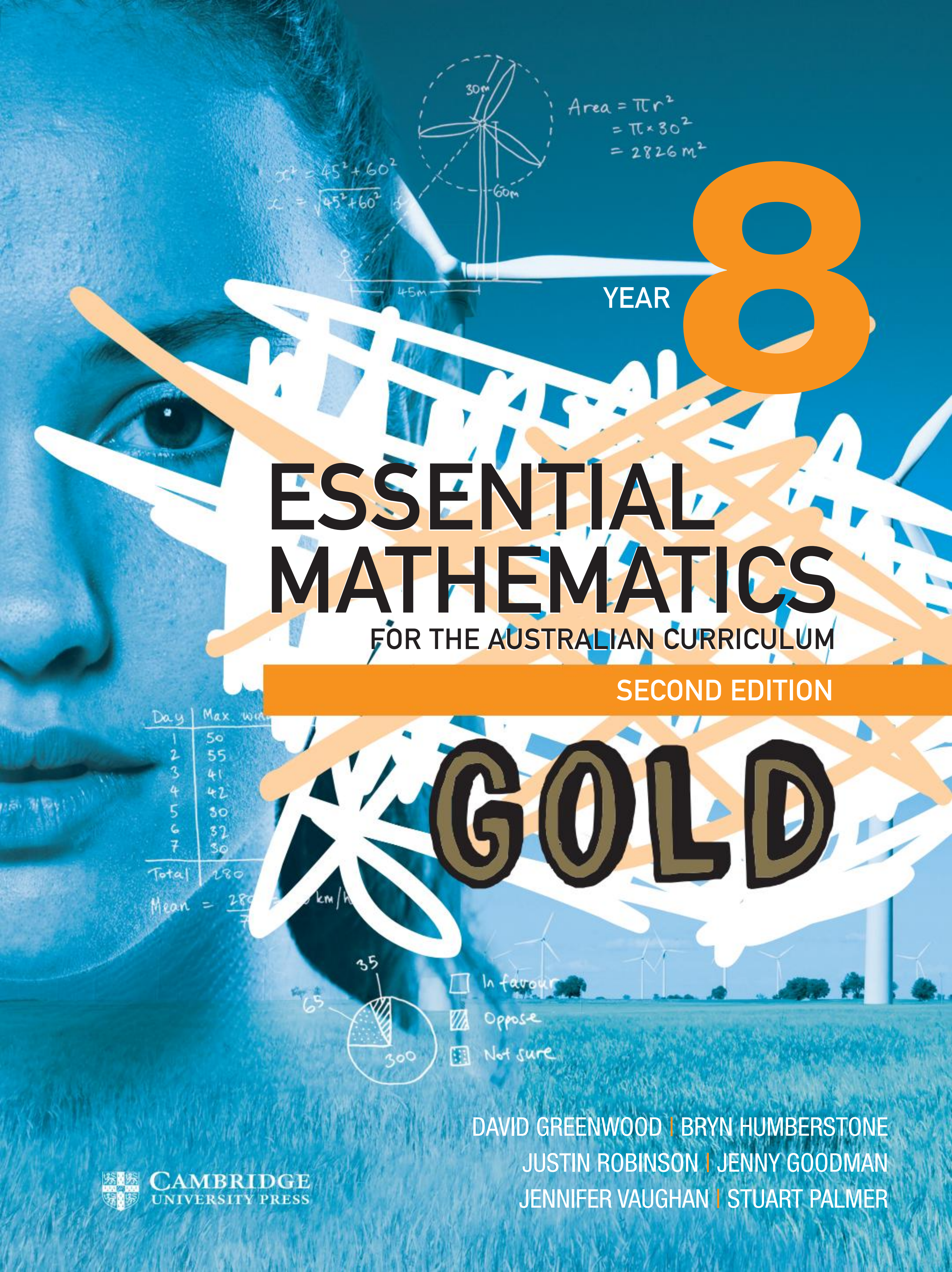

YEAR 8

ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM

SECOND EDITION

GOLD

DAVID GREENWOOD | BRYN HUMBERSTONE
JUSTIN ROBINSON | JENNY GOODMAN
JENNIFER VAUGHAN | STUART PALMER

CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

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www.cambridge.org
Information on this title: www.cambridge.org/9781316644119

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First published 2012
Second Edition 2017
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover and text designed by Sardine Design
Typeset by diacriTech
Printed in China by C & C Offset Printing Co. Ltd.

A Cataloguing-in-Publication entry is available from the catalogue of the National Library of Australia at www.nla.gov.au

ISBN 978-1-316-64411-9 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

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Contents

About the authors

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has more than 20 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, is currently teaching both junior and senior mathematics in Victoria and is the Head of Mathematics (Senior School) at Caulfield Grammar School Wheelers Hill. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.

Justin Robinson is the inaugural Director of the Institute of Positive Education, based at Geelong Grammar School, where he leads a team of staff dedicated to promoting the theory and practice of Positive Education. Prior to this, he spent 20 years teaching mathematics, covering all levels of secondary education and with significant experience teaching VCE, IB and A-Levels. Justin is passionate about challenging students within a safe learning environment and instils in his students a growth mindset. He has a keen interest in promoting student engagement through a wide variety of effective teaching methods and materials.

Jenny Goodman has worked for 20 years in comprehensive state and selective high schools in New South Wales and has a keen interest in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible hence, facilitating student confidence, achievement and enjoyment of maths.

Stuart Palmer has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over New South Wales and beyond. He is a Project Officer with the Mathematical Association of New South Wales, and also works with pre-service teachers at The University of Sydney and The University of Western Sydney.

Acknowledgements

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Introduction and how to use this book

This second edition of *Essential Mathematics Gold for the Australian Curriculum* sees the Gold series revised, updated, redeveloped and seamlessly blended with HOTmaths to create a comprehensive learning package that is specially designed for students who require additional support in mathematics.

An Interactive Textbook (complimentary) for the student and Online Teaching Suite (available separately) for the teacher are new for the second edition. The print textbooks have been carefully revised and updated. New topics have been added to reflect changes to the Australian Curriculum and incorporate selected parts of new state syllabuses, particularly the Victorian Curriculum and the New South Wales Syllabus.

The following four themes have guided the second edition:

Differentiation in the classroom

- All non-core sections are labelled as 'Consolidating' (indicating a revision section) or with a gold star (indicating a topic that could be considered challenging) to help teachers decide on the most suitable way of approaching the course for their class or for individual students.
- Building and Progressing working programs are suggested for all exercises (see 'Guide to the working programs' on the following page for information).
- Two auto-marked quizzes for each section in the Interactive Textbook are aligned with the Building and Progressing learning pathways.
- Two chapter tests per chapter in the Online Teaching Suite have also been aligned to the Building and Progressing learning pathways.
- The Test Generator inside the Online Teaching Suite provides a high level of flexibility for the teacher to create tests from questions containing four difficulty levels.

Mathematical literacy

- In conjunction with margin definitions in the print book, 'enhanced' pop-up definitions in the Interactive Textbook feature visual and audio aids to help students learn definitions in a multi-dimensional way.
- In the Interactive Textbook, simple drag-and-drop and fill-the-gap activities in each chapter allow students to get comfortable using mathematical language and to consolidate definitions.
- In the Online Teaching Suite, maths literacy worksheets featuring a wide range of activities can be downloaded to be used as classwork or homework.

Practical application of mathematics

- A new feature in each chapter called 'Maths@Work' provides case studies to show how the maths learnt in the chapter is relevant to the workplace and to everyday life.

Using technology

- 'Maths@Work' activities feature a section dedicated to giving students practise at using technology such as spreadsheets and calculators in ways they will encounter in the workforce and in everyday life.
- Throughout the textbook, questions best suited to be completed on a calculator are indicated with a calculator icon. 🖩
- In the Interactive Textbook, a fully functional scientific calculator pops up when students click or tap on the calculator icon.

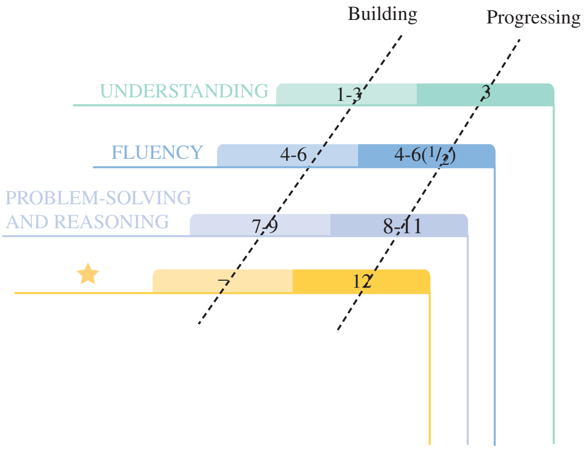
While not part of the ACARA curriculum, Algorithmics is an exciting, powerful way of applying technology to mathematics (and vice versa) and one that is relevant and accessible to students of all ages and ability levels. We have therefore included a new appendix chapter on Algorithmics in each book that takes an introductory, activity-based approach to the subject.

Guide to the working programs

New to the second edition, the working programs that were previously only available to teachers in separate supporting documents have been updated, refined and subtly embedded in the exercises. The suggested working programs provide two pathways through the book to allow differentiation for Building and Progressing students.

As with the first edition, each exercise is structured in subsections that match the Australian Curriculum proficiency strands (with Problem-solving and Reasoning combined into one section to reduce exercise length), as well as ‘Gold Star’ (★). The questions* suggested for each pathway are listed in two columns at the top of each subsection.

- The left column (lightest shade) shows the questions in the Building working program.
- The right column (darkest shade) shows the questions in the Progressing working program.



Gradients within exercises and proficiency strands

The working programs make use of two gradients that have been carefully integrated into the exercises. A gradient runs through the overall structure of each exercise – where there’s an increasing level of sophistication required as a student progresses through the proficiency strands and then to the ‘Gold star’ question(s) – but also within each proficiency strand; the first few questions in Fluency are easier than the last few, for example, and the first few Problem-solving and Reasoning questions are easier than the last few.

The right mix of questions

Questions in the working programs have been selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway are given extra practise at Understanding and basic Fluency and only the easiest Problem-solving and Reasoning questions. The Progressing pathway, while not meant to be challenging, spends a little less time on basic Understanding questions and a little more on Fluency, Problem-solving and Reasoning. The Progressing pathway also includes the ‘Gold star’ question(s).

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works best for them.

If required, the chapter pre-tests can be used as a diagnostic tool. The following are recommended guidelines:

- A student who gets 40% or lower should heavily revise core concepts before doing the Building questions, and may require further assistance.
- A student who gets between 40% and 75% should do the Building questions.
- A student who gets 75% and higher should do the Progressing questions.

For schools that have classes grouped according to ability, teachers may wish to set either the Building or Progressing pathways as the default pathway for an entire class and then make individual alternations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : complete none of the questions in this section.

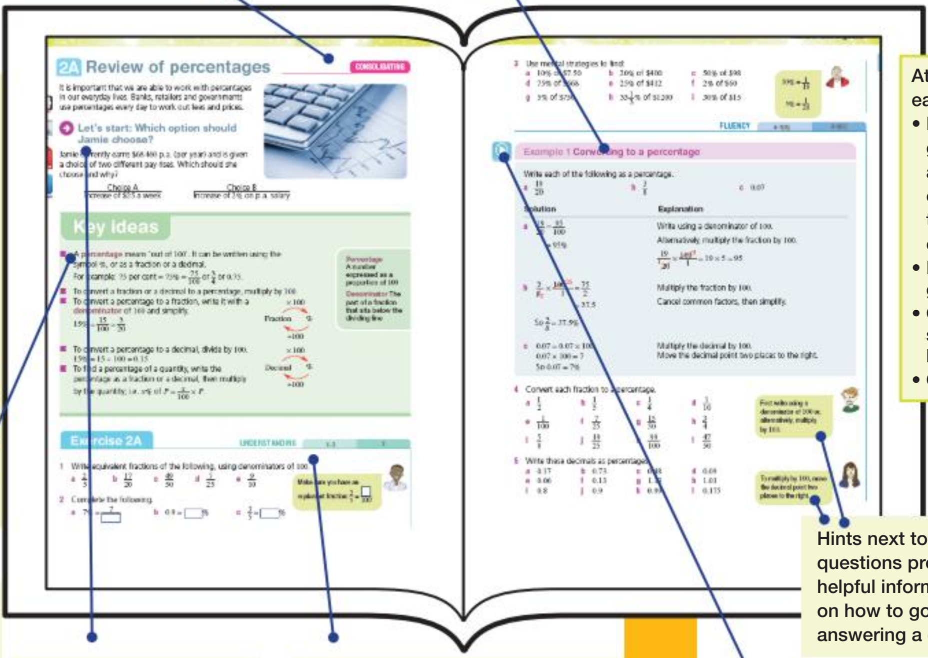
An overview of the *Essential Mathematics Gold for the Australian Curriculum* complete learning suite

For more detail, see the guide in the online Interactive Textbook

Non-core topics in each chapter are labelled as ‘Consolidating’ or ‘Gold Star’ to assist with course planning.

Worked examples with Solution and Explanation columns are placed inside the exercises next to the most relevant questions.

PRINT TEXTBOOK



At the end of each chapter:
- Maths@Work gives practical applications of maths on-the-job and in everyday life.
- Puzzles and games
- Chapter summary of key knowledge
- Chapter review

Hints next to exercise questions provide helpful information on how to go about answering a question.

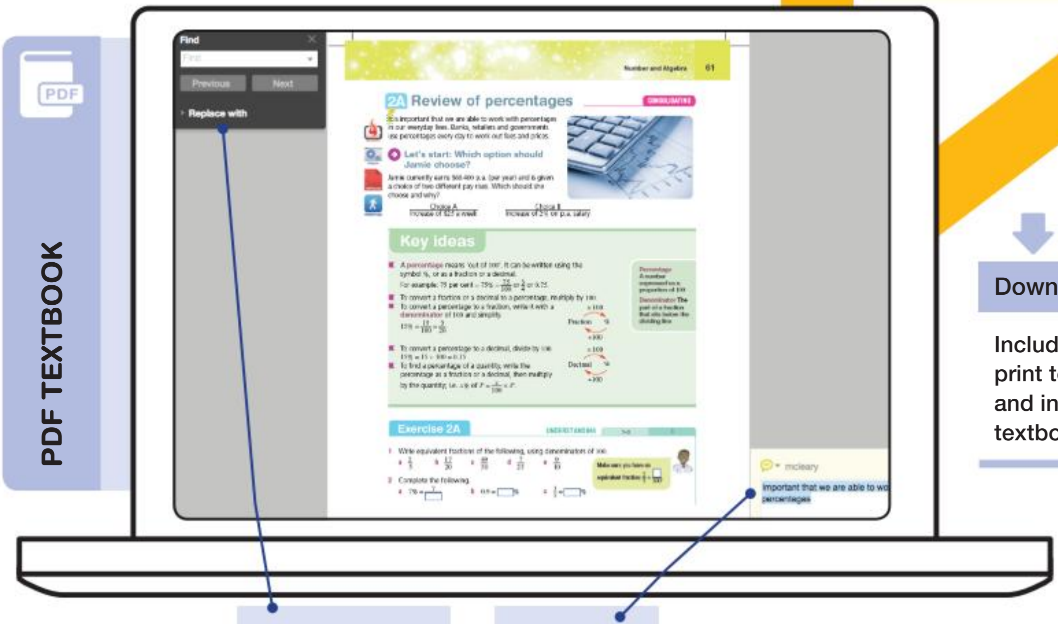
Key Ideas summarises the knowledge and skills required for the lesson

Let’s start activities provide an engaging way to start the lesson.

Building and Progressing working programs in every exercise provide two learning pathways depending on student need.

All worked examples in the print book feature a video demonstration in the Interactive Textbook

PDF TEXTBOOK



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Note-taking

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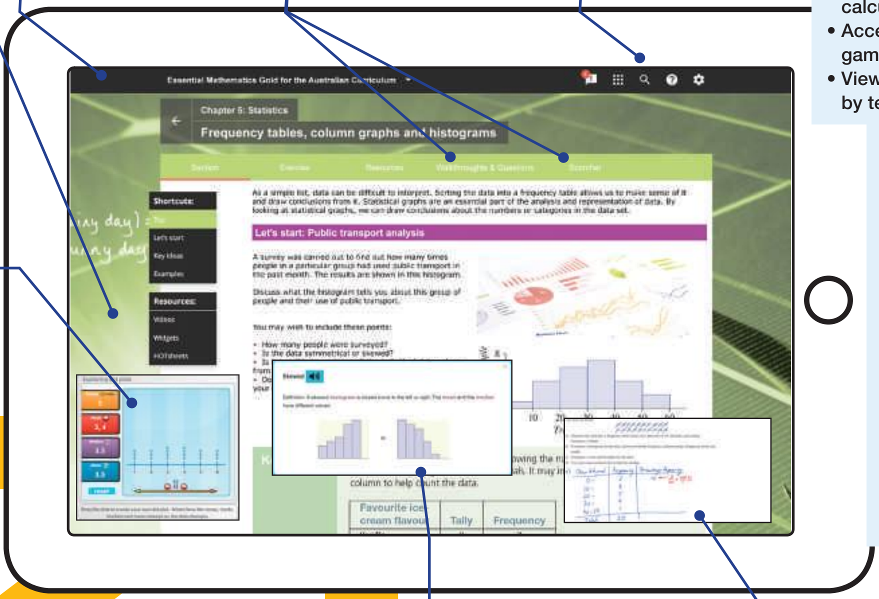

Easy navigation within sections without scrolling

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- Pop-up scientific calculator
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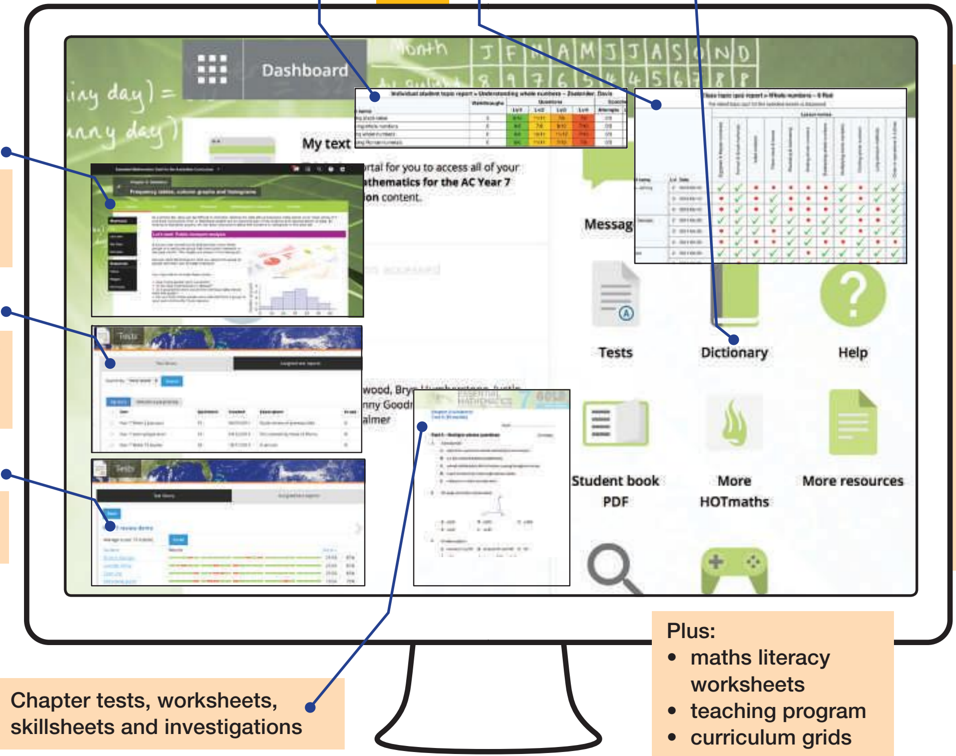

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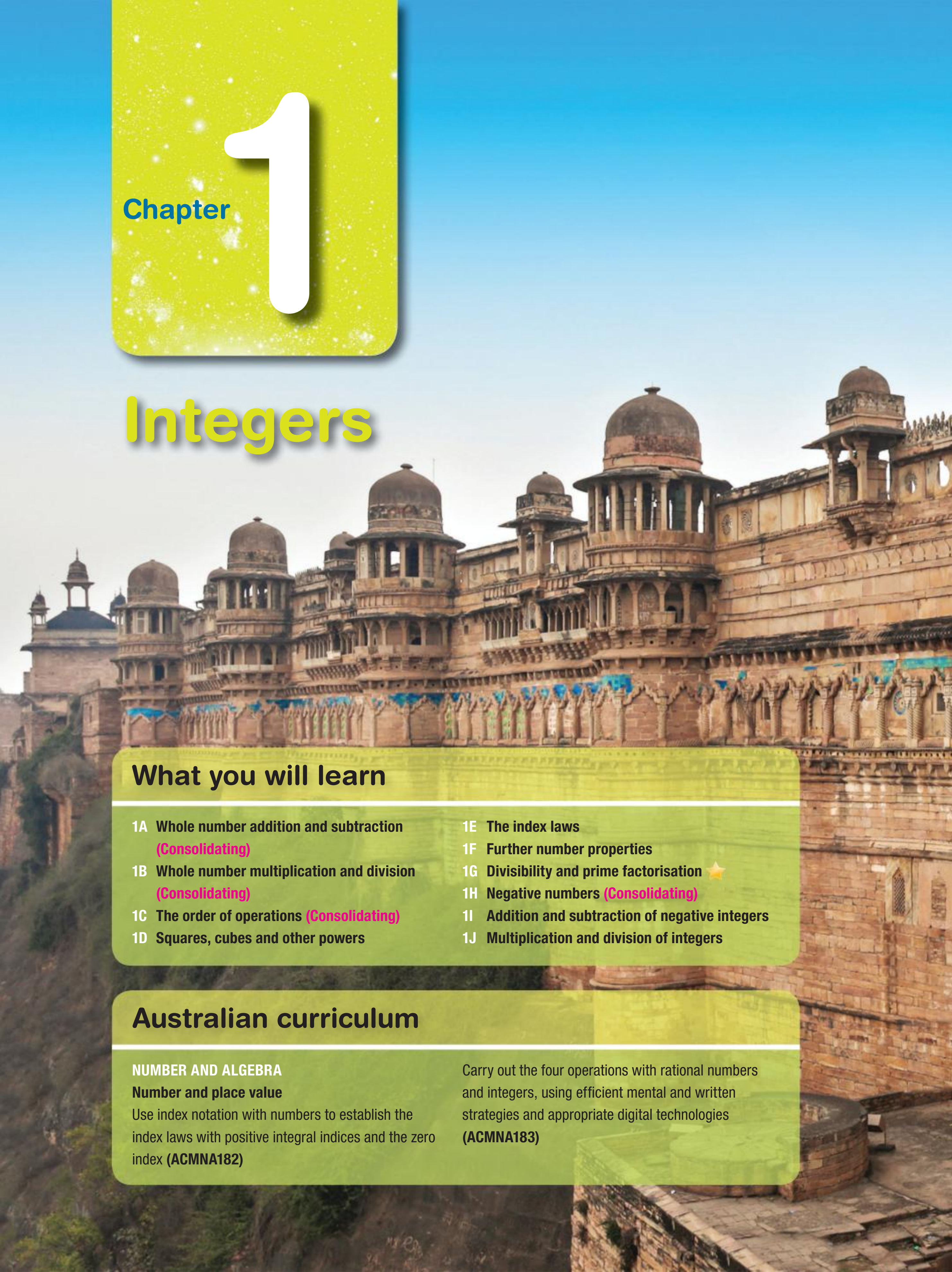


Chapter 1

Integers

What you will learn

- **1A Whole number addition and subtraction (Consolidating)**
- **1B Whole number multiplication and division (Consolidating)**
- **1C The order of operations (Consolidating)**
- **1D Squares, cubes and other powers**
- **1E The index laws**
- **1F Further number properties**
- **1G Divisibility and prime factorisation** ★
- **1H Negative numbers (Consolidating)**
- **1I Addition and subtraction of negative integers**
- **1J Multiplication and division of integers**

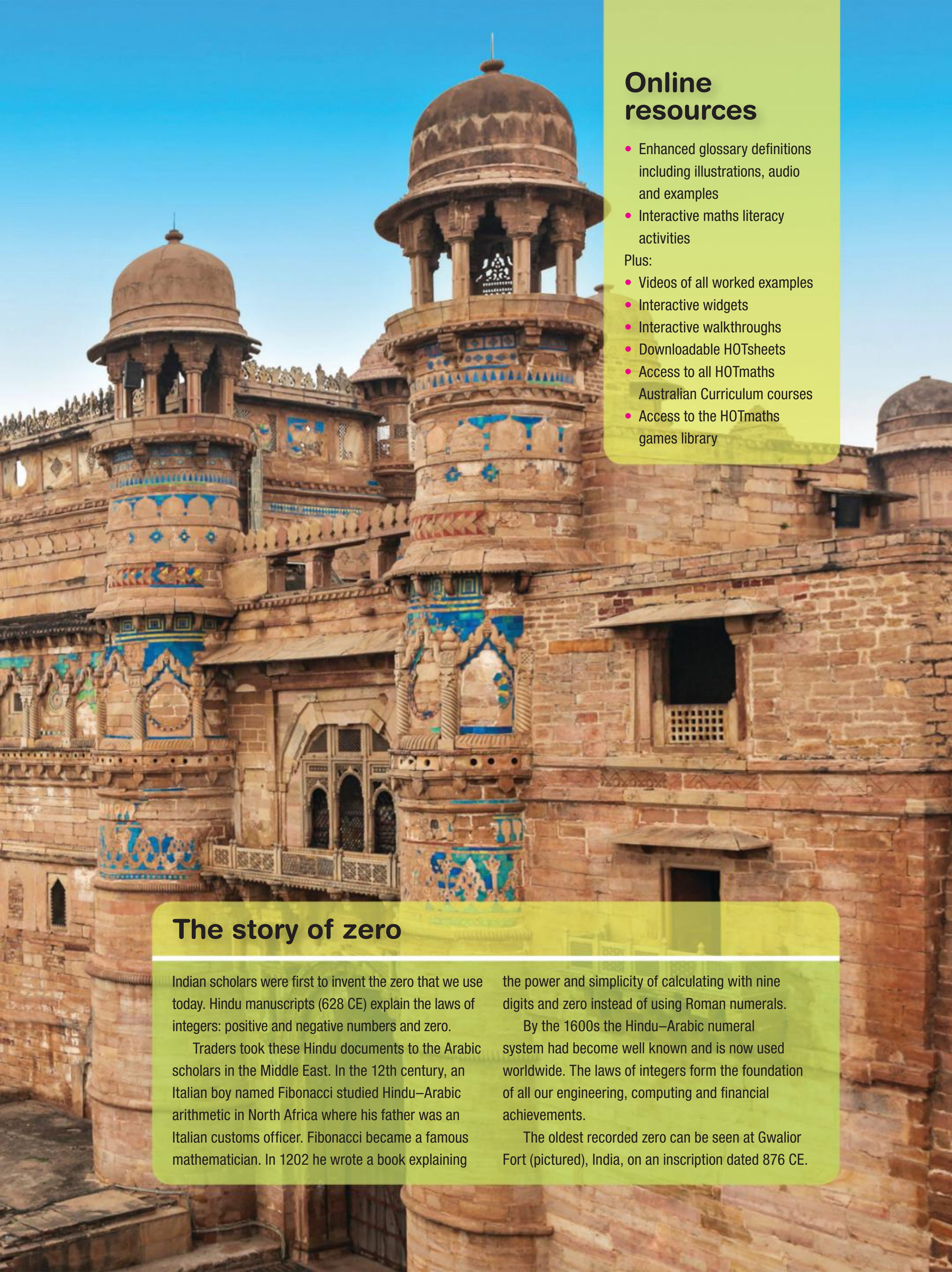
Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Use index notation with numbers to establish the index laws with positive integral indices and the zero index **(ACMNA182)**

Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies **(ACMNA183)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

The story of zero

Indian scholars were first to invent the zero that we use today. Hindu manuscripts (628 CE) explain the laws of integers: positive and negative numbers and zero.

Traders took these Hindu documents to the Arabic scholars in the Middle East. In the 12th century, an Italian boy named Fibonacci studied Hindu–Arabic arithmetic in North Africa where his father was an Italian customs officer. Fibonacci became a famous mathematician. In 1202 he wrote a book explaining the power and simplicity of calculating with nine digits and zero instead of using Roman numerals.

By the 1600s the Hindu–Arabic numeral system had become well known and is now used worldwide. The laws of integers form the foundation of all our engineering, computing and financial achievements.

The oldest recorded zero can be seen at Gwalior Fort (pictured), India, on an inscription dated 876 CE.

Pre-test

1 Decide if the following expressions relate to A: addition (+), S: subtraction (–), M: multiplication (×) or D: division (÷).

a total sum **b** difference **c** 10 more than 7
d 10 less than 13 **e** 5 groups of 3 **f** how many 3s in 18
g the product **h** the quotient **i** increase by 6

2 Complete these additions.

a $12 + 7$ **b** $50 + 19$ **c** $42 + 31$
d $146 + 213$ **e** $15 + 19 + 23$ **f** $\begin{array}{r} 123 \\ +\ 39 \\ \hline \end{array}$

3 Complete these subtractions.

a $12 - 8$ **b** $50 - 28$ **c** $47 - 29$
d $12 - 6 - 6$ **e** $784 - 163$ **f** $\begin{array}{r} 336 \\ -289 \\ \hline \end{array}$

4 Complete these multiplications.

a 9×4 **b** 5×8 **c** 12×11
d 15×5 **e** $\begin{array}{r} 121 \\ \times\ \ 9 \\ \hline \end{array}$ **f** $\begin{array}{r} 338 \\ \times\ 14 \\ \hline \end{array}$

5 Complete these divisions.

a $28 \div 4$ **b** $99 \div 3$ **c** $18 \div 6$
d $72 \div 12$ **e** $3\overline{)453}$ **f** $7\overline{)364}$

6 **a** List the first 5 multiples of 6.
b List the first 4 multiples of 9.
c What is the lowest common multiple (LCM) of 6 and 9?

7 **a** List all the factors of 12.
b List all the factors of 15.
c What is the highest common factor (HCF) of 12 and 15?

8 Prime numbers have exactly two factors. Of the first 15 positive integers (listed below), list the numbers which are prime. Write your primes in ascending (increasing) order. The first prime is circled.

1 ② 3 4 5 6 7 8 9 10 11 12 13 14 15

9 Answer the following as true (T) or false (F).

a $2 + 3 \times 4 = 2 + 12$ **b** $10 - 8 \div 2 = 10 - 4$ **c** $(5 - 2) \times 7 = 3 \times 7$
d $9 \times 3 + 5 = 9 \times 8$ **e** $9 \times (3 + 5) = 9 \times 8$ **f** $12 \div 3 \times 4 = 1$

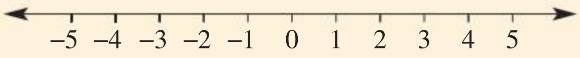
10 State the missing numbers for each part in this table.

a	$2 \times 2 =$ ☐	$\sqrt{4} = 2$	**e**	$9 \times 9 =$ ☐	$\sqrt{☐} = 9$
b	$3 \times 3 =$ ☐	$\sqrt{9} =$ ☐	**f**	$10 \times 10 =$ ☐	$\sqrt{☐} = 10$
c	$4 \times 4 =$ ☐	$\sqrt{16} =$ ☐	**g**	☐ × ☐ $= 49$	$\sqrt{49} =$ ☐
d	$6 \times 6 =$ ☐	$\sqrt{36} =$ ☐	**h**	☐ × ☐ $= 144$	$\sqrt{144} =$ ☐

11 What are the next two numbers in each of these patterns?

a 3, 2, 1, ___, ___ **b** 2, 0, –2, ___, ___ **c** –9, –10, –11, ___, ___

12 Use this number line to help find the answer.



a $2 - 5$ **b** $0 - 3$ **c** $-4 + 6$ **d** $-2 + 7$

1A Whole number addition and subtraction

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

The number system that we use today is called the Hindu–Arabic or decimal system. It uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The value of each digit depends on its place in the number, so, for example, the 4 in 3407 has a place value of 400. Whole numbers include 0 (zero) and the counting (natural) numbers 1, 2, 3, 4, We can add or subtract whole numbers to find sums and differences.

Let's start: Sum and difference

Use a guess-and-check method to try to find a pair of numbers described by these sentences.

- The sum of two numbers is 41 and their difference is 11.
- The sum of two numbers is 41 and their difference is 1.

Describe the meaning of the words 'sum' and 'difference'. Discuss how you found the pair of numbers in each case.

Key ideas

- You can add in any order.
  e.g. $7 + 5 = 5 + 7$
  $9 + 3 + 1 = 9 + 1 + 3$
  - This is called the **commutative law** for addition.
- You cannot subtract in any order.
  e.g. $7 - 5 \neq 5 - 7$
- If the numbers are large, write numbers in columns and use known algorithms to calculate the answer.

$$\begin{array}{r} {}^{1}431 \\ +\ \ 895 \\ \hline 1326 \end{array} \qquad \begin{array}{r} \overset{2}{\cancel{3}}\overset{1}{2}4 \\ -\ 172 \\ \hline 152 \end{array}$$

Commutative law When adding and multiplying, the order in which two numbers are combined does not matter

Exercise 1A

UNDERSTANDING 1–4 4

1 Match each of the questions in the left-hand column (**a**, **b**, **c** and **d**) to the working out in the right-hand column (**I**, **II**, **III**, **IV**).

a the total of 156, 94 and 6

b take 856 away from 2491

c 945 more than 2491

d 945 less 863

I $\begin{array}{r} 2491 \\ +\ 945 \\ \hline \end{array}$

II $2491 - 856$

III $156 + 94 + 6$

IV $\begin{array}{r} 945 \\ -863 \\ \hline \end{array}$

2 Write each of the following using an addition (+) or a subtraction (–) sign instead of the words.

a 26 plus 17
b 43 take away 9
c 134 minus 23
d 451 add 50
e the sum of 19 and 29
f the sum of 111 and 236
g the difference between 59 and 43
h the difference between 339 and 298
i 36 more than 8
j 142 more than 421
k 32 less than 49
l 120 less than 251

3 Copy and complete.

a

+	2	5	7	10	12
5					
0					
18					
58					

b

+	3	9		
15				30
		10		
6			24	
2				

4 Are these sums and differences true (T) or false (F)?

a $15 + 6 = 6 + 15$
b $29 - 6 = 6 - 29$
c $95 + 0 = 95$
d $81 - 81 = 0$
e $15 + 6 + 4 = 15 + 10$
f $41 - 6 + 4 = 41 - 10$

FLUENCY 5–8(½) 6–8(½)

Example 1 Using mental arithmetic

Evaluate this difference and these sums mentally.

a 347 – 39
b 125 + 127
c 28 + 13

Solution	Explanation	
a 347 – 39 = 308	$347 - 39 = 347 - 40 + 1$ $= 307 + 1$ $= 308$	This method is called compensating.
b 125 + 127 = 252	$125 + 127 = 2 \times 125 + 2$ $= 250 + 2$ $= 252$	This method is called doubling.
c 28 + 13 = 41	$28 + 13 = 28 + 12 + 1$ $= 40 + 1$ $= 41$	This method is called counting on.

5 Complete these sums.

a 21 + 5
b 3 + 14
c 17 + 13
d 298 + 2
e 35 + 11
f 16 + 19
g 21 + 5
h 6 + 18

Do these without a calculator or algorithm.

6 Complete these differences.

a 5 – 2
b 16 – 4
c 16 – 14
d 21 – 21
e 16 – 3
f 45 – 13
g 52 – 12
h 52 – 14

7 Evaluate these sums and differences mentally.

a $94 - 62$ **b** $146 + 241$ **c** $1494 - 351$ **d** $36 + 19$

e $138 + 25$ **f** $251 - 35$ **g** $99 - 20$ **h** $441 - 50$

i $350 + 351$ **j** $115 + 114$ **k** $80 - 41$ **l** $320 - 159$

Example 2 Using an algorithm

Use an algorithm to find this sum and difference.

a $\begin{array}{r} 938 \\ +\ 217 \\ \hline \end{array}$ **b** $\begin{array}{r} 141 \\ -\ 86 \\ \hline \end{array}$

Solution	**Explanation**
a $\begin{array}{rrrrr} & & 9 & {}^{1}3 & 8 \\ + & & 2 & 1 & 7 \\ \hline & 1 & 1 & 5 & 5 \end{array}$	$8 + 7 = 15$ (carry the 1 to the tens column) $1 + 3 + 1 = 5$ $9 + 2 = 11$
b $\begin{array}{rrrr} & \cancel{1} & {}^{3}\cancel{4} & {}^{1}1 \\ - & & 8 & 6 \\ \hline & & 5 & 5 \end{array}$	Borrow from the tens column then subtract 6 from 11. Now borrow from the hundreds column and then subtract 8 from 13.

8 Use an algorithm to find these sums and differences.

a $\begin{array}{r} 128 \\ +\ 46 \\ \hline \end{array}$ **b** $\begin{array}{r} 94 \\ +\ 337 \\ \hline \end{array}$ **c** $\begin{array}{r} 9014 \\ +\ 927 \\ +\ 421 \\ \hline \end{array}$

d $\begin{array}{r} 814 \\ +\ 1439 \\ +\ 326 \\ \hline \end{array}$ **e** $\begin{array}{r} 94 \\ -\ 36 \\ \hline \end{array}$ **f** $\begin{array}{r} 421 \\ -\ 204 \\ \hline \end{array}$

g $\begin{array}{r} 1726 \\ -\ 1699 \\ \hline \end{array}$ **h** $\begin{array}{r} 14072 \\ -\ 328 \\ \hline \end{array}$ **i** $\begin{array}{r} 428 \\ +\ 314 \\ +\ 107 \\ +\ 29 \\ \hline \end{array}$

j $\begin{array}{r} 1004 \\ +\ 2407 \\ +\ 9116 \\ +\ 10494 \\ \hline \end{array}$ **k** $\begin{array}{r} 3017 \\ -\ 2942 \\ \hline \end{array}$ **l** $\begin{array}{r} 10024 \\ -\ 936 \\ \hline \end{array}$

Carry the 1 for sums larger than 9 and borrow ‘ten’ for subtraction.

PROBLEM-SOLVING AND REASONING	9, 10	10–12

9 A racing bike’s odometer shows 21 432 km at the start of a race and 22 110 km at the end of the race. How far was the race?

10 Kristian has \$246 more than Sally. David has \$56 less than Sally. If Sally has \$492, how much do Kristian and David have?

11 Callum walks 15 km on Monday and 3 km more each day. How many kilometres does Callum walk on Thursday?

12 The sum of two numbers is 39 and their difference is 5. What is the larger number?

Filling the gap and magic triangles

— 13

13 **a** Write the digit missing from these sums and differences.

i
$$\begin{array}{cccc} & 2 & 3 & 7 \\ + & & 4 & \square \\ \hline & 2 & 7 & 9 \end{array}$$

ii
$$\begin{array}{ccc} & 4 & 9 \\ + & 3 & 8 \\ \hline & 8 & \square \end{array}$$

iii
$$\begin{array}{cccc} & 4 & 9 & 3 \\ + & 2 & 1 & 4 \\ \hline & 7 & \square & 7 \end{array}$$

iv
$$\begin{array}{cccc} & 1 & \square & 4 \\ + & 3 & 9 & 2 \\ \hline & 5 & 5 & 6 \end{array}$$

v
$$\begin{array}{ccc} & 3 & 8 \\ - & 1 & 9 \\ \hline & 1 & \square \end{array}$$

vi
$$\begin{array}{cccc} & 1 & 2 & 8 \\ - & & 8 & \square \\ \hline & & 3 & 9 \end{array}$$

vii
$$\begin{array}{cccc} & 3 & \square & 4 \\ - & 1 & 6 & 2 \\ \hline & 1 & 4 & 2 \end{array}$$

viii
$$\begin{array}{cccc} & 2 & 5 & 1 \\ - & 1 & \square & 4 \\ \hline & & 8 & 7 \end{array}$$

b Find the missing digits in these sums and differences.

i
$$\begin{array}{cccc} & 2 & 3 & \square \\ + & \square & 9 & 4 \\ \hline & 6 & \square & 1 \end{array}$$

ii
$$\begin{array}{cccc} & \square & 3 & \square \\ + & & \square & 2 \\ \hline & 2 & 1 & 9 \end{array}$$

iii
$$\begin{array}{cccc} & \square & 3 & 7 \\ + & 4 & 9 & \square \\ \hline & 7 & \square & 2 \end{array}$$

iv
$$\begin{array}{ccc} & \square & 3 \\ - & 2 & 9 \\ \hline & 6 & \square \end{array}$$

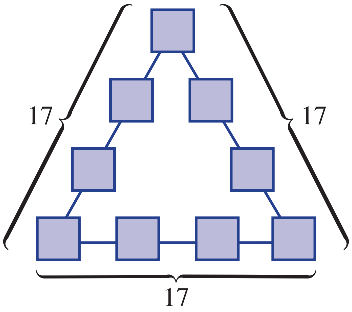
v
$$\begin{array}{cccc} & 3 & \square & 2 \\ - & \square & 3 & \square \\ \hline & 1 & 0 & 4 \end{array}$$

vi
$$\begin{array}{ccccc} & 2 & \square & \square & 5 \\ - & & 6 & 8 & \square \\ \hline & \square & 3 & 1 & 8 \end{array}$$

c The sides of a magic triangle all sum to the same total.

i Show how it is possible to arrange all the digits from 1 to 9 so that each side adds to 17.

ii Show how it is possible to arrange the same digits to a different total. How many different totals can you find?



1B Whole number multiplication and division

CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthrough

Multiplying and dividing are two key operations in mathematics and are useful in many practical situations such as finding the cost of 9 tickets at $\$109$ each or the number of trucks needed to carry 280 tonnes of coal.

A typical large mining truck has a capacity of 140 tonnes.

Let's start: Multiplication or division?

In solving many problems it is important to know whether multiplication or division should be used. Decide if the following situations require the use of multiplication or division. Discuss them in a group or with a partner.

- The number of cookies 4 people get if a packet of 32 cookies is shared equally between them.
- The cost of paving 30 square metres of courtyard at a cost of $\$41$ per square metre.
- The number of sheets of paper in a shipment of 4000 boxes of 5 reams each (1 ream is 500 sheets).
- The number of hours I can afford a plumber at $\$75$ per hour if I have a fixed budget of $\$1650$.

Make up your own situation that requires the use of multiplication and another for division.

Key ideas

- Another word for multiplication is **product**.
- You need to know your multiplication tables.
- Multiplication can be done:
  - mentally
    e.g. $6 \times 5 = 30$
  - algorithm
    e.g.

$$\begin{array}{r l}
217 & \\
\times\ \ 26 & \\
\hline
1302 & \leftarrow 217 \times 6 \\
4340 & \leftarrow 217 \times 20 \\
\hline
5642 & \leftarrow 1302 + 4340
\end{array}$$

- You can multiply numbers in any order.
  e.g. $6 \times 5 = 30$ and $5 \times 6 = 30$
  - This is the commutative law for multiplication.
- The **distributive law** is helpful when multiplying.
  e.g.

$$\begin{aligned} 5 \times 34 &= 5 \times (30 + 4) \\ &= 5 \times 30 + 5 \times 4 \\ &= 150 + 20 \\ &= 170 \end{aligned}$$

- Using division results in finding a **quotient** and a **remainder**.

  e.g. $38 \div 11 = 3$ and 5 remainder or $38 \div 11 = 3\frac{5}{11}$

  dividend divisor quotient
- Division can be done:
  - mentally
    e.g. $56 \div 8 = 7$
  - algorithm

$$\begin{array}{r l} & \ \ 7\ \ 3\ \ 2 \\ 7 & \overline{)\,5\ 1\ {}^{2}2\ {}^{1}4} \end{array}$$

Product
The result of multiplication

Quotient
The result of division

Remainder
The amount left over after division, when one number cannot be divided exactly into another

Distributive law
Adding numbers *then* multiplying the total gives the same answer as multiplying each number first *then* adding the products

Exercise 1B

UNDERSTANDING 1–4 4

1 Match each of the questions to the working out on the right.

a	the product of 9 and 6	**I**	15×12
b	36 divided by 12	**II**	$15 \div 5$
c	15 lots of 12	**III**	9×6
d	the quotient when 15 is divided by 5	**IV**	$15 \div 12$
e	divide 12 into 15	**V**	$36 \div 12$

2 Copy and complete these multiplication grids.

a

×	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

b

×	2	5	7	9
	6			
		20		
			63	
				90

3 Use your knowledge of the multiplication table to answer the following.

a 5×8 **b** 11×9 **c** 6×7 **d** 9×8

e 11×6 **f** 12×11 **g** 8×4 **h** 7×9

i $100 \div 10$ **j** $88 \div 8$ **k** $121 \div 11$ **l** $144 \div 12$

m $56 \div 7$ **n** $33 \div 3$ **o** $65 \div 5$ **p** $78 \div 6$

You should know most of these off by heart.

4 Are these simple equations true (T) or false (F)?

a $4 \times 13 = 13 \times 4$ **b** $2 \times 7 \times 9 = 7 \times 9 \times 2$

c $6 \div 3 = 3 \div 6$ **d** $60 \div 20 = 30 \div 10$

e $14 \div 2 \div 7 = 7 \div 2 \div 14$ **f** $51 \times 7 = (50 \times 7) + (1 \times 7)$

g $79 \times 13 = (80 \times 13) - (1 \times 13)$ **h** $93 \div 3 = (90 \div 3) + (3 \div 3)$

FLUENCY 5–7(½) 5–8(½)

Example 3 Using mental strategies for multiplication

Use a mental strategy to evaluate the following.

a 5×160 **b** 7×89 **c** $5 \times 43 \times 2$

Solution	**Explanation**
a $5 \times 160 = 800$	To multiply by 5 you can multiply by 10 then halve the result. $160 \times 10 = 1600$, $1600 \div 2 = 800$
b $7 \times 89 = 623$	$89 = 90 - 1 \therefore 7 \times 89 = 7 \times 90 - 7 \times 1 = 630 - 7 = 623$ (this is the distributive law)
c $5 \times 43 \times 2 = 430$	$5 \times 43 \times 2 = \underbrace{5 \times 2}_{\text{look for easy pairs}} \times 43$ $= 10 \times 43$ $= 430$

5 Use a mental strategy to evaluate the following.

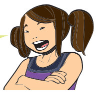
a 15×3 **b** 18×4 **c** $6 \times 5 \times 2$ **d** 7×20

e 16×4 **f** 99×7 **g** 79×3 **h** 42×5

i $5 \times 13 \times 2$ **j** $2 \times 26 \times 5$ **k** 4×35 **l** 17×4

m 17×1000 **n** 136×100 **o** 59×7 **p** 119×6

q 9×51 **r** 6×61 **s** 4×252 **t** 998×6



Do these mentally.

Example 4 Using mental strategies for division

Use a mental strategy to evaluate the following.

a $464 \div 4$ **b** $480 \div 5 \div 2$

Solution	**Explanation**
a $464 \div 4 = 116$	To divide by 4 you can divide by 2 twice. $464 \div 4 = 464 \div 2 \div 2$ ($\div 2$ is the same as halving the number) $= 232 \div 2$ $= 116$
b $480 \div 5 \div 2 = 48$	Dividing by 5 and then by 2 is the same as dividing by 10. $480 \div 10 = 48$

6 Use a mental strategy to evaluate the following.

a $64 \div 2$ **b** $64 \div 4$ **c** $640 \div 4$ **d** $492 \div 4$

e $185 \div 5$ **f** $1980 \div 5 \div 2$ **g** $128 \div 8$ **h** $252 \div 4$

i $123 \div 3$ **j** $508 \div 4$ **k** $96 \div 6$ **l** $1016 \div 8$

Choose one of the mental strategies described above.

Example 5 Using an algorithm for multiplication and division

Use an algorithm to evaluate the following.

a $\begin{array}{r} 412 \\ \times\ 25 \end{array}$ **b** $938 \div 13$

Solution	**Explanation**
a $\begin{array}{r} 412 \\ \times\ \ 25 \\ \hline 2060 \\ 8240 \\ \hline 10300 \end{array}$	$412 \times 5 = 2060$ and $412 \times 20 = 8240$ Add these two products to get the final answer.
b $\begin{array}{r} 7\ \ 2 \\ 13\overline{)9\ 3\ {}^{2}8} \end{array}$ Rem 2	$93 \div 13 = 7$ and 2 remainder $28 \div 13 = 2$ and 2 remainder
So $938 \div 13 = 72$ and 2 remainder.	
$938 \div 13 = 72\frac{2}{13}$	We write remainders as fractions $72\frac{2}{13}$.

7 Use an algorithm to evaluate the following.

a $\begin{array}{r} 67 \\ \times 9 \\ \hline \end{array}$ **b** $\begin{array}{r} 129 \\ \times\ \ 4 \\ \hline \end{array}$ **c** $\begin{array}{r} 294 \\ \times\ 13 \\ \hline \end{array}$ **d** $\begin{array}{r} 1004 \\ \times\ \ 90 \\ \hline \end{array}$

e $\begin{array}{r} 690 \\ \times\ 14 \\ \hline \end{array}$ **f** $\begin{array}{r} 96 \\ \times 12 \\ \hline \end{array}$ **g** $\begin{array}{r} 58 \\ \times 24 \\ \hline \end{array}$ **h** $\begin{array}{r} 163 \\ \times 52 \\ \hline \end{array}$

Use the setting out described in Example 5.

8 Use the short division algorithm to evaluate the following. Write your answer using fractions if there is a remainder.

a $3\overline{)85}$ **b** $7\overline{)214}$ **c** $10\overline{)4167}$ **d** $15\overline{)207}$

e $6\overline{)15084}$ **f** $3\overline{)1236}$ **g** $12\overline{)2520}$ **h** $12\overline{)8892}$

PROBLEM-SOLVING AND REASONING

9–11 10–13

9 A university student earns $550 for 20 hours work. What is the student's pay rate per hour?

10 Packets of biscuits are purchased by a supermarket in boxes of 12. The supermarket orders 220 boxes and sells 89 boxes in one day. How many boxes are left? How many packets of biscuits remain in the supermarket?

11 Riley buys a fridge, which he can pay for by the following options.

A 9 payments of $183

B $1559 up front

Which option is cheaper and by how much?

12 The shovel of a giant excavator can move 6 tonnes of rock in each load. How many loads are needed to shift 750 tonnes of rock?

13 Tom saves $362 a week. How much will he save in 52 weeks?

Maximum tickets

— 14

14 A child ticket to a theatre is $7 and an adult ticket is $12.

a Find the cost of 2 adults and 3 children tickets.

b Find the cost of 1 adult and 5 children tickets.

c Gen spends exactly $90 to buy child tickets and adult tickets. Find the maximum number of tickets that Gen could purchase.

1C The order of operations

CONSOLIDATING

When working with brackets and more than one operation, including multiplication, division, addition and subtraction, a particular order needs to be followed.

Let us look at the simple calculation $5 + 4 \times 5 = 25$.

If we did the addition first, then $5 + 4 \times 5 = 9 \times 5 = 45$, but we know that this is not true. We need to be consistent with our order of operations to ensure we all get the same answer for each problem.

We must all use the same rules with calculations so that we can agree on the results.

Let's start: How many?

How many ways can you express $36 - 20 = 16$?

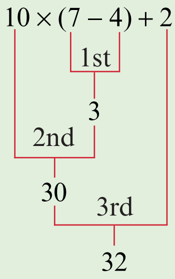
See if you can come up with at least five different statements using the four operations $(+ - \times \div)$ and brackets that give the same subtraction above. One example is $9 \times 4 - (24 - 4)$.

Key ideas

Order of operations

- Deal with the **grouping symbols** or brackets first.
- Do any multiplication $(\times)$ and division $(\div)$ next, working from left to right.
- Do any addition $(+)$ and subtraction $(-)$ next, again working from left to right.

NOTE: Within any brackets the order of operations still needs to be followed.



Grouping symbols
Parentheses ( ), brackets [ ] and braces { } are used to collect terms and operations together

Exercise 1C

UNDERSTANDING 1–4 4

1 Copy each question into your books. By following the order of operations, underline the operation that needs to be done first.

a $2 + 3 \times 9$

b $10 - 2 \div 2$

c $1 \times 3 + 5$

d $6 \times (9 - 6)$

e $(12 + 6) \div 2$

2 Match each of the questions on the left to the correct working on the right.

- **a** $10 + 7 \times 3$
- **b** $15 - 9 \div 3$
- **c** $(9 - 4) \times 6$
- **d** $(9 - 4) - (10 - 6)$
- **e** $18 \div 9 + 5 \times 2$

- **I** $10 + 21$
- **II** $5 - 4$
- **III** $15 - 3$
- **IV** $2 + 10$
- **V** 5×6

Example 6 Combining two operations

Evaluate the following.

- **a** $10 + 5 \times 3$
- **b** $18 \div 6 \times 2$
- **c** $15 - (7 - 3)$

Solution	**Explanation**
a $10 + 5 \times 3 = 10 + 15$ $= 25$	Multiplication $(\times)$ is done BEFORE addition $(+)$. $5 \times 3 = 15$
b $18 \div 6 \times 2 = 3 \times 2$ $= 6$	Division $(\div)$ and multiplication $(\times)$ are done as they appear from left to right. $18 \div 6$ is done first then $\times 2$ last.
c $15 - (7 - 3) = 15 - 4$ $= 11$	Brackets need to be done first $(7 - 3) = 4$. Then do the subtraction $15 - 4$.

3 Find the answers to each of the following.

- **a** $12 + 5 \times 2$
- **b** $24 - 6 \times 3$
- **c** $10 \times 2 + 6$
- **d** $15 \div 3 - 2$
- **e** $(9 - 2) \times 4$
- **f** $18 - (12 - 8)$
- **g** $28 \div (2 \times 7)$
- **h** $56 - 5 \times 10$
- **i** $120 + 200 \div 5$
- **j** $88 \times 2 \div 8$
- **k** $12 \div (18 \div 6)$
- **l** $16 - 18 \div 9$
- **m** $55 \div 11 \times 5$
- **n** $55 - 25 \div 5$
- **o** $240 \div 10 \times 2$
- **p** $58 + 100 \div 20$
- **q** $100 - 25 \div 5$
- **r** $(24 - 9) \times 3$

First: brackets
Next: × or ÷
Last: + or –

4 Find the answers to these problems by first writing the sentence using numbers and symbols.

- **a** Double the sum of 3 and 7
- **b** Double the quotient of 24 and 8
- **c** The product of 5 and 7 plus 4
- **d** 8 more than the product of 12 and 5
- **e** 10 less than the quotient of 66 and 3
- **f** Triple the difference between 18 and 12

Example 7 Using more than two steps

Evaluate the following.

a $4 \times 5 - 3 \times 2$

b $(7 + 2) \times 5 - 6$

c $10 + (2 \times (6 - 4))$

Solution	Explanation
a $4 \times 5 - 3 \times 2$ $= 20 - 6$ $= 14$	Both sets of multiplication (×) need to be done first. Then do the subtraction (–).
b $(7 + 2) \times 5 - 6$ $= 9 \times 5 - 6$ $= 45 - 6$ $= 39$	Do the brackets first $(7 + 2)$. Next do the multiplication 9×5. Then the subtraction $45 - 6$.
c $10 + (2 \times (6 - 4))$ $= 10 + (2 \times 2)$ $= 10 + 4$ $= 14$	Start with the inner most brackets $(6 - 4)$. Finish working with the brackets – we follow the order of operations within the brackets (2×2). Then the addition $10 + 4$.

FLUENCY 5–6(½) 5–6(½), 7

5 Find the answers to the following.

a $2 \times 4 - 4 \div 2$ **b** $13 + 4 \times 5 - 3$ **c** $(14 - 12) \times 4 + 11$

d $(12 - 5) \times (6 + 3)$ **e** $5 \times 6 + 12 \times 3$ **f** $25 - 20 \div 5 + 2$

g $25 - 20 \div 5 + 2 \times 5$ **h** $(10 + 10) \div (25 - 5)$ **i** $(10 \times 10 + 5) \div 5$

Show steps of working as in the examples.

6 Simplify.

a $5 \times 4 + 8 \times 4$ **b** $24 \div 4 \times 6 - 8$ **c** $(15 - 5) \times 8 + 200$

d $6 \times 4 - 2 \times 6 + 12$ **e** $96 \div (12 \times 8)$ **f** $5 + (12 \times (23 - 6))$

g $1 + 4 + 3 \times (8 - 5)$ **h** $(12 - 5) \times (22 - 12)$ **i** $12 + (18 - (12 - 5))$

7 Evaluate.

a $56 - 4 \times 6$ **b** $96 \div 4 + 3 \times 6$

c $150 - (7 \times (10 - 3 \times 2))$ **d** $(12 \times (13 - 8) \times (24 - 18))$

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 True (T) or false (F)?

a $5 + 9 = 5 + 3 \times 3$ **b** $10 + 2 \times 7 = 12 + 7$ **c** $18 - 6 + 5 = 12 + 5$

d $3 \times 5 \times 6 = 15 \times 6$ **e** $120 \div 6 \times 2 = 20 \times 2$ **f** $(5 + 3) \times 9 = 8 \times 9$

9 Insert brackets into each of the following statements to make it true.

a $12 - 8 \times 2 = 8$ **b** $4 \times 5 + 6 = 44$

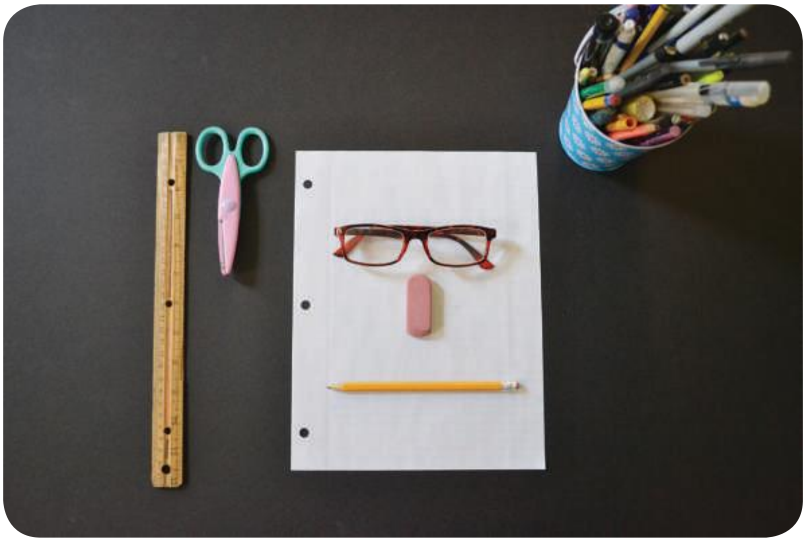
c $16 \div 2 \times 8 = 1$ **d** $6 \times 2 + 6 \times 1 = 48$

10 Insert operation symbols $(+, -, \times, \div)$ between the numbers to make each of the following statements true.

- **a** $5\_\_\_4\_\_\_9 = 0$
- **b** $5\_\_\_4\_\_\_9 = 11$
- **c** $5\_\_\_4\_\_\_9 = 41$

11 Write each of the following situations into mathematical symbols and numbers, and then calculate.

- **a** Murray receives four dollars from his mum and seven dollars from his dad as pocket money each week for 12 weeks. How much money does he have at the end of the 12 weeks?
- **b** A raffle prize consists of \$5000 cash and 6 shopping vouchers each worth \$500. What is the total value of the raffle prize?
- **c** Sally has fifty dollars. She buys four pens at two dollars each and eight exercise books at three dollars each. How much change does Sally get?



12 Decide if the brackets in each of the following are really needed.

- **a** $10 + (9 \times 8)$
- **b** $12 + (3 + 4)$
- **c** $12 - (3 + 4)$
- **d** $25 \times (3 - 1)$
- **e** $(100 - 4 \times 3)$

Make ten from four

— 13

13 Can you make the first 10 counting numbers (1, 2, 3, 4, 5, 6, 7, 8, 9 and 10) using only the four digits 1, 2, 3 and 4 (once each), brackets and any of the four operations?

1D Squares, cubes and other powers

In mathematics there are many ways to abbreviate expressions.

Using repeated addition, $4 + 4 + 4 + 4 + 4$ can be written a 5×4 using multiplication.

Using repeated multiplication, $3 \times 3 \times 3 \times 3$ can be written as 3^4 using index notation.

We read 3^4 as 3 to the power of 4.

Interactive
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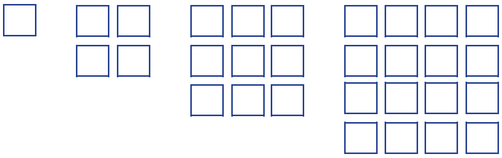
Walkthrough

Let's start: Square numbers



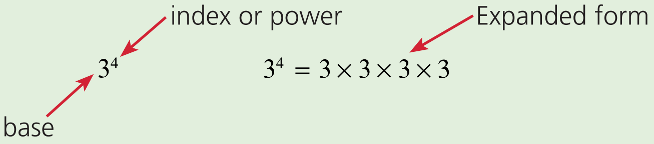
Can you explain why we call the numbers 1, 4, 9 and 16 square numbers?

Draw diagrams for the next two square numbers.

Use centicubes to build the first three cube numbers. Write down the next cube number.

Key ideas

- **Index** notation



The **base** of 3 shows the factor that is repeating in multiplication and the **power** or **index** is the number of times it appears.

- The **square** of a number is written a^2 and it means $a \times a$.
  e.g. 5^2 means 5×5 (we say 5 squared, the square of 5, or 5 to the power of 2)
- The opposite of squaring is the **square root** of a number. The symbol $\sqrt{}$ means square root.
  e.g. $\sqrt{9} = 3$ as $3^2 = 9$
  - The square root of a number is always positive or zero.
- The **cube** of a number a is $a^3 = a \times a \times a$.
  e.g. $5^3 = 5 \times 5 \times 5$ (we say 5 cubed, or, 5 to the power of 3)
- The opposite of cubing is taking the **cube root** of a number. The symbol for cube root is $\sqrt[3]{}$.
  e.g. $\sqrt[3]{8} = 2$ as $2^3 = 2 \times 2 \times 2 = 8$

Base
The number or pronumeral that is being raised to a power

Index
The number of times the base number is repeated under multiplication

Square
To multiply a number by itself

Square root
The opposite operation of squaring

Exercise 1D

UNDERSTANDING 1–5 5

1 Write each of the following using index notation.

a 2×2 **b** 4×4 **c** 5×5

d $5 \times 5 \times 5$ **e** $6 \times 6 \times 6 \times 6$ **f** $7 \times 7 \times 7$

2 Match each expression in words to an expression in symbols, given on the right.

a The square of 10

b The cube of 1

c The square of 12

d The square root of 1

e The cube root of 1

f The square root of 16

I $\sqrt{16}$

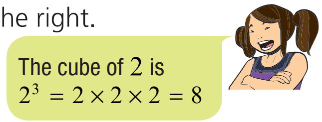
II $\sqrt[3]{1}$

III $\sqrt{1}$

IV 10^2

V 1^3

VI 12^2



3 Copy and complete.

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 =$
$4^2 =$
$5^2 =$
$6^2 =$
$7^2 =$
$8^2 =$
$9^2 =$
$10^2 =$

4 Copy and complete.

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 =$
$4^3 =$
$5^3 =$
$6^3 =$

Example 8 Using index notation

Write each product using index notation.

a $8 \times 8 \times 8$ **b** $7 \times 7 \times 7 \times 7 \times 7 \times 7$

Solution	Explanation
a $8 \times 8 \times 8 = 8^3$	The number 8 appears 3 times. We write 8 to the power of 3.
b $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$	The 7 appears 6 times. We write 7 to the power of 6.

5 Write each of the following products using index notation.

a $7 \times 7 \times 7$ **b** $10 \times 10 \times 10 \times 10$ **c** 8×8

d $4 \times 4 \times 4$ **e** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **f** $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

g 12×12 **h** $5 \times 5 \times 5 \times 5 \times 5 \times 5$ **i** 6

Example 9 Using expanded notation

a Write 5^4 in expanded form.

b Find the value of 5^4.

Solution	**Explanation**
a $5^4 = 5 \times 5 \times 5 \times 5$	The power of 4 tells us that the number 5 appears 4 times. $5^4 = 5 \times 5 \times 5 \times 5$
b $5^4 = 625$	$5^4 = 5 \times 5 \times 5 \times 5$ $= 25 \times 5 \times 5$ $= 125 \times 5$ $= 625$

FLUENCY 6–8(½) 6–9(½)

6 Write each index notation in expanded form.

a 8^5 **b** 3^4 **c** 9^2

d 4^4 **e** 2^8 **f** 11^2

$5 \times 5 \times 5$ is the expanded form of 5^3

7 Find the value of the following by first writing them in expanded form.

a 2^3 **b** 2^4 **c** 3^3

d 10^4 **e** 5^3 **f** 1^4

Example 10 Finding squares, cubes, square roots and cube roots

Evaluate the following.

a 6^2 **b** $\sqrt{81}$ **c** 2^3 **d** $\sqrt[3]{64}$

Solution	**Explanation**
a $6^2 = 6 \times 6$ $= 36$	Find the product of 6 with itself.
b $\sqrt{81} = 9$	$9^2 = 9 \times 9 = 81$ so $\sqrt{81} = 9$
c $2^3 = 2 \times 2 \times 2$ $= 8$	In general $x^3 = x \times x \times x$.
d $\sqrt[3]{64} = 4$	$4^3 = 4 \times 4 \times 4 = 64$ so $\sqrt[3]{64} = 4$

8 Evaluate these squares and square roots.

a 4^2 **b** 10^2 **c** 13^2 **d** 15^2
e 100^2 **f** 20^2 **g** $\sqrt{25}$ **h** $\sqrt{49}$
i $\sqrt{121}$ **j** $\sqrt{900}$ **k** $\sqrt{1600}$ **l** $\sqrt{256}$

$3^2 = 9$ and
$\sqrt{9} = 3$

9 Evaluate these cubes and cube roots.

a 2^3 **b** 4^3 **c** 7^3 **d** 5^3
e 6^3 **f** 10^3 **g** $\sqrt[3]{27}$ **h** $\sqrt[3]{8}$
i $\sqrt[3]{125}$ **j** $\sqrt[3]{512}$ **k** $\sqrt[3]{729}$ **l** $\sqrt[3]{1\,000\,000}$

PROBLEM-SOLVING AND REASONING

10, 11 10–12

10 Decide which of the following is larger.

a 2^3 or 3^2
b 2^4 or 3^2
c 2^5 or 5^2

11 Copy and complete.

a If $13^2 = 169$, then $\sqrt{169} = \boxed{}$
b If $15^2 = 225$, then $\sqrt{225} = \boxed{}$
c If $\sqrt{625} = 25$, then $25^2 = \boxed{}$
d If $9^3 = 729$, then $\sqrt[3]{729} = \boxed{}$
e If $\sqrt[3]{1331} = 11$, then $11^3 = \boxed{}$

12 Given $5 \times 5 \times 5 \times 4 \times 4$ is written as $5^3 \times 4^2$ (the different bases of 5 and 4 are kept separate), write each of the following in index form.

a $6 \times 6 \times 7 \times 7 \times 7 \times 7$ **b** $5 \times 5 \times 5 \times 5 \times 2 \times 2$
c $3 \times 3 \times 8 \times 8$ **d** $11 \times 9 \times 9 \times 9 \times 9$
e $12 \times 12 \times 4 \times 4 \times 4$ **f** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Algebraic bases

— 12

13 Write each of the following in index form. Remember, different bases are kept separate.

a $m \times m \times m$
b $a \times a \times a \times a \times a$
c $n \times n \times n \times n \times n \times n \times n$
d $p \times p \times p \times p \times p \times p \times p \times p \times p \times p$
e $p \times p \times p \times q \times q$
f $a \times a \times a \times a \times b \times b$
g $a \times a \times b \times b \times b \times b$
h $x \times x \times x \times x \times x \times y$

$$\underbrace{a \times a}\times\underbrace{b \times b \times b}$$
$$\underbrace{a^2 \times b^3}$$
$$a^2b^3$$

1E The index laws

When working with one or more numbers written using index form there are a number of rules that help to simplify expressions.

These rules are called the index laws.

Index notation has wide application, particularly in modelling growth and decay, in science, economics and computer applications.

Let's start: Investigating the first two rules

Write out 3^7 in expanded notation, that is, as seven copies of 3 being multiplied.

Now write out 3^4 in expanded notation.

What do you get when 3^7 is multiplied by 3^4? How many times does the base of 3 appear in this product?

What do you get when 3^7 is divided by 3^4? How many times does the base of 3 appear in this quotient, when you cancel any common factors?

Key ideas

Index notation
Method of writing numbers that are multiplied by themselves

- **Index law 1:** $a^m \times a^n = a^{m+n}$
  Use when multiplying numbers written in **index notation**. If the base is the same, you keep the base and add the powers together.
  - e.g. $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2)$
    $= 2^5$ (here the base of 2 appears 5 times $(3 + 2)$)
- **Index law 2:** $a^m \div a^n = a^{m-n}$
  Use when dividing numbers written in index notation. If the base is the same, you keep the base and subtract the powers together.
  - e.g. $2^6 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2)$
    $= \dfrac{2 \times 2 \times 2 \times 2 \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2}}$
    $= 2^4$ (here the base of 2 appears 4 times $(6 - 2)$)
- **Index law 3:** $(a^m)^n = a^{m \times n}$
  Use when a number written in index notation is raised to another power.
  The base remains the same and the two powers (indices) are multiplied together.
  - e.g. $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$
    $= 2^{3+3+3+3}$
    $= 2^{12}$ (here the base of 2 appears in total 12 times (3×4))
- **The zero power:** $a^0 = 1$
  Any non-zero number raised to the power of zero gives an answer of one.
  - e.g. $2^0 = 1$
  - e.g. $2^3 \div 2^3 = 2^{3-3} = 2^0$ (but $2^3 \div 2^3 = 1$ so we observe that $2^0 = 1$)

Exercise 1E

UNDERSTANDING 1–4 4

1 Which of the following is the same as $4^3 \times 4^4$?

A $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
B $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$
C 16^7
D 16^{12}

2 Which of the following is equal to $3^6 \div 3^2$?

A $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
B $3 \times 3 \times 3$
C $3 \times 3 \times 3 \times 3$
D 1^4

3 Write the following using index notation.

a 6 raised to the power of 2
b 7 raised to the power of 0
c $(5 \times 5 \times 5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)$
d $(6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6) \div (6 \times 6 \times 6)$

4 Which of the following is the same as $(2^2)^3$?

A 2^5
B 4^5
C $(2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$
D 2^{23}

FLUENCY 5–10(½) 5–10(½)

Example 11 Using the first two index laws

Simplify each of these, leaving your answer in index form.

a $6^4 \times 6^7$ **b** $5^7 \div 5^4$

Solution	**Explanation**
a $6^4 \times 6^7 = 6^{11}$	Use index law 1: $a^m \times a^n = a^{m+n}$ (keep the base and add the powers) $6^4 \times 6^7$ (the base of 6 appears 4 times in the first term and 7 times in the next term) The base of 6 appears 11 times in the product.
b $5^7 \div 5^4 = 5^3$	Use index law 2: $a^m \div a^n = a^{m-n}$ $5^7 \div 5^4 = 5^{7-4}$ $= 5^3$

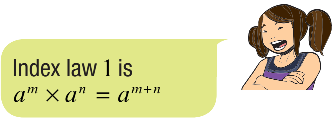
5 Copy and complete the following.

a $7^4 \times 7^2 = 7^{\square}$
b $8^2 \times 8^1 = 8^{\square}$
c $9^6 \times 9^3 = 9^{\square}$
d $5^4 \times 5^3 = 5^{\square}$
e $2^{10} \times 2^3 = 2^{\square}$
f $2^{\square} \times 2^9 = 2^{15}$
g $5^8 \div 5^2 = 5^{\square}$
h $6^4 \div 6^1 = 6^{\square}$
i $2^{12} \div 2^8 = 2^{\square}$
j $1^{16} \div 1^{13} = 1^{\square}$
k $8^{\square} \div 8^4 = 8^2$
l $10^7 \div 10^{\square} = 10^2$

$a^m \times a^n = a^{m+n}$
$a^m - a^n = a^{m-n}$

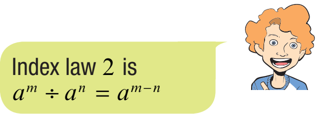
6 Simplify each of the following using the index law for multiplication.

a $3^4 \times 3^2$ **b** $2^2 \times 2^3$ **c** $10^3 \times 10^1$
d $9^6 \times 9^4$ **e** $4^4 \times 4$ **f** $2^3 \times 2^9$
g $8^7 \times 8^3$ **h** $12^9 \times 12$ **i** $16^5 \times 16^3$



7 Simplify each of the following using the index law for division.

a $3^4 \div 3^2$ **b** $2^7 \div 2^5$ **c** $9^6 \div 9^2$
d $4^5 \div 4^2$ **e** $17^{26} \div 17^{20}$ **f** $11^9 \div 11^3$



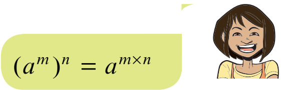
Example 12 Using the third index law

Simplify $(4^5)^2$.

Solution	Explanation
$(4^5)^2 = 4^{10}$	Use index law 3: $(a^m)^n = a^{m \times n}$ $(4^5)^2 = 4^{5 \times 2}$ The base of 4 stays the same and the powers are multiplied together.

8 Copy and complete.

a $(2^3)^4 = 2^{\square}$ **b** $(3^2)^5 = 3^{\square}$ **c** $(5^2)^2 = 5^{\square}$
d $(2^4)^3 = 2^{\square}$ **e** $(7^3)^2 = 7^{\square}$ **f** $(8^4)^5 = 8^{\square}$



9 Simplify the following.

a $(7^2)^2$ **b** $(2^5)^4$ **c** $(3^7)^2$ **d** $(8^4)^2$
e $(3^4)^2$ **f** $(10^6)^5$ **g** $(9^2)^7$ **h** $(5^5)^3$

Example 13 Using the power of zero

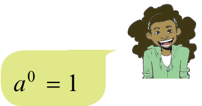
Simplify.

a 9^0 **b** $(3 \times 2)^0$ **c** 4×5^0

Solution	Explanation
a $9^0 = 1$	A number (except zero) raised to the power of zero equals one.
b $(3 \times 2)^0 = 6^0$ $= 1$	As the overall power on the brackets is zero – the expression equals one.
c $4 \times 5^0 = 4 \times 1$ $= 4$	$5^0 = 1$ so the product of 4 and 5^0 is the same as 4×1.

10 Simplify the following.

a 5^0 **b** 6^0 **c** 19^0 **d** 15^0
e $(27 \times 25)^0$ **f** $5^0 + 7$ **g** $8 - 3^0$ **h** 10×2^0
i $5^0 \times 6^0$ **j** $5^0 + 6^0$ **k** $6^0 + 5$ **l** $12^0 \times 3$



PROBLEM-SOLVING AND REASONING 11 11, 12

11 Complete the following.

- **a** Given $4 = 2^2$, write the product $2^7 \times 4$ as $2^{\square}$.
- **b** Write $5^4 \times 25$ as $5^{\square}$.
- **c** Write down the numerical value of $6^{14} \div 6^{12}$.
- **d** What do you notice about $(3^4)^2$ and $(3^2)^4$?
- **e** Write down the numerical value of $4^2 \times 3^2$. Is it the same as 7^2 or 12^2?

12 Simplify the following.

- **a** $2^7 \times 2^4 \div 2^3$
- **b** $(2^3)^3 \times 2^4$
- **c** $10^7 \div 10^2 \div 10^2$
- **d** $7^9 \times 7^3 \times 7^2$
- **e** $6^4 \times 6^5 \div 6^8$
- **f** $3^7 \times 3 \times 3$

Combine the index laws where required.

Algebraic bases — 13, 14

13 Use the index laws to complete these index law questions involving pronumeral bases.

- **a** $a^7 \times a^4$
- **b** $m^4 \times m^3$
- **c** $a^5 \times a^4$
- **d** $x^5 \times x^8$
- **e** $n^7 \times n^4$
- **f** $m^6 \times m^7 \times m$
- **g** $n^9 \div n^3$
- **h** $a^{10} \div a^7$
- **i** $m^6 \div m^4$
- **j** $a^7 \times a^2 \times a^3$
- **k** $w^{12} \div w^3$
- **l** $p^8 \times p^2 \div p^6$

Remember, the base stays the same.

$m^{20} \times m^4$
$= m^{20+4}$
$= m^{24}$

14 Simplify these using the given hint.

- **a** $5m^4 \times m^3$
- **b** $6m^2 \times 4m^6$
- **c** $8m^6 \times 2m^4$
- **d** $3a^2 \times 4a^7$
- **e** $7x^3 \times 3x^4$
- **f** $5x^9 \times 4x^3$

$5x^7 \times 3x^2$
$= 5 \times 3 \times x^7 \times x^2$
$= 15 \times x^{7+2}$
$= 15x^9$

1F Further number properties

Interactive

Widgets

HOTsheets

Walkthrough

Knowing the properties of numbers helps us with our problem-solving work.

A prime number, for example, only has two factors, and can help us with our division.

What number properties do you remember from last year?

Into how many equal groups could these people be divided?

Let's start: How many in 60 seconds?

In 60 seconds, write down as many numbers as you can that fit each description.

- Multiples of 7
- Factors of 144
- Prime numbers

Compare your lists with the results of the class. What is the largest prime number that the class came up with?

Key ideas

- A **multiple** of a number is obtained by multiplying the number by the **counting numbers** $1, 2, 3, \ldots$
  e.g. Multiples of 9 include $9, 18, 27, 36, 45, \ldots$ (think of your multiplication tables)
- The **lowest common multiple** (LCM) is the smallest multiple of two or more numbers that is common.
  e.g. Multiples 3 are $3, 6, 9, 12, \textcircled{15}, 18, \ldots$
  e.g. Multiples of 5 are $5, 10, \textcircled{15}, 20, 25, \ldots$
  The LCM of 3 and 5 is therefore 15.
- A **factor** of a number has a remainder of zero when divided into the given number.
  e.g. 11 is a factor of 77 since $77 \div 11 = 7$ with 0 remainder.
- The **highest common factor** (HCF) is the largest factor of two or more numbers that is common.
  - Factors of 24 are $1, 2, 3, 4, 6, 8, \textcircled{12}, 24$.
  - Factors of 36 are $1, 2, 3, 4, 6, 9, \textcircled{12}, 18, 36$.

  The HCF of 24 and 36 is therefore 12.

Multiple
The multiple of a number is the product of that number and any other whole number

Counting numbers
The set of whole numbers starting at 1

Factor
A whole number that will divide into another number exactly

- **Prime numbers** have only two factors, the number itself and 1.
  - 2, 13 and 61 are examples of prime numbers.
  - 1 is not considered to be a prime number. (It has only **one** factor)
- **Composite numbers** have more than two factors.
  - 6, 20 and 57 are examples of composite numbers.

Prime number
An integer greater than 1 that only has two factors, itself and 1

Composite number
A number that has more than two factors

Exercise 1F

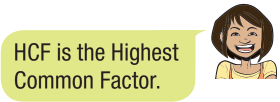
UNDERSTANDING 1–4 4

1 Write down the factors of each number.

a 4 **b** 6 **c** 12 **d** 15 **e** 20

2 Write down the next term in each of these patterns.

a 2, 4, 6, 8, __ **b** 3, 6, 9, 12, __ **c** 5, 10, 15, 20, 25, __
d 7, 14, 21, __ **e** 6, 12, 18, __ **f** 11, 22, 33, 44, __



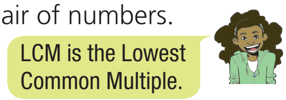
3

The factors of 16 are 1, 2, 4, 8, 16.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
The factors of 18 are 1, 2, 3, 6, 9, 18.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30.
The factors of 8 are 1, 2, 4, 8.

Using the information given in the table, write down the highest common factor (HCF) of each pair of numbers.

a 16 and 24 **b** 24 and 30 **c** 18 and 30 **d** 16 and 8
e 24 and 18 **f** 8 and 24 **g** 16 and 18 **h** 18 and 8

4 Use the first six multiples of the numbers given to find the LCM of each pair of numbers.



Number	Multiples
2	2, 4, 6, 8, 10, 12
4	4, 8, 12, 16, 20, 24
3	3, 6, 9, 12, 15, 24
5	5, 10, 15, 20, 25, 30
6	6, 12, 18, 24, 30, 36

a 2 and 4 **b** 4 and 3 **c** 3 and 6
d 4 and 6 **e** 4 and 5 **f** 5 and 6

FLUENCY 5–7, 8(½) 5, 6, 7–8(½)

Example 14 Working with primes and composites

Decide whether each of the following is a prime number or a composite number.

a 29 **b** 63

Solution	Explanation
a 29 is a prime number	29 has only 2 factors 1 and 29. It is a **prime** number.
b 63 is a composite number	63 has factors 1, 3, 7, 9, 21, 63

5 Decide whether each of the following numbers is prime or composite.

a 7 **b** 12 **c** 27 **d** 69
e 105 **f** 28 **g** 15 **h** 11
i 31 **j** 37 **k** 49 **l** 99

Primes have exactly two factors, composites have more than two factors.

6 Copy the following list of the first 30 counting numbers and circle the prime numbers.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

Example 15 Finding the LCM

Find the LCM of 6 and 8.

Solution	Explanation
Multiples of 6 are: 6, 12, 18, **24**, 30, … Multiples of 8 are: 8, 16, **24**, 32, 40, … The LCM is 24.	First, list some multiples of 6 and 8. Continue the lists until there is at least one in common. Choose the smallest number that is common to both lists.

7 Find the LCM of these pairs of numbers.

a 2, 3 **b** 5, 9
c 8, 12 **d** 4, 8
e 25, 50 **f** 4, 18
g 8, 60 **h** 12, 20
i 5, 7 **j** 10, 15
k 4, 12 **l** 12, 18

Example 16 Finding the HCF

Find the HCF of 36 and 48.

Solution	Explanation
Factors of 36 are: 1, 2, 3, 4, 6, 9, **12**, 18, 36	First, list factors of 36 and 48.
Factors of 48 are: 1, 2, 3, 4, 6, 8, **12**, 16, 24, 48	
The HCF is 12.	Choose the largest number that is common to both lists.

8 Find the HCF of these pairs of numbers.

a 6, 8 **b** 18, 9 **c** 16, 24 **d** 24, 30 **e** 7, 13 **f** 19, 31
g 72, 36 **h** 108, 64 **i** 6, 4 **j** 6, 12 **k** 8, 24 **l** 15, 25

PROBLEM-SOLVING AND REASONING

9, 10 9, 11, 12

9 Find:

a the LCM of 8, 12 and 6
b the LCM of 7, 3 and 5
c the HCF of 20, 15 and 10
d the HCF of 32, 60 and 48

10 A teacher has 64 students to divide into equal groups of greater than 2 with no remainder. In how many ways can this be done?

11 Three sets of traffic lights (A, B and C) all turn red at 9.00 am exactly. Light set A turns red every 2 minutes, light set B turns red every 3 minutes and light set C turns red every 5 minutes. How long does it take for all three lights to turn red again at the same time?

12 Below are the numbers 1 to 100. List all the prime numbers. How many numbers are prime numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Goldbach's conjecture and twin primes

— 13, 14

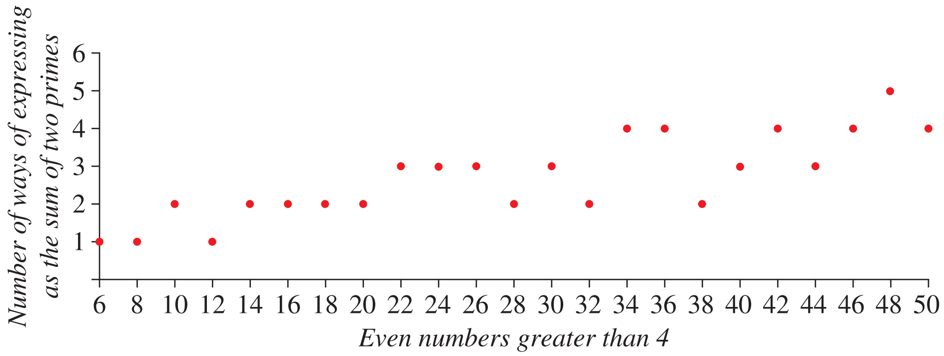
13 *Goldbach's conjecture* is a famous mathematical statement that says that every even number greater than two can be written as the sum of two prime numbers.

The even numbers 4, 6 and 8 have been written as the sum of two primes.

Show how the even numbers 10 to 30 can be written as the sum of two primes. Some can be done in more than one way.

$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 =$
$12 =$
$14 =$
$16 =$
$18 =$
$20 =$
$22 =$
$24 =$
$26 =$
$28 =$
$30 =$

The first ten prime numbers.



A graph illustrating Goldbach's conjecture up to and including 50, is obtained by plotting the number of ways of expressing even numbers greater than 4 as the sum of two primes.

14 Twin primes are pairs of prime numbers that differ by 2. It has been suggested that there are infinitely many twin primes. Use the table of primes you created in question **12** of this exercise and list the pairs of twin primes less than 100.

1G Divisibility and prime factorisation

Every whole number greater than 1 can be written as a product of prime numbers, e.g. $6 = 3 \times 2$ and $20 = 2 \times 2 \times 5$.
Writing numbers as a product of prime numbers can help to simplify expressions and determine other properties of numbers or pairs of numbers.

Prime numbers can be thought of as the building blocks or foundations of all other whole numbers.

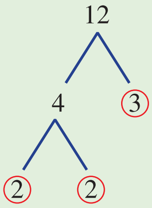
Let's start: Remembering divisibility tests

To test if a number is divisible by 2, we simply need to see if the number is even or odd. All even numbers are divisible by 2. Try to remember the divisibility tests for each of the following. As a class, can you describe tests for any of the following?

- Divisible by 3
- Divisible by 4
- Divisible by 5
- Divisible by 6
- Divisible by 8
- Divisible by 9
- Divisible by 10

Key ideas

- **Prime factorisation** uses a **factor tree**, or similar, to write a number as a product of its prime factors.
  e.g. $12 = 2 \times 2 \times 3$ or $2^2 \times 3$ (using indices)



- The **Highest Common Factor** (HCF) can be found using prime factors.
  The **HCF** = All common primes raised to the smallest power
  e.g. $12 = 2^2 \times 3 \quad 20 = 2^2 \times 5 \quad \therefore$ HCF $= 2^2$ or 4.
- The **Lowest Common Multiple** (LCM) can be found using prime factors.
  The LCM = All different primes raised to the highest power
  e.g. $12 = 2^2 \times 3 \quad 20 = 2^2 \times 5 \quad \therefore$ LCM $= 2^2 \times 3 \times 5$
- **Divisibility tests**
  A number is:
  - divisible by **2** if it is even (ends with the digit $0, 2, 4, 6$ or 8), e.g. 24
  - divisible by **3** if the sum of all the digits is divisible by 3
    e.g. 162 where $1 + 6 + 2 = 9$, which is divisible by 3
  - divisible by **4** if the number formed by the last two digits is divisible by 4
    e.g. 148 where 48 is divisible by 4
  - divisible by **5** if the last digit is a 0 or 5
    e.g. 145 or 2090

Factor tree
An illustrated breakdown of a number into its prime factors

- divisible by **6** if it is divisible by both 2 and 3
  e.g. 456 where 6 is even and $4 + 5 + 6 = 15$, which is divisible by 3
- divisible by **8** if the number formed from the last 3 digits is divisible by 8
  e.g. 2112 where 112 is divisible by 8
- divisible by **9** if the sum of all the digits are divisible by 9
  e.g. 3843 where $3 + 8 + 3 + 3 = 18$ which is divisible by 9
- divisible by **10** if the last digit is a 0
  e.g. 4230

There is no simple test for 7.

Exercise 1G

UNDERSTANDING 1–4 4

1 Write down all the factors of these numbers.

a 15 **b** 24 **c** 40 **d** 84

2 Write down the first 10 prime numbers. Note that 1 is not a prime number.

3 Write using powers.

a $3 \times 3 \times 3 \times 3$ **b** 5×5 **c** $7 \times 7 \times 7 \times 7$

d $2 \times 2 \times 3 \times 3 \times 3$ **e** $2 \times 2 \times 5 \times 5$ **f** $2 \times 2 \times 3 \times 3 \times 5$

4 Evaluate.

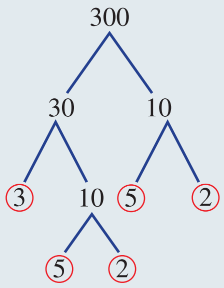
a $2^2 \times 3$ **b** $2 \times 3^2 \times 5$ **c** $2^3 \times 5 \times 7$ **d** $3^3 \times 7$

FLUENCY 5–7(½), 8, 9 5–7(½), 8, 9, 10(½)

Example 17 Finding prime factor form

Use a factor tree to write 300 as a product of prime factors.

Solution



$$300 = 2 \times 2 \times 3 \times 5 \times 5$$
$$= 2^2 \times 3 \times 5^2$$

Explanation

First, divide 300 into the product of **any** two factors. Choose the easiest pair $300 = 30 \times 10$.

Continue dividing numbers into two factors until the factors are prime.

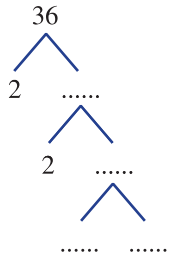
Circle the prime factors.

Write the factors in ascending order.

Use index notation (powers) to abbreviate your answer.

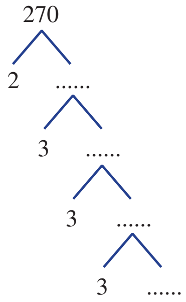
5 Copy and complete these factor trees to help write the prime factor form of the given numbers.

a



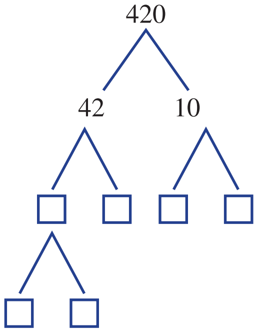
$\therefore 36 = 2^2 \times ......$

b



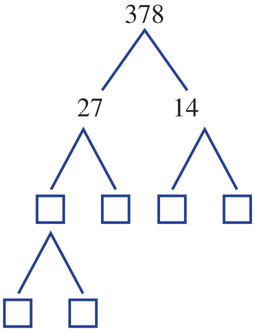
$\therefore 270 = 2 \times ...... \times ......$

c



420 = ____

d



378 = _______

6 Use a factor tree to find the prime factor form of these numbers.

a 20 **b** 28 **c** 40 **d** 90

e 280 **f** 196 **g** 360 **h** 660

Example 18 Testing for divisibility

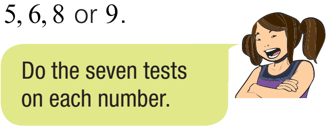
Use divisibility tests to decide if the number 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.

Solution	**Explanation**
Not divisible by **2** since 7 is odd.	The last digit needs to be even.
Divisible by **3** since $6 + 2 + 7 = 15$ and this is divisible by 3.	The sum of all the digits needs to be divisible by 3.
Not divisible by **4** as 27 is not divisible by 4.	The number formed from the last two digits needs to be divisible by 4.
Not divisible by **5** as the last digit is not a 0 or 5.	The last digit needs to be a 0 or 5.
Not divisible by **6** as it is not divisible by 2.	The number needs to be divisible by both 2 and 3.
Not divisible by **8** as the last 3 digits together are not divisible by 8.	The number formed from the last three digits needs to be divisible by 8.
Not divisible by **9** as $6 + 2 + 7 = 15$ is not divisible by 9.	The sum of all the digits needs to be divisible by 9.

7 Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 51 **b** 126 **c** 248 **d** 387

e 315 **f** 517 **g** 894 **h** 3107



Example 19 Finding the LCM and HCF

Find the LCM and HCF of 105 and 90, using prime factorisation.

Solution	Explanation
$105 = 3 \times 5 \times 7$ $90 = 2 \times 3^2 \times 5$	First, express each number in prime factor form. Note that 3 and 5 are common primes.
$\text{LCM} = 2 \times 3^2 \times 5 \times 7$ $= 630$	For the LCM include all the different primes, raising the common primes to their highest power.
$\text{HCF} = 3 \times 5$ $= 15$	For the HCF include only the common primes raised to the smallest power. 105 and 90 **both** have one 3 and one 5.

8 Copy and complete this table of LCM and HCF.

	Number 1	Number 2	LCM	HCF
a	$\mathbf{48} = 2^4 \times 3$	$\mathbf{30} = 2 \times 3 \times 5$		
b	$\mathbf{250} = 2 \times 5^3$	$\mathbf{900} = 2^2 \times 3^2 \times 5^2$		
c	$\mathbf{54} = 2 \times 3^3$	$\mathbf{96} = 2^5 \times 3$		
d	$\mathbf{245} = 5 \times 7^2$	$\mathbf{350} = 2 \times 5^2 \times 7$		
e	$\mathbf{198} = 2 \times 3^2 \times 11$	$\mathbf{693} = 3^2 \times 7 \times 11$		

9 Find the highest common prime factors of these pairs of numbers.

a 10, 45 **b** 42, 72 **c** 24, 80 **d** 539, 525

10 Find the LCM and the HCF of these pairs of numbers, using prime factorisation.

a 10, 12 **b** 14, 28 **c** 15, 24 **d** 12, 15

e 20, 28 **f** 13, 30 **g** 42, 9 **h** 270, 420

PROBLEM-SOLVING AND REASONING 11 11, 12

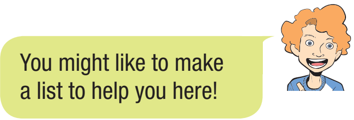
11 What is the smallest number that can be divided, without giving a remainder, by all of the following four numbers?

a 2, 3, 4 and 6

b 2, 6, 8 and 9

c 2, 5, 15 and 6

12 Nana Magoo's two grandchildren love to visit her. Lachlan visits her every 8 days while Bryce visits every 18 days. They both visited her last Monday. How many days will it be from that visit before they both visit her on the same day again?



Find the missing digit

— 13

13 Use the divisibility rules given to you at the start of this section to find the missing digit for each of the following.

a 2☐6 if the number is divisible by 3 (can you have more than one answer?)
b 1☐35 if the number is divisible by 9.
c 4☐3 if the number is divisible by 3.
d 4☐3 if the number is divisible by 3 and 9.
e 276☐ if the number is divisible by 2.
f 276☐ if the number is divisible by 2 and 5.

Progress quiz

1A **1** Evaluate these sums and these differences mentally.

a $55 + 38$ **b** $215 + 219$ **c** $146 - 25$ **d** $770 - 249$

1A **2** Use an algorithm to find these sums and these differences

a $\begin{array}{r} 785 \\ +\ 438 \\ \hline \end{array}$ **b** $\begin{array}{r} 68 \\ +\ 215 \\ +\ 187 \\ +\ 11 \\ \hline \end{array}$ **c** $\begin{array}{r} 513 \\ -\ 378 \\ \hline \end{array}$ **d** $\begin{array}{r} 8139 \\ -\ 964 \\ \hline \end{array}$

1B **3** Use a mental strategy to evaluate the following.

a $5 \times 36 \times 2$ **b** 4×79 **c** $342 \div 3$ **d** $600 \div 4$

1B **4** Use an algorithm to evaluate the following.

a $\begin{array}{r} 72 \\ \times\ 31 \\ \hline \end{array}$ **b** $3720 \div 12$

1C **5** Find the answers to each of the following.

a $12 + 6 \times 3$

b $21 - (18 - 13)$

c $8 \times 7 - 5 \times 6$

d $(15 + 5) \times 2 \div 8 + 3 \times 2$

1D **6** Write each product in index notation.

a $7 \times 7 \times 7 \times 7$

b $5 \times 5 \times 2 \times 2 \times 2$

c $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

1D **7** Evaluate the following.

a 5^2 **b** 2^6 **c** $\sqrt{100}$ **d** $\sqrt[3]{27}$

1E **8** Simplify each of these, leaving your answers in index form.

a $8^5 \times 8^4$ **b** $3^4 \times 3^3$ **c** $2^9 \div 2^3$ **d** $21^{15} \div 21^5$

1E **9** Simplify.

a $(5^2)^3$ **b** $(2^5)^4$ **c** 12^0 **d** $6^0 + 4 - 2^0$

1F **10** Find the LCM of these pairs of numbers.

a 4, 6 **b** 9, 15

1F **11** Find the HCF of these pairs of numbers.

a 20, 35 **b** 11, 17 **c** 48, 72

1G ★ **12** Use a factor tree to write 240 as a product of prime factors.

1G ★ **13** Use divisibility tests to decide if 72 is divisible by 2, 3, 4, 5, 6, 8, or 9.

1G ★ **14** Find the LCM and HCF of 40 and 110, using prime factorisation.

1H Negative numbers

CONSOLIDATING

Interactive

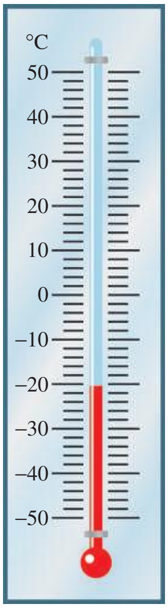
Widgets

HOTsheets

Walkthrough

The Indian mathematician Brahmagupta set out rules for negative numbers in the 7th century.

Today, negative numbers are used in science, engineering and business. They help us describe opposites such as left and right, up and down, profit and loss, and temperatures above and below freezing.



Let's start: A negative world

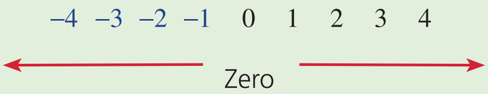
Describe how to use negative numbers in these situations.

- 6°C below zero
- A loss of \$4200
- 150 m below sea level
- A turn of 90° anticlockwise
- The solution to the equation $x + 5 = 3$

Can you describe another situation in which you might make use of negative numbers?

Key ideas

- **Negative numbers** are numbers **less than zero**.
- The **integers** are $\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots$
- These include positive integers (natural numbers), zero and negative integers. These are illustrated clearly on a number line.



negative numbers

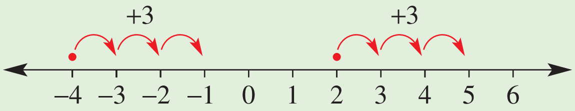
Values decreases as you move to the left along the number line

positive numbers

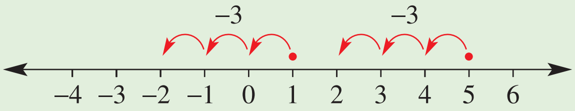
Values increase as you move to the right along the number line

Integers The set of positive and negative whole numbers, including zero

- Adding or subtracting a positive integer can result in a positive or negative number.
  - Adding a positive integer
    e.g. $2 + 3 = 5$
    $-4 + 3 = -1$



  - Subtracting a positive integer
    e.g. $1 - 3 = -2$
    $5 - 3 = 2$



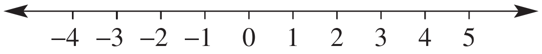
Exercise 1H

UNDERSTANDING 1–4 4

1 Write down the number suggested by:

a 2 above zero **b** 5 above zero **c** 3 below zero
d 10 below zero **e** 1 below zero

2 Copy the number line below and mark (with a dot) the integers $-3, -1, 1, 3$ and 5.



3 Write the symbol < (less than) or > (greater than) to make these statements true.

a 5 ___ -1 **b** -3 ___ 4 **c** -10 ___ 3 **d** -1 ___ -2
e -20 ___ -24 **f** -62 ___ -51 **g** 2 ___ -99 **h** -61 ___ 62

4 What is the final temperature?

a 10°C is reduced by 12°C **b** 32°C is reduced by 33°C
c −11°C is increased by 2°C **d** −4°C is increased by 7°C

FLUENCY 5–7(½) 5–7(½), 8

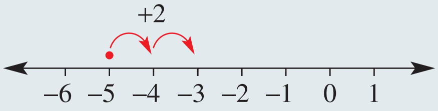
Example 20 Adding a positive integer

Evaluate the following.

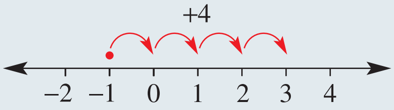
a $-5 + 2$ **b** $-1 + 4$

Solution **Explanation**

a $-5 + 2 = -3$



b $-1 + 4 = 3$



5 Evaluate the following.

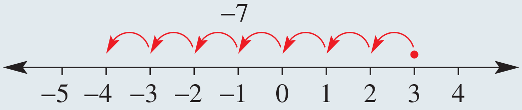
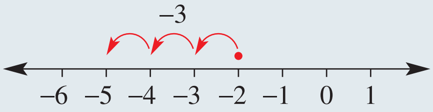
a $-1+2$ **b** $-3+7$ **c** $-10+11$ **d** $-4+12$
e $-20+35$ **f** $-6+4$ **g** $-7+2$ **h** $-15+8$
i $-26+19$ **j** $-38+24$ **k** $-10+15$ **l** $-2+9$
m $-7+3$ **n** $-7+7$ **o** $-6+9$ **p** $-6+1$

Start with the left number and move right on the number line.

Example 21 Subtracting a positive integer

Evaluate the following.

a $3-7$ **b** $-2-3$

Solution	**Explanation**
a $3-7=-4$	
b $-2-3=-5$	

6 Evaluate the following.

a $4-5$ **b** $10-15$ **c** $0-26$ **d** $14-31$
e $6-8$ **f** $10-9$ **g** $-4-7$ **h** $-11-20$
i $-14-15$ **j** $-10-100$ **k** $-11-6$ **l** $0-12$
m $-15-5$ **n** $3-12$ **o** $8-4$ **p** $-8-4$

Start with the left number and move left on the number line.

7 Evaluate the following.

a $-9+6$ **b** $-9-6$ **c** $-12+12$ **d** $-12-12$
e $-7-7$ **f** $-7+0$ **g** $15-14$ **h** $15-16$
i $-9-10$ **j** $-9+10$ **k** $9-15$ **l** $-20+10$
m $100-101$ **n** $-50-50$ **o** $-5+25$ **p** $-9+40$

8 Work from left to right to evaluate the following.

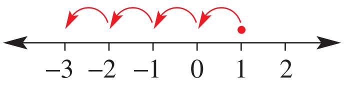
a $-3+4-8+6$ **b** $0-10+19-1$ **c** $26-38+14-9$ **d** $9-18+61-53$

PROBLEM-SOLVING AND REASONING

9, 10 10–12

9 Write the sum (e.g. $-3+4=1$) or difference (e.g. $1-5=-4$) to match these number lines.

a



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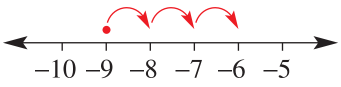
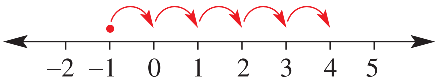
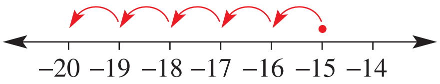


c



d



10 Write the missing number.

a $-1 + \_\_\_ = 5$ **b** $\_\_\_ + 30 = 26$ **c** $\_\_\_ + 11 = -3$ **d** $-32 + \_\_\_ = -21$

e $5 - \_\_\_ = -10$ **f** $\_\_\_ - 17 = -12$ **g** $\_\_\_ - 4 = -7$ **h** $-26 - \_\_\_ = -38$

11 In a high-rise building there are 8 floors above ground level and 6 floors below ground level. A lift starts at the 2nd floor and moves 4 floors up, then 7 floors down before moving down a further 3 floors.

At what floor does the lift finish?

12 On Monday Milly borrows $35 from a friend. On Tuesday she pays her friend $40. On Friday she borrows $42 and pays back $30 that night. How much does Milly owe her friend then?

Budgets and zero

— 13, 14

13 **a** Complete Suzanne's account for the week shown.
A credit is an addition (+) and a debit is a subtraction (–).

Spending and earning	Credit (+)	Debit (–)	Balance
opening balance			$500
pays 1 week's rent of $375		375	
earns $80 babysitting			
receives $100 from her parents for her birthday			
buys a pair of jeans for $90			
buys a top for $45			
pays her monthly mobile phone bill $49			
gives $25 to charity			

b How much would Suzanne need to deposit (credit) into her account so that she can pay the rent for the next week?

14 Find what positive integer needs to be added or subtracted to each so that the end result is always zero.

a -6 ____ $= 0$

b -8 ____ $= 0$

c 16 ____ $= 0$

d $10 - 7$ ____ $= 0$

e $-9 + 7$ ____ $= 0$

f $-9 - 7 - 2$ ____ $= 0$

1I Addition and subtraction of negative integers

Interactive

Widgets

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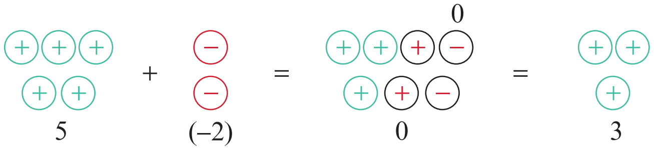
Walkthrough

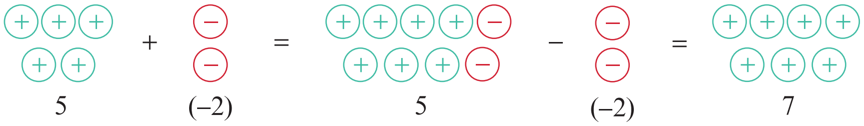
If ⊕ represents +1 and ⊖ represents –1 then ⊕ ⊖ added together has a value of zero.

Using these symbols 5 + (–2) = 3 could be illustrated as the addition of 2 ⊖, leaving a balance of 3.



So 5 + (–2) is the same as 5 – 2.

Also 5 – (–2) = 7 could be illustrated first as 7 ⊕ and 2 ⊖ together then subtracting the 2 ⊖.



So 5 – (–2) is the same a 5 + 2.

When adding or subtracting negative integers we follow the rules set out by the above two illustrations, as well as the patterns below.

Let’s start: Looking at patterns for adding and subtracting negative numbers

Copy and complete.

A

6 + 4	10	
6 + 3	9	
6 + 2	8	
6 + 1		
6 + 0		
6 + (–1)		→ same as 6 – 1 = 5
6 + (–2)		→ same as 6 ☐ 2 =
6 + (–3)		→ same as 6 ☐ 3 =
6 + (–4)		→ same as 6 ☐ 4 =

B

6 – 4	2	
6 – 3	3	
6 – 2	4	
6 – 1		
6 – 0		
6 – (–1)		→ same as 6 + 1 =
6 – (–2)		→ same as ☐
6 – (–3)		→ same as ☐
6 – (–4)		→ same as ☐

Key ideas

- Adding a negative number is the same as subtracting its opposite.
  e.g. 2 + (–3) = 2 – 3 = –1 two opposite signs give a subtraction/minus
  –4 + (–7) = –4 – 7 = –11
- Subtracting a negative number is the same as adding its opposite.
  e.g. 2 – (–5) = 2 + 5 = 7 two like signs give an addition/plus
  –6 – (–4) = –6 + 4 = –2

Exercise 1I

UNDERSTANDING 1–4 | 4

1 -3 and 3 are opposites. Write down the opposites of these numbers.

a -6 **b** 10 **c** 38 **d** -46

e -32 **f** 88 **g** 673 **h** -349

2 Write the words 'add' or 'subtract' to suit each sentence.

a To add a negative number ____________ its opposite.

b To subtract a negative number ____________ its opposite.

3 Are the following statements true (T) or false (F)?

a $5+(-2)=5+2$ **b** $3+(-4)=3-4$ **c** $-6+(-4)=-6-4$

d $-1+(-3)=1-3$ **e** $8-(-3)=8+3$ **f** $2-(-3)=2-3$

g $-3-(-1)=3+1$ **h** $-7-(-5)=-7+5$ **i** $-6-(-3)=6+3$

4 Rewrite each of the following with only a (+) or (–) between the two numbers.

a $7+(-3)$ **b** $10+(-5)$ **c** $8-(-1)$

d $6-(-8)$ **e** $15+(-20)$ **f** $-3-(-4)$

g $-9-(-9)$ **h** $0+(-5)$ **i** $18+(-18)$

$6+(-9)=6-9$

$3-(-7)=3+7$

FLUENCY 5–6(½) | 5–7(½)

Example 22 Adding negative numbers

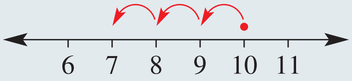
Evaluate the following.

a $10+(-3)$ **b** $-3+(-5)$

Solution | **Explanation**

a $10+(-3)=10-3$
$=7$

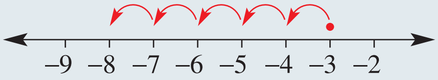
Adding -3 is the same as subtracting 3.



$\boxed{+}$ and $\boxed{-}$ = $\boxed{-}$

b $-3+(-5)=-3-5$
$=-8$

Adding -5 is the same as subtracting 5.



$\boxed{+}$ and $\boxed{-}$ = $\boxed{-}$

5 Evaluate the following.

a $6+(-2)$ **b** $4+(-1)$ **c** $7+(-12)$ **d** $20+(-5)$

e $2+(-4)$ **f** $26+(-40)$ **g** $-3+(-6)$ **h** $-16+(-5)$

i $-18+(-20)$ **j** $-36+(-50)$ **k** $-83+(-22)$ **l** $-120+(-10)$

m $7+(-8)$ **n** $-9+(-12)$ **o** $6+(-12)$ **p** $-6+(-12)$

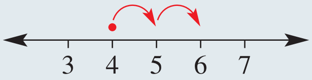
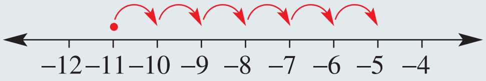
q $-8+(-8)$ **r** $5+(-5)$ **s** $-70+(-15)$ **t** $-100+(-6)$

To add a negative, subtract its opposite.

Example 23 Subtracting negative numbers

Evaluate the following.

a $4-(-2)$ **b** $-11-(-6)$

Solution	**Explanation**
a $4-(-2) = 4+2$ $= 6$	Subtracting -2 is the same as adding 2.  $\boxed{-}$ and $\boxed{-} = \boxed{+}$
b $-11-(-6) = -11+6$ $= -5$	Subtracting -6 is the same as adding 6.  $\boxed{-}$ and $\boxed{-} = \boxed{+}$

6 Evaluate the following.

a $2-(-3)$ **b** $4-(-4)$
c $15-(-6)$ **d** $24-(-14)$
e $59-(-13)$ **f** $147-(-320)$
g $-5-(-3)$ **h** $-8-(-10)$
i $-13-(-16)$ **j** $-10-(-42)$
k $-88-(-31)$ **l** $-125-(-5)$
m $60-(-5)$ **n** $-60-(-5)$
o $-12-(-12)$ **p** $-10-(-18)$
q $41-(-41)$ **r** $48-(-52)$
s $-46-(-8)$ **t** $-170-(-12)$

To subtract a negative, add its opposite.

7 Evaluate the following.

a $46-50$ **b** $46+(-50)$ **c** $9-12$
d $9+(-12)$ **e** $-8+6$ **f** $-8-(-6)$
g $81-15$ **h** $81+(-15)$ **i** $7+(-7)$

PROBLEM-SOLVING AND REASONING 8–10 9–12

8 Write down the missing number.

a $4 + \_\_\_ = 1$ **b** $6 + \_\_\_ = 0$
c $-2 + \_\_\_ = -1$ **d** $\_\_\_ + (-8) = 2$
e $\_\_\_ + (-5) = -3$ **f** $\_\_\_ + (-3) = -17$
g $12 - \_\_\_ = 14$ **h** $8 - \_\_\_ = 12$
i $-1 - \_\_\_ = 29$ **j** $\_\_\_ - (-7) = 2$
k $\_\_\_ - (-2) = -4$ **l** $\_\_\_ - (-436) = 501$

9 An ice cube is removed from a freezer at –25°C and placed into a glass of juice at 7°C.
What is the difference between the two temperatures?

10 Kelvin owes the bank $\$450\,000$. What must he deposit into his account to only owe $\$270\,000$?

11 What must be added or subtracted to each of the following to obtain an answer of zero?

a $-6 + \square = 0$

b $7 - \square = 0$

c $-18 - \square = 0$

12 If $a = -5$ and $b = -3$, find the value of:

a $a + (-3)$

b $a - (-2)$

c $b - (-4)$

d $a + b$

e $a - b$

f $b - a$

Replace the pronumeral in the statement with the number it represents.
e.g. $a = -2$
then $a + (-5)$
$= -2 + (-5)$
$= -2 - 5$
$= -7$

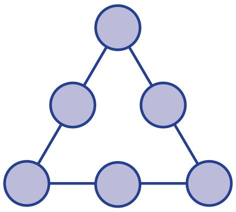
Puzzles with negatives

— 13, 14

13 Place the integers from –3 to 2 in this magic triangle so that each side adds to the given number.

a -3

b 0



14 A magic square has each row, column and main diagonal adding to the same magic sum. Complete these magic squares.

a

		1
0	–2	–4

b

–12		
	–15	
	–11	–18

1J Multiplication and division of integers

Interactive
Widgets
HOTsheets
Walkthrough

As a repeated addition, the product $3 \times (-2)$ can be written as $-2 + (-2) + (-2) = -6$. So $3 \times (-2) = -6$ and, since $a \times b = b \times a$ for all numbers a and b, then -2×3 is also equal to -6.

For division we can write the product $3 \times 2 = 6$ as a quotient $6 \div 2 = 3$.

So, if $3 \times (-2) = -6$ then $-6 \div (-2) = 3$.

Also if $-2 \times 3 = -6$ then $-6 \div 3 = -2$.

The quotient of two negative numbers results in a positive number. The product or quotient of two numbers of opposite sign is a negative number.

$6 \div (-2) = -3$ can also be rearranged to $-3 \times (-2) = 6$. The product of two negative numbers is a positive number.

Let's start: Repeated additions

Rewrite each of these as a multiplication and then find the value of each.

- $7 + 7 + 7 + 7$
- $(-7) + (-7) + (-7) + (-7)$
- $3 + 3 + 3 + 3 + 3 + 3$
- $(-3) + (-3) + (-3) + (-3) + (-3) + (-3)$
- $(-10) + (-10) + (-10)$

Quotients. Complete these statements.

- If $10 \div (-2) = (-5)$, then $(-2) \times (-5) =$ ☐
- If $18 \div (-6) = (-3)$, then $(-3) \times (-6) =$ ☐

What do these observations tell us about multiplying and dividing positive and negative numbers?

Key ideas

- The product **or** quotient of two integers of the **same sign is a positive integer**.
  - Positive × Positive = Positive
  - Positive ÷ Positive = Positive
  - Negative × Negative = Positive
  - Negative ÷ Negative = Positive
- The product **or** quotient of two integers of **opposite signs is a negative integer**.
  - Positive × Negative = Negative
  - Positive ÷ Negative = Negative
  - Negative × Positive = Negative
  - Negative ÷ Positive = Negative

Exercise 1J

UNDERSTANDING 1–4 4

1 Write the missing numbers in these tables. You should create a pattern in the third column.

a

□	△	□ × △
3	5	15
2	5	
1	5	
0	5	
–1	5	
–2	5	
–3	5	

b

□	△	□ × △
3	–5	–15
2	–5	–10
1	–5	
0	–5	
–1	–5	
–2	–5	
–3	–5	

2 Write the missing numbers in these sentences. Use the tables in question **1** to help.

a $3 \times 5 =$ ___ so $15 \div 5 =$ ___

b $-3 \times 5 =$ ___ so $-15 \div 5 =$ ___

c $3 \times (-5) =$ ___ so $15 \div (-5) =$ ___

d $-3 \times (-5) =$ ___ so $15 \div (-5) =$ ___

3 Without finding the answer to these products decide if the answer would be positive or negative.

a 109×4 **b** -76×5 **c** $15 \times (-9)$

d $-6 \times (-13)$ **e** 89×104 **f** -74×8

g $-94 \times (-5)$ **h** $80 \times (-7)$ **i** -37×-3

4 Without finding the answer to these quotients decide if the answer would be positive or negative.

a $16 \div 2$ **b** $24 \div (-3)$ **c** $78 \div (-2)$

d $-56 \div 2$ **e** $-81 \div 9$ **f** $-99 \div (-11)$

FLUENCY 5-7(½) 5-8(½)

Example 24 Finding products of integers

Evaluate the following.

a $3 \times (-7)$ **b** $-4 \times (-12)$

Solution	Explanation
a $3 \times (-7) = -21$	The product of two numbers of opposite sign is negative. $\boxed{+} \times \boxed{-} = \boxed{-}$
b $-4 \times (-12) = 48$	–4 and –12 are both negative and so the product will be positive. $\boxed{-} \times \boxed{-} = \boxed{+}$

5 Evaluate the following.

a $4 \times (-5)$
b $6 \times (-9)$
c -4×10
d -11×9
e $-2 \times (-3)$
f -6×7
g -9×8
h $-11 \times (-9)$
i $20 \times (-2)$
j -16×4
k $-5 \times (-7)$
l $8 \times (-4)$
m $-10 \times (-6)$
n $44 \times (-1)$
o $-9 \times (-1)$
p -5×12

Example 25 Finding quotients of integers

Evaluate the following.

a $-63 \div 7$
b $-121 \div (-11)$

Solution	**Explanation**
a $-63 \div 7 = -9$	The two numbers are of opposite sign so the answer will be negative. $\boxed{-} \div \boxed{+} = \boxed{-}$
b $-121 \div (-11) = 11$	-121 and -11 are both negative so the quotient will be positive. $\boxed{-} \div \boxed{-} = \boxed{+}$

6 Evaluate the following.

a $-10 \div 2$
b $-38 \div 19$
c $-60 \div 15$
d $-120 \div 4$
e $32 \div (-16)$
f $-6 \div 2$
g $6 \div (-2)$
h $-6 \div (-2)$
i $-12 \div 6$
j $-24 \div (-3)$
k $-45 \div 5$
l $-45 \div (-9)$
m $-66 \div (-6)$
n $-5 \div (-5)$
o $-8 \div 1$
p $-8 \div (-1)$

Example 26 Using order of operations

a $-7 + 6 \times (-5)$
b $-4 \times 6 \div (-2)$

Solution	**Explanation**
a $-7 + 6 \times (-5)$ $= -7 + (-30)$ $= -7 - 30$ $= -37$	The order of operation multiplication first $\boxed{+} \times \boxed{-} = \boxed{-}$ $6 \times (-5) = -30$ Lastly, addition of a negative = subtraction $-7 + (-30) = -7 - 30$
b $-4 \times 6 \div (-2)$ $= -24 \div (-2)$ $= 12$	Multiplication and division work from left to right -4×6 *First* $\boxed{-} \times \boxed{+} = \boxed{-}$ $-24 \div -2$ *Last* $\boxed{-} \div \boxed{-} = \boxed{+}$

7 Follow the order of operation to find the following.

a $10 + (-6) \times 5$
b $15 - 3 \times (-2)$
c $18 \times (-2) \div 3$
d $-9 \times 2 + (-5)$
e $45 - 50 \div (-10)$
f $9 - 6 \times 3$
g $-10 \div (-2) \times (-3)$
h $9 \times 3 - 6 \times (-2)$
i $18 \div (-3) + 3 \times (-4)$
j $-9 \times (-2) + (-10)$

8 If $(-2)^2 = -2 \times -2 = 4$, find the value of the following.

a $(-5)^2$ **b** $(-6)^2$ **c** $(-7)^2$

d $(-8)^2$ **e** $(-9)^2$ **f** $(-10)^2$

PROBLEM-SOLVING AND REASONING

9, 10 | 10–12

9 Write the missing number.

a $\_\_\_ \times 3 = -9$ **b** $\_\_\_ \times (-7) = 35$ **c** $\_\_\_ \times (-4) = -28$

d $-3 \times \_\_\_ = -18$ **e** $-19 \times \_\_\_ = 57$ **f** $\_\_\_ \div (-9) = 8$

g $\_\_\_ \div 6 = -42$ **h** $85 \div \_\_\_ = -17$ **i** $-150 \div \_\_\_ = 5$

10 Will $(-2)^3$ give a positive or negative answer?

11 Insert a × sign and/or ÷ sign to make these equations true.

a $-2 \_\_\_ 3 \_\_\_ (-6) = 1$ **b** $10 \_\_\_ (-5) \_\_\_ (-2) = 25$

c $6 \_\_\_ (-6) \_\_\_ 20 = -20$ **d** $-14 \_\_\_ (-7) \_\_\_ (-2) = -1$

12 The product of two numbers is –24 and their sum is –5. What are the two numbers?

Further substitution with integers using brackets

— | 13–17

13 Evaluate these expressions using $a = -2$ and $b = 1$.

a $a + b$ **b** $a - b$ **c** $2a - b$ **d** $b - a$

e $a - 4b$ **f** $3b - 2a$ **g** $b \times (2 + a)$ **h** $(2b + a) - (b - 2a)$

14 Evaluate these expressions using $a = -3$ and $b = 5$.

a ab **b** ba **c** $a + b$ **d** $a - b$

e $b - a$ **f** $3a + 2b$ **g** $(a + b) \times (-2)$ **h** $(a + b) - (a - b)$

15 Evaluate these expressions using $a = -3$ and $b = 5$.

a $a + b^2$ **b** $a^2 - b$ **c** $b^2 - a$ **d** $b^3 + a$

e $a^3 - b$ **f** $a^2 - b^2$ **g** $b^3 - a^3$ **h** $(b - a^2)^2$

16 Evaluate these expressions using $a = -4$ and $b = -3$.

a $3a + b$ **b** $b - 2a$ **c** $4b - 7a$

d $-2a - 2b$ **e** $4 + a - 3b$ **f** $ab - 4a$

g $-2 \times (a - 2b) + 3$ **h** $ab - ba$ **i** $3a + 4b + ab$

j $a^2 - b$ **k** $a^2 - b^2$ **l** $b^3 - a^3$

17 Insert brackets in these statements to make them true.

a $-2 + 1 \times 3 = -3$ **b** $-10 \div 3 - (-2) = -2$

c $-8 \div (-1) + 5 = -2$ **d** $-1 - 4 \times 2 + (-3) = 5$

e $-4 + (-2) \div 10 + (-7) = -2$ **f** $20 + 2 - 8 \times (-3) = 38$

g $1 - (-7) \times 3 \times 2 = 44$ **h** $4 + (-5) \div 5 \times (-2) = -6$

Maths@Work: Retailer of loungeroom furniture

A furniture retailer needs many skills including being a good communicator and a successful salesperson. They need to apply the mathematics of money management to stock orders, delivery costs, insurance and pay rates. It is important that they know their products and have options for clients. A successful retailer relates to customers in a confident, friendly, cheerful and helpful manner.

1 A loungeroom furniture business advertises that all lounges are reduced by $250. What is the sale price on the following lounges currently in stock?

- **a** 2-seater leather seat marked at $2340
- **b** 2.5-seat leather lounge with chaise marked at $2599
- **c** 3-seater + 2-seater sofa set marked at $2099
- **d** 2-seat Recliner lounge in fabric $2249
- **e** 7-seat corner lounge in fabric $4130

A chaise has an extended area for full leg rest.

2 Floor stock is a term describing furniture that has been displayed in the showroom for customers to try out. It is often discounted for a quick sale and is usually available for immediate delivery.

Complete the table below to find the savings on each of these lounges and other available products from the wholesale centre.

Model	Original price	Floor stock price	Savings
Recliner 3-piece	$4499	$2250	
Corner suite	$3299	$1999	
2.5-seat leather chaise	$2295	$1999	
Outdoor sofa	$1120	$895	
Occasional chair	$369	$149	

3 The models on display are available in either leather or fabric.

a Find out the difference in the prices of each lounge suite in leather compared with fabric.

b On average, how much more does the retailer charge for the leather models?

Model	Leather	Fabric	Difference in price
ER 2 + chaise	\$2099	\$1499	
DW 3450 R	\$2350	\$1890	
Ebony 3 + 2	\$3495	\$2599	
Recliner and console 3	\$1149	\$799	
Victa EL + 1	\$3297	\$2599	

4 Jen and Brad decide to buy a grey Boston Chaise lounge suite. They investigate their options from two different outlets to find the best overall price including delivery.

Which is the best buy and by how much?

Option 1: Custom Sofas	Option 2: Leisure Lounges
Leather lounge \$1999 Upgrade on colour of leather to grey \$160 Delivery \$100	Lounge grey leather \$2199 Delivery \$70

Using technology

- To increase a price by 25%, multiply it by 1.25.
- To fill formulas down a column, drag the 'fill handle' down.
- Cell G3 formula = C3*D3 + E3*F3

5 A lounge suite business, Luxury Lounges, uses a spreadsheet for orders.

a Copy the following Excel spreadsheet. Format all the number cells to Number with 0 d.p. and all price and cost cells to Currency with 0 d.p.

b If leather furniture costs 25% more than fabric, enter formulas into the 'Price in leather' column to calculate these prices. See the hint box for extra clues.

c Enter formulas in column G to calculate 'Cost of furniture' and the 'Total cost of order'. Costs will be \$0 until the numbers of items are entered from part **b**.

	A	B	C	D	E	F	G
1	Luxury Lounges						
2	Catalogue item number	Furniture description	Price in fabric	Number in fabric	Price in leather	Number in leather	Cost of furniture
3	021-A	Cayman corner lounge with chaise	\$2859				
4	021-D	Sophie 3-piece cream lounge with 4 recliners	\$1390				
5	021-G	Tuscan red L-shaped sofa set	\$2190				
6	054-B	Sapphire blue lounge with 2-seater sofa and ottoman	\$3499				
7	054-F	Onyx home theatre entertainment lounge with 4 recliners	\$2675				
8	079-L	Mandarin orange recliner chair	\$548				
9	079-M	Zara 2-seater sofa	\$1099				
10	079-R	Corfu outdoor corner suite	\$1563				
11					Total cost of order		

d Use your spreadsheet to calculate the total cost of each of the following orders.

Catalogue item number		021-A	021-D	021-G	054-B	054-F	079-L	079-M	079-R
March order	Fabric	2		1	1			2	1
	Leather		3		1	2	4	1	1
June order	Fabric		1		2		2		3
	Leather	1	2	1	1	1	2	2	

Puzzles and games

1 Hey, do you know what a wisecracker is?

A	$-6-4$	**R**	$-8-(-2)$	**I**	$-4\times7+10$	**Y**	$-17-6$
E	$8-10$	**M**	$-6-7-4$	**S**	$20-7$	**K**	$16-(-6)$
O	$6-(-4)$	**C**	$46+(-6)-8$	**V**	$12+(-3)-6$	**T**	$-13-7-6+8$

Complete the sums above to unlock the puzzle code.

–10		3	–2	–6	–23

13	–17	–10	–6	–18

32	10	10	22	–7	–2

2 What explosive event was in the year 1000 CE?

Answer the following directed number multiplications and divisions to work out the puzzle code. Write your answer on another sheet of paper.

K	-3×4	**N**	$8\div-4$
A	-1×6	**S**	$\frac{-36}{-6}$
C	$100\div-5$	**U**	-9×-7
L	-8×-6	**G**	$\frac{10}{-2}$
W	$40\div8\times-2$	**D**	-2×2
H	4×-4	**O**	$-12+5$
R	$(-10)^2$	**P**	$(-4)^2$
E	0×-5	**V**	-5×-4
M	$-16\div-8$	**F**	$(-3)^2$
I	$24\div8$	**T**	$-3\times-2\times-4$

–20	–16	3	–2	–6		–4	0	20	0	48	–7	16	6

–5	63	–2	16	–7	–10	–4	0	100		–24	–16	3	6

48	0	–6	–4	6		–24	–7		–24	–16	0

2	–6	–2	63	9	–6	–20	–24	63	100	0

–7	9		9	3	100	0	–10	–7	100	–12	6

3 Using the symbols +, –, ×, ÷, make as many sums as you can that have –5 as their answer.

Chapter summary

Integers
{..., –3, –2, –1, 0, 1, 2, 3, ...}

Whole numbers
0, 1, 2, 3, ...

Addition and subtraction

$$\begin{array}{r} 2\,{}^{1}4\,7 \\ +\,1\,0\,8 \\ \hline 3\,5\,5 \end{array} \qquad \begin{array}{r} {}^{8}\cancel{9}\,\overset{13}{\cancel{4}}\,2^{1} \\ -\,3\,6\,8 \\ \hline 5\,7\,4 \end{array}$$

Multiplication and division

$$\begin{array}{rl} 167 & \\ \times\,15 & \\ \hline 835 & 5 \times 167 \\ 1670 & 10 \times 167 \\ \hline 2505 & 835 + 1670 \end{array} \qquad \begin{array}{r} 5\;9 \\ 7\overline{)4\;1{}^{6}6} \end{array}$$

$416 \div 7 = 59$
and 3 remainder

Mental strategies

- $156 + 79 = 156 + 80 - 1 = 235$
- $45 + 47 = 45 + 45 + 2 = 92$
- $3 \times 22 = 3 \times 20 + 3 \times 2 = 66$
- $4 \times 88 = 2 \times 176 = 352$
- $164 \div 4 = 82 \div 2 = 41$
- $297 \div 3 = (300 \div 3) - (3 \div 3) = 99$

Negative number operations

$4 - (-3) = 4 + 3$
$4 + (-3) = 4 - 3$
Multiplication and division

$\boxed{+} \times \boxed{+} = \boxed{+}$
$\boxed{-} \times \boxed{-} = \boxed{+}$
$\boxed{+} \times \boxed{-} = \boxed{-}$
$\boxed{-} \times \boxed{+} = \boxed{-}$
$\boxed{+} \div \boxed{+} = \boxed{+}$
$\boxed{-} \div \boxed{-} = \boxed{+}$
$\boxed{+} \div \boxed{-} = \boxed{-}$
$\boxed{-} \div \boxed{+} = \boxed{-}$

Substitution

$a = -2,\ b = 5,\ c = -4$
$2c - ab = 2 \times (-4) - (-2) \times 5$
$= -8 - (-10)$
$= 2$
use brackets with negatives

Order of operations

- Brackets, × and ÷ then + and –

$10 \times (-3) + 7$
$= -30 + 7$
$= -23$

Properties

Primes (two factors)

2, 3, 5, 7, 11, 13 ...

Composite

more than
two factors
4, 6, 8, 12

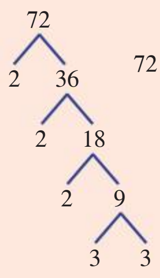
Lowest Common Multiple
LCM

Multiples of 3: 3, 6, 9, **12** ...
Multiples of 4: 4, 8, **12** ...
∴ LCM = **12**

Highest Common Factor
HCF

Factors of 8: 1, 2, **4**, 8
Factors of 28: 1, 2, **4**, 7, 14, 28
∴ HCF = **4**

Prime factorisation ★



$72 = 2^3 \times 3^2$

Powers and indices

$9 \times 9 \times 9 \times 9 = 9^4$

Index laws

$4^2 \times 4^3 = 4^5$ $\quad 4^9 \div 4^3 = 4^6$
$(2^4)^3 = 2^{12}$ $\quad 5^0 = 1$

Squares and cubes

$4^2 = 16,\ \sqrt{16} = 4$
$3^3 = 27,\ \sqrt[3]{27} = 3$

Divisibility ★

- 2 Even number.
- 3 Sum of digits divisible by 3.
- 4 Number from last 2 digits divisible by 4.
- 5 Last digit 0 or 5.
- 6 Divisible by 2 and 3.
- 8 Number from last 3 digits divisible by 8.
- 9 Sum of digits divisible by 9.
- 10 Last digit 0.

Multiple-choice questions

1C **1** $400 \div 5 \times 2$ is the same as:
A $400 \div 10$ **B** 80×2 **C** 16 **D** $400 \div 2 \times 5$

1A **2** The sum and difference of 97 and 49 are:
A 146 and 58 **B** 246 and 48 **C** 136 and 58 **D** 146 and 48

1G **3** 561 is divisible by:
A 5 **B** 2 **C** 3 **D** 9

1B **4** 89×5 is the same as:
A 90×4 **B** $90 \times 5 - 1 \times 5$ **C** $89 \times 10 \times 2$ **D** 178×10

1D **5** $2 \times 2 \times 2 \times 2 \times 5 \times 5$ is:
A $2^4 \times 5^2$ **B** $2 \times 4 + 5 \times 2$ **C** $2^4 + 5^2$ **D** 10^7

1F **6** The LCM of $2^2 \times 3 \times 5$ and 2×7 is:
A 2 **B** $2^2 \times 3 \times 5 \times 7$ **C** $2 \times 3 \times 5 \times 7$ **D** $2^3 \times 3 \times 5 \times 7$

1E **7** $6^9 \div 6^2$ equals:
A 6^{11} **B** 1^7 **C** 12^{11} **D** 6^7

1I **8** $-6 + (-4)$ is the same as:
A $-6 - 4$ **B** $-6 + 4$ **C** $-4 + 6$ **D** $6 + 4$

1D **9** If $18^2 = 324$, then $\sqrt{324}$ equals:
A 162 **B** 102976 **C** 18 **D** 9

1E **10** $16^3 \times 16^2$ equals:
A 32^5 **B** 16^6 **C** 16^5 **D** 256^5

Short-answer questions

1A **1** Use a mental strategy to evaluate the following.
a $324 + 173$ **b** $592 - 180$ **c** $89 + 40$ **d** $135 - 68$
e $55 + 57$ **f** $280 - 141$ **g** $1001 + 998$ **h** $10\,000 - 4325$

1A **2** Use a mental strategy to find these sums and differences.
a $\begin{array}{r} 392 \\ +\,147 \\ \hline \end{array}$ **b** $\begin{array}{r} 1031 \\ +\,999 \\ \hline \end{array}$ **c** $\begin{array}{r} 147 \\ -\,86 \\ \hline \end{array}$ **d** $\begin{array}{r} 3970 \\ -\,896 \\ \hline \end{array}$

1B **3** Use a mental strategy for these products and quotients.
a $2 \times 17 \times 5$ **b** 3×99 **c** 8×42 **d** 141×3
e $164 \div 4$ **f** $357 \div 3$ **g** $618 \div 6$ **h** $1005 \div 5$

1B **4** Find these products and quotients using setting out.
a $\begin{array}{r} 139 \\ \times\,12 \\ \hline \end{array}$ **b** $\begin{array}{r} 507 \\ \times\,42 \\ \hline \end{array}$ **c** $3\overline{)843}$ **d** $7\overline{)854}$

1B **5** Find the remainder when 673 is divided by these numbers.
a 5 **b** 3 **c** 7 **d** 9

1D **6** Write using powers.

a $6 \times 6 \times 6$
b $8 \times 8 \times 8 \times 8$
c $2 \times 2 \times 5 \times 5 \times 5 \times 5$

1F **7** Evaluate.

a $\sqrt{81}$ **b** $\sqrt{121}$ **c** 7^2 **d** 20^2
e $\sqrt[3]{27}$ **f** $\sqrt[3]{64}$ **g** 5^3 **h** 10^3

1E **8** Simplify these powers.

a $4^9 \times 4^2$ **b** $3^4 \div 3^2$ **c** 5^0 **d** $(3^4)^5$

1F **9** **a** Find all the factors of 60.
b Find all the multiples of 7 between 110 and 150.
c Find all the prime numbers between 30 and 60.
d Find the LCM of 8 and 6.
e Find the HCF of 24 and 30.

1G **10** Write these numbers in prime factor form. You may wish to use a factor tree.

a 36 **b** 84 **c** 198

1G **11** Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 84 **b** 155 **c** 124 **d** 621

1G **12** Write the numbers 20 and 38 in prime factor form and then use this to help find the following.

a LCM of 20 and 38 **b** HCF of 20 and 38

1I **13** Evaluate.

a $-6 + 9$ **b** $-24 + 19$ **c** $5 - 13$ **d** $-7 - 24$
e $-62 - 14$ **f** $-194 - 136$ **g** $-111 + 110$ **h** $-328 + 426$

1I **14** Evaluate.

a $5 + (-3)$ **b** $-2 + (-6)$ **c** $-29 + (-35)$ **d** $162 + (-201)$
e $10 - (-6)$ **f** $-20 - (-32)$ **g** $-39 - (-19)$ **h** $37 - (-55)$

1J **15** Evaluate.

a -5×2 **b** $-11 \times (-8)$ **c** $9 \times (-7)$ **d** $-100 \times (-2)$
e $-10 \div (-5)$ **f** $48 \div (-16)$ **g** $-32 \div 8$ **h** $-81 \div (-27)$

1J **16** Evaluate using the order of operations.

a $2 + 3 \times (-2)$ **b** $-3 \div (11 + (-8))$
c $-2 \times 3 + 10 \div (-5)$ **d** $-20 \div 10 - 4 \times (-7)$

1J **17** Let $a = -2$, $b = 3$ and $c = -5$ and evaluate these expressions.

a $ab + c$ **b** $a^2 - b$ **c** $ac - b$ **d** $a + b + c$

1J **18** Copy and complete.

a $1^2 =$ _____ **b** $(-1)^2 =$ _____ **c** $2^2 =$ _____
d $(-2)^2 =$ _____ **e** $3^2 =$ _____ **f** $(-3)^2 =$ _____

Extended-response questions

1 A monthly bank account shows deposits as positive numbers and purchases and withdrawals (P + W) as negative numbers.

Details	P + W	Deposits	Balance
Opening balance	–	–	\$250
Water bill	–\$138	–	a
Cash withdrawal	–\$320	–	b
Deposit	–	c	\$115
Supermarket	d	–	–\$160
Deposit	–	\$400	e

a Find the values of a, b, c, d and e.

b If the water bill amount was \$150, what would be the new value for letter e?

c What would the final deposit need to be if the value for e was \$0? Assume the original water bill amount is \$138 as in the table above.

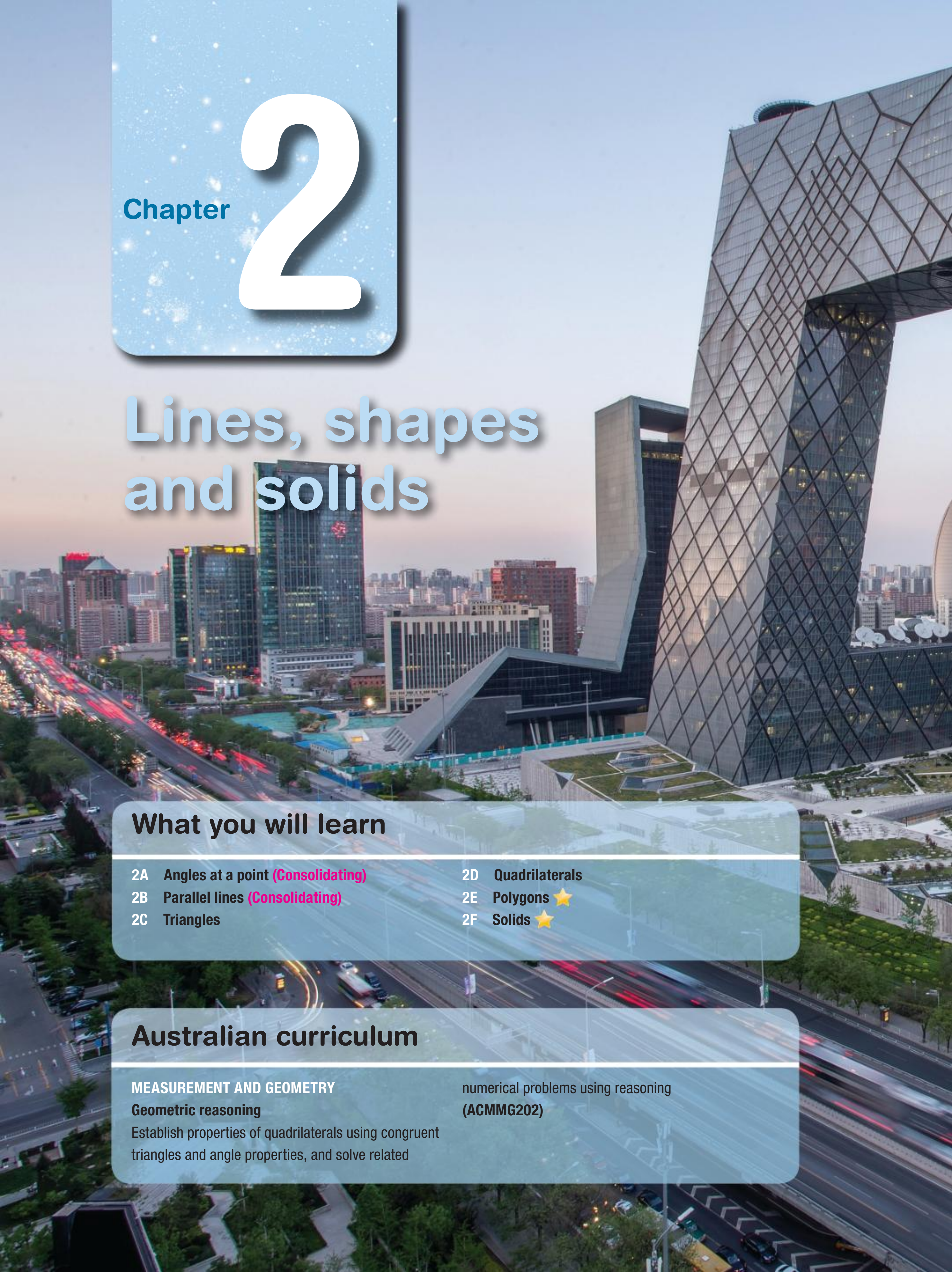
2 Two teams compete at a club games night. Team A has 30 players while team B has 42 players.

a How many players are there in total?

b Write both 30 and 42 in prime factor form.

c Find the LCM and HCF of the number of players representing the two teams.

d Teams are asked to divide into groups with equal numbers of players. What is the largest group size possible if team A and team B must have groups of the same size?



Chapter 2

Lines, shapes and solids

What you will learn

Australian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning **(ACMMG202)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
  Downloadable HOTsheets
  Australian Curriculum courses
- Access to all HOTmaths
- Access to the HOTmaths games library

The China Central Television (CCTV) tower

Geometry, modern computing power and new engineering techniques with glass and steel made it possible for architects to design the unique, 234 m high CCTV tower in Beijing.

Instead of making a single tall building, this construction joins 6 box shapes to form a 3D 'bent tube'. The two vertical towers, each 44 storeys high, are joined at the top and at ground level by giant horizontal glass 'tunnels', each many storeys high.

The CCTV tower is huge and it provides space for 10 000 employees who access it using 75 elevators. All the stages of a TV production can be completed in this one building.

The outer network of steel bracing is arranged in sloping, parallel lines to resist the forces of gravity, wind and earthquakes. Glass panels of various geometrical shapes, such as rhombuses, kites, parallelograms and polygons, are attached in-between the steel mesh.

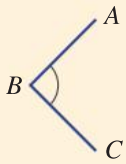
Pre-test

1 Name these objects. Choose from: A: line AB, B: segment AB, C: point A or D: angle ABC.

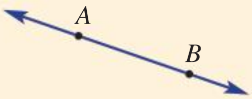
a

b

c

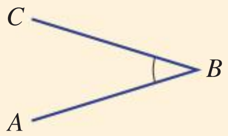


d

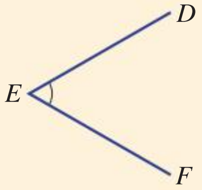


2 Choose a correct angle name from A: $\angle DEF$, B: $\angle STU$ or C: $\angle ABC$ for each given diagram.

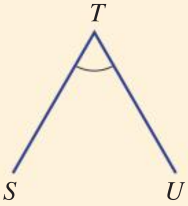
a



b



c



3 Name these angles as A: *acute*, B: *right*, C: *obtuse*, D: *straight*, E: *reflex* or F: *revolution*.

a 360°
b 90°
c 37°
d 149°
e 180°
f 301°

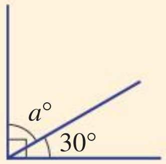
4 Name the triangle that fits the description. Choose from A: *scalene*, B: *isosceles*, C: *equilateral*, D: *acute*, E: *right* or F: *obtuse*. Draw an example of each triangle to help.

a One obtuse angle
b 2 equal length sides
c All angles acute
d 3 different side lengths
e 3 equal 60° angles
f one right angle

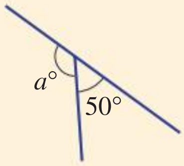
5 Name the six special quadrilaterals with four sides. Choose from A: *circle*, B: *square*, C: *parallelogram*, D: *line*, E: *triangle*, F: *rectangle*, G: *rhombus*, H: *hectagon*, I: *kite*, J: *trapezium*, K: *tetrahedron*.

6 Find the value of a in these diagrams.

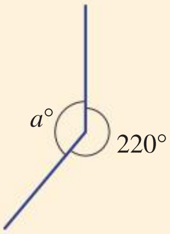
a



b

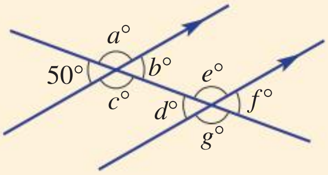


c



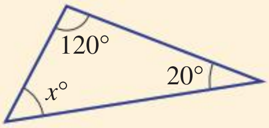
7 This diagram includes a pair of parallel lines and a third line (transversal).

a What is the value of a?
b Which pronumerals (b, c, d, e, f or g) are equal to a? List in alphabetical order.
c Which pronumerals (b, c, d, e, f or g) are equal to 50? List in alphabetical order.

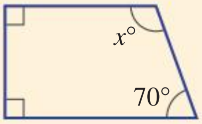


8 Find the value of x in these shapes, using the given angle sum.

a Angle sum = 180°



b Angle sum = 360°



2A Angles at a point

CONSOLIDATING

Interactive

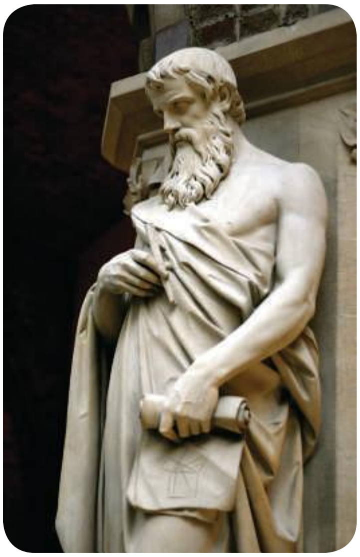
Widgets

HOTsheets

Walkthrough

From three simple objects – point, line and plane – we can develop all the elements of Geometry, just as the Greek mathematician Euclid did about 2300 years ago.

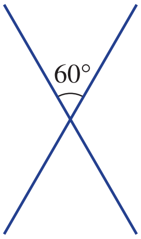
We can start by looking at the angles formed when lines meet at a point.



Let's start: How many angles?

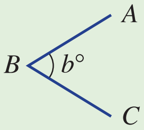
When two lines cross, different angles are formed, like in this example.

- Is there another 60° angle? Why?
- What is the size of one of the obtuse angles? How did you work this out?
- Are there any straight angles in the diagram?
- Are there any reflex angles in the diagram?
- What is a revolution angle?

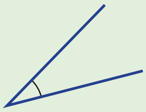


Key ideas

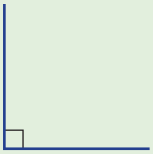
- The angle at right could be named $\angle ABC$, $\angle CBA$, $\angle B$ or $A\hat{B}C$ and has size $b°$.
- Types of angles



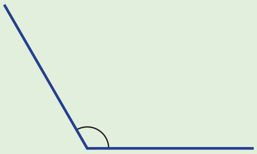
Acute $(0 - 90°)$



Right $(90°)$

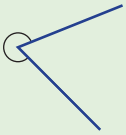


Obtuse $(90 - 180°)$

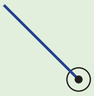


Straight $(180°)$

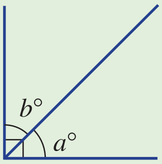
Reflex $(180 - 360°)$



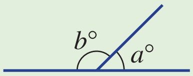
Revolution $(360°)$



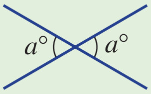
- Special pairs of angles at a point include:
  - **Complementary** angles (sum to 90°)
  - **Supplementary** angles (sum to 180°)
  - **Vertically opposite** angles (equal)



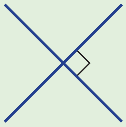
$a + b = 90$

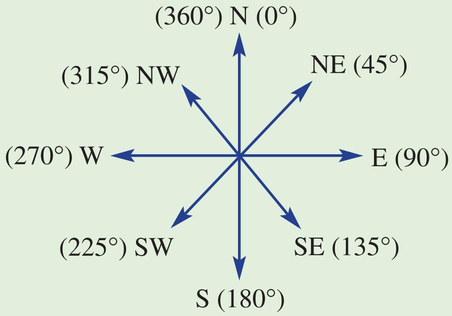


$a + b = 180$



- Angles in a **revolution** sum to 360°.
- Two lines are **perpendicular** if they intersect at right angles (90°).
- 8 point compass bearing
  - Bearings are usually measured clockwise from north.





Exercise 2A

UNDERSTANDING 1–5 5

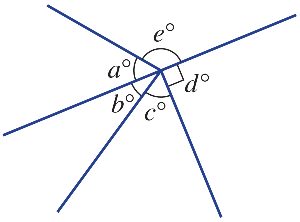
1 Write the missing word. Choose from: *equal, supplementary, complementary and perpendicular*.
  - a Angles that add to 90° are called ___________ angles.
  - b Angles that add to 180° are called ___________ angles.
  - c If two lines meet at right angles (90°), then they are said to be ___________.
  - d Vertically opposite angles are ___________.

2 What type of angle are the following?

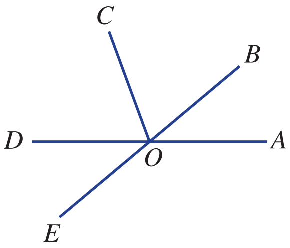
a 27°	b 317°
c 180°	d 90°
e 360°	f 139°

Choose from: *acute*, *right*, *obtuse*, *straight*, *reflex* or *revolution*.

3 Complete these sentences for this diagram.
  - a $b°$ and $c°$ are ________________ angles.
  - b $a°$ and $e°$ are ________________ angles.
  - c $a°, b°, c°, d°$ and $e°$ form a ________________.



4 Estimate the size of these angles.
  - a $\angle AOB$
  - b $\angle AOC$
  - c Reflex $\angle AOE$



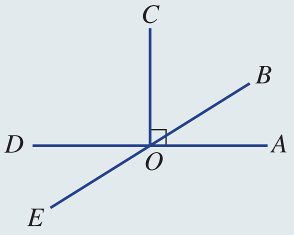
Example 1 Naming angles in relation to other angles

Name an angle which is:

a vertically opposite to $\angle DOE$

b complementary to $\angle COB$

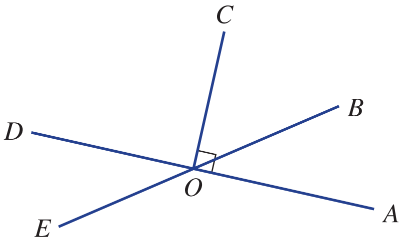
c supplementary to $\angle EOA$



Solution	Explanation
a $\angle AOB$	$\angle DOE$ and $\angle AOB$ are equal and sit opposite each other.
b $\angle BOA$	$\angle COB$ and $\angle BOA$ add to 90°.
c $\angle DOE$ (or $\angle AOB$)	Pairs of angles on a straight line are supplementary (add to 180°).

5 Name an angle which is:

- **a** vertically opposite to $\angle DOE$.
- **b** complementary to $\angle COB$.
- **c** supplementary to $\angle EOA$.



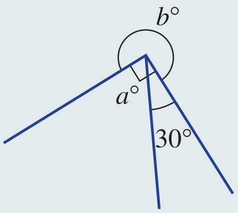
Vertically opposite angles are opposite and equal. Complementary angles add to 90°. Supplementary angles add to 180°.

FLUENCY 6, 7(½) 6–8(½)

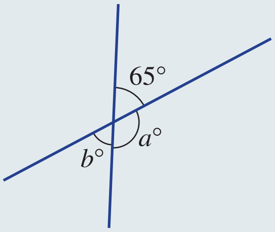
Example 2 Finding angles at a point

Determine the value of the pronumerals in these diagrams.

a



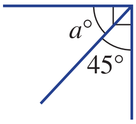
b



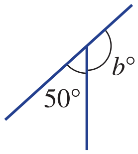
Solution	Explanation
a $a + 30 = 90$ $a = 60$ $b + 90 = 360$ $b = 270$	a° and 30° make a complementary pair of angles adding to 90°. Angles in a revolution add to 360°.
b $a + 65 = 180$ $a = 115$ $b = 65$	a° and 65° make a supplementary pair of angles adding to 180°. b° is vertically opposite the 65° angle.

6 State the value of the pronumeral (letter) in these diagrams.

a



b



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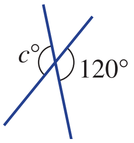
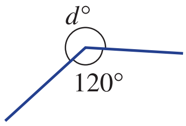
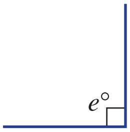


d



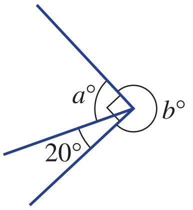
e



f

7 Determine the unknown angles marked in these diagrams.

a



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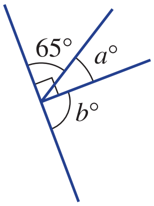
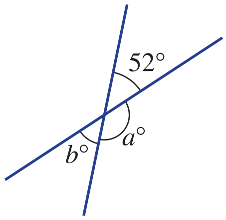
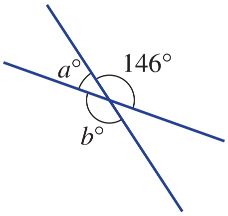


c

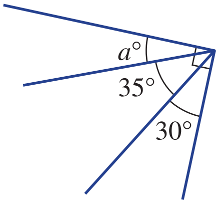


Angles in a right angle add to 90°. Angles on a straight line add to 180°. Angles in a revolution add to 360°.

d



e



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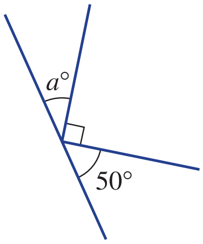
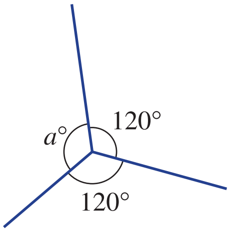
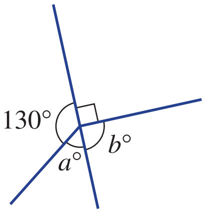


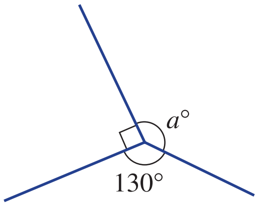
g



h



i



8 Give the compass bearing, in degrees, for these directions.

a West (W)
b East (E)
c North (N)
d South (S)
e NW
f SE
g SW
h NE

PROBLEM-SOLVING AND REASONING

9, 10 | 9, 10–11(½), 12

9 A round birthday cake is cut into sectors for nine friends (including Jack) at Jack's birthday party. After the cake is cut there is no cake remaining. What will be the angle at the centre of the cake for Jack's piece if:

a everyone receives an equal share?

b Jack receives twice as much as everyone else? (In parts **b**, **c** and **d** assume his friends have equal shares of the rest.)

c Jack receives four times as much as everyone else?

d Jack receives ten times as much as everyone else?

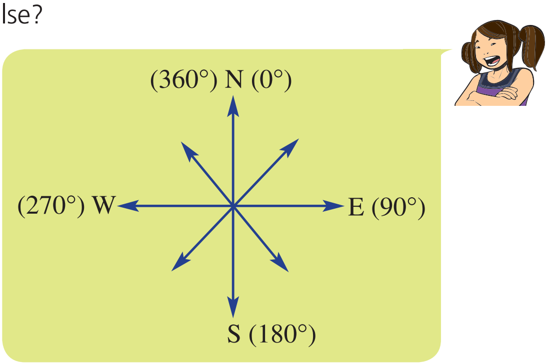
10 In which direction (e.g. north-east or NE) would you be walking if you were headed on these compass bearings?

a 180° **b** 360°

c 270° **d** 90°

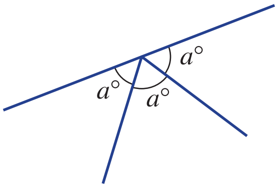
e 45° **f** 315°

g 225° **h** 135°

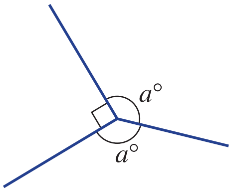


11 Find the value of the pronumerals in these diagrams.

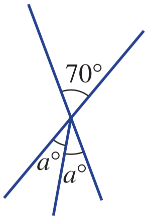
a



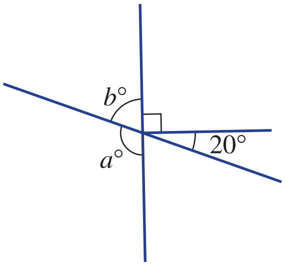
b



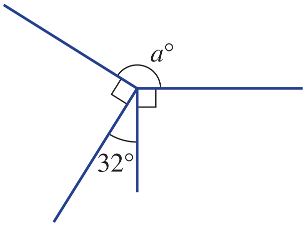
c



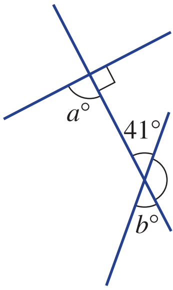
d



e

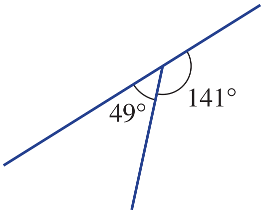


f

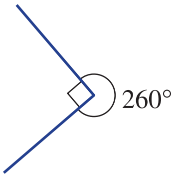


12 Explain, with reasons, what is wrong with these diagrams.

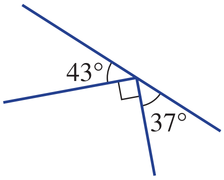
a



b



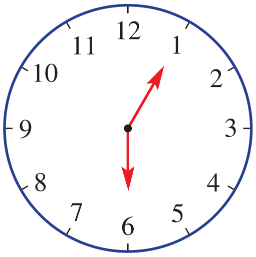
c



Clock geometry

—	13, 14

13 Here is a clock face with an hour hand (short arrow) and minute hand (long arrow). The time shown is 6:05.



a How many degrees does the hour hand turn in:

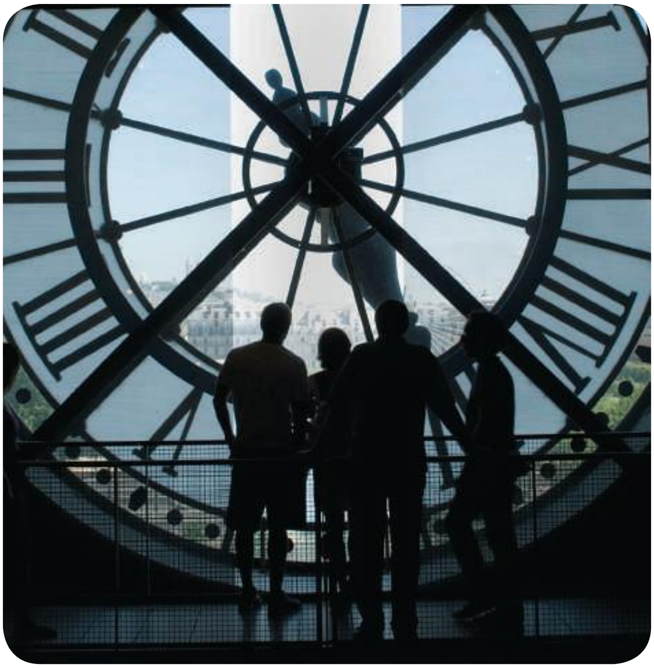
- **i** 6 hours?
- **ii** 12 hours?
- **iii** 1 hour?
- **iv** 3 hours?

b How many degrees does the minute hand turn in:

- **i** 1 hour?
- **ii** 30 minutes?
- **iii** 5 minutes?
- **iv** 20 minutes?

14 What is the angle between the hour hand and minute hand on a clock at these times?

- **a** 2:30 pm
- **b** 5:45 am
- **c** 1:40 am
- **d** 10:20 pm
- **e** 2:35 am
- **f** 12:05 pm
- **g** 4:48 pm
- **h** 10:27 am



2B Parallel lines

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

In simple language, Euclid's 5th axiom says that parallel lines do not intersect or meet.

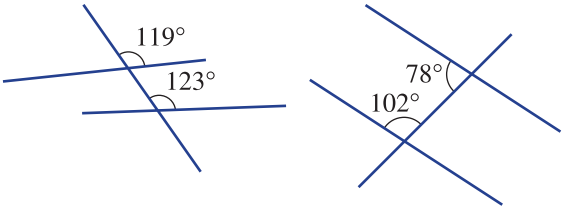
All sorts of shapes and solids both in the theoretical and practical worlds can be constructed using parallel lines. If two lines are parallel and are cut by a third line called a transversal, special pairs of angles are created.

Parallel lines never intersect.

All sorts of shapes and solids can be constructed using parallel lines and transversals.

Let's start: Are they parallel?

Here are two diagrams that show a pair of lines crossed by a third line called a transversal. Two angles are given.



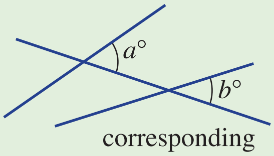
- Do you think that each diagram contains a pair of parallel lines?
- Can you determine all the other angles in the diagrams?
- How many different angles are there in each diagram?

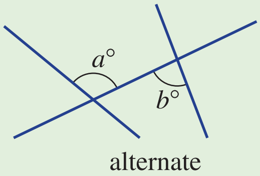
Key ideas

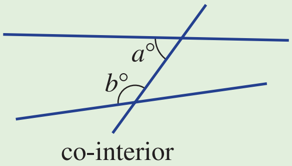
Transversal
A line that cuts two or more lines

A **transversal** is a line cutting at least two other lines.

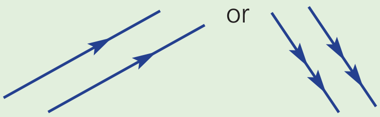
- Pairs of angles formed by transversals can be:
  - **corresponding** (in corresponding positions)
  - **alternate** (on opposite sides of the transversal and inside the other two lines)
  - **co-interior** (on the same side of the transversal and inside the other two lines).





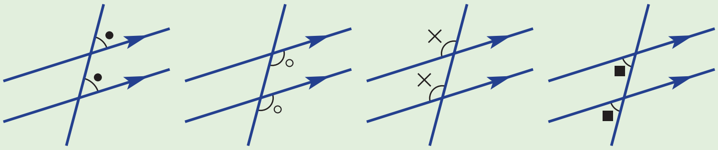


- Lines are **parallel** if they do not intersect.
  - Parallel lines are marked with the same number of arrows.

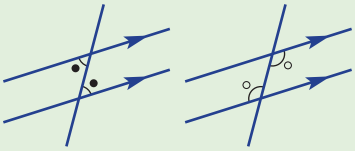


Parallel lines
Lines in the same plane that are the same distance apart and never intersect

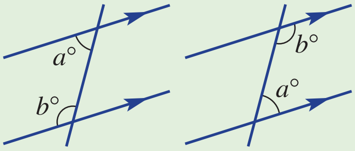
- If two parallel lines are cut by a transversal:
  - the corresponding angles are equal (4 pairs)



  - the alternate angles are equal (2 pairs)



  - the co-interior angles are supplementary (sum to 180°) (2 pairs).



$a + b = 180$ $\quad$ $a + b = 180$

Exercise 2B

UNDERSTANDING 1–3 3

1 Two parallel lines are cut by a transversal. Write the missing word.

a Corresponding angles are ________________.

b Co-interior angles are ________________.

c Alternate angles are ________________.

Choose from: *equal* or *supplementary*.

2 Name the angle that is:

a corresponding to $\angle ABF$

b corresponding to $\angle BCG$

c alternate to $\angle FBC$

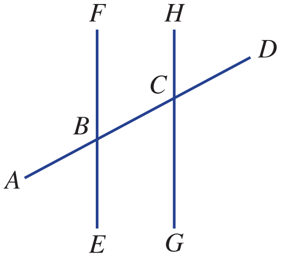
d alternate to $\angle CBE$

e co-interior to $\angle HCB$

f co-interior to $\angle EBC$

g vertically opposite to $\angle ABE$

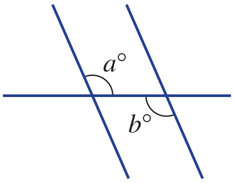
h vertically opposite to $\angle HCB$



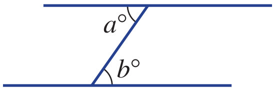
Name angles like this: $\angle ABC$ or $\angle DEF$

3 State whether the following marked angles are corresponding, alternate or co-interior.

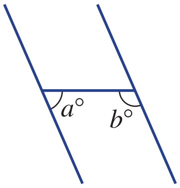
a



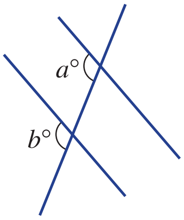
b



c



d



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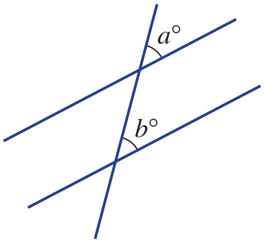
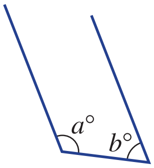


f

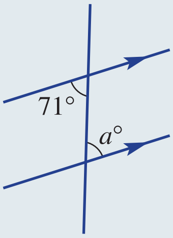


FLUENCY 4–5(½) 4–5(½)

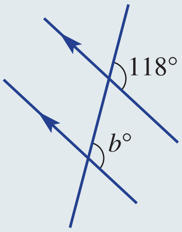
Example 3 Working with parallel lines

Find the value of the pronumerals in these diagrams. Give a reason for each answer.

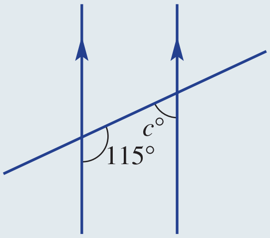
a



b



c

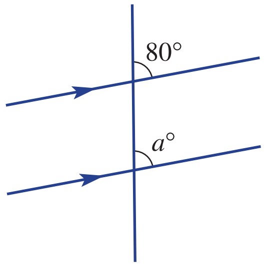


Solution	**Explanation**
a $a = 71$, alternate angles in parallel lines.	Alternate angles in parallel lines are equal.
b $b = 118$, corresponding angles in parallel lines.	Corresponding angles in parallel lines are equal.
c $c = 180 - 115 = 65$, co-interior angles in parallel lines.	Co-interior angles in parallel lines add to 180°.

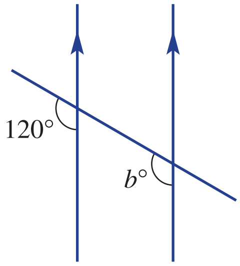
4 Find the value of the pronumerals in these diagrams. Give a reason for each answer.

Corresponding angles are equal in parallel lines. Alternate angles are equal in parallel lines. Co-interior angles in parallel lines are supplementary (add to 180°).

a



b



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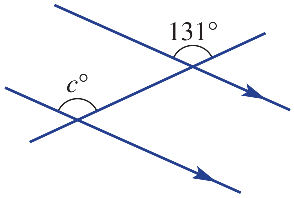

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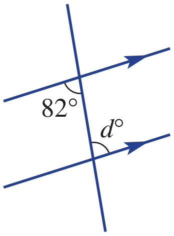

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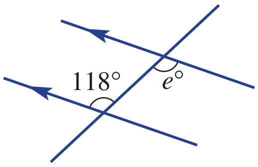

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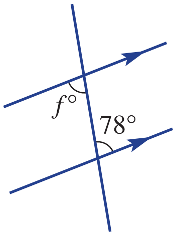

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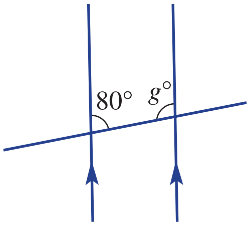
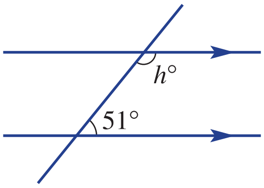
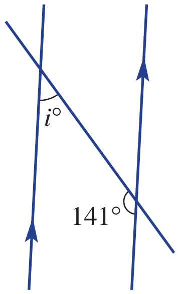


h

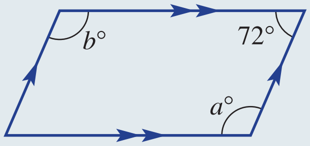


i



Example 4 Using parallel lines in shapes

Find the value of the pronumerals in this diagram, stating reasons.

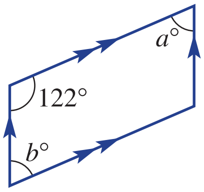


Solution	Explanation
$a + 72 = 180$ $a = 108$	The pairs of angles are co-interior, which are supplementary if the lines are parallel.
$b + 72 = 180$ $b = 108$	This shows that opposite angles in a parallelogram are equal.

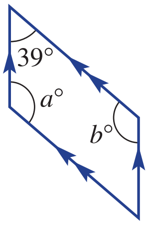
Co-interior angles in parallel lines are supplementary.

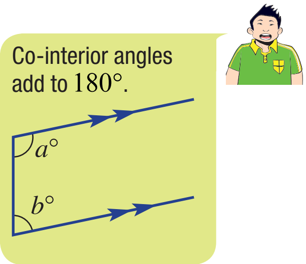
5 Find the value of the pronumerals in these diagrams, stating reasons.

a

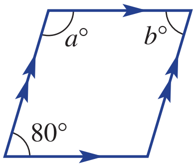


b




Co-interior angles add to $180°$.

c



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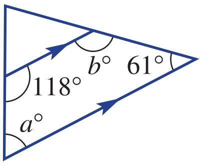

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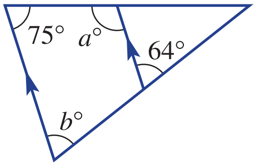

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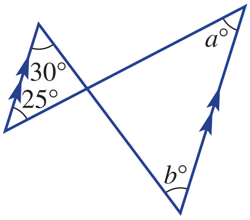
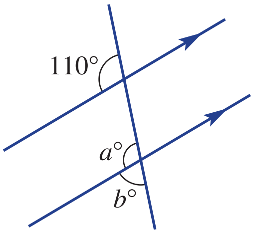


PROBLEM-SOLVING AND REASONING

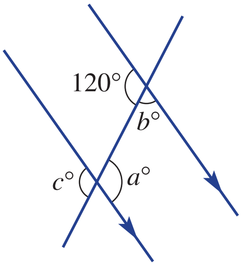
6, 7 | 6–9

6 Find the value of the pronumerals in these diagrams, stating reasons.

a



b



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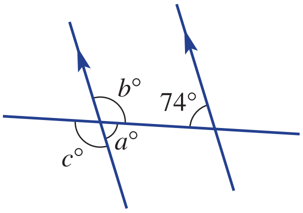

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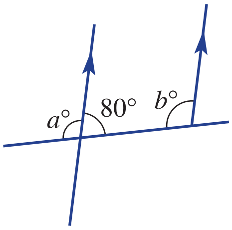

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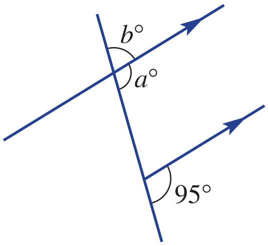

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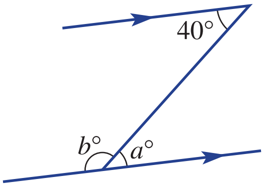
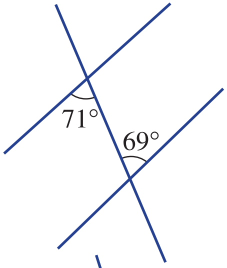


7 Decide if the following diagrams include a pair of parallel lines. Give a reason for each answer.

a



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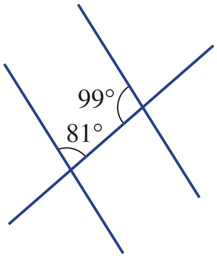

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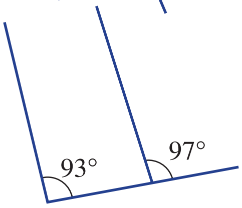
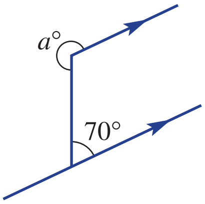


8 Find the value of a in these diagrams.

a



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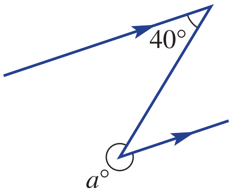
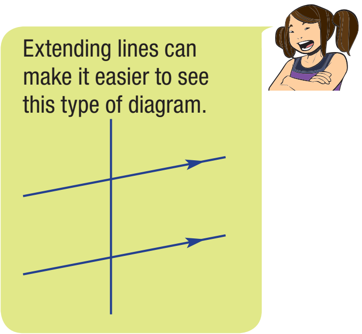
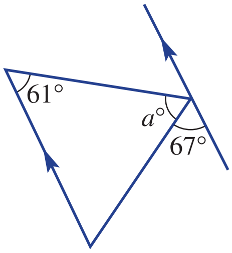




Extending lines can make it easier to see this type of diagram.

c



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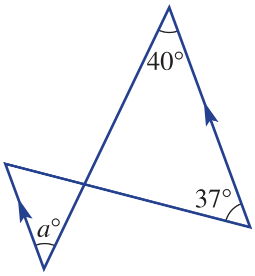

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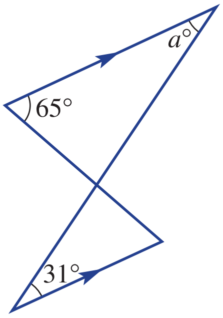

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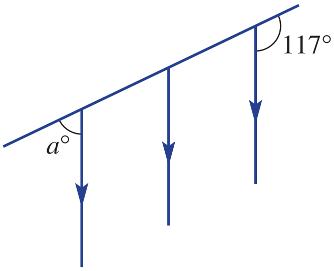
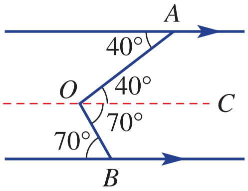
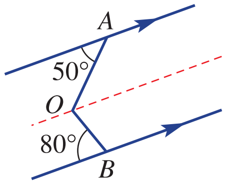


9 Sometimes parallel lines can be added to a diagram to help find an unknown angle. For example, $\angle AOB$ can be found in this diagram by first drawing the dashed line and finding $\angle AOC$ $(40°)$ and $\angle COB$ $(70°)$. So $\angle AOB = 40° + 70° = 110°$.

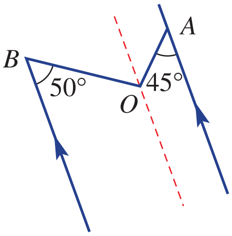


Apply a similar technique to find $\angle AOB$ in these diagrams.

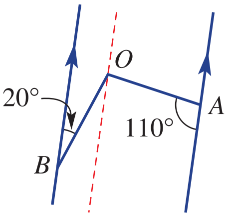
a



b



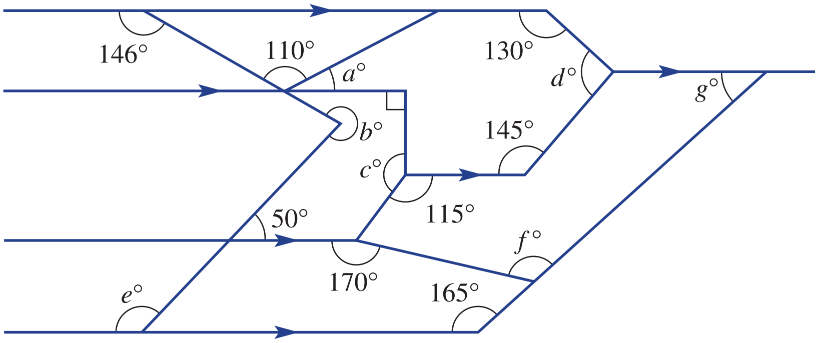
c



Pipe networks

—	10

10 A plan for a natural gas plant includes many intersecting pipelines, some of which are parallel. Help the designers finish the plans by calculating the size of the angles marked a, b etc.



2C Triangles

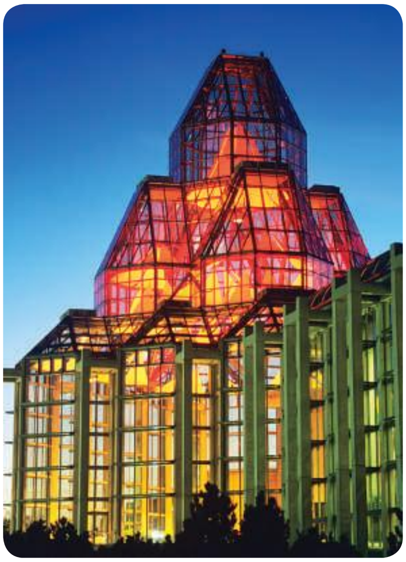
Interactive

Widgets

HOTsheets

Walkthrough

A triangle is a shape with three straight sides. The triangle is a very rigid shape and this leads to its use in the construction of houses and bridges. It is one of the most commonly used shapes in design and construction.

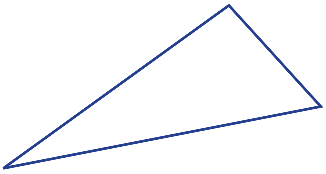


Triangular shapes are often used to striking effect in architecture, as shown by part of the National Gallery of Canada.

Let's start: Illustrating the angle sum

You can complete this task using a pencil and ruler or using dynamic geometry software.

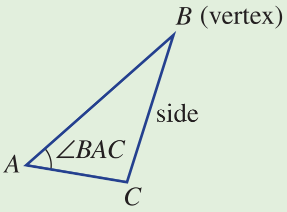
- Draw any triangle and measure each interior angle.
- Add all three angles to find the angle sum of your triangle.
- Compare your angle sum with the results of others. What do you notice?



If dynamic geometry is used, drag one of the vertices to alter the interior angles. Now check to see if your conclusions remain the same.

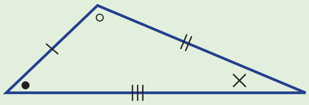
Key ideas

- A **triangle** has:
  - 3 sides
  - 3 vertices (singular: vertex)
  - 3 interior angles.

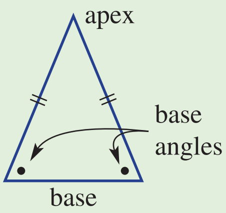


- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

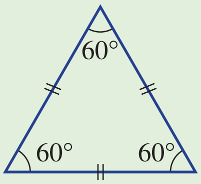
Scalene



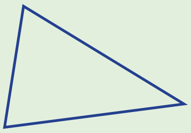
Isosceles

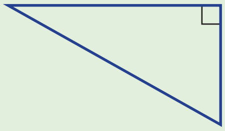


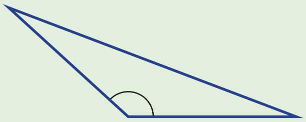
Equilateral



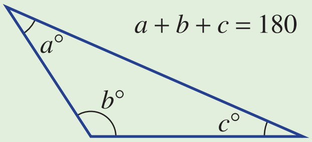
- Triangles are classified by interior angles:
  - **acute** (All angles acute)
  - **right** (1 right angle)
  - **obtuse** (1 obtuse angle)



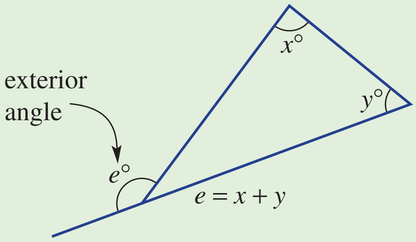




- The angle sum of a triangle is 180°.



- The **exterior angle theorem:**
  The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



Exercise 2C

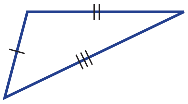
UNDERSTANDING 1–3 3

1 Give the common name of a triangle with these properties. Refer to the key ideas in this section for help.

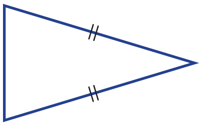
a One right angle
b 2 equal side lengths
c All angles acute
d All angles 60°
e One obtuse angle
f 3 equal side lengths
g 2 equal angles
h 3 different side lengths

2 State whether these triangles are scalene, isosceles or equilateral.

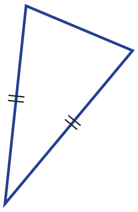
a



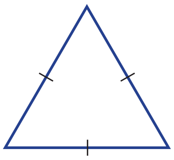
b



c



d



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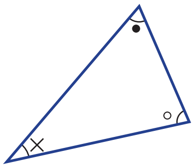

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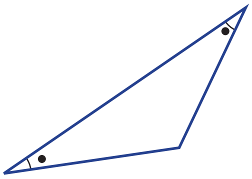
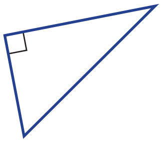


3 State whether these triangles are acute, right or obtuse.

a



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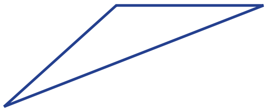

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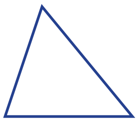
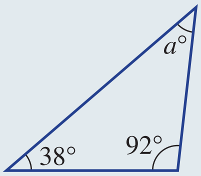


FLUENCY 4, 5, 6(½) 4–6(½)

Example 5 Using the angle sum of triangle

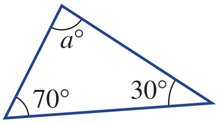
Find the value of a in this triangle.



Solution	**Explanation**
$a + 38 + 92 = 180$ $a + 130 = 180$ $a = 50$	The angle sum of the three interior angles of a triangle is 180°. Also $38 + 92 = 130$ and $180 - 130 = 50$.

4 Use the angle sum of a triangle to help find the unknown angle in these triangles.

a



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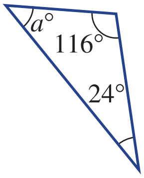

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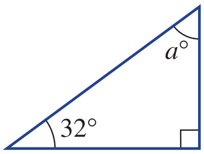
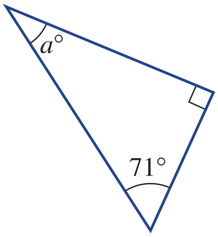
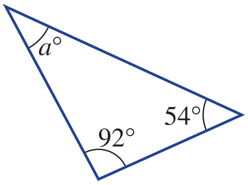


d



e



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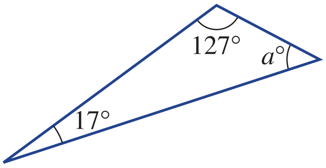
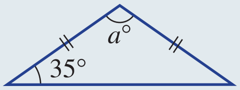


For each one use a mental strategy or start with an equation like $a + 36 + 48 = 180$

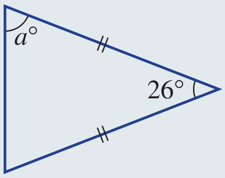
Example 6 Working with isosceles triangles

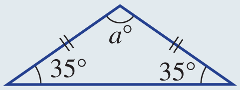
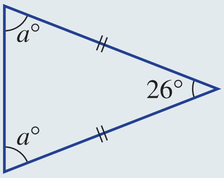
Find the value of a in these isosceles triangles.

a



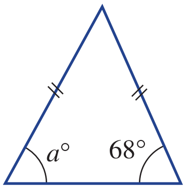
b



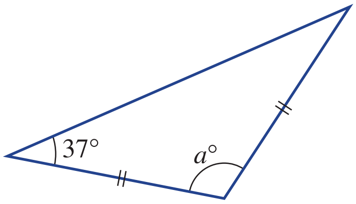
Solution	**Explanation**
a $a + 35 + 35 = 180$ $a + 70 = 180$ $a = 110$	The two base angles are equal. 
b $2a + 26 = 180$ $2a = 154$ $a = 77$	The two base angles in an isosceles triangle are equal. 

5 These triangles are isosceles. Find the value of a.

a

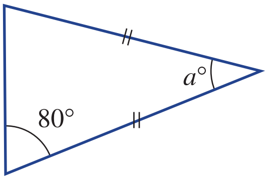


b

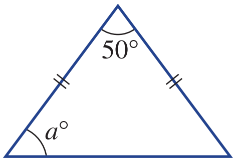


The two base angles in an isosceles triangle are equal.

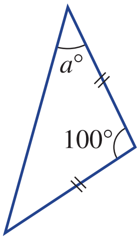
c



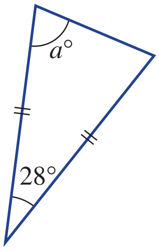
d



e

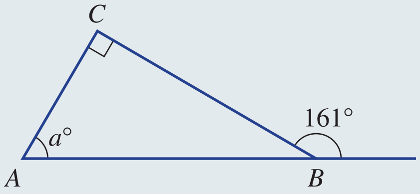


f



Example 7 Using the exterior angle theorem

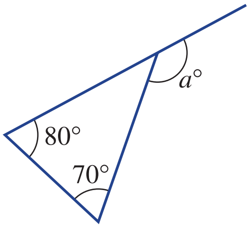
Find the size of the unknown angle.



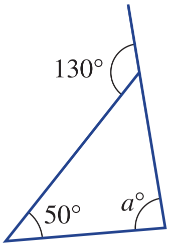
Solution	Explanation
$a + 90 = 161$ $a = 161 - 90 = 71$	Use the exterior angle theorem for a triangle. The exterior angle $(161°)$ is equal to the sum of the two opposite interior angles.

6 Find the value of a.

a



b



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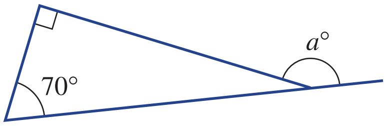

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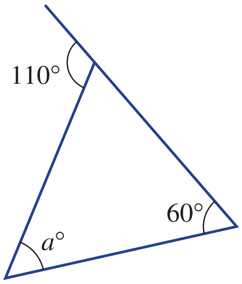

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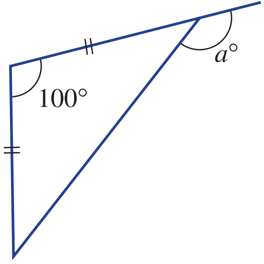

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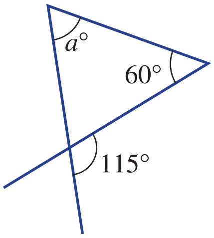
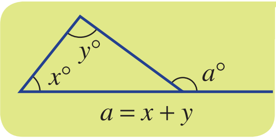




$a = x + y$

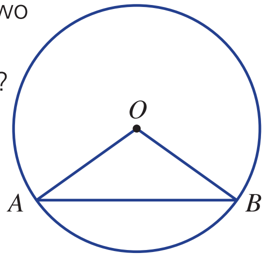
PROBLEM-SOLVING AND REASONING 7, 8 7–9

7 Decide if it is possible to draw a triangle with the given description. Draw a diagram to support your answer.

- **a** Right and scalene
- **b** Obtuse and equilateral
- **c** Right and isosceles
- **d** Acute and isosceles
- **e** Acute and equilateral
- **f** Obtuse and isosceles

8 A triangle is constructed using a circle and two radius lengths.

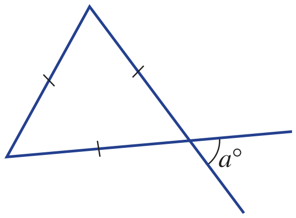
- **a** What type of triangle is ΔAOB and why?
- **b** Name two angles that are equal.
- **c** Find $\angle ABO$ if $\angle BAO$ is $30°$.
- **d** Find $\angle AOB$ if $\angle OAB$ is $36°$.
- **e** Find $\angle ABO$ if $\angle AOB$ is $100°$.



What can you say about the lengths OA and OB?

9 Find the value of a in these diagrams.

a



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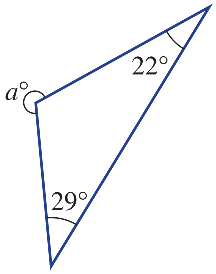
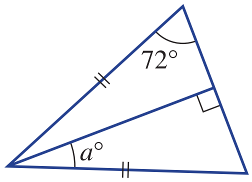


What is the size of all the angles in an equilateral triangle?

c



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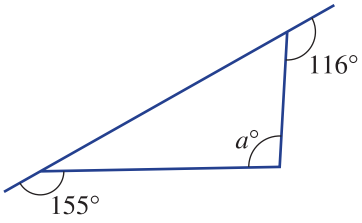

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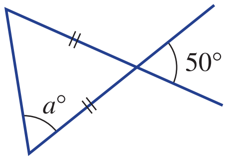

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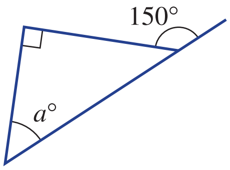
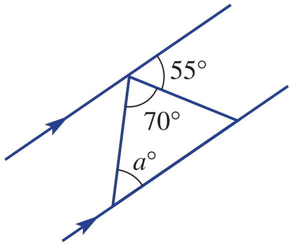


Parallel lines and triangles

— 10–11

10 Use your knowledge of parallel lines and triangles to find the unknown angle a.

a



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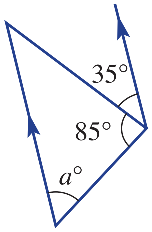

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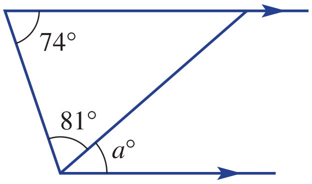
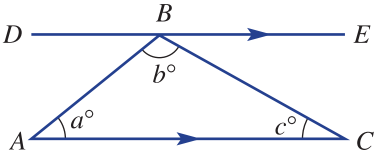


11 To prove that the angle sum of a triangle is 180°, work through these steps with the given diagram.



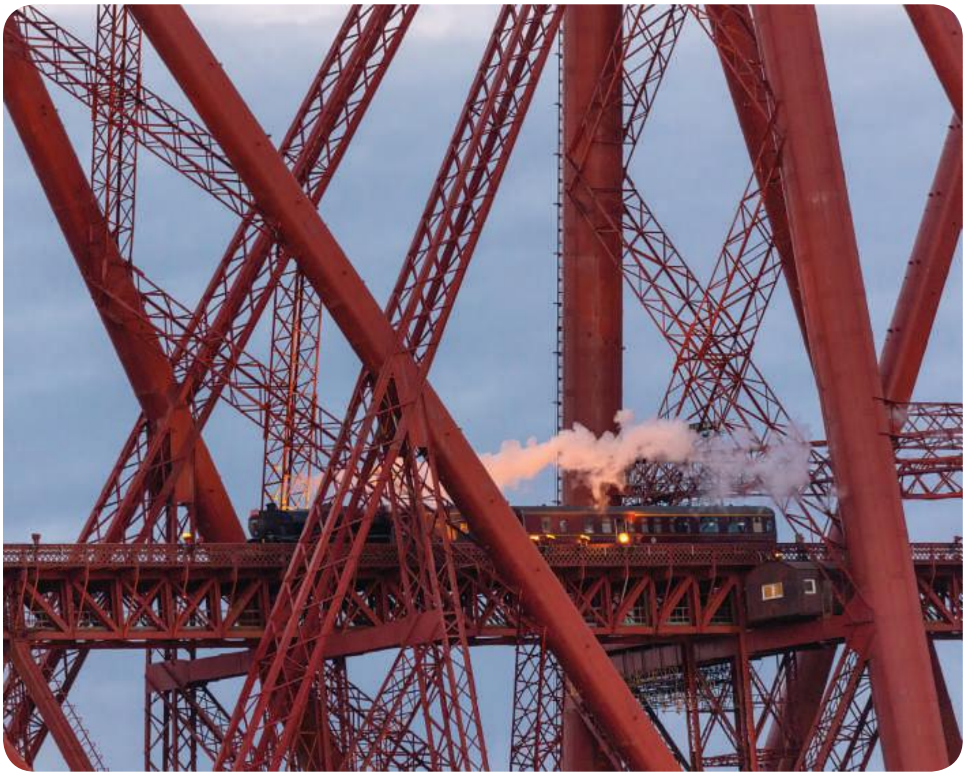
Think back to the parallel line rules from section 2B.

a Using the pronumerals a, b or c, give the value of these angles and state a reason.

- **i** $\angle ABD$
- **ii** $\angle CBE$

b What is true about the three angles $\angle ABD$, $\angle ABC$ and $\angle CBE$ and why?

c What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the angle sum of the triangle ABC ?



Steel girders arranged in triangular shapes combine strength and lightness in load-bearing structures.

2D Quadrilaterals

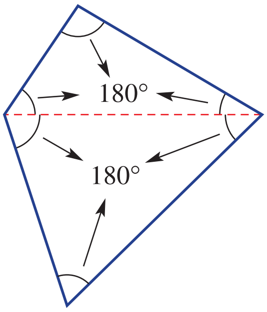
Interactive

Widgets

HOTsheets

Walkthrough

Shapes with four sides are called quadrilaterals. All quadrilaterals have the same angle sum. Their other properties depend on such things as pairs of sides of equal length, parallel sides and lengths of diagonals. All quadrilaterals can be divided into two triangles. Since the six angles inside the two triangles make up the four angles of the quadrilateral, the angle sum is $2 \times 180^\circ = 360^\circ$.



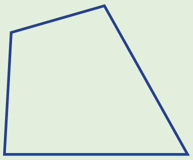
Let's start: Which quadrilaterals suit?

Name all the different quadrilaterals you can think of that have the properties listed below. There may be more than one quadrilateral for each property listed. Draw each quadrilateral to illustrate the shape and its features.

- 4 equal length sides
- 2 pairs of parallel sides
- Equal length diagonals
- 1 pair of parallel sides
- 2 pairs of equal length sides
- 2 pairs of equal opposite angles

Key ideas

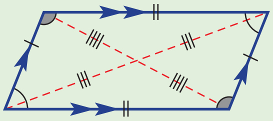
- **Quadrilaterals** are four-sided shapes.



Quadrilateral
A four-sided plane figure with straight sides

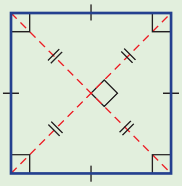
- Parallelograms are quadrilaterals with two pairs of parallel sides.

Parallelogram

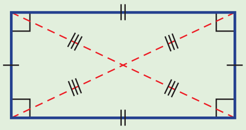


- Special parallelograms

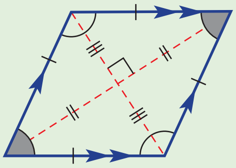
Square



Rectangle

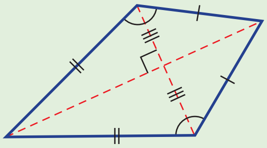


Rhombus

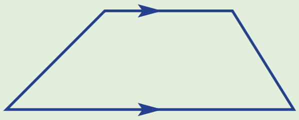


- Other special quadrilaterals

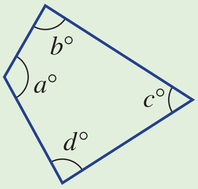
Kite



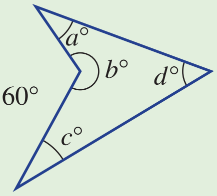
Trapezium



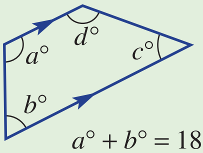
- The angle sum of any quadrilateral is 360°.



$$a^\circ + b^\circ + c^\circ + d^\circ = 360^\circ$$



- Quadrilaterals with parallel sides include two pairs of co-interior angles.



$c^\circ + d^\circ = 180^\circ$

$a^\circ + b^\circ = 180^\circ$

Exercise 2D

UNDERSTANDING 1–3 3

1 What are the six special types of quadrilaterals?

2 Write the missing number or word. Choose from: *90°*, *equal*, *360°* or *two*.

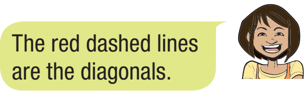
a The angle sum of a quadrilateral is _______________.

b The side lengths of a rhombus are _______________ in length.

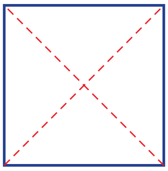
c A kite has _______________ pairs of equal sides.

d The diagonals of squares, rhombuses and kites intersect at _______________.

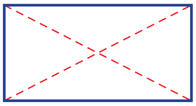
3 Answer true (T) or false (F) to these statements. Refer to the diagrams in the key ideas or accurately draw your own shapes, including the diagonals.



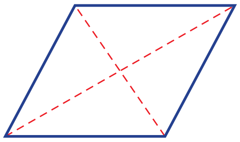
a Square
- **i** All sides are of equal length.
- **ii** Diagonals are not equal in length.
- **iii** All sides are parallel to each other.
- **iv** Diagonals intersect at right angles.



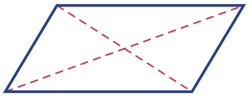
b Rectangle
- **i** Diagonals intersect at right angles.
- **ii** All interior angles are $90°$.
- **iii** All sides are of equal length.
- **iv** There are two pairs of parallel sides.



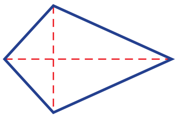
c Rhombus
- **i** All interior angles are equal.
- **ii** All sides are of equal length.
- **iii** Diagonals intersect at right angles.



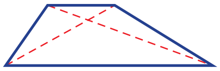
d Parallelogram
- **i** There are two pairs of equal length and parallel sides.
- **ii** Diagonals are equal in length.
- **iii** Diagonals intersect at right angles.



e Kite
- **i** There are two pairs of sides of equal length.
- **ii** There are two pairs of parallel sides.
- **iii** Diagonals intersect at right angles.



f Trapezium
- **i** Diagonals are equal in length.
- **ii** There are two pairs of parallel sides.

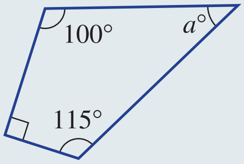


FLUENCY 4, 5 4–5(½)

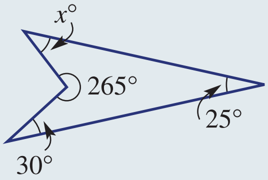
Example 8 Using the angle sum of a quadrilateral

Find the value of the pronumerals in these quadrilaterals.

a



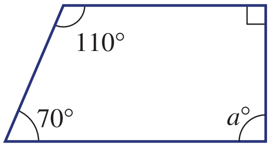
b



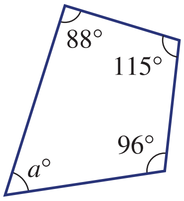
Solution	**Explanation**
a $a + 100 + 90 + 115 = 360$ $a + 305 = 360$ $a = 55$	The sum of angles in a quadrilateral is 360°. Use a mental strategy or solve the equation.
b $x + 265 + 30 + 25 = 360$ $x + 320 = 360$ $x = 40$	Use the angle sum of a quadrilateral.

4 Use the quadrilateral angle sum to find the value of a in these quadrilaterals.

a

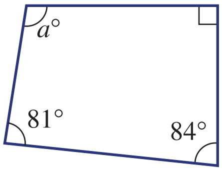


b



∟ is a 90° angle. The angle sum of a quadrilateral is 360°

c



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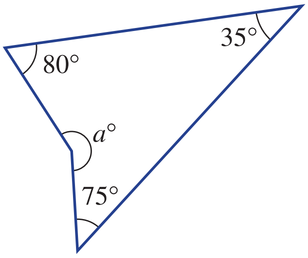

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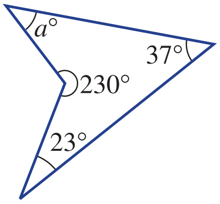

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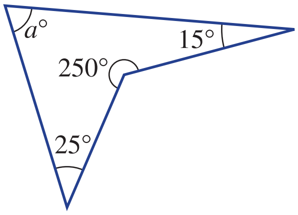
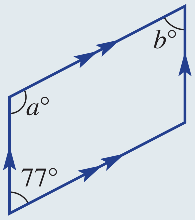


Example 9 Working with parallelograms

Find the value of a and b in this parallelogram.

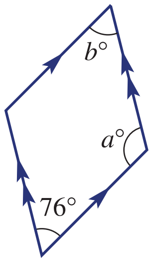


Solution	Explanation
$a + 77 = 180$	Two angles inside parallel lines are co-interior and therefore sum to $180°$.
$a = 103$	
$b = 180 - 103 = 77$	Note that opposite angles in a parallelogram are equal.

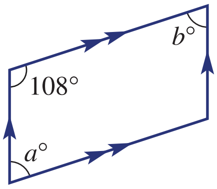
5 Find the value of the pronumerals in these quadrilaterals.

Opposite angles in a parallelogram are equal. Other pairs of angles are co-interior (add to $180°$).

a



b



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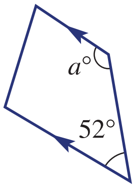

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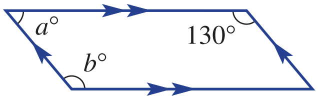

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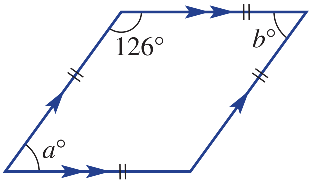

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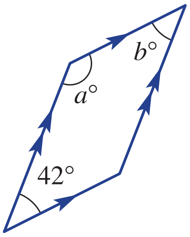


PROBLEM-SOLVING AND REASONING 6, 7 6–9

6 Name the special quadrilaterals which have:

a all sides of equal length

b one pair of parallel lines

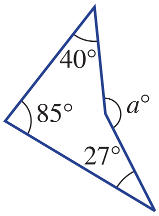
c two pairs of equal sides

d diagonals meeting at right angles

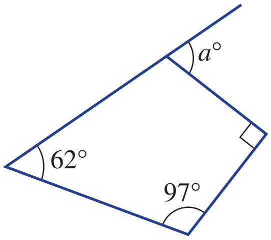
e diagonals of equal length.

7 Use your knowledge of geometry from the previous sections to find the values of a.

a

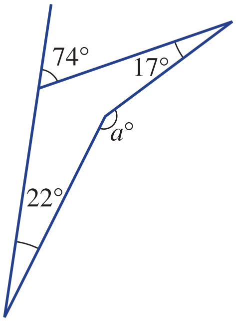


b



Angles in a revolution add to 360°. Angles on a straight line add to 180°. Vertically opposite angles are equal.

c



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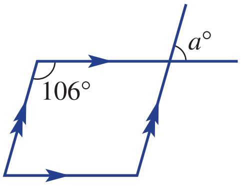

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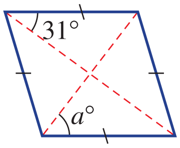

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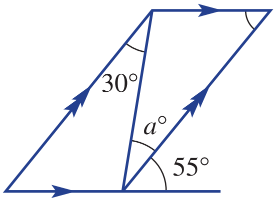


8 Consider the properties of special quadrilaterals. Decide if the following are true (T) or false (F).

a A square is a type of rectangle.

b A rectangle is a type of square.

c A square is a type of rhombus.

d A rectangle is a type of parallelogram.

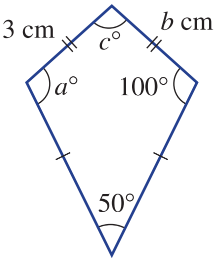
e A parallelogram is a type of square.

f A rhombus is a type of parallelogram.

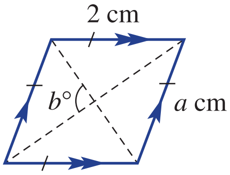
9 Consider the properties of the given quadrilaterals. Give the values of the pronumerals.

a A kite
b A rhombus
c A parallelogram

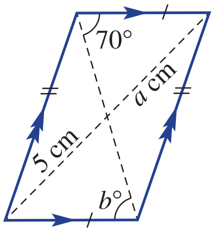
a



b



c

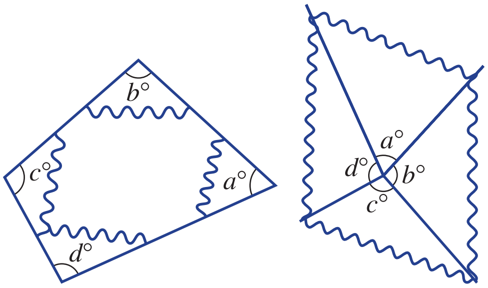


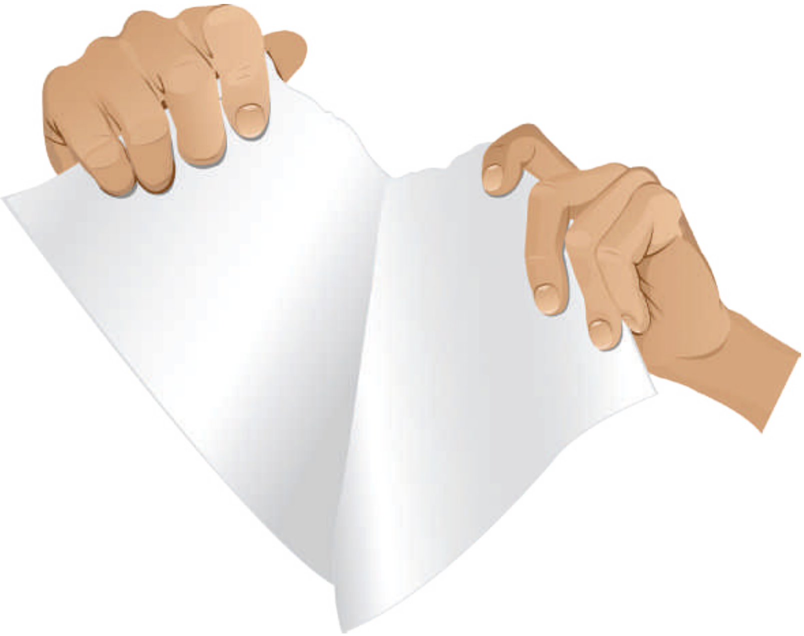
The 'tear off' proof

— 10

10 You can confirm that the angle sum of a quadrilateral is 360° by tearing off the corners of any cut out quadrilateral.

- **a** Use a ruler to draw and cut out any quadrilateral.
- **b** Tear off the four corners.
- **c** Arrange the four pieces so the corners (vertices of quadrilateral) all meet at one point as shown. What do you notice?
- **d** What does this tell you about the angle sum of quadrilaterals?



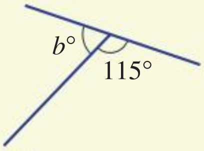


Progress quiz

2A

1 Determine the value of the pronumerals in these diagrams.

a



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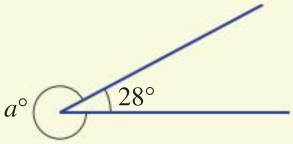
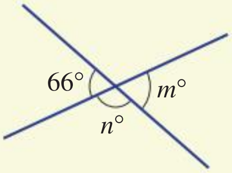
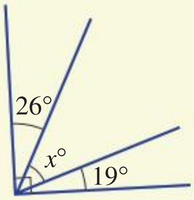


c



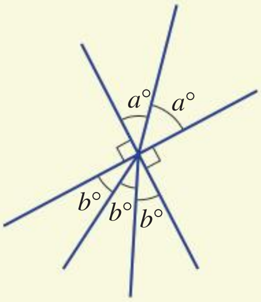
d



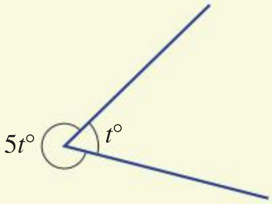
2A

2 Find the value of the pronumerals in these diagrams.

a



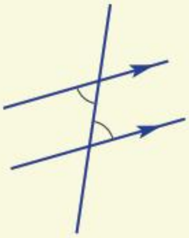
b



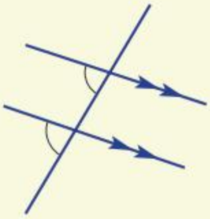
2B

3 Which of the following diagrams have a pair of corresponding angles marked?

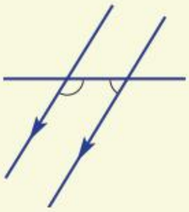
a



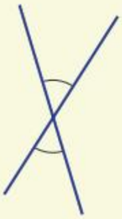
b



c



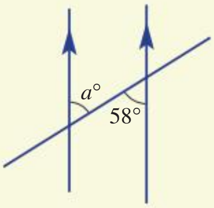
d



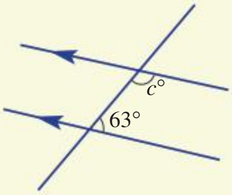
2B

4 Find the value of the pronumerals in these diagrams. Give a reason for each answer.

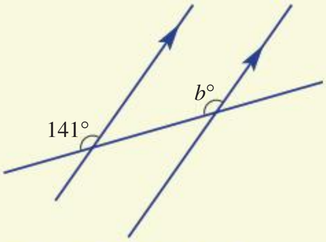
a



b

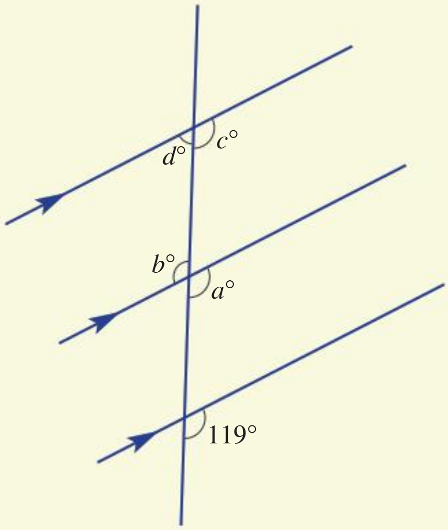


c

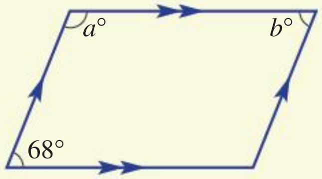


2B **5** Find the value of the pronumerals in these diagrams, stating reasons.

a

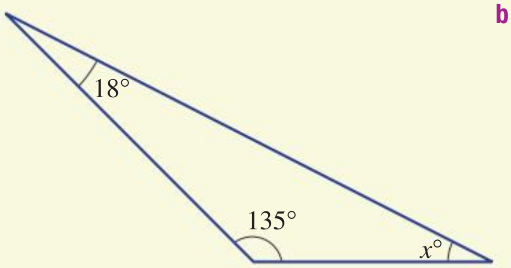


b

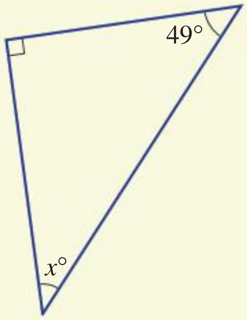


2C **6** Use the angle sum of a triangle to help find the unknown angle in these triangles.

a

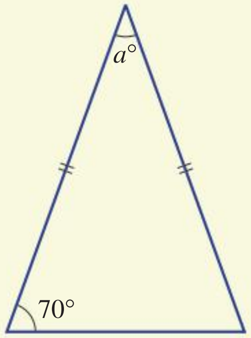


b



2C **7** Find the size of the unknown angles.

a



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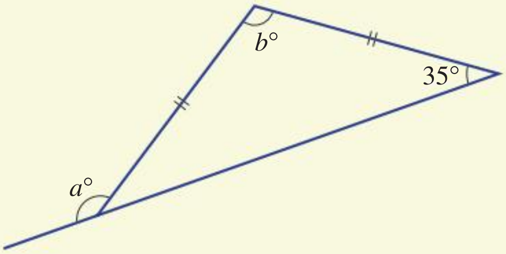
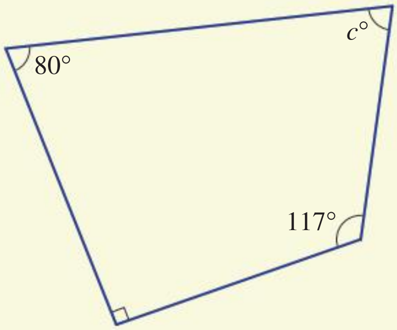


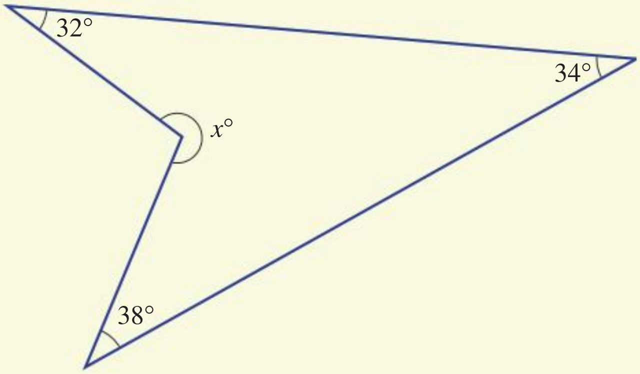
2D

8 Find the value of the pronumerals in these quadrilaterals.

a



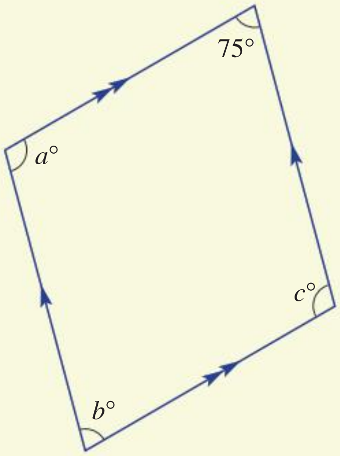
b



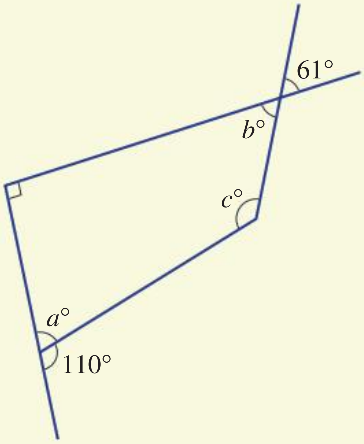
2D

9 Find the value of the pronumerals in these quadrilaterals.

a



b



2D

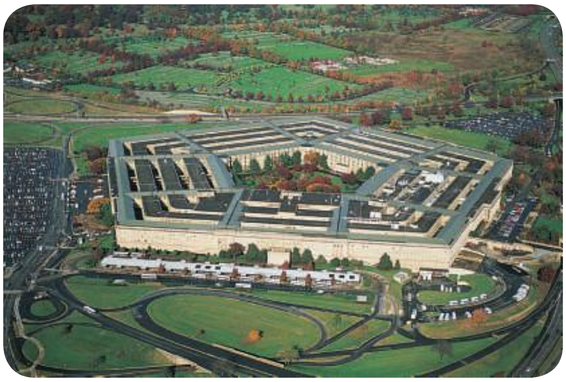
10 Decide if the following are true or false.

- **a** All squares are rectangles.
- **b** All quadrilaterals are rectangles.
- **c** All rectangles are rhombuses.

2E Polygons

Interactive
Widgets
HOTsheets
Walkthrough

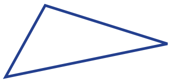
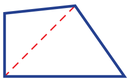
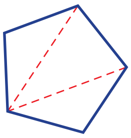
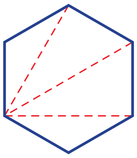
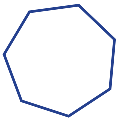
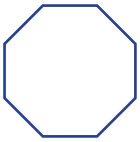
Triangles and quadrilaterals are both examples of polygons. The word ‘polygon’ comes from the Greek words *poly*, meaning ‘many’, and *gonia*, meaning ‘angles’. The number of interior angles equals the number of sides. The angle sum of each type of polygon depends on this number. In this section we will explore the rule for the angle sum of a polygon with n sides.



The Pentagon is a famous government office building in Washington, USA.

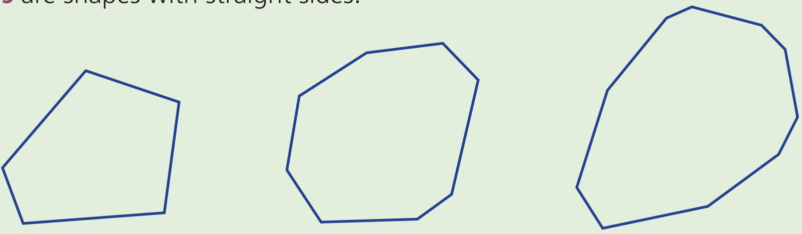
Let’s start: Developing the rule

The following procedure uses the fact that the angle sum of a triangle is 180°. Complete the table and try to write the general rule in the final row.

Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180^\circ = 180^\circ$
Quadrilateral 	4	2	$\_\_\_ \times 180^\circ = \_\_\_$
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		$(\_\_\_) \times 180^\circ$

Key ideas

- **Polygons** are shapes with straight sides.



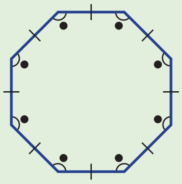
- Polygons are named according to their number of sides.

Number of sides	Name	Angle sum
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1080°
9	Nonagon	1260°
10	Decagon	1440°
11	Undecagon	1620°
12	Dodecagon	1800°

- The angle sum S of a polygon with n sides is given by the rule: $S = (n - 2) \times 180°$.
- A **regular polygon** has sides of equal length and equal interior angles.

Polygon A two-dimensional shape where three or more straight lines are joined together to form a closed figure

Regular polygon A polygon with all sides equal and all angles equal


A regular octagon

Exercise 2E

UNDERSTANDING 1–4 4

1 Name the polygon with the following number of sides.

a 7 **b** 3 **c** 8 **d** 9
e 12 **f** 10 **g** 4 **h** 11

2 State the number of sides on these polygons.

a Hexagon **b** Quadrilateral **c** Decagon
d Heptagon **e** Pentagon **f** Dodecagon

3 Evaluate $(n - 2) \times 180°$ if:

a $n = 6$ **b** $n = 10$ **c** $n = 22$

4 What is the common name given to these polygons?

a Regular quadrilateral **b** Regular triangle

FLUENCY 5–6(½), 7, 8(½) 5–6(½), 7, 8

Example 10 Finding the angle sum

Find the angle sum of a heptagon.

Solution	**Explanation**
$S = (n-2)\times 180^\circ$ $= (7-2)\times 180^\circ$ $= 5\times 180^\circ$ $= 900^\circ$	A heptagon has 7 sides so $n = 7$. Simplify $(7-2)$ before multiplying by 180°.

5 Find the angle sum of these polygons.

a Pentagon $(n = 5)$

b Octagon $(n = 8)$

c Decagon $(n = 10)$

d Hexagon

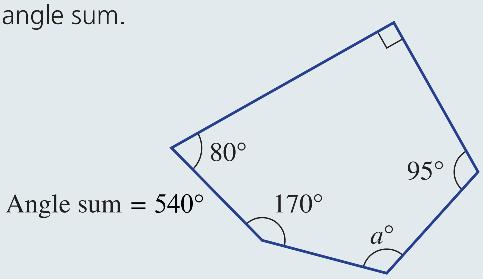
e Nonagon

f Heptagon

Use $S = (n-2)\times 180^\circ$

Example 11 Finding angles in polygons

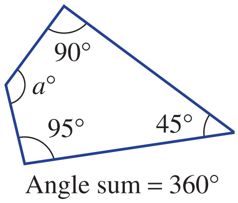
Find the value of a in this pentagon by using the given angle sum.



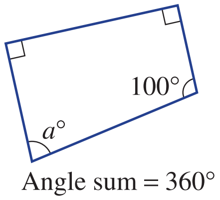
Solution	**Explanation**
$a + 170 + 80 + 90 + 95 = 540$ $a + 435 = 540$ $a = 105$	Sum all the angles and set this equal to the angle sum of 540°. Then simplify and solve for a or use a mental strategy.

6 Find the value of a in these polygons, by using the given angle sum.

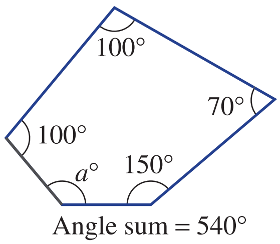
a



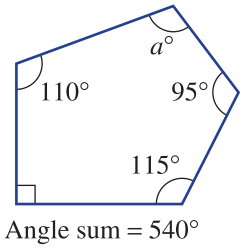
b



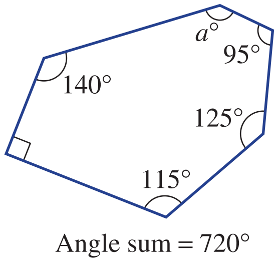
c



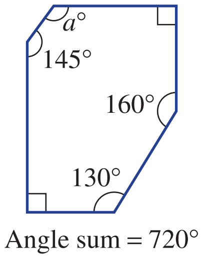
d



e



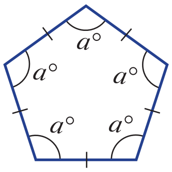
f



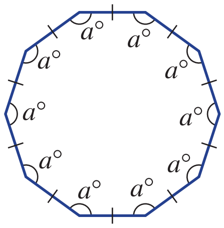
Write an equation using the given angle sum, then find the value of a.

7 Regular polygons have equal interior angles. Find the size of an interior angle for these regular polygons with the given angle sum.

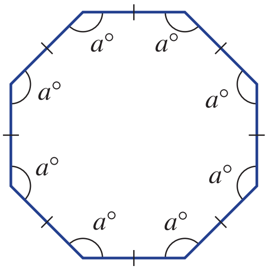
a Pentagon (540°)



b Decagon (1440°)



c Octagon (1080°)



First find the angle sum then divide by the number of sides.

Example 12 Finding interior angles of regular polygons

Find the size of an interior angle in a regular octagon by first finding the angle sum.

Solution	**Explanation**
$S = (n-2) \times 180°$ $= (8-2) \times 180°$ $= 6 \times 180° = 1080°$	First calculate the angle sum of a octagon using $n = 8$ and $S = (n-2) \times 180°$
Angle size $= 1080 \div 8$ $= 135°$	All 8 angles are equal in size so divide the angle sum by 8.

8 Find the size of an interior angle of these regular polygons by firstly finding the angle sum. Round the answer to one decimal place where necessary.

a Regular pentagon

b Regular heptagon

c Regular hexagon

d Regular decagon

e Regular octagon

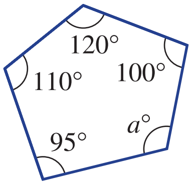
f Regular undecagon

PROBLEM-SOLVING AND REASONING

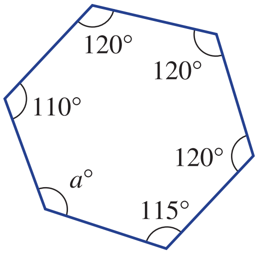
9, 10 | 9–12

9 Find the value of a in these shapes by firstly finding the angle sum.

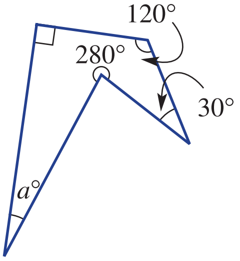
a



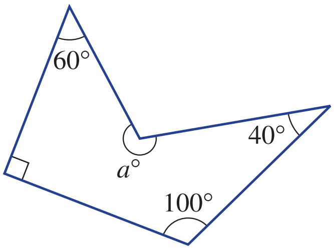
b



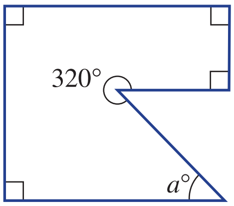
c



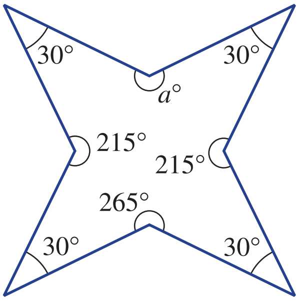
d



e



f



10 Find the number of sides of a polygon with the given angle sums.

a 1260° **b** 2340° **c** 3420° **d** $29\,700^\circ$

The angle sum rule is
$S = (n - 2) \times 180^\circ$

11 Find the number of sides of a regular polygon if each interior angle is:

a 120° **b** 162° **c** $147.272\,727\ldots^\circ$

12 Consider a regular polygon with a very large number of sides (n).

a What shape does this polygon look like?

b Is there a limit to the size of a polygon angle sum or does it increase to infinity as n increases?

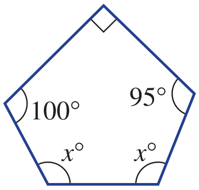
c What size does each interior angle approach as n increases?

Angle sum challenge

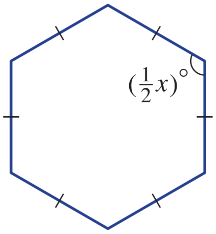
— 13

13 Find the value of x in these diagrams.

a

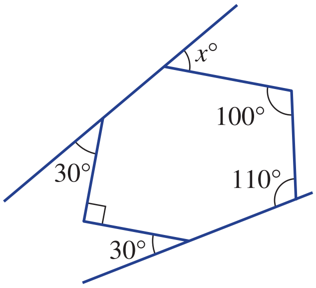


b

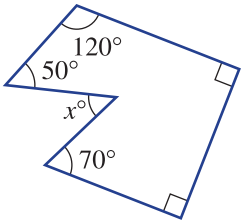


Regular hexagon

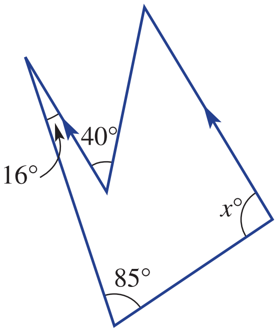
c



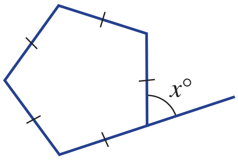
d



e



f



Regular pentagon

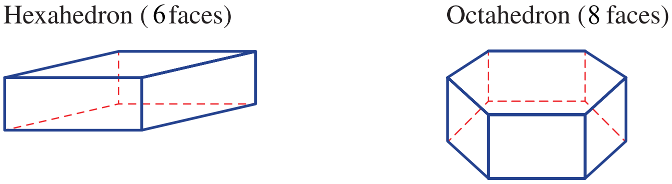
2F Solids

A solid is a an object that occupies three-dimensional space. The outside surfaces could be flat or curved. A solid with all flat surfaces is called a polyhedron, plural *polyhedra* or *polyhedrons*. The word 'polyhedron' comes from the Greek words *poly*, meaning 'many', and *hedron*, meaning 'faces'.


The top of this Canary Wharf building in London (left) is a large, complex polyhedron. Polyhedra also occur in nature, particularly in rock or mineral crystals such as quartz (right).

Let's start: Naming challenge

Solids can be named by their number of faces. Here are two examples.

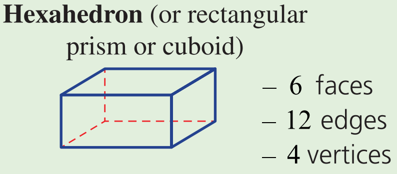


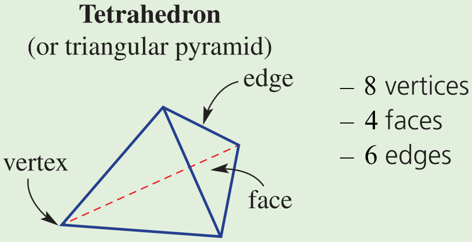
Name and draw a solid with the following number of faces.

- 7 faces
- 5 faces
- 10 faces

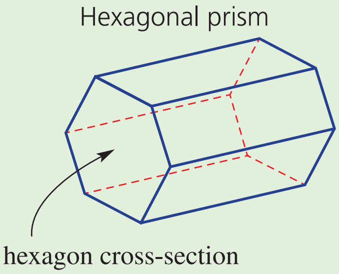
Key ideas

- A **polyhedron** (plural: polyhedra) is a closed solid with flat surfaces (faces), vertices and edges.
  - Polyhedra can be named by their number of faces.

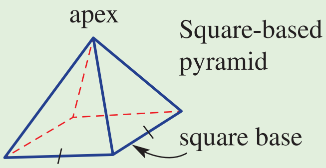




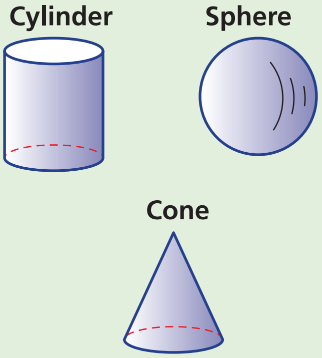
- **Prisms** are polyhedra with two identical (congruent) ends. The congruent ends define the **cross-section** of the prism and also its name. The other faces are parallelograms. If these faces are rectangles, as shown, then the solid is a **right prism.**



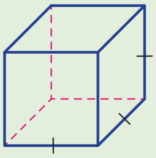
- **Pyramids** are polyhedra with a base face and all other faces meeting at the same vertex point called the apex. They are named by the shape of the base.



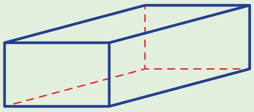
- Some solids have **curved** surfaces. Common examples include:



- A cube is a hexahedron with six square faces.



- Another name for a rectangular prism is cuboid.



Polyhedron
A three-dimensional figure made by joining polygons at their edges

Prism
A solid where each cross-section in a particular direction is exactly the same and all faces are polygons

Pyramid
A solid where the base is a polygon and the other faces are formed by triangles with a common vertex

Natural hexagonal prisms of rock at the Giant's Causeway in Northern Ireland formed when lava cooled quickly and cracked in semi-regular patterns.

Exercise 2F

UNDERSTANDING 1–4 4

1 Write the missing word or number in these sentences. Choose from: *congruent*, *cube*, *six*, *seven*, *vertices*, *octagonal*, *circle*, *seven*.

a A hexahedron has _____________ faces.

b The flat face at the base of a cylinder is a _____________.

c A hexahedron with six square faces is also called a _____________.

d A polyhedron has faces, _____________ and edges.

e A heptahedron has ______ faces.

f A prism has two _____________ ends.

g A pentagonal prism has _____________ faces.

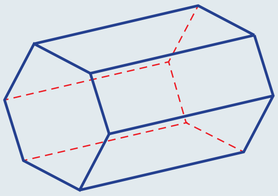
h The base of a pyramid has 8 sides. The pyramid is called an _____________ pyramid.

2 Name three solids that have curved surfaces.

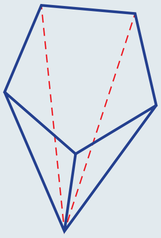
Example 13 Counting faces, vertices and edges

State the number of faces, vertices and edges for these solids.

a Octahedron (or Hexagonal prism)



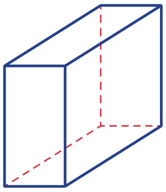
b Hexahedron (or Pentagonal pyramid)



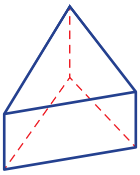
Solution	**Explanation**
a 8 faces	Faces are the flat surfaces.
12 vertices	Vertices are the corners.
18 edges	Edges are the lines on the diagram.
b 6 faces	There is one pentagonal face and five triangular faces.
6 vertices	Five vertices are on the base plus one apex.
10 edges	Five edges are on the base and five meet the apex.

3 Count the number of faces, vertices and edges (in that order) on these polyhedra.

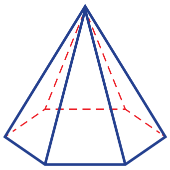
a



b



c



Faces are flat surfaces, vertices are corners and edges are the lines drawn on the diagram.

4 Which of these solids are polyhedra (i.e. have only flat surfaces)?

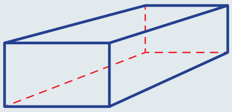
A Cube
B Pyramid
C Cone
D Sphere
E Cylinder
F Rectangular prism
G Tetrahedron
H Hexahedron

FLUENCY 5–7 5–6(½), 7, 8

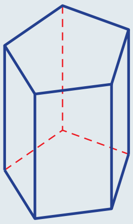
Example 14 Classifying solids using faces

Classify these solids by considering the number of faces.

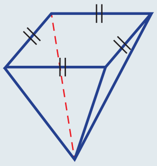
a



b



c



Solution	**Explanation**
a Hexahedron	The solid has 6 faces.
b Heptahedron	The solid has 7 faces.
c Pentahedron	The solid has 5 faces.

5 Name the polyhedron that has the given number of faces.

a 6 **b** 4 **c** 5 **d** 7
e 9 **f** 10 **g** 11 **h** 12

Use names such as pentahedron, hexahedron.

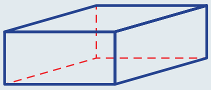
6 How many faces do you think these polyhedra have?

a Octahedron **b** Hexahedron **c** Tetrahedron **d** Pentahedron
e Heptahedron **f** Nonahedron **g** Decahedron **h** Undecahedron

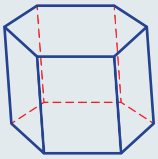
Example 15 Naming prisms

Name these solids as a type of prism.

a



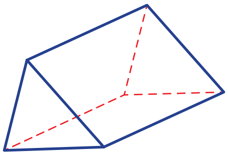
b



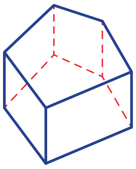
Solution	**Explanation**
a Rectangular prism	The cross-section is a rectangle.
b Hexagonal prism	The cross-section is a hexagon.

7 Name these prisms.

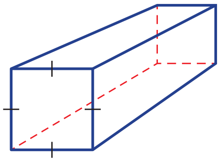
a



b



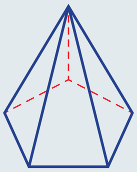
c



Name using the shape of the cross-section.

Example 16 Naming pyramids

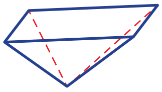
Name this solid as a type of pyramid.



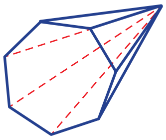
Solution	**Explanation**
Pentagonal pyramid	The base is a pentagon

8 Name these pyramids.

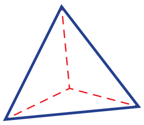
a

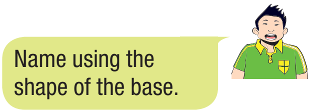


b



c

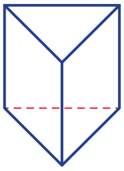




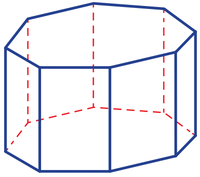
PROBLEM-SOLVING AND REASONING 9, 10 9–12

9 Name each of these solids in two different ways.

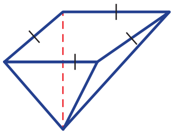
a

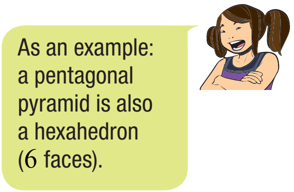


b



c





10 Decide if the following statements are true (T) or false (F). Make drawings to help.

a A tetrahedron is a pyramid.

b All solids with curved surfaces are cylinders.

c A cube and a rectangular prism are both hexahedrons.

d A hexahedron can be a pyramid.

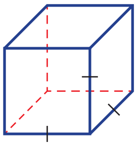
e There are no solids with 0 vertices.

f There are no polyhedra with 3 surfaces.

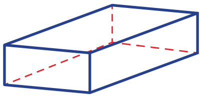
g All pyramids will have an odd number of faces.

11 Name each of these solids in three different ways.

a



b



12 Investigate if this statement is true (T) or false (F).
For all pyramids, the number of faces is equal to the number of vertices.

Euler's rule

— 13, 14

13 **a** Copy and complete this table.

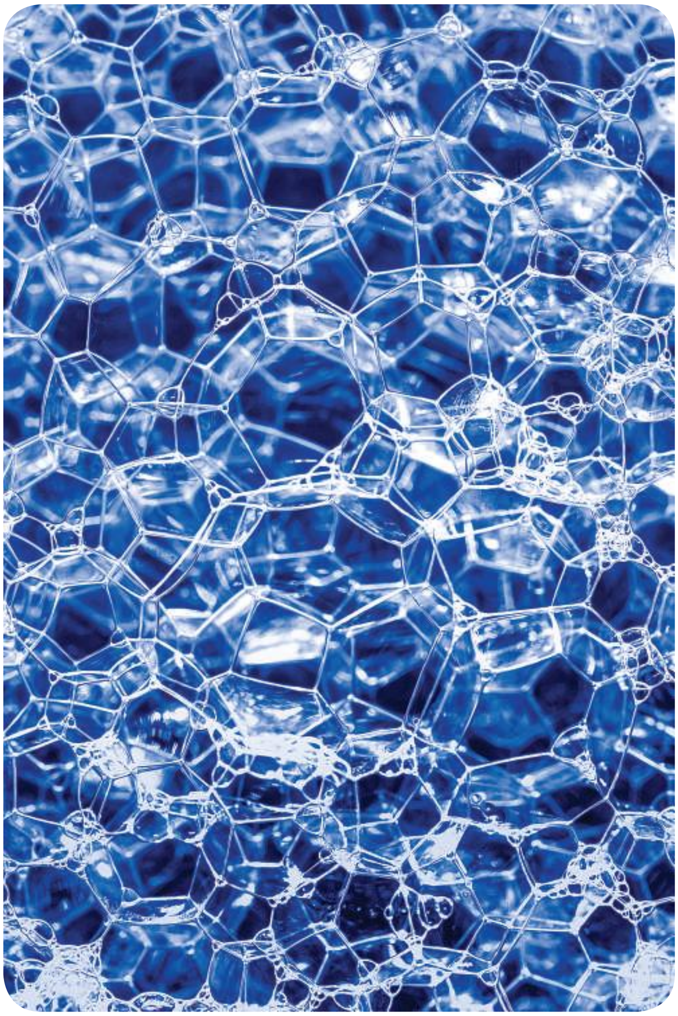
Solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)	$F + V$
Cube				
Square pyramid				
Tetrahedron				

b Compare the number of edges (E) with the value $F + V$ for each polyhedron. What do you notice?

14 **a** A polyhedron has 16 faces and 12 vertices. How many edges does it have?

b A polyhedron has 18 edges and 9 vertices. How many faces does it have?

c A polyhedron has 34 faces and 60 edges. How many vertices does it have?



Maths@Work: Jewellery designer

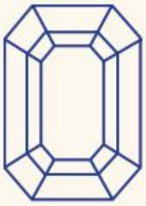
A designer of jewellery is a creative person who needs excellent eyesight, a steady hand and good fine motor skills for their day-to-day work. They like working with materials such as gold, silver and gemstones, and creating something new and interesting. Designers need to understand the chemistry of their materials and the geometry of design. The angles at which gem stones, such as diamonds, are cut can account for price variation and can even be the difference between a beautiful, sparkling diamond or one damaged by a chip or crack.

1 The way that gem stones are cut creates many different designs. The cuts form shapes and angles which contribute to the brightness, sparkle and value of the gem or diamond. Look at the following top views and list all the geometrical shapes that you can see in each design.

a

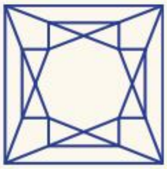
Magna

b



Cabochon

c



Princess

d

Trilliant

e

Standard round

f

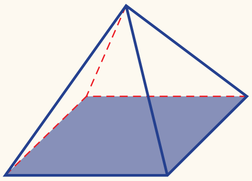
Oval

2 Gemstones and diamonds are made from minerals and crystals which naturally form many different geometrical shapes. Choose the correct name for each solid below from the following list:

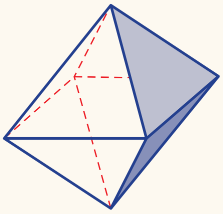
Tetrahedron (4 faces); Icosahedron (20 triangular faces); Heptahedron (7 faces); Octahedron (8 faces); Pentahedron (5 faces); Dodecahedron (12 pentagonal faces).

a

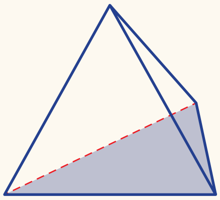
b



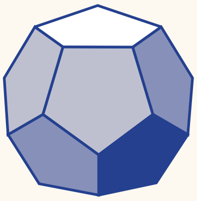
c



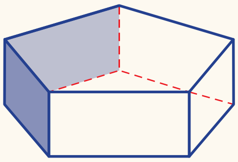
d



e



f

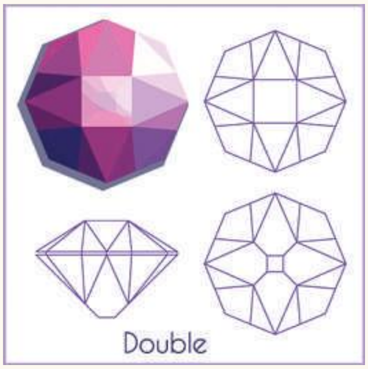


Using technology

3 Use geometry software to digitally draw the top view and a side view of a diamond cut design. You could research diamond cut designs or choose one from the images below.

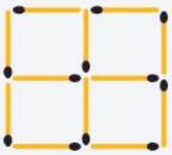

Half dutch rose


Pentagon


Double


English round cut

Puzzles and games



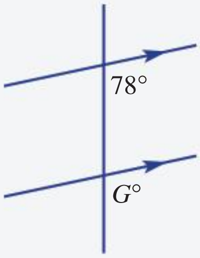
1 This shape includes 12 matchsticks. (To solve these puzzles all matches remaining must connect to other matches at both ends.)

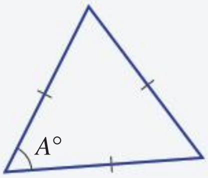
- **a** Remove 2 matchsticks to form 2 squares.
- **b** Move 3 matchsticks to form 3 squares.

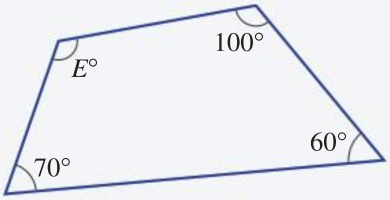
2 **a** Use 9 matchsticks to form 5 equilateral triangles.

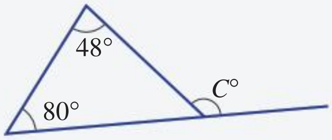
- **b** Use 6 matchsticks to form 4 equilateral triangles.

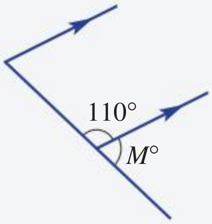
3 Who am I?
I was a female mathematician famous for her work and publications on geometry.
Use your answers to the following to unlock the puzzle code.

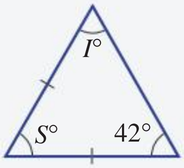


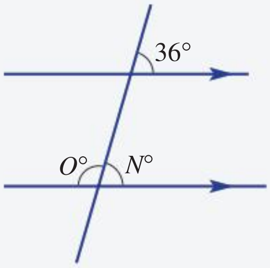


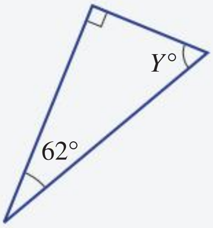


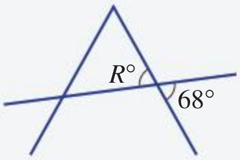


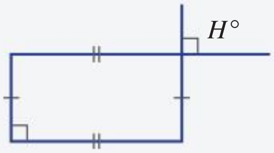


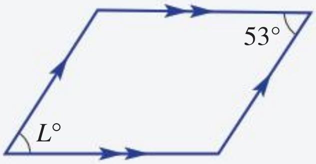


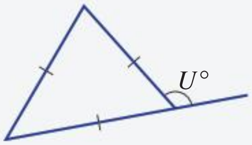












$\overline{78}$ $\overline{68}$ $\overline{60}$ $\overline{128}$ $\overline{130}$ $\overline{128}$ $\overline{90}$ $\overline{42}$ $\overline{96}$ $\overline{90}$ $\overline{144}$ $\overline{53}$ $\overline{70}$ $\overline{28}$ $\overline{144}$ $\overline{120}$ $\overline{36}$ $\overline{78}$

4 Find the angle between the hour and minute hands on a clock at these times.

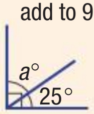
- **a** 11:30 am
- **b** 7:45 pm
- **c** 6:55 pm
- **d** 2:34 am

Chapter summary

Angles at a point

Complementary angles

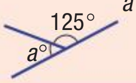
add to 90°



$a + 25 = 90$
$a = 65$

Supplementary angles

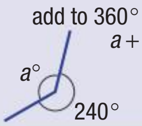
add to 180°



$a + 125 = 180$
$a = 55$

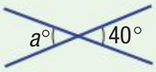
Revolution

add to 360°



$a + 240 = 360$
$a = 120$

Vertically opposite angles

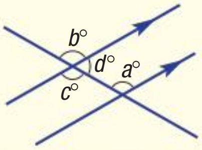


$a = 40$

Lines, shapes and solids

Parallel lines

- Corresponding angles are equal ($a = b$)
- Alternate angles are equal ($a = c$)
- Co-interior angles are supplementary ($a + d = 180$)

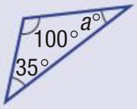


Triangles

(Scalene, Isosceles, Equilateral, Acute, Right, Obtuse)

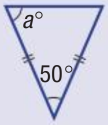
Angle sum = 180°

Scalene



$a + 135 = 180$
$a = 45$

Isosceles



$2a + 50 = 180$
$2a = 130$
$a = 65$

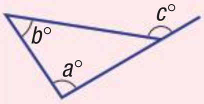
Polygons

$S = (n - 2) \times 180^\circ$
$= (5 - 2) \times 180^\circ$
$= 540^\circ$

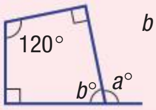
Regular pentagon

$a = \frac{540^\circ}{5} = 108^\circ$

Exterior angles



$c = a + b$



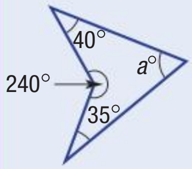
$b + 300 = 360$
$b = 60$
$a = 120$

Quadrilaterals

(Square, Rectangle, Rhombus, Parallelogram, Kite, Trapezium)

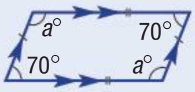
Angle sum = 360°

Quadrilateral



$a + 315 = 360$
$a = 45$

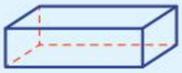
Parallelogram



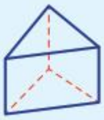
$a + 70 = 180$
$a = 110$

Solids

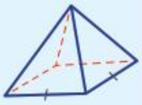
Rectangular prism
(or cuboid or hexahedron)



Triangular prism
(or pentahedron)



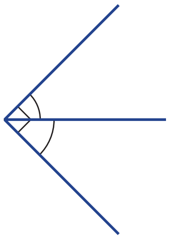
Square pyramid
(or pentahedron)



Multiple-choice questions

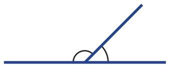
2A **1** What is the name given to two angles that sum to 90°?

A Right **B** Supplementary
C Revolutionary **D** Complementary
E Vertically opposite



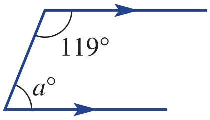
2A **2** Two angles on a straight line add to:

A 180° **B** 90° **C** 45°
D 270° **E** 360°



2B **3** The value of a in this diagram is equal to:

A 45 **B** 122 **C** 241
D 119 **E** 61



2B **4** A pair of alternate angles in parallel lines:

A are not equal **B** are vertically opposite
C are equal **D** are complementary
E are supplementary

2E **5** The rule for the angle sum S of a polygon with n sides is:

A $S = n \times 180^\circ$ **B** $S \times n = 180^\circ$
C $S = (n - 1) \times 180^\circ$ **D** $S = (n - 2) \times 180^\circ$
E $S = (n + 2) \times 180^\circ$

2A **6** The compass bearing for north-east is:

A 90° **B** 45° **C** 305°
D 60° **E** 180°

2E **7** The name given to an eleven-sided polygon is:

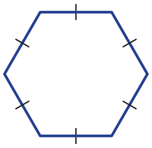
A heptagon **B** elevenagon **C** decagon
D dodecagon **E** undecagon

2E **8** The angle sum (using $S = (n - 2) \times 180^\circ$) of a hexagon is:

A 720° **B** 540° **C** 900°
D 1080° **E** 360°

2E **9** The size of one interior angle of a regular hexagon is:

A 135° **B** 180° **C** 120°
D 720° **E** 108°



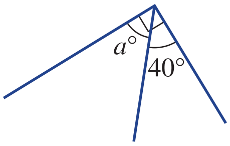
2F **10** How many edges does a rectangular prism have?

A 10 **B** 4 **C** 6
D 12 **E** 8

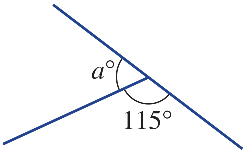
Short-answer questions

2A **1** Find the value of a in these simple diagrams.

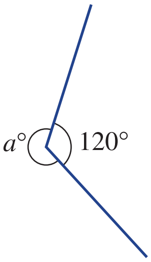
a



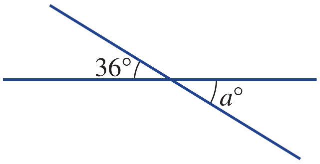
b



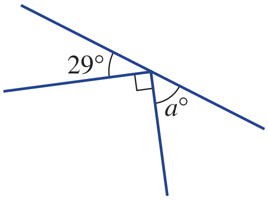
c



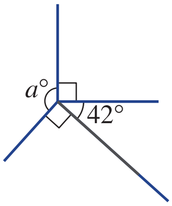
d



e

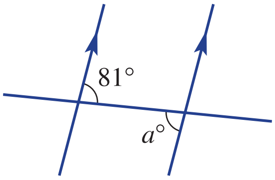


f

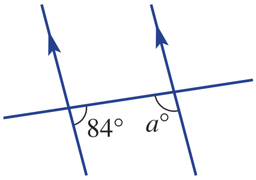


2B **2** These diagrams include parallel lines. Find the value of a.

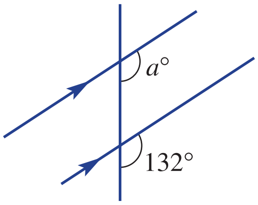
a



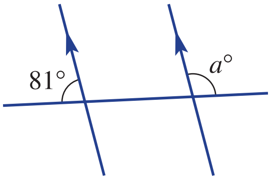
b



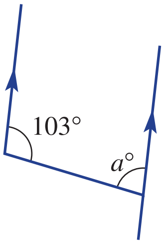
c



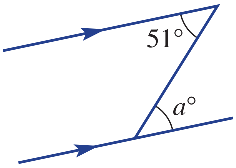
d



e

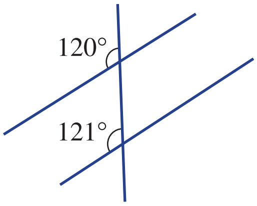


f

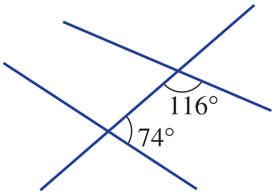


2B **3** Decide if these diagrams include a pair of parallel lines. Give reasons.

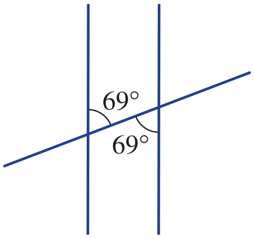
a



b

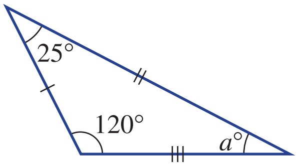


c

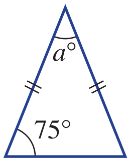


2C **4** Give a name for each triangle and find the value of a.

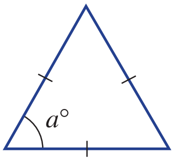
a



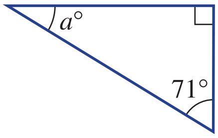
b



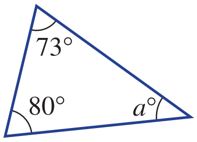
c



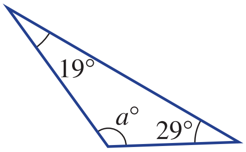
d



e

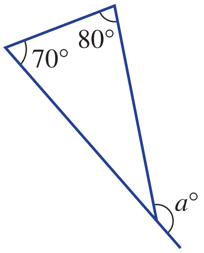


f

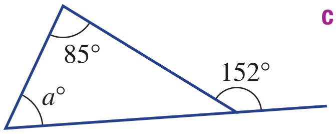


2C **5** These triangles include exterior angles. Find the value of a.

a



b



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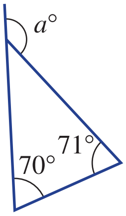
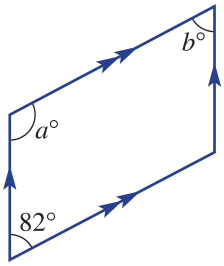


2D **6** Name the quadrilateral(s) which have:

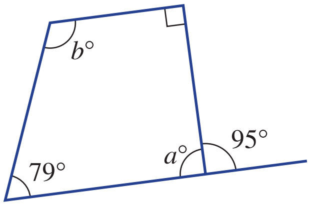
- **a** all sides equal in length.
- **b** one pair of parallel lines.
- **c** two pairs of equal length sides.
- **d** diagonals intersecting at right angles.
- **e** equal length diagonals.

2D **7** Find the value of a and b in these quadrilaterals.

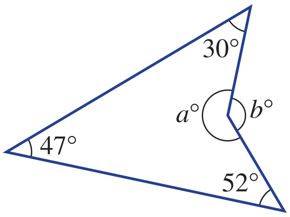
a



b



c



2E **8** Find the angle sum of these polygons using $S = (n - 2) \times 180°$.

a Heptagon **b** Nonagon **c** Dodecagon

2E **9** Find the size of an interior angle of these regular polygons by firstly finding the angle sum.

a Regular pentagon **b** Regular dodecagon

2F

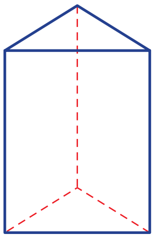
10 Name the polyhedron that has:

a 6 faces **b** 10 faces **c** 11 faces

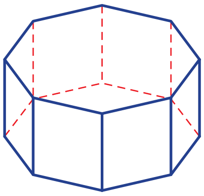
2F

11 What type of prism or pyramid are these solids?

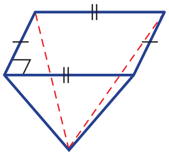
a



b

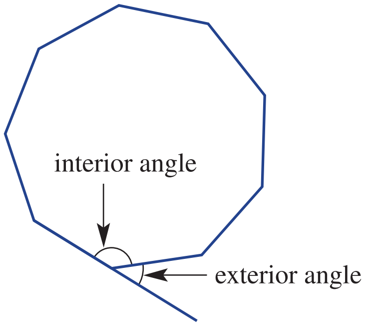


c



Extended-response questions

1 This regular polygon has 9 sides.



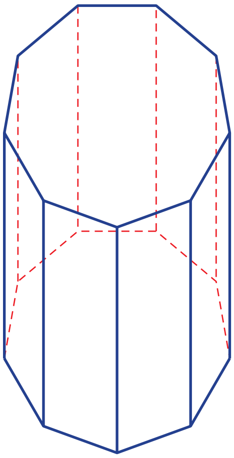
a Find the angle sum using $S = (n-2)\times 180^\circ$.

b Find the size of its interior angles correct to the nearest degree.

c Find the size of its exterior angles correct to the nearest degree.

d The polygon is used to form the ends of a prism. For this prism find the number of:

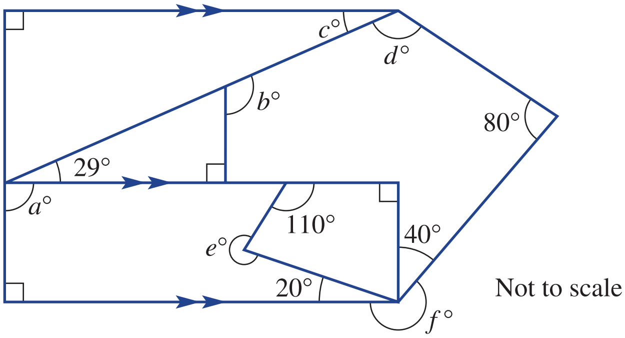
- **i** faces
- **ii** vertices
- **iii** edges.



2 A modern house plan is shown here.

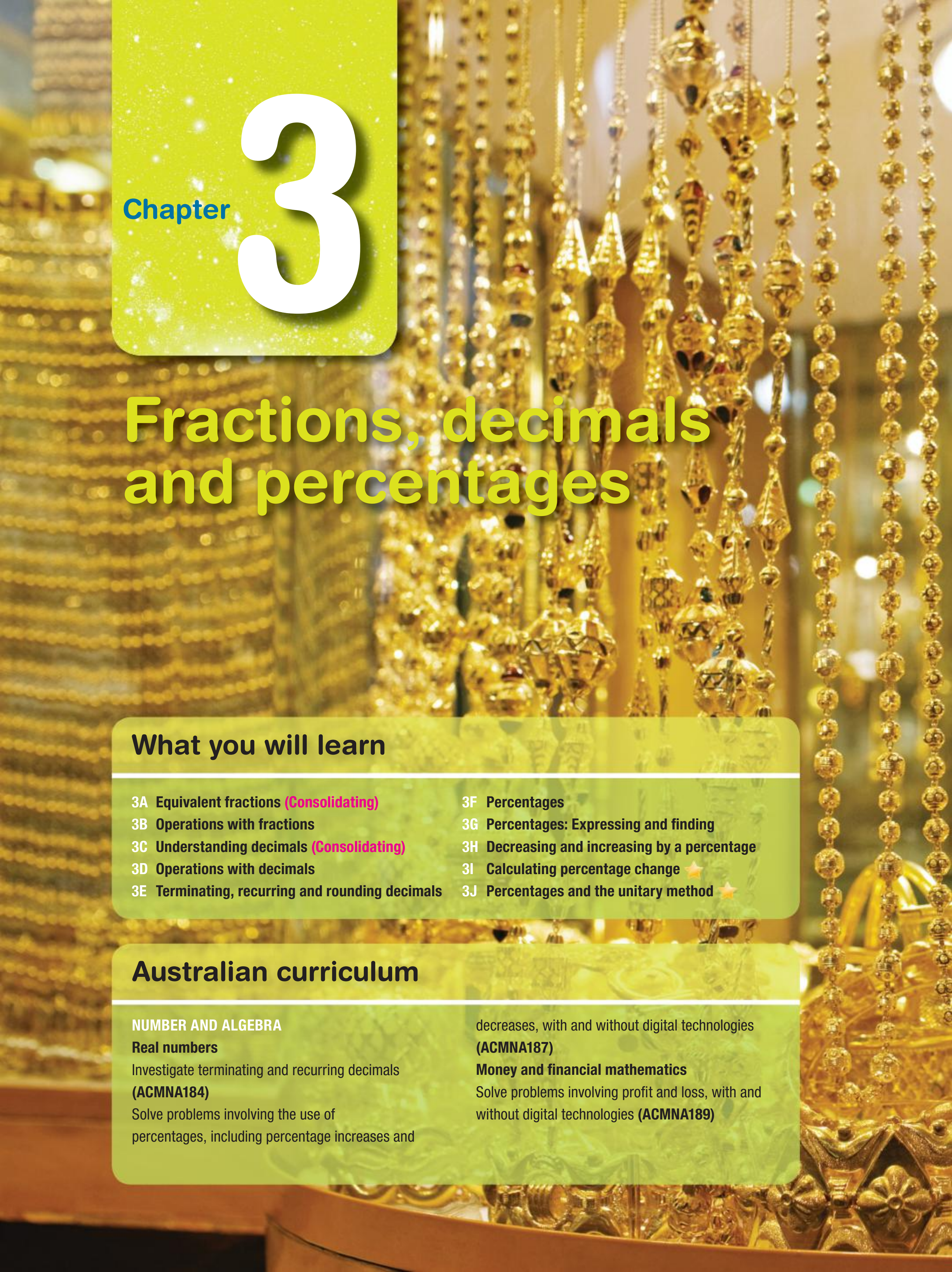
a List the names of at least three different polygons that you see.

b Find the values of the pronumerals a–f.



Hints:

- a is an angle inside a rectangle
- For b, first find the angle next to b inside the small triangle
- c is alternate to 29° inside parallel lines
- d is inside a hexagon, so first use $S = (n - 2) \times 180°$
- e is outside a quadrilateral (angle sum is 360°) and angles in a revolution add to 360°
- Try f without a hint!



Chapter 3

Fractions, decimals and percentages

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

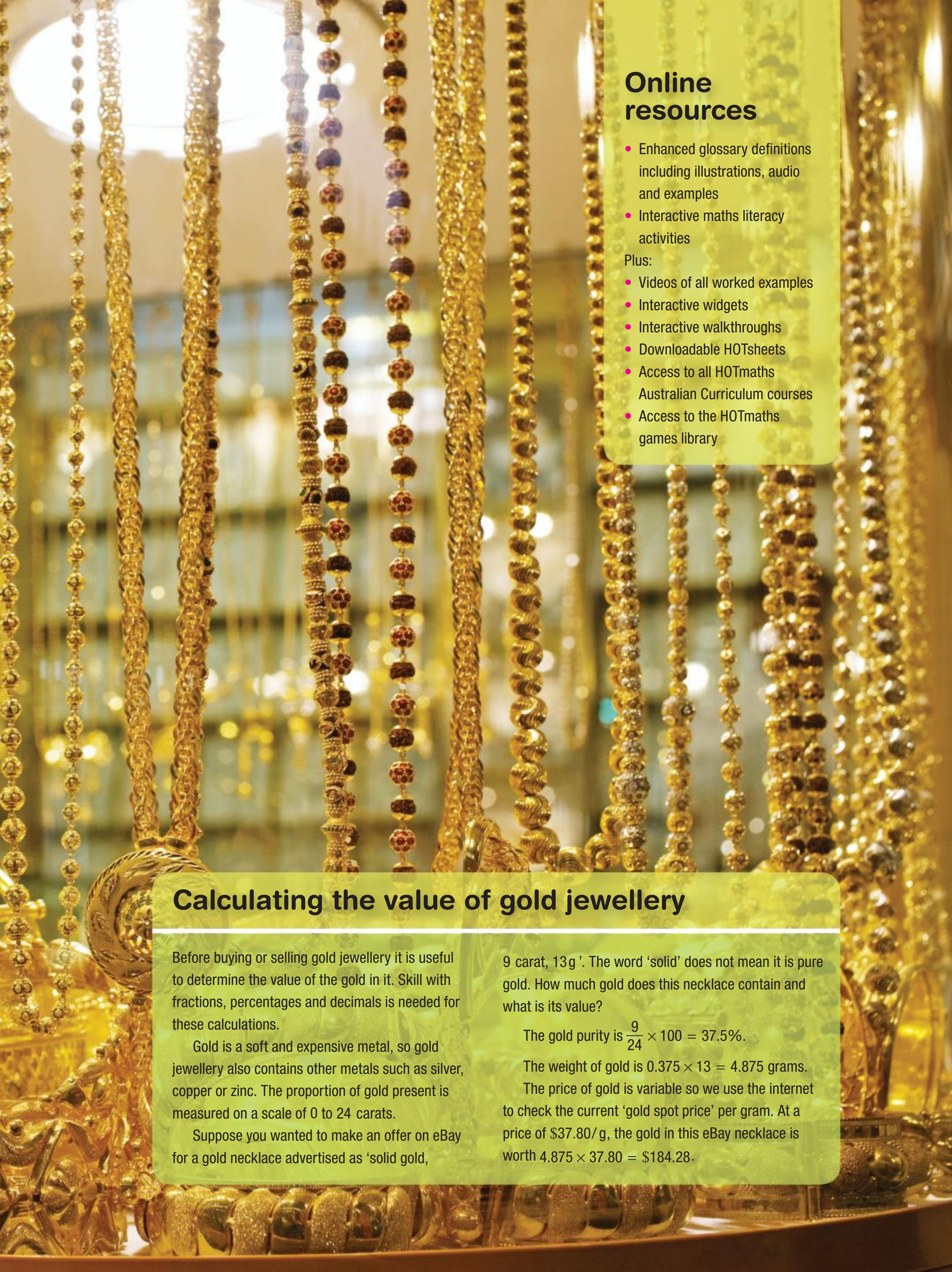
Real numbers

Investigate terminating and recurring decimals **(ACMNA184)**

Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies **(ACMNA187)**

Money and financial mathematics

Solve problems involving profit and loss, with and without digital technologies **(ACMNA189)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Calculating the value of gold jewellery

Before buying or selling gold jewellery it is useful to determine the value of the gold in it. Skill with fractions, percentages and decimals is needed for these calculations.

Gold is a soft and expensive metal, so gold jewellery also contains other metals such as silver, copper or zinc. The proportion of gold present is measured on a scale of 0 to 24 carats.

Suppose you wanted to make an offer on eBay for a gold necklace advertised as 'solid gold, 9 carat, 13 g'. The word 'solid' does not mean it is pure gold. How much gold does this necklace contain and what is its value?

The gold purity is $\frac{9}{24} \times 100 = 37.5\%$.

The weight of gold is $0.375 \times 13 = 4.875$ grams.

The price of gold is variable so we use the internet to check the current 'gold spot price' per gram. At a price of \$37.80/g, the gold in this eBay necklace is worth $4.875 \times 37.80 = \$184.28$.

Pre-test

1 Match the following words to the types of fractions:
A: whole number, B: improper fraction, C: proper fraction, D: mixed number

a $1\frac{2}{5}$ **b** $\frac{3}{7}$ **c** $\frac{10}{2}$ **d** $\frac{7}{4}$

2 How many quarters are in:

a 1 whole? **b** 2 wholes? **c** 5 wholes?

3 Complete the following.

a $1\frac{1}{2} = \frac{\square}{2}$ **b** $2\frac{1}{4} = \frac{\square}{4}$ **c** $1\frac{2}{3} = \frac{5}{\square}$ **d** $1\frac{3}{5} = \frac{\square}{5}$

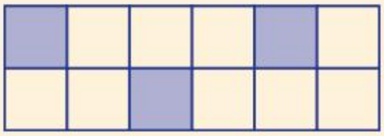
4 Fill in the blanks.

a $\frac{3}{4} = \frac{75}{\square}$ **b** $\frac{3}{6} = \frac{\square}{2}$ **c** $\frac{2}{3} = \frac{\square}{6}$ **d** $\frac{20}{100} = \frac{\square}{5}$

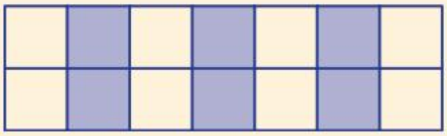
e $\frac{3}{10} = \frac{\square}{100}$ **f** $\frac{3}{5} = \frac{\square}{100}$ **g** $\frac{7}{20} = \frac{35}{\square}$ **h** $\frac{1}{25} = \frac{\square}{100}$

5 What fraction is shaded?

a



b



6 Match the fractions on the left-hand side to their decimal form on the right.

a $\frac{1}{2}$	**A** 3.75
b $\frac{1}{100}$	**B** 0.25
c $\frac{3}{20}$	**C** 0.01
d $3\frac{3}{4}$	**D** 0.5
e $\frac{1}{4}$	**E** 0.15

7 Find:

a $\frac{1}{2} + \frac{1}{4}$ **b** $0.5 + \frac{1}{2}$ **c** $3 - 1\frac{1}{3}$ **d** $0.3 + 0.2 + 0.1$ **e** $2.4 \div 2$ **f** 0.5×6

8 Write as **i** simple fractions **ii** decimals.

a 10% **b** 25% **c** 50% **d** 75%

9 Find 10% of:

a \$50 **b** \$66 **c** 8 km **d** 6900 m

10 Find:

a 25% of 40 **b** 75% of 24 **c** 90% of \$1

11 Copy and complete the following table.

Fraction	$\frac{3}{4}$			$\frac{2}{5}$				2
Decimal		0.2			0.99		1.6	
Percentage			15%			100%		

3A Equivalent fractions

CONSOLIDATING

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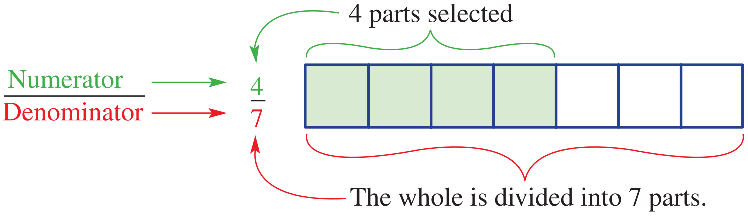
Fractions are made when whole numbers are broken into parts.

This diagram shows the parts of a fraction.

4 → **n**umerator: parts taken from the whole

7 → **d**enominator: number of equal parts the whole is broken into

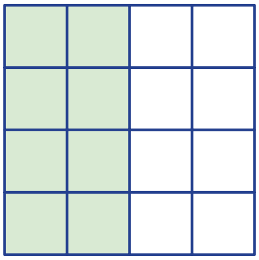
Think ‘**u**’ for ‘**u**p the top’ and ‘**d**’ for ‘**d**own the bottom.



Aerial view of farmland. The paddocks show the farmer’s land divided into parts. Each paddock is a particular fraction of the farmer’s land.

There are 7 equal parts in the whole and 4 of them are shaded.

Equivalent fractions are fractions that represent equal portions of a whole amount and so are equal in value. The skill of generating equivalent fractions is needed whenever you add or subtract fractions with different denominators.



$$\underbrace{\frac{8}{16} = \frac{2}{4} = \frac{1}{2}}_{\text{These equivalent fractions have the same value.}}$$

Fractions are very important whenever we measure or compare. Chefs use them when baking. Builders use them when mixing concrete. Even a musician can use fractions when composing music.

Let’s start: Know your terminology

It is important to know and understand key terms associated with the study of fractions.

As a class give a definition or example of each of the following key terms.

- Numerator
- Equivalent fraction
- Improper fraction
- Multiples
- Lowest common multiple
- Lowest common denominator
- Denominator
- Proper fraction
- Mixed number
- Factors
- Highest common factor

$1\frac{1}{2} = \frac{3}{2}$

mixed number improper fraction

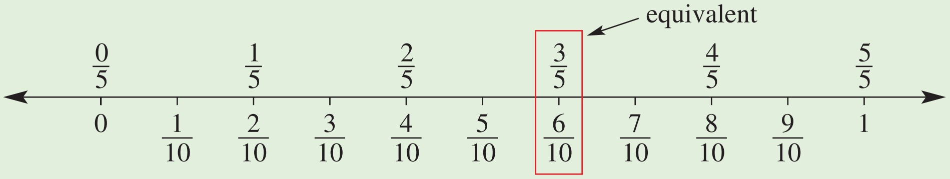
7 ← numerator

3 ← denominator

This is an improper fraction. Do you know why?

Key ideas

- **Equivalent** fractions are equal in value. They mark the same place on a number line. For example, $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent fractions.



- Equivalent fractions are made by multiplying or dividing the numerator *and* denominator by the same number.

e.g. $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$

$$\begin{tikzcd}\frac{2}{7} \arrow[rr, bend left, "\times 5"] \arrow[rr, bend right, "\times 5"'] & = & \frac{10}{35}\end{tikzcd} \qquad \begin{tikzcd}\frac{6}{21} \arrow[rr, bend left, "\div 3"] \arrow[rr, bend right, "\div 3"'] & = & \frac{2}{7}\end{tikzcd}$$

- A fraction can be *cancelled down* (**simplified**) if the top (numerator) *and* bottom (denominator) have a common factor, other than one.

e.g. $\frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{2}{3}$

$$\begin{tikzcd}\frac{12}{18} \arrow[rr, bend left, "\div 6"] \arrow[rr, bend right, "\div 6"'] & = & \frac{2}{3}\end{tikzcd}$$

6 is the HCF of 12 and 18

$\frac{6}{6}$ 'cancels' to 1 because $6 \div 6 = 1$

Equivalent Equal in value

Simplify Find the simplest possible expression for

- The **simplest form** of a fraction is when the numerator and denominator have no common factors other than one.
  - Two fractions are equivalent if they have the same simplest form.

Exercise 3A

UNDERSTANDING 1–4 4

1 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $$\begin{tikzcd}\frac{3}{5} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] \arrow[rrrr, bend left=60, "\times 3"] \arrow[rrrr, bend right=60, "\times 3"'] & = & \frac{\square}{\square} & = & \frac{\triangle}{\triangle}\end{tikzcd}$$

b $$\begin{tikzcd}\frac{4}{7} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{8}{\square} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{\square}{28} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{\square}{\square}\end{tikzcd}$$

c $\frac{50}{100} = \frac{25}{\square} = \frac{10}{\square} = \frac{\square}{10} = \frac{1}{\square}$

d $\frac{1}{3} = \frac{2}{\square} = \frac{3}{\square} = \frac{4}{\square}$

Remember always × or ÷ the top and the bottom by the same number! $\frac{a}{a} = 1$

2 Which of these two fractions is equivalent to $\frac{2}{3}$: $\frac{10}{15}$ or $\frac{3}{4}$?

3 Which of the following fractions can be cancelled down (simplified)?

a $\frac{3}{7}$ **b** $\frac{10}{12}$ **c** $\frac{8}{6}$ **d** $\frac{5}{9}$ **e** $\frac{3}{9}$

4 Are the following statements true (T) or false (F)?

a $\frac{1}{2}$ and $\frac{1}{4}$ are equivalent fractions.

b $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions.

c The fraction $\frac{8}{9}$ is written in its simplest form.

d $\frac{14}{21}$ can be simplified to $\frac{2}{3}$.

e $\frac{11}{99}$ and $\frac{1}{9}$ and $\frac{2}{18}$ are all equivalent fractions.

f $\frac{4}{5}$ can be simplified to $\frac{2}{5}$.

FLUENCY 5–8(½) 5–8(½)

Example 1 Generating equivalent fractions

Rewrite the following fractions with a denominator of 40.

a $\frac{3}{5}$ **b** $\frac{1}{2}$ **c** $\frac{36}{120}$

Solution	**Explanation**
a $\frac{3}{5} = \frac{24}{40}$	$\overset{\times 8}{\underset{\times 8}{\frac{3}{5} = \frac{\square}{40}}}$ Multiply the fraction by $\frac{8}{8}$.
b $\frac{1}{2} = \frac{20}{40}$	$\overset{\times 20}{\underset{\times 20}{\frac{1}{2} = \frac{\square}{40}}}$ Multiply numerator and denominator both by 20.
c $\frac{36}{120} = \frac{12}{40}$	$\overset{\div 3}{\underset{\div 3}{\frac{36}{120} = \frac{\square}{40}}}$ Divide top and bottom both by 3.

5 Write the missing number in these fractions with a denominator of 24.

a $\frac{1}{3} = \frac{\square}{24}$ **b** $\frac{2}{8} = \frac{\square}{24}$ **c** $\frac{1}{2} = \frac{\square}{24}$ **d** $\frac{5}{12} = \frac{\square}{24}$

e $\frac{5}{6} = \frac{\square}{24}$ **f** $\frac{5}{1} = \frac{\square}{24}$ **g** $\frac{3}{4} = \frac{\square}{24}$ **h** $\frac{7}{8} = \frac{\square}{24}$

Multiply or divide top and bottom by the same number.

6 Write the missing number in these fractions with a denominator of 30.

a $\frac{1}{5} = \frac{\square}{30}$ **b** $\frac{2}{6} = \frac{\square}{30}$ **c** $\frac{5}{10} = \frac{\square}{30}$ **d** $\frac{3}{1} = \frac{\square}{30}$

e $\frac{2}{3} = \frac{\square}{30}$ **f** $\frac{22}{60} = \frac{\square}{30}$ **g** $\frac{5}{2} = \frac{\square}{30}$ **h** $\frac{150}{300} = \frac{\square}{30}$

7 Find the missing value to make equivalent fractions.

a $\frac{1}{5} = \frac{\square}{10}$
b $\frac{1}{5} = \frac{\square}{100}$
c $\frac{2}{5} = \frac{4}{\square}$
d $\frac{3}{4} = \frac{\square}{40}$
e $\frac{2}{3} = \frac{12}{\square}$
f $\frac{3}{2} = \frac{6}{\square}$
g $\frac{15}{10} = \frac{\square}{2}$
h $\frac{90}{100} = \frac{\square}{10}$
i $\frac{2}{5} = \frac{\square}{15}$
j $\frac{7}{9} = \frac{14}{\square}$
k $\frac{7}{14} = \frac{1}{\square}$
l $\frac{21}{30} = \frac{\square}{10}$
m $\frac{4}{3} = \frac{\square}{21}$
n $\frac{8}{5} = \frac{80}{\square}$
o $\frac{3}{12} = \frac{\square}{60}$
p $\frac{7}{11} = \frac{28}{\square}$

Example 2 Converting to simplest form

Write the following fractions in simplest form.

a $\frac{8}{20}$
b $\frac{25}{15}$

Solution	**Explanation**
a $\frac{8}{20} = \frac{2 \times \cancel{4}^{1}}{5 \times \cancel{4}_{1}} = \frac{2}{5}$	The HCF of 8 and 20 is 4. $8 = 2 \times 4$ $20 = 5 \times 4$ the HCF of 4 is cancelled in the denominator and numerator.
b $\frac{25}{15} = \frac{5 \times \cancel{5}^{1}}{3 \times \cancel{5}_{1}} = \frac{5}{3}$	The HCF of 25 and 15 is 5. The 5 is 'cancelled' from the numerator and the denominator.

8 Write the following fractions in simplest form.

a $\frac{2}{4}$
b $\frac{3}{6}$
c $\frac{8}{10}$
d $\frac{14}{20}$
e $\frac{3}{9}$
f $\frac{4}{8}$
g $\frac{10}{12}$
h $\frac{15}{18}$
i $\frac{11}{44}$
j $\frac{12}{20}$
k $\frac{16}{18}$
l $\frac{25}{35}$
m $\frac{15}{9}$
n $\frac{22}{20}$
o $\frac{120}{100}$
p $\frac{64}{48}$

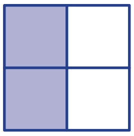
Cancel the HCF of the numerator or denominator.

PROBLEM-SOLVING AND REASONING

9, 10 9–12

9 Each diagram below shows a fraction of the whole. Write each fraction shaded in more than one way.

a



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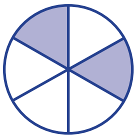
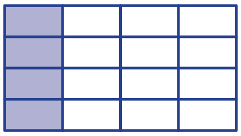
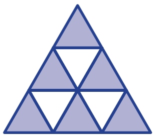


c



d

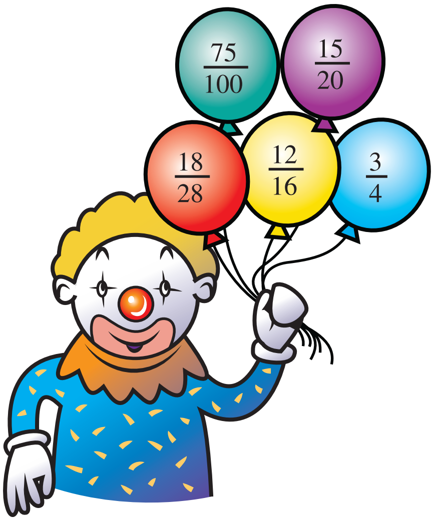


10 **a** Thomas ate $\frac{1}{4}$ of a 250 gram block of chocolate. Mary ate $\frac{3}{6}$ of her 250 gram block. Who ate the most chocolate?

b A pizza is cut into eight equal pieces. Sian ate 2 slices of pizza, Callum had 4 slices. What fraction of the pizza is left?

11 Write down four fractions that when simplified equal $\frac{1}{5}$.

12 Bozzo the Clown is holding five balloons with fractions in them. Which balloon is the odd one out?



Calculate the simplest

— 13

13 Many calculators give the simplified fraction after you press '='. Use a calculator to simplify these fractions.

a $\frac{36}{40}$ **b** $\frac{16}{12}$ **c** $\frac{14}{56}$ **d** $\frac{28}{52}$ **e** $\frac{32}{48}$ **f** $\frac{156}{312}$

3B Operations with fractions

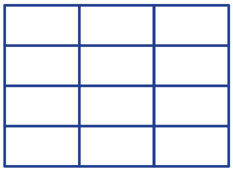
Interactive
Widgets
HOTsheets
Walkthrough

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing fractions.

Proper fractions, improper fractions and mixed numbers will be considered for each of the four mathematical operations.

Let's start: Shading fractions

In pairs, draw and shade this grid to help work out the following fraction questions.



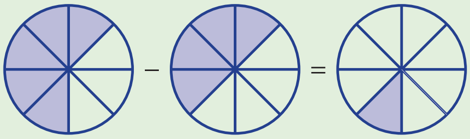
- $\frac{1}{2}+\frac{1}{3}$
- $\frac{7}{12}-\frac{1}{3}$
- $\frac{1}{2}$ of $\frac{1}{2}$

What rules do you know about adding, subtracting, multiplying and dividing fractions?

Key ideas

- Adding and subtracting fractions
  - To add or subtract fractions, convert to the same denominator. When the denominator is the same, just add or subtract the numerator.

e.g.



$$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$$

  - The **lowest common multiple** of the denominators is used if the denominators are different. This is called the **lowest common denominator** (LCD).

e.g. $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$

$=\frac{5}{6}$

Lowest common multiple (LCM) The smallest number that two or more numbers divide into evenly

- Multiplying fractions
  - Convert to improper fractions
  - Multiply the numerators together
  - Multiply the denominators together
  - Simplify your answer

e.g. $1\frac{1}{2}\times\frac{4}{7}=\frac{3}{2}\times\frac{4}{7}$

$=\frac{3\times4}{2\times7}$

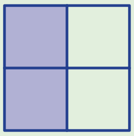
$=\frac{12}{14}$

$=\frac{6}{7}$

- Dividing fractions
  - To divide by a fraction, remember to turn the fraction after the division sign upside down then multiply.

e.g. $\frac{1}{2}\div\frac{1}{4}=\frac{1}{2}\times\frac{4}{1}$

$=2$



There are 2 quarters in one half.

- To turn a fraction $\frac{a}{b}$ upside down to become $\frac{b}{a}$ is called taking its **reciprocal.**

Reciprocals
Two values or terms that multiply to give 1

Exercise 3B

UNDERSTANDING 1–4 4

1 **a** Which two operations require the denominators to be the same before proceeding?
b Which two operations do not require the denominators to be the same before proceeding?

2 State the lowest common denominator for the following pairs of fractions.

a $\frac{1}{5}+\frac{3}{4}$ **b** $\frac{2}{9}+\frac{5}{3}$ **c** $\frac{11}{25}+\frac{7}{10}$ **d** $\frac{5}{12}+\frac{13}{8}$

3 Rewrite the following equations and fill in the empty boxes.

a $\frac{2}{3}+\frac{1}{4}$
$=\frac{8}{12}+\frac{\square}{12}$
$=\frac{11}{\square}$

b $\frac{7}{8}-\frac{9}{16}$
$=\frac{\square}{16}-\frac{9}{16}$
$=\frac{\square}{16}$

c $1\frac{4}{7}\times\frac{3}{5}$
$=\frac{\square}{7}\times\frac{3}{5}$
$=\frac{\square}{35}$

d $\frac{5}{7}\div\frac{2}{3}$
$=\frac{5}{7}\square\frac{3}{2}$
$=\frac{15}{\square}=\square\frac{\square}{14}$

4 State the reciprocal of the following fractions.

a $\frac{5}{8}$ **b** $\frac{3}{2}$ **c** $3\frac{1}{4}$ **d** $1\frac{1}{11}$

Reciprocal means to turn upside down. Convert mixed numbers to improper fractions first!

FLUENCY 5–7(½), 8, 9(½) 5–7(½), 8, 9(½), 10

Example 3 Adding and subtracting fractions

Evaluate.

a $\frac{3}{5}+\frac{4}{5}$ **b** $\frac{5}{3}-\frac{3}{4}$

Solution	**Explanation**
a $\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$ $=1\frac{2}{5}$	The denominators are the same, simply add the numerators. Three **fifths** plus four **fifths** equals seven **fifths**. The final answer can be written as a mixed number.
b $\frac{5}{3}-\frac{3}{4}=\frac{20}{12}-\frac{9}{12}$ $=\frac{11}{12}$	LCM of 3 and 4 is 12. Write equivalent fractions with a LCD of 12. $\frac{5}{3}=\frac{5\times4}{3\times4}, \frac{3}{4}=\frac{3\times3}{4\times3}$ The denominators are now the same, so subtract the numerators.

5 Evaluate.

a $\frac{1}{3}+\frac{1}{3}$

b $\frac{1}{3}+\frac{1}{6}$

c $\frac{7}{12}-\frac{1}{2}$

d $\frac{11}{10}-\frac{7}{10}$

e $\frac{1}{5}+\frac{2}{5}$

f $\frac{7}{9}-\frac{2}{9}$

g $\frac{5}{8}+\frac{7}{8}$

h $\frac{24}{7}-\frac{11}{7}$

i $\frac{3}{4}+\frac{2}{5}$

j $\frac{3}{10}+\frac{4}{5}$

k $\frac{5}{7}-\frac{2}{3}$

l $\frac{11}{18}-\frac{1}{6}$

Convert to a common denominator if required.

Example 4 Adding and subtracting mixed numbers

Evaluate.

a $3\frac{5}{8}+2\frac{3}{4}$

b $2\frac{1}{2}-1\frac{5}{6}$

Solution	Explanation
a $3\frac{5}{8}+2\frac{3}{4}=\frac{29}{8}+\frac{11}{4}$ $=\frac{29}{8}+\frac{22}{8}$ $=\frac{51}{8}=6\frac{3}{8}$	Convert mixed numbers to improper fractions. The LCM of 8 and 4 is 8. Write equivalent fractions with LCD. Add numerators together, denominator remains the same. Convert the answer back to a mixed number if required.
b $2\frac{1}{2}-1\frac{5}{6}=\frac{5}{2}-\frac{11}{6}$ $=\frac{15}{6}-\frac{11}{6}$ $=\frac{4}{6}=\frac{2}{3}$	Convert mixed numbers to improper fractions. The LCD of 2 and 6 is 6. Write equivalent fractions with LCD. Subtract numerators and simplify the answer.

6 Evaluate. Write final answers as mixed numbers.

a $3\frac{1}{7}+1\frac{3}{7}$

b $7\frac{2}{5}+2\frac{1}{5}$

c $3\frac{5}{8}-1\frac{2}{8}$

d $8\frac{5}{11}-7\frac{3}{11}$

e $5\frac{1}{3}+4\frac{1}{6}$

f $17\frac{5}{7}+4\frac{1}{2}$

g $6\frac{1}{2}-2\frac{3}{4}$

h $4\frac{2}{5}-2\frac{5}{6}$

You can add the wholes first if you prefer.

$$3\frac{5}{8}+2\frac{3}{4}=3+2+\frac{5}{8}+\frac{3}{4}$$
$$=5+\frac{5}{8}+\frac{6}{8}$$
$$=5+\frac{11}{8}=6\frac{3}{8}$$

Example 5 Multiplying fractions

Evaluate.

a $\frac{2}{5}\times\frac{3}{7}$

b $\frac{8}{5}\times1\frac{3}{4}$

Solution	**Explanation**
a $\frac{2}{5}\times\frac{3}{7}=\frac{2\times3}{5\times7}$	Multiply the numerators together. Multiply the denominators together.
$=\frac{6}{35}$	The answer is in simplest form.
b $\frac{8}{5}\times1\frac{3}{4}=\frac{\cancel{8}^{2}}{5}\times\frac{7}{\cancel{4}_{1}}$	Use improper fractions, $1\frac{3}{4}=\frac{7}{4}$ Cancel any numerator with any denominator.
$=\frac{14}{5}$	Multiply the numerators $2\times7=14$ Multiply the denominators $5\times1=5$
$=2\frac{4}{5}$	

7 Evaluate.

a $\frac{3}{5}\times\frac{1}{4}$ **b** $\frac{2}{9}\times\frac{5}{7}$

c $\frac{7}{5}\times\frac{6}{5}$ **d** $\frac{5}{3}\times\frac{8}{9}$

e $\frac{4}{9}\times\frac{3}{8}$ **f** $\frac{12}{10}\times\frac{5}{16}$

g $\frac{12}{9}\times\frac{2}{5}$ **h** $\frac{24}{8}\times\frac{5}{3}$

Look to cancel first if possible.

8 Evaluate.

a $2\frac{3}{4}\times1\frac{1}{3}$ **b** $3\frac{2}{7}\times\frac{1}{3}$ **c** $4\frac{1}{6}\times3\frac{3}{5}$ **d** $10\frac{1}{2}\times3\frac{1}{3}$

Convert to improper fractions first.

Example 6 Dividing fractions

Evaluate.

a $\frac{2}{5}\div\frac{3}{7}$

b $2\frac{1}{4}\div1\frac{1}{3}$

Solution	**Explanation**
a $\frac{2}{5}\div\frac{3}{7}=\frac{2}{5}\times\frac{7}{3}$	Change ÷ sign to a × sign and invert the divisor. Multiply by the reciprocal.
$=\frac{14}{15}$	Proceed as for multiplication. Multiply numerators together and multiply denominators together.
b $2\frac{1}{4}\div1\frac{1}{3}=\frac{9}{4}\div\frac{4}{3}$	Convert mixed numbers to improper fractions.
$=\frac{9}{4}\times\frac{3}{4}$	Change ÷ sign to × sign and invert the divisor using the reciprocal. The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$.
$=\frac{27}{16}=1\frac{11}{16}$	Multiply and simplify.

9 Evaluate.

a $\frac{2}{9} \div \frac{3}{5}$

b $\frac{1}{3} \div \frac{2}{5}$

c $\frac{8}{7} \div \frac{11}{2}$

d $\frac{11}{3} \div \frac{5}{2}$

e $\frac{3}{4} \div \frac{6}{7}$

f $\frac{10}{15} \div \frac{1}{3}$

g $\frac{6}{5} \div \frac{9}{10}$

h $\frac{22}{35} \div \frac{11}{63}$

Multiply by the reciprocal.

$\frac{2}{9} \div \frac{3}{5} = \frac{2}{9} \times \frac{5}{3}$

10 Evaluate.

a $1\frac{4}{7} \div 1\frac{2}{3}$

b $3\frac{1}{5} \div 8\frac{1}{3}$

c $3\frac{1}{5} \div 2\frac{2}{7}$

d $6\frac{2}{4} \div 2\frac{1}{6}$

Convert to improper fractions first.

PROBLEM-SOLVING AND REASONING

11 | 11, 12

11 There are 30 students in Miss Mac's Maths class. Find out how many students there are in each of the following groups.

a $\frac{1}{3}$ of the class had brown hair.

b $\frac{1}{2}$ of the class came to school by bus.

c $\frac{5}{6}$ of the class spoke English at home.

d $\frac{1}{10}$ of the class liked Maths.

12 Max and Tanya are painting two adjacent walls of equal area.

Max has painted $\frac{3}{7}$ of his wall and Tanya has painted $\frac{2}{5}$ of her wall.

a What fraction of the two walls have Max and Tanya painted in total?

b What fraction of the two walls remains to be painted?

Multiple operations

— | 13

13 Use a calculator to find the answers to the following.

a **i** $\frac{2}{3} \times \frac{1}{4} \div 1\frac{1}{2}$ **ii** $1\frac{2}{3} + 4\frac{4}{5} - \frac{3}{8}$ **iii** $1\frac{1}{4} \div \frac{2}{3} - \frac{5}{7}$ **iv** $\left(1\frac{1}{2} + 2\frac{2}{3}\right) \times \frac{4}{5}$

b What is the lowest common denominator for:

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$?

3C Understanding decimals

CONSOLIDATING

Interactive

Widgets

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Walkthrough

Decimals also represent 'parts of a whole'. They represent fractions with denominators of 10, 100, 1000… The decimal point is used to separate the whole number and the fraction part.

$$3\frac{17}{100} = 3.17$$

In this section, we go over some decimal concepts from Year 7.

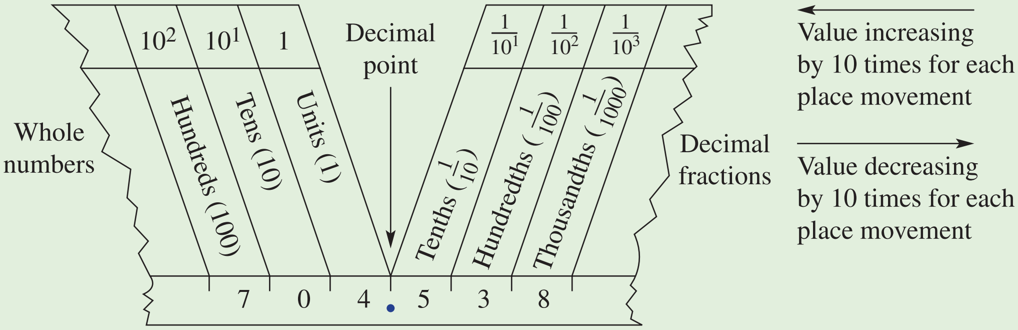
Let's start: Decimals around us

In pairs, list at least five places where decimals are used.

Give a specific example for each one giving a decimal number.

Key ideas

- The **Place value** table is extended for decimals.



$$704.538 = 704\frac{538}{1000} = 7 \times 100 + 4 \times 1 + \frac{5}{10} + \frac{3}{100} + \frac{8}{1000}$$

Place value
The value of a digit in a number as determined by its position

- **Comparing and ordering decimals**
  To compare two decimal numbers with digits in the same place value columns, compare the left-most digits first. Continue comparing digits as you move from left to right until you find two digits that are different.
  e.g. Compare 362.581 and 362.549.

  362.581

  362.549

  the 1st digit that is different: $8 > 4$

  So $362.581 > 362.549$

Converting decimals to fractions

- Count the number of decimal places used.
- This is the number of zeroes that you must place in the denominator.
- Simplify the fraction if required.

e.g. $0.64 = \frac{64}{100} = \frac{16}{25}$

Converting fractions to decimals

- If the denominator is a power of 10, simply change the fraction directly to a decimal from your knowledge of its place value.

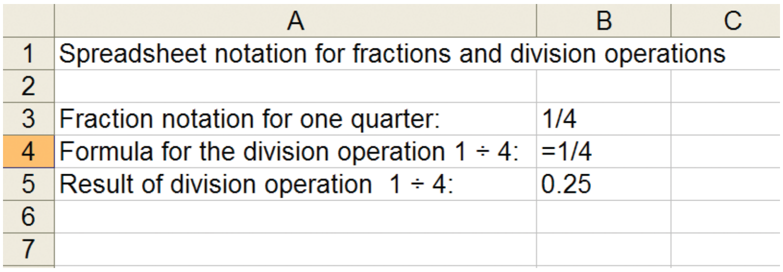
  e.g. $\frac{239}{1000} = 0.239$

- If the denominator is not a power of 10, try to find an equivalent fraction for which the denominator is a power of 10 and then convert to a decimal.

  e.g. $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15$

- If the above two methods are not suitable, divide the bottom (denominator) into the top (numerator).

  e.g. $\frac{1}{8} = \begin{array}{r} 0.\,1\,2\,5 \\ 8\overline{)1.{}^{1}0{}^{2}0{}^{4}0} \end{array}$



	A	B	C
1	Spreadsheet notation for fractions and division operations		
2			
3	Fraction notation for one quarter:	1/4	
4	Formula for the division operation 1 ÷ 4:	=1/4	
5	Result of division operation 1 ÷ 4:	0.25	
6			
7			

A fraction is equivalent to a division operation. In spreadsheets and some calculators fractions and division operations are typed the same way, using a slash (/) between the numbers.

Exercise 3C

UNDERSTANDING 1–4 4

1 Which of the following is the mixed number equivalent of 8.17?

A $8\frac{1}{7}$ **B** $8\frac{17}{10}$ **C** $8\frac{1}{17}$ **D** $8\frac{17}{1000}$ **E** $8\frac{17}{100}$

2 Which of the following is the mixed number equivalent of 5.75?

A $5\frac{75}{10}$ **B** $5\frac{25}{50}$ **C** $5\frac{3}{4}$ **D** $5\frac{15}{25}$ **E** $5\frac{75}{1000}$

Example 7 Comparing decimals

Which is larger?

57.89342 or 57.89631

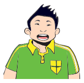
Solution	Explanation
57.89631 is larger.	Write underneath each other. $57.89\textcircled{3}42$ $57.89\textcircled{6}31$ $\uparrow$ 1st digit different from left to right $6 > 3$ $\frac{3}{1000} < \frac{6}{1000}$

3 Write down the larger decimal in each pair.

a 36.485 37.123 **b** 21.953 21.864

c 0.0372 0.0375 **d** 4.21753 4.21809

e 65.4112 64.8774 **f** 9.5281352 9.5281347



Compare digits from left to right.

4 Decide if each of the following statements is true (T) or false (F).

a $\frac{3}{5} = 0.6$ **b** $\frac{11}{20} = \frac{55}{100} = 5.5$ **c** $\frac{1}{100} = 0.01$

d $3.6 < 0.36$ **e** $0.504 > 0.54$ **f** $0.65 < 0.645$

FLUENCY 5–6(½), 7 5–7(½), 8

Example 8 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.725 **b** 5.12

Solution	Explanation
a $\frac{725}{1000} = \frac{29}{40}$	Three decimal places, therefore three zeroes in denominator. $0.725 = 725$ thousandths.
b $5\frac{12}{100} = 5\frac{3}{25}$	Two decimal places, therefore two zeroes in denominator. $0.12 = 12$ hundredths.

5 Convert the following decimals to fractions in their simplest form.

a 0.31 **b** 0.537 **c** 0.815 **d** 0.96

e 5.35 **f** 8.22 **g** 26.8 **h** 8.512

i 0.052 **j** 6.125 **k** 317.06 **l** 0.424

Don't forget to simplify!

Example 9 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{239}{100}$ **b** $\frac{9}{25}$

Solution	**Explanation**
a $\frac{239}{100} = 2\frac{39}{100} = 2.39$	Convert improper fraction to a mixed number. Denominator is a power of 10.
b $\frac{9}{25} = \frac{36}{100} = 0.36$	$\frac{9}{25} = \frac{9 \times 4}{25 \times 4} = \frac{36}{100}$

6 Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $\frac{301}{1000}$ **c** $\frac{45}{100}$ **d** $\frac{6}{10}$

e $\frac{67}{100}$ **f** $\frac{674}{1000}$ **g** $\frac{15}{100}$ **h** $\frac{79}{100}$

i $\frac{7}{10}$ **j** $\frac{17}{10}$ **k** $\frac{118}{100}$ **l** $\frac{41}{1000}$

7 Convert the following fractions to decimals.

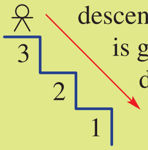
a $\frac{3}{25}$ **b** $\frac{7}{20}$ **c** $\frac{5}{2}$ **d** $\frac{7}{4}$

e $\frac{11}{40}$ **f** $\frac{3}{8}$ **g** $\frac{17}{25}$ **h** $\frac{29}{125}$

Convert to a denominator of 10, 100 or 1000 .

8 Convert the following mixed numbers to decimals and then place them in descending order.

$2\frac{2}{5}, 2\frac{1}{4}, 2\frac{9}{50}, 2\frac{3}{10}$



descending is going down

from largest to smallest

PROBLEM-SOLVING AND REASONING

9, 10 | 9–12

9 The distances from Nam's locker to his six different classrooms are listed below:

- Locker to room B5 (0.186 km)
- Locker to room A1 (0.119 km)
- Locker to room P9 (0.254 km)
- Locker to gym (0.316 km)
- Locker to room C07 (0.198 km)
- Locker to BW Theatre (0.257 km)

List Nam's six classrooms in order of distance of his locker from the closest classroom to the one furthest away.

10 The Prime Minister's approval rating is 0.35, while the Opposition Leader's approval rating is $\frac{3}{5}$. Which leader is ahead in the popularity polls and by how much?

11 Jerome wishes to dig a hole 1.5 metres deep.

Michael has dug a hole $1\frac{3}{4}$ metres deep.

a Whose hole will be the deepest?

b How many centimetres difference is there in the depth of each hole?

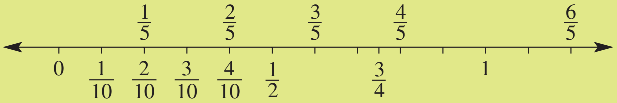
Remember
1 m = 100 cm

12 Write down a *decimal* that lies between the pairs of fractions.

a $\frac{1}{2}$ and $\frac{9}{10}$ **b** $\frac{3}{4}$ and 1

c $\frac{1}{4}$ and $\frac{3}{4}$ **d** $\frac{1}{5}$ and $\frac{7}{10}$

A number line may help



Magic squares

— | 13

13 Complete the following magic squares using a mixture of fractions and decimals.

a

2.6		$1\frac{4}{5}$
	$\frac{6}{2}$	
4.2		

b

0.8	1.8		3.2
3.0		2.0	0.6
2.8			
0.2			2.6

Each row, column and diagonal add to the same number in each magic square. It's the MAGIC number!

3D Operations with decimals

Interactive

Widgets

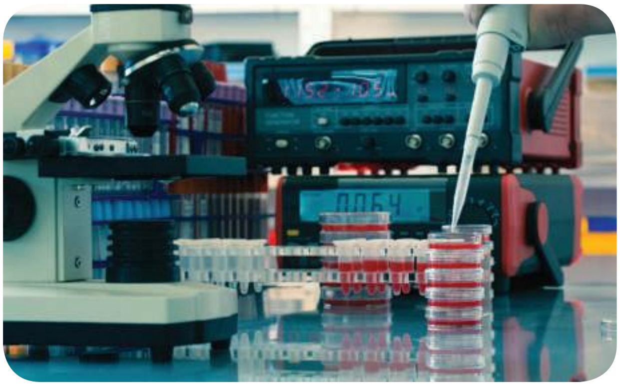
HOTsheets

Walkthrough

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing decimals.

Let's start: Match the phrases

There are seven different sentence beginnings and seven different sentence endings below. Your task is to match each sentence beginning with its correct ending. When you have done this, write the seven correct sentences in your work book.



Precision electronic measuring instruments give a decimal read-out.

Sentence beginnings	Sentence endings
When adding or subtracting decimals	the decimal point moves two places to the right.
When multiplying decimals	the decimal point in the quotient goes directly above the decimal point in the dividend.
When multiplying decimals by 100	make sure you line up the decimal points.
When dividing decimals by decimals	the number of decimal places in the question must equal the number of decimal places in the answer.
When multiplying decimals	the decimal point moves two places to the left.
When dividing by 100	start by ignoring the decimal points.
When dividing decimals by a whole number	we start by changing the question so that the divisor is a whole number.

Key ideas

- **Adding and subtracting decimals**
  - Ensure digits are correctly aligned in similar place value columns.
  - Ensure the decimal points are lined up directly under one another.

$37.56 + 5.231$ $\quad \begin{array}{r} 37.560 \\ \underline{+\ \ 5.231} \end{array}$ ✓ $\quad \begin{array}{l} 37.56 \\ \underline{5.231} \end{array}$ ✗

- **Multiplying and dividing decimals by powers of 10**
  - When multiplying, the decimal point moves to the *right* the same number of places as there are zeroes in the multiplier.

e.g. $13.753 \times 100 = 1375.3$ $\quad 13.7\overset{\frown}{5}\overset{\frown}{3}$

Multiply by 10 twice.

- When dividing, the decimal point moves to the *left* the same number of places as there are zeroes in the divisor.
  e.g. $586.92 \div 10 = 58.692$ $\quad$ 586.92
  Divide by 10 once.

Multiplying decimals

- Initially ignore the decimal points and multiply the wholes.
- Place the decimal point into the answer using the rule:
  'The number of decimal places in the answer must equal the total number of decimal places in the question.'
  e.g. $5.73 \times 8.6 \longrightarrow \begin{array}{r} 573 \\ \times \quad 86 \\ \hline 49278 \end{array} \longrightarrow 5.\underline{73} \times 8.\underline{6} = 49.\underline{278}$
  (3 decimal places in question, 3 decimal places in answer)

Dividing decimals

The decimal point in the quotient goes directly above the decimal point in the **dividend**.

e.g. $56.34 \div 3$

$$\begin{array}{r} 18.78 \longleftarrow \text{Quotient (answer)} \\ \text{Divisor} \longrightarrow 3\overline{)56.34} \longleftarrow \text{Dividend} \end{array}$$

Divisor
The number you are dividing by

Dividend
The number being divided

We avoid dividing decimals by other decimals. Instead we change the **divisor** into a whole number. Whatever change we make to the divisor we must also make to the dividend, so it is equivalent to multiplying by 1 and the value of the question is not changed.

We avoid $\longrightarrow$ $27.354 \div 0.02$ $\qquad$ $27.354 \div 0.02 = 2735.4 \div 2$

Preferring to do $\longrightarrow$ $2735.4 \div 2$

Exercise 3D

UNDERSTANDING 1–4 4

1 Which of the following is the correct set-up for the following addition problem?
$5.386 + 53.86 + 538.6$

A $\begin{array}{r} 5.386 \\ 53.86 \\ +\ 538.6 \\ \hline \end{array}$ **B** $\begin{array}{r} 5.386 \\ 53.860 \\ +538.600 \\ \hline \end{array}$ **C** $\begin{array}{r} 5.386 \\ 53.86 \\ +\quad 538.6 \\ \hline \end{array}$ **D** $\begin{array}{r} 538 + 53 + 5 \\ +\ 0.386 + 0.86 + 0.6 \\ \hline \end{array}$

2 The correct answer to the problem $2.731 \div 1000$ is:
A 2731 **B** 27.31 **C** 2.731
D 0.02731 **E** 0.002731

The key ideas section answers all these questions.

3 If $56 \times 37 = 2072$, the correct answer to the problem 5.6×3.7 is:
A 207.2 **B** 2072 **C** 20.72 **D** 2.072 **E** 0.2072

4 Which of the following divisions would provide the same answer as the division question $62.5314 \div 0.03$?
A $625.314 \div 3$ **B** $6253.14 \div 3$ **C** $0.625314 \div 3$ **D** $625314 \div 3$

FLUENCY | 5–9(½) | 5–9(½), 10

Example 10 Adding and subtracting decimals

Calculate:

a $23.07 + 9.8$ **b** $9.7 - 2.86$

Solution **Explanation**

a
$$\begin{array}{r} 23.07 \\ +\ \ 9.80 \\ \hline 32.87 \end{array}$$

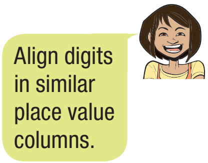
Line up the decimal points. Fill in any zeroes then add vertically.

b
$$\begin{array}{r} {}^{8}9.{}^{16}7\,{}^{1}0 \\ -\ \ 2.\ 8\ 6 \\ \hline 6.\ 8\ 4 \end{array}$$

Align decimal points directly under one another and fill in missing decimal places with zeroes.
Carry out subtraction following the same procedure as for subtraction of whole numbers.

5 Calculate the following.

a $5.6 + 1.2$ **b** $8.4 + 2.1$ **c** $18.6 + 3.3$
d $4.9 + 5.3$ **e** $8.1 + 8.2$ **f** $9.3 + 3.9$
g $23.57 + 39.14$ **h** $64.28 + 213.71$ **i** $5.623 + 18.34$
j $92.3 + 1.872$ **k** $56.3 + 4.41$ **l** $0.61 + 6.1$



6 Calculate the following.

a $5.6 - 1.2$ **b** $8.4 - 2.1$ **c** $18.6 - 3.3$
d $7.9 - 3.8$ **e** $15.6 - 9.5$ **f** $10.4 - 6.4$
g $38.52 - 24.11$ **h** $76.74 - 53.62$ **i** $123.8 - 39.21$
j $14.57 - 9.8$ **k** $96.3 - 4.2$ **l** $85.631 - 5.22$

Example 11 Multiplying and dividing by powers of 10

Calculate:

a $9.753 \div 100$ **b** $27.58 \times 10\,000$

Solution **Explanation**

a $9.753 \div 100 = 0.09753$

Dividing by 100 (2 zeroes), therefore the decimal point must move two places to the left. Additional zeroes are inserted as necessary.

$$.09.753$$

b $27.58 \times 10\,000 = 275\,800$

Multiplying by 10 000 (4 zeroes), therefore the decimal point must move four places to the right. Additional zeroes are inserted as necessary.

$$27.5800.$$

7 Calculate.

a 9.61×10 **b** 9.61×100 **c** 15.463×1000 **d** $19.4 \div 10$
e $19.4 \div 100$ **f** $27.4 \div 10$ **g** 27.4×1000 **h** 1.6×1000
i 36.5173×100 **j** 0.08155×1000 **k** $7.5 \div 10$ **l** $3.812 \div 100$
m $634.8 \times 10\,000$ **n** $1.0615 \div 1000$ **o** $0.003 \times 10\,000$ **p** $0.452 \div 1000$

Move left or right by the number of zeroes.

Example 12 Multiplying decimals

Calculate 25.7×0.3

Solution

$$\begin{array}{r} {}^{1}2{}^{2}57 \\ \times \quad 3 \\ \hline 7\;71 \end{array}$$

$25.7 \times 3 = 7.71$

Explanation

Perform multiplication ignoring the decimal point.
$(257 \times 3 = 771)$
There are two decimal places in the question, so two decimal places in the answer.

Decimal numbers are frequently encountered when dealing with money.

8 Calculate.

a 0.8×7 **b** 0.8×0.7 **c** 15×0.1 **d** 0.4×0.3
e 15.4×2 **f** 1.2×0.3 **g** 0.8×0.4 **h** 0.8×0.04
i 15×0.2 **j** 24.5×0.2 **k** 0.9×9 **l** 1.2×1.2

First ignore the decimal point.

Example 13 Dividing decimals

Calculate:

a $35.756 \div 4$ **b** $64.137 \div 0.03$

Solution

a 8.939

$$\begin{array}{r} 8.\,9\;3\;9 \\ 4\overline{)\,35.{}^{3}7{}^{1}5{}^{3}6} \end{array}$$

Explanation

Carry out division, remembering that the decimal point in the answer is placed directly above the decimal point in the dividend.

Solution

b $64.137 \div 0.03$
$= 6413.7 \div 3$
$= 2137.9$

$$\begin{array}{r} 2\;1\;3\;7.\;9 \\ 3\overline{)\,64{}^{1}1{}^{2}3.{}^{2}7} \end{array}$$

Explanation

Instead of dividing by 0.03, multiply both the divisor and the dividend by 100, to get a whole number, 3.
(Move **each** decimal point two places to the right.) Carry out the division question $6413.7 \div 3$.

9 Calculate.

a $24.54 \div 2$ **b** $17.64 \div 3$ **c** $0.0485 \div 5$ **d** $347.55 \div 7$

e $133.44 \div 12$ **f** $4912.6 \div 11$ **g** $2.58124 \div 8$ **h** $17.31 \div 5$

10 Complete these divisions by filling in the missing numbers.

a $15.6 \div 0.3 = 156 \div 3 = \boxed{}$

b $12.4 \div 0.02 = 1240 \div 2 = \boxed{}$

c $15.06 \div 0.2 = \boxed{} \div 2 = \boxed{}$

d $45.9 \div 0.03 = 4590 \div \boxed{} = \boxed{}$

e $0.484 \div 0.4 = \boxed{} \div 4 = \boxed{}$

PROBLEM-SOLVING AND REASONING

11 11, 12

11 The heights of Mrs Buchanan's five grandchildren are 1.34 m, 1.92 m, 0.7 m, 1.5 m, and 1.66 m. If the grandchildren laid down in a row head to toe, how long would the row be?

12 Look closely at the table of canteen prices below.

Canteen prices		
pie $2.80	chips $1.70	juice $3.40
cola $3.20	chocolate $2.20	sandwich $2.60
sauce $0.60	apple $0.50	milk $1.85

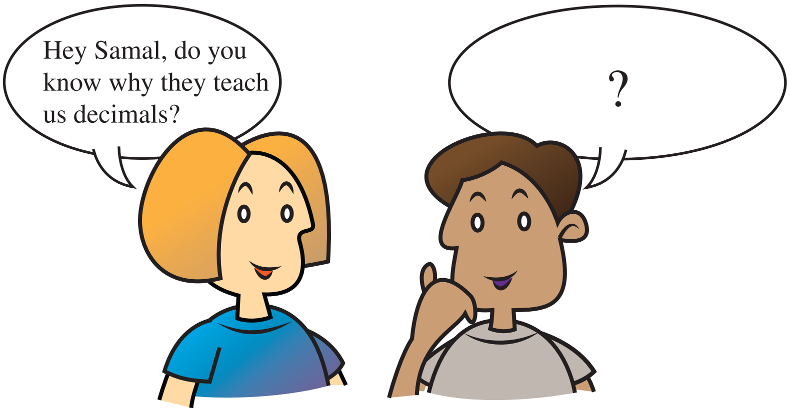
a Find the cost of each person's lunch.

Vaughn	Charlotte	Reece
1 pie	1 sandwich	1 pie
1 sauce	1 chocolate	2 colas
1 apple	1 juice	1 sandwich
2 milks		1 chips

b Who had the most change from $20?

Secret code

— 13



13 Answer each of the 12 questions below to unlock the code and find out how Samal answers Sally's question.

20.7	12.2	4.4

4.4	0.3

12.2	4.75	14.4	12.2

3.2	160

24.2	0.3	12.2	4.75

1.32	160	12.2

12.2	4.75	160

17.97	0.3	20.7	0.72	12.2

!

I $3.2 + 17.5$	O 1.5×0.2	H $47.5 \div 10$	E $96 \div 0.6$
A 1.2×12	T $15.8 - 3.6$	N 0.9×0.8	B $5.9 + 18.3$
W $9.6 \div 3$	P $18.57 - 0.6$	S $9 - 4.6$	G $1.2 + 0.12$

3E Terminating, recurring and rounding decimals

Interactive
Widgets
HOTsheets
Walkthrough

Not all fractions convert to the same type of decimal.
For example:

$\frac{1}{2} = 1 \div 2 = 0.5$ (only has one decimal place)

$\frac{1}{3} = 1 \div 3 = 0.33333\ldots$ (keeps going and going)

$\frac{1}{7} = 1 \div 7 = 0.142857\ldots$ (the pattern repeats)

Decimals that stop (or terminate) are known as terminating decimals, whereas decimals that continue on forever with some form of pattern are known as repeating or recurring decimals.

The fraction of space occupied by each solar panel could be written using a type of decimal.

Let's start: Decimal patterns

Use a calculator to perform these divisions. Can you see a pattern?

- $\frac{1}{9} = 1 \div 9 = 0.1111\ldots\ldots$
- $\frac{2}{9}$
- $\frac{3}{9}$
- $\frac{4}{9}$

Without your calculator, write down $\frac{5}{9}$ and $\frac{6}{9}$ as decimals. What do we call these types of decimals?

Key ideas

- A **terminating decimal** has a fixed number of decimal places (i.e. it terminates).

  e.g. $\frac{5}{8} = 5 \div 8 = 0.625$ $\begin{array}{r} 0.\ 6\ \ 2\ \ 5 \\ 8\overline{)5.{}^{5}0{}^{2}0{}^{4}0} \end{array}$ ← Terminating decimal (3 decimal places only)

- A **recurring decimal** (or **repeating decimal**) keeps going and the decimal places repeat.

  e.g. $\frac{1}{3} = 1 \div 3 = 0.333\ldots$ $\begin{array}{r} 0.\ 3\ 3\ 3\ldots \\ 3\overline{)1.{}^{1}0{}^{1}0{}^{1}0{}^{1}0} \end{array}$ ← Recurring decimal

- A convention is to use dots placed above the digits to show the start and finish of a repeating cycle of digits.

  e.g. $0.55555\ldots = 0.\dot{5}$ and $0.3412412412\ldots = 0.3\dot{4}1\dot{2}$

- Another convention is to use a horizontal line placed above the digits to show the repeating cycle of digits.

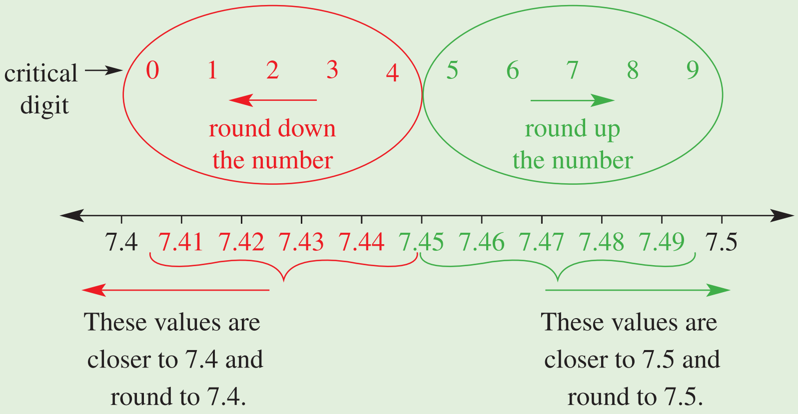
  e.g. $0.55555\ldots = 0.\overline{5}$ and $0.3412412412\ldots = 0.3\overline{412}$

Terminating decimal A decimal that contains a fixed number of digits

Recurring decimal A decimal in which a single digit, or a group of digits, is endlessly repeated

- **Rounding decimals**

  Decimals can be written with fewer decimal places by rounding.
  To round we must look at the digit immediately after the number of places we want.
  (It's the *critical* digit!)



Exercise 3E

UNDERSTANDING 1–4 4

1 State whether the following are terminating decimals (T) or recurring decimals (R).

a 5.47 **b** $3.1541\dot{5}\ldots$ **c** $8.\dot{6}$ **d** 7.1834

e 0.333 **f** $0.\dot{5}3\dot{4}$ **g** 0.5615 **h** $0.32727\ldots$

2 Express the following recurring decimals using the convention of dots or a bar to indicate the start and finish of the repeating cycle.

a $0.33333\ldots$ **b** $6.21212121\ldots$

c $8.5764444\ldots$ **d** $2.135635635\ldots$

e $11.2857328573\ldots$ **f** $0.003523523\ldots$

Write $0.7555\ldots$ as $0.7\dot{5}$.

3 Write down the 'critical' digit (the digit immediately after the rounding digit) for each of the following.

a 3.5724 (rounding to 3 decimal places)

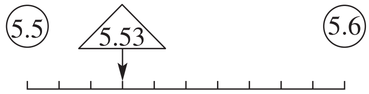
b 15.89154 (rounding to 1 decimal place)

c 0.004571 (rounding to 4 decimal places)

d 5432.726 (rounding to 2 decimal places)

4 For each line given, which circled decimal is the decimal in the triangle closest to?

a



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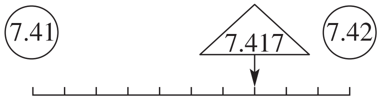
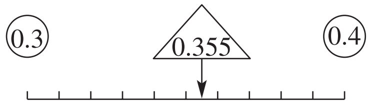
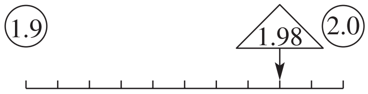


c



d



FLUENCY 5–8(½) 5–8(½), 9

Example 14 Writing terminating decimals

Convert the following fractions to decimals.

a $\frac{1}{4}$ **b** $\frac{7}{8}$

Solution	Explanation
a $\frac{1}{4} = 0.25$	$\begin{array}{r} 0.\ 2\ \ 5 \\ 4\overline{)\,1.\,{}^{1}0\,{}^{2}0} \end{array}$ Write 1 as 1.00. Divide the bottom (denominator) into the top (numerator).
b $\frac{7}{8} = 0.875$	$\begin{array}{r} 0.\ 8\ \ 7\ \ 5 \\ 8\overline{)\,7.\,{}^{7}0\,{}^{6}0\,{}^{4}0} \end{array}$ Write 7 as 7.000. Divide the bottom (denominator) into the top (numerator).

5 Convert the following fractions to decimals.

a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{1}{8}$ **d** $\frac{11}{20}$

e $\frac{1}{2}$ **f** $\frac{4}{5}$ **g** $\frac{1}{25}$ **h** $\frac{9}{50}$

These are all terminating decimals.

Example 15 Writing recurring decimals

Express the following fractions as recurring decimals.

a $\frac{2}{3}$ **b** $3\frac{5}{7}$

Solution	Explanation
a $\frac{2}{3} = 0.\dot{6}$	$\begin{array}{r} 0.\ 6\ \ 6... \\ 3\overline{)\,2.\,{}^{2}0\,{}^{2}0\,{}^{2}0} \end{array}$ This pattern continues, it is a repeating decimal.
b $3\frac{5}{7} = 3.\dot{7}1428\dot{5}$ **or** $3.\overline{714285}$	$\begin{array}{r} 0.\ 7\ \ 1\ \ 4\ \ 2\ \ 8\ \ 5\ \ 7... \\ 7\overline{)\,5.\,{}^{5}0\,{}^{1}0\,{}^{3}0\,{}^{2}0\,{}^{6}0\,{}^{4}0\,{}^{5}0\,{}^{1}0} \end{array}$ This pattern continues.

6 Express the following fractions as recurring decimals.

a $\frac{1}{3}$ **b** $\frac{5}{9}$ **c** $\frac{5}{6}$ **d** $\frac{7}{9}$

e $\frac{3}{7}$ **f** $\frac{1}{6}$ **g** $\frac{4}{3}$ **h** $1\frac{6}{7}$

Remember to use the repeating notation. $0.444\ldots = 0.\dot{4}$.

Example 16 Rounding decimals

a Round 14.258 to 1 decimal place.

b Round 0.671 to 2 decimal places.

Solution	Explanation
a 14.3	14.2⑤8 1 decimal place, look at next digit (5). Critical digit is 5. Round up 14.258 ≈ 14.3
b 0.67	0.67①2 decimal places, look at the next digit (1). Critical digit is 1. Round down 0.671 ≈ 0.67.

7 Round each of the following decimals to one decimal place.

a 0.57 **b** 0.83 **c** 1.49

d 8.16 **e** 9.47 **f** 8.33

g 1.487 **h** 3.444 **i** 0.333

The first decimal place is also called the tenths column.

8 Write each of the following decimals correct to 2 decimal places (or the nearest hundredth).

a 0.783 **b** 0.666 **c** 1.478

d 0.893 **e** 15.488 **f** 9.035

g 9.4163 **h** 8.7499 **i** 1.7891

9 **a** Choose the correct answer to each of the following.

- **i** Is 7.9 closer to 7 or 8?
- **ii** Is 7.99 closer to 7.9 or 8.0?
- **iii** Is 4.96 closer to 4.9 or 5.0?

b Round the following to 1 decimal place.

i 4.96 **ii** 8.941 **iii** 5.999

PROBLEM-SOLVING AND REASONING

10, 11 | 10–12

10 Copy and complete this table for recurring decimals.

Fraction	Decimal	One dec pl	Two dec pl
$\frac{1}{3}$	0.333 …	0.3	0.33
$\frac{2}{3}$			
$\frac{5}{9}$			
$\frac{1}{6}$			

11 Simone and Greer are two very good junior sprinters. Simone ran 100 m in 12.83 seconds, while Greer ran it in 12.77 seconds.

a Who came first, and by how much?

b Round each time to one decimal place. Can you still decide who came first?

12 Write down three different decimals that when written to two decimal places become 3.45.

Calculators and rounding

—	13

13 **a** Find out how your calculator rounds. Use it to round each of the following decimals to the number of decimal places given in the brackets.

i 0.76581 (3)	**ii** 9.4582 (1)	**iii** 6.9701 (1)	**iv** 21.513426 (4)
v 0.9457 (2)	**vi** 17.26 (0)	**vii** 8.5974 (2)	**viii** 8.10552 (3)

b Petrol is sold at 147.9 cents per litre. Find, correct to the nearest cent, the cost of:

- **i** 1 litre
- **ii** 5 litres
- **iii** 12 litres
- **iv** 39.7 litres

c Write the number phi, $1.61803399\ldots$, correct to:

- **i** three decimal places
- **ii** two decimal places
- **iii** one decimal place.

3F Percentages

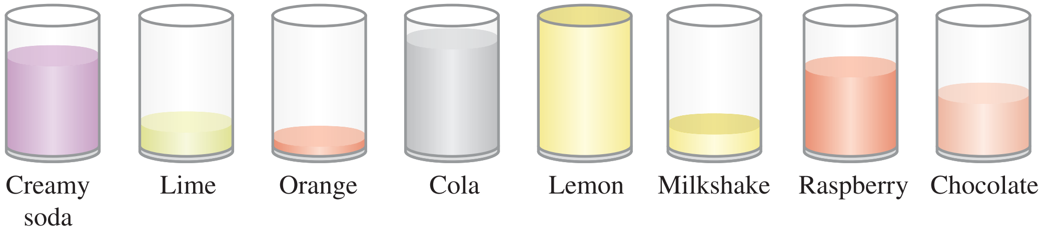
Interactive
Widgets
HOTsheets
Walkthrough

A **percentage** is a particular fraction where the denominator is always 100.
Per cent is Latin for ‘out of 100’.

$$\boxed{7\% = 7 \text{ per cent} = 7 \text{ out of } 100 = \frac{7}{100} = 0.07}$$

People use percentages everyday in banking, sales and school tests.

Let’s start: Estimating percentages



- List the drinks in order from the most to the least amount left in the glass.
- Estimate the percentage of drink remaining in each of the glasses shown above.
- Discuss your estimates with a partner.

Key ideas

- A per cent sign (%) means ‘on one hundred’.

  $23\% = \frac{23}{100}$

- Percentages can be converted to fractions and decimals.

  e.g. $35\% \rightarrow \frac{35}{100} = \frac{7}{20}$ (fraction)

  $35\% \rightarrow 35 \div 100 = 0.35$ (decimal)

- Fractions and decimals can be converted to percentages.

  e.g. $\frac{1}{4}$ or 0.25 as a percentage $\frac{1}{4} \times 100 = 25$ and $0.25 \times 100 = 25$

  so $\frac{1}{4} = 0.25 = 25\%$

  Common percentages and their equivalent fractions are shown in the table below. It is helpful to know these.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	1
Decimal	0.5	$0.\dot{3}$	0.25	0.2	0.125	$0.\dot{6}$	0.75	1
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	$12\frac{1}{2}\%$	$66\frac{2}{3}\%$	75%	100%

Exercise 3F

UNDERSTANDING 1–4 4

1 The fraction equivalent of 27% is:

A $\frac{2}{7}$ **B** $\frac{27}{100}$ **C** 2700 **D** $\frac{13.5}{10}$

Remember if you see a % sign it means 'out of 100'.

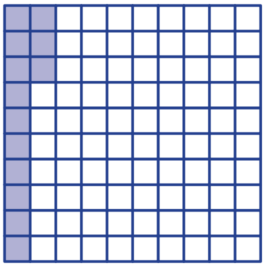
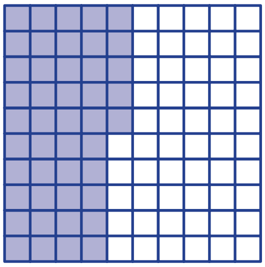
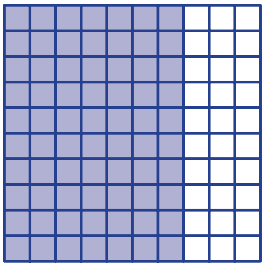
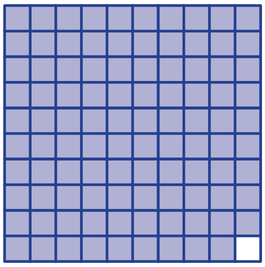
2 The decimal equivalent of 37% is:

A 0.037 **B** 0.37 **C** 3.7 **D** 37.00

3 The percentage equivalent of $\frac{47}{100}$ is:

A 0.47% **B** 4.7% **C** 47% **D** 470%

4 Copy and complete the table or discuss as a group.

		Fraction shaded	Decimal in words	Decimal in figures	Per cent in words	Per cent in figures
a		$\frac{13}{100}$	thirteen hundredths			
b				0.45		
c					seventy per cent	
d						99%

FLUENCY 5–7(½) 5–8(½)

Example 17 Converting percentages to fractions and decimals

Write 72% as:

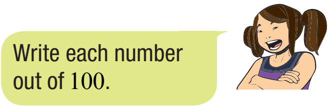
a a simple fraction **b** a decimal

Solution	Explanation
a $72\% = \frac{72}{100}$ $= \frac{18 \times 4}{25 \times 4}$ $= \frac{18}{25}$	Change % sign to a denominator of 100. Cancel the Highest Common Factor of 4 to simplify.
b $72\% = 72 \div 100$ $= 0.72$	To change a % to a decimal divide by 100.

5 Write these percentages as simple fractions.

a 39% **b** 11% **c** 17% **d** 99%

e 20% **f** 70% **g** 75% **h** 55%

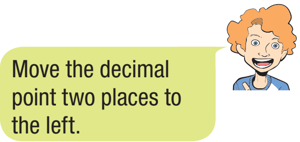


6 Write these percentages as decimals.

a 39% **b** 11% **c** 17% **d** 99%

e 20% **f** 70% **g** 75% **h** 55%

i 7% **j** 1% **k** 10% **l** 47%



Example 18 Converting fractions and decimals to percentages

Write the following as percentages.

a $\frac{4}{5}$ **b** $\frac{3}{8}$ **c** 0.81

Solution	Explanation
a $\frac{4}{5} \times 100 = 80$ So $\frac{4}{5} = 80\%$	Multiply by 100 to express as a percentage
b $\frac{3}{8} \times 100 = \frac{300}{8} = 37\frac{1}{2}$ So $\frac{3}{8} = 37\frac{1}{2}\%$	Multiply the fraction by 100. Simplify $300 \div 8 = 37$ remainder 4
c $0.81 \times 100 = 81$ So $0.81 = 81\%$	Multiply the decimal by 100. $0.81 = 81\%$

7 Write these fractions as percentages.

a $\frac{77}{100}$ **b** $\frac{49}{100}$ **c** $\frac{3}{4}$ **d** $\frac{4}{5}$

e $\frac{7}{25}$ **f** $\frac{9}{20}$ **g** $\frac{11}{20}$ **h** $\frac{19}{50}$

i $\frac{47}{50}$ **j** $\frac{1}{8}$ **k** $\frac{5}{8}$ **l** $\frac{2}{3}$

Multiply by 100% and simplify.

8 Write these decimals as percentages.

a 0.16 **b** 0.79 **c** 0.83 **d** 0.97

e 0.03 **f** 0.33 **g** 0.91 **h** 0.09

i 0.56 **j** 0.22 **k** 1.00 **l** 1.01

PROBLEM-SOLVING AND REASONING

9, 10 9–12

9 Complete the following.

a $7\frac{1}{2}\% = \frac{7\frac{1}{2}}{\square} \times \frac{2}{2}$

$= \frac{15}{\square}$

$= \frac{3}{40}$

b $5\frac{3}{4}\% = \frac{\square}{100} \times \frac{4}{4}$

$= \frac{\square}{400}$

c $\frac{1}{3} = \frac{1}{3} \times 100\%$

$= \frac{\square}{3}\%$

$= 33\frac{\square}{3}\%$

10 **a** If $\frac{1}{5} = 20\%$, what does $\frac{3}{5}$ equal as a percentage?

b If $\frac{1}{8} = 12.5\%$, what does $\frac{7}{8}$ equal as a percentage?

c If $\frac{1}{3} = 33\frac{1}{3}\%$, what does $\frac{2}{3}$ equal as a percentage?

Think: What can I multiply each fraction by?

11 Complete the following conversion tables involving common fractions, decimals and percentages.

a

Fraction	Decimal	%
$\frac{1}{4}$		
$\frac{2}{4}$		
$\frac{3}{4}$		
$\frac{3}{4}$		

b

Fraction	Decimal	%
		20%
		40%
		60%
		80%
		100%

c

Fraction		$\frac{3}{10}$		$\frac{1}{3}$
Decimal	0.15			
%			90%	

12 The Sharks team has won 13 out of 17 games for the season to date. The team still has three games to play. What is the smallest and the largest percentage of games the Sharks could win for the season?

Money and percentages

— 13

13 Copy and complete this table. Can you see a connection?

Cents per 100 cents	Cents in the dollar	Percentage
5c	\$0.05	
10c		
	\$0.09	
		17%
		25%
	\$0.70	
		90%
75c		
100c		
	\$2	

One dollar equals 100 *cents*
One *century* is 100 years.

3G Percentages: Expressing and finding

Interactive
Widgets
HOTsheets
Walkthrough

Percentages are useful when comparing: discounts, interest rates and even marks in a test. For example, Huen's report card could be written as marks out of each total or in percentages:

French test $\frac{14}{20}$
German test $\frac{54}{75}$

French test 70%
German test 72%

In this section we look at expressing a number as a percentage of another number as well as finding a percentage of an amount.

Let's start: What percentage has passed?

Answer the following questions.

- What percentage of your day has passed?
- What percentage of the current month has passed?
- What percentage of the current season has passed?
- What percentage of your school year has passed?
- What percentage of your school education has passed?
- If you live to an average age, what percentage of your life has passed?
- When you turned 5, what percentage of your life was 1 year?
- When you are 40, what percentage of your life will 1 year be?

Key ideas

- **To express one quantity as a percentage of another**

  1 Write the quantities as a fraction (the 'whole' amount is always the denominator).

  2 Multiply this fraction by 100.

  e.g. Express a test score of 14 out of 20 as a percentage.

$$\frac{14}{20}\times 100 = \frac{14}{{}_{1}\cancel{20}}\times\frac{{}^{5}\cancel{100}}{1} = 70$$

$$\text{So } \frac{14}{20} = 70\%$$

$$\frac{14 \leftarrow \text{part of the whole}}{20 \leftarrow \text{whole amount}}$$

- **To find a certain percentage of a quantity**

  1 Express the required percentage as a fraction. [You can use decimals]

  2 Change the 'of' to a multiplication sign.

  3 Express the number as a fraction.

  4 Follow the rules for multiplication of fractions.

  e.g. find 20% of 80.

$$20\% \text{ of } 80 = \frac{20}{100}\times\frac{80}{1} = \frac{{}^{4}\cancel{20}}{{}_{1\,\cancel{5}}\cancel{100}}\times\frac{{}^{4}\cancel{80}}{1} = 16$$

Exercise 3G

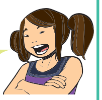
UNDERSTANDING 1–4 4

1 The correct working line to express 42 as a percentage of 65 is:

A $\frac{42}{100}\times 65\%$ **B** $\frac{65}{42}\times 100\%$ **C** $\frac{100}{42}\times 65\%$ **D** $\frac{42}{65}\times 100\%$

2 The correct working line to find 42% of 65 is:

A $\frac{42}{100}\times 65$ **B** $\frac{65}{42}\times 100$ **C** $\frac{100}{42}\times 65$ **D** $\frac{42}{65}\times 100$



The number with the per cent sign is written with the 100 in the denominator.

3 What percentage is meant by:

a a mark of 20 out of 40?
b a mark of 0 out of 10?
c a mark of 50 out of 50?

4 Copy and complete the following sentences.

a Finding 1% of a quantity is the same as dividing the quantity by _______.
b Finding 10% of a quantity is the same as dividing the quantity by _______.
c Finding 20% of a quantity is the same as dividing the quantity by _______.
d Finding 50% of a quantity is the same as dividing the quantity by _______.
e Finding 25% of a quantity is the same as dividing the quantity by _______.

FLUENCY 5(½), 6, 7, 8(½) 5–8(½)

Example 19 Expressing one quantity as a percentage of another

Express 34 out of 40 as a percentage.

Solution

$$\begin{aligned}\frac{34}{40}\times\frac{100}{1}&=\frac{17}{20}\times\frac{100}{1}\\&=\frac{17}{1}\times\frac{5}{1}\\&=85\end{aligned}$$

So $\frac{34}{40} = 85\%$

Explanation

Write as a fraction, with the first quantity as the numerator and second quantity as the denominator. Multiply by 100. Cancel and simplify.

5 Express each of the following as a percentage.

a 20 out of 25	**b** 13 out of 20	**c** 39 out of 50
d 17 out of 25	**e** 12 out of 20	**f** 49 out of 50
g 7 out of 10	**h** 12 out of 30	**i** 15 out of 20
j 32 out of 40	**k** 54 out of 90	**l** 18 out of 24

Multiply by 100.

Example 20 Converting units before expressing as a percentage

Express 60c as a percentage of $5.

Solution

$$\begin{aligned}\frac{60}{500}\times\frac{100}{1}&=\frac{60}{5}\\&=12\end{aligned}$$

So $\frac{60}{500} = 12\%$

Explanation

Units need to be the same.
Convert $5 to 500 cents.
Write quantities as a fraction and multiply by 100. Cancel and simplify.

6 Express:

- **a** 40c as a percentage of $8
- **b** 50c as a percentage of $2
- **c** 3 mm as a percentage of 6 cm
- **d** 400 m as a percentage of 1.6 km
- **e** 200 g as a percentage of 5 kg
- **f** 200 m as a percentage of 8 km

Remember:
1 km = 1000 m
1 cm = 10 mm
1 kg = 1000 g
$1 = 100 cents

7 Express each quantity as a percentage of the total.

- **a** 28 laps of a 50 lap race completed
- **b** Saved $450 towards a $600 guitar
- **c** 172 fans in a train carriage of 200 people
- **d** Level 7 completed of a 28 level video game
- **e** 36 students absent out of 90 total
- **f** 21 km mark of a 42 km marathon

Example 21 Finding a certain percentage of a quantity

Find 25% of 48.

Solution	Explanation
25% of $48 = \frac{25}{100} \times \frac{48}{1}$	Write the percentage as a fraction over 100. 'of' means multiply.
$= \frac{1}{4} \times \frac{48}{1} = 12$	Cancel and simplify.

8 Find:

a 50% of 36	**b** 10% of 80	**c** 30% of 500
d 9% of 200	**e** 20% of 40	**f** 20% of 60
g 75% of 80	**h** 25% of 88	**i** 50% of 25
j 5% of 60	**k** 5% of 6000	**l** 1% of 720
m 2% of 150	**n** 99% of 200	**o** 75% of 960

50% of 36
$= \frac{50}{100} \times \frac{36}{1}$

PROBLEM-SOLVING AND REASONING 9(½), 11 9(½), 10, 12

9 Find:

a 10% of \$750

b 5% of 2km

c 30% of 150kg

d 20% of 90 minutes

e 10% of 5 litres

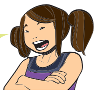
f 25% of one hour

g 50% of \$6.50

h 2% of \$8

i 7% of $\frac{1}{2}$ kg

You may like to change the units in the question to make it easier to work with
3% of 1 km
= 3% of 1000 metres.



Remember to put the units in your answer
10% of \$50
$= \frac{10}{100} \times \$50$
$= \$5$

Remember:
$33\frac{1}{3}\% = \frac{1}{3}$

10 Find:

a $33\frac{1}{3}\%$ of 15 litres of orange juice

b $66\frac{2}{3}\%$ of 3000 marbles

c $12\frac{1}{2}\%$ of a \$64 pair of jeans

d 37.5% of 120 donuts

11 Copy and complete the table of sporting choices.

Sport	Number of students	Fraction of total	Percentage
Tennis	40		
Golf	30		
AFL	70		
NRL	50		
Swimming	10		
Total	200	1	100%

12 Most banks require a 10% deposit before lending you any money. Ashlee and Matt have 7% of the $\$450\,000$ their home costs.

- **a** How much do Ashlee and Matt have as their deposit?
- **b** How much do the banks need them to have?
- **c** How much more do they need to save?
- **d** If they get a government grant of $\$14\,000$, will they have the 10% needed?

Calculator percentages

— 13

13 Calculators make working with percentages easier. Use a calculator to answer these questions.

- **a** Find 8% of $\$8.40$.
- **b** Find 13% of 2 km.
- **c** Find $7\frac{1}{4}\%$ of $\$500$.
- **d** Find 24% of 1 hour.
- **e** Find 31.5% of $\$45\,960$.
- **f** 4% of a class of 25 students are away with the flu. How many students are at school?
- **g** 49.5% of babies born at the local hospital are girls. Of the 200 born in the month, how many were boys?
- **h** Sean pays 42% of his $\$86\,400$ income in tax. How much is left after he pays his tax?

Progress quiz

3A **1** Write the following fractions in simplest form.

a $\frac{4}{10}$ **b** $\frac{6}{9}$ **c** $\frac{16}{20}$ **d** $\frac{45}{25}$

3B **2** Evaluate.

a $\frac{5}{11}+\frac{8}{11}$ **b** $\frac{7}{8}-\frac{3}{4}$ **c** $2\frac{1}{5}+3\frac{7}{10}$ **d** $4\frac{2}{3}-2\frac{1}{2}$

3B **3** Evaluate.

a $\frac{2}{3}\times\frac{5}{7}$ **b** $1\frac{3}{5}\times\frac{3}{4}$ **c** $\frac{3}{7}\div\frac{5}{11}$ **d** $1\frac{1}{6}\div\frac{5}{12}$

3C **4** Convert the following decimals to fractions in their simplest form.

a 0.35 **b** 5.25 **c** 12.8 **d** 456.14

3C **5** Convert the following fractions to decimals.

a $\frac{7}{10}$ **b** $\frac{36}{100}$ **c** $\frac{17}{50}$ **d** $\frac{9}{4}$

3D **6** Calculate.

a $9.5+12.3$ **b** $5.78+12.915$
c $35.8-23.6$ **d** $76.813-56.685$

3D **7** Calculate.

a 6.5734×1000 **b** $12.754\div10\,000$ **c** 0.6×0.9
d 45.23×0.5 **e** $23.845\div5$ **f** $84.561\div0.03$

3E **8** Convert the following fractions to decimals, expressing your answers as recurring decimals if necessary.

a $\frac{3}{8}$ **b** $\frac{5}{4}$ **c** $\frac{5}{3}$ **d** $\frac{11}{7}$

3E **9** Round each of the following decimals to two decimal places.

a 0.789 **b** 0.415 **c** 26.14812 **d** 379.01099

3F **10** Write these percentages as decimals and as simple fractions.

a 45% **b** 84% **c** 2% **d** 109%

3F **11** Write the following as percentages.

a $\frac{4}{5}$ **b** $\frac{3}{8}$ **c** 0.43 **d** 0.94

3G **12** Express each of the following as a percentage.

a 32 out of 40
b 17 out of 50
c \$12 out of \$60
d 560 km out of 800 km

3G **13** Find:

a 20% of \$50
b 5% of 500 kg
c 75% of one hour
d 1% of \$12 500

3H Decreasing and increasing by a percentage

Interactive

Widgets

HOTsheets

Walkthrough

Percentages are used every day, often when dealing with money, to make increases or decreases.

The original amount of something is 100%.

Let's start: What does it mean?

In pairs, answer the following:

- What does it mean to buy a pair of shoes 'on sale'?
- What does it mean if the sale is '20% off'?
- What does it mean to 'pay the marked price'?
- What does it mean to you to buy an item on sale? Is $\$10$ off better than 10% off? Discuss.
- When would you pay more than the original price?
- What current sales are being advertised in today's paper?

Key ideas

- To increase by a given percentage:
  - find the percentage of the amount
  - **add** this amount to the original
- To decrease by a given percentage:
  - find the percentage of the amount
  - **subtract** this amount from the original
- Key words:
  - Decrease: reduction, discount, sale, percentage off, loss
  - Increase: mark-up, profit
  - Selling price = cost price + profit *or* cost price – loss
  - **GST**: Goods and Services Tax (in Australia this is a 10% mark-up)

GST Goods and Services Tax

Exercise 3H

UNDERSTANDING	1–3	3

1 Decide if each of these shows an increase or a decrease.

a Mark's $\$1650$ return airfare to Los Angeles was reduced by 10%.

b Sonya made 15% profit when she sold her house.

c The shop discounted all of its computers by 10%.

d Thomas received a pay rise of 5% on his wage of $\$570$ per week.

e A tax of 15% is added to the cost of everything in the United Kingdom.

2 Add or subtract these percentages.

- **a** $100\% + 20\%$
- **b** $100\% + 15\%$
- **c** $100\% - 10\%$
- **d** $100\% - 15\%$

3 Calculate the new price when:

- **a** an item marked at $15 is discounted by $3
- **b** an item marked at $25.99 is marked up by $8
- **c** an item marked at $17 is reduced by $2.50
- **d** an item marked at $180 is increased by $45

FLUENCY 4–5(½), 6, 7–8(½) 4–5(½), 6, 8(½), 9, 10(½)

Example 22 Finding new values: increasing

Find the new value when $160 is increased by 40%.

Solution	Explanation
40% of $\$160 = \frac{40}{100} \times \frac{160}{1} = \$64$	Calculate 40% of $160. Cancel and simplify.
New price $= \$160 + \$64$ $= \$224$	New price = original price + increase

4 Find the new value when:

- **a** $400 is increased by 10%
- **b** $240 is increased by 10%
- **c** $250 is increased by 10%
- **d** $700 is increased by 20%
- **e** $500 is increased by 1%
- **f** $800 is increased by 25%
- **g** $84 is increased by 25%
- **h** $90 is increased by 50%

Add the increase to the original amount.

Example 23 Finding new values: decreasing

Find the new value when $63 is decreased by 20%.

Solution	Explanation
20% of $\$63 = \frac{20}{100} \times \frac{63}{1} = \$12.60$	Calculate 20% of $63. Cancel and simplify.
New price $= \$63 - \$12.60$ $= \$50.40$	New price = original price – decrease

5 Find the new value when:

- **a** \$400 is decreased by 10%
- **b** \$240 is decreased by 10%
- **c** \$250 is decreased by 10%
- **d** \$90 is decreased by 20%
- **e** \$200 is decreased by 15%
- **f** \$840 is decreased by 25%
- **g** \$1000 is decreased by 50%
- **h** \$60 is decreased by 15%

6 **a** Find 8% of \$2500
- **b** Increase \$2500 by 8%
- **c** Decrease \$2500 by 8%

Example 24 Calculating discounts

Find the cost of a \$860 television that has been discounted by 25%.

Solution	Explanation
Discount = 25% of \$860	Calculate 25% discount.
$= \frac{25}{100} \times \frac{860}{1} = \$215$	Cancel and simplify.
Selling price = \$860 – \$215 = \$645	Selling price = cost price – discount

7 Find the cost of the following.

- **a** A \$600 television that has been discounted by 20%
- **b** A \$150 lipstick that has been reduced by 15%
- **c** A \$52 jumper that has depreciated by 25%

8 Calculate the selling prices of the following items if they are to be reduced by 25%.

- **a** \$16 thongs
- **b** \$32 sunhat
- **c** \$50 sunglasses
- **d** \$85 bathers
- **e** \$130 boogie board
- **f** \$6.60 surfboard wax

Example 25 Calculating mark-ups

Find the cost of a \$250 microwave oven that has been marked up by 12%.

Solution	Explanation
Mark-up = 12% of \$250	Calculate 12% of \$250.
$= \frac{12}{100} \times \frac{250}{1} = \$30$	Cancel and simplify.
Selling price = \$250 + \$30 = \$280	Selling price = cost price + mark-up

9 Find the cost of the following.
- **a** A \$80 framed Pink poster that has been marked up by 30%
- **b** A \$14 meal that has been increased by 10%
- **c** A \$420 stereo that has been marked up by 50%

10 Calculate the selling prices of the following items if they need to have 10% GST added to them.
- **a** \$35 T-shirt
- **b** \$75 backpack
- **c** \$42 massage
- **d** \$83 fishing rod
- **e** \$52.50 toaster
- **f** \$149.99 cricket bat

Remember 10% GST adds/increases the price of an item.

PROBLEM-SOLVING AND REASONING

11 11, 12

11 Answer the following problems involving percentages.
- **a** Anne's annual salary was \$86000. Her new salary is 5% more. What is Anne's new salary?
- **b** The state government increases the cost of a \$9.60 train trip by 5%. What is the new fare?
- **c** A car worth \$47000 dropped in value by 20% during the year. What is the car now worth?
- **d** The 10% GST needs to be added to the cost of the meal. What does a \$74 meal cost once the GST is added in?
- **e** Tax of 40% reduces Saul's wage of \$1600. What amount does Saul receive?
- **f** Sally makes a 24% profit on her house. She paid \$500000. What did she sell it for?

12 Two shops advertise the same bike. Both have a recommended retail price of \$1800. Shop one offers a 10% discount. Shop two offers \$200 off all bikes.
- **a** How much discount does shop one offer on this bike?
- **b** How much do you pay for the bike at each shop?
- **c** Which shop would you recommend and why?
- **d** If the same deal applies to each of the following bikes, would you still buy it from the same shop?
  - **i** \$2000 bike
  - **ii** \$2200 bike

Find the price of each bike at shop one and two before answering part **d**. Are you surprised by your answers?

Depreciation

— 13

13 The word *depreciation* is used when the value of an item, such as a car, boat or a set of golf clubs, reduces in value each year.

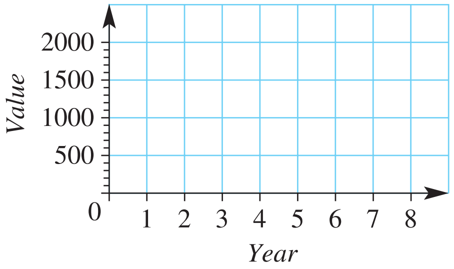
a Rick's set of golf clubs worth $2000 depreciates at a rate of $250 a year.

i Copy and complete the table showing how the value of the clubs changes over time.

End of year	Value
0	2000
1	1750
2	
3	
4	

End of year	Value
5	
6	
7	
8	

ii Draw up a set of axes (like those shown below) and graph the values shown in the table.



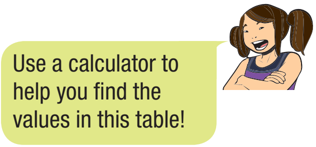
iii What shape is your graph?

iv After how many years is the value of the clubs zero?

b Rick's wife has a set of golf clubs, also valued at $2000, which depreciate at $12\frac{1}{2}\%$ each year.

i Complete a similar table showing how the value of her clubs changes.

End of year	Value
0	2000
1	1750
2	1531.25
3	
4	



ii Will her clubs ever be worthless?

3I Calculating percentage change ★

When selling something, everyone likes to make a **profit**. This is when you sell it for more than you paid for it.

Unfortunately, people often do the opposite and make a **loss**.

The percentage change depends on what the item is originally worth. For example:

Car bought for $1000	Car bought for $16000
Car sold for $200	Car sold for $15200

Loss $800, percentage loss 80%

Loss $800, percentage loss 5%

Let's start: Hang on!

Discuss how you could check if the correct price for the jeans had been paid.

Key ideas

- **Profit** = selling price – cost price
- **Loss** = cost price – selling price
- Calculating a percentage change involves the technique of expressing one quantity as a percentage of another (see Section 3G).

Percentage change $= \dfrac{\text{change}}{\text{original value}} \times 100\%$

Percentage profit $= \dfrac{\text{profit}}{\text{original value}} \times 100\%$

Percentage loss $= \dfrac{\text{loss}}{\text{original value}} \times 100\%$

Profit The amount of money made by selling for more than the cost

Loss The amount of money lost by selling for less than the cost

Exercise 3I

UNDERSTANDING 1–4 4

1 Decide whether each of the following represents a profit or a loss.

a

bought = $250 000
sold = $280 000

b

bought = $795
sold = $210

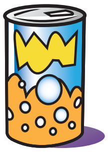
c

bought = $1200
sold = $500

d

bought = $2000
sold = $4500

e



bought = $1.40
sold = $3.20

2 Calculate the profit made in each of the following situations.

a Cost price = $14, Sale price = $21

b Cost price = $75, Sale price = $103

c Cost price = $25.50, Sale price = $28.95

d Cost price = $499, Sale price = $935

Profit = selling price − cost price

3 Calculate the loss made in each of the following situations.

a Cost price = $22, Sale price = $9

b Cost price = $92, Sale price = $47

c Cost price = $71.10, Sale price = $45.20

d Cost price = $1121, Sale price = $874

4 Which of the following is the correct formula for working out percentage change?

A $\% \text{ change} = \dfrac{\text{change}}{\text{original value}}$

B $\% \text{ change} = \dfrac{\text{original value}}{\text{change}} \times 100\%$

C $\% \text{ change} = \text{change} \times 100\%$

D $\% \text{ change} = \dfrac{\text{change}}{\text{original value}} \times 100\%$

FLUENCY 5–8 5–6(½), 7–9

Example 26 Calculating percentage change: profit

Calculate the percentage profit when $25 becomes $32.

Solution

Profit = $7

$\% \text{ Profit} = \dfrac{7}{25} \times \dfrac{100}{1}\%$

$= 28\%$

Explanation

This is percentage **profit** because it was sold for more than the original $25.

Profit = $32 − $25

$\text{Percentage profit} = \dfrac{\text{profit}}{\text{original value}} \times 100\%$

5 Find the percentage profit when:

- **a** \$20 becomes \$36
- **b** \$10 becomes \$13
- **c** \$40 becomes \$50
- **d** \$25 becomes \$30
- **e** \$12 becomes \$20
- **f** \$8 becomes \$11
- **g** \$10 becomes \$15
- **h** \$6 becomes \$12

$$\% \text{ change} = \frac{\text{change}}{\text{original}} \times \frac{100}{1}$$

Example 27 Calculating percentage change: loss

Calculate the percentage loss when \$60 becomes \$48.

Solution	Explanation
Loss = \$12	This is percentage **loss** because it was sold for less than the original \$60. Loss = \$60 – \$48
$\% \text{ Loss} = \frac{12}{60} \times \frac{100}{1}\%$ $= 20\%$	$\text{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$

6 Find the percentage loss when:

- **a** \$40 becomes \$30
- **b** \$25 becomes \$21
- **c** \$6 becomes \$3
- **d** \$8 becomes \$2
- **e** \$12 becomes \$8
- **f** \$10 becomes \$9
- **g** \$25 becomes \$20
- **h** \$20 becomes \$18

$$\% \text{ loss} = \frac{\text{loss}}{\text{original}} \times \frac{100}{1}$$

Example 28 Solving worded problems

Ross buys a ticket to a concert for \$125, but is later unable to go. He sells it to his friend for \$75. Calculate the percentage loss Ross made.

Solution	Explanation
Loss = \$125 – \$75 = \$50	Loss = Cost price – Selling price
$\% \text{ Loss} = \frac{50}{125} \times \frac{100}{1}\%$ $= 40\%$	$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$

Ross made a 40% loss on the concert ticket.

7 Copy and complete the tables below.

a

Cost price ($)	Selling price ($)	Profit ($)	% profit
4	5		
10	12		
24	30		
100	127		

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

b

Cost price ($)	Selling price ($)	Loss ($)	% loss
10	7		
16	12		
50	47		
100	93		

$$\% \text{ loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

8 Find the percentage change (increase or decrease) when:

a 15 kg becomes 18 kg

b 18 kg becomes 15 kg

c 4 kg becomes 24 kg

d 12 kg becomes 30 kg

9 Find the percentage change in population when:

a a town of 4000 becomes a town of 5000

b a city of 750 000 becomes a city of 900 000

c a country of 5 000 000 becomes a country of 12 000 000

PROBLEM-SOLVING AND REASONING

10, 11 | 10–13

10 Gari buys a ticket to a concert for $90, but is unable to go. He sells it to his friend for $72. Calculate the percentage loss Gari made.

11 Xavier purchased materials for $48 and made a dog kennel. He later sold the dog kennel for $84.

a Calculate the profit Xavier made.

b Calculate the percentage profit Xavier made.

12 Gemma purchased a $400 foal, which she later sold for $720.

a Calculate the profit Gemma made.

b Calculate the percentage profit Gemma made.

13 Lee-Sen purchased a $5000 car, which she later sold for $2800.

a Calculate the loss Lee-Sen made.

b Calculate the percentage loss Lee-Sen made.

c What should Lee-Sen sell the car for to make a 10% profit?

Growth rate for Australia

— 14

14 The Australian Bureau of Statistics tracks the population growth of the country and of each individual state and territory.

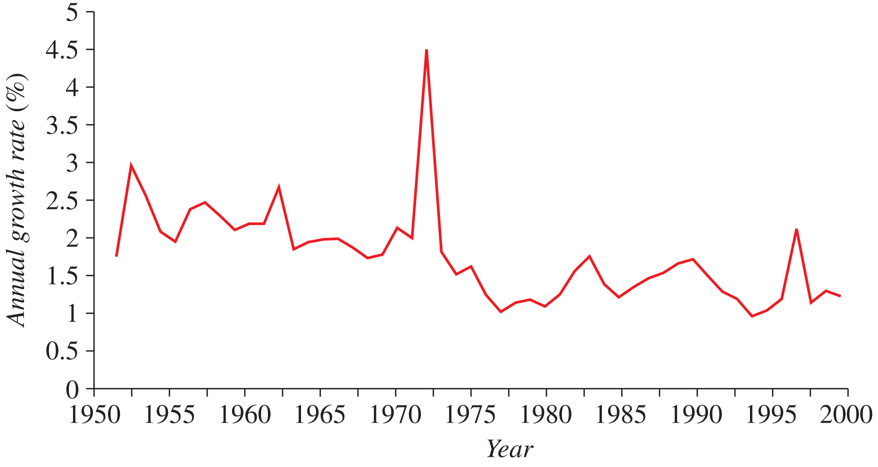
a Copy and complete the table below.

	March 2016	Change in the past 12 months	% change
NSW	7 704 300	103 200	
VIC	6 039 100	114 900	
QLD	4 827 000	61 800	
SA	1 706 500	9 700	
WA	2 613 700	29 800	
TAS	518 500	2 200	
NT	244 000	1 000	
ACT	395 200	5 000	
AUSTRALIA	24 051 400	327 600	

Use a calculator to help you with this question.

b Research the current growth rate of Australia and one other country of your choice.

Population growth in Australia, 1950–2000



3J Percentages and the unitary method

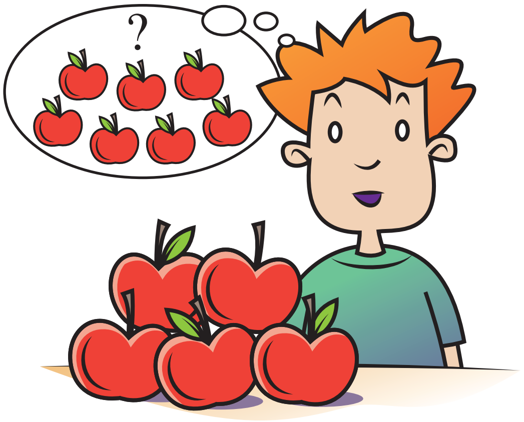
Interactive

Widgets

HOTsheets

Walkthrough

You probably did problems like this in primary school.
If five apples cost $6, what is the cost of 7 apples?



With questions like this, we find the cost of one apple first.

This is helpful in percentages as well. Once we know what 1% is worth, we can find any percentage amount. This is called the **unitary method**.

Let's start: Using the unitary method

- Four tickets to a concert cost $100. What does one ticket cost? How much will three tickets cost?
- Ten workers can dig 40 holes in an hour. How many can one worker dig in an hour? How many holes can seven workers dig in an hour?
- Six small pizzas cost $54. What does one pizza cost? How much would ten pizzas cost?
- If eight pairs of socks cost $64, how much would 11 pairs of socks cost?
- Five passionfruit cost $2.00. How much will nine passionfruit cost?
- If a worker travels 55 km in 5 trips from home to the worksite, how far will he travel in 7 trips?

Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to answer the question.
- When dealing with percentages, finding 'one unit' means finding one per cent (1%).
- Once the value of 1% of an amount is known, it can be multiplied to find the value of any desired percentage.

Unitary method
Calculating the value of one unit of an item and then using this to calculate the value of a number of items

Exercise 3J

UNDERSTANDING 1–3 3

1 **a** What do you divide by to go from 8% to 1%?
b What do you divide by to go from 25% to 1%?
c What do you multiply by to go from 1% to 100%?
d What do you multiply by to go from 1% to 50%?

2 If 1% of an amount is \$3, what is:
a 2% of the amount?
b 10% of the amount?
c 100% of the amount?

3 If 1% of an amount is \$8, what is:
a 10% of the amount?
b 100% of the amount?

FLUENCY 4–6 4(½), 6, 7

4 Copy and complete:

4% of an amount = \$16
then 1% of an amount = ☐
and 100% of an amount = ☐

Example 29 Using the unitary method to find the full amount

If 8% of an amount of money is \$48, what is the full amount of money?

Solution	**Explanation**
÷8 8% of amount is \$48 ÷8 1% of amount is \$6 ×100 100% of amount is \$600 ×100 Full amount of money is \$600.	Remember to find 1% first. Divide by 8 to find the value of 1% $(48 \div 8 = 6)$. Multiply by 100 to find the value of 100% $(6 \times 100 = 600)$.

5 Calculate the full amount of money for each of the following.
a 3% of an amount of money is \$27
b 5% of an amount of money is \$40
c 12% of an amount of money is \$132
d 60% of an amount of money is \$300
e 8% of an amount of money is \$44
f 6% of an amount of money is \$15

Example 30 Using the unitary method to find a new percentage

If 11% of the food bill was $77, how much is 25% of the food bill?

Solution	Explanation
11% of food bill is \$77 ÷11 ↓ ÷11 1% of food bill is \$7 ×25 ↓ ×25 25% of food bill is \$175	Find 1% first. Divide by 11 to find the value of 1% $(77 \div 11 = 7)$. Multiply by 25 to find the value of 25% $(7 \times 25 = 175)$.

6 If 4% of the total bill is $12, how much is 30% of the bill?

First find the value of 1% of the bill.

7 Calculate:

- **a** 20% of the bill, if 6% of the total bill is $36
- **b** 80% of the bill, if 15% of the total bill is $45
- **c** 3% of the bill, if 40% of the total bill is $200
- **d** 7% of the bill, if 25% of the total bill is $75

PROBLEM-SOLVING AND REASONING

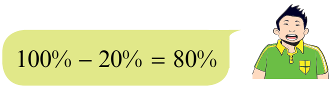
8, 9 9(½), 10–12

Example 31 Using the unitary method to find the original price

A pair of shoes has been discounted by 20%. If the sale price was $160, what was the original price of the shoes?

Solution	Explanation
Only paying 80% of original price:	20% discount, so paying $(100 - 20)\%$.
∴ 80% of original price is \$160	We pay **80%** after the 20% discount.
∴ 1% of original price is \$2	Divide by 80 to find the value of 1% $(160 \div 80 = 2)$.
∴ 100% of original price is \$200	Multiply by 100 to find the value of 100%
The original price of the shoes was \$200.	$(2 \times 100 = 200)$.

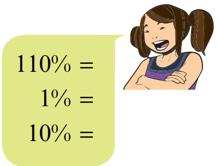
8 A necklace in a jewellery store has been discounted by 20%. If the sale price is \$240, what was the original price of the necklace?

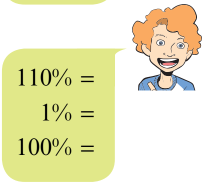


9 Find the original price of the following items.

- **a** A pair of jeans discounted by 40% has a sale price of \$30 (you pay 60%).
- **b** A hockey stick discounted by 30% has a sale price of \$105 (you pay 70%).
- **c** A second-hand computer discounted by 85% has a sale price of \$90 (you pay 15%).
- **d** A second-hand textbook discounted by 80% has a sale price of \$6.
- **e** A standard rose bush discounted by 15% has a sale price of \$8.50.
- **f** A motorbike discounted by 25% has a sale price of \$1500.

10 Once the GST of 10% is added to a bill the price is 110%. If the price of a meal at a cafe, including the GST, is \$55, how much GST is paid?





11 A pair of jeans, including 10% GST, comes to \$88. What is the cost without the GST?

12 If 22% of an amount is \$8540, which of the following would give the value of 1% of the amount?

A $\$8540 \times 100$ **B** $\$8540 \div 100$ **C** $\$8540 \times 22$ **D** $\$8540 \div 22$

Real-life receipts

—	13

13 Here are three real-life receipts (with the names of shops changed). GST rate is 10%. Answer the following based on each one.

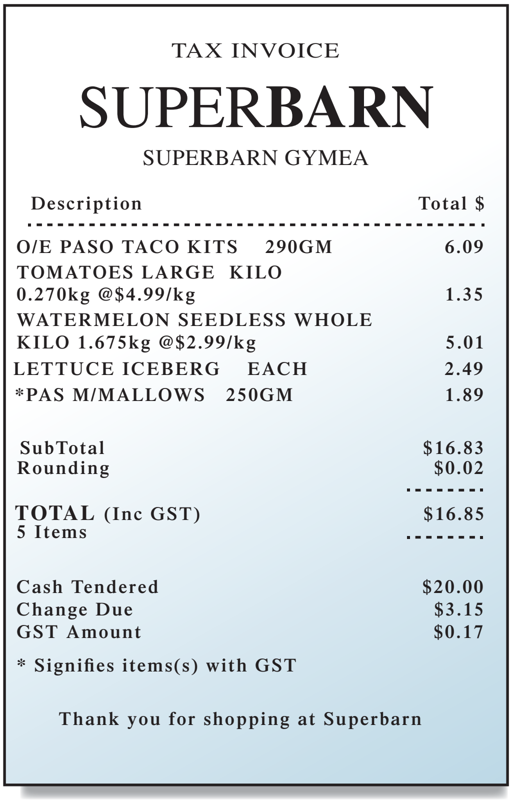
Superbarn

a How much was spent at Superbarn?

b How many kilograms of tomatoes were bought?

c Which item included the GST, and how do you tell by looking at the receipt?

d What was the cost of the item if the GST is not included?



TAX INVOICE

SUPERBARN

SUPERBARN GYMEA

Description	Total $
O/E PASO TACO KITS 290GM	6.09
TOMATOES LARGE KILO 0.270kg @$4.99/kg	1.35
WATERMELON SEEDLESS WHOLE KILO 1.675kg @$2.99/kg	5.01
LETTUCE ICEBERG EACH	2.49
*PAS M/MALLOWS 250GM	1.89
SubTotal	$16.83
Rounding	$0.02
TOTAL (Inc GST) 5 Items	$16.85
Cash Tendered	$20.00
Change Due	$3.15
GST Amount	$0.17

* Signifies items(s) with GST

Thank you for shopping at Superbarn

Gymea Fruit Market

a What was the cost of bananas per kilogram?

b On what date was the purchase made?

c What does ROUND mean?

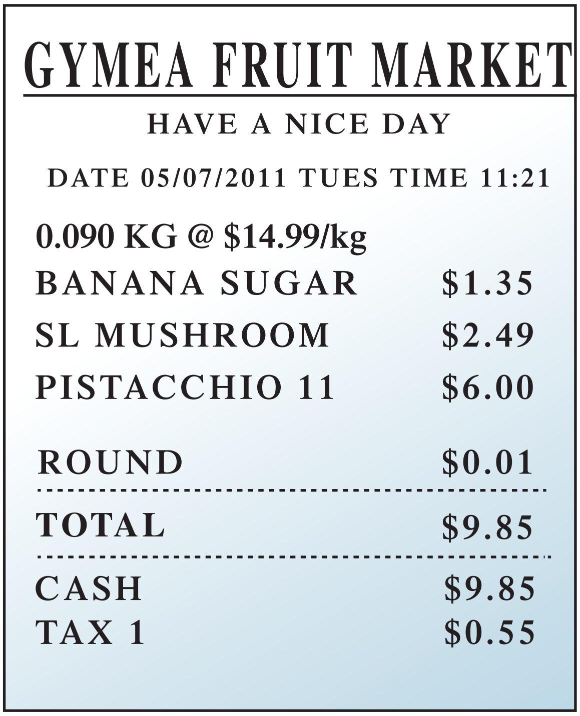
d What was the total paid for the items?

e How much tax was included in the bill?

f What percentage of the bill was the tax?

Xmart

- **a** How many toys were purchased?
- **b** What was the cost of the most expensive item?
- **c** Which of the toys attracted GST?
- **d** How much GST was paid in total?
- **e** What percentage of the total bill was the GST?



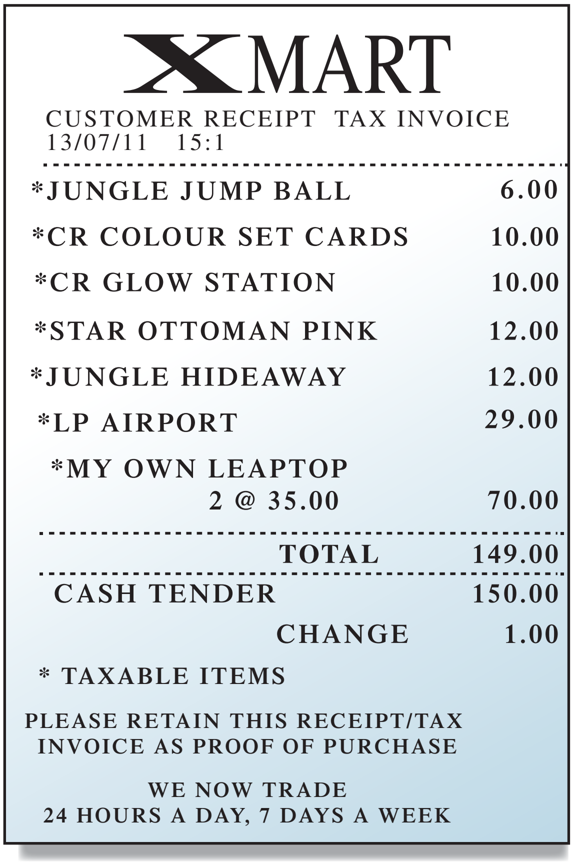
GYMEA FRUIT MARKET

HAVE A NICE DAY

DATE 05/07/2011 TUES TIME 11:21

0.090 KG @ $14.99/kg

BANANA SUGAR	$1.35
SL MUSHROOM	$2.49
PISTACCHIO 11	$6.00
ROUND	$0.01
TOTAL	$9.85
CASH	$9.85
TAX 1	$0.55



XMART

CUSTOMER RECEIPT TAX INVOICE
13/07/11 15:1

*JUNGLE JUMP BALL	6.00
*CR COLOUR SET CARDS	10.00
*CR GLOW STATION	10.00
*STAR OTTOMAN PINK	12.00
*JUNGLE HIDEAWAY	12.00
*LP AIRPORT	29.00
*MY OWN LEAPTOP 2 @ 35.00	70.00
TOTAL	149.00
CASH TENDER	150.00
CHANGE	1.00

* TAXABLE ITEMS

PLEASE RETAIN THIS RECEIPT/TAX
INVOICE AS PROOF OF PURCHASE

WE NOW TRADE
24 HOURS A DAY, 7 DAYS A WEEK

Maths@Work: Owner and manager of a fruit and vegetable shop

Owning and running your own business requires hard work and long hours. The skills you need are varied, from being well organised and having good communication skills, to being able to manage staff and stock. Perseverance is also a requirement, as many businesses fail within their first 3 years.

Skill with numbers is important for the day-to-day running of a fruit and vegetable shop to ensure a profit. Stock needs to be ordered and kept fresh. Enough stock needs to be sold to cover wages, store rental, delivery costs and, if possible, also some profit. Prices must be adjusted for seasonal price fluctuations as well as unexpected costs.

1 Imagine that you are starting up a fruit and vegetable shop in the town or suburb where you live.
  a List some vegetables and fruits that could be supplied from your local markets or farms.
  b What costs would your business have, other than buying produce?
  c What do you think would be the main challenges to successfully running your shop?

2 When ordering, or buying produce from markets, prices per kg or per item are usually stated. Calculate the unit price or cost per kilogram, for each of the following.
  a 15 kg bag of potatoes costs \$14.55
  b 25 kg box of apples costs \$34.50
  c Box of lettuce containing 20 heads of lettuce costs \$18
  d 8 kg bag of carrots costs \$6.40
  e 20 kg box of navel oranges costs \$25.40

3 Fresh produce is generally bought at a wholesale price from suppliers and sold at a higher retail price to cover costs and provide a profit to the store owner. Find the retail price/kg of the items below, given the retail price is 250% of the wholesale price. Round to the nearest cent.
  a Fresh asparagus wholesale price \$1.42 per kg
  b Beans wholesale price is \$2.18 per kg
  c Broccoli wholesaling at \$2.39 per kg
  d Apples 2.5 kg bag wholesaling at \$1.83
  e Celery wholesaling at 85c per kg

4 Find the cost of an individual piece of fruit using the information in this table. Round to the nearest cent.

	Fruit	Cost per kg	Average number of pieces per kg	Cost per piece
a	Gala apples	\$5.00	6	
b	Red Delicious apples	\$4.99	5	
c	Bartlett pears	\$3.99	5	
d	Apricots	\$7.95	12	
e	Peaches	\$10.99	9	

5 Find the cost of the following order for a customer.

$\frac{1}{2}$ kg of pears at \$3.99/kg	2 heads of lettuce at \$2.50 each
1.2 kg of apples at \$5/kg	1 avocado at \$3.50
2 kg bag of potatoes at \$2.99/kg	2.2 kg of tomatoes at \$5.99/kg

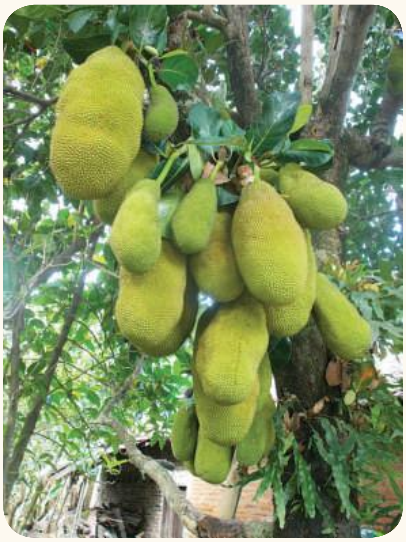
6 Compare the cost of 100 g for each option **A** and **B** and state which is the better buy.

A A tray of Pink Lady apples with a weight 600 grams for \$2.94 or

B 0.75 kg of loose Pink Lady apples for \$3.30.

7 A restaurant in Daintree, north Queensland, ordered the following exotic fruits from a nearby tropical fruit shop. Calculate the total cost of this order including a \$45 packing and delivery fee.

Quantity	Fruit	Price
12	Dragon fruit	\$5.49 each
$2\frac{3}{4}$ kg	Lychees	\$19.90/kg
1	Jackfruit weighing 15.38 kg	\$3.45/kg
16	Star apple fruits	\$2.35 each
4	Custard apples	\$4.55 each
100 g	Soursop dried leaves	\$25 per $\frac{1}{4}$ kg
1900 g	Ice-cream beans	\$2.54 per 100 g
$3\frac{1}{2}$ kg	Chocolate pudding fruits	\$13.99/kg
3.8 kg	purple Mangosteens	\$10.99/kg



A single jackfruit can weigh up to 35 kg.

Using technology

8 Many people work overseas and wish to compare living costs. Set up the following Excel spreadsheet to convert USA produce prices per lb (pound) to Australian dollars (AUD) per kg.

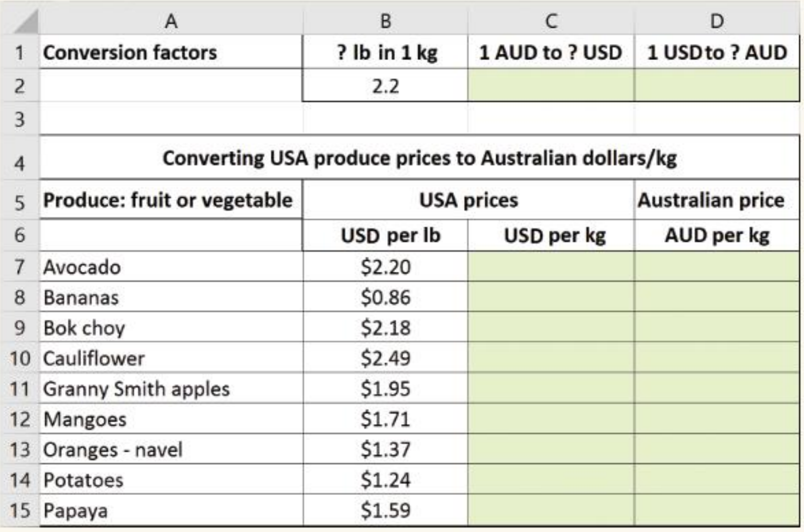
a Use these conversion factors:

- In cell B2 enter 2.2 as there are 2.2 lb per kg.
- In cell C2 enter 0.75 (i.e. 1 AUD = 0.75 USD) or use the current exchange rate.
- In cell D2 enter the formula = 1/C2. This gives the number of AUD for USD.

b In column C, multiply the prices in USD/lb by 2.2 to give USD/kg. Use $ signs to anchor the B2 cell value as in the hint.

c In column D convert USD to AUD. Multiply by cell D2 (i.e. the number of AUD for 1USD).

- Formula cell C7 = B7*B2
- Formula cell D7 = C7*D2
- To fill formulas down a column, drag down the 'fill handle'.



	A	B	C	D
1	**Conversion factors**	**? lb in 1 kg**	**1 AUD to ? USD**	**1 USD to ? AUD**
2		2.2		
3				
4	**Converting USA produce prices to Australian dollars/kg**			
5	**Produce: fruit or vegetable**	**USA prices**		**Australian price**
6		**USD per lb**	**USD per kg**	**AUD per kg**
7	Avocado	$2.20		
8	Bananas	$0.86		
9	Bok choy	$2.18		
10	Cauliflower	$2.49		
11	Granny Smith apples	$1.95		
12	Mangoes	$1.71		
13	Oranges - navel	$1.37		
14	Potatoes	$1.24		
15	Papaya	$1.59		

d Select any five of the above items and compare the converted Australian prices/kg to current Australian online supermarket prices/kg. What observation can you make? Give some possible reasons for the difference in prices.

Puzzles and games

1 Write down four decimals that when rounded to 2 decimal places give 2.67.

2 Jill has five coins in her pocket: a \$2 coin, \$1 coin, 50c piece, 20c piece, and one 10 cent coin. If Jill chooses just two coins from her pocket without looking at them, or noticing their size or shape, how many different amounts could she arrive at?

3 Write one half in ten different ways.

4 Complete these magic squares. All rows, columns and the two diagonals sum to the same total.

a

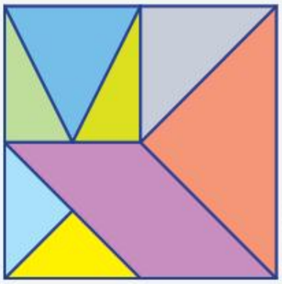
$\frac{4}{3}$		
1	$1\frac{2}{3}$	
$2\frac{2}{3}$		

b

$\frac{5}{3}$		
	$\frac{11}{6}$	$2\frac{1}{6}$
		2

5 A tangram consists of seven geometric shapes (tans) as shown. The tangram puzzle is precisely constructed using vertices, midpoints and straight edges.

- **a** Write each of the separate tan pieces as a percentage, a fraction and a decimal amount of the entire puzzle.
- **b** Check your seven tans add up to a total of 100%.
- **c** Starting with a square, make a new version of a 'modern' tangram puzzle. You must have at least six pieces in your puzzle.
  An example of a modern puzzle is shown.



- **d** Write each of the separate pieces of your new puzzle as a percentage, a fraction and a decimal amount of the whole puzzle.
- **e** Separate pieces of tangrams can be arranged to make more than 300 creative shapes and designs, some of which are shown. You may like to research tangrams and attempt to make some of the images.

Chapter summary

Fractions

What is a fraction?

A fraction is part of a whole.

$1 \rightarrow$ parts from the whole

$4 \rightarrow$ denominator parts in the whole

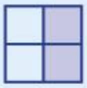
Simplifying fractions

Simplify fractions by dividing the numerator and denominator by their HCF.

$\frac{8}{20} = \frac{4 \times 2}{4 \times 5} = \frac{2}{5}$

Equivalent fractions

- represent the same amount, they are the same decimal.



$\frac{2}{4} = \frac{1}{2} = 0.5$

- create equivalent fractions by multiplying or dividing the top (numerator) and bottom (denominator) by the same number.

$$\begin{tikzcd}\frac{2}{5} \arrow[r, bend left, "\times 3"] \arrow[r, bend right, "\times 3"'] & \frac{6}{15}\end{tikzcd}$$

Mixed and improper

$$\begin{tikzcd}\text{Mixed } 1\frac{1}{3} \arrow[r, bend left, "\frac{1 \times 3 + 1}{3}"] & \text{Improper } \frac{4}{3} \arrow[l, bend left, "4 \div 3 = 1 \text{ rem } 1"]\end{tikzcd}$$

Adding or subtracting fractions

To add or subtract fractions you should use the lowest common denominator.

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$

$= \frac{5}{6}$

Multiplying fractions

- use proper or improper fractions $\frac{3}{2} \times \frac{1}{4}$
- cancel any numerator with any denominator
- multiply the numerators $\frac{3 \times 1}{2 \times 4}$
- multiply the denominators
- simplify $\frac{3}{8}$

Reciprocal

To take the reciprocal of a proper or improper fraction is to turn it upside down (invert).

Dividing fractions

- keep the first
- change the ÷ to ×
- invert the following fraction (its reciprocal) and multiply.

Chapter summary

Decimals

Fractions to decimals

(1) Change denominator to 10, 100 or 1000.

or (2) Divide the denominator into the numerator.

Types of decimals

Decimals can terminate (stop).

0.5, 0.75, 0.125

Decimals can repeat/recur.

$0.3333... = 0.\dot{3}$

$0.616161... = 0.\overline{61}$

Multiplying by tens

The decimal moves the same number of places as the number of zeroes.

$6.413 \times 100 = 641.3$

Adding and subtracting

When adding and subtracting with decimals make sure the decimal points line up.

Dividing by tens

$71.3 \div 100 = 0.713$

Multiplying decimals

The number of decimal places in the question is the same as in the answer

$0.04 \times 0.3 = 0.012$

Rounding decimals

If the digit after the place you want is 0, 1, 2, 3 or 4 round down; if the digit is 5, 6, 7, 8 or 9 round up.

Dividing decimals

Multiply both numbers by 10, 100, 1000 etc. This is dividing by a whole number.

$$\frac{12.66}{0.3} \overset{\times 10}{\underset{\times 10}{=}} \frac{126.6}{3} = 42.2$$

Percentages out of 100

Percentages and decimals

0.45 = 4.5%

20% = 0.2

Unitary method

$$\begin{aligned} 4\% &= \\ 1\% &= \\ 100\% &= \end{aligned} \quad \begin{aligned} &\div 4 \\ &\times 100 \end{aligned}$$

Percentage change

$$\text{Percentage change} = \frac{\text{change}}{\text{original value}} \times \frac{100}{1}\%$$

Percentages and fractions

$$\frac{3}{5} = \frac{3}{5} \times 100\%$$

$$= 60\%$$

$$35\% = \frac{35}{100} = \frac{7}{20}$$

Common conversions

$\frac{1}{2} = 0.5 = 50\%$

$\frac{1}{10} = 0.1 = 10\%$

$\frac{1}{100} = 0.01 = 1\%$

$\frac{1}{4} = 0.25 = 25\%$

$\frac{3}{4} = 0.75 = 75\%$

$\frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\%$

$\frac{2}{3} = 0.\dot{6} = 66\frac{2}{3}\%$

Of means ×

7% of 400

$= \frac{7}{100} \times 400$

% of

$7\% \text{ of } 20 = \frac{7}{100} \times \frac{20}{1}$

$= 1.4$

Increase

Find the % and add to the original mark up profit.

Decrease

Find the % and subtract from the original discount, sale, depreciate.

Multiple-choice questions

3C **1** 0.36 expressed as a fraction is:

A $\frac{36}{10}$ **B** $\frac{36}{100}$ **C** $\frac{3}{6}$ **D** $\frac{9}{20}$

3B **2** $\frac{1}{8}+\frac{5}{8}$ is equal to:

A $\frac{6}{64}$ **B** $\frac{6}{16}$ **C** $\frac{15}{8}$ **D** $\frac{6}{8}$

3D **3** When 21.63 is multiplied by 13.006, the number of decimal places in the answer is:

A 2 **B** 3 **C** 4 **D** 5

3A **4** $2\frac{1}{3}$ is the same as:

A 7 **B** $\frac{3}{7}$ **C** $\frac{7}{3}$ **D** 2.3

3B **5** The reciprocal of $\frac{3}{4}$ is:

A $\frac{4}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $1\frac{1}{2}$

3C **6** Which decimal has the largest value?

A 6.0061 **B** 6.06 **C** 6.016 **D** 6.0006

3D **7** 9.46×1000 is:

A 94 600 000 **B** 9460 **C** 94 605 **D** 0.000 0946

3G **8** 75% of 84 is the same as:

A $\frac{84}{4}\times 3$ **B** $\frac{84}{3}\times 4$ **C** $84\times 100 \div 75$ **D** $\frac{(0.75\times 84)}{100}$

3J **9** If 1% equals 8, then 5% equals:

A 800 **B** 80 **C** 40 **D** 4

3H **10** \$790 increased by 10% gives:

A \$79 **B** \$880 **C** \$771 **D** \$869

Short-answer questions

3A **1** Copy and complete.

a $\frac{7}{20}=\frac{\square}{60}$

b $\frac{25}{40}=\frac{5}{\square}$

c $\frac{4}{7}=\frac{\square}{21}$

3A **2** Simplify.

a $\frac{25}{45}$ **b** $\frac{36}{12}$ **c** $\frac{16}{12}$

3B **3** Evaluate.

a $\frac{1}{4}+\frac{1}{4}$ **b** $\frac{5}{6}-\frac{4}{6}$ **c** $\frac{7}{8}-\frac{4}{8}$ **d** $\frac{7}{10}+\frac{1}{10}$

e $\frac{7}{8}-\frac{3}{4}$ **f** $\frac{1}{4}+\frac{1}{2}$ **g** $\frac{5}{12}+\frac{1}{4}$ **h** $\frac{3}{5}+\frac{7}{10}$

3B **4** Find:

a $3-1\frac{1}{4}$ **b** $1\frac{1}{2}+2\frac{1}{2}$ **c** $10-3\frac{1}{2}$ **d** $3\frac{4}{5}+1\frac{2}{5}$

3B **5** Find:

a $\frac{2}{3}$ of 6 **b** $\frac{1}{5}$ of 10 **c** $\frac{2}{3}\times 12$ **d** $\frac{3}{5}\times 20$

3B **6** Find:

a $\frac{1}{2}\times\frac{1}{3}$ **b** $\frac{2}{5}\times\frac{1}{4}$ **c** $\frac{7}{8}\times\frac{2}{5}$ **d** $1\frac{1}{2}\times\frac{2}{9}$

3B **7** Calculate these divisions.

a $6\div\frac{1}{2}$ **b** $\frac{2}{3}\div\frac{1}{3}$ **c** $\frac{4}{5}\div\frac{1}{2}$ **d** $1\frac{1}{2}\div\frac{3}{4}$

3C **8** Convert these fractions to decimals.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$ **d** $\frac{117}{1000}$

3C **9** Write these decimals as simple fractions.

a 0.6 **b** 0.12 **c** 0.04 **d** 0.95

3D **10** Evaluate.

a $12.6+7.4$ **b** $8.59+5.6$

c $9.4-1.2$ **d** $10-5.4$

e $9.6+10.1+3.21$ **f** $12.4-6.22$

3D **11** Evaluate.

a 3×2 **b** 0.3×0.2

c 1.2×4 **d** 0.12×0.4

e 1.5×0.4 **f** 7.164×100

g 9.6×10 **h** 0.06×7

3D **12** Find:

a $12\div 0.3$ **b** $18.6\div 3$ **c** $14.22\div 0.2$

3E **13** Round these decimals to 3 decimal places.

a 0.666... **b** 3.57964 **c** 0.00549631

3F **14** Copy and complete this table of conversions.

0.1					0.75		
	$\frac{1}{100}$			$\frac{1}{4}$		$\frac{1}{3}$	$\frac{1}{8}$
		5%	50%				

3G **15** Find:

a 10% of \$50
b 25% of \$64
c 5% of 700 g

3G **16** Express each of the following as a percentage.

a \$35 out of \$40
b 6 out of 24
c \$1.50 out of \$1
d 16 cm out of 4 m

3H **17** **a** Increase \$560 by 10%.
b Decrease \$4000 by 15%.

3H **18** If 6% of an amount is \$18, what is the amount?

3I **19** Toni bought a \$194 dress on sale for 20% off.
What did Toni pay for the dress?

3I/J **20** Sally earned \$84 000 last year. This year she got 5% more.
What did Sally earn this year?

Extended-response questions

1 The following table shows the value of A$1 (one Australian dollar) in foreign currency.

Genevieve is planning an extended holiday to Asia. She plans on visiting India, Singapore, Phuket and Hong Kong.

Currency	A$1
Indian rupee (INR)	42
Singapore dollar (SGD)	1.25
Thai baht (THB)	30
Hong Kong dollar (HKD)	7

a She has decided to change some Australian dollars to each of the above currencies before she flies out. How much of each currency will she receive if she changes A$500 to each currency?

b If she spent 70% of her Thai baht on hotels, how much does she have left to spend?

c After visiting Hong Kong, Genevieve has $42 HKD left. What does this convert back to in Australian dollars?

Chapter 4

Measurement

What you will learn

- 4A Length and perimeter (Consolidating)
- 4B Circumference of a circle
- 4C Area
- 4D Area of special quadrilaterals
- 4E Area of a circle
- 4F Volume and capacity
- 4G Volume of prisms
- 4H Time

Australian curriculum

MEASUREMENT AND GEOMETRY

Real numbers

Investigate the concept of irrational numbers, including π **(ACMNA186)**

Using units of measurement

Choose appropriate units of measurement for area and volume and convert from one unit to another **(ACMMG195)**

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites **(ACMMG196)**

Investigate the relationship between features of circles such as circumference, area, radius and diameter. Use formulas to solve problems involving circumference and area **(ACMMG197)**

Develop formulas for volumes of rectangular and triangular prisms and prisms in general. Use formulas to solve problems involving volume **(ACMMG198)**

Solve problems involving duration, including using 12- and 24-hour time within a single time zone **(ACMMG199)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

The box that changed cargo shipping

Thousands of cargo ships navigate the world's vast oceans, transporting goods from one country to another. Loading and unloading millions of items was time consuming and costly until the introduction of shipping containers, steel boxes of fixed dimensions. The uniform size of containers ensures constant volume for costings, organised stacking on ships and docks and efficient land transport by truck or train.

Standard shipping container Imperial dimensions are 8 foot by 8 foot 6 inches by 20 foot or 40 foot. Using the conversion factor of 1 foot = 0.3048 metres, gives metric dimensions of 2.44 m by 2.59 m by 6.1 m or 12.19 m.

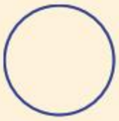
There are now over 6000 purpose-built container ships transporting more than 20 million shipping containers of cargo between countries. The largest container ships can carry over 19 000 containers and are 400 m long (equal in length to 4 football fields end-to-end).

Pre-test

1 For each of the following shapes choose the matching name from options A through H.

A: triangle, B: rhombus, C: square, D: parallelogram, E: circle, F: trapezium, G: rectangle, H: kite

a



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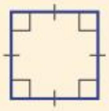

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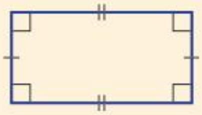

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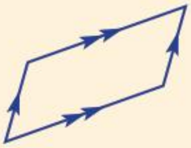
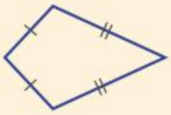
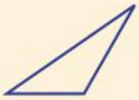


e

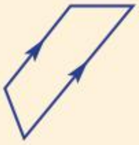
f



g

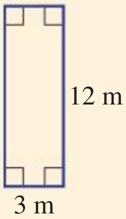


h

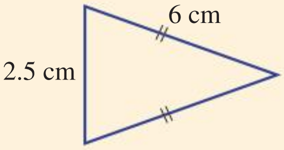


2 Find the perimeter (distance around the outside) of these shapes.

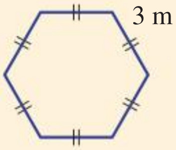
a



b



c



3 Evaluate the following.

a $\frac{1}{2}\times 5\times 4$

b $\frac{1}{2}(2+7)\times 6$

c 5^2

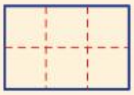
d 11^2

4 Convert these measurements to the units shown in the brackets.

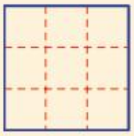
a 3 m (cm)
b 20 cm (mm)
c 1.8 km (m)
d 0.25 m (cm)
e 35 mm (cm)
f 4200 m (km)
g 500 cm (m)
h 100 mm (m)
i 2 minutes (seconds)
j 3 L (mL)
k 4000 mL (L)
l 3000 g (kg)

5 Count squares to find the area of these shapes.

a



b

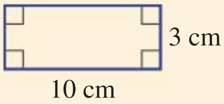


c

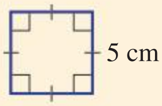
6 Find the area of these rectangles and triangles.

Remember: Area (rectangle) $= l \times w$ and Area (triangle) $= \frac{1}{2}bh$

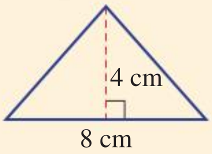
a



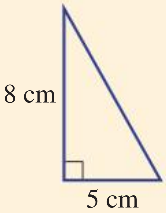
b



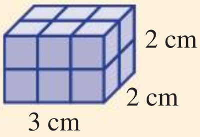
c



d



7 Count cubes to find the volume of this solid.



4A Length and perimeter

CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthrough

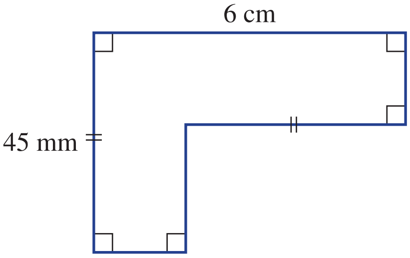
Developed in France in the 1790s, the metric system for measurement includes length units such as millimetre, centimetre, metre and kilometre.

We use such units to describe, for example, the distance between two towns, the perimeter of a block of land, the depth of the ocean or the length of a racetrack.

Let's start: Provide the perimeter

In this diagram some of the lengths are given. Three students were asked to find the perimeter.

- Will says that you cannot work out some lengths and so the perimeter cannot be found.
- Sally says that there is enough information and the answer is $9 + 12 = 21$ cm.
- Greta says that there is enough information but the answer is $90 + 12 = 102$ cm.



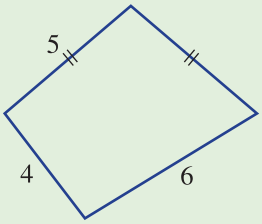
Who is correct?
Discuss how each person arrived at their answer.

Key ideas

- The common metric units of length include:
  - **kilometre** (km) $1 \text{ km} = 1000 \text{ m}$
  - **metre** (m) $1 \text{ m} = 100 \text{ cm}$
  - **centimetre** (cm) $1 \text{ cm} = 10 \text{ mm}$
  - **millimetre** (mm)

$$\begin{tikzcd}
\text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"]
\end{tikzcd}$$

- **Perimeter** is the distance around a closed shape.
  - All units must be of the same type when calculating the perimeter.
  - Sides with the same type of markings (dashes) are of equal length.



$$\begin{aligned} P &= 2 \times 5 + 4 + 6 \\ &= 20 \end{aligned}$$

Perimeter
The total distance (length) around the outside of a figure

Exercise 4A

UNDERSTANDING 1–4 | 4

1 Write the missing words.

- **a** The commonly used measurement system used today is called the ______ system.
- **b** The common metric units for length include millimetres, __________, __________ and __________.

2 Evaluate the following.

- **a** 2×100
- **b** 5.2×1000
- **c** 7.8×10
- **d** $840 \div 100$
- **e** $9610 \div 10$
- **f** $41\,200 \div 1000$

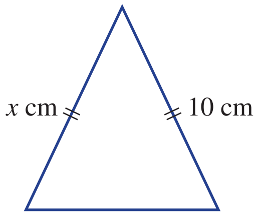
Move the decimal point to the right for × and left for ÷.

3 Write the missing number in these sentences.

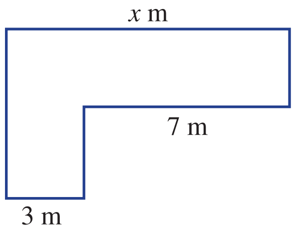
- **a** There are ___ mm in 1 cm.
- **b** There are ___ cm in 1 m.
- **c** There are ___ m in 1 km.
- **d** There are ___ cm in 1 km.
- **e** There are ___ mm in 1 m.
- **f** There are ___ mm in 1 km.

4 Find the value of x in these diagrams.

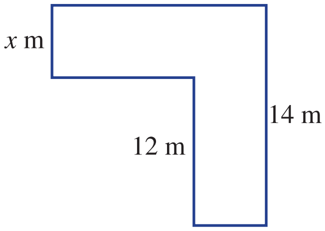
a



b



c



FLUENCY 5–6(½) | 5–6(½), 7

Example 1 Converting length measurements

Convert these lengths to the units shown in the brackets.

- **a** 5.2 cm (mm)
- **b** 2400 m (km)

Solution	Explanation
a $5.2\text{ cm} = 5.2 \times 10$ $= 52\text{ mm}$	$1\text{ cm} = 10\text{ mm}$ so multiply by 10. $\text{cm} \xrightarrow{\times 10} \text{mm}$
b $2400\text{ m} = 2400 \div 1000$ $= 2.4\text{ km}$	$1\text{ km} = 1000\text{ m}$ so divide 1000.

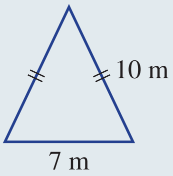
5 Convert these measurements to the units shown in the brackets.

- **a** 3 cm (mm)
- **b** 6.1 m (cm)
- **c** 8.93 km (m)
- **d** 3 m (cm)
- **e** 0.0021 km (m)
- **f** 320 mm (cm)
- **g** 19620 m (km)
- **h** 38 000 cm (m)
- **i** 48 mm (cm)
- **j** 0.2 cm (mm)
- **k** 4.2 cm (m)
- **l** 0.4 m (cm)
- **m** 3700 m (km)
- **n** 600 m (km)
- **o** 0.71 km (m)
- **p** 0.02 m (cm)

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

Example 2 Finding perimeters

Find the perimeter of this triangle.

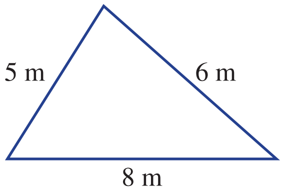


Solution	**Explanation**
$P = 2 \times 10 + 7$ $= 27$ m	There are two equal 10 m lengths and one 7 m length to add up.

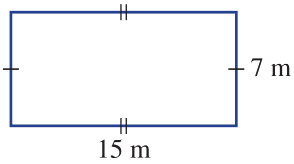
Sides with the same markings have the same length.

6 Find the perimeter of these shapes.

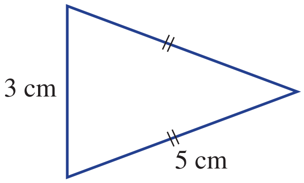
a



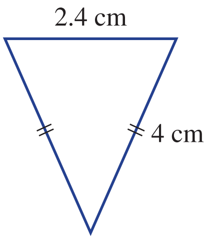
b



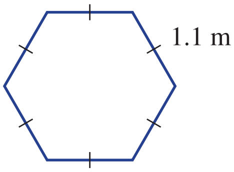
c



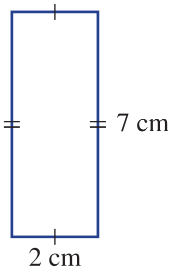
d



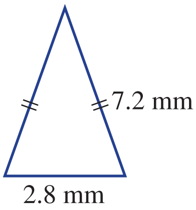
e



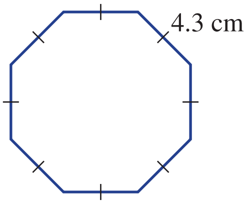
f



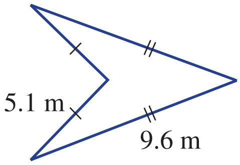
g



h

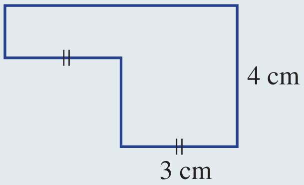


i



Example 3 Finding perimeters of rectangular shapes

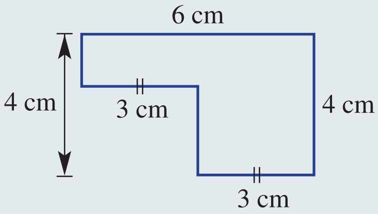
Find the perimeter of this shape.



Solution

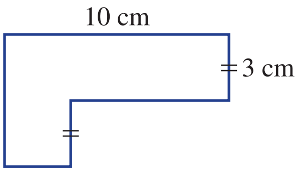
$P = 2 \times (3 + 3) + 2 \times 4$
$= 12 + 8$
$= 20$ cm

Explanation

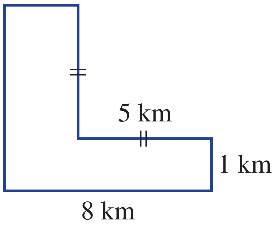


7 Find the perimeter of these shapes.

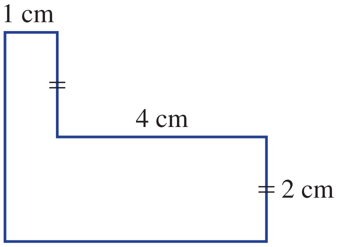
a



b



c



PROBLEM-SOLVING AND REASONING | 8–9(½), 10 | 8–9(½), 11–13

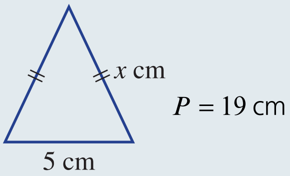
8 Convert these measurements to the units shown in the brackets.

- **a** 0.0043 m (mm)
- **b** 0.0204 km (cm)
- **c** 23 098 mm (m)
- **d** 342 000 cm (km)
- **e** 194 300 mm (m)
- **f** 10 000 mm (km)
- **g** 0.024 03 m (mm)
- **h** 994 000 mm (km)
- **i** 0.000 01 km (cm)

You will need to multiply or divide by at least two factors e.g.
× 100 × 10 or
÷ 1000 ÷ 100

Example 4 Finding an unknown length

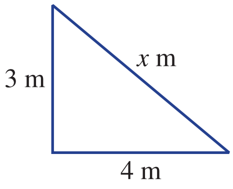
Find the unknown value x in this triangle if the perimeter is 19 cm.



Solution	Explanation
$2x + 5 = 19$ $2x = 14$ $x = 7$	$2x + 5$ makes up the perimeter, which is 19. Solve the equation algebraically or use a guess and check method to find the value of x.

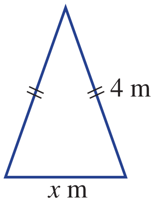
9 Find the unknown value x in these shapes with the given perimeter (P).

a



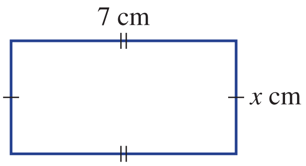
$P = 12$ m

b



$P = 10$ m

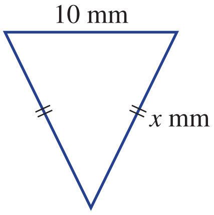
c



$P = 22$ cm

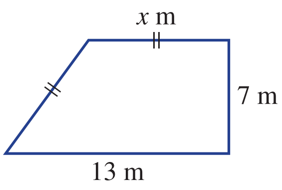
Use the given perimeter to find the value of x.

d



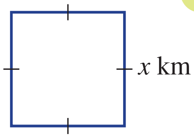
$P = 46$ mm

e



$P = 39$ m

f



$P = 26$ km

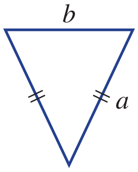
10 Jennifer needs to fence her country house block to keep her dog in. The block is a rectangle with length 50 m and width 42 m. Fencing costs $13 per metre. What will be the total cost of fencing?

11 Gillian can jog 100 metres in 24 seconds. How long will it take her to jog 2 km? Give your answer in minutes. (There are 60 seconds in one minute.)

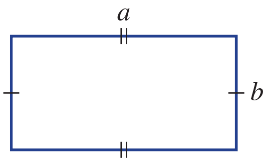
12 A rectangular picture of length 65 cm and width 35 cm is surrounded by a frame of width 5 cm. What is the perimeter of the framed picture?

13 Write down rules using the given letters for the perimeter of these shapes, e.g. $P = a + 2b$.

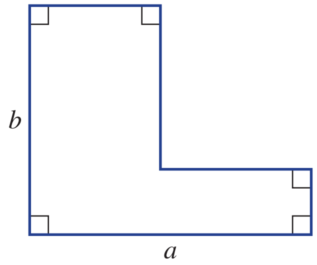
a



b



c

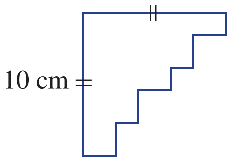


Perimeter challenge

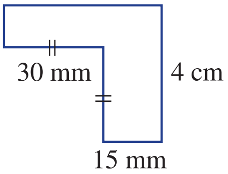
—	14

14 Find the perimeter of these shapes. Give your answers in cm.

a

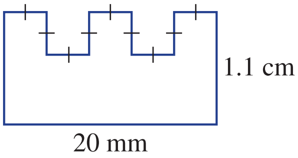


b

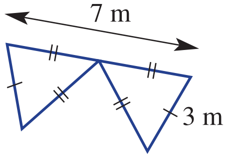


Check to make sure the units are all the same.

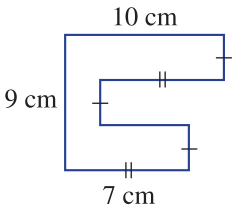
c



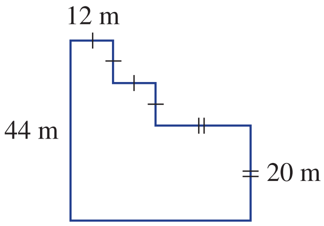
d



e



f



4B Circumference of a circle

Interactive

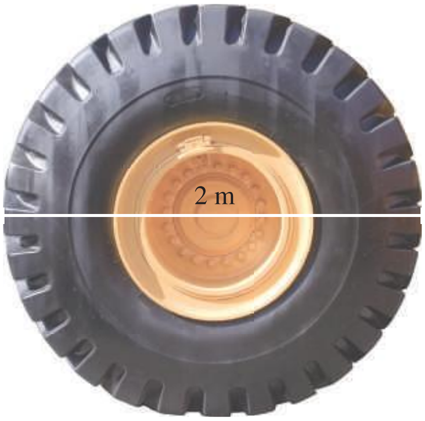
Widgets

HOTsheets

Walkthrough

The distance around the outside of a circle, known as the circumference, is connected to the diameter through a special number called **pi**.

The symbol for pi is π, and as a decimal $\pi = 3.14159 \ldots$. There is no fraction that approximates pi exactly, which is why we often use calculators when working with this number.



Circumference = $2\pi r = 6.2831.....$

Let's start: Discovering pi

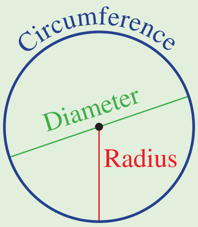
Here are the diameters and circumferences for three circles, correct to two decimal places. Use a calculator to work out the value of Circumference ÷ Diameter and put your results in the third column. Add your own circle measurements by measuring the diameter and circumference of circular objects such as a can or a wheel.

- What do you notice about the numbers $C \div d$ in the third column?
- Why might the numbers in the third column vary slightly from one set of measurements to another?
- What rule can you write down that links C with d?

Diameter d (mm)	Circumference C (mm)	$C \div d$
2.23	7.01	
5.94	18.66	
20.65	64.87	
Add your own	Add your own	

Key ideas

- Features of a circle
  - **Diameter** (d) is the distance across the centre of a circle.
  - **Radius** (r) is the distance from the centre to the circle. Note $d = 2r$.
- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi r$ or $C = \pi d$
- **Pi** (π) ≈ 3.14159 (correct to five decimal places)
  - Common approximations include 3.14 and $\frac{22}{7}$.
  - A more precise estimate for pi can be found on most calculators or on the internet.



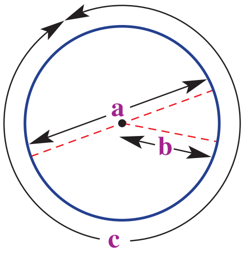
Radius The distance from the centre of a circle to its outside edge

Circumference The curved boundary of a circle

Exercise 4B

UNDERSTANDING 1–4 4

1 Name the features of the circle as shown in the diagram.



2 **a** Find the diameter of a circle if its radius is:
  i 5 m **ii** 11 cm **iii** 2.3 mm
b Find the radius of a circle if its diameter is:
  i 12 cm **ii** 31 mm **iii** 0.42 m

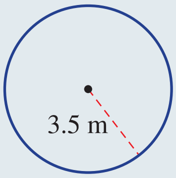
3 Write down the value of π correct to:
a one decimal place **b** two decimal places
c three decimal places

4 Evaluate the following using a calculator and round to two decimal places.
a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

FLUENCY 5, 6(½) 5–6(½)

Example 5 Finding the circumference using the radius

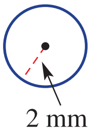
Find the circumference of this circle correct to two decimal places. Use a calculator for the value of pi.



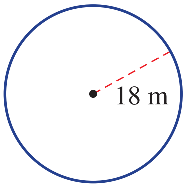
Solution	Explanation
$C = 2\pi r$ $= 2 \times \pi \times 3.5$ $= 7\pi$ $= 21.99$ m (to 2 d.p.)	Since r is given, you can use $C = 2\pi r$.

5 Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of pi.

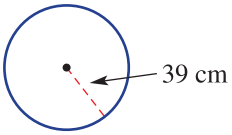
a



b

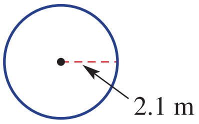


c

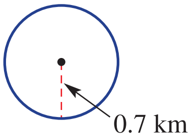


Use the rule $C = 2\pi r$ and substitute the value of r.

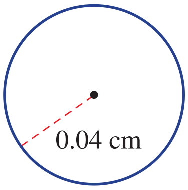
d



e

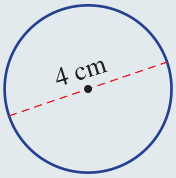


f



Example 6 Finding the circumference using the diameter

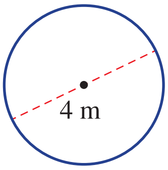
Find the circumference of this circle, correct to two decimal places.



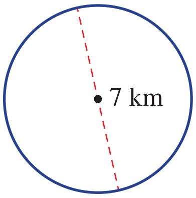
Solution	Explanation
$C = \pi d$ $= \pi \times 4$ $= 4\pi$ $= 12.57$ cm (to 2 d.p.)	Substitute $d = 4$ into the rule $C = \pi d$ or use $C = 2\pi r$ with $r = 2$.

6 Find the circumference of these circles, correct to two decimal places.

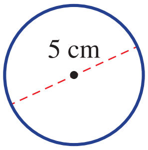
a



b

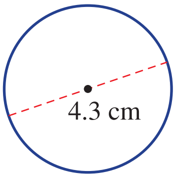


c



Use the rule $C = \pi d$ and substitute the value of d.

d



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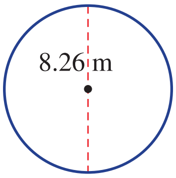

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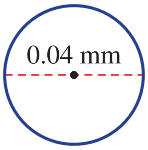


PROBLEM-SOLVING AND REASONING 7–10 10–13

7 The diameter of the circular face of a metal drum is 80 cm. Find its circumference, correct to the nearest whole centimetre.

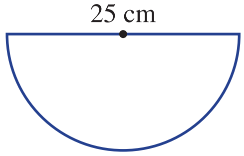
8 A water tank has a diameter of 3.5 m. Find its circumference, correct to one decimal place.

9 A wheel of radius 28 cm rolls one full turn. Find how far it rolls correct to the nearest centimetre.

10 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

11 These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to two decimal places.

a



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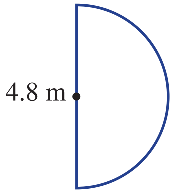

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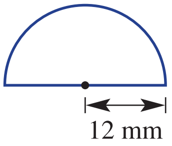
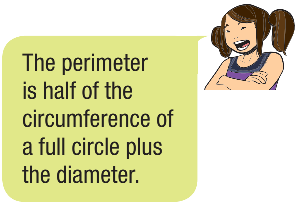




12 Here are some student's approximate circle measurements. Which students have incorrect measurements?

	r	C
Mick	4 cm	25.1 cm
Svenya	3.5 m	44 m
Andre	1.1 m	13.8 m

13 Explain why the rule $C = 2\pi r$ is equivalent (i.e. the same as) $C = \pi d$.

Memorising pi

— 14

14 The box shows π correct to 100 decimal places. The Guinness World record for the most number of digits of π recited from memory is held by Rajveer Meena from India. He recited 70 000 digits non-stop over a 10-hour period.

3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679

Challenge your friends to see who can remember the most number of digits in the decimal representation of π.

Number of digits memorised	Report
10+	A good show
20+	Great effort
35+	Superb
50+	Amazing memory

4C Area

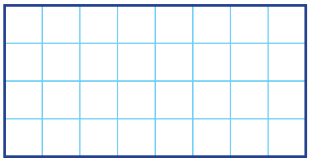
Interactive
Widgets
HOTsheets
Walkthrough

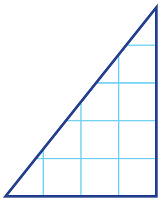
The amount of space on a surface is called area. Area is measured in square units and the common metric units are square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), square kilometres (km^2) and hectares (ha).

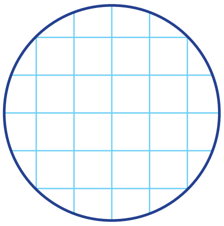
The hectare is often used to describe area of land, since the square kilometre for such areas is considered to be too large a unit and the square metre too small. A school football oval might be about 1 hectare, for example, and a small forest might be about 100 hectares.

Let's start: Estimating area

By counting squares, or by using an estimate, you can find the area of a shape.
For the following shapes find or estimate their area. Explain your method for each one.







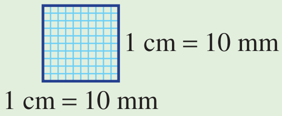
Key ideas

- The common metric units for area include:
  - square millimetres (mm^2)
  - square centimetres (cm^2)
  - square metres (m^2)
  - square kilometres (km^2)
  - hectares (ha)

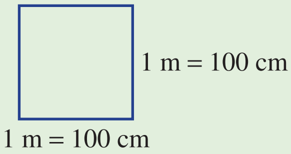
$$\begin{tikzcd}[column sep=huge]
km^2 \arrow[r, bend left, "{\times 1000^2 = 1\,000\,000}"] & m^2 \arrow[l, bend left, "{\div 1000^2 = 1\,000\,000}"] \arrow[r, bend left, "{\times 100^2 = 10\,000}"] & cm^2 \arrow[l, bend left, "{\div 100^2 = 10\,000}"] \arrow[r, bend left, "{\times 10^2 = 100}"] & mm^2 \arrow[l, bend left, "{\div 10^2 = 100}"]
\end{tikzcd}$$

$$\begin{tikzcd}[column sep=huge]
ha \arrow[r, bend left, "\times 10\,000"] & m^2 \arrow[l, bend left, "\div 10\,000"]
\end{tikzcd}$$

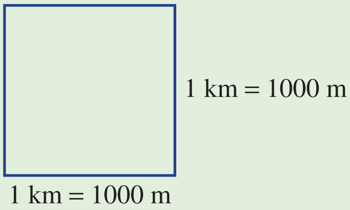
$$1\,cm^2 = 10 \times 10 = 100\,mm^2$$



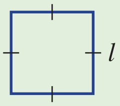
$$1\,m^2 = 100 \times 100 = 10\,000\,cm^2$$



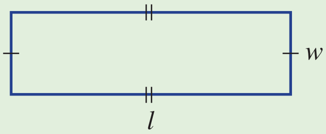
$$1\,km^2 = 1000 \times 1000 = 1\,000\,000\,m^2$$



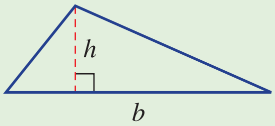
- Area of squares, rectangles and triangles
  - Square $A = l \times l = l^2$



  - Rectangle $A = l \times w = lw$



  - Triangle $A = \frac{1}{2} \times b \times h = \frac{1}{2}bh$



  - The dashed line which gives the height is **perpendicular** (at right angles) to the base.

Perpendicular
At right angles to another line or surface

Exercise 4C

UNDERSTANDING 1–3 3

1 Write the rules for the area of these shapes.

a rectangle **b** square **c** triangle

2 By considering the given diagrams answer the questions.

a **i** How many mm^2 in $1\ cm^2$?

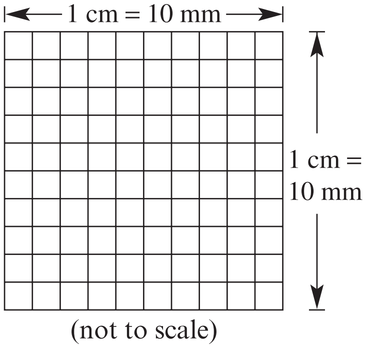
ii How many mm^2 in $4\ cm^2$?

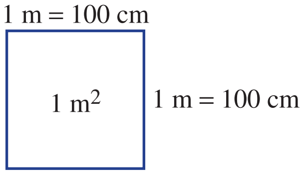
iii How many cm^2 in $300\ mm^2$?

b **i** How many cm^2 in $1\ m^2$?

ii How many cm^2 in $7\ m^2$?

iii How many m^2 in $40\,000\ cm^2$?

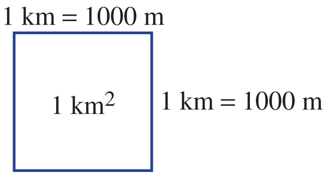




c **i** How many m^2 in $1\ km^2$?

ii How many m^2 in $5\ km^2$?

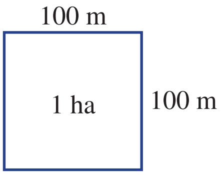
iii How many km^2 in $2\,500\,000\ m^2$?



d **i** How many m^2 in 1 ha?

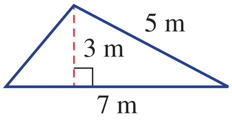
ii How many m^2 in 3 ha?

iii How many ha in $75\,000\ m^2$?

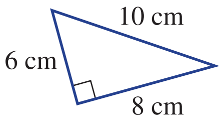


3 Which length measurements would be used for the *base* and the *height* (in that order) to find the area of these triangles?

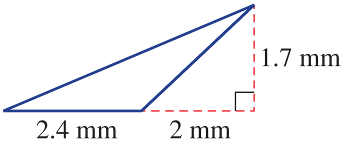
a



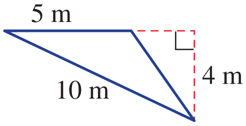
b



c



d



Recall that the base and height are perpendicular (at 90°).

FLUENCY 4–6(½) 4–6(½)

Example 7 Converting units of area

Convert these area measurements to the units shown in the brackets.

a 0.248 m^2 (cm^2) **b** 3100 mm^2 (cm^2)

Solution	**Explanation**
a $0.248\ m^2 = 0.248 \times 10\,000$ $= 2480\ cm^2$	$1\ m^2 = 100^2\ cm^2 = 10\,000\ cm^2$ Multiply since you are changing to a smaller unit. $m^2 \xrightarrow{\times 100^2} cm^2$
b $3100\ mm^2 = 3100 \div 100$ $= 31\ cm^2$	$1\ cm^2 = 10^2\ mm^2 = 100\ mm^2$ Divide since you are changing to a larger unit. $cm^2 \xleftarrow[\div 10^2]{} mm^2$

4 Convert these area measurements to the units shown in the brackets.

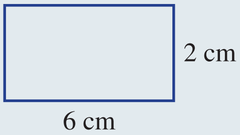
a 2 cm^2 (mm^2) **b** 7 m^2 (cm^2)
c 0.5 km^2 (m^2) **d** 3 ha (m^2)
e 0.34 cm^2 (mm^2) **f** 700 cm^2 (m^2)
g 3090 mm^2 (cm^2) **h** 0.004 km^2 (m^2)
i 2000 cm^2 (m^2) **j** 450 000 m^2 (km^2)
k 4000 m^2 (ha) **l** 3210 mm^2 (cm^2)
m 320 000 m^2 (ha) **n** 0.0051 m^2 (cm^2)
o 0.043 cm^2 (mm^2) **p** 4802 cm^2 (m^2)
q 19 040 m^2 (ha) **r** 2933 m^2 (ha)
s 0.0049 ha (m^2) **t** 0.77 ha (m^2)

$1\ cm^2 = 10 \times 10$
$= 100\ mm^2$
$1\ m^2 = 100 \times 100$
$= 10\,000\ cm^2$
$1\ km^2 = 1000 \times 1000$
$= 1\,000\,000\ m^2$
$1\ ha = 100 \times 100$
$= 10\,000\ m^2$

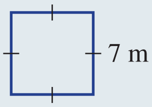
Example 8 Finding the area of rectangles and squares

Find the area of these shapes.

a



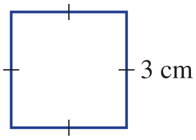
b



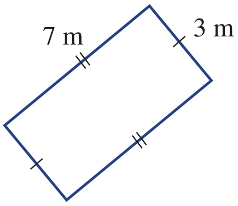
Solution	Explanation
a $A = lw$ $= 6 \times 2$ $= 12\ \text{cm}^2$	Write the formula for the area of a rectangle and substitute $l = 6$ and $w = 2$.
b $A = l^2$ $= 7^2$ $= 49\ \text{m}^2$	For a square, multiply the length of a side by itself to get the area.

5 Find the areas of these squares and rectangles.

a



b



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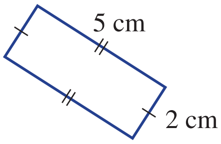

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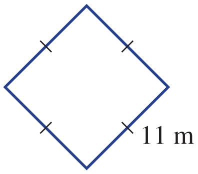

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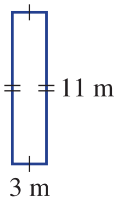

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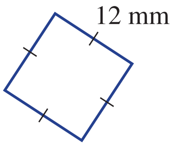
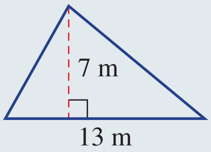


Use $A = l \times w$ or $A = l^2$.

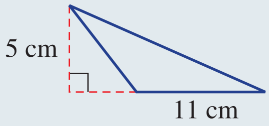
Example 9 Finding the area of triangles

Find the area of these triangles.

a



b

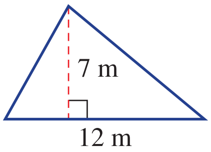


Solution	Explanation
a $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 13 \times 7$ $= 45.5\ \text{m}^2$	Remember that the height is measured using a line that is perpendicular to the base.

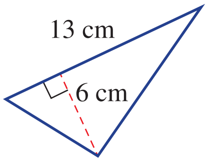
Solution	**Explanation**
b $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 11 \times 5$ $= 27.5\ \text{cm}^2$	The base is 11 cm and the height is 5 cm so use $b = 11$ and $h = 5$.

6 Find the area of these triangles.

a

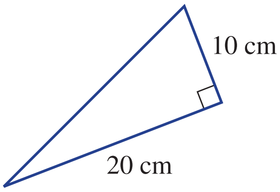


b



Use $A = \frac{1}{2}bh$ and choose the base and height so they are perpendicular (at 90°).

c



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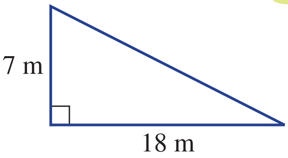

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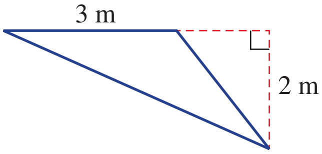

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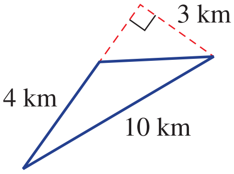


PROBLEM-SOLVING AND REASONING 7–9 9–12

7 A rectangular park has a length of 100 m and an area of 5000 m². What is its width?

8 A triangle has area 20 cm² and base 4 cm. Find its height.

9 Find the side length of a square if its area is:

- **a** 36 m²
- **b** 2.25 cm²

10
- **a** Find the area of a square if its perimeter is 20 m.
- **b** Find the area of a square if its perimeter is 18 cm.
- **c** Find the perimeter of a square if its area is 49 cm².
- **d** Find the perimeter of a square if its area is 169 m².

First find the side length of the square.

4C

11 Paint costs \$12 per litre and can only be purchased in a full number of litres. One litre of paint covers an area of $10\ \text{m}^2$. A rectangular wall is 6.5 m long and 3 m high and needs two coats of paint. What will be the cost of paint for the wall?

12 Use your knowledge of area units to change these measurements to the units shown in the brackets.

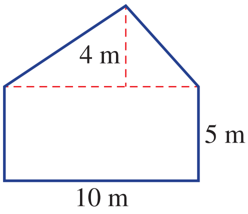
a $0.2\ \text{m}^2$ (mm^2)
b $0.000\,043\ \text{km}^2$ (cm^2)
c $374\,000\ \text{cm}^2$ (km^2)
d $10\,920\ \text{mm}^2$ (m^2)
e $0.000\,000\,2$ ha (cm^2)
f $1\,000\,000\,000\ \text{mm}^2$ (ha)

Composite shapes

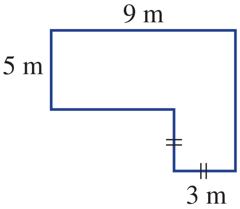
13

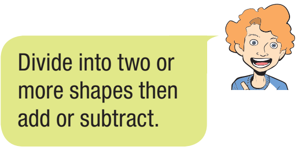
13 Find the area of these composite shapes by using addition or subtraction.

a



b





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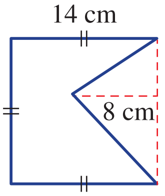

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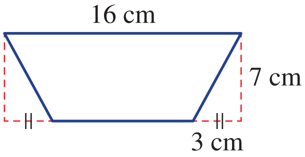

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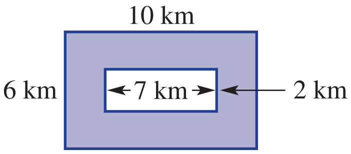
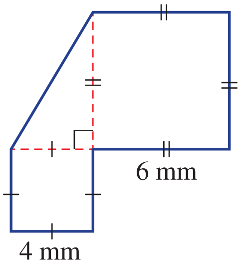


(Find the area of the shaded region)

f



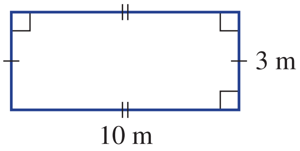
4D Area of special quadrilaterals

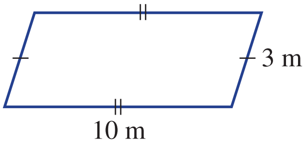
We have used formulas to work out the area of rectangles ($A = lw$), squares ($A = l^2$) and triangles ($A = \frac{1}{2}bh$).

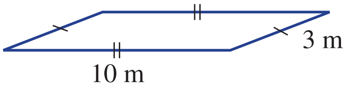
In this section, we will develop and use formulas for a set of special quadrilaterals including the parallelogram, rhombus, kite and trapezium.

Let's start: How is a parallelogram like a rectangle?

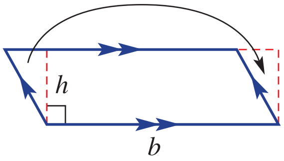
Do all parallelograms, including rectangles, with the same side lengths have the same area? Use these diagrams to help in your discussion.





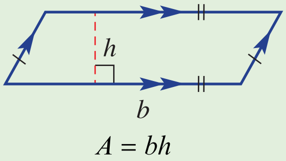


- How does this diagram help you work out the area of a parallelogram?
- Can you write the rule for the area of a parallelogram?

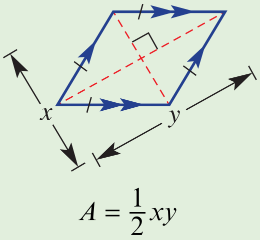


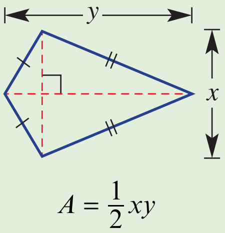
Key ideas

- Area of a **parallelogram**
  Area = base × perpendicular height
  or $A = bh$

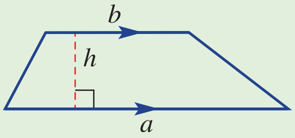


- Area of a **rhombus** and **kite**
  Area $= \frac{1}{2} \times$ diagonal $x \times$ diagonal y
  or $A = \frac{1}{2}xy$





- Area of a **trapezium**
  Area $= \frac{1}{2} \times$ sum of parallel sides × perpendicular height
  or $A = \frac{1}{2}(a+b)h$ or $A = \frac{h}{2}(a+b)$



Parallelogram
A quadrilateral with both pairs of opposite sides parallel

Rhombus
A quadrilateral with both pairs of opposite sides parallel and all sides equal

Kite
A quadrilateral with two pairs of adjacent sides equal

Trapezium
A quadrilateral with at least one pair of parallel sides

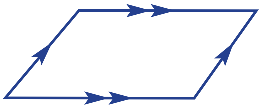
Exercise 4D

UNDERSTANDING 1–3 3

1 Match each formula with a shape.

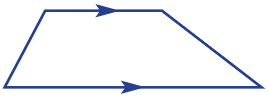
a $A = lw$ **b** $A = \frac{1}{2}xy$ **c** $A = bh$ **d** $A = \frac{1}{2}(a+b)h$

A

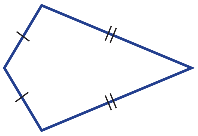


B

C



D



2 Find the value of A using these formulas and given values. Substitute the given values into the formulas.

a $A = bh \ (b = 2, h = 3)$

b $A = \frac{1}{2}xy \ (x = 5, y = 12)$

c $A = \frac{1}{2}(a+b)h \ (a = 2, b = 7, h = 3)$

d $A = \frac{1}{2}(a+b)h \ (a = 7, b = 4, h = 6)$

3 Complete these sentences.

a A perpendicular angle is __________ degrees.

b In a parallelogram, you find the area using a base and the ________________.

c The two diagonals in a kite or a rhombus are ________________.

d To find the area of a trapezium you multiply $\frac{1}{2}$ by the sum of the two ________________ sides and then by the ________________ height.

e The two special quadrilaterals that have the same area formula using diagonal lengths x and y are the ________________ and the ________________.

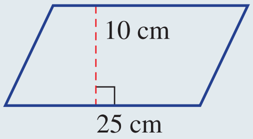
Choose from:
height, 90, parallel, kite, rhombus, perpendicular.

FLUENCY 4–5(½), 6 4–5(½), 6

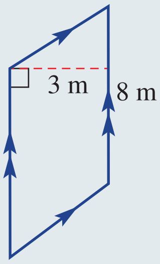
Example 10 Finding the area of parallelograms

Find the area of these parallelograms.

a



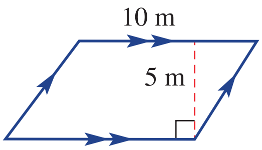
b



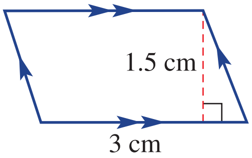
Solution	**Explanation**
a $A = bh$ $= 25 \times 10$ $= 250\text{ cm}^2$	Use $A = bh$ with $b = 25$ and $h = 10$
b $A = bh$ $= 8 \times 3$ $= 24\text{ m}^2$	The height is measured at right angles to the base.

4 Find the area of these parallelograms.

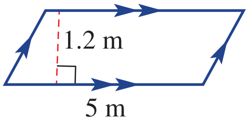
a



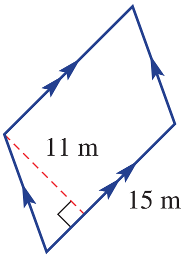
b



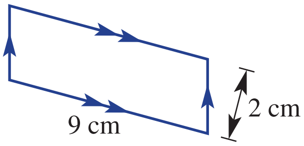
c



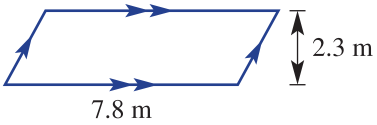
d



e



f

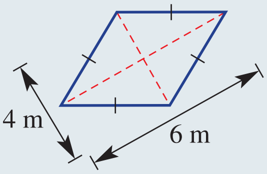


Use $A = bh$ and choose your base and perpendicular height.

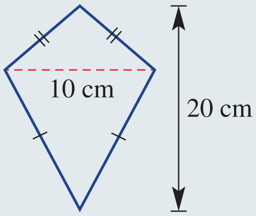
Example 11 Finding the area of rhombuses and kites

Find the areas of this rhombus and kite.

a



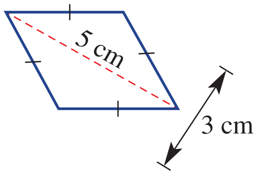
b



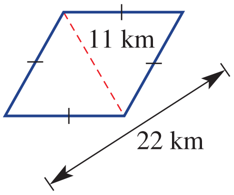
Solution	**Explanation**
a $A = \frac{1}{2}xy$ $= \frac{1}{2} \times 6 \times 4$ $= 12\text{ m}^2$	Use $A = \frac{1}{2}xy$ when the diagonals are given with $x = 6$ and $y = 4$ (or vice versa)
b $A = \frac{1}{2}xy$ $= \frac{1}{2} \times 10 \times 20$ $= 100\text{ cm}^2$	Use the formula $A = \frac{1}{2}xy$ since both diagonals are given. This formula can also be used for a rhombus.

5 Find the area of these rhombuses and kites.

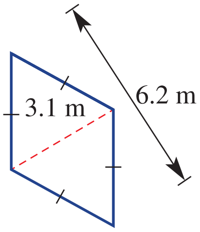
a



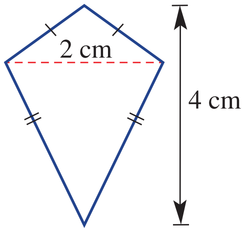
b



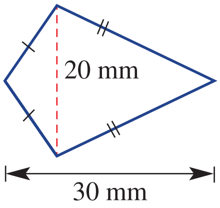
c



d



e



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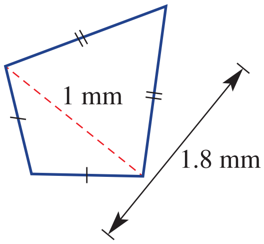
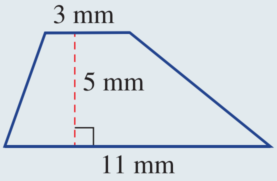


Recall that $A = \frac{1}{2}xy$ for both rhombuses and kites with x and y as the diagonals.

Example 12 Finding the area of trapeziums

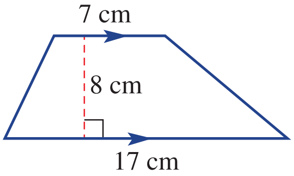
Find the area of this trapezium.



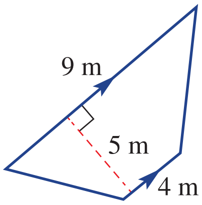
Solution	Explanation
$A = \frac{1}{2}(a + b)h$ $= \frac{1}{2} \times (11 + 3) \times 5$ $= \frac{1}{2} \times 14 \times 5$ $= 35\text{ mm}^2$	The two parallel sides are 11 mm and 3 mm in length. The perpendicular height is 5 mm.

6 Find the area of these trapeziums.

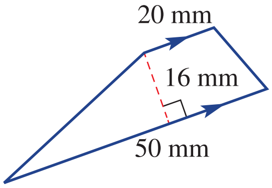
a



b



c



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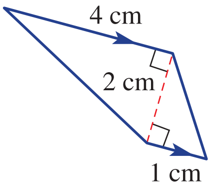
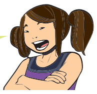
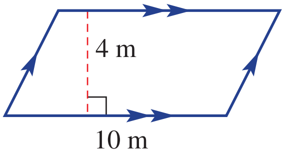




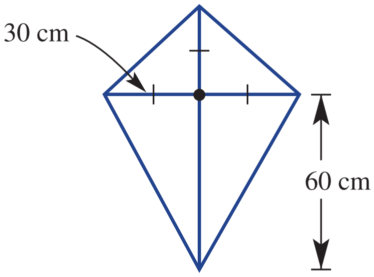
In $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides.

PROBLEM-SOLVING AND REASONING 7–9 9–12

7 A special type of paint costs $3 per square metre and is to be used to paint a wall in the shape of a parallelogram with base 10 m and height 4 m. How much does it cost to paint the wall?



8 A flying kite is made from four centre rods all connected near the middle of the kite as shown. Three of the rods are 30 cm in length and one is 60 cm as shown. What area of plastic, in square metres, is needed to cover the kite?

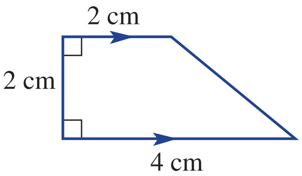


9 A parallelogram has an area of $26\,\text{m}^2$ and its base length is $13\,\text{m}$. What is its perpendicular height?

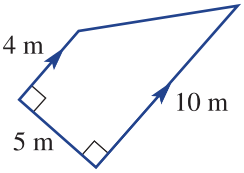
10 A landscape gardener charges $20 per square metre of lawn. A lawn area is in the shape of a rhombus and its diagonals are $8\,\text{m}$ and $14.5\,\text{m}$. What would be the cost of laying this lawn?

11 These trapeziums have one side at right angles to the two parallel sides. Find the area of each.

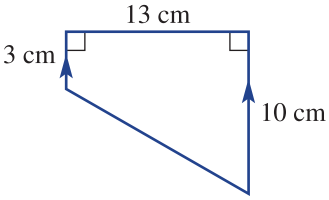
a



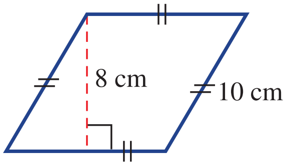
b



c



12 Would you use the formula $A = \frac{1}{2}xy$ to find the area of this rhombus? Explain.



Proving formulas

— 13

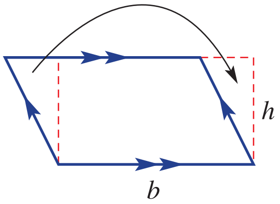
13 Copy and complete these proofs to give the formula for the area of a parallelogram, a rhombus and a trapezium.

a Parallelogram

$A = \text{length} \times \text{width}$

$= \underline{\qquad\quad} \times \underline{\qquad\quad}$

$= \underline{\qquad\quad}$



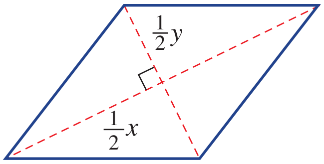
b Rhombus

$A = 4 \text{ triangle areas}$

$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 4 \times \frac{1}{2} \times \underline{\qquad\quad} \times \underline{\qquad\quad}$

$= \underline{\qquad\quad}$



c Trapezium

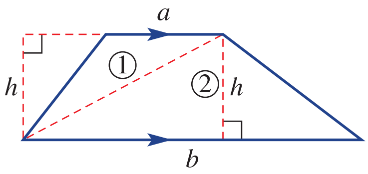
$A = \text{Area (triangle 1)} + \text{Area (triangle 2)}$

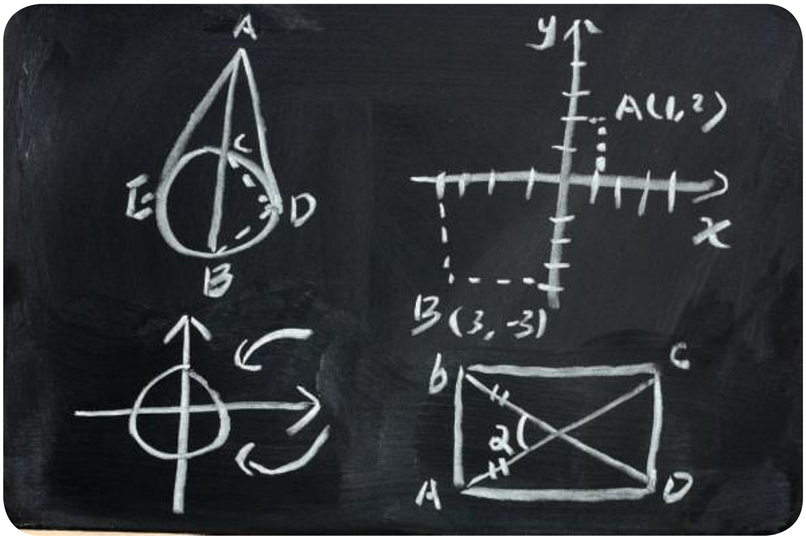
$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$

$= \frac{1}{2} \times \underline{\qquad\quad} \times \underline{\qquad\quad} + \frac{1}{2} \times \underline{\qquad\quad} \times \underline{\qquad\quad}$

$= \underline{\qquad\quad} + \underline{\qquad\quad}$

$= \underline{\qquad\quad}$





4E Area of a circle

Interactive

Widgets

HOTsheets

Walkthrough

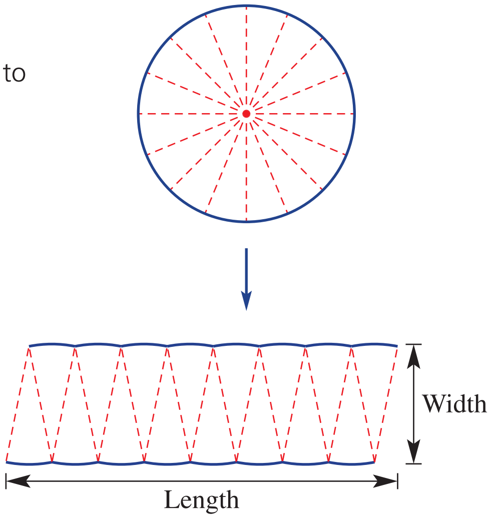
Like the circumference of a circle, the area of a circle is linked to the number pi (π).

One way to consider the area of a circle is to divide it into sectors, then arrange them into a rectangular shape. If very thin sectors are used, then the arrangement will be close to a rectangle with a length that is half the circumference of the circle, or $\frac{1}{2} \times 2\pi r = \pi r$ and width r.

This leads to the area formula: $A = \text{length} \times \text{width}$

$$= \pi r \times r$$

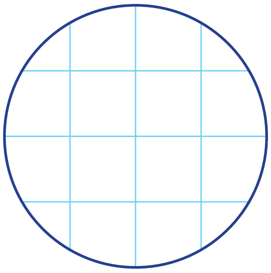
$$= \pi r^2$$



Let's start: Just count squares

To find an estimate for the area of a circle you can count the number of squares.

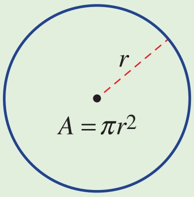
- Count squares to estimate the area of this circle in cm².
- Ask your teacher to give you an accurate measure of its area. Who was the closest?



Key ideas

- The area of a circle is given by the formula $A = \pi r^2$.
  - The diameter is twice the radius: $d = 2r$
  - Substitute the radius into the formula to find the area.
    e.g. If $r = 2$ then

$$A = \pi \times 2^2$$
$$= \pi \times 4$$
$$= 12.57 \text{ (to 2 d.p.)}$$

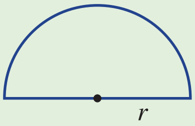


Semicircle
Half a circle

Quadrant
A sector which is one quarter of a circle

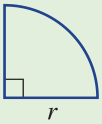
- A half circle is called a **semicircle**.

$$A = \frac{1}{2}\pi r^2$$



- A quarter circle is called a **quadrant**.

$$A = \frac{1}{4}\pi r^2$$



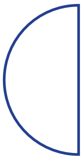
Exercise 4E

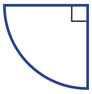
UNDERSTANDING 1–4 4

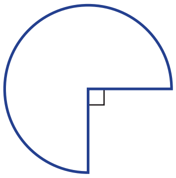
1 Write the rule for:
a the circumference of a circle.
b the area of a circle.

2 Use a calculator to evaluate these to two decimal places.
a $\pi \times 5^2$
b $\pi \times 13^2$
c $\pi \times 3.1^2$
d $\pi \times 9.8^2$

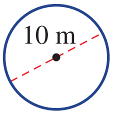
3 What fraction of a full circle is shown here?

a


b


c


4 What is the length of the radius in these shapes?

a


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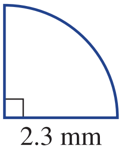
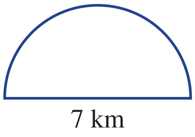


c


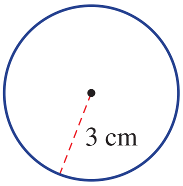
FLUENCY 5–6(½) 5–6(½), 7

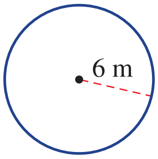
Example 13 Finding circle areas using a radius

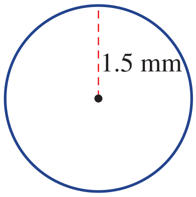
Find the area of this circle correct to two decimal places.

Solution	Explanation
$A = \pi r^2$ $= \pi \times 4^2$ $= 50.27$ cm^2 (to 2 d.p.)	Use the π button on the calculator and enter $\pi \times 4^2$ or $\pi \times 16$.

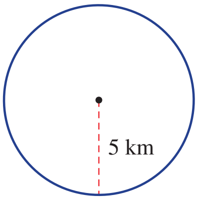
5 Find the area of these circles, correct to two decimal places.

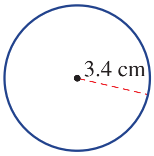
a


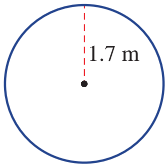
b


c


Substitute the radius for r in $A = \pi r^2$.

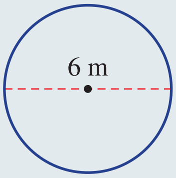
d


e


f


Example 14 Finding circle areas using a diameter

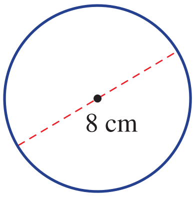
Find the area of this circle correct to two decimal places.



Solution	**Explanation**
$r = d \div 2 = 6 \div 2 = 3$	First work out the radius as half of the diameter.
$A = \pi r^2$ $= \pi \times 3^2$ $= 28.27\text{ m}^2$ (to 2 d.p.)	Substitute $r = 3$ into the rule, then round to two decimal places.

6 Find the area of these circles, correct to two decimal places.

a



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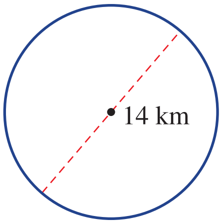

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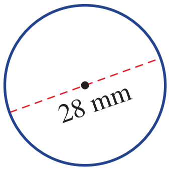
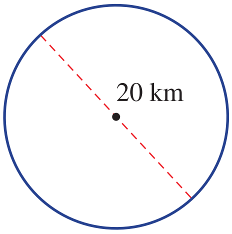
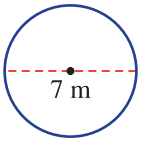


d



e



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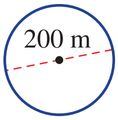
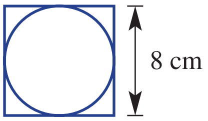


7 Find the area of the circle inside these shapes. Round to two decimal places.

a



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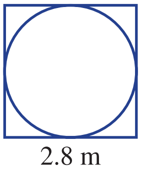

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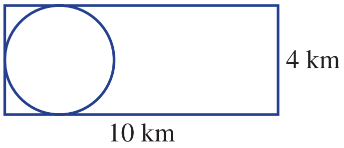


PROBLEM-SOLVING AND REASONING

8–10 10–13

8 A pizza tray has a diameter of 30 cm. Calculate its area to the nearest whole number of cm^2.

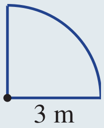
9 A tree trunk is cut to show a circular cross-section of radius 60 cm. Is the area of the cross-section more than 1 m^2 and, if so, by how much? Round your answer to the nearest whole number of cm^2.

10 A circular oil slick has a diameter of 1 km. The newspaper reported an area of more than 1 km^2. Is the newspaper correct?

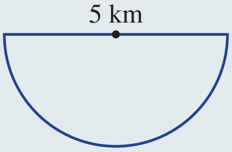
Example 15 Finding the area of quadrants and semicircles

Find the area of this quadrant and semicircle correct to two decimal places.

a



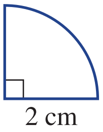
b



Solution	Explanation
a $A = \frac{1}{4} \times \pi r^2$ $= \frac{1}{4} \times \pi \times 3^2$ $= 7.07\text{ m}^2$ (to 2 d.p.)	The area of a quadrant is $\frac{1}{4}$ the area of a circle with the same radius.
b $r = \frac{5}{2} = 2.5$ $A = \frac{1}{2} \times \pi r^2$ $= \frac{1}{2} \times \pi \times 2.5^2$ $= 9.82\text{ km}^2$ (to 2 d.p.)	The radius is half the diameter. The area of a semicircle is $\frac{1}{2}$ the area of a circle with the same radius.

11 Find the area of these quadrants and semicircles, correct to two decimal places.

a



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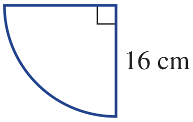

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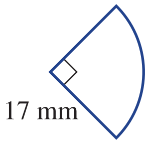

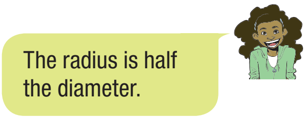

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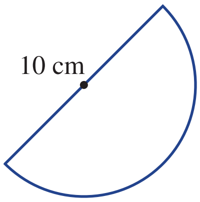

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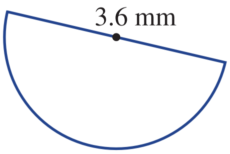

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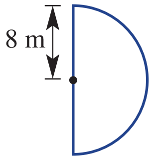


12 Two circular plates have radii 12 cm and 13 cm. Find the difference in their area, correct to two decimal places.

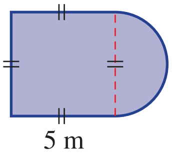
13 A square of side length 10 cm has a hole in the middle. The diameter of the hole is 5 cm. What is the area remaining? Round the answer to the nearest whole number.

Composite problems

— 14

14 Find the areas of the shaded region of these composite shapes using addition or subtraction. Round the answer to two decimal places.

a



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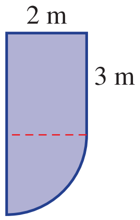

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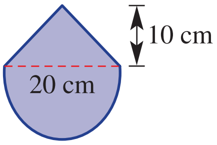
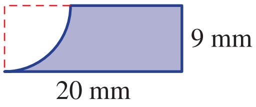


d



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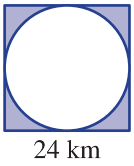

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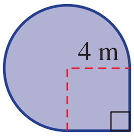
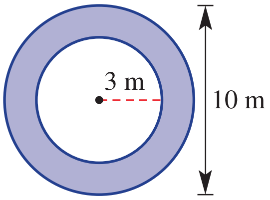


g



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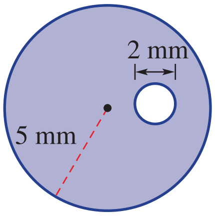
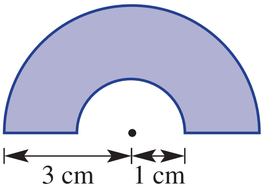


i



Progress quiz

4A

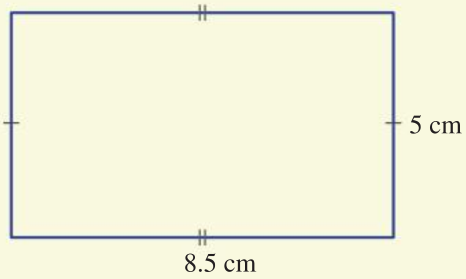
1 Convert these measurements to the units shown in the brackets.

a 12 cm (m)
b 585 mm (cm)
c 6.2 m (mm)
d 2.57 km (m)

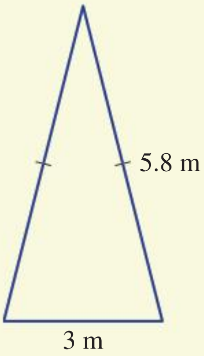
4A

2 Find the perimeter of these shapes.

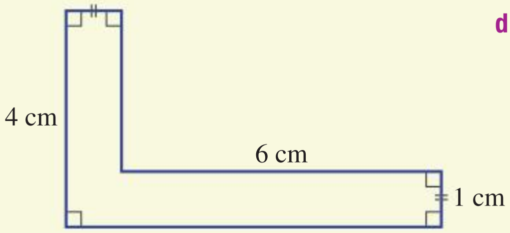
a



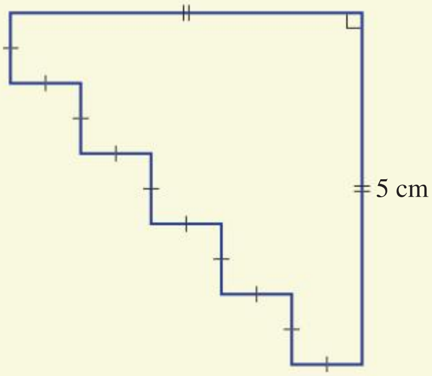
b



c



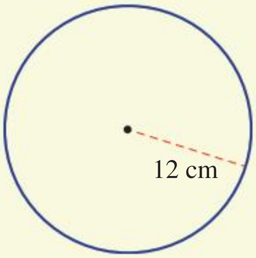
d



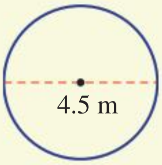
4B

3 Find the circumference of these circles, correct to two decimal places. Use a calculator for the value of pi.

a



b



4B

4 Find the perimeter of a mathematics protractor, which is the shape of a semicircle, with a base of 14 cm. Round the answer to two decimal places.

4B

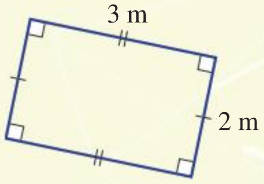
5 Convert these area measurements to the units shown in the brackets.

- **a** 7 cm^2 (mm^2)
- **b** 4500 mm^2 (cm^2)
- **c** 0.0034 m^2 (mm^2)
- **d** 30 km^2 (ha)

4C

6 Find the area of the following shapes.

a



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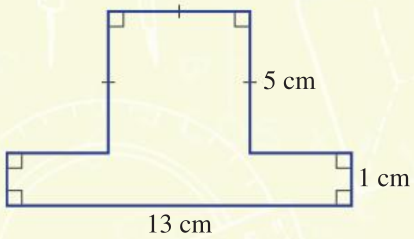

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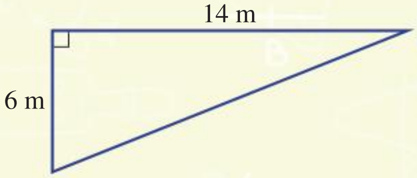

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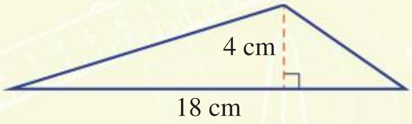
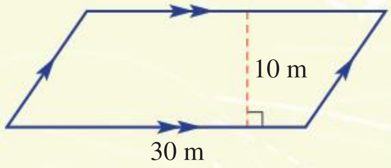


4D

7 Find the area of the following special quadrilaterals.

a



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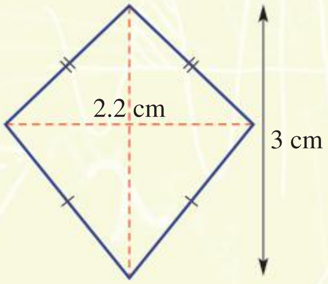

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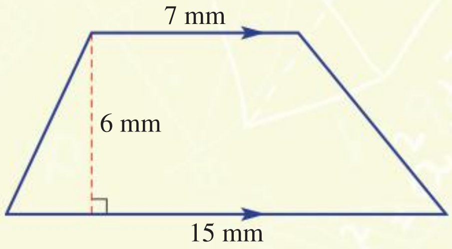


d

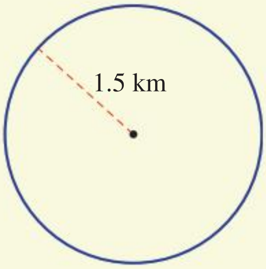
4D

8 A parallelogram has an area of 40 m^2 and its perpendicular height is 8 m. What is the length of its base?

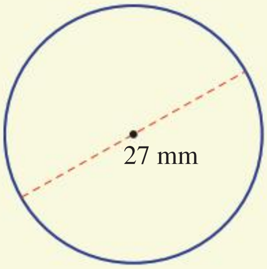
4E

9 Find the area of these circles, correct to two decimal places.

a



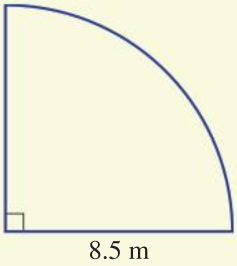
b



4E

10 Find the area of this quadrant and semicircle, correct to two decimal places.

a



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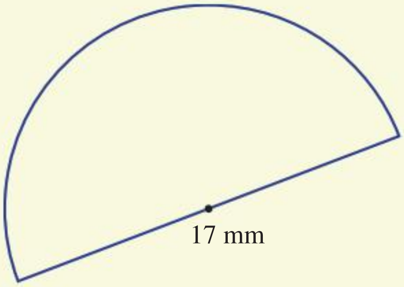


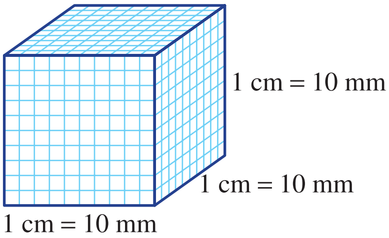
4F Volume and capacity

Volume is a measure of the space occupied by a three-dimensional object. It is measured in cubic units. Common metric units for volume given in abbreviated form include mm^3, cm^3, m^3 and km^3. We also use mL, L, kL and ML to describe volumes of fluids or gas. The volume of space occupied by a room in a house for example might be calculated in cubic metres (m^3) or the capacity of fuel tanker might be measured in litres (L) or kilolitres (kL).

Let's start: Why are there 1000 mm^3 in 1 cm^3?

Shown here is a 1 cm^3 cube (not to scale) that is also divided up into 1 mm^3 cubes.

- How many 1 mm^3 blocks sit along one edge?
- How many 1 mm^3 blocks sit on one layer?
- How many layers of 1 mm^3 blocks make up the full 1 cm^3?
- Now try to explain why there are 1000 mm^3 in 1 cm^3.
- How many cm^3 are in 1 m^3? How many m^3 are there in 1 km^3? Give reasons.



Key ideas

- **Volume** is measured in cubic units. Common metric units are:
  - cubic millimetres (mm^3)
  - cubic centimetres (cm^3)
  - cubic metres (m^3)
  - cubic kilometres (km^3)

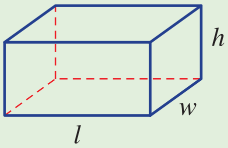
$$\begin{tikzcd}
\text{km}^3 \arrow[r, bend left, "\times 1000^3"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

- **Capacity** is the volume of fluid or gas that a container can hold. Common metric units are:
  - millilitre (mL)
  - litre (L)
  - kilolitre (kL)
  - megalitre (ML)

$$\begin{tikzcd}
\text{ML} \arrow[r, bend left, "\times 1000"] & \text{kL} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{mL} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

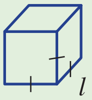
- Some common conversions are:
  - 1 mL = 1 cm^3
  - 1 L = 1000 mL = 1000 cm^3
  - 1 kL = 1000 L = 1 m^3
- Volume of a rectangular prism
  - Volume = length × width × height

    $V = lwh$



- Volume of a cube

  $V = l^3$



Volume The amount of three-dimensional space in an object

Capacity How much liquid a container can hold

Exercise 4F

UNDERSTANDING | 1–3 | 3

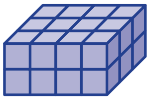
1 State if the following are units for length, area or volume.

a cm **b** cm^2 **c** cm^3 **d** mm^2 **e** m^3

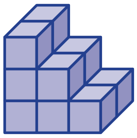
f mm **g** km^2 **h** mm^3 **i** m^2 **j** km

2 Count how many cubic units are shown in these cube stacks.

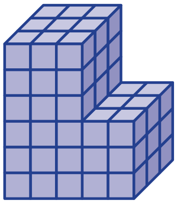
a



b



c



3 Write the missing number in the following unit conversions.

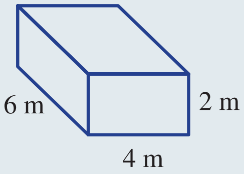
a 1 L = ___ mL **b** ___ kL = 1000 L **c** 1000 kL = ___ ML

d 1 mL = ___ cm^3 **e** 1000 cm^3 = ___ L **f** 1 cm^3 = ___ mm^3

FLUENCY | 4, 5(½), 6 | 4–6(½)

Example 16 Finding the volume of a rectangular prism

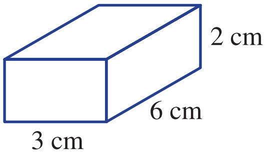
Find the volume of this rectangular prism.



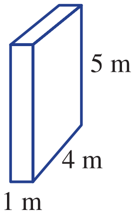
Solution	Explanation
$V = lwh$ $= 6 \times 4 \times 2$ $= 48\ m^3$	First write the rule and then substitute for the length, width and height. Any order will do since $6 \times 4 \times 2 = 4 \times 6 \times 2 = 2 \times 4 \times 6$ etc.

4 Find the volume of these rectangular prisms.

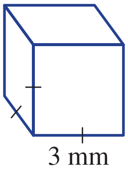
a



b

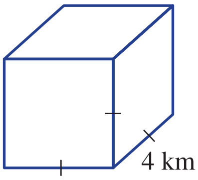


c

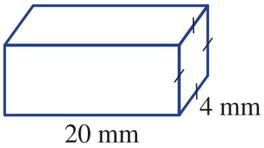


Use $V = lwh$ or use $l^3 = l \times l \times l$ for cubes.

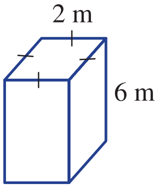
d



e



f



Example 17 Converting units

Convert these measurements to the units in the brackets.

a 0.5 L (mL) **b** 6400 kL (ML) **c** 3500 cm^3 (L)

Solution	Explanation
a $0.5\text{ L} = 0.5 \times 1000$ $= 500\text{ mL}$	There are 1000 mL in 1 L so multiply by 1000.
b $6400\text{ kL} = 6400 \div 1000$ $= 6.4\text{ ML}$	1 ML = 1000 kL and ML is the larger unit so divide by 1000.
c $3500\text{ cm}^3 = 3500 \div 1000$ $= 3.5\text{ L}$	1 L = 1000 mL and 1 mL = 1 cm^3 so 1 L = 1000 cm^3.

5 Convert the measurements to the units shown in the brackets.

a 2 L (mL) **b** 5 kL (L) **c** 0.5 ML (kL)
d 3000 mL (L) **e** 4 mL (cm^3) **f** 50 cm^3 (mL)
g 2500 cm^3 (L) **h** 5.1 L (cm^3) **i** 1 m^3 (L)

1 L = 1000 mL = 1000 cm^3
1 kL = 1000 L
1 ML = 1000 kL

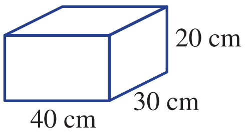
Example 18 Finding capacity

Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.

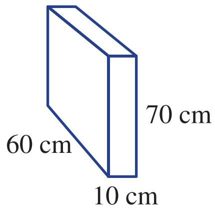
Solution	Explanation
$V = lwh$ $= 20 \times 10 \times 15$ $= 3000\text{ cm}^3$ $= 3000 \div 1000$ $= 3\text{ L}$	First calculate the volume of the container in cm^3. Then convert to litres using 1 L = 1000 cm^3.

6 Find the capacity of these containers, converting your answer to litres.

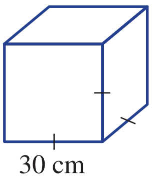
a



b

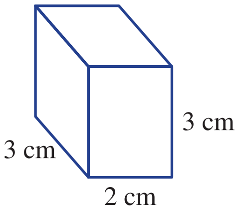


c

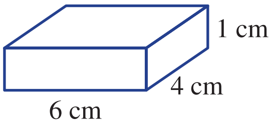


First find the volume in cm^3 using $V = lwh$, then divide by 1000 to convert to litres.

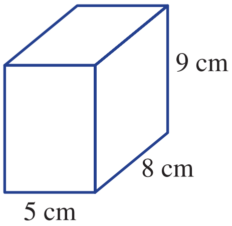
d



e



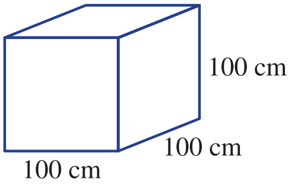
f



PROBLEM-SOLVING AND REASONING

7–9 9–12

7 Here is a 1 m^3 cube with each edge 100 cm.

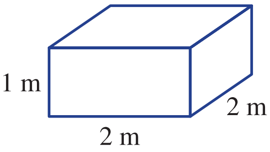


- **a** Find its volume in cm^3.
- **b** Complete this statement: 1 m^3 = _______ cm^3
- **c** How many cm^3 make 1 L?
- **d** Complete this statement: 1 m^3 = _______ L

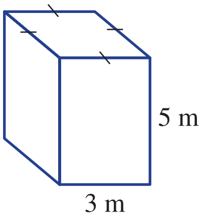
8 Find the capacity of these rectangular prisms in litres.

First find the volume in m^3, then use 1 m^3 = 1000 L.

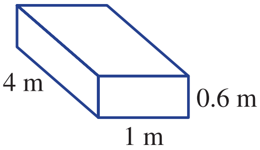
a



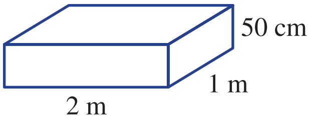
b



c



d



9 An oil tanker has a capacity of 60 000 m^3.

Use 1 m^3 = 1000 L.

- **a** What is the ship's capacity in:
  - **i** litres?
  - **ii** kilolitres?
  - **iii** megalitres?
- **b** If the ship leaks oil at a rate of 300 000 litres per day, how long will it take for all the oil to leak out?

10 If 1 kg is the mass of 1 L of water, what is the mass of water in a full container that is a cube with side length 2 m?

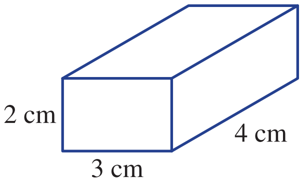
11 Water is being poured into a fish tank at a rate of 2 L every 10 seconds. The tank is 1.2 m long by 1 m wide by 80 cm high. How long will it take to fill the tank? Give the answer in minutes.

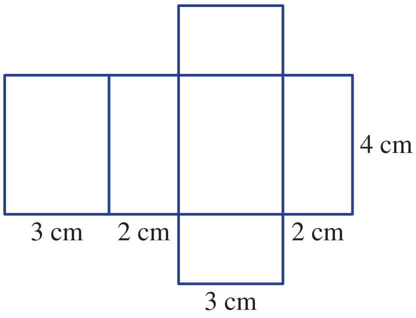
12 How many cubic containers (with side lengths that are a whole number of centimetres) have a capacity of less than 1 litre?

Surface area of rectangular prisms

— 13

13 You can find the total surface area of solids by adding all the areas of all the outside surfaces. Here is an example.





Total surface area

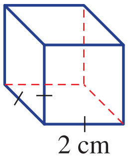
$= 2 \times (3 \times 4) + 2 \times (3 \times 2) + 2 \times (2 \times 4)$

$= 24 + 12 + 16$

$= 52 \text{ cm}^2$

Find the surface area of these rectangular prisms.

a



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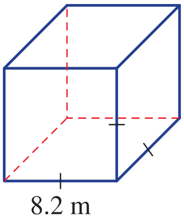

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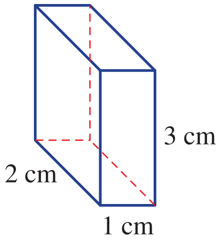
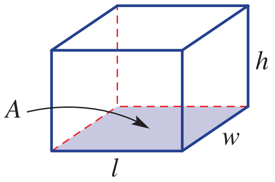


4G Volume of prisms

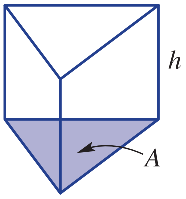
Interactive
Widgets
HOTsheets
Walkthrough

We know that for a rectangular prism, its volume V is given by the rule $V = lwh$. Length × width (lw) gives the area of the base A. So $V = lwh$ could also be written as $V = Ah$.

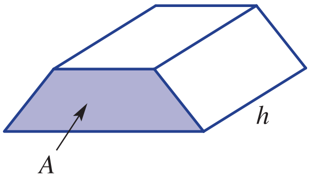


The rule $V = Ah$ can also be applied to prisms that have different shapes as their bases. One condition, however, is that the area of the base must represent the area of the cross-section of the solid. The height h is measured perpendicular to the cross-section.

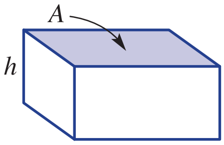
Here are some examples of prisms with A and h marked.



Cross-section is a triangle



Cross-section is a trapezium



Cross-section is a rectangle

Let's start: Drawing prisms

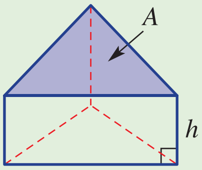
Try to draw prisms that have the following shapes as their cross-sections.

- Rectangle
- Trapezium
- Parallelogram
- Triangle
- Pentagon
- Kite

The cross-section of a prism should be the same size and shape along the entire length of the prism. Check this property on your drawings.

Key ideas

- A **prism** is a solid with a constant (uniform) cross-section.
  - Its sides between the two congruent ends are parallelograms.
  - A right prism has rectangular sides between the congruent ends.
- Volume of a prism = Area of cross-section × perpendicular height or $V = Ah$.



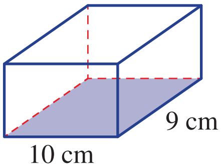
$V = Ah$

Exercise 4G

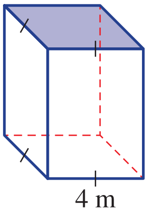
UNDERSTANDING 1–3 3

1 What is the name of the shape of the cross-section in these prisms (shaded)?

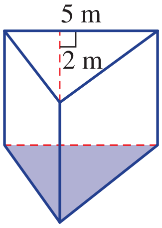
a



b



c



2 What is the area of the shaded cross-sections in question **1**? You will need the formulas: $A = lw$, $A = l^2$ and $A = \frac{1}{2}bh$.

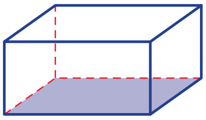
3 For these solids below:

i state whether or not it is a prism.

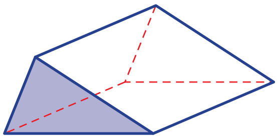
ii if it is a prism, state the shape of its cross-section.

A prism has a constant cross-section.

a



b



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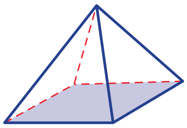
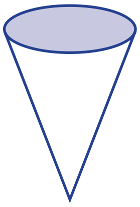
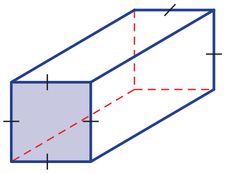


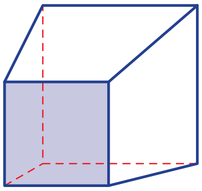
d



e



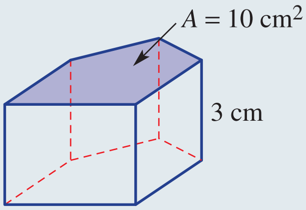
f



FLUENCY 4, 5 4–5(½), 6

Example 19 Finding the volumes of prisms given the cross-section

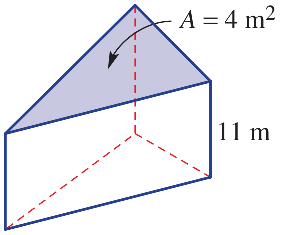
Find the volume of this prism using $V = Ah$.



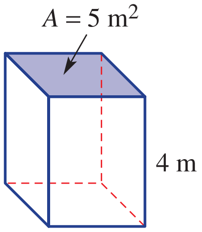
Solution	Explanation
$V = Ah$ $= 10 \times 3$ $= 30\text{ cm}^3$	Write the rule and substitute the given values of A and h, where A is the area of the cross-section.

4 Find the volume of these solids using $V = Ah$.

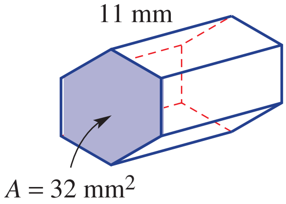
a



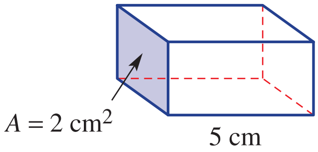
b



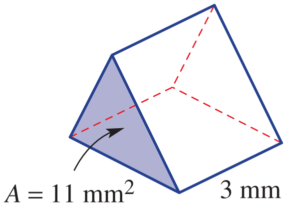
c



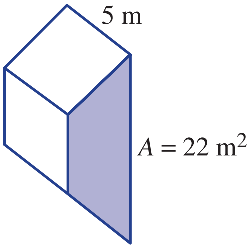
d



e

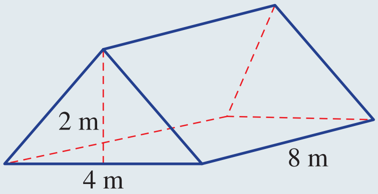


f



Example 20 Finding the volumes of prisms

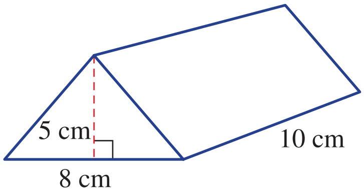
Find the volume of this prism.



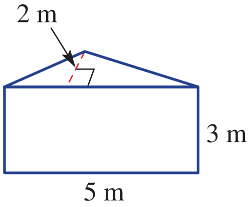
Solution	Explanation
$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 4 \times 2$ $= 4\ \text{m}^2$	The cross-section is a triangle, so use $A = \frac{1}{2}bh$ with base 4 m and height 2 m.
$V = Ah$ $= 4 \times 8$ $= 32\ \text{m}^3$	Then multiply by 8 using $V = Ah$ with $h = 8$.

5 Find the volume of these prisms.

a

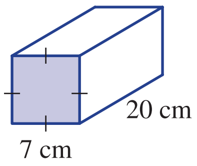


b

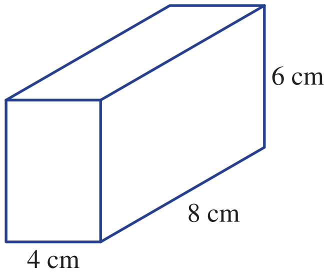


First find the area of the cross-section then multiply by h.

c



d



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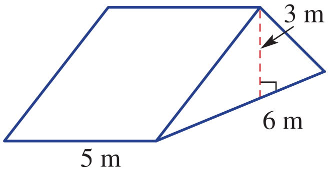

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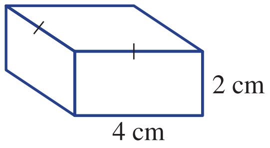


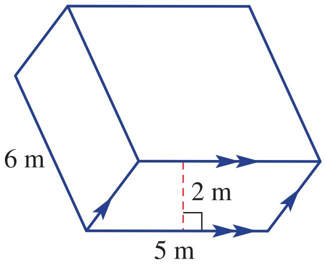
6 A rectangular drain pipe has a cross-sectional area of $4\ \text{m}^2$ and is 10 m long. Find its volume.

PROBLEM-SOLVING AND REASONING

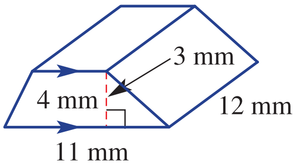
7	7, 8

7 These solids have cross-sections which are parallelograms, trapeziums, rhombuses or kites. Find their volume.

a



b



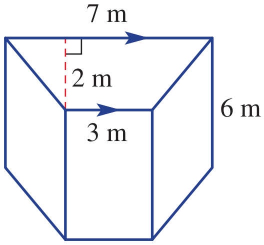
First find the area of the cross-section using

$A = bh$

$A = \frac{1}{2}(a + b)h$ or

$A = \frac{1}{2}xy$

c



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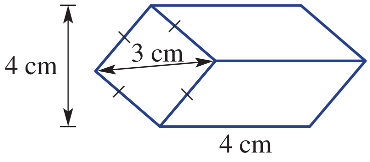

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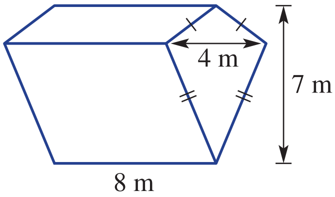

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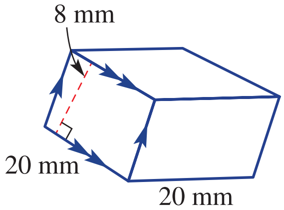
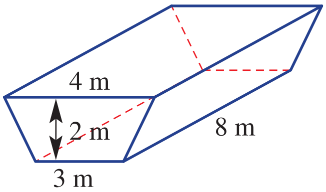


8 A swimming pool is a prism with a cross-section that is a trapezium. The pool is being filled at a rate of 1000 litres per hour.

a Find the capacity of the pool in litres.

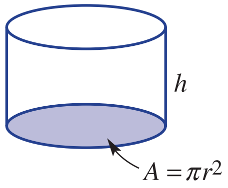
b How long will it take to fill the pool?



Volume of a cylinder

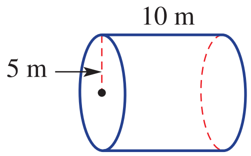
— 13

9 Although a cylinder is not a prism, the volume of a cylinder can be calculated using $V = Ah$ where $A = \pi r^2$ so $V = \pi r^2 h$.



Find the volume of these cylinders. Round the answer to two decimal places.

a



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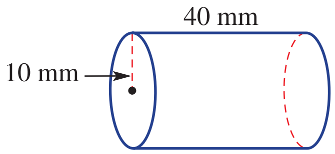

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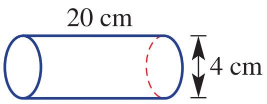

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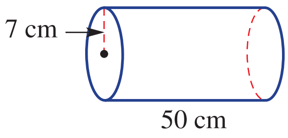

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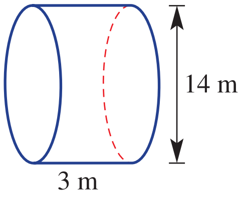

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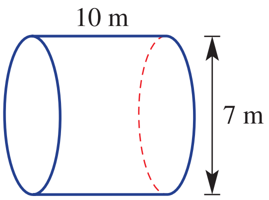


4H Time

Interactive
Widgets
HOTsheets
Walkthrough

The origin of the units seconds and minutes dates back to the ancient Babylonians, who used a base 60 number system. The 24-hour day dates back to the ancient Egyptians, who described the day as 12 hours of day and 12 hours of night. Today, we use am (*ante meridiem*, which is Latin for 'before noon') and pm (*post meridiem*, which is Latin for 'after noon') to represent the hours before and after noon (midday).

During the rule of Julius Caesar, the ancient Romans introduced the solar calendar, which recognised that the Earth takes about $365\frac{1}{4}$ days to orbit the Sun. This gave rise to the leap year, which includes one extra day (in February) every 4 years.

The Earth takes 1 year to orbit the Sun.

Let's start: Time starter

In less than five seconds per question, see if you can write the answers to the following:

- How many seconds in a minute?
- How many hours in two days?
- How many months in a year?
- How many seconds in an hour?
- Which months have 31 days?
- What do BCE (or BC) and CE (or AD) mean on time scales?

Key ideas

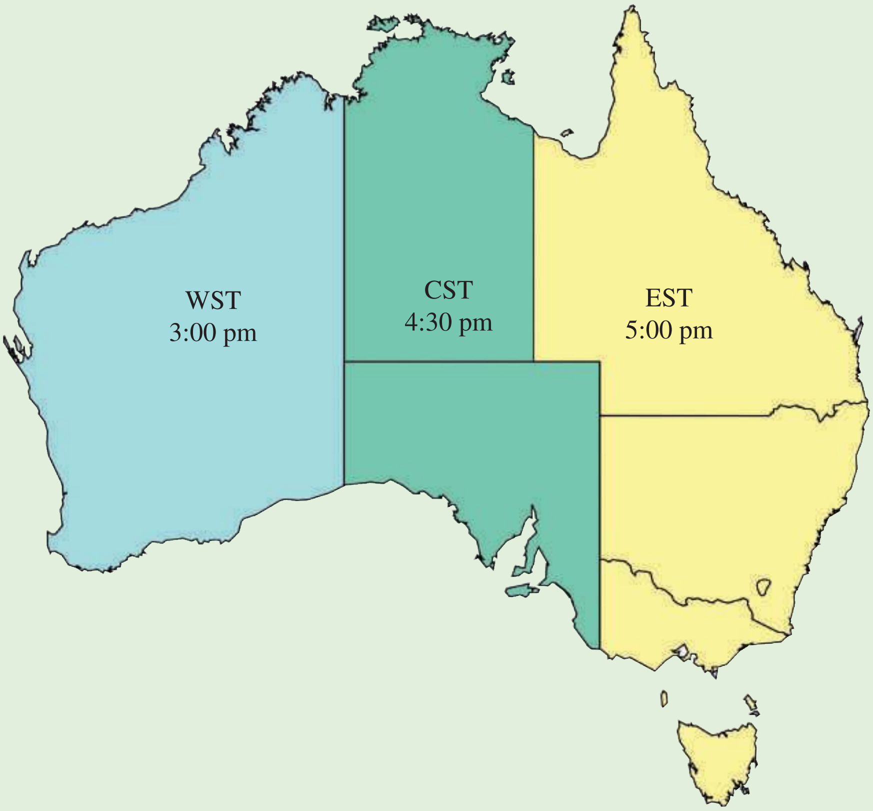
- The standard unit of time is the **second** (s).
- Units of time include:
  - 1 **minute** (min) = 60 **seconds** (s)
  - 1 **hour** (h) = 60 **minute** (min)
  - 1 **day** = 24 **hours** (h)
  - 1 **week** = 7 **days**
  - 1 **year** = 12 **months**

$$\begin{tikzcd} \text{day} \arrow[r, bend left, "\times 24"] & \text{hour} \arrow[l, bend left, "\div 24"] \arrow[r, bend left, "\times 60"] & \text{minute} \arrow[l, bend left, "\div 60"] \arrow[r, bend left, "\times 60"] & \text{second} \arrow[l, bend left, "\div 60"] \end{tikzcd}$$

- am or pm is used to describe the 12 hours before and after noon (midday).
- **24-hour time** shows the number of hours and minutes after midnight.
  - 0330 is 3:30 am.
  - 1530 is 3:30 pm.
  - 1121 is 11:21 am.
  - 2247 is 10:47 pm.
- The Earth is divided into 24 major **time zones** (one for each hour) and several minor time zones.
  - Twenty-four 15° lines of longitude divide the Earth into its time zones. Time zones also depend on a country's borders and how close it is to other countries. (See map on pages 222–223 for details.)
  - Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
  - Places east of Greenwich are ahead in time.
  - Places west of Greenwich are behind in time.

Time zone Any geographic region of the world in which the same standard time is kept

- Australia has three **time zones**:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.



Exercise 4H

UNDERSTANDING 1–3 3

1 Write the missing number.

a 1 minute = ____ seconds

b ____ days = 1 week

c ____ hours = 1 day

d 2 hours = ____ minutes

e 240 seconds = ____ minutes

f March has ____ days

2 Find the number of:

a seconds in 2 minutes

b minutes in 180 seconds

c hours in 120 minutes

d minutes in 4 hours

e hours in 3 days

f days in 48 hours

g weeks in 35 days

h days in 40 weeks

3 What is the time difference between these times?

a 12:00 noon to 6:30 pm

b 12:00 midnight to 10:45 am

c 12:00 midnight to 4:20 pm

d 11:00 am to 3:30 pm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ICELAND
ALASKA
CANADA
UNITED STATES
MEXICO
CUBA
VENEZUELA
COLUMBIA
PERU
BRAZIL
BOLIVIA
CHILE
ARGENTINA
NORWAY
UNITED KINGDOM
IRELAND
GERMAN
FRANCE
PORTUGAL
SPAIN
ITAL
MOROCCO
ALGERIA
MAURITANIA
MALI
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L Greenwich
M Johannesburg
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Q Vancouver
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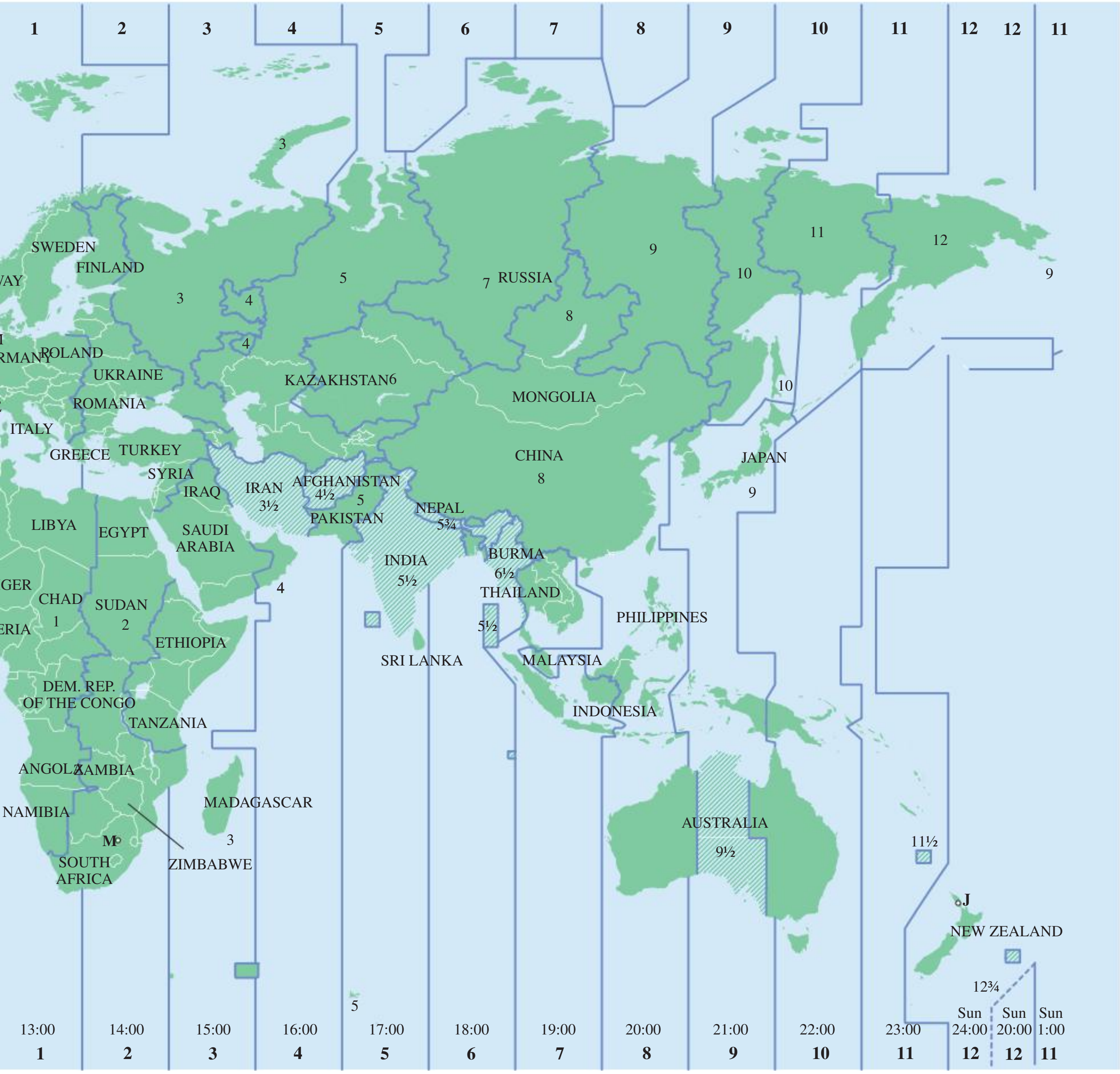
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UKRAINE
ROMANIA
ITALY
GREECE
TURKEY
SYRIA
IRAQ
IRAN
3½
AFGHANISTAN
4½
5
PAKISTAN
NEPAL
5¾
INDIA
5½
SRI LANKA
BURMA
6½
THAILAND
5½
MALAYSIA
INDONESIA
PHILIPPINES
KAZAKHSTAN
6
MONGOLIA
CHINA
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RUSSIA
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JAPAN
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LIBYA
EGYPT
SAUDI ARABIA
4
GER
CHAD
1
ERIA
SUDAN
2
ETHIOPIA
DEM. REP. OF THE CONGO
TANZANIA
ANGOLA
ZAMBIA
NAMIBIA
M
SOUTH AFRICA
ZIMBABWE
MADAGASCAR
3
5
AUSTRALIA
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NEW ZEALAND
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Sun 24:00
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Sun 20:00
12
Sun 1:00
11

FLUENCY 4(½), 5, 6(½), 7, 8 | 4–6(½), 7, 8–9(½)

Example 21 Converting units of time

Convert these times to the units shown in brackets.

a 3 days (minutes) **b** 30 months (years)

Solution	Explanation
a $3 \text{ days} = 3 \times 24 \text{ h}$ $= 3 \times 24 \times 60 \text{ min}$ $= 4320 \text{ min}$	1 day = 24 hours 1 hour = 60 minutes
b $30 \text{ months} = 30 \div 12 \text{ years}$ $= 2\frac{1}{2} \text{ years}$	There are 12 months in 1 year.

4 Convert these times to the units shown in brackets.

- **a** 2 min (s)
- **b** 48 h (days)
- **c** 21 days (weeks)
- **d** 3 h (min)
- **e** 10.5 min (s)
- **f** 240 s (min)
- **g** 90 min (h)
- **h** 6 days (h)
- **i** 72 h (days)
- **j** 1 week (h)
- **k** 1 day (min)
- **l** $3\frac{1}{2}$ h (min)

1 min = 60 sec
1 hr = 60 min
1 day = 24 hrs
1 week = 7 days

5 Write the time for these descriptions.

- **a** 4 hours after 2:30 pm
- **b** 10 hours before 7:00 pm
- **c** $3\frac{1}{2}$ hours before 10:00 pm
- **d** $7\frac{1}{2}$ hours after 9:00 am
- **e** $6\frac{1}{4}$ hours after 11:15 am
- **f** $1\frac{3}{4}$ hours before 1:25 pm

Example 22 Using 24-hour time

Write these times using the system given in brackets.

a 4:30 pm (24-hour time) **b** 1945 (am/pm)

Solution	Explanation
a $4:30 \text{ pm} = 1200 + 0430$ $= 1630 \text{ hours}$	Since the time is pm, add 12 hours to 0430 hours.
b $1945 \text{ hours} = 7:45 \text{ pm}$	Since the time is after 1200 hours, subtract 12 hours.

6 Write these times using the system shown in brackets.

- **a** 1:30 pm (24-hour)
- **b** 8:15 pm (24-hour)
- **c** 10:23 am (24-hour)
- **d** 11:59 pm (24-hour)
- **e** 0630 hours (am/pm)
- **f** 1300 hours (am/pm)
- **g** 1429 hours (am/pm)
- **h** 1938 hours (am/pm)
- **i** 2351 hours (am/pm)
- **j** 0426 hours (am/pm)
- **k** 6:47 pm (24-hour)
- **l** 4:32 am (24-hour)

6:00 am is 0600 hours
12:00 noon is 1200 hours
6:00 pm is 1800 hours

7 Round these times to the nearest hour.

- **a** 1:32 pm
- **b** 5:28 am
- **c** 1219 hours
- **d** 1749 hours

Example 23 Using time zones

Use the world time zone map (on pages 222–223) to answer the following.

a When it is 2:00 pm EST (Eastern Standard Time), find the time in these places.

- **i** Adelaide
- **ii** Perth
- **iii** Queensland
- **iv** Phillipines

b When it is 9:35 am in Western Australia, Australia, find the time in these places.

- **i** Alice Springs
- **ii** Tasmania
- **iii** Brisbane
- **iv** China

Solution	Explanation
a i 1:30 pm	Adelaide is in the Central Standard Time zone, which is $\frac{1}{2}$ hour behind Eastern Standard Time.
ii 12:00 noon	Perth is in the WST zone, 2 hours behind EST.
iii 2:00 pm	Queensland is in the Eastern Standard Time zone.
iv 12:00 noon	Phillipines is in the same zone as Western Australia.
b i 11:05 am	Alice Springs uses Central Standard Time, which is $1\frac{1}{2}$ hours ahead of Western Standard Time.
ii 11:35 am	Tasmania uses Eastern Standard Time, which is 2 hours ahead Western Standard Time.
iii 11:35 am	Brisbane is in the EST zone 2 hours ahead of WST.
iv 9:35 am	China is in the same zone as Western Australia.

8 Use the time zone map on pages 222–223 to find the time in the following places, when it is 10:00 am EST (Eastern Standard Time).

- **a** Melbourne
- **b** Darwin
- **c** Adelaide
- **d** Perth
- **e** Sydney
- **f** Tasmania
- **g** China
- **h** Papua New Guinea

CST is $\frac{1}{2}$ an hour behind EST,
WST is 2 hours behind EST.

9 Use the time zone map on pages 222–223 to find the time in these places, when it is 3:30 pm in Perth.

- **a** Melbourne
- **b** Phillipines
- **c** Sydney
- **d** China
- **e** Hobart
- **f** Queensland
- **g** Alice Springs
- **h** New Zealand
- **i** Japan

PROBLEM-SOLVING AND REASONING 10–13 13–16

10 From options **A** to **F**, match up the time units with the most appropriate description.

a	single heartbeat	**A**	1 hour
b	40 hours of work	**B**	1 minute
c	duration of a university lecture	**C**	1 day
d	bank term deposit	**D**	1 week
e	200-m run	**E**	1 year
f	flight from Australia to the UK	**F**	1 second

11 What is the time difference between these time periods?

a 10:30 am and 1:20 pm
b 9:10 am and 3:30 pm
c 2:37 pm and 5:21 pm
d 10:42 pm and 7:32 am
e 1451 and 2310 hours
f 1940 and 0629 hours

12 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

13 Adrian arrives at school at 8:09 am and leaves at 3:37 pm. How many hours and minutes is Adrian at school?

14 On a flight to Europe, Janelle spends 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, Malaysia, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

15 A pre-paid phone plan charges 11 cents per 30 seconds. The 11 cents are added to the bill at the beginning of every 30-second block of time.

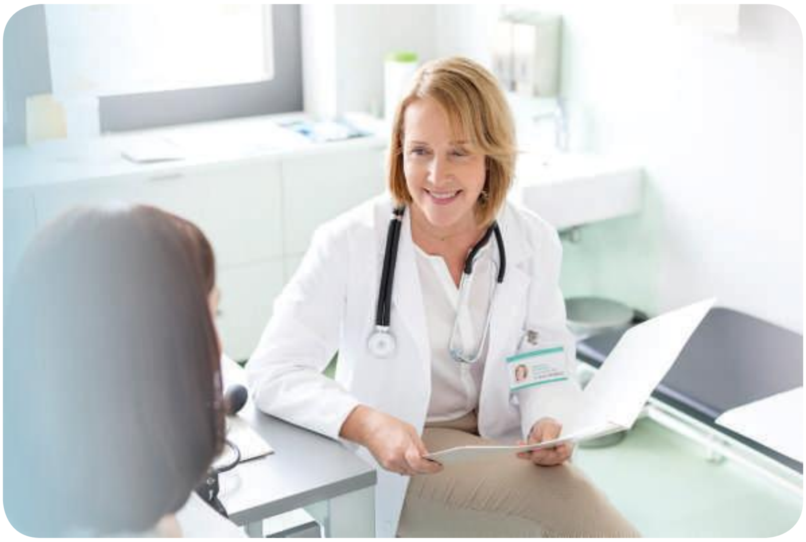
a What is the cost of a 70-second call?

b What is the cost of a call that lasts 6 minutes and 20 seconds?

16 A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

a per day **b** per hour **c** per minute

d per second (in cents)



World time zones

— 17–19

17 Use the time zone map to find the time in the following places if it is 3:30 pm in Victoria.

a United Kingdom **b** Libya **c** Sweden

d Perth **e** Japan **f** Central Greenland

g Alice Springs **h** New Zealand

18 Use the time zone map to find the time in the following places if it is 10:00 am UTC in England.

a Spain **b** Turkey **c** Tasmania

d Darwin **e** Argentina **f** Peru

g Alaska **h** Portugal

19 **a** Explain why you gain time when you travel from Australia to Europe.

b Explain why you lose time when you travel from Germany to Australia.

c Explain what happens to the date when you fly from Australia to Canada across the International Date Line.

Maths@Work: Hairdresser

Hairdressing is a demanding job, with long hours on your feet, a high level of concentration and good communication skills, all part of a hairdresser's day. Being good at maths is also important. Colour combinations need to be weighed and mixed in correct volumes, temperatures maintained as well as being able to manage times. Clients do not like to be kept waiting or to have their experience rushed due to scheduling issues.

1 Tubes of colour are kept in each salon and their volume is usually quoted in cc.

cc means cubic centimetres
$1\text{ mL} = 1\text{ cc} = 1\text{ cm}^3$

Convert the following from cubic centimetres (cc) into millilitres (mL).

a 10 cc **b** 20 cc **c** 45 cc **d** 100 cc **e** 1000 cc

2 Before it is applied, hair colour is mixed with a chemical called developer. The volume of developer is twice the volume of the colour. Determine the volume of developer, in cc, that is needed for:

a 10 cc of colour **b** 30 cc of colour **c** 50 cc of colour

3 A client, Chloe, has thick, long hair. Her salon records show:

Chloe's July appointment:
- mixed and used 3 amounts of colour
- standard (10 cc) + standard + half of a standard volume

On Chloe's next visit the hairdresser decided to mix enough colour all in one batch.

a What volume of colour should be used?

b What volume of developer should be used?

c What is the total volume of mixture that is applied to Chloe's hair?

d Chloe books her appointments every 10 weeks on a Friday. Her last appointment was on Friday the 26th August. What is the date of her next appointment?

e It takes 15 minutes for the consultation, 45 minutes for applying the colour, 20 minutes for the wash and 45 minutes for the cut and blow dry. If Chloe's appointment was for 9.15 am what time would she expect to finish?

f If the same stylist consults and cuts Chloe's hair, while another hairdresser applies the colour and does the wash, how much time between the consultation and the cut is available for the stylist to work on another client?

4 'Happy Hair' is a busy salon where Amelia and Layla are the hairdressers. Answer the following questions about this day's appointments.

Happy Hair appointment book		
Tuesday, March 18		
Start times	**Amelia**	**Layla**
9:00 am	Jessica: Long hair, colour, cut, wash, head massage, blow-dry and style	Liam: Men's hair cut
9:30 am		Mrs White: short hair, cut and style
10:00 am		
10:30 am	Ella: Girl's hair cut	Mrs Davis: Short hair, colour, cut, wash, eye-brows, blow-dry and style
11:00 am	Jessica, cont'd.	
11:30 am	Mrs Williams: short hair, cut, wash and style.	
12:00 noon		Lunch
12:30 pm		Mrs Davis, cont'd.
1:00 pm	Max: Men's hair cut	Joel: Boy's hair cut
1:30 pm	Lunch	Chelsea: Short haircut and style
2:00 pm	Mrs Babb: Short hair, colour, cut, wash, eye-brows, head massage, blow-dry and style	
2:30 pm		Zoe: Braiding
3:00 pm		
3:30 pm		Ruby: Deep conditioning and wash
4:00 pm		
4:30 pm		Holly: Half-head of foils and wash.
5:00 pm	Luke: Boy's hair cut	
5:30 pm		

Amelia

a How many customers does Amelia have on this Tuesday?

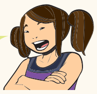
b How long is Jessica's appointment?

c At what times does Jessica start and finish her appointment?

d What could Jessica be doing while Amelia cuts Ella's hair?

e If Max was running 10 minutes late could Amelia still fit him in?

f If Gary rang in the morning and wanted a hair trim that same day, at what times could Amelia fit him in?



The times are start times, e.g. Ella's hair-cut starts at 10:30 am and finishes by 11 am.

Layla

g How many customers does Layla have on this Tuesday?

h At what times does Mrs Davis start and finish her appointment?

i How much time is allocated for Zoe's braiding?

j If Holly's foils went overtime by 20 minutes, at what time would Layla finish her appointments?

Using technology

5 For this task, use digital technology such as: a table in Word, an Excel spreadsheet or a day diary from a digital calendar. Imagine that you are a hairdresser in a hairdressing salon.

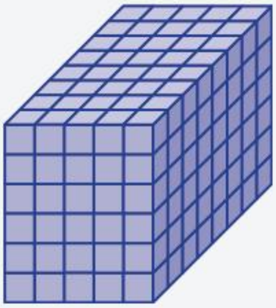
  a You are to fill in appointment times for your customers, as listed below, who are all coming on one day. Include the salon's name, a day and date, your name and the name of each customer.
    - Two long, 2.5-hour appointments for ladies with long hair: colour, cut, wash, head massage, eye-brows, blow-dry and style.
    - Four short, 30-minute appointments for haircuts for two men, a boy and a girl.
    - A 90-minute appointment for foils, wash and blow-dry.

  b Some digital diaries have the option of attaching extra notes to an entry, e.g. in Excel, right-click/insert comment. Suggest what other information could be included in a hairdresser's digital appointment diary.

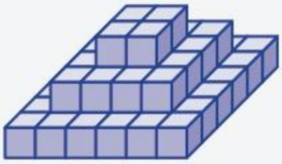
Puzzles and games

1 How many cubes are in each solid stack?

a



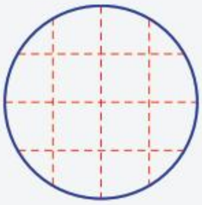
b



c

2 Estimate the area of these shapes by counting squares.

a



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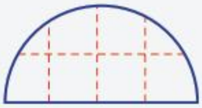

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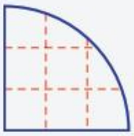
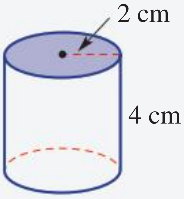


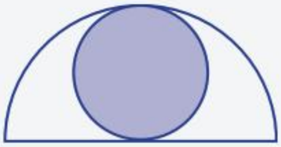
3 A cube has capacity 1 L. What are its dimensions in cm?

4 A fish tank is 60 cm long, 30 cm wide, 40 cm high and contains 70 L of water. Rocks with a volume of 3000 cm^3 are placed into the tank. Will the tank overflow?

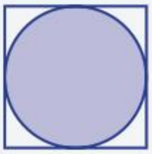
5 Find the total surface area of this cylinder.



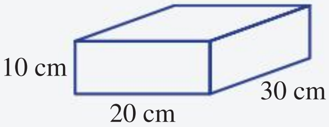
6 What proportion (fraction or percentage) of the semicircle does the full circle occupy?



7 A circle just fits inside a square. What percentage of the square is occupied by the circle?



8 1.8 L of water is poured into this container. What will be the depth of the water?

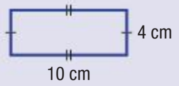


Chapter summary

Measurement

Length

Perimeter



$P = 2 \times 10 + 2 \times 4$

$= 28$ cm

Units

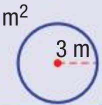
1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

Circumference

$C = 2\pi r$ or πd

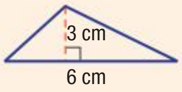
$= 2 \times \pi \times 3$

$= 18.85$ m^2



Area

Triangle



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 3$

$= 9$ cm^2

Units

$\times 1000^2 \quad \times 100^2 \quad \times 10^2$

km^2 m^2 cm^2 mm^2

$\div 1000^2 \quad \div 100^2 \quad \div 10^2$

1 ha = 10 000 m^2

Quadrilaterals

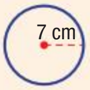
- Square $A = l^2$
- Rectangle $A = lw$
- Parallelogram $A = bh$
- Rhombus $A = \frac{1}{2}xy$
- Kite $A = \frac{1}{2}xy$
- Trapezium $A = \frac{1}{2}(a + b)h$

Circle

$A = \pi r^2$

$= \pi \times 7^2$

$= 153.94$ cm^2



Volume

Units

km^3, m^3, cm^3, mm^3

$\times 1000 \quad \times 1000 \quad \times 1000$

mL kL L mL

$\div 1000 \quad \div 1000 \quad \div 1000$

1 mL = 1 cm^3

1 m^3 = 1000 L

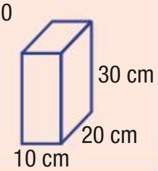
Rectangular prism

$V = lwh$

$= 10 \times 20 \times 30$

$= 6000$ cm^3

$= 6$ L

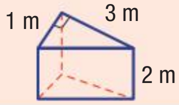


Triangular prism

$V = Ah$

$= \frac{1}{2} \times 3 \times 1 \times 2$

$= 3$ m^3



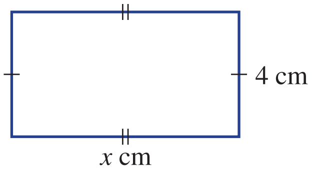
Time

1 min = 60 s
1 h = 60 min
0311 is 03:11 am
2049 is 08:49 pm

Multiple-choice questions

4A **1** The perimeter of this rectangle is 20 cm. The unknown value x is:

A 4 **B** 16 **C** 5
D 10 **E** 6

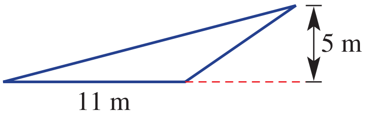


4B/E **2** A wheel has a diameter of 2 m. Its circumference and area (in that order) are given by:

A π, π^2 **B** $2\pi, \pi$ **C** $4\pi, 4\pi$ **D** 2, 1 **E** 4, 4

4C **3** The area of this triangle is:

A 27.5 m² **B** 55 m **C** 55 m²
D 110 m **E** 16 m²



4E **4** Using $\pi = 3.14$, the area of a circular oil slick with radius 100 m is:

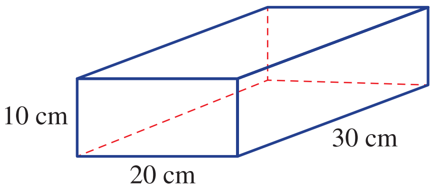
A 7850 m **B** 314 m² **C** 31 400 m² **D** 78.5 m² **E** 628 m²

4F **5** 2.5 L is the same as:

A 250 mL **B** 2500 cm³ **C** 1 ML **D** 0.025 kL **E** 25 000 mL

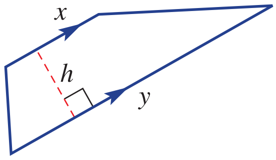
4F **6** The volume of this rectangular prism is:

A 60 L **B** 60 cm **C** 6 m³
D 600 cm³ **E** 6000 cm³



4D **7** The rule for the area of the trapezium shown is:

A $\frac{1}{2}xh$ **B** $\frac{1}{2}(x+y)$ **C** $\frac{1}{2}xy$
D πxy^2 **E** $\frac{1}{2}(x+y)h$

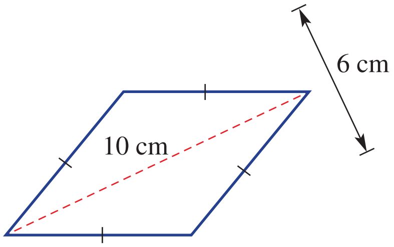


4F **8** The volume of a rectangular prism is 48 cm³. If its width is 4 cm and height 3 cm, its length would be:

A 3 cm **B** 4 cm **C** 2 cm **D** 12 cm **E** 96 cm

4D **9** The diagonals of a rhombus measure 10 cm and 6 cm. Its area is:

A 120 cm² **B** 16 cm² **C** 15 cm²
D 30 cm² **E** 60 cm²



4D **10** A square has area 49 m². Its side length is:

A 5 m **B** 8 m **C** 49 m
D 7 m **E** 4 m

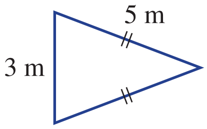
Short-answer questions

4A/C/F **1** Convert these measurements to the units given in the brackets.

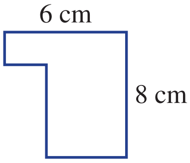
a 2 m (mm) **b** 50 000 cm (m) **c** 320 m (km) **d** 0.04 km (m)
e 3 cm² (mm²) **f** 4000 cm² (m²) **g** 0.01 km² (m²) **h** 350 mm² (cm²)
i 4000 mL (L) **j** 3 cm³ (mm³) **k** 400 cm³ (L) **l** 4300 kL (ML)

4B/C **2** Find the perimeter/circumference of these shapes. Round the answer to two decimal places where necessary.

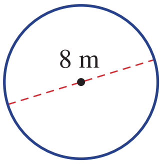
a



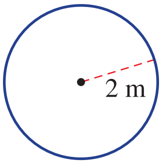
b



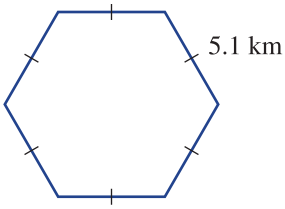
c



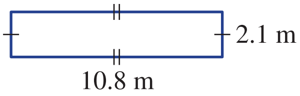
d



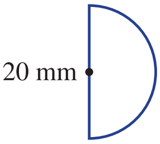
e



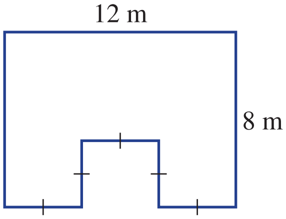
f



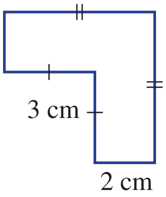
g



h

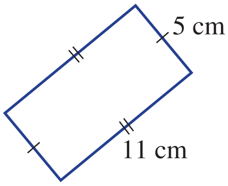


i

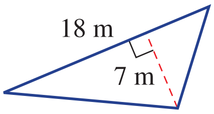


4C/D/E **3** Find the area of these shapes. Round the answer to two decimal places where necessary.

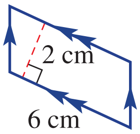
a



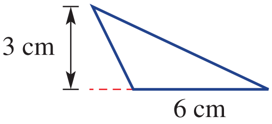
b



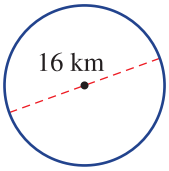
c



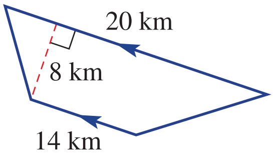
d



e



f



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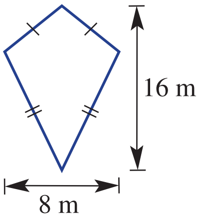
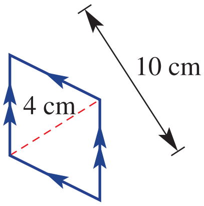
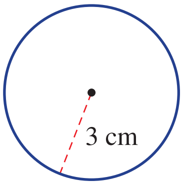


h

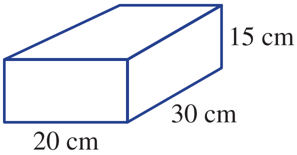


i

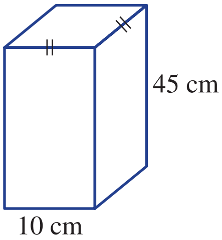


4F **4** Find the volume of these rectangular prisms in litres. Recall 1 L = 1000 cm^3.

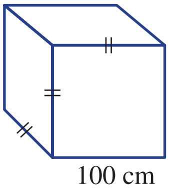
a



b

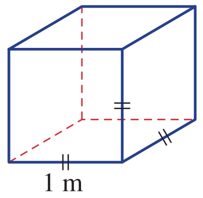


c



4G **5** Find the volume of each prism.

a



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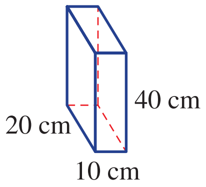

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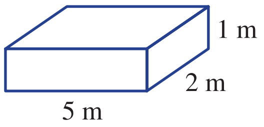

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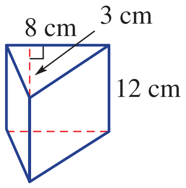
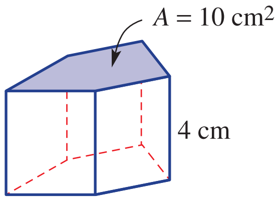
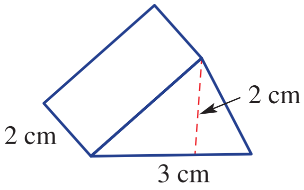


e



f



4H **6** An oven is heated from 23°C to 310°C in 18 minutes and 37 seconds. It then cools by 239°C in 1 hour, 20 minutes and 41 seconds.

- **a** Give the temperature: **i** increase **ii** decrease
- **b** What is the total time taken to heat and cool the oven?
- **c** How much longer does it take for the oven to cool down than heat up?

4H **7** **a** What is the time difference between 4:20 am and 2:37 pm?

- **b** Write 2145 hours in am/pm time.
- **c** Write 11:31 pm in 24-hour time.

4H **8** When it is 4:30 pm in Western Australia, state the time in each of these places.

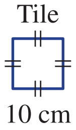
- **a** New South Wales
- **b** Adelaide
- **c** Darwin
- **d** China
- **e** Perth
- **f** Phillipines
- **g** New Zealand
- **h** Tasmania
- **i** Queensland

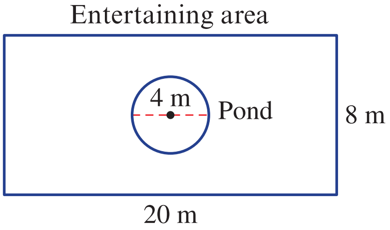
Extended-response questions

1 A rectangular entertaining area is to be tiled.

The tiles are 10 cm square and the entertaining area is 20 m by 8 m. A circular pond of diameter 4 m is to be built in the centre.

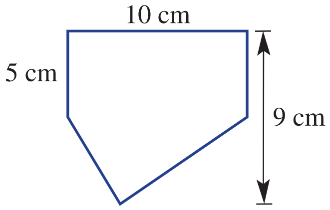
- **a** Find the total area of the entertaining area in m^2.
- **b** Find the perimeter of the entertaining area.
- **c** Find the area of the pond correct to two decimal places.
- **d** Find the area to be tiled (not including the pond area) correct to two decimal places.
- **e** Find the area of one tile in: **i** cm^2 **ii** m^2
- **f** Find the minimum number of tiles required for the job.
- **g** Why would a tiler need more tiles than the minimum number?



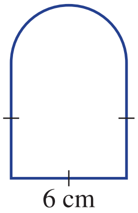


2 Find the area of these composite shapes.

a



b



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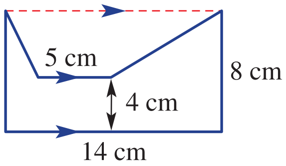


Chapter 5

Algebra

What you will learn

- 5A The language of algebra (Consolidating)
- 5B Substitution and equivalence
- 5C Adding and subtracting terms
- 5D Multiplying and dividing terms
- 5E Expanding brackets
- 5F Factorising expressions ★
- 5G Applying algebra ★

Australian curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Extend and apply the distributive law to the expansion of algebraic expressions **(ACMNA190)**

Factorise algebraic expressions by identifying numerical factors **(ACMNA191)**

Simplify algebraic expressions involving the four operations **(ACMNA192)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Algebra and sport

Algebra is a powerful tool for studying sport performance. By varying the input values in an algebraic equation, virtual reality experiments can discover how to improve an athlete's technique and equipment for better performance.

Sea kayaks, white-water kayaks and competitive canoes each have a different hull shape for overcoming the water drag-force that acts on the underwater surface area. A short kayak is easier to manoeuvre in white-water rapids but in calmer water longer kayaks are always faster. This is because longer kayaks have a faster maximum speed before the prow (i.e. the front) rises up as it tries to climb over its own bow wave, making sustained paddling impossible.

Algebraic formulas and geometric models are built into virtual reality systems that can then trial various designs for a kayak's speed and stability.

Pre-test

1 Evaluate:

a $8 + 4 \times 6$

b $4 \times 5 - 2 \times 3$

c $12 - (6 + 2) + 8$

d $3(6 + 4)$

2 Evaluate:

a the sum of 7 and 10

b the product of 2 and 6

c the sum of 12, 10 and 8

d half of 24

3 If $\square = 10$, write the value of:

a $\square + 2$ **b** $\square \times 7$ **c** $\square - 3$ **d** $\square + \square$

4 Find the value of $\square \times \square$ if:

a $\square = 4$ **b** $\square = 2$ **c** $\square = 11$ **d** $\square = 100$

5 Write an expression for:

a 5 more than x

b 7 less than m

c the product of x and y

d half of w

6 If $y = 2x + 5$, find the value of y when $x = 10$.

7 Complete the tables using the given rules.

a $M = 2A + 3$

A	0	3	7	10
M				

b $y = x + 12$

x	1	3	11	0
y				

8 Substitute $x = 6$ and $y = 2$ into each expression and then evaluate.

a $x + y$

b xy

c $3x - y$

d $2x + 3y$

9 Write down the HCF (highest common factor) of:

a 24 and 36

b 15 and 36

c 48 and 96

5A The language of algebra

CONSOLIDATING

A **pronumeral** (or **variable**) is a letter that can represent any number. For instance, x could represent the number of goals a particular football player scored last year.

Interactive
Widgets
HOTsheets
Walkthrough

Let's start: Algebra sort

Consider the four expressions $x+2$, $x\times 2$, $x-2$ and $x\div 2$.

- If you know that x is 10, can you sort the four expressions from lowest to highest?
- Give an example of a value of x that would make $x\times 2$ less than $x+2$.

The expression $6x + y$ gives the total number of points in an AFL game for a team which kicks x goals and y behinds.

Key ideas

- In algebra, letters can be used to represent numbers. These letters are called **variables** or **pronumerals**.
- $a\times b$ is written ab and $a\div b$ is written $\frac{a}{b}$.
- An **expression** is a combination of numbers and pronumerals combined with mathematical operations, e.g. $3x+2yz$ and $8\div(3a-2b)+41$ are expressions.
- A **term** is a part of an expression with only pronumerals, numbers, multiplication and division, e.g. $9a, 10cd$ and $\frac{3x}{5}$ are all terms.
- A term that does not contain any pronumerals is called a **constant term**.
- A **coefficient** is the number in front of a pronumeral. If the term is being subtracted, the coefficient is a negative number, and if there is no number in front, the coefficient is 1.
  - For the expression $3x+y-7z$, the coefficient of x is 3, the coefficient of y is 1 and the coefficient of z is -7. There are three terms in the expression.
- Certain English words have mathematical meaning.

Words	Symbols
sum	+
difference	–
product	×
quotient	÷

Pronumeral
A letter or a symbol used to represent a number

Expression
A group of mathematical terms containing no equals sign

Coefficient
A numeral placed before a pronumeral to indicate that the pronumeral is to be multiplied by that factor

5A

Exercise 5A

UNDERSTANDING	1–4	4

1 Find the following values.

- **a** The sum of 3 and 5
- **b** The product of 2 and 6
- **c** The difference between 10 to 7
- **d** Double the value of 5

Example 1 Using the language of algebra

a List the individual terms in the expression $4a+b-12c+5$.

b In the expression $4a+b-12c+5$ state the coefficients of a, b and c.

c What is the constant term in $4a+b-12c+5$?

Solution	Explanation
a There are four terms: $4a, b, 12c$ and 5.	Each part of an expression is a term. Terms get added (or subtracted) to make an expression.
b The coefficient of a is 4. The coefficient of b is 1. The coefficient of c is –12.	The coefficient is the number in front of a variable. For b the coefficient is 1 because b is the same as $1 \times b$. For c the coefficient is -12 because this term is being subtracted.
c 5	A constant term is any term that does not have a variable in it.

2 The expression $3a+2b+5c$ has three terms.

- **a** List the terms.
- **b** State the coefficient of:
  - **i** a
  - **ii** b
  - **iii** c
- **c** Write another expression with three terms.

3 **a** List the terms in $4x+6y+2z$.

- **b** Which of the variables (x, y or z) has a coefficient of 6?

4 The expression $5a+7b+c-3ab+6$ has five terms.

- **a** State the constant term.
- **b** State the coefficient of:
  - **i** a
  - **ii** b
  - **iii** c
- **c** Write another expression that has five terms.

A constant term has no pronumerals.

FLUENCY	5–9	5–6(½), 7–10

5 State how many terms there are in each expression.

- **a** $7a+2b+c$
- **b** $19y-52x+32$
- **c** $a+2b$
- **d** $7u-3v+2a+123c$
- **e** $10f+2be$
- **f** $9-2b+4c+d+e$

6 For each of the following expressions, state the coefficient of b.

a $3a + 2b + c$ **b** $3a + b + 2c$

c $4a + 9b + 2c + d$ **d** $3a - 2b + f$

e $5a - 6b + c$ **f** $7a - b + c$

Coefficients are negative if the term is subtracted.

Example 2 Creating expressions from a description

Write an expression for each of the following.

a The sum of 3 and k **b** The product of m and 7

c 5 is added to one half of k **d** The sum of a and b is doubled

Solution	**Explanation**
a $3 + k$	The word 'sum' means +.
b $m \times 7$ or $7m$	The word 'product' means ×.
c $\frac{1}{2}k + 5$ or $\frac{k}{2} + 5$	One half of k can be written $\frac{1}{2} \times k$ (because 'of' means ×), or $\frac{k}{2}$ because k is being divided by two.
d $(a + b) \times 2$ or $2(a + b)$	The values of a and b are being added and the result is multiplied by 2. Brackets are required to multiply the whole result by two and not just the value of b.

7 Match each of the following worded statements with the correct mathematical expression.

a The sum of x and 7

b 3 less than x

c x is divided by 2

d x is tripled

e x is subtracted from 3

f x is divided by 3

A $3 - x$

B $\frac{x}{3}$

C $x - 3$

D $3x$

E $\frac{x}{2}$

F $x + 7$

8 Write an expression for each of the following.

a 7 more than y **b** 3 less than x

c The sum of a and b **d** The product of 4 and p

e Half of q is subtracted from 4 **f** One third of r is added to 10

g The sum of b and c multiplied by 2 **h** The sum of b and twice the value of c

9 Describe each of the following expressions in words.

a $3 + x$ **b** $a + b$ **c** $2 \times k$ **d** $\frac{m}{2}$

10 Describe each of the following expressions in words.

a $4 \times b \times c$ **b** $2a + b$ **c** $(4 - b) \times 2$ **d** $4 - 2b$

PROBLEM-SOLVING AND REASONING

11, 12 | 11–13

11 Write an expression for:

a the total cost of buying 10 litres of petrol at $\$x$ per litre.

b the time spent shopping if you spend A minutes in the supermarket and B minutes in the department store.

c the difference, in age, between Oliver, who is 22 years old, and his younger cousin, Ben, who is k years old.

d the volume of water left in a 50-litre vat after x litres are removed.

If petrol is \$2 per litre, then the cost is \$20.

12 Marcela buys 7 plants from the local nursery.

a If the cost is \$10 for each plant, what is the total cost?

b If the cost is $\$x$ for each plant, write an expression for the total cost in dollars.

c If the cost of each plant is decreased by \$3 during a sale, write an expression for:

- **i** the new cost per plant in dollars
- **ii** the new total cost in dollars of the 7 plants.

13 Francine earns $\$p$ per week for her job. She works for 48 weeks each year. Write an expression for the amount she earns:

a in a fortnight

b in one year (of 48 weeks)

c in one year if her wage is increased by \$20 per week after she has already worked 30 weeks in the year.

DVD Dilemma

— | 14

14 Tom would like to purchase some DVDs of two television shows.

a Write an expression for the total cost of:

- **i** 4 seasons of Numbers
- **ii** 7 seasons of Proof by Induction
- **iii** 5 seasons of both shows
- **iv** all 7 seasons of both shows, if the final price is halved in a sale.

b If a is 20 and b is 30, how many seasons could he buy for \$200 without getting any duplicates?

5B Substitution and equivalence

Interactive

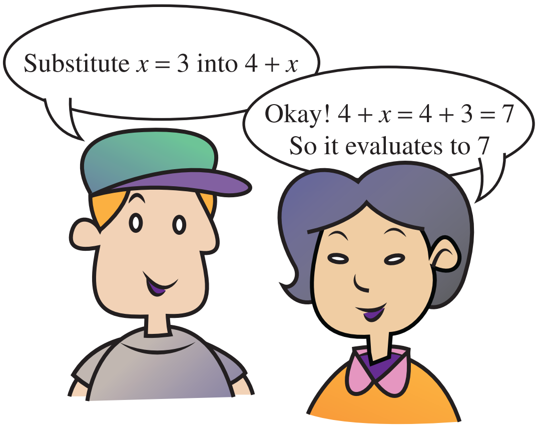
Widgets

HOTsheets

Walkthrough

Replacing pronumerals with numbers is called **substitution.** We can **evaluate** (find the value of) an expression once we substitute in numbers.

If two expressions always evaluate to the same number, they are called **equivalent.** For instance, $4 + x$ and $x + 4$ are equivalent.



Let's start: AFL algebra

In Australian Rules football, the final team score is given by $6x + y$, where x is the number of goals and y is the number of behinds scored.

- State the score if $x = 3$ and $y = 4$.
- If the score is 29, what are the values of x and y? Try to list all the possibilities.
- If $y = 9$ and the score is a 2-digit number, what are the possible values of x?

Key ideas

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value.
  e.g. If $a = 3$, then we can evaluate the expression $7a + 13$:
  $$\begin{aligned}7a + 13 &= 7 \times 3 + 13\\ &= 21 + 13\\ &= 34\end{aligned}$$
- Two expressions are **equivalent** if they have equal values regardless of the number that is substituted for each pronumeral.

Evaluate Find (calculate) the numerical value of

Substitute Replace pronumerals with numerical values

Exercise 5B

UNDERSTANDING 1–4 4

1 State the value of:
  a $5 + 3 \times 2$
  b $5 \times 3 + 2$
  c $17 - 2 \times 4$
  d $20 \div 5 + 3$

Brackets first then division and multiplication, then addition and subtraction.

2 If $\square = 6$, determine the value of each expression.
  a $\square + 5$
  b $\square \times 2$
  c $\square - 3$
  d $\square \div 2$

3 Find the value of $\square + 11$ if:
  a $\square = 5$
  b $\square = 10$
  c $\square = 100$
  d $\square = 59$

4 Fill in the blanks.
  Two expressions that are always equal are called __________ __________.

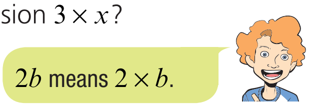
FLUENCY 5–10 5–8, 9(½), 10, 11

Example 3 Substituting for a pronumeral

Substitute $x = 3$ to evaluate $5x$.

Solution	Explanation
$5x = 5 \times 3$ $= 15$	Substitute 3 for x and note that $5x$ means $5 \times x$.

5 **a** What number is obtained when $x = 5$ is substituted into the expression $3 \times x$?
  b What is the result of evaluating $20 - b$ if b is equal to 12?
  c What is the value of $2b$ if b is equal to 10?



6 **a** State the value of $4 + 2x$ if $x = 5$.
  b State the value of $40 - 2x$ if $x = 5$.
  c Are $4 + 2x$ and $40 - 2x$ equivalent expressions?

7 Substitute the following values of x into the expression $7x + 2$.
  a 4 **b** 5 **c** 2 **d** 8

8 If $y = 4$, find the value of:
  a $y + 3$ **b** $9 - y$ **c** $3y - 2$ **d** $5y + 3$

Example 4 Substituting for multiple pronumerals

Substitute $x = 3$ and $y = 6$ to evaluate $3x + 2y$.

Solution	Explanation
$3x + 2y = 3 \times 3 + 2 \times 6$ $= 9 + 12$ $= 21$	Replace all the pronumerals by their values and remember the order in which to evaluate (multiplication before addition).

9 If $a = 4$ and $b = 7$, evaluate:
  a $3a + 2$ **b** $2b - 1$ **c** $a + b$ **d** $6 + ab$
  e $3a + b$ **f** $2a + 3b$ **g** $b - a$ **h** $3b - a$

10 Evaluate the expression $2x - 3y$ when:
  a $x = 10$ and $y = 4$
  b $x = 4$ and $y = 2$

Example 5 Deciding if expressions are equivalent

a Are $x - 3$ and $3 - x$ equivalent expressions?

b Are $a + b$ and $b + a$ equivalent expressions?

Solution	Explanation
a No	The two expressions are equal if $x = 3$ (both equal zero). But if $x = 7$ then $x - 3 = 4$ and $3 - x = -4$. Because they are not equal for every single value of x, they are not equivalent.
b Yes	Regardless of the values of a and b substituted, the two expressions are equal. This is because it does not matter the order in which numbers are added.

11 For the following state whether they are equivalent (E) or not (N).

- **a** $x + y$ and $y + x$
- **b** $3 \times x$ and $x \times 3$
- **c** $4a + b$ and $4b + a$
- **d** $4 + 2x$ and $2 + 4x$
- **e** $\frac{1}{2} \times a$ and $\frac{a}{2}$
- **f** $3 + 6y$ and $3(2y + 1)$

Try different values to see if the expressions are always equal.

PROBLEM-SOLVING AND REASONING 12 12, 13

12 **a** A number is substituted for k in the expression $7k$ and the result is 56. What is the value of k?

b The variable m is chosen so that $4m$ is a two-digit number and $4 + m$ is a single-digit number. List the possible values of m.

13 The expressions ab and $a + b$ are not equivalent.

- **a** Explain why they are not equivalent.
- **b** If $a = 0$ and $b = 0$, the two expressions are equal. Give an example of another pair of values that make them equal.
- **c** Explain why $a + 2$ and $a - 2$ are not equivalent.
- **d** Will $a + 2$ and $a - 2$ ever evaluate to the same number? Why/why not?

Find values for a and b where ab and $a + b$ are not equal.

Missing values

— 14

14 Copy and complete the following table.

x	3				4	2
y	8	7		−3		
$x + y$		12	5			
$x - 2y$			−4		8	
xy				0		12

5C Adding and subtracting terms

Interactive

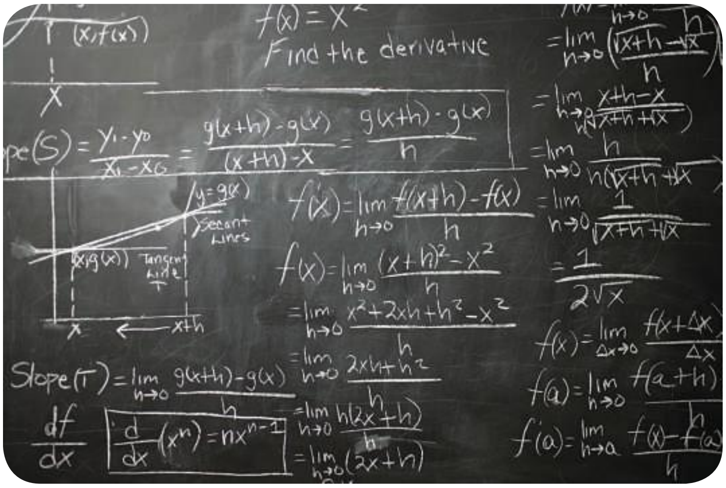
Widgets

HOTsheets

Walkthrough

Two terms with the same pronumerals are called **like terms**, and they can be collected and combined. For example, $2a + 6a$ can be simplified to $8a$ because $2a$ and $6a$ are like terms.

The order of the pronumerals does not matter, so $3ab$ and $5ba$ are like terms because they both include a and b.



Collecting like terms is important even in the most complex mathematics.

Let's start: Like terms

The terms $2abc$ and $5cab$ are like terms, and $2abc + 5cab = 7abc$.
In a short amount of time, see how many ways you can fill in the boxes:

$$\square + \square = 7abc$$

Can you explain why abc and cab are equivalent?

Key ideas

- **Like terms** contain exactly the same pronumerals with the same powers; the pronumerals do not need to be in the same order, e.g. $4ab$ and $7ba$ are like terms.
- Like terms can be combined when they are added or subtracted to simplify an expression, e.g. $3xy + 5xy = 8xy$.

  – sign stays with following term

  $$3x + 7y \boxed{-2x} + 3y + x \boxed{-4y}$$
  $$= 3x - 2x + x + 7y + 3y - 4y$$
  $$= 2x + 6y$$
- A subtraction sign stays in front of a term even when it is moved.

Like terms Terms with the same pronumerals and same powers

Exercise 5C

UNDERSTANDING 1–5 5

1 For each term below, list all the pronumerals that occur in it.

a $7a$ **b** $4ac$ **c** $2xy$ **d** $3wz$

2 Fill in the blanks.

a Two terms with exactly the same pronumerals are called __________ __________.

b If two expressions are always equal when evaluated, they are called __________ expressions.

3 **a** If $x = 3$, evaluate $5x + 2x$.
  b If $x = 3$, evaluate $7x$.
  c $5x + 2x$ is equivalent to $7x$. True or false?

4 **a** If $x = 3$ and $y = 4$, evaluate $5x + 2y$.
  b If $x = 3$ and $y = 4$, evaluate $7xy$.
  c $5x + 2y$ is equivalent to $7xy$. True or false?

5 **a** List the pronumerals that occur in $3abc$.
  b List the pronumerals that occur in $7bca$.
  c Are $3abc$ and $7bca$ like terms?

Like terms have the same pronumerals, possibly in a different order.

FLUENCY 6-8, 9(½) 6, 7-9 (½), 10

Example 6 Identifying like terms

Classify the following pairs as like terms (L) or not like terms (N).

a $3x$ and $12x$ **b** $5y$ and $7z$

Solution	Explanation
a L	Both $3x$ and $12x$ have the same pronumeral (x) so they are like terms.
b N	$5y$ and $7z$ have different pronumerals so they are not like terms.

6 Classify the following pairs as like terms (L) or not like terms (N).
  a $5x$ and $2x$ **b** $5x$ and $2y$ **c** $3k$ and $4k$ **d** $2q$ and $7x$

Example 7 Identifying like terms with multiple pronumerals

Classify the following pairs as like terms (L) or not like terms (N).

a $2ab$ and $3ba$ **b** $4x$ and $2xy$

Solution	Explanation
a L	They have the same pronumerals (order does not matter).
b N	$4x$ has the pronumeral x. $2xy$ has the pronumerals x and y. Since the terms have different pronumerals they are not like terms.

7 Classify the following pairs as like terms (L) or not like terms (N).
  a $4pq$ and $3pq$ **b** $2ab$ and $5bc$ **c** $7rs$ and $12sr$ **d** $5ab$ and $6a$
  e $7abc$ and $2cba$ **f** $8x$ and $8xy$ **g** $12ab$ and $14ba$ **h** $8xyz$ and $9yzx$

Example 8 Simplifying by combining like terms

Simplify the following by combining like terms.

a $7t + 2t - 3t$ **b** $4x + 3y + 2x + 7y$ **c** $8b + 7ac - 5b + 2ca$

Solution	Explanation
a $7t + 2t - 3t = 6t$	These are like terms, so they can be combined: $7 + 2 - 3 = 6$.
b $4x + 3y + 2x + 7y$ $= 4x + 2x + 3y + 7y$ $= 6x + 10y$	Move the like terms next to each other. Combine the pairs of like terms.
c $8b + 7ac - 5b + 2ca$ $= 8b - 5b + 7ac + 2ca$ $= 3b + 9ac$	Move like terms together. The subtraction sign stays in front of $5b$ when it is moved. $8 - 5 = 3$ and $7 + 2 = 9$

8 Simplify the following by combining like terms.

a $3x + 2x$ **b** $7a + 12a$ **c** $15x - 6x$ **d** $9y - 2y$

e $4xy + 3xy$ **f** $16uv - 3uv$ **g** $10ab + 4ba$ **h** $3pq + 12pq$

9 Simplify the following by combining like terms.

a $7f + 2f + 8 + 4$ **b** $10x + 3x + 5y + 3y$

c $2a + 5a + 13b - 2b$ **d** $10a + 5b + 3a + 4b$

e $10 + 5x + 2 + 7x$ **f** $10a + 3 + 4b - 2a - b$

g $10x + 31y - y + 4x$ **h** $11a + 4 - 2a + 12a$

i $2b + 4c + 3b + 5c$ **j** $3a - b + 4b - a$

k $2qr + 3q + 4qr + 6rq$ **l** $12xy - 5yx + 3x + 6x$

m $10ab - 4b - 6ba + 11b$ **n** $20kl + 10kl - 7lk + 2l$

Pair up the like terms
Note: $ab = ba$

10 For each expression choose an equivalent expression from the options listed.

a $7x + 2x$ **A** $10y + 3x$

b $12y + 3x - 2y$ **B** $9xy$

c $3x + 3y$ **C** $9x$

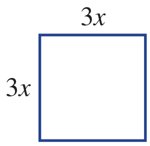
d $8y - 2x + 6y - x$ **D** $3y + 3x$

e $4xy + 5yx$ **E** $14y - 3x$

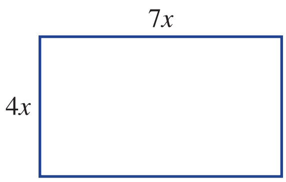
PROBLEM-SOLVING AND REASONING 11, 12 11–13

11 Write expressions for the perimeters of the following shapes in simplest form.

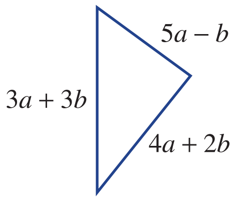
a



b



c



Perimeter = total distance around shape

12 Towels cost $\$c$ each at a shop.

- **a** John buys 3 towels, Mary buys 6 towels and Naomi buys 4 towels. Write a fully simplified expression for the total amount spent on towels.
- **b** On another occasion, Chris buys n towels, David buys twice as many as Chris and Edward buys 3 times as many as David. Write a simplified expression for the total amount they spent on towels.

13
- **a** Make a substitution to prove that $4a + 3b$ is not equivalent to $7ab$.
- **b** Is $4a + 3b$ ever equal to $7ab$? Try to find some values of a and b to make $4a + 3b = 7ab$ a true equation.
- **c** Is $4a + 3a$ ever not equal to $7a$? Explain your answer.

Filling the blanks

— 14

14 The expression $4a + 7b + 6a$ is equivalent to $10a + 7b$.
- **a** Give another way to fill in the blanks to make this statement true:

  $\boxed{}a + \boxed{}b + \boxed{}a = 10a + 7b$
- **b** Assuming the blanks above must be filled by positive integers, how many ways could they be filled to make a true statement?

5D Multiplying and dividing terms

Interactive

Widgets

HOTsheets

Walkthrough

Recall that $4ab$ is shorthand for $4 \times a \times b$. Observing this helps us to see how we can multiply terms.

$$\begin{aligned} 4ab \times 3c &= 4 \times a \times b \times 3 \times c \\ &= 4 \times 3 \times a \times b \times c \\ &= 12abc \end{aligned}$$

Division is written as a fraction so $\frac{12ab}{9ad}$ means $(12ab) \div (9ad)$. To simplify a division we look for common factors.

$$\frac{{}^{4}\cancel{12} \times \cancel{a} \times b}{{}^{3}\cancel{9} \times \cancel{a} \times d} = \frac{4b}{3d}$$

$a \div a = 1$ for any value of a except 0 so $\frac{a}{a}$ cancels to 1.

Let's start: Multiple ways

Multiplying $4a \times 6b$ gives you $24ab$.

- In how many ways can positive integers fill the blanks in $\square a \times \square b = 24ab$?
- Can you explain why there are more ways to fill in the blanks for $\square a \times \square b = 24ab$ than for $\square a \times \square b = 25ab$?

Key ideas

- $12abc$ means $12 \times a \times b \times c$.
- When multiplying, the order is not important: $2 \times a \times 4 \times b = 2 \times 4 \times a \times b$.
- x^2 means $x \times x$
- When dividing, cancel any common factors.

  For example: $\frac{{}^{3}\cancel{15}\, x\, \cancel{y}}{{}^{4}\cancel{20}\, \cancel{y}\, z} = \frac{3x}{4z}$

Exercise 5D

UNDERSTANDING 1–4 4

1 Are the following true (T) or false (F)?

- **a** $3 \times a$ can be written as $3a$.
- **b** $k \times 5$ can be written as $5k$.
- **c** $2x$ is short for $2 + x$.
- **d** $4ab$ could also be written as $4a \div b$.
- **e** $q \times q$ can be written as q^2.

2 Which is the correct way to write $3 \times a \times b \times b$?

A $3ab$ **B** $3ab^2$ **C** ab^3 **D** $3a^2b$

3 Simplify these fractions.

a $\frac{12}{20}$ **b** $\frac{5}{15}$ **c** $\frac{12}{8}$ **d** $\frac{15}{25}$

4 Write these without multiplication signs.

a $3 \times x \times y$ **b** $5 \times a \times b \times c$ **c** $12 \times a \times b \times b$ **d** $4 \times a \times c \times c \times c$

FLUENCY 5–7(½), 8 5–8(½)

Example 9 Multiplying terms

Simplify $7a \times 2bc \times 3d$.

Solution	Explanation
$7a \times 2bc \times 3d = 7 \times a \times 2 \times b \times c \times 3 \times d$ $= 7 \times 2 \times 3 \times a \times b \times c \times d$	Write the expression with multiplication signs and bring the numbers to the front.
$= 42abcd$	Simplify: $7 \times 2 \times 3 = 42$ and $a \times b \times c \times d = abcd$

5 Simplify the following.

a $7d \times 9$ **b** $5a \times 2$ **c** $3 \times 12x$

d $4k \times 6$ **e** $3 \times 2q$ **f** $3x \times 10y$

g $4a \times 2b \times cd$ **h** $3a \times 10bc \times 2d$ **i** $4a \times 6de \times 2b$

Example 10 Multiplying terms with repeated pronumerals

Simplify $3xy \times 5xz$.

Solution	Explanation
$3xy \times 5xz = 3 \times x \times y \times 5 \times x \times z$ $= 3 \times 5 \times x \times x \times y \times z$	Write the expression with multiplication signs and bring the numbers to the front.
$= 15x^2yz$	Simplify, remembering that $x \times x = x^2$.

6 Simplify the following.

a $x \times x$ **b** $a \times a$ **c** $3d \times d$ **d** $5d \times 2d \times e$

e $7x \times 2y \times x$ **f** $5xy \times 2x$ **g** $4xy \times 2xz$ **h** $4abc \times 2abd$

i $12xy \times 4x$ **j** $9ab \times 2a$ **k** $3xy \times 2x \times 4y$ **l** $2ab \times 4a \times 3b$

7 Write each expression without a division sign.

a $k \div 4$ **b** $x \div 5$ **c** $2q \div 5$ **d** $3k \div 10$

e $5 \div a$ **f** $a \div b$ **g** $x \div y$ **h** $12 \div g$

$\frac{k}{4}$ is the same as $k \div 4$.

Example 11 Dividing terms

Simplify $\frac{10ab}{15bc}$

Solution	**Explanation**
$\frac{10ab}{15bc} = \frac{{}^{2}\cancel{10} \times a \times \cancel{b}}{{}^{3}\cancel{15} \times \cancel{b} \times c}$ $= \frac{2a}{3c}$	Write the numerator and denominator in full, with multiplication signs. Cancel any common factors and remove the multiplication signs.

8 Simplify the following divisions by cancelling any common factors.

a $\frac{5a}{10a}$ **b** $\frac{7x}{14y}$ **c** $\frac{10xy}{12y}$ **d** $\frac{ab}{4b}$

e $\frac{7xyz}{21yz}$ **f** $\frac{2}{12x}$ **g** $\frac{4xy}{7x}$ **h** $\frac{3abc}{6b}$

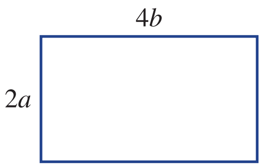
Cancel numbers and pronumerals where possible.

PROBLEM-SOLVING AND REASONING

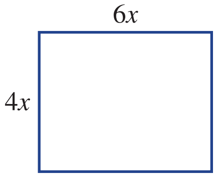
9, 10 9–12

9 Write a simplified expression for the area of the following shapes. Recall that rectangle area = width × length.

a



b



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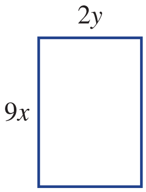


10 Simplify the following completely.

a $2a \times 3b + 5ab$ **b** $6q \times 2r + 4q \times 3r$ **c** $10x \times 2y - 3y \times 6x$

You can combine any like terms.

11 Fill in the missing terms to make the following equivalences true.

a $3x \times \square \times z = 6xyz$ **b** $4a \times \square = 12ab$ **c** $\frac{\square}{4r} = 7s$ **d** $\frac{\square}{2ab} = 4b$

12 Joanne claims that the following three expressions are equivalent: $\frac{2a}{5}, \frac{2}{5} \times a, \frac{2}{5a}$.

- **a** Is she right? Try different values of a.
- **b** Which two expressions are equivalent?
- **c** There are two values of a that make all three expressions are equal. State one of them.

Missing coefficients

— 13

13 **a** Simplify $2a \times 3b + 5b \times 2a$ to a single term.

- **b** State another way to fill in the blanks to make the simplification correct:
  $\square a \times \square b + \square b \times \square a = 16ab$
- **c** Give an example of an even longer expression that is equivalent to $16ab$.

5E Expanding brackets

Interactive
Widgets
HOTsheets
Walkthrough

Two expressions can look different and still be equivalent, like $x + x$ and $2x$.

Note that $3 \times (7 + a) = 3(7 + a)$, which is equivalent to 3 groups of $7 + a$, so

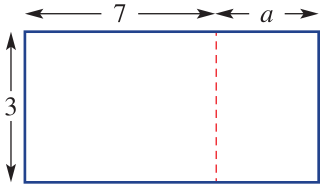
$$\begin{aligned} 3(7 + a) &= 7 + a + 7 + a + 7 + a \\ &= 21 + 3a \end{aligned}$$

This means that $3(7 + a)$ and $21 + 3a$ are equivalent.

Mathematical expressions only make sense if brackets are placed in the correct position.

Let's start: Equivalent areas

What is the total area of the rectangle shown to the right? Try to write two expressions: one with brackets and the other without brackets.

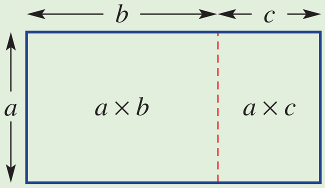


Key ideas

- **Expanding** brackets involves writing an equivalent expression without brackets:

$$\begin{aligned} 2(a + b) &= a + b + a + b \\ &= 2a + 2b \end{aligned} \quad \text{or} \quad \begin{aligned} 2(a + b) &= 2 \times a + 2 \times b \\ &= 2a + 2b \end{aligned}$$

- To eliminate brackets, you can use the **distributive law**, which states that:
    - $a(b + c) = ab + ac$
    - $a(b - c) = ab - ac$
- The distributive law can be demonstrated by considering rectangle areas: $a(b + c) = ab + ac$



$$\left.\begin{aligned} \text{Area} &= a(b + c) \\ \text{Area} &= ab + ac \end{aligned}\right\} a(b + c) = ab + ac$$

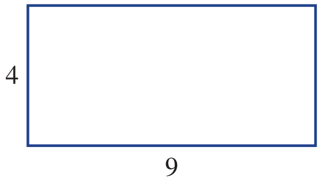
Expand Remove grouping symbols (brackets)

Exercise 5E

UNDERSTANDING 1–4 4

1 What is the area of each of the following rectangles?

a



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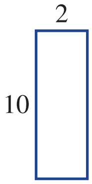

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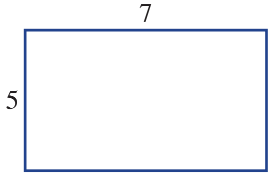
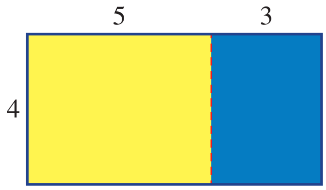


2 The rectangle shown has width 4 and length $5 + 3 = 8$.

a What is the area of the yellow rectangle?

b What is the area of the blue rectangle?

c What is the total combined area?



Example 12 Expanding brackets by simplifying repeated terms

Write the expression $3(2m + 5)$ in full without brackets and simplify the result.

Solution	**Explanation**
$3(2m + 5) = 2m + 5 + 2m + 5 + 2m + 5$	Three copies of the expression $2m + 5$.
$= 6m + 15$	Simplify by collecting the like terms.

3 The expression $3(a + 2)$ can be written as $(a + 2) + (a + 2) + (a + 2)$.

a Simplify this expression by collecting like terms.

b Write $2(x + y)$ out in full without brackets and simplify the result.

c Write $4(p + 1)$ out in full without brackets and simplify the result.

d Write $3(4a + 2b)$ out in full without brackets and simplify the result.

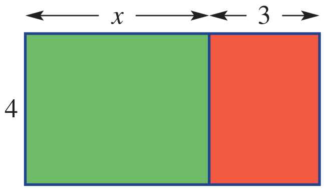
4 The area of the rectangle shown can be written as $4(x + 3)$.

a What is the area of the green rectangle?

b What is the area of the red rectangle?

c Write the total area as an expression without using brackets.

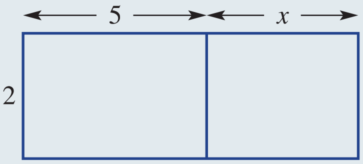
d Fill in the blank: The expressions $4(x + 3)$ and $4x + 12$ are _______ expressions.



FLUENCY 5–7, 8(½) 5–7, 8(½), 9

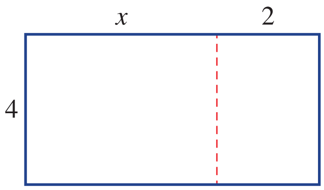
Example 13 Expanding brackets using rectangle areas

Write two equivalent expressions for the total area of the rectangle shown: one with brackets and the other without brackets.



Solution	Explanation
Using brackets: $2(5+x)$	The whole rectangle has height 2 and width $5+x$.
Without brackets: $10+2x$	The smaller rectangles have area $2\times 5=10$ and $2\times x=2x$, so they are added.

5 For each of the following rectangles, write two equivalent expressions for the total area.

a


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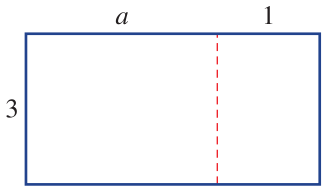
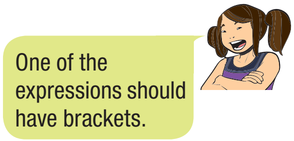
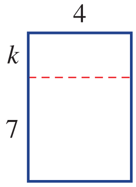
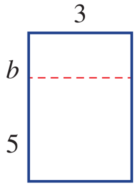




c


d


Example 14 Expanding using the distributive law

Expand the following expressions.

a $5(x+3)$ **b** $3(a-4)$ **c** $2(3p-7q)$

Solution	Explanation
a $5(x+3)=5x+5\times 3$ $=5x+15$	Using the distributive law $5(x+3)=5\times x+5\times 3$ Simplify the result.
b $3(a-4)=3a-3\times 4$ $=3a-12$	Using the distributive law $3(a-4)=3\times a-3\times 4$ Simplify the result.
c $2(3p-7q)=2\times 3p-2\times 7q$ $=6p-14q$	Using the distributive law $2(3p-7q)=2\times 3p-2\times 7q$ Simplify the result, remembering $2\times 3p=6p$ and $2\times 7q=14q$.

6 Use the distributive law to expand the following.

a $6(y+8)$ **b** $7(l+4)$ **c** $9(a+7)$ **d** $2(t+6)$

7 Use the distributive law to expand the following.

a $2(m-10)$ **b** $8(y-3)$ **c** $3(e-7)$ **d** $7(e-3)$

8 Use the distributive law to expand the following.

a $10(6g-7)$ **b** $5(3e-8)$ **c** $5(7w+10)$ **d** $5(2u+5)$
e $7(8x-2)$ **f** $3(9v-4)$ **g** $7(2q-4)$ **h** $4(5c-v)$
i $4(2+5x)$ **j** $3(7+2y)$ **k** $8(9-3x)$ **l** $11(2-4k)$

9 Fill in the missing number in the following expansions.

a $4(x+5) = 4x + \boxed{}$ **b** $3(x+2) = 3x + \boxed{}$
c $5(3a+2) = 15a + \boxed{}$ **d** $7(4x-2) = 28x - \boxed{}$

PROBLEM-SOLVING AND REASONING

10, 11 11–13

10 The perimeter of a rectangle is given by the expression $2(l+w)$ where l is the length and w is the width. What is an equivalent expression for this?

11 Expand the brackets in the following and then simplify the result.

a $3(x+2)+4x$ **b** $4(a+3)-2a$ **c** $5(3b-2)+10$ **d** $6(2c+4)-2c$

You can combine like terms.

12 Write an expression for each of the following and then expand it.

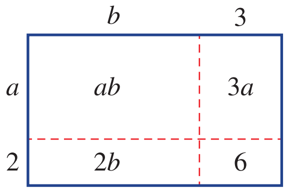
a A number x has 3 added to it and the result is multiplied by 5.
b A number b has 6 added to it and the result is doubled.
c A number z has 4 subtracted from it and the result is multiplied by 3.
d A number y is subtracted from 10 and the result is multiplied by 7.

13 When expanded, $4(2a+6b)$ gives $8a+24b$. Find two other expressions that expand to $8a+24b$.

Bigger expansions

— 14

14 The diagram below helps to demonstrate that $(a+2)(b+3) = ab+2b+3a+6$.



Use a diagram like the one above to expand the following expressions.

a $(a+4)(b+2)$
b $(x+3)(y+5)$
c $(2a+5)(3c+2)$
d $(4a+1)(5b+3)$

Progress quiz

5A **1** For each of the following expressions, state the coefficient of b.

a $8a + 5b - 2c$ **b** $12 - 4b$ **c** $15a + 7c - 11b$ **d** $a + b - 3d$

5A **2** Match each of the following worded statements with the correct mathematical expression.

a The sum of x and y

b 4 is subtracted from x

c x is quadrupled

d x is divided by 3 and y is added

e x is subtracted from 4

f x is halved and 6 is added

A $x - 4$

B $\frac{x}{3} + y$

C $4 - x$

D $x + y$

E $\frac{x}{2} + 6$

F $4x$

5B **3** If $a = 5$, find the value of:

a $11a$ **b** $24 - 3a$

c $a + 2a + 3a$ **d** $100 - a^2 + 2a$

5B **4** Evaluate the expression $4x - 3y$ when:

a $x = 8$ and $y = 5$ **b** $x = 2$ and $y = 3$

c $x = 11$ and $y = 0$ **d** $x = 100$ and $y = 1$

5C **5** Classify the following pairs as like terms (L) or not like terms (N).

a $5p$ and 5 **b** $12p$ and $17pq$

c $40x$ and $5x$ **d** $21ft$ and $2tf$

5C **6** Simplify the following by combining like terms.

a $5h + 8h - 3h$ **b** $12t + 7r - 3t$

c $4x + 4xy - 5y + 3xy$ **d** $9kt - 5k + 6k - 4tk$

5D **7** Simplify the following.

a $5w \times 3$ **b** $6y \times 3z$ **c** $2a \times 3b \times 4c$ **d** $5ef \times 11 \times 2m$

5D **8** Simplify the following.

a $y \times y$ **b** $4t \times 3t$ **c** $5h \times 3jh$ **d** $6g \times 3f \times 2f \times g$

5D **9** Simplify the following.

a $\frac{3f}{12}$ **b** $\frac{15xy}{5y}$ **c** $\frac{3ac}{9bcd}$ **d** $\frac{14xy^2}{21xy}$

5E **10** Expand the following expressions.

a $4(x + 6)$ **b** $2(5y - 7)$ **c** $5(4m - 3n)$ **d** $x(8 - 3x)$

5E **11** Expand the brackets in the following and then simplify the result.

a $6(x + 3) - 2x$ **b** $2(5 - 3x) + 7$ **c** $4(3x - 2y) - 8x$ **d** $x(x + 3) + 7x$

5F Factorising expressions ★

Factorising is the opposite procedure to expanding. Because $3(2x+5)$ expands to $6x+15$, this means that a factorised form of $6x+15$ is $3(2x+5)$.

Let's start: Expanding gaps

- Try to fill in the gaps to make the following equivalence true: $\square(\square+\square) = 12x+24$.
- In how many ways can this be done? Try to find as many ways as possible.
- If the aim is to make the term outside the brackets as large as possible, what is the best possible solution to the puzzle?

Key ideas

- The **highest common factor** (HCF) of two terms is the largest factor that divides into each term.
  e.g. HCF of $15x$ and $21y$ is 3.
  HCF of $10a$ and $20c$ is 10.
  HCF of $12x$ and $18xy$ is $6x$.
- To **factorise** an expression, first take the HCF of the terms outside the brackets and divide each term by it, leaving the result in brackets.
  e.g. $10x+5y$
  HCF $= 5$
  Result $\underset{\text{HCF}}{5}(\underset{10x \div 5}{2x}+\underset{15y \div 5}{3y})$

Highest common factor (HCF) The largest term that is a factor of all the given numbers

Factorise To write an expression as a product

Exercise 5F

UNDERSTANDING 1–5 5

1 Fill in the blanks to make these simple equations true.

a $5 \times \square = 20$ **b** $\square \times 4 = 12$ **c** $10 \times \square = 20$ **d** $\square \times 4 = 24$

2 The factors of 14 are $1, 2, 7$ and 14. The factors of 26 are $1, 2, 13$ and 26. What is the highest factor that these two numbers have in common?

Example 15 Finding the highest common factor (HCF) of numbers

Find the highest common factor (HCF) of 20 and 35.

Solution	Explanation
5	5 is the largest number that divides into 20 and 35. See Example 16 in Chapter 1 for more information.

3 Find the highest common factor of the following pairs of numbers.

a 12 and 18 **b** 15 and 25 **c** 40 and 60 **d** 24 and 10

4 Fill in the blanks.

a $5x \times \square = 15x$ **b** $7 \times \square a = 28a$ **c** $3 \times \square = 6b$ **d** $2 \times \square = 14x$

5 Fill in the blanks to make these expansions correct.

a $3(4x+1) = \square x + 3$ **b** $5(7-2x) = \square - 10x$

c $6(2+5y) = \square + \square y$ **d** $7(2a-3b) = \square - \square$

e $3(2a+\square) = 6a+21$ **f** $4(\square - 2y) = 12 + -8y$

g $7(\square + \square) = 14 + 7q$ **h** $\square(2x+3y) = 8x+12y$

$a(b+c) = ab + ac$

FLUENCY 6, 7, 8(½) 6–9(½)

Example 16 Finding the highest common factor (HCF) of terms

Find the highest common factor (HCF) of:

a $12k$ and 20 **b** $18x$ and $24xy$

Solution	**Explanation**
a 4	There are no pronumerals in common so choose the highest common factor of 12 and 20.
b $6x$	6 is the largest number that divides into 18 and 24, and x is in both terms.

6 Find the highest common factor (HCF) of the following pairs of terms.

a 15 and $10x$ **b** $20a$ and 12 **c** 27 and $9b$

d $7y$ and $14x$ **e** $3a$ and $6b$ **f** $12x$ and $18y$

The HCF can include pronumerals.

7 Find the HCF of the following pairs of terms.

a $12x$ and $18xy$ **b** $8a$ and $16ab$ **c** $9bc$ and $12b$

d $36xy$ and $24y$ **e** $10q$ and $12qr$ **f** $8p$ and $20pq$

Example 17 Factorising expressions

Factorise the following expressions.

a $6x+15$ **b** $12a+18ab$ **c** $21x-14y$

Solution	**Explanation**
a $6x+15 = 3(2x+5)$	HCF of $6x$ and 15 is 3. $6x \div 3 = 2x$ and $15 \div 3 = 5$
b $12a+18ab = 6a(2+3b)$	HCF of $12a$ and $18ab$ is $6a$. $12a \div 6a = 2$ and $18ab \div 6a = 3b$
c $21x-14y = 7(3x-2y)$	HCF of $21x$ and $14y$ is 7. $21x \div 7 = 3x$ and $14y \div 7 = 2y$

8 Factorise the following by first finding the highest common factor.

a $3x+6$ **b** $8v+40$ **c** $15x+35$ **d** $10z+25$

e $40+4w$ **f** $5j-20$ **g** $9b-15$ **h** $12-16f$

i $5d-30$ **j** $10x+5$ **k** $6k-12$ **l** $18p+20$

9 Factorise the following.

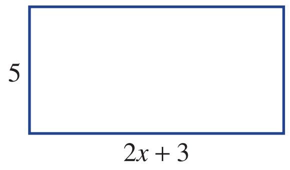
a $10cn+12n$ **b** $24y+8ry$ **c** $14jn+10n$ **d** $24g+20gj$

e $10h+4z$ **f** $30u-20n$ **g** $21p-6c$ **h** $12a+15b$

PROBLEM-SOLVING AND REASONING

10, 11 | 10–12

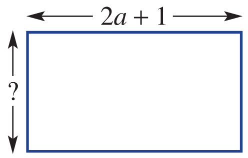
10 The rectangle shown to the right has an area of $10x+15$.
Draw a rectangle that would have an area $12x+16$.



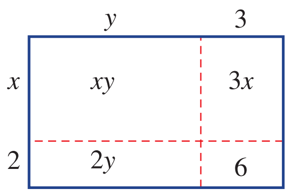
11 The area of the rectangle shown to the right is $10a+5$.
One side's measurement is unknown.

a What is the value of the unknown measurement?

b Write an expression for the perimeter of the rectangle.



12 Consider the diagram shown to the right.
What is the factorised form of $xy+3x+2y+6$?



The factorising photographer

— | 13

13 A group of students lines up for a photo. They are in 6 rows with x students in each row. Another 18 students join the photo.

a Write an expression for the total number of students in the photo.

b Factorise the expression above.

c How many students would be in each of the 6 rows now? Write an expression.

d If the photographer wanted just 3 rows, how many students would be in each row? Write an expression.

e If the photographer wanted just 2 rows, how many students would be in each row? Write an expression.

5G Applying algebra ★

Interactive

Widgets

HOTsheets

Walkthrough

The skills of algebra can be applied to many situations involving unknown or varying quantities.

Let's start: Carnival conundrum

Alwin, Bryson and Calvin have each been offered special deals for the local carnival.

- Alwin can pay \$50 to go on all the rides all day.
- Bryson can pay \$20 to enter the carnival and then pay \$2 per ride.
- Calvin can enter the carnival at no cost and then pay \$5 per ride.

- Which of them do you think has the best deal?
- In the end, they each went on 12 rides. Who paid the most? Who paid the least?

Algebra can be applied to both the engineering of a carnival ride and the price of tickets.

Key ideas

- Different situations can be **modelled** with algebraic expressions.
- To apply a rule, the variables should first be clearly defined.
  e.g. total cost is $2 \times n + 3 \times d$

  n = number of minutes d = distance in km

Exercise 5G

UNDERSTANDING 1–4 4

1 The cost of a newspaper is \$2 and the cost of an ice-cream is \$3. Find the cost of:

a 5 newspapers **b** 4 ice-creams **c** 10 newspapers and 2 ice-creams

2 An episode of Joshua's favourite television program lasts 30 minutes.

a How long would it take him (in minutes) to watch:

i 2 episodes? **ii** 5 episodes? **iii** 10 episodes?

b Which of the following expressions gives the total time to watch n episodes?

A $n + 30$ **B** $30n$ **C** $n \div 30$ **D** $30 - n$

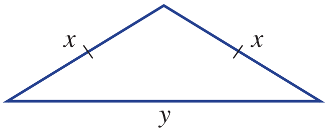
3 Evaluate the expression $3d + 5$ when:

a $d = 10$ **b** $d = 12$ **c** $d = 0$

4 Consider the isosceles triangle shown.

a Write an expression for the perimeter of the triangle.

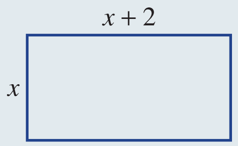
b Find the perimeter when $x = 3$ and $y = 2$.



FLUENCY 5–7 5–8

Example 18 Writing expressions from descriptions

Write an expression for the following situations.



a The total cost of k bottles if each bottle cost $4

b The perimeter of a rectangle if its length is 2 cm more than its width and its width is x cm

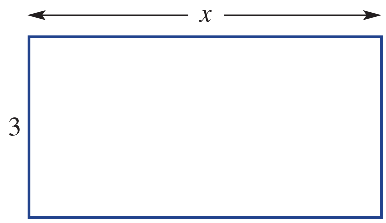
c The total cost of hiring a plumber for n hours if he charges $40 call-out fee and $70 per hour.

Solution	**Explanation**
a $4 \times k = 4k$	Each bottle costs $4 so the total cost is $4 multiplied by the number of bottles purchased.
b $x+x+2+x+x+2=4x+4$	width $= x$ so length $= x+2$. The perimeter is width + length + width + length.
c $40+70n$	$70 per hour means that the cost to hire the plumber would be $70 \times n$. Additionally, $40 is added for the call-out fee, which is charged regardless of how long the plumber stays.

5 **a** Write an expression for the total perimeter of the shape shown.

b If $x = 9$, what is the perimeter?

c Write an expression for the area.

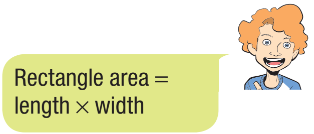


6 Pens cost $3 each.$3

a How much would 10 pens cost?

b Write an expression for the total cost of n pens.

c If $n = 12$, find the total cost.



7 An electrician charges a call-out fee of $30 and $90 per hour.

a How much does a 2-hour visit cost?

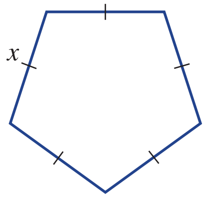
b Which of the following represents the total cost for x hours?

A $x(30+90)$ **B** $30x+90$ **C** $30+90x$ **D** $120x$

8 **a** Give an expression for the perimeter of this regular pentagon.

b If each side length were doubled, what would the perimeter be?

c If each side length were increased by 3, write a new expression for the perimeter.



PROBLEM-SOLVING AND REASONING

9, 10 10–12

9 An indoor soccer pitch costs \$40 per hour to hire plus a \$30 booking fee.
  a Write an expression for the cost of hiring the pitch for x hours.
  b What is the cost of hiring the pitch for an 8-hour tournament?

10 A plumber says that the cost in dollars to hire her for x hours is $50 + 60x$.
  a What is her call-out fee?
  b How much does she charge per hour?
  c How much does a 3-hour visit cost?

11 A repairman says the cost in dollars to hire his services for x hours is $20(3 + 4x)$.
  a How much would it cost to hire him for 1 hour?
  b Expand the expression he has given you.
  c What is:
    i his call-out fee
    ii the amount he charges per hour?

$20(3 + 4x)$
$= \_\_\_\_\_ + \_\_\_\_\_$

12 Tamir notes that whenever he hires an electrician, they charge a call-out fee \$$F$ and an hourly rate of \$$H$ per hour.
  a Write an expression for the cost of hiring an electrician for one hour.
  b Write an expression for the cost of hiring an electrician for two hours.
  c Write an expression for the cost of hiring an electrician for 30 minutes.

Your expressions should involve *F* and/ or *H*.

Ticket sales

— 13

13 Three deals are available at a fair.

Deal 1: Pay \$10, rides cost \$4/each.

Deal 2: Pay \$20, rides cost \$1/each.

Deal 3: Pay \$30, all rides are free.

  a Write an expression for the total cost of n rides using deal 1. (The total cost includes the entry fee of \$10.)
  b Write an expression for the total cost of n rides using deal 2.
  c Write an expression for the total cost of n rides using deal 3.
  d Which of the three deals is best for someone going on just two rides?
  e Which of the three deals is best for someone going on 20 rides?
  f Fill in the gaps:
    i Deal 1 is best for people wanting up to _____ rides.
    ii Deal 2 is best for people wanting between _____ and _____ rides.
    iii Deal 3 is best for people wanting more than _____ rides.

Maths@Work: Pharmacist

Pharmacists complete a university degree, enjoy science and maths and want to work in the medical industry. They need to understand and accurately fill prescriptions, and know multiple safe drug combinations and side effects so that patient's lives are not at risk.

Pharmacists talk with customers about their health and life style and can suggest vitamin supplements, weight loss products, measure blood pressure and blood sugar levels and, if needed, advise people to see their doctor for further treatment.

Compounding pharmacists have the skills and apparatus to mix their own products using various formulas rather than just supply pre-packaged medications. For example, a doctor may prescribe an iron supplement for a baby and the pharmacist will prepare the correct dose in a flavour the child will accept.

1 Chemists can use the Body Mass Index (BMI) as an indication of a client's health. BMI for adults is calculated by dividing a person's weight (in kg) by the square of their height (in m). A healthy range BMI = 20–25 kg/m^2.

Find the nearest whole number BMI for these people and state if it is within the healthy range.

- **a** Sally, weight 56 kg, 1.54 m tall.
- **b** Ahmed, weight 67 kg, 1.73 m tall.
- **c** Ainslie, weight 55 kg, 1.6 m tall.
- **d** Blake, weight 72 kg, 1.75 m tall.
- **e** Dominic, weight 105 kg, 1.9 m tall.

$$BMI = \frac{weight\,(kg)}{[height\,(m)]^2}$$

2 A pharmacist needs to precisely dilute various solutions. The concentration C (grams per litre, g/L) decreases as the volume V (litres, L) increases. This formula is used for these procedures:

$$C_1 \times V_1 = C_2 \times V_2$$

Original concentration and volume

Final concentration and volume

a Hari has a solution with an original concentration $C_1 = 4$ g/L. He dilutes it to create 8 litres of a 2 g/L solution. Using the formula above, calculate the original volume V_1 of solution.

Write rule: $C_1 \times V_1 = C_2 \times V_2$
Substitute: $4 \times V_1 = 2 \times 8$
Find unknown amount: $4 \times V_1 = 16$
$4 \times ? = 16$
$V_1 = ?$ *litres*

b Mal has a solution with a concentration of 5 g/L. He dilutes it to create 10 Litres of a 2 g/L solution. What original volume V_1 of solution did Mal use?

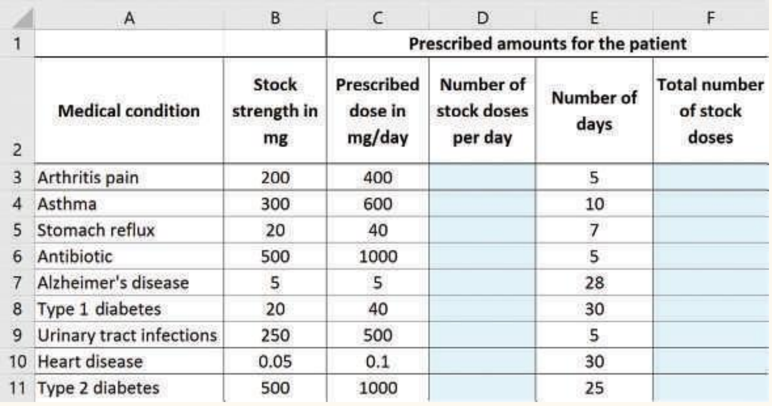
c Kelsey has $\frac{1}{2}$L of a 4 g/L solution and she adds $1\frac{1}{2}$ litres of distilled water. What is the volume V_2 and concentration C_2 of the final solution?

3 A baby is given an antibiotic in oral form. The prescribed amount is 2.5 mL three times a day for 5 days. How many mL does the chemist need to make up for the parents to ensure the doctor's instructions are fulfilled?

Using technology

4 Medications come in certain 'stock' strengths and a chemist needs to calculate the number of 'stock' doses per day that a patient has been prescribed.

Set up the Excel spreadsheet shown below and enter formulas to calculate the prescribed stock dosage numbers.



	A	B	C	D	E	F
1			Prescribed amounts for the patient			
2	Medical condition	Stock strength in mg	Prescribed dose in mg/day	Number of stock doses per day	Number of days	Total number of stock doses
3	Arthritis pain	200	400		5	
4	Asthma	300	600		10	
5	Stomach reflux	20	40		7	
6	Antibiotic	500	1000		5	
7	Alzheimer's disease	5	5		28	
8	Type 1 diabetes	20	40		30	
9	Urinary tract infections	250	500		5	
10	Heart disease	0.05	0.1		30	
11	Type 2 diabetes	500	1000		25	

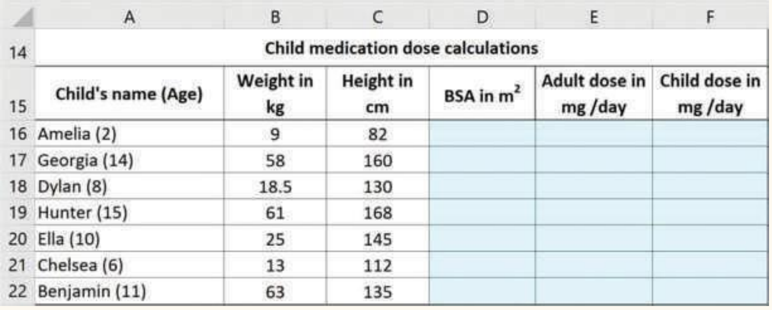
5 A person's body surface area (BSA) is used for calculating medication dose amounts. BSA is calculated from weight (in kg) and height (in cm) using this algebra rule:

$$BSA = \sqrt{\left(\frac{weight(kg) \times height(cm)}{3600}\right)}$$

Use SQRT for $\sqrt{\ }$
Formula cell D16
= SQRT(B16 * C16/3600)

a Set up the Excel spreadsheet shown below for calculating medication dose amounts.

b In column D, enter formulas to calculate the BSAs to 4 decimal places.



	A	B	C	D	E	F
14		Child medication dose calculations				
15	Child's name (Age)	Weight in kg	Height in cm	BSA in m^2	Adult dose in mg /day	Child dose in mg /day
16	Amelia (2)	9	82			
17	Georgia (14)	58	160			
18	Dylan (8)	18.5	130			
19	Hunter (15)	61	168			
20	Ella (10)	25	145			
21	Chelsea (6)	13	112			
22	Benjamin (11)	63	135			

c Children have smaller bodies than adults, so they need smaller doses of medication. The average adult body surface area is $1.7\ \text{m}^2$. This formula calculates child dose amounts:

$$\textit{Child dose} = \textit{adult dose} \times \frac{\textit{child BSA}}{1.7}$$

Enter this formula in column F, e.g. cell F16 formula $= \text{E16}*\text{D16}/1.7$. Format cells to 0 d. p. Column F will have answers of zero until you have completed the next question.

6 Use your Excel spreadsheet from question **5** to find the following prescribed dose amounts. Enter values into column E and the spreadsheet will calculate the answers you need.

- **a** An antibiotic, Keflex, has an adult dose of 500 mg/day. What is the Keflex dose that sisters Amelia and Georgia are prescribed?
- **b** Dylan and Hunter are prescribed Amoxil for a chest infection. If the adult dose is 250 mg, state their prescribed dosages. Why is Hunter's dose almost the same as the adult dose?
- **c** Ella is 4 years older than her sister Chelsea. They are both prescribed a Nuelin for asthma which has 200 mg/day adult dose. How much more does Ella take per dose than Chelsea?
- **d** A blood pressure medication called Tenormin has an adult dose of 50 mg/day. What is the dose that Benjamin is prescribed? What percentage is this of an adult's dose?

Puzzles and games

1 How many matchsticks would be needed to make 1000 squares?

$n = 1$	$n = 2$	$n = 3$	… $n = 1000$
4 sticks	7 sticks	10 sticks	? sticks

2 Find the values of A, B and C so that the rows and columns add up correctly.

A	B	C	**Sum** $= 14$
A	C	B	**Sum** $= 14$
A	C	B	**Sum** $= 14$
Sum $= 15$	**Sum** $= 16$	**Sum** $= 11$	

3 Fill in the missing expressions to make the six equivalences true.

$3x$	$+$		$=$	$7x + 3y + 1$
$+$		$+$		$+$
	$+$		$=$	
$=$		$=$		$=$
$2y + 3x$	$+$		$=$	$7x + 6y + 1$

4 Think of a number n.
Double it and add 4.
Triple the result and subtract 12.
You now have 6 times the original number.
Use algebra to see if this was just a coincidence.
Design a puzzle like this and try it on your friends.

5 Find the largest value for each of the following.

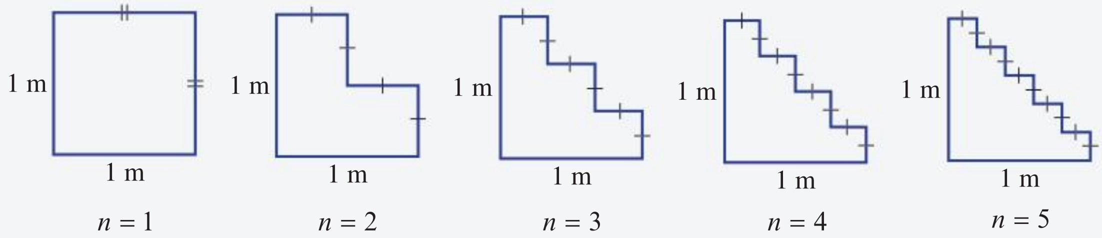
a If b can be any number, what is the largest value of $b \times (10 - b)$?

b If m can be any number, what is the largest value that $10 - m(m + 5)$ could have?

c If $x + y$ evaluates to 15, what is the largest value that $x \times y$ could have?

d If a and b are chosen so that $a^2 + b^2$ is equal to $(a + b)^2$, what is the largest value of $a \times b$?

6 Consider the following pattern.



The perimeter for the shape when $n = 1$ is given by the expression 4 m and the area is 1 m^2.

a Calculate the perimeter and area of the other shapes shown above and try to find a pattern.

b If $n = 1000$, state the perimeter and give the approximate area.

Think about what the shape looks like.

Chapter summary

Algebra

Pronumeral: a letter that stands for a numerical value

Language

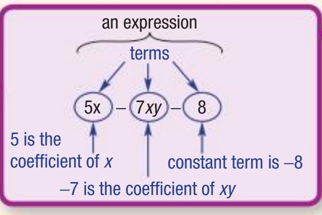
+
sum
more than
added
increased

−
difference
less than
minus
decreased

×
product
times
double (2×)
twice (2×)
triple (3×)

÷
quotient
divide
one third
one half
quarter

× ÷
not used in algebraic expressions



Substitution

'evaluate' 'substitute'

Replace pronumerals with numbers and calculate answer

If $a = 3$ then

$$\begin{aligned} 5a + 7 &= 5 \times 3 + 7 \\ &= 15 + 7 \\ &= 22 \end{aligned}$$

Equivalent expressions

Always evaluate to the same number
e.g. $2x$ and $x + x$

Multiplying and dividing terms

$$\begin{aligned} 3a \times 2b &= 3 \times a \times 2 \times b \\ &= 6 \times a \times b \\ &= 6ab \end{aligned}$$

$$\frac{\overset{2}{\cancel{12}}a}{\underset{3}{\cancel{18}}b} = \frac{2a}{3b}$$

Like terms

Have the same pronumerals
$6x$ and $5x$ ✓
$3x$ and $7y$ ✗
can be in different order
$6ab$ and $12ba$

Adding and subtracting like terms

$\boxed{6a}\ \boxed{+12b}\ \boxed{+3a}\ \boxed{-7b}$

$$\begin{aligned} &= 6a + 3a + 12b - 7b \\ &= 9a + 5b \end{aligned}$$

Expanding

$3(2x + 5y) = 6x + 15y$

$2a(5 - 7b) = 10a - 14ab$

Factorising ★

$$\begin{aligned} &12x + 6 \quad (\text{HCF} = 6) \\ &= 6 \times 2x + 6 \times 1 \\ &= 6(2x + 1) \end{aligned}$$

Multiple-choice questions

5A **1** The sum of x and y can be written as:

A $2x$ **B** $2xy$ **C** $x + y$ **D** $x - y$ **E** xy

5A **2** Consider the expression $5a - 3b + 8$. Which one of the following statements is true?

A The coefficient of a is 5.
B It has 5 terms.
C The constant term is -8.
D The coefficient of b is 3.
E The coefficient of a is 10.

5A **3** If n is a number, which of the following represents one third of n?

A $\frac{3}{n}$ **B** $0.3n$ **C** $3n$ **D** $\frac{n}{3}$ **E** $n - 3$

5B **4** If $a = 2$, then $17 + 2a$ is:

A 3 **B** -3 **C** 21 **D** 11 **E** 13

5D **5** $3 \times x \times y$ is equivalent to:

A $3x + y$ **B** xy **C** $3 + x + y$ **D** $3x + 3y$ **E** $3xy$

5D **6** $\frac{12ab}{24a}$ can be simplified to:

A $2ab$ **B** $\frac{2a}{b}$ **C** $\frac{b}{2a}$ **D** $\frac{ab}{2}$ **E** $\frac{b}{2}$

5E **7** The expanded form of $2(3 + 5y)$ is:

A $6x + 5y$ **B** $3x + 5y$ **C** $6x + 5xy$ **D** $6 + 10y$ **E** $6x + 10xy$

5D **8** Simplifying $3a \div 6b$ gives:

A 2 **B** $\frac{a}{b}$ **C** $\frac{2a}{b}$ **D** $\frac{ab}{2}$ **E** $\frac{a}{2b}$

5C **9** When like terms are combined, $3a + 4b + 2a - 2b$ simplifies to:

A $5a + 6b$ **B** $7ab$ **C** $11ab$ **D** $5a + 2b$ **E** $a + 6b$

5F **10** The factorised form of $3a - 6ab$ is:

A $3a(1 - 2b)$ **B** $3a(a - 2b)$ **C** $3a(a - b)$ **D** $6a(a - b)$ **E** $3(a - 2ab)$

Short-answer questions

5A **1** State whether each of the following is true (T) or false (F).

a The constant term in the expression $5x + 7$ is 5.
b $16xy$ and $5yx$ are like terms.
c The coefficient of d in the expression $6de + 7d + 8abd + 3$ is 7.
d The highest common factor of $12abc$ and $16c$ is $2c$.
e The coefficient of x in $5y - 3x$ is -3.

5A **2** For the expression $6xy + 2x - 4y + 3$, state:

a the coefficient of x

b the constant term

c the number of terms

d the coefficient of xy

5B **3** Substitute the following values of a to evaluate the expression $12 - 2a$.

a 1 **b** 2 **c** 4 **d** 6

5B **4** Substitute $A = 2$ and $B = 5$ into the following expressions.

a $10A$ **b** $A + B$ **c** $B - A$ **d** $3A + 2B$

5B **5** Substitute $x = 2$ and $y = 3$ into each of the following.

a $2y + 3$ **b** $3x + y$ **c** $xy + y$ **d** $4x - 2y$

5C **6** Simplify each of these expressions by collecting like terms.

a $7m + 9m$ **b** $3a + 5b - a$ **c** $3y - x + y + 1$

d $5x + 3y + 2x + 4y$ **e** $7x - 4xy + 5xy + 2x$ **f** $7m - 2n + 3m - 4n$

5D **7** Simplify.

a $9a \times 4b$ **b** $30 \times x \times y$ **c** $2x \times 5y \times 3z$

5D **8** Simplify.

a $\frac{10x}{5}$ **b** $\frac{12ab}{4b}$ **c** $\frac{4xz}{20xy}$

5E **9** Expand and simplify when necessary.

a $3(x - 4)$ **b** $2(5 + x)$ **c** $3(2y + 4)$ **d** $10(2x + 7)$

e $3(x - 5)$ **f** $11(z - 2)$ **g** $4(3a - 11)$ **h** $2(6b - 3)$

5F ★ **10** Find:

a the HCF of $12x$ and 16 **b** the HCF of $14ab$ and $21a$

5F ★ **11** Factorise fully.

a $2x + 6$ **b** $24 - 16g$ **c** $12x + 3xy$ **d** $7a + 14ab$

5G ★ **12** If apricots cost \$$a$ each and pears cost \$$p$ each, write an expression for:

a the cost of 5 apricots

b the cost of 3 pears

c the cost of 5 apricots and 3 pears

5G ★ **13** Greg runs 10 km each day.

a How far (in km) does he run in one week (7 days)?

b Write an expression for how far he runs in n days.

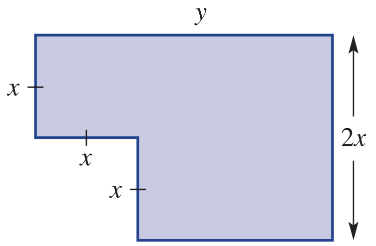
Extended-response questions

1 Two bus companies have different pricing structures. Each charge a call-out fee to cover business running costs. The price per hour pays for the driver's wages.

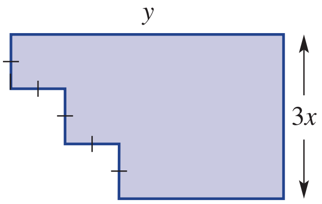
Company A $120 call-out fee, plus $80 per hour	Company B $80 call-out fee, plus $100 per hour

- **a** Write an expression for the total cost of travelling for n hours with company A.
- **b** Write an expression for the total cost of travelling for n hours with company B.
- **c** What is the cost of travelling for 3 hours with each company?
- **d** For how long would you need to hire a bus to make company A the cheaper option?
- **e** If a school hired one bus from each company for n hours, what would the total cost be?

2 Consider the floor plan shown.



- **a** Write an expression for the floor's area in terms of x and y.
- **b** Using that expression, find the floor's area if $x = 3$ metres and $y = 7$ metres.
- **c** Write an expression for the floor's perimeter in terms of x and y.
- **d** Using that expression, find the floor's perimeter if $x = 3$ metres and $y = 7$ metres.
- **e** Another floor plan is shown at right. Write an expression for the floor's area and an expression for its perimeter.



Integers

Multiple-choice questions

1 $156 \div 4$ is the same as:
A $156 \div 2 \times 2$ **B** $156 \div 2 \div 2$ **C** $312 \div 2$ **D** $156 \times 2 \div 2$

2 $-24 + 6 \times (-3)$ is equal to:
A 6 **B** 42 **C** -42 **D** -6

3 What is the smallest number that can be added to 1923 to make the answer divisible by 9?
A 1 **B** 2 **C** 3 **D** 4

4 $(-15)^2$ equals:
A 225 **B** 30 **C** -30 **D** -225

5 $6^4 \times 6^3$ is the same as:
A 36^7 **B** 6^7 **C** 36 **D** 6^1

Short-answer questions

1 Evaluate, without using a calculator.
a $4973 + 196$ **b** $1506 - 156$ **c** -96×3
d 139×5 **e** 14×99 **f** $14 \times 99 + 14 \times 101$
g 9^2 **h** 4^3 **i** $-9 - 7 - 3$

2 Evaluate.
a $10 - 6 \times 4$ **b** $15 \times 4 \div 2$ **c** $24 \div 2 \times 6$
d $-3 + (-10 - (-6))$ **e** $-81 \div (-3) \times 2$ **f** $73 - 72 - 7$

3 Find the HCF of:
a 24 and 42
b 35 and 42
c 100 and 60
d 15 and 45

4 Write down the LCM of:
a 24 and 42
b 8 and 9
c 100 and 60
d 15 and 45

5 Write using powers.
a $7 \times 7 \times 7$
b 8×8
c $3 \times 3 \times 3 \times 3 \times 3$

6 Simplify using index laws.
a $3^4 \times 3^5$ **b** $4^7 \div 4^5$
c 6^0 **d** $(5^3)^4$

7 Write down the value of:
a 9^2 **b** $\sqrt{49}$
c 5^3 **d** $\sqrt[3]{8}$

8 If $a = 5$ and $b = -7$, what is the value of:
a $a + b$ **b** $a - b$ **c** $a \times b$
d $15 - b$ **e** a^2 **f** b^2

Extended-response questions

1 The weather for a November day is given for different cities around the world.

	Minimum (°C)	Maximum (°C)
Amsterdam	3	12
Auckland	11	18
LA	8	14
Hong Kong	16	28
Moscow	6	8
Beijing	–3	0
New York	8	10
Paris	6	13
Tel Aviv	16	23
Wollongong	18	22

a Which city recorded the highest temperature on the day shown in the table?

b Which two cities only had a 2° difference in temperature between minimum and maximum temperature?

c Which city had the largest difference in temperature on this November day?

d What is the difference in the minimum temperatures of Beijing and Auckland?

Lines, shapes and solids

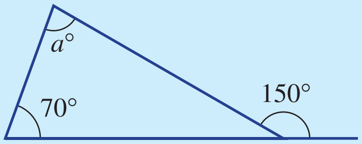
Multiple-choice questions

1 The supplementary angle to 80° is:

A 10° **B** 100° **C** 280° **D** 20°

2 In this diagram a equals:

A 150 **B** 220 **C** 70 **D** 80

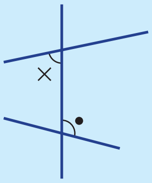


3 The angle sum of a regular pentagon is:

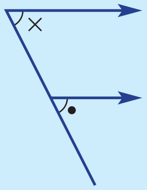
A 72° **B** 108° **C** 540° **D** 120°

4 Which diagram shows equal alternate angles?

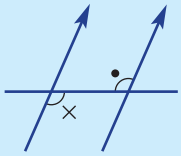
A



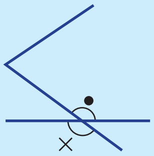
B



C

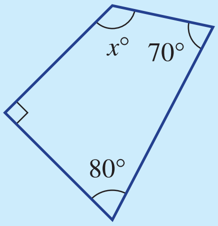


D



5 The value of x in this quadrilateral is:

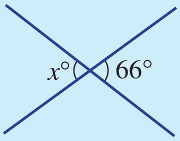
A 150° **B** 240°
C 120° **D** 300°



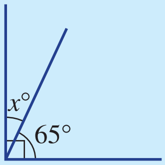
Short-answer questions

1 Find the value of x.

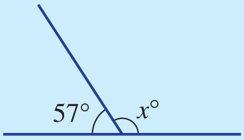
a



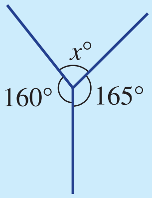
b



c



d



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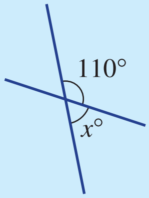

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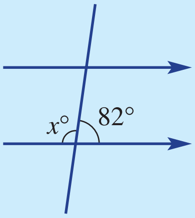
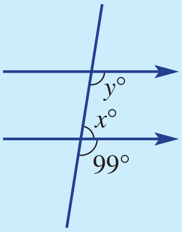


2 Find the value of each pronumeral.

a



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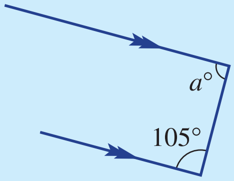

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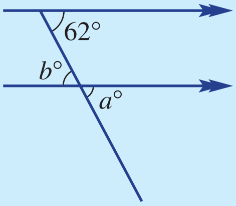

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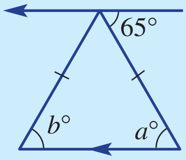

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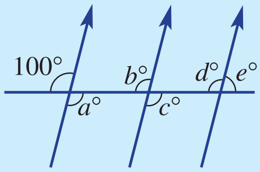

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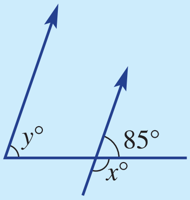
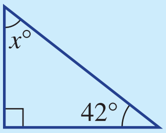


3 Find the value of the pronumeral in these triangles.

a



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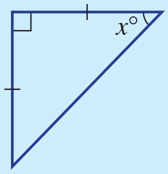
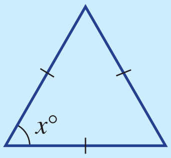


c



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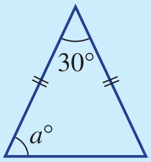

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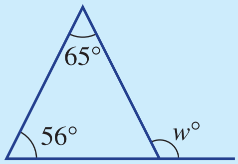

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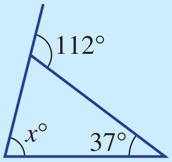
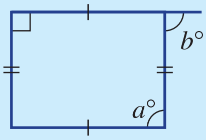


4 Find the value of a and b in these quadrilaterals.

a



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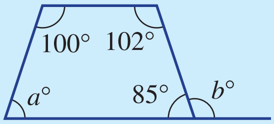

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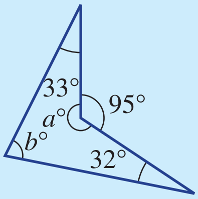
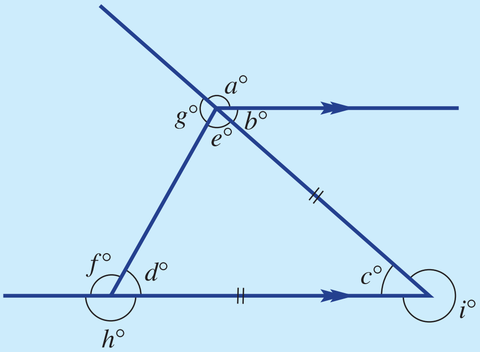


5 Find the angle sum of:

a a pentagon **b** an octagon

Extended-response questions



1 **a** If $a = 115$, find the size of each angle marked. Give a reason for each answer. Write your answers in the order you found them.

b Is the order the same for everybody in the class? Discuss any differences and the reasons associated with each.

Fractions, decimals and percentages

Multiple-choice questions

1 $\frac{150}{350}$ simplifies to:

A $\frac{6}{14}$ **B** $\frac{3}{70}$ **C** $\frac{15}{35}$ **D** $\frac{3}{7}$

2 Sienna spends $\frac{3}{7}$ of her \$280 income on clothes and saves the rest. She saves:

A \$470 **B** \$120 **C** \$160 **D** \$2613

3 0.008×0.07 is equal to:

A 0.056 **B** 0.0056 **C** 0.00056 **D** 56

4 0.24 expressed as a fraction is:

A $\frac{1}{24}$ **B** $\frac{6}{25}$ **C** $\frac{12}{5}$ **D** $\frac{24}{10}$

5 If 5% of x is 8, then 10% of x equals:

A 4 **B** 16 **C** 64 **D** 80

Short-answer questions

1 Copy and complete these equivalent fractions.

a $\frac{3}{5} = \frac{\square}{30}$ **b** $\frac{\square}{11} = \frac{5}{55}$ **c** $1\frac{4}{6} = \frac{\square}{3}$

2 Evaluate each of the following.

a $\frac{3}{4}-\frac{1}{2}$ **b** $\frac{4}{5}+\frac{3}{5}$ **c** $1\frac{1}{2}+1\frac{3}{4}$

d $\frac{4}{7}-\frac{2}{3}$ **e** $\frac{4}{9}\times\frac{3}{4}$ **f** $1\frac{1}{2}\times\frac{3}{5}$

3 Write the reciprocal of:

a $\frac{2}{5}$ **b** 8 **c** $4\frac{1}{5}$

4 Evaluate.

a $2\frac{1}{2}\times1\frac{4}{5}$ **b** $1\frac{1}{2}\div2$ **c** $3-2\frac{1}{3}$

5 Calculate each of the following.

a $3.84+3.09$ **b** $10.85-3.27$ **c** $12.09\div3$

d $6.59-0.08$ **e** 96.37×40 **f** $15.84\div0.02$

6 Evaluate.

a 5.3×100 **b** 9.6×1000 **c** $61.4\div100$

7 Copy and complete this table of decimals, fractions and percentages.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$				
Decimal								0.99	0.005
Percentage						80%	95%		

8 Find:

a 10% of 56 **b** 12% of 98 **c** 15% of 570 m

d 99% of \$2 **e** 25% of \$840 **f** 50% of 8500 g

9 **a** Increase \$560 by 25%.

b Decrease \$980 by 12%.

10 A \$348 Charlie Brown dress was bought for \$261.

a How much money did the buyer save?

b What percentage off does this equal?

Extended-response questions

1 A laptop decreases in value by 20% a year.

a Find the value of a \$2000 laptop at the end of:

i 1 year **ii** 2 years **iii** 3 years

b After how many years is the laptop worth less than \$800?

c Is the laptop ever going to have a value of zero dollars? Explain.

Measurement

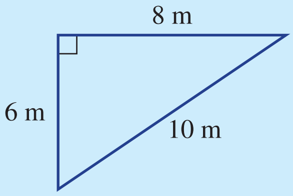
Multiple-choice questions

1 A cube has a volume of 8 cubic metres. The side length of the cube is:

A 8 m **B** 4 m **C** 2 m **D** 16 m

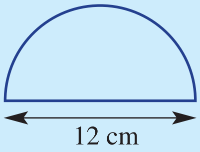
2 The area of this triangle is:

- **A** 48 m^2
- **B** 24 m^2
- **C** 30 m^2
- **D** 40 m^2



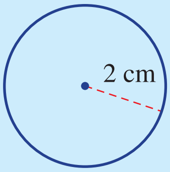
3 The perimeter of this semicircle is closest to:

- **A** 38 cm
- **B** 30 cm
- **C** 19 cm
- **D** 31 cm



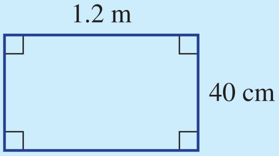
4 The area of this circle is given by:

- **A** 8π cm^2
- **B** 4π cm^2
- **C** 2π cm^2
- **D** π cm^2



5 The area of this rectangle is:

- **A** 48 m^2
- **B** 48 000 cm^2
- **C** 480 cm^2
- **D** 0.48 m^2



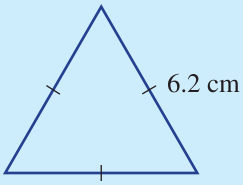
Short-answer questions

1 Complete these conversions.

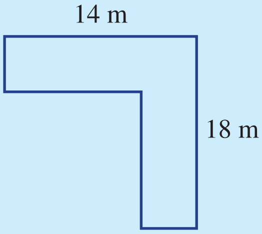
a 5 m = ______ cm
b 1.8 m = ______ cm
c 9 m^2 = ______ cm^2
d 1800 mm = ______ cm
e 4 L = ______ cm^3
f 0.01 km^2 = ______ m^2

2 Find the perimeter of these shapes.

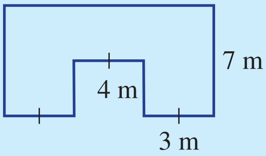
a



b



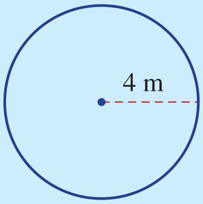
c



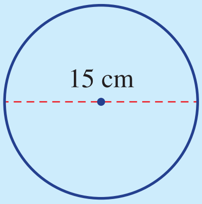
3 Find, correct to two decimal places:

i the circumference
ii the area

a



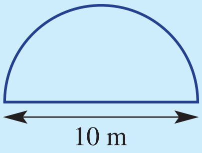
b



4 Find, correct to two decimal places:

i the perimeter **ii** the area

a



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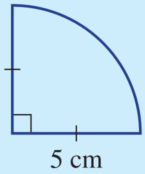
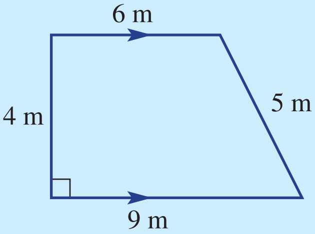


5 Find the area of these shapes.

a



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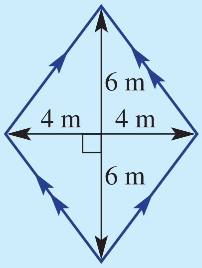

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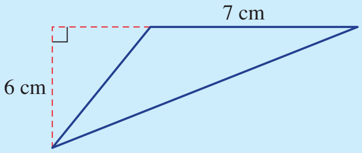
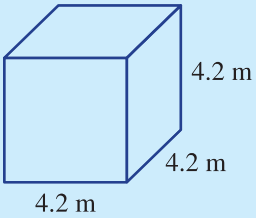


6 Find the volume of these solids.

a



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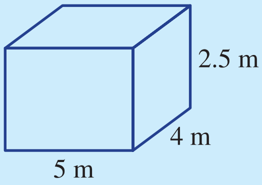
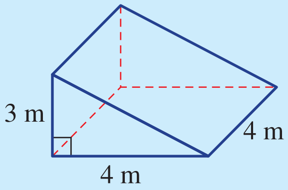


c

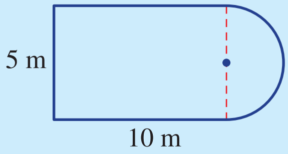


7 Write these times using 24-hour time.

a 3:30 pm **b** 7:35 am

Extended-response questions

1 A paved area is in the shape of a rectangle with a semicircular end as shown.



a What is the radius of the semicircle?

b What is the area of the semicircle, correct to two decimal places?

c What is the total area of the paved area, correct to two decimal places?

d A special brick border is to go around the perimeter of the area. Find this length, correct to the nearest metre.

Algebra

Multiple-choice questions

1 If $x = 3$, then $7x + 2$ equals:

A 21 **B** 75 **C** 73 **D** 23

2 $4x + 5 + 3x$ simplifies to:

A $7x + 5$ **B** $12x$ **C** $12 + x^2$ **D** $2x + 12$

3 $2(6x + 5)$ expands to:

A $12x + 5$ **B** $12x + 10$ **C** $6x + 10$ **D** $60x$

4 $12m + 18$ factorises to:
A $2(6m - 9)$ **B** $-6(2m - 3)$ **C** $6(3 - 2m)$ **D** $6(2m + 3)$

5 The coefficient of x in $12 - 3x + 4y$ is:
A 12 **B** 3 **C** -3 **D** 4

Short-answer questions

1 Write an expression for:
a the sum of p and q
b the product of p and 3
c half the square of m
d the sum of x and y, divided by 2

2 Find the value of $7k - 2$ if:
a $k = 3$ **b** $k = 10$ **c** $k = 5$ **d** $k = 100$

3 If $a = 6$, $b = 4$ and $c = 1$, evaluate:
a $a + b + c$ **b** $ab - c$ **c** $a(b - c)$ **d** $3a + 2b$ **e** abc **f** $a - 2b + 3c$

4 Simplify each algebraic expression.
a $4 \times 6k$ **b** $a + a + a$ **c** $a \times a \times a$ **d** $7p \div 14$
e $3ab + 2 + 4ab$ **f** $7x + 9 - 6x - 10$ **g** $18xy \div 9x$ **h** $m + n - 3m + n$

5 Simplify.
a $\frac{5xy}{5}$ **b** $\frac{30x}{21y}$ **c** $\frac{2w}{10}$ **d** $\frac{17abc}{5bc}$

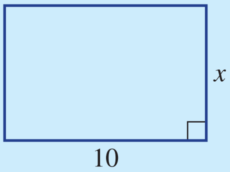
6 Expand, and simplify where necessary.
a $2(x + 5)$ **b** $6(2m - 3)$ **c** $10 + 2(m - 3)$

7 State the HCF of:
a $12x$ and $6y$ **b** $15k$ and $20kl$ **c** $120ab$ and $100bc$

8 Factorise.
a $18a - 12$ **b** $6mn + 12m$ **c** $8x + 12$

9 Write an expression for the rectangle's:
a perimeter **b** area

10 If pens cost \$2 each and notepads cost \$3 each, write expressions for:
a the cost of x pens
b the cost of y notepads
c the total cost of x pens and y notepads.



Extended-response questions

1 A repairman charges a \$60 call-out fee plus \$80 per hour.
a Find the cost of a 2-hour visit.
b Write an expression for the cost of an n-hour visit.
c Another repairman charges no call-out fee but \$100 per hour.
  i Write an expression for this repairman's total cost.
  ii For how many hours were they hired if the total cost was the same for both repairmen?

Chapter 6

Ratios and rates

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Real numbers

Solve a range of problems involving rates and ratios, with and without digital technologies **(ACMNA188)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Gear ratios and the F1 Grand Prix

Formula 1 racing is big business with enormous profits and prize money, giving teams and drivers huge incentives to perform.

Gear ratios play a vital role in the performance of a racing car. Gears are designed to transfer power from the engine to the wheel axle. Gears are simply wheels with teeth. The motor rotates the 'drive' gear which in turn rotates a 'driven' gear. If the drive gear (e.g. with 30 teeth) makes two revolutions for every one revolution made by a driven gear (e.g. with 60 teeth), the gear ratio is 2 : 1.

In Formula 1 racing, high performance engines cannot make the car go faster than the gear ratios allow. The world-famous McLaren P1 has 7 gears with a first gear ratio of 3.98 : 1 giving a maximum speed of 80 km/h in first gear. In 7th gear, the gear ratio is 0.69 : 1 and the top speed becomes 460 km/h.

Pre-test

1 State the missing number.

a $\underset{\div 2}{\overset{\div 2}{\frac{2}{4} = \frac{1}{\square}}}$ **b** $\underset{\div 5}{\overset{\div 5}{\frac{15}{20} = \frac{\square}{4}}}$ **c** $\underset{\div ?}{\overset{\div ?}{\frac{12}{15} = \frac{\square}{5}}}$

2 State the missing number.

a $\times 2 \begin{array}{c} 2 : 5 \\ 4 : \square \end{array} \times 2$ **b** $\div 4 \begin{array}{c} 20 : 28 \\ \square : 7 \end{array} \div 4$ **c** $\times ? \begin{array}{c} 3 : 2 \\ 12 : \square \end{array} \times ?$

3 

Write the ratio of:
- **a** squares to circles
- **b** circles to triangles
- **c** triangles to total shapes

4 Convert:

a 5 m = ____ cm **b** 6 km = ____ m
c 500 cm = ____ m **d** 80 mm = ____ cm
e 120 cm = ____ m **f** 15000 m = ____ km

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

5 Write these ratios in the same units and simplify.

a 3 cm : 15 mm **b** 45 cm : 1 m
c 45 minutes : 1 hour **d** 10 minutes : $\frac{1}{2}$ hour
e 2 km : 500 m **f** 40 m : $\frac{1}{2}$ km

6 If 4 mangoes cost $6:
- **a** how much would one mango cost?
- **b** how much would 12 mangoes cost?

÷4 4 mangoes cost $6 ÷4
1 mango costs $?

7 The cost of 1 kg of bananas is $4.99. Find the cost of:

a 2 kg **b** 5 kg **c** 10 kg **d** $\frac{1}{2}$ kg

8 Kevin walks 3 km in one hour. How far did he walk in 30 minutes?

9 Tao earns $240 for working 8 hours. How much did Tao earn each hour?

10 A car travels at an average speed of 60 km per hour.
- **a** How far does it travel in the following times?
  - **i** 2 hours **ii** 5 hours **iii** $\frac{1}{2}$ hour
- **b** How long would it take to travel the following distances? (Answer in fractions of hours.)
  - **i** 180 km **ii** 90 km **iii** 20 km
- **c** If the car's speed was 70 km per hour, how many minutes would it take to travel 7 km?

÷10 70 km in 60 minutes ÷10
7 km in ? minutes

6A Introducing ratios

Interactive
Widgets
HOTsheets
Walkthrough

Ratios are regularly used in everyday life. They are used to show the relationship between two (or more) related quantities.

Here are five common uses of ratios:

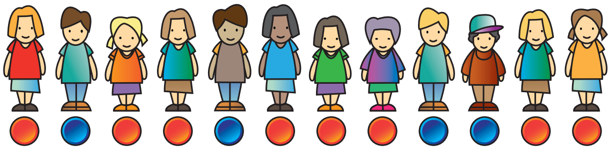
- **Ingredients** – the ratio of different ingredients in a recipe (cooking, medicines, industrial)
- **Maps** – most maps include a scale which is written as a ratio
- **Sporting success** – showing a team's win to loss ratio, or the ratio of kicking goals to points
- **Comparing size** – the ratio of length, area or volume of different shapes
- **Legal requirements** – minimum standards of supervision, staff to student ratios.

When dealing with ratios, the order in which the ratio is written is very important. For example, a team's win : loss ratio of $5:2$ is very different to a team's win : loss ratio of $2:5$.

Ratios compare quantities of the same type with the same unit. Therefore a ratio is not generally written with a unit of measurement.

When we say a ratio, we use the word 'to' or 'is to' for the colon (:). So $3:5$ is spoken "3 to 5" or "3 is to 5".

Let's start: Student teams



In pairs, work through this activity using different coloured counters for girls and boys. Discuss the answers to these questions.

Ms D's class of 12 students has 8 girls and 4 boys.

- Imagine the students lined up with the girls on the left and then the boys on the right. What is the ratio of girls to boys?
- Now consider boys on the left and then the girls on the right. What is the ratio of boys to girls?
- Divide Ms D's class into halves making two equal teams so each team has the same number of girls and boys.

? girls
? boys

? girls
? boys

For one team, what are the ratios of *boys to girls* = ☐ : ☐ and *girls to boys* = ☐ : ☐?

- Divide Ms D's class into quarters by arranging the 8 girls and 4 boys into four equal teams so each team has the same number of girls and boys.

? girls
? boys

? girls
? boys

? girls
? boys

? girls
? boys

For one team, what are the ratios of *boys to girls* = ☐ : ☐ and *girls to boys* = ☐ : ☐?

- List these ratios for girls : boys from the groups above.
  Ms D's whole class = ☐ : ☐
  one team (half class) = ☐ : ☐
  one team (quarter class) = ☐ : ☐

These are equivalent ratios because the proportion of girls and boys is the same for each group.

Key ideas

- A **ratio** shows the relationship between two (or more) amounts of the same type.
- Each quantity must first be written with the same units and then the ratio is written without units.
  e.g. 23 minutes to 1 hour = 23 minutes : 60 minutes = 23 : 60
- The **colon** : is the mathematical symbol used to represent ratios.
- The written ratio of a : b is read as the ratio of '**a to b**' or '**a is to b**'.
- The order in which the quantities are written a ratio is important.
  e.g. cars : bikes = 11 : 2 means 11 cars for every 2 bikes.
- If each number in a ratio is multiplied or divided by the same amount an **equivalent ratio** is formed.
  ×2
  e.g. 1 : 3 and 2 : 6 are equivalent ratios.
  ×2

Ratio A way of comparing quantities of the same kind that are measured in the same units

Exercise 6A

UNDERSTANDING 1–4 4

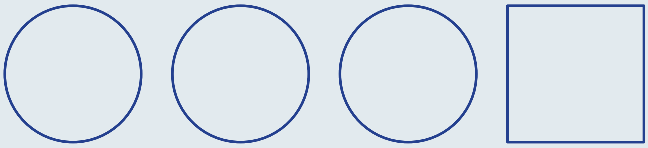
Example 1 Understanding the order of numbers in a ratio

Draw four shapes (circles and squares) to show these ratios.

a Ratio of circles to squares = 3 : 1 **b** Ratio of squares to circles = 3 : 1

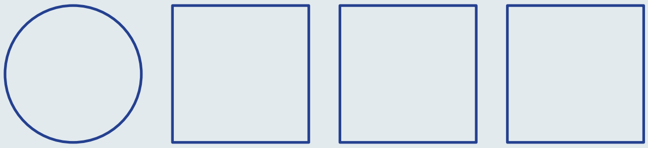
Solution **Explanation**

a



3 circles and 1 square

b



1 circle and 3 squares

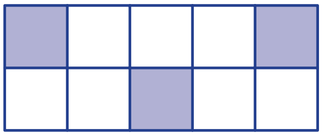
1 Draw 7 shapes (circles and squares) to show each of these ratios.

a Ratio of squares to circles = 5 : 2

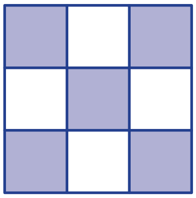
b Ratio of circles to squares = 5 : 2

2 Write down the ratio of shaded parts to unshaded parts for each grid.

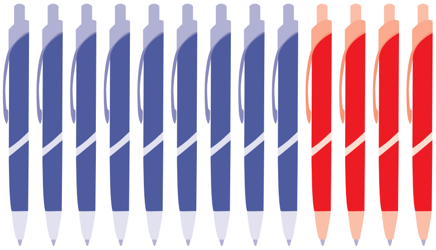
a



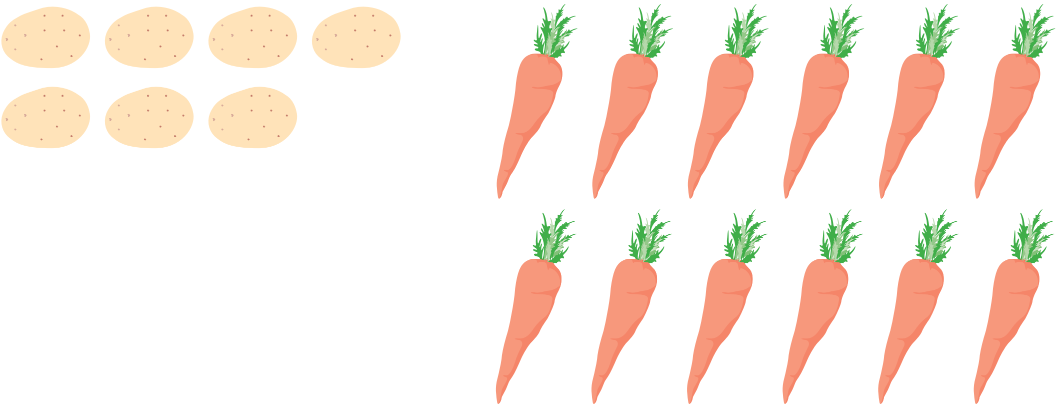
b



3 **a** What is the ratio of blue pens to red pens?



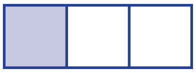
b What is the ratio of potatoes to carrots?



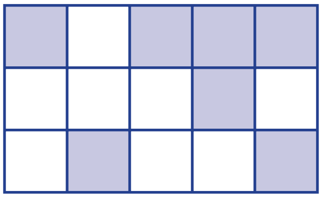
c What is the ratio of cordial to water?

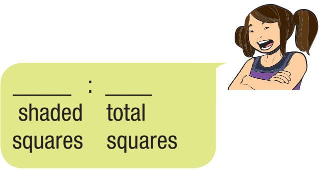
4 Write down the ratio of shaded parts to total parts for each grid.

a



b





FLUENCY 5, 7, 8(½) 6, 7, 8(½)

Example 2 Writing ratios

A sample of mixed nuts contains 5 cashews and 12 peanuts.
Write down:

a the ratio of cashews to peanuts

b the total number of nuts

c the ratio of cashews to the total number of nuts

Solution	Explanation
a $5:12$	cashews : peanuts
b 17	$5 + 12 = 17$
c $5:17$	5 cashews, total nuts 17

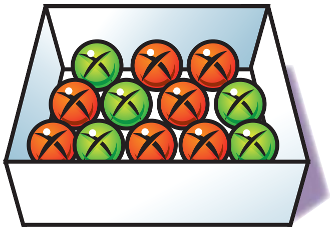
5 A box contains 5 green and 7 red marbles.

a Write the ratio of green marbles to red marbles.

b What is the total number of marbles?

c Write the ratio of green marbles to the total number of marbles.

Remember the order of numbers is important in a ratio.



6 Over the past fortnight, it has rained on eight days and it has snowed on three days.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
fine	fine	rain	rain	snow	snow	snow
rain	rain	rain	rain	rain	rain	fine

Write down the ratio of:

a rainy days to snowy days

b snowy days to total days

c fine days to rainy and snowy days

d rainy days to non-rainy days

7 In a box of 40 flavoured icy poles there were 13 green, 9 lemonade, 11 raspberry and 7 orange icy poles. Write down the ratio of:

a green icy poles to orange icy poles

b raspberry icy poles to lemonade icy poles

c the four different flavours of icy poles; green : lemonade : raspberry : orange

d green and orange icy poles to raspberry and lemonade icy poles

Example 3 Producing equivalent ratios

Complete each pair of equivalent ratios.

a $4 : 9 = 16 : \square$ **b** $30 : 15 = \square : 5$ **c** $2 : 4 : 7 = \square : 12 : \square$

Solution	Explanation
a $\begin{tikzcd} 4 \arrow[rrrr, bend left, "\times 4"] & : & 9 \arrow[rrrr, bend right, "\times 4"'] & = & 16 & : & 36 \end{tikzcd}$	$4 \times 4 = 16$ $9 \times 4 = 36$
b $\begin{tikzcd} 30 \arrow[rrrr, bend left, "\div 3"] & : & 15 \arrow[rrrr, bend right, "\div 3"'] & = & 10 & : & 5 \end{tikzcd}$	$15 \div 3 = 5$ $30 \div 3 = 10$
c $2 : 4 : 7 = 6 : 12 : 21$	$4 \times 3 = 12$ so multiply each number by 3.

8 Copy and complete each pair of equivalent ratios.

a $1 : 3 = 4 : \square$ **b** $1 : 7 = 2 : \square$ **c** $2 : 5 = \square : 10$

d $3 : 7 = \square : 21$ **e** $5 : 10 = 1 : \square$ **f** $12 : 16 = 3 : \square$

g $12 : 18 = \square : 3$ **h** $20 : 50 = \square : 25$ **i** $2 : 3 : 5 = 4 : \square : \square$

j $4 : 12 : 16 = \square : 6 : \square$ **k** $0.5 : 3 = 1 : \square$ **l** $0.1 : 100 = \square : 1000$

PROBLEM-SOLVING AND REASONING

9, 10 10–12

9 Write three equivalent ratios for each of the following ratios.

a $1 : 2$ **b** $2 : 5$ **c** $8 : 6$ **d** $9 : 3$

Multiply or divide both parts of the ratio by the same number.

10 Sort the following ratios into three pairs of equivalent ratios.

$2 : 5,\quad 6 : 12,\quad 7 : 4,\quad 1 : 2,\quad 4 : 10,\quad 70 : 40$

11 Write the ratio of vowels to consonants for each of the following words.

a Queensland **b** Canberra

c Wagga Wagga **d** Australia

The vowels are a, e, i, o, u.

12 There are four groups of students with 12 students in each group. From the ratios given below, work out the number of boys and the number of girls in each group.

a **Group A** boys : girls = 2 : 1

b **Group B** girls : boys = 2 : 1

c **Group C** girls : boys = 1 : 3

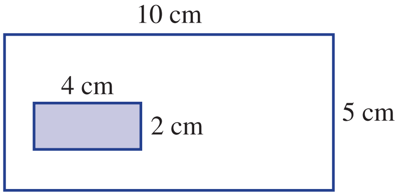
d **Group D** boys : girls = 1 : 5

For group A, find how many lots of 2 boys and 1 girl are needed to make a group of 12 children.

Area ratios

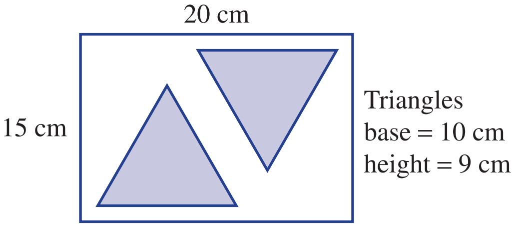
13 Using the dimensions provided, find the ratio of the shaded area to the unshaded area for each of the following diagrams.

a



Area of a triangle $= \dfrac{\text{base} \times \text{height}}{2}$

b



14 Use your ruler to measure the length and width of some rectangular objects that are on your desk. Measure in mm and round each answer to the nearest 5 mm.

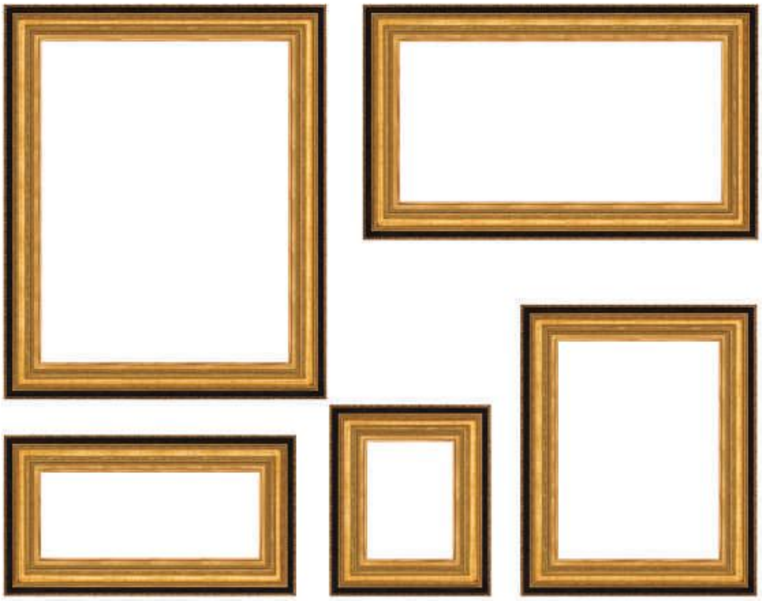
For example: calculator, pencil case, exercise book, textbook and desk.

a For each object, find these ratios and simplify.

  i Length : Width

  ii Length : Area

b What do you notice about your answers to part **ii** when each ratio is simplified?



Rectangular picture frames have various length to width ratios.

6B Simplifying ratios

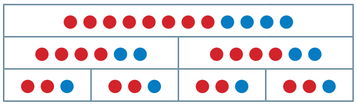
Interactive
Widgets
HOTsheets
Walkthrough

In a similar way to fractions, ratios are simplified by dividing each term by a common factor.

A ratio is said to be in its simplest form when it contains whole numbers only and the highest common factor (HCF) between the terms in the ratio is 1.

Ms D's class had 8 girls and 4 boys. Equal teams (same number of boys and girls in each team) were made by dividing up the class first into halves and then into quarters.

Ratio of girls : boys
Ms D's whole class = $8 : 4$
one team (half of class) = $4 : 2$
one team (quarter of class) = $2 : 1$



So in Ms D's class the ratio of girls : boys = $8 : 4 = 4 : 2 = 2 : 1$.

The smallest ratio $2 : 1$ is called the **simplest form** of these equivalent ratios. This simplest form ratio shows that the proportion of girls to boys, in Ms D's class, is 2 girls for every 1 boy.

Let's start: Class ratios

Look around your classroom and write down the following ratios.

a Ratio of girls to boys
b Ratio of teachers to students
c Ratio of left-handed to right-handed students
d Ratio of white socks to black socks
e Ratio of textbooks open to textbooks closed
f Ratio of not having a pencil case to having a pencil case
g Ratio of blonde hair to brown hair to black hair
h Ratio of blue eyes to brown eyes to other colour eyes

Design your own ratio question for your class or classroom.
Can any of your ratio answers be simplified?

Key ideas

- Before ratios are simplified the quantities must be expressed in the same unit.
- **Simplifying ratios**
  A ratio is simplified by dividing both numbers in the ratio by their highest common factor (HCF).
  e.g. the ratio $15 : 25$ can be simplified to $3 : 5$.

$$\begin{tikzcd} 15:25 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 3:5 \end{tikzcd}$$

- Ratios are usually written in their simplest form.
- Ratios in simplest form use whole numbers only.
- If a ratio is expressed with fractions, it is simplified by converting the quantities to whole numbers. This is generally done by multiplying by the lowest common denominator (LCD).

Exercise 6B

UNDERSTANDING 1–4 4

1 Copy and complete writing these fractions in simplest form.

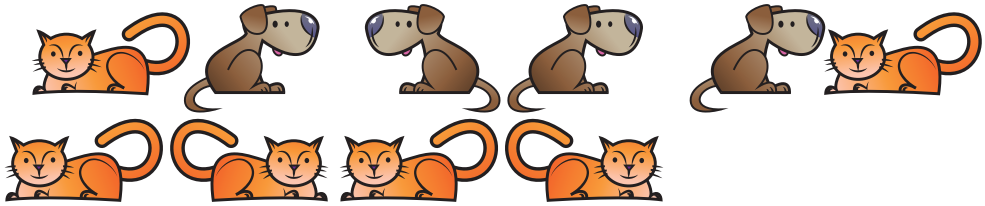
a $\dfrac{5}{10} \underset{\div 5}{\overset{\div 5}{=}} \dfrac{\square}{\square}$ **b** $\dfrac{12}{20} \underset{\div 4}{\overset{\div 4}{=}} \dfrac{\square}{\square}$ **c** $\dfrac{6}{18} \underset{?}{\overset{?}{=}} \dfrac{\square}{\square}$ **d** $\dfrac{15}{35} \underset{?}{\overset{?}{=}} \dfrac{\square}{\square}$ **e** $\dfrac{80}{50} \underset{?}{\overset{?}{=}} \dfrac{\square}{\square}$

2 Copy and complete writing these ratios in simplest form.

a $\div 5 \begin{array}{c} 5:10 \\ = \square : \square \end{array} \div 5$ **b** $? \begin{array}{c} 12:20 \\ = \square : \square \end{array} ?$ **c** $? \begin{array}{c} 6:18 \\ = \square : \square \end{array} ?$

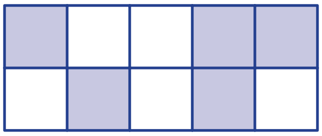
d $? \begin{array}{c} 15:35 \\ = \square : \square \end{array} ?$ **e** $? \begin{array}{c} 80:50 \\ = \square : \square \end{array} ?$

3 Write the ratio of cats to dogs in simplest form.

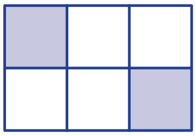


4 Write down the ratio of shaded parts to unshaded parts for each of the following in simplest form.

a



b



purple squares : white squares
$\square : \square$

FLUENCY 5–6(½) 5–7(½)

Example 4 Simplifying ratios

Simplify the following ratios.

a $7:21$ **b** $450:200$

HCF means highest common factor.

Solution	**Explanation**
a $\div 7 \begin{array}{c} 7:21 \\ 1:3 \end{array} \div 7$	HCF of 7 and 21 is 7. Divide both quantities by 7.

Solution	Explanation
b $\begin{tikzcd} 450 : 200 \arrow[d, bend right, "\div 50"'] \arrow[d, bend left, "\div 50"] \\ 9 : 4 \end{tikzcd}$	HCF of 450 and 200 is 50. Divide both quantities by 50. Alternatively, divide both numbers by 10 first and then by 5.

5 Simplify the following ratios.

a 2 : 8
b 10 : 50
c 4 : 24
d 6 : 18
e 8 : 10
f 25 : 40
g 21 : 28
h 24 : 80
i 18 : 14
j 26 : 13
k 45 : 35
l 81 : 27
m 51 : 17
n 20 : 180
o 300 : 550
p 150 : 75
q 1200 : 100
r 70 : 420
s 200 : 125
t 90 : 75

Divide both numbers by the highest common factor.

6 Simplify the following ratios.

a 2 : 4 : 6
b 12 : 21 : 33
c 42 : 60 : 12
d 85 : 35 : 15
e 12 : 24 : 36
f 100 : 300 : 250
g 270 : 420 : 60
h 24 : 48 : 84

Divide all three numbers by the HCF.

Example 5 Simplifying ratios that have different units

First change the quantities to the same unit, by changing the larger unit to the smaller unit. Then express each pair of quantities as a ratio in simplest form.

a 4 mm to 2 cm

b 25 minutes to 2 hours

Solution	Explanation
a 4 mm to 2 cm = 4 mm to 20 mm = 4 : 20 = 1 : 5	2 cm = 20 mm Once in same unit, write as a ratio. Simplify ratio by dividing by HCF of 4.
b 25 minutes to 2 hours = 25 minutes to 120 minutes = 25 : 120 = 5 : 24	2 hours = 120 minutes Once in same unit, write as a ratio. Simplify ratio by dividing by HCF of 5.

7 First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 12 mm to 3 cm **b** 7 cm to 5 mm **c** 120 m to 1 km
d 60 mm to 2.1 m **e** 3 kg to 450 g **f** 200 g to 2.5 kg
g 2 tonnes to 440 kg **h** 1.25 L to 250 mL **i** 400 mL to 1 L
j 20 minutes to 2 hours **k** 3 hours to 15 minutes **l** 3 days to 8 hours
m 180 minutes to 2 days **n** 8 months to 3 years **o** 4 days to 4 weeks
p 8 weeks to 12 days **q** 50 cents to $4 **r** $7.50 to 25 cents

Change the larger unit to the smaller unit.

1 tonne = 1000 kg
1 L = 1000 mL
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

PROBLEM-SOLVING AND REASONING

8–10 10–13

8 To express the ratio 4 : 16 in simplest form, you would:

A multiply both quantities by 2
B subtract 4 from both quantities
C divide both quantities by 2
D divide both quantities by 4

Find the ratio that has a common factor.

9 Decide which of the following ratios is not written in simplest form.

A 1 : 5 **B** 3 : 9 **C** 2 : 5 **D** 11 : 17

Find the ratio that does not have a common factor.

10 Decide which of the following ratios is written in simplest form.

A 2 : 28 **B** 15 : 75 **C** 14 : 45 **D** 13 : 39

The ratio in part **a** will have 7 numbers.

11 When Lisa makes fruit salad for her family, she uses 5 bananas, 5 apples, 2 passionfruit, 4 oranges, 3 pears, 1 lemon (for juice) and 20 strawberries.

a Write the ratio of the fruits in Lisa's fruit salad (in the same order as given in the question).

b Lisa wanted to make four times the amount of fruit salad to take to a party. Write an equivalent ratio that shows how many of each fruit Lisa would need.

c Write these ratios in simplest form.

i bananas to strawberries
ii strawberries to other fruits

12 Andrew incorrectly simplified 12 cm to 3 mm as a ratio of 4 : 1. What was Andrew's mistake and what is the correct simplified ratio?

First write 12 cm : 3 mm with the same units.

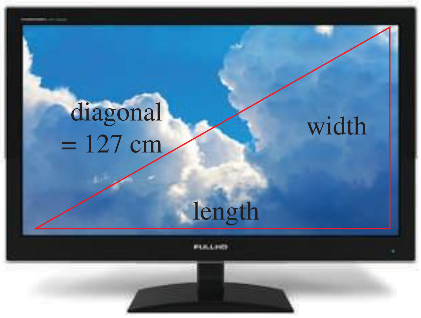
13 **a** Write two quantities of time, in different units, which have a ratio of 2 : 5.

b Write two quantities of distance, in different units, which have a ratio of 4 : 3.

2 hours : 5 hours
2 hours : $\boxed{}$ minutes

Aspect ratios

14 Aspect ratio is the relationship between the width and height of the image as displayed on a screen. The aspect ratio of a rectangle is the ratio of the length to the width.



Size of TV = diagonal length of image

Aspect ratio = length : width

Investigate aspect ratios and create a poster or PowerPoint showing:

a how to calculate the aspect ratio for a rectangular image

b examples of aspect ratios.

Research these examples of aspect ratios.

1 Use the internet or a newspaper to find an advertisement for the various enlargements available from a local photo print shop. State the aspect ratio for each of these enlargements.

2 What is the difference between the size of a television (e.g. 127 cm) and the aspect ratio of the television?

3 Find out the aspect ratio of:
- analog televisions
- high-definition digital televisions
- old cinema movies
- modern cinema movies
- widescreen movies shown on television

4 Calculate the aspect ratio for different-sized newspapers.

6C Dividing a quantity in a given ratio

Interactive
Widgets
HOTsheets
Walkthrough

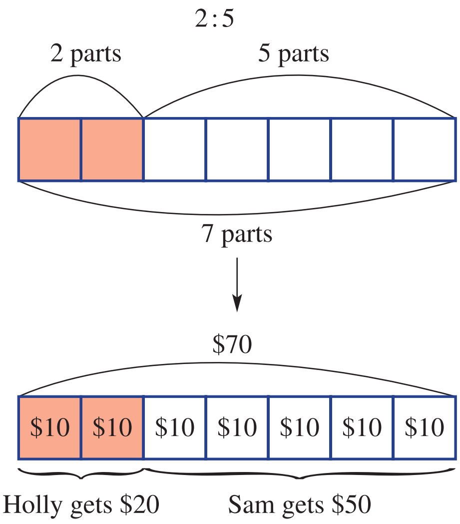
Some ways that ratios can be used are to:

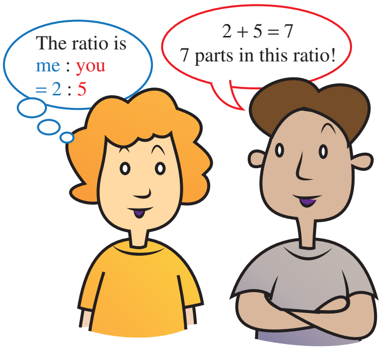
- share money between people
- divide any quantity into certain proportions
- find the correct amount of each portion in a mixture.

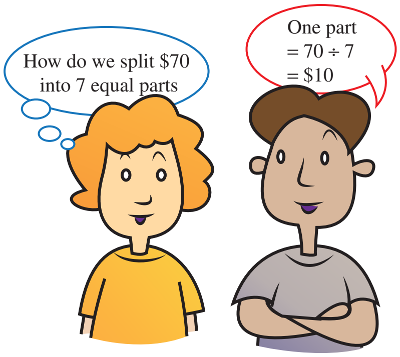
For example:
Each weekend Holly and Sam deliver junk mail.

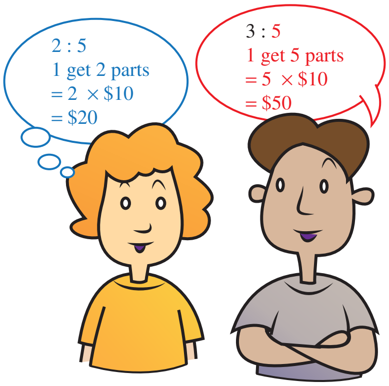
Holly works for 2 hours and Sam works for 5 hours and altogether they earn \$70.

A student worked out how to divide up \$70 in the ratio 2 : 5 using a diagram like this:







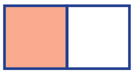


Let's start: Sharing money

With a partner, work out how to share these amounts using the given ratios.

- How can you check that your answers are correct?

\$60 is shared between Lucy and Bronte in the ratio 1 : 1.



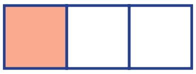
What is the total number of parts?

What is the value of one part?

How much does Lucy get?

How much does Bronte get?

$60 is shared between Jess and Teresa in the ratio 1 : 2.



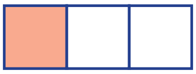
What is the total number of parts?

What is the value of one part?

How much does Jess get?

How much does Teresa get?

$120 is shared between Andrew and Matt in the ratio 1 : 2.



What is the total number of parts?

What is the value of one part?

How much does Andrew get?

How much does Matt get?

$50 is shared between Leos, Djarrin and Nalina in the ratio 3 : 2 : 5.

What is the total number of parts?

What is the value of one part?

How much does Leos get?

How much does Djarrin get?

How much does Nalina get?

$120 is shared between Christine, Prue and Karol in the ratio 3 : 2 : 5.

What is the total number of parts?

What is the value of one part?

How much does Christine get?

How much does Prue get?

How much does Karol get?

Key ideas

- Think of a ratio in terms of '**parts**'.
  A ratio of 2 : 3 has 2 parts of one quantity for every 3 parts of another quantity and a total of 5 parts.
- Using the **unitary method** to divide a quantity in a given ratio:
  - find the total number of parts in the ratio.
  - find the value of one part.
  - find the value of the number of parts required in the ratio.

e.g. Share $20 in ratio of 2 : 3.
Think of sharing $20 into 2 parts and 3 parts.
Total number of parts = 2 + 3 = 5.
Value of one part = $20 ÷ 5 = $4.
Therefore 2 parts = $8, and 3 parts = $12.

$$\begin{tikzcd}
\$\,20 = 5\text{ parts} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
\$\,4 = 1\text{ parts} \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
\$\,8 = 2\text{ parts}
\end{tikzcd}$$

- To find a total quantity from a given ratio:
  - Find the value of one part and then the value of the total parts can be calculated.

Or

  - Use equivalent ratios to find the value of each quantity in the total ratio then add the numbers together to find the total.

Exercise 6C

UNDERSTANDING 1–4 | 4

1 Find the total number of parts in the following ratios.

a 3 : 7 **b** 1 : 5 **c** 11 : 3 **d** 2 : 3 : 4

Add the numbers in the ratio to find the total parts.

2 Marta and Joshua earned $25 between them. They want to share it in the ratio Marta : Joshua = 3 : 2. Copy and complete these steps.

a In the ratio 3 : 2, the total parts = ___+ ___ = ___

b ___ parts = $25, so 1 part = ___

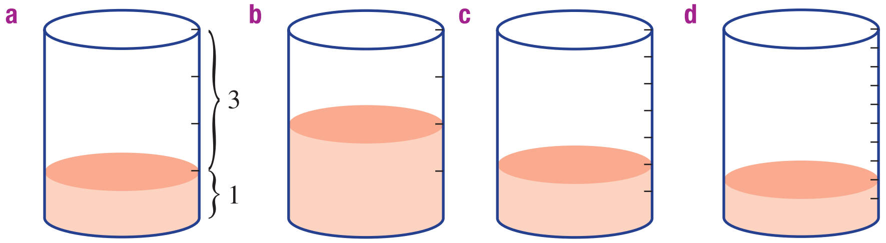
c Marta gets 3 parts, so Marta gets 3 × $___ = $___

d Joshua gets 2 parts, so Joshua gets 2 × $___ = $___

e Total amount = $___+ ___ = $___

3 The diagram shows four glasses that contain different amounts of cordial. Water is then added to fill each glass right to the top. For each drink shown, what is the ratio of cordial to water?

Cordial : water
☐ : 3



Write ratios in simplest form.

4 What is the total number of parts for each ratio in question **3**?

FLUENCY 5(½), 6, 7(½) | 5(½), 6, 7(½), 8

Example 6 Dividing a quantity in a particular ratio

Divide 54 m in a ratio of 4 : 5.

Solution

Total number of parts = 9

$$\begin{tikzcd} 9 \text{ parts} = 54 \text{ m} \arrow[d, bend right, "\div 9"'] \arrow[d, bend left, "\div 9"] \\ 1 \text{ parts} = 6 \text{ m} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ 4 \text{ parts} = 24 \text{ m} \end{tikzcd} \qquad \begin{tikzcd} 1 \text{ parts} = 6 \text{ m} \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ 5 \text{ parts} = 30 \text{ m} \end{tikzcd}$$

The amounts are 24 m and 30 m.

Explanation

Total number of parts = 4 + 5 = 9
Value of 1 part = 54 m ÷ 9 = 6 m

Check numbers add to total:
24 + 30 = 54
Write the answers with units.

5 Divide:

a \$60 in the ratio of 2 : 3
b \$110 in the ratio of 7 : 4
c \$1000 in the ratio of 3 : 17
d 48 kg in the ratio of 1 : 5
e 14 kg in the ratio of 4 : 3
f 360 kg in the ratio of 5 : 7
g 72 m in the ratio of 1 : 2
h 40 m in the ratio of 3 : 5
i 155 m in the ratio of 4 : 1

6 Share \$400 in the ratio:

a 1 : 3
b 2 : 3
c 3 : 5
d 9 : 11

Start by finding the:
- total parts
- value of one part.

Example 7 Dividing a quantity in a ratio with three numbers

Divide \$300 in the ratio of 2 : 1 : 3.

Solution

Total number of parts = 6

$$\begin{tikzcd} 6\text{ parts} = \$300 \arrow[d, bend right, "\div 6"'] \arrow[d, bend left, "\div 6"] \\ 1\text{ part} = \$50 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ 2\text{ parts} = \$100 \end{tikzcd}$$

$$\begin{tikzcd} 1\text{ part} = \$50 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 3\text{ parts} = \$150 \end{tikzcd}$$

The three amounts are \$100, \$50 and \$150.

Explanation

Total number of parts
= 2 + 1 + 3 = 6

Value of 1 part = \$300 ÷ 6 = \$50

Check numbers add to total:
\$100 + \$50 + \$150 = \$300

7 Divide:

a \$200 in the ratio of 1 : 2 : 2
b \$400 in the ratio of 1 : 3 : 4
c 12 kg in the ratio of 1 : 2 : 3
d 88 kg in the ratio of 2 : 1 : 5
e 320 kg in the ratio of 12 : 13 : 15
f \$50 000 in the ratio of 1 : 2 : 3 : 4

What is the total number of parts?

8 Share 600 lollies in the ratio:

a 1 : 9
b 2 : 1 : 3
c 2 : 5 : 5
d 12 : 7 : 8 : 3

Write units in the answers.

PROBLEM-SOLVING AND REASONING 9, 10 10–12

9 Evergreen Fertiliser is made up of the three vital nutrients nitrogen, potassium and phosphorus in a ratio of 4 : 5 : 3. How much of each nutrient is in a 1.5 kg bag?

First change 1.5 kg to g.

10 The angles of a triangle are in the ratio of 2 : 3 : 4. Find the size of each angle.

The angles in a triangle add to 180°.

Example 8 Finding a total quantity from a given ratio

The ratio of boys to girls at Birdsville College is 2 : 3. If there are 246 boys at the school, how many students attend Birdsville College?

Solution	**Explanation**
Unitary method	
$$\begin{array}{rll} & 2\text{ parts} = 246 & \\ \div 2 & 1\text{ part} = 123 & \div 2 \\ \times 5 & 5\text{ parts} = 615 & \times 5 \end{array}$$	Ratio of boys : girls is 2 : 3 Boys have '2 parts' = 246 Value of 1 part = 246 ÷ 2 = 123 Total parts = 2 + 3 = 5 parts 5 parts = 5 × 123 = 615
615 students attend Birdsville College.	
Equivalent ratios method	Use equivalent ratios.
$$\begin{array}{rll} & \text{boys : girls} & \\ \times 123 & = 2 : 3 & \times 123 \\ & = 246 : 369 & \end{array}$$	2 × 123 = 246 boys, so 3 × 123 = 369 girls Total number of students = 246 boys + 369 girls = 615
615 students attend Birdsville College.	

11 In Year 8, the ratio of boys to girls is 5 : 7. If there are 140 girls in Year 8, what is the total number of students in Year 8?

Think: How many parts of the total equals 140 girls?

12 A textbook has three chapters and the ratio of pages in these chapters is 3 : 2 : 5. If there are 24 pages in the smallest chapter, how many pages are in the textbook?

The smallest chapter = 2 parts of the total.

Changing ratios

— 13, 14

13 The ratio of the cost of a shirt to the cost of a jacket is 2 : 5. If the jacket cost $240 more than the shirt, find the cost of the shirt and the cost of the jacket.

Try out some amounts for one part of the ratio.

14 In a class of 24 students the ratio of girls to boys is 1 : 2.

- **a** On one day, the ratios of girls to boys was 3 : 7. How many boys and how many girls were absent?
- **b** If 4 more girls and 4 more boys joined the original class, what would be the new ratio of girls : boys?

How many boys and girls are in the original class?

6D Scale drawings

Interactive
Widgets
HOTsheets
Walkthrough

A **scale drawing** is used when the actual object has measurements too large to fit on a page.

For example, scale drawings are used for:

- house plans
- maps
- drawings of large objects, like a car or plane.

This computer enhanced house plan has a scale of $1 : 200$.

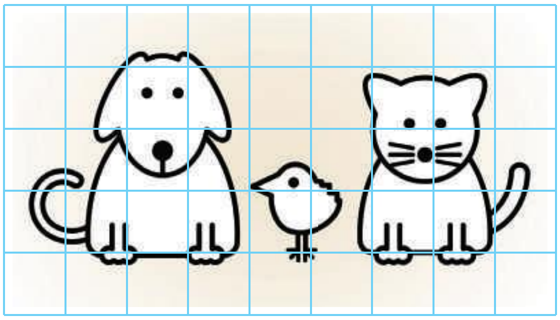
A scale drawing is also used for very small objects, so we can see the details clearly. For example, scale drawings could show a 'close-up' of a:

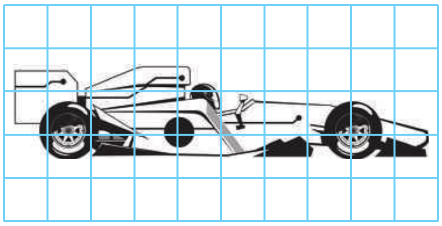
- flea
- strand of hair
- computer chip.

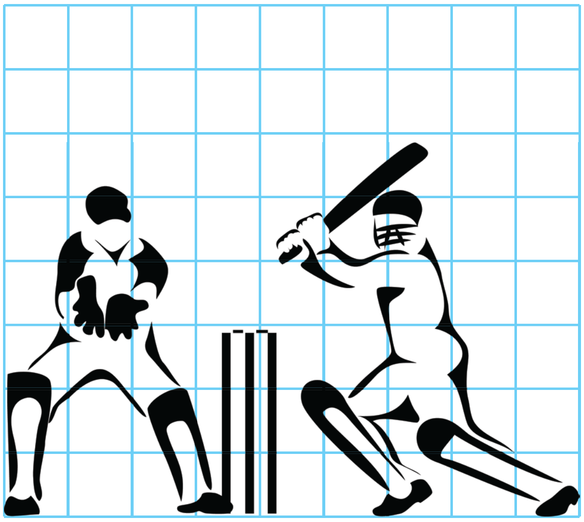
This picture of a dragonfly is 5 times larger than a real dragonfly. The real dragonfly is $\frac{1}{5}$ of the size of this picture. How wide is the real dragonfly's head?

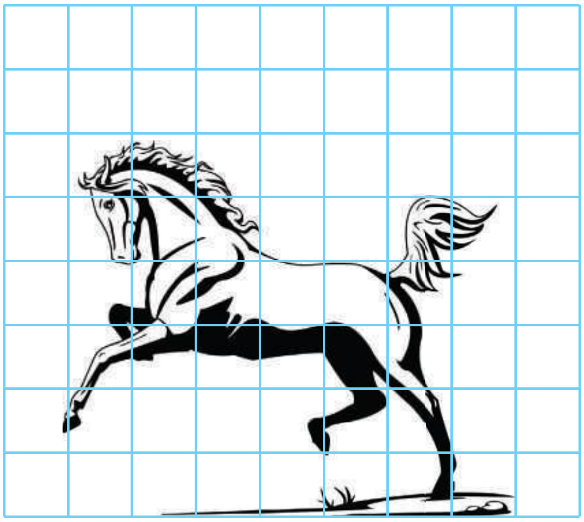
Let's start: Enlarging pictures

Choose one of these pictures and enlarge it to a full page. Complete the enlargement by copying the picture one square at a time onto a full page grid that has the same number of squares as on the picture.









Now, calculate the enlargement factor of your drawing by dividing the width of your drawing with the width of the original picture.

For example:

$$\frac{\text{width of Jack's drawing}}{\text{width of original picture}} = \frac{35\text{ cm}}{7\text{ cm}} = 5 \text{ times larger}$$

Jack's picture is 5 times larger than the original.
Enlargement factor = 5.

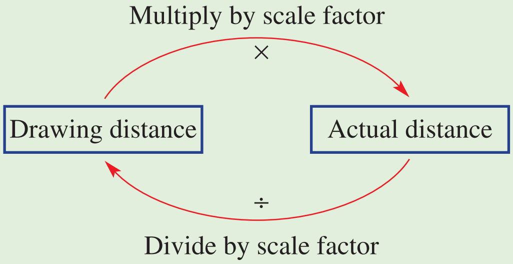
Key ideas

- A scale drawing has exactly the same shape as the original object, but it is a different size.
- The **scale** on a drawing is written as a scale ratio of the drawing length : actual (real) length. e.g. a scale ratio of $1:100$ means that the real object is 100 times larger than the drawing.
- Scales should begin with 1. Then the second number in the ratio is called the **scale factor**.

  e.g.

$$\begin{tikzcd} 5 : 25\,000 \arrow[d, bend right=60, "\div 5"'] \arrow[d, bend left=60, "\div 5"] \\ 1 : 5\,000 \end{tikzcd}$$

The scale factor is 5000.
The real object is 5000 times larger than the drawing.



Scale
A ratio that compares a drawing or a model to the real object

Scale factor
The number you multiply each side length by to enlarge or reduce a shape

- To change a drawing distance to an actual (real) distance, multiply by the scale factor.
- Multiplying by a scale factor does not change the units.

e.g. On a house plan with scale $1:200$ a room is 2 cm wide.

$$\begin{aligned} \text{The real room will be } 2\text{ cm} \times 200 &= 400\text{ cm wide} \\ &= 400 \div 100\text{ m} \\ &= 4\text{ m wide} \end{aligned}$$

- To convert an actual (real) distance to a drawing distance you divide by the scale factor.
- Dividing by a scale factor does not change the units.

e.g. A real house is 12 m wide and the house plan has scale $1:200$.

$$\begin{aligned} \text{The house plan will be } 12\text{ m} \div 200 &= 0.06\text{ m wide} \\ &= 0.06 \times 100\text{ cm} \\ &= 6\text{ cm wide} \end{aligned}$$

- Conversion of units. It is important to remember how to correctly convert measurements of length when dealing with scales.

$$\begin{tikzcd} \text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"] \end{tikzcd}$$

Exercise 6D

UNDERSTANDING 1–4 4

1 **a** Convert $10\,000$ cm to:

i mm **ii** m **iii** km

b Convert 560 m to:

i km **ii** cm **iii** mm

Example 9 Comparing scale distance to real distance

A model of a lounge room has a toy lounge chair that is 2 cm wide. The real lounge chair is 1 m wide.

a Write both toy width and real chair width in cm.

b How much bigger is the real chair compared to the toy chair?

c What is the scale ratio for toy chair : real chair?

Solution	Explanation
a 2 cm, 100 cm	$1\text{ m} \times 100 = 100\text{ cm}$
b 50 times bigger	$2\text{ cm} \times 50 = 100\text{ cm}$
c $1:50$	A scale ratio always starts with 1. The second number shows how many times larger the real object is.

2 Here are pictures of a real classic racing car and a model racing car.

real car length = 4 m

model car length = 4 cm

a Write the model car length in cm and real car length in cm.

b How many times bigger is the real car compared to the model?

c What is the scale ratio for the model car : real car?

3 A model ship is 60 cm long and the real ship is 300 m long.

a Write model length in cm and real ship length in cm.

b How many times bigger is the real ship compared to the model?

c What is the scale ratio for the model ship : real ship?

Example 10 Calculating the real distance

A map has a scale ratio of $1:20\,000$.

a What is the scale factor?

b What actual real distance in cm would 1 cm on the map represent?

c What actual real distance in m would $5\,\text{mm}$ on the map represent?

Solution	**Explanation**
a $20\,000$	The scale factor is the second number in the scale ratio
b $20\,000$ cm	$1\text{ cm}\times 20\,000 = 20\,000\text{ cm}$
c 100 m	$5\text{ mm}\times 20\,000 = 100\,000\text{ mm} = 100\text{ m}$

4 A map has a scale ratio of $1:50\,000$.

a What is the scale factor?

b What actual real distance in cm would 1 cm on the map represent?

c What actual real distance in m would 1 m on the map represent?

d What actual real distance in mm would 5 mm on the map represent?

e What actual real distance in cm would 5 cm on the map represent?

Multiply by the scale factor.

FLUENCY 5–7 5, 6, 7(½), 8

Example 11 Converting scale distance to actual distance

A map has a scale of $1:20\,000$.
Find the actual distance in m for each scaled distance (map distance).

a 2 cm **b** 5 mm

Solution	**Explanation**
a Actual distance $= 2 \text{ cm} \times 20\,000$ $= 40\,000 \text{ cm}$ $= 400 \text{ m}$	Scale factor $= 20\,000$ Multiply scaled distance by scale factor. $\div 100$ to convert cm to m.
b Actual distance $= 5 \text{ mm} \times 20\,000$ $= 100\,000 \text{ mm}$ $= 100 \text{ m}$	Multiply scaled distance by scale factor. 5 mm times scale factor gives the answer in mm. $\div 10 \div 100$ to convert mm to cm to m.

5 Find the actual distance for each of the following scaled distances. Give your answer in the unit that is in brackets after each question.

a Scale $1:2$	**i** 310 cm (cm)	**ii** 2.5 mm (mm)
b Scale $1:10\,000$	**i** 2 cm (m)	**ii** 4 mm (m)
c Scale $1:20\,000$	**i** 80 cm (km)	**ii** 1.25 m (km)
d Scale $1:400$	**i** 16 mm (m)	**ii** 72 cm (m)
e Scale $1:0.01$	**i** 3 cm (mm)	**ii** 0.815 m (mm)

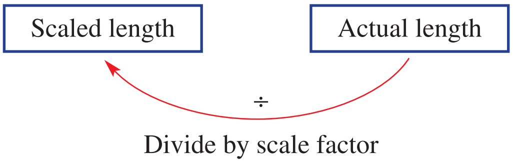
- Multiply by the scale factor.
- Keep the units the same as the question.
- Then convert to the required units.

Example 12 Converting actual distance to scaled distance

A model boat has a scale of $1:500$.
Find the scaled length in mm for each of these actual lengths.

a 50 m (mm) **b** 4550 mm (mm)

Solution	**Explanation**
a Scaled distance $= 50 \text{ m} \div 500$ $= 0.1 \text{ m}$ $= 10 \text{ cm}$ $= 100 \text{ mm}$	Divide by the scale factor, 500. $50 \text{ m} \div 500$ gives the answer in m. $\times 100$ to convert m to cm. $\times 10$ to convert cm to mm.
b Scaled distance $= 4550 \text{ mm} \div 500$ $= 45.5 \text{ mm} \div 5$ $= 9.1 \text{ mm}$	Divide actual distance by scale factor. Shortcut: $\div 100$, then $\div 5$ (or vice versa) The answer is in mm.



6 Find the scaled length for each of these actual lengths. Give your final answer in the unit that is in brackets after each question.

a Scale 1 : 200 **i** 200 m (m) **ii** 4 km (m)
b Scale 1 : 500 **i** 10 000 m (m) **ii** 1 km (m)
c Scale 1 : 10 000 **i** 1350 m (cm) **ii** 736.5 m (cm)
d Scale 1 : 250 000 **i** 5000 m (mm) **ii** 1250 m (mm)
e Scale 1 : 0.05 **i** 7.5 cm (m) **ii** 8.2 mm (m)

- Divide by the scale factor.
- Keep the units the same as the question.
- Then convert to the required units.

7 Change the two measurements provided in each scale into the same unit and then write the scale as a ratio of two numbers in simplest form.

a 2 cm : 200 m **b** 5 mm : 500 cm **c** 12 mm : 360 cm
d 4 mm : 600 m **e** 4 cm : 5 m **f** 1 cm : 2 km
g 28 mm : 2800 m **h** 3 cm : 0.6 mm **i** 1.1 m : 0.11 mm

- Convert larger unit to smaller unit
- Divide by HCF

Example 13 Determining the scale factor

State the scale factor in the following situations.

a 4 mm on a scale drawing represents an actual distance of 50 cm.

b An actual length of 0.1 mm is represented by 3 cm on a scaled drawing.

Solution	**Explanation**
a Scale ratio = 4 mm : 50 cm	Write the ratio drawing length : actual length.
= 4 mm : 500 mm	Convert to 'like' units.
Scale ratio = 4 : 500	Write the scale ratio without units.
= 1 : 125	Divide both numbers by 4. (HCF = 4)
Scale factor = 125	Ratio is now in the form 1 : scale factor. The actual size is 125 times larger than the scaled drawing.
b Scale ratio = 3 cm : 0.1 mm	Write the ratio drawing length : actual length.
= 30 mm : 0.1 mm	Convert to 'like' units.
Scale ratio = 30 : 0.1	Write the scale ratio without units.
= 300 : 1	Multiply both numbers by 10.
$= 1 : \frac{1}{300}$	Divide both numbers by 300. Ratio is now in the form 1 : scale factor.
Scale factor = $\frac{1}{300}$	The actual size is 300 times smaller than the scaled drawing.

8 Find the scale ratio and the scale factor for each of the following.

a 2 mm on a scale drawing represents an actual distance of 50 cm.

b 4 cm on a scale drawing represents an actual distance of 2 km.

c 1.2 cm on a scale drawing represents an actual distance of 0.6 km.

d 5 cm on a scale drawing represents an actual distance of 900 m.

e An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.

f An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

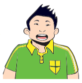
- Same units
- Scale ratios start with 1
- Write scale factors as whole numbers or fractions

PROBLEM-SOLVING AND REASONING 9, 10 10–12

9 A model city has a scale ratio of $1 : 1000$.

a Find the actual height in m of a skyscraper that has a scaled height of 8 cm.

b Find the scaled length in cm of a train platform that is 45 m long in real life.



Scaled height means model height.

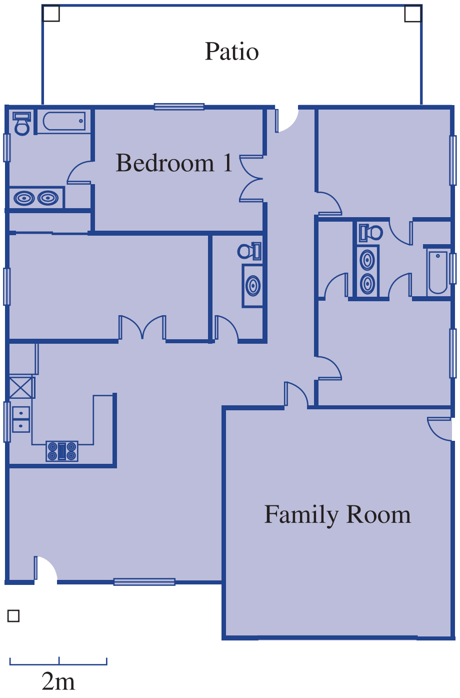
10 Blackbottle and Toowoola are 17 cm apart on a map with a scale of $1 : 50\,000$. How many km apart are the towns in real life?

11 This house plan has a scale of $1 : 150$. For each room listed below do these two steps.

- Use a ruler to measure its length and width in mm.
- Use the scale to calculate the real dimensions in m to one decimal place.

a Bedroom 1 **b** Family room **c** Patio

Length in mm × scale factor = real length in mm



Scale 1 : 150

12 For each question below do these two steps.

- Use a ruler to measure the straight line map distance in cm to one decimal place.
- Use the scale to calculate the real distance and give each answer to the nearest 100 km.

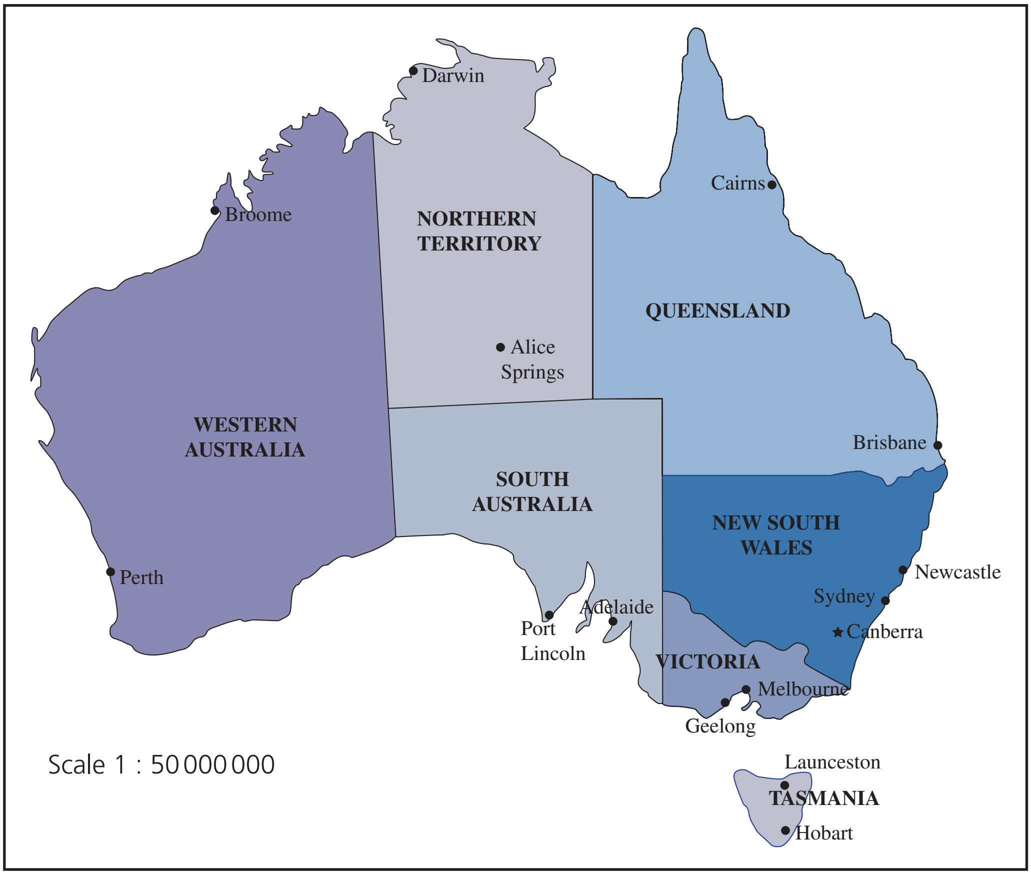
a Hobart to Cairns

b Perth to Sydney

c Darwin to Adelaide

d Brisbane to Melbourne

e Australia's furthest west point to furthest east point.



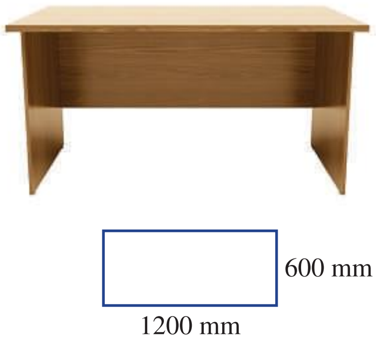
Design a bedroom

— 13

13 For this activity you will design and draw the bedroom of a house plan. Builders use millimetres for units so keep all units in millimetres for this activity.
This bedroom has length = 3000 mm and width = 4000 mm.

The furniture in the bedroom is illustrated on the next page. These pictures are **not** shown to scale. The real dimensions are given for the 'top view'. The 'top view' is how it is seen looking down from above.

How many times larger is the bedroom length than your page length? Choose a whole number for the scale factor.
$scale = 1 : scale\ factor$



You are to design a scaled drawing of a bedroom including this furniture.

a Find a scale that will allow the drawing of this bedroom to fit on one page.

b Use your scale to change all the real dimensions to scaled lengths and widths in mm.

c Draw a scaled rectangle for the bedroom.

d Choose where each piece of furniture will be placed in the bedroom and draw the scaled top view of each.

e Choose where a window and a door will be placed in the bedroom and draw the scaled top view of each.

f Label dimensions with the real measurements in mm.

g Write the scale next to your bedroom plan.

Progress quiz

6A

1 A group of mixed balls contains 7 tennis balls and 5 basketballs. Write down:

- **a** the ratio of tennis balls to basketballs.
- **b** the total number of balls
- **c** the ratio of basketballs to the total number of balls.

6A

2 Complete each pair of equivalent ratios.

- **a** 12 : 15 : 24 = 24 : ☐ : 48
- **b** 35 : 25 = ☐ : 5
- **c** 3 : 5 : 8 = 12 : ☐ : ☐

6B

3 Simplify the following ratios

- **a** 6 : 27
- **b** 100 : 25
- **c** 42 : 28 : 7

6B

4 First change the quantities to the same unit, by changing the larger unit to the smaller unit. Then express each pair of quantities as a ratio in simplest form.

- **a** 15 minutes to $1\frac{1}{2}$ hours
- **b** 8 mm to 1.2 cm
- **c** $6.00 to 40 cents

6B

5 Decide which of the following ratios is not written in simplest form.

- **A** 13 : 15
- **B** 24 : 41
- **C** 34 : 15
- **D** 51 : 17

6C

6 Divide:

- **a** 50 kg in the ratio of 2 : 3
- **b** 18 m in the ratio of 5 : 1
- **c** $4000 in the ratio of 7 : 13
- **d** 4 hours in the ratio of 1 : 5.

6C

7 Divide $1000 in the ratio:

- **a** 3 : 7
- **b** 19 : 1
- **c** 65 : 35
- **d** 23 : 27.

6C

8 The ratio of goals to behinds scored by the U15 Newtown Eagles is 3 : 5. If the team scored 248 times for the season, how many goals did the Eagles scores for the season?

6D

9 A map has a scale ratio of 1 : 50 000.

- **a** What is the scale factor?
- **b** What actual real distance in m would 3 cm on the map represent?
- **c** What actual real distance in km would 8 cm on the map represent?

6D

10 A model car has a scale of 1 : 200. Find the scaled length in mm for each of these actual lengths.

- **a** 8 m
- **b** 4600 mm

6E Introducing rates

If you monitored what you said each day, you might find that you speak about rates many times!

A **ratio** shows the relationship between the same type of quantities with the same units, but a **rate** shows the relationship between two different types of quantities with different units.

The following are all examples of rates:

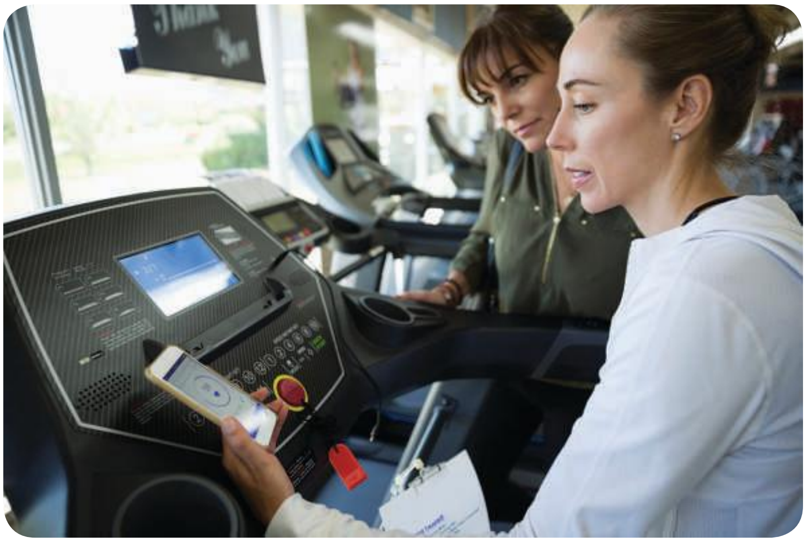
- Cost of petrol was $\$1.45$ per litre.
- Rump steak was on special for $\$18$/kg.
- Dad drove to school at an average speed of 52 km/h.
- After the match, your heart rate was 140 beats/minute.

A **ratio** compares two amounts of the same type and units, so a ratio does not include units.

- For example: the ratio of girls to boys in a group was $4:5$.

A **rate** compares different types of quantities so both units must be shown.

- For example: the average rate of growth of a teenage boy is 6 cm/year.



A knowledge of heart rates is important for understanding fitness.

Let's start: State the rate

For each of the following statements, write down a corresponding rate.

- The Lodges travelled 400 km in 5 hours. What is their speed in km/h?
- Gary was paid $\$98$ for a 4-hour shift at work. What is the rate of pay in $/h?
- Felicity spent $\$600$ on a two-day shopping spree. What is Felicity's spending rate in $/day?
- Max had grown 9 cm in the last three months. What is Max's growth rate in cm/month?
- Vuong paid $\$37$ for half a cubic metre of crushed rock. What is the cost in $/cubic metre?
- Paul cycled a total distance of 350 km for the week. At what rate did Paul cycle in km/day?

What was the rate (in questions/minute) at which you answered these questions?

Key ideas

- **Rates** compare quantities measured in different units.
- All rates must include two different units.
- The two different units are separated by a slash '/', which is the mathematical symbol for 'per'.
  e.g. 20 km/h = 20 km per hour = 20 km for each hour
- It is usual to write rates in their simplest form. This involves writing the rate for only one unit of the second quantity.

$$\begin{aligned} \text{e.g. earned \$45 in 3 hours} &= \text{\$45 in 3 hours} \quad \leftarrow \quad \text{Non simplified rate} \\ \div 3 \qquad & \qquad \div 3 \\ &= \text{\$15 in 1 hour} \\ &= \text{\$15 / h} \quad \leftarrow \quad \text{simplified rate} \end{aligned}$$

Rate The number of units of one quantity for each single unit of another quantity

- The **average rate** is calculated by dividing the total change in the first quantity by the total change in the second quantity.
  e.g. reading a 400-page book in 4 days

$$\begin{aligned} \text{Average reading rate} &= \text{400 pages in 4 days} \\ \div 4 \qquad & \qquad \div 4 \\ &= \text{100 pages in 1 day} \\ \text{Average reading rate} &= \text{100 pages/day} \end{aligned}$$

Exercise 6E

UNDERSTANDING 1–3 3

1 Which of the following are examples of rates?

A \$5.50 **B** 180 mL/min **C** \$60/h **D** $\frac{5}{23}$

E 4.2 runs/over **F** 0.6 g/L **G** 200 cm^2 **H** 84 c/L

Remember that rates have two different units.

2 Match each rate in the first column with its most likely rate in the second column.

a	Employee's wage	90 people/day
b	Speed of a car	\$2100/m^2
c	Cost of building a new home	68 km/h
d	Population growth	64 beats/min
e	Resting heart rate	\$15/h

3 Select from this list the most typical units for each of the following rates.

\$/L mg/tablet \$/kg kJ/serve runs/over words/minute goals/shots (on goal) L/minute

a Price of sausages

b Petrol costs

c Typing speed

d Goal conversion rate

e Energy nutrition information

f Water usage in the shower

g Pain relief medication

h Cricket team's run rate

FLUENCY 4(½), 5 | 4(½), 5, 6

Example 14 Writing simplified rates

Express each of the following as a simplified rate.

a 12 students for two teachers **b** \$28 for 4 kilograms

Solution	**Explanation**
a 12 students/2 teachers = 6 students/teacher	Divide both quantities by the second amount. $12 \div 2 = 6$ students for 1 teacher. Include both units separated by /
b \$28/4 kg = \$7/kg	$28 \div 4 = \$7$ for 1 kg When writing a cost rate, the \$ sign is written before the number.

4 Write each of the following as a simplified rate.

a 12 days in 4 years
b 15 goals in 3 games
c \$180 in 6 hours
d \$17.50 for 5 kilograms
e \$126 000 to purchase 9 acres
f 36 000 cans in 8 hours
g 12 000 revolutions in 10 minutes
h 80 mm rainfall in 5 days
i 60 minutes to run 15 kilometres
j 15 kilometres run in 60 minutes

Divide both amounts by the second number. The answer includes both units separated by (/).

Example 15 Finding average rates

Find the average rate of change in each situation.

a 15 000 revolutions in 5 minutes **b** 30 minutes to run 6 km

Solution	**Explanation**
a Average rate = 15 000 revs/5 mins = 3000 revs/min	Divide both quantities by the second amount $15\,000 \div 5 = 3000$ revs for 1 minute on average
b Average rate = 30 minutes/6 km = 5 minutes/km	$30 \div 6 = 5$ minutes for 1 km on average Include both units separated by a /

5 Find the average rate of change for each situation.

a Relma drove 6000 kilometres in 20 days.
b Holly saved \$420 over 3 years.
c A cricket team scored 78 runs in 12 overs.
d Saskia grew 120 centimetres in 16 years.
e Russell gained 6 kilograms in 4 years.
f The temperature dropped 5°C in 2 hours.

In your answer, write the units in the same order as the question.

Example 16 Finding average rates

Tom was 120 cm tall when he turned 10 years old. He was 185 cm tall when he turned 20 years old. Find Tom's average rate of growth per year between 10 and 20 years of age.

Solution	**Explanation**
Average rate = 65 cm/10 years	Growth = 185 – 120 = 65 cm
= 6.5 cm/year	Divide both numbers by 10.

6 **a** Liam was 150 cm tall at 10 years old and 188 cm tall when 20 years old. Find Liam's average rate of growth per year between 10 and 20 years of age.

b Brittany was 140 cm tall at 10 years old and 164 cm at 18 years old. Find Brittany's average rate of growth per year between 10 and 18 years of age.

PROBLEM-SOLVING AND REASONING 7–9 9–12

7 A dripping tap filled a 9 litre bucket in 3 hours.

a What was the dripping rate of the tap in litres/hour?

b How long would it take the tap to fill a 21 litre bucket?

×? ? litres in 1 hour ×?
21 litres in ? hour

8 Martine grew at an average rate of 6 cm/year for the first 18 years of her life. If Martine was 50 cm long when she was born, how tall was Martine when she turned 18?

×? 6 cm in 1 year ×?
? cm in 18 year

9 If 30 salad rolls were bought to feed 20 people at a picnic, and the total cost was $120, find the following rates.

a Salad rolls/person **b** Cost/person **c** Cost/roll

10 Harvey finished a 10 kilometre race in 37 minutes and 30 seconds. Jacques finished a 16 kilometre race in 53 minutes and 20 seconds. Calculate the running rate of each runner in min/km. Which runner had a faster running pace?

First write the distance in metres and the time in minutes.

11 The Tungamah Football Club had 12 000 members. After five successful years they now have 18 000 members.

- **a** What has been the average rate of membership growth per year for the past 5 years?
- **b** If this membership growth rate continues, how many more years will it take for the club to have 32 400 members?

12
- **a** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:
  - **i** km/L
  - **ii** L/km (give answer as a fraction)
- **b** How can you convert km/L to L/km?

Start with 3 : 216 km uses 24 litres.

Target 155

— | 13

13 In Victoria, due to repeated drought experiences, the state government has urged all residents to save water. The goal was set for each person to use no more than 155 litres of water per day.

- **a** How many people live in your household?
- **b** According to the Victorian government, how many litres of water can your household use per day?

If you live in a different state of Australia, find the target volume of water use per person for your state and determine how many litres of water that your household can use each day.

Use the following rates of water flow for the questions below.

Shower rate (10 L/min)
Hose (24 L/min)
Running tap (16 L/min)
Dishwasher (20 L/wash)
Washing machine (100 L/load)
Toilet (4.5 L/flush, 3 L/half flush)
Drinking water (3 L/day)
Water for food preparation (15 L/day)

Draw a table.

- **c** Estimate the average daily rate of water usage for your household.
- **d** Ask your parents for a recent water bill and find out what your family household water usage rate was for the past three months.
- **e** What is the rate at which your family is charged for its water?

Before the water saving plan, Victorians were using an average of 164 litres/day/person. Twelve months later, Victorians were using 151 litres/day/person.

- **f** How much water per year for the state of Victoria does this saving of 13 litres/day/person represent?

The population of Victoria is about 6 million.

6F Speed and applications of other rates ★

We are interested in how things change over a period of time. A rate that we come across almost every day is speed. Speed is the rate of distance travelled per unit of time.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

A snail can move at 1 mm/second, an Olympic sprint runner can run at a speed of 10 m/second and the Earth travels around the Sun at a speed of around 30 km/second.

- How far does the Earth travel in an hour?
- Can you estimate the speed of a passenger jet in m/second?

Let's start: Racing rates

Work with a partner and help each other to calculate these rates.

One day while at a school camp, students completed an adventure race. Students kayaked down a river, cycled along a country road and finally jogged back to camp.

Piper kayaked 16 km in 2 hours.

- On average, at what speed was Piper kayaking in km/h?
  In a 2 minute period Piper counted 80 paddle strokes.
- What rate was Piper paddling in paddle strokes/minute?

Summer cycled 36 km in 2 hours.

- On average, what speed was Summer cycling in km/h? Cycling uphill, Summer counted 150 pedal turns in 3 minutes.
- What was Summer pedalling rate in pedal turns/minute?
- At this rate, how many pedal turns would Summer make in 7 minutes of uphill cycling?

Luca jogged the final section of the race at a speed of 200 m/minute.

- At this rate, how long did it take Luca to run 1000 m?
- At this rate how far would Luca run in 60 minutes?
- What speed did Luca jog at in km/h?
- Running at this speed, how long did it take Luca to complete the 3 km run back to the camp?

Key ideas

- When a rate is provided, a change in one quantity implies that an equivalent change must occur in the other quantity.
  e.g. Patrick earns $20/hour. How much will he earn in 6 hours?

$$\times 6 \left(\begin{array}{l} \$20 \text{ for 1 hour} \\ \$120 \text{ for 6 hours} \end{array} \right) \times 6$$

  e.g. Patrick earns $20/hour. How long will it take him to earn $60?

$$\times 3 \left(\begin{array}{l} \$20 \text{ in 1 hour} \\ \$60 \text{ in 3 hours} \end{array} \right) \times 3$$

- Carefully consider the units involved in each question and answer.
- **Speed** is a measure of how fast an object is travelling.
- If the speed of an object does not change over time, the object is travelling at a **constant speed**. 'Cruise control' helps a car travel at a constant speed.
- When speed is not constant, due to acceleration or deceleration, we are often interested to know the **average speed** of the object.

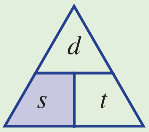
Speed A measure of how fast an object is moving

- Average speed is calculated by the formula:

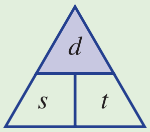
$$\textbf{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \quad \text{or} \quad s = \frac{d}{t}$$

- Depending on the unknown value, the above formula can be rearranged to make d or t the subject. The three formulas involving $s, d,$ and t are:

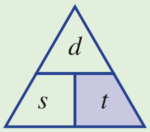
$s = \frac{d}{t}$



$d = s \times t$



$t = \frac{d}{s}$



- Care must be taken with units for speed, and on occasions units will need to be converted. The most common units of speed are m/s and km/h.

Exercise 6F

UNDERSTANDING 1–4 4

1 Fill in the gaps.

a $\times 3 \left(\begin{array}{l} \text{60 km in 1 hour} \\ \text{180 km in } \underline{\hspace{1cm}} \end{array} \right) \times 3$

b $\times 5 \left(\begin{array}{l} \text{\$25 in 1 hour} \\ \text{\$125 in } \underline{\hspace{1cm}} \end{array} \right) \underline{\hspace{1cm}}$

c $\underline{\hspace{1cm}} \left(\begin{array}{l} \text{7 questions in 3 minutes} \\ \text{70 questions in } \underline{\hspace{1cm}} \end{array} \right) \underline{\hspace{1cm}}$

d $\underline{\hspace{1cm}} \left(\begin{array}{l} \text{120 litres in 1 minute} \\ \underline{\hspace{1cm}} \text{ in 6 minutes} \end{array} \right) \underline{\hspace{1cm}}$

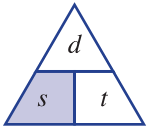
2 Fill in the gaps.

a $\begin{array}{l} \div 3 \left(\begin{array}{l} \text{\$36 for 3 hours} \\ \underline{\hspace{1cm}} \text{ for 1 hour} \end{array} \right) \div 3 \\ \times 5 \left(\begin{array}{l} \underline{\hspace{1cm}} \text{ for 1 hour} \\ \underline{\hspace{1cm}} \text{ for 5 hours} \end{array} \right) \underline{\hspace{1cm}} \end{array}$

b $\begin{array}{l} \underline{\hspace{1cm}} \left(\begin{array}{l} \text{150 rotations in 5 minutes} \\ \underline{\hspace{1.5cm}} \text{ in 1 minute} \end{array} \right) \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \left(\begin{array}{l} \underline{\hspace{1.5cm}} \text{ in 1 minute} \\ \underline{\hspace{1.5cm}} \text{ in 7 minutes} \end{array} \right) \underline{\hspace{1cm}} \end{array}$

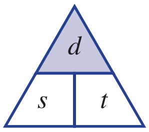
3 Copy and complete by writing in the missing words.

a



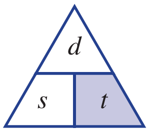
$\text{speed} = \frac{?}{\text{time}}$

b



$\text{distance} = \underline{\quad ? \quad} \times \text{time}$

c



$\text{time} = \frac{?}{?}$

Use the triangles to help work out each rule.

Units of speed have a length unit and a time unit.

4 Which of the following is not a unit of speed?

A m/s
B km/h
C cm/h
D L/kg
E m/min

FLUENCY 5–8 5–9

Example 17 Solving rate problems

a Rachael can touch type at 74 words/minute. How many words can she type in 15 minutes?

b Leanne works in a donut van and sells on average 60 donuts every 15 minutes. How long is it likely to take her to sell 800 donuts?

Solution	**Explanation**
a ×15 74 words in 1 minute ×15 1110 words in 15 minutes Rachael can type 1110 words in 15 minutes.	Calculate $74 \times 15 = 1110$
b ÷15 60 donuts in 15 minutes ÷15 4 donuts in 1 minute ×200 800 donuts in 200 minutes ×200 Leanne is likely to take 3 hours and 20 minutes to sell 400 donuts.	Selling rate = 60 donuts/15 minutes Divide both quantities by 15. Multiply both quantities by 200. Convert answer to hours and minutes.

5 **a** Lewis can touch type at 80 words/minute. How many words can he type in 20 minutes?

b Robbie works at a bakery and, on average, he sells 4 loaves of bread every 10 minutes. How long it will it take him to sell 20 loaves of bread?

6 A factory produces 40 plastic bottles/minute.

a How many bottles can the factory produce in 60 minutes?

b How many bottles can the factory produce in an 8 hour day of operation?

☐ bottles in 1 hour.
☐ bottles in 8 hours.

Example 18 Finding average speed

Find the average speed in km/h of:

a a cyclist who travels 140 km in 5 hours

b a runner who travels 3 km in 15 minutes.

Solution

a

$$s = \frac{d}{t}$$

$$= \frac{140\text{ km}}{5\text{ h}}$$

$$= 28\text{ km/h}$$

Alternative unitary method

$$\begin{tikzcd} 140\text{ km in }5\text{ hours} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 28\text{ km in }1\text{ hour} \end{tikzcd}$$

Average speed = 28 km/h

Explanation

The unknown value is speed.
Write the formula for speed.
Distance travelled = 140 km
Time taken = 5 h. Calculate 140 ÷ 5.
Speed unit is km/h.

Write down the rate provided in the question.
Divide both quantities by 5.

Solution

b

$$s = \frac{d}{t}$$

$$= \frac{3\text{ km}}{15\text{ min}}$$

$$= \frac{1}{5}\text{ km/min} \qquad = 12\text{ km/h}$$

Alternative unitary method

$$\begin{tikzcd} 3\text{ km in }15\text{ minutes} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ 12\text{ km in }60\text{ minutes} \end{tikzcd}$$

Average speed = 12 km/h

Explanation

Distance travelled = 3 km

$$\begin{tikzcd} \frac{1}{5}\text{ km in }1\text{ minute} \arrow[d, bend right, "\times 60"'] \arrow[d, bend left, "\times 60"] \\ 12\text{ km in }60\text{ minutes} \end{tikzcd}$$

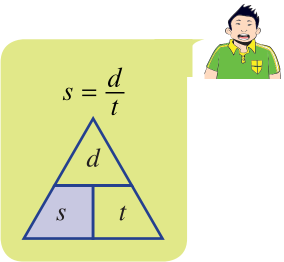
Write down the rate provided in the question.

15 × 4 = 60 minutes = 1 hour

Multiply both quantities by 4.

7 Find the average speed of:

- **a** a sprinter running 200 m in 20 seconds (in m/s)
- **b** a skateboarder travelling 840 m in 120 seconds (in m/s)
- **c** a car travelling 180 km in 3 hours (in km/h)
- **d** a truck travelling 400 km in 8 hours (in km/h)
- **e** a train travelling 60 km in 30 minutes (in km/min and km/h)
- **f** a tram travelling 15 km in 20 minutes (in km/min and km/h).



Example 19 Finding the distance travelled

Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.

Solution

$$d = s \times t$$

$$= 95\text{ km/h} \times 15\text{ h}$$

$$= 1425\text{ km}$$

Alternative unitary method

$$\begin{tikzcd} 95\text{ km in }1\text{ hour} \arrow[d, bend right, "\times 15"'] \arrow[d, bend left, "\times 15"] \\ 1425\text{ km in }15\text{ hours} \end{tikzcd}$$

Truck travels 1425 km in 15 hours.

Explanation

The unknown value is distance.
Write the formula for distance.
Distance unit is km.

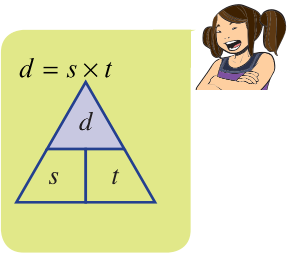
Write the rate provided in the question.

Multiply both quantities by 15.

8 Find the distance travelled by:

- **a** a cyclist travelling at 12 m/s for 90 seconds
- **b** an ant travelling at 2.5 cm/s for 3 minutes
- **c** a bushwalker who has walked for 8 hours at an average speed of 4.5 km/h
- **d** a tractor ploughing fields for 2.5 hours at an average speed of 20 km/h.

$d = s \times t$



Example 20 Finding the time taken

Find the time taken for a hiker walking at 4 km/h to travel 15 km.

Solution

$$t = \frac{d}{s}$$

$$= \frac{15\text{ km}}{4\text{ km/h}}$$

$$= 3.75\text{ h}$$

$$= 3\text{ h } 45\text{ min}$$

Explanation

The unknown value is time.
Write the formula with t as the subject.
The time unit is h. Leave answer as a decimal or convert to hours and minutes.
$0.75\text{ h} = 0.75 \times 60 = 45\text{ min}$

Alternative unitary method

$$\begin{tikzcd} 4\text{ km in }1\text{ hour} \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ 1\text{ km in }\frac{1}{4}\text{ hour} \arrow[d, bend right, "\times 15"'] \arrow[d, bend left, "\times 15"] \\ 15\text{ km in }\frac{15}{4}\text{ hours} \end{tikzcd}$$

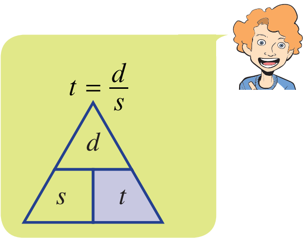
Express the rate as provided in the question.
Divide both quantities by 4.
Multiply both quantities by 15.

It takes 3 h 45 min to travel 15 km.

9 Find the time taken by:

- **a** a sports car to travel 1200 km at an average speed of 150 km/h
- **b** a bus to travel 14 km at an average speed of 28 km/h
- **c** a plane to fly 6900 km at a constant speed of 600 km/h
- **d** a ball moving through the air at a speed of 12 m/s to travel 84 m

$t = \frac{d}{s}$



PROBLEM-SOLVING AND REASONING 10, 11 11–13

10 Putra is an elite rower. When training, his goal is a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.

- **a** How many times does Putra's heart beat during a 30 minute workout?
- **b** How many times does Putra's heart beat during 30 minutes of 'rest'?
- **c** If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

Putra is training when he has a workout.

11 A plane is flying at a cruising speed of 900 km/h. How far will the plane travel from 11:15 am to 1:30 pm on the same day?

12 The wheels on Charlie's bike have a circumference of 1.5 m. When Charlie is riding fastest, the wheels turn at a speed of five turns per second.

a What is the fastest speed Charlie can ride his bike, in km/h ?

b How far would Charlie travel in 5 minutes at his fastest speed?

13 The Ghan train is an Australian icon. You can board the Ghan in Adelaide and 2979 km later, after travelling via Alice Springs, you arrive in Darwin. For these questions, round the answers correct to one decimal place.

a If you board the Ghan in Adelaide on Sunday at $2:20$ pm and arrive in Darwin on Tuesday at $5:30$ pm, what is the average speed of the train journey?

b There are two major rest breaks. The train stops for $4\frac{1}{4}$ hours at Alice Springs and 4 hours at Katherine. Taking these breaks into account, what is the average speed of the train when it is moving?

Speed research

— 14

14 Carry out research to find answers to the following questions.

Light and sound

a What is the speed of sound in m/s?

b What is the speed of light in m/s?

c How long would it take sound to travel 100 m?

d How long would it take light to travel 100 km?

e How many times quicker is the speed of light than the speed of sound?

f What is a Mach number?

Spacecraft

g What is the escape velocity needed by a spacecraft to 'break free' of Earth's gravitational pull? Give this answer in km/h and also km/s.

h What is the orbital speed of planet Earth around the Sun? Give your answer in km/h and km/s.

i What was the average speed of a space shuttle on a journey from Earth to the International Space Station?

Knots

Wind speed and boat speed are often given in terms of knots (kt).

j What does a knot stand for?

k What is the link between nautical miles and a system of locating positions on Earth?

l How do you convert a speed in knots to a speed in km/h?

Maths@Work: Development officer for a fragrance company

Many small businesses are taking advantage of the increasing popularity of the 'wellbeing' industry and the desire for natural products. A group of interested people can form a business producing and distributing fragrance products such as perfume, candles, soaps, body and incense oils and various lotions. The fragrances can be mixed at home and the products sold at community markets and online. Such businesses are taking an increasing share of profits in the global economy.

Understanding and applying the science and mathematics of mixing oils and fragrances is vital to the success of such businesses. To be a qualified development officer in this field you need to have a good sense of smell, a detailed knowledge of fragrances and oils and the skills to safely mix chemicals in the correct ratios.

Note to the teacher: This activity is intended to get students thinking about ratios and rates in a practical context, but is not intended as a practical activity. However, these are real recipes and the creation of these perfumes could be done in class in a safe laboratory environment.

1 To make a perfume for a summer candle 1 part coconut is mixed with 3 parts vanilla. How much vanilla is needed for:

a 10 mL of coconut? **b** 15 mL of coconut? **c** 240 mL of mixture?

2 To make Apple Jack essence, mix 1 drop of apple fragrance with 1 drop of cinnamon fragrance to 2 drops of grapefruit fragrance.

a How many drops of grapefruit fragrance are needed for:

i 2 drops of apple? **ii** 5 drops of cinnamon?

b If 10 mL of apple essence is used how much cinnamon is needed?

3 To make a Charleston candle fragrance 10 parts sandalwood fragrance are mixed with 2 parts cinnamon fragrance.

- **a** Write this as a simplified ratio.
- **b** How much cinnamon is needed for:
  - **i** 10 drops of sandalwood
  - **ii** 25 drops of sandalwood
  - **iii** 120 drops of scent.

4 A 'Quiet time' candle uses a perfume of chamomile and spearmint mixed in the ratio of 1 to 3.

- **a** How many grams of chamomile are needed for:
  - **i** 6 grams of spearmint
  - **ii** 12 grams of spearmint
  - **iii** 30 grams of spearmint
  - **iv** total 240 grams of fragrance.
- **b** How many grams of spearmint are needed for:
  - **i** 20 grams of chamomile
  - **ii** 100 grams of chamomile
  - **iii** total 360 grams of fragrance.

5 In a certain perfume, 3 drops of cedarwood are mixed with 10 drops of lavender to 5 drops of bergamot. These are then mixed with alcohol to form the final product.

- **a** How many drops of lavender are needed for:
  - **i** 15 drops of Bergamot?
  - **ii** 12 drops of cedarwood?
  - **iii** 15 drops of cedarwood?
- **b** How many drops of cedarwood are needed for 40 drops of lavender?
- **c** If one drop is approximately 0.065 mL, how many millilitres are in:
  - **i** 10 drops of lavender?
  - **ii** 20 drops of cedarwood?
  - **iii** 24 drops of bergamot?
- **d** 360 drops of fragrance are used in total. How many drops of each of the three individual scents are used?

6 A seaweed face mask lotion can be made from 3 parts seaweed powder, 6 parts sweet almond oil (or jojoba oil), 1 part aloe vera gel and 1 part honey.

- **a** Given that 1 tablespoon is equal in volume to 3 teaspoons, rewrite the recipe above using spoon measurements.
- **b** Josie grinds 2 sheets of dried seaweed and that amount fills 2 tablespoons. State the spoon quantities of the remaining ingredients that Josie needs to make her face mask if she uses jojoba oil.

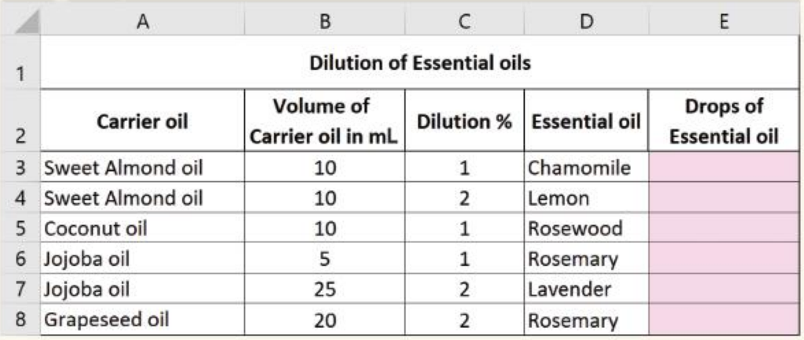
Using technology

An Essential Oils Development Officer would instruct pupils about safety. For example, undiluted essential oils or fragrances must never be put on skin or near eyes, never swallowed and always kept away from children. An important mathematical method to teach is the procedure for mixing Essential oils with Carrier oils to achieve the recommended 1% or 2% dilution.

7 When diluting Essential oils or fragrances with Carrier oils the following rates apply:

- For 1% dilution of Essential oil use 1 drop/5 mL of Carrier oil
- For 2% dilution of Essential oil use 2 drops/5 mL of Carrier oil

a Set up the Excel spreadsheet shown below and enter formulas to calculate the number of drops needed for these dilutions.



	A	B	C	D	E
1	**Dilution of Essential oils**				
2	**Carrier oil**	**Volume of Carrier oil in mL**	**Dilution %**	**Essential oil**	**Drops of Essential oil**
3	Sweet Almond oil	10	1	Chamomile	
4	Sweet Almond oil	10	2	Lemon	
5	Coconut oil	10	1	Rosewood	
6	Jojoba oil	5	1	Rosemary	
7	Jojoba oil	25	2	Lavender	
8	Grapeseed oil	20	2	Rosemary	

b Use more rows in your spreadsheet to find the number of drops of Essential oil needed to make:

- 1% dilution of Peppermint Essential oil with 25 mL of Sweet Almond oil
- 2% dilution of Geranium Essential oil with 20 mL of Coconut oil
- 1% dilution of Lavender Essential oil with 10 mL of Jojoba oil
- 2% dilution of Lemon Grass Essential oil with 15 mL of Sunflower oil

Puzzles and games

1 Write these ratios in simplest form to solve the riddles below.

A	$4 : 8$	**C**	$4 : 16$	**E**	$6 : 10$	**F**	$4 : 12$	**H**	$8 : 12$
I	$20 : 16$	**K**	$10 : 4$	**L**	$12 : 3$	**O**	$9 : 6$	**P**	$15 : 5$
R	$25 : 15$	**S**	$20 : 10$	**T**	$35 : 25$	**V**	$2 : 12$		

a What do termites eat for dessert?

$7 : 5$ $3 : 2$ $3 : 2$ $7 : 5$ $2 : 3$ $3 : 1$ $5 : 4$ $1 : 4$ $5 : 2$ $2 : 1$

b Where do geologists go to have a good time?

$7 : 5$ $3 : 2$ $5 : 3$ $3 : 2$ $1 : 4$ $5 : 2$

$1 : 3$ $3 : 5$ $2 : 1$ $7 : 5$ $5 : 4$ $1 : 6$ $1 : 2$ $4 : 1$ $2 : 1$

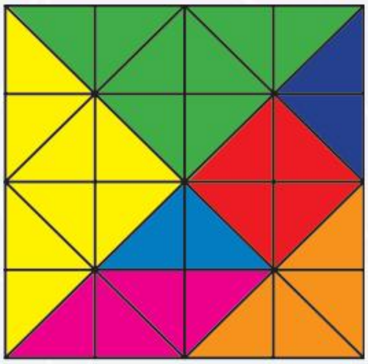
2 The ancient Chinese puzzle known as a tangram consists of 7 geometric shapes (tans) as shown.

a Write the ratio of the areas of the seven shapes in this tangram. Write the ratio in simplest form in ascending order.

b The pieces (tans) of a tangram can be arranged to make many creative shapes and designs. For the shapes shown here, find:

i the ratio of the yacht's sails to the boat hull.

ii the ratio of the cat's head to the rest of the body.



3 Hannah is 14 years old and her brother is 9 years old. Find their ages when the ratio of Hannah's age to Blake's age is:

a $3 : 2$ **b** $5 : 4$ **c** $11 : 10$

4 This diagram is made up of 8 equal-sized squares.

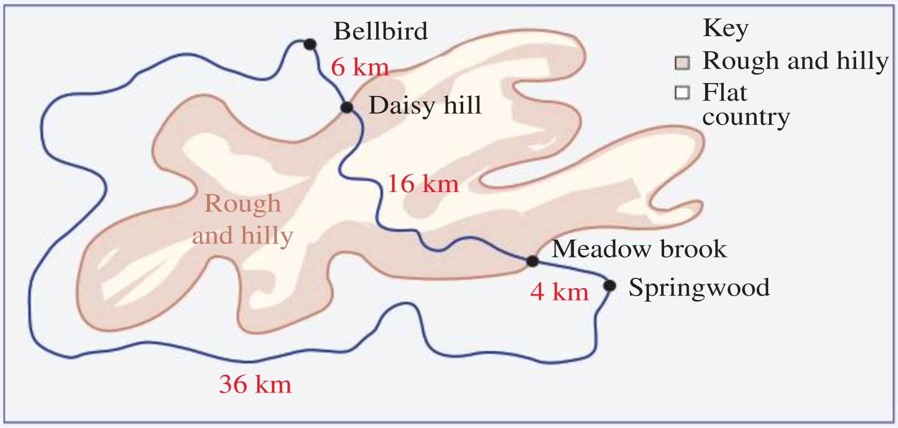
How many squares need to be shaded if the ratio of shaded squares to unshaded squares is:

a $1 : 3$ **b** $2 : 3$ **c** $1 : 2$

Give each answer as a mixed number if necessary.

5 Bottle A has 1 L of cordial drink with a cordial to water ratio of $3 : 7$. Bottle B has 1 L of cordial drink with a cordial to water ratio of $1 : 4$. The drink from both bottles is combined to form a 2 L drink. What is the new cordial to water ratio?

6 A group of cyclists decide to have a race from Springwood to Bellbird. The towns and distances are shown on the sketch map below. Over flat country a cyclist averages 20 km/h but through rough and hilly country the average is 12 km/h. Which route would be fastest and by how much?



7 Brothers Marco and Matthew start riding from home into town, which is 30 km away. Marco rode at 10 km/h and Matthew took 20 minutes longer to complete the trip. Assuming that they both rode at a constant speed, how fast was Matthew riding?

8 Solve the questions below to find the answer to the riddle:
Why did the monkey put a steak under himself?

___ ___ ___ ___ ___ ___ ___ ___ ___
5 m 4 m 8 m 2 m 1 : 8000 1 : 4000 4 m 70 cm 4 m

___ ___ ___ ___ ___ ___ ___
1 : 3000 70 cm 1 m 1 : 8000 90 cm 70 cm 1 : 3000

___ ___ ___ ___ ___ ___
70 cm 4 m 1 : 80 2 m 1 : 4000 2 m

___ ___ ___ ___ ___ ___ ___
90 cm 1 : 500 10 cm 25 cm 25 cm 4 m 1 : 500

If the scale is 1 : 100, find the real length in metres shown by:

a 2 cm **b** 5 cm **c** 8 cm **d** 6 cm **e** 4 cm

If the scale is 1 : 10, find the real length shown by:

f 3 cm **g** 9 cm **h** 7 cm **i** 1 cm **j** 15 cm

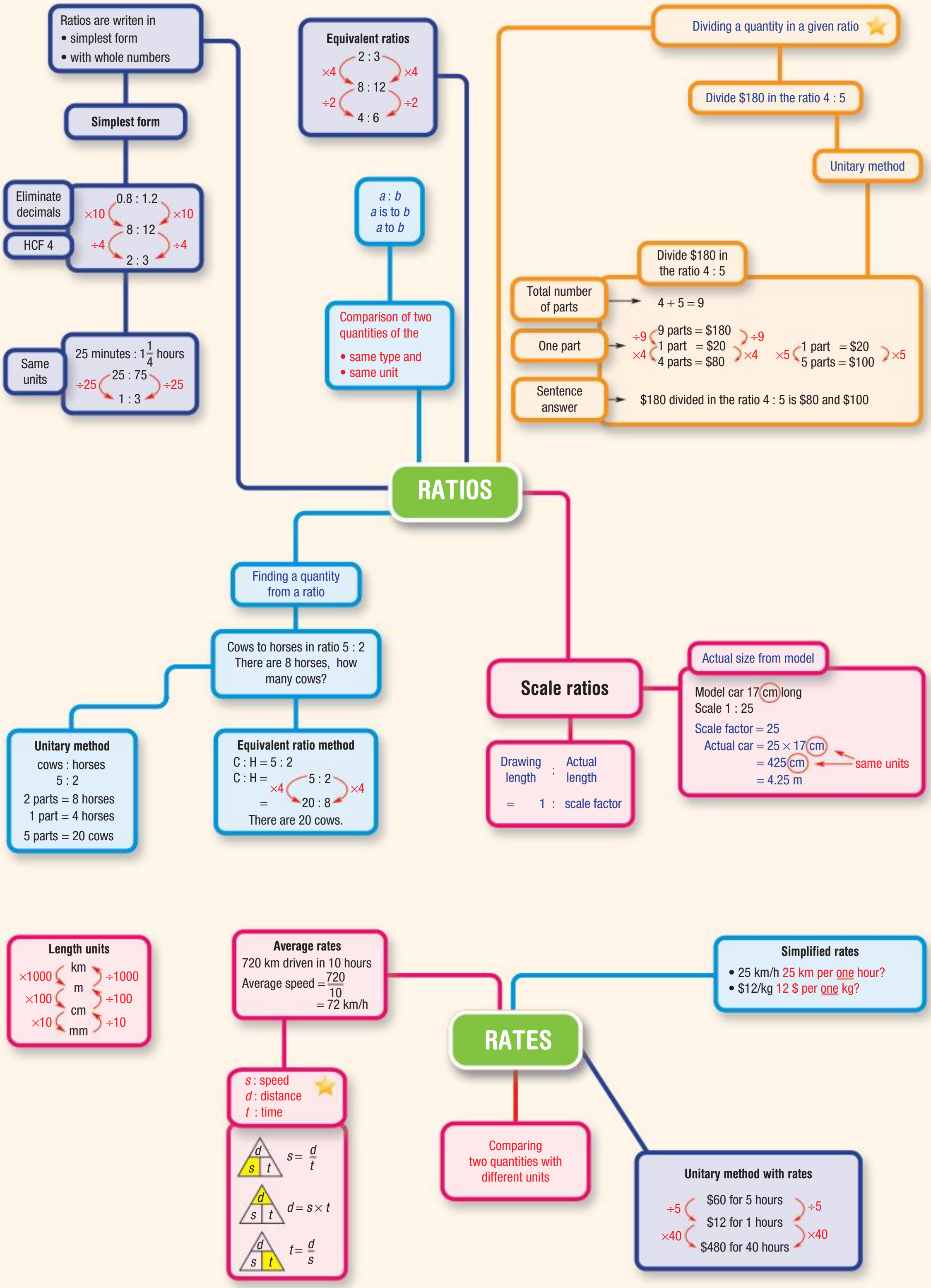
If the scale is 1 : 5, find the real length shown by:

k 3 cm **l** 5 cm **m** 10 cm **n** 30 cm **o** 20 cm

Write each scale in the simplest ratio form:

p 1 cm to 2 m **q** 1 cm to 10 m **r** 1 cm to 5m **s** 1m to 4km
t 1 m to 3 km **u** 1 m to 8 km **v** 1 mm to 3 cm **w** 1 mm to 8 cm
x 1 mm to 15 cm **y** 1 mm to 6 cm **z** 1 mm to 2 cm

Chapter summary



RATIOS

Ratios are writen in
- simplest form
- with whole numbers

Simplest form

Eliminate decimals

$0.8 : 1.2 \xrightarrow{\times 10} 8 : 12$

HCF 4

$8 : 12 \xrightarrow{\div 4} 2 : 3$

Same units

25 minutes : $1\frac{1}{4}$ hours

$25 : 75 \xrightarrow{\div 25} 1 : 3$

Equivalent ratios

$2 : 3 \xrightarrow{\times 4} 8 : 12 \xrightarrow{\div 2} 4 : 6$

a : *b*
a is to *b*
a to *b*

Comparison of two quantities of the
- same type and
- same unit

Dividing a quantity in a given ratio

Divide $180 in the ratio 4 : 5

Unitary method

Divide $180 in the ratio 4 : 5

Total number of parts → $4 + 5 = 9$

One part →

$9 \text{ parts} = \$180 \xrightarrow{\div 9} 1 \text{ part} = \$20 \xrightarrow{\times 4} 4 \text{ parts} = \$80$

$1 \text{ part} = \$20 \xrightarrow{\times 5} 5 \text{ parts} = \$100$

Sentence answer → $180 divided in the ratio 4 : 5 is $80 and $100

Finding a quantity from a ratio

Cows to horses in ratio 5 : 2
There are 8 horses, how many cows?

Unitary method

cows : horses
5 : 2
2 parts = 8 horses
1 part = 4 horses
5 parts = 20 cows

Equivalent ratio method

C : H = 5 : 2

C : H = $5 : 2 \xrightarrow{\times 4} 20 : 8$

There are 20 cows.

Scale ratios

Drawing length : Actual length
= 1 : scale factor

Actual size from model

Model car 17 cm long
Scale 1 : 25
Scale factor = 25
Actual car = 25 × 17 cm
= 425 cm
= 4.25 m

same units

RATES

Length units

$\text{km} \xrightarrow{\times 1000} \text{m} \xrightarrow{\times 100} \text{cm} \xrightarrow{\times 10} \text{mm}$

$\text{mm} \xrightarrow{\div 10} \text{cm} \xrightarrow{\div 100} \text{m} \xrightarrow{\div 1000} \text{km}$

Average rates

720 km driven in 10 hours

$$\text{Average speed} = \frac{720}{10} = 72 \text{ km/h}$$

s : speed
d : distance
t : time

$s = \frac{d}{t}$

$d = s \times t$

$t = \frac{d}{s}$

Comparing two quantities with different units

Simplified rates
- 25 km/h 25 km per one hour?
- $12/kg 12 $ per one kg?

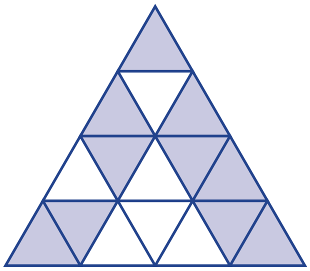
Unitary method with rates

$\$60 \text{ for } 5 \text{ hours} \xrightarrow{\div 5} \$12 \text{ for } 1 \text{ hours} \xrightarrow{\times 40} \$480 \text{ for } 40 \text{ hours}$

Multiple-choice questions

6A/B **1** A school has 315 boys, 378 girls and 63 teachers. The ratio of students to teachers is:
A 11 : 1 **B** 1 : 11 **C** 5 : 6 **D** 6 : 5

6A/B **2** Find the ratio of the shaded area to the unshaded area in this triangle.



A 3 : 5 **B** 8 : 5 **C** 5 : 3 **D** 5 : 8

6B **3** The ratio 500 mm to 20 cm is the same as:
A 50 : 2 **B** 2500 : 1 **C** 2 : 5 **D** 5 : 2

6B **4** The ratio 1 hour : 30 minutes simplifies to:
A 2 : 1 **B** 1 : 2 **C** 1 : 30 **D** 4 : 3

6C **5** $750 is divided in the ratio 1 : 3 : 2. The smallest share is:
A $250 **B** $125 **C** $375 **D** $750

6C **6** The ratio of the areas of two triangles is 5 : 2. The area of the larger triangle is 60 cm^2. What is the area of the smaller triangle?
A 12 cm^2 **B** 24 cm^2 **C** 30 cm^2 **D** 17 cm^2

6E/F **7** Callum fills his car with 28 litres of petrol at 142.7 cents per litre. His change from $50 cash is:
A $10 **B** $39.95 **C** $10.05 **D** $40

6F **8** Madison cycled 20 km in 1.25 hours. Her average speed was:
A 25 km/h **B** 20 km/h **C** 16 km/h **D** 18.75 km/h

6D **9** A house plan has a scale of 1 : 200. On the plan, the lounge room is 25 mm in length. The real length of the lounge room would be:
A 50 m **B** 5 m **C** 50 cm **D** 8 m

6D **10** On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?
A 1 : 5
B 1 : 5000
C 1 : 50 000
D 1 : 5 000 000

Short-answer questions

6A **1** In Lao's pencil case there are 6 coloured pencils, 2 black pens, 1 red pen and 3 lead pencils. Find the ratio of:

- **a** lead pencils to coloured pencils
- **b** black pens to red pens
- **c** all pens to all pencils

6B **2** True (T) or false (F)?

- **a** $1 : 4 = 3 : 6$
- **b** The ratio $2 : 3$ is the same as $3 : 2$.
- **c** The ratio $3 : 5$ is written in simplest form.
- **d** 40 cm : 1 m is written as $40 : 1$ in simplest form.

6B **3** Copy and complete.

- **a** $4 : 50 = 2 : \square$
- **b** $3 : 7 = \square : 21$
- **c** $\square : 12 = 8 : 3$
- **d** $1 : \square : 5 = 5 : 15 : 25$

6B **4** Simplify the following ratios.

- **a** $10 : 40$
- **b** $36 : 24$
- **c** $75 : 100$
- **d** $8 : 64$
- **e** $27 : 9$
- **f** $5 : 25$
- **g** $6 : 4$
- **h** $52 : 26$
- **i** $6 : 9$
- **j** $8 : 4 : 20$

6B **5** Simplify the following ratios by first changing to the same units.

- **a** 2 cm : 8 mm
- **b** 5 mm : 1.5 cm
- **c** 3 L : 7500 mL
- **d** 30 min : 1 h
- **e** 400 kg : 2 tonnes
- **f** 6 h : 1 day
- **g** 120 m : 1 km
- **h** 45 min : $2\frac{1}{2}$ h

6C **6** Divide:

- **a** \$80 in the ratio $7 : 9$
- **b** 200 kg in the ratio $4 : 1$
- **c** 40 m in the ratio $6 : 2$
- **d** \$1445 in the ratio $4 : 7 : 6$
- **e** \$100 in the ratio $3 : 1 : 1$

6C **7** Orange juice, pineapple juice and guava juice are mixed in the ratio $4 : 3 : 2$. If 250 mL of guava juice is used, how many litres of drink does this make?

6D **8** A map has a scale of $1 : 20\,000$. Find the real distance for each of these scaled distances.

- **a** 3 cm (answer in m)
- **b** 12 cm (answer in km)

6D **9** For each of these situations, find the scale ratio and also state the scale factor.

- **a** 5 mm on a scale drawing represents a real length of 1 m.
- **b** 4 cm on a map represents an actual length of 10 km.

6D **10** Two towns are 5 km apart. How many millimetres apart are they on a map that has a scale of $1 : 100\,000$?

6E **11** Express each rate in simplest form.

- **a** 10 km in 2 hours (? km/h)
- **b** \$650 for 13 hours (\$?/h)
- **c** 2800 km in 20 days (? km/day)

6E

12 Copy and complete.

a $$\times ?\left(\begin{array}{l}\text{7 km uses 1 L of fuel}\\ \text{280 km uses ? L of fuel}\end{array}\right)\times ?$$

b $$\times ?\left(\begin{array}{l}\text{60 words typed in 1 minute}\\ \text{? words typed in 10 minutes}\end{array}\right)\times ?$$

6F

13 a A truck uses 12 litres of petrol to travel 84 km. How far will it travel on:

- **i** 1 L of petrol?
- **ii** 42 L of petrol?

b Samira earns $67.20 for a 12-hour shift. How much will she earn for:

- **i** 1 hour?
- **ii** 7 hours?

6F

14 a Sandra drives to her mother's house. It takes 2 hours. Calculate Sandra's average speed in km/h if her mother lives 150 km away.

b How long does it take Ari to drive 180 km along the freeway to work if he manages to average 100 km/h for the trip? Give your answer in hours.

c How far does Siri ride his bike if he rides at 12 km/h for 45 minutes?

Extended-response questions

1 From Canberra, ACT, to Melbourne, Victoria, it is 660 km. Two families, the Harrisons and the Nguygens, both leave Canberra at 8 am to drive to Melbourne.

The Harrison family's trip

- The Harrison's 17-year-old son drives for the first 2 hours at an average speed of 80 km/h.
- Then they stop for a rest of 1.5 hours.
- Mr Harrison drives the rest of the way to Melbourne with no more stops.
  - **a** How far did the Harrison's son drive?
  - **b** How far did Mr Harrison drive?
  - **c** At what time did the Harrison family finish their morning rest break?
  - **d** If the Harrisons arrive in Melbourne at 4:30 pm, for how long did Mr Harrison drive?
  - **e** What was Mr Harrison's average speed?

The Nguygen family's trip

- The Nguygen family drove to Melbourne with one 30-minute break.
- It took them $8\frac{1}{4}$ hours in total.

f At what time did the Nguygen family arrive in Melbourne?

g Calculate the average speed that the Nguygen family drove at, not counting the break.

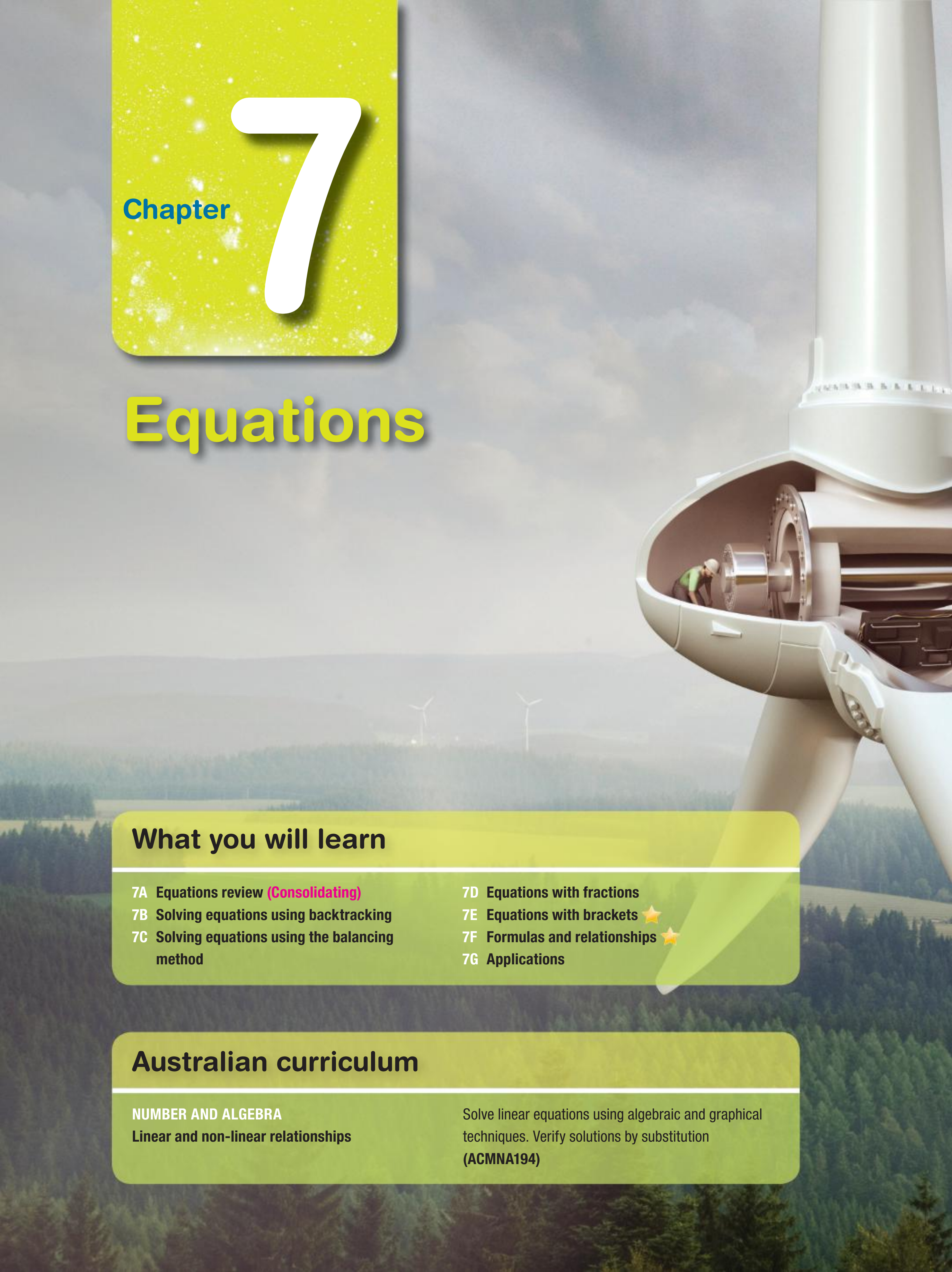
Comparing the cost of each trip

h Using the information below, calculate the cost of each car's fuel for the trip.

Petrol costs 152.7 cents/L.

The Harrison family's car uses 8 L/100km.

The Nguygen family's car uses 11 L/100 km.



Chapter 7

Equations

What you will learn

Australian curriculum

NUMBER AND ALGEBRA **Linear and non-linear relationships**	Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution **(ACMNA194)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Maths and wind power

Powered freely by the wind, the giant blades of a wind turbine rotate at 30–60 rpm (revolutions per minute). A gear box drives the motor shaft at 1000–1800 rpm, generating electricity.

Using mathematical equations, engineers calculated that a turbine with three blades is the most efficient for capturing wind energy. The shape, thickness and total area of the blades must be designed with enough strength to survive huge wind forces as well as extracting as much wind energy as possible. The world's largest wind turbines reach 220 m high and can each power up to 7500 homes from the rotation of their enormous, 80 m long blades (equal to the wingspan of an Airbus A380 jumbo jet).

Wind power is a low-cost, clean, renewable energy and its use helps to reduce global warming.

Pre-test

1 Fill in the missing number in these equations.

a $5+7=\square$ **b** $3\times 9=\square$ **c** $12\div 4=\square$ **d** $5\times 2=\square$

2 Find the value of $\square$ to make these equations true.

a $4+\square=12$ **b** $6\times\square=12$ **c** $\square+14=19$ **d** $\square-4=11$

3 If $x=6$, find the value of:

a $x+2$ **b** $x\times 7$ **c** $x-2$ **d** $8-x$

4 Simplify these algebraic expressions.

a $9m+2m$
b $4a-3a$
c $7n+3n-n$
d $8a+2a-10$
e $4x+2+7x$
f $5b+4+3b$

5 Expand these algebraic expressions using the distributive law.

a $3(m+4)$ **b** $2(a+6)$ **c** $3(x+7)$ **d** $4(k-6)$

6 I think of a number, double it, and then add three to get 27. What is the number?

7 If $x=5$, are the following equations true (T) or false (F)?

a $x+2=7$ **b** $3x=35$ **c** $x-1=6$ **d** $2x=10$

8 Solve each of the following equations by inspection or using guess and check.

a $x+8=12$ **b** $4x=32$ **c** $m-6=-2$ **d** $3m=18$

9 State the opposite operation of each of the following. Choose from: A: +3, B: –2, C: ÷5 or D: ×3

a × 5 **b** +2 **c** ÷3 **d** –3

10 The sum of k and 3 is written as $k+3$. Write expressions for the following. Choose from: A: $q-6$, B: 2z, C: $p+10$ or D: $4x$

a the sum of p and 10
b the product of 4 and x
c double z
d 6 less than q

11 Copy and complete.

a

x	–2	–1	0	1	2	3
$3x-1$	–7					

b

x	–2	–1	0	1	2	3
$2(x+3)$			6			

7A Equations review

CONSOLIDATING

Interactive

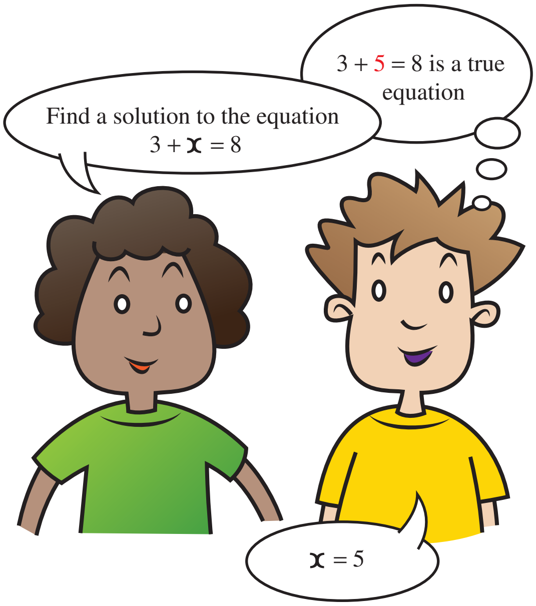
Widgets

HOTsheets

Walkthrough

Equations are mathematical statements saying that two things are equal. For example, $2 + 2 = 4$ is an equation.

If there is a pronumeral involved, then a solution is a value for that pronumeral that makes the equation true.



Let's start: What's missing?

Rory has erased a number in each of the equations below.

- If the equations were originally true, find the missing values:
  $10 + \square = 57$ $\square - 31 = 40$ $2 \times \square + 5 = 19$
- In one equation he erased two numbers to get $\square \times 2 = \square$.
- Is it possible to find the missing values? Why or why not?

Key ideas

- An **equation** is a mathematical statement that two expressions are equal, such as $3 \times 5 = 15$ (which is true) or $2 + 2 = 100$ (which is false).
- The parts of an equation are: $3 + 7 = 2 \times 5$

  left-hand side LHS

  equals sign

  right-hand side RHS
- A **solution** to an equation is a value for a pronumeral that makes an equation true. The process of finding a solution is called **solving**.

Equation A mathematical statement that two expressions (numeric or algebraic) have the same value

Solution The value/s that give a true statement when substituted for the unknown in an equation

Exercise 7A

UNDERSTANDING 1–5 5

1 State the value of:

a $7 + 12$

b 5×3

c 2×8

d $10 - 4$

e $2 \times 5 + 1$

f $3 + 5 \times 2$

g $(3 + 2) \times 7$

h $5 \div (4 + 1)$

Remember: Brackets Order then Division and Multiplication then Addition and Subtraction

Example 1 Classifying equations as true or false

For each of the following equations, state whether they are true or false.

a $3 + 8 = 15 - 4$ **b** $7 \times 3 = 20 + 5$

Solution	Explanation
a True	Left-hand side (LHS) is $3 + 8$, which is 11. Right-hand side (RHS) is $15 - 4$, which is also 11. Since LHS equals RHS, the equation is true.
b False	LHS $= 7 \times 3 = 21$ RHS $= 20 + 5 = 25$ Since LHS and RHS are different, the equation is false.

2 Classify these equations as true (T) or false (F).

a $5 \times 3 = 15$ **b** $7 + 2 = 12$ **c** $5 + 3 = 16 \div 2$

d $8 - 6 = 6$ **e** $4 \times 3 = 12 \times 1$ **f** $2 = 8 - 3 - 3$

3 Find the value of $A + 5$ if:

a $A = 3$ **b** $A = 7$ **c** $A = 10$ **d** $A = 40$

4 If the value of x is 3, what is the value of the following?

a $10 + x$ **b** $3x$ **c** $5 - x$ **d** $6 \div x$

$3x$ means $3 \times x$.

5 State the value of the missing number to make the following equations true.

a $5 + \square = 12$ **b** $10 \times \square = 90$ **c** $\square - 3 = 12$ **d** $3 + 5 = \square$

FLUENCY 6–8(½) 6–9(½)

Example 2 Classifying equations involving pronumerals as true or false

If $x = 10$, is the equation $x + 20 = 3 \times x$ true or false?

Solution	Explanation
True	LHS $= x + 20 = 10 + 20 = 30$. RHS $= 3 \times x = 3 \times 10 = 30$. LHS equals RHS, so the equation is true.

6 If $x = 2$, state whether the following equations are true (T) or false (F).

a $x + 4 = 6$ **b** $10x = 5$ **c** $8 = 10 - x$

d $7x = 8 + 3x$ **e** $10 - x = 4x$ **f** $3x = 5 - x$

7 If $a = 3$, state whether the following equations are true (T) or false (F).

a $7 + a = 10$ **b** $2a + 4 = 12$ **c** $8 - a = 5$

d $4a - 3 = 9$ **e** $7a + 2 = 8a$ **f** $a = 6 - a$

8 For each equation below, choose the correct solution from the bubble on the right.

a $x + 12 = 20$ **b** $10x + 5 = 35$ **c** $12 = x + 5$

d $10 + x = 3x + 2$ **e** $3 + 2x = 5$

$x = 7$ $x = 1$ $x = 4$ $x = 3$ $x = 8$

Example 3 Stating a solution to an equation

State a solution to each of the following equations.

a $4 + x = 25$ **b** $5y = 45$

Solution	**Explanation**
a $x = 21$	We need to find a value of x that makes the equation true. If $4 + 21 = 25$ is a true equation, $x = 21$ is a solution.
b $y = 9$	If $y = 9$ then $5y = 5 \times 9 = 45$, so the equation is true.

9 State a solution to each of the following equations.

a $5 + x = 12$ **b** $3 = x - 10$ **c** $4u = 28$

d $17 = p - 2$ **e** $10x = 20$ **f** $77 = 7k$

PROBLEM-SOLVING AND REASONING 10, 11 11–13

Example 4 Writing equations from a description

Write equations for the following.

a The number k is doubled, then three is added and the result is 52.

b Akira works n hours, earning \$12 per hour. The total she earned was \$156.

Solution	**Explanation**
a $2k + 3 = 52$	The number k is doubled, giving $k \times 2$. This is the same as $2k$. If 3 is added, the left-hand side is $2k + 3$, which must be equal to 52 according to the description.
b $12n = 156$	If Akira works n hours at \$12 per hour, the total amount earned is $12 \times n$, or $12n$.

10 ‘A number x is tripled and the result is 12.’ Which of the following equations describes this?

A $x + 3 = 12$ **B** $12x = 3$ **C** $3x = 12$ **D** $12 - x = 3$

11 Write equations to describe the following problems. You do not need to solve the equations.

- **a** The number k is increased by 4 and the result is 20.
- **b** A number x is doubled and then 7 is added.
  The result is 10.
- **c** The sum of x and half of x is 12.
- **d** Fel's height is h cm and her brother Pat is 30 cm taller.
  Pat's height is 147 cm.
- **e** Coffee costs $\$c$ per cup and tea costs $\$3$.
  Four cups of coffee and two cups of tea cost a total of $\$22$.
- **f** Chairs cost $\$c$ each. To purchase 8 chairs and a $\$2000$ table costs a total of $\$3600$.

12 Find the value of the number for the following problems.

- **a** A number is tripled to obtain the result 21.
- **b** Half of a number is 21.
- **c** Six less than a number is 7.
- **d** A number is doubled and the result is 52.

13 Berkeley buys x kg of oranges at $\$3.20$ per kg. He spends a total of $\$9.60$.

- **a** Write an equation involving x to describe this situation.
- **b** State a solution to this equation.

More than one unknown

— 14

14 **a** There are six equations in the square below. Find the values of a, b, c, d and e to make all six equations true.

$$\begin{array}{ccccc} \boxed{a} & + & \boxed{12} & = & \boxed{22} \\ \times & & \div & & - \\ \boxed{2} & \times & \boxed{b} & = & \boxed{c} \\ = & & = & & = \\ \boxed{d} & \div & \boxed{e} & = & \boxed{10} \end{array}$$

b If the four numbers above $(2, 10, 12, 22)$ are doubled, what would the values of a, b, c, d and e become?

7B Solving equations using backtracking

Interactive

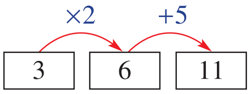
Widgets

HOTsheets

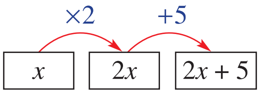
Walkthrough

Backtracking using flowcharts is one way to solve simple equations. An expression can be built up from a single pronumeral.

Performing operations to a number

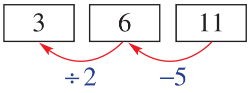


Performing operations to a pronumeral

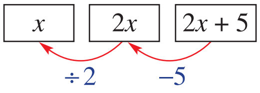


The arrows can also be reversed to break down the expression but the opposite operation is used.

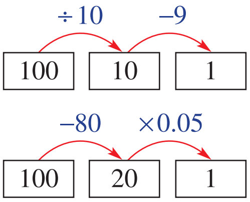
Reversing operations on a number



Reversing operations on a pronumeral



Let's start: One percenter

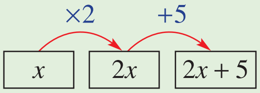


Starting with the number 100, you can get to the number 1 in many different ways (two are shown).

- How could you get from 100 to 1 in just one step?
- Describe how you could do it using 10 steps. Draw a flowchart.

Key ideas

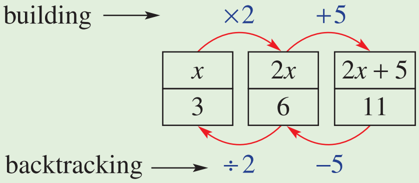
- Expressions like $2x + 5$ can be built from a single pronumeral, like x.



Equations can be solved by reversing the arrows and using the **opposite** operation.

e.g. Equation
$2x + 5 = 11$

Solution
$x = 3$



Operation	Opposite
+3	−3
×5	÷5
−10	+10
÷7	×7

Exercise 7B

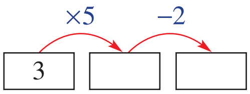
UNDERSTANDING 1–3 3

1 Fill in the gaps with the appropriate word.

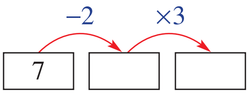
- **a** The opposite of adding 6 is _____ 6.
- **b** The opposite of multiplying by 3 is _____ by 3.
- **c** The opposite of subtracting 20 is _____ 20.
- **d** The opposite of dividing by 12 is _____ by 12.

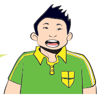
2 Copy and complete the flowcharts below.

a



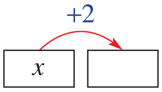
b



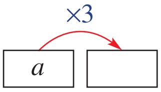


Remember 5 + 2 is written as $3a$.

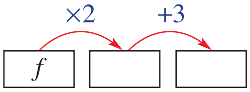
c



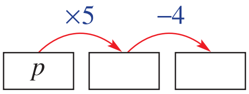
d



e

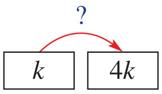


f

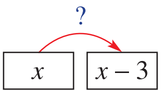


3 State the operation used (e.g. ×4) on the arrow for the flowcharts below.

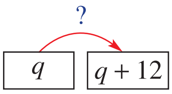
a



b



c

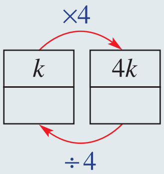


FLUENCY 4–10 6–9, 10–11(½)

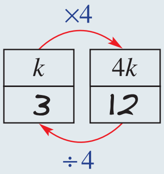
Example 5 Using backtracking to solve simple equations

Use backtracking to solve the equation $4k = 12$.

Solution **Explanation**



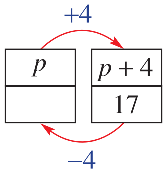
First set up a flowchart for $4k$, showing the opposite operation on the bottom arrow.



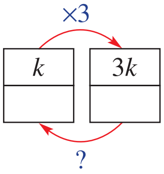
Put the number 12 in below $4k$ and follow the arrow back to find k.

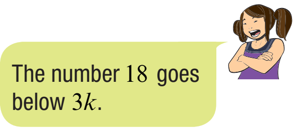
Solution: $k = 3$

4 **a** Copy and complete the flowchart shown for the equation $p + 4 = 17$.
b What is the solution to the equation $p + 4 = 17$?



5 **a** Copy and complete the flowchart below for the equation $3k = 18$.

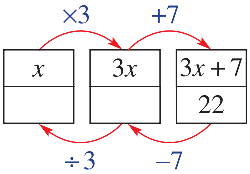




b What is the solution to $3k = 18$?

6 Solve the following equations by first making a flowchart.
a $3k = 30$ **b** $p + 4 = 30$ **c** $r - 12 = 30$
d $5x = 40$ **e** $w \times 12 = 132$ **f** $s \div 3 = 10$

7 **a** Copy and complete the flowchart below for the equation $3x + 7 = 22$.



b What is the solution to $3x + 7 = 22$?

Example 6 Using backtracking to solve two-step equations

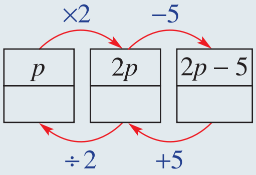
Solve the equation $2p - 5 = 15$ using backtracking.

Solution

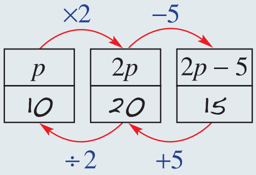
Solution: $p = 10$.

Explanation

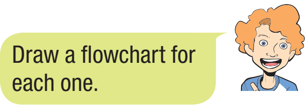
Step 1
Make a flowchart for $2p - 5$



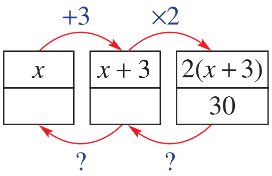
Step 2
Put 15 in and follow the arrows to p



8 Solve the following equations using backtracking.
a $2p - 5 = 25$ **b** $10x + 3 = 43$
c $3q + 7 = 25$ **d** $5r - 11 = 24$
e $6 + 10u = 26$ **f** $3 + 2p = 45$



9 **a** Copy and complete the flowchart below for the equation $2(x + 3) = 30$.



b What is the solution to $2(x + 3) = 30$?

10 Solve the following equations using backtracking.

a $2(x + 5) = 16$ **b** $4(q + 3) = 20$ **c** $3r + 7 = 22$

d $7t - 10 = 39$ **e** $10(s - 20) = 60$ **f** $10s - 20 = 60$

11 The following equations involve negative numbers. Use backtracking to find the solutions.

a $3x = -15$ **b** $p + 10 = 4$ **c** $5x + 12 = -13$

d $4(r - 3) = -20$ **e** $3(n + 40) = 30$ **f** $7u - 10 = -31$

PROBLEM-SOLVING AND REASONING

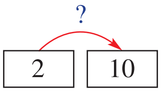
12, 13 13, 14

12 Oliver doubles a number and then adds 7. The result is 59.

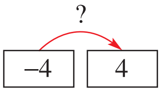
a If x is the number that he started with, draw a flowchart to describe this situation.

b Use backtracking to find the value of x.

13 **a** Give two separate operations that could be used to fill in the question mark below.



b If the same operation can be used for the flowchart below, what is the operation?



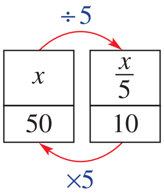
14 **a** Draw flowcharts to solve the equations $2x + 4 = 10$ and $2(x + 4) = 10$.

b Describe how the flowcharts differ from each other.

Fractional flowcharts

— 15

15 Remember that $\frac{x}{5}$ means $x \div 5$. This can be used to solve the equation $\frac{x}{5} = 10$.



Use flowcharts to solve the equations using fractions.

a $\frac{x}{5} = 7$ **b** $\frac{r}{2} = 9$

c $\frac{x}{10} + 4 = 11$ **d** $\frac{y}{3} - 2 = 7$

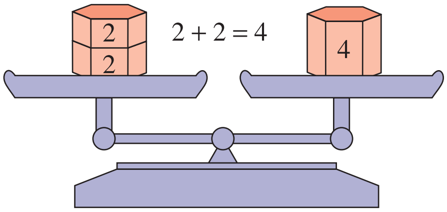
e $2\left(\frac{x}{5}\right) + 3 = 7$ **f** $4\left(\frac{m}{3}\right) - 1 = 7$

g $\frac{x + 4}{5} = 10$ **h** $\frac{r - 3}{2} = 4$

7C Solving equations using the balancing method

Interactive
Widgets
HOTsheets
Walkthrough

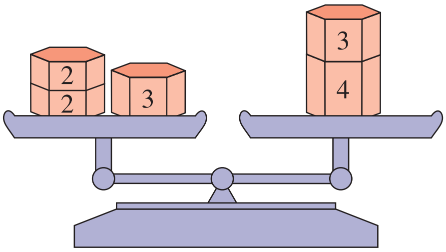
Sometimes it is helpful to think of an equation as two weights balancing on scales.



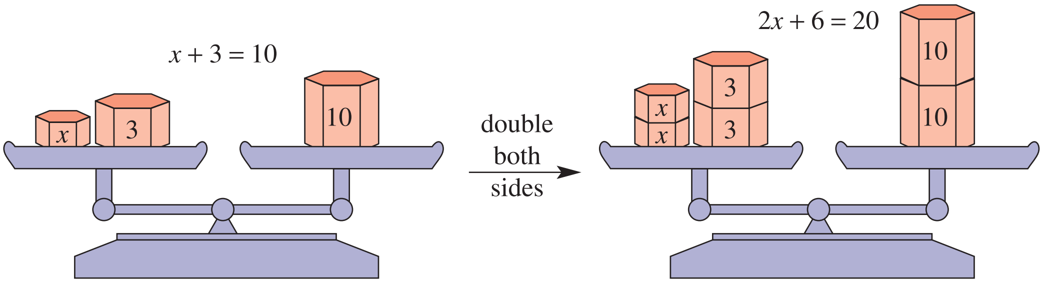
$$2 + 2 = 4$$

If the same weight is added to both sides, the scales still balance.

$$2 + 2 + 3 = 4 + 3$$



We can also subtract a value from both sides, or multiply/divide both sides by the same value, and the scales will still balance.



$$x + 3 = 10$$

$$2x + 6 = 20$$

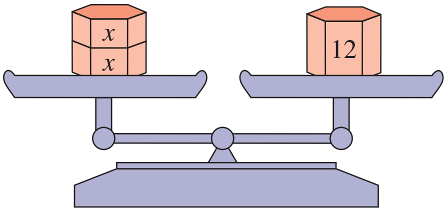
Equations are called **equivalent** if you can get from one to the other by performing the same operations on both sides.

The operations are written next to arrows, like this:

$$\begin{tikzcd} x+3=10 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ 2x+6=20 \end{tikzcd}$$

Let's start: Equivalent equations

- In 60 seconds, write as many equations as you can that are equivalent to $2x = 12$.
- For one equation that you wrote down, show it as a pair of scales like this diagram.



- Show one of them with arrows like this diagram.

$$\begin{tikzcd} 2x=12 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ \_\_ = \_\_ \end{tikzcd}$$

- What is the simplest (smallest) equation that is equivalent to $2x = 12$?

Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side and right-hand sides of the equation
- To solve an equation using the balance method, you should repeatedly find an equivalent equation that is simpler. For example:

$$\begin{tikzcd} 5x+2=32 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 5x=30 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x=6 \end{tikzcd}$$

Check: LHS $= 5 \times 6 + 2$ RHS $= 32$
$= 32$

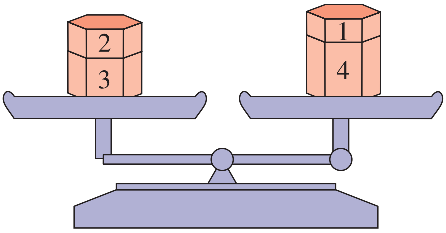
- Check that your solution is correct by substituting into the original equation to see if LHS = RHS.

Exercise 7C

UNDERSTANDING 1–5 5

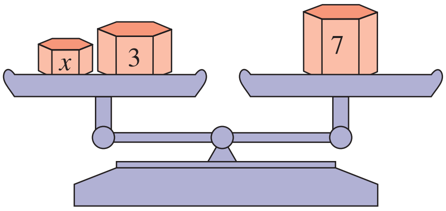
1 Write an equation for each of the balancing scales below.

a

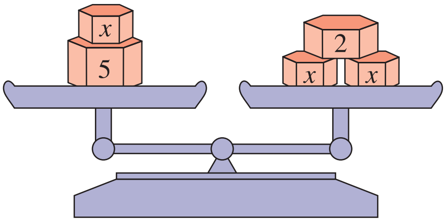


An example could be
$5 + 2 = 6 + 1$ or
$3x + 1 = x + 4$

b



c



2 Write the equivalent equations to $2x = 12$ by filling in the blanks.

a $$\begin{array}{ccc} & 2x = 12 & \\ \times 3 \downarrow & & \downarrow \times 3 \\ & \_\_ = \_\_ & \end{array}$$

b $$\begin{array}{ccc} & 2x = 12 & \\ +1 \downarrow & & \downarrow +1 \\ & \_\_ = \_\_ & \end{array}$$

c $$\begin{array}{ccc} & 2x = 12 & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & \_\_ = \_\_ & \end{array}$$

3 For each equation fill in the blank to get an equivalent equation.

a $$\begin{array}{ccc} & 5x = 10 & \\ +2 \downarrow & & \downarrow +2 \\ & 5x + 2 = \_\_ & \end{array}$$

b $$\begin{array}{ccc} & 10 - 2x = 20 & \\ +5 \downarrow & & \downarrow +5 \\ & 15 - 2x = \_\_ & \end{array}$$

c $$\begin{array}{ccc} & 3q + 4 = 16 & \\ -4 \downarrow & & \downarrow -4 \\ & 3q = \_\_ & \end{array}$$

4 Consider the equation $4x = 32$.

a Copy and complete the following working.

$$\begin{array}{ccc} & 4x = 32 & \\ \div 4 \downarrow & & \downarrow \div 4 \\ & x = \_\_ & \end{array}$$

A solution is a value of x that makes the equation true.

b What is the solution to the equation $4x = 32$?

5 To solve the equation $10x + 5 = 45$, which of the following operations would you first apply to both sides?

A Divide by 5
B Subtract 5
C Divide by 10
D Subtract 45

FLUENCY 6–9(½) 6–10(½)

Example 7 Finding equivalent equations

Show the result of applying the given operation to both sides of these equations.

a $8y = 40\,[\div 8]$ **b** $10 + 2x = 36\,[-10]$ **c** $5a - 3 = 12\,[+3]$

Solution	Explanation
a $\begin{tikzcd} 8y = 40 \arrow[d, bend right=60, "\div 8"'] \arrow[d, bend left=60, "\div 8"] \\ y = 5 \end{tikzcd}$	Write the equation out and then divide both sides by 8. $40 \div 8$ is 5 and $8y \div 8$ is y.
b $\begin{tikzcd} 10 + 2x = 36 \arrow[d, bend right=60, "-10"'] \arrow[d, bend left=60, "-10"] \\ 2x = 26 \end{tikzcd}$	Write the equation out and then subtract 10 from both sides. $36 - 10$ is 26 $10 + 2x - 10$ is $2x$
c $\begin{tikzcd} 5a - 3 = 12 \arrow[d, bend right=60, "+3"'] \arrow[d, bend left=60, "+3"] \\ 5a = 15 \end{tikzcd}$	Write the equation out and then add 3 to both sides. $12 + 3$ is 15 $5a - 3 + 3$ is $5a$

6 For each equation, show the result of applying the given operation to both sides.

a $10 + 2x = 30\,[-10]$ **b** $4 + q = 12\,[-2]$ **c** $13 = 12 - q\,[+5]$

d $4x = 8\,[\times 3]$ **e** $7p = 2p + 4\,[+6]$ **f** $3q + 1 = 2q + 1\,[-1]$

7 Copy and complete the following to solve the given equations using the balancing method.

a
$$\begin{tikzcd} 10x = 30 \arrow[d, bend right=60, "\div 10"'] \arrow[d, bend left=60, "\div 10"] \\ x = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} q + 5 = 12 \arrow[d, bend right=60, "-5"'] \arrow[d, bend left=60, "-5"] \\ \_\_ = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} k - 3 = 8 \arrow[d, bend right=60, "+3"'] \arrow[d, bend left=60, "+3"] \\ \_\_ = \_\_ \end{tikzcd}$$

d
$$\begin{tikzcd} 4x + 2 = 22 \arrow[d, bend right=60, "-2"'] \arrow[d, bend left=60, "-2"] \\ 4x = \_\_ \arrow[d, bend right=60, "\div 4"'] \arrow[d, bend left=60, "\div 4"] \\ \_\_ = \_\_ \end{tikzcd}$$

e
$$\begin{tikzcd} 7p + 2 = 30 \arrow[d, bend right=60, "-2"'] \arrow[d, bend left=60, "-2"] \\ \_\_ = \_\_ \arrow[d, bend right=60, "\square"'] \arrow[d, bend left=60, "\square"] \\ \_\_ = \_\_ \end{tikzcd}$$

f
$$\begin{tikzcd} 10x - 4 = 26 \arrow[d, bend right=60, "+4"'] \arrow[d, bend left=60, "+4"] \\ \_\_ = \_\_ \arrow[d, bend right=60, "\square"'] \arrow[d, bend left=60, "\square"] \\ \_\_ = \_\_ \end{tikzcd}$$

Example 8 Solving equations using the balancing method

Solve the following equations and check your solution using substitution.

a $x-4=16$ **b** $2u+7=17$ **c** $10=3k-11$

Solution | **Explanation**

a

$$\begin{array}{ccc} & x-4=16 & \\ +4 & & +4 \\ & x=20 & \end{array}$$

By adding 4 to both sides of the equation, we get an equivalent equation.
So the solution is $x=20$.

Check: LHS $=20-4$ RHS $=16$
$=16$

Check your solution by substituting $x=20$ into the LHS.

b

$$\begin{array}{ccc} & 2u+7=17 & \\ -7 & & -7 \\ & 2u=10 & \\ \div 2 & & \div 2 \\ & u=5 & \end{array}$$

To get rid of the +7, we subtract 7 from both sides.
Finally, we divide by 2 to reverse the $2u$.
Remember that $2u$ means $2\times u$.
So the solution is $u=5$.

Check: LHS $=2\times 5+7$ RHS $=17$
$=17$

Check your solution using substitution.

c

$$\begin{array}{ccc} & 10=3k-11 & \\ +11 & & +11 \\ & 21=3k & \\ \div 3 & & \div 3 \\ & 7=k & \end{array}$$

First add 11 to 'undo' the -11

Then divide by 3 since $3k$ means $k\times 3$.
Swap the $7=k$ to put the pronumeral first in our solution.
So the solution is $k=7$.

Check: LHS $=10$ RHS $=3\times 7-11$
$=10$

Check that both sides of $10=3k-11$ are equal using $k=7$.

8 Solve the following equations and check your solution using substitution.

a $a+5=8$ **b** $t\times 2=14$ **c** $q-2=7$ **d** $k+2=11$
e $x+9=19$ **f** $3h=30$ **g** $9l=36$ **h** $g\div 3=3$

9 Solve the following equations and check your solution using substitution.

a $9h+5=32$ **b** $9u-6=30$ **c** $5s-2=13$ **d** $3w-6=18$
e $8+5x=28$ **f** $6+10w=56$ **g** $8a-8=8$ **h** $4y-8=40$

10 Solve the following equations and check your solution using substitution.

a $10=5x$ **b** $12=k+7$ **c** $30=x-12$ **d** $5=x\div 4$
e $32=4k+4$ **f** $50=2x-10$ **g** $12=3y-6$ **h** $14=x\div 2+4$

PROBLEM-SOLVING AND REASONING 11, 12 11–14

11 The solutions to the following equations are negative numbers. Solve the equations to find them.

a $x + 10 = 4$ **b** $7a = -21$ **c** $3x + 4 = -26$ **d** $2k + 20 = 10$

e $7 = 2k + 15$ **f** $1 = 7p + 8$ **g** $-2 = p \div 8$ **h** $-3 = 2x + 7$

12 For each of the following, write an equation and solve.

a The sum of p and 8 is 15.

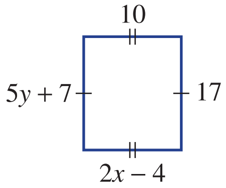
b The product of q and 3 is 12.

c 4 is subtracted from double the value of k and the result is 18.

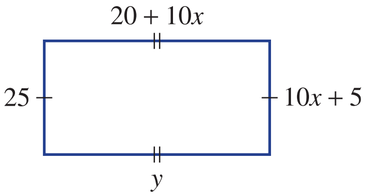
d When r is tripled and 4 is added the result is 34.

13 The following shapes are rectangles. By solving equations, find the value of the variables.

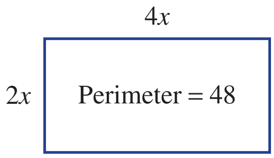
a



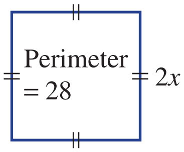
b



c



d



14 Solve the following equations. More than two steps are involved.

a $14 \times (4x + 2) = 140$ **b** $8 = (10x - 4) \div 2$ **c** $3 + (2x + 1) \times 4 = 47$

From solutions to equations — 15

15 A student has taken the equation $x = 5$ and performed some operations to both sides:

$$\begin{tikzcd}
x = 5 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\
4x = 20 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\
4x + 3 = 23 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
(4x + 3) \times 2 = 46
\end{tikzcd}$$

a Solve $(4x + 3) \times 2 = 46$.

b Describe how the steps you used in your solution compare with the steps the student used.

c Give an example of another equation that has $x = 5$ as its solution.

7D Equations with fractions

Interactive

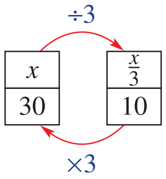
Widgets

HOTsheets

Walkthrough

Recall from algebra that a fraction such as $\frac{x}{3}$ represents $x \div 3$. This means that to solve an equation with $\frac{x}{3}$ on one side, we should first multiply both sides by 3. For example:

$$\begin{tikzcd} \frac{x}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x = 30 \end{tikzcd}$$



Let's start: Practising with fractions

- If $x = 10$, find out what each of these expressions would equal:

$\frac{2x+1}{2}$ $\quad 2\left(\frac{x}{2}+1\right)$ $\quad \frac{2}{x+1}$ $\quad \frac{2+2x}{2}$ $\quad 2\left(x+\frac{1}{2}\right)$

- Which of the above expressions are equal if $x = 0$?

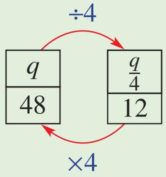
Key ideas

- $\frac{a}{b}$ means $a \div b$.
- To solve an equation with a fraction on one side, multiply both sides by the denominator.

Using equivalent equations

$$\begin{tikzcd} \frac{q}{4} = 12 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ q = 48 \end{tikzcd}$$

Using backtracking



Solution: $q = 48$

Exercise 7D

UNDERSTANDING 1–4 4

1 Which of the following expressions represents 'x divided by 5'?

A $x+5$ **B** $\frac{x}{5}$ **C** $\frac{5}{x}$ **D** $5x$

2 If $x = 20$, state whether the following equations are true (T) or false (F).

a $\frac{x}{4} = 5$ **b** $\frac{x}{2} = 40$ **c** $\frac{x}{5} = 5$ **d** $\frac{x}{10} = 2$

3 **a** If $x = 4$, find the value of $\frac{x}{2} + 6$.

b If $x = 4$, find the value of $\frac{x+6}{2}$.

c Are $\frac{x}{2} + 6$ and $\frac{x+6}{2}$ equivalent expressions?

Expressions are equivalent if they are always equal.

4 Fill in the missing steps to solve these equations.

a
$$\begin{tikzcd} \frac{x}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} \frac{m}{5} = 2 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ m = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} 11 = \frac{q}{2} \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ \_\_ = q \end{tikzcd}$$

d
$$\begin{tikzcd} \frac{p}{10} = 7 \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ p = \_\_ \end{tikzcd}$$

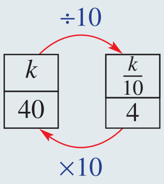
FLUENCY 5–8(½), 9 6–8(½), 9, 10(½)

Example 9 Solving equations with fractions

Solve the following equations.

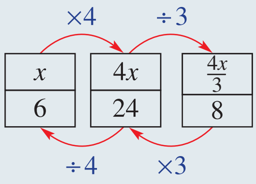
a $\frac{k}{10} = 4$ **b** $\frac{4x}{3} = 8$

Solution	**Explanation**
a $$\begin{tikzcd} \frac{k}{10} = 4 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ k = 40 \end{tikzcd}$$	Multiplying both sides by 10 removes the denominator of 10. Alternatively, backtracking can be used.



The solution is $k = 40$.

Solution	**Explanation**
b $$\begin{tikzcd} \frac{4x}{3} = 8 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 4x = 24 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x = 6 \end{tikzcd}$$	Multiplying both sides by 3 removes the denominator of 3. Both sides are divided by 4 to solve the equation. Alternatively, backtracking can be used.



The solution is $x = 6$.

5 Solve the following equations.

a $\frac{b}{5} = 4$ **b** $\frac{g}{10} = 2$ **c** $\frac{a}{5} = 3$ **d** $\frac{k}{6} = 3$

6 Solve the following equations.

a $\frac{2l}{5} = 8$ **b** $\frac{7w}{10} = -7$ **c** $\frac{3s}{2} = -9$ **d** $\frac{5v}{4} = 15$

e $\frac{3m}{7} = 6$ **f** $\frac{3n}{7} = 6$ **g** $\frac{-6j}{5} = 6$ **h** $\frac{-6f}{5} = -24$

Multiply both sides by a chosen number.

Example 10 Solving more complex equations with fractions

Solve the equation: $\frac{4y+15}{9} = 3$

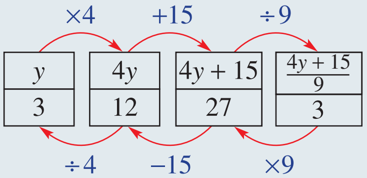
Solution

$$\begin{tikzcd}
\frac{4y+15}{9} = 3 \arrow[d, bend right=60, "\times 9"'] \arrow[d, bend left=60, "\times 9"] \\
4y+15 = 27 \arrow[d, bend right=60, "-15"'] \arrow[d, bend left=60, "-15"] \\
4y = 12 \arrow[d, bend right=60, "\div 4"'] \arrow[d, bend left=60, "\div 4"] \\
y = 3
\end{tikzcd}$$

Explanation

Multiplying both sides by 9 removes the denominator of 9.

The equation $4y+15 = 27$ is solved in the usual fashion (subtract 15, divide by 4).
Alternative solution using backtracking:

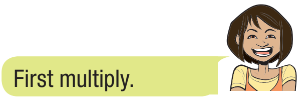


The solution is $y = 3$.

7 Solve the following equations.

a $\frac{t-8}{2} = 10$ **b** $\frac{h+10}{3} = 4$ **c** $\frac{a+12}{5} = 5$

d $\frac{c-7}{2} = 5$ **e** $\frac{s-2}{8} = 1$ **f** $\frac{5j+6}{8} = 2$



Example 11 Solving more equations with fractions

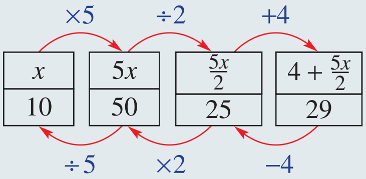
Solve the equation: $4 + \frac{5x}{2} = 29$

Solution

$$\begin{tikzcd}
4 + \frac{5x}{2} = 29 \arrow[d, bend right=60, "-4"'] \arrow[d, bend left=60, "-4"] \\
\frac{5x}{2} = 25 \arrow[d, bend right=60, "\times 2"'] \arrow[d, bend left=60, "\times 2"] \\
5x = 50 \arrow[d, bend right=60, "\div 5"'] \arrow[d, bend left=60, "\div 5"] \\
x = 10
\end{tikzcd}$$

Explanation

We must subtract 4 first because we do not have a fraction by itself on the left-hand side. Once there is a fraction by itself, multiply by the denominator (2).
Alternative solution using backtracking:



The solution is $x = 10$.

8 Solve the following equations.

a $\frac{v}{10} + 3 = 5$ **b** $2 + \frac{x}{4} = 7$ **c** $\frac{y}{2} - 6 = 1$

d $\frac{2x}{5} + 6 = 10$ **e** $\frac{6p}{7} - 4 = 2$ **f** $9 + \frac{3k}{2} = 18$

9 Match each of these equations with the correct first step to solve it.

a $\frac{x}{4}=7$ **b** $\frac{x-4}{2}=5$ **c** $\frac{x}{2}-4=7$ **d** $\frac{x}{4}+4=3$

A Multiply both sides by 2.
B Add 4 to both sides.
C Multiply both sides by 4.
D Subtract 4 from both sides.

10 Solve the following equations.

a $\frac{g-3}{5}=1$ **b** $\frac{2x}{7}=4$ **c** $\frac{k}{3}+1=6$ **d** $\frac{x}{4}=9$ **e** $3=\frac{q}{2}-2$

f $15=\frac{3+x}{2}$ **g** $2=\frac{5p}{15}$ **h** $\frac{2x+7}{3}=3$ **i** $9=\frac{2r}{4}-1$

PROBLEM-SOLVING AND REASONING 11, 12 11–13

11 For the following puzzles, write an equation and solve it to find the unknown number.

a A number x is divided by 5 and the result is 7.
b Half of y is 12.
c A number p is doubled and then divided by 7. The result is 4.
d Four is added to x. This is halved to get a result of 10.
e x is halved and then 4 is added to get a result of 10.
f A number k is doubled and then 6 is added. This result is halved to obtain 14.

12 The average of two numbers can be found by adding them and then dividing the result by 2.

a Find the average of 9 and 5.
b If the average of x and 5 is 12, what is x? Solve the equation $\frac{x+5}{2}=12$ to find out.
c The average of 7 and p is 5. Find p by writing and solving an equation.
d The average of a number and double that number is 18. What is that number?
e The average of $4x$ and 6 is 19. What is the average of $6x$ and 4?

Find x first.

13 A restaurant bill is to be paid. Blake puts in \$40, which is one-third of the amount in his wallet.

a Write an equation to describe this situation, if b represents the amount in Blake's wallet before he pays.
b Solve the equation to find out how much money Blake has in his wallet.

Variable denominators — 14

14 To solve an equation with a variable on the denominator we can first multiply both sides by that variable.

Use this method to solve the equations.

a $\frac{12}{x}=2$ **b** $\frac{15}{x}=5$ **c** $\frac{20}{x}=4$

d $4+\frac{20}{x}=14$ **e** $\frac{16}{x}+1=3$ **f** $\frac{12}{x}=1$

$$\begin{tikzcd}
\frac{30}{x}=10 \arrow[d, bend right=60, "\times x"'] \arrow[d, bend left=60, "\times x"] \\
30=10x \arrow[d, bend right=60, "\div 10"'] \arrow[d, bend left=60, "\div 10"] \\
3=x
\end{tikzcd}$$

7E Equations with brackets

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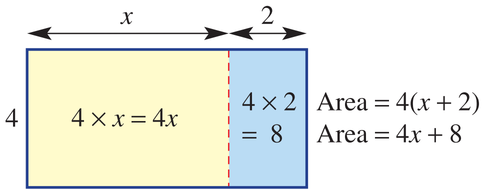
HOTsheets

Walkthrough

In Chapter 5 it was noted that expressions with brackets could be expanded by considering rectangle areas.

So $4(x+2) = 4x+8$



Demonstration that $4x+8$ and $4(x+2)$ are equivalent

Let's start: T-shirts and shorts

Harrison buys two sporting outfits at a shop where shorts cost $5 more than T-shirts.

- If each pair of shorts is $10, how much does the outfit cost?
- If the two outfits cost $60 in total, can you give the cost of each item?
- Try to find an expression for the total cost of the outfits.

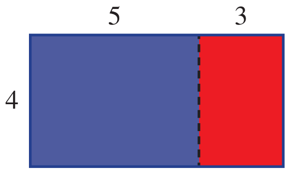
Key ideas

- To expand brackets, use the **distributive law**, which states that
  - $a(b+c) = ab+ac$. e.g. $3(x+4) = 3x+12$.
  - $a(b-c) = ab-ac$. e.g. $4(b-2) = 4b-8$.
- **Like terms** are terms that contain exactly the same pronumerals and can be collected to simplify expressions. For example, $5x+10+7x$ can be simplified to $12x+10$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

Exercise 7E

UNDERSTANDING 1–5 5

1 The rectangle shown has a blue region and a red region.
  - **a** What is the blue area?
  - **b** What is the red area?
  - **c** What is the total area?
  - **d** True (T) or False (F)? $4(5+3) = 4 \times 5 + 4 \times 3$.



Example 12 Expanding brackets

Expand the brackets for:

a $2(5x + 3)$ **b** $3(k - 4)$

Solution **Explanation**

a $2(5x + 3) = 10x + 6$ $2 \times 5x = 10x$ and $2 \times 3 = 6$

b $3(k - 4) = 3k - 12$ $3 \times k = 3k$ and $3 \times 4 = 12$

Note the minus sign remains

2 Fill in the missing numbers.

a $4(y + 3) = 4y + \boxed{}$

b $7(2p - 5) = \boxed{}p - 35$

c $2(4x + 5) = \boxed{}x + \boxed{}$

d $10(5 + 3q) = \boxed{} + \boxed{}q$

3 Match each expression (**a**–**d**) with its expanded form (**A**–**D**).

a	$2(x + 4)$	**A**	$4x + 8$
b	$4(x + 2)$	**B**	$2x + 4$
c	$2(2x + 1)$	**C**	$2x + 8$
d	$2(x + 2)$	**D**	$4x + 2$

4 If $x = 5$, state whether the following equations are true (T) or false (F).

a $3(x + 1) = 18$ **b** $4(x - 2) = 16$ **c** $2(2x + 1) = 22$ **d** $5(x - 1) = 20$

5 Rolf is unsure whether $4(x + 3)$ is equivalent to $4x + 12$ or $4x + 3$.

a Fill out the table below.

x	0	1	2
$4(x + 3)$			
$4x + 12$			
$4x + 3$			

b What is the correct expansion of $4(x + 3)$?

FLUENCY 6, 7(½) 6-8(½)

6 Simplify the following expressions by collecting like terms.

a $4x + 3x$ **b** $7p + 2p + 3$ **c** $8x - 2x + 4$

d $2k + 4 + 5k$ **e** $3x + 6 - 2x$ **f** $7k + 21 - 2k$

Like terms have the same pronumerals.

Example 13 Solving equations with brackets

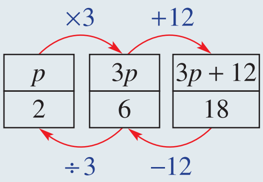
Solve the following equations by first expanding any brackets.

a $3(p+4) = 18$

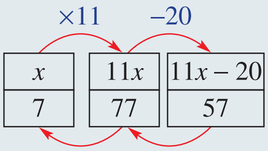
b $4(2x-5)+3x = 57$

Solution	**Explanation**
a $3(p+4)=18$ $3p+12=18$ -12 $3p=6$ $\div 3$ $p=2$	Use the distributive law to expand the brackets. Alternative solution using backtracking: The solution is $p = 2$
b $4(2x-5)+3x=57$ $8x-20+3x=57$ $11x-20=57$ $+20$ $11x=77$ $\div 11$ $x=7$	Use the distributive law to expand the brackets. Combine the like terms: $8x + 3x = 11x$. Alternative solution using backtracking: The solution is $x = 7$

$$\begin{array}{rcl} & 3(p+4) = 18 & \\ -12 & 3p+12 = 18 & -12 \\ \div 3 & 3p = 6 & \div 3 \\ & p = 2 & \end{array}$$



$$\begin{array}{rcl} & 4(2x-5)+3x = 57 & \\ & 8x-20+3x = 57 & \\ +20 & 11x-20 = 57 & +20 \\ \div 11 & 11x = 77 & \div 11 \\ & x = 7 & \end{array}$$



7 Solve the following equations by first expanding the brackets.

a $4(x+1)=24$ **b** $3(k+5)=18$ **c** $2(r-7)=20$

d $2(4u+2)=52$ **e** $3(3j-4)=15$ **f** $5(2p-4)=40$

g $15=5(2m-5)$ **h** $2(5n+5)=60$ **i** $26=2(3a+4)$

8 Solve the following equations by expanding and combining like terms.

a $2(x+3)+x=30$ **b** $3(x-1)+2x=47$

c $5(r-2)+r=50$ **d** $4(3y+2)+2y=50$

e $5(2l-5)+3l=1$ **f** $4(5+3w)+5=49$

g $49=5(3c+5)-3c$ **h** $28=4(3d+3)-4d$

i $58=4(2w-5)+5w$ **j** $23=4(2p-3)+3$

k $44=5(3k+2)+2k$ **l** $49=3(2c-5)+4$

First expand then solve using backtracking or using the balancing method.

PROBLEM-SOLVING AND REASONING 9, 10 10–12

9 A number is increased by 5 and then the result is doubled.

a If the number is n, write an expression for the final result.

b If the final result equals 40, which of the following equations describes this?

A $n+5 \quad \times 2 = 40$ **B** $2(n+5)=40$ **C** $2n+5=40$ **D** $40(n+2)=5$

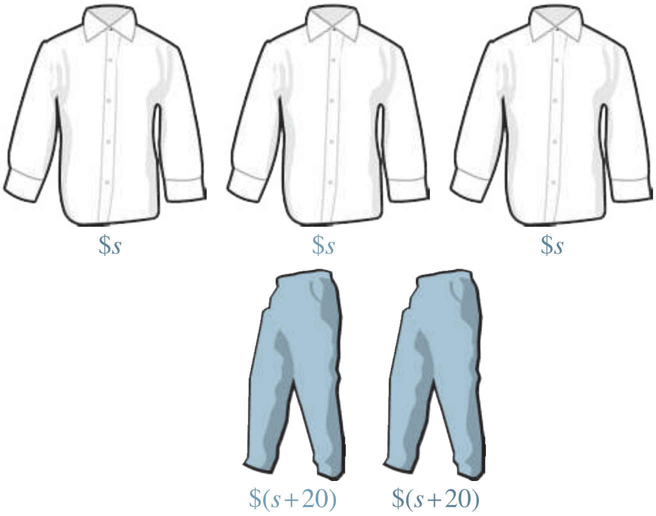
c What was the original number?

10 Desmond notes that in 4 years' time his age when doubled will give the number 50. Desmond's current age is d.

- **a** Write an expression for Desmond's age in 4 years' time.
- **b** Write an expression for double his age in 4 years' time.
- **c** Write an equation to describe the situation described above.
- **d** Solve the equation to find his current age.

11 Amos buys 3 shirts and 2 pairs of trousers for a total of \$225. Each pair of trousers costs \$20 more than a shirt.

- **a** Explain why the total cost is $3s + 2(s + 20)$ if $\$s$ is the cost of one shirt.
- **b** Solve the equation $3s + 2(s + 20) = 225$
- **c** How much does one shirt cost?
- **d** How much does one pair of trousers cost?
- **e** What would the total cost be for 5 shirts and 3 pairs of trousers?



12 Rahda's usual hourly wage is $\$w$. She works for 5 hours at this wage and then 3 more hours at an increased wage of $\$(w + 4)$.

- **a** Write an expression for the total amount Rahda earns for the 8 hours.
- **b** Rahda earns \$104 for the 8 hours. Write and solve an equation to find her usual hourly wage.

Negative brackets

— 13

13 The following equations involve negative numbers. Use the methods from the previous page to solve them.

- **a** $2(x + 1) = -10$
- **b** $3(p - 2) = -18$
- **c** $10(q + 9) = -100$
- **d** $-2(r + 1) = -10$
- **e** $-5(r + 6) = -40$
- **f** $2(x + 5) = -12$
- **g** $3(k + 1) + k = -37$
- **h** $-10(s - 5) = 50$

$$-10(s - 5) = -10s + 50$$
$$(-10 \times -5)$$

Progress quiz

7A **1** If $x = 8$, is the equation $2x + 22 = 5x - 4$ true (T) or false (F)?

7A **2** State a solution to each of the following equations.

a $18 - x = 13$
b $7x = 56$
c $x + 6 = 40$

7A **3** Write equations for the following.

a The number t is tripled, then four is added and the result is 19.
b Jack sleeps for n hours on week nights and 8 hours on weekend nights. The total he sleeps in a week is 61 hours.

7B **4** Solve the following equations using backtracking.

a $4f + 7 = 27$
b $5k + 19 = 34$
c $5 + 3p = 65$
d $7d - 12 = 37$

7B **5** Jennifer quadruples a number and then adds 5. The result is 49.

a If x is the number that she started with, draw a flowchart to describe this situation.
b Use backtracking to find the value of x.

7C **6** Solve the following equations.

a $m - 12 = 3$
b $h + 21 = 40$
c $3b - 6 = 39$
d $99 = 11g - 22$

7C **7** The solutions to the following equations are negative numbers. Solve the equations to find them.

a $p + 14 = 9$
b $j + 100 = 40$
c $4b - 4 = -12$
d $-8 = 3y + 7$

7D **8** Solve the following equations.

a $\frac{x}{5} = 20$ **b** $\frac{g}{11} = 8$ **c** $\frac{3f}{5} = 12$ **d** $\frac{2g}{-5} = -4$

7D **9** Solve the following equations.

a $\frac{p+11}{3} = 5$ **b** $\frac{3p-4}{2} = 7$ **c** $\frac{p}{5} - 3 = 10$ **d** $7 + \frac{2p}{5} = 19$

7E **10** Expand the brackets for:

a $3(6y - 4)$
b $11(y - 7)$
c $(8 + 3y)5$

7E **11** Solve the following equations by first expanding the brackets.

a $5(x - 3) = 10$
b $3(3n + 12) = 63$
c $71 = 8(2d - 6) + 7$

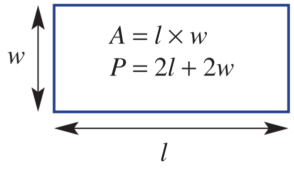
7F Formulas and relationships ★

Formulas occur in many areas of maths and science.
The famous formula $E = mc^2$ relates to energy, mass and the speed of light.
Formulas are a special type of equation that relate to two or more variables.

Let's start: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = l \times w$ and $P = 2l + 2w$.

- If $l = 10$ and $w = 7$ find the perimeter and the area.
- If $l = 2$ and $w = 8$ find the perimeter and the area.
- Notice that sometimes the area is bigger than the perimeter and sometimes the number for the area is less than the number for the perimeter. If $l = 10$, is it possible to make the numbers for the area and the perimeter equal?
- If $l = 2$ can you make the numbers for the area and the perimeter equal? Discuss.



Two formulas used in measurement

Key ideas

- The **subject** of an equation is a pronumeral (or variable) that occurs by itself on the left-hand side, e.g. V is the subject of $V = 3x + 2y$.
- A **formula** or **rule** is an equation containing two or more variables, one of which is the subject of the equation.
- To use a formula, substitute all the known values and then solve the equation to find the final value.

Formula A general rule for finding the value of one quantity given the values of others

Exercise 7F

UNDERSTANDING 1–5 5

1 Fill in the blanks: Choose from: *area*, *formula* or *subject*.

  a A _________ or rule is an equation relating two or more variables.

  b A variable by itself on the left-hand side of an equation is called the _________.

  c The formula $A = l \times w$ is used to find the _________ of a rectangle.

2 **a** Substitute $x = 4$ into the expression $x + 7$.

  b Substitute $a = 2$ into the expression $3a$.

  c Substitute $p = 5$ into the expression $2p - 3$.

  d Substitute $r = -4$ into the expression $7r$.

3 If you substitute $l = 5$ and $w = 3$ into the formula $A = l \times w$, which of the following equations would you get?

A $A = 5 + 3$ **B** $A = 53$ **C** $A = 5 \times 3$ **D** $A = 5 - 3$

4 If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?

A $10 = 6 + x$ **B** $10 = 3m + 2$ **C** $2 = 3m + 10$ **D** $P = 30 + 2$

5 If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?

A $12 = 40 + Q$ **B** $L = 40 + 12$ **C** $12 = 410 + Q$ **D** $10 = 48 + Q$

FLUENCY 6–9 6–10

Example 14 Applying a formula

Apply the formula for a rectangle's perimeter $P = 2l + 2w$ to find:

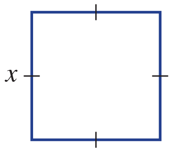
a P when $l = 4$ and $w = 7$ **b** l when $P = 40$ and $w = 3$

Solution	**Explanation**
a $P = 2l + 2w$	Write the formula.
$P = 2 \times 4 + 2 \times 7$	Substitute in the values for l and w.
$P = 22$	Simplify the result.
b $P = 2l + 2w$	Write the formula.
$40 = 2l + 2 \times 3$	Substitute in the values for P and w to obtain an equation.
$40 = 2l + 6$ $-6 \quad 34 = 2l \quad -6$ $\div 2 \quad 17 = l \quad \div 2$ $\therefore \; l = 17$	Solve the equation to obtain the value of l.

6 Look at the rule $A = 4p + 7$.

a Find A if $p = 3$. **b** Find A if $p = 11$. **c** Find A if $p = 0$. **d** Find A if $p = 100$.

7 The perimeter of a square is given by $P = 4x$, where x is the width.



a Find the value of P if x is:

i 10 **ii** 3 **iii** 7.5

b Solve the equation $44 = 4x$.

c If $P = 44$, what is the width of the square?

8 Look at the rule $U = 8a + 4$.

a Find the value of a if $U = 20$. Set up and solve an equation.

b Find a if $U = 44$. Set up and solve an equation.

c Find a if $U = 92$. Set up and solve an equation.

9 Look at the relationship $y = 2x + 4$.

a Find y if $x = 3$.

b By solving an appropriate equation, find the value of x that makes $y = 16$.

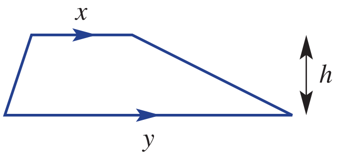
c Find the value of x if $y = 0$.

Your answer will be a negative number.

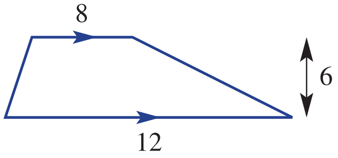
10 Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

PROBLEM-SOLVING AND REASONING 11, 12 11, 13

11 The formula for the area of a trapezium is $A = \frac{h(x + y)}{2}$.

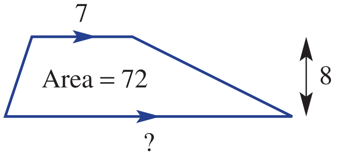


a Find the area of the trapezium shown below.



b Find the value of h if $A = 20$, $x = 3$ and $y = 7$.

c Find the missing value in the trapezium shown below.



12 The cost $\$C$ to hire a taxi for a trip of length d km is $C = 3 + 2d$.

a Find the cost of a 10 km trip (i.e. for $d = 10$).

b A trip has a total cost of $\$161$.

i Set up an equation by substituting $C = 161$.

ii Solve the equation algebraically.

iii How far did the taxi travel? (Give your answer in km.)

13 Look at the rule $G = 120 - 4p$.

a If p is between 7 and 11, what is the largest value of G?

b Is it possible to make G equal to zero? What would p equal?

7F

Mobile phone plans

— 14

14 Two companies have pre-paid mobile phone plans where the cost of a call depends on how much time (t minutes) you talk for.

Company A's cost in dollars: $A = 0.1 + 0.05t$

Company B's cost in dollars: $B = 0.06t$

Use trial and error to solve this equation.

a Find the cost of a 20-minute call with each company.

b If company A charged $\$0.30$ for a call, how long did it take?

c If company B charged $\$0.30$ for a call, how long did it take?

d How long would a call have to be if the cost for company A and company B is the same?

7G Applications

Interactive
Widgets
HOTsheets
Walkthrough

Most problems in science, engineering, finance and other fields can be solved by setting up and solving equations. One critical element of the process is to define the unknowns and then set up an appropriate equation.

Let's start: Sibling sum

John and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 32, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if x is used for John's age?
- How would the equation change if the **product** of their ages is 32?

Problems involving two people's ages can be expressed as an equation.

Key ideas

- An equation can be used to describe any situation in which two values are equal.
- To solve a problem follow these steps.
  1. Define variables to stand for unknown numbers.
  2. Write an equation to link the facts to the question.
  3. Solve the equation if possible.
  4. Make sure you answer the original question, including the correct units (e.g. dollars, years, cm).

1 Let $a =$ John's current age.

2 $a + a + 4 = 32$

3

$$\begin{array}{rcl} & 2a + 4 = 32 & \\ -4 & & -4 \\ & 2a = 28 & \\ \div 2 & & \div 2 \\ & a = 14 & \end{array}$$

4 John is 14 years old.

Exercise 7G

UNDERSTANDING 1–4 4

1 For each of the following, choose the best variable definition to start solving the problem.

a Frank grew by 10 cm and is now 107 cm. How tall was Frank last year?

A Let $f =$ Frank.
B Let $f =$ Frank's height this year.
C Let $f =$ Frank's age.
D Let $f =$ Frank's height last year.

b Waleed worked for 20 hours and earned $300. How much does he earn per hour?

A Let $w =$ Waleed's height.
B Let $w = 300$.
C Let $w =$ Waleed's hourly wage.
D Let $w = 20$.

c Louise spent $400 on 12 identical calculators for her class. How much does a calculator cost?

A Let $c =$ cost of one calculator.
B Let $c =$ number of calculators.
C Let $l =$ Louise.
D Let $l =$ Louise's income.

2 Match each of these worded descriptions with an appropriate expression.

a	The sum of x and 3	**A**	$2x$
b	The cost of 2 apples if they cost $\$x$ each	**B**	$x+1$
c	The cost of x oranges if they cost $\$1.50$ each	**C**	$3x$
d	Triple the value of x	**D**	$x+3$
e	One more than x	**E**	$1.5x$

3 For the following problems choose the equation to describe them.

- **a** The sum of x and 5 is 11.
  - **A** $5x = 11$ **B** $x + 5 = 11$ **C** $x - 5 = 11$ **D** $11 - 5$
- **b** The cost of 4 pens is \$12. Each pen costs $\$p$.
  - **A** $4 = p$ **B** $12p$ **C** $4p = 12$ **D** $12p = 4$
- **c** Josh's age next year is 10. His current age is j.
  - **A** $j + 1 = 10$ **B** $j = 10$ **C** 9 **D** $j - 1 = 10$
- **d** The cost of n pencils is \$10. Each pencil costs \$2.
  - **A** $n \div 10 = 2$ **B** 5 **C** $10n = 2$ **D** $2n = 10.$

4 Solve the following equations.

- **a** $5p = 30$
- **b** $5 + 2x = 23$
- **c** $12k - 7 = 41$
- **d** $10 = 3a + 1$

FLUENCY 5–8 7–9

5 The combined age of twin girls is 26. Let a = the age of one girl.

- **a** Solve the equation $a + a = 26$.
- **b** How old is each girl?

Example 15 Solving a problem using equations

The weight of 6 identical books is 1.2 kg. What is the weight of one book?

Solution	Explanation
Let b grams = weight of one book. $6b = 1.2$	**1** Define a variable to stand for the unknown number. **2** Write an equation to link the facts in the question.
$\div 6\left(\begin{aligned} 6b &= 1.2 \\ b &= 0.2 \end{aligned}\right)\div 6$	**3** Solve the equation.
The books weigh 0.2 kg each, or 200 g each.	**4** Answer the original question. It is not enough to give a final answer as 0.2; this is not the weight of a book, it is just a number.

Remember to include the \$ sign in answer.

6 Jerry buys 4 cups of coffee for \$14.

- **a** Choose a variable to stand for the cost of one cup of coffee.
- **b** Write an equation to link the facts to the question.
- **c** Solve the equation.
- **d** What is the cost of one cup of coffee?

7 A plumber charges a \$70 call-out fee and \$80 per hour. The total cost of a particular visit was \$310.

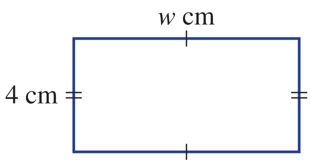
- **a** Define a variable to stand for the length of the visit in hours.
- **b** Write an equation to link the facts to the question.
- **c** Solve the equation.
- **d** What is the length of the plumber's visit?

8 When 6 chairs are bought, a "bulk buy" discount reduces the final price by \$200. The total becomes \$1300.

- **a** Define a variable for the cost of one chair.
- **b** Write an equation to link the facts to the question.
- **c** Solve the equation.
- **d** What is the cost of one chair?

9 The perimeter of this rectangle is 72 cm. Let w cm be the width.

- **a** Write an equation using the given diagram.
- **b** Solve the equation.
- **c** What is the width of the rectangle?

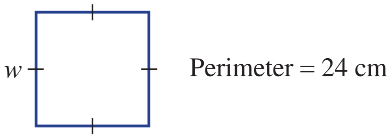


PROBLEM-SOLVING AND REASONING

10, 11 | 11, 12

10 A square has a perimeter of 24 cm.

- **a** Solve an equation to find its width.
- **b** What is the area of the square?



Example 16 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

Solution	Explanation
Let l = Luke's age.	**1** Define a variable for the unknown. Once Luke's age is found, we can double it to find Jane's age.
$l + 2l = 60$	**2** Write an equation to link the facts in the question. Note that Jane's age is $2l$ because she is twice as old as Luke.
$\begin{tikzcd} 3l = 60 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ l = 20 \end{tikzcd}$	**3** Solve the equation by first combining like terms.
Luke is 20 years old and Jane is 40 years old.	**4** Answer the original question. Include units and write a sentence answer.

11 Alison and Flynn's combined age is 40. Flynn is 4 years older than Alison.

- **a** Define a variable, write an equation and solve it to find Alison's age.
- **b** How old is Flynn?

Add the units in your final answer

12 The length of a rectangular pool is 5 metres longer than the width. The perimeter of the pool is 58 metres.

- **a** Draw a diagram of this situation.
- **b** Use an equation to find the pool's width.
- **c** What is the area of the pool?

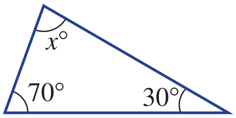
Equational geometry

—	13

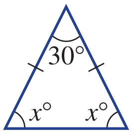
13 The sum of angles in a triangle is 180° and the sum of angles in a quadrilateral is 360°. Find the value of x in the shapes below by first solving an equation.

$x + 70 + 30 = 180$

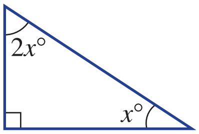
a



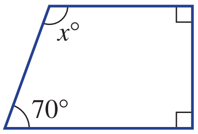
b



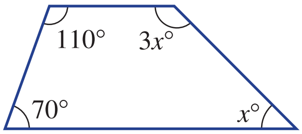
c



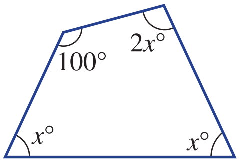
d



e



f



Maths@Work: Financial officers at a local council

The financial officers who are employed by local councils need to follow instructions and work as part of a team, understand accounts, use Excel spreadsheets and pay attention to detail. They require mathematical skill with the equations and formulas that councils use to calculate rates.

All Australian property owners pay rates to their local council for services such as waste management, public transport, roads and bridges, parks, libraries and future planning. The rates, (i.e. a \$ amount), that councils charge, vary with the value and category of each property. Each local council determines the rates due by using an equation that calculates the revenue (i.e. total money collected) needed to cover its expenditure (i.e. the money spent by a council).

Councils supply rate-payers with yellow lid recycling bins.

1 'Rate in the dollar' is cents/\$ of property value that councils charge in rates. For the following property values, calculate the council rates p.a. using 'Rate in the dollar' $= 0.46$ cents/\$.

a $\$350\ 000$

b $\$425\ 000$

c $\$784\ 000$

Rates calculation

$= \$\text{property} \times c / \$ \div 100$
$= 350\,000 \times 0.46 \div 100$
$= \$1610$

2 Local councils can use CIV for the property values when calculating the annual rates payable.
CIV stands for the Capital Improved Value and is the total market value of the land and buildings and any improvements. One council uses the following equation to calculate its rates:

$$\text{RATES } (\$) = \text{CIV} \times \text{Rate in the dollar}$$

$$\text{RATES } (\$) = \text{CIV} \times 0.5739 \text{ cents/\$}$$

a Use this equation to find the council rates on properties with CIV of:

i $\$500\,000$ **ii** $\$560\,000$ **iii** $\$675\,000$

iv $\$750\,000$ **v** $\$1\ 000\ 000$

b Write an equation to calculate quarterly rates, Q, payable from the annual rates, R.

3 Another council calculates its rates using:

- 'Rate in the dollar' $= 0.482075$ cents/\$, charged on land value, LV.
- plus, a fixed charge of \$456.30 for domestic waste services.

a Write an equation for the rates, R, payable on a property with land value, LV.

b Find the annual rates payable on a property with land value of \$345 000.

c Thomas pays \$428.70/quarter. Using your formula, find the current land value, to the nearest \$, for Thomas' property.

4 A different system of rate calculation uses the following equation:

$$\text{Annual Rates}(\$) = AUV \times P + \$765$$

AUV in \$ Average Unimproved Value of land	P Percentage rating factor
1–150 000	0.2746%
151 000–300 000	0.39%
300 001–450 000	0.48%
450 001–600 00	0.54%
600 001 and over	0.575%

Calculation example
Annual Rates
$= AUV \times P + 765$
$= 26\,0000 \times 0.0039 + 765$
$= \$1779$

a Calculate the annual rates payable to council on a property with

i $AUV = \$295\,000$ **ii** $AUV = \$547\,000$

b Write an equation for annual rates payable for a property with $AUV = \$x$, where x lies between \$450 001 and \$600 000.

c If a property has $AUV = \$500\,000$, use your equation from part **b** to find the annual rates payable on this property.

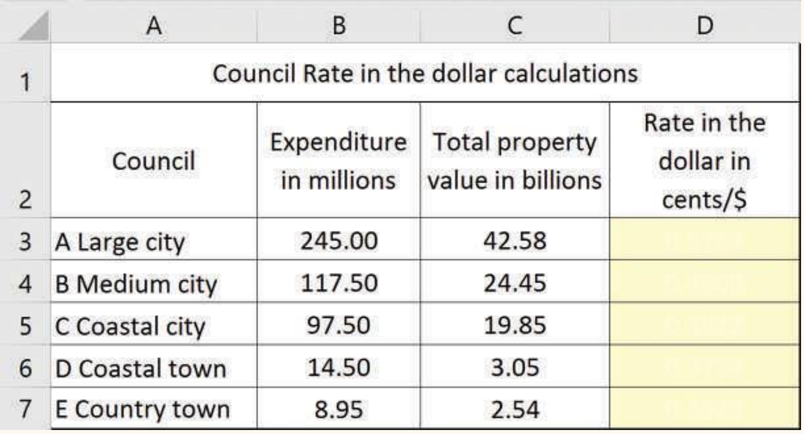
d Calculate the quarterly rates on a property with $AUV = \$700\,000$.

e Write an equation that can be used to find the quarterly rates payable on a property with $AUV = \$y$ where y is over \$600 001.

Using technology

5 Each local council in Australia determines, in advance, its total expenditure for the next financial year. The council must collect enough revenue from rates to pay for all the expenses.

- **a** Set up the following Excel spreadsheet. Format columns B and C to Number (2 decimal places) and column D to Number (4 decimal places)
- **b** Enter formulas to calculate Rate in the dollar in cents/\$. It equals the Expenditure divided by the Total of property values.



	A	B	C	D
1	Council Rate in the dollar calculations			
2	Council	Expenditure in millions	Total property value in billions	Rate in the dollar in cents/\$
3	A Large city	245.00	42.58	
4	B Medium city	117.50	24.45	
5	C Coastal city	97.50	19.85	
6	D Coastal town	14.50	3.05	
7	E Country town	8.95	2.54	

In the column D formulas: Use brackets and multiply millions by $1\,000\,000$, billions by $1\,000\,000\,000$, and multiply by 100 to give the final answer in cents/\$.

The Gold Coast City Council uses rates revenue to run trams as a public transport service.

Puzzles and games

1 Find the value of □, Δ and ○ using the following clues.

- □ × Δ = 24
- ○ + ○ + ○ + ○ = 36
- □ − Δ = ○ + 1
- □ + Δ + Δ + Δ = ○ + ○

2 Find the unknown value in the following puzzles.

a A number is halved, then halved again, then halved again. The result is 11.

b A number is tripled, then it is added to its original value. The result is 24.

c A number is increased by 2, then doubled, then increased by 3 and then tripled. The result is 99.

d The price of a shirt is increased by 10% for GST and then decreased by 10% on a sale. The new price is \$44. What was the original price?

e The average of a number and double that number is 50.

3 Consider the following solution that appears to show that $0 = 1$.

$$\begin{tikzcd} 2x+5 \arrow[d, bend right, "-5"'] & = & 3x+5 \arrow[d, bend left, "-5"] \\ 2x \arrow[d, bend right, "\div x"'] & = & 3x \arrow[d, bend left, "\div x"] \\ 2 \arrow[d, bend right, "-2"'] & = & 3 \arrow[d, bend left, "-2"] \\ 0 & = & 1 \end{tikzcd}$$

a Which step caused the problem in this working? (Hint: Consider the actual solution to the equation.)

b Prove that $0 = 1$ is equivalent to the equation $22 = 50$ by adding, subtracting, multiplying and dividing both sides.

4 Consider the expressions below.

$4x + 2 \qquad 2(x + 4) \qquad 2x + 4 \qquad 4(x + 2) \qquad 4\left(x + \frac{1}{2}\right)$

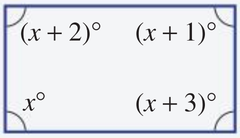
a If $x = 0$, which pairs are equal?

b Use two of the expressions above to form an equation that is always true.

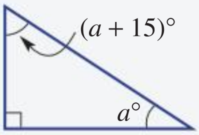
c Use two of the expressions to form an equation that is never true.

5 Find the unknown in these geometric figures.

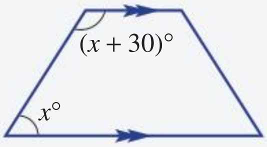
a



b



c



6 A certain pair of scales only registers weights between 100 kg and 150 kg, but it allows more than one person to get on at a time.

a If three people weigh themselves in pairs and the first pair weighs 117 kg, the second pair weighs 120 kg and the third pair weighs 127 kg, what are their individual weights?

b If another three people weigh themselves in pairs and get weights of 108 kg, 118 kg and 130 kg, what are their individual weights?

c A group of four children who all weigh less than 50 kg, weigh themselves in groups of three, getting the weights 122 kg, 128 kg, 125 kg and 135 kg. How much do they each weigh?

Chapter summary

Backtracking

$3x = 12$

$$\begin{tikzcd} x \arrow[r, bend left, "\times 3"] & 3x \\ 4 & 12 \arrow[l, bend left, "\div 3"] \end{tikzcd}$$

Solution: $x = 4$

$4k + 6 = 42$

$$\begin{tikzcd} k \arrow[r, bend left, "\times 4"] & 4k \arrow[r, bend left, "+6"] & 4k + 6 \\ 9 & 36 \arrow[l, bend left, "\div 4"] & 42 \arrow[l, bend left, "-6"] \end{tikzcd}$$

Solution: $k = 9$

Solving equations

- Choose variables to make equation true

e.g. $x + 5 = 12$

Solution: $x = 7$

Balancing

- Same operation to both sides

$$\begin{tikzcd} 3x = 12 \arrow[d, "\div 3"'] \arrow[d, phantom, "\div 3", shift left=8] \\ x = 4 \end{tikzcd}$$

$$\begin{tikzcd} 4k + 6 = 42 \arrow[d, "-6"'] \arrow[d, phantom, "-6", shift left=8] \\ 4k = 36 \arrow[d, "\div 4"'] \arrow[d, phantom, "\div 4", shift left=8] \\ k = 9 \end{tikzcd}$$

Checking solutions

Put variables in to see if equation is true.

Is $x = 3$ a solution to

$4x + 2 = 14$?

$4 \times 3 + 2 = 14 \rightarrow$ true

Equations

- A statement that two values are equal

e.g. $2 + 2 = 4$

LHS Equal sign RHS

Equations with fractions

- Multiply by denominator

$$\begin{tikzcd} \frac{x}{4} = 10 \arrow[d, "\times 4"'] \arrow[d, phantom, "\times 4", shift left=8] \\ x = 40 \end{tikzcd}$$

$$\begin{tikzcd} \frac{k+3}{7} = 9 \arrow[d, "\times 7"'] \arrow[d, phantom, "\times 7", shift left=8] \\ k + 3 = 63 \arrow[d, "-3"'] \arrow[d, phantom, "-3", shift left=8] \\ k = 60 \end{tikzcd}$$

Formulas ⭐

- Equations with 2 or more variables, one as the subject on the LHS.

e.g. $F = ma$

$S = 2x + 3$

- Substitute known values to find unknowns

Equations with brackets ⭐

Expand using distributive law

$2(x + 4) = 14$

$$\begin{tikzcd} 2x + 8 = 14 \arrow[d, "-8"'] \arrow[d, phantom, "-8", shift left=8] \\ 2x = 6 \arrow[d, "\div 2"'] \arrow[d, phantom, "\div 2", shift left=8] \\ x = 3 \end{tikzcd}$$

Combine like terms after expanding.

$4(x + 3) + 2x$ becomes

$4x + 12 + 2x \Rightarrow 6x + 12$

Applications

Whenever two things are equal

1. Define variables	Let \$$c$ = car cost
2. Write equation	$2c = 60000$
3. Solve equation	$c = 30000$
4. Answer question	A car costs \$30 000.

Multiple-choice questions

7A **1** If $x = 3$, then the value of $2x + 5$ is:

A 28 **B** 11 **C** 7 **D** 25 **E** 1

7A **2** If $a = 10$, which one of the following equations is true?

A $a + 5 = 10$ **B** $10 - a = 20$ **C** $a + a = 20$

D $3 = a - 5$ **E** $10 = a + 10$

7A **3** Which one of the following equations does not have the solution $x = 9$?

A $4x = 36$ **B** $x + 7 = 16$ **C** $\frac{x}{3} = 3$

D $x + 9 = 0$ **E** $14 - x = 5$

7B **4** The solution to the equation $6 = 2x$ is:

A $x = 12$ **B** $x = 3$ **C** $x = 6$ **D** $x = 4$ **E** $x = 8$

7C **5** The solution to the equation $3a + 8 = 29$ is:

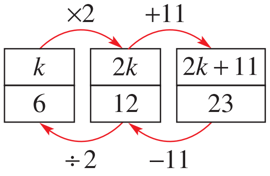
A $a = 21$ **B** $a = 12\frac{1}{3}$ **C** $a = 7$ **D** $a = 18$ **E** $a = 3$

7D **6** 'Three less than half a number is 4' can be expressed as an equation by:

A $\frac{x}{2} - 3 = 4$ **B** $\frac{(x - 3)}{2} = 4$ **C** $2x - 3 = 4$

D $\frac{x}{2} + 3 = 4$ **E** $\frac{x + 3}{2} = 4$

7B **7** A flowchart is used to solve an equation.



What equation is being solved?

A $k = 6$ **B** $2k + 11 = 6$ **C** $2k = 6$

D $2k + 11 = 23$ **E** $2k = 12$

7E **8** The solution to the equation $3(m + 4) + m = 24$ is:

A $m = 7$ **B** $m = 8$ **C** $m = 4$ **D** $m = 1$ **E** $m = 3$

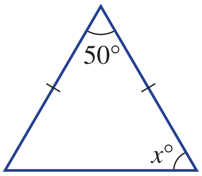
7F **9** Using the formula $F = 3k + b$, if $b = 7$ and $F = 34$ then k equals:

A 27 **B** 3 **C** 9 **D** 14 **E** 13

7G **10** An equation that could be used to find x in this isosceles triangle is:

A $50 + x = 180$ **B** $50 + 2x = 180$ **C** $2x = 180$

D $x = x$ **E** $50 = x$



Short-answer questions

7A **1** Are the following equations true (T) or false (F)?

a If $x = 3$, then $3x = 6$.

b If $a = 21$, then $a - 14 = \frac{a}{3}$.

c $5 \times 4 = 10 + 10$

7A **2** State the solutions to these equations.

a $4m = 16$ **b** $m + 5 = 11$ **c** $20 = 4q$ **d** $z - 10 = 40$

7A **3** Write an equation to represent each of the following statements. You do not need to solve the equations.

a Double m plus 3 equals 27.

b The sum of n and four is tripled; the answer is 18.

c The sum of two consecutive numbers, the first being x, is 7.

7B **4** Use backtracking to solve the following equations.

a $3x + 2 = 14$ **b** $4u + 5 = 21$ **c** $3d - 5 = 13$

d $2b - 1 = 13$ **e** $6f - 2 = 16$ **f** $12k + 3 = 27$

7C **5** Copy and complete the following equivalent equations.

a
$$\begin{tikzcd} 3x + 2 = 14 \arrow[d, bend right=60, "-2"'] \arrow[d, bend left=60, "-2"] \\ \_\_ = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} 2b - 1 = 13 \arrow[d, bend right=60, "+1"'] \arrow[d, bend left=60, "+1"] \\ \_\_ = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} 4x = 20 \arrow[d, bend right=60, "\div 4"'] \arrow[d, bend left=60, "\div 4"] \\ \_\_ = \_\_ \end{tikzcd}$$

7C **6** For each equation below, state the first operation you would apply to both sides.

a $15 + 2x = 45$ **b** $\frac{x}{2} - 5 = 6$ **c** $\frac{3a + 1}{2} = 11$

7C **7** Solve the following equations (using the balancing method).

a $7a + 3 = 38$ **b** $4b - 10 = 14$ **c** $2n + 9 = 41$

d $12 = 4c + 4$ **e** $12 = 3 + x$ **f** $10 = 8x - 6$

7D **8** Solve the following equations.

a $\frac{m}{3} = 2$ **b** $\frac{5x}{2} = 20$ **c** $5 = \frac{k}{6}$

d $\frac{2y}{3} = 12$ **e** $\frac{k + 3}{11} = 5$ **f** $10 = \frac{x - 2}{3}$

7E **9** Expand the brackets to simplify the following expressions.

a $2(x + 5)$ **b** $3(q - 10)$ **c** $4(2r + 3)$

d $5(x + 3) + 2x$ **e** $4(z + 2) + 10$ **f** $3(q - 5) + 2q$

7E **10** Solve the following equations by first expanding the brackets.

a $2(x + 5) = 16$ **b** $3(x + 1) = 9$ **c** $5(p + 2) + p = 46$

d $18 = 2(2x - 1)$ **e** $3(2x + 1) + 4 = 67$ **f** $5(k - 2) + 2k = 74$

7F **11** Look at the formula $F = ma$, relating force, mass and acceleration.

a Find F, if $m = 10$ and $a = 3$.

b Find m, if $F = 20$ and $a = 5$.

c If $F = 100$ and $a = 100$, what is the value of m?

7F **12** **a** If $P = 2(I + b)$, find I when $P = 48$ and $b = 3$.

b If $M = \dfrac{f}{f - d}$, find M when $f = 12$ and $d = 8$.

c If $F = \dfrac{5c}{2} + 20$, find c when $F = 30$.

7G **13** Hugo buys 4 mangoes and a $\$20$ gift voucher from the supermarket, giving a total cost of $\$26$.

a Let $m =$ the cost of a mango. Which of the following equations describes this situation?

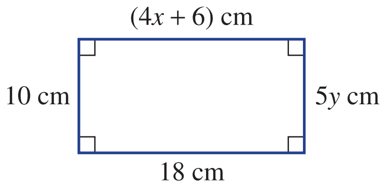
A $m = 20$ **B** $20m + 4 = 26$ **C** $4m = 20$

D $4m + 20 = 26$ **E** $4m + 26 = 20$

b Solve the equation chosen in part **a**.

c What is the cost of a mango?

7G **14** **a** Find the value of x and y for this rectangle.



b The sum of three consecutive numbers is 39. First write an equation and then find the value of the smallest number.

c The difference between a number and three times that number is 17. What is the number?

Extended-response questions

1 At a theme park, customers pay $\$10$ entry fee and then $\$5$ for each ride.

a Write an expression for the total cost to go on n rides.

b Inga spent a total of $\$55$ one afternoon at the theme park.

- **i** Write an equation to link the cost of n rides with how much Inga spent.
- **ii** Solve the equation.
- **iii** How many rides did Inga go on?

A parent and three children visit the park together. The parent does not go on any rides so a formula for the total cost is:

$$T = 3(\underbrace{5n + 10}_{\text{ride plus entry cost for each child}}) + 10 \longleftarrow \text{parent entry}$$

c If the children go on 4 rides together $(n = 4)$, what is the total cost?

d If the total cost was $\$145$, how many rides did the children go on?

2 To upload an advertisement to the www.searches.com.au website costs $\$20$ and then 12 cents whenever someone clicks on it.

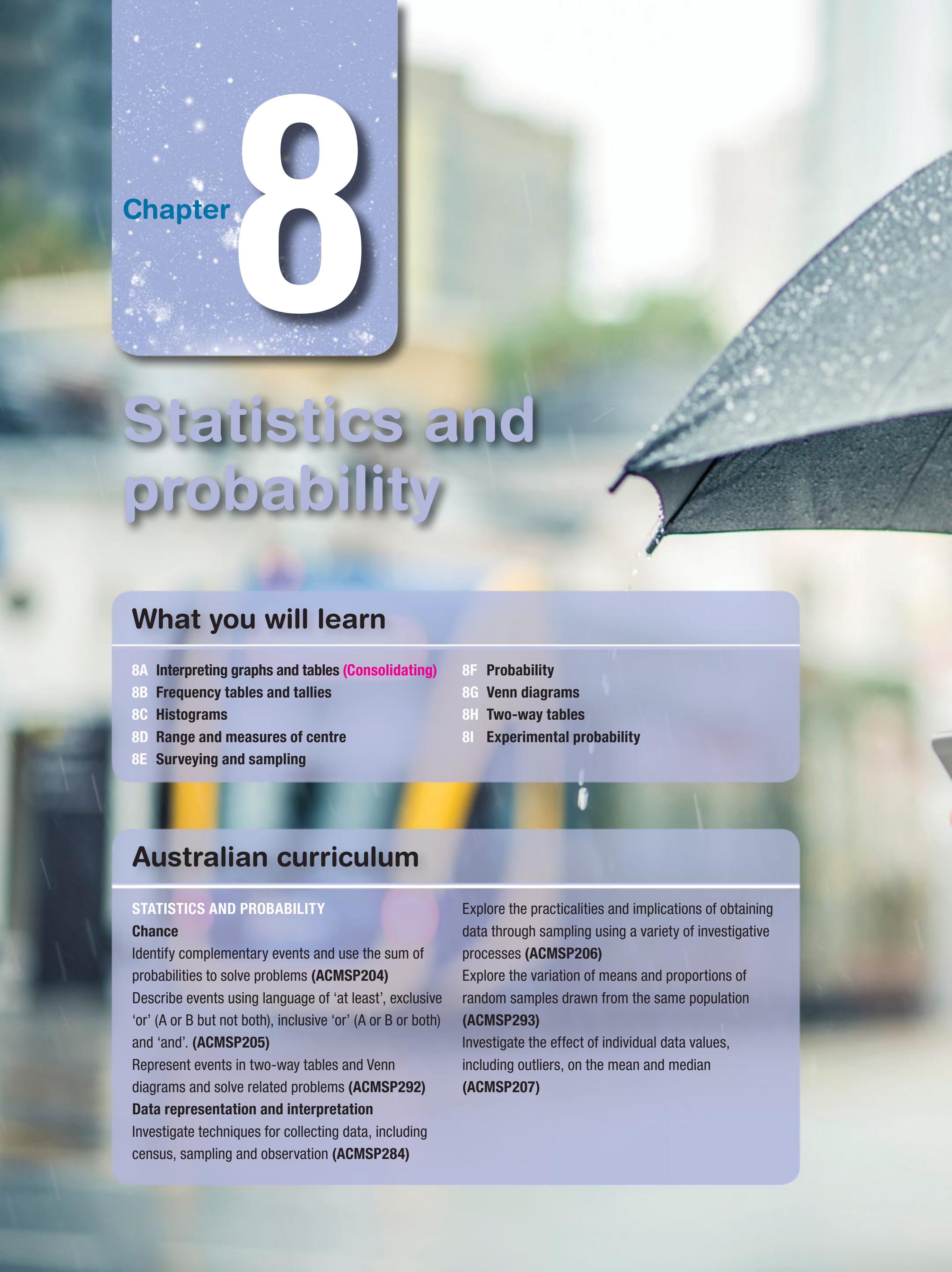
a Write a formula relating the total cost $(\$S)$ and the number of clicks (n) on the advertisement.

b If the total cost is $\$23.60$, write and solve an equation to find out how many times the advertisement has been clicked on.

c To upload to the www.yousearch.com.au website costs $\$15$ initially and then 20 cents for every click. Write a formula for the total cost $\$Y$ when the advertisement has been clicked n times.

d If a person has at most $\$20$ to spend, what is the maximum number of clicks they can afford on their advertisement at yousearch.com.au?

e Use trial and error to find the minimum number of clicks for which the total cost of posting an advertisement to searches.com.au is less than the cost of posting to yousearch.com.au.



Chapter 8

Statistics and probability

What you will learn

Australian curriculum

STATISTICS AND PROBABILITY

Chance

Identify complementary events and use the sum of probabilities to solve problems **(ACMSP204)**

Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’. **(ACMSP205)**

Represent events in two-way tables and Venn diagrams and solve related problems **(ACMSP292)**

Data representation and interpretation

Investigate techniques for collecting data, including census, sampling and observation **(ACMSP284)**

Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes **(ACMSP206)**

Explore the variation of means and proportions of random samples drawn from the same population **(ACMSP293)**

Investigate the effect of individual data values, including outliers, on the mean and median **(ACMSP207)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Statistics and probability in social media advertising

With social media usage increasing, businesses are devising new ways to advertise other than free TV, print media and the internet. Companies are spending time and resources investigating the best technique to pitch their products to individual consumers. Ads on social media platforms like Facebook and Twitter and those embedded in free games on smartphones have a far higher click-rate than normal web ads.

Social media outlets collect data from their users and use it to target them with ads in their newsfeeds, for things that they might be specifically interested in. Before investing money on targeted advertising, companies estimate the probability that a user will 'click' on their ad and the chance that they will then also follow through with an enquiry or purchase.

Pre-test

1 Arrange the following in ascending order.

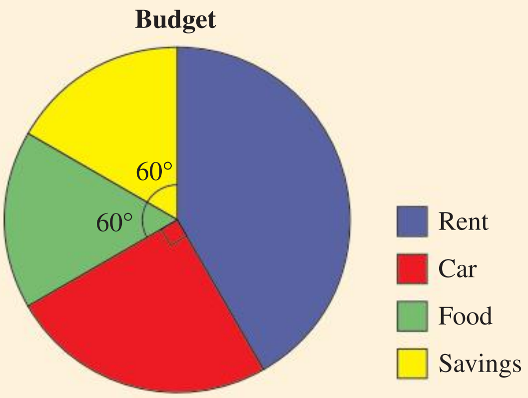
- **a** 2, 4, 10, 7, 1, 0, 6, 14, 9
- **b** 101, 20, 30.6, 204, 36, 100
- **c** 1.2, 1.9, 2.7, 1.7, 3.5, 3.2

2 Write down the total and the average (mean) as a decimal for each of the sets below.

- **a** 4, 6, 8, 10 and 12
- **b** 15, 17, 19, 19 and 24
- **c** 0.6, 0.6, 0.6, 0.7 and 0.8

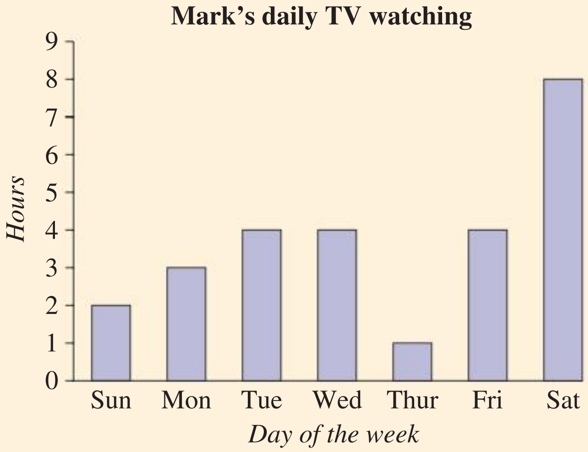
3 Use the information in the sector graph to answer the following questions.

- **a** What fraction of the income was spent on food?
- **b** What is the size of the angle for the rent sector?
- **c** If $420 is saved each month, find how much is spent on:
  - **i** food?
  - **ii** the car?



4

- **a** How many hours of television were watched on Wednesday?
- **b** How many hours of television were watched on Monday?
- **c** On which day was the most TV watched?
- **d** How many hours of TV were watched over the week shown?
- **e** What fraction of Saturday was spent watching TV?



5 If a fair die is rolled, state the probability as a fraction that:

- **a** the number 2 is rolled
- **b** an even number is rolled
- **c** a number less than 5 is rolled
- **d** the number 8 is rolled.

8A Interpreting graphs and tables

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

Statistics give us a way to understand information about our world, from weather patterns to outcomes of horse races. Graphs and tables are the most commonly used ways to represent data that has been collected.

The statistics of a horse's past performance can help predict whether or not it will win a race.

Let's start: Movement graphs

This is a whole class activity.

Two volunteers are needed: the *walker* who completes a journey, and the *grapher* who graphs the journey on the whiteboard. All other students in the class are *support graphers* and draw their own graph of the journey in their books.

A graph can show the distance walked relative to the time taken.

1 To set up your graph, draw two axes: the vertical axis labelled 'Distance from the front of the room' and horizontal labelled 'Time'. No numbers are required.

2 The walker starts from the front of the room, walks steadily to the back of the classroom, stops for a few seconds and then walks steadily back to the front of the room.

3 As the walker moves, all graphers draw a line on their graphs to show the walker's distance from the front of the room versus time. No numbers are required.

4 At the end of the journey, discuss the graph that has been drawn on the whiteboard.
   - Do you agree with this graph?
   - How does the graph show that the walker stopped for a short time?
   - The total distance walked is increasing so why does the graph start and end with zero distance?

5 Make up some other journeys and repeat this activity.
Some ideas you could try are: walking slowly then quickly, stopping and then reversing direction for a few steps, starting the 'journey' in the middle of the room or at the back of the room.

6 Now complete this activity in reverse. A graph is drawn on the white board and a volunteer walks to match the graph. The class checks to make sure the 'walker' is following the graph correctly.

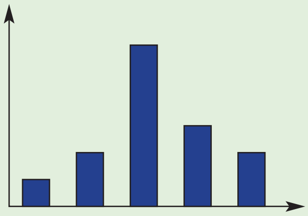
Key ideas

- Data can be represented as a graph or a table.

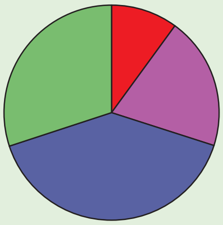
  The numbers in a table link the row and column headings.
  For example:

	Boys	Girls
Year 8	53 (Year 8 boys)	50 (Year 8 girls)
Year 9	49 (Year 9 boys)	52 (Year 9 girls)

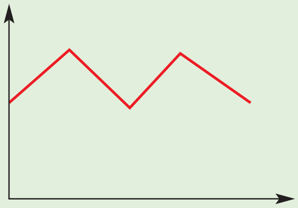
- Common types of graphs include:



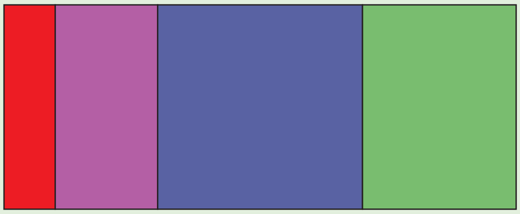
Column graphs



Pie charts (also called sector graphs)



Line graphs



Divided bar graphs

Exercise 8A

UNDERSTANDING 1, 2 2

1 The following table shows the population of some small towns over a 10-year period.

Year	Expton	Calcville	Statsland
2010	400	200	300
2015	320	240	310
2020	180	270	290

a What was the population of Expton in 2015?
b What was the population of Calcville in 2020?
c What was the population of Statsland in 2010?
d Which town's population kept decreasing over time?
e Which town's population kept increasing over time?

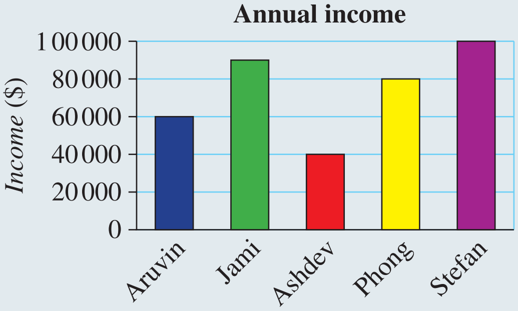
Example 1 Interpreting column graphs

This column graph represents the annual income of five different people.

a What is Aruvin's annual income?

b What is the difference between Jami's income and Ashdev's income?

c Who earns the most?



Solution	Explanation
a $60 000	Reading off the graph, Aruvin earns $60 000
b 90 000 – 40 000 = $50 000	Jami earns $90 000 and Ashdev earns $40 000, so the difference is $50 000.
c Stefan earns the most.	With the highest column, Stefan earns the most ($100 000).

2 The graph at right shows the age of 5 children.

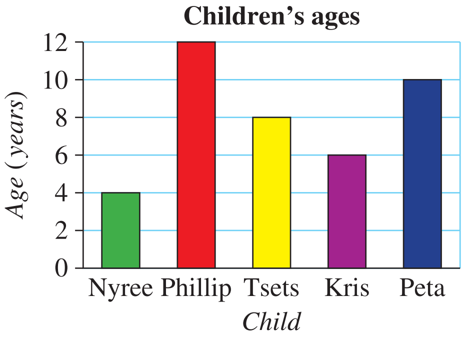
a How old is Peta?

b How old is Kris?

c Who is the oldest of the five children?

d Who is the youngest of the five children?

e What is the difference in age between Tsets and Nyree?



FLUENCY 3–6 4–7

3 Six Year 8 classes are asked to vote for which sport they would like to do next in Physical Education. Their results are shown in the table.

Sport	8A	8B	8C	8D	8E	8F
Badminton	3	5	7	0	8	12
Water polo	9	9	8	14	11	9
Handball	12	10	11	11	7	3

a How many students in 8B want to do water polo?

b How many students in 8A want to do handball?

c What is the most popular sport in 8F?

d Find the total number of students that chose each sport.

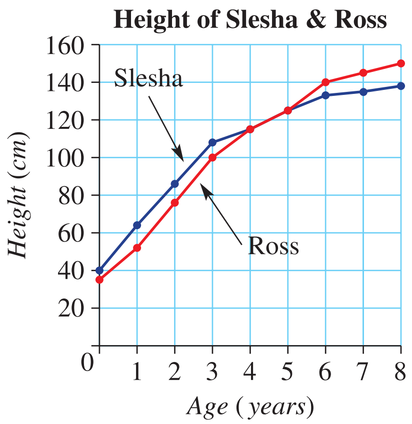
e Which sport had the most votes in total?

For **a**, look in the 8B column.

For **d**, add the numbers in the rows.

4 The line graph shows the height of Slesha and her twin brother Ross from the time they were born.

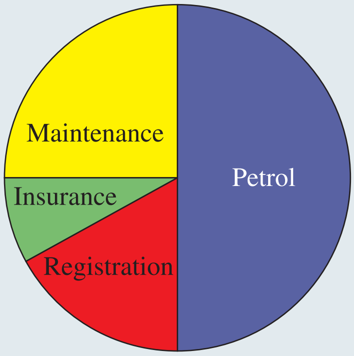
- **a** Which of the children was taller on their first birthday?
- **b** Which of the children was taller on their eighth birthday?
- **c** On which birthdays were the twins the same height?



Example 2 Interpreting sector graphs

A car owner graphs the amount of money spent per year on car-related expenses.

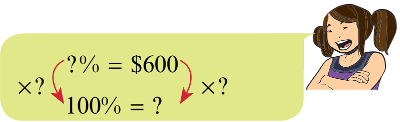
- **a** What is the largest expense each year?
- **b** What percentage of the car's expenses is devoted to maintenance?
- **c** If the car owner spends $\$3000$ per year on petrol, what is the total amount spent on the car each year?

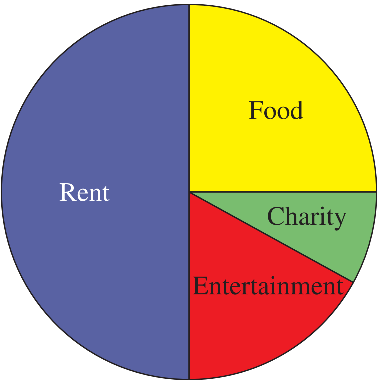


Solution	Explanation
a Petrol	Since petrol occupies the largest area of the graph, it is the largest expense.
b 25%	Maintenance occupies $\frac{1}{4}$ of the chart's area, which equals 25%.
c $\times 2 \left(\begin{array}{l} 50\% \text{ of expenses} = \$3000 \\ 100\% \text{ of expenses} = \$6000 \end{array} \right) \times 2$ The car owner spends $\$6000$ each year.	Petrol occupies half the chart's area, which is 50%. This is doubled to find the total amount spent.

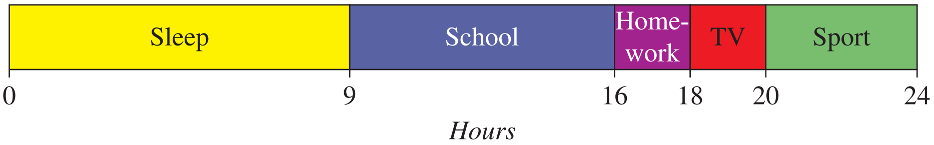
5 This sector graph shows one person's spending in a month.

- **a** What is the largest expense in that month?
- **b** What is the smallest expense in that month?
- **c** What percentage of the month's spending was on rent?
- **d** What percentage was spent on food?
- **e** If the person spent a total of $\$600$ on food in the month, what was their total spending?





6 A student has recorded how she spent her time on one day, shown in the divided bar graph below.

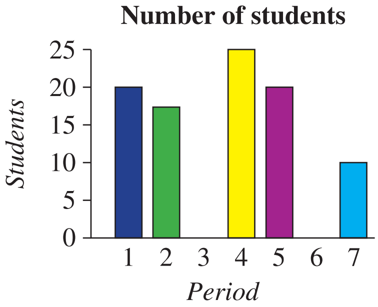


- **a** How much time did she spend doing homework on that day?
- **b** How much time was spent at school during that day?
- **c** What did she spend the most time doing?
- **d** What fraction of her day was spent playing sport?

Use the scale to find the width of each section.

7 A teacher records the number of students in her room during a 7-period day in a column graph.

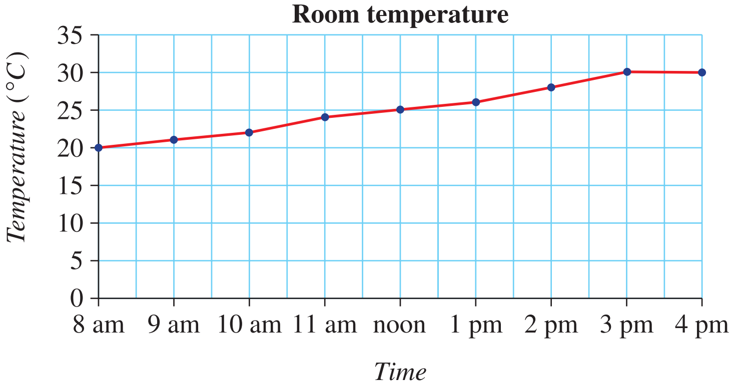
- **a** How many students were in the room during period 1?
- **b** During which periods was the classroom empty?
- **c** During which period was the smallest class in the room?
- **d** The same class was in the room twice on that day. In which periods was that class in the room?



PROBLEM-SOLVING AND REASONING

8, 9 | 9–11

8 The temperature in a classroom is graphed over an eight-hour period.

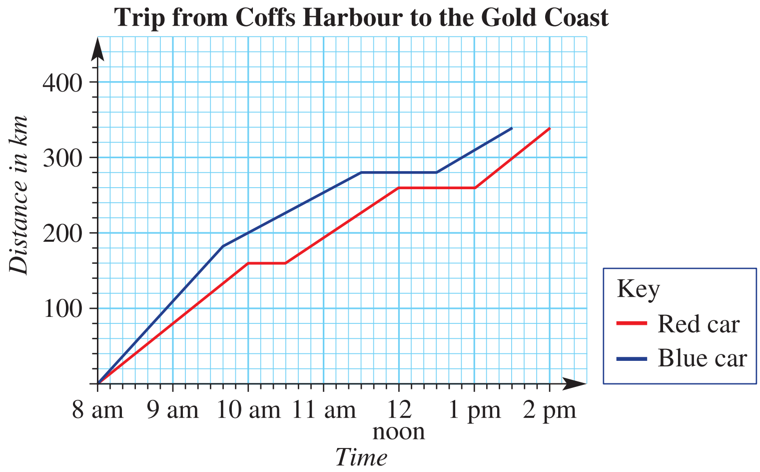


First state the temperature at 8 am and at 4 pm.

- **a** What was the temperature at 8 am?
- **b** By how much did the temperature increase in the eight-hour period from 8 am to 4 pm?
- **c** Students complain that it is uncomfortably hot when the temperature is 25°C or greater. At what time does it become uncomfortably hot?

9 Two families, the Red family driving a red car and the Blue family driving a blue car, leave Coffs Harbour together at 8 am and drive to the Gold Coast.

Does the distance or time change along the flat part of the graph?



a Why does a flat part of the graph show the car is stopped?

b When did the Red family stop for a morning tea break?

c How far had the blue car travelled when the Blue family stopped for lunch?

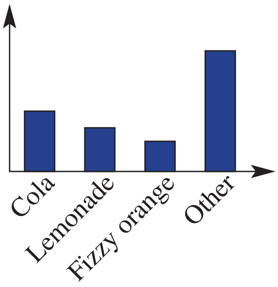
d How long did the Blue family stop for lunch?

e What distance had each family travelled at 10 am?

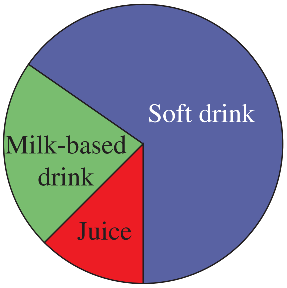
f What was the total driving time for each family? (exclude stops)

10 Three different surveys are conducted to establish whether soft drinks should be sold in the school canteen.

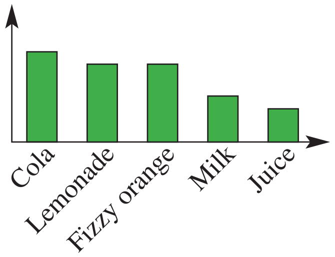
Survey 1: Favourite drink



Survey 2: Favourite type of drink



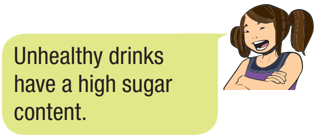
Survey 3: Sugar content per drink



a Which graph could be used to show the financial benefit to the canteen of selling soft drinks?

b Which graph could be used to show there was not much desire for Fizzy orange?

c Which graph could be used to show how unhealthy soft drink is?



11 The population of three nearby towns is shown over a 10-year period.

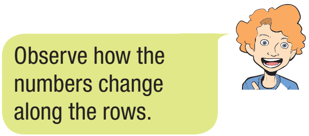
	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Town A	1414	1277	1204	1118	1026	1083	1171	1254	1317	1417
Town B	1062	1137	1188	1285	1371	1447	1502	1571	1665	1728
Town C	1042	1100	1174	1250	1312	1176	1075	992	895	783

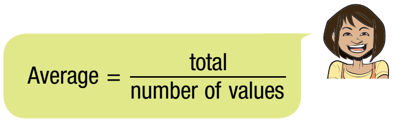
a Match these statements with the correct town.

- **1** The population increased then decreased.
- **2** The population decreased then increased.
- **3** The population kept increasing.

b Find the average population of the three towns in 2012.
Round your answer to the nearest whole number.

c Find the average population for town C over the 10-year period.
Round your answer to the nearest whole number.

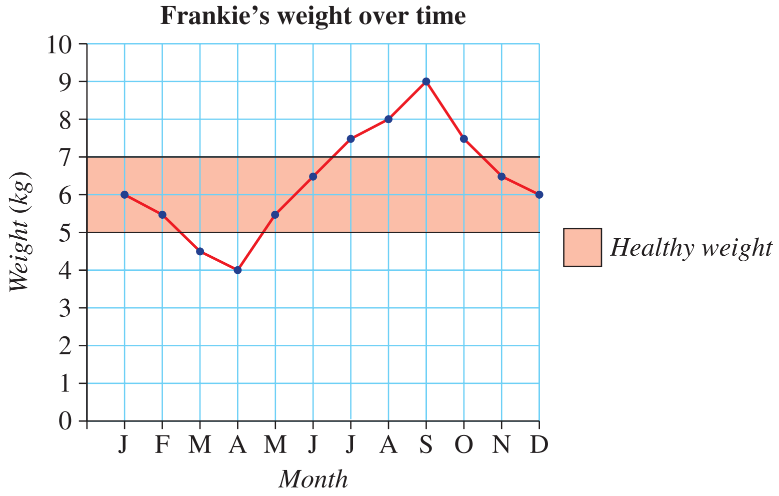




Weight over time

— 12

12 Frankie is a dog whose owner has graphed his weight over the year.



a If a healthy weight for the dog is between 5 kg and 7 kg, fill out the following table.

	Underweight	Normal weight	Overweight	Total
Number of months				12
Fraction of 12				
Angle				

b Represent the results of the table as a pie chart.

c What is the advantage of a line graph over a pie chart?

d What is the advantage of a pie chart over a line graph?

e Draw a line graph showing another dog's weight over 12 months, given that the dog is underweight for 2 months, overweight for 3 months and the normal weight for 7 months.

Sector angle = fraction × 360°

8B Frequency tables and tallies

Interactive

Widgets

HOTsheets

Walkthrough

Often the actual values in a set of data are not required – just knowing how many numbers fall into different categories is often all the information that is needed. A **frequency table** allows us to do this by listing how often the different values occur.

Frequency tables can be used for listing particular values or ranges of values.

Number of cars	Frequency
0	10
1	12
2	5
3	3

Age	Frequency
0–4	7
5–9	12
10–14	10
15–19	11

Let's start: Subject preferences

- Survey a group of peers to find their favourite school subject out of Maths, English, Science, Music and Sport.
- Represent your results in a table like the one below.

	Maths	English	Science	Music	Sport
Tally	𝍸	𝍸|	𝍸|||	||||	||
Frequency	5	6	8	4	2

- How would you expect the results to differ for different classes at your school, or for different schools?

Key ideas

- A **tally** is a tool used for counting as results are gathered. Numbers are written as vertical lines with every 5th number having a cross through a group of lines. e.g. 4 is |||| and 7 is 𝍸||.
- A **frequency table** has a column for the items (values or categories) and another column for the frequency of each item.
  The frequency shows how often each item occurs. A tallying column is also often used as data is gathered.
- The items can be individual values or intervals of values.
  – Individual scores

Hours worked	4	5	6	7	8	9
Tally	||	||||	𝍸	𝍸|	𝍸||	|||
Frequency	2	4	5	6	7	3

  – Scores in intervals or groups

Hours worked	4–6	7–9
Tally	𝍸 𝍸 |	𝍸 𝍸 |||| |
Frequency	11	16

Tally marks Line strokes used to record data, made in groups of 5

Frequency table A table summarising data by showing all possible scores from lowest to highest in one column, and the frequency of each score in another column

Exercise 8B

UNDERSTANDING 1–4 4

1 The table below shows survey results for students' favourite colours.

Colour	Frequency
Red	5
Green	2
Orange	7
Blue	3

Are the following true (T) or false (F)?

a 5 people chose red as their favourite colour.

b 9 people chose orange as their favourite colour.

c Blue is the favourite colour of 3 people.

d More people chose green than orange as their favourite colour.

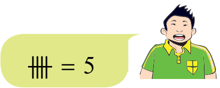
2 Fill in the blanks.

a The tally |||| represents the number _____.

b The tally 𝍸|| represents the number _____.

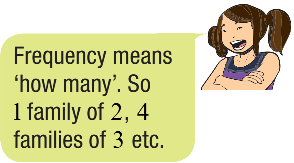
c The tally _____ represents the number 2.

d The tally _____ represents the number 11.



3 Make a list of all the family sizes for this Year 8 class, starting with 1 family of 2.

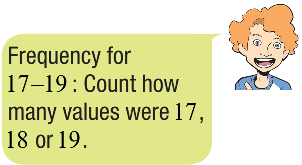
Family size	2	3	4	5	6	7
Frequency	1	4	10	8	2	1



4 This is a list of some students' handspans measured in cm.

19, 18, 20, 17, 22, 19, 22, 20, 24, 18, 20, 19

Copy and complete each of these frequency tables.



a

Handspans	Frequency
17	
18	
19	
20	
21	
22	
23	
24	

b

Handspans	Frequency
17–19	
20–22	
23–25	

FLUENCY 5–7 6–8

Example 3 Interpreting tallies

a The different car colours along a quiet road are noted. Convert the following tally into a frequency table.

White	Black	Blue	Red	Yellow
𝍷𝍷𝍷	𝍸 𝍸 𝍷𝍷𝍷	𝍸 𝍸 𝍸 𝍷𝍷	𝍸 𝍷	𝍸 𝍷𝍷𝍷𝍷

b How many red cars were seen?

c What was the total number of cars seen?

Solution

a

Colour	White	Black	Blue	Red	Yellow
Frequency	3	13	17	6	9

b 6 red cars were seen.

c 48 cars seen

Explanation

Each tally is converted into a frequency. For example, black is two groups of 5 plus 3, giving $10 + 3 = 13$.

This can be read directly from the table.

$3 + 13 + 17 + 6 + 9 = 48$
Add the frequencies to find the total.

5 A basketball player's performance in one game is recorded in the following table.

	Passes	Shots at goal	Shots that go in	Steals
Tally	𝍷𝍷𝍷	𝍸 𝍸 𝍷𝍷	𝍸 𝍷𝍷𝍷	𝍷𝍷
Frequency				

a Copy and complete the table, filling in the frequency row.

b How many shots did the player have at the goal?

c How many shots went in?

d How many steals did the player have during the game?

Example 4 Constructing tables from data

Put the following data into a frequency table: $1, 4, 1, 4, 1, 2, 3, 4, 6, 1, 5, 1, 2, 1$.

Solution

Number	1	2	3	4	5	6
Tally	𝍸 𝍷	𝍷𝍷	𝍷	𝍷𝍷𝍷	𝍷	𝍷
Frequency	6	2	1	3	1	1

Explanation

Construct the tally as you read through the list.
Then go back and convert the tally to frequencies.

6 A student surveys her class to ask how many people are in their family. The results are:

6, 3, 3, 2, 4, 5, 4, 5, 8, 5, 4, 8, 6, 7, 6, 5, 8, 4, 7, 6

a Construct a frequency table. Include a row for family size, a tally row and a frequency row.

b How many students have exactly 5 people in their family?

c How many students have at least 6 people in their family?

Check that the number of scores in the list equals the total of the frequencies in the table.

7 Braxton surveys a group of people to find out how much time they spend watching television each week. They give their answers rounded to the nearest hour.

Add the frequencies to find the total number surveyed.

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–166
Tally	卌	|||	卌 卌 ||	卌 卌 卌	卌 ||||	||||	||

a Draw a frequency table of his results, converting the tallies to frequencies.

b How many people altogether did Braxton survey?

c How many people spend $15-19$ hours per week watching television?

d How many people watch television for less than 5 hours per week?

e How many people watch television 2 hours *per day* or less on average?

Less than 5 hours doesn't include 5 hours. So 0–1 and 2–4

8 The heights of a group of 21 people are shown below, given to the nearest cm.

174 179 161 132 191 196 138 165 151 178 189

147 145 145 139 157 193 146 169 191 145

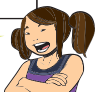
a Copy and complete the frequency table below.

Height (cm)	130–139	140–149	150–159	160–169	170–179	180–189	190 +
Tally							
Frequency							

b How many people are in the range $150-159$ cm?

c How many people are 180 cm or taller?

d How many people are between 140 cm and 169 cm tall?



180 cm or taller means $180-189$ and 190 +

PROBLEM-SOLVING AND REASONING

9, 10 | 10–12

9 A tennis player records the number of double faults they serve per match during one month.

Double faults	0	1	2	3	4	5
Frequency	4	2	1	0	2	1

a How many matches did they play in total during the month?

b How many times did they serve exactly 1 double fault?

c In how many matches did they serve no double faults?

d How many double faults did they serve in total during the month?

Add the frequencies to find the total number of matches played.

10 Match each of these data sets with the correct column (A , B , C or D) in the frequency table shown below.

a 1, 1, 2, 3, 3 **b** 1, 2, 2, 2, 3

c 1, 1, 1, 2, 3 **d** 1, 2, 3, 3, 3

Number	A Frequency	B Frequency	C Frequency	D Frequency
1	3	2	1	1
2	1	1	1	3
3	1	2	3	1

11 Five different classes are in the same building in different rooms at the same time. The ages of students in each room are recorded in the frequency table below.

Age	Room A Frequency	Room B Frequency	Room C Frequency	Room D Frequency	Room E Frequency
12	3	2	0	0	0
13	20	18	1	0	0
14	2	4	3	0	10
15	0	0	12	10	11
16	0	0	12	10	11
17	0	0	0	1	0

a How many students are in room C?

b How many students are in the building?

c How many 14 -year-olds are in the building?

d What is the average (mean) age of students in room B? Answer to one decimal place.

e Make a frequency table showing age and the number of each age group in the building.

12 Some exam results are presented in the frequency table below.

0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–100
0	0	3	1	2	5	8	12	10	2

a Redraw the table so that the intervals are of width 20 rather than 10 (i.e. so the first column is 0–19, the second is 20–39, and so on).

b Redraw the table with the intervals 0–29, 30–59, 60–89, 90–100.

8B

Homework puzzle

— 13

13 Priscilla records the numbers of hours of homework she completes each evening from Monday to Thursday. Her results are shown in this frequency table.

Number of hours	Frequency
1	1
2	1
3	2

a On how many nights did Priscilla do 3 hours of homework?

b One possibility is that she worked 3 hours on Monday, 2 hours on Tuesday, 3 hours on Wednesday and 1 hour on Thursday. Copy and complete this table to show other ways her time could have been allocated for the four nights.

Monday	Tuesday	Wednesday	Thursday
☐ hours	☐ hours	☐ hours	☐ hours
☐ hours	☐ hours	☐ hours	☐ hours
☐ hours	☐ hours	☐ hours	☐ hours

c Priscilla's brother Joey did homework on all five nights. On two nights he worked for 1 hour, on two nights he worked for 2 hours and on one night he worked for 3 hours. Show three ways that the table below could be filled in to match his description.

Monday	Tuesday	Wednesday	Thursday	Friday
☐ hours	☐ hours	☐ hours	☐ hours	☐ hours

d Calculate the average hours of homework per night for Priscilla and Joey.

e How many hours more homework per week would Joey have to do over 5 nights to make his average per night equal to Priscilla's average?

8C Histograms

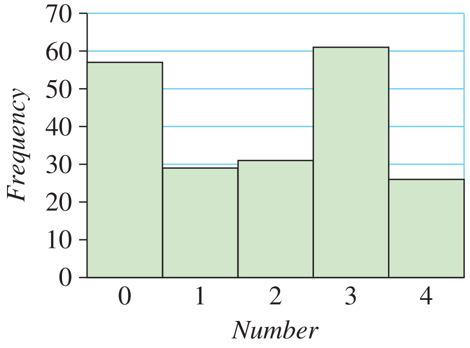
A **histogram** is a graphical representation of a frequency table so that patterns can be observed more easily.

For example, the data below is represented as a frequency table and as a histogram.

As a table

Number	Frequency
0	57
1	29
2	31
3	61
4	26

As a histogram

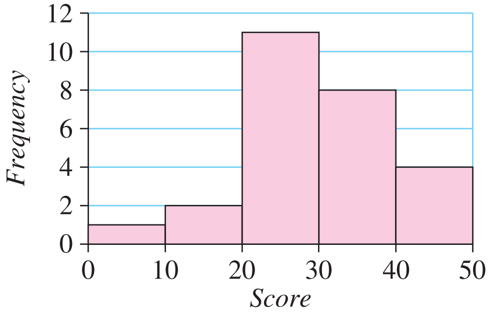


At a glance you can see from the histogram that 0 and 3 are about twice as common as the other values. This is harder to read straight from the table. A histogram makes comparisons of frequency easier.

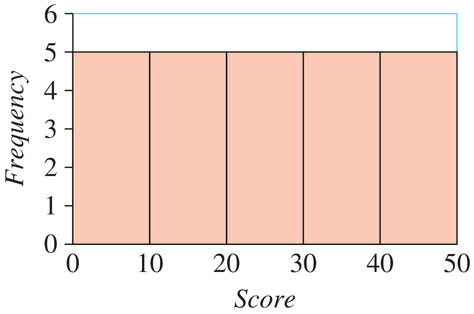
Let's start: Test analysis

The results for some end-of-year tests are shown for four different classes in four different histograms.

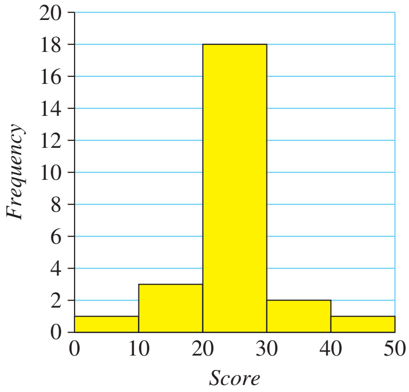
Class 1



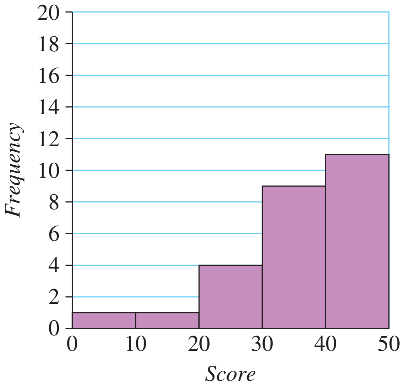
Class 2



Class 3



Class 4



Work with a classmate and discuss the answers to these questions.

1 Choose which class has results that can be described as:
  - **a** a few low scores, a few high scores and a lot of scores around the middle.
  - **b** equal numbers of students getting low, middle and high scores.
  - **c** more students getting high scores than low scores.
  - **d** more students getting middle scores than either high or low scores.

2 Which class has the highest average score?

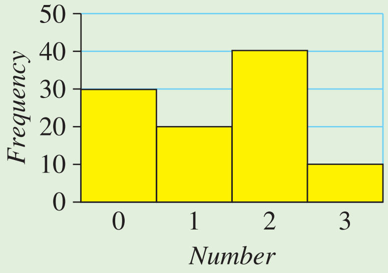
3 Which class has the highest overall score?

4 Which class would be the easiest to teach and which would be the hardest, do you think?

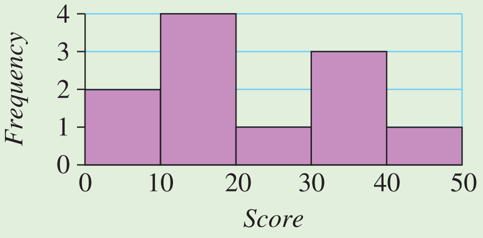
Key ideas

- A **histogram** is a graphical representation of a frequency table. It can be used when the items are numerical.
- The vertical axis (y -axis) is used to represent the frequency of each item.
- Columns are placed next to one another with no gaps in between.
  e.g.

Histogram with individual values



Histogram with intervals or groups



- A half column width space is sometimes placed between the vertical axis and the first column of the histogram if the first vertical bar does not start at zero.

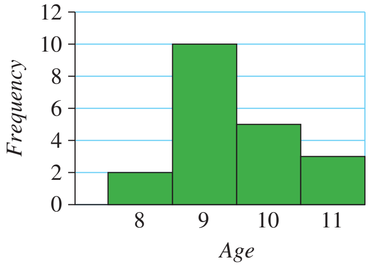
Histogram
A special type of column graph for quantitative data with no gaps between the columns

Exercise 8C

UNDERSTANDING 1–4 4

1 This histogram below shows the ages of people in an Art class.
  - **a** How many 8-year-olds are in this class?
  - **b** What is the most common age for students in this class?
  - **c** What is the age of the oldest person in the class?

Ages of students in Art class



Frequency here is the number of students of that age.

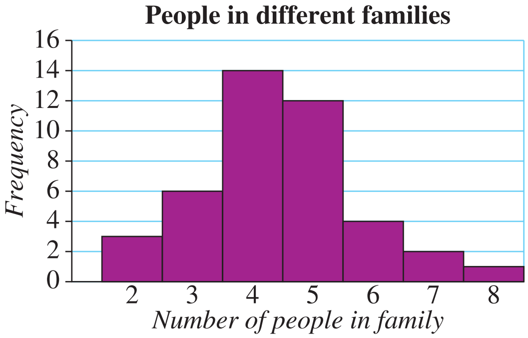
2 A survey is conducted of the number of people in different families. The results are shown in this histogram.

a What is the most likely number of people in a family, on the basis of this survey?

b How many people responding to the survey said they had a family of 6?

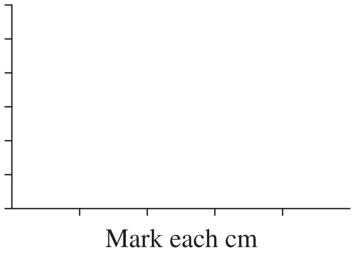
c What is the least likely number (from 2 to 8) of people in a family, on the basis of this survey?

The frequency shows 'how many' of each family size.

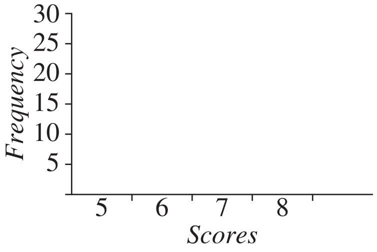


3 Follow these steps to draw and copy this histogram.

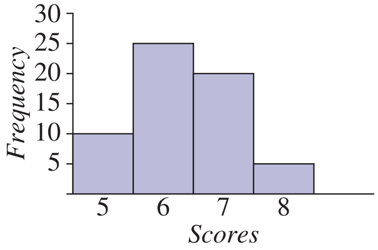
Step 1 Rule two lines and mark with even scales.



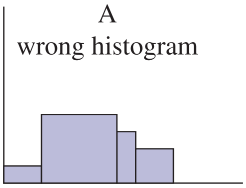
Step 2 Label the scales.

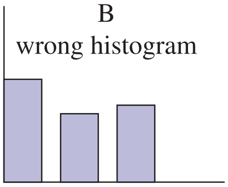


Step 3 Rule the columns of the histogram.



4 A student draws two incorrect histograms like this.





a What mistake has been made with the columns in histogram A? How could this error be prevented?

b What mistake has been made with the columns in histogram B?

c What is missing from both histograms?

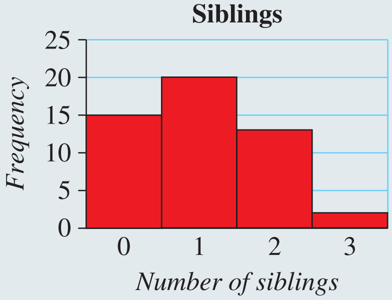
FLUENCY 5–6(½), 7 5–7(½)

Example 5 Constructing histograms from frequency tables with individual labels

Represent this frequency table below as a histogram.

Number of siblings	Frequency
0	15
1	20
2	13
3	2

Solution



Explanation

The scale $0-25$ is chosen to fit the highest frequency (20).

Each different number of siblings in the frequency table is given a column in the graph.

5 Represent the following frequency tables as histograms.

a

Number	Frequency
0	5
1	3
2	5
3	2
4	4

b

Number	Frequency
0	3
1	9
2	3
3	10
4	7

Remember to rule up even scales.

c

Age	Frequency
12	15
13	10
14	25
15	20
16	28

d

Number of cars	Frequency
0	4
1	5
2	4
3	2

6 For the following sets of data:

  i create a frequency table

  ii draw a histogram from the frequency table.

  a 1, 2, 5, 5, 3, 4, 4, 4, 5, 5, 5, 1, 3, 4, 1

  b 5, 1, 1, 2, 3, 2, 2, 3, 3, 4, 3, 3, 1, 1, 3

  c 4, 3, 8, 9, 7, 1, 6, 3, 1, 1, 4, 6, 2, 9, 7, 2, 10, 5, 5, 4

  d 60, 52, 60, 59, 56, 57, 54, 53, 58, 56, 58, 60, 51, 52, 59, 59, 52, 60, 50, 52

Frequency table

number	tally	frequency

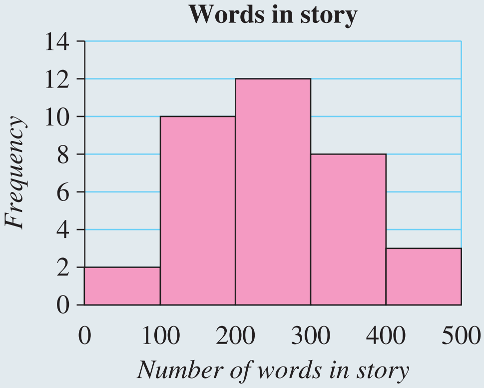
Frequency shows how many of each number.

Example 6 Constructing histograms from frequency tables using intervals

Draw the frequency table below as a histogram.

Number of words in story	Frequency
0–99	2
100–199	10
200–299	12
300–399	8
400–500	3

Solution



Explanation

The scale 0–14 is chosen to fit the highest frequency (12).
The different intervals (0–99 words, 100–199 words etc.) are displayed on the horizontal axis.

7 Represent the following frequency tables as histograms.

a

Score	Frequency
0–19	1
20–39	4
40–59	10
60–79	12
80–100	5

b

Age	Frequency
0–5	5
6–10	12
11–15	14
16–20	11
21–25	5
26–30	8
31–35	2
36–40	1

Frequency is on the vertical axis.

Mark even scales.

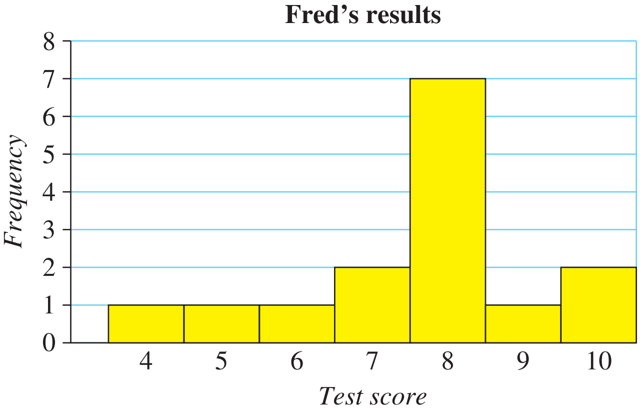
PROBLEM-SOLVING AND REASONING

8–10 9–12

8 Edwin records the results for his spelling tests out of 10. They are 3, 9, 3, 2, 7, 2, 9, 1, 5, 7, 10, 6, 2, 6, 4.

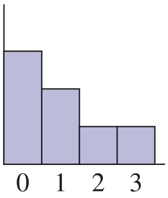
a Draw a histogram for his results.

b Fred's results are given by the histogram shown at right. Is Edwin a better or a worse speller generally than Fred? Give a reason for your answer.

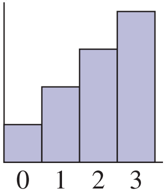


Higher columns means more tennis matches.

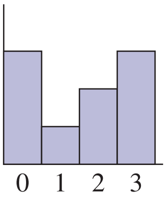
9 Some tennis players count the number of aces served in different matches. Match up the histograms with the descriptions.



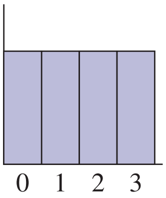
a



b



c



d

A Often serves aces.

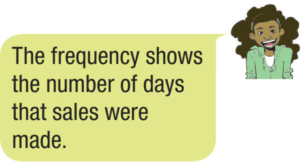
B Generally serves 3 aces or 0 aces.

C Serves a different number of aces in each match.

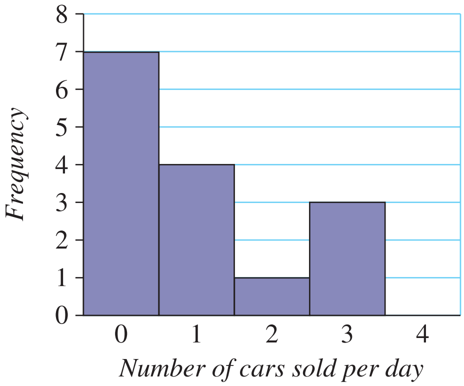
D Serves very few aces.

0 1 2 3 shows the number of aces per match.

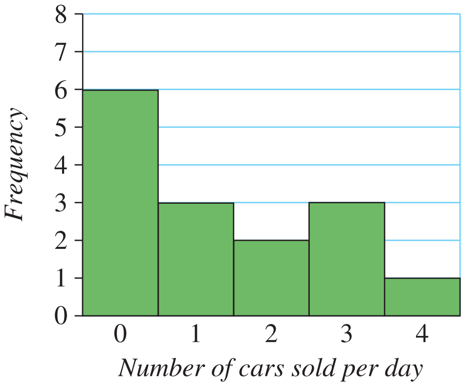
10 A car dealership records the number of sales each salesperson makes per day over three weeks.



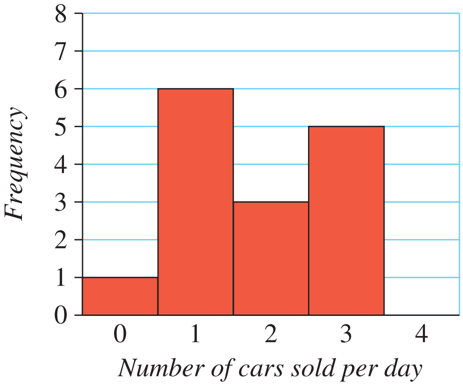
The frequency shows the number of days that sales were made.



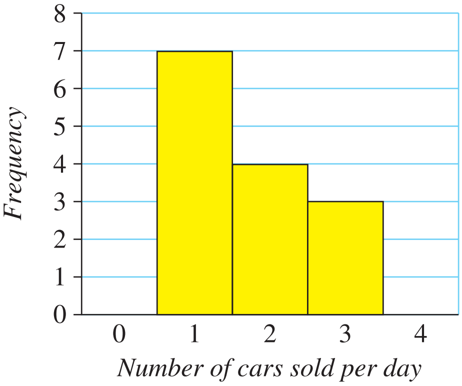
Bill's sales



Marie's sales



Frank's sales



Con's sales

a On how many days did Bill not make any sales?

b For how many days did Bill sell one car per day?

c What is the record for the greatest number of sales in one day and who holds this record?

d Which salesperson made at least one sale every day?

e Over the whole period, which salesperson made the most sales in total? How many cars did they sell?

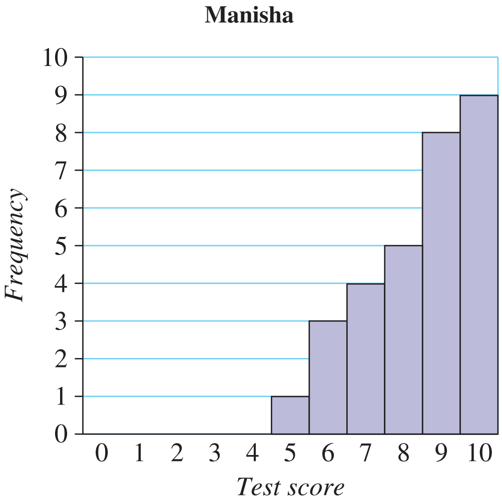
f Over the whole period, which salesperson made the fewest sales in total? How many cars did they sell?

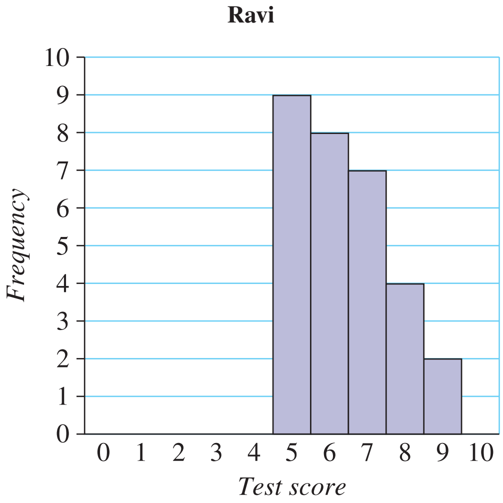
11 This histogram shows the ages of a group of people in a room.

- **a** Which part of this histogram would change if a histogram is drawn for the ages of the same group of people in exactly 12 years' time?
- **b** How would this histogram look if it showed the ages of the same group of people exactly 12 years ago?

The frequency shows the number of people of each age.

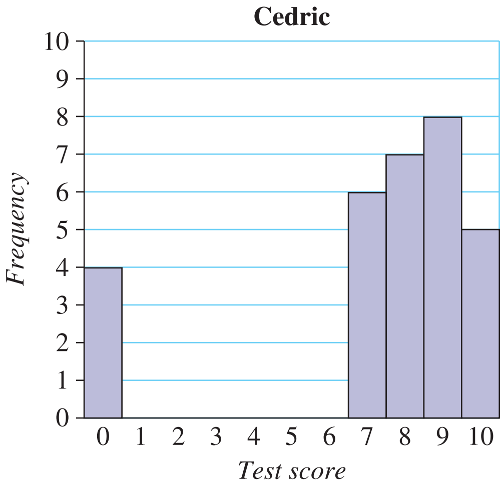
12 Two students have each graphed a histogram that shows their results for a number of spelling tests. Each test is out of 10 and there has been one test per week for 30 weeks.





- **a** Manisha's scores started very high but have got worse during the year. Give an example of a list of scores that Manisha might have received over the 30 weeks.
- **b** Ravi's spelling has actually improved consistently over the course of the year. Give an example of a list of the scores he might have received for the 30 weeks.
- **c** A third student, Cedric, has the following results. What is a likely explanation for the '0' results?

The frequency shows the number of tests for each result.



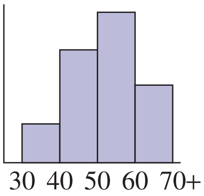
Heights, weights and ages mix-up

13 Three students survey different groups of people to find out their heights, weights and ages. Unfortunately they have mixed up all the graphs they obtained.

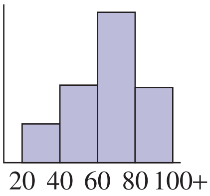
a Copy and complete the table below, stating which graph corresponds to which set of data.

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4		
Shopping centre			
Teachers' common room			

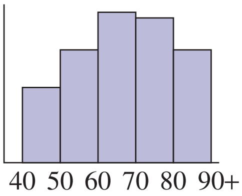
Graph 1



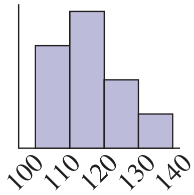
Graph 2



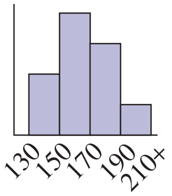
Graph 3



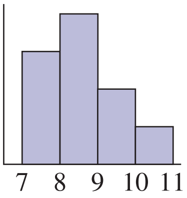
Graph 4



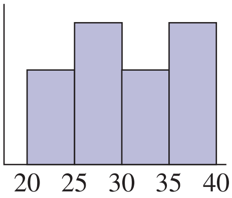
Graph 5



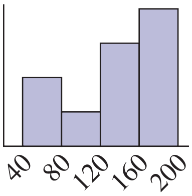
Graph 6



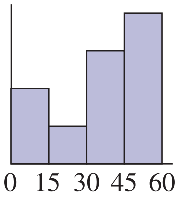
Graph 7



Graph 8



Graph 9



b Show with rough sketches how the age graphs would look for:

- **i** people in a retirement village
- **ii** students at a secondary school
- **iii** guests at a 30-year high school reunion.

8D Range and measures of centre

Interactive
Widgets
HOTsheets
Walkthrough

It is sometimes useful to summarise a large group of data as a single value. The concept of 'average' is familiar to most people, but more precise mathematical terms to use are 'mean', 'median' and 'mode'.

Let's start: Family heights

Each New Year, the Green family measure and record their heights. This year their height measurements are:

Georgia 78 cm, Emily 130 cm, Amy 130 cm, Ethan 188 cm, Mrs Green 165 cm, Mr Green 182 cm.

Work with a classmate to help each other to complete these activities.

Range

1 Who is the shortest and who is the tallest person in the Green family?
2 What is the range (the difference) between the shortest and tallest heights in the Green family?
3 The shortest person in the world is 55 cm and the tallest person in the world is 251cm. What is the current range of heights for all adult humans?
4 The Green family had a snow-skiing holiday. One morning it is –8°C and that afternoon it is 5°C. What was the range of temperature that day?

Median

1 List the heights of the Green family in ascending (increasing) order.
2 What are the two middle heights?
3 Find the median (middle of these two central heights).
4 If the tallest man in the world, height 251 cm, is added into the Green family heights list, what is the median (middle) height now?
5 By how much has the median height changed by adding the tallest man into the list?
6 Does the median value always have to be one of the scores in the list?

Mean

1 Add up the total of all the heights of the Green family.
2 Now find the mean height. (mean = total of heights divided by the number of heights)
3 If the tallest man in the world is included with the Green family heights, what is the mean height now?
4 By how much has the mean height changed by including the tallest man into the list?
5 Does the mean value always have to be one of the scores in the list?

Mode

1 The Green family has twins. Who are they and what is their height?
2 What is the mode (most common) of the Green family heights?
3 The Pink family has heights: 125 cm, 142 cm, 142 cm, 142 cm, 160 cm and 178 cm. What is the height of the Pink triplets?
4 What is the mode of the Pink family heights?
5 Does the mode value always have to be one of the scores in the list?

Key ideas

- The **range** of a set of data is given by:
  Range = highest number – lowest number
- The **mean** (sometimes called the average) of a set of numbers is given by:
  Mean = (sum of all the values) ÷ (total number of values)
  e.g. $7 + 8 + 1 + 10 + 2 + 1 + 6 = 35$
  Mean $= 35 \div 7 = 5$
- The **median** is the middle value if the values are in order (ascending or descending). If there are two middle values then the average of them is taken, by adding them together and dividing by 2.
  e.g. $1 \quad 1 \quad 2 \quad \boxed{6} \quad 7 \quad 8 \quad 10$
  Middle ⇒ Median = 6
  e.g. $2 \quad 3 \quad \boxed{5 \quad 9} \quad 10 \quad 12$
  $\dfrac{5+9}{2}$ ⇒ Median = 7
- The **mode** is the most common value, i.e. the score with the highest frequency. There can be more than one mode.
  e.g. $1 \quad 1 \quad 2 \quad 6 \quad 7 \quad 8 \quad 10$
  Mode = 1
  For data sorted into categories, the most common category is called the **modal category**.
  e.g. bananas 25, oranges 20, mangoes 30
  The modal category is mangoes.
- An **outlier** is a score that is much larger or smaller than the rest of the data.
  - The median and mode are generally unaffected by outliers whereas the mean can be affected significantly by an outlier.

Range The difference between the highest score and the lowest score

Mean The sum of the scores divided by the number of scores

Median The middle score when the data is arranged in order

Mode The most frequently occurring value in a set of data

Outlier Any value that is much larger or much smaller than the rest of the data in a set

Exercise 8D

UNDERSTANDING 1–5 5

1 Fill in the blanks. Choose from: *range, outlier, mode, mean* or *median.*

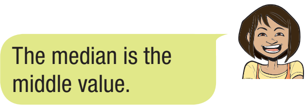
- **a** The most common value in a set of data is called the __________.
- **b** The sum of all values, divided by the number of values is called the __________.
- **c** The __________ can be calculated by finding the middle value(s) of the numbers placed in ascending order.
- **d** The difference between the highest and lowest values is called the __________.
- **e** A value that is much larger or smaller than the other values is called an __________.

2 Use the set of numbers 1, 7, 1, 2, 4.

- **a** Find the sum of these numbers.
- **b** How many numbers are listed?
- **c** Hence find the mean.

$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

3 Use the values $5, 2, 1, 7, 9, 4, 6$.
- **a** Sort these numbers from smallest to largest.
- **b** What is the middle value in your sorted list?
- **c** What is the median of this set?



4 Use the set $1, 5, 7, 9, 10, 13$.
- **a** State the two middle values.
- **b** Find the sum of the two middle values.
- **c** Divide your answer by 2 to find the median of the set.

5 Use the set of numbers $1, 3, 2, 8, 5, 6$.
- **a** State the largest number.
- **b** State the smallest number.
- **c** Hence state the range, by finding the difference of these two values.

FLUENCY 6–8(½), 9 6-8(½), 9–10

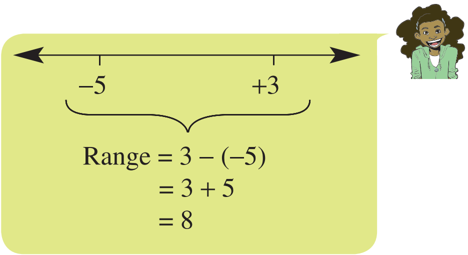
Example 7 Finding the range

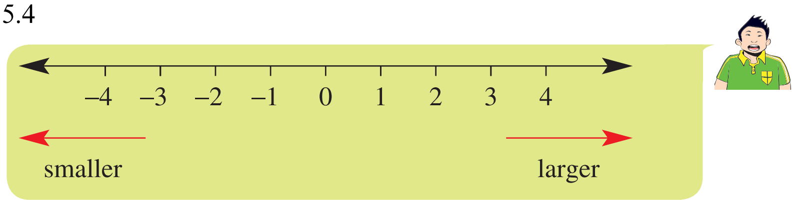
Find the range of the following sets of data.

a $1, 5, 2, 3, 8, 12, 4$ **b** $-6, -20, 7, 12, -24, 19$

Solution	**Explanation**
a Range $= 12 - 1$ $= 11$	Maximum: 12, minimum: 1 Range = maximum – minimum
b Range $= 19 - (-24)$ $= 43$	Maximum: 19, minimum: -24 Range $= 19 - (-24) = 19 + 24$

6 Find the range of the following sets.
- **a** $5, 1, 7, 9, 10, 3, 10, 6$
- **b** $9, 3, 9, 3, 10, 5, 0, 2$
- **c** $4, 13, 16, 9, 1, 6, 5, 8, 11, 10$
- **d** $16, 7, 17, 13, 3, 12, 6, 6, 3, 6$
- **e** $-7, 4, 12, -5, -18, -16, 7, 9$
- **f** $16, -3, -5, -6, 18, -4, 3, -9$
- **g** $3.5, 6.9, -9.8, -10.0, 6.2, 0.8$
- **h** $-4.6, 2.6, -6.1, 2.6, 0.8, -5.4$





Example 8 Finding the mean and the mode

For the set of numbers 10, 2, 15, 1, 15, 5, 11, 19, 4, 8 find:

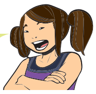
a the mean **b** the mode

Solution	Explanation
a $10 + 2 + 15 + 1 + 15 + 5 + 11 + 19 + 4 + 8 = 90$	The numbers are added to find the total.
Mean $= 90 \div 10 = 9$	The mean is found by dividing the total by the number of items (in this case 10).
b Mode $= 15$	The most common value is 15, so this is the mode.

7 For each of the following sets find:

i the mean **ii** the mode

a 5, 6, 3, 4, 4, 8
b 2, 2, 1, 2, 1, 4, 2
c 4, 3, 3, 10, 10, 2, 3
d $-10, -4, 0, 0, -2, 0, -5$
e $3, 4, 5, -9, 6, -9$
f $3, -6, 7, -4, -3, 3$
g 13, 15, 7, 7, 20, 9, 15, 15, 11, 17
h 20, 12, 15, 11, 20, 3, 18, 2, 14, 16
i 18, 12, 12, 14, 12, 3, 3, 16, 5, 16
j 18, 5, 14, 5, 19, 12, 13, 5, 10, 3
k $-15, -6, -6, 16, 6, 13, 3, 2, 19, -8$
l $-13, -6, -6, -13, -6, 10, -15, 6, 7, 2$



Mean $= \dfrac{\text{sum of scores}}{\text{number of scores}}$

The mode is the most common score.

Example 9 Finding the median

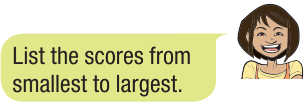
Find the median of:

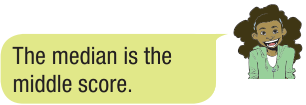
a 16, 18, 1, 13, 14, 2, 11 **b** 7, 9, 12, 3, 15, 10, 19, 3, 19, 1

Solution	Explanation
a Sorted: $\boxed{1,2,11},\ \textcircled{13},\ \boxed{14,16,18}$ Median $= 13$	Sort the numbers from smallest to largest. Split the list into two equal halves. The middle value is 13.
b Sorted: $\boxed{1,3,3,7},\ \textcircled{9\ ,\ 10},\ \boxed{12,15,19,19}$ Median $= \dfrac{9+10}{2} = 9.5$	Sort the numbers from smallest to largest. Split the list into two equal halves. There are two middle values (9 and 10) so we add them and divide by 2.

8 For each of the following sets calculate the median.

- **a** 3, 5, 6, 8, 10
- **b** 3, 4, 4, 6, 7
- **c** 1, 2, 4, 8, 10, 13, 13
- **d** 2, 5, 5, 5, 8, 12, 14
- **e** 14, 15, 7, 1, 11, 2, 8, 7, 15
- **f** 4, 14, 5, 7, 12, 1, 12, 6, 11
- **g** 2, 2, 4, 6, 7, 9
- **h** 1, 1, 2, 9, 9, 10
- **i** 1, 3, 5, 7, 8, 10, 13, 14
- **j** 0, 1, 9, 13, 1, 10, 7, 12, 9, 2
- **k** 12, 17, 7, 10, 2, 17, −2, 15, 11, −8
- **l** −2, −1, −3, 15, 13, 11, 14, 17, 1, 14



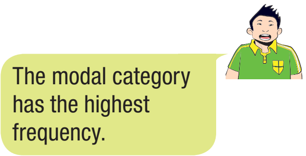


9 Bernie writes down how many hours he works each day for one week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of hours	8	10	8	7	9

- **a** What is the mean number of hours Bernie worked each day?
- **b** What is the median number of hours Bernie worked each day?
- **c** What is the mode number of hours Bernie worked each day?

10 State the modal category for the following frequency tables.



a Colours of cars are noted as they drive past

Colour	Red	Blue	Orange	White	Green	Black
Frequency	21	14	3	42	7	25

b Pizza preferences are noted within a group of teenagers

Hawaiian	Meat-lovers	Vegetarian	Cheese
5	7	4	2

c The favourite day of the week of a group of people

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Frequency	4	12	41	16	28

d The number of gymnasts in different states

New South Wales	Queensland	South Australia	Tasmania	Victoria	Western Australia
152	135	193	86	144	159

PROBLEM-SOLVING AND REASONING

11, 12	12–14

11 Federica is in a dancing competition and each week she is rated out of 10. Her results for one term are shown in the frequency table below.

Score	7	8	9	10
Frequency	3	0	3	4

- **a** In how many weeks did she get 7 out of 10?
- **b** What score did she receive the most often?
- **c** List out all the scores.
- **d** What is her mean dancing score for the 10 weeks?
- **e** What is her median dancing score for the 10 weeks?
- **f** What is the range of Federica's dancing scores?

12 Business A pays wages of $42 000, $48 000, $50 000, $50 000 and the boss gets $70 000.

Business B pays wages of $42 000, $48 000, $50 000, $50 000 and the boss gets $200 000.

- **a** Which group of wages includes an outlier? What is its value?
- **b** Find the mean wage of each business.
- **c** How much larger is the mean wage of Business B than the mean wage of Business A?
- **d** State the median wage of each business.
- **e** Has the outlier affected the median wage?
- **f** Which measure better shows how much the workers are paid in each business, the mean or median? Give a reason for your answer.

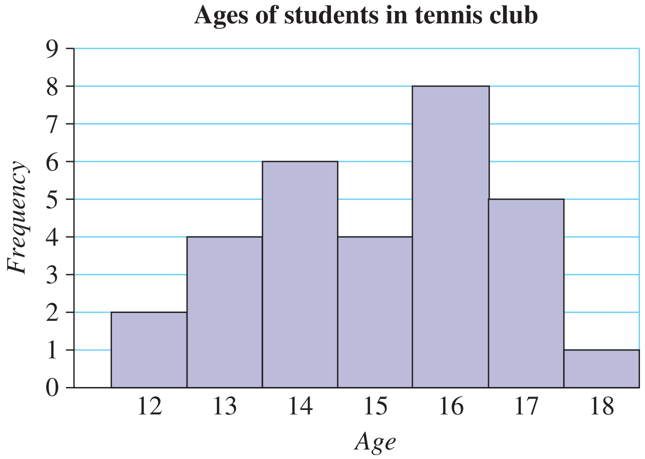
An outlier is a value much larger (or smaller) than the other values.

13 Gary and Nathan compare the number of runs they score in cricket over a number of weeks.

Gary: $17, 19, 17, 8, 11, 20, 5, 13, 15, 15$
Nathan: $39, 4, 26, 28, 23, 18, 37, 18, 16, 20$

- **a** Calculate Gary's range.
- **b** Calculate Nathan's range.
- **c** Who has the greater range?
- **d** Which cricketer is more consistent, on the basis of their ranges only?

14 The histogram at right shows the ages of all students in a school's tennis club.



a List all the ages from smallest to largest.

b What is the range of the ages in the tennis club?

c What is the most common age?

d Calculate the mean age correct to two decimal places.

e Calculate the median age.

f Now include the teacher's age of 52 in the list of ages.

- **i** Find the new mean age.
- **ii** Find the new median age.
- **iii** Which measure has changed the most, the mean or the median?

House for sale

— 15

15 The prices of all the houses in School Court are recorded: \$520 000, \$470 000, \$630 000, \$580 000, \$790 000, \$540 000, \$710 000, \$8.4 million, \$660 000.

a What is the mean house price in School Court, correct to the nearest dollar?

b What is the median house price in School Court?

c What effect does having a single \$8.4 million mansion in School Court have on the mean?

d What effect does having a single \$8.4 million mansion in School Court have on the median?

e Why might 'median house price' be a more useful measure than 'mean house price' when people are looking at living in a particular area?

8E Surveying and sampling

Interactive

Widgets

HOTsheets

Walkthrough

To find information about a large number of people it is generally not possible to ask everybody to complete a survey, so instead a sample of the population is chosen and surveyed. It is hoped that the information given by this smaller group is representative of the larger group of people. Choosing the right sample size and obtaining a representative sample are harder than many people realise.

Let's start: Average word length

To decide how hard the language is in a book, you could try to calculate the average length of the words in it. Because books are generally too large to record the length of every word, instead you can choose a smaller sample. For this exercise, you must decide or be assigned to the 'small sample', 'medium sample' or 'large sample' group. Then:

1 Pick a page from the book at random.

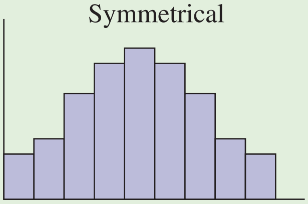
2 Find the average (mean) length of any English words on this page, choosing the first 10 words if you are in the 'small sample' group, the first 30 words for the 'medium sample' group and the first 50 words for the 'large sample' group.

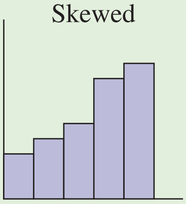
Discuss as a class:

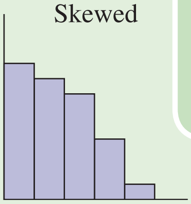
- Which of the groups would have the best estimate for the average word length in the book?
- What are the advantages and disadvantages of choosing a large sample?
- Does this sample help to determine the average length of words in the English language?
- How could the results of a whole class be combined to get the best possible estimate for average word length in the book?
- If all students are allowed to choose the page on which to count words, rather than choosing one at random, how could this bias the results?

Key ideas

- A **population** is the entire group that we are interested in. For example, if we want to find the average height of 14-year-old girls in Australia the population is all the 14-year-old girls in Australia.
- A **sample** is a small group randomly selected out of a population. For example, a sample could be 100 randomly chosen 14-year-old Australian girls.
- A **survey** is a set of questions used in a sample to obtain information about a larger group.
- The accuracy of the survey's conclusion can be affected by:
  - the **sample size** (number of participants or items considered)
  - whether the sample is **representative** of the larger group, or **biased**
  - whether there were any **measurement errors**, which could lead to **outliers** – values that are noticeably different from the other values.
- Data represented as a histogram can be seen as **symmetrical** or **skewed**.







- If a data distribution is symmetrical, the mean and the median are approximately equal.

Population The entire group is selected

Sample A small group randomly selected out of the population

Survey A set of questions that are asked of the people in a sample

Exercise 8E

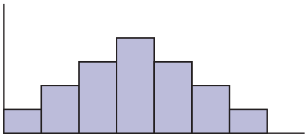
UNDERSTANDING 1–4 4

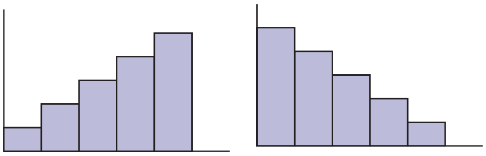
1 Write down the missing word from each statement. Choose from: *sample, symmetrical, skewed, survey* or *biased*.

a A __________ is a set of questions.

b A small group out of a population is called a __________.

c A __________ sample doesn't represent the population.

d  This histogram has a __________ shape.

e  These histograms have a __________ shape.

2 Marieko wishes to know the average age of drivers in her city. She could survey 10 of her friends, or survey 1000 randomly selected drivers.

a Which of these options would give a more accurate result?

b Which would be easier for Marieko to perform?

Example 10 Calculating population numbers from random sample data

Out of a random sample of 10 Tasmanian devils, there are 7 that have a facial tumour.

a What proportion of this population has facial tumours?

b If there are 200 Tasmanian devils in this region, on the basis of this sample, how many would you expect to have facial tumours?

c If there are 100 Tasmanian devils in this region, how many would you expect not to have a facial tumour?

Solution	Explanation
a $\frac{7}{10}$	7 have tumours out of a total of 10.
b $\frac{7}{10} \times 200 = 140$	The sample proportion × 200.
c $\frac{3}{10} \times 100 = 30$	3 out of 10 don't have a tumour.

3 Ajith looks at a random sample of penguins and notes that of the 50 he sees, 20 of them have spots on their bodies.

a What proportion of the population has spots?

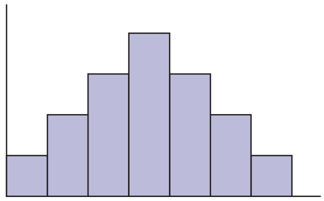
b If there are 5000 penguins in a region, on the basis of this sample how many would you expect to have spots on their bodies?

c If there are 500 penguins in a region, how many would you expect to not have spots on their bodies?

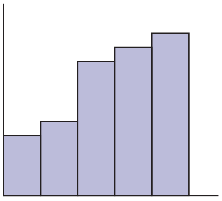
A proportion can be written as a fraction.

4 Classify the following distributions as symmetrical or skewed.

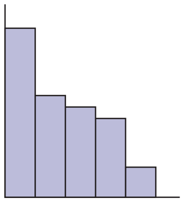
a



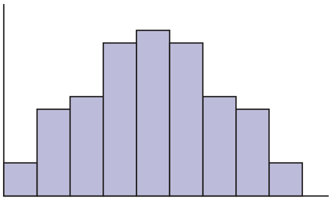
b



c



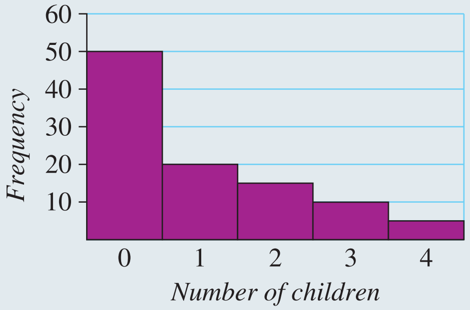
d



FLUENCY 5–7 5, 7, 8

Example 11 Interpreting survey results

A survey is conducted asking 100 randomly selected adults how many children they have. You can assume that this sample is representative of the adult population. The results are shown in this histogram:



a Is this distribution symmetrical or skewed?

b What proportion of the adult population has two or more children?

c In a group of 9000 adults, how many would you expect to have 4 children?

d Which of the following methods of conducting the survey could lead to bias? Give a reason why.

Method 1	Asking people waiting outside a childcare centre
Method 2	Randomly selecting people at a night club
Method 3	Choosing 100 adults at random from the national census and noting how many children they claimed to have.

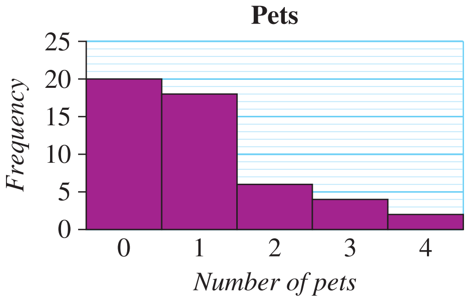
Solution	**Explanation**
a Skewed	Many more people have 0 children, so the distribution is not symmetrical.
b $\frac{3}{10}$	$15 + 10 + 5 = 30$ adults have two or more children Proportion $= \frac{30}{100} = \frac{3}{10}$
c $\frac{1}{20} \times 9000 = 450$	In the survey $\frac{5}{100} = \frac{1}{20}$ of the population have four children.

d Method 1 could lead to bias. If someone is waiting outside a childcare centre they are more likely to have at least one child.
Method 2 could lead to bias. If someone is at a night club they are likely to be a younger adult, and so less likely to have a child.

5 A survey is conducted asking 50 people how many pets they own. You can assume it is a representative sample of the population. The results are shown in this histogram.

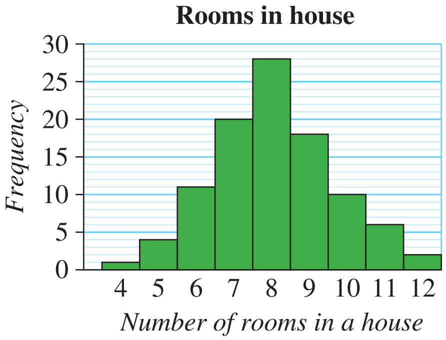
Expected number = proportion × total

- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people had no pets?
- **c** Of a group of 1000 people, how many of them would you expect to have no pets?
- **d** What proportion of people had 2 or more pets?
- **e** Of a group of 5000 people, how many of them would you expect to have 2 or more pets?
- **f** Why would conducting this survey outside a veterinary clinic cause a bias in the results?



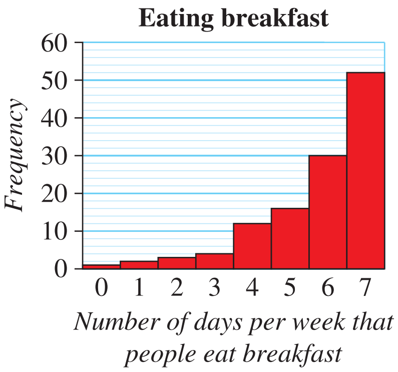
6 A survey of 100 randomly selected people, who live in a house, asked how many rooms were in their house. You can assume that it is a representative sample of the population. The results are shown in this histogram.

- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people live in an 8-room house?
- **c** In a group of 1500 people, how many would you expect to live in an 8-room house?
- **d** What proportion of people live in a house with 5, 6 or 7 rooms?
- **e** In a group of 3000 people, how many would you expect to live in a house with 5, 6 or 7 rooms?
- **f** Why would conducting this survey in a wealthy suburb cause bias in the results?



7 A survey of 120 randomly selected people asked how many days per week each person ate breakfast. You can assume that it is a representative sample of the population. The results are shown in this histogram.

- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people eat breakfast 7 days a week?
- **c** In a group of 36 000 people, how many would you expect to eat breakfast 7 days a week?
- **d** What proportion of people eat breakfast 4 or 5 days a week?
- **e** In a group of 4800 people, how many would you expect to eat breakfast 4 or 5 days a week?
- **f** Why would conducting this survey on a 6 am suburban train to the city cause bias in the results?



8 In a factory producing chocolate bars, a sample of bars is taken and automatically weighed to check whether they are between 50 and 55 grams. The results are shown in a frequency table.

Weight (g)	49	50	51	52	53	54	55	108
Frequency	2	5	10	30	42	27	11	1

a Which weight value is an outlier?

b If you leave out the 108 gram result, is this distribution skewed or symmetrical?

c What proportion of chocolate bars are 53g, 54g or 55g?

d In a batch of 800 chocolate bars how many would be expected to be 53g, 54g or 55g?

e What proportion of chocolate bars are less than 52g?

f In a batch of 2048 chocolate bars, how many would be expected to be less than 52g?

Add up the frequencies to find the total number of chocolate bars in the sample.

PROBLEM-SOLVING AND REASONING

9, 10 | 10–12

9 Fred attempts to find a relationship between people's ages and their incomes. He is considering some questions to put in his survey. For each question, decide whether it should be included in the survey, giving a brief explanation.

a What is your current age in years?

b Are you rich?

c Are you old?

d How much money do you have?

e What is your name?

f How much money did you earn in the past year?

g How much money did you receive today?

10 For each of the following survey questions, give an example of an unsuitable location and time to conduct the survey if you wish to avoid a bias.

a A survey to find the average number of children in a car

b A survey to find how many people are happy with the current prime minister

c A survey to find the proportion of Australians who are vegetarians

d A survey to find the average cost of supermarket groceries.

11 A survey is being conducted to decide how many adults use Mathematics later in life.

a If someone wanted to make it seem that most adults do not use Mathematics, where and when could they conduct the survey?

b If someone wanted to make it seem that most adults use Mathematics a lot, where and when could they conduct the survey?

c How could the survey be conducted to provide less biased results?

12 Robert wishes to find out how much time high school students spend on homework.

a Give some reasons why surveying just his Maths class might introduce bias.

b Why would surveying just the people on his soccer team introduce bias?

c Give a reason why surveying 10 students would not be a representative sample.

d He decides to choose 50 people from across the whole school. Who should he choose in order to minimise the bias? Justify your answer.

8E

Design a survey and graph sample results

—	13

13 **Task 1**

Design survey questions to find out the following information.

- The mean number of siblings of the students in your class.
- The mean number of car trips made to school each week by families in your class.
- The mean number of computers owned by families in your class.

Task 2

Write down how you will choose an unbiased sample of students for your survey.
Run the survey on your chosen sample students. Keep a record of all results.

Task 3

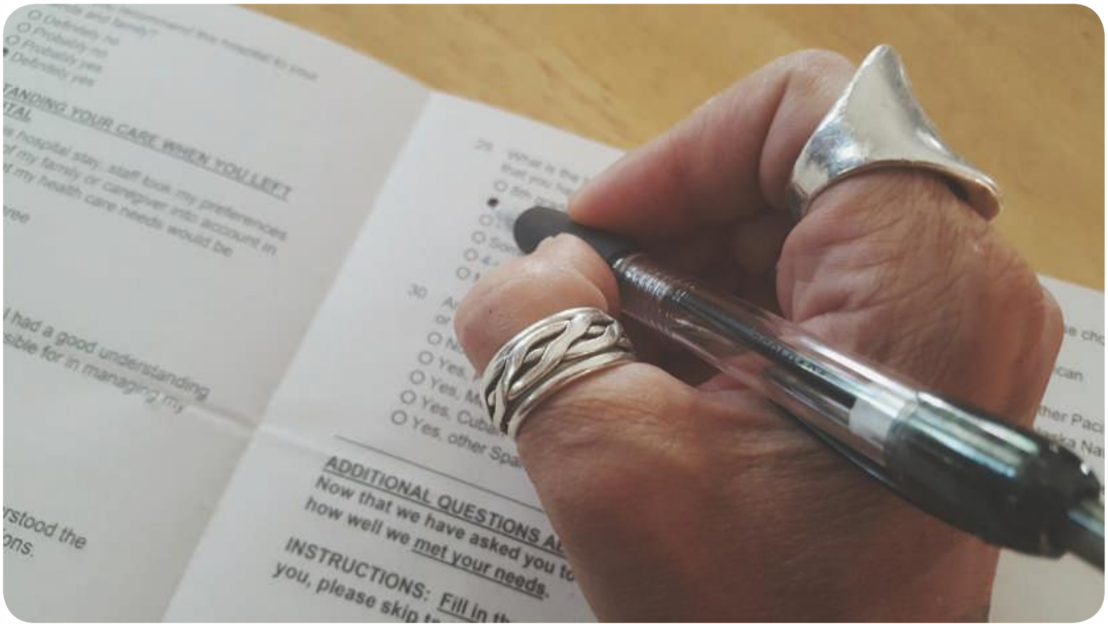
In an Excel spreadsheet, record your results as tables showing the frequency of each answer. Use Excel to draw a histogram for each table. Comment on whether each set of data is symmetrical or skewed.

Task 4

In each table add a column for the proportions and enter the proportion that each frequency is of the total.
Multiply these proportions by the total number of students in Year 8 at your school to find the expected numbers from your year level.

Task 5

For each set of data, use an Excel spreadsheet to help you to find the expected mean for the students in your year level. Write your conclusions in sentences.



8F Probability

Interactive

Widgets

HOTsheets

Walkthrough

Most people would agree that being hit by lightning and getting rained upon are both possible when going outside, but that rain is more likely. **Probability** gives us a way to describe how much more likely one event is than another. A probability is a number between 0 and 1 where 0 means 'impossible' and 1 means 'certain'.

If the outcomes are equally likely, we find the probability of an event by counting the ways it can happen and dividing by the total number of outcomes.

Let's start: Estimating probabilities

Try to estimate the probability of the following events, giving a number between 0 and 1. Compare your answers with other students in the class and discuss any differences.

1. Flipping a 'tail' on a 50-cent coin.
2. An albino whale is born.
3. Rolling three 6s in a row on a fair die.
4. Correctly guessing a number between 1 and 10.
5. Tomorrow being a rainy day.
6. Seeing a wombat in the Australian bush.

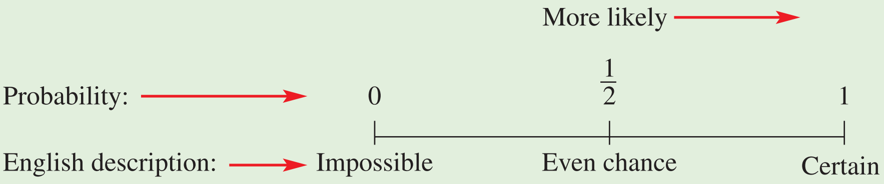
Are there some events for which there is more than one correct answer?

Key ideas

- An **experiment** is a situation involving chance which leads to a set of results.
- A **trial** is a process which can be repeated to produce results. Examples could be flipping a coin, rolling a die or spinning a spinner.
- An **outcome** is a possible result from an experiment; for example, 'rolling a 3' or 'flipping tails on the coin'.
- An **event** is a single outcome (e.g. rolling a 3) or a collection of outcomes (e.g. rolling a 3, 4 or 5).
- The **probability** of an event is a number between 0 and 1 that represents the chance that the event occurs. If all the outcomes are equally likely:

$$\text{Pr(Event)} = \frac{\text{number of outcomes where the event occurs}}{\text{total number of outcomes}}$$

- Probabilities are often written as fractions, but can also be written as decimals or percentages.



Trial One run of an experiment

Outcome One of the possibilities from an event

Event A collection of possible outcomes

Probability A measure of the likelihood that an event will occur

- The **sample space** is the set of possible outcomes of a trial or event. For example, the sample space for the roll of a die is $1,\ 2,\ 3,\ 4,\ 5,\ 6$.
- The **complement** of some event E is written E' (or not E). E' is the event that E does not occur. For example, the complement of 'rolling the number 3' is 'rolling a number other than 3'. Note: $\text{Pr}(E) + \text{Pr}(E') = 1$.
- The following language is also commonly used in probability.
  - 'at least', for example, 'at least 3' means $3,\ 4,\ 5, \ldots$.
  - 'at most', for example, 'at most 7' means $\ldots,\ 5,\ 6,\ 7$.
  - 'or', for example, 'rolling an even number or a 5' means rolling $2,\ 4,\ 5$ or 6.
  - 'and', for example, 'rolling an even number and a prime number' means rolling a 2.

Sample space The list of all the possible outcomes of an event

Complement A set containing the elements that are not in a given set

Exercise 8F

UNDERSTANDING 1–4 4

1 Write the missing word from each statement. Choose from: *outcomes*, *A'*, *complement*, *trial* or *sample space*.

a An example of a __________ is flipping a coin.

b After rolling a die the possible __________ are $1,\ 2,\ 3,\ 4,\ 5$ and 6.

c The set of all possible outcomes from a trial is called the __________ __________.

d The __________ of an event is the opposite of that event.

e If an event is called A then the complement is written as __________.

2 Match each experiment with the set of possible outcomes.

a Flipping a coin

b Choosing a number between 1 and 5

c Choosing a letter of the word MATHS

d Rolling a die

A $1,\ 2,\ 3,\ 4,\ 5,\ 6$ **B** Heads, Tails **C** $1,\ 2,\ 3,\ 4,\ 5$ **D** M, A, T, H, S

3 The following events are shown with their probabilities.

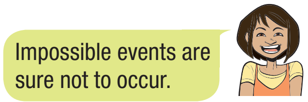
Event A: 0 Event B: 0.9 Event C: 1 Event D: 0.5

a Which of the four events is most likely to occur?

b Which of the four events is sure not to occur?

c Which is more likely – event B or event D?

d Which event is sure to occur?



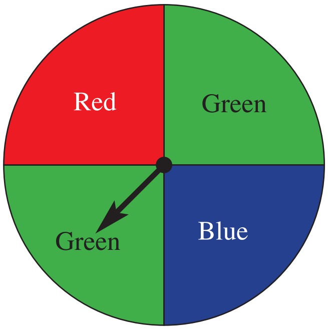
4 The spinner is spun and could land with the pointer on any of the four sections. Answer true (T) or false (F):

a Red and blue are equally likely outcomes.

b Green is less likely to occur than blue.

c The probability of it landing orange is 0.

d Red is less likely to occur than green.



FLUENCY	5–7	5, 6, 8

Example 12 Working with probabilities

The letters of the word PRINCE are written onto 6 equally sized cards and one is chosen at random.

a State the sample space.

b Find Pr(the letter N is chosen).

c What is the sample space of the event $V =$ choosing a vowel?

d Find Pr(V).

e State the sample space of the complement of choosing a vowel, written V'.

f Find Pr(V').

Solution	**Explanation**
a P, R, I, N, C, E	The sample space is all the possible outcomes when a single card is chosen. In this case each of the letters in the word.
b $\text{Pr}(N) = \frac{1}{6}$	There are 6 equally likely cards and 1 of them has the letter N.
c I, E	The sample space V includes all the vowels in the word PRINCE.
d $\text{Pr}(V) = \frac{2}{6}$ $= \frac{1}{3}$	There are 2 cards with vowels, so probability $= 2 \div 6$.
e V' includes P, R, N, C	The complement of V (V') is all the outcomes that are not in V, i.e. all the letters that are not vowels.
f $\text{Pr}(V') = \frac{4}{6}$ $= \frac{2}{3}$	There are 4 cards that do not have vowels, so $\text{Pr}(V') = 4 \div 6$.

5 The letters of the word PIANO are written on 5 cards and then one card is drawn from a hat at random.

Pr means probability.

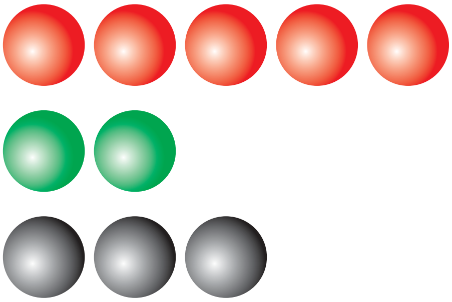
Write probability answers as fractions.

- **a** List the sample space.
- **b** Find Pr(the letter A is chosen).
- **c** Find Pr(a vowel is chosen).
- **d** Find Pr(a consonant is drawn).
- **e** Find Pr(the letter chosen is not an N).
- **f** State the sample space of the complement of choosing a vowel, written V'.
- **g** Find Pr(V').

6 A fair die is rolled.

- **a** List the sample space.
- **b** Find Pr(5). That is, find the probability that a 5 is rolled.
- **c** Find Pr(even number).
- **d** State the sample space of the complement of 'rolling a 5'.
- **e** State the probability that a 5 is not rolled.
- **f** What is the probability of rolling a 14?

7 There are five red marbles, two green marbles and three black marbles. The 10 marbles are placed into a hat and one is picked out.



- **a** What is Pr(red)? That is, what is the probability that the picked marble is red?
- **b** Find Pr(green).
- **c** Find Pr(black).
- **d** Find Pr(a black or a red marble is drawn).
- **e** Find Pr(red′), that is find the probability of the complement of choosing a red marble.
- **f** Find Pr(black′).
- **g** Give an example of an event that has a probability of 0.

8 The numbers 1 to 10 are written on cards. A card is chosen at random.

- **a** List the sample space.
- **b** Find the probability of choosing a 5.
- **c** Find Pr(7 or 9).
- **d** Find Pr(a multiple of 3 is chosen).
- **e** Find Pr(prime number).
- **f** Find Pr(a factor of 24).

A factor of 24 divides into 24 with no remainder. A prime has 2 factors. 1 is not prime.

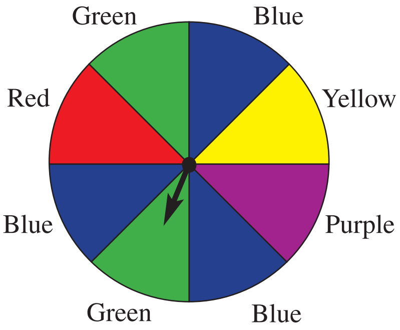
PROBLEM-SOLVING AND REASONING

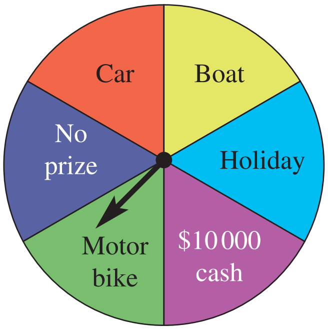
9–11 10–13

9 A spinner has the arrangement of colours as shown.

- **a** List the sample space when this spinner is spun.
- **b** Find Pr(red).
- **c** State Pr(green).
- **d** Find Pr(blue).
- **e** List the sample space of the complement of 'spinner landing on blue'.
- **f** What is Pr(not blue)?
- **g** Find Pr(red or green or blue).
- **h** What is an event that is equally likely to 'spinning red'?
- **i** Give an example of an event that has a probability of 0.

List the colour as many times as it is on the spinner.





10 On a game show, a wheel is spun for a prize with the options as shown.

- **a** Joan wants to go on a $10 000 holiday so she is happy with the cash or the holiday. What is the probability she will get what she wants?
- **b** What is the probability of getting a prize that is not the cash?
- **c** What is Pr(car or motorbike)?
- **d** What is the probability of winning a prize?

11 Each of the numbers 1 to 10 are written on 10 cards and one card is chosen at random. Find the following probabilities.

- **a** Pr(even)
- **b** Pr(3 or even)
- **c** Pr(3 and even)
- **d** Pr(at least 6)
- **e** Pr(at most 5)
- **f** Pr(prime or even)

12 Six counters coloured red, purple or orange are placed in a pocket. You are told that

$$\text{Pr(red or orange)} = \frac{1}{2} \text{ and Pr(red or purple)} = \frac{2}{3}$$

- **a** How many counters of each colour are there?
- **b** State Pr(red).
- **c** Find Pr(purple).
- **d** Find Pr(orange′).

Change the probabilities to have a common denominator.

13 Draw a spinner that has $\text{Pr(red)} = \frac{1}{8}$, $\text{Pr(blue)} = \frac{5}{8}$ and $\text{Pr(green)} = \frac{1}{4}$.

First divide a circle into 8 equal sectors.

Changing probabilities

— 14

14 In a large bucket there are 2 red balls and 8 blue balls.

- **a** State Pr(red).
- **b** One of each colour is added. What is the new Pr(red)?
- **c** The procedure of adding a red ball and a blue ball is repeated several times. How many balls are in the bucket when $\text{Pr(red)} = \frac{1}{3}$?
- **d** Imagine the procedure is repeated many times. What value does Pr(red) eventually approach as more balls are added? It might be helpful to imagine 1000 balls of each colour are added and use decimals.

Make a table.

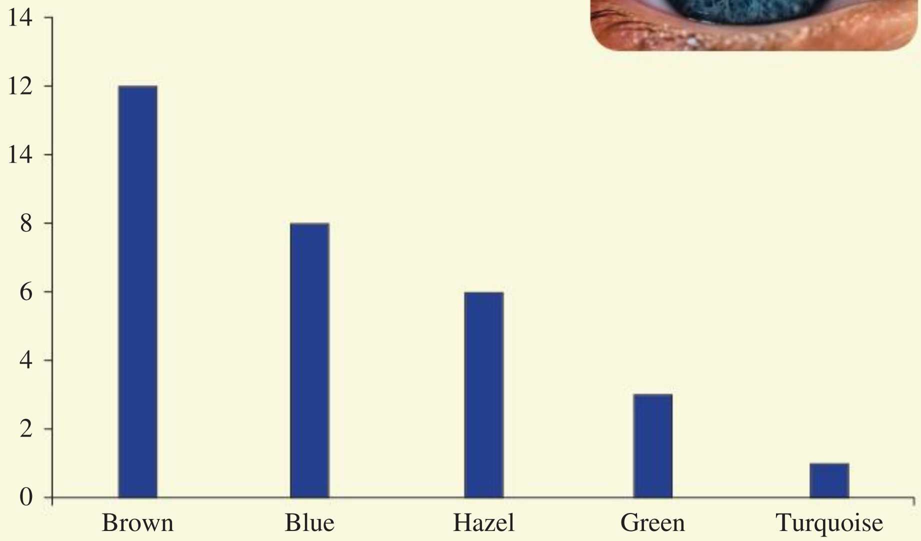
Progress quiz

8A **1** A class of Year 8 students was surveyed for their eye colour and the results are shown in the column graph below.

a How many students are in the class?

b How many students have blue or hazel eyes?

c How many more students have brown eyes than blue eyes?



8B **2** Put the following data into a frequency table.

2, 5, 3, 3, 3, 5, 1, 4, 2, 5, 2, 1, 1, 3, 2, 2, 2, 5, 2, 4

8C **3** Represent this frequency table below as a histogram.

Number of broken bones	Frequency
0	12
1	7
2	2
3	4
4	1

8D **4** Find the range of the following sets of data.

a 2, 11, 3, 6, 7, 15, 3, 4, 8

b 12, 7, –10, –6, 29, 32, 3, 0, –11, –3, 1, 16, 18

8D **5** For each of the following sets find: **i** the mean **ii** the mode

a 1, 2, 5, 4, 4, 3, 2, 2, 2, 5

b –5, –4, –8, –1, 0, 0, –3, –4, –2, –7, –4, 2

Progress quiz

8D

6 For each of the following sets calculate the median.

a 1, 4, 6, 12, 15, 17, 23

b 53, 56, 57, 57, 61, 67, 68, 85

c 3, 11, 6, 4, 5

d –5, –8, 1, 15, –2, –4

8D

7 Paschmal cares for 8 hens that generally lay an egg each day. Paschmal has recorded, in the frequency table below, the number of eggs he has collected over the past 4 weeks.

Number of eggs	5	6	7	8
Frequency	11	7	9	1

a In how many days did each hen lay an egg?

b What was the most common number of eggs Paschmal collected?

c List out all the number of eggs Paschmal collected over the 28 days.

d What is the mean number of eggs collected over the 28 days?

e What is the median number of eggs collected over the 28 days?

f What is the range of the number of eggs Paschmal collected over the 28 days?

8E

8 Marcia spends an hour bird watching and notes that of the 25 she sees, 15 of them are a type of wren.

a From this sample, what proportion of the population of birds in this area do we think are wrens?

b If there are 1000 birds in this area, on the basis of this sample how many would be wrens?

c If there are 30 000 birds in this area, on the basis of this sample how many birds that are not wrens are there?

8F

9 The letters of the word RESILIENCE are written on 10 cards and placed in a hat. One card is then drawn from the hat at random.

a List the sample space.

b Find Pr(the letter E is chosen).

c Find Pr(a consonant is chosen).

d Find Pr(the letter I is not chosen).

e State the probability that a letter with some curved lines is not drawn.

8F

10 Matty has a bowl full of 50 coloured lollies—20 red, 5 yellow, 6 orange, 10 green and the remainder are purple.

a How many purple lollies are in the bowl?

b Matty eats two lollies of each of the different colours. How many lollies are left in the bowl?

c Matty then gives his friend 5 of his favourite red lollies. How many red lollies are now left in the bowl?

d Matty now decides to close his eyes and pick a lolly at random from the bowl. What is the probability Matt chooses a purple one?

8G Venn diagrams

When two events are being considered a Venn diagram gives another way to view the probabilities. They are especially useful when survey results are being considered and converted into probabilities.

Let's start: Free dress day

Work with a classmate and help each other to answer the questions in each activity.

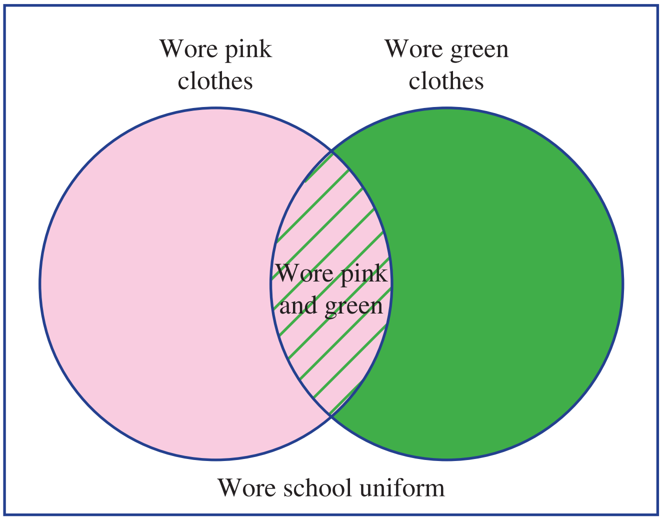
On a free dress day a Year 8 class decided to wear either pink only or green only or both pink and green. A few students came in their school uniform.

This is how the students dressed:

- 9 wore pink only
- 5 wore green only
- 8 wore both pink and green
- 4 wore school uniform.

In Maths class that day, the students drew a Venn diagram showing the colours that the students dressed in on free dress day.

1 Copy this Venn diagram.



2 Write the number of students in each area that matches the colours worn.

3 How many students in total wore pink casual clothes?

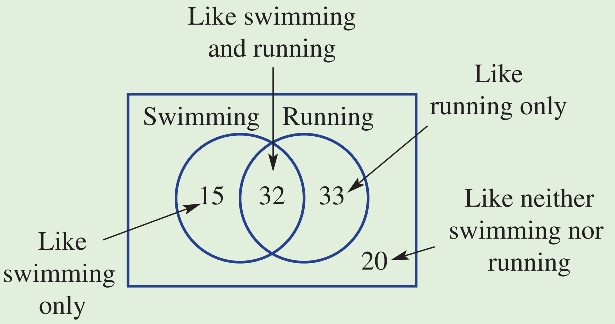
4 How many students in total wore green casual clothes?

5 How many students in total wore green casual clothes or pink casual clothes or both?

6 How many students altogether are in this class?

Key ideas

- A **Venn diagram** is a pictorial representation using overlapping circles showing the number of objects in two or more **categories**.
- The words ‘and’ and ‘or’ usually have the following meanings in the area of probability. For two events A and B then:
  - an outcome belongs to ‘A and B’ if it belongs to both
  - an outcome belongs to ‘A or B’ if it belongs to A or B or both.
- Venn diagrams can be used to find probabilities.



Pr(swimming) = $\frac{47}{100}$

Pr(swimming and running) = $\frac{32}{100}$

Pr(swimming or running) = $\frac{80}{100}$

Venn diagram A diagram using circles to show the relationships between two or more sets

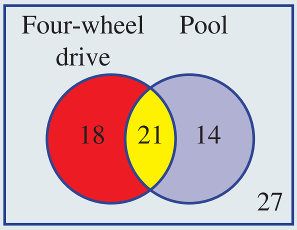
Categories Different types of groups

Exercise 8G

UNDERSTANDING 1–3 3

Example 13 Interpreting a Venn diagram

This Venn diagram shows the results of a survey that asked whether a family has a four-wheel drive car, a pool or both or neither.



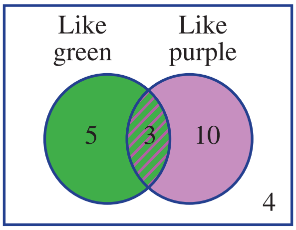
a How many families own a four-wheel drive car?
b How many families own a pool?
c How many families own both a four-wheel drive car and a pool?
d How many families own a pool or a four-wheel drive car or both?
e How many families don’t own either a four-wheel drive car or a pool?
f How many families were in this survey?

Solution	**Explanation**
a 39 families own a four-wheel drive	Add all the numbers in the four-wheel drive circle, $18 + 21 = 39$
b 35 families own a pool	Add all the numbers in the pool circle, $21 + 14 = 35$

Solution	Explanation
c 21 families own both	21 is where the circles overlap. So 21 families own a pool and a four wheel drive
d 53 families own either or both	Add all the numbers across both circles, $18 + 21 + 14 = 53$
e 27 families do not own either	27 is on the outside of the circles.
f 80 families in total	Add all the numbers in the Venn diagram $18 + 21 + 14 + 27 = 80$

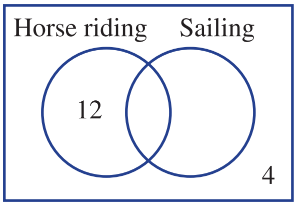
1 Here is a Venn diagram showing how many students like green or purple as their favourite colour.

- **a** How many students like green?
- **b** How many students like purple?
- **c** How many students like both green and purple?
- **d** How many students like either green or purple or both?
- **e** How many students don't like either green or purple?
- **f** How many students were in this survey?



2 A class of Year 8 students were asked who liked sailing and who liked horse riding. Some of the results of this survey are in this Venn diagram.

- **a** Copy this Venn diagram and complete it by writing these numbers in the correct parts.
  - **i** 6 students liked both horse riding and sailing.
  - **ii** 8 students liked sailing but didn't like horse riding.
- **b** How many students in total were in this class?

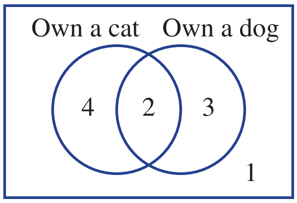


The overlap of the circles shows students who like both horse riding and sailing.

3 Look at the Venn diagram representing cat and dog ownership. State the missing number (1, 2, 3 or 4) to make the following statements true.

- **a** The number of people who own both a cat and a dog is _____.
- **b** The number of people who own a cat but do not own a dog is _____.
- **c** The number of people who own neither a cat nor a dog is _____.
- **d** The number of people who own a dog but do not own a cat is _____.

The number outside of the circles shows the people who don't own either a cat or a dog.



FLUENCY 4–7 5–8

4 A survey asked students if they liked oranges or bananas. Draw and label a Venn diagram showing the results of this survey as listed here.

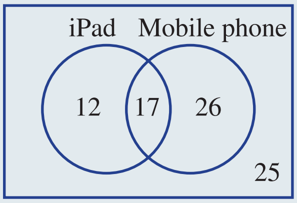
15 students liked only oranges
12 students liked both oranges and bananas
8 students liked only bananas
6 students prefer other fruit

5 In a group of 30 students it is found that 10 play both cricket and soccer, 5 play only cricket and 7 play only soccer.

- **a** How many people do not play cricket or soccer?
- **b** Represent the survey findings in a Venn diagram.
- **c** How many of the people surveyed play cricket?
- **d** How many of the people surveyed play either cricket or soccer or both?

Example 14 Using Venn diagrams to find probabilities

This Venn diagram shows the results of asking some Year 8 students whether they owned an iPad or a mobile phone.



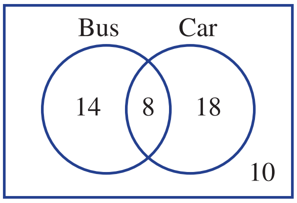
- **a** How many students in total were surveyed?
- **b** What proportion of students owned an iPad only?
- **c** What proportion of students owned both an iPad and a mobile phone?
- **d** What is the probability of choosing a student who owns a mobile phone?
- **e** What is the probability of choosing a student who owns an iPad or a mobile phone or both?

Solution	**Explanation**
a Total = 80	Add all the numbers so $12 + 17 + 26 + 25 = 80$
b $\frac{12}{80} = \frac{3}{20}$	$\frac{\text{iPad only}}{\text{total}}$ $= \frac{\text{number in the iPad circle but not in the overlap}}{\text{total}}$ $= \frac{12}{80} = \frac{3}{20}$
c $\frac{17}{80}$	$\frac{\text{iPad and mobile}}{\text{total}} = \frac{\text{number in the overlap of the circles}}{\text{total}}$ $= \frac{17}{80}$

Solution	**Explanation**
d Pr(mobile phone) = $\frac{43}{80}$	$\frac{\text{total number in the mobile phone circle}}{\text{total}} = \frac{43}{80}$
e Pr(mobile phone or iPad) $= \frac{55}{80}$ $= \frac{11}{16}$	$\frac{\text{total of mobile phone circle and iPad circle and overlap}}{\text{total}}$ $= \frac{12 + 17 + 26}{80} = \frac{55}{80} = \frac{11}{16}$

6 The Venn diagram below shows the results of surveying some Year 8 students about whether they travel to school by bus or car.

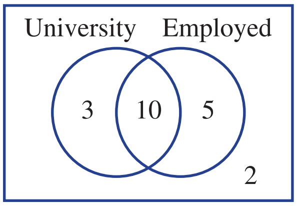
- **a** How many students in total were surveyed?
- **b** What proportion of students travelled by bus only?
- **c** What proportion of students travelled by both car and bus?
- **d** What is the probability of randomly choosing a student who travels by car?
- **e** What is the probability of randomly choosing a student who travels either by car or by bus or both?



Add all the numbers in the rectangle to find the total number of people surveyed.

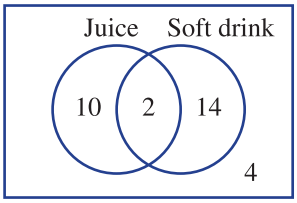
All proportions and probabilities are fractions out of the total number surveyed.

7 Look at this Venn diagram showing the number of people who have a university degree and the number who are now employed.



- **a** What is the total number of people surveyed?
- **b** What is the total number of people in the survey who are employed?
- **c** What proportion of people in the survey are employed?
- **d** What is the number of people who have a university degree and are also employed?
- **e** What is the probability of randomly choosing a person who has a university degree and is also employed?

8 The Venn diagram shows the number of people who like juice and/or soft drinks.



Add the numbers outside of the 'like juice' circle to find how many do not like juice.

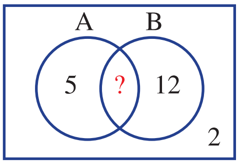
- **a** What is the total number of people who like neither juice nor soft drink?
- **b** What is the probability that a randomly selected person likes neither juice nor soft drink?
- **c** What is the probability that a randomly selected person likes either juice or soft drink or both?
- **d** What is the probability that a randomly selected person does not like juice?

PROBLEM-SOLVING AND REASONING

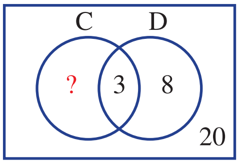
9, 10	9, 11, 12

9 Find all of the missing numbers (?) in each of these Venn diagrams.

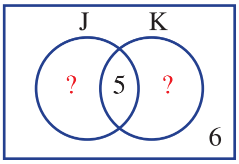
a Overall Total = 20



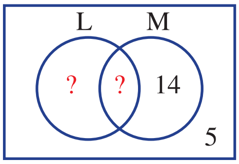
b Overall Total = 40

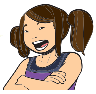


c Total of J = 18
Total of K = 21



d Total of L = 26
Total of M = 21

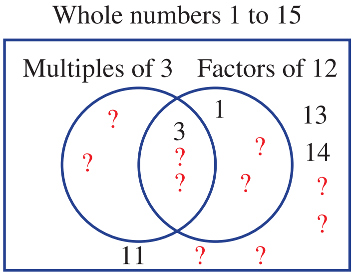




Remember that all the numbers in the Venn diagram add to the overall total.

The total in a circle is the sum of the two numbers in that circle.

10 **a** Copy and complete this Venn diagram by writing in the numbers missing from whole numbers 1 to 15.



Multiples of 3 are found when 3 is multiplied by whole numbers. So 3, 6 ….

b How many numbers that are multiples of 3 are also factors of 12?

c How many factors of 12 are not multiples of 3?

d Out of the numbers 1 to 15, what proportion (fraction) are multiples of 3?

e What is the probability of choosing a factor of 12 out of the numbers 1 to 15?

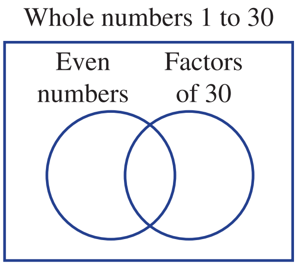
Factors of 12 are numbers that divide into 12 with no remainder.

11 For this question you will select from the numbers from 1 to 30.

a List the even numbers.

b List the factors of 30.

c Make a large copy of this Venn diagram and copy the *values* from your lists into the correct parts of the diagram.



d How many even numbers are also factors of 30?

e What is the probability of choosing an even number that is also a factor of 30 out of the numbers 1 to 30?

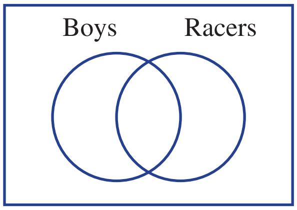
f How many even numbers are there in the Venn diagram?

g What is the probability of choosing an even factor of 30 out of the even numbers?

$\frac{?\,(\text{even factors of } 30)}{?\,(\text{even numbers})}$

12 In Year 8 at a school there are 40 boys, half of whom are in the racing club. Of the 100 students in Year 8, 35 are in the racing club.

a Copy and complete the Venn diagram below to describe the situation.



b What is the probability, as a percentage, that a randomly selected person in Year 8:

- **i** is a boy in the racing club?
- **ii** is a girl in the racing club?
- **iii** is not a boy?
- **iv** is not in the racing club?

c What proportion of Year 8 racers are boys?

d What proportion of Year 8 racers are girls?

e What is the probability of randomly choosing a Year 8 racer out of the Year 8 boys?

Write proportions as fractions in simplest form.

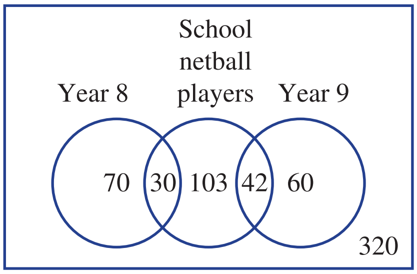
Triple Venn diagrams

— 13

13 This Venn diagram shows the numbers in Years 8 and 9 and the netball players in a school.

a How many students in total are at this school?

b What is the probability of randomly choosing a student in this school who is:
  i a Year 8 netball player?
  ii a Year 9 netball player?



c How many netball players in total are there at this school?

d Out of the netball players only, what is the probability of choosing a:
  i Year 8 netball player?
  ii Year 9 netball player?

e If a student is randomly chosen out of Year 8, what is the probability that the student is a netball player?

f In Year 10, there are 105 students in total and 32 play netball. Copy and complete the Venn diagram above including a new circle for Year 10s. Be careful to enter the numbers correctly.

g If a student is randomly chosen out of the netball students, what is the probability it is a Year 10 student?

8H Two-way tables

When two events are being considered, two-way tables give another way to view events and calculate the probabilities. They can be used alongside or instead of a Venn diagram.

Let's start: Are English and Mathematics enemies?

Conduct a poll among students in the class, asking whether they like English and whether they like Maths. Use a tally like the one shown.

	Like Maths	Do not like Maths
Like English	卌 |	卌 ||||
Do not like English	卌 卌 |	卌 ||

Use your survey results to debate these questions:

- Are the students who like English more or less likely to enjoy Maths?
- If you like Maths does that increase the probability that you will like English?
- Which is the more popular subject within your class?

Key ideas

A **two-way table** lists the number of outcomes or people in different categories, with the final row and column being the total of the other entries in that row or column. For example:

	Like Maths	Do not like Maths	Total
Like English	28	33	61
Do not like English	5	34	39
Total	33	67	100

Two-way table
A way of listing the number of outcomes in different categories

- A two-way table can be used to find probabilities:

  e.g. Pr(like Maths) = $\frac{33}{100}$, Pr(like Maths and not English) = $\frac{5}{100}$

Exercise 8H

UNDERSTANDING 1–4 4

Example 15 Completing and Interpreting a two-way table

a Copy and complete this two-way table.

	Like gardening	Dislike gardening	Total
Like cooking	24	16	40
Dislike cooking	12	4	
Total			

b How many people like cooking and like gardening?
c How many people like cooking and dislike gardening?
d How many people dislike cooking and dislike gardening?
e How many people like gardening?
f How many people were interviewed in this survey?

Solution	Explanation
a (table below)	Add the numbers in each column to get the column totals $24 + 12 = 36, 16 + 4 = 20$ Add the numbers in each row to get the row totals $24 + 16 = 40, 12 + 4 = 16$ The column totals add to 56 $(36 + 20 = 56)$ The row totals also add to 56 $(40 + 16 = 56)$
b 24 like cooking and gardening.	From the 'like cooking' row, move to the 'like gardening' heading, so 24.
c 16 like cooking but dislike gardening.	From the 'like cooking' row, move to the 'dislike gardening' heading, so 16.
d 4 dislike both cooking and gardening.	From the 'dislike cooking' row, move to the 'dislike gardening' heading, so 4.
e 36 like gardening.	Add the total under the like gardening heading, $24 + 12 = 36$
f 56 people in total.	The total number of people is 56, which is in the bottom corner.

	Like gardening	Dislike gardening	Total
Like cooking	24	16	40
Dislike cooking	12	4	16
Total	36	20	56

1 **a** Copy and complete the two-way table by writing in the missing totals.

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	
Total		35	75

b How many people like both apples and bananas?

c How many people dislike apples and dislike bananas?

d How many people were surveyed?

2 Copy and complete the two-way table below using this information:

- 23 students like Anzac biscuits and also like lamingtons
- 14 students like Anzac biscuits but dislike lamingtons
- 12 students like lamingtons but dislike Anzac biscuits
- 3 students dislike lamingtons and also dislike Anzac biscuits

	Like lamingtons	Dislike lamingtons	Total
Like Anzac biscuits			
Dislike Anzac biscuits			
Total			

3 Here is a two-way table showing the results of surveying some teenagers about exercise. Use the numbers in this table to answer the questions below.

	Like jogging	Dislike jogging	Total
Like cycling	15	10	25
Dislike cycling	12	3	15
Total	27	13	40

a How many teenagers like jogging?

b How many teenagers like both jogging and cycling?

c How many teenagers like jogging but dislike cycling?

d How many teenagers like cycling?

e How many teenagers like cycling but dislike jogging?

f How many teenagers were surveyed?

The number who 'likes jogging' is equal to the total of the column 'like jogging'.

4 Answer the questions below about this two-way table:

	Like skateboards	Dislike skateboards	Total
Like BMX bikes	24	12	36
Dislike BMX bikes	15	9	24
Total	39	21	60

If a student is chosen randomly from this group, find these probabilities and simplify your answers.

- **a** Pr(like skateboards but dislike BMX bikes)
- **b** Pr(like BMX bikes and like skateboards)
- **c** Pr(dislike skateboards but like BMX bikes)
- **d** Pr(dislike BMX bikes and also dislike skateboards).

Write probabilities as a fraction out of the total (60) and then simplify this fraction.

FLUENCY 5–7 5–8

Example 16 Completing a two-way table

Copy and complete this two-way table.

	Like Macs	Dislike Macs	Total
Like PCs			
Dislike PCs		3	13
Total	35	40	75

Solution

	Like Macs	Dislike Macs	Total
Like PCs	25	37	62
Dislike PCs	10	3	13
Total	35	40	75

Explanation

Start with a row or column that has only one number missing.

'dislike PCs' row, $10 + 3 = 13$

'dislike Macs' column, $37 + 3 = 40$

Now complete 'like Macs' column, $25 + 10 = 35$

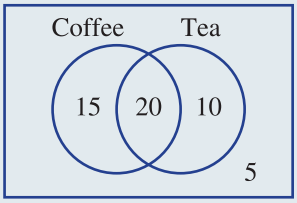
The total in the 'like PCs' row is $25 + 37 = 62$

5 Copy and complete this two-way table.

	Like surfing	Dislike surfing	Total
Like hiking			
Dislike hiking		5	30
Total	70	15	85

Example 17 Constructing two-way tables from Venn diagrams

Consider this Venn diagram showing the number of people who like coffee and who like tea.



a Represent the survey findings in a two-way table.

b How many people liked neither tea nor coffee?

c How many people surveyed like tea?

d How many people like both coffee and tea?

e How many people like coffee or tea (or both)?

Solution | **Explanation**

a

	Like coffee	Dislike coffee	Total
Like tea	20	10	30
Dislike tea	15	5	20
Total	35	15	50

The two-way table has the four numbers from the Venn diagram and also a 'total' column (e.g. $20 + 10 = 30, 15 + 5 = 20$) and a 'total' row. Note that 50 in the bottom corner is both $30 + 20$ and $35 + 15$.

b 5 do not like either tea or coffee.

$50 - 20 - 15 - 10 = 5$ people who do not like either.

c $20 + 10 = 30$ like tea.

10 people like tea but not coffee, but 20 people like both. In total 30 people like tea.

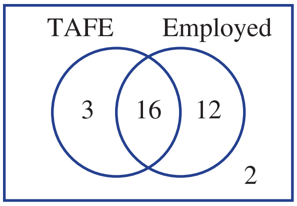
d 20 like both coffee and tea.

20 out of 50 people like both coffee and tea.

e 45 like tea or coffee or both.

$15 + 20 + 10 = 45$ people like either coffee or tea or both.

6 Look at this Venn diagram showing the number of people who have a TAFE degree and the number who are now employed.



a Copy and complete the two-way table shown below.

	Employed	Unemployed	Total
TAFE degree			
No TAFE degree			
Total			

b How many people were unemployed with no TAFE degree?

c How many people surveyed were employed?

d How many people had a TAFE degree and were also employed?

e How many people had a TAFE degree or were employed (or both)?

Example 18 Using two-way tables to calculate probabilities

Consider the two-way table below showing the eating and sleeping preferences of different animals at the zoo.

	Eats meat	No meat	Total
Sleeps during day	20	12	32
Only sleeps at night	40	28	68
Total	60	40	100

For a randomly selected animal find:

a Pr(sleeps only at night)

b Pr(eats meat or sleeps during day)

Solution

a Pr(sleeps only at night)

$$= \frac{68}{100}$$

$$= \frac{17}{25}$$

Explanation

The total of animals that sleep at night is 68.

So $\frac{68}{100} = \frac{17}{25}$.

Solution

b Pr(eats meat or sleeps during day)

$$= \frac{72}{100}$$

$$= \frac{18}{25}$$

Explanation

$20 + 12 + 40 = 72$ animals eat meat or sleep during the day (or both). $\frac{72}{100} = \frac{18}{25}$.

7 The two-way table below shows the results of a poll conducted of a group of students who own mobile phones to see who pays their own bills.

	Boys	Girls	Total
Pay own bill	4	7	11
Do not pay own bill	8	7	15
Total	12	14	26

a How many people participated in this poll?

b How many boys were surveyed?

c How many of the people surveyed pay their own bill?

d Find the probability that a randomly selected person:

  i is a boy who pays his own bill

  ii is a girl who pays her own bill

  iii is a girl

  iv does not pay their own bill.

8 The two-way table shows the results of a survey on car and home ownership at a local supermarket.

	Own car	**Do not own car**	**Total**
Own home	8	2	10
Do not own home	17	13	30
Total	25	15	40

a Find Pr(randomly selected person owns a car and a home).

b Find Pr(randomly selected person owns a car but not a home).

c What is the probability that a randomly selected person owns their own home?

d What is the probability that a randomly selected person does not own a car?

Pr means probability.

PROBLEM-SOLVING AND REASONING 9, 10 10–12

9 Copy and complete the following two-way tables.

a

	B	**Not B**	**Total**
A	20		70
Not A			
Total		60	100

b

	B	**Not B**	**Total**
A		5	
Not A		3	7
Total	10		

10 A car salesman notes that among his 40 cars, there are 15 automatic cars and 10 sports cars. Only two of the sports cars are automatic.

a Create a two-way table of this situation.

b What is the probability that a randomly selected car will be a sports car that is not automatic?

c What is the probability that a randomly selected car will be an automatic car that is not a sports car?

11 A car hire firm has 60 cars for hire. There are 33 four-wheel drive cars (4WD) and 26 automatic cars. There are 10 cars that are neither automatic nor four-wheel drive.

a Copy and complete this two-way table.

	Automatic car	**Not automatic**	**Total**
4WD			
Not 4WD			
Total			

b If a customer randomly selects a car for hire, find these probabilities:

- **i** Pr(automatic and 4WD)
- **ii** Pr(4WD)
- **iii** Pr(automatic)
- **iv** Pr(not 4WD)
- **v** Pr(not automatic)
- **vi** Pr(neither automatic nor 4WD).

Example 19 Using two-way tables in worded problems

A total of 50 students were asked whether they liked or disliked skiing and snowboarding. Of these students, 25 liked both skiing and snowboarding and 12 disliked skiing but liked snowboarding. The total number of students who disliked skiing was 17.

a Complete a two-way table showing this information.

b If a student is randomly chosen from this group, what is the probability that they dislike skiing but like snowboarding?

c How many students like skiing?

d What proportion of students who like skiing also like snowboarding?

e If a student is selected out of those that like snowboarding, what is the probability that they dislike skiing?

Solution | **Explanation**

a

	Like skiing	Dislike skiing	Total
Like snowboarding	25	12	37
Dislike snowboarding	8	5	13
Total	33	17	50

First fill in the given numbers
Use the totals to find any missing values.

b Pr(dislike skiing but like snowboarding)

$= \frac{12}{50} = \frac{6}{25}$

12 dislike skiing but like snowboarding out of a total of 50. Use Pr( ) for all probability answers.

c 33 students like skiing

33 is the total in the like skiing column.

d $\frac{25}{33}$

Out of the 33 who like skiing there are 25 who like snowboarding.

e Pr(dislike skiing given that like snowboarding) $= \frac{12}{37}$

Out of the 37 who like snowboarding there are 12 who dislike skiing.

12 A total of 33 students were asked whether they liked or disliked volleyball and tennis. Of these students, 12 liked both volleyball and tennis and 6 disliked volleyball but liked tennis. The total number of students who liked volleyball was 23.

a Copy and complete a two-way table.

	Like volleyball	**Dislike volleyball**	**Total**
Like tennis			
Dislike tennis			
Total			

b If a student is randomly chosen from this group, what is the probability that they dislike tennis but like volleyball?

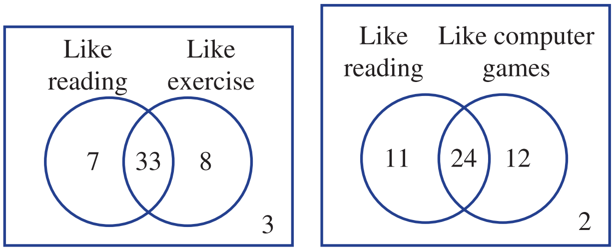
c How many students like tennis?

d What proportion of tennis students also like volleyball?

e If a student is selected out of those who like tennis, what is the probability that they dislike volleyball?

Two-way table from two Venn diagrams

13 Two surveys of two different groups of people showed how many students liked reading compared to how they like exercise and computer games. The results are shown in these Venn diagrams.

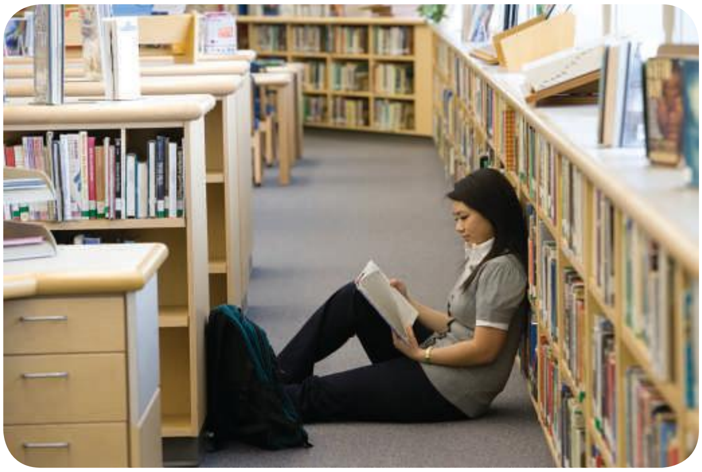


a Copy and complete this two-way table showing this information.

	Like reading	Don't like reading	Total
Like exercise			
Don't like exercise			
Like computer games			
Don't like computer games			
Totals			

b If a student is selected out of those who like exercise, what is the probability that they like reading also?

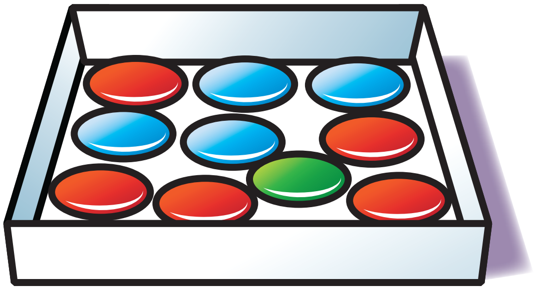
c If a student is selected out of those who don't like exercise, what is the probability that they don't like reading also?



8I Experimental probability

Sometimes the probability of an event is unknown or cannot be determined using the techniques learnt earlier. An experiment or survey results can be used to estimate an event's probability and this estimate is called an **experimental probability**.

Let's start: A horse race



Work in pairs, small groups or as a class for this activity.

Equipment: Container with 5 red counters, 4 blue counters and 1 green counter.

1. Use the colour of each counter in the name of your horses, e.g. Red racer, Blue beauty, Green lightning
2. Copy the table below to track the progress of each horse.
3. Randomly select a counter. The horse with that colour in its name moves forward 100 m. Shade in a cell in the table to show that horse has moved forward 100 m.
4. Return the counter to the container.
5. Continue selecting a counter, moving that horse colour forward, and replacing the counter.
6. The winning horse is the first to reach the finish at 1000 m.
7. Stop playing when a horse has won.

Horse	100 m	200 m	300 m	400 m	500 m	600 m	700 m	800 m	900 m	1000 m
Red Racer										
Blue Beauty										
Green Lightning										

Discussion questions

Discuss these questions with your classmate and write down the answers.

1. Which 'horse' colour won your race?
2. How many counters were selected in total until this horse won?
3. What proportion of the total number of selected counters did the winning colour have? Name this the experimental probability.
4. Use this experimental probability to calculate how many times you would expect to select your winning colour if you selected a counter and replaced it 800 times.
5. What is the actual probability of selecting the colour of your winning horse?
6. Use the actual probability to find out how many times you would expect to select each colour if you selected a counter and replaced it 800 times.
7. Why do you think there can be different expected numbers depending on whether you use experimental probability or actual probability? Give one reason.

Key ideas

- The **experimental probability** of an event based on a particular experiment is defined as $\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$.
- The **expected number** of occurrences = probability × number of trials.
- Complex events can be simulated. A *simulation* is conducted using random devices such as coins, dice, spinners or random number generators.

Experimental probability
An estimate of an outcome's probability based on an experiment

Exercise 8I

UNDERSTANDING 1–4 4

1 A spinner is spun 10 times and the colour shown is recorded:
blue, blue, green, red, blue, green, blue, red, blue, blue
 a How many times was green shown?
 b What is the experimental probability of green being spun?
 c What is the experimental probability of blue being spun?

The experimental probability of green is the proportion (fraction) of times that green has occurred.

2 A coin is tossed 10 times and the result shown is recorded.
Head, Tail, Tail, Head, Head, Tail, Head, Tail, Head, Head
 a How many times did heads appear?
 b What is the experimental probability of a head appearing?
 c What is the experimental probability of a tail appearing?

Example 20 Working with experimental probability

A coin is tossed 20 times and the results are shown in this table:

Outcome	**Head**	**Tail**
Frequency	8	12

a What is the experimental probability of a head?
b Using this experimental probability, calculate the expected number of heads you would get if you tossed this coin 500 times.
c What is the actual probability of a head? (Assume the coin is equally likely to fall on heads as tails.)
d Using the actual probability, calculate the expected number of heads you would get if you tossed this coin 500 times.

Solution	**Explanation**
a $\Pr(\text{H}) = \frac{8}{20} = \frac{4}{5}$	A head came up 8 times out of a total of 20 throws.
b $\frac{8}{20} \times 500 = 200$	Expected number = experimental probability times number of trials
c $\Pr(\text{H}) = \frac{1}{2}$	The coin is equally likely to fall on heads as tails so a 1 in 2 chance for heads.
d $\frac{1}{2} \times 500 = 250$	Expected number = actual probability times number of trials

3 A coin is tossed 50 times and the results are shown in this table:

Outcome	Head	Tail
Frequency	27	23

a What is the experimental probability of a head?

b Using this experimental probability, calculate the expected number of heads you would get if you tossed this coin 700 times.

c What is the actual probability of a head? (Assume the coin is equally likely to fall on heads as tails.)

d Using the actual probability, calculate the expected number of heads you would get if you tossed this coin 700 times.

4 A die is tossed 100 times and the results are shown in this table:

Outcome	Even number	Odd number
Frequency	55	45

a What is the experimental probability of an even number?

b Using this experimental probability, calculate how many times you would expect to get an even number if you tossed this die 1000 times.

c What is the actual probability of obtaining an even number when a die is tossed?

d Using the actual probability, calculate how many times you would expect to get an even number if you tossed this die 1000 times.

FLUENCY 5–8 5, 7–9

Example 21 Working with probabilities and expected numbers

A number of red, white and orange marbles are placed in a jar. Repeatedly, a marble is taken out, its colour is noted and the marble is replaced in the jar. The results are tallied in the table.

Colour	Red	White	Orange
Tally	𝍸 |||	𝍸 𝍸 ||	𝍸 𝍸
Frequency	8	12	10

a What is the experimental probability of a red marble being chosen next?

b What is the experimental probability of a red or a white marble being chosen?

c If the experiment is done 600 times, what is the expected number of times that an orange marble is selected?

Solution	Explanation
a $\text{Pr(red)} = \frac{8}{30} = \frac{4}{15}$	Experimental probability $= \frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$
b $\text{Pr(red or white)} = \frac{20}{30} = \frac{2}{3}$	Red or white marbles were selected 20 times out of the 30 trials.
c $\text{Expectednumber} = \frac{1}{3} \times 600 = 200$	$\text{Pr(orange)} = \frac{1}{3}$ Expected number = probability × number of trials

5 A spinner is spun 50 times and the results are shown in the frequency table below.

Colour	Red	Blue	White	Purple
Tally	卌 卌 卌 卌 卌 卌	卌	||	卌 卌 |||
Frequency	30	5	2	13

Expected number = probability × number of trials

a What is the experimental probability of red?

b What is the experimental probability of blue?

c What is the experimental probability of red or purple?

d If the spinner were spun 1000 times, what is the expected number of times that white would be spun?

6 A group of households are surveyed on how many cars they own. The results are shown.

0 cars	1 car	2 cars	3 cars	4 cars
卌 卌 ||	卌 卌 卌 卌 卌 卌 卌 ||	卌 卌 卌 卌 卌 卌 卌 卌 |	卌 |||	||

a Write the tallied results as a frequency table, with headings 'number of cars' and 'frequency'.

b How many households in total were surveyed?

c What is the experimental probability that a randomly chosen household owns no cars?

d What is the experimental probability that a randomly chosen household owns 2 or more cars?

7 A die is painted so that 3 faces are blue, 2 faces are red and 1 face is green.

a What is the actual probability that it will display red on one roll?

b Using the actual probability how many times would you expect it to display red on 600 rolls?

c Using the actual probability how many times would you expect it to display blue on 600 rolls?

8 A spinner displays the numbers 1, 2, 3 and 4 on four sectors of different sizes. It is spun 20 times and the results are $1, 3, 1, 2, 2, 4, 1, 1, 3, 1, 2, 4, 4, 2, 4, 3, 1, 1, 3, 2$.

Write these probabilities as decimals.

a Give the experimental probability that the spinner will land on:

i 1 **ii** 2 **iii** 3 **iv** 4

b On the basis of this experiment, what is the expected number of times in 1000 trials that the spinner will land on 3?

9 A fair die is rolled 100 times and the number 5 occurs 19 times.

a What is the experimental probability of a 5 being rolled? Give a decimal answer.

b What is the actual probability of a 5 being rolled on a fair die? Round the answer to two decimal places.

c For this experiment, which is greater: the experimental probability or the actual probability?

PROBLEM-SOLVING AND REASONING 10, 11 11-13

10 A basketball player has a 1 in 2 chance of getting a shot in from the free throw line. To simulate these use a coin: heads represents the shot going in, tails represents missing.

a Flip a coin 20 times and write down the results.

b Based on your experiment, what is the experimental probability that a shot will go in?

c Based on the actual probability of $\frac{1}{2}$, how many of 20 throws are expected to go in?

d Is it possible that this basketball player could have 20 shots from the free throw line and 18 go in? Could all 20 go in?

11 A number of marbles are placed in a bag – some are red and some are green. A marble is selected from the bag and then replaced after its colour is noted. The results are shown in the table.

Red	Green
28	72

Based on the experiment, give the most likely answer to the following questions.

a If there are 10 marbles in the bag, how many are red?

b If there are 6 marbles in the bag, how many are red?

c If there are 50 marbles, how many are red?

d If there are 4 marbles, how many are green?

e If there are 14 green marbles in the bag, how many marbles are there in total?

f If there are 3 red marbles, how many green marbles are there?

12 Four coins are tossed together and the number of tails is noted. This is repeated 11 times.

Number of tails	0	1	2	3	4
Frequency	1	3	5	2	0

a Based on this experiment, what is the experimental probability of obtaining 4 tails?

b Based on this experiment, what is the experimental probability of obtaining 3 heads?

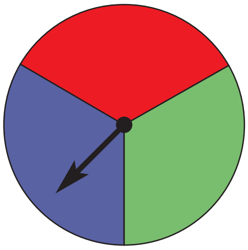
c True (T) or False (F)? If the experimental probability is 0 then the actual probability is 0.

d If 4 coins are tossed at the same time, what is the actual probability of obtaining 5 tails?

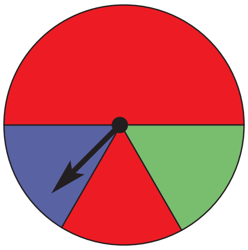
e True (T) or False (F)? If the actual probability is 0 then the experimental probability is 0.

13 In a probability simulation the probability of the outcomes must stay the same. For example, it is possible to simulate (model) a coin using a die by using the numbers 1–3 to stand for tails (probability $= \frac{1}{2}$) and 4–6 to stand for heads (probability $= \frac{1}{2}$). Which of the following spinners could be simulated using a single roll of a die? Explain your answer.

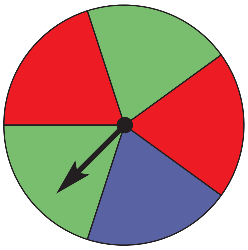
a

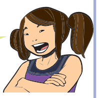


b



c





Explain which numbers on the die correspond to the colours on the spinner.

Running a simulation

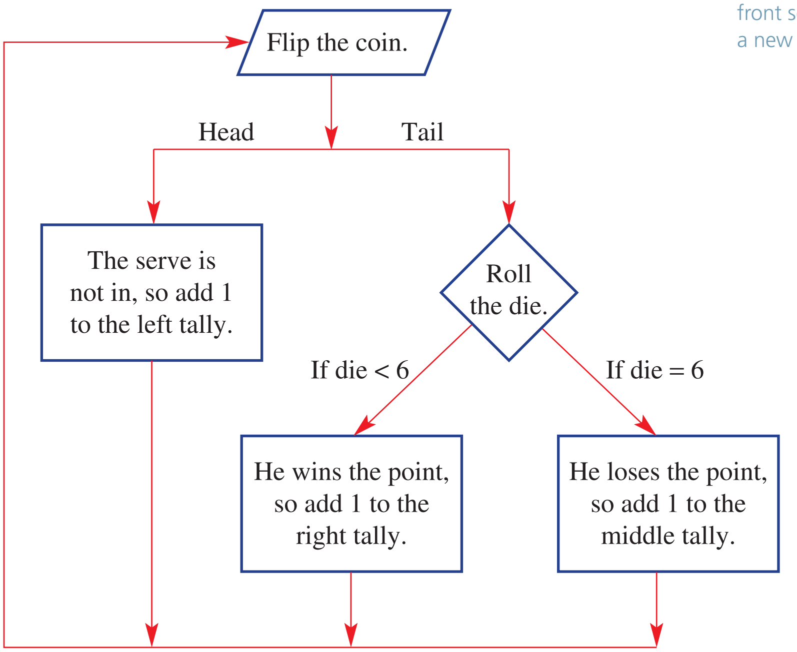
14 A tennis player has a 1 in 2 chance of getting his first serve in. When the serve goes in, he has a 5 in 6 chance of winning the point and a 1 in 6 chance of losing the point.

a Copy this table and keep a record of results from your 'tennis' simulation.

	Serve not in	Serve in but lose point	Serve in and win point
Tally			
Frequency			

A serve is when the ball is hit into the front section of the opposite court and a new rally is started for a point.

b Using a coin and a die, run the simulation outlined below at least 20 times.



c From your results, state your experimental probabilities for:

- **i** serve not in
- **ii** serve in but lose a point
- **iii** serve in and win a point.

d Using these experimental probabilities, find the expected number of points won in a match for a player who makes 120 serves.

Maths@Work: Student Representative Council (SRC) coordinator

Statistics and probability is an area of mathematics used in many occupations and individuals on a daily basis. Statistics refers to the collection and analysis of numerical data. The Australian government uses statistics about population growth, people's health, economic performance and the state of our environment, etc. Federal, state and local governments all use statistics to determine which issues need addressing with new policies. Counting election votes and deciding final results all falls under the banner of statistics.

At a school level, SRC elections are held each year and the coordinating teacher uses statistics to determine which students are elected. SRC students often use statistics to find out which issues to focus on within their school.

1 The Year 7 SRC needs 5 students. Fourteen students were nominated, then every Year 7 student voted for 5 SRC students. The results of this vote are in the table below.

Student	Votes	Student	Votes
Charlie K	83	Sara H	34
Tahlia P	120	William B	45
Hicham J	94	Vivaan P	51
Aanya N	72	Braydon B	29
Eden V	102	Austin C	37
Scarlett K	19	Monique H	11
Josiah G	45	Elizabeth S	8

a How many students are in the year group?

b What was the highest number of votes that a nominated student received?

c List the names of the 5 SRC members for the year.

d What percentage of the total vote (relative frequency) did each SRC member receive?

e Explain why this method of voting is not used on a larger scale, such as in federal government elections.

2 The results from the Year 8 SRC election are shown below as a percentage of the total votes cast. A total of 750 votes were cast.

Student	Percentage
Alex P	44%
Jia Hao N	22%
Kelly Y	14%
Samantha W	12%
Nelson C	8%

a How many votes did each of the final five SRC members receive?

b Which of the SRC elected students had more than 150 votes?

3 The SRC decided to run a survey to determine the usage of the school's computer system in the library at lunch time. The survey question asked: 'How many times a week do you use the library computers at lunch time?'. The results are recorded below:

Usage of library computers	
Number of times per week	Frequency
0	80
1	138
2	234
3	245
4	123
5	65

a Given that every student in the school participated in the survey, how many students are in the school?

b What percentage of the students never used the library computers? Round to the nearest whole per cent.

c What is the mean number of times that any student used the library computers in a typical week? State the answer to 2 d.p.

d What percentage of the student body used the computers more than twice a week? Round the answer to a whole number.

e Based on the results of the survey, should the SRC run a lunch time computer competition in the library?

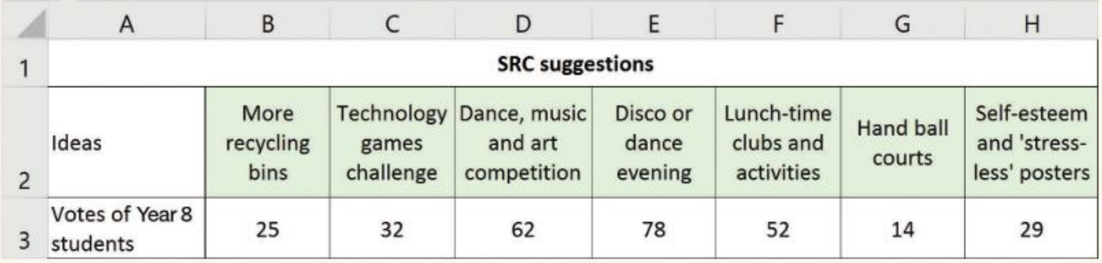
f What other survey questions could the SRC ask to help them accurately determine if the computer competition would be a success?

g Carry out a similar survey in your own class/school.

Using technology

4 The Year 8 SRC students came up with the suggestions shown in the table below. They then conducted a survey where all Year 8 students selected their two highest preferences. The results are shown below.

a Copy this table into an Excel spreadsheet.



	A	B	C	D	E	F	G	H
1	**SRC suggestions**							
2	Ideas	More recycling bins	Technology games challenge	Dance, music and art competition	Disco or dance evening	Lunch-time clubs and activities	Hand ball courts	Self-esteem and 'stress-less' posters
3	Votes of Year 8 students	25	32	62	78	52	14	29

b Choose one or more of the charts below and display the SRC results. You need to first select rows 2 and 3 of the table, choose 'Insert', then follow the selected instructions.

Column charts

i Click 'Insert Column or Bar Chart' and select either the vertical '2D (or 3D) Clustered Column' or the horizontal '2D (or 3D) Clustered Bar'.

ii Select the chart, choose Design and select the colours and style that you want.

A treemap and divided bar graph

iii Click 'Insert Hierarchy Chart', select 'Treemap'.

iv The 'key' is optional as names are within each rectangular area.

v Title the Treemap and choose your colours and its overall rectangular shape.

vi One long rectangle makes it into a divided bar graph.

Puzzles and games

1 John Venn was an English mathematician who invented Venn diagrams to make sorting data and probability calculations easier. He was also a fan of cricket. Solve the questions below to find the answer to this question:

What did John Venn invent that, in 1909, clean bowled (i.e. bowled out) one of our best Australian cricket batsmen four times?

20

7	30	6	E′	23	F	12

5	20	15	14	23	F	T

A. Find the mean of these scores: 26, 25, 13, 24, 12 and 20.

B. What is the median of these scores: 2, 4, 4, 7, 8, 9, 10?

C. What is the range of these scores: 13, 18, 5, 7, 16, 3?

E. True (T) or False (F): A histogram has no spaces between the columns.

G. What frequency does this tally represent? 𝍸 𝍸 ||

H. If 8 students have a cat only and 6 have both a cat and a dog and 9 a dog only, how many have a cat?

I. If 8 students have a cat only and 6 have both a cat and a dog and 9 a dog only, how many have a cat or a dog or both?

L. How is the complement of the event E written?

M. If Amy got 7 marks for 5 tests in a term, which of these numbers is the frequency?

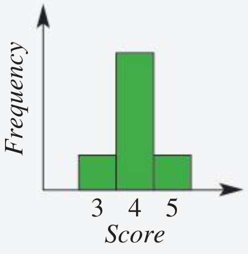
N. True (T) or False (F): A skewed histogram has its highest frequency in the middle.

O. How many times would we expect to throw either a 4 or a 5 if a die was tossed 90 times?

W. How many of these ages are in the interval 12–15 years?
12, 13, 16, 11, 12, 15, 19, 19, 14, 16, 14

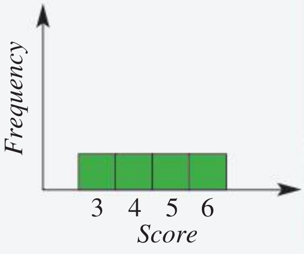
2 The following histograms are drawn to scale but the frequency scale has been omitted. Determine the median for each one.

a



b

c



3 At the local sports academy, everybody plays netball or tennis or both. Given that:

- 10 people play both netball and tennis
- half the tennis players also play netball
- one-third of the netballers also play tennis,
  what is the probability that a randomly chosen person at the academy plays both netball and tennis?

Puzzles and games

4 Choose a word from this list that matches each of the following descriptions. FLUFF, DING, OFF, PERSONALITY, TEN, PROBABILITY, MOON, STUMBLE, TOY, TRY

a Pr(vowel) = $\frac{1}{2}$

b Pr(F) = $\frac{2}{3}$

c Pr(vowel) = $\frac{1}{4}$ and Pr(D) = $\frac{1}{4}$

d Pr(I) = $\frac{2}{11}$ and Pr(consonant) = $\frac{7}{11}$.

e Pr(M) = $\frac{1}{7}$ and Pr(T) = $\frac{1}{7}$ and Pr(S) = $\frac{1}{7}$.

f Pr(vowel) = 0 and Pr(T) = $\frac{1}{3}$

5 In a certain town of 100 adults, there are 22 women who can cook and 18 men who cannot cook. Given that half the town is male and 54% of the town can cook, how many men in the town can cook?

6 Monopoly mystery.
In a board game, two dice are rolled and the player moves forward by the sum of the numbers on each die. E.g. a throw of a 4 and a 6 means the player would move by 10 places.
Are you as likely to obtain a 9 as the sum of two dice as any other number for the total? Explain with an example.

7 Jayden is an AFL player and notes the number of points he scores in his 22-week season. His results are shown in this table. Jayden said he scored a different number of points for each of the 22 games. Is he correct? Use an example to explain your answer.

Points	Frequency
0–4	3
5–9	11
10–14	6
15–19	1
20–25	1

Chapter summary

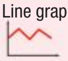
Representing Data

Graphs

Graphs

- Column graph

- Line graph

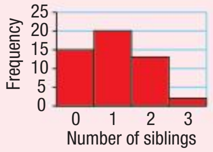


- Sector graph

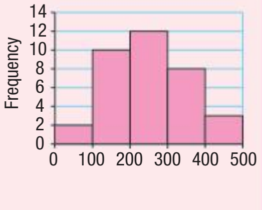
- Divided bar graph

Histograms

Individual scores



Scores in intervals



Tables

Frequency tables

Individual scores

Number of children Score	Number of families Tally	 Frequency
0	|||	3
1	𝍸	5
2	|||| 𝍸 |	11
3	𝍸	5
4	𝍸|	6
		$n = 30$

Scores in intervals

Age	Frequency
0–4	3
5–9	7
10–14	5
	$n = 15$

Measures of centre

numerical order

0, 1, 4, 4, 5, 6, 9, 30

Median (middle) $= \frac{4+5}{2} = 4.5$

Mode (most frequent) = 4

Mean $= \frac{59}{8} = 7.4$

Measure of spread

1 5 7 9 11 15

Range = 15 – 1
= 14

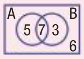
Probability

Probabilities are written as
- fractions
- percentage
- decimals

100% 1 Certain
Likely
0.5 50% $\frac{1}{2}$ Even chance (50–50)
Unlikely
0% 0 Impossible

Venn diagrams and two-way tables

These organise data from two or more categories



Venn diagram

	A	not A	
B	7	3	10
not B	5	6	11
	12	9	21

6 in neither category,
7 in both categories

$\Pr(A) = \frac{12}{21} = \frac{4}{7}$

$\Pr(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$

Theoretical probability (actual probability)

Trial: Roll a fair die
Sample space (possible outcomes)
{ 1, 2, 3, 4, 5, 6 }

$\Pr(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

Trial: Select a playing card and note its suit.
Sample space: spade, diamond, club, heart

Theoretical probabilities

$\Pr(\text{black}) = \frac{26}{52} = \frac{1}{2}$

$\Pr(\text{heart}) = \frac{13}{52} = \frac{1}{4}$

$\Pr(\text{not spade}) = \frac{39}{52} = \frac{3}{4}$

$\Pr(\text{either red or a spade}) = \frac{39}{52} = \frac{3}{4}$

$\Pr(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$

Experimental probability

Playing card selected and replaced 20 times, and its suit noted.

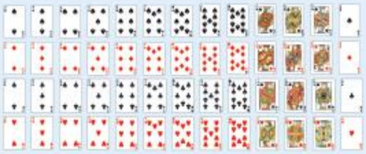
Outcome	Frequency	Experimental probability
Heart	4	$\frac{4}{20}$
Diamond	5	$\frac{5}{20}$
Club	4	$\frac{5}{20}$
Spade	7	$\frac{5}{20}$
	$n = 20$	

Expected number of outcomes

Outcome	Theoretical probability	Expected number in 20 trials
Heart	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Diamond	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Club	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Spade	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$

Playing cards

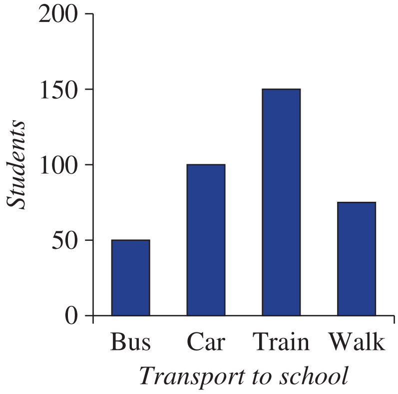
Spades
Diamonds
Clubs
Hearts



Multiple-choice questions

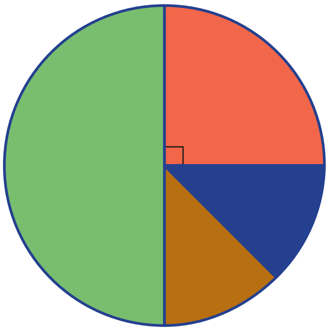
8A **1** Using the information in the column graph, how many students do not walk to school?

A 75 **B** 150 **C** 300 **D** 375 **E** 100



8A **2** The chocolates in a bag are grouped by colour and the proportions shown in the sector graph. If there are equal numbers of blue and brown chocolates, how many are blue, given that the bag contains 28 green ones?

A 112 **B** 7 **C** 56
D 14 **E** 28



8B **3** The table below shows the number of goals scored by a soccer team over a season.

Goals	0	1	2	3	4
Tally for number of games	𝍸 ||	𝍸	|||	|||	||

The total number of games played by the soccer team is:

A 28 **B** 10 **C** 20 **D** 13 **E** 5

8B **4** For the soccer results in the table in question **3**, the total number of goals scored in the season is:

A 28 **B** 10 **C** 4 **D** 20 **E** 13

8D **5** Which is the best description of the mode in a set of test scores?

A The average of the scores
B The score in the middle
C The score with the highest frequency
D The difference between the highest and lowest score
E The lowest score

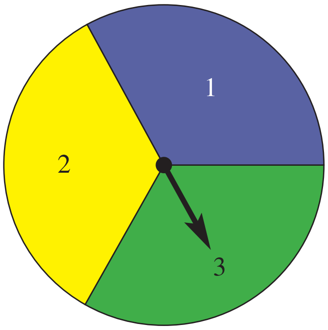
8D **6** For the set of data 1, 5, 10, 12, 14, 20 the range is:

A 1 **B** 19 **C** 4 **D** 11 **E** 6

8F **7** The letters of the word STATISTICS are placed on 10 different cards and placed into a hat. If a card is drawn at random, the probability that it will show a vowel is:

A 0.2 **B** 0.3 **C** 0.4 **D** 0.5 **E** 0.7

8F **8** The spinner shown is spun. The probability that the spinner will display an odd number is:



A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{2}{3}$ **D** $\frac{1}{2}$ **E** 1 and 3

8I **9** A coin is tossed 30 times. The expected number of tails is:

A 29 **B** 30 **C** $\frac{1}{2}$ **D** 15 **E** 25

8I **10** An experiment is conducted in which a die is rolled 300 times. The results are shown in this frequency table.

Outcome	1	2	3	4	5	6
Frequency	48	53	44	55	51	49

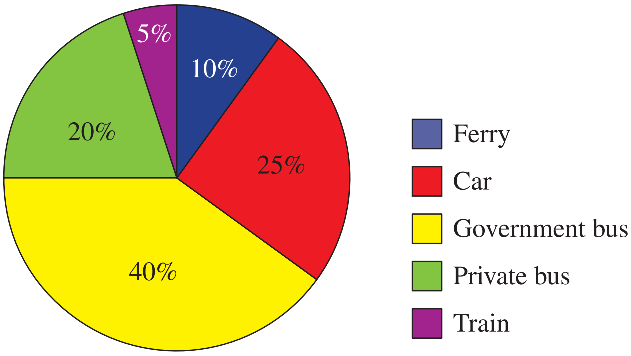
Based on this, the experimental probability of obtaining an even number is:

A $\frac{48}{300}$ **B** $\frac{55}{300}$ **C** $\frac{50}{300}$

D $\frac{150}{300}$ **E** $\frac{157}{300}$

Short-answer questions

8A **1** The sector graph shows the type of transport office workers use to get to work every day.



a Which type of transport is the most popular?

b Which type of transport is the least popular?

c What percentage of office workers did not travel by car?

d If 20 000 workers were surveyed, how many people travelled to work each day by train?

e The year after this survey was taken, it was found that the number of people using government buses had decreased. Give a reason why this could have occurred.

8B **2** Some students were asked how many hours of study they did before their half-yearly Maths exam. Their responses are represented in a tally.

0 hours	1 hour	2 hours	3 hours	4 hours
||	卌 |	|||	|||	卌 |||

a How many students are in the class?

b Convert the tally above into a frequency table.

c Draw a histogram to represent the results of the survey.

d What proportion of the class did no study for the exam?

e Find the total number of hours of study done by this group.

f Calculate the mean number of hours per student in the class spent studying for the exam, giving the answer correct to one decimal place.

8D **3** **a** Rewrite the following data in ascending order:

56 52 61 63 43 44 44 72 70 38 55
60 62 59 68 69 74 84 66 53 71 64

b What is the mode?

c What is the median for these scores?

d Calculate the range.

8D **4** The ages of boys in an after-school athletics squad are shown in the table below.

a State the total number of boys in the squad.

b List the ages of all these boys in ascending order.

c Calculate the mean age of the squad, correct to two decimal places.

d What is the median age of the boys in the squad?

Age	Frequency
10	2
11	3
12	4
13	8
14	10

8C **5** A group of teenagers were weighed and their weights recorded to the nearest kilogram. The results are as follows:

56 64 72 81 84 51 69 69 63 57 59 68 72 73 72 80 78 61 61 70

57 53 54 65 61 80 73 52 64 66 66 56 50 64 60 51 59 69 70 85

a Find the highest and lowest weights and the range.

b Create a grouped frequency distribution table using the groups (intervals) of $50–54$, $55–59$, $60–64$ etc.

c Find the modal group.

d Why is this sample not representative of the whole human population?

8D **6** **a** Use the data $5, 1, 7, 9, 1, 6, 4, 10, 12, 14, 6, 3$ to find:

i the mean **ii** the median **iii** the range.

b An extra score of 52 is added into the list in part **a**. Calculate the new median and mean, and state which measure has changed the most.

c What is the name for a score that is much larger than all the other values in a list?

8E **7** In an attempt to find the average number of hours of homework that a Year 8 student completes, Samantha asks 10 of her friends in Year 8 how much homework they do.

a Explain two ways in which Samantha's sampling is inadequate for representing the population of Years 8's in her school.

b If Samantha wished to convey to her parents that she did more than the average, how could she choose 10 people to bias the results in this way?

8F **8** An eight-sided die has the numbers $1, 2, 3, 4, 5, 6, 7, 8$ on its faces.

a Find the probability that the number 4 is rolled.

b What is the probability that the number rolled is odd?

c What is the probability that the number rolled is both even and greater than 5?

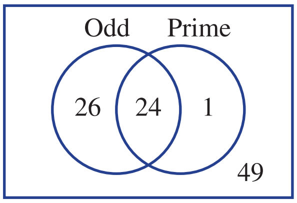
d If P is the event that a prime number is rolled, state the sample space of P', the complement of P.

e What is Pr(P')?

8F **9** The letters of the word MATHEMATICIAN are written on 13 cards. The letters are placed in a bag and one card is drawn at random.

- **a** State the sample space.
- **b** Find the probability of choosing the letter M.
- **c** Find the probability of a vowel being drawn.
- **d** What is the probability of a consonant being drawn?
- **e** What is the probability that the letter chosen will be a letter in the word THEMATIC?

8G **10** The Venn diagram below shows how many numbers between 1 and 100 are odd and how many are prime.



Consider the numbers 1–100.

- **a** How many are odd?
- **b** How many prime numbers are there?

For parts **c**, **d**, **e**, **f** give answers both as a percentage and a simplified fraction.

- **c** What is the probability that a randomly selected number will be odd and prime?
- **d** What is the probability that a randomly selected number will be prime but not odd?
- **e** What is the probability that a prime number is chosen out of all the odd numbers?
- **f** What is the probability that an odd number is chosen out of all the prime numbers?

8H **11** Of 20 workshops in a town, 12 fit tyres (T), 10 fit mufflers (M), and 8 fit both tyres and mufflers.

- **a** Represent this information in a two-way table.

	Mufflers	**No mufflers**	**Total**
Tyres			
No tyres			
Total			

- **b** How many workshops fit neither tyres nor mufflers?
- **c** Find the probability that a randomly selected workshop fits:
  - **i** tyres
  - **ii** tyres and mufflers
  - **iii** tyres or mufflers
  - **iv** mufflers only
  - **v** neither tyres nor mufflers

8I **12** Of 50 people surveyed, 20 said that they intend to send their children to a private school.

- **a** Find the experimental probability that a person intends to send their children to private school.
- **b** 200 people are selected from the population. What is the expected number who intend to send their children to private school?

Extended-response questions

1 The two-way table below shows the results of a survey on car ownership and public transport usage. You can assume the sample is representative of the population.

	Uses public transport	Does not use public transport	Total
Own a car	20	80	
Do not own a car	65	35	
Total			

a Copy and complete the table.

b How many people were surveyed in total?

c What is the probability that a randomly selected person will have a car?

d What is the probability that someone does not use public transport?

e What is the probability that someone will use public transport and also own a car?

f Out of the people who own cars, what is the probability that someone will use public transport?

g In what ways could the survey produce biased results if it had been conducted:

- **i** outside a train station?
- **ii** in regional New South Wales?

2 A spinner is made using the numbers 1, 3, 5 and 10 in four sectors. The spinner is spun 80 times, and the results obtained are shown in the table.

Number on spinner	Frequency
1	30
3	18
5	11
10	21
	80

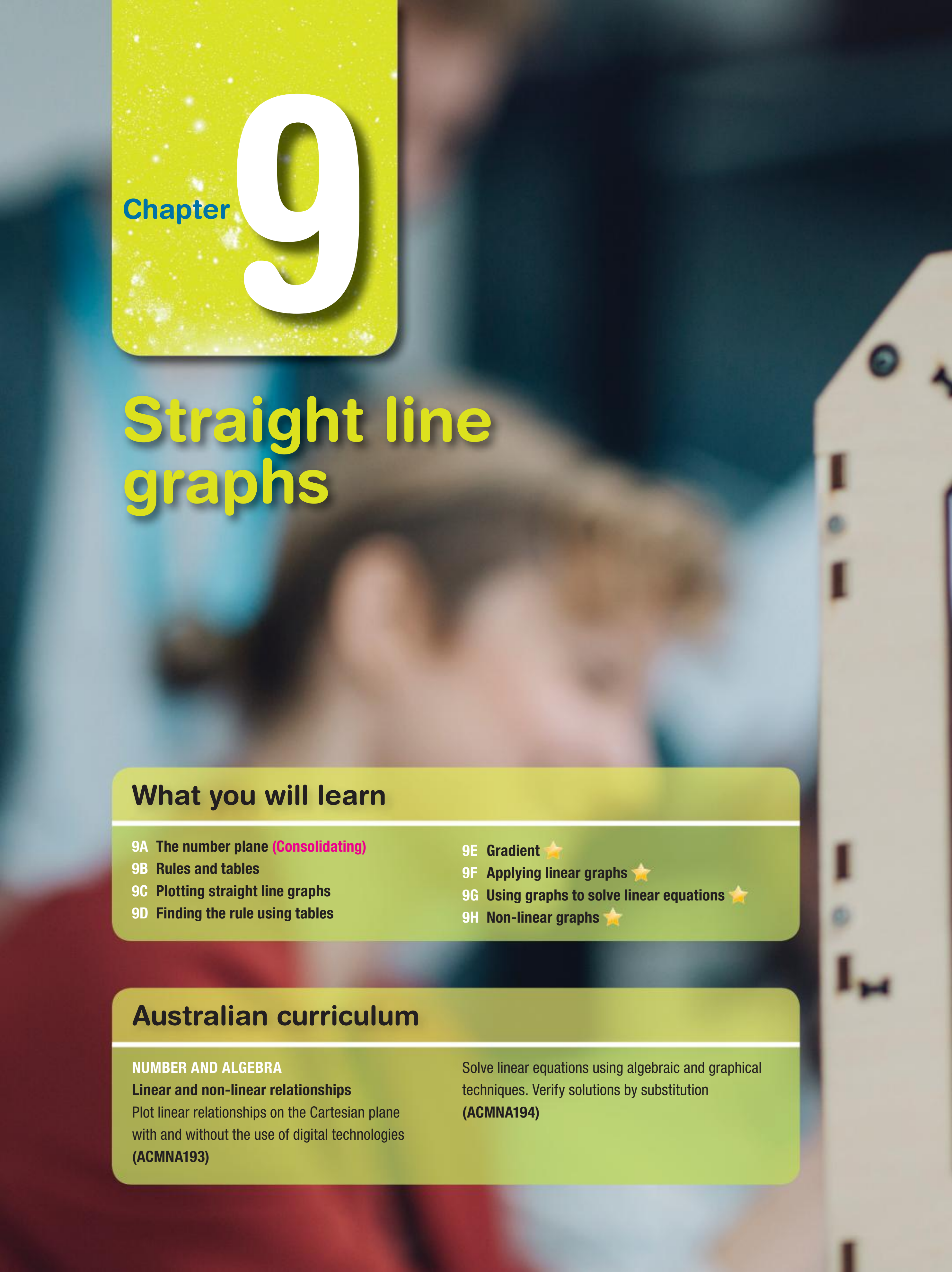
a Display the data as a frequency histogram.

b Which sector on the spinner occupies the largest area? Explain.

c Two sectors of the spinner have the same area. Which two numbers do you think have equal areas, and why?

d What is the experimental probability of obtaining a 1 on the next spin?

e Draw an example of what you think the spinner might look like, in terms of the area covered by each of the four numbers.



Chapter 9

Straight line graphs

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Plot linear relationships on the Cartesian plane with and without the use of digital technologies **(ACMNA193)**

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution **(ACMNA194)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

3-dimensional printers and the number plane

Number plane geometry is the basis for 3D printing. Imagine a virtual number plane with positive x- and y-axes drawn on the print platform of a 3D printer. Computer code instructs the printer head or nozzle how far across and how far up to move on this 2D number plane. A line or curve about 0.2 mm thick is drawn, forming a layer.

After one full layer, the print platform moves downward slightly and the computer directs the nozzle where to plot the next layer. Each layer is called a ‘slice’ and can be slightly different in shape from the previous slice. Thousands of slices slowly build a 3D object.

Objects made using 3D printing include: structural body parts (e.g. bone, cartilage), anatomical structures for a surgeon to study before a delicate operation (e.g. a patient’s brain), scale models of buildings, 3D maps, replica fossils, models of engineering projects and movie characters.

Pre-test

1 This graph relates distance and time for a journey in a train.

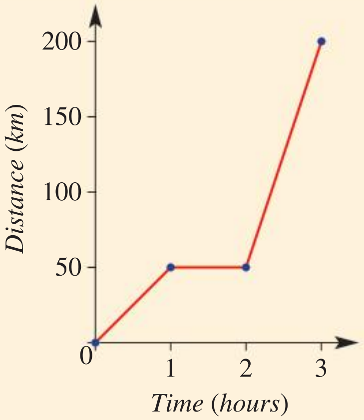
a How far did the train travel:

- **i** in the first hour?
- **ii** in the second hour?
- **iii** in the third hour?

b What was the total distance travelled?

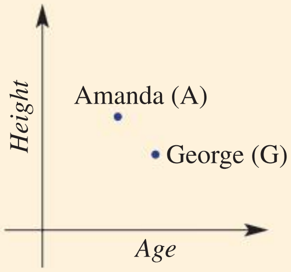
c During which hour was the train at rest?

d During which hour was the train travelling the fastest?



2 This graph shows the relationship between age and height of two people Amanda (*A*) and George (*G*).

a Who is older? (*A* or *G*) **b** Who is taller? (*A* or *G*)



3 Write the missing numbers.

a −3, −2, __, 0, 1, __, __ **b** −7, __, −3, −1, 1, __, __

c 12, 7, __, −3, __, __, −18 **d** −31, __, −13, −4, __, __

4 The x-coordinate in (2, −3) is 2. Write the x-coordinate for these points.

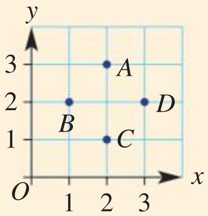
a (1, 2) **b** (1, 5) **c** (0, −1) **d** (−3, 0)

5 The y-coordinate in (2, −3) is −3. Write the y-coordinate for these points.

a (1, 6) **b** (−4, −1) **c** (−3, 0) **d** (−4, 2)

6 The coordinates of the point A on this graph are (2, 3). What are the coordinates of these points?

a B **b** C **c** D



7 If $\square = \Delta + 4$, find the value of $\square$ when:

a $\Delta = 3$ **b** $\Delta = 0$ **c** $\Delta = -2$

8 If $\square = 2 \times \Delta - 5$, find the value of $\square$ when:

a $\Delta = 2$ **b** $\Delta = 0$ **c** $\Delta = -3$

9 Complete the tables for the given rules.

a $\square = 2 \times \Delta$

Δ	−2	−1	0	1	2
$\square$					

b $\square = 3 \times \Delta - 4$

Δ	−2	−1	0	1	2
$\square$					

9A The number plane

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthrough

To describe a relationship between two variables, like water volume and time, we might use a graph. The graph would include a pair of axes and a set of points. The points can be joined to form a line or curve.

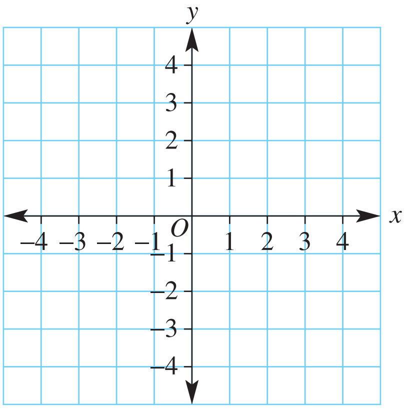
In mathematics, a pair of axes defines a number plane. The number plane is also called the Cartesian plane after its inventor, Rene Descartes, who lived in France in the 17th century. The horizontal axis (x) and vertical axis (y) of a number plane can extended to include negative numbers. The point where these axes cross over is called the origin and it provides a reference point for all other points on the plane.

Let's start: Make the shape

In groups or as a class, see if you can remember how to plot points on a number plane. Then decide what type of shape is formed by each set.

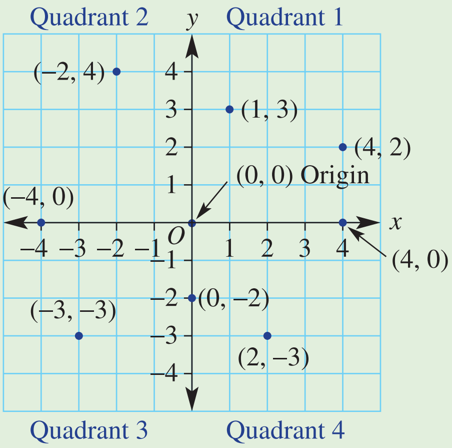
- $A(0, 0), B(3, 1), C(0, 4)$
- $A(-2, 3), B(-2, -1), C(-1, -1), D(-1, 3)$
- $A(-3, -4), B(2, -4), C(0, -1), D(-1, -1)$

Discuss the basic rules for plotting points on a number plane.



Key ideas

- A **number plane** (or **Cartesian plane**) includes a vertical y-axis and a horizontal x-axis intersecting at right angles.
  - There are 4 **quadrants** labelled as shown.
- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first followed by the y-coordinate.
- The point $(0, 0)$ is called the **origin**, and is often labelled O.



Number plane
A plane on which every point is related to a pair of numbers called coordinates

Coordinates
Numbers or letters used to give a location or position

Exercise 9A

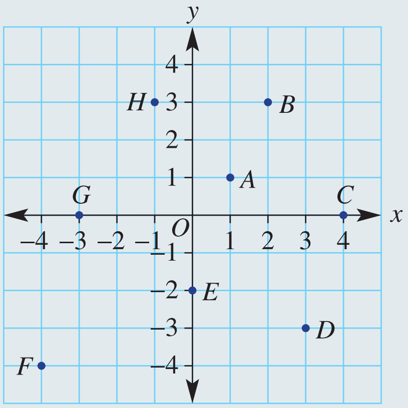
UNDERSTANDING 1–3 3

1 Complete these sentences.

- **a** The x-coordinate in $(3, -4)$ is _______.
- **b** The x-coordinate in $(-4, 7)$ is _______.
- **c** The y-coordinate in $(2, 5)$ is _______.
- **d** The y-coordinate in $(-4, -8)$ is _______.
- **e** The coordinates of the origin are ______.
- **f** The vertical axis is called the ___-axis.

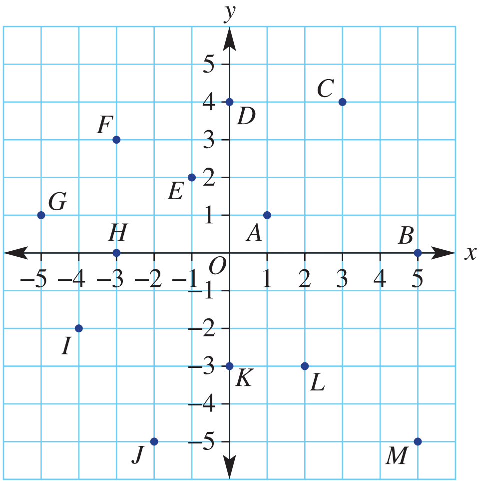
Example 1 Writing coordinates

Write down the coordinates of the points A to H on this number plane.



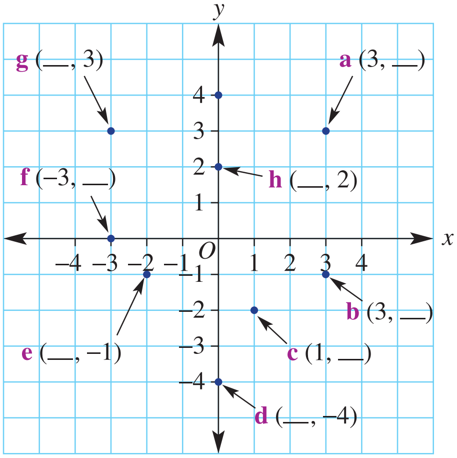
Solution		Explanation
$A(1, 1)$	$E(0, -2)$	First write the x-coordinate
$B(2, 3)$	$F(-4, -4)$	(positive on the right and negative on the left).
$C(4, 0)$	$G(-3, 0)$	Then write the y-coordinate
$D(3, -3)$	$H(-1, 3)$	(positive above and negative below).

2 Write the coordinates of the points labelled A to M.



Write a pair of coordinates (x, y). x is positive on the right and y is positive on the upper side.

3 Write the missing number for the coordinates of the points **a**–**h**.

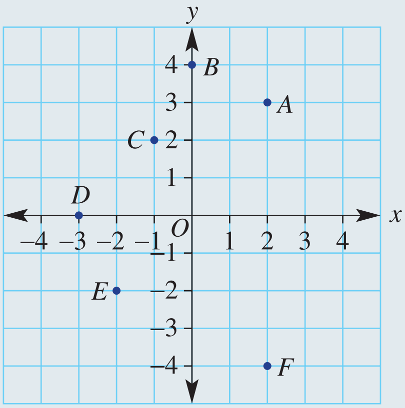


FLUENCY	4, 5	4, 5

Example 2 Plotting points

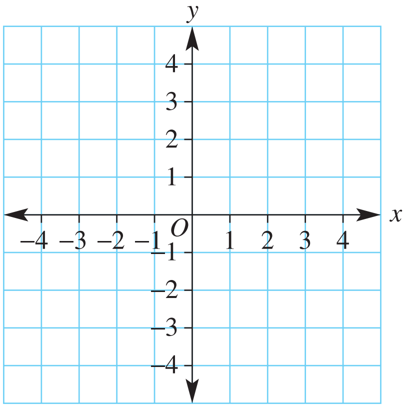
Draw a number plane extending from –4 to 4 on both axes then plot and label these points.

a $A(2, 3)$ **b** $B(0, 4)$ **c** $C(-1, 2)$ **d** $D(-3, 0)$ **e** $E(-2, -2)$ **f** $F(2, -4)$

Solution	**Explanation**
	The x-coordinate is listed first followed by the y-coordinate. For each point start at the origin $(0, 0)$ and move left or right or up and down to suit both x- and y-coordinates. For point $C(-1, 2)$, for example, move 1 to the left and 2 up.

4 Draw a number plane extending from –4 to 4 on both axes and then the plot and label these points. Or you could use this grid.

a $A(4, 1)$ **b** $B(2, 3)$
c $C(0, 1)$ **d** $D(-1, 3)$
e $E(-3, 3)$ **f** $F(-2, 0)$
g $G(-3, -1)$ **h** $H(-1, -4)$
i $I(0, -2)$ **j** $J(0, 0)$
k $K(3, -1)$ **l** $L(1, -4)$



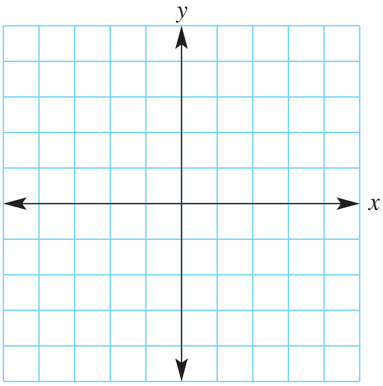
First move horizontally for x then vertically for y.

5 Plot each set of points. What do you notice about each graph?

a $(-4, -4), (-3, -3), (-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2), (3, 3), (4, 4)$

b $(-2, 4), (-1, 2), (0, 0), (1, -2), (2, -4)$

c $(-4, 2), (-2, 1), (0, 0), (2, -1), (4, -2)$



PROBLEM-SOLVING AND REASONING

6, 7 | 6, 8, 9

6 Complete these sentences.

a The point $(2, 4)$ is in the _______ quadrant.

b The point $(1, -5)$ is in the _______ quadrant.

c The point $(-3, 6)$ is in the _______ quadrant.

d The point $(-7, -20)$ is in the _______ quadrant.

e The quadrant that has positive coordinates for both x and y is the _______ quadrant.

f The quadrant that has negative coordinates for both x and y is the _______ quadrant.

Answer as first, second, third or fourth

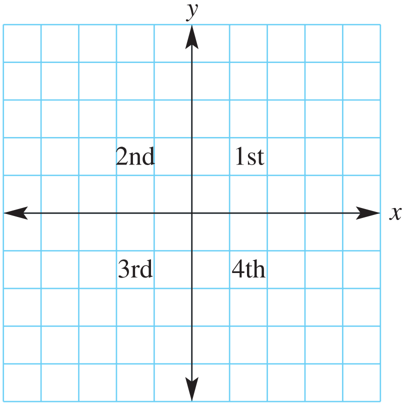
7 One point in each set is not 'in line' with the other points. Plot the points on this grid then name the point not in line with the others in the same set.

a $A(1, 2), B(2, 4), C(3, 4), D(4, 5), E(5, 6)$

b $A(-5, 3), B(-4, 1), C(-3, 0), D(-2, -3), E(-1, -5)$

c $A(-4, -3), B(-2, -2), C(0, -1), D(2, 0), E(3, 1)$

d $A(6, -4), B(0, -1), C(4, -3), D(3, -2), E(-2, 0)$



8 Each set of points forms a basic shape. Describe the shape without drawing a graph if you can.

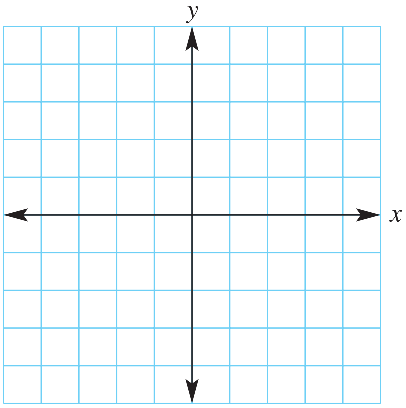
a $A(-2, 4), B(-1, -1), C(3, 0)$

b $A(-3, 1), B(2, 1), C(2, -6), D(-3, -6)$

c $A(-4, 2), B(3, 2), C(4, 0), D(-3, 0)$

d $A(-1, 0), B(1, 3), C(3, 0), D(1, -9)$

9 A set of points has coordinates $(0, y)$ where y is any number. What does this set of points represent?



Plotting pictures

10 Using a scale extending from −5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

a $(-2, -2), (2, -2), (2, 2), (1, 3), (1, 4), \left(\frac{1}{2}, 4\right), \left(\frac{1}{2}, 3\frac{1}{2}\right), (0, 4), (-2, 2), (-2, -2)$

b $(2, 1), (0, 3), (-1, 3), (-3, 1), (-4, 1), (-5, 2), (-5, -2), (-4, -1), (-3, -1), (-1, -3), (0, -3), (2, -1), (1, 0), (2, 1)$

9B Rules and tables

Interactive
Widgets
HOTsheets
Walkthrough

When two variables like *number of kilograms of bananas* and *cost* are related, we can use a rule to describe this relationship. For example, if bananas are $5 per kg, then a rule connecting the number of kilograms (n kg) and the cost ($$C$) would be $C = 5n$.

This rule can then be used to create a table of values. This can then be used to create a graph, which will be studied in the next section.

Let's start: Weigand's emu eggs

Weigand has a small emu farm and sells emu eggs for $50 each. It costs him $5000 per year to run the farm, so the rule connecting his profit ($$P$) and number of emu eggs sold (n) is

$$P = 50n - 5000$$

- Find Weigand's profit if he sells 120 eggs.
- Find Weigand's profit if he sells 300 eggs.
- Find Weigand's profit if he sells 50 eggs. What do you notice, and what does this mean?
- What values are missing from this table?

n	0	50	100	150	
P	–5000			2500	5000

- What is Weigand's 'break even' point?

Key ideas

- A **rule** is an equation that connects two or more variables.
  Examples: $d = 50t$, $y = 3x - 2$, $V = 100 - 10t$, $C = 100n - 1000$
- A rule can be used to construct a table of values.
  - Substitute the values of one variable into the rule to find the value of the other variables.
    Rule: $C = 2n + 3$

n	–1	0	1	2	3
C	1	3	5	7	9

If $n = 3, C = 2 \times 3 + 3 = 9$
If $n = -1, C = 2 \times -1 + 3 = 1$

Rule
An equation connecting two or more variables

Exercise 9B

UNDERSTANDING 1-3 3

1 Write the missing values in each table for the given rules.

a $\square = \Delta + 2$

Δ	0	1	2	3
$\square$	2		4	

b $\square = 2 \times \Delta$

Δ	0	1	2	3
$\square$		2	4	

c $\square = \Delta - 7$

Δ	4	5	6	7
$\square$		–2		0

d $\square = 2 \times \Delta + 4$

Δ	0	2	4	6
$\square$	4		12	

e $\square = 3 \times \Delta - 8$

Δ	0	1	2	3
$\square$	–8			1

f $\square = \Delta \div 2 - 1$

Δ	0	2	4	6
$\square$	–1			2

2 If $y = 2x - 5$, find the value of y for these values of x.

a $x = 4$ **b** $x = 3$
c $x = 2$ **d** $x = 1$
e $x = 0$ **f** $x = -1$
g $x = -2$ **h** $x = -3$

Remember: A positive times a negative equals a negative e.g. $2 \times (-1) = -2$

3 If $y = -3x + 1$, find the value of y for these values of x.

a $x = 0$ **b** $x = 1$
c $x = 4$ **d** $x = 7$
e $x = -1$ **f** $x = -3$
g $x = -10$ **h** $x = -50$

Remember: A negative times a negative equals a positive e.g. $-3 \times (-1) = 3$

FLUENCY 4, 6(½) 5, 6(½)

Example 3 Constructing tables using positive numbers

The rule connecting the distance travelled (d km) and time (t hours) is $d = 60t$.

a Construct a table of values using the following values of t: 0, 1, 2, 3, 4

b What distance is travelled after 3 hours?

c How long does it take to travel 90 km?

Solution | **Explanation**

a

t	0	1	2	3	4
d	0	60	120	180	240

Substitute each value of t into the rule $d = 60t$. For example, when $t = 3, d = 60 \times 3 = 180$.

b 180 km

The value of d at $t = 3$ is 180.

c 1.5 hours

60 km is travelled every hour, so 1.5 hours is required for 90 km.

For every hour 40 km is travelled.

4 The rule connecting the distance travelled (d km) and time (t hours) is given by $d = 40t$.

a Construct a table of values using the following values of t: $0, 1, 2, 3, 4$.

b What distance is travelled after 3 hours?

c How long does it take to travel 80 km?

5 The rule connecting the volume of water in a tank (V litres) after t minutes is given by $V = 20t + 1000$.

a Construct a table of values using the following values of t: $0, 1, 2, 3, 4, 5$.

b What is the volume after 4 minutes?

c How long does it take for the volume to reach 1100 litres?

Example 4 Constructing tables using negative numbers

For the given rules, construct a table of values for x values from –2 to 2.

a $y = 2x - 3$ **b** $y = -x + 4$

Solution

a

x	–2	–1	0	1	2
y	–7	–5	–3	–1	1

b

x	–2	–1	0	1	2
y	6	5	4	3	2

Explanation

If $y = 2x - 3$ then when $x = -2$, $y = 2 \times (-2) - 3$
$= -4 - 3 = -7$

Repeat for other values of x.

If $y = -x + 4$ then when $x = -2$, $y = -(-2) + 4$
$= 2 + 4 = 6$

Repeat for other values of x.

6 For the given rules complete the given tables.

a $y = 3x$

x	–2	–1	0	1	2
y					

b $y = x - 2$

x	–2	–1	0	1	2
y					

A positive times a negative is a negative.

c $y = 2x + 1$

x	–2	–1	0	1	2
y					

d $y = 2x - 3$

x	–2	–1	0	1	2
y					

A negative times a negative is a positive.

e $y = -x + 2$

x	–2	–1	0	1	2
y					

f $y = -x - 1$

x	–2	–1	0	1	2
y					

g $y = -2x - 1$

x	-2	-1	0	1	2
y					

h $y = -4x + 2$

x	-2	-1	0	1	2
y					

i $y = -6x - 11$

x	-2	-1	0	1	2
y					

PROBLEM-SOLVING AND REASONING

7, 8 7–9

7 To hire a car costs \$70 per day so the rule for the cost (\$$C$) for n days is $C = 70n$.

a What is the cost for:

i $n = 2$? **ii** $n = 10$?

b What does it cost to hire the car for 2 weeks?

c How long can you hire the car if you have \$280 to spend?

8 A rule linking x and y is given by $y = 3x - 4$.

a What is the value of y when:

i $x = 2$? **ii** $x = 1$?

b What x-value gives a y-value of:

i 5? **ii** -4?

c What is the smallest whole number for x which makes y positive?

d What is the largest whole number for x which makes y negative?

9 The profit equation for a small company producing watches is given by $P = 15n - 15\,000$, where \$$P$ is the profit and n is the number of watches sold.

a What is the profit if:

i $n = 2000$? **ii** $n = 1000$?

b How many watches need to be sold to make a profit of:

i \$30 000? **ii** \$45 000? **iii** \$75 000?

c Explain what happens to the profit if $n < 1000$. i.e. less than 1000 watches are sold.

d What is the loss if only 200 watches are sold?

The sum shortcut

— 10

10 To sum the first 3 consecutive positive whole number would mean to calculate $1 + 2 + 3$ which equals 6. Note also that $(3 \times 4) \div 2 = 6$. In a similar way, $1 + 2 + 3 + 4 + 5$ can be calculated as $(5 \times 6) \div 2 = 15$. In general, the rule is: Sum $= \frac{n \times (n+1)}{2}$ so for $n = 5$, Sum $= \frac{5 \times 6}{2} = 15$.

a Use the rule Sum $= \frac{n \times (n+1)}{2}$ for:

i $n = 4$ **ii** $n = 8$ **iii** $n = 10$

b Use the rule to find the sum of:

i the first six positive whole numbers ($n = 6$)

ii the first twelve positive whole numbers.

c Use the rule to calculate:

i $1 + 2 + 3 + \ldots + 7 \quad (n = 7)$ **ii** $1 + 2 + 3 + \ldots + 20$

iii $1 + 2 + 3 + \ldots + 100$

9C Plotting straight line graphs

Interactive

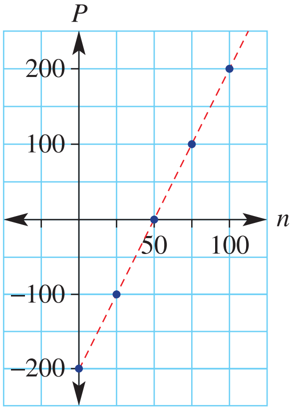
Widgets

HOTsheets

Walkthrough

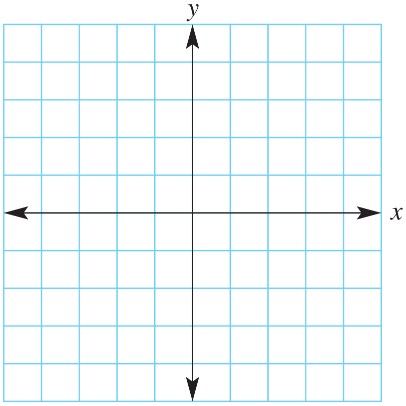
The next step in illustrating the relationship between two variables is to plot points to form a graph. The points are taken from a table of values and plotted on a number plane. If the points form a single straight line, the relationship is said to be linear.

For example, if the profit (\$$P$) from producing and selling n kg of raspberries is given by $P = 4n - 200$, then the graph would look like the following:



Let’s start: Which set of points form a straight line?

Consider the set of points (x, y) from these three tables of values.



A. $y = 1 \div x$

x	−2	−1	0	1	2
y	−0.5	−1	╳	1	0.5

B. $y = x^2$

x	−2	−1	0	1	2
y	4	1	0	1	4

C. $y = 2x$

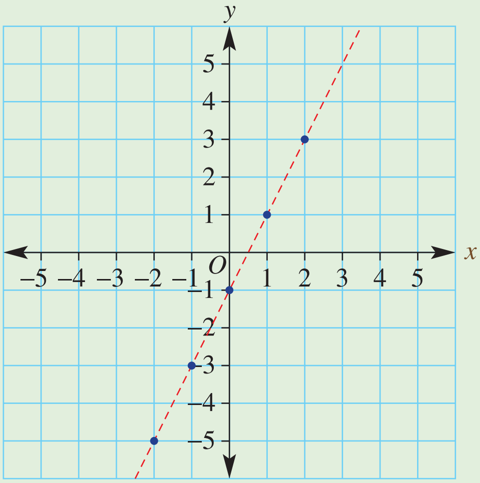
x	−2	−1	0	1	2
y	−4	−2	0	2	4

- If each set is plotted on a number plane, which set would form a single straight line?
- What do you notice about the values in the table that gives a straight line graph?

Key ideas

- A **linear** relationship gives a straight line graph.
  For example, $y = 2x - 1$

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



Linear
A linear graph in two dimensions is a straight line

- To draw a linear graph using a rule:
  - Construct a table of values finding a y-coordinate for each given x-coordinate. Substitute each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

Exercise 9C

UNDERSTANDING 1–3 3

1 For the rule $y = 2x + 3$ find the y-coordinate for these x-coordinates.

a 1	**b** 2	**c** 0	**d** -1
e -5	**f** -7	**g** 11	**h** -12

2 Write the missing number in these tables for the given rules.

a $y = 2x$

x	0	1	2	3
y	0		4	6

b $y = x - 3$

x	-1	0	1	2
y	-4	-3		-1

c $y = 5x + 2$

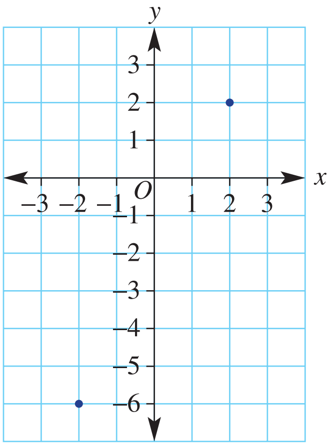
x	-3	-2	-1	0
y		-8	-3	2

3 Complete the graph to form a straight line from the given rule and table. Two points have been plotted for you.

For $(-1, 4)$ move 1 left and 4 down.
For $(0, -2)$ just move 2 down from the origin $(0, 0)$.

$y = 2x - 2$

x	−2	−1	0	1	2
y	−6	−4	−2	0	2



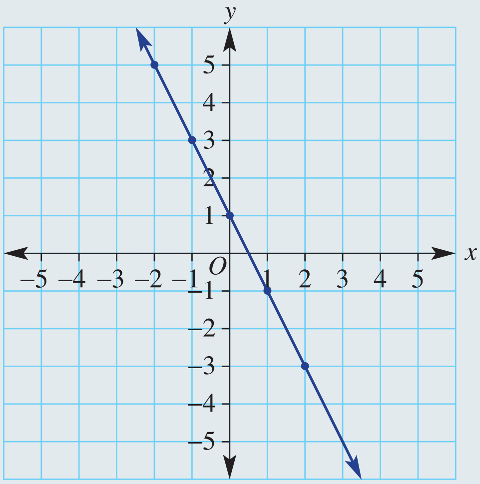
FLUENCY 4, 5(½) 4-5(½)

Example 5 Plotting a graph from a table

Plot a graph from this tables of values.

x	−2	−1	0	1	2
y	5	3	1	−1	−3

Solution



Explanation

Plot the five points $(-2, 5), (-1, 3), (0, 1), (1, -1)$ and $(2, -3)$. Then join to form a straight line.

4 Plot a graph from these tables of values.

a

x	–2	–1	0	1	2
y	2	1	0	–1	–2

b

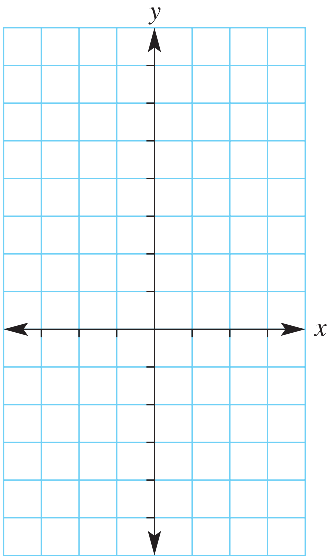
x	–2	–1	0	1	2
y	–3	–2	–1	0	1

c

x	–3	–2	–1	0	1	2	3
y	–3	–2	–1	0	1	2	3

d

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7

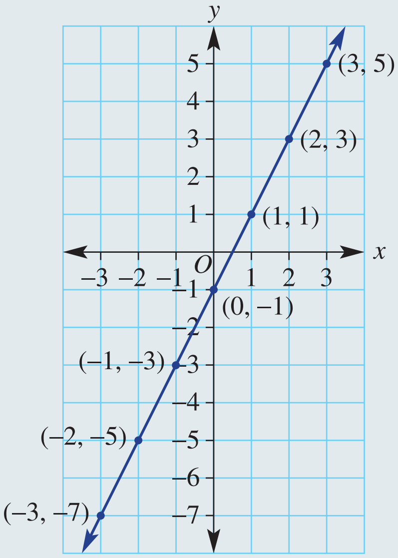


Example 6 Plotting a graph from a rule

For the rule $y = 2x - 1$ construct a table and draw a graph.

Solution

x	–3	–2	–1	0	1	2	3
y	–7	–5	–3	–1	1	3	5



Explanation

Substitute each x-coordinate in the table into the rule to find the y-coordinate.

Plot each point (–3, –7), (–2, –5) … and join them to form the straight line graph.

5 For each rule construct a table then plot and draw a graph. Use a table like the one shown here for each rule.

x	–3	–2	–1	0	1	2	3
y							

Substitute each value of x into the rule to find the value of y. Then plot this pair on a graph.

a $y = x + 1$ **b** $y = x - 2$ **c** $y = 2x - 3$ **d** $y = 2x + 1$

e $y = -2x + 3$ **f** $y = -3x - 1$ **g** $y = -x$ **h** $y = -x + 4$

PROBLEM-SOLVING AND REASONING 6, 7 6–9

6 Decide if the following tables of values would give a straight line graph.

a

x	0	1	2	3	4
y	0	2	4	6	8

b

x	0	1	2	3	4
y	0	3	6	9	12

c

x	0	1	2	3	4
y	2	0	1	5	2

d

x	0	1	2	3	4
y	7	0	4	1	6

e

x	–1	0	1	2	3
y	6	–2	4	1	8

f

x	–2	1	0	1	2	3
y	2	1	0	–1	–2	–3

7 The distance a car travelled (d km) is given by the rule $d = 80t$ where t is in hours.

a Complete this table of values.

t	0	1	2	3
d	0			

b Draw a graph which illustrates the rule and table. Use t for time on the horizontal axis.

c How far does the car travel after 4 hours?

d How long would it take for the car to travel 400 km?

8 a What is the minimum number of points needed to draw a graph of a straight line?

b Draw the graph of these rules by plotting only two points. Use $x = 0$ and $x = 1$.

i $y = \frac{1}{2}x$ **ii** $y = 2x - 1$

9 The graphs of $y = x$, $y = 3x$ and $y = -2x$ all pass through the origin $(0, 0)$. Explain why.

The intersection point

— 10

10 It is possible to find the intersection point of two graphs by plotting each graph and observing the intersection. For each pair of rules, follow these steps.

1 Construct a table of values for each rule.

2 Plot to form two straight line graphs.

3 Give the coordinates of the intersection point (if any).

a $y = x + 2$
$y = -x + 2$

b $y = 2x - 3$
$y = -x + 3$

c $y = 2x - 2$
$y = 2x + 1$

9D Finding the rule using tables

Interactive
Widgets
HOTsheets
Walkthrough

A mathematical rule is a convenient way of describing a relationship between two variables. While a table and a graph are limited by the number of points they show, a rule can be used to find any value of y for any given x-value quickly. Finding such a rule from a collection of points on a graph or table is an important step in the development and application of mathematics.

For example, if a straight fence requires two upright posts per metre, then the rule for the number of posts P is $P = 2n + 1$, where n is the number of metres. An extra post is required for the ends.

Let's start: What's my rule?

Each of the tables here describe a linear relationship between y and x.

x	y
0	4
1	5
2	6
3	7
4	8

x	y
–3	–5
–2	–3
–1	–1
0	1
1	3

x	y
–2	5
–1	4
0	3
1	2
2	1

Rules to choose from:
$y = 2x + 1$
$y = -x + 3$
$y = x + 4$

- For each table match one of the rules to each table.
- Discuss your strategy for finding the three different rules. What patterns did you notice and how did these patterns help determine the rule?

Key ideas

- A rule must satisfy every pair of coordinates (x, y) in a table.
  - Most linear rules can be written in the form $y = \underset{\text{insert a number here}}{\Delta} x + \underset{\text{insert a number here}}{\square}$

    Consider a linear rule of the form $y = \Delta \times x + \square$.
  - The value of Δ will be the increase in y as x increases by 1. If there is a decrease in y then Δ will be negative.

x	–2	–1	0	1	2
y	–1	1	3	5	7

+2 +2 +2 +2

$y = 2x + 3$

x	–2	–1	0	1	2
y	1	0	–1	–2	–3

–1 –1 –1 –1

$y = -x - 1$

  - The value of $\square$ will be the value of y when $x = 0$.

Exercise 9D

UNDERSTANDING 1–3 3

1 Match the rules **A**, **B** and **C** with the tables **a**, **b** and **c**.

a

x	-1	0	1	2	3
y	-5	-4	-3	-2	-1

b

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

c

x	-2	-1	0	1	2
y	2	1	0	-1	-2

A $y = 2x - 3$

B $y = -x$

C $y = x - 4$

2 By how much does y increase for each increase by 1 in x? If y is decreasing give a negative answer.

a

x	-2	-1	0	1	2
y	-1	1	3	5	7

b

x	-3	-2	-1	0	1
y	4	3	2	1	0

c

x	-4	-3	-2	-1	0
y	4	2	0	-2	-4

d

x	-3	-2	-1	0	1
y	-6	-3	0	3	6

3 For each of the tables in question **2**, state the value of y when $x = 0$.

FLUENCY 4-5(½) 4-5(½)

Example 7 Finding rules from tables

Find the rule for these tables of values.

a

x	-2	-1	0	1	2
y	0	1	2	3	4

b

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

Solution

a $\Delta = 1$, $\square = 2$

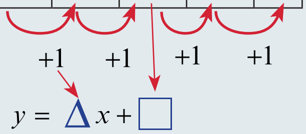
$y = x + 2$

Explanation

x	-2	-1	0	1	2
y	0	1	2	3	4

$\Delta = 1$

$\square = 2$



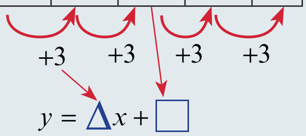
b $\Delta = 3$, $\square = -2$

$y = 3x - 2$

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

$\Delta = 3$

$\square = -2$



4 Find the rule for these tables of values.

a

x	–1	0	1	2	3
y	0	1	2	3	4

b

x	–2	–1	0	1	2
y	–4	–2	0	2	4

c

x	–2	–1	0	1	2
y	0	2	4	6	8

d

x	–2	–1	0	1	2
y	–7	–4	–1	2	5

e

x	–2	–1	0	1	2
y	–8	–4	0	4	8

f

x	–2	–1	0	1	2
y	–3	0	3	6	9

In $y = \Delta x + \square$

Δ = increase in y

$\square$ = y value at $x = 0$

Example 8 Finding rules when y is decreasing

Find the rule for these tables of values.

a

x	–1	0	1	2	3
y	1	0	–1	–2	–3

b

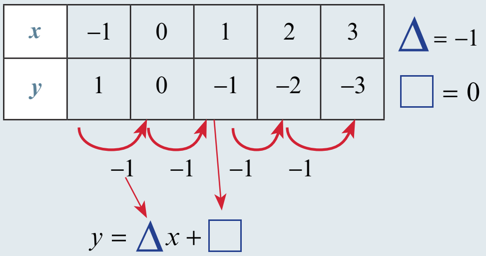
x	–2	–1	0	1	2
y	5	3	1	–1	–3

Solution

a $\Delta = -1$ and $\square = 0$

$y = -x$

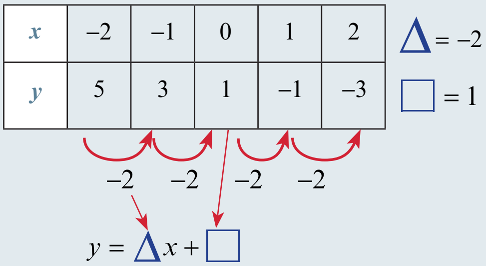
Explanation



x	–1	0	1	2	3
y	1	0	–1	–2	–3

b $\Delta = -2$ and $\square = 1$

$y = -2x + 1$



x	–2	–1	0	1	2
y	5	3	1	–1	–3

5 Find the rule for these tables of values.

a

x	–2	–1	0	1	2
y	2	1	0	–1	–2

b

x	–2	–1	0	1	2
y	1	0	–1	–2	–3

c

x	–3	–2	–1	0	1
y	4	3	2	1	0

d

x	–1	0	1	2	3
y	8	6	4	2	0

e

x	–2	–1	0	1	2
y	4	2	0	–2	–4

f

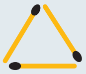
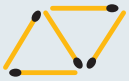
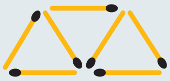
x	–3	–2	–1	0	1
y	10	7	4	1	–2

Since y is decreasing Δ will be negative.

PROBLEM-SOLVING AND REASONING 6, 7 6–8

Example 9 Finding rules for patterns

If x = number of triangles and y = number of matchsticks, use a table to help find a rule for this pattern.

shape 1	shape 2	shape 3	shape 4
			

Solution

x	1	2	3	4
y	3	5	7	9

$y = 2x + 1$

Explanation

x is the number of triangles and y is the number of matchsticks.

x	1	2	3	4
y	3	5	7	9

+2 +2 +2

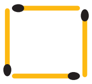
$y = 2x + \square$

$\square = 1$ since $2 \times 1 + 1 = 3$

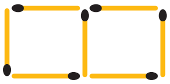
6 Write a rule for these matchstick patterns.

a x = number of squares and y = number of matchsticks

Shape 1

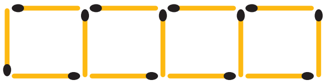


Shape 2



Shape 3

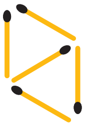
Shape 4



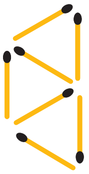
b x = number of triangles and y = number of matchsticks

Shape 1

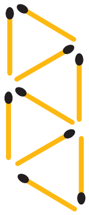
Shape 2



Shape 3



Shape 4

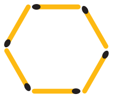


First draw a table and fill it in.

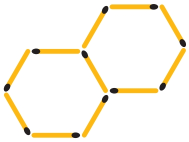
x	1	2	3	4
y	4	7		

c x = number of hexagons and y = number of matchsticks

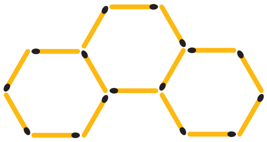
Shape 1



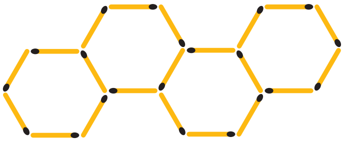
Shape 2



Shape 3

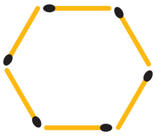


Shape 4

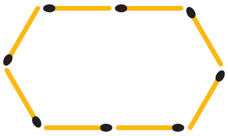


d x = number of matchsticks on top row and y = number of matchsticks

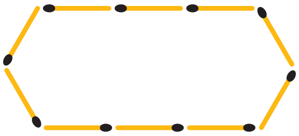
Shape 1



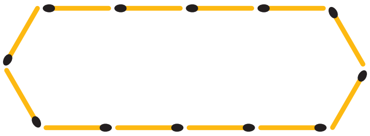
Shape 2



Shape 3



Shape 4



7 A rule is of the form $y = 3x + \square$. Find the value of $\square$, if a pair of (x, y) is:

a $(1, 4)$ **b** $(-1, 0)$ **c** $(-2, 1)$ **d** $(0, 0)$

8 Look at this table of values.

x	–2	0	2	4
y	–4	–2	0	2

a The increase in y for each unit increase in x is not 2. Explain why.

b If the pattern is linear, state the increase in y for each increase by 1 in x.

c Write the rule for the relationship.

d Find the rule for these tables.

i

x	–4	–2	0	2	4
y	–5	–1	3	7	11

ii

x	–6	–3	0	3	6
y	15	9	3	–3	–9

iii

x	–3	–1	1	3	5
y	–10	–4	2	8	14

iv

x	–10	–8	–6	–4	–2
y	20	12	4	–4	–12

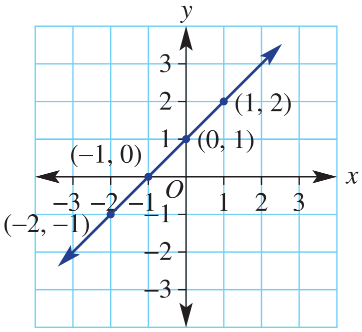
9D

Rules from graphs

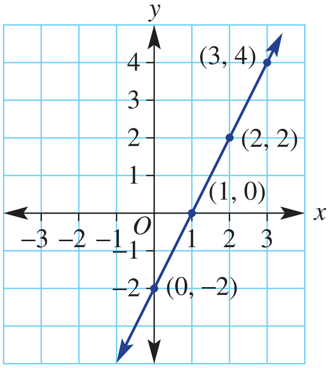
— 9

9 Find the rule for these graphs by first constructing a table of (x, y) values.

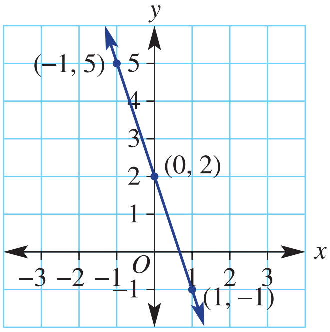
a



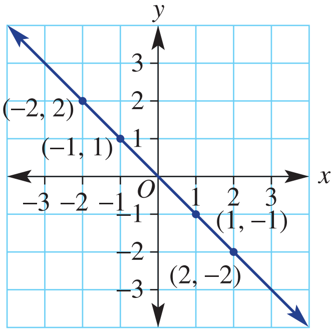
b



c

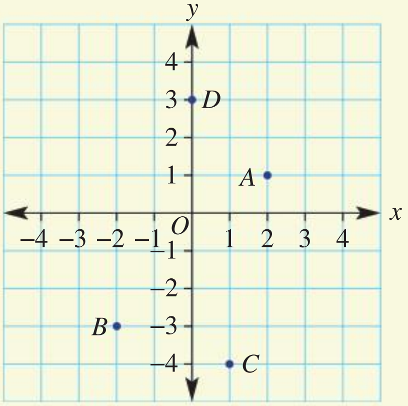


d



Progress quiz

9A **1** Write down the coordinates of the points A to D on this number plane.



9A **2** The following set of points forms a basic shape. Describe the shape, with or without drawing a graph.

$A(-3, 4)\ B(-1, 4)\ C(-3, -3)\ D(-1, -3)$

9B **3** If $y = 3x + 4$, find the y value for these values of x.

a $x = 4$ **b** $x = 1$ **c** $x = -3$ **d** $x = 11$

9B **4** For the given rules complete the given tables.

a $y = 4x$

x	-2	-1	0	1	2
y					

b $y = -2x + 5$

x	-2	-1	0	1	2
y					

9B **5** To rent a surfboard costs $12 per hour, so the rule for the cost $(\$C)$ for h hours is $C = 12h$.

a What is the cost for:

i $h = 3$? **ii** $h = 7$?

b What would it cost to hire the surfboard for 4 hours?

c A secondhand surfboard is available to purchase for $230. How many hours of surfing would you need to do for it to be better value to purchase the secondhand surfboard rather than keep renting a board?

9C **6** Plot a graph from this table of values and then join the dots to form a straight line.

x	–2	–1	0	1	2
y	–1	1	3	5	7

9C **7** For the rule $y = 2x - 2$ construct a table and draw a graph.

9C **8** For the following tables of values, fill in the missing numbers that would give a straight line graph.

a

x	–2	–1	0	1	2
y	5	3		–1	

b

x	–2	–1	0	1	2
y		0		6	9

9D **9** Find the rules for these tables of values.

a

x	–2	–1	0	1	2
y	–3	–2	–1	0	1

b

x	–2	–1	0	1	2
y	8	6	4	2	0

c

x	–2	–1	0	1	2
y	–4	–1	2	5	8

9D **10** A rule is of the form $y = 2x - \square$. Find the value of $\square$, if a pair of (x, y) is:

a $(2, 3)$

b $(1, -3)$

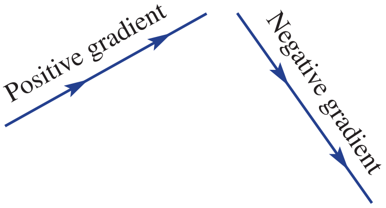
c $(7, 14)$

9E Gradient

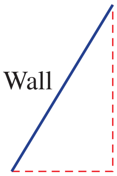
Interactive
Widgets
HOTsheets
Walkthrough

When we think about how two variables (like population and time) are related, we are often concerned about how quickly one variable changes with respect to the other.

Graphically, we use the concept of **gradient** to describe this change. This is illustrated by the slope of a line. The steepness or slope of a line depends on how far it rises or falls over a given horizontal distance. This is why the gradient is calculated by dividing the vertical rise by the horizontal run between two points. Lines that rise (from left to right) have a positive gradient and lines that fall (from left to right) have a negative gradient.



Let's start: Which is the steepest?

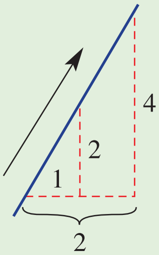


At a children's indoor climbing centre there are three types of sloping walls to climb. The blue wall rises 2 metres for each metre across. The red wall rises 3 metres for every 2 metres across and the yellow wall rises 7 metres for every 3 metres across.

- Draw a diagram showing the slope of each wall.
- Label your diagrams with the information given above.
- Discuss which wall might be the steepest giving reasons.
- Discuss how it might be possible to accurately compare the slope of each wall.

Key ideas

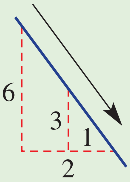
- The **gradient** is a measure of slope.
- **Gradient** $= \frac{\textbf{rise}}{\textbf{run}}$
  - Rise = change in y
  - Run = change in x
  - The run is always considered to be positive when moving from left to right.



Positive gradient

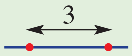
$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2 \quad \text{or} \quad \text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$$

- A gradient is negative if y decreases as x increases. The rise is considered to be negative.
- The gradient of a horizontal line is 0.
- The gradient of a vertical line is undefined.

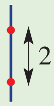


Negative gradient

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-6}{2} = -3 \quad \text{or} \quad \text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{1} = -3$$



$$\text{Gradient} = \frac{0}{3} = 0$$



$$\text{Gradient} = \frac{2}{0} = \text{undefined}$$

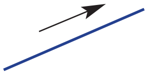
Gradient
A measure of slope

Exercise 9E

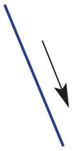
UNDERSTANDING 1–4 4

1 Decide if the gradients of these lines are positive or negative.

a



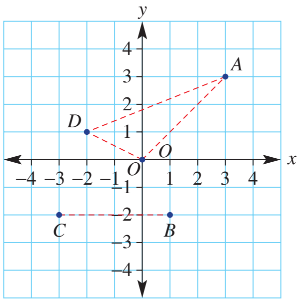
b



c

d

2 This graph shows four points A, B, C and D.



a Write down the coordinates of the four points A, B, C and D.

b What is the rise between these pairs of points?

- **i** From O to A
- **ii** From C to B
- **iii** From D to O
- **iv** From D to A

c What is the run between these pairs of points?

- **i** From O to A
- **ii** From C to B
- **iii** From D to O
- **iv** From D to A

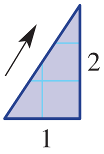
3 Simplify these fractions.

a $\frac{4}{2}$ **b** $\frac{12}{4}$ **c** $\frac{6}{4}$ **d** $\frac{14}{7}$

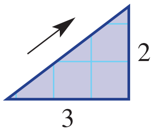
e $-\frac{6}{3}$ **f** $-\frac{20}{4}$ **g** $-\frac{16}{6}$ **h** $-\frac{15}{9}$

4 Write down the gradient, using $\frac{\text{rise}}{\text{run}}$ for each of these slopes.

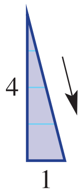
a



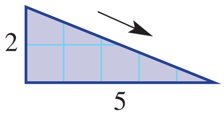
b



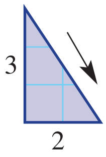
c



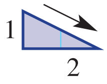
d



e

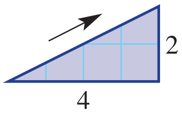


f

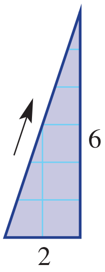


Remember that the rise is negative when the arrow is pointing downwards.

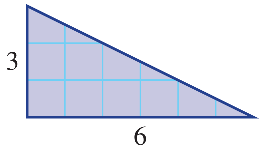
g



h



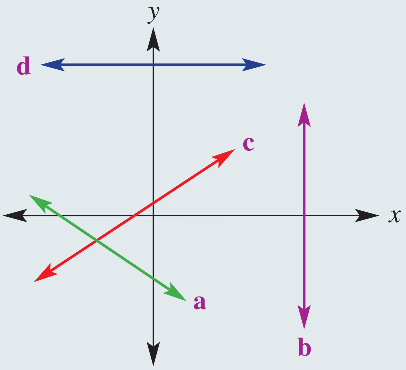
i



FLUENCY | 5–7 | 5, 6–7(½)

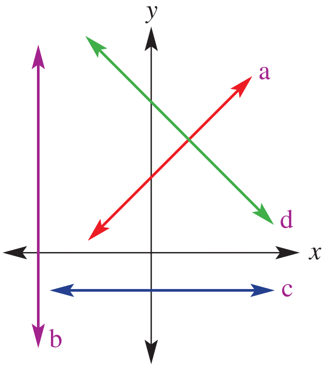
Example 10 Deciding a type of gradient

Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



Solution	Explanation
a Negative gradient	As x increases y decreases.
b Undefined gradient	The line is vertical.
c Positive gradient	y increases as x increases.
d Zero gradient	There is no increase or decrease in y as x increases.

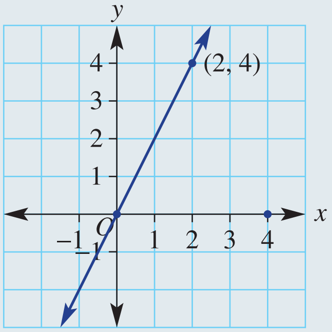
5 Decide if these lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



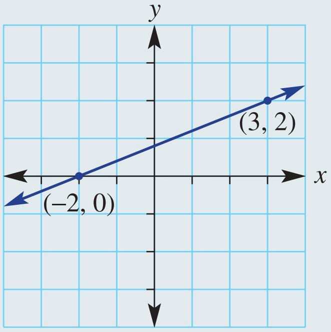
Example 11 Finding a positive gradient from a graph

Find the gradient of these lines.

a



b



Solution

a Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{4}{2} = 2$

b Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{2}{5}$

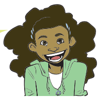
Explanation

The rise is 4 for every 2 across to the right. So rise = 4 and run = 2.

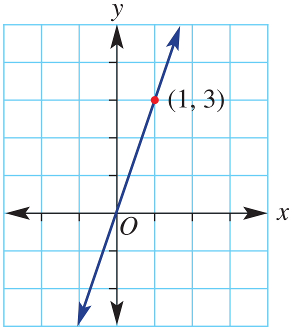
From $(-2, 0)$ to $(3, 2)$ the rise is 2 and the run is 5. Leave your answer as a fraction.

6 Find the gradient of these lines. Use Gradient $= \frac{\text{rise}}{\text{run}}$.

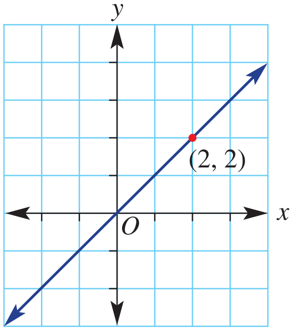
The run is always positive.



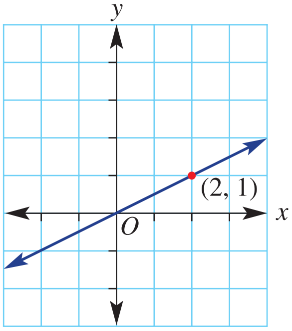
a



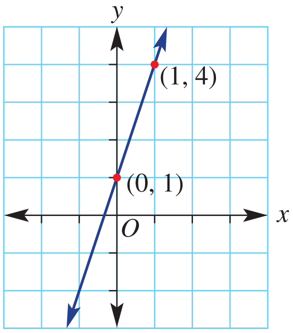
b



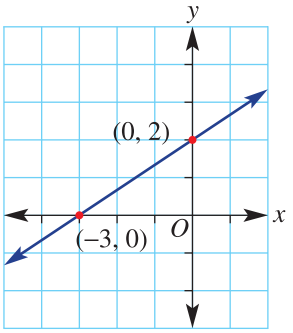
c



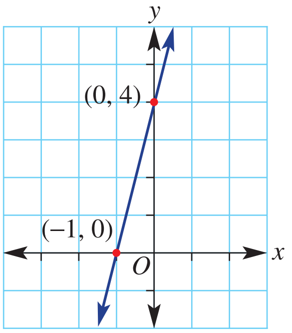
d



e



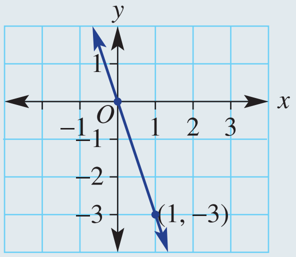
f



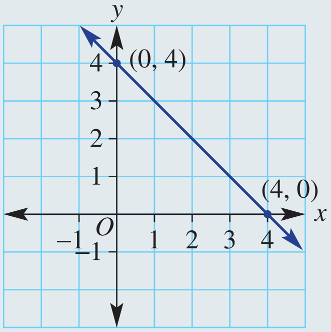
Example 12 Finding a negative gradient

Find the gradient of these lines.

a



b

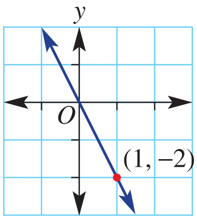


Solution	Explanation
a Gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{-3}{1}$ $= -3$	Between $(0, 0)$ and $(1, -3)$ the rise is -3 and the run is 1.
b Gradient $= \frac{\text{rise}}{\text{run}}$ $= \frac{-4}{4}$ $= -1$	The y value falls 4 units while the x value increases by 4.

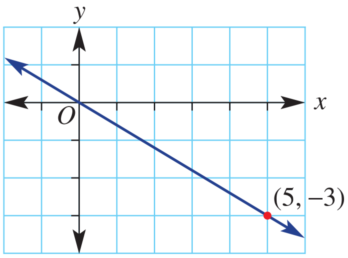
The run is positive and the rise will be negative for these lines.

7 Find the gradient of these lines. Use Gradient $= \frac{\text{rise}}{\text{run}}$.

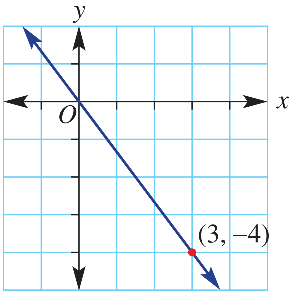
a



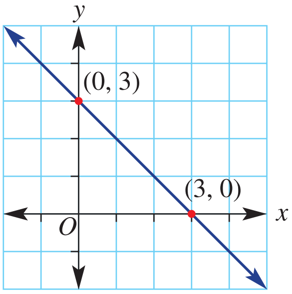
b



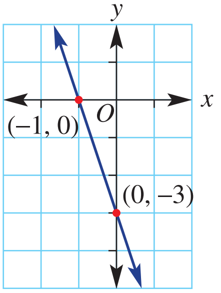
c



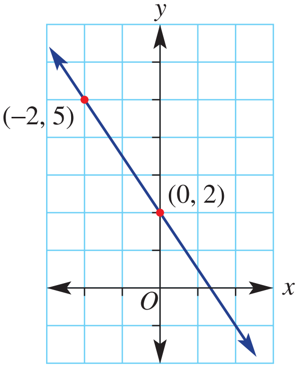
d



e



f

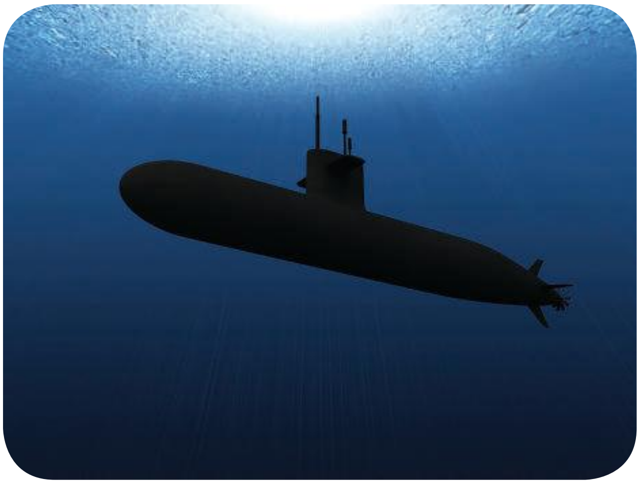


PROBLEM-SOLVING AND REASONING

8, 9	8, 10, 11

8 Abdullah climbs a rocky slope that rises 12 m for each 6 metres across. His friend Jonathan climbs a nearby grassy slope that rises 25 m for each 12 m across. Which slope is steeper?

9 A submarine falls 200 m for each 40 m across and a torpedo falls 420 m for each 80 m across in pursuit of the submarine. Which has the steeper gradient, the submarine or torpedo?



10 **a** Explain why the rise between the two points $(-1, 1)$ and $(3, 7)$ is 6.
b Explain why the rise between the two points $(-4, 2)$ and $(3, -5)$ is -7.
c Explain why the run between the two points $(-1, 1)$ and $(3, 7)$ is 4.
d Explain why the run between the two points $(-4, 2)$ and $(3, -5)$ is 7.

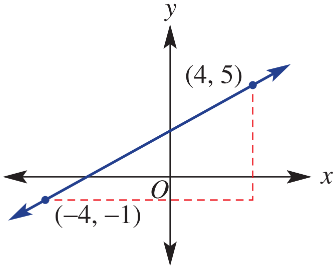
11 A line joins the point $(0, 0)$ with the point (a, b) with a gradient of 2.
a If $a = 1$, find b. **b** If $a = 5$, find b.

Gradients without grids

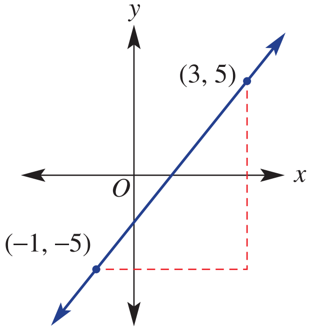
— 12

12 Find the gradient of these lines. You will need to first calculate the rise and the run.

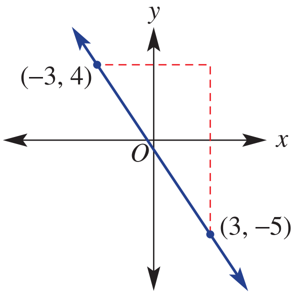
a



b



c



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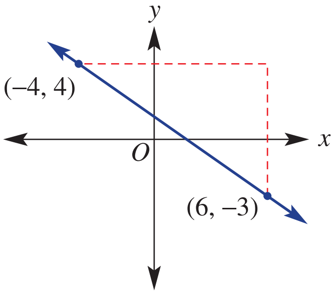


13 Find the gradient of the line joining these pairs of points.

a $(0, 2)$ and $(2, 7)$

b $(0, -1)$ and $(3, 4)$

c $(-3, 7)$ and $(0, -1)$

d $(-5, 6)$ and $(1, 2)$

e $(-2, -5)$ and $(1, 3)$

f $(-5, 2)$ and $(5, -1)$

9F Applying linear graphs ★

Interactive

Widgets

HOTsheets

Walkthrough

Rules, tables and graphs can be applied to many practical situations where a relationship exists between two variables. It is also often the case that this relationship is linear, meaning that the graph is a straight line. For example, if a pile of coal being emptied out of a pit increases at a rate of 12 tonnes per hour, then the graph of the mass of dirt over time would be a straight line. For every hour, the mass of dirt increases by 12 tonnes.

A constant rate of excavation creates a linear relationship and a straight line graph.

Let's start: Mining for coal

A coal mining pit produces 12 tonnes of coal per hour.

- Describe the two related variables in this situation.
- Discuss whether or not the relationship between the two variables is linear. Give reasons.
- Use a table and a graph to illustrate the relationship.
- Find a rule that links the two variables and discuss how your rule might be used to find the mass of coal at a given time.

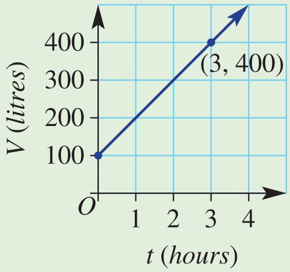
Key ideas

- If the rate of change of one variable with respect to another is constant, then the relationship between the two variables is **linear**.
- When applying straight line graphs, choose letters to replace x and y to suit the variables.

  For example, V for volume and t for time.

t	0	1	2	3
V	100	200	300	400

$V = 100t + 100$



Exercise 9F

UNDERSTANDING 1–4 4

1 Find a rule linking d and t in these tables.

a

t	0	1	2	3
d	0	3	6	9

b

t	0	1	2	3
d	2	3	4	5

c

t	0	1	2	3
d	4	3	2	1

Put d on the left by itself as in $d = 2t$ or $d = -t + 3$.

2 A rule linking distance d and time t is given by $d = 10t + 5$. Use this rule to find the value of d for the given values of t.

a $t = 1$ **b** $t = 4$ **c** $t = 0$ **d** $t = 12$

3 The height (in cm) of fluid in a flask increases at a rate of 30 cm every minute starting at 0 cm. Find the height of fluid in the flask at these times.

a 2 minutes **b** 5 minutes **c** 11 minutes

4 The volume of gas in a tank decreases from 30 L by 2 L every second. Find the volume of gas in the tank at these times.

a 1 second **b** 3 seconds **c** 10 seconds

FLUENCY 5, 7 6, 7

Example 13 Linking distance with time

A hiker walks at a constant rate of 4 kilometres per hour for 4 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 4.
b Draw a graph by plotting the points given in the table in part **a**.
c Write a rule linking d with t.
d Use your rule to find the distance travelled for 2.5 hours of walking.
e Use your rule to find the time taken to travel 8 km.

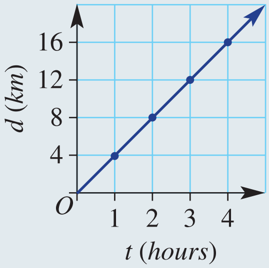
Solution **Explanation**

a

t	0	1	2	3	4
d	0	4	8	12	16

d increases by 4 for every increase in t by 1.

b



Plot the points on a graph using a scale that matches the numbers in the table.

Solution | **Explanation**

c

t	0	1	2	3	4
d	0	4	8	12	16

$+4 \quad +4 \quad +4 \quad +4$

For $d = \Delta t + \square$, $\Delta = 4$ and $\square = 0$.

d $d = 4t$
$= 4 \times 2.5$
$= 10$

The distance is 10 km after 2.5 hours of walking.

Substitute $t = 2.5$ into your rule and find the value for d.

e $d = 4t$
$8 = 4t$
$2 = t$

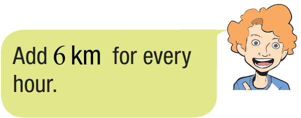
It takes 2 hours to travel 8 km.

Substitute $d = 8$ into your rule then divide both sides by 4. Check by looking at your graph from part **b**.

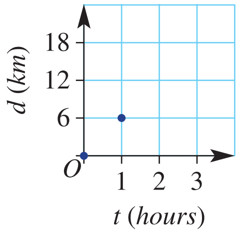
5 A jogger runs at a rate of 6 kilometres per hour for 3 hours.

a Complete this table of values using t for time in hours and d for distance in kilometres.

t	0	1	2	3
d	0	6		



b Complete this graph by plotting the points given in the table in part **a**.



c Write a rule linking d with t.

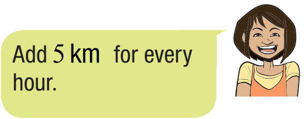
d Use your rule to find the distance travelled for 1.5 hours of jogging.

e Use your rule to find how long it takes to travel 12 km. Check by looking at your graph from part **b**.

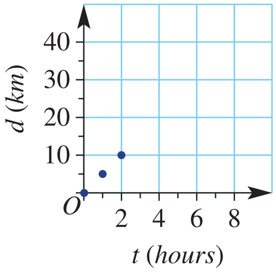
6 A paddle steamer moves up the Murray River at a constant rate of 5 kilometres per hour for 8 hours.

a Complete this table of values using t for time in hours and d for distance in kilometres.

t	0	1	2	3	4	5	6	7	8
d	0	5	10						



b Complete this graph by plotting the points given in the table in part **a**.



c Write a rule linking d with t.

d Use your rule to find the distance travelled after 4.5 hours.

e Use your rule to find how long it takes to travel 20 km. Check by looking at your graph from part **b**.

Example 14 Applying graphs when the rate is negative

The initial volume of water in a dish in the sun is 300 mL. The water evaporates and the volume decreases by 50 mL per hour for 6 hours.

a Draw a table of values using t for time in hours and V for volume in millilitres.
b Draw a graph by plotting the points given in the table in part **a**.
c Write a rule linking V with t.
d Use your rule to find the volume of water in the dish after 4.2 hours in the sun.
e Use your rule to find the time taken for the volume to reach 75 mL.

Solution

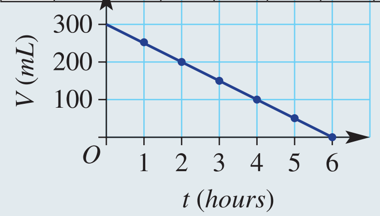
a

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

Explanation

The volume starts at 300 millilitres and decreases by 50 millilitres every hour.

b



Use numbers from 0 to 300 on the V-axis and 0 to 6 on the t-axis to accommodate all the numbers in the table.

c $V = -50t + 300$

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

−50 −50 −50 −50 −50

$V = \Delta t + \square$, $\Delta = -50$ and $\square = 300$.

d

$$\begin{aligned} V &= -50t + 300 \\ &= -50 \times 4.2 + 300 \\ &= 90 \end{aligned}$$

The volume of water in the dish is 90 millilitres after 4.2 hours.

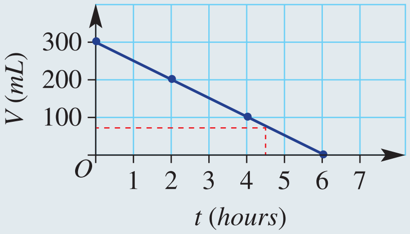
Substitute $t = 4.2$ into your rule to find V.

e

$$\begin{aligned} V &= -50t + 300 \\ 75 &= -50t + 300 \\ -225 &= -50t \\ 4.5 &= t \end{aligned}$$

It takes 4.5 hours for the volume to reach 75 mL. Check by looking at your graph from part **b**.

Substitute $V = 75$ into your rule.
Subtract 300 from both sides.
Divide both sides by −50.
Check by looking at your graph from part **b**.

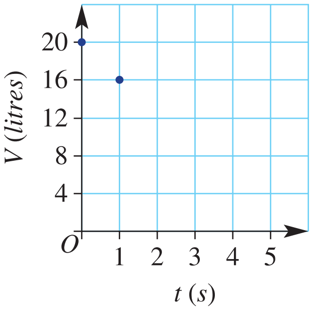


7 The volume of water in a sink is 20 L. The plug is pulled out and the volume decreases by 4 L per second for 5 seconds.

a Complete this table of values using t for time in seconds and V for volume in litres.

t	0	1	2	3	4	5
V	20	16				

Complete this graph by plotting the points given in the table in part **a**.



b Write a rule linking V with t.

c Use your rule to find the volume of water in the sink 2.2 seconds after the plug is pulled.

d Use your rule to find how long it takes for the volume to fall to 8 L. Check by looking at your graph.

PROBLEM-SOLVING AND REASONING 8, 9 9, 10

8 A weather balloon at a height of 500 m starts to descend at a rate of 125 m per minute for 4 minutes.

a Draw a table of values using t for time in minutes and h for height in metres.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking h with t.

d Use your rule to find the height of the balloon after 1.8 seconds.

e Use your rule to find how long it takes for the balloon to fall to a height of 125 m. Check by looking at your graph from part **b**.

9 A BBQ gas bottle starts with 3.5 kg of gas. Gas is used at a rate of 0.5 kg per hour for a long lunch.

a Write a rule for the mass of gas M in terms of time t.

b How long will it take for the gas bottle to empty?

c How long will it take for the mass of the gas in the bottle to reduce to 1.25 kg?

Write the rule in the form $M = \Delta t + \square$.

10 A cyclist races $50\,\text{km}$ at an average speed of $15\,\text{km}$ per hour.

- **a** Write a rule for the distance travelled d in terms of time t.
- **b** How long will it take the cyclist to travel $45\,\text{km}$?
- **c** How long will the cyclist take to complete the 50-km race? Give your answer in hours and minutes.

Danger zone

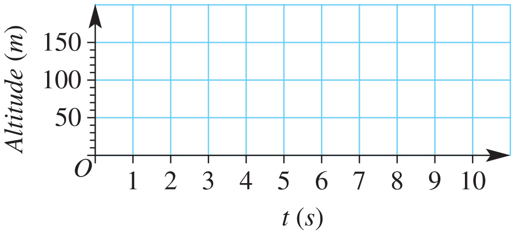
— 11

11 Two small planes take off and land at the same airfield. One plane takes off from the runway and gains altitude at a rate of 15 metres per second. At the same time, the second plane flies near the runway and reduces its altitude from 100 metres at rate of 10 metres per second.

- **a** Complete this table of values using t between 0 and 10 seconds and h for height in metres of both planes.

t(s)	0	1	2	3	4	5	6	7	8	9	10
h_1(m)	0	15									
h_2(m)	100	90									

- **b** On the same set of axes draw a graph of the height of each plane during the 10-second period.



- **c** How long does it take for the second plane to touch the ground?
- **d** Write a rule for the height of each plane.
- **e** At what time are the planes at the same height?
- **f** At what time is the first plane at a height of $37.5\,\text{m}$?
- **g** At what time is the second plane at a height of $65\,\text{m}$?

9G Using graphs to solve linear equations ★

Interactive

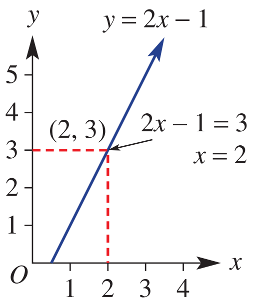
Widgets

HOTsheets

Walkthrough

The rule for a straight line shows the connection between the x- and y-coordinate of each point on the line. We can substitute a given x-coordinate into the rule to calculate the y-coordinate. When we substitute a y-coordinate into the rule, it makes an equation that can be solved to give the x-coordinate. So, for every point on a straight line, the value of the x-coordinate is the solution to a particular equation.

The point of intersection of two straight lines is the shared point where the lines cross over each other. This is the only point with coordinates that satisfy both equations; that is, makes both equations true (LHS = RHS).



For example, the point $(2, 3)$ on the line $y = 2x - 1$ shows us that when $2x - 1 = 3$ the solution is $x = 2$

Let's start: Matching equations and solutions

When a value is substituted into an equation and it makes the equation true (LHS = RHS), then that value is a solution to that equation.

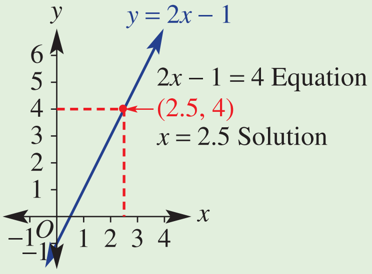
- From the lists below, match each equation with a solution. Some equations have more than one solution.

Equations	
$2x - 4 = 8$	$y = x + 4$
$3x + 2 = 11$	$y = 2x - 5$
$y = 10 - 3x$	$5x - 3 = 2$

Possible solutions			
$x = 1$	$(1, 5)$	$x = 2$	$(3, 1)$
$x = -1$	$x = 6$	$(2, -1)$	$(2, 6)$
$(-2, -9)$	$(-2, 16)$	$x = 3$	$(2, 4)$

- Which two equations share the same solution and what is this solution?
- List the equations that have only one solution. What is a common feature of these equations?
- List the equations that have more than one solution. What is a common feature of these equations?

Key ideas



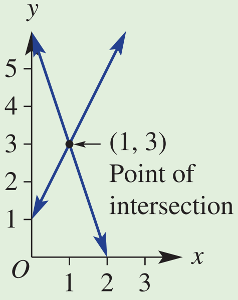
- The x-coordinate of each point on the graph of a straight line is a solution to a particular linear equation.
  - A particular linear equation is formed by substituting a chosen y-coordinate into a linear relationship.

  For example, if $y = 2x - 1$ and $y = 4$, then the linear equation is $2x - 1 = 4$.
  - The solution to this equation is the x-coordinate of the point with the chosen y-coordinate.

  For example, the point $(2.5, 4)$ shows that $x = 2.5$ is the solution to $2x - 1 = 4$.
- A point (x, y) is a solution to the equation for a line if its coordinates make the equation true.
  - An equation is true when LHS = RHS after the coordinates are substituted.
  - Every point on a straight line is a solution to the equation for that line.
  - Every point that is not on a straight line is a not a solution to the equation for that line.

- The point of intersection of two straight lines is the only solution that satisfies both equations.
  - The point of intersection is the shared point where two straight lines cross each other.
  - This is the only point with coordinates that make both equations true. For example, $(1, 3)$ is the only point that makes both $y = 6 - 3x$ and $y = 2x + 1$ true.
    Substituting $(1, 3)$

$$\begin{aligned} y &= 6 - 3x \\ 3 &= 6 - 3 \times 1 \\ 3 &= 3 \text{ (True)} \end{aligned} \qquad \begin{aligned} y &= 2x + 1 \\ 3 &= 2 \times 1 + 1 \\ 3 &= 3 \text{ (True)} \end{aligned}$$



Exercise 9G

UNDERSTANDING 1–3 3

1 Use the given rule to complete this table and then plot and join the points to form a straight line. $y = 2x - 1$.

x	-2	-1	0	1	2	3
y						

2 State the coordinates (x, y) of the point on this graph of $y = 2x$ where:

a $2x = 4$ (i.e. $y = 4$)

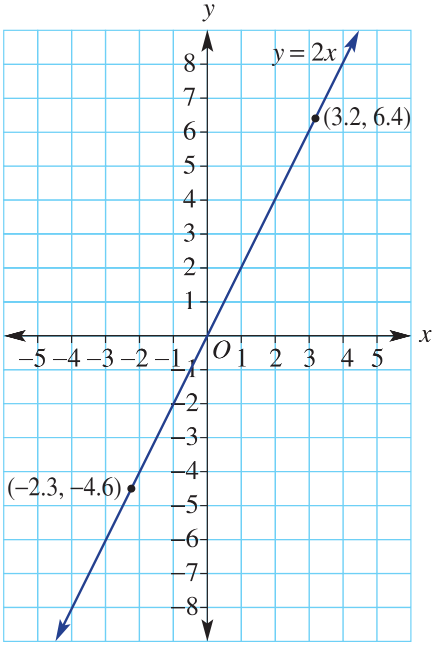
b $2x = 6.4$ (i.e. $y = 6.4$)

c $2x = -4.6$

d $2x = 7$

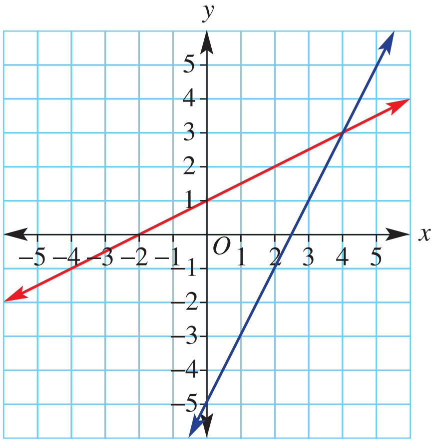
e $2x = -14$

f $2x = 2000$

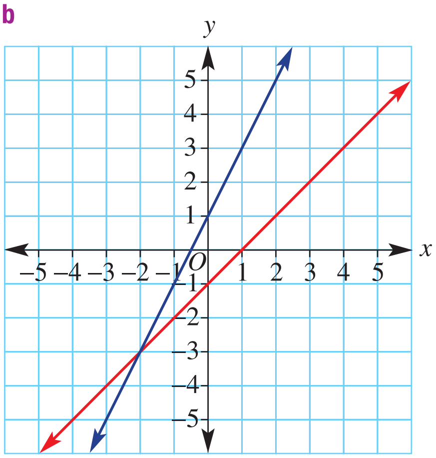


3 For each of these graphs write down the coordinates of the point of intersection (i.e. the point where the lines cross over each other).

a



b



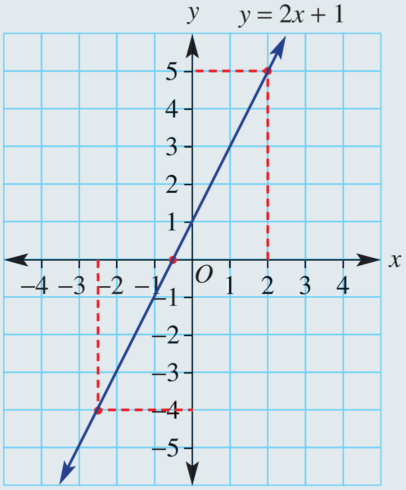
Example 15 Using a linear graph to solve an equation

Use the graph of $y = 2x + 1$ shown here to solve each of the following equations:

a $2x + 1 = 5$

b $2x + 1 = 0$

c $2x + 1 = -4$



Solution	Explanation
a $x = 2$	Locate the point on the line with y-coordinate 5. The x-coordinate of this point is 2 so $x = 2$ is the solution to $2x + 1 = 5$.
b $x = -0.5$	Locate the point on the line with y-coordinate 0. The x-coordinate of this point is -0.5 so $x = -0.5$ is the solution to $2x + 1 = 0$.
c $x = -2.5$	Locate the point on the line with y-coordinate -4. The x-coordinate of this point is -2.5 so $x = -2.5$ is the solution to $2x + 1 = -4$.

FLUENCY | 4–6 | 4, 6, 7(½)

4 Use the graph of $y = 2x - 1$, shown here, to find the solution to each of these equations.

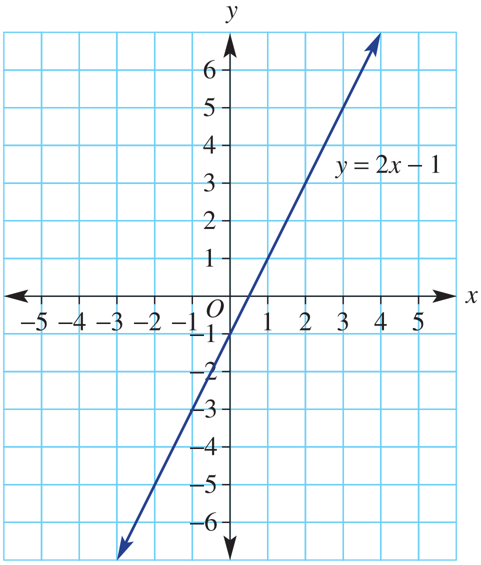
a $2x - 1 = 3$

b $2x - 1 = 0$

c $2x - 1 = 5$

d $2x - 1 = -6$

e $2x - 1 = -4$



5 Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.

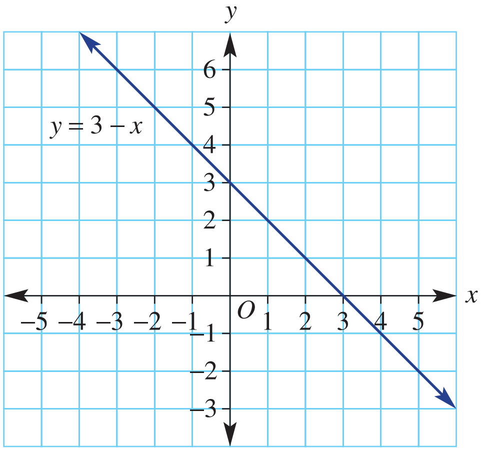
a $3 - x = 5.5$

b $3 - x = 0$

c $3 - x = 3.5$

d $3 - x = -1$

e $3 - x = -2$



Example 16 Using the point of intersection of two lines to solve an equation

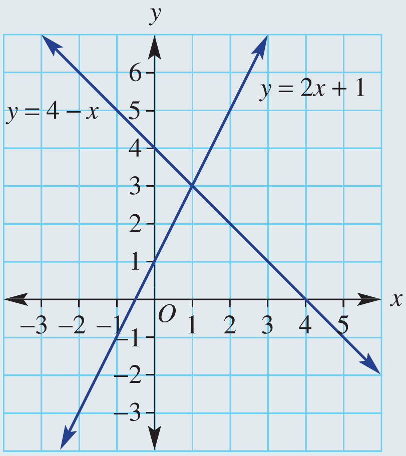
Use the graph of $y = 4 - x$ and $y = 2x + 1$, shown here, to answer these questions.

a Write four solutions (x, y) for the line with equation $y = 4 - x$.

b Write four solutions (x, y) for the line with equation $y = 2x + 1$.

c Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.

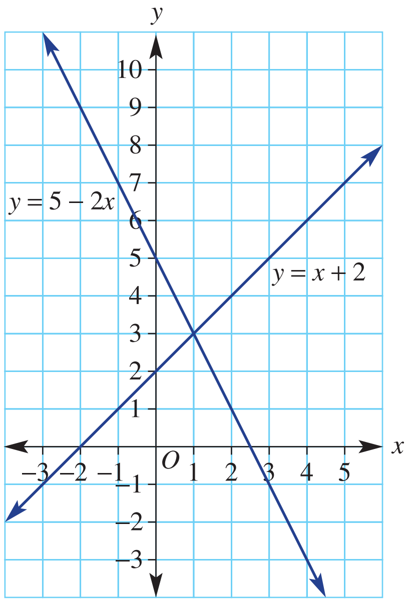
d Solve the equation $4 - x = 2x + 1$.



	Solution	Explanation
a	$(-2, 6), (-1, 5), (1, 3), (4, 0)$	Many correct answers. Each point on the line $y = 4 - x$ is a solution to the equation for that line.
b	$(-2, -3), (0, 1), (1, 3), (2, 5)$	Many correct answers. Each point on the line $y = 2x + 1$ is a solution to the equation for that line.
c	$(1,3)$ $y = 4 - x$ $3 = 4 - 1$ $3 = 3$ (True) $(1,3)$ $y = 2x + 1$ $3 = 2 \times 1 + 1$ $3 = 3$ (True)	The point of intersection $(1, 3)$ is the solution that satisfies both equations. Substitute $(1, 3)$ into each equation and show that it makes a true equation (LHS = RHS).
d	$x = 1$	The solution to $4 - x = 2x + 1$ is the x-coordinate at the point of intersection. The value of both rules is equal for this x-coordinate.

6 Use the graph of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.

- **a** Write four solutions (x, y) for the equation $y = 5 - 2x$.
- **b** Write four solutions (x, y) for the equation $y = x + 2$.
- **c** Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.
- **d** Solve the equation $5 - 2x = x + 2$ from the graph.



7 Graph each pair of lines on the same set of axes and read off the point of intersection.

a $y = 2x - 1$

x	−2	−1	0	1	2	3
y						

$y = x + 1$

x	−2	−1	0	1	2	3
y						

b $y = -x$

x	−2	−1	0	1	2	3
y						

$y = x + 2$

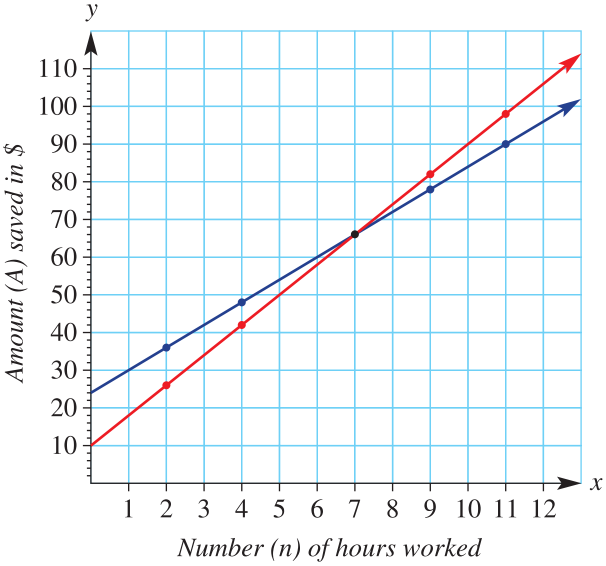
x	−2	−1	0	1	2	3
y						

PROBLEM-SOLVING AND REASONING | 8, 10 | 8–10

8 Jayden and Ruby are saving all their money for the school ski trip.

- Jayden has saved $\$24$ and earns $\$6$ per hour mowing lawns.
- Ruby has saved $\$10$ and earns $\$8$ per hour babysitting.

This graph shows the total Amount (A) in dollars of their savings for the number (n) of hours worked.



a Here are rules for calculating the Amount (A) saved for working for n hours.
$A = 10 + 8n$ and $A = 24 + 6n$
Which rule applies to Ruby and which to Jayden? Explain why.

b Use the appropriate line on the above graph to find the solution to the following equations.

i $10 + 8n = 42$ **ii** $24 + 6n = 48$ **iii** $10 + 8n = 66$

iv $24 + 6n = 66$ **v** $10 + 8n = 98$ **vi** $24 + 6n = 90$

c From the graph write three solutions (n, A) that satisfy $A = 10 + 8n$.

d From the graph write three solutions (n, A) that satisfy $A = 24 + 6n$.

e Write the solution (n, A) that is true for both Ruby's and Jayden's equations and show that it satisfies both equations.

f From the graph find the solution to the equation: $10 + 8n = 24 + 6n$ (i.e. find the value of n that makes Ruby's and Jayden's savings equal to each other).

g Explain how many hours have been worked and what their savings are at the point of intersection of the two lines.

9 This graph shows two lines with equations $y = 11–3x$ and $y = 2x + 1$.

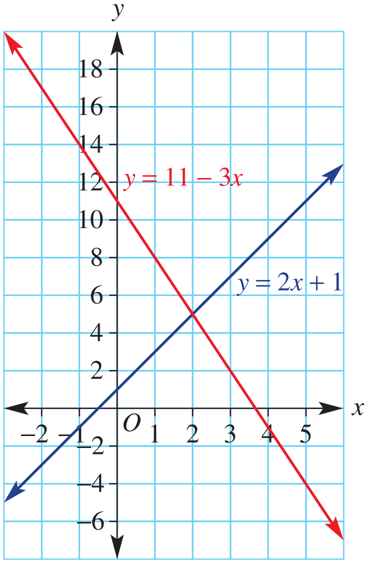
a Copy and complete the coordinates of each point that is a solution for the given linear equation.

i $y = 11–3x$
(–2,?) , (–1,?) (0, ?) (0, ?) (2, ?) (3, ?) (4, ?) (5, ?)

ii $y = 2x + 1$
(–2,?) , (–1,?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)

b State the coordinates of the point of intersection.

c Explain why the point of intersection is the only solution that satisfies both equations.



10 Use digital technology to sketch a graph of $y = 1.5x – 2.5$ for x and y values between –7 and 7. Use the graph to solve each of the following equations. Round answers to two decimal places.

a $1.5x – 2.5 = 3$
b $1.5x – 2.5 = –4.8$
c $1.5x – 2.5 = 5.446$

— 11

The Max and Jessica race

11 Jessica and Max have a 10 second running race.

- Max runs at 6 m/second.
- Jessica is given a 10 m head-start and runs at 4 m/second.

a Copy and complete this table showing the distance run by each athlete.

Time (t) in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance (d) in metres	0										
Jessica's distance (d) in metres	10										

b Plot these points on a distance-time graph and join to form two straight lines labelling them 'Jessica' and 'Max'.

c Find the rule linking distance d and time t for Max.

d Using the rule for Max's race, write an equation that has the solution:

i $t = 3$
ii $t = 5$
iii $t = 8$

e Find the rule linking distance d and time t for Jessica.

f Using the rule for Jessica's race, write an equation that has the solution:

- **i** $t = 3$
- **ii** $t = 5$
- **iii** $t = 8$

g Write the solution (t, d) that is true for both distance equations and show that it satisfies both equations.

h Explain what is happening in the race at the point of intersection and for each athlete state the distance from the starting line and time taken.

9H Non-linear graphs ★

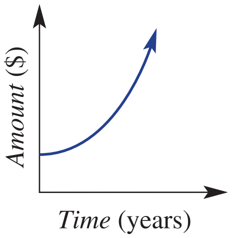
Interactive

Widgets

HOTsheets

Walkthrough

Not all relationships between two variables are linear. The amount of money invested in a compound interest account, for example, will not increase at a constant rate. Over time, the account balance will increase more rapidly, meaning that the graph of the relationship between *Amount and Time* will be a curve and not a straight line.



Let's start: The fixed perimeter play-pen

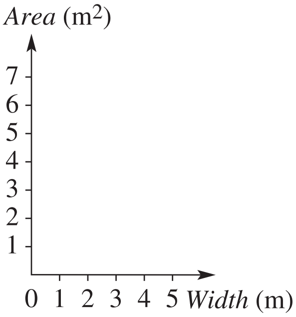
Imagine you have 10 metres of fencing material to make a rectangular play-pen.

- List some possible dimensions of your rectangle.
- What is the area of the play-pen for some of your listed dimensions?
- Complete this table showing all the positive integer dimensions of the play-pen.

Width (m)	1	2	3	4
Length (m)		3		
Area (m²)		6		

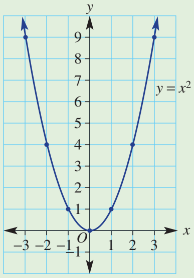
- Plot the *Area* against *Width* to form a graph.
- Discuss the shape of your graph.
- Discuss the situation and graphical points when the width is 1 m or 4 m.
- What dimensions would deliver a maximum area? Explain how your graph helps determine this.

Compound interest means your account balance increases at a more rapid pace over time.



Key ideas

- To plot **non-linear** curves given their rule, follow these steps.
  - Construct a table of values using the rule.
  - Plot the points on a set of axes.
  - Join the plotted points to form a smooth curve.
- The graph of $y = x^2$ is an example of a non-linear graph called a **parabola**.

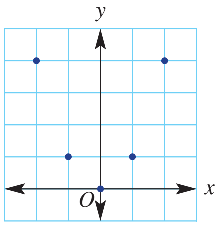


Exercise 9H

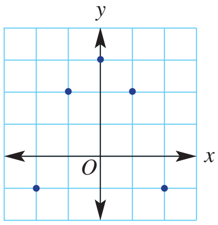
UNDERSTANDING 1–3 1–3

1 Copy these graphs then join the points to form smooth curves.

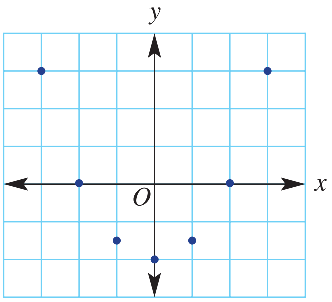
a



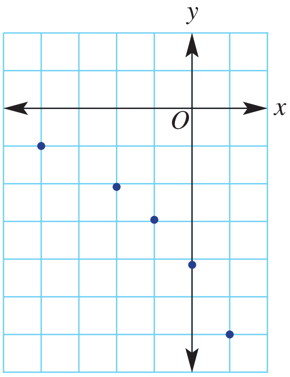
b



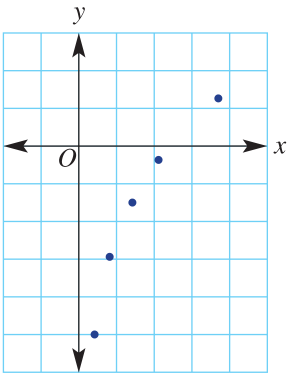
c



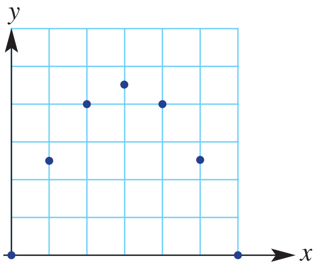
d



e



f



2 If $y = x^2 - 1$, find the value of y for these x values.

a $x = 0$ **b** $x = 3$ **c** $x = 2$ **d** $x = -4$

3 Decide if the following rules would give a non-linear graph.

a $y = x^2$ **b** $y = x$ **c** $y = 2x$ **d** $y = x^2 - 1$

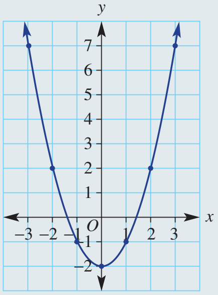
FLUENCY 4, 5 4(½), 5

Example 17 Plotting a non-linear relationship

Plot points to draw the graph of $y = x^2 - 2$ using a table.

Solution

x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7



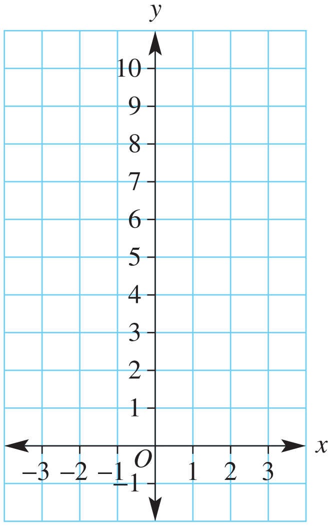
Explanation

Find the value of y by substituting each value of x into the rule. Plot the points and join with a smooth curve. The curve is called a parabola.

4 Plot points to draw the graph of each of the given rules. Use the table and set of axes as a guide.

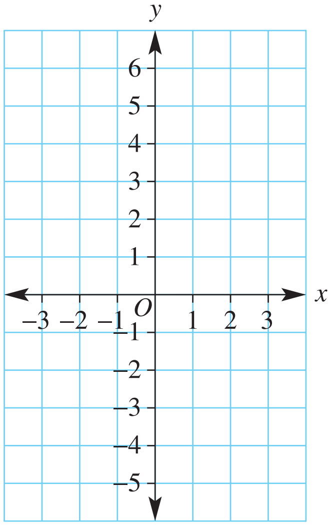
a $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9						



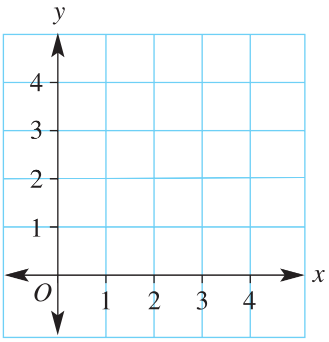
b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5						



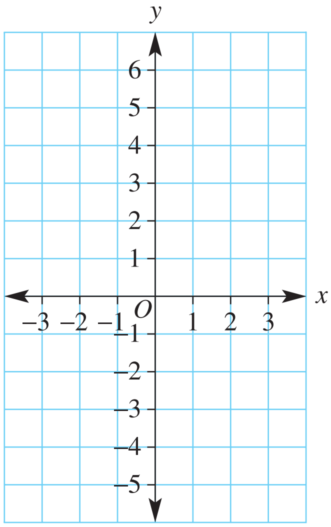
c $y = x(4 - x)$

x	0	1	2	3	4
y					



d $y = 5 - x^2$

x	−3	−2	−1	0	1	2	3
y		1					



5 The behaviour of the Australian dollar against the British pound over a 6 month period is summarised by the data in this table.

Time	0	1	2	3	4	5	6
AUD	0.69	0.64	0.61	0.6	0.61	0.64	0.69

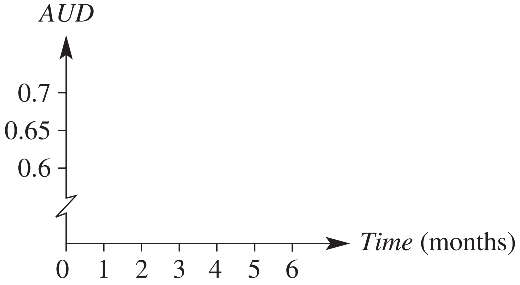
a Plot the data on the given graph and join to form a smooth curve.

b Describe the shape of your graph.

c By how much has the Australian dollar:

- **i** decreased in the first month?
- **ii** increased in the fifth month?

d Estimate the value of the Australian dollar after 7 months.



PROBLEM-SOLVING AND REASONING 6, 7 6–8

6 James has 8 cm of string to form a rectangular space.

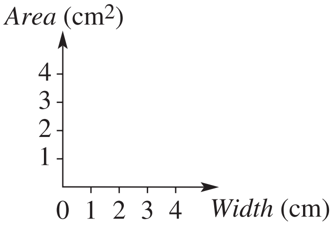
a For the given width values, complete this table of values.

Width (cm)	0	1	2	3	4
Length (cm)			2		
Area (cm^2)			4		

b Plot the *Area* against the *Width* to form a graph.

c Describe the shape of your graph.

d What rectangle dimensions appear to provide the maximum area?



7 Explain why the graph of the rule $y = x^2$ is curved and not straight.

8 By choosing and plotting a selection of points, decide if the following rules would deliver linear or non-linear curves.

a $y = 5x$

b $y = 1 - x$

c $y = x^2 + 2$

d $y = \frac{1}{x}$

e $y = \frac{2}{x}$

f $y = x \times (x + 1)$

Lara's toy paint

— 9

9 Lara has enough toy paint to cover 12 cm^2 of space. She intends to paint a rectangular area.

Width (cm)	1	2	3		6	
Length (cm)				3		
Perimeter (cm)						26

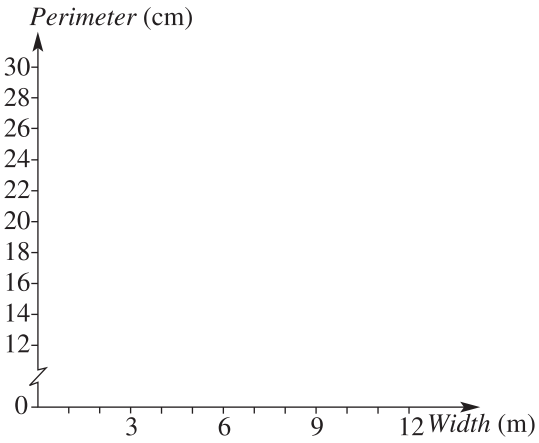
a For the given values, complete the table above.

b Plot the *Perimeter* against *Width* to form a graph.

c Would you describe the curve to be linear or non-linear?

d Look at the point where there is a minimum perimeter.

- **i** Estimate the width of the rectangle at this point.
- **ii** Estimate the perimeter at this point.



Maths@Work: Economists and household expenditure

Economists study how the wealth of a nation is created and shared between governments, businesses, people and households. Economists usually have a university degree and have studied mathematics.

Economists use many equations and formulae in tables, spreadsheets and graphs. They study relationships such as supply and demand, profit and loss and household income and expenditure. Economists work in teams to write reports interpreting data and advising of future trends for governments and the boards of large corporations.

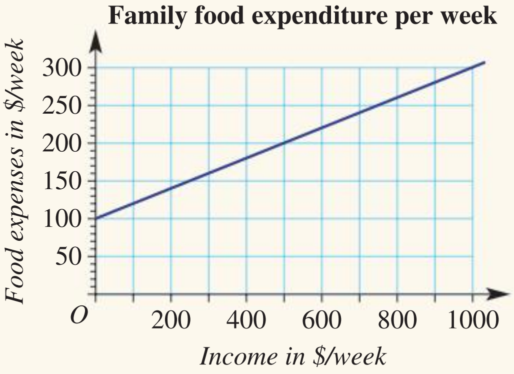
1 Positive gradients represent growth and negative gradients show a decline over time. For each of the following, state whether it has a positive or negative gradient.

- **a** Petrol price went from $\$1.34$/litre to $\$1.37$/litre in a week.
- **b** An electricity bill was $\$789$ last quarter and is $\$684.35$ this quarter.
- **c** The Australian milk price per litre has been steadily decreasing over the last decade.
- **d** Each year more tourists visit the Great Barrier Reef than in the previous year.
- **e** Over the last year, Australia's unemployment rate has increased.

2 This straight line graph shows the relation between how much money ($\$y$/week) a family spends on food and family income ($\$x$/week).

Answer the questions below about this graph.

- **a** Food is essential and can be bought using savings or a credit card. How much is spent weekly on food even if there is zero weekly income?
- **b** How much is spent weekly on food by a family with a $\$1000$/week income?
- **c** State the change in food expenditure for each $\$100$ increase in family income?
- **d** Is the gradient of this line positive or negative?
- **e** What is the gradient of the line?
- **f** Write a rule for this straight line.



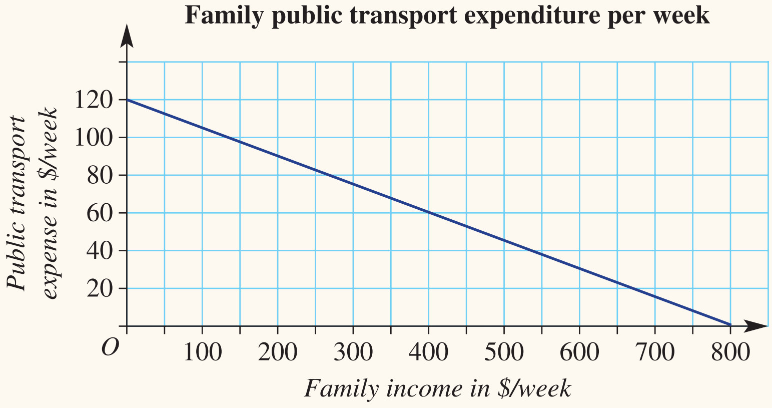
Rule $y = \Delta x + \square$

Δ = the gradient

$\square$ = y value when $x = 0$

3 This straight line graph shows a simple relationship between a family's public transport expenses ($\$y$/week) and a family's income ($\$x$/week).

Answer the questions below about this graph.



- **a** How much is spent on public transport for families with:
  - **i** zero weekly income?
  - **ii** $\$400$ weekly income?
- **b** State the change in public transport expenditure, in $\$$, per $\$100$ increase in family income.
- **c** Is the gradient of this graph positive or negative?
- **d** Find the gradient of this line.
- **e** Write the rule for this straight line.
- **f** At what family income is the public transport expense zero?
- **g** Do you think this graph is realistic? Can you think of why families with higher income might use less public transport, or more?

4 Economists use the terms below for types of 'goods' (goods are objects or products).

- Normal goods: an *increasing* income (x) results in *increasing* expenditure (y), e.g. food.
- Inferior goods: an *increasing* income (x) results in *decreasing* expenditure (y), e.g. cheap mass-produced clothing.

Which of these equations represent Normal goods, and which represent Inferior goods?

a $y = 150 + 0.2x$ **b** $y = 200 - 0.5x$ **c** $y = 250 + 0.05x$ **d** $y = -0.02x + 60$

Using technology

Format cells: Currency/0d.p.
$y = 0.25x - 120$ In cell B3, this rule is $= 0.25 \times \text{B2} - 120$

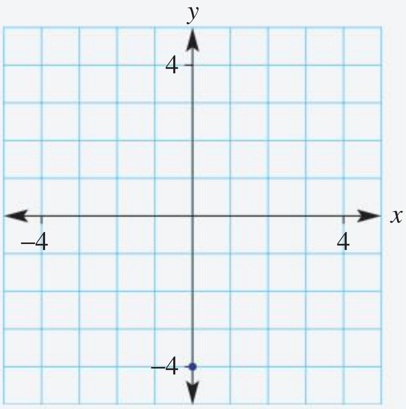
5 The rule $y = 0.25x - 120$ calculates family restaurant spending $(\$y/\text{week})$ from family income $(\$x/\text{week})$.

- **a** Set up the following Excel spreadsheet and enter the given data.
- **b** Enter formulas to calculate restaurant expenditure from income.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Restaurant expenditure vs Income											
2	Income $/week	$0	$200	$400	$600	$800	$1,000	$1,200	$1,400	$1,600	$1,800	$2,000
3	Restaurant expenditure $/week											

- **c** Follow these instructions for inserting a graph:
  - **i** Select rows 2 and 3 of the table.
  - **ii** Choose Insert/Scatter graph/icon for 'Scatter with Straight Lines'.
  - **iii** Right-click on each graph axes and select 'Add minor gridlines'.
  - **iv** Choose Design and select one of the 'Quick layout' formats and add a title and label both axes. (A key is not required and can be deleted.)
- **d** Economists call Restaurant spending a 'luxury good' as only families with higher weekly incomes consume restaurant meals. Above what weekly income does a family start spending money on restaurant meals?
- **e** What is the weekly income of a family that spends $\$200$/week on restaurant meals?

Puzzles and games



1 Plot and join the set of points in order to form the picture. What is the picture of?

$(4, 1), (2, 1), (1, -1), (0, 1), (-3, 1), (-3, 3), (-2, 2), (3, 2), (4, 1)$

2 Find the rule linking y and x.

a

x	–2	–1	0	1	2
y	–15	–11	–7	–3	1

b

x	1	2	3	4	5
y	10	9	8	7	6

c

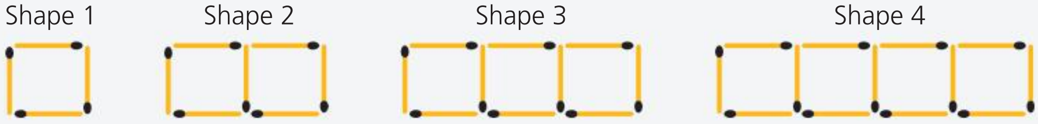
x	–10	–9	–8	–7	–6
y	–100	–95	–90	–85	–80

d

x	0	3	6	9	12
y	–10	–7	–4	–1	2

3 How many matchsticks would be needed for the 10th shape in each pattern?

a

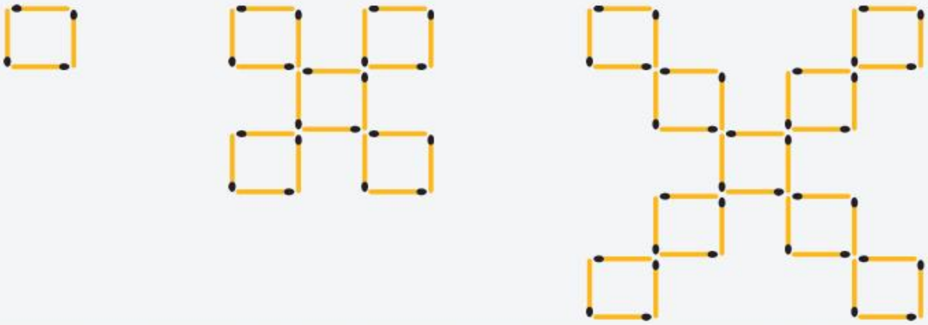


b

4 A trekker hikes along a track at 3 km per hour. Two hours later, a second trekker sets off on the same track at 5 km per hour. How long is it before the second trekker to catches up with the first?

5 Two cars travel toward each other on a 100 km stretch of road. One car travels at 80 km per hour and the other at 70 km per hour. If they set off at the same time, how long will it be before the cars meet?

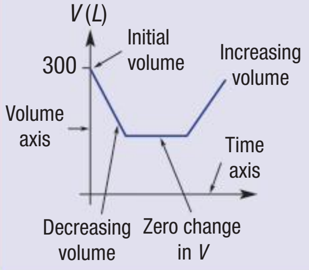
6 Find the number of matchsticks needed in the 100th diagram in the pattern given below. The first three diagrams in the pattern are given.



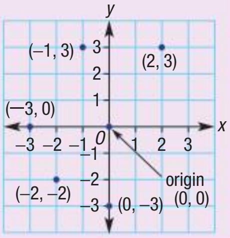
Chapter summary

Straight line graphs

Graphs



Number/Cartesian plane



Rules and tables

$y = -2x + 3$

x	–2	–1	0	1	2
y	7	5	3	1	–1

If $x = -2$

$y = -2 \times (-2) + 3$

$= 4 + 3$

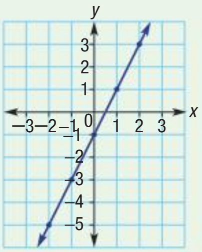
$= 7$

Plotting straight line graphs

Rule $y = 2x - 1$

Table

x	–2	–1	0	1	2
y	–5	–3	–1	1	3



Finding the rule from tables

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

+3 +3 +3 +3

$y = \triangle \times x + \square$

$= 3x + (-2)$

$= 3x - 2$

Applications

- Distance increases by 20 km per hour.

t	0	1	2	3
d	0	20	40	60

$d = 20t$

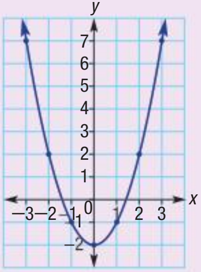
- Volume decreases from 1000 L by 200 L per minute.

t	0	1	2	3	4	5
V	1000	800	600	400	200	0

$V = -200t + 1000$

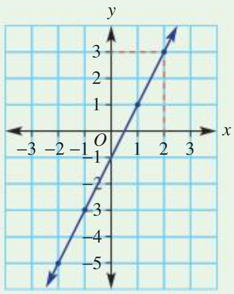
Non-linear graphs

$y = x^2 - 2$



Using graphs to solve linear equations

Solve $2x - 1 = 3$

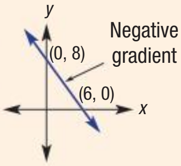


The solution is $x = 2$

Gradient

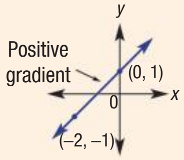
$\frac{\text{rise}}{\text{run}} = -\frac{8}{6}$

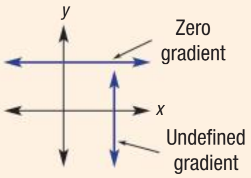
$= -\frac{4}{3}$



$\frac{\text{rise}}{\text{run}} = \frac{2}{2}$

$= 1$

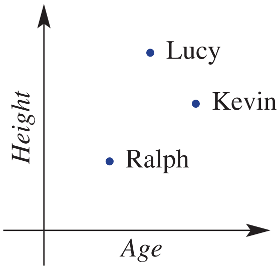




Multiple-choice questions

9A **1** This graph shows the relationship between the height and age of three people. Who is the tallest person?

A Ralph **B** Lucy **C** Kevin
D Lucy and Ralph together
E Kevin and Lucy together



9A **2** The name of the point $(0, 0)$ on a number (Cartesian) plane is:

A y-intercept **B** gradient **C** origin
D axis **E** x-intercept

9A **3** Which point is not in line with the other points?

$A(-2, 3), B(-1, 2), C(0, 0), D(1, 0), E(2, -1)$

A A **B** B **C** C **D** D **E** E

9B **4** If $d = 10t$, then the value of d when $t = 6$ is:

A 600 **B** 6 **C** 10 **D** 60 **E** 100

9B **5** The rule for this table of values is:

x	-2	-1	0	1	2
y	-1	0	1	2	3

A $y = x$ **B** $y = x + 1$ **C** $y = -x$ **D** $y = -x - 1$ **E** $y = 1$

9E **6** The gradient of a line joining the two points $(0, 0)$ and $(1, -6)$ is:

A 3 **B** 6 **C** 1 **D** -6 **E** -3

9D **7** The rule for this table of values is:

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5

A $y = -x - 3$ **B** $y = x - 3$ **C** $y = -x + 3$ **D** $y = x + 3$ **E** $y = -(x - 1)$

9E **8** A vertical line has what type of gradient?

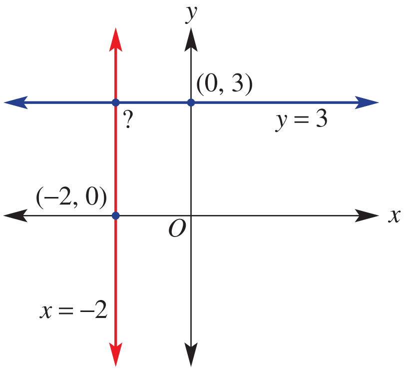
A fraction **B** positive **C** negative **D** zero **E** undefined

9G **9** The coordinates of the point of intersection of the graphs of $y = 3$ and $x = -2$ are:

A $(0, 3)$ **B** $(2, -3)$ **C** $(-2, -3)$ **D** $(-2, 3)$ **E** $(2, 3)$

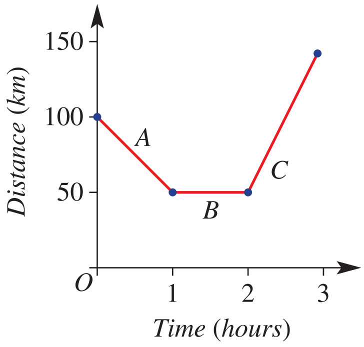
9F **10** The water level in a dam starts at 300 cm deep and decreases by 5 cm every day for 10 days. The water level after 7 days would be:

A 35 cm
B 275 cm
C 230 cm
D 160 cm
E 265 cm



Short-answer questions

9A **1** The graph shows how far a bird is from its nest.



a How far was the bird from its nest initially (at the start)?

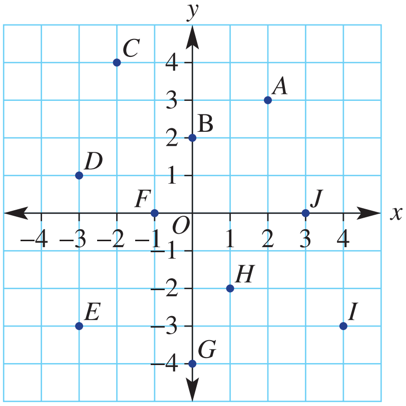
b For how long did the bird rest?

c How far did the bird travel in:

i section A? **ii** section C?

d During which section did the bird fly the fastest?

9A **2** Write the coordinates of all the points $A–J$ in the graph below.



9B **3** Use the given rules to find the missing values in the tables.

a $y = x - 1$

x	−2	−1	0	1	2
y	−3				

b $y = 2x$

x	−2	−1	0	1	2
y		−2			

c $y = 3x + 1$

x	−2	−1	0	1	2
y		−2			

d $y = -x + 1$

x	−2	−1	0	1	2
y		2			

9B **4** For each rule create a table using x values from −3 to 3 and plot to draw a straight line graph.

a $y = 2x$ **b** $y = 3x - 1$ **c** $y = 2x + 2$

d $y = -x + 1$ **e** $y = -2x + 3$ **f** $y = 3 - x$

x	−3	−2	−1	0	1	2	3
y							

9D **5** Write the rule for these tables of values.

a

x	−2	−1	0	1	2
y	−3	−1	1	3	5

b

x	−2	−1	0	1	2
y	−4	−1	2	5	8

c

x	3	4	5	6	7
y	6	7	8	9	10

d

x	–3	–2	–1	0	1
y	4	3	2	1	0

e

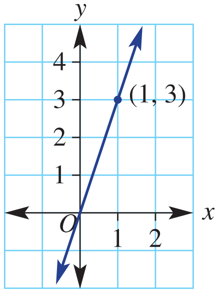
x	–1	0	1	2	3
y	3	–1	–5	–9	–13

f

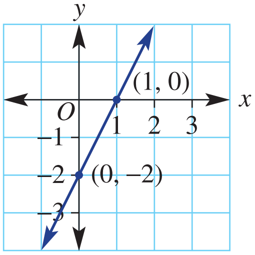
x	0	1	2	3	4
y	8	7	6	5	4

9E **6** Find the gradient of each of these lines.

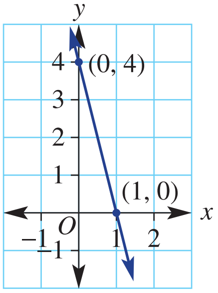
a



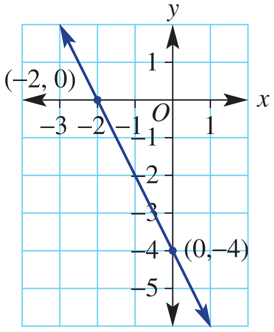
b



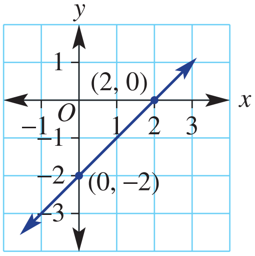
c



d



e



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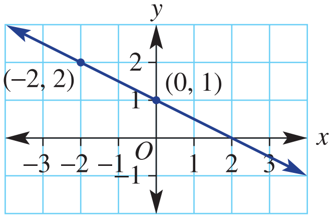
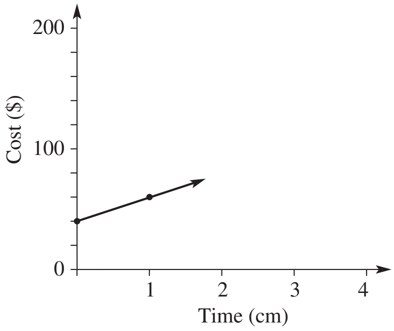


9F/G **7** A bicycle hire company has the following cost structure: \$40 upfront plus \$20 per hour after that.



a Copy and complete this graph showing the relationship between total cost (\$$C$) and time ($t$ hours) for 4 hours.

b Use your graph to find the cost of hiring a bike for 2.5 hours.

c Use your graph to find how long you can hire a bike using \$110.

d Write a rule connecting C and t.

e Use your rule to confirm your answers to parts ***b*** and ***c*** above.

f Use your graph to solve:

i $80 = 20t + 40$

ii $70 = 20t + 40$

Another company offers hire at \$30 per hour and no upfront fee.

g Sketch a graph on the same set of axes from part ***a*** to illustrate this company's cost structure.

h Use your graph to determine the number of hours of hire for which the cost for both companies is equal.

9H **8** Plot a graph of the rule $y = 2 - x^2$ for x values from –2 to 2.

Extended-response questions

1 A seed sprouts and the plant grows 3 millimetres per day in height for 6 days.

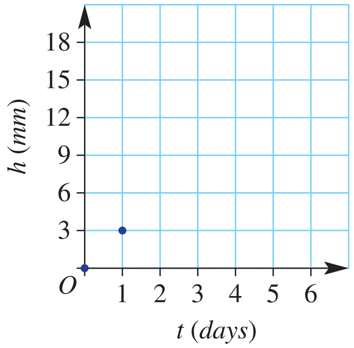
a Complete this table of values using t for time in days and 0 for height in millimetres.

t	0	1	2	3	4	5	6
h	0						

b Complete this graph using the points from your table.

c Find a rule linking h with t.

d Use your rule to find the height of the plant after 3.5 days.



e If the linear pattern continued, what would be the height of the plant after 10 days?

f How long will it be before the plant grows to $15\,\text{mm}$ in height?

2 A speed boat at sea is initially $12\,\text{km}$ from a distant rock. The boat travels towards the rock at a rate of $2\,\text{km}$ per minute. The distance between the boat and the rock will therefore decrease over time.

a Complete this table showing t for time in minutes and d for distance to the rock in kilometres.

t	0	1	2	3	4	5	6
d	12	10					

b Draw a graph using the points from your table. Use t on the horizontal axis.

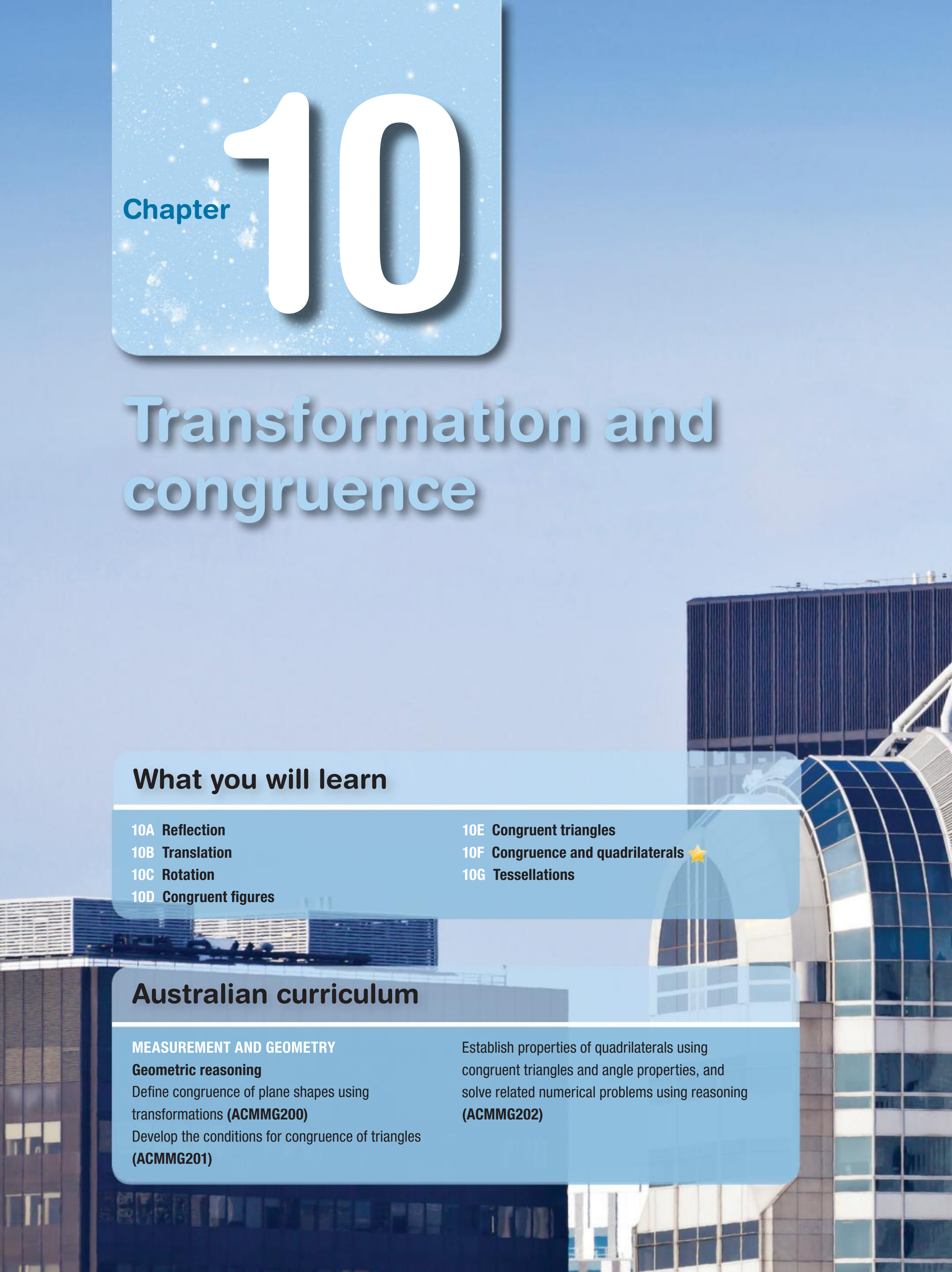
c How long does it take the speed boat to reach the rock?

d What is the gradient of the line drawn in part **b**?

e Find a rule linking d with t.

f Use your rule to find the distance from the rock at the 2.5 minute mark.

g How long does it take for the distance to reduce to $3.5\,\text{km}$?Fluency



Chapter 10

Transformation and congruence

What you will learn

Australian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Define congruence of plane shapes using transformations **(ACMMG200)**

Develop the conditions for congruence of triangles **(ACMMG201)**

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning **(ACMMG202)**

Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Modern architecture and geometry

The British architecture firm, Foster and Partners, is famous for designing unique high-tech buildings that are also environmentally friendly.

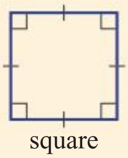
The Hearst Tower in New York City has a complicated structure of glass and steel in its design, using congruent triangles to create a diagonal steel-grid framework called a ‘diagrid’. This design provides both strength and rigidity and uses 20% less steel than the traditional framework of a skyscraper.

Another of their famous buildings is the London skyscraper nicknamed ‘the Gherkin’ or ‘the London Egg’ (pictured). It also uses congruent triangles, creating diamond-shaped glass and steel panels. The Gherkin is a very energy-efficient building with spiralling shafts that allow natural light to enter and fresh air to flow. These shafts are visible on the outside as a swirling striped pattern. The Gherkin has received many prizes for its architecture and energy-efficient design.

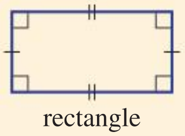
Pre-test

1 How many lines of symmetry are there in these shapes?

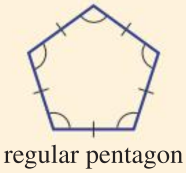
a



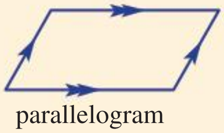
b



c



d



2 What is the order of rotational symmetry for the shapes in question **1**?

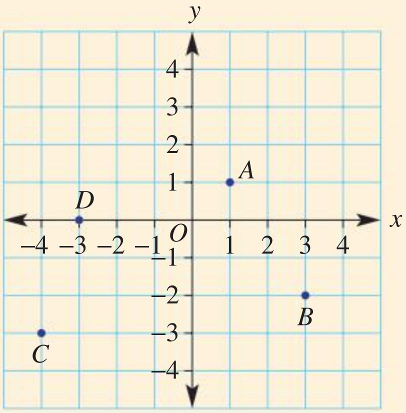
3 This number plane shows four points A, B, C, D.

a State the coordinates of the points A, B, C, D.

b What would be the coordinates of point A if:

- **i** it were shifted left by 1 unit?
- **ii** it were shifted right by 2 units and 1 unit down?
- **iii** it were shifted left by 5 units and 3 units down?

c What would be the coordinates of point C if it were reflected in the x-axis?

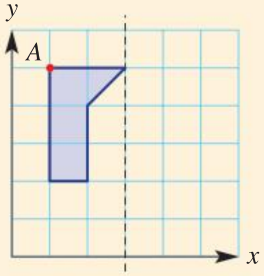


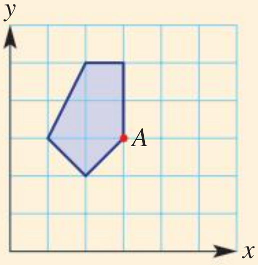
4 Complete the simple transformations of the given shapes as instructed then state the coordinates of the image of point A after the given transformation.

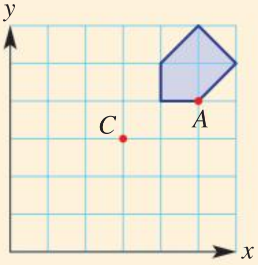
a Reflect this shape over the mirror line.

b Shift this shape 2 units to the right and 1 unit down.

c Rotate this shape 180° around point C.

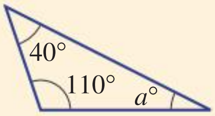




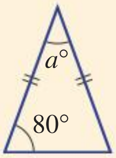


5 Find the value of a in these triangles.

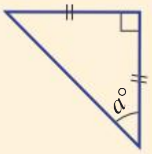
a



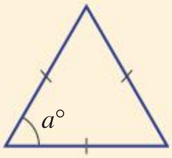
b



c



d



6 Which of the special quadrilaterals fit the description?

A Square	**B** Rectangle	**C** Rhombus
D Parallelogram	**E** Kite	**F** Trapezium

a Opposite sides are of equal length.

b It has at least one pair of equal opposite angles.

c Diagonals are of equal length.

d Diagonals intersect at right angles.

10A Reflection

When an object is shifted from one position to another, rotated about a point, reflected over a line or enlarged by a scale factor, we say the object has been **transformed**. The names of these types of transformations are **reflection**, **translation**, **rotation** and **dilation**.

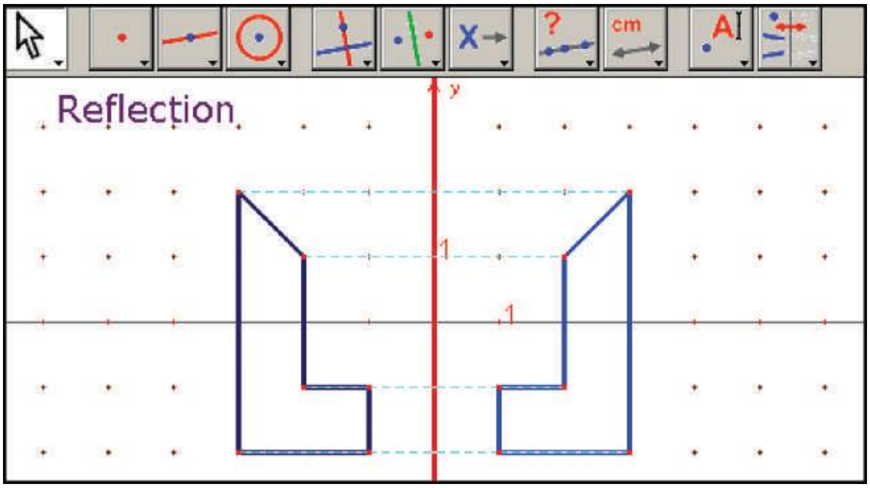
The first three of these transformations, studied in this chapter, are called **isometric transformations** because the object's geometric properties are unchanged and the transformed object will be congruent to the original object. This means that the size and shape of the object are not altered. The word 'isometric' comes from the Greek words *isos* meaning 'equal' and *metron* meaning 'measure'.

Reflection creates an image reversed as in a mirror or in water.

Let's start: Visualising the image

This activity could be done by hand on a page, in a group using a whiteboard or using dynamic geometry projected onto a whiteboard.

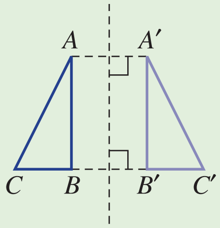
- Draw any shape with straight sides.
- Draw a vertical or horizontal mirror line outside the shape.
- Try to draw the reflected image of the shape in the mirror line.
- If dynamic geometry is used, reveal the precise image (the answer) using the Reflection tool to check your result.
- For a further challenge, redraw or drag the mirror line so it is not horizontal or vertical. Then try to draw the image.

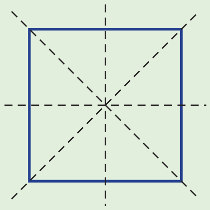


Dynamic geometry software provides a reflection tool.

Key ideas

- **Reflection** is a **transformation** in which the size and shape of the object is unchanged.
- The **image** of a point A is denoted A'.
- Each point is reflected at right angles to the **mirror line**.
- The distance from a point A to the mirror line is equal to the distance from the image point A' to the mirror line.
- **Lines of symmetry** are mirror lines that result in an image being reflected onto itself.
  - A square has four lines of symmetry.





Reflection The result of flipping a geometrical figure across a line

Image The result of a transformation

Mirror line The line over which a figure is reflected

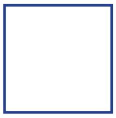
Line of symmetry A line that divides a figure into two identical parts

Exercise 10A

UNDERSTANDING 1–3 3

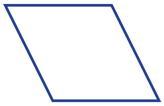
1 Draw in all the lines of symmetry for these shapes.

a



square

b

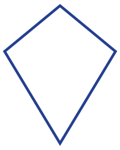


rhombus

c

rectangle

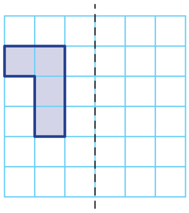
d



kite

2 Use the grid to precisely reflect each shape in the given mirror line.

a



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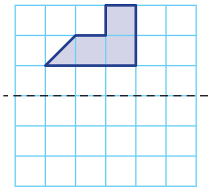
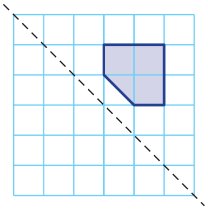


c



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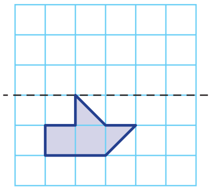

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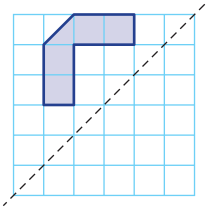

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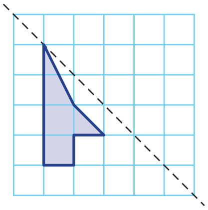
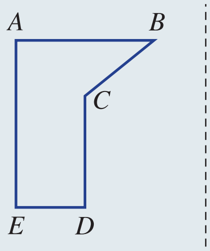
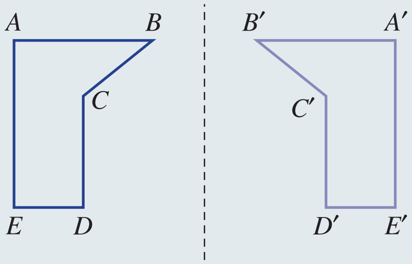


Example 1 Drawing simple reflected images

Copy the diagram and draw the reflected image over the given mirror line.



Solution

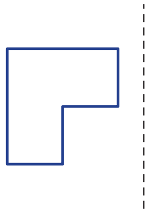


Explanation

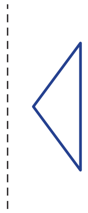
Reflect each vertex point at right angles to the mirror line. Join the image points to form the final image.
Use A' as the image point of A.

3 Copy the diagram and draw the reflected image over the given mirror line.

a

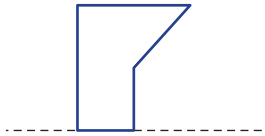


b

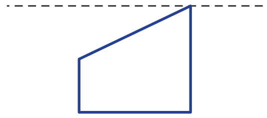


Start by reflecting each vertex point at 90° across the mirror line. Then join these points to form the shape.

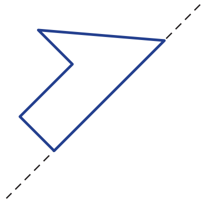
c



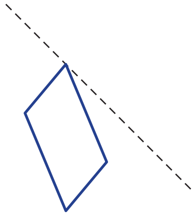
d



e



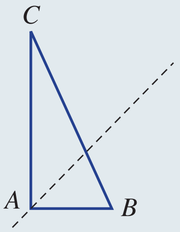
f

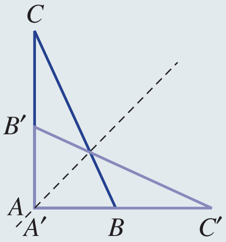


FLUENCY 4–5(½), 6 4–5(½), 7

Example 2 Drawing more complex reflected images

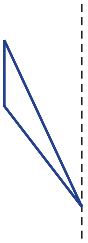
Copy and reflect over the mirror line.



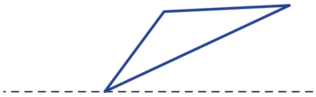
Solution	**Explanation**
	Reflect points A, B and C at right angles to the mirror line to form A', B' and C'. Note that A' is in the same position as A as it is on the mirror line. Join the image points to form the image triangle.

4 Copy the diagram and draw the reflected image over the given mirror line.

a

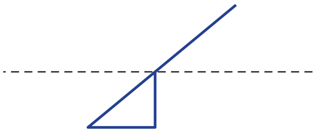


b

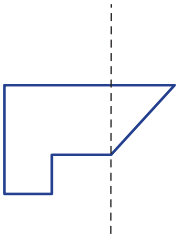


Reflect the vertex points first. Then join the points to finish.

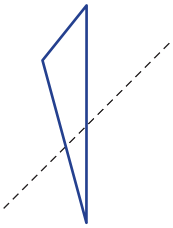
c



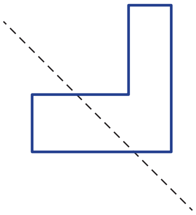
d



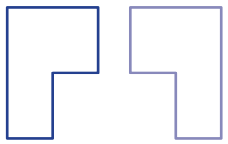
e

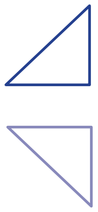


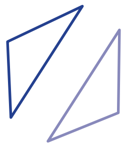
f

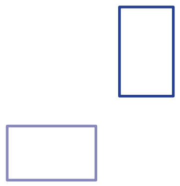


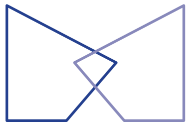
5 Copy the diagram and accurately locate and draw the mirror line. Alternatively, pencil in the line on this page.

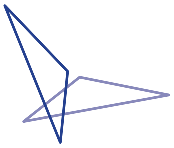
a


b


c


d


e


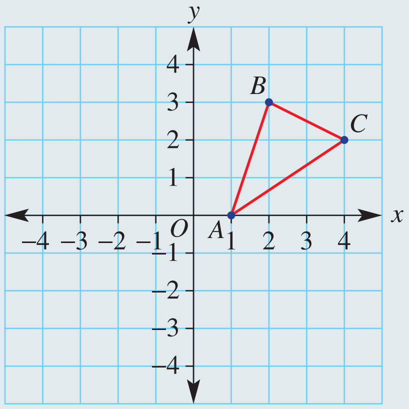
f


Example 3 Using coordinates in reflection

State the coordinates of the vertices A', B' and C' after this triangle is reflected in the given axes.

a x-axis

b y-axis



Solution

a $A' = (1, 0)$

$B' = (2, -3)$

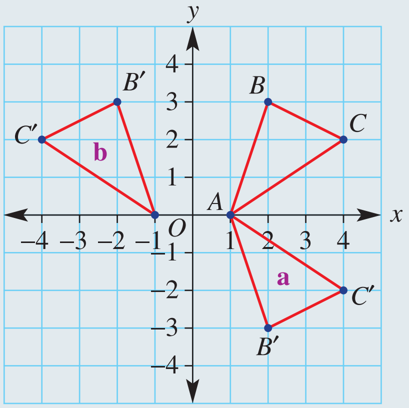
$C' = (4, -2)$

b $A' = (-1, 0)$

$B' = (-2, 3)$

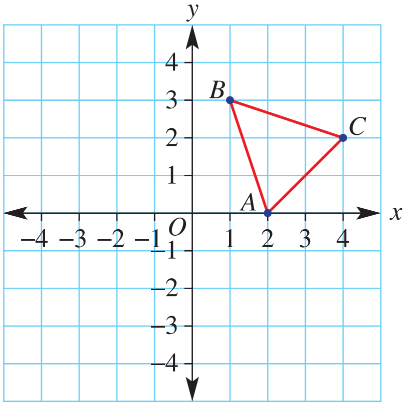
$C' = (-4, 2)$

Explanation



6 State the coordinates of the vertices x and y after the triangle (right) is reflected in the given axes.

a x-axis **b** y-axis

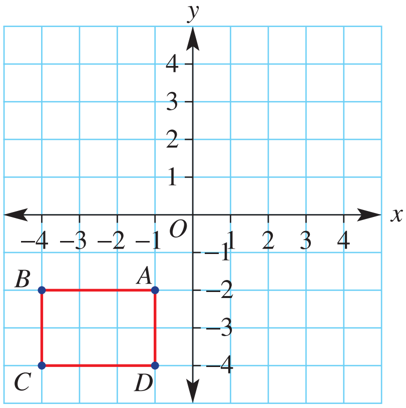


Pencil in the reflection then look at the position of the image points. The x-axis is the horizontal axis and the y-axis is the vertical axis.

7 State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

a x-axis

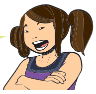
b y-axis



PROBLEM-SOLVING AND REASONING

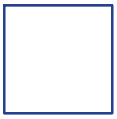
8–10 9–12

8 How many lines of symmetry do these shapes have?



Be careful: Not all diagonals are lines of symmetry.

a

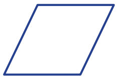


Square

b

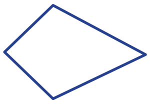
Rectangle

c



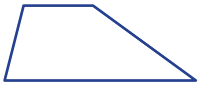
Rhombus

d



Kite

e

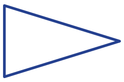


Trapezium

f

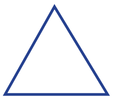
Parallelogram

g



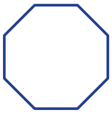
Isosceles triangle

h



Equilateral triangle

i



Regular octagon

9 Explain why a parallelogram in general has no lines of symmetry but a rhombus has two lines of symmetry.

10 A shape with area 10 m^2 is reflected in a line. What is the area of the image shape? Give a reason for your answer.

11 How many lines of symmetry does a regular polygon with n sides have? Write an expression.

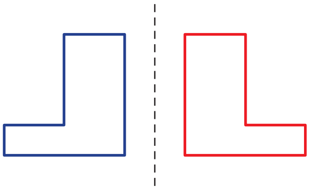
12 A point is reflected in the x-axis then in the y-axis and finally in the x-axis again. What single reflection could replace all three reflections?

Computer reflection

— 13

13 Use computer geometry to construct a shape and a mirror line.

- **a** Reflect your shape in the mirror line.
- **b** Drag the mirror line. What do you notice?
- **c** Drag your original shape. What do you notice?
- **d** Drag the mirror line across the middle of your original shape. What do you notice?



10B Translation

Translation is a shift of every point on an object in a given direction and by the same distance. The direction and distance is best described by the use of a translation vector. This vector describes the overall direction using a horizontal component (for shifts left and right) and a vertical component (for shifts up and down). Negative numbers are used for translations to the left and down.

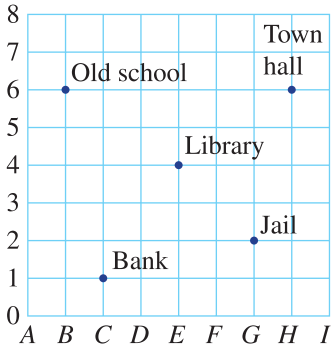
Designers of animated movies translate images in many of their scenes. Computer software is used and translation vectors help to define the specific movement of the objects and characters on the screen.

Animated characters move through a series of translations.

Let's start: City walking tour

The position of places in a city square is given by this grid. For example, the Town Hall is H6.

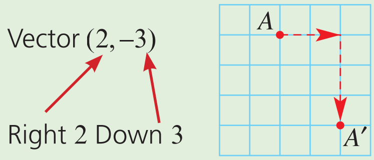
On a walking tour, vectors are used to describe the walk between two places. For example, the vector $(-3, -2)$ takes you from the Town Hall (H6) to the Library (E4). The $(-3, -2)$ vector means move 3 blocks west and 2 blocks south.

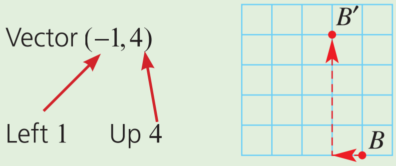


- What is the translation vector that takes you from:
  - the Town Hall to the Jail?
  - the Town Hall to the Old School?
  - the Bank to the Library?
  - the Old School to the Jail?
- Write down, in order, the places you would visit if you started at the Town Hall and followed these vectors: $(-3, -2)$, $(-2, -3)$, $(4, 1)$ and $(-5, 4)$
- Write down the vector that takes you from:
  - B1 to F3
  - H4 to D7
  - D3 to A8.

Key ideas

- **Translation** is a transformation that involves a shift by a given distance in a given direction.
- A **vector** (x, y) is used to describe the distance and direction of a translation.





  - If x is positive you shift to the right.
  - If x is negative you shift to the left.
  - If y is positive you shift up.
  - If y is negative you shift down.
- The image of a point A is denoted A'.

Translation Movement of an object a certain distance in a given direction

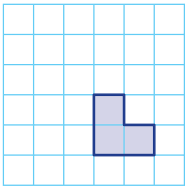
Vector A pair of numbers used to describe a translation

Exercise 10B

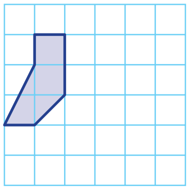
UNDERSTANDING 1–3 3

1 Pencil in the image of these shapes for the given translations.

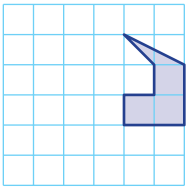
a Shift 2 left and 1 up (vector $(-2, 1)$)



b Shift 3 right and 2 down (vector $(3, -2)$)



c Shift 3 left and 1 down (vector $(-3, -1)$)



2 Use the words left, right, up or down, to complete these sentences.

a The vector $(2, 4)$ means to move 2 units to the ______ and 4 units ______.

b The vector $(-5, 6)$ means to move 5 units to the _____ and 6 units _____.

c The vector $(3, -1)$ means to move 3 units to the _____ and 1 unit _____.

d The vector $(-10, -12)$ means to move 10 units to the _____ and 12 units _____.

As an example the vector $(2, -1)$ means to move 2 right and 1 down.

3 Write the vector (x, y) that describes these transformations.

a 5 units to the right and 2 units down

b 2 units to the left and 6 units down

c 7 units to the left and 4 units up

d 9 units to the right and 17 units up

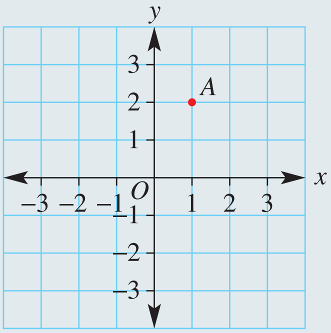
FLUENCY 4, 5(½), 6 4, 5–6(½)

Example 4 Translating points

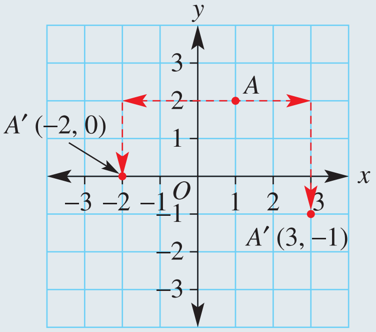
Give the coordinates of the image of the point A if it is translated by these vectors.

a vector $(2, -3)$

b vector $(-3, -2)$



Solution	**Explanation**
a (3, –1) **b** (–2, 0)	

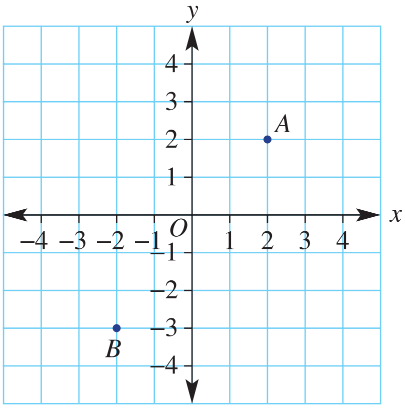


4 **a** Give the coordinates of the image of the point A if it is translated by these vectors.

- **i** vector $(1, 2)$
- **ii** vector $(-3, 1)$
- **iii** vector $(-4, -2)$
- **iv** vector $(-5, -3)$

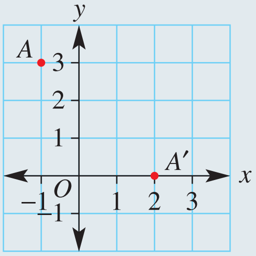
b Give the coordinates of the image of the point B if it is translated by these vectors.

- **i** vector $(1, 4)$
- **ii** vector $(5, 3)$
- **iii** vector $(-1, 4)$
- **iv** vector $(0, 6)$



Example 5 Finding the translation vector

State the translation vector that moves the point $A(-1, 3)$ to $A'(2, 0)$.



Solution	**Explanation**
$(3, -3)$	To shift A to A' move 3 units to the right and 3 units down.

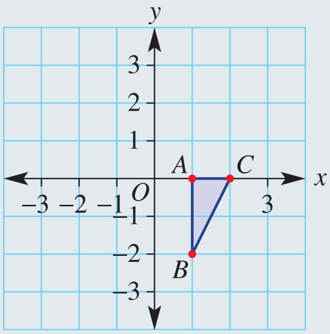
5 Write the vector that takes each point to its image. Use a grid to help you.

a $A(2, 3)$ to $A'(3, 2)$
b $B(1, 4)$ to $B'(4, 3)$
c $C(-2, 4)$ to $C'(0, 2)$
d $D(-3, 1)$ to $D'(-1, -3)$
e $E\left(-2, -4\right)$ to $E'(1, 3)$
f $F(1, -3)$ to $F'(-2, 2)$
g $G(0, 3)$ to $G'(2, 0)$
h $H(-3, 5)$ to $H'(0, 0)$
i $I(5, 2)$ to $I'(-15, 10)$
j $J(-3, -4)$ to $J'(-12, -29)$

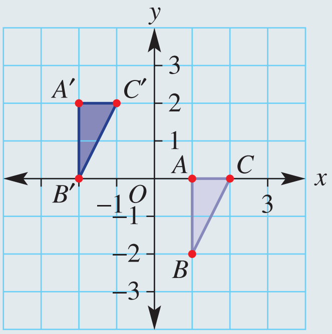
The point and its image are given, so write the vector which takes A to A' and B to B' etc.

Example 6 Drawing images using translation

Draw the image of the triangle ABC after a translation by the vector $(-3, 2)$.



Solution

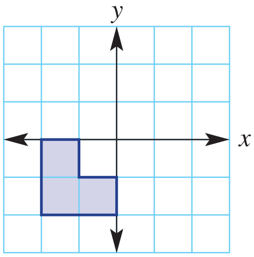


Explanation

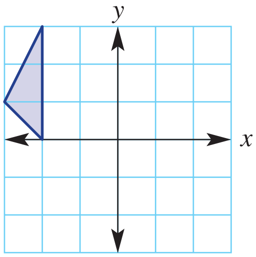
First translate each vertex, A, B and C, 3 spaces to the left, and then 2 spaces up. Then join the image vertices A', B' and C'.

6 Copy the diagrams and draw the image of the shapes translated by the given vectors.

a vector $(2, 3)$

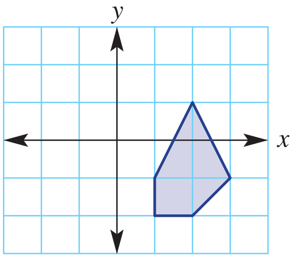


b vector $(4, -2)$

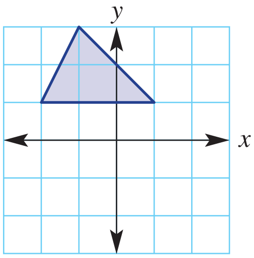


First move each vertex (corner) then join to form the image shape.

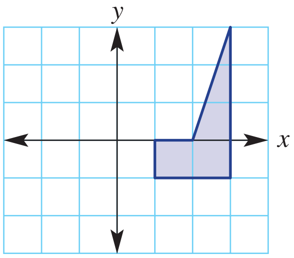
c vector $(-3, 1)$



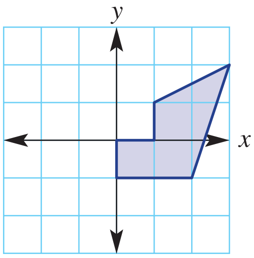
d vector $(0, -3)$



e vector $(-4, -1)$



f vector $(-3, 0)$



PROBLEM-SOLVING AND REASONING

7, 8 7–9

7 Decide if these vectors describe vertical or horizontal translation.

a $(2, 0)$ **b** $(0, 7)$ **c** $(0, -4)$ **d** $(-6, 0)$

8 Write the coordinates of the image of the point $A(13, -1)$ after a translation by the given vectors.

a $(2, 3)$ **b** $(8, 0)$ **c** $(0, 7)$

d $(-4, 3)$ **e** $(-2, -1)$ **f** $(-10, 5)$

g $(-2, -8)$ **h** $(6, -9)$

Always start at $(13, -1)$ then move to the image point using the given vector.

9 A reverse vector takes a point in the reverse direction by the same distance. For example, the reverse vector of $(2, -3)$ is $(-2, 3)$ Write the reverse vectors of these vectors.

a $(3, -2)$ **b** $(-5, 0)$ **c** (x, y) **d** $(-x, -y)$

City street vectors

— 10

10 A car makes its way around a city street grid. A vector $(2, 3)$ represents travelling 200 m east and 300 m north.

a What vector would be used to describe travelling:

i 100 m east and 200 m south?

ii 300 m west and 400 m north?

iii 300 m south only?

b Find how far the car travels in total if it follows these vectors in order.
$(2, 3), (-5, 1), (3, -3)$ and $(-2, -4)$

c What vector takes the car back to the origin $(0, 0)$, assuming it started at the origin and used the travel vectors in part **b**?

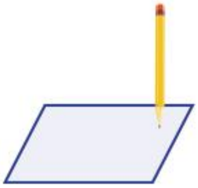
10C Rotation

When the arm of a crane moves left, right, up or down, it undergoes a rotation about a fixed point. This type of movement is a transformation called a **rotation**. The pivot point on a crane would be called the **centre of rotation** and all other points on the crane's arm move around this point by the same angle in a circular arc.

Every point on this crane's arm rotates around the pivot point when it moves.

Let's start: Parallelogram centre of rotation

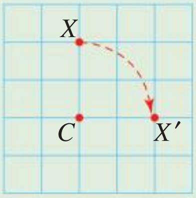
This activity will need a pencil, paper, ruler and scissors.



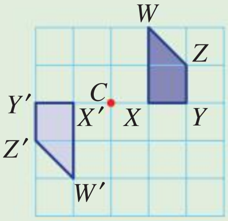
- Accurately draw a large parallelogram on a separate piece of paper and cut it out.
- Place the tip of a pencil at any point on the parallelogram and spin the shape around the pencil.
- At what position do you put the pencil tip to produce the largest circular arc?
- At what position do you put the pencil tip to produce the smallest circular arc?
- Can you rotate the shape by an angle of less than 360° so that it exactly covers the area of the shape in its original position? Where would you put the pencil to achieve this?

Key ideas

- **Rotation** is a transformation about a centre point and by a given angle.
- An object can be rotated clockwise ↻ or anticlockwise ↺.
- Each point is rotated on a circular arc about the **centre of rotation** C.



x is rotated 90° clockwise about C

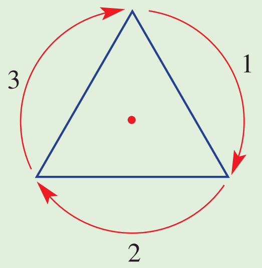


Shape $XYZW$ is rotated 180° about C

Rotation A turn around a centre point or axis

Centre of rotation Fixed point about which a figure rotates

- A shape has **rotational symmetry** if it can be rotated about a centre point to produce an exact copy covering the entire area of the original shape.
  - The number of times the shape can make an exact copy in a 360° rotation is called the **order of rotation**. If the order of rotation is 1, then it is said that the shape has no rotational symmetry.
  - This equilateral triangle has rotational symmetry of order 3.



Exercise 10C

UNDERSTANDING 1–2 2

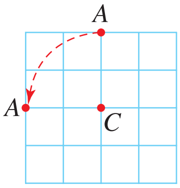
1 Match each description **a**, **b** and **c** with each diagram **A**, **B** and **C**.

a rotation 90° clockwise

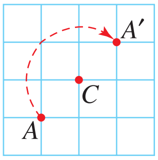
b rotation 90° anticlockwise

c rotation 180° clockwise

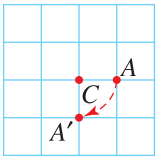
A



B

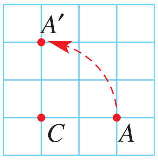


C

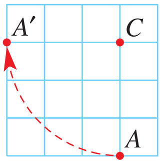


2 Point A has been rotated to its image point A'. For each part state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

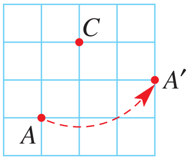
a



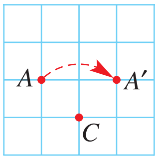
b



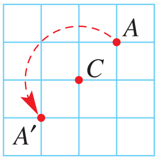
c



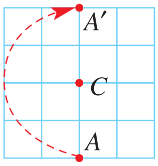
d



e



f



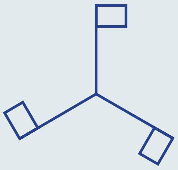
FLUENCY 3–5 3–6(½)

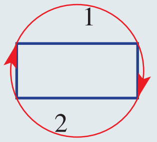
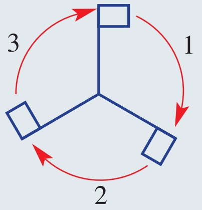
Example 7 Finding the order of rotational symmetry

Find the order of rotational symmetry for these shapes.

a

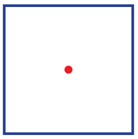
b



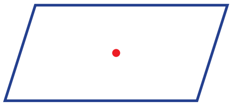
Solution	**Explanation**
a Order of rotational symmetry $= 2$	
b Order of rotational symmetry $= 3$	

3 Find the order of rotational symmetry for these shapes.

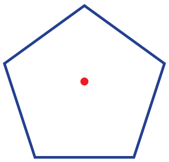
a



b

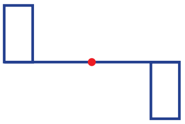


c

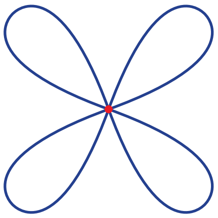


How many times can you make an exact copy of the original in a 360° turn?

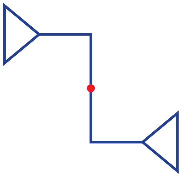
d



e

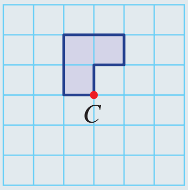


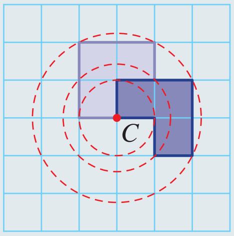
f



Example 8 Rotating a shape by 90°

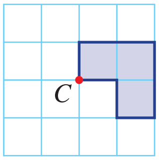
Rotate this shape about C clockwise by 90°.



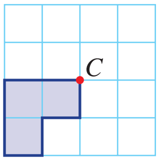
Solution	**Explanation**
	Take each vertex point and rotate about C by 90°, but it may be easier to visualise a rotation of some of the sides first. Horizontal sides will rotate to vertical sides in the image and vertical sides will rotate to horizontal sides in the image.

4 Rotate these shapes about the point C by 90° in the given direction.

a Clockwise

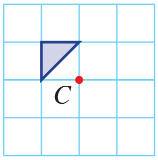


b Anticlockwise

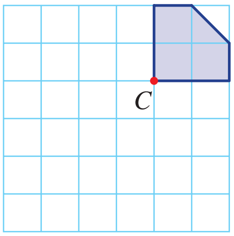


Try just rotating a point or a side first. Then join to form the image shape.

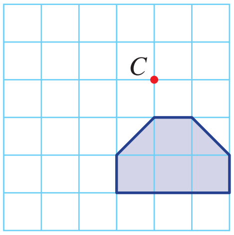
c Anticlockwise



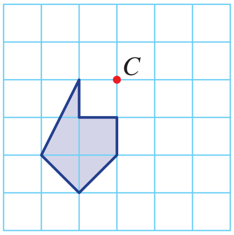
d Clockwise



e Clockwise

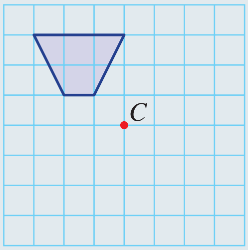


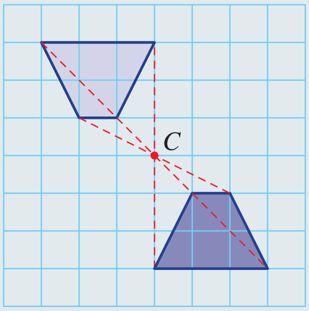
f Anticlockwise



Example 9 Rotating a shape by 180°

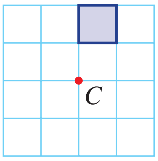
Rotate this shape about point C by 180°.



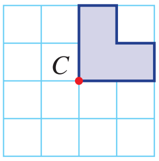
Solution	Explanation
	You can draw a dashed line from each vertex through C to a point at an equal distance on the opposite side.

5 Rotate these shapes about the point C by $180°$.

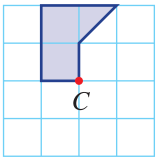
a



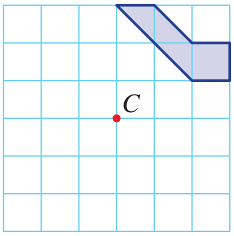
b



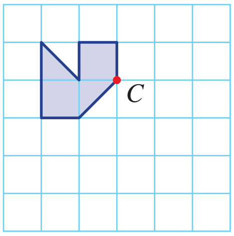
c



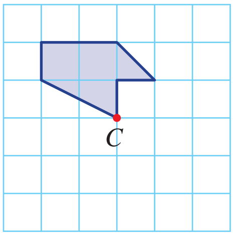
d



e



f



6 The point $A(4, 3)$ is rotated about the origin $C(0, 0)$ by the given angle and direction. Give the coordinates of A'.

a $180°$ clockwise

b $180°$ anticlockwise

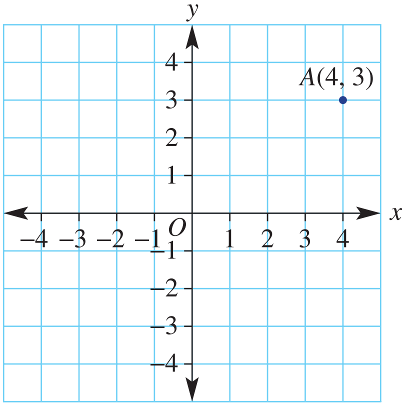
c $90°$ clockwise

d $90°$ anticlockwise

e $270°$ clockwise

f $270°$ anticlockwise

g $360°$ clockwise



PROBLEM SOLVING AND REASONING

7, 8 8–10

7 Complete these sentences.

a A rotation clockwise by $90°$ is the same as a rotation anticlockwise by ______.

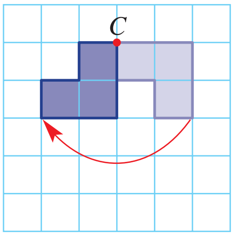
b A rotation anticlockwise by $180°$ is the same as a rotation clockwise by ______.

c A rotation anticlockwise by ______ is the same as a rotation clockwise by $58°$.

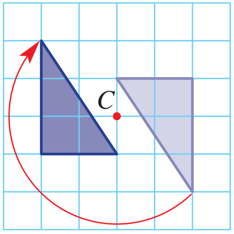
d A rotation clockwise by ______ is the same as a rotation anticlockwise by $296°$.

8 By how many degrees have these shapes been rotated?

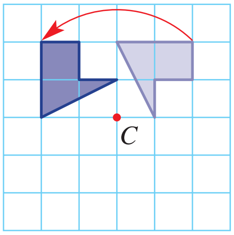
a



b



c

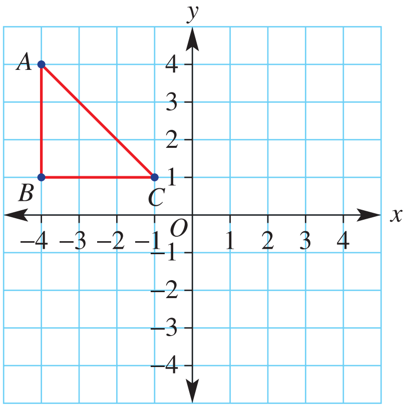


9 The triangle shown below is rotated about $(0, 0)$ by the given angle and direction. Give the coordinates of the image points A', B' and C'.

a $180°$ clockwise

b $90°$ clockwise

c $90°$ anticlockwise



10 Which capital letters of the alphabet (A, B, C … Z) have rotational symmetry of order 2 or more?

Combining line and rotational symmetry

— 11

11 Draw an example of a shape that has these properties:

a Rotational symmetry of order 4 with 4 lines of symmetry.

b Rotational symmetry of order 2 with no line symmetry.

c Rotational symmetry of order 6 with 6 lines of symmetry.

d Rotational symmetry of order 4 with no line symmetry.

e No rotational symmetry with 1 line of symmetry.

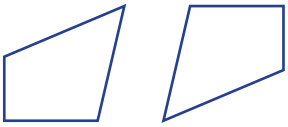
10D Congruent figures

If two objects are identical and have the same size and shape, we say they are congruent. The images on the front cover of your Year 8 maths textbook, for example, would be congruent to the images on the front of another Year 8 textbook, assuming it's the same type of book. Even if one text was flipped over, shifted or rotated you would still say the images on the books were congruent.

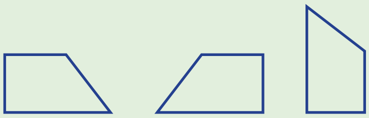
Let's start: Are they congruent?

Here are two shapes. To be congruent they need to be exactly the same shape and size.

- Do you think they look congruent? Give reasons.
- What measurements could be taken to help establish whether or not they are congruent?
- Can you just measure angles or do you need to measure lengths as well? Discuss.

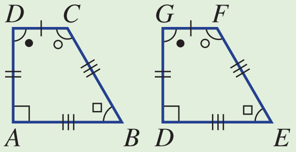


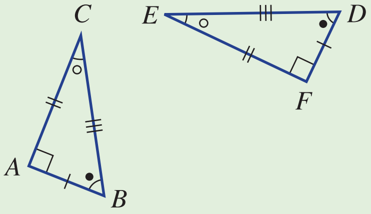
Key ideas



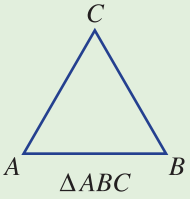
- A **figure** is a shape, diagram or illustration.
- **Congruent figures** have the same size and shape.
- The image of a figure that is reflected, translated or rotated is congruent to the original figure.
- Corresponding (matching) parts of a figure have the same geometric properties. For example:
  - Vertex B corresponds to vertex E.
  - Side CD corresponds to side FG.
  - Angle $\angle C$ corresponds to $\angle F$.
  - Vertex C corresponds to vertex E.
  - Side AB corresponds to side FD.
  - Angle $\angle B$ corresponds to $\angle D$.

Congruent figures Figures that are exactly the same size and shape





- The symbol for 'triangle' is Δ.



Exercise 10D

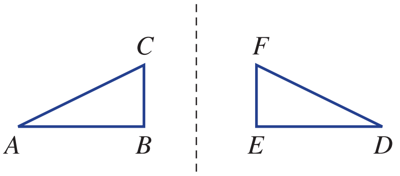
UNDERSTANDING 1–4 4

1 Answer true (T) or false (F).

- **a** Congruent shapes can be of different size.
- **b** Congruent shapes have equal matching sides.
- **c** Congruent shapes have equal matching angles.
- **d** The image of a shape after reflection is congruent to the original shape.
- **e** The image of a shape after rotation is congruent to the original shape.
- **f** The image of a shape after translation is congruent to the original shape.

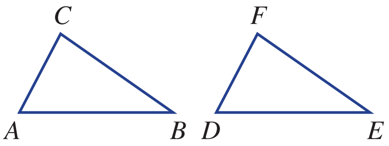
2 In this diagram ΔABC has been reflected to give the image triangle ΔDEF.

Choose the matching vertex (corner), side or angle on the opposite triangle.



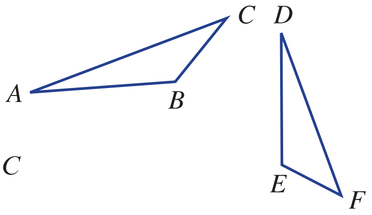
- **a** Is ΔDEF congruent to ΔABC?
- **b** Name the vertex on ΔDEF which corresponds to:
  - **i** vertex A **ii** vertex B **iii** vertex C
- **c** Name the side on ΔDEF which corresponds to:
  - **i** side AB **ii** side BC **iii** side AC
- **d** Name the angle in ΔDEF which corresponds to:
  - **i** $\angle B$ **ii** $\angle C$ **iii** $\angle A$

3 In this diagram ΔABC has been translated (shifted) to give the image triangle ΔDEF.



- **a** Is ΔDEF congruent to ΔABC?
- **b** Name the vertex on ΔDEF which corresponds to:
  - **i** vertex A **ii** vertex B **iii** vertex C
- **c** Name the side on ΔDEF which corresponds to:
  - **i** side AB **ii** side BC **iii** side AC
- **d** Name the angle in ΔDEF which corresponds to:
  - **i** $\angle B$ **ii** $\angle C$ **iii** $\angle A$

4 In this diagram ΔABC has been rotated to give the image triangle ΔDEF.



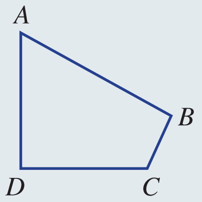
- **a** Is ΔDEF congruent to ΔABC?
- **b** Name the vertex on ΔDEF which corresponds to:
  - **i** vertex A **ii** vertex B **iii** vertex C
- **c** Name the side on ΔDEF which corresponds to:
  - **i** side AB **ii** side BC **iii** side AC
- **d** Name the angle in ΔDEF which corresponds to:
  - **i** $\angle B$ **ii** $\angle C$ **iii** $\angle A$

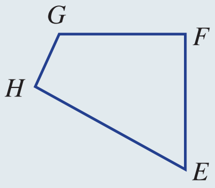
FLUENCY 5 5, 6

Example 10 Naming corresponding pairs

These two quadrilaterals are congruent. Name the objects in quadrilateral $EFGH$ that correspond to these objects in quadrilateral $ABCD$.

a vertex C **b** side AB **c** $\angle C$



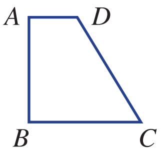


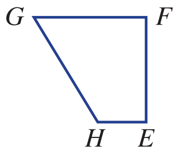
Solution	Explanation
a vertex G	C sits opposite A and $\angle A$ is the smallest angle. G sits opposite E and $\angle E$ is also the smallest angle.
b side EH	Sides AB and EH are both the longest sides of their respective shapes. A corresponds to E and B corresponds to H.
c $\angle G$	$\angle C$ and $\angle G$ are both the largest angle in their corresponding quadrilateral.

5 These two quadrilaterals are congruent. Name the object in quadrilateral $EFGH$ which corresponds to these objects in quadrilateral $ABCD$.

- **a** **i** vertex A **ii** vertex D
- **b** **i** side AD **ii** side CD
- **c** **i** $\angle C$ **ii** $\angle A$

Corresponding angles will be equal and corresponding sides are the same length.



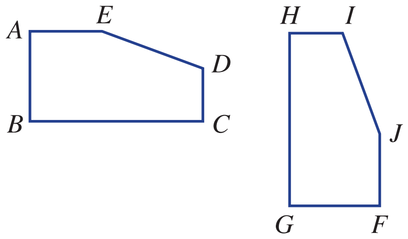


6 These two pentagons are congruent. Name the object in pentagon $FGHIJ$ which corresponds to these objects in pentagon $ABCDE$.

a **i** vertex A **ii** vertex D

b **i** side AE **ii** side CD

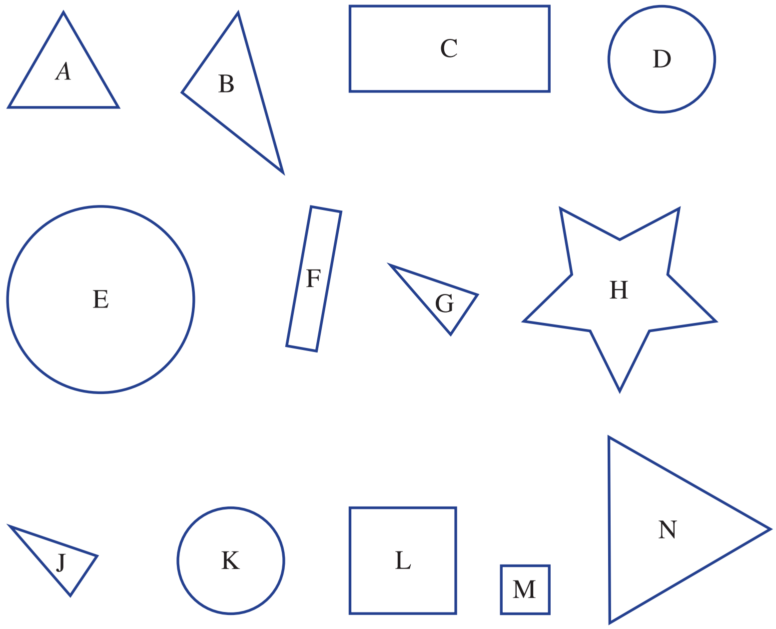
c **i** $\angle C$ **ii** $\angle E$



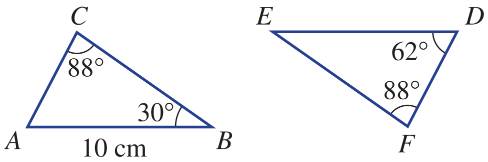
PROBLEM-SOLVING AND REASONING

7–9 8–11

7 From all the shapes shown here, find 3 pairs that look congruent.



8 These triangles are congruent.



a Which side on ΔDEF corresponds to side AB?

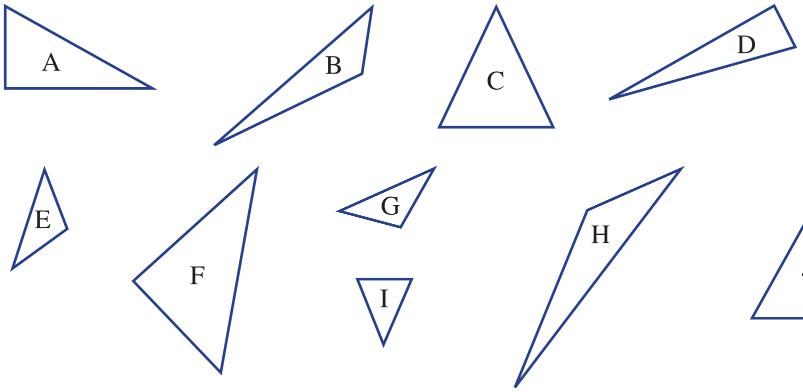
b Which angle on $\angle ABC$ corresponds to $\angle E$?

c What is the length DE?

d What is the size of $\angle A$?

e What is the size of $\angle E$?

9 List the pairs of the triangles below that look congruent.

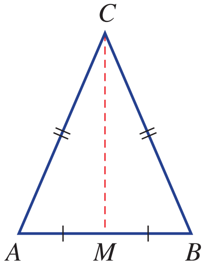


Look for three pairs.

10 An isosceles triangle is cut as shown, using the midpoint of AB.

a Name the two triangles formed.

b Will the two triangles be congruent? Give reasons.



11 If a parallelogram is cut by either diagonal will the two triangles be congruent?

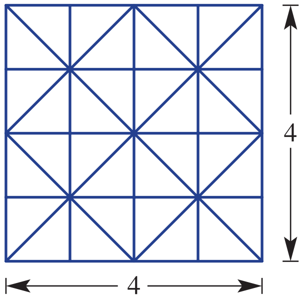
Counting triangles

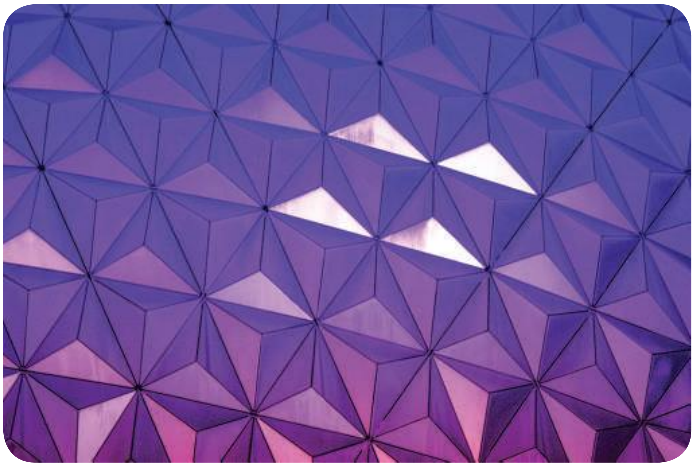
—	12

12 How many congruent triangles are there in this diagram with:

a area $\frac{1}{2}$ **b** area 1? **c** area 2?

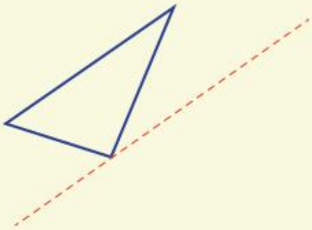
d area 4? **e** area 8?





Progress quiz

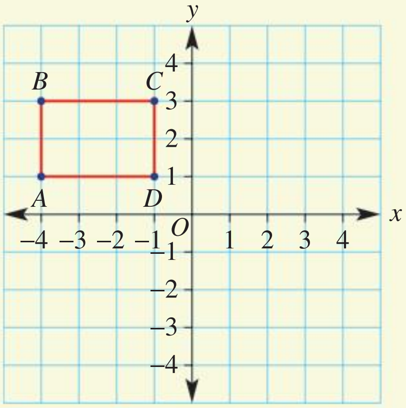
10A **1** Copy the diagram and draw the reflected image over the given mirror line.



10A **2** State the coordinates of the vertices A', B', C' and D' after this rectangle is reflected in the given axes.

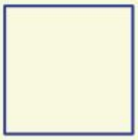
a x-axis

b y-axis



10A **3** How many lines of symmetry do these quadrilaterals have?

a Square



b Rectangle

c Parallelogram

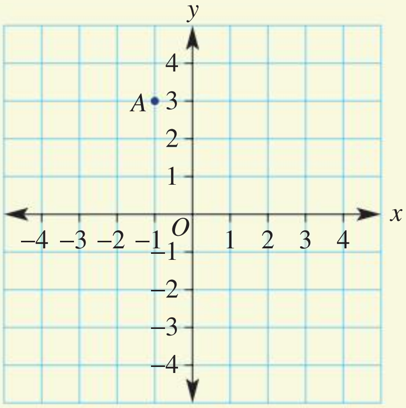
10B **4** Write the vector (x, y) that describes the following transformations.

a 4 units to the left and 3 units up

b 6 units to the right and 2 units down

10B **5** Give the coordinates of the image of the point A if it is translated by these vectors.

a $(2, 3)$ **b** $(-3, 1)$ **c** $(5, 0)$

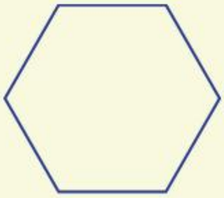


10B **6** Write the vector that takes each point to its image. Use a grid to help you.

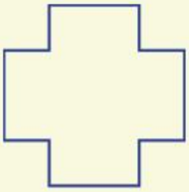
a $A(3, 4)$ to $A'(2, -1)$ **b** $A(-1, 0)$ to $A'(-5, 3)$

10C **7** Find the order of rotational symmetry for the following shapes.

a



b



10C **8** The point A(3, −4) is rotated about the origin $O\,(0, 0)$ by the given angle and direction. Give the coordinates of A':

a 180° clockwise

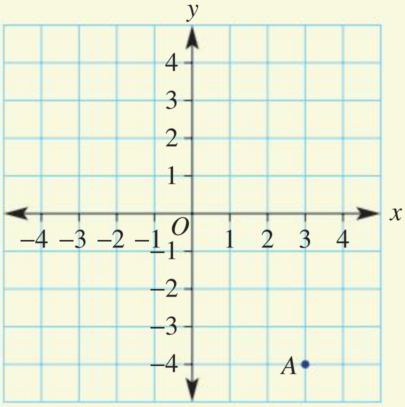
b 180° anticlockwise

c 90° clockwise

d 90° anticlockwise

e 360° clockwise

f 270° anticlockwise



10D **9** These two quadrilaterals are congruent.
Name the object in quadrilateral $MNOP$ which corresponds to these objects in quadrilateral $ABCD$.

a Vertex A

b Vertex C

c Side AB

d Side BC

e $\angle D$

f $\angle B$

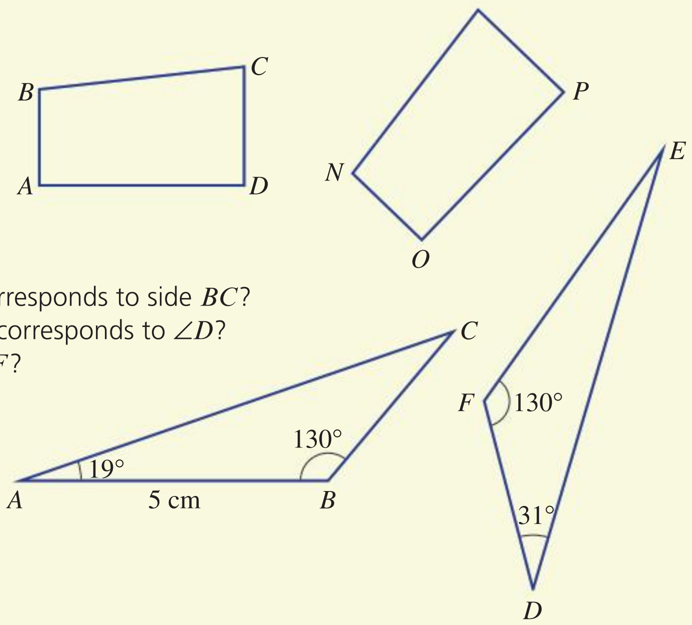
10D **10** These triangles are congruent.

a Which side on ΔDEF corresponds to side BC?

b Which angle on ΔABC corresponds to $\angle D$?

c What is the length of EF?

d What is the size of $\angle C$?



10E Congruent triangles

Interactive

Widgets

HOTsheets

Walkthrough

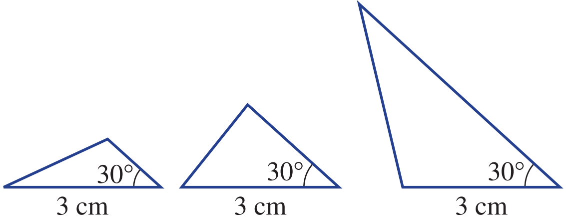
Imagine the sorts of design and engineering problems we would face if we could not guarantee that two objects such as window panes or roof truss frames were not the same size or shape. Also, it might not be possible to measure every length and angle to test for congruence. In the case of triangles, it is possible to consider only a number of pairs of sides or angles to test whether or not they are congruent. This leads to a special set of minimum conditions (tests) which can be used establish that two triangles are congruent.

Let's start: How much information is enough?

Given one corresponding angle (say 30°) and one corresponding equal side length (say 3 cm), it is clearly not enough information to say two triangles are congruent. This is because more than one triangle can be drawn with the given information.

Decide if the following information is enough to determine if two triangles are congruent. If you can draw two non-identical triangles then there is not enough information. You could use a ruler and a protractor or simply try this by hand labelling vertices, sides and angles as you go.

- ΔABC with $AC = 4$ cm and $\angle C = 40^\circ$
- ΔABC with $AB = 5$ cm and $AC = 4$ cm
- ΔABC with $AB = 5$ cm, $AC = 4$ cm and $\angle A = 45^\circ$
- ΔABC with $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm
- ΔABC with $AB = 4$ cm, $\angle A = 40^\circ$ and $\angle B = 60^\circ$

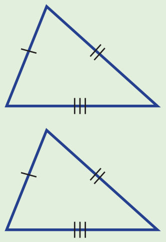


Knowing one corresponding side and one corresponding angle is not enough to say that two triangles will be congruent.

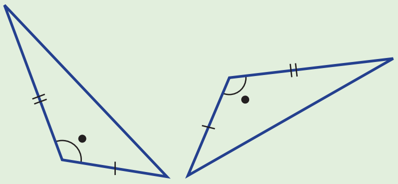
This wall of a building at Federation Square in Melbourne includes many congruent triangles.

Key ideas

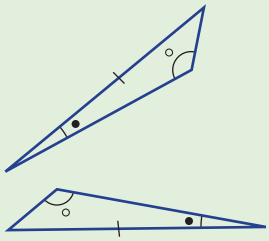
- Two triangles are congruent if one of these four sets of conditions is satisfied.
  - **SSS**
    3 equal corresponding sides



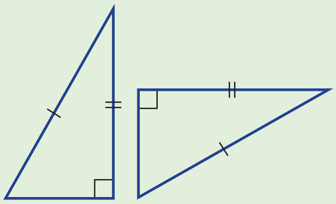
  - **SAS**
    2 equal corresponding sides and 1 equal corresponding angle between them. This angle is called the **included angle**.



  - **AAS**
    2 equal corresponding angles and 1 equal corresponding side. Any order is accepted AAS, ASA, SAA.



  - **RHS**
    2 right-angled triangles with equal hypotenuse lengths and 1 other pair of equal corresponding sides.



- If triangle ABC is congruent to triangle DEF, we write $\Delta ABC \equiv \Delta DEF$.
  – This is called a congruence statement.
  – Letters are usually written in matching order.

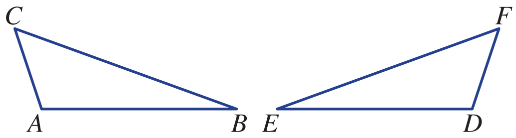
Exercise 10E

UNDERSTANDING 1–2 2

1 Are the following tests for congruent triangles? Answer Yes or No.

a SSS **b** SAS **c** AAA
d AAS **e** RHS **f** SSA

2 Look at this pair of congruent triangles.



a Which vertex on ΔDEF corresponds to (matches) these vertices on ΔABC?
i vertex C **ii** vertex A **iii** vertex B

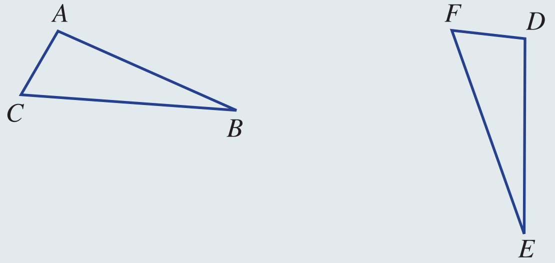
b Which angle on ΔABC corresponds to these angles on ΔDEF?
i $\angle D$ **ii** $\angle F$ **iii** $\angle E$

c Which side on ΔDEF corresponds to these sides on ΔABC?
i AB **ii** CA **iii** BC

FLUENCY 3, 4 3–5

Example 11 Writing a congruence statement

Write a congruence statement for this pair of congruent triangles.

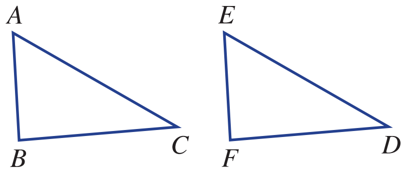


Solution	Explanation
$\Delta ABC \equiv \Delta DEF$	Given the size of the angles and the side lengths, it appears that A matches D, B matches E and C matches F.

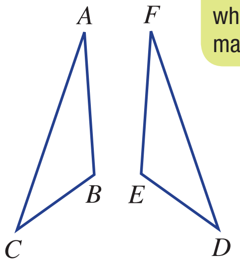
3 Write a congruence statement (e.g. $\Delta ABC \equiv \Delta DEF$) for these pairs of congruent triangles. Try to match vertices.

Match vertices which have the same matching angles.

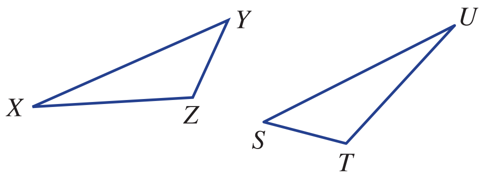
a



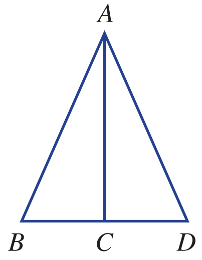
b



c



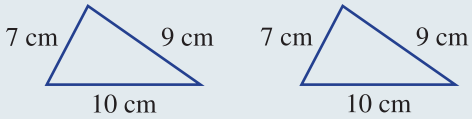
d



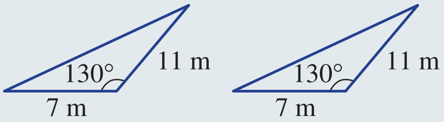
Example 12 Deciding on a test for congruence

Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these pairs of triangles?

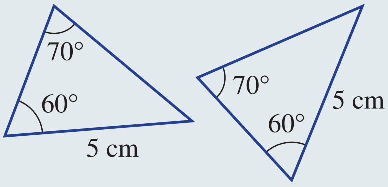
a



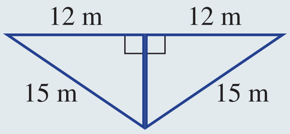
b



c



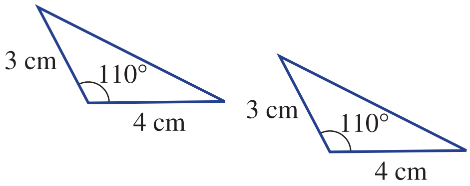
d



Solution	Explanation
a SSS	There are 3 equal corresponding pairs of sides.
b SAS	There are 2 equal corresponding pairs of sides and 1 equal angle between them.
c AAS	There are two equal angles and 1 pair of equal corresponding sides. The side that is 5 cm is adjacent to the 60° angle on both triangles.
d RHS	There is a pair of right angles with hypotenuses of equal lengths. A second pair of corresponding sides are also of equal length.

4 Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these triangles?

a



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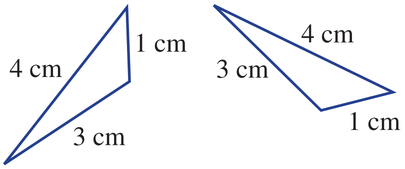
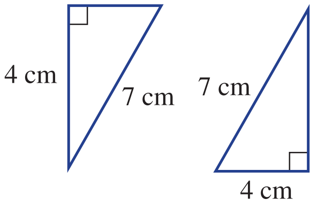


c



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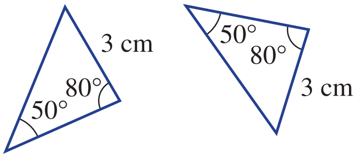
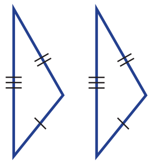


5 Pick the congruence test (SSS, SAS, AAS or RHS) that matches each pair of congruent triangles.

a



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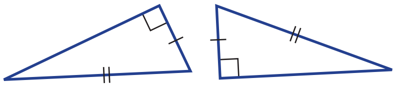

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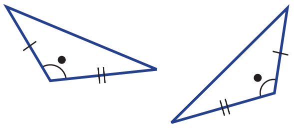

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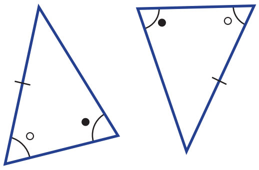
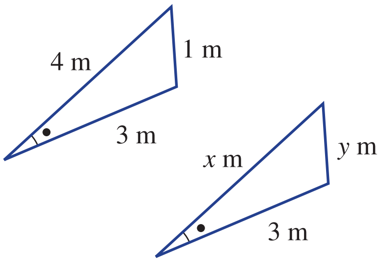


PROBLEM-SOLVING AND REASONING

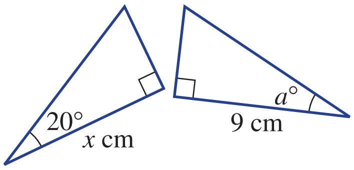
6, 7 6–9

6 These pairs of triangles are congruent. Find the values of the pronumerals.

a

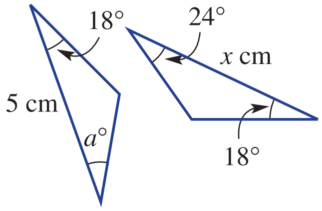


b

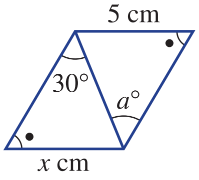


Matching sides will be equal and matching angles will be equal.

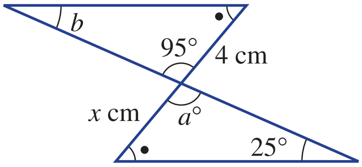
c



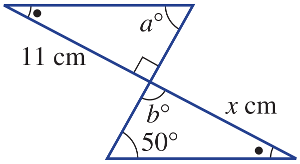
d



e



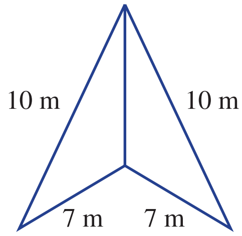
f



7 Which of SSS, SAS, AAS or RHS would you choose to say that each pair of triangles is congruent?

Use the information given in the diagram.

a



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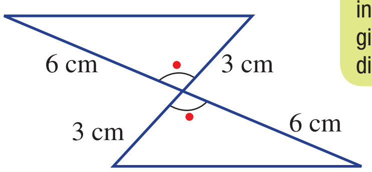

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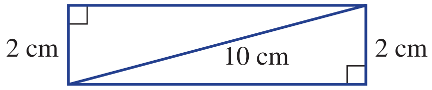

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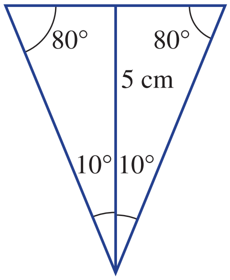
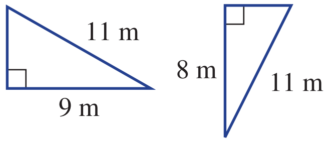
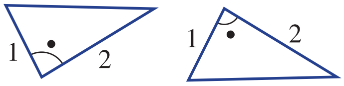


8 Are these pairs of triangles congruent? If they are, give a reason.

a



b



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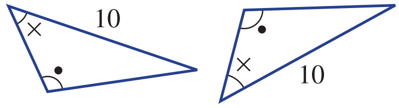

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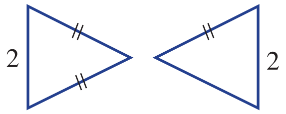


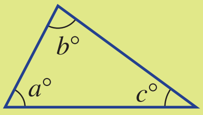
9 Explain why AAA is not sufficient to prove that two triangles are congruent. Draw diagrams to show your reasoning.

Using the triangle angle sum

— 10

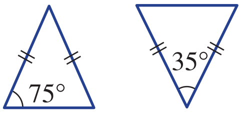
10 Decide if each pair of triangles is congruent. You may first need to use the angle sum of a triangle to help calculate some of the angles.

First work out all the missing angles in the triangles.

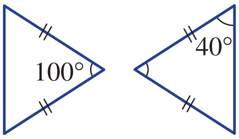


$a + b + c = 180$

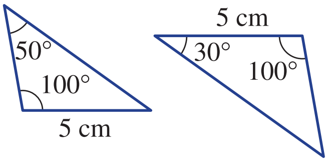
a



b



c



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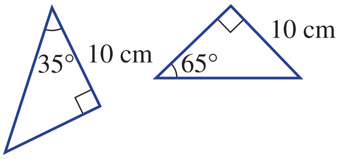


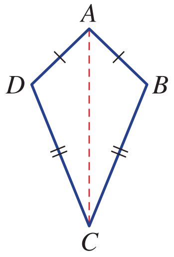
10F Congruence and quadrilaterals

The properties of special quadrilaterals, including the parallelogram, rhombus, rectangle, square, trapezium and kite, can be examined more closely using congruence. By drawing the diagonals and using the tests for the congruence of triangles we can prove many of the properties of these special quadrilaterals.

Let's start: Why is one pair of opposite angles in a kite equal?

A kite with two pairs of equal length sides can be divided into two triangles, as shown.

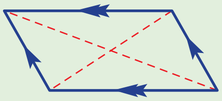
- Are these two triangles congruent?
- Which congruent triangle test (SSS, SAS, AAS, RHS) would be used to confirm this?
- What does this say about $\angle B$ and $\angle D$?



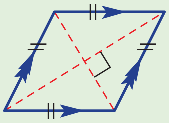
Key ideas

This is a summary of the properties of the special quadrilaterals. Many of the proofs of these properties will be considered in the following exercise.

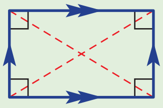
- **Parallelogram** – A quadrilateral with two pairs of parallel sides.
  - Opposite sides are equal.
  - Opposite angles are equal.
  - Diagonals bisect each other.



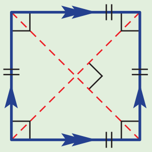
- **Rhombus** – A parallelogram with all sides equal.
  - Opposite angles are equal.
  - Diagonals bisect each other at 90°.
  - Diagonals bisect the interior angles.



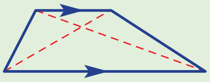
- **Rectangle** – A parallelogram with all interior angles 90°.
  - Opposite sides are equal.
  - Diagonals bisect each other.
  - Diagonals are equal in length.



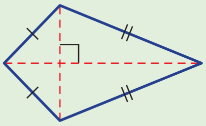
- **Square** – A rectangle with all sides equal.
  - Diagonals bisect each other at 90°.
  - Diagonals are equal in length.



- **Trapezium** – A quadrilateral with one pair of parallel sides.



- **Kite** – A quadrilateral with two pairs of equal sides.
  - One pair of opposite angles are equal.
  - Diagonals bisect each other at right angles.
  - One of the diagonals is bisected by the other.



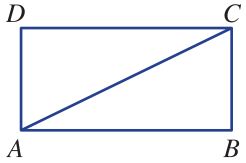
Exercise 10F

UNDERSTANDING 1–4 4

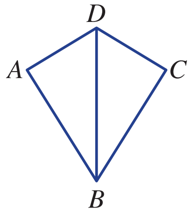
1 SSS is one test for congruence of triangles. Write down the other three.

2 Name the side (e.g. AB) that is common to both triangles in each diagram.

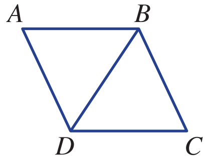
a



b

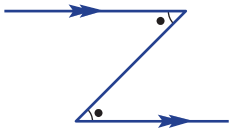


c



3 Give the reason why the two marked angles are equal.

a



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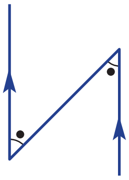

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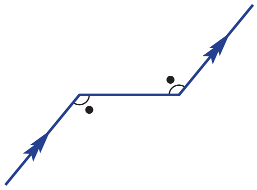
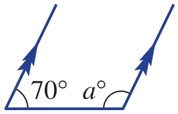


Recall that in parallel lines:
- corresponding angles are equal
- alternate angles are equal
- cointerior angles add to 180°

4 Give the reason why the two marked angles add to 180° and then state the value of a.

a



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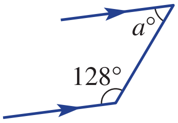


FLUENCY 5(½) 5–6(½)

Use the information in the key ideas to help.

5 Answer true (T) or false (F).

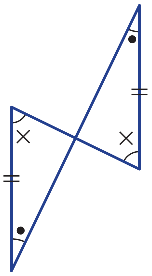
- **a** Opposite sides of a parallelogram are parallel.
- **b** Opposite sides of a kite are equal.
- **c** A trapezium has two pairs of parallel sides.
- **d** The diagonals of a rectangle are equal.
- **e** The diagonals of a kite are equal.
- **f** The diagonals of a parallelogram are equal.
- **g** The diagonals of a trapezium are equal.
- **h** The diagonals of a rhombus are equal.
- **i** The diagonals of a square are equal.
- **j** All angles inside a square are 90°.
- **k** Opposite angles in a kite are equal.
- **l** The diagonals of a parallelogram intersect at right angles.
- **m** The diagonals of a rhombus intersect at right angles.
- **n** The diagonals of a kite intersect at right angles.
- **o** The diagonals of a rhombus bisect each other.
- **p** The diagonals of a parallelogram bisect each other.
- **q** The diagonals of a rectangle bisect each other.

Bisect means to cut in half.

6 Which of the four tests for congruence of triangles would be used to prove that each pair of triangles is congruent? Angles and sides with the same markings are equal.

Two are SSS, one is SAS, two are AAS and one is RHS.

a



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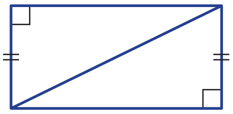

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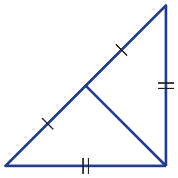

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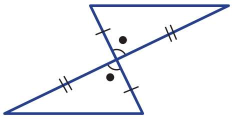

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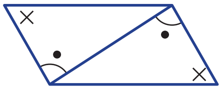
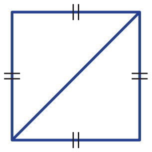
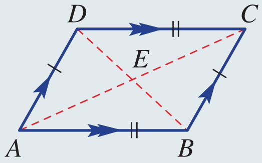


f



PROBLEM-SOLVING AND REASONING 7–9 9–12

Example 13 Exploring the diagonals of a parallelogram



Answer these questions regarding the diagonals of this parallelogram.

a Are the diagonals equal in length?

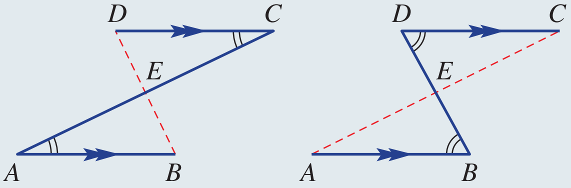
b What can be said about $\angle BAE$ and $\angle DCE$?

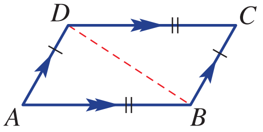
c What can be said about $\angle ABE$ and $\angle CDE$?

d Does $AB = DC$?

e Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABE \equiv \Delta CDE$?

f Why do parallelogram diagonals bisect each other?

Solution	**Explanation**
a No.	Alternate angles in parellel lines are equal.
b They are equal.	 D C E A B
c They are equal.	
d Yes.	
e AAS.	
f $\Delta ABE \equiv \Delta CDE$ so $BE = DE$ and $AE = CE$	The two triangles are congruent using the AAS test. Since ΔABE and ΔCDE are congruent the corresponding sides are equal.



7 Answer these questions about angles in this parallelogram.

a List two triangles formed by the diagonal.

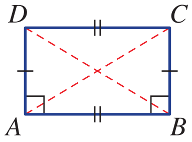
b What can be said about sides AB and DC?

c What can be said about sides AD and BC?

d Which side is common to both triangles?

e Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABD \equiv \Delta CDB$?

f Why is $\angle A = \angle C$?



8 Answer these questions about diagonals in this rectangle.

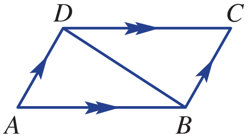
a Locate ΔABD and ΔBAC. Is $\angle A = \angle B$?

b Is $AD = BC$?

c Is $AB = CD$?

d Which reason (SSS, SAS, AAS, RHS) explain why $\Delta ABC \equiv \Delta BAD$?

e Why is $AC = BD$?



9 A parallelogram $ABCD$ has two pairs of parallel sides.

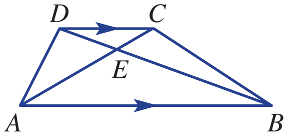
a What can be said about $\angle ABD$ and $\angle CDB$ and give a reason?

b What can be said about $\angle BDA$ and $\angle DBC$ and give a reason?

c Which side is common to both ΔABD and ΔCDB?

d Which congruence test would be used to show that $\Delta ABD \equiv \Delta CDB$?

e If $\Delta ABD \equiv \Delta CDB$, what can be said about the opposite sides of a parallelogram?

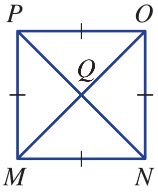


10 A trapezium $ABCD$ has one pair of parallel sides.

a Which angle is equal to $\angle BAE$?

b Which angle is equal to $\angle ABE$?

c Explain why ΔABE is not congruent to ΔCDE.

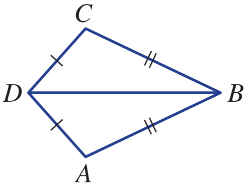


11 For this square assume that $MQ = QO$ and $NQ = PQ$.

a Give reasons why $\Delta MNQ \equiv \Delta ONQ$.

b Give reasons why $\angle MQN = \angle OQN = 90°$.

c Give reasons why $\angle QMN = 45°$.



12 Use the information in this kite to prove these results.

a $\Delta ABD \equiv \Delta CBD$ **b** $\angle DAB = \angle DCB$ **c** $\angle ADB = \angle CDB$

Writing a formal proof

13

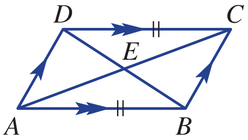
13 Prove by giving reasons that the diagonals in a parallelogram bisect each other. Opposite sides are equal, so use $AB = CD$. Complete the proof by following these steps.

Step 1. List the pairs of equal angles in ΔABE and ΔCDE giving reasons why they are equal.

Step 2. List the pairs of equal sides in ΔABE and ΔCDE giving reasons why they are equal.

Step 3. Write $\Delta ABE \equiv \Delta CDE$ and give the reason SSS, SAS, AAS or RHS.

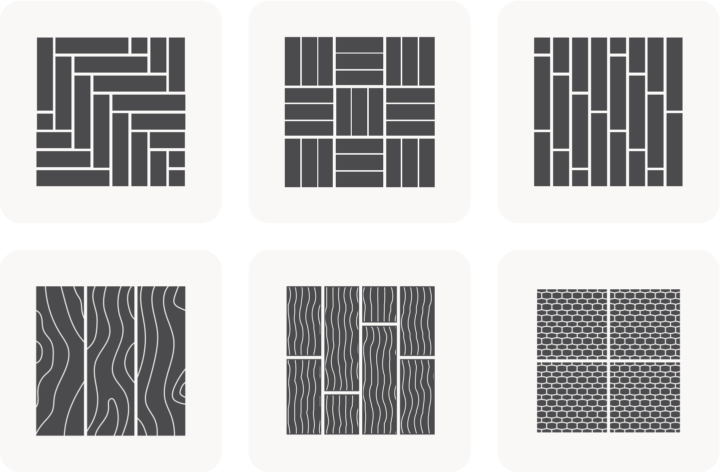
Step 4. State that $BE = DE$ and $AE = CE$ and give a reason.



10G Tessellations

Interactive
Widgets
HOTsheets
Walkthrough

Architects, builders and interior designers have great interest in arranging basic congruent shapes to create interesting patterns within a new home. These patterns are often formed using tiles or pavers and can be found on bathroom walls, interior floors or exterior courtyards.

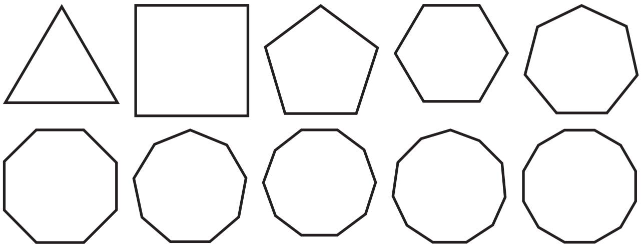


A common range of paving patterns that show how a rectangle can be arranged without leaving any gaps or holes. Which is your preferred pattern if you were wanting to pave an outdoor entertaining area?

The words *tessellate* and *tessellation* originate from the Latin noun, *tessera*, referring to a small tile used in the construction of a mosaic. Tessellated tile designs are commonly used throughout history in the fields of Art and Design and continue to be extensively employed today. It is most likely, that various tessellations exist within your home and your school.

Let’s start: To tessellate or not to tessellate?

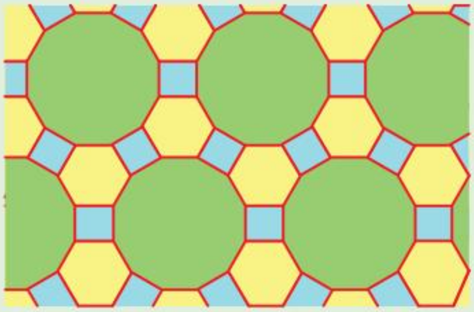
In Section 2E you were introduced to the concept of *regular polygons* as shapes with sides of equal length and equal interior angles. Can you remember the name given to the first ten polygons?



Johann Kepler, back in 1619, was the first mathematician to prove that there are only three polygons that will tessellate by themselves. Working with a partner, can you determine which polygons these are?

Key ideas

- A **tessellation** is a pattern made up of shapes that fit together without any gaps and without any overlaps.
- Isometric transformations, such as **reflections, translations** and **rotations**, are used with appropriate shapes to produce tessellated patterns.
- **Regular tessellations** are formed by arranging multiple copies of one regular polygon. There are only three regular polygons that tessellate by themselves: triangle, square and hexagon.
- **Semi-regular tessellations** are formed by arranging multiple copies of two or more regular polygons. There are eight distinct semi-regular tessellations.
- Regular and semi-regular tessellations can be **named** by counting the number of sides each regular polygon has at any of the identical vertices.
  For example: the following semi-regular tessellation consists of squares, hexagons and dodecagons. It can be named as a 4.6.4.12 tessellation.



- Other tessellated patterns can be formed by any combination of shapes (regular, irregular, composite). Curved shapes and images can also be used to form tessellated patterns, like the one shown.

Exercise 10G

UNDERSTANDING 1–3 3

When asked to construct tessellations you can do these by hand using grid or dot paper; alternatively, you may prefer to use geometry software.

1 Which of the following phrases best describes what a *tessellation* is?

A A group of shapes joined together

B A group of shapes all stacked on top of one another to form a totem pole

C A group of shapes arranged together without any overlaps or any gaps

D A group of shapes positioned in such a way to form an attractive pattern

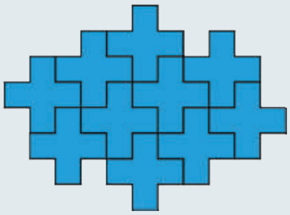
2 Which of the following words best matches the mathematical term *congruence*?

A Parallel **B** Similar **C** Related **D** Identical

Example 14 Tessellating plus signs

Using the following 'plus sign' shape  draw ten identical plus signs to show that this shape will tessellate.

Solution

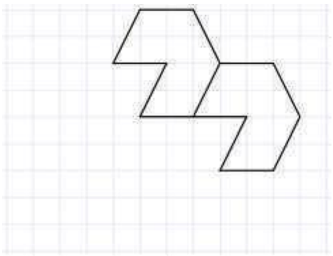


Explanation

Translate each identical plus sign to make sure it fits without leaving any gaps and without any overlaps.

Note that the final shape does not need to be a neat rectangle to be classified as a tessellation. The only requirement is that the pattern can continue to grow without leaving any holes.

3 The following image shows the start of a tessellation, with two identical shapes joined together. Continue the tessellation by adding on another six identical shapes.



FLUENCY 4–6, 8 5–8

4 Using the following trapezium shape  draw ten identical shapes to show that a trapezium will tessellate. You can translate, rotate or reflect this shape to form your tessellation.

5 Draw regular tessellations using only:

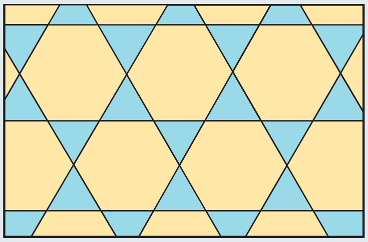
a triangles

b squares

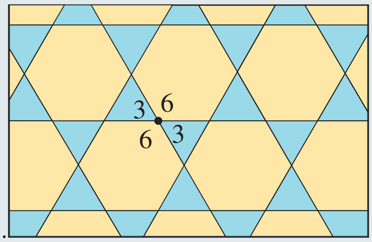
c hexagons

Example 15 Naming tessellations

By considering any vertex, name the following semi-regular tessellation.



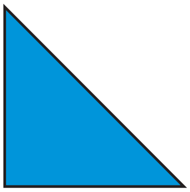
Solution	Explanation
(3.6.3.6)	Select any vertex and as you go around the vertex, count the number of sides each polygon has.



6 By looking at vertices, label each of the tessellations drawn in Question **5**.

7 Which of the following shapes tessellate by themselves? Reflections, rotations and translations of the original shape can be used.

a



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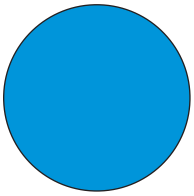

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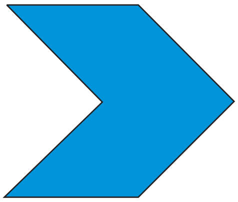

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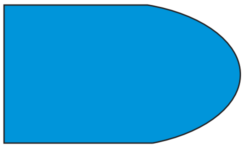

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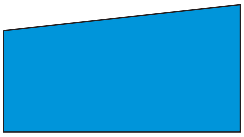
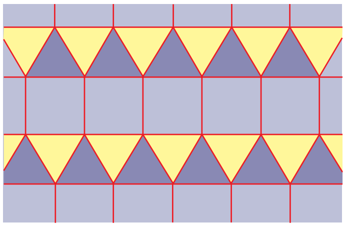


8 By considering any vertex, name the following semi-regular tessellations.

a



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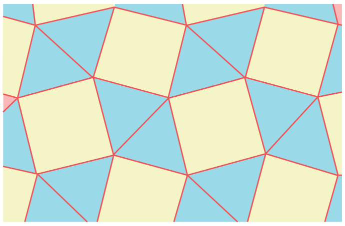

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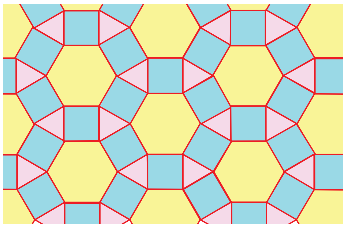

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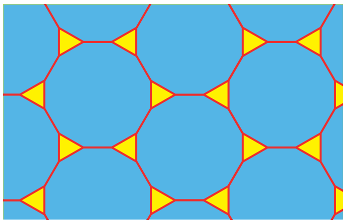
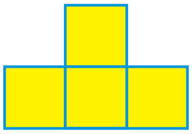


PROBLEM-SOLVING AND REASONING 9, 10 10, 11

9 Design a tessellation using the following. Use rotations, reflections and translations if needed.

a only the following shape

b only the following shape

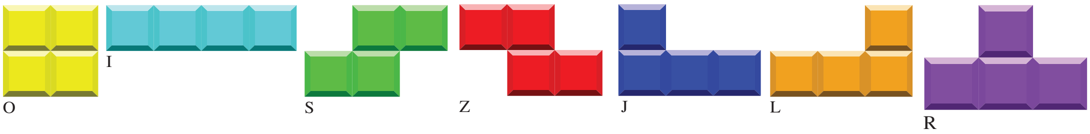


c any combination of the above two shapes

10 Produce a tessellation using only regular octagons and squares.

11 The object of the game *Tetris* is to produce rows with no gaps, or in other words to produce a tessellation with the tiles as they appear. Using 1 cm grid paper, draw large rectangle of width = 10 cm and height = 20 cm.

The following image shows the seven different *Tetris pieces*, with each small cube representing a 1 cm × 1 cm square.



a How many *Tetris* pieces will be needed to completely fill the 10 cm × 20 cm rectangle?

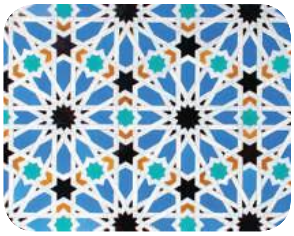
b Using at least three of each piece, design a tessellated pattern to fill the 10 cm × 20 cm rectangle.

Ancient and modern tessellations

— 12, 13

12 **Ancient tessellations**

During the Middle Ages the Moorish people, particularly of Spain, were well known for their distinctive and elaborate tile designs. Several images are shown below.



a Carry out research on Moorish tile designs and print two of your favourite tessellations.

b Using grid paper, design your own intricate 10×10 tile, consisting of a range of simple coloured shapes which tessellate and completely cover the tile.

c Either by hand, or using appropriate geometry software, repeatedly draw your intricate tile to show how it tessellates and see how effective it looks as a design that could go in a modern home.

13 Modern tessellations

The Dutch artist M.C. Escher (1898–1972) is famous for making irregular tessellations involving repeated images which gradually change form. An example of Escher's work is shown above.

a Carry out research on M.C. Escher and print two of your favourite Escher designs.

b Either by hand, or using appropriate geometry software, design your own irregular tessellation consisting of the one repeated image.

The Alcazar, a royal palace built for the Moorish Muslim kings in Seville, Spain is regarded as one of the most intricate and beautiful palaces in the world. It is the oldest royal palace still in use in Europe today and is a very popular tourist site for visitors to Spain. *The Alcazar* showcases many amazing mosaic and tessellating tiles.

Maths@Work: Fashion designer

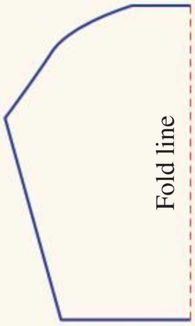
The fashion industry is a glamorous but extremely competitive market where many talented designers fail to survive. A degree in Fashion Design provides up-to-date technical skills and the opportunity to meet industry professionals.

Aspiring designers should study fashion trends, take art classes, be inventive with colour and learn to sew a variety of fabrics. Designers need to budget money to survive without income at first. A good, working relationship with a manufacturer is vital to success as well as having resilience, as designers often face critique.

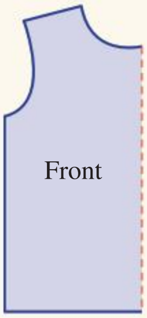
Mathematics is an essential tool in fashion, both in the business aspect and especially in the technical package development. Garment pattern and fabric placement, cutting angles, accurate, detailed measurements and design geometry, are all examples of how mathematics is embedded in fashion.

1 Many patterns use symmetry in their designs. Copy (e.g. trace) and complete each design below, using each dashed line as the axis of symmetry.

a



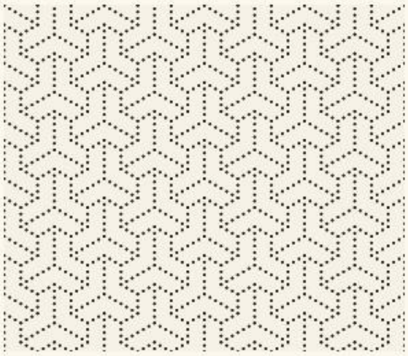
b



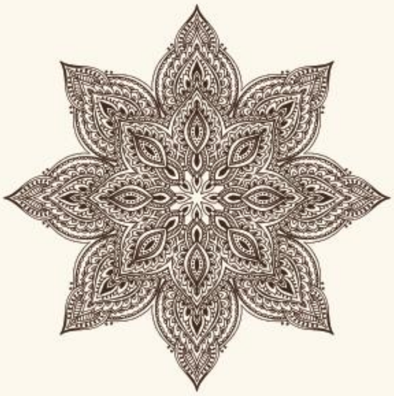
c

2 Designers often use transformations when designing their own fabrics. Describe the transformations used in each of the following fabric designs.

a

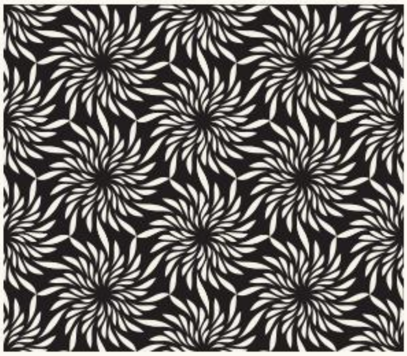


b



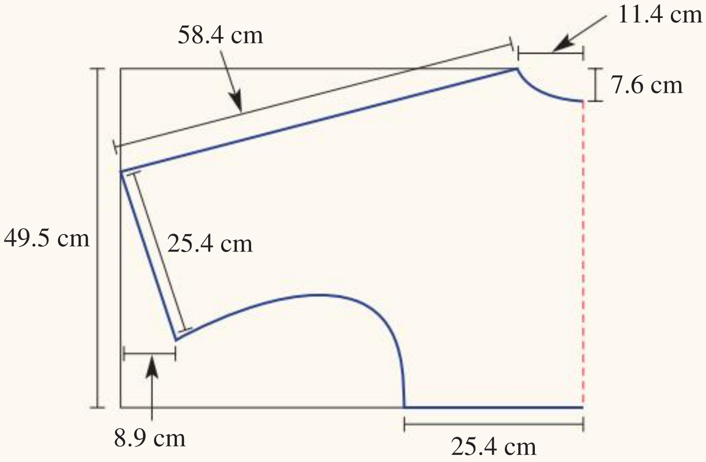
c

d



3 Garment pattern makers develop a designer's sketch for the manufacturer. It is a very technical and detailed process resulting in a pattern that can be successfully sewn by numerous workers in a large factory. Garments need to be manufactured in identical (i.e. congruent) sets of each size as well as scaled copies (i.e. similar sets) that are the various sizes.

Reproduce an exact copy of the scale drawing below of a garment pattern. First make accurate measurements of lengths and angles. Check that all students in the class have congruent patterns at the end of the activity.



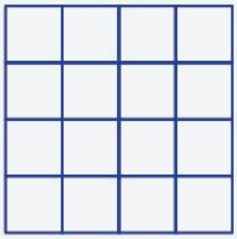
Using technology

4 Use geometry software to digitally design a fabric pattern using geometrical transformations. Colour the fabric in three different combinations for presentation to a commercial fabric manufacturer.

Explain the transformations you have used in your design.

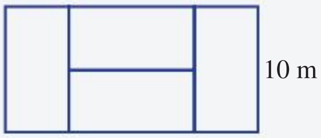
Puzzles and games

1 How many squares are there in this diagram?

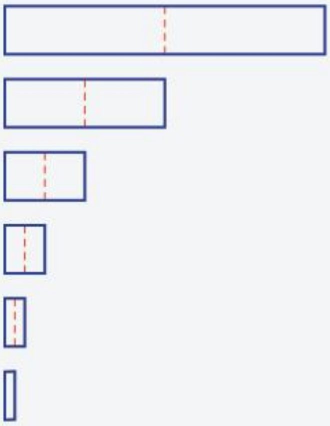


2 How many triangles are there in this diagram?

3 The four rectangles inside this diagram are congruent. What is the perimeter of each rectangle?

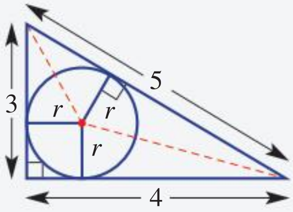


4 A strip of paper is folded 5 times in one direction only. How many creases will there be in the original strip when it is folded out?



5 Can you fit the shapes in the two smaller squares into the largest square? Try drawing or constructing the design and then use scissors to cut out each shape.

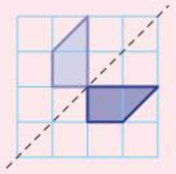
6 Use congruent triangles to find the radius r in this diagram.



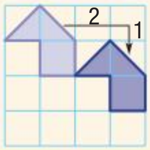
Chapter summary

Transformation and congruence

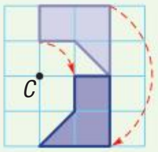
Reflection



Translation vector (2, –1)

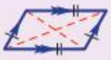


Rotation clockwise by 90°



Quadrilaterals and congruence

- Parallelogram



- Rhombus

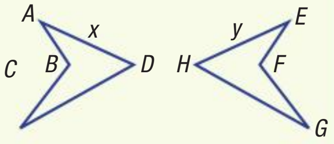
- Rectangle

- Square

- Trapezium

- Kite

Congruent figures same size and shape



$ABCD \equiv EFGH$ $x = y$

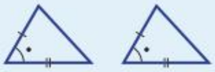
$AB = EF$

$\angle C = \angle G$

Tests for congruent triangles

- SSS

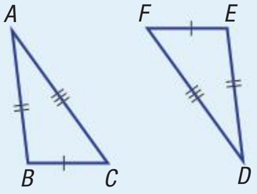
- SAS



- AAS

- RHS

Congruent triangles



$\Delta ABC \equiv \Delta DEF$

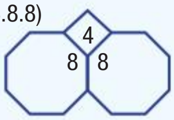
$AB = DE$, $BC = EF$, $AC = DF$

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Tessellations

- Patterns including shapes with no gaps.
- Named by counting the number of sides of each polygon at a vertex.

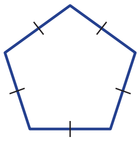
Name: (4.8.8)



Multiple-choice questions

10A **1** The number of lines of symmetry in a regular pentagon is:

A 10 **B** 5 **C** 2
D 1 **E** 0



10B **2** Which vector describes a translation of 5 units to the left and 3 units up?

A $(3, -5)$ **B** $(-3, 5)$ **C** $(5, -3)$ **D** $(-5, 3)$ **E** $(-5, -3)$

10B **3** The point $A(-3, 4)$ is translated to the point A' by the vector $(6, -4)$. The coordinates of point A' are:

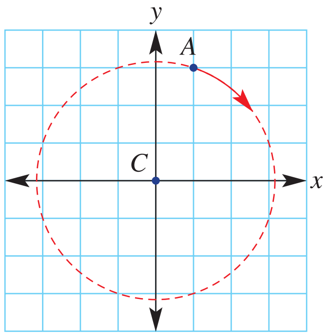
A $(3, 8)$ **B** $(-9, 8)$ **C** $(3, 0)$ **D** $(0, 3)$ **E** $(-9, 0)$

10C **4** An anticlockwise rotation of 125° about a point is the same as a clockwise rotation about the same point of:

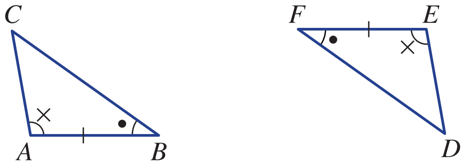
A 235° **B** 65° **C** 55° **D** 135° **E** 245°

10C **5** Point $A(1, 3)$ is rotated clockwise about C by $90°$ to A'. The coordinates of A' are:

A $(3, 0)$ **B** $(3, -1)$ **C** $(-3, 1)$
D $(-1, 3)$ **E** $(3, 1)$



Questions **6**, **7**, and **8** relate to this pair of congruent triangles.



10D **6** The angle on ΔDEF that corresponds to $\angle A$ is:

A $\angle C$ **B** $\angle B$ **C** $\angle F$ **D** $\angle D$ **E** $\angle E$

10D **7** If $AC = 5$ cm then ED is equal to:

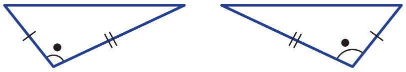
A 5 cm **B** 10 cm **C** 2.5 cm **D** 15 cm **E** 1 cm

10E **8** A congruent statement with ordered vertices for the triangles is:

a $\Delta ABC \equiv \Delta FED$ **b** $\Delta ABC \equiv \Delta EDF$ **c** $\Delta ABC \equiv \Delta DFE$
d $\Delta ABC \equiv \Delta DEF$ **e** $\Delta ABC \equiv \Delta EFD$

10E **9** Which of the four tests (SSS, SAS, AAS, RHS) would be chosen to show that these two triangles are congruent?

A AAS **B** RHS **C** AAA
D SAS **E** SSS



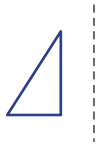
10F **10** Which of the following has equal length diagonals?

A Trapezium **B** Parallelogram **C** Kite
D Rhombus **E** Rectangle

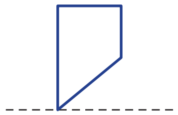
Short-answer questions

10A **1** Copy these shapes and draw the reflected image over the mirror line.

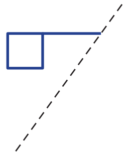
a



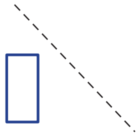
b



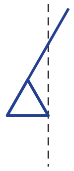
c



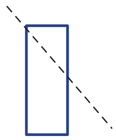
d



e



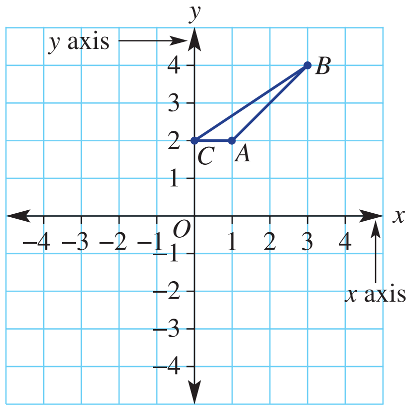
f



10A **2** The triangle $A(1, 2)$, $B(3, 4)$, $C(0, 2)$ is reflected in the given axis. State the coordinates of the image points A', B' and C'.

a x-axis

b y-axis



10A **3** How many lines of symmetry do these shapes have?

a Square

b Isosceles triangle

c Rectangle

d Kite

e Regular hexagon

f Parallelogram

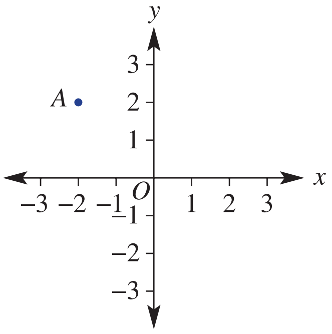
10B **4** Give the coordinates of the image of the point A if it is translated by these vectors.

a $(2, 1)$

b $(4, -3)$

c $(-1, -1)$

d $(-1, -4)$



10B **5** Write the vectors that translate each point A to its image A'.

a $A(2, 5)$ to $A'(3, 9)$

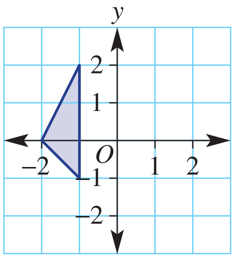
b $A(-1, 4)$ to $A'(2, -2)$

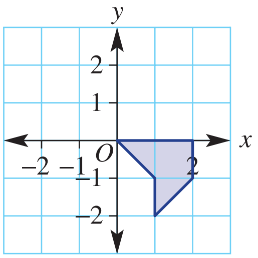
c $A(0, 7)$ to $A'(-3, 0)$

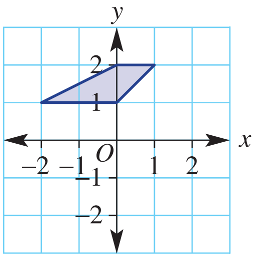
d $A(-4, -6)$ to $A'(0, 0)$

10B **6** Copy these shapes and draw the translated image using the given translation vector.

a vector $(3, -1)$ **b** vector $(-2, 2)$ **c** vector $(0, -3)$





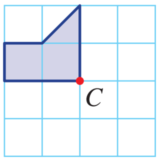


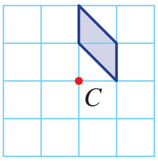
10C **7** What is the order of rotational symmetry of these shapes?

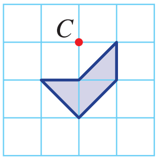
a Equilateral triangle **b** Parallelogram **c** Isosceles triangle

10C **8** Rotate these shapes about the point C by the given angle.

a Clockwise 90° **b** Clockwise 180° **c** Anticlockwise 270°





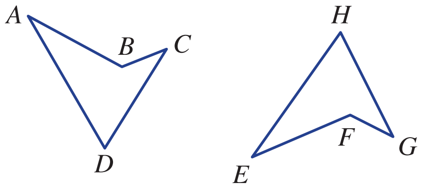


10D **9** For these congruent quadrilaterals, name the object in quadrilateral $EFGH$ that corresponds to the given object in quadrilateral $ABCD$.

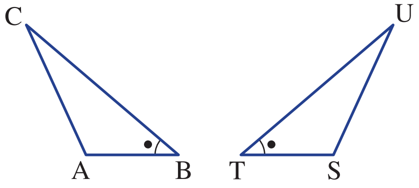
a **i** Vertex B **ii** Vertex C

b **i** Side AD **ii** Side BC

c **i** $\angle C$ **ii** $\angle A$

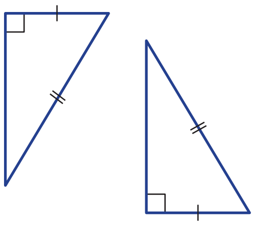


10E **10** Write a congruence statement for these congruent triangles.

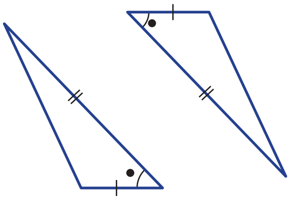


10E **11** Which of the tests SSS, SAS, AAS or RHS would you choose to explain the congruence of these pairs of triangles? Sides or angles with the same markings are equal.

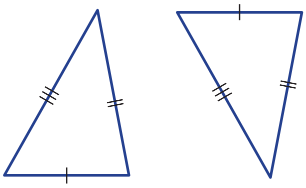
a



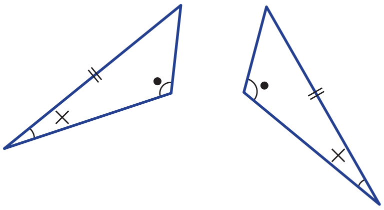
b



c

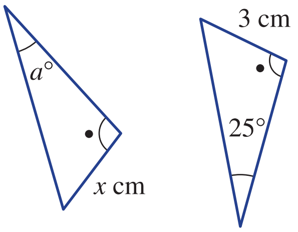


d

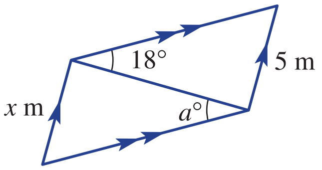


10E **12** Find the values of the pronumerals for these congruent triangles.

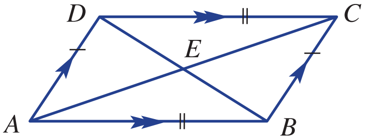
a



b



10F **13** This quadrilateral is a parallelogram with 2 pairs of parallel sides. You can assume that $AB = DC$ as shown.



a Is $\angle BAE = \angle DCE$? Give a reason.

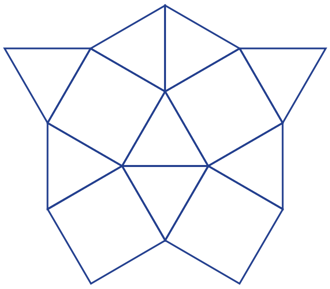
b Is $\angle ABE = \angle CDE$? Give a reason.

c Is $AB = DC$?

d Which reason (SSS, SAS, AAS, RHS) would be used to explain that $\Delta ABE \equiv \Delta CDE$?

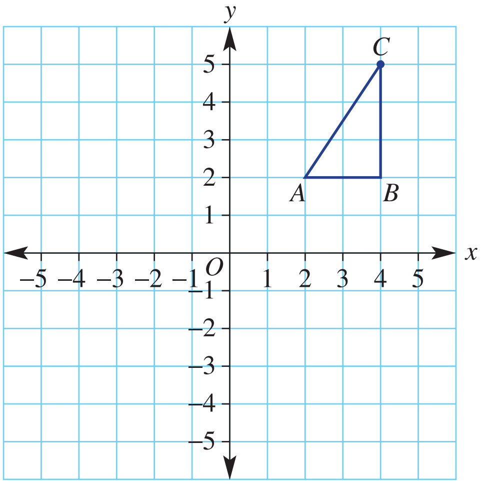
e Explain why BD and AC bisect each other.

10G **14** Name this semi-regular tessellation.



Extended-response questions

1 The shape on this set of axes is to be transformed by a succession of transformations. The image of the first transformation is used to start the next transformation. For each set of transformations write down the coordinates of the vertices A', B' and C' of the final image.



Parts **a** and **b** are to be treated as separate questions.

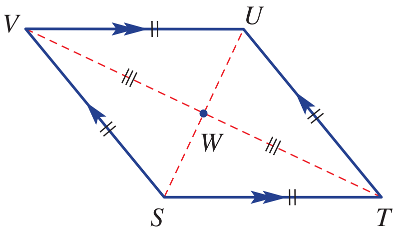
a Set 1
- **i** Reflection in the x-axis.
- **ii** Translation by the vector $(-2, 1)$.
- **iii** Rotation about $(0, 0)$ by 180°.

b Set 2
- **i** Rotation about $(0, 0)$ clockwise by 90°.
- **ii** Reflection in the y-axis.
- **iii** Translation by the vector $(5, 3)$.

2 For this rhombus note that $VW = WT$.

a Give reasons why $\Delta VWU \equiv \Delta TWU$.

b Give reasons why $\angle VWU = \angle TWU = 90°$.

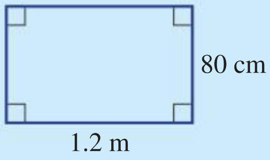


Ratios and rates

Multiple-choice questions

1 The ratio of the length to the width of this rectangle is:

A 12 : 80 **B** 3 : 20
C 3 : 2 **D** 20 : 3



2 Which of the following ratios is not written in simplest form?

A 2 : 3 **B** 5 : 10 **C** 11 : 3 **D** 3 : 7

3 \$18 is divided in the ratio 2 : 3. The larger part is:

A \$3.60 **B** \$7.20 **C** \$10.80 **D** \$12

4 Calvin spent \$3 on his mobile phone plan for every \$4 he spent on his internet connection. Calvin spent \$420 on his phone last year. How much did he spend for his internet connection the same year?

A \$140 **B** \$315 **C** \$560 **D** \$240

5 A boat sailed 30 kilometres in 90 minutes. What was the average speed of the boat?

A 15 km/h **B** 45 km/h **C** 3 km/h **D** 20 km/h

Short-answer questions

1 Simplify these ratios.

a 24 to 36 **b** 15 : 30 : 45
c 0.6 m to 70 cm **d** 15 cents to \$2
e 2 kg to 400 g **f** 30 seconds to $1\frac{2}{3}$ minutes

2 **a** Divide 960 cm in the ratio of 3 : 2.
b Divide \$4000 in the ratio of 3 : 5.
c Divide \$8 in the ratio of 2 : 5 : 3.

3 A business has a ratio of profit to costs of 5 : 8. If the costs were \$12 400, how much profit was made?

4 A map has a scale 1 : 10 000. Find the real distance in cm and also in m between two towns that are 6 cm apart on the map.

5 Write each of the following as a simplified rate.

a 84 mm rainfall in 7 days **b** 18 goals in 6 games
c \$15 for 750 g of meat

6 A shop sells $1\frac{1}{2}$ kg bags of apples for \$3.40. Find the cost of a one kilogram at this rate.

7 A family travels the 1070 km road from Rockhampton to Cairns, in 12.5 hours. Calculate their average speed.

Extended-response questions

1 A small car uses 30 litres of petrol to travel 495 km.

a What is the average distance travelled per litre?
b At this rate, what is the maximum distance a small car can travel on 45 litres of petrol?
c Find the number of litres used to travel 100 km, correct to one decimal place.
d Petrol costs 117.9 cents/litre. Find the cost of petrol for the 495-km trip.
e A larger car uses 42 litres of petrol to travel 378 km. The smaller car holds 36 litres of petrol while the larger car holds 68 litres. How much further can the larger car travel on a full tank of petrol?

Equations

Multiple-choice questions

1 If $x = 5$, which one of these equations is true?

A $x + 3 = 2$ **B** $7x = 75$ **C** $7 - x = 2$ **D** $2x = 20$

2 The sum of a number and three is doubled. The result is 12. This can be written as:

A $x + 3 \times 2 = 12$ **B** $2(x + 3) = 12$ **C** $2x + 3 = 12$ **D** $x + 3 = 24$

3 The solution to the equation $2m - 4 = 48$ is:

A $m = 8$ **B** $m = 22$ **C** $m = 20$ **D** $m = 26$

4 The solution to the equation $4k + 3 = 39$ is:

A $k = 36$ **B** $k = 9$ **C** $k = 10$ **D** $k = 4$

5 The solution to $\frac{x}{4} - 3 = 4$ is:

A $x = 20$ **B** $x = 28$ **C** $x = 4$ **D** $x = 24$

Short-answer questions

1 Solve each of these equations.

a $3w = 27$ **b** $12 = m + 5$ **c** $2x - 1 = 9$

d $4a + 2 = 10$ **e** $2w + 6 = 32$ **f** $4 = 6x - 2$

2 Solve each of these equations.

a $\frac{x}{3} = 10$ **b** $\frac{2q}{5} = 4$ **c** $3 = \frac{p}{5}$

d $\frac{x + 2}{4} = 3$ **e** $\frac{r - 3}{12} = 1$ **f** $2 = \frac{3a - 4}{10}$

3 Solve the following equations.

a $2(x + 3) = 16$ **b** $4(2k + 1) = 84$ **c** $3(r + 2) + r = 6$ **d** $10(z - 4) + 2z = 80$

4 Double a number less three is the same as 9. What is the number?

5 The formula $S = 6g + b$ relates an AFL score (S) to the number of goals (g) and behinds (b).

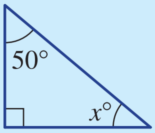
a Find S if $g = 3$ and $b = 2$.

b Find b if $S = 62$ and $g = 10$.

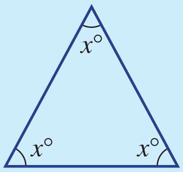
c Find g if $S = 50$ and $b = 8$.

6 Using the fact that angles of a triangle add to 180°, find x in the following triangles.

a



b



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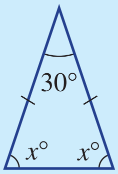


Extended-response questions

1 EM Publishing has fixed costs of $1500 and production costs of $5 per book.

a Write an expression for the cost of producing n books.

b The total costs for one year were $2000. Use an equation to find how many books were produced.

c Write an expression for the money made by selling n books, if they sell each book for $20.

d If the total revenue is $1000, find the number of books sold.

e Given that the profit is given by the formula, $P = 15n - 1500$, find:

- **i** the profit if 200 books are sold.
- **ii** the profit if 1000 books are sold.
- **iii** the number of books sold if the profit is $0.

f If $n = 50$, the profit is $ –750. Explain what this means for EM Publishing.

Probability and statistics

Multiple-choice questions

1 For the set of numbers 3, 2, 1, 3, 5, 1, 3, 9, 3, 5 the mode is:

A 3 **B** 3.5 **C** 8 **D** 35

2 Look at the set of numbers 8, 9, 10, 10, 16, 19, 20, 20. Which of the following statements is true?

A Median = 13 **B** Mean = 13 **C** Mode = 13 **D** Range = 13

3 In a bag there are 5 green marbles, 6 blue marbles, 2 red marbles and 3 purple marbles.

The probability of randomly selecting a blue marble is:

A $\frac{6}{10}$ **B** $\frac{2}{16}$ **C** $\frac{6}{10}$ **D** $\frac{3}{8}$

4 A die is rolled 60 times. How many times would you expect to throw an even number?

A 10 times **B** 15 times **C** 30 times **D** 45 times

5 A die is rolled 60 times. The number 4 appears exactly 24 times. The experimental probability of obtaining the number 4 is:

A 0 **B** $\frac{2}{5}$ **C** $\frac{2}{3}$ **D** $\frac{1}{6}$

Short-answer questions

1 Find: **i** the mean, **ii** the median and **iii** the range of these data sets.
- **a** 10, 15, 11, 14, 14, 16, 18, 12
- **b** 1, 8, 7, 29, 36, 57
- **c** 1.5, 6, 17.2, 16.4, 8.5, 10.4

2 Draw a histogram for this frequency table.

Score	Frequency
10	2
11	3
12	5
13	1

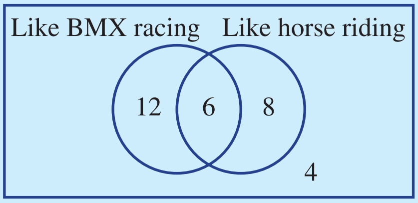
3 A bag contains 16 balls of equal size and shape. Of these balls 7 are yellow, 1 is blue and the rest are black. If one ball is chosen from the bag at random, find the probability that it is:
- **a** yellow
- **b** blue
- **c** not blue
- **d** black
- **e** pink

4 The ages of 50 people at a party are shown in the table below.

Ages	0–9	10–19	20–29	30–39	40–49	50–59	Over 60
Frequency	3	7	1	28	6	2	3

If one person is chosen at random to a prepare a speech, find the probability that the person is aged:
- **a** 0 – 9
- **b** 30 or older
- **c** in their twenties
- **d** not in their fifties

5 The Venn diagram to the right shows the results of a survey asking students who liked BMX racing and who liked horse riding.



- **a** How many students like BMX racing?
- **b** How many students like both BMX racing and horse riding?
- **c** How many students like horse riding but not BMX racing?
- **d** How many students don't like horse riding and also don't like BMX racing?
- **e** How many students don't like horse riding?

6 Answer the questions below about this two-way table:

	Likes rugby	Dislikes rugby	Total
Likes soccer	20	4	24
Dislikes soccer	9	3	12
Total	29	7	36

If a student is chosen randomly from this group, find these probabilities and simplify your answers.
- **a** Pr(likes soccer but dislikes rugby)
- **b** Pr(likes soccer and likes rugby)
- **c** Pr(dislikes soccer but likes rugby)
- **d** Pr(dislikes soccer and also dislikes rugby).

Extended-response questions

1 Two groups of students have their pulse rates recorded as beats per minute. The results are listed here:

Group A: 65, 70, 82, 81, 67, 74, $\square$, 81, 88, 84, 72, 65, 66, 81, 72, 68, 86, 86
Group B: 83, 88, 78, 60, 81, 89, 91, 76, 78, 72, 86, 80, 64, 77, 62, 74, 87, 78

a How many students are in group B?

b If the median pulse rate for group A is 76, what number belongs in the $\square$?

c What is the median pulse rate for group B?

d Which group has the largest range?

Straight line graphs

Multiple-choice questions

1 The value of y in the rule $y = 2x - 1$ when $x = -1$ is:

A 3 **B** 1 **C** -1 **D** -3

2 The coordinates of the point 3 units directly above the origin is:

A (0, 0) **B** (0, 3) **C** (0, −3) **D** (3, 0)

3 The rule for the table of values shown is:

A $y = 2x$ **B** $y = 2x + 2$
C $y = 2(x + 2)$ **D** $y = x + 4$

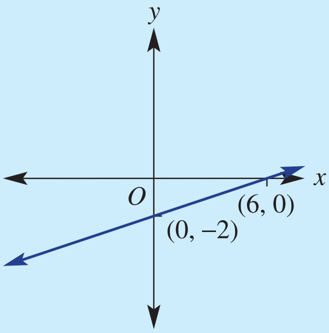
x	0	2	4
y	4	8	12

4 The gradient of the line through $A(4, 7)$ and $B(8, -1)$ is:

A $-\frac{1}{2}$ **B** 2 **C** $\frac{1}{2}$ **D** -2

5 Which equation suits the given graph?

A $y = 6x - 2$
B $y = 3x - 2$
C $y = x - 2$
D $3y = x - 6$



Short-answer questions

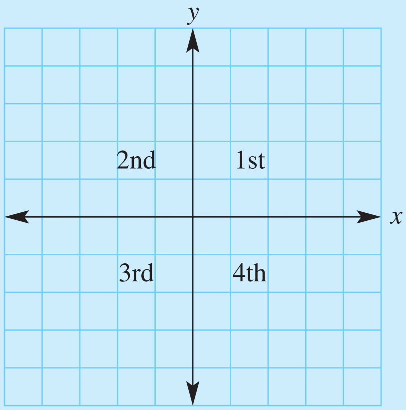
1 In which quadrant does each point lie?

a $(5, 1)$

b $(-3, 4)$

c $(-5, -1)$

d $(8, -3)$



2 **a** Complete these tables of values.

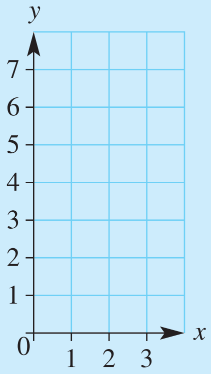
i $y = 2x + 1$

x	0	1	2	3
y				

ii $y = 4 - x$

x	0	1	2	3
y				

b Plot the points from both tables and join to form two graphs.



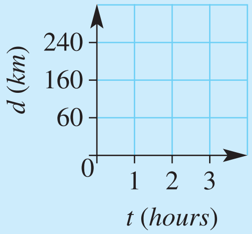
3 Decide if the gradient for each graph in question **2** is zero, positive, negative or infinite.

4 The distance a car travels (d km) over t hours is given by $d = 80t$.

a Complete this table of values.

t	0	1	2	3
d				

b Plot a graph using your table.



c How far does the car travel after 3 hours?

d How long would it take for the car to travel 320 km?

5 For each rule, complete a table of values like the one shown and plot to form a graph.

a $y = 2x - 2$

b $y = -x + 1$

x	−3	−2	−1	0	1	2	3
y							

Extended-response questions

1 The cost $(\$C)$ of running a coffee shop is given by the rule $C = 400 + 5n$, where n is the number of customers on any given day. The revenue (income) is $\$R$ and is given by $R = 13n$.

a Complete this table.

n	0	10	20	30	40	50	60
C							
R							

b Plot a graph for both C and R on the same set of axes.

c What is the 'break even' point for the coffee shop i.e. where does the cost = revenue?

d If they are particularly busy on a Saturday, and serve 100 people, calculate the shop's profit (profit = revenue – cost).

Transformation and congruence

Multiple-choice questions

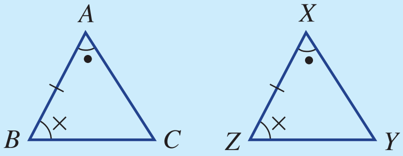
1 The number of lines of symmetry in a square is/are:

A 0 **B** 2 **C** 4 **D** 6

2 The side AC corresponds to:

A XZ **B** XY

C ZY **D** BC



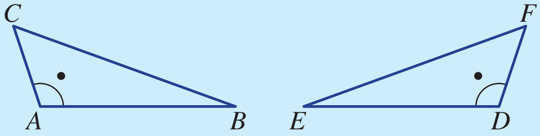
3 A congruency statement for these triangles is:

A $\Delta ABC \equiv \Delta DFE$

B $\Delta ABC \equiv \Delta EFD$

C $\Delta ABC \equiv \Delta EDF$

D $\Delta ABC \equiv \Delta DEF$



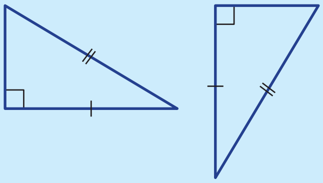
4 Which test is used to show triangle ABC congruent to triangle ADC?

A SSS

B SAS

C AAS

D RHS



5 Which of the following codes is not enough to prove congruency for triangles?

A SSS **B** AAS **C** AAA **D** SAS

Short-answer questions

1 How many lines of symmetry does each of these shapes have?

a Scalene triangle **b** Rhombus **c** Rectangle **d** Semicircle

2 Write the vectors that translate each point P to its image P'.

a $P(1, 1)$ to $P'(3, 3)$ **b** $P(-1, 4)$ to $P'(-2, 2)$

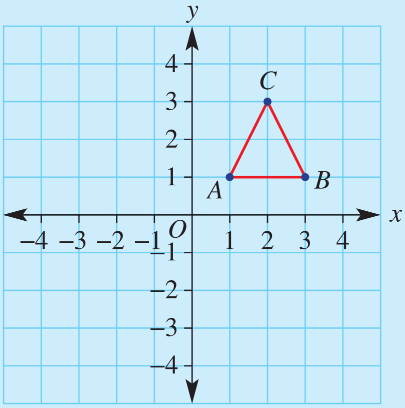
3 Triangle ABC is on a Cartesian plane as shown. List the coordinates of the image points A', B' and C' after:

a a reflection in the x-axis

b translation by the vector $(-4, -2)$

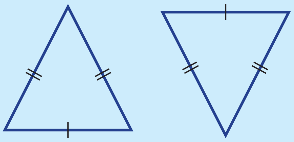
c a rotation 90° clockwise about $(0, 0)$

d a rotation 180° about $(0, 0)$.

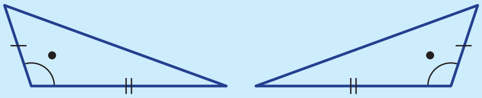


4 Which congruency test (SSS, SAS, AAS or RHS) would be used to prove the following pairs of triangles are congruent?

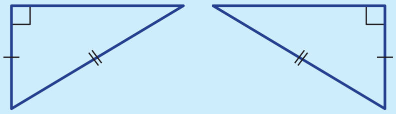
a



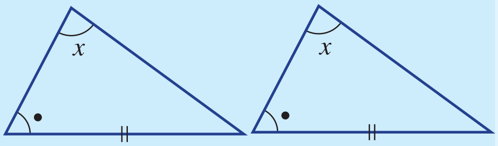
b



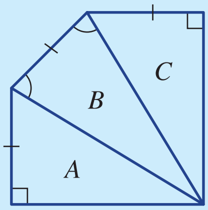
c



d



5 Which two triangles are congruent?



Extended-response questions

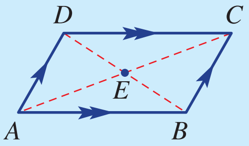
1 For this parallelogram with $AB = DC$, answer the following.

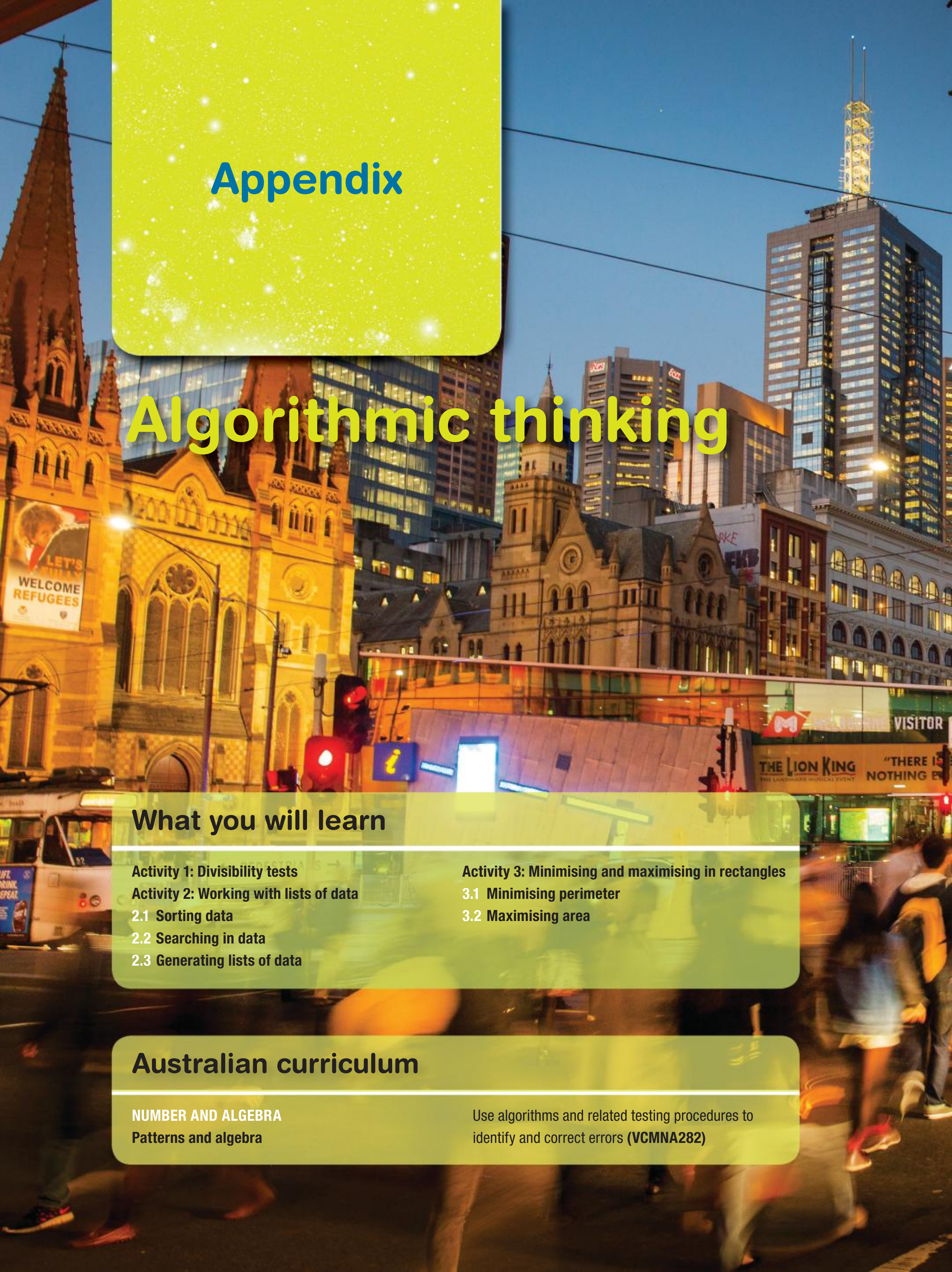
a Why is $\angle BAE = \angle DCE$?

b Why is $\angle ABE = \angle CDE$?

c Give the reason (SSS, SAS, AAS, RHS) why $ABE \equiv \Delta CDE$.

d Explain why the diagonals bisect each other.





Appendix

Algorithmic thinking

What you will learn

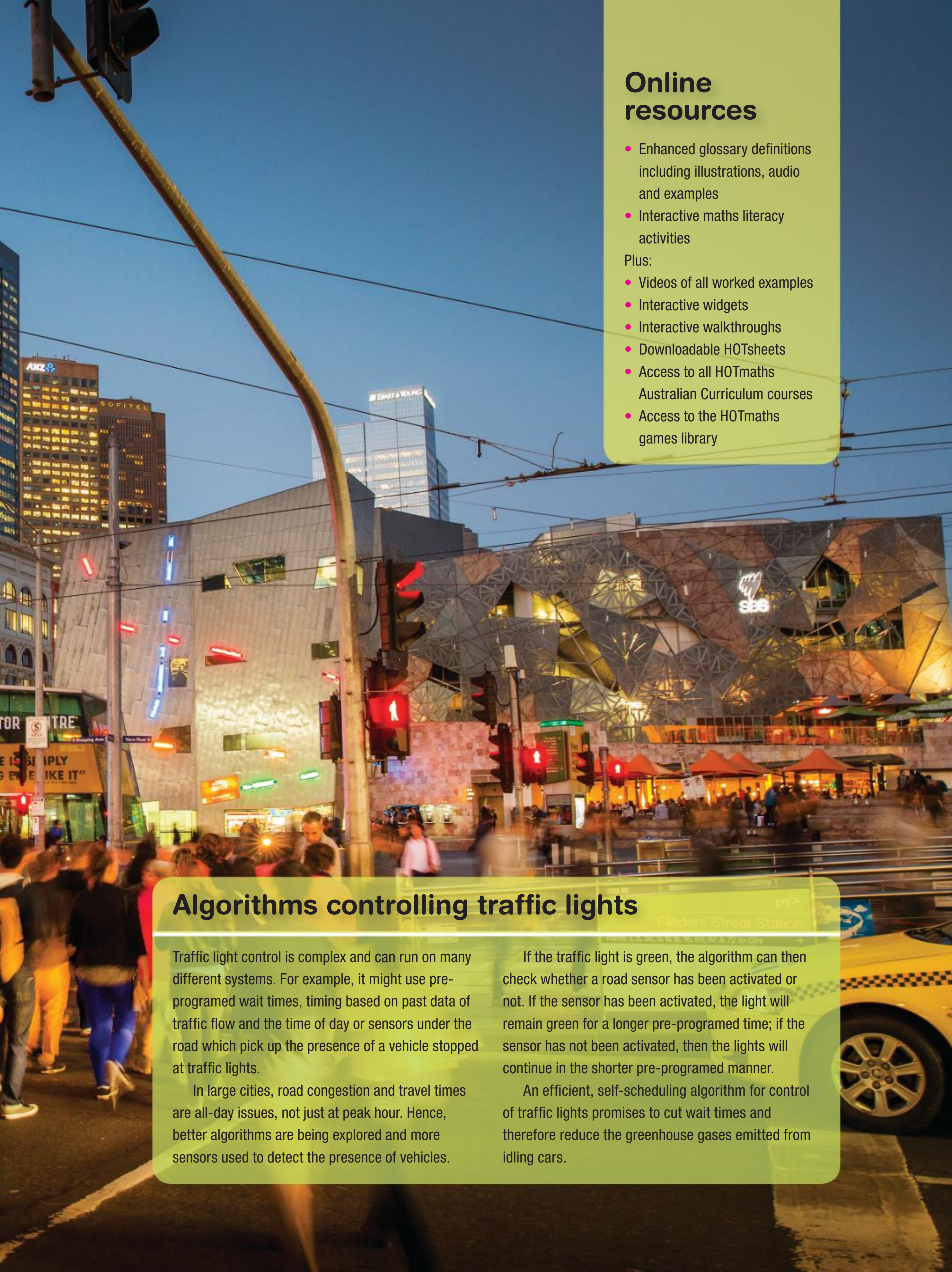
Activity 1: Divisibility tests
Activity 2: Working with lists of data
- **2.1 Sorting data**
- **2.2 Searching in data**
- **2.3 Generating lists of data**

Activity 3: Minimising and maximising in rectangles
- **3.1 Minimising perimeter**
- **3.2 Maximising area**

Australian curriculum

NUMBER AND ALGEBRA
Patterns and algebra

Use algorithms and related testing procedures to identify and correct errors **(VCMNA282)**



Online resources

- Enhanced glossary definitions including illustrations, audio and examples
- Interactive maths literacy activities

Plus:

- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

Algorithms controlling traffic lights

Traffic light control is complex and can run on many different systems. For example, it might use pre-programed wait times, timing based on past data of traffic flow and the time of day or sensors under the road which pick up the presence of a vehicle stopped at traffic lights.

In large cities, road congestion and travel times are all-day issues, not just at peak hour. Hence, better algorithms are being explored and more sensors used to detect the presence of vehicles.

If the traffic light is green, the algorithm can then check whether a road sensor has been activated or not. If the sensor has been activated, the light will remain green for a longer pre-programed time; if the sensor has not been activated, then the lights will continue in the shorter pre-programed manner.

An efficient, self-scheduling algorithm for control of traffic lights promises to cut wait times and therefore reduce the greenhouse gases emitted from idling cars.

Introduction

An **algorithm** is a procedure involving a number of steps that eventually leads to the answer of a problem. There should be clearly defined inputs and outputs.
For example:

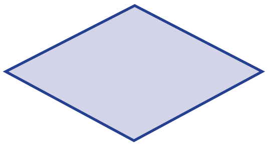
- The highest common factor (HCF) algorithm has two inputs like 20 and 50, and gives the output 10.
- The decimal multiplication algorithm has two inputs like 0.8 and 0.2 and gives the output 0.16.

Algorithms occur in mathematics and computing, and in simple areas of daily life such as following a recipe.

The algorithms in the following activities will be described through the use of spreadsheets, flow charts, pseudocode (an informal programming language) and simulations. The following symbols will be used in the flow charts with arrows used to connect each stage.

For input/output stages

For process stages

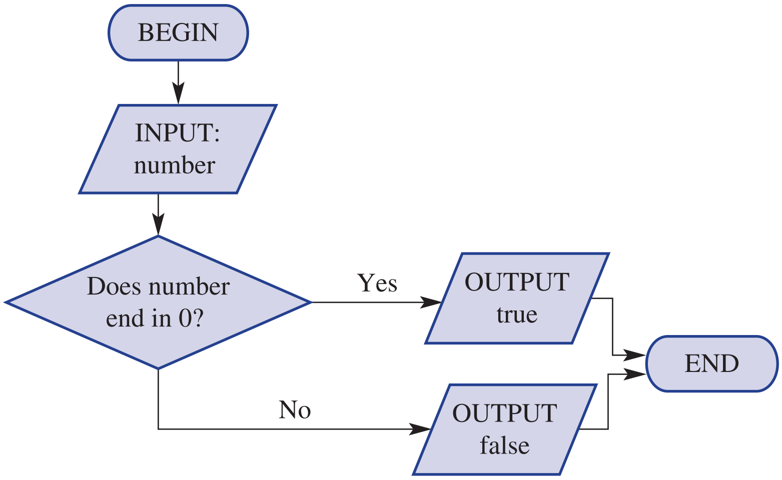


For decision stages

Activity 1: Divisibility tests

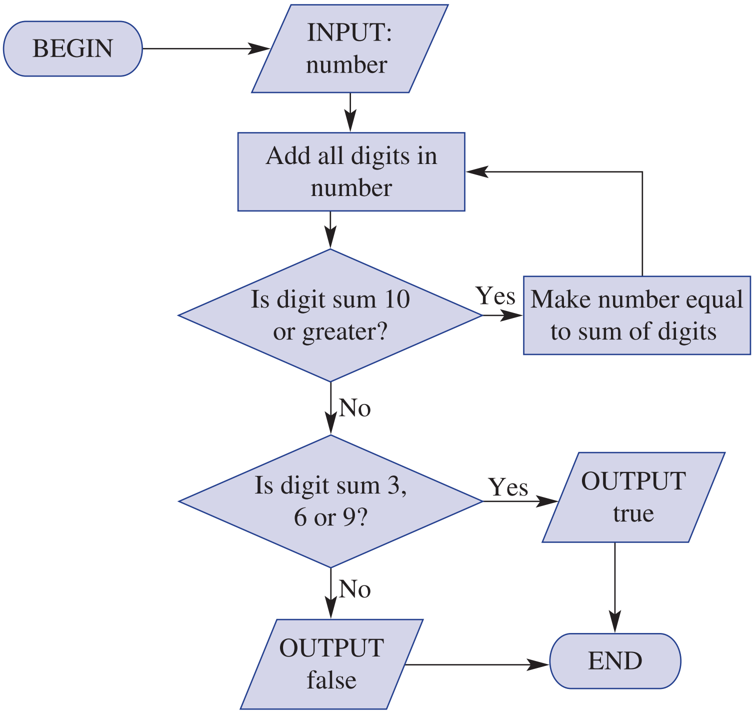
Number and Algebra

a The flowchart below takes a number and tells you whether it is divisible by 10 or not.



i What changes are required to the original flowchart to test if a number is divisible by 5?
ii What changes are required to the original flowchart to test if a number is divisible by 2?
iii Jonty has attempted to test if a number is divisible by 4, so he has changed the conditional step to the following test 'Does number end in 0, 4 or 8?' Explain what is wrong with this.

b **i** Testing for divisibility by 3 involves looping. Describe what happens if the input number is 209 in the flowchart below.



ii Describe what happens if the input number is 831.

iii Draw a flowchart for the algorithm that tests to see if a number is divisible by 9. Recall that divisibility by 9 involves adding up the digits to see if the sum is divisible by 9.

c You can assume now you have algorithms to test if a number is divisible by the numbers 2, 3, 5, 9 and 10.

i Describe how you could combine these algorithms to test if a number is divisible by 6.

ii Describe how you could combine these algorithms to test if a number is divisible by 45.

iii Describe how you could combine these algorithms to test if a number is divisible by 18.

iv What other numbers can you test for divisibility by combining these algorithms?

Activity 2: Working with lists of data

Probability and Statistics

2.1 Sorting data

In order to find the median or interquartile range of a list of numbers we first need to sort the list in ascending order.

Sample input: $[12,1,3,8,2,7]$

Sample output: $[1,2,3,7,8,12]$

One algorithm, called 'Selection Sort' is as follows:

Step 1. Make a new (empty) output list.
Step 2. Find the smallest number in the input list.
Step 3. Remove it from the input list and put it at the end of the output list.
Step 4. Repeat steps 2 and 3 until the input list is empty.

In a high-level programming language, it might look something like the following.

Line	Command	Explanation
1	`output ← []`	Set the output list to be empty
2	`loop:`	Perform lines 3 to 6 forever (until you hit `break`)
3	`  current ← min(input)`	Find the minimum value in the input list
4	`  input.remove(current)`	Remove one copy of current minimum from input
5	`  output.append(current)`	Put current minimum at the end of the output list
6	`  if input = []: break`	If we reach an empty input list, we leave this loop
7	`display output`	At the end we need to display the output

A computer can run this program line by line. The execution is shown below on the input list $[7,\ 1,\ 5]$ and takes 15 steps.

Line	Command	Result
1	`output ← []`	`output` is set to the be the empty list
2	`loop:`	Get ready to repeat lines 3 to 6
3	`current ← min(input)`	`current` is now 1
4	`input.remove(current)`	`input` is now $[7,\ 5]$
5	`output.append(current)`	`output` is now $[1]$
6	`if input = []: break`	No effect, because `input` is not empty.
3	`current ← min(input)`	`current` is now 5
4	`input.remove(current)`	`input` is now $[7]$
5	`output.append(current)`	`output` is now $[1,\ 5]$
6	`if input = []: break`	No effect, because `input` is not empty.

Line	Command	Result
3	`current ← min(input)`	`current` is now 7
4	`input.remove(current)`	`input` is now [ ]
5	`output.append(current)`	`output` is now $[1,\ 5,\ 7]$
6	`if input = []: break`	Exit the loop, because `input` is empty.
7	`display output`	Displays the list `output` which is $[1,\ 5,\ 7]$

a Show the step-by-step result of running the program on the input list $[5, 8, 2]$. Your results should be shown in a table like the one below.

Line	Result
1	`output` is [ ]
2	Begin loop
3	`current` is 2
4	`input` is $[5,\ 8]$
5	`output` is $[2]$
6	No action
3	`current` is 5
…	…

b Estimate how many execution steps would be required on an input list with 100 numbers in it.

c A 'bug' in a program or algorithm is a step that means the algorithm no longer gives the desired results. For each of the following bugs, explain the effect of it by first demonstrating what the algorithm will do with a sample input list like $[5, 8, 2]$.

- **i** Describe the effect of changing the algorithm so that it puts the current minimum at the *start* of the output list rather than at the end.
- **ii** Describe the effect of changing the algorithm so that it does not remove the current minimum from the input list.
- **iii** Describe the effect of changing the variable input in line 6 to output.

2.2 Searching in data

In a set of data, it is sometimes necessary to search through for a particular value.

a Consider the following algorithm to search for the number 100 in the input list.

```
Step 1. Look at the first item in the list.
Step 2. If it is 100 then output the value 'Found'.
Step 3. Look at the next item in the list.
Step 4. Repeat steps 2 and 3 until you get to the end of the list or until you find 100.
Step 5. Output 'Not Found' if you have not found the number 100.
```

- **i** If the input list is $[3, 9, 100, 2, 17, 52]$, describe what this algorithm does.
- **ii** If the input list is $[2, 6, 1, 7, 3, 5]$, describe what this algorithm does.
- **iii** What will be the output if the input list is $[100, 3, 50, 100, 75]$?

b With a large set of data, it is usually helpful to have the data sorted first in ascending order. For instance, a dictionary or a large phone list is sorted alphabetically to allow people to find values quickly. Page numbers in a book are also sorted, which can help you find a page easily. Describe how you could quickly find the page 381 in a book that was 1000 pages long.

c Here is an algorithm to find page 381 in a book.

Line	Command	Explanation
1	`lower ← 0`	The page must be greater than 0
2	`upper ← 1000`	The page is at most 1000
3	`loop:`	Performs lines 4 to 7 indefinitely (until you hit `break`)
4	`  page = round((lower + upper) / 2)`	Find the middle page in the range
5	`  if page = 381: break`	If we find the page, leave the loop
6	`  if page > 381: upper = page`	If we are too high then this becomes the new upper bound
7	`  if page < 381: lower = page`	If we are too low then this becomes the new lower bound
8	`display contents of page`	Outside the loop, display the contents of the page we found

i Construct a table like the one below to show how the bounds change each loop.

Loop number	lower	upper
0	0	1000
1	0	500
2	250	500
…	…	…

ii Describe how this algorithm performs compared to an algorithm that looks at each page from 1 to 381. You should include a comparison of roughly how many steps each takes.

d Emily has written an algorithm to search for page 210 in a 400 page book.

Line	Command
1	`lower ← 0`
2	`upper ← 400`
3	`loop:`
4	`  item = round((lower + upper) / 2)`
5	`  if page = 210: break`
6	`  if page < 210: upper = page`
7	`  if page > 210: lower = page`
8	`display contents of page`

i Construct a table like the one below to show how the bounds change each loop.

Loop number	lower	upper
0	0	400
…	…	…

ii Explain what the error is in Emily's algorithm and how to fix it.

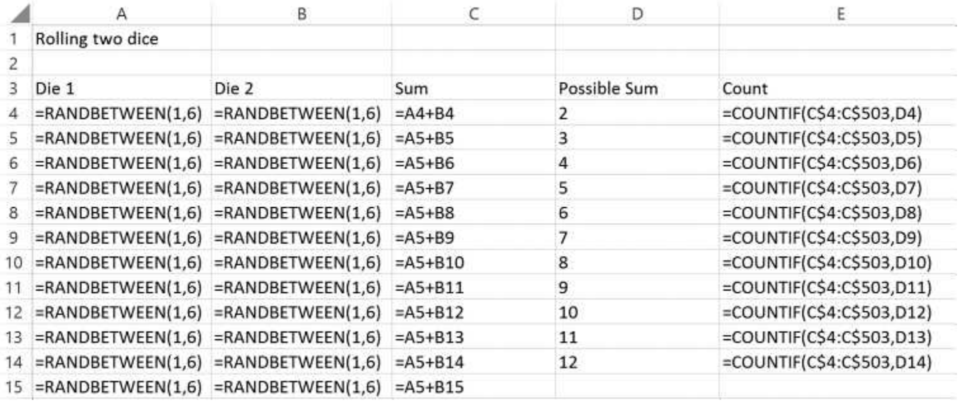
2.3 Generating lists of data (using simulation)

When investigating the probability of an event occurring, it is often helpful to use a simulation. For example, if two dice are rolled, we might want to know the probability of obtaining a sum of 10.
An algorithm to do this is:

Step 1. Roll two dice 500 times.
Step 2. For each pair of dice, find the sum of the faces.
Step 3. Count how many times the number 10 occurs.
Step 4. Divide this by 500 to estimate the probability.

Spreadsheets allow us to do this easily.

a Set up a table in a spreadsheet to randomly generate 500 outcomes for tossing two dice like the one below. Note the \$ signs are necessary for the formulas in column E.



	A	B	C	D	E
1	Rolling two dice				
2					
3	Die 1	Die 2	Sum	Possible Sum	Count
4	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A4+B4	2	=COUNTIF(C$4:C$503,D4)
5	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B5	3	=COUNTIF(C$4:C$503,D5)
6	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B6	4	=COUNTIF(C$4:C$503,D6)
7	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B7	5	=COUNTIF(C$4:C$503,D7)
8	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B8	6	=COUNTIF(C$4:C$503,D8)
9	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B9	7	=COUNTIF(C$4:C$503,D9)
10	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B10	8	=COUNTIF(C$4:C$503,D10)
11	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B11	9	=COUNTIF(C$4:C$503,D11)
12	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B12	10	=COUNTIF(C$4:C$503,D12)
13	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B13	11	=COUNTIF(C$4:C$503,D13)
14	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B14	12	=COUNTIF(C$4:C$503,D14)
15	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B15		

b Write out a frequency table of the different sums from 2 to 12.

c Comment on the number of times different sums occurred. For instance, which sums are most likely and which sums are least likely? Why do you think this is?

d Add a column in to calculate the experimental probability by dividing the count by the number of trials. To count the number of trials you could use a formula like = COUNT(A : A)

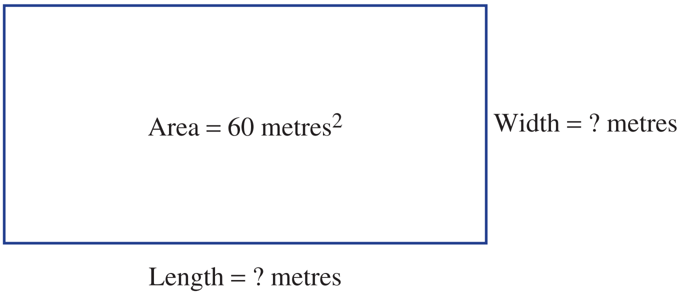
e Modify your spreadsheet to conduct a large simulation (1000 or more rows) of rolling three dice and noting the sums; it should show the experimental probability of each sum from 3 to 18. Use the spreadsheet software to generate a frequency column graph of these experimental probabilities and comment on what you notice about these probabilities.

Activity 3: Minimising and maximising in rectangles

Measurement and Geometry

3.1 Minimising perimeter

If a rectangle has an area of 60 metres^2, what is the smallest perimeter it can have?



One algorithm that could be used to solve this problem is:

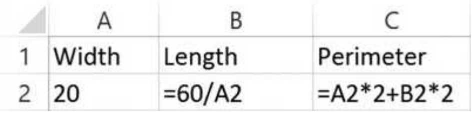
Step 1. Guess a value for the width.
Step 2. Calculate the length by dividing 60 by the width.
Step 3. Calculate the perimeter of this rectangle (2 × width + 2 × length).
Step 4. If this perimeter is smaller than the best answer found so far (or if this is the first time completing Step 4), make this the new 'best answer'.
Step 5. Reduce or increase the value for width by a small amount and repeat. When the perimeter cannot get smaller when you change the value of width in either direction, stop looping and give this as the minimum perimeter.

a Copy and complete the following table that reflects a few iterations of this algorithm.

Width	Length	Perimeter
20	3	46
10	6	
2		
5		
7.5		

Because a spreadsheet includes dynamic and interactive calculation abilities, it is a good tool to implement this algorithm.

b Fill out the following in a spreadsheet, using the initial guess of the width of 20. Change the width value to check your answer(s) in part **a**.



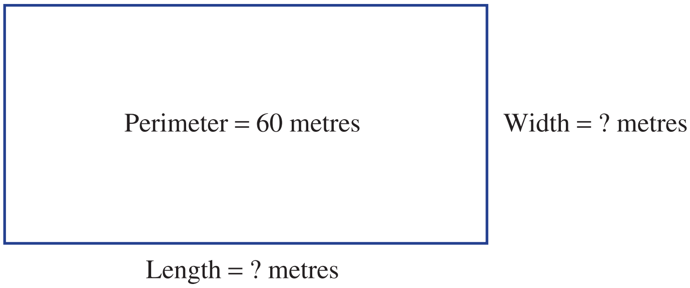
	A	B	C
1	Width	Length	Perimeter
2	20	=60/A2	=A2*2+B2*2

c Using trial and error, find the minimum perimeter. Draw a picture of this rectangle roughly to scale. (Remember you are allowed to have a width and length that are decimals; keep going until you get the minimum possible perimeter.)

d Repeat the procedure to find the minimum perimeter for a rectangle, if its area is 80 metres^2.

3.2 Maximising area

If a rectangle has a perimeter of 60 metres, what is the maximum area it can have?

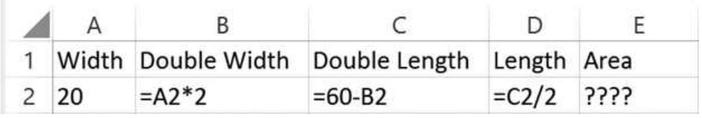


a. **i.** If the width is 20 metres, what is the length of the rectangle? Remember that the perimeter includes two lots of the length and two lots of the width.

  ii Describe an algorithm for calculating the length of the rectangle once its width is known.

  iii Describe an algorithm for calculating the area of the rectangle once its width is known.

b Set up a spreadsheet as follows. What is the formula that needs to go in E2 to calculate the rectangle's area?



	A	B	C	D	E
1	Width	Double Width	Double Length	Length	Area
2	20	=A2*2	=60-B2	=C2/2	????

c Using trial and error, find the maximum area of a rectangle with perimeter 60 metres. Draw a diagram of the rectangle that is formed.

d Repeat the procedure to find the maximum area of a rectangle if its perimeter is 80 metres.

A

Algorithm a procedure involving a number of steps to find the answer of a problem

B

Base the number or pronumeral that is being raised to a power

C

Capacity how much liquid a container can hold

Categories different types of groups

Centre of rotation fixed point about which a figure rotates

Circumference the curved boundary of a circle

Coefficient a numeral placed before a pronumeral to indicate that the pronumeral is to be multiplied by that factor

Commutative law when adding and multiplying, the order in which two numbers are combined does not matter

Complement a set containing the elements that are not in a given set

Composite number a number that has at least three factors

Congruent figures figures that are exactly the same size and shape

Coordinates numbers or letters used to give a location or position

Counting numbers the set of whole numbers starting at 1

D

Distributive law adding numbers *then* multiplying the total gives the same answer as multiplying each number first *then* adding the products

Dividend the number being divided

Divisor the number you are dividing by

E

Equation a mathematical statement that two expressions (numeric or algebraic) have the same value

Equivalent equal in value

Evaluate find (calculate) the numerical value of

Event a collection of possible outcomes

Expand remove grouping symbols

Experimental probability an estimate of an outcome's probability based on an experiment

Expression a group of mathematical terms containing no equal sign

F

Factor a whole number that will divide into another number exactly

Factor tree an illustrated breakdown of a number into its prime factors

Factorise to write an expression as a product

Formula a general rule for finding the value of one quantity given the values of others

Frequency table a table summarising data by showing all possible scores from lowest to highest in one column, and the frequency of each score in another column

G

Gradient a measure of slope

Grouping symbols parentheses $()$, brackets [ ] and braces $\{\}$ used to collect terms and operations together

GST Goods and Services Tax

H

Highest common factor (HCF) the largest number that is a factor of all the given numbers

Histogram a special type of column graph for quantitative data with no gaps between the columns

I

Image the result of a reflection

Index a number multiplied by itself one or more times

Index notation method of writing numbers that are multiplied by themselves

Integers the set of positive and negative whole numbers, including zero

K

Kite a quadrilateral with two pairs of adjacent sides equal

L

Like terms terms with the same pronumerals and same powers

Line of symmetry a line that divides a figure into two identical parts

Linear a linear graph in two dimensions is a straight line

Loss the amount of money lost by selling for less than the cost

Lowest common multiple (LCM) the smallest number that two or more numbers divide into evenly

M

Mean the average value of the scores

Median the middle score when the data is arranged in order

Mirror line the line over which a figure is reflected

Mode the most frequently occurring value in a set of data

Multiple the multiple of a number is the product of that number and any other whole number

N

Number plane a plane on which every point is related to a pair of numbers called coordinates

O

Outcome one of the possibilities from an event

Outlier any value that is much larger or much smaller than the rest of the data in a set

P

Parallel lines lines in the same plane that are the same distance apart and never intersect

Parallelogram a quadrilateral with both pairs of opposite sides parallel

Perimeter the total distance (length) around the outside of a figure

Perpendicular at right angles to another line or surface

Place value the value of a digit in a number as determined by its position

Polygon a two-dimensional shape where three or more straight lines are joined together to form a closed figure

Polyhedron a three-dimensional figure made by joining polygons at their edges

Population the entire group is selected

Prime number an integer greater than 1 that only has two factors, itself and 1

Prism a solid where each cross-section in a particular direction is exactly the same and all faces are polygons

Probability a measure of the likelihood that an event will occur

Product the result of multiplication

Profit the amount of money made by selling for more than the cost

Pronumeral a letter or a symbol used to represent a number

Pyramid a solid where the base is a polygon and the other faces are formed by triangles with a common vertex

Q

Quadrant a sector which is one quarter of a circle

Quadrilateral a four-sided plane figure with straight sides

Quotient the result of division

R

Radius the distance from the centre of a circle to its outside edge

Range the difference between the highest score and the lowest score

Rate the number of units of one quantity for each single unit of another quantity

Ratio a way of comparing quantities of the same kind that are measured in the same units

Reciprocals two values or terms that multiply to give 1

Recurring decimal a decimal in which a single digit, or a group of digits, is endlessly repeated

Reflection the result of flipping a geometrical figure across a line

Regular polygon a polygon with all sides equal and all angles equal

Remainder the amount left over after division, when one number cannot be divided exactly into another

Rhombus a quadrilateral with both pairs of opposite sides parallel and all sides equal

Rotation a turn around a centre point or axis

Rule an equation connecting two or more variables

S

Sample a small group randomly selected out of the population

Sample space the list of all the possible outcomes of an event

Scale a ratio that compares a drawing or a model to the real object

Scale factor the number you multiply each side length by to enlarge or reduce a shape

Semicircle half a circle

Simplify find the simplest possible expression for

Solution the value/s that give a true statement when substituted for the unknown in an equation

Speed a measure of how fast an object is moving

Square to multiply a number by itself

Square root the opposite operation of squaring

Substitute replace pronumerals with numerical values

Survey a set of questions that are asked of the people in a sample

T

Tally marks line strokes used to record data, made in groups of 5

Terminating decimal a decimal that contains a fixed number of digits

Time zone any geographic region of the world in which the same standard time is kept

Translation movement of an object a certain distance in a given direction

Transversal a line that cuts two or more lines

Trapezium a quadrilateral with at least one pair of parallel sides

Trial one run of an experiment

Two-way table a way of listing the number of outcomes in different categories

U

Unitary method calculating the value of one unit of an item and then using this to calculate the value of a number of items

V

Vector a pair of numbers used to describe a translation

Venn diagram a diagram using circles to show the relationships between two or more sets

Volume the amount of three-dimensional space in an object

Answers

Chapter 1

Pre-test

1 **a** A **b** S **c** A
d S **e** M **f** D
g M **h** D **i** A
2 **a** 19 **b** 69 **c** 73
d 359 **e** 57 **f** 162
3 **a** 4 **b** 22 **c** 18
d 0 **e** 621 **f** 47
4 **a** 36 **b** 40 **c** 132
d 75 **e** 1089 **f** 4732
5 **a** 7 **b** 33 **c** 3
d 6 **e** 151 **f** 52
6 **a** 6, 12, 18, 24, 30 **b** 9, 18, 27, 36 **c** 18
7 **a** 1, 2, 3, 4, 6, 12 **b** 1, 3, 5, 15 **c** 3
8 2, 3, 5, 7, 11, 13
9 **a** T **b** T **c** T
d F **e** T **f** F
10 **a** 4 **b** 9, 3 **c** 16, 4 **d** 36, 6
e 81, 81 **f** 100, 100 **g** 7, 7, 7 **h** 12, 12, 12
11 **a** 0, – 1 **b** –4, – 6 **c** –12, – 13
12 **a** –3 **b** –3 **c** 2 **d** 5

Exercise 1A

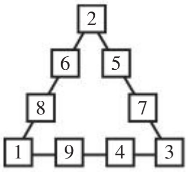
1 **a** III **b** II **c** I **d** IV
2 **a** 26 + 17 **b** 43 – 9 **c** 134 – 23 **d** 451 + 50
e 19 + 29 **f** 111 + 236 **g** 59 – 43 **h** 339 – 298
i 8 + 36 **j** 421 + 142 **k** 49 – 32 **l** 251 – 120
3 **a**

+	2	5	7	10	12
5	7	10	12	15	17
0	2	5	7	10	12
18	20	23	25	28	30
58	60	63	65	68	70

b

+	3	9	18	15
15	18	24	33	30
1	4	10	19	16
6	9	15	24	21
2	5	11	20	17

4 **a** T **b** F **c** T
d T **e** T **f** F
5 **a** 26 **b** 17 **c** 30 **d** 300
e 46 **f** 35 **g** 26 **h** 24
6 **a** 3 **b** 12 **c** 2 **d** 0
e 13 **f** 32 **g** 40 **h** 38
7 **a** 32 **b** 387 **c** 1143 **d** 55
e 163 **f** 216 **g** 79 **h** 391
i 701 **j** 229 **k** 39 **l** 161
8 **a** 174 **b** 431 **c** 10 362 **d** 2579
e 58 **f** 217 **g** 27 **h** 13 744
i 878 **j** 23 021 **k** 75 **l** 9088
9 678 km
10 David has $436 and Kristian has $738
11 24 km
12 22
13 **a** **i** 2 **ii** 7 **iii** 0 **iv** 6
v 9 **vi** 9 **vii** 0 **viii** 6
b Answers given from top row down and from left to right.
i 7,3,3 **ii** 1,7,8 **iii** 2,5,3
iv 9,4 **v** 4,2,8 **vi** 0,0,7,1
c **i**



ii 5 totals, 17, 19, 20, 21 and 23

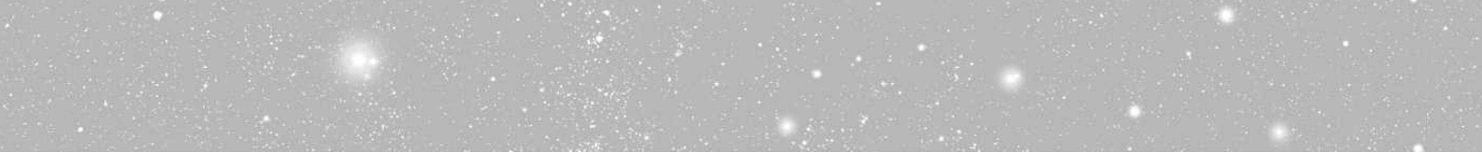
Exercise 1B

1 **a** III **b** V **c** I **d** II **e** IV
2 **a**

×	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7
2	2	4	6	8	10	12	14
3	3	6	9	12	15	18	21
4	4	8	12	16	20	24	28
5	5	10	15	20	25	30	35
6	6	12	18	24	30	36	42
7	7	14	21	28	35	42	49

b

×	2	5	7	9
3	6	15	21	27
4	8	20	28	36
9	18	45	63	81
10	20	50	70	90



3 **a** 40 **b** 99 **c** 42 **d** 72
e 66 **f** 132 **g** 32 **h** 63
i 10 **j** 11 **k** 11 **l** 12
m 8 **n** 11 **o** 13 **p** 13
4 **a** T **b** T **c** F **d** T
e F **f** T **g** T **h** T
5 **a** 45 **b** 72 **c** 60 **d** 140
e 64 **f** 693 **g** 237 **h** 210
i 130 **j** 260 **k** 140 **l** 68
m 17 000 **n** 13 600 **o** 413 **p** 714
q 459 **r** 366 **s** 1008 **t** 5988
6 **a** 32 **b** 16 **c** 160 **d** 123
e 37 **f** 198 **g** 16 **h** 63
i 41 **j** 127 **k** 16 **l** 127
7 **a** 603 **b** 516 **c** 3822 **d** 90 360
e 9660 **f** 1152 **g** 1392 **h** 8476
8 **a** 28 rem 1 or $28\frac{1}{3}$ **b** 30 rem 4 or $30\frac{4}{7}$
c 416 rem 7 or $416\frac{7}{10}$ **d** 13 rem 12 or $13\frac{12}{15} = 13\frac{4}{5}$
e 2514 **f** 412
g 210 **h** 741
9 $27.50 an hour
10 131 boxes; 1572 packets
11 Option B by $88
12 125 loads
13 $18 824
14 **a** 45 **b** 47
c 4 adults and 6 kids = 10 tickets

Exercise 1C

1 **a** 3×9 **b** $2 \div 2$ **c** 1×3
d $(9 - 6)$ **e** $(12 + 6)$
2 **a** I **b** III **c** V
d II **e** IV
3 **a** 22 **b** 6 **c** 26 **d** 3
e 28 **f** 14 **g** 2 **h** 6
i 160 **j** 22 **k** 4 **l** 14
m 25 **n** 50 **o** 48 **p** 63
q 95 **r** 45
4 **a** $2 \times (3 + 7) = 20$ **b** $2 \times (24 \div 8) = 6$ **c** $5 \times 7 + 4 = 39$
d $12 \times 5 + 8 = 68$ **e** $66 \div 3 - 10 = 12$ **f** $3 \times (18 - 12) = 18$
5 **a** 6 **b** 30 **c** 19 **d** 63
e 66 **f** 23 **g** 31 **h** 1
i 21
6 **a** 52 **b** 28 **c** 280 **d** 24
e 1 **f** 209 **g** 14 **h** 70
i 23
7 **a** 32 **b** 42 **c** 122 **d** 360
8 **a** T **b** F **c** T **d** T
e T **f** T
9 **a** $(12 - 8) \times 2$ **b** $4 \times (5 + 6)$
c $16 \div (2 \times 8)$ **d** $6 \times (2 + 6) \times 1$
10 **a** $5 + 4 - 9$ **b** $5 \times 4 - 9$ **c** $5 + 4 \times 9$
11 **a** $(4 + 7) \times 12 = \$132$
b $5000 + 6 \times 500 = \$8000$
c $50 - (4 \times 2 + 8 \times 3) = \$18$
12 **a** no **b** no **c** yes **d** yes **e** no

13 Some suggestions include:

1	$(3 + 2 - 4) \div 1$
2	$1 + 2 + 3 - 4$
3	$(1 + 2 \times 3) - 4$
4	$(1 + 2) \div 3 \times 4$
5	$(1 + 2) \div 3 + 4$
6	Change of order gives $4 \times 3 \div 2 \times 1$
7	$(4 + 3) \times (2 - 1)$
8	$(1 + 3) \times 4 \div 2$
9	$1 \times 2 + 3 + 4$
10	$1 + 2 + 3 + 4$

Exercise 1D

1 **a** 2^2 **b** 4^2 **c** 5^2
d 5^3 **e** 6^4 **f** 7^3
2 **a** IV **b** V **c** VI
d III **e** II **f** I
3 $3 \times 3 = 9$
$4 \times 4 = 16$
$5 \times 5 = 25$
$6 \times 6 = 36$
$7 \times 7 = 49$
$8 \times 8 = 64$
$9 \times 9 = 81$
$10 \times 10 = 100$
4 $3 \times 3 \times 3 = 27$
$4 \times 4 \times 4 = 64$
$5 \times 5 \times 5 = 125$
$6 \times 6 \times 6 = 216$
5 **a** 7^3 **b** 10^4 **c** 8^2
d 4^3 **e** 2^7 **f** 6^7
g 12^2 **h** 5^6 **i** 6^1
6 **a** $8 \times 8 \times 8 \times 8 \times 8$ **b** $3 \times 3 \times 3 \times 3$
c 9×9 **d** $4 \times 4 \times 4 \times 4$
e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **f** 11×11
7 **a** 8 **b** 16 **c** 27
d 10000 **e** 125 **f** 1
8 **a** 16 **b** 100 **c** 169 **d** 225
e 10000 **f** 400 **g** 5 **h** 7
i 11 **j** 30 **k** 40 **l** 16
9 **a** 8 **b** 64 **c** 343 **d** 125
e 216 **f** 1000 **g** 3 **h** 2
i 5 **j** 8 **k** 9 **l** 100
10 **a** 3^2 **b** 2^4 **c** 2^5
11 **a** 13 **b** 15 **c** 625 **d** 9 **e** 1331
12 **a** $6^2 \times 7^4$ **b** $5^4 \times 2^2$ **c** $3^2 \times 8^2$
d $9^4 \times 11$ **e** $4^3 \times 12^2$ **f** $2^6 \times 3^3$
13 **a** m^3 **b** a^5 **c** n^7 **d** p^{10}
e p^3q^2 **f** a^4b^2 **g** a^2b^4 **h** x^4y

Exercise 1E

1 A **2** C
3 **a** 6^2 **b** 7^0 **c** $5^6 \times 5^8$ **d** $6^9 \div 6^3$
4 C
5 **a** 6 **b** 3 **c** 9 **d** 7
e 13 **f** 6 **g** 6 **h** 3
i 4 **j** 3 **k** 6 **l** 5

6 a 3^6 b 2^5 c 10^4
d 9^{10} e 4^5 f 2^{12}
g 8^{10} h 12^{10} i 16^8
7 a 3^2 b 2^2 c 9^4
d 4^3 e 17^6 f 11^6
8 a 12 b 10 c 4
d 12 e 6 f 20
9 a 7^4 b 2^{20} c 3^{14} d 8^8
e 3^8 f 10^{30} g 9^{14} h 5^{15}
10 a 1 b 1 c 1 d 1
e 1 f 8 g 7 h 10
i 1 j 2 k 6 l 3
11 a 2^9 b 5^6 c $6^2 = 36$
d the same e $144 = 12^2$
12 a 2^8 b 2^{13} c 10^3
d 7^{14} e f 3^9
13 a a^{11} b m^7 c a^9 d x^{13}
e n^{11} f m^{14} g n^6 h a^3
i m^2 j a^{12} $6^1 = 6$ k w^9 l p^4
14 a $5m^7$ b $24m^8$ c $16m^{10}$
d $12a^9$ e $21x^7$ f $20x^{12}$

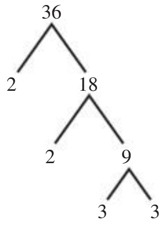
Exercise 1F

1 a 1, 2, 4 b 1, 2, 3, 6 c
d 1, 3, 5, 15 e
2 a 10 b 15 c 30
d 28 e 24 f 55
3 a 8 b 6 c 6 1, 2, 3, 4, 6, 12 d 8
e 6 f 8 1, 2, 4, 5, 10, 20 g 2 h 2
4 a 4 b 12 c 6
d 12 e 20 f 30
5 a prime (P) b composite (C)
c C d C e C f C
g C h P i P j P
k C l C
6 2,3,5,7,11,13,17,19,23,29
7 a 6 b 45 c 24 d 8
e 50 f 36 g 120 h 60
i 35 j 30 k 12 l 36
8 a 2 b 9 c 8 d 6
e 1 f 1 g 36 h 4
i 2 j 6 k 8 l 5
9 a 24 b 105 c 5 d 4
10 4 ways
11 30 minutes
12 25:2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
13 $10 = 5 + 5$ $12 = 5 + 7$ $14 = 7 + 7$
$16 = 5 + 11$ $18 = 5 + 13$ $20 = 3 + 17$
$22 = 5 + 17$ $24 = 5 + 19$ $26 = 7 + 19$
$28 = 5 + 23$ $30 = 7 + 23$
14 (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)

Exercise 1G

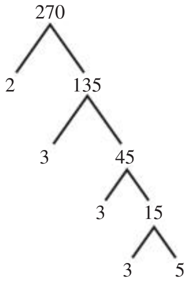
1 a 1, 3, 5, 15
b 1, 2, 3, 4, 6, 8, 12, 24
c 1, 2, 4, 5, 8, 10, 20, 40
d 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
2 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
3 a 3^4 b 5^2 c 7^4
d $2^2 \times 3^3$ e $2^2 \times 5^2$ f

4 a 12 b 90 c 280 d 189
5 a



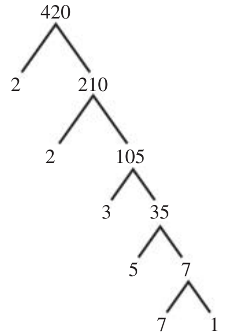
$\therefore 36 = 2^2 \times 3^2$

b

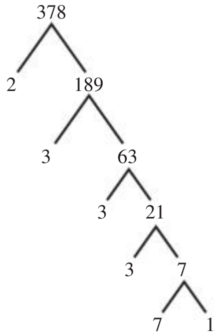


$\therefore 270 = 2 \times 3^3 \times 5$

c



d



$420 = 2^2 \times 3 \times 5 \times 7$ $378 = 2 \times 3^3 \times 7$

6 a $2^2 \times 5$ b $2^2 \times 7$ c $2^3 \times 5$
d $2 \times 3^2 \times 5$ e $2^3 \times 5 \times 7$ f $2^2 \times 7^2$
g $2^3 \times 3^2 \times 5$ h $2^2 \times 3 \times 5 \times 11$
7 a divisible by 3 b divisible by 2, 3, 6, 9
c divisible by 2, 4, 8 d divisible by 3, 9
e divide by 3, 5, 9 f none
g 2, 3, 6 h none
8 a LCM $= 2^4 \times 3 \times 5$ HCF $= 2 \times 3 = 6$
b LCM $= 2^2 \times 3^2 \times 5^3$ HCF $= 2 \times 5^2 = 50$
c LCM $= 2^5 \times 3^3$ HCF $= 2 \times 3 = 6$
d LCM $= 2 \times 5^2 \times 7^2$ HCF $= 5 \times 7 = 35$
e LCM $= 2 \times 3^2 \times 7 \times 11$ HCF $= 3^2 \times 11 = 99$
9 a 5 b 3 c 2 d 7
10 a 60, 2 b 28, 14 c 120, 3 d 60, 3
e 140, 4 f 390, 1 g 126, 3 h 3780, 30
11 a 12 b 72 c 30
12 72 days
13 a 1, 4, 7 b 0, 9 c 2, 5, 8
d 2 e 0, 2, 4, 6, 8, f 0

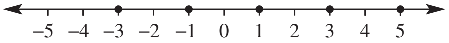
Progress Quiz – Chapter 1 – Integers

1 a 93 b 434 c 121 d 521
2 a 1223 b 481 c 135 d 7175
3 a 360 b 316 c 114 d 150
4 a 2232 b 310
5 a 30 b 16 c 26 d 11
6 a 7^4 b $5^2 \times 2^3$ c 1^8
7 a 25 b 64 c 10 d 3
8 a 8^9 b 3^7 c 2^6 d 21^{10}
9 a 5^6 b 2^{20} c 1 d 4
10 a 12 b 45
11 a 5 b 1 c 24
12 $2^4 \times 3 \times 5$
13 72 is divisible by 2, 3, 4, 6, 8 and 9. 72 is not divisible by 5
14 LCM = 440, HCF = 10

Exercise 1H

1 a 2 **b** 5 **c** –3 **d** –10 **e** –1

2



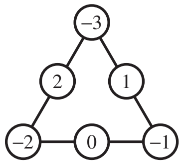
3 a > **b** < **c** < **d** >
e > **f** < **g** > **h** <
4 a –2°C **b** –1°C **c** –9°C **d** 3°C
5 a 1 **b** 4 **c** 1 **d** 8
e 15 **f** –2 **g** –5 **h** –7
i –7 **j** –14 **k** 5 **l** 7
m 4 **n** 0 **o** 3 **p** –5
6 a –1 **b** –5 **c** –26 **d** –17
e –2 **f** 1 **g** –11 **h** –31
i –29 **j** –110 **k** –17 **l** –12
m –20 **n** –9 **o** 4 **p** –12
7 a –3 **b** –15 **c** 0 **d** –24
e –14 **f** –7 **g** 1 **h** –1
i –19 **j** 1 **k** –6 **l** –10
m –1 **n** –100 **o** 20 **p** 31
8 a –1 **b** 8 **c** –7 **d** –1
9 a 1 – 4 = –3 **b** –9 + 3 = –6
c –1 + 5 = 4 **d** –15 – 5 = –20
10 a 6 **b** –4 **c** –14 **d** 11
e 15 **f** 5 **g** –3 **h** 12
11 4 floors below ground level (–4)
12 $7
13 a

+	–	Balance
		500
	375	125
80		205
100		305
	90	215
	45	170
	49	121
	25	96

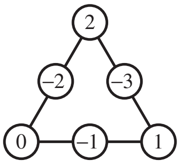
b
14 a +6 **b** +8 **c** –16
d –3 **e** +2 **f** +18

Exercise 1I

1 a 6 **b** –10 **c** –38 **d** 46
e 32 **f** –88 **g** –673 **h** 349
2 a subtract **b** add
3 a F **b** T **c** T
d F **e** T **f** F
g F **h** T **i** F
4 a 7 – 3 **b** 10 – 5 **c** 8 + 1
d 6 + 8 **e** 15 – 20 **f** –3 + 4
g –9 + 9 **h** 0 – 5 **i** 18 – 18
5 a 4 **b** 3 **c** –5 **d** 15
e –2 **f** –14 **g** –9 **h** –21
i –38 **j** –86 **k** –105 **l** –130
m –1 **n** –21 **o** –6 **p** –18
q –16 **r** 0 **s** –85 **t** –106
6 a 5 **b** 8 **c** 21 **d** 38
e 72 **f** 467 **g** –2 **h** 2
i 3 **j** 32 **k** –57 **l** –120
m 65 **n** –55 **o** 0 **p** 8
q 82 **r** 100 **s** –38 **t** –158
7 a –4 **b** –4 **c** –3
d –3 **e** –2 **f** –2
g 66 **h** 66 **i** 0
8 a –3 **b** –6 **c** 1 **d** 10
e 2 **f** –14 **g** –2 **h** –4
i –30 **j** –5 **k** –6 **l** 65
9 32°
10 $180 000
11 a 6 **b** 7 **c** –18
12 a –8 **b** –3 **c** 1
d –8 **e** –2 **f** 2
13 a



b



14 a

–1	–6	1
0	–2	–4
–5	2	–3

b

–12	–19	–14
–17	–15	–13
–16	–11	–18

Exercise 1J

1 a

□	△	□×△
3	5	15
2	5	10
1	5	5
0	5	0
–1	5	–5
–2	5	–10
–3	5	–15

b

□	△	□×△
3	–5	–15
2	–5	–10
1	–5	–5
0	–5	0
–1	–5	5
–2	–5	10
–3	–5	15

2 a 15, 3 **b** –15, –3 **c** –15, –3 **d** 15, –3
3 a + **b** – **c** –
d + **e** + **f** –
g + **h** – **i** +

4 **a** + **b** – **c** –
d – **e** – **f** +
5 **a** –20 **b** –54 **c** –40 **d** –99
e 6 **f** –42 **g** –72 **h** 99
i –40 **j** –64 **k** 35 **l** –32
m 60 **n** –44 **o** 9 **p** –60
6 **a** –5 **b** –2 **c** –4 **d** –30
e –2 **f** –3 **g** –3 **h** 3
i –2 **j** 8 **k** –9 **l** 5
m 11 **n** 1 **o** –8 **p** 8
7 **a** –20 **b** 21 **c** –12 **d** –23
e 50 **f** –9 **g** –15 **h** 39
i –18 **j** 8
8 **a** 25 **b** 36 **c** 49
d 64 **e** 81 **f** 100
9 **a** –3 **b** –5 **c** 7
d 6 **e** –3 **f** –72
g –252 **h** –5 **i** –30
10 negative
11 **a** ×, ÷ **b** ×, ÷ **c** ÷, × **d** ÷, ÷
12 –8 and 3
13 **a** –1 **b** –3 **c** –5 **d** 3
e –6 **f** 7 **g** 0 **h** –5
14 **a** –15 **b** –15 **c** 2 **d** –8
e 8 **f** 1 **g** –4 **h** 10
15 **a** 22 **b** 4 **c** 28 **d** 122
e –32 **f** –16 **g** 152 **h** 16
16 **a** –15 **b** 5 **c** 16 **d** 14
e 9 **f** 28 **g** –1 **h** 0
i –12 **j** 19 **k** 7 **l** 37
17 **a** $(-2+1)\times 3=-3$ **b** $-10\div(3-(-2))=-2$
c $-8\div(-1+5)=-2$ **d** $(-1-4)\times(2+(-3))=5$
e $(-4+-2)\div(10+(-7))=-2$ **f** $20+(2-8)\times(-3)=38$
g $(1-(-7)\times 3)\times 2=44$ **h** $(4+-5\div 5)\times(-2)=-6$

Maths@work: Retailer of loungeroom furniture

1 **a** $2090 **b** $2349 **c** $1849 **d** $1999 **e** $3880

2

Model	Savings
Recliner 3 piece	$2249
Corner suite	$1300
2.5 seat leather chaise	$296
Outdoor sofa	$225
Occasional chair	$220

3 **a**

Model	Difference in price
ER 2 + chaise	$600
DW 3450 R	$460
Ebony 3 + 2	$896
Recliner and console 3	$350
Victa EL +1	$698

b $600.80

4 Custom Sofas is cheaper by $10.

5a–d

March order		
Catalogue item number	Price in leather	Cost of furniture
021-A	$3574	$5718
021-D	$1738	$5213
021-G	$2738	$2190
054-B	$4374	$7873
054-F	$3344	$6688
079-L	$685	$2740
079-M	$1374	$3572
079-R	$1954	$3517
Total cost of order		$37 509

June order		
Catalogue item number	Price in leather	Cost of furniture
021-A	$3574	$3574
021-D	$1738	$4865
021-G	$2738	$2738
054-B	$4374	$11 372
054-F	$3344	$3344
079-L	$685	$2466
079-M	$1374	$2748
079-R	$1954	$4689
Total cost of order		$35 794

Puzzles and games

1 Answers will vary.
2 Answers will vary.
3 Answers will vary.

Multiple-choice questions

1 B **2** D **3** C **4** B **5** A
6 B **7** D **8** A **9** C **10** C

Short-answer questions

1 **a** 497 **b** 412 **c** 129 **d** 67
e 112 **f** 139 **g** 1999 **h** 5675
2 **a** 539 **b** 2030 **c** 61 **d** 3074
3 **a** 170 **b** 297 **c** 336 **d** 423
e 41 **f** 119 **g** 103 **h** 201
4 **a** 1668 **b** 21 294 **c** 281 **d** 122
5 **a** 3 **b** 1 **c** 1 **d** 7
6 **a** 6^3 **b** 8^4 **c** $2^2\times 5^4$
7 **a** 9 **b** 11 **c** 49 **d** 400
e 3 **g** 125
f 4 **h** 1000

8 **a** 4^{11} **b** 3^2 **c** 1 **d** 3^{20}
9 **a** 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 **b** 112, 119, 126, 133, 140, 147
c 31, 37, 41, 43, 47, 53, 59 **d** 24
e 6
10 **a** $2^2 \times 3^2$ **b** $2^2 \times 3 \times 7$ **c** $2 \times 3^2 \times 11$
11 **a** divisible by 2, 3, 4, 6 **b** divisible by 5
c divisible by 2, 4 **d** divisible by 3, 9
12 **a** 380 **b** 2
13 **a** 3 **b** –5 **c** –8 **d** –31
e –76 **f** –330 **g** –1 **h** 98
14 **a** 2 **b** –8 **c** –64 **d** –39
e 16 **f** 12 **g** –20 **h** 92
15 **a** –10 **b** 88 **c** –63 **d** 200
e 2 **f** –3 **g** –4 **h** 3
16 **a** –4 **b** –1 **c** –8 **d** 26
17 **a** –11 **b** 1 **c** 7 **d** –4
18 **a** 1 **b** 1 **c** 4
d 4 **e** 9 **f** 9

Extended-response questions

1 **a** a = \$112, b = –\$208, c = \$323, d = –\$275, e = \$240
b \$228 **c** \$160
2 **a** 72
b $30 = 2 \times 3 \times 5, 42 = 2 \times 3 \times 7$
c LCM = 210, HCF = 6
d 6

Chapter 2

Pre-test

1 **a** B: segment AB **b** C: point A
c D: angle ABC **d** A: line AB
2 **a** C: $\angle ABC$ **b** A: $\angle DEF$ **c** B: $\angle STU$
3 **a** F: revolution **b** B: right **c** A: acute
d C: obtuse **e** D: straight **f** E: reflex
4 **a** F: obtuse **b** B: isosceles **c** D: acute
d A: scalene **e** C: equilateral **f** E: right
5 B, C, F, G, I, J
6 **a** 60 **b** 130 **c** 140
7 **a** 130 **b** (c, e, g) **c** (b, d, f)
8 **a** 40 **b** 110

Exercise 2A

1 **a** complementary **b** supplementary
c perpendicular **d** equal
2 **a** acute **b** reflex **c** straight
d right **e** revolution **f** obtuse
3 **a** complementary **b** supplementary **c** revolution
4 **a** 40° **b** 110° **c** 220°
5 **a** $\angle AOB$ **b** $\angle BOA$ (or $\angle DOE$)
c $\angle AOB$ (or $\angle EOD$)
6 **a** 45 **b** 130 **c** 120
d 240 **e** 90 **f** 180
7 **a** $a = 70, b = 270$ **b** $a = 25, b = 90$ **c** $a = 128, b = 52$
d $a = 34, b = 146$ **e** $a = 25$ **f** $a = 40$
g $a = 120$ **h** $a = 50, b = 90$ **i** $a = 140$

8 **a** 270° **b** 90° **c** 0° (or 360°) **d** 180°
e 315° **f** 135° **g** 225° **h** 45°
9 **a** 40° **b** 72° **c** 120° **d** 200°
10 **a** S **b** N **c** W **d** E
e NE **f** NW **g** SW **h** SE
11 **a** 60 **b** 135
c 35 **d** $a = 110, b = 70$
e $a = 148$ **f** $a = 90, b = 41, c = 139$
12 **a** Supplementary angles should add to 180°.
b Angles in a revolution should add to 360°.
c Angles on straight line should add to 180°.
13 **a** **i** 180° **ii** 360° **iii** 30° **iv** 90°
b **i** 360° **ii** 180° **iii** 30° **iv** 120°
14 **a** 105° **b** 97.5° **c** 170° **d** 170°
e 132.5° **f** 27.5° **g** 144° **h** 151.5°

Exercise 2B

1 **a** equal **b** supplementary **c** equal
2 **a** $\angle BCH$ **b** $\angle ABE$ **c** $\angle GCB$ **d** $\angle BCH$
e $\angle FBC$ **f** $\angle GCB$ **g** $\angle FBC$ **h** $\angle DCG$
3 **a** alternate **b** alternate
c co-interior **d** corresponding
e corresponding **f** co-interior
4 **a** 80 (corresponding) **b** 120 (corresponding)
c 131 (corresponding) **d** 82 (alternate)
e 118 (alternate) **f** 78 (alternate)
g 100 (co-interior) **h** 129 (co-interior)
i 39 (co-interior)
5 **a** $a = 58, b = 58$ (both co-interior to 122°)
b $a = 141, b = 141$ (both co-interior to 39°)
c $a = 100$ (co-interior to 80°), $b = 80$ (co-interior to a°)
d $a = 62$ (co-interior to 118°), $b = 119$ (co-interior to 61°)
e $a = 105$ (co-interior to 75°), $b = 64$ (corresponding to 64°)
f $a = 25$ (alternate to 25°), $b = 30$ (alternate to 30°)
6 All reasons assume that lines are parallel.
a $a = 110$ (corresponding to 110°), $b = 70$ (supplementary to a°)
b $a = 120$ (alternate to 120°), $b = 60$ (co-interior to a°), $c = 120$ (corresponding to 120°)
c $a = 74$ (alternate to 74°), $b = 106$ (co-interior to 74°), $c = 106$ (supplementary to a°)
d $a = 100$ (supplementary to 80°), $b = 100$ (co-interior to 80°)
e $a = 95$ (corresponding to 95°), $b = 85$ (supplementary to a°)
f $a = 40$ (alternate to 40°), $b = 140$ (co-interior to 40°)
7 **a** No, the alternate angles are not equal.
b Yes, the co-interior angles are supplementary.
c No, the corresponding angles are not equal.
8 **a** 250 **b** 320 **c** 52
d 40 **e** 31 **f** 63
9 **a** 130° **b** 95° **c** 90°
10 $a = 36, b = 276, c = 155, d = 85, e = 130, f = 155, g = 15$

Exercise 2C

1 **a** right-angled triangle **b** isosceles triangle
c acute-angled triangle **d** equilateral triangle
e obtuse-angled triangle **f** equilateral triangle
g isosceles triangle **h** scalene triangle
2 **a** scalene **b** isosceles **c** isosceles
d equilateral **e** scalene **f** isosceles
3 **a** right **b** obtuse **c** acute
4 **a** 80 **b** 40 **c** 58
d 19 **e** 34 **f** 36

5 **a** 68 **b** 106 **c** 20
d 65 **e** 40 **f** 76
6 **a** 150 **b** 80 **c** 160
d 50 **e** 140 **f** 55
7 **a** yes **b** no **c** yes
d yes **e** yes **f** yes
8 **a** Isosceles, the two radii are of equal length.
b $\angle OAB, \angle OBA$ **c** 30°
d 108° **e** 40°
9 **a** 60 **b** 231 **c** 18
d 91 **e** 65 **f** 60
10 **a** 55 **b** 60 **c** 25
11 **a** **i** a, alternate angles in parallel lines
ii c, alternate angles in parallel lines
b They add to 180°, they are on a straight line.
c $a + b + c = 180$, angles in a triangle add to 180°.

Exercise 2D

1 square, rectangle, parallelogram, rhombus, kite, trapezium
2 **a** 360° **b** equal **c** 2 **d** 90°
3 **a** **i** T **ii** F **iii** F **iv** T
b **i** F **ii** T **iii** F **iv** T
c **i** F **ii** T **iii** T
d **i** T **ii** F **iii** F
e **i** T **ii** F **iii** T
f **i** F **ii** F
4 **a** 90 **b** 61 **c** 105
d 170 **e** 70 **f** 70
5 **a** $a = 104, b = 76$ **b** $a = 72, b = 72$ **c** $a = 128$
d $a = 50, b = 130$ **e** $a = 54, b = 54$ **f** $a = 138, b = 42$
6 **a** square, rhombus **b** trapezium
c rectangle, parallelogram, kite **d** square, rhombus, kite
e square, rectangle
7 **a** 152 **b** 69 **c** 145
d 74 **e** 59 **f** 30
8 **a** T **b** F **c** T
d T **e** F **f** T
9 **a** $a = 100, b = 3, c = 110$ **b** $a = 2, b = 90$
c $a = 5, b = 70$
10 **a–c** Answers will vary. **d** Angle sum = 360°

Progress Quiz – Chapter 2 – Lines, shapes and solids

1 **a** 65 **b** 332
c $m = 66, n = 114$ **d** $x = 45$
2 **a** $a = 45, b = 30$ **b** $t = 60$
3 b
4 **a** $a = 58$ (alternate) **b** $c = 117$ (co-interior)
c $b = 141$ (corresponding)
5 **a** $a = 119$ (corresponding), $b = 119$ (vertically opposite), $c = 119$ (corresponding to a, alternate to b), $d = 61$ (co-interior to b, or supplementary to c)
b $a = 112$ (co-interior), $b = 68$ (co-interior to a)
6 **a** $x = 27$ **b** $x = 41$
7 **a** $a = 40$ **b** $a = 145, b = 110$
8 **a** $c = 73$ **b** $x = 256$
9 **a** $a = 105, b = 75, c = 105$ **b** $a = 70, b = 61, c = 139$
10 **a** T **b** F **c** F

Exercise 2E

1 **a** heptagon **b** triangle **c** octagon
d nonagon **e** dodecagon **f** decagon
g quadrilateral **h** undecagon
2 **a** 6 **b** 4 **c** 10
d 7 **e** 5 **f** 12
3 **a** 720° **b** 1440° **c** 3600°
4 **a** square **b** equilateral triangle
5 **a** 540° **b** 1080° **c** 1440°
d 720° **e** 1260° **f** 900°
6 **a** 130 **b** 80 **c** 120
d 130 **e** 155 **f** 105
7 **a** 108° **b** 144° **c** 135°
8 **a** 108° **b** 128.6° **c** 120°
d 144° **e** 135° **f** 147.3°
9 **a** 115 **b** 135 **c** 20
d 250 **e** 40 **f** 265
10 **a** 9 **b** 15 **c** 21 **d** 167
11 **a** 6 **b** 20 **c** 11
12 **a** circle **b** Increases to infinity. **c** 180°
13 **a** 127.5 **b** 240 **c** 60
d 60 **e** 79 **f** 72

Exercise 2F

1 **a** six **b** circle **c** cube
d vertices **e** seven **f** congruent
g seven **h** octagonal
2 cylinder, sphere, cone
3 **a** 6, 8, 12 **b** 5, 6, 9 **c** 7, 7, 12
4 A, cube; B, pyramid; F, rectangular prism; G, tetrahedron; H, hexahedron
5 **a** hexahedron **b** tetrahedron **c** pentahedron
d heptahedron **e** nonahedron **f** decahedron
g undecahedron **h** dodecahedron
6 **a** 8 **b** 6 **c** 4 **d** 5
e 7 **f** 9 **g** 10 **h** 11
7 **a** triangular prism **b** pentagonal prism
c square prism
8 **a** rectangular pyramid **b** heptagonal pyramid
c triangular pyramid
9 **a** pentahedron, triangular prism
b octagonal prism, decahedron
c square pyramid, pentahedron
10 **a** T **b** F **c** T **d** T
e F (sphere) **f** T **g** F
11 **a** cube, square prism, hexahedron
b cuboid, rectangular prism, hexahedron
12 T
13 **a**

Solid	Faces (F)	Vertices (V)	Edges (E)	F + V
Cube	6	8	12	14
Square pyramid	5	5	8	10
Tetrahedron	4	4	6	8

b $F + V$ is 2 more than E.
14 **a** 26 **b** 11 **c** 28

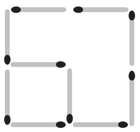
Maths@Work: Jewellery designer

1 **a** Magna: octagon, square, isosceles triangle, scalene triangle, kite.
b Cabochon: octagon, trapezium.
c Princess: Square, rhombus, octagon, isosceles triangle, kite, trapezium.
d Trilliant: equilateral triangle, isosceles triangle, right angled triangle, quadrilateral.

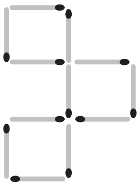
e Standard round: octagon, circle, isosceles triangle, parallelogram or rhombus.
f Oval: square, octagon, right angled triangle, kite, isosceles triangle.
2 **a** Icosahedron **b** Pentahedron
c Octahedron **d** Tetrahedron
e Dodecahedron **f** Heptahedron
3 Answers will vary.

Puzzles and games

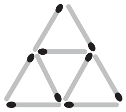
1 **a**



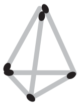
b



2 **a**



b



3 GRACE CHISHOLM YOUNG
4 **a** 165° **b** 37.5° **c** 122.5° **d** 127°

Multiple-choice questions

1 D **2** A **3** E **4** C **5** D
6 B **7** E **8** A **9** C **10** D

Short-answer questions

1 **a** 50 **b** 65 **c** 240
d 36 **e** 61 **f** 138
2 **a** 81 **b** 96 **c** 132
d 99 **e** 77 **f** 51
3 **a** No – corresponding angles are not equal.
b No – cointerior angles do not add to 180°.
c Yes – alternate angles are equal.
4 **a** scalene or obtuse, 35
b isosceles or acute, 30
c equilateral or acute, 60
d right angle or scalene, 19
e acute or scalene, 27
f obtuse or scalene, 132
5 **a** 150 **b** 67 **c** 141
6 **a** square, rhombus
b trapezium
c rectangle, kite, parallelogram
d square, rhombus, kite
e square, rectangle
7 **a** $a = 98, b = 82$ **b** $a = 85, b = 106$
c $a = 231, b = 129$
8 **a** 900° **b** 1260° **c** 1800°
9 **a** 108° **b** 150°
10 **a** hexahedron **b** decahedron
c undecahedron
11 **a** triangular prism **b** octagonal prism
c rectangular pyramid

Extended-response questions

1 **a** 1260° **b** 140° **c** 40°
d **i** 11 **ii** 18 **iii** 27
2 **a** triangle, quadrilateral, pentagon, hexagon
b $a = 90, b = 119, c = 29, d = 121, e = 270, f = 230$

Chapter 3

Pre-test

1 **a** D: mixed number **b** C: proper
c B: improper **d** B: improper
2 **a** 4 **b** 8 **c** 20
3 **a** 3 **b** 9 **c** 3 **d** 8
4 **a** 100 **b** 1 **c** 4 **d** 1
e 30 **f** 60 **g** 100 **h** 4
5 **a** $\frac{1}{4}$ **b** $\frac{3}{7}$
6 **a** D **b** C **c** E **d** A **e** B
7 **a** $\frac{3}{4}$ **b** 1 **c** $1\frac{2}{3}$
d 0.6 **e** 1.2 **f** 3
8 **a** **i** $\frac{1}{10}$ **ii** 0.1
b **i** $\frac{1}{4}$ **ii** 0.25
c **i** $\frac{1}{2}$ **ii** 0.5
d **i** $\frac{3}{4}$ **ii** 0.75
9 **a** \$5 **b** \$6.60 **c** 0.8 km **d** 690 m
10 **a** 10 **e** 18 **f** 90 cents

11

Fraction	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{3}{20}$	$\frac{2}{5}$	$\frac{99}{100}$	1	$\frac{8}{5}$	2
Decimal	0.75	0.2	0.15	0.4	0.99	1.0	1.6	2.0
Percentage	75%	20%	15%	40%	99%	100%	160%	200%

Exercise 3A

1 **a** $\frac{6}{10}, \frac{9}{15}$ **b** $14, 16, \frac{32}{56}$ **c** 50, 20, 5, 2 **d** 6, 9, 12
2 $\frac{10}{15}$
3 b, c and e
4 **a** F **b** T **c** T
d T **e** T **f** F
5 **a** 8 **b** 6 **c** 12
d 10 **e** 20 **f** 120
g 18 **h** 21
6 **a** 6 **b** 10 **c** 15
d 90 **e** 20 **f** 11
g 75 **h** 15
7 **a** 2 **b** 20 **c** 10 **d** 30
e 18 **f** 4 **g** 3 **h** 9
i 6 **j** 18 **k** 2 **l** 7
m 28 **n** 50 **o** 15 **p** 44
8 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{4}{5}$ **d** $\frac{7}{10}$
e $\frac{1}{3}$ **f** $\frac{1}{2}$ **g** $\frac{5}{6}$ **h** $\frac{5}{6}$
i $\frac{1}{4}$ **j** $\frac{3}{5}$ **k** $\frac{8}{9}$ **l** $\frac{5}{7}$
m $\frac{5}{3}$ **n** $\frac{11}{10}$ **o** $\frac{6}{5}$ **p** $\frac{4}{3}$
9 **a** $\frac{1}{2}$ or $\frac{2}{4}$ **b** $\frac{1}{3}$ or $\frac{2}{6}$
c $\frac{1}{4}, \frac{4}{16}$ or $\frac{2}{8}$ **d** $\frac{2}{3}$ or $\frac{6}{9}$

10 a Mary ate the most (125 grams) **b** $\frac{1}{4}$

11 Answers vary, some include: $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{10}{50}, \frac{20}{100}$

12 $\frac{18}{28}$ as it does not simplify to $\frac{3}{4}$

13 a $\frac{9}{10}$ **b** $1\frac{1}{3}$ **c** $\frac{1}{4}$ **d** $\frac{7}{13}$
e $\frac{2}{3}$ **f** $\frac{1}{2}$

Exercise 3B

1 a $+, -$ **b** $\times, \div$
2 a 20 **b** 9 **c** 50 **d** 24
3 a 3, 12 **b** 14, 5 **c** 11, 33 **d** $\times$, 14, 1, 1
4 a $\frac{8}{5}$ **b** $\frac{2}{3}$ **c** $\frac{4}{13}$ **d** $\frac{11}{12}$
5 a $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{12}$ **d** $\frac{2}{5}$
e $\frac{3}{5}$ **f** $\frac{5}{9}$ **g** $1\frac{1}{2}$ **h** $1\frac{6}{7}$
i $1\frac{3}{20}$ **j** $1\frac{1}{10}$ **k** $\frac{1}{21}$ **l** $\frac{4}{9}$
6 a $4\frac{4}{7}$ **b** $9\frac{3}{5}$ **c** $2\frac{3}{8}$ **d** $1\frac{2}{11}$
e $9\frac{1}{2}$ **f** $22\frac{3}{14}$ **g** $3\frac{3}{4}$ **h** $1\frac{17}{30}$
7 a $\frac{3}{20}$ **b** $\frac{10}{63}$ **c** $1\frac{17}{25}$ **d** $1\frac{13}{27}$
e $\frac{1}{6}$ **f** $\frac{3}{8}$ **g** $\frac{8}{15}$ **h** 5
8 a $3\frac{2}{3}$ **b** $1\frac{2}{21}$ **c** 15 **d** 35
9 a $\frac{10}{27}$ **b** $\frac{5}{6}$ **c** $\frac{16}{77}$ **d** $1\frac{7}{15}$
e $\frac{7}{8}$ **f** 2 **g** $1\frac{1}{3}$ **h** $3\frac{3}{5}$
10 a $\frac{33}{35}$ **b** $\frac{48}{125}$ **c** $1\frac{2}{5}$ **d** 3
11 a 10 **b** 15 **c** 25 **d** 3
12 a $\frac{29}{70}$ **b** $\frac{41}{70}$
13 a i $\frac{1}{9}$ **ii** $6\frac{11}{120}$ **iii** $1\frac{9}{56}$ **iv** $3\frac{1}{3}$
b 60

Exercise 3C

1 E
2 C
3 a 37.123 **b** 21.953 **c** 0.0375
d 4.218 09 **e** 65.4112 **f** 9.528 135 2
4 a T **b** F **c** T
d F **e** F **f** F
5 a $\frac{31}{100}$ **b** $\frac{537}{1000}$ **c** $\frac{163}{200}$ **d** $\frac{24}{25}$
e $5\frac{7}{20}$ **f** $8\frac{11}{50}$ **g** $26\frac{4}{5}$ **h** $8\frac{64}{125}$
i $\frac{13}{250}$ **j** $6\frac{1}{8}$ **k** $317\frac{3}{50}$ **l** $\frac{53}{125}$
6 a 0.17 **b** 0.301 **c** 0.45 **d** 0.6
e 0.67 **f** 0.674 **g** 0.15 **h** 0.79
i 0.7 **j** 1.7 **k** 1.18 **l** 0.041
7 a 0.12 **b** 0.35 **c** 2.5 **d** 1.75
e 0.275 **f** 0.375 **g** 0.68 **h** 0.232
8 2.18, 2.25, 2.3, 2.4

9 A1, B5, C07, P9, BW Theatre, gym
10 Opposition leader by 0.25.
11 a Michael's by 0.25 m **b** 25 cm
12 Answers vary, one possible is given for each:
a 0.7 **b** 0.8 **c** 0.5 **d** 0.6

13 a

2.6	4.6	$1\frac{4}{5}$
2.2	$\frac{6}{2}$	3.8
4.2	1.4	$3\frac{2}{5}$

b

0.8	1.8	1.0	3.2
3.0	1.2	2.0	0.6
2.8	1.4	2.2	0.4
0.2	2.4	1.6	2.6

Exercise 3D

1 B **2** E **3** C **4** B
5 a 6.8 **b** 10.5 **c** 21.9 **d** 10.2
e 16.3 **f** 13.2 **g** 62.71 **h** 277.99
i 23.963 **j** 94.172 **k** 60.71 **l** 6.71
6 a 4.4 **b** 6.3 **c** 15.3
d 4.1 **e** 6.1 **f** 4
g 14.41 **h** 23.12 **i** 84.59
j 4.77 **k** 92.1 **l** 80.411
7 a 96.1 **b** 961 **c** 15 463 **d** 1.94
e 0.194 **f** 2.74 **g** 27 400 **h** 1600
i 3651.73 **j** 81.55 **k** 0.75 **l** 0.038 12
m 6 348 000 **n** 0.0010615 **o** 30 **p** 0.000452
8 a 5.6 **b** 0.56 **c** 1.5 **d** 0.12
e 30.8 **f** 0.36 **g** 0.32 **h** 0.032
i 3 **j** 4.9 **k** 8.1 **l** 1.44
9 a 12.27 **b** 5.88 **c** 0.0097 **d** 49.65
e 11.12 **f** 446.6 **g** 0.322 655 **h** 3.462
10 a 52 **b** 620 **c** 150.6;75.3
d 3;1530 **e** 4.84;1.21
11 7.12 m

12 a

Vaughn	Charlotte	Reece
cost \$7.60 change = \$12.40	cost \$8.20 change = \$11.80	cost \$13.50 change = \$6.50

b Vaughn had the most change from \$20
13 Answer comes from the puzzle – ask your teacher if your answer does not make sense.

Exercise 3E

1 a T **b** R **c** R **d** T
e T **f** R **g** T **h** R
2 a $0.\dot{3}$ **b** $6.\dot{2}\dot{1}$ or $6.\overline{21}$
c $8.576\dot{4}$ **d** $2.1\dot{3}5\dot{6}$ or $2.1\overline{356}$
e $11.\dot{2}857\dot{3}$ or $11.\overline{28573}$ **f** $0.00\dot{3}5\dot{2}$ or $0.00\overline{352}$
3 a 4 **b** 9 **c** 7 **d** 6
4 a 5.5 **b** 7.42 **c** 0.4 **d** 2.0
5 a 0.6 **b** 0.75 **c** 0.125 **d** 0.55
e 0.5 **f** 0.8 **g** 0.04 **h** 0.18
6 a $0.\dot{3}$ **b** $0.\dot{5}$ **c** $0.8\dot{3}$ **d** $0.\dot{7}$
e $0.\overline{428571}$ **f** $0.1\dot{6}$ **g** $1.\dot{3}$ **h** $1.\dot{8}5714\dot{2}$

7 **a** 0.6 **b** 0.8 **c** 1.5 **d** 8.2
e 9.5 **f** 8.3 **g** 1.5 **h** 3.4
i 0.3
8 **a** 0.78 **b** 0.67 **c** 1.48 **d** 0.89
e 15.49 **f** 9.04 **g** 9.42 **h** 8.75
i 1.79
9 **a** **i** 8 **ii** 8 **iii** 5
b **i** 5.0 **ii** 8.9 **iii** 6.0

10

Fraction	Decimal	1 dec pl	2 dec pl
$\frac{1}{3}$	0.333…	0.3	0.33
$\frac{2}{3}$	0.666…	0.7	0.67
$\frac{5}{9}$	0.555…	0.6	0.56
$\frac{1}{6}$	0.166…	0.2	0.17

11 **a** Greer by 0.06 of a second
b 12.8 for both, as they are the same to one decimal place you can't tell who came first.
12 Answers vary, some include: 3.451, 3.446, 3.450 71, 3.449, 3.451
13 **a** **i** 0.766 **ii** 9.5 **iii** 7.0 **iv** 21.5134
v 0.95 **vi** 17 **vii** 8.60 **viii** 8.106
b **i** \$1.48 **ii** \$7.40 **iii** \$17.75 **iv** \$58.72
c **i** 1.618 **ii** 1.62 **iii** 1.6

Exercise 3F

1 B
2 B
3 C
4

	Fraction	Decimal in words	Decimal in figures	Percent in words	Percent in figures
a	$\frac{13}{100}$	thirteen hundredths	0.13	thirteen percent	13%
b	$\frac{45}{100}$	forty five hundredths	0.45	forty five percent	45%
c	$\frac{70}{100}$	seven tenths	0.7	seventy percent	70%
d	$\frac{99}{100}$	ninety nine hundredths	0.99	ninety nine percent	99%

5 **a** $\frac{39}{100}$ **b** $\frac{11}{100}$ **c** $\frac{17}{100}$ **d** $\frac{99}{100}$
e $\frac{1}{5}$ **f** $\frac{7}{10}$ **g** $\frac{3}{4}$ **h** $\frac{11}{20}$
6 **a** 0.39 **b** 0.11 **c** 0.17 **d** 0.99
e 0.2 **f** 0.7 **g** 0.75 **h** 0.55
i 0.07 **j** 0.01 **k** 0.1 **l** 0.47
7 **a** 77% **b** 49% **c** 75% **d** 80%
e 28% **f** 45% **g** 55% **h** 38%
i 94% **j** 12.5% **k** 62.5% **l** $66.\dot{6}\%$
8 **a** 16% **b** 79% **c** 83% **d** 97%
e 3% **f** 33% **g** 91% **h** 9%
i 56% **j** 22% **k** 100% **l** 101%
9 **a** 100, 200 **b** $5\frac{3}{4}$, 23 **c** 100, 1
10 **a** $3 \times 20\% = 60\%$ **b** $7 \times 12.5\% = 87.5\%$ **c** $66\frac{2}{3}\%$

11 **a**

Fraction	Decimal	%
$\frac{1}{4}$	0.25	25%
$\frac{2}{4}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{4}{4}$	1	100%

b

Fraction	Decimal	%
$\frac{1}{5}$	0.2	20%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{4}{5}$	0.8	80%
$\frac{5}{5}$	1	100%

c

$\frac{3}{20}$	$\frac{3}{10}$	$\frac{9}{10}$	$\frac{1}{3}$
0.15	0.3	0.9	$0.\dot{3}$
15%	30%	90%	$33\frac{1}{3}\%$

12 65%, 80%
13

Cent per 100 cents	Cents in the dollar	Percentage
5c	\$0.05	5%
10c	\$0.10	10%
9c	\$0.09	9%
17c	\$0.17	17%
25c	\$0.25	25%
70c	\$0.70	70%
90c	\$0.90	90%
75c	\$0.75	75%
100c	\$1.00	100%
200c	\$2	200%

Exercise 3G

1 D **2** A
3 **a** half the test correct, 50%
b no answers correct, 0%
c every answer correct, 100%
4 **a** 100 **b** 10 **c** 5 **d** 2 **e** 4
5 **a** 80% **b** 65% **c** 78% **d** 68%
e 60% **f** 98% **g** 70% **h** 40%
i 75% **j** 80% **k** 60% **l** 75%
6 **a** 5% **b** 25% **c** 5%
d 25% **e** 4% **f** 2.5%
7 **a** 56% **b** 75% **c** 86%
d 25% **e** 40% **f** 50%

8 **a** 18 **b** 8 **c** 150 **d** 18
e 8 **f** 12 **g** 60 **h** 22
i 12.5 **j** 3 **k** 300 **l** 7.2
m 3 **n** 198 **o** 720
9 **a** \$75 **b** 100 m **c** 45 kg
d 18 minutes **e** 500 mL **f** 15 minutes
g \$3.25 **h** 16 cents **i** 35 g
10 **a** 5 L **b** 2000 marbles
c \$8 **d** 45 donuts
11

$\frac{1}{5}$	20%
$\frac{3}{20}$	15%
$\frac{7}{20}$	35%
$\frac{1}{4}$	25%
$\frac{1}{20}$	5%

12 **a** \$31500 **b** \$45 000 **c** \$13 500
d Yes \$500 more
13 **a** 67 c **b** 260 m **c** \$36.25
d 14 min 24 s (14.4 min) **e** \$14 477.40
f 24 **g** 101 **h** \$50 112

Progress Quiz – Chapter 3 Fractions and decimals percentages

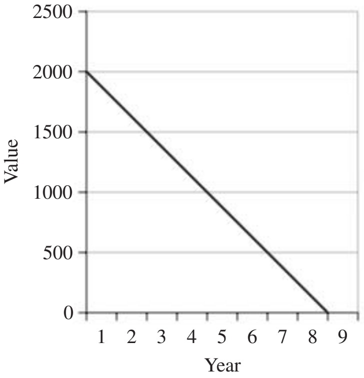
1 **a** $\frac{2}{5}$ **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $\frac{9}{5}$ or $1\frac{4}{5}$
2 **a** $1\frac{2}{11}$ **b** $\frac{1}{8}$ **c** $5\frac{9}{10}$ **d** $2\frac{1}{6}$
3 **a** $\frac{10}{21}$ **b** $1\frac{1}{5}$ **c** $\frac{33}{35}$ **d** $2\frac{4}{5}$
4 **a** $\frac{7}{20}$ **b** $5\frac{1}{4}$ **c** $12\frac{4}{5}$ **d** $456\frac{7}{50}$
5 **a** 0.7 **b** 0.36 **c** 0.34 **d** 2.25
6 **a** 21.8 **b** 18.695 **c** 12.2 **d** 20.128
7 **a** 6573.4 **b** 0.0012754 **c** 0.54
d 22.615 **e** 4.769 **f** 2818.7
8 **a** 0.375 **b** 1.25 **c** $1.\dot{6}$ **d** $1.\dot{5}7142\dot{8}$
9 **a** 0.79 **b** 0.42 **c** 26.15 **d** 379.01
10 **a** 0.45, $\frac{9}{20}$ **b** 0.84, $\frac{21}{25}$ **c** 0.02, $\frac{1}{50}$ **d** 1.09, $1\frac{9}{100}$
11 **a** 80% **b** 37.5% **c** 43% **d** 94%
12 **a** 80% **b** 34% **c** 20% **d** 70%
13 **a** \$10 **b** 25 kg **c** 45 minutes **d** \$125

Exercise 3H

1 **a** decrease **b** increase **c** decrease
d increase **e** increase
2 **a** 120% **b** 115% **c** 90% **d** 85%
3 **a** \$12 **b** \$33.99 **c** \$14.50 **d** \$225
4 **a** \$440 **b** \$264 **c** \$275 **d** \$840
e \$505 **f** \$1000 **g** \$105 **h** \$135
5 **a** \$360 **b** \$216 **c** \$225 **d** \$72
e \$170 **f** \$630 **g** \$500 **h** \$51
6 **a** \$200 **b** \$2700 **c** \$2300
7 **a** \$480 **b** \$127.50 **c** \$39
8 **a** \$12 **b** \$24 **c** \$37.50
d \$63.75 **e** \$97.50 **f** \$4.95
9 **a** \$104 **b** \$15.40 **c** \$630
10 **a** \$38.50 **b** \$82.50 **c** \$46.20
d \$91.30 **e** \$57.75 **f** \$164.99
11 **a** \$90 300 **b** \$10.08 **c** \$37 600
d \$81.40 **e** \$960 **f** \$620 000
12 **a** \$180
b shop 1 = \$1620, shop 2 = \$1600
c 2 as the bike is cheaper
d **i** Bikes are the same price so either shop is recommended
ii Shop 1 is now cheaper \$1980 versus \$2000
13 **a** **i**

End of year	Value
0	2000
1	1750
2	1500
3	1250
4	1000
5	750
6	500
7	250
8	0

ii



iii straight line **iv** after 8 years
b **i**

3	\$1339.84
4	\$1172.36

ii never

Exercise 3I

1 **a** profit **b** loss **c** loss
d profit **e** profit
2 **a** \$7 **b** \$28 **c** \$3.45 **d** \$436
3 **a** \$13 **b** \$45 **c** \$25.90 **d** \$247
4 D
5 **a** 80% **b** 30% **c** 25% **d** 20%
e $66\frac{2}{3}$% **f** $37\frac{1}{2}$% **g** 50% **h** 100%
6 **a** 25% **b** 16% **c** 50% **d** 75%
e $33\frac{1}{3}$% **f** 10% **g** 20% **h** 10%

7 a

4	5	1	25%
10	12	2	20%
24	30	6	25%
100	127	27	27%

b

10	7	3	30%
16	12	4	25%
50	47	3	6%
100	93	7	7%

8 a 20% increase **b** $16\frac{2}{3}$% decrease
c 500% increase **d** 150% increase
9 a 25% increase **b** 20% increase **c** 140% increase
10 20% loss
11 a $36 **b** 75% profit
12 a $320 **b** 80%
13 a $2200 **b** 44% loss **c** $5500
14 a

	March 2016	Change in the past 12 months	% Change
NSW	7 704 300	103 200	1.3
VIC	6 039 100	114 900	1.9
QLD	4 827 000	61800	1.3
SA	1706 500	9700	0.6
WA	2 613 700	29 800	1.1
TAS	518 500	2200	0.4
NT	244 000	1000	0.4
ACT	395 200	5000	1.3
AUSTRALIA	24 051400	327 600	1.4

b Answers will vary.

Exercise 3J

1 a 8 **b** 25 **c** 100 **d** 50
2 a $6 **b** $30 **c** $300
3 a $80 **b** $800
4 $4, $400
5 a $900 **b** $800 **c** $1100
d $500 **e** $550 **f** $250
6 $90
7 a $120 **b** $240 **c** $15 **d** $21
8 $300
9 a $50 **b** $150 **c** $600
d $30 **e** $10 **f** $2000
10 $5 **11** $80 **12** D
13

Superbarn	Gymea fruit market	Xmart
a $16.85 b 0.27 kg c marshmallows – marked with * d 1.89 – 0.17 = $1.72	a $14.99 per kg b 5th July 2011 c taking the cash amount to the nearest 5 cents d $9.85 e 55 cents f 5.6% (1 dec pl)	a 8 b $35 c all toys d $13.55 e 9.09%

Maths@Work: Owner and manager of a fruit and vegetable shop

1 a Answers will vary.
b Answers could include: rent; wages; shop fit-out e.g. shelves and fridges; a vehicle; home delivery service; and advertising.
c Keeping produce fresh and also including a big variety despite seasonal availability.
2 a $0.97 / kg **b** $1.38 / kg **c** $0.90 / head
d $0.80 / kg **e** $1.27 / kg
3 a $3.55 **b** $5.45 **c** $5.98
d $1.83 **e** $2.13
4 a $0.83 / apple **b** $1.00 / apple **c** $0.80 / pear
d $0.66 / apricot **e** $1.22 / peach
5 $35.65
6 Option A: $0.49 / 100 g; Option B: $0.44 / 100 g, the best buy.
7 $378.45
8 a, **b** and **c** using 1 AUD = $0.75 US

Produce: fruit or vegetable	USA prices		Australian price
	per lb	per kg	per kg
Avocado	$2.20	$4.84	$3.63
Bananas	$0.86	$1.89	$1.42
Bok choy	$2.18	$4.80	$3.60
Cauliflower	$2.49	$5.48	$4.11
Granny Smith apples	$1.95	$4.29	$3.22
Mangoes	$1.71	$3.76	$2.82
Oranges - navel	$1.37	$3.01	$2.26
Potatoes	$1.24	$2.73	$2.05
Papaya	$1.59	$3.50	$2.62

d Australian fresh produce prices are generally dearer than USA prices. Possible reasons include Australia's smaller population and larger distances for transporting produce from farms to shops.

Puzzles and games

1 Answers vary, some include: 2.6701, 2.666, 2.668, 2.6712...
2 10
3 $\frac{1}{2}$, 50%, 0.5, $\frac{2}{4}$, $\frac{10}{20}$ etc.
4a

$\frac{4}{3}$	3	$\frac{2}{3}$
1	$1\frac{2}{3}$	$\frac{7}{3}$
$2\frac{2}{3}$	$\frac{1}{3}$	2

b

$\frac{5}{3}$	$\frac{5}{2}$	$\frac{4}{3}$
$\frac{3}{2}$	$\frac{11}{6}$	$2\frac{1}{6}$
$\frac{7}{3}$	$\frac{7}{6}$	2

5 See teacher if your answer to the puzzle does not make sense.

Multiple-choice questions

1 B 2 D 3 D 4 C 5 A
6 B 7 B 8 A 9 C 10 D

Short-answer questions

1 a 21 b 8 c 12
2 a $\frac{5}{9}$ b 3 c $1\frac{1}{3}$
3 a $\frac{1}{2}$ b $\frac{1}{6}$ c $\frac{3}{8}$ d $\frac{4}{5}$
e $\frac{1}{8}$ f $\frac{3}{4}$ g $\frac{2}{3}$ h $1\frac{3}{10}$
4 a $1\frac{3}{4}$ b 4 c $6\frac{1}{2}$ d $5\frac{1}{5}$
5 a 4 b 2 c 8 d 12
6 a $\frac{1}{6}$ b $\frac{1}{10}$ c $\frac{7}{20}$ d $\frac{1}{3}$
7 a 12 b 2 c $1\frac{3}{5}$ d 2
8 a 0.5 b 0.25 c 0.6 d 0.117
9 a $\frac{3}{5}$ b $\frac{3}{25}$ c $\frac{1}{25}$ d $\frac{19}{20}$
10 a 20 b 14.19 c 8.2 d 4.6
e 22.91 f 6.18
11 a 6 b 0.06 c 4.8 d 0.048
e 0.6 f 716.4 g 96 h 0.42
12 a 40 b 6.2 c 71.1
13 a 0.667 b 3.580 c 0.005
14

0.1	0.01	0.05	0.5	0.25	0.75	$0.\dot{3}$	0.125
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{8}$
10%	1%	5%	50%	25%	75%	$33\frac{1}{3}$%	12.5%

15 a $5 b $16 c 35 g
16 a 87.5% b 25% c 150% d 4%
17 a $616 b $3400
18 $300 19 $155.20 20 $88 200

Extended-response question

1 a Indian = 21 000 INR, SINGAPORE = $625 SGD Thai = 15 000 THB, Hong Kong = $3500 HKD
b 4500 THB
c $6 Australian

Chapter 4

Pre-test

1 a E b C c G d D
e B f H g A h F
2 a 30 m b 14.5 cm c 18 m
3 a 10 b 27 c 25 d 121
4 a 300 cm b 200 mm c 1800 m d 25 cm
e 3.5 cm f 4.2 km g 5 m h 0.1 m
i 120 s j 3000 mL k 4 L l 3 kg
5 a 6 b 9 c 4
6 a 30 cm^2 b 25 cm^2 c 16 cm^2 d 20 cm^2
7 12 cm^3

Exercise 4A

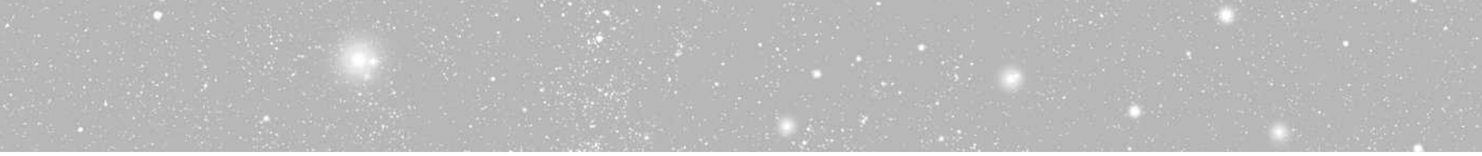
1 a metric
b centimetres, metres, kilometres
2 a 200 b 5200 c 78
d 8.4 e 961 f 41.2
3 a 10 b 100 c 1000
d 100 000 e 1000 f 1000 000
4 a 10 b 10 c 2
5 a 30 mm b 610 cm c 8930 m d 300 cm
e 2.1m f 32 cm g 19.62 km h 380 m
i 4.8 cm j 2 mm k 0.042 m l 40 cm
m 3.7 km n 0.6 km o 710 m p 2 cm
6 a 19 m b 44 m c 13 cm d 10.4 cm
e 6.6 m f 18 cm g 17.2 mm h 34.4 cm
i 29.4 m
7 a 32 cm b 28 km c 18 cm
8 a 4.3 mm b 2040 cm c 23.098 m d 3.42 km
e 194.3 m f 0.01km g 24.03 mm h 0.994 km
i 1 cm
9 a 5 b 2 c 4
d 18 e 9.5 f 6.5
10 $2392 11 8 min 12 240 cm
13 a $P = 2a + b$ b $P = 2a + 2b$ c $P = 2a + 2b$
14 a 40 cm b 17 cm c 7.8 cm
d 2000 cm e 46 cm f 17 600 cm

Exercise 4B

1 a diameter b radius c circumference
2 a i 10 m ii 22 cm iii 4.6 mm
b i 6 cm ii 15.5 mm iii 0.21m
3 a 3.1 b 3.14 c 3.142
4 a 15.71 b 40.84 c 18.85 d 232.48
5 a 12.57 mm b 113.10 m c 245.04 cm
d 13.19 m e 4.40 km f 0.25 cm
6 a 12.57 m b 21.99 km c 15.71cm
d 13.51cm e 25.95 m f 0.13 mm
7 251cm 8 11.0 m 9 176 cm 10 12 566 m
11 a 64.27 cm b 12.34 m c 61.70 mm
12 Svenya and Andre
13 $d = 2r$, so $2\pi r$ is the same as πd.
14 Answers will vary.

Exercise 4C

1 a $A = lw$ b $A = l^2$ c $A = \frac{1}{2}bh$
2 a i 100 ii 400 iii 3
b i 10 000 ii 70 000 iii 4
c i 1000 000 ii 5 000 000 iii 2.5
d i 10 000 ii 30 000 iii 7.5
3 a 7 m, 3 m b 8 cm, 6 cm (or other way around)
c 2.4 mm, 1.7 mm d 5 m, 4 m
4 a 200 mm^2 b 70 000 cm^2 c 500 000 m^2 d 30 000 m^2
e 34 mm^2 f 0.07 m^2 g 30.9 cm^2 h 4000 m^2
i 0.2 m^2 j 0.45 km^2 k 0.4 ha l 32.1cm^2
m 32 ha n 51cm^2 o 4.3 mm^2 p 0.4802 m^2
q 1.904 ha r 0.2933 ha s 49 m^2 t 7700 m^2
5 a 9 cm^2 b 21m^2 c 10 cm^2 d 121m^2
e 33 m^2 f 144 mm^2
6 a 42 m^2 b 39 cm^2 c 100 cm^2
d 63 m^2 e 3 m^2 f 6 km^2
7 50 m 8 10 cm
9 a 6 m b 1.5 cm
10 a 25 m^2 b 20.25 cm^2 c 28 cm d 52 m
11 $48



12 a 200 000 mm^2 **b** 430 000 cm^2 **c** 0.000 037 4 km^2
d 0.010 92 m^2 **e** 20 cm^2 **f** 0.1 ha
13 a 70 m^2 **b** 54 m^2 **c** 140 cm^2
d 91 cm^2 **e** 46 km^2 **f** 64 mm^2

Exercise 4D

1 a B **b** D **c** A **d** C
2 a 6 **b** 30 **c** 13.5 **d** 33
3 a 90° **b** Height
c perpendicular **d** parallel, perpendicular
e rhombus, kite
4 a 50 m^2 **b** 4.5 cm^2 **c** 6 m^2
d 165 m^2 **e** 18 cm^2 **f** 17.94 m^2
5 a 7.5 cm^2 **b** 121 km^2 **c** 9.61 m^2
d 4 cm^2 **e** 300 mm^2 **f** 0.9 mm^2
6 a 96 cm^2 **b** 32.5 m^2 **c** 560 mm^2 **d** 5 cm^2
7 \$120 **8** 0.27 m^2 **9** 2 m **10** \$1160
11 a 6 cm^2 **b** 35 m^2 **c** 84.5 cm^2
12 No, use formula for parallelogram $A = bh$, as we already know these lengths.
13 a
$$\begin{aligned} A &= \text{length} \times \text{width} \\ &= b \times h \\ &= bh \end{aligned}$$
b
$$\begin{aligned} A &= 4 \text{ triangle areas} \\ &= 4 \times \frac{1}{2} \times \text{base} \times \text{height} \\ &= 4 \times \frac{1}{2} \times \frac{1}{2}x \times \frac{1}{2}y \\ &= \frac{1}{2}xy \end{aligned}$$
c
$$\begin{aligned} A &= \text{Area (triangle 1)} + \text{Area (triangle 2)} \\ &= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2 \\ &= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h \\ &= \frac{1}{2}ah + \frac{1}{2}bh \\ &= \frac{1}{2}(a+b)h \end{aligned}$$

Exercise 4E

1 a $c = 2\pi r$ or $c = \pi d$ **b** $A = \pi r^2$
2 a 78.54 **b** 530.93 **c** 30.19 **d** 301.72
3 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
4 a 5 m **b** 2.3 mm **c** 3.5 km
5 a 28.27 cm^2 **b** 113.10 m^2 **c** 7.07 mm^2
d 78.54 km^2 **e** 36.32 cm^2 **f** 9.08 m^2
6 a 50.27 cm^2 **b** 153.94 km^2 **c** 615.75 mm^2
d 314.16 km^2 **e** 38.48 m^2 **f** 31415.93 m^2
7 a 50.27 cm^2 **b** 6.16 m^2 **c** 12.57 km^2
8 707 cm^2
9 yes, by 1310 cm^2
10 no ($A = 0.79$ km^2)
11 a 3.14 cm^2 **b** 201.06 cm^2 **c** 226.98 mm^2
d 39.27 cm^2 **e** 5.09 mm^2 **f** 100.53 m^2
12 78.54 cm^2 **13** 80 cm^2
14 a 34.82 m^2 **b** 9.14 m^2 **c** 257.08 cm^2
d 116.38 mm^2 **e** 123.61 km^2 **f** 53.70 m^2
g 50.27 m^2 **h** 75.40 mm^2 **i** 12.57 cm^2

Progress Quiz – Chapter 4 – Measurement

1 a 0.12 m **b** 58.5 cm **c** 6200 mm **d** 2570 m
2 a 27 cm **b** 14.6 m **c** 22 cm **d** 20 cm
3 a 75.40 cm **b** 14.14 m
4 35.99 cm
5 a 700 mm^2 **b** 45 cm^2 **c** 3400 mm^2 **d** 3000 ha
6 a 6 m^2 **b** 38 cm^2 **c** 42 m^2 **d** 36 cm^2
7 a 300 m^2 **b** 3.3 cm^2 **c** 66 mm^2 **d** 160 cm^2
8 5 m
9 a 7.07 km^2 **b** 572.56 mm^2
10 a 56.75 m^2 **b** 113.49 mm^2

Exercise 4F

1 a length **b** area **c** volume
d area **e** volume **f** length
g area **h** volume **i** area
j length
2 a 24 **b** 12 **c** 72
3 a 1000 **b** 1 **c** 1
d 1 **e** 1 **f** 1000
4 a 36 cm^3 **b** 20 m^3 **c** 27 mm^3
d 64 km^3 **e** 320 mm^3 **f** 24 m^3
5 a 2000 mL **b** 5000 L **c** 500 kL
d 3 L **e** 4 cm^3 **f** 50 mL
g 2.5 L **h** 5100 cm^3 **i** 1000 L
6 a 24 L **b** 42 L **c** 27 L
d 0.018 L **e** 0.024 L **f** 0.36 L
7 a 1 000 000 cm^3 **b** 1 000 000
c 1000 **d** 1000
8 a 4000 L **b** 45000 L **c** 2400 L **d** 1000 L
9 a i 60 000 000 L **ii** 60 000 kL **iii** 60 ML
b 200 days
10 8000 kg **11** 80 minutes **12** 9
13 a 24 cm^2 **b** 403.44 m^2 **c** 22 cm^2

Exercise 4G

1 a rectangle **b** square **c** triangle
2 a 90 cm^2 **b** 16 m^2 **c** 5 m^2
3 a i prism **ii** rectangle
b i prism **ii** triangle
c i not a prism (pyramid)
d i not a prism (cone)
e i prism **ii** square
f i not a prism (truncated pyramid)
4 a 44 m^3 **b** 20 m^3 **c** 352 mm^3
d 10 cm^3 **e** 33 mm^3 **f** 110 m^3
5 a 200 cm^3 **b** 15 m^3 **c** 980 cm^3
d 192 cm^2 **e** 45 m^3 **f** 32 cm^3
6 40 m^3
7 a 60 m^3 **b** 270 mm^3 **c** 60 m^3
d 24 cm^3 **e** 112 m^3 **f** 3200 mm^3
8 a 56 000 L **b** 56 hours
9 a 785.40 m^3 **b** 12566.37 mm^3 **c** 251.33 cm^3
d 7696.90 cm^3 **e** 461.81 m^3 **f** 384.85 m^3

Exercise 4H

1 a 60 **b** 7 **c** 24
d 120 **e** 4 **f** 31
2 a 120 s **b** 3 min **c** 2 h **d** 240 min
e 72 h **f** 2 days **g** 5 weeks **h** 280 days
3 a 6 h 30 min **b** 10 h 45 min **c** 16 h 20 min **d** 4 h 30 min
4 a 120 s **b** 2 days **c** 3 weeks **d** 180 min
e 630 s **f** 4 min **g** 1.5 h **h** 144 h
i 3 days **j** 168 h **k** 1440 min **l** 210 min

5 **a** 6:30 pm **b** 9:00 am **c** 6:30 pm
d 4:30 pm **e** 5:30 pm **f** 11:40 am
6 **a** 1330 h **b** 2015 h **c** 1023 h **d** 2359 h
e 6:30 am **f** 1:00 pm **g** 2:29 pm **h** 7:38 pm
i 11:51 pm **j** 4:26 am **k** 1847 h **l** 0432 h
7 **a** 2:00 pm **b** 5:00 am **c** 1200 hours **d** 1800 hours
8 **a** 10:00 am **b** 9:30 am **c** 9:30 am
d 8:00 am **e** 10:00 am **f** 10:00 am
g 8:00 am **h** 10:00 am
9 **a** 5:30 pm **b** 3:30 pm **c** 5:30 pm
d 3:30 pm **e** 5:30 pm **f** 5:30 pm
g 5:00 pm **h** 7:30 pm **i** 4:30 pm
10 **a** F **b** D **c** A
d E **e** B **f** C
11 **a** 2 h 50 min **b** 6 h 20 min **c** 2 h 44 min
d 8 h 50 min **e** 8 h 19 min **f** 10 h 49 min
12 17 min 28 s **13** 7 h 28 min **14** 23 h 15 min
15 **a** 33c **b** 143c or $1.43
16 **a** $900 **b** $90 **c** $1.50 **d** 2.5c
17 **a** 5:30 am **b** 6:30 am **c** 6:30 am **d** 1:30 pm
e 2:30 pm **f** 2:30 am **g** 3:00 pm **h** 5:30 pm
18 **a** 11:00 am **b** 12 noon **c** 8:00 pm **d** 7:30 pm
e 7:00 am **f** 5:00 am **g** 1:00 am **h** 10:00 am
19 **a** You have to turn your clock back.
b You have to turn your clock forward.
c You adjust the date back one day.

Maths@work: Hairdresser

1 **a** 10 mL **b** 20 mL **c** 45 mL
d 100 mL **e** 1000 mL
2 **a** 20 cc of developer **b** 60 cc of developer
c 100 cc of developer
3 **a** 25 cc **b** 50 cc **c** 75 cc
d Friday, November 4 **e** 11:20 am **f** 65 minutes
4 **a** 6 customers
b 2.5 hours
c Start 9 am, finish 11:30 am
d Under the hair dryer.
e Yes, she could work into her lunch time.
f Around 3:30 or 4 pm while Mrs Babb's hair dried or 5:30 pm
g 8 customers
h Start 10:30 am, finish 1 pm
i 1hour **j** 6:20 pm
5 **a** Various answer similar to this page of appointments.

Salon appointment book	
Day and date	
Start Time	Hairdresser's name
9:00 am	Name: long hair, colour, cut, wash, head massage, eyebrows, blow-dry and style.
9:30 am	
10:00 am	
10:30 am	Name: men's hair cut
11:00 am	First customer, cont'd.
11:30 am	Name: foils wash and blow-dry
12:00 noon	
12:30 pm	
1:00 pm	Name: men's hair cut
1:30 pm	Lunch
2:00 pm	Name: long hair, colour, cut, wash, head massage, eyebrows, blow-dry and style
2:30 pm	
3:00 pm	
3:30 pm	Name: boy's hair cut
4:00 pm	After lunch customer, cont'd.
4:30 pm	Name: girl's hair cut

b Customers phone numbers, colour mixing records and price charged.

Puzzles and games

1 **a** 240 **b** 56 **c** 50
2 **a** 12.5 **b** 6.3 **c** 7
3 10 cm each side
4 Yes, 1 L will overflow.
5 75.4 cm^2 **6** $\frac{1}{2}$ **7** 78.5% **8** 3 cm

Multiple-choice questions

1 E **2** B **3** A **4** C **5** B
6 E **7** E **8** B **9** D **10** D

Short-answer questions

1 **a** 2000 mm **b** 500 m **c** 0.32 km **d** 40 m
e 300 mm^2 **f** 0.4 m^2 **g** 10 000 m^2 **h** 3.5 cm^2
i 4 L **j** 3000 mm^3 **k** 0.4 L **l** 4.3 ML
2 **a** 13 m **b** 28 cm **c** 25.13 m
d 12.57 m **e** 30.6 km **f** 25.8 m
g 51.42 mm **h** 48 m **i** 20 cm
3 **a** 55 cm^2 **b** 63 m^2 **c** 12 cm^2
d 9 cm^2 **e** 201.06 km^2 **f** 136 km^2
g 64 m^2 **h** 20 cm^2 **i** 28.27 cm^2
4 **a** 9 L **b** 4.5 L **c** 1000 L
5 **a** 1m^3 **b** 8000 cm^3 **c** 10 m^3
d 144 cm^3 **e** 40 cm^3 **f** 6 cm^3
6 **a** **i** 287°C **ii** 239°C
b 1h 39 min 18 s **c** 1h 2 min 4 s
7 **a** 10 h 17 min **b** 9:45 pm **c** 2331hours
8 **a** 6:30 pm **b** 6:00 pm **c** 6:00 pm
d 4:30 pm **e** 4:30 pm **f** 4:30 pm
g 8:30 pm **h** 6:30 pm **i** 6:30 pm

Extended-response questions

1 **a** 160 m^2 **b** 56 m **c** 12.57 m^2 **d** 147.43 m^2
e **i** 100 cm^2 **ii** 0.01m^2
f 14 744 tiles
g Some will break and more are needed to go around the pond.
2 **a** 70 cm^2
b 50.14 cm^2
c 74 cm^2

Chapter 5

Pre-test

1 a 32 **b** 14 **c** 12 **d** 30
2 a 17 **b** 12 **c** 30 **d** 12
3 a 12 **b** 70 **c** 7 **d** 20
4 a 16 **b** 4 **c** 121 **d** 10000
5 a $x+5$ **b** $m-7$ **c** xy **d** $\frac{w}{2}$
6 25
7 a

A	0	3	7	10
M	3	9	17	23

b

X	1	3	11	0
Y	13	15	23	12

8 a 8 **b** 12 **c** 16 **d** 18
9 a 12 **b** 3 **c** 48

Exercise 5A

1 a 8 **b** 12 **c** 3 **d** 10
2 a $3a, 2b, 5c$ **b i** $a:3$ **ii** $b:2$ **iii** $c:5$
c $2x+5y+8z$. Other answers are possible.
3 a $4x, 6y, 2z$ **b** y
4 a 6 **b i** $a:5$ **ii** $b:7$ **iii** $c:1$
c $x+2y+3z+4w+91k$. Other answers are possible.
5 a 3 **b** 3 **c** 2
d 4 **e** 2 **f** 5
6 a 2 **b** 1 **c** 9
d –2 **e** –6 **f** –1
7 a F **b** C **c** E
d D **e** A **f** B
8 a $y+7$ **b** $x-3$ **c** $a+b$ **d** $4p$
e $4-\frac{q}{2}$ **f** $10+\frac{r}{3}$ **g** $2(b+c)$ **h** $b+2c$
9 a the sum of 3 and x **b** the sum of a and b
c double k **d** half m
10 a the product of $4, b$ and c **b** double a is added to b
c b is subtracted from 4 and the result is doubled.
d b is doubled and the result is subtracted from 4.
11 a $10x$ **b** $A+B$ **c** $22-k$ **d** $50-x$
12 a $70 **b** $7x$ **c i** $x-3$ **ii** $7(x-3)$
13 a $2p$ **b** $48p$ **c** $30p+18(p+20)$
14 a i $4a$ **ii** $7b$ **iii** $5a+5b$ **iv** $\frac{7a+7b}{2}$
b 7 numbers, 2 Proof by Induction $\Rightarrow$ total = 9 DVDs

Exercise 5B

1 a 11 **b** 17 **c** 9 **d** 7
2 a 11 **b** 12 **c** 3 **d** 3
3 a 16 **b** 21 **c** 111 **d** 70
4 equivalent expressions
5 a 15 **b** 8 **c** 20
6 a 14 **b** 30 **c** no
7 a 30 **b** 37 **c** 16 **d** 58
8 a 7 **b** 5 **c** 10 **d** 23
9 a 14 **b** 13 **c** 11 **d** 34
e 19 **f** 29 **g** 3 **h** 17
10 a 8 **b** 2
11 a E **b** E **c** N **d** N **e** E **f** E
12 a 8 **b** 3, 4, 5
13 a If $a=3$ and $b=4$ $3+4=7, 3\times 4=12$
b $a=2$ and $b=2$
c Not equal if $a=10\,(12\neq 8)$
d No, always 4 apart.
14

x	3	5	2	0	4	2
y	8	7	3	–3	–2	6
x + y	11	12	5	–3	2	8
x – 2y	–13	–9	–4	6	8	–10
xy	24	35	6	0	–8	12

Exercise 5C

1 a a **b** a, c **c** x, y **d** w, z
2 a like terms **b** equivalent
3 a 21 **b** 21 **c** true
4 a 23 **b** 84 **c** false
5 a a, b, c **b** a, b, c **c** Yes
6 a L **b** N **c** L **d** N
7 a L **b** N **c** L **d** N
e L **f** N **g** L **h** L
8 a $5x$ **b** $19a$ **c** $9x$ **d** $7y$
e $7xy$ **f** $13uv$ **g** $14ab$ **h** $15pq$
9 a $9f+12$ **b** $13x+8y$ **c** $7a+11b$ **d** $13a+9b$
e $12+12x$ **f** $8a+3b+3$ **g** $14x+30y$ **h** $21a+4$
i $5b+9c$ **j** $2a+3b$ **k** $12qr+3q$ **l** $7xy+9x$
m $4ab+7b$ **n** $23lk+2l$
10 a C **b** A **c** D **d** E **e** B
11 a $12x$ **b** $22x$ **c** $12a+4b$
12 a $13c$ **b** $9nc$
13 a If $a=1, b=2: 4a+3b=10, 7ab=14$. Other answers are possible.
b Yes, for example if $a=0$ and $b=0$.
c No, they are equivalent.
14 a $5a+7b+5a$. Other answers are possible.
b 9 ways

Exercise 5D

1 a T **b** T **c** F
d F **e** T
2 B
3 a $\frac{3}{5}$ **b** $\frac{1}{3}$ **c** $\frac{3}{2}$ **d** $\frac{3}{5}$
4 a $3xy$ **b** $5abc$ **c** $12ab^2$ **d** $4ac^3$
5 a $63d$ **b** $10a$ **c** $36x$
d $24k$ **e** $6q$ **f** $30xy$
g $8abcd$ **h** $60abcd$ **i** $48abde$
6 a x^2 **b** a^2 **c** $3d^2$ **d** $10d^2e$
e $14x^2y$ **f** $10x^2y$ **g** $8x^2yz$ **h** $8a^2b^2cd$
i $48x^2y$ **j** $18a^2b$ **k** $24x^2y^2$ **l** $24a^2b^2$
7 a $\frac{k}{4}$ **b** $\frac{x}{5}$ **c** $\frac{2q}{5}$ **d** $\frac{3k}{10}$
e $\frac{5}{a}$ **f** $\frac{a}{b}$ **g** $\frac{x}{y}$ **h** $\frac{12}{g}$
8 a $\frac{1}{2}$ **b** $\frac{x}{2y}$ **c** $\frac{5x}{6}$ **d** $\frac{a}{4}$
e $\frac{x}{3}$ **f** $\frac{1}{6x}$ **g** $\frac{4y}{7}$ **h** $\frac{ac}{2}$

9 a $8ab$ **b** $24x^2$ **c** $18xy$
10 a $11ab$ **b** $24qr$ **c** $2xy$
11 a $2y$ **b** $3b$ **c** $28rs$ **d** $8ab^2$
12 a no **b** $\frac{2a}{5}$ and $\frac{2}{5} \times a$ **c** $a = 1$ or $a = -1$
13 a $16ab$ **b** 2,5,6,1 others possible
c $2a \times 3b + 3a \times 2b + 4a \times b$. Others possible.

Exercise 5E

1 a 36 **b** 20 **c** 35
2 a 20 **b** 12 **c** 32
3 a $3a + 6$ **b** $2x + 2y$ **c** $4p + 4$ **d** $12a + 6b$
4 a $4x$ **b** 12 **c** $4x + 12$ **d** equivalent
5 a $4(x + 2) = 4x + 8$ **b** $3(a + 1) = 3a + 3$
c $4(k + 7) = 4k + 28$ **d** $3(b + 5) = 3b + 15$
6 a $6y + 48$ **b** $7l + 28$ **c** $9a + 63$ **d** $2t + 12$
7 a $2m - 20$ **b** $8y - 24$ **c** $3e - 21$ **d** $7e - 21$
8 a $60g - 70$ **b** $15e - 40$ **c** $35w + 50$ **d** $10u + 25$
e $56x - 14$ **f** $27v - 12$ **g** $14q - 28$ **h** $20c - 4v$
i $8 + 20x$ **j** $21 + 6y$ **k** $72 - 24x$ **l** $22 - 44k$
9 a 20 **b** 6 **c** 10 **d** 14
10 $2l + 2w$
11 a $7x + 6$ **b** $2a + 12$ **c** $15b$ **d** $10c + 24$
12 a $5(x + 3) = 5x + 15$ **b** $2(b + 6) = 2b + 12$
c $3(z - 4) = 3z - 12$ **d** $7(10 - y) = 70 - 7y$
13 $2(4a + 12b)$ and $8(a + 3b)$. Others possible.
14 a $ab + 4b + 2a + 8$ **b** $xy + 3y + 5x + 15$
c $6ac + 15c + 4a + 10$ **d** $20ab + 5b + 12a + 3$

Progress Quiz – Chapter 5 – Algebra

1 a 5 **b** –4 **c** –11 **d** 1
2 a D **b** A **c** F
d B **e** C **f** E
3 a 55 **b** 9 **c** 30 **d** 85
4 a 17 **b** –1 **c** 44 **d** 397
5 a N **b** N **c** L **d** L
6 a 10h **b** $9t + 7r$
c $4x - 5y + 7xy$ **d** $5kt + k$
7 a $15w$ **b** $18yz$ **c** $24abc$ **d** $110efm$
8 a y^2 **b** $12t^2$ **c** $15jh^2$ **d** $36f^2g^2$
9 a $\frac{f}{4}$ **b** $3x$ **c** $\frac{a}{3bd}$ **d** $\frac{2y}{3}$
10 a $4x + 24$ **b** $10y - 14$ **c** $20m - 15n$ **d** $8x - 3x^2$
11 a $4x + 18$ **b** $17 - 6x$ **c** $4x - 8y$ **d** $x^2 + 10x$

Exercise 5F

1 a 4 **b** 3 **c** 2 **d** 6
2 2
3 a 6 **b** 5 **c** 20 **d** 2
4 a 3 **b** 4 **c** $2b$ **d** $7x$
5 a 12 **b** 35 **c** 12, 30 **d** $14a, 21b$
e 7 **f** 3 **g** $2, q$ **h** 4
6 a 5 **b** 4 **c** 9
d 7 **e** 3 **f** 6
7 a $6x$ **b** $8a$ **c** $3b$
d $12y$ **e** $2q$ **f** $4p$
8 a $3(x + 2)$ **b** $8(v + 5)$ **c** $5(3x + 7)$
d $5(2z + 5)$ **e** $4(10 + w)$ **f** $5(j - 4)$
g $3(3b - 5)$ **h** $4(3 - 4f)$ **i** $5(d - 6)$
j $5(2x + 1)$ **k** $6(k - 2)$ **l** $2(9p + 10)$
9 a $2n(5c + 6)$ **b** $8y(3 + r)$ **c** $2n(7j + 5)$
d $4g(6 + 5j)$ **e** $2(5h + 2z)$ **f** $10(3u - 2n)$
g $3(7p - 2c)$ **h** $3(4a + 5b)$
10 For example: length = 2, width = $6x + 8$. Other answers are possible.
11 a 5 **b** $4a + 12$
12 $(x + 2)(y + 3)$
13 a $6x + 18$ **b** $6(x + 3)$ **c** $x + 3$
d $2x + 6$ **e** $3x + 9$

Exercise 5G

1 a \$10 **b** \$12 **c** \$26
2 a i 60 mins **ii** 150 mins **iii** 300 mins
b B
3 a 35 **b** 41 **c** 5
4 a $2x + y$ **b** 8
5 a $2x + 6$ **b** 24 **c** $3x$
6 a \$30 **b** $3n$ **c** \$36
7 a \$210 **b** C
8 a $5x$ **b** $10x$ **c** $5(x + 3)$ or $5x + 15$
9 a $30 + 40x$ **b** \$350
10 a \$50 **b** \$60 **c** \$230
11 a \$140 **b** $60 + 80x$
c i \$60 **ii** \$80
12 a $F + H$ **b** $F + 2H$ **c** $F + \frac{H}{2}$
13 a $10 + 4n$ **b** $20 + n$ **c** 30 **d** deal 1
e deal 3 **f i** 3 **ii** 4,10 **iii** 9

Maths@work: Pharmacist

1 a 24, healthy **b** 22, healthy **c** 21, healthy
d 24, healthy **e** 29, unhealthy
2 a $V_1 = 4$ litres **b** $V_1 = 4$ litres
c $V_2 = 2$ litres, C2 = 1 g/L
3 37.5 mL
4

		Prescribed amounts for the patient			
Medical condition	**Stock strength in mg**	**Prescribed dose in mg/day**	**Number of stock doses per day**	**Number of days**	**Total number of stock doses**
Arthritis pain	200	400	2	5	10
Asthma	300	600	2	10	20
Stomach reflux	20	40	2	7	14
Antibiotic	500	1000	2	5	10
Alzheimer's disease	5	5	1	28	28
Type 1 diabetes	20	40	2	30	60
Urinary tract infections	250	500	2	5	10
Heart disease	0.05	0.1	2	30	60
Type 2 diabetes	500	1000	2	25	50

5 a–c

Child medication dose calculations					
Child's name (Age)	**Weight in kg**	**Height in cm**	**BSA in m²**	**Adult dose in mg/day**	**Child dose in mg/day**
Amelia (2)	9	82	0.4528	500	133
Georgia (14)	58	160	1.6055	500	472
Dylan (8)	18.5	130	0.8173	250	120
Hunter (15)	61	168	1.6872	250	248
Ella (10)	25	145	1.0035	200	118
Chelsea (6)	13	112	0.6360	200	75
Benjamin (11)	63	135	1.5370	50	45

6 a Amelia 133 mg/day; Georgia 472 mg/day
b Dylan 120 mg/day; Hunter 248 mg/day; Hunter's BSA = 1.6872 which is almost the same as the adult BSA of 1.7.
c Ella takes 43 mg/day more than Chelsea
d Benjamin 45 mg/day; 90%

Puzzles and games

1 3001 sticks
2 $A = 5, B = 2, C = 7$
3

$$\begin{array}{ccccc} \boxed{3x} & + & \boxed{4x+3y+1} & = & \boxed{7x+3y+1} \\ + & & + & & + \\ \boxed{2y} & + & \boxed{y} & = & \boxed{3y} \\ = & & = & & = \\ \boxed{2y+3x} & + & \boxed{4x+4y+1} & = & \boxed{7x+6y+1} \end{array}$$

4 $3(2n+4) - 12$ simplifies to $6n \rightarrow$ Not a coincidence
5 a 25 **b** 16.25 **c** 56.25 **d** 0
6 a All perimeters = 4 m

Areas: 1 m², $\frac{3}{4}$ m², $\frac{6}{9}$ m², $\frac{10}{16}$ m², $\frac{15}{25}$ m²

b Perimeter = 4 m

Area approximately $\frac{1}{2}$ m².

Multiple-choice questions

1 C **2** A **3** D **4** C **5** E
6 E **7** D **8** E **9** D **10** A

Short-answer questions

1 a F **b** T **c** T **d** F **e** T
2 a 2 **b** 3 **c** 4 **d** 6
3 a 10 **b** 8 **c** 4 **d** 0
4 a 20 **b** 7 **c** 3 **d** 16
5 a 9 **b** 9 **c** 9 **d** 2
6 a $16m$ **b** $2a + 5b$ **c** $4y - x + 1$
d $7x + 7y$ **e** $9x + xy$ **f** $10m - 6n$
7 a $36ab$ **b** $30xy$ **c** $30xyz$
8 a $2x$ **b** $3a$ **c** $\frac{z}{5y}$
9 a $3x - 12$ **b** $10 + 2x$ **c** $6y + 12$
d $20x + 70$ **e** $3x - 15$ **f** $11z - 22$
g $12a - 44$ **h** $12b - 6$
10 a 4 **b** $7a$
11 a $2(x + 3)$ **b** $8(3 - 2g)$
c $3x(4 + y)$ **d** $7a(1 + 2b)$

12 a $5a$ **b** $3p$ **c** $5a + 3p$
13 a 70 km **b** $10n$

Extended-response questions

1 a $120 + 80n$ **b** $80 + 100n$
c A costs $360, B costs $380.
d any more than two hours
e $200 + 180n$
2 a $2xy - x^2$ **b** 33 m² **c** $4x + 2y$
d 26 m **e** Area = $3xy - 3x^2$; Perimeter = $6x + 2y$

Semester review 1

Integers

Multiple-choice questions

1 B **2** C **3** C **4** A **5** B

Short-answer questions

1 a 5169 **b** 1350 **c** –288
d 695 **e** 1386 **f** 2800
g 81 **h** 64 **i** –19
2 a –14 **b** 30 **c** 72
d –7 **e** 54 **f** –6
3 a 6 **b** 7 **c** 20 **d** 15
4 a 168 **b** 72 **c** 300 **d** 45
5 a 7^3 **b** 8^2 **c** 3^5
6 a 3^9 **b** 4^2 **c** 1 **d** 5^{12}
7 a 81 **b** 7 **c** 125 **d** 2
8 a –2 **b** 12 **c** –35
d 22 **e** 25 **f** 49

Extended-response questions

1 a Hong Kong **b** Moscow, New York
c Hong Kong **d** 14°

Lines, shapes and solids

Multiple-choice questions

1 B **2** D **3** C **4** C **5** C

Short-answer questions

1 a 66 **b** 25 **c** 123
d 35 **e** 70 **f** 98
2 a $x = 81, y = 99$ **b** $a = 75$
c $a = 62, b = 62$ **d** $a = 65, b = 65$
e $a = b = c = d = 100, e = 80$ **f** $x = 95, y = 85$
3 a 48 **b** 45 **c** 60
d 75 **e** 121 **f** 75
4 a $a = b = 90$ **b** $a = 73, b = 95$
c $a = 265, b = 30$
5 a 540° **b** 1080°

Extended-response questions

1 a $b = 65$ (supplementary to a)
$c = 65$ (alternate to b)
$d = e = 57.5$ (isosceles triangle)
$f = 122.5$ (supplementary to d)
$g = 122.5$ (revolutionangle = 360)
$h = 180$ (straight angle)
$i = 295$ (revolution)
b Answers will vary.

Fractions, decimals and percentages

Multiple-choice questions

1 D **2** C **3** C
4 B **5** B

Short-answer questions

1 a 18 **b** 1 **c** 5
2 a $\frac{1}{4}$ **b** $\frac{7}{5}$ **c** $3\frac{1}{4}$
d $-\frac{2}{21}$ **e** $\frac{1}{3}$ **f** $\frac{9}{10}$
3 a $\frac{5}{2}$ **b** $\frac{1}{8}$ **c** $\frac{5}{21}$
4 a $\frac{9}{2}$ **b** $\frac{3}{4}$ **c** $\frac{2}{3}$
5 a 6.93 **b** 7.58 **c** 4.03
d 6.51 **e** 3854.8 **f** 792
6 a 530 **b** 9600 **c** 0.614

7

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{19}{20}$	$\frac{99}{100}$	$\frac{1}{200}$
Decimal	0.25	0.5	0.2	$0.\dot{3}$	$0.\dot{6}$	0.8	0.95	0.99	0.005
Percentage	25%	50%	20%	$33.\dot{3}\%$	$66.\dot{6}\%$	80%	95%	99%	0.5%

8 a 5.6 **b** 11.76 **c** 85.5m
d \$1.98 **e** \$210 **f** 4250g
9 a \$700 **b** \$862.40
10 a \$87 **b** 25%

Extended-response questions

1 a i \$1600
ii \$1280
iii \$1024
b 5 years
c No, there will always be 80% of the previous value.

Measurement

Multiple-choice questions

1 C **2** B **3** D **4** B **5** D

Short-answer questions

1 a 500 cm **b** 180 cm **c** 90000 cm^2
d 180 cm **e** 4000 cm^3 **f** 10000 m^2
2 a 18.6 cm **b** 64 m **c** 40 m
3 a i 25.13 m **ii** 50.27 m^2
b i 47.12 cm **ii** 176.71 cm^2
4 a i 25.71 m **ii** 39.27 m^2
b i 17.85 cm **ii** 19.63 cm^2
5 a 30 m^2 **b** 48 m^2 **c** 21 cm^2
6 a 74.088 m^3 **b** 50 m^3 **c** 24 m^3
7 a 1530
b 0735

Extended-response questions

1 a 2.5 m **b** 9.82 m^2 **c** 59.82 m^2 **d** 32.85 m

Algebra

Multiple-choice questions

1 D **2** A **3** B **4** D **5** C

Short-answer questions

1 a $p+q$ **b** $3p$ **c** $\frac{m^2}{2}$ **d** $\frac{x+y}{2}$
2 a 19 **b** 68 **c** 33 **d** 698
3 a 11 **b** 23 **c** 18
d 26 **e** 24 **f** 1
4 a $24k$ **b** $3a$ **c** a^3 **d** $\frac{p}{2}$
e $7ab+2$ **f** $x-1$ **g** $2y$ **h** $2n-2m$
5 a xy **b** $\frac{10x}{7y}$ **c** $\frac{w}{5}$ **d** $\frac{17a}{5}$
6 a $2x+10$ **b** $12m-18$ **c** $4+2m$
7 a 6 **b** $5k$ **c** $20b$
8 a $6(3a-2)$ **b** $6m(n+2)$ **c** $4(2x+3)$
9 a $2x+20$ **b** $10x$
10 a $2x$ **b** $3y$ **c** $2x+3y$

Extended-response questions

1 a \$220 **b** $60+80n$
c i $100n$ **ii** 3hrs

Chapter 6

Pre-test

1 **a** 2 **b** 3 **c** 4
2 **a** 10 **b** 5 **c** 8
3 **a** 3 : 4 **b** 4 : 5 **c** 5 : 12
4 **a** 500 **b** 6000 **c** 5
d 8 **e** 1.2 **f** 15
5 **a** 2 : 1 **b** 9 : 20 **c** 3 : 4
d 1 : 3 **e** 4 : 1 **f** 2 : 25
6 **a** $1.50 **b** $18.00
7 **a** $9.98 **b** $24.95 **c** $49.90 **d** $2.50
8 1.5 km
9 $30 / h
10 **a** **i** 120 km **ii** 300 km **iii** 30 km
b **i** 3 hours **ii** $1\frac{1}{2}$ hours **iii** $\frac{1}{3}$ hour
c 6 minutes

Exercise 6A

1 **a** □□□□□○○ **b** □□○○○○○
2 **a** 3 : 7 **b** 5 : 4
3 **a** 9 : 4 **b** 7 : 12 **c** 10 : 75 (or 2 : 15)
4 **a** 1: 3 **b** 7 : 15
5 **a** 5 : 7 **b** 12 **c** 5 : 12
6 **a** 8 : 3 **b** 3 : 14 **c** 3 : 11 **d** 8 : 6 or 4 : 3
7 **a** 13 : 7 **b** 11 : 9
c 13 : 9 : 11 : 7 **d** 20 : 20 or 1 : 1
8 **a** 12 **b** 14 **c** 4 **d** 9
e 2 **f** 4 **g** 2 **h** 10
i 4 : 6 : 10 **j** 2 : 6 : 8 **k** 6 **l** 1
9 Answers may vary, some include:
a 2 : 4, 3 : 6, 5 : 10 **b** 4 : 10, 20 : 50, 200 : 500
c 4 : 3, 16 : 12, 40 : 30 **d** 3 : 1, 6 : 2, 18 : 6
10 2 : 5 and 4 : 10, 6 : 12 and 1: 2, 7 : 4 and 70 : 40
11 **a** 4 : 6 **b** 3 : 5 **c** 4 : 6 **d** 5 : 4
12 **a** 8 boys, 4 girls **b** 4 boys, 8 girls
c 9 boys, 3 girls **d** 2 boys, 10 girls
13 **a** 4 : 21 **b** 3 : 7
14 **a** **i and ii** Answers will vary.
b Each ratio length : area simplifies to 1: width.

Exercise 6B

1 **a** $\frac{1}{2}$ **b** $\frac{3}{5}$ **c** $\frac{1}{3}$ **d** $\frac{3}{7}$ **e** $\frac{8}{5}$
2 **a** 1: 2 **b** 3 : 5 **c** 1: 3 **d** 3 : 7 **e** 8 : 5
3 3 : 2
4 **a** 1: 1 **b** 1: 2
5 **a** 1: 4 **b** 1: 5 **c** 1: 6 **d** 1: 3
e 4 : 5 **f** 5 : 8 **g** 3 : 4 **h** 3 : 10
i 9 : 7 **j** 2 : 1 **k** 9 : 7 **l** 3 : 1
m 3 : 1 **n** 1: 9 **o** 6 : 11 **p** 2 : 1
q 12 : 1 **r** 1: 6 **s** 8 : 5 **t** 6 : 5
6 **a** 1: 2 : 3 **b** 4 : 7 : 11 **c** 7 : 10 : 2 **d** 17 : 7 : 3
e 1: 2 : 3 **f** 2 : 6 : 5 **g** 9 : 14 : 2 **h** 2 : 4 : 7
7 **a** 2 : 5 **b** 14 : 1 **c** 3 : 25 **d** 1: 35
e 20 : 3 **f** 2 : 25 **g** 50 : 11 **h** 5 : 1
i 2 : 5 **j** 1: 6 **k** 12 : 1 **l** 9 : 1
m 1: 16 **n** 2 : 9 **o** 1: 7 **p** 14 : 3
q 1: 8 **r** 30 : 1
8 D
9 B
10 C
11 **a** 5 : 5 : 2 : 4 : 3 : 1 : 20 **b** 20 : 20 : 8 : 16 : 12 : 4 : 80
c **i** 1: 4 **ii** 1: 1
12 Andrew did not convert amounts to the same units. Correct ratio is 40 : 1.
13 Answers may vary, some include:
a 2 hours to 300 minutes, 24 minutes to 1 hour
b 4 kilometres to 3000 metres, 2 kilometres to 1500 metres
14 Answers will vary.

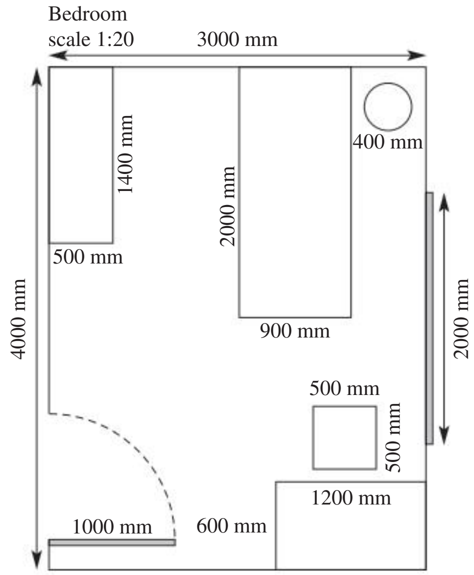
Exercise 6C

1 **a** 10 **b** 6 **c** 14 **d** 9
2 **a** In the ratio 3 : 2 the total parts = 2 + 3 = 5
b 5 parts = $25, so 1 part = $5
c Marta gets 3 parts, so Marta gets 3 × $5 = $15
d Joshua gets 2 parts, Joshua gets 2 × $5 = $10
e Total amount = $15 + $10 = $25
3 **a** 1: 3 **b** 1: 1 **c** 2 : 5 **d** 1: 4
4 **a** 4 **b** 2 **c** 7 **d** 5
5 **a** $24 and $36 **b** $70 and $40
c $150 and $850 **d** 8 kg and 40 kg
e 8 kg and 6 kg **f** 150 kg and 210 kg
g 24 m and 48 m **h** 15 m and 25 m
i 124 m and 31 m
6 **a** $100 and $300 **b** $160 and $240
c $150 and $250 **d** $180 and $220
7 **a** $40, $80, $80 **b** $50, $150, $200
c 2 kg, 4 kg, 6 kg **d** 22 kg, 11 kg, 55 kg
e 96 kg, 104 kg, 120 kg
f $5000, $10 000, $15 000, $20 000
8 **a** 60, 540 **b** 200, 100, 300
c 100, 250, 250 **d** 240, 140, 160, 60
9 nitrogen: 500 g, potassium: 625 g, phosphorus: 375 g
10 40°, 60°, 80°
11 240 students
12 120 pages
13 shirt $160, jacket $400
14 **a** 2 boys and 2 girls were absent or 5 girls and 9 boys.
b 3 : 5

Exercise 6D

1 **a** **i** 100 000 mm **ii** 100 m **iii** 0.1 km
b **i** 0.56 km **ii** 56 000 cm **iii** 560 000 mm
2 **a** 4 cm, 400 cm
b real car 100 × bigger
c 1: 100
3 **a** 60 cm, 30000 cm
b real ship 500 × bigger
c 1: 500
4 **a** 50000 **b** 50000 cm **c** 50000 m
d 250000 mm **e** 250000 cm
5 **a** **i** 620 cm **ii** 5 mm
b **i** 200 m **ii** 40 m
c **i** 16 km **ii** 25 km

d **i** 6.4 m **ii** 288 m
e **i** 0.3 mm **ii** 8.15 mm
6 **a** **i** 1 m **ii** 20 m
b **i** 20 m **ii** 2 m
c **i** 13.5 cm **ii** 7.365 cm
d **i** 20 mm **ii** 5 mm
e **i** 1.5 m **ii** 0.164 m
7 **a** 1 : 10000 **b** 1 : 1000 **c** 1 : 300
d 1 : 150000 **e** 1 : 125 **f** 1 : 200000
g 1 : 100000 **h** 50 : 1 **i** 10000 : 1
8 **a** 1 : 250 **b** 1 : 50000
c 1 : 50000 **d** 1 : 18000
e 7 : 1 or 1 : $\frac{1}{7}$ **f** 600 : 1 or 1 : $\frac{1}{600}$
9 **a** 80 m **b** 4.5 cm
10 8.5 km
11 **a** 3.8 m × 2.7 m **b** 5 m × 5 m **c** 8.3 m × 2.1 m
12 **a** 2800 km **b** 3300 km **c** 2500 km
d 1300 km **e** 3900 km
13 **a–g** Note: Different furniture arrangements also correct.



Progress Quiz – Chapter 6 – Ratios and rates

1 **a** 7 : 5 **b** 12 **c** 5 : 12
2 **a** 30 **b** 7 **c** 20,32
3 **a** 2 : 9 **b** 4 : 1 **c** 6 : 4 : 1
4 **a** 1 : 6 **b** 2 : 3 **c** 15 : 1
5 D
6 **a** 20 kg : 30 kg **b** 15 m : 3 m
c $1400 : $2600 **d** 40min : 200min
7 **a** $300 : $700 **b** $950 : $50
c $650 : $350 **d** $460 : $540
8 93 goals
9 **a** 50 000 **b** 1500 m **c** 4 km
10 **a** 40 mm **b** 23 mm

Exercise 6E

1 B, C, E, F, H
2 **a** employee's wage: $15/h
b speed of a car: 68 km/h
c cost of building new home: $2100/m^2
d population growth: 90 people/day
e resting heart rate: 64 beats/min

3 **a** $/kg **b** $/L
c words per minute **d** goals/shots on goal
e kJ/serve **f** L/min
g mg/tablet **h** runs/over
4 **a** 3 days/year **b** 5 goals/game
c $30/h **d** $3.50/kg
e $14000/acre **f** 4500 cans/hour
g 1200 revs/min **h** 16 mm/day
i 4min/km **j** 0.25 km/min or 250 m/min
5 **a** 300 km/day **b** $140/year
c 6.5 runs/over **d** 7.5 cm/year
e 1.5 kg/year **f** dropped 2.5°C/h or –2.5°C/h
6 **a** 3.8 cm/year **b** 3 cm/year
7 **a** 3 L/h **b** 7 hours
8 158 cm
9 **a** 1.5 rolls/person **b** $6/person **c** $4/roll
10 Harvey: 3.75 min/km, Jacques: 3.33 min/km; Jacques
11 **a** 1200 members/year **b** 12 years
12 **a** **i** 9 km/L **ii** $\frac{1}{9}$ L/km
b Find the reciprocal.
13 Answers will vary.

Exercise 6F

1 **a** 3 hours **b** 5 hours, ×5
c ×10, 30 minutes, ×10 **d** ×6, 720 litres, ×6
2 **a** $12, $60, ×5
b ÷5, 30 rotations, ÷5, ×7, 210 rotations, ×7
3 **a** speed = $\frac{\text{distance}}{\text{time}}$ **b** distance = speed × time
c time = $\frac{\text{distance}}{\text{speed}}$
4 D
5 **a** 1600 words **b** 50 minutes
6 **a** 2400 bottles **b** 19200 bottles
7 **a** 10 m/s **b** 7 m/s **c** 60 km/h **d** 50 km/h
e 2 km/min, 120 km/h **f** 0.75 km/min, 45 km/h
8 **a** 1080 m **b** 4.5 m **c** 36 km **d** 50 km
9 **a** 8 hours **b** $\frac{1}{2}$ hour or 30 minutes
c 11.5 hours **d** 7 seconds
10 **a** 3750 beats **b** 1380 beats **c** 80 minutes
11 2025 km
12 **a** 27 km/h **b** $2\frac{1}{4}$ km
13 **a** 58.2 km/h **b** 69.4 km/h
14 **a** 343 m/s
b 299792458 m/s
c 0.29 s
d 0.0003 s
e 874030
f How many times the speed of sound (mach 1 = speed of sound)
g 40000 km/h, 11.2 km/s
h 107218 km/h, 29.78 km/s
i 7.7 km/s
j–l Answers vary.

Maths@work: Development officer for a body and fragrance company

1 **a** 30 mL **b** 45 mL **c** 180 mL
2 **a** **i** 4 drops **ii** 10 drops
b 10 mL
3 **a** 5 : 1
b **i** 2 drops **ii** 5 drops **iii** 20 drops
4 **a** **i** 2 g **ii** 4 g
iii 10 g **iv** 60 g
b **i** 60 g **ii** 300 g **iii** 270 g
5 **a** **i** 30 drops **ii** 40 drops **iii** 50 drops
b 12 drops
c **i** 0.65 mL **ii** 1.3 mL **iii** 1.56 mL
d 60 drops of cedarwood; 200 drops of lavender; 100 drops of bergamot
6 **a** 1 tablespoon of seaweed powder
2 tablespoons of sweet almond oil (or jojoba)
1 teaspoon of aloe vera gel
1 teaspoon of honey.
b 2 tablespoons of seaweed powder
4 tablespoons of jojoba oil
2 teaspoons of aloe vera gel
2 teaspoons of honey.

7 **a**

Dilution of Essential oils				
Carrier oil	**Volume of Carrier oil in mL**	**Dilution %**	**Essential oil**	**Drops of Essential oil**
Sweet Almond Oil	10	1	Chamomile	2
Sweet Almond Oil	10	2	Lemon	4
Coconut oil	10	1	Rosewood	2
Jojoba oil	5	1	Rosemary	1
Jojoba oil	25	2	Lavender	10
Grapeseed oil	20	2	Rosemary	8

b

Dilution of Essential oils				
Carrier oil	**Volume of Carrier oil in mL**	**Dilution %**	**Essential oil**	**Drops of Essential oil**
Sweet Almond oil	25	1	Peppermint	5
Coconut oil	20	2	Geranium	8
Jojoba oil	10	1	Lavender	2
Sunflower oil	15	2	Lemon Grass	6

Puzzles and games

1 **a** toothpicks
b to rock festivals
2 **a** 1 : 1 : 2 : 2 : 2 : 4 : 4
b **i** 13 : 3 **ii** 1 : 3
3 **a** Hannah 15, Blake 10
b Hannah 25, Blake 20
c Hannah 55, Blake 50
4 **a** 2 **b** $3\frac{1}{5}$ **c** $2\frac{2}{3}$
5 1 : 3
6 A flat route (1 h 48 min) faster by 2 minutes.
7 9 km/h
8 because he thought he was a griller.

Multiple-choice questions

1 A **2** C **3** D **4** A **5** B
6 B **7** C **8** C **9** B **10** D

Short-answer questions

1 **a** 1 : 2 **b** 2 : 1 **c** 1 : 3
2 **a** F **b** F **c** T **d** F
3 **a** 25 **b** 9 **c** 32 **d** 3
4 **a** 1 : 4 **b** 3 : 2 **c** 3 : 4 **d** 1 : 8
e 3 : 1 **f** 1 : 5 **g** 3 : 2 **h** 2 : 1
i 2 : 3 **j** 2 : 1 : 5
5 **a** 5 : 2 **b** 1 : 3 **c** 2 : 5 **d** 1 : 2
e 1 : 5 **f** 1 : 4 **g** 3 : 25 **h** 3 : 10
6 **a** \$35,\$45 **b** 160 kg,40 kg
c 30 m,10 m **d** \$340,\$595,\$510
e \$60, \$20, \$20
7 1.125 L or $1\frac{1}{8}$L
8 **a** 600 m **b** 2.4 km
9 **a** scale ratio = 1 : 200
scale factor = 200
b scale ratio = 1 : 250 000
scale factor = 250000
10 50 mm
11 **a** 5 km/h **b** \$50/h **c** 140 km/day
12 **a**

$$\begin{tikzcd}
\text{7 km uses 1 L} \arrow[d, bend right, "\times 40"'] \arrow[d, bend left, "\times 40"] \\
\text{280 km uses 40 L}
\end{tikzcd}$$

b

$$\begin{tikzcd}
\text{60 words typed in 1 minute} \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\
\text{600 words typed in 10 minutes}
\end{tikzcd}$$

13 **a** **i** 7 km **ii** 294 km
b **i** \$5.60 **ii** \$39.20
14 **a** 75 km/h **b** 1.8 hours **c** 9 km

Extended-response questions

1 **a** 160 km **b** 500 km
c 11 : 30 am **d** 5 hours
e 100 km/h **f** 4 : 15 pm
g 85.2 km/h **h** Harrison's cost \$80.63
Nguyen's cost\$110.86

Chapter 7

Pre-test

1 **a** 12 **b** 27 **c** 3 **d** 10
2 **a** 8 **b** 2 **c** 5 **d** 5
3 **a** 8 **b** 42 **c** 4 **d** 2
4 **a** $11m$ **b** a **c** $9n$
d $10a-10$ **e** $11x+2$ **f** $8b+4$
5 **a** $3m+12$ **b** $2a+12$ **c** $3x+21$ **d** $4k-24$
6 12
7 **a** T **b** F **c** F **d** T
8 **a** $x=4$ **b** $x=8$ **c** $m=4$ **d** $m=6$
9 **a** $\div 5$ **b** -2 **c** $\times 3$ **d** $+3$
10 **a** $p+10$ **b** $4x$ **c** $2z$ **d** $q-6$
11 **a**

x	–2	–1	0	1	2	3
$3x-1$	–7	–4	–1	2	5	8

b

x	–2	–1	0	1	2	3
$2(x+3)$	2	4	6	8	10	12

Exercise 7A

1 **a** 19 **b** 15 **c** 16 **d** 6
e 11 **f** 13 **g** 35 **h** 1
2 **a** T **b** F **c** T
d F **e** T **f** T
3 **a** 8 **b** 12 **c** 15 **d** 45
4 **a** 13 **b** 9 **c** 2 **d** 2
5 **a** 7 **b** 9 **c** 15 **d** 8
6 **a** T **b** F **c** T
d T **e** T **f** F
7 **a** T **b** F **c** T
d T **e** F **f** T
8 **a** $x=8$ **b** $x=3$ **c** $x=7$
d $x=4$ **e** $x=1$
9 **a** $x=7$ **b** $x=13$ **c** $u=7$
d $p=19$ **e** $x=2$ **f** $x=11$
10 C
11 **a** $k+4=20$ **b** $2x+7=10$
c $x+\dfrac{x}{2}=12$ **d** $h+30=147$
e $4c+6=22$ **f** $8c+2000=3600$
12 **a** 7 **b** 42 **c** 13 **d** 26
13 **a** $3.2x=9.6$ **b** $x=3$
14 **a** $a=10,\ b=6,\ c=12,\ d=20,\ e=2$
b $a=20,\ b=6,\ c=24,\ d=80,\ e=4$

Exercise 7B

1 **a** subtracting **b** dividing
c adding **d** multiplying

2 **a**
$$\begin{tikzcd}\boxed{3} \arrow[r, bend left, "\times 5"] & \boxed{15} \arrow[r, bend left, "-2"] & \boxed{13}\end{tikzcd}$$
b
$$\begin{tikzcd}\boxed{7} \arrow[r, bend left, "-2"] & \boxed{5} \arrow[r, bend left, "\times 3"] & \boxed{15}\end{tikzcd}$$
c
$$\begin{tikzcd}\boxed{x} \arrow[r, bend left, "+2"] & \boxed{x+2}\end{tikzcd}$$
d
$$\begin{tikzcd}\boxed{a} \arrow[r, bend left, "\times 3"] & \boxed{3a}\end{tikzcd}$$
e
$$\begin{tikzcd}\boxed{f} \arrow[r, bend left, "\times 2"] & \boxed{2f} \arrow[r, bend left, "+3"] & \boxed{2f+3}\end{tikzcd}$$
f
$$\begin{tikzcd}\boxed{p} \arrow[r, bend left, "\times 5"] & \boxed{5p} \arrow[r, bend left, "-4"] & \boxed{5p-4}\end{tikzcd}$$

3 **a** $\times 4$ **b** -3 **c** $+12$

4 **a**
$$\begin{tikzcd}\frac{p}{13} \arrow[r, bend left, "+4"] & \frac{p+4}{17} \arrow[l, bend left, "-4"]\end{tikzcd}$$
b $p=13$

5 **a**
$$\begin{tikzcd}\frac{k}{6} \arrow[r, bend left, "\times 3"] & \frac{3k}{18} \arrow[l, bend left, "\div 3"]\end{tikzcd}$$
b $k=6$

6 **a** $k=10$ **b** $p=26$ **c** $r=42$
d $x=8$ **e** $w=11$ **f** $s=30$

7 **a**
$$\begin{tikzcd}\frac{x}{5} \arrow[r, bend left, "\times 3"] & \frac{3x}{15} \arrow[l, bend left, "\div 3"] \arrow[r, bend left, "+7"] & \frac{3x+7}{22} \arrow[l, bend left, "-7"]\end{tikzcd}$$
b $x=5$

8 **a** $p=15$ **b** $x=4$ **c** $q=6$
d $r=7$ **e** $u=2$ **f** $p=21$

9 **a**
$$\begin{tikzcd}\frac{x}{12} \arrow[r, bend left, "+3"] & \frac{x+3}{15} \arrow[l, bend left, "-3"] \arrow[r, bend left, "\times 2"] & \frac{2(x+3)}{30} \arrow[l, bend left, "\div 2"]\end{tikzcd}$$
b $x=12$

10 **a** $x=3$ **b** $q=2$ **c** $r=5$
d $t=7$ **e** $s=26$ **f** $s=8$
11 **a** $x=-5$ **b** $p=-6$ **c** $x=-5$
d $r=-2$ **e** $n=-30$ **f** $u=-3$

12 **a**
$$\begin{tikzcd}\boxed{x} \arrow[r, bend left, "\times 2"] & \boxed{2x} \arrow[r, bend left, "+7"] & \boxed{\frac{2x+7}{59}}\end{tikzcd}$$
b $x=26$

13 **a** $\times 5,\ +8$ **b** $+8$
14 **a** $x=3,\ x=1$ **b** order of $\times 2+4$ is reversed
15 **a** $x=35$ **b** $r=18$ **c** $x=70$ **d** $y=27$
e $x=10$ **f** $m=6$ **g** $x=46$ **h** $r=11$

Exercise 7C

1 **a** $2+3=1+4$ **b** $x+3=7$ **c** $x+5=2x+2$
2 **a** $6x=36$ **b** $2x+1=13$ **c** $x=6$
3 **a** 12 **b** 25 **c** 12
4 **a** 8 **b** $x=8$
5 B
6 **a** $2x=20$ **b** $2+q=10$
c $18=17-q$ **d** $12x=24$
e $7p+6=2p+10$ **f** $3q=2q$
7 **a** $x=3$ **b** $q=7$ **c** $k=11$
d $4x=20,\ x=5$
e $7p=28,\ p=4$ (missing operation $\div 7$)
f $10x=30,\ x=3$ (missing operation $\div 10$)
8 **a** $a=3$ **b** $t=7$ **c** $q=9$ **d** $k=9$
e $x=10$ **f** $h=10$ **g** $l=4$ **h** $g=9$
9 **a** $h=3$ **b** $u=4$ **c** $s=3$ **d** $w=8$
e $x=4$ **f** $w=5$ **g** $a=2$ **h** $y=12$
10 **a** $x=2$ **b** $k=5$ **c** $x=42$ **d** $x=20$
e $k=7$ **f** $x=30$ **g** $y=6$ **h** $x=20$

11 a $x=-6$ **b** $a=-3$ **c** $x=-10$ **d** $k=-5$
e $k=-4$ **f** $p=-1$ **g** $p=-16$ **h** $x=-5$
12 a $p+8=15, p=7$ **b** $3q=12, q=4$
c $2k-4=18, k=11$ **d** $3r+4=34, r=10$
13 a $x=7, y=2$ **b** $x=2, y=40$ **c** $x=4$ **d** $x=3.5$
14 a $x=2$ **b** $x=2$ **c** $x=5$
15 a $x=5$
b Opposite operations from bottom to top.
c For example, $7-3x=-8$

Exercise 7D

1 B
2 a T **b** F **c** F **d** T
3 a 8 **b** 5 **c** no
4 a 30 **b** 10 **c** ×2,22 **d** ×10,70
5 a $b=20$ **b** $g=20$ **c** $a=15$ **d** $k=18$
6 a $l=20$ **b** $w=-10$ **c** $s=-6$ **d** $v=12$
e $m=14$ **f** $n=14$ **g** $j=-5$ **h** $f=20$
7 a $t=28$ **b** $h=2$ **c** $a=13$
d $c=17$ **e** $s=10$ **f** $j=2$
8 a $v=20$ **b** $x=20$ **c** $y=14$
d $x=10$ **e** $p=7$ **f** $k=6$
9 a C **b** A **c** B **d** D
10 a $g=8$ **b** $x=14$ **c** $k=15$
d $x=36$ **e** $q=10$ **f** $x=27$
g $p=6$ **h** $x=1$ **i** $r=20$
11 a $x=35$ **b** $y=24$ **c** $p=14$
d $x=16$ **e** $x=12$ **f** $k=11$
12 a 7 **b** 19 **c** 3 **d** 12
e 26
13 a $\frac{b}{3}=40$ **b** $120
14 a $x=6$ **b** $x=3$ **c** $x=5$ **d** $x=2$
e $x=8$ **f** $x=12$

Exercise 7E

1 a 20 **b** 12 **c** 32 **d** T
2 a 12 **b** 14 **c** 8,10 **d** 50,30
3 a C **b** A **c** D **d** B
4 a T **b** F **c** T **d** T
5 a

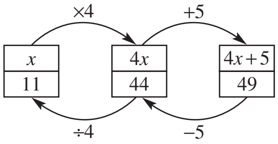
x	0	1	2
$4(x+3)$	12	16	20
$4x+12$	12	16	20
$4x+3$	3.	7	11

b $4x+12$
6 a $7x$ **b** $9p+3$ **c** $6x+4$
d $7k+4$ **e** $x+6$ **f** $5k+21$
7 a $x=5$ **b** $k=1$ **c** $r=17$
d $u=6$ **e** $j=3$ **f** $p=6$
g $m=4$ **h** $n=5$ **i** $a=3$
8 a $x=8$ **b** $x=10$ **c** $r=10$ **d** $y=3$
e $l=2$ **f** $w=2$ **g** $c=2$ **h** $d=2$
i $w=6$ **j** $p=4$ **k** $k=2$ **l** $c=10$
9 a $2(n+5)$ **b** B **c** 15
10 a $d+4$ **b** $2(d+4)$
c $2(d+4)=50$ **d** 21

11 a $3\times$ cost of shirt $+2\times$ cost of trouser
b $s=37$ **c** $37 **d** $57 **e** $356
12 a $5w+3(w+4)$ **b** $11.50
13 a $x=-6$ **b** $p=-4$ **c** $q=-19$ **d** $r=4$
e $r=2$ **f** $x=-11$ **g** $k=-10$ **h** $s=0$

Progress Quiz – Chapter 7 – Equations

1 F
2 a 5 **b** 8 **c** 34
3 a $3t+4=19$ **b** $5n+16=61$
4 a $f=5$ **b** $k=3$ **c** $p=20$ **d** $d=7$
5 a $4x+5=49, x=11$
b



6 a $m=15$ **b** $h=19$ **c** $b=15$ **d** $g=11$
7 a $p=-5$ **b** $j=-60$ **c** $b=-2$ **d** $y=-5$
8 a 100 **b** 88 **c** 20 **d** 10
9 a 4 **b** 6 **c** 65 **d** 30
10 a $18y-12$ **b** $11y-77$ **c** $40+15y$
11 a $x=5$ **b** $n=3$ **c** $d=7$

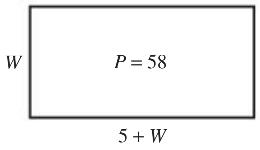
Exercise 7F

1 a formula **b** subject **c** area
2 a 11 **b** 6 **c** 7 **d** –28
3 C
4 B
5 A
6 a $A=19$ **b** $A=51$ **c** $A=7$ **d** $A=407$
7 a i 40 **ii** 12 **iii** 30
b $x=11$ **c** 11 units
8 a $a=2$ **b** $a=5$ **c** $a=11$
9 a $y=10$ **b** $x=6$ **c** $x=-2$
10 $m=5.5$
11 a $A=60$ **b** $h=4$ **c** 11
12 a $23
b i $161=3+2d$ **ii** $d=79$ **iii** 79 km
13 a 92 **b** Yes, if $p=30$
14 a A : $1.10, B : $1.20 **b** 4 minutes
c 5 minutes **d** 10 minutes

Exercise 7G

1 a D **b** C **c** A
2 a D **b** A **c** E **d** C **e** B
3 a B **b** C **c** A **d** D
4 a $p=6$ **b** $x=9$ **c** $k=4$ **d** $a=3$
5 a $a=13$ **b** 13 years old
6 a Let $$c$ = cost of one cup.
b $4c=14$ **c** $c=3.5$ **d** $3.50
7 a Let t = time spent (hours)
b $70+80t=310$ **c** $t=3$ **d** 3 hours
8 a Let $$c$ = cost of one chair.
b $6c-200=1300$ **c** $c=250$ **d** $250
9 a $2(4+w)=72$ or $8+2w=72$
b $w=32$
c 32 cm

10 a $4w = 24, w = 6$, so width = 6 cm.
b 36 cm^2
11 a $a + a + 4 = 40, a = 18$, so Alison is 18 years old.
b 22 years old
12 a



b 12 m **c** 204 m^2
13 a $x = 80$ **b** $x = 75$ **c** $x = 30$
d $x = 110$ **e** $x = 45$ **f** $x = 65$

Maths@work: Financial officers at a local council

1 a \$1610 **b** \$1955 **c** \$3606.40
2 a i \$2869.50 **ii** \$3213.84 **iii** \$3873.83
iv \$4304.25 **v** \$5739.00
b $Q = \frac{R}{4}$
3 a $R = 0.0048207 \times LV + 456.3$
b \$2119.46
c \$261 059
4 a i \$191.50 **ii** \$3226.50
b $R = 0.0054x + 765$
c \$866.25
d \$1197.50
e $R = \frac{(0.00575y + 765)}{4}$
5 b 0.575, 0.481, 0.491, 0.475, 0.352

Puzzles and games

1 □ = 12, △ = 2, ○ = 9
2 a 88 **b** 6 **c** 13
d \$44.44 **e** $33\frac{1}{3}$
3 a 2nd step or 3rd line (can't divide by 0)
b

$$\begin{tikzcd} 0 = 1 \arrow[d, bend right, "\times 28"'] \arrow[d, bend left, "\times 28"] \\ 0 = 28 \arrow[d, bend right, "+22"'] \arrow[d, bend left, "+22"] \\ 22 = 50 \end{tikzcd}$$

4 a $4x + 2$ and $4\left(x + \frac{1}{2}\right)$, $2(x + 4)$ and $4(x + 2)$
b $4x + 2 = 4\left(x + \frac{1}{2}\right)$
c $2(x + 4) = 2x + 4$
5 a $x = 88.5$ **b** $a = 37.5$ **c** $x = 75$
6 a 65 kg, 62 kg, 55 kg
b 70 kg, 60 kg, 48 kg
c 35 kg, 42 kg, 45 kg, 48 kg

Multiple-choice questions

1 B **2** C **3** D **4** B **5** C
6 A **7** D **8** E **9** C **10** B

Short-answer questions

1 a F **b** T **c** T
2 a $m = 4$ **b** $m = 6$ **c** $q = 5$ **d** $z = 50$
3 a $2m + 3 = 27$ **b** $3(n + 4) = 18$
c $x + x + 1 = 7$ (or $2x + 1 = 7$)
4 a $x = 4$ **b** $u = 4$ **c** $d = 6$
d $b = 7$ **e** $f = 3$ **f** $k = 2$
5 a $3x = 12$ **b** $2b = 14$ **c** $x = 5$
6 a subtract 15 **b** add 5 **c** multiply by 2
7 a $a = 5$ **b** $b = 6$ **c** $n = 16$
d $c = 2$ **e** $x = 9$ **f** $x = 2$
8 a $m = 6$ **b** $x = 8$ **c** $k = 30$
d $y = 18$ **e** $k = 52$ **f** $x = 32$
9 a $2x + 10$ **b** $3q - 30$ **c** $8r + 12$
d $7x + 15$ **e** $4z + 18$ **f** $5q - 15$
10 a $x = 3$ **b** $x = 2$ **c** $p = 6$
d $x = 5$ **e** $x = 10$ **f** $k = 12$
11 a $F = 30$ **b** $m = 4$ **c** $m = 1$
12 a $I = 21$ **b** $M = 3$ **c** $c = 4$
13 a D **b** $m = 1.5$ **c** \$1.50
14 a $x = 3, y = 2$
b $x + x + 1 + x + 2 = 39, x = 12$
c 8.5

Extended-response questions

1 a $10 + 5n$
b i $10 + 5n = 55$ **ii** $n = 9$ **iii** 9 rides
c \$100 **d** 7 rides
2 a $S = 20 + 0.12n$ **b** 30 times
c $Y = 15 + 0.2n$ **d** 25
e 63 is the minimum number.

Chapter 8

Pre-test

1 a 0, 1, 2, 4, 6, 7, 9, 10, 14
b 20, 30.6, 36, 100, 101, 204
c 1.2, 1.7, 1.9, 2.7, 3.2, 3.5
2 a Total = 40, average = 8
b Total = 94, average = 18.8
c Total = 3.3, average = 0.66
3 a $\frac{1}{6}$ **b** 150° **c i** \$420 **ii** \$630
4 a 4 **b** 3 **c** Saturday **d** 26
e $\frac{1}{3}$
5 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** 0

Exercise 8A

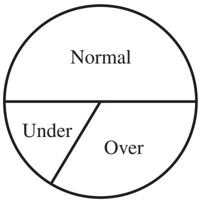
1 a 320 **b** 270 **c** 300
d Expton **e** Calcville
2 a 10 **b** 6 **c** Phillip
d Nyree **e** 4 years
3 a 9 **b** 12 **c** Badminton
d Badminton 35 **e** Water polo
Water polo 60
Hand ball 54
4 a Slesha **b** Ross **c** 4th and 5th
5 a rent **b** charity **c** 50% **d** 25% **e** \$2400

6 **a** 2 hours **b** 7 hours **c** sleeping **d** $\frac{1}{6}$
7 **a** 20 **b** periods 3 and 6
c period 7 **d** periods 1 and 5
8 **a** 20°C
b 10°C
c midday
9 **a** The distance doesn't change for a period of time.
b 10 am to 10 : 30 am
c 280 km
d 1 hour
e Red 160 km, Blue 200 km
f Red $4\frac{1}{2}$ hours, Blue $4\frac{1}{2}$ hours
10 **a** Survey 2 **b** Survey 1 **c** Survey 3
11 **a** 1 Town C 2 Town A 3 Town B
b 1235 people/town **c** 1080 people

12 **a**

	Underweight	Normal weight	Overweight	Total
Number of months	2	6	4	12
Fraction of 12	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	1
Angle	60°	180°	120°	360°

b



c Can see how weight changes over time.
d Can see how much of the year the dog was underweight, overweight and normal weight.
e Answers will vary.

Exercise 8B

1 **a** T **b** F **c** T **d** F
2 **a** 4 **b** 7 **c** II **d** 𝍸 𝍸 I
3 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 7
4 **a**

Handspan	Frequency
17	1
18	2
19	3
20	3
21	0
22	2
23	0
24	1

b

Handspan	Frequency
17–19	6
20–22	5
23–25	1

5 **a**

	Passes	Shots at goal	Shots that go in	Steals
Frequency	3	12	8	2

b 12 **c** 8 **d** 2
6 **a**

People in family	2	3	4	5	6	7	8
Tally	I	II	IIII	IIII	IIII	II	III
Frequency	1	2	4	4	4	2	3

b 4 **c** 9
7 **a**

Number of hours	0–1	2–4	5–9	10–14	15–19	20–24	25–166
Frequency	5	3	12	15	9	4	2

b 50 **c** 9 **d** 8 **e** 35
8 **a**

Height (cm)	Tally	Frequency
130–139	III	3
140–149	𝍸	5
150–159	II	2
160–169	III	3
170–179	III	3
180–189	I	1
190 +	IIII	4

b 2 **c** 5 **d** 10
9 **a** 10 **b** 2 **c** 4 **d** 17
10 **a** B **b** D **c** A **d** C
11 **a** 28 **b** 130 **c** 19
d 13.1 years old
e

Age	12	13	14	15	16	17
Frequency	5	39	19	33	33	1

12 **a**

Score	0–19	20–39	40–59	60–79	80–100
Frequency	0	4	7	20	12

b

Score	0–29	30–59	60–89	90–100
Frequency	3	8	30	2

13 **a** 2
b Any arrangement of 3, 3, 2, 1 will be correct.
c All arrangements of 3, 2, 2, 1, 1 will be correct.
d Priscilla = 2.25 hrs/night, Joey 1.8 hrs/night
e 2.25 hours more homework.

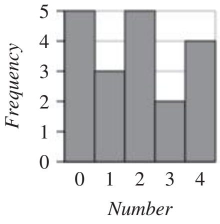
Exercise 8C

1 **a** 2 **b** 9 **c** 11 years old
2 **a** 4 **b** 4 **c** 8
4 **a** The mistake is the columns are not of equal width. Use a ruler to mark an even scale.

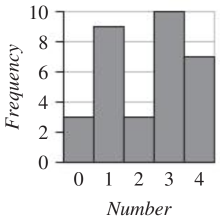
b The mistake is that there are gaps between the columns.

c Both histograms do not have numbers and words labelling each scale.

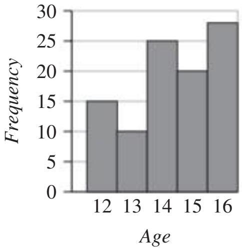
5 a



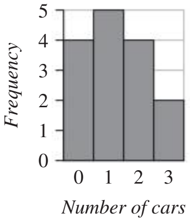
b



c



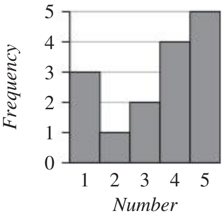
d



6 a i

Number	Tally	Frequency
1	III	3
2	I	1
3	II	2
4	IIII	4
5	𝍸	5

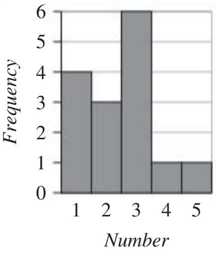
ii



b i

Number	Tally	Frequency
1	IIII	4
2	III	3
3	𝍸 I	6
4	I	1
5	I	1

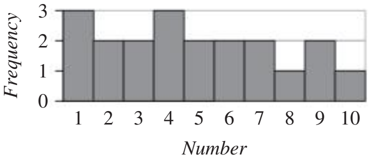
ii



c i

Number	Tally	Frequency
1	III	3
2	II	2
3	II	2
4	III	3
5	II	2
6	II	2
7	II	2
8	I	1
9	II	2
10	I	1

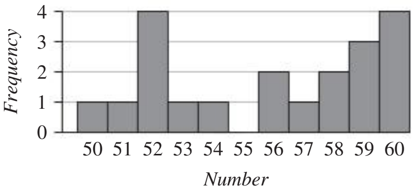
ii



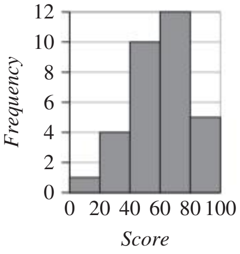
d i

Number	Tally	Frequency
50	I	1
51	I	1
52	IIII	4
53	I	1
54	I	1
55		0
56	II	2
57	I	1
58	II	2
59	III	3
60	IIII	4

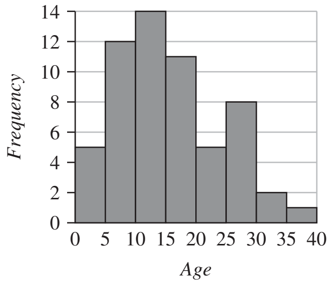
ii



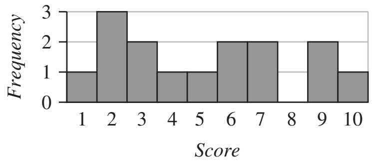
7 a



b



8 a



b Edwin is worse than Fred as most of Fred's scores are 8 or higher.

9 a D **b** A **c** B **d** C

10 a 7 days
b 4 days
c 4 cars in one day sold by Marie
d Con
e Frank who sold 27 cars
f Bill who sold 15 cars

11 a It would look identical but the age labels would start at 22 and go to 26.
b It would look just like the right half (12, 13, 14) but with the age axis labelled 0, 1, 2.

12 a 9 weeks of 10, 8 weeks of 9, 5 weeks of 8, 4 weeks of 7, 3 weeks of 6, 1 week of 5 (any list with the higher scores coming first and the lowest scores last is correct).
b 9 weeks of 5, then 8 weeks of 6, then 7 weeks of 7, then 4 weeks of 8, then 2 weeks of 9 out of 10
c They were absent from the tests perhaps.

13 a

Survey location	Height graph (cm)	Weight graph (kg)	Age graph (years)
Primary school classroom	Graph 4	Graph 7	Graph 6
Shopping centre	Graph 8	Graph 2	Graph 9
Teachers common room	Graph 5	Graph 3	Graph 1

b **i–iii** Answers will vary.

Exercise 8D

1 a mode **b** mean **c** median
d range **e** outlier
2 a 15 **b** 5 **c** 3
3 a 1, 2, 4, 5, 6, 7, 9 **b** 5 **c** 5
4 a 7 and 9 **b** 16 **c** 8
5 a 8 **b** 1 **c** 7
6 a 9 **b** 10 **c** 15 **d** 14
e 30 **f** 27 **g** 16.9 **h** 8.7
7 a i 5 **ii** 4 **b i** 2 **ii** 2
c i 5 **ii** 3 **d i** –3 **ii** 0
e i 0 **ii** –9 **f i** 0 **ii** 3
g i 12.9 **ii** 15 **h i** 13.1 **ii** 20
i i 11.1 **ii** 12 **j i** 10.4 **ii** 5
k i 2.4 **ii** –6 **l i** –3.4 **ii** –6
8 a 6 **b** 4 **c** 8 **d** 5
e 8 **f** 7 **g** 5 **h** 5.5
i 7.5 **j** 8 **k** 10.5 **l** 12
9 a 8.4 **b** 8 **c** 8
10 a white **b** meat-lovers
c Wednesday **d** South Australia
11 a 3 **b** 10 **c** 7, 7, 7, 9, 9, 9, 10, 10, 10, 10
d 8.8 **e** 9 **f** 3
12 a Business B, \$200 000
b Mean A = \$52 000, mean B = \$78 000
c \$26 000 larger
d \$50 000 for both A and B
e No
f The median, \$50 000 as it is not affected by the outlier.
13 a 15 **b** 35 **c** Nathan **d** Gary
14 a 12, 12, 13, 13, 13, 13, 14, 14, 14, 14, 14, 15, 15, 15, 15, 16, 16, 16, 16, 16, 16, 16, 16, 17, 17, 17, 17, 17, 18
b 6 years **c** 16 years old
d 15.03 years old **e** 15
f i 16.23 years **ii** 15 years
iii The mean has increased the most. The median is unchanged.
15 a \$1477 778 **b** \$630 000
c A strong effect – it makes the mean significantly higher.
d Only increases the median by a small amount.
e Median is not easily distorted by a few very large values.

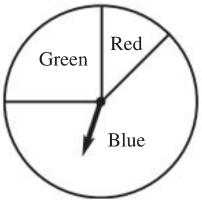
Exercise 8E

1 a survey **b** sample **c** biased
d symmetrical **e** skewed
2 a surveying 1000 randomly selected people
b surveying 10 friends
3 a $\frac{2}{5}$ **b** 2000 **c** 300
4 a symmetrical **b** skewed
c skewed **d** symmetrical
5 a skewed **b** $\frac{2}{5}$ **c** 400 **d** $\frac{6}{25}$ **e** 1200
f More likely that people will have pets if near a vet clinic.
6 a symmetrical **b** $\frac{28}{100} = \frac{7}{25}$ **c** 420
d $\frac{35}{100} = \frac{7}{20}$ **e** 1050
f In a wealthy suburb the houses are more likely to be larger.
7 a skewed **b** $\frac{52}{120} = \frac{13}{30}$ **c** 15 600
d $\frac{28}{120} = \frac{7}{30}$ **e** 1120
f The people on this train probably start work early and are less likely to eat breakfast.
8 a 108 g **b** symmetrical **c** $\frac{5}{8}$
d 500 **e** $\frac{17}{128}$ **f** 272
9 a Yes, it is required information.
b No, it is too vague or personal.
c No, it is too vague or personal.
d No, it addresses wealth but not income.
e No, it is irrelevant.

f Yes, it can be used to decide income.
g No, if it is not a pay day then results will be distorted.
10 a at midday on a Thursday on a major road
b outside a political party office
c in a butcher's shop
d at 11 pm, when people will buy just a few items
11 a at a professional dance studio in the afternoon
b in a Bank
c Choose a large random sample.
12 a Only one year level. Possibly streamed class, so similar work ethic.
b Only males would be surveyed, also same age.
c Sample size too small.
d A range of students in age, gender and results.
13 Answers will vary.

Exercise 8F

1 a trial **b** outcomes **c** sample space
d complement **e** A'
2 a B **b** C **c** D **d** A
3 a Event C **b** Event A **c** Event B **d** Event C
4 a T **b** F **c** T **d** T
5 a P, I, A, N, O **b** $\frac{1}{5}$ **c** $\frac{3}{5}$ **d** $\frac{2}{5}$
e $\frac{4}{5}$ **f** $V' = P, N$ **g** $\Pr(V') = \frac{2}{5}$
6 a 1, 2, 3, 4, 5, 6 **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** 1, 2, 3, 4, 6
e $\frac{5}{6}$ **f** 0
7 a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{3}{10}$ **d** $\frac{4}{5}$
e $\frac{1}{2}$ **f** $\frac{7}{10}$ **g** choosing a purple marble
8 a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 **b** $\frac{1}{10}$ **c** $\frac{1}{5}$
d $\frac{3}{10}$ **e** $\frac{2}{5}$ **f** $\frac{3}{5}$
9 a red, green, blue, yellow, purple, blue, green, blue
b $\frac{1}{8}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$
e green, green, red, yellow, purple
f $\frac{5}{8}$ **g** $\frac{3}{4}$
h spinning purple (or spinning yellow)
i spinning orange
10 a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$ **d** $\frac{5}{6}$
11 a $\frac{1}{2}$ **b** $\frac{3}{5}$ **c** 0 **d** $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** $\frac{4}{5}$
12 a 1 red, 2 orange, 3 purple
b $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$
13



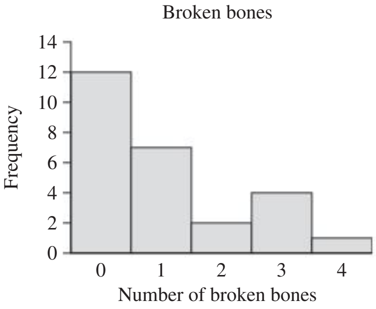
14 a $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** 18
d It approaches $\frac{1}{2}$ or 0.5.

Progress Quiz – Chapter 8 – Statistics and probability

1 a 30 **b** 14 **c** 4
2

Number	1	2	3	4	5
Frequency	3	7	4	2	4

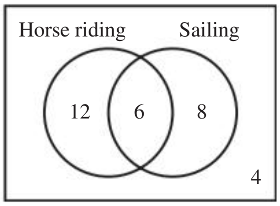
3



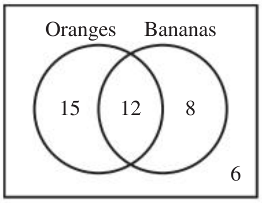
4 a 13 **b** 43
5 a i 3 **ii** 2 **b i** –3 **ii** –4
6 a 12 **b** 59 **c** 5 **d** –3
7 a 1 **b** 5
c 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 8
d 6 **e** 6 **f** 3
8 a $\frac{3}{5}$ **b** 600 **c** 12000
9 a R, E, S, I, L, I, E, N, C, E **b** $\frac{3}{10}$ **c** $\frac{1}{2}$
d $\frac{4}{5}$ **e** $\frac{7}{10}$
10 a 9 **b** 40 **c** 13 **d** $\frac{1}{5}$

Exercise 8G

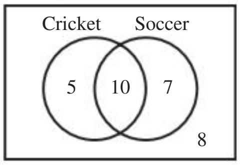
1 a 8 **b** 13 **c** 3 **d** 18 **e** 4 **f** 22
2 a i–iii



b Total = 12 + 6 + 8 + 4 = 30 students
3 a 2 **b** 4 **c** 1 **d** 3
4



5 a 8 **b**



c 15 **d** 22

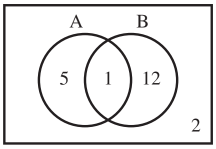
6 **a** $14 + 8 + 18 + 10 = 50$ **b** $\frac{14}{50} = \frac{7}{25}$

c $\frac{8}{50} = \frac{4}{25}$ **d** $\frac{26}{50} = \frac{13}{25}$ **e** $\frac{40}{50} = \frac{4}{5}$

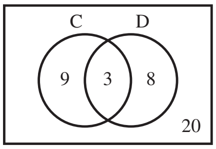
7 **a** 20 **b** 15 **c** $\frac{15}{20} = \frac{3}{4}$ **d** 10 **e** $\frac{10}{20} = \frac{1}{2}$

8 **a** 4 **b** $\frac{2}{15}$ **c** $\frac{13}{15}$ **d** $\frac{9}{15}$

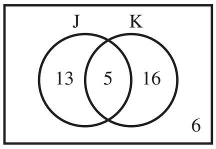
9 **a**



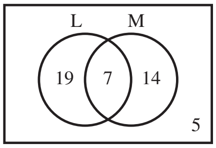
b



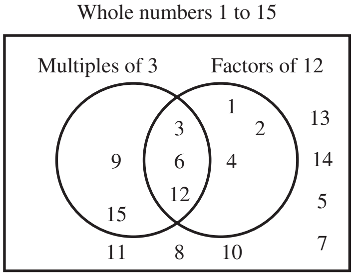
c



d



10 **a**

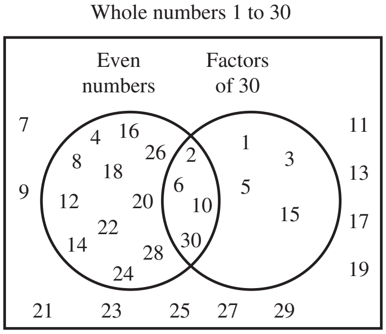


b 3 **c** 3 **d** $\frac{5}{15} = \frac{1}{3}$ **e** $\frac{6}{15} = \frac{2}{5}$

11 **a** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30

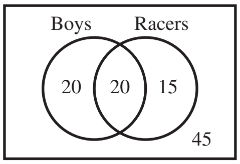
b 1, 2, 3, 5, 6, 10, 15, 30

c



d 4 **e** $\frac{4}{30} = \frac{2}{15}$ **f** 15 **g** $\frac{4}{15}$

12 **a**



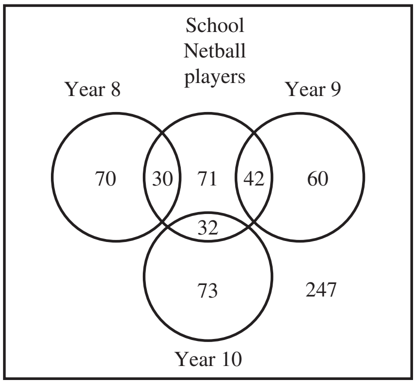
b **i** 20% **ii** 15% **iii** 60% **iv** 65%

c $\frac{4}{7}$ **d** $\frac{3}{7}$ **e** $\frac{1}{2}$

13 **a** 625 **b** **i** $\frac{6}{125}$ **ii** $\frac{42}{625}$ **c** 175

d **i** $\frac{6}{35}$ **ii** $\frac{6}{25}$ **e** $\frac{3}{10}$

f



g $\frac{32}{175}$

Exercise 8H

1 **a**

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	30
Total	40	35	75

b 30 **c** 20 **d** 75

2

	Like lamingtons	Dislikes lamingtons	Total
Like Anzacs	23	14	37
Dislikes Anzacs	12	3	15
Total	35	17	52

3 **a** 27 **b** 15 **c** 12
d 25 **e** 10 **f** 40

4 **a** $\frac{15}{60} = \frac{1}{4}$ **b** $\frac{24}{60} = \frac{2}{5}$ **c** $\frac{12}{60} = \frac{1}{5}$ **d** $\frac{9}{60} = \frac{3}{20}$

5

	Like surfing	Dislike surfing	Total
Like hiking	45	10	55
Dislike hiking	25	5	30
Total	70	15	85

6 **a**

	Employed	Unemployed	Total
TAFE degree	16	3	19
No TAFE degree	12	2	14
Total	28	5	33

b 2 **c** 28 **d** 16 **e** 31

7 a 26 **b** 12 **c** 11

d i $\frac{2}{13}$ **ii** $\frac{7}{26}$ **iii** $\frac{7}{13}$ **iv** $\frac{15}{26}$

8 a $\frac{1}{5}$ **b** $\frac{17}{40}$ **c** $\frac{1}{4}$ **d** $\frac{3}{8}$

9 a

	B	Not B	Total
A	20	50	70
Not A	20	10	30
Total	40	60	100

b

	B	Not B	Total
A	6	5	11
Not A	4	3	7
Total	10	8	18

10 a

	Sports	Not Sports	Total
Automatic	2	13	15
Not Automatic	8	17	25
Total	10	30	40

b $\frac{1}{5}$ **c** $\frac{13}{40}$

11 a

	Automatic car	Not automatic	Total
4WD	9	24	33
Not 4WD	17	10	27
Total	26	34	60

b i $\Pr(\text{Automatic and 4WD}) = \frac{9}{60} = \frac{3}{20}$

ii $\Pr(\text{4WD}) = \frac{33}{60} = \frac{11}{20}$

iii $\Pr(\text{automatic}) = \frac{26}{60} = \frac{13}{30}$

iv $\Pr(\text{not4WD}) = \frac{27}{60} = \frac{9}{20}$

v $\Pr(\text{not automatic}) = \frac{34}{60} = \frac{17}{30}$

vi $\Pr(\text{neither automatic nor 4WD}) = \frac{10}{60} = \frac{1}{6}$

12 a

	Like volleyball	Dislike volleyball	Total
Like tennis	12	6	18
Dislike tennis	11	4	15
Total	23	10	33

b $\frac{11}{33} = \frac{1}{3}$

c 18

d $\frac{12}{18} = \frac{2}{3}$

e $\frac{6}{18} = \frac{1}{3}$

13 a

	Like reading	Don't like reading	Totals
Like exercise	33	8	41
Don't like exercise	7	3	10
Like computer games	24	12	36
Don't like computer games	11	2	13
Total	75	25	100

b $\frac{33}{41}$ **c** $\frac{3}{10}$

Exercise 8I

1 a 2 **b** $\frac{1}{5}$ **c** $\frac{3}{5}$

2 a 6 **b** $\frac{6}{10} = \frac{3}{5}$ **c** $\frac{4}{10} = \frac{2}{5}$

3 a $\frac{27}{50}$ **b** 378 **c** $\frac{1}{2}$ **d** 350

4 a $\frac{55}{100} = \frac{11}{20}$ **b** 550 **c** $\frac{1}{2}$ **d** 500

5 a $\frac{3}{5}$ **b** $\frac{1}{10}$ **c** $\frac{43}{50}$ **d** 40

6 a

No. of cars	0	1	2	3	4
Frequency	12	37	41	8	2

b 100 **c** 0.12 **d** 0.51

7 a $\frac{1}{3}$ **b** 200 **c** 300

8 a i 0.35 **ii** 0.25 **iii** 0.2 **iv** 0.2
b 200

9 a 0.19 **b** 0.17 **c** experimental

10 a Answers will vary. **b** Answers will vary.
c 10 **d** Possible but unlikely.

11 a 3 **b** 2 **c** 14 **d** 3 **e** 19 **f** 8

12 a 0 **b** $\frac{3}{11}$ **c** F **d** 0 **e** T

13 a Could be (Red: 1, 2, Blue: 3, 4, Green: 5, 6)
b Could be (Red: 1, 2, 3, 4, Blue: 5, Green: 6)
c Could not be; probability of $\frac{1}{5}$ cannot be achieved with single die roll.

14 a–d Answers will vary.

Maths@work: Student Representative Council (SRC) coordinator

1 a 150 **b** 120
c Tahlia P, Eden V, Hicham J, Charlie K, Aanya N
d Tahlia P 16%, Eden V 14%, Hicham J 13%, Charlie K 11%, Aanya N 10%
e Answers will vary.

2 a Alex P 330, Jia Hao N 165, Kelly Y 105, Samantha W 90, Nelson C 60
b Alex P and Jia Hao N

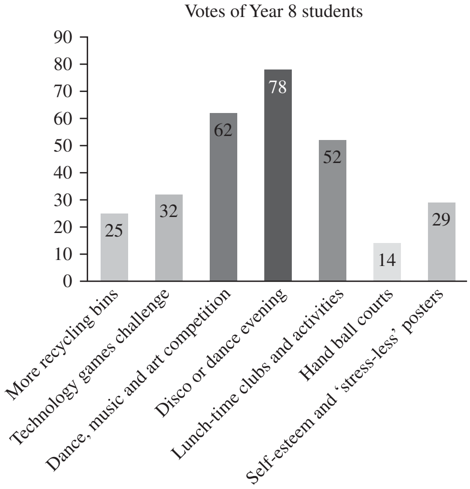
3 a 885 students **b** 9%
c 2.44 times per week **d** 49%

e From the survey results it appears that there are students working on library computers every lunchtime. Whether a competition could be run depends on how many students will be in the competition and how many library computers are spare.

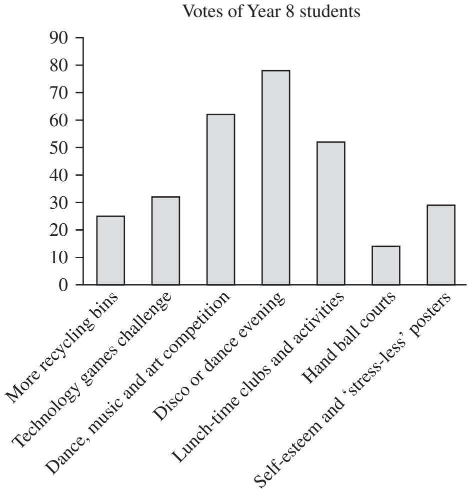
f Answers will vary. Possible questions could include:
'Which days do you regularly work on the library computers at lunchtime?'
'Would you be interested being part of a lunch-time computer competition in the library?'

g Answers will vary.

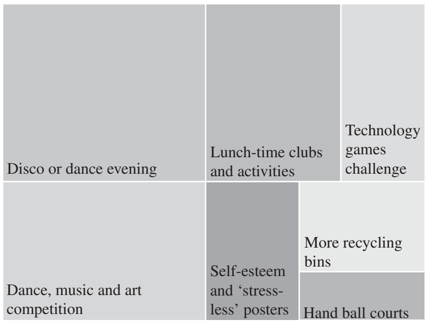
4 Horizontal Column chart



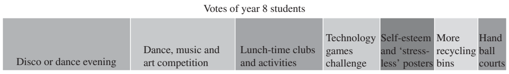
Vertical Column chart



Treemap



Divided bar chart



Puzzles and games

1 A BOWLING MACHINE

2 **a** 4 **b** 11 **c** 4.5

3 0.25

4 **a** MOON **b** OFF **c** DING
d PROBABILITY **e** STUMBLE **f** TRY

5 32

6 No, many reasons e.g. sum of 12 only from 6, 6 but sum of 9 from 4, 5 : 5, 4 : 6, 3 : 36 so sum of 9 more likely than sum of 12.

7 No, must have repeated points in 5 – 9 and 10 – 14.

Multiple-choice questions

1 C **2** B **3** C **4** A **5** C
6 B **7** B **8** C **9** D **10** E

Short-answer questions

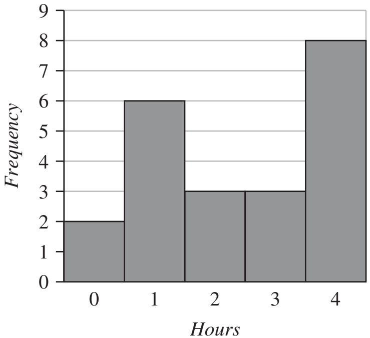
1 **a** government bus **b** train **c** 75% **d** 1000
e Example: Prices went up for government buses.

2 **a** 22

b

Hours	0	1	2	3	4
Frequency	2	6	3	3	8

c



d $\frac{1}{11}$ **e** 53 hours **f** 2.4

3 **a** 38, 43, 44, 44, 52, 53, 55, 56, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 74, 84
b 44 **c** 61.5 **d** 46

4 **a** 27 **b** 10, 10, 11, 11, 11, 11, 12, 12, 12, 12, 13, 13, 13, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14
c 12.78 years **d** 13 years

5 **a** Lowest: 50 kg, highest: 85 kg, range = 35 kg

b

Weight	Frequency
50 – 54	6
55 – 59	6
60 – 64	8
65 – 69	7
70 – 74	7
75 – 79	1
80 – 85	5

c 60 – 64 kg
d Only teenagers were chosen, not including children or adults.

6 a i 6.5 **ii** 6 **iii** 13

b median = 6
mean = 10
The mean changes the most.

c Outlier.

7 a Not enough people, and her friends might work harder (or less hard) than other students.

b She could choose 10 people who worked less hard than her.

8 a $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** 1, 4, 6, 8 **e** $\frac{1}{2}$

9 a M, A, T, H, E, M, A, T, I, C, I, A, N **b** $\frac{2}{13}$

c $\frac{6}{13}$ **d** $\frac{7}{13}$ **e** $\frac{12}{13}$

10 a 50 **b** 25 **c** $24\% = \frac{6}{25}$

d $1\% = \frac{1}{100}$ **e** $\frac{12}{25} = 48\%$ **f** $\frac{24}{25} = 96\%$

11 a

	Mufflers	No mufflers	Total
Tyres	8	4	12
No tyres	2	6	8
Total	10	10	20

b 6

c i $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{7}{10}$ **iv** $\frac{1}{10}$ **v** $\frac{3}{10}$

12 a $\frac{2}{5}$ **b** 80

Extended-response questions

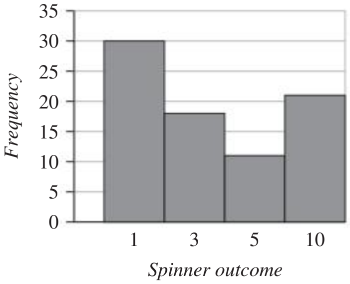
1 a

	Uses public transport	Does not use public transport	Total
Own a car	20	80	100
Do not own a car	65	35	100
Total	85	115	200

b 200 **c** $\frac{1}{2}$ **d** $\frac{23}{40}$ **e** $\frac{1}{10}$ **f** $\frac{1}{5}$

g i More public transport users expected.
ii People less likely to use public transport in regional area.

2 a

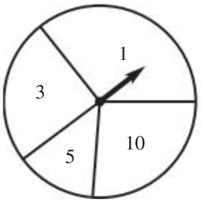


b 1, it has the most occurrences.

c 3 and 10, as they occur a similar number of times.

d $\frac{3}{8}$

e



Chapter 9

Pre-test

1 a i 50 km **ii** 0 km **iii** 150 km
b 200 km **c** 2nd hour **d** 3rd hour

2 a George **b** Amanda

3 a −1, 2, 3 **b** −5, 3, 5 **c** 2, − 8, − 13 **d** −22, 5, 14

4 a 1 **b** 1 **c** 0 **d** −3

5 a 6 **b** −1 **c** 0 **d** 2

6 a (1, 2) **b** (2, 1) **c** (3, 2)

7 a 7 **b** 4 **c** 2

8 a −1 **b** −5 **c** −11

9 a

x	−2	−1	0	1	2
y	−4	−2	0	2	4

b

x	−2	−1	0	1	2
y	−10	−7	−4	−1	2

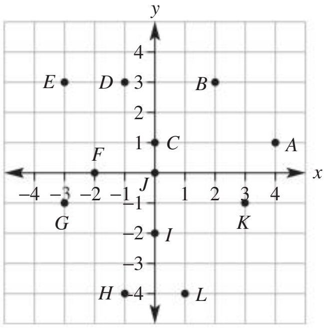
Exercise 9A

1 a 3 **b** −4 **c** 5 **d** −8 **e** (0, 0) **f** y

2 $A(1, 1)$, $B(5, 0)$, $C(3, 4)$, $D(0, 4)$, $E(-1, 2)$, $F(-3, 3)$, $G(-5, 1)$, $H(-3, 0)$, $I(-4, -2)$, $J(-2, -5)$, $K(0, -3)$, $L(2, -3)$, $M(5, -5)$

3 a 3 **b** −1 **c** −2 **d** 0
e −2 **f** 0 **g** −3 **h** 0

4



5 All give straight lines, passing through origin.

6 a first **b** fourth **c** second
d third **e** first **f** third

7 a B **b** C **c** E **d** D

8 a triangle **b** rectangle
c parallelogram **d** kite

9 a line on the y-axis

10 a house **b** fish

Exercise 9B

1 a 3, 5 **b** 0, 6 **c** −3, −1
d 8,16 **e** −5, −2 **f** 0, 1

2 a 3 **b** 1 **c** −1 **d** −3
e −5 **f** −7 **g** −9 **h** −11

3 a 1 **b** −2 **c** −11 **d** −20
e 4 **f** 10 **g** 31 **h** 151

4 a

t	0	1	2	3	4
d	0	40	80	120	160

b 120 km **c** 2 hours

5 a

t	0	1	2	3	4	5
v	1000	1020	1040	1060	1080	1100

b 1080 L **c** 5 minutes

6 a

x	–2	–1	0	1	2
y	–6	–3	0	3	6

b

x	–2	–1	0	1	2
y	–4	–3	–2	–1	0

c

x	–2	–1	0	1	2
y	–3	–1	1	3	5

d

x	–2	–1	0	1	2
y	–7	–5	–3	–1	1

e

x	–2	–1	0	1	2
y	4	3	2	1	0

f

x	–2	–1	0	1	2
y	1	0	–1	–2	–3

g

x	–2	–1	0	1	2
y	3	1	–1	–3	–5

h

x	–2	–1	0	1	2
y	10	6	2	–2	–6

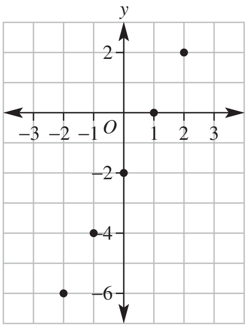
i

x	–2	–1	0	1	2
y	1	–5	–11	–17	–23

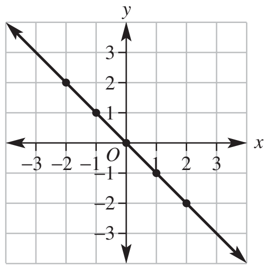
7 a i \$140 **ii** \$700 **b** \$980 **c** 4 days
8 a i 2 **ii** –1 **b i** 3 **ii** 0
c 2 **d** 1
9 a i \$15 000 **ii** \$0
b i 3000 **ii** 4000 **iii** 6000
c A loss is made **d** \$12 000
10 a i 10 **ii** 36 **iii** 55
b i 21 **ii** 78
c i 28 **ii** 210 **iii** 5050

Exercise 9C

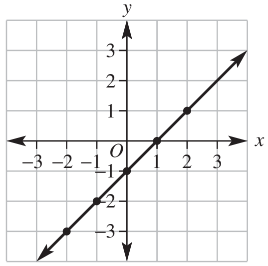
1 a 5 **b** 7 **c** 3 **d** 1
e –7 **f** –11 **g** 25 **h** –21
2 a 2 **b** –2 **c** –13
3



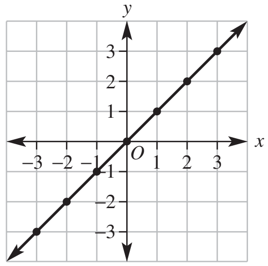
4 a



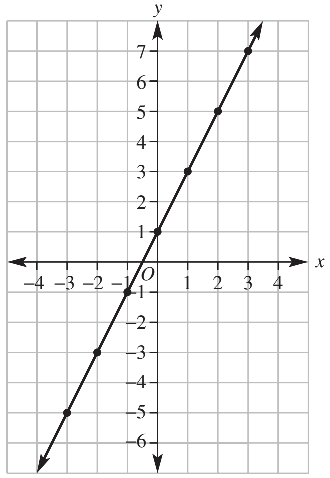
b



c

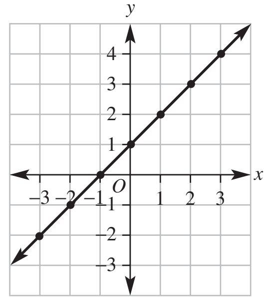


d



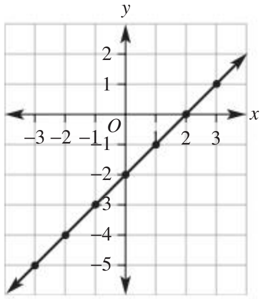
5 a

x	–3	–2	–1	0	1	2	3
y	–2	–1	0	1	2	3	4



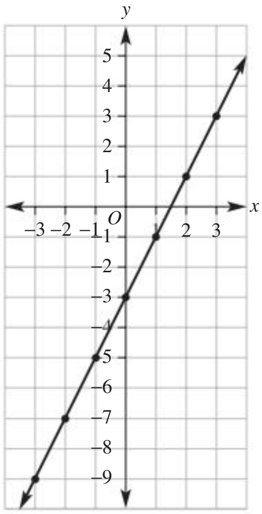
b

x	–3	–2	–1	0	1	2	3
y	–5	–4	–3	–2	–1	0	1



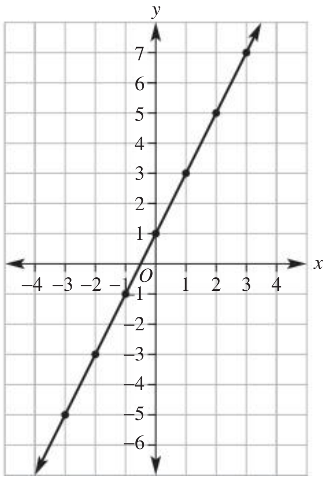
c

x	–3	–2	–1	0	1	2	3
y	–9	–7	–5	–3	–1	1	3



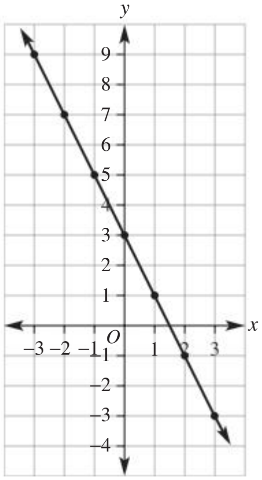
d

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7



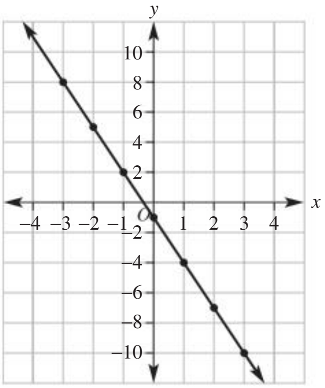
e

x	–3	–2	–1	0	1	2	3
y	9	7	5	3	1	–1	–3



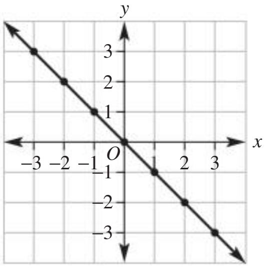
f

x	–3	–2	–1	0	1	2	3
y	8	5	2	–1	–4	–7	–10



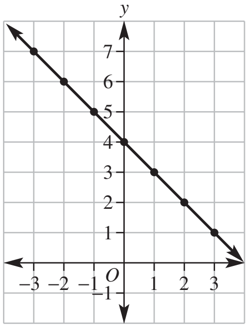
g

x	–3	–2	–1	0	1	2	3
y	3	2	1	0	–1	–2	–3



h

x	–3	–2	–1	0	1	2	3
y	7	6	5	4	3	2	1

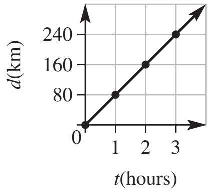


6 **a** yes **b** yes **c** no
d no **e** no **f** yes

7 **a**

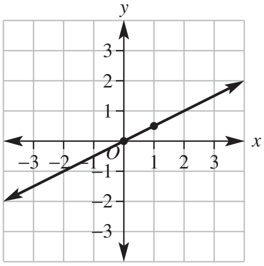
t	0	1	2	3
d	0	80	160	240

b

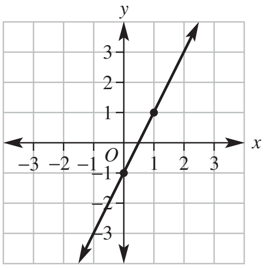


c 320 km
d 5 hours

8 **a** 2
b **i**



ii



9 When $x = 0, y = 0$ for all rules.
10 Intersection points are:
a (0, 0) **b** (2, 1) **c** No intersection

Exercise 9D

1 **a** C **b** A **c** B
2 **a** 2 **b** –1 **c** –2 **d** 3
3 **a** 3 **b** 1 **c** –4 **d** 3
4 **a** $y = x + 1$ **b** $y = 2x$ **c** $y = 2x + 4$
d $y = 3x - 1$ **e** $y = 4x$ **f** $y = 3x + 3$
5 **a** $y = -x$ **b** $y = -x - 1$ **c** $y = -x + 1$
d $y = -2x + 6$ **e** $y = -2x$ **f** $y = -3x + 1$
6 **a** $y = 3x + 1$ **b** $y = 2x + 1$
c $y = 5x + 1$ **d** $y = 2x + 4$
7 **a** 1 **b** 3 **c** 7 **d** 0
8 **a** x is not increasing by 1. **b** 1 **c** $y = x - 2$
d **i** $y = 2x + 3$ **ii** $y = 3x - 1$
iii $y = -2x + 3$ **iv** $y = -4x - 20$
9 **a** $y = x + 1$ **b** $y = 2x - 2$
c $y = -3x + 2$ **d** $y = -x$

Progress Quiz – Chapter 9 – Straight line graphs

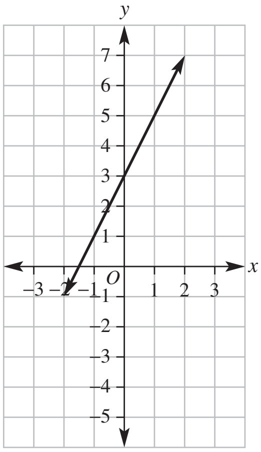
1 $A(2, 1), B(-2, -3), C(1, -4), D(0, 3)$
2 Rectangle
3 **a** 16 **b** 7 **c** –5 **d** 37
4 **a**

x	–2	–1	0	1	2
y	–8	–4	0	4	8

b

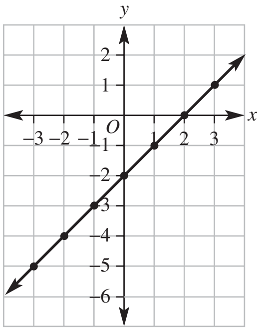
x	–2	–1	0	1	2
y	9	7	5	3	1

5 **a** **i** \$36 **ii** \$84 **b** \$48 **c** 20
6



7

x	–2	–1	0	1	2
y	–6	–4	–2	0	2



8 **a**

x	–2	–1	0	1	2
y	5	3	1	–1	–3

b

x	–2	–1	0	1	2
y	–3	0	3	6	9

9 **a** $y = x - 1$ **b** $y = -2x + 4$ **c** $y = 3x + 2$
10 **a** 1 **b** 5 **c** 0

Exercise 9E

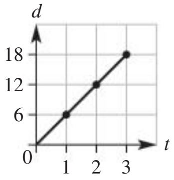
1 **a** positive **b** negative **c** negative **d** positive
2 **a** $A(3, 3), B(1, -2), C(-3, -2), D(-2, 1)$
b **i** 3 **ii** 0 **iii** –1 **iv** 2
c **i** 3 **ii** 4 **iii** 2 **iv** 5
3 **a** 2 **b** 3 **c** $\frac{3}{2}$ **d** 2
e –2 **f** –5 **g** $-\frac{8}{3}$ **h** $-\frac{5}{3}$
4 **a** 2 **b** $\frac{2}{3}$ **c** –4 **d** $\frac{-2}{5}$ **e** $\frac{-3}{2}$
f $\frac{-1}{2}$ **g** $\frac{1}{2}$ **h** 3 **i** $\frac{-1}{2}$
5 **a** positive **b** undefined **c** zero **d** negative
6 **a** 3 **b** 1 **c** $\frac{1}{2}$
d 3 **e** $\frac{2}{3}$ **f** 4
7 **a** –2 **b** $\frac{-3}{5}$ **c** $\frac{-4}{3}$
d –1 **e** –3 **f** $\frac{-3}{2}$
8 grassy slope **9** torpedo
10 **a** The y-value rises from 1 to 7.
b The y-value falls from 2 to –5.
c The x-value increases from –1 to 3.
d The x-value increases from –4 to 3.
11 **a** 2 **b** 10
12 **a** $\frac{3}{4}$ **b** $\frac{5}{2}$ **c** $\frac{-3}{2}$ **d** $\frac{-7}{10}$
13 **a** $\frac{5}{2}$ **b** $\frac{5}{3}$ **c** $\frac{-8}{3}$
d $\frac{-2}{3}$ **e** $\frac{8}{3}$ **f** $\frac{-3}{10}$

Exercise 9F

1 **a** $d = 3t$ **b** $d = t + 2$ **c** $d = -t + 4$
2 **a** 15 **b** 45 **c** 5 **d** 125
3 **a** 60 cm **b** 150 cm **c** 330 cm
4 **a** 28 L **b** 24 L **c** 10 L
5 **a**

t	0	1	2	3
d	0	6	12	18

b

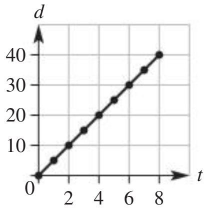


c $d = 6t$ **d** 9 km **e** 2 hours

6 **a**

t	0	1	2	3	4	5	6	7	8
d	0	5	10	15	20	25	30	35	40

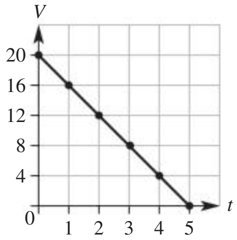
b



c $d = 5t$ **d** 22.5 km **e** 4 hours
7 **a**

t	0	1	2	3	4	5
d	20	16	12	8	4	0

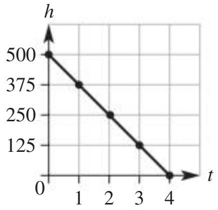
b



c $V = -4t + 20$ **d** 11.2 L **e** 3 seconds
8 **a**

t	0	1	2	3	4
h	500	375	250	125	0

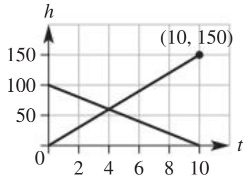
b



c $h = -125t + 500$
d 275 m
e 3 minutes
9 **a** $M = -0.5t + 3.5$ **b** 7 hours **c** 4.5 hours
10 **a** $d = 15t$ **b** 3 hours **c** 3 hours 20 minutes
11 **a**

t	0	1	2	3	4	5	6	7	8	9	10
h_1	0	15	30	45	60	75	90	105	120	135	150
h_2	100	90	80	70	60	50	40	30	20	10	0

b



c 10 seconds **d** $h = 15t, h = -10t + 100$
e at 4 seconds **f** at 2.5 seconds
g at 3.5 seconds

Exercise 9G

1

x	–2	–1	0	1	2
y	–5	–3	–1	1	3

2 **a** (2, 4) **b** (3.2, 6. 4) **c** (– 2.3,–4.6)
d (3.5, 7) **e** (–7,–14) **f** (1000, 2000)

3 **a** (4,3) **b** (– 2, – 3)

4 **a** $x = 2$ **b** $x = 0.5$ **c** $x = 3$
d $x = –2.5$ **e** $x = –1.5$

5 **a** $x = –2.5$ **b** $x = 3$ **c** $x = –0.5$
d $x = 4$ **e** $x = 5$

6 **a** Any point that lies on the line is correct, e.g. (–2, 9) (0, 5) (1, 3) (2, 1)
b Any point that lies on the line is correct, e.g. (–2, 0) (0, 2) (1, 3) (3, 5)
c (1, 3)
$y = x + 2$ $y = 5–2x$
$3 = 1 + 2$ $3 = 5–2 \times 1$
$3 = 3$ True $3 = 3$ True
d $x = 1$

7 **a** (2, 3) **b** (–1, 1)

8 **a** $A = 10 + 8n$ applies to Ruby as she has \$10 to start with and adds to her savings by \$8 times the number (n) of hours worked. $A = 24 + 6n$ applies to Jayden as he has \$24 to start with and increases his savings by \$6 times the number (n) of hours worked.
b **i** $n = 4$ **ii** $n = 4$ **iii** $n = 7$ **iv** $n = 7$
v $n = 11$ **vi** $n = 11$
c Many solutions e.g. (2, 26) (4, 42) (7, 66) (9, 82) (11, 98)
d Many solutions e.g. (2, 36) (4, 48) (7, 66) (9, 78) (11, 90)
e (7, 66) (7, 66)
$A = 10 + 8n$ $A = 24 + 6n$
$66 = 10 + 8 \times 7$ $66 = 24 + 6 \times 7$
$66 = 10 + 56$ $66 = 24 + 42$
$66 = 66$ True $66 = 66$ True
f $n = 7$
g Ruby and Jayden have both worked 7 hours and both have \$66 saved.

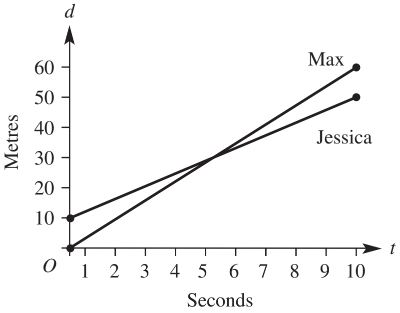
9 **a** **i** (–2, 17) (–1, 14) (0, 11) (1, 8) (2, 5) (3, 2) (4, –1)(5, –4)
ii (–2, –3), (–1, –1) (0, 1) (1, 3) (2, 5) (3, 7) (4, 9)(5, 11)
b (2, 5)
c It is the only shared point.

10 **a** $x = 3.67$ **b** $x = –1.53$ **c** $x = 5.30$

11 **a**

Time in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance in metres	0	6	12	18	24	30	36	42	48	54	60
Jessica's distance in metres	10	14	18	22	26	30	34	38	42	46	50

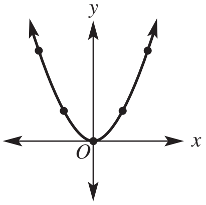
b



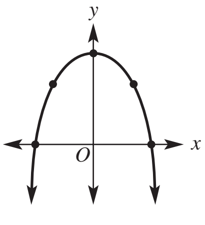
c $d = 6t$
d **i** $6t = 18$ **ii** $6t = 30$ **iii** $6t = 48$
e $d = 10 + 4t$
f **i** $10 + 4t = 22$ **ii** $10 + 4t = 30$ **iii** $10 + 4t = 42$
g (5, 30) (5, 30)
$d = 6t$ $d = 10 + 4t$
$30 = 6 \times 5$ $30 = 10 + 4 \times 5$
$30 = 30$ True $30 = 30$ True
h Max catches up to Jessica. They are both 30 m from the starting line and have each run for 5 seconds.

Exercise 9H

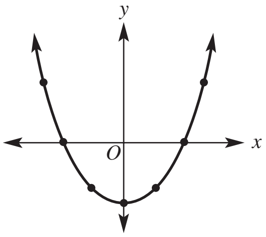
1 **a**



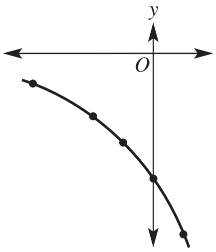
b



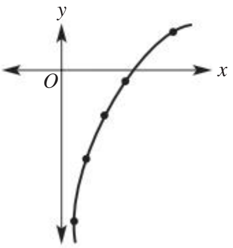
c



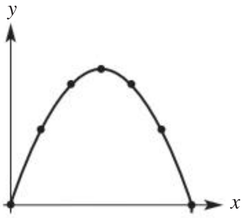
d



e



f

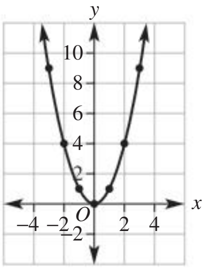


2 **a** −1 **b** 8 **c** 3 **d** 15

3 **a** Yes **b** No **c** No **d** Yes

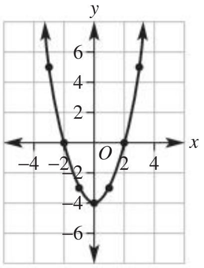
4 **a** $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



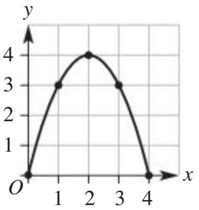
b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5



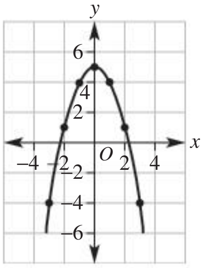
c $y = x(4 - x)$

x	0	1	2	3	4
y	0	3	4	3	0

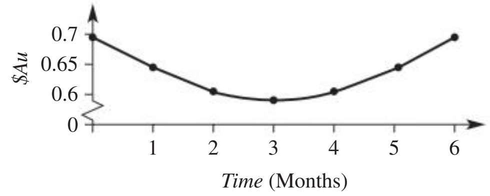


d $y = 5 - x^2$

x	−3	−2	−1	0	1	2	3
y	−4	1	4	5	4	1	−4



5 **a**



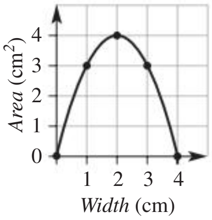
b non-linear (parabolic)

c **i** \$0.05 **ii** \$0.03 **d** ≈ 0.76

6 **a**

Width (cm)	0	1	2	3	4
Length (cm)	4	3	2	1	0
Area (cm²)	0	3	4	3	0

b



c non-linear (parabolic) **d** 2 cm by 2 cm

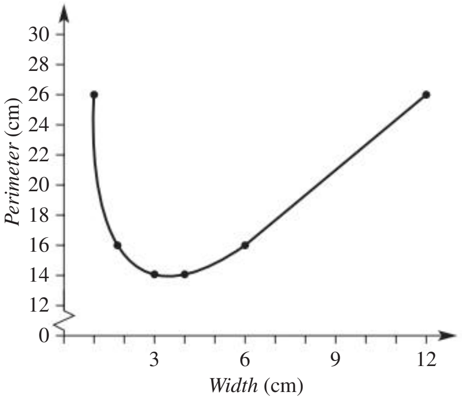
7 For each unit change in x there are variable changes in y.

8 **a** linear **b** linear **c** non-linear **d** non-linear
e non-linear **f** non-linear

9 **a**

Width (cm)	1	2	3	4	6	12
Length (cm)	12	6	4	3	2	1
Perimeter (cm)	26	16	14	14	16	26

b



c non-linear

d **i** ≈ 3.5 cm **ii** ≈ 13.9 cm

Maths@Work: Economists and household expenditure

1 **a** Positive **b** Negative **c** Negative
d Positive **e** Positive

2 **a** \$100/week **b** \$300/week **c** \$20 increase
d Positive **e** 0.2 **f** $y = 0.2x + 100$

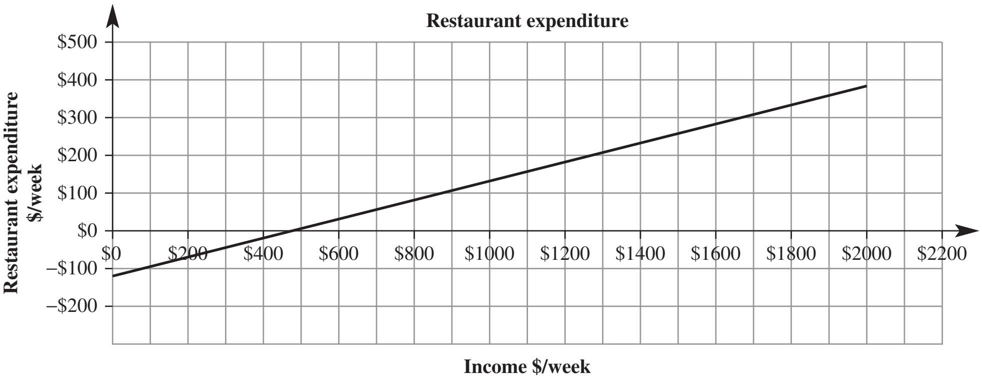
3 **a** **i** \$120/week **ii** \$60/week
b \$15 less or – \$15 **c** Negative **d** –0.15
b $y = -0.15x + 120$ **f** \$800 / week **g** Answers will vary.

4 **a** Normal **b** Inferior **c** Normal **d** Inferior

5 **a** and **b**

Weekly Income	\$0	\$200	\$400	\$600	\$800	\$1000	\$1200	\$1400	\$1600	\$1800	\$2000
Restaurant Expenditure	–\$120	–\$70	–\$20	\$30	\$80	\$130	\$180	\$230	\$280	\$330	\$380

c i–iv



d \$480 **e** \$1280

Puzzles and games

1 Aeroplane

2 **a** $y = 4x - 7$ **b** $y = -x + 11$
c $y = 5x - 50$ **d** $y = x - 10$

3 **a** 31 **b** 165

4 3 hours **5** 40 min **6** 1588

Multiple-choice questions

1 B	**2** C	**3** C	**4** D	**5** B
6 D	**7** A	**8** E	**9** D	**10** E

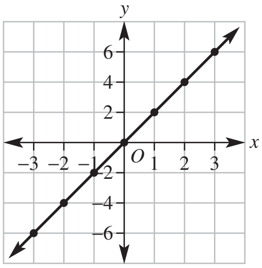
Short-answer questions

1 **a** 100 km **b** 1 hour **c** **i** 50 km **ii** 100 km
d Section C

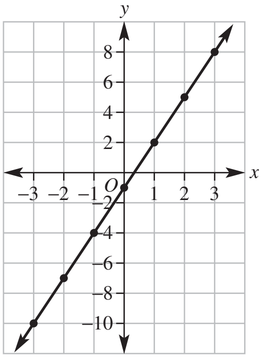
2 $A(2, 3), B(0, 2), C(-2, 4), D(-3, 1), E(-3, -3),$
$F(-1, 0), G(0, -4), H(1, -2), I(4, -3), J(3, 0)$

3 **a** −2,−1, 0, 1 **b** −4, 0, 2, 4 **c** −5, 1, 4, 7 **d** 3, 1, 0,−1

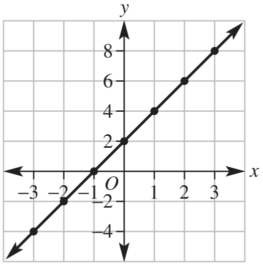
4 **a**



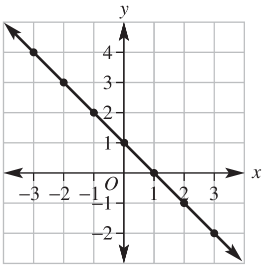
b



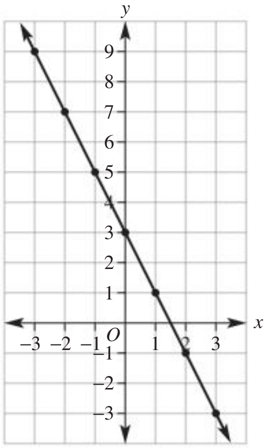
c



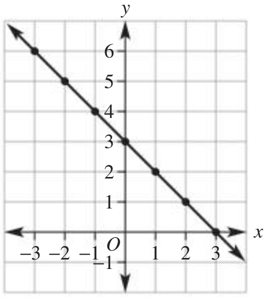
d



e



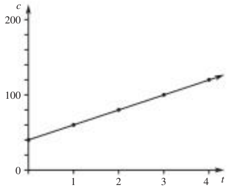
f



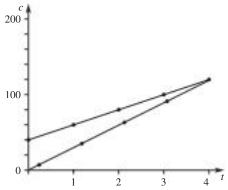
5 **a** $y = 2x + 1$ **b** $y = 3x + 2$
c $y = x + 3$ **d** $y = -x + 1$
e $y = -4x - 1$ **f** $y = -x + 8$

6 **a** 3 **b** 2 **c** −4
d −2 **e** 1 **f** $-\frac{1}{2}$

7 **a**

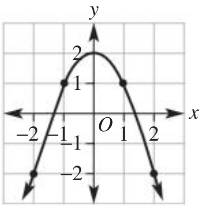


b $90 **c** 3.5 hours **d** $C = 20t + 40$
e $90 and 3.5 hours
f **i** $t = 2$ **ii** $t = 1.5$
g



h 4 hours

8

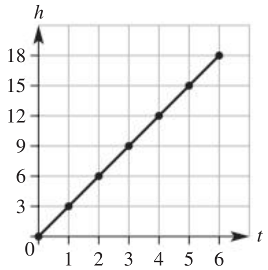


Extended-response questions

1 **a**

t	0	1	2	3	4	5	6
h	0	3	6	9	12	15	18

b

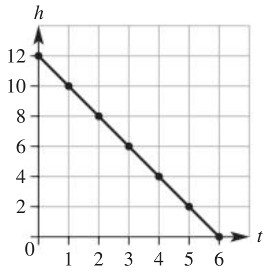


c $h = 3t$ **d** 10.5 mm
e 30 mm **f** 5 days

2 **a**

t	0	1	2	3	4	5	6
h	12	10	8	6	4	2	0

b



c 6 minutes **d** −2 **e** $h = -2t + 12$
f 7 km **g** 4 minutes 15 seconds

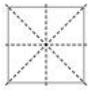
Chapter 10

Pre-test

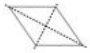
1 **a** 4 **b** 2 **c** 5 **d** 0
2 **a** 4 **b** 2 **c** 5 **d** 2
3 **a** $A(1,1), B(3,-2), C(-4,-3), D(-3,0)$
b **i** $(0,1)$ **ii** $(3,0)$ **iii** $(-4,-2)$
c $(-4,3)$
4 **a** $(5,5)$ **b** $(5,2)$ **c** $(1,2)$
5 **a** 30° **b** 20° **c** 45° **d** 60°
6 **a** A, B, C, D **b** A, B, C, D, E
c A, B **d** A, C, E

Exercise 10A

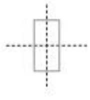
1 **a**



b

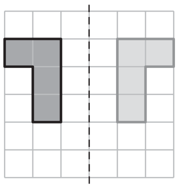


c

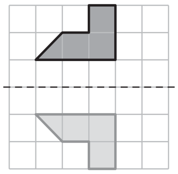


d

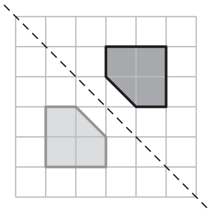
2 a



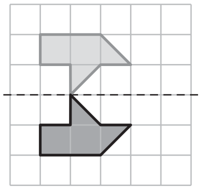
b



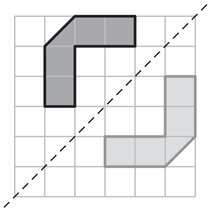
c



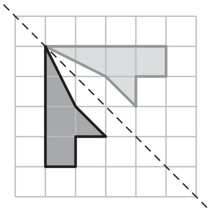
d



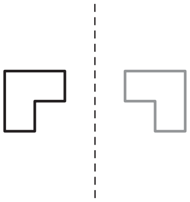
e



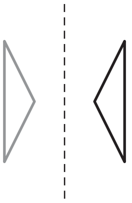
f



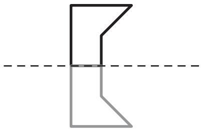
3 a



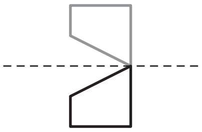
b



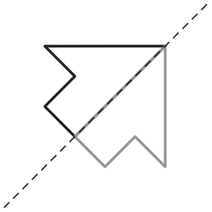
c



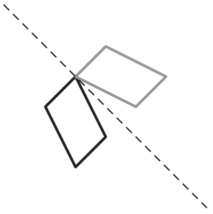
d



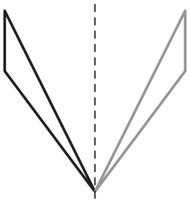
e



f



4 a



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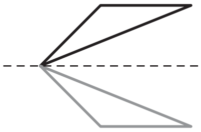

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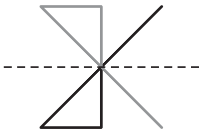

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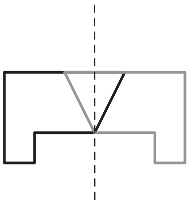

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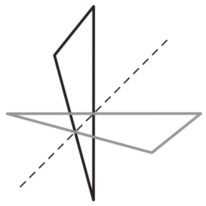

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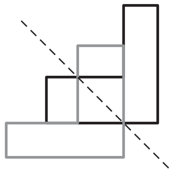
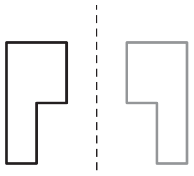


5 a



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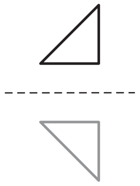

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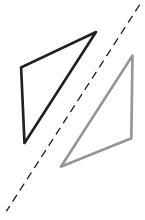

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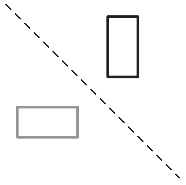

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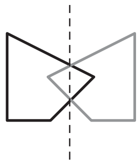

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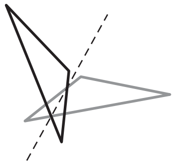


6 a $A'(2,0)$, $B'(1,-3)$, $C'(4,-2)$
b $A'(-2,0)$, $B'(-1,3)$, $C'(-4,2)$

7 a $A'(-1,2)$, $B'(-4,2)$, $C'(-4,4)$, $D'(-1,4)$
b $A'(1,-2)$, $B'(4,-2)$, $C'(4,-4)$, $D'(1,-4)$

8 a 4 **b** 2 **c** 2 **d** 1 **e** 0
f 0 **g** 1 **h** 3 **i** 8

9 A parallelogram cannot be reflected onto itself over a mirror line but a rhombus can do this in two ways.

10 10 m^2, the area is unchanged after reflection.

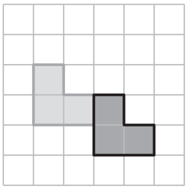
11 n

12 Reflection in the y-axis.

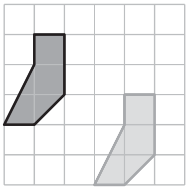
13 a–d Computer geometry required, answers will vary.

Exercise 10B

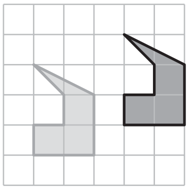
1 a



b



c



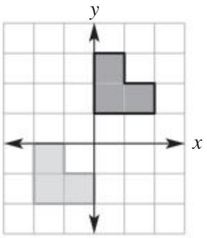
2 a right, up **b** left, up **c** right, down
d left, down

3 **a** (5, −2) **b** (−2, −6) **c** (−7, 4) **d** (9, 17)

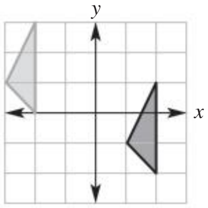
4 **a** **i** (3, 4) **ii** (−1, 3) **iii** (−2, 0) **iv** (−3, −1)
b **i** (−1, 1) **ii** (3, 0) **iii** (−3, 1) **iv** (−2, 3)

5 **a** (1, −1) **b** (3, −1) **c** (2, −2) **d** (2, −4)
e (3, 7) **f** (−3, 5) **g** (2, −3) **h** (3, −5)
i (−20, 8) **j** (−9, −25)

6 **a**



b



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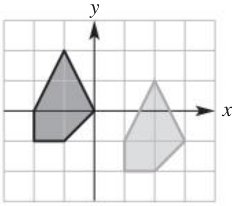

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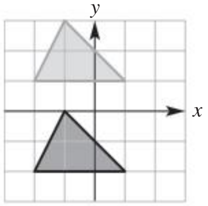
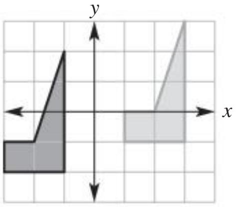


e



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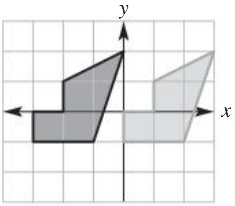


7 **a** horizontal **b** vertical
c vertical **d** horizontal

8 **a** (15, 2) **b** (21, −1) **c** (13, 6) **d** (9, 2)
e (11, −2) **f** (3, 4) **g** (11, −9) **h** (19, −10)

9 **a** (−3, 2) **b** (5, 0) **c** $(-x, -y)$ **d** (x, y)

10 **a** **i** (1, −2) **ii** (−3, 4) **iii** (0, −3)
b 2.3 km (or 2300 m) **c** (2, 3)

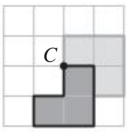
Exercise 10C

1 **a** C **b** A **c** B

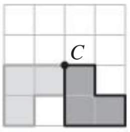
2 **a** anticlockwise, 90° **b** clockwise, 90°
c anticlockwise, 90° **d** clockwise, 90°
e anticlockwise, 180° **f** clockwise, 180°

3 **a** 4 **b** 2 **c** 5 **d** 2
e 4 **f** 2

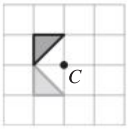
4 **a**



b



c



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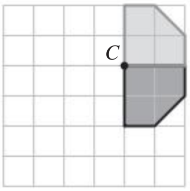

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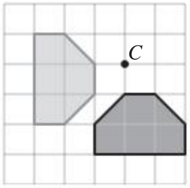
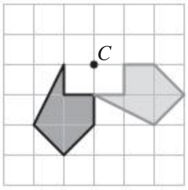
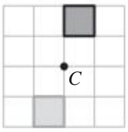


f



5 **a**



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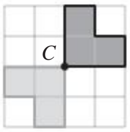

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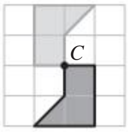

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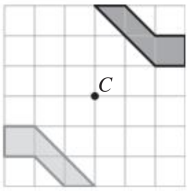

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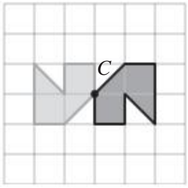

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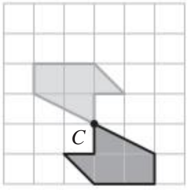


6 **a** (−4, −3) **b** (−4, −3) **c** (3, −4) **d** (−3, 4)
e (−3, 4) **f** (3, −4) **g** (4, 3)

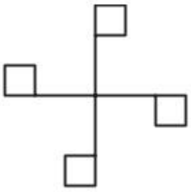
7 **a** 270° **b** 180° **c** 302° **d** 64°

8 **a** 90° **b** 180° **c** 90°

9 **a** $A'(4, -4)$, $B'(4, -1)$, $C'(1, -1)$
b $A'(4, 4)$, $B'(1, 4)$, $C'(1, 1)$
c $A'(-4, -4)$, $B'(-1, -4)$, $C'(-1, -1)$

10 H, I, N, O, S, X and Z

11 Answers may vary. Examples are:
a square **b** parallelogram **c** regular hexagon
d



e kite

Exercise 10D

1 **a** F **b** T **c** T
d T **e** T **f** T

2 **a** yes
b **i** D **ii** E **iii** F
c **i** DE **ii** EF **iii** DF
d **i** $\angle E$ **ii** $\angle F$ **iii** $\angle D$

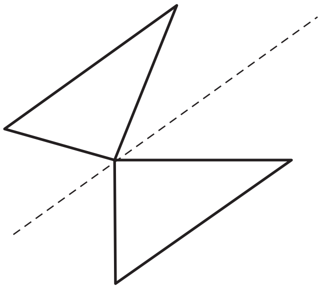
3 **a** yes
b **i** D **ii** E **iii** F
c **i** DE **ii** EF **iii** DF
d **i** $\angle E$ **ii** $\angle F$ **iii** $\angle D$

4 **a** yes
b **i** D **ii** E **iii** F
c **i** DE **ii** EF **iii** DF

d **i** $\angle E$ **ii** $\angle F$ **iii** $\angle D$
5 **a** **i** E **ii** H
b **i** EH **ii** GH
c **i** $\angle G$ **ii** $\angle E$
6 **a** **i** F **ii** I
b **i** FJ **ii** HI
c **i** $\angle H$ **ii** $\angle J$
7 (J, G), (D, K), (C, I)
8 **a** DE **b** $\angle B$ **c** 10 cm **d** 62° **e** 30°
9 (A, J), (C, K), (E, G)
10 **a** $\Delta AMC, \Delta BMC$
b Yes, all corresponding sides and angles will be equal.
11 Yes
12 **a** 32 **b** 24 **c** 20 **d** 8 **e** 4

Progress Quiz – Chapter 10 – Transformation and congruence

1



2 **a** $A'(-4,-1)\, B'(-4,-3)\, C'(-1,-3)\, D'(-1,-1)$
b $A'(4,1)\, B'(4,3)\, C'(1,3)\, D'(1,1)$
3 **a** 4 **b** 2 **c** 0
4 **a** $(-4,3)$ **b** $(6,-2)$
5 **a** $(1,6)$ **b** $(-4,4)$ **c** $(4,3)$
6 **a** $(-1,-5)$ **b** $(-4,3)$
7 **a** 6 **b**
8 **a** $(-3,4)$ **b** $(-3,4)$ **c** $(-4,-3)$
d $(4,3)$ **e** $(3,-4)$ **f** $(-4,-3)$
9 **a** Vertex N **b** Vertex P **c** Side NO
d Side OP **e** $\angle M$ **f** $\angle O$
10 **a** FD **b** $\angle C$ **c** 5 cm **d** 31°

Exercise 10E

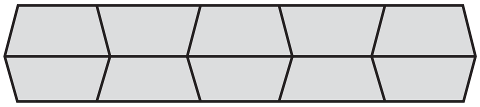
1 **a** yes **b** yes **c** no
d yes **e** yes **f** no
2 **a** **i** F **ii** D **iii** E
b **i** $\angle A$ **ii** $\angle C$ **iii** $\angle B$
c **i** DE **ii** FD **iii** EF
3 **a** $\Delta ABC \equiv \Delta EFD$ **b** $\Delta ABC \equiv \Delta FED$
c $\Delta XYZ \equiv \Delta UST$ **d** $\Delta ABC \equiv \Delta ADC$
4 **a** SAS **b** SSS
c RHS **d** AAS
5 **a** SSS **b** RHS
c SAS **d** AAS
6 **a** $x = 4, y = 1$ **b** $x = 9, a = 20$
c $x = 5, a = 24$ **d** $x = 5, a = 30$
e $x = 4, a = 95, b = 25$ **f** $x = 11, a = 50, b = 90$
7 **a** SSS **b** SAS
c RHS **d** AAS
8 **a** no **b** yes, SAS
c yes, AAS **d** no
9 You can draw an infinite number of triangles with the same shape but of different size.
10 **a** no **b** yes **c** yes **d** no

Exercise 10F

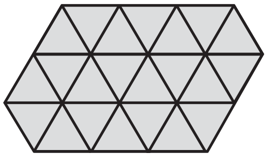
1 SAS, AAS and RHS
2 **a** AC **b** BD **c** DB
3 **a** alternate angles in parallel lines
b alternate angles in parallel lines
c alternate angles in parallel lines
4 **a** co-interior angles in parallel lines, $a = 110$
b co-interior angles in parallel lines, $a = 52$
5 **a** T **b** F **c** F **d** T
e F **f** F **g** F **h** F
i T **j** T **k** F **l** F
m T **n** T **o** T **p** T
q T
6 **a** AAS **b** RHS **c** SSS
d SAS **e** AAS **f** SSS
7 **a** $\Delta ABD, \Delta CDB$ **b** equal **c** equal
d BD **e** SSS
f Corresponding angles in congruent triangles.
8 **a** yes (90°) **b** yes **c** yes **d** SAS
e Corresponding sides in congruent triangles.
9 **a** equal (alternate angles in parallel lines)
b equal (alternate angles in parallel lines)
c BD
d AAS
e They must be equal.
10 **a** $\angle DCE$
b $\angle CDE$
c There are no pairs of equal sides.
11 **a** SSS (3 equal sides)
b They are equal and add to 180° so each must be 90°.
c Since ΔQMN is isosceles and $\angle MQN$ is 90° then $\angle QMN = 45$.
12 **a** $AB = CB, AD = CD$ and BD is common, so $\Delta ABD \equiv \Delta CBD$ by SSS.
b $\Delta ABD \equiv \Delta CBD$ so $\angle DAB \equiv \angle DCB$
c $\Delta ABD \equiv \Delta CBD$ so $\angle ADB \equiv \angle CDB$
13 $\angle ABE \equiv \angle CDE$ (alternate angles in parallel lines)
$\angle BAE \equiv \angle DCE$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\angle ABE \equiv \angle CDE$ (AAS)
$BE = DE$ and $AE = CE$ because corresponding sides on congruent triangles are equal.

Exercise 10G

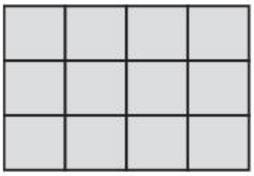
1 C
2 D
3 Answers will vary.
4 Answers will vary. Example:



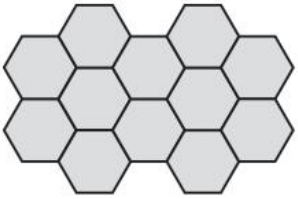
5 Answers will vary. Examples:
a



b



c



6 **a** 3.3.3.3.3.3 **b** 4.4.4.4 **c** 6.6.6

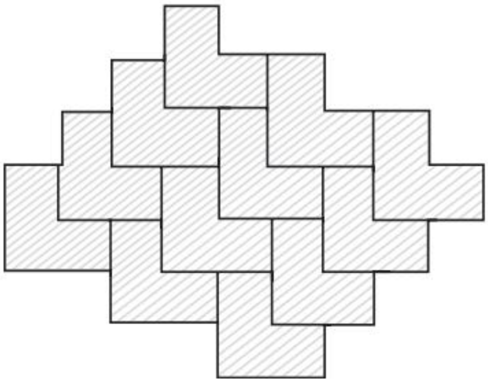
7 **a** yes **b** no **c** yes

d no **e** yes

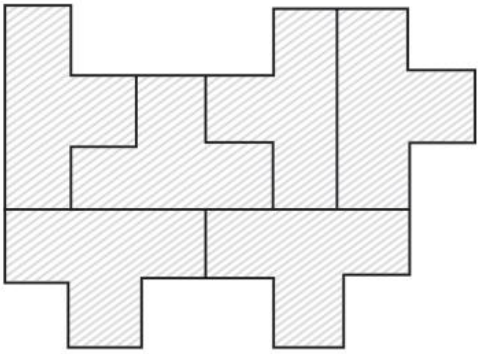
8 **a** 3.3.3.4.4 **b** 3.3.4.3.4 **c** 3.4.6.4 **d** 3.12.12

9 Answers will vary. Examples:

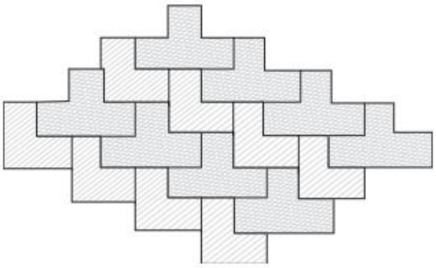
a



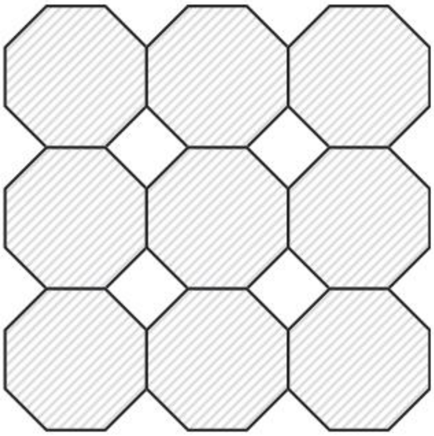
b



c



10



11 **a** 50

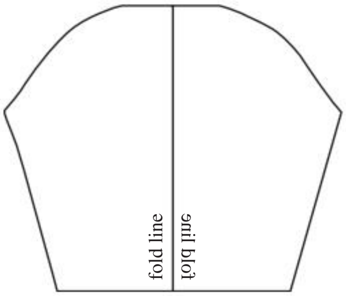
b Answers will vary.

12 **a–b** Answers will vary.

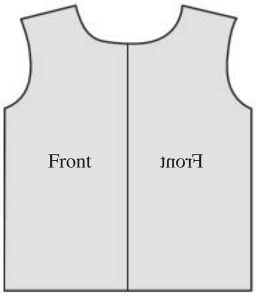
13 **a and b** Answers will vary.

Maths@Work: Fashion designer

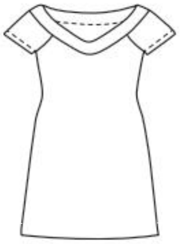
1 **a**



b



c



2 **a** Rotation by 120°, translation

b Rotation by 45°, reflection in the 8 axes of symmetry

c Reflection in the vertical and horizontal axes of symmetry

d Rotation by 30°, translation

3 A drawing congruent to the image in the question

4 Answers will vary.

Puzzles and games

1 30 **2** 27 **3** 30 m **4** 31

5 Yes, illustrates Pythagoras' theorem using areas

6 $(3-r)+(4-r)=5$, so $r=1$

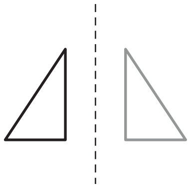
Multiple-choice questions

1 B **2** D **3** C **4** A **5** B

6 E **7** A **8** E **9** D **10** E

Short-answer questions

1 a



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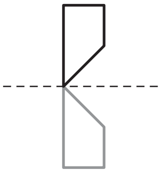
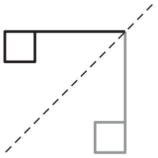
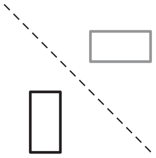


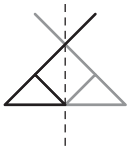
c



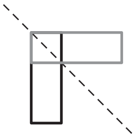
d



e



f



2 a $A'(1,-2), B'(3,-4), C'(0,-2)$

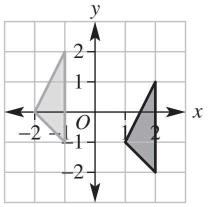
b $A'(-1,2), B'(-3,4), C'(0,\ 2)$

3 a 4 **b** 1 **c** 2 **d** 1 **e** 6 **f** 0

4 a $(0,3)$ **b** $(2,-1)$ **c** $(-3,1)$ **d** $(-3,-2)$

5 a $(1,4)$ **b** $(3,-6)$ **c** $(-3,-7)$ **d** $(4,6)$

6 a



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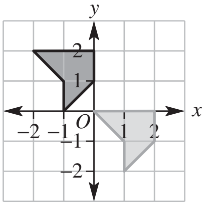
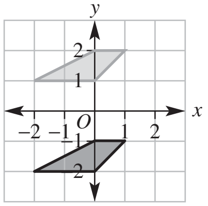


c

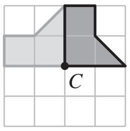


7 a 3

b 2

c no rotational symmetry

8 a



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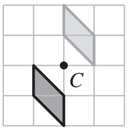

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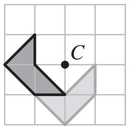


9 a i F **ii** G

b i EH **ii** FG

c i $\angle G$ **ii** $\angle E$

10 $\Delta ABC \equiv \Delta STU$

11 a RHS **b** SAS **c** SSS **d** AAS

12 a $x=3, a=25$ **b** $x=5, a=18$

13 a yes – alternate angles

b yes – alternate angles

c yes – given

d AAS

e $AE=CE$ (matching sides of congruent triangles), $BE=DE$ (matching sides of congruent triangles), therefore AC and BD bisect each other.

14 (3.3.4.3.4)

Extended-response questions

1 a $A'(0,1)$, $B'(-2,1)$, $C'(-2,4)$

b $A'(3,1)$, $B'(3,-1)$, $C'(0,-1)$

2 a SSS

b $\angle VWU = \angle TWU$ and $\angle VWU + \angle TWU = 180°$ so $\angle VWU = \angle TWU = 90°$

Semester review 2

Ratios and rates

Multiple-choice questions

1 C **2** B **3** C **4** C **5** D

Short-answer questions

1 a 2 : 3 **b** 1 : 2 : 3 **c** 6 : 7

d 3 : 40 **e** 5 : 1 **f** 3 : 10

2 a 576 cm, 384 cm **b** \$1500, \$2500 **c** \$1.60, \$4, \$2.40

3 \$7750

4 60 000 cm, 600 m

5 a 12 mm/day **b** 3 goals/game

c 2 cents/g or \$0.02 /g or \$20/kg

6 \$2.27

7 85.6 km/h

Extended-response questions

1a 16.5 km **b** 742.5 km **c** 6.1 L

d \$35.37 **e** 18 km

Equations

Multiple-choice questions

1 C **2** B **3** D **4** B **5** B

Short-answer questions

1 **a** $w = 9$ **b** $m = 7$ **c** $x = 5$
d $a = 2$ **e** $w = 13$ **f** $x = 1$
2 **a** $x = 30$ **b** $q = 10$ **c** $p = 15$
d $x = 10$ **e** $r = 15$ **f** $a = 8$
3 **a** $x = 5$ **b** $k = 10$
c $r = 0$ **d** $z = 10$
4 6
5 **a** 20 **b** 2 **c** 7
6 **a** $x = 40$ **b** $x = 60$ **c** $x = 75$

Extended-response questions

1a $1500 + 5n$ **b** 100 books **c** $20n$
d 50 books
e **i** \$1500 **ii** \$13 500 **iii** 100 books
f They make a \$750 loss

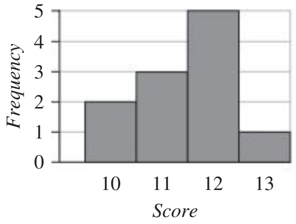
Probability and statistics

Multiple-choice questions

1 A **2** A **3** D **4** C **5** B

Short-answer questions

1 **a** **i** 13.75 **ii** 14 **iii** 8
b **i** 23 **ii** 18.5 **iii** 56
c **i** 10 **ii** 9.45 **iii** 15.7
2



3 **a** $\frac{7}{16}$ **b** $\frac{1}{16}$ **c** $\frac{15}{16}$ **d** $\frac{1}{2}$ **e** 0
4 **a** $\frac{3}{50}$ **b** $\frac{39}{50}$ **c** $\frac{1}{50}$ **d** $\frac{24}{25}$
5 **a** 18 **b** 6 **c** 8 **d** 4 **e** 16
6 **a** $\frac{1}{9}$ **b** $\frac{5}{9}$ **c** $\frac{1}{4}$ **d** $\frac{1}{12}$

Extended-response questions

1a 18 **b** 78 **c** 78 **d** Group B

Straight line graphs

Multiple-choice questions

1 D **2** B **3** C **4** D **5** D

Short-answer questions

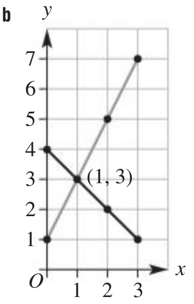
1 **a** 1st **b** 2nd **c** 3rd **d** 4th
2 **a** **i**

x	0	1	2	3
y	1	3	5	7

ii

x	0	1	2	3
y	4	3	2	1

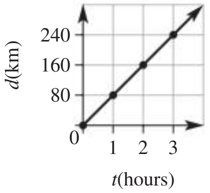
b



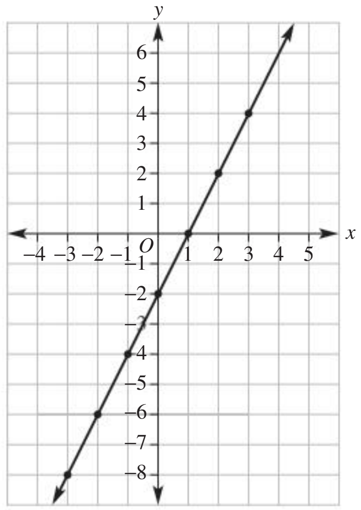
3 **i** positive **ii** negative
4 **a**

t	0	1	2	3
d	0	80	160	240

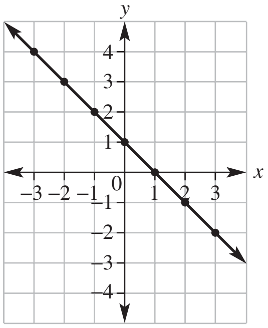
b



c 240 km **d** 4 hours
5 **a**



b

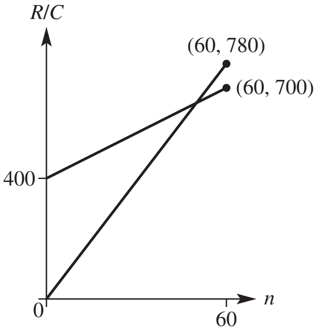


Extended-response questions

1a

n	0	10	20	30	40	50	60
C	400	450	500	550	600	650	700
R	0	130	260	390	520	650	780

b



c (50,650)
d $400

Transformation and congruence

Multiple-choice questions

1 C	**2** B	**3** D	**4** D	**5** C

Short-answer questions

1 **a** 0 **b** 2 **c** 2 **d** 1
2 **a** (2,2) **b** (−1,−2)
3 **a** A'(1,−1), B'(3,−1), C'(2,−3)
b A'(−3,−1), B'(−1,−1), C'(−2, 1)
c A'(1,−1), B'(1,−3), C'(3,−2)
d A'(−1,−1), B'(−3,−1), C'(−2,−3)
4 **a** SSS **b** SAS **c** RHS **d** AAS
5 A,C

Extended-response questions

1a Alternate angles in parallel lines.
b Alternate angles in parallel lines.
c AAS
d $\Delta ABE \equiv \Delta CDE$ so $AE = CE$ and $BE = DE$.1