative numbers.



SC 1 I can write any natural number raised to a power with a negative integer exponent as a power with a positive exponent

To explain a negative integer exponent, we draw on the patterns seen in the division of powers with the same base.

When two powers with the same base are divided, some of the factors cancel out. When the number of factors in the numerator is less than the number of factors in the denominator, some factors remain in the denominator.

For $\frac{a^m}{a^n} = a^{m-n}$ where $m - n$ is negative, the difference between the number of factors gives the number of factors remaining in the denominator.

The exponent law for negative exponents is given as $a^{-n} = \frac{1}{a^n}$.

This law is also known as the negative exponent law.

Worked example

Using the exponent law for division to explain negative exponents

(a) Simplify $\frac{2^3}{2^6}$ by writing the numerator and denominator in expanded form.

THINKING	WORKING
Write the numerator and denominator in expanded form. Cancel common factors. Write the answer.	$\frac{2^3}{2^6} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{2 \times 2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}}$ $= \frac{1}{2^3}$

(b) Simplify $\frac{2^3}{2^6}$ using the exponent law for division.

THINKING	WORKING
Recall the exponent law for division.	$a^m \div a^n = a^{m-n}$
Use this law to simplify the division of terms with the same base. Write the answer.	$\frac{2^3}{2^6} = 2^{3-6} = 2^{-3}$

(c) What do you conclude from your answers to parts **(a)** and **(b)**?

THINKING	WORKING
Compare the results from parts **(a)** and **(b)**.	$\frac{2^3}{2^6} = \frac{1}{2^3}$ and $\frac{2^3}{2^6} = 2^{-3}$
Write the answer.	Hence $2^{-3} = \frac{1}{2^3}$

Practice

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SC 1 I can write any natural number raised to a power with a negative integer exponent as a power with a positive exponent

1 Copy and complete the following table.

Question	Positive power in the answer	Exponent law for division	Single power answer
$7^4 \div 7^6 =$	$\frac{1}{7^2}$	7^{4-6}	7^{-2}
$7^3 \div 7^7 =$			
$7^2 \div 7^5 =$			
$7^4 \div 7^5 =$			

2 **(a)** Copy and complete this table.

2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
4	2	1	$\frac{1}{2}$		

(b) Describe the pattern in the exponent values.

(c) Describe the pattern in the value of the numbers.

(d) What does this tell you about the value of 2^{-2} and 2^{-3}?

3 Evaluate, expressing your answer in fractional form.

(a) $5^2 \times 2^{-3}$ **(b)** $3^3 \times 5^{-2}$ **(c)** $4^{-2} \times 3^4 \times 5^{-4}$ **(d)** $10^{-3} \times 11^{-2}$

4 Simplify, by first expressing your answer in terms of powers of prime numbers with positive exponents, then as a single fraction.

(a) $\frac{81}{729}$ **(b)** $\frac{5}{25^2}$ **(c)** $\frac{2401}{49^4}$ **(d)** $\frac{4^2}{32 \times 8^2}$

5 Simplify, expressing your answer in terms of powers of prime numbers with positive exponents.

(a) $\frac{10 \times 45}{150 \times 242 \times 27}$ **(b)** $\frac{14 \times 28}{49 \times 16 \times 70}$ **(c)** $\frac{45 \times 8}{250 \times 90 \times 80}$ **(d)** $\frac{25^2 \times 36}{15^4 \times 8^2}$

6 Two students in a class are chatting and decide it would be funny to whisper a joke. This is the first round of the joke.

The two students then each tell two other people the joke. This is the second round of the joke.

(a) If the people who were told the joke in the second round each tell two other people the joke, and so on, how many people in total will know the joke at the tenth round?

(b) How many rounds will it be until the whole class of 30 students know the joke? (Assume that no one is told the joke twice.)

Apply exponent laws to simplify numerical expressions

2.6

Learning intention: To be able to apply exponent laws to simplify numerical expressions

Success criteria:

SC 1 I can represent decimals and fractions in expanded form, using negative exponents where necessary.

SC 2 I can simplify and evaluate numerical expressions involving positive and negative exponents.

Lesson warm-up

Positive or negative



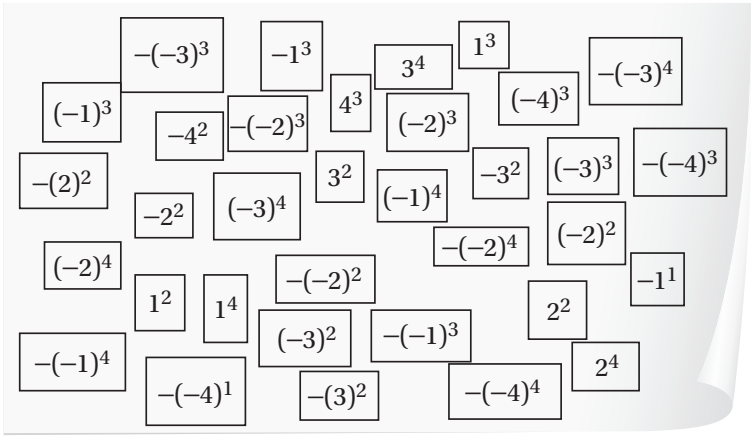
Team Positive is matched up against Team Negative for a game of 'Balance of powers'.

Both teams have 1 minute to write as many different terms containing exponents onto pieces of paper as they can and put them into a box, one at a time.

Each team can only use positive exponents and values from –4 to 4, excluding 0.

Each unique positive-valued term earns a point for Team Positive, and each unique negative-valued term earns a point for Team Negative.

The terms emptied out of the box following a game are shown below.



Which team won and by how many points?

To help keep track, list the negative and positive values as you discover them.

SC 1 I can represent decimals and fractions in expanded form using negative exponents where necessary

A number written in exponent form consists of two parts, the base and the exponent. Using 2^5 as an example, the 2 is referred to as the base and the 5 is the exponent.

A negative exponent can be made positive by swapping the term from the numerator to the denominator position. For example, $5^{-3} = \frac{1}{5^3}$.

Another of the exponent rules states that any base, except 0, raised to the power of 0, is equal to 1. For example, $5^0 = 1$.

Exponent form is a useful way to write numbers expressed in decimal form.

The decimal number 34.297 can be put into a place value table as shown.

Tens	Ones	Tenths	Hundredths	Thousandths
$10 = 10^1$	$1 = 10^0$	$\frac{1}{10} = 10^{-1}$	$\frac{1}{100} = 10^{-2}$	$\frac{1}{1000} = 10^{-3}$
3	4	2	9	7

So, $34.297 = 3 \times 10 + 4 \times 1 + 2 \times \frac{1}{10} + 9 \times \frac{1}{100} + 7 \times \frac{1}{1000}$

Because $\frac{1}{10} = 10^{-1}$, $\frac{1}{100} = 10^{-2}$, $\frac{1}{1000} = 10^{-3}$,

$34.297 = 3 \times 10^1 + 4 \times 10^0 + 2 \times 10^{-1} + 9 \times 10^{-2} + 7 \times 10^{-3}$

Fractions can also be written in exponent form:

$\frac{1}{5} = 5^{-1}$, $\frac{1}{23} = 23^{-1}$ or $\frac{2}{17} = 2 \times 17^{-1}$

Fractions such as $\frac{1}{16}$ can be written with negative exponents:

$$\begin{aligned}\frac{1}{16} &= \frac{1}{4 \times 4} \\ &= \frac{1}{2 \times 2 \times 2 \times 2} \\ &= \frac{1}{2^4} \\ &= 2^{-4}\end{aligned}$$

Worked example

Expressing fractions and decimals in exponent form

Express the following in exponent form.

(a) 29.04

THINKING	WORKING
Write the value in a place value table.	<table><tr><th>Tens</th><th>Ones</th><th>Tenths</th><th>Hundredths</th></tr><tr><td>2</td><td>9</td><td>0</td><td>4</td></tr></table>
Write the value in exponent form.	$29.04 = 2 \times 10 + 9 \times 10^0 + 0 \times 10^{-1} + 4 \times 10^{-2}$
Rewrite, leaving out any zero terms.	$29.04 = 2 \times 10 + 9 \times 10^0 + 4 \times 10^{-2}$

(b) $\frac{1}{8}$

THINKING	WORKING
Identify the base and exponent.	$8 = 2 \times 2 \times 2 = 2^3$
Write the fraction with a positive exponent.	$\frac{1}{8} = \frac{1}{2 \times 2 \times 2}$ $= \frac{1}{2^3}$
Write the answer with a negative exponent.	$\frac{1}{8} = 2^{-3}$

(c) $\frac{3}{4}$

THINKING	WORKING
Identify the base and exponent of the denominator.	$4 = 2 \times 2 = 2^2$
Write the fraction with a positive exponent.	$\frac{3}{4} = \frac{3}{2 \times 2}$ $= \frac{3}{2^2}$
Write the answer with a negative exponent.	$\frac{3}{4} = 3 \times 2^{-2}$

SC 2 I can simplify and evaluate numerical expressions involving positive and negative exponents

A summary of all the exponent laws covered so far will help you simplify numerical expressions involving exponents.

Exponent law	Example
Multiplying terms with the same base $a^m \times a^n = a^{m+n}$	$5^3 \times 5^2 = 5^{(3+2)} = 5^5$
Dividing terms with the same base $\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^6}{7^4} = 7^{(6-4)} = 7^2$
Raising a term in exponent form to another power $(a^m)^n = a^{m \times n} = a^{mn}$	$(3^7)^2 = 3^{7 \times 2} = 3^{14}$
Raising a product to a power $(a \times b)^m = a^m \times b^m$	$(3 \times 7)^4 = 3^4 \times 7^4$

Raising a quotient to a power $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
Raising a value to a negative exponent $a^{-m} = \frac{1}{a^m}, \frac{1}{a^{-m}} = a^m$	$3^{-3} = \frac{1}{3^3}, \frac{1}{3^{-2}} = 3^2$
Raising to the power of zero $a^0 = 1, a \neq 0$	$5^0 = 1$ Note: 0^0 is classed as undefined.

Worked example

Using the exponent laws to simplify numerical expressions

Simplify each of the following expressions, leaving your answers in exponent form.

(a) $3^2 \times 3^7$

THINKING	WORKING
Recall the exponent law for multiplying terms with the same base.	$a^m \times a^n = a^{m+n}$
Apply the appropriate exponent law.	$3^2 \times 3^7 = 3^{(2+7)}$ $= 3^9$

(b) $\frac{3^7}{3^2}$

THINKING	WORKING
Recall the exponent law for dividing terms with the same base.	$\frac{a^m}{a^n} = a^{m-n}$
Apply the appropriate exponent law.	$\frac{3^7}{3^2} = 3^{(7-2)}$ $= 3^5$

(c) $(3^2)^7$

THINKING	WORKING
Recall the exponent law for raising a term in exponent form to another power.	$(a^m)^n = a^{m \times n} = a^{mn}$
Apply the appropriate exponent law.	$(3^2)^7 = 3^{2 \times 7}$ $= 3^{14}$

(d) $(3^2 \times 5^4)^7$

THINKING	WORKING
Recall the exponent law for raising a product to a power.	$(a \times b)^m = a^m \times b^m$
Apply the appropriate exponent law.	$(3^2 \times 5^4)^7 = 3^{2\times 7} \times 5^{4\times 7}$ $= 3^{14} \times 5^{28}$

Practice

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SC 1 I can represent decimals and fractions in expanded form using negative exponents where necessary

1 Write the following decimal values in simplest exponent form.

(a) 13.287 **(b)** 1.098 **(c)** 105.78 **(d)** 1.0085

2 Write the following as decimals.

(a) $2 \times 10^1 + 4 \times 10^0 + 3 \times 10^{-1} + 1 \times 10^{-2} + 9 \times 10^{-3}$

(b) $5 \times 10^3 + 6 \times 10^1 + 2 \times 10^{-1} + 7 \times 10^{-3} + 4 \times 10^{-4}$

(c) $7 \times 10^5 + 4 \times 10^3 + 1 \times 10^1 + 4 \times 10^{-1} + 6 \times 10^{-3}$

(d) $6 \times 10^3 + 3 \times 10^2 + 1 \times 10^1 + 7 \times 10^{-3} + 4 \times 10^{-5}$

3 Express the following fractions in exponent form.

(a) $\frac{1}{11}$ **(b)** $\frac{2}{13}$ **(c)** $\frac{3}{29}$ **(d)** $\frac{5}{71}$

4 Express the following fractions in exponent form using the smallest base number possible for the exponent term.

(a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{1}{16}$

(e) $\frac{1}{32}$ **(f)** $\frac{1}{9}$ **(g)** $\frac{5}{16}$ **(h)** $\frac{7}{125}$

5 Write the following as fractions.

(a) 7^{-1} **(b)** 7^{-2} **(c)** 3^{-3}

(d) 3^{-4} **(e)** 6×5^{-3} **(f)** 9×2^{-5}

6 Complete the following by writing the number written in exponent form as:

- a fraction using exponent notation
- a fraction written in expanded form, and
- a fraction.

(a) 7^{-4} **(b)** 11^{-3} **(c)** 2^{-8} **(d)** 3^{-6}

SC 2 I can simplify and evaluate numerical expressions involving positive and negative exponents

1 Simplify each of the following, leaving your answers in exponent form.

(a) $2^3 \times 2^5$ **(b)** $(2 \times 11)^4$ **(c)** $\left(\frac{3}{7}\right)^6$ **(d)** $\frac{2^8}{2^3}$

2 Calculate the value of each of the following after first applying the appropriate exponent law.

(a) $5^3 \times 5^2$ **(b)** $\frac{7^3}{7^3}$ **(c)** $(3 \times 5)^3$ **(d)** $(5^2)^3$

3 Simplify each of the following, leaving your answers in exponent form with positive exponents.

(a) $(2^5)^3 \div (2^3)^4$ **(b)** $\left(\frac{3}{7}\right)^3 \div \left(\frac{3}{7}\right)^5$ **(c)** $\frac{(3\times 2)^3 \times \left(2^3\right)^4}{8^4}$ **(d)** $\frac{\left(\frac{5}{3}\right)^2 \times \left(\frac{5}{3}\right)^3}{\left(\frac{3}{5}\right)^2}$

4 Calculate the value of the following, after first applying the appropriate exponent rules.

(a) $2^3 \div 4^2 + (3^6)^0$ **(b)** $\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{2}{5}\right)^2} \times 5^{-3}$ **(c)** $\frac{\left(3\times 5^2\right)^2}{\left(3^3 \times 7\right)^3} \times \frac{1}{\left(3^{-2}\right)^4}$ **(d)** $\frac{\left(\frac{7^2}{3^3}\right)^4 \times \left(\frac{7}{3^2}\right)^{-1}}{\left(3^{-2}\right)^2}$

5 Complete the following by determining the missing exponent.

(a) $\frac{5^2}{5^{\square}} = \frac{1}{5}$ **(b)** $\frac{5^2}{5^{\square}} = 125$ **(c)** $\frac{5^{\square}}{5^4} = \frac{1}{25}$ **(d)** $\frac{5^{\square}}{5^2} = 625$

6 Answer the following questions about this pattern.

$5^1 \times 5^1 = 5^2 = 25$

$5^0 \times 5^0 = 5^0 = 1$

$5^{-1} \times 5^{-1} = 5^{-2} = \frac{1}{25}$

(a) What has happened to the final numerical value as you go down this pattern?

(b) What will be the result for $5^{\frac{1}{2}} \times 5^{\frac{1}{2}}$? Express this in exponent form and as a numerical value.

(c) What does this mean the value of $5^{\frac{1}{2}}$ must be?

(d) So, what is the value of $5^{-\frac{1}{2}}$?

Choose and apply efficient strategies to exponent laws of numerical expressions

2.7

Learning intention: To be able to choose and apply efficient strategies to exponent laws of numerical expressions

Success criteria:

SC 1 I can connect the calculation of numerical expressions involving exponents to the exponent laws.

SC 2 I can apply the exponent laws to simplify expressions involving products, quotients and powers of constants.

Lesson warm-up

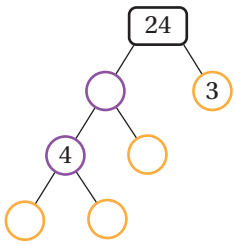
Prime products review



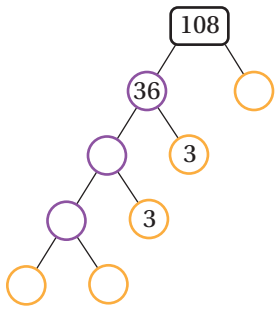
Complete these factor trees.

Purple circles contain composite values and yellow circles contain prime values.

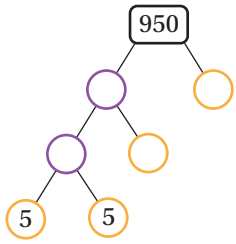
Write each composite number as a product of its prime factors.



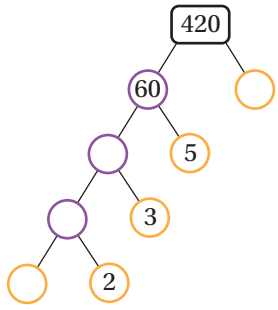
$24 = __ \times __$



$108 = __ \times __$



$950 = __ \times __ \times __$



$420 = __ \times __ \times __ \times __$

SC 1 I can connect the calculation of numerical expressions involving exponents to the exponent laws

The exponent laws are useful when comparing values given in exponent form. In many cases, difficult numerical calculations can be avoided.

As an example, how many times bigger than 6^4 is $2^4 \times 3^5$?

The strategy here is to recognise that $6^4 = (3 \times 2)^4 = 3^4 \times 2^4$.

So, $2^4 \times 3^5$ is three times bigger than 6^4.

2

Worked example

Manipulating expressions written in exponent form to compare values

Compare the values of $\frac{8^3}{4^4}$ and 2^3 without calculating either value.

THINKING	WORKING
Use the exponent laws to simplify expressions where possible.	$\frac{8^3}{4^4} = \frac{8^3}{4^3 \times 4}$ $= \frac{8^3}{4^3} \times \frac{1}{4}$ $= \left(\frac{8}{4}\right)^3 \times \frac{1}{4}$ $= 2^3 \times \frac{1}{4}$
Make the comparison between the values.	$\frac{8^3}{4^4}$ is one quarter of the value of 2^3.
As a check, compare the numerical values using a calculator.	$\frac{8^3}{4^4} = 2$ and $2^3 = 8$ $2 \div 8 = \frac{1}{4}$

SC 2 I can apply the exponent laws to simplify expressions involving products, quotients and powers of constants

Expressing numbers as products of prime factors can make final calculations much simpler because factors can be cancelled more easily.

Worked example

Using prime factors in exponent form to simplify calculations

Express $\frac{18\times36\times1000}{2500\times324}$ using products of prime factors, then evaluate the answer, expressing it as a fraction in simplest form.

THINKING	WORKING
Rewrite the expression using a product of prime factors.	$\frac{18\times36\times1000}{2500\times324}$ $=\frac{2\times9\times6^2\times10^3}{25\times10^2\times4\times81}$ $=\frac{2\times3^2\times2^2\times3^2\times2^3\times5^3}{5^2\times2^2\times5^2\times2^2\times3^4}$ $=\frac{2^6\times3^4\times5^3}{2^4\times3^4\times5^4}$
Use the exponent laws to simplify this expression.	$=\frac{2^2}{5}$ $=\frac{4}{5}$

Practice

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SC 1 I can connect the calculation of numerical expressions involving exponents to the exponent laws

1 Compare the following values without calculating either value.

(a) 12^7 and $4^6\times3^7$ **(b)** 16^6 and $8^6\times2^5$ **(c)** 20^7 and $5^7\times4^5$ **(d)** 20^7 and $5^5\times4^7$

2 Compare the following values without calculating either value.

(a) 36^{10} and $2^{10}\times3^9\times6^{11}$ **(b)** 24^8 and $2^6\times3^7\times4^8$

3 Compare the following values without calculating either value.

(a) $\frac{24^6}{12^5}$ and 2^5 **(b)** $\frac{24^5}{12^6}$ and 2^5 **(c)** $\frac{33^8}{3^6}$ and 11^8 **(d)** $\frac{5^4}{15^3}$ and 3^{-3}

4 Complete the following statements.

(a) $12^3=(4\times3)^{\square}$ **(b)** $(5\times3)^6=__^6$ **(c)** $\left(\frac{16}{8}\right)^7=__^7$ **(d)** $\left(\frac{24}{8}\right)^{\square}=__^3$

5 Express the following in simplest exponent form.

(a) $9\times27\times81$ **(b)** $2\times8\times32$ **(c)** $5\times125\times625$

6 Express the following as products of prime factors, using exponent notation as required.

(a) 18×27 **(b)** $24\times36\times50$ **(c)** $200\times35\times77$

SC 2 I can apply the exponent laws to simplify expressions involving products, quotients and powers of constants

1 Express the following as products of primes, then evaluate the answers, expressed in simplest fraction form if necessary.

(a) $\frac{50\times243\times150}{135\times54}$ **(b)** $\frac{600\times225\times162}{108\times900\times5}$ **(c)** $\frac{88\times320\times810}{486\times121\times216\times32}$

2 Copy and complete the following.

$2^6 = 64$

$2^5 = ___$

$2^4 = ___$

$2^3 = ___$

$2^2 = ___$

$2^1 = ___$

$2^0 = ___$

$2^{-1} = \frac{1}{2} = 0.5$

$2^{-2} = ___$

$2^{-3} = ___$

$2^{-4} = ___$

$2^{-5} = ___$

$2^{-6} = ___$

Now use the values above to write the following in exponent form.

(a) 0.03125×8 **(b)** $16 \times 0.25 \times 0.0625$ **(c)** $\frac{128}{0.25\times32}$

(d) $(0.125)^3$ **(e)** $\frac{64}{(0.5)^5}$ **(f)** $(0.25)^6 \times (64)^{-4}$

3 Determine the value of each of the following, expressing your answers in fraction form if necessary.

(a) $\left(\frac{3}{4}\right)^3 \times 2^4 + 3^0$ **(b)** $(16)^2 \times 2^{-5} \times \left(\frac{1}{2}\right)^{-3}$

(c) $(50)^2 \div 5^5 - (5^{-2})^2$ **(d)** $\frac{32}{250} \times \left(\frac{5}{2}\right)^4 + \left(\frac{5^{-1}}{2^{-3}}\right)^2$

4 Insert =, < or > to make correct statements.

(a) $3^4 \times 3^{-2} ___ 3^2$ **(b)** $(3^4)^{-2} ___ 3^{-6}$ **(c)** $\frac{3^4}{3^2} ___ 3^6$ **(d)** $\frac{3^4}{3^{-2}} ___ 3^2$

Apply exponent laws to simplify algebraic expressions

2.8

Learning intention: To be able to apply exponent laws to simplify algebraic expressions

Success criteria:

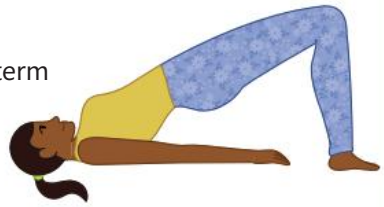
SC 1 I can verify the exponent laws as applied to variables.

SC 2 I can simplify algebraic expressions using exponent laws.

SC 3 I can simplify expressions that use exponent form and involve two or more variables.

Lesson warm-up

Back to base 2



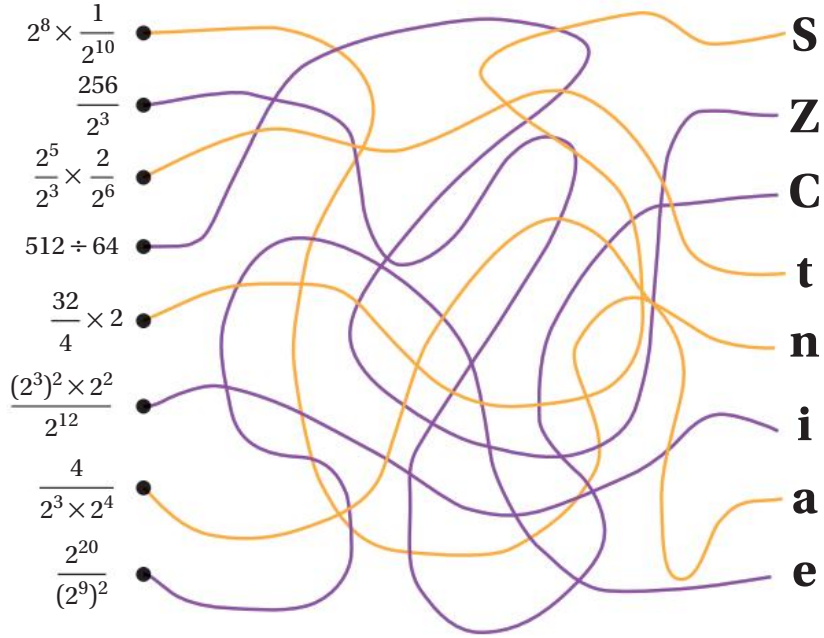
Simplify each of the expressions on the left-hand side to a single term with base 2.

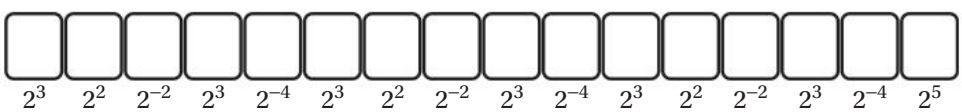
Follow the jumbled pathways to the letter that each base 2 value represents.

Match the letters to the terms under the answer boxes to discover this fact:

This now obsolete word was once used to represent 'a number raised to the power of 8'.

Be careful as not all letters shown here are used in the word.





SC 1 I can verify the exponent laws as applied to variables

A summary of all the exponent laws will help you simplify numerical expressions involving exponents.

Exponent law	Example
Multiplying terms with the same base $a^m \times a^n = a^{m+n}$	$5^3 \times 5^2 = 5^{(3+2)} = 5^5$
Dividing terms with the same base $\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^6}{7^4} = 7^{(6-4)} = 7^2$
Raising a term in exponent form to another power $(a^m)^n = a^{m \times n} = a^{mn}$	$(3^7)^2 = 3^{7 \times 2} = 3^{14}$
Raising a product to a power $(a \times b)^m = a^m \times b^m$	$(3 \times 7)^4 = 3^4 \times 7^4$
Raising a quotient to a power $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
Raising a value to a negative exponent $a^{-m} = \frac{1}{a^m}, \frac{1}{a^{-m}} = a^m$	$3^{-3} = \frac{1}{3^3}, \frac{1}{5^{-2}} = 5^2$
Raising to the power of zero $a^0 = 1, a \neq 0$	$5^0 = 1$ Note: 0^0 is classed as undefined.

Worked example

Verifying an exponent law

Verify the rule $a^m \times a^n = a^{m+n}$ using $a^5 \times a^2 = a^{(5+2)} = a^7$.

THINKING	WORKING
Write the statement that needs to be proved.	$a^5 \times a^2 = a^{(5+2)} = a^7$
Write the left-hand side (LHS) statement in expanded form.	LHS: $a^5 \times a^2$ $= (a \times a \times a \times a \times a) \times (a \times a)$
Simplify this expanded form.	$= a^7$
Show that this is the same as the right-hand side.	= RHS
Make a final statement.	Since the left-hand side can be shown to be the same as the right-hand side of the original equation, the proof is complete.

SC 2 I can simplify algebraic expressions using exponent laws

Algebraic terms seen in the context of the exponent laws usually consist of a numerical coefficient and a variable raised to a power. Terms such as $3x^2$, $2y^{-2}$ and $24z^6$ are examples of this format.

When applying the exponent laws to terms such as these, the numerical coefficients need to be considered separately from the parts of the terms that are raised to a power.

It is important to recognise that $2x^2$ and $(2x)^2$ are not equivalent terms.

$(2x)^2 = 2x \times 2x$

$= 4x^2$

$\neq 2x^2$

Worked example

Applying the exponent laws to simplify algebraic expressions

Use relevant exponent laws to simplify the following.

(a) $2a^2 \times 6a^4$

THINKING	WORKING
Write the expression with the coefficients separated from the variable part.	$2a^2 \times 6a^4 = 2 \times a^2 \times 6 \times a^4$
Rewrite, grouping the coefficients and the variable components.	$= 2 \times 6 \times a^2 \times a^4$
Multiply the coefficients together.	$= 12 \times a^2 \times a^4$
Apply the relevant exponent law and write in simplest algebraic form.	$= 12 \times a^6$ $= 12a^6$

(b) $\left(\frac{5a}{3}\right)^3$

THINKING	WORKING
Apply the relevant exponent law to write the expression in expanded form.	$\left(\frac{5a}{3}\right)^3 = \frac{5a}{3} \times \frac{5a}{3} \times \frac{5a}{3}$
Group the variables and the numbers.	$= \frac{5 \times 5 \times 5 \times a \times a \times a}{3 \times 3 \times 3}$
Complete the multiplication and write the answer in simplest algebraic form.	$= \frac{125a^3}{27}$

SC 3 I can simplify expressions that use exponent form and involve two or more variables

Algebraic expressions often contain more than one variable. This also occurs when the exponent laws are involved. No real change in approach is required – it is simply a matter of treating each variable separately.

It is a mathematical convention that a numerical coefficient appears at the start of a term, then variables are listed in alphabetical order. While $3a^2b^3$ is mathematically equivalent to $3b^3a^2$, it is preferable to use the first representation.

Worked example

Applying the exponent laws to algebraic terms that involve more than one variable

Use relevant exponent laws to simplify each of the following.

(a) $\dfrac{(3a^2b)^2}{2ab}$

THINKING	WORKING
Use an appropriate exponent law to remove brackets.	$\dfrac{(3a^2b)^2}{2ab} = \dfrac{3a^2b \times 3a^2b}{2ab}$
Use an appropriate exponent law to simplify the numerator.	$= \dfrac{9a^4b^2}{2ab}$
Apply an appropriate exponent law to simplify the expression.	$= \dfrac{9a^{(4-1)}b^{(2-1)}}{2}$ $= \dfrac{9a^3b}{2}$

(b) $(ab^2)^3 \times 2(a^2b)^2$

THINKING	WORKING
Use an appropriate exponent law to remove any brackets.	$(ab^2)^3 \times 2(a^2b)^2 = a^3b^6 \times 2a^4b^2$
Simplify the expression using appropriate exponent laws.	$= 2a^7b^8$

Practice

ANSWERS Page 613

SC 1 I can verify the exponent laws as applied to variables

1 Verify the rule $\frac{a^m}{a^n} = a^{m-n}$ using $\frac{a^6}{a^4} = a^{(6-4)} = a^2$.

2 Verify the rule $(a^m)^n = a^{m \times n} = a^{mn}$ using $(a^3)^2 = a^{3 \times 2} = a^6$.

3 Verify the rule $(a \times b)^m = a^m \times b^m$ using $(5 \times 2)^3 = 5^3 \times 2^3$.

4 Verify the rule $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ using $\left(\frac{10}{2}\right)^3 = \frac{10^3}{2^3}$.

5 Verify the rule $a^0 = 1, a \neq 0$.

SC 2 I can simplify algebraic expressions using exponent laws

1 Use an exponent law to simplify the following algebraic expressions.

(a) $2x^2 \times 3x^6$ **(b)** $5x^4 \times x^2$ **(c)** $6x^4 \times 2x^{-2}$ **(d)** $8x^{-4} \times 2x^{-3}$

2 Use an exponent law to simplify the following algebraic expressions.

(a) $\frac{6x^5}{2x^2}$ **(b)** $\frac{24x^3}{x}$ **(c)** $\frac{16x^4}{4x^{-2}}$ **(d)** $\frac{9x^{-3}}{3x^{-4}}$

3 Use an exponent law to simplify the following algebraic expressions.

(a) $(3x^2)^5$ **(b)** $(-3x)^2$ **(c)** $(2x^{-4})^3$ **(d)** $(3x^2)^{-4}$

4 One of the questions on the exponents test asked students to simplify $\left(\frac{-3a^3}{2}\right)^3$. Students A and B gave the following incorrect answers. Explain the errors made.

Student A	Student B
$\left(\frac{-3a^3}{2}\right)^3 = -\frac{3}{2}a^9$	$\left(\frac{-3a^3}{2}\right)^3 = \frac{27a^9}{8}$

5 Complete the following.

(a) $(x^a)^a = x^{\square}$ **(b)** $(x^a)^{-a} = x^{\square}$ **(c)** $x^a \times x^{-a} = x^{\square}$ **(d)** $\frac{x^a}{x^{-a}} = x^{\square}$

6 Simplify each of the following, expressing your answers in exponent form.

(a) $(x^3)^2 \times (x^4)^{-2}$ **(b)** $\frac{(5x^3)^2}{x^{-2}} \times \left(\frac{x}{5}\right)^3$ **(c)** $2(a^3)^2 \div 4(a^2)^3$ **(d)** $(2a^3)^2 \div (4a^2)^3$

7 Determine the value of x in each of the following.

(a) $a^{x-8} = a$ **(b)** $a^{5+x} = a$ **(c)** $a^{2x-3} = a^5$ **(d)** $(a^{2x})^3 = a^{12}$

SC 3 I can simplify expressions that use exponent form and involve two or more variables

1 Use an exponent law to simplify each of the following.

(a) $2a^3 \times 3b^2 \times 4a^2b$

(b) $-2a^2 \times 2b^4 \times 3a^3b^2$

(c) $-3a^{-2} \times 2ab \times (-5a^3b^{-2})$

(d) $4a^3 \times 5a^2 \times 3a^{-3}b^{-1}$

2 Use an exponent law to simplify each of the following.

(a) $\dfrac{4x^6 \times 3a^2}{6ax^3}$

(b) $\dfrac{2a^2 \times 5b^{-2}}{15a \times 4b^3}$

(c) $\dfrac{7ab^2 \times 2a^2b}{21a^5b^2}$

(d) $\dfrac{-2a^3b^2 \times 6a^{-1}b}{12a^4b}$

3 Use an exponent law to simplify each of the following.

(a) $(2ab^2)^3$

(b) $2(ab^2)^3$

(c) $2(ab^2)^{-3}$

(d) $-2(ab^2)^3$

(e) $(-2ab^2)^3$

4 Simplify each of the following.

(a) $4a^3b^0$

(b) $(4a^3b)^0$

(c) $(4a^3)^0 + b^0$

(d) $(4a^3)^0 \times b^0$

5 Simplify each of the following.

(a) $(xy)^2 \times (2xy)^{-1} \div 3x^2y$

(b) $\dfrac{(3xy)^{-1}}{2xy^2} \times \dfrac{(5xy)^2}{5\left(x^2y\right)^2}$

(c) $\dfrac{4abc^2 \times 3ab^2c \times 2a^2bc}{(2abc)^2}$

(d) $\left(\dfrac{3ab}{c}\right)^2 \div \dfrac{2\left(ab^2\right)^0}{c^3}$

TOPIC REVIEW

Multiple choice

ANSWERS Page 661

1 What is the exponent notation form of $3\times3\times3\times3$?

A 4×3 **B** 4^3 **C** 81 **D** 3^4

2 The expression $a^2\times a^7$ is the same as:

A 9 **B** 14 **C** a^9 **D** a^{14}

3 In the expression $3\times3\times8\times8\times8\times8\times8$, how many factors of 3 and 8, respectively, are present?

A 3 and 8 **B** 2 and 5 **C** 8 and 3 **D** 5 and 2

4 The simplest exponent form of the expression $2\times2^4\times x^2\times y^3\times x^5\times y^6$ is:

A $4^4\times7x\times9y$ **B** $2^4x^{10}y^{18}$ **C** $2^5x^5y^9$ **D** $2^5x^7y^9$

5 The expression $\frac{2^7}{2}$ simplifies to:

A 2^6 **B** $\frac{2^6}{2}$ **C** 2^7 **D** None of the above

6 The best simplified form of $\frac{c^7d^8}{c^3d^7}$ is:

A $c^{10}d^{15}$ **B** c^4d^1 **C** c^4d **D** $c^{10}d$

7 The expression $\left(4^2\right)^9$ is the same as:

A 4^{18} **B** 4^{11} **C** 4^7 **D** 4^7

8 The expression $\left(x^2\right)^7$ is the same as:

A x^9 **B** x^5 **C** $14x$ **D** x^{14}

9 The expression $\frac{2^3}{2^3}$ is the same as:

A 0 **B** 1 **C** 2^3 **D** All of the above

10 The expression $3^{-2}\div3^2$ is the same as:

A 3^{-4} **B** $\frac{1}{81}$ **C** $\frac{1}{3^4}$ **D** All of the above

11 The expression $12x^{-5}y^3$ simplified to a single fraction with positive exponents is:

A $12x^2$ **B** $\frac{1}{12x^5y^3}$ **C** $12xy^{-2}$ **D** $\frac{12y^3}{x^5}$

12 The exponent expression $1\times10^1+5\times10^0+2\times10^{-1}+2\times10^{-2}+9\times10^{-3}$ is the same as:

A 15229.00 **B** 15.229 **C** 1.5229 **D** 1522.90

13 The expression $\left(3^3\right)^3\div\left(3^3\right)^6$ is the same as:

A 3^{-9} **B** 3^9 **C** 3^{-3} **D** 3^3

14 The expression $3a^4 \times 4a^3$ is the same as:

A $7a^7$ **B** $12a^{12}$ **C** a **D** $12a^7$

15 The expression $\frac{(5m^5n^4)^2}{5m^3n^6}$ is the same as:

A $5m^7n^{-4}$ **B** $5m^4$ **C** $5m^2n^{-2}$ **D** $5m^7n^2$

Short answer

1 Simplify $13 \times 13 \times 3 \times 3 \times 3$.

2 Write the following expression in expanded notation, then write it in its simplest form.

$4^3 \times 4 \times 4^5$

3 Write the following expressions in terms of powers of prime numbers.

(a) 25×27 **(b)** 625×8^2

4 Simplify the following.

(a) $2x \times 2x^2 \times 5y^3 \times y^8$ **(b)** $8a^2 \times 2b^3 \times a \times 3b^2$

5 Write the expressions $\frac{2^3 \times 3^7}{2^2 \times 3^3}$ in simplest exponent form.

6 For each of the following expressions, write the power of a power in expanded form, then write in simplified exponent form.

(a) $(2^4)^2$ **(b)** $(4^3)^3$

7 Consider the expression $(6^2)^4$.

(a) Simplify in exponent form.

(b) Answer the following: There are ___ factors of ___.

8 Solve for x in each of the following equations.

(a) $\frac{3^x \times 3^2}{3^5 \times 3^4} = 1$ **(b)** $\frac{4^3 \times 4^x}{(4^2)^3} = 1$

9 Simplify the following.

(a) $2^4 \div 2^4$ **(b)** $150^8 \div 150^8$ **(c)** $2Z^0 \times 3Z^0$

10 Simplify the expression $\frac{5^3}{5^7}$, leaving your answer in exponent form with a positive exponent.

11 Write the following decimal values in expanded form in terms of powers of 10.

(a) 206.7009 **(b)** 0.00124 **(c)** 1.00961 **(d)** 25.55

12 Solve for x.

(a) $14^4 = (7 \times 2)^x$ **(b)** $\left(\frac{18}{2}\right)^6 = x^6$ **(c)** $4 \times 16 \times 64 = 2^x$

13 Are the expressions $4a^3$ and $(4a)^3$ equivalent? Explain your answer.

14 Simplify the expression $3x^2y^3 \times 5y^{-2} \times 6x^{-5}y^2$, leaving your answer in exponent form with a positive exponent.

Extended response

1 Use your understanding of exponent notation and multiplying and dividing powers with the same base to answer the following.

(a) Consider the expression $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$.

(i) Write using exponent notation.

(ii) Copy and complete: The base is ___ and the exponent is ___ .

(b) Consider the expression $5 \times 5^3 \times (a \times a \times a \times a) \times (b \times b \times b) \times (b \times b)$.

(i) Write the exponent law for multiplying two powers with the same base.

(ii) Write the expression in exponent notation, including the numerical factors.

(iii) Copy and complete: The exponent notation shows that there are ___ factors of ___, ___ factors of ___ and___factors of ___.

(c) Consider the expression $\dfrac{7^5 \times 2^3}{7^2 \times 2}$.

(i) Write the exponent law for dividing two powers with the same base.

(ii) Write the expression in expanded form.

(iii) Simplify the expression, leaving your answer in exponent form.

(d) Write the following expressions with the numbers and variables in their simplest exponent form.

(i) $\dfrac{3^9 p^3 \times 3^4 p^2}{3p^3}$ **(ii)** $\dfrac{4^6 a^2 b^7}{4^4 ab^4}$

2 Use your understanding of the zero exponent and raising a power to a power to answer the following.

(a) Demonstrate the rule $a^0 = 1$, where $a \neq 0$, using the expression $\dfrac{2^2}{2^2}$.

(b) Consider the expression $\left(4^3\right)^2$.

(i) Write the expression in expanded form, then simplify.

(ii) Write the exponent law for raising a power to a power.

(iii) Apply the exponent law to simplify the expression.

(c) Complete the following table using your knowledge of exponent laws for raising a power to a power.

Expression	Expansion	Simplified form
$\left(3^5\right)^3$	$3^5 \times 3^5 \times 3^5$	3^{15}
$\left(4^7\right)^2$		
$\left(13^6\right)^4$		
$\dfrac{\left(2^2\right)^5}{2^3}$		
$\dfrac{m^4\left(n^3\right)^2}{m^2 n}$		

(d) Solve for x in each of the following equations.

(i) $\dfrac{2^3 \times m^4 \times m^3}{m^5 \times m^x} = 8$ **(ii)** $\dfrac{4\left(a^4\right)^3}{2a^x} = 2$

3 Use your understanding of negative exponents, exponent form and simplifying fractions using exponents to answer the following.

(a) Use negative exponents to express the following fractions in exponent form.

(i) $\dfrac{1}{13}$ **(ii)** $\dfrac{3}{31}$

(b) Write each of the following expressions as a number in decimal form.

(i) $3\times10^1+7\times10^0+2\times10^{-1}+5\times10^{-2}$ **(ii)** $9\times10^2+6\times10^1+5\times10^0+6\times10^{-1}$

(c) Write each of the following decimals in expanded form in terms of powers of 10.

(i) 1.3206 **(ii)** 1001.4001

(d) Express the following fractions as a single power using the smallest base possible.

(i) $\dfrac{1}{16}$ **(ii)** $\dfrac{1}{64}$

(e) For the power 8^{-3}, write as a fraction:

(i) using exponent notation **(ii)** in expanded form **(iii)** in simplest form.

4 Use prime factors as the base to simplify the calculations and expressions in each of the following.

(a) How many times bigger than 8^5 is $4^6 \times 2^5$?

(b) How many times smaller is 6^8 when compared to $2^9 \times 3^8$?

(c) Calculate the value of the fraction $\dfrac{16\times27\times625}{4\times729\times25}$.

(d) Simplify the expression $\dfrac{12x^4 \times 2z^6}{6xz^4}$.

TOPIC
3

Algebra (binomials)

Recall 66

3.1 Understand and use the distributive law 67

3.2 Recognise, expand and factorise special binomial products 74

3.3 Understand algebraic factorisation 82

3.4 Factorise simple quadratics 88

Topic review 91

Why learn this?

Learning algebra, particularly binomials, provides a foundation for understanding more complex mathematical concepts. This topic introduces the distributive law, which simplifies calculations and aids in understanding patterns in mathematics. The area model is also covered by this topic and can be used to visualise multiplying out brackets in algebraic expressions.

Knowledge gained from studying binomials can be applied in fields such as engineering, physics and computer science, where algebraic equations are often used to solve problems. This knowledge also enhances problem-solving skills and logical thinking, and helps with making connections between different concepts.

RECALL

ANSWERS Page 614

I can simplify algebraic expressions by collecting like terms

1 Simplify each of the following expressions.

(a) $15p - 5p$ **(b)** $-3q + 7q$ **(c)** $-8x - 4x$

(d) $6c - 2c - 8c$ **(e)** $-20d + 14d - 3d$ **(f)** $-3a + 2a - 5a$

2 Simplify each of the following expressions by grouping first.

(a) $7b - 8a + 5b + 10a$ **(b)** $-3c + 4d - 4c - 8d$ **(c)** $2x^2 + 5y - 4x^2 + 7y$

I can simplify algebraic expressions involving the multiplication of terms

1 Simplify each of the following.

(a) $5a \times 12b$ **(b)** $2a \times 60b$ **(c)** $4y \times 11y$

2 Simplify each of the following.

(a) $3a \times (-4b) \times (-2c)$ **(b)** $12x \times (-y) \times 3z$ **(c)** $8m \times 4n \times 2m$

I can expand and simplify algebraic expressions with like terms

1 Expand each of these expressions.

(a) $7(b + 4)$ **(b)** $3(2c - 1)$ **(c)** $-2(3x + 5)$

2 Expand and simplify each of these expressions.

(a) $3(2a + 3b - 4a)$ **(b)** $-2(x + 2y - 6x)$ **(c)** $3(5a + b + 3a - 7b)$

I can find the highest common factor for numbers

1 Write the largest number that divides into the following number groups with no remainder.

(a) 24, 36, 48 **(b)** 35, 63, 42 **(c)** 108, 54, 81

I can factorise simple algebraic expressions

1 Factorise each of the following.

(a) $7x + 14$ **(b)** $9x - 45$ **(c)** $18x - 27$

(d) $24 + 8c$ **(e)** $54 - 24m$

2 Write the greatest common negative factor for each expression, then factorise.

(a) $-6a - 18$ **(b)** $-14x - 35$ **(c)** $30 - 3n$

(d) $-8k + 16$ **(e)** $-48 - 18d$

Understand and use the distributive law

3.1

Learning intention: To be able to understand and use the distributive law to expand simple algebraic expressions

Success criteria:

SC 1 I can describe and illustrate the distributive law using the area model.

SC 2 I can apply the distributive law to calculate a range of numerical expressions.

SC 3 I can apply the distributive law to multiply out the brackets of algebraic expressions.

Lesson warm-up

Patterns

Observe the patterns in these two images.

$3 \times \square = \square + \square + \square$

$= 3\square$

$3 \times (\square + \bigcirc) = (\square + \bigcirc) + (\square + \bigcirc) + (\square + \bigcirc)$

$= \square + \square + \square + \bigcirc + \bigcirc + \bigcirc$

$= 3\square + 3\bigcirc$

1 Apply the patterns above to complete these two images.

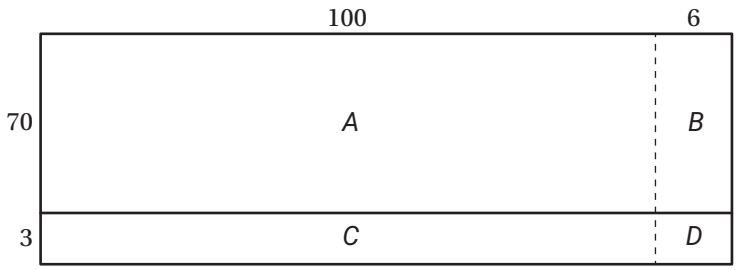
(a) $4 \times (\triangle + 1) =$

(b) $2 \times (\bigcirc + \triangle + \square) =$

2 Is there a quick way to go straight to the last line in each pattern? Explain.

SC 1 I can describe and illustrate the distributive law using the area model

If you are asked to calculate 10×7, you should be able to quickly write the answer as 70. However, more difficult multiplications are more challenging to calculate, for example 106×73. Using the distributive law can help simplify such calculations. Consider the following representation.



The area of each rectangle can be calculated as shown.

A: $100 \times 70 = 7000$

B: $6 \times 70 = 420$

C: $100 \times 3 = 300$

D: $6 \times 3 = 18$

The total area is $7000 + 420 + 300 + 18 = 7738$ units^2.

You can use you calculator to check: $106 \times 73 = 7738$.

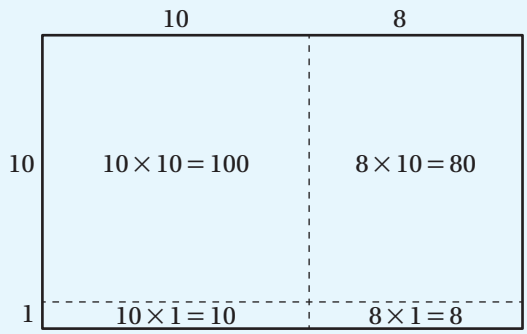
Each of the four smaller calculations can be completed using mental arithmetic skills, so this method helps make a difficult calculation easier to complete with just mental arithmetic skills. This is known as the area model.

3

Worked example

Using the area model

Use the area model to calculate the result for 18×11.

THINKING	WORKING
Decide on the way to break up the two values. Use multiples of tens, hundreds and thousands where possible.	$18 \times 11 = (10 + 8) \times (10 + 1)$
Draw a diagram to show this and complete the multiplications.	
Add the results to determine the product.	$18 \times 11 = 100 + 80 + 10 + 8$ $= 198$
Check the result using a calculator.	$18 \times 11 = 198$

SC 2 I can apply the distributive law to calculate a range of numerical expressions

The area model is one way to model the distributive law.

The distributive law shows how to simplify expressions by removing brackets.

The process can be demonstrated as follows.

$2(3 + 4) = 2 \times 3 + 2 \times 4$

$= 6 + 8$

$= 14$

You can easily verify that this is the correct solution: $2 \times 7 = 14$.

The process can be extended to two sets of brackets.

$$\begin{aligned}(1 + 2)(3 + 4) &= 1 \times (3 + 4) + 2 \times (3 + 4) \\ &= 1 \times 3 + 1 \times 4 + 2 \times 3 + 2 \times 4 \\ &= 3 + 4 + 6 + 8 \\ &= 21\end{aligned}$$

Again, the solution can be easily verified: $(1 + 2) \times (3 + 4) = 3 \times 7 = 21$.

The process described in words is: multiply each of the terms inside the first bracket by each of the terms in the second bracket.

Worked example

Using the distributive law on a numerical expression

Use the distributive law to calculate:

(a) $5 \times (7 + 6)$

THINKING	WORKING
Apply the distributive law. Multiply the first term by each term in the bracket.	$5 \times (7 + 6) = 5 \times (7 + 6)$ $= 5 \times 7 + 5 \times 6$ $= 35 + 30$ $= 65$
Check the result by simplifying the bracket.	$5 \times (7 + 6) = 5 \times 13$ $= 65$

(b) $3 \times (7 + 6)$

THINKING	WORKING
Apply the distributive law. Multiply the first term by each term in the bracket.	$3 \times (7 + 6) = 3 \times (7 + 6)$ $= 3 \times 7 + 3 \times 6$ $= 21 + 18$ $= 39$
Check the result by simplifying the bracket.	$3 \times (7 + 6) = 3 \times 13$ $= 39$

(c) $(5 + 3)(7 + 6)$

THINKING	WORKING
Apply the distributive law. Multiply each term in the first bracket by each term in the second bracket.	$(5 + 3)(7 + 6) = 5 \times (7 + 6) + 3 \times (7 + 6)$ $= 5 \times 7 + 5 \times 6 + 3 \times 7 + 3 \times 6$ $= 35 + 30 + 21 + 18$ $= 65 + 39$ $= 104$
Check the result by simplifying each bracket.	$(5 + 3)(7 + 6) = 8 \times 13$ $= 104$

(d) $(5 - 3)(7 + 6)$

THINKING	WORKING
Apply the distributive law. Multiply each term in the second bracket by each term in the first bracket. Note the subtraction in the first bracket.	$(5 - 3)(7 + 6) = 5 \times (7 + 6) - 3 \times (7 + 6)$ $= 5 \times 7 + 5 \times 6 - 3 \times 7 - 3 \times 6$ $= 35 + 30 - 21 - 18$ $= 65 - 39$ $= 26$
Check the result by simplifying each bracket.	$(5 - 3)(7 + 6) = 2 \times 13$ $= 26$

SC 3 I can apply the distributive law to multiply out the brackets of algebraic expressions

Using the distributive law with numerical expressions is one way of learning the method. In most cases, multiplication strategies could also help you to calculate the value.

The real application of the distributive law is with algebraic expressions.

The process is the same, except with algebra some variables are involved.

When you multiply variables, it is important to remember that:

- you don't need to write the multiplication symbol
- generally variables are written in alphabetical order
- when a variable is multiplied by a number, the number (coefficient) is written before the variable. For example, $a \times b$ is written as ab and $b \times 2$ is written as $2b$.

Worked example

Using the distributive law on an algebraic expression

Use the distributive law to expand the following algebraic expressions.

(a) $(x + 3)(y + 4)$

THINKING	WORKING
Apply the distributive law.	$(x + 3)(y + 4) = x(y + 4) + 3(y + 4)$ $= x \times y + x \times 4 + 3 \times y + 3 \times 4$ $= xy + 4x + 3y + 12$
Check whether any like terms are present that can be simplified.	There are no like terms, so this is the simplest form of the expression.

(b) $(x + 3)(x + 4)$

THINKING	WORKING
Apply the distributive law.	$(x + 3)(x + 4) = x(x + 4) + 3(x + 4)$ $= x \times x + x \times 4 + 3 \times x + 3 \times 4$ $= x^2 + 4x + 3x + 12$
Check whether any like terms are present that can be simplified.	$= x^2 + (4x + 3x) + 12$ $= x^2 + 7x + 12$

(c) $(2x - 3)(x + 4)$

THINKING	WORKING
Apply the distributive law.	$(2x - 3)(x + 4) = 2x(x + 4) - 3(x + 4)$ $= 2x \times x + 2x \times 4 - 3 \times x - 3 \times 4$ $= 2x^2 + 8x - 3x - 12$
Check whether any like terms are present that can be simplified.	$= 2x^2 + (8x - 3x) - 12$ $= 2x^2 + 5x - 12$

(d) $(2x - 3)(5x + 4)$

THINKING	WORKING
Apply the distributive law.	$(2x - 3)(5x + 4) = 2x(5x + 4) - 3(5x + 4)$ $= 2x \times 5x + 2x \times 4 - 3 \times 5x - 3 \times 4$ $= 10x^2 + 8x - 15x - 12$
Check whether any like terms are present that can be simplified.	$= 10x^2 + (8x - 15x) - 12$ $= 10x^2 - 7x - 12$

Practice

ANSWERS Page 615

SC 1 I can describe and illustrate the distributive law using the area model

1 Use the area model to determine the result for each of the following multiplications.

(a) 14×13 **(b)** 21×18 **(c)** 106×42 **(d)** 103×21

2 Write an expanded form (using multiples of ten, one hundred, one thousand) that you would use with the area model to complete the following calculations. There is no need to complete the calculation.

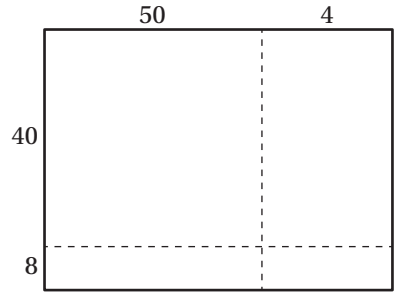
(a) 53×42 **(b)** 105×14 **(c)** 1003×53 **(d)** $10\,005 \times 1006$

3 Use an area model to calculate the result for each of the following multiplications.

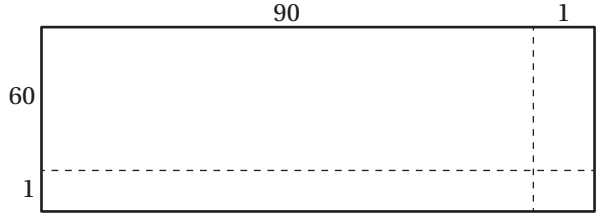
(a) 110×101 **(b)** 120×110 **(c)** 115×103 **(d)** 204×103

4 Write the simplified question represented by each of the following area diagrams and state the product it represents.

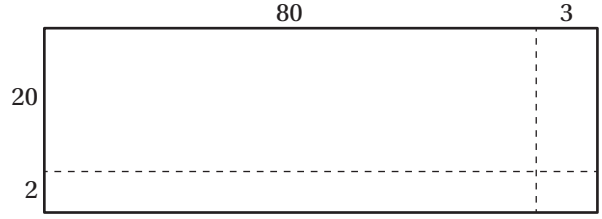
(a)



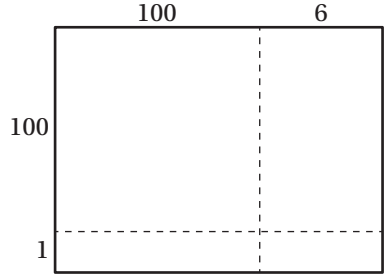
(b)



(c)



(d)



SC 2 I can apply the distributive law to calculate a range of numerical expressions

1 Use the distributive law to calculate the value of each of the following.

(a) $(3 + 9)(4 + 6)$ **(b)** $(7 + 9)(6 + 7)$ **(c)** $(10 + 4)(20 + 5)$ **(d)** $(20 + 7)(11 + 4)$

2 Use the distributive law to calculate each of the following expressions.

(a) $(6 + 8)(4 - 1)$ **(b)** $(9 + 7)(9 - 3)$ **(c)** $(6 - 4)(8 + 2)$ **(d)** $(12 - 5)(11 - 4)$

3 Demonstrate the distributive law for the following multiplications using multiples of ten and

(i) addition of values

(ii) an addition and a subtraction.

(a) 42×17 **(b)** 35×19

4 Erasmus was attempting the following question: Calculate 47×8.

(a) Erasmus's first step was to write: $47 \times 8 = (50 - 3)(10 - 2)$. Is this step correct?

(b) The second step was to produce the following working:

$$47 \times 8 = (50 - 3)(10 - 2)$$
$$= 50 \times 10 + 50 \times (-2) + (-3) \times 10 + (-3) \times (-2)$$
$$= 500 - 100 - 30 - 6$$
$$= 364$$

His teacher marked this answer as incorrect. Locate and correct any mistakes you can find in the working.

SC 3 I can apply the distributive law to multiply out the brackets of algebraic expressions

1 Use the distributive law to expand each of the following.

(a) $(x + 4)(y + 2)$ **(b)** $(x - 5)(y + 6)$ **(c)** $(x + 6)(y - 4)$ **(d)** $(x - 7)(y - 8)$

2 Use the distributive law to expand each of the following.

(a) $(3a + 2)(2a + 3)$ **(b)** $(5b - 2)(2b + 3)$ **(c)** $(6c + 5)(3c - 4)$ **(d)** $(10d - 4)(5d - 3)$

3 Use the distributive law to expand each of the following.

(a) $(3a + 2b)(4a + 5b)$ **(b)** $(6a - 3b)(4a + 2b)$

(c) $(8a + 3b)(6a - 4b)$ **(d)** $(9a - 3b)(3a - 9b)$

4 Express the following in simplest form.

(a) Determine the area of a rectangle with a length of $(2x - 6)$ cm and a width of $(3x + 5)$ cm.

(b) Determine the area of a square with a side length of $(6x - 4)$ cm.

(c) Determine the area of a right-angled triangle with a base of $(3x - 5)$ cm and a perpendicular height of $(8 - 2x)$ cm. Determine the area for $x = 2$.

3.2 Recognise, expand and factorise special binomial products

Learning intention: To be able to recognise, expand and factorise special binomial products

Success criteria:

SC 1 I can expand the sum or difference of perfect squares.

SC 2 I can expand the difference of two squares.

SC 3 I can factorise perfect squares or the difference of two squares.

Lesson warm-up

Depth discovery

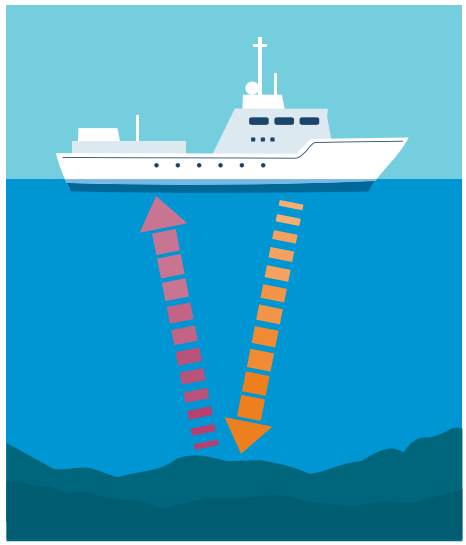
A survey boat is recording the depth of the ocean every 5 km when sailing from a popular shipping port. The data for the past 30 km was requested by the captain, who was given the following answer:

All depths have been recorded to the nearest 10 m.

Each depth is different.

Each depth value is a factor of 240 and all the depths have a sum of 350 m.

What depths were recorded?



SC 1 I can expand the sum or difference of perfect squares

When using the distributive law with algebraic expressions, some patterns become apparent.

The first of these is the perfect square expansion.

An algebraic perfect square means that the two factors are the same.

$(x + a)(x + a) = (x + a)^2$ is an example of a perfect square.

The perfect square rule is written in these forms:

$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$

$(x - a)^2 = (x - a)(x - a) = x^2 - 2ax + a^2$

Worked example

Expanding a perfect square

Use the distributive law to expand the following.

(a) $(x + 3)^2 = (x + 3)(x + 3)$

THINKING	WORKING
Use the distributive law to expand the brackets.	$(x + 3)^2 = (x + 3)(x + 3)$ $= x \times x + x \times 3 + 3 \times x + 3 \times 3$ $= x^2 + 3x + 3x + 9$ $= x^2 + 6x + 9$

(b) $(x - 3)^2 = (x - 3)(x - 3)$

THINKING	WORKING
Use the distributive law to expand the brackets.	$(x - 3)^2 = (x - 3)(x - 3)$ $= x \times x - x \times 3 - 3 \times x + (-3) \times (-3)$ $= x^2 - 3x - 3x + 9$ $= x^2 - 6x + 9$

(c) $(3x + y)^2 = (3x + y)(3x + y)$

THINKING	WORKING
Use the distributive law to expand the brackets.	$(3x + y)^2 = (3x + y)(3x + y)$ $= 3x \times 3x + 3x \times y + 3x \times y + y \times y$ $= 9x^2 + 3xy + 3xy + y^2$ $= 9x^2 + 6xy + y^2$

(d) $(3x - y)^2 = (3x - y)(3x - y)$

THINKING	WORKING
Use the distributive law to expand the brackets.	$(3x - y)^2 = (3x - y)(3x - y)$ $= 3x \times 3x + 3x \times (-y) + 3x \times (-y) + (-y) \times (-y)$ $= 9x^2 - 3xy - 3xy + y^2$ $= 9x^2 - 6xy + y^2$

SC 2 I can expand the difference of two squares

The next pattern to consider is the difference of two squares pattern. It is sometimes referred to as DOTS (**d**ifference **o**f **t**wo **s**quares). This pattern occurs when the two factors differ only in the operation that separates the two terms, such as $(a + b)(a - b)$.

Using the distributive law shows that $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$.

The reason for the name can be seen in the result of the expansion. It produces two terms separated by a subtraction operation.

As with the perfect square rule, you should commit this rule, or pattern, to memory.

Worked example

Expanding by using the difference of two squares (DOTS)

Expand the following using the difference of two squares (DOTS) rule.

(a) $(x - 3)(x + 3)$

THINKING	WORKING
Recall the DOTS rule.	$(a + b)(a - b) = a^2 - b^2$
Assign the terms in the expression to the variables in the expression.	$a = x$ $b = 3$
Substitute into the rule and simplify.	$(x - 3)(x + 3) = (x)^2 - (3)^2$ $= x^2 - 9$

(b) $(x - 3y)(x + 3y)$

THINKING	WORKING
Recall the DOTS rule.	$(a + b)(a - b) = a^2 - b^2$
Assign the terms in the expression to the variables in the expression.	$a = x$ $b = 3y$
Substitute into the rule and simplify.	$(x - 3y)(x + 3y) = (x)^2 - (3y)^2$ $= x^2 - 9y^2$

(c) $(2x - 3y)(2x + 3y)$

THINKING	WORKING
Recall the DOTS rule.	$(a + b)(a - b) = a^2 - b^2$
Assign the terms in the expression to the variables in the expression.	$a = 2x$ $b = 3y$
Substitute into the rule and simplify.	$(2x - 3y)(2x + 3y) = (2x)^2 - (3y)^2$ $= 4x^2 - 9y^2$

SC 3 I can factorise perfect squares or the difference of two squares

Factorising is the opposite to expanding. When expanding an expression, the task is to remove the brackets. When factorising, the task is to write an equivalent expression that uses brackets.

The two perfect square patterns are $a^2 \pm 2ab + b^2$:

$(a + b)(a + b) = a^2 + 2ab + b^2$

$(a - b)(a - b) = a^2 - 2ab + b^2$

If an expanded expression fits this pattern, then the factorised form can be written immediately.

An expression like $a^2 + 2ab + b^2$ is called a quadratic trinomial. 'Trinomial' means there are three terms, and 'quadratic' means the highest power is a square.

Worked example

Factorising using the perfect square pattern

Factorise the following.

(a) $x^2 + 4x + 4$

THINKING	WORKING
If possible, write the expression in the form $a^2 \pm 2ab + b^2$.	$x^2 + 4x + 4 = x^2 + 2 \times x \times 2 + (2)^2$
Recall the perfect square rule.	$(a + b)(a + b) = a^2 + 2ab + b^2$
Write the expression in factorised form.	$x^2 + 4x + 4 = (x + 2)(x + 2)$ $= (x + 2)^2$

(b) $x^2 - 4x + 4$

THINKING	WORKING
If possible, write the expression in the form $a^2 \pm 2ab + b^2$.	$x^2 - 4x + 4 = x^2 - 2 \times x \times 2 + (2)^2$
Recall the perfect square rule.	$(a - b)(a - b) = a^2 - 2ab + b^2$
Write the expression in factorised form.	$x^2 - 4x + 4 = (x - 2)(x - 2)$ $= (x - 2)^2$

(c) $x^2 + 4xy + 4y^2$

THINKING	WORKING
If possible, write the expression in the form $a^2 \pm 2ab + b^2$.	$x^2 + 4xy + 4y^2 = x^2 + 2 \times x \times 2y + (2y)^2$
Recall the perfect square rule.	$(a + b)(a + b) = a^2 + 2ab + b^2$
Write the expression in factorised form.	$x^2 + 4xy + 4y^2 = (x + 2y)(x + 2y)$ $= (x + 2y)^2$

Worked example

Factorising using the difference of perfect squares pattern

Factorise the following.

(a) $9x^2 - y^2$

THINKING	WORKING
Rewrite the expression in the form $(ax)^2 - (by)^2$.	$9x^2 - y^2 = (3x)^2 - (y)^2$
Use the DOTS pattern to write the expression in factorised form.	$9x^2 - y^2 = (3x)^2 - (y)^2$ $= (3x - y)(3x + y)$

(b) $x^2 - 16y^2$

THINKING	WORKING
Rewrite the expression in the form $(ax)^2 - (by)^2$.	$x^2 - 16y^2 = (x)^2 - (4y)^2$
Use the DOTS pattern to write the expression in factorised form.	$x^2 - 16y^2 = (x)^2 - (4y)^2$ $= (x - 4y)(x + 4y)$

(c) $9x^2 - 16y^2$.

THINKING	WORKING
Rewrite the expression in the form $(ax)^2 - (by)^2$.	$9x^2 - 16y^2 = (3x)^2 - (4y)^2$
Use the DOTS pattern to write the expression in factorised form.	$9x^2 - 16y^2 = (3x)^2 - (4y)^2$ $= (3x - 4y)(3x + 4y)$

Practice

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SC 1 I can expand the sum or difference of perfect squares

1 Expand each of the following perfect squares.

(a) $(x + 5)^2$ **(b)** $(x + 2)^2$ **(c)** $(x + b)^2$ **(d)** $(x + 5b)^2$

(e) $(2x + 5)^2$ **(f)** $(2x + b)^2$ **(g)** $(2x + 5b)^2$

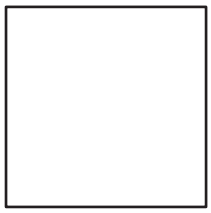
2 Expand each of the following perfect squares.

(a) $(x - 4)^2$ **(b)** $(x - b)^2$ **(c)** $(x - 4b)^2$

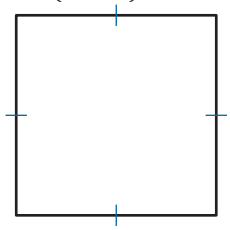
(d) $(3x - 4)^2$ **(e)** $(3x - b)^2$ **(f)** $(3x - 4b)^2$

3 Write a simplified algebraic expression for the area of each of the following squares.

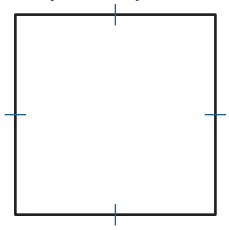
(a)



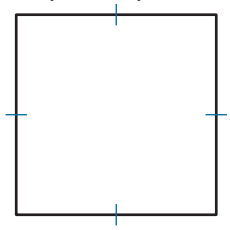
(b)



(c)



(d)



4 Complete the following steps to calculate 21^2 without using a calculator.

(a) Write 21^2 in the form $(a + b)^2$, where a is the largest multiple of 10 that is smaller than 21.

(b) Use the perfect square formulation to complete the calculation.

(c) Use a similar method to calculate the value of 19^2.

SC 2 I can expand the difference of two squares

1 Expand each of the following using the DOTS pattern.

(a) $(3 - x)(3 + x)$ **(b)** $(x - 3)(x + 3)$ **(c)** $(7x - 3)(7x + 3)$

(d) $(3 - 7x)(3 + 7x)$ **(e)** $(t + 2)(t - 2)$ **(f)** $(2 + t)(2 - t)$

(g) $(2 + 5t)(2 - 5t)$ **(h)** $(5t + 2)(5t - 2)$ **(i)** $(k - 3)(k + 3)$

(j) $(k - p)(k + p)$ **(k)** $(k - 3p)(k + 3p)$ **(l)** $(5 + n)(5 - n)$

(m) $(m + n)(m - n)$ **(n)** $(5m + n)(5m - n)$

2 Expand each of the following using the DOTS pattern, or otherwise.

(a) $(4k - 6t)(4k + 6t)$ **(b)** $(5p + 11q)(5p - 11q)$

(c) $(8x - 9y)(8x + 9y)$ **(d)** $(12r + 11s)(12r - 11s)$

3 One of the questions on the Expanding Brackets test was 'Which of the special expansion rules can be used to expand $(a + b)(b - a)$?'

(a) What is the answer to that question?

(b) Show the expansion of $(a + b)(b - a)$ using the distributive law.

4 Sergio provided the following template to determine the value of 59×61.

(a) Copy and complete the product.

$59 \times 61 = (60 - 1)(60 ___ 1)$

$= 60^2 - __^2$

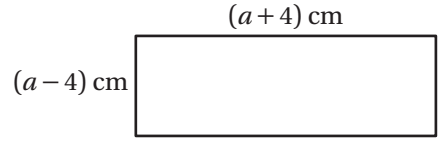
$= 3600 - ___$

$= ___$

Now use the same method to calculate the following products.

(b) 72×68 **(c)** 99×101 **(d)** 44×36

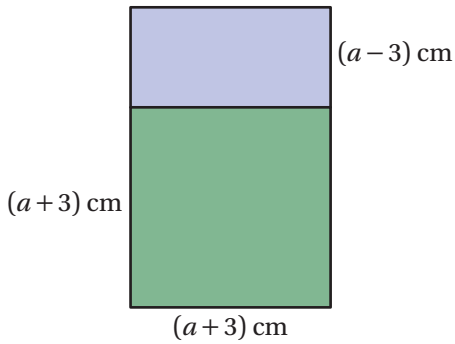
5 Consider the following rectangle.



(a) In simplest form, what is the area of this rectangle?

(b) Write an inequality describing the possible values of a.

6 Consider the following diagram.



(a) In expanded form, determine the expression for the area of the blue rectangle.

(b) In expanded form, determine the expression for the area of the green square.

(c) In simplified form, state the area of the complete shape.

(d) What can be said about the value of a?

SC 3 I can factorise perfect squares or the difference of two squares

1 Factorise the following quadratic trinomials using the perfect square pattern.

(a) $x^2 + 6x + 9$ **(b)** $x^2 - 6x + 9$ **(c)** $4x^2 + 12x + 9$

(d) $9x^2 + 12x + 4$ **(e)** $9x^2 - 6xy + y^2$ **(f)** $9x^2 - 12xy + 4y^2$

(g) $25 + 20x + 4x^2$ **(h)** $36x^2 - 60xy + 25y^2$

2 Factorise the following quadratics using the DOTS pattern.

(a) $81 - z^2$ **(b)** $z^2 - 81$ **(c)** $81 - 9z^2$ **(d)** $t^2 - 49$

(e) $144t^2 - 49$ **(f)** $k^2 - 16p^2$ **(g)** $25k^2 - p^2$ **(h)** $25k^2 - 16p^2$

(i) $100t^2 - 121s^2$ **(j)** $121s^2 - 100t^2$

3 Copy and complete the following to factorise $4a^4 - 16a^8$.

$4 = (__)^2$

$a^4 = (__)^2$

$4a^4 = (__)^2$

$16a^8 = (__)^2$

So

$4a^4 - 16a^8 = (__)^2 - (__)^2$

$= (__ - __)(__ + __)$

3.3 Understand algebraic factorisation

Learning intention: To be able to understand algebraic factorisation

Success criteria:

SC 1 I can factorise algebraic expressions containing both numerical and algebraic common factors.

SC 2 I can factorise algebraic expressions in which the common factor contains brackets.

Lesson warm-up

What is in common?



Each emoji pair exists more than once in the image.

Identify all the different emoji pairings.

Identify how many of each matching pair there are.

Identify and group the matching pairs that have an emoji in common.

How could you group the emojis in a way that shows the common emoji and the two others it is paired with?

SC 1 I can factorise algebraic expressions containing both numerical and algebraic common factors

When factorising algebraic expressions, the task is to determine factors that are common to all terms in the expression, then put this common factor outside a set of brackets. These common factors can be numerical, algebraic or a combination of both. It is important to understand that a factorised expression must contain brackets.

In the first worked example, the factor must be numerical as both terms are numerical.

Worked example

Factorising when the factor is numerical

Determine the highest common factor of 12 and 20.

THINKING	WORKING
List the factors of each number.	12: 1, 2, 3, 4, 6, 12 20: 1, 2, 4, 5, 10, 20
Identify the highest common factor (HCF).	HCF: 4

Use the highest common factor to do the following.

(a) Factorise $20x + 12$

THINKING	WORKING
Write each term in the expression as a product using the highest common factor.	$20x + 12 = 4 \times 5x + 4 \times 3$
Factorise the expression by writing the highest common factor in front of the bracket.	$= 4(5x + 3)$
Check your answer by expanding the bracket.	$4(5x + 3) = 4 \times 5x + 4 \times 3$ $= 20x + 12$

(b) Factorise $12x - 20$

THINKING	WORKING
Write each term in the expression as a product using the highest common factor.	$12x - 20 = 4 \times 3x - 4 \times 5$
Factorise the expression.	$= 4(3x - 5)$
Check your answer by expanding the bracket.	$4(3x - 5) = 4 \times 3x - 4 \times 5$ $= 12x - 20$

(c) Factorise $12 - 20x$

THINKING	WORKING
Write each term in the expression as a product using the highest common factor.	$12 - 20x = 4 \times 3 - 4 \times 5x$
Factorise the expression.	$= 4(3 - 5x)$
Check your answer by expanding the bracket.	$4(3 - 5x) = 4 \times 3 - 4 \times 5x$ $= 12 - 20x$

Worked example

Factorising when the factor is algebraic

Factorise the following.

(a) $m + m^2$

THINKING	WORKING
Identify the algebraic component of the HCF.	Algebraic HCF: m
Write each term in the expression as a product using the HCF, then factorise.	$m + m^2 = m \times 1 + m \times m$ $= m(1 + m)$
Check your answer by expanding the bracket.	$m(1 + m) = m \times 1 + m \times m$ $= m + m^2$

(b) $27m^2 + 6m$

THINKING	WORKING
List the factors of each of the numeric coefficients.	6: 1, 2, 3, 6 27: 1, 3, 9, 27
Identify the numeric component of the HCF.	Numeric HCF: 3
Identify the algebraic component of the HCF.	Algebraic HCF: m
Write each term in the expression as a product using the highest common factor and factorise by writing the highest common factor in front of the bracket.	$27m^2 + 6m = 3m \times 9m + 3m \times 2$ $= 3m(9m + 2)$
Check your answer by expanding the bracket.	$3m(9m + 2) = 3m \times 9m + 3m \times 2$ $= 27m^2 + 6m$

(c) $6mn^2 - 27m^2n^2$

THINKING	WORKING
List the factors of each of the numeric coefficients.	6: 1, 2, 3, 6 27: 1, 3, 9, 27
Identify the numeric component of the HCF.	Numeric HCF: 3
Identify the algebraic component of the HCF.	Algebraic HCF: mn^2
Write each term in the expression as a product using the highest common factor and factorise by writing the highest common factor in front of the bracket.	$6mn^2 - 27m^2n^2 = 3mn^2 \times 2 - 3mn^2 \times 9m$ $= 3mn^2(2 - 9m)$
Check your answer by expanding the bracket.	$3mn^2(2 - 9m) = 3mn^2 \times 2 - 3mn^2 \times 9m$ $= 6mn^2 - 27m^2n^2$

SC 2 I can factorise algebraic expressions in which the common factor contains brackets

Worked example

Factorising when the common factor is an expression

Factorise the following.

(a) $2(2y - 3) + 5(2y - 3)$

THINKING	WORKING
Identify the common factor.	Each term is written as a product using $2y - 3$.
Write each term in the expression as a product using the highest common factor and factorise by writing the highest common factor in front of the bracket.	$2(2y - 3) + 5(2y - 3) = (2 + 5)(2y - 3)$ $= 7(2y - 3)$
Check your answer.	2 groups of $(2y - 3)$ plus 5 more groups of $(2y - 3)$ is equal to 7 groups of $(2y - 3)$.

(b) $2x(2y - 3) + 5x(2y - 3)$

THINKING	WORKING
Identify the common factor.	$2y - 3$
Write each term in the expression as a product using the highest common factor and factorise by writing the highest common factor in front of the bracket.	$2x(2y - 3) + 5x(2y - 3) = (2x + 5x)(2y - 3)$ $= 7x(2y - 3)$
Check your answer.	$2x$ groups of $(2y - 3)$ plus $5x$ more groups of $(2y - 3)$ is equal to $7x$ groups of $(2y - 3)$.

(c) $2(2y - 3) + 5x(2y - 3)$

THINKING	WORKING
Identify the common factor.	$2y - 3$
Write each term in the expression as a product using the highest common factor and factorise by writing the highest common factor in front of the bracket.	$2(2y - 3) + 5x(2y - 3) = (2 + 5x)(2y - 3)$

(d) $2z(2y - 3) + 5x(2y - 3)$

THINKING	WORKING
Identify the common factor.	$2y - 3$
Write each term in the expression as a product using the highest common factor and factorise by writing the highest common factor in front of the bracket.	$2z(2y - 3) + 5x(2y - 3)$ $= (2z + 5x)(2y - 3)$

Practice

ANSWERS Page 616

SC 1 I can factorise algebraic expressions containing both numerical and algebraic common factors

1 Determine the highest common factor for each of the following expressions. There is no need to complete the factorisation.

(a) $12a + 3$ **(b)** $28 - 8b$ **(c)** $c^2 + 5c$

(d) $6d + 12d^2$ **(e)** $24x - 36xy$ **(f)** $52x - 24$

(g) $110 - 33y^2$ **(h)** $46xyz + 20xy - 12xz$ **(i)** $a^2b^3c^4 + a^3b^2c^4 - a^4b^3c^2$

2 Factorise each of the following expressions by taking out the numerical common factor.

(a) $6x + 12$ **(b)** $6x - 12$ **(c)** $12 - 6x$ **(d)** $12x - 6y$

(e) $6x^2 + 12y$ **(f)** $6x - 120$ **(g)** $14x + 49$ **(h)** $49 - 14x^3$

(i) $100x - 200y$ **(j)** $16xy + 48z$

3 Factorise each of the following expressions by taking out a common factor.

(a) $a^2 + a$ **(b)** $a^2 + ab$ **(c)** $2a^2 - 4ab$

(d) $2a^2b - 4ab^2$ **(e)** $3ab^2 + 9a^2b$ **(f)** $24abc + 18ad$

(g) $90b^3c - 30b^2c^2$ **(h)** $27a^3b^2c + 57a^2bc^3$

4 Fully factorise each of the following.

(a) $3abc - 6ab + 12bc$ **(b)** $11az + 22ak - 44k$

(c) $24a^2b - 16abc + 64b^2c$ **(d)** $5a^2bc^3 + 25abc - 105ab^2c$

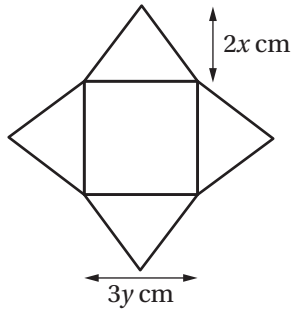
5 Write expressions in expanded form for each of the following, then factorise. Use x for the first variable and y for the second.

(a) Twelve times the product of two numbers is subtracted from twenty times the first number.

(b) Half the product of two numbers is added to five times the first number.

(c) Ten times the first number is subtracted from fifteen times the product of the two numbers.

6 Consider the following diagram.



(a) Write an expression for the area of this shape.

(b) Factorise the expression.

(c) Given $x = 2$ cm and $y = 1$ cm, substitute these values into both expressions and confirm that the expressions are equivalent.

SC 2 I can factorise algebraic expressions in which the common factor contains brackets

1 Write each of the following in fully factorised form.

(a) $3(2x - 1) + 5(2x - 1)$

(b) $3z(2x - 1) + 5(2x - 1)$

(c) $3z(2x - 1) + 5y(2x - 1)$

(d) $8(4x + 3) - 3(4x + 3)$

(e) $8(4x + 3) - 3z(4x + 3)$

(f) $8y(4x + 3) - 3z(4x + 3)$

(g) $11(2y - 9) + 7(2y - 9)$

(h) $11x(2y - 9) + 7(2y - 9)$

(i) $11x(2y - 9) + 7z(2y - 9)$

(j) $14(3m - 2n) - 9(3m - 2n)$

(k) $14(3m - 2n) - 9p(3m - 2n)$

(l) $14k(3m - 2n) - 9p(3m - 2n)$

2 Write each of the following in fully factorised form.

(a) $4x(3p - 2t) + 7xy(3p - 2t)$

(b) $5t(2s + 5r) - 15(2s + 5r)$

(c) $8ky(11x - 6z) + 11y(11x - 6z)$

(d) $4x^2(5a + 6b) - x^2y(5a + 6b)$

3 Write each of the following in fully factorised form.

(a) $5x(2p - 3q) + 25xy(2p - 3q)$

(b) $4xy(3a + 4b) - 10x(3a + 4b)$

(c) $9x^2y(7a - 3b) + 15x^3y(7a - 3b)$

(d) $14x^2y^2(9b + 5c) + 21x^2(9b + 5c)$

4 One of the questions on the topic test was to fully factorise $x^2(2y - 3z) - y^2(2y - 3z)$. Pierre wrote $(2y - 3z)(x^2 - y^2)$ and was surprised when he did not get full marks for his answer.

(a) Explain to Pierre why his answer is incomplete.

(b) Write the correct answer.

(c) Substitute $x = 1$, $y = 2$ and $z = 3$ into the original expression, Pierre's answer and the fully factorised answer to confirm that all three give the same value.

5 Fully factorise each of the following.

(a) $z^2(3y + 2x) - k^2(3y + 2x)$

(b) $3p^2(4x - 3y) - 3q^2(4x - 3y)$

3.4 Factorise simple quadratics

Learning intention: To be able to factorise simple quadratics

Success criteria:

SC 1 I can factorise simple quadratics.

Lesson warm-up

Sums and products

Copy and complete the table.



Number	Number	Sum	Product
2	7	9	14
		5	6
		10	25
		12	32
6		11	
	12		48
−3	−2	−5	6
−6	4		
		4	−5
3	−1		−3
−3		2	
		−3	−54
		−17	30

SC 1 I can factorise simple quadratics

Look at the expansion of $(x + a)(x + b)$:

$$(x + a)(x + b) = x(x + b) + a(x + b)$$
$$= x^2 + bx + ax + ab$$
$$= x^2 + (a + b)x + ab$$

For a quadratic trinomial where the coefficient of x^2 is 1, you can find the factors of the numeric term that add to the coefficient of the x term. It is important to understand that not every quadratic trinomial has integer (whole number) factors.

Worked example

Factorising a simple quadratic

Factorise the following:

(a) $x^2 + x - 6$

THINKING	WORKING
Recall the form of a simple quadratic.	$(x + a)(x + b) = x^2 + (a + b)x + ab$
Identify the two conditions that need to be met by the values of a and b.	$a \times b = -6$ $a + b = 1$
Determine the sign of the variables required.	The product is negative, so either a or b is positive, and the other is negative. The sum is positive, so the larger value is positive, and in this case, one more than the size of the other.
Determine the values for the variables, demonstrating why they are correct.	The values are 3 and –2. $3 \times (-2) = -6$ $3 + (-2) = 1$
Write the factorised expression.	$x^2 + x - 6 = (x + 3)(x - 2)$
Check your answer by writing the quadratic expression in the form $x^2 + ax + bx + ab$ and regrouping.	$x^2 + 3x - 2x - 6 = x(x + 3) - 2(x + 3)$ $= (x + 3)(x - 2)$

(b) $x^2 - 5x - 6$

THINKING	WORKING
Recall the form of simple quadratics.	$(x + a)(x + b) = x^2 + (a + b)x + ab$
Identify the two conditions that need to be met by the values of a and b.	$a \times b = -6$ $a + b = -5$
Determine the sign of the variables required.	The product is negative, so either a or b is positive, and the other is negative. The sum is negative, so the larger value is negative, and in this case, five more than the size of the other.
Determine the values for the variables, demonstrating why they are correct.	The values are –6 and 1. $1 \times (-6) = -6$ $1 + (-6) = -5$
Write the factorised expression.	$x^2 - 5x - 6 = (x - 6)(x + 1)$
Check your answer by writing the quadratic expression in the form $x^2 + ax + bx + ab$ and regrouping.	$x^2 - 6x + 1x - 6 = x(x - 6) + 1(x - 6)$ $= (x + 1)(x - 6)$

Practice

ANSWERS Page 617

SC 1 I can factorise simple quadratics

1 Factorise the following quadratic trinomials.

(a) $x^2 + 7x + 12$ **(b)** $x^2 + 8x + 12$ **(c)** $x^2 + 13x + 12$

(d) $x^2 - 7x + 12$ **(e)** $x^2 - 8x + 12$ **(f)** $x^2 - 13x + 12$

(g) $x^2 - 11x - 12$ **(h)** $x^2 + 11x - 12$ **(i)** $x^2 - 4x - 12$

(j) $x^2 - x - 12$ **(k)** $x^2 + 4x - 12$ **(l)** $x^2 + x - 12$

2 Factorise the following quadratic trinomials.

(a) $x^2 + 12x + 11$ **(b)** $x^2 + 12x + 27$ **(c)** $x^2 + 12x + 36$

(d) $x^2 + 12x + 32$ **(e)** $x^2 + 12x + 20$ **(f)** $x^2 + 12x + 35$

3 Factorise the following quadratic trinomials.

(a) $x^2 - 12x + 11$ **(b)** $x^2 - 12x - 13$ **(c)** $x^2 - 12x + 27$

(d) $x^2 - 12x - 45$ **(e)** $x^2 - 12x + 35$ **(f)** $x^2 - 12x - 64$

4 Factorise the following.

(a) $x^2 - 5x - 24$ **(b)** $x^2 - 15x + 44$

(c) $x^2 + 16x + 60$ **(d)** $x^2 + 5x - 84$

5 Eloise factorised $x^2 - 4x - 12$.

(a) Determine the values for a and b.

(b) Eloise wasn't sure which value to put first when she wrote the expression in the form $x^2 + bx + ax + ab$. Show that the order doesn't matter.

6 Copy and complete the following to determine the factors of $2x^2 - 7x - 4$.

The key values in the expansion of $ax^2 + bx + c$ are ac and b.

In this case, $ac =$ ___ and $b =$ ___.

Look for two numbers that multiply to ac and add to b.

In this case, the values will have the same/different signs.

The values are -8 and ___, since $-8 \times$ ___ $= -8$ and $-8 +$ ___ $= -7$.

$$2x^2 - 7x - 4 = 2x^2 - 8x + ___ - 4$$
$$= 2x(x - 4) + ___(x - 4)$$
$$= (x - 4)(2x + ___)$$

7 Factorise each of the following quadratic trinomials using the process outlined in the previous question.

(a) $3x^2 - 5x - 2$ **(b)** $5x^2 - 23x + 12$

(c) $4x^2 - 7x - 15$ **(d)** $2x^2 + 5x - 7$

TOPIC REVIEW

Multiple choice

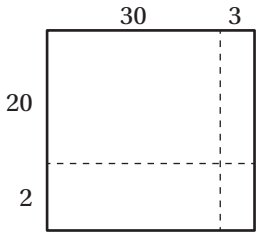
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1 The multiplication 103×82 can be written in which of the following expanded forms?

A $(100 + 2) \times (80 + 3)$ **B** $(100 + 3) \times (80 + 2)$

C $100 + 3 \times 80 + 2$ **D** $(100 + 80) \times (3 + 2)$

2 The area of the overall rectangle, in square units, is:



A 546 **B** 660 **C** 719 **D** 726

3 Evaluating $(3 + 7)(11 + 2)$ gives:

A 130 **B** 126 **C** 116 **D** 97

4 Evaluating $(6 + 2)(15 + (-2))$ gives:

A 104 **B** 112 **C** 128 **D** 136

5 Expanding $(x + 2)(x - 6)$ gives:

A $x^2 + 4x - 12$ **B** $x^2 + 4x + 12$ **C** $x^2 - 4x - 12$ **D** $x^2 - 8x - 12$

6 Expanding $(x - 11)(x - 4)$ gives:

A $x^2 - 7x - 44$ **B** $x^2 - 15x + 44$ **C** $x^2 - 3x - 44$ **D** $x^2 - 15x - 44$

7 Determine the missing term in the following expansion $(3k - 5)^2 = 9k^2 + ___ + 25$.

A $-30k$ **B** $-15k$ **C** $15k$ **D** $30k$

8 Which of the following is the best representation of $(5x - 2y)^2$?

A $5x^2 - 10xy + 2y^2$ **B** $25x^2 + 20xy - 4y^2$ **C** $25x^2 - 20xy + 4y^2$ **D** $5x^2 - 10xy + 2y^2$

9 When expanded, $(3 - 7x)(3 + 7x)$ is equivalent to:

A $9 - 42x + 49x^2$ **B** $9 - 42x - 49x^2$ **C** $9 + 49x^2$ **D** $9 - 49x^2$

10 Which of the following is the factorised form of $16x^2 - 24xy + 9y^2$?

A $(4x - 3y)(4x + 3y)$ **B** $(4x + 3y)(4x + 3y)$ **C** $(4x - 3y)(4x - 3y)$ **D** $(4x + 3y)(4x - 3y)$

11 Which of the following is the factorised form of $81k^2 - 121$?

A $(9k - 11)(9k + 11)$ **B** $(9k - 11)(9k - 11)$ **C** $(9k + 11)(9k + 11)$ **D** $(9k - 121)(9k + 1)$

12 Which of the following is the fully factorised expression for $24a^2 - 36$?

A $12a(2a - 3)$ **B** $4(6a^2 - 9)$ **C** $-12(2a^2 + 3)$ **D** $12(2a^2 - 3)$

13 Which of the following is the fully factorised expression for $6a^3bc + 27a^2bc^2$?

A $3abc(2a + 9c)$ **B** $3a^2bc(2a + 9c)$ **C** $3a^3bc(2 + 9c)$ **D** $6a^2bc(a + 4c)$

14 Which of the following is the fully factorised form of $3x(2y - 5z) + 6(2y - 5z)$?

A $3(2y - 5z)(x + 2)$ **B** $3x(2y - 5z)(2 - x)$ **C** $3(2y + 5z)(x - 2)$ **D** $(2y - 5z)(3x + 6)$

15 Which of the following are a pair of possible factors for the factorisation of $7x^2 + 25x - 12$?

A $(7x - 3)$ and $(x - 4)$ **B** $(7x + 3)$ and $(x + 4)$

C $(7x - 3)$ and $(x + 4)$ **D** $7(x - 3)$ and $(x + 4)$

16 Which of the following is the correct factorisation of $x^2 + x - 20$?

A $(x - 4)(x - 5)$ **B** $(x + 4)(x + 5)$ **C** $(x + 4)(x - 5)$ **D** $(x - 4)(x + 5)$

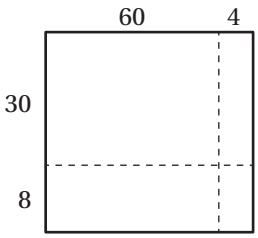
17 Which of the following is the correct factorisation of $2x^2 + 7x - 15$?

A $(2x + 3)(x - 5)$ **B** $(2x - 3)(x + 5)$ **C** $(2x + 3)(x + 5)$ **D** $(2x - 3)(x - 5)$

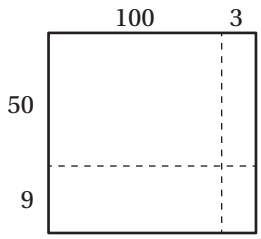
Short answer

1 Write the binomial product represented by each of the following area diagrams, the simplified product and the result.

(a)



(b)



2 Use the area model to complete and determine the product for each of the following multiplications.

(a) 42×31 **(b)** 106×43

3 Use the distributive law to evaluate each of the following.

(a) $(16 + 4)(100 + 2)$ **(b)** $(31 - 3)(10 + 9)$

(c) $(22 - 3)(11 - 2)$ **(d)** $(7 + 11)(8 - 3)$

4 Estelle was attempting to evaluate 39×42.

(a) Her first step was to write $39 \times 42 = (40 + 1)(40 + 2)$. Is this correct? If not, what needs to change?

(b) Her complete working is as follows.

$$39 \times 42 = (40 + 1)(40 + 2)$$
$$= 40 \times 40 + 40 \times 2 + 1 \times 40 + 1 \times 2$$
$$= 1600 + 80 + 40 + 2$$
$$= 1722$$

Comment on this working and then calculate the correct answer, if necessary.

5 Use the distributive law to expand each of the following, writing the answers in simplest form.

(a) $(4a + 3)(2a + 5)$ **(b)** $(7a + 2)(3a - 1)$

(c) $(6a - 5)(4a + 3)$ **(d)** $(10a - 7)(5a - 4)$

6 State, in expanded form, the area for each of the following shapes.

(a) A square with a side length of $(2a - 3)$ cm.

(b) A rectangle with dimensions $(3a + 2)$ cm and $(3a - 4)$ cm.

(c) A rectangle with a width of $(6x - 5)$ cm and a length double this.

7 State the missing term in each of the following expansions.

(a) $(3a - 2)^2 = 9a^2 + __ + 4$ **(b)** $(3a + 2)^2 = 9k^2 + __ + 4$

(c) $(4a - 3b)^2 = 16a^2 + __ + 9b^2$ **(d)** $(4a + 3b)^2 = 16a^2 + __ + 9b^2$

8 Fully expand the following, writing the answers in simplest form.

(a) $(x - 3)(x + 1)^2$ **(b)** $(x + 2)(x - 3)^2$

9 Show, by expanding the base to two terms, how the perfect square rules can be used to calculate each of the following.

(a) 23^2 **(b)** 29^2

10 Expand each of the following.

(a) $(5 - 12x)(5 + 12x)$ **(b)** $(7x + 2)(7x - 2)$

11 Expand each of the following.

(a) $(5m - 6n)(5m + 6n)$ **(b)** $(10s + 12t)(10s - 12t)$

12 Show how DOTS can be used to calculate the product 38×42.

13 Factorise the following quadratic trinomials using the prefect square pattern.

(a) $x^2 - 10x + 25$ **(b)** $9x^2 + 12x + 4$

14 Factorise the following quadratics using the DOTS pattern.

(a) $64 - 81z^2$ **(b)** $400k^2 - 121t^2$

15 Fully factorise each of the following.

(a) $15x - 300$ **(b)** $14a^2b + 21ab^2$ **(c)** $12abc + 16ab^2c - 24abc^2$

16 Fully factorise each of the following.

(a) $4k(3t - 1) - 6(3t - 1)$ **(b)** $5t^2(6k + 5) - 15t(6k + 5)$ **(c)** $25a^2b(4c - 3) + 125a^2b^2(4c - 3)$

17 Complete the following factorisations.

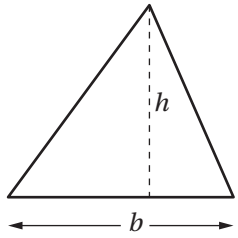
(a) $8x^2 + 14x - 15 = (4x - 3)(_____)$ **(b)** $15t^2 - 26t + 8 = (5t - 2)(____)$

18 Factorise the following quadratic trinomials.

(a) $x^2 + 3x - 10$ **(b)** $x^2 - 13x + 42$ **(c)** $2x^2 - 7x - 4$

Extended response

1 The rule for the area of a triangle is $A = \frac{1}{2} \times b \times h$.



(a) Write an expression, in factor form and expanded form, for the area of a triangle with a base length of $(x + 2)$ cm and a height of $(x - 2)$ cm.

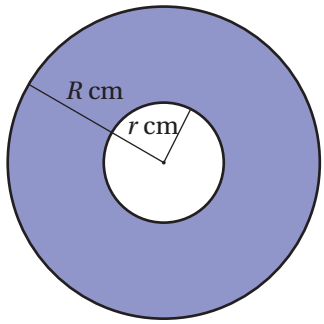
(b) Calculate the area of the triangle when $x = 12$ cm.

(c) Write an inequality describing the restricted value of x.

(d) The base is now extended by 3 cm and the height is doubled. Write an expression for the new area in simplest factor form, and in expanded form.

(e) Calculate the area of this new triangle when $x = 10$ cm.

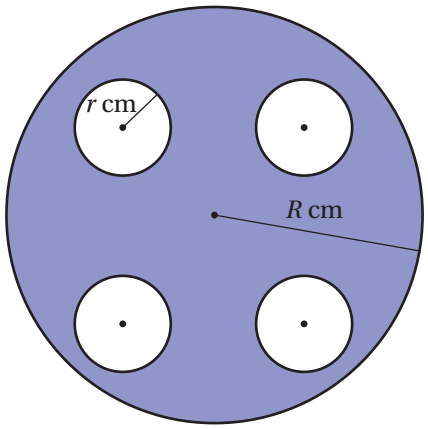
2 Consider the following diagram.
The area of a circle is given by the rule $A = \pi \times \text{radius}^2$.



(a) Write an expression for the shaded area in factorised form.

(b) In terms of π, calculate the shaded area when $R = 5$ cm and $r = 1$ cm.

Now consider the following diagram.



(c) Write an expression for the shaded area in factorised form.

(d) In terms of π, calculate the shaded area if $R = 10$ cm and $r = 3$ cm.

(e) What is the next number of circles that can be cut from the big circle that gives a 'neat' answer for the shaded area?

(f) Write an expression for the shaded area in factorised form for the number of cut out circles given in **(e)**.

3 Janelle's teacher always likes to put a rather difficult question as the last question on a topic test.

On the algebra test it was 'Fully factorise $x^8 + x^4 - 42$'.

Complete the following questions, using $a^2 + 2ab + b^2 = (a + b)^2$, to see if you can find the answer to this question.

(a) Factorise $x^2 + 2x + 1$.

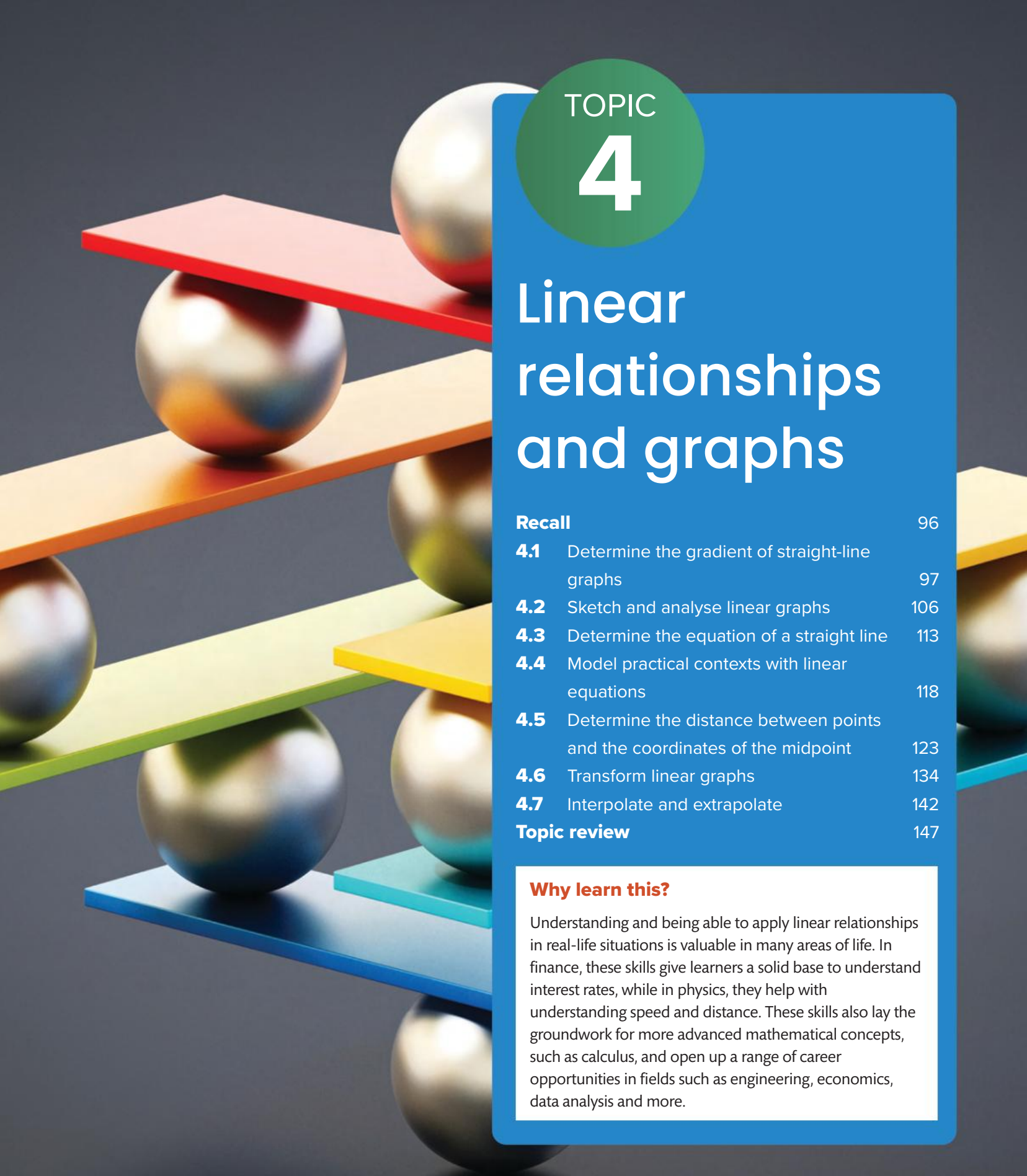
(b) Factorise $x^4 + 2x^2 + 1$ by making the substitution $a = x^2$.

(c) Factorise $x^6 + 2x^3 + 1$ by making the substitution $a = x^3$.

(d) Factorise $x^2 + x - 42$.

(e) What substitution will enable you to factorise the original expression, $x^8 + x^4 - 42$?

(f) Show the substitution and the working to factorise the original expression.



TOPIC
4

Linear relationships and graphs

Recall 96

4.1 Determine the gradient of straight-line graphs 97

4.2 Sketch and analyse linear graphs 106

4.3 Determine the equation of a straight line 113

4.4 Model practical contexts with linear equations 118

4.5 Determine the distance between points and the coordinates of the midpoint 123

4.6 Transform linear graphs 134

4.7 Interpolate and extrapolate 142

Topic review 147

Why learn this?

Understanding and being able to apply linear relationships in real-life situations is valuable in many areas of life. In finance, these skills give learners a solid base to understand interest rates, while in physics, they help with understanding speed and distance. These skills also lay the groundwork for more advanced mathematical concepts, such as calculus, and open up a range of career opportunities in fields such as engineering, economics, data analysis and more.

RECALL

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I can evaluate equations following substitution

1 Calculate the value of a in each of the following.

(a) $a = 8b - 2$ when $b = 4$

(b) $a = 5 - 3b$ when $b = 2$

(c) $a = 2b + 4$ when $b = -3$

(d) $a = 12 - 14b$ when $b = \frac{1}{2}$

(e) $a = \frac{b}{3} + 15$ when $b = -12$

I can evaluate linear equations using substitution

1 Determine the value of y where $y = 3x - 2$, and:

(a) $x = 7$ **(b)** $x = -4$ **(c)** $x = 5$

2 Determine the value of I for $I = 6 - 5m$, and:

(a) $m = 3$ **(b)** $m = -1$ **(c)** $m = 12$

I can generate a table of values

1 Complete the table of values for the following equations.

(a) $p = 3m - 5$

m	−2	−1	0	1	2
p					

(b) $y = \frac{x}{2} - 1$

x	−2	−1	0	1	2
y					

(c) $a = \frac{3b - 1}{2}$

b	−2	−1	0	1	2
a					

(d) $y = 2(x + 7)$

x	−2	−1	0	1	2
y					

(e) $m = 13p + 27$

p	−2	−1	0	1	2
m					

I can identify the gradient and y-intercept of a line from the equation

1 Write the gradient and y-intercept of the line for each of the following equations.

(a) $y = 3x + 7$ **(b)** $y = \frac{x}{5} - 6$ **(c)** $y = \frac{x + 7}{4}$

(d) $6x + 2y = 12$ **(e)** $3x - y = 8$

I can calculate the gradient of a line between two points

1 Calculate the gradient of the line joining the points:

(a) (3, 2) and (4, −13) **(b)** (2, −4) and (−3, 1) **(c)** (−4, −5) and (7, −14) **(d)** (7, 10) and (−5, 3)

Determine the gradient of straight-line graphs

4.1

Learning intention: To be able to determine the gradient of straight-line graphs

Success criteria:

SC 1 I understand the basic ideas related to gradient.

SC 2 I can determine the gradient of a straight line and link its connection to the variable m in the straight line $y = mx + b$.

Lesson warm-up

Stairway approval

Australia's occupational industry has specific standards for building platforms and landings.

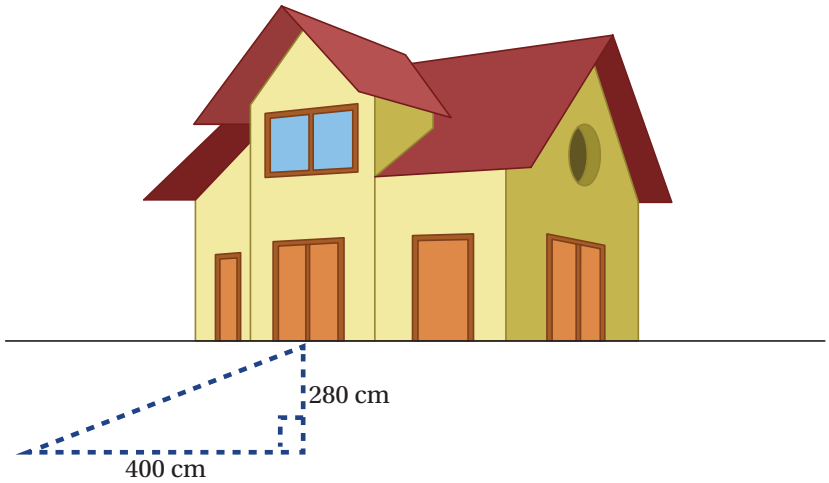
Different angles of steepness require specific types of structures (walkways, stairways, step ladders, ladders, etc.) for people to use.

The requirement for using stairways is that the path/ground must have a slope of between 20° and 45°.

A 20° slope means that for every 10 cm moved horizontally, the slope rises vertically by about 3.6 cm.

A 45° slope means that for every 10 cm moved horizontally, the slope rises vertically by 10 cm.

You have been given plans for an office entry and need to recommend what type of structure is required.



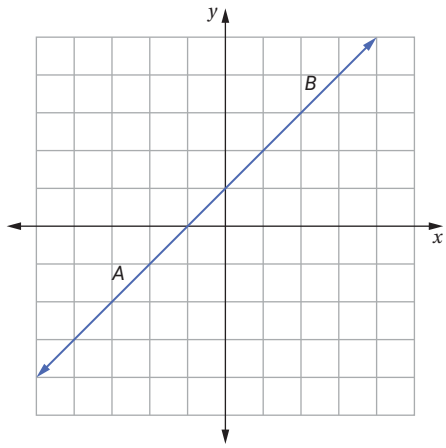
The slope starts 400 cm horizontally away from the entry door and rises vertically 280 cm to the bottom of the door.

Would you approve building a stairway to the office entry?

Briefly give your reasons.

SC 1 I understand the basic ideas related to gradient

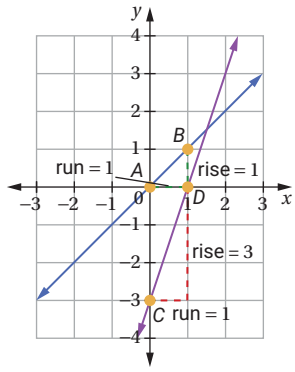
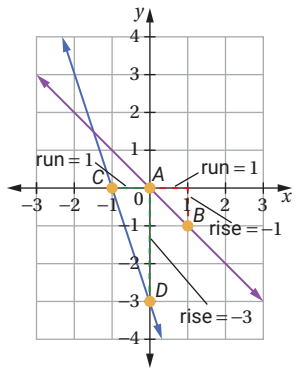
A straight line has no beginning and no end. When it is represented on a graph, it should have an arrow on each end, signalling that the line continues.

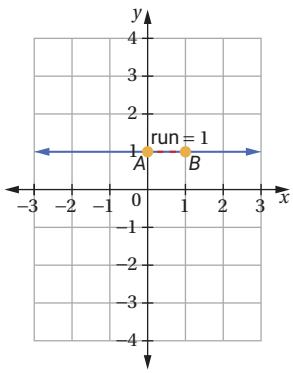
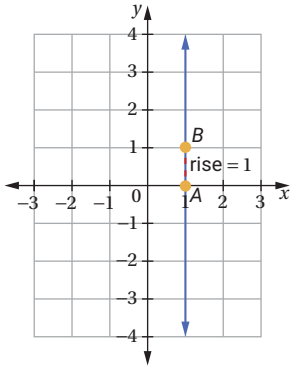


A straight line is named using any two points through which the line passes. A bar is placed above the letters to show that it is a line. The line above could be named $\overline{AB}$.

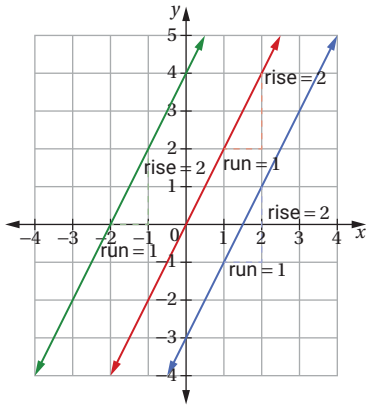
Often it is only part of a line that is of interest. A part of a line with defined start and end points is called a line segment or a line interval. In the above diagram, there is a line segment that could be called $\overline{AB}$. You should note that the line above the letters has no arrows. This means, in this case, that the line segment starts at A and ends at B. In many cases, the line above is omitted and written as AB instead of $\overline{AB}$ for simplicity.

The gradient of a straight line is a measure of how steep it is.

Explanation	Example
A line that slopes from bottom left to top right, as in the graph on the right, has a positive gradient. The steeper the line, the greater the gradient. CD is steeper than AB.	
A line that slopes from top left to bottom right has a negative gradient. The steeper the line, the greater the magnitude of the negative value of the gradient. CD is steeper than AB.	

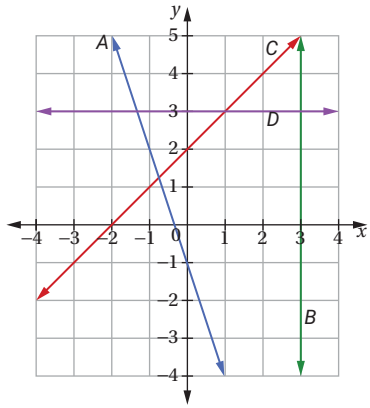
A horizontal line has zero gradient (a rise of 0).	
A vertical line has an undefined gradient (a run of 0). Gradient is technically the ratio of rise to run for the line. A vertical line has zero run, hence the ratio is undefined because you cannot divide by zero.	

Parallel lines are different lines that have the same gradient. Consider the following straight-line graphs. For every increase of 1 unit horizontally, there is an increase of 2 vertically.



Worked example

Describing gradients



Describe the gradients of each line as positive, negative, zero or undefined.

THINKING	WORKING
Recall the definition of a positive gradient and identify the line with a positive gradient.	A positive gradient slopes up from left to right. As the run increases, the rise increases. Line C has a positive gradient.
Recall the definition of a negative gradient and identify the line with a negative gradient.	A negative gradient slopes down from left to right. As the run increases, the rise decreases. Line A has a negative gradient.
Recall the definition of a zero gradient and identify the line with a zero gradient.	A zero gradient is horizontal. As the run increases, the rise does not change, so it is 0. Line D has a zero gradient.
Recall the definition of an undefined gradient and identify the line with an undefined gradient.	An undefined gradient is vertical. The run is equal to 0. Line B has an undefined gradient.

SC 2 I can determine the gradient of a straight line and link its connection to the variable m in the straight line $y = mx + b$

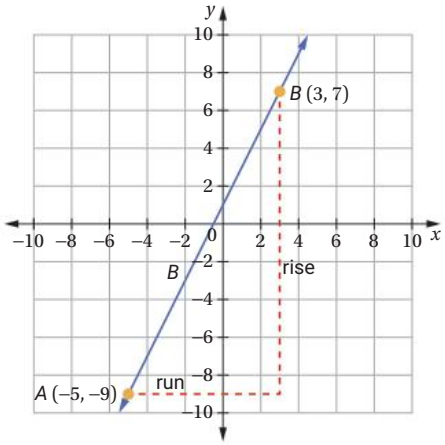
Gradient is defined as $\frac{\text{rise}}{\text{run}}$, where rise refers to the vertical change between two points and run refers to the horizontal change between those two points. The variable m is used to represent the gradient.

To calculate the gradient, given two points on the graph, substitute the values into the most convenient form of the linear equation for the gradient.

For example, $(x_1, y_1) = (-5, -9)$ and $(x_2, y_2) = (3, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7-(-9)}{3-(-5)}$$
$$= \frac{7+9}{3+5}$$
$$= \frac{16}{8}$$
$$= 2$$

The gradient of this straight line is 2.

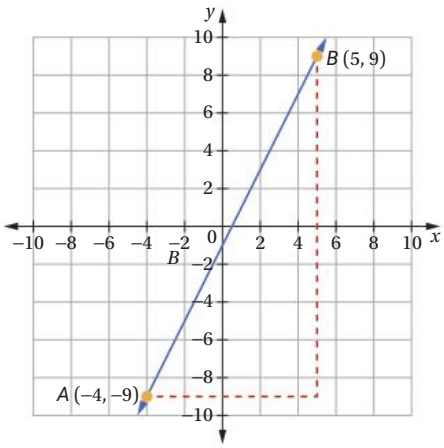


Worked example

Determining the gradient from a graph

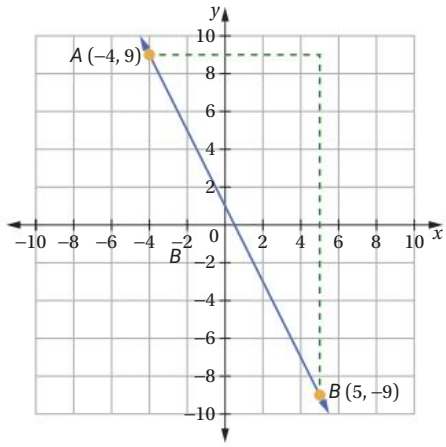
Determine the gradient of the lines shown below.

(a)



THINKING	WORKING
Allocate the given coordinates to (x_1, y_1) and (x_2, y_2).	$(x_1, y_1) = (-4, -9)$ $(x_2, y_2) = (5, 9)$
Substitute the values into $m = \frac{y_2 - y_1}{x_2 - x_1}$ and calculate the gradient. The gradient is a ratio, so it has no units.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9-(-9)}{5-(-4)}$ $= \frac{9+9}{5+4}$ $= \frac{18}{9}$ $= 2$
Write the answer.	The gradient of the line AB is 2. For every increase of 1 unit horizontally, there is an increase of 2 units vertically.

(b)



THINKING	WORKING
Allocate the given coordinates to (x_1, y_1) and (x_2, y_2).	$(x_1, y_1) = (-4, 9)$ $(x_2, y_2) = (5, -9)$
Substitute the values into $m = \frac{y_2 - y_1}{x_2 - x_1}$ and calculate the gradient. The gradient is a ratio, so it has no units.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-9 - 9}{5 - (-4)}$ $= \frac{-18}{5 + 4}$ $= \frac{-18}{9}$ $= -2$
Write the answer.	The gradient of the line AB is -2. For every increase of 1 unit horizontally, there is a decrease of 2 units vertically.

Worked example

Calculating the gradient from a pair of values

Calculate the gradient of the line $y = 3x + 2$.

THINKING	WORKING	
Determine two coordinate pairs for the linear equation by substituting values for x.	For $x = 0$, $y = 3(0) + 2$ $= 2$	For $x = 1$, $y = 3(1) + 2$ $= 3 + 2$ $= 5$
Allocate the given coordinates to (x_1, y_1) and (x_2, y_2).	$(x_1, y_1) = (0, 2)$ $(x_2, y_2) = (1, 5)$	

Substitute the values into $m = \frac{y_2 - y_1}{x_2 - x_1}$ and calculate the gradient.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5-2}{1-0}$ $= \frac{3}{1}$ $= 3$
Write the answer.	The gradient is 3.

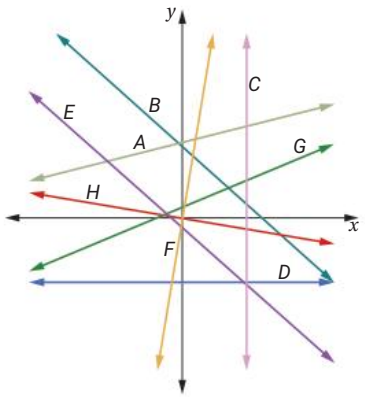
When you have been given the linear equation, the calculation above is not actually necessary. In this case, the linear equation was given in the form $y = mx + b$ where m is the gradient. So the gradient value of 3 can be taken directly from the equation, as long as it is presented in this form.

Practice

ANSWERS Page 618

SC 1 I understand the basic ideas related to gradient

1 Consider the following graph showing a number of different straight-line graphs.



(a) Which of the lines have a positive gradient?

(b) Which of the lines have a negative gradient?

(c) Which lines are parallel (have the same gradient)?

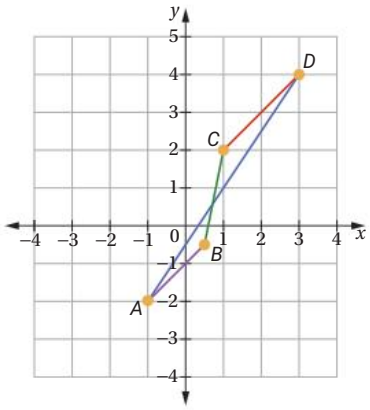
(d) Which line has an undefined gradient?

(e) Which line has a zero gradient?

(f) Which lines have the highest negative gradient?

(g) Which line has the highest positive gradient?

2 Consider the following graph.



Describe the following statements as True or False.

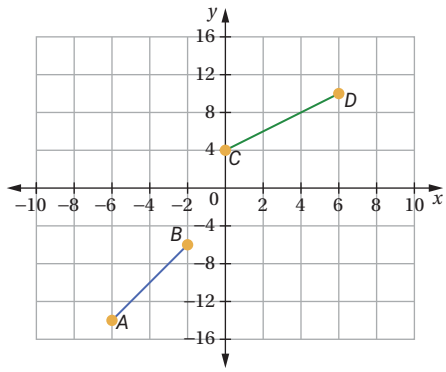
(a) The gradient of AB is the same as the gradient of CD.

(b) The line AD has the same gradient as the line AB.

(c) The line segments AB and CD are parallel.

(d) The gradient of BC is greater than the gradient of AD.

3 Consider the following graph that shows two separate line segments.



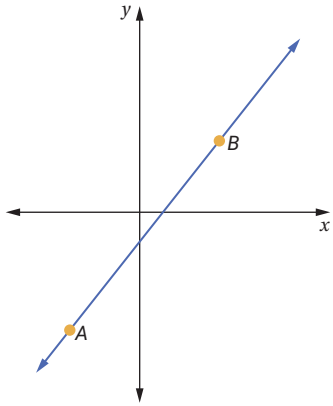
(a) Keeping A anchored, what horizontal shift needs to be made to B to make AB a line that has an undefined gradient?

(b) Keeping C anchored, what vertical shift needs to be made to D to make CD a line that has a zero gradient?

(c) Describe two different simple shifts that could be made to CD to make it parallel to AB.

(d) Describe two different simple shifts that could be made to AB to make it parallel to CD.

4 Consider the following graph.



(a) If A is anchored, what happens to the gradient as B is moved vertically up?

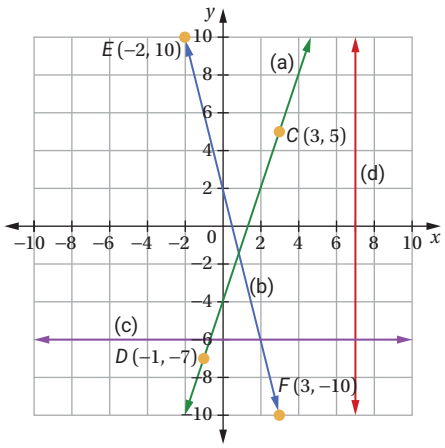
(b) If A is anchored, what happens to the gradient as B is moved vertically down?

(c) What happens to the gradient if B is moved vertically so that it is in line with A?

(d) What happens to the gradient if B is moved vertically so that it is lower than A?

SC 2 I can determine the gradient of a straight line and link its connection to the variable m in the straight line $y = mx + b$

1 Determine the gradient for each of the following straight-line graphs.



2 Determine the gradient for the straight line represented by each of the following tables of values.

(a)

x	−3	1	4
y	−15	13	34

(b)

x	−4	−1	3
y	7	1	−7

(c)

x	−6	−3	−1
y	−26	−17	−11

(d)

x	1	5	11
y	3	−13	−37

3 Determine the gradient for each of the following straight lines.

(a) $y = 5x - 2$ **(b)** $y = 6 - 7x$ **(c)** $y = \dfrac{2x + 3}{4}$ **(d)** $2x + 3y = -4$

4 Min Woo was confused about which of the two coordinate pairs should be treated as (x_1, y_1). His sister, Minjee, said it didn't matter. Use the points (−3, −23) and (4, 21) to show that Minjee is correct.

5 Complete the following to check whether this table of values represents a straight line.

x	−4	−1	3
y	−23	−8	18

(a) Determine the gradient of the straight line connecting the first point to the second point.

(b) Determine the gradient of the straight line connecting the second point to the third point.

(c) What do these results tell you?

(d) Change the y-coordinate for the third point to make the three points form a straight line.

4.2 Sketch and analyse linear graphs

Learning intention: To be able to sketch and analyse linear graphs

Success criteria:

SC 1 I can sketch a linear graph by determining the coordinates of two points.

SC 2 I can sketch a linear graph by determining the x- and y-intercepts.

SC 3 I can sketch a linear graph written in any form including general form and $y = mx + b$ form.

Lesson warm-up

Straight crystals

It is rare to find perfectly straight lines occurring in nature. One such instance is in crystal formations.

In 2000, a crystal cave like no other was discovered in Mexico. It is called Naica's crystal cave. The gypsum crystals there have taken millions of years of slow growth to get so large. The edges of many of these crystals are razor sharp.



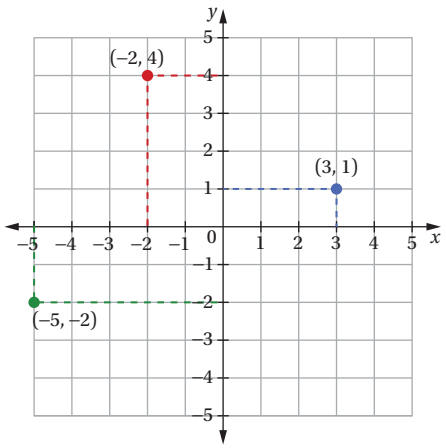
Programming a virtual walk-through of a cave similar to this would require the application of many linear equations to represent crystal formations in a 3D coordinate space.

Can you think of any other natural environments on Earth that would require applying many linear relationships to recreate them in a digital space?

SC 1 I can sketch a linear graph by determining the coordinates of two points

A straight line can be drawn on a set of Cartesian axes, given that any two different points on the graph are known. Sometimes a third point will be calculated and plotted just to make sure, but this is not necessary.

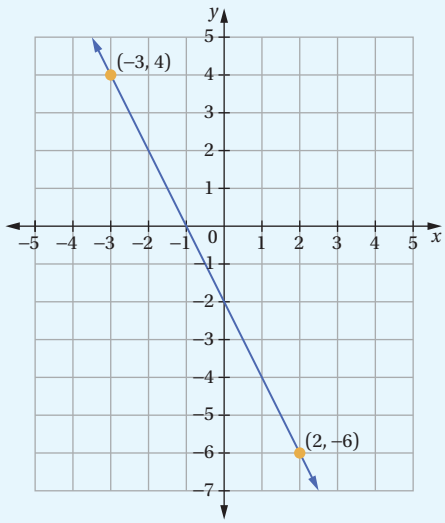
When plotting points, recall that an ordered pair (x, y) consists of an x-value that tells how far the point is horizontally from the origin, and a y-value that tells how far vertically the point is from the origin.



Worked example

Plotting points to draw a straight line

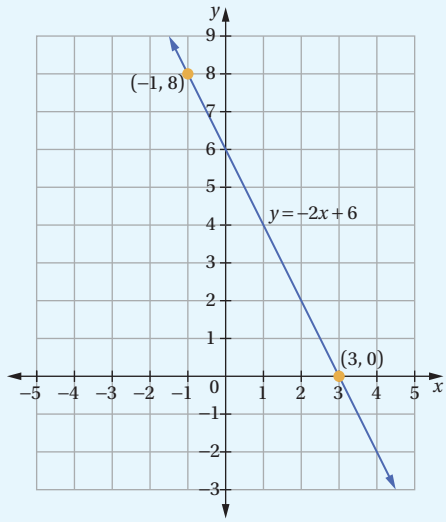
Draw a straight line that passes through the points (−3, 4) and (2, −6).

THINKING	WORKING
Draw a set of Cartesian axes, plot the points and join them with a straight line. Make the line continue in each direction.	

Worked example

Calculating the points before plotting

Draw the straight line $y = -2x + 6$ by calculating two points that lie on the line, then plotting them.

THINKING	WORKING
Pick any two values for x and calculate the corresponding values for y. (It is a good idea to choose two values that are not too close together.)	For $x = -1$: $y = -2x + 6$ $= -2 \times (-1) + 6$ $= 2 + 6$ $= 8$ The coordinates are $(-1, 8)$. For $x = 3$: $y = -2x + 6$ $= -2 \times (3) + 6$ $= -6 + 6$ $= 0$ The coordinates are $(3, 0)$.
Draw a set of Cartesian axes, plot the points and join them with a straight line. Make the line continue in each direction.	

SC 2 I can sketch a linear graph by determining the x- and y-intercepts

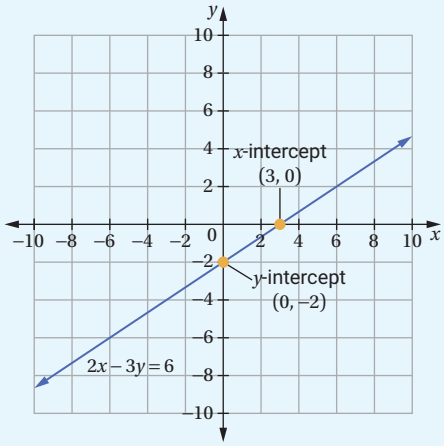
As has been seen, a straight line can be drawn on a set of Cartesian axes, given that any two different points on the graph are known. Usually, the simplest points to calculate are the x- and y-intercepts. The x-intercept is where the line crosses the x-axis, so occurs when $y = 0$ and the y-intercept is where the line crosses the y-axis, so occurs when $x = 0$.

At this stage, it is necessary to understand that a straight line can be written in several different ways. The equations $y = -2x + 3$, $2x + y = 3$ and $2x + y - 3 = 0$ all represent the same straight line.

Worked example

Sketching a linear graph using the intercepts

Determine the intercepts for the straight line $2x - 3y = 6$, then plot the points and draw the graph.

THINKING	WORKING
Determine the x-intercept by substituting $y = 0$ and solving the equation for x.	$2x - 3y = 6$ for $y = 0$: $2x - 3 \times 0 = 6$ $2x - 0 = 6$ $2x = 6$ $x = 3$ The coordinates of the x-intercept are (3, 0).
Determine the y-intercept by substituting $x = 0$ and solving the equation for y.	$2x - 3y = 6$ for $x = 0$: $2 \times 0 - 3y = 6$ $-3y = 6$ $y = \frac{6}{-3}$ $y = -2$ The coordinates of the y-intercept are (0, –2).
Plot the points and draw the graph by connecting the points.	

SC 3 I can sketch a linear graph written in any form including general form and $y = mx + b$ form

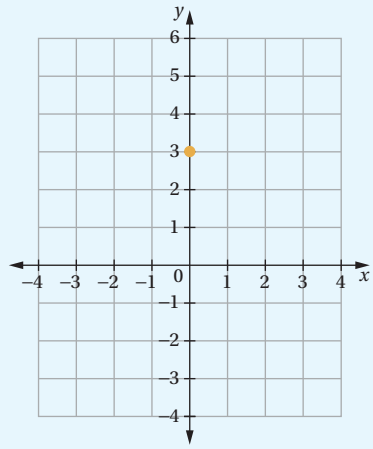
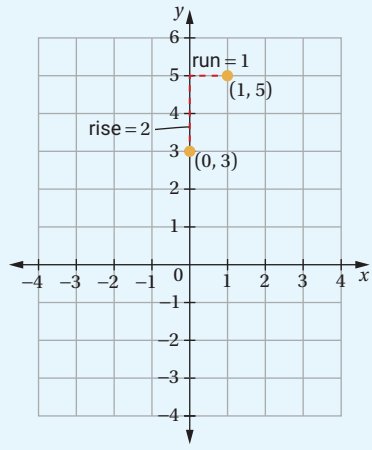
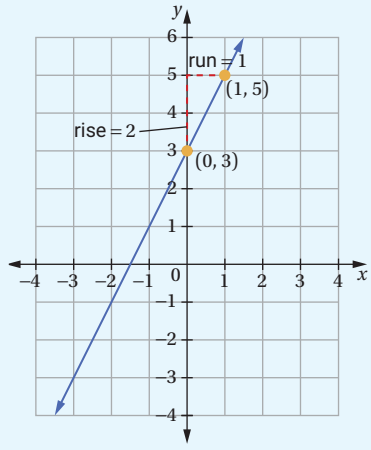
In the previous sections, several ways were shown for obtaining two coordinate pairs so that a linear equation could be sketched. There is another way of sketching the graph for an equation in the form $y = mx + b$.

In this method, the first step is to mark the y-intercept on the graph. Then, recall the gradient $m = \frac{\text{rise}}{\text{run}}$ and use this to determine a second point. Start at the y-intercept and 'rise' the value of the numerator in the gradient, then 'run' the value of the denominator in the gradient. If the gradient is a whole number, the rise will be the whole number value and the run will be 1.

Worked example

Using the gradient-intercept method to sketch a graph

Draw the graph for $y = 2x + 3$ using the gradient and y-intercept.

THINKING	WORKING
Draw up a set of Cartesian axes and plot the y-intercept.	The coordinates of the y-intercept are (0, 3). 
Write the value of the gradient as a fraction.	$m = \frac{2}{1}$
Start at the y-intercept and plot an additional point by moving vertically the value of the rise, and horizontally the value of the run.	
Join the points with a straight line graph.	

Practice

ANSWERS Page 618

SC 1 I can sketch a linear graph by determining the coordinates of two points

1 Calculate two coordinate pairs for each equation, then plot each straight line on a set of Cartesian axes that have x-values from -5 to 10 and y-values from -10 to 10.

(a) $y = 2x - 7$ **(b)** $y = -3x + 2$

(c) $y = 6x + 1$ **(d)** $y = -4x + 5$

2 Complete the ordered pairs for the following straight lines.

(a) For $y = 7x - 3$, the ordered pairs $(-4, ___)$ and $(5, ___)$ could be used.

(b) For $y = -3x + 3$, the ordered pairs $(-4, ___)$ and $(5, ___)$ could be used.

(c) For $y = 11x + 6$, the ordered pairs $(-4, ___)$ and $(5, ___)$ could be used.

(d) For $y = -9x - 7$, the ordered pairs $(-4, ___)$ and $(5, ___)$ could be used.

3 For each of the following pairs of coordinates, state whether the line joining them will have a positive or a negative gradient. Try to do this without plotting the points.

(a) $(-2, -31)$ and $(-3, -19)$ **(b)** $(4, 16)$ and $(2, 32)$

(c) $(-2, -15)$ and $(3, -6)$ **(d)** $(6, 4)$ and $(-5, 11)$

4 Sketch the following straight-line graphs.

(a) The horizontal line that passes through the point $(2, 3)$

(b) The vertical line that passes through $(-2, -3)$

(c) How many points are needed to sketch horizontal or vertical lines?

SC 2 I can sketch a linear graph by determining the x- and y-intercepts

1 Calculate the coordinates of the intercepts for each of the following.

(a) $3x + 2y = 24$ **(b)** $-4x + 5y = 60$

(c) $7x - 3y = 21$ **(d)** $5x - 7y = -70$

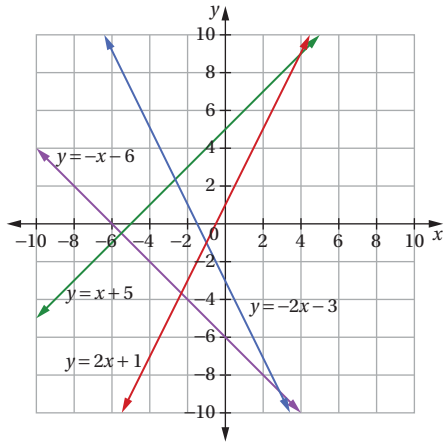
2 Calculate the coordinates of the intercepts for each of the following.

(a) $2x + 3y + 4 = 0$ **(b)** $3x - 2y + 1 = 0$

(c) $5x - 3y - 6 = 0$ **(d)** $-2x + 5y - 7 = 0$

3 Determine the coordinates of the intercepts of the graph of $4x - 2y = -8$ and use them to sketch the graph.

4 A number of different straight-line graphs are shown on the following axes.



(a) For each of the equations, state the coordinates of the y-intercept.

(b) What do you notice about your answers compared to the equation? This form of the equation is sometimes referred to as the gradient-intercept form.

(c) Use what you have discovered to state the coordinates of the y-intercept of the following equations.

(i) $y = -2x + 3$ **(ii)** $y = \frac{5x - 6}{2}$

(iii) $y = \frac{4 - 3x}{5}$ **(iv)** $y = \frac{5x}{3} + \frac{1}{7}$

5 The gradient-intercept form is often represented as $y = mx + b$. Transpose each of the following equations into this form and hence state both the gradient and the coordinates of the y-intercept for each equation.

(a) $2x - 3y = 7$ **(b)** $-4x + 7y = -2$ **(c)** $3x + 11y - 4 = 0$ **(d)** $7x + 2y - 5 = 0$

SC 3 I can sketch a linear graph written in any form including general form and $y = mx + b$ form

1 Sketch each of the following graphs, using the gradient–intercept method.

(a) $y = 2x - 6$ **(b)** $y = 3x + 4$ **(c)** $y = 4x + 2$ **(d)** $y = 3x - 2$

2 Sketch each of the following on the same set of axes, using the gradient–intercept method.

(a) $y = -2x + 4$ **(b)** $y = -x - 3$ **(c)** $y = -3x - 2$ **(d)** $y = -5x + 1$

3 For each of the following equations, state the two different coordinate pairs that could be used in conjunction with the y-intercept if the gradient–intercept method is being used to sketch the graph.

(a) $y = -7x + 2$ **(b)** $y = -4x - 6$

4 Sketch the graph of $2x - 3y + 12 = 0$ by determining the intercepts. First state the coordinates of the intercepts, then sketch the graph.

5 Sketch the graph of $3x - 4y + 24 = 0$ by determining the coordinates for $x = -3$ and $x = 4$. First state the coordinates, then sketch the graph.

6 Determine the coordinates of the intercepts for each of the following equations.

(a) $y = \frac{5x - 2}{3}$ **(b)** $5x + 2y = 12$ **(c)** $3x - 4y - 7 = 0$

Determine the equation of a straight line

4.3

Learning intention: To be able to determine the equation of a straight line

Success criteria:

SC 1 I can identify the y-intercept graphically and calculate it using the gradient and a coordinate.

SC 2 I can use the gradient and y-intercept to determine the equation for a straight line.

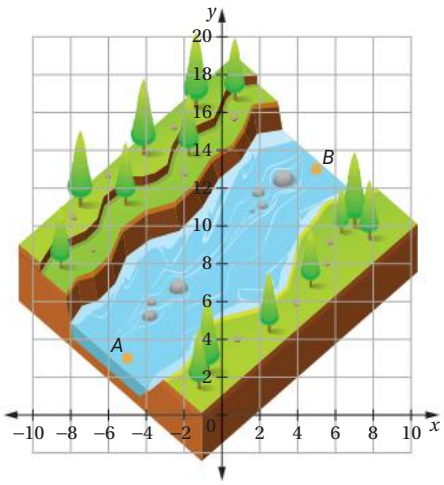
Lesson warm-up

Paddling from A to B

A digital video is required to show the loop of a canoe moving back and forth between point A and point B.

The grid information shown here is provided to help plan the video.

What information about the straight path between the two points can you observe that might help with how to program the exact path the canoe is to move along?



SC 1 I can identify the y-intercept graphically and calculate it using the gradient and a coordinate

The y-intercept is the place where a graph crosses the y-axis.

If the gradient is known, and a point on the graph is known, then the coordinates of the y-intercept can be calculated.

The gradient is calculated by using $m = \frac{y_2 - y_1}{x_2 - x_1}$ and recognising that the coordinate pair for the y-intercept of the graph $y = mx + b$ is $(0, b)$.

Worked example

Determining the y-intercept using the gradient and a coordinate

Determine the value of the y-intercept of a straight line given the gradient is 3 and the line passes through the point (4, 6).

THINKING	WORKING
Allocate the given point to (x_2, y_2) and $(0, b)$ to (x_1, y_1) and identify the value of m.	(x_1, y_1): $(0, b)$ (x_2, y_2): (4, 6) $m = 3$
Substitute the values into $m = \frac{y_2 - y_1}{x_2 - x_1}$ and determine the value of the y-intercept.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $3 = \frac{6 - b}{4 - 0}$ $3 = \frac{6 - b}{4}$ $12 = 6 - b$ $12 + b = 6$ $b = 6 - 12$ $b = -6$
Write the answer.	The coordinates of the y-intercept are $(0, -6)$.

SC 2 I can use the gradient and y-intercept to determine the equation for a straight line

The equation of a straight line can be written as $y = mx + b$, so if the value of the gradient is known, and the value of the y-intercept is known, then the equation can be written.

Worked example

Determining the equation of a straight line where m and b are known

State the equation of the straight line with a gradient of 4 that has a y-intercept at $(0, -3)$.

THINKING	WORKING
Identify the values of m and b.	$m = 4$ $b = -3$
Substitute the values into the rule $y = mx + b$, simplifying if necessary.	$y = mx + b$ $y = 4x + (-3)$ $y = 4x - 3$

Worked example

Calculating the gradient and intercept to determine the equation of a straight line

Determine the equation of the straight line that passes through (–2, 8) and (5, –13).

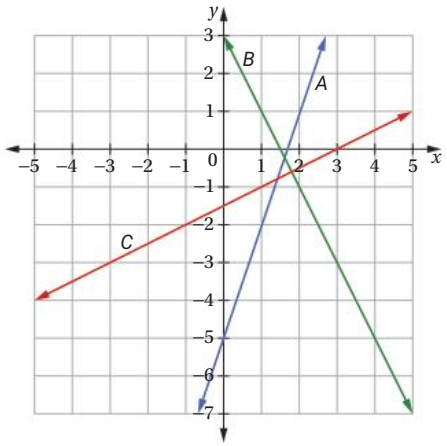
THINKING	WORKING
State the given coordinates.	$(x_1, y_1) = (-2, 8)$ $(x_2, y_2) = (5, -13)$
Use the given points and the rule $m = \frac{y_2 - y_1}{x_2 - x_1}$ to determine the gradient.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-13 - 8}{5 - (-2)}$ $= \frac{-21}{5 + 2}$ $= \frac{-21}{7}$ $= -3$
Use the value for m and one of the two original points to determine the value of the y-intercept.	$y = mx + b$ $-13 = -3(5) + b$ $-13 = -15 + b$ $b = -13 + 15$ $b = 2$ The coordinates of the y-intercept are (0, 2).
Write the answer.	$y = -3x + 2$

Practice

ANSWERS Page 620

SC 1 I can identify the y-intercept graphically and calculate it using the gradient and a coordinate

1 State the coordinates for the y-intercept for each straight line shown on the graph.



2 For each of the following straight lines, determine the coordinates of the y-intercept.

(a) A gradient of 3 passing through (2, 7)

(b) A gradient of -4 passing through $(-1, 3)$

(c) A gradient of 2 passing through $(-3, -4)$

(d) A gradient of -3 passing through $(2, -8)$

3 For each of the following straight lines, state the coordinate pair for the y-intercept.

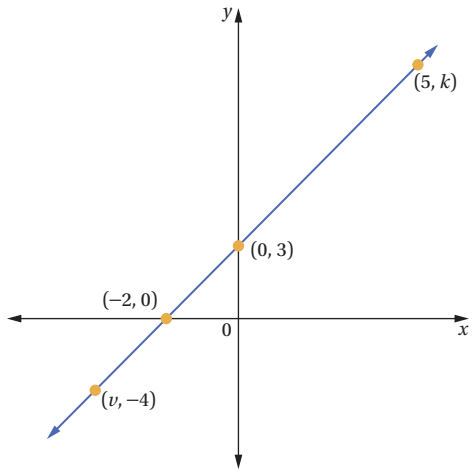
(a) A gradient of $\frac{2}{3}$ passing through $(-3, 2)$

(b) A gradient of $-\frac{4}{7}$ passing through $(-2, -2)$

(c) A gradient of $\frac{2}{5}$ passing through $\left(\frac{1}{2}, \frac{2}{5}\right)$

(d) A gradient of $-\frac{5}{6}$ passing through $\left(\frac{2}{3}, \frac{1}{6}\right)$

4 Consider the following graph.



(a) Determine the value of the gradient.

(b) Determine the value of k.

(c) Determine the value of v.

5 Complete the following.

(a) Sketch the graph of $y = 5x - 4$, using the gradient–intercept method.

(b) What will be the gradient of any graph that is parallel with this one?

(c) Determine the y-intercept of the parallel graph that passes through (2, 2).

(d) Sketch the graph of **(c)** on the same axes.

SC 2 I can use the gradient and y-intercept to determine the equation for a straight line

1 Determine the equation for each of the following straight lines.

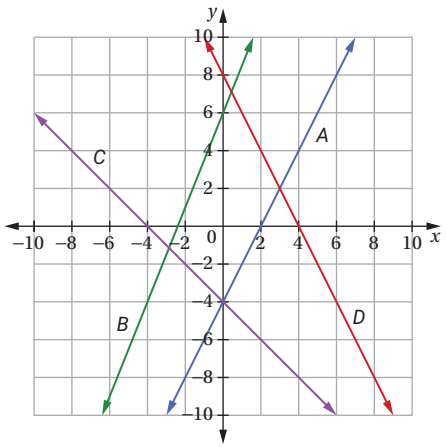
(a) A gradient of -4 with a y-intercept at (0, 6)

(b) A gradient of $\frac{2}{3}$ with a y-intercept at (0, 3)

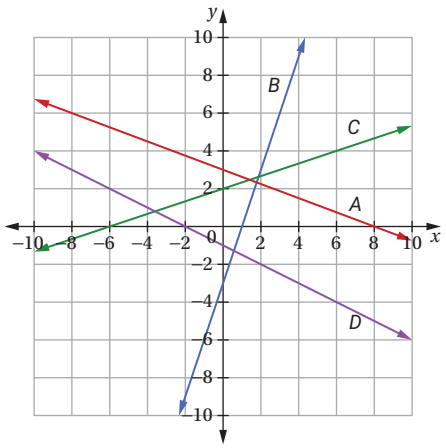
(c) A gradient of -2 with a y-intercept at $\left(0, \frac{1}{4}\right)$

(d) A gradient of $-\frac{3}{5}$ with a y-intercept at $\left(0, -\frac{1}{5}\right)$

2 Identify the y-intercept and gradient from the graph to determine the equation of each of the straight lines shown on the following axes.



3 State the equation of each of the straight lines shown on the following axes.



4 State the equation of each of the following straight lines in the form $cx + dy + e = 0$, where c, d and e are integers (whole numbers).

(a) A gradient of $-\frac{2}{5}$ passing through $\left(0, \frac{3}{10}\right)$ **(b)** A gradient of $\frac{4}{7}$ passing through $\left(0, \frac{5}{7}\right)$

(c) A gradient of $\frac{3}{4}$ passing through $\left(\frac{3}{2}, -\frac{1}{5}\right)$ **(d)** A gradient of $-\frac{2}{7}$ passing through $\left(-\frac{1}{2}, -\frac{1}{3}\right)$

5 Complete the following steps to determine the equation of the straight line that has a gradient of $\frac{3}{4}$ and passes through the point $\left(\frac{1}{2}, 0\right)$.

(a) Substitute the value of the gradient into the formula $y = mx + b$.

(b) Substitute the x- and y-value of the given point into the formula from part **(a)**.

(c) Solve the equation for b.

(d) Write the equation of the line.

6 Use the process outlined above to determine the equation of the straight line with a gradient of $-\frac{3}{8}$ that passes through $\left(\frac{15}{8}, 0\right)$.

Hint: Be careful with the signs for $\frac{\text{rise}}{\text{run}}$.

4.4 Model practical contexts with linear equations

Learning intention: To be able to model practical contexts with linear equations

Success criteria:

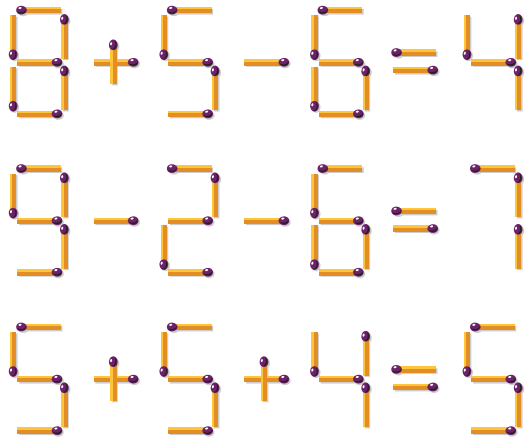
SC 1 I can solve linear equations.

SC 2 I can represent practical contexts using linear models.

Lesson warm-up

What's the problem?

In each number sentence, move one matchstick so that the expression on the left of the equals sign is equivalent to the expression on the right of the equals sign.



SC 1 I can solve linear equations

To solve an equation means to determine the value of the unknown that makes the equation true.

Consider the example $x + 1 = 5$. To solve this equation, you need to determine the value of x that can be added to 1 to get a total of 5. The answer, or solution, is $x = 4$. In this case, the solution can be found by observation or inspection. However, a linear equation can generally be solved by using the process of 'inverse operations'. This means the equation is deconstructed by using the inverse operations to those used to build it up.

To solve $\frac{2x-5}{3} = 5$, the following steps are followed. (Note: A full amount of detail is shown here, but you will quickly find yourself using shortcuts.)

$$\frac{2x-5}{3}=5$$
$$\frac{2x-5}{3}\times 3=5\times 3$$
$$2x-5=15$$
$$2x-5+5=15+5$$
$$2x=20$$
$$\frac{2x}{2}=\frac{20}{2}$$
$$x=10$$

You should notice that the same operation is performed on both sides of the equals sign.

Worked example

Solving a simple equation

Solve $\frac{4x}{3}+7=11$.

THINKING	WORKING
The constructed expression is on the LHS of the equation. Determine the order in which the operations will be undone.	Subtract 7 Multiply by 3 Divide by 4
Complete the processes in the order determined.	$\frac{4x}{3}+7=11$ $\frac{4x}{3}+7-7=11-7$ $\frac{4x}{3}=4$ $\frac{4x}{3}\times 3=4\times 3$ $4x=12$ $\frac{4x}{4}=\frac{12}{4}$ $x=3$
Check the solution by substituting the value of x into the original equation. If LHS $\neq$ RHS, then the solution is wrong.	$\text{LHS}=\frac{4x}{3}+7$ $=\frac{4\times 3}{3}+7$ $=\frac{12}{3}+7$ $=4+7$ $=11$ $=\text{RHS}$
Write the answer.	The solution is $x = 3$.

The order of undoing the operations is not always unique. For instance, in the above example, the multiplication and division can be swapped.

Worked example

Solving a more complex equation

Solve the equation $5(x - 2) = 3(x + 4)$.

THINKING	WORKING
The unknown appears on both sides of the equation. Expand the brackets and rearrange so that the unknown terms are on the same side of the equation. (This is usually the LHS, but it is not essential.)	$5(x-2)=3(x+4)$ $5x-10=3x+12$ $5x-10-3x=3x+12-3x$ $2x-10=12$
Determine the order in which the remaining operations will be undone.	Add 10 Divide by 2
Complete the processes in the order determined.	$2x-10=12$ $2x-10+10=12+10$ $2x=22$ $\frac{2x}{2}=\frac{22}{2}$ $x=11$
Check the solution by substituting the value into the original equation. If LHS $\neq$ RHS, then the solution is wrong.	$5(x-2)=3(x+4)$ $5(11-2)=3(11+4)$ $5\times9=3\times15$ $45=45$ LHS = RHS
Write the answer.	The solution is $x = 11$.

It is important to note that the solution to a linear equation can be a positive or negative whole number or zero, or a positive or negative fraction. Answers that are fractions can also be written in decimal form.

SC 2 I can represent practical contexts using linear models

Many practical situations can be modelled using a linear equation. It is important that there is clarity regarding what the variables represent. An equation needs to be established, then solved for the variable used. The question then needs to be reread to ensure the actual question asked has been answered.

Worked example

Using a linear equation in a practical situation

A rectangle with a perimeter of 60 cm is such that the length is five times the width. Determine the length and width of this rectangle.

THINKING	WORKING
Write the general rule to be used in this context.	$P = 2(L + W)$
State the definitions for any variables used.	P: perimeter, L: length, W: width
State any relationships between the variables.	$L = 5W$
Rewrite the general rule using this relationship and simplify.	$P = 2(L + W)$ $= 2(5W + W)$ $= 2 \times 6W$ $= 12W$
Substitute any known values and solve for the unknown.	$60 = 12W$ $\frac{60}{12} = \frac{12W}{12}$ $5 = W$
Write the answer.	The width of the rectangle is 5 cm and its length is 25 cm.

Practice

ANSWERS Page 620

SC 1 I can solve linear equations

1 Solve the following linear equations.

(a) $3x + 9 = -2$ **(b)** $4x - 6 = 5$ **(c)** $4 - 7x = 8$ **(d)** $6 - 2x = 5$

2 Solve the following linear equations.

(a) $\frac{5x}{2} - 2 = 4$ **(b)** $\frac{6x}{7} + 4 = -2$ **(c)** $4 - \frac{3x}{5} = 6$ **(d)** $9 - \frac{5x}{7} = -3$

3 Solve the following linear equations.

(a) $4(3a + 2) = 2(a - 3)$ **(b)** $5(2a + 4) = -3(3a - 4)$

(c) $3(4 - 5a) = -2(2a + 5)$ **(d)** $-2(3a + 4) = 6(2 - 3a)$

4 Solve the following equations.

(a) $\frac{3t - 4}{5} = \frac{2t + 3}{10}$ **(b)** $\frac{4(t - 2)}{3} = 4(3t + 5)$

(c) $\frac{5(3t - 2)}{4} = \frac{6(3 - 5t)}{5}$ **(d)** $\frac{6(4 - 3t)}{7} - \frac{2}{3} = \frac{5(2t + 1)}{21}$

5 Convert each of the following into equation form, then solve for the unknown, k.

(a) Seven times a certain number is the same as four times that number subtracted from six.

(b) Two-thirds of a certain number added to ten is the same as five-sixths of that number subtracted from fifteen.

(c) Three less than five times a certain number, then all multiplied by four-sevenths, gives the same result as twelve less than two-thirds of that number.

(d) Six more than five-eighths of a certain number is the same as five times that number subtracted from eight.

SC 2 I can represent practical contexts using linear models

1 Determine the length and width of each of the following rectangles.

(a) The length is three times the width and the perimeter is 72 cm.

(b) The length is seven times the width and the perimeter is 176 cm.

2 Charles is one-third the age of his mother Elizabeth, while his brother Arthur is two-thirds the age of Charles. Their total age is 98 years.

(a) Write an equation that could be written to model this situation. Let E represent Elizabeth's age.

(b) State the age of each person.

3 A local plumber charges a $70 call-out fee to come to the house, then $55/hour for the work completed.

(a) Write an equation to represent this situation, using C to represent the total cost, and n to represent the number of hours worked.

(b) What would be the total cost for a job that takes 6 hours to complete?

(c) How long does the plumber spend working on a job that has a total cost of $482.50?

(d) If this equation was used to draw a graph, where C was placed on the vertical axis, what would the call-out cost of $70 represent?

4 The internal angle sum of a pentagon is 540°. Determine the size of each angle in a particular pentagon, given that the angles are measured as $(2x + 3)°$, $(4x - 9)°$, $(3x + 5)°$, $(2x - 9)°$ and $(3x + 4)°$, respectively.

5 Consecutive numbers are whole numbers that are listed in order, such as 4, 5 and 6.

(a) Considering the sequence 4, 5, 6, if you let $5 = x$, write expressions involving x that could be used to represent 4 and 6.

(b) In terms of x, what is the sum of the three terms?

(c) Now consider another sequence of three consecutive whole numbers. This time, they sum to 336. What are the three numbers?

(d) Now consider a sequence of three consecutive whole odd numbers. Write an expression that could be used to find these three odd numbers, given that they add to 231, and state the three numbers.

(e) Now consider a sequence of three consecutive whole even numbers. Write an expression that could be used to find these three even numbers, given that they add to 132, and state the three numbers.

Determine the distance between points and the coordinates of the midpoint

4.5

Learning intention: To be able to determine the distance between points and the coordinates of the midpoint

Success criteria:

SC 1 I can find the distance between two points for a linear equation.

SC 2 I can determine the midpoint of a line segment between two coordinates.

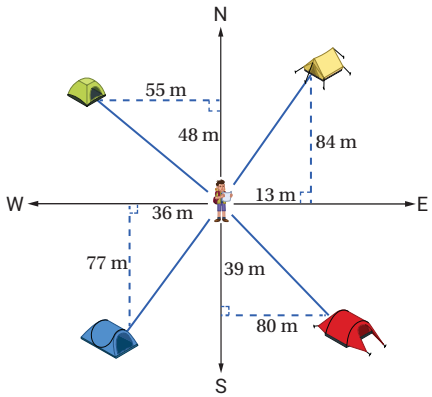
SC 3 I can determine an unknown coordinate of a line segment, using the midpoint and a coordinate.

Lesson warm-up

Decision dilemma

As part of a scavenger hunt, a participant needs to find clues in a particular order to unlock a combination lock. The clues are located inside four different tents.

They are given a map, showing how far the four tents in the north, east, south and west are away from the current location (note the map is not to scale).



The instruction is: *The code for the combination lock is found by visiting the tents in the order of ascending distance from your current location.*

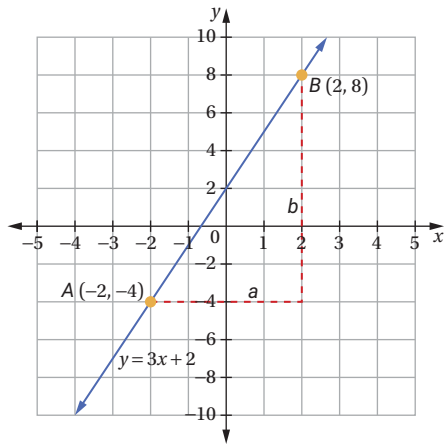
This has left the participant with a dilemma.

What is the problem faced?

SC 1 I can find the distance between two points for a linear equation

Any two points on a straight line can be framed by a right-angled triangle, with the distance between the points being represented by the hypotenuse of the right-angled triangle. Hence, the distance between the points can be found using Pythagoras' theorem.

Using Pythagoras' theorem, the length of line segment AB can be found.



Let the two shorter sides be $a = 4$ and $b = 12$. The hypotenuse $c = AB$ is unknown.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ (AB)^2 &= 4^2 + 12^2 \\ &= 16 + 144 \\ &= 160 \\ AB &= \sqrt{160} \\ &= 4\sqrt{10} \\ &= 12.649\ldots \\ &= 12.65 \ (2 \text{ d.p.}) \end{aligned}$$

So, the distance between the points $A(-2, -4)$ and $B(2, 8)$ is $\sqrt{160} = 12.65 \ (2 \text{ d.p.})$ units.

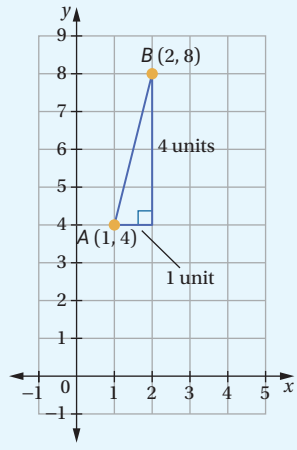
There is no need to draw a graph, or have a graph, to complete these calculations. Looking at the graph above, if $(-2, -4)$ is designated as (x_1, y_1) and $(2, 8)$ as (x_2, y_2), then $a = (x_2 - x_1)$ and $b = (y_2 - y_1)$, which then allows the equation: $c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to be used.

Worked example

Determining the distance between two points on a straight line

Determine the distance between the points $A(1, 4)$ and $B(2, 8)$

(a) using a diagram

THINKING	WORKING
Plot the points on the Cartesian plane and determine the horizontal and vertical distance between the points.	

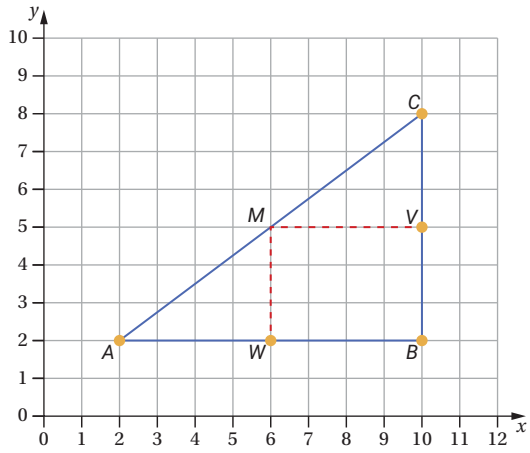
Recall Pythagoras' theorem.	$c^2 = a^2 + b^2$ where $a = 1$, $b = 4$ and $c = AB$
Substitute the values into the rule and calculate the length of the hypotenuse.	$c^2 = a^2 + b^2$ $(AB)^2 = 1^2 + 4^2$ $= 1 + 16$ $= 17$ $AB = \sqrt{17}$ $= 4.123\ldots$ $= 4.12$ (2 d.p.)
Write the answer.	The distance between the two points is $\sqrt{17}$ units.

(b) using the formula.

THINKING	WORKING
Allocate the points to (x_1, y_1) and (x_2, y_2). (It doesn't matter which is which.)	$(x_1, y_1) = (1, 4)$ $(x_2, y_2) = (2, 8)$
Recall the formula for the distance between two points.	$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Substitute the values into the formula and calculate.	$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2-1)^2 + (8-4)^2}$ $= \sqrt{1^2 + 4^2}$ $= \sqrt{1+16}$ $= \sqrt{17}$
Write the answer.	The distance between the two points is $\sqrt{17}$ units.

SC 2 I can determine the midpoint of a line segment between two coordinates

Consider the following diagram.



Using Pythagoras' theorem, you can determine the length of $\overline{AC}$.

The coordinates of the points are $A(2, 2)$, $B(10, 2)$ and $C(10, 8)$.

Horizontal (run) $\overline{AB} = 8$ units and vertical (rise) $\overline{BC} = 6$ units.

$$\left(\overline{AC}\right)^2 = 8^2 + 6^2$$
$$\overline{AC} = \sqrt{8^2 + 6^2}$$
$$= \sqrt{64 + 36}$$
$$= \sqrt{100}$$
$$= 10$$

In the diagram, W is the midpoint of $\overline{AB}$ and V is the midpoint of $\overline{BC}$, so M must have the coordinates (6, 5). From the graph, it appears that this point lies on $\overline{AC}$, but this should be confirmed. To do this, determine the equation of the line that passes through A and C.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{8-2}{10-2}$$
$$= \frac{6}{8}$$
$$= \frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{3}{4}(x - 2)$$
$$4(y - 2) = 3(x - 2)$$
$$4y - 8 = 3x - 6$$

This could be rearranged into a neater form, but this is not necessary to confirm the coordinate pair.

When $x = 6$:

$4y - 8 = 3x - 6$

$4y - 8 = 3(6) - 6$

$4y - 8 = 18 - 6$

$4y - 8 = 12$

$4y = 20$

$y = 5$

This confirms that the point (6, 5) lies on $\overline{AC}$.

Now determine the length of $\overline{AM}$.

For $A(2, 2)$ and $M(6, 5)$,

$$\sqrt{(6-2)^2 + (5-2)^2} = \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

So, the length of $\overline{AM} = 5$, which is half the length of $\overline{AC}$. Hence, M is the midpoint of $\overline{AC}$.

In general terms, this is expressed in the following way.

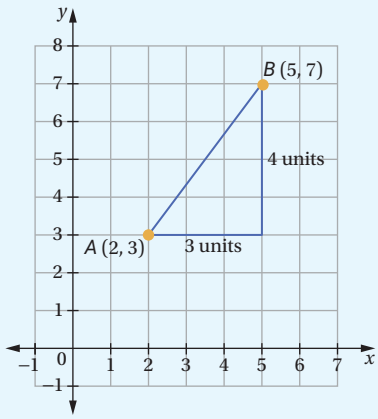
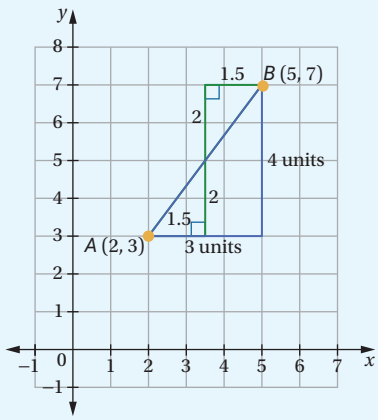
The midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) has coordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Worked example

Locating the midpoint of a line segment

Determine the coordinates of the midpoint of the line segment joining the points (2, 3) and (5, 7)

(a) using a graph

THINKING	WORKING
Plot the points on the Cartesian plane and determine the horizontal and vertical distance between the points.	
Determine the midpoint by determining half the horizontal and vertical distance.	(Half) horizontal distance: $3 \div 2 = 1.5$ (Half) vertical distance: $4 \div 2 = 2$ 
Determine the midpoint.	From point A: $(2 + 1.5, 3 + 2) = (3.5, 5)$. Check against point B: $(5 - 1.5, 7 - 2) = (3.5, 5)$
Write the answer.	The midpoint of the line segment lies at the point $\left(\frac{7}{2}, 5\right)$.

(b) using a rule.

THINKING	WORKING
Recall the rule to determine the coordinates of the midpoint.	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
Allocate the points to (x_1, y_1) and (x_2, y_2).	$(x_1, y_1) = (2, 3)$ $(x_2, y_2) = (5, 7)$
Substitute the values into the rule and simplify.	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+5}{2}, \frac{3+7}{2}\right)$ $= \left(\frac{7}{2}, \frac{10}{2}\right)$ $= \left(\frac{7}{2}, 5\right)$
Write the answer.	The midpoint of the line segment lies at the point $\left(\frac{7}{2}, 5\right)$.

SC 3 I can determine an unknown coordinate of a line segment, using the midpoint and a coordinate

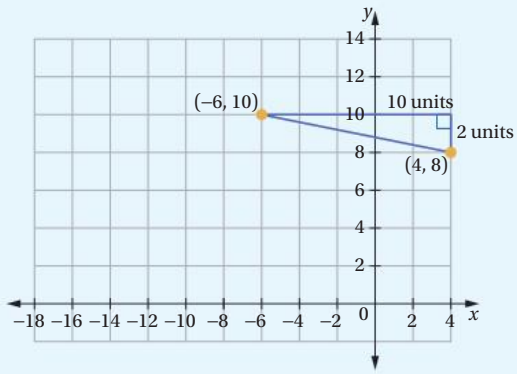
As with other algebra topics, the midpoint formula can be rearranged to determine one of the coordinates given the other coordinate and the midpoint. For the purposes of these calculations, the midpoint will be represented by (x_m, y_m).

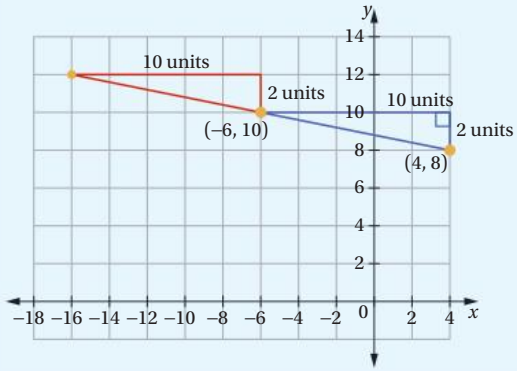
Worked example

Determining a coordinate pair for an interval endpoint given the midpoint and one endpoint

Determine the coordinate pair for the second endpoint of an interval that starts at (4, 8) and has its midpoint at (−6, 10)

(a) using a diagram

THINKING	WORKING
Plot the points on the Cartesian plane and determine the horizontal and vertical distance between them.	

Extend the same distance from the midpoint to the other endpoint.	
Determine the coordinates of the endpoint by adding the horizontal and vertical distances to the coordinates of the midpoint.	(–6 + (–10), 10 + 2) = (–16, 12)

(b) using the rule.

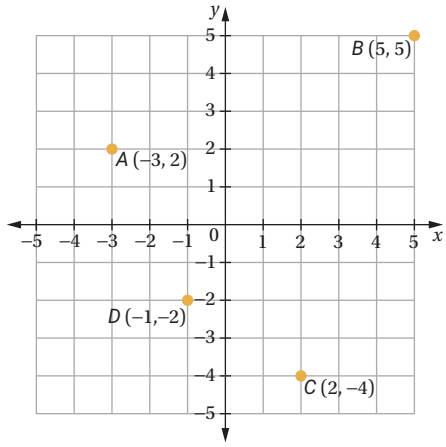
THINKING	WORKING
Allocate the given endpoint to (x_1, y_1).	$(x_1, y_1) = (4, 8)$
Substitute this point, and the midpoint into the rule $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.	$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $(-6, 10) = \left(\frac{4 + x_2}{2}, \frac{8 + y_2}{2}\right)$
Solve for x_2.	$-6 = \frac{4 + x_2}{2}$ $-12 = 4 + x_2$ $-16 = x_2$
Solve for y_2.	$10 = \frac{8 + y_2}{2}$ $20 = 8 + y_2$ $12 = y_2$
Write the answer.	$(x_2, y_2) = (-16, 12)$

Practice

ANSWERS Page 621

SC 1 I can find the distance between two points for a linear equation

1 Determine the length of the following line segments.



(a) $\overline{AB}$ **(b)** $\overline{BC}$ **(c)** $\overline{CD}$ **(d)** $\overline{DA}$

2 Consider the straight line represented by $y = -4x + 6$. Determine the lengths of the following line segments that are part of this straight line.

(a) The line segment connecting the points that pass through $x = 4$ and $x = -3$

(b) The line segment connecting the points that pass through $x = -3$ and $x = 0$

(c) The line segment connecting the points that pass through $y = -2$ and $y = -26$

(d) The line segment connecting the points that pass through $y = 6$ and $y = -18$

3 Show that the order in which the points are allocated makes no difference to the calculation for the length of a line segment. Use the points (2, 5) and (4, 17).

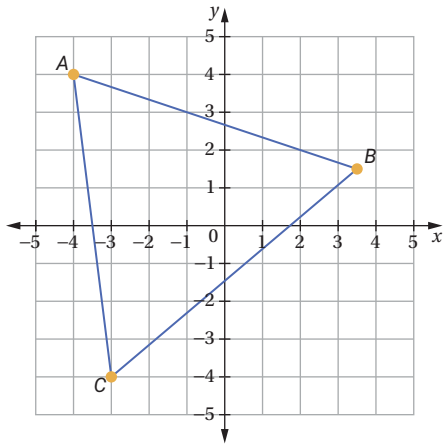
4 Answer the following questions.

(a) The length of the line segment joining (1, 1) and (13, y) is 13 units. Determine two possible values for y.

(b) The length of the line segment joining (1, −2) and (9, y) is 17 units. Determine two possible values for y.

(c) The length of the line segment joining (−3, 4) and (x, −20) is 25 units. Determine two possible values for x.

5 Follow the steps to determine the perimeter of the shape shown on the grid below. Give the answers correct to 2 decimal places.



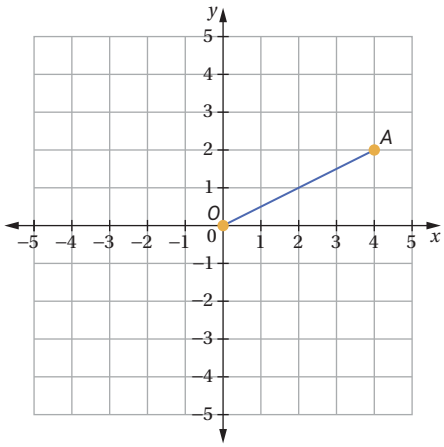
(a) Determine the length of $\overline{AB}$.

(b) Determine the length of $\overline{BC}$.

(c) Determine the length of $\overline{CA}$.

(d) Hence, calculate the perimeter of ΔABC.

6 Consider the length of the line segment $\overline{OA}$ shown on the grid below.



(a) Determine the exact length of the line segment shown.

(b) Determine the exact length of the line segment $\overline{OB}$, where B is located at (2, 4).

(c) State the whole number coordinates of another six points such that the exact length of the interval from the origin to that point is the same as the length $\overline{OA}$.

SC 2 I can determine the midpoint of a line segment between two coordinates

1 Determine the coordinates of the midpoints of the line segments joining the following pairs of points.

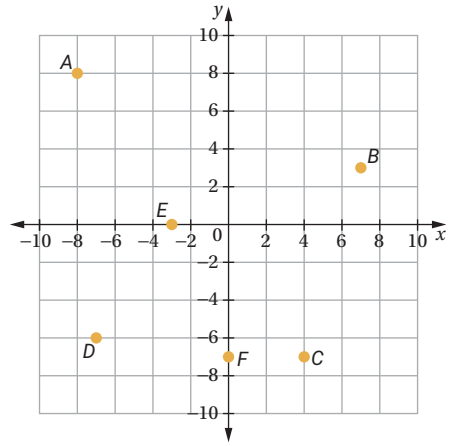
(a) (2, 5) and (4, 7)

(b) (–1, 4) and (5, 6)

(c) (–2, –5) and (2, –3)

(d) (–4, –6) and (5, 9)

2 Determine the coordinates of the midpoint for each of the intervals based on the grid shown.



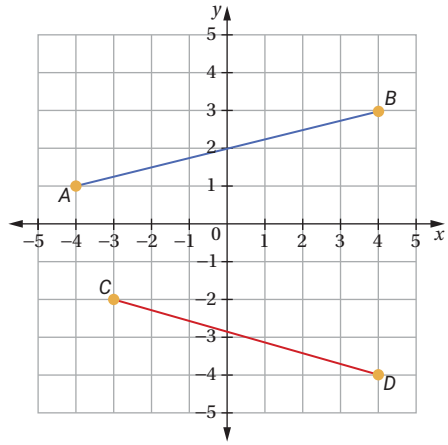
(a) $\overline{AC}$

(b) $\overline{BF}$

(c) $\overline{ED}$

(d) $\overline{EF}$

3 Consider the following diagram showing two separate line intervals.



(a) Determine the midpoint of $\overline{AB}$ and call it R.

(b) Determine the midpoint of $\overline{CD}$ and call it S.

(c) Determine the length of $\overline{RS}$.

(d) Determine the coordinates of the midpoint of $\overline{RS}$.

4 Complete the following steps.

(a) Plot the following points on a grid: $A(2, 3)$, $B(5, 8)$ and $C(7, 5)$.

(b) Determine the coordinates of the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ and call them D, E, F, respectively.

(c) Plot these points on the grid.

(d) Now join $\overline{AE}$, $\overline{BF}$ and $\overline{CD}$.

(e) What do you notice about the line segments formed in part **(d)**?

SC 3 I can determine an unknown coordinate of a line segment, using the midpoint and a coordinate

1 For each of the following, state the coordinates of the second endpoint.

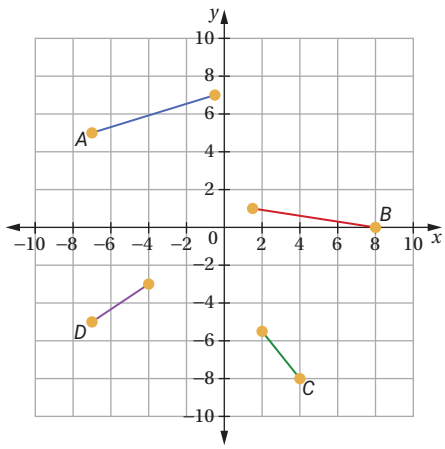
(a) An interval starts at (–3, 2) and has its midpoint at (1, 4).

(b) An interval starts at (3, –2) and has its midpoint at (–1, 4).

(c) An interval starts at (–3, –5) and has its midpoint at (–6, 3).

(d) An interval starts at (–2, –6) and has its midpoint at (5, 3).

2 On the following graph, the letters A–D refer to the starting point of some interval.
The other point shown is the midpoint of the interval.
In each case, state the coordinates for the other endpoint of the segment beginning at:



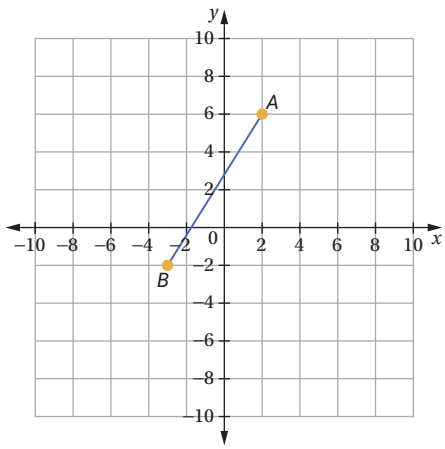
(a) A

(b) B

(c) C

(d) D

3 The following graph shows an interval. It is not known which of the points shown is an endpoint and which is the midpoint. Determine the two possible locations for the second endpoint.



4 The points $A(-3, -5)$ and $B(2, 7)$ represent an endpoint and the midpoint of an interval, but it is not known which is which.

(a) Given that A is the midpoint, find the coordinates of the other endpoint. Call this point C.

(b) Given that B is the midpoint, find the coordinates of the other endpoint. Call this point D.

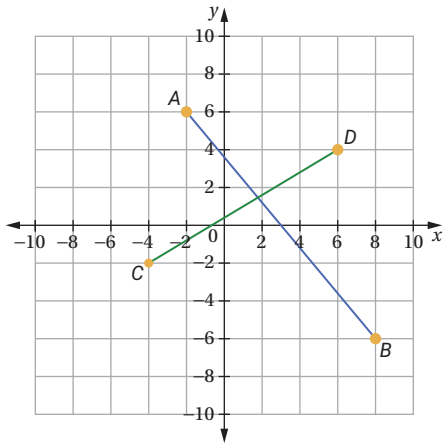
(c) Find the coordinates of the midpoint of $\overline{CD}$ and compare that point with the midpoint of $\overline{AB}$. What do you notice?

5 The following graph shows two line intervals.

(a) Determine the coordinates of the midpoint of $\overline{AB}$.

(b) Determine the coordinates of the midpoint of $\overline{CD}$.

(c) Determine the equation of the line that passes through the two midpoints.



4.6 Transform linear graphs

Learning intention: To be able to transform linear graphs

Success criteria:

SC 1 I can interpret and analyse dilation and reflection as related to the equation of a straight line.

SC 2 I can interpret and analyse translation as related to the equation of a straight line.

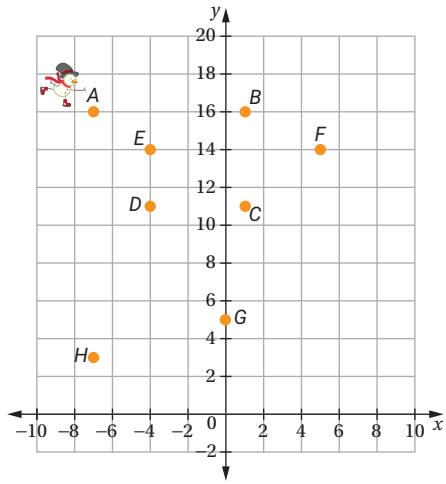
SC 3 I can interpret and analyse combined transformations on a straight line.

Lesson warm-up

Frosty fun

Our frosty friend is having some fun skating around an icy coordinate plane.
The path skated follows the dots in alphabetical order (and at each dot the skater does a little pirouette for added style).

1. Describe the translations of the skater from each point to the next.
2. At what location did the skater pirouette on the y-axis?
3. What is the overall translation of the skater from the starting position?



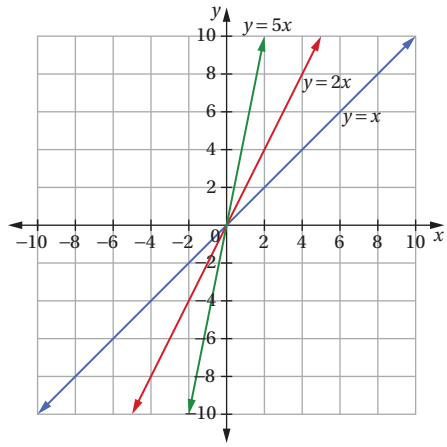
SC 1 I can interpret and analyse dilation and reflection as related to the equation of a straight line

The graph below shows $y = x$, $y = 2x$, and $y = 5x$. All three graphs pass through the origin.

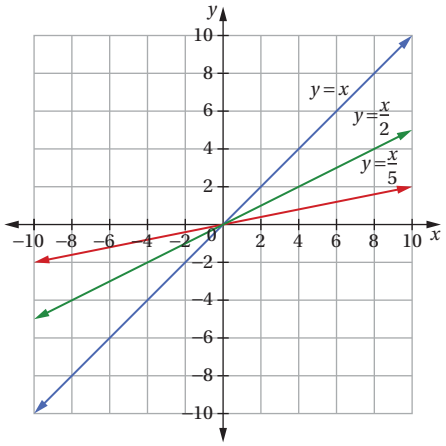
Compare the table of values for these equations.

x	−3	−1	0	1	3
$y = x$	−3	−1	0	1	3
$y = 2x$	−6	−2	0	2	6
$y = 5x$	−15	−5	0	5	15

The effect of the value in front of the x (the coefficient) changes the steepness of the graph (stretches vertically).



What happens if the coefficient is a fraction between 0 and 1?



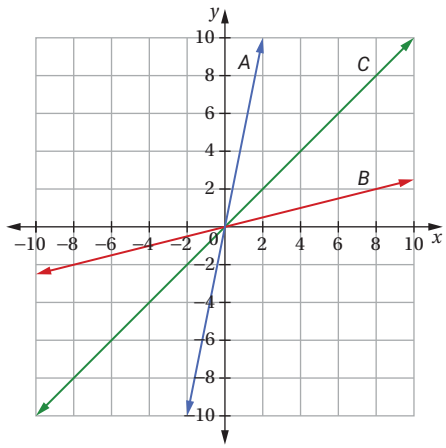
Now the graphs are flatter (compressed vertically).

This stretching and compressing is technically called dilation.

Worked example

Identifying stretch and compression

The graphs for three equations are shown on the following axes. They represent $y = 5x$, $y = \frac{x}{4}$ and $y = x$. Which is which?



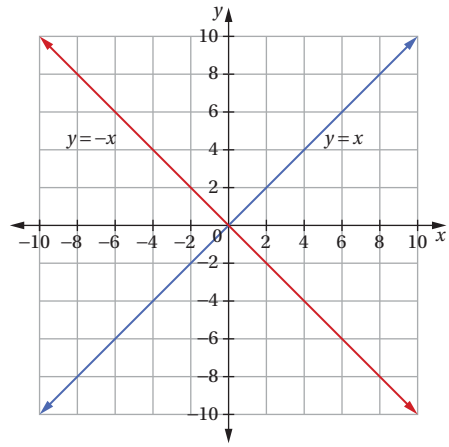
THINKING	WORKING
Check that the three rules are possible matches for the graphs shown.	All three graphs need to pass through the origin. This is the case, so the rules are possible.
The effect of a coefficient greater than 1 is to stretch vertically up and for a fraction between 0 and 1 it is to compress vertically. Use this to identify the 'standard' equation.	Line C must be $y = x$.
Now identify the other two equations.	Line A is $y = 5x$ and Line B is $y = \frac{x}{4}$.

Consider the following graph.

The graph of $y = -x$ is the reflection of the graph of $y = x$ in the x-axis.

The table of values for these two graphs is as follows.

x	-3	-1	0	1	3
$y = x$	-3	-1	0	1	3
$y = -x$	3	1	0	-1	-3



To reflect a graph in the x-axis is to multiply all the y-values by -1, which changes the signs (from positive to negative or from negative to positive).

Worked example

Reflecting in the x-axis

Complete the table of values for the graph of $y = -3x$ as compared to the graph of $y = 3x$.

x	-3	-1	0	1	3
$y = 3x$	-9	-3	0	3	9
$y = -3x$					

THINKING

Substitute the values into the rule one after the other.

Note that $-3x$ is the same as $-1 \times 3x$.

WORKING

x	-3	-1	0	1	3
$y = 3x$	-9	-3	0	3	9
$y = -3x$	9	3	0	-3	-9

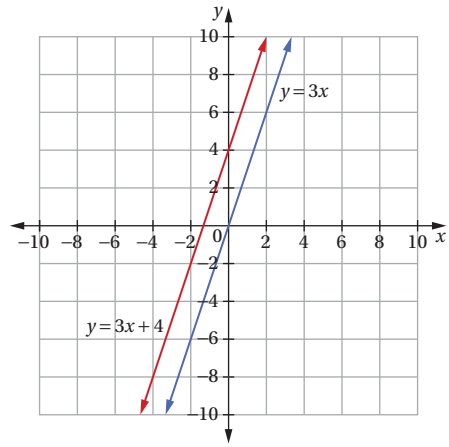
The graph of $y = -3x$ is the reflection of the graph of $y = 3x$ in the x-axis.

SC 2 I can interpret and analyse translation as related to the equation of a straight line

Consider the following graphs.

These two equations have the same gradient, represented by the coefficient of the x-term. Because they have the same gradient, the lines are parallel. If the graph of $y = 3x$ has been drawn, then the second graph could be drawn by laying a ruler along the graph of $y = 3x$ and carefully pushing it vertically 4 units.

When a graph is transformed using a vertical translation, the only change to the original rule is for the size of the vertical translation to be added to the b term in $y = mx + b$.



Worked example

Identifying the size of a vertical translation of a straight line

The graph $y = 2x - 3$ is translated vertically to give a graph with equation $y = 2x + 4$. Determine the size of the vertical translation.

THINKING	WORKING
Check that the graphs have the same gradient.	In both cases, the gradient is $m = 2$, so vertical translation is the only transformation to be considered.
Determine the value that is added to the first equation to get the second.	$y = 2x - 3 \rightarrow y = 2x + 4$ $2x - 3 + t = 2x + 4$ $t = 2x + 4 - (2x - 3)$ $t = 7$
Write the answer.	The vertical translation is +7 units.

SC 3 I can interpret and analyse combined transformations on a straight line

In this section, the three transformations – dilation, reflection in the x-axis and vertical translation – will be combined and the equation of the transformed graph obtained.

As with arithmetic operations, the order in which transformations are applied can affect the outcome. As a simple example, consider the following statements.

Statement A	Statement B
The graph of $y = x$ is stretched by a factor of 2, then translated 3 units vertically up. $y = x \rightarrow y = 2x \rightarrow y = 2x + 3$	The graph of $y = x$ is translated 3 units vertically up, then stretched by a factor of 2. $y = x \rightarrow y = x + 3 \rightarrow y = 2(x + 3)$

As you can see, $y = 2x + 3$ is not the same as $y = 2(x + 3) = 2x + 6$.

To avoid these differences, when you are given a transformed equation and are asked to describe the transformations that have been applied, it will be assumed that the transformations are applied in the order: dilation → reflection → transformation. This is often shortened to DRT.

To be able to identify the order of the transformations, the equation needs to be written in the form $y = mx + b$, where the value of m tells you the size of the dilation as well as whether the graph has been reflected in the x-axis, and the value of b tells you the size and direction of the translation, assuming the base graph was $y = x$. At this level, that will always be the case.

Note, however, that if you are being given instructions on how to transform an equation, this order will not always be followed.

Worked example

Combining transformations

The graph of $y = x$ has been transformed to give $y = 5x - 10$. Using DRT, state the transformations that have been applied and the order in which they were applied.

THINKING	WORKING
In the same way as is done with backtracking, build up the RHS of the equation.	x $5x$ $5x - 10$
State the first transformation that was applied.	Dilation by a factor of 5
State the next transformation.	Translated 10 units vertically down

Practice

ANSWERS Page 622

SC 1 I can interpret and analyse dilation and reflection as related to the equation of a straight line

1 The graph of $y = x$ is altered in the following ways. Write the rule that describes the altered graph.

(a) The graph is stretched by a factor of 3.

(b) The graph is compressed by a factor of $\frac{1}{6}$.

(c) The graph is stretched by a factor of 10.

(d) The graph is compressed by a factor of $\frac{2}{3}$.

2 The following tables of values are provided. In each case, state the value of k.

(a)

x	−5	−1	0	1	5
$y = kx$	−1	−0.2	0	0.2	1

(b)

x	−6	−1	0	1	6
$y = kx$	−2	$-\frac{1}{3}$	0	$\frac{1}{3}$	2

(c)

x	−2	−1	0	1	2
$y = kx$	10	5	0	−5	−10

(d)

x	−3	−1	0	1	3
$y = kx$	30	10	0	−10	−30

3 Describe the following statement as 'sometimes true', 'always true' or 'never true' and use examples to justify your answer.

A reflection through an axis produces a pair of perpendicular lines.

4 Draw the following pairs of graphs on the same axes.

(a) $y = x$ and $y = -x$

(b) $y = \frac{x}{2}$ and $y = -2x$

(c) $y = -\frac{x}{3}$ and $y = 3x$

(d) $y = \frac{2x}{5}$ and $y = -\frac{5x}{2}$

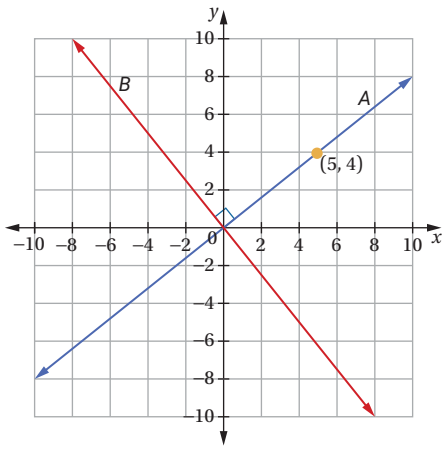
(e) State the product of the gradients of each pair of graphs.

(f) Complete the following to describe the relationship between the pairs of graphs.
If the product of the gradients of two graphs is equal to ___, then the lines are ___.

5 Determine the equation of the graph in the form $y = mx$ that is perpendicular to each of the following.

(a) $y = 7x$ **(b)** $y = -11x$ **(c)** $y = \frac{x}{10}$ **(d)** $y = -\frac{5x}{6}$

6 Consider the following graph.



(a) Determine the equation for A.

(b) What does the symbol between the two graphs indicate?

(c) Hence, determine the equation of B.

SC 2 I can interpret and analyse translation as related to the equation of a straight line

1 Consider the base equation $y = 2x - 4$. For each of the following, state the transformed equation after the given translation.

(a) Translated 3 units vertically down

(b) Translated 3 units vertically up

(c) Translated 10 units vertically up

(d) Translated 7 units vertically down

2 State the size and direction of the following translations.

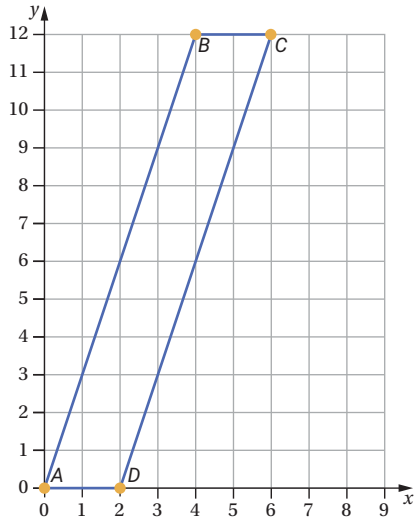
(a) $y = -3x + 6 \rightarrow y = -3x - 2$

(b) $y = 2x + 6 \rightarrow y = 2x - 10$

(c) $y = -5x - 3 \rightarrow y = -5x$

(d) $y = 6x + 4 \rightarrow y = 6x + 5$

3 The following graph shows a parallelogram with a vertex at the origin.



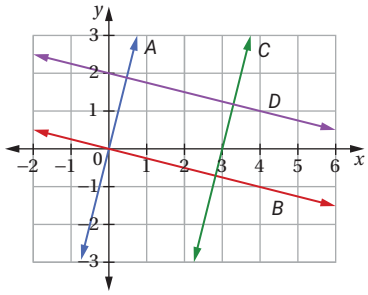
(a) State the equation of the line that includes $\overline{AD}$.

(b) State the equation of the line that includes $\overline{BC}$.

(c) State the equation of the line that includes $\overline{AB}$.

(d) State the equation of the line that includes $\overline{CD}$.

4 Consider the following graph that shows four lines that intersect to make a rectangle.



You are told that the equation for A is $y = 4x$.

(a) State the equation for B and explain how you get your answer.

(b) State the equation for D and explain how you get your answer using translation.

(c) Graph C is graph A translated, but the size of the translation cannot be seen on the graph. How do you find its equation? State the answer.

SC 3 I can interpret and analyse combined transformations on a straight line

1 In each of the following, the graph of $y = x$ has been transformed in a number of ways. State the equation of the transformed graph after all transformations have been completed.

(a) Dilated by a factor of 10 and translated 3 units vertically down

(b) Reflected in the x-axis and translated 5 units vertically up

(c) Dilated by a factor of 2 and reflected in the x-axis

(d) Dilated by a factor of 4, reflected in the x-axis and translated 1 unit vertically up

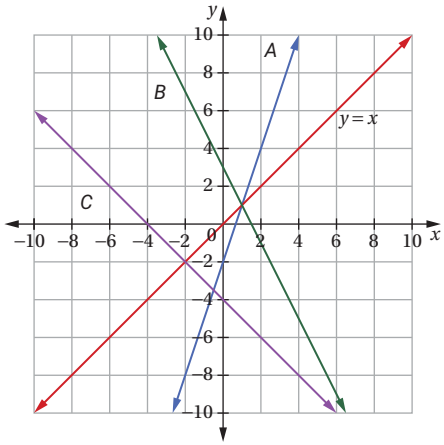
2 The graph of $y = x$ has been transformed in each of the following examples using DRT. State the transformations applied in the correct order.

(a) $y = 6x - 2$ **(b)** $y = -2x + 4$ **(c)** $y = -3x + 6$ **(d)** $y = -5x - 2$

3 The graph of $y = x$ has been transformed in each of the following examples using DRT. State the transformations applied in the correct order.

(a) $y = 7 - 3x$ **(b)** $y = 8 - x$ **(c)** $y = -2(x + 3)$ **(d)** $y = -3(5 - x)$

4 Consider the following graph.



(a) Determine the equation of A and state the transformations applied to the graph of $y = x$ to obtain the graph.

(b) Determine the equation of B and state the transformations applied to the graph of $y = x$ to obtain the graph.

(c) Determine the equation of C and state the transformations applied to the graph of $y = x$ to obtain the graph.

5 Compare the following sets of transformations by drawing the resulting graph on the same set of axes.

(a) RT: $y = x \rightarrow y = -x \rightarrow y = -x + 2$ and TR: $y = x \rightarrow y = x + 2 \rightarrow y = -(x + 2)$

(b) DT: $y = x \rightarrow y = 2x \rightarrow y = 2x - 3$ and TD: $y = x \rightarrow y = x - 3 \rightarrow y = 2(x - 3)$

(c) DR: $y = x \rightarrow y = 2x \rightarrow y = -2x$ and RD: $y = x \rightarrow y = -x \rightarrow y = 2(-x)$

(d) Which pair of transformations can be performed in either order and give the same result?

4.7 Interpolate and extrapolate

Learning intention: To be able to interpolate and extrapolate

Success criteria:

SC 1 I can interpolate and extrapolate to predict values.

Lesson warm-up

Archery, fish-style

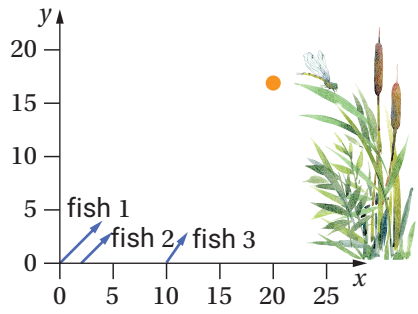
Archer fish spit a straight stream of water at insects they spy above the water's surface. They hope to knock the insect off its perch, so that it lands in the water, providing a tasty meal.

Observe the early trajectory of the streams of water for three different archer fish aiming for the same dragonfly.

Will any of the three fish successfully hit the dragonfly?

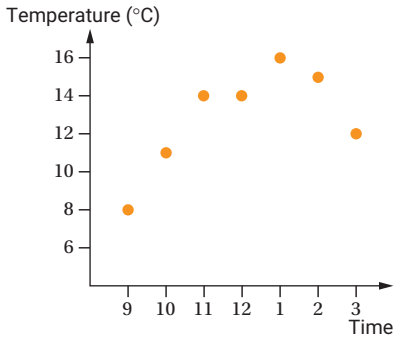
Write down the reasons for your answer.





SC 1 I can interpolate and extrapolate to predict values

When a situation is modelled using a linear equation, the assumption is that there is good reason to think that the change between known points is consistent. In real-world situations, it is often not the case that things are so precise. As an example, consider the daily temperature. This may be recorded every hour, then an estimate is made to state the temperature at times other than those officially recorded.



Looking at the plot, what was the maximum temperature reached in the time period shown? The answer may be 16°C, but this is not certain. What is certain, is that the maximum temperature reached was no less than 16°C.

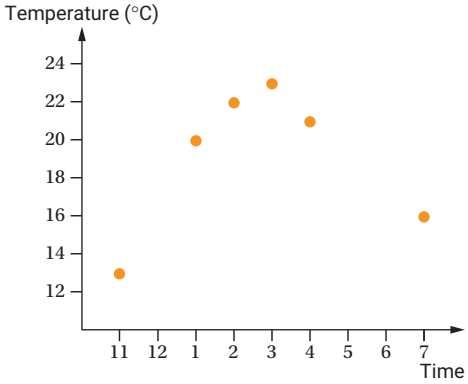
What was the temperature at 11:30 am? Assuming that a linear relationship exists between 11 am and noon, then the answer would be 14°C. This is an example of interpolation. This means that the estimate is being made within the set of known values.

If the question was to estimate the temperature at 4 pm, then this is an example of extrapolation. This means the estimate is being made outside the set of known values. The estimate is made on the assumption that the previous relationship will continue.

Worked example

Interpolating and extrapolating

Use the following graph to answer the questions.



(a) Determine the temperature at 1 pm.

THINKING	WORKING
Locate the point.	The coordinates of the point are (1, 20).
Interpret the coordinate pair to answer the question.	At 1 pm, the temperature was 20°C.

(b) Estimate the temperature at 5 pm.

THINKING	WORKING
Locate the nearest points.	The points are $(x_1, y_1) = (4, 21)$ and $(x_2, y_2) = (7, 16)$.
Establish and interpret the gradient of the straight line between the nearest known values on either side of the desired point.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{16 - 21}{7 - 4}$ $= -\frac{5}{3}$ $= -1.666\ldots$ Between 4 pm and 7 pm, for every hour that passes, the temperature changes on average $-1.666\ldots$°C.
Use the gradient and a known point to estimate the coordinates of the required point.	$\left(4 + 1, 21 - \frac{5}{3}\right) = \left(5, 19\frac{1}{3}\right)$
Write the answer.	The temperature at 5 pm (assuming a linear decrease in temperature) is approximately 19.3°C.

(c) Estimate the temperature at 8 pm.

THINKING	WORKING
Write an assumption on which to base your estimation.	The assumption is that the temperature will continue to fall at the rate of $\frac{5}{3}$°C/h as calculated in part **(b)**.
Use the gradient and a known point to estimate the coordinates of the required point.	$\left(7 + 1, 16 - \frac{5}{3}\right) = \left(8, 14\frac{1}{3}\right)$
Write the answer.	The temperature at 8 pm (assuming a linear decrease in temperature, that is consistent from 4 pm to 8 pm) is approximately 14.3°C.

(d) In which estimate do you have more confidence, 5 pm or 8 pm?

THINKING	WORKING
In general, estimates that fall within known values are considered to be more accurate.	The value for 5 pm is the value in which more confidence could be placed.

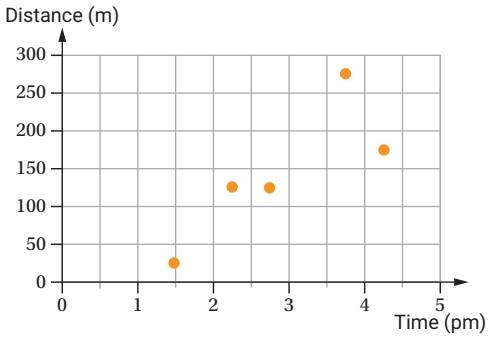
It was not possible to establish the rule in the form $y = mx + b$ since the horizontal axis labelling does not fit the standard pattern. (This would be possible with some manipulation of the values.)

Practice

ANSWERS Page 624

SC 1 I can interpolate and extrapolate to predict values

1 The following graph shows the distance from home for Tom at several times during the afternoon.



(a) Tom stopped at a cafe during this time. When is it most likely this happened?

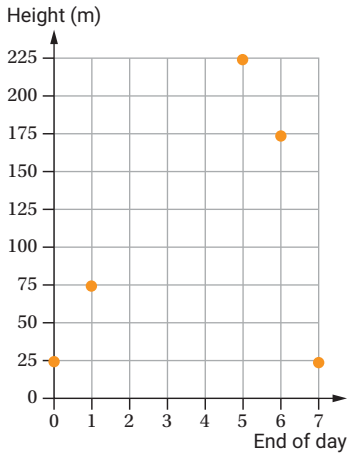
(b) What is the maximum distance Tom is recorded as being from his home?

(c) Is it certain that this is the actual maximum distance? Write a statement that is certainly true regarding his distance from home.

(d) Estimate Tom's distance from home at 3:30 pm.

(e) Estimate the time at which Tom arrived back to his home.

2 As part of the end-of-year activities, a group of students participate in a cycling tour. The following points show their height above sea level at the end of some of the days.



(a) The tour started from the school. How high above sea level is the school located?

(b) Enrico, one of the students on the tour, told his parents when he got home that the highest they got above sea level was 250 m. Assuming that Enrico was telling the truth, explain how this is possible.

(c) What evidence is there on the plot that the trip finished after seven days?

(d) Estimate the height above sea level at the end of day 4.

(e) Estimate the height above sea level half-way through day 6.

(f) Write a rule that could be used to describe the journey on day 7.

3 After a period of very low official interest rates, the Reserve Bank of Australia started lifting the rate at its May 2022 meeting. The following table shows the official interest rate after the Reserve Bank board meeting of each month.

Month (2022)	Rate (%)
April	0.1
May	0.35
June	0.85
July	1.35
August	1.85
September	2.35
October	2.60
November	2.85
December	3.1

(a) The change at the May 2022 meeting is described as an increase of 25 basis points. Describe the change at the September 2022 meeting.

(b) Determine the average monthly increase over the time shown in the table.

(c) Use this average value to predict the interest rate after the March 2023 meeting.

(d) Use the last two known values to predict the interest rate after the March 2023 meeting.

(e) Are the predictions made in **(c)** and **(d)** examples of interpolation or extrapolation?

4 In August 1906, the official world record for the men's 100 m run was 10.6 s. In August 2009, it was 9.58 s.

(a) Use these values to calculate the rate at which the record has changed, assuming the rate is linear. State your answer correct to 4 decimal places.

(b) Use this rate to predict the record in August 1988.

(c) The actual record at that time was 9.92 s. What do you think of your estimate?

(d) Now use the original rate to predict the record in August 2050.

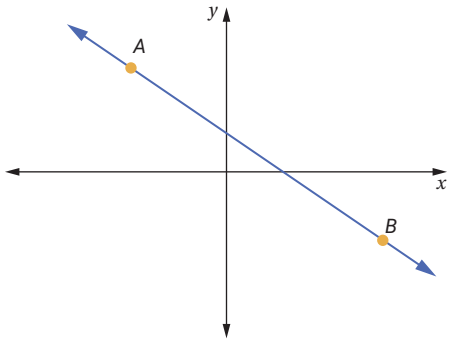
(e) Describe any concerns you have about this prediction.

TOPIC REVIEW

Multiple choice

ANSWERS Page 663

1 Consider the following graph.



If A is anchored when B is moved vertically down, the gradient of $\overline{AB}$:

A becomes a larger positive number.

B becomes a larger magnitude negative number.

C stays the same.

D cannot be determined.

2 The equation $5x - 3y + 4 = 0$ produces a straight-line graph. What is the gradient of this line?

A $m = -\frac{3}{4}$ **B** $m = -3$ **C** $m = \frac{5}{3}$ **D** $m = 5$

3 To draw the graph of $y = -2x - 4$, two coordinate pairs are calculated. Which of the following represents a correct pair of points for this equation?

A (–6, –8) and (–2, –14)

B (–2, 0) and (5, –14)

C (–6, 8) and (7, –10)

D (–2, 0) and (7, –10)

4 Which of the following represents the intercepts for the graph of $4x - 3y - 12 = 0$?

A x-intercept: (–3, 0); y-intercept: (0, –4)

B x-intercept: (0, – 4); y-intercept: (–3,0)

C x-intercept: (0, –4); y-intercept: (3, 0)

D x-intercept: (3, 0); y-intercept: (0,–4)

5 For the equation $2x - 5y - 7 = 0$, which of the following is correct?

A The gradient is $m = \frac{2}{5}$ and the y-intercept occurs at $\left(0, -\frac{7}{5}\right)$.

B The gradient is $m = 2$ and the y-intercept occurs at (0, – 7).

C The gradient is $m = 2$ and the y-intercept occurs at (0, – 5).

D The gradient is $m = -\frac{2}{5}$ and the y-intercept occurs at $\left(0, -\frac{7}{5}\right)$.

6 Consider the graph for the equation $y = -2x + 4$.

To sketch this using the gradient-intercept method, which of the following sets of steps should be followed?

A Plot the point (4, 0) and then go down 2 units and across 1 unit and plot the point (2, 1).

B Plot the point (0, 4) and then go up 2 units and across 1 unit and plot the point (1, 6).

C Plot the point (0, 4) and then go down 2 units and across 1 unit and plot the point (1, 2).

D Plot the point (0, 4) and then go down 2 units and across 1 unit and plot the point (–2, 5).

7 For a straight-line graph that has a gradient of –3 and that passes through the point (–2, 6), the coordinate pair for the y-intercept is:

A (0, –12) **B** (0, 16) **C** (0, 0) **D** (0, –20)

8 Which of the following represents the equation of a straight line that passes through (0, 6) and has a gradient of $\frac{1}{2}$?

A $y = \frac{x}{2} - 6$ **B** $x - 2y + 12 = 0$ **C** $2y = x - 6$ **D** $x - 2y + 6 = 0$

9 The solution to $2(x + 1) = 4(x + 2)$ is:

A $x = -3$ **B** $x = -1$ **C** $x = -\frac{1}{2}$ **D** $x = 3$

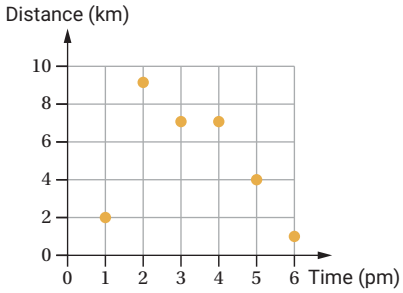
10 Arthur is six times the age of his daughter Stephanie. After another 8 years the sum of their ages will be 65. How old are Arthur and Stephanie at the moment?

A Arthur is 36 and Stephanie is 6. **B** Arthur is 54 and Stephanie is 9.

C Arthur is 42 and Stephanie is 7. **D** Arthur is 30 and Stephanie is 5.

Use the following information to answer Questions **11** and **12**.

The following graph shows Audrey's distance from home during the afternoon.



11 Which of the following best estimates Audrey's distance from home at 1.30 pm?

A 4.5 km **B** 5 km **C** 5.5 km **D** 6 km

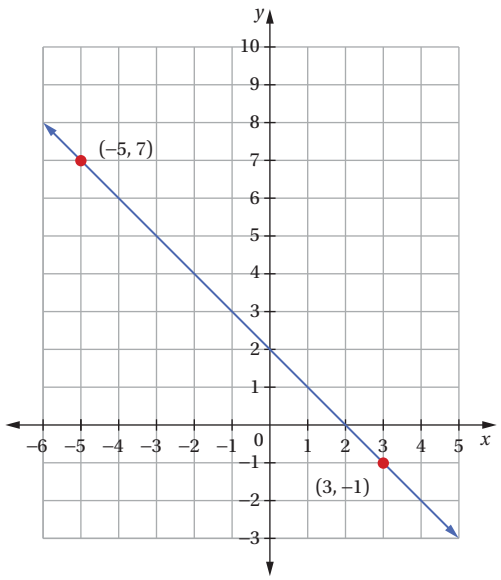
12 If Audrey travels home at the same rate from 4 pm, which of the following best estimates the time Audrey arrived home?

A 6:20 pm **B** 6:30 pm **C** 7 pm **D** 7:15 pm

13 Which of the following represents the distance between the points (–2,3) and (4,7)?

A $\sqrt{10}$ units **B** 10 units **C** $2\sqrt{13}$ units **D** 52 units

14 Which of the following represents the coordinates of the midpoint of the line segment shown?

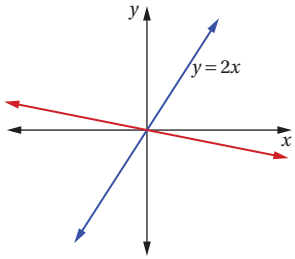


A (4, 4) **B** (–1, 3) **C** (0, 2) **D** (–2, 4)

15 An interval starts at (–3, 4) and has its midpoint at (5, –6). The other endpoint is at:

A (1, –2) **B** (2, –2) **C** (13, –16) **D** (8, 10)

16 The following axes show the graphs resulting from $y = 2x$ and another graph. Which of the following is a possible equation for the second graph?



A $y = 4x$ **B** $y = -2x$ **C** $y = \frac{x}{4}$ **D** $y = -\frac{x}{4}$

17 Which of the following is the equation of a line perpendicular to $y = -\frac{3x}{5}$?

A $y = \frac{3x}{5}$ **B** $y = \frac{5x}{3}$ **C** $y = -\frac{5x}{3}$ **D** $y = -\frac{3x}{5} - 1$

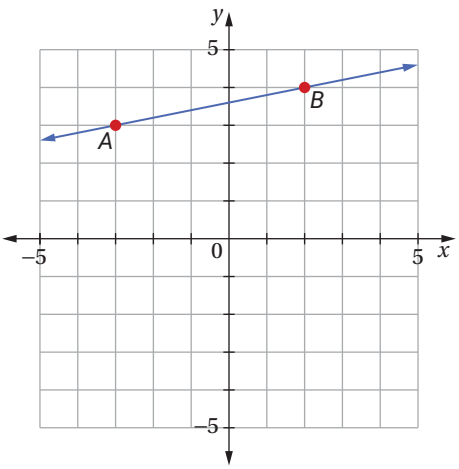
18 The graph of $y = 5x - 9$ is translated vertically to give $y = 5x + 2$. Which of the following best describes this translation?

A 11 units vertically down **B** 7 units vertically down

C 11 units vertically up **D** 7 units vertically up

Short answer

1 Consider the following graph.



If A is anchored, and B is moved vertically down 7 units, the gradient of $\overline{AB}$ will become ____.

2 The following table of values is supposed to represent a straight line.

x	–3	0	4
y	–11	–2	14

(a) Calculate the gradient of the straight line connecting the first coordinate pair to the second coordinate pair.

(b) Calculate the gradient of the straight line connecting the second coordinate pair to the third coordinate pair.

(c) What do these results tell you?

(d) Change the y-coordinate of the third coordinate pair to make the table represent a straight-line graph.

3 Complete the ordered pairs for the following straight lines.

(a) For $y = 3x - 2$, the ordered pairs (–4, ___) and (3, ___) could be used.

(b) For $y = \frac{5x - 3}{2}$, the ordered pairs (–3, ___) and (5, ___) could be used.

(c) For $3x - 2y + 1 = 0$, the ordered pairs (1, ___) and (7, ___) could be used.

4 Consider the equation $5x - 3y + 15 = 0$.

(a) State the coordinates of the x-intercept.

(b) State the coordinates of the y-intercept.

(c) Use these points to sketch the graph.

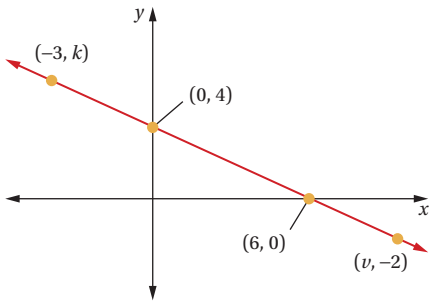
(d) State the gradient of the graph.

5 For each of the following equations, state the two different coordinate pairs that could be used in conjunction with the y-intercept if the gradient-intercept method is being used to sketch the graph.

(a) $y = 3x - 4$

(b) $y = -\frac{2x}{3} + 5$

6 Consider the following graph.



(a) State the value of the gradient.

(b) State the value of k.

(c) State the value of v.

7 State the equation, in the form $y = mx + b$, for the following straight lines.

(a) Passes through (0, 2) with a gradient of −2

(b) Passes through $\left(0, \frac{2}{5}\right)$ with a gradient of $-\frac{1}{2}$

(c) Passes through $(1, 3)$ and $(7, -2)$

8 Solve each of the following linear equations.

(a) $5 - 6x = 2$

(b) $\frac{3x}{7} - 2 = 5$

(c) $2(3x - 1) = 4(3 - 5x)$

(d) $\frac{2x - 1}{3} = \frac{5x - 2}{4}$

9 A local electrician charges $65 to attend a house-call and then $50 an hour for the work completed.

(a) Write an equation to represent this situation, using C to represent the total cost in dollars and n to represent the number of hours worked.

(b) What would be the total cost for a job that takes 7 hours to complete?

(c) How long did the electrician spend working on a job that had a total cost of $265?

(d) If this equation was used to draw a graph, where C was placed on the vertical axis, what would the call-out cost of $65 represent?

10 The world record for the 100 m run was 12.8 seconds in 1922 and 10.9 seconds in 1963.

(a) At what average rate has the record changed over that period, correct to 4 decimal places?

(b) At this rate, what predicted time would you give for 1988?

(c) The record in 1988 was actually 10.49 seconds. Comment on the prediction considering this.

(d) Use the two most recent stated times to predict the record in 2022.

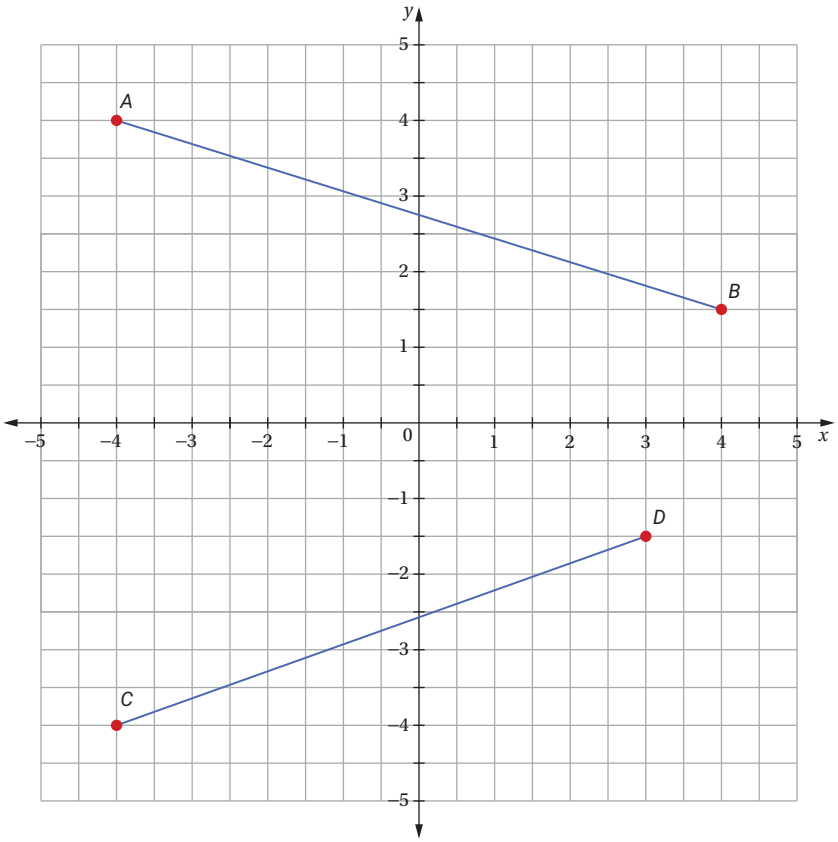
(e) In 2022 the record was still 10.49 seconds. Make a comment about this.

11 Consider the straight line represented by the equation $y = 3x - 5$. Calculate the lengths of the following line segments, which are part of this straight line.

(a) The line segment connecting the points where $x = 3$ and $x = -4$.

(b) The line segment connecting the points where $y = -8$ and $y = 10$.

12 Consider the following diagram showing two separate line segments.



(a) Calculate the midpoint of $\overline{AB}$ and call it R.

(b) Calculate the midpoint of $\overline{CD}$ and call it S.

(c) Calculate the length of $\overline{RS}$.

(d) Calculate the coordinates of the midpoint of $\overline{RS}$.

13 The points $A(2, 9)$ and $B(6, 15)$ represent an endpoint and the midpoint of an interval, but it is not known which is which. Find the two possible coordinate pairs for the other endpoint.

14 State the size and direction of the following translations.

(a) $y = -2x + 2 \rightarrow y = -2x - 4$

(b) $y = 3x - 3 \rightarrow y = 3x + 2$

15 The graph of $y = x$ has been transformed to give graphs represented by the following equations. Using dilation, reflection and translation, in that order, state the transformations applied.

(a) $y = -x + 4$

(b) $y = 2x - 3$

(c) $y = -4(x - 2)$

(d) $y = \frac{5x - 4}{6}$

Extended response

1 Complete the following.

(a) Sketch the graph of the line that passes through (2, 5) and (–2, 3).

(b) Write the equation of this line in the form $cx + dy + e = 0$.

(c) Write the coordinate pair that represents the y-intercept.

(d) Write the coordinate pair that represents the x-intercept.

(e) Calculate the length of the line interval that joins these intercepts.

2 A taxi fare is made up of a flagfall fee and a price per kilometre. In a particular city, the flagfall fee is $5.60 and the price per kilometre is $1.75.

(a) Calculate the cost of a 20 km trip in that city.

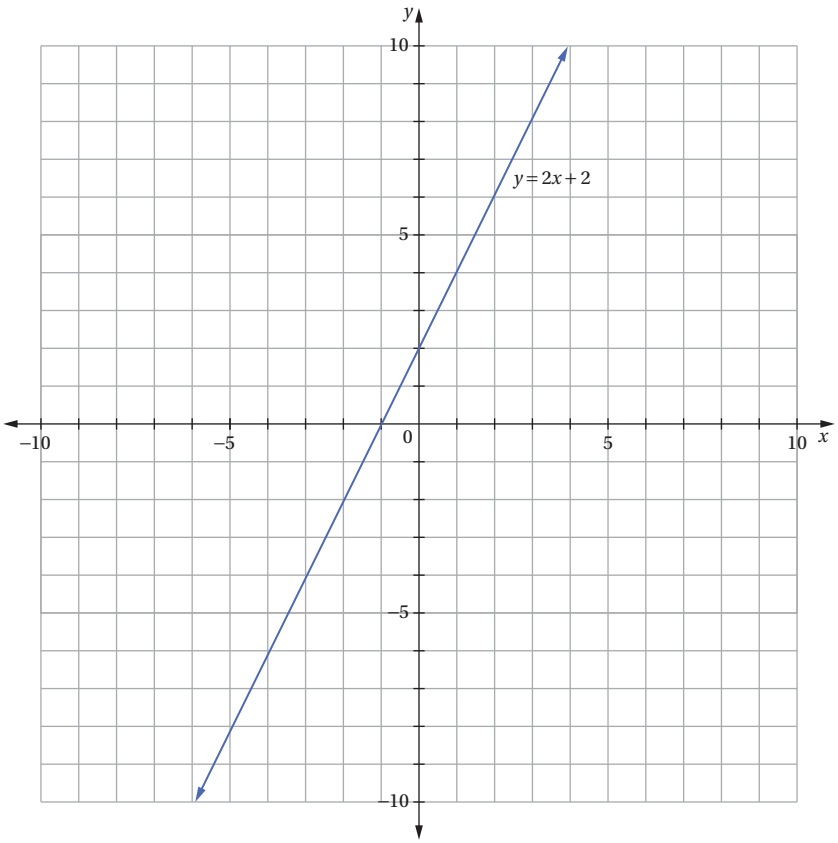
(b) If there was no flagfall fee, what would the per-kilometre price need to be for the trip to cost the same?

(c) In the situation where there is a flagfall fee, what distance was travelled if the total cost was $62.90? State your answer correct to 2 decimal places.

(d) What percentage of this total cost is the flagfall fee? Give your answer correct to 2 decimal places.

(e) For a trip where the flagfall fee represents 15% of the total cost, what distance was travelled? Give your answer correct to 2 decimal places.

3 Consider the following graph.



(a) What is the gradient of a line perpendicular to $y = 2x + 2$?

(b) Determine the equation of the line perpendicular to $y = 2x + 2$ that has the same y-intercept.

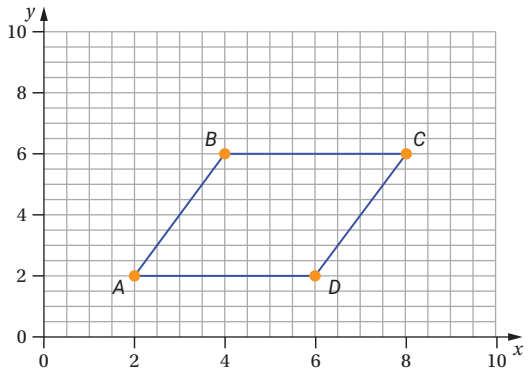
(c) Determine the equation of the line perpendicular to $y = 2x + 2$ that has the same x-intercept.

(d) Now determine the equation of the line parallel to $y = 2x + 2$ that has the same y-intercept as the graph from **(c)**.

(e) Draw all four graphs on the same axes, using the same scale on each axis.

(f) What shape is formed by these lines?

4 Consider the following graph.



(a) State the coordinates of the four points shown.

(b) Calculate the coordinates of the midpoints for $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$.

(c) Plot these points on the graph. Join the two midpoints on the sloping sides and join the two midpoints on the horizontal sides. Also join A to C and B to D.

(d) What do you notice on your graph?

5 Consider the equation $y = \dfrac{-2(3-4x)}{5}$.

(a) Express the equation in the form $y = mx + b$.

(b) Using DRT, state the transformations that have been applied to the graph of $y = x$ to become the graph of the equation $y = \dfrac{-2(3-4x)}{5}$.

(c) Sketch the graphs of $y = \dfrac{-2(3-4x)}{5}$ and $y = x$ on the same set of axes.

(d) For the point (3,3) on the graph of $y = x$, determine the coordinates of the corresponding point on the other graph with the same x-value.

(e) Substitute $x = 3$ into the equation $y = \dfrac{-2(3-4x)}{5}$. Explain the result.

TOPIC

5

Quadratic relationships and graphs

Recall 157

5.1 Understanding solutions to simple quadratic equations 158

5.2 Solve monic quadratic equations using the Null Factor Law 165

5.3 Identify and plot the key features of quadratic graphs 173

5.4 Determine transformations of parabolas 184

5.5 Solve quadratic equations graphically 199

5.6 Model and solve practical problems with quadratic equations 205

Topic review 212

Why learn this?

Understanding quadratic relationships and their graphical representations is fundamental in various contexts. The ability to solve quadratic equations enables the prediction and analysis of situations involving parabolic patterns, such as projectile motion or market trends.

These skills are applicable in fields such as physics, economics, and any area requiring optimisation. They also lay the groundwork for further study in mathematics and related disciplines.

RECALL

ANSWERS Page 624

I can perform translations of shapes on the Cartesian plane

1 A square drawn on a number plane has vertices at (0, 3), (0, 7), (4, 7) and (4, 3). Write down the new coordinates for the vertices, given that the square is translated two units to the left and four units down.

2 A right-angled triangle has vertices located at (–1, 1), (3, 1), and (3, 4). Write down the new coordinates for this triangle, given that it is reflected over the x-axis.

I can evaluate quadratic equations using substitution

1 If $y = x^2 + 5x - 3$, complete the following by determining the value of y for the given values of x.

(a) $x = 3$ **(b)** $x = 0$ **(c)** $x = -2$

2 If $y = (x - 3)(x + 2)$, find the value of y for the given values of x.

(a) $x = 2$ **(b)** $x = 0$ **(c)** $x = -2$

I can expand brackets and simplify using the distributive law

1 Expand and simplify each of the following expressions.

(a) $x(x + 3)$ **(b)** $3x(x - 4)$ **(c)** $-2x(x - 3)$ **(d)** $x(5x - 6) - 4x$

I can expand and simplify the product of two brackets

1 Expand and simplify each of the following expressions.

(a) $(x + 3)(x + 5)$ **(b)** $(x + 2)(x - 3)$ **(c)** $(x - 5)(1 + x)$ **(d)** $(x - 6)(x - 3)$

I can identify the highest common factor

1 Determine the highest common factor (HCF) of each of the following pairs of numbers.

(a) 15 and 60 **(b)** 27 and 63 **(c)** 16 and 24 **(d)** 44 and 66

I can solve linear equations

1 Solve each of the following equations.

(a) $7a - 8 = 20$ **(b)** $83 - d = -13$ **(c)** $-9 - e = 10$ **(d)** $-7g + 23 = 44$

I can identify the two solutions for a perfect square

1 Write the two possible values of x for each of the following.

(a) $x^2 = 4$ **(b)** $x^2 = 1$ **(c)** $x^2 = 16$ **(d)** $x^2 = 64$

5.1 Understanding solutions to simple quadratic equations

Learning intention: To understand solutions to simple quadratic equations

Success criteria:

SC 1 I can recognise and use square numbers to solve quadratic equations in the form $x^2 = a$.

SC 2 I can recognise and use square numbers to solve quadratic equations in the form $ax^2 = a$.

SC 3 I understand that a quadratic equation can have zero, one or two solutions.

Lesson warm-up

Review: Solving linear equations using inverse operations



Equations are mathematical sentences that contain two expressions with an equals (=) sign between them. The expression on the left-hand side of the equals sign must be equal to the expression on the right-hand side, otherwise the statement is not true.
For example:

$5x + 7 = 32$

This statement means that the value of $5x + 7$ must be equal to 32.

When solving an equation, the aim is to determine the value of the unknown (the variable or pronumeral) that makes the sentence true. In the example above, when $x = 5$, the statement is true:

$5 \times 5 + 7 = 32$

In previous lessons, you used inverse operations to solve for the unknown. Recall the inverse operations for each of the following.

The inverse operation to *addition* is _____.

The inverse operation to *subtraction* is _____.

The inverse operation to *multiplication* is _____.

The inverse operation to *division* is _____.

Solve the following linear equations using your knowledge of inverse operations:

(a) $x + 12 = 21$ **(b)** $x - 15 = 3$ **(c)** $5x = 30$ **(d)** $\frac{x}{3} = 8$

SC 1 I can recognise and use square numbers to solve quadratic equations in the form $x^2 = a$

Recall that:

- the square of an integer means that the integer is multiplied by itself
- the square of an integer will always result in a positive number

- there are two integers that produce a positive square number.
  $5^2 = 5 \times 5 = 25$
  and
  $(-5)^2 = (-5) \times (-5) = 25$

The inverse operation to squaring is to take the square root $\left(\sqrt{\ }\right)$. By taking the square root, you will determine the original integer/s that formed the square number.

If the square root is not one you recognise, you can use a calculator to find an approximate value.

For example, using a calculator, you could determine that an approximate value for $\sqrt{2}$ is 1.41... and $-\sqrt{2}$ is $-1.41...$

Knowing that taking the square root is the inverse operation to squaring will help you to solve equations where the unknown (pronumeral) is squared.

Worked example

Solving simple quadratic equations using inverse operations

Solve the following by applying the inverse operation. Express your answer to 2 decimal places where appropriate.

(a) $x^2 = 36$

THINKING	WORKING
Recall that the solution to $x^2 = a$. Solve for the unknown.	$x^2 = 36$ $x = \pm\sqrt{36}$ $x = \pm 6$

(b) $x^2 = 28$

THINKING	WORKING
Recall that the solution to $x^2 = a$. Solve for the unknown.	$x^2 = 28$ $x = \pm\sqrt{28}$ $x = \pm 5.291...$
Write the answer to two decimal places.	$x = \pm 5.29$ (2 d.p.)

(c) $x^2 = -16$

THINKING	WORKING
Recall that the solution to $x^2 = a$. Solve for the unknown.	$x^2 = -16$ $x = \pm\sqrt{-16}$
Recall the number of solutions for an equation in the form $x^2 = a$ when $a < 0$.	As $a < 0$, there are no real solutions.
Write the answer.	There are no real solutions, as there are no real values of x that can make the equation true.

SC 2 I can recognise and use square numbers to solve quadratic equations in the form $ax^2 = a$

Worked example

Solving quadratic equations of the form $ax^2 = a$

Solve the following using inverse operations. Express your answers to 3 decimal places where appropriate.

(a) $4x^2 = 64$

THINKING	WORKING
Write the equation in the form $x^2 = a$.	$4x^2 = 64$ $\frac{4x^2}{4} = \frac{64}{4}$ $x^2 = 16$
Recall that the solution to $x^2 = a$. Solve for the unknown.	$x = \pm\sqrt{16}$ $x = \pm 4$

(b) $2x^2 = 480$

THINKING	WORKING
Write the equation in the form $x^2 = a$.	$2x^2 = 480$ $\frac{2x^2}{2} = \frac{480}{2}$ $x^2 = 240$
Recall that the solution to $x^2 = a$. Solve for the unknown.	$x = \pm\sqrt{240}$ $x = \pm 15.491\ 9...$
Express the answer to 3 decimal places.	$x = \pm 15.492$ (3 d.p.)

SC 3 I understand that a quadratic equation can have zero, one or two solutions

Recall that, when determining the integers that form a square number, there are often two possible solutions.

For example, $7^2 = 49$ and $(-7)^2 = 49$. There are always two solutions to an equation in the form $x^2 = a$ when a is a positive number.

If instead of 49, you were asked to find the integer/s that, when squared, form -49, would it be possible? Consider the table below.

Integer	Square number
4	$4^2 = 4 \times 4 = 16$
3	$3^2 = 3 \times 3 = 9$
2	$2^2 = 2 \times 2 = 4$
1	$1^2 = 1 \times 1 = 1$
0	$0^2 = 0 \times 0 = 0$
−1	$(-1)^2 = (-1) \times (-1) = 1$
−2	$(-2)^2 = (-2) \times (-2) = 4$
−3	$(-3)^2 = (-3) \times (-3) = 9$
−4	$(-4)^2 = (-4) \times (-4) = 16$

Notice that all the square numbers are positive, or zero. There is no possible way that squaring an integer will result in a negative number. So, there is no real solution to an equation in the form $x^2 = a$ when a is a negative number.

There are some instances when only one solution is possible when solving equations in the form $x^2 = a$. These instances are limited to equations where $a = 0$.

Worked example

Determining the number of solutions to simple quadratic equations

Determine the number of solutions to each of the following. State the solution, if it exists.

(a) $x^2 = 9$ **(b)** $x^2 = -9$ **(c)** $x^2 = 0$

THINKING	WORKING
Recall the number of solutions for an equation in the form $x^2 = a$.	If $a < 0$, there are no solutions. If $a = 0$, there is one solution. If $a > 0$, there are two solutions.
Determine the number of solutions, check the reasonableness of your answer and write the answer.	**(a)** In $x^2 = 9$, the square number is positive, and therefore two solutions exist. $3^2 = 3 \times 3 = 9$ $(-3)^2 = (-3) \times (-3) = 9$ There are two solutions to the equation: $x = -3$ or $x = 3$. **(b)** In $x^2 = -9$, the square number is negative, and therefore there are no solutions. **(c)** In $x^2 = 0$, the square number is zero, and therefore one solution exists. $0^2 = 0 \times 0 = 0$ There is one solution to this equation: $x = 0$.

Worked example

Using square numbers to solve worded problems

Determine the side length of a square with area $25\,\text{cm}^2$.

THINKING	WORKING
Write the formula for the area of a square.	$A = l^2$
Substitute known values into the equation.	$25\,\text{cm}^2 = l^2$
Recall the positive integer that multiplies by itself to give the square number.	$25\,\text{cm}^2 = 5^2$
Write the answer. (Note: in this situation, a negative answer is not reasonable. There is only one solution.)	The side length of the square is 5 cm.

Practice

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SC 1 I can recognise and use square numbers to solve quadratic equations in the form $x^2 = a$

1 State the inverse operations for each of the following.

(a) The inverse operation to *addition* is _____.

(b) The inverse operation to *subtraction* is _____.

(c) The inverse operation to *multiplication* is _____.

(d) The inverse operation to *division* is _____.

(e) The inverse operation to *squaring* is _____.

2 Solve the following by applying the inverse operation.

(a) $x^2 = 81$ **(b)** $x^2 = 121$ **(c)** $x^2 = 1$

(d) $x^2 = 0$ **(e)** $x^2 = 49$

3 Use a calculator to solve the following equations by applying the inverse operation.

(a) $x^2 = 2500$ **(b)** $x^2 = 900$ **(c)** $x^2 = 625$

(d) $x^2 = 400$ **(e)** $x^2 = 2025$

4 Use a calculator to solve the following by applying the inverse operation. Express your answers to 2 decimal places.

(a) $x^2 = 24$ **(b)** $x^2 = 60$ **(c)** $x^2 = 39$

(d) $x^2 = 2$ **(e)** $x^2 = 130$

5 Use a calculator to solve the following by applying the inverse operation. Express your answers to 2 decimal places where appropriate.

(a) $x^2 = 2.5$ **(b)** $x^2 = -100$ **(c)** $x^2 = -49$

(d) $x^2 = 0.09$ **(e)** $x^2 = -0.04$

6 Determine the side length of a square with the following areas. Express all answers to 2 decimal places.

(a) $120\,cm^2$ **(b)** $30\,m^2$ **(c)** $532\,mm^2$ **(d)** $2025\,cm^2$

7 Draw a square with an area $201.64\,cm^2$. Label the side length of the square.

8 Draw a cube whose total surface area is $384\,cm^2$. Label the side length of the cube.

9 Mia was asked to solve the following equation, to 3 decimal places: $x^2 = 15$

This is Mia's solution:

$x^2 = 15$

$\sqrt{x^2} = \sqrt{15}$

$x = 7.5$

(a) Determine the error in Mia's working out and solution.

(b) If the working out and/or solution are incorrect, rewrite them so that they are correct.

SC 2 I can recognise and use square numbers to solve quadratic equations in the form $ax^2 = a$

1 Express each of the following equations in the form $x^2 = a$.

(a) $7x^2 = 28$ **(b)** $4x^2 = 36$ **(c)** $2x^2 = 128$

(d) $3x^2 = 75$ **(e)** $2x^2 = 32$

2 Solve the following equations by applying the inverse operation.

(a) $7x^2 = 28$ **(b)** $4x^2 = 36$ **(c)** $2x^2 = 128$

(d) $3x^2 = 75$ **(e)** $2x^2 = 32$

3 Solve the following equations.

(a) $2x^2 = 288$ **(b)** $4x^2 = 484$ **(c)** $12x^2 = 12$

(d) $8x^2 = 72$ **(e)** $4x^2 = 16$

4 Use a calculator to solve the following equations. Express your answers to 3 decimal places.

(a) $5x^2 = 25$ **(b)** $6x^2 = 36$ **(c)** $9x^2 = 180$

(d) $11x^2 = 55$ **(e)** $13x^2 = 39$

5 Solve the following equations.

(a) $2.5x^2 = 62.5$ **(b)** $3.6x^2 = 230.4$ **(c)** $8.2x^2 = 131.2$

(d) $1.8x^2 = 145.8$ **(e)** $6.7x^2 = 328.3$

6 Explain why there is a solution to the equation $-4x^2 = -36$, yet there is no real solution to the equation $4x^2 = -36$.

7 The area of a circle is measured to be $196\pi\,cm^2$. Determine the radius of this circle. (Hint: the formula for calculating the area of a circle is $A = \pi r^2$.)

SC 3 I understand that a quadratic equation can have zero, one or two solutions

1 Complete these statements about the number of real solutions to the simple quadratic equation: $x^2 = a$.

(a) If $a < 0$, there are _____ solutions.

(b) If $a = 0$, there is _____ solution.

(c) If $a > 0$, there are _____ solutions.

2 Without solving, determine the number of real solutions to each of the following.

(a) $x^2 = -20$ **(b)** $x^2 = 100$ **(c)** $x^2 = -49$

(d) $x^2 = 0$ **(e)** $x^2 = 35$

3 Determine the number of real solutions to each of the following. State the solution, if it exists.

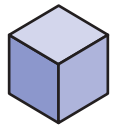
(a) $x^2 = 36$ **(b)** $x^2 = -25$ **(c)** $x^2 = 64$ **(d)** $x^2 = 0$

(e) $x^2 = -4$ **(f)** $x^2 = -18$ **(g)** $x^2 = 121$

4 Determine the side length of a square with area:

(a) $144\,\text{cm}^2$ **(b)** $10\,000\,\text{cm}^2$ **(c)** $169\,\text{cm}^2$ **(d)** $225\,\text{cm}^2$

5 Consider a cube with a total surface area of $726\,\text{cm}^2$ as shown.

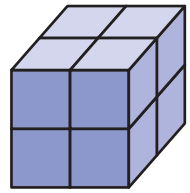


total surface area $= 726\ \text{cm}^2$

(a) What is the area of one face of the cube?

(b) What is the length of one side of the cube?

Imagine eight of these same cubes, stacked to form a larger cube as shown.



(c) What is the length of each side of this larger cube?

(d) What is the area of each face of this larger cube?

(e) What is the total surface area of this larger cube?

Solve monic quadratic equations using the Null Factor Law

5.2

Learning intention: To be able to solve monic quadratic equations using the Null Factor Law

Success criteria:

SC 1 I can use the Null Factor Law to solve simple quadratic equations.

SC 2 I can factorise quadratic expressions using perfect squares and solve equations using the Null Factor Law.

SC 3 I can factorise quadratic expressions using the difference of two squares and solve equations using the Null Factor Law.

SC 4 I can factorise monic quadratic expressions and solve equations using the Null Factor Law.

Lesson warm-up

A product of zero?



For each of the following rectangles, determine the value of the variable/s that make the area of the rectangle equal to 0 square units. Is there more than one way to make an area of 0 square units?

(a)

x units

3 units

(b)

$(x + 1)$ units

3 units

(c)

$(x - 1)$ units

3 units

(d)

$(x - 1)$ units

$3x$ units

(e)

$(x - 1)$ units

$(3x + 2)$ units

SC 1 I can use the Null Factor Law to solve simple quadratic equations

You will recall that $5 \times 0 = 0 \times 5 = 0$. In algebra, this is formalised as the Null Factor Law, which states that if a product of terms is equal to zero, then at least one of the terms must be equal to zero.

Consider, $(x + 3)(x - 4) = 0$.

Applying the Null Factor Law means $x + 3 = 0$ or $x - 4 = 0$. These are both simple linear equations that can be solved to determine the value of x.

$$\begin{aligned} x + 3 &= 0 \\ x + 3 - 3 &= 0 - 3 \\ x &= -3 \end{aligned}$$

$$\begin{aligned} x - 4 &= 0 \\ x - 4 + 4 &= 0 + 4 \\ x &= 4 \end{aligned}$$

So, the solution to $(x + 3)(x - 4) = 0$ is $x = -3$ or $x = 4$.

You can check this by substituting the values into the expression.

For $x = -3$:

$$\begin{aligned} (x + 3)(x - 4) &= (-3 + 3) \times (-3 - 4) \\ &= 0 \times (-7) \\ &= 0 \end{aligned}$$

For $x = 4$:

$$\begin{aligned} (x + 3)(x - 4) &= (4 + 3) \times (4 - 4) \\ &= 7 \times 0 \\ &= 0 \end{aligned}$$

Worked example

Solving simple equations using the Null Factor Law

Determine the solution to each of the following.

(a) $4(x - 3) = 0$

THINKING	WORKING
Identify the terms in the product.	The terms in the product are 4 and $(x - 3)$.
Recall the Null Factor Law.	The Null Factor Law states that if a product of terms is equal to zero, then at least one of the terms must be equal to zero.
Write an equation to let the factors be equal to zero.	$x - 3 = 0$ $x - 3 + 3 = 0 + 3$ $x = 3$
Write the answer.	For $4(x - 3) = 0$, $x = 3$.

(b) $4x(x-3)=0$

THINKING	WORKING
Identify the terms in the product.	The terms in the product are $4x$ and $(x-3)$.
Apply the Null Factor Law to write an equation to let the term/s in the product be equal to zero.	The first term is equal to zero: $4x=0$ $\frac{4x}{4}=\frac{0}{4}$ $x=0$ The second term is equal to zero: $x-3=0$ $x-3+3=0+3$ $x=3$
Write the answer.	For $4x(x-3)=0$, $x=0$ or $x=3$.

(c) $4x(x-3)+8(x-3)=0$

THINKING	WORKING
Factorise by grouping to write the expression as a product.	$4x(x-3)+8(x-3)=0$ $(4x+8)(x-3)=0$
Identify the terms in the product.	The terms in the product are $(4x+8)$ and $(x-3)$.
Apply the Null Factor Law to write an equation to let the term/s in the product be equal to zero.	The first term is equal to zero: $4x+8=0$ $4x=-8$ $x=-2$ The second term is equal to zero: $x-3=0$ $x=3$
Write the answer. Remember, you can check your answer by substituting it back into the equation.	For $4x(x-3)+8(x-3)=0$, $x=-2$ or $x=3$.

SC 2 I can factorise quadratic expressions using perfect squares and solve equations using the Null Factor Law

An algebraic perfect square means that the two factors are the same.

$(x+a)(x+a)=(x+a)^2$ is an example of a perfect square.

The perfect square rule is usually written in the form:

$(x+a)^2=(x+a)(x+a)=x^2+2ax+a^2$

$(x-a)^2=(x-a)(x-a)=x^2-2ax+a^2$

Perfect squares have a term multiplied by itself, so using the Null Factor Law gives a single solution:

$(x + a)^2 = 0, x = -a$

$(x - a)^2 = 0, x = a$

Worked example

Recognising, factorising and solving with perfect squares

Solve each of the following equations using perfect squares.

(a) $x^2 + 6x + 9 = 0$

THINKING	WORKING
Recall the matching perfect squares form.	$(x + a)^2 = (x + a)(x + a)$ $= x^2 + 2ax + a^2$
Factorise the quadratic expression.	$x^2 + 6x + 9 = x^2 + 2 \times 3x + 3^2$ $= (x + 3)^2$
Use the Null Factor Law to solve the equation.	$(x + 3)^2 = 0, x = -3$

(b) $x^2 - 6x + 9 = 0$

THINKING	WORKING
Recall the matching perfect squares form.	$(x - a)^2 = (x - a)(x - a)$ $= x^2 - 2ax + a^2$
Factorise the quadratic expression.	$x^2 - 6x + 9 = x^2 - 2 \times 3x + 3^2$ $= (x - 3)^2$
Use the Null Factor Law to solve the equation.	$(x - 3)^2 = 0, x = 3$

SC 3 I can factorise quadratic expressions using the difference of two squares and solve equations using the Null Factor Law

Recall the difference of two squares (DOTS) method to expand and factorise quadratics.

$(x + a)(x - a) = x^2 + ax - ax - a^2$

$= x^2 - a^2$

Solving the factorised form using the Null Factor Law results in both positive and negative values of a.

$x + a = 0$ or $x - a = 0$

$x = -a$ $x = a$

Worked example

Recognising, factorising and solving with the difference of two squares

Use the difference of two squares to factorise the expression and the Null Factor Law to solve for x.

(a) $x^2 - 3^2 = 0$

THINKING	WORKING
Use the DOTS pattern $x^2 - a^2 = (x + a)(x - a)$ to factorise the quadratic expression.	$x^2 - 3^2 = 0$ $(x + 3)(x - 3) = 0$
Use the Null Factor Law to solve the equation.	$x + 3 = 0$ or $x - 3 = 0$ $x = -3$ $x = 3$
Write the answer.	For $x^2 - 3^2 = 0$, $x = 3$ or $x = -3$.

(b) $x^2 - 16 = 0$

THINKING	WORKING
Rewrite the expression in the form $x^2 - a^2$.	$x^2 - 16 = x^2 - 4^2$
Use the DOTS pattern $x^2 - a^2 = (x + a)(x - a)$ to factorise the quadratic expression.	$x^2 - 4^2 = (x - 4)(x + 4)$
Use the Null Factor Law to solve the equation.	$x + 4 = 0$ or $x - 4 = 0$ $x = -4$ $x = 4$
Write the answer.	For $x^2 - 16 = 0$, $x = 4$ or $x = -4$.

SC 4 I can factorise monic quadratic expressions and solve equations using the Null Factor Law

Look at the expansion of $(x + a)(x + b)$.

$$\begin{aligned}(x + a)(x + b) &= x(x + b) + a(x + b)\\ &= x^2 + bx + ax + ab\\ &= x^2 + (a + b)x + ab\end{aligned}$$

This means that for a quadratic trinomial where the coefficient of x^2 is 1, the factors, if they exist, can be found by finding the factors of the numeric term that add to the coefficient of the x term.

It is important to understand that not every quadratic trinomial has rational factors.

Worked example

Factorising monic quadratic expressions and solving equations using the Null Factor Law

Factorise the following quadratic expressions, then solve the equation using the Null Factor Law.

(a) $x^2 + x - 6 = 0$

THINKING	WORKING
Recall the form of a simple quadratic.	$(x + a)(x + b) = x^2 + (a + b)x + ab$
Identify the two conditions that need to be met by the values of a and b.	$a \times b = -6$ $a + b = 1$
Determine the values for the variables that multiply to give the constant and add to give the coefficient of x.	The values are 3 and -2. $3 \times (-2) = -6$ $3 + (-2) = 1$
Write the factorised expression.	$x^2 + x - 6 = 0$ $x^2 + 3x - 2x + 3 \times (-2) = 0$ $x(x + 3) - 2(x + 3) = 0$ $(x + 3)(x - 2) = 0$
Use the Null Factor Law to solve the equation.	$x + 3 = 0$ or $x - 2 = 0$ $x = -3$ $x = 2$
Write the answer.	For $x^2 + x - 6 = 0$, $x = -3$ or $x = 2$.

(b) $x^2 - 5x - 6 = 0$

THINKING	WORKING
Recall the form of a simple quadratic.	$(x + a)(x + b) = x^2 + (a + b)x + ab$
Identify the two conditions that need to be met by the values of a and b.	$a \times b = -6$ $a + b = -5$
Determine the values for the variables that multiply to give the constant and add to give the coefficient of x.	The values are -6 and 1. $1 \times (-6) = -6$ $1 + (-6) = -5$
Write the factorised expression.	$x^2 - 5x - 6 = 0$ $x^2 - 6x + 1x + (-6) \times 1 = 0$ $x(x - 6) + 1(x - 6) = 0$ $(x - 6)(x + 1) = 0$
Use the Null Factor Law to solve the equation.	$x - 6 = 0$ or $x + 1 = 0$ $x = 6$ $x = -1$
Write the answer.	For $x^2 - 5x - 6 = 0$, $x = 6$ or $x = -1$.

Practice

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SC 1 I can use the Null Factor Law to solve simple quadratic equations

1 Use the Null Factor Law to determine the solution for each of the following linear equations.

(a) $2(x-3)=0$ **(b)** $2(x-2)=0$ **(c)** $2(x-1)=0$ **(d)** $2x=0$

(e) $2(x+1)=0$ **(f)** $2(x+2)=0$ **(g)** $2(x+3)=0$

2 Use the Null Factor Law to determine the solution for each of the following quadratic equations.

(a) $x(x-8)=0$ **(b)** $2x(x-5)=0$ **(c)** $3x(x-1)=0$ **(d)** $5x^2=0$

(e) $x(x+1)=0$ **(f)** $3x(x+5)=0$ **(g)** $5x(x+8)=0$

3 Determine the solution for each of the following quadratic equations.

(a) $(x+1)(x-2)=0$ **(b)** $(x+2)(x-3)=0$

(c) $(x+3)(x-4)=0$ **(d)** $(x+4)(x-5)=0$

4 Determine the solution for each of the following quadratic equations.

(a) $(x-4)(x+4)=0$ **(b)** $(x-7)(x+7)=0$ **(c)** $(x-3)^2=0$ **(d)** $(x+6)^2=0$

5 Use the Null Factor Law to solve the following equations.

(a) $(3a+2)(2a+3)=0$ **(b)** $(5b-2)(2b+3)=0$

(c) $(6c+5)(3c-4)=0$ **(d)** $(10d-4)(5d-3)=0$

6 For each of the following solutions:

(i) write a factorised quadratic expression equal to zero that satisfies the conditions stated

(ii) write the equation in expanded form, $x^2+bx+c=0$.

(a) The equation has solutions $x=-3$ or $x=6$. **(b)** The equation has solutions $x=-5$ or $x=9$.

(c) The equation has the lone solution $x=10$. **(d)** The equation has the lone solution $x=-8$.

SC 2 I can factorise quadratic expressions using perfect squares and solve equations using Null Factor Law

1 For each of the following quadratic equations:

(i) factorise the quadratic expressions using $(x+a)^2=(x+a)(x+a)=x^2+2ax+a^2$

(ii) solve the equation using the Null Factor Law.

(a) $x^2+2x+1=0$ **(b)** $x^2+8x+16=0$ **(c)** $x^2+12x+36=0$

(d) $x^2+14x+49=0$ **(e)** $x^2+20x+100=0$ **(f)** $x^2+30x+225=0$

2 For each of the following quadratic equations:

(i) factorise the quadratic expression using $(x-a)^2=(x-a)(x-a)=x^2-2ax+a^2$

(ii) solve the equation using the Null Factor Law.

(a) $x^2-4x+4=0$ **(b)** $x^2-8x+16=0$ **(c)** $x^2-10x+25=0$

(d) $x^2-16x+64=0$ **(e)** $x^2-22x+121=0$ **(f)** $x^2-26x+169=0$

SC 3 I can factorise quadratic expressions using the difference of two squares and solve equations using the Null Factor Law

1 For each of the following quadratic equations:

(i) factorise the quadratic expressions using $x^2 - a^2 = (x + a)(x - a)$

(ii) solve the equation using the Null Factor Law.

(a) $x^2 - 1 = 0$ **(b)** $x^2 - 9 = 0$ **(c)** $x^2 - 36 = 0$

(d) $x^2 - 64 = 0$ **(e)** $x^2 - 144 = 0$ **(f)** $x^2 - 289 = 0$

SC 4 I can factorise monic quadratic expressions and solve equations using the Null Factor Law

1 For each of the following monic quadratics in the form $(x + a)(x + b)$, where $a \times b = 18$,

(i) factorise the quadratic expression

(ii) solve the equation for x.

(a) $x^2 + 11x + 18 = 0$ **(b)** $x^2 + 9x + 18 = 0$ **(c)** $x^2 + 19x + 18 = 0$

(d) $x^2 - 17x - 18 = 0$ **(e)** $x^2 - 7x - 18 = 0$ **(f)** $x^2 - 3x - 18 = 0$

(g) $x^2 + 17x - 18 = 0$ **(h)** $x^2 + 3x - 18 = 0$ **(i)** $x^2 + 7x - 18 = 0$

2 For each of the following monic quadratics in the form $(x + a)(x + b)$, where $a + b = 18$,

(i) factorise the quadratic expression

(ii) solve the equation for x.

(a) $x^2 + 18x + 32 = 0$ **(b)** $x^2 + 18x + 77 = 0$ **(c)** $x^2 + 18x + 80 = 0$

(d) $x^2 + 18x + 81 = 0$ **(e)** $x^2 + 18x + 56 = 0$

3 For each of the following monic quadratics in the form $(x + a)(x + b)$, where $a + b = -18$,

(i) factorise the quadratic expression

(ii) solve the equation for x.

(a) $x^2 - 18x - 19 = 0$ **(b)** $x^2 - 18x - 40 = 0$ **(c)** $x^2 - 18x + 80 = 0$

(d) $x^2 - 18x + 96 = 0$ **(e)** $x^2 - 18x + 45 = 0$

4 Solve the following equations.

(a) $x^2 - 4x - 21 = 0$ **(b)** $x^2 - 8x + 12 = 0$

(c) $x^2 + 14x + 45 = 0$ **(d)** $x^2 + 2x - 80 = 0$

Identify and plot the key features of quadratic graphs

5.3

Learning intention: To be able to identify and plot the key features of quadratic graphs

Success criteria:

SC 1 I can generate a table of values from a quadratic equation and plot the points.

SC 2 I can identify and plot key features of quadratic graphs.

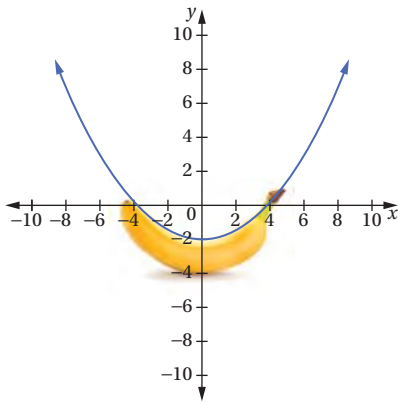
Lesson warm-up

Curvy bananas

When plants first grow, regardless of the orientation of the pot plant, the stems will grow away from the ground (against gravity) and extend upwards towards the sun. This phenomenon is called negative geotropism.



When bananas initially shoot from the flower, they are pointed towards the ground. However, they then continually grow against gravity and towards the sunlight, giving them their well-known curved appearance. This curve in bananas is just one of the known natural occurrences of a parabolic shape.



What do you notice about the parabolic curve on the banana in terms of the x-intercepts?

Does the curve appear to have symmetry?

SC 1 I can generate a table of values from a quadratic equation and plot the points

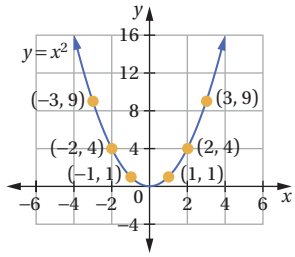
Generating a table of values

The graphs of quadratic functions are called parabolas.

To sketch a quadratic from a table of values, plot the points and join them with a smooth curve.

To draw the basic quadratic function $y = x^2$, generate a table of values, then plot the points and join them with a continuous and smooth curve.

x	-3	-2	-1	0	1	2	3
$y = x^2$	$(-3)^2 = 9$	$(-2)^2 = 4$	$(-1)^2 = 1$	$0^2 = 0$	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$
(x, y)	$(-3, 9)$	$(-2, 4)$	$(-1, 1)$	$(0, 0)$	$(1, 1)$	$(2, 4)$	$(3, 9)$



A quadratic graph requires more points than this so that the nature of the relationship can be correctly represented in the graph. At the very least, enough points need to be plotted to ensure that the minimum or maximum point can be approximated.

To determine whether the table of values covers this value, look for a change of direction in the values of the dependent variable. Also, if the same value for the dependent variable appears in two different positions in the table, then it is certain that the maximum or minimum point has been included.

Checking a table of values for a quadratic pattern

To check that your table of values is accurate, check that the second difference is consistent.

x	-3	-2	-1	0	1	2	3
$y = x^2$	$(-3)^2 = 9$	$(-2)^2 = 4$	$(-1)^2 = 1$	$0^2 = 0$	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$
Difference in consecutive y-values		$9 - 4 = 5$	$4 - 1 = 3$	$1 - 0 = 1$	$0 - 1 = -1$	$1 - 4 = -3$	$4 - 9 = -5$
Second difference			$5 - 3 = 2$	$3 - 1 = 2$	$1 - (-1) = 2$	$-1 - (-3) = 2$	$-3 - (-5) = 2$

Worked example

Generating a table of values and plotting a parabola

Consider the quadratic equation $y = x^2 - 4x - 5$.

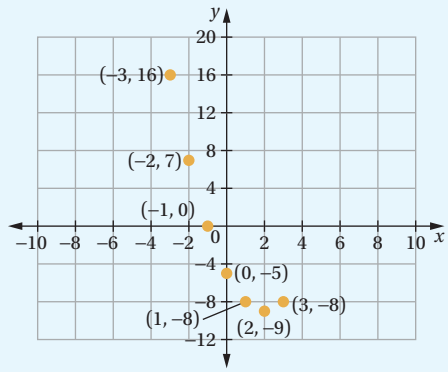
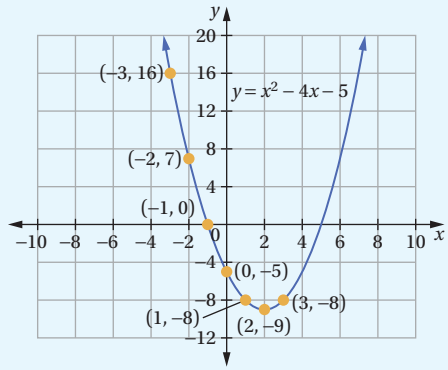
(a) Generate a table of values between $x = -3$ and $x = 3$.

THINKING	WORKING
Draw a table with the required x-values.	x | -3 | -2 | -1 | 0 | 1 | 2 | 3 y | | | | | | |
Substitute the x-values into the quadratic equation to calculate the corresponding y-values.	For example, for $x = -3$, $y = (-3)^2 - 4(-3) - 5$ $= 9 + 12 - 5$ $= 16$ x | -3 | -2 | -1 | 0 | 1 | 2 | 3 y | 16 | 7 | 0 | -5 | -8 | -9 | -8
Add a row to the table and list the coordinates ready to plot.	x | -3 | -2 | -1 | 0 | 1 | 2 | 3 y | 16 | 7 | 0 | -5 | -8 | -9 | -8 (x, y) | $(-3, 16)$ | $(-2, 7)$ | $(-1, 0)$ | $(0, -5)$ | $(1, -8)$ | $(2, -9)$ | $(3, -8)$

(b) Demonstrate that the pattern generated is a quadratic by showing that the second difference is constant.

THINKING	WORKING
Determine the difference in consecutive y-values.	x | -3 | -2 | -1 | 0 | 1 | 2 | 3 y | 16 | 7 | 0 | -5 | -8 | -9 | -8 | | $16 - 7 = 9$ | $7 - 0 = 7$ | $0 - (-5) = 5$ | $-5 - (-8) = 3$ | $-8 - (-9) = 1$ | $-9 - (-8) = -1$
Determine the second difference (the difference in consecutive differences).	x | -3 | -2 | -1 | 0 | 1 | 2 | 3 y | 16 | 7 | 0 | -5 | -8 | -9 | -8 | | $16 - 7 = 9$ | $7 - 0 = 7$ | $0 - (-5) = 5$ | $-5 - (-8) = 3$ | $-8 - (-9) = 1$ | $-9 - (-8) = -1$ | | | $9 - 7 = 2$ | $7 - 5 = 2$ | $5 - 3 = 2$ | $3 - 1 = 2$ | $1 - (-1) = 2$
Write the answer.	The second difference is consecutive, so the pattern generated in the table is a quadratic.

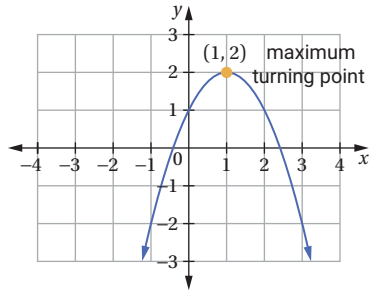
(c) Plot the graph.

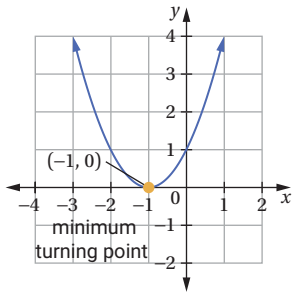
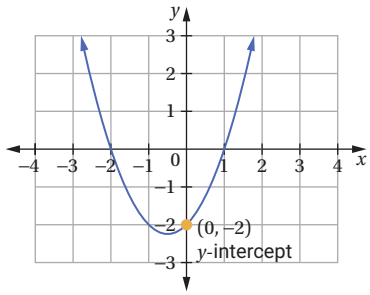
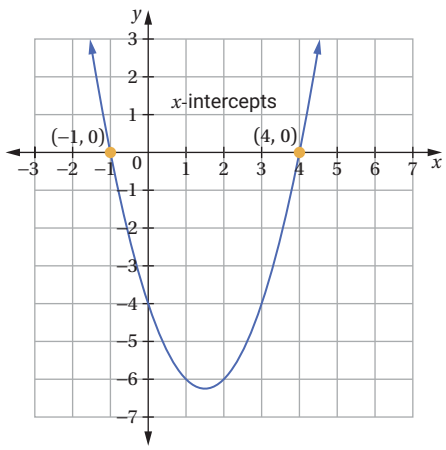
THINKING	WORKING
Draw a Cartesian plane and plot the points (from the coordinates found in part **(a)**). Check that the points show the maximum or minimum value.	
Now join the points using a smooth curve. You may have to estimate the location of the minimum or maximum point. (In this case, the symmetry of a parabola indicates that the x-value of this point must be 2 because there are two values with the same y-value either side of $x = 2$.)	

SC 2 I can identify and plot key features of quadratic graphs

The minimum or maximum point on a parabola is called a turning point. This point occurs exactly half way between the x-intercepts.

For the quadratic $y = ax^2 + bx + c$, the turning point is at $x = -\dfrac{b}{2a}$.

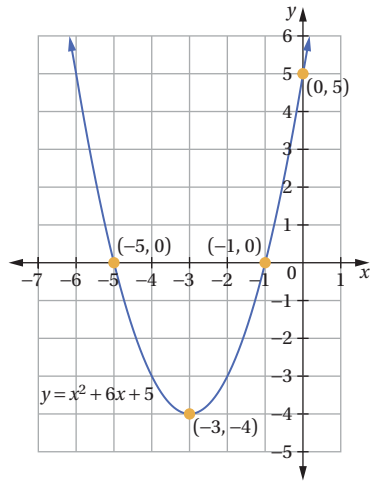
Feature	Algebraic representation	Graphical representation
Maximum turning point	For $y = -x^2 + 2x + 1$, the turning point is at: $x = -\dfrac{2}{2 \times (-1)}$ $= -\dfrac{2}{-2}$ $= 1$ At $x = 1$, $y = -(1)^2 + 2 \times (1) + 1$ $= -1 + 2 + 1$ $= 2$ The coordinates of the turning point are (1, 2).	

Feature	Algebraic representation	Graphical representation
Minimum turning point	For $y = x^2 + 2x + 1$, the turning point is at: $x = -\frac{2}{2 \times 1}$ $= -\frac{2}{2}$ $= -1$ At $x = -1$, $y = (-1)^2 + 2 \times (-1) + 1$ $= 1 - 2 + 1$ $= 0$ The coordinates of the turning point are $(-1, 0)$.	
y-intercept	The y-intercept occurs at $x = 0$. From a graph, it can be found at the point where the parabola crosses the y-axis. From the equation, the y-intercept can be determined by substituting $x = 0$ and calculating the value of y. For $y = x^2 + x - 2$ at $x = 0$: $y = (0)^2 + (0) - 2$ $= -2$ The coordinates of the y-intercept are $(0, -2)$.	
x-intercept/s	The x-intercept/s occur at $y = 0$. From a graph, it can be found at the point where the parabola touches or crosses the x-axis. From the equation, the x-intercept can be determined by substituting $y = 0$ and solving to determine the value of x. You can use the Null Factor Law to solve for $y = 0$. For $y = x^2 - 3x - 4$ at $y = 0$: $0 = x^2 - 3x - 4$ $= (x + 1)(x - 4)$ The solutions are $x = -1$ or $x = 4$. The coordinates of the x-intercepts are $(-1, 0)$ and $(4, 0)$.	

Worked example

Identifying and verifying key features from quadratic graphs

The graph of $y = x^2 + 6x + 5$ is shown. Identify the following features and verify their location algebraically.



(a) The coordinates of the y-intercept

THINKING	WORKING
Substitute $x = 0$ and calculate the value of y.	At $x = 0$, $y = (0)^2 + 6(0) + 5$ $= 0 + 0 + 5$ $= 5$
Write the answer.	The coordinates of the y-intercept are (0, 5).

(b) The coordinates of the x-intercepts

THINKING	WORKING
Substitute $y = 0$.	$0 = x^2 + 6x + 5$
Factorise the quadratic expression.	$0 = (x + 5)(x + 1)$
Use the Null Factor Law to solve the equation.	$x = -5$ or $x = -1$
Write the answer.	The coordinates of the x-intercepts are (−5, 0) and (−1, 0).

(c) The coordinates of the turning point

THINKING	WORKING
For the quadratic $y = ax^2 + bx + c$, determine the value of $x = -\frac{b}{2a}$.	$x = -\frac{b}{2a}$ $= -\frac{6}{2 \times 1}$ $= -\frac{6}{2}$ $= -3$
Alternatively, the x-value of the turning point is half way between the x-intercepts.	The x-intercepts are at $x = -5$ and $x = -1$. The turning point occurs at $x = \frac{-5 + (-1)}{2}$ $= -\frac{6}{2}$ $= -3$
Substitute the value of x into the equation to calculate the value of y.	For $x = -3$, $y = (-3)^2 + 6 \times (-3) + 5$ $= 9 - 18 + 5$ $= -9 + 5$ $= -4$
Write the answer.	The coordinates of the turning point are $(-3, -4)$.

Practice

ANSWERS Page 626

SC 1 I can generate a table of values from a quadratic equation and plot the points

1 For each of the following quadratic equations

(i) complete the table of values

x	-2	-1	0	1	2
y					

(ii) plot the points and sketch the shape of the graph on a Cartesian plane.

(a) $y = x^2 + 1$ **(b)** $y = x^2 - 1$ **(c)** $y = (x + 1)^2$ **(d)** $y = (x - 1)^2$

2 Complete the tables of values and use them to plot the graph for each of the following quadratic relationships.

(a) $y = x^2 - 4x - 12$

x	−1	0	1	2	3	4
y						

(b) $y = 3x^2 + 3x - 6$

x	−3	−2	−1	0	1	2
y						

(c) $y = -x^2 + 2x + 8$

x	−3	−2	0	1	2	4
y						

(d) $y = -2x^2 - 10x - 8$

x	−5	−4	−3	−2	−1	0
y						

3 For each of the following quadratics, copy and complete the table of values and determine the first and second differences to show that the values represent a quadratic pattern.

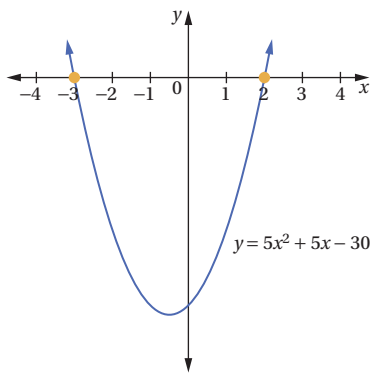
x	−2	−1	0	1	2	3
y						
First difference						
Second difference						

(a) $y = x^2 - 3$ **(b)** $y = -x^2 + 4x$

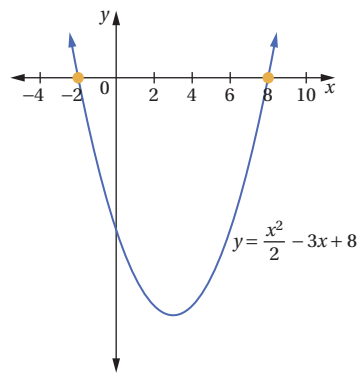
(c) $y = 2x^2 - 3x - 5$ **(d)** $y = (x - 2)^2$

4 For each of the following graphs, use the symmetry of a parabola and the equation to determine the coordinates of the maximum or minimum point.

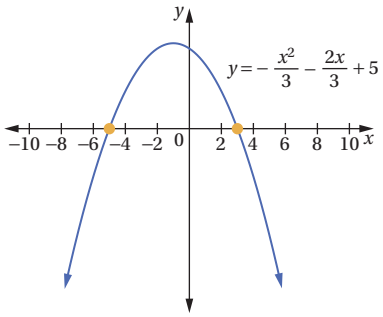
(a)



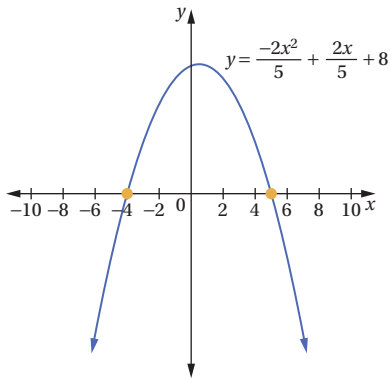
(b)



(c)



(d)



5 Using the symmetry of a parabola, complete the following tables of values.

(a)

x	0	1	3	4	5	6	8	9
y	18		0	−2		0	10	

(b)

x	−10	−6	−4	3	5	9
y	−78		0	0	−18	

(c)

x	−20		−9		−5	−3	0	
y	187	27	0	−8	−8	0	27	187

6 Complete the tables of values and use them to plot the graph for each of the following quadratic relationships.

(a) $y = (x - 6)(x + 4)$

x	−4	0	1	2	6
y					

(b) $y = -(x + 3)(x - 7)$

x	−3	−2	−1	0	2	4	7
y							

(c) $y = \frac{1}{2}(x-4)(x+8)$

x	−8	−4	−2	0	4
y					

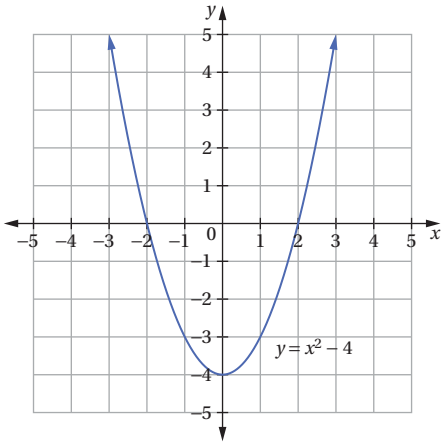
(d) $y = -\frac{1}{2}(x+5)(x-7)$

x	−5	−3	1	5	7
y					

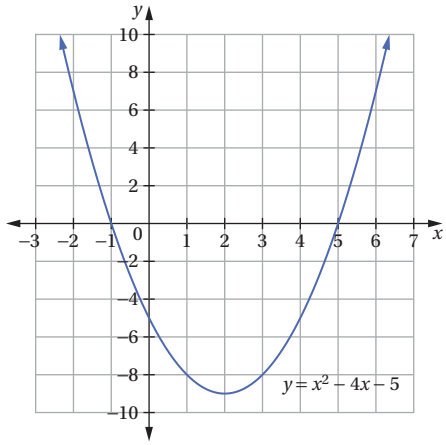
SC 2 I can identify and plot key features of quadratic graphs

1 For each of the following quadratic graphs, state the coordinates of the y-intercept, the turning point and the x-intercept (if they exist). Also, state whether the parabola has a maximum or minimum turning point.

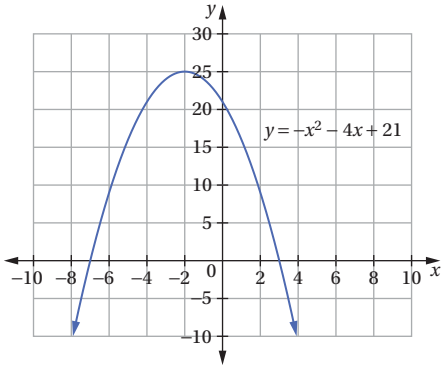
(a) $y = x^2 - 4$



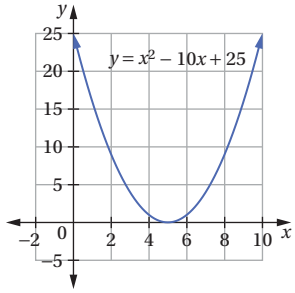
(b) $y = x^2 - 4x - 5$



(c) $y = -x^2 - 4x + 21$

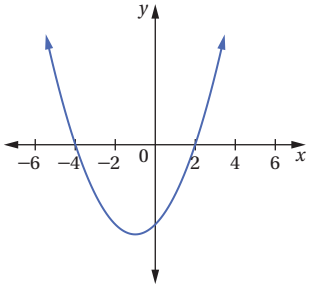


(d) $y = x^2 - 10x + 25$

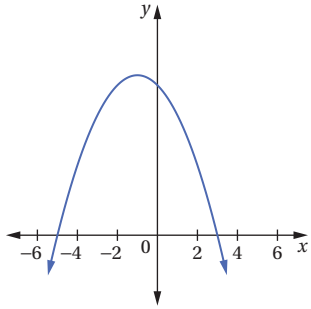


2 For each of the following graphs, use the symmetry of a parabola and the equation to determine the coordinate pair for the turning point.

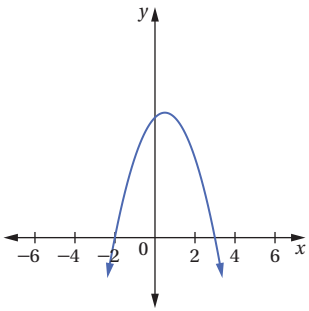
(a) $y = x^2 + 2x - 8$



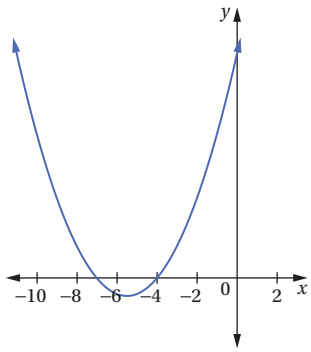
(b) $y = -x^2 - 2x + 15$



(c) $y = -x^2 + x + 6$



(d) $y = x^2 + 11x + 28$



3 For each of the following quadratic equations:

(i) determine the coordinates of the x-intercepts

(ii) determine the coordinates of the y-intercept

(iii) determine the coordinates of the turning point

(iv) sketch the graph.

(a) $y = (x - 8)(x + 10)$

(b) $y = -(x - 5)(x + 11)$

(c) $y = \frac{1}{2}(x-2)(x+6)$

(d) $y = -\frac{1}{4}(x-8)(x+3)$

4 Each of the following quadratic equations have a single x-intercept. State the coordinates for the x-intercept, the y-intercept and state whether the graph would have a maximum or minimum turning point.

(a) $y = (x - 2)^2$ **(b)** $y = (3x + 2)^2$ **(c)** $y = -(5x - 1)^2$ **(d)** $y = (3 - 2x)^2$

5.4 Determine transformations of parabolas

Learning intention: To be able to determine transformations of parabolas

Success criteria:

SC 1 I can analyse and interpret both horizontal and vertical translations.

SC 2 I can analyse and interpret dilations and determine reflections in the x-axis.

SC 3 I can analyse and interpret combined transformations.

Lesson warm-up

Maze navigation

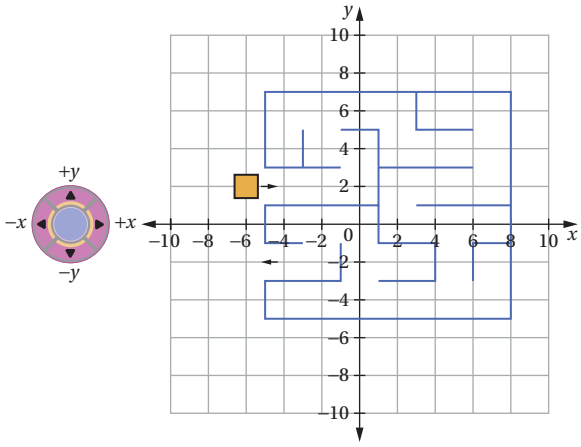


A square is to be navigated through the maze shown.

The rule for the movement is:

Movement = x + (← or →) and, y + (↑ or ↓)

Each time a directional key is pressed on a remote, the square translates one unit in the selected direction. Write a summary of the button presses you would use to navigate the cube through the maze (e.g. 6 × → would be the first instruction).



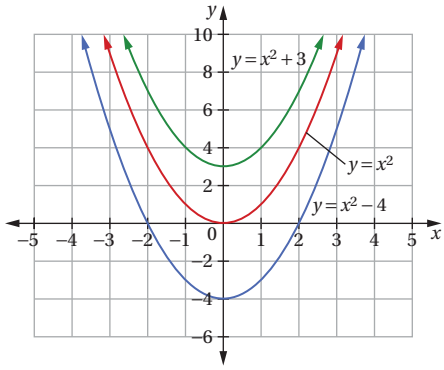
SC 1 I can analyse and interpret both horizontal and vertical translations

Vertical translations

Consider the following graphs.

x	-2	-1	0	1	2
$y = x^2$	$(-2)^2 = 4$	$(-1)^2 = 1$	$0^2 = 0$	$1^2 = 1$	$2^2 = 4$
$y = x^2 - 4$	$(-2)^2 - 4$ $= 4 - 4$ $= 0$	$(-1)^2 - 4$ $= 1 - 4$ $= -3$	$0^2 - 4$ $= 0 - 4$ $= -4$	$1^2 - 4$ $= 1 - 4$ $= -3$	$2^2 - 4$ $= 4 - 4$ $= 0$
$y = x^2 + 3$	$(-2)^2 + 3$ $= 4 + 3$ $= 7$	$(-1)^2 + 3$ $= 1 + 3$ $= 4$	$0^2 + 3$ $= 0 + 3$ $= 3$	$1^2 + 3$ $= 1 + 3$ $= 4$	$2^2 + 3$ $= 4 + 3$ $= 7$

The only difference between the three graphs is that, compared to the graph of $y = x^2$, the graph of $y = x^2 + 3$ has been translated 3 units vertically up, and the graph of $y = x^2 - 4$ has been translated 4 units vertically down.



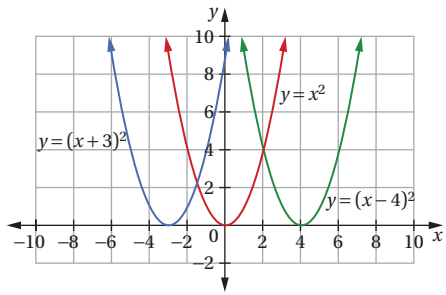
The general form for vertical translation is $y = x^2 + k$. The translation is k units vertically.

An equation such as $y = x^2 - 5$ represents a translation of 5 units vertically down or -5 units vertically.

Horizontal translations

Consider the following graphs.

x	-5	-1	0	1	5
$y = x^2$	$(-5)^2 = 25$	$(-1)^2 = 1$	$0^2 = 0$	$1^2 = 1$	$5^2 = 25$
$y = (x - 4)^2$	$(-5 - 4)^2$ $= (-9)^2$ $= 81$	$(-1 - 4)^2$ $= (-5)^2$ $= 25$	$(0 - 4)^2$ $= (-4)^2$ $= 16$	$(1 - 4)^2$ $= (-3)^2$ $= 9$	$(5 - 4)^2$ $= 1^2$ $= 1$
$y = (x + 3)^2$	$(-5 + 3)^2$ $= (-2)^2$ $= 4$	$(-1 + 3)^2$ $= 2^2$ $= 4$	$(0 + 3)^2$ $= 3^2$ $= 9$	$(1 + 3)^2$ $= 4^2$ $= 16$	$(5 + 3)^2$ $= 8^2$ $= 64$



The only difference between the three graphs is that, compared to the graph of $y = x^2$, the graph of $y = (x + 3)^2$ has been translated 3 units horizontally to the left, and the graph of $y = (x - 4)^2$ has been translated 4 units horizontally to the right.

The general form for horizontal translation is $y = (x - h)^2$. The translation is h units to the right.

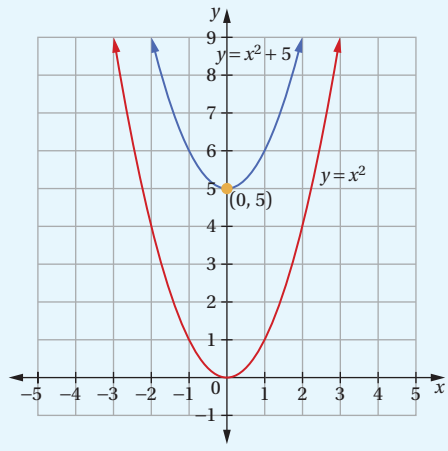
The graph of $y = (x + 3)^2$ can be thought of as $y = [x - (-3)]^2$ to give a translation of -3 units to the right, or 3 units to the left.

Worked example

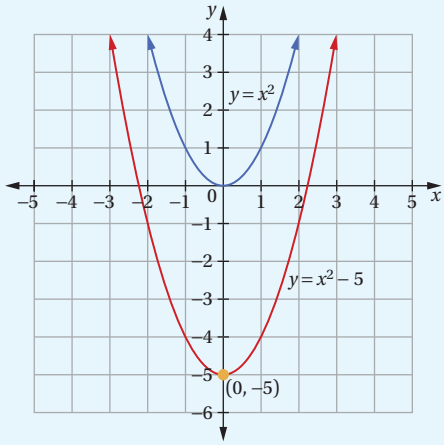
Describing and sketching vertical translations of a parabola

Describe the following translations, state the coordinates of the turning point and sketch the graph.

(a) The graph of $y = x^2$ is translated to $y = x^2 + 5$.

THINKING	WORKING
Recall the general form of vertical translations.	A vertical translation is in the form $y = x^2 + k$, where k represents the vertical translation.
Determine the direction of the translation.	The graph of $y = x^2 + 5$ represents a vertical translation where $k = 5$. The value of k is positive, so the translation is up.
Describe the translation.	Compared to the graph of $y = x^2$, the graph of $y = x^2 + 5$ has been translated 5 units vertically up.
Determine the coordinates of the turning point.	The coordinates of the turning point are (0, 5).
Sketch the graph.	

(b) The graph of $y = x^2$ is translated to $y = x^2 - 5$.

THINKING	WORKING
Determine the direction of the translation. You can rewrite the equation to match the general form.	The graph of $y = x^2 - 5$ or $y = x^2 + (-5)$ represents a vertical translation where $k = -5$. The value of k is negative, so the translation is down.
Describe the translation.	Compared to the graph of $y = x^2$, the graph of $y = x^2 - 5$ has been translated 5 units vertically down.
Determine the coordinates of the turning point.	The coordinates of the turning point are (0, –5).
Sketch the graph.	

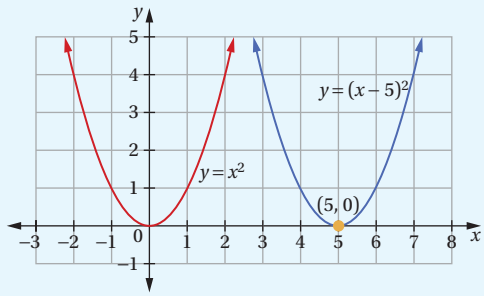
Worked example

Describing and sketching horizontal translations of a parabola

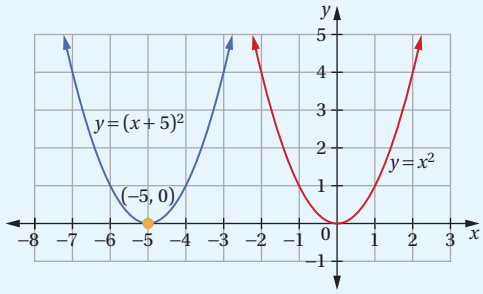
Describe the following translations, state the coordinates of the turning point and sketch the graph.

(a) The graph of $y = x^2$ is translated to $y = (x - 5)^2$.

THINKING	WORKING
Recall the general form of horizontal translations.	A horizontal translation is in the form $y = (x - h)^2$, where h represents the horizontal translation to the right.
Determine the direction of the translation.	The graph of $y = (x - 5)^2$ represents a horizontal translation where $h = 5$. The translation is to the right.
Describe the translation.	Compared to the graph of $y = x^2$, the graph of $y = (x - 5)^2$ has been translated 5 units to the right.
Determine the coordinates of the turning point.	The coordinates of the turning point are (5, 0).

Sketch the graph.	

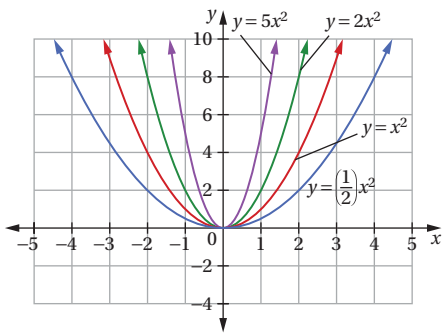
(b) The graph of $y = x^2$ is translated to $y = (x + 5)^2$.

THINKING	WORKING
Determine the direction of the translation. You can rewrite the equation to match the general form.	The graph of $y = (x + 5)^2$ or $y = [x - (-5)]^2$ represents a horizontal translation where $h = -5$. The translation is to the left.
Describe the translation.	Compared to the graph of $y = x$, the graph of $y = (x + 5)^2$ has been translated 5 units to the left.
Determine the coordinates of the turning point.	The coordinates of the turning point are (−5, 0).
Sketch the graph.	

SC 2 I can analyse and interpret dilations and determine reflections in the x-axis

Vertical dilations

Consider the following graph.



All of the graphs have the same turning point, so there has been no translation involved.
The differences between the graphs are the result of stretching or compressing the base graph, $y = x^2$ in the direction of the y-axis. This is also known as vertical dilation.

The tables of values will provide more information about this effect.

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4
$y = 2x^2$	8	2	0	2	8
$y = 5x^2$	20	5	0	5	20
$y = \frac{1}{2}x^2$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2

It can be seen that the y-values for $y = x^2$ have been multiplied by the coefficient of x^2 in each case. The value of this coefficient is the dilation factor in the direction of the y-axis (or vertically).

The general form for vertical dilation is $y = ax^2$.

The dilation $0 < a < 1$ compresses the graph of $y = x^2$ vertically; it becomes flatter or wider.

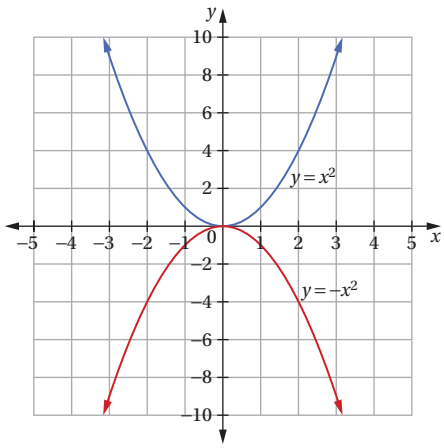
The dilation $a > 1$ vertically stretches the graph of $y = x^2$; it becomes steeper or narrower.

Reflections

Quadratic graphs can be reflected in the x-axis.

Consider the following graphs.

x	-2	-1	0	1	2
$y = x^2$	$(-2)^2 = 4$	$(-1)^2 = 1$	$0^2 = 0$	$1^2 = 1$	$2^2 = 4$
$y = -x^2$	$-(-2)^2 = -4$	$-(-1)^2 = -1$	$-0^2 = 0$	$-1^2 = -1$	$-2^2 = -4$

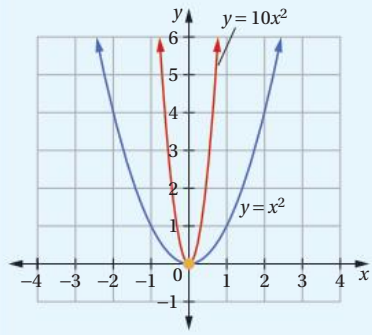


Worked example

Describing and sketching vertical dilations of a parabola

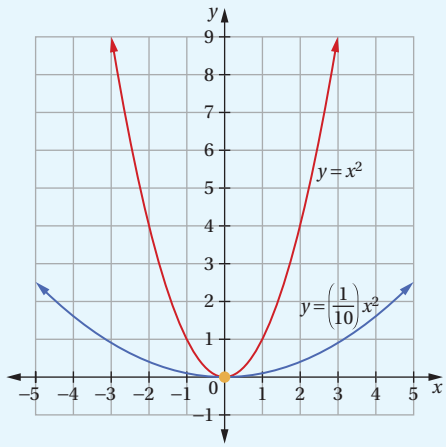
Describe the following dilations, state the coordinates of the turning point and sketch the graph.

(a) The graph of $y = x^2$ is dilated to $y = 10x^2$.

THINKING	WORKING
Recall the general forms of vertical dilations.	The dilation is in the form $y = ax^2$, where a represents the vertical dilation.
Determine the shape of the dilation.	$a > 1$ represents a vertical stretching (steeper or narrower) compared to the graph of $y = x^2$.
Describe the dilation.	The graph of $y = 10x^2$ has a dilation factor of 10 and is narrower than the graph of $y = x^2$.
Determine the coordinates of the turning point.	The coordinates of the turning point are (0, 0).
Sketch the graph.	

(b) The graph of $y = x^2$ is dilated to $y = \frac{1}{10}x^2$.

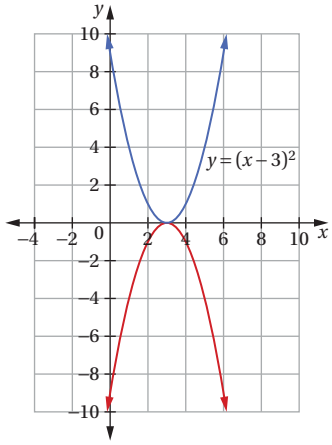
THINKING	WORKING
Determine the shape of the dilation.	$0 < a < 1$ represents a vertical compression (steeper or narrower) compared to the graph of $y = x^2$.
Describe the dilation.	The graph of $y = \frac{1}{10}x^2$ has a dilation factor of $\frac{1}{10}$ and is flatter than the graph of $y = x^2$.
Determine the coordinates of the turning point.	The coordinates of the turning point are (0, 0).

Sketch the graph.	

Worked example

Describing reflections of a parabola

Consider the graphs shown below.



Determine the equation of the reflected graph.

THINKING	WORKING
Describe the reflection.	The graphs have the same turning point and the shape of one is the shape of the other reflected in the x-axis.
Write the answer.	The equation for the second graph is $y = -(x - 3)^2$.

SC 3 I can analyse and interpret combined transformations

It is now time to look at graphs that have had the whole range of transformations covered so far applied to them.

To recap, two types of translation have been looked at:

- horizontal translation (translation in the direction of the x-axis), which leads to an equation of the form $y = (x - h)^2$, where h is the translation to the right.
- vertical translation (translation in the direction of the y-axis), which leads to an equation of the form $y = x^2 + k$, where k is the translation up.

Dilation by a factor of a leads to the equation $y = ax^2$.

Reflection in the x-axis gives the equation $y = -x^2$.

One of the general forms for a quadratic can, therefore, be written as $y = a(x - h)^2 + k$. This is usually referred to as 'turning point form' because it shows the turning point (h, k) directly.

Using this framework, it is important to understand that every quadratic equation can be described as a series of transformations from the standard quadratic $y = x^2$.

Worked example

Calculating the coordinates of the turning point in a transformed parabola

State the coordinates of the turning point on the graph representing the following equations.

(a) $y = (x - 1)^2 + 2$

THINKING	WORKING
Recall the turning point form.	$y = a(x - h)^2 + k$, where the coordinates of the turning point are (h, k).
Identify the values of h and k and use them to describe the translation that has taken place.	$h = 1$: a translation of 1 unit to the right. $k = 2$: a translation of 2 units up.
Write the answer.	The coordinates of the turning point are (1, 2).

(b) $y = \frac{(x+2)^2}{3} + 5$

THINKING	WORKING
Rewrite the equation in turning point form, $y = a(x - h)^2 + k$.	$y = \frac{(x+2)^2}{3} + 5$ $= \frac{1}{3}\left[x - (-2)\right]^2 + 5$
Identify the values of h and k and use them to describe the translations that have taken place from the graph of $y = x^2$.	$h = -2$: a translation of 2 units to the left. $k = 5$: a translation of 5 units up.
Write the answer.	The coordinates of the turning point are (–2, 5).

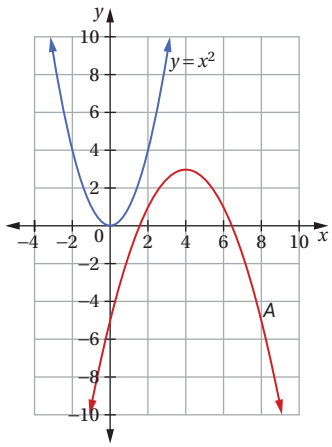
(c) $y = -\dfrac{(x+5)^2 - 14}{7}$.

THINKING	WORKING
Rewrite the equation in turning point form, $y = a(x - h)^2 + k$.	$y = -\dfrac{(x+5)^2 - 14}{7}$ $= -\dfrac{1}{7}(x+5)^2 - \dfrac{14}{7}$ $= -\dfrac{1}{7}[x-(-5)]^2 - 2$
Identify the values of h and k and use them to describe the translations that have taken place from the graph of $y = x^2$.	$h = -5$: a translation of 5 units to the left. $k = -2$: a translation of 2 units down.
Write the answer.	The coordinates of the turning point are $(-5, -2)$.

Worked example

Determining the equation for a transformed graph

Write the equation for the graph labelled on the following set of axes.



THINKING	WORKING
Recall the turning point form.	$y = a(x - h)^2 + k$
Identify the coordinates of the turning point (h, k) for the graph labelled A.	The coordinates of the turning point are (4, 3). The equation of the graph is in the form: $y = a(x - 4)^2 + 3$
Determine whether a reflection in the x-axis has taken place.	Reflection has occurred. $y = -a(x - 4)^2 + 3$

Determine the coordinates of a known point and solve for a.	The coordinates of a known point are (6, 1). $1 = -a(6-4)^2 + 3$ $1 = -a \times 2^2 + 3$ $1 - 3 = -4a$ $-2 = -4a$ $a = \frac{-2}{-4}$ $a = \frac{1}{2}$.
Write the answer.	$y = -\frac{1}{2}(x-4)^2 + 3$

Practice

ANSWERS Page 629

SC 1 I can analyse and interpret both horizontal and vertical translations

1 Describe the translation of each of the following equations from $y = x^2$.

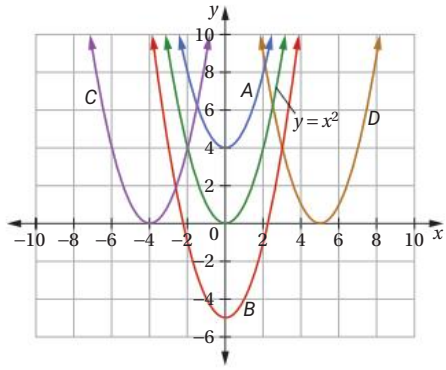
(a) $y = x^2 + 6$ **(b)** $y = x^2 - 3$ **(c)** $y = (x + 6)^2$ **(d)** $y = (x - 3)^2$

2 On the same set of axes, sketch the graphs for the following equations.

(a) $y = x^2$ **(b)** $y = x^2 + 6$ **(c)** $y = x^2 - 3$

(d) $y = (x + 6)^2$ **(e)** $y = (x - 3)^2$

3 State the equation for each of the graphs shown below. The graph of $y = x^2$ is provided for comparison.



4 For each of the following equations, state the coordinates of the turning point.

(a) $y = x^2 - 7$ **(b)** $y = x^2 - 5$ **(c)** $y = x^2 + 2$ **(d)** $y = x^2 + 6$

5 For each of the following equations, state the coordinates of the turning point.

(a) $y = (x - 7)^2$ **(b)** $y = (x - 5)^2$ **(c)** $y = (x + 2)^2$ **(d)** $y = (x + 6)^2$

SC 2 I can analyse and interpret dilations and determine reflections in the x-axis

1 Write the equation that represents the following vertical dilations applied to the graph of $y = x^2$.

(a) 2 **(b)** 5 **(c)** 10

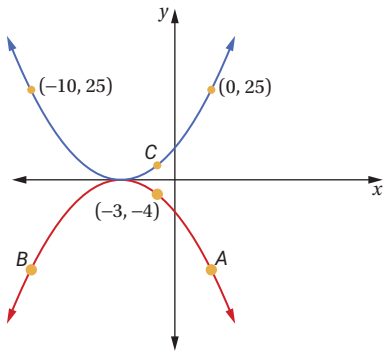
(d) $\frac{1}{2}$ **(e)** $\frac{1}{5}$ **(f)** $\frac{1}{10}$

2 Determine the equation of the reflected graph when the following are reflected through the x-axis.

(a) $y = x^2$ **(b)** $y = -x^2$ **(c)** $y = 2x^2$

(d) $y = \frac{1}{2}x^2$ **(e)** $y = -2x^2$ **(f)** $y = -\frac{1}{2}x^2$

3 The two graphs shown are the reflection of each other in the x-axis. State the coordinates for the points labelled A, B and C.



4 Consider the graphs of $y = x^2$, $y = \frac{1}{5}x^2$ and $y = 5x^2$.

(a) Complete the tables of values to show each of the following transformations.

x	−10	−5	0	5	10
$y = x^2$					
$y = \frac{1}{5}x^2$					
$y = 5x^2$					

(b) Sketch the graphs on the same Cartesian plane.

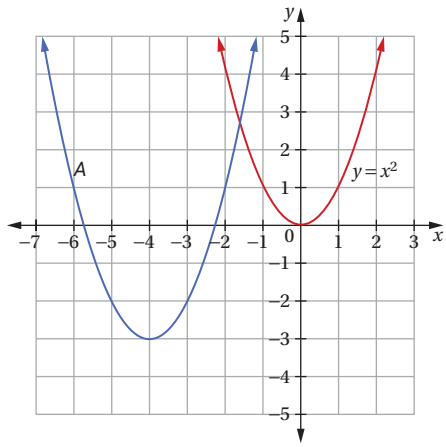
(c) Describe the transformation on each of the graphs when compared to the graph of $y = x^2$.

(d) Write the corresponding equations for the graphs when they are reflected in the x-axis.

(e) Sketch the graphs for the equations in part **(d)**.

SC 3 I can analyse and interpret combined transformations

1 Consider the following graph with an equation in the form $y = (x - h)^2 + k$.

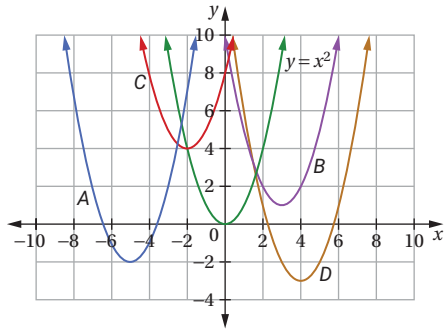


(a) Determine the coordinates of the turning point (h, k) for the graph labelled A.

(b) Complete the following sentence by filling in the blanks and choosing the correct words: Compared to the graph of $y = x^2$, the graph labelled A has been translated ___ units to the right/left and ___ units vertically up/down.

(c) State the equation for the graph labelled A.

2 Write the equation for each of the graphs shown below, in the form $y = (x - h)^2 + k$, except for $y = x^2$, which is provided for reference.



3 Determine the coordinates of the turning points for the graphs that represent each of the following equations.

(a) $y = (x - 3)^2$ **(b)** $y = 2(x - 3)^2$ **(c)** $y = \dfrac{(x-3)^2}{3}$ **(d)** $y = -(x - 3)^2$

4 State the equation that would represent the graph if the following transformations were applied to the graph of $y = x^2$.

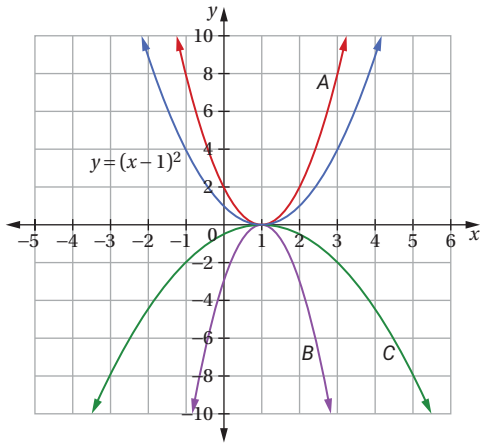
(a) A vertical dilation of factor 3 and a translation of 5 units in the positive direction of the x-axis.

(b) A reflection in the x-axis, followed by a translation of 6 units to the left and 3 units up.

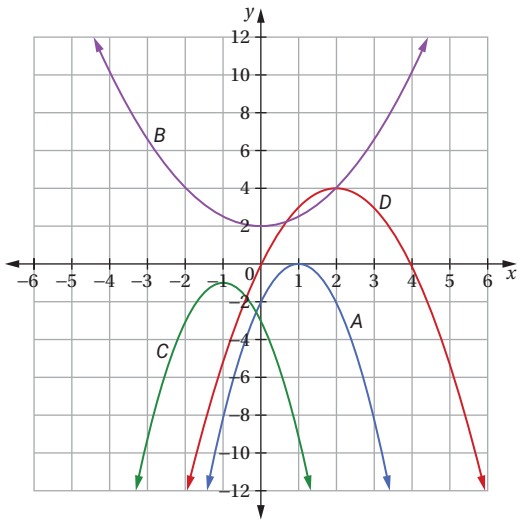
(c) A vertical dilation by a factor of $\frac{2}{3}$, a reflection in the x-axis, a translation of 2 units to the right and a translation of 7 units down.

(d) A vertical dilation by a factor of 7, a reflection in the x-axis, and a translation of 6 units up.

5 State the equations that represent each of the graphs labelled A–C on the following set of axes.



6 Write the equations for each of the graphs labelled A–D.



7 Complete the following on the same set of axes. (If possible, use technology to complete this question.)

(a) Draw the graph of $y = x^2$.

(b) Draw the graph that results from this graph being translated 2 units vertically up and label it A.

(c) Now reflect A in the x-axis and label it B.

(d) Draw the graph that represents $y = x^2$ reflected in the x-axis and label it C.

(e) Now translate C vertically up 2 units and label it D.

(f) What conclusion can you draw about graphs B and D?

8 Complete the following on the same set of axes. (If possible, use technology to complete this question.)

(a) Draw the graph of $y = x^2$.

(b) Draw the graph that results from this graph being translated 2 units to the right and label it A.

(c) Now reflect A in the x-axis and label it B.

(d) Draw the graph that represents $y = x^2$ reflected in the x-axis and label it C.

(e) Now translate C to the right 2 units and label it D.

(f) What conclusion can you draw about graphs B and D?

9 Angela was a bit confused about the order in which transformations should be described. Her teacher told her to think back to the order of operations. There, the first operation applied was anything in brackets. Angela's teacher said she could consider transformations in the same way.

(a) Looking at the turning point form of the quadratic $y = a(x - h)^2 + k$, which transformation is described first?

(b) What would come next? Remember, the value of a possibly involves two transformations.

(c) What transformation comes last?

(d) Compare this to what you found in Questions **7** and **8** above.

10 Describe the transformations applied to the graph of $y = x^2$ to produce graphs with the following equations. State the transformations in the order: dilation, reflection, translation (DRT).

(a) $y = (x - 4)^2 + 3$

(b) $y = -(x + 2)^2 + 1$

(c) $y = 4(x + 5)^2 - 2$

(d) $y = -\frac{2(x-1)^2}{5} + 1$

Solve quadratic equations graphically

5.5

Learning intention: To be able to solve quadratic equations

Success criteria:

SC 1 I can determine the number of solutions to a quadratic equation (zero, one or two).

SC 2 I can approximate solutions to quadratic equations from a graph.

Lesson warm-up

Puzzled pathways



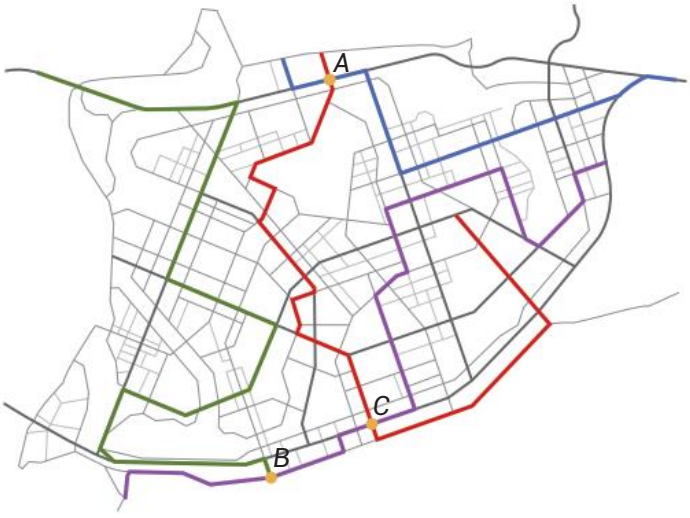
Four teams (Red, Blue, Green and Purple) have been tasked to solve a puzzle cooperatively.

Each team needs to solve riddles along their allocated routes, then share their information with all the other teams in order to complete the puzzle.

Each team can only exchange clues and solutions where their paths intersect with another team at points A, B and C.

Given that the Green team can only solve their riddle once they receive information from the Blue team, at which location will the puzzle finally be solved?

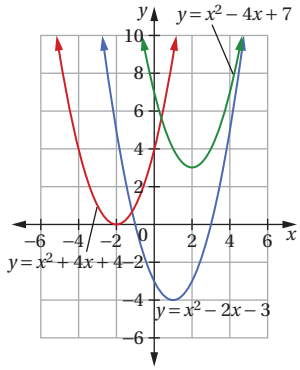
Give reasons for your answer, explaining how information has been shared by the four teams.



SC 1 I can determine the number of solutions to a quadratic equation (zero, one or two)

You can solve an equation in the form $ax^2 + bx + c = 0$ by determining the x-intercepts ($y = 0$) of the parabola with the equation $y = ax^2 + bx + c$.

Look at the following series of graphs.



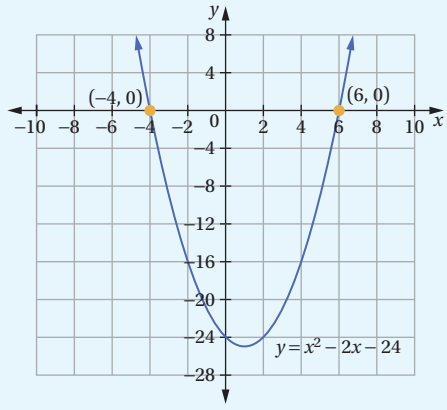
Consider the following additional information.

Equation	Factorised form	Solution ($y = 0$)	x-intercepts
$y = x^2 - 2x - 3$	$y = (x - 3)(x + 1)$	$0 = (x - 3)(x + 1)$ $x = -1$ or $x = 3$	Two x-intercepts $(-1, 0)$ and $(3, 0)$
$y = x^2 + 4x + 4$	$y = (x + 2)^2$	$0 = (x + 2)^2$ $x = -2$	One x-intercept $(-2, 0)$
$y = x^2 - 4x + 7$	-	-	No x-intercepts

Worked example

Using a graph to determine the number of solutions to a quadratic equation

Use technology to determine the number of solutions to $x^2 - 2x - 24 = 0$.

THINKING	WORKING
Use technology to draw the graph by letting the quadratic equation be equal to y.	Graph $y = x^2 - 2x - 24$ and determine the coordinates of the x-intercepts ($y = 0$). 

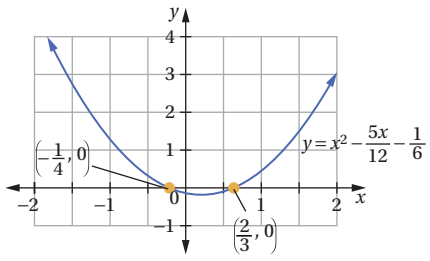
Identify the number of x-intercepts from the graph.	There are two x-intercepts.
Read off the values for the x-intercepts.	The coordinates of the x-intercepts are (–4, 0) and (6, 0).
Check that the values of the intercepts satisfy the equation when $y = 0$.	For $x = -4$: $(-4)^2 - 2(-4) - 24 = 16 + 8 - 24$ $= 24 - 24$ $= 0$ For $x = 6$: $(6)^2 - 2(6) - 24 = 36 - 12 - 24$ $= 24 - 24$ $= 0$
Write the answer.	There are two real solutions to the equation $0 = x^2 - 2x - 24$. The solutions are $x = -4$ or $x = 6$. Graphically, there are two x-intercepts of the equation $y = x^2 - 2x - 24$ at (–4, 0) and (6, 0).

SC 2 I can approximate solutions to quadratic equations from a graph

Not every quadratic equation that has x-intercepts crosses the x-axis at an integer value, so it is not easy to find the factors from the expanded form. However, a table of values can be used to sketch the graph, then the x-intercepts can be approximated from the graph. Of course, these values will also be the solutions to the quadratic equation.

Even better than using a table of values would be to use technology to produce the graph. Some forms of technology will even approximate the value of the x-intercept for you.

Consider the following graph.



The form of technology used here has given the values for the x-intercepts. However, these could have been approximated from the scale on the horizontal axis. (You might like to write the equation in factorised form and expand to prove to yourself that the stated equation is correct.)

Using the solutions, you can determine the linear factors:

$x = -\frac{1}{4}$ and $x = \frac{2}{3}$

$4x = -1$ $3x = 2$

$4x + 1 = 0$ $3x - 2 = 0$

$y = a(4x + 1)(3x - 2)$

$y = a(12x^2 - 5x - 2)$

To determine the dilation factor a, you need another point.

For example, given a y-intercept of $\left(0, -\frac{1}{6}\right)$,

$$-\frac{1}{6} = a\left[12\times(0)^2 - 5\times(0) - 2\right]$$

$$-\frac{1}{6} = -2a$$

$$a = \frac{1}{12}$$

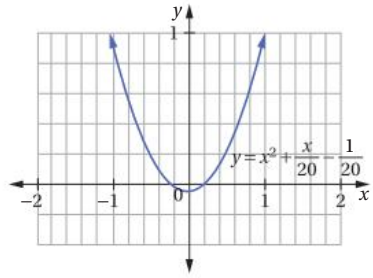
$$y = \frac{1}{12}\left(12x^2 - 5x - 2\right)$$

$$= x^2 - \frac{5}{12}x - \frac{1}{6}$$

Worked example

Approximating solutions to a quadratic equation from a graph

Use the following graph to approximate solutions for the quadratic equation $0 = x^2 + \frac{x}{20} - \frac{1}{20}$.



THINKING	WORKING
Identify the size of the small scale markings.	The space from 0 to 1 is divided into 5 equal pieces, so each scale mark represents $\frac{1}{5}$.
The right-hand intercept looks like it passes through one of the scale marks. State its value.	The right-hand intercept occurs at approximately $x = \frac{1}{5}$.
State the bounds for the left-hand intercept.	The left-hand intercept is bigger than $-\frac{2}{5}$, but smaller than $-\frac{1}{5}$ and is much closer to $-\frac{1}{5}$.
Write an inequality statement in decimal form for the left-hand intercept.	$-0.4 < x < -0.2$
Make an estimate for this intercept.	Because the value is close to −0.2, an estimate of −0.25 seems reasonable.
State the estimated solutions.	The two solutions are approximately $x = -0.25 = -\frac{1}{4}$ and $x = -\frac{1}{5}$.

Practice

ANSWERS Page 631

SC 1 I can determine the number of solutions to a quadratic equation (zero, one or two)

1 Use technology to draw the following graphs to determine the number of solutions, and state these solutions.

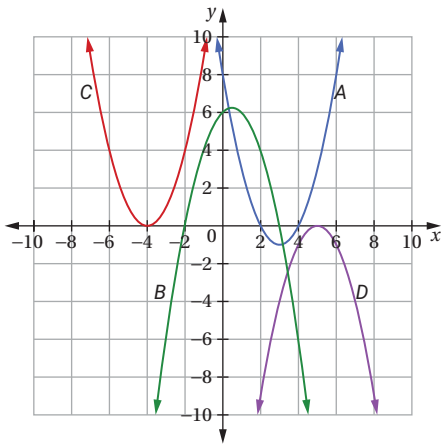
(a) $x^2 - 4x - 12 = 0$ **(b)** $x^2 - 10x + 25 = 0$

(c) $-x^2 - 6x - 9 = 0$ **(d)** $-x^2 + 4x - 8 = 0$

2 For each of the graphs labelled A–D on the set of axes below, state:

(i) the equation in factorised form as well as expanded form, $y = x^2 + bx + c$

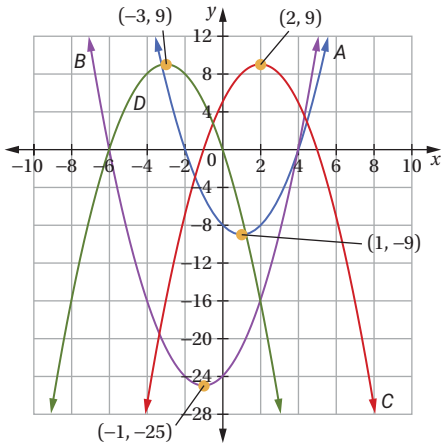
(ii) the number of x-intercepts **(iii)** the coordinates of the x-intercepts.



3 Each of the following graphs are translations of the graph $y = x^2$.

(i) State the equation in turning point form, $y = (x - h)^2 + k$ (or $y = -(x - h)^2 + k$).

(ii) Hence, determine the solution to the equations in the form $(x - h)^2 + k = 0$.

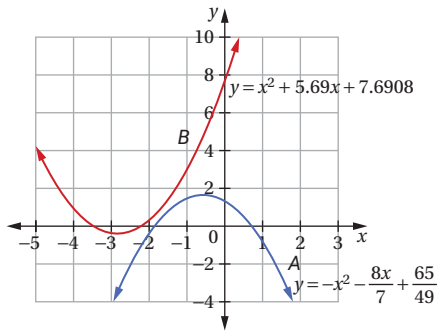


4 **(a)** A monic quadratic equation has two solutions and the turning point occurs at (5, −4). State the equation in expanded form and state the position of the two x-intercepts.

(b) A monic quadratic equation is reflected in the x-axis to give a maximum turning point at (4, 9). State the equation of the reflected graph and state the position of the x-intercepts.

SC 2 I can approximate solutions to quadratic equations from a graph

1 For each of the graphs on the axes below, approximate the solutions to 1 decimal place.



2 The table of values below is for the equation $y = x^2 + 3x - 5$.

x	−6	−4	−2	0	2	4
y	13	−1	−13	−5	5	23

(a) How can you use the table of values to approximate the x-intercepts?

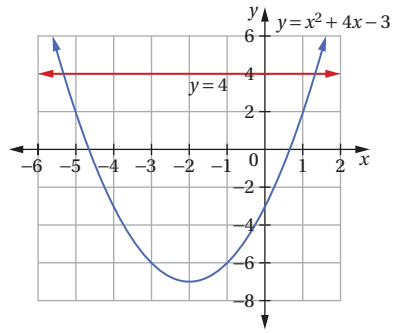
(b) State the pairs of numbers between which the x-intercepts occur.

(c) Do some additional calculations, using the equation, to find estimates correct to 1 decimal place for the x-intercepts.

(d) Will this quadratic have a maximum or a minimum turning point?

3 Complete the following steps to determine approximations for when $x^2 + 4x - 3 > 4$.

(a) The graph shows both $y = x^2 + 4x - 3$ and $y = 4$. Approximate the two points of intersection of the graphs, correct to 1 decimal place.



(b) Write two inequalities that satisfy $x^2 + 4x - 3 > 4$.

(c) Use technology to determine the values for these points of intersection. A different way to do this is to simplify the inequality from $x^2 + 4x - 3 > 4$ to $x^2 - 4x - 7 > 0$.

(d) Use technology to determine the two inequalities that satisfy this. What values do you need to look for here?

(e) Did the two methods give the same result?

4 Find the exact value for the second solution in each of the following cases.

(a) For $y = x^2 + \frac{5x}{2} - \frac{39}{16}$, the solutions are $x = \frac{3}{4}$ and $x =$ _____.

(b) For $y = -x^2 + \frac{9x}{5} + \frac{22}{25}$, the solutions are $x = -\frac{2}{5}$ and $x =$ _____.

(c) For $y = -x^2 + 7.66x - 9.0045$, the solutions are $x = 6.21$ and $x =$ _____.

(d) For $y = x^2 + 4.322x + 3.582072$, the solutions are $x = -3.204$ and $x =$ _____.

Model and solve practical problems with quadratic equations

5.6

Learning intention: To be able to model and solve practical problems with quadratic equations

Success criteria:

SC 1 I can use tables and graphs to represent practical applications of simple quadratic functions and interpret the results.

SC 2 I can determine the reasonableness of a solution.

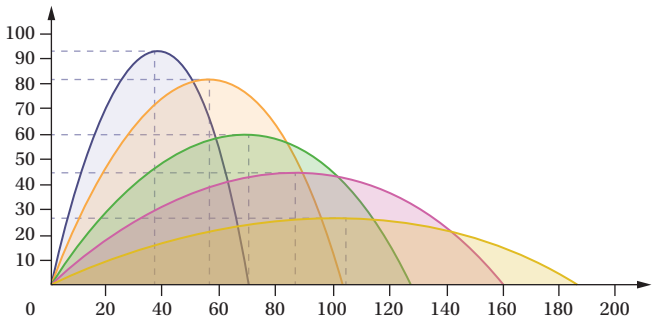
SC 3 I can use digital tools to represent practical applications of simple quadratic functions and interpret the results.

Lesson warm-up

Archery, Barcelona Olympics

When firing a projectile, like an arrow from a bow, the path of flight can take the form of a parabola.

Higher (steeper) angles of projection will lead to a higher flight of the arrow, while lower angles of projection can lead to further distances, for identical 'launch speeds'.



At the opening ceremony of the 1992 Barcelona Summer Olympic games, the Olympic cauldron was lit by an archer shooting a flaming arrow from a great distance.



Imagine the pressure on the archer (Antonio Rebollo) to get the arrow to fly right into the Olympic Cauldron, reaching just the right height, in front of millions of viewers.

Using the graph shown above as a guide, explain what would have happened if the angle of the arrow shot was too steep or too low.

SC 1 I can use tables and graphs to represent practical applications of simple quadratic functions and interpret the results

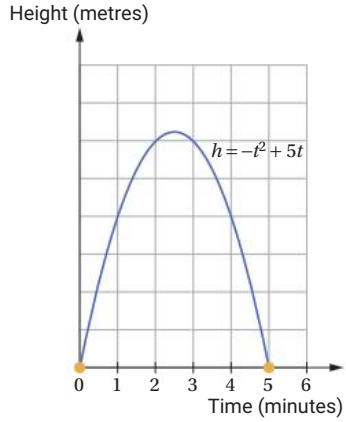
The skills covered in the previous sections can now be brought together in the solution of practical applications involving quadratic equations.

Worked example

Interpreting a quadratic graph

The following graph represents the height of a chair above the entry/exit platform on a Ferris wheel over a period of 5 minutes.

Determine the maximum height above the platform reached by the Ferris wheel.



THINKING	WORKING
Identify the two points where the chair is at the platform level.	The Ferris wheel is at platform level at the points (0, 0) and (5, 0).
Calculate the x-value of the turning point. It is midway between the two x-values.	$\frac{0+5}{2} = \frac{5}{2}$
Substitute this value into the equation.	For $t = \frac{5}{2}$, $h = -t^2 + 5t$ $= -\left(\frac{5}{2}\right)^2 + 5 \times \frac{5}{2}$ $= -\frac{25}{4} + \frac{25}{2}$ $= \frac{25}{4}$
Write the answer.	The maximum height above the platform reached by the Ferris wheel is $\frac{25}{4} = 6.25$ m.

SC 2 I can determine the reasonableness of a solution

There are several reasons why a solution may be deemed unreasonable.

- The solution may be given to a degree of accuracy that does not fit the situation.
- The solution may be mathematically correct, but impossible in the practical situation. For instance, a question using the area of a rectangle might produce negative side lengths.
- It may be that the solution doesn't make sense in the practical world.

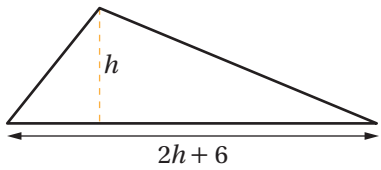
In this section, the focus will be on solutions that are impossible in the practical situation.

Worked example

Checking the reasonableness of a solution to a quadratic equation

A triangle has the following relationship between its height and base length.

Given that the area of the triangle is 130 m^2, determine its height and base length.



THINKING	WORKING
Recall the rule for the area of a triangle.	$A = \frac{1}{2} \times \text{base} \times \text{height}$
Identify the values given for this triangle.	$A = 130$ height $= h$ base $= 2h + 6$
Substitute the values into the rule and simplify.	$A = \frac{1}{2} \times (2h + 6) \times h$ $130 = (h + 3) \times h$ $130 = h^2 + 3h$ $h^2 + 3h - 130 = 0$
Factorise the expression on the left-hand side of the equation.	Multiply to give −130. Add to give 3. $h^2 + 13h - 10h - 130 = 0$ $h(h + 13) - 10(h + 13) = 0$ $(h + 13)(h - 10) = 0$
Use the Null Factor Law to determine the mathematical) solutions.	$h + 13 = 0$ $\quad$ $h - 10 = 0$ $h + 13 - 13 = 0 - 13$ $\quad$ $h - 10 + 10 = 0 + 10$ $h = -13$ $\quad$ $h = 10$ The solutions to the equation are $h = -13$ or $h = 10$.

Check the reasonableness of each solution.	Because h represents a measurement of distance, a negative value is not reasonable.
Check that the other solution gives a correct answer.	For $h = 10$, $2h + 6 = 2 \times 10 + 6$ $= 26$ $A = \frac{1}{2} \times 26 \times 10$ $= 13 \times 10$ $= 130$
Write the answer.	The height of the triangle is 10 m and its base length is 26 m.

SC 3 I can use digital tools to represent practical applications of simple quadratic functions and interpret the results

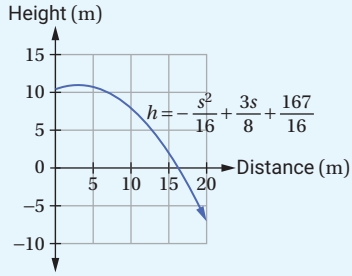
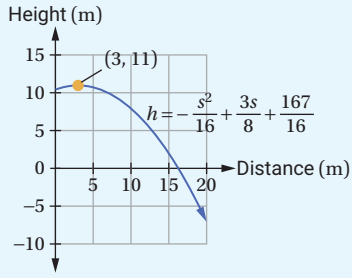
Using technology makes it possible to consider quadratic functions that are not as simple and neat as the ones looked at so far.

5

Worked example

Determining the turning point using technology

The path of a ball flicked off a balcony can be described by the equation $h = -\frac{s^2}{16} + \frac{3s}{8} + \frac{167}{16}$, where h is the height in metres above ground level and s is the horizontal distance in metres from the balcony. Determine the maximum height above ground level reached by the ball.

THINKING	WORKING
Use technology to draw the graph, using a domain starting at $s = 0$.	
Use the appropriate process for the technology used to identify the coordinates of the turning point.	
Interpret the value obtained in the context of the question.	The turning point is (3, 11), so the maximum height reached by the ball is 11 m above ground level.

Practice

ANSWERS Page 632

SC 1 I can use tables and graphs to represent practical applications of simple quadratic functions and interpret the results

1 The speed of a train was tracked by a drone over a period of 10 minutes. It was found that the speed followed the equation $s = t^2 - 10t + 100$, where s is the speed in km/h and t is the time in minutes.

(a) Complete the following table of values for this time period.

x	0	2	4	6	8	10
s						

(b) Over this time, will the graph have a maximum or a minimum turning point?

(c) Determine the coordinates of this turning point.

(d) State the coordinates of any x-intercepts.

(e) Sketch the graph for the 10-minute recording period.

2 The temperature in a deep freeze unit follows the rule $t = h^2 - 8h - 9$, where t is the temperature in °C and h is the number of hours after noon one day.

(a) What was the temperature in the unit at noon that day?

(b) Factorise the right-hand side of the equation and use it to sketch the graph.

(c) What was the minimum temperature reached, and when did it occur?

(d) The electricity supply failed at some stage in the afternoon. When do you think that occurred?

(e) When did the temperature reach 0°C?

3 The product of a positive integer and a number 14 more than the integer is 120.

(a) Write an equation to determine the value of the numbers.

(b) Explain whether the answer would be different if the question just said integer and not positive integer.

4 An arrow shot into the sky at a steep angle follows a path described by the equation $h = -x^2 + 20x$, where h is the height in metres above the ground and x is the horizontal distance from the point of fire.

(a) Write the equation in factorised form.

(b) What are the solutions to this equation?

(c) What was the maximum height reached by the arrow and how far was this from the point of fire?

(d) Use technology, if possible, to estimate when the arrow was above 40 m.

5 There are two positive whole numbers, a and b, $a < b$ such that the product of the sum of the numbers and their difference is equal to 25.

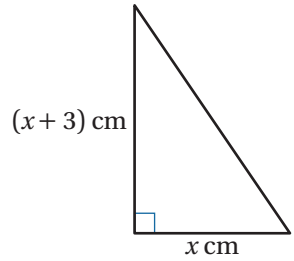
(a) Write a simplified expression for the product described.

(b) Using trial and error, if necessary, what are the values of a and b?

(c) If the condition that the numbers be positive was removed, what other solution is possible?

SC 2 I can determine the reasonableness of a solution

1 Consider the right-angled triangle shown in the following diagram, which has an area of 90 cm^2.



(a) Write an equation that can be solved to determine the value of x.

(b) Solve this equation and state the two mathematically correct solutions.

(c) State the only reasonable solution in this context.

(d) Use this solution and state the lengths of the two shorter sides of the triangle.

2 An equation has been found for the area of a rectangle, where x is the length, in centimetres, of the rectangle. The rule is $250 = x(x - 5)$.

(a) Express the rule in the form $x^2 + bx + c = 0$.

(b) Using technology, if possible, sketch the graph, showing the intercepts.

(c) State approximations, correct to 1 decimal place, for each of the mathematical solutions obtained.

(d) State whether both solutions are possible in the practical situation.

(e) Check that the solution gives an area approximately equal to 250 cm^2.

3 A rectangle has a length 3 cm longer than its width. Its perimeter has the same numerical value as its area.

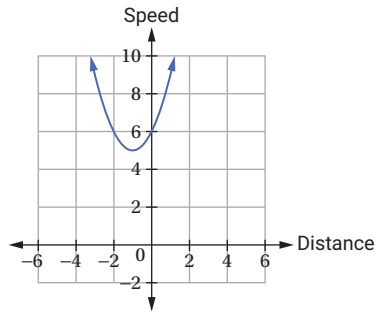
(a) Write an equation that can be solved to determine the value of x.

(b) Solve this equation and state the two mathematically correct solutions.

(c) State the only reasonable solution in this context.

(d) Use this solution and state the length and width of the rectangle.

4 Estelle was trying to determine the speed at which a bike would need to travel to meet certain distance conditions that had been specified. As part of her solution process, she drew the following graph.

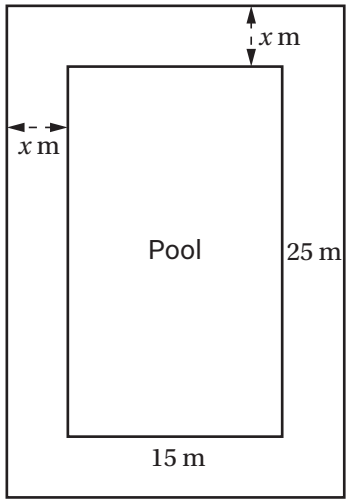


(a) How many mathematical solutions are possible from this graph?

(b) What would you say to Estelle?

SC 3 I can use digital tools to represent practical applications of simple quadratic functions and interpret the results

1 A swimming pool has the dimensions shown in the diagram below. Around the pool there is a paved area.



(a) Write an expression for the area of the outer rectangle. (This includes the pool.)

(b) What is the area of the pool?

(c) Write an expression for the area of the paved area only.

(d) Given that the paved area is 500 m^2, state the equation that needs to be solved to determine the width of the paved area.

(e) Draw the graph of this function (using technology).

(f) State the width of the paved area.

2 The sum of the squares of three consecutive whole numbers is 302.

(a) Given that x is the smallest of the three numbers, write expressions for each of the other two numbers.

(b) Now write an equation that can be solved to determine the value of x.

(c) Solve this equation, using technology to graph the equation if possible. State the sequence of three whole numbers.

(d) Would there be another answer if the question asked about consecutive integers? If so, what would it be?

3 Consider the equation $(4x - 2)^2 = 16$.

(a) Using technology, on the same set of axes plot $y = (4x - 2)^2$ and $y = 16$.

(b) How can you use this graph to find the solutions for x? What are the solutions?

(c) Now expand and simplify the original equation and write it in the form $y = ax^2 + bx + c$.

(d) If this graph was plotted, what feature(s) on the graph would give the solutions for x?

(e) Check that this graph gives the same solutions.

4 This question initially refers to the equation $y = x^2 - 2x + 5$.

(a) Use technology to draw the graph.

(b) How many x-intercepts does it have?

(c) What is the turning point of this graph?

(d) How many units do you need to translate this graph vertically up or down so that it has exactly one solution?

(e) What is this solution?

(f) What is the equation of this translated graph?

TOPIC REVIEW

Multiple choice

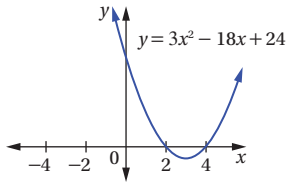
ANSWERS Page 665

1 The following table of values represents a quadratic relationship. What are the missing values?

x	−2	−1	0	1	2	3	4
y	7	−2	−5	−2	7	22	A
First difference		−9	−3	3	B	15	21
Second difference			6	6	6	6	6

A $A = 6; B = 6$ **B** $A = 43; B = 9$ **C** $A = 28; B = -6$ **D** $A = 9; B = 43$

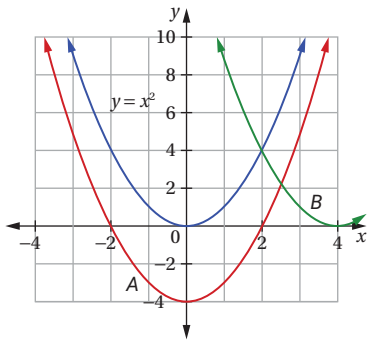
2 The graph shows $y = 3x^2 - 18x + 24$.



The turning point occurs at:

A (0, 24) **B** (3, 0) **C** (3, −3) **D** (3.5, −3)

Use the following set of axes to answer Questions **3** and **4**.



3 Which of the following represents graph A on the axes?

A $y = x^2 + 4$ **B** $y = x^2 - 4$ **C** $y = (x - 4)^2$ **D** $y = (x + 4)^2$

4 Which of the following represents graph B on the axes?

A $y = x^2 + 4$ **B** $y = x^2 - 4$ **C** $y = (x - 4)^2$ **D** $y = (x + 4)^2$

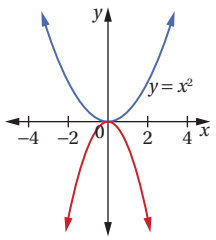
5 The equation for the unlabeled parabola could be:

A $y = -3x^2$

B $y = -\dfrac{x^2}{3}$

C $y = x^2 - 3$

D $y = (x-3)^2$



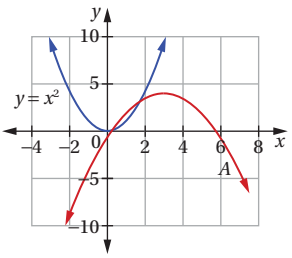
6 Which of the following is the equation for the graph labelled A on the following set of axes?

A $y = -2(x-3)^2 + 4$

B $y = -2(x+3)^2 + 4$

C $y = -\dfrac{(x+3)^2}{2} + 4$

D $y = -\dfrac{(x-3)^2}{2} + 4$



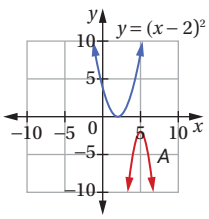
7 Which of the following is the equation for the graph labelled A on the following set of axes?

A $y = -3(x-2)^2 - 2$

B $y = -3(x-5)^2 - 2$

C $y = -\dfrac{(x-5)^2}{3} - 2$

D $y = -\dfrac{(x+5)^2}{3} - 2$



8 Determine the solution for $y = -2(x-3)(x+4)$ when $y = 0$.

A $x = 6$ or $x = -8$ **B** $x = 3$ or $x = -4$ **C** $x = -3$ or $x = 4$ **D** $x = -6$ or $x = 8$

9 Determine the solution for $y = (2x-5)(3x+2)$ when $y = 0$.

A $x = \dfrac{5}{2}$ or $x = -\dfrac{2}{3}$ **B** $x = \dfrac{2}{5}$ or $x = -\dfrac{3}{2}$ **C** $x = -\dfrac{5}{2}$ or $x = \dfrac{2}{3}$ **D** $x = -\dfrac{2}{5}$ or $x = \dfrac{3}{2}$

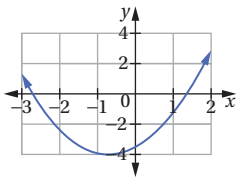
10 For the graph shown below, the solutions to $y = 0$ are closest to which pair of values?

A $x = -2.35$ or $x = 1.5$

B $x = -2.8$ or $x = 1.4$

C $x = -2.7$ or $x = 1.3$

D $x = -2.8$ or $x = 1.2$

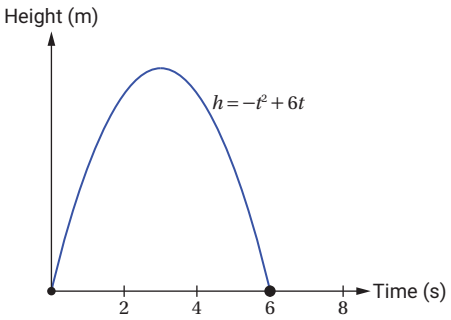


11 An isosceles right-angled triangle has its equal sides of length $(x-3)$ cm. The area of the triangle is 50 cm^2. The value of x is:

A 7 or 13 **B** 7 **C** 13

D Unable to be determined from the information given.

12 Part of a rollercoaster ride follows the equation $h = -t^2 + 6t$, where h is the height in metres and t is the time in seconds. This part of the ride is shown on the graph below.



The maximum height reached by the rollercoaster during this time is:

A 3 m **B** 6 m

C 9 m **D** 27 m

13 The following table shows some of the points on a graph representing a large open stormwater drain. The equation is $h = \frac{1}{12}(x^2 - 13x)$, where x is the distance in metres from the left-hand side of the drain and h is the height in metres relative to ground level.

x	0	1	2	11	12	13
h	0	-1	$-1\frac{5}{6}$	$-1\frac{5}{6}$	-1	0

The maximum depth of the drain is:

A $1\frac{5}{6}$ m **B** $3\frac{25}{48}$ m **C** $6\frac{1}{2}$ m **D** 7 m

Use the following information to answer Questions **14** and **15**.

An item falls from a high ledge. The height h of the item in metres, t seconds after falling can be represented by the relationship $h = 50 - 4.9t^2$.

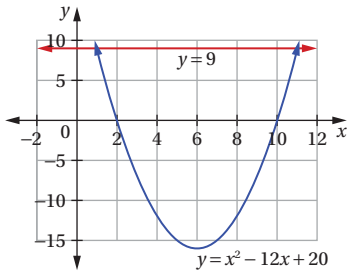
14 The height of the ledge is:

A 4.9 m **B** 45.1 m **C** 50 m **D** 54.9 m

15 The item takes how many seconds to reach the ground?

A 3.2 s **B** 4.9 s **C** 10.2 s **D** 50 s

Use the following pair of graphs to answer Questions **16** and **17**.



16 The graph is used to show that an equation has solutions $x = 1$ or $x = 11$. What is that equation?

A $y = 9$ **B** $x^2 - 12x + 20 = 0$

C $x^2 - 12x + 29 = 0$ **D** $x^2 - 12x + 11 = 0$

17 Which of the following shows when $x^2 - 12x + 20 < 9$?

A $x < 1 \cup x > 11$ **B** $x \leq 1 \cup x \geq 11$

C $1 < x < 11$ **D** $1 \leq x \leq 11$

Short answer

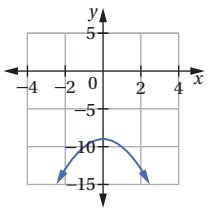
1 For each of the following quadratics, complete the table of values and find the first and second differences to confirm that each is a quadratic relationship.

x	−2	−1	0	1	2	3	4
y							
First difference							
Second difference							

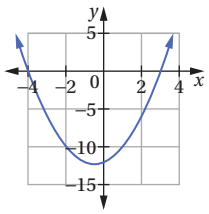
(a) $y = 3(x - 1)^2 + 2$ **(b)** $y = -2x^2 + 3x - 4$

2 For each of the following quadratic graphs, state the coordinates of the y-intercept and the x-intercepts (if they exist). Also, state the coordinates of the turning point and whether this represents a maximum or minimum value.

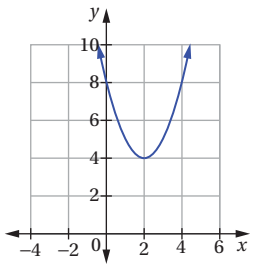
(a) $y = -x^2 - 9$



(b) $y = x^2 + x - 12$



(c) $y = (x - 2)^2 + 4$



3 Use the symmetry of a parabola to complete the following tables of values.

(a)

x	0	1	2	3	4	5	6
y		3	−3	−5	−3		13

(b)

x	−4	−3	−2	−1	0	1	2
y		−5	3	6		−5	−21

4 On the same set of axes, sketch the graphs for the following equations.

(a) $y = x^2$ **(b)** $y = x^2 + 2$ **(c)** $y = x^2 - 3$

(d) $y = (x - 2)^2$ **(e)** $y = (x + 3)^2$

5 For each of the following equations, state the coordinates of the turning point.

(a) $y = (x + 4)^2$ **(b)** $y = -x^2 + 2$

(c) $y = (x - 2)^2$ **(d)** $y = -x^2 - 3$

6 State the equation that would represent the graph if the following transformations were applied to the graph of $y = x^2$.

(a) Dilated vertically by a factor of $\frac{1}{2}$ and reflected in the x-axis

(b) Dilated vertically by a factor of 2 and translated 3 units vertically up

7 State the y-intercept, as a value, for the graph of each of the following equations.

(a) $y = (x - 3)^2$

(b) $y = 2(x - 3)^2$

(c) $y = -3(x - 3)^2$

(d) $y = \frac{1}{2}(x - 3)^2$

8 State the equation that would represent the graph if the following transformations were applied to the graph of $y = x^2$.

(a) A vertical dilation factor of $\frac{1}{2}$, a translation 4 units in the negative direction of the x-axis and a translation 3 units vertically up

(b) A vertical dilation factor of 2, a reflection in the x-axis and a translation 6 units vertically down

9 Find the solution for each of the following equations when $y = 0$.

(a) $y = x^2 - 64$

(b) $y = x^2 + 25$

(c) $y = x^2 - 12x + 36$

(d) $y = -x^2 + 2x + 8$

10 Use technology to draw the graphs for the following equations and state the solutions when $y = 0$.

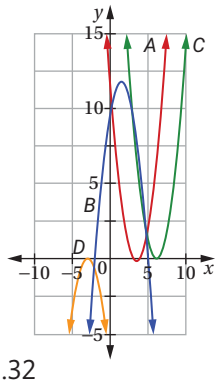
(a) $y = x^2 - 5x + \frac{25}{4}$

(b) $y = x^2 + \frac{x}{5} - \frac{6}{25}$

(c) $y = x^2 - \frac{9}{16}$

(d) $y = -x^2 - 6x - \frac{65}{9}$

11 For each of the graphs labelled A to D on the set of axes below, state the equation in factorised form as well as expanded form, $y = x^2 + bx + c$.



12 For the following turning points, determine the quadratic equation in expanded form and factorised form, and state the values of the x-intercepts. Both quadratics have two x-intercepts and a coefficient of x^2 of 1 or −1.

(a) (1, 36)

(b) (2, −1)

13 For each of the following equations, determine the second exact solution, given the first exact solution.

(a) $x^2 + 0.17x - 1.775 = 0$; $x = 1.25$

(b) $x^2 + 3.66x + 3.0888 = 0$; $x = -1.32$

14 A rectangle has side lengths $(x - 3)$ cm and $(x + 2)$ cm. Its area is 24 cm^2.

(a) Write an equation that can be solved to determine the value of x.

(b) Solve the equation.

(c) What are the dimensions of the rectangle?

15 The temperature in a freezer unit follows the rule $t = h^2 - 7h - 30$, where t is the temperature in °C and h is the number of hours after noon one day.

(a) What was the temperature at noon?

(b) Factorise the right-hand side of the rule.

(c) When does the freezer reach 0°C?

(d) What was the minimum temperature reached, and when was it reached?

16 The speed of a car was found to follow the relationship $s = -5t^2 + 38t$, where s is the speed in km/h and t is the time in minutes after starting a journey, until it stops. Use technology in the solution of this question.

(a) What was the maximum speed reached and when did this occur?

(b) When did the car stop?

(c) Correct to 1 decimal place, between what times was the car travelling at greater than 50 km/h?

Extended response

1 This question refers to the quadratic equations and their graphs.

(a) Expand the quadratic expressions in each of the following.

(i) $y = (x - 2)(x + 4)$ **(ii)** $y = (x + 3)(x + 5)$ **(iii)** $y = (x - 6)(x - 2)$

(b) State the coordinates of the y-intercept in each graph of the equations in **(a)**.

(c) Expand the quadratic expression in $y = (x - a)(x - b)$.

(d) What will be the coordinates of the y-intercept in the graph of the equation in **(c)**?

(e) What will be the coordinates of the turning point of the graph for $y = (x - a)(x - b)$?

(f) Check your solution to **(e)** is correct by determining the turning point for the graph of $y = (x - 2)(x - 6)$ and checking by substituting into the expressions in **(e)**.

2 A quadratic graph is the graph of $y = x^2$ translated so that the new turning point is $(-3,4)$.

(a) Write the equation for this quadratic graph.

(b) The graph in **(a)** is reflected in the x-axis. Write its new equation.

(c) The graph in **(a)** is vertically dilated by a factor of 2. Write its new equation.

(d) The graph in **(a)** is translated 5 units to the right. Write its new equation.

(e) The graph in **(a)** is translated 6 units vertically up. Write its new equation.

(f) If all the transformations described in **(b)** to **(e)** were applied in sequence, starting with the graph in **(a)**, what would be the equation of the resulting graph?

(g) Sketch the graphs of the equations from **(a)** and **(f)** on the same set of axes.

3 The temperature in a cool room is measured over a period of time after 8 am one day. The temperature follows the rule $T = -0.75h^2 + 5h - 2$, where T is measured in °C and h is measured in hours.

(a) Use technology to sketch the graph and use its features to answer the following questions.

(b) What was the temperature at 8 am?

(c) What was the maximum temperature, correct to 1 decimal point, reached in the room, and when did it occur?

(d) Correct to the nearest minute, between which times was the temperature above 0°C?

(e) People working in the room need to leave the room if the temperature falls below −5°C. To the nearest minute, when does this first occur?

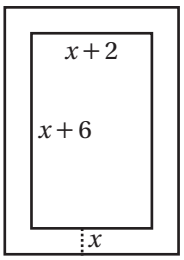
4 A swimming pool is surrounded by a paved area of constant width, as shown in the diagram below, where all measurements are in metres.

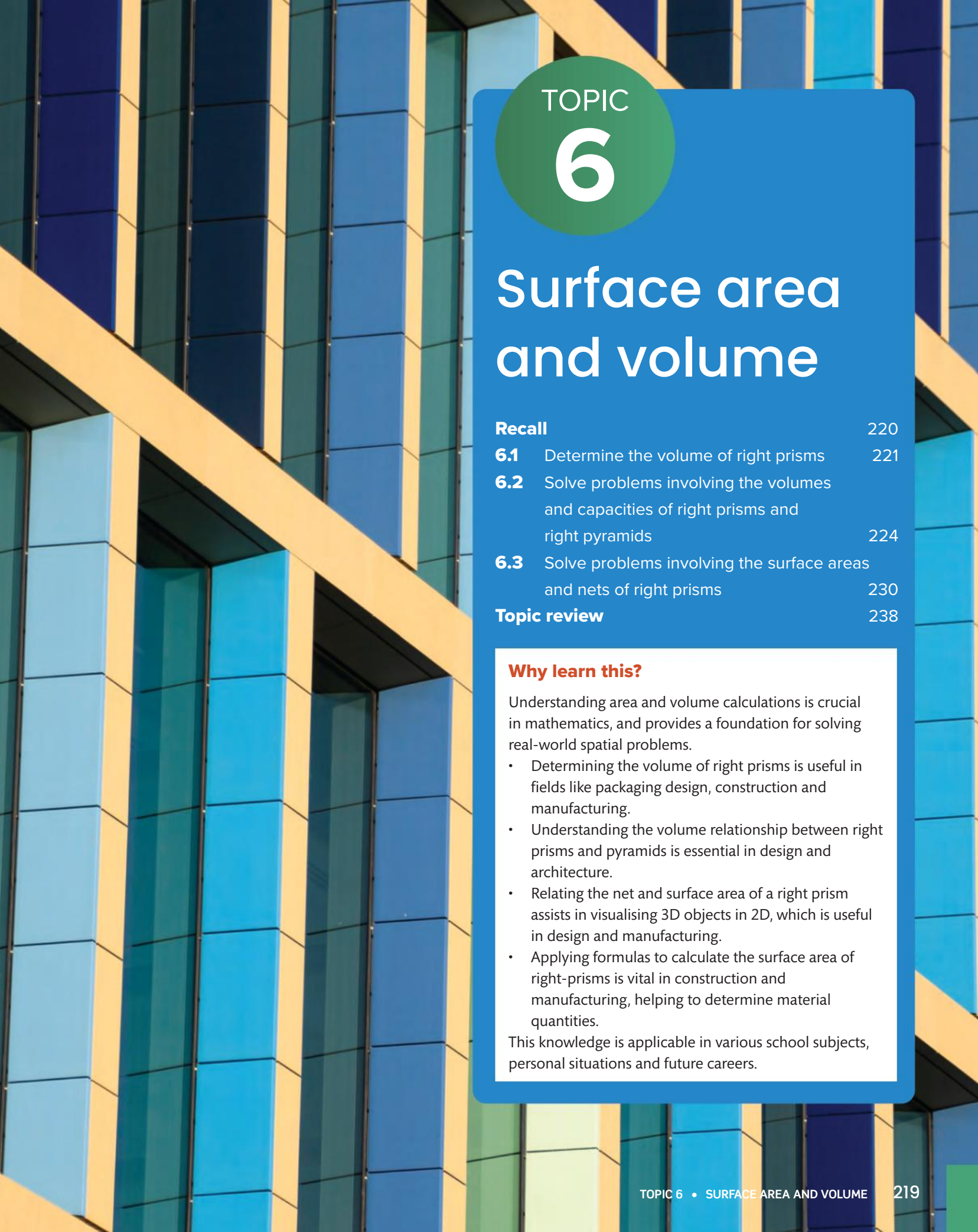
(a) Write an expression for the area of the pool in square metres.

(b) Write an expression for the combined area of the pool and paved area, in square metres.

(c) Write an expression for the paved area in square metres.

(d) If the paved area has an area of 42 m^2, determine the dimensions of the pool.





TOPIC
6

Surface area and volume

Recall 220

6.1 Determine the volume of right prisms 221

6.2 Solve problems involving the volumes and capacities of right prisms and right pyramids 224

6.3 Solve problems involving the surface areas and nets of right prisms 230

Topic review 238

Why learn this?

Understanding area and volume calculations is crucial in mathematics, and provides a foundation for solving real-world spatial problems.

- Determining the volume of right prisms is useful in fields like packaging design, construction and manufacturing.
- Understanding the volume relationship between right prisms and pyramids is essential in design and architecture.
- Relating the net and surface area of a right prism assists in visualising 3D objects in 2D, which is useful in design and manufacturing.
- Applying formulas to calculate the surface area of right-prisms is vital in construction and manufacturing, helping to determine material quantities.

This knowledge is applicable in various school subjects, personal situations and future careers.

RECALL

ANSWERS Page 634

I can multiply and divide decimals by powers of 10

1 Write the value of the following multiplications.

(a) 81.4 × 10 **(b)** 816.43 × 1000 **(c)** 0.0934 × 10 000 **(d)** 0.0074 × 100

2 Write the value of the following divisions.

(a) 6173.2 ÷ 1000 **(b)** 6.154 ÷ 100 **(c)** 693.5 ÷ 1000 **(d)** 0.0024 ÷ 10

I can convert between units of length

1 Complete each of the following conversions to the units given.

(a) 5000 m to km **(b)** 2.04 cm to mm **(c)** 65 cm to m **(d)** 90.5 mm to cm

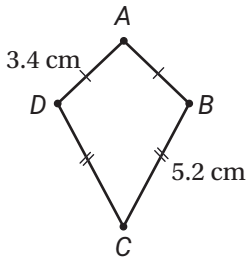
2 Complete each of the following conversions to the units given.

(a) 6000 cm to km **(b)** 4 mm to m **(c)** 0.057 m to mm **(d)** 0.25 km to cm

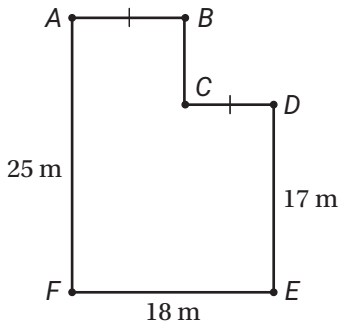
I can find the unknown length of a plane shape

1 Determine the length of the side/s indicated in each of these shapes:

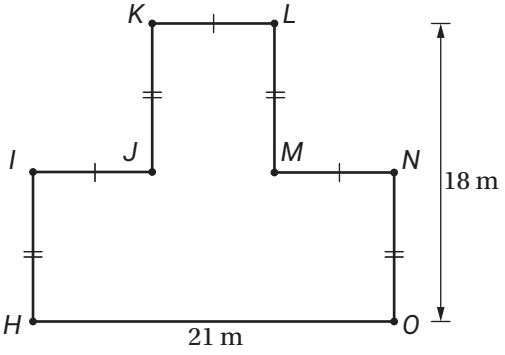
(a) Side AB



(b) Sides AB and BC



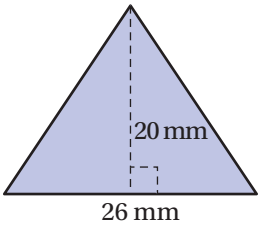
(c) Sides KL and LM



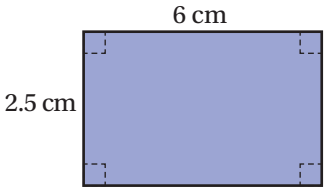
I can calculate the area of plane shapes

1 Calculate the area of the following shapes.

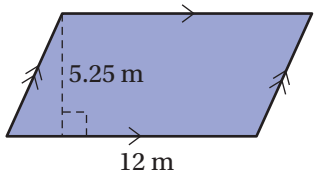
(a)



(b)



(c)



Determine the volume of right prisms

6.1

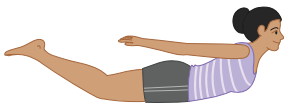
Learning intention: To be able to determine the volume of right prisms

Success criteria:

SC 1 I can determine the volume of right prisms.

Lesson warm-up

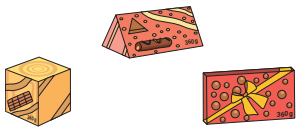
Chocolate boxes



Three chocolate boxes, each containing 360 g of chocolates, are sold in prism packaging.

Which packaging is the biggest?

Which packaging is the best?



SC 1 I can determine the volume of right prisms

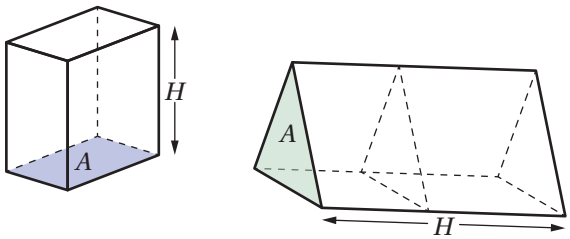
The volume of a solid is the amount of space occupied by the object.

Volume is usually measured in cubic centimetres (cm^3) or cubic metres (m^3) and sometimes cubic millimetres (mm^3) or cubic kilometres (km^3).

Remember that linear conversion factors must be cubed to become volume conversion factors. For example, because 100 cm = 1 m, this means $(100\,cm)^3$ or $1\,000\,000\,cm^3 = 1\,m^3$.

Prisms are three-dimensional objects with the uniform cross-section of a polygonal shape (triangle, quadrilateral, etc.).

Right prisms are prisms in which the joining edges are at right-angles to the polygon-shaped bases.



The formula for the volume of any prism is $V = AH$, where

A = area of base

H = perpendicular height of prism.

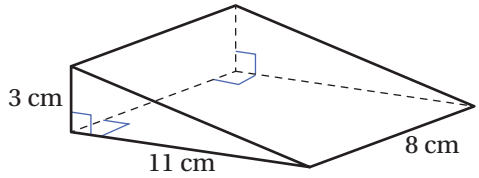
- If it is a right prism, the perpendicular height is the length of one of the edges.
- If it is a prism, but not a right prism, the formula still applies, but you need to find the perpendicular height first.

This formula for the volume of a prism actually applies to any shape with a uniform cross-sectional base (for instance a cylinder).

Worked example

Determining the volume of a right prism

Determine the volume of the right prism.



THINKING	WORKING
Write the formula for the volume of any prism.	$V = AH$, where $H = 8$ cm
Identify the shape of the base of the prism and write the formula for its area.	Triangular base: $A = \frac{bh}{2}$, where $b = 11$ cm, $h = 3$ cm
Substitute the area formula into the volume formula.	$V = \frac{bh}{2} \times H$
Substitute in the dimensions and calculate the volume, attaching volume units to the answer.	$V = \frac{11 \times 3}{2} \times 8$ $= 132 \text{ cm}^3$
Write the answer.	The volume of the triangular prism is 132 cm^3.

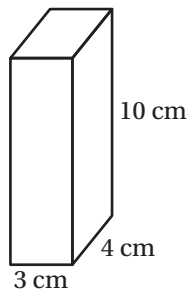
Practice

ANSWERS Page 634

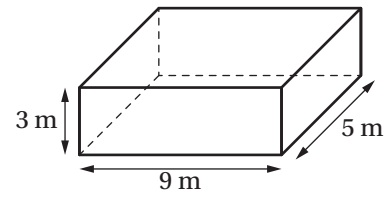
SC 1 I can determine the volume of right prisms

1 Determine the volume of the rectangular prisms.

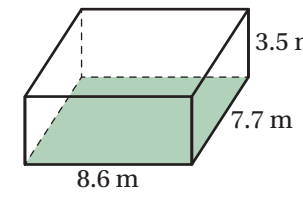
(a)



(b)

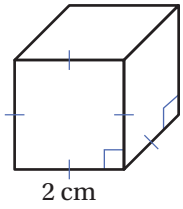


(c)

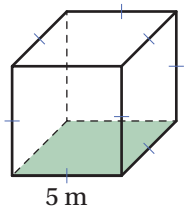


2 Determine the volume of the cubes.

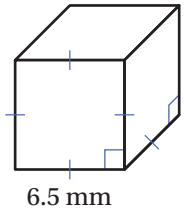
(a)



(b)

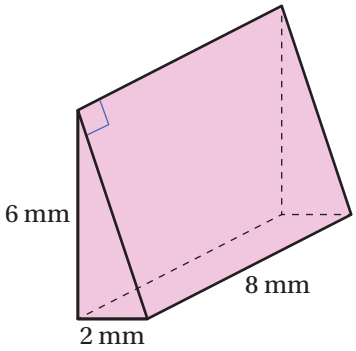


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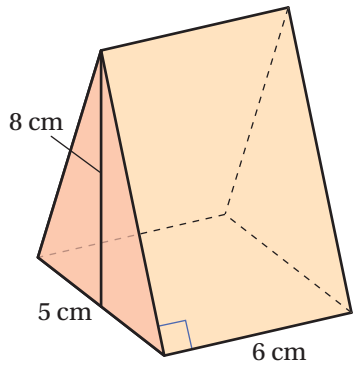


3 Determine the volume of the triangular right prisms.

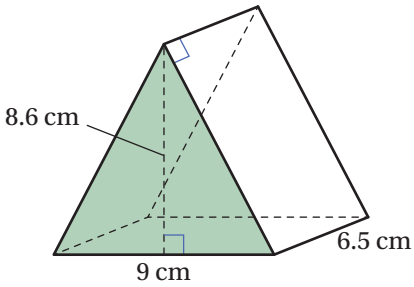
(a)



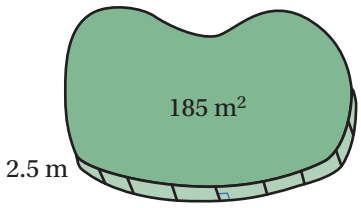
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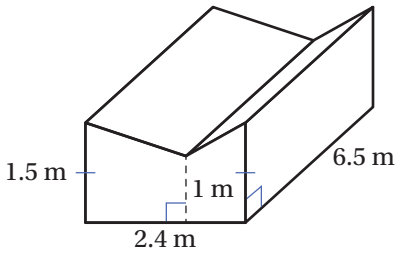
(c)



4 Determine the volume of the solid whose cross-sectional area and height are given.



5 Determine the volume of the pentagon-based right prism.



6.2 Solve problems involving the volumes and capacities of right prisms and right pyramids

Learning intention: To be able to solve problems involving the volumes and capacities of right prisms and right pyramids

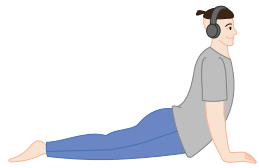
Success criteria:

SC 1 I can explain the relationships between the volumes of right prisms and pyramids.

SC 2 I can solve a variety of problems involving the volume and capacity of prisms and pyramids.

Lesson warm-up

Fill it up



- Explore the relationship between the capacity of a prism and a pyramid with the same base area and height.



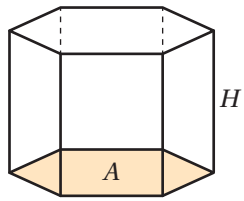
- Find objects with identical bases where one is a prism (or cylinder) and the other is a pyramid (or cone).
- Fill the pyramid with water (or rice, or small pasta).
- Pour the water, rice or pasta from the pyramid into its matching prism.
- Report on your findings.

SC 1 I can explain the relationships between the volumes of right prisms and pyramids

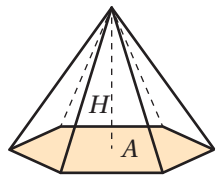
A pyramid takes up one-third of the space of a right prism with an identical base and perpendicular height.

The volume formula for any prism is $V = AH$.

Hence the volume formula for any pyramid is $V = \frac{1}{3}AH$.



Prism: $V = AH$

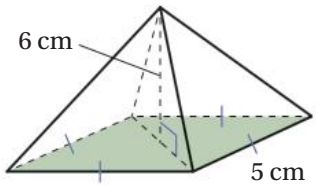


Pyramid: $V = \frac{1}{3}AH$

Worked example

Determining the volume of a pyramid

Determine the volume of the square-based pyramid.



THINKING	WORKING
Recall the formula for the volume of a pyramid.	$V = \frac{1}{3}AH$, where $H = 6$ cm
Identify the shape of the base of the pyramid and recall the formula for its area.	Square base: $A = l^2$, where $l = 5$ cm
Substitute the area formula into the volume formula.	$V = \frac{1}{3}l^2H$
Substitute in the dimensions and calculate the volume, attaching volume units to the answer.	$V = \frac{1}{3} \times 5^2 \times 6$ $= 50 \text{ cm}^3$
Write the answer.	The volume of the square-based pyramid is 50 cm^3.

SC 2 I can solve a variety of problems involving the volume and capacity of prisms and pyramids

Volume and capacity are closely related because the capacity of a container is the same as the volume of material the container can hold.

Determining capacity from the dimensions of an object (assumed to be the internal dimensions) involves calculating volume, then converting the result to suitable units of capacity.

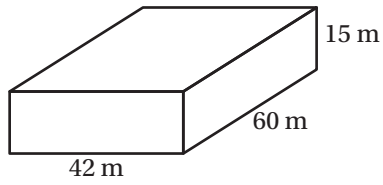
Volume	Capacity
1 cm^3	1 mL
1000 cm^3	1 L
1 m^3	1 kL

Also: 1000 mL = 1 L, 1000 L = 1 kL (1 kilolitre) and 1000 kL = 1 ML (1 megalitre).

Worked example

Calculating capacity

Determine the capacity of the rectangular-prism water reservoir in megalitres.



THINKING	WORKING
Calculate the volume.	$V = lwh$ $= 42 \times 60 \times 15$ $= 37\,800\,\text{m}^3$
Convert to capacity units. $1\,\text{m}^3 = 1\,\text{kL}$	$37\,800\,\text{m}^3 = 37\,800\,\text{kL}$
Convert to the required capacity units.	$37\,800\,\text{kL} = 37.8\,\text{ML}$
Write the answer.	The capacity of the reservoir is 37.8 ML.

6

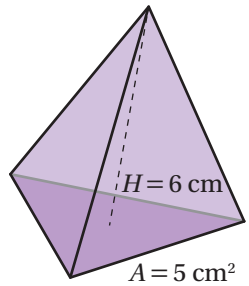
Practice

ANSWERS Page 634

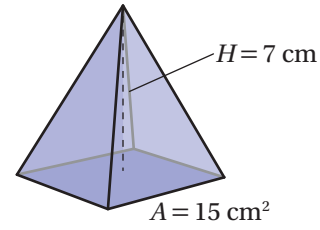
SC 1 I can explain the relationships between the volumes of right prisms and pyramids

1 Determine the volume of each of the pyramids. Round your answers to 2 decimal places where required.

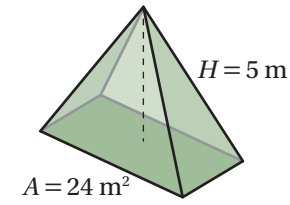
(a)



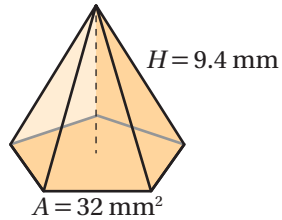
(b)



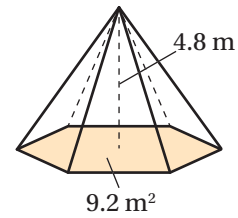
(c)



(d)

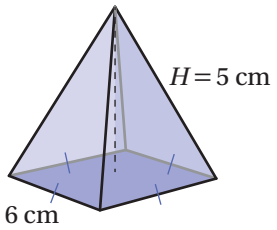


(e)

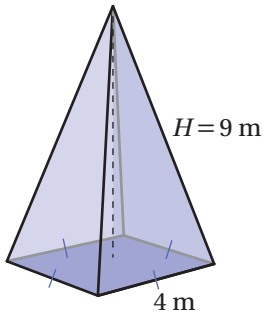


2 Determine the volume of the square-based pyramids. Round your answers to 2 decimal places where required.

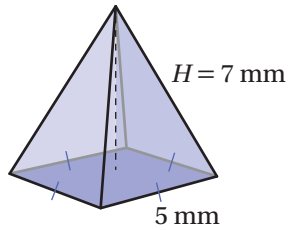
(a)



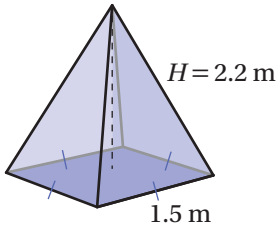
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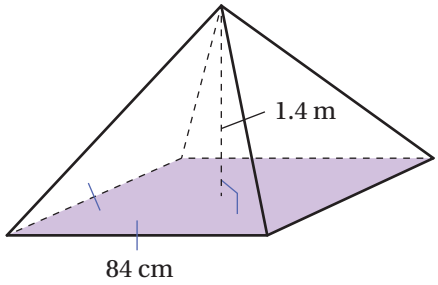
(c)



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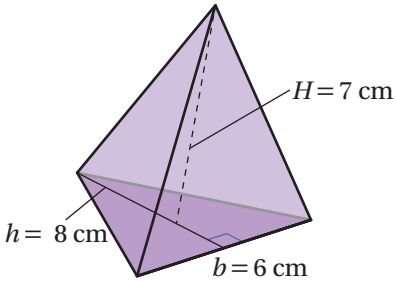


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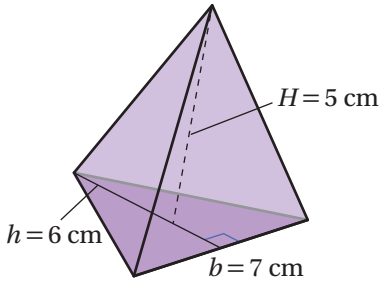


3 Determine the volume of the triangle-based pyramids. Round your answers to 2 decimal places where required.

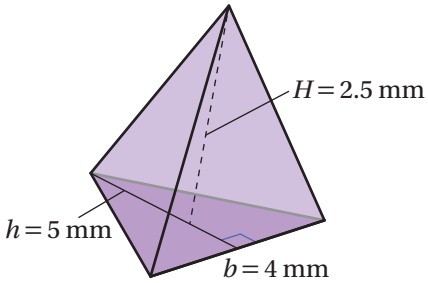
(a)



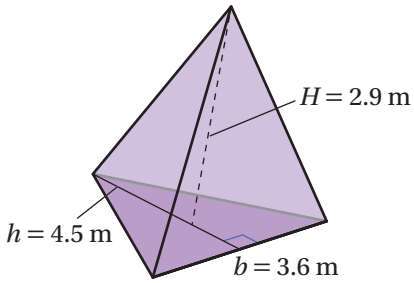
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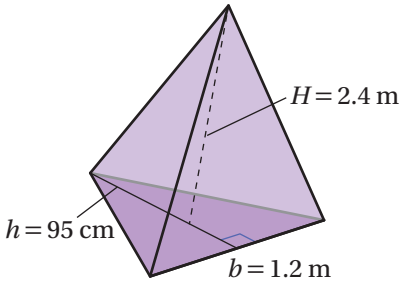
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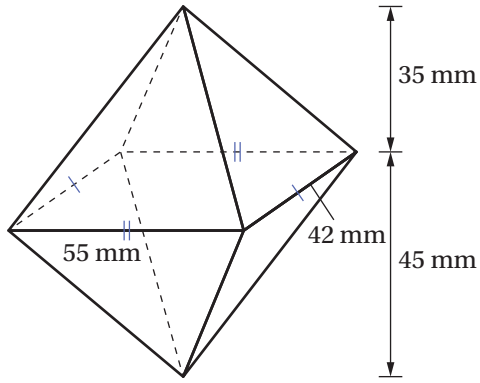
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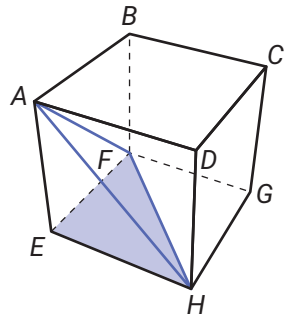
(e)



4 Determine the volume of the octahedron made of two rectangular pyramids with a shared base.



5 Consider the cube $ABCDEFGH$ of edge length 15 cm.



(a) Determine the volume of the cube.

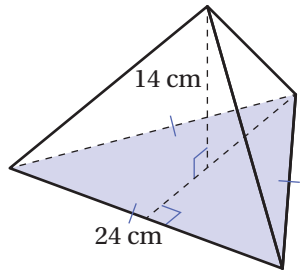
(b) A cut is made from the vertex A down to the baseline FH such that a small portion of the cube is removed. Identify the solid formed by the removed portion, including its dimensions.

(c) Determine the volume of the removed portion.

(d) Compare the volume of the removed portion to the volume of the original cube.

(e) Describe a prism that has three times the volume of the removed portion.

6 Determine the volume of the equilateral-triangle-based pyramid by following the steps below.



(a) Use Pythagoras' theorem to calculate the height of the triangular base, correct to 2 decimal places.

(b) Use the rounded answer from **(a)** to calculate the area of the triangular base.

(c) Calculate the volume of the pyramid.

SC 2 I can solve a variety of problems involving the volume and capacity of prisms and pyramids

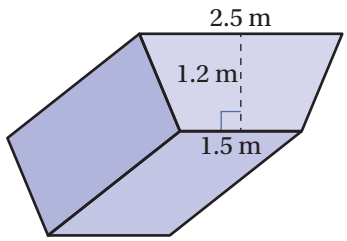
1 A square-based milk carton has a base edge length of 13 cm. How much milk is in the carton when it is filled to a depth of 16 cm? Answer in both millilitres and litres.



2 A trench with a rectangular cross-section 50 cm wide and 85 cm deep is dug. Given that the trench goes for 1.2 km, how many cubic metres of soil are removed?

3 Industrial bins in the shape of trapezoidal prisms are provided to homes undergoing renovations.

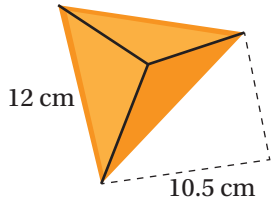
The area of a trapezium can be calculated using $A = \frac{1}{2}(a+b)h$.



Determine the length of the bin, given the following bin volumes or capacities.

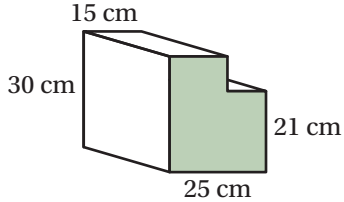
(a) 600 L **(b)** 6 m^3

4 A frozen pack of orange juice is in the shape of an irregular tetrahedron.



Determine the height of the tetrahedron, to the nearest millimetre, given that it holds 250 mL of orange juice.

5 The volume of the L-based prism is 26 400 cm^3. Determine the height of the prism.



6.3 Solve problems involving the surface areas and nets of right prisms

Learning intention: To be able to solve problems involving the surface areas and nets of right prisms

Success criteria:

SC 1 I can relate the net and the surface area of a right prism.

SC 2 I can understand and apply formulas to calculate the surface area of right prisms.

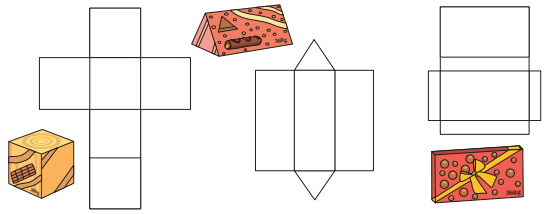
SC 3 I can solve a variety of problems involving the nets and surface areas of right prisms.

Lesson warm-up

Investigate packaging

Investigate the most cost-efficient options for packaging.

What other factors should you consider when selling chocolate?

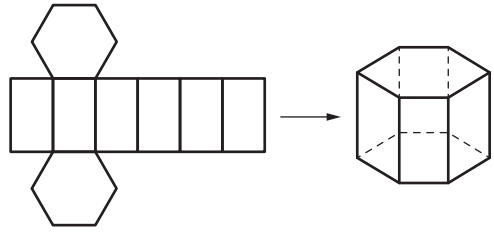




SC 1 I can relate the net and the surface area of a right prism

Drawing the net of a polyhedron is a way to transform a 3D object to a 2D representation of its surface.

Right prisms, having a uniform cross-section, always have identical faces at each end, joined by rectangles. The edges of the rectangles joining the end faces must, of course, have matching lengths.

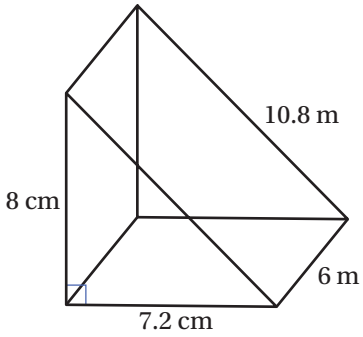


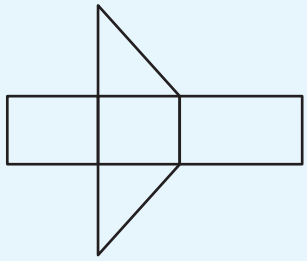
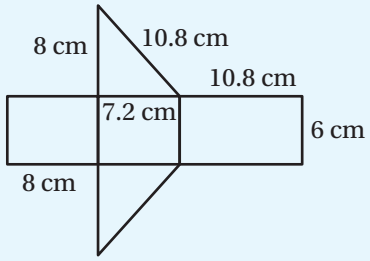
Imagine the hexagonal faces of the net being flipped up and the strip of small rectangles wrapping around to form the hexagonal prism.

Worked example

Drawing nets of right prisms

Draw a net to represent the right-angled triangle-based right prism. Include measurements for clarity.

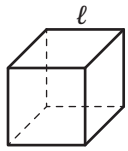


THINKING	WORKING
Sketch a net where the end shapes (triangles) are flipped down and the vertical and oblique rectangles are folded down flat.	
Write in the dimensions, ensuring that matching lengths can fold back to make the original shape.	

SC 2 I can understand and apply formulas to calculate the surface area of right prisms

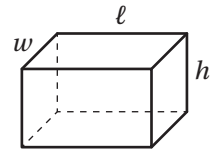
It is not essential that a net be drawn in order to calculate the surface area of a prism.

A cube has six identical square faces. Given that the edge length is l, the area of each face is l^2.



Hence, the surface area of a cube has the formula $S = 6l^2$.

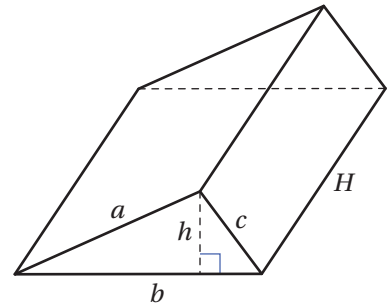
A rectangular prism has six rectangular faces with top and bottom faces matching, left and right ends matching, and front and back faces matching.



For a rectangular prism of length l, width w and height h, the area of the three different faces is lw, lh and wh, where there are two of each face.

Hence the surface area of a rectangular prism is given by $S = 2lw + 2lh + 2wh$.

For a prism whose cross-section is not a rectangle (or square), the area of one end is calculated, then doubled to account for the other end. The other surfaces are all rectangles, so their areas are easily calculated. The areas are added together to determine the total area of the surfaces.



The surface area of the triangular prism above could be found using the formula:

$$S = \left(\frac{bh}{2}\right) \times 2 + aH + bH + cH$$

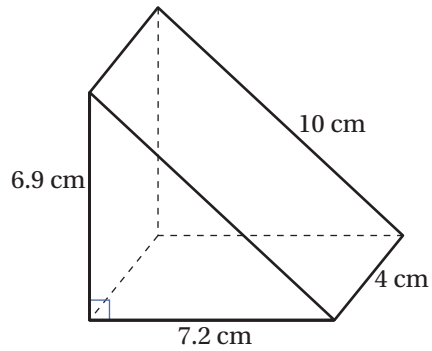
or $S = \left(\frac{bh}{2}\right) \times 2 + (a + b + c) \times H$

This matches the worded formula: S = (area of base) × 2 + (perimeter of base) × H, where (perimeter of base) × H gives the area of the strip of rectangles from the net diagram.

Worked example

Determining the surface area of a prism

Use your knowledge of the nets of prisms to calculate the surface area of the right-angled triangle-based prism.



THINKING	WORKING
Write the worded formula for the surface area of a prism.	S = (area of base) × 2 + (perimeter of base) × H
Identify the given dimensions.	Area of base: $\frac{1}{2} \times 7.2 \times 6.9$ Perimeter of base: 7.2 + 6.9 + 10 Depth or $H = 4$
Substitute values into the formula and calculate each part.	$S = \left(\frac{7.2 \times 6.9}{2}\right) \times 2 + (7.2 + 6.9 + 10) \times 4$ $= 24.84 \times 2 + 24.1 \times 4$ $= 49.68 + 96.4$ $= 146.08\ \text{cm}^2$
Write the answer.	The surface area of the prism is 146.08 cm^2, where each end has an area of 24.84 cm^2 and the wrap-around area of the rectangles is 96.4 cm^2.

SC 3 I can solve a variety of problems involving the nets and surface areas of right prisms

6

Worked example

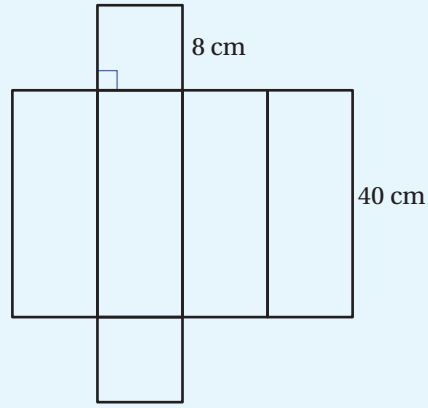
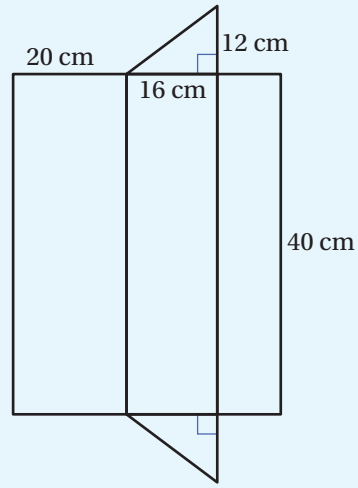
Solving problems involving the surface area of right prisms

A square-based prism and a right-angled triangle-based prism both have a height of 40 cm. The side length of the square base is 8 cm. The right-angled triangle base has a hypotenuse of 20 cm and one shorter side length of 16 cm.

(a) Calculate the unknown side length of the triangular base.

THINKING	WORKING
Recall Pythagoras' theorem.	$b^2 = c^2 - a^2$
Use Pythagoras' theorem to calculate the unknown side length in the right-angled triangle.	c = 20 cm and a = 16 cm (or use 3 : 4 : 5 Pythagorean triple) $b^2 = 20^2 - 16^2$ $= 144$ $\therefore b = 12$
Write the answer.	The third side of the triangle is 12 cm.

(b) Sketch a net of both prisms.

THINKING	WORKING
Use the net method of a strip of rectangles whose lengths wrap around the base; their width matches the height of the prism. Then attach the base shapes to protrude from opposite sides of a matching edge.	Net of square-based prism:  Net of right-angled triangle-based prism: 

(c) Determine which prism has the greater surface area. What is the difference in the surface area of the prisms?

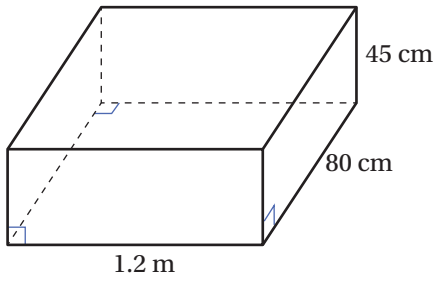
THINKING	WORKING
Calculate the surface area of the square-based prism.	Square-based prism $S = (8^2) \times 2 + (4 \times 8) \times 40$ $= 1408\,\text{cm}^2$
Calculate the surface area of the right-angled triangle-based prism.	Right-angled triangle-based prism $S = \left(\frac{16 \times 12}{2}\right) \times 2 + (20 + 16 + 12) \times 40$ $= 2112\,\text{cm}^2$
Use subtraction to compare the areas.	$2112 - 1408 = 704$
Write the answer.	The right-angled triangle-based prism has the greater surface area by $704\,\text{cm}^2$.

Practice

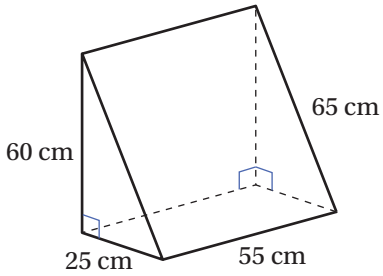
ANSWERS Page 634

SC 1 I can relate the net and the surface area of a right prism

1 Draw a net of the rectangular prism.



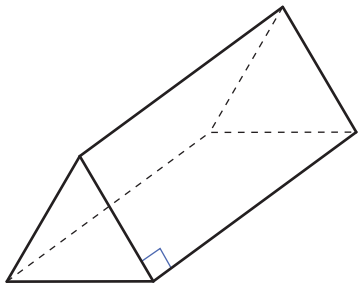
2 Draw a net of the right-angled triangle-based right prism.



3 For the equilateral triangle-based right prism,

(a) sketch four possible nets involving a strip of rectangles

(b) explain why such a variety of nets is possible for this solid.



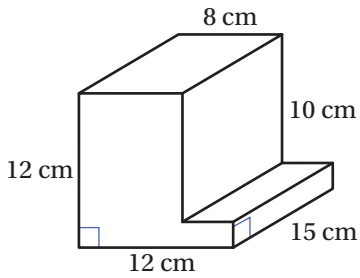
4 A net of the L-shaped prism is to be drawn as a strip of rectangles. The first rectangle from the left on the strip has the vertical left-hand edge of the prism as one of its sides.

(a) What is the width of the strip of rectangles?

(b) Determine the total length of the strip of rectangles.

(c) In the second net, the rectangle with one side of 8 cm is drawn by itself before all the other rectangles. List the length of each rectangle from left to right along the strip: 8 cm, 12 cm, etc.

(d) Describe the relative position and orientation of the second net compared to the first net.

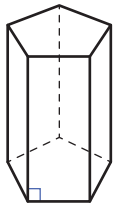


5 A regular pentagon-based right prism has rectangular faces 6 cm by 15 cm. A net is to be drawn, beginning with a strip of five rectangles.

(a) Determine the overall dimensions of the strip of rectangles.

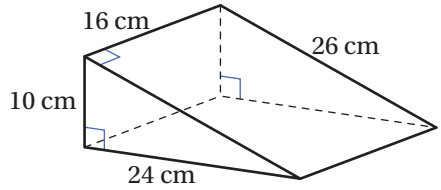
(b) If the diagram was not provided, give a second possible answer to **(a)**.

(c) Describe the possible positions of the regular pentagons on the net.

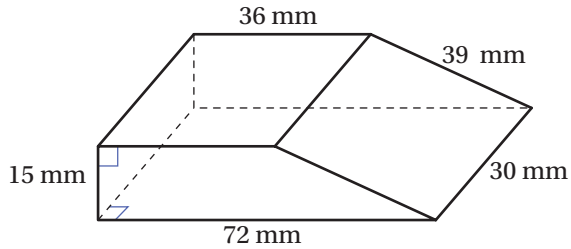


SC 2 I can understand and apply formulas to calculate the surface area of right prisms

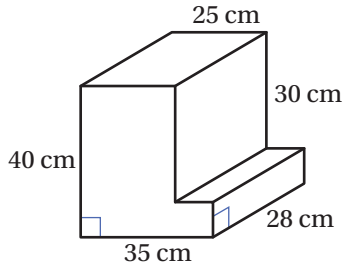
1 Determine the surface area of the right-angled triangle-based right prism.



2 Determine the surface area of the trapezium-based prism.



3 Determine the surface area of the L-shape-based prism.



4 A rectangular prism has dimensions 10 cm by 20 cm by 30 cm.
Consider the net of a prism where the base shapes protrude from a strip of rectangles.

(a) Considering the 10 cm by 20 cm rectangle as the base,

(i) determine the area of the base

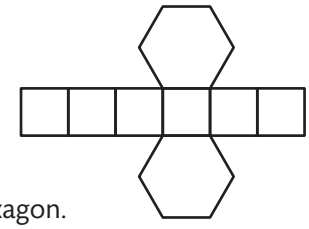
(ii) determine the area of the strip of rectangles.

(iii) Hence calculate the surface area of the prism.

(b) Repeat part **(a)**, letting the 10 cm by 30 cm rectangle be the base.

(c) Repeat part **(a)**, letting the 20 cm by 30 cm rectangle be the base.

5 A particular regular hexagon-based prism has edge lengths that are all 10 cm. Answer each part of the question to determine the surface area of the prism.



(a) The hexagon at the base is broken into six equilateral triangles.
What is the side length of each triangle?

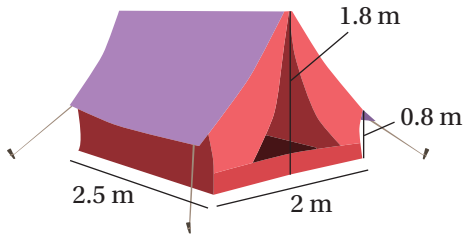
(b) Use Pythagoras' theorem to calculate the height of the equilateral triangle, correct to 2 decimal places.

(c) Using the rounded value for height, determine the area of one hexagon.

(d) Hence determine the surface area of the prism.

SC 3 I can solve a variety of problems involving the nets and surface areas of right prisms

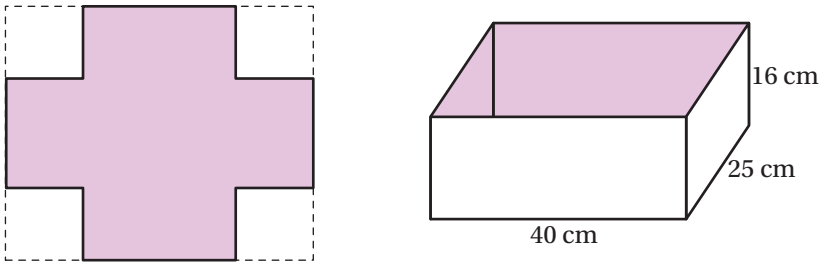
1 A symmetrical tent is in the shape of a double-trapezium-based prism.



(a) Calculate the length of the sloping edge of the roof, correct to 2 decimal places.

(b) Determine the amount of material used to make the tent, including the floor. Ignore any overlaps and seam allowances.

2 An open box is made from a rectangular sheet of cardboard.



(a) Determine the dimensions of the original piece of cardboard.

(b) State the shape and dimensions of the removed corners of the cardboard.

(c) Calculate the area of cardboard used to make the box.

3 A wooden cube has a side length of 6 cm. The cube is sawn into many cubes of length 2 cm.

(a) Calculate the surface area of the original cube.

(b) Calculate the surface area of one of the small cubes.

(c) Determine how many times larger the total surface area of the smaller cubes is compared to the original surface area.

4 A square-based pyramid has identical 20 cm edges.

(a) Sketch a net of the pyramid with the square base in the middle and a triangle attached to each side.

(b) Calculate the height of each triangle, correct to 2 decimal places.

(c) Use the rounded height from part **(b)** to determine the surface area of the pyramid.

5 A wooden cube has all of its faces painted red before being sliced in three in each of three dimensions to give many smaller cubes. How many of the smaller cubes have exactly:

(a) 3 red faces? **(b)** 2 red faces? **(c)** 1 red face? **(d)** no red faces?

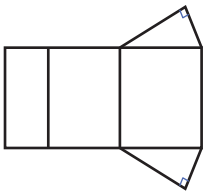
TOPIC REVIEW

Multiple choice

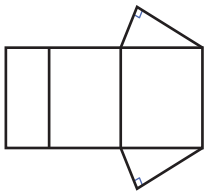
ANSWERS Page 667

1 Which of the diagrams below could *not* be a net for a right-angled triangle-based prism?

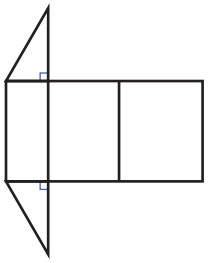
A



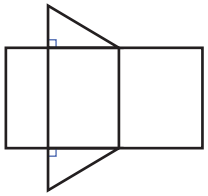
B



C



D



2 What is the surface area of a cube of edge length 2 m?

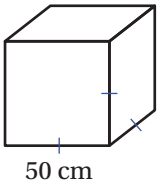
A $8\,m^2$ **B** $16\,m^2$ **C** $24\,m^2$ **D** $32\,m^2$

3 What is the surface area of a rectangular prism with dimensions 60 mm, 45 mm and 30 mm?

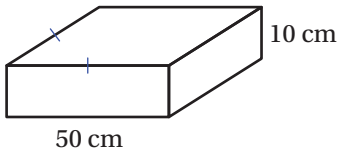
A $4050\,mm^2$ **B** $5400\,mm^2$ **C** $5850\,mm^2$ **D** $11\,700\,mm^2$

4 Which of the right prisms has the greatest volume?

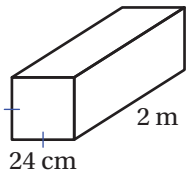
A



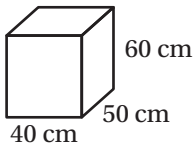
B



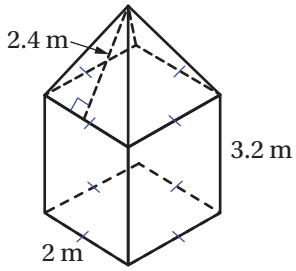
C



D

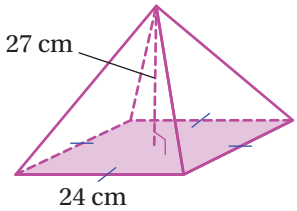


5 What is the surface area of this solid?



A $13.6\,m^2$ **B** $33.6\,m^2$ **C** $39.2\,m^2$ **D** $47.2\,m^2$

6 What is the volume of this square-based pyramid?



A 5184 cm^3 **B** 5616 cm^3 **C** 7776 cm^3 **D** 15 552 cm^3

7 What is the volume of a triangular prism of height 10 cm, where the base and height of the triangle are 4 cm and 3 cm, respectively?

A 120 cm^3 **B** 62 cm^3 **C** 60 cm^3 **D** 40 cm^3

8 Which pair(s) of objects have the same volume?

I A rectangular prism with $l = 30$ cm, $w = 20$ cm, $H = 10$ cm

II A rectangular pyramid with $l = 30$ cm, $w = 10$ cm, $H = 40$ cm

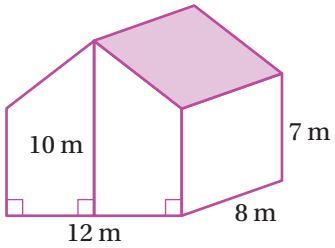
III A rectangular prism with $l = 50$ cm, $w = 8$ cm, $H = 10$ cm

IV A rectangular pyramid with $l = 60$ cm, $w = 30$ cm, $H = 10$ cm

A I and III only **B** I and IV only

C I and III as well as II and IV **D** I and IV as well as II and III

9 What is the volume of this symmetrical prism?



A 165 m^3 **B** 176 m^3 **C** 816 m^3 **D** 960 m^3

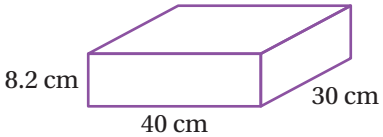
10 Which value is equal to 750 cm^3?

A 7.5 L **B** 750 mL **C** 75 mL **D** 7.5 m^3

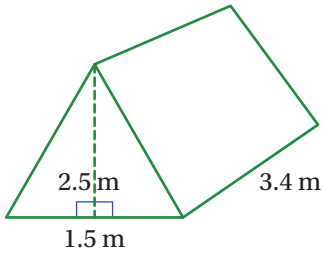
Short answer

1 Calculate the volume of the right prisms.

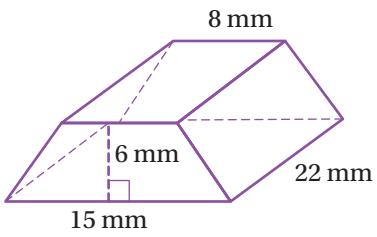
(a)



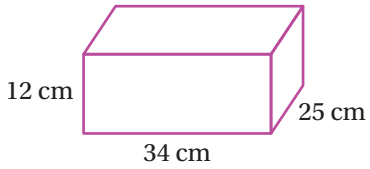
(b)



(c)

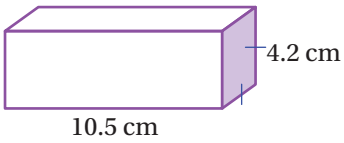


2 Determine the capacity of the rectangular-prism-shaped tank in litres.

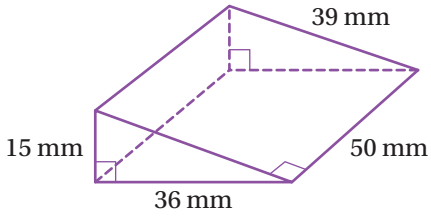


3 Calculate the surface area and volume of each of the right prisms.

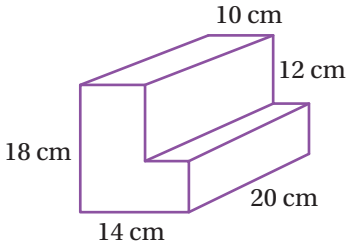
(a)



(b)



4 Consider this L-shaped prism.

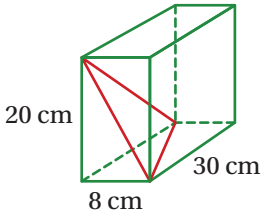


(a) Calculate the area of one end.

(b) Determine the dimensions of a rectangle that would cover the rectangular faces.

(c) Hence, calculate the surface area of the prism.

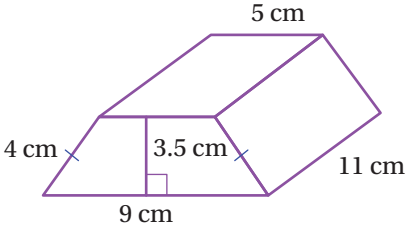
5 A right-angled triangle-based pyramid is formed from a rectangular prism, as shown.



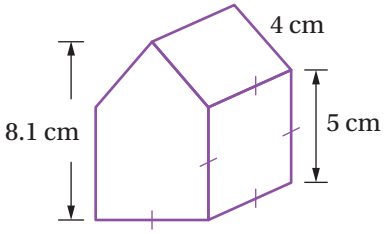
Calculate the volume of the pyramid.

6 Determine the surface area of each of the right prisms.

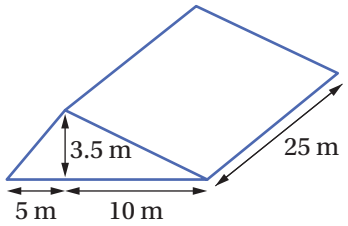
(a)



(b)



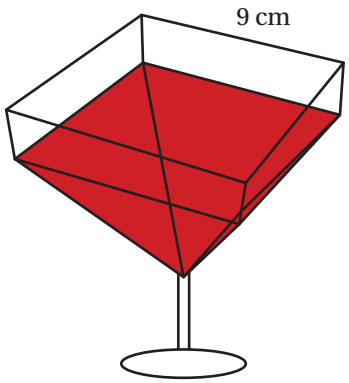
7 Consider this triangular-prism-shaped shelter.



(a) Use Pythagoras' theorem to calculate the sloping-side lengths of the roof, correct to 2 decimal places.

(b) Determine the exposed surface area (all but the ground level), using the rounded values from **(a)**.

8 A catering company chooses square-based pyramid glasses with a square-based prism lip. The lip is 3 cm deep and the liquid, filled as shown, is 7 cm deep.



(a) Determine the capacity of the glass in millilitres if the liquid is intended to reach just to the top of the pyramid as shown.

(b) Determine the capacity of the glass in millilitres if it is filled to overflowing.

(c) How many 2 L bottles of soft drink should be on hand so that a party of 50 people can drink up to three glasses each, filled as shown?

9 A cube has a surface area of 138.24 cm^2. Determine the edge length of the cube.

10 A square-based prism has a volume of 12 960 cm^3. Determine the surface area of the prism given that its height is 40 cm.

11 Determine the surface area of an equilateral-triangle-based prism where the edges of the base are all 1.5 m and the height is 70 cm. Write your answer in square metres, correct to 2 decimal places.

12 A wooden rectangular prism has edge lengths 15 cm, 12 cm and 8 cm. The prism is sawn into rectangular prisms each 3 cm by 2 cm by 4 cm.

(a) Calculate the surface area of the original prism.

(b) Calculate the surface area of one of the smaller prisms.

(c) Determine how many times the total surface area of the smaller prisms is compared to the original surface area.

Extended response

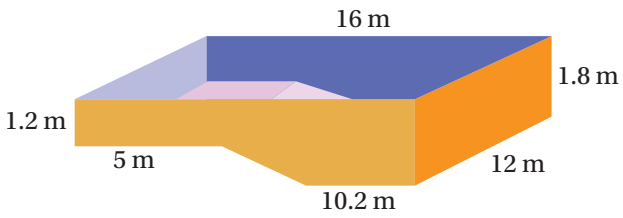
1 Two backyard pools both have vertical sides to a depth of 1.4 m. The area of the top of the smaller pool is 20 m^2. The larger pool, while the same cross-sectional shape, has surface dimensions 1.3 times the dimensions of the smaller pool.

(a) Determine the area of the top of the larger pool.

(b) Determine the volume of each pool.

(c) If 8 mm evaporates from the top of both pools on a summer's day, how many litres are needed to refill each pool?

2 A local pool is to be tiled then filled with water.



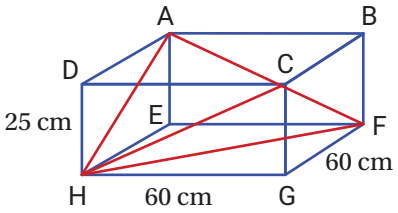
(a) Determine the length of the sloping edge of the pool.

(b) Calculate the area to be tiled (bottom and sides).

(c) Determine the number of packs of 100 40 cm by 40 cm tiles that need to be bought for the tiling job.

(d) Determine the amount of water, in kilolitres, needed to fill the pool.

3 A square-based prism, 25 cm by 60 cm by 60 cm, has lines drawn from two of the top vertices (as shown) to form triangles with a common diagonal along the base. Hence, the original object has been divided into two identical, symmetrical, right-angled triangle-based pyramids (i.e. ACDH and CFGH) and a symmetrical 7-faced polyhedron.



(a) Determine the volume and surface area of the original prism.

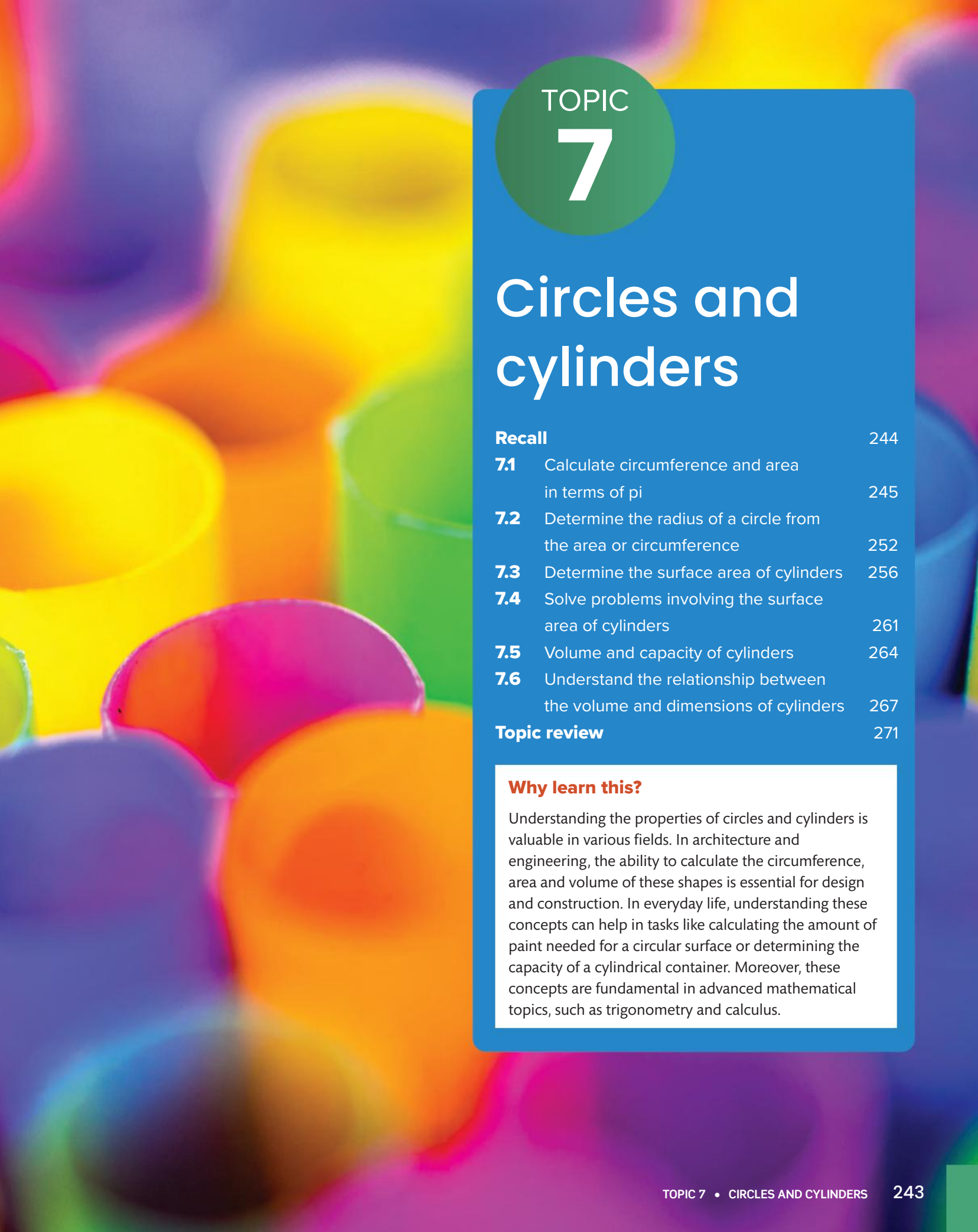
(b) Calculate the volume of one of the pyramids.

(c) Hence, calculate the volume of the 7-faced polyhedron, and say what fraction of the original volume it takes up.

(d) Describe the faces of one of the pyramids.

(e) Describe the faces of the 7-faced polyhedron.

(f) Determine the increase in surface area due to breaking the object into three separate parts.



TOPIC 7

Circles and cylinders

Recall 244

7.1 Calculate circumference and area in terms of pi 245

7.2 Determine the radius of a circle from the area or circumference 252

7.3 Determine the surface area of cylinders 256

7.4 Solve problems involving the surface area of cylinders 261

7.5 Volume and capacity of cylinders 264

7.6 Understand the relationship between the volume and dimensions of cylinders 267

Topic review 271

Why learn this?

Understanding the properties of circles and cylinders is valuable in various fields. In architecture and engineering, the ability to calculate the circumference, area and volume of these shapes is essential for design and construction. In everyday life, understanding these concepts can help in tasks like calculating the amount of paint needed for a circular surface or determining the capacity of a cylindrical container. Moreover, these concepts are fundamental in advanced mathematical topics, such as trigonometry and calculus.

RECALL

ANSWERS Page 635

I can round decimals for different purposes

1 Round each of the following numbers to the number of decimal places stated in the brackets.

(a) 8.386 (2) **(b)** 83.71 (0) **(c)** 8.5985 (3) **(d)** 0.999 (2)

I can multiply decimals

1 Evaluate the following.

(a) 3.5×2.5 **(b)** 10.15×0.4 **(c)** 6.25×4.25 **(d)** 8.21×0.029

I can convert between units of length

1 Complete each of the following length conversions.

(a) 700 cm to m **(b)** 5.31 cm to mm **(c)** 185 mm to cm

(d) 3575 mm to m **(e)** 0.094 m to cm **(f)** 0.001 km to cm

I can calculate the area of squares and rectangles

1 Calculate the area of the following.

(a) A square with sides of length 8 cm

(b) A rectangle with length 14.5 m and width 6 m

I can calculate the circumference and area of circles to 1 decimal place

1 Calculate the circumference of the circles with the following dimensions, correct to 1 decimal place.

(a) Radius 10 cm **(b)** Diameter 12 mm

2 Calculate the area of the circles with the following dimensions using $A = \pi r^2$, correct to 1 decimal place.

(a) Radius 4 cm **(b)** Diameter 6 m

I can calculate the volume of prisms

1 Calculate the volume of the following prisms.

(a) A rectangular prism of height 2.5 m with a cross-sectional area of 15 m^2

(b) A triangular prism with a cross-sectional area of 24 cm^2 and a perpendicular height of 30 cm.

(c) A cube with a cross-sectional area of 36 mm^2.

Calculate circumference and area in terms of pi

7.1

Learning intention: To be able to calculate circumference and area in terms of pi

Success criteria:

SC 1 I can calculate circumference as a value in terms of pi.

SC 2 I can calculate the area of a circle as a value in terms of pi.

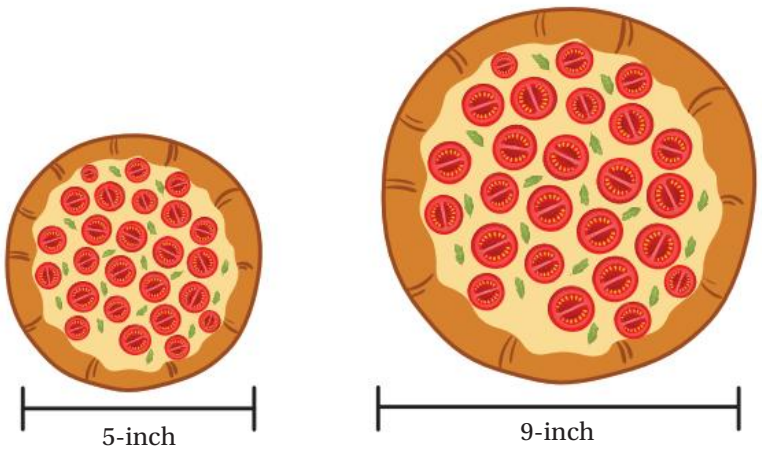
Lesson warm-up

Is this a good deal?

In June 2022 a Twitter user @cretiredroy wrote:

I ordered a 9-inch pizza. After a while, the waiter brought two 5-inch pizzas and said, 'The 9-inch pizza is not available, so I am giving you two 5-inch pizzas.'

When I asked about the difference, the waiter replied, 'You can have the extra 1 inch for free.'

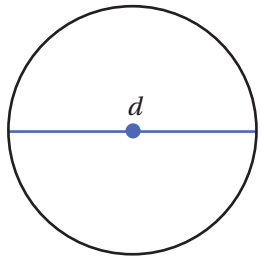
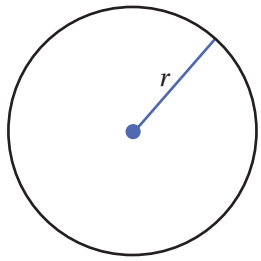


Mathematically, what should have actually happened in this scenario?

SC 1 I can calculate circumference as a value in terms of pi

Circle calculations nearly always involve rounding values to a required degree of accuracy. It sometimes makes better sense to give an 'exact' value by leaving π as part of the answer. Comparisons between distances and area may be clearer as a result, and rounding errors no longer apply.

Recall the two formulas for the circumference (perimeter) of a circle, given either the length of the diameter d or the radius r.

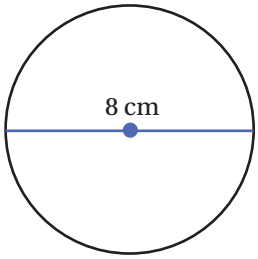
Circle measurement	Circumference formula
	$C = \pi d$
	$C = 2\pi r$

Worked example

Calculating circumference in terms of pi

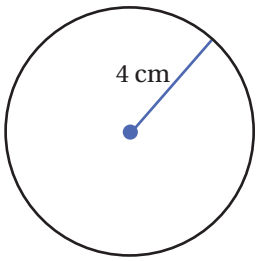
Calculate the circumference of the circles in terms of pi.

(a)



THINKING	WORKING
Recall the formula for circumference using the diameter.	$C = \pi d$
Substitute the length of the diameter into the circumference formula.	The diameter is $d = 8$ cm. $C = \pi \times 8$
Simplify, treating π as an algebraic factor, and writing it after the number factors.	$C = 8\pi$
Write the answer.	The circumference is 8π cm.

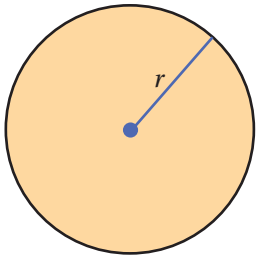
(b)



THINKING	WORKING
Recall the formula for circumference using the radius.	$C = 2\pi r$
Substitute the length of the radius into the circumference formula.	The radius is $r = 4$ cm. $C = 2\pi \times 4$
Simplify, multiplying the numbers (other than π) together.	$C = 2 \times 4 \times \pi$ $= 8\pi$
Write the answer.	The circumference is 8π cm.

SC 2 I can calculate the area of a circle as a value in terms of pi

Remember the formula for the area of a circle, given the length of the radius r.



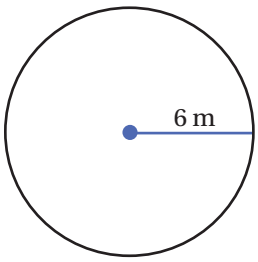
$A = \pi r^2$, with the units for area being square units.

Worked example

Calculating the area of a circle in terms of pi

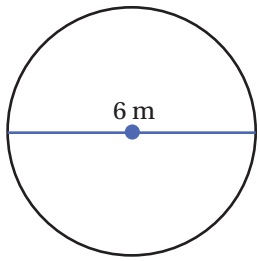
Calculate the area of the circles in terms of pi.

(a)



THINKING	WORKING
Recall the formula for the area of a circle.	$A = \pi r^2$
Determine the length of the radius.	The length of the radius $r = 6$ m.
Substitute the length of the radius into the area formula.	$A = \pi \times 6^2$
Simplify the expression.	$A = \pi \times 6 \times 6$ $= \pi \times 36$ $= 36\pi$
Write the answer.	The area of the circle is 36π m^2.

(b)



THINKING	WORKING
Recall the formula for area of a circle.	$A = \pi r^2$
Determine the length of the radius and substitute it into the area formula.	The length of the diameter is $d = 6$ m. $r = \frac{d}{2}$ $= \frac{6}{2}$ $= 3$ m Hence, the length of the radius is $r = 3$ m.
Substitute the length of the radius into the area formula.	$A = \pi \times 3^2$
Simplify the expression.	$A = \pi \times 3 \times 3$ $= \pi \times 9$ $= 9\pi$
Write the answer.	The area of the circle is 9π m^2.

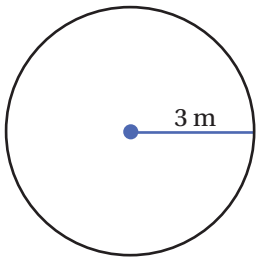
Practice

ANSWERS Page 636

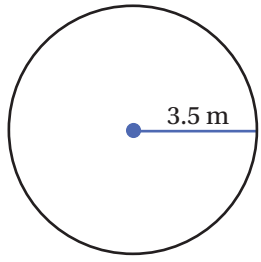
SC 1 I can calculate circumference as a value in terms of pi

1 Calculate the circumference of each circle in terms of pi.

(a)

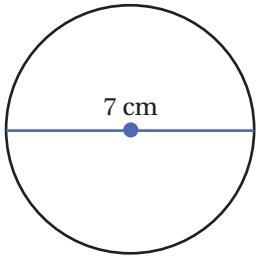


(b)

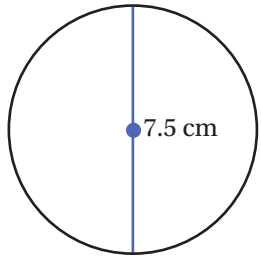


2 Calculate the circumference of each circle in terms of pi.

(a)

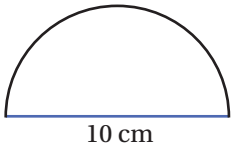


(b)

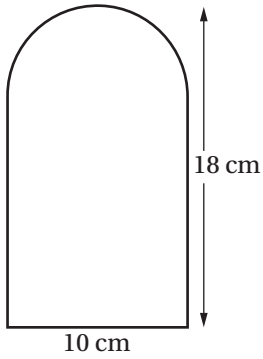


3 Calculate the exact perimeter of each shape that contains a semicircle. Write your answer in terms of pi.

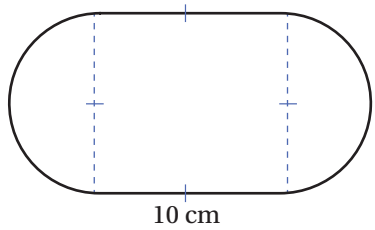
(a)



(b)

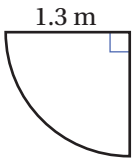


(c)

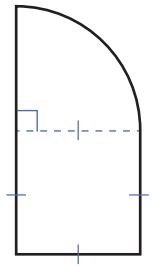


4 Calculate the exact perimeter of each shape that contains a quarter of a circle. Write your answer in terms of pi.

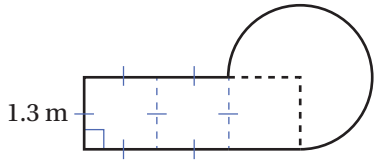
(a)



(b)

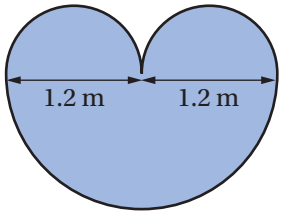


(c)

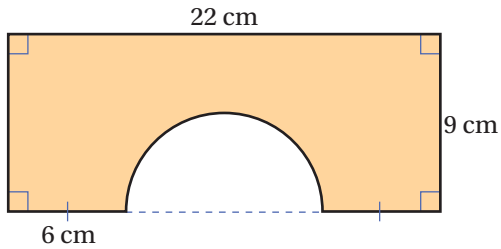


5 Calculate the perimeter of each shape.

(a)

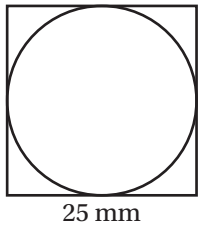


(b)

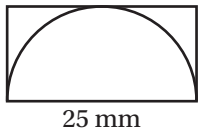


6 Determine the exact ratio of the perimeter of each shape.

(a) The circle to the square



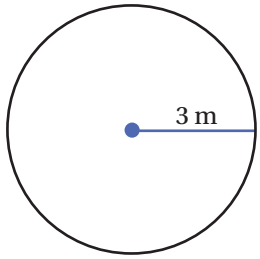
(b) The semicircle to the rectangle



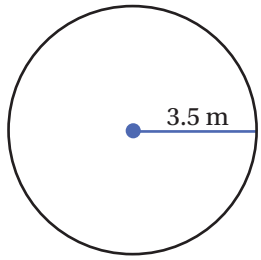
SC 2 I can calculate the area of a circle as a value in terms of pi

1 Calculate the area of each circle in terms of pi.

(a)

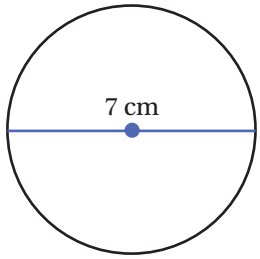


(b)

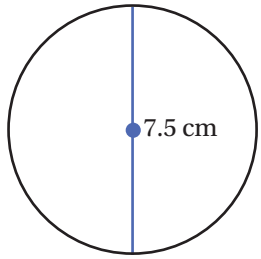


2 Calculate the area of each circle in terms of pi. Determine the length of the radius first.

(a)

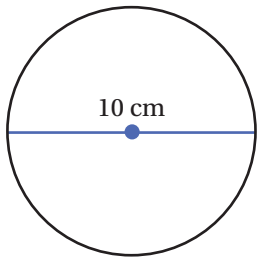


(b)

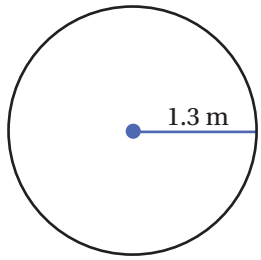


3 Calculate the area of each circle. Write your answer in terms of pi.

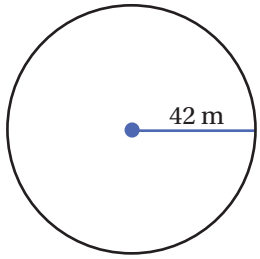
(a)



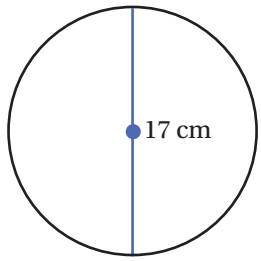
(b)



(c)

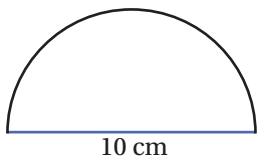


(d)

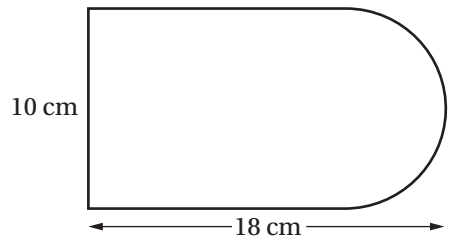


4 Calculate the exact area of each shape that contains a semicircle. Write your answer in terms of pi.

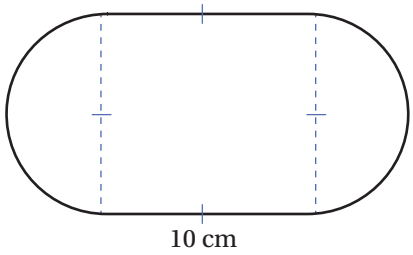
(a)



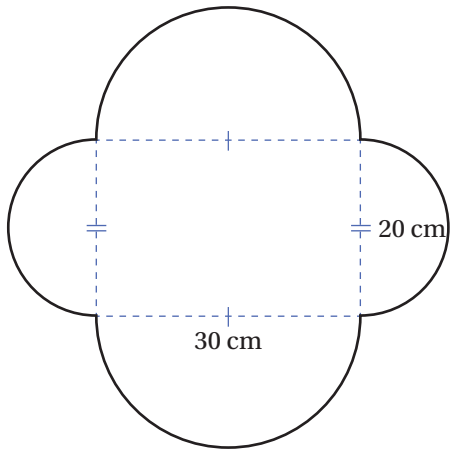
(b)



(c)

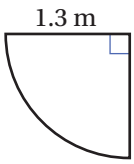


(d)

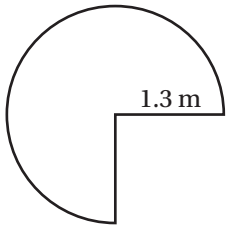


5 Calculate the exact area of each shape that contains a quarter of a circle. Write your answer in terms of pi.

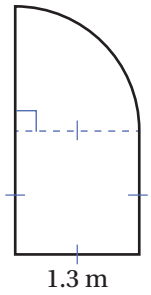
(a)



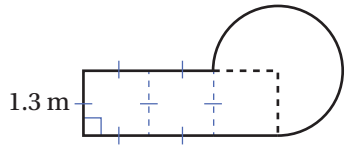
(b)



(c)

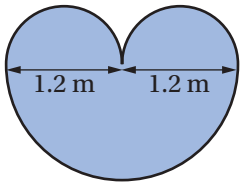


(d)

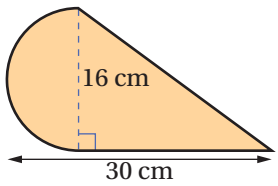


6 Calculate the exact shaded area.

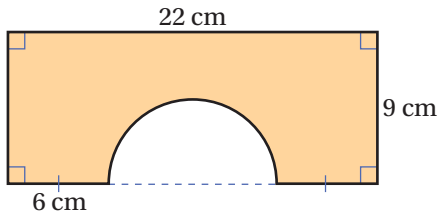
(a)



(b)

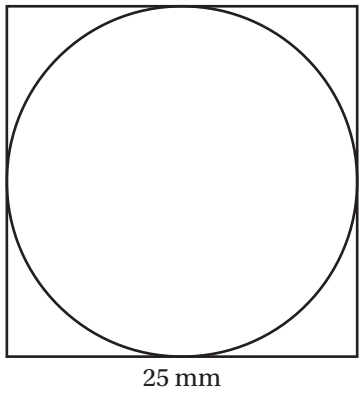


(c)

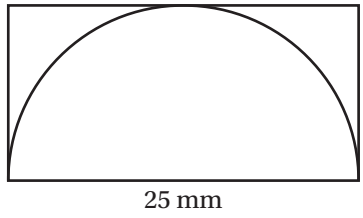


7 Determine the exact ratio of the area of each shape.

(a) The circle to the square



(b) The semicircle to the rectangle



7.2 Determine the radius of a circle from the area or circumference

Learning intention: To be able to determine the radius of a circle from the area or circumference

Success criteria:

SC 1 I can determine the radius of a circle given the circumference.

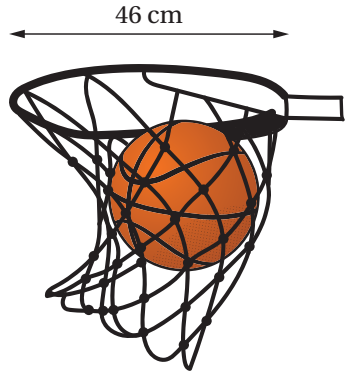
SC 2 I can determine the radius of a circle given the area.

Lesson warm-up

Room for error?

During the half-time entertainment at a professional basketball game, a performer bounced off a trampoline, dunked a basketball, then fell through the ring.

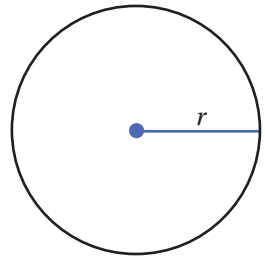
How much room for error is there when shooting a goal in basketball? Note that the basketball has a circumference of approximately 74 cm.



SC 1 I can determine the radius of a circle given the circumference

Backtracking can be used to determine the length of the radius r when the circumference C is known.

$$C = 2\pi r$$
$$C = 2 \times \pi \times r$$
$$\frac{C}{2\pi} = \frac{2 \times \pi \times r}{2\pi}$$
$$r = \frac{C}{2\pi}$$



Worked example

Determining radius from the circumference

Determine the length of the radius of a circle, in terms of pi, given the following information.

(a) The circumference is 24π cm.

THINKING	WORKING
Recall the formula for circumference.	$C = 2\pi r$
Identify the circumference.	$C = 24\pi$ cm
Substitute the circumference into the formula.	$24\pi = 2\pi r$
Solve the equation for the radius, leaving pi as part of the answer.	$r = \frac{24\pi}{2\pi}$ $= \frac{24}{2}$ $= 12$
Write the answer.	The radius of the circle is 12 cm.

(b) The circumference is 40 cm.

THINKING	WORKING
Recall the formula for circumference.	$C = 2\pi r$
Identify the circumference.	$C = 40$ cm
Substitute the circumference into the formula.	$40 = 2\pi r$
Solve the equation for the radius, leaving pi as part of the answer.	$r = \frac{40}{2\pi}$ $= \frac{20}{\pi}$
Write the answer.	The radius of the circle is $\frac{20}{\pi}$ cm.

SC 2 I can determine the radius of a circle given the area

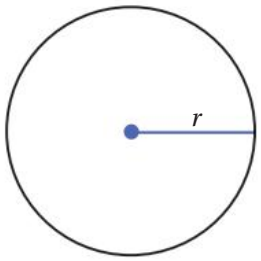
Backtracking can be used to determine the length of the radius r when the area A is known.

$A = \pi r^2$

$\frac{A}{\pi} = \frac{\pi \times r^2}{\pi}$

$r^2 = \frac{A}{\pi}$

$\therefore r = \sqrt{\frac{A}{\pi}}$



Worked example

Determining radius from area

Determine the length of the radius of a circle, in terms of pi, given the following area.

(a) The area of the circle is $16\pi\,\text{cm}^2$.

THINKING	WORKING
Recall the formula for the area of a circle.	$A = \pi r^2$
Identify the area of the circle.	$A = 16\pi\,\text{cm}^2$
Substitute the area into the formula.	$16\pi = \pi r^2$
Solve the equation for the variable r. Given that r represents a length, $r > 0$, so the negative root can be ignored.	$\frac{16\pi}{\pi} = \frac{\pi \times r^2}{\pi}$ $r^2 = 16$ $r = \sqrt{16}$ $r = 4$
Write the answer.	The radius of the circle is 4 cm.

(b) The area of the circle is $85\,\text{cm}^2$.

THINKING	WORKING
Recall the formula for the area of a circle.	$A = \pi r^2$
Identify the area of the circle.	$A = 85\,\text{cm}^2$
Substitute the area into the formula.	$85 = \pi r^2$
Solve the equation for the variable r. Given that r represents a length, $r > 0$, so the negative root can be ignored.	$\frac{85}{\pi} = \frac{\pi r^2}{\pi}$ $r^2 = \frac{85}{\pi}$ $r = \sqrt{\frac{85}{\pi}}$ $r = 5.20157\ldots$
Write the answer.	The exact radius of the circle is $\sqrt{\frac{85}{\pi}}$ cm.

Practice

ANSWERS Page 636

SC 1 I can determine the radius of a circle given the circumference

1 Determine the radius for each circle, given the circumference.

(a) 10π mm **(b)** 15π mm **(c)** 20π mm **(d)** 25π mm

2 Determine the radius for each circle, given the circumference. Write your answer in terms of pi.

(a) 10 mm **(b)** 15 mm **(c)** 20 mm **(d)** 25 mm

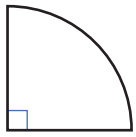
3 Describe the relationship between the radius and the diameter.

4 Determine the exact diameter and radius for each circle, given the circumference.

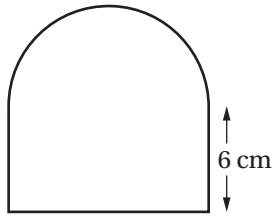
(a) 140 mm **(b)** 18 cm **(c)** 1.35 m **(d)** 26.8 cm

5 Determine the exact radius of the partial circle in each shape, given the perimeter.

(a) $P = 55$ cm



(b) $P = 40$ cm



SC 2 I can determine the radius of a circle given the area

1 Determine the radius for each circle, given the area.

(a) 4π mm^2 **(b)** 9π mm^2 **(c)** 16π mm^2 **(d)** 25π mm^2

2 Determine the radius for each circle, given the area. Write your answer in terms of pi.

(a) 10 mm^2 **(b)** 15 mm^2 **(c)** 20 mm^2 **(d)** 25 mm^2

3 Describe the relationship between the radius and the diameter.

4 Determine the exact radius and diameter for each circle, given the area.

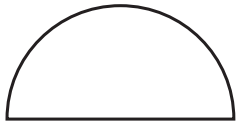
(a) 49π mm^2 **(b)** 81π cm^2 **(c)** 10π mm^2 **(d)** 20 m^2 **(e)** 26.8 cm^2

5 Determine the exact radius for each circle, then the approximate radius correct to 2 decimal places, using the given information of circumference C or area A.

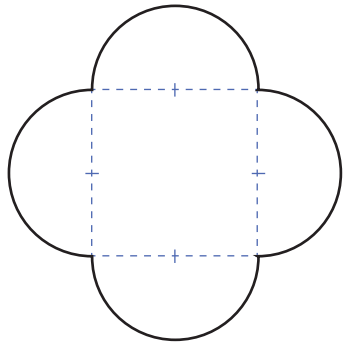
(a) $C = 45$ cm **(b)** $A = 350$ cm^2 **(c)** $C = 40$ m **(d)** $A = 62$ m^2

6 Determine the exact radius of the partial circle in each shape, given the area.

(a) $A = 50$ cm^2



(b) $A = 30$ m^2



7.3 Determine the surface area of cylinders

Learning intention: To be able to determine the surface area of cylinders

Success criteria:

SC 1 I can relate the net and the surface area of a cylinder.

SC 2 I can determine the surface area of a cylinder.

Lesson warm-up

Can you describe the net and surface area of a cylinder?

The net of a solid can be imagined as unfolding the 3D solid into its 2D shapes.

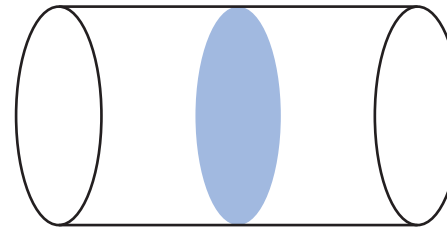


Draw the net of a cylinder.

Describe each of the dimensions of the 2D shapes that make up the net.

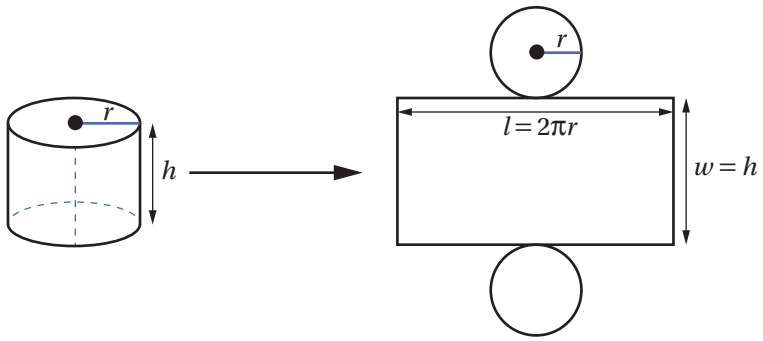
SC 1 I can relate the net and the surface area of a cylinder

Like right prisms, cylinders have a uniform cross-section: a circle.



Cylinders have two faces that are identical circles and a curved surface that, when imagined to be cut at right angles to the base and flattened out, becomes a rectangle.

- The length of the rectangle is the same as the circumference of the circle.
- The width of the rectangle is the same as the height of the cylinder.



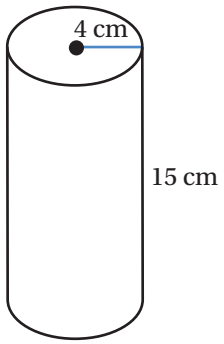
The 'net' of a cylinder is not a true net since the circles theoretically just touch the rolled-out rectangle at a single point, but it is still usual to draw the shapes as though they are joined.

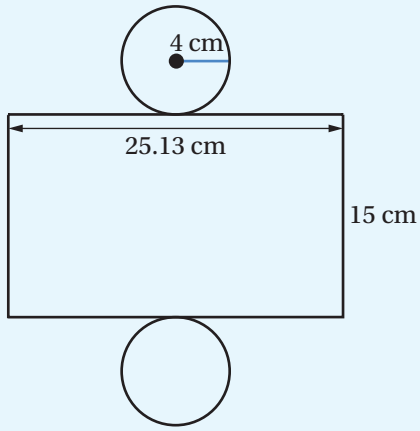
The circles may be placed anywhere along the curved-edge side of the rectangle, but are usually placed opposite each other.

Worked example

Drawing nets of cylinders

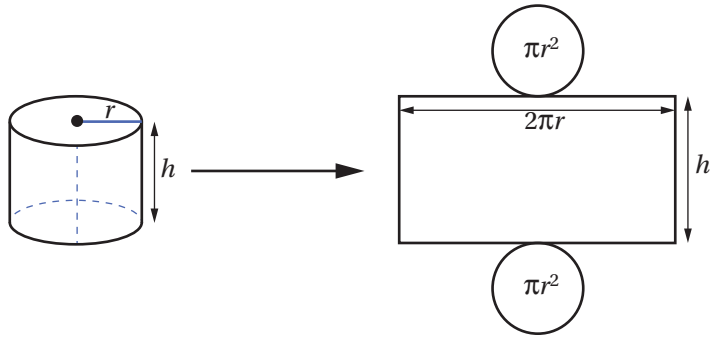
Draw a net to represent the cylinder below.



THINKING	WORKING
Determine the circumference of an end circle, correct to 2 decimal places.	$C = 2\pi r$ where $r = 4$ cm $C = 2\pi \times 4$ $= 8\pi$ ≈ 25.13 cm
Draw the net by placing two circles, the same as the ends of the cylinder, along opposite sides of a rectangle. The length of the rectangle should match the circumference of the circles. The width of the rectangle should match the height of the cylinder.	

SC 2 I can determine the surface area of a cylinder

The net of a cylinder is made up of two circular faces and a curved rectangular surface.
Each circle has an area of πr^2. The dimensions of the rectangle are the circumference $2\pi r$ and the height of the cylinder h.



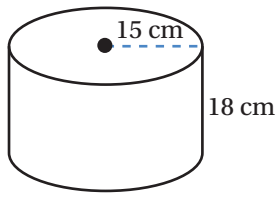
Hence, the surface area of a cylinder has the formula:
$S = 2 \times (\pi r^2) + (2\pi r \times h)$
$= 2\pi r^2 + 2\pi rh$
where r is the radius of the circular ends and h is the height of the cylinder.

This matches the general formula for surface area of a prism:
S = (area of base) × 2 + (perimeter of base) × h

Worked example

Determine the surface area of a cylinder

Determine the surface area of the cylinder, correct to 2 decimal places.



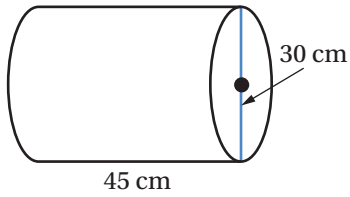
THINKING	WORKING
Write the formula for the surface area of a cylinder.	$S = 2\pi r^2 + 2\pi rh$
Substitute values for the working and calculate in terms of pi.	$S = 2 \times \pi \times (15)^2 + 2 \times \pi \times 15 \times 18$ $= 450\pi + 540\pi$ $= 990\pi$ $= 3110.176 \ldots$
Write the answer, rounding appropriately.	The surface area of the cylinder is $3110.18\,\text{cm}^2$, correct to 2 decimal places.

Practice

ANSWERS Page 636

SC 1 I can relate the net and the surface area of a cylinder

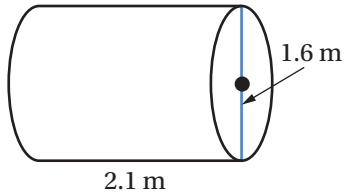
1 Consider the cylinder shown.



(a) Draw the net and label the radius and the height.

(b) Determine the exact dimensions of the rectangle in the net.

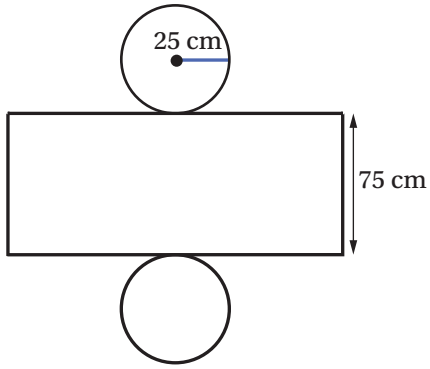
2 Consider the cylinder shown.



(a) Draw the net and label the radius and the height.

(b) Determine the exact dimensions of the rectangle in the net.

3 From the net of the cylinder, draw and label the dimensions of the cylinder.



4 A rectangle, 60 cm by 20 cm, is wrapped around the curved surface of a cylinder, with the longer sides wrapping around the circular ends.

(a) State the height of the cylinder.

(b) Determine the exact radius of the circular base.

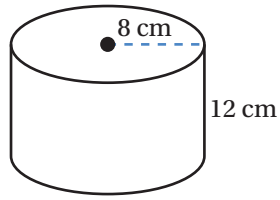
5 A square of side length 80 mm is wrapped around the curved surface of a cylinder.

(a) State the height of the cylinder.

(b) Determine the exact diameter of the circular base.

SC 2 I can determine the surface area of a cylinder

1 Determine the exact surface area of a cylinder of height 8 cm and radius of 12 cm.



2 For a cylinder of height 26.1 cm and radius 9.6 cm, determine the following.

(a) The exact area of a circular end

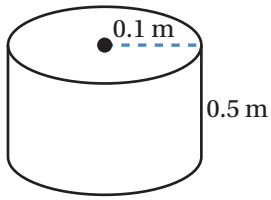
(b) The exact circumference of a circular end

(c) The exact curved surface area

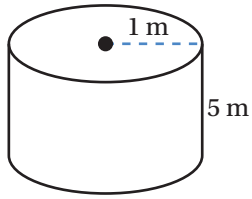
(d) The exact surface area of the cylinder

3 Determine the exact surface area of each cylinder.

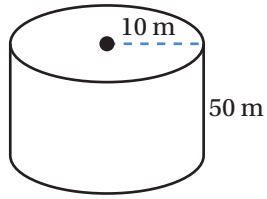
(a)



(b)

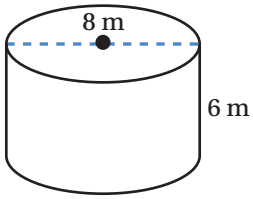


(c)

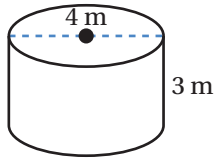


4 Determine the exact surface area of each cylinder.

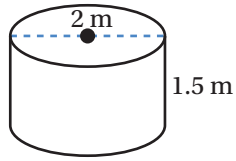
(a)



(b)



(c)



5 The area of a circular end of a cylinder is 1156π cm^2 and the curved surface area is 3944π cm^2. Determine the following.

(a) The length of the radius of the cylinder.

(b) The height of the cylinder.

6 Icing can be used to decorate cakes. The cake shown has a diameter of 25 cm and a depth of 10 cm.



(a) Describe the difference between the surface area of the cake and the surface area of the cake that is to be iced.

(b) Determine the surface area of the cake to be iced, correct to 2 decimal places.

Solve problems involving the surface area of cylinders

7.4

Learning intention: To be able to solve problems involving the surface area of cylinders

Success criteria:

SC 1 I can solve a variety of problems involving the nets and surface areas of cylinders.

Lesson warm-up

Painting curved surfaces

'The mural shown at Sheep Hills (below) represents the passing on of knowledge and local Indigenous history from Elders to the next generation within the community; as well as depicting elements of the local dreaming and the passing of time. The artist, Adnate, spent three weeks with the Barenji Gadjin community to conceive and complete the mural.'



In Victoria, the Silo Art Trail began at a small rural town called Brim. A small community project to paint the cylindrical grain silos resulted in international media attention and an influx of visitors to the tiny town.

The silos are 30 m high and each of the six silos has a diameter of approximately 6 m.

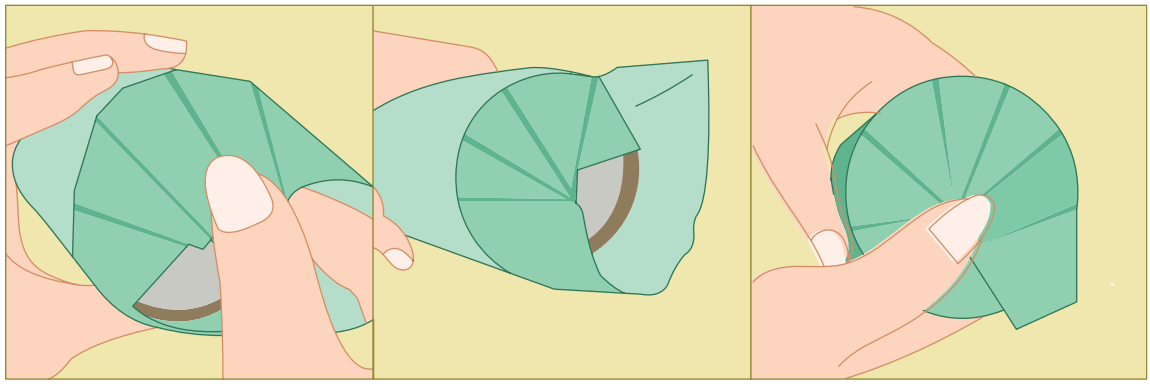
Determine the size of the murals.

SC 1 I can solve a variety of problems involving the nets and surface areas of cylinders

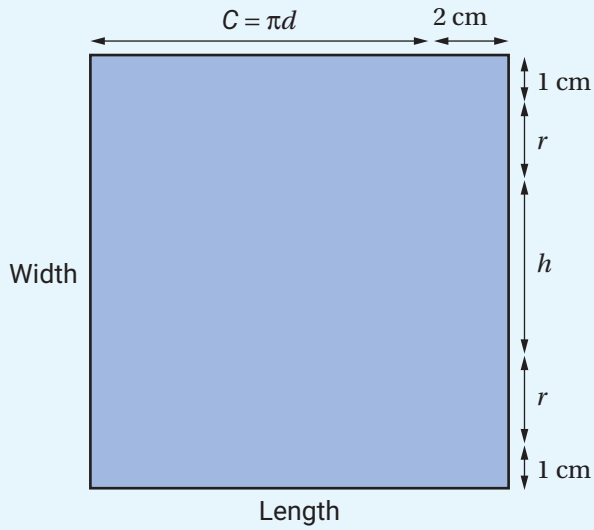
Worked example

Solving problems involving surface area of cylinders

A cylindrical gift of height 25 cm and diameter 11 cm is to be wrapped with a rectangular piece of paper. There needs to be a 2 cm overlap of the paper wrapped around the middle of the cylinder. The paper will be folded to cover the circular ends, so each end needs an additional radius length plus a 1 cm overlap. This will allow the paper at the ends to be folded over, as in the pictures below.



Determine the smallest dimensions possible for the wrapping paper, rounded up to the nearest centimetre.

THINKING	WORKING
Draw a diagram of the rectangular paper, labelling all side lengths.	
Determine the length of the curved surface (rectangle).	The length is the circumference of the circle, plus the overlap. $\pi d + 2 = \pi \times 11 + 2$ $= 36.5\ldots$ The length of the rectangle must be at least 37 cm, rounded up to the nearest whole number.

Determine the width of the curved surface (rectangle).	One radius length plus a 1 cm overlap will be added at both the top and the bottom of the height. This extra length will be used to fold over the top and bottom of the cylinder. $r = \frac{11}{2} = 5.5$ cm The width of the rectangle will equal: $h + (r + 1) + (r + 1)$ $= 25 + (5.5 + 1) + (5.5 + 1)$ $= 38$ The width of the rectangle must be at least 38 cm.
Write the answer.	The wrapping paper must be at least 37 cm by 38 cm.

Practice

ANSWERS Page 637

SC 1 I can solve a variety of problems involving the nets and surface areas of cylinders

1 Determine the height of a cylinder that has a radius of 80 cm and a surface area of 5 m^2. Give your answer in centimetres, to the nearest millimetre.

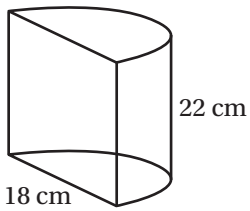
2 Determine the dimensions of the cardboard rectangle required to cut the curved and circular ends for a cylinder of height 10 cm and radius 6 cm. Choose the option that ensures the rectangle has the smallest possible area. Write your answer correct to the nearest millimetre.

3 A cylindrical container has no lid.

(a) Write a formula for the external surface area of the container.

(b) Determine the surface area, correct to 2 decimal places, given that the cylinder is 30 cm wide and 30 cm high.

4 A semi-cylindrical container with no lid has a diameter of 18 cm and a height of 22 cm. Determine the outer surface area of the container, in terms of pi.



5 Cans of baked beans are 7 cm high and 6.5 cm in diameter. The paper labels for the cans wrap around the curved surface, with a 1 cm overlap where they are glued. Determine the number of labels that will fit onto a 1.5 m by 1 m rectangular sheet of paper.

6 Determine the surface area of a quarter cylinder of height and radius 16 cm, in terms of pi.

7.5 Volume and capacity of cylinders

Learning intention: To be able to determine the volume and capacity of cylinders

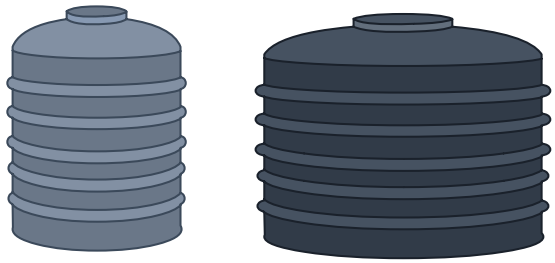
Success criteria:

SC 1 I can apply the cylinder volume formula.

SC 2 I can solve a variety of problems involving the volumes and capacities of cylinders.

Lesson warm-up

Water tanks for sale



Cylindrical water tanks sold through a hardware shop are priced as shown.

The larger tank has a capacity of 5000 L and is sold for $1040. The smaller tank has a capacity of 2000 L and costs $837.

The smaller water tank holds less than half the amount of water, but costs 80% of the price of the larger tank. Explain why this is reasonable or unreasonable.

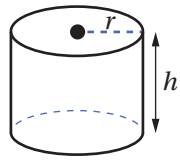
SC 1 I can apply the cylinder volume formula

The formula for the volume of any prism holds for shapes with constant cross-sections, even for bases that have curves.

V = Area of cross-section × height

In the case of a cylinder, the cross-section is a circle, with area πr^2.

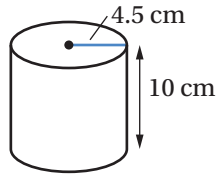
The formula for the volume of a cylinder is: $V = \pi r^2 h$



Worked example

Calculating the volume of a cylinder

Determine the volume of the cylinder, correct to the nearest whole number.



THINKING	WORKING
Recall the formula for the volume of a cylinder.	$V = \pi r^2 h$
Identify the radius and height of the cylinder.	$r = 4.5$ cm, $h = 10$ cm
Substitute the lengths of the radius and height into the formula and calculate.	$V = \pi \times 4.5^2 \times 10$ $= 636.1\ldots$
Write the answer, rounded as directed, with appropriate units.	The volume is 636 cm^3 to the nearest whole number.

SC 2 I can solve a variety of problems involving the volumes and capacities of cylinders

Volume and capacity equivalences:

$1\text{ cm}^3 = 1\text{ mL}$

$1000\text{ cm}^3 = 1\text{ L}$

$1\text{ m}^3 = 1\text{ kL}$

Also, 1000 mL = 1 L and 1000 L = 1 kL

Worked example

Determining the capacity of a cylinder

Determine the capacity of an approximately cylindrical jar of juice, to the nearest millilitre, given that its diameter is 8 cm and its height is 18 cm.



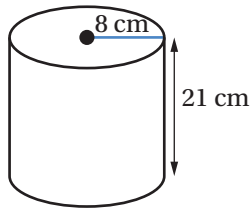
THINKING	WORKING
Recall the formula for the volume of a cylinder.	$V = \pi r^2 h$
Determine the length of the radius and the height.	$d = 8$ cm, so $r = 4$ cm $h = 18$ cm
Substitute into the volume formula and calculate.	$V = \pi \times 4^2 \times 18$ $= 904.7...$
Round the volume to the nearest cubic centimetre.	The volume of space in the can is about 905 cm^3.
Recall the volume capacity equivalence.	$1\text{ cm}^3 = 1\text{ mL}$
Write the answer.	The capacity of the can is 905 mL.

Practice

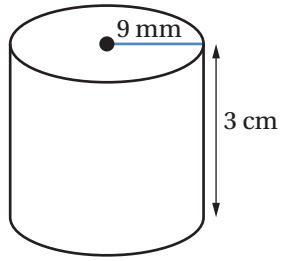
ANSWERS Page 637

SC 1 I can apply the cylinder volume formula

1 A cylinder has a height of 8 cm and a diameter of 21 cm. Determine the volume of the cylinder in cubic centimetres, correct to 2 decimal places.



2 A cylinder has a radius of 9 mm and a height of 3 cm. Convert the radius to centimetres, then determine the volume of the cylinder in cubic centimetres, correct to 2 decimal places.



3 A cylinder just fits inside a cubic box of internal edge length 15 cm. Determine the volume of the cylinder to the nearest whole number.

4 Six cylindrical cans of diameter of 8 cm and height 16 cm are packed into a box in 2×3 formation, with no room to spare. Determine the following.

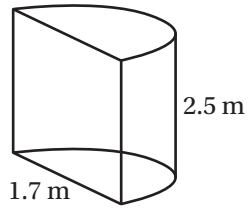
(a) The internal dimensions of the box.

(b) The volume of the box.

(c) The total volume of the six cans, to the nearest cubic centimetre.

(d) The percentage of the interior of the box that is empty space, correct to 1 decimal place.

5 Determine the volume of the half cylinder, correct to 2 decimal places.



7

SC 2 I can solve a variety of problems involving the volumes and capacities of cylinders

1 A semi-cylindrical water trough for a horse is 2.5 m long and 1.8 m wide (both measured internally). Determine the number of kilolitres of water needed to fill the trough, correct to 2 decimal places.

2 Cylinder 1 has a height of 12 cm and radius of 10 cm. Other cylinders have the same volume as Cylinder 1. Determine the following correct to 2 decimal places.

(a) The height of Cylinder 2, given that its radius is 2 cm less than the radius of Cylinder 1

(b) The radius of Cylinder 3, given that its height is 2 cm less than the height of Cylinder 1

3 Ginger beer is sold in small cans of 200 mL. Given that the diameter of the cans is 5 cm, determine their height, to the nearest millimetre.

4 A copper pipe has a diameter of 20 mm and is 2 mm thick. Determine the following.



(a) The amount of copper in a 2 m length of the pipe, to the nearest cubic centimetre

(b) The number of millilitres of water held in the 2 m length of pipe, to the nearest whole number

Understand the relationship between the volume and dimensions of cylinders

7.6

Learning intention: To understand the relationship between the volume and dimensions of cylinders

Success criteria:

SC 1 I can use values in terms of pi to demonstrate the effect of change in linear dimensions on volume.

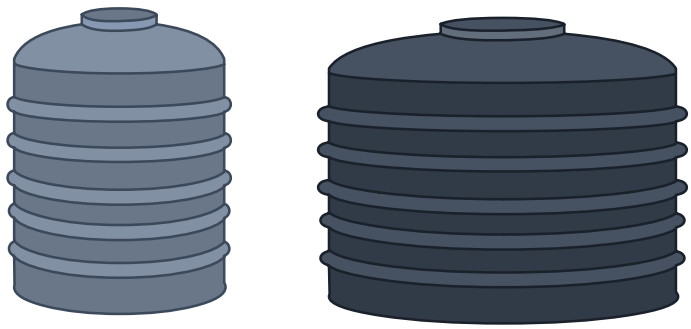
Lesson warm-up

What size?

A cylindrical tank has a volume of 80 m^2.

What could its dimensions be?

What are reasonable values for each dimension (radius, height)?



SC 1 I can use values in terms of pi to demonstrate the effect of change in linear dimensions on volume

The formula for the volume of a cylinder, $V = \pi r^2 h$, can be used to show that increasing the height by a factor produces the same factor increase in the volume; however, because the radius value is squared, increasing the radius (or diameter) by a factor produces a square of the factor increase in the volume.

Worked example

Demonstrating the effect of an increase in the height of a cylinder on its volume

A cylinder has a height of 30 cm and a radius of 12 cm.

(a) Calculate the volume in terms of pi.

THINKING	WORKING
Recall the formula for the volume of a cylinder.	$V = \pi r^2 h$, where $r = 12$ cm, $h = 30$ cm
Substitute the length of the radius and height into the formula and calculate in terms of pi.	$V = \pi \times 12^2 \times 30$ $= 4320\pi \text{ cm}^2$

(b) Determine the new height, given that it is increased by 20%.

THINKING	WORKING
Determine the scale factor.	100% + 20% = 120% 120% = 1.2
Multiply the original height by the scale factor.	New height: 30 cm × 1.2 = 36 cm

(c) Calculate the new volume in terms of pi.

THINKING	WORKING
Substitute the new height into the volume formula and calculate the volume in terms of pi.	New volume, using $h = 36$ cm: $V = \pi \times 12^2 \times 36$ $= 5184\pi \text{ cm}^2$

(d) Determine the percentage increase in volume and reflect on the answer.

THINKING	WORKING
Write the new volume as a percentage of the old volume.	New volume compared to old: $\frac{5184\pi}{4320\pi} \times 100\% = 120\%$
Subtract 100% to determine the percentage increase and compare with the percentage increase in height.	120% − 100% = 20% The 20% increase in height produces the same percentage increase in volume.

Worked example

Demonstrating the effect of an increase in the radius of a cylinder on its volume

A cylinder has a height of 30 cm and a radius of 12 cm.

(a) Calculate the volume in terms of pi.

THINKING	WORKING
Recall the volume for a cylinder.	$V = \pi r^2 h$, where $r = 12$ cm, $h = 30$ cm
Substitute the length of the radius and height into the formula and calculate in terms of pi.	$V = \pi \times 12^2 \times 30$ $= 4320\pi$ cm^3

(b) Increase the radius by 20%.

THINKING	WORKING
Determine the scale factor.	100% + 20% = 120%, 120% = 1.2
Multiply the original height by the scale factor.	New height: 12 cm × 1.2 = 14.4 cm

(c) Calculate the new volume in terms of pi.

THINKING	WORKING
Substitute the new height into the volume formula and calculate the volume in terms of pi.	New volume, using $r = 14.4$ cm: $V = \pi \times 14.4^2 \times 30$ $= 6220.8\pi$ cm^3

(d) Determine the percentage increase in volume and reflect on the answer.

THINKING	WORKING
Write the new volume as a percentage of the old volume.	New volume compared to old: $\frac{6220.8\pi}{4320\pi} \times 100\% = 144\%$
Subtract 100% to determine the percentage increase and compare with the percentage increase in radius.	144% − 100% = 44% The 20% increase in radius produces a 44% increase in volume.

(e) Determine the ratio of the new volume to the old volume.

THINKING	WORKING
State the ratio of the new radius to the old radius.	100% + 20% = 120%, 120% = 1.2 The new to old ratio of radii is 1.2.
Determine the ratio of the new volume to the old volume.	In the formula for volume $V = \pi r^2 h$, the radius is squared. If the radius is increased by 1.2, then the volume will be increased by $1.2^2 = 1.44$.

Practice

ANSWERS Page 637

SC 1 I can use values in terms of pi to demonstrate the effect of change in linear dimensions on volume

1 A cylinder has a height of 3.2 m and a radius of 1.4 m.

(a) Calculate the volume in terms of pi.

(b) Calculate the new volume, in terms of pi, when the height is doubled.

(c) Compare the new volume to the original volume using ratios.

2 A cylinder has a height of 50 cm and a radius of 10 cm.

(a) Calculate the volume in terms of pi.

(b) Calculate the new volume, in terms of pi, when the radius is doubled.

(c) Compare the new volume to the original volume using ratios.

3 A cylinder has a height of 58 mm and a diameter of 90 mm.

(a) Calculate the volume in terms of pi.

(b) Calculate the new volume, in terms of pi, when the height is doubled.

(c) Compare the new volume to the original volume using ratios.

4 A cylinder has a height and diameter of 1 m.

(a) Calculate the volume, in cubic metres, in terms of pi.

(b) Calculate the new volume, in cubic metres, in terms of pi, when the diameter is doubled.

(c) Compare the new volume to the original volume using ratios.

5 Determine the effect on the volume for each of the following changes.

(a) The height is increased by a factor of 10.

(b) The radius is increased by a factor of 10.

(c) The diameter is increased by a factor of 6.

(d) The height is increased by 40%.

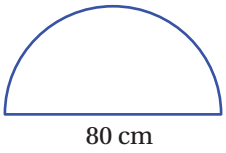
(e) The radius is increased by 40%.

TOPIC REVIEW

Multiple choice

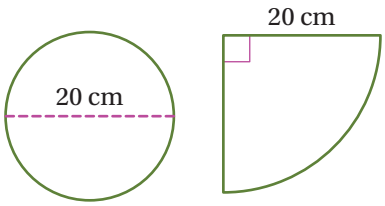
ANSWERS Page 668

1 The perimeter of the semicircle in centimetres is:



A 80π **B** $20\pi + 40$ **C** $80\pi + 80$ **D** $40\pi + 80$

2 The difference between the perimeters of the circle and quarter circle, in centimetres, is:



A $40 - 10\pi$ **B** $10\pi - 20$ **C** $20\pi - 40$ **D** $40\pi - 40$

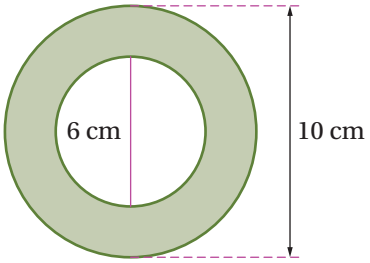
3 The exact area of a circle with a diameter of 25 m is:

A $\frac{625\pi^2}{4}\text{ m}^2$ **B** $\frac{625\pi}{4}\text{ m}^2$ **C** $625\pi^2\text{ m}^2$ **D** $\frac{625\pi}{2}\text{ m}^2$

4 The exact area of a circle with a radius of 75 mm is:

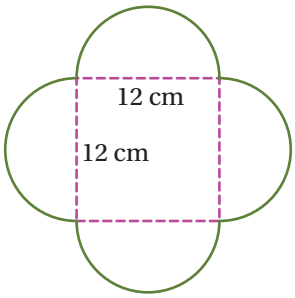
A $75\pi\text{ mm}^2$ **B** $150\pi\text{ mm}^2$ **C** $5625\pi\text{ mm}^2$ **D** $22\,500\pi\text{ mm}^2$

5 The shaded area of this shape, in square centimetres, is:



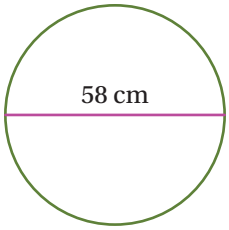
A 64π **B** 16π **C** 8π **D** 4π

6 The area of this shape, in square centimetres, is:



A 24π **B** 72π **C** $48 + 24\pi$ **D** $144 + 72\pi$

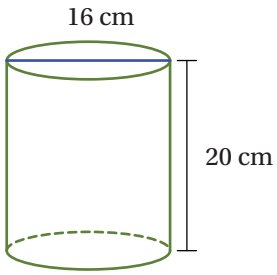
7 The exact circumference of this circle is:



A 58π cm **B** 29π cm **C** 3364π cm **D** 841π cm

Use the following information to answer Questions **8** to **10**.

A net is to be drawn, consisting of two circles and a rectangle, to represent the cylinder below.



8 The area of each circle will be:

A 256π cm^2 **B** 64π cm^2 **C** 32π cm^2 **D** 16π cm^2

9 The shorter side of the rectangle will be:

A 8 cm **B** 16 cm **C** 20 cm **D** 40 cm

10 The longer side of the rectangle will be:

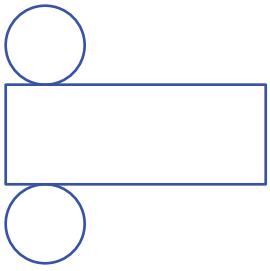
A 256π cm **B** 64π cm **C** 16π cm **D** 32π cm

11 A cylindrical can full of water has a diameter of 30 cm and a height of 20 cm. The number of smaller cylinders of diameter 10 cm and height 5 cm that can be filled from the larger cylinder is:

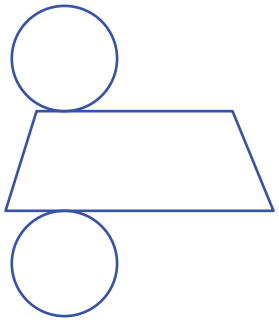
A 12 **B** 48 **C** 144 **D** 36

12 The simplest nets of cylinders involve rectangles, but other shapes are possible. The net that could *not* form a cylinder is:

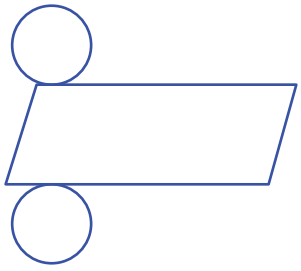
A



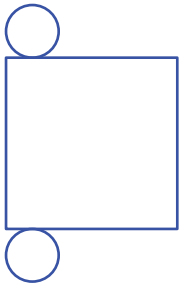
B



C

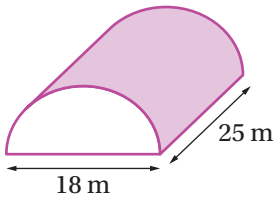


D



Use the following information to answer Questions **13** and **14**.

Consider the semi-cylinder below.



13 The volume, in cubic metres, of the semi-cylinder is:

A 1012.5π **B** 4050π **C** 2025π **D** 225π

14 The surface area, in square metres, of the semi-cylinder is:

A $265.5\pi + 450$ **B** $387\pi + 450$ **C** $531\pi + 450$ **D** $306\pi + 450$

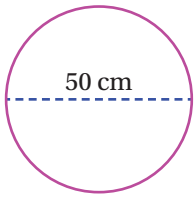
15 The volume of a cone is one third of the volume of a cylinder with the same radius and height. The radius of a cone, 20 cm high, with a volume of 40 000 cm^3 is closest to:

A 1910 cm **B** 25 cm **C** 44 cm **D** 637 cm

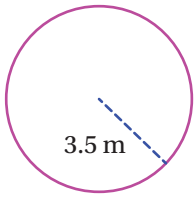
Short answer

1 Calculate the circumference of each circle in terms of pi.

(a)

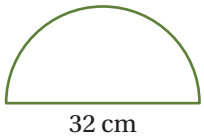


(b)

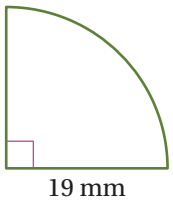


2 Determine the perimeter of each shape in terms of pi.

(a)

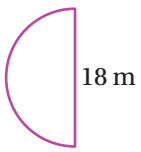


(b)

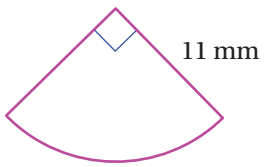


3 Determine the area of each shape in terms of pi.

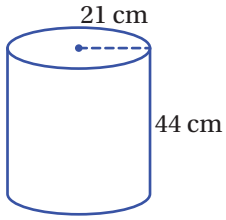
(a)



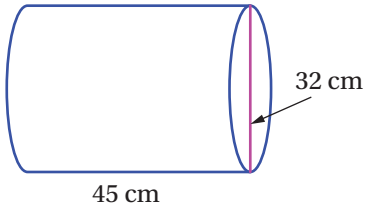
(b)



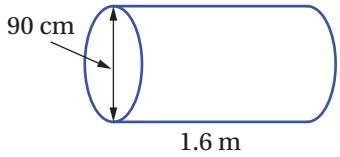
4 Determine the surface area and volume of the cylinder, correct to the nearest whole number each time.



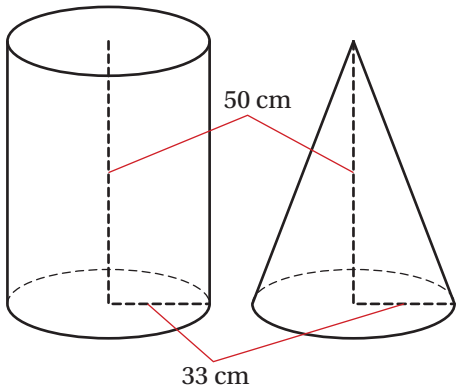
5 Determine the surface area of the cylinder in terms of pi.



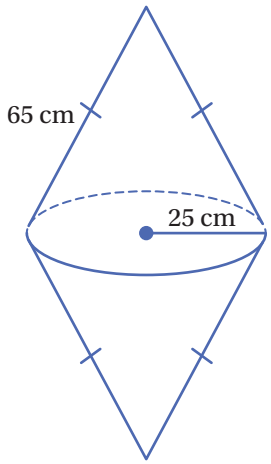
6 Determine the volume of the cylinder, in cubic metres, in terms of pi.



7 Recall that the volume of a cone is one third of the volume of a cylinder with the same radius and height. Calculate the volume of the cylinder in terms of pi, and hence determine the volume of the cone.



8 A double-cone sculpture is made of identical cones.



(a) Use Pythagoras' theorem to determine the height of each cone.

(b) Determine the exact volume of the sculpture.

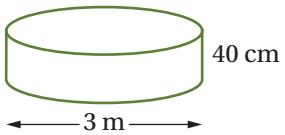
(c) If a cylinder has the same radius and volume as the sculpture, determine its height.

(d) If a cylinder has the same height and volume as the sculpture, determine its exact radius.

9 Determine the capacity of a jar of pickles, to the nearest millilitre, that is 8 cm wide and 12 cm high, internally.

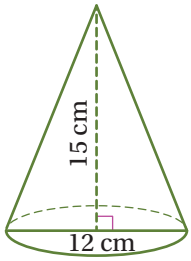
10 A cylinder is to be made from an A4 sheet of coloured cardboard 210 mm by 297 mm. If the height and diameter of the cylinder are both 9 cm, what area of coloured cardboard will be wasted, to the nearest square centimetre?

11 Determine the number of metric cups, 250 mL each, it takes to fill the water tank below.

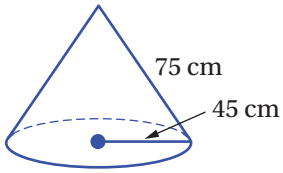


12 Recall that the volume of a cone is one third of the volume of a cylinder with the same radius and height. Determine the volume of each cone, correct to the nearest whole number.

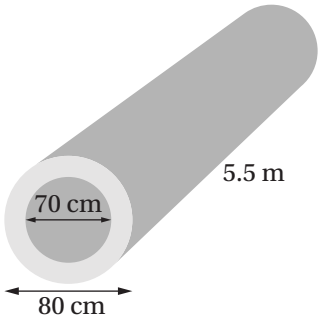
(a)



(b)

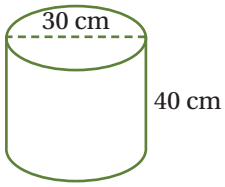


13 Determine the volume of material in the pipe, in cubic metres, correct to 2 decimal places.



Extended response

1 Consider the cylinder below.



(a) Determine the volume in terms of pi.

(b) Determine the volume of a cone with the same diameter and height.

(c) Determine the volume of a square-based prism with the same width and height.

(d) Write a simple ratio of the three volumes, in the order calculated.

2 A celebration cake is to be made using three square-based pans of base lengths 50 cm, 40 cm and 30 cm, respectively. The depth of each tier is to be 15 cm.



(a) Calculate the total volume of the un-iced cake.

(b) Determine the height, to the nearest centimetre, of a single-tier round cake made from the same amount of cake mix in a cake tin of diameter 40 cm.

(c) If both cakes are to be iced on the visible surfaces only, determine the area to be iced for each cake, rounding to the nearest hundred square centimetres, where necessary.

3 Grain silos like the one below can be up to 90 m high, with diameters as much as 27 m.



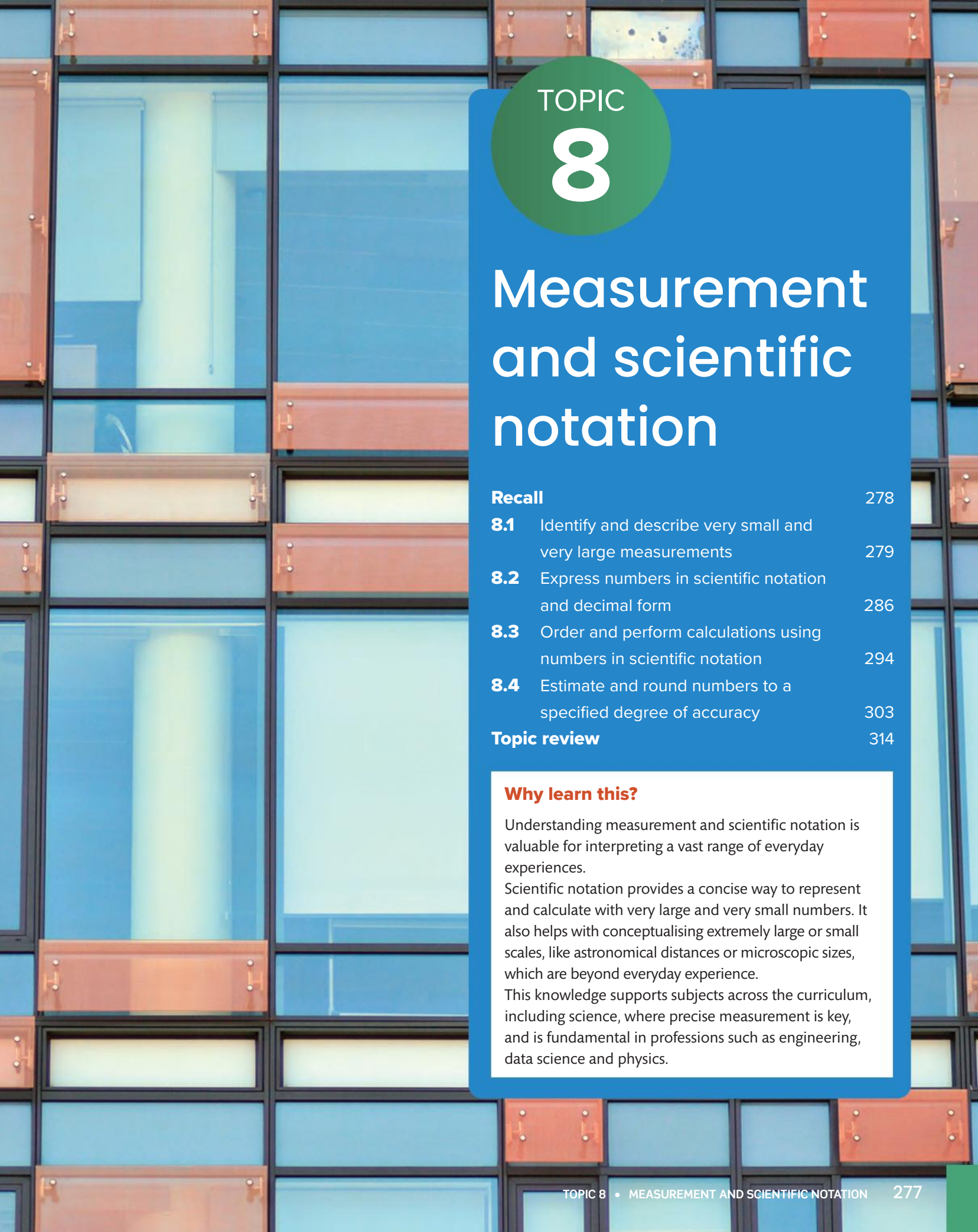
(a) If the cylindrical part of the silo is 30 m high and 10 m wide, determine the volume of grain held in this section, in cubic metres, correct to 2 decimal places.

(b) If the conical parts of the silo have heights of 1 m and 1.5 m, respectively, determine the total volume of grain held in the silo, rounding as before. Use the fact that the volume of a cone is one third of the volume of a cylinder with the same radius and height.

Trucks with trays in the shape of rectangular prisms are to take the grain to various markets.

(c) Determine the volume of grain a truck can carry for each load if the tray dimensions are 2 m by 4 m by 3 m.

(d) Determine the number of loads needed to empty the silo.



TOPIC

8

Measurement and scientific notation

Recall 278

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8.2 Express numbers in scientific notation and decimal form 286

8.3 Order and perform calculations using numbers in scientific notation 294

8.4 Estimate and round numbers to a specified degree of accuracy 303

Topic review 314

Why learn this?

Understanding measurement and scientific notation is valuable for interpreting a vast range of everyday experiences.

Scientific notation provides a concise way to represent and calculate with very large and very small numbers. It also helps with conceptualising extremely large or small scales, like astronomical distances or microscopic sizes, which are beyond everyday experience.

This knowledge supports subjects across the curriculum, including science, where precise measurement is key, and is fundamental in professions such as engineering, data science and physics.

RECALL

ANSWERS Page 638

I can write large powers of 10 using exponent notation

1 Write these values using exponent notation.

(a) 100 **(b)** 1000 **(c)** 1 000 000 **(d)** 1 000 000 000

2 Write the number represented by these powers of 10.

(a) 10^2 **(b)** 10^4 **(c)** 10^5 **(d)** 10^7

I can multiply and divide decimals by powers of 10

1 Write the value of the following multiplications.

(a) 4.85×10 **(b)** 24.768×100 **(c)** 79.6×1000

(d) $0.003\,49 \times 100$ **(e)** 0.8003×1000 **(f)** $0.0104 \times 10\,000$

I can order decimal values

1 Arrange these decimals in ascending order.

3.02, 3.43, 3.76, 3.14, 2.98, 0.39, 3.36

2 Arrange these decimals in descending order.

0.684, 0.667, 0.585, 0.633, 0.701, 0.643, 0.611

I can multiply and divide decimals, rounding the results

1 Evaluate the following, rounding answers correct to 2 decimal places.

(a) 0.76×8.4 **(b)** 65.1×9.35 **(c)** 0.629×6.25 **(d)** 1.05×0.901

2 Evaluate the following, rounding answers correct to 3 decimal places.

(a) $96.8 \div 16.9$ **(b)** $9.12 \div 24.8$ **(c)** $6.17 \div 44.6$ **(d)** $0.36 \div 21.5$

I can estimate the product of decimals

1 Estimate the nearest whole value result for the following products.

(a) 0.9×3.1 **(b)** 2.8×5.8 **(c)** 5.6×7.3 **(d)** 12.09×5.299

Identify and describe very small and very large measurements

8.1

Learning intention: To be able to identify and describe very small and very large measurements

Success criteria:

SC 1 I can identify and describe common measurement prefixes.

SC 2 I can recall prefixes for very small and very large measurement values.

Lesson warm-up

Space is huge

When distances are so large that using standard units makes their comprehension difficult, other units of measure are used.

Distances between locations in space, for example, are so large they are measured using astronomical units, such as 'light years'. 1 light year is the distance light travels over a whole year.

Light travels approximately 300 000 km every second! After a whole year, light can have travelled a distance of 9.46 trillion kilometres (or approximately 9 trillion kilometres).

Find out the following distances in light years.

1. The approximate distance across the Milky Way
2. The approximate distance between Earth and the Andromeda Galaxy (our closest neighbour)
3. The approximate distance between Earth and Sirius (the brightest star in the night sky)

How would these distances be communicated in kilometres?



SC 1 I can identify and describe common measurement prefixes

Prefixes help to communicate quantities more efficiently. Attaching a prefix to values allows measurements of different magnitude to be clearly communicated.

The nearest power of 10 that any number rounds to is referred to as the order of magnitude for that number.

Metric prefixes identify the order of magnitude (power of 10) applied to the base unit of measurement.

Commonly used prefixes in measurement are milli (m), centi (c), kilo (k) and mega (M). These prefixes are placed in front of base units, which include the following.

metres	m	measurements of length/distance
grams	g	measurements of mass
litres	L	measurements of capacity
bytes	B	measurements of data
seconds	s	measurements of time duration

For example, for the base length of 1 metre, the following prefixes may be applied.

1 megametre (1 Mm): the prefix 'mega' means 'one million'	$1\text{ Mm} = 10^6 \times 1\text{ metre}$ $= 1\,000\,000 \times 1\text{ metre}$ $= 1\,000\,000\text{ metres}$	1 Mm rounds to 10 raised to the power of 6 metres, so the order of magnitude is 6.
1 kilometre (1 km): the prefix 'kilo' means 'one thousand'	$1\text{ km} = 10^3 \times 1\text{ metre}$ $= 1000 \times 1\text{ metre}$ $= 1000\text{ metres}$	1 km rounds to 10 raised to the power of 3 metres, so the order of magnitude is 3.
1 centimetre (1 cm): the prefix 'centi' means 'one hundredth'	$1\text{ cm} = 10^{-2} \times 1\text{ metre}$ $= \frac{1}{10^2} \times 1\text{ metre}$ $= \frac{1}{100} \times 1\text{ metre}$ $= 0.01 \times 1\text{ metre}$ $= 0.01\text{ metres}$	1 cm rounds to 10 raised to the power of −2 metres, so the order of magnitude is −2.
1 millimetre (1 mm): the prefix 'milli' means 'one thousandth'	$1\text{ mm} = 10^{-3} \times 1\text{ metre}$ $= \frac{1}{10^3} \times 1\text{ metre}$ $= \frac{1}{1000} \times 1\text{ metre}$ $= 0.001 \times 1\text{ metre}$ $= 0.001\text{ metres}$	1 mm rounds to 10 raised to the power of −3 metres, so the order of magnitude is −3.

Worked example

Identifying and describing common prefixes

Identify the value and prefix associated with these orders of magnitude (powers of 10).

(a) 10^3

THINKING	WORKING
Identify the magnitude of the number.	10^3 is 1000.
Identify the prefix associated with this multiplier.	$\times 10^3$ (or $\times$ 1000) represents the prefix kilo.
Write the answer.	10^3 represents the multiplier $\times$ 1000 and is denoted by the prefix kilo (k).

(b) 10^{-3}

THINKING	WORKING
Identify the magnitude of the number.	10^{-3} is $\frac{1}{10^3} = \frac{1}{1000}$ or 0.001.
Identify the prefix associated with this multiplier.	$\times 10^{-3}$ (or $\times$ 0.001) represents the prefix milli.
Write the answer.	10^{-3} represents the multiplier $\times$ 0.001 and is denoted by the prefix milli (m).

SC 2 I can recall prefixes for very small and very large measurement values

For very small or very large measurements, there are many different prefixes used relating to specific power of 10 multipliers.

Each multiplication of 10 with a *positive* power shifts the value one place value *higher* (in front of the decimal place). The value of the power indicates the number of place values in front of the decimal.

For example:

$$5.0 \times 10^2 = 5.0 \times 10 \times 10$$
$$= 500.0$$

The value 5 is shifted 2 place values higher.

Each multiplication of 10 with a *negative* power shifts the value one place value *lower* (after the decimal place). The value of the power indicates the number of place values lower than the decimal place.

For example:

$$5.0 \times 10^{-2} = 5.0 \times \frac{1}{10 \times 10}$$
$$= 5.0 \times 0.01$$
$$= 0.05$$

The value 5 is shifted 2 place values lower.

Prefix	Symbol		Scientific notation
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^{9}
mega	M	1 000 000	10^{6}
kilo	k	1 000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
--	--	1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}

This table shows how the prefixes with powers of 10 up to ± 12 shift the place value for each unit measurement.

Prefixes like 'giga' and 'tera' are used often in computing to denote the amount of data storage available.

Worked example

Recalling the meaning of prefixes for measurements

1 Identify how many place values higher/lower these prefixes shift any base unit measurement.

(a) Mega (M)

THINKING	WORKING
Identify the multiplier associated with the prefix.	mega (M) = (× 1 000 000)
Identify how this multiplier shifts the measurement value.	× 1 000 000 shifts the base unit measurement 6 place values higher. ($10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1\,000\,000$)
Write the answer.	Mega (M) shifts the base unit measurement 6 place values higher to 1 000 000 m.

(b) Micro (μ)

THINKING	WORKING
Identify the multiplier associated with the prefix.	Micro (μ) = (× 10^{-6}) or (× 0.000 001).
Identify how this multiplier shifts the measurement value.	× 0.000 001 shifts the base measurement 6 place values lower. ($10^{-6} = \frac{1}{10\times10\times10\times10\times10\times10} = 0.000\,001$)
Write the answer.	Micro (μ) shifts the base unit measurement 6 place values lower to 0.000 001 m.

2 Identify the magnitude of the prefix multiplier for each of these measurements and express their value without a prefix.

(a) 3 GB

THINKING	WORKING
Identify the prefix and base units.	3 GB is 3 gigabytes. The prefix is giga, the base unit is bytes.
Identify the magnitude of the multiplier associated with the prefix.	Giga is $10^9 = 1\,000\,000\,000$
Write the answer.	The magnitude of the multiplier associated with the prefix for 3 GB is 10^9 (or 1 000 000 000) bytes. $3\text{ GB} = 3 \times 1\,000\,000\,000\text{ B}$ $= 3\,000\,000\,000\text{ B}$

(b) 3 milliseconds

THINKING	WORKING
Identify the prefix and base units.	3 milliseconds. The prefix is milli (m), and the base unit is seconds.
Identify the magnitude of the multiplier associated with the prefix.	Milli is $10^{-3} = \frac{1}{1000} = 0.001$
Write the answer.	The magnitude of the multiplier associated with the prefix for 3 milliseconds is 10^{-3}(or 0.001) seconds. 3 milliseconds = 3 ms $= 3 \times 0.001$ s $= 0.003$ s

3 Identify the most appropriate prefix to use for these measurements.

(a) 350 000 g

THINKING	WORKING
Identify the prefixes the measurement lies between.	350 000 g is between 1 000 g (1 kg) and 1 000 000 g (1 Mg).
Choose the lower prefix and identify the value needed to make it equal the measurement.	$350\,000\text{ g} = 350 \times 1000\text{ g}$ $= 350 \times 1\text{ kg}$ $= 350\text{ kg}$
Write the answer.	The most appropriate prefix for the measurement 300 000 g is kg.

(b) 0.005 s

THINKING	WORKING
Identify the prefixes the measurement lies between.	0.005 s is between 0.001 s (1 ms) and 0.01 s (1 cs)
Choose the lower prefix and identify the value needed to make it equal the measurement.	$0.005\text{ s} = 5 \times 0.001\text{ s}$ $= 5 \times 1\text{ ms}$ $= 5\text{ ms}$
Write the answer.	The most appropriate prefix for the measurement 0.005 s is ms.

Practice

ANSWERS Page 638

SC 1 I can identify and describe common measurement prefixes

1 Name the prefix used for each of these measurements.

(a) 100 mL **(b)** 10 kg **(c)** 1000 cm **(d)** 10 ML

2 Write the multiplier associated with each of these prefixes.

(a) kilo **(b)** mega

3 Write the multiplier associated with each of these prefixes.

(a) centi **(b)** milli

4 Write each of these large measurements in simpler form using prefixes and units.

(a) 1000 grams **(b)** 1 000 000 bytes

5 Write each of these small measurements in simpler form using prefixes and units.

(a) 0.01 metres **(b)** 0.001 seconds

6 Write the order of magnitude for the following measurements.

(a) 1 mm **(b)** 1 cm **(c)** 1 km **(d)** 1 Mm

7 Identify the prefix that would be used to represent each of these measurement multipliers.

(a) 10^3 L **(b)** 10^{-3} s **(c)** $\frac{1}{10^2}$ m

8 Arrange these distance measurements in order from shortest to longest:

10^2 m, 1 km, 1 mm, 10 000 m, 10 m, 1 cm, 0.1 m, 1 ML

9 Use the prefix multipliers to show that 1 000 000 cm is equal to 10 km.

SC 2 I can recall prefixes for very small and very large measurement values

1 Write the number of place values (higher or lower) these multipliers shift any base unit measurement.

(a) $\times 10^3$ or $\times (10 \times 10 \times 10)$

(b) $\times 10^6$ or $\times (10 \times 10 \times 10 \times 10 \times 10 \times 10)$

(c) $\times 10^{-3}$ or $\times \left(\frac{1}{10 \times 10 \times 10}\right)$

(d) $\times 10^{-6}$ or $\times \left(\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10}\right)$

2 Write the number of place values higher these prefixes shift any base unit measurement.

(a) giga (G) **(b)** tera (T)

3 Write the number of place values lower these prefixes shift any base unit measurement.

(a) nano (n) **(b)** pico (p)

4 Write the magnitude of the prefix multiplier applied to these measurements.

(a) 2 megabytes **(b)** 5 microseconds **(c)** 4.5 gigalitres **(d)** 32 nanometres

5 Write the value of these measurements without a prefix.

(a) 2 megabytes **(b)** 5 microseconds **(c)** 4.5 gigalitres **(d)** 32 nanometres

6 Write down the prefix that would be most appropriate to use for measurements between these pairs of multipliers.

(a) 10^3 and 10^6 **(b)** 10^6 and 10^9 **(c)** 10^9 and 10^{12}

(d) 10^{-6} and 10^{-3} **(e)** 10^{-9} and 10^{-6} **(f)** 10^{-12} and 10^{-9}

7 Write the prefix that is the most appropriate to use for these measurements.

(a) 25 000 metres **(b)** 480 000 grams **(c)** 128 000 000 bytes

8 Write the prefix that is the most appropriate to use for these measurements.

(a) 0.02 litres **(b)** 0.002 metres **(c)** 0.000 02 seconds

8.2 Express numbers in scientific notation and decimal form

Learning intention: To be able to express numbers in scientific notation and decimal form

Success criteria:

SC 1 I can represent very small and very large numbers using scientific notation.

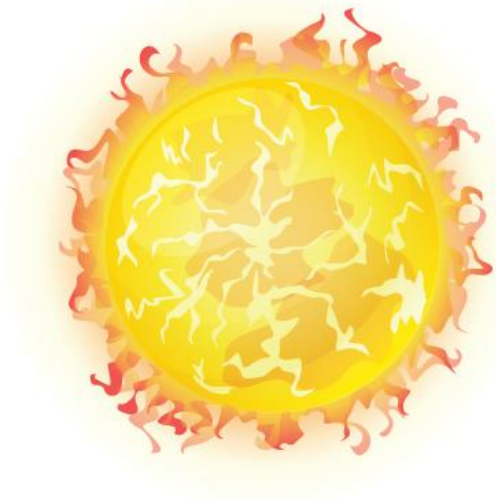
SC 2 I can convert numbers written using scientific notation into decimal form.

Lesson warm-up

Earth is very small

The Milky Way – the galaxy in which our Solar System sits – is one of billions in the known universe. The Sun is the largest object in our Solar System.

Drawings like this are common, however they will never truly show the difference in scale.



Sun



Earth

The Sun

If the Sun were hollow, it could hold approximately 1 300 000 Earths by volume inside it.

The Milky Way

By volume, the entire Milky Way could hold approximately
6 700 000 000 000 000 000 000 000 000 000 000 000 000 000 Earths!!

Saying this the other way, Earth is 0.000 000 000 000 000 000 000 000 000 000 000 000 000 000 149 the volume of the Milky Way.

The numbers involved in comparing celestial objects are extremely large, and tiring to write out. How could we write these numbers in a simpler way?

SC 1 I can represent very small and very large numbers using scientific notation

Scientific notation is a useful way of presenting very small or very large numbers without writing all the numerals of insignificant place values.

Scientific notation consists of powers of 10 combined with the significant figures for the given value.

Scientific notation for very large numbers

Large numbers are written as a decimal between 1 and less than 10, multiplied by a positive power of 10.

The decimal is formed using the significant figures of the large number.

For example, the significant figures for the number 46 000 are 46. The first non-zero digits reading from the left are where the significant figures for large values start.

These significant figures written as a value from 1 to less than 10, is 4.6.

To make 4.6 equal to 46 000, it is multiplied by 10 000, or 10^4.

46 000 written using scientific notation is 4.6×10^4.

The positive power of 10 shows the number of higher place values the decimal value (4.6) shifts to get to the original value (46 000).

$4.6 \times 10^1 = 46$

$4.6 \times 10^2 = 460$

$4.6 \times 10^3 = 4600$

$4.6 \times 10^4 = 46\,000$

Worked example

Writing very large numbers using scientific notation

Write the following large numbers using scientific notation.

(a) 35 000 000

THINKING	WORKING
Identify the significant figures for the large number.	For 35 000 000, the digits 3 and 5 are significant.
Write the significant figures as a decimal value from 1 to less than 10.	3.5
Identify the positive power of 10 that the significant figures are multiplied by to equal the original large number.	$35\,000\,000 = 3.5 \times 10\,000\,000$ $= 3.5 \times 10^7$
Write the answer.	35 000 000 written using scientific notation is 3.5×10^7.

(b) 1 075 000

THINKING	WORKING
Identify the significant figures for the large number.	For 1 075 000, the digits 1, 0, 7 and 5 are significant.
Write the significant figures as a decimal value from 1 to less than 10.	1.075
Identify the positive power of 10 that the significant figures are multiplied by to equal the original large number.	$1\,075\,000 = 1.075 \times 1\,000\,000$ $= 1.075 \times 10^6$
Write the answer.	1 075 000 written using scientific notation is 1.075×10^6.

Scientific notation for very small numbers

Scientific notation for small numbers is written as a decimal from 1 to less than 10, multiplied by a negative power of 10.

The decimal is formed using the significant figures of the small number.

For example, the significant figures for the number 0.000 46 are 4 and 6. The first non-zero digits reading from the left are where the significant figures for small decimal values start.

These significant figures written as a value from 1 to less than 10 is 4.6.

To make 4.6 equal to 0.000 46, it is multiplied by 0.0001, or 10^{-4}.

Therefore, 0.000 46 written using scientific notation is 4.6×10^{-4}.

The negative power of 10 shows the number of lower place values the significant figures (4.6) shift to get to the original value (0.000 46).

$4.6 \times 10^{-1} = 0.46$

$4.6 \times 10^{-2} = 0.046$

$4.6 \times 10^{-3} = 0.0046$

$4.6 \times 10^{-4} = 0.000\,46$

Worked example

Writing very small numbers using scientific notation

Write the following small numbers using scientific notation.

(a) 0.000 000 35

THINKING	WORKING
Identify the significant figures for the small number.	For 0.000 000 35, the digits 3 and 5 are significant.
Write the significant figures as a decimal value from 1 to less than 10.	3.5

Identify the negative power of 10 that the significant figures are multiplied by to equal the original small number.	$0.000\,000\,35 = 3.5 \times 0.000\,000\,1$ $= 3.5 \times 10^{-7}$
Write the answer.	0.000 000 35 written using scientific notation is 3.5×10^{-7}.

(b) 0.000 107 5

THINKING	WORKING
Identify the significant figures for the small number.	For 0.000 107 5, the digits 1, 0, 7 and 5 are significant.
Write the significant figures as a decimal value from 1 to less than 10.	1.075
Identify the negative power of 10 that the significant figures are multiplied by to equal the original small number.	$0.000\,107\,5 = 1.075 \times 0.0001$ $= 1.075 \times 10^{-4}$
Write the answer.	0.000 107 5 written using scientific notation is 1.075×10^{-4}.

Worked example

Writing very large and very small numbers in context using scientific notation

Write the following measurements using scientific notation.

(a) 35 ML

THINKING	WORKING
Identify the prefix and the units.	35 ML is 35 megalitres.
Identify the multiplier associated with the prefix.	35 megalitres = 35×10^6 litres
Identify the significant figures for the large measurement.	For 35×10^6 (35 000 000), the digits 3 and 5 are significant.
Write the significant figures as a decimal value from 1 to less than 10.	3.5
Identify the positive power of 10 that the decimal value is multiplied by to equal the original large number.	$35\,000\,000 = 3.5 \times 10\,000\,000$ $= 3.5 \times 10^7$
Write the answer.	35 ML written using scientific notation is 3.5×10^7 L.

(b) 107 μs

THINKING	WORKING
Identify the prefix and the units.	107 μs is 107 microseconds.
Identify the multiplier associated with the prefix.	107 microseconds = 107×10^{-6} seconds
Identify the significant figures for the small measurement.	For 107×10^{-6} (0.000 107), the digits 1, 0 and 7 are significant.
Write the significant figures as a decimal value from 1 to less than 10.	1.07
Identify the negative power of 10 the decimal value is multiplied by to equal the original small number.	$0.000\,107 = 1.07 \times 0.0001$ $= 1.07 \times 10^{-4}$
Write the answer.	107 μs written using scientific notation is 1.07×10^{-4}s.

SC 2 I can convert numbers written using scientific notation into decimal form

Values written using scientific notation include multiplication by a power of 10. The magnitude of the power (n) indicates the number of places the significant figures shift.

Positive powers of 10 shift the value n place values higher.

Negative powers of 10 shift the value n place values lower.

Positive powers of 10

Express 2.0×10^3 in its equivalent decimal form.

The power of 10 is 3 (positive).

Therefore, the significant figures are shifted 3 place values higher.

$2.0 \times 10^3 = 2.0 \times 1000$

$= 2000$

This gives the appearance that the decimal point has moved 3 places to the right.

$2.0 \times 10^3 = 2\,0\,0\,0.$ (3)

$= 2000$

Importantly, it is not the decimal point that moves. Instead, the significant figure 2 increases by a factor of 1000, shifting it three place values higher to become 2000.

Negative powers of 10

Express 2.0×10^{-3} in its equivalent decimal form.

The power of 10 is –3 (negative).

Therefore, the significant figures are shifted 3 place values lower.

$$\begin{aligned} 2.0 \times 10^{-3} &= 2.0 \times \frac{1}{1000} \\ &= 2.0 \times 0.001 \\ &= 0.002 \end{aligned}$$

This gives the appearance that the decimal point has moved 3 places to the left.

$$\begin{aligned} 2.0 \times 10^{-3} &= 0.\overset{3}{\curvearrowleft}002 \\ &= 0.002 \end{aligned}$$

Importantly, it is not the decimal point that moves. Instead, the significant figure 2 decreases by a factor of 1000, shifting it three place values lower to become 0.002.

Always remember to include the 0 in the ones place for values less than 1.

Worked example

Converting numbers from scientific notation into decimal form

Convert these numbers written using scientific notation into decimal form.

(a) 1.005×10^6

THINKING	WORKING
Identify the magnitude and sign for the power of 10 multiplier.	$\times 10^6$, power is +6
Interpret how this power will adjust the significant figures.	$\times 10^6$ shifts the significant figures (1.005), 6 place values higher.
Adjusting the significant figures 6 place values higher gives the appearance of the decimal point moving 6 places to the right.	$1.005 \times 10^6 = 1005000.\overset{6}{\curvearrowleft}$
Write the answer.	$1.005 \times 10^6 = 1\,005\,000$ in decimal form

(b) 1.005×10^{-6}

THINKING	WORKING
Identify the magnitude and sign for the power of 10 multiplier.	$\times 10^{-6}$, power is –6
Interpret how this power will adjust the value of the significant figures.	$\times 10^{-6}$ shifts the significant figures (1.005), 6 place values lower.
Adjusting the significant figures 6 place values lower gives the appearance of the decimal point moving 6 places to the left. Ensure that a 0 is written in the ones place for the decimal form for all values less than 1.	$1.005 \times 10^{-6} = 0.000001005$ (6)
Write the answer.	$1.005 \times 10^{-6} = 0.000\,001\,005$ in decimal form

Practice

ANSWERS Page 638

SC 1 I can represent very small and very large numbers using scientific notation

1 Write these large numbers using scientific notation.

(a) 60 000 **(b)** 600 000 **(c)** 6 000 000 **(d)** 60 000 000

2 Write these small numbers using scientific notation.

(a) 0.0004 **(b)** 0.000 04 **(c)** 0.000 004 **(d)** 0.000 000 4

3 Write these numbers using scientific notation.

(a) 750 000 **(b)** 19 500 000 **(c)** 307 100 000

4 Write these numbers using scientific notation.

(a) 0.000 55 **(b)** 0.000 083 2 **(c)** 0.000 004 509

Use this prefix table to help with Questions **5** to **8**.

Prefix	Symbol		Scientific notation
tera	T	1 000 000 000 000	10^{12}
giga	G	1 000 000 000	10^{9}
mega	M	1 000 000	10^{6}
kilo	k	1 000	10^{3}
hecto	h	100	10^{2}
deca	da	10	10^{1}
--	--	1	10^{0}
deci	d	0.1	10^{-1}
centi	c	0.01	10^{-2}
milli	m	0.001	10^{-3}
micro	μ	0.000 001	10^{-6}
nano	n	0.000 000 001	10^{-9}
pico	p	0.000 000 000 001	10^{-12}

5 Write each of these large measurements using scientific notation and the base unit.

(a) 20 km **(b)** 50 GB

6 Write each of these small measurements using scientific notation and the base unit.

(a) 20 mL **(b)** 50 nm

7 Write each of these large measurements using scientific notation and the base unit.

(a) 2050 ML **(b)** 8903 Gs **(c)** 0.05 Tm

8 Write each of these small measurements using scientific notation and the base unit.

(a) 955 mg **(b)** 1005 ps **(c)** 0.05 μm

SC 2 I can convert numbers written using scientific notation into decimal form

1 How many place values higher will each of the significant figures for these scientific notation values move when converted into decimal form?

(a) 5.0×10^3 **(b)** 5.0×10^6 **(c)** 5.0×10^9

2 Convert each of these values from scientific notation into decimal form.

(a) 5.0×10^3 **(b)** 5.0×10^6 **(c)** 5.0×10^9

3 How many place values lower will each of the significant figures for these scientific notation values move when converted into decimal form?

(a) 4.0×10^{-2} **(b)** 4.0×10^{-5} **(c)** 4.0×10^{-7}

4 Convert each of these values from scientific notation into decimal form.

(a) 4.0×10^{-2} **(b)** 4.0×10^{-5} **(c)** 4.0×10^{-7}

5 Convert each of these large values from scientific notation into decimal form.

(a) 3.4×10^5 **(b)** 6.05×10^7 **(c)** 1.1052×10^9

6 Convert each of these large values from scientific notation into decimal form.

(a) 4.5×10^{-3} **(b)** 8.91×10^{-5} **(c)** 2.905×10^{-9}

8.3 Order and perform calculations using numbers in scientific notation

Learning intention: To be able to order and perform calculations using numbers in scientific notation

Success criteria:

SC 1 I can order numbers written in scientific notation using practical contexts.

SC 2 I can perform calculations involving numbers written using scientific notation.

Lesson warm-up

From tiny to mighty

Order the objects below from smallest to largest.



SC 1 I can order numbers written in scientific notation using practical contexts

Scientific notation is used to represent a wide range of measurements, from very small through to very large.

The distance of objects from a common point can be arranged into ascending and descending order of magnitude quickly by comparing the measurements using scientific notation.

To order values given in scientific notation by magnitude, first compare the power of 10 multipliers (their order of magnitude). The greater the power, the larger the number.

If two numbers have the same power of 10, then compare their significant figures. Consider, for example, 5.0×10^4 and 5.5×10^4.

Worked example

Comparing and ordering large measurements given using scientific notation

(a) Arrange the following three values into ascending order.

8.25×10^7, 8.52×10^6, 2.85×10^8

THINKING	WORKING
Identify the order of magnitude for each number using the power of 10 multipliers.	8.25×10^7: order of magnitude is 7 8.52×10^6: order of magnitude is 6 2.85×10^8: order of magnitude is 8
Check whether any of the values have the same order of magnitude.	No repeated orders of magnitude.
Arrange the numbers in ascending order by their order of magnitude. Ascending means from smallest to highest.	8.52×10^6, 8.25×10^7, 2.85×10^8
Write the answer.	The three values arranged into ascending order are: 8.52×10^6, 8.25×10^7, 2.85×10^8.

(b) Arrange the following three values into descending order.
8.25×10^7, 8.52×10^7, 2.85×10^8

THINKING	WORKING
Identify the order of magnitude for each number using the power of 10 multipliers.	8.25×10^7: order of magnitude is 7 8.52×10^7: order of magnitude is 7 2.85×10^8: order of magnitude is 8
Check whether any of the values have the same order of magnitude.	Two of the numbers have the same order of magnitude.
Arrange the numbers in descending order by their order of magnitude. Descending means from highest to smallest.	2.85×10^8, 8.25×10^7, 8.52×10^7
Arrange the numbers with matching order of magnitude in descending order by their significant figures.	2.85×10^8, 8.52×10^7, 8.25×10^7
Write the answer.	The three values arranged into descending order are: 2.85×10^8, 8.52×10^7, 8.25×10^7.

Worked example

Comparing and ordering small measurements given using scientific notation

(a) Arrange the following three values into ascending order.
8.25×10^{-7}, 8.52×10^{-6}, 2.85×10^{-8}

THINKING	WORKING
Identify the order of magnitude for each number by comparing the power of 10 multipliers.	8.25×10^{-7}: order of magnitude is -7 8.52×10^{-6}: order of magnitude is -6 2.85×10^{-8}: order of magnitude is -8
Check whether any of the values have the same order of magnitude.	No repeated orders of magnitude.
Arrange the numbers in ascending order by their order of magnitude. Note: $10^{-7} > 10^{-8}$ Ascending means from smallest to highest.	2.85×10^{-8}, 8.25×10^{-7}, 8.52×10^{-6}
Write the answer.	The three values arranged into ascending order are: 2.85×10^{-8}, 8.25×10^{-7}, 8.52×10^{-6}.

(b) Arrange the following three values into descending order:
8.25×10^{-7}, 8.52×10^{-8}, 2.85×10^{-8}

THINKING	WORKING
Identify the order of magnitude for each number by comparing the power of 10 multipliers.	8.25×10^{-7}: order of magnitude is -7 8.52×10^{-8}: order of magnitude is -8 2.85×10^{-8}: order of magnitude is -8
Check whether any of the values have the same order of magnitude.	Two of the numbers have the same order of magnitude.
Arrange the numbers in descending order by their order of magnitude. Note: $10^{-8} < 10^{-7}$ Descending means from highest to smallest.	8.25×10^{-7}, 8.52×10^{-8}, 2.85×10^{-8}
Arrange the numbers with matching order of magnitude in descending order by their significant figures.	8.25×10^{-7}, 8.52×10^{-8}, 2.85×10^{-8}
Write the answer.	The three values arranged into descending order are: 8.25×10^{-7}, 8.52×10^{-8}, 2.85×10^{-8}.

Comparing integer values in scientific notation

Some measurements have both magnitude and direction (such as temperature and water level).

Their order of magnitude is compared in the same manner; however, if one or more values represent negative numbers, then the interpretation changes slightly when ordering.

For example:

$-5.0 \times 10^4 < -5.0 \times 10^2$, because $-50\,000 < -500$

The order of magnitude for the first value is greater; however, the negative sign out the front makes it represent a value further away from 0 in the negative direction than the second value. That makes the first value less than the second value.

Worked example

Comparing and ordering integer values given using scientific notation

Arrange the following integer values into ascending order:
8.0×10^6, -8.0×10^8, 7.0×10^7, -5.0×10^6

THINKING	WORKING
Identify the sign for each number (negative or positive).	$-8.0 \times 10^8 < 0$, $-5.0 \times 10^6 < 0$ $8.0 \times 10^6 > 0$, $7.0 \times 10^7 > 0$
Identify the order of magnitude for each number using the power of 10 multipliers.	-8.0×10^8: order of magnitude is 8 -5.0×10^6: order of magnitude is 6 8.0×10^6: order of magnitude is 6 7.0×10^7: order of magnitude is 7
For the positive and negative numbers, arrange the numbers in ascending order by their order of magnitude. Ascending means from smallest to highest. For a negative number, the higher the order of magnitude, the smaller the value.	Arrange the negative numbers: -8.0×10^8, -5.0×10^6 Arrange the positive numbers: 8.0×10^6, 7.0×10^7
Write the answer.	The integers arranged into ascending order are: -8.0×10^8, -5.0×10^6, 8.0×10^6, 7.0×10^7.

SC 2 I can perform calculations involving numbers written using scientific notation

The exponent laws are used to estimate the value of calculations containing scientific notation.

When multiplying two terms written in scientific notation, the significant values and powers of 10 are multiplied separately first.

The significant number parts are multiplied, and the exponent laws are used to simplify the powers.

If the product of the significant number parts is 10 or greater, the number is re-written to align with scientific notation rules.

When dividing two terms written in scientific notation, the same approach is used.

Worked example

Multiplying numbers written using scientific notation

Perform these calculations, leaving the answer in scientific notation.

(a) $(2.0 \times 10^3) \times (3.5 \times 10^4)$

THINKING	WORKING
Rearrange the multiplication so that the significant number parts are together, and the powers of 10 are together.	$(2.0 \times 10^3) \times (3.5 \times 10^4)$ $= (2.0 \times 3.5) \times (10^3 \times 10^4)$
Multiply the significant number parts and simplify the powers of 10. $(a^n \times a^m = a^{n+m})$	$= 7.0 \times 10^{(3+4)}$ $= 7.0 \times 10^7$
Write the answer.	$(2.0 \times 10^3) \times (3.5 \times 10^4) = 7.0 \times 10^7$

(b) $(8.2 \times 10^3) \times (3.5 \times 10^4)$

THINKING	WORKING
Rearrange the multiplication so that the significant number parts are together, and the powers of 10 are together.	$(8.2 \times 10^3) \times (3.5 \times 10^4)$ $= (8.2 \times 3.5) \times (10^3 \times 10^4)$
Multiply the significant number parts and simplify the powers of 10. $(a^n \times a^m = a^{n+m})$	$= 28.7 \times 10^{(3+4)}$ $= 28.7 \times 10^7$
Adjust this answer to meet the requirements for scientific notation.	$= 2.87 \times 10^1 \times 10^7$ $= 2.87 \times 10^{(1+7)}$ $= 2.87 \times 10^8$
Write the answer.	$(8.2 \times 10^3) \times (3.5 \times 10^4) = 2.87 \times 10^8$

Worked example

Dividing numbers written using scientific notation

Perform these calculations, leaving the answer in scientific notation.

(a) $(4.8 \times 10^5) \div (2.0 \times 10^3)$

THINKING	WORKING
Rearrange the expression, pairing the significant figures and the powers of 10.	$(4.8 \times 10^5) \div (2.0 \times 10^3)$ $= (4.8 \div 2.0) \times (10^5 \div 10^3)$
Divide the significant number parts and simplify the powers of 10. $(a^n \div a^m = a^{n-m})$	$= 2.4 \times 10^{(5-3)}$ $= 2.4 \times 10^2$
Write the answer.	$(4.8 \times 10^5) \div (2.0 \times 10^3) = 2.4 \times 10^2$

(b) $(1.7 \times 10^9) \div (6.8 \times 10^6)$

THINKING	WORKING
Rearrange the expression, pairing the significant figures and the powers of 10.	$(1.7 \times 10^9) \div (6.8 \times 10^6)$ $= (1.7 \div 6.8) \times (10^9 \div 10^6)$
Divide the significant number parts and simplify the powers of 10. $(a^n \div a^m = a^{n-m})$	$= 0.25 \times 10^{(9-6)}$ $= 0.25 \times 10^3$
Adjust this answer to meet the requirements for scientific notation.	$= 2.5 \times 10^{-1} \times 10^3$ $= 2.5 \times 10^{(-1+3)}$ $= 2.5 \times 10^2$
Write the answer.	$(1.7 \times 10^9) \div (6.8 \times 10^6) = 2.5 \times 10^2$

Worked example

Calculating with real numbers in scientific notation

Perform these calculations, leaving the answer in scientific notation.

(a) $5 \times (8.5 \times 10^3)$

THINKING	WORKING
Rewrite as the product of two values in scientific notation.	$5 \times (8.5 \times 10^3)$ $= (5 \times 10^0) \times (8.5 \times 10^3)$
Rearrange the expression, pairing the significant figures and the powers of 10.	$= (5 \times 8.5) \times (10^0 \times 10^3)$
Multiply the significant number parts and simplify the powers of 10. $(a^n \times a^m = a^{n+m})$	$= 42.5 \times 10^{(0+3)}$ $= 42.5 \times 10^3$
Adjust this answer to meet the requirements for scientific notation.	$= 4.25 \times 10^1 \times 10^3$ $= 4.25 \times 10^{(1+3)}$ $= 4.25 \times 10^4$
Write the answer.	$5 \times (8.5 \times 10^3) = 4.25 \times 10^4$

(b) $(4.0 \times 10^4) \div 5$

THINKING	WORKING
Rewrite as the quotient of two values in scientific notation.	$(4.0 \times 10^4) \div 5$ $= (4.0 \times 10^4) \div (5 \times 10^0)$
Rearrange the expression, pairing the significant figures and the powers of 10.	$= (4.0 \div 5) \times (10^4 \div 10^0)$
Divide the significant number parts and simplify the powers of 10. $(a^n \div a^m = a^{n-m})$	$= 0.8 \times 10^{(4-0)}$ $= 0.8 \times 10^4$
Adjust this answer to meet the requirements for scientific notation.	$= 8.0 \times 10^{-1} \times 10^4$ $= 8.0 \times 10^{(-1+4)}$ $= 8.0 \times 10^3$
Write the answer.	$(4.0 \times 10^4) \div 5 = 8.0 \times 10^3$

Practice

ANSWERS Page 638

SC 1 I can order numbers written in scientific notation using practical contexts

1 Write the correct symbol '>' or '<' on the line between each pair of numbers.

(a) 6.0×10^3 ___ 6.0×10^2

(b) 4.0×10^4 ___ 4.0×10^5

(c) 8.5×10^5 ___ 1.5×10^6

(d) 8.5×10^8 ___ 9.5×10^7

2 Write the correct symbol '>' or '<' on the line between each pair of numbers.

(a) 5.0×10^3 ___ 7.5×10^3

(b) 8.3×10^6 ___ 9.0×10^6

(c) 7.2×10^5 ___ 4.0×10^5

(d) 2.0×10^{10} ___ 1.5×10^{10}

3 Write the correct symbol '>' or '<' on the line between each pair of numbers.

(a) 5.0×10^{-3} ___ 5.0×10^{-2}

(b) 4.0×10^{-6} ___ 4.0×10^{-7}

(c) 2.5×10^{-2} ___ 5.0×10^{-3}

(d) 3.8×10^{-5} ___ 1.5×10^{-4}

4 Write the correct symbol '>' or '<' on the line between each pair of numbers.

(a) -5.0×10^3 ___ -5.0×10^2

(b) -2.5×10^3 ___ -3.0×10^3

(c) 1.0×10 ___ -2.5×10^2

(d) -1.0×10^2 ___ -2.5×10^2

5 Write the correct symbol '>' or '<' on the line between each pair of numbers.

(a) 4.0×10^4 ___ 5.0×10^{-5}

(b) -4.0×10^4 ___ -5.0×10^5

(c) -4.0×10^4 ___ -5.0×10^{-5}

(d) -4.0×10^{-4} ___ -5.0×10^{-5}

6 Arrange the following approximate planet distances from Earth into ascending order.

Mercury: 8.25×10^7 km Saturn: 1.20×10^9 km Venus: 3.98×10^7 km

Jupiter: 5.91×10^8 km Mars: 5.57×10^7 km

7 Arrange these moons' recorded surface temperatures into descending order.

Io: -1.36×10^2 °C Titan: -1.79×10^2 °C Callisto: -1.39×10^2 °C

Miranda: -2.04×10^2 °C Europa: -1.43×10^2 °C

8 Arrange each of these microorganisms into ascending order by size.

Paramecium: 2.5×10^{-4} m Rhinovirus: 3.0×10^{-8} m

Lactobacillus: 2.0×10^{-6} m Rabies: 1.5×10^{-7} m 0.15 μm (micro (μ) is $\times 10^{-6}$)

Red blood cell: 8.0×10^{-6} m Poliomyelitis: 50 nm (nano (n) is $\times 10^{-9}$)

Influenza: 1.0×10^{-7} m Pollen: 0.09 mm (milli (m) is $\times 10^{-3}$)

SC 2 I can perform calculations involving numbers written using scientific notation

1 Calculate these products and write the answers using scientific notation.

(a) $(2.0 \times 10^2) \times (3.0 \times 10^3)$ **(b)** $(4.0 \times 10^2) \times (2.0 \times 10^3)$ **(c)** $(3.0 \times 10^5) \times (3.0 \times 10^2)$

2 Calculate these quotients and write the answers using scientific notation.

(a) $(6.0 \times 10^5) \div (2.0 \times 10^3)$ **(b)** $(8.0 \times 10^5) \div (2.0 \times 10^3)$ **(c)** $(9.0 \times 10^7) \div (3.0 \times 10^5)$

3 Calculate these products and write the answers using scientific notation.

(a) $(1.4 \times 10^3) \times (2.5 \times 10^7)$ **(b)** $(2.5 \times 10^4) \times (3.5 \times 10^3)$ **(c)** $(4.8 \times 10) \times (1.2 \times 10^5)$

4 Calculate these quotients and write the answers using scientific notation.

(a) $(9.6 \times 10^7) \div (3.2 \times 10^3)$ **(b)** $(7.2 \times 10^3) \div (1.2 \times 10)$ **(c)** $(8.4 \times 10^6) \div (2.0 \times 10^3)$

5 Write the value of these calculations in scientific notation.

(a) $(3.0 \times 10^5) \times (5.0 \times 10^3)$ **(b)** $(2.4 \times 10^3) \times (7.5 \times 10^7)$ **(c)** $(5.8 \times 10^6) \times (9.1 \times 10^2)$

(d) $(1.5. \times 10^5) \div (7.5 \times 10^2)$ **(e)** $(1.1 \times 10^8) \div (1.6 \times 10^3)$ **(f)** $(2.3 \times 10^2) \div (9.2 \times 10^5)$

6 Write the value of these calculations in scientific notation.

(a) $3 \times (1.5 \times 10^5)$ **(b)** $(4.2 \times 10^6) \times 8$ **(c)** $(8.2 \times 10^4) \div 2$ **(d)** $15 \div (5 \times 10^{-2})$

7 Calculate the distances.

(a) How many leaps of 2.5×10^2 mm will it take a flea to travel a total distance of 2.0×10^3 mm?

(b) How far has the flea travelled (in scientific notation) after 15 of the same distanced leaps?

8 A passenger jet completes the journey between Melbourne airport and Los Angeles airport (1.28×10^7 m) 24 times over a 2-week period. Answer the following.

(a) How many metres did it fly in total for those 24 flights, in scientific notation?

(b) How many more flights did it take for the plane to travel 4.608×10^8 m?

Estimate and round numbers to a specified degree of accuracy

8.4

Learning intention: To be able to estimate and round numbers to a specified degree of accuracy

Success criteria:

SC 1 I can use the language of estimation and round numbers to a specified number of significant figures.

SC 2 I can compare the effect that truncating or rounding of values during calculations can have on the accuracy of the results.

Lesson warm-up

Waiting on a friend

You and a friend have arranged a catch up at a local park to go for a walk together.

Upon texting them that you have arrived, they reply with 'see you in a bit'.

Is this a precise way to communicate their likely arrival time?

How long would you expect to be waiting for your friend following that information? Hours, minutes, seconds?

Assuming your phone battery has now died, how long do you think you would wait for your friend based on that last communication before deciding to start walking?

What factors might affect your decision?



SC 1 I can use the language of estimation and round numbers to a specified number of significant figures

You do not always need to be exact when communicating measurements like distances or times. Approximations often provide a 'near enough' estimation in different contexts.

Terms such as 'approximately', 'roughly', 'about' and 'close to' are all ways to indicate that a measurement being communicated is not an exact value.

When measurements are communicated using this language, a degree of rounding has taken place.

Only the information that is significant is needed because the finer, more precise details are not necessary.

For example, for a parcel that is expected to arrive in approximately 48 hours time, knowing the actual nearest second or minute that it will arrive does not significantly alter the timeframe you will be waiting to receive it.



8

The significant figures are the number of important digits required in any measurement communication for determining accuracy or precision. There are some rules that must be followed when determining the number of significant figures in a value.

- All non-zero digits are significant.
- All zeros between non-zero digits are significant.
- All leading zeros in decimal values are not significant.
- All trailing zeros in decimal values to the right of the decimal point are significant.
- All trailing zeros for whole numbers are not significant, unless followed by a decimal point.

For scientific notation, the numbers multiplied by the power of 10 are all significant.

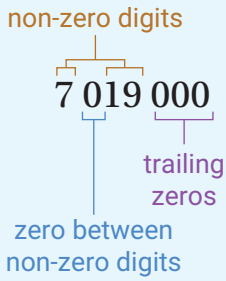
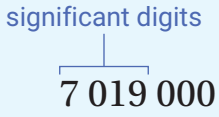
The trailing zeros for a number written scientific notation are not significant.

Worked example

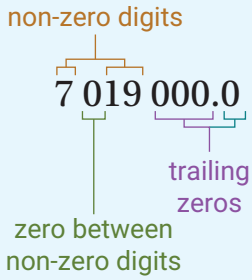
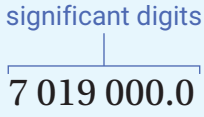
Identifying and counting significant figures

Identify the number of significant figures within each of the following.

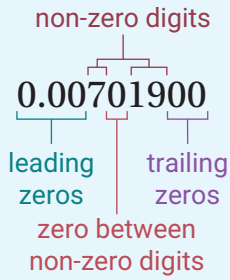
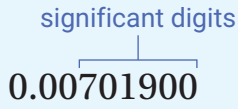
(a) 7 019 000

THINKING	WORKING
There are 3 non-zero numbers and 1 zero in between the non-zero numbers. There are 3 trailing zeros and no decimal point.	
Identify and count the significant figures.	 4 of the digits are significant.
Write the answer.	The number 7 019 000 has 4 significant figures.

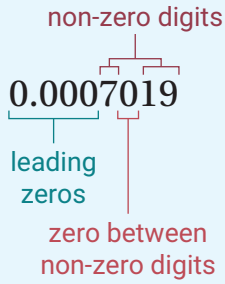
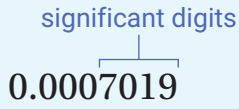
(b) 7 019 000.0

THINKING	WORKING
There are 3 non-zero numbers and 1 zero in between the non-zero numbers. There are 4 trailing zeros and a decimal point.	
Identify and count the significant figures.	 All 8 of the digits are significant.
Write the answer.	The number 7 019 000.0 has 8 significant figures.

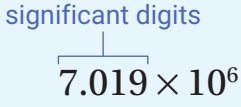
(c) 0.007 019 00

THINKING	WORKING
There are 3 leading zeros, 3 non-zero numbers, 1 zero in between the non-zero numbers, and 2 trailing zeros.	
Identify and count the significant figures.	 6 of the digits are significant.
Write the answer.	The number 0.007 019 00 has 6 significant figures.

(d) 0.000 701 9

THINKING	WORKING
There are 4 leading zeros, 3 non-zero numbers, 1 zero in between the non-zero numbers, and 0 trailing zeros.	
Identify and count the significant figures.	 4 of the digits are significant.
Write the answer.	The number 0.000 701 9 has 4 significant figures.

(e) 7.019×10^6

THINKING	WORKING
For scientific notation, all the numbers in the first term are significant.	 4 of the digits are significant.
Write the answer.	The number 7.019×10^6 has 4 significant figures.

Lower place values for significant digits in a measurement can sometimes be considered as not important. These measurements can be rounded to a certain number of significant figures.

The distance 223 000 500 m has 7 significant figures, however the place value of the '5' can be considered not important. The measurement 2.23 billion metres is not affected much by ignoring the 500 metres part. This distance can be estimated to be approximately 2.23 billion metres, making the first three non-zero numbers the only significant figures.

Therefore, 223 000 500 m = 223 000 000 m (rounded to 3 significant figures).

The rules for rounding measurements to a certain number of significant figures are as follows.

- Count the number of significant figures required for the measurement, then look at the next significant value.
- If the next value is < 5, the last significant value remains unchanged (round down).
- If the next value is ≥ 5, the last significant value increases by 1 (round up).

Worked example

Rounding measurements to certain significant figures

Round each measurement to the given number of significant figures.

(a) 3 405 020 km to four significant figures

THINKING	WORKING
Identify the first significant figure in the number.	3 405 020 ↑ 1st significant digit
Count four significant digits from the first significant digit to identify the fourth significant digit.	3 405 020 ↑ 4th significant digit
Look at the value in the fifth significant place. If it is < 5, leave the fourth significant value as written (or round down). If it is ≥ 5, round the fourth significant value up. Change the remaining place values to 0.	3 405 020 ↑ 5th significant digit < 5, round down
Write the answer.	The value 3 405 020 km rounds down to 3 405 000 km when rounded to four significant figures.

(b) 0.003 405 02 g to three significant figures

THINKING	WORKING
Identify the first significant figure in the number.	0.00340502 ↑ 1st significant digit
Count three significant digits from the first significant digit to identify the third significant digit.	0.00340502 ↑ 3rd significant digit
Look at the value in the fourth significant place. If it is < 5, leave the third significant value as written (round down). If it is ≥ 5, round the third significant value up. Leave all remaining trailing zeros off.	0.00340502 ↑ 4th significant digit ≥ 5, round up
Write the answer.	The value 0.003 405 02 g is 0.003 41 g when rounded to three significant figures.

SC 2 I can compare the effect that truncating or rounding of values during calculations can have on the accuracy of the results

Rounding numbers

Rounding numbers changes their value to a required degree of accuracy.

The rules for rounding numbers to a particular place value are as follows.

- Count the number of decimal places – or locate the place value – required for the measurement accuracy, then look at the value in the next lowest place value.
- If the next place value is < 5, the last value remains unchanged (round down)
- If the next place value is ≥ 5, the last value increases by 1 (round up).

Truncating numbers

Truncating numbers involves writing all the values up to the significant place value for the degree of accuracy required, then using the following rules to complete the truncation.

- If any fractional/decimal place values remain beyond the required degree of accuracy, they are removed.
- If any power of 10 place values remain beyond the required degree of accuracy, they are changed to 0, regardless of their size.

Worked example

Rounding and truncating numbers

1 Round the following decimals to the given degree of accuracy.

(a) 0.012 345 6 accurate to the nearest hundredth

THINKING	WORKING
Locate the hundredths place value (second decimal place).	0.0123456 ↑ hundredth place value
Look at the thousandths place value (third decimal place). If it is < 5, leave the second decimal place value as written (or round down). If it is ≥ 5, round the second decimal place value up.	0.0123456 ↑ thousandth place value The third decimal place value is < 5, so round down.
Write the answer.	0.012 345 6 accurate to the nearest hundredth is 0.01.

(b) 7.012 345 6 accurate to 5 decimal places

THINKING	WORKING
Locate the fifth decimal place by counting the place values starting from the decimal point.	7.0123456 ↑ 5th decimal place
Look at the value in the sixth decimal place. If it is < 5, leave the fifth decimal place value as written (or round down). If it is ≥ 5, round the fifth decimal place value up.	7.0123456 ↑ 6th decimal place ≥ 5, round up 6th decimal place is ≥ 5, so round up
Write the answer.	7.012 345 6 accurate to 5 decimal places is 7.012 35.

2 Truncate the following values to the given degree of accuracy.

(a) 543 212.3567 accurate to 1 decimal place

THINKING	WORKING
Locate the first decimal place by counting the place values starting from the decimal point.	543 212.3567 ↑ 1st decimal place
Remove all values in lower decimal place values.	543 212.3~~567~~
Write the answer.	543 212.3567 truncated to 1 decimal place is 543 212.3. Note that 543 212.3567 rounded to 1 decimal place is 543 212.4.

(b) 543 987.3456 accurate to the thousands

THINKING	WORKING
Locate the thousands place value in the number.	543 987.3456 ↑ thousands place value
Remove all decimal values and change digits in lower power of 10 place values to 0.	543 000.~~3456~~
Write the answer.	543 987.3456 truncated to the thousands place is 543 000. Note that 543 987.3456 rounded to the thousands place is 544 000.

When performing calculations involving measurements requiring a certain degree of accuracy, it is important to only round or truncate the last calculation. Rounding or truncating the values prior to calculating will greatly affect the final value.

For example:

You complete four short walks of distances 20.3 m, 32.8 m, 45.7 m and 50.9 m.

The total distance walked is (20.5 + 32.8 + 45.7 + 50.9) m = 149.9 m.

Rounding this to the nearest metre gives 150 m, which is representative of the actual distance.

However, rounding each walk to the nearest metre before calculating the total distance, the calculation becomes (21 + 33 + 46 + 51) m = 151 m, which is less representative of the actual total distance.

Truncating the actual distance after the ones value gives 149 m, which is representative of the actual distance.

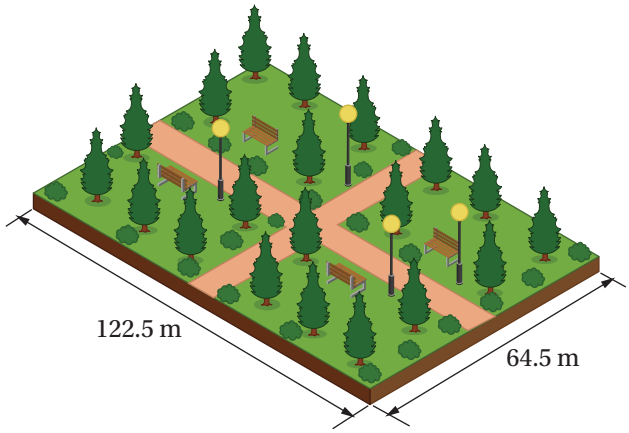
Truncating after the ones value before calculating the total distance, the calculation becomes (20 + 32 + 45 + 50) m = 144 m. This is even less representative of the actual total distance.

For larger values, or for calculations involving multiplication – such as area and volume – the error can become quite significant if numbers are rounded or truncated too early.

Worked example

Understanding the effect of rounding or truncating during calculations on the accuracy

A rectangular park has the dimensions 64.5 m and 122.5 m.



(a) Calculate the area of the park accurate to 1 decimal place.

THINKING	WORKING
Substitute dimensions into the area formula for a rectangle and calculate.	$A = 122.5\,\text{m} \times 64.5\,\text{m}$ $= 7901.25\,\text{m}^2$
Round the answer accurate to 1 decimal place.	$7901.25\,\text{m}^2 = 7901.3\,\text{m}^2$
Write the answer.	The area of the park accurate to 1 decimal place is $7901.3\,\text{m}^2$.

(b) Truncate the area of the park after the ones value.

THINKING	WORKING
Identify the ones value and truncate all other digits in lower place values.	The digit in the ones place value is 1. $7901.3\,\text{m}^2 = 7901\,\text{m}^2$
Write the answer.	The truncated area of the park is $7901\,\text{m}^2$.

(c) Truncate the park dimensions after the ones value, then calculate the area of the park.

THINKING	WORKING
Truncate the two dimensions after the ones value.	Length: $122.5\,\text{m} = 122\,\text{m}$ Width: $64.5\,\text{m} = 64\,\text{m}$
Calculate the area of the rectangle by multiplying the dimensions.	$A = 122\,\text{m} \times 64\,\text{m}$ $= 7808\,\text{m}^2$
Write the answer.	The area of the park using the truncated dimensions is $7808\,\text{m}^2$.

(d) Compare and comment on the results for **(b)** and **(c)**.

THINKING	WORKING
Recall the difference in values.	Truncated area = 7901 m^2 Area using truncated dimensions = 7808 m^2
Compare the values and comment.	There is a difference of 93 m^2 (almost 100 m^2) between the area values, which is quite substantial because each dimension changed by only 0.5 m. The truncated area is much more representative of the actual calculation, whereas the area calculated by truncating the dimensions first is not at all accurate and should not be used.

Practice

ANSWERS Page 639

SC 1 I can use the language of estimation and round numbers to a specified number of significant figures

1 Write the number of significant figures for each of these numbers.

(a) 100 **(b)** 1200 **(c)** 12 300 **(d)** 12 340

2 Write the number of significant figures for each of these numbers.

(a) 0.1 **(b)** 0.20 **(c)** 0.300 **(d)** 0.4000

3 Write the number of significant figures for each of these numbers.

(a) 1010 **(b)** 10 200 **(c)** 102 030 **(d)** 1 020 304

4 Write the number of significant figures for each of these numbers.

(a) 0.010 **(b)** 0.0020 **(c)** 0.000 30 **(d)** 0.000 040

5 Write the number of significant figures for each of these numbers.

(a) 1010.0 **(b)** 10 200.00 **(c)** 102 030.000 **(d)** 1 020 304.0000

6 Write each of these numbers accurate to 3 significant figures.

(a) 1234 **(b)** 12 340 **(c)** 123 400 **(d)** 1 234 000

7 Write each of these numbers accurate to 2 significant figures.

(a) 1234 **(b)** 23 456 **(c)** 345 678 **(d)** 4 567 890

8 Write each of these numbers accurate to 5 significant figures.

(a) 5.1234 **(b)** 5.012 34 **(c)** 5.001 234 **(d)** 5.000 123 4

9 Write each of these numbers accurate to 4 significant figures.

(a) 10.203 04 **(b)** 20.304 05 **(c)** 30.405 06 **(d)** 40.506 07

10 Write the number of significant figures for each of these numbers.

(a) 1.23×10^5 **(b)** 1.203×10^5 **(c)** 1.200×10^5 **(d)** 10.020×10^5

11 Write each of these numbers accurate to 2 significant figures.

(a) 1.234×10^3 **(b)** 1.023×10^3 **(c)** 3.456×10^3 **(d)** 3.045×10^3

SC 2 I can compare the effect that truncating or rounding of values during calculations can have on the accuracy of the results

1 Round these measurements accurate to 2 decimal places.

(a) 10.123 kg **(b)** 10.234 kg **(c)** 10.345 kg **(d)** 10.456 kg

2 Round these values to the nearest ten thousand.

(a) 12 345 **(b)** 23 456 **(c)** 34 567 **(d)** 45 678

3 Truncate these measurements after the hundredths value.

(a) 10.1234 kg **(b)** 10.2345 kg **(c)** 10.3456 kg **(d)** 10.4567 kg

4 Truncate these numbers after the ten thousands place value.

(a) 123 456 **(b)** 234 567 **(c)** 345 678 **(d)** 456 789

5 A runner's five lap times (in minutes) around a cross-country course were recorded:

15.464, 17.455, 20.887, 14.478, 16.959.

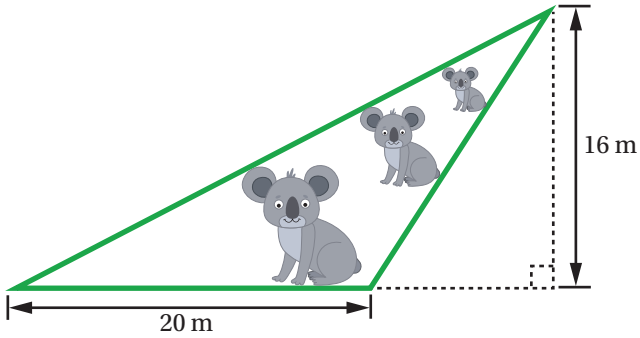
(a) Calculate the total time taken to complete the five laps.

(b) Round each lap time to the nearest minute, then calculate the total time taken.

(c) Truncate each lap time to minute place values, then calculate the total time taken.

(d) Comment on how the rounding and truncating affected the accuracy of the times.

6 A painter was provided the following dimensions of a triangular mural 'Koala highway' to be painted onto a wall.



Just enough paint was ordered to cover the area of the mural for the given dimensions.

(a) Calculate the wall area that the paint was to cover.

(b) While completing the mural, the painter was told the dimensions had been rounded to the nearest metre. The actual dimensions were 20.49 m and 16.40 m.

(c) Will the painter have enough paint to complete the mural?

(d) Comment on how the rounding of the dimensions before calculating the area affected the painter.

TOPIC REVIEW

Multiple choice

ANSWERS Page 669

1 The prefix 'milli' has been used in which of the following quantities?

A 5.3 ML **B** 6.7 km **C** 16.04 mg **D** 48 cm

2 The order of magnitude, in metres, expressed by 1 cm is:

A 10^2 **B** 2 **C** -2 **D** 10^{-2}

3 The prefix 'micro' (μ) shifts any base measurement:

A 12 places lower **B** 9 places lower **C** 6 places lower **D** 3 places lower

4 The scientific notation for 603 000 000 is:

A 0.603×10^9 **B** 6.03×10^8 **C** 60.3×10^7 **D** 603×10^6

5 The decimal form for 5.10×10^{-4} is:

A 0.000 510 **B** 0.000 51 **C** 0.000 051 **D** 0.000 051 0

6 The correct order, smallest to largest, for this list of numbers is:

A 6.7×10^3, 8.6×10^3, 5.3×10^4, 1.6×10^5

B 1.6×10^5, 5.3×10^4, 6.7×10^3, 8.6×10^3

C 6.7×10^3, 5.3×10^4, 1.6×10^5, 8.6×10^3

D 5.3×10^4, 6.7×10^3, 1.6×10^5, 8.6×10^3

7 The rounded scientific notation answer to the calculation $(6.3 \times 10^4) \times (2.9 \times 10^5)$ is:

A 1.8×10^9 **B** 1.8×10^{10} **C** 18×10^9 **D** 18×10^5

8 The rounded scientific notation answer to the calculation $(2.6 \times 10^5) \div (3.8 \times 10^8)$ is:

A 6.8×10^4 **B** 6.8×10^3 **C** 6.8×10^{-3} **D** 6.8×10^{-4}

9 Which number does *not* have 3 significant figures?

A 0.0625 **B** 4.02×10^5 **C** 1.280×10^{-4} **D** 7.60×10^{-2}

10 2546 mL accurate to the nearest 100 mL is:

A 2400 mL **B** 2500 mL **C** 2550 mL **D** 2600 mL

11 1876 truncated after the tens place is:

A 1000 **B** 1800 **C** 1870 **D** 1880

Short answer

1 Write each of the following using symbols for the prefixes and base units.

(a) 1000 litres **(b)** 0.001 metres

2 Write the number of place values higher or lower that these prefixes shift any base unit measurement.

(a) mega (M) **(b)** deci (d) **(c)** pico (p)

3 Write the value of these measurements without using a prefix.

(a) 5.2 km **(b)** 7 ms **(c)** 2.8 TB

4 Write these numbers using scientific notation.

(a) 0.0601 **(b)** 0.000 012 **(c)** 0.000 008 203

5 Write each of these measurements using scientific notation and the base unit.

(a) 350 kL **(b)** 35 nm **(c)** 0.35 μs **(d)** 0.035 MB

6 Convert each of these values from scientific notation into decimal form.

(a) 7.8×10^{7} **(b)** 4.03×10^{3} **(c)** 2.1030×10^{8}

7 Write the correct symbol, ' > ' or ' < ', between each pair of numbers.

(a) 8.3×10^{-5} ___ 8.3×10^{-4} **(b)** 3.4×10^{-6} ___ 5.2×10^{-7} **(c)** 6.1×10^{-3} ___ 2.4×10^{-4}

8 Write the correct symbol, ' > ' or ' < ', between each pair of numbers.

(a) -3.1×10^{6} ___ -3.1×10^{7} **(b)** -6.4×10^{5} ___ -4.8×10^{6}

(c) -2.3×10^{-3} ___ -2.3×10^{-4} **(d)** -1.5×10^{-5} ___ -2.7×10^{-4}

9 Write the value of each calculation in scientific notation, correct to 2 decimal places.

(a) $7 \times (2.9 \times 10^{4})$ **(b)** $(3.5 \times 10^{8}) \times 6$

(c) $(1.4 \times 10^{5}) \div 18$ **(d)** $22 \div (6.4 \times 10^{-4})$

10 Write each of these numbers accurate to the required number of significant figures given in the brackets.

(a) 864 201 (2) **(b)** 42 051 (3) **(c)** 63.149 (4) **(d)** 7.217 (2)

11 Write each of these numbers in scientific notation, accurate to 2 significant figures.

(a) 3.604×10^{2} **(b)** 1.2503×10^{7} **(c)** 6.516×10^{-3} **(d)** 4.022×10^{-5}

12 Truncate these measurements after the hundredths value.

(a) 47.0258 km **(b)** 1.0281 mL **(c)** 60.5443 g

Extended response

1 Convert:

(a) 14 mm to

(i) centimetres **(ii)** metres **(iii)** kilometres

(b) 38 400 mm to

(i) centimetres **(ii)** metres **(iii)** kilometres

(c) 5.02 m to

(i) millimetres **(ii)** centimetres **(iii)** kilometres

(d) 6.8 m^2 to

(i) square millimetres **(ii)** square centimetres **(iii)** square kilometres.

2 The table gives some facts in scientific notation about the planets of our solar system: diameter in kilometres, mass in kilograms, period for rotation in hours and period of orbit around the Sun in days.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Diameter	4.9×10^3	1.2×10^4	1.3×10^4	6.8×10^3	1.4×10^5	1.2×10^5	5.1×10^4	5.0×10^4
Mass	3.3×10^{23}	4.9×10^{24}	6.0×10^{24}	6.4×10^{23}	1.9×10^{27}	5.7×10^{26}	8.7×10^{25}	1.0×10^{26}
Rotation	1.4×10^3	5.8×10^3	2.4×10^1	2.5×10^1	9.9×10^0	1.1×10^1	1.7×10^1	1.6×10^1
Orbit	8.80×10^1	2.25×10^2	3.65×10^2	6.87×10^2	4.33×10^3	1.07×10^4	3.06×10^4	5.98×10^4

Write the names of the planets listed in order to show each of the following statistics from smallest to largest.

(a) Diameter

(b) Mass

(c) Period for rotation

(d) Period of orbit around the Sun

3 The Earth has an equatorial diameter of 1.2756×10^4 km and is, on average, 1.497×10^8 km from the Sun. Venus has an equatorial diameter of 1.2104×10^4 km and is, on average, 1.08×10^8 km from the Sun. Answer the following using scientific notation, correct to 3 significant figures.

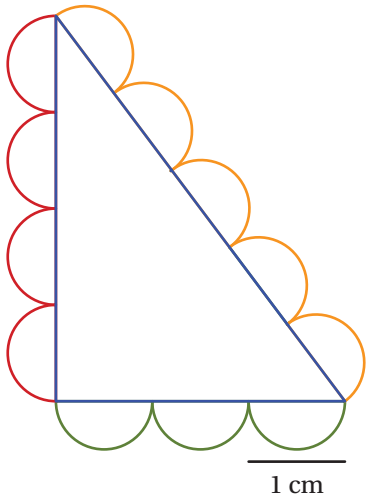
(a) Estimate how close Earth and Venus could be to each other.

(b) Estimate how far apart Earth and Venus could be from each other.

(c) How many Earths could fit side-by-side between Earth and the Sun?

(d) How many Venuses could fit side-by-side between Venus and the Sun?

4 A shape is made from a right-angled triangle of sides 3 cm, 4 cm and 5 cm with semicircles of diameter 1 cm along each of the sides.



(a) Use the formula $\frac{C}{2} = \frac{\pi d}{2}$ to determine the exact length of one semicircle.

(b) Calculate the exact perimeter of the figure.

(c) Write the perimeter correct to 1 decimal place.

(d) Write the length of one semicircle correct to 1 decimal place.

(e) Use the rounded value in **(d)** to calculate the perimeter.

(f) Write the length of one semicircle truncated after the tenths place.

(g) Use the rounded value in **(f)** to calculate the perimeter.

(h) Use the formula $A_{\text{semicircle}} = \frac{\pi r^2}{2}$ to determine the exact area of one semicircle.

(i) Calculate the exact area of the figure.

(j) Write the area correct to 1 decimal place.

(k) Write the area of one semicircle correct to 1 decimal place.

(l) Use the rounded value in **(k)** to calculate the area of the figure.

(m) Write the area of one semicircle truncated after the tenths place.

(n) Use the rounded value in **(m)** to calculate the area of the figure.



TOPIC

9

Percentage error in measurement

Recall 319

9.1 Determine the precision and absolute error of measurements 320

9.2 Estimate the degree of accuracy 327

9.3 Determine the percentage error in measurements and calculations 332

Topic review 341

Why learn this?

Being able to determine the precision and absolute error of measurements offers several benefits.

It helps with understanding the limitations of measuring tools and the potential for error in measurements, and it promotes accuracy in measurements, including in scientific, mathematical and everyday contexts. It also supports informed decision-making in contexts where precise measurements are required, such as in construction, engineering or cooking.

This knowledge lays the groundwork for more advanced mathematical concepts, including relative and percentage errors, and provides a basis for critical evaluation of data in research findings, news reports and other information sources.

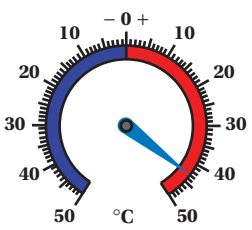
RECALL

ANSWERS Page 640

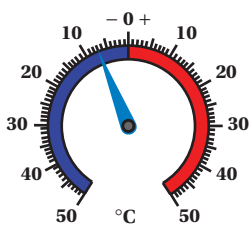
I can read measuring devices

1 Write the temperature recorded in each of the following.

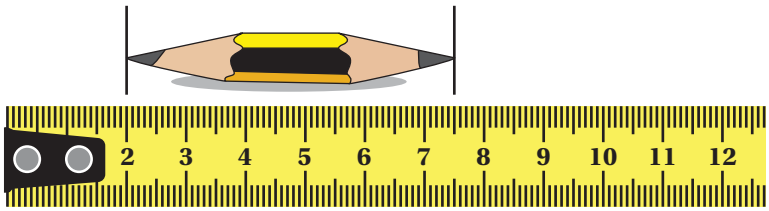
(a)



(b)



2 Write the length recorded in the following in centimetres.



I can identify the magnitude of calculations

1 Write the magnitude for each of the following.

(a) $4 + (-2)$ **(b)** $-5 - 8$ **(c)** -2×3 **(d)** $7 - 2 \times 4$

I can write fractions with a power of 10 as a denominator in decimal form

1 Convert the following fractions to decimals.

(a) $\frac{8}{10}$ **(b)** $\frac{23}{10}$ **(c)** $\frac{171}{1000}$ **(d)** $\frac{7}{1000}$ **(e)** $\frac{91}{10\,000}$

I can convert between units of length

1 Complete each of the following length conversions.

(a) 2500 cm to m **(b)** 75.45 mm to m **(c)** 0.2315 km to cm

I can round decimals for different purposes

1 Round each of the following measurements to the number of decimal places stated in the brackets.

(a) 5.55 cm (1) **(b)** 3.843 km (2) **(c)** 5.795 m (2) **(d)** 25.254 °C (1)

I can calculate the percentage of amounts

1 Calculate the following percentages.

(a) 5 as a percentage of 100 **(b)** 4 as a percentage of 40 **(c)** 24 as a percentage of 32

9.1 Determine the precision and absolute error of measurements

Learning intention: To be able to determine the precision and absolute error of measurements

Success criteria:

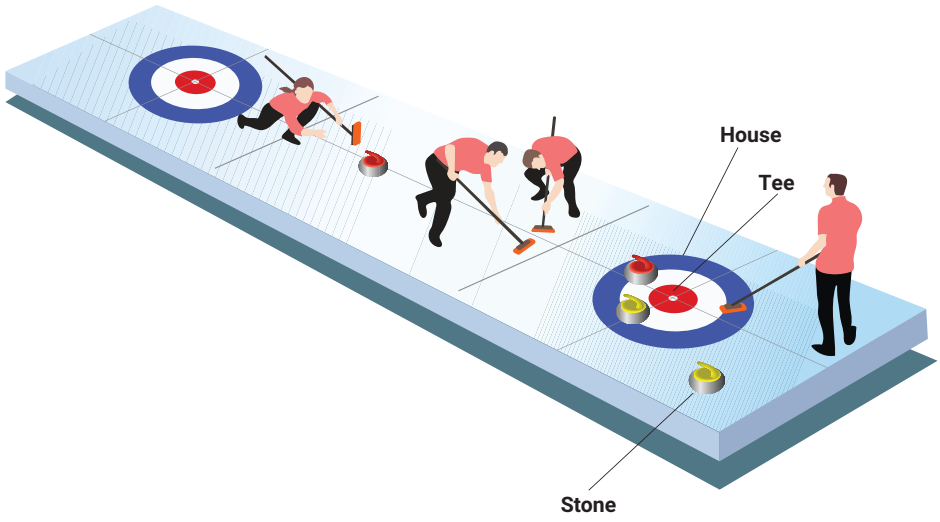
SC 1 I can use the precision of measuring tools to calculate the absolute error for measurements.

Lesson warm-up

Accuracy and precision in sport



Curling is a competitive winter team sport. The objective is to get your team's stones closer to the centre of the target house (called the tee) than the other team's stones. The team with the stone closest to the centre at the end of a round scores points.

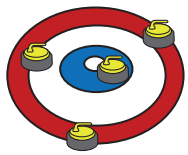


Apply the following two definitions to describe each attempt at aiming to land four stones closest to the tee.

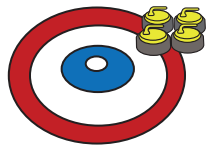
Accuracy: How close all the attempts are to the intended target.

Precision: How close all the attempts are compared to each other.

Attempt 1



Attempt 2



Attempt 3



Attempt 4



SC 1 I can use the precision of measuring tools to calculate the absolute error for measurements

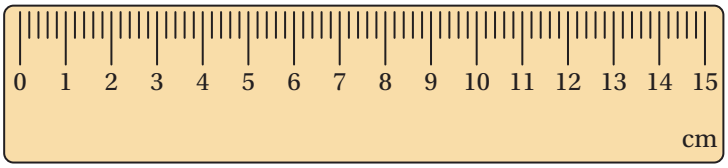
Tools used to perform measurements are rarely 100% accurate. This can be due to many different factors, including:

- precision of the measuring tool
- physical limitations of the tool used
- parallax error (the apparent shift in an object's position as it is viewed from different angles)
- irregularity of the shape being measured
- user error.

Precision

The precision of a measuring tool refers to the smallest units that can be read on that tool. The smallest units are those whose increments are the closest together. This is also called the limit of reading.

This ruler has centimetre (cm) and millimetre (mm) markings. The millimetre marks are the smallest units of distance measurable by this ruler. Since the ruler can only precisely measure to the nearest millimetre, the limit of reading is millimetres (mm).



Absolute error

The accuracy of a measuring tool can be limited by many factors. Therefore, most measurements provide close estimates of the actual (true) value.

The absolute error is the greatest expected error between the measured value and the actual (true) measure.

Given that the actual value is known, the absolute error is:
Absolute error = $|$measured value − actual value$|$

Taking the absolute value of the difference ensures the magnitude of the error is calculated.
If the actual value is unknown, the absolute error for measurements is the precision of the tool used. This provides the greatest possible error because the actual (true) value is not available for comparison.

$$\text{Absolute error} = \frac{1}{2} \times \text{precision}$$

$$= \frac{1}{2} \times \text{the limit of reading of the measuring tool}$$

For the ruler shown above, the limit of reading is 1 mm.

Therefore, the absolute error for any measurement using this ruler is $\frac{1}{2} \times 1\text{ mm} = 0.5\text{ mm}$.

Any measurement recorded using this ruler could differ from the true value by up to 0.5 mm.

Worked example

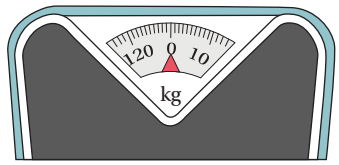
Determining precision and absolute error

(a) Calculate the absolute error when a flower petal that is 12.1 cm long is measured by a student to be 12.4 cm.

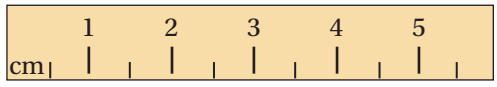
THINKING	WORKING
Identify the actual and measured values for the length of the petal.	Actual length of the flower petal is 12.1 cm. Measured length of the flower petal is 12.4 cm.
Calculate the absolute value of the difference to get the absolute error.	Absolute error $= \lvert 12.4 - 12.1 \rvert$ cm $= \lvert 0.3 \rvert$ cm $= 0.3$ cm
Write the answer.	The absolute error for the measured flower is 0.3 cm.

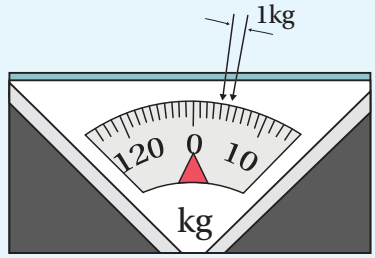
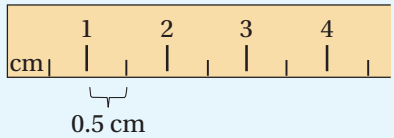
(b) Identify the precision (limit of reading) for the scales and wooden ruler pictured.

Scales



Wooden ruler



THINKING	WORKING
Identify the smallest units of measure for each instrument.	Scales: Smallest increments are kilograms (kg).  Wooden ruler: Smallest increments are half a centimetre (0.5 cm). 

Write the answer.	The precision (limit of reading) for the scales is 1 kg. The precision (limit of reading) for the wooden ruler is 0.5 cm.

(c) Calculate the absolute error for measurements recorded using the scales and wooden ruler from **(b)**.

THINKING	WORKING
Recall the definition of precision.	The precision is the limit of reading.
Multiply the precision of each instrument by one half.	Scales: Absolute error is $\frac{1}{2} \times 1 \text{ kg} = 0.5 \text{ kg}$. Wooden ruler: Absolute error is $\frac{1}{2} \times 0.5 \text{ cm} = 0.25 \text{ cm}$.
Write the answer.	The absolute error for measurements recorded using the scales rule is 0.5 kg. The absolute error for measurements recorded using the wooden ruler is 0.25 cm.

The precision (limit of reading) and absolute error for recorded measurements can be estimated given the precision of the measurement.

A measurement of 10.4 cm has been recorded accurate to the nearest tenth of a centimetre. Therefore, the precision of the tool used to measure the distance is 0.1 cm.

The absolute error for this measurement is $\frac{1}{2} \times 0.1 \text{ cm} = 0.05 \text{ cm}$ (or 0.5 mm).

Another method will lead to the same outcome.

The measurement of 10.4 cm recorded accurate to the nearest tenth of a centimetre is actually 10 cm and 4 mm.

This measurement has been recorded accurate to the nearest 1 mm, which is the same as the nearest tenth of a centimetre (0.1 cm).

This indicates that the precision of the tool used to measure the distance was millimetres, or 1 mm.

Therefore, the absolute error for this measurement is $\frac{1}{2} \times 1 \text{ mm} = 0.5 \text{ mm}$ (or 0.05 cm).

Worked example

Identifying precision and absolute error

Identify the precision (limit of reading) and absolute error for each measurement.

(a) 21 cm recorded accurate to the nearest whole centimetre.

THINKING	WORKING
Identify the units for the given measurement.	The measurement is in centimetres (cm).
Identify the precision for which the measurement has been recorded.	The precision of the measurement is 1 cm.
Calculate the absolute error.	Absolute error is $\frac{1}{2} \times 1 \text{ cm} = 0.5 \text{ cm}$.
Write the answer.	The measurement of 21 cm has a precision of 1 cm and an absolute error of 0.5 cm.

(b) 9.9 g recorded accurate to the nearest tenth of a gram.

THINKING	WORKING
Identify the units for the given measurement.	The measurement is in grams (g).
Identify the precision for which the measurement has been recorded.	The precision of the measure is one tenth of a gram (0.1 g).
Calculate the absolute error.	Absolute error is $\frac{1}{2} \times 0.1 \text{ g} = 0.05 \text{ g}$.
Write the answer.	The measurement 9.9 g has a precision of 0.1 g and an absolute error of 0.05 g (or 5 milligrams (5 mg)).

(c) 150 m recorded accurate to the nearest 10 m.

THINKING	WORKING
Identify the units for the given measurement.	The measurement is in metres (m).
Identify the precision for which the measurement has been recorded.	The precision of the measurement is 10 m.
Calculate the absolute error.	Absolute error is $\frac{1}{2} \times 10 \text{ m} = 5 \text{ m}$.
Write the answer.	The measurement 150 m has a precision of 10 m and an absolute error of 5 m.

Practice

ANSWERS Page 640

SC 1 I can use the precision of measuring tools to calculate the absolute error for measurements

1 Calculate the absolute error of the measured value for the following measurement pairs.

(a) Measured value: 24 cm; actual value: 25 cm

(b) Measured value: 25 °C; actual value: 24 °C

(c) Measured value: 2.5 mm; actual value: 2.4 mm

(d) Measured value: 0.24 kg; actual value: 0.25 kg

2 Calculate the absolute error of the measured value for the following measurement pairs.

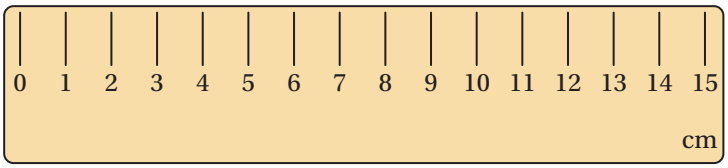
(a) Measured value: 15.8 cm; actual value: 15.5 cm

(b) Measured value: 1.55 g; actual value: 1.58 g

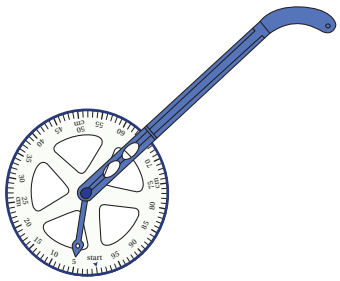
(c) Measured value: 155.0 km; actual value: 158.5 km

3 Identify the precision of the following measuring instruments.

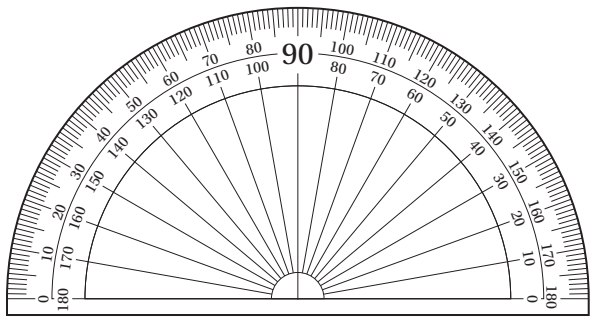
(a) 15 cm ruler with 1 cm markings



(b) 100 cm trundle wheel with 1 cm markings

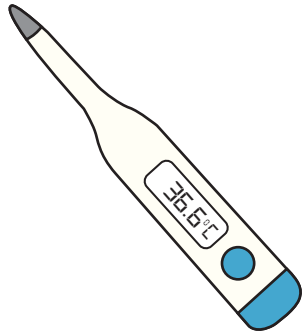


(c) 180° protractor with 1° increments

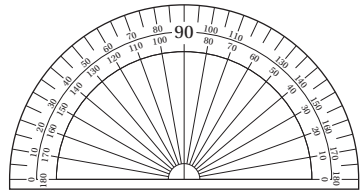


4 Identify the precision of the following measuring instruments.

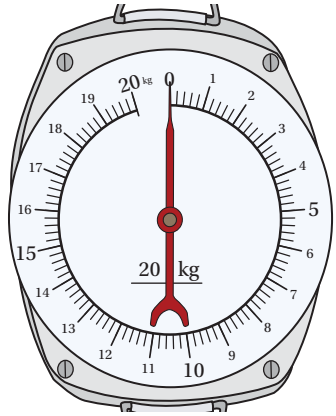
(a) Medical thermometer with temperature increments of 0.1 °C



(b) Protractor with 5° increments



(c) Hanging scale with 5 smaller increments every 1 kg



5 What is the precision of the following recorded measurements?

(a) 45 kg recorded accurate to the nearest kilogram

(b) 45 kg recorded accurate to the nearest 5 kilograms

(c) 450 kg recorded accurate to the nearest 10 kilograms

(d) 4.5 kg recorded accurate to the nearest tenth of a kilogram

6 What is the absolute error for each of the following recorded measurements?

(a) 30 cm recorded accurate to the nearest centimetre

(b) 30 cm recorded accurate to the nearest 2 centimetres

(c) 30 cm recorded accurate to the nearest 5 centimetres

(d) 30.0 cm recorded accurate to the nearest tenth of a centimetre

7 Identify the absolute error for the measurements recorded using the measuring instruments from Question **4**.

Estimate the degree of accuracy

9.2

Learning intention: To be able to estimate the degree of accuracy

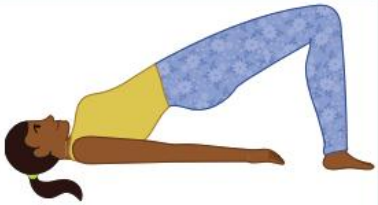
Success criteria:

SC 1 I can use absolute error to estimate degrees of accuracy by identifying the upper and lower bounds.

Lesson warm-up

Precision of measuring instruments

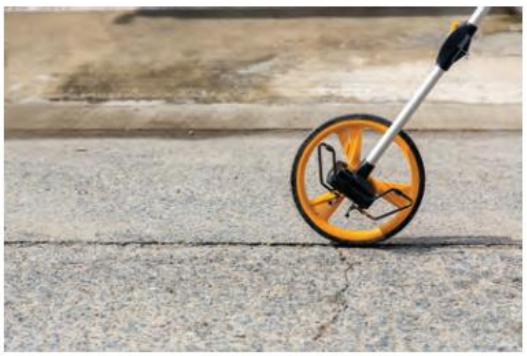
For each of the following, describe the precision of the tool used and give an example of something you might measure with the tool.



Odometer



Trundle wheel



Measuring tape



Micrometer



9

SC 1 I can use absolute error to estimate degrees of accuracy by identifying the upper and lower bounds

Degree of accuracy

This is a range of values between which a measured value has a higher chance of matching the actual (true) measurement.

Degree of accuracy = measured value ± the absolute error

Or

Degree of accuracy = measured value ± half the precision (limit of reading)

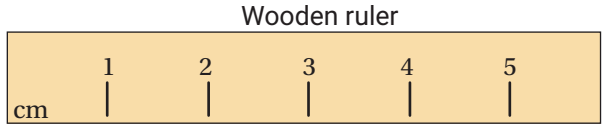
Lower and upper bounds

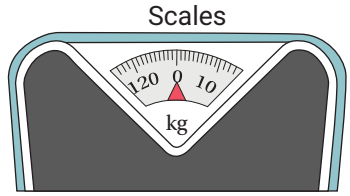
Lower and upper bound values are on either side of the measured value. They are calculated by applying ± the absolute error to the measured value.

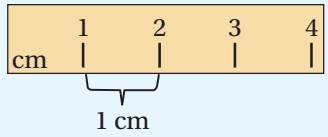
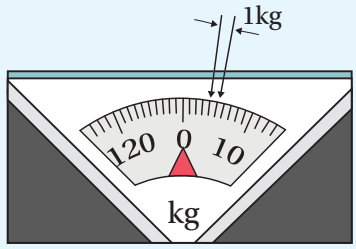
Worked example

Identifying and calculating the degree of accuracy of measuring devices

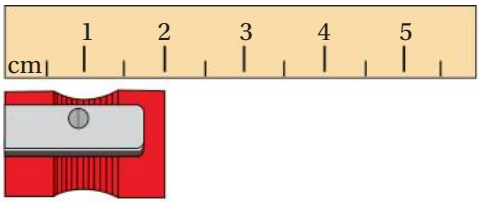
(a) Identify the limit of reading for each measuring device pictured.



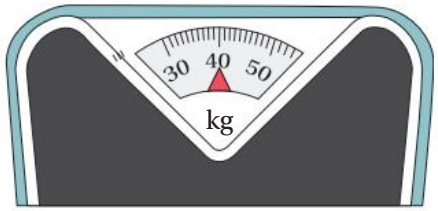


THINKING	WORKING
Identify the smallest units of measure for each instrument.	Wooden ruler: Smallest increments are centimetres (cm).  Scales: Smallest increments are kilograms (kg). 
Write the answer.	The limit of reading for the wooden ruler is 1 cm. The limit of reading for the scales is 1 kg.

(b) Calculate the degree of accuracy for each of these measurements.



Pencil sharpener



THINKING	WORKING
Identify the measurement recorded on each tool.	Wooden ruler: The measured length of the pencil sharpener is 2 cm. Scales: The measured mass on the scales is 40 kg.
Identify ± the degree of accuracy for each instrument. The degree of accuracy is ± half the limit of reading.	Wooden ruler: The limit of reading is 0.5 cm. $\frac{1}{2} \times 0.5 \text{ cm} = 0.25 \text{ cm}$ Therefore, the degree of accuracy is ± 0.25 cm. Scales: The limit of reading is 1 kg. $\frac{1}{2} \times 1 \text{ kg} = 0.5 \text{ kg}$ Therefore, the degree of accuracy is ± 0.5 kg.
Write the answer.	The degree of accuracy for the length of the pencil sharpener is ± 0.25 cm. The degree of accuracy for the mass recorded is ± 0.5 kg.

(c) Calculate the upper and lower bounds for each of the measurements recorded in **(b)**.

THINKING	WORKING
Write each measured value including the degree of accuracy.	The length of the pencil sharpener is 2 cm ± 0.25 cm. The mass recorded by the scales is 40 kg ± 0.5 kg.
Calculate the lower and upper bound values for each measurement. Subtract/add the absolute error from/to the measured value.	Pencil sharpener length: Lower bound is 2 cm − 0.25 cm = 1.75 cm Upper bound is 2 cm + 0.25 cm = 2.25 cm Mass recorded by the scale: Lower bound is 40 kg − 0.5 kg = 39.5 kg Upper bound is 40 kg + 0.5 kg = 40.5 kg
Write the answer.	The true length of the pencil sharpener is a value from 1.75 cm to 2.25 cm. The true mass recorded by the scales is a value from 39.5 kg to 40.5 kg.

Practice

ANSWERS Page 640

SC 1 I can use absolute error to estimate degrees of accuracy by identifying the upper and lower bounds

1 Write the degree of accuracy for the following measurements.

(a) A length measurement using a ruler with a precision of 1 mm

(b) A distance measurement using a trundle wheel with a precision of 1 m

(c) A temperature reading from a thermometer with a precision of 1 °C

(d) A weight measurement using kitchen scales with a precision of 1 g

2 Write the degree of accuracy for the following measurements.

(a) A length measurement using a tape with a precision of 5 cm

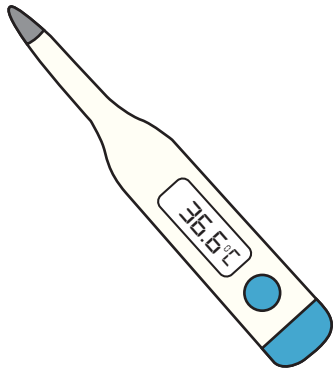
(b) A distance measurement using a trundle wheel with a precision of 0.5 m

(c) A temperature reading from a digital thermometer with a precision of 0.1 °C

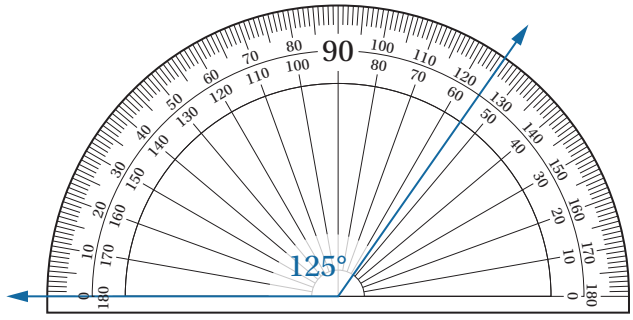
(d) An angle measurement using a clinometer with a precision of 2°

3 Write the measurement and degree of accuracy for each recorded value using these measuring instruments.

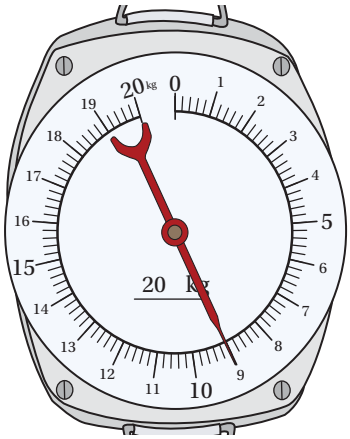
(a) Digital thermometer with 0.1 °C increments, recording a temperature of 36.6 °C



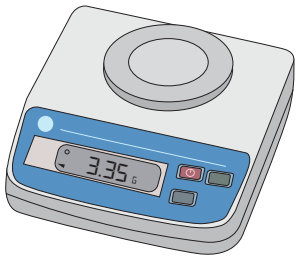
(b) This protractor with 1° increments, measuring an angle as 125°



(c) This hanging scale with 0.2 kg increments, recording a mass of 9 kg



(d) This digital laboratory scale with 0.01 g increments, recording a mass of 3.35 g



4 Write the lower and upper bounds for these measurements.

(a) A length measured as 10 mm using a ruler with a precision of 1 mm

(b) A distance measured as 15 m using a trundle wheel with a precision of 1 m

(c) A temperature reading of 20 °C from a thermometer with a precision of 1 °C

(d) A weight measure of 200 g using kitchen scales with a precision of 10 g

5 Write the lower and upper bounds for these measurements.

(a) A length measurement of 95 cm using a tape with a precision of 5 cm

(b) A measured distance of 8 m using a trundle wheel with a precision of 0.5 m

(c) A recorded temperature of 22 °C using a digital thermometer accurate to 0.1 °C

(d) A distance of 250 km recorded on an odometer with a precision of 0.1 km

6 Write the lower and upper bounds for each recorded value using the measuring instruments from Question **3**.

7 The lower and upper bound limits are calculated the same way for measurements involving negative values. Write the lower and upper bounds for these measurements.

(a) A recorded temperature of −10 °C using a thermometer with a precision of 2 °C

(b) A recorded depth of −24 m using a weighted rope with a precision of 1 m

(c) A temperature of −15 °C recorded on a thermometer with a precision of 0.1 °C

(d) A difference in time of −2.5 s recorded on a stopwatch with a precision of 0.1 s

9.3 Determine the percentage error in measurements and calculations

Learning intention: To be able to determine the percentage error in measurements and calculations

Success criteria:

SC 1 I can calculate the relative and percentage error in measurements.

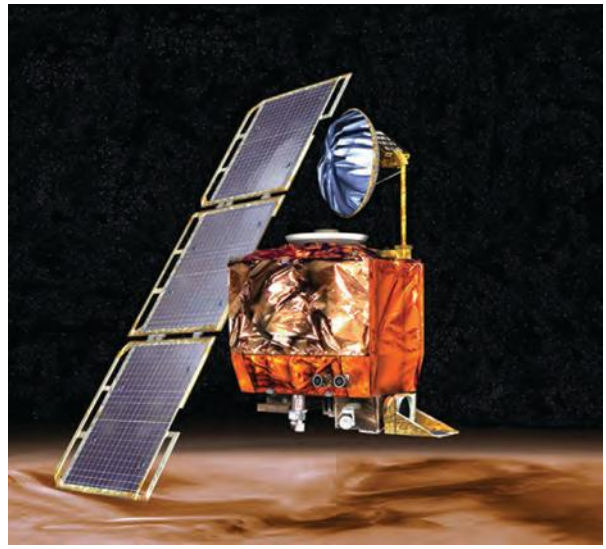
SC 2 I can calculate the relative and percentage error between measured and actual values in context.

Lesson warm-up

From little errors, big errors grow

Sending science probes to other planets in the solar system requires a lot of very accurate calculations. Even the smallest errors can be catastrophic.

In 1999, the Mars Climate Orbiter (MCO) was sent to Mars to study its atmosphere. Due to a miscommunication, the software on the orbiter was programmed to perform a calculation using different units to those being used in the calculations back on Earth.



This meant that after launch, each of the four (barely noticeable) course corrections made to keep the orbiter on target, were calculated differently by the orbiter's computer.

By the time the orbiter reached Mars, these small errors had accumulated enough to put the orbiter too far off target (by a couple hundred kilometres) and the mission failed. This small mistake cost $327 million.

Describe a situation where small errors can compound to have a large effect.

SC 1 I can calculate the relative and percentage error in measurements

All measurements are estimations due to various factors. This means all measurements recorded have an associated error affecting their accuracy. The accuracy of the reading is often dependent on the precision and associated absolute error of the instrument being used to perform the measurement.

Absolute error as a measure of precision

When using measuring instruments, the absolute error is one-half of the instrument's precision (limit of reading).

$$\text{Absolute error} = \frac{1}{2} \times \text{precision (limit of reading)}$$

Relative error as a measure of precision

The relative error is the error that is relative to the size of the attribute being measured (length, time, mass, etc.).

The relative error of any measurement is the magnitude of the absolute error divided by the measured value.

$$\text{Relative error (precision)} = \left| \frac{\text{absolute error}}{\text{measured value}} \right|$$

The absolute value of this calculation is used because only the magnitude of the error is required.

Both the absolute error and measured value must be in the same units.

Percentage error as a measure of precision

The percentage error is simply the relative error expressed as a percentage, by multiplying it by 100.

Percentage error = relative error × 100%

The smaller the percentage error, the more precise the measurement is.

The larger the percentage error, the less precise the measurement is.

Worked example

Calculating relative error and percentage error in measurements

(a) Calculate the relative error and percentage error when a 25 cm length is measured using a tape measure with an absolute error of 0.5 cm.

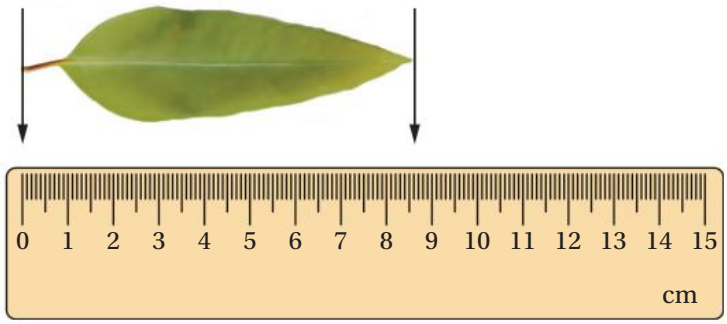
THINKING	WORKING
Calculate the relative error. Remember, it is the magnitude of the absolute error divided by the measured value.	$\text{Relative error} = \left\| \frac{\text{absolute error}}{\text{measured value}} \right\|$ $= \left\| \frac{0.5}{25} \right\|$ $= 0.02$
Convert the relative error to a percentage.	Percentage error = relative error × 100% = 0.02 × 100% = 2%
Write the answer.	The relative error for the measurement is 0.02. The percentage error for the measurement is 2%.

(b) A stopwatch with an absolute error of 0.5 seconds was used to record a time of 2 minutes and 15 seconds. Calculate the relative error and percentage error for a recorded time.



THINKING	WORKING
Identify the limit of reading units for the stopwatch.	The limit of reading is seconds.
Convert the time recorded to the same units. 1 minute = 60 seconds	Time recorded: 2 minutes and 15 seconds $2 \times 60 \text{ s} + 15 \text{ s} = 135 \text{ s}$
Use the formula to calculate the relative error.	$\text{Relative error} = \left\lvert \frac{\text{absolute error}}{\text{measured value}} \right\rvert$ $= \left\lvert \frac{0.5}{135} \right\rvert$ $= 0.0037$ correct to 4 decimal places
Convert the relative error to a percentage.	$\text{Percentage error} = \text{relative error} \times 100\%$ $= 0.0037 \times 100\%$ $= 0.37\%$
Write the answer.	The relative error for the precision of the measurement using this stopwatch is 0.0037, correct to 4 decimal places. The percentage error associated with the precision of this measurement is 0.37%.

(c) The length of a eucalyptus leaf was measured using the ruler shown. Calculate the relative error and percentage error for the 8.6 cm measurement, correct to 4 decimal places.



THINKING	WORKING
Identify the precision (limit of reading) of the ruler.	10 smaller increments for every 1 cm. Smallest increments each represent 1 cm ÷ 10 = 0.1 cm. Therefore, the precision (limit of reading) of the ruler is 0.1 cm.
Determine the absolute error of the measurement.	$\text{Absolute error} = \frac{1}{2} \times \text{precision}$ $= \frac{1}{2} \times 0.1 \text{ cm}$ $= 0.05 \text{ cm}$
Use the formula to calculate the relative error.	$\text{Relative error} = \left\lvert \frac{\text{absolute error}}{\text{measured value}} \right\rvert$ $= \left\lvert \frac{0.05}{8.6} \right\rvert$ $= 0.0058 \ (4 \text{ d.p.})$
Convert the relative error to a percentage.	Percentage error = relative error × 100% = 0.0058 × 100% = 0.58%
Write the answer.	The relative error for the measurement is 0.0116, correct to 4 decimal places. The percentage error for the measurement of 8.6 cm with an absolute error of 0.05 cm is 0.58%.

SC 2 I can calculate the relative and percentage error between measured and actual values in context

Absolute error for measures of accuracy

When the actual (true) value of a measurement is known, the absolute error calculation compares the accuracy of the measured value to the actual value. The absolute error is the difference between the measured value and the actual value in the same units.

$\text{Absolute error} = |\text{measured value} - \text{actual value}|$

For these cases, the absolute value of the relative error is used.

$$\text{Relative error} = \left\lvert \frac{\text{absolute error}}{\text{actual value}} \right\rvert$$

Relative error as a measure of accuracy

Recall that the relative error is the error that is relative to the actual size of the attribute being measured (length, time, mass, etc.)

The relative error in this case is the absolute error (calculated as measured value – actual value) divided by the actual value.

$$\text{Relative error (accuracy)} = \frac{\text{measured value} - \text{actual value}}{\text{actual value}}$$

In the above equation, the measured value is the value obtained through guessing, measuring, approximating, or predicting using a computer model.

The actual value is the known quantity, accepted as fact (e.g. the standard reference measurement, real attendance numbers, product content claims or actual sales values). It is the real value to which the measured value is compared.

The absolute value is not used for this calculation because the sign of the error is useful for interpreting the result.

Percentage error as a measure of accuracy

Percentage error is useful for determining how close an expected/estimated/measured value is to the actual stated/known value.

The smaller the percentage error, the closer together (more accurate) the two values are.

The larger the percentage error, the further away (less accurate) the expected/estimated/measured value is from the actual value.

$$\begin{aligned}\text{Percentage error} &= \frac{\text{measured value} - \text{actual value}}{\text{actual value}} \times 100\% \\ &= \text{relative error} \times 100\%\end{aligned}$$

If the expected/estimated/measured value is greater than the actual value, then the expected value was too high, and the percentage error will be positive.

If the expected/estimated/measured value is less than the actual value, then the expected value was too low, and the percentage error will be negative.

Context examples

Attendance at events

Event planning includes, among many other things, predicting the number of people expected to attend. This helps organisers provide enough resources/services to cater for all who attend. Good planning ensures that the difference between the expected and actual number of people who attend is at a minimum.

Food packaging information

Most packaged food products sold will include information on the expected/estimated number of items or mass of the product contained within the package.

Percentage error calculations help ensure that manufacturers maintain accurate product descriptions. The values are routinely tested to ensure consumers are not being misled. Manufacturers will also regularly perform the same checks to ensure they are not putting more product into the packaging than what consumers are paying for.

Both examples use a percentage error calculation to compare the estimated/expected values to the actual/known values.

Worked example

Applying relative and percentage errors in context

(a) A trundle wheel records the distance across a garden to be 22.4 m. Calculate the absolute relative error and percentage error of this measurement, given that the true distance is known to be 2040 cm.

THINKING	WORKING
Convert the measured value given in metres to the same smaller units of the true distance (cm).	Measured distance $= 22.4\,\text{m}$ $= 22.4 \times 100\,\text{cm}$ $= 2240\,\text{cm}$
Calculate the absolute error.	Absolute error $= \lvert\text{measured value} - \text{actual value}\rvert$ $= \lvert 2240 - 2040 \rvert\,\text{cm}$ $= \lvert 200 \rvert\,\text{cm}$ $= 200\,\text{cm}$
Relative error is the absolute error divided by the actual value.	$\text{Relative error} = \dfrac{\text{absolute error}}{\text{actual value}}$ $= \dfrac{200}{2040}$ $= 0.0980\ldots$
Convert the relative error to a percentage.	Percentage error $=$ relative error $\times 100\%$ $= 0.0980 \times 100\%$ $\approx 9.8\%$
Write the answer.	The relative error for the distance measured using the trundle wheel is 0.098, which is a percentage error of 9.8%.

(b) A packet of frozen peas shows it contains 500 g of product. One packet was randomly selected, and the mass of the contents was measured to be 518 g. Calculate the percentage error and comment on the results.

THINKING	WORKING
Use the formula to calculate the percentage error between the actual and expected values.	$\text{Percentage error} = \dfrac{\text{measured value} - \text{actual value}}{\text{actual value}} \times 100\%$ $= \dfrac{518\,\text{g} - 500\,\text{g}}{500\,\text{g}} \times 100\%$ $= 0.03448\ldots \times 100\%$ $\approx 3.4\%$

Compare the measured and expected values.	Measured value (518 g) > actual value (500 g)
Write the answer.	The percentage error between the actual and estimated value is 3.4%. The positive percentage error indicates that the selected item contained 3.4% more product than the actual indicated amount.

(c) A local event prepared for a total of 400 patrons to attend. During the event, 360 people were counted as the total attendance. Calculate the percentage error and comment on the results.

THINKING	WORKING
Use the formula to calculate the percentage error between the actual and expected values.	$\text{Percentage error} = \frac{\text{measured value} - \text{actual value}}{\text{actual value}} \times 100\%$ $= \frac{360 - 400}{400} \times 100\%$ $= -0.1 \times 100\%$ $= -10\%$
Compare the measured and expected values.	Measured value (360) < actual value (400)
Write the answer.	The percentage error between the counted and actual value is −10%. The negative result indicates that less people were present at the event than had been planned for.

Practice

ANSWERS Page 640

SC 1 I can calculate the relative and percentage error in measurements

1 Calculate the relative error for each of these measurements recorded using a thermometer with an absolute error of 0.5 °C.

(a) 10 °C **(b)** 20 °C **(c)** 40 °C

2 Calculate the percentage error for each of these measurements recorded using a thermometer with an absolute error of 0.5 °C.

(a) 10 °C **(b)** 20 °C **(c)** 40 °C

3 Calculate the relative error for these mass measurements recorded using a set of scales with an absolute error of 0.01 kg.

(a) 200 g **(b)** 400 g **(c)** 800 g

4 Calculate the percentage error for these mass measurements recorded using a set of scales with an absolute error of 0.01 kg.

(a) 200 g **(b)** 400 g **(c)** 800 g

5 Calculate the relative error associated with each of the following percentage errors.

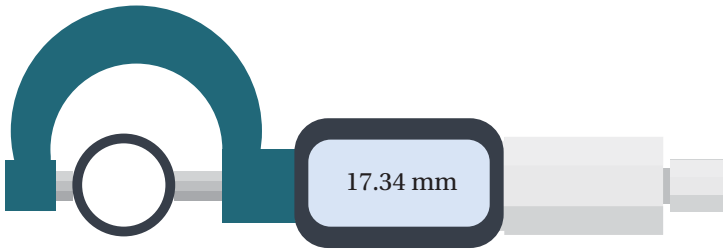
(a) 10% **(b)** 2% **(c)** 2.5% **(d)** 0.5%

6 Calculate the percentage error for these measurements.

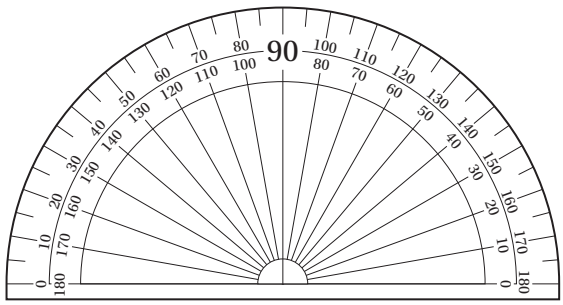
(a) A measuring cylinder with an absolute error of 5 mL recording 0.2 L of water.

(b) A stopwatch with an absolute error of 0.1 s recording a time of $\frac{2}{3}$ of a minute.

7 A digital micrometer measured the width of a pipe to be 17.34 mm. Calculate the percentage error associated with this measurement, correct to 3 decimal places, given that this device has an absolute error of 0.005 mm.



8 An obtuse angle was measured to be 155° using this protractor. Calculate the percentage error associated with this measurement, correct to 1 decimal place.



SC 2 I can calculate the relative and percentage error between measured and actual values in context

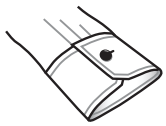
1 Calculate the percentage error for the following pairs of values.

(a) Measured value: 33 marbles; actual value: 30 marbles

(b) Measure value: 11.66 m; actual value: 10.60 m

(c) Measured value: 14.7 kg; actual value: 14.0 kg

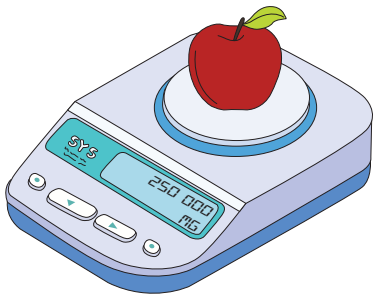
2 A shirt cuff circumference was measured to be 16 cm. The actual circumference of the cuff is 168 mm.



(a) Calculate the absolute relative error of the measurement, correct to 3 decimal places.

(b) Calculate the percentage error of this measurement.

3 A digital scale records the mass of an apple to be 250 000 mg. The true mass of the apple is 240 g.



(a) Calculate the absolute relative error of the measurement, correct to 2 decimal places.

(b) Calculate the percentage error of this measurement.

4 The lower and upper bound limits are given for each of these values. Calculate the absolute percentage error.

(a) A temperature of 10 °C ± 2 °C

(b) A height of 20 m ± 0.5 m

(c) The number of football supporters at a local game is 400 ± 30

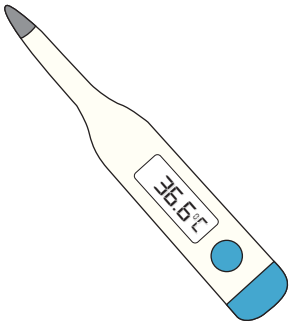
(d) The liquid of contents in a vial is 75 ml ± 3 ml

5 While preparing for a social catch-up at a park, you believe that 12 friends will attend. On the day, a total of 14 friends attend. Calculate the percentage error and comment on the sign of the result.

6 A student claims to be able to solve 5 puzzle cubes within 60 seconds. After 60 seconds, the student has solved 4 puzzle cubes. Calculate the percentage error this represents and comment on the sign of the percentage error.

7 A quality assurance officer conducts tests on production line scales in a factory. If the percentage error is less than or equal to 2%, the equipment is performing adequately. A machine programmed to pour 450 g of cereal into a bag is tested and the value of the first bag filled is measured to be 445.5 g. Calculate the percentage error and comment on the officer's findings for that machine.

8 A tester checks the accuracy of digital thermometers by using them to measure a liquid with a set temperature of 36 °C. The measured value taken by this thermometer is 36.6 °C.



Determine whether the tester passed or failed this thermometer, given that the error can only be a maximum of 1%.

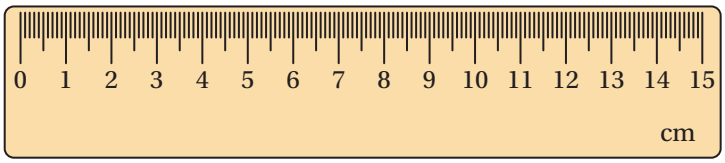
TOPIC REVIEW

Multiple choice

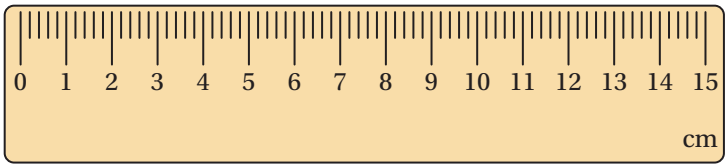
ANSWERS Page 670

1 The ruler that has a precision of 5 mm is:

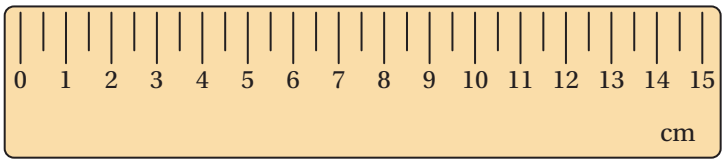
A



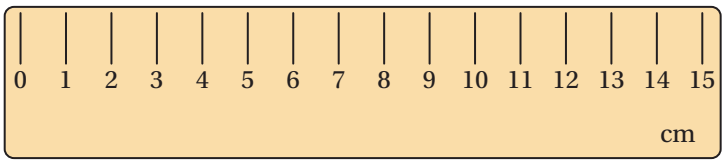
B



C

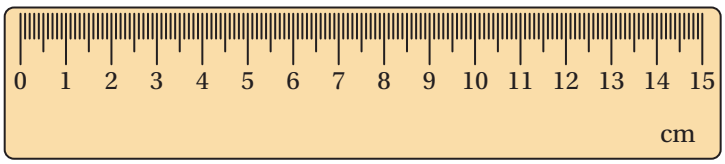


D

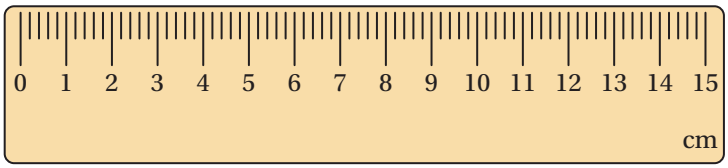


2 The ruler that will measure lengths with an absolute error of $\frac{1}{2}$ mm is:

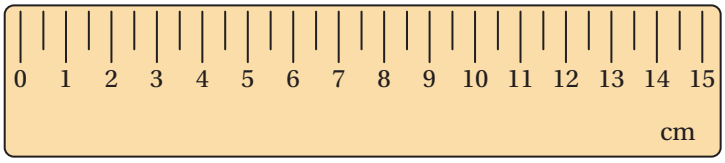
A



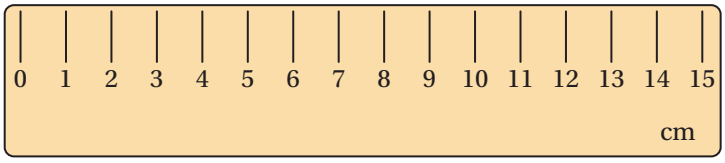
B



C



D



3 The absolute error for a measurement of 1340 km recorded to the nearest 10 km is:

A 20 km **B** 10 km **C** 5 km **D** 2.5 km

4 Which of the following shows the degree of accuracy for a child's temperature measured as 38.2 °C using a thermometer with a precision of 0.1 °C?

A 38.2 °C ± 1 °C **B** 38.2 °C ± 0.5 °C **C** 38.2 °C ± 0.1 °C **D** 38.2 °C ± 0.05 °C

5 The lower and upper bounds for a reading of 243 g taken from kitchen scales with a precision of 1 g are:

A 242.75 g and 243.25 g **B** 242.5 g and 243.5 g

C 242 g and 244 g **D** 241 g and 245 g

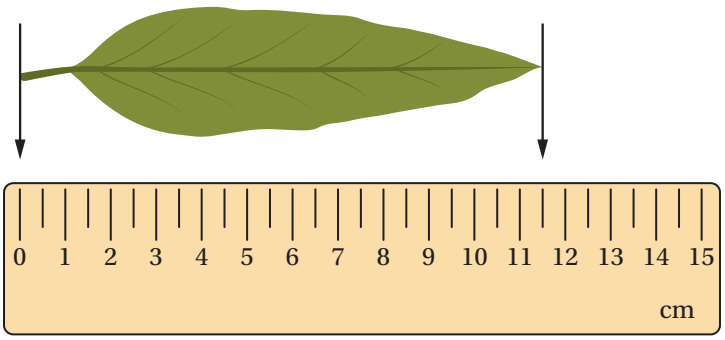
6 The relative error for a mass measured to be 2.5 kg using a scale with an absolute error of 0.05 kg is:

A 0.04 **B** 0.02 **C** 0.01 **D** 0.005

7 The percentage error for a measuring cylinder with an absolute error of 2.5 mL when it is filled to the 250 mL mark is:

A 1% **B** 0.5% **C** 0.2% **D** 0.1%

8 The length of a eucalyptus leaf was measured using the ruler shown.



The percentage error for the measurement is nearest to:

A 0.54% **B** 1.09% **C** 2.17% **D** 4.35%

9 The relative error (precision) for a distance measured on a map as 25 ± 0.5 mm is:

A 0.005 **B** 0.01 **C** 0.02 **D** 0.04

10 A fish and chip shop claims that each of its minimum serves of chips has 80 chips. A customer counted her chips and found there were 77. What was the percentage error?

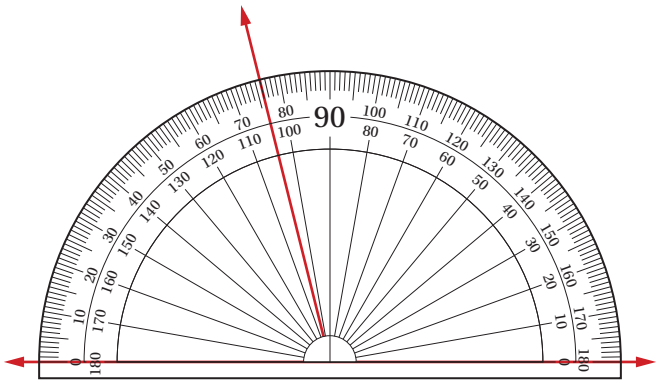
A 3.75% **B** 1.875% **C** −1.875% **D** −3.75%

11 Cans of soft drink are labelled 330 mL. One can was measured to contain 332 mL. What was the percentage error?

A −0.606% **B** −0.602% **C** 0.602% **D** 0.606%

Short answer

1 For the angle drawn on the protractor, determine:



(a) the angle being measured

(b) the precision of the protractor

(c) the absolute error for the measurement.

2 Identify the precision of the following measuring instruments.

(a) A tape measure with five divisions between each centimetre marking

(b) A measuring cylinder with ten divisions between the 20 mL mark and the 25 mL mark

(c) A digital scale giving a reading of 56.3 kg

3 Determine the absolute error for each of the following recorded measurements.

(a) 124 cm, recorded accurate to the nearest centimetre

(b) 63.5 g, recorded accurate to the nearest 0.5 g

(c) 580 km, recorded accurate to the nearest 10 km

4 Write the degree of accuracy for the following measurements in the form ± absolute error.

(a) A length measurement using a tape with a precision of 2 mm

(b) A maximum temperature for the day with a precision of 1 °C

(c) A mass measurement using a kitchen scale with a precision of 50 g

5 Write the measurement with its degree of accuracy for each of the following.

(a) An oven temperature dial with 10 °C increments set to a temperature of 180 °C

(b) A weight scale with 50 g increments showing a mass of 1.65 kg

(c) A ruler with 2 mm increments showing a length of 7.0 cm

6 Write the lower and upper bounds for the following measurements.

(a) A weight measured as 320 g using kitchen scales with a precision of 50 g

(b) A length measured as 65 mm using a ruler with a precision of 2 mm

(c) A temperature reading of −7 °C in Norway from a thermometer with a precision of 1 °C

7 Calculate the relative error, correct to 3 decimal places, for each of the following measurements, taken using a tape with a precision of 1 cm.

(a) A height of 169 cm

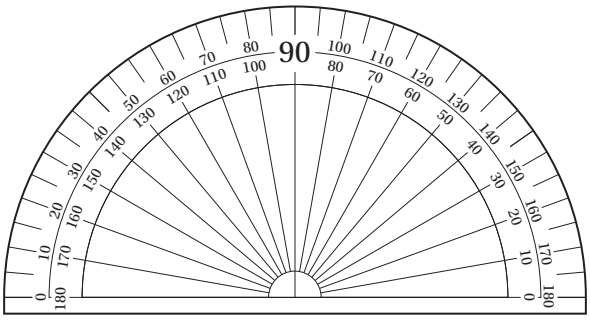
(b) A length of 12 cm

8 Calculate the percentage error, correct to 1 decimal place, for each of the following measurements, taken using bathroom scales with a precision of 0.1 kg.

(a) A suitcase weighing 17.1 kg

(b) A child weighing 38.7 kg

9 Each of the following angles was measured using this protractor.



Calculate the percentage error associated with each measurement, correct to 1 decimal place.

(a) 135° **(b)** 20°

10 Calculate the percentage error, correct to 1 decimal place, for the following pairs of values.

(a) 51 matches counted in a box marked as 50 matches

(b) A measured mass of 125 g of tuna in a can marked 130 g

(c) A measured height of 15.85 m for an overpass marked 15.5 m

11 The waist on a pair of trousers was measured to be 83 cm. According to the manufacturer, the waist size is 80 cm.

(a) Calculate the relative error of the measurement.

(b) Calculate the percentage error of this measurement.

(c) Interpret your answer to **(b)**.

12 The following measurements are given with their degrees of accuracy. Calculate the percentage error (precision) for each measurement, correct to 1 decimal place.

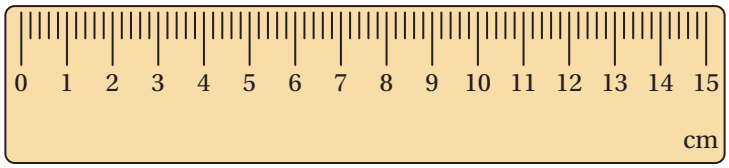
(a) The liquid in a bottle is 128 mL ± 2 mL.

(b) The temperature in an oven is 200 °C ± 5 °C.

13 A tester checks the accuracy of laboratory scales by placing standard weights on the tray totalling 520 g. The scale reading is 517.6 g. Determine whether the tester passed/failed this scale if the maximum allowed absolute error is 1%.

Extended response

1 Answer the following questions with reference to a 15 cm ruler.



(a) Determine the precision of the ruler.

(b) Calculate the absolute error of measurements made with the ruler.

(c) Determine the degree of accuracy for measurements made with the ruler.

(d) Write the measurement 8.6 cm to include the degree of accuracy.

(e) Write the measurement 8.6 cm as a range of values according to the degree of accuracy.

(f) Determine the relative error (precision) for the measurement 8.6 cm, correct to 3 decimal places.

(g) Determine the percentage error for the measurement.

2 A teacher cuts a piece of ribbon to be as close as possible to 16.2 cm long. Ten groups of students estimate the length of the ribbon, without using a ruler. The results are as follows: 10.5 cm, 18 cm, 15 cm, 19.5 cm, 14 cm, 17 cm, 14.5 cm, 18.5 cm, 20 cm and 11.5 cm.

(a) Determine the absolute error of each measurement.

(b) Determine the relative error of each measurement, correct to 2 decimal places.

(c) Determine the average (mean) of the estimated measurements.

(d) Determine the relative error of the average measurement, correct to 2 decimal places.

(e) Determine the average of the relative errors.

(f) Determine the average of the absolute errors.

3 Students are asked to draw a rectangle 12.5 cm by 8.2 cm. Assume that all students take special care to make the angles exactly right.

(a) If the rulers used by the students have a precision of 1 mm, determine the range of values for:

(i) the length **(ii)** the width **(iii)** the perimeter **(iv)** the area.

(b) Determine the percentage error (precision), correct to 2 decimal places, for:

(i) the length **(ii)** the width **(iii)** the perimeter.

TOPIC
10

Pythagoras and geometry

Recall 347

10.1 Apply Pythagoras' theorem to solve problems 348

10.2 Calculate the distance between two points on a Cartesian plane 352

10.3 Understand when right-angled triangles are similar 359

10.4 Use the triangle inequality theorem 369

Topic review 373

Why learn this?

Understanding Pythagoras' theorem and its applications in geometry is valuable in fields such as architecture, engineering and computer graphics, where spatial reasoning and being able to take accurate measurements are crucial.

The ability to apply the theorem to solve problems, calculate distances between points on a Cartesian plane, and identify the properties of similar right-angled triangles all enhance logical thinking and problem-solving. Additionally, the triangle inequality theorem helps with recognising the conditions necessary for a triangle's existence, improving understanding of geometric principles.

RECALL

ANSWERS Page 640

I can round decimals for different purposes

1 Write the following values, correct to 2 decimal places.

(a) 2.018 **(b)** 0.435 **(c)** 1500.156

(d) 345.765 92 **(e)** 19.9958

I can perform calculations involving squaring numbers

1 Calculate each of the following, correct to 2 decimal places.

(a) 3.56^2 **(b)** 9.13^2 **(c)** 15.27^2

(d) 0.45^2 **(e)** 0.04^2

2 Calculate each of the following, correct to 2 decimal places.

(a) $4.83^2 + 5.79^2$ **(b)** $54.36^2 + 26.65^2$ **(c)** $6.57^2 + 95.78^2$

(d) $55.17^2 - 41.34^2$ **(e)** $234.54^2 - 165.72^2$

I can use a calculator to calculate square roots

1 Use your calculator to determine the exact value of each of the following.

(a) $\sqrt{196}$ **(b)** $\sqrt{1225}$ **(c)** $\sqrt{0.0009}$ **(d)** $\sqrt{0.473344}$

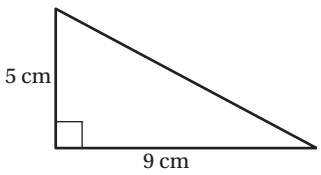
2 Use your calculator to approximate the value of each of the following, correct to 2 decimal places.

(a) $\sqrt{17}$ **(b)** $\sqrt{50}$ **(c)** $\sqrt{170}$ **(d)** $\sqrt{5.25}$

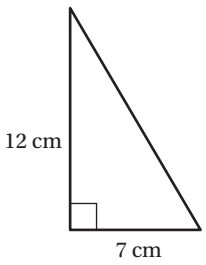
I can use Pythagoras' theorem to find the length of unknown sides in right-angled triangles

1 Use Pythagoras' theorem to calculate the length of the hypotenuse, correct to 2 decimal places.

(a)

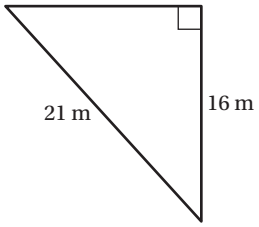


(b)

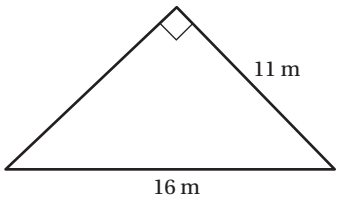


2 Use Pythagoras' theorem to calculate the length of the unknown side, correct to 2 decimal places.

(a)



(b)



10.1 Apply Pythagoras' theorem to solve problems

Learning intention: To be able to apply Pythagoras' theorem to solve problems

Success criteria:

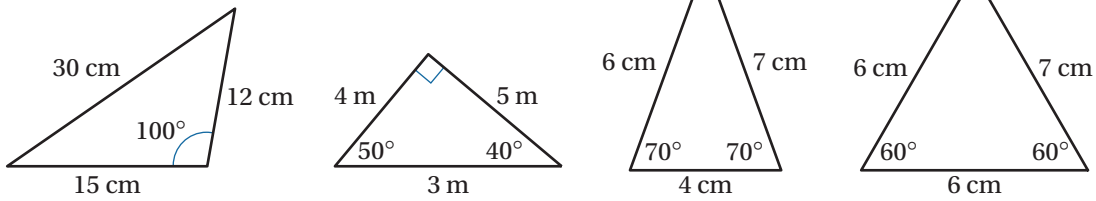
SC 1 I can apply Pythagoras' theorem to surveying problems.

SC 2 I can apply Pythagoras' theorem to problems in a design context.

Lesson warm-up

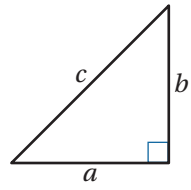
Odd one out

Consider each of the following and find what is wrong in each of the triangles. Explain your reasoning for each.



SC 1 I can apply Pythagoras' theorem to surveying problems

Recall that in a right-angled triangle, the square of the length of the longest side the (hypotenuse) is equal to the sum of the squares of the two shorter sides.



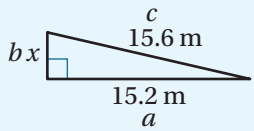
For this diagram: $c^2 = a^2 + b^2$ is used to calculate the length of the hypotenuse; while either $a^2 = c^2 - b^2$ or $b^2 = c^2 - a^2$ are used to calculate the length of one of the shorter sides.

Pythagoras' theorem lends itself to use in surveying, where right angles naturally occur between horizontal and vertical lines.

Worked example

Using Pythagoras' theorem in surveying

A surveyor uses a taut string to measure a line up a sloping piece of land as 15.6 m. A laser level determines the horizontal distance covered by this line to be 15.2 m. Calculate the change in height along this line, to the nearest centimetre.

THINKING	WORKING
Sketch the situation using a right-angled triangle. Mark the required distance with a variable.	

Recall Pythagoras' theorem.	To determine the length of a shorter side: $b^2 = c^2 - a^2$
Use Pythagoras' theorem to solve for the unknown.	$x^2 = 15.6^2 - 15.2^2$ $x^2 = 12.32$ $x = \sqrt{12.32}$ $x = 3.509\ldots$
Interpret the answer, rounding as directed.	The change in height is 3.51 m (2 d.p.).

SC 2 I can apply Pythagoras' theorem to problems in a design context

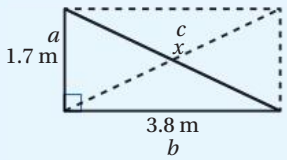
In design applications, right angles naturally occur between horizontal and vertical lines, and where corners are chosen to be perpendicular.

Worked example

Using Pythagoras' theorem in design

A wooden gate, 3.8 m by 1.7 m, is to be made from wooden planks and braced with planks of wood across the diagonals.

(a) Calculate the length of wood needed for each brace, to the nearest centimetre.

THINKING	WORKING
Sketch the situation using a right-angled triangle. Mark the required distance with a variable.	
Recall Pythagoras' theorem.	To determine the length of the hypotenuse: $c^2 = a^2 + b^2$
Use Pythagoras' theorem to solve for the unknown.	$x^2 = 1.7^2 + 3.8^2$ $x^2 = 17.33$ $x = \sqrt{17.33}$ $x = 4.162\ldots$
Interpret the answer, rounding as directed.	The length of wood needed for each brace is 4.16 m (2 d.p.).

(b) Determine the total length of wood needed to make the gate.

THINKING	WORKING
Add the lengths of timber, using multiplication for doubles.	$(1.7 + 3.8 + 4.16) \times 2 = 19.32$
Interpret the answer.	The total length of wood needed is 19.32 m.

Practice

ANSWERS Page 641

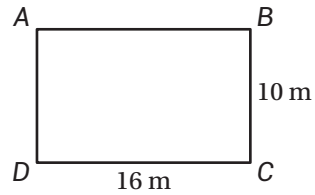
SC 1 I can apply Pythagoras' theorem to surveying problems

1 A trundle wheel is used to measure a sloping stretch of land, where a house is to be built, as 24.7 m. A laser level shows the horizontal distance to be 24.5 m. Determine the change in height for this stretch of land, to the nearest centimetre.

2 A sloping piece of land measures 25.5 m along the ground with the endpoints having an elevation difference of 1.9 m. Determine the distance between the end positions, measured horizontally, correct to 1 decimal place.

3 A map shows the distance between two places as 15.8 km. Given that the elevation difference between the places is 800 m, determine, correct to the nearest metre, the distance along the road, assuming that there are no bends and there is a consistent slope.

4 The footprint of a rectangular house is given.



The soil needs to be excavated to match the level at A, the lowest point.

Round all answers to the questions below to the nearest centimetre.

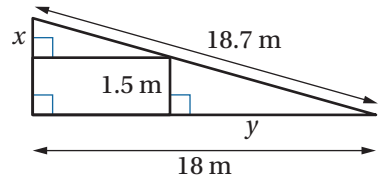
(a) Determine the elevation of B compared to A, given that a taut line between the positions, before excavation, is 16.23 m long.

(b) Determine the distance along the ground, from A to D, assumed to be of even slope, given that the elevation of D compared to A is 2.41 m.

(c) The elevation of C compared to A is 1.58 m. Assuming an even slope, determine the distance along the ground from:

(i) C to D **(ii)** B to C.

5 The 18 m side of a house is planned along a piece of sloping ground, as shown.



The lower part is to be filled with soil to a maximum depth of 1.5 m and the higher part is to be excavated so that the side of the house is all at the same level.

Round all answers to the questions below to the nearest centimetre.

(a) Determine the maximum depth of soil to be excavated.

(b) Determine the distance, measured horizontally, of the section to be filled.

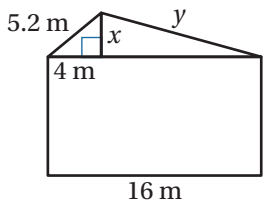
SC 2 I can apply Pythagoras' theorem to problems in a design context

1 A wooden rectangular gate, 2.8 m (horizontal) by 1.6 m (vertical), is to be braced with a horizontal plank, halfway up the gate, and two diagonal planks, one across each of the two equal halves of the gate.

(a) Determine the total length of wood needed, after rounding each length to the nearest centimetre, for the three pieces of bracing.

(b) Determine how much less timber is required with an alternative design where the horizontal plank is replaced by a vertical plank in the middle of the gate, and two diagonals, one across each of the two equal halves of the gate as before.

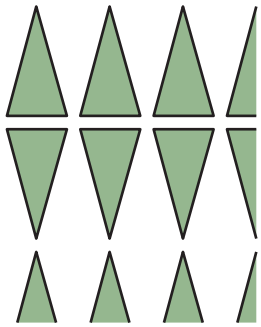
2 The side view of a house with sloping roof sections is shown. Correct to the nearest centimetre:



(a) calculate the maximum height of the roof above the ceiling

(b) determine the length of the sloping roof shown on the right of the diagram.

3 A symmetrical pattern of isosceles triangles is to be used for wrapping paper. Each triangle has longer sides of 4.8 cm and a short side of 2.2 cm. A gap of 3 mm is left between the bases, between the tips and between the triangles at the base. The wrapping paper has a border of 5 mm on all sides.



(a) Calculate the height of each triangle, to the nearest millimetre.

(b) Use the answer from **(a)** to determine the dimensions of the wrapping paper, given that there are 20 triangles along its width and 500 triangles along its length.

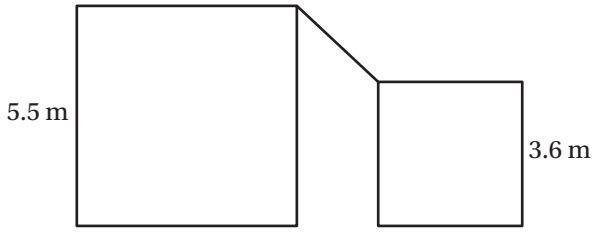
4 A rectangular room measuring 5.2 m by 3.5 m has a sloping ceiling that is 2.5 m high along one of the longer sides and 3.2 m high along the other of the longer sides.

(a) Write the dimensions of the two rectangular walls.

(b) Determine the length of the sloping sides of the matching trapezium-shaped walls, to the nearest centimetre.

(c) Write the dimensions of the ceiling.

5 A new house and shed are to be built with a cable running from the ceiling of the house to the ceiling of the shed. Determine, to the nearest centimetre:



(a) the length of the cable, given that the house and shed are 2.8 m apart

(b) the extra length of cable needed, given that the house and shed are placed an extra metre apart.

10.2 Calculate the distance between two points on a Cartesian plane

Learning intention: To be able to calculate the distance between two points on a Cartesian plane

Success criteria:

SC 1 I can calculate the vertical and horizontal distances between two points plotted on Cartesian axes.

SC 2 I can calculate the distance between any pair of points given in coordinate form.

SC 3 I can apply distance calculations on a Cartesian plane to embedded similar triangles.

Lesson warm-up

Adding squares

Find two square numbers that add to give a third square number.

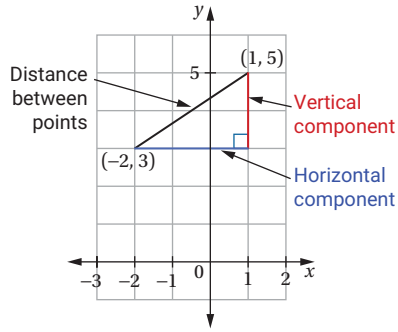


SC 1 I can calculate the vertical and horizontal distances between two points plotted on Cartesian axes

René Descartes (1596–1650) was a French mathematician and philosopher. His best known philosophical quote is 'I think, therefore I am'.

Descartes invented the Cartesian plane in the 17th century, and this revolutionised mathematics by providing the first link between geometry and algebra. Thus, the Cartesian coordinate system is named after him.

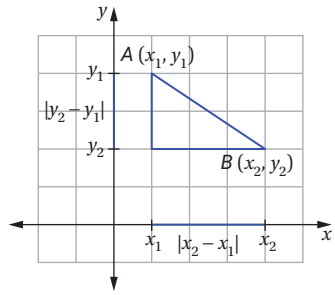
On a grid system, it is easy to break down the distance between points into their horizontal and vertical components.



For point (–2, 3) and (1, 5), the horizontal distance is 1 – (–2) = 3 and the vertical distance is 5 – 3 = 2. These form the side lengths of the right-angled triangle.

Formulas may be used for the horizontal and vertical distances. Using the subscripted points $A(x_1, y_1)$ and $B(x_2, y_2)$:

The horizontal distance is the difference between the x-coordinates: $|x_2 - x_1|$ and the vertical distance is the difference between the y-coordinates: $|y_2 - y_1|$. The absolute value must be used in the formula in case the first value is smaller than the second in the subtraction, and distances are lengths without direction.

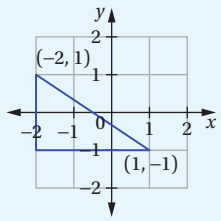


Worked example

Calculating the vertical and horizontal distances between two points plotted on a Cartesian plane

Determine the horizontal and vertical distances between the following points.

(a) (–2, 1) and (1, –1) from a diagram

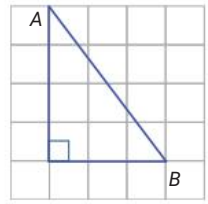
THINKING	WORKING
Plot both points, forming a right-angled triangle, with a line joining the points and the grid lines.	
Use the grid to determine the horizontal and vertical distances between the points.	Horizontal distance: 3 units Vertical distance: 2 units

(b) (6, –2) and (3, 2) from formulas

THINKING	WORKING
Write the formulas for horizontal and vertical distances.	Horizontal distance: $\lvert x_2 - x_1 \rvert$ Vertical distance: $\lvert y_2 - y_1 \rvert$
Identify the values for the formulas.	(6, –2): $x_1 = 6, y_1 = -2$ (3, 2): $x_2 = 3, y_2 = 2$
Substitute and calculate the distances.	$\lvert x_2 - x_1 \rvert = \lvert 3 - 6 \rvert$ $= \lvert -3 \rvert$ $= 3$ $\lvert y_2 - y_1 \rvert = \lvert 2 - (-2) \rvert$ $= \lvert 4 \rvert$ $= 4$
Write the answers in words, with units of length.	Horizontal distance: 3 units Vertical distance: 4 units

SC 2 I can calculate the distance between any pair of points given in coordinate form

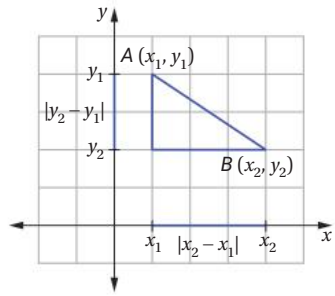
Pythagoras' theorem can be used to calculate the distance between two points.



$$AB^2 = 4^2 + 3^2$$
$$= 25$$
$$AB = \sqrt{25}$$
$$= 5$$

The distance between the points is 5 units.

By incorporating the formulas for the horizontal and vertical distances between two points, a formula may be used to calculate the distance between the points.



$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, or $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where d is the distance between the two points.

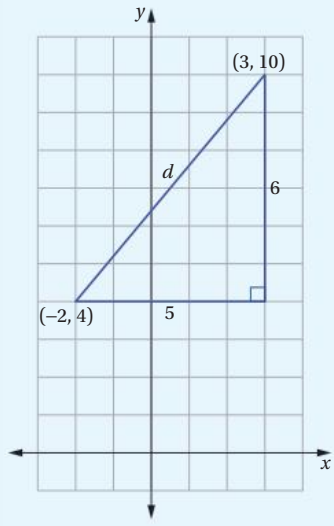
Note: the absolute value symbols are not required in this formula. Because the results of the subtraction are squared, any negative signs will be cancelled out by the squaring. For example, $3^2 = 9$ and $(-3)^2 = 9$.

Worked example

Calculating the distance between points on a Cartesian plane

Calculate the exact distance between the following.

(a) (–2, 4) and (3, 10) from a diagram

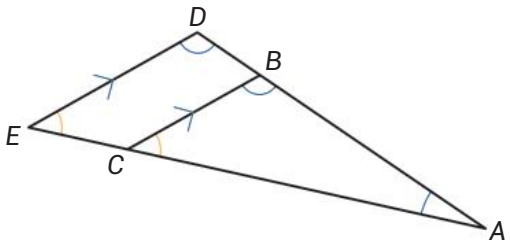
THINKING	WORKING
Plot the points on a Cartesian plane. Form a right-angled triangle and write the lengths on it. Mark the required distance with a variable.	
Calculate the distance using Pythagoras' theorem.	$d^2 = 5^2 + 6^2$ $d^2 = 61$ $d = \sqrt{61}$
Write the answer in words, with units of length.	The distance between the points is $\sqrt{61}$ units.

(b) (6, 20) and (9, 12) using a formula

THINKING	WORKING
Write the distance formula.	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Identify the values for the formula.	(6, 20): $x_1 = 6$, $y_1 = 20$ (9, 12): $x_2 = 9$, $y_2 = 12$
Substitute the values into the formula and calculate the distance.	$d = \sqrt{(9-6)^2 + (12-20)^2}$ $= \sqrt{(3)^2 + (-8)^2}$ $= \sqrt{9+64}$ $= \sqrt{73}$
Write the answer in words, with units of length.	The distance between the points is $\sqrt{73}$ units.

SC 3 I can apply distance calculations on a Cartesian plane to embedded similar triangles

Embedded triangles with a pair of parallel sides have three equal pairs of corresponding angles.



Given that three pairs of corresponding angles in a pair of triangles are equal, the triangles will be similar.

The corresponding sides in similar triangles are in the same proportion.

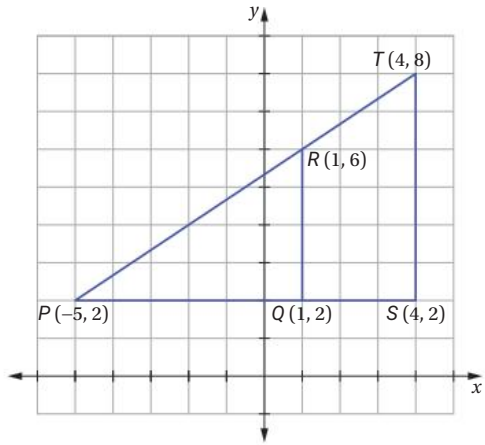
So, $\frac{DE}{BC} = \frac{DA}{BA} = \frac{EA}{CA}$ with the proportion being the scale ratio between the triangles.

In this case, the comparison is larger : smaller, so the scale ratio will be greater than 1.

Worked example

Using the Cartesian plane to demonstrate that embedded, similar, right-angled triangles have pairs of corresponding sides in the same proportion

By calculating the ratio of each pair of corresponding sides, show that the similar right-angled triangles have pairs of corresponding sides in proportion.



THINKING	WORKING
Write the lengths of the horizontal sides, then calculate the ratio of longer to shorter length.	Horizontal lengths: $PS = 9$ units, $PQ = 6$ units $\frac{PS}{PQ} = \frac{9}{6}$ The ratio is $\frac{3}{2}$ or 1.5.
Write the lengths of the vertical sides, then calculate the ratio of longer to shorter length.	Vertical lengths: $TS = 6$ units, $RQ = 4$ units $\frac{TS}{RQ} = \frac{6}{4}$ The ratio is $\frac{3}{2}$ or 1.5.
Calculate the length of the hypotenuse for each triangle.	$PT = \sqrt{9^2 + 6^2}$ $= \sqrt{117}$ units $PR = \sqrt{6^2 + 4^2}$ $= \sqrt{52}$ units
Calculate the ratio of the hypotenuses, longer to shorter length.	$\frac{PT}{PR} = \frac{\sqrt{117}}{\sqrt{52}}$ $= \sqrt{\frac{117}{52}}$ $= \sqrt{2.25}$ $= 1.5$
Summarise, referring to the scale factor.	The corresponding pairs of sides all have the same ratio, longer to shorter length, of 1.5. This is the scale factor.

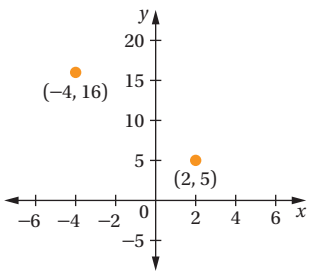
Practice

ANSWERS Page 641

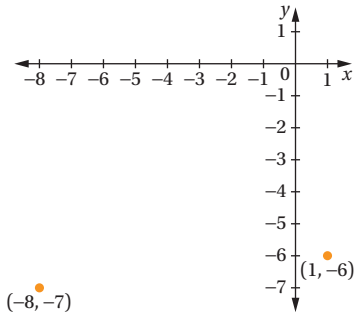
SC 1 I can calculate the vertical and horizontal distances between two points plotted on Cartesian axes

1 Determine the horizontal and vertical distances between the pairs of points on the Cartesian plane.

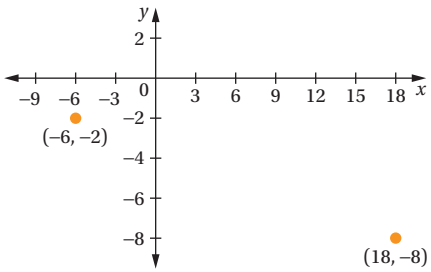
(a)



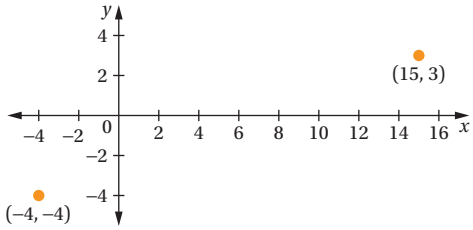
(b)



(c)



(d)



2 Use formulas to calculate the horizontal and vertical distances between each pair of coordinates.

(a) (11, 22) and (1, 8)

(b) (12, −5) and (−24, 5)

(c) (14, −4) and (−10, 16)

(d) (−10, −20) and (0, 15)

3 Determine the coordinates of all points that have a horizontal distance of 3 units and a vertical distance of 7 units from each of the given points.

(a) (4, 5)

(b) (−2, 3)

(c) (−6, −1)

(d) (7, −6)

4 A circle has its centre at (3, 1). Determine the length of the radius when each of the following points lies on the circumference.

(a) (5, 1)

(b) (−4, 1)

(c) (3, 6)

(d) (3, −5)

5 A rectangle $ABCD$ has vertices $A(-2, 3)$ and $B(4, 3)$.

(a) Determine the side length AB.

(b) Determine both possible sets of coordinates for C and D such that the length of the rectangle is 8 units.

SC 2 I can calculate the distance between any pair of points given in coordinate form

1 Draw a diagram to calculate the exact distance between each pair of points.

(a) (2, 10) and (5, 6)

(b) (−4, −4) and (1, 8)

(c) (−2, −5) and (0, 8)

(d) (−4, 12) and (3, −2)

(e) (−2, 7) and (10, −4)

(f) (−9, −8) and (−3, −2)

2 Use a formula to calculate the exact distance between each pair of points.

(a) (3, 2) and (11, 8) **(b)** (−4, 3) and (6, −21) **(c)** (−1, −2) and (3, 8)

(d) (0, −2) and (11, −6) **(e)** (4, 5) and (9, 7) **(f)** (−5, 12) and (−3, −10)

3 A circle has its centre at (3, 1). Calculate the exact length of the radius, given each of the following points on the circumference.

(a) (7, 4) **(b)** (−2, 13) **(c)** (−3, −2) **(d)** (8, −4)

4 Parallelogram $ABCD$ has points $A(-1, 2)$, $B(2, 4)$ and $C(6, 1)$. Determine:

(a) the coordinates of D **(b)** the exact lengths of the sides

(c) the exact lengths of the diagonals.

5 Rectangle $ABCD$ has points $A(3, -2)$ and $B(5, -2)$. Determine possible pairs of coordinates for C and D, given that the length of AC is $\sqrt{20}$ units or $2\sqrt{5}$ units.

SC 3 I can apply distance calculations on a Cartesian plane to embedded similar triangles

1 A Cartesian plane has three given points: $A(0, 1)$, $B(4, 1)$ and $C(4, 3)$.

(a) On grid paper, draw and label ΔABC. **(b)** Extend AB to D so that $\frac{AD}{AB} = \frac{3}{2}$.

(c) Place F so that ΔABC is embedded in ΔADF. **(d)** Show, by calculation, that $AC = 2CF$.

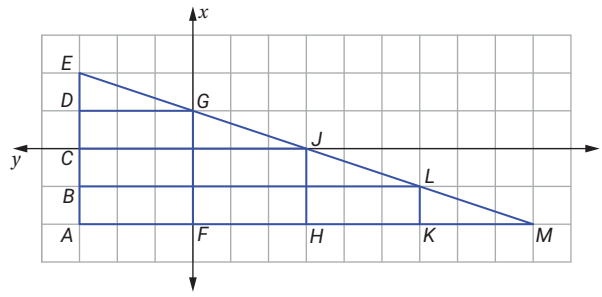
2 A Cartesian plane has three given points: $P(-3, -2)$, $Q(5, -2)$ and $R(5, 4)$.

(a) On grid paper, draw and label ΔPQR.

(b) Place T on RQ and S on PR so that ΔSTR is embedded in ΔPQR with a scale factor of 2.

(c) Show, by calculation, that $PS = SR$.

Use the image below to answer Questions **3**, **4** and **5**.



3 Write the large to small scale ratios, as whole numbers or fractions, for each pair of similar triangles.

(a) ΔEDG and ΔEAM **(b)** ΔECJ and ΔEBL **(c)** ΔMFG and ΔMAE

4 Name the triangle that has ΔECJ embedded, with a scale factor of $\frac{3}{2}$, between the two similar triangles.

5 By calculating the lengths of all sides of ΔMFG and ΔMAE, show that the triangles are similar with a scale factor of $\frac{4}{3}$.

Understand when right-angled triangles are similar

10.3

Learning intention: To be able to understand when right-angled triangles are similar

Success criteria:

SC 1 I can identify pairs of similar right-angled triangles.

SC 2 I can calculate an unknown side given enough information using similar triangle properties.

SC 3 I can use the scale factor between two similar right-angled triangles to calculate the area of one triangle from the area of the other.

Lesson warm-up

Egyptian rope

How might the ancient Egyptians have used this piece of rope to build the pyramids?



SC 1 I can identify pairs of similar right-angled triangles

Similar triangles have three pairs of corresponding angles that are equal and three pairs of corresponding sides in the same proportion (referred to as the scale factor).

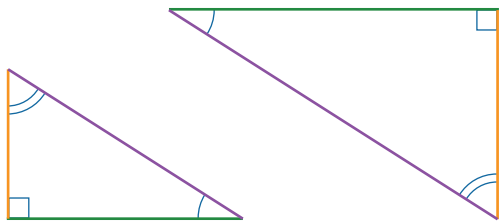
All right-angled triangles have one angle of 90°, with the other two angles being complementary (that is, they add to 90°, such as 70° and 20°).

Thus, a pair of right-angled triangles are similar when an acute angle in one triangle is the same as an acute angle in the other, or an acute angle in one is complementary to an acute angle in the other.

Given a right-angled triangle with two known sides, the third 'fixed' side can be solved using Pythagoras' theorem.

Thus, a pair of right-angled triangles are similar when any two pairs of corresponding sides are in the same proportion.

Unless a right-angled triangle is isosceles, the corresponding sides and angles are simply the largest pair (the right angles and the hypotenuses), the middle-sized pair (the larger of the acute angles and the sides opposite these angles) and the smallest pair (the smallest angles and the sides opposite these angles).



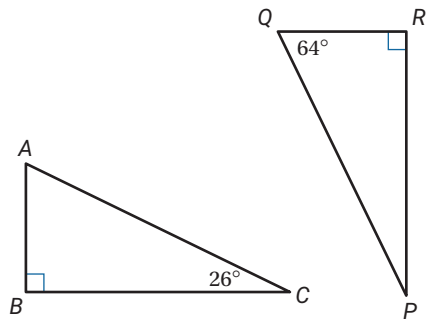
Worked example

Identifying pairs of similar right-angled triangles

In each of the following pairs of triangles:

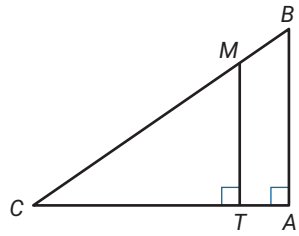
- determine whether or not the second triangle is certain to be similar to ΔABC
- explain why or why not
- where applicable, write the similarity statement matching the corresponding vertices.

(a)



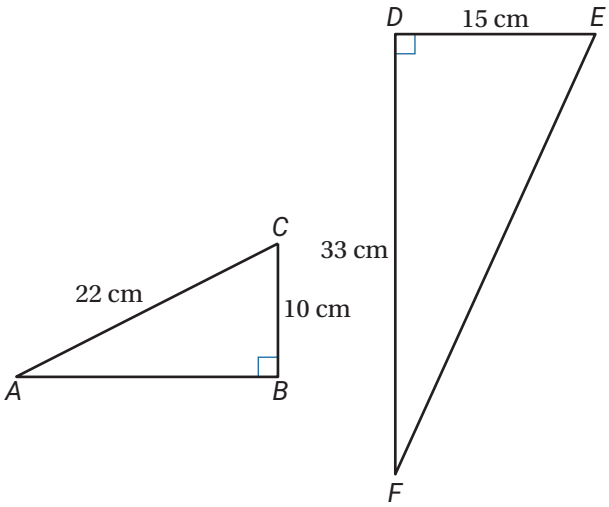
THINKING	WORKING
Check whether the given angles are complementary.	$26° + 64° = 90°$
Two triangles are similar when all corresponding angles are equal.	Hence $\angle C = \angle P$ and $\angle A = \angle Q$. A pair of equal angles, as well as the corresponding right angles, is sufficient to be certain of similarity.
Match the vertices of the right angles, the smallest angles and the middle-sized angles.	$\Delta ABC \sim \Delta QRP$

(b)



THINKING	WORKING
Identify a pair of equal angles other than the right angles.	$\angle C$ is a shared angle.
Two triangles are similar when all corresponding angles are equal.	Hence, with the right angles, there are two pairs of equal angles, so the triangles are similar. Note: the third pair of angles must be equal because the angle sum of a triangle is a constant 180°.
Match the vertices of the right angles, the smallest angles and the middle-sized angles.	$\Delta ABC \sim \Delta TMC$

(c)



THINKING	WORKING
Check whether the sides with given lengths are corresponding pairs of sides.	AC is the hypotenuse of ΔABC, but the length of EF, the hypotenuse of ΔDEF, is not known.
Check the proportions of any matching pairs of sides.	$\frac{DE}{CB} = \frac{15}{10} = 1.5$ Check the other side lengths. The hypotenuse EF would be in the same proportion: $AC \times 1.5 = 22 \times 1.5$ $= 33\text{ cm}$ Given that the shorter side DF is 33 cm, the hypotenuse EF is longer than 33 cm.
State whether the information is insufficient to be certain of similarity, or whether the triangles are definitely not similar.	The triangles are not similar.

SC 2 I can calculate an unknown side given enough information using similar triangle properties

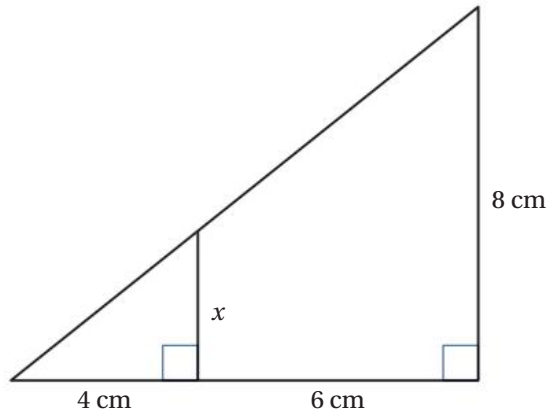
In right-angled triangles, Pythagoras' theorem can be used to calculate a third side when two sides are known.

With similar triangles, the scale factor, k, can be calculated whenever a pair of corresponding side lengths are known. The scale factor can then be used to calculate an unknown side length from its corresponding side in the other triangle.

Worked example

Calculating scale factor and solving unknown lengths in pairs of similar right-angled triangles

For the embedded right-angled triangles:



(a) identify the corresponding sides for those sides that are marked

THINKING	WORKING
Trace over the two triangles with your finger, then match the corresponding sides, starting with the smaller one each time.	$x \leftrightarrow 8\,\text{cm}$ $4\,\text{cm} \leftrightarrow (4 + 6)\,\text{cm}$

(b) calculate the scale factor, k using the larger to smaller ratio

THINKING	WORKING
Use the correspondence where both sides are known. Write the ratio in the order large to small.	Scale factor: $\frac{10}{4}$ Hence $k = \frac{5}{2}$ or 2.5.

(c) calculate the length of the unknown side.

THINKING	WORKING
The unknown side is on the smaller triangle, so divide the corresponding side by the scale factor.	$x = \frac{8}{k}$ $= \frac{8}{2.5}$ $= 3.2$
Write the answer.	The unknown side is 3.2 cm.

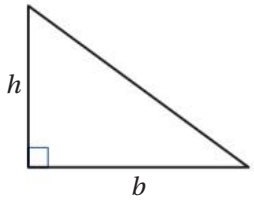
SC 3 I can use the scale factor between two similar right-angled triangles to calculate the area of one triangle from the area of the other

The area of any triangle can be found from the formula $A = \frac{bh}{2}$.

For right-angled triangles, the base, b, and height, h, are the two sides forming the right angle. When only one of these sides is given, but the hypotenuse is known, Pythagoras' theorem can be used to calculate the unknown side.

For any pair of similar triangles connected by a scale factor of k, the area calculation for the larger triangle (assuming $k > 1$) involves multiplying both the base and the height of the smaller triangle by k.

$$A_{\text{larger triangle}} = \frac{(b \times k)(h \times k)}{2}$$
$$= k^2 \times \frac{bh}{2}$$
$$= k^2 \times A_{\text{smaller triangle}}$$

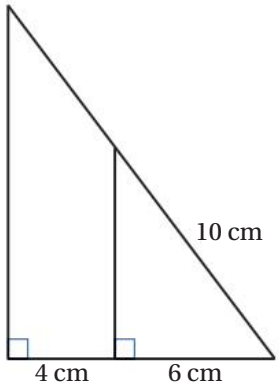


Hence, the factor connecting the areas of the triangles is k^2.

Worked example

Calculating the area of similar right-angled triangles from the scale factor

For the embedded right-angled triangles, calculate:



(a) the scale factor in the order larger to smaller

THINKING	WORKING
For a pair of corresponding sides where both lengths are known, divide the larger by the smaller.	Horizontal sides: $(4 + 6)$ cm $\leftrightarrow$ 6 cm $k = \frac{10}{6}$
Write the answer.	The scale factor is $\frac{5}{3}$.

(b) the height of the smaller triangle

THINKING	WORKING
Use the subtraction version of Pythagoras' theorem for calculating the length of a shorter side. Note: Pythagorean triads could have been used here.	$\sqrt{10^2 - 6^2} = \sqrt{64}$ $= 8$
Write the answer.	The height of the smaller triangle is 8 cm.

(c) the area of the smaller triangle

THINKING	WORKING
Recall the formula for the area of a triangle.	$A = \frac{bh}{2}$
Substitute the base and height to calculate the area.	For $b = 6$ cm, $h = 8$ cm $A = \frac{6 \times 8}{2}$ $= 24 \text{ cm}^2$
Write the answer.	The area of the smaller triangle is 24 cm^2.

(d) the exact area of the larger triangle from the area of the smaller triangle, using the scale factor.

THINKING	WORKING
Recall how to use the scale factor to calculate area of similar shapes.	Multiply the area of the smaller triangle by k^2.
Calculate the area of the larger triangle by multiplying by the square of the scale factor.	$A_{\text{larger triangle}} = k^2 \times A_{\text{smaller triangle}}$ $= \left(\frac{5}{3}\right)^2 \times 24$ $= \frac{200}{3}$ $= 66\frac{2}{3} \text{ cm}^2$
Write the answer.	The area of the larger triangle is $66\frac{2}{3} \text{ cm}^2$.

Practice

ANSWERS Page 642

SC 1 I can identify pairs of similar right-angled triangles

1 In each of the following triangles, determine whether the given triangle is certain to be similar to ΔDEF, where $\angle D$ is a right angle and $\angle E = 37°$. Where applicable, write the similarity statement matching the corresponding vertices.

(a) ΔABC, where $\angle A$ is a right angle and $\angle B = 37°$

(b) ΔGHJ, where $\angle H$ is a right angle and $\angle G = 63°$

(c) ΔKLM, where $\angle L = 37°$ and $\angle K = 53°$

(d) ΔPQR, where $\angle P = 37°$ and $\angle Q = 57°$

2 In each of the following triangles, determine whether the given triangle is certain to be similar to ΔJKL, where $\angle L$ is a right angle, $JK = 15\,\text{cm}$ and $JL = 12\,\text{cm}$.

For similar triangles, write the scale factor (using the large to small ratio) and the similarity statement matching the corresponding vertices.

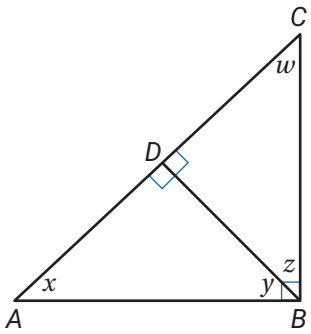
(a) ΔABC, where $\angle C$ is a right angle, $AB = 5\,\text{cm}$ and $AC = 4\,\text{cm}$

(b) ΔPQR, where $\angle Q$ is a right angle, $PR = 24\,\text{cm}$ and $QR = 18\,\text{cm}$

(c) ΔXYZ, where $XY = 12\,\text{cm}$, $XZ = 15\,\text{cm}$ and $YZ = 18\,\text{cm}$

(d) ΔMNT, where $\angle M$ is a right angle, $MN = 45\,\text{cm}$ and $MT = 36\,\text{cm}$

3 Consider the diagram on the right.



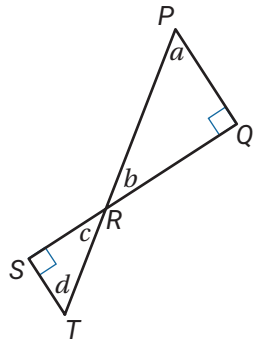
(a) Solve for x, y and z, given:

(i) $w = 51°$

(ii) $w = 47°$.

(b) Write two similarity statements that begin: $\Delta ABC \sim$.

4 Consider the diagram on the right.



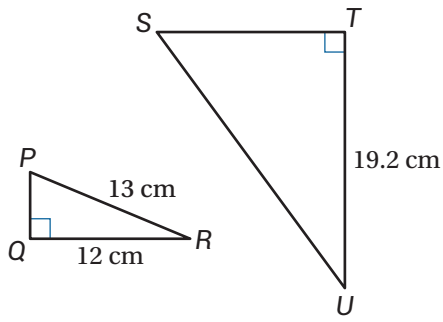
(a) Solve for b, c and d, given:

(i) $a = 48°$

(ii) $a = 53°$.

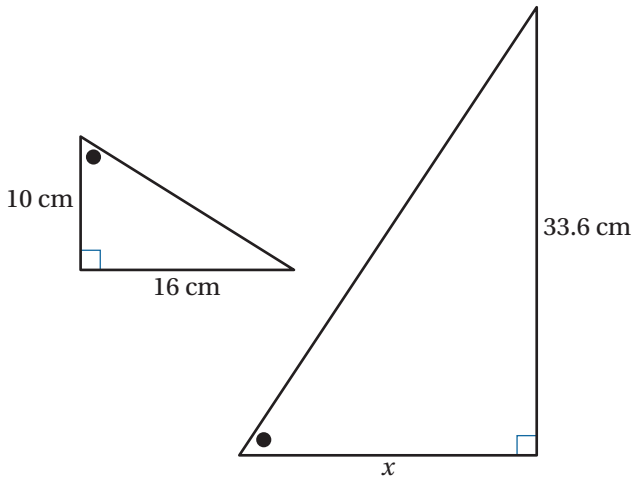
(b) Write a similarity statement that begins: $\Delta PQR \sim$.

5 In the right-angled triangles below, $\Delta PQR \sim \Delta STU$. Determine the side lengths ST and SU.



SC 2 I can calculate an unknown side given enough information using similar triangle properties

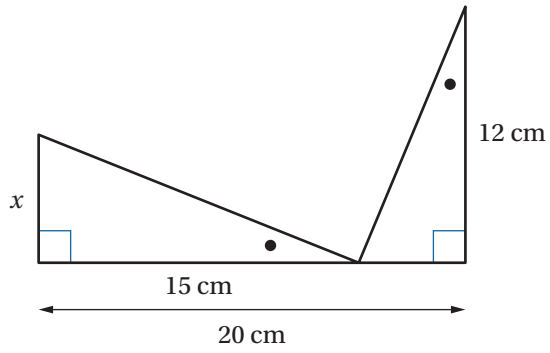
1 For the similar right-angled triangles:



(a) calculate the scale factor, k, using the larger to smaller ratio

(b) calculate the length of the unknown side.

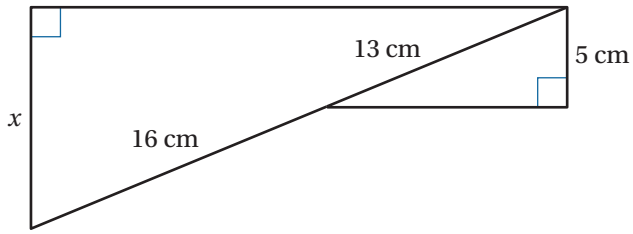
2 For the similar right-angled triangles:



(a) calculate the scale factor, k, using the larger to smaller ratio

(b) calculate the length of the unknown side.

3 For the pair of right-angled triangles:



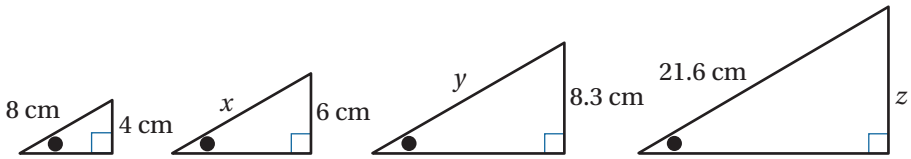
(a) identify a pair of acute angles that are equal, giving reasons

(b) write the corresponding sides from those that are marked

(c) calculate the exact scale factor, k, using the larger to smaller ratio

(d) calculate the exact length of the unknown side.

4 Consider the set of four similar right-angled triangles below.

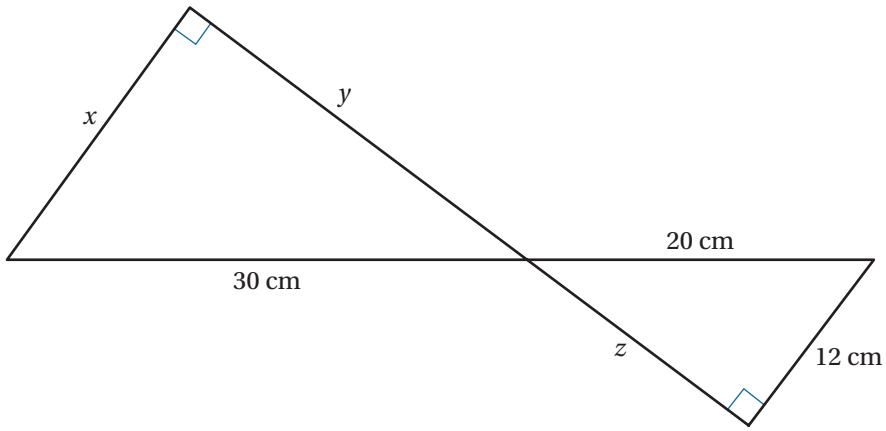


(a) Calculate the lengths of the unknown sides.

(b) Write the values of the three different scale factors, comparing side lengths with the first triangle on the left.

(c) Describe the relationship between the hypotenuse and the vertical side in each of the triangles.

5 For the pair of right-angled triangles:



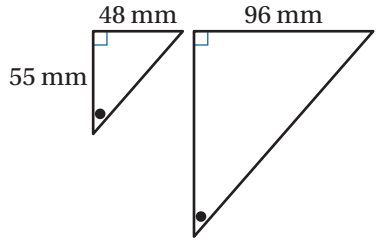
(a) identify a pair of acute angles that are equal, giving reasons

(b) calculate the scale factor, k, using the larger to smaller ratio

(c) calculate the exact lengths of the unknown sides.

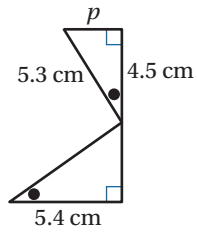
SC 3 I can use the scale factor between two similar right-angled triangles to calculate the area of one triangle from the area of the other

1 For the similar right-angled triangles on the right, calculate:



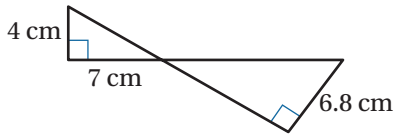
- **(a)** the scale factor in the order larger to smaller
- **(b)** the area of the smaller triangle
- **(c)** the area of the larger triangle.

2 For the similar right-angled triangles on the right, calculate:



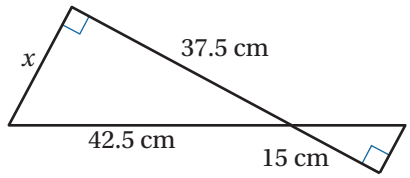
- **(a)** the scale factor in the order larger to smaller
- **(b)** the horizontal side of the smaller triangle
- **(c)** the area of the smaller triangle
- **(d)** the area of the larger triangle.

3 For the pair of right-angled triangles, calculate:



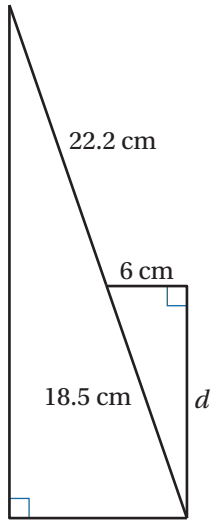
- **(a)** the scale factor in the order larger to smaller
- **(b)** the area of the smaller triangle
- **(c)** the area of the larger triangle.

4 For the pair of right-angled triangles, calculate:



- **(a)** the scale factor in the order larger to smaller
- **(b)** the unknown side of the larger triangle
- **(c)** the area of the larger triangle
- **(d)** the area of the smaller triangle.

5 For the pair of right-angled triangles, calculate:



- **(a)** the scale factor in the order larger to smaller
- **(b)** the height of the smaller triangle
- **(c)** the area of the smaller triangle
- **(d)** the area of the larger triangle.

Use the triangle inequality theorem 10.4

Learning intention: To be able to use the triangle inequality theorem

Success criteria:

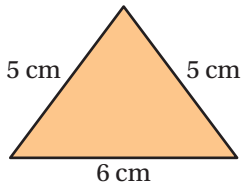
SC 1 I can apply the triangle inequality theorem to practical situations involving right-angled triangles.

SC 2 I can extend the theorem to acute- and obtuse-angled triangles.

Lesson warm-up

Increase the base

Consider the triangle below.



The base of the triangle is increased to 8 cm and the other sides are kept the same.

What happens to the area of the triangle?

SC 1 I can apply the triangle inequality theorem to practical situations involving right-angled triangles

The inequality theorem for triangles says that any side cannot be longer than (or even 'as long as') the other two sides combined.

Triangles with shorter sides of 3 units and 4 units cannot have a third side of 7 units or more.

With right-angled triangles, the side lengths are further constrained by Pythagoras' theorem.

Worked example

Understanding constraints in triangles

Two side lengths of a triangle are 8 cm and 12 cm.

(a) Determine the constraints on the third side if it is:

(i) the shortest side **(ii)** the second-shortest side **(iii)** the longest side.

THINKING	WORKING
The shortest side must be shorter than both of the given sides and longer than the difference between them.	The shortest side must be between 4 cm and 8 cm.
The second-shortest side must be between the values of the given sides.	The second-shortest side must be between 8 cm and 12 cm.
The longest side must be longer than both of the given sides, but less than their total.	The longest side must be between 12 cm and 8 + 12 = 20 cm.

(b) Determine both possible lengths for the third side, to the nearest millimetre, given that the triangle is to be right-angled. Write your answer correct to one decimal place.

THINKING	WORKING
Use Pythagoras' theorem to determine the unknown length if it was the hypotenuse of the right-angled triangle.	$\sqrt{8^2+12^2} = \sqrt{208}$ $= 14.42\ldots$ $= 14.4\ (1\text{ d.p.})$
Use Pythagoras' theorem to determine the unknown length if it was a shorter side of the right-angled triangle.	$\sqrt{12^2-8^2} = \sqrt{80}$ $= 8.94\ldots$ $= 8.9\ (1\text{ d.p.})$
Write the answer.	The third side could be a hypotenuse of 14.4 cm or the second-shortest side, 8.9 cm. It cannot be the shortest side.

SC 2 I can extend the theorem to acute- and obtuse-angled triangles

The two smallest angles in any triangle must be acute, while the largest angle could be either acute, right or obtuse. Hence, triangles are named for their largest angle.

Worked example

Calculating the range of side lengths for acute- and obtuse-angled triangles

The two shorter sides of a triangle are given as 18 cm and 25 cm. Determine the range of lengths for the longest side if the triangle is:

(i) acute-angled **(ii)** obtuse-angled.

THINKING	WORKING
Determine the range for the longest side.	The longest side must be greater than 25 cm, but shorter than 25 + 18 = 43 cm.
Determine the hypotenuse value for a right-angled triangle.	$\sqrt{18^2+25^2} = \sqrt{949}$ $= 30.80\ldots$ $= 30.8\,(1\text{ d.p.})$ If the triangle was right-angled, then the third side would be exactly $\sqrt{949}$ or about 30.8 cm
State the range for the longest side in an acute-angled triangle.	If the triangle is acute-angled, the range for the length of the longest side is between 25 cm and 30.8 cm.
State the range for the longest side in an obtuse-angled triangle.	If the triangle is obtuse-angled, the range for the length of the longest side is between 30.8 cm and 43 cm.

Practice

ANSWERS Page 643

SC 1 I can apply the triangle inequality theorem to practical situations involving right-angled triangles

1 Two side lengths of a triangle are 5.6 cm and 7.7 cm.

(a) Determine the constraints on the third side if it is:

(i) the shortest side **(ii)** the second-shortest side **(iii)** the longest side.

(b) Determine both possible lengths for the third side, to the nearest millimetre, given that the triangle is to be right-angled.

2 In a design, a right-angled triangle is to have a hypotenuse of 3.7 m.

(a) Write all possible pairs of shorter side lengths where one side is 0.5 m, 1.5 m, 2.5 m or 3.5 m. Round the other correct to the nearest centimetre.

(b) For each pair of shorter side lengths from **(a)**, determine the range of the longest side lengths that are possible for any triangle.

3 Two side lengths of a triangle are 40 cm and 50 cm.

(a) Determine the range of possible values for the third side.

(b) Explain why a right-angled triangle cannot be built with a third side between 40 cm and 50 cm.

4 Consider sets of three whole-number values (with no repeats) that represent the side lengths in a triangle.

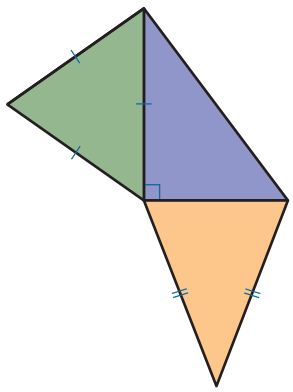
(a) List all such triples in which the three values lie in the range 11 cm to 15 cm.

(b) Using any pair from the triple, try to determine a side length in the required range (but not necessarily a whole number) that could form a right-angled triangle. Comment on what you have found.

5 A design is to have a scalene right-angled triangle with an equilateral and an isosceles triangle attached.

- The sides of the equilateral and isosceles triangles are to be whole numbers of centimetres from 2 cm to 6 cm.
- The base of the isosceles triangle must be shorter than its equal sides.
- The base of the isosceles triangle must be shorter than the sides of the equilateral triangle.
- The hypotenuse does not have to have whole-number lengths, but must not be more than 6 cm.

Determine all possible sets of side lengths for the triangles in the design, including the exact length of each hypotenuse.



SC 2 I can extend the theorem to acute- and obtuse-angled triangles

1 An isosceles triangle has equal sides of 40 cm. Determine the range of lengths for the third side, to the nearest millimetre, for the triangle to be:

(a) acute-angled **(b)** obtuse-angled.

2 The two shorter sides of a triangle are given as 1.5 m and 2.9 m. Determine the range of lengths for the longest side, to the nearest centimetre, for the triangle to be:

(a) acute-angled **(b)** obtuse-angled.

3 A triangle is to be constructed with side lengths of 6 cm and 10 cm.

(a) State the range of values possible for the third side.

(b) Write the pair of values, rounded to the nearest millimetre where necessary, that would result in a right-angled triangle.

(c) Break the range of possible values for the third side into three subgroups, using the right-angled triangle values as the limits of the middle group. These subgroups will lead to all acute-angled triangles or all obtuse-angled triangles.

(d) Use whole number values to test each subgroup to see whether all the triangles in the group are acute-angled or obtuse-angled.

4 A triangle is to be constructed with side lengths of 12 cm and 16 cm.

(a) State the range of values possible for the third side.

(b) Write the pair of values, rounded to the nearest millimetre where necessary, that would result in a right-angled triangle.

(c) Write the range of values that would result in an acute-angled triangle.

(d) Write the range of values that would result in an obtuse-angled triangle.

5 A triangle is to be constructed with side lengths of 5 cm and 12 cm.

(a) State the range of values possible for the third side.

(b) Write a list of all possible whole-number lengths that would result in:

(i) an acute-angled triangle **(ii)** an obtuse-angled triangle.

TOPIC REVIEW

Multiple choice

ANSWERS Page 671

1 If two sides of a right-angled triangle are 10 cm and 14 cm, the hypotenuse, to the nearest millimetre, could be:

A 9.8 cm or 17.2 cm **B** 14 cm or 17.2 cm **C** 17.2 cm or 24 cm **D** 14 cm or 24 cm

2 If the two shorter sides of a triangle are 3.5 m and 4.5 m, the longest side, must be between:

A 4.5 m and 5 m **B** 6 m and 8 m **C** 3.5 m and 4.5 m **D** 4.5 m and 8 m

3 An isosceles triangle has equal sides of 22 cm. The triangle could be:

A acute-angled or obtuse-angled only

B right-angled or obtuse-angled only

C acute-angled or right-angled only

D acute-angled, right-angled or obtuse-angled

4 The longest side of an isosceles triangle is 25 cm. The shortest side must be:

A less than 25 cm **B** between 12.5 cm and 25 cm

C between 12.5 cm and 17.7 cm **D** between 17.7 cm and 25 cm

5 A piece of string pulled taut gives the distance between two positions on a sloping block to be 12.63 m, with the endpoints having a height difference of 3.66 m. The distance between the two positions, measured horizontally, is closest to:

A 9.0 m **B** 10.0 m **C** 12.1 m **D** 13.1 m

6 A wooden gate, 3 m by 1.3 m, is to be made from wooden planks and braced with a pair of planks placed diagonally. The total length of wood needed to make the gate is closest to:

A 11.9 m **B** 15.1 m **C** 7.6 m **D** 10.8 m

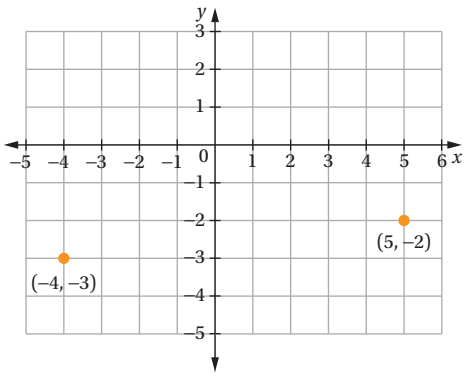
7 The horizontal and vertical distances between the points are:

A Horizontal: 1 unit; vertical: 9 units

B Horizontal: 9 units; vertical: 1 unit

C Horizontal: 5 units; vertical: 9 units

D Horizontal: 1 unit; vertical: 1 unit



8 The horizontal and vertical distances between (3, 6) and (0, –1) are:

A Horizontal: –3 units; vertical: –7 units

B Horizontal: 0 units; vertical: 5 units

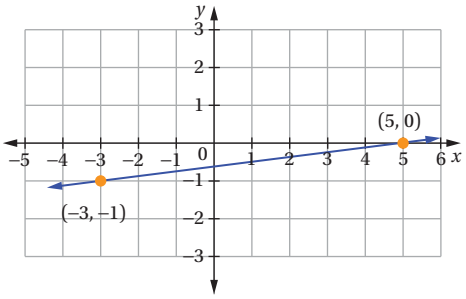
C Horizontal: 3 units; vertical: 5 units

D Horizontal: 3 units; vertical: 7 units

9 The point that has a horizontal distance of 4 units and a vertical distance of 2 units from (–5,8) is:

A (–9,10) **B** (–1,4) **C** (–3,6) **D** (–7,12)

10 The exact distance between the points is:

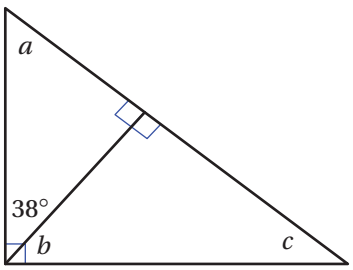


A 9 units **B** $\sqrt{68}$ units **C** $\sqrt{65}$ units **D** $\sqrt{5}$ units

11 The distance between the points (–10,4) and (10,–17) is:

A $\sqrt{569}$ units **B** 13 units **C** 29 units **D** $\sqrt{41}$ units

12 For the diagram below, determine the angles a, b and c.



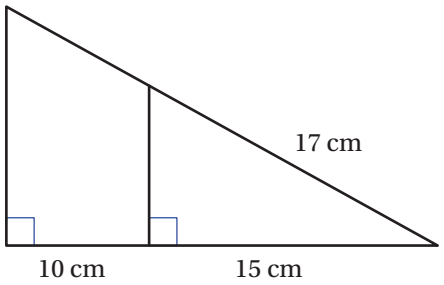
A $a = 52°$, $b = 52°$, $c = 38°$

B $a = 52°$, $b = 38°$, $c = 52°$

C $a = 38°$, $b = 52°$, $c = 38°$

D $a = 38°$, $b = 52°$, $c = 52°$

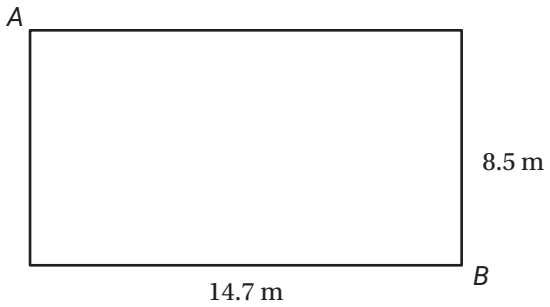
13 For the embedded right-angled triangles, the exact area of the larger triangle is:



A $\frac{125\sqrt{514}}{6}$ cm^2 **B** $166\frac{2}{3}$ cm^2 **C** 375 cm^2 **D** 135 cm^2

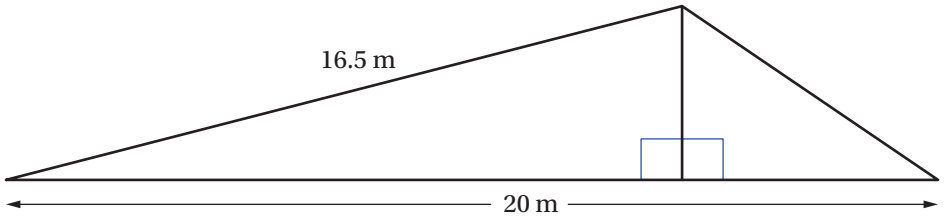
Short answer

1 The footprint of a rectangular house is given.



Before excavation of soil to flatten the block, the level at A is 1.84 m above the level at B. Determine the distance along the sloping ground, from A to B, to the nearest centimetre, assuming an even slope.

2 The side view of a sloping roof section of a house is shown. Given that the roof rises to a maximum of 2.1 m above ceiling height, determine the unknown length of the sloping roof on the right, correct to the nearest centimetre.



3 A rectangle, $PQRS$, has given vertices $P(1,4)$ and $Q(1,-2)$. Determine both possible sets of coordinates for R and S such that the width of the rectangle is 3 units.

4 Calculate the exact distance between each pair of points.

(a) (6,5) and (8,1) **(b)** (–7,2) and (1,10)

5 Calculate the distance between each pair of points.

(a) (–3,–6) and (5,0) **(b)** (–20,2) and (15,–10)

6 A circle has its centre at (1,2), with (3,–3) on the circumference.

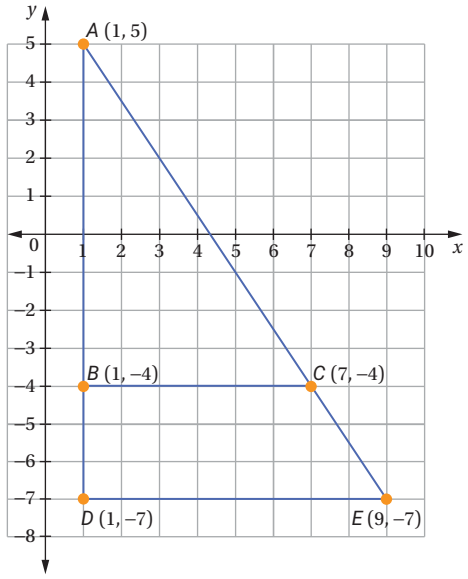
(a) Calculate the exact length of the radius.

(b) Determine the whole-number coordinates of three other points on the circumference.

7 By calculating the lengths of all sides of ΔABC and ΔADE, as shown on the axes below:

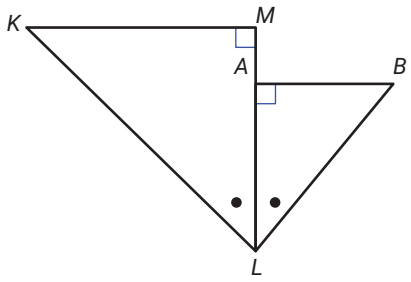
(a) show that the triangles are similar

(b) state the scale factor.

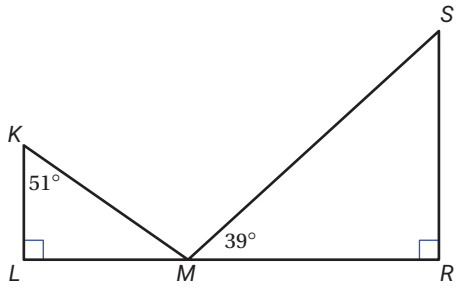


8 For each of the following pairs of triangles, write the similarity statement beginning $\Delta KLM \sim$, matching the corresponding vertices.

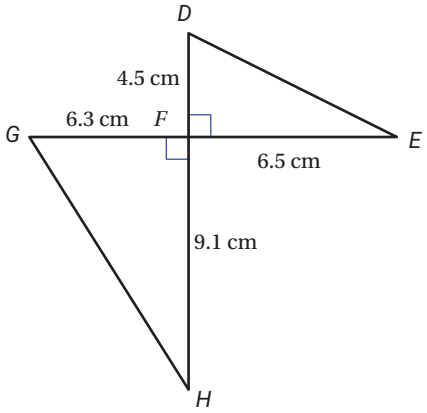
(a)



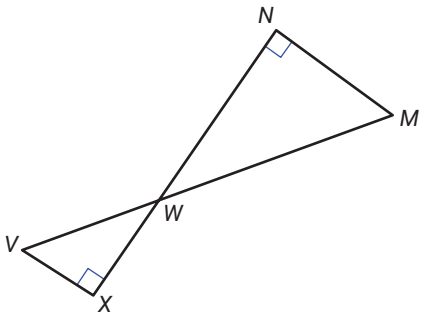
(b)



9 For the pair of right-angled triangles, write the scale factor and the similarity statement, matching the corresponding vertices.

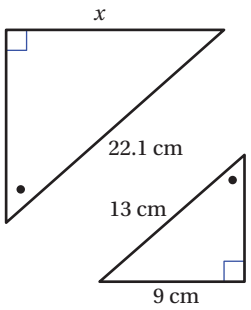


10 Write a similarity statement for the pair of right-angled triangles.

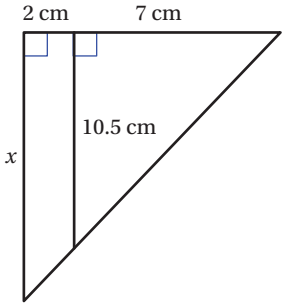


11 For each pair of right-angled triangles, calculate the value of x.

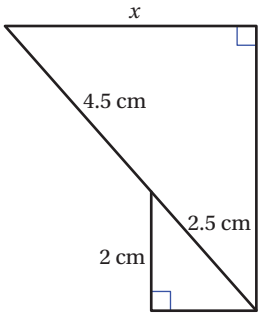
(a)



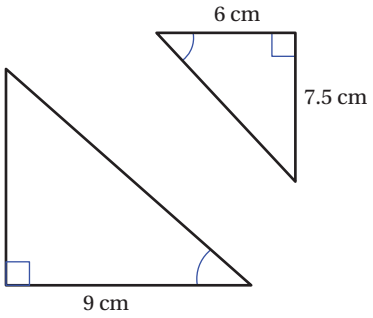
(b)



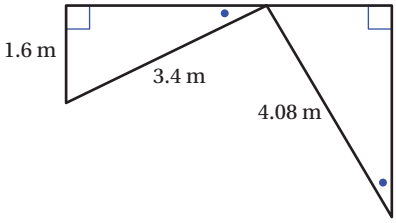
(c)



12 Calculate the areas of both similar right-angled triangles.



13 Calculate the areas of both similar right-angled triangles.



14 The two shorter sides of a triangle are given as 33 mm and 56 mm. Determine the range of lengths for the longest side if the triangle is to be acute-angled.

Extended response

1 A rectangular room, 4.8 m by 4.1 m, has a sloping ceiling that is 2.6 m high along one of the longer sides of the room and 2.8 m high along the other longer side of the room.

(a) Write the dimensions of the two rectangular walls.

(b) Determine the length of the sloping sides of the matching trapezium-shaped walls, to the nearest millimetre.

(c) Write the dimensions of the ceiling.

(d) Determine the total area of the walls and ceiling, using the rounded answers.

2 Parallelogram $ABCD$ has $B(-1,0)$ and $D(3,3)$.

(a) Determine the length of the diagonal BD.

(b) Two of the sides of the parallelogram are vertical and 2 units long. For each of the two possible parallelograms, determine:

(i) the coordinates of A and C

(ii) the exact lengths of the sides BC and AD

(iii) the exact lengths of the other diagonal, AC .

3 ABC, ADE and AFG are a set of embedded triangles.

(a) Determine the length of the horizontal sides for each triangle.

(b) Calculate the ratio of the lengths of the horizontal sides, as smallest : middle : largest.

(c) Determine the length of the vertical sides for each triangle.

(d) Calculate the ratio of the lengths of the vertical sides, as smallest : middle : largest .

(e) Determine the exact length of the hypotenuse for each triangle.

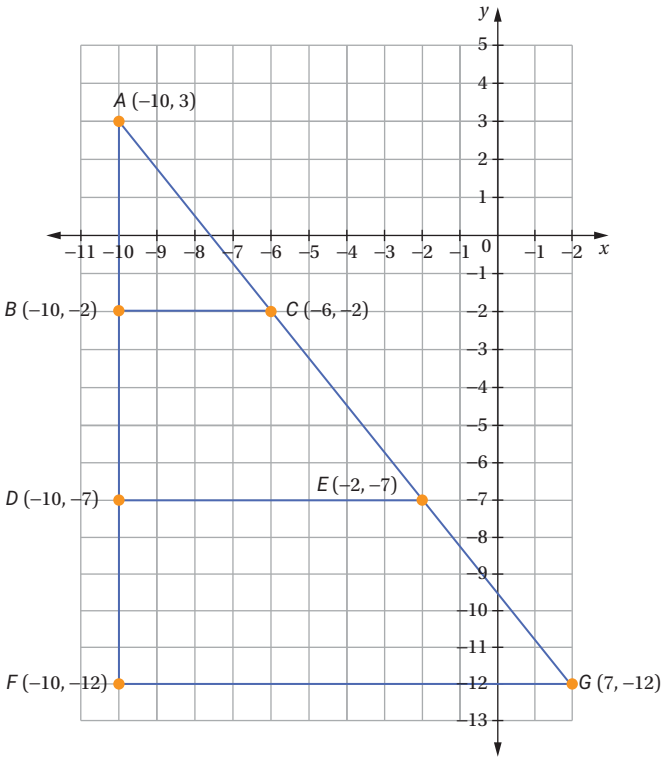
(f) Calculate the ratio of the lengths of the hypotenuses, as smallest : middle : largest.

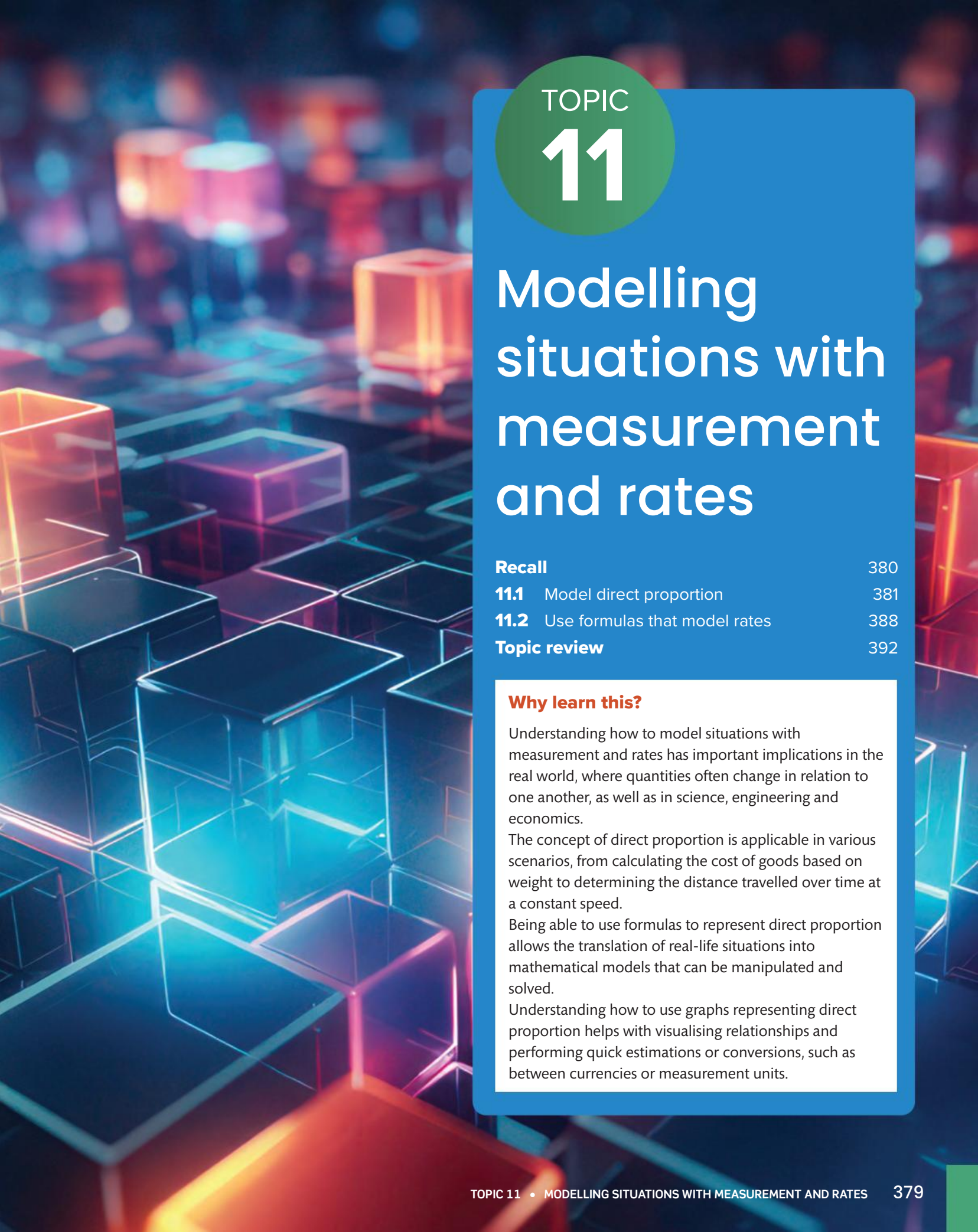
(g) State the scale ratio, in the order larger to smaller, between triangles:

(i) ABC and ADE

(ii) ABC and AFG

(iii) ADE and AFG.





TOPIC
11

Modelling situations with measurement and rates

Recall 380
11.1 Model direct proportion 381
11.2 Use formulas that model rates 388
Topic review 392

Why learn this?

Understanding how to model situations with measurement and rates has important implications in the real world, where quantities often change in relation to one another, as well as in science, engineering and economics.

The concept of direct proportion is applicable in various scenarios, from calculating the cost of goods based on weight to determining the distance travelled over time at a constant speed.

Being able to use formulas to represent direct proportion allows the translation of real-life situations into mathematical models that can be manipulated and solved.

Understanding how to use graphs representing direct proportion helps with visualising relationships and performing quick estimations or conversions, such as between currencies or measurement units.

RECALL

ANSWERS Page 643

I can determine the rule from a table of values

1 Determine the rule in the form $y = x + c$ for each table of values.

(a)

x	−2	−1	0	1	2
y	3	4	5	6	7

(b)

x	−2	−1	0	1	2
y	−4	−3	−2	−1	0

2 Determine the rule in the form $y = ax$ for each table of values.

(a)

x	−2	−1	0	1	2
y	−8	−4	0	4	8

(b)

x	−2	−1	0	1	2
y	1	0.5	0	0.5	1

I can determine the gradient of linear graphs

1 Write the gradient described by the following lines.

(a) Rises 8 units for every 4 units that it moves horizontally

(b) Descends 12 units for every 3 units that it moves across

(c) Rises 15 units for every 10 units that it moves horizontally

(d) Descends 6 units for every 18 units that it moves across

I can interpret percentages in sentences

1 Write the percentage amount, correct to 2 decimal places, represented by these rate statements.

(a) 4 out of every 24 taxi passengers were senior citizens.

(b) 5 out of 7 university students lived away from home.

(c) 15 out of 35 tennis shots landed out of court.

(d) 3 out of 11 guitars in a shop were out of tune.

(e) 14 out of 18 hikers stayed on the well-worn trail.

I can substitute values into formulas

1 If $y = x - 3$, find y when:

(a) $x = 5$ **(b)** $x = -1$ **(c)** $x = 3$

2 If $m = 3n + 1$, find m when:

(a) $n = 2$ **(b)** $n = -2$ **(c)** $n = 5$

3 If $I = 5 - 2m$, find I when:

(a) $m = 3$ **(c)** $m = 4$

(b) $m = -2$

Model direct proportion

11.1

Learning intention: To be able to model direct proportion

Success criteria:

SC 1 I can model direct proportion with formulas.

SC 2 I can model direct proportion with graphs.

Lesson warm-up

Stacking shelves



Tilly has a job stacking shelves at the supermarket. She takes cans out of a cardboard box and puts them on the shelf. Each box holds 24 cans.

This table shows the relationship between the number of boxes emptied and the number of cans stacked.

Number of boxes, b	1	2	3	
Number of cans, c	24	48		240

1. Complete the table.
2. What is the ratio between the number of boxes and the number of cans, $b : c$?
3. By how much does the number of cans increase for every box emptied? Write your answer as a rate.
4. Write an equation linking the variables.

SC 1 I can model direct proportion with formulas

Two quantities are in direct proportion if they stay in the same ratio as one quantity increases; when one increases, the other increases too. For example, if tomatoes cost \$6/kg, buying twice the number of kilograms will cost you twice as many dollars. The total cost of the tomatoes is in direct proportion to the number of kilograms bought.

Number of kg bought, n	1	2	5	10
Total cost at \$6/kg, T	\$6	\$12	\$30	\$60

The symbol $\propto$ means 'is in direct proportion to'. So, total cost $\propto$ number of kg, or $T \propto n$.

Another way of describing the relationship between the total cost and the number of kilograms uses the rate (\$6/kg). If you define the total cost as T and the number of kilograms as n, the equation for the total cost is: $T = 6n$.

The statements $T = 6n$ and $T \propto n$ both indicate direct proportion. In this relationship, 6 is called the 'constant of proportionality'.

In general, if two quantities y and x are in direct proportion, you can write $y \propto x$ and $y = kx$, where k is the constant of proportionality. k is the constant value of the ratio for that relationship.

Some other important notes:

In the example above, 0 kg of tomatoes costs \$0. If $y \propto x$, when x is zero, y is also zero.

The constant of proportionality, k, is always positive for direct proportion, so that as one variable increases, the other increase as well.

If $y \propto x$, then it is also true that $x \propto y$. The equation can be written as $y = kx$ or as $x = \frac{1}{k}y$.

Worked example

Writing equations for direct proportion

(a) Are these variables in direct proportion? If so, write a proportionality statement and an equation.

a	1	2	3	4
b	3	6	9	12

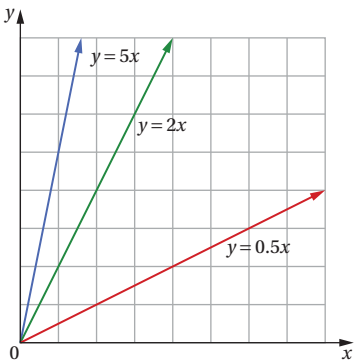
THINKING	WORKING
Determine the ratio between the two variables. Does it change or is it the same?	$\frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4} = 3$ The ratio is constant (stays the same), so the variables are in direct proportion.
Write a proportionality statement.	$b \propto a$
Write an equation connecting the variables.	$b = 3a$
Check that the equation makes sense.	This equation says that if $a = 1$, $b = 3$. This matches the data given.

(b) Mei is paid \$31 per hour. How is her total pay related to the number of hours she works? Write a proportionality statement and an equation.

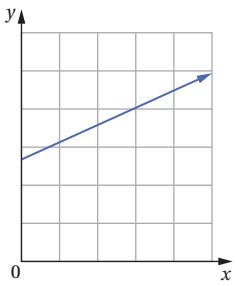
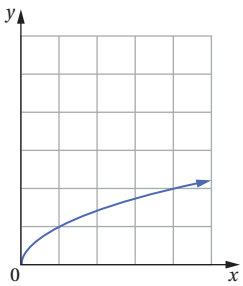
THINKING	WORKING
Define the variables.	Let Mei's total pay in dollars be T, and the number of hours she works be n.
Decide whether the variables are in direct proportion.	Mei is paid a constant amount per hour. If she works triple the number of hours, then she gets paid triple the amount. The variables are in direct proportion.
Write a proportionality statement.	$T \propto n$
Write an equation connecting the variables.	$T = 31n$
Check that the equation makes sense.	This equation says that if Mei works 2 hours, then she gets paid \$62, which matches the data given.

SC 2 I can model direct proportion with graphs

If a graph represents direct proportion, then it must be a straight line that passes through the origin. The equation is of the form $y = kx$, where $k > 0$. The constant of proportionality, k, is the gradient of the line. Some examples are shown below.



The following graphs do not represent direct proportion.

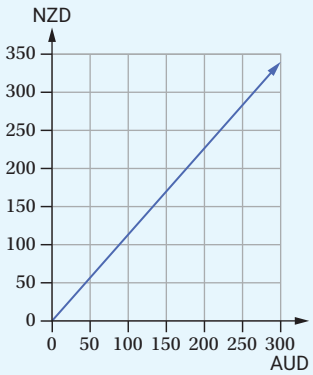
	
Not direct proportion. Does not pass through the origin (0, 0).	Not direct proportion. Not a straight line.

Worked example

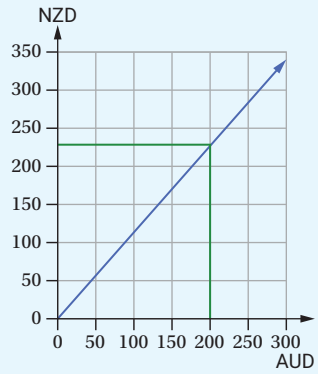
Drawing and using a conversion graph

The exchange rate between Australian dollars and New Zealand dollars for a particular day is 1 AUD ↔ 1.13 NZD.

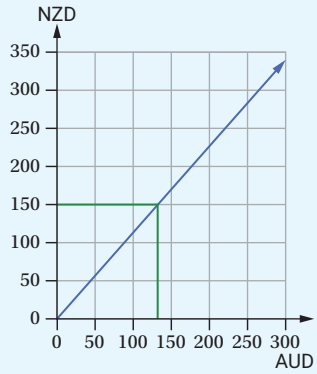
(a) Draw a graph that shows the direct proportion between AUD and NZD.

THINKING	WORKING
Calculate one point on the line.	If 1 AUD ↔ 1.13 NZD, then 300 AUD ↔ 1.13 × 300 NZD, which is 339 NZD. This is represented by the point (300, 339), assuming that AUD is on the horizontal axis.
Plot the origin and one other point and draw the line.	

(b) Use the graph to convert 200 AUD to NZD.

THINKING	WORKING
Draw a line up from the required value, then across to the vertical axis.	
Identify the coordinates of the point of intersection.	The coordinates are approximately (200, 230).
Write the answer.	200 AUD is approximately 230 NZD.

(c) Use the graph to convert 150 NZD to AUD.

THINKING	WORKING
Draw a line across from the required value, then read down to the horizontal axis.	
Identify the coordinates of the point of intersection.	The coordinates are approximately (130, 150).
Write the answer.	150 NZD is about 130 AUD.

Practice

ANSWERS Page 643

SC 1 I can model direct proportion with formulas

1 For each of the following tables, determine whether the variables are in direct proportion. If they are, write a statement of proportionality and determine the equation.

(a)

a	0	3	5	7	9
b	0	18	30	42	54

(b)

a	0	1	4	6	8
b	0	0.5	2	3	4

(c)

a	0	1	4	6	9
b	0	4	17	25	54

(d)

a	0	2	3	6	7
b	0	10.4	15.6	31.2	36.4

2 The variables in the following tables are in direct proportion. Determine the missing values.

(a)

d		1	3		10
t	0		12	24	40

(b)

m	0	3	8	15	
V		1		5	20

3 8 kg of apples costs $27.60.

(a) Use the given information to write an equation in the form cost = $k \times$ number of kilograms.

(b) Why is it reasonable to assume direct proportion exists here?

(c) Determine the cost, to the nearest cent, of 4.75 kg.

(d) Determine the mass of apples in kg, correct to 2 decimal places, that can be purchased for $12.50.

4 For two variables p and q, $q \propto p$ and when $p = 6$, $q = 4$.

(a) Determine the value of k, the constant of proportionality.

(b) Write an equation connecting the variables.

(c) Determine the value of q when $p = 33$.

(d) Determine the value of p when $q = 10$.

5 Gianni is normally paid $16.68/hr at his casual job at the barber. On public holidays, he gets paid $33.35/hr.

(a) Write an equation that relates his normal hours worked with his pay for those hours.

(b) Write an equation that relates his public holiday hours worked with his pay for those hours.

(c) To earn $100, how many normal hours would he need to work?

(d) To earn $100, how many public holiday hours would he need to work?

(e) Write an equation for his total pay, including normal and public holiday hours.

(f) Do you think the equation in part **(e)** shows direct proportion?

6 For a person travelling at constant speed, the distance travelled, d, is in direct proportion to the time spent travelling, t.

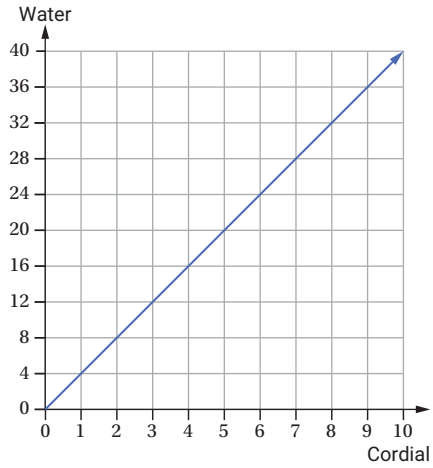
(a) Write an equation to represent this, using k as the constant of proportionality.

(b) Given that the person travels 120 km in 8 hours, determine the value of k. Include units in your answer.

(c) What does k represent?

SC 2 I can model direct proportion with graphs

1 Use the following conversion graph to answer the questions below.



(a) If you used 6 cups of cordial concentrate, how many cups of water would be required?

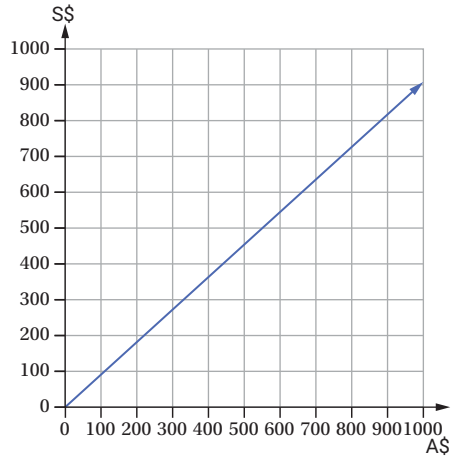
(b) How much cordial mixture would you now have?

(c) If you used 32 L of water, how much cordial concentrate would you need?

(d) How much cordial mixture would you now have?

(e) Does it matter what unit you use to measure the water or cordial concentrate?
Explain your answer.

2 The following conversion graph relates the value of Australian dollars and Singapore dollars on a particular day.



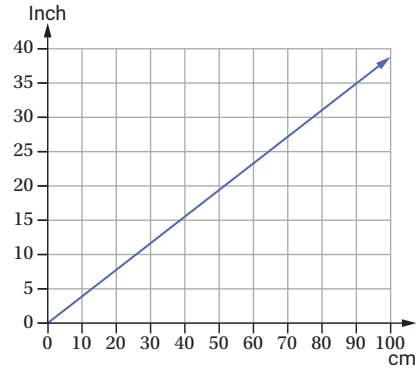
(a) State the approximate worth of S$600 in A$.

(b) State the approximate worth of A$900 in S$.

(c) A meal cost S$24. Use your earlier approximation to convert this to the equivalent cost in A$.

(d) Explain why this method is better than just doing an approximation directly from the conversion graph.

3 When Australia converted from the imperial measurement system to the metric system, there were some quick conversion values used. One of these was that 1 inch ≈ 2.54 cm. The following conversion graph relates measurements in centimetres with measurements in inches.



(a) Using this approximation, what would 20 inches be approximately equal to?

(b) Does the conversion graph confirm the conversion rate 1 inch ≈ 2.54 cm?

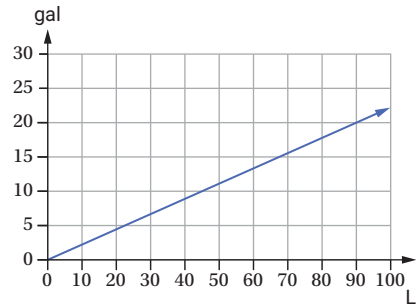
(c) A ruler that a student uses is 30 cm long. How long do you think the equivalent ruler would have been using imperial measurements?

(d) What is the approximate equivalent for 100 cm according to the conversion graph?

(e) By what do you need to multiply 100 cm to get 100 m?

(f) What do you think the equivalent distance would have been at the Olympics for the 400 m run, given that 12 inches = 1 foot and 3 feet = 1 yard?

4 The following graph enables conversion between litres (L) and imperial gallons (gal).



(a) According to the graph, what is the approximate equivalent for 40 L? Give your answer to the nearest gallon.

(b) According to the graph, what is the approximate equivalent for 15 gal? Give your answer to the nearest litre.

(c) At which coordinates (apart from (0, 0)) does the line get closest to the intersection of gridlines?

(d) Use the coordinate pair from part **(c)** to approximate the conversion relationship.

11.2 Use formulas that model rates

Learning intention: To be able to use formulas that model rates

Success criteria:

SC 1 I can use formulas involving direct proportion and rates in science contexts.

Lesson warm-up

Speeding up



Speed, distance and time are related by the equation:

$\text{Speed} = \frac{\text{distance}}{\text{time}}$.

Rearranged, this is:

Distance = speed × time.

Think about how speed, distance and time are related in each of these scenarios.

1. You are cycling at a constant speed. If you have further to go, does it take a longer or shorter time?
2. You are cycling a fixed distance from home to school. If you cycle faster, does the time taken get larger or smaller?
3. You have exactly 30 minutes to cycle to the shop and back. If there are two shops and you choose to go to the one that is further away, will you have to cycle faster or slower?

Can you use the equations above to explain your answers?

SC 1 I can use formulas involving direct proportion and rates in science contexts

Direct proportion can be seen in many science contexts. Some examples are below. The equations have a constant multiplied by a variable.

Context	Variables and constant	Equation
Motion	For a constant speed, increasing the time increases the distance.	$d = st$
Electricity	For a constant resistance, increasing the voltage increases the current.	$V = IR$
Springs	For a constant spring stiffness, increasing the force increases the extension.	$F = kx$ (Hooke's law)
Motion	For a constant mass, increasing the velocity increases the momentum, p.	$p = mv$
Density	For constant density, ρ, increasing the volume increases the mass.	$m = \rho V$

In your science class, you may have used a spring scale, such as shown here. This scale is used to measure forces. For example, hanging a mass on the scale can measure the force due to gravity on the mass. The spring inside the scale extends when a mass is hung on it. A bigger force will extend the spring more.



Hooke's law says that the force on a spring, F, is proportional to the distance the spring will extend, x. As an equation, this is $F = kx$. Each spring has its own 'spring constant', k, which is also called the stiffness.

Note that not all situations involving rates have direct proportion. When one variable decreases as the other increases, this is a different kind of relationship. For example, if you have a fixed (constant) distance to travel, and increase your speed, it decreases the time taken.

Worked example

Using Hooke's law

When a 500 g mass is hung on a particular spring scale, the spring extends 1.3 cm.

(a) Determine the spring constant for this spring.

THINKING	WORKING
Recall the formula for force.	$F = mg$
Calculate the force, F, due to gravity, g, on the mass.	Use $g = 9.8\,\text{m/s}^2$ and write the mass in kg. $F = mg$ $= 0.5 \times 9.8$ $= 4.9\,\text{N}$
Recall the formula for Hooke's law.	$F = kx$ where F represents the force and x represents the distance that the spring extends in metres.
Substitute the known pair of values.	$4.9 = k \times 0.013$
Determine the value of the constant.	$k = 377\,\text{N/m}$
Write the answer.	The spring constant is 377 N/m.

(b) How far will the spring extend if an 800 g mass is hung on it?

THINKING	WORKING
Calculate the force due to gravity on the mass.	Use $g = 9.8\,\text{m/s}^2$ and write the mass in kg. $F = mg$ $= 0.8 \times 9.8$ $= 7.84\,\text{N}$
Recall the formula for Hooke's law.	$F = kx$
Substitute the known pair of values.	$7.84 = 377 \times x$
Determine the value of the constant.	$x = 0.0208\,\text{m}$ (4 d.p.) $= 2.08\,\text{cm}$
Write the answer.	The extension will be 2.08 cm.

Practice

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SC 1 I can use formulas involving direct proportion and rates in science contexts

1 When a 300 g mass is placed on a spring scale, it extends the spring 1.4 cm. Use this spring scale to answer the following questions, using the formula $F = kx$, where k is the stiffness of the spring.

(a) What is the value of the stiffness of this spring?

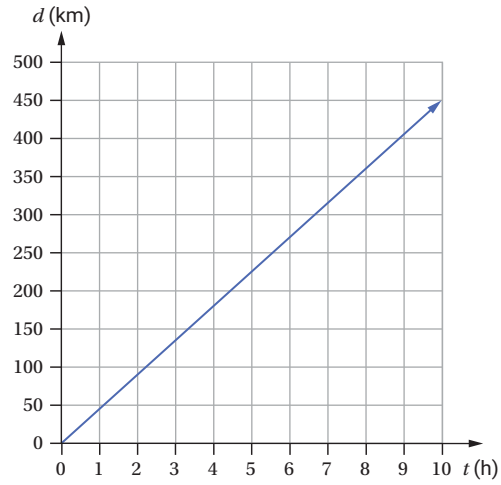
(b) How far would the spring be extended if a 190 g mass was used?

(c) How far would the spring be extended if a 460 g mass was used?

(d) What was the mass used if the spring extended 0.75 cm?

(e) What was the mass used if the spring extended 1.95 cm?

2 The following graph relates the distance travelled and the time spent travelling at a constant speed.



(a) To the nearest kilometre, what distance was travelled in 1 h?

(b) How long did it take to travel 100 km? Answer as a half-hour interval.

(c) There is a coordinate pair that occurs on the intersection of two of the grid lines. What is it?

(d) Use this coordinate pair to determine the constant speed.

3 Farrah tests a circuit that has a voltage of 3 volts across a resistor. The current through the resistor is 0.1 amps. The equation relating voltage, V, resistance, R, and current, I, is $V = IR$.

(a) Calculate the resistance of the resistor (in ohms).

(b) Write the equation that relates the voltage and current for this resistor. (A resistor has a constant resistance.)

(c) Farrah changes the voltage to 12 volts. What will the current now be?

4 The momentum, p, of an object is the product of its mass, m, and its velocity, v. The equation is: $p = mv$ (the more momentum an object has, the harder it is to stop). For the following calculations, the mass of the object is a constant 1.5 kg. Give your answers correct to 1 decimal place.

(a) Given that the object has a momentum of 10 kg·m/s, what is its velocity?

(b) Given that the object has a velocity of 5 m/s, what is its momentum?

(c) What happens to the momentum if the velocity is doubled?

(d) What happens to the speed if the momentum is divided by three?

5 The density of steel is around 7.9 g/cm^3, and every cubic centimetre of steel has a mass of 7.9 g. The equation that relates the mass of an object to its density and volume is: $m = \rho V$

(a) Calculate the volume of steel that would have a mass of 1 kg (1000 g).

(b) Determine the mass of a cube of steel that is 1000 cm^3 (dimensions 10 × 10 × 10 cm).

(c) How many 1000 cm^3 cubes fit into a 1 m^3 cube (dimensions 100 × 100 × 100 cm)?

(d) How much would a 1 m^3 cube of steel weigh?

Exploration Task

Population and natural resources

Population growth and sustainability is a complex issue all over the world. To investigate this, research the current population of 5–10 countries of your choice and then identify:

- each country's average life expectancy (ALE)
- the average number of children born per person (birth rate or BR)
- the migration rate (MR).

By varying the future average life expectancy and birth rates of each country, how will its future population change? Will this create problems for the country's resources, economy, industry and so on? You will need to investigate the country's size and existing resources, and the effects of population change. What strategies can you think of to help solve the potential problems?.

TOPIC REVIEW

Multiple choice

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1 For the following table of values, choose the correct equation.

a	0	1	4	6	9
b	0	6	24	36	54

A $b = 5a$ **B** $b = 5.5a$ **C** $b = 6a$ **D** $b = 9a$

2 For the following table of values, choose the correct equation.

a	0	2	4	6	12
b	0	5	10	15	30

A $b = 2a$ **B** $b = 2.5a$ **C** $b = 3a$ **D** $b = 6a$

3 For the following table of values, choose the correct equation.

a	0	1	2	3	4
b	0	2	8	18	32

A $b = 2a$ **B** $b = 4a$ **C** $b = a^2$ **D** $b = 2a^2$

4 For the following table of values, choose the correct equation.

a	0	1	4	9	25
b	0	4	8	12	20

A $b = 4a$ **B** $b = 2a$ **C** $b = 4\sqrt{a}$ **D** $b = \frac{\sqrt{a}}{4}$

5 It is known that the following table of values represents variables that display direct proportion. What is the missing value in the table?

q	0	4	49	100
r	0	20		500

A 10 **B** 49 **C** 245 **D** 490

6 It is known that the following table of values represents variables that display direct proportion. What is the missing value in the table?

q	0	50	200	
r	0	10	40	200

A 5 **B** 10 **C** 40 **D** 1000

7 It is known that $q \propto \sqrt[3]{p}$ and that when $p = 64$, $q = 20$. The constant of proportionality is:

A 2.5 **B** 4 **C** 5 **D** 16

8 Assuming direct proportion applies, if 3.75 kg of apples costs \$15.75, then 2.9 kg of the same apples costs:

A \$4.20 **B** \$5.43 **C** \$10.87 **D** \$12.18

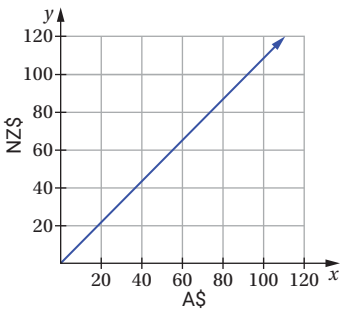
9 When a 250 g weight is placed on a particular spring scale, the spring extends 3.2 cm. Which of the following best represents how far this spring will extend if a 310 g weight is placed on it?

A 2.58 cm **B** 3.968 cm **C** 9.92 cm **D** 78.125 cm

10 The rule for the perimeter of a regular polygon is $P = ns$, where n is the number of sides and s is the length of each side. If a regular 12-sided polygon has a perimeter of 21 cm, then each side length is:

A 0.57 cm **B** 1.75 cm **C** 2.52 cm **D** 9 cm

11 For the following currency conversion graph, which is the best estimate for the value of NZ$110?



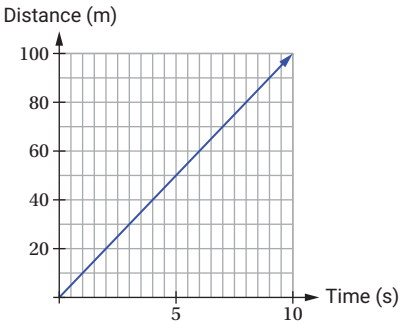
A A$98

B A$102

C A$110

D A$119

12 Consider the following graph.



From this graph it can be concluded that:

A time = 10 × distance **B** constant speed is 0.1 m/s

C constant speed is 10 m/s **D** distance $= \frac{\text{time}}{10}$

Short answer

1 State the value of the constant of proportionality in each of the following.

(a) $P = \frac{3Q}{5}$ **(b)** $A = \frac{\pi d}{2}$ **(c)** $P = 4l$ **(d)** $h = \frac{3f}{2}$

2 Write each of the following as an equation, using k as the constant of proportionality.

(a) m varies directly as the square of x. **(b)** t varies directly as the cube of p.

(c) x varies directly as y.

3 It is known that $q \propto r^2$ and that when $r = 6$, $q = 6$.

(a) Calculate the constant of proportionality. **(b)** State the equation connecting the variables.

(c) Determine the value of q when $r = 15$. **(d)** Determine the value of r when $q = 66\frac{2}{3}$.

4 The cost of a particular type of mulch is \$19.90 m^3.

(a) Use the information given to write a rule in the form cost = $k \times$ number of m^3.

(b) Find the cost for 25 m^3 of this mulch.

(c) Find the volume of this mulch purchased if it cost \$445.76.

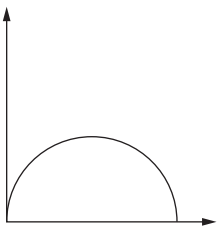
5 The momentum (p kg·m/s) of an object is the product of its mass (m kg) and its velocity (v m/s): $p = mv$. For the following calculations, the mass of the object is held constant at 1.4 kg. Calculate:

(a) the momentum when the velocity is 2.25 m/s

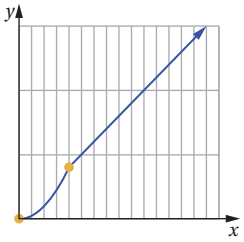
(b) the velocity when the momentum is 5.74 kg·m/s.

6 For each of the following graphs, state whether the graph might represent direct proportion. For any that do not represent direct proportion, give a reason why.

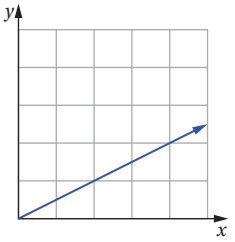
(a)



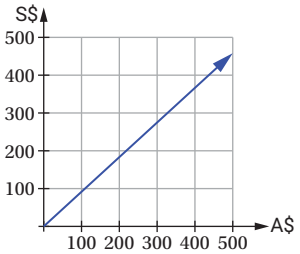
(b)



(c)



7 The following conversion graph connects the worth of Australian dollars (A\$) and Singapore dollars (S\$) on a particular day.



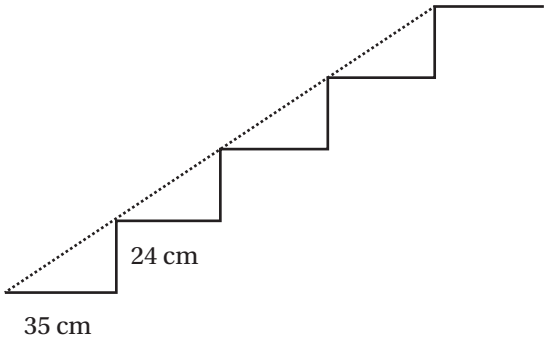
(a) State the approximate worth of S\$300 in A\$.

(b) State the approximate worth of A\$450 in S\$.

(c) A meal cost S\$41.50. Use your earlier approximation to convert this to the equivalent cost in A\$.

(d) Explain why using an earlier approximation, like you did in **(c)**, is better than just doing an approximation directly from the conversion graph.

8 A set of stairs is shown below. The dotted line represents the slope, or gradient, of the stairs.



(a) What is the gradient of the stairs?

(b) Another set of stairs is to be built with the same gradient. If these stairs have a vertical rise of 30 cm, what will be the horizontal depth of the stairs?

(c) Another set of stairs is to be built with the same gradient. If these stairs have a horizontal depth of 28 cm, what will be the vertical rise of the stairs?

9 For each of the following tables of values, calculate the constant of proportionality, k, and use it to determine the missing values.

(a) $b \propto \sqrt{a}$

a	1	9	25	
b	4	12		40

(b) $d \propto c^3$

c	2	3	6	
d	4.8	16.2		437.4

Extended response

1 The volume of a cylinder of a fixed radius is directly proportional to its height. The table shows the height and volume for some values of h.

h (cm)	2	5	9
V (cm^3)	18π	45π	81π

(a) Use the first two coordinate pairs to determine the exact value of the gradient of the line joining these points.

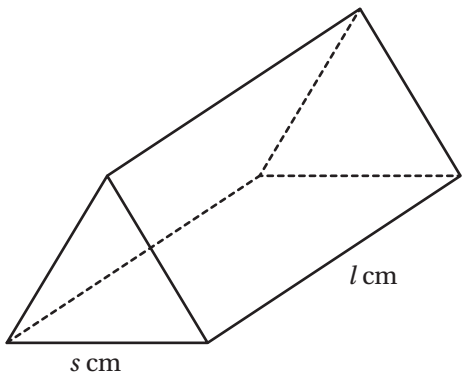
(b) Use the second and third coordinate pairs to determine the exact value of the gradient of the line joining these points.

(c) How do these answers confirm the presence of direct proportionality?

(d) The rule for the volume of a cylinder is $V = \pi r^2 h$. What is the value of the fixed radius considered here?

(e) Determine the height of a cylinder with the same fixed radius and a volume of 180π cm^3.

2 Consider the equilateral triangular prism shown.



(a) Write an expression for the total edge length, T cm, of this prism.

(b) Explain why $T \propto l$ is not true for a fixed value of s.

The volume of this prism can be written as:

$$V = \frac{s^2\sqrt{3}}{4} \times l$$

(c) Explain why $V \propto l$ is true for a fixed value of s.

(d) For $s = 5$ cm, calculate, correct to 2 decimal places:

(i) V, when $l = 6$ cm **(ii)** l, when $V = 200$ cm^3.

TOPIC
12

Trigonometry

Recall 397

12.1 Understand right-angled triangle trigonometry 398

12.2 Use trigonometry to solve an unknown side length in a right-angled triangle 405

12.3 Use trigonometry to solve an unknown angle in a right-angled triangle 414

12.4 Define a right-angled triangle 421

Topic review 426

Why learn this?

Understanding trigonometry in right-angled triangles is a fundamental skill in mathematics. It provides a basis for further study in various fields, including physics, engineering and computer science. This knowledge allows you to calculate distances and angles, solve real-world problems and make accurate predictions.

- Understanding and identifying the sides of a right-angled triangle (hypotenuse, adjacent and opposite) is the foundation for applying trigonometric ratios.
- Calculating sine, cosine and tangent ratios enables the determination of the relationship between the sides and angles of a triangle.
- Using trigonometry to solve unknown side lengths and angles in a right-angled triangle is a practical application of this knowledge, used in navigation, construction and physics.
- Defining a right-angled triangle with minimal information allows us to solve complex problems using a simple geometric shape.

This knowledge can be applied in a variety of school subjects, personal situations and future careers. For example, it can help in understanding the physics of motion, estimating distances in map reading or designing structures in architecture and engineering.

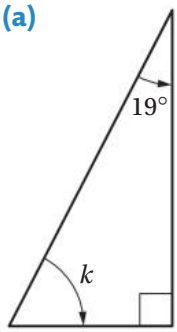
RECALL

ANSWERS Page 644

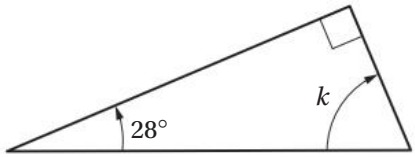
I can calculate the internal angles of a right-angled triangle

1 Determine the value of k in each of the following triangles.

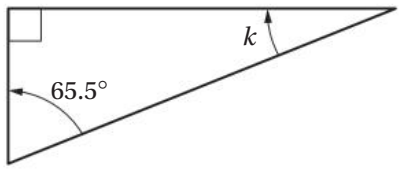
(a)



(b)



(c)



I can solve linear equations

1 Solve each of the following equations for x.

(a) $2x + 14 = 50$ **(b)** $5x - 17 = 28$ **(c)** $\frac{6x}{8} = -3$

I can calculate the value of formulas following substitution

1 In the formula $v = u + at$, calculate:

(a) u, if $v = 10$, $a = 4$ and $t = 6$ **(b)** a, if $v = 6$, $u = 2$ and $t = 5$ **(c)** t, if $v = 10$, $u = 4$ and $a = 6$.

2 In the formula $E = mc^2$, find:

(a) E, if $m = 30$ and $c = 3$ **(b)** m, if $E = 200$ and $c = 5$ **(c)** c, if $E = 900$ and $m = 4$.

I can simplify fractions

1 Write each of these fractions in simplest form.

(a) $\frac{5}{20}$ **(b)** $\frac{3}{18}$ **(c)** $\frac{6}{45}$ **(d)** $\frac{18}{4}$ **(e)** $\frac{56}{16}$

I can simplify ratios of lengths

1 Simplify each of these ratios.

(a) 18 cm : 3 cm **(b)** 7.5 cm : 60 cm **(c)** 45 mm : 180 mm

(d) 6 m : 1.5 m **(e)** 0.25 cm : 7.5 cm

12.1 Understand right-angled triangle trigonometry

Learning intention: To be able to understand right-angled triangle trigonometry

Success criteria:

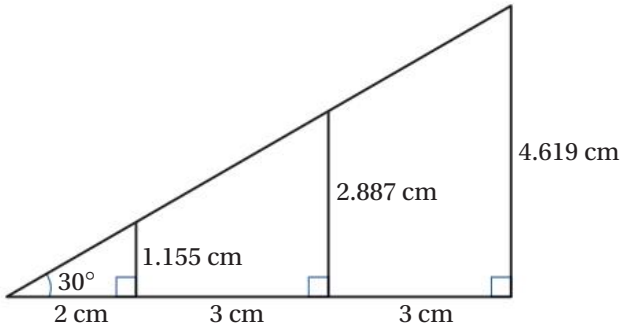
SC 1 I can correctly identify the hypotenuse, adjacent and opposite sides in a right-angled triangle.

SC 2 I can calculate sine and cosine ratios.

SC 3 I can calculate tangent ratios.

Lesson warm-up

Estimating the height of a building





This shows three right-angled triangles, one large, one medium and one small. Find the ratio $\frac{\text{height}}{\text{base}}$ for each of the triangles.

What do you notice?

Tom is 400 m from the base of a new apartment building. To see to the top of the building, he must look up 30°.

Using what you learned from the triangle above, estimate the height of the building.

SC 1 I can correctly identify the hypotenuse, adjacent and opposite sides in a right-angled triangle

Pythagoras' theorem gives the relationship between the sides of a right-angled triangle.

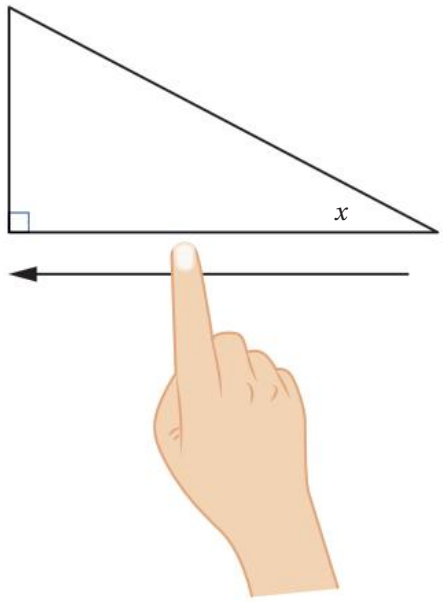
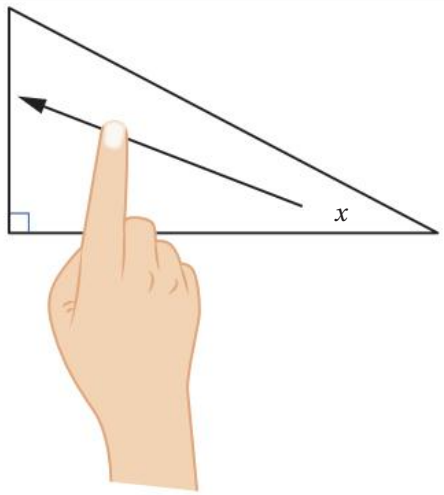
Trigonometry develops the relationships between the sides and angles of right-angled triangles.

The ancient Greeks developed trigonometry, and as such, it is common to use Greek letters such as α, β, θ and ϕ (alpha, beta, theta and phi) to represent the size of unknown angles.

The words 'opposite' and 'adjacent' can be used to describe the position of the sides of a right-angled triangle and the right angle.

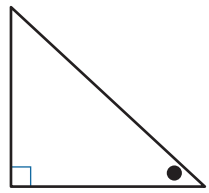
- The hypotenuse is opposite the right angle.
- Both the shorter sides are adjacent to (beside) the right angle.

In trigonometry, the words 'opposite' and 'adjacent' are used in reference to a particular acute angle, the angle of interest.

Side length	Example
Adjacent To identify the adjacent side, put your finger on the angle of interest and move it across to the right angle. (Note that the hypotenuse is also adjacent to this angle, but already has its own identity.)	
Opposite To identify the opposite side, put your finger on the angle of interest and move it across the triangle (in the same way that the hypotenuse is opposite the right angle).	

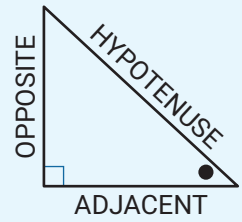
Worked example

Identifying the opposite and adjacent sides of an acute angle in a right-angled triangle



In this right-angled triangle, the acute angle of interest is marked with a dot.

Identify the hypotenuse, the side adjacent to the angle of interest and the side opposite the angle of interest.

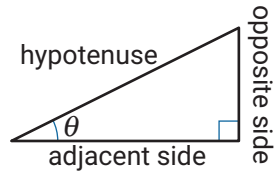
THINKING	WORKING
Identify the side opposite the right angle, and label it with the word *hypotenuse* on the diagram. Identify the side adjacent to the angle of interest. Write the word *adjacent* on this side. Identify the side opposite the angle of interest. Write the word *opposite* on this side.	

SC 2 I can calculate sine and cosine ratios

Each triangle has three side lengths. The sine and cosine ratios for an angle θ in a right-angled triangle compare one of the shorter sides with the hypotenuse.

The sine ratio is $\frac{\text{side opposite angle } \theta}{\text{hypotenuse}}$, shortened to $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$.

The cosine ratio is $\frac{\text{side adjacent to angle } \theta}{\text{hypotenuse}}$, shortened to $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$.

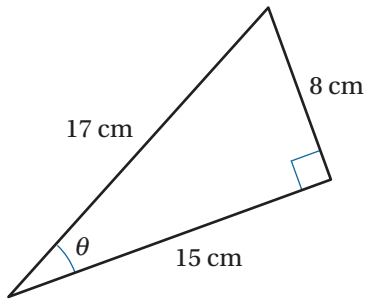


Because the hypotenuse is the longest side in any right-angled triangle, the sine and cosine ratios of an acute angle are always less than 1.

Worked example

Calculating sine and cosine ratios

Determine the values of $\sin(\theta)$ and $\cos(\theta)$ as fractions in simplest form, then as decimals, correct to 4 decimal places.



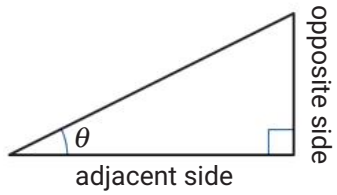
THINKING	WORKING
Identify the lengths of the hypotenuse, the adjacent side and the opposite side.	Hypotenuse: 17 cm Side adjacent to θ: 15 cm Side opposite θ: 8 cm
Write the formula for the sine ratio. Substitute in the lengths. Write the answer as a fraction and a decimal, rounding as required. Note: ratios have no units.	$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$ $= \frac{8}{17}$ $= 0.4706 \ (4 \text{ d.p.})$
Write the formula for the cosine ratio. Substitute in the lengths. Write the answer as a fraction and a decimal, rounding as required.	$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$ $= \frac{15}{17}$ $= 0.8824 \ (4 \text{ d.p.})$

SC 3 I can calculate tangent ratios

The tangent ratio involves the two shorter sides of the right-angled triangle.

The tangent ratio is $\frac{\text{side opposite angle } \theta}{\text{side adjacent to angle } \theta}$, shortened to:

$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$.



If the opposite side is longer than the adjacent side, the tangent ratio will be greater than 1.

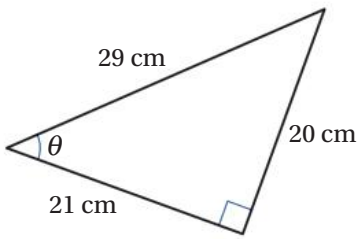
If the opposite side is shorter than the adjacent side, the tangent ratio will be less than 1.

For an isosceles right-angled triangle, the tangent ratio is equal to 1.

Worked example

Calculating tangent ratios

Determine the value of $\tan(\theta)$ as a fraction in simplest form, then as a decimal, correct to 4 decimal places.



THINKING	WORKING
Identify the lengths of the adjacent side and the opposite side.	Side adjacent to θ: 21 cm Side opposite θ: 20 cm
Write the formula for the tangent ratio. Substitute in the lengths. Write the answer as a fraction and a decimal, rounding as required.	$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$ $= \frac{20}{21}$ $= 0.9524 \ (4 \text{ d.p.})$

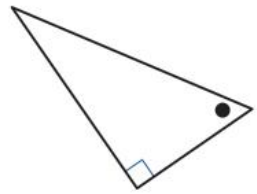
Practice

ANSWERS Page 645

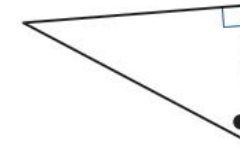
SC 1 I can correctly identify the hypotenuse, adjacent and opposite sides in a right-angled triangle

1 In each of the right-angled triangles, the angle of interest is marked with a dot. Copy each triangle and label the sides: H for hypotenuse, A for the adjacent side and O for the opposite side.

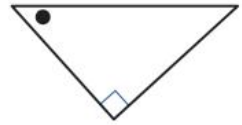
(a)



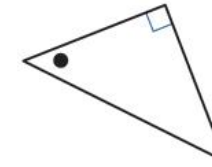
(b)



(c)

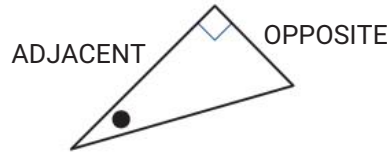


(d)

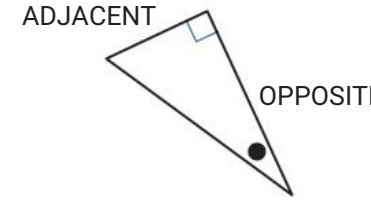


2 Decide whether the triangles are labelled correctly for the angle marked with a dot. Write 'correct' or 'incorrect' for each one.

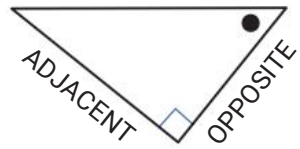
(a)



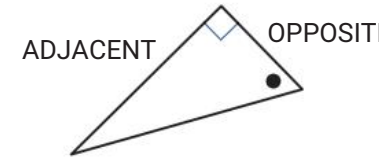
(b)



(c)



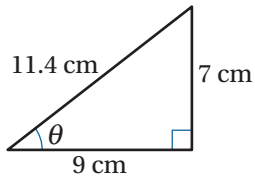
(d)



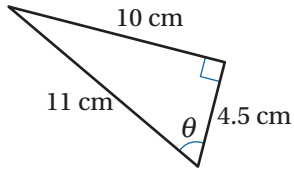
3 The following right-angled triangles have the angle of interest marked with the Greek letter θ. For each triangle, write the length of:

(i) the hypotenuse **(ii)** the side adjacent to θ **(iii)** the side opposite θ.

(a)

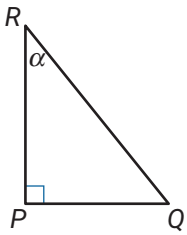


(b)

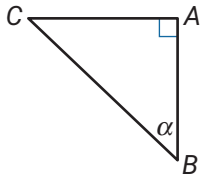


4 Identify each named side as either the side adjacent to α or the side opposite α.

(a) PQ

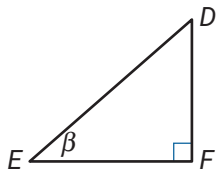


(b) AB

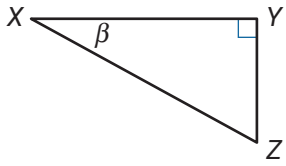


5 Use the letters at the ends of the line segment to name each of the following sides.

(a) The side opposite β

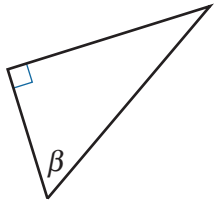


(b) The side adjacent to β

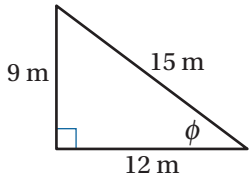


SC 2 I can calculate sine and cosine ratios

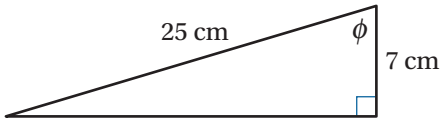
1 From the appearance of the diagram, which is larger: $\sin(\beta)$ or $\cos(\beta)$? Explain your reasoning.



2 Determine the values of $\sin(\phi)$ and $\cos(\phi)$ as fractions in simplest form.



3 Determine the values of $\sin(\phi)$ and $\cos(\phi)$ as decimals.



4 Sketch a right-angled triangle for which $\sin(\theta) = \frac{5}{11}$.

(a) Determine the exact value of $\cos(\theta)$.

(b) Write the value of $\cos(\theta)$, correct to 4 decimal places.

5 Sketch a right-angled triangle for which $\cos(\theta) = 0.63$.

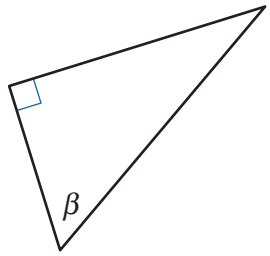
(a) Determine the exact value of $\sin(\theta)$.

(b) Write the value of $\sin(\theta)$ correct to 4 decimal places.

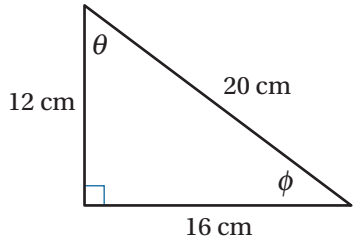
12

SC 3 I can calculate tangent ratios

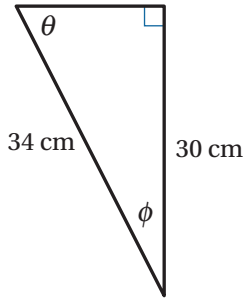
1 From the appearance of the diagram, say whether $\tan(\beta) < 1$ or $\tan(\beta) > 1$. Explain your reasoning.



2 Determine the values of $\tan(\phi)$ and $\tan(\theta)$ as fractions in simplest form.



3 Determine the values of $\tan(\phi)$ and $\tan(\theta)$ as decimals, correct to 4 decimal places, if necessary.



4 Sketch a right-angled triangle with acute angles α and β, in which $\tan(\alpha) = \frac{6}{7}$.

(a) Determine the exact value of $\tan(\beta)$.

(b) Write the value of $\tan(\beta)$ correct to 4 decimal places.

5 Sketch a right-angled triangle with acute angles α and β for which $\tan(\alpha) = 2.4$. Determine the exact values of:

(a) $\tan(\beta)$ **(b)** $\sin(\alpha)$ **(c)** $\cos(\alpha)$.

Use trigonometry to solve an unknown side length in a right-angled triangle

12.2

Learning intention: To be able to use trigonometry to solve an unknown side length in a right-angled triangle

Success criteria:

SC 1 I can identify the ratio needed to solve an unknown side length in a right-angled triangle.

SC 2 I can solve the unknown side in a right-angled triangle given one side and an angle.

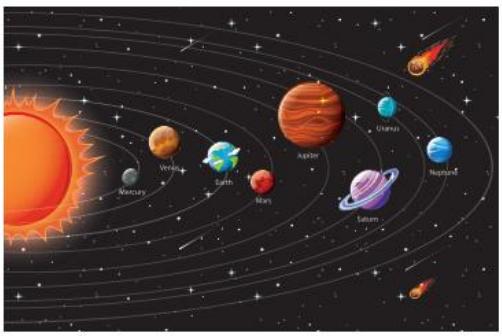
SC 3 I can solve problems using trigonometry involving unknown side lengths in a right-angled triangle.

Lesson warm-up

Mnemonics

A mnemonic is a memory tool that helps learners to recall larger pieces of information.





To help remember the order of the planets, you might remember,

My Very Educated Mother Just Served Us Noodles

instead of

Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune.

Write your own mnemonic to help you remember the order of the planets.

SC 1 I can identify the ratio needed to solve an unknown side length in a right-angled triangle

Scientific calculators have stored values of the sine, cosine and tangent values for all angles. The trigonometric ratios are functions of the angle of interest.

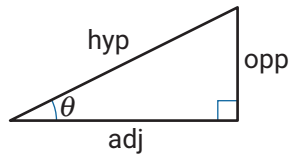
Always ensure that your calculator is in degrees (DEG) mode when using these function keys for right-angled triangles.

When solving the length of a side in a right-angled triangle using trigonometric ratios, one side length must be known and one of the acute angles must be given.

Identify, for both the known side and the side to be solved, whether they are:

- the hypotenuse
- the angle opposite the given angle
- the angle adjacent to the given angle.

SOHCAHTOA is a handy mnemonic for remembering the rules for the ratios.

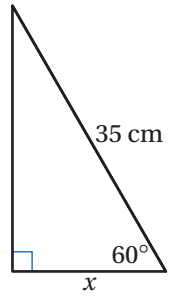


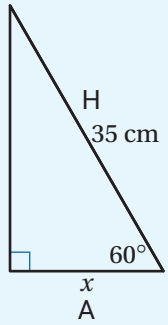
SOH means: $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$; CAH means: $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$; TOA means: $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$.

Worked example

Using trigonometric ratios to calculate a side length in a right-angled triangle

Determine which trigonometric ratio should be used to solve for x.



THINKING	WORKING
Identify the side to be solved as the hypotenuse, adjacent or opposite. Also, identify given sides.	 x is the side adjacent to 60°. 35 cm is the hypotenuse.

Use SOHCAHTOA to identify the ratio.	SOHCAHTOA: $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$
Write the answer.	The only ratio that involves the adjacent side and the hypotenuse is the cosine ratio.

SC 2 I can solve the unknown side in a right-angled triangle given one side and an angle

A scientific calculator should be used to provide the sine, cosine and tangent ratios of the given acute angles. Calculators must be set in degrees mode for these function values.

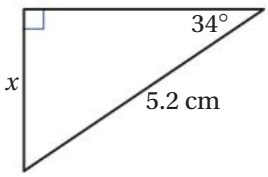
(Note: in Excel, the default is for radians, so a conversion to degrees needs to be part of the formula.)

Worked example

Determining the unknown side of a right-angled triangle given one side and an acute angle

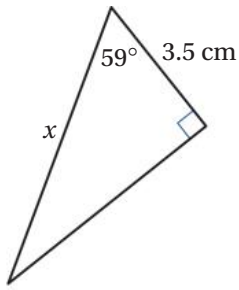
Solve for x, correct to 2 decimal places.

(a)



THINKING	WORKING
Identify the given angle and side lengths.	$\theta = 34°$ The hypotenuse, $H = 5.2$ cm The opposite, $O = x$ cm
Recall the trigonometric ratio.	$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$
Substitute the unknown and known sides and use a calculator to provide the value of sine, cosine or tangent as a decimal.	$\sin(34°) = \frac{x}{5.2}$
Calculate the value of the unknown side: in this case, multiply the given length by the trigonometric ratio.	$x = 5.2 \times \sin(34°)$ $= 2.907\ldots$ $= 2.91$ (2 d.p.)
Write the answer.	$x = 2.91$ cm

(b)

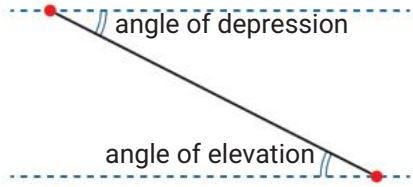


THINKING	WORKING
Identify the given angle and side lengths.	$\theta = 59°$ The hypotenuse, $H = x$ cm The adjacent, $A = 3.5$ cm
Recall the trigonometric ratio.	$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$
Substitute the unknown and known sides and use a calculator to provide the value of sine, cosine or tangent as a decimal.	$\cos(59°) = \frac{3.5}{x}$
Calculate the value of the unknown side: in this case, divide the given length by the trigonometric ratio.	$x = \frac{3.5}{\cos(59°)}$ $= 6.795\ldots$ $= 6.80$ (2 d.p.)
Write the answer.	$x = 6.80$ cm

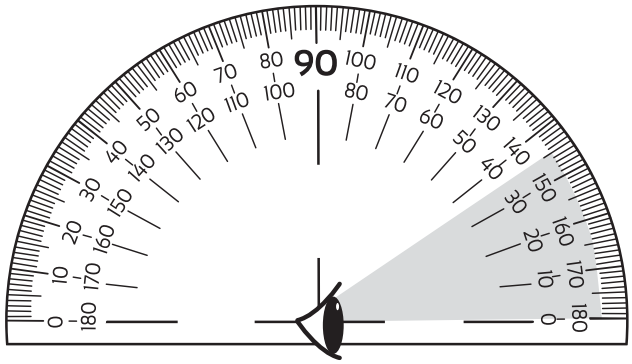
SC 3 I can solve problems using trigonometry involving unknown side lengths in a right-angled triangle

Right-angled triangle trigonometry can be used to calculate lengths and distances whenever a right-angled triangle can be formed to represent a situation, and at least one length and one angle are known.

Common instances of the use of trigonometry involve angles of elevation and angles of depression, both referring to the angle made with the horizontal.

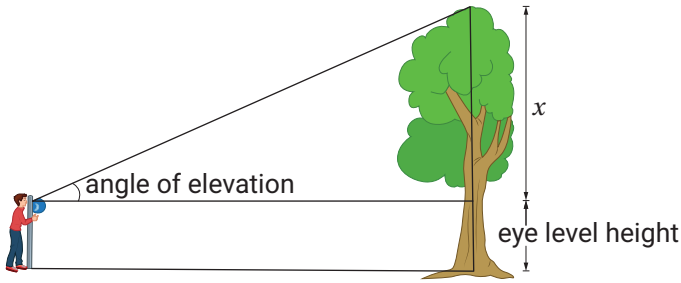


To estimate the height of a tree using an angle of elevation, a simple clinometer can be made from a protractor, using a straw and a weight hanging from the centre point attached to some thread. Alternatively, estimate the line of sight from the horizontal as shown below.

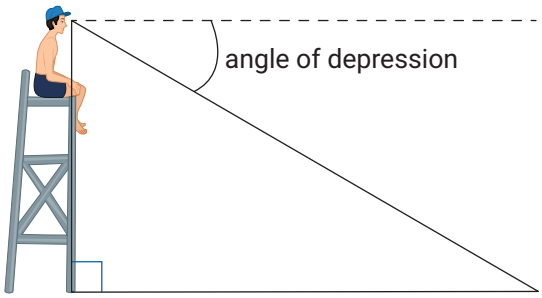


Use a long ruler to measure your eye level height above the ground.

Use a trundle wheel to click out the distance from the tree, the further the better, hopefully on level ground.



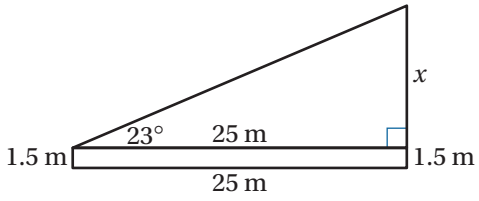
The angle of depression can be used whenever the viewer is elevated, such as a surf lifesaver in a tower, looking out to sea.

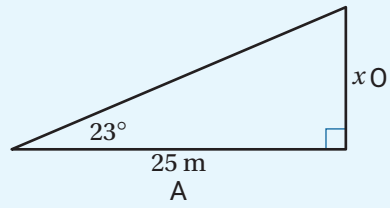


Worked example

Determining the height of a tree from clinometer readings and trundle wheel distances

Determine the height of a tree, correct to 1 decimal place, given an angle of elevation of 23°, an eye level height of 1.5 m and a distance from the tree of 25 m.



THINKING	WORKING
Draw and label the right-angled triangle.	
Calculate the height of the right-angled triangle.	$\frac{x}{25} = \tan(23°)$ $x = 25 \times \tan(23°)$ $= 10.61\ldots$ $= 10.6 \ (1\text{ d.p.})$
Add the eye-level height to the height of the triangle.	10.6 + 1.5 = 12.1
Write the answer in words.	The height of the tree is estimated to be 12.1 m.

12

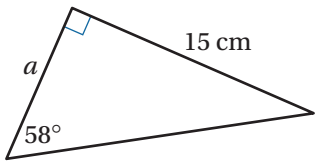
Practice

ANSWERS Page 645

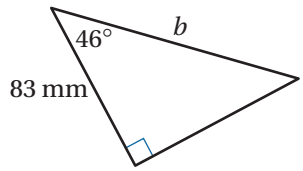
SC 1 I can identify the ratio needed to solve an unknown side length in a right-angled triangle

1 Determine which trigonometric ratio should be used for solving the unknown side marked with a variable.

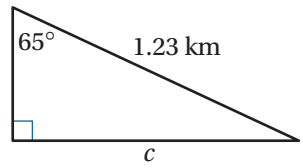
(a)



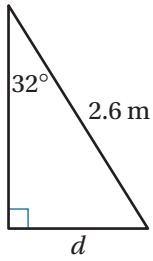
(b)



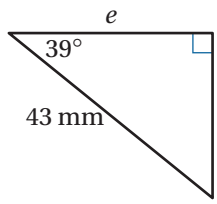
(c)



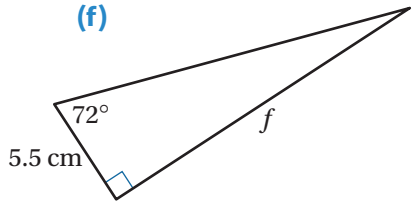
(d)



(e)



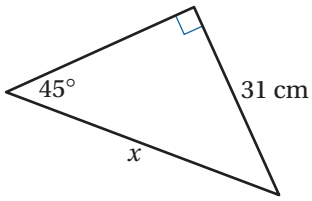
(f)



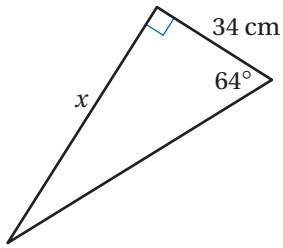
2 Given that one of the short sides in a triangle is known and the other short side is to be determined, which trigonometry ratio should be used?

3 State the trigonometric function that should be used to solve for the unknown side length.

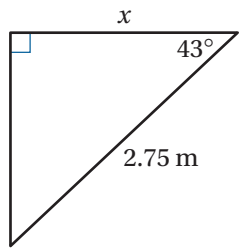
(a)



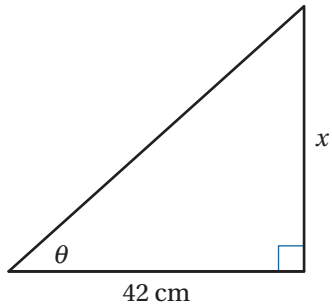
(b)



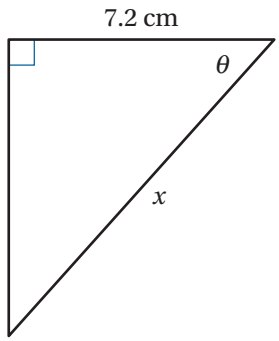
(c)



4 Given that $\sin(\theta)=\frac{20}{29}$, $\cos(\theta)=\frac{21}{29}$ and $\tan(\theta)=\frac{20}{21}$, solve for x.



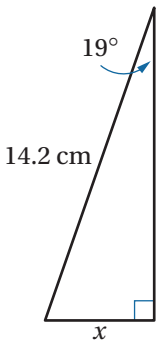
5 Given that $\sin(\theta)=\frac{4}{5}$, $\cos(\theta)=\frac{3}{5}$ and $\tan(\theta)=\frac{4}{3}$, solve for x.



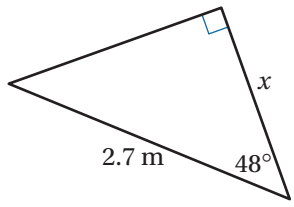
SC 2 I can solve the unknown side in a right-angled triangle given one side and an angle

1 Solve for x, correct to 2 decimal places.

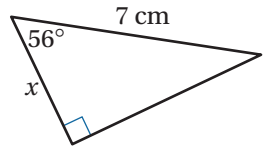
(a)



(b)

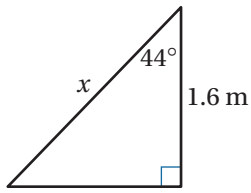


(c)

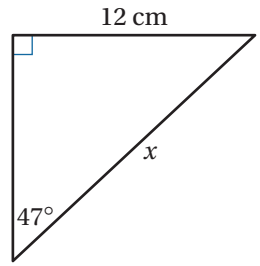


2 Solve for x, correct to 2 decimal places.

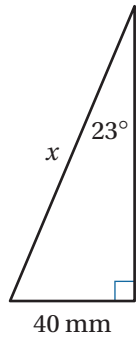
(a)



(b)

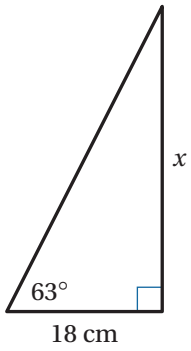


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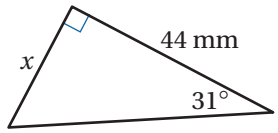


3 Solve for x, correct to 2 decimal places.

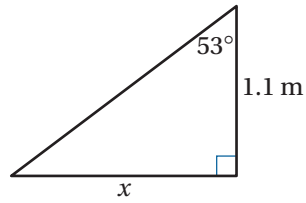
(a)



(b)

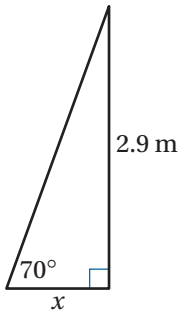


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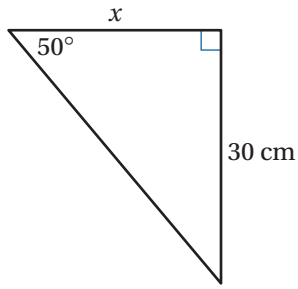


4 Solve for x, correct to 2 decimal places.

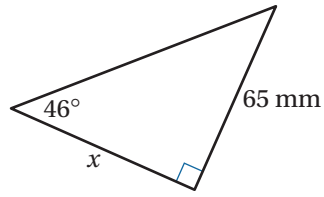
(a)



(b)

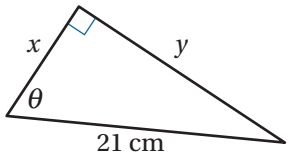


(c)

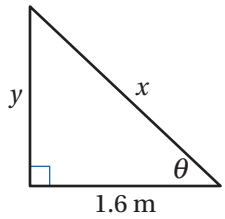


5 Use the given function value to solve for either x or y, whichever is appropriate.

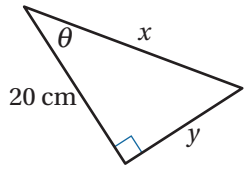
(a) $\sin(\theta) = \frac{5}{7}$



(b) $\cos(\theta) = \frac{8}{11}$

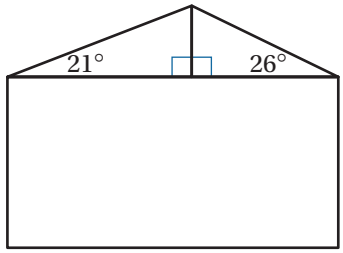


(c) $\tan(\theta) = \frac{4}{5}$

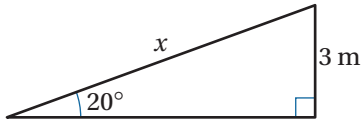


SC 3 I can solve problems using trigonometry involving unknown side lengths in a right-angled triangle

1 The side view of a roof on a house is shown. Given that the highest point of the roof above the ceiling is 2.8 m, how wide is the house, correct to 1 decimal place?

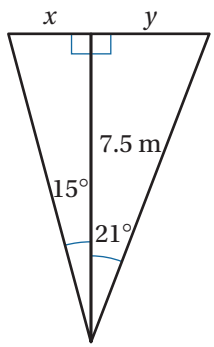


2 Ramps connect the floors of a multistorey car park, where each floor is 3 m above the previous floor. Determine the length of the ramps, to the nearest centimetre.

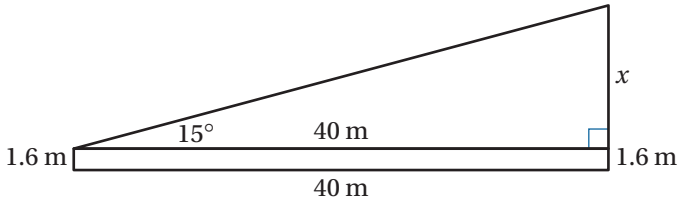


3 Ash, who is standing on the top of a cliff, sees two yachts in a direct line in front of her at angles of depression of 24.7° and 41.9°. Given that Ash is 75 m above water level, determine the distance between the yachts, correct to 1 decimal place.

4 The canopy of a new hotel stands 7.5 m above the architect who designed the hotel, with one end of the canopy at an angle of 15° to the vertical and the other at an angle of 21°, as shown. Determine the length of the canopy, to the nearest centimetre.



5 The height of a tree is calculated from the diagram below. Give your answers to the questions below correct to 1 decimal place.



(a) Calculate the height of the tree.

(b) Given that the angle could be off by anything up to 1°, give a range of values for the height of the tree.

(c) Given that the angle could be off by anything up to 3°, give a range of values for the height of the tree.

12.3 Use trigonometry to solve an unknown angle in a right-angled triangle

12

Learning intention: To be able to use trigonometry to solve an unknown angle in a right-angled triangle

Success criteria:

SC 1 I can identify the ratio needed to solve an unknown angle in a right-angled triangle.

SC 2 I can solve an unknown angle in a right-angled triangle given two of the sides.

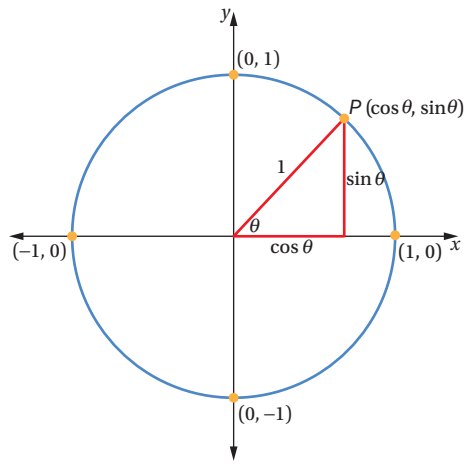
SC 3 I can solve problems using trigonometry involving unknown angles in a right-angled triangle.

Lesson warm-up

Why's that?

Draw a circle with a radius of one unit around the origin, then $\cos(\theta)$ is the x-coordinate.

Explain why.

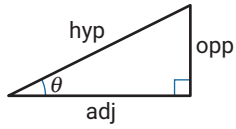


SC 1 I can identify the ratio needed to solve an unknown angle in a right-angled triangle

Given two sides of any right-angled triangle, the third side can be calculated using Pythagoras' theorem. Hence, the shape of the triangle is fixed.

The sizes of the acute angles can be determined from the functions of sine, cosine and tangent stored in a scientific calculator.

The choice of trigonometric ratio to use depends on which sides of the triangle are known.

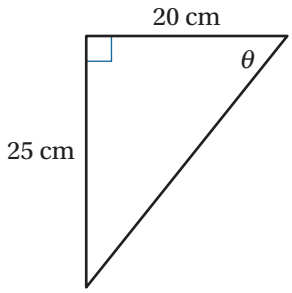


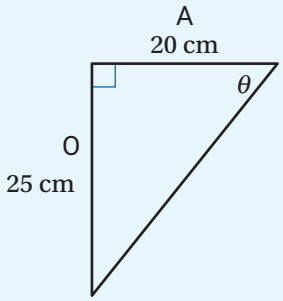
To solve for the angle θ, use $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$, $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$ or $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$.

Worked example

Identifying which trigonometric ratio to use for determining an angle in a right-angled triangle

Determine which trigonometric ratio should be used to solve for θ.



THINKING	WORKING
Label the known sides as hypotenuse, adjacent or opposite to the angle to be solved.	 20 cm is the side adjacent to θ. 25 cm is the side opposite θ.
Use SOHCAHTOA to identify the ratio.	SOHCAHTOA: $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$
Write the answer.	The only ratio that involves the adjacent side and the opposite side is the tangent ratio.

SC 2 I can solve an unknown angle in a right-angled triangle given two of the sides

Using the sine, cosine and tangent functions on a scientific calculator is a straightforward process, for example, sin(30°) = 0.5.

Backtracking to find the angle that has 0.5 as its sine ratio requires the inverse sine function, often written above the sine key, and thus requiring the shift key.

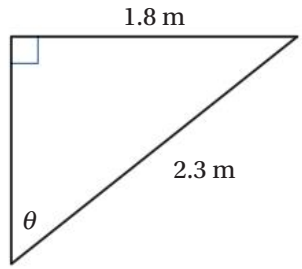
$\sin^{-1}(0.5) = 30°$

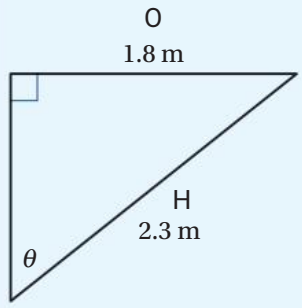
Note that the superscripted –1 is not a power, but an indication of an inverse function.

Worked example

Solving angles using inverse trigonometric functions

Determine the value of θ, to the nearest degree.



THINKING	WORKING
Label and identify the known side lengths.	 1.8 m is the side opposite θ. 2.3 m is the hypotenuse.
Use SOHCAHTOA to recall the ratio and write the rule.	$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$
Substitute in the lengths.	$\sin(\theta) = \frac{1.8}{2.3}$
Use the inverse function to calculate the value of the angle, rounding as directed.	$\theta = \sin^{-1}\left(\frac{1.8}{2.3}\right)$ $= 51.5\ldots$
Write the answer.	The size of the angle, θ, is approximately 52° (to the nearest degree).

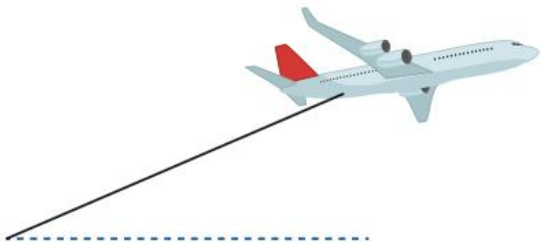
SC 3 I can solve problems using trigonometry involving unknown angles in a right-angled triangle

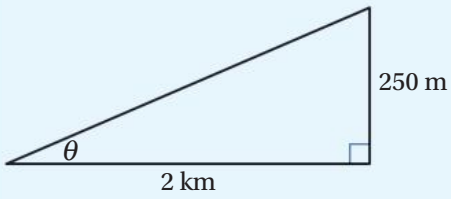
When solving angles using trigonometry, ensure that a right-angled triangle can be drawn to represent the situation and that two of the sides are known.

Worked example

Solving angles in worded problems, using trigonometry

A plane takes off, reaching a height of 250 m by the time it is 2 km from the airport. Determine the angle of take-off, to the nearest degree, assuming that the initial path of the plane is a straight line.



THINKING	WORKING
Sketch the situation, forming a right-angled triangle. Mark the known side lengths. Mark the required angle with a variable.	 The angle of take-off is marked θ. The side opposite angle θ is 250 m. The side adjacent to angle θ is 2 km.
Use SOHCAHTOA to recall the ratio and write the rule.	$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$
Convert the units so that they are the same.	2 km = 2000 m
Substitute the values into the rule.	$\tan(\theta) = \frac{250}{2000}$
Solve for the angle.	$\theta = \tan^{-1}\left(\frac{250}{2000}\right)$ $= 7.1\ldots$
Write the answer.	The angle of take-off is approximately 7°.

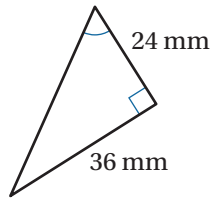
Practice

ANSWERS Page 645

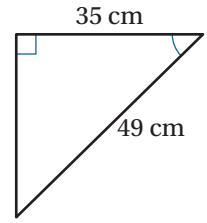
SC 1 I can identify the ratio needed to solve an unknown angle in a right-angled triangle

1 Determine which trigonometric ratio should be used for calculating the marked angle.

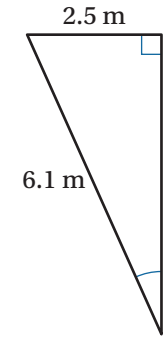
(a)



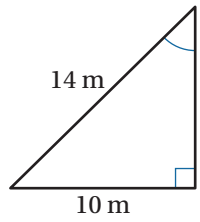
(b)



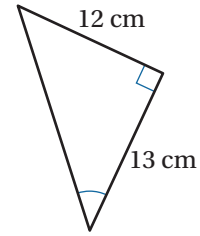
(c)



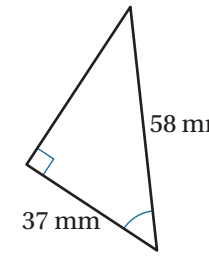
(d)



(e)



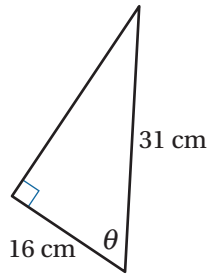
(f)



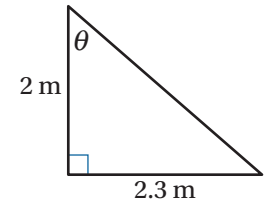
2 Given that the hypotenuse and one short side in a triangle are known, which trigonometry ratios could be used?

3 State the trigonometric equation that should be used to solve for θ.

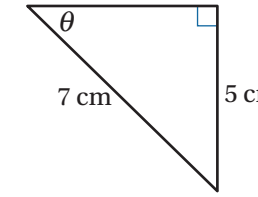
(a)



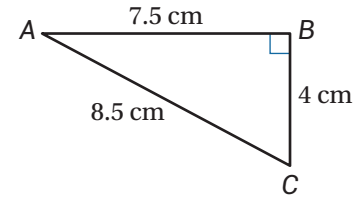
(b)



(c)



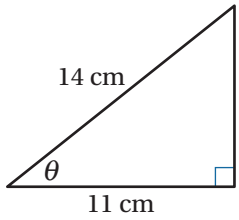
4 Given that $\tan(\theta) = 1.875$:



(a) determine which of the acute angles is θ

(b) determine the values of $\sin(\theta)$ and $\cos(\theta)$, correct to 4 decimal places.

5 Given that cos(38°) = 0.788, cos(39°) = 0.777, cos(40°) = 0.766, cos(41°) = 0.755, cos(42°) = 0.743, use consecutive whole numbers of degrees to complete the inequality: ____ $< \theta <$ ____



SC 2 I can solve an unknown angle in a right-angled triangle given two of the sides

1 Write the inverse function equation used to solve for θ in each of the following.

(a) $\sin(\theta) = 0.8922$ **(b)** $\cos(\theta) = 0.71$ **(c)** $\tan(\theta) = 0.65$

(d) $\sin(\theta) = 0.3$ **(e)** $\cos(\theta) = 0.1414$ **(f)** $\tan(\theta) = 2.5$

2 Write the inverse function equation used to solve for θ in each of the following.

(a) $\sin(\theta) = \frac{6}{17}$ **(b)** $\cos(\theta) = \frac{3}{20}$ **(c)** $\tan(\theta) = \frac{8}{15}$

(d) $\tan(\theta) = \frac{20.3}{11.25}$ **(e)** $\sin(\theta) = \frac{1.4}{2.3}$ **(f)** $\cos(\theta) = \frac{8.6}{10.5}$

3 Determine the value of θ, to the nearest degree, given the following.

(a) $\tan(\theta) = 2.33$ **(b)** $\sin(\theta) = 0.45$ **(c)** $\cos(\theta) = 0.6892$

(d) $\sin(\theta) = 0.85$ **(e)** $\tan(\theta) = 0.276$ **(f)** $\cos(\theta) = 0.5$

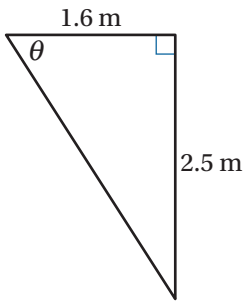
4 Determine the value of θ, to the nearest degree, given the following.

(a) $\cos(\theta) = \frac{2.6}{10}$ **(b)** $\sin(\theta) = \frac{3.9}{4.5}$ **(c)** $\tan(\theta) = \frac{26}{12}$

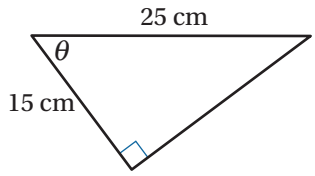
(d) $\cos(\theta) = \frac{1.87}{3.5}$ **(e)** $\tan(\theta) = \frac{4.2}{5.8}$ **(f)** $\sin(\theta) = \frac{6}{25}$

5 Determine the value of θ, to the nearest degree.

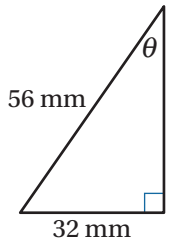
(a)



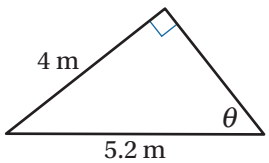
(b)



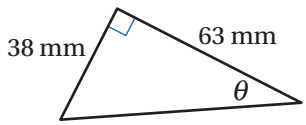
(c)



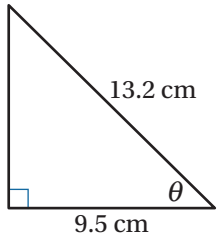
(d)



(e)

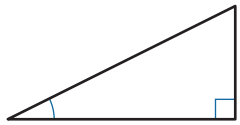


(f)



SC 3 I can solve problems using trigonometry involving unknown angles in a right-angled triangle

1 Determine the angle of elevation, to the nearest degree, from 10 m away to the top of:



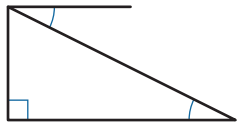
(a) a 5.8 m high lamppost

(b) a 15 m high eucalyptus tree

(c) a 60 m high building

(d) a 2 m high fence.

2 Determine the angle of depression, to the nearest degree, from a 35 m high tenth floor apartment to (measuring horizontally):



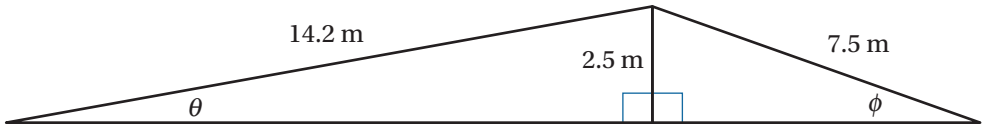
(a) a car 50 m away

(b) the riverbank 250 m away

(c) a pedestrian 10 m away

(d) a restaurant 35 m away.

3 The side view of a roof is shown. Determine the pitch of the roof on each side, to the nearest degree each time, by solving for θ and ϕ.



4 At 9 am and 11 am, the shadow of a person, 165 cm tall, shrinks from 230 cm to 140 cm. Determine the change in the angle of elevation of the Sun, to the nearest degree.

5 Jay climbed a tree to a height of 9.25 m. His dog runs away from the base of the tree at 8 m/s. Determine the angle of depression from Jay to his dog after 2 s, to the nearest degree.

Define a right-angled triangle

12.4

Learning intention: To be able to recognise and use minimal information to define a right-angled triangle

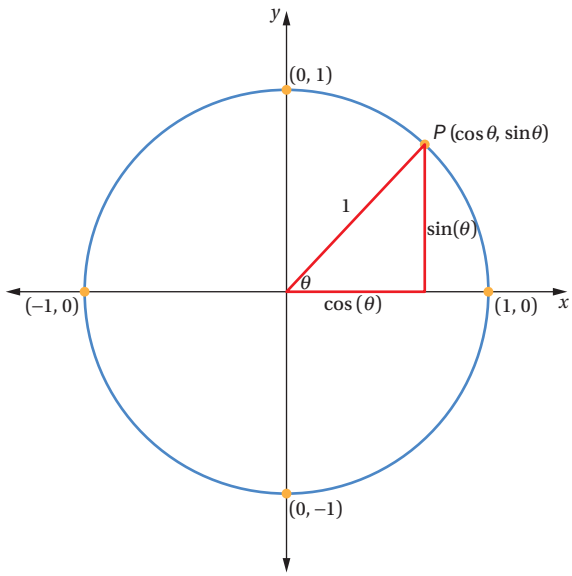
Success criteria:

SC 1 I can recognise the minimal information required to define a right-angled triangle and write a strategy to determine any missing values.

SC 2 I can solve the missing side lengths and angle sizes in right-angled triangles.

Lesson warm-up

Trigonometric identity





Determine the value of $\sin(60°)^2 + \cos(60°)^2$.

Try some other angles in the triangle shown to determine the value of the expression $\cos(\theta)^2 + \sin(\theta)^2$.

Can you explain why this is the case?

SC 1 I can recognise the minimal information required to define a right-angled triangle and write a strategy to determine any missing values

For a right-angled triangle to have a fixed size and shape, two pieces of information (other than the right angle) must be provided, including at least one length.

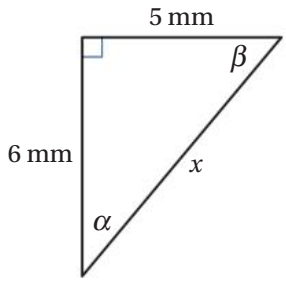
Solving a right-angled triangle could involve Pythagoras' theorem, complementary angles and the trigonometric functions of the acute angles to determine the lengths of sides and sizes of angles not known at the start.

There is usually more than one strategy possible.

Worked example

Writing a strategy to solve a right-angled triangle

Write a strategy to solve the right-angled triangle.



THINKING	WORKING
Since two sides are known, Pythagoras' theorem could be used to calculate the third.	Use Pythagoras' theorem to solve for x: $x^2 = 5^2 + 6^2$
Any of the inverse trigonometric functions can be used to solve either of the angles, now that all sides are known. It is recommended to use the given sides, since no rounding will have occurred.	Use the inverse of the tangent function. Either $\tan(\alpha) = \frac{5}{6}$ or $\tan(\beta) = \frac{6}{5}$
Once one acute angle is known, the easiest way to solve for the second acute angle is to use complementary angles.	$\alpha + \beta = 90°$

SC 2 I can solve the missing side lengths and angle sizes in right-angled triangles

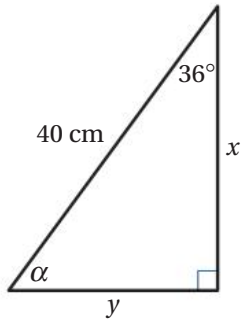
When solving right-angled triangles, choose relationships where two of the three values are known, so it is possible to solve for the third.

If you are using a value that you have previously solved, use the unrounded, calculator version so that rounding errors do not invalidate your work. Round answers as a last step.

Worked example

Solving right-angled triangles

Determine any unknown sides and angles in the triangle shown. Round your answers correct to 1 decimal place.



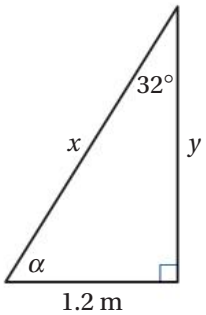
THINKING	WORKING
Any of the three unknowns can be solved first. Choose the angle, and solve it using complementary angles.	$\alpha + 36° = 90°$ $\alpha = 90° - 36°$ $\alpha = 54°$
From here, either acute angle could be used for either unknown side. Where possible, always use given values.	$\cos(36°) = \frac{x}{40}$ $x = 40 \times \cos(36°)$ $x = 32.4 \text{ (1 d.p.)}$
The final side can be found in the same way, but now that two sides are known, Pythagoras' theorem can be used to calculate the third. Remember to use the unrounded value of x in your calculator.	$40^2 = x^2 + y^2$ $y^2 = 40^2 - \left[40\cos(36°)\right]^2$ $y^2 = 552.78\ldots$ $y = \sqrt{552.78\ldots}$ $y = 23.5 \text{ (1 d.p.)}$

Practice

ANSWERS Page 646

SC 1 I can recognise the minimal information required to define a right-angled triangle and write a strategy to determine any missing values

1 For the triangle shown, determine a strategy for the problems below.

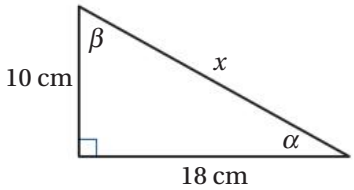


(a) Write an equation to solve α as a first step.

(b) Without using the value for α, write an equation to solve x as a first step.

(c) Without using the value for α, write an equation to solve y as a first step, using the value of x.

2 For the triangle shown, determine a strategy for the problems below.

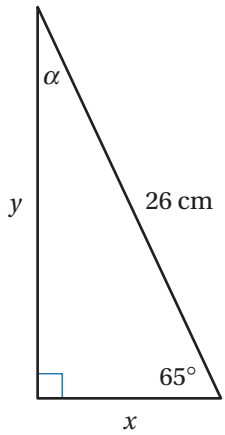


(a) Write an equation to solve x as a first step.

(b) Without using the value for x, write an equation to solve α as a first step.

(c) Without using the value for x, write an equation to solve β using the value of α.

3 For the triangle shown, determine a strategy for the problems below.

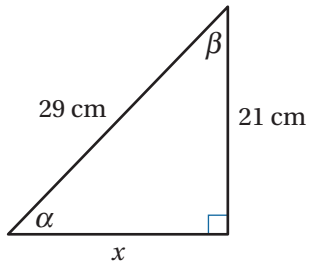


(a) Write an equation to solve α as a first step.

(b) Without using the value for α, write an equation to solve x as a first step.

(c) Without using the value for α, write an equation to solve y as a first step, using the value of x.

4 For the triangle shown, determine a strategy for the problems below.

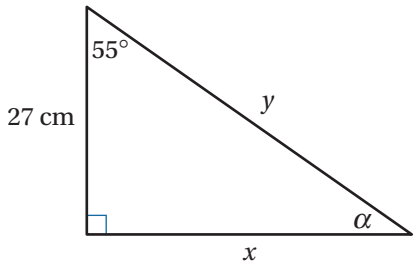


(a) Write an equation to solve x as a first step.

(b) Without using the value for x, write an equation to solve α as a first step.

(c) Without using the value for x, write an equation to solve β as a first step, using the value of α.

5 For the triangle shown, determine a strategy for the problems below.



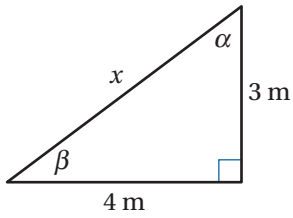
(a) Write an equation to solve α as a first step.

(b) Without using the value for α, write an equation to solve x as a first step.

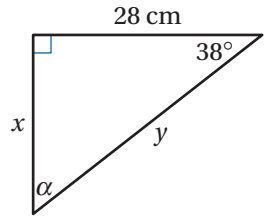
(c) Without using the value for α, write an equation to solve y as a first step, using the value of x.

SC 2 I can solve the missing side lengths and angle sizes in right-angled triangles

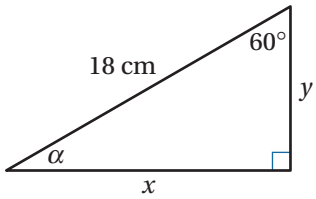
1 Determine any unknown sides and angles in the triangle shown. Round angles to the nearest degree.



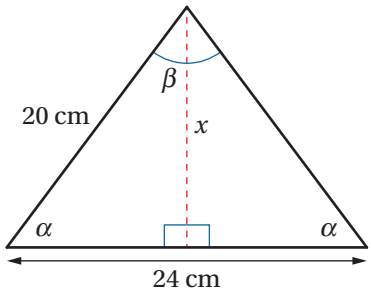
2 Determine any unknown sides and angles in the triangle shown. Round sides correct to 1 decimal place.



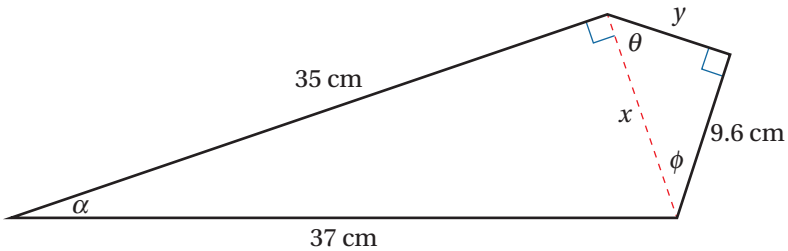
3 Determine any unknown sides and angles in the triangle shown. Round sides correct to 1 decimal place where necessary.



4 Determine the unknown angles and height of the isosceles triangle shown. Round angles to the nearest degree.



5 Determine any unknown sides and angles in the quadrilateral shown. Some variables have been added to assist your working. Round angles to the nearest degree.



TOPIC REVIEW

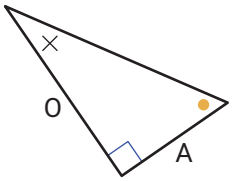
Multiple choice

ANSWERS Page 672

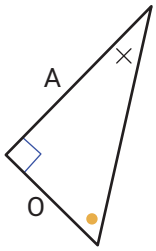
Use the following information to answer Questions **1** and **2**. In each of the following triangles, O means the side opposite the angle marked with a cross and A means the side adjacent to the angle marked with a cross.

1 Identify the correctly labelled triangles.

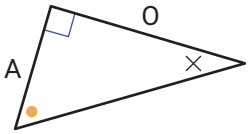
I



II



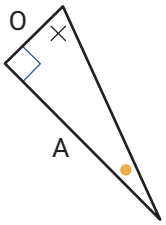
III



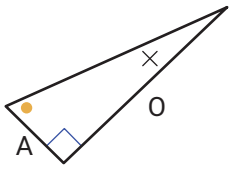
A I only **B** II only **C** III only **D** I and III

2 Identify the correctly labelled triangles.

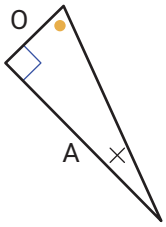
I



II



III



A I only **B** II only **C** III only **D** II and III

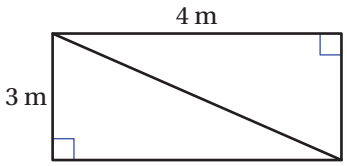
3 The string of a kite makes an angle of 54° with the horizontal. If the length of the string is 32 m, the height of the kite in metres is:

A $\frac{32}{\tan(54°)}$ **B** $\frac{32}{\sin(54°)}$ **C** $32 \times \tan(54°)$ **D** $32 \times \sin(54°)$

4 The shadow of a 5 m tree at 1 pm is 6 m long. If the shadow increases in length by an average of 50 cm/h, the angle of elevation from the end of the shadow to the treetop at 4 pm will be closest to:

A 43° **B** 50° **C** 40° **D** 34°

5 A rectangular gate is strengthened by a diagonal support. The smaller of the angles the diagonal makes with the sides of the gate is closest to:

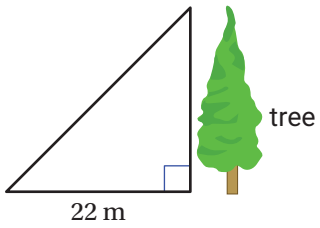


A 53° **B** 37° **C** 49° **D** 41°

6 A mine shaft 120 m long is dug at an angle of 25.8° to the vertical. The depth of the end of the shaft is closest to:

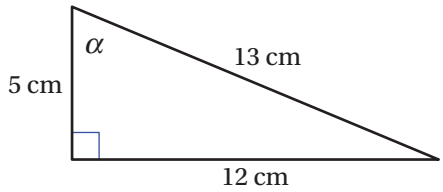
A 108 m **B** 96 m **C** 51 m **D** 59 m

7 The angle of elevation of the top of a tree from a point 22 m away is 24°. The height of the tree in metres is closest to:



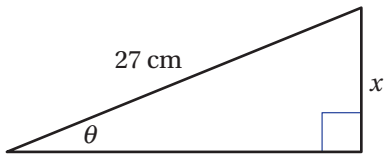
A 20.10 **B** 8.95 **C** 20.08 **D** 9.80

8 For the triangle shown, which of the following represents the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for the acute angle that is *not* marked α?



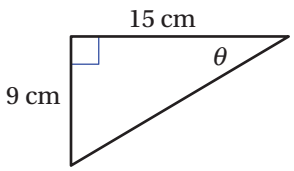
A $\frac{5}{13}$ **B** $\frac{5}{12}$ **C** $\frac{12}{13}$ **D** $\frac{13}{12}$

9 If the sine ratio for the angle marked α is $\frac{1}{3}$, what is the value of x?



A 3 cm **B** 9 cm **C** 15 cm **D** 27 cm

10 Which equation can be used to solve for θ?

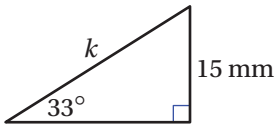


A $\cos(\theta) = \frac{9}{15}$ **B** $\sin(\theta) = \frac{9}{15}$ **C** $\tan(\theta) = \frac{9}{15}$ **D** $\tan(\theta) = \frac{15}{9}$

11 A 9.5 m tree casts a shadow across the ground. If the angle of elevation of the sun is 49°, the length of the shadow is closest to:

A 1.2 m **B** 6.2 m **C** 7.2 m **D** 8.3 m

12 For the following triangle, the value of k, in millimetres, could be calculated by the equation:



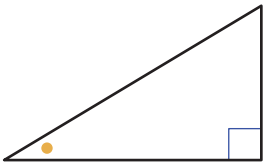
A $\sin(33°) = \frac{15}{k}$ **B** $\cos(33°) = \frac{k}{15}$ **C** $\sin(33°) = \frac{k}{15}$ **D** $\tan(33°) = \frac{15}{k}$

12

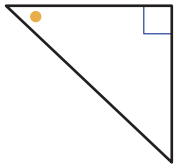
Short answer

1 Copy each diagram and label the hypotenuse. Using the angle marked with a dot as the angle of interest, label the remaining two sides as adjacent or opposite.

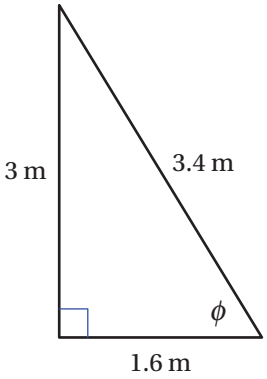
(a)



(b)



2 Determine the values of $\sin(\phi)$, $\cos(\phi)$ and $\tan(\phi)$ as fractions in simplest form.

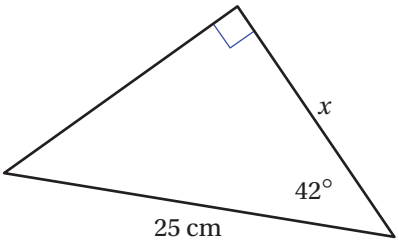


3 Write the inverse-function equation used to solve for θ in each of the following.

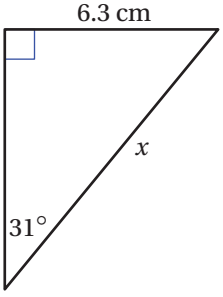
(a) $\sin(\theta) = 0.65$ **(b)** $\tan(\theta) = 1.2425$ **(c)** $\cos(\theta) = \frac{28.6}{45.1}$

4 Solve for x, correct to the nearest millimetre.

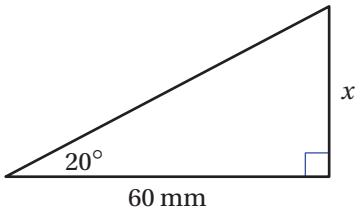
(a)



(b)

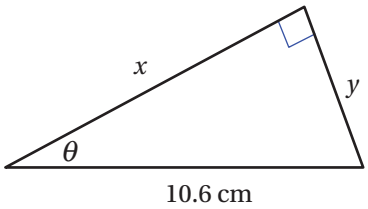


(c)

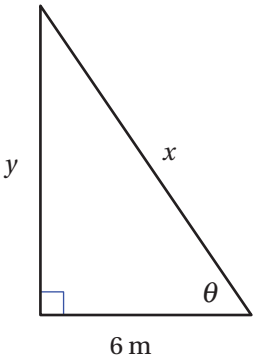


5 Use the given function value to solve for x, y and θ. Round angles to the nearest degree.

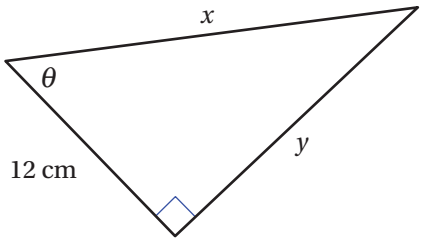
(a) $\sin(\theta) = \frac{28}{53}$



(b) $\cos(\theta) = \frac{12}{37}$



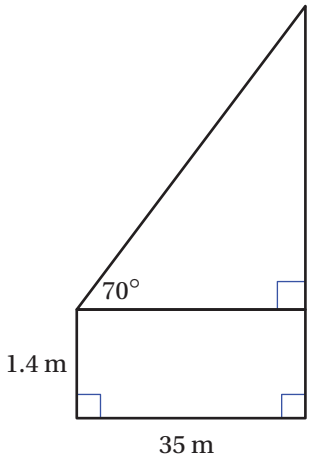
(c) $\tan(\theta) = \frac{55}{48}$



6 A plank of wood allows a construction worker to wheelbarrow loads of wet cement up a rise of 35 cm. If the plank is angled at 3° to the horizontal, determine its length, in metres, correct to 2 decimal places.

7 A surf lifesaver, sitting in an observation tower, sees that two swimmers make a single line with the tower. If the angles of depression from the line of sight 3 m above the current sea level are 10.4° and 5.8°, for the two swimmers respectively, determine the distance between the swimmers, correct to 1 decimal place.

8 The height of a building can be calculated using the diagram below. Calculate the height, correct to 1 decimal place.

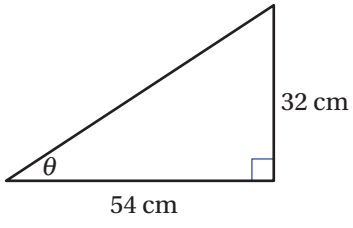


9 Determine the value of θ, to the nearest degree, given:

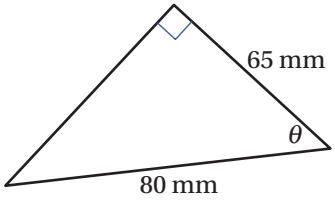
(a) $\tan(\theta) = 0.68$ **(b)** $\sin(\theta) = 0.7778$ **(c)** $\cos(\theta) = \frac{8.5}{10.2}$

10 Determine the value of θ, to the nearest degree, in each of the following.

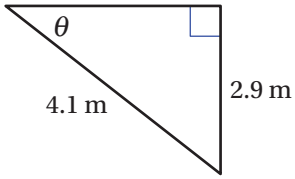
(a)



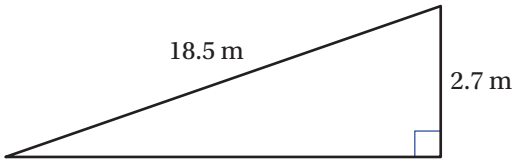
(b)



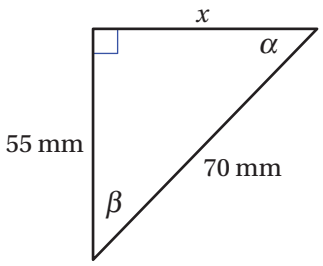
(c)



11 The side view of a roof is shown. Determine the pitch of the roof, correct to 1 decimal place.



12 For this triangle, choose a strategy and write an equation to:

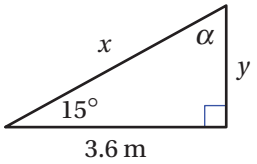


(a) solve x as a first step

(b) solve α without using the value of x

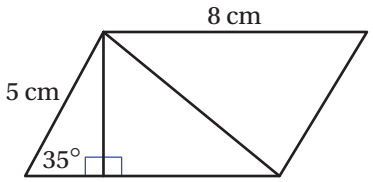
(c) solve β using the value of α.

13 Solve the triangle for any unknown sides and angles. Write side lengths in metres, correct to 2 decimal places.



Extended response

1 For the parallelogram below, calculate, to the nearest millimetre:



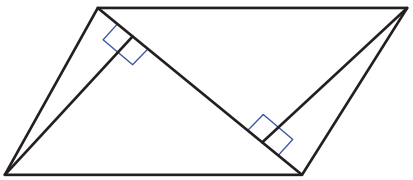
(a) the perimeter

(b) the perpendicular height

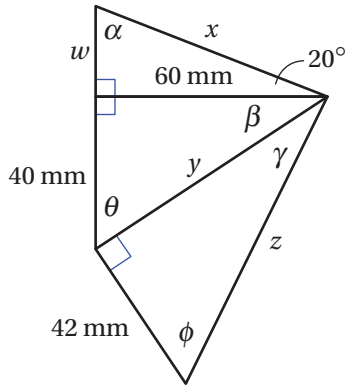
(c) the area, correct to 2 decimal places

(d) the length of the shorter diagonal

(e) the height of each triangle formed by the shorter diagonal of the parallelogram. Use the figure below to help you.



2 Consider the figure below.

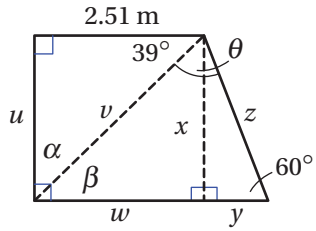


(a) Solve each unknown length and angle, to the nearest whole number.

(b) Classify the overall shape.

(c) List the side lengths and angles for the overall shape.

3 Consider the trapezium below.



(a) Solve each labelled unknown length and angle, to the nearest centimetre or degree, respectively.

(b) List the side lengths and angles for the trapezium.

TOPIC
13

Ratios and proportion in space

Recall 433

13.1 Understand the effect of the enlargement transformation 434

13.2 Calculate the area of a shape or the volume of an object using a scale factor 443

13.3 Use images and proportion to estimate lengths 452

13.4 Ascertain compliance involving horizontal and vertical distances 459

13.5 Recognise image editing 463

Topic review 467

Why learn this?

Understanding ratios and proportion in space provides valuable skills for interpreting the world geometrically. Mastery of enlargement transformations, identifying similar shapes, and calculating areas and volumes using scale factors equips learners with tools to:

- accurately scale drawings and models
- apply mathematical reasoning to real-life problems
- interpret and evaluate the use of proportion in media.

These competencies form a foundation for advanced mathematical concepts and can be applied across a range of subjects, careers and everyday tasks.

RECALL

ANSWERS Page 646

I can convert between units of length

1 Complete each of the following length conversions.

(a) 2375 cm to m **(b)** 5.04 cm to mm **(c)** 58.35 mm to cm

(d) 8.55 mm to m **(e)** 0.975 m to cm **(f)** 0.0045 km to cm

I can simplify ratios of lengths in different units

1 Express these ratios in the same units, then simplify.

(a) 2 mm : 4 cm **(b)** 500 m : 1.5 km **(c)** 6.5 cm : 9.75 m

(d) 250 cm : 10 m **(e)** 40 mm : 320 cm

I can calculate the area of plane shapes

1 Calculate the area of these plane shapes.

(a) A square with sides of length 4.5 cm

(b) A rectangle of length 2.25 m and width 1.6 m

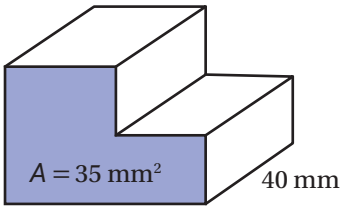
(c) A right-angled triangle with shorter sides of length 10 cm and 6.8 cm

(d) A rhombus with diagonals of length 1.2 m and 0.8 m

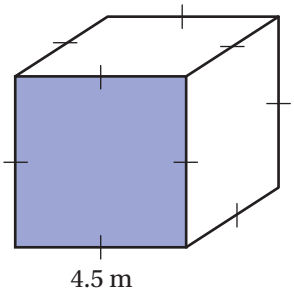
I can calculate the volume of solids

1 Calculate the volume of these solids.

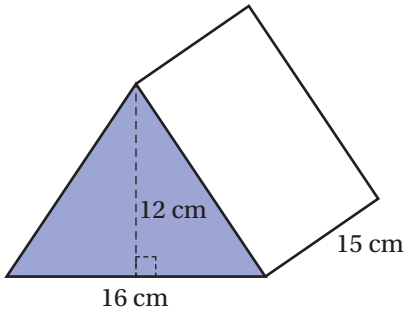
(a)



(b)



(c)



13.1 Understand the effect of the enlargement transformation

Learning intention: To understand the effect of the enlargement transformation

Success criteria:

SC 1 I can apply the enlargement transformation to shapes and objects.

SC 2 I can identify aspects of similar shapes that are the same and those that change.

SC 3 I can identify aspects of similar objects that are the same and those that change.

Lesson warm-up

Twice the dice

The dice on the right is an enlarged version of the dice on the left.

For each of the following attributes, decide whether it is the same, larger or smaller.



Attribute	Right dice compared to left dice
side length	larger
height	
width	
number of faces	
shape of faces	
angles	
surface area	
volume	

SC 1 I can apply the enlargement transformation to shapes and objects

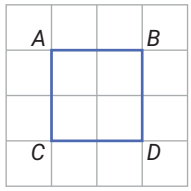
An enlargement changes the size of a shape (or a 3D solid). The enlarged shape has the same angles as the original, but the side lengths are increased by a scale factor (e.g. a scale factor of 2 means that every side is twice the original length).

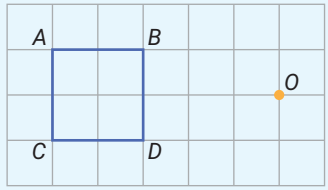
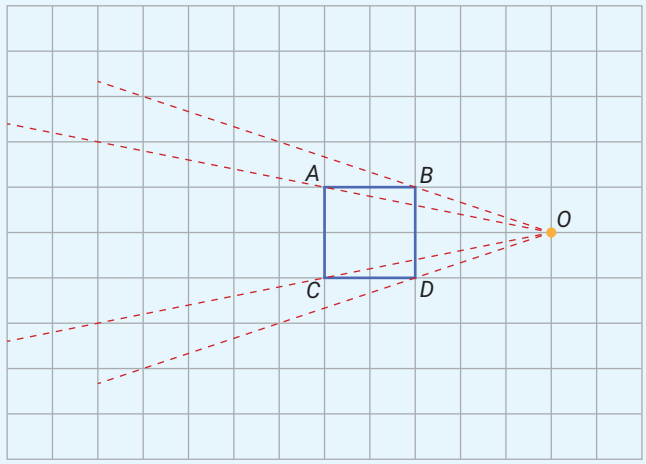
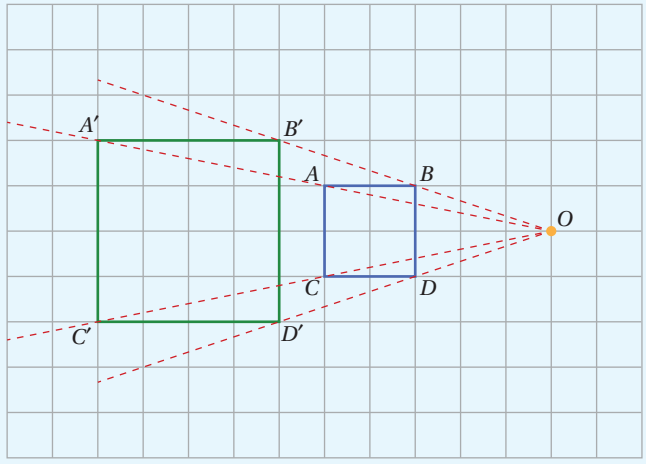
One way of enlarging a shape is to use a centre of enlargement. This can be done on paper or using a dynamic geometry model.

Worked example

Enlarging a shape using a centre of enlargement

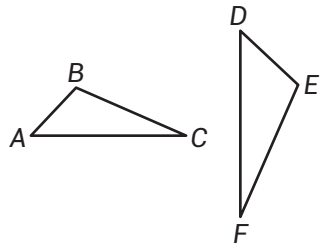
Enlarge the square by a scale factor of 2, using a centre of enlargement.



THINKING	WORKING
Choose a centre of enlargement, anywhere outside (or inside) the shape.	
Rule lines from the centre of enlargement through each vertex and out the other side.	
Measure the distances from the centre to each vertex (or use the grid), multiply by the scale factor and draw the image vertices. Draw the image.	
Check that the lengths of the sides have been increased by the scale factor.	The side lengths have doubled as intended.

SC 2 I can identify aspects of similar shapes that are the same and those that change

When two shapes are similar, they look the same, except that one is an enlargement of the other. The corresponding pairs of angles are equal and the corresponding sides are in proportion.



$\Delta ABC \sim \Delta DEF$ means that:

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and

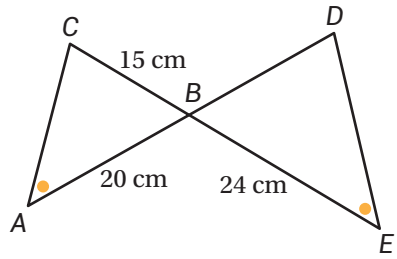
$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

13

Worked example

Identifying equal angles and calculating side lengths in similar shapes

For the pair of similar triangles:



(a) determine the angle equal to $\angle ABC$

THINKING	WORKING
Determine the corresponding angle. Write the angle using three letters with the vertex in the middle.	$\angle ABC = \angle EBD$ (vertically opposite angles)

(b) write the statement of similarity

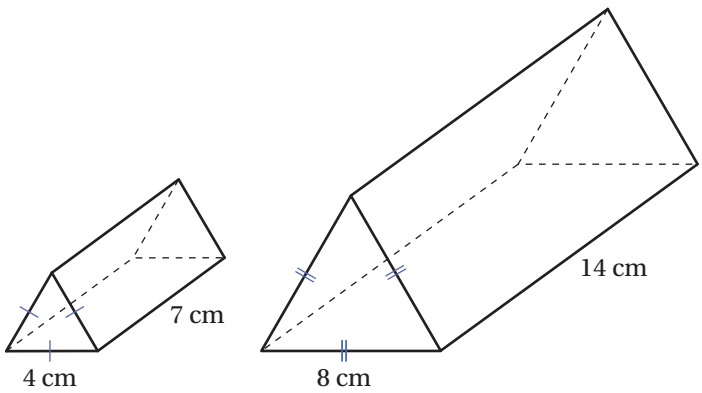
THINKING	WORKING
Write the letters for one triangle in any order, then for the other triangle in matching order.	$\Delta ABC \sim \Delta EBD$

(c) calculate the length of BD.

THINKING	WORKING
Write the proportion statement, comparing matching sides on the two triangles. Put the unknown in the numerator, and choose other sides that have known values.	$\Delta ABC \sim \Delta EBD$
Substitute the known values, then calculate the required value.	$\frac{BD}{15\text{ cm}} = \frac{24\text{ cm}}{20\text{ cm}}$ $\frac{BD}{15\text{ cm}} = 1.2$ $BD = 15\text{ cm} \times 1.2$ $BD = 18\text{ cm}$

SC 3 I can identify aspects of similar objects that are the same and those that change

When two 3D objects are similar, they look the same except that one is an enlargement of the other. The corresponding pairs of angles are equal and the corresponding edges are in proportion. The faces of the enlarged object are the same shape, but larger.



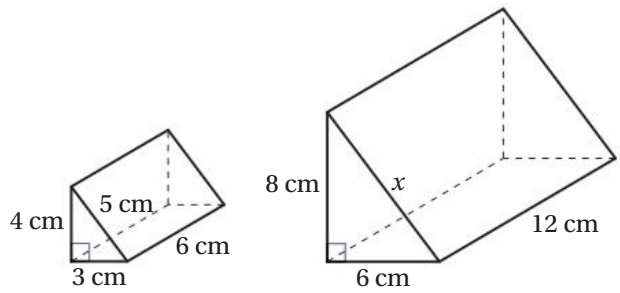
For example, for these similar triangular prisms, each pair of matching faces are similar shapes.

The surface area and volume of the enlarged shape are much larger than the original. For example, for the prisms above, you could stack 8 of the smaller prisms inside the larger prism.

Worked example

Comparing similar objects

For the pair of similar triangular prisms:



(a) determine the scale factor

THINKING	WORKING
Determine the scale factor by choosing a matching pair of edges, and dividing.	$\frac{6}{3} = 2$
Write the answer.	The scale factor is 2.

(b) calculate the length of the unknown edge

THINKING	WORKING
Multiply the known length by the scale factor to determine the length on the enlarged solid.	$x = 2 \times 5$ $= 10\,\text{cm}$
Write the answer.	The unknown side length is 10 cm.

(c) calculate the volumes and compare as a ratio.

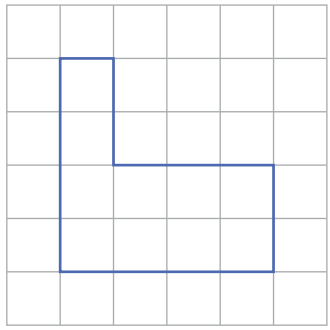
THINKING	WORKING
Recall the formula for the volume of a prism.	$V = AH$ where $A = \frac{1}{2}bh$
Substitute the side lengths to calculate the volume of the smaller prism.	$V = \left(\frac{1}{2} \times 3 \times 4\right) \times 6$ $= 36\,\text{cm}^3$
Substitute the side lengths to calculate the volume of the larger prism.	$V = \left(\frac{1}{2} \times 6 \times 8\right) \times 12$ $= 288\,\text{cm}^3$
Write as a ratio and reduce to a unit ratio.	Ratio of volumes = 36 : 288 = 1 : 8
Write your answer.	The volumes are in the ratio 1 : 8 or $1 : 2^3$.

Practice

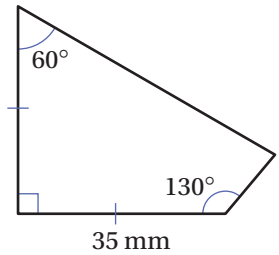
ANSWERS Page 646

SC 1 I can apply the enlargement transformation to shapes and objects

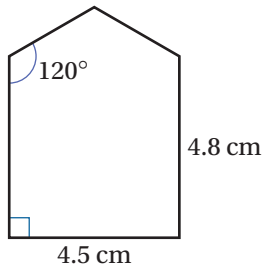
1 Copy this shape onto grid or dot paper, then enlarge it using a scale factor of 2. Use any method.



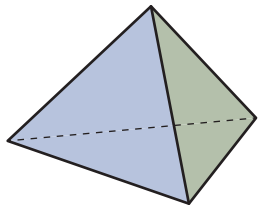
2 Enlarge using a scale factor of 1.2.



3 Draw the original symmetrical pentagon, given that it was enlarged by a factor of 1.5 to result in this image.



4 A tetrahedron is a 3D object with 4 faces that are equilateral triangles.



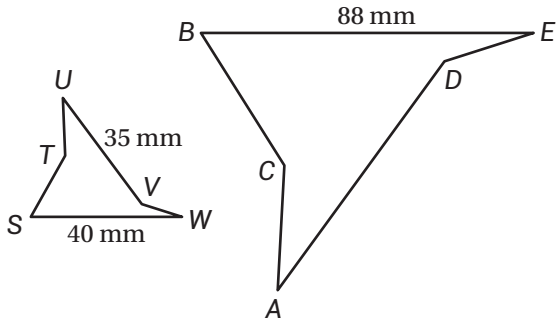
(a) Copy the drawing and enlarge it using a centre of enlargement and a scale factor of 3.

(b) Consider using the same method to enlarge a 3D model of a tetrahedron (i.e. building an enlarged 3D object). How would you adapt the method?

(c) Can you think of an easier way to make an enlarged model of a tetrahedron?

SC 2 I can identify aspects of similar shapes that are the same and those that change

1 For the pair of pentagons below:

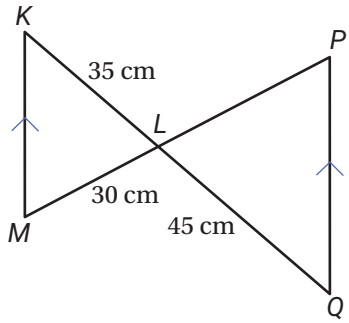


(a) identify the angle equal to $\angle V$

(b) write a similarity statement beginning with $STUVW$

(c) calculate the length DA.

2 For the pair of triangles below:

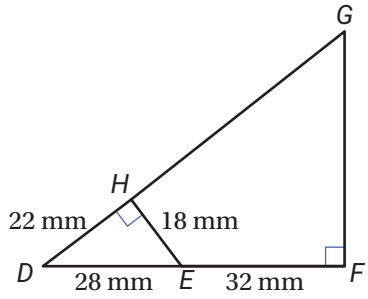


(a) determine the angle equal to $\angle KML$

(b) write a similarity statement beginning with ΔKLM

(c) calculate the length PL to the nearest centimetre.

3 For the pair of triangles below:



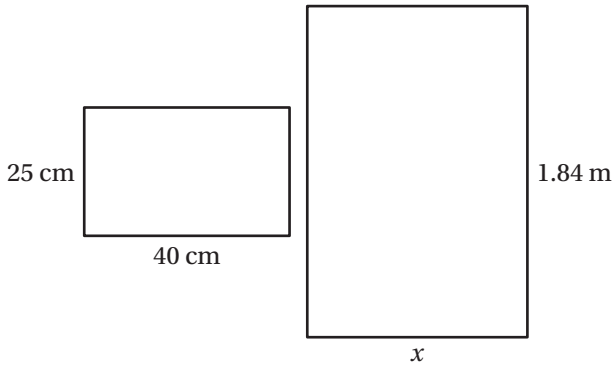
(a) write a similarity statement beginning with ΔDEH

(b) complete the equality: $\frac{FG}{HE} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

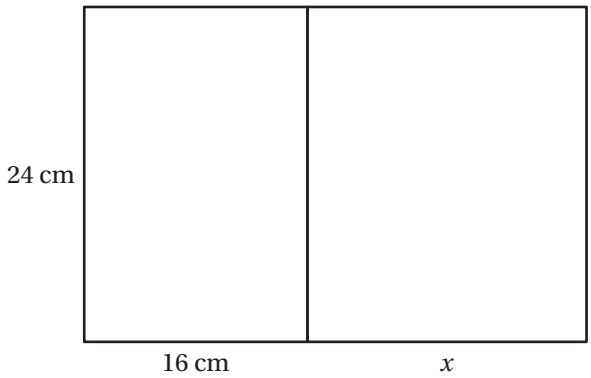
(c) calculate the length GF, to the nearest millimetre.

4 For each pair of similar rectangles determine the value of x.

(a)

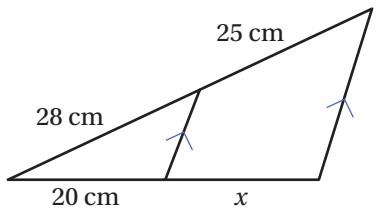


(b) Note, the smaller rectangle is sitting within the enlarged rectangle.

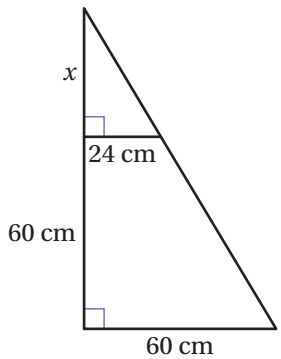


5 For each diagram, use similar triangles to determine the value of x. Round to the nearest centimetre where necessary.

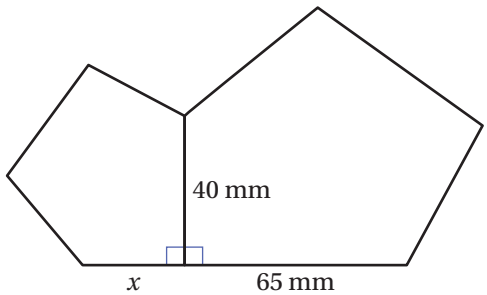
(a)



(b)



6 Given that the two pentagons are similar, calculate the value of x to the nearest millimetre.



SC 3 I can identify aspects of similar objects that are the same and those that change

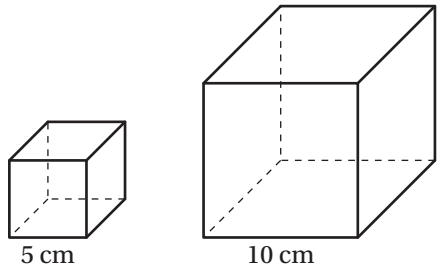
1 When a 3D object is enlarged, some attributes change and some stay the same. Decide whether the following statements are true or false.

(a) The enlarged object has more faces.

(b) The faces of the enlarged object are similar to the original faces.

(c) The angles at the vertices get larger.

2 For these cubes with edge lengths of 5 cm and 10 cm, calculate the following.



(a) The surface area of the original cube

(b) The surface area of the enlarged cube

(c) The ratio of the surface areas

(d) The volume of the original cube

(e) The volume of the enlarged cube

(f) The ratio of the volumes

3 Decide whether the following are true or false.

(a) All cubes are similar.

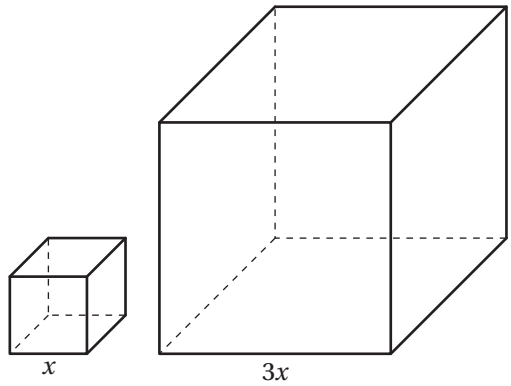
(b) All hexagonal prisms are similar.

(c) All spheres are similar.

(d) All cylinders are similar.

4 An ice cream cone has a diameter of 6 cm and a height of 15 cm. To make an ice cream cone sculpture that is 1.2 m tall, what should the diameter be?

5 For these cubes with edge lengths x and $3x$, calculate the following.



(a) The surface area of the original cube

(b) The surface area of the enlarged cube

(c) The ratio of the surface areas

(d) The volume of the original cube

(e) The volume of the enlarged cube

(f) The ratio of the volumes

Calculate the area of a shape or the volume of an object using a scale factor

13.2

Learning intention: To be able to calculate the area of a shape or the volume of an object using a scale factor

Success criteria:

SC 1 I can calculate the area of a triangle from a similar triangle.

SC 2 I can use scale factor to calculate the area of any shape from a similar shape.

SC 3 I can solve problems involving length, area and volume of similar shapes and objects.

Lesson warm-up

Estimating area and volume

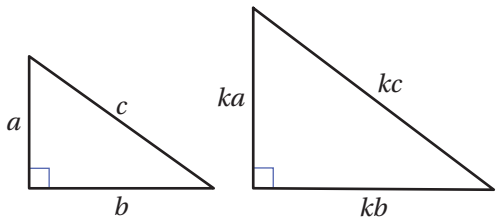


The local swimming pool has a large pool and a smaller pool. The large pool is 50 m long and 25 m wide. The smaller pool is 20 m long and 20 m wide, and not as deep. Jonas says that the smaller pool is about half the size of the larger pool. Gina says it is probably closer to a quarter of the size.

1. Who do you agree with, and why?
2. A teacher wants to know how many people can safely fit in the larger pool compared to the smaller pool. Is Jonas's or Gina's estimate more useful here?
3. The pool staff need to estimate how much more water is in the large pool compared to the smaller pool. Is Jonas's or Gina's estimate more useful here?

SC 1 I can calculate the area of a triangle from a similar triangle

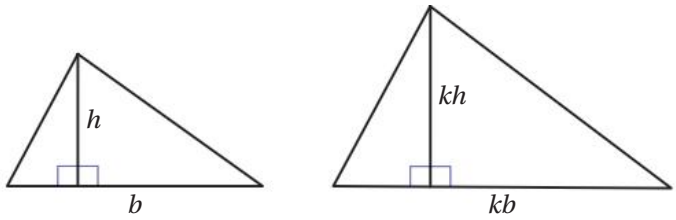
The right-angled triangle below has been enlarged by a factor of k.



The original triangle has an area of $\frac{ba}{2}$. Its enlargement has an area of $\frac{kb \times ka}{2} = k^2 \frac{ba}{2}$.

Hence, the enlargement factor for the area is k^2.

When a triangle is enlarged by a scale factor of k, all corresponding dimensions are enlarged by the same factor, including the perpendicular height.



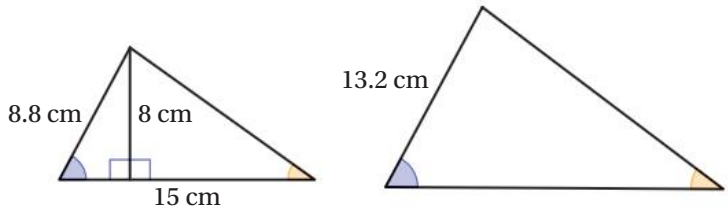
The original triangle has an area of $\frac{bh}{2}$. Its enlargement has an area of $\frac{kb \times kh}{2} = k^2 \frac{bh}{2}$.
Hence, the enlargement factor for the area is k^2.

13

Worked example

Calculating the area of any enlarged triangle from the original area and the scale factor

The triangle on the left has been enlarged to give the triangle on the right.



(a) Calculate the area of the original triangle.

THINKING	WORKING
Use the area of a triangle formula.	$A = \frac{bh}{2}$ $= \frac{15 \times 8}{2}$ $= 60\text{ cm}^2$
Write the answer.	The area of the original triangle is 60 cm^2.

(b) Calculate the scale factor.

THINKING	WORKING
Calculate the ratio of any length on the larger triangle to the corresponding length on the original triangle.	$k = \frac{13.2}{8.8}$ $= 1.5$
Write the answer.	The scale factor is 1.5 or $\frac{3}{2}$.

(c) Calculate the area of the enlarged triangle.

THINKING	WORKING
Multiply the area of the original triangle by the scale factor squared.	$60 \times 1.5^2 = 135\,\text{cm}^2$
Write the answer.	The area of the enlarged triangle is $135\,\text{cm}^2$.

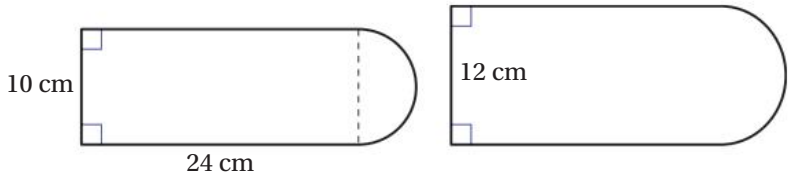
SC 2 I can use scale factor to calculate the area of any shape from a similar shape

For any pair of similar shapes, dimensions on one can be calculated from corresponding dimensions on the other by multiplying or dividing by the scale factor, or enlargement factor, k. The area of one can be calculated from the area of the other by multiplying or dividing by the square of the scale factor, or enlargement factor, k^2.

Worked example

Calculating the area of a shape from the area of a similar shape using the scale factor

The figure on the left has been enlarged to give the figure on the right.



(a) Calculate the exact area of the original figure.

THINKING	WORKING
Add the area of a rectangle to the area of a semicircle.	$A = lw + \frac{\pi r^2}{2}$ $= 24 \times 10 + \frac{\pi \times 5^2}{2}$ $= (240 + 12.5\pi)\,\text{cm}^2$
Write the answer.	The exact area of the original figure is $(240 + 12.5\pi)\,\text{cm}^2$.

(b) Calculate the scale factor.

THINKING	WORKING
Calculate the ratio of any dimension on the larger figure to the corresponding length on the original figure.	$k = \frac{12}{10}$ $= 1.2$
Write the answer.	The scale factor is 1.2 or $\frac{6}{5}$.

(c) Calculate the exact area of the enlarged figure.

THINKING	WORKING
Multiply the area of the original figure by k^2.	$(240 + 12.5\pi) \times 1.2^2 = (345.6 + 18\pi)$ cm^2
Write the answer.	The exact area of the enlarged figure is $(345.6 + 18\pi)$ cm^2.

SC 3 I can solve problems involving length, area and volume of similar shapes and objects

The transformation of enlarging a shape keeps the shape looking the same, with all angles preserved and all lengths in the same proportion.

If the scale factor of an enlargement (or reduction) of k is applied:

- all lengths of sides, perpendicular heights, curves and perimeters will be k times as long as the original length
- all areas, including surface areas, will be k^2 times as large as the original areas
- all volumes will be k^3 times as large as the original volumes.

Worked example

Demonstrating the effect of enlargement on length, area and volume

1 For a rectangle measuring 6 cm by 4 cm, what effect does doubling the side lengths have on each of the following? Show your calculations.

(a) Perimeter

THINKING	WORKING
Calculate the original perimeter.	Original $P = 2l + 2w$ $= 2 \times 6 + 2 \times 4$ $= 20$ cm
Double each of the lengths.	Enlarged rectangle Length: 6 cm $\times$ 2 = 12 cm Width: 4 cm $\times$ 2 = 8 cm
Calculate the perimeter of the enlarged rectangle.	$P = 2l + 2w$ $= 2 \times 12 + 2 \times 8$ $= 40$ cm
Write the perimeters as a ratio and simplify.	20 cm : 40 cm = 1 : 2
Write the answer.	Doubling the side lengths caused the perimeter to double.

(b) Area

THINKING	WORKING
Calculate the original area.	Original $A = lw$ $= 6 \times 4$ $= 24\,\text{cm}^2$
Calculate the area of the enlarged rectangle.	Enlarged rectangle $A = lw$ $= 12 \times 8$ $= 96\,\text{cm}^2$
Write the areas as a ratio and simplify.	$24\,\text{cm}^2 : 96\,\text{cm}^2 = 1 : 4$
Write the answer.	Doubling the side lengths caused the area to quadruple, i.e. it is 2^2 times as large.

2 For a rectangular prism measuring 5 cm by 4 cm by 3 cm, what effect does doubling the side lengths have on each of the following? Show your calculations.

(a) Surface area

THINKING	WORKING
Calculate the original surface area.	Original $S = 2lw + 2lh + 2wh$ $= 2 \times 5 \times 4 + 2 \times 5 \times 3 + 2 \times 4 \times 3$ $= 94\,\text{cm}^2$
Double each of the lengths.	Enlarged rectangular prism Length: $5\,\text{cm} \times 2 = 10\,\text{cm}$ Width: $4\,\text{cm} \times 2 = 8\,\text{cm}$ Height: $3\,\text{cm} \times 2 = 6\,\text{cm}$
Calculate the surface area of the enlarged rectangular prism.	$S = 2lw + 2lh + 2wh$ $= 2 \times 10 \times 8 + 2 \times 10 \times 6 + 2 \times 8 \times 6$ $= 376\,\text{cm}^2$
Write the surface areas as a ratio and simplify.	$94\,\text{cm}^2 : 376\,\text{cm}^2 = 1 : 4$
Write the answer.	Doubling the side lengths caused the surface area to quadruple, i.e. it is 2^2 times as large.

(b) Volume

THINKING	WORKING
Calculate the original volume.	Original $V = lwh$ $= 5 \times 4 \times 3$ $= 60\,\text{cm}^3$
Calculate the volume of the enlarged rectangular prism.	Enlarged rectangular prism $V = lwh$ $= 10 \times 8 \times 6$ $= 480\,\text{cm}^3$
Write the volumes as a ratio and simplify.	$60\,\text{cm}^2 : 480\,\text{cm}^2 = 1 : 8$
Write the answer.	Doubling the side lengths caused the volume to become 8 times as large, i.e. it is 2^3 times as large.

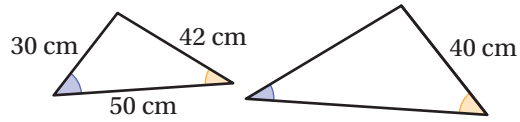
Practice

ANSWERS Page 647

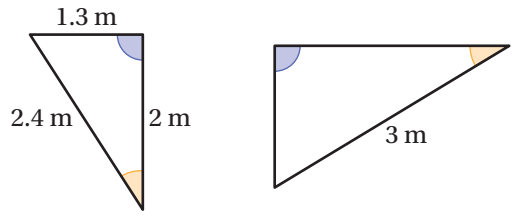
SC 1 I can calculate the area of a triangle from a similar triangle

1 Calculate the enlargement factor for each pair of similar triangles. What number would you multiply the smaller area by to get the larger area?

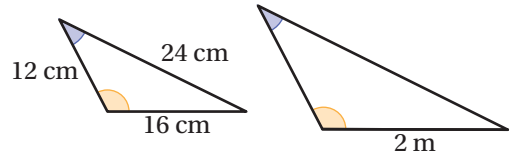
(a)



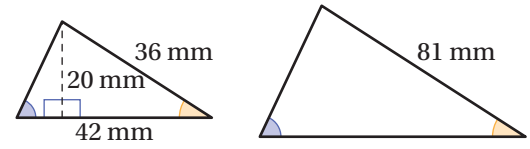
(b)



(c)



2 For the pair of similar triangles, calculate:

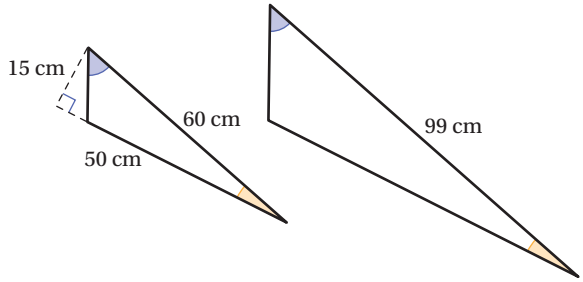


(a) the area of the original triangle

(b) the scale factor

(c) the area of the enlarged triangle, to the nearest square millimetre.

3 For the pair of similar triangles, calculate:

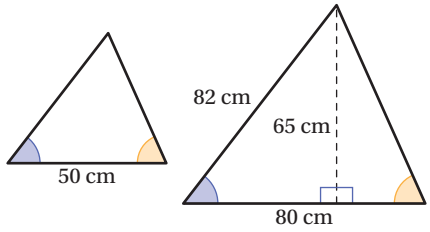


(a) the area of the original triangle

(b) the scale factor

(c) the area of the enlarged triangle, to the nearest square centimetre.

4 For the pair of similar triangles, calculate:

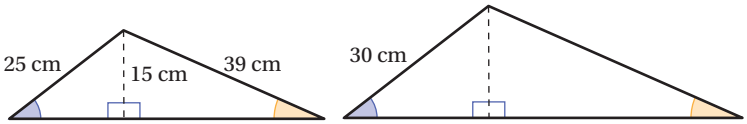


(a) the area of the enlarged triangle

(b) the scale factor

(c) the area of the smaller triangle, to the nearest square centimetre.

5 For the pair of similar triangles, calculate:



(a) the length of the base of the smaller triangle

(b) the area of the smaller triangle

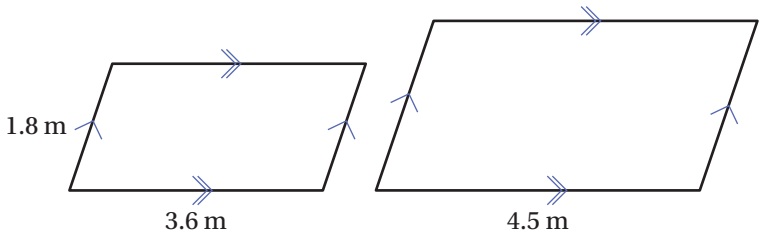
(c) the scale factor

(d) the area of the enlarged triangle, to the nearest square centimetre.

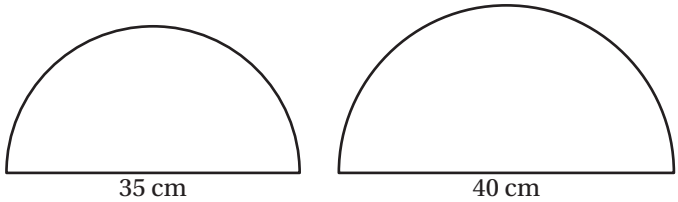
SC 2 I can use scale factor to calculate the area of any shape from a similar shape

1 Calculate the enlargement factor for each pair of similar shapes. What number would you multiply the smaller area by to get the larger area?

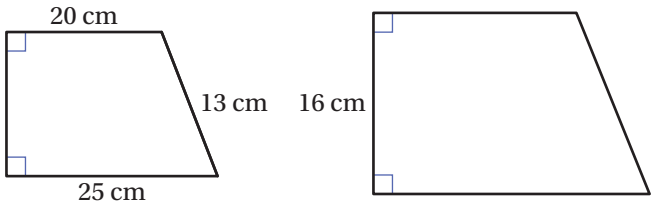
(a)



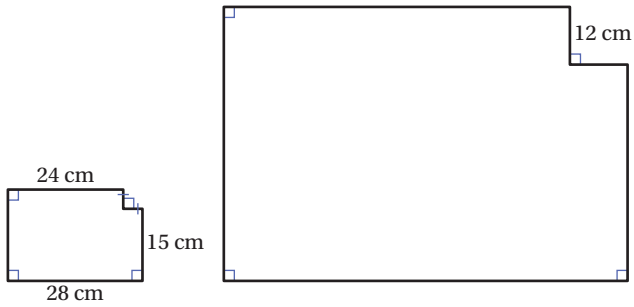
(b)



(c)



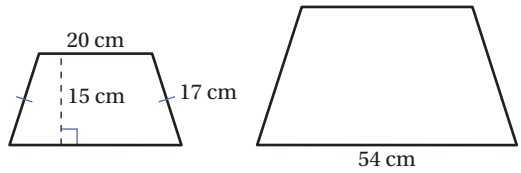
2 For the pair of similar shapes, calculate:



(a) the area of the original figure **(b)** the scale factor

(c) the area of the enlarged figure.

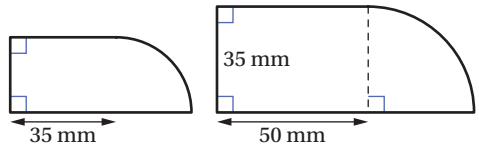
3 For the pair of similar isosceles trapeziums, calculate:



(a) the area of the original trapezium **(b)** the scale factor

(c) the area of the enlarged trapezium.

4 For the pair of similar shapes, calculate:

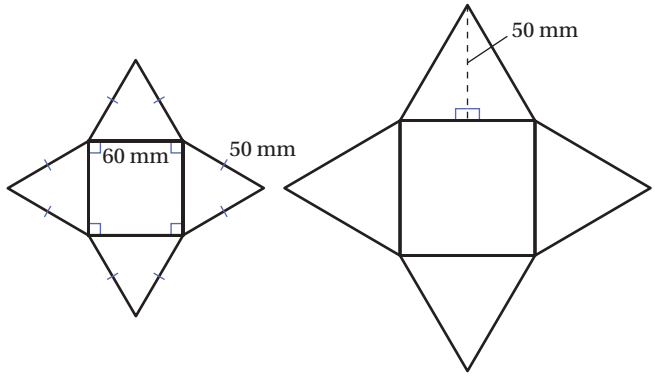


(a) the exact area of the enlarged shape

(b) the scale factor

(c) the area of the smaller shape, correct to the nearest square millimetre.

5 For the pair of similar shapes, calculate:



(a) the height of the smaller isosceles triangles

(b) the area of the smaller figure

(c) the scale factor

(d) the area of the enlarged figure.

SC 3 I can solve problems involving length, area and volume of similar shapes and objects

1 Show by calculation the effect of tripling the side lengths on the perimeter and area of the following.

(a) A rectangle 10 cm by 4 cm

(b) A rectangle 8 cm by 3 cm

(c) A rectangle 2 m by 1.5 m

(d) A rectangle 42 mm by 25 mm

2 Show by calculation the effect of tripling the side lengths on the surface area and volume of the following.

(a) A rectangular prism 10 cm by 6 cm by 4 cm

(b) A rectangular prism 7 cm by 2 cm by 1 cm

(c) A rectangular prism 24 cm by 18 cm by 12 cm

(d) A rectangular prism 3.6 m by 3.1 m by 1.4 m

3 Show by calculation the effect of increasing the side lengths by a factor of 1.6 on the following.

(a) Perimeter of a rectangle 15 cm by 10 cm

(b) Area of a rectangle 9 m by 5 m

(c) Surface area of a rectangular prism 18 cm by 10 cm by 5 cm

(d) Volume of a rectangular prism 2.6 m by 2.2 m by 2 m

4 Show by calculation the effect of reducing the side lengths by a factor of 0.7 on the following.

(a) Perimeter of a rectangle 28 m by 25 m

(b) Area of a rectangle 4.6 cm by 3.5 cm

(c) Surface area of a rectangular prism 90 mm by 52 mm by 35 mm

(d) Volume of a rectangular prism 20 cm by 15 cm by 10 cm

5 Describe, and justify, the effect of choosing to farm a piece of land 40% larger (on all linear dimensions), but the same shape as the original land, in terms of:

(a) cost of a border fence

(b) cost of planting seed.

6 Describe, and justify, the effect of choosing to package rice in containers 10% larger (on all linear dimensions), but the same shape as an original package, in terms of:

(a) area of packaging material needed

(b) mass of contents able to be packaged.

13.3 Use images and proportion to estimate lengths

Learning intention: To be able to use images and proportion to estimate lengths

Success criteria:

SC 1 I can estimate heights from images of objects that are side-by-side.

SC 2 I can estimate heights from images of objects that are in the foreground and the background.

Lesson warm-up

Who is tallest?

Which person in this photo is the tallest? Why do you think that?





SC 1 I can estimate heights from images of objects that are side-by-side

2D images can be used to estimate the heights of 3D objects, but to get a good estimate there are some things to remember. For example, to estimate the height of one person using the height of another, both people would have to be standing up straight, the same distance from the camera.



Worked example

Estimating the height of a person from a photo

The family photo below hangs on a wall. In the photo, the woman in green is 7 cm high and the child in blue standing at the front is 6 cm high (from heel to top of head). The woman is known to be 158 cm tall.



(a) Estimate the height, to the nearest centimetre, of the child in blue at the front.

THINKING	WORKING
Calculate a scale factor, actual height to photo height.	$\frac{158\text{ cm}}{7\text{ cm}} = 22.57\ldots$
Multiply the child's photo height by the scale factor.	$6\text{ cm} \times 22.57\ldots \approx 135\text{ cm}$
Alternatively, solve using ratios.	Let the child's height be x. $x : 158 = 6 : 7$ $\frac{x}{158} = \frac{6}{7}$ $x = 158 \times \frac{6}{7}$ $x \approx 135\text{ cm}$
Write the answer.	The child is about 135 cm tall.

(b) Are there any reasons why this estimate might not be an accurate one?

THINKING	WORKING
Consider the distance to the camera, and whether anyone in the photo might be taller or shorter than they appear.	The woman in green could be wearing high heels or standing on her toes: her heels can't be seen in the photo. The people at the back are a bit further from the camera.

SC 2 I can estimate heights from images of objects that are in the foreground and the background

Images that appear smaller in photos are taken further away from the camera. The smaller the image is, the further away it was when the photo was taken.



If the size and distance of a near object are known, then the size of a faraway object can be estimated, as long as its distance from the camera is known.

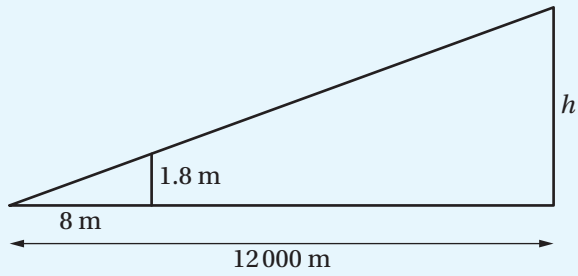
Worked example

Estimating the height of a background object in an image

A man has his photo taken in front of an iconic mountain.



(a) Given that the man is 180 cm tall and stands 8 m from the camera, determine the height of a mountain that would match his height in the photo, at a distance of 12 km.

THINKING	WORKING
Draw an embedded triangle diagram. Mark all lengths in the same units to make calculations easier.	

Match corresponding sides in the embedded triangles to solve for the unknown height.	$\frac{h}{12000} = \frac{1.8}{8}$ $h = 12000 \times \frac{1.8}{8}$ $= 2700\,\text{m}$
Write the answer.	The man's height would match a mountain of height 2700 m.

(b) A particular enlargement of the photo has the man as 4.2 cm high and the mountain with a height of 5.7 cm. Estimate the actual height of the mountain above the road level shown in the photo, to the nearest 10 m.

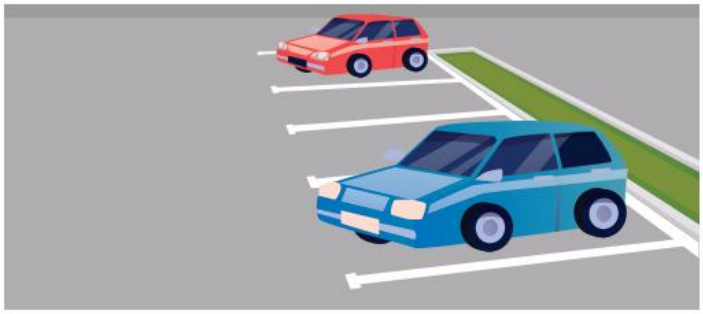
THINKING	WORKING
Use proportion to enlarge or reduce this calculated height to give the actual height of the mountain.	The man appears smaller than the mountain, so increase the previously calculated value. $2700 \times \frac{5.7}{4.2} = 3664.2\ldots\,\text{m}$
Write the answer.	The mountain in the photo is about 3660 m higher than the road.

Practice

ANSWERS Page 648

SC 1 I can estimate heights from images of objects that are side-by-side

1 The cars in the photo are parked parallel to each other.

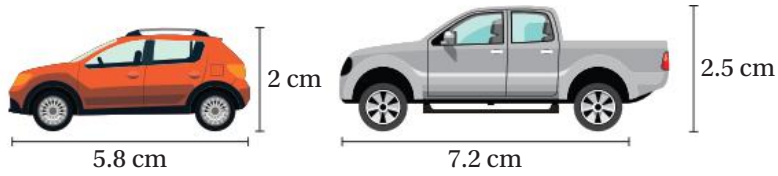


(a) Is it possible that both cars are the same length? Explain.

(b) For estimating the length of one car from the known length of another, does it matter that the cars are not photographed from the side?

(c) Explain how you would determine a valid estimate of the length of one car, given that the actual heights of both cars were known and the length of the other car was known.

2 The hatch and ute have been drawn to the correct side-view proportions.



On a blown-up version of this illustration, the lengths are 5.8 cm and 7.2 cm, respectively, while the heights are 2 cm and 2.5 cm, respectively.

Given that an average hatchback is 3252 mm long, estimate to the nearest millimetre:

(a) the length of the average ute

(b) the height of the average hatchback

(c) the height of the average ute.

3 A family photo has the heights of four people as 9.5 cm, 6.8 cm, 6.2 cm and 10.3 cm, left to right. Assume that the figure on the left is the father and the figure on the right is the mother.



(a) Explain why the mother's height in the photo has been exaggerated in comparison to the father's.

(b) Given that the father is 185 cm tall, estimate, to the nearest centimetre, the height of the child nearer to him.

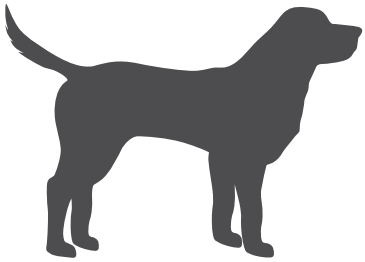
(c) Given that the mother is 176 cm tall, estimate, to the nearest centimetre, the height of the child nearer to her.

4 A two-storey and one-storey house stand side-by-side in the street, with a photo showing the images of their heights as 6.4 cm and 3.9 cm, respectively.

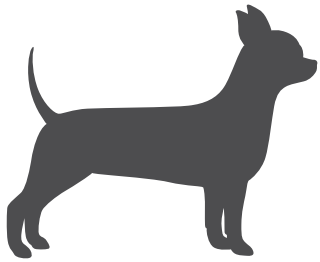


If the average height of a one-storey house is 4.5 m, estimate the average height of the two-storey house, correct to 1 decimal place.

5 This illustration could be helpful for estimating lengths and heights of dog breeds, but only if the drawings are in the correct proportion to each other.



Labrador



Chihuahua

Chihuahuas grow to an average height of 28 cm with a body length of 31 cm. Labradors grow to an average height of 56 cm with a body length of 60 cm.

(a) Explain how you can tell that the silhouettes in the illustration are not drawn to the same scale.

(b) If the illustrations used the same scale, determine, to the nearest millimetre, the body length of the labrador illustration when the chihuahua illustration has a body length of 10 cm.

(c) If the illustrations used the same scale, determine, to the nearest millimetre, the height of the chihuahua illustration when the labrador illustration has a height of 5 cm.

SC 2 I can estimate heights from images of objects that are in the foreground and the background

1 A person has their photo taken in front of a mountain range.



(a) Given that the person is 163 cm tall and stands 15 m from the camera, determine the height of a mountain, to the nearest metre, that would match their height in the photo, at a distance of 9.3 km.

(b) Given that a particular enlargement of the photo has the person at 4.5 cm high and the highest mountain at 6 cm, estimate the actual height of the mountain above the level shown in the photo, to the nearest 10 m.

2 Mount Coot-tha Lookout, 5 km from Brisbane's CBD, is an excellent place to take photos of the city skyline across the suburb of Toowong, 1.5 km away in the foreground.



(a) The government building, 1 William, standing by the river in the city, shows up as 5 cm high on a scenic photo taken at the lookout. Determine how tall the building would look in the photo, to the nearest millimetre, if it was located in Toowong.

(b) Given that the 1 William building is 260 m tall, determine the image height, to the nearest millimetre, of a 20 m apartment block in Toowong in the same photo.

3 A large dog is photographed at the front of its house, with both the house and dog appearing as 5.8 cm high in the photograph.



Given that the dog is actually 65 cm tall in this position, and is sitting 1.5 m from the camera with the house 10 m from the camera,

(a) estimate the height of the house, in metres, correct to 1 decimal place

(b) express any concerns for the validity of the method for this photograph.

4 A sedan is photographed in front of the owner's house. The side of the car nearest the photographer is 6 m away, with the front of the house a further 7 m away.



(a) Given that the car appears as 8.1 cm long in the photo, determine what the image length would have been, to the nearest millimetre, if the car had been as far away as the house.

(b) Given that the actual length of the car is 3.5 m and the image width of the house is 6.4 cm, estimate the actual width of the house in metres, correct to 1 decimal place.

Ascertain compliance involving horizontal and vertical distances

13.4

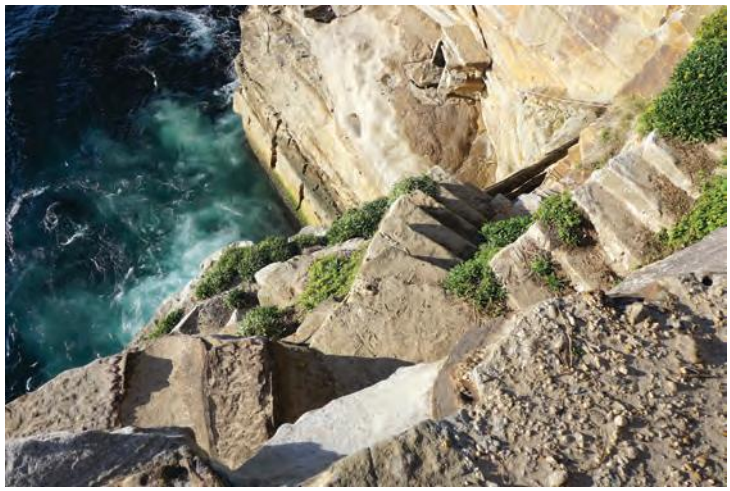
Learning intention: To be able to ascertain compliance involving horizontal and vertical distances

Success criteria:

SC 1 I can calculate ratios of horizontal and vertical distances to check compliance.

Lesson warm-up

Unsafe stairs



What do you think makes these stairs dangerous?

Have you ever used stairs that didn't feel safe? Why did they feel unsafe?

SC 1 I can calculate ratios of horizontal and vertical distances to check compliance

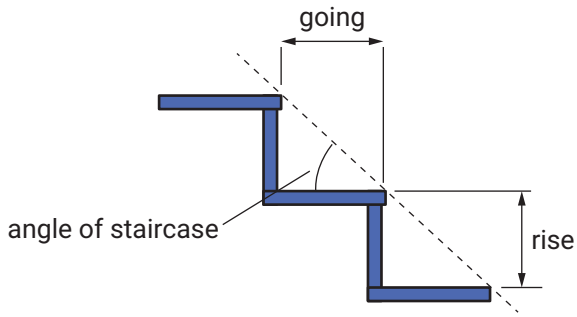
When roads, ramps, stairs and escalators are built, there are rules about how steep they can be.

The steepness allowed for a road depends on how the road will be used. A suburban street can be steeper than a freeway. A steep dirt road in wet weather might only be safe for four-wheel drive vehicles.



Stairs, escalators and ramps must be compliant with both Australian and State regulations. Regulations are based on the intended use, including people on foot, in wheelchairs, in prams and in cars.

One area of compliance is the vertical to horizontal ratio (steepness) of the slopes used. There are other safety requirements such as type of surface, width of the path, and distances a staircase can go until a landing is required.



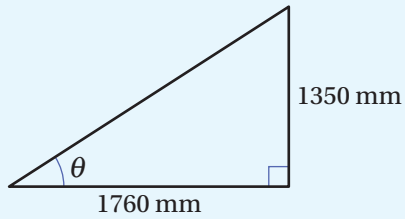
Builders often use millimetres instead of centimetres, rounding to the nearest millimetre.

Worked example

Compliance in staircase design

A staircase is to be built joining two levels of a split-level house. The height difference between the floors is 1350 mm and the horizontal distance allocated for the staircase is 1760 mm.

(a) Given that the staircase must have an angle between 20° and 45°, and the recommended steepness is between 30° and 38°, determine whether or not the staircase is compliant and at a recommended angle.

THINKING	WORKING
Draw a diagram showing the vertical and horizontal distance.	
Use the tangent ratio to determine the angle.	$\tan(\theta) = \frac{1350}{1760}$ $\theta = \tan^{-1}\left(\frac{1350}{1760}\right)$ $\theta = 37.4\ldots°$
Check the angle against the compliance range and the recommended range.	$20° < 37° < 45°$ so the staircase is compliant. $30° < 37° < 38°$ so the staircase is at a recommended angle.

(b) Given that the risers (step heights) must be between 130 mm and 225 mm, how many steps could there be?

THINKING	WORKING
To determine the range of the allowed numbers of stairs using the riser limitations, divide the height by the minimum and maximum riser height.	$\frac{1350\text{ mm}}{130\text{ mm}} = 10.3\ldots$ $\frac{1350\text{ mm}}{225\text{ mm}} = 6$
Write the answer.	From 6 to 10 stairs would be allowed.

(c) Check that the number of steps complies with the requirement that the goings (horizontal distance between two steps) must be between 215 mm and 355 mm.

THINKING	WORKING
To determine the range of the allowed numbers of stairs using the goings limitations, divide the horizontal distance by the minimum and maximum goings lengths.	$\frac{1760\text{ mm}}{215\text{ mm}} = 8.1\ldots$ $\frac{1760\text{ mm}}{355\text{ mm}} = 4.9\ldots$ From 5 to 8 stairs would be allowed.
Combine the requirements.	To comply with both riser and goings requirements, the allowed number of stairs is 6, 7 or 8.

Practice

ANSWERS Page 648

SC 1 I can calculate ratios of horizontal and vertical distances to check compliance

1 Compliance gradients can be written as ratios (vertical to horizontal), percentages or angles (made with the horizontal). Copy and complete the table for the compliance gradients, rounding answers correct to 1 decimal place.

	Ratio	Angle	Percentage
Maximum cross-slope on a pathway	1 : 40	1.4°	2.5%
Maximum cross-slope on a bitumen pathway	1 : 33		
Minimum steepness for landing needed after 15 m	1 : 20		
Maximum steepness for ramp > 1900 mm	1 : 14		

2 A staircase is to be built joining two storeys of a house. The height difference between the floors is 2450 mm and the horizontal distance allocated for the staircase is 3420 mm.

(a) Given that the staircase must have an angle between 20° and 45°, and the recommended steepness is between 30° and 38°, determine whether the staircase is compliant and at a recommended angle.

(b) Given that the risers must be between 130 mm and 225 mm, how many steps could there be?

(c) Check that the number of steps complies with the requirement that the goings must be between 215 mm and 355 mm.

3 A single flight of stairs is allowed to have from 2 to 18 risers; any more than that and landings are needed between flights.

The stairs are placed between levels 8 m apart vertically.

(a) Determine the minimum number of flights needed.

(b) Write three possible combinations of numbers of risers per flight for the number of flights calculated in part **(a)**.

4 Concrete driveways (especially flat ones) are given a cross-gradient to assist drainage of rainfall.

(a) For a driveway 5500 mm wide, determine the height difference across the driveway, to the nearest millimetre, to comply with the 0.5% minimum gradient rule for drainage.

(b) For a driveway 5500 mm wide, determine the height difference across the driveway, to the nearest millimetre, to comply with the maximum transverse slope of 5% rule.

(c) Convert each of the percentages in the rules to angles, correct to 1 decimal place, made with the horizontal.

5 For safety reasons, the steps on a staircase should all be the same. The tolerance (maximum variation) allowed between the heights of steps on a staircase is ±5 mm. The same rule exists for the lengths of the treads (goings).

For a staircase of height 2750 mm and horizontal distance 3520 mm, twelve risers of 212 mm and twelve goings of 271 mm have been built so far.

(a) Determine the height of the final riser.

(b) State whether the tolerance is acceptable for the final riser.

(c) Determine the length of the final going.

(d) State whether the tolerance is acceptable for the final going.

(e) Suggest simple adjustments in design to address any unacceptable tolerance breaches.

Recognise image editing

13.5

Learning intention: To be able to recognise image editing

Success criteria:

SC 1 I can recognise and produce distortion of an image.

Lesson warm-up

Looking through the glass

A glass full of water can distort an image by magnifying and stretching.

Look at different objects through a bottle of water.

Describe the distortions: what changed and what stayed the same?



SC 1 I can recognise and produce distortion of an image

Artists use distortion to produce interesting effects. If an image doesn't look 'real', the scale used in one part of the picture might be different to the scale in the rest.

A simple image distortion involves use of a different scale factor in either the horizontal or vertical direction.



Social media filters can use distortion to alter images. This can produce unrealistic images of people's faces and bodies. It can be useful to recognise when an image has been altered, although it is not always possible to tell.

Worked example

Recognising distortion and reproducing the effect

This image of a face has been distorted.

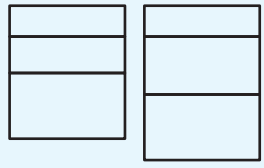


(a) Describe the distortion and explain its effect.

THINKING	WORKING
Describe how the face is different from a real face.	The section from under the eyes to above the mouth has been enlarged in the vertical direction only. The nose and cheeks appear to be very long.

(b) Perform, by sketch or digitally, a similar distortion to the image below.



THINKING	WORKING
Draw horizontal lines below the eyes and above the mouth. Remove this piece. Insert grids that elongate the removed section.	
Stretch the removed section and replace.	

Practice

ANSWERS Page 648

SC 1 I can recognise and produce distortion of an image

1 The image of a 16 cm wide by 10 cm tall painting has been reproduced to each of the following dimensions. Determine whether each image will appear to be stretched vertically, stretched horizontally or not distorted.

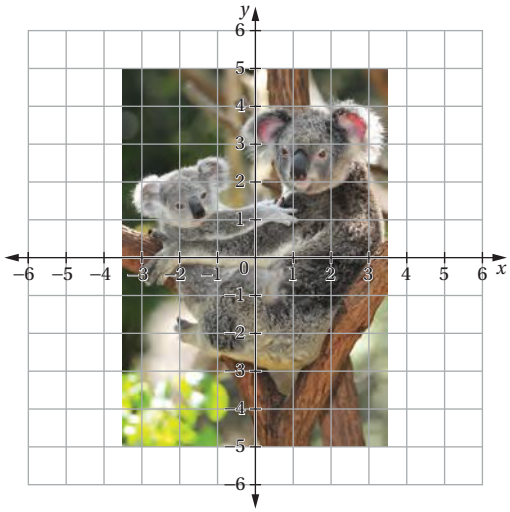
(a) 10 cm wide by 16 cm tall

(b) 20 cm square

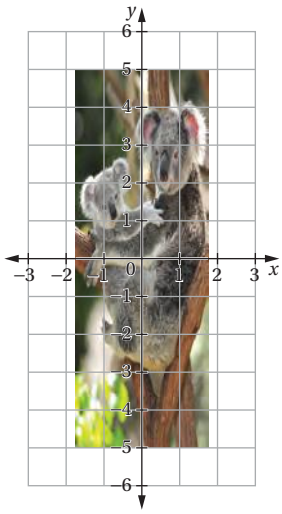
(c) 24 cm wide by 15 cm tall

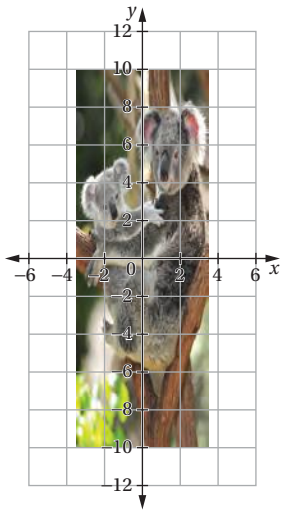
(d) 32 cm wide by 19 cm tall

2 The photo shows a mother and baby koala.

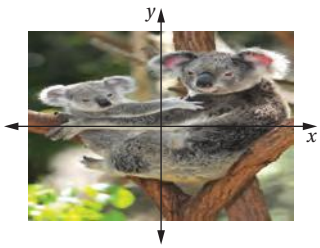


(a) Describe the distortion that produces each of the images shown below.





(b) Describe two possible distortions that would create the image below.



3 For the side-on face image below, or using an image of your choice:



(a) describe a distortion in the horizontal direction that will exaggerate the size of the nose

(b) perform the distortion described in **(a)**

(c) describe a distortion in the vertical direction that will exaggerate the size of the nose

(d) perform the distortion described in **(c)**.

4 Use this photograph, or your own, to show that enlargement in the vertical direction and reduction in the horizontal direction, by the same factor, have the same effect on the proportion of an image.



5 Describe a distortion in the horizontal direction, as a percentage, that would give the same image proportion as each of the following distortions in the vertical direction. Write answers to the nearest whole percent.

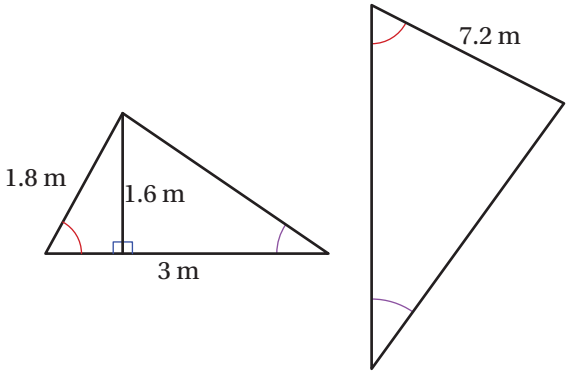
(a) 200% **(b)** 40% **(c)** 150% **(d)** 30%

TOPIC REVIEW

Multiple choice

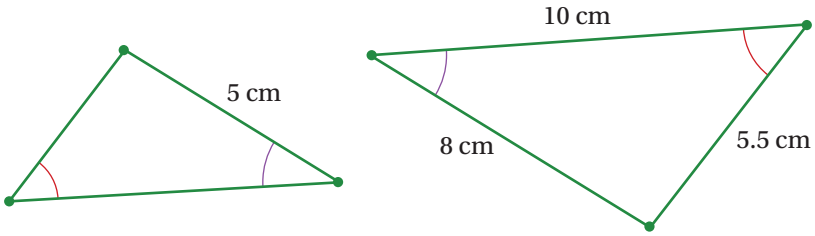
ANSWERS Page 672

1 The area of the larger triangle is:



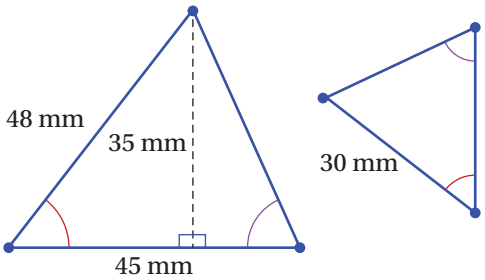
A $48.6\,m^2$ **B** $43.2\,m^2$ **C** $38.4\,m^2$ **D** $9.6\,m^2$

2 For the pair of similar triangles, the enlargement factor for area is:



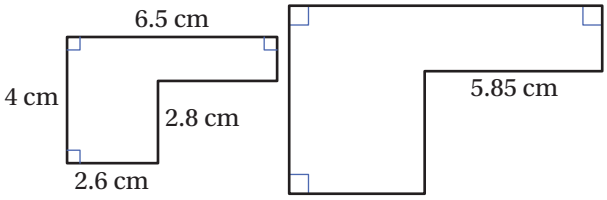
A $\frac{121}{100}$ **B** $\frac{64}{25}$ **C** $\frac{11}{10}$ **D** $\frac{8}{5}$

3 The area of the smaller triangle is closest to:



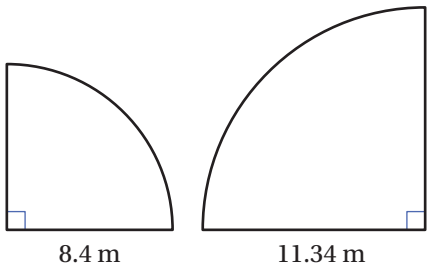
A $307.6\,mm^2$ **B** $421.9\,mm^2$ **C** $492.2\,mm^2$ **D** $2016\,mm^2$

4 For the pair of similar figures, the enlargement factor for area is:



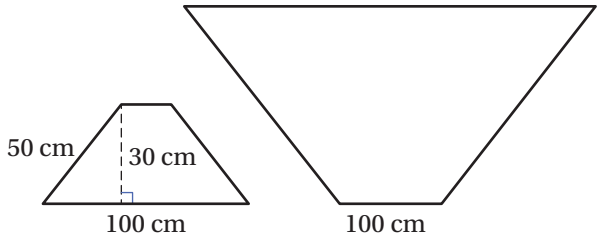
A 1.2 **B** 1.44 **C** 1.5 **D** 2.25

5 The enlargement factor for area is:



A 1.35 **B** 1.35^2 **C** 2.94 **D** 2.94^2

6 Below is a pair of similar isosceles trapeziums. The area of the larger trapezium is:



A $450\,m^2$ **B** $9\,m^2$ **C** $4.5\,m^2$ **D** $0.9\,m^2$

7 In a photo of a woman and a boy, the boy, who is known to be 145 cm tall, is 7.2 cm high and the woman is 8.1 cm high.

The best estimate of the woman's actual height is:

A 129 cm **B** 146 cm **C** 153 cm **D** 163 cm

8 An assumption in making the estimate in Question **7** was:

A the boy was standing in front of the woman.

B both the boy and the woman had their knees bent.

C the woman is taller than the boy.

D the woman and the boy were standing the same distance from the camera.

9 A girl has her photo taken in front of a mountain, in such a way that she appears to be the same height as the mountain.

If the girl is 168 cm tall and stands 4.5 m from the camera and the mountain is 15 km away, the best estimate for the height of the mountain is:

A 5600 m **B** 40 000 m **C** 560 m **D** 4000 m

10 A boy has his photo taken in front of a distant skyscraper.

If the boy is 150 cm tall and stands 8 m from the camera, what is the height of the skyscraper, in metres, if it matches the boy's height in the photo and is 2.4 km away?

A $\frac{h}{2.4} = \frac{150}{8}$ **B** $\frac{h}{2400} = \frac{1.5}{8}$ **C** $\frac{h}{2400} = \frac{150}{8}$ **D** $\frac{h}{2400} = \frac{8}{1.5}$

11 If a particular enlargement of the photo from Question **10** has the boy at 10 cm high and the skyscraper at only 6 cm high, the actual height of the skyscraper, in metres, can be calculated by:

A $h \times \frac{6}{10}$ **B** $h \times \frac{10}{6}$ **C** $h \times \frac{6}{10} \times 100$ **D** $h \times \frac{10}{6} \times 100$

12 Escalators are to be installed to link two floors, 4.2 m apart. If the escalator angle must not exceed 30°, the minimum amount of horizontal distance allocated to the sloping part of the escalator, in metres, can be estimated by the calculation:

A $4.2 \times \sin(30°)$ **B** $\frac{4.2}{\sin(30°)}$ **C** $4.2 \times \tan(30°)$ **D** $\frac{4.2}{\tan(30°)}$

For Questions **13** to **15**, convert the compliance gradient to an angle made with the horizontal.

13 Convert 1:10:

A 36° **B** 10° **C** 5.7° **D** 9°

14 Convert 1:5:

A 1.8° **B** 11.5° **C** 11.3° **D** 40°

15 Convert 4%:

A 21.8° **B** 2.3° **C** 23.6° **D** 3.6°

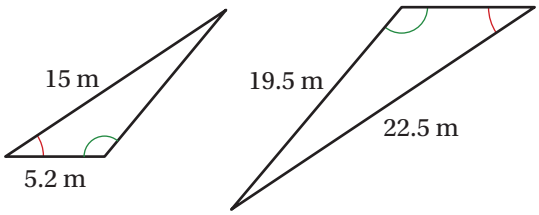
16 A distortion in the vertical direction that would give the closest look to a 210% distortion in the horizontal direction is closest to:

A 48% **B** 91% **C** 90% **D** 110%

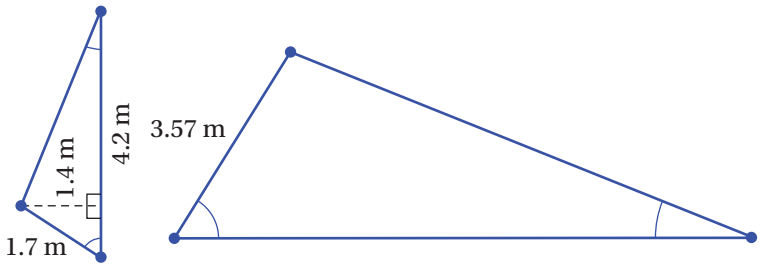
Short answer

1 For the pair of similar triangles, where the original is on the left, calculate:

(a) the scale factor **(b)** the enlargement factor for area.



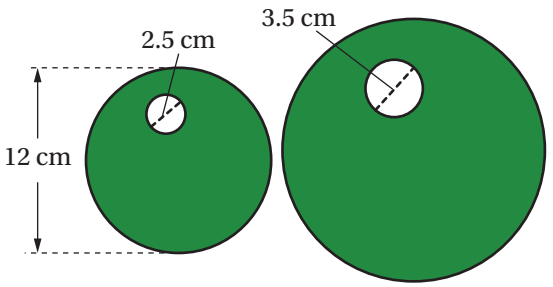
2 For the pair of similar triangles, calculate:



(a) the area of the smaller triangle **(b)** the scale factor

(c) the area of the larger triangle.

3 The figure on the left has been enlarged to give the figure on the right.



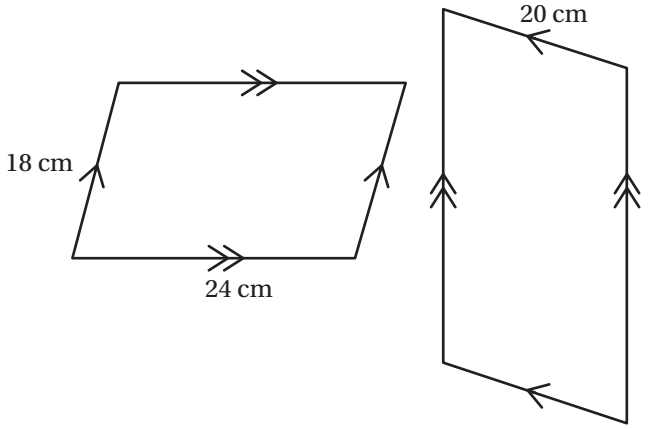
Calculate:

(a) the exact shaded area of the original figure

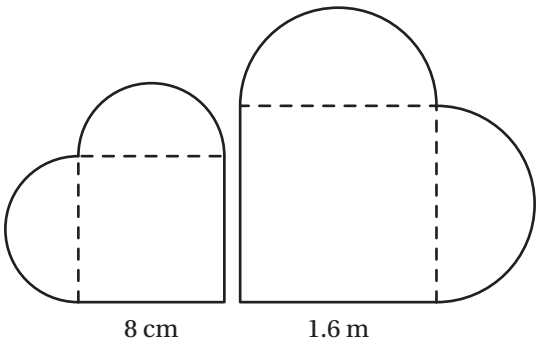
(b) the scale factor

(c) the exact shaded area of the enlarged figure.

4 Calculate the exact enlargement factor for area for the pair of similar parallelograms.



5 For the pair of similar shapes, calculate:



(a) the exact area of the smaller shape in square centimetres

(b) the enlargement factor for area

(c) the exact area of the larger shape, in square centimetres

(d) the exact area of the larger shape, in square metres.

6 Three children are photographed, side-by-side, with an enlarged print giving image heights of the three as 7.5 cm, 8.8 cm and 6.3 cm, respectively. If the tallest child is actually 162 cm tall, estimate, to the nearest centimetre, the heights of the other children.

7 A cricketer is photographed in the background, 40 m from the camera, with a set of wickets in the foreground just 5 m in front of the camera.

(a) If the wickets stand 71 cm high, determine the height the cricketer would have to be to match the height in the photo of the wickets.

(b) If the cricketer appears to be a third of the height of the wickets, estimate the cricketer's height to the nearest centimetre.

8 Concrete driveways require a cross-gradient of between $\frac{1}{2}$% and 5% to assist drainage of rainfall. For a driveway 3500 mm wide, determine the range of height differences acceptable across the driveway.

9 Sloping sections of a concrete path are not to exceed a steepness of 1 in 20, or 5%.

Determine whether each of the following vertical, horizontal pairs represent acceptable steepness for the path.

(a) 1000 mm, 25 m **(b)** 1400 mm, 30 m **(c)** 2200 mm, 40 m

10 The image of a 40 cm wide by 55 cm tall painting has been reproduced to each of the following dimensions. Determine whether each image will appear to be stretched vertically or stretched horizontally.

(a) 16 cm wide by 24 cm tall **(b)** 50 cm square **(c)** 60 cm wide by 80 cm tall

11 The photo of a wombat on the left has been distorted to give the image on the right. Without giving actual proportions:



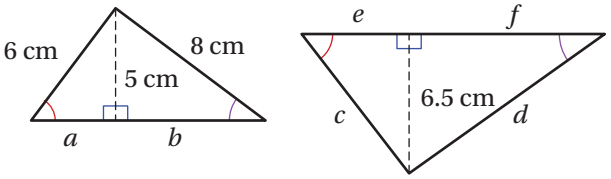


(a) describe the distortion

(b) describe a different distortion that would have the same effect on the look of the image.

Extended response

1 For the pair of similar triangles, calculate:



(a) the exact values of a and b

(b) the exact area of the smaller triangle

(c) the scale factor

(d) the exact values of c, d, e and f

(e) the exact area of the larger triangle.

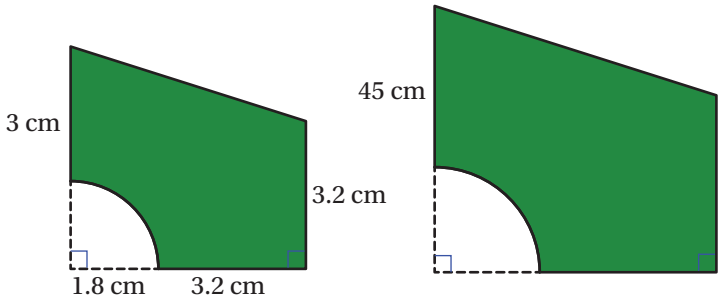
2 For the pair of similar figures, where the original is on the left, calculate:

(a) the exact area of the original shape

(b) the scale factor

(c) the enlargement factor for area

(d) the exact area of the enlarged shape.



3 The length of Car A is known to be 2020 mm.

Various photos of Car A are available with other cars in used car lots.

(a) Use the knowledge that Car A is known to be 2020 mm long to calculate the apparent length, to the nearest millimetre, of each of the other cars sharing a photo with Car A, given the image lengths:

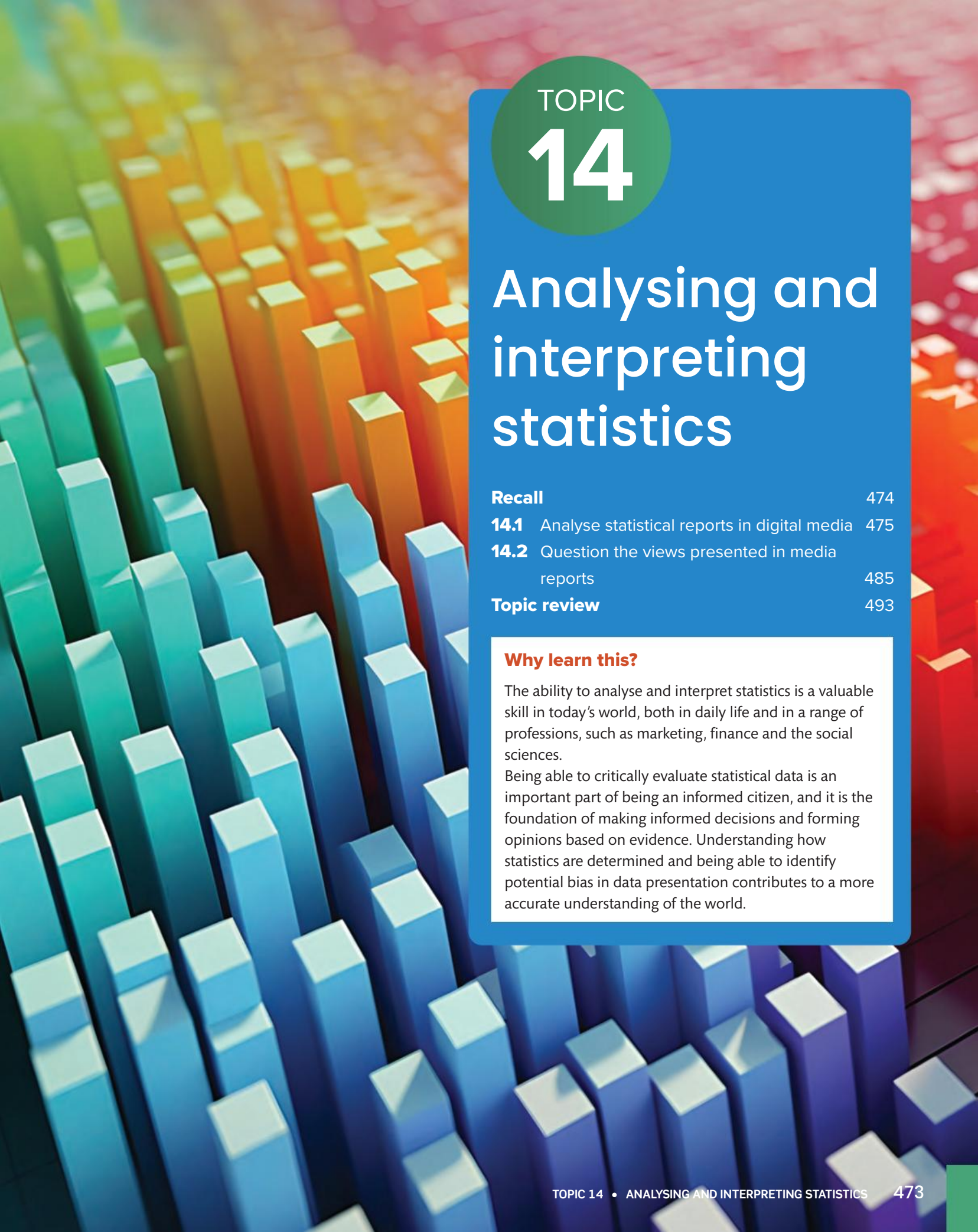
(i) Car A: 3.2 cm; Car B: 8.5 cm

(ii) Car A: 5.6 cm; Car C: 11.9 cm

(iii) Car A: 4.1 cm; Car D: 7.8 cm

(b) What assumptions were made in using the measurements for the calculations in **(a)**?

(c) The actual lengths of the other vehicles are Car B: 4840 mm; Car C: 4284 mm; Car D: 4625 mm. Give a valid explanation for any discrepancies with your calculations in **(a)**.



TOPIC
14

Analysing and interpreting statistics

Recall 474
14.1 Analyse statistical reports in digital media 475
14.2 Question the views presented in media reports 485
Topic review 493

Why learn this?

The ability to analyse and interpret statistics is a valuable skill in today's world, both in daily life and in a range of professions, such as marketing, finance and the social sciences.

Being able to critically evaluate statistical data is an important part of being an informed citizen, and it is the foundation of making informed decisions and forming opinions based on evidence. Understanding how statistics are determined and being able to identify potential bias in data presentation contributes to a more accurate understanding of the world.

RECALL

ANSWERS Page 649

I can identify the source of collected data as primary or secondary

1 Describe the difference between primary and secondary sourced data.

2 Identify whether the following descriptions represent primary or secondary sourced data.

(a) Recording the number of likes each short video you post receives

(b) Listening to a podcast to learn which musical artist is currently on top in sales each week

(c) Recording the most viewed movies from a streaming site's 'most viewed' display

(d) Counting the number of students wearing hats when entering the school grounds each morning

I can identify the collected data as randomly or non-randomly sourced

1 Describe the difference between random and non-random data.

2 Identify whether the following descriptions represent randomly or non-randomly sourced data

(a) People who listened to a Jazz music podcast were surveyed on what was the most enjoyable genre of music.

(b) Various people walking through a mall in the city were asked to complete a survey.

(c) Every third person wearing the same team jersey at a sports event was given a survey to complete.

(d) The winners of a raffle prize were drawn one at a time from a box full of tickets.

Exploration Task

Lies and statistics

In the late 1800s, then British Prime Minister Benjamin Disraeli famously said that there are two types of lies – lies and statistics.

Clearly, he recognised the dangers of misusing statistics, over 140 years ago.

Five of the following statistics were reported in the media in 2022, and the other two are made up.

- 8% of people pronounce Wi-Fi as 'wiffy'.
- Over 50% of red-headed people are also left-handed.
- 80% of Americans are surprised to find that Australians speak English.
- 99% of all species who have ever lived are now extinct.
- 15% of Google searches have never been searched.
- Left-handed people earn 10% less than right-handed people.
- 20% of songs on Spotify have never been played.

Which two do you think are made up?

Analyse statistical reports in digital media

14.1

Learning intention: To be able to analyse statistical reports in digital media

Success criteria:

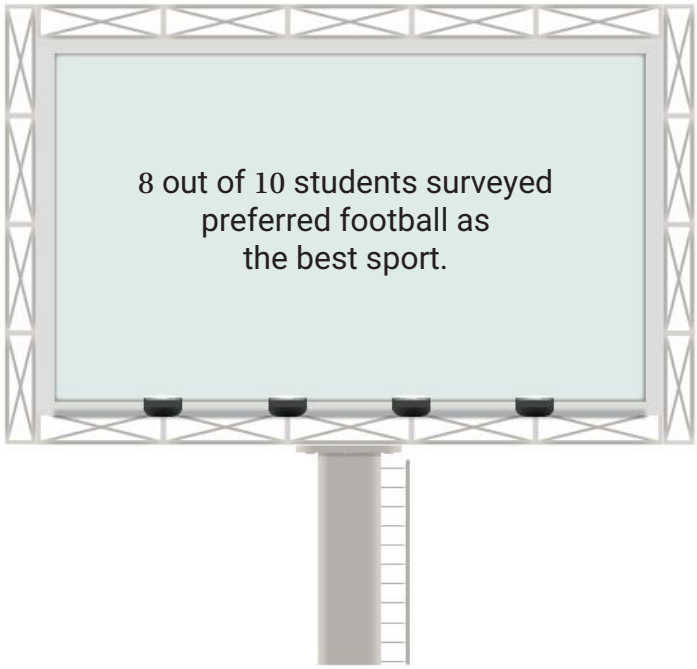
SC 1 I can identify the processes by which statistics have been estimated.

SC 2 I can analyse reports published in the media.

Lesson warm-up

Billboard claims

Read the claim advertised on the billboard.



Discuss whether you think this is misleading or informative.

SC 1 I can identify the processes by which statistics have been estimated

When a sample is used to make inferences about the underlying population, it is important that there is an understanding of the way the sample was selected. If the sample does not seem to be representative of the underlying population, then the conclusions reached, or inferences made, about the whole population need to be treated with caution.

Worked example

Identifying the sample used

Teachers don't have enough time to prepare well for class. We have a solution

Published: January 30, 2022 9.04pm AEDT

Almost all teachers (92%) in our study out today said they don't have enough time to prepare effectively for classroom teaching – the core of their job.

The Grattan Institute surveyed 5,442 Australian teachers and school leaders across all states and territories, primary and secondary schools, and government and non-government schools. The survey was about teachers' use of time.

Teachers told us they are too stretched to do everything we ask of them. When teachers aren't supported to do their jobs well, teaching quality suffers and students lose out.

Beyond preparation for effective teaching, 86% of teachers reported they didn't have time for high-quality lesson planning.

According to the Australian Bureau of Statistics, in 2022, there were 151 021 teachers in secondary schools across Australia.

(a) What percentage of all secondary teachers are reported to have participated in this survey? Give your answer correct to 1 decimal place.

THINKING	WORKING
Calculate the percentage. Round the answer as directed.	$\frac{5442}{151\,021} \times 100\% = 3.6035\ldots\%$
Write the answer.	About 3.6% of the population is included in the survey.

(b) How do you think the data was gathered for this report? Do you think the sample is representative of the population of teachers? Give a brief reason for your response.

THINKING	WORKING
How do you think the data was gathered for this report?	The data was gathered in the form of a survey.
Do you think the sample is representative of the population?	According to the report, the survey included 'teachers and school leaders across all states and territories, primary and secondary schools and government and non-government schools'. It is likely that it is representative of the population.

SC 2 I can analyse reports published in the media

When reading reports in the media, it is important to look beyond the headlines and the opening sentences that can be statistically biased. In some cases, the claims made in these opening sentences are not fully backed by the statistics provided, and some qualifications are included further into the article. In other cases, some crucial terms may not be clearly defined, encouraging the reader to apply their own meaning to such terms, possibly making the claims misleading.

Worked example

Going beyond a headline

The following are the first two lines of a press release from the Office of the Premier of South Australia.

SA SETS RECORD EXPORT HIGH AS GROWTH OUTSTRIPS NEIGHBOURS

South Australian exports have topped $17 billion for the first time in the State's history, with local export growth outpacing neighbouring states and territories.

The record figure represents a 27% increase in the 12 months to February 2023, smashing the previous record of $16.5 billion set in the year ending January 2023. Export value from South Australia is worth $3.6 billion more than at the same time last year.

Source: The Government of South Australia website is licensed under a CC BY 4.0 licence.
© Copyright 2022. Link to licence: https://creativecommons.org/licenses/by/4.0/

(a) What was the increase in the value of trade between January 2023 and February 2023?

THINKING	WORKING
Find the difference between the figures for January 2023 and February 2023.	Using $17 billion as the February 2023 figure gives a difference of $0.5 billion.

(b) What is this expressed as a percentage? Round your answer correct to 1 decimal place.

THINKING	WORKING
Convert to a percentage.	$\frac{0.5}{16.5} \times 100\% = 3.0303...\%$, say 3.0%.

(c) What was the value of trade for the year ending February 2022?

THINKING	WORKING
Use $17 billion as the 2023 figure and use the stated increase to find the 2022 figure.	$(17 − 3.6) = $13.4 billion

(d) What is the annual percentage change in the yearly figure to February 2023? Round your answer correct to 1 decimal place.

THINKING	WORKING
Convert to a percentage in the usual manner.	$\frac{3.6}{13.4} \times 100\% = 26.865...\%$, say 26.9%.

(e) Do you think the release has made clear what the change actually is?

THINKING	WORKING
Explain your answer based on your opinion.	The 27% is an accurate representation of the change from the year ending February 2022 to the year ending February 2023. However, putting this figure so close to the $16.5 billion figure has blurred the relationship between the numbers and which change is being described.

Practice

ANSWERS Page 649

SC 1 I can identify the processes by which statistics have been estimated

1 For many years there has been a suggestion that every adult should complete 10 000 steps per day. Consider the following extract from a newspaper article.

TIME TO STEP OFF THE 10 000 GOAL

If you're struggling to reach 10 000 steps in a day, here's some good news: the latest science suggests fewer daily steps may be the sweet spot for many of us, depending on our age, fitness and health goals.

There is nothing magical or evidence-based about 10 000 steps a day. The notion stems from a marketing ploy: as the 1964 Tokyo Summer Olympic approached, a Japanese researcher decided to nudge his nation to be more active by offering pedometers with a name that loosely translated as '10 000-step meter'. (The Japanese character for the number 10 000 looks a little like a person walking.)

In the past few years, multiple large-scale studies have looked into how many steps we probably need for our health and longevity. In the largest, published last year in Lancet Public Health, dozens of global researchers pooled data from 15 earlier step-count studies, some unpublished, covering 47 471 adults of all ages, and compared their daily step count to their longevity.

The sweet spot for step counts was not 10 000 or more. In general, the pooled data showed that for men and women younger than age 60, the greatest relative reductions in the risk of dying prematurely came with step counts of between about 8000 and 10 000 per day.

Source: Nine: From The Age digital edition 'Time to step off the goal' by Gretchen Reynolds, 15 June 2023.

(a) How would you describe the way the original value of 10 000 steps was obtained?

(b) What would be the benefit of pooling the data from many different studies?

(c) What is the maximum percentage reduction in recommended steps indicated by the pooled research for adults younger than 60?

(d) The report goes on to state that, 'For people older than age 60, the sweet spot in terms of reduced mortality risk came at between 6000 and 8000 steps a day'. What is the range of percentage reduction in the number of steps for these older adults?

2 The following table and article appeared in The Age newspaper and on their website.

HOMELESSNESS MORE THAN DOUBLES IN SOME PARTS OF VICTORIA

A surging number of homeless people identified in Victoria has prompted a call for urgent investment in support and public housing, with the problem growing fastest in some regional areas.

The Australian Bureau of Statistics recorded 30 660 homeless people in Victoria on the night of the August 2021 census, which was a 24 per cent increase since 2016 and a jump almost five times the national average.

Council to Homeless Persons chief executive Deborah Di Natale said the data had uncovered previously hidden cases because the census took place during a COVID-19 lockdown, when homeless people were offered emergency accommodation in hotels, making them easier to identify.

The census data is almost two years old, but Di Natale said the problem had not improved, based on demand for her organisation's frontline services and the growing public housing waiting list, which blew out to a record 58 131 applications in March.

HOMELESSNESS SURGE			
Areas with fastest growing homelessness	2016	2021	Increase %
South Barwon (Geelong)	148	838	465%
Melton	170	398	134%
Eureka (Ballarat)	166	354	113%
Pakenham	122	260	113%
Cranbourne	240	506	111%
Bendigo East	159	330	107%
Oakleigh	633	1269	101%
Mildura	264	518	96%
Box Hill	333	625	88%
Morwell (Gippsland)	205	380	85%

Source: Council to Homeless Persons, ABS

(a) Where did the figure of 30 660 homeless people in Victoria come from?

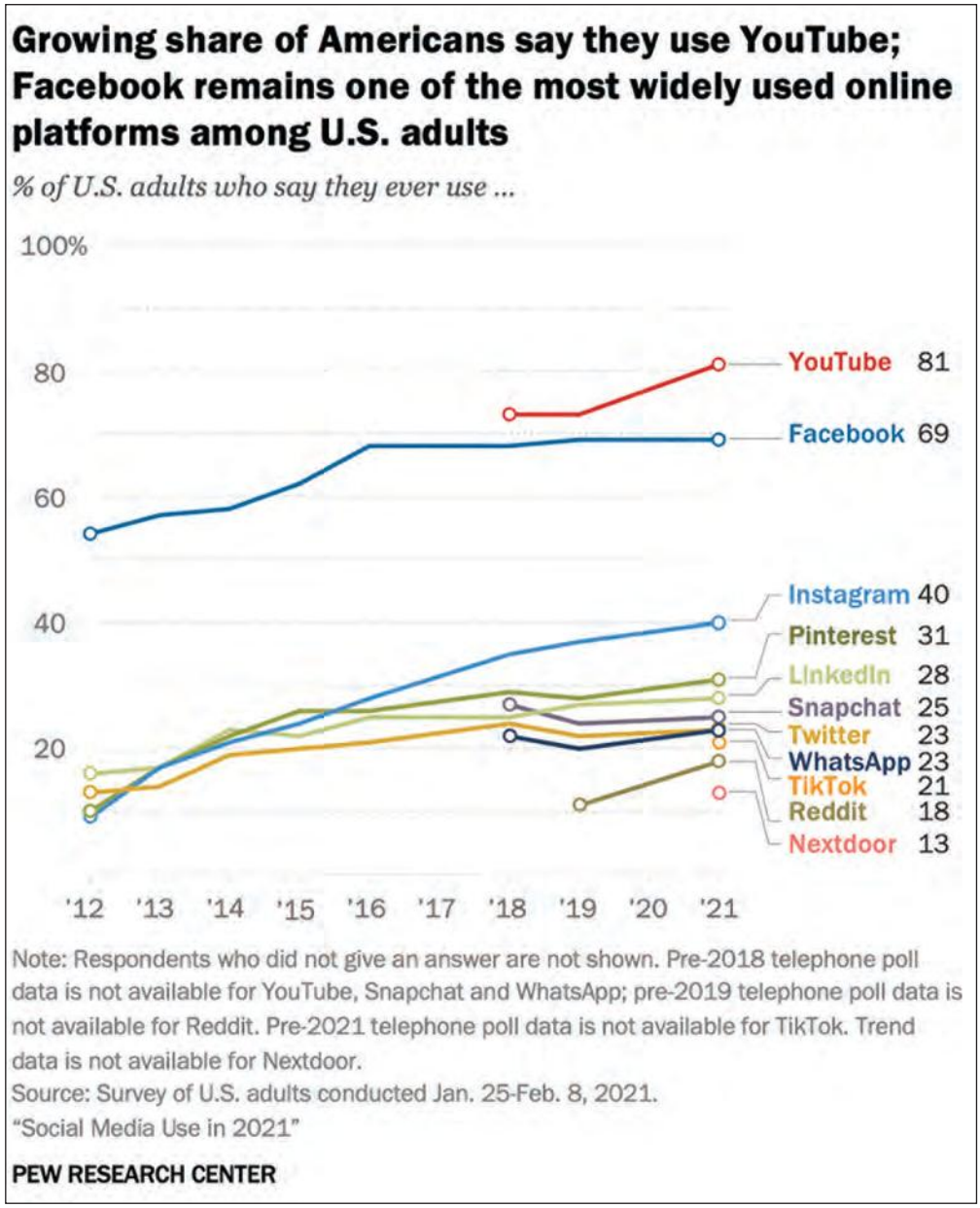
(b) Is this likely to be an accurate figure or an estimate?

(c) Why are the years 2016 and 2021 the years listed in the table?

(d) What part did the COVID-19 lockdowns play in the increase in numbers?

(e) What does the article suggest has happened since the last census?

3 The US-based Pew Research Center published a report with the headline 'Social Media Use in 2021'. The following graph was included as part of this report.



(a) What type of survey was this?

(b) What crucial information is not included with the graph?

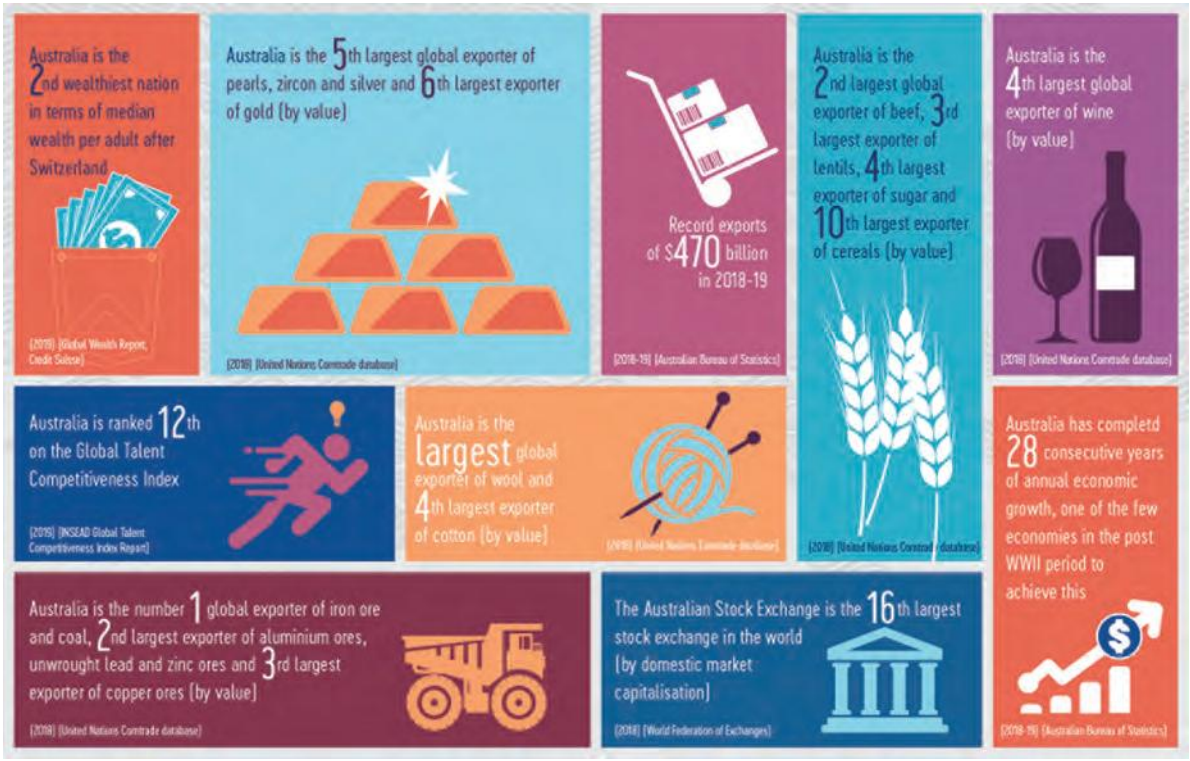
(c) Do you think this is a biased sample? Give a reason for your answer.

The report goes on to say that the nationally representative survey was of 1502 US adults conducted via telephone between 25 January and 8 February 2021.

(d) What do you think about this sample number?

SC 2 I can analyse reports published in the media

1 The following graphic comes from a webpage produced by the Australian Department of Foreign Affairs and Trade (DFAT).



(a) Look at the purple section about wine exports. What does 'by value' mean? What other measurement could be referred to regarding wine exports?

(b) Which country has the highest median wealth per adult?

(c) When the chart was compiled, how many consecutive years of annual growth had been recorded?

(d) What do you think the purpose of such a chart would be?

2 The following item was released by headspace National Youth Mental Health Foundation.

YOUNG PEOPLE WANT TO DISCONNECT FROM SOCIAL MEDIA – BUT FOMO WON'T LET THEM

21/06/2023

Key points:

- One in three young people surveyed experience problematic social media use (33%)
- Half of young people (51%) want to disconnect from social media but experience a fear of missing out
- headspace is offering a range of support for young people looking to improve their relationship with social media

New research released today from headspace National Youth Mental Health Foundation shows half of young people surveyed wanted to disconnect from social media but a fear of missing out (FOMO) may be stopping them, with close to a third feeling pressured to keep up with everything from politics to gossip on social media.

The headspace National Youth Mental Health Survey surveyed 3107 young people and found 51 per cent of young people have thought about logging off social media, but FOMO keeps them online. This is despite almost half (44 per cent) of respondents agreeing the content they see is more negative than positive.

In fact, a third (35 per cent) of young people also felt an urge to use social media more and more, despite them knowing its negative impacts.

Participants told headspace they felt pressure to use social media to keep up to date with news and current events (32 per cent).

Ultimately, one in three young people who took part in the survey were found to experience problematic social media use (33 per cent).

headspace CEO Jason Trethowan said: "We know young people can have really positive experiences using these platforms.

"Social media can be used to connect with others, as a creative outlet, to learn new things and to pursue interests. Overall, 41 per cent of young people we surveyed said the information they can find on social media is empowering, and two in five reported social media is also a good place to meet new friends.

Social media has also given young people the opportunity to find their voice; almost half of young people agreed it is easier to express their opinions online than in person (44 per cent).

"However, the amount of content and the kind of content on social media can be very overwhelming.

"The more time young people spend on social media, the less time they have for other important parts of life, like being active, getting into life, sleeping and eating well. It's also the reasons they're choosing to use social media that need to be addressed.

"Young people thinking of logging off might experience a fear of missing out on news, popular culture or conversations with friends. They may also worry about how going offline could impact their status or influence. It's similar to the feeling young people might experience if they missed out on a party or social event.

"Social media can also lead us to make unhelpful comparisons between our life and the lives of others, whether they be friends, celebrities or social media influencers.

"The research shows us young people are aware of how social media can impact their mental health now and into the future, with 55 per cent of survey participants agreeing the content they post today will impact their job prospects and relationships going forward. They are telling us that they are aware of the risks and they want to switch off.

"But that's easier said than done when most platforms are designed to keep us scrolling.

"That's why the responsibility to foster healthy social media habits can't rest solely on the shoulders of young people or their families. It is important that social media companies and governments put in place the mechanisms needed to ensure users can have a safe and healthy experience."

The National Youth Mental Health Survey also found a majority of young people who were surveyed believed not enough was being done in terms of regulation and laws surrounding social media (55 per cent).

Source: © headspace National Youth Mental Health Foundation Ltd. "Young people want to disconnect from social media – but FOMO won't let them" published on June 26, 2023.

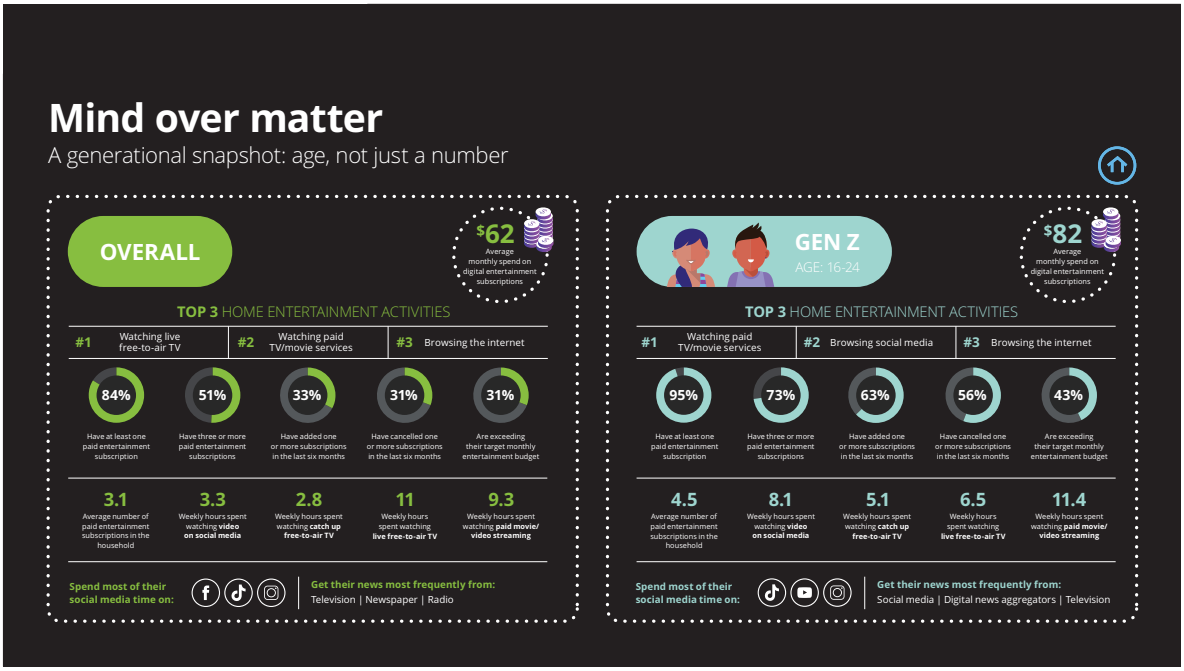
(a) How many people were involved in the survey?

(b) What fraction of the respondents felt pressured to keep up with everything from politics to gossip on social media?

(c) What percentage of the respondents were aware that what they post today could impact job prospects and their relationships going forward?

(d) Do you think the thrust of the article is positive towards the use of social media, negative, or balanced?

3 In 2022, the consulting firm Deloitte conducted its eleventh Media Consumer Survey. The infographic below summarises some of the findings from that survey.



(a) What is the stated age range for Gen Z?

(b) What is the difference in average monthly spend on digital entertainment subscriptions for Gen Z compared to all respondents?

(c) On average, how many paid subscriptions do Gen Z individuals have?

(d) In the Overall category, which social media site is most popular?

(e) In the Gen Z category, which social media site is most popular?

(f) Do you think Gen Z consumers pay for their own social media subscriptions?

4 The following article was published in the media.

LIFE EXPECTANCY INCREASES, BUT MORE MEN NEED TO WATCH WHAT THEY EAT

Men are living longer, exercising more but eating fewer vegetables and are more obese than five years ago, new national data reveals.

Released today by the Australian Institute of Health and Wellness, the comparative report between men and women reveals an increase in life expectancy, with women now expected to reach an average age of 85.4 compared with 84.6 in 2017, while men can expect to live to 81.3 years, up from 80.5 years in 2017.

When it comes to diet, there has been no improvement for men, with only 4 per cent meeting the vegetable guideline of at least five serves a day, while only 13 per cent of women are hitting their leafy greens target, up from 6.67 per cent in 2017.

Males and females in lower socioeconomic areas experienced twice the rate of potentially avoidable deaths in 2020 compared with higher-socioeconomic areas, the data also reveals.

In 2021–22, 79 per cent of men visited a general practitioner, compared with 88 per cent of women.

The report also reveals that 2 559 400 or 25 per cent, of the female population in 2021–2022 said they experienced mental health or behavioural conditions. Back problems were the most common chronic ailments for men in the same reporting period, with 1 884 500, or 19 per cent of the male population, indicating they were sufferers of the condition.

Source: Nine: Extract from The Age 'Life expectancy increases, but more men need to watch what they eat', by Laura Banks, page 13, 27 June 2023.

(a) The life expectancy for both males and females has increased since 2017. What is the percentage increase for both women and men? State your answer correct to 2 decimal places.

(b) What percentage of men meet the vegetable guideline of at least five serves per day?

(c) For males and females in lower socioeconomic areas, what was the rate of potentially avoidable deaths in 2020 compared to those in higher socioeconomic areas?

(d) The report states that 1 884 500 males indicated they suffered from back problems. Does this indicate that the report was based on a census or a sample? Give a brief reason for your answer.

Question the views presented in media reports

14.2

Learning intention: To be able to question the views presented in media reports

Success criteria:

SC 1 I can identify how particular visual displays can affect the perception of results.

SC 2 I can identify potential bias in media statistical reports.

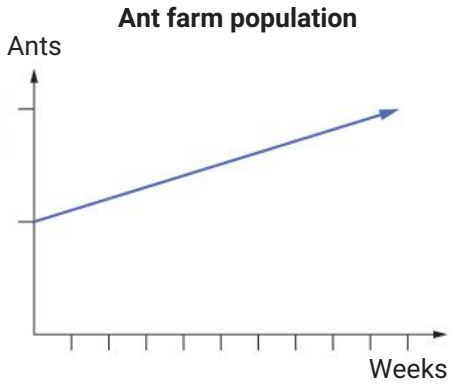
Lesson warm-up

Dubious headlines



An ant farm was started in a science classroom.





The number of ants in the colony was counted each week over a 10-week period, and the results were graphed as shown.

The school newspaper decided to present a report on the growing colony with the excited heading: 'Science department ant colony doubles in size each week since starting'.

Write a comment on whether you agree with the headline or not.

Give reasons for your response.

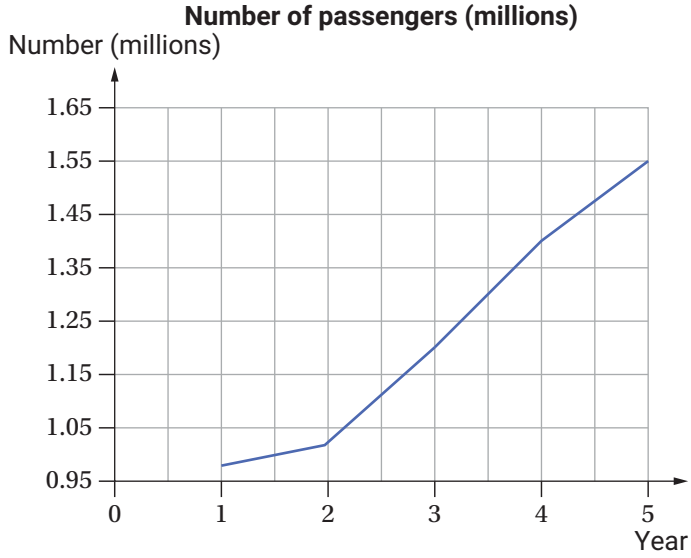
SC 1 I can identify how particular visual displays can affect the perception of results

The old saying 'a picture tells a thousand words' means that a good visual presentation can take the place of lots of words. This is also true when presenting statistical information, but care needs to be taken as some visual representations can be misleading.

Worked example

Identifying a misleading graph

The following graph appeared on a classroom test.



The graph represents the number of passengers carried on a particular airline over a five-year period.

The graph appeared on a test with a question asking the students to estimate what the number of passengers in year 1 needed to be multiplied by to get the number of passengers in year 5. Estelle estimated the answer as about 5. She was very surprised when she was marked incorrect and found the correct answer was only about 1.6.

(a) How do you think Estelle got her answer?

THINKING	WORKING
Look at the graph and count how many grid lines are between the two values.	There are 6 grid lines between the two values, but the first point is almost at the first line. So Estelle estimated the answer to be about 5 (for 5 intervals).

(b) What information do you need to work out the actual answer?

THINKING	WORKING
What does the vertical axis represent?	The vertical axis represents the number of passengers, so the actual vertical values are needed.

(c) Estimate these values and do the calculation to find the answer.

THINKING	WORKING
Estimate the values by looking at the scale markings on the vertical axis.	Year 1: 0.98 Year 5: 1.55
Calculate the difference between the values.	$1.55 - 0.98 = 0.57$
Determine the percentage change.	$\frac{0.57}{0.98} \times 100\% = 0.5816\ldots \times 100\%$ $\approx 60\%$
Write the answer.	The year 5 number is about 60% bigger than the year 1 figure. So, the multiplying factor is about 1.6.

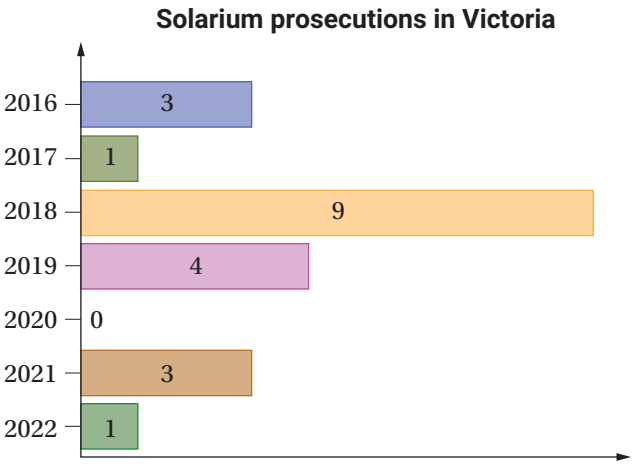
SC 2 I can identify potential bias in media statistical reports

Newspapers, and other media outlets, including social media sites, aim to attract customers. However, there should be a social responsibility to provide accurate information in an unbiased manner. Misleading graphs are sometimes used to justify the headlines used, or the conclusions drawn in articles. A statistically literate individual must always be on the lookout for articles and items that are not as ethically and accurately written as they should be.

Worked example

Checking for the proper use of data

The following graph appeared in the print media. As background, in Australia, commercial solariums have been banned since 2015, but private use is still allowed.



(a) Is this graph accurate, assuming the figures used are accurate?

THINKING	WORKING
As far as scale is concerned, does the graph appear to be accurate?	Yes, the graph appears to be accurate.

(b) What is it about the graph that makes the problem seem worse than it may be?

THINKING	WORKING
When you look at the graph do you think this is a widespread problem? Why is this the case?	The numbers are so small that using a 1 : 1 scale makes the row for 2018, especially, seem quite big.

Practice

ANSWERS Page 650

SC 1 I can identify how particular visual displays can affect the perception of results

1 Two circles are to be used to compare the values shown in the following questions. Given that the smaller amount is represented by a circle of radius 1 cm, what would the radius of the other circle be? State your answers in exact form.

(a) \$400, \$800 **(b)** \$350, \$1050 **(c)** \$125, \$1250 **(d)** \$220, \$1000

2 The table below shows the mean monthly temperatures for 2022 at weather stations located at Maitland Airport, NSW and Hobart Airport, Tasmania.

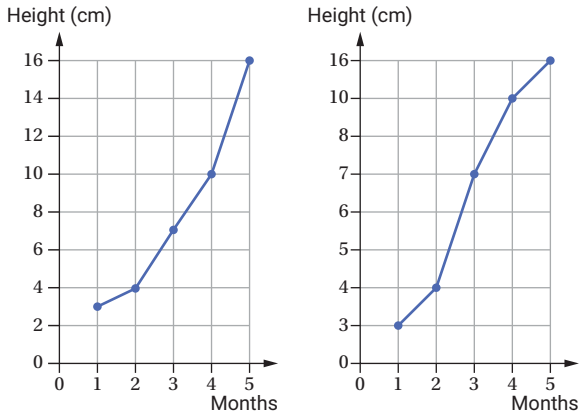
Location/Month	Maitland Airport	Hobart Airport
January	29.7	22.4
February	28.2	22.8
March	26.1	21.8
April	24.6	19.1
May	20.9	15.4
June	17.5	12.4
July	17.2	12.8
August	19.8	14.1
September	21.2	14.8
October	24.0	16.7
November	25.8	18.3
December	27.6	19.9

(a) Plot both data sets on a set of axes where the vertical axis represents temperature and starts at 0°C.

(b) Plot both data sets on a set of axes where the vertical axis represents temperature and starts at 12°C. Make this graph have the same vertical height as the first graph.

(c) Make a comment about the two graphs.

3 Consider the following two graphs that show the same information relating to the height of a small shrub some months after it has been planted.



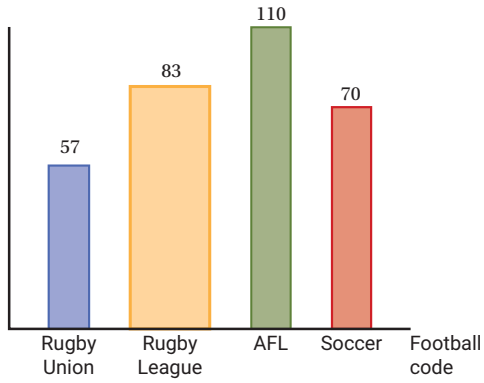
(a) Looking at the left-hand graph, between which two months does the growth rate of the shrub appear to be the greatest?

(b) Looking at the right-hand graph, between which two months does the growth rate appear to be the greatest?

(c) What about the two graphs makes these two answers different?

(d) Which graph is the better representation of the data?

4 Look at the following graph which relates to a survey of 320 adults. The question asked was 'which is your favourite football code?'.

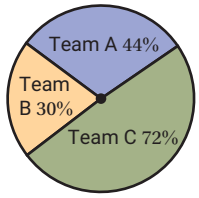


(a) List as many things as you can that you believe are not correct about this graph.

(b) Redraw the graph as a column graph, but make it an accurate representation of the data.

(c) Should this second graph be used to claim that 18% of Australians prefer Rugby Union to all other football codes? Explain your answer.

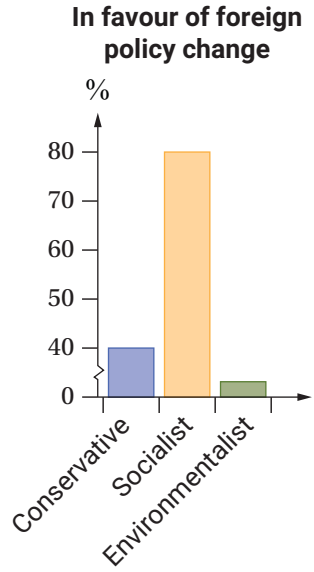
5 The following chart shows information regarding the percentage of players for three different teams in a national sporting competition who were born in the same state as the team they play for.



(a) What is wrong with this graph?

(b) Redraw the graph as a column graph.

6 The following graph shows information about a particular foreign policy question related to the party voted for at the last election.



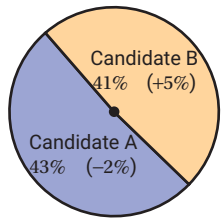
(a) Just looking at the graph, how many times would the blue column fit inside the yellow column?

(b) Does this match the actual figures? What is the connection between the size of the blue column percentage and the yellow column percentage?

(c) Redraw the graph so that it is an accurate representation of the data.

SC 2 I can identify potential bias in media statistical reports

1 Newspapers often run mini surveys where a simple question is asked, and readers are encouraged to respond online. A chart like the one following was published around the time of the last general election. The question was 'Who would make the better Prime Minister?'



(a) What is the problem with this chart?

(b) How is this an example of unethical, or biased reporting?

(c) What do you think would be represented by the missing percentage in this case?

(d) Redraw the chart properly, choosing an appropriate label for the third section.

(e) The numbers in brackets refer to the changes since the last poll. What inference would you take from these bracketed numbers?

2 The following appeared in the printed media.

SOLAR NOT FOR EVERYONE

While experts are encouraging households to beat soaring electricity costs by using solar power, new research reveals that nearly a fifth of Aussies can't get solar at their property.

New findings from Compare the Market show that 18 per cent of respondents can't install solar where they live, while a further 15 per cent want solar but say they can't afford installation costs.

Compare the Market's head of energy Meredith O'Brien said solar wasn't the only way of lowering electricity bills. But she said consumers doing it tough should resist the urge to turn the heating up too high.

"Every degree warmer to set the temperature can add 10 per cent to your electricity consumption, " Ms O'Brien explained.

Source: News Corp Australia: Extract from Herald Sun print 'Solar not for everyone', p. 33, 1 July 2023.

(a) What indication is given about the size of the sample used to reach these conclusions?

(b) What reasons are given to explain why some people cannot install solar power where they live?

(c) The article suggests there are other ways to lower electricity bills. The only method suggested is to not turn the heating up too high. Can you suggest any other ways that consumers might consider?

3 The following item appeared in the print media.

FORGET LUNCH, I'M BUSY

Three out of four office workers skip lunch breaks, according to a new snapshot of lunchtime habits at work.

The research conducted by YouGov on behalf of Youfoodz reveals only 23 per cent of workers say they take their full lunch breaks.

Forty-two per cent of office workers say they don't take lunch breaks as they have too much work to complete.

Twenty-nine percent skip lunch breaks as they have not had enough time to shop for food, and 23 per cent say the lunchtime offerings around their workplace are too expensive.

Work-from-home or remote workers are also not immune from this concerning trend, with 31 per cent of those who work from home saying they are also inclined to skip their lunch breaks.

Source: News Corp Australia: Extract from Herald Sun print 'Forget lunch, I'm busy', p. 9, 29 June 2023.

(a) Youfoodz is a company that packages meals. Do you think this could lead to a risk of bias in the way the survey results are conveyed?

(b) What do you think 'snapshot' implies in the brief article?

(c) The word 'trend' is used. Is any evidence presented to support the idea that these results differ from, or confirm, previous results?

(d) What evidence supports the first claim in the article that 'Three out of four office workers skip lunch breaks'?

4 The following article and graph appeared in the print media.

MILLENNIAL MEN SEEK GREENER PASTURES

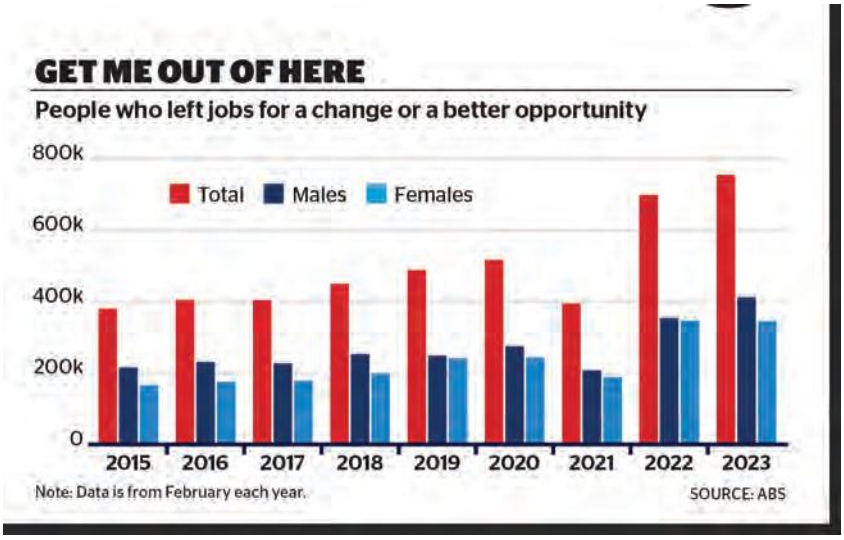
Men almost two decades into their careers staggered to the end of COVID-19 lockdowns, face masks and Zoom meetings and decided they needed a change.

Data from the Australian Bureau of Statistics shows the so-called 'great resignation' evident in some overseas job markets during the pandemic did not occur locally. But many people with years under their career belts have taken advantage of the strong jobs market post pandemic to find a better paying or more fulfilling occupation.

According to the bureau, about 1.3 million people changed jobs in the 12 months to the end of February. That equates to 9.5 per cent of the workforce – the highest job mobility rate since February 2012 but well short of the record 19.5 per cent set in 1989.

In the same period, 2.3 million people left or lost a job, a 200 000 increase from the previous year. Among that group, almost a third, or 751 600, left their job to get a better one or just wanted a change. This was 55 per cent more than the last non-COVID-affected period in the 12 months to the end of February 2019.

Source: Nine: Extract from The Age 'Millennial men look for greener, better-paid pastures' by Shane Wright, 30 June 2023.



(a) What evidence is presented in the article to support the claim made in the first sentence?

(b) In the graph shown, is there ever a year in which the number of men changing jobs for the stated reason was less than the number of women doing the same?

(c) In which year was the number of men and women changing jobs for the stated reason the closest?

(d) What reason is suggested for the high number of career moves?

(e) Do you think the headline is supported by the statistics presented in the article or is it just designed to catch the eye?

TOPIC REVIEW

Multiple choice

ANSWERS Page 674

1 If a random sample of 40 students was selected from Year 9 at Bondi High, NSW, to give their opinion about surfing lessons as part of the curriculum, what is the underlying population whose views are reflected in the survey?

A Year 9s at Bondi High
B Year 9s in NSW
C Year 9s in Australia
D Students at Bondi High

2 Which of the following data could come from a census of the Australian people?

A Diagnosis of diabetes
B Country of birth
C Opinion about housing needs
D Participation in sports

3 If the heading to a story in the media sparks your interest, you should:

A immediately believe what the headline suggests.
B send the article to everyone you know.
C assume the editor is simply trying to get your attention.
D make sure that the heading is truly reflected in the article.

4 A visual display that would affect your perception is:

A using area rather than height for comparison.
B using the horizontal axis to display quantities.
C an uneven scale on the axis showing quantities.
D using different colours for each attribute.

5 Areas of rectangles are used as a visual display to compare quantities. If the width is increased by a factor of 1.2 and the height by a factor of 1.3, then the area is increased by a factor of:

A 1.5 **B** 1.56 **C** 1.6 **D** 2.5

6 Which circumstance would *not* cause you to be wary of bias:

A No evidence of a trend is apparent in the data.
B The sample size of the survey is not known.
C No reason is given for the claims of the reporter.
D The survey was conducted by the company being reported on.

7 If a statistical report arrives at a particular conclusion, you would not be able to:

A trust the results without further reflection.
B find other surveys reaching the same conclusion.
C reach a similar conclusion on reviewing the data presented.
D observe the validity of the results from your own experience.

Short answer

1 The Australian Bureau of Statistics collects data by survey. Data collected referencing 2022 was summarised for a report at the end of 2023. The population of Australia was estimated to be 26 million in 2022.

Health conditions prevalence

Key findings on long-term health conditions and chronic conditions, and prevalence in Australia

Reference period 2022

Released 15/12/2023 **Next release** Unknown **Previous releases**

Key statistics

- Eight in ten (81.4%) people had at least one long-term health condition
- One in two (49.9%) people had at least one chronic condition
- Mental and behavioural conditions (26.1%), back problems (15.7%), and arthritis (14.5%) were the most common chronic conditions

A long-term health condition is one that the person had suffered from, or expected to suffer from, for at least 6 months.

(a) Explain why the percentage of people with long-term health conditions is greater than the percentage with chronic conditions.

(b) For each of the percentages given in brackets in the first two dot points, verify that the simplified wording matches the percentage.

(c) Create your own simplified wording for the number of people with mental and behavioural conditions and back problems.

(d) Estimate the number of arthritis sufferers in Australia in 2022.

2 The following snapshot summary is taken from the website of the Australian Government's Australian Institute of Health and Welfare.

3 in 4 (77%) people aged 14 and over **consumed alcohol** in the previous year in 2022–2023 >





There were around **105,700 national illicit drug seizures** in 2020-21, equivalent to around **one seizure every 5 minutes** >

(a) What percentage of people aged 14 and over did not consume any alcohol in the year prior to the survey?

(b) Explain why 77% is not such a high proportion considering the consumption behaviour included in this category.

(c) Suggest why the alcohol and illicit drug summaries were placed together.

(d) Verify the translation of the number of illicit drug seizures in a year to the simplified rate.

(e) What are the implications for society suggested by the data on the frequency of illicit drug seizures?

3 This table comes from a 2024 report to the federal government on the state of the housing system in Australia. The report claims that housing shortages will continue for at least six more years.

Projections of new housing market supply, demand and net balance

Year	Gross new supply	Net new supply	New demand	Net balance (net new supply less new demand)
2023–24	177,676	171,001	208,280	–37,279
2024–25	173,443	166,926	173,229	–6,303
2025–26	179,507	172,762	180,578	–7,816
2026–27	183,731	176,828	172,290	4,538
2027–28	183,525	176,629	173,705	2,924
2028–29	182,850	175,980	171,220	4,760
Total during forecast period	1,080,733	1,040,126	1,079,302	–39,176
Memo item				
Total during National Housing Accord period	903,056	869,126	871,022	–1,897

Source: Based on Australian Government: National Housing Supply and Affordability Council data

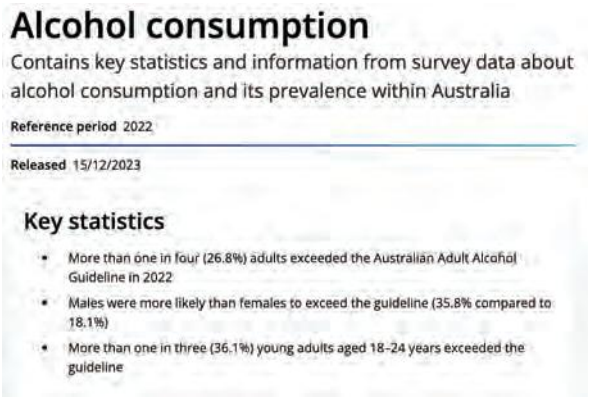
(a) What is the shortage reported for the first year of the report, 2023–24?

(b) How do the shortages for the following two years compare to the first year? Write your answers as a percentage of the figure for 2023–24.

(c) Explain what is forecast to happen over the final three years covered by the report.

(d) What is the forecast shortage at the end of the six years?

4 The Australian Bureau of Statistics regularly produces reports like the one shown here, on a wide range of subjects relevant to the Australian people.



Alcohol consumption

Contains key statistics and information from survey data about alcohol consumption and its prevalence within Australia

Reference period 2022

Released 15/12/2023

Key statistics

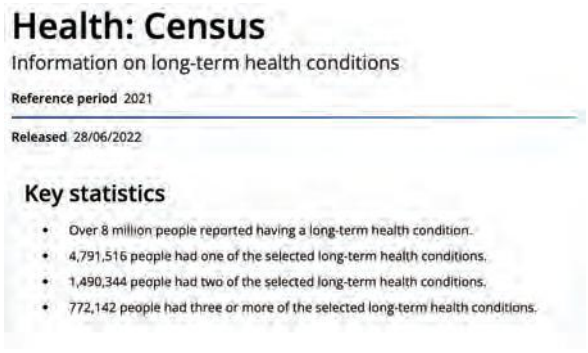
- More than one in four (26.8%) adults exceeded the Australian Adult Alcohol Guideline in 2022
- Males were more likely than females to exceed the guideline (35.8% compared to 18.1%)
- More than one in three (36.1%) young adults aged 18–24 years exceeded the guideline

(a) Why is the report on data for 2022 just coming out at the end of 2023?

(b) For each of the percentages given in brackets, verify that the simplified wording matches the percentage.

(c) The digital report provides a link to further information about the report. What kinds of information would you expect to find by visiting that link? Explain why this information is not included in this Key statistics section of the report.

5 The Australian Bureau of Statistics collects data by survey, but also presents data from each census. In 2021, the census included 25 422 788 people and the population of Australia was estimated to be 25.69 million.



Health: Census

Information on long-term health conditions

Reference period 2021

Released 28/06/2022

Key statistics

- Over 8 million people reported having a long-term health condition.
- 4,791,516 people had one of the selected long-term health conditions.
- 1,490,344 people had two of the selected long-term health conditions.
- 772,142 people had three or more of the selected long-term health conditions.

For the following questions, write percentages correct to the nearest whole number.

(a) What percentage of census participants reported having a long-term health condition?

(b) Explain why it is impossible to tell how many census participants had a long-term health condition that was not listed on the census.

(c) What percentage of census participants reported having a long-term health condition but did not have one of the conditions listed on the census? Consider your answer to **(b)**.

(d) How close a reflection of the actual population would this data be? Explain by saying who was included in the census and who was not.

6 The following snapshot summary is taken from the website of the Australian Government's Australian Institute of Health and Welfare.

In 2022, **83% of adolescents** aged 15-17 and **41% of adults** aged 18 and over **did not meet** the physical activity guideline





More **Indigenous children** aged 5–17 **met the physical activity** guideline than non-Indigenous children aged 5–17

(a) What percentage of adolescents reached the physical activity guidelines?

(b) Suggest some reasons why this percentage is so low.

(c) Explain how the statisticians obtained the data to compare Indigenous children to non-Indigenous children.

(d) Explain why the government collects data specifically about Indigenous Australians.

(e) Explain why the government collects data about people meeting the physical activity guidelines.

7 The following is an online article published on realestate.com.au

Great migration to Qld continues but slows due to housing shortage

By Samantha Healy, Property Journalist, 31 July 2023

REGIONAL Queensland continues to attract the lion's share of Australia's internal movers despite a lack of housing slowing down the great migration from capital cities, a new report has revealed.

The latest Regional Movers Index by the Regional Australia Institute (RAI) and the Commonwealth Bank revealed that the Sunshine Coast (17%) and Gold Coast (9%) recorded the biggest share of net internal migration during the 12 months to June, but both recorded falling growth (7% and 55% respectively) compared to the same period a year ago.

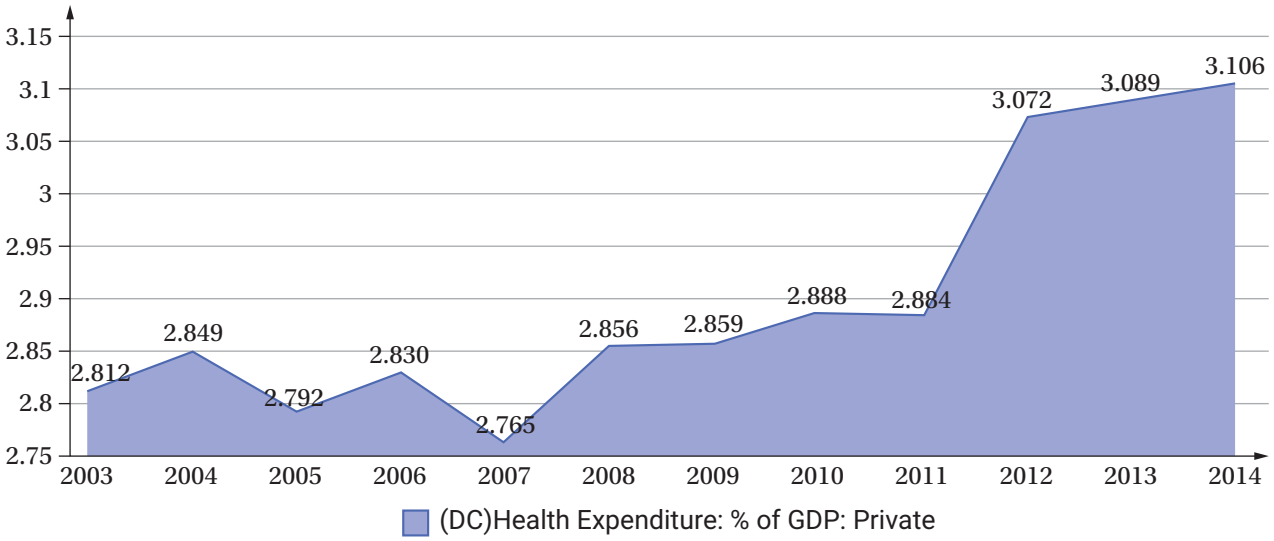
(a) Does the article give evidence of the great migration and the slowdown?

(b) Does the article give evidence that the housing shortage is the reason for the slowdown?

(c) Sunshine Coast and Gold Coast are referred to as the showing the greatest regional migration, so what part of Queensland would not constitute a regional area?

(d) Calculate the previous year's percentage growth for the Sunshine Coast and the Gold Coast before the fall that resulted in this year's figures.

8 The following chart shows the percentage of Government expenditure on public health as a percentage of GDP, over a period of 11 years.



(a) Compare the height of the graph at the trough in 2009 to the peak in 2011.

(b) Write the actual percentages for 2009 and 2011.

(c) Explain the contrast in the answers to **(a)** and **(b)**.

(d) Suggest a reason for presenting the data in this way.

(e) Explain how the large contrast suggested in **(a)** is put into perspective by the other features of the data presentation.

Extended response

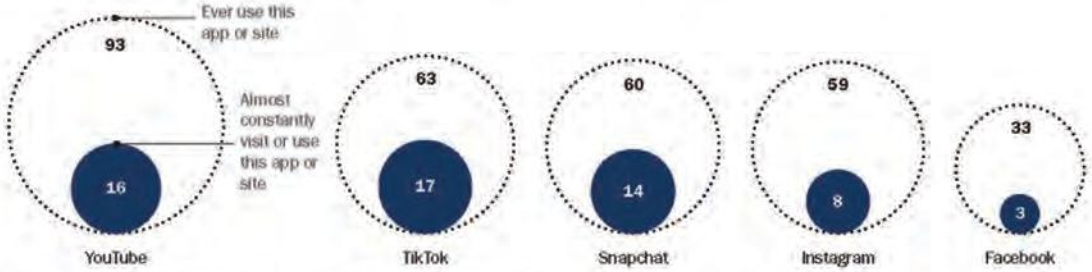
1 The US-based Pew Research Center published the following report in 2023.

How often do teens visit online platforms?

In addition to asking teens about the types of platforms they use, we also asked them how often they use five specific platforms: YouTube, TikTok, Snapchat, Instagram and Facebook.

Nearly 1 in 5 teens say they're on YouTube, TikTok 'almost constantly'

% of U.S. teens ages 13 to 17 who say they ...



Note: Those who did not give an answer or gave other responses are not shown.
Source: Survey of U.S. teens conducted Sept. 26-Oct. 23, 2023.
"Teens, Social Media and Technology 2023"

PEW RESEARCH CENTER

YouTube, the most widely used platform measured in the survey, is also frequently visited by its users. About seven-in-ten teens say they visit the video-sharing platform daily, including 16% who report being on the site almost constantly.

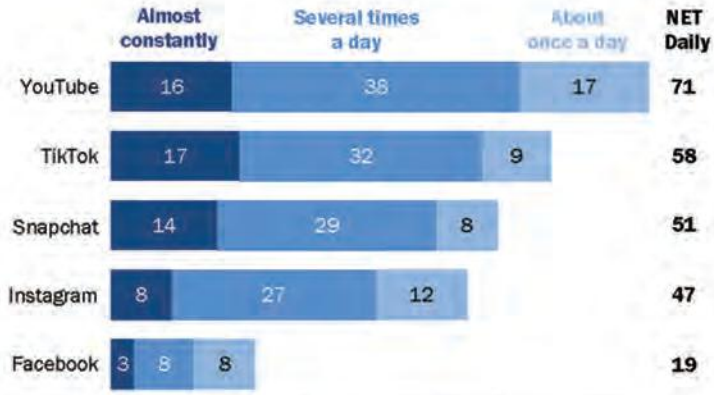
At the same time, 58% of teens are daily users of TikTok. This includes 17% who describe their TikTok use as almost constant.

About half of teens use Snapchat and Instagram daily. A somewhat larger share reports using Snapchat almost constantly compared with Instagram (14% vs. 8%).

Far fewer teens say they use Facebook on a daily basis (19%), with only 3% saying they are on the site almost constantly.

A majority of teens visit YouTube, TikTok daily

% of U.S. teens ages 13 to 17 who say they visit or use the following apps or sites ...



Note: Those who did not give an answer or gave other responses are not shown.
Source: Survey conducted Sept. 26-Oct. 23, 2023.
"Teens, Social Media and Technology 2023"

PEW RESEARCH CENTER

(a) What piece of information appears on both graphs?

(b) The article says 'YouTube, the most widely used platform measured in the survey'. Where is this statement supported in the graphs.

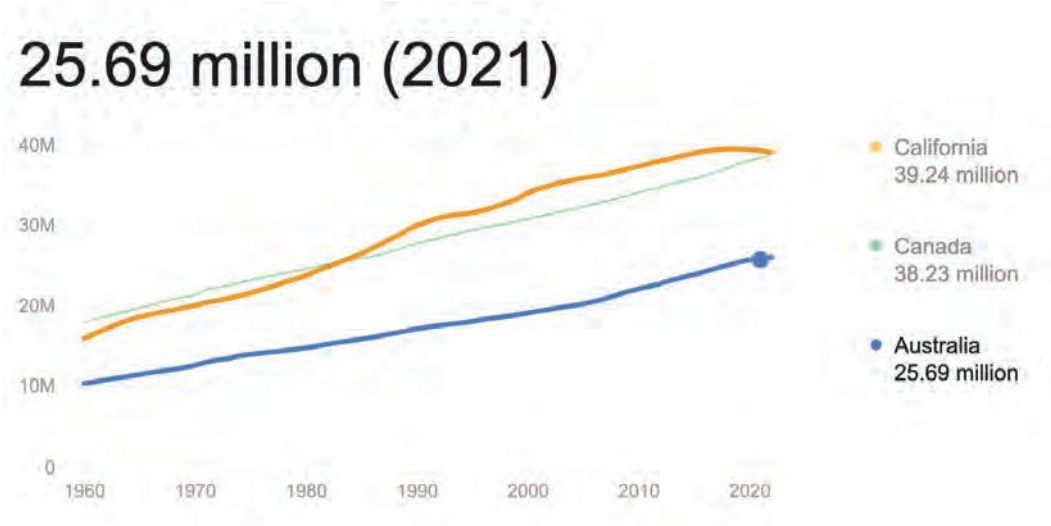
(c) What total percentage of teens describe themselves as being 'almost constantly' on one of the platforms included in the survey?

(d) Could there be some overlap in the figures used to calculate **(c)**? What does this say about the validity of the answer?

(e) What data suggests that Snapchat is currently more addictive than Instagram.

(f) Suggest why a smaller percentage of teens are on Facebook compared to the other platforms.

2 The following graph was published online.



(a) Suggest a possible reason for comparing the population trend of Australia, Canada and California.

(b) What is extraordinary about including California with the other two?

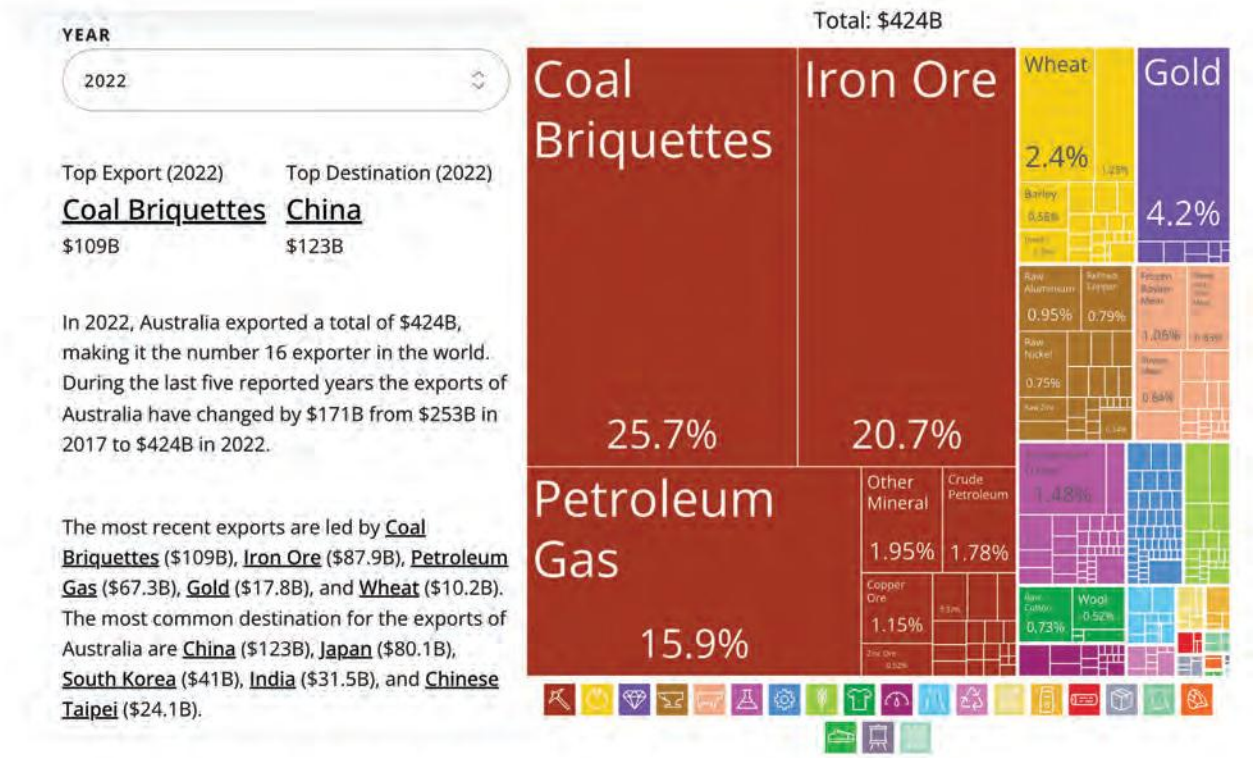
(c) Often zero is not shown on the vertical axis when showing changes or trends. Why do you think zero was included in this case, and how is this helpful?

(d) Summarise the trends simply by comparing the visual aspects of the lines.

(e) By estimating the figures for 1960, calculate the percentage growth in population for the three places over the 60 or so years shown.

(f) Write a summary of your calculations in simpler language.

3 The Observatory of Economic Complexity (OEC), is known for its visual reports on trade, such as the one shown here.



(a) Perhaps you have heard that Australia's most lucrative export is iron ore. How can you reconcile this with coal taking top spot on the OEC infographic?

(b) Comment on the use of the category name Coal Briquettes.

(c) Verify that the percentages for Coal, Wheat and Gold are correctly represented by area.

(d) Verify that the percentages for Iron Ore and Petroleum Gas match the values given in the summary.

(e) Calculate the percentages of our exports that go to China, Japan and South Korea.

(f) Suggest a reason for the colour coding on the graph.

TOPIC

15

Displaying and comparing statistical data

Recall 503

15.1 Represent data using statistical displays 504

15.2 Justify the selection of particular data displays 517

Topic review 532

Why learn this?

Studying this topic offers valuable insights into how to represent data using statistical displays, such as bar charts, histograms and stem-and-leaf plots.
It also helps with understanding how to interpret and compare data sets using measures of central tendency like mean and median.
With these skills, you can analyse and interpret real-world data, such as the number of students in a class, the stock of paints an artist has, or the median house price in different cities. This knowledge also enables a deeper understanding of trends and patterns in various fields, from business to the social sciences, and even in everyday personal situations. The ability to interpret data displays such as infographics is a particularly important skill in today's data-driven world.

RECALL

ANSWERS Page 651

I can complete a frequency distribution table

1 Complete the frequency table for this data showing the number of each special salad pack (A to E) sold during lunch hour at a store.
C, B, A, A, B, D, E, D, C, B, A, C, D, A, B, A, C, D, E, B

Salad packs	Tally	Frequency
A		
B		
C		
D		
E		
Total		

I can calculate the mean score for a set of data

1 Calculate the mean of each of the following sets of numbers.

(a) 2, 3, 7, 6, 5, 4, 3

(b) 8, 11, 12, 9, 6, 3

(c) 11, 15, 14, 18, 17, 20, 16, 15

I can calculate the median score for a set of data

1 Determine the median of each of the following sets of numbers.

(a) 7, 9, 11, 8, 15, 14, 12

(b) 2, 1, 21, 31, 4, 52

(c) 101, 112, 117, 118, 119, 104

I can identify the mode for a set of data

1 Identify the modal score for each of the following sets of data.

(a) 1, 3, 4, 2, 4, 1, 2, 4, 3, 2, 2, 3, 2, 4

(b) 20, 28, 24, 22, 25, 23, 21, 23, 26, 25, 23, 20

(c) 108, 110, 109, 111, 109, 108, 110, 111, 108, 112

I can calculate the range of a set of data

1 Calculate the range for each of the following sets of data.

(a) 3, 7, 8, 9, 11, 15 **(b)** 6, 7, 2, 1, 8, 19, 7 **(c)** 81, 27, 16, 42

15.1 Represent data using statistical displays

Learning intention: To be able to represent data using statistical displays

Success criteria:

SC 1 I can describe the underlying shape of a distribution.

SC 2 I can draw and interpret comparative stem-and-leaf plots.

SC 3 I can draw and interpret comparative bar charts and histograms.

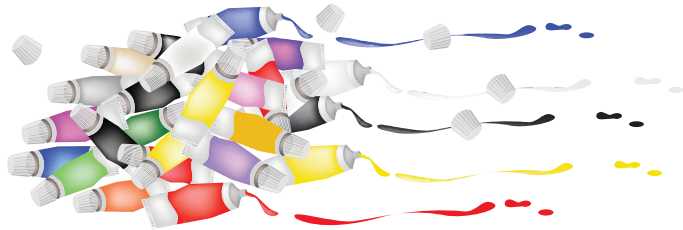
Lesson warm-up

Artist stock-take

An artist has been commissioned to paint a large mural called 'Shifting Colours'.

This art project is to blend incremental shades of similar colours together before merging into new ones.

The complete stock of different paints owned by the artist, which are scattered around the studio, need to be categorised by both similar colours and quantity of each colour available, then arranged in a display before work can start.

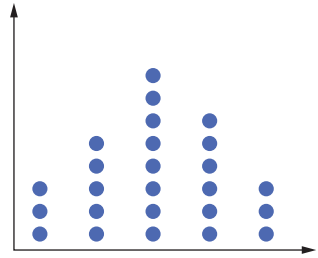


How would you categorise, sort and display the available paint colours and quantities in a meaningful way to help the artist be aware of what is available to use?

SC 1 I can describe the underlying shape of a distribution

Data sets can be displayed in different ways. However, histograms, stem-and-leaf plots and dot plots all enable a statement to be made about the shape of the distribution. The following descriptions are based on dot plots, but the same statements can be made using the other display types.

Symmetrical distribution

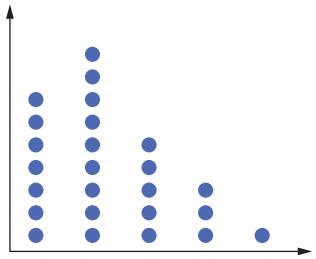


The graph of a data set is symmetrical if a vertical line of symmetry exists for the graph. This does not have to be exact. If the graph of a data set is symmetrical, then the mean, median and mode will be close in value.

The distribution shown here is close to perfectly symmetrical, but there is one extra dot in the column to the right of the middle. This is fine; the distribution would still be regarded as symmetrical.

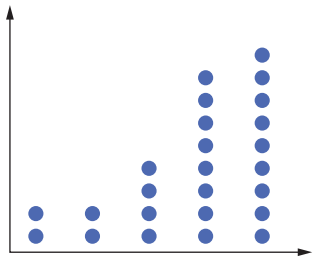
The graph of a data set is said to display positive skew if most of the results fall on the left of the graph. In a data set like this, in most cases, the mode will occur to the left of the median and the mean will be to the right of the median: mode < median < mean. The larger the number of data values the more likely this order will apply.

Positive skew



The graph of a data set is said to display negative skew if most of the results fall on the right of the graph. In a data set like this, in most cases, the mode will occur to the right of the median and the mean will be to the left of the median: mean < median <mode. The larger the number of data values the more likely this order will apply.

Negative skew



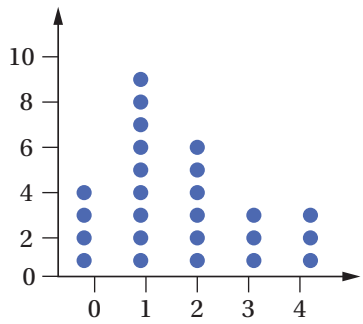
There is another type of distribution that needs to be mentioned. This is a bimodal distribution. As the name suggests, this is a data set that has two modes. These values do not have to be exactly the same.

Worked example

Identifying the shape of a statistical distribution

Consider the following dot plot.

Number of siblings



(a) Identify the shape of the distribution.

THINKING	WORKING
If the shape is not symmetrical, determine on which side of the middle most of the values lie. If to the left, then it has a positive skew.	The distribution is not symmetrical and more values are to the left side than the right. So, the distribution shows positive skew.

(b) Calculate the mean.

THINKING	WORKING
To calculate the mean $(\bar{x})$, add together all the values $(\sum x)$ and divide by the number of values (n).	$\sum x = 4\times0+9\times1+6\times2+3\times3+4\times3$ $= 42$ $n = 25$ $\bar{x} = \frac{42}{25}$ $= 1.68$

(c) Calculate the median.

THINKING	WORKING
The median is the value in the middle of the ordered data set. For an odd number of values, the median is the $\frac{n+1}{2}$ th value.	There are 25 values, the median is the 13th value. The dot plot makes it easy to find the median. In this case it is 1.

(d) Calculate the mode.

THINKING	WORKING
The mode is the most frequently occurring value.	The mode is 1.

(e) Put the three measures of centre in order using < symbols.

THINKING	WORKING
Look at the values and write an appropriate statement.	mode = median < mean

SC 2 I can draw and interpret comparative stem-and-leaf plots

In statistics it is common to want to compare one data set with another data set. This is especially useful when the two data sets are closely related. For instance, you may be interested in comparing the heights of boys to the heights of girls of the same age.

A back-to-back stem-and-leaf plot is one way of making such comparisons. In this type of display, there is a central column representing the common stem values. One data set appears on the left of the stem, increasing in size the further the value is from the stem. The other data set appears on the right of the stem, also increasing in size the further the value is from the stem.

An example is shown below.

Leaf	Stem	Leaf
3 1	0	8 9
9 9 6 0	1	5 7 9
8 8 8 6 5 1	2	4 4 4 8
7 4 1	3	2 3 3 3 3 6
8 7 7 3 2 0	4	0 5 7 7
8	5	8 8 9
0	6	5 5

It should be emphasised that the leaf values are always written in order. This makes the calculation of the median a relatively straightforward task.

One of the benefits of a stem-and-leaf plot is that it retains all the detail in the data. However, because of this, it is not an appropriate display for large data sets.

Worked example

Analysing a back-to-back stem-and-leaf plot

The following back-to-back stem-and-leaf plot represents the results in the latest mathematics tests for two different classes.

Leaf Class A	Stem	Leaf Class B
3 1	0	9 9
9 9 6 0	1	3 6 7
8 8 8 6 5 1	2	4 4 7 8
7 4 1	3	2 2 2 2 3 6
8 7 7 3 2 0	4	1 5 7 8
8	5	0 8 9
0	6	4 5

(a) In which class was the lowest mark obtained?

THINKING	WORKING
The lowest mark will be the lowest leaf value in the row representing the lowest stem value.	The lowest result is 1 and it occurs in Class A.

(b) In which class was the highest mark obtained?

THINKING	WORKING
The highest mark will be the highest leaf value in the row representing the highest stem value.	The highest result is 65 and it occurs in Class B.

(c) Compare the median result for each class.

THINKING	WORKING
Calculate the median in the usual way. For an even number of values the median is the mean of the nth and $(n + 1)$th values. For an odd number of values the median is the $\left(\frac{n+1}{2}\right)$ th value.	For Class A, there are 23 values, so the median is the 12th value. Here, this is 28. For Class B there are 24 values, so the median is the mean of the 12th and 13th values. Here, this is $\frac{32+32}{2}=\frac{64}{2}$ $=32$ The median for Class B is higher than the median for Class A.

(d) Compare the mean result for each class. State the answers correct to 1 decimal place.

THINKING	WORKING
Calculate the mean in the usual way: $\bar{x}=\frac{\sum x}{n}$	The values need to be added and divided by the number of values in each class, respectively. Class A: $\bar{x}=\frac{\sum x}{n}$ $=\frac{711}{23}$ $=30.913\ldots$ The mean for Class A is 30.9. Class B: $\bar{x}=\frac{\sum x}{n}$ $=\frac{841}{24}$ $=35.041\ldots$ The mean for Class B is 35.0. The mean for Class B is higher than the mean for Class A.

15

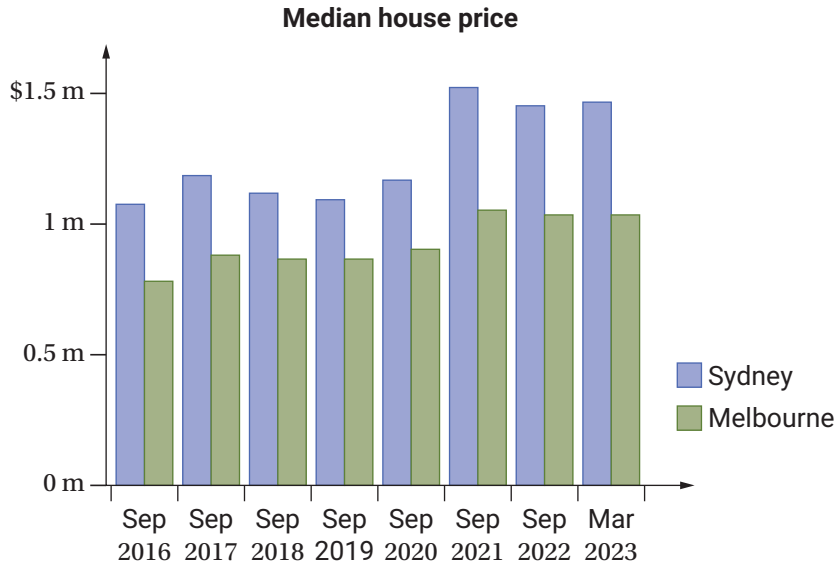
SC 3 I can draw and interpret comparative bar charts and histograms

Bar charts and histograms are closely related statistical displays. A histogram is useful when the range of the data is large, and some grouping of values occurs. Although this makes it easier to draw the display, it does mean that the individual data values are lost.

Worked example

Analysing a comparative bar chart

The following graph shows the median house price in both Sydney and Melbourne from 2016 to 2023.



(a) When was the median house price the highest in Sydney?

THINKING	WORKING
Use the legend to see which colour represents the city of interest, then find the highest column for that colour.	September 2021

(b) When was the median house price the highest in Melbourne?

THINKING	WORKING
Use the legend to see which colour represents the city of interest, then find the highest column for that colour.	September 2021

(c) Describe the trend in Sydney prices.

THINKING	WORKING
Use general terms to describe what you see in the graph.	The price initially rose, but then fell for two years before rising again and surging to the September 2021 figure. After this, it fell, then rose again, but not as high as its previous record.

(d) Describe the trend in Melbourne prices.

THINKING	WORKING
Use general terms to describe what you see in the graph.	The price initially rose, but then fell for one year, then stayed steady for a year before rising to the September 2021 figure, which is the highest it reaches. After this it fell and stayed steady.

(e) Compare the median house price in Sydney to the median house price in Melbourne for the time covered in this graph.

THINKING	WORKING
Compare the two cities shown in the graph.	The Sydney value and the Melbourne value have basically followed the same trend, although the Sydney price is always higher than Melbourne's. The surge in September 2021 was greater in Sydney.

Practice

ANSWERS Page 651

SC 1 I can describe the underlying shape of a distribution

1 Consider the following data set.

Score	Frequency
3	4
4	7
5	11
6	18
7	13
8	6
9	2

(a) Calculate the mean for the data set. State the answer correct to 1 decimal place.

(b) Calculate the median for the data set.

(c) Calculate the mode for the data.

(d) Is the data set symmetrical, or does it show positive skew, or negative skew? How do the values of the mean, median and mode help confirm this?

2 Consider the following data set.

0, 3, 4, 2, 3, 5, 6, 5, 4, 5, 6, 2, 0, 1, 2, 5, 6, 4, 5, 6, 5, 6, 0, 2, 3, 1, 6, 5, 4, 5, 6, 5

(a) Construct a frequency table to show the data.

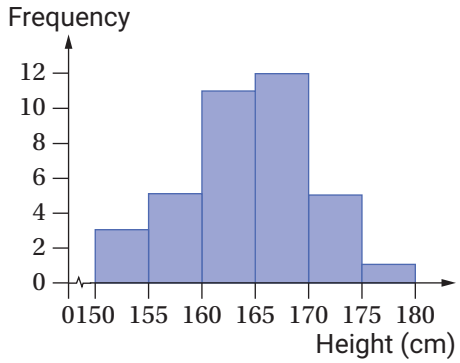
(b) Calculate the mean for the data set. State the answer correct to 1 decimal place.

(c) Calculate the median for the data set.

(d) Calculate the mode for the data set.

(e) Is the data set symmetrical, or does it show positive skew, or negative skew? How do the values of the mean, median and mode help confirm this?

3 Consider the following grouped data histogram.



Height (cm)	Frequency
150–<155	
155–<160	
160–<165	
165–<170	
170–<175	
175–<180	

(a) Complete the frequency table for this data set.

(b) Calculate the mean for this data set. State the answer correct to 1 decimal place.

(c) In which data range does the median occur?

(d) In which data range does the mode occur?

(e) Is this a symmetrical distribution or does it display negative skew or positive skew?

4 Only the following statistics are provided for some data sets. On the basis of these statistics, decide whether the data set is symmetrical or whether it shows positive skew or negative skew.

(a) mean: 101.5
median: 95.5
mode: 93

(b) mean: 115
median: 114.5
mode: 116

(c) mean: 219.6
median: 226
mode: 234

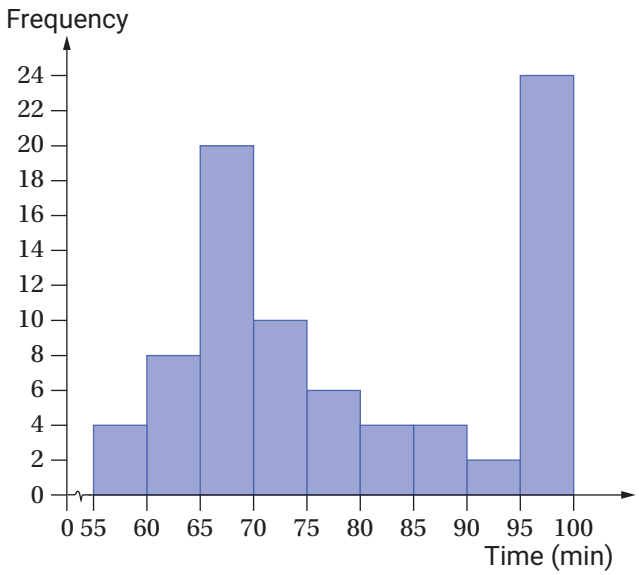
5 The following histogram shows the time taken for the students in Year 9 at a school to complete a cross-country run.

(a) How would you describe the distribution of this data set?

(b) What do you think could cause such a distribution?

(c) Calculate the mean of this data set. State your answer correct to 1 decimal place.

(d) Do you think this value is a fair representation of the data set?



6 The following stem-and-leaf plot concerns marks obtained on a mathematics test that was marked out of 65.

Stem	Leaf
0	8 9
1	5 7 9
2	4 4 4 8
3	2 3 3 3 3 6
4	0 5 7 7
5	8 8 9
6	5 5

(a) What is the mean of this data set? State your answer correct to 1 decimal place.

(b) What is the median of this data set?

(c) What is the mode of this data set?

(d) How would you describe the shape of the distribution of this data set?

SC 2 I can draw and interpret comparative stem-and-leaf plots

1 Consider the following data that shows the scores obtained by students in two different classes on the same test that was marked out of 50.

Class A: 29, 33, 47, 46, 50, 23, 30, 37, 42, 44, 19, 23, 46, 28, 27, 40, 42, 44, 46, 25, 27
Class B: 44, 46, 47, 49, 38, 38, 37, 29, 14, 23, 27, 29, 34, 35, 36, 42, 48, 49, 30, 21, 50, 58, 59, 59

(a) Use the data to construct a back-to-back stem-and-leaf plot for the scores obtained by the students.

(b) Describe the shape of each distribution.

(c) Determine the median for each class.

(d) Determine the mean for each class. State your answers correct to 1 decimal place.

2 The following are the officially listed heights (cm) for players in the 2023 squads for two teams in the NRL.

Melbourne Storm:
185, 179, 185, 187, 193, 183, 183, 178, 196, 180, 187, 196, 181, 193, 194, 182, 190,
185, 187, 190, 195, 193, 177, 174, 190, 190, 187, 200, 193, 190, 192, 194, 195

Brisbane Broncos:
173, 192, 185, 176, 190, 189, 180, 182, 186, 185, 177, 190, 177,
180, 194, 188, 178, 190, 192, 189, 194, 190, 190, 194, 190, 181

(a) Use the data to construct a back-to-back stem-and-leaf plot for the heights of the two teams. As the values are so tightly clustered, use stems like 17_L (170–174) and 17_H (175–179), and so on.

(b) From the plot, which team seems to be the taller team? Keep in mind that the number of players listed for each club is not the same.

(c) Determine the median height for each team.

(d) Determine the mean height for each team. State your answers correct to 1 decimal place.

(e) What is the mode for each team? Does this value seem unusual? Write a brief sentence outlining your thoughts.

3 The following numbers are the scores obtained in the first two rounds in the 2023 NRL season.

Round 1:	12	16	20	12	12	13	31	6	19	18	18	27	28	18	10	22
Round 2:	16	10	26	30	28	16	20	12	20	14	12	26	12	14	32	18

(a) Draw an ordered back-to-back stem-and-leaf plot for the data using a value range of 10.

(b) Now redraw the plot, this time using a data range of 5.

(c) What is the advantage of using a range of 5 compared to a range of 10?

(d) Calculate the median score for each round.

(e) Calculate the mean score for each round. If necessary, write the values correct to 1 decimal place.

(f) In which round would you say the scores were higher?

4 The following data sets were obtained from the Bureau of Meteorology website. The data values represent the maximum daily temperatures recorded at Perth Airport and Adelaide Airport for June 2023.

Perth Airport

19.3	18.9	18.7	21.4	14.6	10.6	15.8	16.8	19.9	19.8	18.0	17.3	20.0	18.7	19.4
19.1	16.5	17.6	18.1	16.3	15.9	17.7	15.0	16.7	16.2	15.1	16.7	18.3	17.1	17.0

Adelaide Airport

18.6	20.3	25.3	22.2	19.3	23.7	21.8	17.1	16.3	16.5	18.1	19.1	15.9	14.9	17.1
18.3	14.8	15.5	13.8	14.9	14.2	13.6	14.9	13.9	14.8	15.6	12.8	14.9	13.6	15.4

Because the data values represent decimals, the stem values will be the whole number part and the leaf values will be the decimal part. So, 19|3 will represent 19.3.

(a) Construct an ordered back-to-back stem-and-leaf plot to represent the data.

(b) Calculate the median daily maximum temperature for each location. State the answers correct to 2 decimal places.

(c) Calculate the mean daily maximum temperature for each location. State the answers correct to 2 decimal places.

(d) Which location seems to be hotter in June?

(e) Would you suggest constructing this plot using a data value range of 5? Give a reason for your answer.

5 The following back-to-back stem-and-leaf plot shows the results for two lots of 50 numbers in the range 20–49 generated by a process that is said to be random.

Leaf Sample 1	Stem	Leaf Sample 2
4 4 3 3 2 2 1 1 1	20_L	3 4
9 9 9 8 7 7 7 6 6 5 5 5	20_H	5 6 6 7 7 7 8 8 8 9
4 3 3 1 1	30_L	2 2 2 2 2 3 3 4
9 9 8 8 8 6 5 5	30_H	6 7 7 8 8 8 9 9 9
3 3 3 3 2 0	40_L	0 0 0 1 1 2 2 2 2 2 3 4 4
9 9 8 8 7 6 6 6 5 5	40_H	6 6 7 7 7 8 9 9

(a) Calculate the median for each sample.

(b) Calculate the mean for each sample.

(c) If these were random numbers, what would be the median and mean?

(d) Does Sample 1 appear to be random?

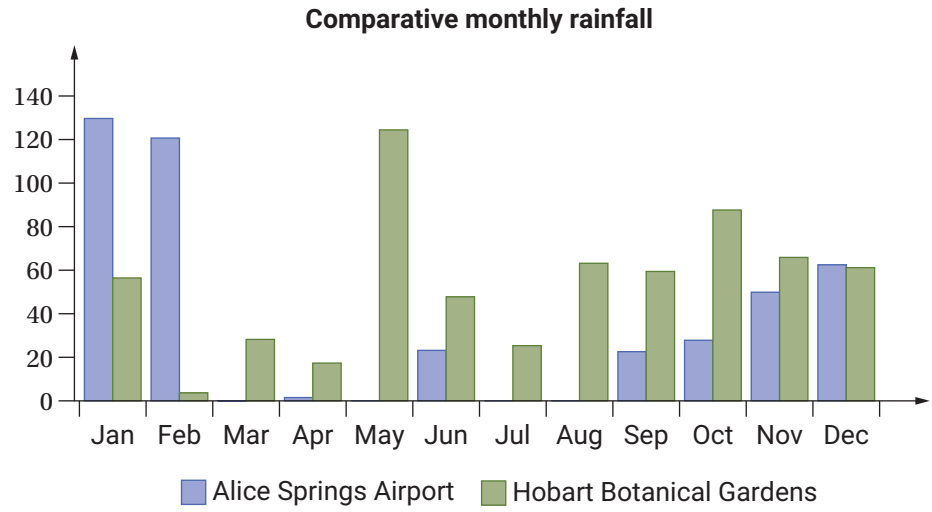
(e) Does Sample 2 appear to be random?

(f) Calculate the median and the mean for the combined sample of 100 numbers.

(g) Do the combined sample statistics support the claim that these are random numbers?

SC 3 I can draw and interpret comparative bar charts and histograms

1 The monthly rainfall for two places in Australia during 2022 is shown in the following graph.



(a) Why would it be difficult to discuss the average monthly rainfall in Alice Springs?

(b) What seems unusual about the rainfall for Hobart?

(c) Which season has the most rain in Alice Springs?

(d) Which season has the most rain in Hobart?

2 The US LPGA Women's Open is one of the major events in women's golf. The table below, which runs over two lines, shows a summary of the scores in the final two rounds of the event in 2023.

Score	66	68	69	70	71	72	73	74	75
Frequency (Round 3)	1	0	0	3	7	9	8	6	13
Frequency (Round 4)	0	3	4	2	9	10	10	8	5

Score	76	77	78	79	80	82	87
Frequency (Round 3)	10	3	4	6	3	0	1
Frequency (Round 4)	6	5	3	2	2	1	0

(a) How many players competed in each of the final two rounds of this event?

(b) Group the data in twos with the first group being 66–67, and construct a frequency table for the data.

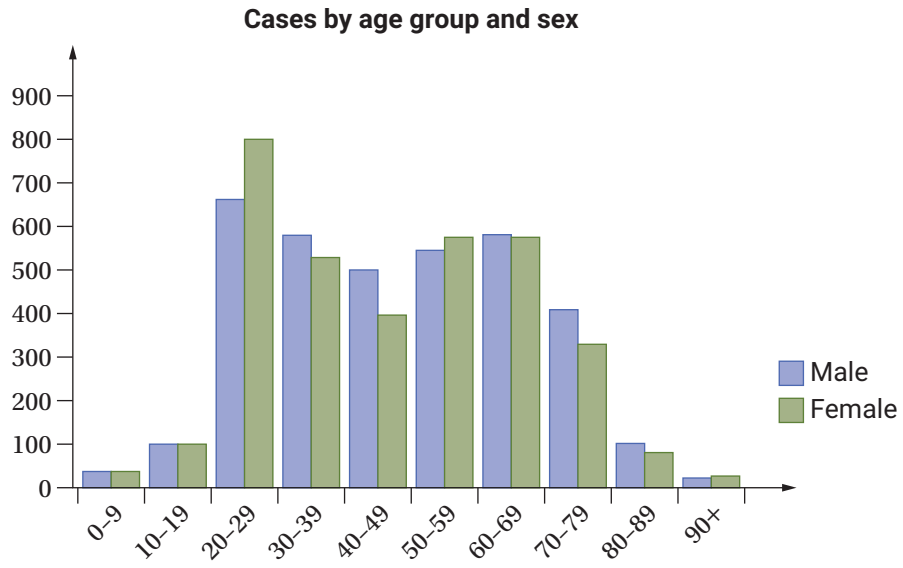
(c) Now draw a back-to back histogram of the data. Be careful with the data because there are some values not included in the table due to nobody on either day getting the score.

(d) Find the mean score for each round. State the answers correct to 1 decimal place.

(e) In golf, the lower the score the better. Which day seemed to have the better scores?

(f) Describe the shape of the data distribution for each of rounds 3 and 4.

3 The following graph relates to the number of cases of COVID-19 in Australia at some time in May 2020. The graph was produced for the Australian Government, Department of Health.



(a) Does it seem as though there are more male or female cases recorded in the graph?

(b) Which age group has the highest number of cases for males?

(c) Which age group has the highest number of cases for females?

(d) What is concerning about the total number of cases in the age group 60–69?

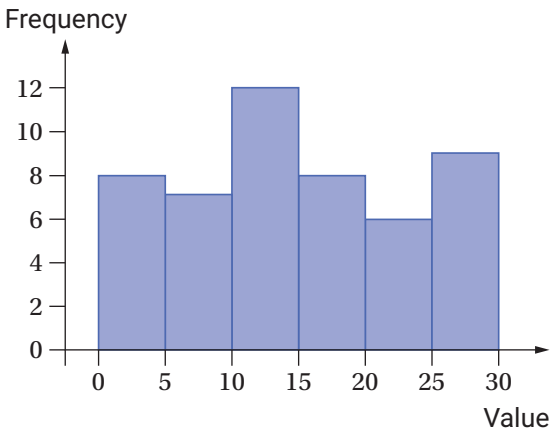
4 Roman and Julia were required to collate the following data into some form of statistical display. The values are random numbers in the range 0–29.

2	2	4	24	13	0	22	19	13	6	22	23	27	24	10	17	22
18	11	24	25	14	4	21	12	26	28	28	14	9	29	0	6	15
22	25	27	17	15	1	10	5	6	5	0	29	15	12	9	29	

Roman chose a stem-and-leaf plot with a range of 5.

Stem	Leaf
0_L	0 0 0 1 2 2 4 4
0_H	5 5 6 6 6 9 9
1_L	0 0 1 2 2 2 3 3 3 4 4 4
1_H	5 5 5 7 7 7 8 9
2_L	1 2 2 2 2 4
2_H	5 5 6 7 8 8 9 9 9

Julia chose a histogram with a range of 5 and the first bracket being 0–<5.



(a) Look at Roman's plot. Can you tell what the highest value recorded was? If so, what was it?

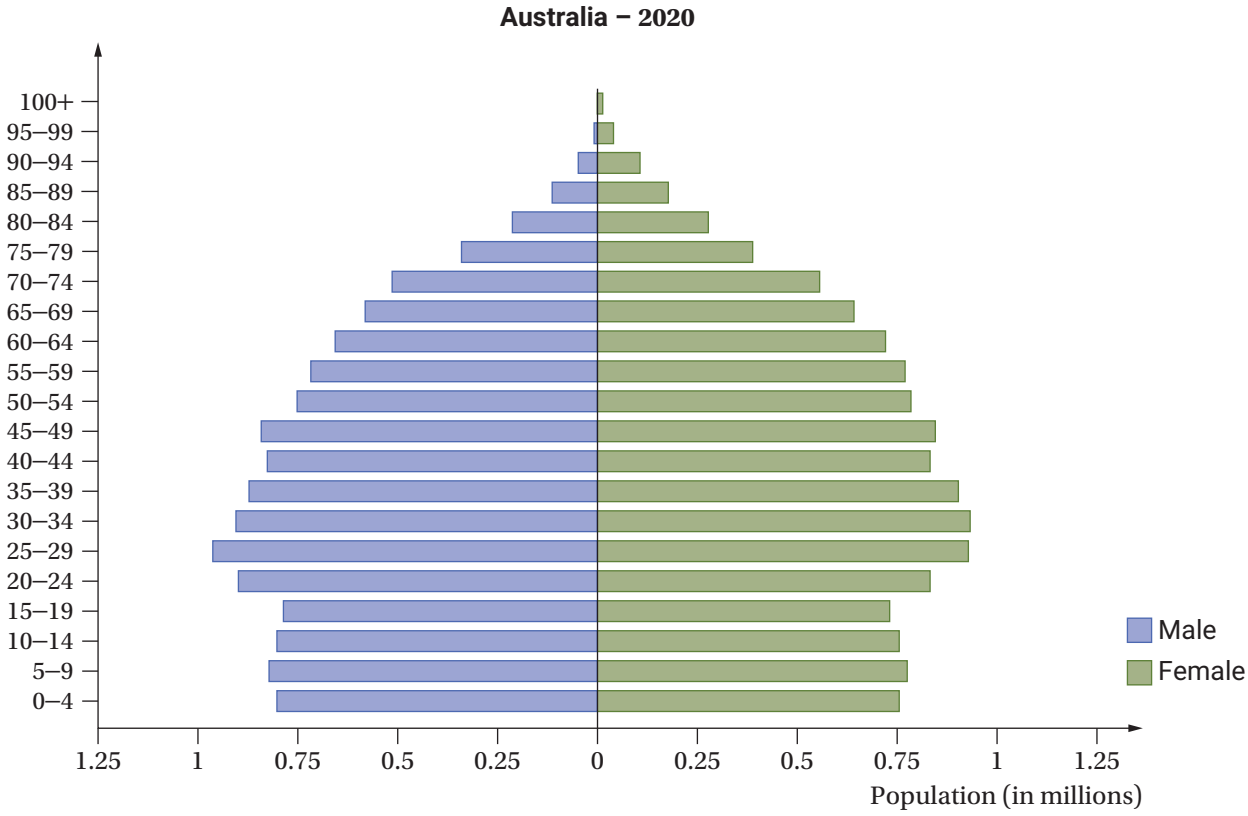
(b) Look at Julia's graph. Can you tell what the lowest value recorded was? What is the best you can do?

(c) Can you identify the mode from either display? If so, which one?

(d) Describe how Roman's plot would change if the range of data values changed from 0–29 to 1–30.

5 Another form of a back-to-back graph is the age-sex pyramid, which is used extensively when looking at the population of a country. On one side of a central column that contains age brackets you will find the proportion of the population that is female and in that age bracket. On the other side will be the equivalent data for males.

The following graph shows this information for Australia in 2020.



(a) Which age group appears to have the most males?

(b) Which age group appears to have the most females?

(c) How can there be more people in the age range 20–24 than in the age range 0–4?

(d) From the definition provided, do you think Australia has a young population, an older population, or neither of these?

Justify the selection of particular data displays

15.2

Learning intention: To be able to justify the selection of particular data displays

Success criteria:

SC 1 I can use mean, median and range to interpret and compare data sets.

SC 2 I can distinguish between the type of display relevant for categorical data and numerical data.

SC 3 I can interpret data displays such as infographics.

Lesson warm-up

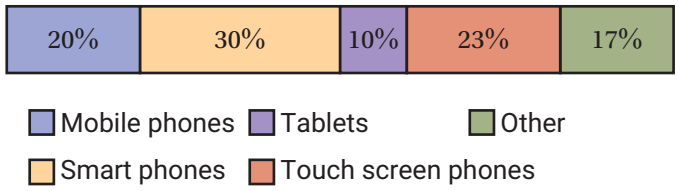
What is it saying?

A school-wide campaign involved students bringing all their old mobile phones and other e-waste items to place into e-waste recycle bins. Students who contributed e-waste to the bin could then enter a competition to guess the total number of discarded phones. A new smartphone was to be given to the student with the closest guess.

At the end of the campaign, the results of the e-waste collection and winning guess were displayed using this chart.



E-waste collection and competition results



Comment on how these statistics for the campaign have been displayed.

Include whether you feel the chart provides enough information.

Give reasons for your response.

SC 1 I can use mean, median and range to compare data sets

Some of this comparison work using these statistics has been completed in earlier sections. Here, the emphasis will be on data sets where the values have been grouped. The technique used to estimate the mean will be outlined and the terms 'median class' and 'modal class' will be used.

The following frequency table uses grouped data. The original data here is continuous data, which is why the score ranges are written as they are. The first range contains every score that is at least 0 but less than 20.

Score	Frequency (f)
0–<20	5
20–<40	10
40–<60	12
60–<80	8
80–<100	3
	$\sum f = 38$

The first thing that can be stated about this data set is that the modal class is 40–<60. This is simply the score range that has the highest frequency associated with it.

The median class is the score range where the middle value will be found. Because there are 38 values here, the key values are the 19th and 20th values. Count down the frequency column until you find these values. In this case, they are both in the score range 40–<60, so this is the median class.

It is not possible to find the exact value for the mean. An estimate is the best that can be found. To make this estimate, the midpoint value of each class range is used. This is found by adding the endpoints together and dividing by 2. So, for the value range 0–<20, the midpoint,
$x_m = \frac{0+20}{2} = \frac{20}{2} = 10$.

A new column is added to the frequency table that shows these values. This is multiplied by the frequency.

The estimate for the mean is then $\bar{x} = \frac{\sum x_m f}{\sum f}$.

This is now shown for the example above.

Score	Frequency (f)	Midpoint (x_m)	$x \times f$
0–<20	5	10	50
20–<40	10	30	300
40–<60	12	50	600
60–<80	8	70	560
80–<100	3	90	270
	$\sum f = 38$		$\sum x_m f = 1780$

So,

$$\bar{x} = \frac{\sum x_m f}{\sum f}$$
$$= \frac{1780}{38}$$
$$= 46.8421\ldots$$

This value would normally be rounded to some specified number of decimal places.

It is also useful to calculate the range of the values. This is simply the difference between the smallest value observed and the largest. With grouped data, it is only possible to use the endpoints. Here, the range is 100 – 0 = 100.

Worked example

Calculating statistics for grouped data

For the grouped data in the frequency table, calculate the following statistics.

Score	Frequency (f)
5–<15	10
15–<25	16
25–<35	8
35–<45	5
45–<55	2
	$\sum f = 41$

(a) State the modal class.

THINKING	WORKING
The modal class is the score range that has the highest frequency.	The modal class is 15–<25.

(b) State the median class.

THINKING	WORKING
The median class is the score range that contains the median value. For an odd number of data values, the median is the $\left(\frac{n+1}{2}\right)$th value.	For 41 values, the median will be the 21st value. Count down the frequency column until you find the required value. Here it is in the 15–<25 score range, so this is the median class.

(c) Calculate an estimate for the range.

THINKING	WORKING
Calculate an estimate for the range by subtracting the lower end of the first score from the upper end of the upper score.	An estimate for the range is 55 – 5 = 50.

(d) Calculate an estimate for the mean. State the answer correct to one decimal place.

THINKING

Add two new columns to the frequency table and use $\bar{x} = \frac{\sum x_m f}{\sum f}$.

Score	Frequency (f)	x_m	$x_m f$
5–<15	10	10	100
15–<25	16	20	320
25–<35	8	30	240
35–<45	5	40	200
45–<55	2	50	100
	$\sum f = 41$		$\sum x_m f = 960$

Calculate an estimate for the mean using:

$$\bar{x} = \frac{\sum x_m f}{\sum f}$$

State the answer correct to the specified number of decimal places.

WORKING

$$\bar{x} = \frac{\sum x_m f}{\sum f}$$
$$= \frac{960}{41}$$
$$= 23.4146\ldots$$

An estimate for the mean is 23.4.

SC 2 I can distinguish between the type of display relevant for categorical data and numerical data

Data can be categorical or numerical. Different data displays are typically used to display these data sets.

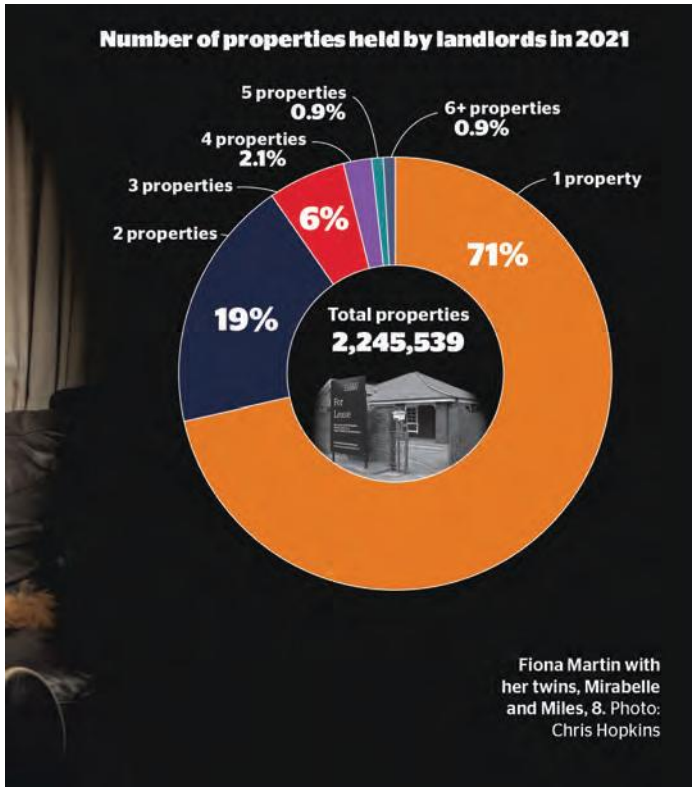
Categorical data is limited in the types of data displays that can be used; pie charts and bar charts/column graphs are typically used. Because a pie chart is generally represented in percentages, a sense of the whole is required. For example, the total number of responses in a survey or the total number of cars in a car park.

Numerical data can be displayed using a wider range of data displays, including dot plots, stem-and-leaf plots and histograms. In some cases, pie charts and bar charts can also be used. Sometimes, it is necessary to group the data because the range is too great to make any real sense of what is being shown. For data analysed over time, a line graph is usually drawn.

Worked example

Analysing graphed data

The following graph, related to the number of rental properties held by landlords in Australia in 2021, was published in the print media.



(a) What percentage of landlords own more than 2 rental properties?

THINKING	WORKING
The questions says 'more than 2'. Add up the percentages for the sectors for 3, 4, 5, 6+ properties.	$(6 + 2.1 + 0.9 + 0.9)\% = 9.9\%$

(b) How many landlords own 1 rental property?

THINKING	WORKING
Determine the required percentage of the total properties (the value in the middle of the graph).	$71\% \times 2\,245\,539 = \frac{71}{100} \times 2\,245\,539$ $= 1594\,332.69$
Round this value to something that makes sense in the context of the question.	The value represents the number of houses, so cannot be a decimal. The nearest whole number is 1 594 333.

(c) Does it make any difference to the way this graph is interpreted, because the middle of the circle is missing?

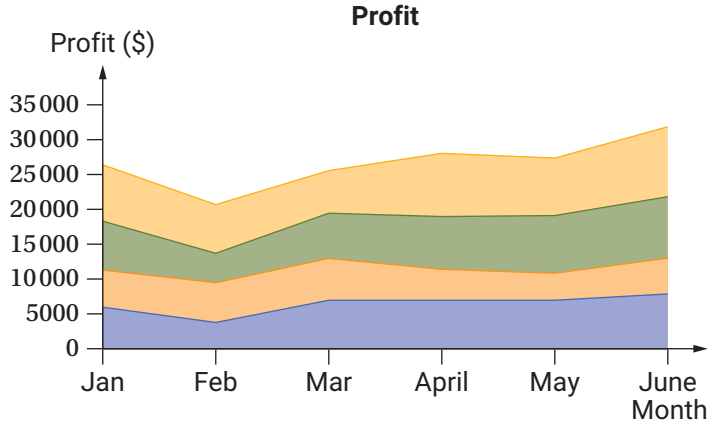
THINKING	WORKING
Consider the effect, if any, of this feature.	No, it makes no difference to the way the graph is interpreted.

(d) What do the pieces of the sector graph add up to? Is this of concern?

THINKING	WORKING
Add together the numbers provided.	(71 + 19 + 6 + 2.1 + 0.9 + 0.9)% = 99.9% This is not of concern; there has been a slight rounding effect.

SC 3 I can interpret data displays such as infographics

- The word 'infographic' is an example of a portmanteau, which is a word that includes elements from two different words. In this case, it is a combination of 'information' and 'graphic' because it presents information in a graphic way. The purpose of an infographic is to display a lot of information simply, using as few words as possible.
- A divided bar chart is another type of data display. This is essentially a pie graph drawn in rectangular form. As with a pie graph, there needs to be some concept of whole for this display to be used.
- An area chart is another data display type. An example is shown below.



An area chart combines elements of a line graph with a stacked bar chart. It is important to note that each value is added to the previous elements in the category. It is not like a line graph where the lines can cross over. That is not possible for this type of graph.

When presenting statistical information, it is important to choose a correct display type to match the data. It also a good idea to make your report as interesting as possible. Using a variety of data displays may assist in this objective.

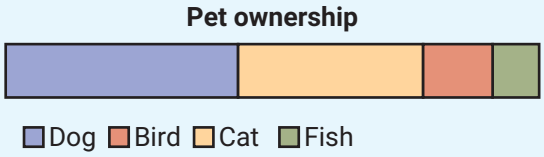
Worked example

Constructing a divided bar chart

Consider the following simplified data set, which shows the number of students in a class who have at least one of the following different types of pets.

Construct a divided bar chart to represent this data.

Pet	Number
Dog	10
Cat	8
Bird	3
Fish	2

THINKING	WORKING
Calculate the total for the categories.	10 + 8 + 3 + 2 = 23
Make a suitable scale choice.	0.5 cm = 1 animal
Draw a rectangle of the required length.	23 cm × 0.5 = 11.5 cm 
Divide the rectangle into the appropriate lengths and include a legend for the graph.	

Practice

ANSWERS Page 654

SC 1 I can use mean, median and range to interpret and compare data sets

1 Consider the following grouped data histogram.

(a) Identify the modal class.

(b) Determine the median class.

(c) Estimate the range for the data set.

(d) Estimate the mean for the data set. State your answer correct to 1 decimal place.

Score	Frequency (f)
75–<80	4
80–<85	12
85–<90	12
90–<95	15
95–<100	6
	$\sum f = 49$

2 Consider the following grouped data histogram.

(a) Identify the modal class.

(b) Determine the median class.

(c) Estimate the range for the data set.

(d) Estimate the mean for the data set. State your answer correct to 1 decimal place.

Score	Frequency (f)
10–13	9
14–17	13
18–21	15
22–25	18
26–29	21
	$\sum f = 76$

3 Refer to the data sets in Questions **1** and **2** to answer this question.

(a) For the data set in Question **1**, place the three measures of central tendency in order from smallest to largest.

(b) What does this suggest about the shape of this data set?

(c) For the data set in Question **2**, place the three measures of central tendency in order from smallest to largest.

(d) What does this suggest about the shape of the distribution?

4 The following data set contains 100 numbers in the range 1–50.

2	25	3	40	36	20	12	26	36	15	17	18	1	15
2	36	2	5	14	38	40	37	46	42	9	38	4	36
24	14	43	33	20	49	22	8	7	3	43	10	44	24
10	23	10	5	5	15	27	30	38	24	33	37	29	41
18	6	28	37	18	36	11	3	23	4	34	5	44	17
49	7	17	23	41	12	45	20	46	16	24	48	12	47
4	15	42	6	45	42	11	31	12	24	27	38	44	42
48	27												

(a) Calculate the mean of this data set.

(b) Construct a frequency table using the class intervals 1–10, 11–20, 21–30, 31–40, 41–50.

(c) Use the frequency table to find an estimate for the mean.

(d) Compare this estimate to the actual value for the mean.

5 The following table shows the scores for the players who played the final two rounds of the 2023 Scottish Golf Open.

Score	Round 3	Round 4
63	1	0
64	1	1
65	5	0
66	5	1
67	16	1
68	11	6
69	12	4
70	11	14
71	6	13
72	5	6
73	1	13
74	1	8
75	0	2
76	1	2
77	1	3
78	0	1
79	0	1
80	0	1

(a) How many players completed each of the final two rounds?

(b) Complete the following table. Write the mean values correct to 2 decimal places.

Statistic	Round 3	Round 4
Mean		
Median		
Mode		
Range		

(c) Put the three measures of centre in ascending order.

(d) Describe the distribution of scores for each of the final two rounds.

(e) In golf, the lower the score, the better. Which round had the better scores?

(f) What does the range suggest?

(g) Draw a back-to-back histogram, with the scores grouped in twos, so that the first range is 63–64.

(h) Does this data display confirm your description of the distribution of the scores, or suggest a different description?

6 Complete each of the following statements by choosing the best response.

(a) When grouping data, it is/is not important that each of the class intervals are the same size.

(b) If the mean, median and mode are all about the same value, then it is reasonable to describe the data set as displaying positive skew/negative skew/symmetry.

(c) If a data set has a range that is bigger than another data set, then it is reasonable to conclude that the first data set is more/less spread out than the second.

(d) If a data set has two different values that each share the highest frequency, then it can/cannot be described as bimodal.

SC 2 I can distinguish between the type of display relevant for categorical data and numerical data

1 The following graph appeared in the print media and shows the trend in the age of landlords in Australia in the years 2001–2021.

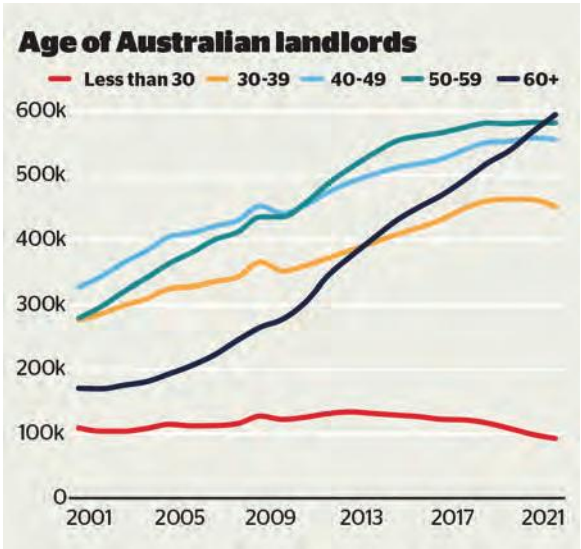
(a) Why is a line graph the best choice to show this information?

(b) Which age group has shown the greatest increase over the period shown?

(c) Suggest a reason why this is the case.

(d) Which age group has fallen in absolute number over this time?

(e) Why are different colours necessary on this graph?



2 Consider the following table that shows the number of cars of various colours in the school car park on a particular day.

Colour	Number
White	18
Red	5
Silver	10
Blue	7
Green	4
Other	22

(a) Why has an 'other' category been included rather than just a longer list of colours?

(b) What is the total number of cars represented?

(c) In a pie graph of this data, exactly how many degrees would represent 1 car?

(d) Why does this make it difficult to draw an accurate graph?

(e) Draw a pie graph of the data.

3 The following table gives the value of the top ten Exports, Goods and Services for the Australian economy for the financial year 2020–21, with the corresponding values for the previous two financial years. The figures are in A$ million. Write answers to 1 decimal place.

Export, Good or Service	2018–19	2019–20	2020–21
Iron ore concentrates	77 520	102 864	152 975
Coal	69 595	54 620	39 196
Natural gas	49 727	47 525	30 427
Education-related travel services	37 824	37 338	27 554
Gold	18 867	24 394	26 105
Beef	9476	11 258	8370
Aluminium ore concentrates	11 358	8875	8155
Copper ore concentrates	5969	6854	7328
Crude petroleum	8494	8568	7001
Wheat	3657	3847	6805

Based on data from Department of Foreign Affairs and Trade website – www.dfat.gov.au. Licensed under CC BY 4.0, https://creativecommons.org/licenses/by/4.0/

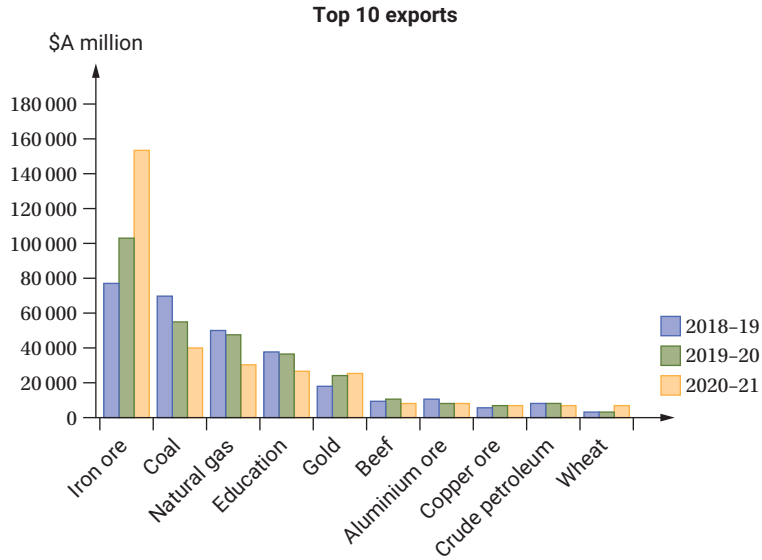
(a) What percentage of the 2018–19 figure for coal is the 2020–21 figure?

(b) What percentage of the 2018–19 figure for wheat is the 2020–21 figure?

(c) What type of data is this?

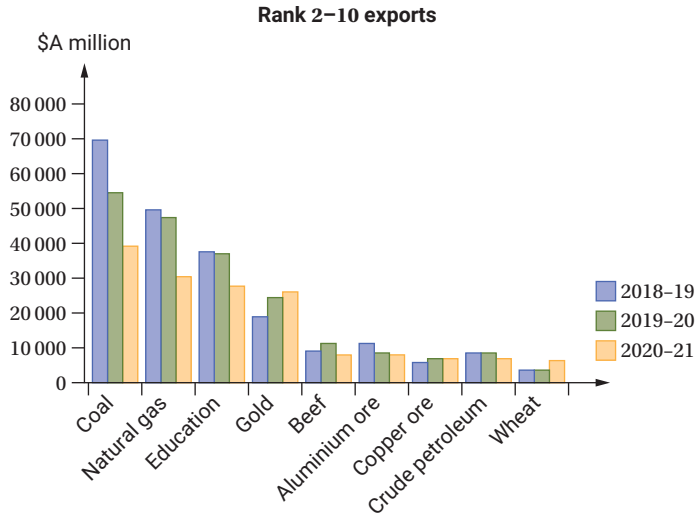
(d) What type of graph would be best to display this data? Why would it be difficult to construct by hand?

The following is a graph based on the data in the table. It has been drawn using technology.



(e) What makes this display difficult to interpret?

The following graph does not include the iron ore data.



(f) Apart from iron ore, which products have seen a rise year on year?

(g) How does this graph make that question easier to answer than the first graph?

4 The following graph appeared in the online media. It shows the trend in the sales of properties held for two years or less across a number of Australian capital cities.



(a) For Sydney, when was the high point on the graph?

(b) Which city shows the lowest value, and when did it occur?

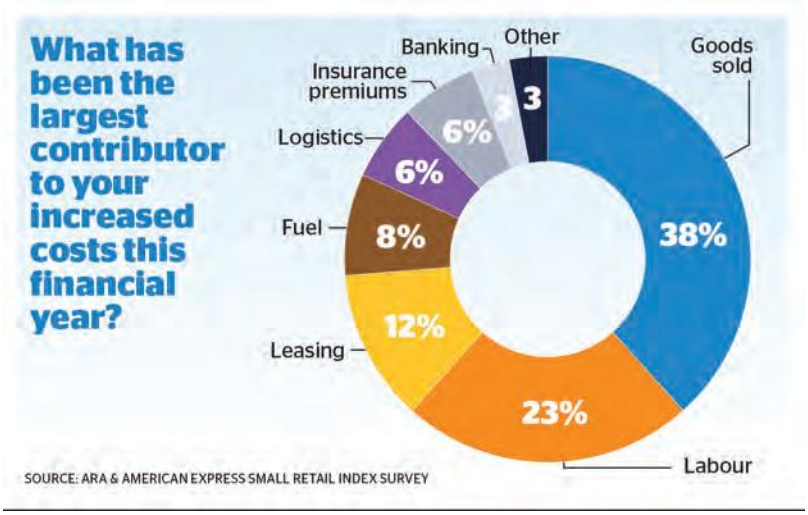
(c) Which city seems to be rising at the fastest rate in 2023?

(d) Which city seems to be rising at the slowest rate in 2023?

(e) What feature on the graph stands out for the period 2015–16?

(f) Suggest a possible reason why a property might be resold within a very short time.

5 The following graph appeared in the print media with a headline 'Small retailers bear brunt of rising costs'.



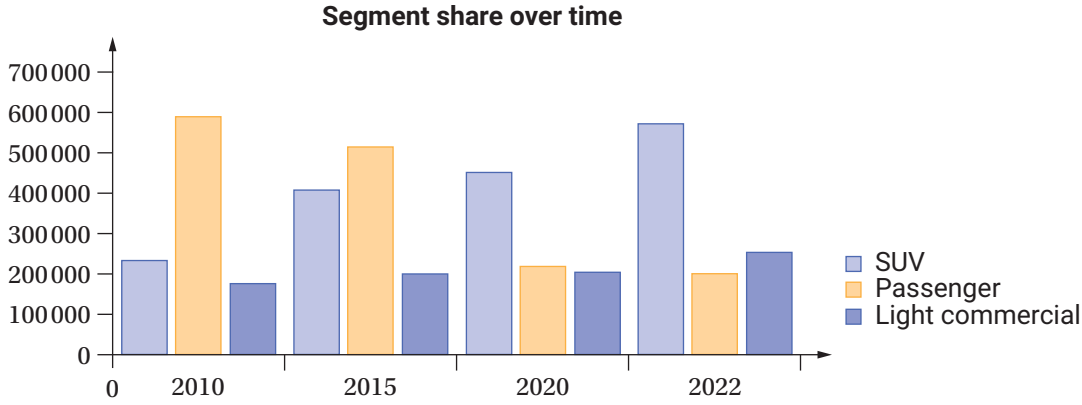
(a) What do the values on the graph add up to? Is this of concern?

(b) How many degrees, to the nearest whole number, does Labour require?

(c) Write a short name for what represents the idea of 'total' for this graph.

(d) If you had the data for two other years, what type of graph would you draw to compare the costs year against year?

6 The following graph shows the number of new vehicle sales in Australia for selected years in the period 2010–22.



(a) What type of data is this?

(b) Which vehicle type, of those shown, has seen the greatest increase in sales?

(c) Which vehicle type, of those shown, has seen the greatest decrease in sales?

(d) Using your best estimates for the numbers, what percentage of 2010 passenger vehicle sales does the 2022 figure represent?

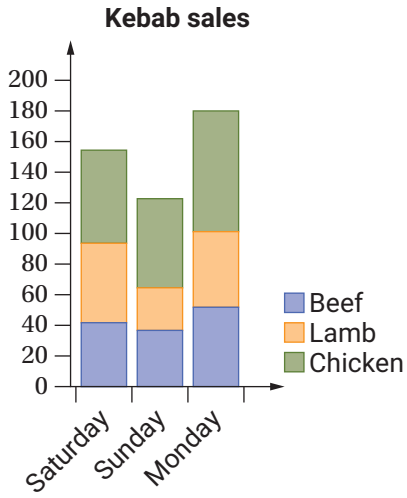
SC 3 I can interpret data displays such as infographics

1 A kebab shop sells three different types: beef, lamb and chicken. The following stacked bar chart shows the sales over the three days of a long weekend.

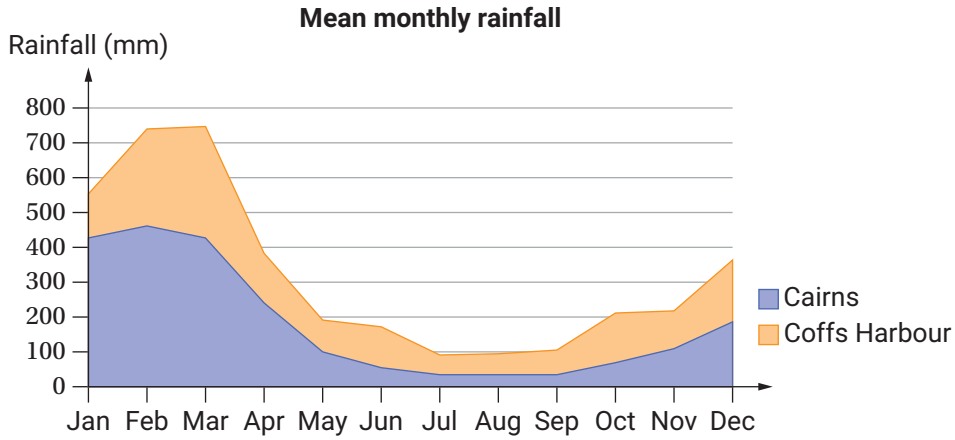
(a) How does this display type make it easy to order the days in ascending order of kebab sales?

(b) On which day were the most beef kebabs sold?

(c) Why is it easier to answer the question about most sales for beef kebabs as opposed to either of the other two types?



2 The following graph shows the mean monthly rainfall for Cairns and Coffs Harbour.



(a) Which month is the wettest in Cairns?

(b) Which month is the wettest in Coffs Harbour?

(c) Which month is the driest in Cairns?

(d) Which month is the driest in Coffs Harbour?

(e) Which month has the most combined rainfall?

(f) Which month has the least combined rainfall?

(g) Explain why some of these questions are easier to answer than others.

3 The following infographic provides data on the number of times different actions or activities took place every minute in 2016.

(a) What is your first impression of this infographic?

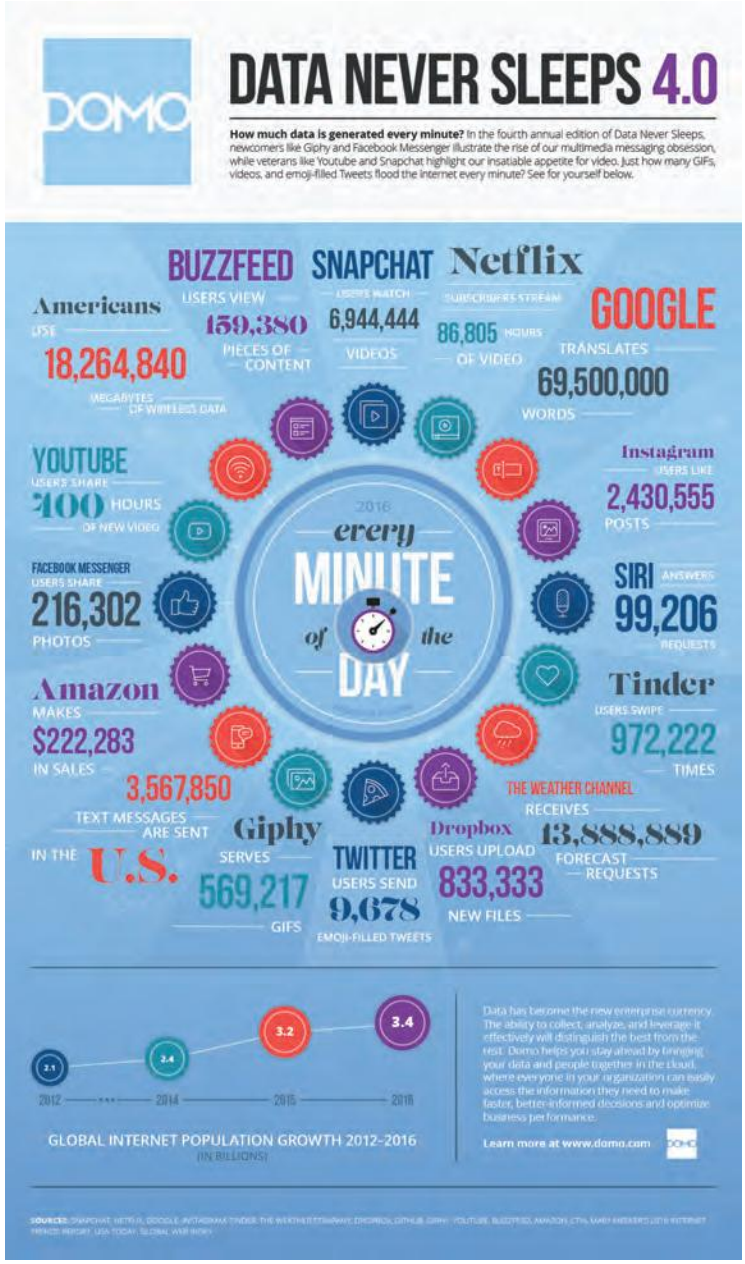
(b) How many weather forecasts did the Weather Channel receive every minute?

(c) How many forecasts is this per day?

(d) How many photographs were shared on Facebook Messenger every minute?

(e) How many photographs is this per week?

(f) How many requests did Siri answer in 2016?



4 At Esme's school, student assessment tasks are graded on the scale: unsatisfactory, satisfactory, good, excellent. The following data display shows how Esme's class was rated on several assessment tasks.

(a) On which task was the number of unsatisfactory grades the highest?

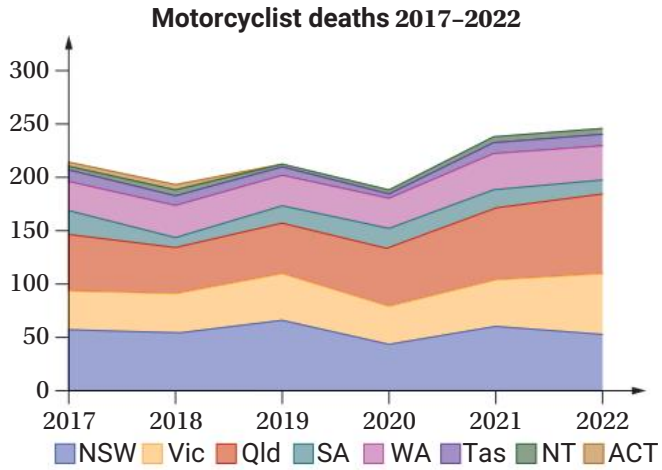
(b) How can you tell there were the same number of students assessed each time?

(c) On which task was the number of good results the greatest?

(d) On which task was the number of excellent results the greatest?



5 The following graph shows the number of motorcycle deaths from 2017–22 for each of the Australian states and territories.



(a) Which year has the lowest number of motorcyclist deaths?

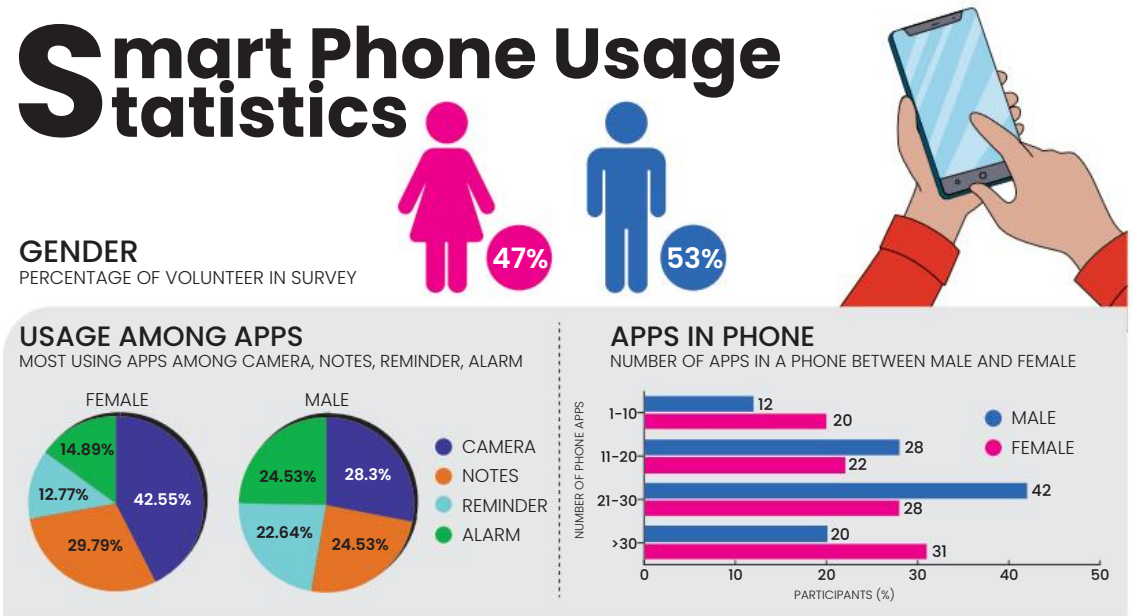
(b) Which year has the most motorcyclist deaths?

(c) Which state or territory had the highest number of motorcyclist deaths in 2021–22?

(d) What can you say about the number of these deaths that occurred in the ACT?

(e) Which state appears to have had the most consistent number of motorcyclist deaths?

6 Consider the following.



(a) Does the display indicate how many people were surveyed to obtain the statistics?

(b) Do men or women in the age range 21–30 have more apps on their phone?

(c) What is the most used app by females?

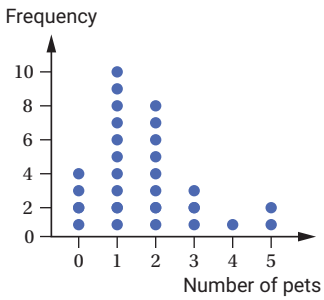
(d) What is the least used app by males?

TOPIC REVIEW

Multiple choice

ANSWERS Page 676

1 The mean, median and mode, respectively, for the data shown are:



A 8.17, 2.5, 10

B 1.75, 1, 1

C 2.5, 2.5, 1.5

D 1.75, 1.5, 1

2 The mean, median and mode of some data sets are given as:

I mean: 16.5, median: 16, mode: 16

II mean: 77.67, median: 85, mode: 85

III mean: 55.2, median: 58, mode: 58

The set(s) displaying a negative skew are:

A I and III **B** II and III **C** III only **D** I and II

3 The mean, median and mode, respectively, for the data shown below are:

A 63.0, 64, 51

B 63.1, 58.5, 55

C 63.0, 58.5, 51

D 63.1, 64, 55

Stem	Leaf
3	8 8 9
4	0 4 7
5	1 1 1 1 2 5
6	2 6 6 8 9
7	4 6 6
8	2 5 7 7
9	2 2

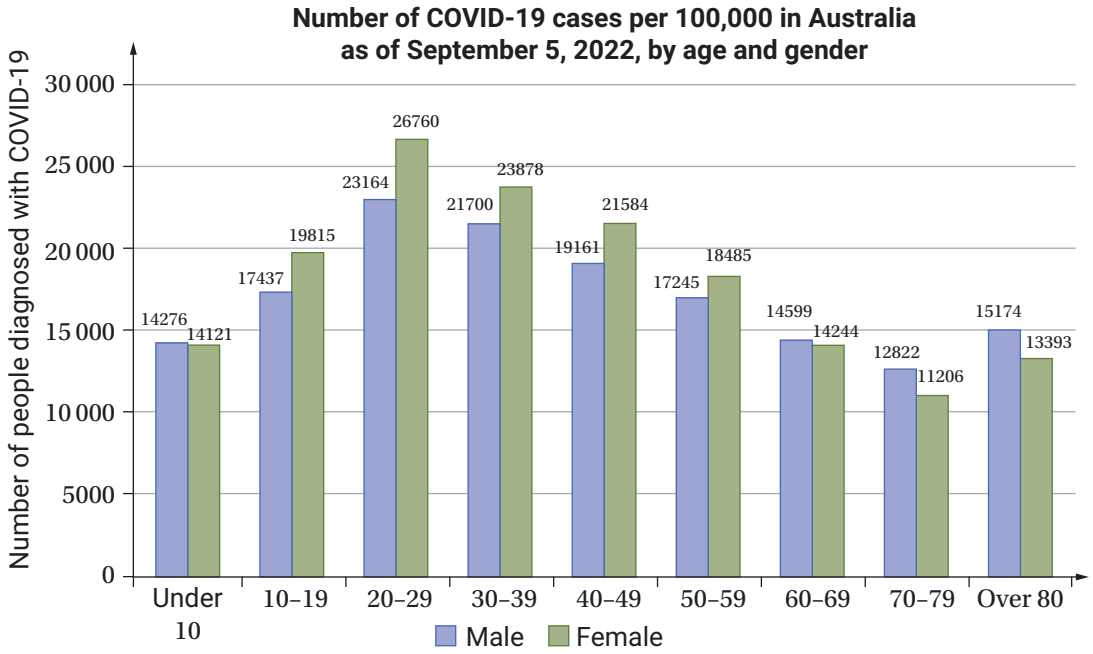
4 The median weights (kg) for 8C and 10B, respectively, are:

Weights (kg)

Leaf 8C	Stem	Leaf 10B
9 9 8 6 5	6_H	
4 4 3 2 2 1 1 1 0 0	7_L	3 4 4
8 7 7 5 5 5 5 5	7_H	5 5 6 7 7 8 8 9 9 9
4 3 2 2 0	8_L	0 0 0 1 2 4 4 4 4 4
5 5	8_H	5 7 9 9
	9_L	2 3

A 74, 79 **B** 74, 79.5 **C** 74.5, 80.0 **D** 75, 80.0

5 The highest number of COVID-19 cases per 100 000 by age group for males and females, respectively, is:



Source: Based on data from © Statista 2024, Statista Research Department, 3 April, 2024

A 23 164 and 26 760 **B** 26 760 and 23 878

C 23 878 and 21 700 **D** 21 700 and 26 760

6 For the grouped data in the frequency table, the median class is:

Score	Frequency (f)
25–<35	1
35–<45	5
45–<55	40
55–<65	30
65–<75	22
	$\sum f = 98$

A 35–<45 **B** 45–<55 **C** 55–<65 **D** 65–<75

7 Which of the following statements is *not* true?

A For grouped data, the intervals should be the same size.

B When the mean, median and mode are all about the same value, the data set is described as symmetrical.

C A greater value for the range means the data is more spread out.

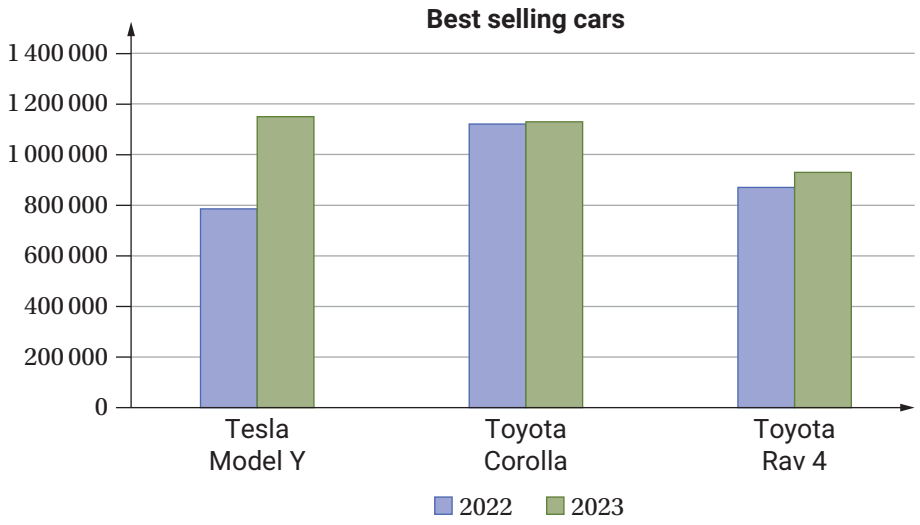
D If a data set has two different values that share the same frequency, then it is described as bimodal.

8 If a pie graph is redrawn as a column graph, what visual effect will be lost?

A The relative amounts for each category **B** How the parts make up the whole

C How the parts compare to each other **D** Any trend in the data

9 The following graph shows the number of new vehicle sales globally for the currently best selling cars in the world.



Source: Based on data from carexpert.com.au

The vehicle types, of those shown, that have seen the greatest increase and the least increase, respectively, in sales are:

A Toyota Rav 4, Toyota Corolla **B** Tesla Model Y, Toyota Corolla

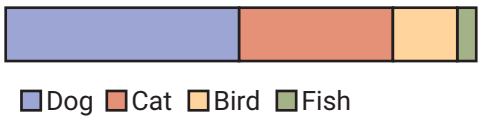
C Toyota Rav 4, Tesla Model Y **D** Tesla Model Y, Toyota Rav 4

10 Consider the following data set that shows the favourite pets of the students in a class.

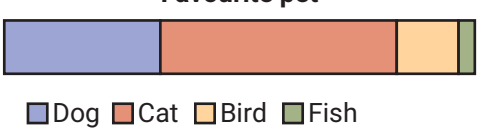
Favourite pet	Frequency
Dog	15
Cat	10
Bird	4
Fish	1

The divided bar chart that matches the data is:

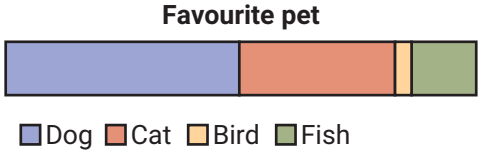
A



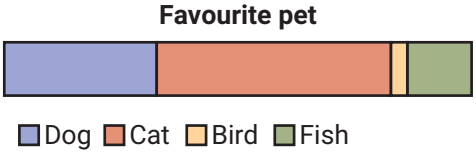
B



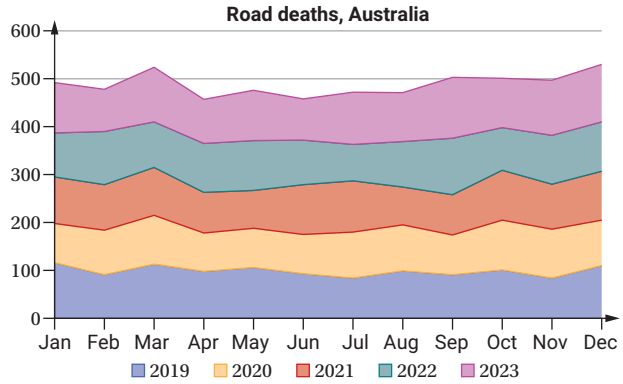
C



D



11 The following graph shows the monthly road deaths for 2019–2023 in Australia.



The smallest number of deaths in a calendar month in the five years was in July 2022, which can be seen as:

A a peak in the top of the 2022 section.

B a dip in the bottom of the 2022 section.

C the lowest point on the graph.

D the shortest vertical distance of any coloured section.

Short answer

1 Consider the following data set:

9, 10, 10, 11, 8, 12, 11, 10, 9, 9, 11, 12, 10, 10, 8, 13, 9, 11, 9, 11, 9, 10, 8, 13, 8, 9, 10, 9, 10, 9, 8, 9, 12, 9, 9, 11, 9, 9, 12, 9

(a) Construct a frequency table to show the data.

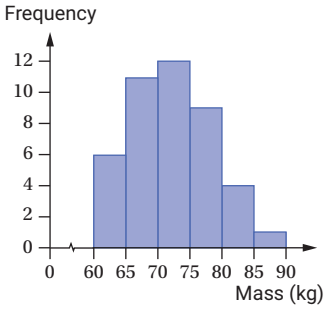
(b) Calculate the mean for the data set.

(c) Calculate the median for the data set.

(d) Calculate the mode for the data set.

(e) Is the data set symmetrical, or does it show positive skew or negative skew? How do the values of the mean, median and mode help to confirm this?

2 Consider the following grouped data histogram, where the first class is 60–<65 kg.



(a) Complete a frequency table for this data set.

(b) Calculate the mean for this data set. State the answer correct to 1 decimal place.

(c) In which data range does the median occur?

(d) What is the modal data range?

(e) Is this a symmetrical distribution or does it display negative skew or positive skew?

3 The following stem-and-leaf plot shows the marks obtained on a mathematics test.

(a) What is the mean of this data set? State your answer correct to 1 decimal place.

(b) What is the median of this data set?

(c) What is the mode of this data set?

(d) How would you describe the shape of the distribution of this data set?

Stem	Leaf
4	8 8
5	4 6
6	5 9
7	3 5 5
8	1 4 4 6
9	8 8 9 9
10	0 0

4 Consider the following data, which shows the scores obtained by students in two different classes on a test marked out of 75.

Class A: 55, 49, 48, 68, 46, 67, 68, 46, 45, 57, 56, 53, 49, 53, 53, 57, 68, 67, 65, 64, 62, 48

Class B: 47, 72, 36, 37, 38, 38, 63, 41, 70, 58, 42, 42, 44, 46, 38, 47, 36, 52, 57, 61, 63, 68, 70

(a) Use the data to construct a back-to-back stem-and-leaf plot for the scores obtained by the students.

(b) Describe the shape of each distribution.

(c) Determine the median for each class.

(d) Calculate the mean for each class. State your answers correct to 1 decimal place.

5 The following are the officially listed heights (cm) for players in the 2024 squads for two teams in the AFL.

Carlton:

194, 188, 180, 184, 188, 186, 188, 180, 195, 200, 190, 201, 193, 180, 187, 188, 192, 184, 173, 189, 186, 193, 195, 186, 183, 202, 183, 187, 194, 201, 190, 202, 194, 183, 184, 182, 199, 187, 202, 194, 178, 188, 179, 210, 184

Lions:

183, 203, 202, 204, 182, 183, 192, 184, 187, 185, 188, 197, 195, 181, 183, 183, 205, 204, 184, 177, 185, 185, 201, 183, 191, 185, 192, 182, 178, 178, 190, 191, 188, 175, 186, 184, 182, 193, 189, 196, 182, 180, 200, 206, 184, 192, 187

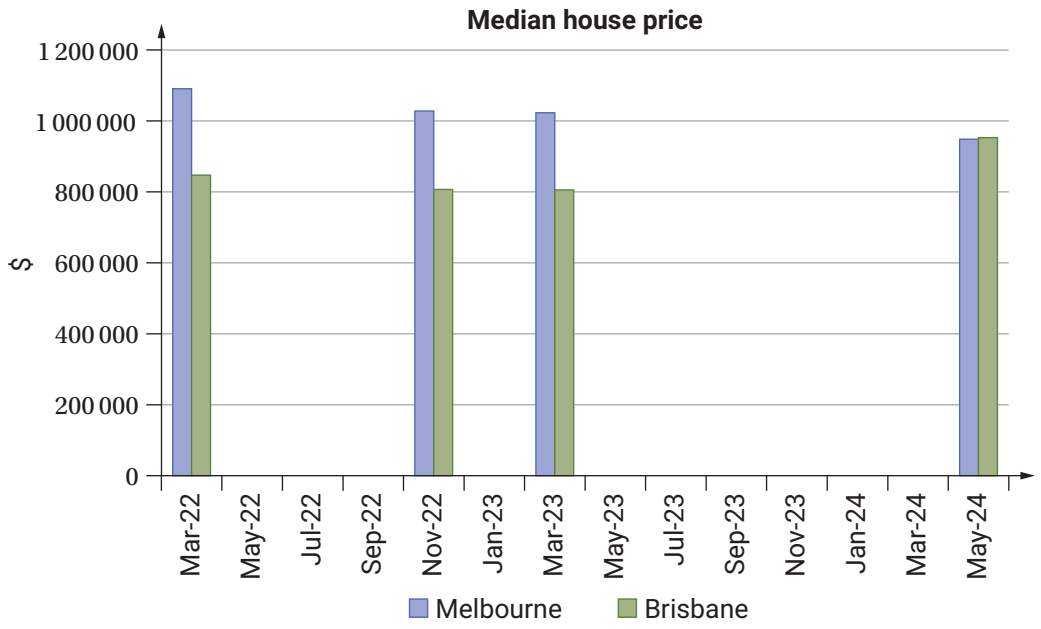
(a) Use the data to construct a back-to-back stem-and-leaf plot for the heights of the two teams. As the values are so tightly clustered, use stems like 17_L (170–174) and 17_H (175–179), and so on.

(b) Determine the median height for each team.

(c) Calculate the mean height for each team. State your answers correct to 1 decimal place.

(d) What is the modal class for each team?

6 The following graph shows the median house prices in both Melbourne and Brisbane from early 2022 to mid-2024.

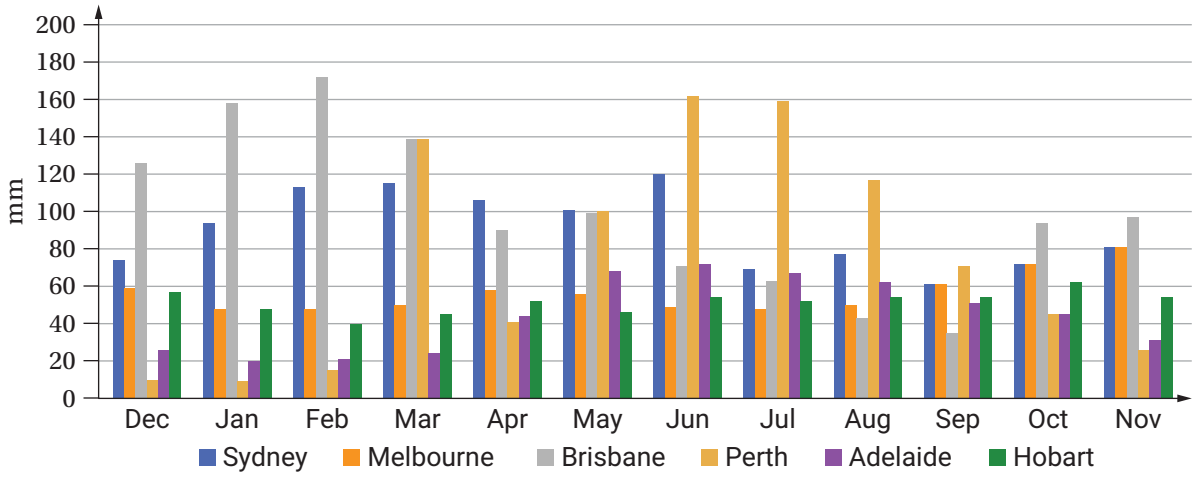


(a) Describe the trend in Melbourne house prices.

(b) Describe the trend in Brisbane house prices.

(c) If the values on the graph for May 2024 are Melbourne: $948 879 and Brisbane: $953 028, write a sentence to describe the comparative values of the median house prices for these two cities over the period shown in the graph.

7 The average monthly rainfall for the state capitals of Australia is shown in the following graph.



(a) Which two cities have a winter wet season?

(b) Which city has a winter dry season?

(c) What four consecutive months give Sydney most of its rain?

(d) Describe the similarity in rainfall patterns for Melbourne and Hobart.

8 Consider the following grouped data histogram.

(a) Identify the modal class.

(b) Determine the median class.

(c) Estimate the range for the data set.

(d) Estimate the mean for the data set. State your answer correct to 1 decimal place.

Score	Frequency (f)
22–25	7
26–29	12
30–33	20
34–37	22
38–41	16
	$\Sigma f = 77$

9 Consider the following table that shows the number of cars of various colours in the school car park on a particular day.

(a) What is the total number of cars represented?

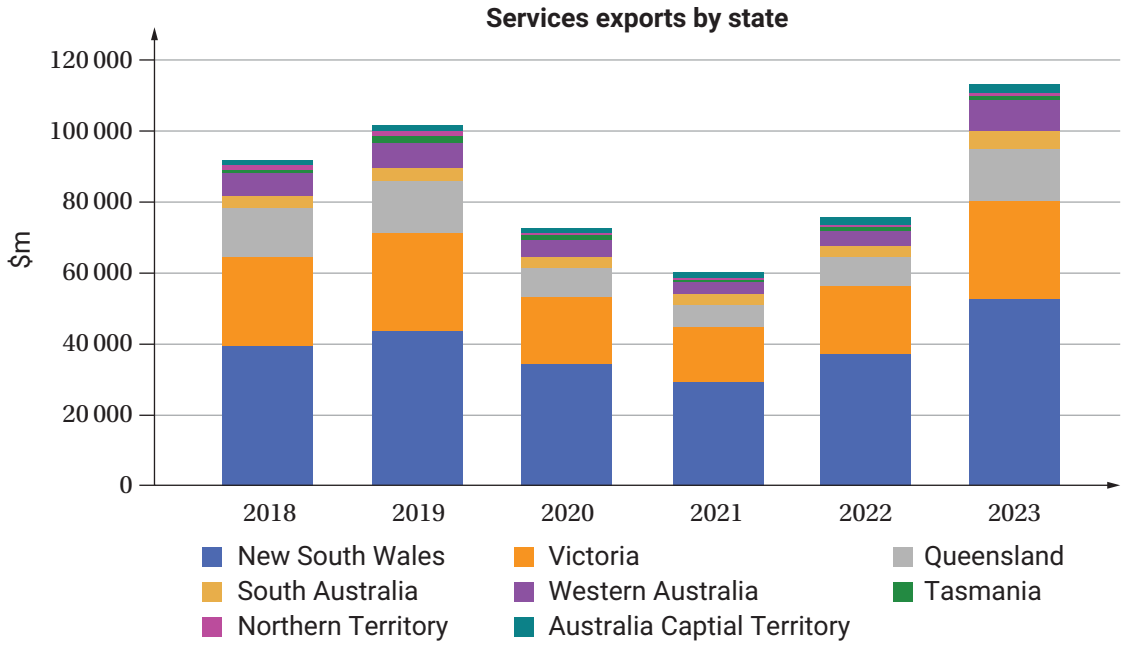
(b) In a pie graph of this data, exactly how many degrees would represent 1 car?

(c) Draw this graph, using technology if possible.

Colour	Number
White	25
Red	10
Silver	15
Blue	6
Green	2
Other	11

15

10 The graph shows the value of services exports by state and territory over six years.

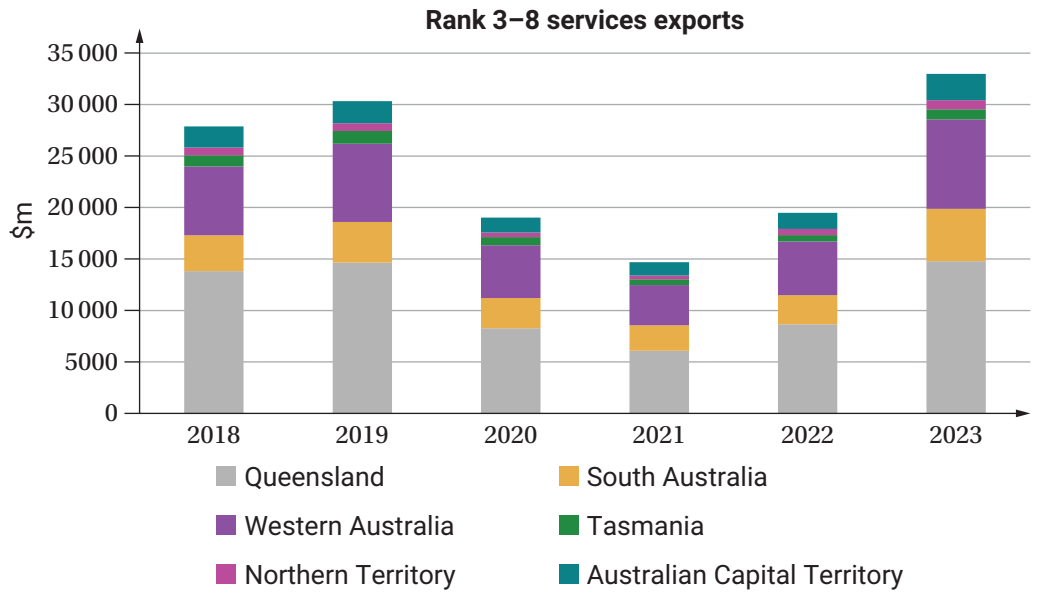


Source: Australian Bureau of Statistics, International Trade

(a) Describe the success overall of Australian services exports over the six years.

(b) In 2023, New South Wales, followed by Victoria, had the highest value of services exports. Express each as a percentage of the Australian figure, to the nearest 5%.

The following graph excludes the data for the two largest services exporting states, to make the remaining values easier to read.

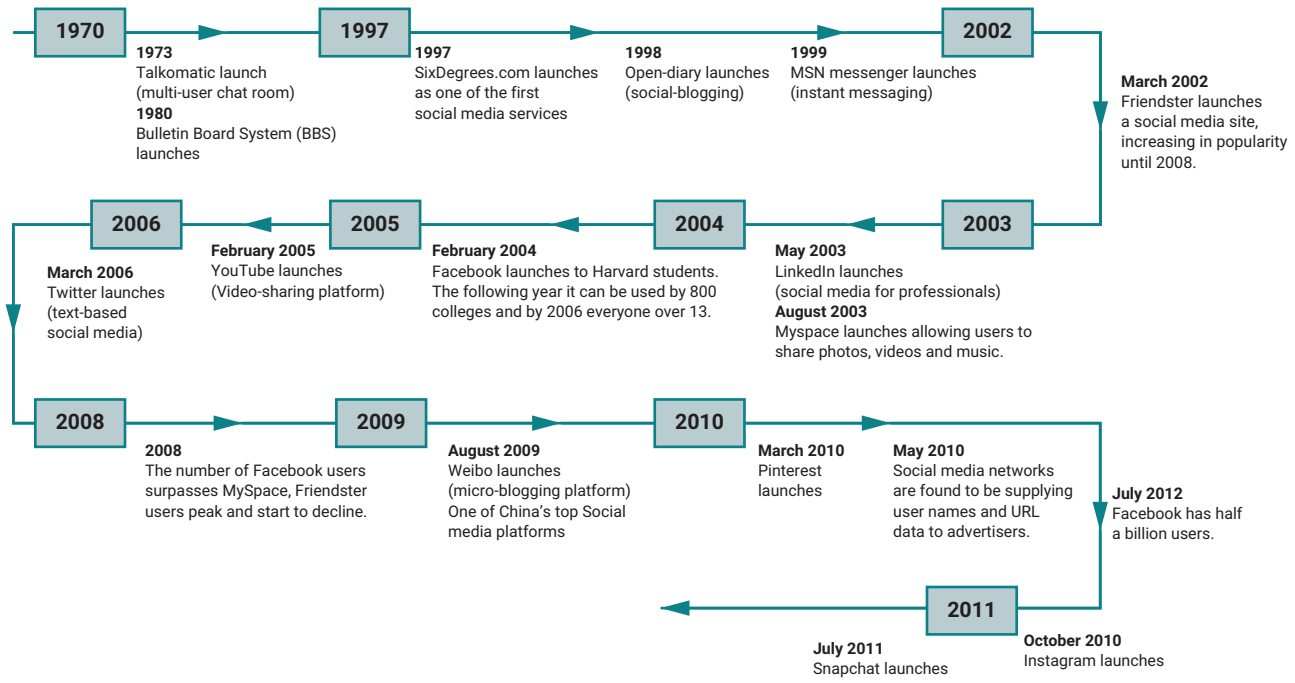


Source: Australian Bureau of Statistics, International Trade

(c) In 2023, all states and territories experienced a recovery in their services exports, with the strongest growth in South Australia and Queensland. Use the graph to estimate the percentage growth in these two states, to the nearest 10%.

11 The following infographic provides data on the history of social media.

The evolution of social media



(a) Comment on how the timeline is to be read.

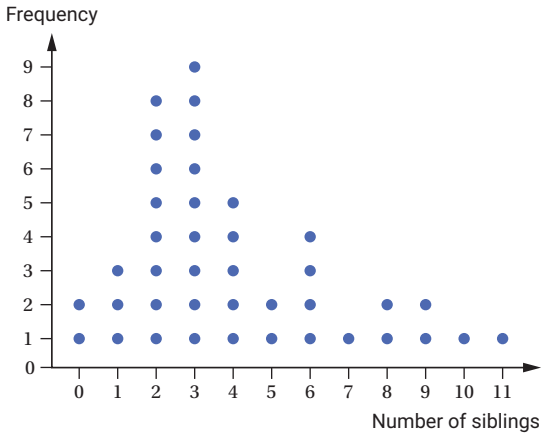
(b) Comment on the scale of the timeline.

(c) What type of service was launched in 1999?

(d) When did Facebook's popularity overtake that of MySpace?

Extended response

1 The following dot plot gives the number of siblings for each student in a class of 40 in the 1960s.



(a) Is the distribution symmetrical, positively skewed or negatively skewed?

(b) Draw a frequency table for the data.

(c) Determine the mode.

(d) Determine the median.

(e) Calculate the mean.

(f) List the raw data, in order from smallest to largest.

2 The following are the average number of disposals per game during the 2024 AFL season for Carlton and the Lions.

Carlton:

12, 12, 21, 15, 17, 13, 27, 14, 17, 15, 22, 8, 10, 9, 16, 31, 6, 16, 6, 10, 14, 23, 2, 15, 13, 22, 13, 13, 15, 9, 18, 9, 11

Lions:

11, 8, 17, 13, 25, 25, 22, 12, 8, 26, 15, 18, 8, 8, 11, 11, 2, 17, 8, 15, 5, 11, 14, 11, 17, 6, 15, 11, 13, 11, 11, 20, 12

(a) Draw an ordered back-to-back stem-and-leaf plot for the data using a value range of 10.

(b) Redraw the plot, this time using a data range of 5.

(c) Determine the median number of disposals for each team.

(d) Calculate the mean number of disposals for each team, correct to 1 decimal place.

3 The following table, which runs over two lines, shows a summary of the scores in the final two rounds for the men's field of the golf Australian Open.

Score	65	66	67	68	69	70	71	72	73
Frequency (Round 3)	3	2	3	5	12	12	11	4	4
Frequency (Round 4)	1	4	2	6	4	9	7	10	7

Score	74	75	76	77	78	79	80	81
Frequency (Round 3)	3	6	0	1	1	0	0	0
Frequency (Round 4)	4	3	4	1	2	0	2	1

(a) How many players competed in each of the final two rounds of this event?

(b) Group the data in threes, with the first group being 65–67, and construct a frequency table for the data.

(c) Draw a back-to-back histogram for the grouped data.

(d) Calculate the mean score for each round, correct to 1 decimal place.

(e) Determine the median score for each round.

(f) In golf, the lower the score the better. Which day seemed to have the better scores?

(g) Describe the shape of the data distribution for each of Rounds 3 and 4.

TOPIC

16

Probability (compound events, relative frequencies)

Recall 543

16.1 Use tables and tree diagrams to represent two- and three-step chance events 544

16.2 Calculate the probability of compound events 555

16.3 Estimate the probability of events 565

16.4 Design and conduct experiments and simulations 576

Topic review 584

Why learn this?

Learning to calculate the probability of compound events makes it easier to analyse situations involving uncertainty and make informed decisions. This knowledge is applicable in various real-life scenarios, such as risk assessment in finance, strategy in games and sports, and interpreting statistical data in scientific studies. Understanding the outcomes of combined events and their likelihood can also be beneficial in technology fields, where algorithms for predictive modeling and artificial intelligence rely on probability concepts. Additionally, the ability to estimate probabilities from experimental or historical data is essential in fields like meteorology, insurance and market research, where predictions and decisions are made based on past occurrences.

RECALL

ANSWERS Page 655

I can write fractions in their simplest form

1 Simplify these fractions.

(a) $\frac{8}{20}$ **(b)** $\frac{50}{75}$ **(c)** $\frac{50}{85}$ **(d)** $\frac{54}{96}$

I can write the product of fractions in simplest form

1 Calculate the product of these fractions, leaving your answer in simplest form.

(a) $\frac{1}{5} \times \frac{2}{3}$ **(b)** $\frac{2}{5} \times \frac{3}{4}$ **(c)** $\frac{1}{8} \times \frac{6}{10}$ **(d)** $\frac{4}{5} \times \frac{4}{20}$

I can identify the complement probability

1 Write the complement in simplest form for each of these probabilities.

(a) $\frac{3}{7}$ **(b)** $\frac{5}{9}$ **(c)** $\frac{2}{8}$ **(d)** $\frac{6}{18}$

I can write fractions in their equivalent decimal form

1 Write these fractions as equivalent decimals.

(a) $\frac{22}{100}$ **(b)** $\frac{12}{50}$ **(c)** $\frac{42}{54}$ **(d)** $\frac{132}{250}$

I can list the sample space for all possible outcomes of an event

1 List the sample space for each of the following.

(a) A standard cube die is rolled.

(b) A tile is drawn from a bag containing plain, patterned and textured tiles.

(c) A spinner with even numbers less than thirteen is spun.

(d) A goldfish is selected from a tank with orange, white and yellow fish.

I can write probabilities

1 A random number generator produces twenty numbers ranging in value from 0 to 9. Write the probability of getting the following, as fractions in simplest form.

(a) The number 8 **(b)** The numbers 2 or 5 **(c)** An odd number

(d) A factor of 24 **(e)** Not a number less than 6

16.1 Use tables and tree diagrams to represent two- and three-step chance events

Learning intention: To be able to use tables and tree diagrams to represent two- and three-step chance events

Success criteria:

SC 1 I can create tables to represent the outcomes of a two-step chance event with replacement.

SC 2 I can use a tree diagram to represent the outcomes of a two- and three-step chance event with replacement.

SC 3 I can use a tree diagram to represent the outcomes of a two- and three-step chance event without replacement.

Lesson warm-up

Does replacement matter?

Sally has a bag of 3 red and 2 green marbles.



Sally has been investigating different probability experiments with drawing out marbles. Tom has told Sally that when drawing out 2 marbles, it doesn't make a difference to the experimental probabilities whether you replace the first marble or not. Your task is to investigate whether Tom's claim is correct by completing the following steps.

1. Create a list with 10 spaces and label it 'Without replacement'. Draw one marble out of the bag and record the colour in your list. Without replacing the first marble, draw another marble out of the bag and record the colour next to the first colour in your list. Replace both marbles. Repeat this process 9 more times to fill out your list of 10 results.
2. Create a list with 10 spaces and label it 'With replacement'. Draw one marble out of the bag and record the colour in your list. Replace the first marble, draw another marble out of the bag and record the colour next to the first colour in your list. Replace the marble. Repeat this process 9 more times to fill out your list of 10 results.
3. List out your probabilities for each unique outcome. You may like to do this as a percentage, decimal or fraction.

Compare your results from both experiments. Does replacing the marble have an impact on your results? Why do you think this is? Therefore, is Tom's claim correct?

SC 1 I can create tables to represent the outcomes of a two-step chance event with replacement

A single-step probability experiment involves only one action, such as flipping one coin or rolling a die, to determine an outcome.

A two-step probability experiment involves two actions, such as flipping two coins, to determine an outcome.

In two-step experiments, such as choosing two cards from a deck, the probability of each outcome depends on whether the cards are drawn with or without replacement. With replacement, the first card is returned to the deck before the second is drawn. Without replacement, the card is not returned. Without replacement means the probabilities in the second event are dependent upon the result of the first event.

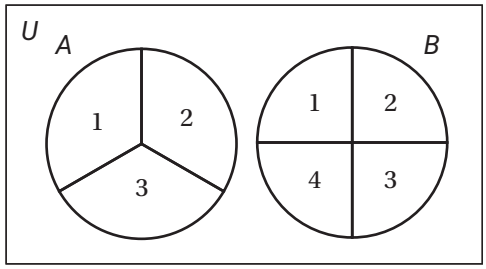
Recall that $\Pr(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$.

Worked example

Calculating the probabilities from two spinners

These two spinners are spun once each, and the product of the results on the spinners is then calculated.

Determine the probability that the product of the two numbers will be odd by creating an array.



THINKING	WORKING
Draw an array (table) with the outcomes for Spinner A across a row and the outcomes for Spinner B down a column.	
Write the product of the row by column in each cell. The highlighted cell is $2 \times 3 = 6$	
Count the total number of outcomes and the number of odd numbers.	There are 4 odd numbers out of a total of 12 numbers.
Write the probability.	$\Pr(\text{an odd product}) = \dfrac{4}{12} = \dfrac{1}{3}$

		Spinner B			
		1	**2**	**3**	**4**
Spinner A	**1**				
	2				
	3				

		Spinner B			
	×	**1**	**2**	**3**	**4**
Spinner A	**1**	1	2	3	4
	2	2	4	6	8
	3	3	6	9	12

16

SC 2 I can use a tree diagram to represent the outcomes of a two- and three-step chance event with replacement

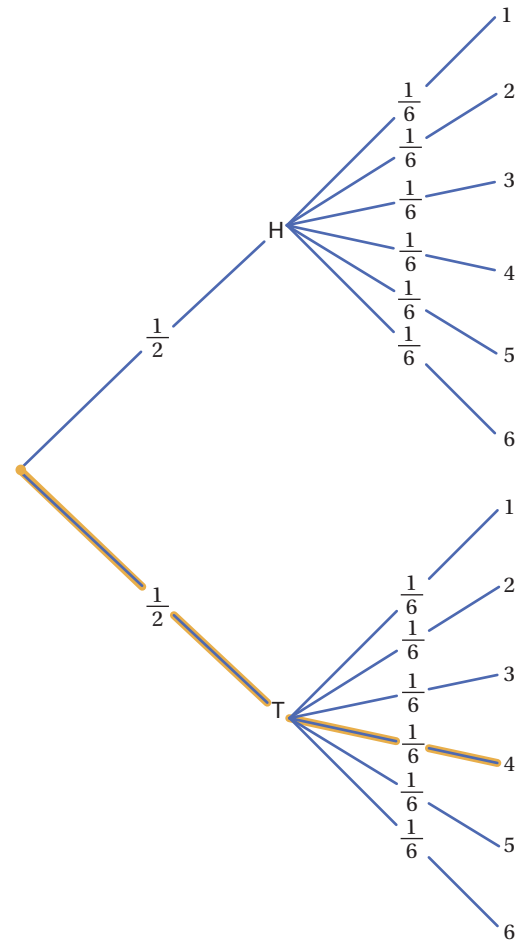
For probability problems with a limited number of outcomes, a tree diagram provides a useful visual aid of the situation.

To draw a tree diagram, follow the instructions below.

1. Start with a node.
2. From the node, draw lines (or branches) that correspond to the number of outcomes in the sample space for the first stage. For example, throwing a coin has two outcomes: a head or a tail.
3. Write the probability of each outcome on the branches.
4. The tree diagram for the first stage is now complete. You follow the same process for the second stage. The second stage is the result of rolling a standard 6-sided die.
5. The probability of each outcome on a fair die is equal, so each number has a probability of $\frac{1}{6}$.

To determine the probability of a tail and a 4, multiply the numbers on the relevant branches.

$$\Pr(\text{T and } 4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

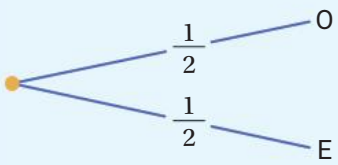
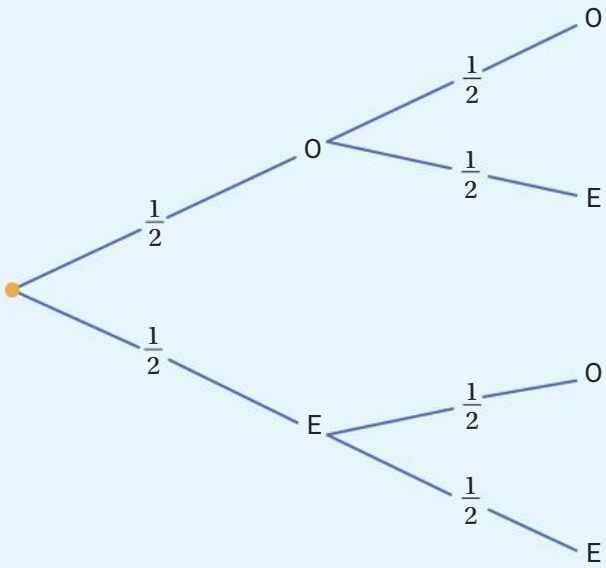


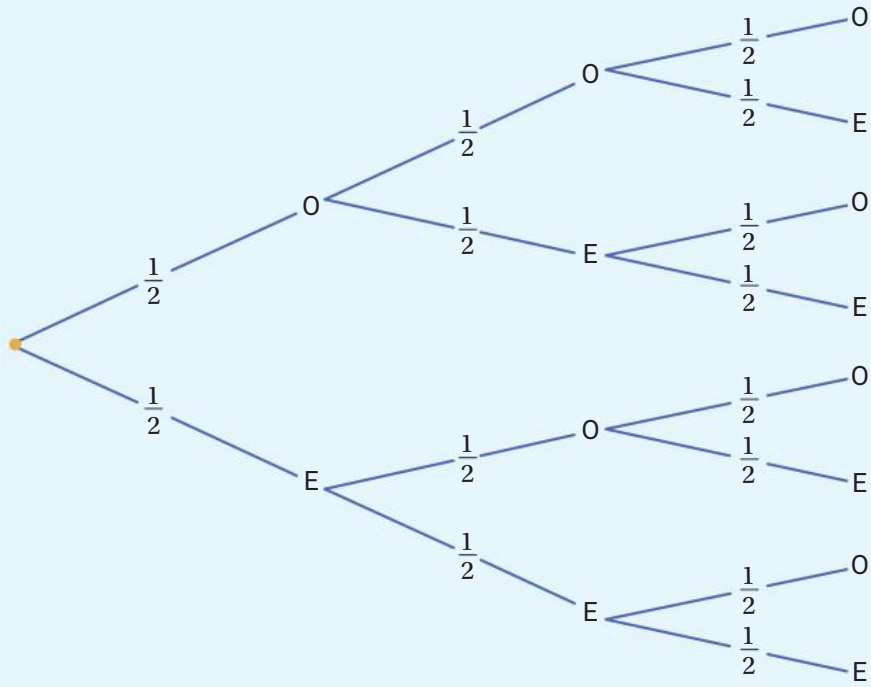
Worked example

Using a tree diagram, with replacement

Three cards are selected from 10 cards numbered 1 to 10 one at a time, with replacement.

(a) Use a tree diagram to determine the probability that all the cards are odd numbers.

THINKING	WORKING
The first draw has 2 possibilities: odd or even, so draw a tree with two branches. Determine the probability of each outcome in the branches.	For the set of 10 cards, the even-numbered cards are 2, 4, 6, 8, 10. There are 5 even-numbered cards, so $\Pr(\text{even}) = \frac{5}{10} = \frac{1}{2}$. Similarly, the odd-numbered cards are 1, 3, 5, 7, 9, so $\Pr(\text{odd}) = \frac{5}{10} = \frac{1}{2}$. 
Add two branches to each existing branch for the draw of the second card. Because the cards are drawn with replacement, the probabilities for drawing odd and even cards remain the same.	

Add two branches to each existing branch for the draw of the third card.	
List the outcomes for each stage of the experiment at the end of each branch.	Multiply the probabilities along each branch. $\Pr(OOO) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
Write the answer.	$\Pr(OOO) = \frac{1}{8}$

(b) Use complementary events to calculate the probability that at least one of the cards is even.

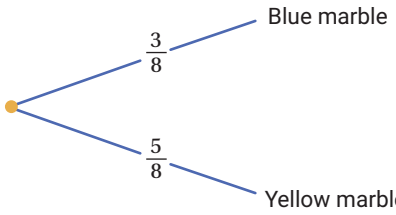
THINKING	WORKING
Identify the complement of the event.	All outcomes contain at least one even card, except for the outcome {OOO}. The complement of the event (at least one card is even) is three odd cards.
Use the complement to calculate the probability of the event.	$\Pr(\text{at least one even}) = 1 - \Pr(OOO)$ $= 1 - \frac{1}{8}$ $= \frac{7}{8}$
Write the answer.	$\Pr(\text{at least one even}) = \frac{7}{8}$

SC 3 I can use a tree diagram to represent the outcomes of a two- and three-step chance event without replacement

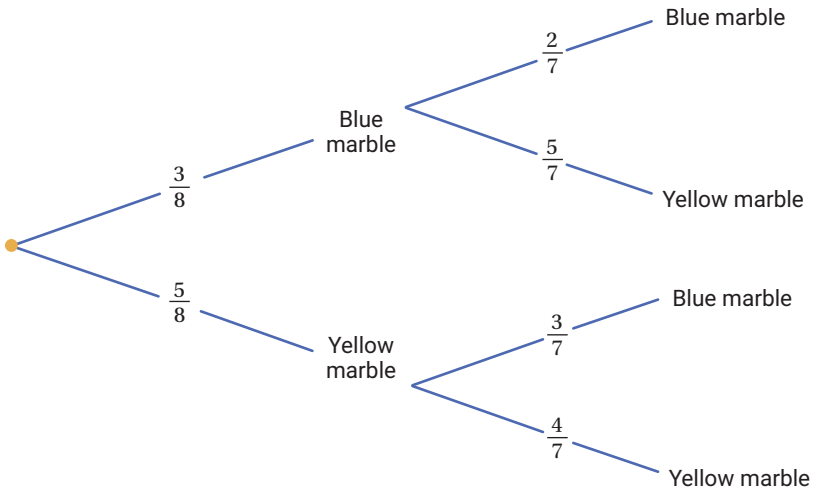
In problems where items are removed and not replaced, you need to account for the reduced number of items in your probability calculations and tree diagram.

Consider a bag with 3 blue marbles and 5 yellow marbles. Two marbles are removed from the bag. What is the probability of getting a blue marble, followed by a yellow marble?

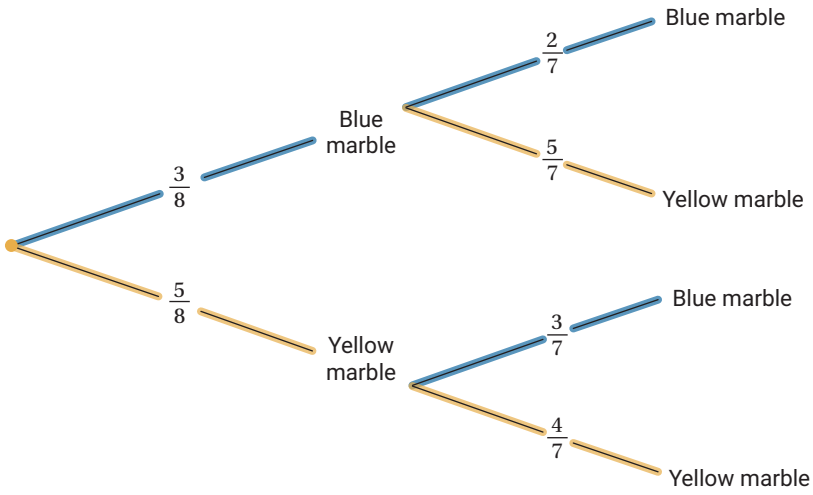
The tree diagram for the first marble is



For the second marble, you now have 7 marbles left in the bag. If you drew a blue marble at the start, you now have 2 blue marbles and 5 yellow marbles. If you drew a yellow marble at the start, you now have 3 blue marbles and 4 yellow marbles.



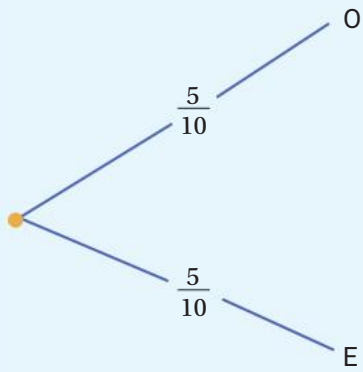
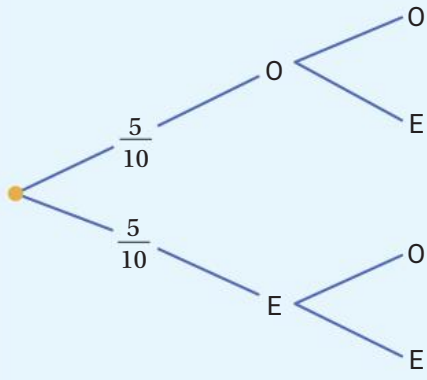
So the probability of getting a blue marble, followed by a yellow marble is $\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$.

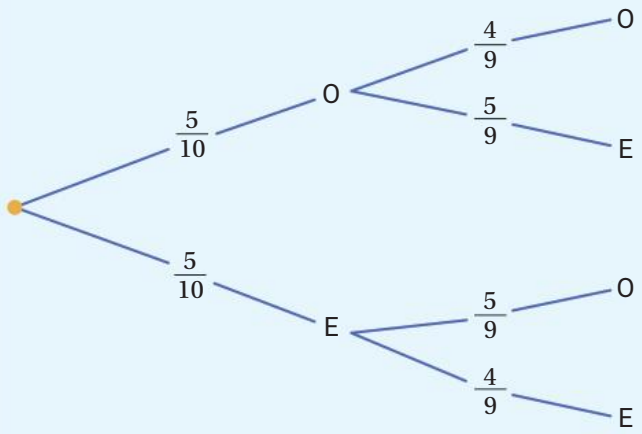


Worked example

Using a tree diagram for drawing cards without replacement

Two cards are selected from 10 cards numbered 1 to 10 without replacement. Use a tree diagram to determine the probability that both cards are odd numbers.

THINKING	WORKING
The first draw has 2 possibilities, odd or even, so draw a tree with two branches. Determine the probability of each outcome in the branches.	For the set of 10 cards, the even-numbered cards are 2, 4, 6, 8, 10. There are 5 even-numbered cards, so $\Pr(\text{even}) = \frac{5}{10}$. Similarly, the odd-numbered cards are 1, 3, 5, 7, 9, so $\Pr(\text{odd}) = \frac{5}{10}$.  Note: In the working above, the fraction $\frac{5}{10}$ is not simplified because you can use the denominator and numerator to keep track of the number of cards you have.
Add two branches to each existing branch for the draw of the second card.	

Write the probability of each event along the associated branch. Without replacement is different to what has been done before and requires some thinking. Firstly, after the first draw, only nine cards remain, so the denominator becomes 9. Secondly, the number of odd or even cards changes depending upon what happened on the first draw. If an odd card came out the first time, then there are 5 even cards and only 4 odd cards remain. If an even card came out the first time, then there are 5 odd cards and only 4 even cards remain.	
Multiply the probabilities along the branches that lead to OO.	$\frac{5}{10} \times \frac{4}{9} = \frac{20}{90} = \frac{2}{9}$
Write the answer.	The probability that both cards are odd numbers is $\frac{2}{9}$.

16

Practice

ANSWERS Page 656

SC 1 I can create tables to represent the outcomes of a two-step chance event with replacement

1 Penny has two pencil cases. Both cases contain a pink, a blue and a yellow highlighter.


Case A


Case B

One highlighter is selected at random from each bag.

(a) Show the sample space using a table.

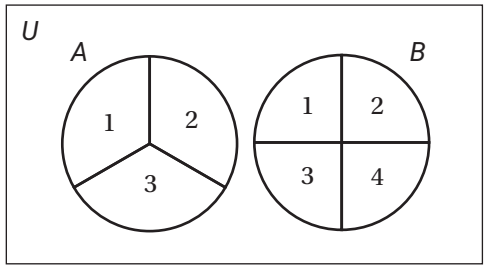
(b) Calculate the probability of drawing two pink highlighters.

(c) Calculate the probability of drawing two blue highlighters.

(d) Calculate the probability of drawing two yellow highlighters.

(e) Calculate the probability of drawing different coloured highlighters.

2 These two spinners are spun once each and the sum of the resulting numbers is recorded.



(a) Copy and complete the table.

		Spinner B			
		1	2	3	4
Spinner A	1				
	2				
	3				

(b) Determine the probability that the sum of the two numbers will be greater than 4.

(c) Determine the probability that the sum of the two numbers will be less than 4.

(d) Determine the probability that the sum of the two numbers will be equal to 4.

(e) What do you notice about the total of your answers from parts **(b)**, **(c)** and **(d)**? Explain why this happened.

3 The letters from the word EACH are placed in a bag. Two letters are drawn, and the first letter is placed back in the bag before the second is drawn.



(a) Show the sample space using an array.

(b) Calculate the probability of drawing two Es.

(c) Calculate the probability of drawing two Hs.

(d) Calculate the probability of drawing two vowels.

(e) Calculate the probability of drawing different letters.

4 The letters from the word MATH are placed in a bag. Two letters are drawn, replacing the first one before drawing the second.



(a) Show the sample space using an array.

(b) Calculate the probability of drawing two Ms.

(c) Calculate the probability of drawing different letters.

(d) Calculate the probability of drawing two vowels.

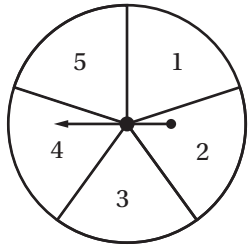
(e) Calculate the probability of not drawing the letter A.

(f) Calculate the probability of drawing an M followed by an A.

(g) Calculate the probability of drawing an A and an M in any order.

SC 2 I can use a tree diagram to represent the outcomes of a two- and three-step chance event with replacement

1 A spinner with the numbers 1 to 5 is spun twice.



(a) Show the sample space using a tree diagram.

(b) Calculate the probability of spinning two 1s.

(c) Calculate the probability of spinning two 4s.

(d) Calculate the probability of spinning two odd numbers.

(e) Calculate the probability of spinning two even numbers.

(f) Why don't the answers to **(d)** and **(e)** add to 1?

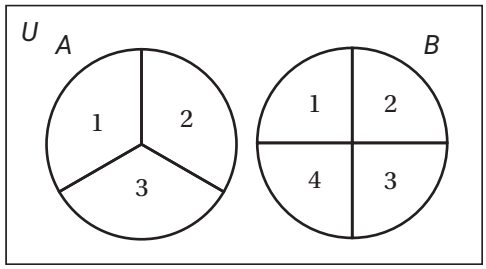
(g) Calculate the probability of not spinning a 5.

(h) Calculate the probability of spinning a 1 followed by a 2.

(i) Calculate the probability of drawing an odd number followed by even number.

(j) Calculate the probability of drawing one odd number and one even number.

2 Spinner A is spun once, then spinner B is spun once. The numbers are then multiplied.



(a) Draw a tree diagram to represent the situation.

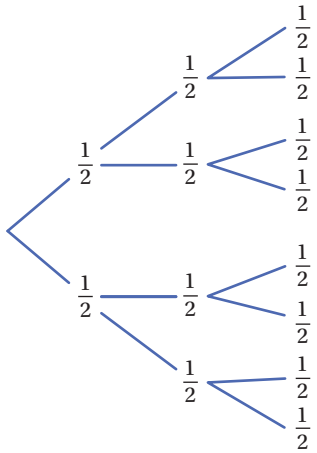
(b) Determine the probability of the product being:

(i) 1 **(ii)** 2

(iii) 4 **(iv)** > 2

(v) 7 **(vi)** 1, 2 or 3.

3 A coin is flipped three times and the tree diagram is shown below.



(a) What is the probability of flipping HTH?

(b) Determine the probability of flipping zero heads.

(c) Determine the probability of flipping exactly one head.

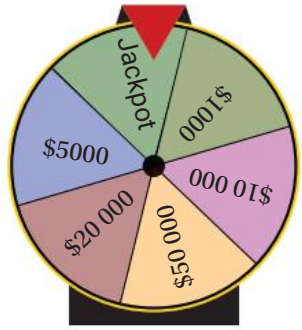
(d) Determine the probability of flipping at least one head.

4 Jai's probability of successfully shooting a free throw in basketball is $\frac{2}{5}$. Just as the buzzer sounds for the end of the game, Jai is awarded 3 free throws. His team is trailing by 2 points. Three successful throws and his team wins; two successes and they draw, and anything less than that they lose. Determine the probability that his team:

(a) wins **(b)** draws **(c)** loses.

SC 3 I can use a tree diagram to represent the outcomes of a two- and three-step chance event without replacement

1 A wheel with 6 prizes is spun. Each prize can only be won once. The jackpot currently stands at $100 000.



(a) Determine the probability of winning the jackpot on the first spin.

(b) If a spin lands on a prize that has already been won, it does not count. Assuming that the jackpot does not come up beforehand, determine the probability of winning the jackpot on:

(i) the second spin **(ii)** the third spin **(iii)** the fourth spin

(iv) the fifth spin **(v)** the final spin.

(c) Given that the wheel is spun only once, determine the probability of:

(i) winning more than $10 000 **(ii)** winning less than $50 000.

2 Alex has 6 red socks, 2 blue socks and 4 green socks in a drawer. Alex randomly takes out 2 single socks without replacement.

(a) Draw a tree diagram to represent the situation.

(b) Determine the probability that she:

(i) selects 2 blue socks **(ii)** selects 2 red socks

(iii) selects 2 green socks **(iv)** selects a matching pair of socks

(v) selects a red sock and a green sock in that order

(vi) selects a red sock and a green sock in any order.

3 A card is taken at random from a standard deck of 52 playing cards and not replaced. A second card is then drawn. Using a tree diagram, determine the probability that:

(a) both cards are clubs **(b)** both cards are Kings **(c)** 1 card is red and 1 card is black.

4 You are choosing 1 marble from the bag, then replacing it. The marble is selected and replaced 3 times.



(a) What is the probability of drawing at least 1 white marble?

(b) The experiment is repeated, but this time there is no replacement of the marble that has been drawn. What is the probability of drawing at least 1 white marble?

Calculate the probability of compound events

16.2

Learning intention: To be able to calculate the probability of compound events

Success criteria:

SC 1 I can use Venn diagrams to calculate the probability of 'and', inclusive 'or' and exclusive 'or' events.

SC 2 I can use two-way tables to calculate the probability of 'and', inclusive 'or' and exclusive 'or' events.

SC 3 I can calculate the probability of 'and' and 'or' events.

Lesson warm-up

Probability Bingo



You will be playing a game of Probability Bingo! Your teacher is going to roll 2 dice. One has 3 green, 2 red and 1 blue sides. The other has 2 yellow, 2 blue and 2 red sides. Complete the following steps to prepare for the game of Bingo:

1. Construct an array of all the possible outcomes from the two dice being rolled.
2. Draw a 5 × 5 grid in your book (25 squares total). In each square, list one outcome from the array. The catch is that you can't have the same outcome in any of the surrounding boxes (directly next to horizontally, vertically or on the diagonal).

Playing the game

When your teacher rolls the dice, they will call out the result. If this is on your grid, you can mark one off.

When you have 5 in a row marked off (horizontally, vertically or diagonally), yell out 'Bingo!' The first person with 5 in a row wins.

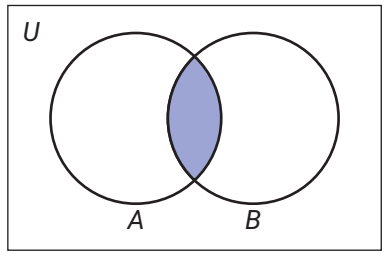
SC 1 I can use Venn diagrams to calculate the probability of 'and', inclusive 'or' and exclusive 'or' events

Independent events are events where the probability of an outcome is not influenced by the outcome of any other event. For example, if a coin is flipped twice and you get a tail followed by another tail, the outcome of each event is independent of the result of the other. People often mistakenly believe that if an event has not occurred recently, then it is more likely to happen next time. In the case of flipping a coin, the coin has no memory of what has happened before, so it is not influenced by previous outcomes.

In probability, the word 'and' means that two events must both occur.

The combined event A and B is written as $A \cap B$.

On a Venn diagram, it is the intersection of the two sets.



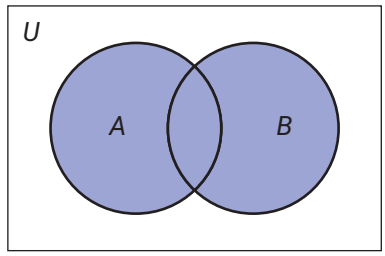
Provided that the events are independent of each other,

then $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

In probability, the word 'or' has two meanings; it can be inclusive or exclusive.

Inclusive 'or'

When 'or' is inclusive, this means A or B, or both.



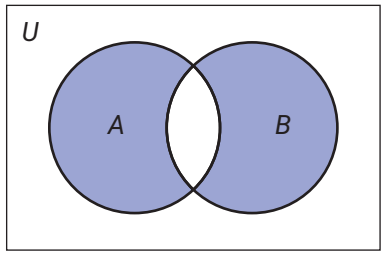
Provided that the events are independent of each other,

then $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$.

Think of this as adding the two sets, but the overlap has been added twice, so must be subtracted.

Exclusive 'or'

When 'or' is exclusive, this means A or B, but not both.



Provided that the events are independent of each other,

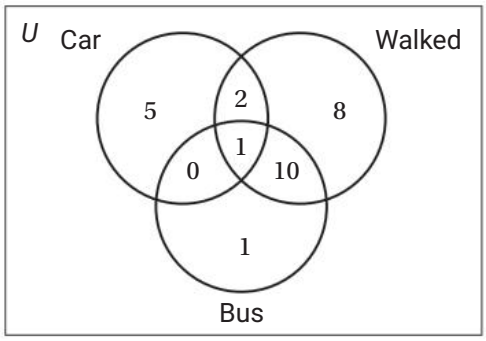
then $\Pr(A \cup B) = \Pr(A) + \Pr(B) - 2\Pr(A \cap B)$.

Think of this as adding the two sets, but the overlap has been added twice, so it must be subtracted twice.

Worked example

Using a Venn diagram to calculate probability

Students in 9A were asked how they travelled to school this morning. The results were collated and are shown in this Venn diagram.

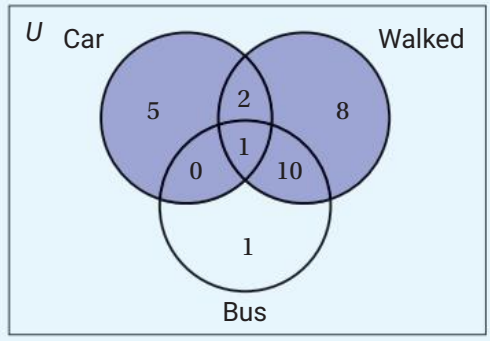


(a) Determine the probability of a student walking and travelling by car.

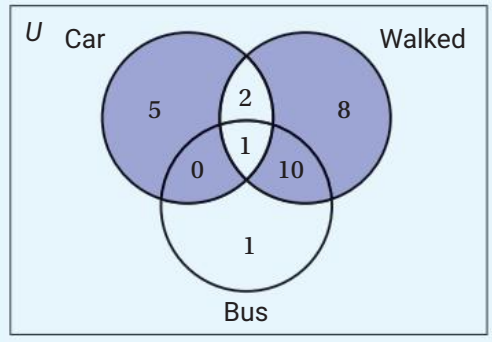
THINKING	WORKING
Work out what the problem is asking you to find.	$\Pr(W \cap C)$
Determine the total number of students.	$5 + 2 + 1 + 8 + 10 + 1 = 27$
Determine the number of students who walked and came by car.	$2 + 1 = 3$ students
Write the probability.	$\Pr(W \cap C) = \frac{3}{27} = \frac{1}{9}$

(b) Determine the probability of a student walking or coming to school by car or both.

THINKING	WORKING
Work out what the problem is asking you to find.	Calculate $\Pr(W \cup C)$ using the formula for inclusive 'or'.
Recall the formula to calculate the probability for inclusive 'or'.	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
Determine the total number of students.	27 students
Determine the number of students who walked, came by car or did both.	21 students walked, 8 came by car, but 3 of them did both.
Write the probabilities.	$\Pr(W) = \frac{21}{27}$ $\Pr(C) = \frac{8}{27}$ $\Pr(W \cap C) = \frac{3}{27} = \frac{1}{9}$

Substitute the probabilities into the formula.	$\Pr(W \cup C) = \Pr(W) + \Pr(C) - \Pr(W \cap C)$ $= \frac{21}{27} + \frac{8}{27} - \frac{3}{27}$ $= \frac{26}{27}$
Verify the result using the Venn diagram.	 $\frac{5+2+1+8+10}{27} = \frac{26}{27}$

(c) What is the probability of a student walking or coming to school by car, but not both?

THINKING	WORKING
Work out what the problem is asking you to solve.	Calculate $\Pr(W \cup C)$ using the formula for exclusive 'or'.
Recall the formula to calculate the probability for exclusive 'or'.	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - 2\Pr(A \cap B)$
Substitute the probabilities into the formula.	$\Pr(W \cup C) = \Pr(W) + \Pr(C) - \Pr(W \cap C)$ $= \frac{21}{27} + \frac{8}{27} - 2 \times \frac{3}{27}$ $= \frac{23}{27}$
Verify the result using the Venn diagram.	 $\frac{5+8+10}{27} = \frac{23}{27}$

SC 2 I can use two-way tables to calculate the probability of 'and', inclusive 'or' and exclusive 'or' events

A two-way table (or contingency table) is simply a way of organising categorical data. It is used specifically for two different categorical variables. This is referred to as bivariate data – 'bi' means two. This kind of table is also called a two-way frequency table because it records the frequencies of responses or counts in each category.

To make it easier to see patterns and relationships between the different categories of data, it is organised into a table with rows and columns representing the two different categories. Two-way tables are particularly useful for representing bivariate data collected from surveys because you can easily compare the responses.

Worked example

Using a two-way table to calculate probability

Students in 9B were surveyed on their opinion regarding the possibility of withdrawing pies from the canteen menu. The results were further broken into groups according to gender and are shown in this two-way table.

	Yes	No	Total
Female	8	3	11
Male	2	12	14
Total	10	15	25

(a) Determine the probability that a student selected at random from the class is female and agrees to removing pies from the menu.

THINKING	WORKING
Determine the total number of students.	25
Determine the number of females who agreed.	8
Write the answer.	The probability that a student selected at random from the class is female and agrees to removing pies from the menu is $\frac{8}{25}$.

(b) Determine the probability that a student selected at random from the class is female or does not agree to removing pies from the menu.

THINKING	WORKING
Determine the total number of students.	25
Determine the number of students who are female.	11
Determine the number of students who disagreed.	15
Work out what the problem is asking you to find.	You need to find Pr(female ∪ disagreed) using the formula for inclusive 'or'.
Use $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	$\Pr(\text{female} \cup \text{disagreed}) = \frac{11}{25} + \frac{15}{25} - \frac{3}{25}$ $= \frac{23}{25}$
Write the answer.	$\frac{23}{25}$

SC 3 I can calculate the probability of 'and' and 'or' events

Not all information is conveniently presented in Venn diagrams and tables. Sometimes you will need to calculate the probability of 'and', 'or' events from worded problems.

16

Worked example

Calculating the probability of 'and' and 'or' events

An experiment uses a special pack of 40 cards. The cards are numbered 1 to 10, and each numbered card comes in four colours: red, blue, green and yellow. A card is drawn at random from the deck.

Determine the probability that:

(a) it is red

THINKING	WORKING
State the number of red cards and the total number cards.	10 red cards in a deck of 40 cards
Write the probability.	$\Pr(\text{red}) = \frac{10}{40} = \frac{1}{4}$

(b) it is red and an even number

THINKING	WORKING
State the number of cards	40 cards
How many cards are red and even?	5
Write the probability.	$\Pr(\text{red and even}) = \frac{5}{40}$ $= \frac{1}{8}$

(c) it is red or an even number.

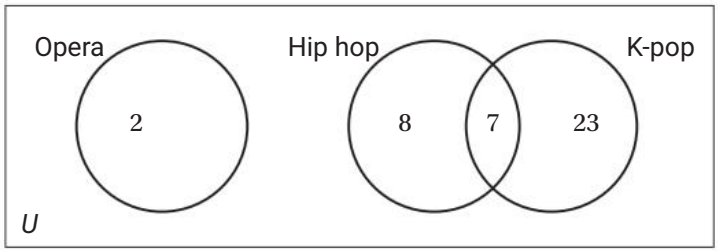
THINKING	WORKING
Write the probability of drawing a red card and the probability of drawing an even card.	10 red cards and 20 even cards $\Pr(\text{red}) = \frac{10}{40}, \Pr(\text{even}) = \frac{20}{40}$
Write the probability of drawing a red card that is also even.	$\Pr(\text{red and even}) = \frac{5}{40}$
Calculate the probability using $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	$\Pr(\text{red or even}) = \Pr(\text{red}) + \Pr(\text{even}) - \Pr(\text{red and even})$ $= \frac{10}{40} + \frac{20}{40} - \frac{5}{40}$ $= \frac{25}{40}$ $= \frac{5}{8}$

Practice

ANSWERS Page 658

SC 1 I can use Venn diagrams to calculate the probability of 'and', inclusive 'or' and exclusive 'or' events

1 The results of a survey of music students' favourite music genres are shown in the Venn diagram below. The students were allowed to choose a maximum of two of the three genres.

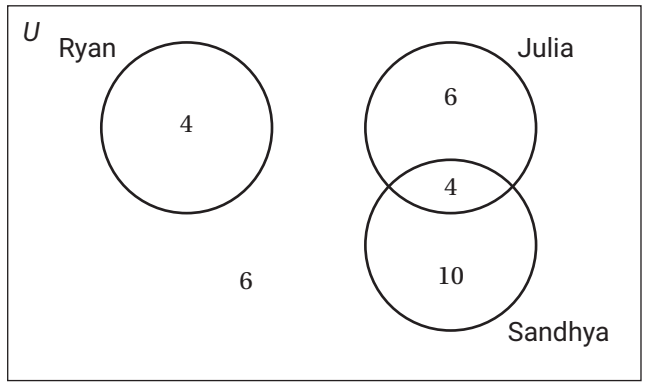


Determine the probability of a student's favourite genre of music being:

(a) opera **(b)** K-pop **(c)** hip-hop

(d) K-pop or hip-hop **(e)** K-pop or hip-hop, but not both.

2 The results of a survey about which actors teenagers really liked in a particular film are shown in the Venn diagram below.



Determine the probability of a teenager preferring:

(a) Ryan **(b)** Julia **(c)** Sandhya

(d) none of the three actors listed **(e)** Sandhya or Julia

(f) Sandhya or Julia, but not both **(g)** Sandhya or Ryan, but not both.

3 After a school concert, 50 students were surveyed about who gave exceptional performances. Students were allowed to vote for more than one performer. Of these 50 students, 20 students declared that Gina was exceptional, 25 thought Mia was exceptional and 15 students did not vote for either Gina or Mia.

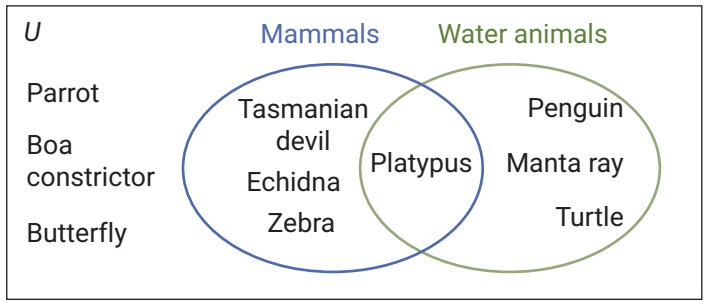
(a) Create a Venn diagram to represent this situation.

(b) Determine the probability of a randomly chosen student:

(i) voting for Gina **(ii)** voting for Mia

(iii) voting for either Mia or Gina **(iv)** voting for either Mia or Gina, but not both.

4 A list of 10 animals is split into the categories 'mammals' and 'water animals'.



An animal was selected at random from the list. Calculate the probability that:

(a) the animal is a mammal and a water animal

(b) the animal is a mammal or a water animal, but not both

(c) the animal is not a mammal and not a water animal.

SC 2 I can use two-way tables to calculate the probability of 'and', inclusive 'or' and exclusive 'or' events

1 Students in 9B were surveyed on their opinion regarding the possibility of withdrawing pies from the canteen menu.

	Yes	No	Total
Female	8	3	11
Male	2	12	14
Total	10	15	25

Use the table to determine the following probabilities.

(a) A student selected at random from the class is male and does not agree to removing pies from the menu.

(b) A student selected at random from the class is female or agrees to removing pies from the menu.

2 Students in Years 9 and 10 at a school were surveyed on their opinion regarding adding shorts to the standard school uniform options. The results were further broken into groups according to their year level and are shown in this two-way table.

	Yes	No	Total
Year 9	35	15	50
Year 10	40	20	60
Total	75	35	110

Determine the probability of a randomly chosen student:

(a) being in Year 10

(b) voting yes

(c) being a Year 9 student who voted no.

3 The newest superhero film is out! The cinema collected the following data on where different age groups sat in the theatre.

	Near the front	Near the back	
Children	17		38
Adults		13	
	26		60

(a) Copy and complete the two-way frequency table.

(b) One person in the cinema is selected at random from this group. What is the probability that they were:

(i) sitting near the front? **(ii)** an adult and sitting near the back?

(iii) not an adult? **(iv)** not a child sitting near the front?

(v) a child sitting near the back or an adult sitting near the front?

SC 3 I can calculate the probability of 'and' and 'or' events

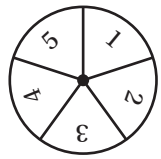
1 A coin is tossed, and a die is rolled. Determine the probability of:

(a) rolling a 3 and tossing a head

(b) rolling a number less than 3 and tossing a head

(c) rolling an odd number and tossing a tail.

2 This spinner is spun twice.



Determine the probability of:

(a) spinning 2 odd numbers

(b) spinning an odd and an even number in that order

(c) spinning an odd and an even number in any order

(d) spinning 2 numbers that add to 4.

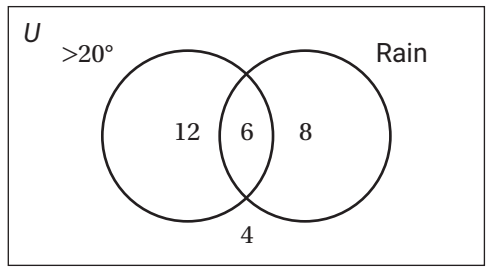
3 Two fair six-sided dice are rolled at the same time. Determine the probability of:

(a) rolling two odd numbers

(b) rolling two numbers greater than 4

(c) rolling exactly one odd and one even number in any order.

4 The temperatures and rainfall in a small town were recorded each day during the month of April. The results are presented in the Venn diagram.



Determine the probability that on any given day, the:

(a) temperature was greater than 20°

(b) temperature was less than 20°

(c) temperature was less than 20° and it was raining

(d) temperature was greater than 20° and it was raining

(e) temperature was greater than 20° or it was raining, or both

(f) temperature was greater than 20° or it was raining, but not both.

Estimate the probability of events

16.3

Learning intention: To be able to estimate the probability of events

Success criteria:

SC 1 I can use the relative frequency of events from large statistical samples or long run experiments to estimate the probability of events.

SC 2 I can use Venn diagrams and two-way tables to estimate frequencies of events involving 'and' and 'or' questions.

Lesson warm-up

Mystery Venn



Use the following clues to draw a three-way Venn diagram to represent the information.

- A group of twelve friends are comparing whether they received homework for Maths, Science and/or Humanities.
- There are seven boys (Ryan, James, Thomas, Wayne, Jack, Aidan and Ash) and five girls (Maddie, Sophie, Ellie, Mya and Katie).
- Ellie has homework for all three subjects.
- Ryan and Jack only have one piece of homework.
- Nobody has only Science homework.
- Mya has two pieces of homework.
- Seven students in total have Maths homework and Mya is not one of them.
- Thomas received homework from Maths and Humanities but not Science.
- James has more homework than Thomas.
- Katie is the only girl with no homework.
- Maddie is one of two people who just have Maths homework.
- Sophie, Ryan and Ash do not have Humanities homework.
- No girl has the exact same homework as another girl.
- Five students have Science homework.
- Wayne and Jack have more homework than Aidan but less than Ash.
- Two of the group just have Humanities homework and they are both male.

What is the probability that one of the students chosen at random has homework from all three subjects?

SC 1 I can use the relative frequency of events from large statistical samples or long run experiments to estimate the probability of events

Statistics kept over a long period of time are used to inform businesses and governments and help them in future planning.

For example:

- Knowing the distribution of yearly rainfall helps governments to know where to improve drainage and water storage.
- Knowing historical attendance figures helps retail businesses employ the correct number of staff.

The data should be collected over a long timeframe to improve the reliability of the estimate.

For long-term data, you can use the **relative frequency** as a measure of the probability.

Probability that is calculated based on the results of a trial is called **experimental probability**.

$$\Pr(\text{Event}) = \frac{\text{Number of times the event occured}}{\text{Total number of trials}}$$

Compare this to **theoretical probability**, which is calculated using

$$\Pr(\text{Event}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

Worked example

Estimating the probabilities from long-term data

For the township of Waterside, the number of days recorded as being over 30°C in December over the last 20 years is 120. Use this information to calculate the probability of it being over 30°C on 25 December this year.

THINKING	WORKING
Calculate the number of December days in 20 years.	$20 \times 31 = 620$
Express the relative frequency.	Relative frequency $= \frac{120}{620} = \frac{6}{31}$
Write the answer.	$\Pr(\text{over } 30°) = \frac{6}{31}$

Worked example

Assessing risk

Insurance companies continually monitor the number and costs of claims so they can adjust their premiums (policy costs) to ensure they are making a profit.

The following information was collected for 18-year-old drivers over a number of years.

	City drivers		Country drivers	
	Male	Female	Male	Female
Probability of a claim	0.15	0.08	0.09	0.04
Mean cost of a claim	$12 000	$12 000	$15 000	$15 000

(a) Do you think the premium should be the same for all drivers regardless of age, gender or location?

THINKING	WORKING
Consider the likelihood of making a claim and the mean cost of that claim.	Males living in the city are four times more likely to have an accident than females in the country, so it would not be fair to charge everyone the same price.

(b) Calculate the premium for an 18-year-old female living in the city, given that the company wishes to make a 30% profit.

THINKING	WORKING
Consider a sample of 100 insured people in this category. Calculate the number who will make a claim.	Probability of a claim = 0.08 Number of claims from 100 people is estimated to be $0.08 \times 100 = 8$
Calculate the cost of those claims.	$8 \times 12\,000 = \$96\,000$
To calculate the premium, add 30% to the claim total for profit and divide by 100 (people).	$\frac{96\,000 + 96\,000 \times 0.3}{100} = \$1248$

SC 2 I can use Venn diagrams and two-way tables to estimate frequencies of events involving 'and' and 'or' questions

16

Worked example

Converting to frequencies

A survey of 100 residents was conducted in Lilydale about the needs of the community. They were asked to vote for a new performing arts centre or a new aquatic playground.

	Performing arts centre	Aquatic playground	Neither	Total
Under 18	5	24	2	31
18 to 40	20	10	1	31
Over 40	30	6	2	38
Total	55	40	5	100

There are 800 000 residents in the community.

(a) Estimate the total number of residents who voted for the aquatic playground.

THINKING	WORKING
Determine the fraction of residents in the survey who voted for the aquatic playground.	Reading from the table, 40 people voted for the aquatic playground. So the fraction of residents is: $\frac{40}{100}$
Multiply the fraction of residents in the survey by the population number to obtain an estimate of the total number of residents who voted for the aquatic playground.	$\frac{40}{100} \times 800\,000 = 320\,000$
Write the answer.	An estimated 320 000 residents voted for the aquatic playground.

(b) Estimate the total number of residents who are between 18 and 40 in the community, or who did not vote for the aquatic playground.

THINKING	WORKING
Determine the fraction of residents who are between 18 and 40 in the community.	Reading from the table, 31 of the people surveyed were between 18 and 40. So the fraction of residents is: $\frac{31}{100}$
Determine the fraction of residents in the survey who did not vote for the aquatic playground.	Reading from the table, 40 people voted for the aquatic playground, so 60 people didn't vote for it. So the fraction of residents is: $\frac{60}{100}$
Use the equation $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ to find the relative frequency required.	The number of people between 18 and 40 who did not vote for the aquatic playground is: $20 + 1 = 21$ So $\Pr(A \cap B) = \frac{21}{100}$ $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ $= \frac{31}{100} + \frac{60}{100} - \frac{21}{100}$ $= \frac{70}{100}$
Multiply the fraction of residents in the survey by the population number.	$\frac{70}{100} \times 800\,000 = 560\,000$
Write the answer.	There are an estimated 560 000 people in the population who are between 18 and 40, or who did not vote for the aquatic playground.

(c) Estimate the total number of residents over 40 who voted for the performing arts centre.

THINKING	WORKING
Determine the fraction of residents over 40 in the survey who voted for the performing arts centre.	Reading from the table, 30 people over 40 in the survey voted for the performing arts centre. So the fraction of residents is: $\frac{30}{100}$
Multiply the fraction of residents in the survey by the population number to obtain an estimate of the total number of residents over 40 who voted for the performing arts centre.	$\frac{30}{100} \times 800\,000 = 240\,000$
Write the answer.	An estimated 240 000 residents over 40 voted for the performing arts centre.

Worked example

Estimating the frequency of an event

The ballot results of the last 60 years for people in an electorate were compiled and the following table of relative frequencies was created.

	Worker's Party	Freedom Party
Male	0.28	0.21
Female	0.26	0.25

(a) Given that 210 000 ballots were cast, how many votes from males did the Worker's Party receive?

THINKING	WORKING
Multiply the total number by the relative frequency in the table.	$210\,000 \times 0.28 = 58\,800$
Write the answer.	The Worker's Party received 58 800 votes from men.

(b) How many of the 210 000 ballots were cast by females or were a vote for the Freedom Party?

THINKING	WORKING
What is the question asking for?	The question asks for the number of female votes, and for the number of votes for the Freedom Party.
Write what you need to do.	Use the equation $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ to find the relative frequency required, then multiply the result by 210 000.

Calculate Pr($A \cup B$).	Let A = ballot cast by a female Let B = vote for the Freedom Party $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ $= (0.26 + 0.25) + (0.21 + 0.25) - 0.25$ $= 0.51 + 0.46 - 0.25$ $= 0.72$
Calculate the number of ballots.	$210\,000 \times 0.72 = 151\,200$ votes from women or for the Freedom Party.

(c) How many of the 210 000 ballots were cast by females and were a vote for the Worker's Party?

THINKING	WORKING
Read the relative frequency of female votes for the Worker's Party from the table.	0.26
Multiply this by the total votes to obtain the answer.	$0.26 \times 210\,000 = 54\,600$

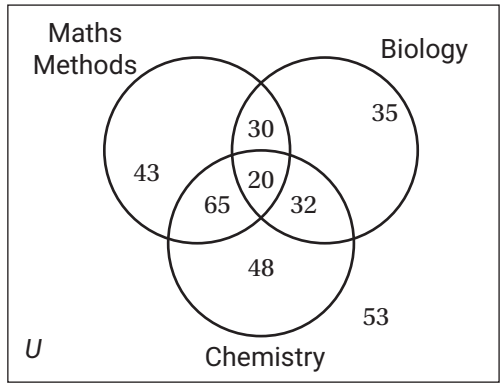
(d) Given the current number of people on the electoral roll is 65 000, how many votes can the Freedom Party expect to receive at the next election?

THINKING	WORKING
Determine the relative frequency of people who would vote for the Freedom Party.	Add the relative frequencies of male and female votes for the party. $0.21 + 0.25 = 0.46$
Multiply this by the number of people on the electoral roll to obtain the answer.	$0.46 \times 65\,000 = 29\,900$ votes

Worked example

Using relative frequency to make estimations

The Venn diagram below shows the number of students studying Year 11 maths and science subjects at a senior secondary school.



(a) How many students are there in Year 11?

THINKING	WORKING
Add all the numbers together.	$43 + 65 + 20 + 30 + 32 + 35 + 48 + 53 = 326$
Write the answer.	There are 326 students in Year 11.

(b) Assuming that the proportion of students studying Maths Methods at this senior secondary school is representative of the proportion of 60 000 Year 11 students in Victoria, estimate the number of Victorian Year 11 students taking Maths Methods.

THINKING	WORKING
Determine the number of students at the school studying Maths Methods.	$43 + 65 + 20 + 30 = 158$
Calculate the relative frequency of those studying Maths Methods.	$\frac{158}{326} = \frac{79}{163}$
Estimate the number of Year 11 students in Victoria studying Maths Methods.	$\frac{79}{163} \times 60\,000 \approx 29\,079$
Write the answer.	There are an estimated 29 079 Year 11 Maths Methods students in Victoria.

(c) Estimate the number of Year 11 students in Victoria who study either Biology or Chemistry, but not both.

THINKING	WORKING
Determine the number of students at the school studying Biology, and state the relative frequency.	$20 + 30 + 32 + 35 = 117$ Relative frequency $= \frac{117}{326}$
Do the same for Chemistry.	$65 + 20 + 32 + 48 = 165$ Relative frequency $= \frac{165}{326}$
Determine the number of students studying both subjects, and state the relative frequency.	$20 + 32 = 52$ Relative frequency $= \frac{52}{326}$

Determine the relative frequency of students at the school who study either Biology or Chemistry, but not both. Use $\Pr(A \cup B) = \Pr(A) + \Pr(B) - 2\Pr(A \cap B)$	$\Pr(B \cup C) = \Pr(B) + \Pr(C) - 2\Pr(B \cap C)$ $= \frac{117}{326} + \frac{165}{326} - 2 \times \frac{52}{326}$ $= \frac{178}{326}$ $= \frac{89}{163}$
Estimate the number of Year 11 students in Victoria studying either Biology or Chemistry, but not both.	$\frac{89}{163} \times 60\,000 \approx 32\,760$
Write your answer.	There are an estimated 32 760 Year 11 students in Victoria studying either Biology or Chemistry, but not both.

(d) Estimate the number of Year 11 students in Victoria who study either Maths Methods or Chemistry.

THINKING	WORKING
Use the relative frequencies calculated in parts **(b)** and **(c)** for Maths Methods and Chemistry.	Relative frequency (Maths Methods) $= \frac{158}{326}$ Relative frequency (Chemistry) $= \frac{165}{326}$
Determine the number of students studying both subjects, and state the relative frequency.	$65 + 20 = 85$ Relative frequency (Chemistry and Maths) $= \frac{85}{326}$
Determine the relative frequency of students at the school who study either Maths Methods or Chemistry. Use $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	$\Pr(M \cup C) = \Pr(M) + \Pr(C) - \Pr(M \cap C)$ $= \frac{158}{326} + \frac{165}{326} - \frac{85}{326}$ $= \frac{238}{326}$ $= \frac{119}{163}$
Estimate the number of Year 11 students in Victoria who study either Maths Methods or Chemistry.	$\frac{119}{163} \times 60\,000 \approx 43\,803$
Write your answer.	There are an estimated 43 803 Year 11 students in Victoria studying either Maths Methods or Chemistry.

(e) Estimate the number of Year 11 students in Victoria who study Maths Methods and Biology.

THINKING	WORKING
Determine the number of students at the school studying Maths Methods and Biology, and state the relative frequency.	$20 + 30 = 50$ Relative frequency (Maths Methods and Biology) $= \frac{50}{326} = \frac{25}{163}$
Estimate the number of Year 11 students in Victoria who study Maths Methods and Biology.	$\frac{25}{163} \times 60\,000 \approx 9202$
Write your answer.	There are an estimated 9202 Year 11 students in Victoria studying Maths Methods and Biology.

Practice

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SC 1 I can use the relative frequency of events from large statistical samples or long run experiments to estimate the probability of events

1 Dale's annual bowling average is shown in this table.

Year	2016	2017	2018	2019	2020	2021
Average	146	152	147	150	146	147

(a) Calculate Dale's long-term bowling average.

(b) Estimate his 2022 bowling average.

(c) Justify how you arrived at this estimate.

2 The attendance at a tourist attraction has been above 1000 people on a Sunday 125 times over the last 3 years. What is the probability that the attendance will be over 1000 next Sunday? Write your answer correct to 2 decimal places.

3 The following information about 30-year-old drivers was collected over a number of years.

30-year-old drivers	City drivers		Country drivers	
	Male	Female	Male	Female
Probability of a claim	0.06	0.05	0.04	0.03
Mean cost of a claim	$14 000	$14 000	$18 000	$18 000

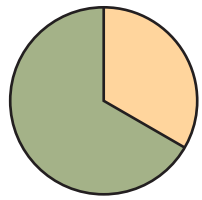
The company wishes to make a 40% profit.

(a) Calculate the premiums for 30-year-old drivers and complete the table below.

	City drivers		Country drivers	
30-year-old drivers	Male	Female	Male	Female
Premium				

(b) Why do you think city drivers have to pay more than country drivers?

4 A probability experiment using 10 000 spins of this spinner is to be run.



(a) Estimate the fraction of the area covered by each colour.

(b) Estimate how many of each colour will be spun in 10 000 trials.

(c) The frequency of each colour is shown in this table after 100, 500 and 10 000 trials.

Spins	Yellow	Green
100	25	75
500	150	350
10 000	3340	6660

(i) Do the results support your estimation of the fractions of each colour? Explain.

(ii) How did the number of spins affect the reliability of the results?

(d) Draw a different spinner that will produce similar results.

5 An experiment is to be conducted drawing random numbers from 1 to 10.

(a) What is the theoretical probability of drawing a multiple of 5? Give your answer to 2 decimal places.

(b) What is the expected number of times the number will be divisible by 5 if an experiment is run with 50 trials?

A random number generator ran 50 trials of this simulation. The results are recorded in this table.

	Frequency
Divisible by 5	9
Not divisible by 5	41
Total	50

(c) From the results provided in the table, estimate the probability of drawing a number divisible by 5. Use 2 decimal places.

(d) Do the results provided in the table support your answers to **(a)** and **(b)**?

(e) What impact would running 10 000 trials instead of 50 have on the estimated probability?

SC 2 I can use Venn diagrams and two-way tables to estimate frequencies of events involving 'and' and 'or' questions

1 A sample of 50 students were surveyed about the colour of the school uniform. Of these, 40% said they were happy with it, 45% said they wanted to change the colour and the rest had no opinion. Given that the school population was 800 students, estimate the number of students who would like to see a change in the uniform.

2 60 students were surveyed about changing the name of the school newsletter. Seven said they were happy with it as is, 45 had no opinion and the rest said they wanted to change it, but could not agree on a new name.

(a) Given that the school population is 850 students, estimate the number of students who would like to see a change in the name of the newsletter.

(b) Should the school change the name of the newsletter? Use mathematics to justify your answer and suggest what could be done to get clearer data.

3 A sample survey of 200 residents was conducted about the needs of the community. They were asked to vote for a new shopping precinct or a new childcare centre.

	Shopping precinct	Childcare centre	Neither	Total
Under 18	24	50	4	78
18 to 40	28	24	2	54
Over 40	58	6	4	68
Total	110	80	10	200

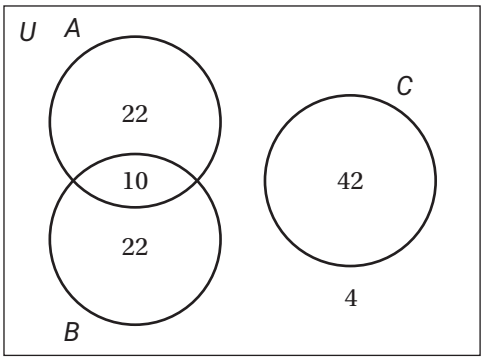
There are 600 000 residents in the community.

(a) Estimate the number of residents who voted for the shopping precinct.

(b) Estimate the number of under 18s who voted for the shopping precinct.

(c) Estimate the total number of residents who are under 18 in the community, or who voted for the childcare centre.

4 A probability experiment was conducted with 100 trials. The results are shown in this Venn diagram.



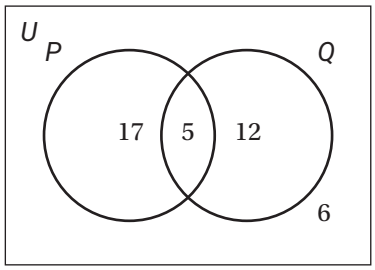
A second experiment is planned with 100 000 trials.

Estimate the results of the second experiment for the following.

(a) A, B or C **(b)** C **(c)** A only

(d) A and B **(e)** A or B **(f)** A or B but not both

5 Look at the information in this Venn diagram.



(a) Determine the number of people in P and Q.

(b) Determine the number of people in P or Q, but not in both.

(c) The Venn diagram represents the interview responses of 40 people. Given that 1000 interviews were held, estimate the number of people not in P or Q.

16.4 Design and conduct experiments and simulations

Learning intention: To be able to design and conduct experiments and simulations

Success criteria:

SC 1 I can conduct experiments and simulations for single and compound events.

SC 2 I can use the results of a probability experiment to predict future events and evaluate the outcomes.

Lesson warm-up

Dice game



In groups of four, play the following dice game.

- Each player starts with 10 coloured discs.
- One person is in charge of the 'bank' and rolls the dice.
- At the start of the game, each player puts 5 discs in the 'bank'.
- The banker throws the pair of dice.
- If the dice totals:
  - 5, 6, 7, 8 or 9, each player puts another disc in the bank
  - 3, 4, 10 or 11, each player gets a disc back
  - 2 or 12, each player gets 2 discs back.

The game ends when either the players or banker have no discs remaining. If the players lose all their discs, the banker wins. If the banker loses all their discs, the players win.

SC 1 I can conduct experiments and simulations for single and compound events

Worked example

Simulated AFL game

AFL teams score points from goals (6 points) and behinds (1 point). The game is played over four quarters and scores from each quarter are added. The team with the highest number of points wins.

A game can be simulated by rolling a six-sided die twice. The first roll represents the number of goals scored and the second roll represents the number of behinds. Note that 5.2 means 5 goals and 2 behinds not 5.2 goals.

Fred and Barney score as shown in this table.

	Fred	Barney
Quarter 1	2.4	3.6
Quarter 2	3.1	6.1
Quarter 3	6.2	1.5
Quarter 4	4.4	2.4
Total score		
Points		
_____ won the game by _____ points		

Complete the table.

THINKING	WORKING
To find the total score, add the numbers in each column. Remember, these are not decimal numbers, just add the number of goals and behinds.	<table><tr><th></th><th>Fred</th><th>Barney</th></tr><tr><td>Quarter 1</td><td>2.4</td><td>3.6</td></tr><tr><td>Quarter 2</td><td>3.1</td><td>6.1</td></tr><tr><td>Quarter 3</td><td>6.2</td><td>1.5</td></tr><tr><td>Quarter 4</td><td>4.4</td><td>2.4</td></tr><tr><td>Total score</td><td>15.11</td><td>12.16</td></tr><tr><td>Points</td><td></td><td></td></tr><tr><td colspan="3">_____ won the game by _____ points</td></tr></table>
To find the number of points, multiply the number of goals by 6 and add it to the number of behinds.	<table><tr><th></th><th>Fred</th><th>Barney</th></tr><tr><td>Quarter 1</td><td>2.4</td><td>3.6</td></tr><tr><td>Quarter 2</td><td>3.1</td><td>6.1</td></tr><tr><td>Quarter 3</td><td>6.2</td><td>1.5</td></tr><tr><td>Quarter 4</td><td>4.4</td><td>2.4</td></tr><tr><td>Total score</td><td>15.11</td><td>12.16</td></tr><tr><td>Points</td><td>$15 \times 6 + 11 = 101$</td><td>$12 \times 6 + 16 = 88$</td></tr><tr><td colspan="3">_____ won the game by _____ points</td></tr></table>
The highest score wins. Subtract to find the winning margin.	Fred won by 13 points.

You can use technology to generate random numbers for testing.

Microsoft Excel has two functions that will help you generate random numbers: RAND() and RANDBETWEEN().

RAND()

RAND() will give you decimal values between 0 and 1.

fx =RAND()

RANDBETWEEN()

RANDBETWEEN() will generate a random integer between two other integers. To generate a random integer between 1 and 1000, in the active cell, type

=RANDBETWEEN(1, 1000)

and hit 'enter'.

fx =RANDBETWEEN(1,1000)

You can copy the formula down and across to generate many random numbers in one go.

	=RANDBETWEEN(1,1000)				
	729	369	374	156	624
	61	574	762	323	293
	52	265	927	322	120
	681	437	428	826	143
	310	133	444	325	494
	851	547	39	548	770
	902	955	562	599	730
	847	485	138	764	787
	449	497	818	105	630
	25	224	124	553	97

Worked example

Using a virtual dice roller to simulate single and multiple rolls of the dice

A game involves rolling an eight-sided die and guessing the number that the die will land on.

Each game costs $1 to play. What should the prize be so that the game is fair?

THINKING	WORKING
Determine what the question needs you to do.	In a fair game, the profits would equal the losses. So you need to balance the losses (the cost of playing the game) with the wins.
Write an equation for the situation.	Since there are 8 equally likely outcomes, the probability of each outcome is $\frac{1}{8}$. So, $\Pr(\text{win}) = \frac{1}{8}$. The expected gain or loss over x games is: $\frac{1}{8} \times x \times (\text{prize money}) - x \times 1 = \frac{x}{8}(\text{prize money}) - x$
In a fair game, there is no gain or loss.	$\frac{x}{8}(\text{prize money}) - x = 0$ $\frac{x}{8}(\text{prize money}) = x$ $\frac{1}{8}(\text{prize money}) = 1$ $\text{prize money} = 8$
Write the answer.	The prize for guessing the right number should be $8.

Note: to simulate this game, you can use =RANDBETWEEN(1, 8) in Excel to generate as many results as you would like.

SC 2 I can use the results of a probability experiment to predict future events and evaluate the outcomes

Collecting a large data sample is vital to obtaining accurate estimations for probability. Small samples may lead to considerable variation in the values found.

Worked example

Experimenting with lotto games

90% of money invested in a carnival game must be returned in prize money in the long run. Consider a very simple machine where the possible outcomes are the numbers 1 to 10. The numbers 1 to 9 are winning numbers and 10 is a loss.

Use this table of random numbers to simulate playing 100 spins. Comment upon your results.

Table of random numbers

36518	36777	89116	05542	29705	83775	21564	81639	27973	62413	85652	62817	57881
46132	81380	75635	19428	88048	08747	20092	12615	35046	67753	69630	10883	13683
31841	77367	40791	97402	27569	90184	02338	39318	54936	34641	95525	86316	87384
84180	93793	64953	51472	65358	23701	75230	47200	78176	85248	90589	74567	22633
78435	37586	07015	98729	76703	16224	97661	79907	06611	26501	93389	92725	68158
41859	94198	37182	61345	88857	53204	86721	59613	67494	17292	94457	89520	77771
13019	07274	51068	93129	40386	51731	44254	66685	72835	01270	42523	45323	63481
82448	72430	29041	59208	95266	33978	70958	60017	39723	00606	17956	19024	15819
25432	96593	83112	96997	55340	80312	78839	09815	16887	22228	06206	54272	83516
69226	38655	03811	08342	47863	02743	11547	38250	58140	98470	24364	99797	73498
25837	68821	66426	20496	84843	18360	91252	99134	48931	99538	21160	09411	44659
38914	82707	24769	72026	56813	49336	71767	04474	32909	74162	50404	68562	14088
04070	60681	64290	26905	65617	76039	91657	71362	32246	49595	50663	47459	57072
01674	14751	28637	86980	11951	10479	41454	48527	53868	37846	85912	15156	00865
70294	35450	39982	79503	34382	43186	69890	63222	30110	56004	04879	05138	57476
73903	98066	52136	89925	50000	96334	30773	80571	31178	52799	41050	76298	43995
87789	56408	77107	88452	80975	03406	36114	64549	79244	82044	00202	45727	35709
92320	95929	58545	70699	07679	23296	03002	63885	54677	55745	52540	62154	33314
46391	60276	92061	43591	42118	73094	53608	58949	42927	90993	46795	05947	01934
67090	45063	84584	66022	48268	74971	94861	61749	61085	81758	89640	39437	90044
11666	99916	35165	29420	73213	15275	62532	47319	39842	62273	94980	23415	64668
40910	59068	04594	94576	51187	54796	17411	56123	66545	82163	61868	22752	40101
41169	37965	47578	92180	05257	19143	77486	02457	00985	31960	39033	44374	28352
76418												

THINKING	WORKING
Select 100 numbers and count the number of zeroes in your sample.	84843 18360 91252 99134 56813 49336 71767 04474 65617 76039 91657 71362 11951 10479 41454 48527 34382 43186 69890 63222 5 zeroes only

Calculate the winning percentage.	With 95 wins and 5 losses, the player gets a return of 95% for this sample.
Comment on the results.	The machine must return 90% in the long run, but 100 trials is not enough to reflect this. A large number of trials are required.

Practice

ANSWERS Page 659

SC 1 I can conduct experiments and simulations for single and compound events

1 Use a die or die simulator to play a game of AFL against a partner. Keep score in a table like this.

	You	Partner
Quarter 1		
Quarter 2		
Quarter 3		
Quarter 4		
Total score		
Points		
_____ won the game by _____ points.		

2 A batter's innings in cricket can be simulated by rolling a single die and recording the number rolled. The number of runs scored with each ball is given by the number rolled. Since 5 runs are rarely scored, the batter is out, and their innings comes to an end when a 5 is rolled. A team score can be simulated by using ten batters. For a shorter game, fewer batters can be used.

(a) Find a partner and simulate your own game of cricket.

(b) Determine the probability of:

(i) a batter being out first ball **(ii)** a batter being out second ball

(iii) a batter being out third ball **(iv)** a batter facing fewer than 4 balls.

(c) Determine the mean expected score per batter.

(d) Can you suggest an improvement to this simulation to make it more realistic?

3 The rules for the game 'How high can you go?' are that all students stand up and the teacher rolls a six-sided die. Any number from 1 to 5 scores points according to the number rolled. A 6 being rolled means that everyone standing scores zero for that round.

After the first roll, students must decide to sit down and take the score they have or stay standing and risk getting zero if a 6 is rolled.

The die is rolled again and those students who chose to stand add that number onto their score for the round. Once again, students must decide either to sit down and take the score they have or stay standing and risk getting zero if a 6 is rolled.

This continues until all students are eliminated or are sitting down. A second and third round are played until a player reaches a score of 100.

(a) What is the probability of being eliminated by a six being rolled on any turn?

(b) The first 4 rolls of the die add to 9, should you sit down or stay standing? Why?

(c) The die has been rolled 7 times without a 6 coming up. What is the probability of a 6 coming up on the next roll?

4 Take a sport that you are familiar with and design a dice simulation for that game. Trial your game with a partner.

5 The game of two-up is a big part of the ANZAC tradition. It involves throwing two coins in the air and betting on the outcome.

(a) Write the sample space for the outcomes from throwing two coins.

(b) Determine the probability of:

(i) getting two heads **(ii)** getting two tails **(iii)** getting a head and a tail.

The person who throws the coins is known as the 'spinner' and the entire group of people placing bets standing around the spinner is known as the 'school'. Bets are made between people in the school on whether two heads or two tails will come up.

If the spinner throws two heads, they have won and continue to be the spinner for the next throw after all the bets have been settled and new ones made in the school.

If the spinner throws a split, head/tail, then all bets stay as they are and they go again.

If the spinner throws two tails, they have lost, and a new spinner is appointed for the next game after all the bets have been settled and new ones made.

(c) Determine the probability of:

(i) a spinner losing in the first throw of a game

(ii) a spinner losing in the second throw of a game

(iii) a spinner losing in the third throw of a game.

(d) Use the coin simulator to check your answers for **(b)** and **(c)**. Write a few sentences to describe your results.

SC 2 I can use the results of a probability experiment to predict future events and evaluate the outcomes

1 Use the table of random numbers from the Worked example to answer the following questions.

(a) Take four more samples of 100 values from the table of random numbers and calculate the percentage returned to the player.

(b) Determine the mean of the five percentages you have found. Comment on whether this figure is closer to the 90% the government regulations demand.

2 The owners of carnival games know the long-term percentage of success for each of the games they offer. For instance, in the basketball throwing game the balls and rings are smaller than normal and the distance from the ring and height of the ring are all unfamiliar. In this new set of circumstances, they know that people shooting a basketball into the ring will be successful about 1 in 3 throws.

Rules of this game:
Buy three shots. Cost $1 each for a total of $3.
Shoot each of your three basketballs and count the number of successes.
Your prize is dependent upon the number of successes from your three shots.
The prizes increase in value as the number of successes increases.
The long-term average of success is $\frac{1}{3}$ and the average of failure is $\frac{2}{3}$.

(a) Use a tree diagram to find the sample space for success (S) and failure (F) on the three throws. How many outcomes are there?

(b) Are each of the outcomes equally likely? Explain your answer.

(c) Complete this table.

Number of successes	List of different configurations to achieve the number of successes	Calculation of probability
3		
2		
1		
0		
	Total	

(d) What does the total of the probability tell you about your calculations?

(e) In the long run, i.e. after a large number of trials, the number of times a contestant wins each prize is shown below.

Number of successes	Wins per 27 trials
3	1
2	6
1	12
0	8

(i) What is the income from the 27 games?

(ii) Assuming the operator wants to make $27 from the 27 games, how much do they spend on prizes for the players?

(f) How does knowing the long-term statistics for the game help the operator make a profit?

3 Another popular carnival game is putting table tennis balls into a clown's mouth. The balls land in a slot numbered 1 to 6. Not all games are the same, but this is representative of the way it works.

For this game, it costs $2 to play and the prizes are listed on the board for the total of your 5 balls.

Winning numbers

Top shelf prize	5 or 30
Middle shelf prize	6, 7, 28, 29
Bottom shelf prize	9, 10, 11, 12, 21, 22, 23, 24, 25, 26, 27



For example, for balls landing on 5, 3, 6, 3, 4, the score is: $5 + 2 \times 3 + 6 + 4 = 21$.

(a) What is the maximum and minimum total that can be achieved? How does this compare to the prize list?

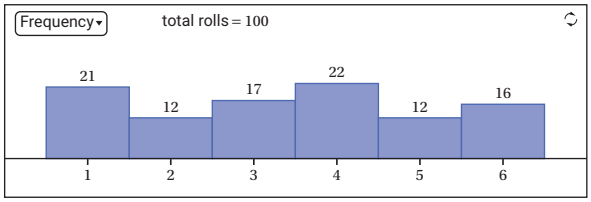
This is a good game for the operator because there are 17 numbers that have a prize attached to them out of 26 possibilities, so it sounds like there is a good chance of winning. In fact, players have a much lower chance of getting the totals needed to win a prize.

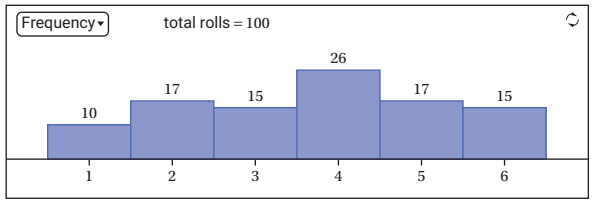
Knowing the mathematics in advance, you can be aware of the long-term outcomes and set the prizes to ensure a profit.

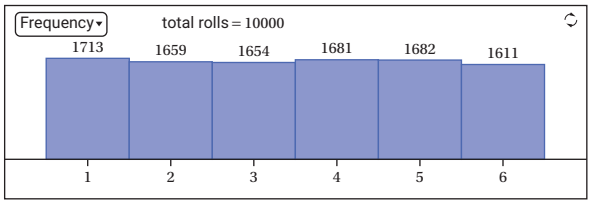
(b) Explain how the top, middle and bottom shelf prizes are decided upon.

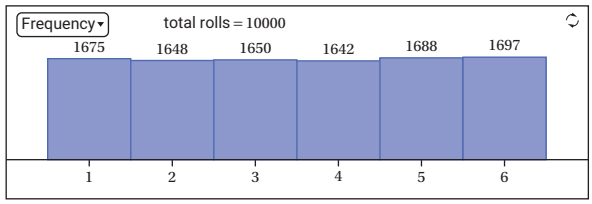
4 There is a branch of mathematics called actuarial studies that looks at the statistics related to insurance claims. How would an understanding of the long-term statistics help insurance companies?

5 Consider the results from these four experiments.









What do these results show in relation to the number of trials and the reliability of results?

TOPIC REVIEW

Multiple choice

ANSWERS Page 678

1 What could this array represent?

		Event A			
		1	**2**	**3**	**4**
Event B	**1**	1	2	3	4
	2	2	4	6	8
	3	3	6	9	12
	4	4	8	12	16
	5	5	10	15	20
	6	6	12	18	24

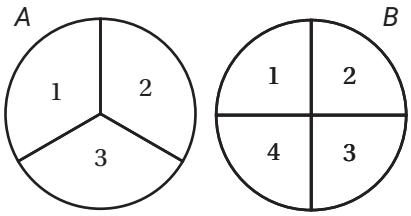
A The sum of rolling two 6-sided dice

B The sum of rolling a 4-sided and a 6-sided dice

C The product of rolling two 6-sided dice

D The product of rolling a 4-sided and a 6-sided dice

2 The correct array to show the sample space for spinning these two spinners is:



A

		Spinner B			
		1	**2**	**3**	**4**
Spinner A	**1**	(1,1)	(1,2)	(1,3)	(1,4)
	2	(2,1)	(2,2)	(2,3)	(2,4)
	3	(3,1)	(3,2)	(3,3)	(3,4)

B

		Spinner B			
		1	**2**	**3**	**4**
Spinner A	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7

C

		Spinner B		
		1	**2**	**3**
Spinner A	**1**	(1,1)	(1,2)	(1,3)
	2	(2,1)	(2,2)	(2,3)
	3	(3,1)	(3,2)	(3,3)
	4	(4,1)	(4,2)	(4,3)

D

		Spinner B		
		1	**2**	**3**
Spinner A	**1**	1	2	3
	2	2	4	6
	3	3	6	9
	4	4	8	12

3 The letters of the word 'FAR' are placed in a bag. Two letters are drawn, replacing the first before drawing the second. The sample space is:

A {FF, FA, FR, AF, AA, AR, RF, RA, RR}

B {F, A, R}

C {FA, FR, AF, AR, RF, RA}

D {FAR, FRA, AFR, ARF, RAF, RFA}

4 The students in 9A were surveyed on their opinion about whether a new lunchtime computer gaming club should be introduced. The results were further broken into groups according to gender and are shown in this two-way table.

	Yes	No	Total
Female	8	3	11
Male	10	4	14
Total	18	7	25

The probability of a student selected at random being a female agreeing to the introduction of a computer gaming club is:

A $\frac{8}{11}$ **B** $\frac{3}{8}$ **C** $\frac{10}{25}$ **D** $\frac{8}{25}$

5 A box contains 2 red pencils and 5 blue pencils. One pencil is taken out at random, and it is red. The pencil is placed back in the box and another pencil is taken out at random. The probability that the second pencil is also red is:

A $\frac{2}{7}$ **B** $\frac{5}{7}$ **C** $\frac{1}{6}$ **D** $\frac{2}{6}$

6 A box contains 2 CDs and 5 DVDs. One is taken out at random, and it is a CD. The CD is not placed back in the box. What is the probability of a CD being drawn now?

A $\frac{1}{7}$ **B** $\frac{3}{7}$ **C** $\frac{3}{6}$ **D** $\frac{1}{6}$

7 An experiment is done using a special pack of 40 cards, containing the numbers 1 to 10 in four colours: red, blue, green, yellow. A card is drawn at random from the deck.

The probability that the card is red or an even number is:

A $\frac{25}{40}$ **B** $\frac{10}{40}$ **C** $\frac{5}{40}$ **D** $\frac{1}{5}$

8 Over the 92 days of winter, the township of Pondsy experienced 20 wet days. Use this information to estimate the probability that 1 July was a wet day.

A $\frac{20}{90} = \frac{2}{9}$ **B** $\frac{22}{90} = \frac{11}{45}$ **C** $\frac{20}{92} = \frac{5}{23}$ **D** $\frac{22}{92} = \frac{11}{46}$

9 Insurance companies continually monitor the number and cost of claims so they can adjust their premiums (the cost of their policies) to ensure they are making a profit.

Over a long period of time, the following information is collected for 18-year-old drivers.

	City drivers	
	Male	**Female**
Probability of a claim	0.16	0.1
Mean cost of a claim	$12 500	$11 800

The premium for an 18-year-old female living in the city if the company wishes to make a 40% profit is closest to:

A $475 **B** $845 **C** $1180 **D** $1650

10 An experiment is run using random numbers from 1 to 10. If 500 trials are conducted, how many times will the number drawn be divisible by 5?

A 10 **B** 20 **C** 50 **D** 100

11 An experiment uses a special pack of 40 cards, containing the numbers 1 to 10 in four colours. Two cards are drawn at random from the deck without replacement.

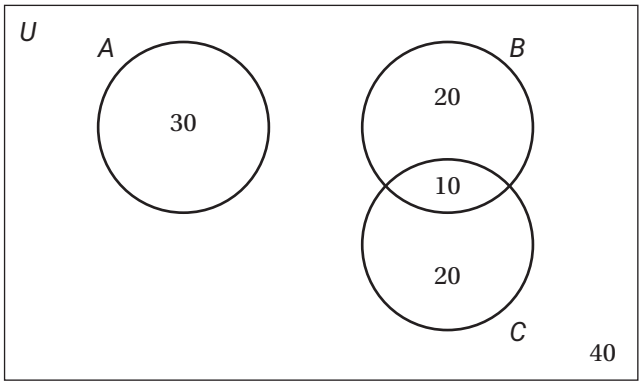
What is the probability that both cards are the number 4?

A $\frac{3}{390}$ **B** $\frac{2}{40}$ **C** $\frac{1}{100}$ **D** $\frac{1}{40}$

12 The letters from the word ANNA are placed in a bag and two are drawn without replacement. What is the probability that the two letters are the same?

A $\frac{1}{4}$ **B** $\frac{3}{8}$ **C** $\frac{1}{3}$ **D** $\frac{1}{2}$

13 The probability of A or B is:



A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** $\frac{3}{4}$ **D** $\frac{3}{8}$

Short answer

1 Bobby and Kai are part of a 10-player squad selected to represent the school in basketball. Determine the probability that:

(a) one of them will be captain

(b) neither of them will be captain.

2 During the 31 days of January, it was over 30°C on 17 days. Determine the probability that:

(a) 1 January was not over 30°C

(b) two days in a row are over 30°C.

3 The letters from the word 'ACHE' are placed in a bag. Two letters are drawn, replacing the first one before drawing the second.

(a) Show the sample space using an array.

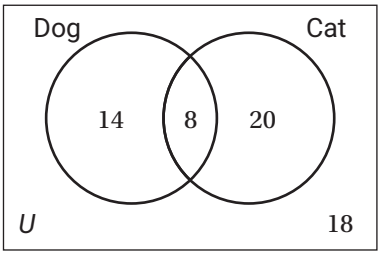
(b) Calculate the probability of drawing two A's.

(c) Calculate the probability of drawing exactly one H.

(d) Calculate the probability of drawing two vowels.

(e) Calculate the probability of drawing different letters.

4 The residents of Peach Street were asked about whether they had a dog or a cat. The results are shown in the Venn diagram.



(a) How many residents were interviewed?

(b) What is the probability of a resident owning a dog?

(c) What is the probability of a resident owning a cat?

(d) What is the probability of a resident owning a dog or a cat?

(e) What is the probability of a resident owning a dog or a cat but not both?

(f) What is the probability of a resident owning both a dog and a cat?

(g) What is the probability of a resident not owning a dog?

5 Year 8 and 9 students were surveyed on their opinion regarding adding a sports jacket to the standard school uniform options. The results were further broken into groups according to year level and are shown in this two-way table.

	Yes	No	No opinion	Total
Year 8	32	11	17	60
Year 9	45	20	25	90
Total	77	31	42	150

Calculate the probability of a randomly chosen student:

(a) being in Year 8

(b) voting no

(c) being a Year 9 student who had no opinion.

6 The attendance at a tourist attraction has been above 1000 people on a Friday 27 times over the last 3 years. What is the probability the attendance will be over 1000 next Friday? Write your answer correct to 2 decimal places.

7 A sample survey of 200 residents was conducted in Pearcedale about the needs of their community. They were asked whether the community needed a new shopping centre or a new aquatic playground.

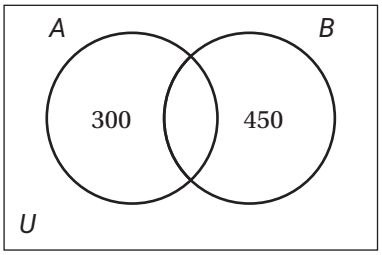
	Shopping centre	Aquatic playground	Neither	Total
Under 18	24	50	4	78
18 to 40	28	24	2	54
Over 40	58	6	4	68
Total	110	80	10	200

There are 600 000 residents in the community.

(a) Estimate the number of residents who want the aquatic playground.

(b) Estimate the number of residents under the age of 18 who want the shopping centre.

8 A probability experiment is conducted with 1000 trials. Some of the results are shown in the Venn diagram.



100 results were neither A nor B.

(a) Calculate the number of results in either A or B or both.

(b) Calculate the number of results in A.

(c) State the number of results in A only.

(d) Calculate the number of results in A and B.

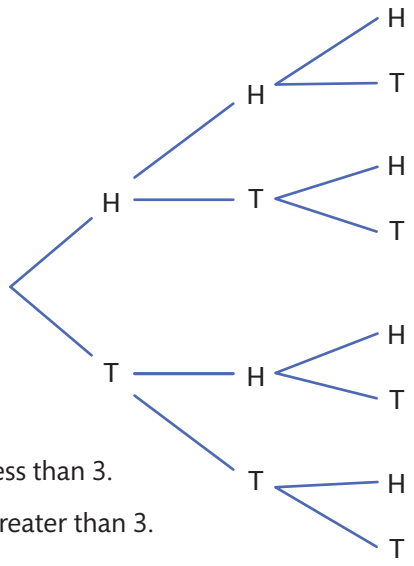
(e) Calculate the number of results in A or B but not both.

(f) Calculate the probabilities, as decimals, of a random result being:

(i) both A and B **(ii)** A or B **(iii)** A or B but not both.

Extended response

1 A coin is tossed three times and the tree diagram is shown below.



(a) What is the probability of tossing HTH?

(b) What is the probability of tossing three tails?

(c) What is the probability of tossing exactly one tail?

(d) What is the probability of tossing at least one tail?

2 A spinner with the numbers 1 to 5 is spun twice.

(a) Show the sample space using a tree diagram.

(b) Calculate the probability of spinning two 2's.

(c) Calculate the probability of spinning two numbers less than 3.

(d) Calculate the probability of spinning two numbers greater than 3.

(e) Why don't the answers to **(c)** and **(d)** add to 1?

(f) Calculate the probability of not spinning a 3.

(g) Calculate the probability of spinning a 1 followed by a 5.

(h) Calculate the probability of spinning an odd number followed by an even number.

(i) Calculate the probability of spinning one odd number and one even number.

3 A probability experiment is run using 1000 spins of this spinner.



(a) State the fraction of the area covered by each colour.

(b) Calculate the expected numbers of each colour in 1000 trials.

(c) The cumulative frequency is shown in this table after 100, 500 and 1000 trials.

Spins	Blue	Green	Red
100	25	35	40
500	135	120	245
1000	244	252	504

(i) Do the results support your estimation of the fractions of each colour? Explain.

(ii) How did the number of spins affect the reliability of the results?

4 Jing drives home from work each night, passing through three major intersections. Each intersection can add up to 5 minutes to the length of her trip. The shortest trip home (i.e. not stopping at any of the intersections) is 24 minutes.

Use a 6-sided die to simulate the extra time Jing will take to get home. Rolling a 1, 2, 3, 4 or 5 corresponds to a wait time of 1, 2, 3, 4 or 5 minutes, respectively. Rolling a 6 means that Jing does not have to wait.

This simulation was run 10 times, as shown in the table.

(a) Complete the table using the information above. (You will then use the table to answer the remaining questions.)

	Numbers rolled	Extra time	Total trip time
Trial 1	6, 3, 2		
Trial 2	4, 1, 6		
Trial 3	5, 3, 1		
Trial 4	5, 1, 4		
Trial 5	3, 3, 6		
Trial 6	1, 5, 2		
Trial 7	2, 5, 2		
Trial 8	4, 3, 1		
Trial 9	6, 1, 5		
Trial 10	4, 5, 2		

(b) Calculate the theoretical maximum travel time.

(c) State the experimental minimum travel time.

(d) State the experimental maximum travel time.

(e) Calculate the experimental mean travel time.

GLOSSARY

acquire data
The process of obtaining data by direct collection or from other sets of data so that it can be sorted, displayed and analysed.

acute angle
An angle that measures between 0 and 90 degrees, i.e. less than one right angle.

addition facts
The results associated with the sums of pairs of natural numbers from 0 to 9. They are foundational to arithmetic.

additive
A situation or relationship that involves addition, subtraction or both, e.g. giving change from a simple money transaction.

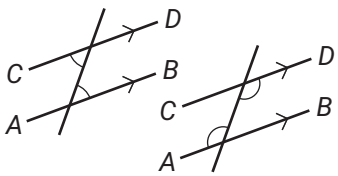
algebra
The process of manipulating variables and constants in a mathematical expression according to laws, properties or rules, e.g. simplifying an expression or solving an equation.

algebraic expression
A mathematical statement formed by combining numbers and algebraic symbols using arithmetic operations, e.g. $a^2 + 3ab - 2b^2$.

algebraically
Carrying out a process symbolically using variables, constants, operations and rules.

algorithm
A process that can be carried out systematically, using a well-defined set of instructions, to perform a particular task or solve a type of problem.

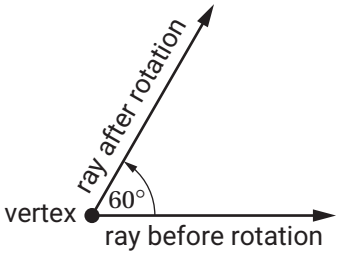
alternate angles
Angles on alternate sides of a transversal of a pair of parallel lines. If the lines AB and CD are parallel, then each pair of alternate angles are equal.



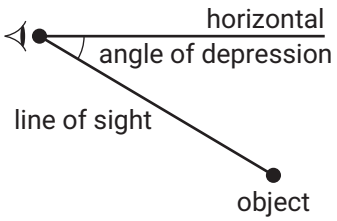
am
Any time before midday when using 12-hour time. An abbreviation of the Latin term *ante meridiem*.

analog clock
A clock with hours marked from 1 to 12 arranged in a circle and moving hands to show you the time.

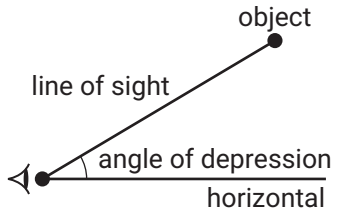
angle
The figure formed at the intersection of two rays in the plane with a common endpoint, called its vertex. It is typically indicated with an arc.



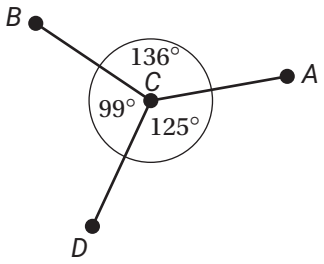
angle of depression
The angle between horizontal and the line of sight from an observer to an object that is lower than the observer.



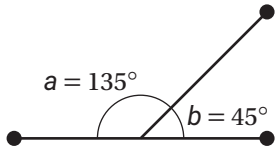
angle of elevation
The angle between horizontal and the line of sight from an observer to an object that is higher than the observer.



angles at a point
The sum of the angles with vertices at a common point in the plane is equivalent to four right angles or 360 degrees. This spatial fact can be applied to determine the size of missing angles.



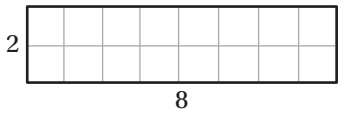
angles on a straight line
The sum of the angles that form a straight line is equivalent to two right angles or 180 degrees. This spatial fact can be applied to determine the size of missing angles.



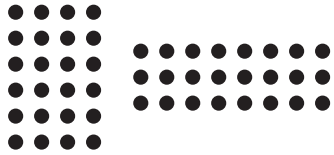
approximate
To obtain or state a value to a particular accuracy.

approximation
A result that is not exact, but is close enough for a given purpose, e.g. giving an approximation of the area of a complex shape by using a combination of basic shapes.

area
The amount of interior space bounded by the perimeter of a shape. It is usually measured in square units. For example, the area of this rectangle is given by $8 \times 2 = 16$ units.



array
An ordered collection of discrete objects or numbers, such as rectangular arrangements of rows and columns. For example, two arrays of dots (4×6 and 8×3) produce two different representations of the number 24.



Asia/Asian
Geographically, the largest continent, bounded by Europe and the Pacific, Arctic and Indian oceans. It can be described in cultural, religious, historical and language boundaries or commonalities.

assign probabilities
The process of providing a qualitative or quantitative value for the likelihood or chance of an event occurring. This may involve judgement based on experience, experiment, counting or measurement.

association(s)
Relationships between pairs of variables for different data types, e.g. in terms of strength, form and direction.

associative
Of or relating to an operation, which, that when applied to any three elements of an expression, is the same regardless of which pair of elements (without changing their order) is combined first.

associative law
In general, the addition or multiplication of a set of real numbers gives the same output, no matter how the numbers are grouped, e.g. $a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$.

attribute
A property of an object or event that can be measured and/or directly or indirectly compared.

back-to-back stem-and-leaf-plot
A method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem'.

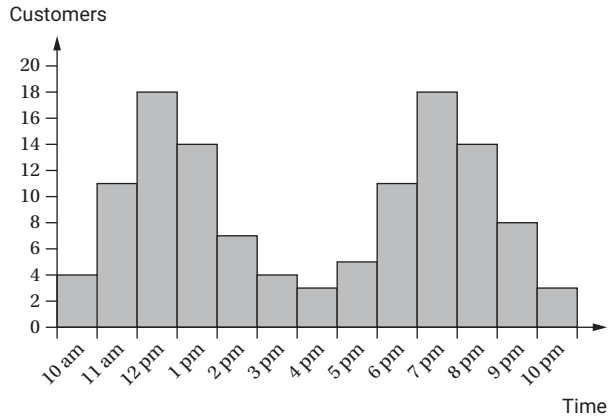
Pulse rate

Before		After
9 8 8 8	6	
8 6 6 4 1 1 0	7	
8 8 6 2	8	6 7 8 8
6 0	9	0 2 2 4 5 8 9 9
4	10	0 4 4
0	11	8
	12	4 4
	13	
	14	6

base-10
A number system which uses the digits 0–9 and the value of the digit is determined by its face value and its place value, e.g. $283 = 2 \times 100 + 8 \times 10 + 3 \times 1$ and $283 = 200 + 80 + 3$.

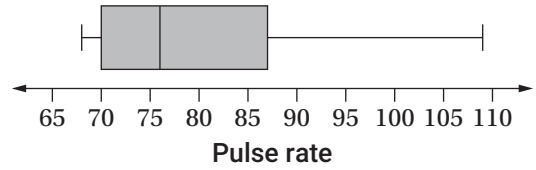
bias (statistics)
A measure being systematically moved away from the true value, leading to overestimating or underestimating that value.

bimodal distribution
The distribution of a set of data that has two modes. The term 'bimodal' is also used if the graph of the distribution has two distinct peak values, which may not be equal.



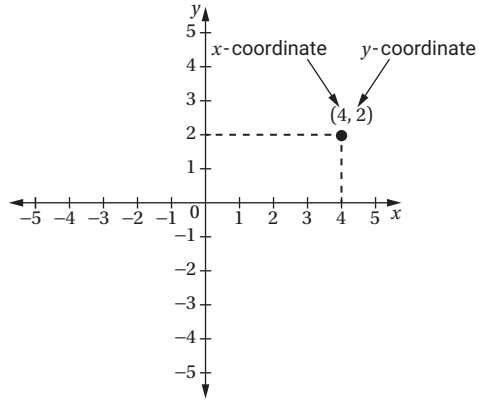
bivariate data
Data relating to measurement of two variables. It can be categorical data, numerical data or a combination of both.

boxplot
A graphical display of a 5-number summary of data. The 'box' covers the middle 50% of scores. The 'whiskers' extend to the maximum and minimum values in the data set.

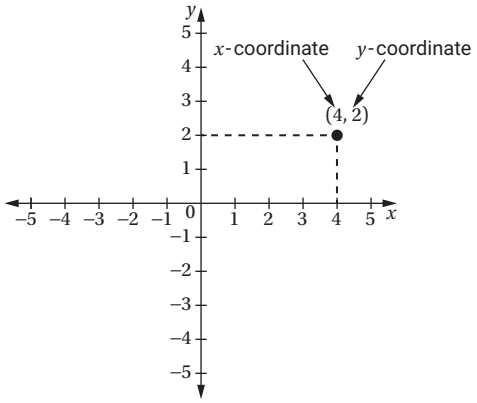


capacity
The amount a container will hold. It is often used in relation to the volume of fluids. Units of capacity (volume of fluids or gases) include litres (L) and millilitres (mL).

Cartesian coordinate system
The positioning system for any point on the Cartesian plane, represented by an ordered pair of real numbers (x, y), which are called the coordinates of the point relative to the origin at (0, 0).



Cartesian plane
A plane divided into 4 regions (quadrants) by 2 perpendicular axes, the x-axis and the y-axis, which intersect at a point called the origin.



categorical data
Data in discrete categories, e.g. data on blood groups with values of type A, B, AB or O.

categorical variable
A variable that takes values from sets of categories. There are two types of categorical variables: nominal and ordinal.

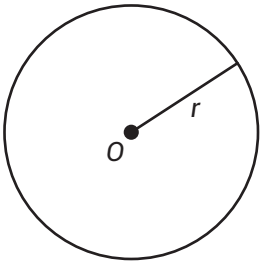
census
Collection of information about the whole population.

centre (space)
The middle point of a shape, object or space.

centre (statistics)
In a set of data, a single value that is used to represent or typify that data. Informally, it is a notion of average.

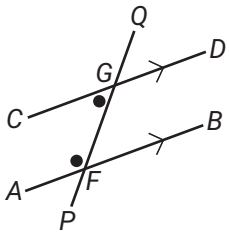
chance experiment (trial)
An investigation process that involves planning and conducting a repeatable experiment involving chance, where more than one possible outcome may occur. Experiments can be single or multi-step and results can vary across trials.

circle
The set of all points in the plane that are a fixed distance (the radius) from a given point (the centre of the circle).



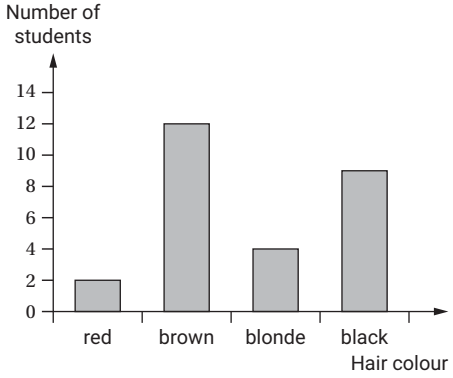
circumference
The boundary (perimeter) of a circle. The length of the circumference C is given by $C = \pi d$, where d is the diameter. Alternatively, it is given by $C = 2\pi r$ where r is the radius.

co-interior angles
A pair of angles that lie between a pair of parallel lines cut by a transversal and has a sum of 180 degrees or two right angles.



colonisation
The act of setting up a colony away from one's place of origin. With humans, it can be seen negatively because it tends to involve an invading culture taking control over an indigenous population.

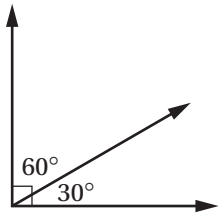
column graph
A graph for organising and displaying categorical data. Equal-width rectangular bars are constructed for each category with height equal to the observed frequency of the category. Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees, is 90°.



commutative
An operation in which the result of applying it to any two elements of a set is the same, regardless of the order of the elements. Addition and multiplication are commutative on the set of natural numbers.

commutative property
In general, the commutative property of addition and multiplication of real numbers is that for all real numbers a and b, $a + b = b + a$ and $a \times b = b \times a$, respectively.

complementary angles
Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees is 90°.



complementary events
Two mutually exclusive outcomes in a probability experiment, such that Pr(A) + Pr(B) = 1, where Pr(A) is the probability of event A and Pr(B) is the probability of event B.

complex shapes and objects
A complex shape is a closed shape comprised of a number of plane shapes. Complex objects are comprised of simpler objects.



composite number
A non-zero natural number that has a factor other than 1 and itself.

composite shape
A shape that is formed by combining other plane shapes. Composite shapes are often described as 'complex' when they are made up of many and different shapes.



compound event
In probability, an event that consists of more than one event occurring at the same time, e.g. tossing a coin and selecting a card.

compound interest
The interest earned by investing a sum of money (the principal) when each successive interest payment is added to the principal for calculating the next interest payment.

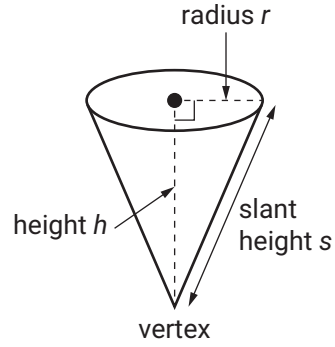
compounding error
The increasing level of error that arises when measurements with errors are used to calculate other quantities.

computation
The action of performing a mathematical calculation. It can be performed mentally or by using an algorithm or a digital tool, such as a calculator.

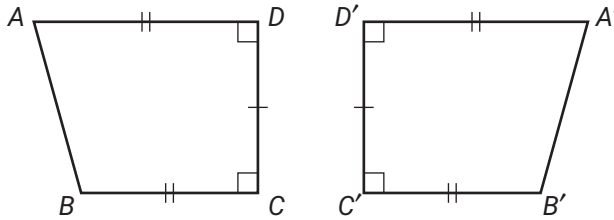
computational thinking
A way of thinking that helps to organise data logically by breaking down problems into parts; defining abstract concepts; and designing and using algorithms, patterns and models.

conditional probability
Consideration of whether the knowledge of the occurrence of one event, A, affects the probability of occurrence of another event, B.

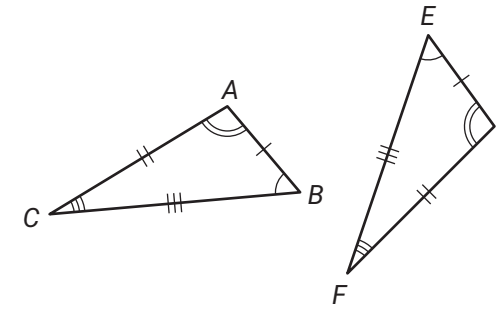
cone
An object that is formed by taking a circular base and a point not in the plane of this circle, called the vertex, and joining the vertex to each point on the circumference of the base.



congruent
An exact match between every part of one figure with the corresponding part of the other figure, i.e. congruent figures can be exactly superimposed on each other.



congruent triangles
Triangles that have the same size and shape. When not all measurements are known, the determination can be made using the four standard congruence tests for triangles (SSS, SAS, AAS, RHS).



conjecture
A statement whose truth or otherwise is not yet determined, but is open to further investigation, e.g. Goldbach's conjecture of 1742 has not yet been either proven to be true or shown to be false.

construct
To draw, make or build a mathematical object such as a shape, pattern, algorithm or proof.

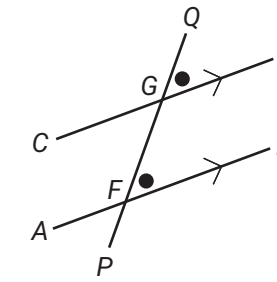
context
The situation or setting of an event, problem or application. Contexts can be real world or constructed.

continuous numerical variable
A numerical variable that can take any value that lies within an interval. In practice, the values taken are recorded to the accuracy of measurement of the instrument used to obtain these values.

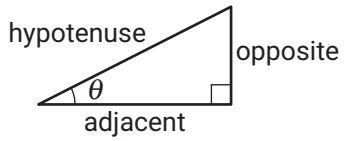
coordinate system
A system for showing a set of values that give an exact position, e.g. Cartesian coordinate system, grid map or a seating plan for a venue.

coordinates
A set of values that define the position of a point or an object in a space. The coordinate system being used will govern the nature of the coordinates.

corresponding angles
Angles that lie in the same relative position at the points of intersection, where a pair of parallel lines is cut by a transversal and which are equal.



cosine ratio
The ratio of the length of the base, b, of a right-angled triangle to the length of its hypotenuse, h, for a given base angle theta, θ. In any right-angled triangle,
$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$, where $0 < \theta < 90°$.

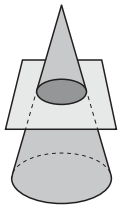


counting
The process of quantifying the number of objects in a set or collection.

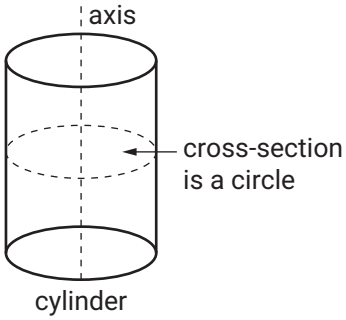
Country
A space mapped out that individuals or groups of First Peoples of Australia occupy and regard as their own and having varying degrees of spirituality. It includes lands, waters and sky.

Country/Place
Spaces mapped out that individuals or groups of Aboriginal and Torres Strait Islander Peoples occupy and regard as their own and that have varying degrees of spirituality. They include lands, waters and sky.

cross-section
The intersection of a solid object with a plane. It may differ for a given solid depending on the orientation of the slicing plane.



cylinder
An object that has parallel circular discs of equal radius at the ends, which are joined by a curved surface.



data
A general term for a set of observations or measurements collected during an investigation. Primary data is generated by the user; secondary data is collected from a secondary source, such as a journal, book or website.

data displays
Visual formats for organising, representing and summarising data. Also called visualisations.

data distributions
Patterns in the values of a set of data. The representation used to show the distribution depends on the type of data and how it is measured.

decay
Decrease in the value of the dependent variable of a function as the value of the independent variable increases.

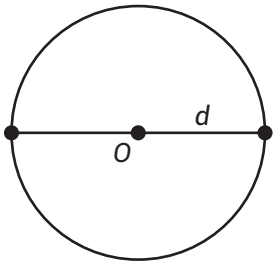
decimal
Used to describe aspects of the base-10 number system. The decimal point (. or ,) separates the whole number part of a number from its decimal part.

degrees
A unit of measurement for measuring an angle. Angles are measured as a proportion of a full turn, which is equivalent to 360 degrees, so that one degree is equal to $\frac{1}{360}$ of a full turn. Written as $\alpha°$.

denominator
In the fraction $\frac{a}{b}$, b is the denominator. It is the number of equal parts into which the whole is divided.

diagonal
A line segment joining two vertices of a polygon that do not share an edge.

diameter
A chord passing through the centre of a circle. Also describes the length of the diameter.



digit
A single symbol that is used to represent a number as a numeral. In the base–10 number system there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

digital tools
Electronic devices, programs, websites or online resources that can be used to support the learning and doing of mathematics.

direct comparison
Comparing objects and events using a particular attribute, e.g. measuring the length of two pencils by placing them side by side or starting two events at the same time to compare duration.

direct proportion
The relationship between corresponding values of two variables in a fixed ratio relative to each other;
i.e. for variables x and y, $\frac{y}{x} = k$ and $y = kx$, where $k \neq 0$.

directional language
Words that communicate position, location and movement within a space.

discrete numerical data
Numerical data that can take only a countable number of values, e.g. the number of people in a car, or clothing sizes.

distributive
An operation that can take priority over the operation used for combination within brackets. Multiplication is distributive over addition, e.g. $8 \times 17 = 136$ as $8 \times 17 = 8 \times (10 + 7)$ and $8 \times (10 + 7) = 8 \times 10 + 8 \times 7$ and $8 \times 10 + 8 \times 7 = 80 + 56$, which equals 136.

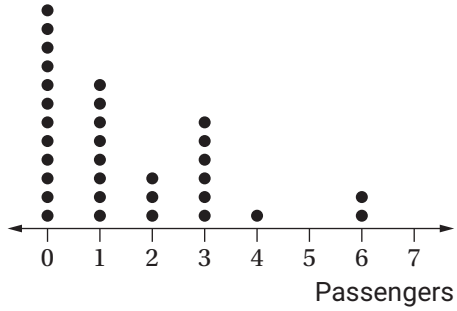
distributive property
In general, the distributive law (property) for multiplication over addition for real numbers states that for all real numbers a, b and c: $a(b + c) = a \times b + a \times c$.

divisible
Able to be divided exactly, i.e. without leaving a remainder.

division
For a finite set, the process of partitioning the set into subsets of equal size. For natural numbers, it expresses a given number as a multiple of a smaller number and any remainder.

division facts
Facts that draw on the inverse relationship between division and multiplication and are directly related to the multiplication facts, e.g. $2 \times 5 = 10$, so $\frac{10}{2} = 5$ and $\frac{10}{5} = 2$.

dot plot
A graph used to display numerical data. Using a number line, it displays a dot for each piece of data and, when there is more than one piece of data with the same value, the dots are stacked vertically.



duration
A measure of time that an event or activity takes to complete.

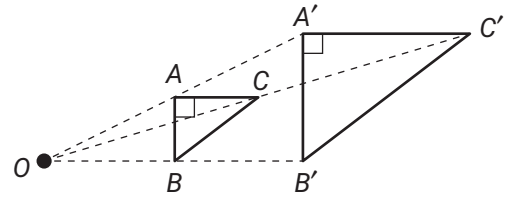
dynamic geometric software
Software that supports the representation, construction and manipulation of geometric shapes and objects, and related measures and other functions in an interactive and dynamic digital environment.

efficient strategies
The choice of suitable approaches (mental, written, using digital tools), and methods within these. The means used for calculation reflect the purpose and context.

element(s)
A member of a set, e.g. 3 is a member of the set of natural numbers $N = \{0, 1, 2, 3, ...\}$. This relation can be written more concisely as $3 \in N$ ('3 is an element of the set N').

enclosed spaces
Space that is enclosed by a boundary, such as the perimeter of a shape or the outer surface area of an object.

enlargement transformation
A scaling of a figure in which the corresponding lengths in the transformed figure are increased or decreased in proportion to the original figure. The two figures are similar.



equal sharing
Dividing a collection, shape or object into equal parts.

equally likely outcomes
An event for which all outcomes have the same probability of occurring.

equation
A statement that includes the '=' symbol. Equations are used to show the equality of two expressions.

equivalent
Equal in value or meaning, such as an expression or statement that is essentially the same, two or more sets that are capable of being mapped in a one-to-one relationship.

equivalent fractions
Fractions that are different representations of the same value, e.g. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

error
The difference between an actual value and its measured or estimated value.

estimation
The skill of conceptualising and mentally manipulating numbers or measurements to find an approximate answer. The capacity to make reasonable adjustments to estimates is essential in estimating.

estimation strategies
Strategies used to estimate or find an approximate value for the amount, extent, duration, position or size of something.

Euler's formula
A relation between the number of faces, vertices and edges of any polyhedron. It is written $F + V = E + 2$, where F is the number of faces, V is the number of vertices and E is the number of edges.

even number
An integer that is divisible by 2. The even natural numbers are {0, 2, 4, 6, ...}

event
A subset of the sample space of a random experiment.

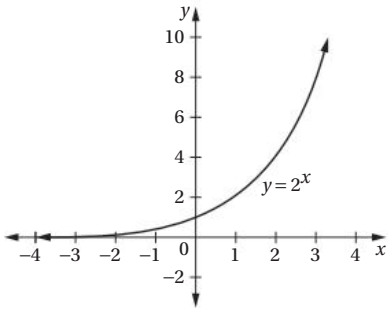
expanded notation
A way of representing a number, separating each digit and representing it in an additive number sentence to show each digit's place value, e.g. $2345 = 2 \times 1000 + 3 \times 100 + 4 \times 10 + 5$.

expected frequencies
The number of times that a particular event or outcome is predicted to occur using theoretical probability.

exponent
The power (or index) to which a number or algebraic expression is raised, shown using superscript, e.g. in the following expressions the exponent is 3: 2^3, $2x^3$, $2(x + 5)^3$.

exponent laws
Rules for manipulating exponents. Also referred to as index laws.

exponential functions
Functions comprising a positive constant raised to the power of the variable; i.e. $f(x) = ax$ where a is a constant and x is the variable.



expression
Two or more numbers or variables connected by operations.

factors
Let a, b and c be natural numbers such that $a \times b = c$, then a and b are factors (or divisors) of c, e.g. $3 \times 4 = 12$, so 3 and 4 are factors (divisors) of 12.

formulating expressions
The process of using mathematical notation, conventions and symbols to represent situations (including informal or worded statements) as mathematical expressions.

fraction
A number of the form $\frac{a}{b}$, where a is an integer and b is a non-zero integer, e.g. $\frac{4}{6}, \frac{11}{5}, -\frac{3}{7}$. A fraction is said to be in simplest form when a and b have no common divisor greater than 1.

frequency
The number of times that a particular value occurs in a data set. For grouped data, it is the number of observations that lie in that group or class interval.

frequency table
A table listing the frequency (number of occurrences) of observations, measurements or outcomes in different ranges, called class intervals.

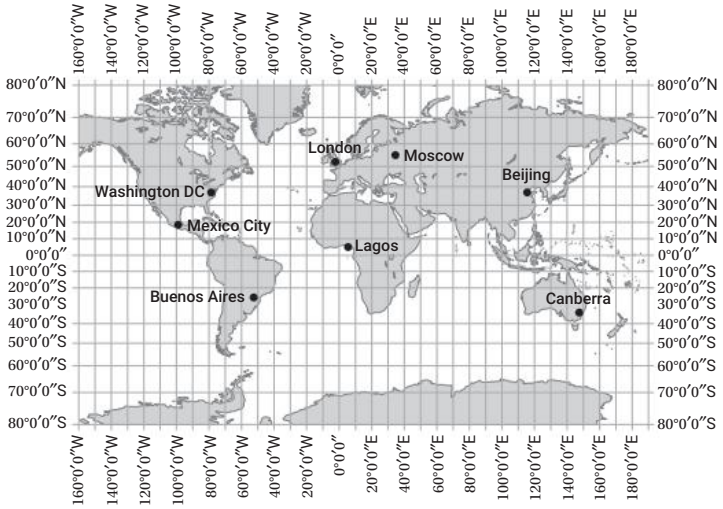
function
A correspondence (map or relation) between the elements of two sets where each element in the first set is mapped to exactly one corresponding element in the second set, either one-to-one or many-to-one.

function machine
An algorithmic process which takes an input, applies an operation (or operations) and results in an output.

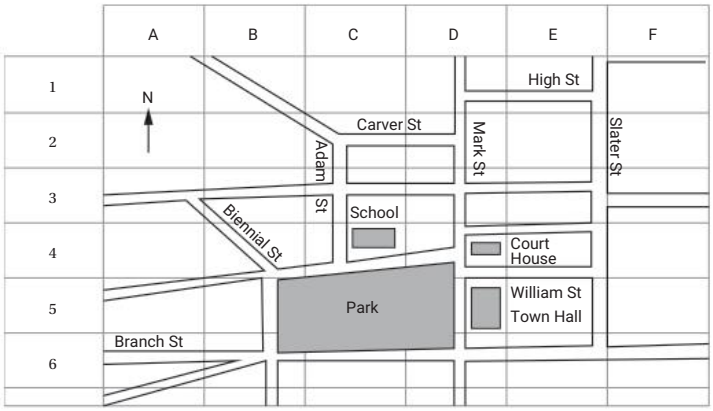
geometric patterns
Patterns formed by a series of shapes. They may be repeating or growing patterns which change in predicted ways that can be generalised. Tessellations are geometric patterns formed by transforming shapes.

gradient
The slope of a line. It is calculated as the gradient of a line segment it contains. The gradient (m) of the line containing $A(x_1, y_1)$ and $B(x_2, y_2)$ is calculated as $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

grid coordinate system
A system for referencing location within a space using labelled horizontal and vertical grid lines. Locations are defined using a set of coordinates which indicate points in the plane where two grid lines meet.



grid reference
A way of identifying a region on a map. Coordinates and grid lines are used to refer to specific features or locations, e.g. in the map shown, the school is located at the grid reference C4.



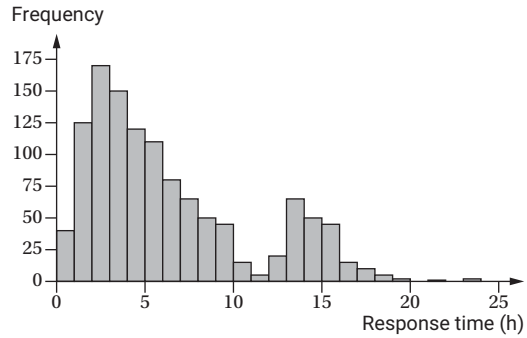
grouping
A process of partitioning a set into subsets according to a given criterion, e.g. a class of students according to eye colour. Equal grouping means each subset has the same number of elements.

growing pattern
A pattern where each term is either an increase or a decrease on the previous term. Change from term to term is predictable if the pattern is growing additively, multiplicatively or exponentially.

growth
A measure of the increase in value of the dependent variable of a function as the value of the independent variable increases, e.g. the length of a spring when weights are added.

guided statistical investigation
A teacher-led, scaffolded approach to the statistical investigation process, including collecting, representing and analysing data, and interpreting and communicating results.

histogram
A statistical graph for displaying the frequency distribution of continuous data. It is also a graphical representation of the information contained in a frequency table.



improper fraction
A fraction where the numerator is greater than or equal to the denominator, e.g. $\frac{7}{5}$ is an improper fraction, whereas $\frac{5}{7}$ is a proper fraction.

inequality
A mathematical expression containing the terms 'less than', 'less than or equal to', 'greater than', or 'greater than or equal to' and their respective symbolic representations $<$, $>$, $\leq$ and $\geq$.

inference (statistics)
The process for determining consequences or conclusions based on assumptions, evidence and reasoning.

informal units
Units whose values are decided on in a given context, e.g. the use of pace to measure distance (non-uniform unit); the use of paperclips to measure length (uniform unit).

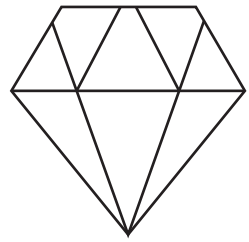
integer
An element of the infinite set of numbers $Z = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$.

interval(s)
A continuous subset of the real number line, e.g. 'the set of all real numbers greater than or equal to 10'.

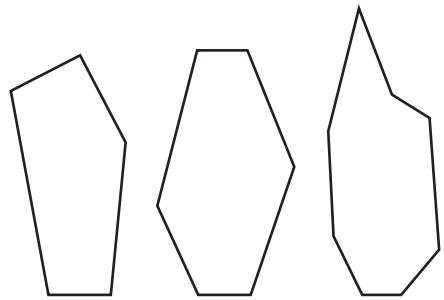
inverse operation
An operation in arithmetic which undoes the effect of another operation. Multiplication and division are inverse operations, as are addition and subtraction.

irrational number
A number that cannot be expressed as a fraction in the form $\frac{m}{n}$, where m and n are integers and n is non-zero. The decimal expansions of such numbers are infinite and non-recurring.

irregular object
An object with faces of varying shapes. Contrast with a regular object, such as a cube.

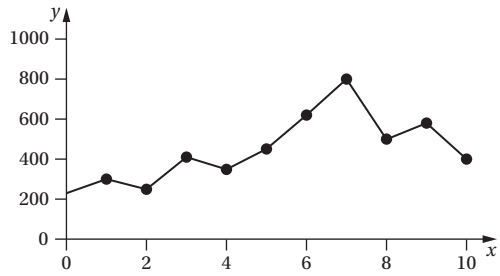

diamond

irregular shape
A shape with sides of varying lengths. Contrast with a regular shape, such as a rhombus or equilateral triangle.



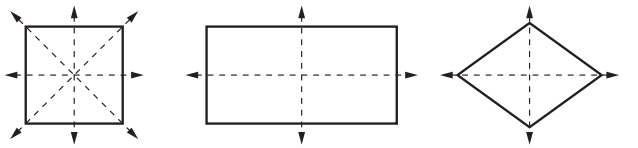
line
A model of what is perceived visually as straight. It has one dimension and extends indefinitely in the plane.

line graphs
Graphs that represent the changing relationship between two variables. They use lines to connect individual data points, and are generally used to show how something changes over time.



line segment (interval)
Let A and B be two distinct points on a line, then the part of the line between and including A and B is a line segment.

line symmetry
A line in which a shape is symmetrical, such that if you were to fold the shape about this line it would map onto itself exactly.



Square
4 lines of symmetry

Rectangle
2 lines of symmetry

Rhombus
2 lines of symmetry

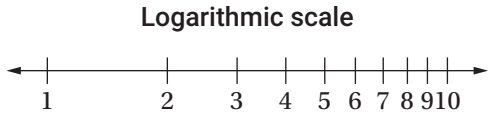
linear equations
Equations involving linear expressions. The general form of a linear equation in one variable is $ax + b = c$, where a, b and c are constants.

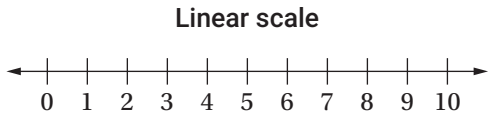
linear expression
An expression involving only variables to the power 1, and constants, e.g. $2x + 3$, $180(n - 2)$, $4x - 5y$.

linear functions
A function which when graphed on a Cartesian plane produces a straight line. It can be reduced to the general form $y = mx + b$.

linear inequalities
Inequalities that involve linear expressions and contain one of the inequality signs: either $<$, $>$, $\leq$ or $\geq$.

logarithmic scales
Scales where successive endpoint values of intervals increase by a constant factor (multiplicatively). Contrast with linear scales in which the increase is a constant amount.





mass
The measure for how much matter an object contains. The SI unit for mass is kilogram (kg).

mathematical modelling
A mathematical process involving the identification of a real-world problem, formulation and calculation of a solution to the problem mathematically using knowledge and understanding of mathematical concepts, structures and relationships, and the interpretation and communication of the mathematical solution(s) in terms of the situation. As students', mathematical modelling skills become more sophisticated, they begin to consider assumptions, choose variables, justify decisions and evaluate the effectiveness of the model.

mean
The sum of values in a data set divided by the total number of values in the data set. Also called the average.

measure
A record of the magnitude of an attribute (such as weight, length, duration and likelihood) associated with an object or event.

measure of central tendency
A statistic that is used to summarise a data set. There are three common measures of centre for a data set: mode, median and mean.

measure of turn
A measurement of rotation from an initial starting position. A turn can be a fraction of a circle (quarter turn) or complete rotation (revolution). It can also be directional (clockwise or anti-clockwise).

median
The value in a set of ordered data that divides the data into two equal parts. It is often called the 'middle value'.

mixed numeral
A form of representation of a rational number (also called a mixed fraction or mixed number) composed of an integer and a proper fraction, i.e. a fraction with a smaller numerator than denominator, e.g. $2\frac{3}{5}$.

mode
The most frequently occurring value in a set of data. There can be more than one mode. When there are two modes, the data set is said to be bimodal.

modelling
Using mathematical concepts, structures and relationships to describe and characterise, or model, a situation in a way that captures its essential features.

multiples
A multiple of a number is the product of that number and an integer. A multiple of a real number x is any number that is a product of x and an integer.

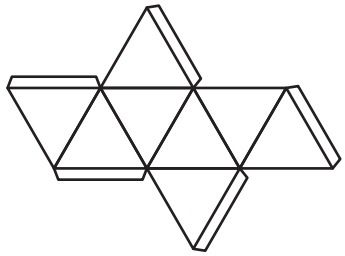
multiplication facts
The results associated with the products of pairs of natural numbers from 0 to 9, associated with reasons. They are foundational to arithmetic.

multiplicative
Problems or contexts that involve multiplication or division, e.g. calculating the number of seats in a theatre that has 30 rows of 24 seats.

natural numbers
The set $N = \{0, 1, 2, 3, ...\}$ or $N = \{1, 2, 3, ...\}$ depending on whether counting is started at 0 or 1. The elements of N are also called the counting numbers, used to count the number of elements in finite sets.

nets

Plane figures that can be folded to form a polyhedron. More specifically, two-dimensional representations comprising joined shapes (the faces) that can be folded (along edges) to form the object.



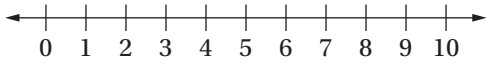
network



A set of points (vertices or nodes), some of which are joined by lines or curves (edges) and sometimes enclose regions (faces), e.g. road networks, a family tree or the edges lining a tennis court.

number line

A line that gives a pictorial representation of real numbers.



number sentence

A statement of equality or inequality using numbers, operations and common symbols, e.g. $8 + 5 = 13$ and $16 - 6 = 10$.

numeral

The designation of a number in a given language, e.g. the number 'three' is designated by the Hindu-Arabic numeral 3, the Roman numeral III, and the Chinese numeral 三.

numerical data

Data associated with a numerical variable. It can be discrete or continuous, e.g. number of siblings in a family or heights of students in the class.

numerical variables

Variables with values that are numbers, and for which measuring and arithmetic processes, such as adding and subtracting or calculating an average, make sense.

object

A solid or three-dimensional (3D) skeletal representation composed of a number of faces, edges, vertices or closed surfaces, such as a prism, pyramid, cylinder or sphere. The term three-dimensional shape is also used.

obtuse angle

An angle that measures between 90 and 180 degrees, i.e. between one and two right angles.

odd number

An integer that is not divisible by 2. The odd numbers are $\{... -5, -3, -1, 1, 3, 5, ...\}$.

online tools

Digital hardware, software, platforms and resources that are connected and support communications between users. Examples of online tools include websites for online shopping and fitness apps.

operation

The process of combining numbers or expressions. Operations are arithmetic – addition, subtraction, multiplication and division – and also include exponentiation and substitution.

order of operations

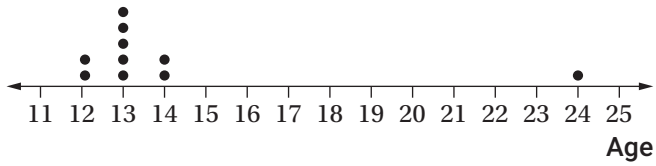
A set of conventions for evaluating expressions involving several operations. Operations in brackets are first, followed by exponents, multiplication/division, then addition/subtraction left to right.

ordinal categorical data

Categorical data that has a ranking or order, such as Likert scales, clothing sizes, placegetters in a race.

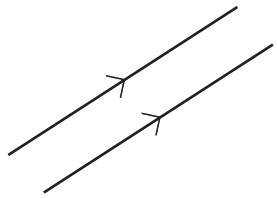
outlier

A data value that appears to stand out from the other members of the data set by being unusually high or low.



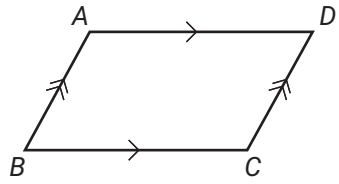
parallel lines

Two lines L and M in a plane that have no point of intersection in that plane, written $L \parallel M$.



parallelogram

A quadrilateral in which opposite sides are parallel and equal in length. The quadrilateral $ABCD$ shown is a parallelogram because $BA \parallel CD$ and $AD \parallel BC$.



parameters

A factor for which a specific value defines a condition or constraint for a function or relation.

partition numbers

Separating numbers additively or multiplicatively into two or more parts, e.g. 10 is 8 + 2 and 8 is 3 + 3 + 2. 12 can be divided into six equal parts of 2 (so $12 = 6 \times 2$ or $12 \div 6 = 2$), or into 3 equal parts of 4 (so $12 = 3 \times 4$ or $12 \div 3 = 4$).

partitioning

The ability to think about numbers as made up of two or more parts. Numbers can be partitioned into standard or non-standard place value partitions, such as 248 as 200 + 40 + 8 or 62 as 50 + 10 + 2.

part-part-whole reasoning

A model used to recognise relationships between a whole number and its parts, e.g. $7 = 5 + 2$ or $7 = 4 + 3$.

percentage

A ratio to 100 or a fraction whose denominator is 100.

perfect square
A number that is the square of an integer, e.g. 169 is a perfect square as $13^2 = 169$. Perfect squares can also be represented pictorially.

perimeter
The total length of the boundary of a plane figure or space.

perpendicular
Two lines, rays, line segments, vectors, planes or other objects that intersect at a 90° angle (a right angle) are considered perpendicular.

pi (π)
The ratio of the circumference of any circle to its diameter. It is denoted by the Greek letter π (pi). Approximate values are 3.14 or $\frac{22}{7}$.

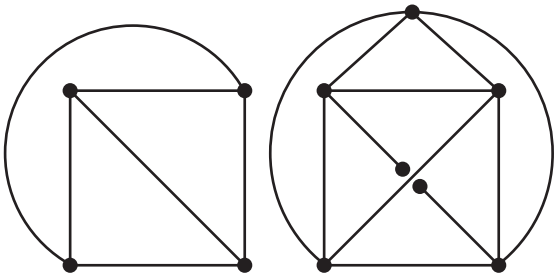
picture graph (pictographs)
A statistical graph for organising and displaying categorical data.

Ball sports played by students in Year 4	
Football	⚽⚽⚽⚽
Basketball	⚽⚽⚽
Netball	⚽⚽⚽⚽⚽
Soccer	⚽⚽
Rugby	⚽⚽⚽
Hockey	⚽⚽

Key ⚽ = 10 students

place value
The value of a digit as determined by its position in a number relative to the ones (or units) place. For integers, the ones place is occupied by the rightmost digit in the number.

planar graph
A connected graph that is drawn within a plane whose edges do not cross.



platonic solids
Three-dimensional, convex and regular objects that have polygonal faces. The five platonic solids are the tetrahedron, cube, octahedron, dodecahedron and icosahedron.

point
Representation of a location in space that has zero dimensions. Lines, curves, shapes, surfaces and objects are constructed from sets of points. Points on the Cartesian plane are specified by coordinates.

polyhedra
Three-dimensional objects whose faces are adjacent polygons. A convex polyhedron is a finite region bounded by planes, in the sense that the region lies entirely on one side of the plane.

population
The complete set of individuals, objects, places, etc. that we want information about. A census collects information about the whole population.

prime number
A natural number that is greater than 1 and its only factors are 1 and itself.

prism
A convex polyhedron that has two congruent and parallel polygonal faces; all its other faces are parallelograms. It is named according to these two congruent faces, e.g. a triangular prism.

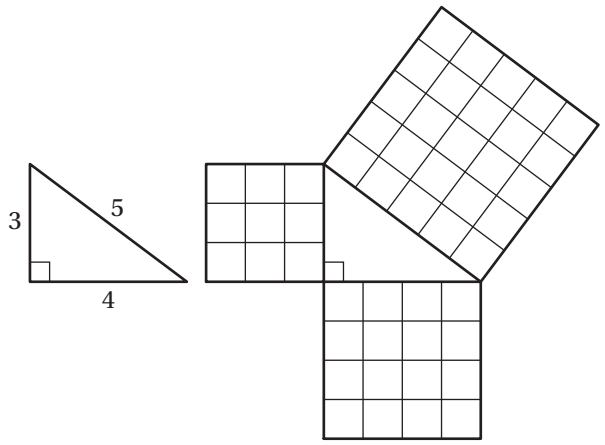
probability
The chance of something happening shown on a scale from 0 to 1 (inclusive), e.g. the probability that a fair coin toss will come up 'heads' is 0.5.

product
The result of multiplying together two or more numbers or algebraic expressions.

proof
A mathematical argument that demonstrates whether a proposition is true. A mathematical statement that has been proved is called a theorem.

proportion
A constant ratio between corresponding elements of two sets, e.g. the circumference and diameter of a circle are in proportion as their ratio is the constant π (pi).

Pythagoras' theorem
The square of the length of the hypotenuse, c, of a right-angled triangle equals the sum of the squares of the lengths of the other two sides, a and b, such that $c^2 = a^2 + b^2$.



quadratic equation
A quadratic expression that has a particular value. The general quadratic equation in one variable is $ax^2 + bx + c = 0$, for constants a, b, c and where $a \neq 0$.

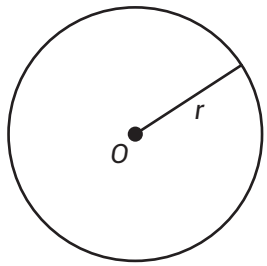
quadratic expression
An expression that contains one or more of the terms in which the variable is raised to the second power, but no variable is raised to a higher power. Its general form is $ax^2 + bx + c$, where $a \neq 0$.

quadratic function
A function in which the rule is a quadratic expression. When graphed on the Cartesian plane it produces a parabola and algebraically it can be represented in the form $y = ax^2 + bx + c$, where $a \neq 0$.

quadrilateral
A four-sided polygon. Examples include square (a regular quadrilateral, all sides and angles equal), rectangle, kite and trapezium.

quotient
The result of dividing one number or algebraic expression by another.

radius
The distance from the centre to the circumference of a circle.



random sample
A subset of the population chosen such that every element of the population has an equal chance of being selected; i.e. their selection is governed by chance, e.g. by using a random number generator.

random variable
A function that assigns real numbers to random events.

range
A set of different things of the same general type, e.g. different representations of the same fraction or a variety of strategies for solving problems involving the basic operations.

range (statistics)
The difference between the largest and smallest numerical values in a data set. It can be used as a measure of spread in a data set. It is sensitive to outliers and should be interpreted with care.

rate
A comparison of two quantities measured in different units, e.g. the rate of distance to time, known as speed, because distance and time are measured in different units (such as kilometres and hours).

ratio
A comparison of magnitudes of sets, quantities of the same kind or algebraic expressions. It is often used as a comparison of the size of two (or more) quantities relative to each other. and is written as $a{:}b$, $a{:}b{:}c$, etc .

rational number
An element of the infinite set of numbers $Q = \frac{m}{n}$, where m and n are integers and $n \neq 0$. It may be expressed in decimal form, e.g. $\frac{1}{8} = 0.125$ and $\frac{4}{9} = 0.444\ldots$

real number line
The representation of real numbers using the infinite set of points on a continuous line.

real numbers
The numbers generally used in mathematics, scientific work and everyday life. They are any of the elements of the infinite set R, which includes all the rational numbers and irrational numbers.

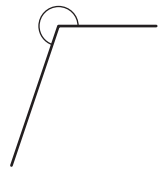
reasonableness (calculation)
Checking the results of calculations or operations to see if they are reasonable in the context, e.g. the sum of two odd natural numbers must be an even number.

rectangle
A quadrilateral in which all angles are right angles.

recurring decimal
A decimal expansion of a number that contains a finite sequence of digits that repeats indefinitely, such as 34.271271271271... The repeating sequence is often designated by a bar over it, such as $34.\overline{271}$.

reflection
A transformation where each point of a shape is mirrored to a position the same distance from an axis of symmetry.

reflex angle
An angle that measures between 180 and 360 degrees, i.e. between two and four right angles.



regular shape
A shape that has all sides of equal length, and all interior angles equal.

related denominators
One of the denominators is a multiple of the other denominator of a pair of fractions.

relation
A correspondence (map) between the elements of two sets, e.g. 'favourite colour' related to the set of students in a class (the domain) and the set of colours (the co-domain).

rename
To express a number according to the relationship between the place value powers of 10, e.g. 263 can be renamed 2 hundreds and 63 ones or 1 hundred, 16 tens and 3 ones.

repeated addition
Adding the same number several times, e.g. $3 + 3 + 3 + 3 = 4 \times 3 = 12$. A strategy sometimes used for multiplication.

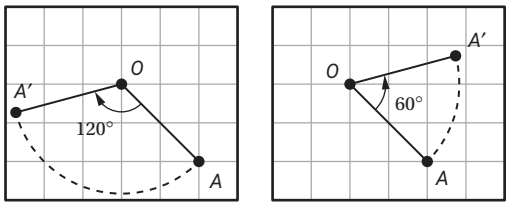
repeating pattern
A pattern where there is an identifiable repeating unit, such as beads arranged in a sequence of red, green, blue, red, green, blue ...; days of the week or months of the year.

revolution
A complete turn around a point.

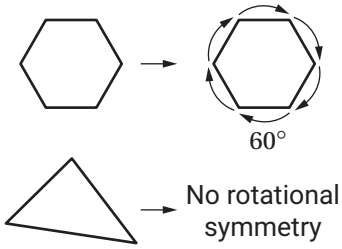
right angle
An angle formed by a quarter turn of a revolution. It is equal to 90 degrees. If two lines are at a right angle, they may also be referred to as being perpendicular.

right prism
Convex polyhedra that have two congruent and parallel faces and all remaining faces are congruent rectangles.

rotation
A transformation where each point in the plane is rotated through a given angle about a fixed point in the plane (the point or centre of rotation). For example, point A is rotated 120° clockwise and 60° anti-clockwise.



rotational symmetry
A quality of some shapes and objects to appear the same when turned around their centre for less than a full turn, e.g. a square has quarter-turn rotational symmetry.



rounding
The process for approximating a value that lies between two known values. It is used to specify a number correct to a given accuracy, e.g. 4.027 becomes 4.03 when rounded to 2 decimal places.

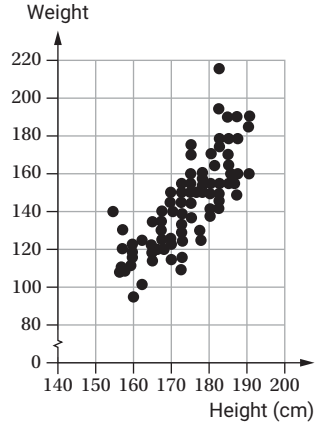
sample
A subset of a population used to estimate characteristics of the population, e.g. a randomly selected group of 8-year-olds (sample) selected to estimate the height of 8-year-olds in Australia (population).

sample space
The set of all possible outcomes of a chance experiment, e.g. the set of outcomes (also called sample points) from tossing two heads is {HH, HT, TH, TT}, where H represents a 'head' and T a 'tail'.

sampling
A process by which a subset of a population is selected for the purposes of data analysis. This subset is called a sample.

scale
Specification of proportion between two measures, e.g. a model of a house may be made on a 1:10 scale of length; on a graph, each interval of the vertical axis corresponds to a given value.

scatter plot
A visual representation of bivariate numerical data using the position of each dot on the horizontal and vertical axis to indicate the numerical values for an individual data point. Scatter plots are used to observe relationships between variables.



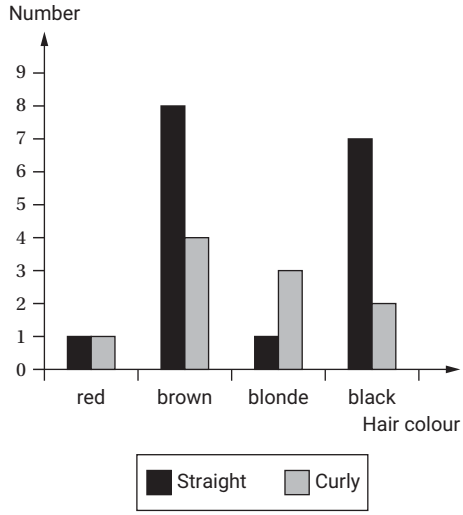
scientific notation
A way of writing numbers that are too big or too small to be written in an accessible way. It uses powers of 10, e.g. the scientific notation for 34 590 is 3.459×104.

set
A collection of objects or elements, usually specified by listing its elements (e.g. {1, 2, 3, 4}), by describing it in words (e.g. 'the set of primes') or by using a rule such as $\{(x, y): y = 2x + 1, x \in N\}$.

shape (space)
A geometric figure in 2-dimensional space, such as a freehand closed curve, a triangle or square. It can be regular or irregular, simple or complex.

shape (statistics)
The form of the visual representation of the distribution of numerical data, which may be symmetrical, skewed or multimodal.

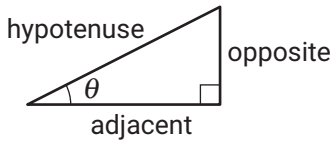
side-by-side column graph
A type of column graph that can be used to organise and display the data that arises when a group of individuals or things are categorised according to two or more criteria.



simulation
The process of simulating an event using technology or a device, e.g. if two players are equally likely to win a game on past performance, then games could be simulated by successive tossing of a fair coin.

simulations
A simulation is a re-creation of a random phenomena used to collect probability data using either physical or virtual manipulatives such as spinners, coins and cards or computer generated simulations, and which can be performed a large number of times.

sine ratio
In any right-angled triangle, $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$, where $0 < \theta < 90°$.



skip counting
Counting from a given starting value using multiples of a fixed natural number, e.g. {2, 4, 6, ...} or {7, 12, 17, ...}.

spread
A statistic that indicates how widely the values of a data set are distributed. Common measures of spread include range, interquartile range, quantiles and percentiles.

square number
The result of multiplying a natural number by itself, e.g. $16 = 4 \times 4 = 42$, and is an element of the set {0, 1, 4, 9, 16, 25}.

square root
A value which, when multiplied by itself gives the original number. The positive square root of a given real number x is the positive real number y, such that $y^2 = x$.

statistical investigation
An investigation process involving a chosen context, area of interest or an identified problem scenario, identifying the subjects and variables to be investigated, planning the approach, collecting data, representing, interpreting and analysing the data, making inferences and communicating the results of the investigation in terms of the context.

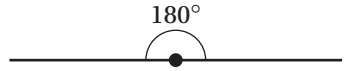
stem-and-leaf plot
A method of organising and displaying numerical data in which each data value is split into two parts: a 'stem' and a 'leaf'. Stem plots give a visual indication of spread.

Pulse rate

Stem	Leaves
6	8 8 8 9
7	0 1 1 4 6 6 8
8	2 6 8 8
9	0 6
10	4
11	0

key $6|8 = 68$ bpm

straight angle
The angle formed by two opposite rays on a straight line from a common point. A straight angle is also formed by the half-turn of a ray about its endpoint. It measures 180 degrees.



subitising
The capacity to visually recognise the size of a small set of objects without counting.

supplementary angles
Two adjacent angles that form a straight angle and their sum is 180 degrees (a straight angle).

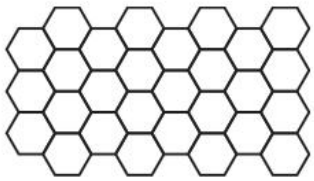
surface area
The measure of the total area of the surface(s) of a three-dimensional shape or object.

survey
A tool for collecting data that comprises a series of questions, which can be constructed to collect quantitative or qualitative data, numerical or categorical responses.

tangent ratio
The ratio of the length of the altitude, a, of a right-angled triangle to the length of its base, b, for a given base angle, theta, θ.

terminating decimal
A decimal expansion of a number which has a finite number of non-zero digits, e.g. 3.125.

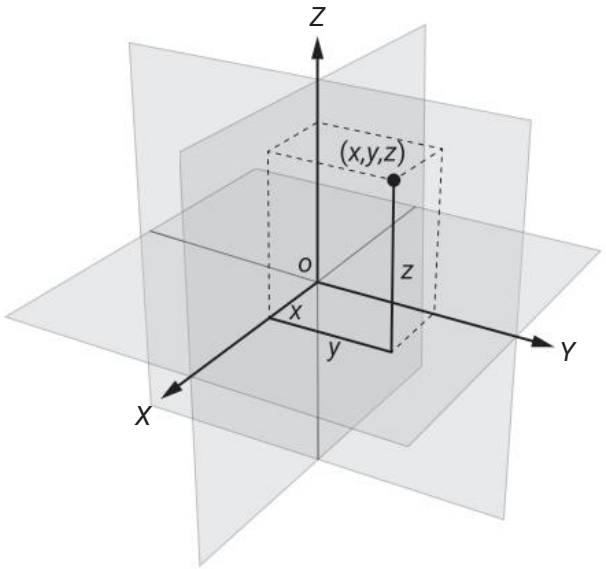
tessellation
A repeated pattern in the plane or on a surface where shapes completely fill all of the space around a given point where their boundaries meet, e.g. a honeycomb is a tessellation using hexagons.



theorem
A statement which has been derived from given statements using agreed rules of deduction (proof).

three-dimensional
An object that has three independent dimensions of length, width (breadth) and depth, or a space in which position is specified by three independent coordinates.

three-dimensional coordinate system
A coordinate system where any point in space is defined using three different dimensions, e.g. longitude, latitude and altitude for locating the position of a drone in the sky.



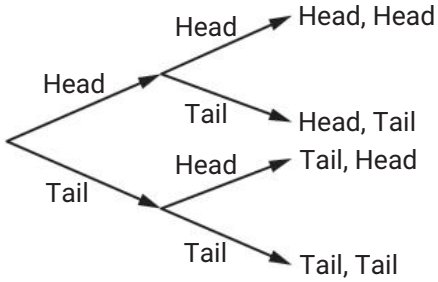
time zones
The 24 divisions of the globe, where each change of 15 degrees longitude corresponds to one hour. As time zones often align with national or regional boundaries, this is an approximate relationship.

transformation
A map of the plane onto itself. Transformations include dilation (enlargements or compressions), reflections, rotations and translations.

translation
Shifting a figure in the plane without turning it. Translations can be specified as a combination of a horizontal shift and a vertical shift.

transversal
A line that intersects two other lines obliquely.

tree diagram
A diagram consisting of line segments (edges) connected to points (vertices) like the branches of a tree. It shows the relationship between sets, events, or the set of outcomes of a multi-step random experiment.



trials
Any repeatable procedures with a well-defined sample space, e.g. the flipping of a coin.

trigonometric ratios
Relationships between the angles and sides of right triangles. The three basic trigonometric ratios are: sine (sin), cosine (cos) and tangent (tan).

two-dimensional
A shape that has two independent dimensions of length and width (breadth) is two-dimensional, as is a space in which position is specified by two independent coordinates, such as the plane.

two-way tables
A common way of displaying the two-way frequency distribution that arises when a group is categorised according to two criteria.

	Curly hair	Straight hair	Total
Red hair	1	1	2
Brown hair	8	4	12
Blonde hair	1	3	4
Black hair	7	2	9
Total	17	10	27

unit fraction
A fraction of the form $\frac{1}{n}$, where n is a non-zero natural number, such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

unit of repeat
In a repeating pattern, the pattern sequence that is repeated within the pattern, e.g. 1234 in the pattern 1234123412341234...

units
The basis for counting or measurement or both, e.g. the number 1 is the unit for counting; the metre is the standard unit for measurement of length in the metric system.

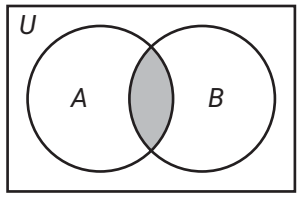
univariate data
Data relating to measurement of a single variable, e.g. shoe size.

validate (data)
The process of checking that data to be analysed is reasonable and fit for purpose. This can involve checking for accuracy, clarity, detail, consistency, correctness and completeness.

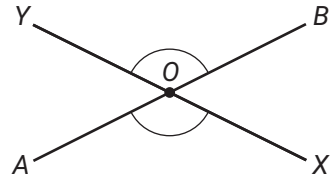
variable
Things that are measurable or observable and are expected to either change over time or between individual observations. They are often designated by symbols, such as n, x and y, to represent members of a set.

variation in data
The characteristic that elements of a data set differ, e.g. the maximum temperature recorded at different suburbs of a town on a given day.

Venn diagrams
Graphical representations, using several typically overlapping circles, showing elements of sets in relation to properties or attributes. They are drawn for some specified universal set.



vertically opposite angles
Two pairs of vertically opposite angles formed at the point of intersection of two lines in the plane. Vertically opposite angles are equal.



virtual materials
Dynamic digital representations of physical materials (sometimes called virtual manipulatives).

visualisations
Visual formats for organising, representing and summarising data. Also called data displays.

volume
The amount of space bounded by the surface of an object. It is usually measured in cubic units.

zero
The number represented by the numeral or the symbol 0, e.g. the origin on a number line. It also functions as an empty place-holder digit in the decimal expansion of numbers, e.g. 205.

Based on Australian Curriculum, Assessment and Reporting Authority (ACARA) materials

ANSWERS

Topic 1: Real numbers

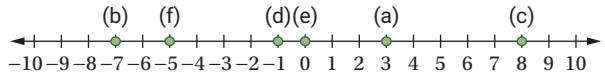
Recall

I can identify prime and composite numbers

1 **(a)** 2, 4, 6, 12, 14, 16, 28 **(b)** 2, 3, 7, 19, 37, 41
(c) 4, 6, 9, 12, 14, 16, 21, 25, 28, 33
(d) 4, 9, 16, 25

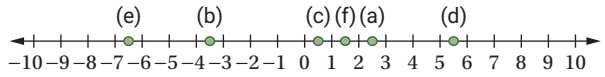
I can locate and plot integers on a number line

1



I can locate and plot mixed numbers on a number line

1



I can round decimals to a required number of decimal places

1 **(a)** 9.25 **(b)** 6.15 **(c)** 0.04
2 **(a)** 0.157 **(b)** 1.207 **(c)** 13.237
3 **(a)** 13 **(b)** 1 **(c)** −4

1.1 Identify and define irrational numbers

SC 1

1 **(a)** $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$; rational
(b) $6e$; irrational because e is a transcendental number
(c) $1\frac{15}{22} = \frac{37}{22}$; rational
(d) $9.\dot{3} = 9.3333\ldots = 9\frac{1}{3} = \frac{28}{3}$
(e) $\sqrt{48}$; irrational because 48 is not a perfect square
(f) $-\sqrt{81} = -\sqrt{9 \times 9} = -9$

2 **(a)** $\frac{195}{99}$ **(b)** $\frac{61}{45}$ **(c)** $\frac{202}{99}$ **(d)** $\frac{41}{333}$

3 0 can be expressed in an infinite number of ways in the form $\frac{a}{b}$. Some of them are $\frac{0}{-3}$, $\frac{0}{2}$ and $\frac{0}{123456}$.

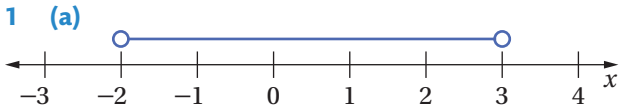
4 $\sqrt[3]{-8} = \sqrt[3]{(-2) \times (-2) \times (-2)} = -2$ because an odd number of negative values multiplied together results in a negative value. However, −4 can only be achieved using two values by multiplying a negative value with a positive value, hence the square root cannot be found. Since the value is impossible given our understanding of numbers, this value is not a real number.

5 **(a)** perfect square
(b) perfect cube; positive or negative
(c) rational

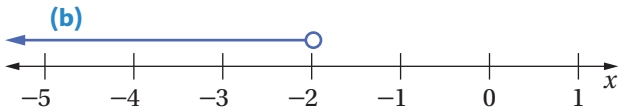
6 **(a)** $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$; irrational because $\sqrt{2}$ is irrational
(b) $\sqrt{\pi^4} = \pi^2$; irrational because π is a transcendental number
(c) $\left(1.\dot{2}\right)^3 = \left(\frac{11}{9}\right)^2 = \frac{121}{81}$; rational
(d) $\sqrt[3]{250} = \sqrt[3]{125 \times 2} = 5\sqrt[3]{2}$; irrational because 2 is not a perfect cube

SC 2

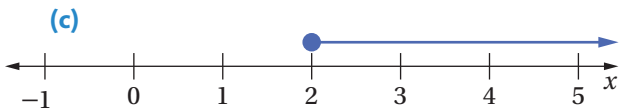
1 **(a)**



(b)



(c)



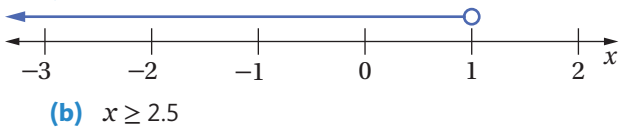
(d)



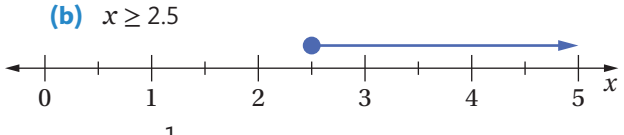
2 **(a)** $-3 < x \leq 0$ **(b)** $2 < x < 4$
(c) $x \geq -1$ **(d)** $x < 4$

3 **(a)** $-0.4 \leq x < 1.3$ **(b)** $x \geq -0.25$
(c) $x < 1.4$ **(d)** $-1.5 < x \leq 1.5$

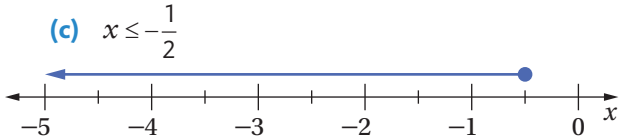
4 **(a)** $x < 1$



(b) $x \geq 2.5$



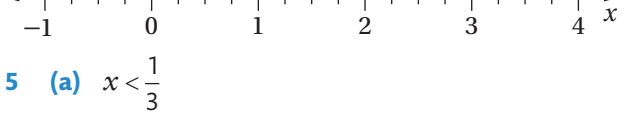
(c) $x \leq -\frac{1}{2}$



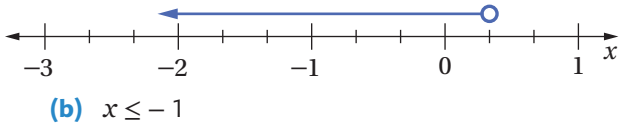
(d) $x > \frac{1}{4}$



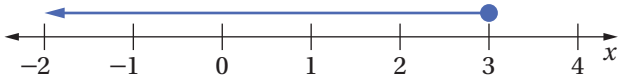
5 **(a)** $x < \frac{1}{3}$



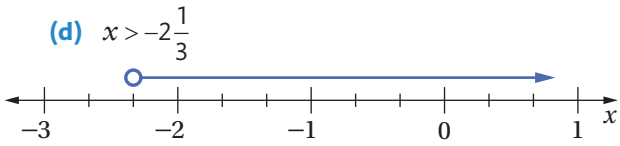
(b) $x \leq -1$



(c) $x \leq 3$



(d) $x > -2\frac{1}{3}$



6 **(a)** does

(b) it is necessary to multiply or divide by a negative number across the inequality sign

(c) is

(d) false

1.2 Apply irrational numbers

SC 1

1 **(a)** **(i)** 3.32 **(ii)** 3.31662

(b) **(i)** 4.69 **(ii)** 4.69042

(c) **(i)** 1.59 **(ii)** 1.58740

(d) **(i)** 2.71 **(ii)** 2.71442

2 **(a)** **(i)** 2.36 **(ii)** 2.35619

(b) **(i)** 4.28 **(ii)** 4.28319

(c) **(i)** 8.15 **(ii)** 8.15485

(d) **(i)** 2.15 **(ii)** 2.14552

3 **(a)** 4 **(b)** 2 **(c)** 9

(d) 3 **(e)** 2, 3, 2

4 **(a)** 5 and 6 **(b)** 6 and 7

(c) 10 and 11 **(d)** 14 and 15

5 **(a)** 2 and 3 **(b)** 4 and 5

(c) 1 and 2 **(d)** 3 and 4

SC 2

1 **(a)** 6π mm **(b)** 17π mm

(c) $\frac{1}{2}\pi$ cm, or $\frac{\pi}{2}$ cm

(d) 2.2π cm, or $\frac{22\pi}{20}$ cm, or $2\frac{2}{10}\pi$ cm

2 **(a)** **(i)** 18π cm **(ii)** 56.55 cm

(b) **(i)** 26π cm **(ii)** 81.68 cm

(c) **(i)** 6.6π mm, or $6\frac{2}{3}\pi$ mm, or $\frac{33\pi}{5}$ mm

(ii) 20.73 mm

(d) **(i)** 3.5π cm, $3\frac{1}{2}\pi$ cm or $\frac{7\pi}{2}$ cm

(ii) 11 cm

3 **(a)** **(i)** 36π cm^2 **(ii)** 113.10 cm^2

(b) **(i)** 289π mm^2 **(ii)** 907.92 mm^2

(c) **(i)** $\frac{1}{4}\pi$ cm^2, or $\frac{\pi}{4}$ cm^2 **(ii)** 0.79 cm^2

(d) **(i)** 4.84π m^2, or $\frac{121\pi}{25}$ m^2 **(ii)** 15.21 m^2

4 **(a)** **(i)** 45π m^3 **(ii)** 141.37 m^3

(b) **(i)** 328π mm^3 **(ii)** 1030.44 mm^3

(c) **(i)** 105π cm^3 **(ii)** 329.87 cm^3

5 Sample answer: $S = 32\pi$ cm^2 is more accurate as writing the value using the symbol for pi represents the exact value. It includes every decimal place of pi in the answer. The answer $S = 100.530\,964\,9$ cm^2, while accurate to many decimal places, is still only an approximation.

6 $c^2 = 4^2 + 4^2$

$= 16 + 16$

$= 32$

$\therefore c = \sqrt{32}$ m

7 $\sqrt{125} = \sqrt{a^2 + 10^2}$

$= \sqrt{a^2 + 100}$

$a^2 = 25$

$\therefore a = 5$ mm

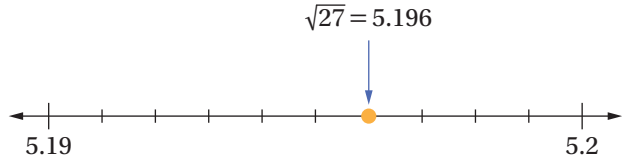
8 **(a)** 4 cm and 6 cm $\left(\sqrt{4^2 + 6^2} = \sqrt{52}\right)$

(b) 6 m and 9 m $\left(\sqrt{6^2 + 9^2} = \sqrt{117}\right)$

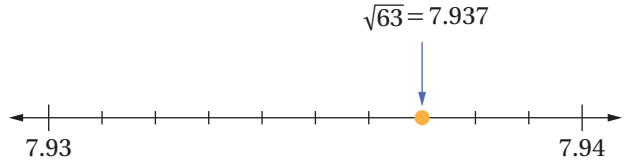
1.3 Plot and locate irrational numbers on a number line by estimation and by construction

SC 1

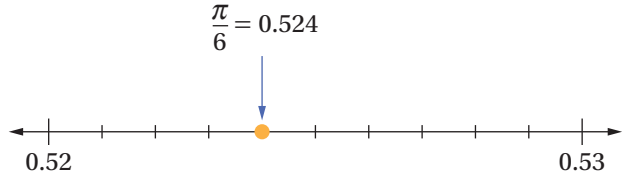
1 **(a)**



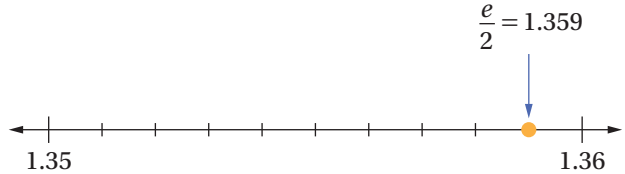
(b)



(c)



(d)



2 **(a)** 3.1415 and 3.1416 **(b)** 2.7182 and 2.7183
(c) 3.9882 and 3.9883 **(d)** −0.9971 and −0.9970

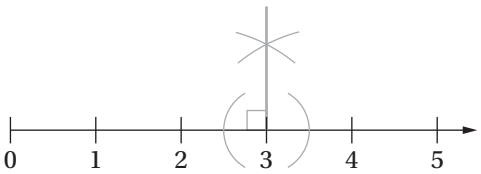
3 **(a)** **(ii)** $3\sqrt{2}$ **(b)** **(iii)** $\sqrt[3]{7}$
(c) **(iv)** $\frac{\pi}{3}$ **(d)** **(iii)** π^2

4 $\frac{\pi}{5}, \frac{2\pi - e}{4}, \sqrt[3]{5}, \sqrt{3}, \frac{2e}{3}$

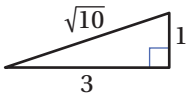
5 $2e - 3\pi, -\frac{3\pi}{4}, \sqrt[3]{-11}, \frac{\sqrt[3]{-20}}{2}, \sqrt{15} - \sqrt{8}$

SC 2

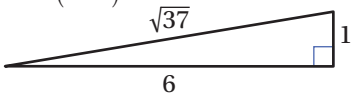
1



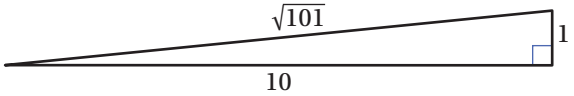
2 **(a)** $\left(\sqrt{10}\right)^2 = 3^2 + 1^2$



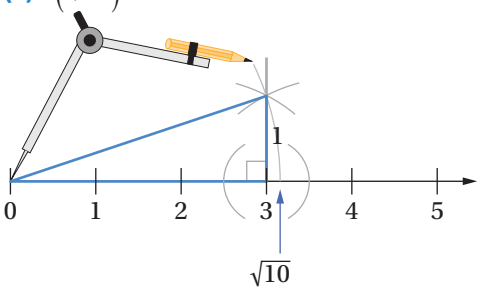
(b) $\left(\sqrt{37}\right)^2 = 6^2 + 1^2$



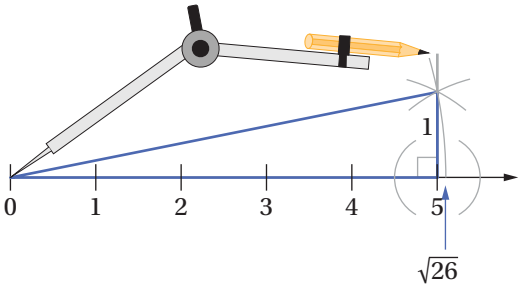
(c) $\left(\sqrt{101}\right)^2 = 10^2 + 1^2$



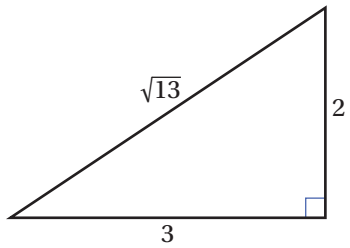
3 **(a)** $\left(\sqrt{10}\right)^2 = 3^2 + 1^2$



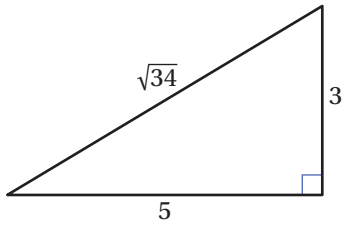
(b) $\left(\sqrt{26}\right)^2 = 5^2 + 1^2$



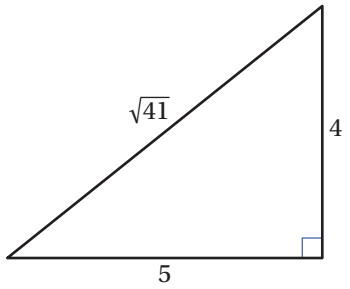
4 **(a)**



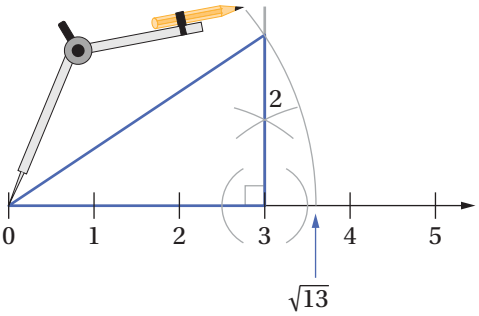
(b)



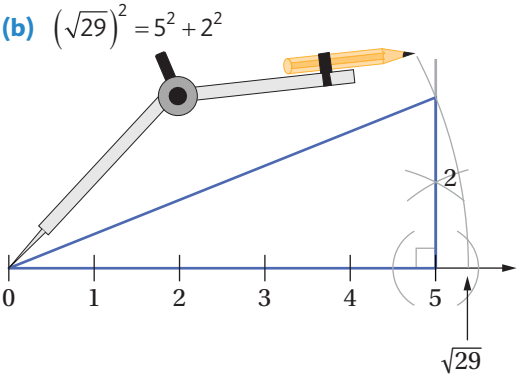
(c)



5 **(a)** $\left(\sqrt{13}\right)^2 = 3^2 + 2^2$



(b) $\left(\sqrt{29}\right)^2 = 5^2 + 2^2$

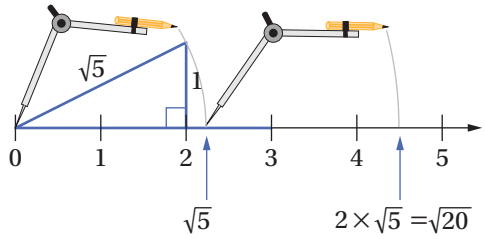


6 **(a)** $\sqrt{98}$ **(b)** $\sqrt{7}$

7 $\sqrt{20} = \sqrt{4} \times \sqrt{5}$

$= 2 \times \sqrt{5}$

Therefore $\sqrt{20}$ can be located by drawing a second $\sqrt{5}$ arc from the position of $\sqrt{5}$ on the number line.



Topic 2: Exponents

Recall

I can write numbers raised to a power in expanded form

1 **(a)** 5×5

(b) $6 \times 6 \times 6 \times 6$

(c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(d) $(-3) \times (-3) \times (-3) \times (-3)$

(e) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

I can write repeated multiplications in exponent form

1 **(a)** 3^4 **(b)** 8^7 **(c)** 11^6

(d) $(-2)^6$ **(e)** $\left(\frac{1}{4}\right)^5$

2 **(a)** $2^3 \times 3^2$ **(b)** $3^4 \times 5^3$ **(c)** $10^4 \times 9^2$

(d) $2^2 \times 3^3 \times 5$ **(e)** $3 \times 4^3 \times 5^4$

I can calculate squares and cubes of numbers

1 **(a)** 16 **(b)** 225 **(c)** 9 **(d)** 8 **(e)** −64

I understand the effect of raising a number to a power

1 **(a)** Largest possible: $5^6 + 3^4 + 2^1 = 15\,708$

(b) Smallest possible: $1^6 + 5^2 + 4^3 = 90$

2.1 Establish and apply the exponent law for multiplication

SC 1

1 **(a)** $(2 \times 2 \times 2 \times 2 \times 2) \times 2 = 2^{(5+1)} = 2^6$

(b) $(2 \times 2 \times 2 \times 2) \times (2 \times 2) = 2^{(4+2)} = 2^6$

(c) $(2 \times 2) \times (2 \times 2 \times 2) = 2^{(2+3)} = 2^5$

(d) $(2 \times 2) \times 2 \times (2 \times 2 \times 2 \times 2) = 2^{(2+1+4)} = 2^7$

2 **(a)** 5×7^4 **(b)** 7^9 **(c)** $5^6 \times 7^2$ **(d)** 5^8

3 **(a)** 2^8 **(b)** 5^4

(c) $2^3 \times 5^3$ **(d)** $2^2 \times 3^2$

(e) $2^4 \times 3^2 \times 5^4$ **(f)** $3^1 \times 5^2 \times 11^1$

4 **(a)** $3^4 \times 5^3$ **(b)** $2^3 \times 3^2 \times 5^5$

(c) $2^{10} \times 3^{10} \times 5^4$ **(d)** $2^6 \times 3^5 \times 5^6 \times 11^2$

SC 2

1 **(a)** $(x \times x \times x \times x \times x) \times x = x^{(4+1)} = x^5$

(b) $(x \times x \times x \times x \times x \times x) \times (x \times x) = x^{(5+2)} = x^7$

(c) $x \times (x \times x \times x \times x) \times (x \times x \times x) = x^{(1+4+3)} = x^8$

(d) $(x \times x) \times (x \times x) \times (x \times x \times x) = x^{(2+2+3)} = x^7$

2 **(a)** x^4y^4 **(b)** $3x^4y^2$ **(c)** $3^8x^2y^4$ **(d)** $3^5x^8y^3$

3 **(a)** 11^6a^5 **(b)** 7^3b^5 **(c)** 3^3c^4 **(d)** 5^2d^5

4 **(a)** $6x^6y^6$ **(b)** $5x^4y^7$ **(c)** $x^2y^3z^2$ **(d)** $3x^4y^4$

2.2 Establish and apply the exponent law for division

SC 1

1 **(a)** $\frac{3 \times 3 \times \cancel{3}}{\cancel{3}} = 3^{3-1} = 3^2$

(b) $\frac{3 \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}} = 3^{8-7} = 3^1$ or 3

(c) $\frac{\left(3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}\right) \times \left(\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}\right)}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}} = 3^{5+4-6} = 3^3$

(d) $\frac{(3 \times 3 \times 3) \times \left(3 \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}\right) \times \left(\cancel{3} \times \cancel{3}\right)}{\left(\cancel{3} \times \cancel{3}\right) \times \left(\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}\right)} = 3^{3+5+2-(2+4)} = 3^4$

2 **(a)** $11^5 \times 13^2$ **(b)** $2^3 \times 7$

(c) $3^5 \times 5^3$ **(d)** $2^5 \times 3^3$

3 **(a)** $3^2 = 9$ **(b)** $2^4 = 16$

(c) $5^4 = 625$ **(d)** $11^3 = 1331$

4 **(a)** $2^2 \times 11^2$ **(b)** $3^4 \times 5 \times 7$

(c) $3 \times 5^2 \times 11$ **(d)** $2^2 \times 3 \times 5^3 \times 11$

SC 2

1 **(a)** $\frac{a \times a \times a \times a \times \cancel{a}}{\cancel{a}} = a^{5-1} = a^4$

(b) $\frac{a \times a \times a \times a \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a}} = a^{6-2} = a^4$

(c) $\frac{a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}} = a^{8-6} = a^2$

(d) $\frac{(a \times a \times a \times a) \times \left(a \times \cancel{a} \times \cancel{a} \times \cancel{a}\right)}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a^{4+4-3} = a^5$

2 **(a)** ab^2 **(b)** c^2d^2 **(c)** e^5f^2 **(d)** g^2h^6

3 **(a)** 2^2a^4 **(b)** $3b^2$ **(c)** 5^2c^2 **(d)** 3^4

4 **(a)** 5^5xy^3 **(b)** $3^2x^2y^5$ **(c)** $2^4x^2y^3$ **(d)** $7y^2$

2.3 Establish and apply the exponent law for raising a power to a power

SC 1

1 **(a)** $(5 \times 5) \times (5 \times 5) = 5^{2 \times 2} = 5^4$

(b) $(5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) = 5^{3 \times 4} = 5^{12}$

(c) $(5 \times 5) \times (5 \times 5) \times (5 \times 5) \times (5 \times 5) \times (5 \times 5) = 5^{2 \times 5} = 5^{10}$

(d) $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^{3 \times 5} = 2^{15}$

(e) $(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^{4 \times 2} = 2^8$

(f) $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) = 2^{5 \times 2} = 2^{10}$

2

Expression	Expansion	Simplified form	Multiplying the exponents
$(7^2)^3$	$7^2 \times 7^2 \times 7^2$	7^6	$2 \times 3 = 6$
$(5^4)^2$	$5^4 \times 5^4$	5^8	$4 \times 2 = 8$
$(11^3)^3$	$11^3 \times 11^3 \times 11^3$	11^9	$3 \times 3 = 9$
$(2^4)^3$	$2^4 \times 2^4 \times 2^4$	2^{12}	$4 \times 3 = 12$

3 The bases are the same, 7, and both yield 24 factors of 7, since $6 \times 4 = 4 \times 6 = 24$, i.e. 4 lots of 6 or 6 lots of 4 factors, so 7^{24} is the simplified form for both expressions.

4 **(a)** 5^{26} **(b)** 5^{13} **(c)** 5^{25} **(d)** 2^{10} **(e)** 2^{15} **(f)** 2^5

5 **(a)** 3^6 **(b)** 3^4 **(c)** 3^7 **(d)** 3^6 **(e)** 1 **(f)** 3^1 or 3

6 **(a)** 2^{17} **(b)** 3^{18} **(c)** 5^{14} **(d)** 7^2 **(e)** 2^4 **(f)** 3^7

SC 2

1 **(a)** $(a \times a \times a \times a \times a \times a \times a) \times (a \times a \times a \times a \times a \times a \times a) = a^{7 \times 2} = a^{14}$

(b) $(b \times b \times b) \times (b \times b \times b) \times (b \times b \times b) = b^{3 \times 3} = b^9$

(c) $(c \times c) \times (c \times c) \times (c \times c) \times (c \times c) \times (c \times c) = c^{2 \times 5} = c^{10}$

(d) $(d \times d \times d \times d) \times (d \times d \times d \times d) = d^{4 \times 2} = d^8$

(e) $(e \times e \times e \times e \times e) \times (e \times e \times e \times e \times e) \times (e \times e \times e \times e \times e) = e^{5 \times 3} = e^{15}$

(f) $(f \times f \times f \times f \times f \times f) \times (f \times f \times f \times f \times f \times f) = f^{6 \times 2} = f^{12}$

2

Expression	Expansion	Simplified form	Multiplying the exponents
$(x^5)^2$	$x^5 \times x^5$	x^{10}	$5 \times 2 = 10$
$(y^4)^3$	$y^4 \times y^4 \times y^4$	y^{12}	$4 \times 3 = 12$
$(p^3)^7$	$p^3 \times p^3 \times p^3 \times p^3 \times p^3 \times p^3 \times p^3$	p^{21}	$3 \times 7 = 21$
$(q^2)^8$	$q^2 \times q^2 \times q^2 \times q^2 \times q^2 \times q^2 \times q^2 \times q^2$	q^{16}	$2 \times 8 = 16$

3 **(a)** a^{11} **(b)** b^{17} **(c)** c^{25} **(d)** d^{34}

4 **(a)** $x^{15}y^{10}$ **(b)** x^6y^6 **(c)** $x^{24}y^7$ **(d)** $x^{15}y^8$

5 **(a)** x^2 **(b)** x^6 **(c)** x^6 **(d)** x^3

6 **(a)** x^2y^4 **(b)** $x^{12}y^6$ **(c)** $x^{10}y^{10}$ **(d)** x^5y^9

2.4 Establish and apply the exponent law for raising to the power of 0

SC 1

1

Question	Calculated value	Exponent form	Simplest exponent form answer
$5 \div 5 =$	1	5^{1-1}	5^0
$3^4 \div 3^4 =$	1	3^{4-4}	3^0
$4^6 \div 4^6 =$	1	4^{6-6}	4^0
$12^3 \div 12^3 =$	1	12^{3-3}	12^0
$6^8 \div 6^8 =$	1	6^{8-8}	6^0

2 **(a)**

2^5	2^4	2^3	2^2	2^1	2^0
32	16	8	4	2	1

(b) From left to right the exponent is decreasing by 1 each time.

(c) Each number is half of the one before.

(d) 2^0 is half of 2^1, so $2^0 = 1$.

3 **(a)** $x = 8$ **(b)** $x = 6$ **(c)** $x = 6$ **(d)** $x = 9$

4 **(a)** $x = 3, y = 6$ **(b)** $x = 4, y = 3$ **(c)** $x = 2, y = 5$ **(d)** $x = 7, y = 3$

SC 2

1 **(a)** 1 **(b)** 2 **(c)** -5 **(d)** 1 **(e)** -1

2 **(a)** 1 **(b)** 1 **(c)** $3^4 = 81$ **(d)** 1 **(e)** 1 **(f)** $4^3 = 64$

3 **(a)**

x^5	x^4	x^3	x^2	x^1	x^0

(b) From left to right the exponent is decreasing by 1 each time.

(c) Each expression can be divided by x to obtain the next expression.

(d) $\frac{x}{x} = 1$, so $x^0 = 1$.

4 **(a)** $x = 2$ **(b)** $x = 3$ **(c)** $x = 20$ **(d)** $x = 4$

5 **(a)** $x = 5, y = 7$ **(b)** $x = 7, y = 3$ **(c)** $x = 2, y = 8$ **(d)** $x = 10, y = 2$

2.5 Apply exponent laws to numerical expressions with negative integer exponents

SC 1

1

Question	Positive power in the answer	Exponent law for division	Single power answer
$7^4 \div 7^6 =$	$\frac{1}{7^2}$	7^{4-6}	7^{-2}
$7^3 \div 7^7 =$	$\frac{1}{7^4}$	7^{3-7}	7^{-4}
$7^2 \div 7^5 =$	$\frac{1}{7^3}$	7^{2-5}	7^{-3}
$7^4 \div 7^5 =$	$\frac{1}{7}$	7^{4-5}	7^{-1}

2 (a)

2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

(b) From left to right the exponent is decreasing by 1 each time.

(c) Each number is half of the one before.

(d) 2^{-2} is half of 2^{-1}, so half of $\frac{1}{2}$ is $\frac{1}{4}$, and 2^{-3} is half of 2^{-2}, so half of $\frac{1}{4}$ is $\frac{1}{8}$.

3 (a) $\frac{25}{8}$ **(b)** $\frac{27}{25}$ **(c)** $\frac{81}{10\,000}$ **(d)** $\frac{1}{121\,000}$

4 (a) $\frac{1}{3^2} = \frac{1}{9}$ **(b)** $\frac{1}{5^3} = \frac{1}{125}$

(c) $\frac{7^4}{7^8} = \frac{1}{7^4} = \frac{1}{2401}$ **(d)** $\frac{1}{2^7} = \frac{1}{128}$

5 (a) $\frac{1}{2 \times 3^2 \times 11^2}$ **(b)** $\frac{1}{2^2 \times 5 \times 7}$

(c) $\frac{1}{2^3 \times 5^4}$ **(d)** $\frac{1}{2^4 \times 3^2}$

6 (a) In the tenth round, $2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$ people will know. This calculates to 2046 people knowing the joke.

(b) In the fourth round: $2^4 + 2^3 + 2^2 + 2^1 = 30$. So, all members of the class will know the joke after the fourth round.

2.6 Apply exponent laws to simplify numerical expressions

SC 1

1 (a) $1 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 8 \times 10^{-2} + 7 \times 10^{-3}$

(b) $1 \times 10^0 + 9 \times 10^{-2} + 8 \times 10^{-3}$

(c) $1 \times 10^2 + 5 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2}$

(d) $1 \times 10^0 + 8 \times 10^{-3} + 5 \times 10^{-4}$

2 (a) 24.319 **(b)** 5060.2074

(c) 704 010.406 **(d)** 6310.00 704

3 (a) 11^{-1} **(b)** 2×13^{-1} **(c)** 3×29^{-1} **(d)** 5×71^{-1}

4 (a) 2^{-1} **(b)** 2^{-2} **(c)** 2^{-3} **(d)** 2^{-4}

(e) 2^{-5} **(f)** 3^{-2} **(g)** 5×2^{-4} **(h)** 7×5^{-3}

5 (a) $\frac{1}{7}$ **(b)** $\frac{1}{49}$ **(c)** $\frac{1}{27}$

(d) $\frac{1}{81}$ **(e)** $\frac{6}{125}$ **(f)** $\frac{9}{32}$

6 (a) $7^{-4} = \frac{1}{7^4} = \frac{1}{7 \times 7 \times 7 \times 7} = \frac{1}{2401}$

(b) $11^{-3} = \frac{1}{11^3} = \frac{1}{11 \times 11 \times 11} = \frac{1}{1331}$

(c) $2^{-8} = \frac{1}{2^8} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{256}$

(d) $3^{-6} = \frac{1}{3^6} = \frac{1}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{729}$

SC 2

1 (a) 2^8 **(b)** $2^4 \times 11^4$ **(c)** $\frac{3^6}{7^6}$ **(d)** 2^5

2 (a) 3125 **(b)** 1 **(c)** 3375 **(d)** 15 625

3 (a) 2^3 **(b)** $\frac{7^2}{3^2}$ **(c)** $3^3 \times 2^3$ **(d)** $\frac{5^7}{3^7}$

4 (a) $1\frac{1}{2}$ **(b)** $\frac{9}{500}$ **(c)** $\frac{1875}{343}$ **(d)** $\frac{823\,543}{729}$

5 (a) $\frac{5^2}{5^3} = \frac{1}{5}$ **(b)** $\frac{5^2}{5^{-1}} = 125$

(c) $\frac{5^2}{5^4} = \frac{1}{25}$ **(d)** $\frac{5^6}{5^2} = 625$

6 (a) The value is divided by 25 each time you move down a line.

(b) $5^1 = 5$ **(c)** $5^{\frac{1}{2}} = \sqrt{5}$ **(d)** $5^{-\frac{1}{2}} = \frac{1}{\sqrt{5}}$

2.7 Choose and apply efficient strategies to exponent laws of numerical expressions

SC 1

1 **(a)** 12^7 is four times bigger than $4^6 \times 3^7$
(b) 16^6 is twice as big as $8^6 \times 2^5$
(c) 20^7 is sixteen times the size of $5^7 \times 4^5$
(d) 20^7 is twenty-five times the size of $5^5 \times 4^7$

2 **(a)** 36^{10} is half the size of $2^{10} \times 3^9 \times 6^{11}$
(b) 24^8 is twelve times the size of $2^6 \times 3^7 \times 4^8$

3 **(a)** $\frac{24^6}{12^5}$ is 24 times the size of 2^5
(b) $\frac{24^5}{12^6}$ is one-twelfth the size of 2^5
(c) $\frac{33^8}{3^6}$ is nine times the size of 11^8
(d) $\frac{5^4}{15^3}$ is five times the size of 3^{-3}

4 **(a)** $12^3 = (4 \times 3)^3$ **(b)** $(5 \times 3)^6 = 15^6$
(c) $\left(\frac{16}{8}\right)^7 = 2^7$ **(d)** $\left(\frac{24}{8}\right)^3 = 3^3$

5 **(a)** 3^9 **(b)** 2^9 **(c)** 5^8

6 **(a)** 2×3^5 **(b)** $2^6 \times 3^3 \times 5^2$
(c) $2^3 \times 5^3 \times 7^2 \times 11$

SC 2

1 **(a)** $\frac{2^2 \times 3^6 \times 5^4}{2 \times 3^6 \times 5} = 2 \times 5^3 = 250$
(b) $\frac{2^4 \times 3^7 \times 5^4}{2^4 \times 3^5 \times 5^3} = 3^2 \times 5 = 45$
(c) $\frac{2^{10} \times 3^4 \times 5^2 \times 11}{2^9 \times 3^8 \times 11^2} = \frac{2 \times 5^2}{3^4 \times 11} = \frac{50}{891}$

2 $2^6 = 64$
$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = 0.5$
$2^{-2} = 0.25$
$2^{-3} = 0.125$
$2^{-4} = 0.0625$
$2^{-5} = 0.031\ 25$
$2^{-6} = 0.015\ 625$
(a) 2^{-2} **(b)** 2^{-2} **(c)** 2^4
(d) 2^{-9} **(e)** 2^{11} **(f)** 2^{-36}

3 **(a)** $\frac{31}{4}$ **(b)** 64 **(c)** $\frac{499}{625}$ **(d)** $7\frac{14}{25}$

4 **(a)** $3^4 \times 3^{-2} = 3^2$ **(b)** $(3^4)^{-2} < 3^{-6}$
(c) $\frac{3^4}{3^2} < 3^6$ **(d)** $\frac{3^4}{3^{-2}} > 3^2$

2.8 Apply exponent laws to simplify algebraic expressions

SC 1

1 $\frac{a^6}{a^4} = a^{6-4} = a^2$
LHS:
$\frac{a^6}{a^4} = \frac{a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}$
$= \frac{a \times a}{1}$
$= \frac{a^2}{1}$
$= a^2$
= RHS
Because LHS = RHS, the proof is complete.

2 $(a^3)^2 = a^{3 \times 2} = a^6$
LHS:
$(a^3)^2 = a^3 \times a^3$
$= (a \times a \times a) \times (a \times a \times a)$
$= a^6$
= RHS
Because LHS = RHS, the proof is complete.

3 $(5 \times 2)^3 = 5^3 \times 2^3$
LHS:
$(5 \times 2)^3 = 10^3$
$= 10 \times 10 \times 10$
$= 1000$
RHS:
$5^3 \times 2^3 = 5 \times 5 \times 5 \times 2 \times 2 \times 2$
$= 125 \times 8$
$= 1000$
= LHS
Because LHS = RHS, the proof is complete.

4 $\left(\frac{10}{2}\right)^3 = \frac{10^3}{2^3}$

LHS:

$\left(\frac{10}{2}\right)^3 = 5^3$

$= 5 \times 5 \times 5$

$= 125$

RHS:

$\frac{10^3}{2^3} = \frac{(2 \times 5)^3}{2^3}$

$= \frac{2^3 \times 5^3}{2^3}$

$= 5^3$

$= 5 \times 5 \times 5$

$= 125$

$=$ LHS

Because LHS = RHS, the proof is complete.

5 $a^0 = 1$

$\frac{a^m}{a^m} = a^{m-m}$

$= a^0$

Let $m = 3$

$\frac{a^m}{a^m} = \frac{a^3}{a^3}$

$= \frac{\cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a}}$

$= \frac{1}{1}$

$= 1$

The two different answers are for the same question, so $a^0 = 1$ has been proved.

SC 2

1 **(a)** $6x^8$ **(b)** $5x^6$ **(c)** $12x^2$ **(d)** $16x^{-7}$

2 **(a)** $3x^3$ **(b)** $24x^2$ **(c)** $4x^6$ **(d)** $3x$

3 **(a)** $243x^{10}$ **(b)** $9x^2$

(c) $8x^{-12}$ **(d)** $3^{-4}x^{-8} = \frac{1}{81x^8}$

4 Student A has not raised $\left(-\frac{3}{2}\right)$ to the power of 3, they only multiplied the variable.

Student B has it all correct, except for the coefficient on the numerator. The calculation required is $-3 \times (-3) \times (-3) = -27$. The product of an odd number of negative terms is always negative.

5 **(a)** $(x^a)^a = x^{a^2}$ **(b)** $(x^a)^{-a} = x^{-a^2}$

(c) $x^a \times x^{-a} = x^0$ **(d)** $\frac{x^a}{x^{-a}} = x^{2a}$

6 **(a)** x^{-2} **(b)** $\frac{x^{11}}{5}$ **(c)** $\frac{1}{2} = 2^{-1}$ **(d)** $\frac{1}{16} = 2^{-4}$

7 **(a)** $x = 9$ **(b)** $x = -4$ **(c)** $x = 4$ **(d)** $x = 2$

SC 3

1 **(a)** $24a^5b^3$ **(b)** $-12a^5b^6$

(c) $30a^2b^{-1}$ **(d)** $60a^2b^{-1}$

2 **(a)** $2ax^3$ **(b)** $\frac{ab^{-5}}{6} = \frac{a}{6b^5}$

(c) $\frac{2a^{-2}b}{3} = \frac{2b}{3a^2}$ **(d)** $-a^{-2}b^2 = -\frac{b^2}{a^2}$

3 **(a)** $8a^3b^6$ **(b)** $2a^3b^6$ **(c)** $2a^{-3}b^{-6} = \frac{2}{a^3b^6}$

(d) $-2a^3b^6$ **(e)** $-8a^3b^6$

4 **(a)** $4a^3$ **(b)** 1 **(c)** 2 **(d)** 1

5 **(a)** $\frac{1}{6x}$ **(b)** $\frac{5}{6x^4y^3}$ **(c)** $6a^2b^2c^2$ **(d)** $\frac{9a^2b^2c}{2}$

Topic 3: Algebra (binomials)

Recall

I can simplify algebraic expressions collecting like terms

1 **(a)** $10p$ **(b)** $4q$ **(c)** $-12x$

(d) $-4c$ **(e)** $-9d$ **(f)** $-6a$

2 **(a)** $2a + 12b$ **(b)** $-7c - 4d$ **(c)** $-2x^2 + 12y$

I can simplify algebraic expressions involving the multiplication of terms

1 **(a)** $60ab$ **(b)** $120ab$ **(c)** $44y^2$

2 **(a)** $24abc$ **(b)** $-36xyz$ **(c)** $64m^2n$

I can expand and simplify algebraic expressions with like terms

1 **(a)** $7b + 28$ **(b)** $6c - 3$ **(c)** $-6x - 10$

2 **(a)** $-6a + 9b$ **(b)** $10x - 4y$ **(c)** $24a - 18b$

I can find the highest common factor for numbers

1 **(a)** 12 **(b)** 7 **(c)** 27

I can factorise simple algebraic expressions

1 **(a)** $7(x + 2)$ **(b)** $9(x - 5)$ **(c)** $9(2x - 3)$

(d) $8(3 + c)$ **(e)** $6(9 - 4m)$

2 **(a)** -6: $-6(a + 3)$ **(b)** -7: $-7(2x + 5)$

(c) -3: $-3(10 + n)$ **(d)** -8: $-8(k - 2)$

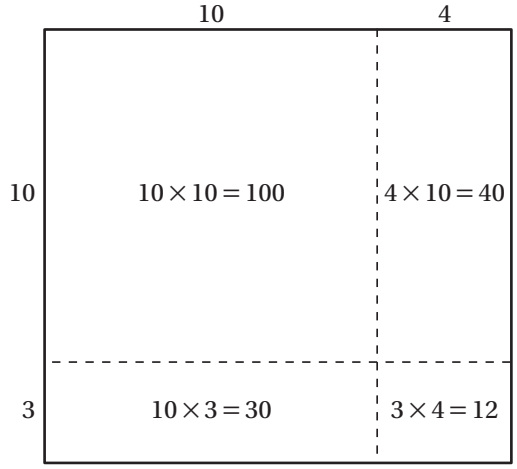
(e) -6: $-6(8 + 3d)$

3.1 Understand and use the distributive law

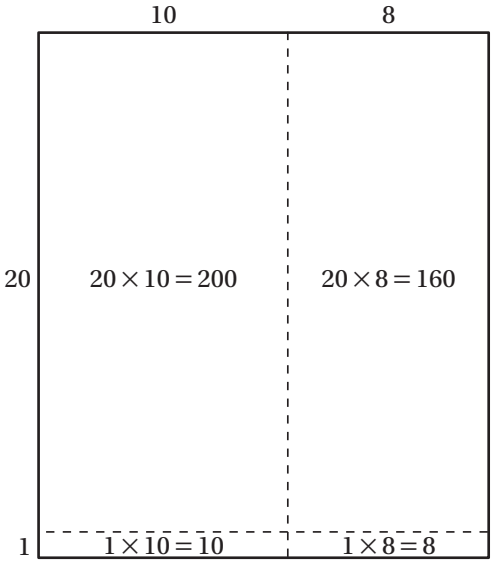
SC 1

1 Sample answers

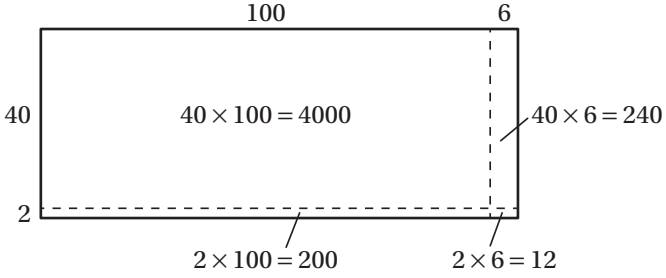
(a) $14 \times 13 = (10 + 4) \times (10 + 3)$
$= 100 + 30 + 40 + 12$
$= 182$



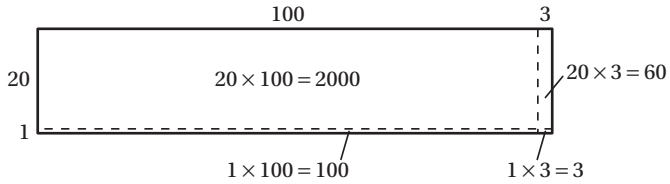
(b) $21 \times 18 = (20 + 1) \times (10 + 8)$
$= 200 + 160 + 10 + 8$
$= 378$



(c) $106 \times 42 = (100 + 6) \times (40 + 2)$
$= 4000 + 200 + 240 + 12$
$= 4452$



(d) $103 \times 21 = (100 + 3) \times (20 + 1)$
$= 2000 + 100 + 60 + 3$
$= 2163$



2 **(a)** $53 \times 42 = (50 + 3) \times (40 + 2)$
(b) $105 \times 14 = (100 + 5) \times (10 + 4)$
(c) $1003 \times 53 = (1000 + 3) \times (50 + 3)$
(d) $10\,005 \times 1006 = (10\,000 + 5) \times (1000 + 6)$

3 **(a)** $110 \times 101 = (100 + 10) \times (100 + 1)$
$= 10\,000 + 100 + 1000 + 10$
$= 11\,110$

(b) $120 \times 110 = (100 + 20) \times (100 + 10)$
$= 10\,000 + 1000 + 2000 + 200$
$= 13\,200$

(c) $115 \times 103 = (100 + 15) \times (100 + 3)$
$= 10\,000 + 300 + 1500 + 45$
$= 11\,845$

(d) $204 \times 103 = (200 + 4) \times (100 + 3)$
$= 20\,000 + 600 + 400 + 12$
$= 21\,012$

4 **(a)** $54 \times 48 = 2592$ **(b)** $91 \times 61 = 5551$
(c) $83 \times 22 = 1826$ **(d)** $106 \times 101 = 10\,706$

SC 2

1 **(a)** $(3 + 9)(4 + 6) = 3 \times (4 + 6) + 9 \times (4 + 6)$
$= 3 \times 4 + 3 \times 6 + 9 \times 4 + 9 \times 6$
$= 12 + 18 + 36 + 54$
$= 120$

(b) 208 **(c)** 350 **(d)** 405

2 **(a)** $(6 + 8)(4 - 1) = 6 \times (4 - 1) + 8 \times (4 - 1)$
$= 6 \times 4 - 6 \times 1 + 8 \times 4 - 8 \times 1$
$= 24 - 6 + 32 - 8$
$= 42$

(b) 96 **(c)** 20 **(d)** 49

3 Sample answers

(a) **(i)** $(40 + 2)(10 + 7)$ **(ii)** $(40 + 2)(20 - 3)$
(b) **(i)** $(30 + 5)(10 + 9)$ **(ii)** $(30 + 5)(20 - 1)$

4 **(a)** This step is correct.
(b) The only error Erasmus made was simplifying $+ (-3) \times (-2)$, which should have been $+ 6$.

SC 3

1 **(a)** $xy + 2x + 4y + 8$ **(b)** $xy + 6x - 5y - 30$
(c) $xy - 4x + 6y - 24$ **(d)** $xy - 8x - 7y + 56$

2 **(a)** $6a^2 + 13a + 6$ **(b)** $10b^2 - 11b - 6$
(c) $18c^2 - 9c - 20$ **(d)** $50d^2 - 50d + 12$

3 **(a)** $12a^2 + 23ab + 10b^2$ **(b)** $24a^2 - 6b^2$
(c) $48a^2 - 14ab - 12b^2$ **(d)** $27a^2 - 90ab + 27b^2$

4 **(a)** $(6x^2 - 8x - 30)\,\text{cm}^2$ **(b)** $(36x^2 - 48x + 16)\,\text{cm}^2$
(c) $(-3x^2 + 17x - 20)\,\text{cm}^2$; $2\,\text{cm}^2$

3.2 Recognise, expand and factorise special binomial products

SC 1

1 **(a)** $x^2 + 10x + 25$ **(b)** $x^2 + 2x + 4$
(c) $x^2 + 2xb + b^2$ **(d)** $x^2 + 10bx + 25b^2$
(e) $4x^2 + 20x + 25$ **(f)** $4x^2 + 4xb + b^2$
(g) $4x^2 + 20bx + 25b^2$

2 **(a)** $x^2 - 8x + 16$ **(b)** $x^2 - 2xb + b^2$
(c) $x^2 - 8bx + 16b^2$ **(d)** $9x^2 - 24x + 16$
(e) $9x^2 - 6xb + b^2$ **(f)** $9x^2 - 24bx + 16b^2$

3 **(a)** $(x^2 - 10x + 25)\,\text{cm}^2$ **(b)** $(4x^2 + 16x + 16)\,\text{cm}^2$
(c) $(9a^2 - 30ab + 25b^2)\,\text{cm}^2$
(d) $(16a^2 + 32ab + 16b^2)\,\text{cm}^2$

4 **(a)** $21^2 = (20 + 1)^2$
(b) $21^2 = (20 + 1)^2$
$= 20^2 + 2 \times 20 \times 1 + 1^2$
$= 400 + 40 + 1$
$= 441$
(c) $19^2 = (20 - 1)^2$
$= 20^2 - 2 \times 20 \times 1 + 1^2$
$= 400 - 40 + 1$
$= 361$

SC 2

1 **(a)** $9 - x^2$ **(b)** $x^2 - 9$ **(c)** $49x^2 - 9$
(d) $9 - 49x^2$ **(e)** $t^2 - 4$ **(f)** $4 - t^2$
(g) $4 - 25t^2$ **(h)** $25t^2 - 4$ **(i)** $k^2 - 9$
(j) $k^2 - p^2$ **(k)** $k^2 - 9p^2$ **(l)** $25 - n^2$
(m) $m^2 - n^2$ **(n)** $25m^2 - n^2$

2 **(a)** $16k^2 - 36t^2$ **(b)** $25p^2 - 121q^2$
(c) $64x^2 - 81y^2$ **(d)** $144r^2 - 121s^2$

3 **(a)** The expansion rule that can be used is Difference of Two Squares (DOTS).
(b) $(a + b)(b - a) = ab - a^2 + b^2 - ab$
$= -a^2 + b^2$
$= b^2 - a^2$

4 **(a)** $59 \times 61 = (60 - 1)(60 + 1)$
$= 60^2 - 1^2$
$= 3600 - 1$
$= 3599$
(b) 4896 **(c)** 9999 **(d)** 1584

5 **(a)** $(a^2 - 16)\,\text{cm}^2$ **(b)** $a > 4\,\text{cm}$

6 **(a)** $(a^2 - 9)\,\text{cm}^2$ **(b)** $(a^2 + 6a + 9)\,\text{cm}^2$
(c) $(2a^2 + 6a)\,\text{cm}^2$ **(d)** $a > 3$

SC 3

1 **(a)** $(x + 3)^2$ **(b)** $(x - 3)^2$
(c) $(2x + 3)^2$ **(d)** $(3x + 2)^2$
(e) $(3x - y)^2$ **(f)** $(3x - 2y)^2$
(g) $(5 + 2x)^2$ **(h)** $(6x - 5y)^2$

2 **(a)** $(9 - z)(9 + z)$ **(b)** $(z + 9)(z - 9)$
(c) $(9 - 3z)(9 + 3z)$ **(d)** $(t - 7)(t + 7)$
(e) $(12t - 7)(12t + 7)$ **(f)** $(k + 16)(k - 16)$
(g) $(5 + p)(5 - p)$ **(h)** $(5k - 4p)(5k + 4p)$
(i) $(10t - 11s)(10t + 11s)$
(j) $(11s - 10t)(11s + 10t)$

3
$4 = (2)^2$
$a^4 = (a^2)^2$
$4a^4 = (2a^2)^2$
$16a^8 = (4a^4)^2$
$4a^4 - 16a^8 = (2a^2)^2 - (4a^4)^2$
$= (2a^2 - 4a^4)(2a^2 + 4a^4)$

3.3 Understand algebraic factorisation

SC 1

1 **(a)** 3 **(b)** 4 **(c)** c **(d)** $6d$ **(e)** $12x$
(f) 4 **(g)** 11 **(h)** $2x$ **(i)** $a^2b^2c^2$

2 **(a)** $6(x + 2)$ **(b)** $6(x - 2)$
(c) $6(2 - x)$ **(d)** $6(2x - y)$
(e) $6(x^2 + 2y)$ **(f)** $6(x - 20)$
(g) $7(2x + 7)$ **(h)** $7(7 - 2x^3)$
(i) $100(x - 2y)$ **(j)** $16(xy + 3z)$

3 **(a)** $a(a + 1)$ **(b)** $a(a + b)$
(c) $2a(a - 2b)$ **(d)** $2ab(a - 2b)$
(e) $3ab(b + 3a)$ **(f)** $6a(4bc + 3d)$
(g) $30b^2c(3b - c)$ **(h)** $3a^2bc(9ab + 19c^2)$

4 **(a)** $3(abc - 2ab + 4bc)$ **(b)** $11(az + 2ak - 4k)$
(c) $8b(3a^2 - 2ac + 8bc)$ **(d)** $5abc(ac^2 + 5 - 21b)$

5 **(a)** $20x - 12xy = 4x(5 - 3y)$
(b) $\frac{xy}{2} + 5x = x\left(\frac{y}{2} + 5\right)$
(c) $15xy - 10x = 5x(3y - 2)$

6 **(a)** $(9y^2 + 12xy)\,\text{cm}^2$ **(b)** $3y(3y + 4x)\,\text{cm}^2$
(c) For $x = 2$ and $y = 1$,
$9y^2 + 12xy = 9(1)^2 + 12 \times 2 \times 1$
$= 9 + 24$
$= 33\,\text{cm}^2$
$3y(3y + 4x) = (3 \times 1)(3 \times 1 + 4 \times 2)$
$= 3(3 + 8)$
$= 3 \times 11$
$= 33\,\text{cm}^2$
Both values are the same, hence the expressions are equivalent.

SC 2

1 **(a)** $8(2x - 1)$ **(b)** $(3z + 5)(2x - 1)$
(c) $(3z + 5y)(2x - 1)$ **(d)** $5(4x + 3)$
(e) $(8 - 3z)(4x + 3)$ **(f)** $(8y - 3z)(4x + 3)$
(g) $18(2y - 9)$ **(h)** $(11x + 7)(2y - 9)$

(i) $(2y - 9)(11x + 7z)$ **(j)** $5(3m - 2n)$
(k) $(14 - 9p)(3m - 2n)$ **(l)** $(3m - 2n)(14k - 9p)$

2 **(a)** $x(3p - 2t)(4 + 7y)$ **(b)** $5(2s + 5r)(t - 3)$
(c) $y(11x - 6z)(8k + 11)$ **(d)** $x^2(5a + 6b)(4 - y)$

3 **(a)** $5x(2p - 3q)(1 + 5y)$ **(b)** $2x(3a + 4b)(2y - 5)$
(c) $3x^2y(7a - 3b)(3 + 5x)$
(d) $7x^2(9b + 5c)(2y^2 + 3)$

4 **(a)** The factor $(x^2 - y^2)$ can be factorised using DOTS.
(b) $(2y - 3z)(x - y)(x + y)$
(c) For $x = 1$, $y = 2$ and $z = 3$,

$$\begin{aligned} x^2(2y - 3z) - y^2(2y - 3z) &= 1^2(2 \times 2 - 3 \times 3) - 2^2(2 \times 2 - 3 \times 3) \\ &= 1(4 - 9) - 4(4 - 9) \\ &= 1 \times (-5) - 4 \times (-5) \\ &= -5 + 20 \\ &= 15 \end{aligned}$$

$$\begin{aligned} (2y - 3z)(x^2 - y^2) &= (2 \times 2 - 3 \times 3)(1^2 - 2^2) \\ &= (4 - 9)(1 - 4) \\ &= -5 \times (-3) \\ &= 15 \end{aligned}$$

$$\begin{aligned} (2y - 3z)(x - y)(x + y) &= (2 \times 2 - 3 \times 3)(1 - 2)(1 + 2) \\ &= (4 - 9)(-1)(3) \\ &= -5 \times (-1) \times 3 \\ &= 15 \end{aligned}$$

All three expressions give the same value, confirming that they are equivalent.

5 **(a)** $(3y + 2x)(z - k)(z + k)$
(b) $3(4x - 3y)(p - q)(p + q)$

3.4 Factorise simple quadratics

SC 1

1 **(a)** $(x + 3)(x + 4)$ **(b)** $(x + 2)(x + 6)$
(c) $(x + 1)(x + 12)$ **(d)** $(x - 3)(x - 4)$
(e) $(x - 2)(x - 6)$ **(f)** $(x - 1)(x - 12)$
(g) $(x + 1)(x - 12)$ **(h)** $(x - 1)(x + 12)$
(i) $(x - 6)(x + 2)$ **(j)** $(x - 4)(x + 3)$
(k) $(x - 2)(x + 6)$ **(l)** $(x - 3)(x + 4)$

2 **(a)** $(x + 1)(x + 11)$ **(b)** $(x + 3)(x + 9)$
(c) $(x + 6)^2$ **(d)** $(x + 4)(x + 8)$
(e) $(x + 2)(x + 10)$ **(f)** $(x + 5)(x + 7)$

3 **(a)** $(x - 1)(x - 11)$ **(b)** $(x + 1)(x - 13)$
(c) $(x - 3)(x - 9)$ **(d)** $(x + 3)(x - 15)$
(e) $(x - 5)(x - 7)$ **(f)** $(x - 16)(x + 4)$

4 **(a)** $(x - 8)(x + 3)$ **(b)** $(x - 4)(x - 11)$
(c) $(x + 6)(x + 10)$ **(d)** $(x + 12)(x - 7)$

5 **(a)** -6 and 2
(b) $x^2 - 6x + 2x - 12 = x(x - 6) + 2(x - 6)$
$= (x - 6)(x + 2)$
$x^2 + 2x - 6x - 12 = x(x + 2) - 6(x + 2)$
$= (x + 2)(x - 6)$
Although the order of the factors is different, these are equivalent expressions so the order of multiplication of terms doesn't matter.

6 The key values in the expansion of $ax^2 + bx + c$ are ac and b.
In this case $ac = -8$ and $b = -7$.
Look for two numbers that multiply to ac and add to b.
In this case, the values will have different signs.
The values are -8 and 1, since $-8 \times 1 = -8$ and $-8 + 1 = -7$.

$$\begin{aligned} 2x^2 - 7x - 4 &= 2x^2 - 8x + 1x - 4 \\ &= 2x(x - 4) + 1(x - 4) \\ &= (x - 4)(2x + 1) \end{aligned}$$

7 **(a)** $(3x + 1)(x - 2)$ **(b)** $(5x - 3)(x - 4)$
(c) $(4x - 5)(x + 3)$ **(d)** $(2x + 7)(x - 1)$

Topic 4: Linear relationships and graphs

Recall

I can evaluate equations following substitution

1 **(a)** $a = 30$ **(b)** $a = -1$ **(c)** $a = -2$
(d) $a = 5$ **(e)** $a = 11$

I can evaluate linear equations using substitution

1 **(a)** 19 **(b)** -14 **(c)** 13
2 **(a)** -9 **(b)** 11 **(c)** -54

I can generate a table of values

1 **(a)** $p = 3m - 5$

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1

(b) $y = \frac{x}{2} - 1$

x	-2	-1	0	1	2
y	-2	-1.5	-1	-0.5	0

(c) $a = \frac{3b - 1}{2}$

x	-2	-1	0	1	2
y	-3.5	-2	-0.5	-1	-2.5

(d) $y = 2(x + 7)$

x	-2	-1	0	1	2
y	10	12	14	16	18

(e) $m = 13p + 27$

x	-2	-1	0	1	2
y	1	14	27	40	53

I can identify the gradient and *y*-intercept of a line from the equation

1 **(a)** $m = 3$, (0,7) **(b)** $m = \frac{1}{5}$, (0,6)
(c) $m = \frac{1}{4}, \left(0, \frac{7}{4}\right)$ **(d)** $m = 3$, (0,6)
(e) $m = 3$, (0,8)

I can calculate the gradient of a line between two points

1 **(a)** -15 **(b)** -1 **(c)** $-\frac{9}{11}$ **(d)** $\frac{7}{12}$

4.1 Determine the gradient of straight-line graphs

SC 1

1 **(a)** A, F, G **(b)** B, E, H **(c)** B, E **(d)** C
(e) D **(f)** E, B **(g)** F
2 **(a)** True **(b)** False **(c)** True **(d)** True
3 **(a)** 4 units to the left
(b) 6 units down
(c) Move C 3 units to the right or move D 6 units up.
(d) Move A 4 units to the left or move B 4 units down.
4 **(a)** The gradient will increase in value.
(b) The gradient will decrease in value.
(c) The gradient becomes zero.
(d) The gradient will become negative.

SC 2

1 **(a)** $m = 3$ **(b)** $m = -4$
(c) $m = 0$ (horizontal line)
(d) Undefined (vertical line)
2 **(a)** $m = 7$ **(b)** $m = -2$ **(c)** $m = 3$ **(d)** $m = -4$
3 **(a)** $m = 5$ **(b)** $m = -7$ **(c)** $m = \frac{1}{2}$ **(d)** $m = -\frac{2}{3}$

4 Using $(-3, -23)$ as (x_1, y_1):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{21 - (-23)}{4 - (-3)}$$
$$= \frac{21 + 23}{4 + 3}$$
$$= \frac{44}{7}$$

Using $(4, 21)$ as (x_1, y_1):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-23 - 21}{-3 - 4}$$
$$= \frac{-44}{-7}$$
$$= \frac{44}{7}$$

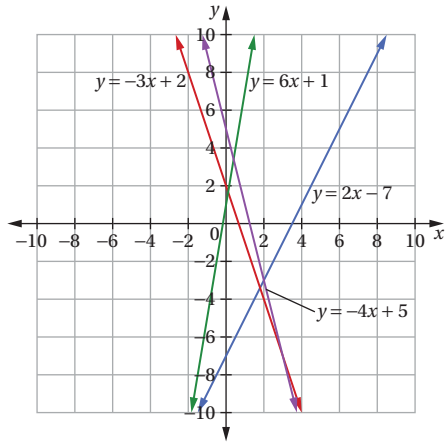
As can be seen, the gradients are the same.

5 **(a)** $m = 5$ **(b)** $m = 6.5$
(c) The three points do not form a straight line.
(d) The coordinate pair should be (3, 12).

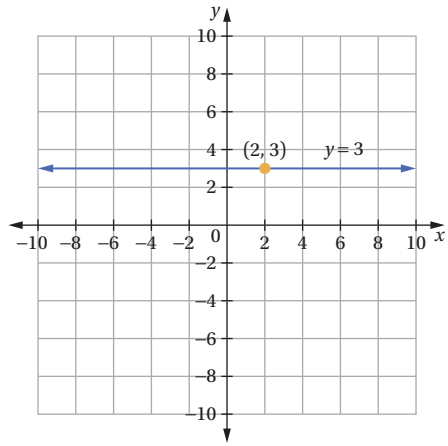
4.2 Sketch and analyse linear graphs

SC 1

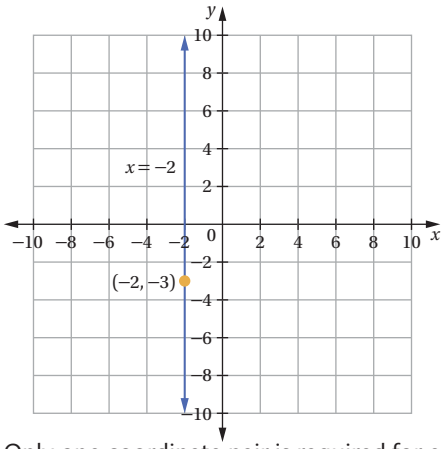
1



2 **(a)** $(-4, -31)$ and $(5, 32)$
(b) $(-4, 15)$ and $(5, -12)$
(c) $(-4, -38)$ and $(5, 61)$
(d) $(-4, 29)$ and $(5, -52)$
3 **(a)** negative **(b)** negative
(c) positive **(d)** negative
4 **(a)**



(b)



(c) Only one coordinate pair is required for each of these lines.

SC 2

1 **(a)** x-intercept (8, 0), y-intercept (0, 12)

(b) x-intercept (−15, 0), y-intercept (0, 12)

(c) x-intercept (3, 0), y-intercept (0, −7)

(d) x-intercept (−14, 0), y-intercept (0, 10)

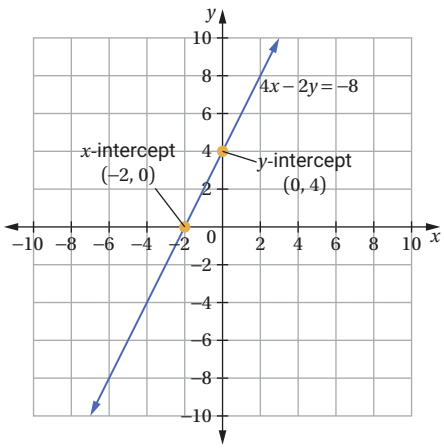
2 **(a)** x-intercept $(-2, 0)$, y-intercept $\left(0, -\frac{4}{3}\right)$

(b) x-intercept $\left(-\frac{1}{3}, 0\right)$, y-intercept $\left(0, \frac{1}{2}\right)$

(c) x-intercept $\left(\frac{6}{5}, 0\right)$, y-intercept $(0, -2)$

(d) x-intercept $\left(-\frac{7}{2}, 0\right)$, y-intercept $\left(0, \frac{7}{5}\right)$

3



4 **(a)** $y = 2x + 1$, (0, 1)
$y = -2x - 3$, (0, −3)
$y = x + 5$, (0, 5)
$y = -x - 6$, (0, −6)

(b) The constant in the equation is the y-intercept.

(c) **(i)** (0, 3) **(ii)** (0, −3) **(iii)** $\left(0, \frac{4}{5}\right)$ **(iv)** $\left(0, \frac{1}{7}\right)$

5 **(a)** $y = \frac{2x}{3} - \frac{7}{3}$: gradient $\frac{2}{3}$, y-intercept $\left(0, -\frac{7}{3}\right)$

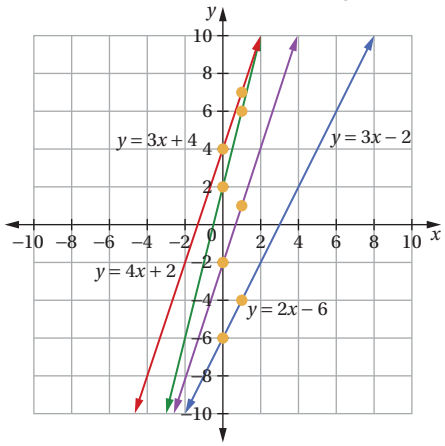
(b) $y = \frac{4x}{7} - \frac{2}{7}$: gradient $\frac{4}{7}$, y-intercept $\left(0, -\frac{2}{7}\right)$

(c) $y = -\frac{3x}{11} + \frac{4}{11}$: gradient $-\frac{3}{11}$, y-intercept $\left(0, \frac{4}{11}\right)$

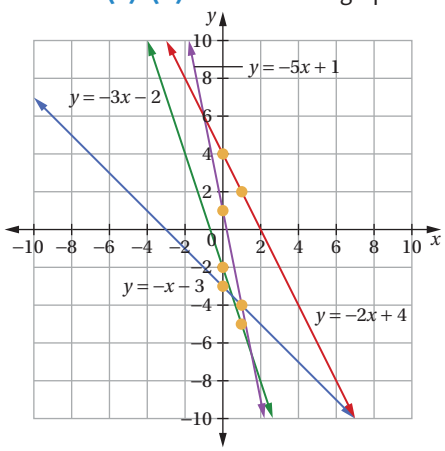
(d) $y = -\frac{7x}{2} + \frac{5}{2}$: gradient $-\frac{7}{2}$, y-intercept $\left(0, \frac{5}{2}\right)$

SC 3

1 Parts **(a)–(d)** shown on the graph below.



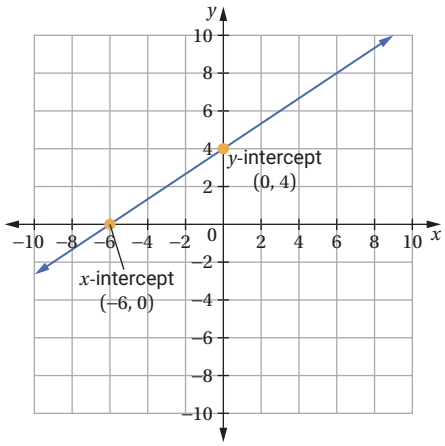
2 Parts **(a)–(d)** shown on the graph below.



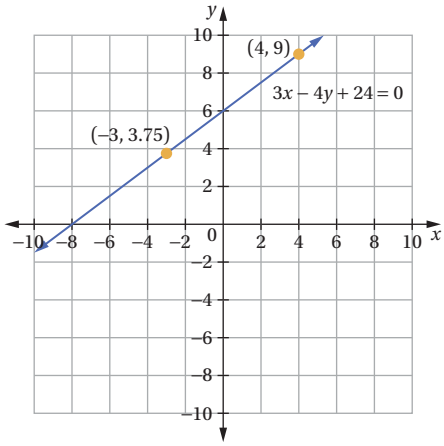
3 **(a)** (−1, 9) and (1, −5)

(b) (−1, −2) and (1, −10)

4 x-intercept (−6, 0), y-intercept (0, 4)



5 $\left(-3, 3\frac{3}{4}\right)$ and (4, 9)



6 **(a)** x-intercept $\left(\frac{2}{5}, 0\right)$, y-intercept $\left(0, -\frac{2}{3}\right)$

(b) x-intercept $\left(\frac{12}{5}, 0\right)$, y-intercept (0, 6)

(c) x-intercept $\left(\frac{7}{3}, 0\right)$, y-intercept $\left(0, -\frac{7}{4}\right)$

4.3 Determine the equation of a straight line

SC 1

1 Graph A: (0, −5)
Graph B: (0, 3)
Graph C: (0, −1.5)

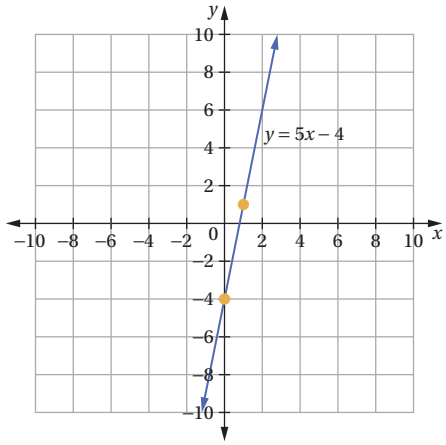
2 **(a)** (0, 1) **(b)** (0, −1) **(c)** (0, 2) **(d)** (0, −2)

3 **(a)** (0, 4) **(b)** $\left(0, -\frac{22}{7}\right)$ **(c)** $\left(0, \frac{1}{5}\right)$ **(d)** $\left(0, \frac{13}{18}\right)$

4 **(a)** $m = \frac{3}{2}$ **(b)** $k = \frac{21}{2} = 10\frac{1}{2}$

(c) $v = -\frac{14}{3} = -4\frac{2}{3}$

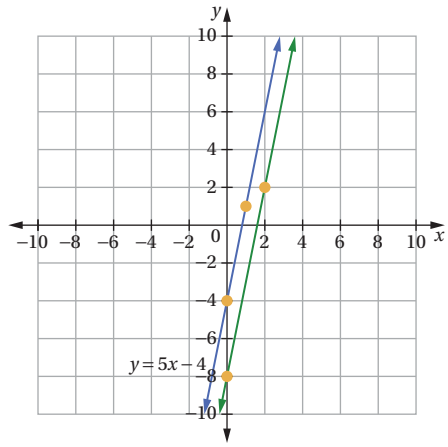
5 **(a)**



(b) The gradient will be 5.

(c) (0, −8)

(d)



SC 2

1 **(a)** $y = -4x + 6$ **(b)** $y = \frac{2x}{3} + 3$

(c) $y = -2x + \frac{1}{4}$ **(d)** $y = \frac{-3x}{5} - \frac{1}{5} = \frac{-(3x+1)}{5}$

2 Line A: $y = 2x - 4$
Line B: $y = 3x + 6$
Line C: $y = -x - 4$
Line D: $y = -2x + 8$

3 Line A: $y = -\frac{3x}{8} + 3$
Line B: $y = 3x - 3$
Line C: $y = \frac{x}{3} + 2$
Line D: $y = -\frac{x}{2} - 1$

4 **(a)** $4x + 10y - 3 = 0$
(b) $-4x + 7y - 5 = 0$
(c) $-30x + 40y + 53 = 0$
(d) $6x + 21y + 10 = 0$

5 **(a)** $y = \frac{3}{4}x + b$ **(b)** $0 = \frac{3}{4} \times \left(\frac{1}{2}\right) + b$

(c) $b = -\frac{3}{8}$ **(d)** $y = \frac{3}{4}x - \frac{3}{8}$

6 $y = -\frac{3x}{8} + \frac{45}{64}$

4.4 Model practical contexts with linear equations

SC 1

1 **(a)** $x = -\frac{11}{3}$ **(b)** $x = \frac{11}{4}$

(c) $x = -\frac{4}{7}$ **(d)** $x = \frac{1}{2}$

2 **(a)** $x = \frac{12}{5}$ **(b)** $x = -7$

(c) $x = -\frac{10}{3}$ **(d)** $x = \frac{84}{5}$

3 **(a)** $a = -\frac{7}{5}$ **(b)** $a = -\frac{8}{19}$
(c) $a = 2$ **(d)** $a = \frac{5}{3}$

4 **(a)** $t = \frac{11}{4}$ **(b)** $t = -\frac{17}{8}$
(c) $t = \frac{112}{195}$ **(d)** $t = \frac{53}{64}$

5 **(a)** $k = \frac{6}{11}$ **(b)** $k = 60$
(c) $k = \frac{144}{37}$ **(d)** $k = \frac{16}{45}$

SC 2

1 **(a)** width : 9 cm; length : 27 cm
(b) width : 11 cm; length : 77 cm

2 **(a)** $E + \frac{1}{3}(E) + \frac{2}{3}\left(\frac{E}{3}\right) = 98$
(b) Elizabeth: 63; Charles: 21; Arthur: 14

3 **(a)** $C = 55n + 70$
(b) \$400
(c) 7.5 hours
(d) This would be the y-intercept value.

4 $x = 39$
In the same order, the angles are 81°, 147°, 122°, 69° and 121°.

5 **(a)** $4 : x - 1; 6 : x + 1$
(b) $3x$
(c) $3x = 336; x = 112$
The numbers are 111, 112, 113.
(d) $(x - 2) + x + (x + 2) = 3x = 231$; 75, 77, 79
(e) $(x - 2) + x + (x + 2) = 3x = 132$; 42, 44, 46

4.5 Determine the distance between points and the coordinates of the midpoint

SC 1

1 **(a)** $\sqrt{73}$ units **(b)** $3\sqrt{10}$ units
(c) $\sqrt{13}$ units **(d)** $2\sqrt{5}$ units

2 **(a)** $\sqrt{133}$ units **(b)** $\sqrt{153}$ units
(c) $2\sqrt{153}$ units **(d)** $2\sqrt{153}$ units

3 Let $(x_1, y_1) = (2, 5)$ and $(x_2, y_2) = (4, 17)$.

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-2)^2 + (17-5)^2}$$
$$= \sqrt{2^2 + 12^2}$$
$$= \sqrt{4 + 144}$$
$$= \sqrt{148}$$
$$= 2\sqrt{37} \text{ units}$$

Let $(x_1, y_1) = (4, 17)$ and $(x_2, y_2) = (2, 5)$.

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2-4)^2 + (5-17)^2}$$
$$= \sqrt{(-2)^2 + (-12)^2}$$
$$= \sqrt{4 + 144}$$
$$= \sqrt{148}$$
$$= 2\sqrt{37} \text{ units}$$

The same answer is obtained; hence, the order of the points is irrelevant.

4 **(a)** $y = -4$ or $y = 6$
(b) $y = -17$ or $y = 13$
(c) $x = -10$ or $x = 4$

5 **(a)** 7.91 units **(b)** 8.51 units
(c) 8.06 units **(d)** 24.48 units

6 **(a)** $2\sqrt{5}$ units **(b)** $2\sqrt{5}$ units
(c) (−2, 4), (−2, −4), (2, −4), (−4, 2), (−4, −2), (4, −2)

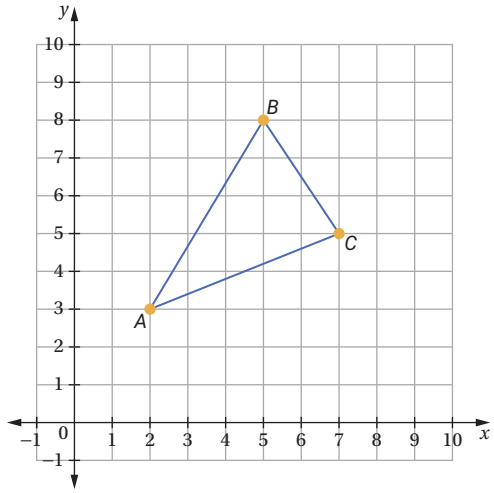
SC 2

1 **(a)** (3, 6) **(b)** (2, 5) **(c)** (0, −4) **(d)** (0.5, 1.5)

2 **(a)** (−1, 0.5) **(b)** (3.5, −2)
(c) (−5, −3) **(d)** (−1.5, −3.5)

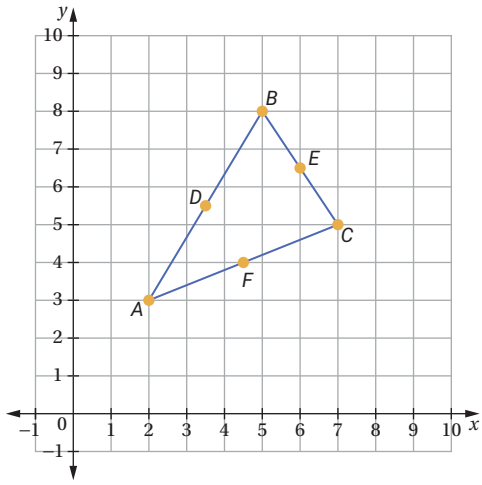
3 **(a)** $R = (0, 2)$ **(b)** $S = (0.5, -3)$
(c) $\sqrt{25.25}$ units **(d)** (0.25, −0.5)

4 **(a)**

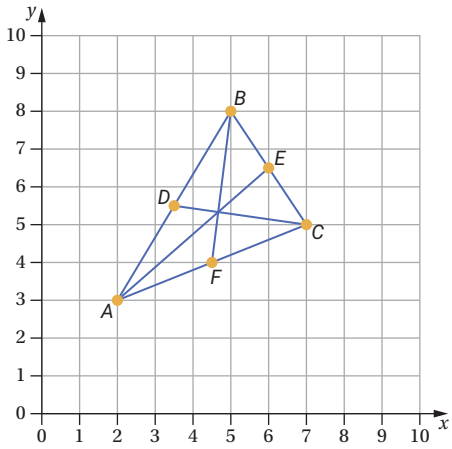


(b) For $\overline{AB}$, the midpoint D occurs at (3.5, 5.5);
for $\overline{BC}$, the midpoint E occurs at (6, 6.5);
for $\overline{CA}$, the midpoint F occurs at (4.5, 4).

(c)



(d)



(e) The three line segments intersect at the same point. (This point of intersection is called the centroid.)

SC 3

1 **(a)** (5, 6) **(b)** (−5, 10) **(c)** (−9, 11) **(d)** (12, 12)

2 **(a)** (6, 9) **(b)** (−5, 2) **(c)** (0, −2) **(d)** (−1, −1)

3 If A is the midpoint, then the other endpoint is at (7, 14), and if B is the midpoint, then the other endpoint is at (−8, −10).

4 **(a)** (−8, −17) **(b)** (7, 19)

(c) The midpoint of $\overline{CD}$ occurs at (−0.5, 1) and the midpoint of $\overline{AB}$ occurs at (−0.5, 1); it is the same point.

5 **(a)** (3, 0) **(b)** (1, 1)

(c) $2y + x - 3 = 0$ or $y = -\frac{1}{2}x + \frac{3}{2}$

4.6 Transform linear graphs

SC 1

1 **(a)** $y = 3x$ **(b)** $y = \frac{x}{6}$

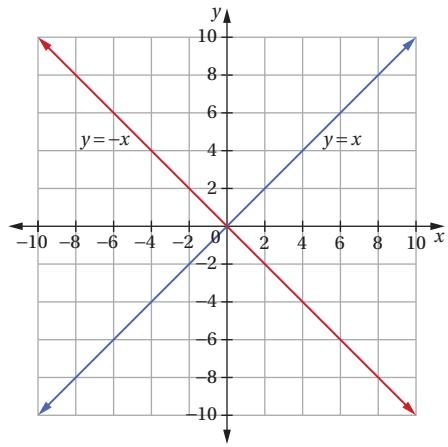
(c) $y = 10x$ **(d)** $y = \frac{2x}{3}$

2 **(a)** $k = \frac{1}{5} = 0.2$ **(b)** $k = \frac{1}{3}$

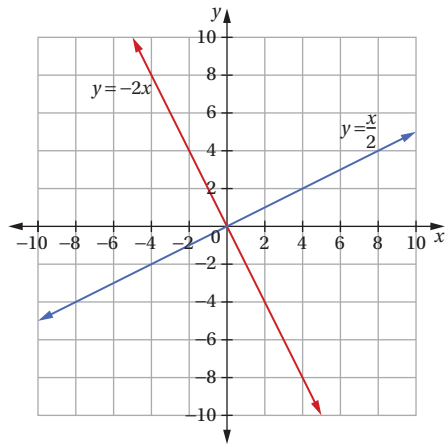
(c) $k = -5$ **(d)** $k = -10$

3 Sometimes true. This is only true for $y = x$ and $y = -x$.

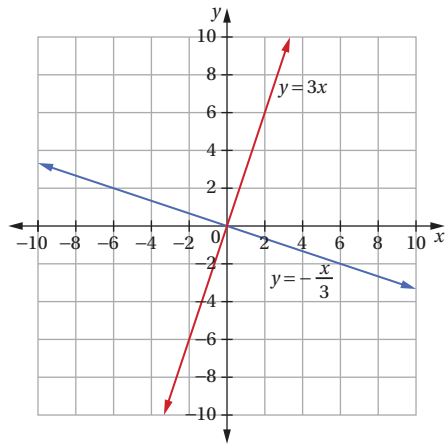
4 **(a)**



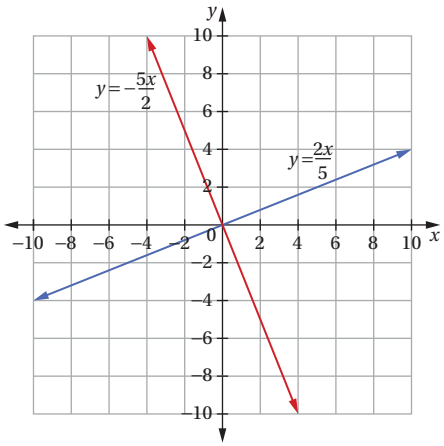
(b)



(c)



(d)



(e) In each case, the product of the gradients is -1.

(f) If the product of the gradients of two graphs is equal to -1, then the lines are perpendicular.

5 **(a)** $y=-\frac{x}{7}$ **(b)** $y=\frac{x}{11}$

(c) $y=-10x$ **(d)** $y=\frac{6x}{5}$

6 **(a)** $y=\frac{5x}{4}$

(b) The two lines are perpendicular.

(c) $y=-\frac{4x}{5}$

SC 2

1 **(a)** $y=2x-7$ **(b)** $y=2x-1$

(c) $y=2x+6$ **(d)** $y=2x-11$

2 **(a)** down 8 units **(b)** down 16 units

(c) up 3 units **(d)** up 1 unit

3 **(a)** $y=0$ **(b)** $y=12$

(c) $y=3x$ **(d)** $y=3x-6$

4 **(a)** The shape is a rectangle, so A and B are perpendicular. Hence, the equation for B is $y=-\frac{x}{4}$.

(b) The graph has been translated 2 units up, so its equation is $y=-\frac{x}{4}+2$.

(c) Graph C is parallel to graph A, so the gradient is 4 and it passes through (3, 0).

$y-y_1=m(x-x_1)$

$y-0=4(x-3)$

$y=4x-12$

SC 3

1 **(a)** $y=10x-3$ **(b)** $y=-x+5$

(c) $y=-2x$ **(d)** $y=-4x+1$

2 **(a)** Dilation by a factor of 6 and translation of 2 units vertically down.

(b) Dilation by a factor of 2, reflection in the x-axis and translation of 4 units vertically up.

(c) Dilation by a factor of 3, reflection in the x-axis and translation of 6 units vertically up.

(d) Dilation by a factor of 5, reflection in the x-axis and translation of 2 units vertically down.

3 **(a)** $y=-3x+7$: dilation by a factor of 3, reflection in the x-axis and translation of 7 units vertically up.

(b) $y=-x+8$: reflection in the x-axis and translation of 8 units vertically up.

(c) $y=-2x-6$: dilation by a factor of 2, reflection in the x-axis. and translation of 6 units vertically down.

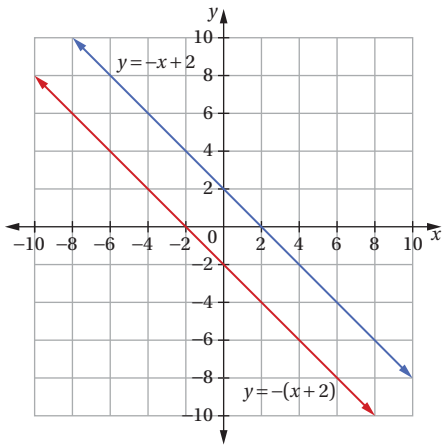
(d) $y=3x-15$: dilation by a factor of 3 and translation of 15 units vertically down.

4 **(a)** $y=3x-2$: dilation by a factor of 3 and translation of 2 units vertically down.

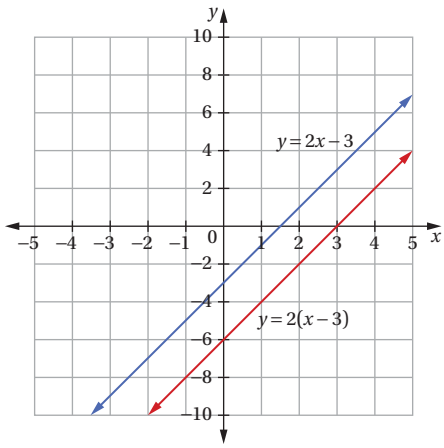
(b) $y=-2x+3$: dilation by a factor of 2, reflection in the x-axis and translation of 3 units vertically up.

(c) $y=-x-4$: reflection in the x-axis and translation of 4 units vertically down.

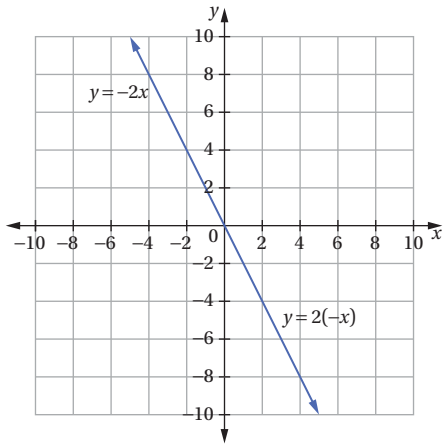
5 **(a)**



(b)



(c)



(d) Dilation and reflection in the x-axis can be applied in either order and yield the same result.

4.7 Interpolate and extrapolate

SC 1

1 (a) Between 2:15 pm and 2:45 pm
(b) 275 m
(c) No, maximum distance ≥ 275 m (maximum distance is no less than 275 m)
(d) 230 − 240 m
(e) Between 5 pm and 5:10 pm

2 (a) The school is 25 m above sea level. This is represented by the vertical intercept.
(b) The highest point on the graph is 225 m, but the students may have gone higher earlier in the day, or at the start of the next day. The points only reflect the height at which they stayed each night.
(c) At the end of day 7 the students are the same height above sea level as the school.
(d) 187.5 m
(e) 200 m
(f) height = − 150 × day + 1075

3 (a) An increase of 50 basis points
(b) An increase of 37.5 basis points
(c) 3.1 + 3 × 0.375 = 4.225%
(d) 3.1 + 3 × 0.25 = 3.85%
(e) Extrapolation

4 (a) 0.0099 seconds per year
(b) 9.7882 s
(c) The estimate was too low because the change in the record was probably most dramatic in the early years of record keeping. In more recent times, only very small improvements are made. The increase is not linear.
(d) 9.1741 s
(e) This is probably much too low. There has to be some limit to the speed at which a person can run. The fact that in 2022 the record was still the same as in 2009 is evidence of this.

Topic 5: Quadratic relationships and graphs

Recall

I can perform translations of shapes on the Cartesian plane

1 (−2, −1), (−2, 3), (2, 3) and (2, −1)
2 (−1, −1), (3, −1), and (3, −4)

I can evaluate quadratic equations using substitution

1 (a) $y = 21$ **(b)** $y = -3$ **(c)** $y = 11$
2 (a) $y = -4$ **(b)** $y = -6$ **(c)** $y = 0$

I can expand brackets and simplify using the distributive law

1 (a) $x^2 + 3x$ **(b)** $3x^2 - 12x$
(c) $6x - 2x^2$ **(d)** $5x^2 - 10x$

I can expand and simplify the product of two brackets

1 (a) $x^2 + 8x + 15$ **(b)** $x^2 - x - 6$
(c) $x^2 - 4x - 5$ **(d)** $x^2 - 9x + 18$

I can identify the highest common factor

1 (a) 15 **(b)** 9 **(c)** 8 **(d)** 22

I can solve linear equations

1 (a) $a = 4$ **(b)** $d = 96$ **(c)** $e = -19$ **(d)** $g = -3$

I can identify the two solutions for a perfect square

1 (a) $x = \pm 2$ **(b)** $x = \pm 1$ **(c)** $x = \pm 4$ **(d)** $x = \pm 8$

5.1 Understanding solutions to simple quadratic equations

SC 1

1 (a) The inverse operation to addition is *subtraction.*
(b) The inverse operation to subtraction is *addition.*
(c) The inverse operation to multiplication is *division.*
(d) The inverse operation to division is *multiplication.*
(e) The inverse operation to squaring is *taking the square root.*

2 (a) $x = \pm 9$ **(b)** $x = \pm 11$ **(c)** $x = \pm 1$
(d) $x = 0$ **(e)** $x = \pm 7$

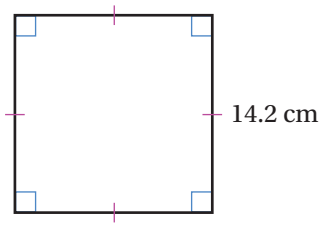
3 (a) $x = \pm 50$ **(b)** $x = \pm 30$ **(c)** $x = \pm 25$
(d) $x = \pm 20$ **(e)** $x = \pm 45$

4 (a) $x = \pm 4.90$ **(b)** $x = \pm 7.75$
(c) $x = \pm 6.24$ **(d)** $x = \pm 1.41$
(e) $x = \pm 11.40$

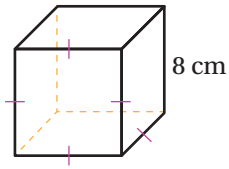
5 **(a)** $x = \pm 1.58$ **(b)** No real solution
(c) No real solution **(d)** $x = \pm 0.3$
(e) No real solution

6 **(a)** 10.95 cm **(b)** 5.48 m
(c) 23.07 mm **(d)** 45.00 cm

7



8



9 **(a)** Mia has shown the correct steps, but instead of calculating the square root of 15, Mia has incorrectly divided 15 by 2.

(b) The correct working out and solution are:
$x^2 = 15$
$\sqrt{x^2} = \sqrt{15}$
$x = 3.873$ (3 d.p.)

SC 2

1 **(a)** $x^2 = 4$ **(b)** $x^2 = 9$ **(c)** $x^2 = 64$
(d) $x^2 = 25$ **(e)** $x^2 = 16$

2 **(a)** $x = \pm 2$ **(b)** $x = \pm 3$ **(c)** $x = \pm 8$
(d) $x = \pm 5$ **(e)** $x = \pm 4$

3 **(a)** $x = \pm 12$ **(b)** $x = \pm 11$ **(c)** $x = \pm 1$
(d) $x = \pm 3$ **(e)** $x = \pm 2$

4 **(a)** $x = \pm 2.236$ **(b)** $x = \pm 2.449$
(c) $x = \pm 4.472$ **(d)** $x = \pm 2.236$
(e) $x = \pm 1.732$

5 **(a)** $x = \pm 5$ **(b)** $x = \pm 8$ **(c)** $x = \pm 4$
(d) $x = \pm 9$ **(e)** $x = \pm 7$

6 Express each equation in the form $x^2 = c$.
After dividing both sides by -4:
$-4x^2 = -36$ becomes $x^2 = 9$
$4x^2 = -36$ becomes $x^2 = -9$
Recall that there are two solutions when $c > 0$, but no solutions for $c < 0$. This is because there is no integer, when squared, that can result in a negative number. Hence, the first equation can be solved, but the second cannot.

7 The radius is 14 cm.

SC 3

1 **(a)** If $c < 0$, there are no solutions.
(b) If $c = 0$, there is one solution.
(c) If $c > 0$, there are two solutions.

2 **(a)** No solutions **(b)** Two solutions
(c) No solutions **(d)** One solution
(e) Two solutions

3 **(a)** Two solutions, $x = \pm 6$ **(b)** No solutions
(c) Two solutions, $x = \pm 8$ **(d)** One solution, $x = 0$
(e) No solutions **(f)** No solutions
(g) Two solutions, $x = \pm 11$

4 **(a)** 12 cm **(b)** 100 cm **(c)** 13 cm **(d)** 15 cm

5 **(a)** 121 cm^2 **(b)** 11 cm **(c)** 22 cm
(d) 484 cm^2 **(e)** 2904 cm^2

5.2 Solve monic quadratic equations using the Null Factor Law

SC 1

1 **(a)** $x = 3$ **(b)** $x = 2$ **(c)** $x = 1$ **(d)** $x = 0$
(e) $x = -1$ **(f)** $x = -2$ **(g)** $x = -3$

2 **(a)** $x = 0$ or $x = 8$ **(b)** $x = 0$ or $x = 5$
(c) $x = 0$ or $x = 1$ **(d)** $x = 0$
(e) $x = 0$ or $x = -1$ **(f)** $x = 0$ or $x = -5$
(g) $x = 0$ or $x = -8$

3 **(a)** $x = -1$ or $x = 2$ **(b)** $x = -2$ or $x = 3$
(c) $x = -3$ or $x = 4$ **(d)** $x = -4$ or $x = 5$

4 **(a)** $x = 4$ or $x = -4$ **(b)** $x = 7$ or $x = -7$
(c) $x = 3$ (only one solution)
(d) $x = -6$ (only one solution)

5 **(a)** $a = -\frac{2}{3}$ or $a = -\frac{3}{2}$ **(b)** $b = \frac{2}{5}$ or $b = -\frac{3}{2}$
(c) $c = -\frac{5}{6}$ or $c = \frac{4}{3}$ **(d)** $d = \frac{2}{5}$ or $d = \frac{3}{5}$

6 **(a)** **(i)** $(x + 3)(x - 6) = 0$
(ii) $x^2 - 3x - 18 = 0$
(b) **(i)** $(x + 5)(x - 9) = 0$
(ii) $x^2 - 4x - 45 = 0$
(c) **(i)** $(x - 10)^2 = 0$
(ii) $x^2 - 20x + 100 = 0$
(d) **(i)** $(x + 8)^2 = 0$
(ii) $x^2 + 16x + 64 = 0$

SC 2

1 **(a)** **(i)** $(x + 1)^2 = 0$ **(ii)** $x = -1$
(b) **(i)** $(x + 4)^2 = 0$ **(ii)** $x = -4$
(c) **(i)** $(x + 6)^2 = 0$ **(ii)** $x = -6$
(d) **(i)** $(x + 7)^2 = 0$ **(ii)** $x = -7$
(e) **(i)** $(x + 10)^2 = 0$ **(ii)** $x = -10$
(f) **(i)** $(x + 15)^2 = 0$ **(ii)** $x = -15$

2 **(a)** **(i)** $(x-2)^2=0$ **(ii)** $x=2$
(b) **(i)** $(x-4)^2=0$ **(ii)** $x=4$
(c) **(i)** $(x-5)^2=0$ **(ii)** $x=5$
(d) **(i)** $(x-8)^2=0$ **(ii)** $x=8$
(e) **(i)** $(x-11)^2=0$ **(ii)** $x=11$
(f) **(i)** $(x-13)^2=0$ **(ii)** $x=13$

SC 3

1 **(a)** **(i)** $(x+1)(x-1)=0$ **(ii)** $x=-1$ or $x=1$
(b) **(i)** $(x+3)(x-3)=0$ **(ii)** $x=-3$ or $x=3$
(c) **(i)** $(x+6)(x-6)=0$ **(ii)** $x=-6$ or $x=6$
(d) **(i)** $(x+8)(x-8)=0$ **(ii)** $x=-8$ or $x=8$
(e) **(i)** $(x+12)(x-12)=0$ **(ii)** $x=-12$ or $x=12$
(f) **(i)** $(x+17)(x-17)=0$ **(ii)** $x=-17$ or $x=17$

SC 4

1 **(a)** **(i)** $(x+2)(x+9)=0$ **(ii)** $x=-2$ or $x=-9$
(b) **(i)** $(x+3)(x+6)=0$ **(ii)** $x=-3$ or $x=-6$
(c) **(i)** $(x+1)(x+18)=0$ **(ii)** $x=-1$ or $x=-18$
(d) **(i)** $(x+1)(x-18)=0$ **(ii)** $x=-1$ or $x=18$
(e) **(i)** $(x+2)(x-9)=0$ **(ii)** $x=-2$ or $x=9$
(f) **(i)** $(x-6)(x+3)=0$ **(ii)** $x=6$ or $x=-3$
(g) **(i)** $(x+18)(x-1)=0$ **(ii)** $x=-18$ or $x=1$
(h) **(i)** $(x+6)(x-3)=0$ **(ii)** $x=-6$ or $x=3$
(i) **(i)** $(x+9)(x-2)=0$ **(ii)** $x=-9$ or $x=2$

2 **(a)** **(i)** $(x+2)(x+16)=0$ **(ii)** $x=-2$ or $x=-16$
(b) **(i)** $(x+7)(x+11)=0$ **(ii)** $x=-7$ or $x=-11$
(c) **(i)** $(x+8)(x+10)=0$ **(ii)** $x=-8$ or $x=-10$
(d) **(i)** $(x+9)^2=0$ **(ii)** $x=-9$
(e) **(i)** $(x+4)(x+14)=0$ **(ii)** $x=-4$ or $x=-14$

3 **(a)** **(i)** $(x+1)(x-19)=0$ **(ii)** $x=-1$ or $x=19$
(b) **(i)** $(x+2)(x-20)=0$ **(ii)** $x=-2$ or $x=20$
(c) **(i)** $(x-10)(x-8)=0$ **(ii)** $x=10$ or $x=8$
(d) **(i)** $(x-12)(x-6)=0$ **(ii)** $x=12$ or $x=6$
(e) **(i)** $(x-15)(x-3)=0$ **(ii)** $x=15$ or $x=3$

4 **(a)** $x=7$ or $x=-3$ **(b)** $x=2$ or $x=6$
(c) $x=-5$ or $x=-9$ **(d)** $x=-10$ or $x=8$

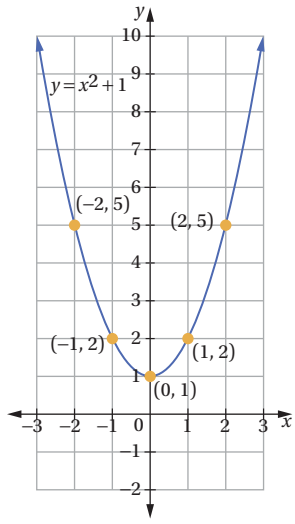
5.3 Identify and plot the key features of quadratic graphs

SC 1

1 **(a)** **(i)**

x	-2	-1	0	1	2
$y=x^2+1$	5	2	1	2	5

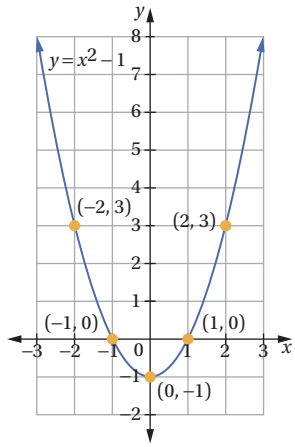
(ii)



(b) **(i)**

x	-2	-1	0	1	2
$y=x^2-1$	3	0	-1	0	3

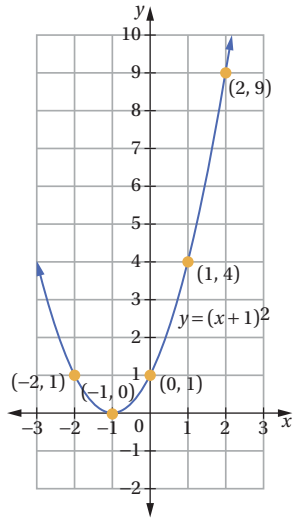
(ii)



(c) **(i)**

x	-2	-1	0	1	2
$y=(x+1)^2$	1	0	1	4	9

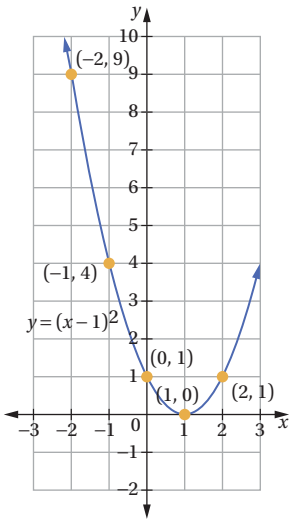
(ii)



(d) **(i)**

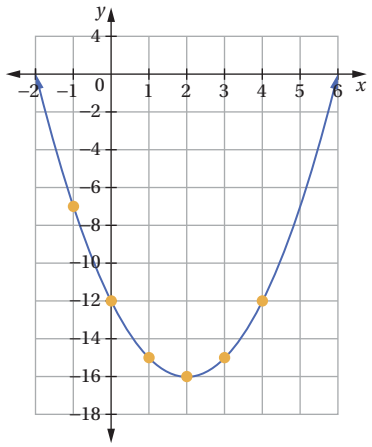
x	−2	−1	0	1	2
$y = (x-1)^2$	9	4	1	0	1

(ii)



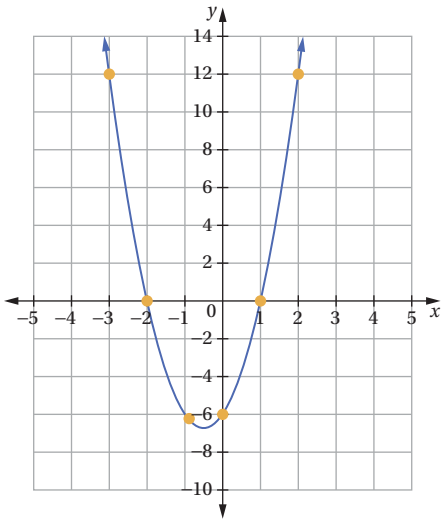
2 **(a)**

x	−1	0	1	2	3	4
y	−7	−12	−15	−16	−15	−12



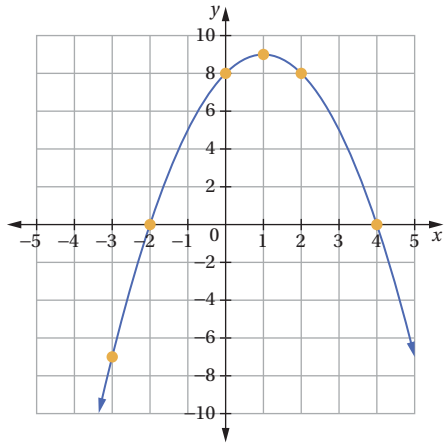
(b)

x	−3	−2	−1	0	1	2
y	12	0	−6	−6	0	12



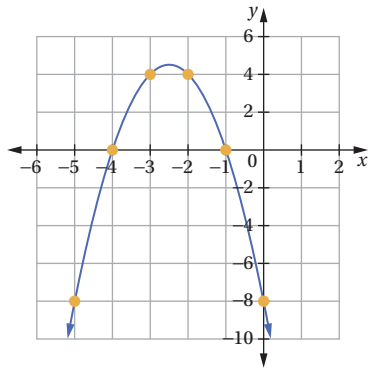
(c)

x	−3	−2	0	1	2	4
y	−7	0	8	9	8	0



(d)

x	−5	−4	−3	−2	−1	0
y	−8	0	4	4	0	−8



3 **(a)** $y = x^2 - 3$

x	−2	−1	0	1	2	3
y	1	−2	−3	−2	1	6
First difference		−3	−1	1	3	5
Second difference			2	2	2	2

(b) $y = -x^2 + 4x$

x	−2	−1	0	1	2	3
y	−12	−5	0	3	4	3
First difference		7	5	3	1	−1
Second difference			−2	−2	−2	−2

(c) $y = 2x^2 - 3x - 5$

x	−2	−1	0	1	2	3
y	9	0	−5	−6	−3	4
First difference		−9	−5	−1	3	7
Second difference			4	4	4	4

(d) $y = (x - 2)^2$

x	−2	−1	0	1	2	3
y	16	9	4	1	0	1
First difference		−7	−5	−3	−1	1
Second difference			2	2	2	2

4 **(a)** $\left(-\frac{1}{2}, -\frac{125}{4}\right)$ **(b)** $\left(3, \frac{7}{2}\right)$

(c) $\left(-1, \frac{16}{3}\right)$ **(d)** $\left(\frac{1}{2}, \frac{81}{10}\right)$

5 **(a)**

x	0	1	3	4	5	6	8	9
y	18	10	0	−2	−2	0	10	18

(b)

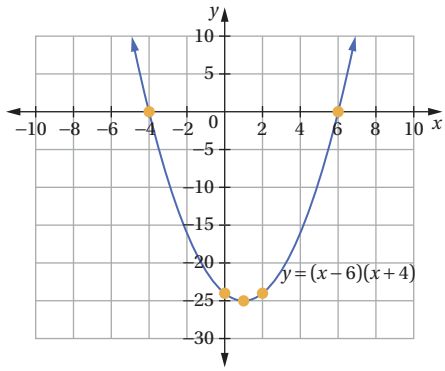
x	−10	−6	−4	3	5	9
y	−78	−18	0	0	−18	−78

(c)

x	−20	−12	−9	−7	−5	−3	0	8
y	187	27	0	−8	−8	0	27	187

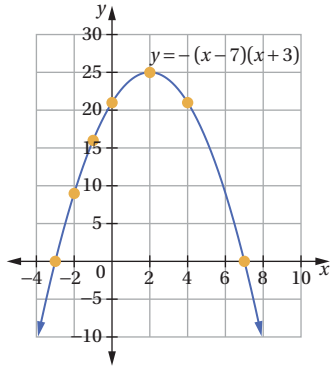
6 **(a)**

x	−4	0	1	2	6
y	0	−24	−25	−24	0



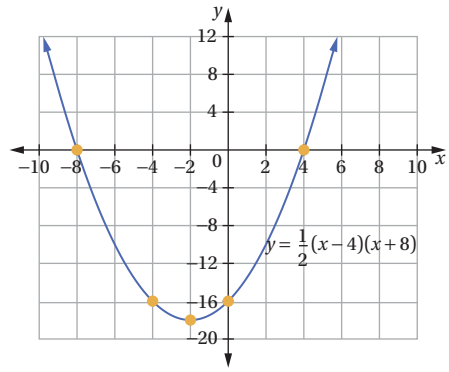
(b)

x	−3	−2	−1	0	2	4	7
y	0	9	16	21	25	21	0



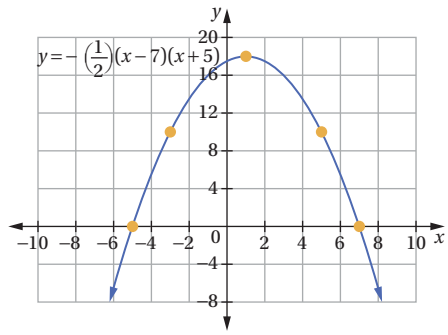
(c)

x	−8	−4	−2	0	4
y	0	−16	−18	−16	0



(d)

x	−5	−3	1	5	7
y	0	10	18	10	0



SC 2

1 **(a)** y-intercept (0, −4), minimum turning point (0, −4)
x-intercepts (−2, 0) and (2, 0)

(b) y-intercept (0, −5), minimum turning point (2, −9),
x-intercepts (−1, 0) and (5, 0)

(c) y-intercept (0, 21), maximum turning point (−2, 25),
x-intercepts (−7, 0) and (3, 0)

(d) y-intercept (0, 25), minimum turning point (5, 0),
x-intercept (5, 0)

2 **(a)** (−1, −9) **(b)** (−1, 16)

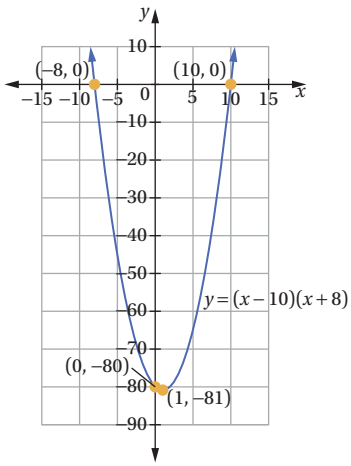
(c) (1, 6) **(d)** $\left(\frac{1}{2}, 6\frac{1}{2}\right)$

3 **(a)** **(i)** x-intercepts (8, 0) and (−10, 0)

(ii) y-intercept (0, −80)

(iii) Turning point (−1, −81)

(iv)

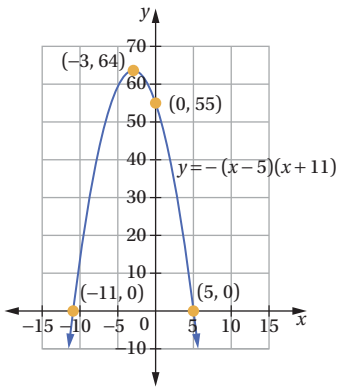


(b) **(i)** x-intercepts (5, 0) and (−11, 0)

(ii) y-intercept (0, 55)

(iii) Turning point (−3, 64)

(iv)

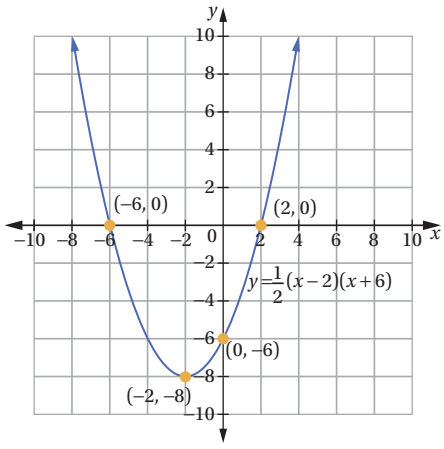


(c) **(i)** x-intercepts (2, 0) and (−6, 0)

(ii) y-intercept (0, −6)

(iii) Turning point (−2, −8)

(iv)

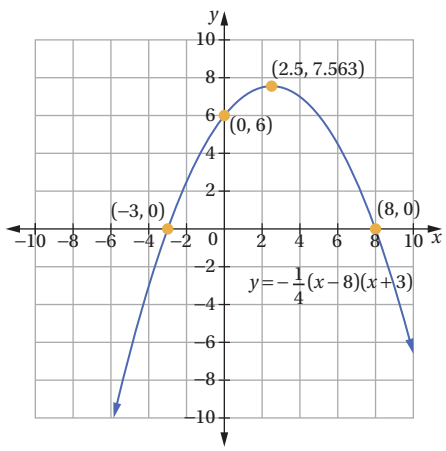


(d) **(i)** x-intercepts (8, 0) and (−3, 0)

(ii) y-intercept (0, 6)

(iii) Turning point $\left(\frac{5}{2}, \frac{121}{16}\right)$

(iv)



4 **(a)** x-intercept (2, 0), y-intercept (0, 4); minimum turning point

(b) x-intercept $\left(-\frac{2}{3}, 0\right)$, y-intercept (0, 4); minimum turning point

(c) x-intercept $\left(\frac{1}{5}, 0\right)$, y-intercept (0, 1); maximum turning point

(d) x-intercept $\left(\frac{3}{2}, 0\right)$, y-intercept (0, 9); minimum turning point

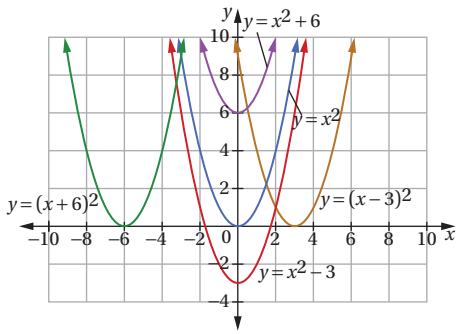
5.4 Determine transformations of parabolas

SC 1

1 **(a)** 6 units up **(b)** 3 units down

(c) 6 units left **(d)** 3 units right

2



3 A: $y = x^2 + 4$

B: $y = x^2 - 5$

C: $y = (x + 4)^2$

D: $y = (x - 5)^2$

4 **(a)** (0, −7) **(b)** (0, −5) **(c)** (0, 2) **(d)** (0, 6)

5 **(a)** (7, 0) **(b)** (5, 0) **(c)** (−2, 0) **(d)** (−6, 0)

SC 2

1 **(a)** $y = 2x^2$ **(b)** $y = 5x^2$ **(c)** $y = 10x^2$
(d) $y = \frac{1}{2}x^2$ **(e)** $y = \frac{1}{5}x^2$ **(f)** $y = \frac{1}{10}x^2$

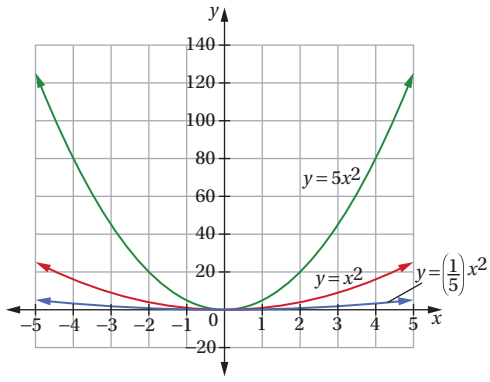
2 **(a)** $y = -x^2$ **(b)** $y = x^2$ **(c)** $y = -2x^2$
(d) $y = -\frac{1}{2}x^2$ **(e)** $y = 2x^2$ **(f)** $y = \frac{1}{2}x^2$

3 A: (0, −25), B: (−10, −25), C: (−3, 4)

4 **(a)**

x	−10	−5	0	5	10
$y = x^2$	$(-10)^2 = 100$	$(-5)^2 = 25$	$0^2 = 0$	$5^2 = 25$	$10^2 = 100$
$y = \frac{1}{5}x^2$	$\frac{1}{5} \times 100 = 20$	$\frac{1}{5} \times 25 = 5$	$\frac{1}{5} \times 0 = 0$	$\frac{1}{5} \times 25 = 5$	$\frac{1}{5} \times 100 = 20$
$y = 5x^2$	5 × 100 = 500	5 × 25 = 125	5 × 0 = 0	5 × 25 = 125	5 × 100 = 500

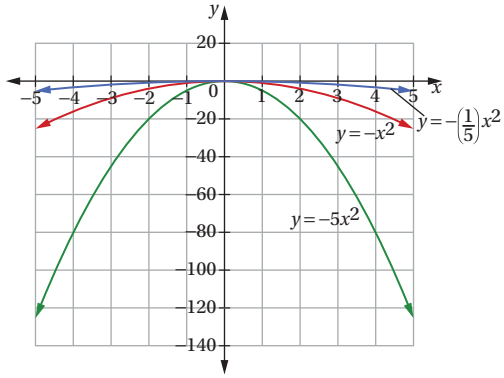
(b)



(c) $y = \frac{1}{5}x^2$ is compressed by a factor of 5, it is flatter than $y = x^2$.
$y = 5x^2$ is enlarged by a factor of 5, it is steeper than $y = x^2$.

(d) $y = x^2$ reflected in the x-axis is $y = -x^2$
$y = \frac{1}{5}x^2$ reflected in the x-axis is $y = -\frac{1}{5}x^2$
$y = 5x^2$ reflected in the x-axis is $y = -5x^2$

(e)



SC 3

1 **(a)** (−4, −3)
(b) 4 units left and 3 units down
(c) $y = (x + 4)^2 - 3$

2 A: $y = (x + 5)^2 - 2$
B: $y = (x - 3)^2 + 1$
C: $y = (x + 2)^2 + 4$
D: $y = (x - 4)^2 - 3$

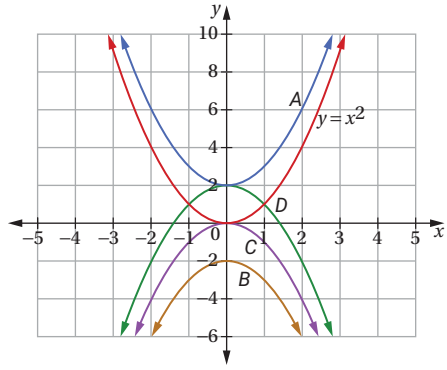
3 **(a)** (3, 0) **(b)** (3, 0) **(c)** (3, 0) **(d)** (3, 0)

4 **(a)** $y = 3(x - 5)^2$ **(b)** $y = -(x + 6)^2 + 3$
(c) $y = -\frac{2(x-2)^2}{3} - 7$ **(d)** $y = -7x^2 + 6$

5 $A: y = 2(x-1)^2$
$B: y = -3(x-1)^2$
$C: y = -\frac{1}{2}(x-1)^2$

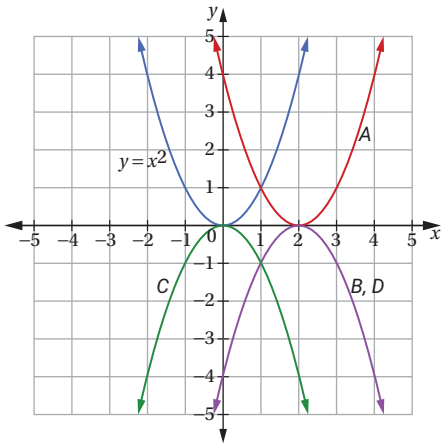
6 $A: y = -2(x-1)^2$
$B: y = \frac{1}{2}x^2 + 2$
$C: y = -2(x+1)^2 - 1$
$D: y = -(x-2)^2 + 4$

7 **(a)-(e)**



(f) The order of transformations is important as these two pairs of transformations result in different outcomes.

8 (a)–(e)



(f) In this case, the order of transformations is not important.

9 (a) The horizontal translation is described first.

(b) Next comes dilation and/or reflection as these transformations are both indicated by the value of a.

(c) The vertical translation comes last.

(d) This does not contradict the result from Questions **6** and **7** because it was found that reflection and horizontal translation could be done in either order.

10 (a) Translation 4 units to the right and 3 units up.

(b) Reflection in the x-axis, translation 2 units to the left and 1 unit up.

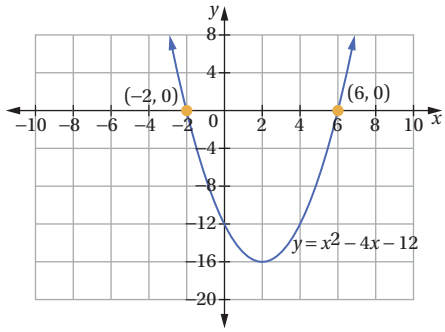
(c) Dilation by a factor of 4 in the direction of the y-axis, translation 5 units to the left and 2 units down.

(d) Dilation by a factor of $\frac{2}{5}$ in the direction of the y-axis, reflection in the x-axis, translation 1 unit to the right and 1 unit up.

5.5 Solve quadratic equations graphically

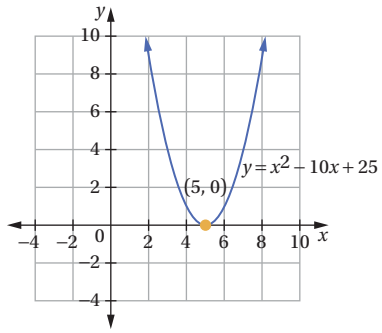
SC 1

1 (a)



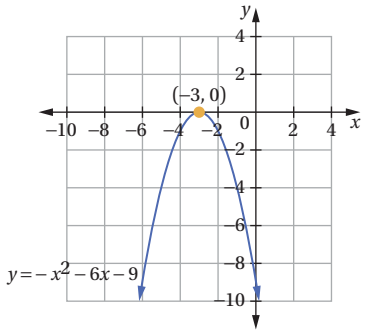
The equation has two solutions, $x = -2$ or $x = 6$.

(b)



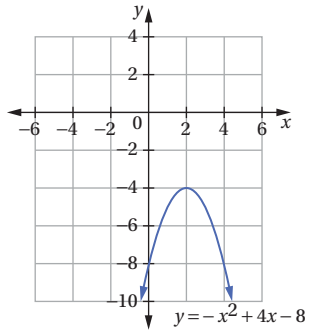
The equation has one solution, $x = 5$.

(c)



The equation has one solution, $x = -3$.

(d)



The equation has zero solutions.

2 *A*: **(i)** $y = (x - 2)(x - 4) = x^2 - 6x + 8$
(ii) Two **(iii)** (2, 0) and (4, 0)
B: **(i)** $y = -(x + 2)(x - 3) = -x^2 + x + 6$
(ii) Two **(iii)** (−2, 0) and (3, 0)
C: **(i)** $y = (x + 4)^2 = x^2 + 8x + 16$
(ii) One **(iii)** (−4, 0)
D: **(i)** $y = -(x - 5)^2 = -x^2 + 10x - 25$
(ii) One **(iii)** (5, 0)

3 *A*: **(i)** $y = (x - 1)^2 - 9$
(ii) $x = -2$ or $x = 4$
B: **(i)** $y = (x + 1)^2 - 25$
(ii) $x = -6$ or $x = 4$
C: **(i)** $y = -(x - 2)^2 + 9$
(ii) $x = -1$ or $x = 5$
D: **(i)** $y = -(x + 3)^2 + 9$
(ii) $x = -6$ or $x = 0$

4 **(a)** $y = x^2 - 10x + 21$ and the x-intercepts are $x = 3$ and $x = 7$.

(b) $y = -x^2 + 8x - 7$ and the x-intercepts are $x = 1$ and $x = 7$.

SC 2

1 A: $x = \frac{5}{7} = 0.7$ (1 d.p.) or $x = -\frac{13}{7} = -1.9$ (1 d.p.)

B: $x = -2.2$ (1 d.p.) or $x = -3.5$ (1 d.p.)

2 **(a)** The x-intercepts will occur between the pairs of x-values where the sign of the y-values change sign.

(b) Between -6 and -4, but much closer to -4 and between 0 and 2 with the value being about midway between the values stated.

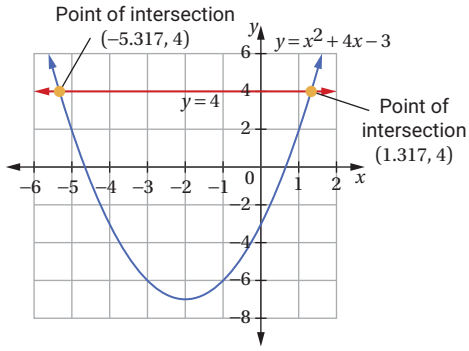
(c) -4.2 and 1.2

(d) The quadratic has a minimum turning point.

3 **(a)** -5.4 and 1.3

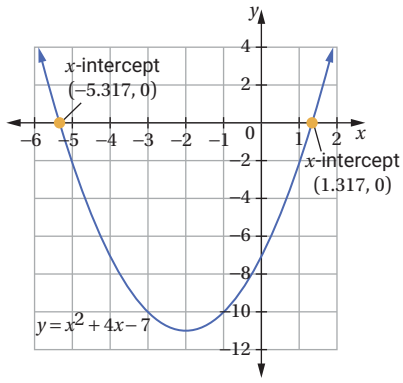
(b) $x < -5.4$ or $x > 1.3$

(c)



$x = -5.317$ and $x = 1.317$

(d)



You need to look for the x-intercepts: $x < -5.317$ or $x > 1.317$.

(e) Yes, the results are the same.

4 **(a)** $x = -\frac{13}{4}$ **(b)** $x = \frac{11}{5}$

(c) $x = 1.45$ **(d)** $x = -1.118$

5.6 Model and solve practical problems with quadratic equations

SC 1

1 **(a)**

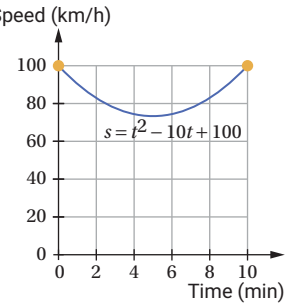
x	0	2	4	6	8	10
s	100	84	76	76	84	100

(b) A minimum turning point

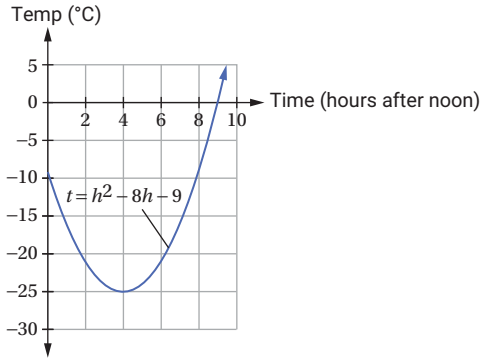
(c) (5, 75)

(d) There are none.

(e)



2 **(a)** -9 °C **(b)** $t = (h - 9)(h + 1)$



(c) -25 °C at 4 pm **(d)** Most likely at 4 pm.

(e) 9 pm

3 **(a)** 20 and 6

(b) In this case, the answer could also be -20 and -6.

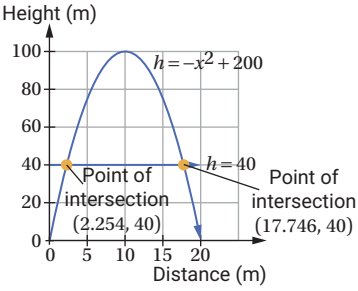
4 **(a)** $h = -x(x - 20)$

(b) $x = 0$ or $x = 20$

These are the locations where the arrow is at ground level.

(c) The maximum height was 100 m at a distance of 10 m from the point of fire.

(d) Height (m)



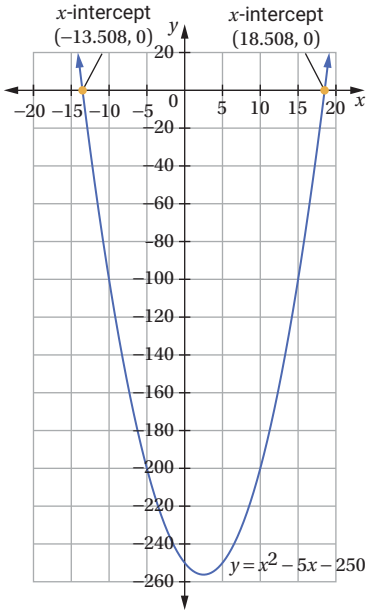
$2.254 < x < 17.746$ metres from the point of fire.

5 **(a)** $a^2 - b^2 = 25$ **(b)** $a = 6, b = 3$
(c) $a = -3, b = -6$

SC 2

1 **(a)** $\frac{1}{2} \times x \times (x+3) = 90$ or $x^2 + 3x - 180 = 0$
(b) $x = -15$ or $x = 12$
(c) $x = 12$
(d) The base is 12 cm and the height is 15 cm.

2 **(a)** $x^2 - 5x - 250 = 0$
(b)



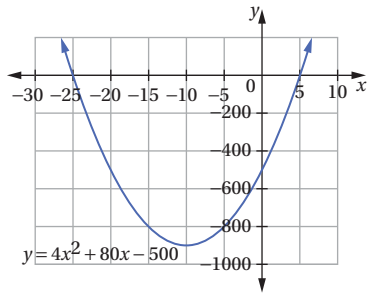
(c) $x = -13.5$ or $x = 18.5$
(d) Only the positive solution is reasonable in this practical situation.
(e) If $x = 18.5$, then $x - 5 = 13.5$. $18.5 \times 13.5 = 249.75$, which is approximately 250 cm^2.

3 **(a)** $x(x + 3) = 4x + 6$ or $x^2 - x + 6 = 0$
(b) $x = -2$ or $x = 3$
(c) $x = 3$
(d) The length is 6 cm and the width is 3 cm.

4 **(a)** There are no solutions possible.
(b) You should tell Estelle to check her work because the graph she drew produces no solutions.

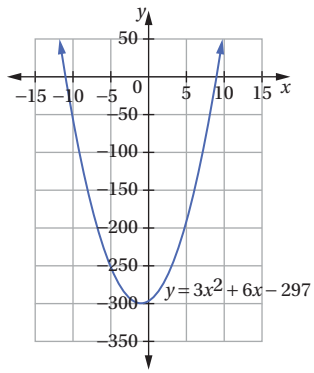
SC 3

1 **(a)** $(2x + 25)(2x + 15) = (4x^2 + 80x + 375)$ m^2
(b) 375 m^2
(c) $4x^2 + 80x$
(d) $4x^2 + 80x = 500$ or $4x^2 + 80x - 500 = 0$
(e)



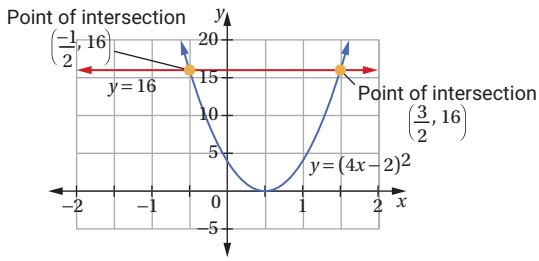
(f) The paved area is 5 m wide.

2 **(a)** $x, x + 1, x + 2$
(b) $x^2 + (x + 1)^2 + (x + 2)^2 = 302$. This can be simplified to give $3x^2 + 6x - 297 = 0$.
(c)



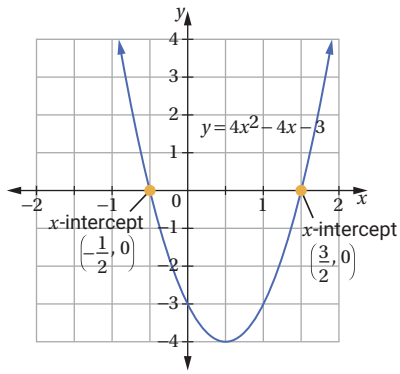
The graph gives $x = 9$ as the positive solution, so the numbers are 9, 10 and 11.
(d) If integers are allowed, then the negative solution, $x = -11$, comes into play. This gives the three integers as −11, −10 and −9.

3 **(a)**



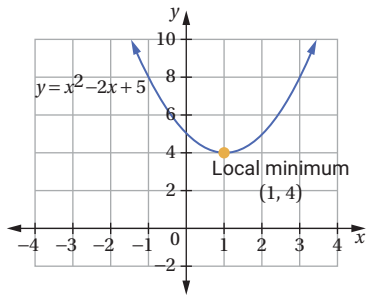
(b) The x-value of the points of intersection give the solutions. (These are marked already on the graph in **(a)**.) The solutions are $x = -\frac{1}{2}$ or $x = \frac{3}{2}$.
(c) $(4x - 2)^2 = 16$ is equivalent to $4x^2 - 4x - 3 = 0$, so the graph to be plotted is $y = 4x^2 - 4x - 3$.
(d) The x-intercepts will give the solutions.

(e)



As can be seen on the graph, the solutions are the same.

4 (a)



(b) The equation has zero x-intercepts.
(c) (1, 4)
(d) 4 units vertically down
(e) $x = 1$
(f) $y = x^2 - 2x + 1$

Topic 6: Surface area and volume

Recall

I can multiply and divide decimals by powers of 10

1 (a) 814 **(b)** 81 643 **(c)** 934 **(d)** 0.74
2 (a) 6.1732 **(b)** 0.061 54 **(c)** 0.6935 **(d)** 0.000 24

I can convert between units of length

1 (a) 5 km **(b)** 20.4 mm
(c) 0.65 m **(d)** 9.05 cm
2 (a) 0.06 km **(b)** 0.004 m
(c) 57 mm **(d)** 25 000 cm

I can find the unknown length of a plane shape

1 (a) $AB = 3.4$ cm **(b)** $AB = 9$ m; $BC = 8$ m
(c) $KL = 7$ m; $LM = 9$ m

I can calculate the area of plane shapes

1 (a) 260 mm^2 **(b)** 15 cm^2 **(c)** 63 m^2

6.1 Determine the volume of right prisms

SC 1

1 (a) 120 cm^3 **(b)** 135 m^3 **(c)** 231.77 m^3
2 (a) 8 cm^3 **(b)** 125 m^3 **(c)** 274.625 mm^3
3 (a) 48 mm^3 **(b)** 120 cm^3 **(c)** 251.55 cm^3
4 462.5 m^3
5 19.5 m^3

6.2 Solve problems involving the volumes and capacities of right prisms and right pyramids

SC 1

1 (a) 10 cm^3 **(b)** 35 cm^3 **(c)** 40 m^3
(d) 100.27 mm^3 **(e)** 14.72 m^3
2 (a) 60 cm^3 **(b)** 48 m^3 **(c)** 58.33 mm^3
(d) 1.65 m^3 **(e)** 0.33 m^3 or 329 280 cm^3
3 (a) 56 cm^3 **(b)** 35 cm^3 **(c)** 8.33 mm^3
(d) 7.83 m^3 **(e)** 0.46 m^3 or 456 000 cm^3
4 61 600 mm^3 or 61.6 cm^3
5 (a) 3375 cm^3
(b) Right-angled triangle-based pyramid with the base and height of the triangle both 15 cm and height of the pyramid also 15 cm.
(c) 562.5 cm^3
(d) $\frac{1}{6}$
(e) Right-angled triangle-based prism such as $ABDEFH$.
6 (a) 20.78 cm **(b)** 249.36 cm^2 **(c)** 1163.68 cm^3

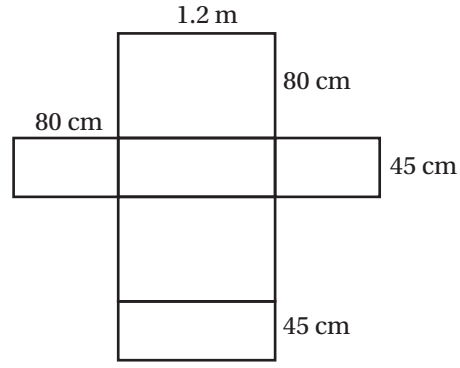
SC 2

1 2704 mL, 2.704 L
2 510 m^3
3 (a) 25 cm or 0.25 m **(b)** 2.5 m
4 11.9 cm
5 40 cm

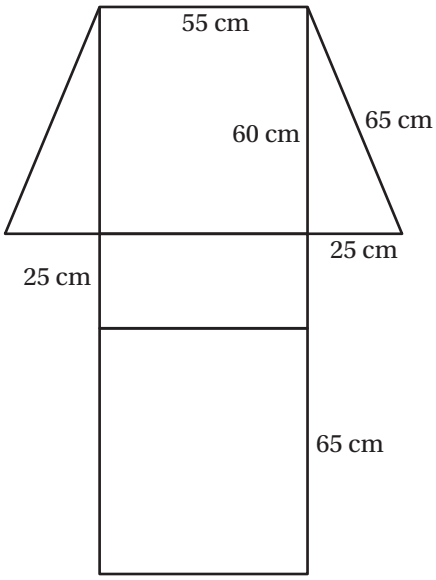
6.3 Solve problems involving the surface areas and nets of right prisms

SC 1

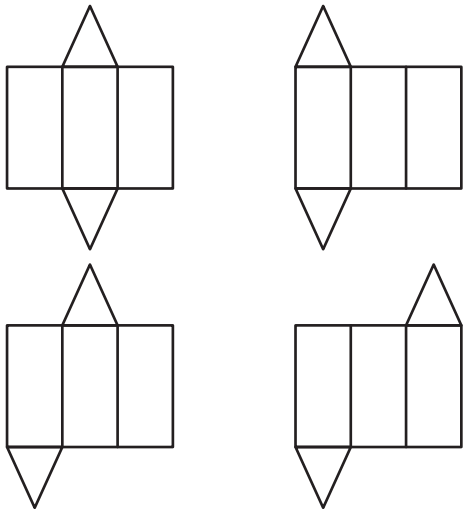
1 A possible net:



2 A possible net:



3 **(a)** Four possible nets:



(b) Because the side lengths of the triangle are equal, the placement of the triangles will match one side of the rectangles in any of three positions on each side of the strip.

4 **(a)** 15 cm **(b)** 48 cm
(c) 8 cm, 12 cm, 12 cm, 2 cm, 4 cm, 10 cm
(d) The 8 cm by 15 cm rectangle is attached to the opposite side of the same rectangle (12 cm, 12 cm, 2 cm, 4 cm, 10 cm) by 15 cm.

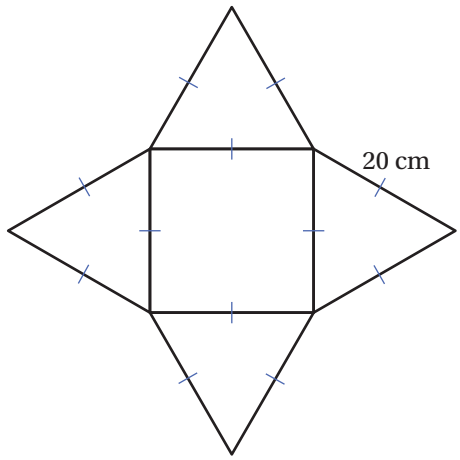
5 **(a)** 30 cm by 15 cm **(b)** 75 cm by 6 cm
(c) The pentagons must be placed one at each side (the side which is not the height of the prism) of the strip with no other restrictions (except that they be attached to an end of a rectangle).

SC 2

1 1200 cm^2
2 6480 mm^2 or 64.8 cm^2
3 6400 cm^2
4 **(a)** **(i)** 200 cm^2 **(ii)** 1800 cm^2
(iii) 2200 cm^2
(b) **(i)** 300 cm^2 **(ii)** 1600 cm^2
(iii) 2200 cm^2
(c) **(i)** 600 cm^2 **(ii)** 1000 cm^2
(iii) 2200 cm^2
5 **(a)** 10 cm **(b)** 8.66 cm
(c) 259.8 cm^2 **(d)** 1119.6 cm^2

SC 3

1 **(a)** 1.41 m **(b)** 21.25 m^2
2 **(a)** 72 cm by 57 cm **(b)** Square, side length 16 cm
(c) 3080 cm^2
3 **(a)** 216 cm^2 **(b)** 24 cm^2 **(c)** 3
4 **(a)**



(b) 17.32 cm **(c)** 1092.8 cm^2
5 **(a)** 8 **(b)** 12 **(c)** 6 **(d)** 1

Topic 7: Circles and cylinders

Recall

I can round decimals for different purposes

1 **(a)** 8.4 **(b)** 84 **(c)** 8.599 **(d)** 1.00

I can multiply decimals

1 **(a)** 8.75 **(b)** 4.06
(c) 26.5625 **(d)** 0.238 09

I can convert between units of length

1 **(a)** 7 m **(b)** 53.1 mm **(c)** 18.5 cm
(d) 3.575 m **(e)** 9.4 cm **(f)** 100 cm

I can calculate the area of squares and rectangles

1 **(a)** 64 cm^2 **(b)** 87 m^2

I can calculate the circumference and area of circles to one decimal place

1 **(a)** 62.8 cm **(b)** 37.7 mm
2 **(a)** 50.3 cm^2 **(b)** 28.3 m^2

I can calculate the volume of prisms

1 **(a)** 37.5 m^3 **(b)** 720 cm^3 **(c)** 216 mm^3

7.1 Calculate circumference and area in terms of pi

SC 1

1 **(a)** 6π m **(b)** 7π m
2 **(a)** 7π cm **(b)** 7.5π cm
3 **(a)** $(5\pi + 10)$ cm **(b)** $(5\pi + 36)$ cm
(c) $(10\pi + 20)$ cm
4 **(a)** $(0.65\pi + 2.6)$ m **(b)** $(0.65\pi + 5.2)$ m
(c) $(1.95\pi + 6.5)$ m
5 **(a)** 3.6π m **(b)** $(5\pi + 52)$ cm
6 **(a)** $\pi : 4$ **(b)** $(\pi + 2) : 6$

SC 2

1 **(a)** 9π m^2 **(b)** 12.25π m^2
2 **(a)** 12.25π cm^2 **(b)** 14.0625π cm^2
3 **(a)** 25π cm^2 **(b)** 1.69π m^2
(c) 1764π mm^2 **(d)** 72.25π cm^2
4 **(a)** 12.5π cm^2 **(b)** $(12.5\pi + 130)$ cm^2
(c) $(25\pi + 100)$ cm^2 **(d)** $(325\pi + 600)$ cm^2
5 **(a)** 0.4225π m^2 **(b)** 1.2675π m^2
(c) $(0.4225\pi + 1.69)$ m^2
(d) $(1.2675\pi + 6.76)$ m^2
6 **(a)** 1.08π m^2 **(b)** $(32\pi + 240)$ cm^2
(c) $\left(198 - \frac{25\pi}{2}\right)$ cm^2
7 **(a)** $\pi : 4$ **(b)** $\pi : 4$

7.2 Determine the radius of a circle from area or circumference

SC 1

1 **(a)** 5 mm **(b)** 7.5 mm **(c)** 10 mm **(d)** 12.5 mm
2 **(a)** $\frac{5}{\pi}$ mm **(b)** $\frac{15}{2\pi}$ mm
(c) $\frac{10}{\pi}$ mm **(d)** $\frac{25}{2\pi}$ mm
3 The length of the diameter is twice the length of the radius. The radius is half the length of the diameter.
4 **(a)** $d = \frac{140}{\pi}$ mm, $r = \frac{70}{\pi}$ mm
(b) $d = \frac{18}{\pi}$ cm, $r = \frac{9}{\pi}$ cm
(c) $d = \frac{1.35}{\pi}$ m or $\frac{27}{20\pi}$ m, $r = \frac{0.675}{\pi}$ m or $\frac{27}{40\pi}$ m
(d) $d = \frac{26.8}{\pi}$ cm or $\frac{134}{5\pi}$ cm, $r = \frac{13.4}{\pi}$ cm or $\frac{67}{5\pi}$ cm
5 **(a)** $r = \frac{110}{\pi + 4}$ cm **(b)** $r = \frac{28}{\pi + 2}$ cm

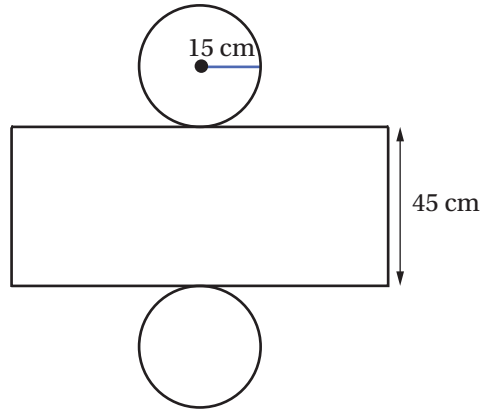
SC 2

1 **(a)** 2 mm **(b)** 3 mm **(c)** 4 mm **(d)** 5 mm
2 **(a)** $\sqrt{\frac{10}{\pi}}$ mm **(b)** $\sqrt{\frac{15}{\pi}}$ mm
(c) $\sqrt{\frac{20}{\pi}}$ mm **(d)** $\frac{5}{\sqrt{\pi}}$ mm
3 The length of the diameter is twice the length of the radius. The radius is half the length of the diameter.
4 **(a)** $r = 7$ mm, $d = 14$ mm
(b) $r = 9$ cm, $d = 18$ cm
(c) $r = \sqrt{10}$ mm, $d = 2\sqrt{10}$ mm
(d) $r = \sqrt{\frac{20}{\pi}}$ m, $d = 2\sqrt{\frac{20}{\pi}}$ m
(e) $r = \sqrt{\frac{26.8}{\pi}}$ cm, $d = 2\sqrt{\frac{26.8}{\pi}}$ cm
5 **(a)** $\frac{45}{2\pi}$ cm, 7.16 cm **(b)** $\sqrt{\frac{350}{\pi}}$ cm, 10.56 cm
(c) $\frac{20}{\pi}$ m, 6.37 m **(d)** $\sqrt{\frac{62}{\pi}}$ m, 4.44 m
6 **(a)** $r = \frac{10}{\sqrt{\pi}}$ cm **(b)** $r = \sqrt{\frac{15}{\pi + 2}}$ m

7.3 Determine the surface area of cylinders

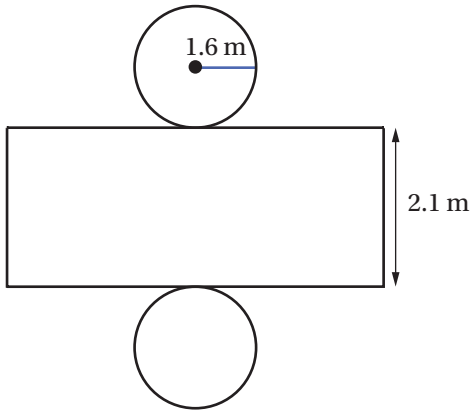
SC 1

1 **(a)**



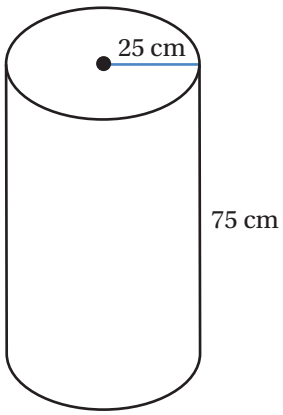
(b) $l = 30\pi$ cm, $w = 45$ cm

2 **(a)**



(b) $l = 3.2\pi$ m, $w = 2.1$ m

3



4 **(a)** $h = 20$ cm **(b)** $r = \frac{30}{\pi}$ cm

5 **(a)** $h = 80$ mm **(b)** $d = \frac{80}{\pi}$ mm

SC 2

1 480π cm^2

2 **(a)** 92.16π cm^2 **(b)** 19.2π cm^2
(c) 501.12π cm^2 **(d)** 685.44π cm^2

3 **(a)** 0.12π m^2 **(b)** 12π m^2 **(c)** 1200π m^2

4 **(a)** 80π m^2 **(b)** 20π m^2 **(c)** 5π m^2

5 **(a)** 34 cm **(b)** 58 cm

6 **(a)** The bottom circle is not required to be iced.
(b) 1276.27 cm^2

7.4 Solve problems involving the surface area of cylinders

SC 1

1 19.5 cm

2 Possibilities: 37.7 cm by 16 cm gives $A = 603.2$ cm^2; 43.7 cm by 12 cm gives $A = 524.4$ cm^2; 49.7 cm by 10 cm gives $A = 497$ cm^2. Hence 49.7 cm by 10 cm is the best option.

3 **(a)** $S = \pi r^2 + 2\pi rh$ **(b)** 3534.29 cm^2

4 $(238.5\pi + 396)$ cm^2

5 $\frac{150}{21.42} \to 7, \frac{100}{7} \to 14; 7 \times 14 = 98$
$\frac{100}{21.42} \to 4, \frac{150}{7} \to 21; 4 \times 21 = 84$
Hence 98 labels will fit.

6 $(256\pi + 512)$ cm^2

7.5 Volume and capacity of cylinders

SC 1

1 4222.30 cm^3

2 7.63 cm^3

3 2651 cm^3

4 **(a)** 16 cm by 16 cm **(b)** 6144 cm^3
(c) 4825 cm^3 **(d)** 21.5%

5 2.84 m^3

SC 2

1 3.18 kL

2 **(a)** 18.75 cm **(b)** 10.95 cm

3 10.2 cm

4 **(a)** 226 cm^3 **(b)** 402 mL

7.6 Understand the relationship between the volume and dimensions of cylinders

SC 1

1 **(a)** 6.272π m^3 **(b)** 12.544π m^3
(c) $\frac{\text{New volume}}{\text{Original volume}} = 2$

2 **(a)** 5000π cm^3 **(b)** $20\,000\pi$ cm^3
(c) $\frac{\text{New volume}}{\text{Original volume}} = 4$

3 **(a)** $117\,450\pi$ mm^3 **(b)** $234\,900\pi$ mm^3
(c) $\frac{\text{New volume}}{\text{Original volume}} = 2$

4 **(a)** 0.25π m^3 **(b)** π cm^3
(c) $\frac{\text{New volume}}{\text{Original volume}} = 4$

5 **(a)** The volume is increased by a factor of 10.
(b) The volume is increased by a factor of 100.
(c) The volume is increased by a factor of 36.
(d) The volume is increased by 40%.
(e) $1.4^2 = 1.96$, so the volume is increased by 96%.

Topic 8: Measurement and scientific notation

Recall

I can write large powers of 10 using exponent notation

1 **(a)** 10^2 **(b)** 10^3 **(c)** 10^6 **(d)** 10^9
2 **(a)** 100 **(b)** 10 000
(c) 100 000 **(d)** 10 000 000

I can multiply and divide decimals by powers of 10

1 **(a)** 48.5 **(b)** 2476.8 **(c)** 79 600
(d) 0.349 **(e)** 800.3 **(f)** 104

I can order decimal values

1 0.39, 2.98, 3.02, 3.14, 3.36, 3.43, 3.76
2 0.701, 0.684, 0.667, 0.643, 0.633, 0.611, 0.585

I can multiply decimals and round the results

1 **(a)** 6.38 **(b)** 608.69 **(c)** 3.93 **(d)** 0.95
2 **(a)** 5.728 **(b)** 0.368 **(c)** 0.138 **(d)** 0.017

I can estimate the product of decimals

1 **(a)** $\approx 1 \times 3 = 3$ **(b)** $\approx 3 \times 6 = 18$
(c) $\approx 6 \times 7 = 42$ **(d)** $\approx 12 \times 5 = 60$

8.1 Identify and describe very small and very large measurements

SC 1

1 **(a)** milli **(b)** kilo **(c)** centi **(d)** mega
2 **(a)** 1000 or 10^3 **(b)** 1 000 000 or 10^6
3 **(a)** 0.01 or 10^{-2} **(b)** 0.001 or 10^{-3}
4 **(a)** 1 kg **(b)** 1 MB
5 **(a)** 1 cm **(b)** 1 ms
6 **(a)** 10^{-3} **(b)** 10^{-2} **(c)** 10^3 **(d)** 10^6
7 **(a)** kilo **(b)** milli **(c)** centi
8 1 mm; 1 cm; 0.1 m; 10 m; 10^2 m; 1 km; 10 000 m; 1 ML
9 $1\text{ cm} = \frac{1}{100}$ m, therefore, there are 100 cm in every 1 m
$1\text{ m} = \frac{1}{1000}$ km, therefore, there are 1000 m in every 1 km
$100\text{ cm} \times 1000\text{ m} = 1\text{ km}$ (in cm)
$= 100\,000$ cm
$10\text{ km} = 10 \times 100\,000$ cm
$= 1\,000\,000$ cm

SC 2

1 **(a)** 3 place values higher **(b)** 6 place values higher
(c) 3 place values lower **(d)** 6 place values lower
2 **(a)** 9 place values higher **(b)** 12 place values higher
3 **(a)** 9 place values lower **(b)** 12 place values lower
4 **(a)** 1 000 000 or 10^6 **(b)** 0.000 001 or 10^{-6}
(c) 1 000 000 000 or 10^9 **(d)** 0.000 000 001 or 10^{-9}
5 **(a)** 2 000 000 bytes **(b)** 0.000 005 seconds
(c) 4 500 000 000 litres **(d)** 0.000 000 032 metres
6 **(a)** kilo (10^3) **(b)** mega (10^6)
(c) giga (10^9) **(d)** tera (10^{12})
(e) milli (10^{-3}) **(f)** micro (10^{-6})
7 **(a)** kilo (25 km) **(b)** kilo (480 kg)
(c) mega (128 MB)
8 **(a)** milli (20 mL) **(b)** milli (200 mm)
(c) micro (20 ms)

8.2 Express numbers in scientific notation and decimal form

SC 1

1 **(a)** 6.0×10^4 **(b)** 6.0×10^5
(c) 6.0×10^6 **(d)** 6.0×10^7
2 **(a)** 4.0×10^{-4} **(b)** 4.0×10^{-5}
(c) 4.0×10^{-6} **(d)** 4.0×10^{-7}
3 **(a)** 7.5×10^5 **(b)** 1.95×10^7 **(c)** 3.071×10^8
4 **(a)** 5.5×10^{-4} **(b)** 8.32×10^{-5} **(c)** 4.509×10^{-6}
5 **(a)** 2.0×10^4 metres **(b)** 5.0×10^{10} bytes
6 **(a)** 2.0×10^{-2} litres **(b)** 5.0×10^{-8} metres
7 **(a)** 2.05×10^9 litres **(b)** 8.903×10^{12} seconds
(c) 5.0×10^{10} metres
8 **(a)** 9.55×10 grams **(b)** 1.005×10^{-9} seconds
(c) 5.0×10^{-11} metres

SC 2

1 **(a)** 3 **(b)** 6 **(c)** 9
2 **(a)** 5000 **(b)** 5 000 000
(c) 5 000 000 000
3 **(a)** 2 **(b)** 5 **(c)** 7
4 **(a)** 0.04 **(b)** 0.000 04 **(c)** 0.000 000 4
5 **(a)** 340 000 **(b)** 60 500 000
(c) 1 105 200 000
6 **(a)** 0.0045 **(b)** 0.000 089 1
(c) 0.000 000 002 905

8.3 Order and perform calculations using numbers in scientific notation

SC 1

1 **(a)** > **(b)** < **(c)** < **(d)** >
2 **(a)** < **(b)** < **(c)** > **(d)** >
3 **(a)** < **(b)** > **(c)** > **(d)** <
4 **(a)** < **(b)** > **(c)** > **(d)** >
5 **(a)** > **(b)** > **(c)** < **(d)** <

6 Venus 3.98×10^7 km
Mars 5.57×10^7 km
Mercury 8.25×10^7 km
Jupiter 5.91×10^8 km
Saturn 1.20×10^9 km

7 Io: -1.36×10^2 °C
Callisto: -1.39×10^2 °C
Europa: -1.43×10^2 °C
Titan: -1.79×10^2 °C
Miranda: -2.04×10^2 °C

8 Rhino virus: 3.0×10^{-8} m
Polio virus: 5.0×10^{-8} m
Influenza: 1.0×10^{-7} m
Rabies: 1.5×10^{-7}
Lactobacillus: 2.0×10^{-6} m
Red blood cell: 8.0×10^{-6} m
Pollen: 9.0×10^{-5} m
Paramecium: 2.5×10^{-4} m

SC 2

1 **(a)** 6.0×10^5 **(b)** 8.0×10^5 **(c)** 9.0×10^7
2 **(a)** 3.0×10^2 **(b)** 4.0×10^2 **(c)** 3.0×10^2
3 **(a)** 3.5×10^{10} **(b)** 8.75×10^7 **(c)** 5.76×10^6
4 **(a)** 3.0×10^4 **(b)** 6.0×10^2 **(c)** 4.2×10^3
5 **(a)** 1.5×10^9 **(b)** 1.8×10^{11} **(c)** 5.278×10^9
(d) 2.0×10^6 **(e)** 6.875×10^4 **(f)** 2.5×10^{-4}
6 **(a)** 4.5×10^5 **(b)** 3.36×10^7
(c) 4.1×10^4 **(d)** 3.0×10^2
7 **(a)** It takes the flea 8 leaps.
(b) After 15 leaps, the flea has travelled 3.75×10^3 mm.
8 **(a)** The plane flew a total of 3.072×10^8 m.
(b) The plane flew a further 12 flights to travel a total of 4.608×10^8 m.

8.4 Estimate and round numbers to a specified degree of accuracy

SC 1

1 **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4
2 **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4
3 **(a)** 4 **(b)** 5 **(c)** 6 **(d)** 7
4 **(a)** 2 **(b)** 2 **(c)** 2 **(d)** 2
5 **(a)** 5 **(b)** 7 **(c)** 9 **(d)** 10
6 **(a)** 1230 **(b)** 12 300
(c) 123 000 **(d)** 1 230 000
7 **(a)** 1200 **(b)** 23 000
(c) 350 000 **(d)** 4 600 000
8 **(a)** 5.1234 **(b)** 5.0123 **(c)** 5.0012 **(d)** 5.0001
9 **(a)** 10.20 **(b)** 20.30 **(c)** 30.41 **(d)** 40.51
10 **(a)** 3 **(b)** 4 **(c)** 4 **(d)** 5
11 **(a)** 1.2×10^3 or 1200 **(b)** 1.0×10^3 or 1000
(c) 3.5×10^3 or 3500 **(d)** 3.0×10^3 or 3000

SC 2

1 **(a)** 10.12 kg **(b)** 10.23 kg
(c) 10.35 kg **(d)** 10.46 kg
2 **(a)** 10 000 **(b)** 20 000
(c) 30 000 **(d)** 50 000
3 **(a)** 10.12 kg **(b)** 10.23 kg
(c) 10.34 kg **(d)** 10.45 kg
4 **(a)** 120 000 **(b)** 230 000
(c) 340 000 **(d)** 450 000
5 **(a)** (15.464 + 17.455 + 20.887 + 14.478 + 16.959) min
= 85.243 min
(b) (15 + 17 + 21 + 14 + 17) min = 84 min
(c) (15 + 17 + 20 + 14 + 16) min = 82 min
(d) Sample answer:
Both rounding and truncating methods led to the total running time appearing to be shorter than the actual time.
Rounding each lap time to the nearest minute meant that almost half a minute for some laps went uncounted, which led to the runner appearing to run the five laps almost $1\frac{1}{4}$ minutes faster than the actual time.
The truncated lap values ignored all the fractions of a minute, and made the runner appear to complete the five laps over 3 minutes faster than the actual time.
6 **(a)** Enough paint to cover exactly:
$\frac{1}{2} \times 20 \text{ m} \times 16 \text{ m} = 160 \text{ m}^2$
(b) Paint needs to cover:
$\frac{1}{2} \times 20.49 \text{ m} \times 16.40 \text{ m} = 168.018 \text{ m}^2$
$168.018 \text{ m}^2 > 160 \text{ m}^2$, therefore no, the painter does not have enough paint.
(c) Sample answer:
Communicating the rounded dimensions to the nearest metre before calculating the area, the painter ordered less paint than was needed, and will be required to get enough paint to cover approximately 8 m^2 more.
(d) If the actual dimensions were given to the painter, if the painter rounded the area of coverage after the calculation, the paint ordered could have possibly been made to stretch just a little bit further to cover the extra 0.018 m^2.

Topic 9: Percentage error in measurement

Recall

I can read measuring devices

1 **(a)** 43°C **(b)** −7°C
2 **(a)** 5.5 cm

I can identify the magnitude of calculations

1 **(a)** 2 **(b)** 13 **(c)** 6 **(d)** 1

I can write fractions with a power of 10 as a denominator in decimal form

1 **(a)** 0.8 **(b)** 2.3 **(c)** 0.171
(d) 0.007 **(e)** 0.0091

I can convert between units of length

1 **(a)** 25 m **(b)** 0.075 45 m **(c)** 23 150 cm

I can round decimals for different purposes

1 **(a)** 5.6 cm **(b)** 3.84 km **(c)** 5.80 m
(d) 25.3°C

I can calculate the percentage of amounts

1 **(a)** 5% **(b)** 10% **(c)** 75%

9.1 Determine the precision and absolute error of measurements

SC 1

1 **(a)** 1 cm **(b)** 1°C **(c)** 0.1 mm **(d)** 0.01 kg
2 **(a)** 0.3 cm **(b)** 0.03 g **(c)** 3.5 km
3 **(a)** 1 cm **(b)** 1 cm **(c)** 1°
4 **(a)** 0.1°C **(b)** 5° **(c)** 0.2 kg
5 **(a)** 1 kg **(b)** 5 kg **(c)** 10 kg **(d)** 0.1 kg
6 **(a)** 0.5 cm **(b)** 1 cm **(c)** 2.5 cm **(d)** 0.05 cm
7 **(a)** 0.05°C **(b)** 2.5° **(c)** 0.1 kg

9.2 Estimate the degree of accuracy

SC 1

1 **(a)** ± 0.5 mm **(b)** ± 0.5 m
(c) ± 0.5°C **(d)** ± 0.5 g
2 **(a)** ± 2.5 cm **(b)** ± 0.25 m
(c) ± 0.05°C **(d)** ± 1°C
3 **(a)** 36.6°C ± 0.05°C **(b)** 125° ± 0.5°
(c) 9 kg ± 0.1 kg **(d)** 3.35 g ± 0.005 g
4 **(a)** 9.5 mm and 10.5 mm **(b)** 14.5 m and 15.5 m
(c) 19.5°C and 20.5°C **(d)** 195 g and 205 g
5 **(a)** 92.5 cm and 97.5 cm **(b)** 7.75 m and 8.25 m
(c) 21.95°C and 22.05°C
(d) 249.95 km and 250.05 km
6 **(a)** 36.55°C and 36.65°C **(b)** 124.5°C and 125.5°C
(c) 8.9 kg and 9.1 kg **(d)** 3.345 g and 3.355 g
7 **(a)** −11°C and −9°C **(b)** −24.5 m and −23.5 m
(c) −15.05°C and −14.95°C
(d) −2.55 s and −2.45 s

9.3 Determine the percentage error in measurements and calculations

SC 1

1 **(a)** 0.05 **(b)** 0.025 **(c)** 0.0125
2 **(a)** 5% **(b)** 2.5% **(c)** 1.25%
3 **(a)** 0.05 **(b)** 0.025 **(c)** 0.0125
4 **(a)** 5% **(b)** 2.5% **(c)** 1.25%
5 **(a)** 0.1 **(b)** 0.02 **(c)** 0.025 **(d)** 0.005
6 **(a)** 2.5% **(b)** 0.25%
7 Associated percentage error is: 0.029% accurate to 3 decimal places.
8 Associated percentage error is: 1.6%.

SC 2

1 **(a)** 10% **(b)** 10% **(c)** 5%
2 **(a)** 0.048 **(b)** 4.8%
3 **(a)** 0.04 **(b)** 4%
4 **(a)** 20% **(b)** 2.5% **(c)** 7.5% **(d)** 4%
5 Percentage error is 16.6% or 16.7% rounded to 1 decimal place.
The error is a positive value, indicating that more friends than the actual planned amount attended. This may have meant not enough party supplies were provided.
6 Percentage error is −20%. The negative percentage error indicates that the timed value of 4 solved puzzle cubes was below the actual goal of 5 solved puzzle cubes.
7 Percentage error is −2%.
The value of −1% is less than 2% away from the actual value being measured. The negative value shows less product was poured in the bag than the required amount, however the error is within the required limits for quality control purposes.
8 The percentage error is 1.7%. This is larger than the allowable maximum of 1%. Therefore, the thermometer failed the test.

Topic 10: Pythagoras and geometry

Recall

I can round decimals for different purposes

1 **(a)** 2.02 **(b)** 0.44 **(c)** 1500.16
(d) 345.77 **(e)** 20.00

I can perform calculations involving squaring numbers

1 **(a)** 12.67 **(b)** 83.36 **(c)** 233.17
(d) 0.20 **(e)** 0.00
2 **(a)** 56.85 **(b)** 3665.23 **(c)** 9216.97
(d) 1334.73 **(e)** 27 545.89

I can use a calculator to calculate square roots

1 **(a)** 14 **(b)** 35 **(c)** 0.03 **(d)** 0.688
2 **(a)** 4.12 **(b)** 7.07 **(c)** 13.04 **(d)** 2.29

I can use Pythagoras' theorem to find the length of unknown sides in right-angled triangles

1 **(a)** 10.30 cm **(b)** 13.89 cm
2 **(a)** 13.60 m **(b)** 11.62 m

10.1 Apply Pythagoras' theorem to solve problems

SC 1

1 3.14 m
2 25.4 m
3 15.820 km
4 **(a)** 2.72 m **(b)** 10.29 m
(c) **(i)** 16.02 m **(ii)** 10.06 m
5 **(a)** Using total elevation change of 5.07 m: 3.57 m
(b) Using $\frac{5.07}{18} = \frac{1.5}{y}$: 5.33 m

SC 2

1 **(a)** 8.62 m **(b)** 2.76 m
2 **(a)** 3.32 m **(b)** 12.45 m
3 **(a)** 4.7 cm
(b) $2.2 \times 20 + 0.3 \times 19 + 0.5 \times 2 = 50.7$ cm by
$4.7 \times 100 + 0.3 \times 99 + 0.5 \times 2 = 500.7$ cm
or about half a metre by 5 m
4 **(a)** 5.2 m by 2.5 m and 5.2 m by 3.2 m
(b) 3.57 m **(c)** 5.2 m by 3.57 m
5 **(a)** 3.38 m **(b)** 87 cm

10.2 Calculate the distance between two points on a Cartesian plane

SC 1

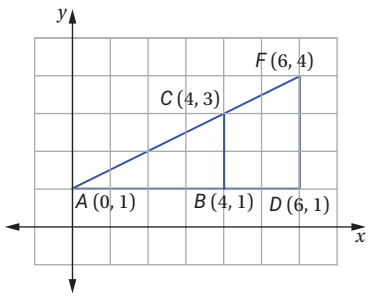
1 **(a)** Horizontal: 6 units; vertical: 11 units
(b) Horizontal: 9 units; vertical: 1 unit
(c) Horizontal: 24 units; vertical: 6 units
(d) Horizontal: 19 units; vertical: 7 units
2 **(a)** Horizontal: 10 units; vertical: 14 units
(b) Horizontal: 36 units; vertical: 10 units
(c) Horizontal: 24 units; vertical: 20 units
(d) Horizontal: 10 units; vertical: 35 units
3 **(a)** (7, 12), (1, −2), (1, 12), (7, −2)
(b) (1, 10), (−5, −4), (1, −4), (−5, 10)
(c) (−3, 6), (−9, −8), (−3, −8), (−9, 6)
(d) (10, 1), (4, −13), (10, −13), (4, 1)
4 **(a)** 2 units **(b)** 7 units **(c)** 5 units **(d)** 6 units
5 **(a)** 6 units
(b) $C(4, 11)$, $D(-2, 11)$; $C(4, -5)$, $D(-2, -5)$

SC 2

1 **(a)** 5 units **(b)** 13 units
(c) $\sqrt{173}$ units **(d)** $\sqrt{245}$ units or $7\sqrt{5}$ units
(e) $\sqrt{265}$ units **(f)** $\sqrt{72}$ units or $6\sqrt{2}$ units
2 **(a)** 10 units **(b)** 26 units
(c) $\sqrt{116}$ units or $2\sqrt{29}$ units
(d) $\sqrt{137}$ units **(e)** $\sqrt{29}$ units
(f) $\sqrt{488}$ units or $2\sqrt{122}$ units
3 **(a)** 5 units **(b)** 13 units
(c) $\sqrt{45}$ units or $3\sqrt{5}$ units
(d) $\sqrt{50}$ units or $5\sqrt{2}$ units
4 **(a)** $D(3, -1)$
(b) $AB = DC = \sqrt{13}$ units, $BC = AD = 5$ units
(c) $AC = \sqrt{50}$ units or $5\sqrt{2}$ units, $BD = \sqrt{26}$ units
5 $C(5, 2)$, $D(3, 2)$; $C(5, -6)$, $D(3, -6)$

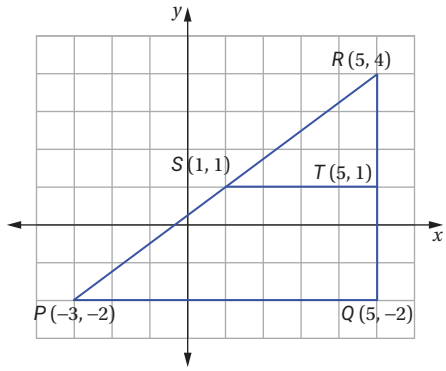
SC 3

1 **(a)–(c)**



(d) $AC = \sqrt{4^2 + 2^2}$
$= \sqrt{20}$ units
$CF = \sqrt{2^2 + 1^2}$
$= \sqrt{5}$ units
$\frac{AC}{CF} = \frac{\sqrt{20}}{\sqrt{5}}$
$= \sqrt{\frac{20}{5}}$
$= \sqrt{4}$
$= 2$ units
Hence, $AC = 2CF$

2 (a), (b)



(c) $PS = \sqrt{4^2 + 3^2}$
$= 5$ units
$SR = \sqrt{4^2 + 3^2}$
$= 5$ units
Hence, $PS = SR$

3 (a) 4 **(b)** $\frac{3}{2}$ **(c)** $\frac{4}{3}$

4 ΔEBL

5 ΔMFG: $FG = 3$ units, $MF = 9$ units
$MG = \sqrt{3^2 + 9^2}$
$= \sqrt{90}$ units
ΔMAE: $AE = 4$ units, $MA = 12$ units
$ME = \sqrt{4^2 + 12^2}$
$= \sqrt{160}$ units
Large to small ratios:
$\frac{AE}{FG} = \frac{4}{3}$
$\frac{MA}{MF} = \frac{12}{9}$
$= \frac{4}{3}$
$\frac{ME}{MG} = \frac{\sqrt{160}}{\sqrt{90}}$
$= \sqrt{\frac{160}{90}}$
$= \sqrt{\frac{16}{9}}$
$= \frac{4}{3}$
Since the ratio between each pair of sides is the same, the triangles are similar. The scale factor is the common ratio, $\frac{4}{3}$.

10.3 Understand when right-angled triangles are similar

SC 1

1 (a) $\Delta DEF \sim \Delta ABC$
(b) The triangles are not similar.
(c) $\Delta DEF \sim \Delta MLK$.
(d) The triangles are not similar.

2 (a) $\Delta JKL \sim \Delta ABC$. The scale factor is 3.
(b) The triangles are not similar.
(c) The triangles are not similar.
(d) The triangles are not similar.

3 (a) (i) $x = 39°, y = 51°, z = 39°$
(ii) $x = 43°, y = 47°, z = 43°$
(b) $\Delta ABC \sim \Delta ADB$, $\Delta ABC \sim \Delta BDC$

4 (a) (i) $b = 42°, c = 42°, d = 48°$
(ii) $b = 37°, c = 37°, d = 53°$
(b) $\Delta PQR \sim \Delta TSR$

5 $SU = 20.8$ cm and $ST = 8$ cm (since $PQ = 5$ cm using Pythagorean triads)

SC 2

1 (a) $k = 2.1$ **(b)** $x = 21$ cm

2 (a) $k = 1.25$ **(b)** $x = 6.25$ cm

3 (a) The bottom-left angle of the larger triangle is equal to the top-right angle of the smaller triangle because they are alternate angles with parallel lines (due to the right angles).
(b) $5 \leftrightarrow x$, $13 \leftrightarrow 29$ **(c)** $k = \frac{29}{13}$
(d) $x = 11\frac{2}{13}$ cm or $x = \frac{145}{13}$ cm

4 (a) $x = 12$ cm, $y = 16.6$ cm, $z = 10.8$ cm
(b) 1.5, $\frac{83}{40}$, $\frac{108}{40}$
(c) The hypotenuse is double the length of the vertical side each time.

5 (a) The pair of angles where the lines cross are vertically opposite angles, so they are equal.
(b) $k = 1.5$ **(c)** $x = 18$ cm

SC 3

1 (a) $k = 2$ **(b)** 1320 mm^2 **(c)** 5280 mm^2

2 (a) $k = 1.2$ **(b)** $p = 2.8$ cm
(c) 6.3 cm^2 **(d)** 9.072 cm^2

3 (a) $k = 1.7$ **(b)** 14 cm^2 **(c)** 40.46 cm^2

4 (a) $k = 2.5$ **(b)** $x = 20$ cm
(c) 375 cm^2 **(d)** 60 cm^2

5 (a) $k = 2.2$ **(b)** $d = 17.5$ cm
(c) 52.5 cm^2 **(d)** 254.1 cm^2

10.4 Use the triangle inequality theorem

SC 1

1 **(a)** **(i)** Between 2.1 cm and 5.6 cm
(ii) Between 5.6 cm and 7.7 cm
(iii) Between 7.7 cm and 13.3 cm
(b) 9.5 cm or 5.3 cm

2 **(a)**, **(b)**
0.5 m, 3.67 m; from 3.67 m to less than 4.17 m
1.5 m, 3.38 m; from 3.38 m to less than 4.88 m
2.5 m, 2.73 m; from 2.73 m to less than 5.23 m
3.5 m, 1.2 m; from 3.5 m to less than 4.7 m

3 **(a)** Between 10 cm and 90 cm.
(b) The third side, if less than 50 cm, is fixed at $\sqrt{50^2 - 40^2} = 30$ cm by Pythagoras' theorem.

4 **(a)** 11 cm, 12 cm, 13 cm; 11 cm, 12 cm, 14 cm; 11 cm, 12 cm, 15 cm; 11 cm, 13 cm, 14 cm; 11 cm, 13 cm, 15 cm; 11 cm, 14 cm, 15 cm; 12 cm, 13 cm, 14 cm; 12 cm, 13 cm, 15 cm
(b) All lengths that could be a hypotenuse are larger than 15 cm. All lengths that could be a shorter side are much smaller than 11 cm.

5 All lengths are in centimetres.

Equilateral	Isosceles	Hypotenuse
(3, 3, 3)	(2, 3, 3) (2, 4, 4) (2, 5, 5) (2, 6, 6)	$\sqrt{13}$
(4, 4, 4)	(2, 3, 3) (2, 4, 4) (2, 5, 5) (2, 6, 6) (3, 4, 4) (3, 5, 5) (3, 6, 6)	$\sqrt{20}$ $\sqrt{20}$ $\sqrt{20}$ $\sqrt{20}$ 5 5 5
(5, 5, 5)	(2, 3, 3) (2, 4, 4) (2, 5, 5) (2, 6, 6) (3, 4, 4) (3, 5, 5) (3, 6, 6)	$\sqrt{29}$ $\sqrt{29}$ $\sqrt{29}$ $\sqrt{29}$ $\sqrt{34}$ $\sqrt{34}$ $\sqrt{34}$

SC 2

1 **(a)** Less than 56.6 cm
(b) Between 56.6 cm and 80 cm

2 **(a)** Between 2.9 m and 3.26 m
(b) Between 3.26 m and 4.4 m

3 **(a)** Between 4 cm and 16 cm
(b) 8 cm and 11.7 cm
(c) Between 4 cm and 8 cm; between 8 cm and 11.7 cm; between 11.7 cm and 16 cm
(d) Using, say 5 cm, a triangle with 5 cm, 6 cm and 10 cm is obtuse-angled, so between 4 cm and 8 cm leads to an obtuse-angled triangle.
Using, say 10 cm, a triangle with 10 cm, 6 cm and 10 cm is acute-angled, so between 8 cm and 11.7 cm leads to an acute-angled triangle.
Using, say 13 cm, a triangle with 13 cm, 6 cm and 10 cm is obtuse-angled, so between 11.7 cm and 16 cm leads to an obtuse-angled triangle.

4 **(a)** Between 4 cm and 28 cm
(b) 10.6 cm and 20 cm
(c) Between 10.6 cm and 20 cm
(d) Between 4 cm and 10.6 cm and between 20 cm and 28 cm

5 **(a)** Between 7 cm and 17 cm
(b) **(i)** 11 cm, 12 cm
(ii) 8 cm, 9 cm, 10 cm, 14 cm, 15 cm, 16 cm

Topic 11: Modelling situations with measurement and rates

Recall

I can find the rule from a table of values

1 **(a)** $y = x + 5$ **(b)** $y = x - 2$

2 **(a)** $y = 4x$ **(b)** $y = \frac{1}{2}x$ or $y = \frac{x}{2}$

I can determine the gradient of linear graphs

1 **(a)** 2 **(b)** −4 **(c)** $\frac{3}{2}$ **(d)** $-\frac{1}{3}$

I can interpret percentages in sentences

1 **(a)** 16.67% **(b)** 71.43% **(c)** 42.86%
(d) 27.27% **(e)** 77.78%

I can substitute values into formulas

1 **(a)** 2 **(b)** −4 **(c)** 0
2 **(a)** 7 **(b)** −5 **(c)** 16
3 **(a)** −1 **(b)** 9 **(c)** −3

11.1 Model direct proportion

SC 1

1 **(a)** $b \propto a$ (or $a \propto b$), $b = 6a$ (or $a = \frac{1}{6}b$)

(b) $b \propto a$ (or $a \propto b$), $b = \frac{1}{2}a$ (or $a = 2b$)

(c) $\frac{1}{4} \neq \frac{4}{17}$. The ratio is not constant so the variables are not in direct proportion.

(d) $b \propto a$ (or $a \propto b$), $b = 5.2a$ (or $a = \frac{1}{5.2}b$)

2 (a)

d	0	1	3	6	10
t	0	4	12	24	40

(b)

m	0	3	8	15	60
V	0	1	$2\frac{2}{3}$	5	20

3 (a) cost = 3.45 × number of kilograms

(b) 0 kg of apples costs \$0, and usually each kilogram costs a set amount (e.g. \$3.45). This is direct proportion.

(c) \$16.39 **(d)** 3.62 kg

4 (a) $k = \frac{2}{3}$ **(b)** $q = \frac{2}{3}p$

(c) $q = 22$ **(d)** $p = 15$

5 (a) Normal pay = 16.68 × normal hours

(b) Public holiday pay = 33.35 × public holiday hours

(c) 6 hours

(d) 3 hours

(e) Total pay = 16.68 × normal hours + 33.35 × public holiday hours

(f) This equation is more complicated than direct proportion. There are 3 variables, and the number of normal hours and the number of holiday hours are not related to each other in any fixed proportion.

6 (a) $d = kt$ **(b)** $k = 15$ km/h

(c) k is the constant speed.

SC 2

1 (a) 24 cups of water

(b) 30 cups of cordial mixture

(c) 8 L of cordial concentrate

(d) 40 L of cordial mixture

(e) The unit does not matter as long as the same unit is used for both measurements. A ratio does not require a unit.

2 (a) Approximately A\$655. (The exact answer is \$660.50.)

(b) Approximately S\$820. (The exact answer is \$817.56.)

(c) Approximately A\$26.20

(d) It is too difficult to make approximations for low values because even a small error will have a significant impact.

3 (a) This would be 50.8 cm.

(b) Yes, the graph gives a value just barely over 50 cm.

(c) The graph gives a value about 12 inches, while it is 11.81 inches using the conversion.

(d) A value very close to 40, say 39.5 inches

(e) Multiply by 100.

(f) This gives 3950 inches = 329.17 feet = 109.72 yards. In fact, the race was run over 110 yards.

4 (a) It is approximately 9 gal.

(b) It is approximately 68 L.

(c) There is a coordinate pair very close to (90, 20).

(d) Using this coordinate pair, the relationship is 1 L = 0.222 gal. (The actual relationship is 1 L = 0.219 969 gal.)

11.2 Use formulas that model rates

SC 1

1 (a) $k = 2.1$ N/m **(b)** 0.9 cm **(c)** 2.1 cm

(d) 161 g **(e)** 418 g

2 (a) It is approximately 45 km.

(b) It is between 2 h and 2 h 30 min.

(c) (5, 225)

(d) 45 km/h

3 (a) 30 ohms **(b)** $V = 30I$ **(c)** 0.4 amps

4 (a) 6.7 kg **(b)** 7.5 kg·m/s

(c) The momentum is doubled.

(d) The speed is divided by three.

5 (a) 127 cm^3 **(b)** 7900 g or 7.9 kg

(c) 1000 **(d)** 7900 kg

Exploration task

Answers will vary.

Topic 12: Trigonometry

Recall

I can calculate the internal angles of a right-angled triangle

1 (a) $k = 71°$ **(b)** $k = 62°$ **(c)** $k = 24.5°$

I can solve linear equations

1 (a) $x = 18$ **(b)** $x = 9$ **(c)** $x = -4$

I can calculate the value of formulas following substitution

1 (a) $u = -14$ **(b)** $a = 0.8$ **(c)** $t = 1$

2 (a) $E = 270$ **(b)** $m = 8$ **(c)** $c = 15$

I can simplify fractions

1 (a) $\frac{1}{4}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{2}{15}$

(d) $\frac{9}{2} = 4\frac{1}{2}$ **(e)** $\frac{7}{2} = 3\frac{1}{2}$

I can simplify ratios of lengths

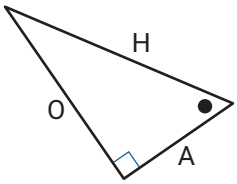
1 (a) 6 : 1 **(b)** 1 : 8 **(c)** 1 : 4

(d) 4 : 1 **(e)** 1 : 30

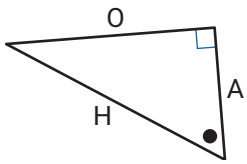
12.1 Understand right-angled triangle trigonometry

SC 1

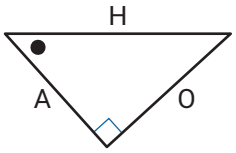
1 **(a)**



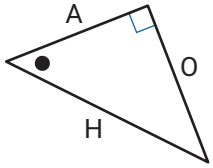
(b)



(c)



(d)



2 **(a)** correct **(b)** incorrect
(c) incorrect **(d)** incorrect

3 **(a)** **(i)** 11.4 cm **(ii)** 9 cm **(iii)** 7 cm
(b) **(i)** 11 cm **(ii)** 4.5 cm **(iii)** 10 cm

4 **(a)** the side opposite α
(b) the side adjacent to α

5 **(a)** DF **(b)** XY

SC 2

1 The side opposite β is longer than the side adjacent to β, hence, $\sin(\beta) > \cos(\beta)$.

2 $\sin(\phi) = \frac{3}{5}, \cos(\phi) = \frac{4}{5}$

3 The opposite side is 24 cm; $\sin(\phi) = 0.96$ and $\cos(\phi) = 0.28$

4 The simplest triangle would have sides of 11 units and 5 units, with the third side solvable as $\sqrt{96}$ units or $4\sqrt{6}$ units.

(a) $\cos(\theta) = \frac{\sqrt{96}}{11}$ or $\frac{4\sqrt{6}}{11}$

(b) 0.8907

5 The simplest triangle would have sides of 100 units and 63 units, with the third side solvable as $\sqrt{6031}$ units.

(a) $\sin(\theta) = \sqrt{0.6031}$ **(b)** 0.7766

SC 3

1 The side opposite β is longer than the side adjacent to β, hence, $\tan(\beta) > 1$.

2 $\tan(\phi) = \frac{3}{4}, \tan(\theta) = \frac{4}{3}$

3 The side opposite ϕ is 16 cm; $\tan(\phi) = 0.5333$ and $\tan(\theta) = 1.875$.

4 The simplest triangle would have short sides of 6 units and 7 units, with 6 units opposite α.

(a) $\tan(\beta) = \frac{7}{6}$ **(b)** 1.1667

5 A simple right-angled triangle could have sides of 5 units, 12 units and 13 units, with 12 units opposite α.

(a) $\tan(\beta) = \frac{5}{12}$ **(b)** $\sin(\alpha) = \frac{12}{13}$ **(c)** $\cos(\alpha) = \frac{5}{13}$

12.2 Use trigonometry to solve an unknown side length in a right-angled triangle

SC 1

1 **(a)** tangent **(b)** cosine **(c)** sine
(d) sine **(e)** cosine **(f)** tangent

2 tangent

3 **(a)** sin (45°) **(b)** tan (64°) **(c)** cos (43°)

4 $x = 40$ cm

5 $x = 12$ cm

SC 2

1 **(a)** 4.62 cm **(b)** 1.81 m **(c)** 3.91 cm

2 **(a)** 2.22 m **(b)** 16.41 cm **(c)** 102.37 mm

3 **(a)** 35.33 cm **(b)** 26.44 mm **(c)** 1.46 m

4 **(a)** 1.06 m **(b)** 25.17 cm **(c)** 62.77 mm

5 **(a)** $y = 15$ cm **(b)** $x = 2.2$ m **(c)** $y = 16$ cm

SC 3

1 13.0 m

2 8.77 m

3 79.5 m

4 4.89 m

5 **(a)** 12.3 m **(b)** 11.6 m to 13.1 m
(c) 10.1 m to 14.6 m

12.3 Use trigonometry to solve an unknown angle in a right-angled triangle

SC 1

1 **(a)** tangent **(b)** cosine **(c)** sine
(d) sine **(e)** tangent **(f)** cosine

2 Sine or cosine

3 **(a)** $\cos(\theta) = \frac{16}{31}$ **(b)** $\tan(\theta) = \frac{2.3}{2}$
(c) $\sin(\theta) = \frac{5}{7}$

4 **(a)** $\angle C$
(b) $\sin(\theta) = 0.8824$, $\cos(\theta) = 0.4706$

5 $38° < \theta < 39°$

SC 2

1 **(a)** $\theta = \sin^{-1}(0.8922)$ **(b)** $\theta = \cos^{-1}(0.71)$
(c) $\theta = \tan^{-1}(0.65)$ **(d)** $\theta = \sin^{-1}(0.3)$
(e) $\theta = \cos^{-1}(0.1414)$ **(f)** $\theta = \tan^{-1}(2.5)$

2 **(a)** $\theta = \sin^{-1}\left(\frac{6}{17}\right)$ **(b)** $\theta = \cos^{-1}\left(\frac{3}{20}\right)$
(c) $\theta = \tan^{-1}\left(\frac{8}{15}\right)$ **(d)** $\theta = \tan^{-1}\left(\frac{20.3}{11.25}\right)$
(e) $\theta = \sin^{-1}\left(\frac{1.4}{2.3}\right)$ **(f)** $\theta = \cos^{-1}\left(\frac{8.6}{10.5}\right)$

3 **(a)** $\theta = 67°$ **(b)** $\theta = 27°$ **(c)** $\theta = 46°$
(d) $\theta = 58°$ **(e)** $\theta = 15°$ **(f)** $\theta = 60°$

4 **(a)** $\theta = 75°$ **(b)** $\theta = 60°$ **(c)** $\theta = 65°$
(d) $\theta = 58°$ **(e)** $\theta = 36°$ **(f)** $\theta = 14°$

5 **(a)** $\theta = 57°$ **(b)** $\theta = 53°$ **(c)** $\theta = 35°$
(d) $\theta = 50°$ **(e)** $\theta = 31°$ **(f)** $\theta = 44°$

SC 3

1 **(a)** 30° **(b)** 56° **(c)** 81° **(d)** 11°

2 **(a)** 35° **(b)** 8° **(c)** 74° **(d)** 45°

3 $\theta = 10°, \phi = 19°$

4 From 35.7° to 49.7°, so an increase of 14°

5 30°

12.4 Define a right-angled triangle

SC 1

1 **(a)** $\alpha + 32° = 90°$
(b) $\sin(32°) = \frac{1.2}{x}$
(c) $x^2 = 1.2^2 + y^2, \frac{y}{x} = \cos(32°)$

2 **(a)** $x^2 = 10^2 + 18^2$
(b) $\tan(\alpha) = \frac{10}{18}$
(c) $\alpha + \beta = 90°$

3 **(a)** $\alpha + 65° = 90°$
(b) $\cos(65°) = \frac{x}{26}$
(c) $26^2 = x^2 + y^2, \frac{y}{x} = \tan(65°)$

4 **(a)** $29^2 = x^2 + 21^2$
(b) $\sin(\alpha) = \frac{21}{29}$
(c) $\alpha + \beta = 90°$

5 **(a)** $\alpha + 55° = 90°$
(b) $\tan(55°) = \frac{x}{27}$
(c) $y^2 = x^2 + 27^2, \sin(55°) = \frac{x}{y}$

SC 2

1 $x = 5\,\text{m}, \alpha = 53°, \beta = 37°$

2 $\alpha = 52°, x = 21.9\,\text{cm}, y = 35.5\,\text{cm}$

3 $\alpha = 30°, x = 15.6\,\text{cm}, y = 9\,\text{cm}$

4 $x = 16\,\text{cm}, \alpha = 53°, \beta = 74°$

5 $x = 12\,\text{cm}, \alpha = 19°, y = 7.2\,\text{cm}, \phi = 37°, \theta = 53°$, the top-left angle is 90 + 53 = 143°, the bottom-right angle is 108°.

Topic 13: Ratios and proportion in space

Recall

I can convert between units of length

1 **(a)** 23.75 m **(b)** 50.4 mm **(c)** 5.835 cm
(d) 0.008 55 m **(e)** 97.5 cm **(f)** 450 cm

I can simplify ratios of lengths in different units

1 **(a)** 1 : 20 **(b)** 1 : 3 **(c)** 1 : 150
(d) 1 : 4 **(e)** 1 : 80

I can calculate the area of plane shapes

1 **(a)** 20.25 cm^2 **(b)** 3.6 m^2 **(c)** 34 cm^2 **(d)** 0.48 m^2

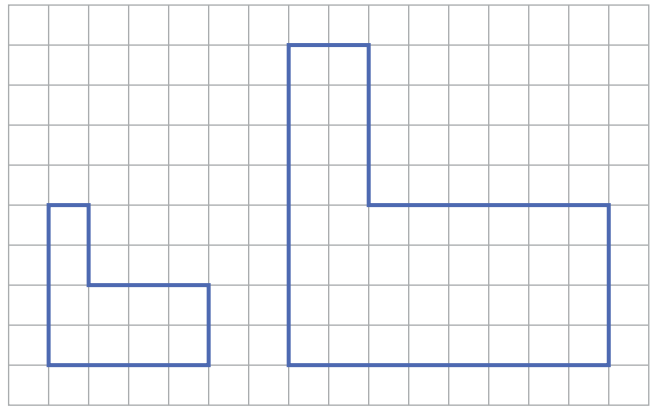
I can calculate the volume of solids

1 **(a)** 1400 mm^3 **(b)** 91.125 m^3 **(c)** 1440 cm^3

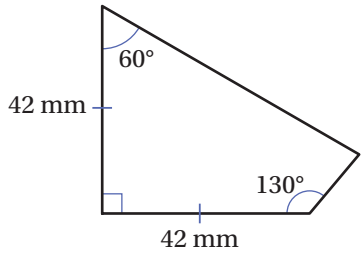
13.1 Understand the effect of the enlargement transformation

SC 1

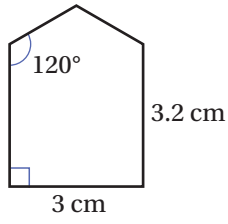
1



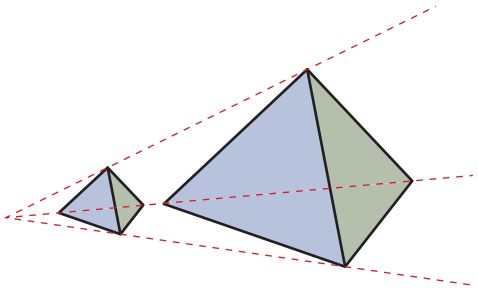
2



3



4 (a)



(b) An adapted method might use bamboo skewers instead of pencil lines, and the enlarged solid would also need to be built from rigid materials such as paper straws.

(c) The enlarged tetrahedron will have edges 3 times the length of the original. It can be built by joining together 4 edges that are 3 times the original length.

SC 2

1 (a) $\angle V = \angle D$ **(b)** $STUVW \sim BCADE$
(c) $DA = 77$ mm

2 (a) $\angle KML = \angle QPL$ **(b)** $\Delta KLM \sim \Delta QLP$
(c) $GF = 38.6$ cm

3 (a) $\Delta DEH \sim \Delta DGF$ **(b)** $\frac{FG}{HE} = \frac{DF}{DH} = \frac{DG}{DE}$
(c) $GF = 49$ mm

4 (a) $x = 1.15$ m **(b)** $x = 20$ cm

5 (a) $x = 17.9$ cm **(b)** $x = 40$ cm

6 $x = 25$ mm

SC 3

1 (a) False **(b)** True **(c)** False

2 (a) 150 cm^2 **(b)** 600 cm^2 **(c)** 1 : 4
(d) 125 cm^3 **(e)** 1000 cm^3 **(f)** 1 : 8

3 (a) True **(b)** False **(c)** True **(d)** False

4 48 cm or 0.48 m

5 (a) $6x^2$ **(b)** $54x^2$ **(c)** 1 : 9
(d) x^3 **(e)** $27x^3$ **(f)** 1 : 27

13.2 Calculate the area of a shape or the volume of an object using a scale factor

SC 1

1 (a) $\frac{4}{3}$ and $\frac{16}{9}$ **(b)** $\frac{5}{4}$ or 1.25, and $\frac{25}{16}$ or 1.5625
(c) 12.5 and 156.25

2 (a) 420 mm^2 **(b)** 2.25 or $\frac{9}{4}$ **(c)** 2126 mm^2

3 (a) 375 cm^2 **(b)** 1.65 or $\frac{33}{20}$ **(c)** 1021 cm^2

4 (a) 2600 cm^2 **(b)** 1.6 or $\frac{8}{5}$ **(c)** 1016 cm^2

5 (a) 20 + 36 = 56 cm **(b)** 420 cm^2
(c) 1.2 or $\frac{6}{5}$ **(d)** 605 cm^2

SC 2

1 (a) 1.25 and 1.5625 **(b)** $\frac{8}{7}$ and $\frac{64}{49}$
(c) Height of smaller trapezium is $\sqrt{13^2 - 5^2} = 12$ cm; $\frac{4}{3}$ and $\frac{16}{9}$

2 (a) Cut out square has sides 4 cm; 516 cm^2
(b) 3 **(c)** 4644 cm^2

3 (a) Base is $20 + 2\sqrt{17^2 - 15^2} = 36$ cm; 420 cm^2
(b) 1.5 **(c)** 945 cm^2

4 (a) $(1750 + 306.25\pi)$ mm^2
(b) $\frac{85}{35} = \frac{17}{7}$ **(c)** 460 mm^2

5 (a) 40 mm **(b)** 8400 mm^2
(c) 1.25 **(d)** 13 125 mm^2

SC 3

1 (a) Perimeter: 28 cm → 84 cm; area: 40 cm^2 → 360 cm^2
(b) Perimeter: 22 cm → 66 cm; area: 24 cm^2 → 216 cm^2
(c) Perimeter: 7 m → 21 m; area: 3 m^2 → 27 m^2
(d) Perimeter: 134 mm → 402 mm; area: 1050 mm^2 → 9450 mm^2
Each perimeter is tripled; each area is 9 (i.e. 3^2) times as large.

2 (a) Surface area: 248 cm^2 → 2232 cm^2; volume: 240 cm^3 → 6480 cm^3
(b) Surface area: 46 cm^2 → 414 cm^2; volume: 14 cm^3 → 378 cm^3
(c) Surface area: 1827 cm^2 → 16 848 cm^2; volume: 5184 cm^3 → 139 968 cm^3
(d) Surface area: 41.08 m^2 → 369.72 m^2; volume: 15.624 m^3 → 421.848 m^3
Each surface area is 9 (i.e. 3^2) times as large; each volume is 27 (i.e. 3^3) times as large.

3 (a) 50 cm → 80 cm; perimeter is 1.6 times as large
(b) 45 m^2 → 115.2 m^2; area is 2.56 (i.e. 1.6^2) times as large.
(c) 640 cm^2 → 1638.4 cm^2; surface area is 2.56 (i.e. 1.6^2) times as large.
(d) 11.44 m^3 → 46.858 24 m^3; volume is 4.096 (i.e. 1.6^3) times as large.

4 (a) 106 m → 74.2 m; perimeter is reduced to 0.7 of the original.
(b) 16.1 cm^2 → 7.889 cm^2; area is reduced to 0.49 (i.e. 0.7^2) of the original.
(c) 19 300 mm^2 → 9457 mm^2; surface area is reduced to 0.49 (i.e. 0.7^2) of the original.
(d) 3000 cm^3 → 1029 cm^3; volume is reduced to 0.343 (i.e. 0.7^3) of the original.

5 **(a)** 40% larger means 140% or 1.4 of the original lengths. The border fence is increased in length by the same proportion, so the cost will increase by 40%. Assumptions: Cost does not include gates (for which the cost may not change at all) and the number of corner posts needed does not impact the cost (because the number may not have increased, hence the cost would increase by less than 40%).

(b) $1.4^2 = 1.96$, so the area increases by 96%, or is almost doubled. The cost of planting seed will be double, unless this cost is reduced by the efficiency of planting a larger amount.

6 **(a)** 10% larger means 110% or 1.1 of the original lengths. $1.1^2 = 1.21$, so the surface area increases by 21%. The area of packaging material will increase by 21% also, assuming the fold-over amounts increase in the same proportion. In reality, the fold-overs needed would not have to increase quite so much.

(b) $1.1^3 = 1.331$ so the volume increases by 33.1%. For substances such as rice or flour, the mass of the contents could be increased by 33.1%, or almost a third.

13.3 Use images and proportion to estimate lengths

SC 1

1 **(a)** Yes, possible since the smaller size of the car at the back could be explained by being further from the camera.

(b) So long as the cars are parallel to each other, the proportion should be maintained. There will be some distortion from the depth, however, but if the cars are close enough to the same length the impact will be minimal.

(c) Write a proportion equation and solve it:

$$\frac{\text{Unknown length (car 1)}}{\text{Known length (car 2)}} = \frac{\text{Known height (car 1)}}{\text{Known height (car 2)}}$$

2 **(a)** 4037 mm **(b)** 1121 mm **(c)** 1402 mm

3 **(a)** The mother is standing closer to the camera.

(b) 132 cm **(c)** 106 cm

4 7.5 m

5 **(a)** If they were to the same scale, the height and length of the chihuahua would be about half the height and length of the labrador. Here they are almost the same.

(b) 19.4 cm **(c)** 2.5 cm

SC 2

1 **(a)** 1011 m **(b)** 1350 m

2 **(a)** 16.7 cm **(b)** 1.3 cm

3 **(a)** 4.3 m

(b) The top of the house is a different distance from the camera from the base of the house, so the actual height would be less than calculated.

4 **(a)** 3.7 cm **(b)** 6.0 m

13.4 Ascertain compliance involving horizontal and vertical distances

SC 1

1

Ratio	Angle	Percentage
1 : 40	1.4°	2.5%
1 : 33	1.7°	3.0%
1 : 20	2.9°	5%
1 : 14	4.1°	7.1%

2 **(a)** 36°, so it is compliant and a recommended steepness.

(b) 11 to 19

(c) 10 to 16 for goings, so 11 to 16 complies with both.

3 **(a)** 40 risers, so 3 flights

(b) From 2 to 18 risers per flight, so: (15, 15, 10), (12, 16, 12) and (10, 14, 16) are possible numbers of risers in each of three flights.

4 **(a)** 27.5 mm **(b)** 275 mm

(c) 0.3° and 2.9°

5 **(a)** 206 mm

(b) Not compliant since 6 mm difference

(c) 268 mm

(d) Compliant since 3 mm difference

(e) Could adjust one riser to 209 mm and make the final one the same.

13.5 Recognise image editing

SC 1

1 **(a)** stretched vertically

(b) stretched vertically

(c) not distorted

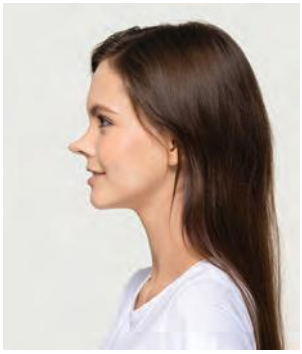
(d) stretched horizontally

2 **(a)** Compress horizontally to half the width or stretch vertically to twice the height.

(b) Compress vertically to half the height or stretch horizontally to twice the width.

3 **(a)** Draw a vertical line where the nose protrudes from the rest of the face and stretch everything horizontally from that line, away from the face.

(b)



(c) Draw horizontal lines below the eyes and below the nose. Stretch the middle section vertically and replace in the enlarged space.

(d)



4



5 **(a)** 50% **(b)** 250% **(c)** 67% **(d)** 333%

Topic 14: Analysing and interpreting statistics

Recall

I can identify the source of collected data as primary or secondary

1 Primary data is collected by the person doing the survey themselves. Secondary data is reading the results of a survey conducted by someone else.

2 **(a)** Primary **(b)** Secondary
(c) Secondary **(d)** Primary

I can identify the collected data as randomly or non-randomly sourced

1 Random data is one where every element has an equal chance of being recorded, while non-random data follows a set method to source the data, so not all data has an equal chance of being recorded.

2 **(a)** Non-random **(b)** Random
(c) Non-random **(d)** Random

Exploration task

The two made up statistics are 'Over 50% of red-headed people are also left-handed' and '80% of Americans are surprised to find that Australians speak English'.
The first doesn't make sense because 50% is too high. It is true that recessive traits often come in pairs, and red-headed people are somewhat more likely to be left-handed; however, to say it is as common as 50% is simply not true.
Likewise, the idea that 80% of Americans don't know that Australians speak English is false. While it is true that some Americans confuse Australians with Austrians and so are surprised to hear that Australians speak English, the true statistic is much lower.

14.1 Analyse statistical reports in digital media

SC 1

1 **(a)** It was made up.
(b) The subjects will be from a broader cross-section of the whole population compared to the individual studies, which most likely use subjects from only one country.
(c) This is a 20% reduction.
(d) This is between a 20% and 40% reduction.

2 **(a)** It is the figure from the August 2021 census.
(b) It is an accurate figure, not an estimate.
(c) These are the years of the two last censuses.
(d) Many homeless people were in emergency accommodation during COVID-19 lockdowns, so they were easier to find and include in the census.
(e) The anecdotal evidence is that the situation has become worse.

3 **(a)** A telephone survey
(b) The number of participants
(c) There is the potential for bias in that people without a telephone, or those who do not answer calls from unknown numbers, will not be included in the sample.
(d) It seems small, but it is approximately the same as the number of people surveyed in national political surveys.

SC 2

1 (a) This refers to the dollar value of the exports. The other option is by volume. No indication is given as to what the by volume ranking might be. Having both figures would make it possible to consider whether Australian wine was seen as a cheap product, or more towards the higher end of the quality scale.

(b) Switzerland

(c) 28 years

(d) This is produced to boost positive feelings about Australia and its place in the world.

2 (a) 3107 people were surveyed.

(b) Close to a third

(c) 55%

(d) The article seems balanced, pointing out some of the advantages as well as some of the disadvantages of social media use.

3 (a) 16–24 years of age

(b) Gen Z spends $20 more per month than the Overall respondents.

(c) 4.5

(d) Facebook

(e) TikTok

(f) Some would, but others would still be living at home, probably being supported by their parents.

4 (a) Females: 0.95% increase; males: 0.98% increase

(b) 4%

(c) Twice the rate

(d) It is almost certainly based on a sample. The numbers are rounded off numbers and it is extremely unlikely that the Australian Institute of Health and Wellness could afford the time or money required for a census.

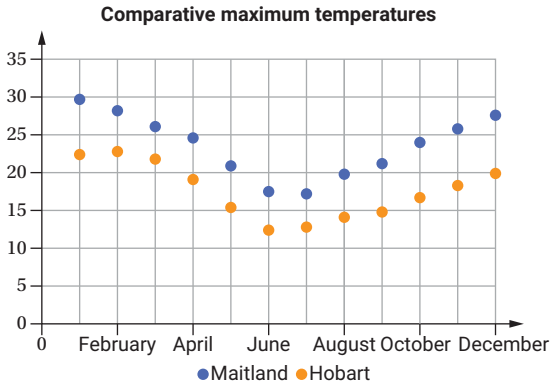
14.2 Question the views presented in media reports

SC 1

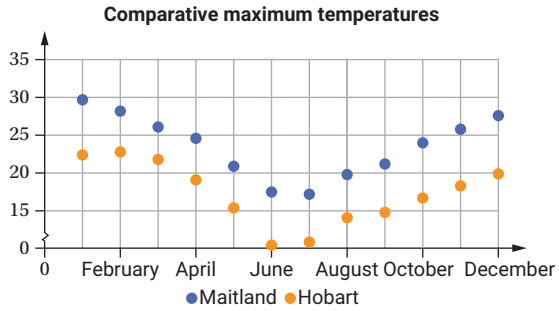
1 (a) $\sqrt{2}$ cm **(b)** $\sqrt{3}$ cm

(c) $\sqrt{10}$ cm **(d)** $10\sqrt{\frac{1}{22}}$

2 (a)



(b)



(c) The second graph makes it look like the temperature differences are a lot larger than they actually are.

3 (a) Between months 4 and 5

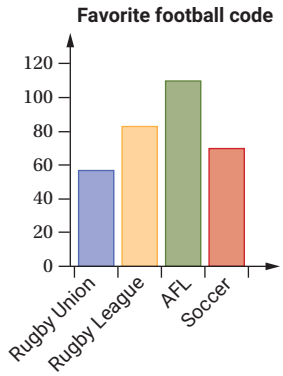
(b) Between months 2 and 3

(c) The vertical scale on the right-hand graph is inconsistent.

(d) The left-hand graph, with its consistent scale, is the better representation of the data.

4 (a) There is no vertical scale given, the widths of the columns are not consistent and the vertical scale is not accurate. It is only because the actual numbers are given that any sense can be made of the data.

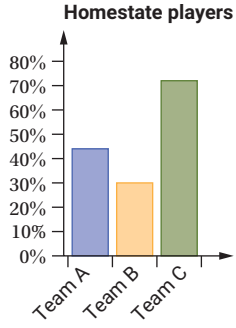
(b)



(c) No. There is no indication of how the sample was obtained. Rugby Union is likely to have higher support in NSW and Queensland, for instance.

5 (a) The sections in a pie chart must add to 100% when they are presented in percentage form.

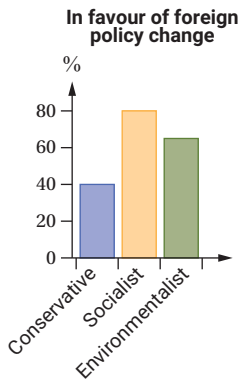
(b)



6 **(a)** It would fit in five times.

(b) No, the blue column percentage is half the red column percentage.

(c)



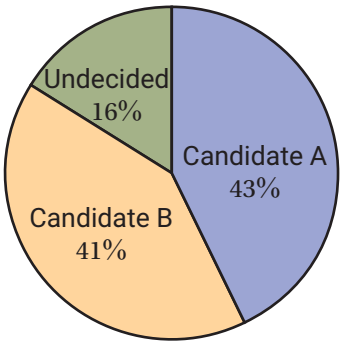
SC 2

1 **(a)** The two sections of the pie chart do not add up to 100%.

(b) The graph overstates the support for Candidate A, making it look like that person enjoys majority support.

(c) These would be undecided voters, or perhaps people who voted for a person other than the two named candidates.

(d) **Preferred Prime Minister**



(e) Candidate B has increasing support, while Candidate A has decreasing support.

2 **(a)** There is no indication given as to sample size.

(b) No reasons are given as to the reason why some houses cannot be connected to solar.

(c) Some ideas might be to turn lights off when nobody is in the room, turn off electrical appliances when they are not in use, wash clothes using the cold water cycle, wear warmer clothes before turning on the heating, draught proof your home.

3 **(a)** Youfoodz could be looking for some statistics to use in a forthcoming advertising campaign targeting office workers.

(b) It possibly means only a small sample was used, or that the focus on office workers means the findings are not applicable to a wider population.

(c) No evidence is presented to suggest these figures are any different to what has always been the case.

(d) None, as the word 'skip' has not been adequately defined. The figures suggest that 77% do not take their full lunch break, but this is very different to skipping lunch.

4 **(a)** There does not appear to be any evidence supporting the reasons for changing jobs.

(b) No

(c) 2022

(d) The very tight labour market, with record job creation.

(e) There is minimal evidence to support the claim; it is most likely just there to catch the eye. The report is not biased, as such, but there is not a strong connection between the content and the headline.

Topic 15: Displaying and comparing statistical data

Recall

I can complete a frequency distribution tables

1

Salad packs	Tally	Frequency
A	𝍸	5
B	𝍸	5
C	𝍷	4
D	𝍷	4
E	𝍵	2
Total		20

I can calculate the mean score for a set of data

1 **(a)** 4.29 **(b)** 8.17 **(c)** 15.75

I can calculate the median score for a set of data

1 **(a)** 11 **(b)** 12.5 **(c)** 114.5

I can identify the mode for a set of data

1 **(a)** 2 **(b)** 23 **(c)** 108

I can calculate the range of a set of data

1 **(a)** 12 **(b)** 18 **(c)** 65

15.1 Represent data using statistical displays

SC 1

1 **(a)** 5.9 **(b)** 6 **(c)** 6

(d) Symmetrical; the values of the mean, median and mode are all very close to each other.

2 **(a)**

Score	Frequency
0	3
1	2
2	4
3	3
4	4
5	9
6	7

(b) 3.8 **(c)** 4.5 **(d)** 5

(e) Negative skew; mean < median < mode confirm it is a negative skew.

3 **(a)**

Height (cm)	Frequency
150–<155	3
155–<160	5
160–<165	11
165–<170	12
170–<175	5
175–<180	1

(b) 164.4 **(c)** 160–<165 **(d)** 165–<170

(e) This is a symmetrical data set.

4 **(a)** positive skew **(b)** symmetrical

(c) negative skew

5 **(a)** Bimodal

(b) A group of students didn't really try hard and possibly just walked the course.

(c) 82.7 min

(d) No, this is not really a fair representation. The large number of high times has dragged the mean higher than it probably should be.

6 **(a)** 35.5 **(b)** 33

(c) 33 **(d)** symmetrical

SC 2

1 **(a)**

Leaf Class A	Stem	Leaf Class B
9	1	4
9 8 7 7 5 3 3	2	1 3 7 9 9
7 3 0	3	0 4 5 6 7 8 8
7 6 6 6 4 4 2 2 0	4	2 4 6 7 8 9 9
0	5	0 8 9 9

(b) Class A would be best described as bimodal and Class B is harder to classify, perhaps symmetrical.

(c) Class A: 37; Class B: 38

(d) Class A: 35.6; Class B: 39.3

2 **(a)**

Leaf Melbourne Storm	Stem	Leaf Brisbane Broncos
4	17_L	3
9 8 7	17_H	6 7 7 8
3 3 2 1 0	18_L	0 0 1 2
7 7 7 7 5 5 5	18_H	5 5 6 8 9 9
4 4 3 3 3 3 2 0 0 0 0 0	19_L	0 0 0 0 0 0 2 2 4 4 4
6 6 5 5	19_H	
0	20_L	

(b) Even considering the difference in the number of values for each team, it would seem that Melbourne Storm are taller. The five tallest players are on their list.

(c) Melbourne Storm: 190 cm; Brisbane Broncos: 188.5 cm

(d) Melbourne Storm: 187.7 cm; Brisbane Broncos: 185.8 cm

(e) For both teams it is 190 cm. It seems unusual that there would be this many players of this height on both teams. Some players may just like to think they are at least 190 cm tall!

3 **(a)**

Leaf Round 1	Stem	Leaf Round 2
6	0	
9 8 8 8 6 3 2 2 2 0	1	0 2 2 2 4 4 6 6 8
8 7 2 0	2	0 0 6 6 8
1	3	0 2

(b)

Leaf Round 1	Stem	Leaf Round 2
6	0_H	
3 2 2 2 0	1_L	0 2 2 2 4 4
9 8 8 8 6	1_H	6 6 8
2 0	2_L	0 0
8 7	2_H	6 6 8
1	3_L	0 2

(c) The width of each row is reduced, and it is easier to see whether the values are at the high end for the stem value or at the low end.

(d) Round 1: 18; Round 2: 17

(e) Round 1: 17.6; Round 2: 19.1

(f) The scores in the two rounds are quite close, but the difference in mean values tips the scale, so Round 2 has higher scores.

4 (a)

Leaf Perth Airport	Stem	Leaf Adelaide Airport
6	10	
	11	
	12	8
	13	6 6 8 9
6	14	2 8 8 9 9 9 9
9 8 1 0	15	4 5 6 9
8 7 7 5 3 2	16	3 5
7 6 3 1 0	17	1 1
9 7 7 3 1 0	18	1 3 6
9 8 4 3 1	19	1 3
0	20	3
4	21	8
	22	2
	23	7
	24	
	25	3

19 | 3 represents 19.3

(b) Perth Airport: 17.45; Adelaide Airport: 15.75

(c) Perth Airport: 17.42; Adelaide Airport: 16.23

(d) The values for Perth Airport are much more clustered in the middle of the table, whereas the values for Adelaide Airport are much more spread out. The median and mean both confirm that Perth Airport, on average, is warmer than Adelaide Airport during June.

(e) No, there would be too many stem values and many of them would have no leaf values attached.

5 (a) Sample 1: 33.5; Sample 2: 38

(b) Sample 1: 34.14; Sample 2: 36.82

(c) They would both be 34.5.

(d) In Sample 1, the values seem to be well spread and both the median and mean are close to the expected values.

(e) In Sample 2, the values are reasonably well spread, although there are fewer than expected in the 20_L stem. Both the median and the mean are slightly higher than expected.

(f) Median: 36.5; Mean: 35.48

(g) Combining the sample makes the 20_L look more like what is expected. It is reasonable to accept that these are randomly generated numbers.

SC 3

1 (a) There are only 9 months when there is rain and in two of these months, there is virtually no rainfall. It would be difficult to state an 'average' or 'typical' value for this location.

(b) The rainfall in May seems a little high. It would be necessary to look at other years to see whether this is a usual pattern.

(c) Summer

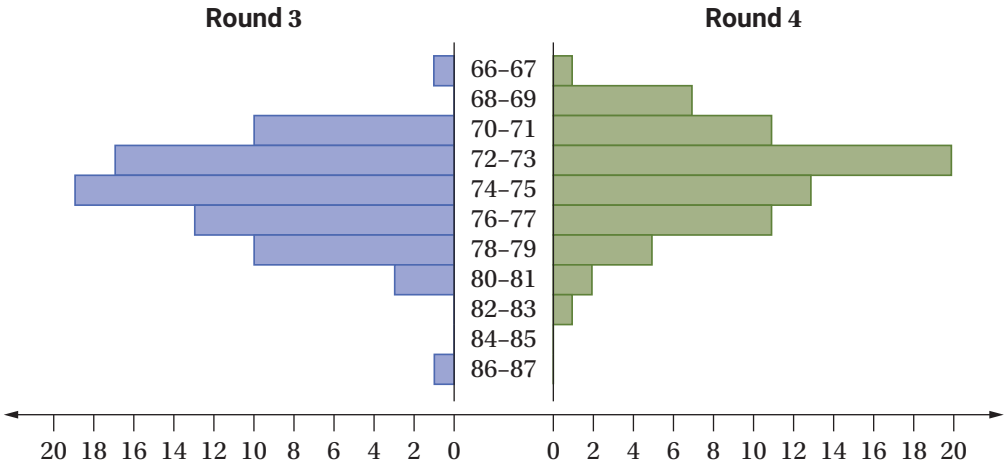
(d) Spring/Summer has the highest total rainfall, however there was a large amount of rain in May. May is the highest total for a single month.

2 (a) 74 players round one. 70 players round two.

(b)

Score	Round 3 frequency	Round 4 frequency
66–67	1	1
68–69	0	7
70–71	10	11
72–73	17	20
74–75	19	13
76–77	13	11
78–79	10	5
80–81	3	2
82–83	0	1
84–85	0	0
86–87	1	0

(c)



(d) Round 3: 74.7; Round 4: 72.7

(e) From the look of the graph, round 4 seems to have lower scores. This is backed up by the values of the mean where round 4 is two shots better.

(f) Round 3 is close to symmetrical, while round 4 shows a slight positive skew.

3 (a) It seems as though there are more female cases.

(b) 20–29

(c) 20–29

(d) As the age groups get higher, the proportion of the total population falls. There are far fewer people aged 60–69 than there are aged 40–49, for instance. This appears to suggest that a greater proportion of people aged 60–69 had COVID-19 relative to their proportion in the total population.

4 (a) Yes, it was 29.

(b) No, the best you can say is that it was in the range 0–<5.

(c) Yes, but only Roman's plot. It was 22.

(d) The lowest leaf is 0 so Roman's plot would require an extra stem value, 3_L.

5 (a) 25–29 **(b)** 25–29 or 30–34

(c) There may be a declining birth rate, although this would probably not account for the seemingly large difference. It is more likely the effect of immigration.

(d) Australia would probably fit the definition of an older population. About 16% of the population is 65+.

15.2 Justify the selection of particular data displays

SC 1

1 (a) 90–<95 **(b)** 85–<90 **(c)** 25 **(d)** 88.2

2 (a) 26–29 **(b)** 22–25 **(c)** 19 **(d)** 21.0

3 (a) mean ≈ median < mode

(b) slight negative skew

(c) mean < median < mode

(d) negative skew

4 (a) 24.38

(b)

Score	Frequency (f)
1–10	23
11–20	22
21–30	17
31–40	18
41–50	20
	$\sum f = 100$

(c) 24.5

(d) The estimate is close to the actual value.

5 (a) 77 players

(b)

Statistic	Round 3	Round 4
Mean	68.65	71.78
Median	68	71
Mode	67	70
Range	14	16

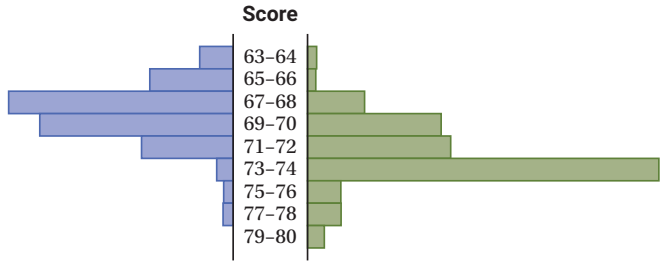
(c) Round 3: mode < median < mean
Round 4: mode < median < mean

(d) The statistics suggest that each data set shows positive skew. However, the values are quite close, so an answer of symmetrical would be quite reasonable.

(e) Round 3 clearly shows better scores.

(f) The range values are close, but the slightly higher value for Round 4 shows the scores are more spread out, reflecting the worse scores obtained.

(g)



(h) The graph confirms Round 3 as showing positive skew, however, Round 4 is less decisive. It looks almost symmetrical with a little bias towards negative skew.

6 (a) is **(b)** symmetry

(c) more **(d)** can

SC 2

1 (a) The line graph is attempting to show trends over time. Being able to follow each age group separately is important.

(b) The 60+ age group has shown the greatest increase.

(c) It could be that these people have held onto an investment property that they have owned for several years, and they have aged along with the property, so that they now fit into this age group.

(d) The 'less than 30' age group has fallen in absolute number.

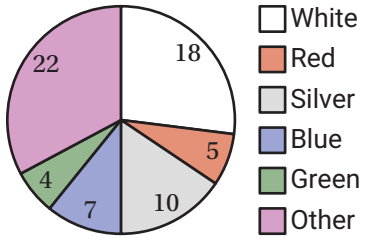
(e) The colours are needed to distinguish between the different age groups. Without them it would be very difficult to interpret the display.

2 (a) There are many colours and shades used for cars. Many of these may have only one representation in the car park. Having too many sections in a pie chart makes it difficult to interpret.

(b) 66 **(c)** $5.\overline{45}°$

(d) It is difficult enough to draw a pie graph with 1° divisions. Having a repeating decimal would be impossible.

(e) **Cars in school car park**



3 **(a)** 56.3% **(b)** 186.6%
(c) It is categorical data.
(d) A side-by-side column graph.
(e) The figures for iron ore are so much larger than any of the other figures that they make the scale difficult.
(f) Gold, copper ore and wheat
(g) The scale is now such that the changes can be seen easily.

4 **(a)** About 9.4%. **(b)** Perth, in late 2021
(c) Brisbane **(d)** Melbourne or Sydney
(e) The percentage for Perth was falling, while all the rest were rising.
(f) The owner may be suffering financial stress and be unable to keep up the payments, or the property prices may be rising rapidly and a quick profit is sought.

5 **(a)** 99, which is not of concern because all the values have been rounded to the nearest whole number.
(b) 83°
(c) Total cost increase factors (or something similar)
(d) A side-by-side column graph (bar graph) would be best.

6 **(a)** Categorical data **(b)** SUV
(c) Passenger **(d)** Approximately $33\frac{1}{3}\%$

SC 3

1 **(a)** The higher the column, the more sales.
(b) This is the base element, so it is simply a matter of lining up against the vertical scale. The most beef kebabs were sold on Monday.
(c) It is necessary to estimate each of the other values because they have not started from the same place.

2 **(a)** February **(b)** March
(c) August (or September) **(d)** May
(e) March **(f)** July
(g) The questions about Cairns are easier to answer than the questions about Coffs Harbour, because the Cairns graph is measured from the bottom, whereas the Coffs Harbour one is measured from the Cairns value. The combined questions are also relatively easy to answer because the answers are just related to where the Coffs Harbour graph ends.

3 **(a)** Sample answer: It is very involved and not easy to read due to the overwhelming amount of information included.
(b) 13 888 889 **(c)** 2×10^{10}
(d) 216 302 **(e)** 2 180 324 160
(f) 5.229×10^{10}

4 **(a)** Task 3
(b) The columns are all the same height.
(c) Task 1 **(d)** Task 2

5 **(a)** 2020
(b) 2022
(c) Queensland
(d) There are few to none each year.
(e) Tasmania, although WA and SA are also fairly consistent.

6 **(a)** No, just the percentage of males and females.
(b) Males **(c)** Camera **(d)** Reminder

Topic 16: Probability (compound events, relative frequencies)

Recall

I can write fractions in their simplest form

1 **(a)** $\frac{2}{5}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{10}{17}$ **(d)** $\frac{9}{16}$

I can write the product of fractions in simplest form

1 **(a)** $\frac{2}{15}$ **(b)** $\frac{3}{10}$ **(c)** $\frac{3}{40}$ **(d)** $\frac{4}{25}$

I can identify the complement probability

1 **(a)** $\frac{4}{7}$ **(b)** $\frac{4}{9}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{2}{3}$

I can write fractions in their equivalent decimal form

1 **(a)** 0.22 **(b)** 0.24 **(c)** $0.\dot{7}$ **(d)** 0.528

I can list the sample space for all possible outcomes of an event

1 **(a)** {1, 2, 3, 4, 5, 6} **(b)** {plain, patterned, textured}
(c) {2, 4, 6, 8, 10, 12} **(d)** {orange, white, yellow}

I can write probabilities

1 **(a)** $\frac{1}{10}$ **(b)** $\frac{1}{5}$ **(c)** $\frac{1}{2}$
(d) $\frac{3}{5}$ **(e)** $\frac{2}{5}$

16.1 Use tables and tree diagrams to represent two- and three-step chance events

SC 1

1 (a)

		Case B		
		Pink	**Blue**	**Yellow**
Case A	**Pink**	PP	PB	PY
	Blue	BP	BB	BY
	Yellow	YP	YB	YY

Sample space is {PP, PB, PY, BP, BB, BY, YP, YB, YY}

(b) $\frac{1}{9}$ **(c)** $\frac{1}{9}$ **(d)** $\frac{1}{9}$ **(e)** $\frac{6}{9} = \frac{2}{3}$

2 (a)

		Spinner B			
		1	**2**	**3**	**4**
Spinner A	**1**	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7

(b) $\frac{6}{12} = \frac{1}{2}$ **(c)** $\frac{3}{12} = \frac{1}{4}$ **(d)** $\frac{3}{12} = \frac{1}{4}$

(e) The probabilities add to 1. The sum of the numbers must be one of < 4, $= 4$ or > 4, so one of these events must occur, hence a probability of 1.

3 (a)

		Letter 2			
		E	**A**	**C**	**H**
Letter 1	**E**	EE	EA	EC	EH
	A	AE	AA	AC	AH
	C	CE	CA	CC	CH
	H	HE	HA	HC	HH

(b) $\frac{1}{16}$ **(c)** $\frac{1}{16}$ **(d)** $\frac{4}{16} = \frac{1}{4}$ **(e)** $\frac{12}{16} = \frac{3}{4}$

4 (a)

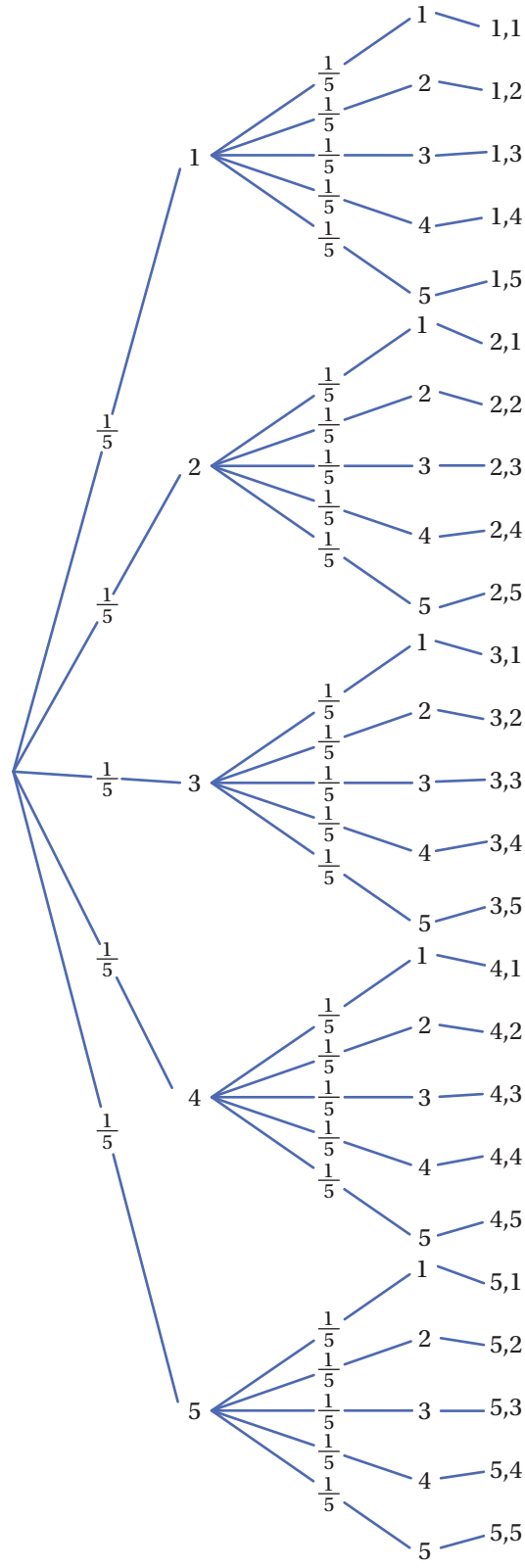
		Letter 2			
		M	**A**	**T**	**H**
Letter 1	**M**	MM	MA	MT	MH
	A	AM	AA	AT	AH
	T	TM	TA	TT	TH
	H	HM	HA	HT	HH

(b) $\frac{1}{16}$ **(c)** $\frac{12}{16} = \frac{3}{4}$ **(d)** $\frac{1}{16}$

(e) $\frac{9}{16}$ **(f)** $\frac{1}{16}$ **(g)** $\frac{2}{16} = \frac{1}{8}$

SC 2

1 (a)

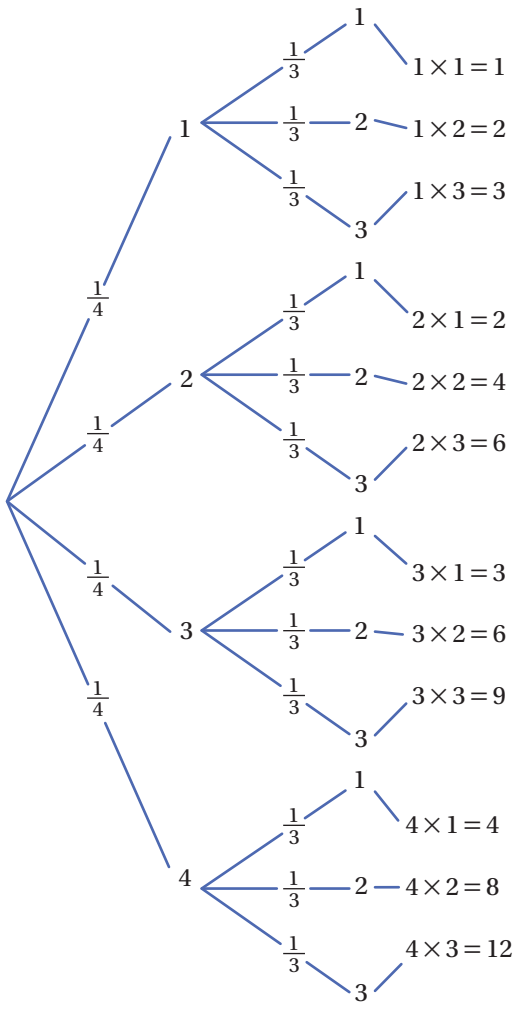


(b) $\frac{1}{25}$ **(c)** $\frac{1}{25}$ **(d)** $\frac{9}{25}$ **(e)** $\frac{4}{25}$

(f) There is also the possibility of obtaining one even and one odd number.

(g) $\frac{16}{25}$ **(h)** $\frac{1}{25}$ **(i)** $\frac{6}{25}$ **(j)** $\frac{12}{25}$

2 (a)



(b) (i) $\frac{1}{12}$ **(ii)** $\frac{2}{12}=\frac{1}{6}$

(iii) $\frac{2}{12}=\frac{1}{6}$ **(iv)** $\frac{9}{12}=\frac{3}{4}$

(v) 0 **(vi)** $\frac{5}{12}$

3 (a) $\frac{1}{8}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{3}{8}$ **(d)** $\frac{7}{8}$

4 (a) $\frac{8}{125}$ **(b)** $\frac{36}{125}$

(c) $\Pr(\text{lose})=1-\left(\frac{36}{125}+\frac{8}{125}\right)=\frac{81}{125}$

SC 3

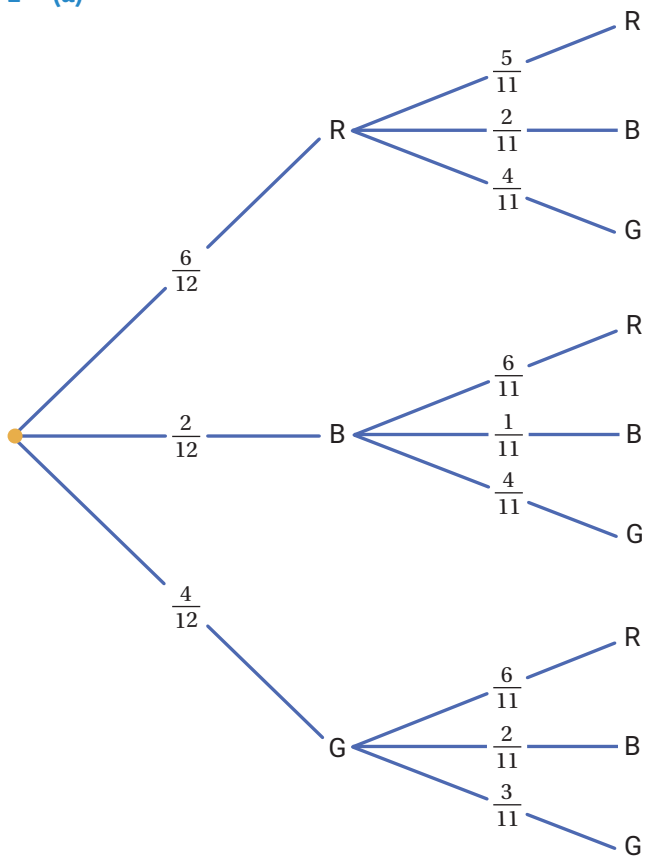
1 (a) $\frac{1}{6}$

(b) (i) $\frac{1}{5}$ **(ii)** $\frac{1}{4}$ **(iii)** $\frac{1}{3}$

(iv) $\frac{1}{2}$ **(v)** 1

(c) (i) $\frac{3}{6}=\frac{1}{2}$ **(ii)** $\frac{4}{6}=\frac{2}{3}$

2 (a)

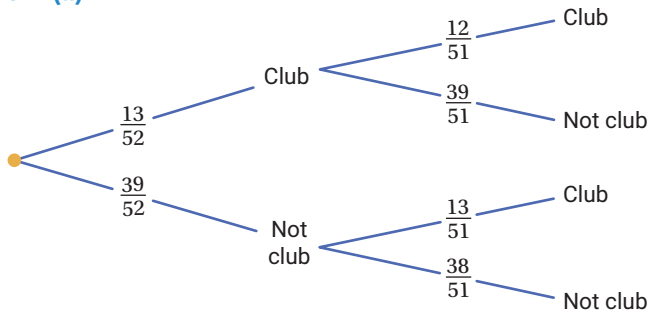


(b) (i) $\frac{2}{132}=\frac{1}{66}$ **(ii)** $\frac{5}{22}$ **(iii)** $\frac{1}{11}$

(iv) $\frac{30}{132}+\frac{2}{132}+\frac{12}{132}=\frac{44}{132}=\frac{1}{3}$ **(v)** $\frac{2}{11}$

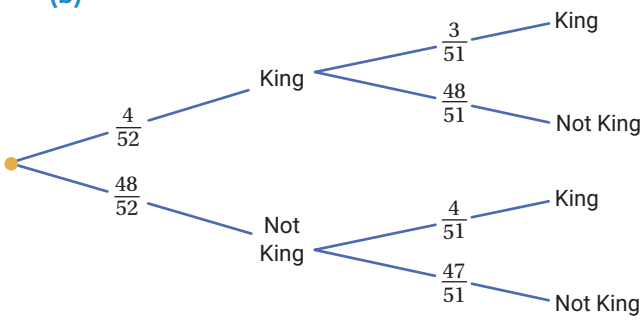
(vi) $\frac{4}{11}$

3 (a)



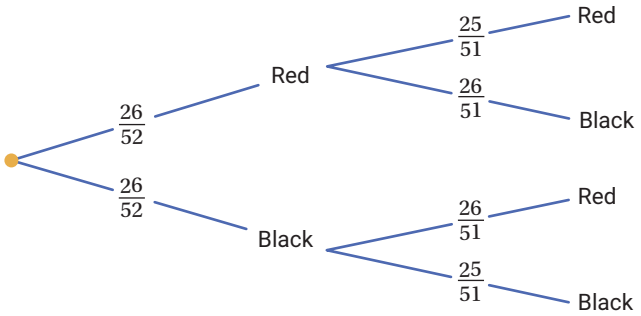
$\Pr(\text{2 clubs})=\frac{13}{52}\times\frac{12}{51}=\frac{156}{2652}=\frac{1}{17}$

(b)



$\Pr(\text{2 Kings}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

(c)



$\Pr(\text{1 red, 1 black}) = \frac{1}{2} \times \frac{26}{51} + \frac{1}{2} \times \frac{26}{51} = \frac{52}{102} = \frac{26}{51}$

4 **(a)** $0.784 = \frac{98}{125}$ **(b)** $\frac{5}{6}$

16.2 Calculate the probability of compound events

SC 1

1 **(a)** $\frac{2}{40} = \frac{1}{20}$ **(b)** $\frac{30}{40} = \frac{3}{4}$ **(c)** $\frac{15}{40} = \frac{3}{8}$

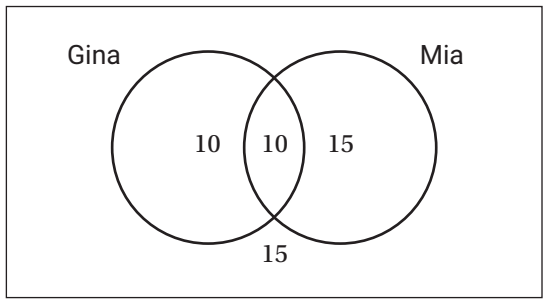
(d) $\frac{38}{40} = \frac{19}{20}$ **(e)** $\frac{31}{40}$

2 **(a)** $\frac{4}{30} = \frac{2}{15}$ **(b)** $\frac{10}{30} = \frac{1}{3}$

(c) $\frac{14}{30} = \frac{7}{15}$ **(d)** $\frac{6}{30} = \frac{1}{5}$

(e) $\frac{20}{30} = \frac{2}{3}$ **(f)** $\frac{16}{30} = \frac{8}{15}$ **(g)** $\frac{18}{30} = \frac{3}{5}$

3 **(a)**



(b) **(i)** $\frac{20}{50} = \frac{2}{5}$ **(ii)** $\frac{25}{50} = \frac{1}{2}$ **(iii)** $\frac{35}{50} = \frac{7}{10}$ **(iv)** $\frac{25}{50} = \frac{1}{2}$

4 **(a)** $\Pr(\text{mammal and water animal}) = \frac{1}{10}$

(b) $\Pr(\text{mammal or water animal but not both}) = \frac{3}{5}$

(c) $\Pr(\text{not a mammal and not a water animal}) = \frac{3}{10}$

SC 2

1 **(a)** $\Pr(\text{male and disagree}) = \frac{12}{25}$

(b) $\Pr(\text{female or agree}) = \frac{13}{25}$

2 **(a)** $\frac{60}{110} = \frac{6}{11}$ **(b)** $\frac{75}{110} = \frac{15}{22}$ **(c)** $\frac{15}{110} = \frac{3}{22}$

3 **(a)**

	Near the front	Near the back	
Children	17	21	38
Adults	9	13	22
	26	34	60

(b) **(i)** $\frac{26}{60} = \frac{13}{30}$ **(ii)** $\frac{13}{60}$ **(iii)** $\frac{38}{60} = \frac{19}{30}$

(iv) $\frac{43}{60}$ **(v)** $\frac{30}{60} = \frac{1}{2}$

SC 3

1 **(a)** $\frac{1}{12}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$

2 **(a)** $\frac{9}{25}$ **(b)** $\frac{6}{25}$ **(c)** $\frac{12}{25}$ **(d)** $\frac{3}{25}$

3 **(a)** $\frac{1}{4}$ **(b)** $\frac{4}{36} = \frac{1}{9}$ **(c)** $\frac{18}{36} = \frac{1}{2}$

4 **(a)** $\frac{18}{30} = \frac{3}{5}$ **(b)** $\frac{8+4}{30} = \frac{12}{30} = \frac{2}{5}$

(c) $\frac{8}{30} = \frac{4}{15}$ **(d)** $\frac{6}{30} = \frac{1}{5}$

(e) $\frac{26}{30} = \frac{13}{15}$ **(f)** $\frac{20}{30} = \frac{2}{3}$

16.3 Estimate the probability of events

SC 1

1 **(a)** 148 **(b)** 148

(c) The long-term average can be used to estimate a future performance.

2 $\frac{125}{156} = 0.80$

3 (a)

	City drivers		Country drivers	
30-year-old drivers	Male	Female	Male	Female
Premium	$1176	$980	$1008	$756

(b) There is less traffic in the country, so there are fewer accidents.

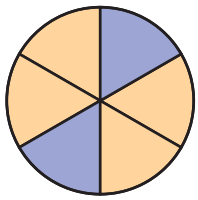
4 (a) $\frac{1}{3}$ yellow, $\frac{2}{3}$ green

(b) 3333 yellow, 6667 green

(c) (i) The long-term results are quite close to my estimation.

(ii) The more trials there are, the more reliable the estimation.

(d) Sample answer:



5 (a) 0.20 **(b)** 10 **(c)** $\frac{41}{50} = 0.82$

(d) Yes. From the results provided, we estimate an experimental probability of a multiple of 5 because $\frac{9}{50} = 0.18$. This value is close to the answer in part **(a)**.

(e) With a large number of trials, the theoretical and estimated probabilities become closer.

SC 2

1 360

2 (a) 113

(b) The survey results show that only 13.3% agree to a change, 11.7% want to keep it as is and 75% of the students have no opinion. Based on that alone, there is no urgency to change the name. The sample size was not huge, so a more extensive survey could be conducted with a larger sample size.

3 (a) Pr(voting for shopping precinct) = $\frac{110}{200} = \frac{11}{20}$

$\frac{110}{200} \times 600\,000 = 330\,000$

(b)

Pr(under 18 and voting for shopping precinct) = $\frac{24}{200} = \frac{6}{50}$

$\frac{6}{50} \times 600\,000 = 72\,000$

(c)

$\Pr(\text{under 18 or voting for childcare centre}) = \frac{108}{200} = \frac{27}{50}$

$\frac{27}{50} \times 600\,000 = 324\,000$

4 (a) 96 000 **(b)** 42 000 **(c)** 22 000

(d) 10 000 **(e)** 54 000 **(f)** 44 000

5 (a) 5 **(b)** 29 **(c)** 150

16.4 Design and conduct experiments and simulations

SC 1

1 Sample answer:

	Me	Toni
Quarter 1	2.5	3.4
Quarter 2	4.1	6.1
Quarter 3	6.2	3.6
Quarter 4	3.4	3.4
Total score	15.12	15.15
Points	102	105
Toni won the game by 3 points.		

2 (a) Answers will vary depending upon the number of batters each player has. Expect a mean score of 16 per batter.

(b) (i) $\frac{1}{6}$ **(ii)** $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

(iii) $\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ **(iv)** $\frac{36}{216} + \frac{30}{216} + \frac{25}{216} = \frac{91}{216}$

(c) $1 + 2 + 3 + 4 + 6 = 16$

(d) Sample answer. The first 5 batters are usually better than the last 5 batters. For the first set of batters, instead of a 5 meaning the batter is out, an appeal can be made to the umpire: 'How's that?' The die is rolled again to get the umpire's decision. An even number is out, and an odd number is not out, and the batter continues.

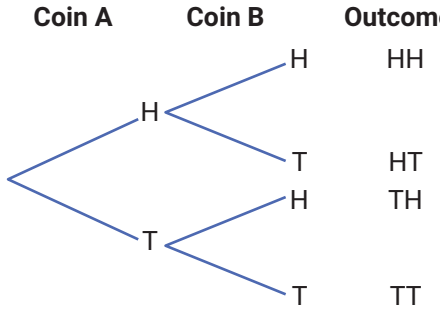
3 (a) $\frac{1}{6}$

(b) Stay standing, because the total is not large.

(c) $\frac{1}{6}$, the die has no memory of what has happened before.

4 Student answers will vary. They could explore sports such as tennis, basketball or athletics, among others.

5 (a)



Sample space is {HH, HT, TH, TT}

(b) (i) $\frac{1}{4}$ **(ii)** $\frac{1}{4}$ **(iii)** $\frac{1}{2}$

(c) (i) $\frac{1}{4}$ **(ii)** $\frac{3}{16}$ **(iii)** $\frac{9}{64}$

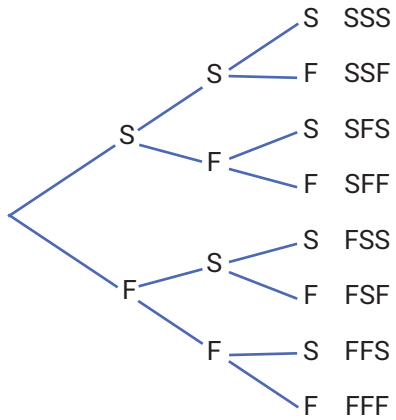
(d) Answers will vary.

SC 2

1 (a) Individual results depending upon which set of numbers were chosen.

(b) The mean should be closer to the 90% because we now have a sample size of 500.

2 (a) eight outcomes



(b) No. the probability of S and F are not the same, so the probability of SSS is different to FFF, for instance.

(c)

Number of successes	List of different configurations to achieve the number of successes	Calculation of probability
3	SSS	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}$
2	SSF or SFS or FSS	$\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}\times 3=\frac{6}{27}$
1	SFF or FSF or FFS	$\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}\times 3=\frac{12}{27}$
0	FFF	$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$
	Total	1

(d) That there is no mistake in the calculations.

(e) (i) The income for 27 trials is $27 \times \$3 = \$81$.

(ii) \$54

(f) It allows them to arrange the prizes so a long-term profit is made.

3 (a) The totals 5 and 30 are the least likely, so have been given the top prize.

(b) Numbers in the middle tier are the second most unlikely and the bottom shelf is the third most unlikely group. The most likely group of numbers wins no prize at all.

4 Sample answer

Knowing the long-term statistics will allow companies to set their premiums (cost for customers), allowing them to pay the claims, but also make a profit.

5 Sample answer

The larger the number of trials, the closer the results get to the theoretical probability.

Topic 1: Real numbers

Topic review

Multiple choice

1 B
2 D
3 A
4 C
5 A
6 D
7 B
8 C
9 D
10 A

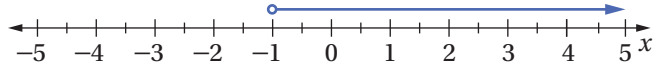
Short answer

1 (a) $\frac{200}{99}$ **(b)** $\frac{91}{6}$ **(c)** $\frac{137}{11}$ **(d)** $\frac{10\,196}{333}$

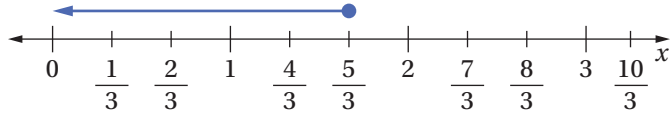
2 (a) $\sqrt[3]{(-3)^3} = -3$: rational **(b)** $5\sqrt{-1}$: not real

(c) $2\sqrt[3]{5}$: irrational **(d)** $12\sqrt{30}$: irrational

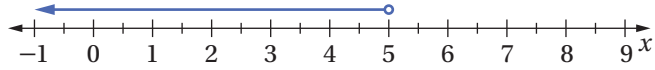
3 (a) $x > -1$



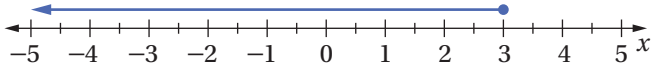
(b) $x \leq \frac{5}{3}$



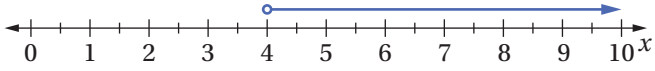
(c) $x < 5$



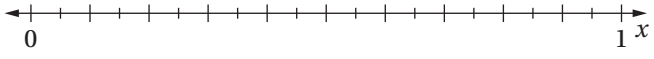
4 **(a)** $x \leq 3$



(b) $x > 4$



(c) $x \geq 0.4$



5 **(a)** 4.359 **(b)** 3.107 **(c)** 2.749 **(d)** 2.099

6 **(a)** 3 and 4 **(b)** 3 and 4 **(c)** 3 and 4

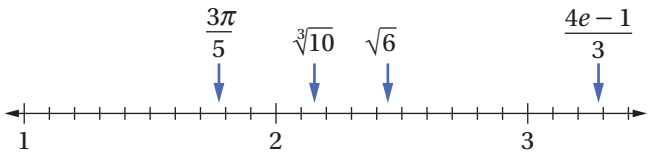
7 **(a)** $\frac{1024\pi}{3}$ cm^3 **(b)** 144 π cm^3

8 **(a)** $\sqrt{500} = 10\sqrt{5}$ cm

(b) $\sqrt{8} = 2\sqrt{2}$ m

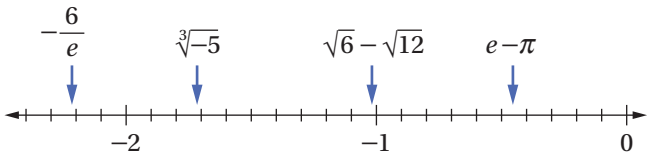
(c) $\sqrt{117} = 3\sqrt{13}$ mm

9 **(a)** $\frac{3\pi}{5} \approx 1.88$, $\sqrt{6} \approx 2.45$, $\sqrt[3]{10} \approx 2.15$, $\frac{4e-1}{3} \approx 3.29$



In order: $\frac{3\pi}{5}, \sqrt[3]{10}, \sqrt{6}, \frac{4e-1}{3}$

(b) $\sqrt{6} - \sqrt{12} \approx -1.01$, $e - \pi \approx -0.42$, $\sqrt[3]{-5} \approx -1.71$, $-\frac{6}{e} \approx -2.21$



In order: $-\frac{6}{e}, \sqrt[3]{-5}, \sqrt{6} - \sqrt{12}, e - \pi$

10 **(a)** 6 units and 1 unit

(b) 4 units and 7 units

(c) 3 units and 7 units

Extended response

1 **(a)** **(i)** $n = 1, 4, 9, 16$

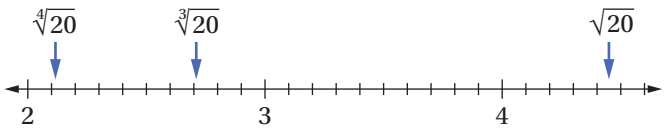
(ii) $n = 64, 81$

(b) **(i)** $n = 216, 343$

(ii) $n = -125$

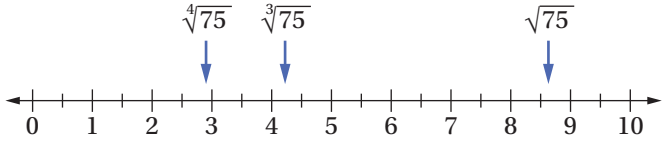
2 **(a)** **(i)** $4.4 < \sqrt{20} < 4.5$, $2.7 < \sqrt[3]{20} < 2.8$, $2.1 < \sqrt[4]{20} < 2.2$

(ii)



(b) **(i)** $8.6 < \sqrt{75} < 8.7$, $4.2 < \sqrt[3]{75} < 4.3$, $2.9 < \sqrt[4]{75} < 3.0$

(ii)



3 **(a)** $a = \sqrt{37}$ cm

(b) $b = \sqrt{38}$ cm

(c) $c = \sqrt{39}$ cm

(d) $d = \sqrt{40}$ cm, $e = \sqrt{41}$ cm

Topic 2: Exponents

Topic review

Multiple choice

1 D
2 C
3 B
4 D
5 A
6 C
7 A
8 D
9 B
10 D
11 D
12 B
13 A
14 D
15 D

Short answer

1 $13^2 \times 3^3$

2 $(4 \times 4 \times 4) \times 4 \times (4 \times 4 \times 4 \times 4 \times 4) = 4^9$

3 **(a)** $5^2 \times 3^3$ **(b)** $5^4 \times 2^6$

4 **(a)** $20x^3y^{11}$ **(b)** $48a^3b^5$

5 2×3^4

6 **(a)** $(2^4) \times (2^4) = (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^8$

(b) $(4^3) \times (4^3) \times (4^3) = (4 \times 4 \times 4) \times (4 \times 4 \times 4) \times (4 \times 4 \times 4) = 4^9$

7 **(a)** 6^8 **(b)** There are 8 factors of 6.

8 **(a)** $x = 7$ **(b)** $x = 3$

9 **(a)** 1 **(b)** 1 **(c)** 6

10 $\frac{1}{5^4}$

11 **(a)** $2 \times 10^2 + 6 \times 10^0 + 7 \times 10^{-1} + 9 \times 10^{-4}$

(b) $1 \times 10^{-3} + 2 \times 10^{-4} + 4 \times 10^{-5}$

(c) $1 \times 10^0 + 9 \times 10^{-3} + 6 \times 10^{-4} + 1 \times 10^{-5}$

(d) $2 \times 10^1 + 5 \times 10^0 + 5 \times 10^{-1} + 5 \times 10^{-2}$

12 **(a)** $x = 4$ **(b)** $x = 9$ **(c)** $x = 12$

13 No. $(4a)^3 = 4a \times 4a \times 4a = 64a^3 \neq 4a^3$, except when $a = 0$.

14 $\dfrac{90y^3}{x^3}$

Extended response

1 **(a)** **(i)** 3^7

(ii) The base is 3 and the exponent is 7.

(b) **(i)** $a^m \times a^n = a^{m+n}$

(ii) $5^4a^4b^5$

(iii) The exponent notation shows that there are four factors of 5, four factors of a and five factors of b.

(c) **(i)** $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$

(ii) $\dfrac{7\times7\times7\times7\times7\times2\times2\times2}{7\times7\times2}$

(iii) $7^3 \times 2^2$

(d) **(i)** $3^{12}p^2$ **(ii)** 4^2ab^3

2 **(a)** $\dfrac{2^2}{2^2} = 1$, $\dfrac{2^2}{2^2} = 2^{2-2} = 2^0$. Hence, $2^0 = 1$.

(b) **(i)** $(4^3)^2 = 4^3 \times 4^3 = 4^6$

(ii) $(a^m)^n = a^{m\times n} = a^{mn}$

(iii) $(4^3)^2 = 4^{3\times2} = 4^6$

(c)

Expression	Expansion	Simplified form
$(3^5)^3$	$3^5 \times 3^5 \times 3^5$	3^{15}
$(4^7)^2$	$4^7 \times 4^7$	4^{14}
$(13^6)^4$	$13^6 \times 13^6 \times 13^6 \times 13^6$	13^{24}
$\dfrac{(2^2)^5}{2^3}$	$\dfrac{2^2\times2^2\times2^2\times2^2\times2^2}{2^3}$	2^7
$\dfrac{m^4(n^3)^2}{m^2n}$	$\dfrac{m^4\times n^3\times n^3}{m^2n}$	m^2n^5

(d) **(i)** $8m^{2-x} = 8 \Rightarrow 2 - x = 0$, so $x = 2$.

(ii) $2a^{12-x} = 2 \Rightarrow 12 - x = 0$, so $x = 12$.

3 **(a)** **(i)** 13^{-1}

(ii) 3×31^{-1}

(b) **(i)** 37.25

(ii) 965.6

(c) **(i)** $1 \times 10^0 + 3 \times 10^{-1} + 2 \times 10^{-2} + 6 \times 10^{-4}$

(ii) $1 \times 10^3 + 1 \times 10^0 + 4 \times 10^{-1} + 1 \times 10^{-4}$

(d) **(i)** 2^{-4}

(ii) 2^{-6}

(e) **(i)** $\dfrac{1}{8^3}$ **(ii)** $\dfrac{1}{8\times8\times8}$ **(iii)** $\dfrac{1}{512}$

4 **(a)** $\dfrac{4^6\times2^5}{8^5} = \dfrac{\left(2^2\right)^6\times2^5}{\left(2^3\right)^5} = \dfrac{2^{2\times6+5}}{2^{3\times5}} = \dfrac{2^{17}}{2^{15}} = 2^2$.

Hence, $4^6 \times 2^5$ is four times greater than 8^5.

(b) $\dfrac{2^9\times3^8}{6^8} = \dfrac{2^9\times3^8}{(2\times3)^8} = \dfrac{2^9\times3^8}{2^8\times3^8} = 2$. Hence, 6^8 is two times smaller when compared to $2^9 \times 3^8$.

(c) $\dfrac{16\times27\times625}{4\times729\times25} = \dfrac{2^4\times3^3\times5^4}{2^2\times3^6\times5^2} = \dfrac{2^2\times5^2}{3^3} = \dfrac{100}{27}$

(d) $\dfrac{12x^4\times2z^6}{6xz^4} = \dfrac{2^2\times3\times2}{2\times3} \times \dfrac{x^4}{x} \times \dfrac{z^6}{z^4} = 4x^3z^2$

Topic 3: Algebra (binomials)

Topic review

Multiple choice

1 B
2 D
3 A
4 A
5 C
6 B
7 A
8 C
9 D
10 C
11 A
12 D
13 B
14 A
15 C
16 D
17 B

Short answer

1 **(a)** $(60+4)(30+8) = 64\times38$
$= 2432$

(b) $(100+3)(50+9) = 103\times59$
$= 6077$

2 **(a)** $42\times31 = (40+2)\times(30+1)$
$= 40\times30 + 40\times1 + 2\times30 + 2\times1$
$= 1200 + 40 + 60 + 2$
$= 1302$

(b) $106\times43 = (100+6)\times(40+3)$
$= 100\times40 + 100\times3 + 6\times40 + 6\times3$
$= 4000 + 300 + 240 + 18$
$= 4558$

3 **(a)** 2040 **(b)** 532 **(c)** 171 **(d)** 90

4 **(a)** This is not correct. The first bracket should be $(40 - 1)$.

(b) There are no additional errors in the working other than the incorrect first step. The correct answer is 1638.

5 **(a)** $8a^2 + 26a + 15$ **(b)** $21a^2 - a - 2$

(c) $24a^2 - 2a - 15$ **(d)** $50a^2 - 75a + 28$

6 **(a)** $(4a^2 - 12a + 9)\,\text{cm}^2$ **(b)** $(9a^2 - 6a - 8)\,\text{cm}^2$

(c) $(72x^2 - 120x + 50)\,\text{cm}^2$

7 **(a)** $-12a$ **(b)** $12a$ **(c)** $-24ab$ **(d)** $24ab$

8 **(a)** $x^3 - x^2 - 5x - 3$ **(b)** $x^3 - 4x^2 - 3x + 18$

9 **(a)** $23^2 = (20+3)^2$

$= 20^2 + 2\times 20\times 3 + 3^2$

$= 400 + 120 + 9$

$= 529$

(b) $29^2 = (30-1)^2$

$= 30^2 - 2\times 30\times 1 + 1^2$

$= 900 - 60 + 1$

$= 841$

10 **(a)** $25 - 144x^2$ **(b)** $49x^2 - 4$

11 **(a)** $25m^2 - 36n^2$ **(b)** $100s^2 - 144t^2$

12 $38\times 42 = (40-2)(40+2)$

$= 40^2 - 2^2$

$= 1600 - 4$

$= 1596$

13 **(a)** $(x-5)(x-5) = (x-5)^2$

(b) $(3x+2)(3x+2) = (3x+2)^2$

14 **(a)** $(8 - 9z)(8 + 9z)$

(b) $(20k - 11t)(20k + 11t)$

15 **(a)** $15(x - 20)$ **(b)** $7ab(2a + 3b)$

(c) $4a\text{bc}(3 + 4\text{b} - 6\text{c})$

16 **(a)** $2(3t - 1)(2k - 3)$ **(b)** $5t(6k + 5)(t - 3)$

(c) $25a^2b(4c - 3)(1 + 5b)$

17 **(a)** $(2x + 5)$ **(b)** $(3t - 4)$

18 **(a)** $(x + 5)(x - 2)$ **(b)** $(x - 6)(x - 7)$

(c) $(2x + 1)(x - 4)$

Extended response

1 **(a)** $A = \frac{1}{2}(x+2)(x-2) = \frac{1}{2}(x^2-4) = \frac{1}{2}x^2 - 2$

(b) $70\,\text{cm}^2$ **(c)** $x > 2$

(d) $A = (x + 5)(x - 2) = x^2 + 3x - 10$

(e) $120\,\text{cm}^2$

2 **(a)** $A = \pi(R - r)(R + r)$ **(b)** $A = 24\pi\,\text{cm}^2$

(c) $A = \pi(R - 2r)(R + 2r)$ **(d)** $A = 64\pi\,\text{cm}^2$

(e) The next number of circles is nine.

(f) $A = \pi(R - 3r)(R + 3r)$

3 **(a)** $x^2 + 2x + 1 = (x + 1)^2$

(b) $x^4 + 2x^2 + 1 = (x^2 + 1)^2$

(c) $x^6 + 2x^3 + 1 = (x^3 + 1)^2$

(d) $x^2 + x - 42 = (x + 7)(x - 6)$

(e) Let $a = x^4$

(f) Let $a = x^4$

$x^8 + x^4 - 42 = a^2 + a - 42$

$= (a + 7)(a - 6)$

Now revert to the original form.

$x^8 + x^4 - 42 = (x^4 + 7)(x^4 - 6)$

Topic 4: Linear relationships and graphs

Topic review

Multiple choice

1 B

2 C

3 B

4 D

5 A

6 C

7 C

8 B

9 A

10 C

11 C

12 A

13 C

14 B

15 C

16 D

17 B

18 C

Short answer

1 zero

2 **(a)** $m = 3$ **(b)** $m = 4$

(c) The table does not produce a straight-line graph.

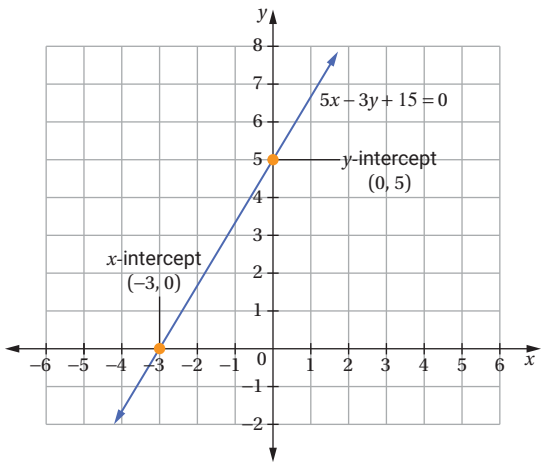
(d) Change 14 to 10.

3 **(a)** (−4,−14) and (3,7) **(b)** (−3,−9) and (5,11)

(c) (1,2) and (7,11)

4 **(a)** (−3,0) **(b)** (0,5)

(c)



(d) $m = \frac{5}{3}$

5 **(a)** $(-1,-7)$ and $(1,-1)$ **(b)** $(-3,7)$ and $(3,3)$

6 **(a)** $m = -\frac{2}{3}$ **(b)** $k = 6$ **(c)** $v = 9$

7 **(a)** $y = -x + 2$ **(b)** $y = -\frac{x}{2} + \frac{2}{5}$

(c) $y = -\frac{5x}{6} + \frac{23}{6}$

8 **(a)** $x = \frac{1}{2}$ **(b)** $x = \frac{49}{3}$ **(c)** $x = \frac{7}{13}$ **(d)** $x = \frac{2}{7}$

9 **(a)** $C = 50n + 65$ **(b)** \$415

(c) 4 hours

(d) The intercept on the vertical axis

10 **(a)** −0.0463 seconds/year

(b) 9.74 seconds

(c) The rapid rate of improvement in record times between 1922 and 1963 was not sustainable.

(d) 9.93 seconds

(e) This is a long time for a record to exist. There may have been some unusual circumstances surrounding it, or the athlete who set the record was quite exceptional.

11 **(a)** $7\sqrt{10}$ units **(b)** $6\sqrt{10}$ units

12 **(a)** $R(0,2.75)$ **(b)** $S(-0.5,-2.75)$

(c) $\sqrt{30.5}$ units or $\frac{\sqrt{122}}{2}$ units

(d) $(-0.25,0)$

13 $(-2,3)$ and $(10,21)$

14 **(a)** 6 units vertically down

(b) 5 units vertically up

15 **(a)** Reflected in the x-axis and translated 4 units vertically up

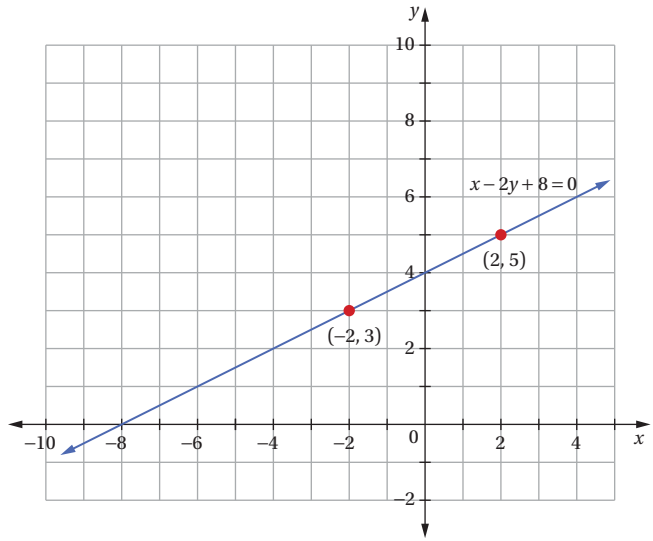
(b) Dilated by a factor of 2 and translated 3 units vertically down

(c) Dilated by a factor of 4, reflected in the x-axis and translated 8 units vertically up

(d) Dilated by a factor of $\frac{5}{6}$ and translated $\frac{2}{3}$ unit vertically down

Extended response

1 **(a)**



(b) $x - 2y + 8 = 0$ **(c)** (0,4)

(d) (8,0) **(e)** $4\sqrt{5}$ units

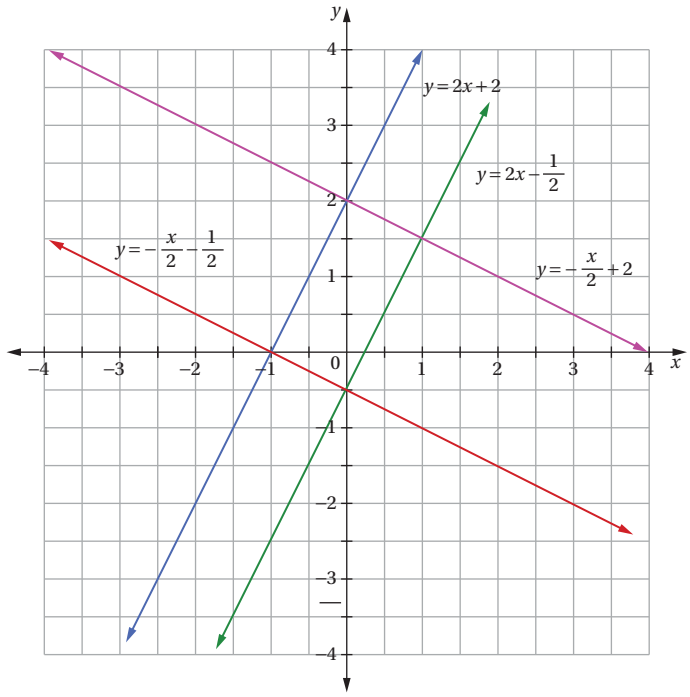
2 **(a)** \$40.60 **(b)** \$2.03/km **(c)** 32.74 km

(d) 8.90% **(e)** 18.13 km

3 **(a)** $m = -\frac{1}{2}$ **(b)** $y = -\frac{x}{2} + 2$

(c) $y = -\frac{x}{2} - \frac{1}{2}$ **(d)** $y = 2x - \frac{1}{2}$

(e)

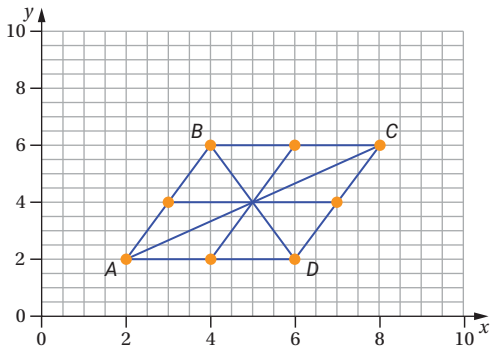


(f) A rectangle is formed.

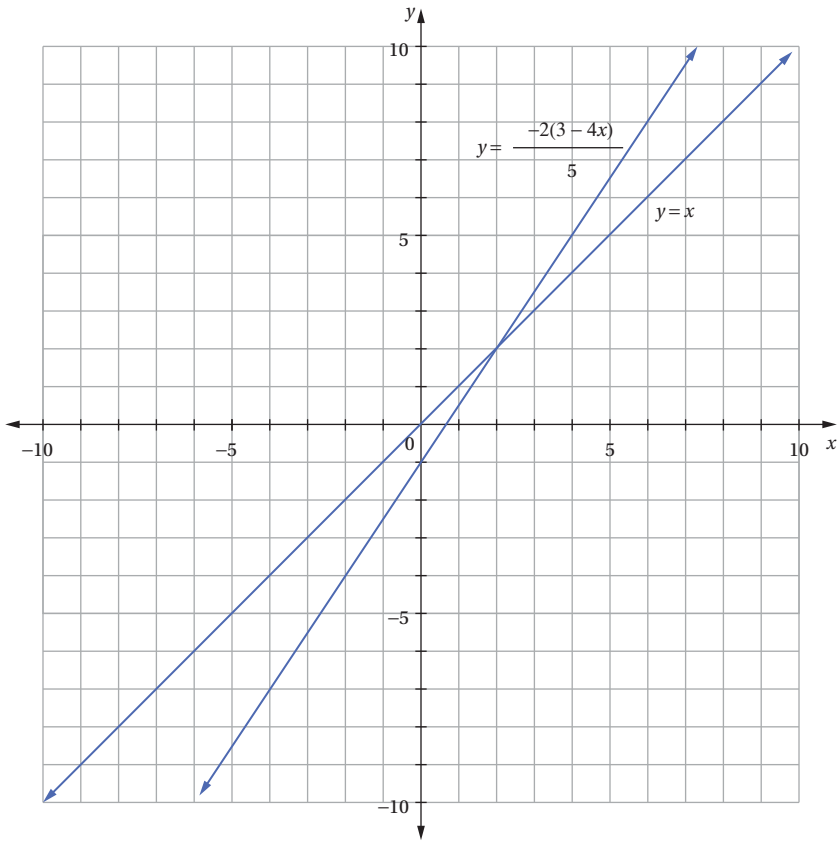
4 **(a)** $A(2,2)$; $B(4,6)$; $C(8,6)$; $D(6,2)$

(b) $\overline{AB}$: (3,4); $\overline{BC}$: (6,6); $\overline{CD}$: (7,4); $\overline{DA}$: (4,2)

(c)



(c)



(d) $(3,3) \rightarrow \left(3, \frac{24}{5}\right) \rightarrow \left(3, \frac{18}{5}\right)$

(e) This gives $y = \frac{18}{5}$, which proves the transformed coordinates given above are correct.

(d) All of the intervals intersect at the same point, (5,4).

5 **(a)** $y = \frac{8x}{5} - \frac{6}{5}$

(b) Dilated by a factor of $\frac{8}{5}$ and translated $\frac{6}{5}$ units vertically down

Topic 5: Quadratic relationships and graphs

Topic review

Multiple choice

1 B
2 C
3 B
4 C
5 A
6 D
7 B

8 B
9 A
10 C
11 C
12 C
13 B
14 C
15 A
16 D
17 C

Short answer

1 (a)

x	-2	-1	0	1	2	3	4
y	29	14	5	2	5	14	29
First difference		-15	-9	-3	3	9	15
Second difference			6	6	6	6	6

(b)

x	-2	-1	0	1	2	3	4
y	-18	-9	-4	-3	-6	-13	-24
First difference		9	5	1	-3	-7	-11
Second difference			-4	-4	-4	-4	-4

2 (a) y-intercept: $(0,-9)$
x-intercepts: none
Turning point: $(0,-9)$, maximum

(b) y-intercept: $(0,-12)$
x-intercepts: $(-4,0)$ and $(3,0)$
Turning point: $\left(-\frac{1}{2},-12\frac{1}{4}\right)$, minimum

(c) y-intercept: $(0,8)$
x-intercepts: none
Turning point: $(2,4)$, minimum

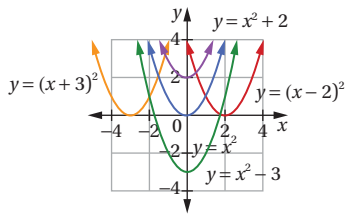
3 (a)

x	0	1	2	3	4	5	6
y	13	3	-3	-5	-3	3	13

(b)

x	-4	-3	-2	-1	0	1	2
y	-21	-5	3	6	3	-5	-21

4



5 (a) $(-4,0)$ **(b)** $(0,2)$ **(c)** $(2,0)$ **(d)** $(0,-3)$

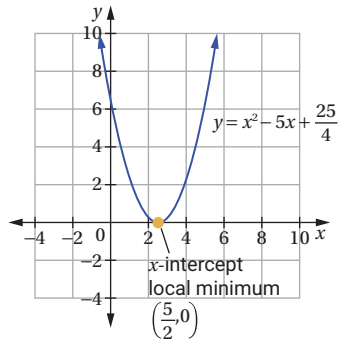
6 (a) $y=-\frac{x^2}{2}$ **(b)** $y=2x^2+3$

7 (a) $y=9$ **(b)** $y=18$
(c) $y=-27$ **(d)** $y=4.5$

8 (a) $y=\frac{(x+4)^2}{2}+3$ **(b)** $y=-2x^2-6$

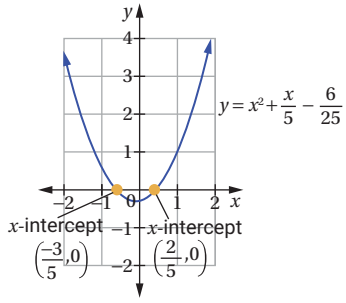
9 (a) $x=-8$ or $x=8$ **(b)** no solutions
(c) $x=6$ **(d)** $x=-2$ or $x=4$

10 (a)



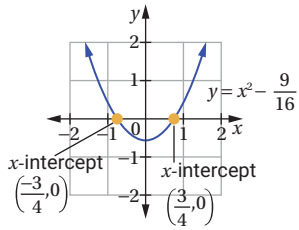
$x=\frac{5}{2}$

(b)



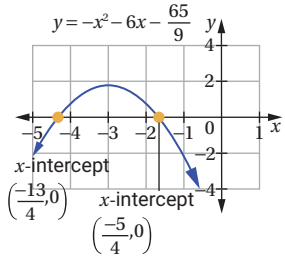
$x=-\frac{3}{5}$ or $x=\frac{2}{5}$

(c)



$x=\pm\frac{3}{4}$

(d)



$x=-\frac{13}{3}$ or $x=-\frac{5}{3}$

11 A: $y=(x-3)(x-4)=x^2-7x+12$
B: $y=-(x+2)(x-5)=-x^2+3x+10$
C: $y=(x-6)^2=x^2-12x+36$
D: $y=-(x+3)^2=-x^2-6x-9$

12 (a) $y = -x^2 + 2x + 35 = -(x + 5)(x - 7)$
x-intercepts: $x = -5$, $x = 7$
(b) $y = x^2 - 4x + 3 = (x - 1)(x - 3)$
x-intercepts: $x = 1$, $x = 3$

13 (a) $x = -1.42$ **(b)** $x = -2.34$

14 (a) $(x - 3)(x + 2) = 24$
(b) $x = 6$ (the solution $x = -5$ is not possible in this practical context)
(c) 3 cm × 8 cm

15 (a) −30°C
(b) $h^2 - 7h - 30 = (h - 10)(h + 3)$
(c) 10 pm
(d) −42.25°C at 3:30 pm

16 (a) The maximum speed was 72.2 km/h, reached 3.8 minutes after starting the journey.
(b) After 7.6 minutes
(c) 1.7 minutes $< t <$ 5.9 minutes

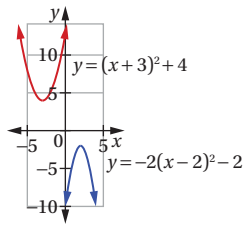
Extended response

1 (a) (i) $y = x^2 + 2x - 8$
(ii) $y = x^2 + 8x + 15$
(iii) $y = x^2 - 8x + 12$
(b) (i) (0,−8)
(ii) (0,15)
(iii) (0,12)
(c) $y = x^2 - (a + b)x + ab$
(d) $(0, ab)$
(e) $\left(\frac{a+b}{2}, -\frac{(a-b)^2}{4}\right)$
(f) Using the values directly: $a = 2$, $b = 6$

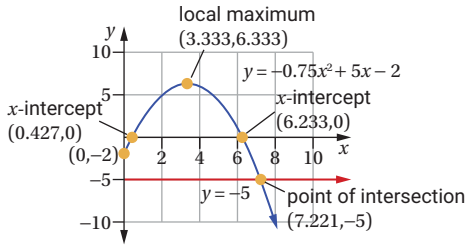
$$x = \frac{2+6}{2} = \frac{8}{2} = 4$$
$$y = (x-2)(x-6)$$
$$y = (4-2)(4-6)$$
$$= 2 \times (-2)$$
$$= -4$$

So, the turning point occurs at (4,−4).
Substituting the values $a = 2$, $b = 6$ into $\left(\frac{a+b}{2}, -\frac{(a-b)^2}{4}\right)$ gives the same result, (4,−4).
This supports the view that the algebraic expressions are correct.

2 (a) $y = (x + 3)^2 + 4$
(b) $y = -(x + 3)^2 - 4$
(c) $y = 2(x + 3)^2 + 8$
(d) $y = (x - 2)^2 + 4$
(e) $y = (x + 3)^2 + 10$
(f) $y = x^2$, $y = (x + 3)^2 + 4$, $y = -(x + 3)^2 - 4$, $y = -2(x + 3)^2 - 8$, $y = -2(x - 2)^2 - 8$, $y = -2(x - 2)^2 - 2$
(g)



3 (a)



(b) −2°C
(c) 6.3°C at 11:20 am
(d) Between 8:26 am and 2:14 pm
(e) 3:13 pm

4 (a) $(x + 2)(x + 6) = x^2 + 8x + 12$
(b) $(3x + 2)(3x + 6) = 9x^2 + 24x + 12$
(c) $8x^2 + 16x$
(d) $8x^2 + 16x = 42$; Solve graphically, or by factorising:

$$8x^2 + 16x - 42 = 0$$
$$4x^2 + 8x - 21 = 0$$
$$(2x + 7)(2x - 3) = 0$$

Hence, $x = 1.5$ is the only possible solution.
Hence, $x + 2 = 3.5$ and $x + 6 = 7.5$.
The path is 1 m wide and the pool is 3.5 m by 7.5 m.

Topic 6: Surface area and volume

Topic review

Multiple choice

1 B
2 C
3 D
4 A
5 C
6 A
7 C
8 D
9 C
10 B

Short answer

1 (a) 9840 cm³ **(b)** 6.375 m³
(c) 1518 mm³ or 1.518 cm³

2 10.2 L

3 (a) 211.68 cm², 185.22 cm³
(b) 5040 mm² or 50.4 cm², 13 500 mm³ or 13.5 cm³

4 (a) 204 cm² **(b)** 64 cm by 20 cm
(c) 1688 cm²

5 $800\,cm^3$

6 **(a)** $291\,cm^2$ **(b)** $180.5\,cm^2$

7 **(a)** 6.10 m and 10.59 m

(b) $469.75\,m^2$

8 **(a)** 189 mL **(b)** 432 mL

(c) 28.35 L, so 15 bottles

9 4.8 cm

10 $3528\,cm^2$

11 $5.10\,m^2$

12 **(a)** $792\,cm^2$ **(b)** $52\,cm^2$

(c) $\frac{52 \times 60}{792} = 3.9393...$, so about 4 times.

Extended response

1 **(a)** $20 \times 1.3^2 = 33.8$, so $33.8\,m^2$

(b) $28\,m^3$; $47.32\,m^3$

(c) $20 \times 0.008 = 0.16\,m^3$, so 160 L; $33.8 \times 0.008 = 0.2704\,m^3$, so 270.4 L

2 **(a)** Short sides of right-angled triangle are 0.6 m and 0.8 m, so the sloping edge is 1 m long.

(b) Floor and ends: $(1.2 + 5 + 1 + 10.2 + 1.8) \times 12 = 230.4$

One side: $1.2 \times 16 + 0.6 \times 10.2 + \frac{0.8 \times 0.6}{2} = 25.56$

Total: $230.4 + 25.26 \times 2$

Area to be tiled: $281.52\,m^2$

(c) $\frac{281.52}{0.4^2} = 1759.5$ tiles, so 18 packs of 100

(d) $25.5 \times 12 = 306.72\,m^3$, so 306.72 kL

3 **(a)** $V = 90\,000\,cm^3$, $SA = 13\,200\,cm^2$

(b) $V = 15\,000\,cm^3$

(c) $V = 60\,000\,cm^3$, $\frac{2}{3}$

(d) Two right-angled triangles of base 60 cm, height 25 cm; an isosceles right-angled triangle of base and height 60 cm; an isosceles triangle of base $\sqrt{60^2 + 60^2} \approx 84.85$ cm and equal sides $\sqrt{60^2 + 25^2} = 65$ cm

(e) A square of side length 60 cm; four right-angled triangles of base 60 cm, height 25 cm; two isosceles triangles of base ~84.85 cm and equal sides 65 cm

(f) The extra area is due to four isosceles triangles of base ~84.85 cm and equal sides 65 cm.

Solving for height: $h = \sqrt{65^2 - 42.425^2} \approx 49.246$ cm

Area of four triangles: $\frac{84.85 \times 49.246}{2} \times 4 = 8356.9...$

The extra surface area is about $8357\,cm^2$.

Topic 7: Circles and cylinders

Topic review

Multiple choice

1 D

2 A

3 B

4 C

5 B

6 D

7 A

8 B

9 C

10 C

11 D

12 B

13 A

14 D

15 C

Short answer

1 **(a)** 50π cm **(b)** 7π m

2 **(a)** $(16\pi + 32)$ cm **(b)** $(9.5\pi + 38)$ mm

3 **(a)** $40.5\pi\,m^2$ **(b)** $30.25\pi\,mm^2$

4 $8577\,cm^2$; $60\,959\,cm^3$

5 $1952\pi\,cm^2$

6 $0.324\pi\,m^3$

7 $54\,450\pi\,cm^3$; $18\,150\pi\,cm^3$

8 **(a)** 60 cm **(b)** $25\,000\pi\,cm^3$

(c) 40 cm **(d)** $\frac{25}{\sqrt{3}}$ cm or $\sqrt{\frac{625}{3}}$ cm

9 603 mL

10 $242\,cm^2$

11 11 310

12 **(a)** $565\,cm^3$ **(b)** $127\,235\,cm^3$

13 $0.65\,m^3$

Extended response

1 **(a)** $9000\pi\,cm^3$ **(b)** $3000\pi\,cm^3$

(c) $36\,000\,cm^3$ **(d)** $3\pi:\pi:12$

2 **(a)** $75\,000\,cm^3$ **(b)** 60 cm

(c) Three-tier: $50^2 + (30 \times 15 + 40 \times 15 + 50 \times 15) \times 4 = 9700$, so $9700\,cm^2$; round: $\pi \times 20^2 + \pi \times 40 \times 60 = 2800\pi$, so $8800\,cm^2$

3 **(a)** $\pi \times 5^2 \times 30 = 750\pi$, so $2356.19\,m^3$

(b) $750\pi + \frac{\pi \times 5^2 \times 1}{3} + \frac{\pi \times 5^2 \times 1.5}{3} = \frac{4625\pi}{6}$, so $2421.64\,m^3$

(c) $24\,m^3$ **(d)** 101

Topic 8: Measurement and scientific notation

Topic review

Multiple choice

1 C
2 D
3 C
4 B
5 A
6 A
7 B
8 D
9 C
10 B
11 C

Short answer

1 (a) 1 kL (b) 1 mm
2 (a) 6 places higher (b) 1 place lower
(c) 12 places lower
3 (a) 5200 m (b) 0.007 s
(c) 2 800 000 000 000 B
4 (a) 6.01×10^{-2} (b) 1.2×10^{-5}
(c) 8.203×10^{-6}
5 (a) 3.5×10^{5} L (b) 3.5×10^{-8} m
(c) 3.5×10^{-7} s (d) 3.5×10^{4} B
6 (a) 78 000 000 (b) 4030
(c) 210 300 000
7 (a) $8.3 \times 10^{-5} < 8.3 \times 10^{-4}$
(b) $3.4 \times 10^{-6} > 5.2 \times 10^{-7}$
(c) $6.1 \times 10^{-3} > 2.4 \times 10^{-4}$
8 (a) $-3.1 \times 10^{6} > -3.1 \times 10^{7}$
(b) $-6.4 \times 10^{5} > -4.8 \times 10^{6}$
(c) $-2.3 \times 10^{-3} < -2.3 \times 10^{-4}$
(d) $-1.5 \times 10^{-5} > -2.7 \times 10^{-4}$
9 (a) 2.0×10^{5} (b) 2.1×10^{9}
(c) 7.8×10^{3} (d) 3.4×10^{4}
10 (a) 860 000 (b) 42 100
(c) 63.15 (d) 7.2
11 (a) 3.6×10^{2} (b) 1.3×10^{7}
(c) 6.5×10^{-3} (d) 4.0×10^{-5}
12 (a) 47.02 km (b) 1.02 mL (c) 60.54 g

Extended response

1 (a) (i) 1.4 cm (ii) 0.014 m
(iii) 0.000 014 km
(b) (i) 3840 cm (ii) 38.4 m
(iii) 0.038 4 km
(c) (i) 5020 mm (ii) 502 cm
(iii) 0.00502 km
(d) (i) 6 800 000 mm^2 (ii) 68 000 cm^2
(iii) 0.000 006 8 km^2
2 (a) 4.9×10^{3}, 6.8×10^{3}, 1.2×10^{4}, 1.3×10^{4}, 5.0×10^{4}, 5.1×10^{4}, 1.2×10^{5}, 1.4×10^{5}
Diameter: Mercury, Mars, Venus, Earth, Neptune, Uranus, Saturn, Jupiter
(b) 3.3×10^{23}, 6.4×10^{23}, 4.9×10^{24}, 6.0×10^{24}, 8.7×10^{25}, 1.0×10^{26}, 5.7×10^{26}, 1.9×10^{27}
Mass: Mercury, Mars, Venus, Earth, Uranus, Neptune, Saturn, Jupiter
(c) 9.9×10^{0}, 1.1×10^{1}, 1.6×10^{1}, 1.7×10^{1}, 2.4×10^{1}, 2.5×10^{1}, 1.4×10^{3}, 5.8×10^{3}
Period for rotation: Jupiter, Saturn, Neptune, Uranus, Earth, Mars, Mercury, Venus
(d) 8.80×10^{1}, 2.25×10^{2}, 3.65×10^{2}, 6.87×10^{2}, 4.33×10^{3}, 1.07×10^{4}, 3.06×10^{4}, 5.98×10^{4}
Period of orbit: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune
3 (a) Closest: $(1.497 \times 10^{8} - 1.08 \times 10^{8}) = 4.17 \times 10^{7}$ km
(b) Furthest: $(1.497 \times 10^{8} + 1.08 \times 10^{8}) = 2.58 \times 10^{8}$ km
(c) Number of Earths: $\frac{1.497 \times 10^{8} \text{ km}}{1.2756 \times 10^{4} \text{ km}} = 1.17 \times 10^{4}$
(d) Number of Venuses: $\frac{1.08 \times 10^{8} \text{ km}}{1.2104 \times 10^{4} \text{ km}} = 8.92 \times 10^{3}$
4 (a) $\frac{C}{2} = \frac{\pi}{2}$ cm
(b) $P = \frac{\pi}{2} \times 12 = 6\pi$ cm
(c) $P = 18.8$ cm (1 d.p.)
(d) $\frac{C}{2} = 1.6$ cm (1 d.p.)
(e) $1.6 \times 12 = 19.2$ cm
(f) $\frac{C}{2} = 1.5$ cm (truncated)
(g) $1.5 \times 12 = 18.0$ cm
(h) $A_{\text{semicircle}} = \frac{\pi \times 0.5^2}{2} = \frac{\pi}{8}$ or 0.125π cm^2
(i) $A = \frac{\pi}{8} \times 12 + \frac{3 \times 4}{2} = \left(\frac{3\pi}{2} + 6\right)$ cm^2
(j) $A = 10.7$ cm^2 (1 d.p.)
(k) $A_{\text{semicircle}} = 0.4$ cm^2 (1 d.p.)
(l) $A = 0.4 \times 12 + 6 = 10.8$ cm^2
(m) $A_{\text{semicircle}} = 0.3$ cm^2 (truncated)
(n) $A = 0.3 \times 12 + 6 = 9.6$ cm^2

Topic 9: Percentage error in measurement

Topic review

Multiple choice

1 C
2 A
3 C
4 D
5 B
6 B
7 A
8 C
9 C
10 D
11 D

Short answer

1 **(a)** 104° **(b)** 1° **(c)** 0.5°
2 **(a)** 2 mm **(b)** 0.5 mL **(c)** 0.1 kg
3 **(a)** 0.5 cm **(b)** 0.25 g **(c)** 5 km
4 **(a)** ± 1 mm **(b)** ± 0.5°C **(c)** ± 25 g
5 **(a)** 180°C ± 5°C **(b)** 1.65 kg ± 0.025 kg
(c) 7.0 cm ± 0.1 cm
6 **(a)** 295 g and 345 g **(b)** 64 mm and 66 mm
(c) −8°C and −6°C
7 **(a)** $\frac{0.5}{169} \approx 0.003$ **(b)** $\frac{0.5}{12} \approx 0.042$
8 **(a)** $\frac{0.05}{17.1} \times 100\% \approx 0.3\%$ **(b)** $\frac{0.05}{38.7} \times 100\% \approx 0.1\%$
9 **(a)** $\frac{2.5}{135} \times 100\% \approx 1.9\%$ **(b)** $\frac{2.5}{20} \times 100\% = 12.5\%$
10 **(a)** $\frac{1}{50} \times 100\% = 2\%$ **(b)** $\frac{-5}{130} \times 100\% \approx -3.8\%$
(c) $\frac{0.35}{15.5} \times 100\% \approx 2.3\%$
11 **(a)** $\frac{3}{80} = 0.0375$ **(b)** 3.75%
(c) The percentage error is positive, so the garment has a waist size 3.75% larger than expected.
12 **(a)** $\frac{2}{128} \times 100\% \approx 1.6\%$ **(b)** $\frac{5}{200} \times 100\% = 2.5\%$
13 $\frac{517.6 - 520}{520} \times 100\% \approx -0.46\%$
$|-0.46|\% = 0.46\% < 1\%$
Therefore, the scale passed the test.

Extended response

1 **(a)** 2 mm or 0.2 cm
(b) 1 mm or 0.1 cm
(c) ±1 mm or ±0.1 cm
(d) (8.6 ± 0.1) cm
(e) 8.5 cm to 8.7 cm
(f) $\frac{0.1}{8.6} \approx 0.116$
(g) 11.6%

2

	10.5 cm	18 cm	15 cm	19.5 cm	14 cm	17 cm	14.5 cm	18.5 cm	20 cm	11.5 cm
(a)	5.7 cm	1.8 cm	1.2 cm	3.3 cm	2.2 cm	0.8 cm	1.7 cm	2.3 cm	3.8 cm	4.7 cm
(b)	−0.35	0.11	−0.07	0.20	−0.14	0.05	−0.10	0.14	0.23	−0.29

(c) $\frac{158.5 \text{ cm}}{10} = 15.85 \text{ cm}$

(d) $\frac{15.85 \text{ cm} - 16.2 \text{ cm}}{16.2 \text{ cm}} = -0.02$

(e) $\frac{-0.22}{10} = -0.022$; This answer would be the same as **(d)** except for the use of rounded values.

(f) $\frac{27.5 \text{ cm}}{10} = 2.75 \text{ cm}$

3 **(a)** **(i)** The length is (12.5 ± 0.05) cm, so 12.45 cm to 12.55 cm
(ii) The width is (8.2 ± 0.05) cm, so 8.15 cm to 8.25 cm
(iii) 12.5 × 2 + 8.2 × 2 = 41.4; 12.45 × 2 + 8.15 × 2 = 41.2; 12.55 × 2 + 8.25 × 2 = 41.6
The perimeter is (41.4 ± 0.2) cm or 41.2 cm to 41.6 cm
(iv) 12.5 × 8.2 = 102.5; 12.45 × 8.15 = 101.4675; 12.55 × 8.25 = 103.5375
The area is 102.5 cm^2 or 101.4675 cm^2 to 103.5375 cm^2

(b) **(i)** $\frac{0.05}{12.5} \times 100\% = 0.4\%$
(ii) $\frac{0.05}{8.2} \times 100\% \approx 0.61\%$
(iii) $\frac{0.2}{41.4} \times 100\% \approx 0.48\%$

Topic 10: Pythagoras and geometry

Topic review

Multiple choice

1 A
2 D
3 D
4 B
5 C
6 B
7 B
8 D
9 A
10 C
11 C
12 A
13 B

Short answer

1 17.08 m
2 4.20 m
3 $R(4,-2)$, $S(4,4)$; $R(-2,-2)$, $S(-2,4)$
4 **(a)** $\sqrt{20}$ units or $2\sqrt{5}$ units
(b) $\sqrt{128}$ units or $8\sqrt{2}$ units
5 **(a)** 10 units **(b)** 37 units
6 **(a)** $\sqrt{29}$ units **(b)** (6,0), (6,4), (3,7)
7 **(a)** ΔABC: $AB = 9$ units, $BC = 6$ units

$AC = \sqrt{9^2 + 6^2}$

$= \sqrt{117}$ units

ΔADE: $AD = 12$ units, $DE = 8$ units

$AE = \sqrt{12^2 + 8^2}$

$= \sqrt{208}$ units

Large to small ratios:

$\frac{AD}{AB} = \frac{12}{9}$

$= \frac{4}{3}$

$\frac{DE}{BC} = \frac{8}{6}$

$= \frac{4}{3}$

$\frac{AE}{AC} = \frac{\sqrt{208}}{\sqrt{117}}$

$= \sqrt{\frac{208}{117}}$

$= \sqrt{\frac{16}{9}}$

$= \frac{4}{3}$

Since the ratio between each pair of sides is the same, the triangles are similar.

(b) $\frac{4}{3}$
8 **(a)** $\Delta KLM \sim \Delta BLA$
(b) $\Delta KLM \sim \Delta SRM$
9 1.4, $\Delta DEF \sim \Delta GHF$
10 $\Delta NMW \sim \Delta XVW$
11 **(a)** 15.3 cm
(b) 13.5 cm
(c) 4.2 cm
12 22.5 cm^2, 50.625 cm^2
13 2.4 m^2, 3.456 m^2
14 Between 56 mm and 65 mm

Extended response

1 **(a)** 4.8 m by 2.6 m and 4.8 m by 2.8 m
(b) 4.105 m
(c) 4.8 m by 4.105 m
(d) 67.74 m^2
2 **(a)** $BD = 5$ units
(b) **(i)** $A(-1,2)$, $C(3,1)$ or $A(-1,-2)$, $C(3,5)$
(ii) $BC = AD = \sqrt{17}$ units or $BC = AD = \sqrt{41}$ units
(iii) $AC = \sqrt{17}$ units or $AC = \sqrt{65}$ units
3 **(a)** $BC = 4$ units, $DE = 8$ units , $FG = 12$ units
(b) 1 : 2 : 3
(c) $AB = 5$ units, $AD = 10$ units, $AF = 15$ units
(d) 1 : 2 : 3
(e) $AC = \sqrt{41}$ units, $AE = \sqrt{164} = 2\sqrt{41}$ units, $AG = \sqrt{369} = 3\sqrt{41}$ units
(f) 1 : 2 : 3
(g) **(i)** 2 **(ii)** 3 **(iii)** $\frac{3}{2}$ or 1.5

Topic 11: Modelling situations with measurement and rates

Topic review

Multiple choice

1 C
2 B
3 D
4 C
5 C
6 D
7 C
8 D
9 B
10 B
11 B
12 C

Short answer

1 **(a)** $\frac{3}{5}$ **(b)** $\frac{\pi}{2}$ **(c)** 4 **(d)** $\frac{3}{2}$

2 **(a)** $m = kx^2$ **(b)** $t = kp^3$ **(c)** $x = ky$

3 **(a)** $k = \frac{1}{6}$ **(b)** $q = \frac{r^2}{6}$

(c) $q = 37.5$ **(d)** $r = 20$

4 **(a)** cost = 19.9 × number of m^3

(b) \$497.50 **(c)** 22.4 m^3

5 **(a)** 3.15 kg·m/s **(b)** 4.1 m/s

6 **(a)** No, because it rises and falls.

(b) No, because the graph is not consistent as it has a curved part and a straight-line part.

(c) Yes, this could be direct proportion.

7 **(a)** Approximately A\$325

(b) Approximately S\$415

(c) This will be about A\$45.

(d) It is difficult to convert small values as a small variation leads to a large conversion error.

8 **(a)** $\frac{24}{35}$ **(b)** 43.75 cm **(c)** 19.2 cm

9 **(a)** $k = 4$

a	1	9	25	100
b	4	12	20	40

(b) $k = 0.6$

c	2	3	6	9
d	4.8	16.2	129.6	437.4

Extended response

1 **(a)** 9π **(b)** 9π

(c) The straight lines connecting these pairs of points have the same gradient, so there is a single straight line that passes through all three points. A cylinder of 0 cm height will have a volume of zero as well, so the straight line also passes through the origin. Hence, the requirements for direct proportion have been satisfied.

(d) $r^2 = 9 \Rightarrow r = 3$. The radius is 3 cm.

(e) $h = 20$ cm

2 **(a)** $T = 6s + 3l$

(b) As an example, let $s = 1$ cm. When $l = 0$ cm, $T \neq 0$ since the triangle still exists. The graph would not pass through the origin, hence direct proportion is not possible.

(c) From the rule, with s constant, then $k = \frac{s^2\sqrt{3}}{4}$, so the rule can be written as $V = kl$, which fits the requirements for direct proportion.

(d) **(i)** $V = 64.95\ \text{cm}^3$ **(ii)** $l = 18.48$ cm

Topic 12: Trigonometry

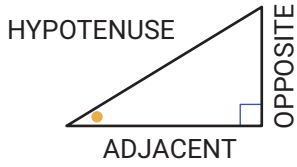
Topic review

Multiple choice

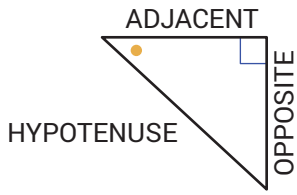
1 B
2 C
3 D
4 D
5 B
6 A
7 D
8 C
9 B
10 C
11 D
12 A

Short answer

1 **(a)**



(b)



2 $\sin(\phi) = \frac{15}{17}, \cos(\phi) = \frac{8}{17}, \tan(\phi) = \frac{15}{8}$

3 **(a)** $\theta = \sin^{-1}(0.65)$ **(b)** $\theta = \tan^{-1}(1.2425)$

(c) $\theta = \cos^{-1}\left(\frac{28.6}{45.1}\right)$

4 **(a)** 18.6 cm **(b)** 12.232 m **(c)** 22 mm

5 **(a)** $x = 9$ cm, $y = 5.6$ cm, $\theta = 32°$

(b) $x = 18.5$ m, $y = 17.5$ m, $\theta = 71°$

(c) $x = 18.25$ m, $y = 13.75$ cm, $\theta = 49°$

6 6.69 m

7 29.53... − 16.34... ≈ 13.2 m

8 97.6 m

9 **(a)** $\theta = 34°$ **(b)** $\theta = 51°$ **(c)** $\theta = 34°$

10 **(a)** $\theta = 31°$ **(b)** $\theta = 36°$ **(c)** $\theta = 45°$

11 8.4°

12 **(a)** $70^2 = x^2 + 55^2$ (Pythagoras' theorem)

(b) $\sin(\alpha) = \frac{55}{70}$ (Trigonometry)

(c) $\alpha + \beta = 90°$ (Angle sum of a triangle)

13 $\alpha = 75°$, $x = 3.73$ m, $y = 0.96$ m

Extended response

1 **(a)** 26 cm
(b) 2.9 cm
(c) 22.94 cm^2
(d) 4.8 cm
(e) Both 4.7 cm

2 **(a)** $w = 22$ mm, $x = 64$ mm, $y = 72$ mm, $z = 83$ mm
$\alpha = 70°, \beta = 34°, \theta = 56°, \phi = 60°, \gamma = 30°$
(b) quadrilateral
(c) $x = 64$ mm, $w + 40 = 62$ mm, 42 mm, $z = 83$ mm
$\alpha = 70°$, $\theta + 90 = 146°$, $\phi = 60°$, $20 + \beta + \gamma = 84°$

3 **(a)** $u = 2.03$ m, $v = 3.23$ m, $w = 2.51$ m, $x = 2.03$ m, $y = 1.17$ m, $z = 2.35$ m
$\alpha = 51°, \beta = 39°, \theta = 81°$
(b) 2.51 m, $u = 2.03$ m, $w + y = 3.68$ m, $z = 2.35$ m
90°, 90°, 60°, $\theta + 39 = 120°$

Topic 13: Ratios and proportion in space

Topic review

Multiple choice

1 C
2 B
3 A
4 D
5 B
6 C
7 D
8 D
9 A
10 B
11 A
12 D
13 C
14 C
15 B
16 A

Short answer

1 **(a)** 1.5
(b) 2.25

2 **(a)** 2.94 m^2
(b) 2.1 or $\frac{21}{10}$
(c) 12.9654 m^2

3 **(a)** 34.4375π cm^2 or $\frac{551\pi}{16}$ cm^2
(b) 1.4 or $\frac{7}{5}$
(c) 67.4975π cm^2 or $\frac{26999\pi}{400}$ cm^2

4 $\frac{100}{81}$

5 **(a)** $(64 + 16\pi)$ cm^2 **(b)** 400
(c) $(25\,600 + 6400\pi)$ cm^2
(d) $(2.56 + 0.64\pi)$ m^2

6 138 cm, 116 cm

7 **(a)** 568 cm **(b)** 189 cm

8 17.5 mm to 175 mm

9 **(a)** 4% , acceptable
(b) $4\frac{2}{3}$%, acceptable
(c) 5.5%, not acceptable

10 **(a)** stretched vertically
(b) stretched horizontally
(c) stretched horizontally

11 **(a)** stretched in the horizontal direction
(b) compressed in the vertical direction

Extended response

1 **(a)** $a = \sqrt{11}$ cm, $b = \sqrt{39}$ cm
(b) $\frac{5\left(\sqrt{11}+\sqrt{39}\right)}{2}$ cm^2 or $2.5\left(\sqrt{11}+\sqrt{39}\right)$ cm^2
(c) 1.3 or $\frac{13}{10}$
(d) $c = 7.8$ cm, $d = 10.4$ cm, $e = 1.3\sqrt{11}$ cm or $e = \frac{13\sqrt{11}}{10}$ cm, $f = 1.3\sqrt{39}$ cm or $f = \frac{13\sqrt{39}}{10}$ cm
(e) $\frac{169\left(\sqrt{11}+\sqrt{39}\right)}{40}$ cm^2 or $4.225\left(\sqrt{11}+\sqrt{39}\right)$ cm^2

2 **(a)** $(20 - 0.81\pi)$ cm^2 **(b)** 15
(c) 225
(d) $(4500 - 182.25\pi)$ cm^2

3 **(a)** **(i)** 5366 mm **(ii)** 4293 mm
(iii) 3843 mm
(b) The cars were all side-on to the camera, or at least parallel. The cars being compared were the same distance from the camera.
(c) 5366 mm > 4840 mm, so Car B was probably closer to the camera than Car A; 4293 mm ≈ 4284 mm, so the Car C estimate is within the expected accuracy resulting from the 2-digit measurements; 3843 mm < 4625 mm, so Car D was probably further from the camera than Car A.

Topic 14: Analysing and interpreting statistics

Topic review

Multiple choice

1 A
2 B
3 D
4 C
5 B
6 A
7 A

Short answer

1 **(a)** Chronic conditions would be a part of long-term health conditions.

(b) $\frac{8}{10} \times 100\% = 80\%$, so 'eight in ten' is similar to 81.4%.

$\frac{1}{2} \times 100\% = 50\%$, so 'one in two' is similar to 49.9%.

(c) 26.1% ≈ 25%, so one in four had mental and behavioural conditions.

15.7% ≈ 16.6%, so one in six had back problems.

(d) $\frac{14.5}{100} \times 26 \text{ million} = 3.77 \text{ million}$, so an estimated 3.77 million had arthritis.

2 **(a)** 23% did not consume any alcohol.

(b) The category includes people who may have only consumed one drink over the entire year.

(c) Consumption of alcohol and illicit drugs are both risk factors for health and wellbeing.

(d) 1 year = 365 × 24 × 60 minutes = 525 600 minutes

$\frac{525\,600 \text{ minutes}}{105\,700 \text{ seizures}} \approx 5.0$ minutes/seizure or one seizure per 5 minutes

(e) The frequency of seizures is an indicator of the prevalence of illicit drugs as well as the cost to the tax payer of policing.

3 **(a)** Shortfall of 37 279

(b) 2024-25: $\frac{6303}{37\,279} \times 100\% \approx 17\%$ of 2023-24;

2025-26: $\frac{7816}{37\,279} \times 100\% \approx 21\%$ of 2023-24

(c) In the final three years, there is predicted to be a surplus compared to new demand, but there will still be an overall shortfall due to previous years' shortfalls being so high.

(d) Shortfall of 39 176

4 **(a)** Data for the whole of 2022 would need to include 31 December 2022. Further, conducting the interviews, analysing the data and producing the report takes time, and none of this can start until after 31 December.

(b) $\frac{1}{4} = 25\%$, so that 26.8% is 'more than one in four' adults exceeded the Australian Adult Alcohol Guideline.

35.8% > 18.1%, so 'Males were more likely than females' to exceed the guideline.

$\frac{1}{3} = 33.\dot{3}\%$, so that 36.1% is 'more than one in three' young adults exceeding the guideline.

(c) Sample size (if this is not a census) and what segments of the population of Australia were involved (cities, country, remote areas). It is good to be able to check on the details separately as otherwise the summary report could become cluttered.

5 **(a)** $\frac{8\,000\,000}{25\,422\,788} \times 100\% \approx 31\%$, so over 31%

(b) If you subtract the numbers with 1, 2 and 3 or more of the listed conditions, then you will be eliminating people with listed conditions that may also have unlisted conditions.

(c) 8 000 000 − 4 791 516 − 1 490 344 − 772 142 = 945 998

$\frac{945\,998}{25\,422\,788} \times 100\% \approx 4\%$, so around 4% reported having a long-term health condition but did not have any of the ones listed on the census.

(d) The census includes any people in a household at the time of the census. This includes people visiting from overseas. These visitors would be included in the figures, even though they are not part of the Australian population.

Further, homeless people are likely to be underrepresented in the data, since their details are difficult to capture in the census. This is a concern for the representativeness of the data on long-term health conditions, since homeless people often have chronic and long-term health problems.

6 **(a)** 17% of adolescents aged 15–17 reached the physical activity guidelines.

(b) Most 15–17 year olds are either in school participating in important years of study or working in entry-level jobs, so it takes commitment to participate in regular physical activity. Social media, and screen time in general, is readily available so adolescents turn to this for entertainment rather than physical activity.

(c) Each participant in the survey would be asked whether or not they identify as First Nations.

(d) The government tracks the health and wellbeing of First Nations people specifically because of their historically poorer health outcomes, with a view to helping to close the gap.

(e) Meeting physical activity guidelines is a factor in producing better health and wellbeing outcomes.

7 (a) Statistics from the Regional Australia Institute and the Commonwealth Bank are given as evidence of both the great migration and the slowdown.

(b) The article suggests that the lack of regional housing is *likely* to be linked to the slow down in migration, but admits to there being no data that explicitly proves this.

(c) Sunshine Coast: 93 parts $\leftrightarrow$ 17%, so

100 parts $\leftrightarrow \frac{100}{93} \times 17\% = 18.3\%$

Gold Coast: 45 parts $\leftrightarrow$ 9%, so

100 parts $\leftrightarrow \frac{100}{45} \times 9\% = 20\%$

8 (a) The height at 2011 is going on for three times the height at 2009.

(b) 2009: 16.779%; 2011: 17.651%

(c) The numbers on the vertical axis do not go down to zero.

(d) The data is presented to accentuate the changes in percentage.

(e) The scale on the vertical axis is given as well as actual values for each data point.

Extended response

1 (a) The 'almost constantly' category.

(b) The first graph says that 93% of teens have ever used YouTube (the next highest is TikTok at 63%). The second graph says that 71% of teens have ever used YouTube (the next highest is TikTok at 58%).

(c) 16% + 17% + 14% + 8% + 3% = 58%

(d) There is likely to be some overlap as it is possible to access multiple platforms almost simultaneously. This suggests that the total percentage found may be higher than the real percentage of teens using social media almost constantly.

(e) Snapchat is being used by around 14% of teens 'almost constantly', compared to 8% of teens 'almost constantly' using Instagram.

(f) Teens are more attracted to the newer platforms, which have often been designed to reflect their interests and lifestyles.

2 (a) All three had populations between 10 million and 20 million in 1960.

(b) California is just one state of 50 whereas the other two are countries.

(c) The range of values is 10 million to 40 million so there was no benefit in not showing the lowest 10 million. It is helpful to picture comparisons such as when a population has doubled.

(d) Australia always had the lowest population of the three while California's population overtook Canada's in the 1980s, with Canada catching up by 2021. Canada's population has increased at a steady rate whereas Australia had more rapid growth in the last 15 years and California's growth slowed to a stop, and then decreased in recent years.

(e) California: $\frac{39.24}{15.5} \times 100\% \approx 253\%$, so 153% growth;

Canada: $\frac{38.23}{18.5} \times 100\% \approx 207\%$, so 107% growth;

Australia: $\frac{25.69}{10} \times 100\% \approx 257\%$, so 157% growth.

(f) California's and Australia's populations are both more than two and a half times what they were in 1960. Canada's population has more than doubled over the same period.

3 (a) You could try to find figures in other reports to determine how the calculations were made. There could be a problem with the interpretation of the data due to the use of different currencies. Or perhaps the price per unit weight of coal was higher in 2022, making it more lucrative during that year.

(b) Coal briquettes are a processed coal product. In Australia, they are mainly used in barbecues. However, in other parts of the world, they are important for heating.

(c) If the overall graph is 10 cm by 9.1 cm then the total area is 91 cm^2.

Coal: $3.8 \times 6 = 22.8\ cm^2$, $\frac{22.8}{91} \times 100\% \approx 25.1\%$

which is close to 25.7%.

Wheat: $1.3 \times 1.7 = 2.21\ cm^2$, $\frac{2.21}{91} \times 100\% \approx 2.4\%$

which is the same as the given percentage.

Gold: $2.8 \times 1.2 = 3.36\ cm^2$, $\frac{3.36}{91} \times 100\% \approx 3.7\%$

which is close to 4.2%.

(d) Iron ore: $\frac{87.9B}{424B} \times 100\% \approx 20.7\%$, as given

Petroleum gas: $\frac{67.3B}{424B} \times 100\% \approx 15.9\%$, as given

(e) China: $\frac{123B}{424B} \times 100\% \approx 29.0\%$;

Japan: $\frac{80.1B}{424B} \times 100\% \approx 18.9\%$;

South Korea: $\frac{41B}{424B} \times 100\% \approx 9.7\%$

(f) The colour coding, confirmed by the key, groups categories such as Cotton and Wool together so that an overall impression of their importance to the export economy can be visualised.

Topic 15: Displaying and comparing statistical data

Topic review

Multiple choice

1 D
2 B
3 A
4 C
5 A
6 C
7 D
8 B
9 B
10 A
11 D

Short answer

1 (a)

Score	Frequency
8	5
9	15
10	8
11	6
12	4
13	2

(b) Mean: $\frac{395}{40} = 9.875$

(c) Median is the average of the 20th and 21st: $\frac{9+10}{2} = 9.5$

(d) Mode: 9

(e) Positive skew; mode < median < mean, confirming this suggestion.

2 (a)

Mass (kg)	Frequency
60–<65	6
65–<70	11
70–<75	12
75–<80	9
80–<85	4
85–<90	1

(b) 72.2 kg **(c)** 70–<75 kg

(d) 70–<75 kg

(e) This is a symmetrical data set.

3 (a) Mean: $\frac{1752}{22} \approx 79.6$

(b) Median: 11th and 12th both 84

(c) There is no mode.

(d) Negatively skewed

4 (a)

Leaf Class A	Stem	Leaf Class B
	3	6 6 7 8 8 8
9 9 8 8 6 6 5	4	1 2 2 4 6 7 7
7 7 6 5 3 3 3	5	2 7 8
8 8 8 7 7 5 4 2	6	1 3 3 8
	7	0 0 2

(b) Class A is symmetrical. Class B is positively skewed.

(c) Class A: 55.5 Class B: 47

(d) Class A: $\frac{1244}{22} \approx 56.5$ Class B: $\frac{1166}{23} \approx 50.7$

5 (a)

Height (cm)

Leaf Carlton	Stem	Leaf Lions
3	17_L	
9 8	17_H	5 7 8 8
4 4 4 4 3 3 3 2 0 0 0	18_L	0 1 2 2 2 2 3 3 3 3 3 4 4 4 4
9 8 8 8 8 8 7 7 7 6 6 6	18_H	5 5 5 5 6 7 7 8 8 9
4 4 4 4 3 3 2 0 0	19_L	0 1 1 2 2 2 3
9 5 5	19_H	5 6 7
2 2 2 1 1 0	20_L	0 1 2 3 4 4
	20_H	5 6
1	21_L	

(b) Carlton: 188 cm; Lions: 186 cm

(c) Carlton: 189.5 cm; Lions: 188.7 cm

(d) Carlton: 185–189 cm; Lions: 180–184 cm

6 (a) Median house prices in Melbourne dropped gradually from early 2022 to mid 2024.

(b) Median house prices in Brisbane dropped gradually from early 2022 to early 2023 and then rose dramatically by mid 2024.

(c) Median house prices in Melbourne were substantially more than Brisbane prices from early 2022 to early 2023, but by mid 2024 the median price in Brisbane had overtaken the median price in Melbourne, making it more affordable to buy a house in Melbourne than in Brisbane.

7 **(a)** Perth and Adelaide **(b)** Brisbane

(c) February to June

(d) Melbourne and Hobart have consistent rainfall all year round, and a moderate average for every month.

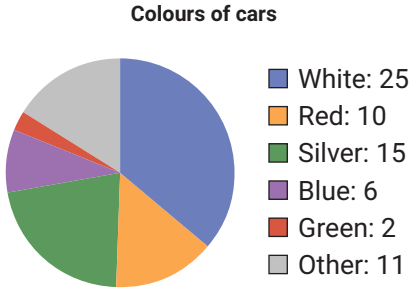
8 **(a)** 34–37 **(b)** 30–33

(c) $41 - 22 = 19$ **(d)** $\frac{2537.5}{77} \approx 33.0$

9 **(a)** 69

(b) $5\frac{5}{23}°$

(c)



10 **(a)** The value of service exports rose from 2018 to 2019, then fell each year to 2021 before recovering each year to 2023.

(b) New South Wales: 45% of the total; Victoria: 25% of the total.

(c) South Australia: up 80%; Queensland: up 70%.

11 **(a)** Starting at top left, read to the right, then go down and read to the left, then go down and repeat to the latest entry.

(b) There is no set scale, simply a regular distance used for each significant entry.

(c) An instant messaging service

(d) April 2008

Extended response

1 **(a)** Positively skewed

(b)

Number of siblings	Frequency
0	2
1	3
2	8
3	9
4	5
5	2
6	4
7	1
8	2
9	2
10	1
11	1

(c) 3 siblings

(d) 3 siblings

(e) 4.05 siblings

(f) 0, 0, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 6, 6, 6, 6, 7, 8, 8, 9, 9, 10, 11

2 **(a)**

Average disposals per person per game

Leaf Carlton	Stem	Leaf Lions
9 9 9 8 6 6 2	0	2 5 6 8 8 8 8 8
8 7 7 6 6 5 5 5 5 4 4 3 3 3 3 2 2 1 0 0	1	1 1 1 1 1 1 1 1 2 2 3 3 4 5 5 5 7 7 7 8
7 3 2 2 1	2	0 2 5 5 6
1	3	

(b)

Average disposals per person per game

Leaf Carlton	Stem	Leaf Lions
2	0_L	2
9 9 9 8 6 6	0_H	5 6 8 8 8 8 8
4 4 3 3 3 3 2 2 1 0 0	1_L	1 1 1 1 1 1 1 1 2 2 3 3 4
8 7 7 6 6 5 5 5 5	1_H	5 5 5 7 7 7 8
3 2 2 1	2_L	0 2
7	2_H	5 5 6
1	3_L	

(c) Carlton: 14; Lions: 11

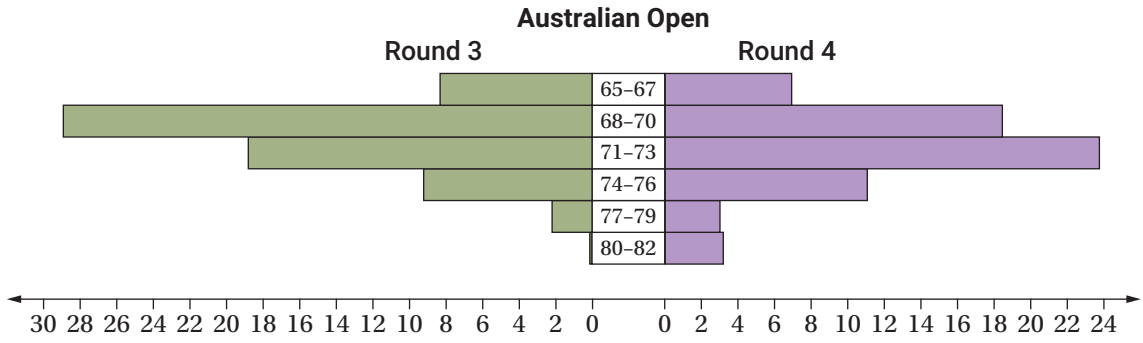
(d) Carlton: 12.9; Lions: 13.2

3 **(a)** 67 players in each round.

(b)

Score	Round 3 frequency	Round 4 frequency
65–67	8	7
68–70	29	19
71–73	19	24
74–76	9	11
77–79	2	3
80–82	0	3

(c)



(d) Round 3: 70.5 Round 4: 71.7

(e) Round 3: 70 Round 4: 72

(f) On average (mean and median), Round 3 has lower (better) scores. This is backed up by the lower modal class.

(g) Both rounds show a slightly positive skew.

Topic 16: Probability (compound events, relative frequencies)

Topic review

Multiple choice

1 D
2 C
3 A
4 D
5 A
6 D
7 A
8 C
9 D
10 D
11 A
12 C
13 B

Short answer

1 **(a)** $\frac{2}{10}=\frac{1}{5}$ **(b)** $\frac{8}{10}=\frac{4}{5}$

2 **(a)** $1-\frac{17}{31}=\frac{14}{31}$ **(b)** $\frac{272}{930}=\frac{136}{465}$

3 **(a)**

		Bag 2			
		E	**A**	**C**	**H**
Bag 1	**E**	EE	EA	EC	EH
	A	AE	AA	AC	AH
	C	CE	CA	CC	CH
	H	HE	HA	HC	HH

(b) $\frac{1}{16}$ **(c)** $\frac{6}{16}=\frac{3}{8}$

(d) $\frac{4}{16}=\frac{1}{4}$ **(e)** $\frac{12}{16}=\frac{3}{4}$

4 **(a)** 60 **(b)** $\frac{22}{60}=\frac{11}{30}$

(c) $\frac{28}{60}=\frac{7}{15}$ **(d)** $\frac{42}{60}=\frac{21}{30}$

(e) $\frac{34}{60}=\frac{17}{30}$ **(f)** $\frac{8}{60}=\frac{2}{15}$

(g) $\frac{38}{60}=\frac{19}{30}$

5 **(a)** $\frac{60}{150}=\frac{2}{5}$ **(b)** $\frac{31}{150}$ **(c)** $\frac{25}{150}=\frac{1}{6}$

6 0.17

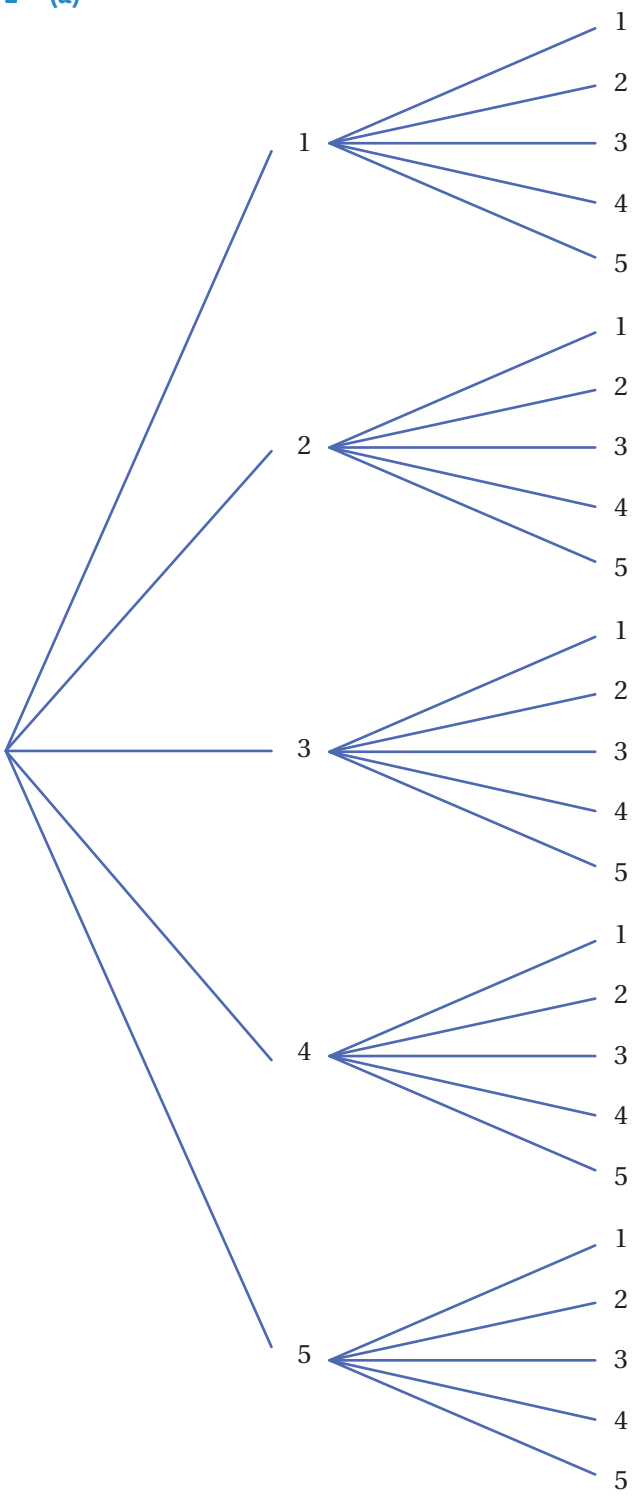
7 **(a)** 240 000 **(b)** 72 000

8 **(a)** 900 **(b)** 450
(c) 300 **(d)** 150
(e) 750
(f) **(i)** 0.15 **(ii)** 0.9 **(iii)** 0.75

Extended response

1 **(a)** $\frac{1}{8}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{3}{8}$ **(d)** $\frac{7}{8}$

2 **(a)**



(b) $\frac{1}{25}$ **(c)** $\frac{4}{25}$ **(d)** $\frac{4}{25}$

(e) There is also the possibility of getting the number 3 with any number, and of spinning one less than 3 or one greater than 3.

(f) $\frac{16}{25}$ **(g)** $\frac{1}{25}$ **(h)** $\frac{6}{25}$ **(i)** $\frac{12}{25}$

3 **(a)** blue: $\frac{1}{4}$; green: $\frac{1}{4}$; red: $\frac{1}{2}$

(b) blue: 250; green: 250; red: 500

(c) **(i)** The long-term results are quite close to the estimation.

(ii) The more trials, the more reliable the estimation.

4 **(a)**

	Numbers rolled	Extra time	Total trip time
Trial 1	6,3,2	5	29
Trial 2	4,1,6	5	29
Trial 3	5,3,1	9	33
Trial 4	5,1,4	10	34
Trial 5	3,3,6	6	30
Trial 6	1,5,2	8	32
Trial 7	2,5,2	9	33
Trial 8	4,3,1	8	32
Trial 9	6,1,5	6	30
Trial 10	4,5,2	11	35

(b) $24 + 3 \times 5 = 39$, so 39 minutes

(c) 29 minutes

(d) 35 minutes

(e) $\frac{317}{10} = 31.7$, so 31.7 minutes

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