MATHEMATICS

OXFORD MATHS 8

STAGE 4

SERIES CONSULTANT:
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NSW CURRICULUM

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Warning to First Nations Australians
Aboriginal and Torres Strait Islander peoples are advised that this publication may include images or names of people now deceased.

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Introducing Oxford Maths 7–10

Oxford Maths 7–10 NSW Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

Key features of Student Books

- Complete access to all digital resources available on Student obook pro.
- Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
- STEAM projects encourage inter-disciplinary thinking.
- Semester reviews provide an opportunity to revise key concepts from each semester.
- An illustrated glossary of newly introduced mathematics terms.

Each chapter opens with:

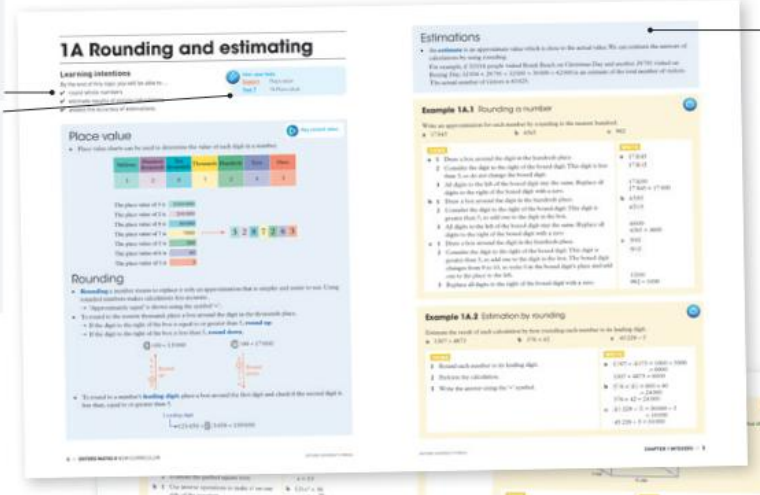
- **Prerequisite skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the NSW Curriculum.
- **Materials** used to complete the exercises.

Learning intentions

- Signpost the foundational skills being developed in each section.

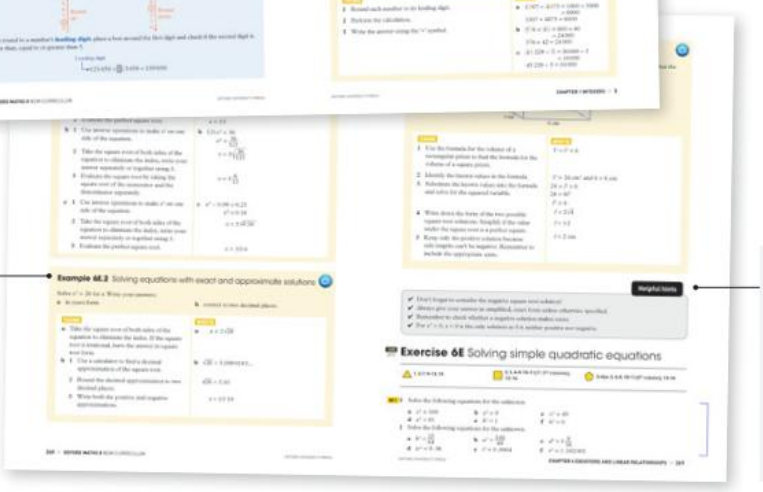
Inter-year links

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.



New theory

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life and aid understanding.



Worked examples

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

Helpful hints

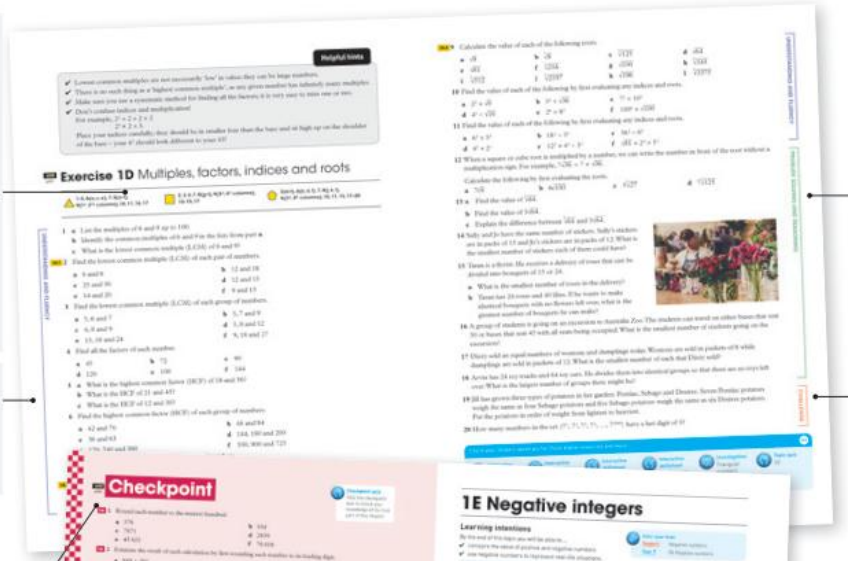
- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

Differentiated learning pathways

- Separate each exercise into three pathways, tailored for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every chapter.

Understanding and fluency

- Basic exercises dedicated to practicing key concepts.

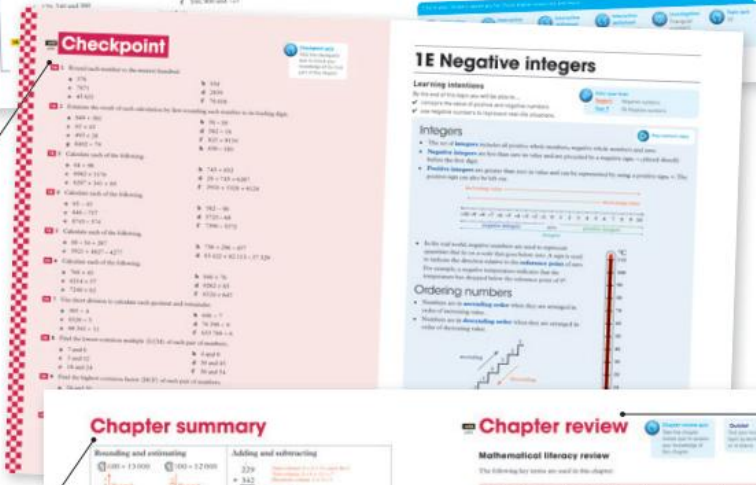


Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

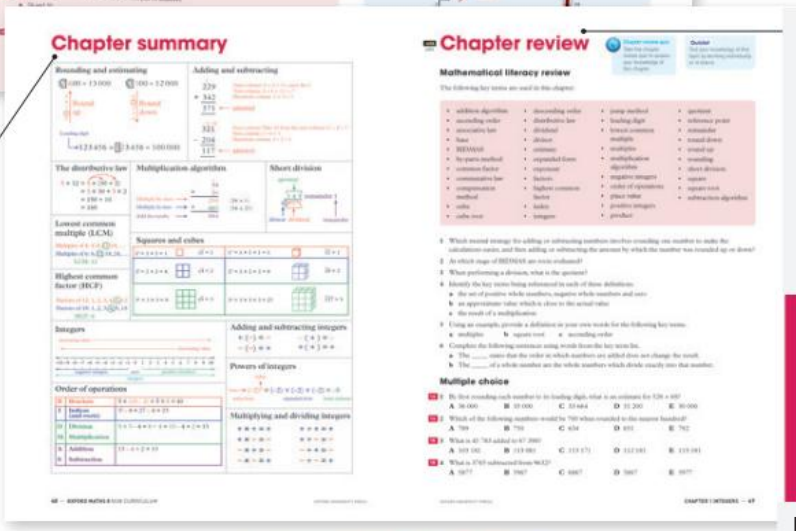
Challenge

- Advanced exercises designed to build engagment and anticipate future learning outcomes.



Checkpoint

- A section at the middle of each chapter dedicated to surmising key skills and encourage memory retention.
- Reference to an online checkpoint quiz to gauge student progress.



Chapter reviews

- Additional practice questions to further consolidate understanding at the end of each chapter.
- Reference to an online chapter review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Chapter summary

- Condense all the theory from each section into one accessible revision page.

Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and HSIE curricula.

Problem solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem solve using design thinking.

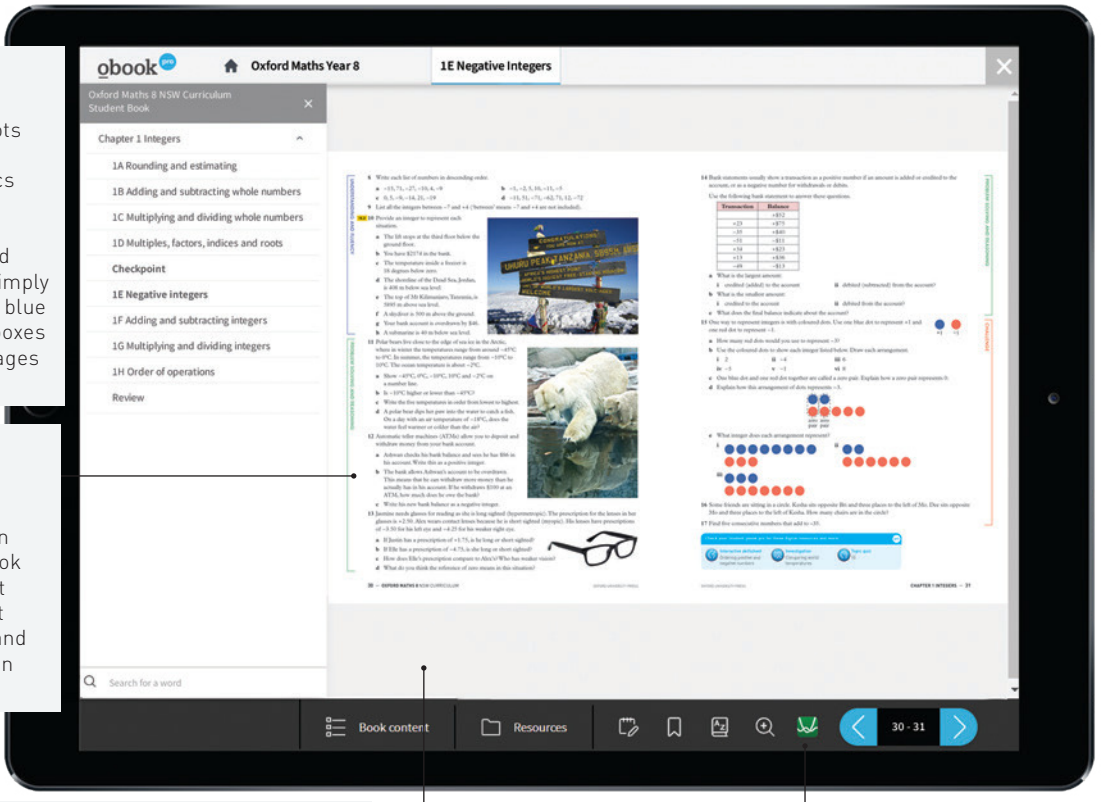
Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

Key features of Student obook pro

obook pro

- Student obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- It offers a complete digital version of the Student Book with interactive note-taking, highlighting and bookmarking functionality, allowing students to revisit points of learning.
- A complete ePDF of the Student Book is also available for download for offline use and read-aloud functionality.



Integrated digital resources

- Integrated hotspots allow students to access diagnostics tests, quizzes, interactive skill sheets, videos and inter-year links simply by clicking on the blue digital resource boxes throughout the pages of the book.

Complete digital version of the Student Book

- The digital version of the Student Book is true to the print version, making it easy to navigate and transition between print and digital.

Toolbar features

- Notes can be added and saved to the text by simply selecting and highlighting.
- Bookmarks can be saved to any page.
- Australian Concise Oxford Dictionary can provide immediate definitions to any word within the text.

Desmos integration

- Our partership with Desmos allows students to access a suite of calculator tools as they read through the text, providing convenient graphical support as well as the opportunity to investigate plane geometry and Cartesian coordinates.

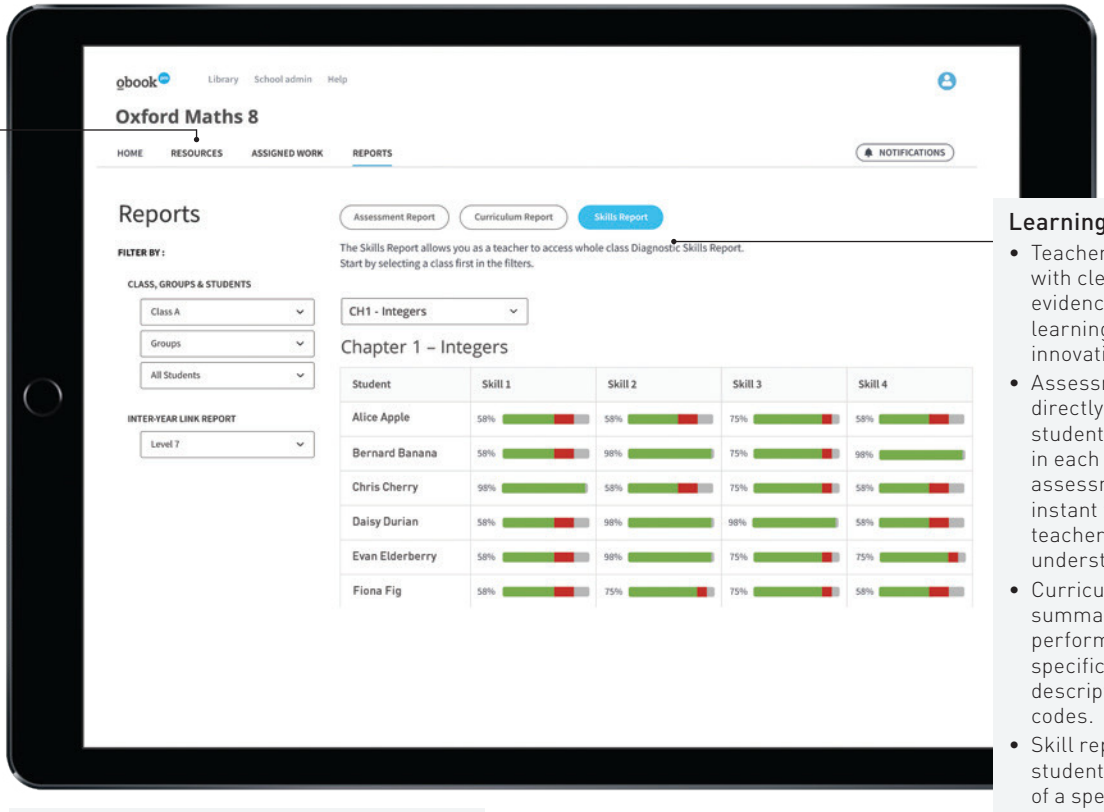
Benefits for students

- Integrated *Australian Concise Oxford Dictionary* look-up feature.
- Targeted instructional videos for every worked example question.
- Groundwork resources to support assumed knowledge.
- Interactive assessments to consolidate understanding.
- Auto-marked practice exam question sets.
- Integrated Quizlet sets, including real-time online quizzes with live leaderboards.
- Access to online assessment results to track progress.

obook pro

Key features of Teacher obook pro

- Teacher obook pro is a completely digital product delivered via Oxford's online learning platform, **Oxford Digital**.
- Each chapter and topic of the Student Book is accompanied by full teaching support, including assessment reporting, worked solutions, chapter tests, detailed teacher notes and lesson plans.
- Teachers can use their Teacher obook pro to share notes and easily assign resources or assessments to students, including due dates and email notifications.



Learning pathway reports

- Teachers are provided with clear and tangible evidence of student learning progress through innovative reports.
- Assessment reports directly show how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding.
- Curriculum reports summarise student performance against specific curriculum content descriptors and curriculum codes.
- Skill reports indicate the students understanding of a specific skill in mathematics.

Additional resources

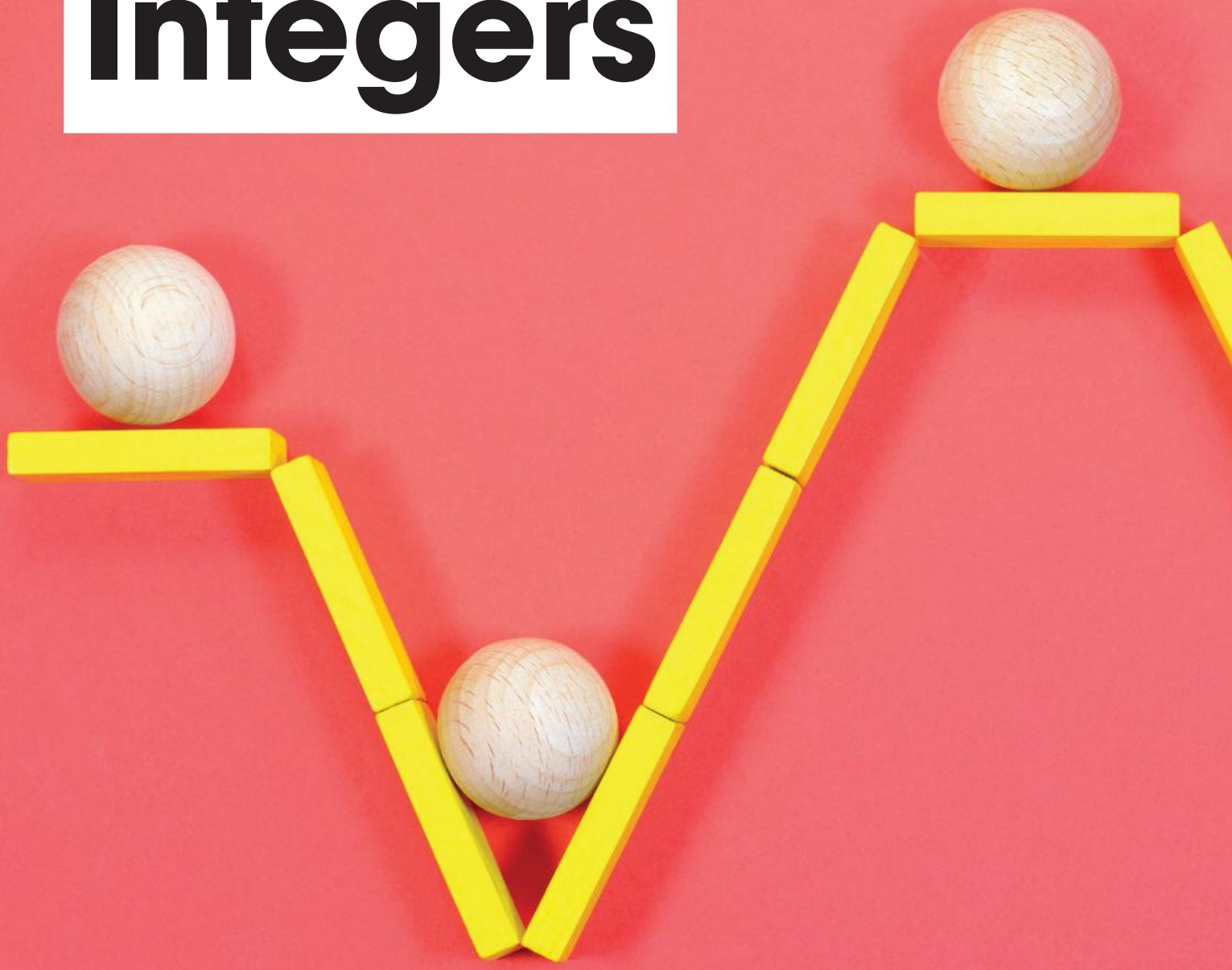
- Each chapter of the Student Book is accompanied by additional interactive skillsheets, worksheets, investigations and topic quizzes to help students progress.

Benefits for teachers

- Diagnostic pre-tests and chapter tests that track students' progress against Study Design key knowledge, providing detailed learning pathway reports that differentiate each student's ability in each skill.
- Assign reading and assessments to students either individually, or in groups – administration is taken care of!
- Set-up classes, monitor student progress and graph results.
- Worked solutions for every Student Book question.
- Detailed teacher notes, teaching programs and lesson plans.

1

Integers



Index

1A Rounding and estimating
1B Adding and subtracting whole numbers
1C Multiplying and dividing whole numbers
1D Multiples, factors, indices and roots
1E Negative integers
1F Adding and subtracting integers
1G Multiplying and dividing integers
1H Order of operations

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Adding and subtracting two-digit numbers
- ✔ Times tables
- ✔ Multiplying and dividing by powers of 10
- ✔ Number lines

Curriculum links

- Compares, orders and calculates with integers to solve problems (MA4-INT-C-01)
  - Multiply and divide positive and negative indices
  - Apply the 4 operations to integers
- Operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws (MA4-IND-C-01)
  - Examine cube roots and square roots

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Materials

✔ Calculator

1A Rounding and estimating

Learning intentions

By the end of this topic you will be able to ...

✔ round whole numbers
✔ estimate results of simple calculations
✔ assess the accuracy of estimations.

Inter-year links
Support — Place value
Year 7 — 1A Place value

Place value

Key content video

- Place value charts can be used to determine the value of each digit in a number.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	8	7	2	6	3

The place value of 3 is	3 000 000
The place value of 2 is	200 000
The place value of 8 is	80 000
The place value of 7 is	7000
The place value of 2 is	200
The place value of 6 is	60
The place value of 3 is	3

→ 3 2 8 7 2 6 3

Rounding

- **Rounding** a number means to replace it with an approximation that is simpler and easier to use. Using rounded numbers makes calculations less accurate.
  → 'Approximately equal' is shown using the symbol '≈'.
- To round to the nearest thousand, place a box around the digit in the thousands place.
  → If the digit to the right of the box is equal to or greater than 5, **round up**.
  → If the digit to the right of the box is less than 5, **round down**.

$1\boxed{2}600 \approx 13\,000$

9
8
7
6
5
Round up

$1\boxed{7}300 \approx 17\,000$

4
3
2
1
0
Round down

- To round to a number's **leading digit**, place a box around the first digit and check if the second digit is less than, equal to or greater than 5.

Leading digit

$123\,456 = \boxed{1}23\,456 \approx 100\,000$

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Estimations

- An **estimate** is an approximate value which is close to the actual value. We can estimate the answers of calculations by using rounding.

  For example, if 32 034 people visited Bondi Beach on Christmas Day and another 29 791 visited on Boxing Day, 32 034 + 29 791 ≈ 32 000 + 30 000 = 62 000 is an estimate of the total number of visitors. The actual number of visitors is 61 825.

Example 1A.1 Rounding a number

Write an approximation for each number by rounding to the nearest hundred.

a 17 845 **b** 4565 **c** 992

THINK	WRITE
a 1 Draw a box around the digit in the hundreds place.	**a** 17 $\boxed{8}$45
2 Consider the digit to the right of the boxed digit. This digit is less than 5, so do not change the boxed digit.	17 $\boxed{8}$45
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.	17 $\boxed{8}$00 17 845 ≈ 17 800
b 1 Draw a box around the digit in the hundreds place.	**b** 4$\boxed{5}$65
2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box.	4$\boxed{5}$65
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.	4$\boxed{6}$00 4565 ≈ 4600
c 1 Draw a box around the digit in the hundreds place.	**c** $\boxed{9}$92
2 Consider the digit to the right of the boxed digit. This digit is greater than 5, so add one to the digit in the box. The boxed digit changes from 9 to 10, so write 0 in the boxed digit's place and add one to the place to the left.	$\boxed{9}$92
3 Replace all digits to the right of the boxed digit with a zero.	1$\boxed{0}$00 992 ≈ 1000

Example 1A.2 Estimation by rounding

Estimate the result of each calculation by first rounding each number to its leading digit.

a 1307 + 4875 **b** 576 × 42 **c** 45 229 ÷ 5

THINK	WRITE
1 Round each number to its leading digit.	**a** $\boxed{1}$307 + $\boxed{4}$875 ≈ 1000 + 5000 = 6000
2 Perform the calculation.	1307 + 4875 ≈ 6000
3 Write the answer using the '≈' symbol.	**b** $\boxed{5}$76 × $\boxed{4}$2 ≈ 600 × 40 = 24 000
	576 × 42 ≈ 24 000
	c $\boxed{4}$5 229 ÷ $\boxed{5}$ ≈ 50 000 ÷ 5 = 10 000
	45 229 ÷ 5 ≈ 10 000

Helpful hints

- ✔ Remember that the place of a digit in a number indicates its value.
  For example, the 4 in 3452 represents 400, not 4.
- ✔ The digit 0 (zero) is important. It shows that there is nothing in that place value position and it keeps all other digits in the correct places.
- ✔ Be careful when rounding to a specific place value. If you place a box around the wrong digit, your answer will not be correct. Use a place value chart if you are unsure.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1 000 000	100 000	10 000	1000	100	10	1

ANS p478

Exercise 1A Rounding and estimating

△ 1–6, 7(a, c, e), 8(1st, 2nd columns), 9–11, 13–15, 18 ■ 3–6, 7(b, d, f), 8(2nd, 3rd columns), 9–10(e–h), 12, 14, 16, 17, 19, 20 ⬠ 3, 4, 6, 8(i–l), 10, 12, 16, 17, 20–22

UNDERSTANDING AND FLUENCY

1 Decide whether each number is closer to 400 or 500.

a 438 **b** 477 **c** 462 **d** 455 **e** 433 **f** 449

2 Decide whether each number is closer to 6000 or 7000.

a 6789 **b** 6306 **c** 6010 **d** 6505 **e** 6880 **f** 6448

1A.1 **3** **a** Round each number to the nearest ten.

i 482 **ii** 6377 **iii** 56 026 **iv** 738 494 **v** 8075 **vi** 904 507

b Round each number in part **a** to the nearest hundred.

4 **a** Round each number to the nearest thousand.

i 36 428 **ii** 7510 **iii** 183 915 **iv** 50 703 **v** 6052 **vi** 825

b Round each number in part **a** to the nearest hundred.

5 Round each number to its leading digit.

a 77 **b** 42 **c** 81 **d** 347 **e** 160

f 555 **g** 909 **h** 2489 **i** 6902 **j** 22 117

6 Round each number to its leading digit.

a 94 **b** 99 **c** 952 **d** 92 949 **e** 9008 **f** 960

7 Give three examples of numbers that round to each of these approximations if you round to the leading digit.

a 40 **b** 700 **c** 3000 **d** 50 000 **e** 800 000 **f** 2 000 000

8 Calculate:

a 900 + 700 **b** 300 − 10 **c** 30 × 20

d 80 ÷ 4 **e** 500 × 40 **f** 800 + 3000

g 200 ÷ 5 **h** 6000 − 500 **i** 40 000 ÷ 20

j 7000 × 300 **k** 20 000 − 9000 **l** 5000 + 10 000

1A.2 **9** Estimate the result of each calculation by first rounding each number to its leading digit.

a 468 + 731 **b** 92 – 38 **c** 27 × 49 **d** 83 ÷ 2

e 582 × 17 **f** 245 + 6379 **g** 4512 ÷ 43 **h** 137 – 51

10 Estimate the result of each calculation by first rounding each number to its leading digit.

a 3694 ÷ 442 **b** 187 × 9364 **c** 7085 – 750 **d** 964 + 5803

e 8277 × 65 234 **f** 13 761 + 8036 **g** 94 113 ÷ 587 **h** 24 905 – 780

11 An office building has 35 782 panes of glass. Write an approximation for this number if you round to:

a the leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten

e the nearest ten thousand.

12 Russia is the largest country in the world, with an area of 17 098 242 km^2. Round this value to:

a its leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten thousand

e the nearest hundred thousand

f the nearest million.

13 Ida is making a budget for her family's groceries. She decides to base her budget on the previous week's grocery bill, in which she spent the following:

- Supermarket $184
- Fish market $37
- Butcher $48
- Fruit and vegetable market $29

a Estimate to the nearest $10 the cost of the groceries for Ida's family last week.

b If Ida uses this figure for her budget, will it be an overestimation or underestimation?

14 Tina thinks 9648 rounded to its leading digit is 10 000. Ruby isn't sure, as she thinks the rounded number should have the same number of digits as the original number. Who is correct? Explain.

15 Daniel is saving for a quad bike. He has saved $60 per month for the past 28 months.

a Write an approximation for the number of months by rounding the value to its leading digit.

b Using your answer to part **a**, estimate the amount of money Daniel has saved.

c Does he have enough money to buy the quad bike?

d If he doesn't have enough money, estimate for how many more months he needs to save.

16 Paving tiles cost $7 each. Antonia needs 385 tiles for her back patio.

a Write an approximation of the number of tiles Antonia needs by rounding to the leading digit.

b Estimate the cost of the tiles, using your answer to part **a**.

c Compare your estimate to the exact cost of the tiles.

PROBLEM SOLVING AND REASONING

17 The school fundraising committee wants to raise money by buying boxes of sunscreen from a wholesaler to sell for a profit. Each box has a wholesale price of $47.

a Estimate the number of boxes the committee could buy with $2000.

b If the committee used this estimate, without performing the exact calculation decide whether there would be money left over or money owed to the wholesaler. Explain your answer.

18 Australia has an area of 7 692 024 km^2 and Indonesia has an area of 1 904 569 km^2.

a Round each value to its leading digit.

b Use your approximations from part **a** to estimate:

i the difference in area between the two countries

ii the area of Australia compared to the area of Indonesia.

c Check how close your estimations from part **b** are to the exact answers.

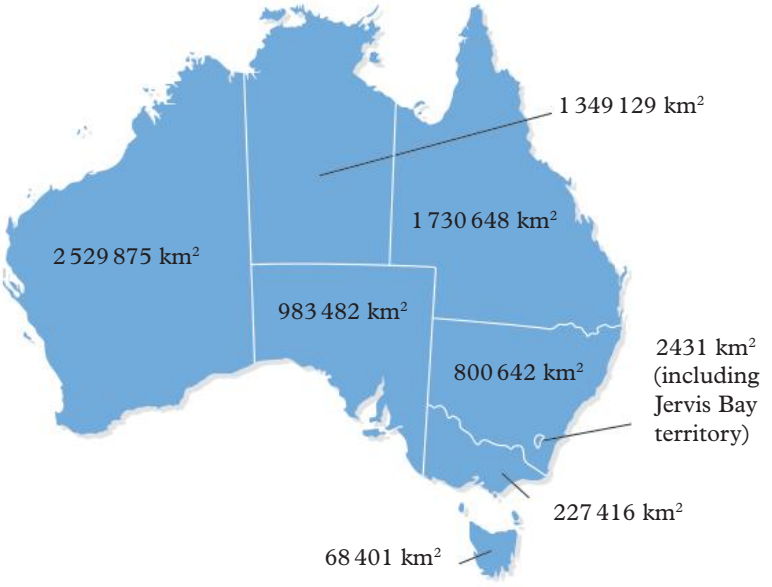
19 People attending a State of Origin rugby league match entered the stadium through one of five gates. The number of people passing through each gate is shown.

Gate	A	B	C	D	E
Number of people	9361	10 758	12 196	8844	11 037

a Estimate the number of people attending the match by rounding the numbers at each gate to the leading digit.

b Calculate the exact number of people attending the match.

20 Australia is the sixth largest country in the world. The areas of the states and territories (including islands) of Australia are shown on the map.



a Round each area to its leading digit.

b Estimate the area of Australia by adding up the approximate values from part **a**.

c The area of NSW, rounded to the nearest thousand, is 801 000 km^2. Round each of the other areas to the nearest thousand.

d Estimate the area of Australia by adding up the approximate values from part **c**.

e Compare the estimates you obtained in parts **b** and **d** with the accepted value for the area of Australia of 7 692 024 km^2.

CHALLENGE

21 The area of Australia is 7 692 024 km^2 and the population at the end of 2020 was 25 694 393.

a Estimate the area per person for the entire population by rounding both numbers to the leading digit.

b New Zealand has an area of 268 021 km^2 and the population at the end of 2020 was 5 106 400. Estimate the area per person for New Zealand by rounding both numbers to the leading digit and compare it with that for Australia.

22 Estimate the number of words you say in a day. Explain how you reached your estimate.

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Place value
- Interactive skillsheet: Rounding integers
- Interactive skillsheet: Estimation by rounding
- Investigation: An Australian road trip
- Topic quiz: 1A

1B Adding and subtracting whole numbers

Learning intentions

By the end of this topic you will be able to ...

✔ use mental strategies for addition and subtraction

✔ use the addition and subtraction algorithms for large numbers.

Inter-year links

Support — Adding whole numbers

Year 7 — 1B Adding whole numbers

Addition properties

Key content video

- The **associative law of addition** states that regardless of how the numbers are grouped in an addition, the result does not change.

  For example, (3 + 5) + 2 = 3 + (5 + 2).
- The **commutative law of addition** states that the order in which numbers are added does not change the result.

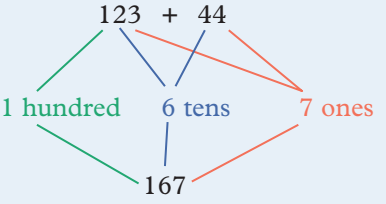
  For example, 2 + 8 = 8 + 2.

Addition (+)	Subtraction (−)
• Sum	• Difference
• Plus	• Take away
• More than	• Less than
• Increase	• Fewer
• Total	• Reduce
• Together	• Minus

Mental strategies

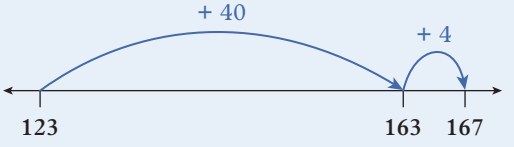
- The **by-parts method** involves adding the digits in each place value separately.

  For example, 123 + 44 = 100 + (20 + 40) + (3 + 4)
  = 100 + 60 + 7
  = 167.



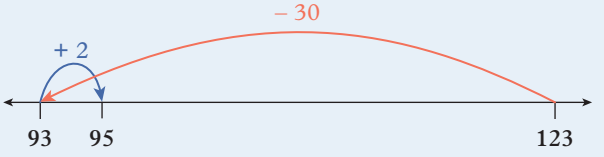
- The **jump method** involves breaking down one of the given numbers, then adding or subtracting each part of the number in stages.

  For example, 123 + 44 = 123 + 40 + 4
  = 163 + 4
  = 167.



- The **compensation method** involves rounding one number to make the calculation easier. The amount by which the number was rounded up or down is then added or subtracted.

  For example, 123 − 28 = 123 − 30 + 2
  = 93 + 2
  = 95.



Addition algorithm

- In the **addition algorithm**, start by adding the digits in the ones column, then add the digits in the tens column, then the hundreds column, and so on.

$$\begin{array}{r} {\scriptstyle 1} \\ 472 \\ +\ 846 \\ \hline 1318 \end{array}$$

Ones column: 2 + 6 = 8
Tens column: 7 + 4 = 11, carry the 10
Hundreds column: 4 + 8 + (1) = 13

← Answer

Subtraction algorithm

- In the **subtraction algorithm**, start by subtracting the digits in the ones column, then subtract the digits in the tens column, then the hundreds column, and so on.
  - → If the subtraction in a particular column cannot be done, take a value from the column to the left.

$$\begin{array}{r} {\scriptstyle 7\ 11} \\ 3\cancel{8}\cancel{1} \\ -\ 145 \\ \hline 236 \end{array}$$

Ones column: Take 10 from the tens column
11 − 5 = 6
Tens column: 7 − 4 = 3
Hundreds column: 3 − 1 = 2

← Answer

Example 1B.1 Using mental strategies to add two numbers

Work out each sum using the mental strategy given in brackets.

a 282 + 317 (by-parts method)
b 51 + 39 (jump method)
c 48 + 68 (compensation method)

THINK

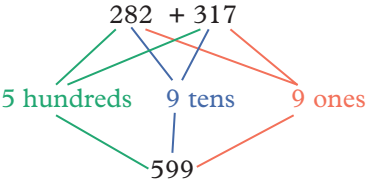
a **1** Add the hundreds together.
2 Add the tens together.
3 Add the ones together.
4 Combine these sums for the final answer.

b **1** Choose a starting number, then break down the second number into tens and ones.
2 Add the tens to the starting number, followed by the ones.

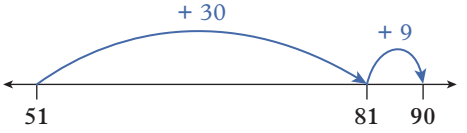
c **1** Choose a starting number, then round the second number to the nearest 10.
2 Add the rounded number to the starting number, then adjust for rounding.

WRITE

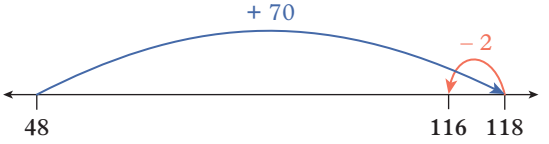
a 282 + 317 = 599



b 51 + 39 = 90



c 48 + 68 = 116



Example 1B.2 Using the addition algorithm

Calculate 781 + 965 using the addition algorithm:

$$\begin{array}{r} 781 \\ +\ 965 \\ \hline \end{array}$$

THINK

1. Add the digits in the ones column: 1 + 5 = 6.
2. Add the digits in the tens column: 8 + 6 = 14.
   Write 4 in the tens column of the answer and write 1 above the hundreds column.
3. Add the digits in the hundreds column: 1 + 7 + 9 = 17.

WRITE

$$\begin{array}{r} {}^{1} \\ 7\,8\,1 \\ +\ 9\,6\,5 \\ \hline 1\,7\,4\,6 \end{array}$$

Example 1B.3 Using mental strategies to subtract two numbers

Work out each difference using the mental strategy given in brackets.

a 112 – 67 (jump method)

b 200 – 89 (compensation method)

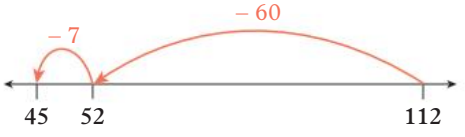
THINK

a 1. Start with the first number, then break down the second number into tens and ones.
2. Subtract the tens from that number, followed by the ones.

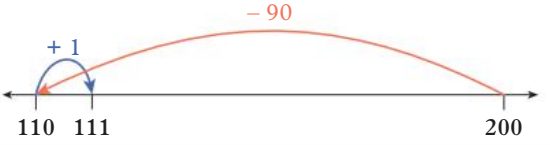
b 1. Start with the first number, then round the second number to the nearest 10.
2. Subtract the rounded number from the starting number, then adjust for rounding.

WRITE

a 112 – 67 is 45.



b 200 – 89 is 111.



Example 1B.4 Using the subtraction algorithm

Calculate 678 – 93 using the subtraction algorithm:

$$\begin{array}{r} 6\,7\,8 \\ -\ \ \ 9\,3 \\ \hline \end{array}$$

THINK

1. Subtract the digits in the ones column: 8 – 3 = 5.
2. Subtract the digits in the tens column: 7 – 9.
   Take 1 from the hundreds column by reducing the hundreds to 5 and increasing the tens to 17.
   Subtract the digits in the tens column: 17 – 9 = 8.
3. Subtract the digits in the hundreds column: 5 – 0 = 5. Recall that if there is no digit in a place then that place has a value of zero.

WRITE

$$\begin{array}{r} {}^{5}\,{}^{17} \\ \cancel{6}\,\cancel{7}\,8 \\ -\ \ \ 9\,3 \\ \hline 5\,8\,5 \end{array}$$

Helpful hints

- ✔ Don't forget that addition is commutative – reordering sums can make them a lot easier!
- ✔ Subtraction is not commutative. Unlike addition problems, you cannot change the order of the numbers in subtraction problems. You must work through subtractions from left to right.
- ✔ Make sure that your place value columns are well aligned so that you add or subtract the correct digits.
- ✔ Read each question carefully. You may find some questions easy to calculate in your head.

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Exercise 1B Adding and subtracting whole numbers

▲ 1–3, 4(a–c), 5(a, c), 6, 7, 8–10(b, c), 11, 13, 15 ■ 1a–d(ii, iii), 3(c, d), 4(d–f), 5(b, d), 6a–d(ii, iii), 8–10, 12, 14, 15, 17, 18(a) ⬟ 1a–d(iv), 4(e, f), 5(c, d), 6a–d(iv), 9(c, d), 10(c, d), 12, 14–19

UNDERSTANDING AND FLUENCY

1B.1 **1** **a** Use the by-parts method to work out each sum.

i 34 + 63 **ii** 89 + 72 **iii** 654 + 325 **iv** 1243 + 9654

b Use the jump method to work out each sum.

i 647 + 72 **ii** 768 + 51 **iii** 456 + 128 **iv** 3452 + 139

c Use the compensation method to work out each sum.

i 24 + 99 **ii** 472 + 56 **iii** 658 + 465 **iv** 7629 + 475

d Use any mental strategy to work out each sum.

i 87 + 90 **ii** 489 + 650 **iii** 8765 + 59 **iv** 5983 + 587

1B.2 **2** Calculate each sum using the addition algorithm.

a $\begin{array}{r} 243 \\ +\ 715 \\ \hline \end{array}$ **b** $\begin{array}{r} 538 \\ +\ 154 \\ \hline \end{array}$ **c** $\begin{array}{r} 758 \\ +\ 461 \\ \hline \end{array}$ **d** $\begin{array}{r} 697 \\ +\ 805 \\ \hline \end{array}$

3 Use the addition algorithm to calculate each of the following.

a 4765 + 476 **b** 7410 + 3065 **c** 16 239 + 2633 **d** 98 543 + 23 987

4 Use the addition algorithm to calculate each of the following.

a 456 + 280 + 487 **b** 876 + 387 + 594 **c** 986 + 386 + 2769

d 56 + 875 + 3987 **e** 6954 + 389 + 23 **f** 7632 + 9520 + 3198

5 Evaluate:

a 56 871 + 34 981 **b** 54 870 + 29 528 + 43 679

c 5498 + 2873 + 8921 + 7629 **d** 397 + 287 + 888 + 396 + 492

1B.3 **6** **a** Use the jump method to work out each difference.

i 93 – 30 **ii** 126 – 39 **iii** 145 – 51 **iv** 764 – 71

b Use the compensation method to work out each difference.

i 87 – 54 **ii** 763 – 68 **iii** 874 – 618 **iv** 433 – 67

c Use any mental strategy to work out each difference.

i 130 – 70 **ii** 5600 – 410 **iii** 390 – 180 **iv** 568 – 456

d Use any mental strategy to work out each problem.

i 79 + 39 – 56 **ii** 750 + 830 – 690 **iii** 78 + 565 – 120 **iv** 350 – 90 + 876

1B.4 **7** Calculate each difference using the subtraction algorithm.

a $\begin{array}{r} 438 \\ -\ 399 \\ \hline \end{array}$ **b** $\begin{array}{r} 852 \\ -\ 576 \\ \hline \end{array}$ **c** $\begin{array}{r} 2356 \\ -\ 976 \\ \hline \end{array}$ **d** $\begin{array}{r} 3765 \\ -\ 387 \\ \hline \end{array}$

8 Use the subtraction algorithm to calculate each of the following.

a 487 – 458 **b** 3865 – 376 **c** 4652 – 234 **d** 6798 – 478

9 Use the subtraction algorithm to calculate each of the following.

a 7654 – 488 **b** 7421 – 6423 **c** 75 638 – 689 **d** 86 408 – 42 766

10 Calculate:

a 566 + 238 – 328 **b** 2776 + 5409 – 3111

c 76 409 + 45 629 – 83 888 **d** 6543 – 2389 + 4319

11 Yiyan is the manager of five city car parks. The capacities of the car parks are 356, 821, 489, 398 and 450 cars. What is the total number of cars that can use Yiyan's car parks?

12 At the 2021 Census the population of NSW was 8 072 163 and the population of Greater Sydney was 5 231 147.

a How many people from NSW live outside Greater Sydney?

b How much does the population of Greater Sydney have to grow for it to become 6 000 000?

c If the population of Greater Sydney decreases by 32 000 in 2022 and increases by 59 000 in 2023, what will the population be at the end of 2023?

13 Two local libraries decide to merge their book collections to form a mega-library. Spring Hill Library has 27 356 books and Dove St Library has 54 237 books.

a How many books are there in total?

b The mega-library finds that there are two copies of 8345 books so it is decided one copy of each will be given away. After the giveaway, how many books will there be in the new library?

14 Raf and Mac go on a road trip around NSW. They combine expenses for the trip and Raf keeps a tally of the distances they travel and the money they spend.

Stage	Distance travelled (km)	Accommodation (2 nights)	Expenses (petrol, food, etc.)
Sydney to Bathurst	201	$292	$128
Bathurst to Gilgandra	262	$320	$175
Gilgandra to Narrabri	187	$228	$184
Narrabri to Muswellbrook	276	$248	$162
Muswellbrook to Sydney	251	$0	$85

a What was the total distance Raf and Mac travelled on their road trip?

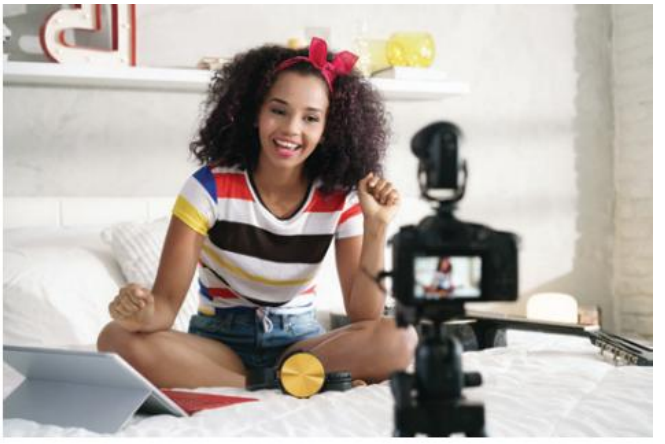
b How much in total did they spend on accommodation?

c What were their total expenses not including accommodation?

d Raf and Mac had each saved up $1000 for the trip. How much was left over when they got back to Sydney?

PROBLEM SOLVING AND REASONING

15 Aya has 2437 subscribers to her YouTube channel and Akuac has 3762 subscribers.



a What is the total number of subscribers between the two girls?

b Aya and Akuac decide to combine their channels into one new channel, thus combining their subscribers.
If there are 1562 people who initially subscribed to both channels, how many subscribers will the new combined channel have? (Assuming channels can be merged and subscribers can't be counted twice.)

16 The Six Foot Track goes from Katoomba to Jenolan Caves through the Blue Mountains National Park. Jai and Rudra walk the track over 3 days in the summer. They walk the following distances: 15 km, 19 km and 10 km.

a How long is the Six Foot Track?

Katoomba is 1017 m above sea level and Jenolan Caves are 828 m above sea level.

b What is the difference in height between the two locations?

17 Shannon's shoe store has enough space for 568 pairs of shoes. If Shannon has 483 pairs of shoes in her store, how many does she need to order to be at full capacity?

CHALLENGE

18 a What is the difference between the sum of the first five odd numbers (starting from 1) and the sum of the first five even numbers (starting from 2)?

b What is the difference between the sum of the first 2000 odd numbers (starting from 1) and the first 2000 even numbers (starting from 2)?

19 A cryptarithm is a type of mathematical puzzle in which numbers have been replaced by letters. There are six solutions to the cryptarithm below. How many of them can you find?

$$\begin{array}{r} \text{SEED} \\ +\text{WATER} \\ \hline \text{FRUIT} \end{array}$$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Adding and subtracting whole numbers

Investigation
Palindromic numbers

Topic quiz
1B

1C Multiplying and dividing whole numbers

Learning intentions

By the end of this topic you will be able to ...

- ✔ use mental strategies for multiplication
- ✔ calculate products and quotients using the multiplication and division algorithms.

Inter-year links

Support	Multiplying whole numbers
Year 7	1D Multiplying whole numbers

Key content video

Multiplication

- Repeated addition can be reduced to multiplication of **factors** to give a **product**.

$$\underbrace{3 + 3}_{\text{repeated addition}} = \underbrace{2 \times 3}_{\text{factors}} = \underbrace{6}_{\text{product}}$$

Multiplication strategies

- The **associative law of multiplication** states that regardless of how the numbers are grouped, the result of a multiplication does not change.

  For example, $(2 \times 4) \times 6 = 2 \times (4 \times 6)$.
- The **commutative law of multiplication** states that the order in which numbers are multiplied does not change the results.

  For example, $112 \times 4 = 4 \times 112$.
- The **distributive law** states that multiplication can be 'distributed' across a bracket. This means that large numbers can be broken down into a sum of smaller numbers, each of which can then be multiplied separately.

  For example,

$$\begin{aligned} 5 \times 32 &= 5 \times (30 + 2) \\ &= 5 \times 30 + 5 \times 2 \\ &= 150 + 10 \\ &= 160. \end{aligned}$$

Multiplication algorithm

- When two numbers are multiplied, the first number is multiplied by the individual digits of the second number, considering the place values of the digits, then the products are added together.

$$\begin{array}{lrll} & {}^{2} & & \\ & 34 & & \\ & \times\quad 26 & & \\ \hline \text{Multiply by ones} \longrightarrow & 204 & (34 \times 6) & \\ \text{Multiply by tens} \longrightarrow & +\quad 680 & (34 \times 20) & \text{Don't forget the zero(es).} \\ \hline \text{Add the results} \longrightarrow & 884 & & \end{array}$$

Division

- When performing a division, the **dividend** is the number that is being divided by another number.
- The **divisor** is the number by which the dividend is divided.
- The **quotient** is the whole-number result of the division.
- The **remainder** is the part of the dividend that is left over when the number is divided by the divisor. The remainder can be zero.

$$13 \div 6 = 2 \text{ remainder } 1$$

dividend divisor quotient remainder

Short division

- **Short division** is an algorithm used to simplify a division problem by breaking it down into easy steps.
- Start by dividing the first digit of the dividend by the divisor.

quotient

$$\begin{array}{r} 1\ 4\ 5 \\ 6\overline{)8^{2}7^{3}1} \end{array} \text{ remainder } 1$$

divisor dividend remainder

Multiplication (×)	Division (÷)
• Product	• Quotient
• Times	• Divide
• Multiply	• Shared
• Groups of	• Distributed

Example 1C.1 Using strategies to multiply numbers

Work out each product using the multiplication strategy given in brackets.

a 9×34 (distributive law)

b $12 \times 4 \times 2$ (associative law)

THINK

a **1** Break down the second number into tens and ones.

2 Multiply each part of the second number by the first number.

3 Add the results together.

b **1** Use brackets to group pairs of numbers that are easy to multiply together and/or produce round numbers.

2 Multiply the numbers grouped in the brackets.

3 Multiply the result by the third number.

WRITE

a $9 \times 34 = 9 \times (30 + 4)$

$= 9 \times 30 + 9 \times 4$

$= 270 + 36$

$= 306$

b $12 \times 4 \times 2 = 12 \times (4 \times 2)$

$= 12 \times 8$

$= 96$

Example 1C.2 Using the multiplication algorithm

Calculate the product using the multiplication algorithm:

$$\begin{array}{r} 359 \\ \times \quad 72 \\ \hline \end{array}$$

THINK

1. Set out the multiplication problem by lining up the digits according to their place value.
2. Multiply the first number by the ones of the second number: 359 by 2 (359 × 2 = 718).
3. Multiply the first number by the tens of the second number: 359 by 70 (359 × 70 = 25 130).
4. Add the products and write the final answer at the bottom of the calculation.

WRITE

$$\begin{array}{rl} & \begin{array}{r} {\scriptstyle 4\ 6} \\ {\scriptstyle 1\ 1} \\ 359 \end{array} \\ \times & 72 \\ \hline & 718 \quad (359 \times 2) \\ + & 25\,130 \quad (359 \times 70) \\ \hline & 25\,848 \end{array}$$

359 × 72 = 25 848

Example 1C.3 Using short division

Use short division to calculate the quotient and remainder of 1257 ÷ 8.

THINK

1. Set up the problem using the division algorithm. The divisor is 8 and the dividend is 1257.
2. How many 8s go into 1? Write a zero above the 1.
3. How many 8s go into 12? Write 1 above the 2 on the quotient line. Put the remainder 4 in front of the next digit, 5.
4. How many 8s go into 45? Write a 5 above the 5. Put the remainder 5 in front of the next digit, 7.
5. How many 8s go into 57? Write 7 above the 7 on the quotient line. Work out the remainder. 7 × 8 = 56, so the remainder is 57 − 56 = 1.

WRITE

$$\begin{array}{r} 0\ 1\ 5\ 7 \text{ remainder } 1 \\ 8\overline{)1\ 2^{4}5^{5}7} \end{array}$$

1257 ÷ 8 = 157 remainder 1

Helpful hints

- ✔ When using the distributive law, don't forget to maintain place values using zeroes.
  For example, 32 × 3 = (30 × 3) + (2 × 3)
  32 × 3 ≠ (3 × 3) + (2 × 3).
- ✔ Carefully line up digits by place value when setting up long multiplication problems to avoid making errors when adding the products together.
- ✔ Remember that division goes from left to right.

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Exercise 1C Multiplying and dividing whole numbers

1–3(1st, 2nd columns), 4(a, c), 5, 6, 7, 8–9(a–c), 10–13, 17

1–3(2nd, 3rd columns), 4(b, d), 6(c, f), 7, 8–9(d–f), 10(c, d), 11, 14, 15, 17, 19

1(j–l), 2(g, i), 4(d), 6(d–f), 8–9(e, f), 10(c, d), 11, 14, 16, 18–21

UNDERSTANDING AND FLUENCY

1C.1 **1** Use the distributive law to calculate:

a 3×42 **b** 6×102 **c** 203×7 **d** 305×9
e 17×98 **f** 83×99 **g** 24×95 **h** 41×19
i 62×101 **j** 29×1001 **k** 11×97 **l** 15×999

2 Use mental strategies to calculate:

a $17 \times 8 \times 3$ **b** $21 \times 5 \times 6$ **c** $3 \times 74 \times 2$
d $66 \times 8 \times 10$ **e** $4 \times 6 \times 25$ **f** $921 \times 5 \times 20$
g $35 \times 4 \times 5$ **h** $29 \times 250 \times 4$ **i** $200 \times 186 \times 5$

3 Find each product.

a 9×70 **b** 12×800 **c** 11×700 **d** 126×50
e 356×900 **f** 2765×3000 **g** 2763×9000 **h** $1674 \times 50\,000$

4 Find each product.

a $60 \times 40 \times 300$ **b** $80 \times 700 \times 8000$ **c** $2400 \times 20 \times 200$ **d** $15\,000 \times 500 \times 50$

1C.2 **5** Multiply the following pairs of numbers using the multiplication algorithm.

a 589×76 **b** 498×49 **c** 4598×23
d 4623×32 **e** 9245×72 **f** 9430×520

6 Multiply the following pairs of numbers using the multiplication algorithm.

a 387×860 **b** 378×598 **c** 638×496
d 3769×458 **e** 4287×352 **f** 5923×5030

7 **a** Estimate these products by first rounding each number to its leading digit, then multiplying.

i 8462×95 **ii** 3299×87 **iii** $95\,687 \times 625$ **iv** $78\,438 \times 8345$

b Check how close your estimations are to the exact results by performing the calculations.

1C.3 **8** Use short division to calculate each quotient and remainder.

a $8\overline{)463}$ **b** $6\overline{)348}$ **c** $7\overline{)160}$
d $5\overline{)375}$ **e** $9\overline{)689}$ **f** $4\overline{)5682}$

9 Use short division to calculate each quotient and remainder.

a $5824 \div 4$ **b** $8399 \div 6$ **c** $4210 \div 3$
d $84\,276 \div 5$ **e** $56\,289 \div 9$ **f** $1489 \div 7$

10 Perform each division using short division.

a $\frac{143}{6}$ **b** $\frac{653}{8}$ **c** $\frac{246}{7}$ **d** $\frac{489}{3}$

11 The average of a set of scores is the total sum of the scores divided by the number of scores. For example, the average of 25, 30 and 23 is $(25 + 30 + 23) \div 3$ or $\frac{25 + 30 + 23}{3}$.

a Find the sum of 19, 22, 24 and 27.

b Divide your answer from part **a** by 4 to find the average of 19, 22, 24 and 27.

12 a The commutative law relates to the order of operations. Calculate:

i $3 + 6$

ii $6 + 3$

b Does the order matter when you add two numbers? Try two more examples.

c Does subtraction obey the commutative law? Try these calculations.

i $6 - 3$

ii $3 - 6$

d Does multiplication obey the commutative law? Try these.

i 6×3

ii 3×6

e Does division obey the commutative law? Try these.

i $6 \div 3$

ii $3 \div 6$

13 a The associative law also relates to addition. Calculate:

i $(5 + 6) + 7$

ii $5 + (6 + 7)$

iii $(5 + 7) + 6$

b Does it matter in which order you add three numbers? Try two more examples.

c Do subtraction, multiplication and division obey the associative law? Try some examples to help you decide.

14 The commutative and associative laws can make mental calculations easier.

a To calculate $54 + 118 + 16$, you can add any two of the numbers together first.

i Which two numbers are easiest to add together first? Explain.

ii Perform the calculation.

b To calculate $87 \times 25 \times 4$, you can multiply any two of the numbers together first.

i Which two numbers are easiest to multiply together first? Explain.

ii Perform the calculation.

c Explain how these laws can make some calculations easier to perform.

15 a Calculate these, then compare your answers.

i $4 \times (5 + 6)$ **ii** $4 \times 5 + 4 \times 6$

b Calculate these, then compare your answers.

i $6 \times (10 - 2)$ **ii** $6 \times 10 - 6 \times 2$

c Parts **a** and **b** are examples of the application of the distributive law. Describe this law.

d Show how the distributive law works by providing two more examples.

16 The distributive law can be used to make some multiplication calculations easier to perform.

a The calculation 16×24 can be written as $16 \times (20 + 4)$ or $16 \times 20 + 16 \times 4$.

i Calculate 16×20.

ii Calculate 16×4.

iii Add your results and write the answer to 16×24.

b The calculation 45×998 can be written as $45 \times (1000 - 2)$ or $45 \times 1000 - 45 \times 2$.

i Calculate 45×1000.

ii Calculate 45×2.

iii Subtract your results and write the answer to 45×998.

c Explain how the distributive law can be used to make some multiplication calculations easier to perform.

PROBLEM SOLVING AND REASONING

17 Ms Lee spends $480 per school year on coffee from The Blue Cup Café.

a If the school year is 40 weeks, how much does Ms Lee spend per week?

b If The Blue Cup Café has a special teacher price of $4 per cup, how many times per week does Ms Lee buy coffee?

Mr Lin prefers the cheaper coffee at the service station over the road. He pays $2 per cup and buys coffee five days per week.

c How much does Mr Lin pay for coffee in one week?

d How much does Mr Lin pay for coffee in a school year?

18 The Year 8 class captains are tasked with providing icy poles for the whole school at the swimming sports carnival. There are 1289 students at the school.

a How many packs of icy poles do the class captains have to buy if there are 20 in each pack, so that no student misses out?

b How many icy poles will be left over if every student has one?

c If the packs of icy poles cost $3 each, how much money do the class captains need?

19 Astrid is planning to hire a car for a road trip from Sydney to Cairns. The distance she plans to travel is 2422 km. The car hire company informs Astrid that the car can travel an average of 7 kilometres per litre of fuel.

a Use this information to calculate how many litres of fuel Astrid will need to complete her journey.

b If Astrid's fuel tank is 50 litres, how many full tanks of fuel will she need for her journey?

c Astrid wants to make sure she overestimates the amount of money she will need to spend on fuel, so she rounds the price of fuel up to $2 per litre. Using this estimate, how much money should Astrid budget for her fuel expenses?

CHALLENGE

20 Darcy makes a number of mini spring rolls. After eating one, she gives half the remainder to her sister. After eating another spring roll, she gives half of what is left to her brother. Darcy now has only five spring rolls left. How many did she start with?

21 A recipe for eight biscuits requires 20 g butter, 30 g sugar and 40 g rolled oats.

a How many grams of sugar do I need if I want to make 16 biscuits?

b How many biscuits can I make if I have 140 g butter, 150 g sugar and 160 g rolled oats?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing whole numbers

Investigation
Factorials

Topic quiz
1C

1D Multiples, factors, indices and roots

Learning intentions

By the end of this topic you will be able to ...

- ✔ find the lowest common multiple and highest common factor of two numbers
- ✔ evaluate squares and cubes of numbers
- ✔ evaluate square roots and cube roots.

Inter-year links

Support	Multiples and factors
Year 7	2C Multiples and the lowest common multiple

Key content video

Multiples

- The **multiples** of a whole number are the numbers produced by multiplying that number by another whole number.
  For example, some multiples of 3 are:
  $3 \times 1 = 3$
  $3 \times 2 = 6$
  $3 \times 3 = 9$.
- The **lowest common multiple (LCM)** of two or more numbers is the common multiple of the numbers with the smallest value.

Multiples of 4: 4, 8, 12, 16, …
Multiples of 6: 6, 12, 18, 24, …
LCM: 12

Factors

- The **factors** of a whole number are the whole numbers which divide exactly into that number.
  For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- The divisibility rules are helpful when looking for factors of large numbers.

÷ 2 The last digit of the number is even.	÷ 3 The sum of all the digits in the number is divisible by 3.	÷ 4 The number made by the last two digits is divisible by 4.
÷ 5 The last digit is 5 or 0.	÷ 6 The number is divisible by 2 and 3.	÷ 7 There is no easy trick for 7.
÷ 8 The number made by the last three digits is divisible by 8.	÷ 9 The sum of all the digits in the number is divisible by 9.	÷ 10 The last digit is 0.

- The **highest common factor (HCF)** of two or more whole numbers is the **common factor** of the numbers with the greatest value.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18
HCF: 6

Indices

- Repeated multiplications can be represented using indices.

index

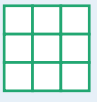
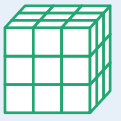
base → $2^3 = 2 \times 2 \times 2 = 8$

index form | expanded form | basic numeral

- The **base** is the number that is repeatedly multiplied.
- The **index** (or **exponent**) represents the number of times the base appears in **expanded form**.

Squares, cubes and roots

- The **square** of a number is the result of multiplying that number by itself. Squaring a number is the same as raising it to a power (or index) of 2 and is indicated by a superscript 2.
  For example, $5^2 = 5 \times 5 = 25$.
- The **square root** of a number is the number that when squared, results in the original number. Square roots are indicated using the symbol $\sqrt{}$.
  For example, $\sqrt{9} = 3$ because $3^2 = 3 \times 3 = 9$.
- For positive numbers, there is an inverse relationship between squares and square roots.
  This means that if $8^2 = 64$, then $\sqrt{64} = 8$.
- The **cube** of a number is the result of multiplying that number by itself twice. Cubing a number is the same as raising it to a power (or index) of 3 and is indicated by a superscript 3.
  For example, $4^3 = 4 \times 4 \times 4 = 64$.
- The **cube root** of a number is the number that when cubed, results in the original number. Cube roots are indicated using the symbol $\sqrt[3]{}$.
  For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$.
- There is an inverse relationship between cubes and cube roots.
  This means that if $5^3 = 125$, then $\sqrt[3]{125} = 5$.
- When multiplying a number by a square or cube root, we can write the number in front of the root without using a multiplication sign.
  For example, $5\sqrt{9}$ means the same as $5 \times \sqrt{9}$.

Squares			Cubes		
$1^2 = 1 \times 1 = 1$		$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$		$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$		$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$		$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$		$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$		$\sqrt[3]{27} = 3$

Example 1D.1 Finding the LCM and HCF

Determine:

a the LCM of 14 and 35

b the HCF of 24 and 38.

THINK

a **1** List the multiples of 14 and the multiples of 35.

2 Draw a box around the multiple with the lowest value that appears in both lists (the LCM).

b **1** List all the factors of 24 and all the factors of 38.

2 Draw a box around the factor with the highest value that appears in both lists (the HCF).

WRITE

a Multiples of 14: 14, 28, 42, 56, $\boxed{70}$, …
Multiples of 35: 35, $\boxed{70}$, 105, 140, …
The LCM is 70.

b Factors of 24: 1, $\boxed{2}$, 3, 4, 6, 8, 12, 24
Factors of 38: 1, $\boxed{2}$, 19, 38
The HCF is 2.

Example 1D.2 Calculating the square and cube

Calculate:

a the square of 5

b the cube of 5.

THINK

1 To 'square' 5, raise 5 to the power of 2. To 'cube' 5, raise 5 to the power of 3.

2 Identify the base and the index and multiply the base by itself as many times as specified by the index.

3 Perform the multiplication.

WRITE

a $5^2 = 5 \times 5$
$= 25$

b $5^3 = 5 \times 5 \times 5$
$= (5 \times 5) \times 5$
$= 25 \times 5$
$= 125$

Example 1D.3 Calculating the square root and cube root

Calculate the value of each of the following roots.

a $\sqrt{36}$

b $\sqrt[3]{64}$

THINK

a Use times tables to determine which number, multiplied by itself, is equal to the number under the square root symbol.

b Use times tables and estimation to determine which number, multiplied by itself three times, is equal to the number under the cube root symbol.

WRITE

a $6 \times 6 = 36$
So, $\sqrt{36} = 6$

b $4^3 = 4 \times 4 \times 4$
$= (4 \times 4) \times 4$
$= 16 \times 4$
$= 64$
So, $\sqrt[3]{64} = 4$

Helpful hints

- ✔ Lowest common multiples are not necessarily 'low' in value; they can be large numbers.
- ✔ There is no such thing as a 'highest common multiple', as any given number has infinitely many multiples.
- ✔ Make sure you use a systematic method for finding all the factors; it is very easy to miss one or two.
- ✔ Don't confuse indices and multiplication!
  For example, $2^3 = 2 \times 2 \times 2$
  $2^3 \neq 2 \times 3$.
  Place your indices carefully; they should be in smaller font than the base and sit high up on the shoulder of the base – your 4^3 should look different to your 43!

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Exercise 1D Multiples, factors, indices and roots

△ 1–5, 6(a, c, e), 7, 8(a–f), 9(1st, 2nd columns), 10, 11, 16, 17 | □ 2, 3, 6, 7, 8(g–l), 9(3rd, 4th columns), 10–15, 17 | ⬠ 3(d–f), 6(b, d, f), 7, 8(j, k, l), 9(3rd, 4th columns), 10, 11, 15, 17–20

UNDERSTANDING AND FLUENCY

1 **a** List the multiples of 6 and 9 up to 100.
b Identify the common multiples of 6 and 9 in the lists from part **a**.
c What is the lowest common multiple (LCM) of 6 and 9?

1D.1 **2** Find the lowest common multiple (LCM) of each pair of numbers.

a 6 and 8 **b** 12 and 18
c 25 and 30 **d** 12 and 15
e 14 and 20 **f** 9 and 15

3 Find the lowest common multiple (LCM) of each group of numbers.

a 5, 6 and 7 **b** 5, 7 and 9
c 6, 8 and 9 **d** 5, 8 and 12
e 15, 18 and 24 **f** 9, 18 and 27

4 Find all the factors of each number.

a 45 **b** 72 **c** 90
d 120 **e** 100 **f** 144

5 **a** What is the highest common factor (HCF) of 18 and 36?
b What is the HCF of 21 and 45?
c What is the HCF of 12 and 30?

6 Find the highest common factor (HCF) of each group of numbers.

a 42 and 70 **b** 48 and 84
c 36 and 63 **d** 144, 180 and 200
e 120, 240 and 360 **f** 500, 900 and 725

7 Find the LCM and the HCF of each group of numbers.

a 24 and 45 **b** 16 and 36 **c** 6, 15 and 20 **d** 7, 12 and 22

1D.2 **8** Calculate the square and the cube of each of the following numbers.

a 3 **b** 2 **c** 10 **d** 7 **e** 6 **f** 12
g 8 **h** 4 **i** 1 **j** 9 **k** 11 **l** 14

1D.3 **9** Calculate the value of each of the following roots.

a $\sqrt{9}$ **b** $\sqrt[3]{8}$ **c** $\sqrt[3]{125}$ **d** $\sqrt{64}$

e $\sqrt{81}$ **f** $\sqrt[3]{216}$ **g** $\sqrt{100}$ **h** $\sqrt[3]{343}$

i $\sqrt[3]{512}$ **j** $\sqrt[3]{2197}$ **k** $\sqrt{196}$ **l** $\sqrt[3]{3375}$

10 Find the value of each of the following by first evaluating any indices and roots.

a $2^3 \times \sqrt{9}$ **b** $3^3 + \sqrt{36}$ **c** $7^2 \times 10^3$

d $4^3 \div \sqrt{16}$ **e** $2^6 \times 8^2$ **f** $100^2 \times \sqrt{100}$

11 Find the value of each of the following by first evaluating any indices and roots.

a $6^2 \times 3^3$ **b** $18^2 \div 3^2$ **c** $36^2 \div 6^3$

d $4^5 \times 2^1$ **e** $12^2 \times 4^2 \div 3^2$ **f** $\sqrt{81} \times 2^4 \times 5^2$

12 When a square or cube root is multiplied by a number, we can write the number in front of the root without a multiplication sign. For example, $7\sqrt{36} = 7 \times \sqrt{36}$.

Calculate the following by first evaluating the roots.

a $7\sqrt{9}$ **b** $6\sqrt{100}$ **c** $5\sqrt[3]{27}$ **d** $7\sqrt[3]{125}$

13 **a** Find the value of $\sqrt[3]{64}$.

b Find the value of $3\sqrt{64}$.

c Explain the difference between $\sqrt[3]{64}$ and $3\sqrt{64}$.

14 Sally and Jo have the same number of stickers. Sally's stickers are in packs of 15 and Jo's stickers are in packs of 12. What is the smallest number of stickers each of them could have?

15 Taran is a florist. He receives a delivery of roses that can be divided into bouquets of 15 or 24.

a What is the smallest number of roses in the delivery?

b Taran has 24 roses and 40 lilies. If he wants to make identical bouquets with no flowers left over, what is the greatest number of bouquets he can make?

16 A group of students is going on an excursion to Australia Zoo. The students can travel on either buses that seat 30 or buses that seat 45 with all seats being occupied. What is the smallest number of students going on the excursion?

17 Dizzy sold an equal numbers of wontons and dumplings today. Wontons are sold in packets of 8 while dumplings are sold in packets of 12. What is the smallest number of each that Dizzy sold?

18 Arvin has 24 toy trucks and 64 toy cars. He divides them into identical groups so that there are no toys left over. What is the largest number of groups there might be?

19 Jill has grown three types of potatoes in her garden: Pontiac, Sebago and Desiree. Seven Pontiac potatoes weigh the same as four Sebago potatoes and five Sebago potatoes weigh the same as six Desiree potatoes. Put the potatoes in order of weight from lightest to heaviest.

20 How many numbers in the set $\{7^1, 7^2, 7^3, 7^4, \ldots, 7^{2000}\}$ have a last digit of 3?

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Lowest common multiple
- Interactive skillsheet: Highest common factor
- Interactive skillsheet: Indices
- Interactive skillsheet: Squares and square roots
- Investigation: Triangular numbers
- Topic quiz: 1D

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

1A **1** Round each number to the nearest hundred.

a 576 **b** 934
c 7871 **d** 2839
e 45 621 **f** 76 618

1A **2** Estimate the result of each calculation by first rounding each number to its leading digit.

a $849 + 381$ **b** $56 - 18$
c 65×43 **d** $582 \div 16$
e 493×28 **f** $825 + 9114$
g $8402 \div 79$ **h** $658 - 180$

1B **3** Calculate each of the following.

a $64 + 98$ **b** $745 + 832$
c $6982 + 5176$ **d** $28 + 743 + 6287$
e $6287 + 341 + 89$ **f** $3916 + 5328 + 6124$

1B **4** Calculate each of the following.

a $63 - 45$ **b** $582 - 96$
c $846 - 757$ **d** $5723 - 68$
e $8743 - 374$ **f** $7398 - 3572$

1B **5** Calculate each of the following.

a $68 - 34 + 287$ **b** $738 + 296 - 457$
c $5921 + 4827 - 4277$ **d** $83\,422 + 82\,113 - 57\,329$

1C **6** Calculate each of the following.

a 768×43 **b** 846×76
c 6214×57 **d** 9262×83
e 7248×82 **f** 6524×645

1C **7** Use short division to calculate each quotient and remainder.

a $395 \div 4$ **b** $646 \div 7$
c $8328 \div 5$ **d** $76\,398 \div 9$
e $68\,341 \div 11$ **f** $653\,789 \div 6$

1D **8** Find the lowest common multiple (LCM) of each pair of numbers.

a 7 and 6 **b** 4 and 9
c 5 and 12 **d** 30 and 45
e 18 and 24 **f** 36 and 54

1D **9** Find the highest common factor (HCF) of each pair of numbers.

a 24 and 30 **b** 27 and 72
c 45 and 75 **d** 120 and 185
e 450 and 600 **f** 288 and 432

1D **10** Find the value of each of the following by first evaluating any indices and roots.

a $6^2 + 4^3$ **b** $9^2 \div 3^2$
c $\sqrt{81} \times 4^2$ **d** $2^6 \div 8$
e $12^2 + 6^2 \div 3^2$ **f** $\sqrt{49} \times 3^3 + 8^2$

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1E Negative integers

Learning intentions

By the end of this topic you will be able to ...

✔ compare the value of positive and negative numbers

✔ use negative numbers to represent real-life situations.

Inter-year links

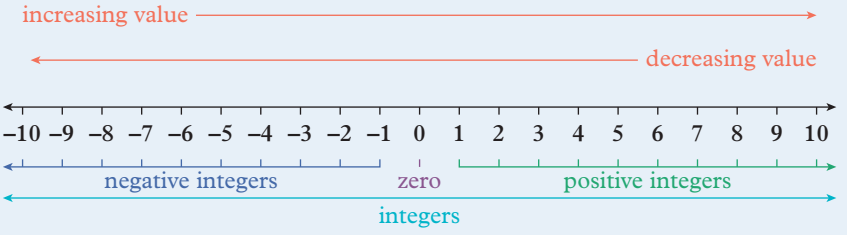
Support — Negative numbers

Year 7 — 5A Negative numbers

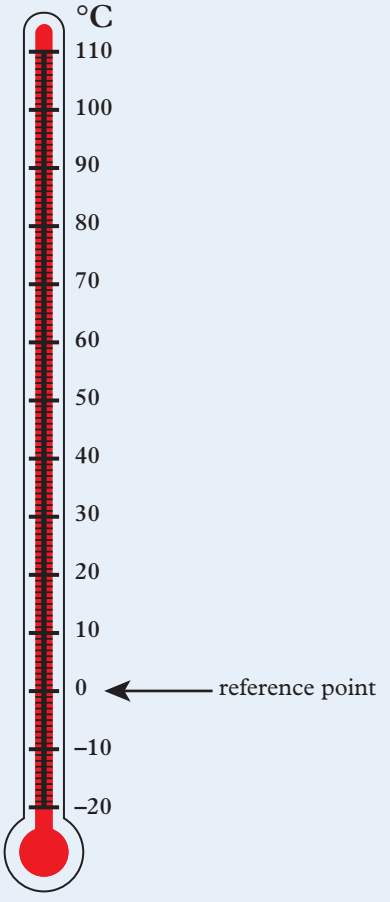
Integers

Key content video

- The set of **integers** includes all positive whole numbers, negative whole numbers and zero.
- **Negative integers** are less than zero in value and are preceded by a negative sign, –, placed directly before the first digit.
- **Positive integers** are greater than zero in value and can be represented by using a positive sign, +. The positive sign can also be left out.

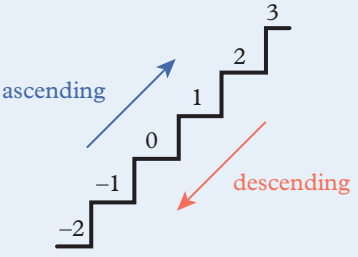


- In the real world, negative numbers are used to represent quantities that lie on a scale that goes below zero. A sign is used to indicate the direction relative to the **reference point** of zero. For example, a negative temperature indicates that the temperature has dropped below the reference point of 0°.



Ordering numbers

- Numbers are in **ascending order** when they are arranged in order of increasing value.
- Numbers are in **descending order** when they are arranged in order of decreasing value.

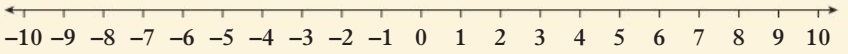


- 'Less than' is shown using the symbol '<'.
- 'Greater than' is shown using the symbol '>'.
- 'Equal to' is shown using the symbol '='.

Example 1E.1 Identifying the greater of two integers

Identify which integer is greater in each pair of numbers. Use the following number line to help you.

a −5 and 2 **b** −4 and −7



THINK

a **1** Locate the two numbers on the number line and compare their positions. 2 is to the right of −5, so 2 is greater than −5.

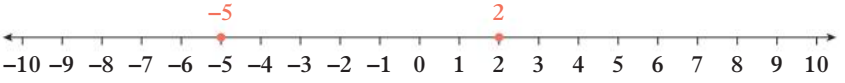
2 Use the 'greater than' symbol, >, in your answer.

b **1** Locate the two numbers on the number line and compare their positions. −4 is to the right of −7, so −4 is greater than −7.

2 Use the 'greater than' symbol, >, in your answer.

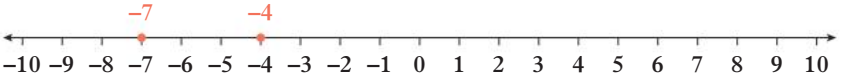
WRITE

a



$2 > -5$

b



$-4 > -7$

Example 1E.2 Representing a situation with an integer

Provide an integer to represent each situation.

a The temperature is 15 degrees above zero.

b A fish swims 4 m below the surface of the water.

c Your bank account is overdrawn by $30.

THINK

a **1** The reference point is 0°.

2 Determine the sign of the integer based on the number's position relative to the reference point. '15 degrees above zero' can be indicated using 15.

b **1** The reference point is the water surface.

2 Determine the sign of the integer based on the number's position relative to the reference point. '4 m below' can be indicated using −4.

c **1** The reference point is $0.

2 Determine the sign of the integer based on the number's position relative to the reference point. 'Overdrawn by $30' can be indicated using −30.

WRITE

a 15

b −4

c −30

Helpful hints

- ✔ The negative equivalent of a positive number is called the additive inverse.
  For example, −4 is the additive inverse of 4.
- ✔ Don't get your 'greater than' and 'less than' symbols mixed up! The 'mouth' of the symbol is always trying to 'eat' the greater number.
  For example, $4 > 3$ and $3 < 4$.

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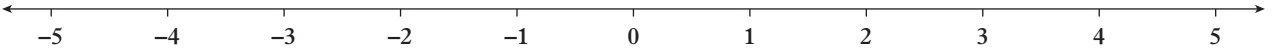
Exercise 1E Negative integers

△ 1–5, 6(a–f), 7(a, c), 8(a, c), 9, 10(a–d), 11, 12 | □ 4, 5, 6(g–l), 7(b, d), 8(b, d), 9, 10(e–h), 12–15 | ⬠ 6(g–l), 7(c, d), 8(c, d), 9, 10(e–h), 13–17

1 For the numbers shown in the following number line, write:

a all the negative integers **b** all the positive integers **c** the integer that is neither positive nor negative.



1E.1 **2** Which integer is greater in each pair of numbers?

a 3 and 9 **b** 0 and 5 **c** −7 and 1
d 5 and −8 **e** 0 and −2 **f** −6 and −4

3 Which integer is smaller in each pair of numbers?

a 5 and 6 **b** 0 and 3 **c** −2 and 7
d 4 and −5 **e** 0 and −1 **f** −8 and −9

4 Which number is greater in each pair of numbers?

a 0 and −38 **b** −42 and 24 **c** 14 and −16
d 125 and −50 **e** −25 and −35 **f** −14 and 0

5 Which number is smaller in each pair of numbers?

a 7 and −70 **b** −21 and −12 **c** 55 and −68
d −40 and −50 **e** −354 and −345 **f** 450 and −450

6 Complete each number statement by writing < (is less than) or > (is greater than) in the space provided. You can draw a number line to help you think.

a −3 ____ 2 **b** −8 ____ −4 **c** 0 ____ −1
d 9 ____ −9 **e** −3 ____ −5 **f** −7 ____ −2
g 4 ____ −6 **h** 5 ____ −12 **i** 6 ____ −4
j −18 ____ −8 **k** −6 ____ −7 **l** −8 ____ 0

7 Write each list of integers in ascending order.

a 14, −20, 10, −7, 8, −11, −12
b −33, 42, −19, −41, 0, 6, 29
c −8, −88, 68, 8, −28, −18, −48
d 73, −82, 3, −140, 104, 145, −126

UNDERSTANDING AND FLUENCY

8 Write each list of numbers in descending order.

a −15, 71, −27, −10, 4, −9

b −1, −2, 5, 10, −11, −5

c 0, 5, −9, −14, 21, −19

d −11, 51, −71, −62, 71, 12, −72

9 List all the integers between −7 and +4 ('between' means −7 and +4 are not included).

1E.2 **10** Provide an integer to represent each situation.

a The lift stops at the third floor below the ground floor.

b You have $2174 in the bank.

c The temperature inside a freezer is 18 degrees below zero.

d The shoreline of the Dead Sea, Jordan, is 408 m below sea level.

e The top of Mt Kilimanjaro, Tanzania, is 5895 m above sea level.

f A skydiver is 500 m above the ground.

g Your bank account is overdrawn by $46.

h A submarine is 40 m below sea level.

PROBLEM SOLVING AND REASONING

11 Polar bears live close to the edge of sea ice in the Arctic, where in winter the temperatures range from around −45°C to 0°C. In summer, the temperatures range from −10°C to 10°C. The ocean temperature is about −2°C.

a Show −45°C, 0°C, −10°C, 10°C and −2°C on a number line.

b Is −10°C higher or lower than −45°C?

c Write the five temperatures in order from lowest to highest.

d A polar bear dips her paw into the water to catch a fish. On a day with an air temperature of −18°C, does the water feel warmer or colder than the air?

12 Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

a Ashwan checks his bank balance and sees he has $86 in his account. Write this as a positive integer.

b The bank allows Ashwan's account to be overdrawn. This means that he can withdraw more money than he actually has in his account. If he withdraws $100 at an ATM, how much does he owe the bank?

c Write his new bank balance as a negative integer.

13 Jasmine needs glasses for reading as she is long sighted (hypermetropic). The prescription for the lenses in her glasses is +2.50. Alex wears contact lenses because he is short sighted (myopic). His lenses have prescriptions of −3.50 for his left eye and −4.25 for his weaker right eye.

a If Justin has a prescription of +1.75, is he long or short sighted?

b If Elle has a prescription of −4.75, is she long or short sighted?

c How does Elle's prescription compare to Alex's? Who has weaker vision?

d What do you think the reference of zero means in this situation?

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14 Bank statements usually show a transaction as a positive number if an amount is added or credited to the account, or as a negative number for withdrawals or debits.

Use the following bank statement to answer these questions.

Transaction	Balance
	+$52
+23	+$75
−35	+$40
−51	−$11
+34	+$23
+13	+$36
−49	−$13

a What is the largest amount:

i credited (added) to the account **ii** debited (subtracted) from the account?

b What is the smallest amount:

i credited to the account **ii** debited from the account?

c What does the final balance indicate about the account?

15 One way to represent integers is with coloured dots. Use one blue dot to represent +1 and one red dot to represent −1.

a How many red dots would you use to represent −3?

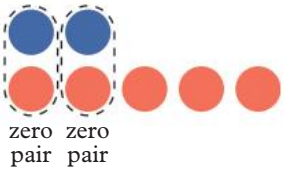
b Use the coloured dots to show each integer listed below. Draw each arrangement.

i 2 **ii** −4 **iii** 6

iv −5 **v** −1 **vi** 8

c One blue dot and one red dot together are called a zero pair. Explain how a zero pair represents 0.

d Explain how this arrangement of dots represents −3.



e What integer does each arrangement represent?

i

ii

iii

16 Some friends are sitting in a circle. Kesha sits opposite Bri and three places to the left of Mo. Dee sits opposite Mo and three places to the left of Kesha. How many chairs are in the circle?

17 Find five consecutive numbers that add to −35.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Ordering positive and negative numbers

Investigation
Comparing world temperatures

Topic quiz
1E

1F Adding and subtracting integers

Learning intentions

By the end of this topic you will be able to ...

✔ solve problems involving addition and subtraction of positive and negative numbers.

Inter-year links

Year 7 5B Adding and subtracting positive numbers

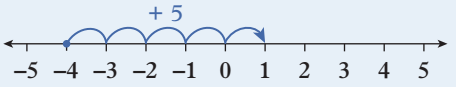
Adding and subtracting positive integers

Key content video

- When adding a positive integer, move to the right on the number line.

  For example, to calculate $-4 + 5$, start at -4 and move 5 steps to the right.

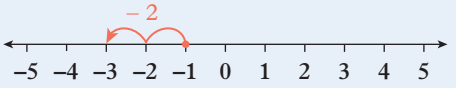
  $-4 + 5 = 1$



- When subtracting a positive integer, move to the left on the number line.

  For example, to calculate $-1 - 2$, start at -1 and move 2 steps to the left.

  $-1 - 2 = -3$

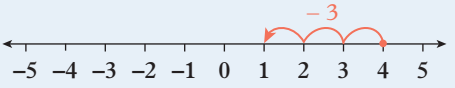


Adding and subtracting negative integers

- When adding a negative integer, move to the left on the number line.

  For example, to calculate $4 + (-3)$, start at 4 and move 3 steps to the left.

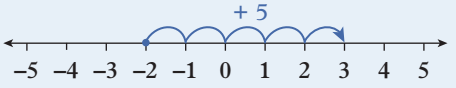
  $4 + (-3) = 4 - 3$
  $= 1$



- When subtracting a negative integer, move to the right on the number line.

  For example, to calculate $-2 - (-5)$, start at -2 and move 5 steps to the right.

  $-2 - (-5) = -2 + 5$
  $= 3$



- Adding a negative number can be simplified to subtracting its value. $+(-) = -$
- Subtracting a negative number can be simplified to adding its value. $-(-) = +$

Example 1F.1 Adding and subtracting positive numbers

Use the number line to calculate:

a $-6 + 8$ **b** $4 - 8$

THINK

a 1 Draw a number line.

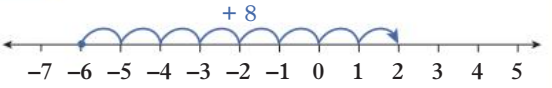
2 Start at the first number, then add the second number by moving right.

b 1 Draw a number line.

2 Start at the first number, then subtract the second number by moving left.

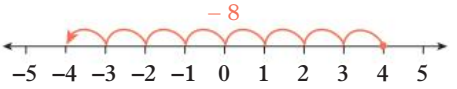
WRITE

a



$-6 + 8 = 2$

b



$4 - 8 = -4$

Example 1F.2 Adding and subtracting negative numbers

Use the number line to calculate:

a $4 + (-7)$ **b** $-2 - (-3)$

THINK

a 1 Simplify the calculation. Remember that $+(-) = -$.

2 Draw a number line.

3 Start at the first number, then subtract the value by moving left.

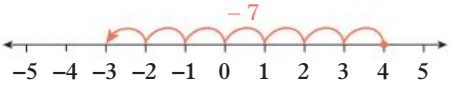
b 1 Simplify the calculation. Write the problem in a simpler equivalent form by replacing $-(-)$ with $+$.

2 Draw a number line.

3 Start at the first number, then add the value by moving right.

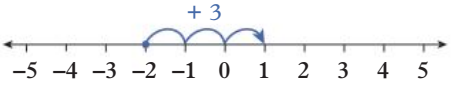
WRITE

a $4 + (-7) = 4 - 7$



$4 + (-7) = -3$

b $-2 - (-3) = -2 + 3$



$-2 - (-3) = 1$

Helpful hints

✔ The negative sign is identical to the minus sign. The function of the symbol depends on its context.

Example	Function of '–'	Read as
-8	Indicates the number is less than zero in value	'negative eight'
$2 - 8$	Indicates the operation of subtraction	'two minus eight'
$-8 + 2$	Indicates the number is less than zero in value	'negative eight plus two'

✔ Don't make assumptions about the sign of the answer – subtraction can produce positive numbers and addition can produce negative numbers!

✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$+(-) = -$ $-(-) = +$ $+(+) = +$ $-(+) = -$

ANS p481

Exercise 1F Adding and subtracting integers

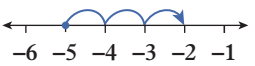
△ 1–3, 4–6(a–e), 7, 8–9(a–e), 10(a, b, d, e), 11(a), 12–14(1st column), 15, 17, 18

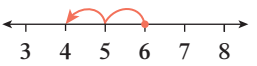
□ 4–6(e–i), 7(a, c, f), 8–10(c–f), 11(b), 12–14(2nd column), 16, 18, 19, 21, 22(a, b)

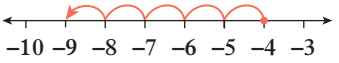
⬠ 8–9(f–i), 10(f–h), 11(b), 12(c, f, i), 13–14(2nd column), 18, 20–23

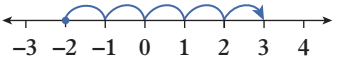
UNDERSTANDING AND FLUENCY

1 Write the addition or subtraction calculation shown in each diagram. Hint: Consider the starting point, the number of steps and the end point.

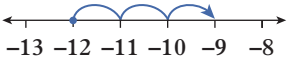
a


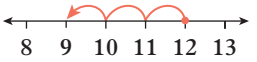
b


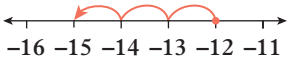
c


d


2 Match each diagram (**I**, **II**, **III**) to the appropriate problem (**a**, **b**, **c**).

I


II


III


a $-12 - 3$ **b** $-12 + 3$ **c** $12 - 3$

3 Calculate each result for question **2**.

1F.1 **4** Use a number line to calculate:

a $-4 + 7$ **b** $1 + 5$ **c** $8 - 3$
d $-2 - 6$ **e** $-9 + 5$ **f** $3 - 6$
g $-5 - 1$ **h** $7 - 2$ **i** $2 - 6$

5 Draw a number line from −20 to 20 and use it to calculate:

a $15 - 12$ **b** $-17 + 11$ **c** $9 - 13$
d $-4 + 20$ **e** $-2 - 17$ **f** $8 - 18$
g $-14 - 3$ **h** $-16 + 14$ **i** $11 - 24$
j $-19 + 19$ **k** $-10 - 10$ **l** $-18 + 20$

6 Decide whether each result is positive, negative or zero.

a $-3 - 7$ **b** $16 + 14$ **c** $8 - 15$
d $-13 + 13$ **e** $-11 + 18$ **f** $19 - 12$
g $-17 + 4$ **h** $-14 - 10$ **i** $17 - 20$

7 Simplify the following calculations by using + or − to fill the gap.

a $-3 - (+7) = -3$ _____ 7 **b** $1 + (+6) = 1$ _____ 6
c $-4 + (-5) = -4$ _____ 5 **d** $2 - (-4) = 2$ _____ 4
e $5 - (+9) = 5$ _____ 9 **f** $-6 - (-8) = -6$ _____ 8

8 Simplify each problem.

a $-1 - (-3)$ **b** $8 - (+4)$ **c** $-5 + (-2)$
d $6 - (-1)$ **e** $4 + (+3)$ **f** $3 - (+9)$
g $-5 - (-8)$ **h** $-3 + (+1)$ **i** $-7 + (-3)$

1F.2 **9** Use a number line to calculate:

a $-4 - (-7)$ **b** $9 + (-3)$ **c** $2 - (+6)$
d $-5 - (-5)$ **e** $-8 + (+7)$ **f** $6 + (-1)$
g $-3 - (-4)$ **h** $4 - (+9)$ **i** $2 - (-7)$

10 Decide whether the result of each problem will be positive, negative or zero.

a 11 – (–8) **b** –16 – (+4) **c** 13 – (+13) **d** 15 – (+18)

e –10 – (–7) **f** –17 – (+11) **g** 8 – (–14) **h** 12 – (+19)

11 Complete each addition table.

a

+	–5	–3	0	1	4
–3				–2	
–2					
–1			–1		
3					
6	1				

b

+	–10	–14	–7	13	19
11					
14					
–10					
–22					
–18					

12 Calculate:

a –45 + 40 **b** –72 – 27 **c** 56 – 88

d –34 + 43 **e** 130 – 170 **f** –213 – 62

g –158 + 400 **h** 286 – 168 **i** –505 + 505

13 Calculate:

a –2 + 5 + 7 **b** 3 – 9 + 2

c 6 – 1 – 8 **d** –10 + 4 – 7

e –12 + 19 – 7 **f** –20 – 11 – 2

g 33 + 22 – 65 **h** 70 – 81 – 15

i –24 – 18 + 12 **j** 59 – 93 + 17

14 Simplify each problem, and then calculate the result.

a –5 – (+7) + (+6) **b** 9 + (–2) – (–8)

c –4 – (–3) + (+1) **d** 8 + (–6) – (–10)

e –22 – (+5) + (+13) **f** –14 – (–8) + (–25)

g 34 + (–19) – (+7) **h** –9 – (+26) – (–37)

15 You enter a lift at the 15th floor and travel down 19 floors. Which level do you finish at? Show how you calculated your answer.

16 Sarah enters a lift at the third floor and travels down nine floors and then up five floors. What floor does she finish at? Show the problem you used to get your answer.

17 What is the difference between a temperature of –18°C and a temperature of 33°C? Show the subtraction problem you used to calculate the result.

18 Jess owes her brother $60.

a Write this amount as an integer.

b She pays him $35. Write an addition problem to work out the amount she still owes him.

c How much does Jess still owe her brother?

PROBLEM SOLVING AND REASONING

19 A bungee jumper dives from a platform (position A) 50 m above the water. He dips 2 m under the water (position B) before rebounding to a height 18 m below the platform (position C).

a If the reference is the surface of the water, write positions A, B and C as integers.

b Write an expression to find the distance between:

i A and B

ii B and C

iii A and C.

c Calculate each distance.

d What is the largest distance for part **c**? Explain why that is the largest distance.

20 A deep sea submersible is lowered into the ocean from a ship's deck 15 m above sea level. The unmanned vessel is on a mission to take photos of deep sea animals.

For each part, write an addition or subtraction problem and then calculate the answer.

a A deep sea anglerfish is 900 m below sea level. What vertical distance has the submersible travelled from the deck of the ship to reach this fish?

b A deep sea amphipod is 2500 m below sea level. How much further has the submersible travelled vertically to reach it?

c What vertical distance does the submersible now need to travel to return to the deck of the ship?

d When the ship gets back to the dock, the crane on board the ship lowers the submersible onto the dock. The crane lifts the submersible 2 m above the ship's deck, moves it horizontally at that height and then lowers it 8 m onto the dock. What is the height of the dock above sea level?

CHALLENGE

21 What number should be subtracted from each of these to give the result of 20?

a 65 **b** −10 **c** 1281

d −347 **e** −1000 **f** −3240

22 Find the missing integer to make each number sentence true.

a $-54 + 33 + ____ = 12$ **b** $121 - ____ - 50 = -6$

c $-300 + 225 - ____ = 50$ **d** $____ - 58 + 73 = -25$

23 At the start of March, Tyra's bank account shows a balance of −$310. At the end of the month, it shows a balance of −$247.

a Calculate the difference between the two account balances. Show the subtraction problem you used to calculate the result.

b Interpret the result to part **a**.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Adding and subtracting with negative numbers

Investigation
Create a subtraction

Topic quiz
1F

1G Multiplying and dividing integers

Learning intentions

By the end of this topic you will be able to ...

✔ solve problems involving multiplication and division of negative numbers

✔ solve problems involving indices of negative numbers.

Inter-year links

Support	Multiplying whole numbers
Year 7	1D Multiplying whole numbers

Key content video

Multiplying integers

- Multiplication is a form of repeated addition. For example, 3×4 is the same as $4 + 4 + 4$.
- Multiplication of negative integers can also be thought of as repeated addition. For example, 2×-5 is the same as $(-5) + (-5)$.
- When multiplying by a negative value, we are taking the negative value of the repeated addition. For example, -2×-5 is the same as $-[(-5) + (-5)]$.
- Multiplying two integers can give a result that is positive or negative (or zero), according to these rules.
  - → The product of two positive integers is a positive integer.
  - → The product of a positive integer and a negative integer is a negative integer.
  - → The product of two negative integers is a positive integer.

$$+ \times + = +$$
$$+ \times - = -$$
$$- \times + = -$$
$$- \times - = +$$

Dividing integers

- Division of integers follows the same rules as multiplication of integers.
  - → The quotient of two positive integers is a positive integer.
  - → The quotient of a positive integer and a negative integer is a negative integer.
  - → The quotient of two negative integers is a positive integer.

$$+ \div + = +$$
$$+ \div - = -$$
$$- \div + = -$$
$$- \div - = +$$

Indices of negative numbers

- When applying indices to negative numbers, the sign needs to be taken into account.

index

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

index form expanded form basic numeral

  - → If the base is negative and the index is an even number, the basic numeral is positive.
  - → If the base is negative and the index is an odd number, the basic numeral is negative.

Example 1G.1 Multiplying integers

Evaluate each product.

a -4×7

b $-9 \times (-5)$

THINK	WRITE
a **1** Determine whether the result is positive or negative. The signs are different, so the result is negative.	**a** $4 \times 7 = 28$
2 Calculate the product.	So, $-4 \times 7 = -28$
b **1** Determine whether the result is positive or negative. The signs are the same, so the result is positive.	**b** $9 \times 5 = 45$
2 Calculate the product.	So, $-9 \times (-5) = 45$

Example 1G.2 Dividing integers

Evaluate each quotient.

a $-30 \div (-5)$

b $28 \div (-7)$

THINK	WRITE
a **1** Decide whether the result is positive or negative. The signs are the same, so the result is positive.	**a** $30 \div 5 = 6$
2 Calculate the quotient.	So, $-30 \div (-5) = 6$
b **1** Decide whether the result is positive or negative. The signs are different, so the result is negative.	**b** $28 \div 7 = 4$
2 Calculate the quotient.	So, $28 \div (-7) = -4$

Example 1G.3 Multiplying three integers

Evaluate the product $-3 \times (-5) \times (-2)$.

THINK	WRITE
1 Work from left to right. Multiply the first two numbers and determine the sign. As the signs are the same, the result is positive.	$-3 \times (-5) = 15$
2 Multiply the result by the third number and determine the sign. As the signs are different, the result is negative.	$15 \times (-2) = -30$ $-3 \times (-5) \times (-2) = -30$

Example 1G.4 Calculating the value of a number in index form

Write each of these in expanded form and calculate their values.

a $(-3)^2$ **b** $(-2)^5$

THINK

a 1 Write the calculation as a repeated multiplication.

2 Determine whether the result will be positive or negative. A negative multiplied by a negative is a positive.

3 Calculate the product.

b 1 Write the calculation as a repeated multiplication.

2 Continue to simplify the repeated multiplication. A negative multiplied by a negative is a positive. A positive multiplied by a negative is a negative.

3 Calculate the product.

WRITE

a $(-3)^2 = (-3) \times (-3)$
$= 9$

b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$
$= 4 \times (-2) \times (-2) \times (-2)$
$= (-8) \times (-2) \times (-2)$
$= 16 \times (-2)$
$= -32$

Helpful hints

✔ Be careful when determining the sign of a result when multiplying or dividing numbers.
If the signs are the same, then the result is positive.
If the signs are different, then the result is negative.

ANS p481

Exercise 1G Multiplying and dividing integers

△ 1, 2–3(1st, 2nd columns), 4–6, 7(a), 8–14, 15(a–c), 16(a, b), 17, 19, 20(a, b, e), 22(a–c)

□ 4–6, 7(b), 8, 9–10(d–f), 13(e–h), 14(c, d), 15(d–f), 16(c, d), 18, 20(c, d, f), 22, 23(a, b)

⬠ 5(i–l), 7(b), 8, 9(g–i), 10(b, d, e), 14(c, d), 15(g–i), 20, 21, 23–26

1G.1 **1** Express these repeated additions as multiplications.

a $9 + 9 + 9 + 9 + 9$ **b** $(-7) + (-7) + (-7) + (-7) + (-7) + (-7)$

c $-(6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6)$ **d** $-[(-3) + (-3) + (-3) + (-3)]$

1G.1 **2** Evaluate each product.

a $3 \times (-4)$ **b** 7×5 **c** -6×2 **d** $-3 \times (-9)$

e -1×8 **f** $4 \times (-6)$ **g** $-8 \times (-9)$ **h** 2×10

i -4×11 **j** $7 \times (-1)$ **k** $-2 \times (-4)$ **l** 9×5

1G.2 **3** Evaluate each quotient.

a $15 \div (-3)$ **b** $-32 \div (-8)$ **c** $24 \div 4$ **d** $-63 \div 9$

e $-17 \div (-1)$ **f** $48 \div (-6)$ **g** $-56 \div 8$ **h** $-81 \div (-9)$

i $42 \div (-7)$ **j** $-12 \div 1$ **k** $-20 \div (-2)$ **l** $72 \div (-8)$

4 Evaluate each quotient.

a $\frac{-48}{6}$ **b** $\frac{60}{-5}$ **c** $\frac{-51}{-3}$ **d** $\frac{150}{10}$ **e** $\frac{-200}{-25}$ **f** $\frac{57}{-19}$

5 Calculate each of the following.

a $5 \times (-7)$ **b** $-9 \div 3$ **c** $-28 \div (-4)$ **d** -10×6

e $36 \div (-9)$ **f** $-15 \times (-4)$ **g** $-100 \div 20$ **h** $18 \times (-2)$

i $-3 \times (-30)$ **j** $25 \div (-25)$ **k** -1×38 **l** $-45 \div (-1)$

6 Complete each statement.

a $-15 \times$ _____ $= 45$ **b** $56 \div$ _____ $= -8$ **c** _____ $\times (-4) = 36$

d _____ $\div (-25) = -4$ **e** $-140 \div$ _____ $= 2$ **f** _____ $\times 16 = -48$

7 Complete each multiplication table.

a

×	−2	−1	0	1	2
−4	8			−4	
−2					
0			0		
2					
4		−4			
6					

b

×		−20	−10		20
7				70	
			−50		
0					
−2	50				
		80			
					−120

8 A short way of writing $-1 \times (-2)$ is $-(-2)$. Write each problem as the product of two numbers and then calculate the result.

a $-(-7)$ **b** $-(+12)$ **c** $-(-88)$

d $-(+25)$ **e** $-(8 - 5)$ **f** $-(-3 + 2)$

1G.3 **9** Evaluate each product.

a $-4 \times 3 \times (-5)$ **b** $3 \times (-2) \times 7$ **c** $6 \times (-1) \times 9$

d $8 \times 2 \times (-3)$ **e** $-5 \times (-2) \times (-13)$ **f** $-7 \times 3 \times (-4)$

g $4 \times 5 \times (-8)$ **h** $-11 \times 5 \times (-2)$ **i** $9 \times (-4) \times 5$

10 Evaluate each statement.

a $-2 \times 2 \times (-3) \times 3$ **b** $20 \div (-2) \div (-2) \div (-1)$ **c** $-4 \times (-1) \times 3 \times (-2)$

d $-18 \div (-2) \times 4 \div (-6)$ **e** $9 \times (-2) \div 3 \times (-5)$ **f** $-7 \times (-4) \div 2 \times (-3)$

11 Which of these five options is the expanded form of $(-4)^5$?

A $4 \times 4 \times 4 \times 4 \times 4$ **B** $5 \times 5 \times 5 \times 5$ **C** $-4 \times (-4) \times 4 \times 4 \times 4$

D $-4 \times (-4) \times (-4) \times (-4)$ **E** $-4 \times (-4) \times (-4) \times (-4) \times (-4)$

12 Without calculating each result, predict whether the basic numeral is positive or negative.

a $(-2)^9$ **b** $(-15)^6$ **c** $(+6)^7$ **d** $(-100)^4$

1G.4 **13** Write each of these in expanded form and calculate their values.

a $(-9)^2$ **b** 8^2 **c** $(-6)^3$ **d** 7^3

e $(-5)^4$ **f** 1^4 **g** 4^5 **h** $(-10)^5$

14 By collecting like terms, write each of these multiplications in index form.

a $-5 \times (-5) \times (-5) \times (-9) \times (-9) \times (-9) \times (-9)$

b $-4 \times (-4) \times (-4) \times (-4) \times (-4) \times (-4) \times 3 \times 3 \times 3$

c $7 \times 7 \times (-6) \times (-6) \times (-6) \times (-6) \times (-6) \times (-6)$

d $-8 \times (-8) \times (-8) \times (-8) \times (-10) \times (-10)$

15 Write each of these in expanded form and calculate their values.

a $(-2)^3 \times (-4)^2$ **b** $(-5)^2 \times 3^4$ **c** $(-10)^3 \times (-2)^5$

d $(-3)^5 \times (-1)^2$ **e** $(-6)^2 \times (-2)^3$ **f** $(-1)^4 \times 3^2$

g $(-7)^2 \times (-1)^5$ **h** $(-3)^3 \times (-2)^2$ **i** $(-1)^3 \times (-2)^5$

16 Complete each set of problems. Describe the pattern you see.

a

$2 \times 5 =$ _____
$2 \times 4 =$ _____
$2 \times 3 =$ _____
$2 \times 2 =$ _____
$2 \times 1 =$ _____
$2 \times 0 =$ _____
$2 \times (-1) =$ _____
$2 \times (-2) =$ _____
$2 \times (-3) =$ _____
$2 \times (-4) =$ _____
$2 \times (-5) =$ _____

b

$-3 \times 5 =$ _____
$-3 \times 4 =$ _____
$-3 \times 3 =$ _____
$-3 \times 2 =$ _____
$-3 \times 1 =$ _____
$-3 \times 0 =$ _____
$-3 \times (-1) =$ _____
$-3 \times (-2) =$ _____
$-3 \times (-3) =$ _____
$-3 \times (-4) =$ _____
$-3 \times (-5) =$ _____

c

$-4 \times (-5) =$ _____
$-4 \times (-4) =$ _____
$-4 \times (-3) =$ _____
$-4 \times (-2) =$ _____
$-4 \times (-1) =$ _____
$-4 \times 0 =$ _____
$-4 \times 1 =$ _____
$-4 \times 2 =$ _____
$-4 \times 3 =$ _____
$-4 \times 4 =$ _____
$-4 \times 5 =$ _____

d

$5 \times (-5) =$ ____
$5 \times (-4) =$ _____
$5 \times (-3) =$ _____
$5 \times (-2) =$ _____
$5 \times (-1) =$ _____
$5 \times 0 =$ _____
$5 \times 1 =$ _____
$5 \times 2 =$ _____
$5 \times 3 =$ _____
$5 \times 4 =$ _____
$5 \times 5 =$ _____

17 Dividing is the inverse operation to multiplying. Complete each statement.

a $2 \times 3 =$ _____ so _____ $\div 2 = 3$ or _____ $\div 3 = 2$

b $2 \times (-3) =$ _____ so _____ $\div 2 = -3$ or _____ $\div (-3) = 2$

c $-2 \times 3 =$ _____ so _____ $\div (-2) = 3$ or _____ $\div 3 = -2$

d $-2 \times (-3) =$ _____ so _____ $\div (-2) = -3$ or _____ $\div (-3) = -2$

18 a Calculate each of these by first writing them in expanded form.

i $(-1)^1$ **ii** $(-1)^2$ **iii** $(-1)^3$

iv $(-1)^4$ **v** $(-1)^5$ **vi** $(-1)^6$

b Which of these problems give a positive result?

c Which of these problems give a negative result?

d Explain your observations.

e Predict the result for:

i $(-1)^{13}$ **ii** $(-1)^{32}$ **iii** $(-1)^{100}$

iv $(-1)^{203}$ **v** $(-1)^{188}$ **vi** $(-1)^{555}$

19 a Using a calculator, determine:

i 8×0 **ii** -15×0 **iii** 0×65 **iv** $0 \times (-72)$

b What do you notice when you multiply any integer by zero?

c Using a calculator, determine:

i $0 \div (-9)$ **ii** $0 \div 26$ **iii** $0 \div (-11)$ **iv** $0 \div 39$

d What do you notice when you divide zero by any integer?

20 Find two integers that have:

a a sum of −9 and a product of 18 **b** a sum of 4 and a product of −45

c a sum of −5 and a product of −66 **d** a difference of 2 and a product of 24

e a difference of 8 and a product of 9 **f** a difference of 18 and a product of −32.

21 The product of 4 integers is 50. What are the possible sums of these 4 integers if they are:

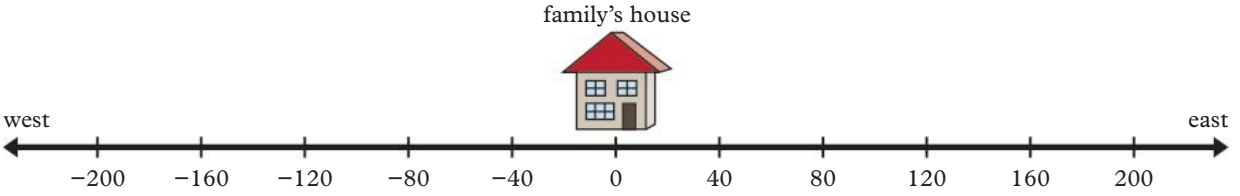
a all positive **b** all negative **c** either positive or negative?

Hint: First find all the factors of 50.

PROBLEM SOLVING AND REASONING

22 Astro the dog has escaped from his yard. His owners decide to search along the road which runs east–west outside the house. This number line can be used to represent the road outside the family's house, with the house at zero.

a Tyler and Rhys walk 120 m west from the house and call Astro's name. What is their position on the number line?



b Sophie and Imogen walk 80 m east from the house and call Astro's name. What is their position on the number line?

c What is the distance between the pairs of searchers?

d Imogen now walks a further 30 m east. What is her new position on the number line?

e Rhys now walks 160 m east. What is his new position on the number line?

f What is the distance between:

i Sophie and Rhys **ii** Tyler and Imogen?

g Natalie starts at the house, then walks twice as far as Sophie but in the same direction as Tyler. What is her position on the number line?

h Hayden starts at the house, then walks one-third of the distance walked by Tyler and in the same direction. What is his position on the number line?

i A neighbour has found Astro in her yard, 60 m west of the family's house. Who is closest to Astro? How far and in which direction does he or she need to walk to collect the dog?

j If each member of the family now walks home, calculate the distance each person walked from the start of the search.

23 Squaring two different numbers can produce the same result. For example, $3^2 = 9$ and $(-3)^2 = 9$.

a Which two numbers, when squared (raised to the power of 2), give 25? Why are there two numbers?

b Is there a number that, when squared, gives −25? Explain.

c How many different numbers can be cubed (raised to the power of 3) to give 8?

d How many different numbers can be cubed (raised to the power of 3) to give −8?

e Predict how many numbers can be raised to the power of 4 to give the same positive result. Hint: Try to raise a number to the power of 4 to give 16. Repeat for a negative result.

f Similarly, predict what would happen for indices of 5, 6 and so on. Provide examples to support your answer.

24 What number(s), when squared, give(s) each result?

a 49 **b** 81 **c** 1 **d** 4 **e** 64 **f** 100

CHALLENGE

25 What number(s), when cubed, give(s) each result?

a 27 **b** −125 **c** 64 **d** −1 **e** −64 **f** −1000

26 Consider $(-3)^2$, 3^2 and -3^2.

a Which of these produce the same result?

b Explain how $(-3)^2$ is different from -3^2.

c Decide whether each pair produces the same result.

i $(-4)^3$ and -4^3 **ii** $(-2)^4$ and -2^4 **iii** $(-1)^6$ and -1^6 **iv** $(-10)^5$ and -10^5

d Describe the general pattern you have seen.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing negative numbers

Interactive skillsheet
Indices of negative numbers

Investigation
Multiplication triangulation

Topic quiz
1G

1H Order of operations

Learning intentions

By the end of this topic you will be able to ...

✔ order operations in calculations involving two or more operations

✔ evaluate expressions including integers using the order of operations.

Inter-year links

Support — Order of operations

Year 7 — 1H Order of operations

Order of operations

Key content video

- **'BIDMAS'** can be used to help remember the **order of operations**.

B	**Brackets**	Operations inside brackets are always performed first. For example, $2^2 \times (4 - 6) - 8 \div (-2) = 2^2 \times (-2) - 8 \div (-2)$.
I	**Indices (and roots)**	Next, evaluate indices and roots. For example, $2^2 \times (-2) - 8 \div (-2) = 4 \times (-2) - 8 \div (-2)$.
D **M**	**Division** **Multiplication**	Then, working, from **left to right**, perform any multiplication or division before adding or subtracting. For example, $4 \times (-2) - 8 \div (-2) = -8 - (-4)$.
A **S**	**Addition** **Subtraction**	Finally, working from **left to right**, perform any addition and subtraction. For example, $-8 - (-4) = -8 + 4 = -4$.

- Operations at the same ranking are performed in order from left to right.

  For example, $5 + 8 - 2 = 13 - 2$
  $= 11$

  and

  $-20 \times \frac{3}{10} = -\frac{60}{10}$
  $= -6$.

- Where there is more than one set of brackets in the problem, perform the operations inside the innermost brackets first.

  For example, $[2 \times (5 - 3)] - 7 = [2 \times (5 - 3)] - 7$
  $= (2 \times 2) - 7$
  $= 4 - 7$
  $= -3$.

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)
• Sum • Plus • More than • Increase • Total • Together	• Difference • Take away • Less than • Fewer • Reduce • Minus	• Product • Times • Multiply • Groups of	• Quotient • Divide • Shared • Distributed

Example 1H.1 Order of operations

Evaluate each of the following.

a $-3 + 4 \times (-2)$ **b** $7 - (-15) \div 5$ **c** $(-2)^3 \times 3 + 5$

THINK

a **1** Perform the multiplication and determine the sign of the result.

2 Simplify the signs and perform the subtraction.

b **1** Perform the division and determine the sign of the result.

2 Simplify the signs and perform the addition.

c **1** Apply the index and determine the sign of the result.

2 Perform the multiplication and determine the sign of the result.

3 Perform the addition.

WRITE

a $-3 + 4 \times (-2) = -3 + (-8)$

$= -3 - 8$

$= -11$

b $7 - (-15) \div 5 = 7 - (-3)$

$= 7 + 3$

$= 10$

c $(-2)^3 \times 3 + 5 = -8 \times 3 + 5$

$= -24 + 5$

$= -19$

Example 1H.2 Order of operations with brackets

Evaluate each of the following.

a $-18 \div (4 - 10) + 2 \times 7$ **b** $6 - [-12 \div (9 - 5)]^2 - 2$

THINK

a **1** Perform any operations inside brackets first.

2 Working from left to right, perform division and multiplication before addition.

3 Perform the addition.

b **1** There are two sets of brackets. Perform the operation in the innermost set of brackets first.

2 Perform the operation in the remaining set of brackets.

3 Apply the index before addition and subtraction.

4 Working from left to right, perform the subtraction.

WRITE

a $-18 \div (4 - 10) + 2 \times 7 = -18 \div -6 + 2 \times 7$

$= 3 + 2 \times 7$

$= 3 + 14$

$= 17$

b $6 - [-12 \div (9 - 5)]^2 - 2 = 6 - [-12 \div 4]^2 - 2$

$= 6 - [-3]^2 - 2$

$= 6 - 9 - 2$

$= -3 - 2$

$= -5$

Helpful hints

- ✔ Remember to use BIDMAS rather than working from left to right.
- ✔ It is helpful to determine the sign at each step of a calculation before progressing.
- ✔ Calculating the value of a number in index form uses the following rules:
  - If the base is negative and the index is an even number, the result is positive.
  - If the base is negative and the index is an odd number, the result is negative.
- ✔ For multiplying and dividing integers, recall the simplification rules for calculations involving two numbers.
  - If the signs are the same, then the result is positive.
  - If the signs are different, then the result is negative.

  $+(-) = -$ $\quad -(-) = +$ $\quad +(+) = +$ $\quad -(+) = -$
- ✔ For adding and subtracting integers, recall the simplification rules. You can also use the number line to help you.

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Exercise 1H Order of operations

△ 1–3, 4(1st column), 5–10, 12, 13(a, b) ▢ 3, 4(2nd column), 5–9, 11, 13, 16 ⬠ 4(2nd column), 5–8, 13, 14, 16–18

1 For each calculation, list the operations in the correct order in which they should be completed.

a $-28 - 3 \times 7 + 11$ **b** $50 + (34 - 19) \div (-5)$ **c** $12 + 3 \times \sqrt{4} - (-41)$

d $-20 - 2 \times 6 + 3^2$ **e** $8 + [9 - (-3 + 2)]$ **f** $24 \div 6 + [5 + (-1)]^2$

1H.1 **2** Evaluate each of the following.

a $16 + 28 \div 4$ **b** $60 - 2 \times 10$ **c** $27 \div 3 + 6 \times 11$

d $22 + 30 \times 2^3 \div 15$ **e** $10 \times 8 - 7 \times \sqrt{9}$ **f** $6^2 - 8 \times 3 + 35$

3 Evaluate each of the following.

a $7 - 2 \times 8$ **b** $-1 + 16 \div 16$ **c** $28 - 3 \times 11$

d $-6 + 5 \times \sqrt[3]{27} - 2$ **e** $-5 + [-9 + (-6 + 4)]$ **f** $42 \div (-7) + [5 \times (-2)^2]$

1H.2 **4** Evaluate each of the following.

a $72 \div (7 + 2) - 2 \times 3$ **b** $(27 - 14) \times (19 + 11)$

c $11 - (4 \times 2 - 19)$ **d** $(-3 \times 7 + 1) \div (8 - 12)$

e $9^2 + 7 \times (34 - 28)$ **f** $10 \times 8 - 4 \times (3^2 + \sqrt[3]{8})$

g $60 \div [6 + (3 \times 5 - 1)] + 48$ **h** $-12 + [-48 \div (28 - 26)]^2 - 9$

5 Evaluate each of the following.

a $(-3)^2 \times (-14 + 10) \div (-6)$ **b** $(-19 + 16)^2 + (-23 + 17)^3$ **c** $(-2)^3 \times (-4) \div (-8)$

d $(-2)^2 \times (-2^2)$ **e** $(120 \div 6) - (-10) \div \sqrt{4}$ **f** $[-3 + (150 \div 50)] \times (-7)^2 \div 5^2$

6 Write an equivalent sum to the following using a single sign. Do not calculate the answers.

a $66 + (-7)$ **b** $415 - (-322)$ **c** $365 - (+109)$

d $1750 - (-3487)$ **e** $4376 + (-237)$ **f** $6548 - (+2399)$

UNDERSTANDING AND FLUENCY

7 Identify the mistakes that have been made in the following calculations.

a $(-10 + 18)^2 = -64$ **b** $-15 + 5 \times 5 = -50$ **c** $-18 \div 3 + \sqrt{36} = -2$

d $(4 - 20) \div (-4) = -4$ **e** $(-4)^3 \div 8 = 8$ **f** $-5 \times 6 + 90 \div 10 = 39$

8 Find the correct answers to the problems in question **7**.

9 In which of the following do the brackets not change the value of $-2^2 + 5 \times 3^2 - 2 \times \sqrt[3]{8} - 7$?

A $(-2)^2 + 5 \times 3^2 - 2 \times \sqrt[3]{8} - 7$ **B** $(-2^2 + 5) \times 3^2 - 2 \times \sqrt[3]{8} - 7$ **C** $-2^2 + (5 \times 3^2) - 2 \times \sqrt[3]{8} - 7$

D $-2^2 + 5 \times (3^2 - 2) \times \sqrt[3]{8} - 7$ **E** $-2^2 + 5 \times 3^2 - 2 \times (\sqrt[3]{8} - 7)$

10 You have \$195 to buy three shirts priced at \$58 each.

a Write a calculation to show how to work out how much money you will have left over.

b Estimate how much money is left over by rounding each number to its leading digit then performing the calculation.

c Perform the calculation in part **a** and compare your estimate to the exact amount of money you have left over.

11 Using the images shown, calculate the cost of buying lunch for the class: 14 salad rolls, 11 samosa packs, 9 flavoured milks, 16 orange juices and 25 apples. Show the numbers and operations you used in the calculation.

\$7 \$6 for three \$4 \$5 \$1

12 When standing with outstretched arms above her head, the height of an Olympic diver is 3 m. She dives from the 10 m platform and touches the bottom of the pool with her outstretched fingers. The depth of the pool is 6 m.

a If the pool surface is the reference point for zero, write the maximum height of the diver above the pool surface as a directed number.

b Write the depth of the pool as a directed number.

c Write a subtraction problem to calculate the vertical distance covered by the diver. Hint: Find the difference between the two positions.

d What vertical distance has the diver covered?

13 The daily maximum and minimum temperatures at Thredbo were recorded over a week.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Maximum temperature (°C)	8	4	−3	−2	−1	0	1
Minimum temperature (°C)	2	−3	−5	−6	−6	−3	0

a Calculate the difference between the minimum and maximum temperatures for each day.

b Which day had the biggest range of temperatures?

c Calculate the average (mean) of:

i the maximum temperatures

ii the minimum temperatures.

d What is the difference between the average minimum and average maximum temperatures?

14 This incomplete statement shows some deposits and withdrawals made to Martin's bank account.

Date	Reference	Transaction	Balance
30 April	–	–	+$289
2 May	Tony's Newsagent	+$132	
15 May	Movie Place	−$17	
21 May	Bicycle City	−$500	
24 May	Tony's Newsagent	+$230	
26 May	DD Online Music Store	−$38	
27 May	Sports and Stuff Pty Ltd		+$69

a Is a transaction of +$132 a deposit or a withdrawal?

b Is a transaction of −$17 a deposit or a withdrawal?

c What does it mean if the balance in Martin's account is:

i positive **ii** negative?

d Find the account balance after the transaction made on 21 May.

e Find the balance after the transaction made on 24 May.

f What transaction is made on 27 May so that the account balance is +$69?

g Was Martin's account overdrawn at any stage? What penalty do banks have for an account that is overdrawn?

15 If the sum of two numbers is zero, is the product of the numbers positive or negative? Explain.

16 Find three integers whose sum is −5 and product is 120.

17 Evaluate the following.

a $6+\cfrac{52}{6+\cfrac{63}{3+\cfrac{48}{3+\cfrac{55}{3+\cfrac{96}{8+\cfrac{40}{8+\cfrac{4}{1+\cfrac{5}{1+\cfrac{8}{2}}}}}}}}}$

b $4-\cfrac{-64}{10-\cfrac{52}{15-\cfrac{132}{-3-\cfrac{-54}{-1-\cfrac{50}{3-\cfrac{7}{3-\cfrac{36}{4-\cfrac{10}{1-\cfrac{24}{8}}}}}}}}}$

18 The 'Four Fours' problem uses four of the digit '4' and a combination of operations and brackets to write calculations that are equal to each integer.

For example, 4 + 4 + 4 + 4 = 16.

a Write a series of calculations, using four '4's and any combination of the operations +, −, ×, ÷ and brackets, that are equal to each of the integers 0 to 20. Are all of these integers possible using these operations?

b If we include the square root operation, $\sqrt{4}$, can you write a calculation for each of the integers 1 to 20? If you couldn't find all the numbers in part **a**, see if you can find them using the square root operation.

c The factorial, $n!$, is an operation that multiplies each of the integers from a number, n, down to 1.

For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

Using the factorial with the above operations, which numbers are now possible?

d Explain how you could use your results above to write calculations that are equal to each of the integers from −20 to −1.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Order of operations

Investigation
Converting temperatures

Topic quiz
1H

Chapter summary

Rounding and estimating

$1\boxed{2}600 \approx 13\,000$ Round up (9, 8, 7, 6, 5)

$1\boxed{2}300 \approx 12\,000$ Round down (4, 3, 2, 1, 0)

Leading digit

$123\,456 = \boxed{1}23\,456 \approx 100\,000$

Adding and subtracting

$$\begin{array}{r} {}^{1}\,2\,2\,9 \\ +\ 3\,4\,2 \\ \hline 5\,7\,1 \end{array}$$

Ones column: 9 + 2 = 11, carry the 1
Tens column: 2 + 4 + (1) = 7
Hundreds column: 2 + 3 = 5

← answer

$$\begin{array}{r} 3\,{}^{1}2\,{}^{11}1 \\ -\ 2\,0\,4 \\ \hline 1\,1\,7 \end{array}$$

Ones column: Take 10 from the tens column 11 – 4 = 7
Tens column: 1 – 0 = 1
Hundreds column: 3 – 2 = 1

← answer

The distributive law

$5 \times 32 = 5 \times (30 + 2)$
$= 5 \times 30 + 5 \times 2$
$= 150 + 10$
$= 160$

Multiplication algorithm

$$\begin{array}{rr} & {}^{2}\,3\,4 \\ \times & 2\,6 \\ \hline \text{Multiply by ones} \rightarrow & 2\,0\,4 \quad (34 \times 6) \\ \text{Multiply by tens} \rightarrow + & 6\,8\,0 \quad (34 \times 20) \\ \hline \text{Add the results} \rightarrow & 8\,8\,4 \end{array}$$

Short division

quotient

$$\begin{array}{r} 1\,4\,5 \\ 6\overline{)8^{2}7^{3}1} \end{array}$$ remainder 1

divisor, dividend, remainder

Lowest common multiple (LCM)

Multiples of 4: 4, 8, 12, 16, …
Multiples of 6: 6, 12, 18, 24, …
LCM: 12

Highest common factor (HCF)

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18
HCF: 6

Squares and cubes

$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$	$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$	$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$	$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$	$\sqrt[3]{27} = 3$

Integers

increasing value →
← decreasing value

–10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7 8 9 10

negative integers | zero | positive numbers

integers

Adding and subtracting integers

$+\,(-) = -$ $\quad -\,(+) = -$
$-\,(-) = +$ $\quad +\,(+) = +$

Powers of integers

index

base → $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

index form | expanded form | basic numeral

Order of operations

B	Brackets	$5 \times (10 - 2) = 5 \times 8 = 40$
I	Indices (and roots)	$3^3 - 4 = 27 - 4 = 23$
D M	Division Multiplication	$5 \times 3 - 4 + 8 \div 4 = 15 - 4 + 2 = 13$
A S	Addition Subtraction	$15 - 4 + 2 = 13$

Multiplying and dividing integers

$+ \times + = +$ $\quad + \div + = +$
$+ \times - = -$ $\quad + \div - = -$
$- \times + = -$ $\quad - \div + = -$
$- \times - = +$ $\quad - \div - = +$

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Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- addition algorithm
- ascending order
- associative law
- base
- BIDMAS
- by-parts method
- common factor
- commutative law
- compensation method
- cube
- cube root
- descending order
- distributive law
- dividend
- divisor
- estimate
- expanded form
- exponent
- factors
- highest common factor
- index
- integers
- jump method
- leading digit
- lowest common multiple
- multiples
- multiplication algorithm
- negative integers
- order of operations
- place value
- positive integers
- product
- quotient
- reference point
- remainder
- round down
- round up
- rounding
- short division
- square
- square root
- subtraction algorithm

1 Which mental strategy for adding or subtracting numbers involves rounding one number to make the calculations easier, and then adding or subtracting the amount by which the number was rounded up or down?

2 At which stage of BIDMAS are roots evaluated?

3 When performing a division, what is the quotient?

4 Identify the key terms being referenced in each of these definitions.

a the set of positive whole numbers, negative whole numbers and zero

b an approximate value which is close to the actual value

c the result of a multiplication

5 Using an example, provide a definition in your own words for the following key terms.

a multiples **b** square root **c** ascending order

6 Complete the following sentences using words from the key term list.

a The _____ states that the order in which numbers are added does not change the result.

b The _____ of a whole number are the whole numbers which divide exactly into that number.

Multiple choice

1A **1** By first rounding each number to its leading digit, what is an estimate for 526 × 68?

A 36 000 **B** 35 000 **C** 33 664 **D** 31 200 **E** 30 000

1A **2** Which of the following numbers would be 700 when rounded to the nearest hundred?

A 789 **B** 750 **C** 634 **D** 651 **E** 792

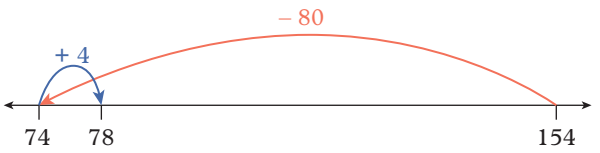
1B **3** What is 45 783 added to 67 398?

A 103 181 **B** 113 081 **C** 113 171 **D** 112 181 **E** 113 181

1B **4** What is 3765 subtracted from 9632?

A 5877 **B** 5967 **C** 6867 **D** 5867 **E** 5977

1B **5** Which calculation is being displayed in this diagram of the compensation method?



A 154 – 76 = 78 **B** 154 – 84 = 78 **C** 154 – 84 = 74
D 74 + 78 = 154 **E** 78 + 84 = 154

1C **6** What is 144 multiplied by 212?
A 356 **B** 30 536 **C** 30 000 **D** 30 528 **E** 28 800

1C **7** What is 672 divided by 12?
A 58 **B** 57 **C** 56 **D** 55 **E** 54

1C **8** What are the quotient and remainder when 1175 is divided by 7?
A quotient = 166, remainder = 1 **B** quotient = 167, remainder = 1
C quotient = 168, remainder = 1 **D** quotient = 166, remainder = 6
E quotient = 167, remainder = 6

1D **9** Which of the following is the HCF of 36 and 90?
A 5 **B** 9 **C** 36 **D** 18 **E** 3

1D **10** Which of the following represents the square root of 36?
A $\sqrt{36}$ **B** $\sqrt{6}$ **C** 36^2 **D** 6^2 **E** 216

1E **11** Which of the following numbers is smaller than –25?
A –18 **B** 2 **C** 25 **D** 0 **E** –30

1E **12** Which list of integers is written in descending order?
A 14, 11, 6, 0, –3, –5, –22 **B** 14, 11, 6, 0, –22, –5, –3
C –22, 14, 11, 6, –5, –3, 0 **D** –22, –5, –3, 0, 6, 11, 14
E 0, 6, 11, 14, –3, –5, –22

1F **13** What integer when added to 12 gives –7?
A –17 **B** –5 **C** 5 **D** 19 **E** –19

1F **14** What is –24 – 20?
A –4 **B** 4 **C** –44 **D** 44 **E** –48

1F **15** What does –35 – (–47) simplify to?
A –35 – 47 **B** 35 – 47 **C** 35 + 47 **D** –35 + 47 **E** –47 + 35

1G **16** What is 8 ÷ (–2)?
A –16 **B** –4 **C** 4 **D** 16 **E** 6

1G **17** What pair of integers gives a sum of –5 and a product of –36?
A –4 and 9 **B** –12 and 3 **C** 4 and 9 **D** –9 and –4 **E** –9 and 4

1G **18** Which gives the largest result?
A $(-2)^6$ **B** 6^2 **C** $(-1)^{10}$ **D** $(-4)^3$ **E** $-(8)^2$

1H **19** Which of these problems does *not* equal 5?
A –4 – 11 + 16 – 2 + 6 **B** 8 – 13 – 1 + 17 – 6 **C** –7 + 5 – 9 + 22 – 6
D 5 – 12 + 3 – 7 + 6 **E** 8 – 9 + 20 – 18 + 4

1H **20** What operation is performed first in $6 \times (12 - 5) + 8^2 \div 4$?
A 12 – 5 **B** 8^2 **C** 6 × 12 **D** 64 ÷ 4 **E** 8 ÷ 4

Short answer

1A **1** Round each number to its leading digit.

a 236 **b** 67 145 **c** 3890 **d** 149 046

1A **2** Estimate the result by first rounding each number to its leading digit.

a 12 345 + 3648 **b** 94 501 − 32 566 **c** 394 × 338 **d** 18 654 ÷ 425

1B **3** Three houses in one street were all sold on the same day. The selling prices were \$765 340, \$875 900 and \$655 000. What was the total price of all three houses?

1B **4** The City of Newcastle has 390 519 residents, while the Blue Mountains region has 78 121 residents. What is the difference in the number of residents between the two local government areas?

1C **5** Quinn sold 14 bouquets of roses today. If each bouquet has 12 roses, how many roses in total did Quinn sell?

1C **6** Tyler has saved up \$165. He intends to spend the money on \$15 T-shirts. How many will he be able to buy?

1D **7** Aida sold equal numbers of jam donuts and cinnamon donuts today. Jam donuts are sold in boxes of six, while cinnamon donuts are sold in packets of nine. What is the smallest number of each that Aida sold?

1D **8** Find the value of the following.

a 6^3 **b** 2^7

c 8 squared **d** 11^2

1D **9** Find the value of the following.

a square root of 4 **b** $\sqrt{81}$ **c** cube root of 27 **d** $\sqrt[3]{125}$

1E **10** Use the correct inequality sign to make each statement true.

a $-5 \square 2$ **b** $-5 \square -2$ **c** $7 \square -4$ **d** $4 \square -7$

1E **11** Write each list of integers in ascending order.

a 24, −40, 20, −14, 16, −22, −24

b −17, 21, −27, −35, 0, 9, 63

c −6, −66, 86, 6, −26, −16, −46

d 65, −87, 3, −300, 190, 45, −409

1F **12** Calculate:

a −5 + (−4) **b** 2 + (7) **c** 22 + (−34) **d** −50 + (69)

1F **13** Tamara's bank account shows a balance of −\$28. If she deposits \$150, what is her new account balance?

1F **14** Overnight, the minimum temperature in Alice Springs was −3°C. By 2 pm, the temperature had climbed to a maximum of 18°C. Calculate the difference between the minimum and maximum temperatures.

1F **15** Calculate:

a −8 + 7 **b** −4 − 6 **c** 17 − 25 **d** −44 + 34

e −66 + 66 **f** −50 − 50

1G **16** Calculate:

a 7 × (−8) **b** −5 × (−9) **c** −36 ÷ 4 **d** −100 ÷ (−20)

e −12 × 0 **f** 4 × (−15) **g** $\frac{18}{-6}$ **h** $\frac{-42}{-3}$

1G **17** Calculate:

a −6 × 3 × (−10) **b** [20 ÷ (−4)] × 7 **c** [−18 ÷ (−3)] × 2 **d** −5 × (−2) × (−4)

1G **18** Calculate:

a $(-2)^5$ **b** $(-3)^2$ **c** $(-1)^9$ **d** $(-10)^4$

1H **19** Calculate:

a $33 + (-40) \times 3 \div 10$

b $75 - 4 \times 11 + 6^2 \div (-4)$

c $-10 \times 7 - 5 \times [(-4)^2 - 4]$

d $-120 \div [-9 + (4 \times 8 - 11)]$

1H **20** Calculate:

a $-15 \times (-4 + 16)$

b $(-4)^2 - (-3)^3$

c $[-6 \times 7 + (-10 + 7)^2] \div 3$

d $[-1 + (-2)^2 + (-3)^3] \div 3$

Analysis

1 Jez is the elephant keeper at the zoo. He has two elephants to take care of: Mali and Man Jai. Mali weighs $3^3 \times 10^2$ kg while Man Jai weighs $2 \times 3^2 \times 10^2$ kg.

a What is the weight of each elephant as a basic numeral?

b What is the difference in weight between the two elephants?

c What is the sum of the weights of the two elephants?

d What is the HCF between the weight of the elephants?

Jez has to divide the food for the elephants according to their weight.

e The elephants need about one-tenth of their weight in food. How much food does Jez have to provide?

f How much food does each elephant need?

Man Jai gets sick and stops eating for a few days. As a result, he loses 5 kg each day. Jez writes the following expression to show Man Jai's weight after his illness: $2 \times 3^2 \times 10^2 - 5^n$ where n is the number of days he is sick. For example, if Man Jai is sick for 2 days, then Jez calculates his weight as $2 \times 3^2 \times 10^2 - 5^2$.

g Calculate Man Jai's weight after his illness using Jez's expression for 1, 2 and 3 days.

h Explain the problem with Jez's expression.

i Rewrite the expression so that it gives the correct weight of Man Jai after his illness.

2 The population figures (rounded to the nearest 100) for each state and territory of Australia at the end of June 2020 are displayed in this table.

State or territory	Population
New South Wales	8 164 100
Victoria	6 694 900
Queensland	5 174 400
Western Australia	2 661 900
South Australia	1 769 300
Tasmania	540 600
Australian Capital Territory	431 100
Northern Territory	246 000

a Determine the actual difference in population between NSW and Victoria.

b Round each population figure to the leading digit.

Use your answers from part **b** to answer the following questions.

c Estimate the difference in population between NSW and Victoria. Compare this to part **a**.

d Estimate the population of Australia.

3 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Minimum temperature (°C)	1	−2	−2	−3	−5	−2	−1
Maximum temperature (°C)	8	5	9	4	−1	5	5

a Which day had:

i the highest temperature

ii the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for each day.

c Which day had the biggest range of temperatures?

d Calculate the average of the minimum and maximum temperatures.

e What is the difference between the average minimum and average maximum temperatures?

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Round whole numbers ☐ Estimate results of simple calculations ☐ Assess the accuracy of estimations	☐ Go back to Topic 1A Rounding and estimating
☐ Use mental strategies for addition and subtraction ☐ Use the addition and subtraction algorithms for large numbers	☐ Go back to Topic 1B Adding and subtracting whole numbers
☐ Use mental strategies for multiplication ☐ Calculate products and quotients using the multiplication and division algorithms	☐ Go back to Topic 1C Multiplying and dividing whole numbers
☐ Find the lowest common multiple and highest common factor of two numbers ☐ Evaluate squares and cubes of numbers ☐ Evaluate square roots and cube roots	☐ Go back to Topic 1D Multiples, factors, indices and roots
☐ Compare the value of positive and negative numbers ☐ Use negative numbers to represent real-life situations	☐ Go back to Topic 1E Negative integers
☐ Solve problems involving addition and subtraction of positive and negative numbers	☐ Go back to Topic 1F Adding and subtracting integers
☐ Solve problems involving multiplication and division of negative numbers ☐ Solve problems involving indices of negative numbers	☐ Go back to Topic 1G Multiplying and dividing integers
☐ Order operations in calculations involving two or more operations ☐ Evaluate expressions including integers using the order of operations	☐ Go back to Topic 1H Order of operations

2

Fractions and decimals

Index

2A Fractions
2B Adding and subtracting fractions
2C Multiplying and dividing fractions
2D Decimals
2E Adding and subtracting decimals
2F Multiplying and dividing decimals
2G Terminating, non-terminating and recurring decimals

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Place value
- ✔ Number lines
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying and dividing whole numbers
- ✔ Order of operations

Curriculum links

- Represents and operates with fractions, decimals and percentages to solve problems (MA4-FRC-C-01)
  - Compare fractions using equivalence
  - Round decimals to a specified degree of accuracy using approximations
  - Identify terminating and recurring decimals
  - Examine the concept of irrational numbers
  - Order and compare the value of fractions, decimals and percentages
  - Solve problems that involve the addition and subtraction of fractions
  - Solve problems that involve the multiplication and division of fractions and decimals
  - Represent one quantity as a fraction, decimal or percentage of another, with and without the use of digital tools

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Materials

✔ Calculator

2A Fractions

Learning intentions

By the end of this topic you will be able to ...

✔ identify and find equivalent fractions
✔ compare fractions
✔ simplify fractions.

Inter-year links
Support Fractions
Year 7 3A Fractions

Fractions

Key content video

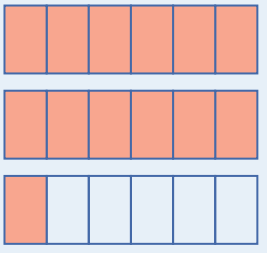
- **Proper fractions** have a **numerator** that is less than the **denominator**.
  Some examples are: $\frac{3}{5}$, $\frac{2}{6}$, $\frac{11}{15}$.
- **Improper fractions** have a numerator that is greater than or equal to the denominator.
  Some examples are: $\frac{13}{6}$, $\frac{5}{5}$, $\frac{31}{8}$.
- **Mixed numbers** consist of both a whole number and a proper fraction.
  For example, $2\frac{1}{6} = 2 + \frac{1}{6}$.

Proper fractions
Numerator < Denominator

$\frac{2}{6}$

Improper fraction
Numerator ≥ Denominator

$\frac{13}{6}$



Mixed number
How many wholes?
How many parts?

$2\frac{1}{6}$

To convert a mixed number to an improper fraction:	To convert an improper fraction to a mixed number:
1 Multiply the denominator by the whole number and add the numerator. **2** Write the result over the denominator. $3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$	**1** Divide the numerator by the denominator. **2** Write down the whole number. Place the remainder over the denominator. **3** Simplify the fraction if possible. $\frac{13}{4} = 13 \div 4 = 3$ remainder 1 $= 3\frac{1}{4}$

- Whole numbers can be written as fractions where the denominator is a factor of the numerator.
  For example, $5 = \frac{5}{1}$, $2 = \frac{2}{1}$, $100 = \frac{100}{1}$ and $\frac{9}{3} = 3$, $\frac{28}{7} = 4$.

Negative fractions

- Just like integers, negative fractions can be added, subtracted, multiplied and divided.
- Fractions with a negative **sign** can be written as shown.

Negative fractions

$-\frac{1}{2} = \frac{-1}{2} = \frac{1}{-2}$

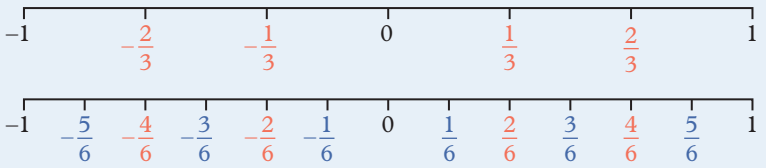
Positive fractions

$\frac{-1}{-2} = \frac{1}{2}$

Equivalent fractions

- **Equivalent fractions** are fractions that have the same numerical value. They mark the same point on the number line.

  For example, $-\frac{1}{3}$ and $-\frac{2}{6}$ mark the same point on the number line, so they are equivalent fractions.



- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.

$$\frac{2}{3} \overset{\times 5}{\underset{\times 5}{=}} \frac{10}{15} \qquad \frac{12}{10} \overset{\div 2}{\underset{\div 2}{=}} \frac{6}{5}$$

Comparing fractions

- When fractions have the same denominator, the bigger the numerator, the bigger the fraction.
- When fractions have the same numerator, the bigger the denominator, the smaller the fraction.
- To change a pair of fractions so they have the **lowest common denominator (LCD)**, find the lowest common multiple of the denominators and then convert each fraction into the equivalent fraction.

The LCM of 2 and 5 is 10.

$$\frac{1}{2} \overset{\times 5}{\underset{\times 5}{=}} \frac{5}{10} \qquad \frac{3}{5} \overset{\times 2}{\underset{\times 2}{=}} \frac{6}{10}$$

Simplifying fractions

- To find an equivalent fraction in its simplest form:
  - → Cancel out common factors until the only common factor between the numerator and denominator is 1.

    For example, $\frac{36}{72} = \frac{\cancel{36}^{18}}{\cancel{72}^{36}}$

    $= \frac{\cancel{18}^{3}}{\cancel{36}^{6}}$

    $= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$

    $= \frac{1}{2}$.
  - → Find the HCF of the numerator and the denominator and then divide the numerator and denominator by the HCF.

    For example, the HCF of 36 and 72 is 36, so $\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}}$

    $= \frac{1}{2}$.
- For mixed numbers, only simplify the fraction part; the whole number stays the same.

  For example, $2\frac{12}{24} = 2\frac{\cancel{12}^{1}}{\cancel{24}^{2}}$

  $= 2\frac{1}{2}$.

Example 2A.1 Writing equivalent fractions

Fill the gaps to produce equivalent fractions.

a $\frac{3}{7} = \frac{18}{}$ **b** $\frac{91}{35} = \frac{}{5}$ **c** $\frac{-2}{11} = \frac{}{33}$

THINK

1. Look for a relationship between the two numerators or the two denominators. Consider if the equivalent fraction has been found by using multiplication or division.
2. Use the same relationship to obtain the denominator or numerator of the second fraction.
3. Ensure that the sign of the equivalent fraction matches the first fraction.

WRITE

a

$$\frac{3}{7} \overset{\times 6}{=} \frac{18}{}$$

$$\frac{3}{7} \underset{\times 6}{\overset{\times 6}{=}} \frac{18}{42}$$

b

$$\frac{91}{35} = \frac{}{5} \quad (\div 7)$$

$$\frac{91}{35} \underset{\div 7}{\overset{\div 7}{=}} \frac{13}{5}$$

c

$$\frac{-2}{11} = \frac{}{33} \quad (\times 3)$$

$$\frac{-2}{11} \underset{\times 3}{\overset{\times 3}{=}} \frac{-6}{33}$$

Example 2A.2 Ordering fractions by finding the lowest common denominator

Determine which fraction is greater. Write your answer as an inequality.

a $\frac{5}{6} \square \frac{7}{9}$ **b** $-\frac{7}{2} \square -\frac{13}{4}$

THINK

1. Find the LCD of the denominators.
2. For each fraction, find the equivalent fraction with a denominator equal to the LCD.
3. Compare the fractions with the same denominator and write the answer using the original fractions.

WRITE

a 18 is the LCD.

$$\frac{5}{6} \underset{\times 3}{\overset{\times 3}{=}} \frac{15}{18} \qquad \frac{7}{9} \underset{\times 2}{\overset{\times 2}{=}} \frac{14}{18}$$

$\frac{15}{18} > \frac{14}{18}$; therefore, $\frac{5}{6} > \frac{7}{9}$.

b 4 is the LCD.

$$-\frac{7}{2} \underset{\times 2}{\overset{\times 2}{=}} -\frac{14}{4}$$

$-\frac{14}{4} < -\frac{13}{4}$; therefore, $-\frac{7}{2} < -\frac{13}{4}$.

Example 2A.3 Simplifying fractions

Write each fraction in its simplest form.

a $\frac{24}{40}$ **b** $2\frac{35}{105}$

THINK

a **1** Find a common factor of 24 and 40. 2 is a common factor.

2 Divide both the numerator and the denominator by 2.

3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

b **1** Find the HCF of the numerator and the denominator of the fraction component.

2 Leave the whole number as it is and divide both the numerator and the denominator by 35.

WRITE

a $\frac{24}{40} = \frac{\cancel{24}^{12}}{\cancel{40}^{20}}$

$= \frac{\cancel{12}^{6}}{\cancel{20}^{10}}$

$= \frac{\cancel{6}^{3}}{\cancel{10}^{5}}$

$= \frac{3}{5}$

b HCF = 35

$2\frac{35}{105} = 2\frac{\cancel{35}^{1}}{\cancel{105}^{3}}$

$= 2\frac{1}{3}$

Helpful hints

- ✔ Remember that 'equivalent' is another way of saying 'equal to'.
- ✔ Each fraction has infinitely many equivalent fractions. If the only common factor between the numerator and the denominator is 1, the fraction is in its simplest form.

  For example, $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ and so on.
- ✔ Use the divisibility rules from Chapter 1. The larger the common factor, the faster you'll find the equivalent fraction in its simplest form.

ANS p484

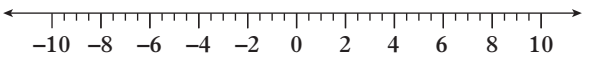
Exercise 2A Fractions

1–3, 4(a, c, e, f, k), 5, 6–9(1st, 2nd columns), 10, 11, 12(1st column), 13, 14, 15(a, b), 16(a, b, f, h), 18	1(b, d, e), 4(b, d, g, h, l), 6–9(2nd, 3rd columns), 10–11(b, c), 12(2nd column), 15(c, d), 16(c, d, e, g), 17, 19, 20(a)	4(i–l), 6(3rd column), 7(g, h), 8–9(3rd column), 10–11(b, c), 12(g–l), 16(e, g, h), 19–21

1 Write all equivalent positive or negative fractions showing the possible positions of the negative sign.

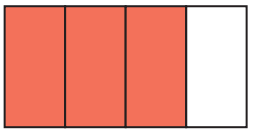
a $\frac{1}{-2}$ **b** $\frac{-10}{3}$ **c** $\frac{-5}{-6}$ **d** $\frac{-2}{-3}$ **e** $-\frac{2}{5}$ **f** $-\frac{-3}{-7}$

2 Show the position of each number on the following number line.



a -3 **b** $\frac{5}{2}$ **c** $-\frac{1}{2}$ **d** 5 **e** $7\frac{3}{4}$ **f** $-4\frac{3}{4}$

3 Consider the rectangle below.



a What fraction of the rectangle has been shaded?

b Draw a rectangle that is divided into 12 equal sections. Shade the rectangle so that it represents an equivalent fraction to that in part **a**.

c Write the equivalent fraction for part **b**.

2A.1 **4** Fill the gaps to produce equivalent fractions.

a $\frac{3}{10} = \frac{6}{}$ **b** $\frac{2}{7} = \frac{}{35}$ **c** $\frac{18}{33} = \frac{}{11}$ **d** $\frac{16}{48} = \frac{2}{}$

e $-\frac{4}{5} = -\frac{}{30}$ **f** $\frac{-8}{36} = \frac{-2}{}$ **g** $-1\frac{3}{8} = \frac{}{40}$ **h** $3\frac{3}{4} = \frac{75}{}$

i $\frac{1}{4} = \frac{2}{} = \frac{}{20} = \frac{8}{} = \frac{20}{} = \frac{}{100}$

j $-\frac{3}{7} = \frac{-15}{} = \frac{}{49} = \frac{}{70} = \frac{-300}{} = -\frac{}{210}$

k $1\frac{1}{5} = \frac{}{10} = \frac{60}{} = \frac{}{250} = \frac{}{45} = \frac{72}{}$

l $-2\frac{1}{3} = -\frac{14}{} = \frac{}{15} = \frac{49}{} = -\frac{}{9} = -\frac{}{300}$

5 From the list below, select which fractions are equivalent to:

a $\frac{2}{5}$ **b** $\frac{4}{3}$.

$\frac{6}{10}, \frac{20}{15}, \frac{8}{20}, \frac{7}{10}, \frac{28}{21}, \frac{22}{55}, \frac{4}{10}, \frac{16}{9}, 1\frac{4}{12}, \frac{10}{25}, \frac{18}{45}, 2\frac{6}{15}$

2A.2 **6** Compare the following fractions with an inequality sign. Hint: Write equivalent fractions with the same denominator.

a $\frac{3}{8} \square \frac{1}{2}$ **b** $\frac{4}{7} \square \frac{5}{14}$ **c** $\frac{-9}{10} \square \frac{-4}{5}$

d $-\frac{5}{12} \square -\frac{2}{3}$ **e** $1\frac{5}{6} \square 1\frac{19}{24}$ **f** $\frac{17}{15} \square \frac{7}{3}$

g $-\frac{2}{3} \square -\frac{3}{4}$ **h** $\frac{5}{6} \square \frac{4}{5}$ **i** $-\frac{11}{8} \square -\frac{5}{3}$

j $2\frac{3}{4} \square \frac{17}{6}$ **k** $-\frac{29}{3} \square -9\frac{1}{2}$ **l** $3\frac{5}{12} \square \frac{25}{8}$

7 Decide which fraction is greater in each pair.

a $\frac{5}{9}, \frac{7}{9}$ **b** $\frac{11}{4}, \frac{9}{4}$ **c** $-2\frac{3}{5}, -\frac{12}{5}$

d $-\frac{43}{6}, -7\frac{5}{6}$ **e** $3\frac{7}{11}, \frac{37}{11}$ **f** $-\frac{29}{2}, -8\frac{1}{2}$

g $4\frac{2}{3}, \frac{16}{3}$ **h** $-\frac{55}{9}, -6\frac{2}{9}$ **i** $3\frac{5}{7}, \frac{25}{7}$

2A.3 **8** Write each fraction in its simplest form.

a $\frac{8}{10}$ **b** $\frac{14}{21}$ **c** $\frac{42}{54}$

d $\frac{45}{72}$ **e** $\frac{40}{12}$ **f** $\frac{55}{30}$

9 Write each fraction in its simplest form.

a $\frac{6}{15}$ **b** $-\frac{33}{77}$ **c** $-\frac{18}{45}$

d $-\frac{30}{18}$ **e** $\frac{32}{36}$ **f** $\frac{18}{42}$

g $3\frac{24}{32}$ **h** $-5\frac{35}{63}$ **i** $4\frac{9}{33}$

j $-1\frac{24}{42}$ **k** $2\frac{15}{40}$ **l** $-7\frac{45}{54}$

10 Write each list of fractions in ascending order.

a $\frac{2}{5}, \frac{7}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$ **b** $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$ **c** $-1\frac{1}{2}, -\frac{2}{5}, -1\frac{9}{10}, -\frac{9}{2}, -\frac{3}{10}, -\frac{9}{5}$

11 Write each list of fractions in descending order.

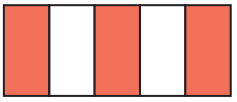
a $\frac{13}{11}, \frac{5}{11}, \frac{19}{11}, \frac{9}{11}, \frac{3}{11}, 1\frac{5}{11}$ **b** $-\frac{2}{3}, -1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, -\frac{4}{3}, -1\frac{1}{4}$ **c** $\frac{5}{6}, -2\frac{1}{6}, -\frac{11}{6}, \frac{1}{6}, -\frac{5}{12}, 1\frac{1}{3}$

12 Write each amount as a fraction of the total amount in simplest form. Hint: Make sure the units are the same before you simplify your fraction.

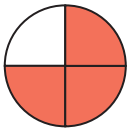
a 15 kg out of 25 kg **b** 80 m out of 360 m
c \$35 out of \$120 **d** 18 seconds out of 60 seconds
e 50c out of \$3 **f** 14 days out of 5 weeks
g 20 mm out of 7 cm **h** 300 g out of 2 kg
i 28 seconds out of 1 minute **j** 75c out of \$4.25
k 20 cm out of 6 m **l** 16 minutes out of 2 hours

13 Match one of the fractions $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$ or $\frac{5}{8}$ to the diagram.

a

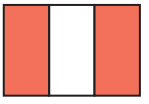


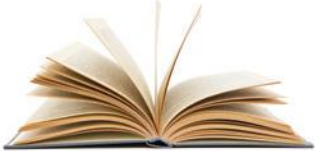
b



c

d





14 If you have read 56 out of 224 pages of a novel, what fraction have you read?

15 Consider these two rows of a multiplication grid.

	1	2	3	4	5	6	7	8	9
× 3	3	6	9	12	15	18	21	24	27
× 4	4	8	12	16	20	24	28	32	36

a Write numbers in each column of the table as a fraction by making the first number the numerator and the second the denominator. For example, the first column will be $\frac{3}{4}$.

b What do you notice in your answer to part **a**?

Consider these two rows.

	1	2	3	4	5	6	7	8	9
× 7	7	14	21	28	35	42	49	56	63
× 5	5	10	15	20	25	30	35	40	45

c Write numbers in each column of the table as a fraction by making the first number the numerator and the second the denominator. For example, the first column will be $\frac{7}{5}$.

d What do you notice in your answer to part **c**?

16 Find the fraction that is halfway between each pair of fractions.

a $\frac{3}{7}$ and $\frac{5}{7}$ **b** $-\frac{5}{11}$ and $-\frac{9}{11}$ **c** $\frac{3}{8}$ and $\frac{5}{8}$ **d** $-\frac{1}{3}$ and $-\frac{5}{9}$
e $\frac{4}{15}$ and $\frac{2}{5}$ **f** $\frac{1}{2}$ and $-\frac{3}{4}$ **g** $-1\frac{1}{4}$ and $1\frac{1}{3}$ **h** $2\frac{2}{3}$ and $3\frac{1}{5}$

PROBLEM SOLVING AND REASONING

17 Brodie scored 25 out of 30 on the first Maths test and 21 out of 28 on the second. On which test did Brodie perform better?

18 Tony and Lisbeth hit a target with a ball with the same level of accuracy. Tony hit the target four times in five shots. Lisbeth hit the target 12 times. Use equivalent fractions to work out how many shots Lisbeth took.

19 Fractions can be compared in several ways:

1 comparing with a common denominator or the lowest common denominator

2 comparing with a common numerator or the lowest common numerator

3 comparing with a benchmark such as 1 or $\frac{1}{2}$.

For each of the following pairs of fractions:

i decide which strategy would be the most efficient to compare the pair and explain why you chose that strategy

ii state which fraction is greater by writing an appropriate inequality.

a $\frac{4}{7}, \frac{7}{15}$

b $\frac{15}{17}, \frac{15}{4}$

c $\frac{99}{100}, \frac{100}{101}$

d $1\frac{1}{8}, 1\frac{1}{11}$

e $1\frac{3}{5}, 1\frac{2}{3}$

f $\frac{77}{10}, \frac{39}{5}$

CHALLENGE

20 The number of faults out of the total number of serves was recorded for each player in a tennis match.

	Naomi	Ash
Total number of serves	119	102
Number of serving faults	14	10

a Who had the better success rate in serving?

b If Naomi served at the same fault rate in her next match, how many faults did she make out of 85 serves?

c If Ash served at the same fault rate in her next match and made 15 faults, what was her total number of serves?

21 a Which one of the following fractions could not be placed between the fractions marked on the number line?

A $\frac{3}{7}$ **B** $\frac{5}{11}$ **C** $\frac{48}{96}$ **D** $\frac{31}{41}$ **E** $\frac{51}{121}$

b Explain how you checked each fraction.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet — Visual representations of fractions

Interactive skillsheet — Equivalent fractions

Investigation — Cross country fractions

Topic quiz — 2A

2B Adding and subtracting fractions

Learning intentions

By the end of this topic you will be able to ...

- ✔ add and subtract fractions with different denominators
- ✔ add and subtract fractions and mixed numbers
- ✔ add and subtract negative fractions and mixed numbers.

Inter-year links

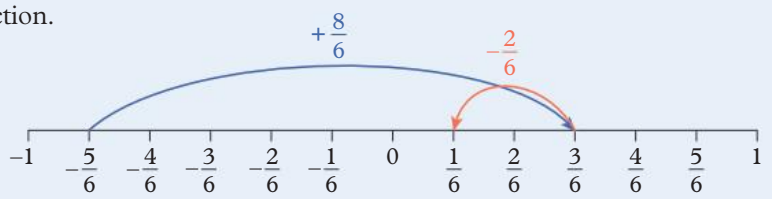
Support	Adding and subtracting fractions
Year 7	3E Adding and subtracting fractions

Adding and subtracting fractions

Key content video

- To add and subtract fractions:
  1. Convert into equivalent fractions with a common denominator.
  2. Add or subtract the numerators and keep the same denominator.
  3. Simplify the fraction.

$$-\frac{5}{6}+\frac{4}{3}-\frac{1}{3}=\frac{-5+8-2}{6}$$
$$=\frac{1}{6}$$



- To add or subtract mixed numbers, first convert the mixed numbers to improper fractions. Convert the sum or difference back to a mixed number and simplify your answer.
- Simplify the calculation by considering the operation and the sign applied to each fraction.

$$+(-)=-$$
$$-(+)=-$$
$$-(-)=+$$
$$+(+)=+$$

Example 2B.1 Adding and subtracting fractions with common denominators

Determine the result of each sum or difference.

a $\frac{2}{7}+\frac{8}{7}$ **b** $\frac{13}{6}-\frac{4}{6}$

THINK

a Add the numerators and keep the same denominator.

b 1 Subtract the numerators and keep the same denominator.

2 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{2}{7}+\frac{8}{7}=\frac{10}{7}$

b $\frac{13}{6}-\frac{4}{6}=\frac{\cancel{9}^{3}}{\cancel{6}_{2}}$

$=\frac{3}{2}$

Example 2B.2 Adding and subtracting fractions with different denominators

Determine the result of each sum or difference.

a $\frac{5}{6}+\frac{1}{2}$

b $\frac{11}{5}-\frac{6}{4}$

THINK

a **1** Convert into equivalent fractions with a common denominator. A common multiple of 2 and 6 is 6.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b **1** Convert into equivalent fractions with a common denominator. A common multiple of 5 and 4 is 20.

2 Subtract the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

WRITE

a $\frac{5}{6}+\frac{1}{2}=\frac{5}{6}+\frac{1\times 3}{2\times 3}$

$=\frac{5}{6}+\frac{3}{6}$

$=\frac{\cancel{8}^{4}}{\cancel{6}_{3}}$

$=\frac{4}{3}$

b $\frac{11}{5}-\frac{6}{4}=\frac{11\times 4}{5\times 4}-\frac{6\times 5}{4\times 5}$

$=\frac{44}{20}-\frac{30}{20}$

$=\frac{\cancel{14}^{7}}{\cancel{20}_{10}}$

$=\frac{7}{10}$

Example 2B.3 Adding and subtracting negative fractions

Determine the result of each sum or difference.

a $-\frac{12}{5}-\left(-\frac{7}{5}\right)$

b $-\frac{2}{3}+\left(-\frac{1}{2}\right)$

THINK

a **1** Simplify the calculation. Remember that subtracting a negative number is the same as adding a positive number.

2 Add the numerators and keep the same denominator.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

b **1** Simplify the calculation. Remember that adding a negative number is the same as subtracting a positive number.

2 Convert into equivalent fractions with a common denominator. A common multiple of 3 and 2 is 6.

3 Subtract the numerators and keep the same denominator.

WRITE

a $-\frac{12}{5}-\left(-\frac{7}{5}\right)=-\frac{12}{5}+\frac{7}{5}$

$=-\frac{\cancel{5}^{1}}{\cancel{5}_{1}}$

$=-1$

b $-\frac{2}{3}+\left(-\frac{1}{2}\right)=-\frac{2}{3}-\frac{1}{2}$

$=-\frac{2\times 2}{3\times 2}-\frac{1\times 3}{2\times 3}$

$=-\frac{4}{6}-\frac{3}{6}$

$=-\frac{7}{6}$

Example 2B.4 Adding and subtracting mixed numbers

Determine the result of each sum or difference.

a $2\frac{1}{3}+\frac{5}{6}$

b $1\frac{3}{4}-2\frac{5}{12}$

THINK

a **1** Convert the mixed numbers into improper fractions.

2 Convert into equivalent fractions with a common denominator. A common multiple of 3 and 6 is 6.

3 Add the numerators and keep the same denominator.

4 Convert the answer into a mixed number and simplify the fraction.

b **1** Convert the mixed numbers into improper fractions.

2 Convert into equivalent fractions with a common denominator. A common multiple of 4 and 12 is 12.

3 Subtract the numerators and keep the same denominator.

4 Simplify the fraction.

WRITE

a $2\frac{1}{3}+\frac{5}{6}=\frac{7}{3}+\frac{5}{6}$

$=\frac{7\times 2}{3\times 2}+\frac{5}{6}$

$=\frac{14}{6}+\frac{5}{6}$

$=\frac{19}{6}$

$=3\frac{1}{6}$

b $1\frac{3}{4}-2\frac{5}{12}=\frac{7}{4}-\frac{29}{12}$

$=\frac{7\times 3}{4\times 3}-\frac{29}{12}$

$=\frac{21}{12}-\frac{29}{12}$

$=-\frac{\cancel{8}^{2}}{\cancel{12}_{3}}$

$=-\frac{2}{3}$

Helpful hints

- ✔ Only add or subtract the numerators of fractions; the denominator stays the same.

$$\frac{3}{5}-\frac{2}{5}=\frac{3-2}{5}$$

- ✔ You can reduce the complexity of an addition or subtraction by finding the lowest common denominator. Always try to simplify first so that your lowest common multiple will be as small as possible.
- ✔ Adding does not always make a number larger. For example, adding a negative number will give a smaller result.
- ✔ Subtracting does not always make a number smaller. For example, subtracting a negative number will give a larger result.
- ✔ When simplifying calculations involving negative numbers, only combine the signs located in between the numbers.

$$+(-)=-$$
$$-(+)=-$$
$$-(-)=+$$
$$+(+)=+$$

ANS p485

Exercise 2B Adding and subtracting fractions

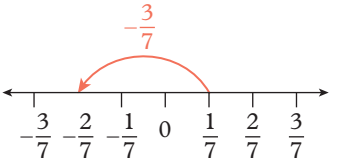
1–7, 8(a, b, c, f), 9, 10(a), 11(a–d), 14, 15 | 4, 5, 7, 8, 9, 10(b), 11(e–h), 12(a, b, c), 13, 16, 18, 19 | 4–5(e–h), 7–8(e, g, h), 9, 10(b, c), 11(g, h), 12(d–f), 16–20

UNDERSTANDING AND FLUENCY

1 Consider $\left(+\frac{1}{7}\right) + \left(-\frac{3}{7}\right)$.

a Explain why this can be simplified to $\frac{1}{7} - \frac{3}{7}$.

b Use this number line to calculate the result.



2B.1 **2** Evaluate the result of each sum or difference.

a $\frac{5}{17} + \frac{8}{17}$ **b** $\frac{7}{10} - \frac{3}{10}$ **c** $\frac{3}{8} + \frac{5}{8}$ **d** $\frac{6}{3} - \frac{5}{3}$

3 Find the lowest common denominator for each pair of fractions.

a $\frac{4}{5}$ and $\frac{2}{3}$ **b** $\frac{3}{7}$ and $\frac{1}{14}$ **c** $\frac{6}{10}$ and $\frac{4}{15}$ **d** $\frac{3}{8}$ and $\frac{5}{12}$

e $\frac{2}{15}$ and $\frac{2}{3}$ **f** $\frac{3}{4}$ and $\frac{2}{5}$ **g** $\frac{1}{6}$ and $\frac{3}{4}$ **h** $\frac{4}{9}$ and $\frac{1}{6}$

2B.2 **4** Evaluate the result of each sum or difference.

a $\frac{5}{22} + \frac{8}{11}$ **b** $\frac{3}{4} - \frac{2}{5}$ **c** $\frac{9}{10} + \frac{2}{3}$ **d** $\frac{3}{8} - \frac{1}{12}$

e $\frac{11}{15} - \frac{7}{10}$ **f** $\frac{5}{6} + \frac{3}{4}$ **g** $\frac{5}{6} - \frac{4}{9}$ **h** $\frac{3}{8} + \frac{2}{5}$

2B.3 **5** Evaluate the result of each sum or difference.

a $\frac{1}{7} - \left(-\frac{3}{7}\right)$ **b** $-\frac{2}{3} + \left(-\frac{5}{3}\right)$ **c** $\frac{5}{8} - \left(+\frac{7}{8}\right)$ **d** $-\frac{4}{5} + \left(+\frac{2}{5}\right)$

e $-\frac{1}{3} + \left(-\frac{4}{3}\right)$ **f** $-\frac{3}{4} - \left(+\frac{1}{2}\right)$ **g** $-\frac{1}{3} - \left(-\frac{5}{9}\right)$ **h** $\frac{2}{5} + \left(-\frac{3}{2}\right)$

6 Evaluate the result of each sum or difference.

a $-\frac{4}{5} + \frac{3}{5}$ **b** $\frac{2}{7} - \frac{5}{7}$ **c** $-\frac{3}{11} - \frac{6}{11}$ **d** $-\frac{4}{3} + \frac{1}{6}$ **e** $\frac{2}{7} - \frac{1}{2}$ **f** $-\frac{3}{4} + \frac{7}{8}$

2B.4 **7** Evaluate each problem and simplify where possible.

a $1\frac{1}{4} - \frac{7}{8}$ **b** $2\frac{3}{5} + \frac{13}{20}$ **c** $2\frac{2}{3} + 1\frac{4}{5}$ **d** $3\frac{7}{11} - 1\frac{1}{2}$

e $4\frac{3}{4} + 2\frac{2}{3}$ **f** $5\frac{5}{6} - 3\frac{2}{15}$ **g** $1\frac{2}{3} - 2\frac{1}{3}$ **h** $-4\frac{3}{5} - 1\frac{1}{10}$

8 Determine the result of each sum and difference.

a $\frac{2}{3} - \frac{4}{3} + \frac{1}{3}$ **b** $-\frac{3}{4} + \frac{1}{4} + \frac{1}{2}$ **c** $\frac{2}{3} - \frac{5}{6} - \frac{1}{2}$ **d** $-\frac{3}{5} - \frac{1}{4} + \frac{7}{10}$

e $\frac{5}{9} - \frac{2}{3} + \frac{1}{2}$ **f** $5\frac{2}{7} - 9\frac{6}{7} + 1\frac{5}{7}$ **g** $-4\frac{3}{4} + 3\frac{5}{8} + 2\frac{7}{12}$ **h** $10\frac{3}{10} - 4\frac{3}{4} - 5\frac{3}{5}$

PROBLEM SOLVING AND REASONING

9 Nine numbers are arranged in a square.

a Add the three numbers in:

i the first row **ii** the second row **iii** the third row.

b Add the three numbers in:

i the first column **ii** the second column **iii** the third column.

c Add the three numbers along each diagonal.

d What do you notice about your answers to parts **a**, **b** and **c**?

e Your answer to part **d** is the reason why the nine numbers form a magic square. The sum in each row, column and diagonal is called the magic sum. What is the magic sum for this set of numbers?

$\frac{4}{5}$	$\frac{1}{10}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{1}{2}$	$\frac{7}{10}$
$\frac{2}{5}$	$\frac{9}{10}$	$\frac{1}{5}$

10 Copy and complete each magic square. Hint: First work out the magic sum.

a

		$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$

b

		$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	
$\frac{1}{3}$		

c

		$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$

11 Consider the following sums and differences. Without performing the calculation, state whether the sum or difference is positive, negative or zero.

a $\left(-\frac{2}{5}\right)+\frac{3}{4}$ **b** $\left(-\frac{7}{10}\right)-\left(-\frac{3}{10}\right)$ **c** $\frac{9}{11}-\left(-\frac{51}{17}\right)$ **d** $\frac{2}{3}+\left(-\frac{4}{6}\right)$

e $\left(-\frac{13}{25}\right)+\left(-\frac{81}{15}\right)$ **f** $\left(-\frac{5}{4}\right)-\left(+\frac{7}{3}\right)$ **g** $\left(-\frac{12}{5}\right)-\left(-\frac{36}{15}\right)$ **h** $-\frac{1}{77}-\frac{9}{15}$

12 We can take advantage of negative fractions to add and subtract mixed numbers without first converting to improper fractions by using the associative, commutative and distributive laws.

For example:
$$\begin{aligned} 2\tfrac{1}{3}-1\tfrac{2}{3} &= \left(2+\tfrac{1}{3}\right)-\left(1+\tfrac{2}{3}\right) \\ &= 2+\tfrac{1}{3}-1-\tfrac{2}{3} \\ &= (2-1)+\left(\tfrac{1}{3}-\tfrac{2}{3}\right) \\ &= 1+\left(-\tfrac{1}{3}\right) \\ &= \tfrac{2}{3} \end{aligned}$$

Calculate the following sums and differences without converting to improper fractions. Give your answer as a mixed number.

a $5\frac{7}{8}-4\frac{3}{8}$ **b** $5\frac{3}{8}-4\frac{7}{8}$ **c** $4\frac{7}{8}-5\frac{3}{8}$

d $4\frac{3}{8}-5\frac{7}{8}$ **e** $5\frac{7}{8}+4\frac{3}{8}$ **f** $-5\frac{7}{8}+\left(-4\frac{3}{8}\right)$

13 In subtraction of fractions problems, when is it better to convert to improper fractions first?

14 Craig wants to combine two open cartons of milk into one 2 L container. If one carton has $\frac{1}{3}$ L and the other has $1\frac{3}{4}$ L, will the 2 L container be large enough? Explain.

15 On Monday evening, Eleanor spent $1\frac{1}{4}$ hours on the computer researching an assignment for school, Adam spent $2\frac{1}{2}$ hours on the computer catching up with friends on Facebook, and their mother spent $\frac{5}{6}$ of an hour typing up a report for work. How much time was spent on the computer on Monday evening?

16 One-fifth of the student population at a local high school ride to school, one-tenth walk to school and two-thirds catch public transport. The remainder is driven to school.

What fraction of students:

a ride or walk to school

b catch public transport, ride or walk to school

c are driven to school?

17 John had a pizza delivered to his house and ate half of it before soccer training. Jen arrived home and ate half of what was left. Oscar then ate one-third of the remaining pizza. What fraction of the original pizza is left over?

PROBLEM SOLVING AND REASONING

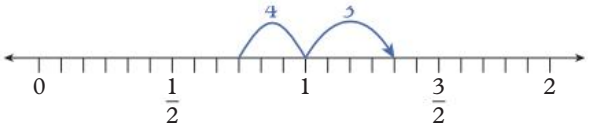
18 Metal screws come in different sizes, with diameters often listed in inches.

a Find the difference in size between two screws with diameters $\frac{3}{50}$ inches and $\frac{41}{250}$ inches.

b Wes has drilled a hole in the wall of diameter $\frac{3}{20}$ inches to hang a painting. Which of these two screws would be best for him to use?

19 a Match each set of calculations with the corresponding number line. There is one set of calculations and one number line that do not have a match. For these, write a corresponding calculation and draw a corresponding number line diagram.

i



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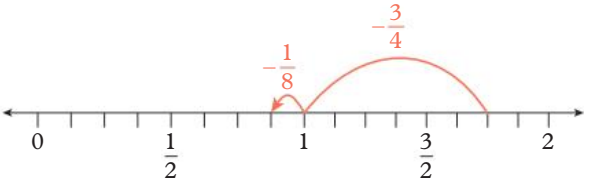

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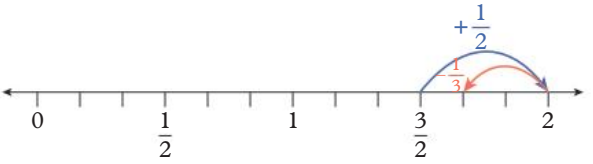

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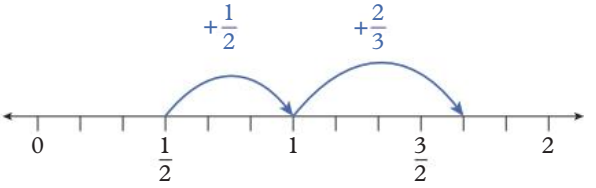

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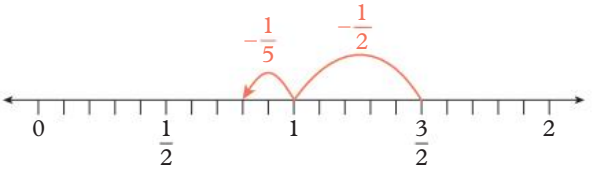

A $\frac{1}{2}+\frac{7}{6}=\left(\frac{1}{2}+\frac{3}{6}\right)+\frac{4}{6}$
$=1+\frac{2}{3}$
$=1\frac{2}{3}$

B $\frac{3}{4}+\frac{7}{12}=\left(\frac{3}{4}+\frac{3}{12}\right)+\frac{4}{12}$
$=1+\frac{1}{3}$
$=1\frac{1}{3}$

C $\frac{3}{2}-\frac{7}{10}=\left(\frac{3}{2}-\frac{5}{10}\right)-\frac{2}{10}$
$=1-\frac{1}{5}$
$=\frac{4}{5}$

D $\frac{7}{4}-\frac{7}{8}=\left(\frac{7}{4}-\frac{6}{8}\right)-\frac{1}{8}$
$=1-\frac{1}{8}$
$=\frac{7}{8}$

E $\frac{3}{4}-\frac{1}{12}=\left(\frac{3}{4}+\frac{3}{12}\right)-\frac{4}{12}$
$=1-\frac{1}{3}$
$=\frac{2}{3}$

b Describe the strategy that was used to evaluate these calculations.

CHALLENGE

20 Consider the following calculations. Determine which one is not equivalent to the others.

A $\left(-\frac{5}{3}\right)+\left(+\frac{4}{7}\right)-\left(-\frac{7}{11}\right)$

B $\left(-\frac{5}{3}\right)-\left(-\frac{4}{7}\right)+\left(+\frac{7}{11}\right)$

C $\left(+\frac{4}{8}\right)-\left(+\frac{5}{3}\right)+\left(+\frac{4}{7}\right)-\left(-\frac{7}{11}\right)+\left(-\frac{33}{66}\right)$

D $\left(-\frac{7}{11}\right)-\left(-\frac{4}{7}\right)+\left(-\frac{5}{3}\right)$

E $\left(\frac{-5}{3}\right)+\left(\frac{-4}{-7}\right)-\left(\frac{7}{-11}\right)$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Adding and subtracting fractions

Investigation
Puzzling pumpkins

Topic quiz
2B

2C Multiplying and dividing fractions

Learning intentions

By the end of this topic you will be able to ...

✔ multiply and divide fractions and mixed numbers
✔ multiply and divide negative fractions and mixed numbers
✔ determine fractions of an amount.

Inter-year links
Year 7 3F Multiplying fractions

Multiplying fractions

Key content video

- To multiply two fractions together:
  1. Look diagonally and vertically for any common factors between the numerators and denominators. Cancel any common factors in the numerators and denominators by dividing both numbers by the highest common factor (HCF).
  2. Multiply the numerators together and the denominators together.

$$\frac{2}{3}\times\frac{3}{4} = \frac{2\times \cancel{3}^{1}}{\cancel{3}^{1}\times 4} \quad \text{Diagonally}$$

$$= \frac{\cancel{2}^{1}}{\cancel{4}^{2}} \quad \text{Vertically}$$

$$= \frac{1}{2}$$

- When multiplying mixed numbers, convert them into improper fractions, multiply across and give the result as a mixed number.
- Whole numbers can be written as fractions with a denominator of 1. For example, $8 = \frac{8}{1}$.
- When asked to find a fraction 'of' a whole number or fraction, replace the 'of' with '×'. For example, $\frac{2}{3}$ of 20 is the same as $\frac{2}{3} \times 20$.

Dividing fractions by fractions

- To find the **reciprocal** of a fraction, invert the fraction by swapping the numerator and the denominator.

Reciprocal: $5 \rightleftarrows \frac{1}{5}$ Reciprocal

Reciprocal: $\frac{3}{4} \rightleftarrows \frac{4}{3}$ Reciprocal

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction. Keep the first fraction the same, and multiply by the reciprocal of the second fraction.

$$\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$$

- For both multiplying and dividing fractions:
  → If the signs are the same, then the result is positive.
  → If the signs are different, then the result is negative.

+ × + = +	+ ÷ + = +
+ × − = −	+ ÷ − = −
− × + = −	− ÷ + = −
− × − = +	− ÷ − = +

Example 2C.1 Multiplying fractions

Determine the result of each product.

a $\frac{8}{9} \times \frac{5}{6}$ **b** $\frac{23}{9} \times 3$ **c** $1\frac{2}{3} \times 2\frac{7}{10}$

THINK

a **1** Simplify the 8 and 6 by dividing both numbers by 2.

2 Multiply the numerators together, then multiply the denominators together.

b **1** Write 3 as a fraction with a denominator of 1.

2 Simplify the 9 and 3 by dividing both numbers by 3.

3 Multiply the numerators together, then multiply the denominators together.

c **1** Convert each mixed number into an improper fraction.

2 Simplify the 5 and 10 by dividing both numbers by 5 and then cancel the 3 and 27 by dividing both numbers by 3.

3 Multiply the numerators together, then multiply the denominators together.

4 Convert the improper fraction to a mixed number.

WRITE

a $\frac{8}{9} \times \frac{5}{6} = \frac{\cancel{8}^{4}}{9} \times \frac{5}{\cancel{6}_{3}}$

$= \frac{4 \times 5}{9 \times 3}$

$= \frac{20}{27}$

b $\frac{23}{9} \times 3 = \frac{23}{9} \times \frac{3}{1}$

$= \frac{23}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{1}$

$= \frac{23 \times 1}{3 \times 1}$

$= \frac{23}{3}$

c $1\frac{2}{3} \times 2\frac{7}{10} = \frac{5}{3} \times \frac{27}{10}$

$= \frac{\cancel{5}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}_{2}}$

$= \frac{1 \times 9}{1 \times 2}$

$= \frac{9}{2}$

$= 4\frac{1}{2}$

Example 2C.2 Dividing fractions

Determine the result of each quotient.

a $\frac{1}{7} \div \frac{3}{5}$ **b** $\frac{4}{5} \div 6$ **c** $3\frac{3}{8} \div 2\frac{1}{4}$

THINK

a **1** Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Multiply the numerators together and then multiply the denominators together.

WRITE

a $\frac{1}{7} \div \frac{3}{5} = \frac{1}{7} \times \frac{5}{3}$

$= \frac{1 \times 5}{7 \times 3}$

$= \frac{5}{21}$

b 1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

2 Cancel out any common factors.

3 Multiply the numerators together, then multiply the denominators together.

c 1 Convert each mixed number to an improper fraction.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel out any common factors.

4 Multiply the numerators together, then multiply the denominators together.

5 Change the improper fraction to a mixed number.

b $\frac{4}{5} \div 6 = \frac{4}{5} \times \frac{1}{6}$

$= \frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{6}^{3}}$

$= \frac{2 \times 1}{5 \times 3}$

$= \frac{2}{15}$

c $3\frac{3}{8} \div 2\frac{1}{4} = \frac{27}{8} \div \frac{9}{4}$

$= \frac{27}{8} \times \frac{4}{9}$

$= \frac{\cancel{27}^{3}}{\cancel{8}^{2}} \times \frac{\cancel{4}^{1}}{\cancel{9}^{1}}$

$= \frac{3 \times 1}{2 \times 1}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

Example 2C.3 Multiplying and dividing negative fractions

Determine the result of each product or quotient.

a $-\frac{2}{9} \times \left(-\frac{13}{4}\right)$ **b** $\frac{12}{7} \div \left(-\frac{3}{14}\right)$ **c** $-2\frac{3}{4} \div \left(-4\frac{1}{2}\right)$

THINK

a 1 Determine whether the result is positive or negative. The signs are the same, so the result is positive.

2 Simplify the 2 and 4 by dividing both numbers by 2.

3 Multiply the numerators together, then multiply the denominators together.

b 1 Determine whether the result is positive or negative. The signs are different, so the result is negative.

2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

3 Cancel out any common factors.

4 Multiply the numerators together, then multiply the denominators together.

WRITE

a $\frac{\cancel{2}^{1}}{9} \times \frac{13}{\cancel{4}^{2}} = \frac{13}{18}$

So, $-\frac{2}{9} \times \left(-\frac{13}{4}\right) = \frac{13}{18}$

b $\frac{12}{7} \div \frac{3}{14} = \frac{\cancel{12}^{4}}{\cancel{7}^{1}} \times \frac{\cancel{14}^{2}}{\cancel{3}^{1}}$

$= \frac{8}{1}$

$= 8$

So, $\frac{12}{7} \div \left(-\frac{3}{14}\right) = -8$

c **1** Determine whether the result is positive or negative. The signs are the same, so the result is positive.

2 Convert each mixed number to an improper fraction.

3 Keep the first fraction the same and multiply by the reciprocal of the second fraction.

4 Cancel out any common factors.

5 Multiply the numerators together, then multiply the denominators together.

c $2\frac{3}{4} \div 4\frac{1}{2} = \frac{11}{4} \div \frac{9}{2}$

$= \frac{11}{\cancel{4}^{2}} \times \frac{\cancel{2}^{1}}{9}$

$= \frac{11}{18}$

So, $-2\frac{3}{4} \div \left(-4\frac{1}{2}\right) = \frac{11}{18}$

Helpful hints

- ✔ You don't need a common denominator when multiplying and dividing fractions.
- ✔ Don't forget to change any whole number into a fraction with a denominator of 1.
- ✔ Make sure to simplify your fractions! You can cancel common factors vertically or diagonally.
- ✔ The phrase 'Keep, Change, Flip' can help you remember how to turn a 'dividing by a fraction' problem into a multiplication problem.
- ✔ When working with fractions, multiplying does not always make a number larger and dividing does not always make a number smaller.

Keep Change Flip

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$$

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Exercise 2C Multiplying and dividing fractions

△ 1–5(1st, 2nd columns), 6–8, 9(a–c), 10–13, 14(a, b), 16(a)

□ 1–5(3rd, 4th columns), 6–8, 9(d–f), 11, 13, 14, 16(a, c, e), 17(a–c), 18(a, b)

⬠ 2(e–h), 5, 6(e, f), 8, 9, 11, 15–17, 18(c, d), 19, 20

UNDERSTANDING AND FLUENCY

1 Write the equivalent multiplication equation for each division equation by using the reciprocal.

a $\frac{9}{20} \div \frac{3}{5}$ **b** $\frac{9}{20} \div \frac{7}{15}$ **c** $\frac{20}{9} \div \frac{15}{7}$ **d** $\frac{20}{9} \div \frac{7}{15}$

e $\frac{2}{9} \div \frac{7}{4}$ **f** $\frac{2}{9} \div \frac{5}{21}$ **g** $\frac{9}{2} \div \frac{21}{5}$ **h** $\frac{9}{2} \div \frac{5}{21}$

2C.1 **2** Determine the result of each product. Hint: Simplify before multiplying.

a $\frac{3}{8} \times \frac{2}{5}$ **b** $\frac{9}{10} \times \frac{4}{3}$ **c** $\frac{18}{25} \times \frac{35}{36}$ **d** $\frac{32}{49} \times \frac{21}{40}$

e $\frac{6}{375} \times \frac{25}{20}$ **f** $\frac{64}{12} \times \frac{27}{16}$ **g** $\frac{10}{77} \times \frac{242}{33}$ **h** $\frac{75}{70} \times \frac{42}{45}$

3 Determine the result of each product.

a $1\frac{7}{8} \times \frac{3}{5}$ **b** $3\frac{1}{5} \times 1\frac{1}{4}$ **c** $9\frac{1}{6} \times 1\frac{1}{11}$ **d** $\frac{5}{18} \times 2\frac{7}{10}$

e $5\frac{1}{3} \times 2\frac{2}{5}$ **f** $\frac{3}{4} \times 8$ **g** $7 \times \frac{3}{8}$ **h** $2\frac{1}{6} \times 15$

i $42 \times 3\frac{9}{21}$ **j** $12 \times 5\frac{1}{60}$ **k** $\frac{12}{23} \times 3\frac{1}{3}$ **l** $1\frac{12}{13} \times \frac{5}{3}$

2C.2 **4** Determine the result of each quotient.

a $\frac{5}{7} \div \frac{3}{2}$ **b** $\frac{3}{14} \div \frac{12}{35}$ **c** $\frac{9}{28} \div \frac{6}{7}$ **d** $\frac{15}{22} \div \frac{5}{33}$

e $\frac{21}{20} \div \frac{3}{20}$ **f** $\frac{17}{42} \div \frac{17}{6}$ **g** $\frac{27}{40} \div \frac{15}{40}$ **h** $\frac{12}{5} \div \frac{3}{50}$

5 Determine the result of each quotient.

a $2\frac{3}{4} \div \frac{5}{12}$ **b** $1\frac{5}{8} \div 2\frac{8}{9}$ **c** $2\frac{1}{3} \div 4\frac{2}{3}$ **d** $\frac{50}{63} \div 3\frac{4}{7}$

e $2\frac{1}{3} \div 1\frac{5}{9}$ **f** $\frac{12}{7} \div 6$ **g** $\frac{17}{4} \div 7$ **h** $12 \div \frac{6}{7}$

i $7 \div \frac{17}{4}$ **j** $20\frac{25}{27} \div 5$ **k** $\frac{2}{13} \div 1\frac{1}{3}$ **l** $2\frac{3}{14} \div \frac{5}{7}$

6 Calculate each amount in the units shown in brackets.

a $\frac{1}{2}$ of 14 hours (hours) **b** $\frac{3}{4}$ of \$48 (dollars) **c** $\frac{2}{5}$ of 35 kg (kg)

d $\frac{4}{9}$ of 72 cm (cm) **e** $\frac{2}{7}$ of 3 weeks (days) **f** $\frac{3}{5}$ of 2 cm (mm)

2C.3 **7** Determine the result of each of the following.

a $-\frac{6}{11} \times \left(-\frac{2}{3}\right)$ **b** $\frac{4}{5} \times \left(-\frac{5}{12}\right)$ **c** $-\frac{15}{8} \times \frac{16}{25}$

d $-\frac{12}{35} \div \left(-\frac{7}{9}\right)$ **e** $-\frac{5}{6} \div \frac{10}{7}$ **f** $\frac{4}{9} \times \left(-\frac{5}{12}\right)$

g $-4\frac{1}{2} \times 3\frac{1}{3}$ **h** $-2\frac{1}{6} \div \left(-3\frac{1}{4}\right)$ **i** $7\frac{2}{3} \div \left(-5\frac{1}{9}\right)$

8 Determine the result of each of the following. Hint: Recall your knowledge of BIDMAS.

a $\left(\frac{6}{11} + \frac{2}{3}\right) \div \frac{5}{6}$ **b** $3\frac{1}{4} - 2\frac{1}{2} \times \frac{7}{10}$ **c** $\frac{8}{9} \times \left(\frac{4}{8} - \frac{3}{4}\right) \div 2\frac{2}{3}$ **d** $2\frac{3}{5} \times 3\frac{1}{3} + \frac{2}{11} \times 1\frac{3}{8}$

9 Calculate:

a $-\frac{5}{9} \times \frac{3}{5} - \frac{2}{3}$ **b** $-\frac{5}{6} \div \frac{10}{9} \times -\frac{4}{7}$ **c** $\frac{1}{2} + \frac{4}{7} \div -\frac{2}{5}$

d $\frac{4}{9} \times \frac{3}{8} - \frac{5}{3} \times \frac{7}{10}$ **e** $-\frac{5}{2} \times \left(\frac{7}{15} - \frac{3}{5}\right)$ **f** $-\frac{2}{3} \times \frac{6}{7} \div 2\frac{2}{3}$

10 The seating capacity of an interstate train carriage is 88 people. If the carriage is three-quarters full, how many people are seated in the train carriage?

11 To bake 12 chocolate cupcakes, you need these ingredients.

Chocolate cupcakes

(makes 12)

100 g butter

$\frac{1}{2}$ cup sugar

2 large eggs

$1\frac{1}{2}$ cups flour

1 teaspoon baking powder

$\frac{1}{2}$ cup milk

$\frac{3}{4}$ cup cocoa powder

1 cup boiling water

Chocolate icing

$1\frac{1}{2}$ cups icing sugar

150 g butter

120 g chocolate

$\frac{1}{2}$ cup thickened cream

a Work out the quantities of each ingredient (excluding the icing) if you want to bake:

i 24 cupcakes **ii** 6 cupcakes **iii** 8 cupcakes.

b Finn bakes 12 cupcakes but before he can ice them, one-quarter are eaten. Adjust the ingredient list for the chocolate icing so it makes only enough for the remaining cupcakes.

12 Grapes are to be sold in $2\frac{1}{2}$ kg bags.

a How many bags must Tricia pack if she has a 20 kg crate of grapes?

b From another crate, Tricia packs 11 bags and has $1\frac{1}{2}$ kg of grapes left over. How many kilograms of grapes were in the second crate?

13 Leanne has 20 red buttons amongst other coloured buttons to make an art piece. Leanne wants the red buttons to be $\frac{5}{6}$ of the entire design.

a How many buttons in total are needed for Leanne's art piece?

b Leanne stumbles across a box containing more red buttons and decides to make the art piece $\frac{1}{4}$ larger than its original size. How many red buttons are needed for the increased art piece?

c What fraction is the number of buttons in the increased art piece out of the number of buttons in the original-size art piece?

14 Mixed numbers can be multiplied without first writing them as improper fraction by using the distributive property.

For example,

$$\begin{aligned} 8 \times 2\frac{1}{2} &= 8 \times \left(2 + \frac{1}{2}\right) \\ &= 8 \times 2 + 8 \times \frac{1}{2} \\ &= 16 + 4 \\ &= 20. \end{aligned}$$

Multiply the following by using the distributive property. Write your answers using mixed numbers where necessary.

a $14 \times 2\frac{4}{7}$ **b** $30 \times 4\frac{9}{10}$ **c** $\frac{20}{3} \times 6\frac{2}{5}$ **d** $8 \times 4\frac{3}{5}$

15 A mixed number can be divided without first writing it as an improper fraction.

For example,

$$\begin{aligned} 3\frac{1}{2} \div \frac{4}{5} &= \left(3 + \frac{1}{2}\right) \div \frac{4}{5} \\ &= \left(3 \div \frac{4}{5}\right) + \left(\frac{1}{2} \div \frac{4}{5}\right) \\ &= \frac{15}{4} + \frac{5}{8} \\ &= 4\frac{3}{8}. \end{aligned}$$

Divide the following by using the distributive property. Write your answers as mixed numbers where necessary.

a $25\frac{10}{19} \div 5$ **b** $6\frac{5}{9} \div \frac{1}{9}$ **c** $20\frac{8}{15} \div \frac{4}{3}$ **d** $5\frac{11}{13} \div \frac{10}{7}$

16 We can evaluate powers of fractions by using repeated multiplication of fractions.

For example,

$\left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1^2}{3^2} = \frac{1}{9}$.

Evaluate the following. Hint: Be careful with powers of negative fractions and write mixed numbers as improper fractions.

a $\left(\frac{2}{5}\right)^3$ **b** $\left(-\frac{2}{3}\right)^4$ **c** $\left(-\frac{2}{3}\right)^3$ **d** $\left(\frac{9}{7}\right)^2$ **e** $\left(3\frac{1}{3}\right)^3$

17 Since repeated multiplication of fractions repeatedly multiplies the numerator and repeatedly multiplies the denominator, we can reverse the process to find the square root of a fraction by finding the square root of the numerator and the square root of the denominator.

For example,

$\sqrt{\frac{4}{9}} = \sqrt{\frac{2^2}{3^2}} = \sqrt{\frac{2}{3} \times \frac{2}{3}} = \frac{2}{3}$.

Evaluate the following. Hint: Write mixed numbers as improper fractions and simplify first.

a $\sqrt{\frac{16}{25}}$ **b** $\sqrt{\frac{49}{9}}$ **c** $\sqrt{\frac{81}{100}}$ **d** $\sqrt{\frac{50}{18}}$ **e** $\sqrt{3\frac{13}{36}}$

18 Multiplying two mixed numbers can be performed by using the distributive law three times.

For example,

$$\begin{aligned}
8\tfrac{1}{2} \times 2\tfrac{1}{2} &= 8\tfrac{1}{2} \times \left(2 + \tfrac{1}{2}\right) \\
&= \left(8\tfrac{1}{2} \times 2\right) + \left(8\tfrac{1}{2} \times \tfrac{1}{2}\right) \\
&= \left(8 \times 2 + \tfrac{1}{2} \times 2\right) + \left(8 \times \tfrac{1}{2} + \tfrac{1}{2} \times \tfrac{1}{2}\right) \\
&= (16 + 1) + \left(4 + \tfrac{1}{4}\right) \\
&= 17 + 4\tfrac{1}{4} \\
&= 21\tfrac{1}{4}.
\end{aligned}$$

Multiply the following by using the distributive property. Write your answers using mixed numbers where necessary.

a $2\frac{3}{4} \times 6\frac{8}{9}$ **b** $14\frac{7}{15} \times 5\frac{3}{7}$ **c** $4\frac{1}{5} \times 9\frac{1}{3}$ **d** $3\frac{7}{12} \times 5\frac{3}{14}$

19 Explain why $3\frac{9}{10} \div 5\frac{2}{3} = \left(3 \div 5\frac{2}{3}\right) + \left(\frac{9}{10} \div 5\frac{2}{3}\right)$ but $3\frac{9}{10} \div 5\frac{2}{3} \neq \left(3\frac{9}{10} \div 5\right) + \left(3\frac{9}{10} \div \frac{2}{3}\right)$ by referring to the distributive property and the reciprocal.

20 a Which of the following operations with fractions is not made easier by using a common denominator?

A Addition **B** Subtraction **C** Multiplication

D Division **E** Raising to a power

b Explain why the answer(s) from part **a** are not made easier by using a common denominator.

c Explain why the other operations are made easier (or not made harder) by using a common denominator.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing fractions

Investigation
Fractions on the farm

Topic quiz
2C

ANS p487

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

2A **1** Fill the gaps to produce equivalent fractions.

a $\frac{6}{} = \frac{48}{80}$ **b** $\frac{}{20} = 2\frac{3}{4}$

c $\frac{63}{18} = \frac{}{8}$ **d** $\frac{6}{5} = \frac{-18}{}$

2A **2** Simplify the following fractions.

a $\frac{12}{15}$ **b** $\frac{60}{45}$

c $8\frac{16}{64}$ **d** $\frac{35}{-63}$

2A **3** Which fraction in each given pair is greater? Write your answer as an inequality.

a $\frac{2}{3} \square \frac{3}{4}$ **b** $\frac{7}{9} \square -\frac{8}{9}$

c $-\frac{17}{10} \square -\frac{9}{5}$ **d** $\frac{30}{36} \square \frac{36}{54}$

2A **4** Determine the LCD for each fraction pair.

a $\frac{3}{4}$ and $\frac{2}{3}$ **b** $\frac{5}{9}$ and $\frac{1}{6}$

c $\frac{3}{8}$ and $\frac{5}{12}$ **d** $\frac{7}{11}$ and $\frac{31}{33}$

2B **5** Evaluate the following sums.

a $\frac{2}{3} + \frac{2}{5}$ **b** $-\frac{7}{9} + 4\frac{2}{3}$

c $\frac{43}{10} + \left(-2\frac{3}{50}\right)$ **d** $-\frac{9}{7} + \left(-\frac{9}{4}\right)$

2B **6** Evaluate the following differences.

a $\frac{2}{3} - \frac{3}{4}$ **b** $-4\frac{1}{5} - 2\frac{8}{15}$

c $\frac{3}{16} - \left(-\frac{17}{24}\right)$ **d** $-3\frac{1}{3} - \left(-2\frac{1}{7}\right)$

2C **7** Evaluate the following products.

a $\frac{12}{5} \times \frac{25}{6}$ **b** $-\frac{4}{3} \times \frac{8}{9}$

c $4\frac{2}{5} \times \left(-\frac{1}{2}\right)$ **d** $-\frac{8}{6} \times \left(-3\frac{1}{3}\right)$

2C **8** Determine the reciprocal of the following.

a $\frac{3}{5}$ **b** $\frac{1}{7}$

c 9 **d** $5\frac{2}{3}$

2C **9** Evaluate the following quotients.

a $\frac{12}{35} \div \frac{4}{7}$ **b** $-2\frac{4}{5} \div \frac{2}{5}$

c $\frac{6}{7} \div \left(-1\frac{7}{8}\right)$ **d** $-\frac{3}{2} \div \left(-\frac{27}{16}\right)$

2B/2C **10** Evaluate the following using the order of operations.

a $\frac{3}{5} + \frac{6}{5} \times \frac{2}{5}$ **b** $-\frac{2}{3} - \frac{11}{15} + \frac{7}{10}$

c $\frac{4}{9} + \left(-\frac{6}{5}\right) \div \frac{8}{15}$ **d** $\frac{9}{10} \div \left(-\frac{5}{3}\right) \times \left(-\frac{2}{7}\right)$

2C **11** Find:

a $\frac{5}{8}$ of 104 GB **b** $\frac{3}{4}$ of 10 L

c $1\frac{1}{2}$ of 500 g **d** $\frac{17}{20}$ of \$5.00

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2D Decimals

Learning intentions

By the end of this topic you will be able to ...

✔ compare and order decimals

✔ round decimals to the appropriate decimal place

✔ convert between fractions and decimals.

Inter-year links

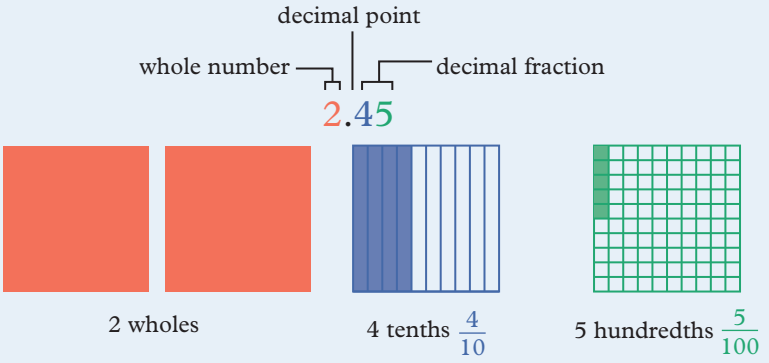
Support — Fractions and decimals

Year 7 — 4A Decimals

Decimals

Key content video

- A **decimal** is a number whose whole number part and fraction part are separated by a decimal point.



- The value of each digit depends on the place or position of the digit in the number.
  For example, the decimal 1234.5678 is shown in the place value table.

Thousands 1000	Hundreds 100	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
1	2	3	4	.	5	6	7	8

→ Each place value to the right is 10 times smaller than the place before.

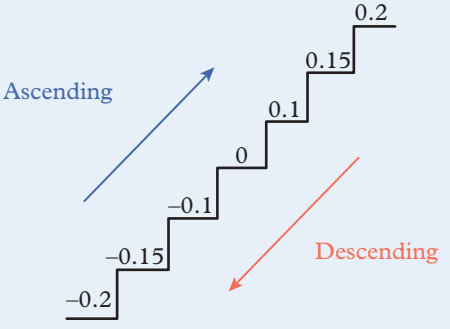
→ Each place value to the left is 10 times larger than the place before.

Ordering numbers

- To order decimals consider the digits from left to right.

  For example, 5.673 and 5.689 have the same whole number and tenth values. 8 hundredths is greater than 7 hundredths, so 5.673 < 5.689.

Ones	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
5	.	6	7	3
5	.	6	8	9



Rounding

- A decimal can be rounded to a given number of decimal places by considering the next digit to the right.

  For example, both numbers shown have been rounded to two decimal places.

$1.23\boxed{3}95 \approx 1.24$

9, 8, 7, 6, 5 — Round up

$1.23\boxed{3}25 \approx 1.23$

4, 3, 2, 1, 0 — Round down

Converting between fractions and decimals

- To convert from a decimal to a fraction place the decimal fraction part of the number as the numerator of the fraction. Write 10, 100, 1000 … with as many zeroes as there are digits after the decimal point in the denominator.

0.5	5 tenths	$\frac{5}{10}$
0.55	55 hundredths	$\frac{55}{100}$
0.555	555 thousandths	$\frac{555}{1000}$

- To convert from a fraction to a decimal, **trailing zeroes** are added following digits after the decimal point in the dividend until there is no remainder. Trailing zeroes do not change the value of the number.

Align decimal points

Dividend → $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ = 0.1 2 5

Divisor → 8

Trailing zeroes

Example 2D.1 Ordering decimals

Arrange the following in ascending order.

5.74, 5.7, 5.47, 5.04, 5.407, 4.57

THINK	WRITE
1 Compare the whole number parts. There are five numbers with a whole number of 5 and one with a whole number of 4.	4.57, 5.74, 5.7, 5.47, 5.04, 5.407
2 Compare the tenths values of the numbers with the same whole number part. 7 tenths is greater than 4 tenths and 4 tenths is greater than 0 tenths.	4.57, 5.04, 5.47, 5.74, 5.407, 5.7
3 Compare the hundredths values of the numbers with the same whole number and tenths parts. 7 hundredths is greater than 0 hundredths and 4 hundredths is greater than 0 hundredths. When a place value does not contain a digit, its value is zero.	4.57, 5.04, 5.47, 5.74, 5.407, 5.7
4 Write the decimals in ascending order, from smallest to largest.	4.57, 5.04, 5.407, 5.47, 5.7, 5.74

Example 2D.2 Rounding decimals

Round each number to two decimal places.

a 5.7323 **b** −12.099 76

THINK	WRITE
a **1** Draw a box around the second decimal place.	**a** $5.7\boxed{3}23$
2 Look at the digit to the right of the box. The digit to the right is a 2. Do not change the digit in the box.	$5.7\boxed{3}23$
3 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.	$5.7323 \approx 5.73$
b **1** Draw a box around the second decimal place.	**b** $-12.0\boxed{9}9\,76$
2 Look at the digit to the right of the box. The digit to the right is a 9. Add 1 to the digit in the box.	$-12.0\boxed{9}9\,76$
3 As the boxed digit changes from 9 to 10, write zero in the boxed digit's place and add one to the place to the left.	$-12.1\boxed{0}$
4 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same. Keep the zero in the second decimal place as the question is asking for two decimal places.	$-12.099\,76 \approx -12.10$

Example 2D.3 Converting a decimal to a fraction

Write the following decimals as fractions.

a 1.4 **b** −0.52 **c** 10.068

THINK	WRITE
1 Identify any whole numbers to the left of the decimal point.	**a** $1.4 = 1\frac{4}{10}$
2 Place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000 … with as many zeroes as there are digits after the decimal point in the decimal fraction part of the number.	$= 1\frac{\cancel{4}^{2}}{\cancel{10}_{5}}$
3 Simplify the fraction by finding common factors or the HCF of the numerator and denominator.	$= 1\frac{2}{5}$
	b $-0.52 = -\frac{52}{100}$
	$= -\frac{\cancel{52}^{13}}{\cancel{100}_{25}}$
	$= -\frac{13}{25}$
	c $10.068 = 10\frac{68}{1000}$
	$= 10\frac{\cancel{68}^{17}}{\cancel{1000}_{250}}$
	$= 10\frac{17}{250}$

Example 2D.4 Converting fractions to decimals

Write $\frac{3}{8}$ as a decimal.

THINK

1. As the denominator of the fraction is not a power of 10, write the fraction as a division problem with a decimal point after the numerator.
2. Work out the division using short division. Include trailing zeroes after the decimal point until you can divide exactly without a remainder. Align the decimal points.

WRITE

$\frac{3}{8} = 3 \div 8$

$$\begin{array}{r} 0.\ 3\ 7\ 5 \\ 8\overline{)3.^{3}0^{6}0^{4}0} \end{array}$$

$\frac{3}{8} = 0.375$

Helpful hints

- ✔ Decimals and fractions are actually two different ways of writing the same thing!
- ✔ Zeroes between non-zero digits in a decimal (called placeholder zeroes) must never be left out, or the value of the number will be changed.
- ✔ Zeroes at the end of a decimal (called trailing zeroes) do not change the value of the number.
- ✔ If you have trouble determining the place value of a digit, you can draw the place value table to help you.

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
1	2	.	3	4	5	6

- ✔ You may think that 0.000 19 is greater than 0.2 because there are more digits shown. Remember to compare from left to right. As 0 tenths is smaller than 2 tenths, 0.2 is greater than 0.000 19.
- ✔ Make sure to consider the real life context before rounding decimals.
  For example, if a chip packet costs \$1 and I have \$1.50, don't tell me that I can buy 1.5 chip packets – I can only buy 1!

ANS p487

Exercise 2D Decimals

△ 1–6, 7(a–c, g, i), 8(a–f), 9(a–d, g), 10(a–f, h), 11(a–e), 12(a–d, g, j), 13, 14, 17, 19

□ 2(d–f), 4–5(b, d), 7(d–i), 8(g–l), 9(e–h), 10(g–l), 11(f–h), 12(e–h, k, l), 15, 16, 18, 20, 21

⬠ 2(e, f), 4(d), 5(c, d), 7–8(3rd column), 9(g, h), 10(j–l), 11(g, h), 12(i–l), 15, 18, 20–22

UNDERSTANDING AND FLUENCY

1 Write the following numbers into the place value chart below.

a 0.0065 **b** 0.6500 **c** 6.005 **d** 60.5

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
		.				

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2 Determine the place value of 5 in each number. Refer to the place value chart to help you.

a 4.705 **b** 59.300 **c** 605.9
d 1740.50 **e** 1.3745 **f** 88.352 00

3 How many decimal places does each number in question **2** have?

2D.1 **4** Write each list of numbers in ascending order.

a 0.214, 0.142, 0.2, 0.41, 0.104, 0.14
b 8.702, 9.87, 8.072, 9.782, 8.2, 8.97
c 150.4, –10.45, 15.04, 10.54, 105.4, –15.40
d –799.8, –800.07, 800.7, –800.8, 799.08, –800.08

5 Write each list of numbers in descending order.

a 13.7, 13.07, 13.173, 13.31, 13.137, 13.713
b 0.658, 0.6058, 0.6508, 0.685, 0.6085, 0.65
c –50.5050, 5.5555, 5.550 00, 555.55, 5.005 00, –50.0505
d 100, –99.90, –100, 999.099, –99.009, 999.90

6 Explain why 9.46 is larger than 9.452 even though 9.452 has more digits.

2D.2 **7** Round each number to two decimal places.

a 25.679 **b** −673.8234 **c** 0.784 231
d −101.0157 **e** 8148.255 **f** −54.2618
g 330.406 **h** −4.854 793 **i** 15.9999

8 Round each decimal to the number of decimal places shown in the brackets.

a 57.234 (2) **b** 8.9137 (3) **c** 92.087 12 (4)
d −0.756 825 (3) **e** −135.152 79 (1) **f** 65.071 226 33 (5)
g 7.510 429 (3) **h** −208.009 862 (4) **i** 1.727 727 727 (5)
j 845.845 845 (1) **k** −0.123 456 78 (4) **l** −0.123 456 78 (7)

2D.3 **9** Write the following decimals as fractions.

a 1.03 **b** 2.9 **c** 3.007 **d** −0.0001
e 10.7 **f** −0.000 03 **g** 12.000 009 **h** −0.000 000 7

10 Write each number as a fraction in simplest form.

a 0.12 **b** −0.54 **c** 0.803
d −0.04 **e** 0.025 **f** −0.0006
g −0.74 **h** 0.252 **i** −0.0015
j −0.048 **k** 0.000 17 **l** −0.85

11 Write each fraction as a decimal.

a $\frac{7}{10}$ **b** $-\frac{29}{100}$ **c** $\frac{451}{1000}$ **d** $-\frac{2047}{10\,000}$
e $\frac{4}{100}$ **f** $-\frac{9}{1000}$ **g** $\frac{5}{10\,000}$ **h** $-\frac{63}{1000}$

2D.4 **12** Write each fraction as a decimal.

a $\frac{4}{5}$ **b** $\frac{5}{8}$ **c** $-\frac{1}{2}$ **d** $\frac{5}{4}$
e $\frac{1}{8}$ **f** $-\frac{2}{5}$ **g** $\frac{7}{20}$ **h** $-\frac{9}{25}$
i $\frac{1}{40}$ **j** $-\frac{3}{5}$ **k** $\frac{11}{50}$ **l** $-\frac{5}{16}$

PROBLEM SOLVING AND REASONING

13 Wendy thinks 8.2735 rounded to three decimal places is 8.273. Rosa thinks it is 8.274. Who is correct? Explain.

14 Jayden thinks that 24.7963 rounded to two decimal places is 24.80. Oscar thinks it is 24.71. Who is correct? Explain.

15 Peter thinks that 15.998 33 rounded to two decimal places is 16. Kota thinks it is 16.00. Who is correct? Explain.

16 Sabine needs to round 2.448 to one decimal place. First, Sabine rounds 2.448 to two decimal places to get 2.45. Sabine then rounds to one decimal place to get 2.5. Explain what Sabine has done wrong.

17 The length of a mosquito is measured to be 0.68 cm. Write this value correct to one decimal place.

18 Six students' heights were measured as 1.651 m, 1.63 m, 1.568 m, 1.615 m, 1.58 m and 1.657 m.

- **a** Which is the smallest measurement?
- **b** Which is the largest measurement?
- **c** Write the list of measurements in ascending order.
- **d** Round each measurement to two decimal places.
- **e** Write your list from part **d** in ascending order. What do you notice?
- **f** Round each measurement to one decimal place.
- **g** Write your list from part **f** in ascending order. What do you notice?

19 The width of a human hair is measured to be 0.018 35 mm. Write this value as an approximation to:

- **a** two decimal places
- **b** three decimal places
- **c** four decimal places.

20 The diameter of an optical fibre is 0.2845 mm.

- **a** Round this value to:
  - **i** one decimal place
  - **ii** two decimal places
  - **iii** three decimal places.
- **b** Write the exact diameter as a fraction in simplest form.

21 Round each amount to the nearest five cents.

- **a** \$42.37
- **b** \$207.62
- **c** \$7.24
- **d** \$15.03
- **e** \$60.71
- **f** \$99.99

CHALLENGE

22 Use each digit from 0 to 9 only once to write a number with nine decimal places such that when rounded to an even number of decimal places, the number rounds up, and when rounded to an odd number of decimal places, the number rounds down.

□.□□□□□□□□□

Check your Student obook pro for these digital resources and more:

- Interactive skillsheet: Ordering decimals
- Interactive skillsheet: Rounding decimals
- Interactive skillsheet: Fractions and decimals
- Investigation: Decimal history
- Topic quiz: 2D

2E Adding and subtracting decimals

Learning intentions

By the end of this topic you will be able to ...

✔ add and subtract positive and negative decimals.

Inter-year links

Support	Adding and subtracting decimals
Year 7	4C Adding and subtracting decimals

Adding and subtracting decimals

Key content video

- Decimals can be added or subtracted in the same way as whole numbers using the vertical addition and subtraction algorithms. Add or subtract digits in each column from right to left.

$$\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \\ \hline \end{array}$$

- Make sure that the decimal points are aligned and the digits are in the correct place value columns.
- When working with negative decimals, simplify the calculation by considering the operation and whether each decimal is positive or negative.

$$+\ (+) = +$$
$$-\ (+) = -$$
$$+\ (-) = -$$
$$-\ (-) = +$$

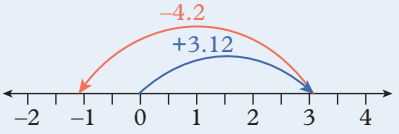
- If a positive number is being subtracted from a negative number, use the distributive law to rewrite the expression as a product of −1 and a sum of two positive numbers. Follow the order of operations to find the value of the original expression.
  - → Use the vertical addition algorithm if necessary to find the sum of the two positive numbers.

$$\begin{aligned} -3.12 - 4.2 &= -1 \times (3.12 + 4.2) \\ &= -1 \times 7.32 \\ &= -7.32 \end{aligned}$$

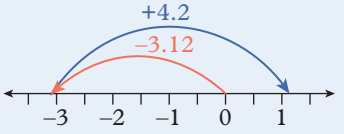
- The **magnitude** of a number is the value of a number ignoring the sign. For example, the magnitude of −3 is 3.
- If a positive number and a negative number are being added, ignore the negative sign and subtract the number smaller in magnitude from the number greater in magnitude. The sign of the result is the same as the sign of the number greatest in magnitude.
  - → Use the vertical subtraction algorithm if necessary to find the difference between the two numbers.

$$\begin{aligned} 3.12 - 4.2 &= -(4.2 - 3.12) \\ &= -1.08 \end{aligned} \qquad \begin{aligned} -3.12 + 4.2 &= +(4.2 - 3.12) \\ &= +1.08 \end{aligned}$$

  - → It may be helpful to use a number line to determine whether the result of an operation is positive or negative.



3.12 − 4.2 = −1.08



−3.12 + 4.2 = (+)1.08

Example 2E.1 Adding and subtracting decimals

Use vertical addition and vertical subtraction to calculate each result.

a 14.507 + 8.63

b 45.93 − 30.498

THINK

1. Estimate the result by rounding to the nearest whole number.
2. Write the numbers underneath each other. Ensure the decimal points and digits are aligned.
3. Replace blank spaces with additional zeroes so that each number contains the same number of decimal places.
4. Add or subtract the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.
5. Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.

WRITE

a 14.507 + 8.63 ≈ 15 + 9
= 24

```
  1 1
  14.507
+ 08.630
  23.137
```

14.507 + 8.63 = 23.137
24 ≈ 23.137

b 45.93 − 30.498 ≈ 46 − 30
=16

```
     8 12 10
  45.9~~3~~~~0~~
− 30.498
  15.432
```

45.93 − 30.498 = 15.432
16 ≈ 15.432

Example 2E.2 Adding decimals with negative signs

Calculate each result.

a −3.256 + 5.89

b −1.68 + (−3.45)

THINK

a
1. Estimate the result by rounding to the nearest whole number.
2. Determine whether to add or subtract the decimals. Since one is negative and one is positive, ignore the minus sign and subtract the smaller decimal from the larger decimal using the vertical subtraction algorithm.
3. Determine the sign of the result. Since the positive decimal is greater in magnitude than the negative decimal, the result is positive.
4. Check your answer against your estimate.

WRITE

a −3.256 + 5.89 ≈ −3 + 6
= 3

```
   8 10
  5.8~~9~~~~0~~
−3.256
 2.634
```

−3.256 + 5.89 = 2.634

3 ≈ 2.634

b **1** Simplify the signs. Estimate the result by rounding to the nearest whole number.

2 Determine whether to add or subtract the decimals. Since both are negative, ignore the minus signs and add the two decimals together using the vertical addition algorithm.

3 Determine the sign of the result. Since both decimals are negative, the result is negative.

4 Check your answer against your estimate.

b $-1.68 + (-3.45) = -1.68 - 3.45$
$\approx -2 - 3$
$= -5$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 1.68 \\ +3.45 \\ \hline 5.13 \end{array}$$

$-1.68 + (-3.45) = -(1.68 + 3.45)$
$= -5.13$

$-5 \approx -5.13$

Example 2E.3 Subtracting decimals with positive and negative signs

Calculate each result.

a $7.58 - 8.6$

b $4.206 - (-13.797)$

THINK

a **1** Estimate the result by rounding to the nearest whole number.

2 Determine whether to add or subtract the decimals. Since one is negative and one is positive, ignore the minus sign and subtract the smaller decimal from the larger decimal using the vertical subtraction algorithm.

3 Determine the sign of the result. Since the negative decimal is greater in magnitude than the positive decimal, the result is negative.

4 Check your answer against your estimate.

b **1** Simplify the signs. Estimate the result by rounding to the nearest whole number.

2 Add the two decimals together using the vertical addition algorithm.

3 Determine the sign of the result. Since both decimals in the simplified expression are positive, the result is positive.

4 Check your answer against your estimate.

WRITE

a $7.58 - 8.6 \approx 8 - 9$
$= -1$

$$\begin{array}{r} {\scriptstyle 5\ 10} \\ 8.\cancel{6}\cancel{0} \\ -\ 7.58 \\ \hline 1.02 \end{array}$$

$7.58 - 8.6 = -1.02$

$-1 \approx -1.02$

b $4.206 - (-13.797) = 4.206 + 13.797$
$\approx 4 + 14$
$= 18$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 13.797 \\ +\ 4.206 \\ \hline 18.003 \end{array}$$

$4.206 - (-13.797) = 4.206 + 13.797$
$= 18.003$

$18 \approx 18.003$

Helpful hints

- ✔ Rounding to the nearest whole number is a good way to check if your result makes sense.
- ✔ Make sure that the decimal points in each number are aligned when adding or subtracting decimals. For example,

```
  100 000.3
–      12.000 003
-----------------
```

- ✔ When adding trailing zeroes after the decimal, you are not changing the number. You are just writing a zero that was already implicitly there.

ANS p488

Exercise 2E Adding and subtracting decimals

△ 1, 2(a–c), 3, 4–7(1st column), 8–11, 12(a, b), 14, 15(a)

□ 1, 2(d–f), 3, 4–7(2nd column), 8(c, d), 11, 12(a, c), 13, 15, 16(a, b)

⬠ 2(d–f), 4–7(2nd column), 8(d), 12(c, d), 13, 16(c, d), 17, 18

UNDERSTANDING AND FLUENCY

2E.1 **1** Use vertical addition algorithm to calculate each result.

a $\begin{array}{r} 5.27 \\ +4.83 \\ \hline \end{array}$ **b** $\begin{array}{r} 18.9 \\ +\ 7.35 \\ \hline \end{array}$ **c** $\begin{array}{r} 0.085 \\ 1.32 \\ +0.647 \\ \hline \end{array}$ **d** $\begin{array}{r} 17.378 \\ 6.003 \\ +\ 5.999 \\ \hline \end{array}$

2 Use vertical addition algorithm to calculate each result.

a 23.82 + 17.13 + 124.06 + 82.95 **b** 5.613 + 9.27 + 11.3856 + 7.5

c 324.7 + 76.29 + 4.93 + 285.064 **d** 74.0106 + 2.93 + 28.738 + 0.9167

e 0.0034 + 0.21 + 0.024 + 0.0402 **f** 99.999 + 14.24 + 4.748 + 0.0284

3 Use vertical subtraction algorithm to calculate each result.

a $\begin{array}{r} 158.29 \\ -\ 75.46 \\ \hline \end{array}$ **b** $\begin{array}{r} 2051.384 \\ -\ 1265.73 \\ \hline \end{array}$ **c** $\begin{array}{r} 0.35 \\ -\ 0.2156 \\ \hline \end{array}$ **d** $\begin{array}{r} 70.308 \\ -\ 44.057 \\ \hline \end{array}$

4 Use vertical subtraction algorithm to calculate each result.

a 9.104 – 6.3728 **b** 265.7 – 88.504

c 5061.03 –1266.735 **d** 73 412 – 8743.56

e 9000 – 0.555 **f** 6.5677 – 4.5678

2E.2 **5** Calculate each result.

a 3.2 + (–5.8) **b** –3.2 + (–5.8)

c –3.2 + 5.8 **d** 51.294 + (–3.028)

e –4.1928 + 0.285 **f** –0.048 + (–495.19)

2E.3 **6** Calculate each result.

a 4.78 – 8.48 **b** –4.78 – 8.48

c –4.78 – (–8.48) **d** –3.541 – 58.242

e –274.28 – (–0.4827) **f** 9.402 – 10.611

7 Evaluate the following.

a 5.8 – 9.9 + 3.2 **b** 19.53 + 3.78 – 32.05

c 100 – 210.15 + 27.3 **d** 0.0032 + 0.1 – 0.352

8 Evaluate the following.

a $(+3.7) - (+5.2) + (-6.1)$

b $(+4.8) + (-9.7) + (+10.11)$

c $(-17.3) - (+12.4) - (-30.8)$

d $(-85.23) + (-0.0573) - (+2.902\,04)$

9 The circuit of a fun run has five different stages. The lengths of the stages are: 1.435 km, 2.790 km, 1.902 km, 2.285 km and 3.613 km. First estimate the distance around the full circuit by rounding each length to the nearest whole number and then calculate the exact distance.

10 Lee Redmond once held the record for the longest fingernails on both hands (see table), after not cutting them for 30 years.

	Fingernail lengths (cm)	
	Right hand	**Left hand**
Thumb	76.4	80.0
Index finger	72.3	76.4
Middle finger	74.1	76.7
Ring finger	73.6	76.2
Little finger	71.6	73.6

a What is the total length of the fingernails on:

i her right hand

ii her left hand?

b Which hand has the longer fingernails? Suggest a reason for this.

c What is the total length of the fingernails on both hands?

11 Consider the calculation $7.2 - 9.4 + 2.3$.

a Which of the following is not equivalent to the original calculation?

A $(7.2 - 9.4) + 2.3$ **B** $(7.2 + 2.3) - 9.4$ **C** $7.2 - (9.4 - 2.3)$

D $7.2 - (9.4 + 2.3)$ **E** $2.3 - (9.4 - 7.2)$

b Explain why the answer to part **a** is not equivalent to $7.2 - 9.4 + 2.3$ and write a similar expression that would be equivalent.

c For any calculation involving only addition and subtraction, if all subtractions are performed first, will the answer always be correct? Explain why or why not.

12 Calculate the following sums and differences by using addition and subtraction of the equivalent fractions.

For example, $12.3 + 4.21 = \frac{123}{10} + \frac{421}{100} = \frac{1230}{100} + \frac{421}{100} = \frac{1651}{100} = 16.51$.

a $-17.32 + 4.0032$

b $9.999 + (-50.91)$

c $-0.003 - (-0.0502)$

d $-90.06 - 73.0012$

13 Jimmy currently has \$32.50 saved up. He wants to buy a new game that costs \$69.99. Jimmy's parents agree to pay the difference if Jimmy pays them back. Jimmy earns \$5.50 a week in pocket money but can increase it by completing various chores. Each chore has a limit of how often Jimmy can complete it for pocket money. The chores and how much Jimmy earns from them are listed below.

- unload and hang the washing up to dry, \$1.10 twice a week
- fold the laundry and put it away, \$0.90 twice a week
- take out the rubbish bin, \$0.45 daily
- dust the shelves and bookcases, \$1.70 weekly
- help prepare dinner, \$2.10 daily

a Can Jimmy pay his parents back in two weeks? State how much Jimmy still owes or how much extra he earns.

b If Jimmy wants to save up for a \$79.99 game after he has paid his parents back, what is the minimum number of additional weeks it will take?

14 Look back over your calculations for adding decimals together. What do you notice about the number of decimal places in the calculation and the number of decimal places in the result? Determine the number of decimal places in the result of a sum where a number with x decimal places is added to a number with y decimal places, where x is greater than or equal to y.

PROBLEM SOLVING AND REASONING

15 a Use the fact that $-(3.12 + 4.2) = -3.12 + (-4.2)$ by the distributive law to help explain why we can add 3.12 and 4.2 to calculate $-3.12 - 4.2$.

b Use the fact that $-(4.2 - 3.12) = -4.2 - (-3.12)$ by the distributive law to explain why we can subtract 3.12 from 4.2 to calculate $3.12 - 4.2$.

16 Subtracting a positive number can be rewritten as adding a negative number. The associative and commutative laws allow us to rearrange the order of calculations. We can then add all the positive numbers together and all the negative numbers together. The result can be determined by finding the difference between the two sums. For example,

$$\begin{aligned} &-3.1 + (-2.4) + 4.9 - (-9.3) - 6.2 \\ &= (-3.1) + (-2.4) + (+4.9) + (+9.3) + (-6.2) \\ &= [(+4.9) + (+9.3)] + [(-3.1) + (-2.4) + (-6.2)] \\ &= 14.2 + (-11.7) \\ &= 2.5 \end{aligned}$$

Write the following as the difference of the total positive and negative sums by making use of the associative and commutative laws, then evaluate.

a $9.2 - 7.2 + (-4.1) - (-4.8) + 7 - 2.6$

b $-28.19 - (-49.18) + 99.24 + (-72.09) - (-13.25)$

c $0.028 - 0.18 - (-0.291) + (-2.59) - 12.01 + 9$

d $-70.18 - 81.8 + 29.09 - (-45.29) - 29.90 + 17.7$

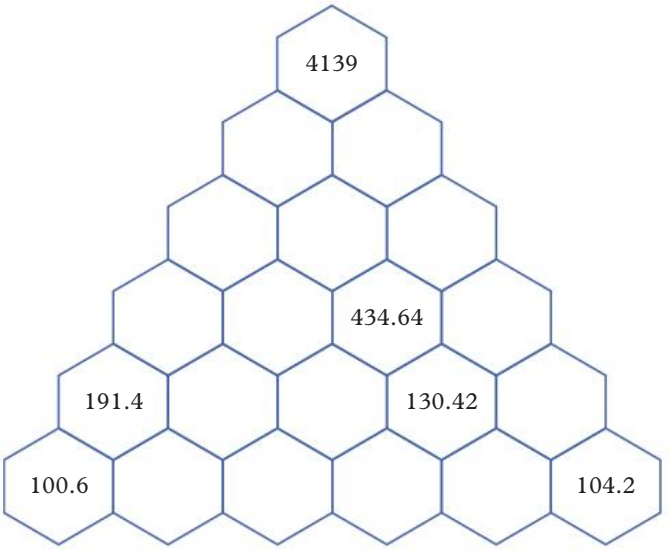
17 You begin by facing North. You turn 197.35° clockwise, then 87.194° anti-clockwise, then 3.813° clockwise, then 99.24° clockwise, then 100.01° anti-clockwise, and finally 32.021° anti-clockwise.

a In what direction (clockwise or anti-clockwise) and by how many degrees are you now facing relative to North?

b How many degrees did you turn in total, regardless of direction?

CHALLENGE

18 Fill in the blank boxes with the correct number. Every number is formed by adding the number left below and right below in the box. Your algebra skills may be useful for solving this problem.



Check your Student obook pro for these digital resources and more:

Interactive skillsheet Adding and subtracting decimals

Investigation How large is your hand?

Topic quiz 2E

2F Multiplying and dividing decimals

Learning intentions

By the end of this topic you will be able to ...

✔ multiply positive and negative decimals

✔ divide positive and negative decimals.

Inter-year links

Support — Multiplying decimals

Year 7 — 4D Multiplying decimals

Multiplying and dividing by powers of 10

Key content video

- Multiplying a number by 10 increases each digit's place value by 1 column. Move the decimal point one place value space to the right and insert a zero where necessary.

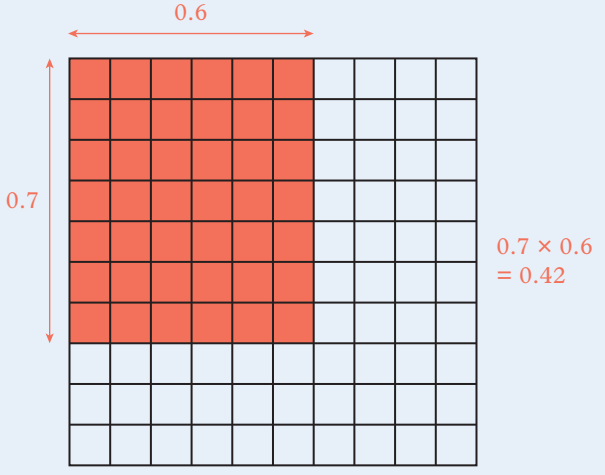
$$5.23 \times 10 = 52.3$$
$$5.23 \times 100 = 523.$$
$$5.23 \times 1000 = 5230.$$

- Dividing a number by 10 decreases each digit's place value by 1 column. Move the decimal point one place value space to the left and insert a zero where necessary.

$$5.23 \div 10 = 0.523$$
$$5.23 \div 100 = 0.0523$$
$$5.23 \div 1000 = 0.00523$$

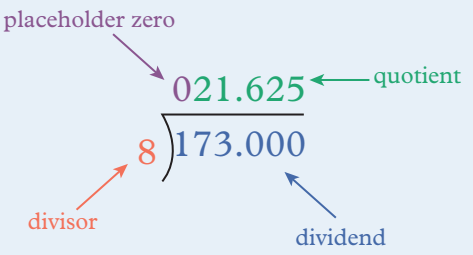
Multiplying decimals

- Decimals can be multiplied in the same way as whole numbers.
- The result has the same number of decimal places as the total number of decimal places in the original problem.
  1. Count the decimal places in the problem.
  2. Multiply the decimals as whole numbers.
  3. Write the answer with this same number of decimal places. Add zeroes where necessary.



Dividing a decimal by a whole number

- Align the decimal point in the answer with the one in the dividend.
- Trailing zeroes are added after the decimal point in the dividend until there is no remainder.



Dividing a decimal by a decimal

- When you divide by a decimal, you can create an equivalent problem by moving the decimal point in both the dividend and divisor the same number of spaces to the right of the decimal point, or multiplying by $\frac{10}{10}, \frac{100}{100}, \frac{1000}{1000}, \ldots$ (You are creating equivalent decimal fractions!)
  1. Convert the divisor to a whole number by moving the decimal point to the right.
  2. Move the decimal point the same number of spaces to the right on the dividend.
  3. Perform short division and add trailing zeroes when necessary. Make sure the decimal points are aligned.

Dividend Divisor

$$1.2 \div 0.4 = 12 \div 4 = 3$$

Example 2F.1 Multiplying and dividing by powers of 10

Calculate:

a 2.27×100

b $3.14 \div 1000$

THINK	WRITE
a When multiplying by 100, move the decimal point two place-value spaces to the right. Don't include the decimal point if there are no digits other than zero after it.	**a** 2.27 $2.27 \times 100 = 227$
b When dividing by 1000, move the decimal point three place-value spaces to the left. Add zeroes where necessary.	**b** 0003.14 $3.14 \div 1000 = 0.003\,14$

Example 2F.2 Multiplying decimals

Use long multiplication to calculate 20.45×-3.9.

THINK	WRITE
1 Estimate the result by rounding to the leading digit. Remember that a positive number multiplied by a negative number is a negative number.	$20.45 \times -3.9 \approx 20 \times -4$ $= -80$
2 Count the decimal places in the problem. There are three decimal places in total.	20.45×3.9
3 Multiply the decimals as whole numbers using long multiplication.	$\begin{array}{r} {\scriptstyle 1\,1} \\ {\scriptstyle 4\,4} \\ 2045 \\ \times \quad 39 \\ \hline 18405 \\ +61350 \\ \hline 79755 \end{array}$
4 Write the answer with this same number of decimal places. Add zeroes where necessary and write the answer with a negative sign.	$79\,755$ $20.45 \times -3.9 = -79.755$
5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.	$-79.755 \approx -80$

Example 2F.3 Dividing a decimal by a whole number

Use short division to calculate $-37.45 \div 4$.

THINK	WRITE
1 Estimate the result by rounding to the leading digit. Remember that a negative divided by a positive is a negative.	$-37.45 \div 4 \approx -40 \div 4$ $= -10$
2 Evaluate the division in the same way as for whole numbers, using short division.	$\begin{array}{r} -09.\,3\;6\;2\;5 \\ 4\overline{)-37.{}^{1}4^{2}5^{1}0^{2}0} \end{array}$
3 Align the decimal point and the digits that have the same place value.	
4 Add trailing zeroes after the last digit in the dividend to obtain an exact decimal answer without a remainder.	
5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position. Don't forget to include the appropriate sign.	$-37.45 \div 4 = -9.3625$ $-10 \approx -9.3625$

Example 2F.4 Dividing by decimals

Use short division to calculate $-9.46 \div -0.5$.

THINK

1. Convert the divisor to a whole number by moving the decimal point to the right.
2. Move the decimal point the same amount of spaces to the right on the dividend. Add a placeholding zero where necessary.
3. Perform short division and add trailing zeroes when necessary. Make sure the decimal points are aligned. Remember that a negative divided by a negative is a positive.

WRITE

$-9.46 \div -0.5 = -94.6 \div -5$

$$\begin{array}{r} 1\ 8.\ 9\ 2 \\ -5\overline{)-9^{4}4.^{4}6^{1}0} \end{array}$$

$-9.46 \div -0.5 = 18.92$

Helpful hints

- ✔ Moving the decimal point of the dividend and the divisor one place to the right is the same as multiplying by $\frac{10}{10}$.
- ✔ Remember that trailing zeroes should be added to the dividend until you can divide exactly, without a remainder.

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Exercise 2F Multiplying and dividing decimals

1–5, 6(a, b, d, e, g), 7, 8(a, c, d), 9(a–c), 12(a, b), 13, 14	1, 2, 3(e–h), 4(e–h), 6(c, d, f, h, i), 7, 8(b, e, f), 9, 10, 11(a, c), 12(c), 15(a, c), 16(a, d)	3–4(c, f, h), 6(d, f, g, i), 7, 8(c, e, f), 9(e, f), 11(b, d), 12(c), 15–18

UNDERSTANDING AND FLUENCY

2F.1 **1** Evaluate the products and quotients.

a 0.0052×10 **b** 10.4×100 **c** $92.1 \div 10$
d $340 \div 10\,000$ **e** $0.057 \div 100$ **f** $45.78 \div 1000$

2F.2 **2** Use multiplication to calculate each result. Remember to estimate your result first, and to compare it with your answer.

a 24.35×8 **b** 305.6×4 **c** $17\,352.7 \times 3$ **d** $0.327\,51 \times 9$

3 Use long multiplication to calculate each result.

a 8.27×5.3 **b** -12.86×2.4 **c** $-235.2 \times (-9.7)$ **d** 62.513×1.8
e $9.2 \times (-0.46)$ **f** $3.8927 \times (-0.52)$ **g** $-56.361 \times (-0.14)$ **h** 157.93×7.12

2F.3 **4** Use short division to calculate each result.

a $72.596 \div 4$ **b** $6.724 \div (-5)$ **c** $-58.2743 \div 7$ **d** $65.0955 \div 3$
e $-4.73 \div (-8)$ **f** $0.627 \div (-4)$ **g** $0.1536 \div 6$ **h** $-138.0714 \div 5$

5 Dividing a decimal by another decimal is easier if you write an equivalent division problem where the divisor is a whole number.

a In $3.568 \div 0.4$, the divisor is 0.4. Should you multiply 0.4 by 10, 100 or 1000 to produce 4?

b Complete: $3.568 \div 0.4 =$ _____ $\div 4$

c Why is it important to multiply each decimal in the division problem by the same number?

d Use your answer to part **b** to work out the division.

6 Write each calculation as an equivalent division problem where the divisor is a whole number.

a $7.24 \div 0.2$ **b** $4.902 \div (-0.6)$ **c** $19.764 \div 0.03$

d $-1.728 \div (-0.08)$ **e** $1.4368 \div 0.004$ **f** $-7.491 \div 0.06$

g $4.872 \div (-1.5)$ **h** $2.808 \div 0.12$ **i** $-0.1638 \div (-0.009)$

2F.4 **7** Calculate the result of each division problem in question **6**.

8 Calculate:

a $(15.73 + 2.9) \times 1.5 - 26.07$ **b** $72.816 \div 8 + 4.9 \times 0.5$

c $6.9 + 0.528 \div 0.04 - 9.3$ **d** $12.08 - \dfrac{3.4 + 0.5}{4} + 8.217$

e $-3.2 \times (-9.51) + (-91.5) \div 1.5$ **f** $\dfrac{0.31 \times 0.031}{5} - \dfrac{5}{-0.25 \times (-0.4)}$

9 Calculate the cost of each item. Write your answers correct to the nearest cent.

a 4 kg of frozen berries at \$7.35 per kg

b 3 kg of watermelon at \$1.95 per kg

c 0.6 kg of snow peas at \$5.62 per kg

d 2.5 kg of potatoes at \$3.95 per kg

e 0.55 kg of bok choy at \$8.99 per kg

f 1.74 kg of green beans at \$6.89 per kg

10 a Estimate the total cost of the items listed in question **9** by first rounding the cost per kg of each item to the nearest dollar. Round your answer to the nearest dollar.

b Find the total cost of these items. Round your answer to the nearest five cents.

c How much change would you receive if you paid with a \$100 note?

11 Calculate the following products and quotients by using multiplication and division of fractions.

For example, $12.3 \times 0.02 = \dfrac{123}{10} \times \dfrac{2}{100} = \dfrac{246}{1000} = 0.246$ and $0.02 \div 0.4 = \dfrac{2}{100} \div \dfrac{4}{10} = \dfrac{2}{100} \times \dfrac{10}{4} = \dfrac{1}{20} = \dfrac{5}{100} = 0.05$

a -0.17×0.0032 **b** $9.999 \times (-5.91)$ **c** $-0.003 \div (-7.5)$ **d** $-9.6 \div 0.0012$

12 a Calculate the cost of one item in each case.

i nine bread rolls cost \$5.40 **ii** four cartons of milk cost \$11.56 **iii** five DVDs cost \$94.75

b Calculate the cost of 1 kg in each case.

i 3 kg of tomatoes costs \$13.95 **ii** 7 kg of sausages costs \$47.95 **iii** 6 kg of dog food costs \$81.66

c Calculate the cost of 100 g in each case.

i 200 g of salami costs \$25.90 **ii** 500 g of cheese costs \$7.95 **iii** 300 g of cashews costs \$5.85

PROBLEM SOLVING AND REASONING

13 A box of 10 coloured pencils costs \$7.50. The pencils can also be bought individually for 72 cents each.

a Find the cost per pencil in each case.

b Which option provides better value (or is the 'best buy')? Explain.

14 William bought a 400 g pack of mushrooms for \$3.69. Kate bought 300 g of loose mushrooms for \$2.98. In each of these calculations, round your answer to the nearest cent.

a Calculate the amount paid by each of them for 100 g of mushrooms.

b Who made the best buy? Explain.

c Why does a store offer both options?

15 We can evaluate powers of decimals by using repeated multiplication of decimals. However, evaluating the power of a decimal can be made easier by first writing the decimal as a fraction. For example,

$(-1.2)^2 = \left(-\frac{12}{10}\right)^2 = \left(-\frac{12}{10}\right) \times \left(-\frac{12}{10}\right) = \frac{144}{100} = 1.44$ and

$(-0.2)^3 = \left(-\frac{2}{10}\right)^3 = \left(-\frac{2}{10}\right) \times \left(-\frac{2}{10}\right) \times \left(-\frac{2}{10}\right) = -\frac{8}{1000} = -0.008$

Evaluate the following powers.

a $(-0.4)^3$ **b** $(-0.001)^3$ **c** $(-0.09)^2$ **d** $(2.5)^5$

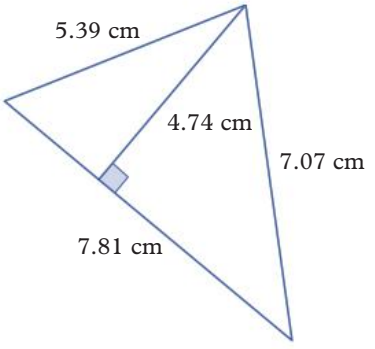
16 Write the following division problems as multiplication by the reciprocal, then evaluate the quotient.

a $6.3 \div 0.5$ **b** $9.2 \div 0.2$ **c** $0.49 \div 0.25$

d $7.4 \div 1.25$ **e** $0.041 \div 2.5$ **f** $10.21 \div 6.25$

CHALLENGE

17 Calculate the area and perimeter of the triangle shown.



18 Use the digits 0 to 9 no more than once each to make the result of this calculation as close to 0 as possible.

□.□□□ − □.□□ × □.□□

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Multiplying and dividing decimals

Investigation
Let's party!

Topic quiz
2F

2G Terminating, non-terminating and recurring decimals

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify decimals as terminating, non-terminating and recurring
- ✔ convert fractions to recurring decimals
- ✔ write non-terminating and recurring decimals using appropriate notation.

Inter-year links

Support	Fractions, decimals and percentages
Year 7	4G Fractions, decimals and percentages

Key content video

Terminating decimals

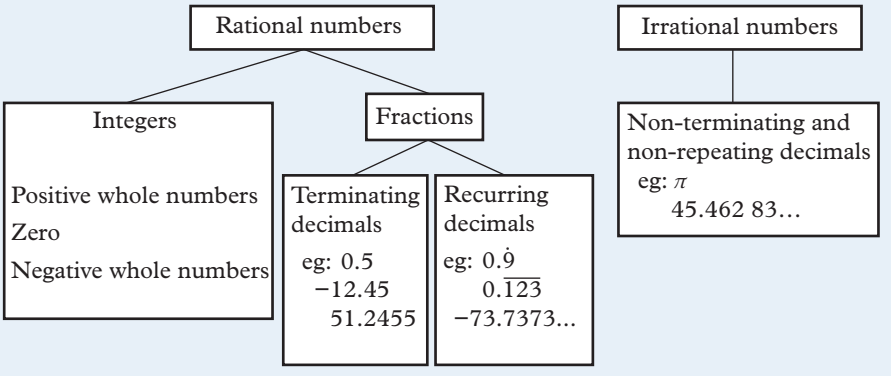
- A **terminating decimal** 'terminates' or stops after a number of decimal places.
  For example: 24.3, 0.49, 5.768 234

Non-terminating decimals

- A **non-terminating decimal** has an endless (infinite) number of decimal places.
  For example: 17.326 35…, 0.567 743…, 0.788 888…
  → The ellipses (three full stops) written after the last decimal place indicate that an endless number of digits follow.

Recurring decimals

- A **recurring decimal** is a special type of non-terminating decimal. The digits after the decimal point form a recurring or repeating pattern.
  For example: 8.999 999…, 42.671 671 671…, 0.582 323 232…
- A recurring decimal can be written in an abbreviated form using dots or a vinculum (bar) over repeating digits.
  → If a single digit is repeated, place a dot or a bar over the digit that repeats forever.
  For example, $8.999\,999... = 8.\dot{9}$
  → If more than one digit is repeated, place a dot over the first and last digits in the repeating pattern, or place a bar over the digits in the repeating pattern.
  For example, $0.582\,323\,23... = 0.58\dot{2}\dot{3}$ and $42.671\,671... = 42.\overline{671}$
- **Rational numbers** are numbers that can be written exactly as fractions, both the numerator and denominator need to be integers. This includes whole numbers, fractions, terminating decimals and recurring decimals.
- **Irrational numbers** are numbers that cannot be written exactly as fractions. These are non-terminating decimals that are not recurring decimals; for example, π (or '**pi**').



Example 2G.1 Writing recurring decimals in abbreviated form

Write each recurring decimal in abbreviated form using dots or a bar.

a 7.682 222 222…

b −24.503 150 315 031…

THINK

a Look for a repeating pattern of digits. Since there is only one digit that repeats, show a dot (or a bar) above the 2.

b Look for a repeating pattern of digits. Show a bar above 5031 (or show a dot above the first and last digit in the pattern).

WRITE

a $7.682\,222\,222\ldots$
$= 7.68\dot{2}$

b $-24.503\,150\,315\,031\ldots$
$= -24.\overline{5031}$ (or $-24.\dot{5}03\dot{1}$)

Example 2G.2 Writing recurring decimals to a number of decimal places

Write each recurring decimal to show twelve decimal places.

a $0.\dot{4}$

b $-159.8\overline{372}$

THINK

a 1 Identify the repeating digit. The digit 4 has a dot above it so it is a repeating digit.

2 Write the decimal to twelve decimal places and indicate that more digits follow.

b 1 Identify the repeating digits. The digits 372 have a bar above them, so all three digits are repeating.

2 Write the decimal to twelve decimal places and indicate that more digits follow.

WRITE

a $0.\dot{4}$
$= 0.444\,444\,444\,444\ldots$

b $-159.8\overline{372}$
$= -159.837\,237\,237\,237\ldots$

Example 2G.3 Expressing fractions as recurring decimals

Convert $\frac{5}{3}$ to a decimal by dividing the numerator by the denominator.

THINK

1 Write the fraction as a division problem with a decimal point after the numerator.

2 Perform short division and add trailing zeroes until a pattern occurs. Make sure the decimal points are aligned.

3 Write the decimal using the abbreviated form with a dot (or a bar) above 6.

WRITE

$\frac{5}{3} = 5 \div 3$

$$\begin{array}{r} 1.\ 6\ \ 6\ \ 6\ldots \\ 3\overline{)5.{}^{2}0{}^{2}0{}^{2}0{}^{2}0} \end{array}$$

$\frac{5}{3} = 1.\dot{6}$

Example 2G.4 Expressing mixed numbers as recurring decimals

Convert $-1\frac{2}{7}$ to a decimal by first converting to an improper fraction and then dividing the numerator by the denominator.

THINK

1. Convert the mixed number to an improper fraction.
2. Write the fraction as a division problem with a decimal point after the numerator.
3. Perform short division and add trailing zeroes until a pattern occurs. Make sure the decimal points are aligned.
4. Write the decimal using the abbreviated form with a bar above the six recurring digits.

WRITE

$-1\frac{2}{7} = -\frac{9}{7}$

$= -9 \div 7$

$$\begin{array}{r} -1.\,2\,8\,5\,7\,1\,4\,2\,8\,5\,7\ldots \\ 8\overline{)-9.{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0} \end{array}$$

$-1\frac{2}{7} = -1.\overline{285714}$

Helpful hints

- ✔ There are only two options for fractions when they are converted to decimals. Either they terminate or they are recurring. There are no fractions with an integer numerator and denominator that do not terminate and have no recurring pattern. Such decimals can never be written as fractions.
- ✔ Some people think that decimals are more 'exact' than fractions. However, mathematicians prefer fractions as you cannot tell if a decimal has been rounded, truncated or only measured to a number of decimal places.
- ✔ Zero is a rational number as $0 = \frac{0}{1}$. In fact, all integers are rational numbers.
- ✔ There is no limit on the number of decimals that can make a repeating pattern in a recurring decimal. For example, $\frac{1}{7} = 0.\overline{142857}$ has a pattern of six repeating digits.

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Exercise 2G Terminating, non-terminating and recurring decimals

△ 1–3, 4–5(a–h), 6, 7(a, c, f), 8–10, 11(a–f), 12, 14, 15, 18

□ 3, 4–5(i–l), 6–8, 9(d–f), 10, 11(g–k), 12, 13, 16–19

⬠ 3, 4(j–l), 5(i–o), 6–7(f, g, h), 9(e, f), 10, 11(g, h, k, l), 12, 16–19

1 State whether each decimal is terminating (T) or non-terminating (N).

a 4.57235…	**b** 56.803	**c** 21.57777…	**d** −8.56139562
e −0.45173069…	**f** 17.5656565…	**g** −0.123412341234…	**h** −0.78787878
i −30.082174…	**j** 291.2912912	**k** 0.999…	**l** −5.043127

2 Which of the numbers in question **1** are recurring decimals?

3 Explain the differences between terminating, non-terminating and recurring decimals.

2G.1 **4** Write each recurring decimal in abbreviated form using dots or a bar.

a 0.999999… **b** −4.3636363… **c** 87.255555… **d** 3.476476476…
e −0.738222222… **f** 16.19666666… **g** 483.88888… **h** −6.729191919…
i −0.183673673673… **j** 1.356283562835… **k** 28.7070707… **l** −9.230714444444…

2G.2 **5** Write each recurring decimal to show twelve decimal places.

a $0.\dot{1}$ **b** $5.3\dot{7}$ **c** $-14.\overline{82}$
d $-21.5\dot{6}\dot{9}$ **e** $8.\overline{104}$ **f** $0.\overline{6513}$
g $-232.68\dot{4}$ **h** $44.761\,\overline{359}$ **i** $-94.57\dot{0}\dot{2}$
j $-1.\overline{73924}$ **k** $11.812\dot{1}$ **l** $-6.9\overline{04}$
m $14.2\dot{4}1\dot{3}$ **n** $0.5\overline{620}$ **o** $-139.\dot{2}74\dot{5}$

2G.3 **6** Convert each fraction to a decimal by dividing the numerator by the denominator.

a $\frac{7}{10}$ **b** $-\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{5}{4}$
e $\frac{2}{11}$ **f** $-\frac{5}{8}$ **g** $\frac{8}{3}$ **h** $-\frac{3}{4}$

2G.4 **7** Convert each fraction to a decimal.

a $-1\frac{1}{8}$ **b** $1\frac{2}{5}$ **c** $2\frac{5}{6}$ **d** $-1\frac{1}{7}$
e $3\frac{3}{5}$ **f** $-4\frac{1}{9}$ **g** $1\frac{8}{11}$ **h** $-2\frac{5}{12}$

8 Consider the fractions and your answers to question **6**.

a Which fractions produce:
  i terminating decimals
  ii non-terminating decimals?
b Which fractions produce recurring decimals?
c Do any of the fractions produce a non-terminating decimal that isn't a recurring decimal?

9 Write each number as a fraction. Hint: Whole numbers can be written with a denominator of 1.

a 0.5 **b** 4 **c** 6.2
d 0.37 **e** 18 **f** 2.509

10 Numbers that can be written exactly as fractions are rational numbers.

a Use your answers to question **9** to explain why whole numbers and terminating decimals are rational numbers.
b Are recurring decimals rational numbers? Use your answers to questions **6** and **8** to help you decide. Explain your decision.

11 Classify each number as rational or irrational.

a 4.6 **b** 0.12121212… **c** 0.5871 **d** 2.35789621…
e 35 **f** 734.94651824… **g** $0.872\,\overline{503}$ **h** 96.73425
i $\frac{3}{17}$ **j** 500 000.000… **k** 0.117 843 26… **l** $10\frac{3}{4}$

12 An irrational number that is commonly used is π (read as pi). This number has intrigued mathematicians for thousands of years and is related to measurements on a circle.
Since it is a decimal with an endless number of decimal places and no obvious repeating pattern, an estimate or approximation of its value is used in calculations.

a Find the symbol π on your calculator. Press the appropriate keys to obtain a decimal approximation for π. Write the number shown on your calculator screen.
b Write an approximation for π to the following number of decimal places.
  i two **ii** four **iii** five **iv** six

13 It is possible to determine if a fraction will have a recurring decimal or not without performing the division.

a Complete the table below.

Unit fraction	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{11}$	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{12}$
Decimal	$0.\dot{1}$	0.1	0.2								
Denominator's prime factorisation	3^2	2×5	5								
Is 2 and/or 5 a factor of the denominator?	No	Yes									
Any prime factors other than 2 and/or 5?	Yes	No									
Is the decimal terminating?	No										

b According to the table, when will a unit fraction have a terminating decimal?

c Use your rule from part **b** to decide if these unit fractions will have a terminating decimal or not. Convert each simplified fraction to a decimal to check your answers.

i $\frac{1}{25}$ **ii** $\frac{1}{24}$ **iii** $\frac{1}{18}$ **iv** $\frac{1}{40}$ **v** $\frac{1}{125}$

d Your rule from part **b** extends to all simplified fractions. Use the rule to decide if these fractions will have a terminating decimal or not. Remember to simplify your fractions first! Convert each simplified fraction to a decimal to check your answers.

i $\frac{24}{7}$ **ii** $\frac{7}{32}$ **iii** $\frac{21}{30}$ **iv** $\frac{4}{26}$ **v** $\frac{42}{15}$

14 Another irrational number is $\sqrt{2}$.

a Write the number shown on your calculator screen for $\sqrt{2}$.

b Write an approximation for $\sqrt{2}$ to the following number of decimal places.

i two **ii** four **iii** six **iv** seven

c Write the number shown on your calculator screen for $\sqrt{3}$.

d Is $\sqrt{3}$ an irrational number?

e Write an approximation for $\sqrt{3}$ to the following number of decimal places.

i two **ii** four **iii** five **iv** six

15 a Convert each fraction to a decimal.

i $\frac{1}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{3}{9}$ **iv** $\frac{4}{9}$

b What type of decimal do you get?

c Use the pattern from part **a** to write these as decimals.

i $\frac{5}{9}$ **ii** $\frac{6}{9}$ **iii** $\frac{7}{9}$ **iv** $\frac{8}{9}$

16 Other fractions, both with recurring and non-recurring decimal expansions, have patterns in their decimal expansion.

a Write down the first ten multiples of 9.

b Convert each fraction to a decimal.

i $\frac{1}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{3}{11}$ **iv** $\frac{4}{11}$

c Describe the pattern in the decimal expansion of the elevenths. Check your pattern holds by writing and then checking the decimal expansion of $\frac{9}{11}$.

PROBLEM SOLVING AND REASONING

17 **a** Write $\frac{1}{3}$ and $\frac{2}{3}$ as decimals, showing at least the first five decimal places.

b Add the two decimals you found in part **a**.

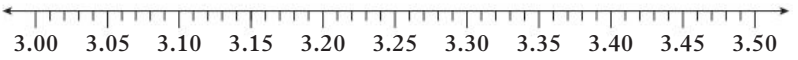
c Calculate $\frac{1}{3} + \frac{2}{3}$ and compare your answer to part **b**.

18 Before the use of calculators and computers, many mathematicians used a fraction as an estimate for the value of π. Some examples are shown in the table below.

Place, person and era of origin	Fraction estimate	Decimal approximation (to six decimal places)
Ancient Egypt (1700 BCE)	$\frac{256}{81}$	
Ancient Greece, Archimedes (287–212 BCE)	$\frac{223}{71}$	
	$\frac{22}{7}$	
China, Zu Chongzhi (429–500 CE)	$\frac{355}{113}$	
India, Bhāskara II (1114–1185 CE)	$\frac{3927}{1250}$	
Italy, Leonardo of Pisa (1170–1250 CE)	$\frac{12\,960}{4126}$	

a Use a calculator to complete the third column of the table.

b Locate π on the following number line.



c Which value is closest to the approximation of π you wrote for question **12b** part **iv**?

d Archimedes proved that π was between $\frac{223}{71}$ and $\frac{22}{7}$. Can you use a computer to write the *exact* value of π as a fraction or a decimal? Explain.

CHALLENGE

19 Recurring decimals can be written as fractions. For example, $0.\dot{6}$ is equivalent to $\frac{2}{3}$. You can verify this using short division or a calculator. To write a recurring decimal as a fraction, there is a series of steps you can follow.

a In the calculation below, match each step (**1–6**) with its corresponding line of working (**I–VI**) and then write the lines of working in the correct order.

Steps:	**Lines of working:**
1 Let x represent the recurring decimal.	**I** $0.\dot{6} = \frac{2}{3}$
2 Multiply the recurring decimal by 10 to find an equation for $10x$.	**II** $x = \frac{6}{9}$
3 Form a new subtraction equation from the two equations for x and $10x$.	**III** $10x = 6.666\,666\ldots$
4 Write the simplified equation that results from the subtraction.	**IV** $x = 0.666\,666\ldots$
5 Divide both sides of the equation by 9 to obtain x on the left-hand side of the equation and a fraction on the right-hand side.	**V** $10x - x$ $= 6.666\,666\ldots - 0.666\,666\ldots$
6 Write the recurring decimal as a fraction, in simplest form.	**VI** $9x = 6$

b Why did you multiply x by 10 in the calculation above? How did this help you obtain a whole number on the right-hand side of the equation in step **4**?

c Use the method in part **a** to write each of these recurring decimals as a faction.

i $0.\dot{2}$ **ii** $0.\dot{7}$ **iii** $2.\dot{3}$ **iv** $5.\dot{6}$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Terminating, non-terminating and recurring decimals

Investigation
Let the cycle begin!

Topic quiz
2G

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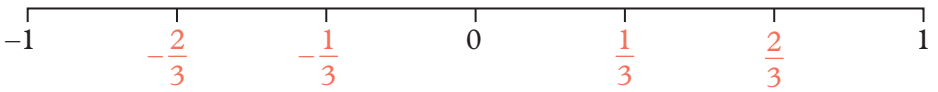
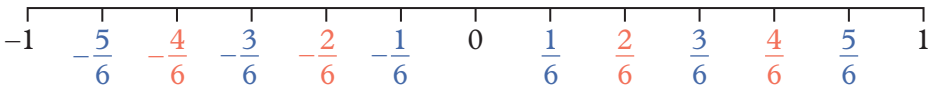
Chapter summary

Equivalent fractions





$\frac{2}{3} = \frac{10}{15}$ (× 5) $\frac{12}{10} = \frac{6}{5}$ (÷ 2)

The HCF of 36 and 72 is 36.

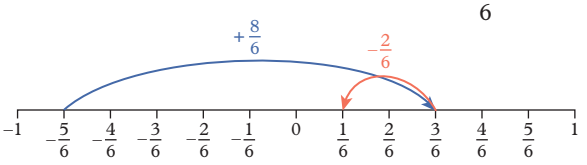
$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}} = \frac{1}{2}$

Adding and subtracting fractions

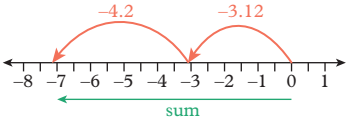
1. Convert into equivalent fractions with a common denominator.
2. Add or subtract the numerators and keep the same denominator.

$-\frac{5}{6} + \frac{4}{3} - \frac{1}{3} = -\frac{5}{6} + \frac{8}{6} - \frac{2}{6}$

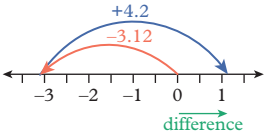
$= \frac{1}{6}$



Adding and subtracting decimals



$$-\left(\begin{array}{r} 3.12 \\ +\ 4.20 \\ \hline 7.32 \end{array}\right)$$



$$+\left(\begin{array}{r} 4.\cancel{2}\,\cancel{0} \\ -\ 3.1\ 2 \\ \hline 1.0\ 8 \end{array}\right)$$

Fractions to decimals

Align decimal points

Dividend → $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ = 0.1 2 5 ← Divisor

Trailing zeroes

Multiplying fractions

$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times \cancel{3}^{1}}{\cancel{3}^{1} \times 4}$

$= \frac{\cancel{2}^{1}}{\cancel{4}^{2}}$

$= \frac{1}{2}$

Diagonally

Vertically

Multiplying decimals

2.75 ← two decimal places
×0.03 ← two decimal places

$$\begin{array}{r} {}^{2\ 1} \\ 275 \\ \times\ \ 3 \\ \hline 825 \end{array}$$

2.75 × 0.03 = 0.0825 ← four decimal places

Dividing by a whole number

$$\begin{array}{r} 19.16 \\ 9\overline{)172.44} \end{array}$$

← quotient (the answer)

divisor (the number we are dividing by)

dividend (the number being divided)

Reciprocals

$\frac{3}{4}$ ↔ $\frac{4}{3}$

Dividing fractions

$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$

Lowest common denominators

The LCM of 2 and 5 is 10.

$\frac{1}{2} = \frac{5}{10}$ (× 5) $\frac{3}{5} = \frac{6}{10}$ (× 2)

Multiplying and dividing by 10s

5.23 × 10 = 52.3

5.23 × 100 = 523.0

5.23 ÷ 10 = 0.523

5.23 ÷ 100 = 0.0523

Dividing by a decimal

Dividend Divisor

1.2 ÷ 0.4 = 12 ÷ 4 = 3

Improper fractions and mixed numbers

$3\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$

$\frac{13}{4} = 13 \div 4 = 3$ remainder 1

$= 3\frac{1}{4}$

Rational and irrational numbers

Rational numbers

- **Integers**
  - Positive whole numbers
  - Zero
  - Negative whole numbers
- **Fractions**
  - **Terminating decimals** — eg: 0.5, −12.45, 51.2455
  - **Recurring decimals** — eg: $0.\dot{9}$, $0.\overline{123}$, −73.7373...

Irrational numbers

- Non-terminating and non-repeating decimals — eg: π, 45.462 83...

ANS p490

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- decimal
- denominator
- equivalent fraction
- fraction
- highest common factor
- improper fraction
- integer
- irrational numbers
- lowest common denominator
- magnitude
- mixed number
- non-terminal decimal
- number line
- numerator
- pi
- proper fraction
- rational numbers
- reciprocal
- recurring decimal
- sign
- terminating decimal
- trailing zeroes

1 Fill in 'lowest' or 'highest' in the following frequently used terms.

a _____ common multiple

b _____ common denominator

c _____ common factor

2 Briefly explain why when simplifying a fraction, the whole number part of a mixed number stays the same.

3 The phrase 'Keep, Change, Flip' can help you to turn a 'dividing by a fraction' problem into a multiplication problem, explain what has been kept, changed and flipped.

4 Identify whether the following statements are true or false. If a statement is false, briefly explain why.

a The reciprocals of whole numbers are unit fractions.

b Multiplying by a fraction makes a number smaller.

c All decimals can be written as fractions.

d All fractions can be written as decimals.

5 Briefly explain the difference between the following terms using examples.

a digit, decimal place

b placeholder zero, trailing zero

c rational number, irrational number

Multiple choice

2A **1** What missing value in $\frac{7}{9} = \frac{49}{}$ will produce an equivalent fraction?

A 51 **B** 63 **C** 81 **D** 441 **E** 7

2A **2** Which of the following is incorrect?

A $\frac{5}{3} > \frac{4}{3}$ **B** $\frac{3}{5} < \frac{3}{4}$ **C** $\frac{2}{3} > \frac{32}{47}$ **D** $\frac{7}{9} < \frac{9}{11}$ **E** $\frac{9}{8} > \frac{13}{12}$

2B **3** What is the lowest common denominator for $\frac{3}{8} - \frac{1}{6}$?

A 48 **B** 16 **C** 24 **D** 2 **E** 18

2B **4** What is $\frac{1}{5} - \frac{4}{5}$?

A -3 **B** $-\frac{3}{5}$ **C** $\frac{3}{5}$ **D** $\frac{5}{5}$ **E** $-\frac{2}{5}$

2B/2C **5** Which problem has a positive answer?

A $-\frac{2}{3} + \left(-\frac{3}{4}\right)$ **B** $-\frac{2}{5} \times \frac{1}{2}$ **C** $-\frac{5}{2} - \left(-\frac{6}{7}\right)$ **D** $-\frac{4}{5} - \left(+\frac{2}{3}\right)$ **E** $-\frac{1}{7} \div \left(-\frac{8}{9}\right)$

2C **6** What is $\frac{9}{10} \div 1\frac{1}{5}$?

A $\frac{9}{2}$ **B** $\frac{3}{4}$ **C** $\frac{27}{25}$ **D** $\frac{21}{10}$ **E** $\frac{4}{3}$

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2D **7** What is −3.125 84 rounded to three decimal places?

A −3.125 **B** −3.13 **C** −3.128 **D** −3.126 **E** −3.584

2E **8** What is −5 + 2.38?

A −7.38 **B** −7.62 **C** −2.62 **D** −2.38 **E** −3.38

2F **9** How many decimal places are there in the answer to 26.018 × 15.4?

A 1 **B** 3 **C** 2 **D** 8 **E** 4

2G **10** Which number is an example of a recurring decimal?

A 32.85 **B** −7.757 575 **C** −6.666... **D** 3.654 864 321... **E** −0.121 232 123 432…

2G **11** Which of the following is *not* equivalent to $\frac{5}{33}$?

A $0.\overline{15}$ **B** $0.\dot{1}\dot{5}$ **C** $0.1\dot{5}$ **D** $\frac{15}{99}$ **E** $0.1\overline{51}$

Short answer

2A **1** Write each fraction in its simplest form.

a $\frac{35}{60}$ **b** $-\frac{30}{16}$ **c** $-3\frac{18}{32}$ **d** $5\frac{48}{72}$

2B **2** Calculate:

a $\frac{3}{7}+\frac{2}{5}$ **b** $\frac{11}{15}-\frac{3}{10}$ **c** $5\frac{2}{3}+\frac{4}{11}$ **d** $6\frac{2}{5}-3\frac{4}{7}$

2C **3** Calculate:

a $\frac{12}{25}\times\frac{5}{14}$ **b** $\frac{7}{18}\div\frac{7}{9}$ **c** $6\frac{1}{4}\times3\frac{2}{5}$ **d** $3\frac{1}{2}\div1\frac{5}{9}$

2C **4** Calculate:

a $-\frac{10}{9}\times\left(-\frac{27}{35}\right)$ **b** $\frac{15}{11}\div\left(-\frac{90}{44}\right)$ **c** $-3\frac{5}{6}\times8\frac{2}{3}$ **d** $-2\frac{4}{5}\div\left(-5\frac{4}{9}\right)$

2B/2C **5** Calculate:

a $-\frac{3}{4}-\left(-\frac{1}{2}\right)$ **b** $\frac{2}{5}-\left(+\frac{1}{3}\right)$ **c** $\frac{3}{7}-1\frac{2}{7}$

d $-2\frac{1}{6}+6\frac{2}{3}$ **e** $\frac{1}{2}\div\frac{7}{4}-\frac{2}{5}$ **f** $-\frac{1}{8}+\frac{2}{3}\times\left(-\frac{9}{8}\right)$

2C **6** Find:

a $\frac{2}{5}$ of 60 km/h **b** $\frac{1}{2}$ of $\frac{2}{3}$ of a cup

c $3\frac{1}{3}$ of 21 metres **d** $\frac{18}{60}$ of 10 minutes

2D **7** Write in ascending order.

a 0.123, 0.120, 0.213, 0.0012, 0.231, 0.2 **b** 8.916, 9.6, 8.16, 8.9, 9.16, 9.916

c 0.05, 5, −0.5, −5, 0.50, −0.05 **d** −0.421, −2.41, −10.42, −14.2, −12.04, −2.14

2D **8** Write in descending order.

a 0.017, 0.7, 0.71, 0.0701, 0.717, 0.107 **b** 2.127, 2.1, 2.721, 2.27, 2.72, 2.7

c −0.89, 89, 90, −9.8, −0.899, 80.9 **d** −0.9, −1, −0.01, −0.99, −0.099, −0.1

2D **9** Round each decimal to the number of decimal places shown in brackets.

a 13.216 51 (2) **b** −0.001 215 (4)

c 54.545 545 (3) **d** −181.182 154 136 (1)

2E **10** Calculate:

a 23.56 + 45.98 + 74.13 **b** 53.15 − 29.67

c 123.5 + 46.12 + 9.0142 + 87.15 **d** 1284.068 − 294.4687

2E **11** Calculate:

a −5.42 + 1.59 **b** −8.30 − (−12.482)

c 49.207 + (−51.52) **d** 99.29 − (−15.490)

2F **12** Calculate:

a 6.3128×4 **b** $65.589 \div (-6)$

c -3.46×5.7 **d** $-468.268 \times (-0.89)$

2G **13** **a** Decide whether each decimal is terminating or non-terminating.

i 0.123123123 **ii** 6.562562...

iii −5.5 **iv** 3.645841235...

b Which of the numbers in part **a** are recurring decimals?

2G **14** Write each decimal in short form using dots or a horizontal bar.

a 0.333333... **b** 5.385385...

c −45.685757575... **d** −0.092451451...

2G **15** Write each number to show twelve decimal places.

a $-0.2\dot{3}$ **b** $3.\overline{27}$ **c** $46.35\dot{8}\dot{7}$ **d** $-17.36\overline{485}$

2G **16** Write each fraction as a recurring decimal using dots or a horizontal bar.

a $\frac{1}{30}$ **b** $-\frac{4}{7}$ **c** $-9\frac{5}{9}$ **d** $\frac{17}{11}$

Analysis

1 The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Min. temp. (°C)	0.7	−2.2	−0.9	−3.4	−5.1	−1.8	0.2
Max. temp. (°C)	6.5	4.1	7.8	2.5	−0.6	3.9	5.3

a Which day had:

i the highest temperature

ii the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for Monday.

c Calculate the difference between the minimum and maximum temperatures for each other day.

d Which day had the biggest range of temperatures?

e Calculate the average of the following, correct to one decimal place.

i the minimum temperatures

ii the maximum temperatures

f What is the difference between the average minimum and average maximum temperatures?

2 Rob, Carina, Belinda and Mario want to buy Alicia a bunch of white roses for her birthday.

a Rob has \$10.35, Carina has \$8.25, Belinda has \$9.90 and Mario has \$10.80. How much money do they have altogether?

b Their local florist, Fantastic Flowers, has white roses for \$2.99 each. How many white roses can they buy?

c How much change will they have left over?

Fantastic Flowers has a special offer that if you buy a dozen white roses you get $\frac{1}{4}$ of the total price off.

d How much would a dozen white roses cost without the discount?

e How much would a dozen white roses cost with the discount?

f Mario bargains with the storekeeper and manages to buy 18 roses for the price of 12 roses. Is this more discount compared with the special offer?

g How much is a single rose with this offer?

i Write you answer as a recurring decimal.

ii Round you answer to two decimal places.

3 A school has four house teams for sports events: Alpha, Beta, Gamma, and Delta, and every student is assigned into one of the four houses. The school tries to have as close to one-quarter of students in each house every year. In 2022, the school had 1596 students.

a In 2022, Alpha house had exactly $\frac{1}{4}$ of the students in the school. How many students were in Alpha house?

b In 2022, Beta house had $\frac{9}{38}$ of the students in the school.

i How many students were in Beta house?

ii If $\frac{11}{21}$ of Beta house are girls, how many girls are in Beta house?

c In 2022, Gamma house had 402 students.

i What fraction of students were in Gamma house?

ii How many students did Gamma house have above or below one-quarter of the students at the school?

d In 2022:

i how many students were in Delta house

ii what fraction of the school were in Delta house?

e In 2023, Delta house had 416 students. If Delta house had $\frac{13}{50}$ of the students at the school that year, how many students went to the school in 2023?

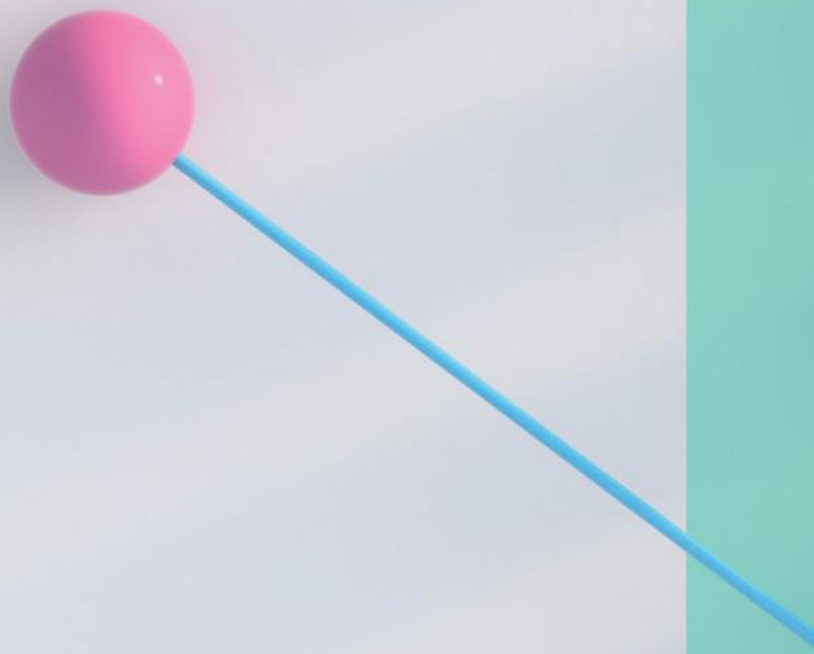
Chapter checklist

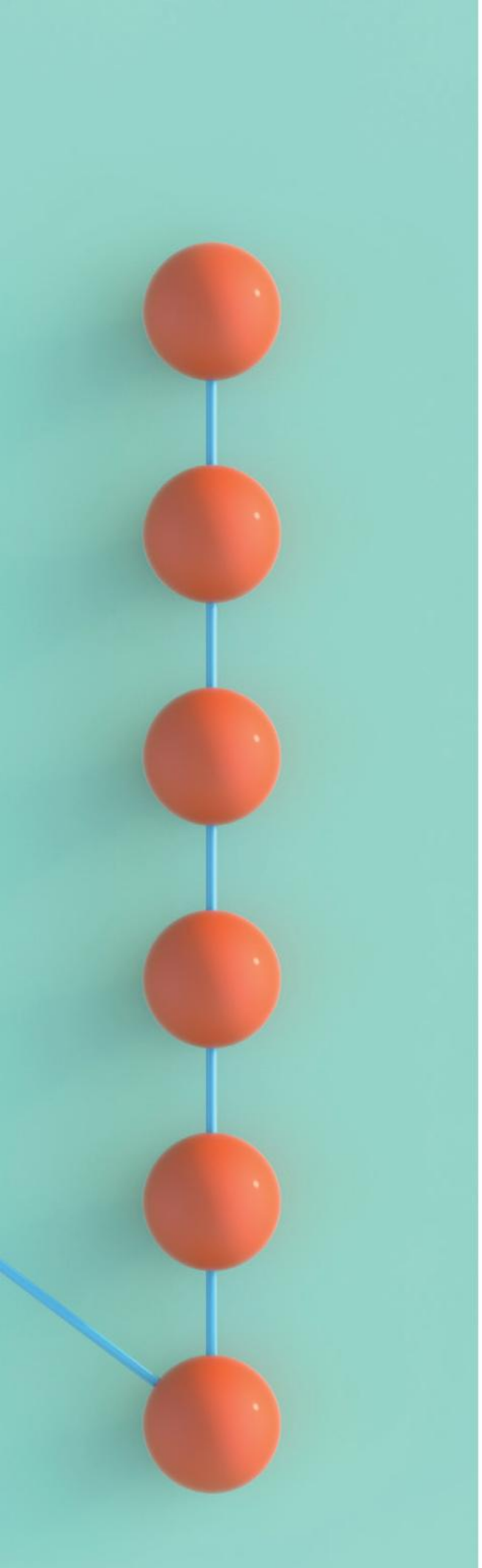
Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify and find equivalent fractions ☐ Compare fractions ☐ Simplify fractions	☐ Go back to Topic 2A Fractions
☐ Add and subtract fractions with different denominators ☐ Add and subtract fractions and mixed numbers ☐ Add and subtract negative fractions and mixed numbers	☐ Go back to Topic 2B Adding and subtracting fractions
☐ Multiply and divide fractions and mixed numbers ☐ Multiply and divide negative fractions and mixed numbers ☐ Determine fractions of an amount	☐ Go back to Topic 2C Multiplying and dividing fractions
☐ Compare and order decimals ☐ Round decimals to the appropriate decimal place ☐ Convert between fractions and decimals	☐ Go back to Topic 2D Decimals
☐ Add and subtract positive and negative decimals	☐ Go back to Topic 2E Adding and subtracting decimals
☐ Multiply positive and negative decimals ☐ Divide positive and negative decimals	☐ Go back to Topic 2F Multiplying and dividing decimals
☐ Identify decimals as terminating, non-terminating and recurring ☐ Convert fractions to recurring decimals ☐ Write non-terminating and recurring decimals using appropriate notation	☐ Go back to Topic 2G Terminating, non-terminating and recurring decimals

3

Percentages, ratios and rates





Index

3A Percentages, fractions and decimals
3B Calculating percentages
3C Mark-ups and discounts
3D Profit and loss
3E Ratios
3F Equivalent ratios
3G Working with ratios
3H Rates
3I Distance–time graphs

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Visual representation of fractions
- ✔ Equivalent fractions
- ✔ Multiplying and dividing fractions
- ✔ Multiplying and dividing decimals
- ✔ Rounding decimals

Curriculum links

- Represents and operates with fractions, decimals and percentages to solve problems (MA4-FRC-C-01)
  - Identify and make use of the relationship between fractions, decimals and percentages to carry out simple conversions
  - Order and compare the value of fractions, decimals and percentages
  - Represent one quantity as a fraction, decimal or percentage of another, with and without the use of digital tools
- Solves problems involving ratios and rates, and analyses distance–time graphs (MA4-RAT-C-01)
  - Solve problems involving ratios
  - Recognise and simplify rates
  - Solve problems involving rates
  - Interpret and construct distance–time graphs from authentic data

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Materials

✔ Calculator

3A Percentages, fractions and decimals

Learning intentions

By the end of this topic you will be able to ...

✔ convert between percentages, fractions and decimals

✔ compare and order percentages, fractions and decimals.

Inter-year links

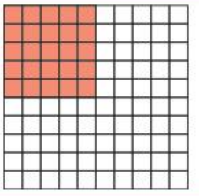
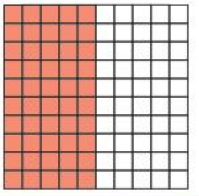
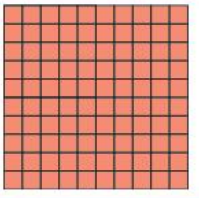
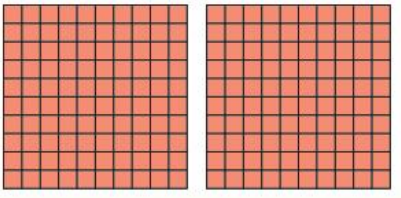
Support Fractions, decimals and percentages

Year 7 4G Fractions, decimals and percentages

Percentages, fractions and decimals

Key content video

- A **percentage** is a proportion of a whole that has been divided into one hundred parts.
- The term 'per cent' means 'per hundred' or 'out of 100'. The symbol for percentage is %.
- Percentages, fractions and decimals are different ways of representing the same value.

Percentage	Fraction	Decimal	
25%	$\frac{25}{100} = \frac{1}{4}$	0.25	
50%	$\frac{50}{100} = \frac{1}{2}$	0.5	
100%	$\frac{100}{100} = 1$	1	
200%	$\frac{200}{100} = 2$	2	

	Fraction	Decimal
Percentage to...	Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100}$ $= \frac{3}{20}$	Divide the percentage by 100. $23.4\% = 23.4 \div 100$ $= 0.234$

	Percentage	Decimal
Fraction to…	Multiply the fraction by 100% and simplify. $\frac{1}{3} \times 100\% = \frac{100}{3}\%$ $= 33\frac{1}{3}\%$	Divide the numerator by the denominator. Add trailing zeroes as required. Align decimal points $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ = 0.1 2 5 Dividend → 1 Divisor → 8 Trailing zeroes

	Percentage	Fraction
Decimal to…	Multiply the decimal by 100 and include the percentage symbol %. 0.65 = 65%	Place the decimal as the numerator of the fraction with a denominator of 10, 100, 1000, … with as many zeroes as there are digits after the decimal point. $0.65 = \frac{65}{100}$

Example 3A.1 Converting from a percentage

Write each percentage as a fraction in its simplest form and as a decimal.

a 220% **b** 25.3%

THINK

1. To convert a percentage to a fraction, divide by 100 and simplify the fraction.
2. To write a percentage as a decimal, divide the percentage by 100.

WRITE

a Fraction: $220\% = \frac{\cancel{220}^{11}}{\cancel{100}^{5}}$

$= \frac{11}{5}$

Decimal: 220% = 2.2

b Fraction: $25.3\% = \frac{25.3}{100}$

$= \frac{253}{1000}$

Decimal: 25.3% = 0.253

Example 3A.2 Converting from a decimal

Write each decimal as a percentage and as a fraction in its simplest form.

a 3.2 **b** 0.0125

THINK

1. To convert a decimal to a percentage, multiply by 100%. 'Move' the decimal point two places to the right and insert a place-holding zero in the empty space if necessary.
2. To convert a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Make the fraction denominator 10, 100, 1000, … with as many zeroes as there are digits after the decimal point and simplify.

WRITE

a Percentage: 3.2 = 320%

Fraction: $3.2 = 3\frac{\cancel{2}^{1}}{\cancel{10}^{5}}$

$= 3\frac{1}{5}$

b Percentage: 0.0125 = 1.25%

Fraction: $0.0125 = \frac{\cancel{125}^{1}}{\cancel{10\,000}^{80}}$

$= \frac{1}{80}$

Example 3A.3 Converting from a fraction

Write each fraction as a percentage and as a decimal.

a $\frac{27}{300}$ **b** $4\frac{2}{3}$

THINK

a **1** Multiply the fraction by 100% and divide the numerator by the denominator to simplify.

2 Convert the fraction to an equivalent fraction with a denominator of 100 and then convert to a decimal by 'moving' the decimal point of the numerator two places to the left.

b **1** For mixed numbers, multiply the whole number and the fraction by 100%. Divide the numerator by the denominator to simplify, then add both values together.

2 Express the mixed number as a whole number plus a fraction. If you cannot easily determine an equivalent fraction with a denominator of 100, use short division to convert the fraction to a decimal. Add both values together again.

WRITE

a Percentage: $\frac{27}{300} = \frac{27}{300} \times 100\%$

$= \frac{2700}{300}\%$

$= 9\%$

Decimal:

$$\frac{27}{300} \overset{\div 3}{=} \frac{9}{100}$$

$= 0.09$

b Percentage: $4\frac{2}{3} = 4\frac{2}{3} \times 100\%$

$= 400\% + \frac{200}{3}\%$

$= 400\% + 66\frac{2}{3}\%$

$= 466\frac{2}{3}\%$

Decimal: $4\frac{2}{3} = 4 + \frac{2}{3}$

$= 4 + 3\overline{)2.^{2}0^{2}0^{2}0}$ (0. 6 6 6...)

$= 4.\dot{6}$

Example 3A.4 Comparing fractions, decimals and percentages

Complete each statement by writing < (is less than) or > (is greater than) in the space provided.

a $\frac{3}{4}$ _____ 0.7 **b** 35% _____ $\frac{2}{5}$

THINK

a **1** Convert one term into the same format as the other term. In this instance we will convert the fraction into a decimal.

2 Compare the two terms in the same format to determine which one is greater.

3 Complete the statement by using either > or <.

b **1** Convert one term into the same format as the other term. In this instance we will convert the fraction into a percentage.

2 Compare the two terms in the same format to determine which one is greater.

3 Complete the statement by using either > or <.

WRITE

a $\frac{3}{4} = \frac{75}{100}$

$= 0.75$

$0.75 > 0.7$

Therefore, $\frac{3}{4} > 0.7$

b $\frac{2}{5} = \frac{2}{5} \times 100\%$

$= \frac{200}{5}\%$

$= 40\%$

$35\% < 40\%$

Therefore, $35\% < \frac{2}{5}$

Helpful hints

- ✔ Remember that one whole equals 100%. If you have a percentage that is greater than 100% you have more than one whole.
- ✔ It's easier to convert from a fraction to a decimal when the fraction has a denominator of 100, before you simplify your answer.
- ✔ Don't forget to simplify your fractions.
- ✔ It's usually quicker to compare and order percentages or decimals than it is to compare and order fractions.
- ✔ As all percentages can be converted to fractions, percentages are also rational numbers.

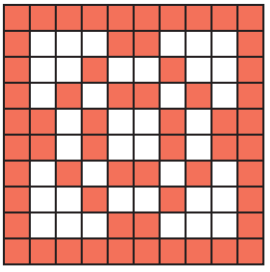
ANS p491

Exercise 3A Percentages, fractions and decimals

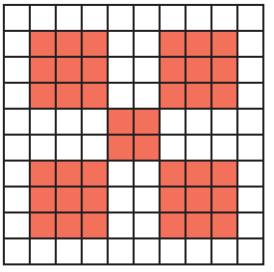
△ 1–13, 14(a–d), 15–17, 20(a–c) □ 3(d–f), 4–7, 8(b, d), 9–14(e–h), 15, 18, 19(a, b), 20–22 ⬠ 5, 7, 9–14(e–h), 19–24

1 What percentage of each large square has been shaded?

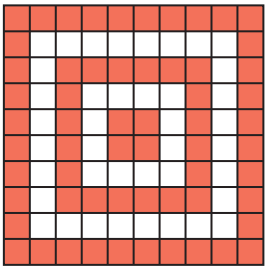
a



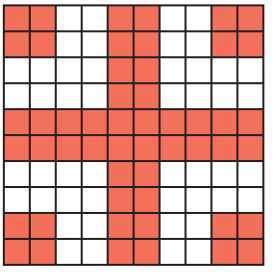
b



c



d



2 Write the percentage of each square in question **1** that is *not* shaded.

3 **a** Huynh has downloaded 76% of a song onto her iPhone. What percentage remains to be downloaded?

b If the selling price of an item is reduced by 23%, what percentage of the original price is to be paid?

c The GWS Giants lose 17% of their games. What percentage do they win (assuming no draws)?

d Forty-two per cent of homes were affected by floods. What percentage of homes were not affected?

e Morgan received 93% on her Japanese test. What percentage of marks did she lose?

f It is estimated that 54% of Australians do no exercise on a regular basis. What percentage of Australians exercise on a regular basis?

3A.1 **4** Write each percentage as a fraction in its simplest form and as a decimal.

a 17% **b** 21% **c** 35% **d** 8%

e 215% **f** 98% **g** 174% **h** 28%

UNDERSTANDING AND FLUENCY

5 Write each percentage as a fraction in its simplest form and as a decimal.

a 7% **b** 3.1% **c** 162% **d** 72.1%

e 347% **f** 81.09% **g** 10.57% **h** 6.78%

3A.2 **6** Write each decimal as a percentage and as a fraction in its simplest form.

a 0.71 **b** 0.28 **c** 0.05 **d** 0.08

e 0.9 **f** 0.31 **g** 0.54 **h** 0.95

7 Write each decimal as a percentage and as a fraction in its simplest form.

a 0.369 **b** 0.248 **c** 10.06 **d** 0.81

e 7.057 **f** 1.859 **g** 12.1 **h** 0.2446

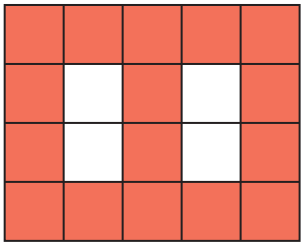
8 For each figure:

i write how many sections are shaded as a fraction of the total number of sections

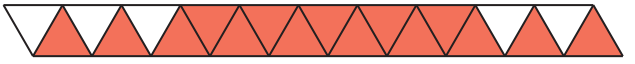
ii write this fraction as an equivalent fraction with a denominator of 100

iii write the percentage of sections that are shaded.

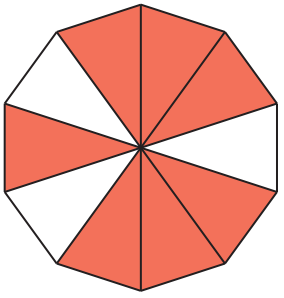
a



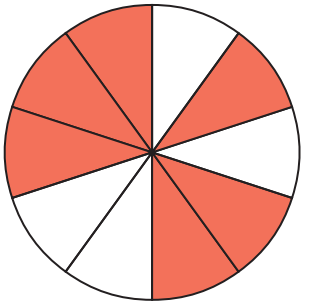
b



c



d



3A.3 **9** Write each fraction as a decimal and percentage by first finding an equivalent fraction that has a denominator of 100.

a $\frac{3}{5}$ **b** $\frac{7}{10}$ **c** $\frac{17}{200}$ **d** $\frac{23}{25}$

e $\frac{13}{10}$ **f** $\frac{3}{2}$ **g** $\frac{11}{400}$ **h** $\frac{7}{2}$

10 Write each mixed number as a percentage. Hint: Whole numbers are multiples of 100%.

a $2\frac{1}{4}$ **b** $6\frac{1}{200}$ **c** $7\frac{3}{10}$ **d** $3\frac{3}{4}$

e $4\frac{7}{25}$ **f** $5\frac{4}{5}$ **g** $1\frac{12}{500}$ **h** $13\frac{13}{50}$

11 Write each fraction as a percentage and a decimal.

a $\frac{5}{8}$ **b** $\frac{27}{40}$ **c** $5\frac{1}{3}$ **d** $\frac{3}{16}$

e $\frac{11}{125}$ **f** $3\frac{2}{9}$ **g** $\frac{11}{15}$ **h** $1\frac{3}{7}$

12 Write each percentage as a fraction in simplest form and as a decimal.

a $\frac{1}{4}$% **b** $\frac{11}{3}$% **c** $\frac{5}{8}$% **d** $2\frac{1}{2}$%

e $8\frac{4}{5}$% **f** $15\frac{2}{3}$% **g** $3\frac{5}{9}$% **h** $13\frac{1}{6}$%

3A.4 **13** Complete each statement by writing < (is less than) or > (is greater than) in the space provided.

a 0.46 ____ $\frac{1}{2}$ **b** $\frac{3}{5}$ ____ 57% **c** 0.95 ____ 71% **d** 59% ____ $\frac{4}{7}$

e $\frac{5}{4}$ ____ 122% **f** 252% ____ 21.2 **g** 13.4 ____ $1\frac{3}{4}$ **h** $2\frac{1}{6}$ ____ 206%

14 Write a percentage to describe these situations by first finding an equivalent fraction that has a denominator of 100.

a Andrew ate three out of five cupcakes.

b Nine out of twenty people went home.

c Seventeen out of twenty-five people voted yes.

d Mark knew 46 people out of 200.

e Steffie obtained a score of 17 out of 20 for the quiz.

f The movie critic rated the movie one out of five stars.

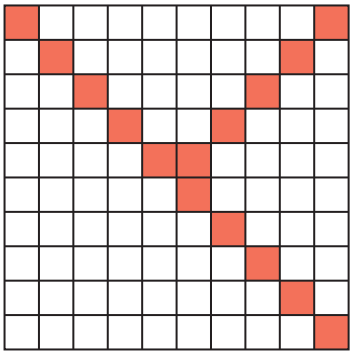
g A dam with a capacity of 600 megalitres currently holds 144 megalitres.

h Seventy-five of the 250 people surveyed were happy with their mobile phone provider.

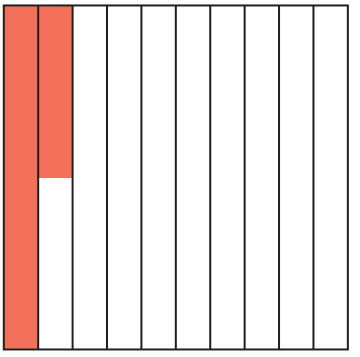
15 What percentage of its original value is a car worth if it depreciates (loses value) by these amounts?

a 20% **b** 19% **c** 8% **d** 12% **e** 28.5% **f** $33\frac{1}{3}$%

16 Thomas and Matthew were asked to draw a diagram representing 15%. Their answers are shown below.



Matthew



Thomas

a Which diagram is correct? Explain your reasoning.

b Show another way of representing 15% in a diagram.

17 These containers each contain 50% of their total capacity.

Explain how each container holds a different volume of liquid, but they are the same percentage full.

18 A rational number is a number that can be written exactly as a fraction with the numerator and denominator both being integers. Explain why all percentages are rational numbers.

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

19 Arrange the following lists in ascending order by first converting all terms to either a fraction, decimal or percentage.

a $\frac{7}{5}$, 0.89, $\frac{3}{4}$, 200%, $\frac{4}{5}$, $1.\dot{3}$

b 4.9, $\frac{13}{2}$, 560%, 7.2, $4\frac{3}{10}$, $\frac{22}{4}$

c $\frac{16}{5}$, $\frac{3}{2}$, 340%, 0.28, $1\frac{4}{5}$, 65%

20 Percentages can be added and subtracted like other numbers. Calculate:

a 17% + 49%

b $23\% + 32\frac{1}{2}\%$

c $53\frac{1}{2}\% + 16\frac{1}{2}\% + 9\frac{2}{3}\%$

d 7.9% + 28.7% + 46.5%

e 92% − 68%

f $16\frac{4}{5}\% - 12\frac{2}{5}\%$

g 37.4% + 52.9% − 15.6%

h 75.95% + 5.28% − 32.13% − 28.66%

21 A packet of gumballs is emptied onto a table. See the gumballs on the right.

a Write the number of blue gumballs as a fraction of the total.

b Convert this to a decimal and round to four decimal places.

c Convert this decimal to a percentage. What percentage of the gumballs are blue?

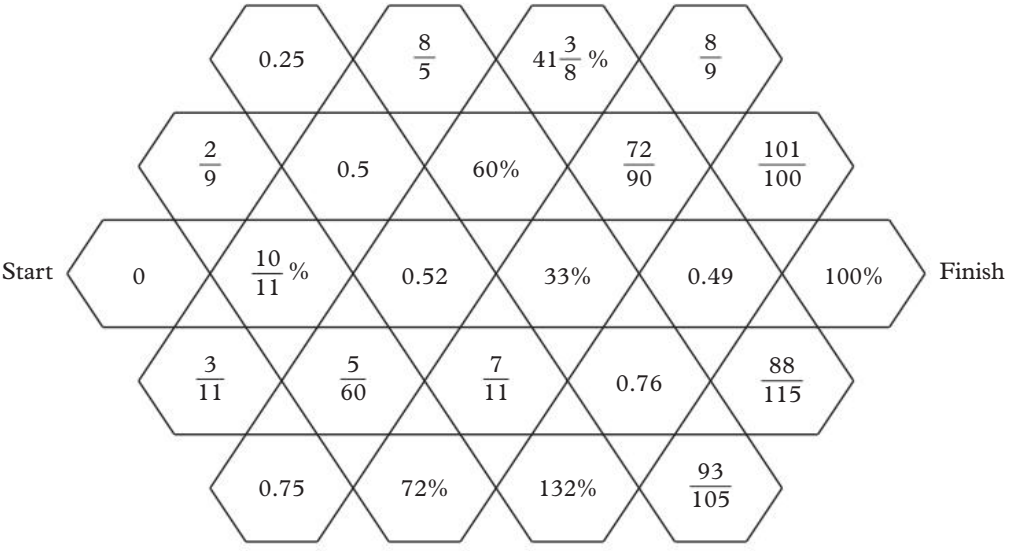
d What percentage of the gumballs, correct to two decimal places, is:

i orange **ii** green **iii** red or yellow **iv** not red?

22 Marigo received a $5\frac{1}{4}\%$ pay rise. What fraction of her salary does this represent?

CHALLENGE

23 Starting at zero in this diagram, find the shortest path to 100%, moving in ascending order.



24 Showing examples, explain how you could represent the percentages below using:

a a rectangle of length 10 cm and width 2 cm

80% 92% 25% 17%

b a grid of 200 squares.

43% 66%

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Fractions, decimals and percentages

Investigation Best one-percentages, decimals or fractions

Topic quiz 3A

3B Calculating percentages

Learning intentions

By the end of this topic you will be able to ...

✔ express a quantity as a percentage of another quantity
✔ calculate a percentage of a quantity
✔ increase and decrease a quantity by a percentage.

Inter-year links

Support	Calculating percentages
Year 7	4H Calculating percentages

Key content video

A quantity as a percentage of another quantity

- To express a quantity as a percentage of another quantity:
  1. Make sure that both quantities are expressed in the same units.
  2. Form a fraction with the numerator as the quantity you want to express as a percentage. Simplify the fraction if possible.
  3. Write the fraction as a percentage.

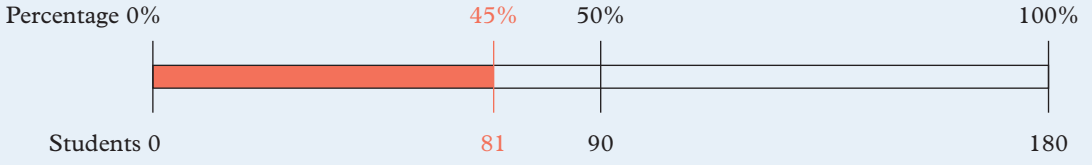
  For example, 81 students as a percentage of 180 students:

$$\frac{81}{180} = \frac{9}{20} = \frac{45}{100} = 45\%$$

Percentage of a quantity

- To calculate a percentage of a quantity, write the percentage as a decimal and multiply by the quantity. For example, 45% of 180 students like ice cream more than frozen yoghurt.

$$45\% \text{ of } 180 = 0.45 \times 180 = 81 \text{ students}$$



Percentage increase and decrease

- A **percentage increase** occurs when an original quantity is increased by a percentage.
- A **percentage decrease** occurs when an original quantity is decreased by a percentage.
- To increase a quantity by a percentage:
  1. Add the percentage to 100% and convert this new percentage to a decimal number.
  2. Multiply the quantity to be increased by the decimal number.

  For example, to increase a quantity by 50% multiply it by 1.5 (100% + 50% = 150%).
- To decrease a quantity by a percentage:
  1. Subtract the percentage from 100% and convert this new percentage to a decimal number.
  2. Multiply the quantity to be decreased by the decimal number.

  For example, to decrease a quantity by 40% multiply it by 0.6 (100% − 40% = 60%).

Percentage error

- The error of a measurement is the positive difference between the true value and the measured value.

  For example, if a person is 120 cm tall and their height is measured to be 123 cm, then the error in the measurement is 3 cm.
- The **percentage error** is the error of a measurement written as a percentage of the true value of the quantity.

  For example, the percentage error from the example above is:

$$\frac{3\text{ cm}}{120\text{ cm}} = \frac{1}{40}$$
$$= \frac{2.5}{100}$$
$$= 2.5\%.$$

Example 3B.1 Expressing a quantity as a percentage of another

a Write 60 as a percentage of 150.

b Write 90 cents as a percentage of \$12.

THINK

a **1** Express the amount as a fraction of the total and simplify.

2 To convert a fraction to a decimal or percentage, write the fraction with a denominator of 100.

3 Write your answer.

b **1** Convert \$12 to 1200 cents and write 90 cents as a fraction of 1200 cents.

2 To convert a fraction to a decimal or percentage, write the fraction with a denominator of 100.

3 Write your answer using the units from the question.

WRITE

a

$$\frac{60}{150} \overset{\div 30}{\underset{\div 30}{=}} \frac{2}{5}$$
$$= \frac{40}{100}$$
$$= 40\%$$

60 is 40% of 150.

b

$$\frac{90}{1200} \overset{\div 12}{\underset{\div 12}{=}} \frac{7.5}{100}$$
$$= 7.5\%$$

90 cents is 7.5% of \$12.

Example 3B.2 Calculating a percentage of a quantity

Calculate 7% of 220.

THINK

1 Convert the percentage to a decimal.

2 Multiply by the quantity.

3 Write your answer.

WRITE

7% of $220 = 0.07 \times 220$

$= 15.4$

7% of 220 is 15.4.

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Example 3B.3 Changing a quantity by a percentage

a Increase 760 by 25%.

b Decrease 9000 by 35%.

THINK

a **1** Add the percentage to 100% and convert the result to a decimal number.

2 Multiply the quantity to be increased by the decimal number.

b **1** Subtract the percentage from 100% and convert the result to a decimal number.

2 Multiply the quantity to be decreased by the decimal number.

WRITE

a $(100\% + 25\%) \times 760 = 125\% \times 760$
$= 1.25 \times 760$
$= 950$

b $(100\% - 35\%) \times 9000 = 65\% \times 9000$
$= 0.65 \times 9000$
$= 5850$

Example 3B.4 Finding the original amount after a change

a After a 30% increase, a number is now 1560. What was the original amount?

b After a 10% decrease, a number is now 207. What was the original amount?

THINK

a **1** To increase a number by 30%, multiply by 130%. To reverse the process, divide by 130%.

2 Convert this new percentage to a decimal number.

3 Divide the new amount by the decimal number to calculate the original amount.

b **1** To decrease a number by 10%, multiply by 90%. To reverse the process, divide by 90%.

2 Convert this new percentage to a decimal number.

3 Divide the new amount by the decimal number to calculate the original amount.

WRITE

a Original amount $= 1560 \div (100\% + 30\%)$
$= 1560 \div 130\%$
$= 1560 \div 1.3$
$= 1200$

b Original amount $= 207 \div (100\% - 10\%)$
$= 207 \div 90\%$
$= 207 \div 0.9$
$= 230$

Helpful hints

✔ You can use fractions or decimals to find the percentage of a quantity. Both methods will give you the same answer. For example, here is an alternative way to solve **Example 3B.1** part **a**.

$$\frac{60}{\cancel{150}^{3}} \times \frac{\cancel{100}^{2}\%}{1} = \frac{\cancel{60}^{20}}{\cancel{3}^{1}} \times \frac{2\%}{1}$$
$$= \frac{20}{1} \times \frac{2}{1}\%$$
$$= 40\%$$

✔ Writing a quantity as a percentage of a total and finding a percentage of a total are the opposite of each other.

20 as a percentage of 40	**50% of 40**
$\frac{20}{40} = 50\%$	$50\% \times 40 = 20$

ANS p492

Exercise 3B Calculating percentages

△ 1–7, 8–9(a–d), 10, 11, 13 □ 3–7, 8(e–h), 9(c–f), 10, 12, 14 ⬠ 3, 5, 8(g–l), 9(c–f), 10, 12, 14, 15

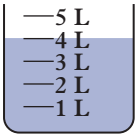
FLUENCY AND UNDERSTANDING

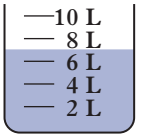
3B.1 **1** Write these amounts.

a 20 as a percentage of 80 **b** 45 as a percentage of 50 **c** 19 as a percentage of 25
d 90 as a percentage of 120 **e** 300 as a percentage of 250 **f** 135 as a percentage of 108

2 Calculate the percentage of the measuring jar:

i that has been filled **ii** that is empty.

a


b


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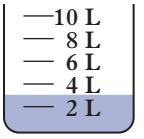

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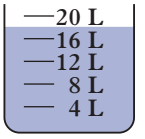


3 Write each proportion:

i as a fractional percentage **ii** as a decimal percentage.

a 140 as a percentage of 800 **b** 72 as a percentage of 36 000 **c** 46 as a percentage of 125
d 57 as a percentage of 200 **e** 58 as a percentage of 80 **f** 282 as a percentage of 48

4 Write each amount as a percentage of the total.

a \$1288 of a \$5600 loan is repaid.
b 10.55 km of the 42.2 km marathon has been completed.
c 42 of the 120 lots of land have been sold.
d 63 likes have been received out of 90 views on YouTube.
e 3 of 40 training sessions have been missed.

5 Write each of these as a percentage.

a 50 g out of 4 kg **b** 48 minutes out of 8 hours **c** 9 months out of 3 years
d 18 hours out of one day **e** 35 cents out of \$3.50 **f** 90 minutes out of 24 hours
g 60 mL out of 2 L **h** 800 mm out of 3.2 m

3B.2 **6** Calculate:

a 10% of 840 **b** 42% of 1800 **c** 6% of 150
d 18% of 6400 **e** 125% of 580 **f** 9% of 1250

7 Calculate:

a 63% of 720 m **b** 150% of 90 seconds **c** 35% of 660 cm
d 12% of 840 L **e** 40% of 190 buttons **f** 10.5% of 112 mL

8 Calculate:

a 30% of \$1200 **b** 25% of \$5000 **c** 40% of \$250 **d** 15% of \$10 500
e 8% of \$200 **f** 21% of \$2745 **g** 13% of \$850 **h** 150% of \$299
i $5\frac{1}{2}$% of \$1260 **j** $10\frac{1}{4}$% of \$1260 **k** 20.75% of \$199 **l** 450% of \$820

3B.3 **9** Increase or decrease each amount by the given percentage, rounding answers to two decimal places where necessary.

a Increase \$3200 by 40% **b** Decrease \$500 by 13%
c Increase 60 kg by 20% **d** Decrease 2.4 km by 35%
e Increase 15 minutes by 200% **f** Decrease 5.6 L by 12.5%

3B.4 **10** Calculate the original amount if:

a after a 20% increase, a number is now 612

b after a 15% decrease, a number is now 544

c after a 75% increase, a number is now 3185

d after a 98% decrease, a number is now 3.2.

11 The tallest horse ever recorded is a Belgian draft horse named Big Jake, who measured 210 cm tall. The smallest horse ever recorded is a dwarf miniature horse named Thumbelina who measured 44.5 cm tall. As a comparison, an average horse is about 165 cm tall and an average Labrador dog is about 60 cm tall.

a Write Thumbelina's height as a fraction of Big Jake's in simplest form.

b Convert the answer from part **a** to a percentage rounded to two decimal places.

c Find the percentage, correct to two decimal places, of:

i Thumbelina's height compared to an average horse's height

ii Thumbelina's height compared to an average Labrador's height

iii Big Jake's height compared to an average horse's height.

The heights of horses and dogs are measured at their withers (shoulders) because their height does not change when the animal moves, like the height of the head can.

d Estimate how tall these two horses would be measured to the top of their heads.

e How would this change the percentage you calculated in part **b**?

12 **a** What is 20% of 50?

b If a platter originally had 50 cupcakes and 20% were eaten, how many cupcakes would be left?

c By the end of the night, 20% of the remaining cupcakes were taken from the platter. How many cupcakes is this?

d Explain why the amounts obtained in parts **a** and **c** are different.

e Find how many cupcakes are still on the platter at the end of the night.

13 A salary of $70 000 was increased by 16%.

a What is the new salary?

b What is the increase as a percentage of the new salary? Give your answer correct to two decimal places.

14 The total land area of New South Wales is approximately 801 150 km^2. Determine the percentage of the total New South Wales land area traditionally owned by the following Aboriginal Nations. Give your answers correct to two decimal places.

a Bundjalung people, 6000 km^2

b Wonnarua people, 5200 km^2

c Djangadi people, 9100 km^2

15 Jamie has 5 kg of flour. He uses 20% of the flour to bake bread, and then he uses 5% of the remaining flour to bake biscuits.

a How many kilograms of flour remained after he baked the bread?

b How much flour did he use to bake the biscuits?

c Calculate the total percentage of the flour that has been used after baking the bread and biscuits.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Calculating percentages

Investigation
Of and As – what's the diff?

Topic quiz
3B

3C Mark-ups and discounts

Learning intentions

By the end of this topic you will be able to ...

✔ calculate mark-ups and discounts including GST

✔ use the unitary method to solve financial problems.

Inter-year links

Support Calculating percentages

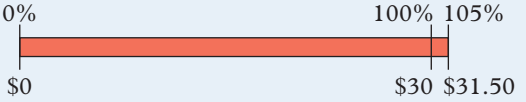
Year 7 4H Calculating percentages

Mark-ups, discounts and GST

Key content video

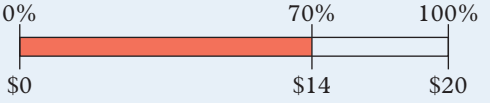
- The **cost price** (wholesale price) is the price paid to create or obtain a good or service before it is sold to a consumer.
- The **selling price** is the price that a product or service is sold at by the seller.
- A **mark-up** is the amount a cost price is increased by to give a profit and is usually expressed as a percentage.
- Finding the price after a mark-up is the same as increasing a quantity by a given percentage.
  1. Add the percentage to 100% and convert this new percentage to a decimal number.
  2. Multiply the cost price by the decimal number.

  Increase \$30 by 5% = (100% + 5%) × \$30
  = 105% × \$30
  = 1.05 × \$30
  = \$31.50

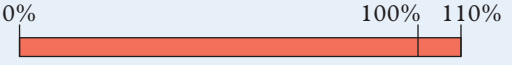


- A **discount** is the amount a selling price is decreased by to sell at a lower price and is usually expressed as a percentage.
- Finding the price after a discount is the same as decreasing a quantity by a given percentage.
  1. Subtract the percentage from 100% and convert this new percentage to a decimal number.
  2. Multiply the selling price by the decimal number.

  Decrease \$20 by 30% = (100% − 30%) × \$20
  = 70% × \$20
  = 0.7 × \$20
  = \$14



- **GST** (Goods and Services Tax) was introduced by the Australian Government in 1999. Currently, the rate is 10% of the selling price of a product or service.
- GST is added to the price of a good or service by performing a percentage increase of 10% to the selling price of a product or service.
  → The process of calculating the total amount paid by a customer is the same as calculating a 10% mark-up on the selling price.



The unitary method

- The **unitary method** can be used to solve percentage increase or decrease questions when the original amount is not known. The unitary method works by first calculating the value of a single unit, in this case 1%, and then using this value to calculate an unknown amount, in this case 100%.

  For example, if after a discount of 15% a television has a retail price of \$765, you can divide the retail price by 85 (the remaining percentage) to calculate 1% of the original price. This can then be multiplied by 100 to give the original price.

  1% of original price $= \frac{\$765}{85}$
  = \$9

  Original price (100%) = \$9 × 100
  = \$900

Example 3C.1 Calculating mark-ups and discounts

Calculate the final price of:

a a \$350 camera with a 25% mark-up

b a \$299 pair of shoes discounted by 30%

c a \$418 monitor with GST.

THINK		WRITE
a 1	Mark-ups are the same as percentage increases. Add the percentage to 100% and convert this new percentage to a decimal number.	**a** final price = (100% + 25%) × \$350 = 125% × \$350
2	Multiply the amount to be increased by the decimal number.	= 1.25 × \$350
3	Write your answer using the correct units.	= \$437.50
b 1	Discounts are the same as percentage decreases. Subtract the percentage from 100% and convert this new percentage to a decimal number.	**b** final price = (100% − 30%) × \$299 = 70% × \$299
2	Multiply the amount to be decreased by the decimal number.	= 0.7 × \$299
3	Write your answer using the correct units.	= \$209.30
c 1	GST is a tax applied to goods increasing the price of a good or service by 10%. Add the percentage to 100% and convert this new percentage to a decimal number.	**c** final price = (100% + 10%) × \$418 = 110% × \$418
2	Multiply the amount to be increased by the decimal number.	= 1.1 × \$418
3	Write your answer using the correct units.	= \$459.80

Example 3C.2 Calculating the original price

Calculate the original price of:

a a mountain bike, if the selling price is \$904 and the mark-up is 60%

b a notebook, if the selling price is \$1125 and the discount is 25%.

THINK		WRITE
a 1	To increase a number by 60%, multiply by 160%. To reverse the process, divide by 160%.	**a** original price = \$904 ÷ (100% + 60%) = \$904 ÷ 160%
2	Convert this new percentage to a decimal number.	= \$904 ÷ 1.6
3	Divide the selling price by the decimal number to calculate the original price.	= \$565
b 1	To decrease a number by 25%, multiply by 75%. To reverse the process, divide by 75%.	**b** original price = \$1125 ÷ (100% − 25%) = \$1125 ÷ 75%
2	Convert this new percentage to a decimal number.	= \$1125 ÷ 0.75
3	Divide the selling price by the decimal number to calculate the original price.	= \$1500

Example 3C.3 Using the unitary method to calculate the original price

After a mark-up of 80%, a television retails for \$1800. Use the unitary method to calculate the original price.

THINK

1. 180% of the original price is \$1800, so divide \$1800 by 180 to calculate 1% of the original price.
2. Multiply the value for 1% of the original price by 100 to determine the original price.

WRITE

a 1% of original price $= \frac{\$1800}{180}$

$= \$10$

b original price $= \$10 \times 100$

$= \$1000$

Helpful hints

✔ A discount will always result in a value less than the original selling price, and a mark-up will always result in a value greater than the cost price.

ANS p492

Exercise 3C Mark-ups and discounts

△ 1, 2–3(1st, 2nd columns), 4, 5, 6(a, c, e), 7, 8(a–e), 9, 10, 12–15, 16(a, c, e, g), 21, 22

□ 1(b, d, f, h), 2-3(2nd, 3rd columns), 4, 6(b, d, f), 7, 8(e–i), 9, 11(a–d), 12–14, 16(b, d, f, h), 17, 18, 20, 21, 23

⬠ 1(g, h), 2–3(3rd column), 4(2nd column), 6(e, f), 7, 8(e–i), 9, 11(e–h), 12, 13, 16(b, d, f, h), 17, 19–21, 23, 24

UNDERSTANDING AND FLUENCY

1 Calculate each of these percentages.

a 10% of \$360.50 **b** 25.25% of \$4200

c 20% of \$550.75 **d** 120% of \$400

e 190% of \$850 **f** 7.3% of \$960

g 32% of \$729 **h** 115.09% of \$2900

3C.1 **2** Calculate the new price after each of these discounts.

a 20% discount on \$150 **b** 15% discount on \$300 **c** 25% discount on \$840

d 40% discount on \$680 **e** 50% discount on \$1238 **f** 12% discount on \$460

g 45% discount on \$855 **h** 30% discount on \$124.50 **i** 70% discount on \$2075

3 Calculate the selling price after each of these mark-ups.

a 20% mark-up on \$420 **b** 50% mark-up on \$668 **c** 65% mark-up on \$120

d 18% mark-up on \$924 **e** 87% mark-up on \$1348 **f** 120% mark-up on \$1600

4 Calculate the final selling prices.

a a 30% mark-up on \$1200 **b** a 25% discount on \$449

c a 40% discount on \$2250 **d** a 75% mark-up on \$149

e a 47% mark-up on \$645 **f** a 38% discount on \$750

g a 54% discount on \$1200 **h** a 25% discount on \$1050

i an 85% mark-up on \$175 **j** a 15% mark-up on \$399

5 Lakmali works at a clothing store that applies a 45% mark-up on all items. What is the final selling price of:

a a pair of jeans with cost price $98
b a t-shirt with cost price $25
c a hoodie with cost price $65
d a shirt with cost price $102?

6 Calculate the prices paid for these items after GST is added, rounding to the nearest cent where appropriate.

a dining table and chairs: $1285
b services provided by a plumber: $240
c insurance purchased for a car: $601.45
d five 3-m lengths of timber at $6.50 per metre
e electricity service and supply charge: $314.65
f membership at a gymnasium at $72.95 per month

7 For each of these, determine:

i the selling price
ii the mark-up or discount amount.

a A camera was purchased for $120 and sold later at a mark-up of 62%.
b A laptop originally marked at $1198 is offered for sale at a discount of 35%.
c Work tools each marked at $49.90 are offered for sale with a 15% discount.

3C.2 **8** Calculate the original price in each of these sales. Where necessary, round your answer to the nearest five cents.

a A mobile phone sells for $450 after a mark-up of 50%.
b A pair of sports shorts sells for $25 after a discount of 20%.
c Eyeliner sells for $11.85 following a 15% discount.
d A hardware store sells an electric chainsaw for $169 after it is marked up by 95%.
e A furniture store offers a leather lounge suite for sale for $9995 after a discount of 12.5%.
f Fitness equipment retails for $1499 following a 140% mark-up.
g The digital copy of a video game is on sale for $4.99 after a 90% discount.
h A jumper is on sale for $31.25 after a 37.5% discount.
i A painting is being sold for $1200 000 after a 250% mark-up.

9 The following items each include GST in the price. Calculate the pre-GST price, rounding to the nearest cent where appropriate.

a telephone and internet services $155.65
b computer accessories purchased for $235.95
c garden maintenance provided for $182
d a necklace bought at a jewellery store for $120.50

10 The selling price of an item is also known as the retail price. Michael plans to buy a new external hard drive to store his games. The hard drive has a retail price of $157.95, but he receives a 12.5% discount because he has a customer loyalty card.

a If the discount is 12.5%, what percentage of the retail price will Michael pay?
b Calculate the price Michael pays after the discount. Round your answer to the nearest five cents.

11 The following represent the original prices and the percentage discount amounts offered on some goods. In each case, calculate:

i the selling price after the discount
ii the discount amount.

Where appropriate, round answers to the nearest cent.

a $500; 12% discount
b $179.50; 15% discount
c $249; 8% discount
d $895.95; 4% discount
e $624.60; 14% discount
f $29 995; 5.5% discount
g $12 680; 12.5% discount
h $1495.99; 17.5% discount

UNDERSTANDING AND FLUENCY

3C.3 **12** Use the unitary method to calculate the original prices. Round your answers to the nearest five cents where necessary.

a a bicycle retailing for $1189 after a mark-up of 45%

b a mobile phone retailing for $792 after a mark-up of 32%

c a box of chocolates retailing for $3.10 after a mark-up of 120%

d a sofa retailing for $1400 after a mark-up of 25%

13 Use the unitary method to calculate the original prices.

a a package holiday retailing for $1500 after a discount of 40%

b a set of headphones retailing for $127.50 after a discount of 15%

c an eyeliner retailing for $8.25 after a discount of 75%

d a pair of shoes retailing for $178.50 after a discount of 30%

PROBLEM SOLVING AND REASONING

14 Melinda makes jewelled earrings and adds an 85% mark-up to her costs when determining the retail prices. Each individual earring contains a metal hook that costs $8.50 and three decorative stones that each cost $4.60.

a How much does it cost Melinda to make each pair of these earrings?

b What is the value of the mark-up?

c How much would Melinda advertise these pairs of jewelled earrings for?

15 A manufacturer advertises their football boots for a wholesale price of $89.90. A sports store plans to sell these boots to the public at a mark-up of 110%.

a If the mark-up is 110%, what percentage of the wholesale price will a member of the public pay for these boots?

b Calculate the retail price for these boots correct to the nearest five cents.

16 The following represent the wholesale prices and the percentage mark-up amounts offered on some goods. In each case, calculate:

i the retail price after the mark-up **ii** the mark-up amount.

Where appropriate, round your answers to the nearest cent.

a $620; 24% mark-up

b $89.95; 45% mark-up

c $1269; 80% mark-up

d $450.50; 85.5% mark-up

e $6250; 140% mark-up

f $350.99; 125% mark-up

g $14 625; 112.5% mark-up

h $2295; 137.5% mark-up

17 A girl's bicycle is reduced to $198 after a discount of 20%. To determine the original price, Jane thinks that she needs to calculate 20% of $198 and add the result to $198. Tim thinks that Jane has it wrong and that the calculation is more complex. Which person do you think is correct? Show working to support your answer.

18 Reconsider the scenario in question **17**. What was the price of the bike before the discount?

19 Julian accidentally discounted a $699 food mixer by 20% instead of the advertised 30%. Once he realised his mistake he took a further 10% off the discounted price. The customer claimed that this would not be the same as reducing the original price of the mixer by 30%. Who is correct? Show workings to support your answer.

20 Switched On Electrics is having a 'Scratch and Dent' sale. Any damaged stock will be discounted by 35%. A refrigerator is scratched and discounted to $1820.

a What was the original price of the refrigerator?

b How much money is saved because of the scratch?

21 A grocery store applies a 30% mark-up on all its goods. A GST of 10% is then added to this marked-up price.

a Calculate the selling price of each item once the mark-up and GST have been added.

i toothpaste, 160 g: cost price \$3.88 **ii** tuna, 95 g: cost price \$1.24

iii instant coffee: cost price \$1.99 **iv** oil, 4 L: cost price \$17.99

v rice crackers, 90 g: originally \$2.49 **vi** raisin toast, 520 g: originally \$4.59

b Compare the cost price with the selling price of each item and calculate the actual percentage increase correct to two decimal places.

c Explain why increasing a price by 30% then by another 10% is different from increasing it by 40%.

22 Consider the two statements: 'All stock discounted by 45%' and 'Stock discounted by up to 45%'. Explain whether these statements mean the same thing.

23 In Australia, workers pay income tax on their income to the government. The amount they pay depends on the amount they earn. The amount of income tax paid is outlined in the following table.

Taxable income	Tax on this income
0 – \$18 200	Nil (\$0)
\$18 201 – \$45 000	19 cents for each \$1 over \$18 200
\$45 001 – \$120 000	\$5092 plus 32.5 cents for each \$1 over \$45 000
\$120 001 – \$180 000	\$29 467 plus 37 cents for each \$1 over \$120 000
\$180 001 and over	\$51 667 plus 45 cents for each \$1 over \$180 000

Source: https://www.ato.gov.au/rates/individual-income-tax-rates/

For example, a worker earning \$46 000 a year would pay the following amount in tax:

$$\begin{aligned} 5092 + 32.5\% \times (46\,000 - 45\,000) &= 5092 + 32.5\% \times 1000 \\ &= 5092 + 0.325 \times 1000 \\ &= 5092 + 325 \\ &= \$5417 \end{aligned}$$

a Explain how the information in the table is used in the above example.

b Calculate the amount of tax paid by someone earning:

i \$30 000 **ii** \$65 000 **iii** \$109 000

iv \$135 000 **v** \$205 000 **vi** \$1 000 000.

c What do the amounts \$5092, \$29 467 and \$51 667 represent? Show how they are calculated.

d For each amount in part **b**, calculate the percentage of the total income paid in tax correct to one decimal place.

e Explain why it is incorrect to say that people who earn over \$180 000 pay 45% income tax.

24 David bought a new computer and paid \$500 after several discounts. The computer was in a 45% off sale and his store membership rewarded him with an extra 10% off purchases more than \$100. He also used a credit of \$75 from the return of a faulty item.

a If the discount percentages were applied sequentially and then the store credit was applied, how much did the computer cost originally?

b If the store credit was applied first and then the discount percentages, how much did it originally cost?

c If the discounts were added and the total percentage was applied, and then the store credit was applied, how much did it cost originally?

d If the percentage discounts were applied sequentially, does it matter in which order they were applied? Explain why or why not.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Mark-ups, discounts and GST

Investigation
Inflating growth

Topic quiz
3C

3D Profit and loss

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate the percentage profit or loss on the cost price
- ✔ calculate the percentage profit or loss on the selling price
- ✔ calculate the profit or loss in a given situation.

Inter-year links

Support	Calculating percentages
Year 7	4I Percentage increases and decreases
Year 9	1B Other forms of income
Year 10	1A Earning money

Profit and loss

Key content video

- A **profit** occurs when the selling price is greater than the cost price.
  Profit = selling price – cost price
- The **percentage profit** usually refers to the profit as a percentage of the cost price.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

For example, Sam made a profit of \$20 when he sold a bike he bought for \$100.

$$\begin{aligned}\text{Percentage profit} &= \frac{\text{profit}}{\text{cost price}}\\ &= \frac{20}{100}\\ &= 20\%\end{aligned}$$

- A mark-up and percentage profit are both calculated as the percentage amount that the cost price is increased by to give the selling price.
- A **loss** occurs when the selling price is less than the cost price. Loss = cost price – selling price
- The **percentage loss** usually refers to the loss as a percentage of the cost price.

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

For example, Greg made a loss of \$120 when he sold the phone he bought for \$499.

$$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{cost price}}\\ &= \frac{120}{499}\\ &= 24.05\%\end{aligned}$$

- A discount is the amount a selling price is decreased by to sell at a lower price, whereas a percentage loss is calculated as the percentage amount that a cost price is decreased by to give the selling price.
- Percentage profit and loss calculations are usually in relation to the cost price, but can also be given in relation to the selling price.
- **Revenue** is the selling price multiplied by the number of items (or services) sold.

Example 3D.1 Calculating profit and loss

A television initially bought for \$800 is later sold for \$950.

a State if a profit or loss has been made and determine the amount.

b Write the profit or loss amount as a percentage of the cost price.

THINK	WRITE
a The selling price is more than the cost price, so a profit has been made. Find the difference.	**a** A profit has been made. Profit = selling price − cost price = \$950 − \$800 = \$150
b Write the profit amount as a percentage of the cost price. Write the comparison as a fraction and convert it to a percentage.	**b** Percentage profit $= \frac{\cancel{150}^{3}}{\cancel{800}^{16}}$ $= \frac{3}{16} \times 100\%$ $= \frac{3}{\cancel{16}^{4}} \times \frac{\cancel{100}^{25}}{1}\%$ = 18.75% The television sold for an 18.75% profit.

Helpful hints

- ✔ Remember that you can only generate a profit when the selling price is greater than the cost price, and you can only generate a loss when the selling price is less than the cost price.
  - → When the result (selling price minus the cost price) is positive, there is a profit.
  - → When the result (selling price minus the cost price) is negative, there is a loss.
- ✔ When writing a percentage, make sure to carefully check what amount you are writing the percentage of.

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Exercise 3D Profit and loss

△ 1–5, 7, 8, 10, 12, 14, 15, 17, 20 □ 2, 3, 6, 7, 9–11, 13, 16, 18, 19, 22 ⬠ 2, 3, 7, 10, 11, 16, 18, 19, 21–23

You can use your calculator for all questions in this section unless otherwise specified.

1 Determine the amount of profit or loss (in dollars) for each of the following.

a cost price: \$35, selling price: \$45

b cost price: \$82, selling price: \$68

c cost price: \$92.50, selling price: \$87.95

d cost price: \$299.98, selling price: \$145.50

3D.1 **2** For each of the following scenarios:

i state whether a profit or loss has been made and determine the amount

ii write the profit or loss amount as a percentage of the cost price, correct to two decimal places where appropriate.

a Shoes are bought for \$240 and later sold for \$180.

b A greengrocer buys a box of cherries for \$2.50 and sells them for \$9.80.

c An investor buys shares for \$5.20 and sells them for \$4.80.

d A car is purchased brand new for \$24 640 and sold for \$19 250.

e Coins are purchased in a set for \$120 and sold for \$350.

f A novel is purchased for \$29.95 and sold for \$8.

3 **a** Re-calculate the percentages in question **2** part **ii** if they are now based on the selling price.

b How do the percentage amounts change if the percentages are based on the selling price instead of the cost price?

UNDERSTANDING AND FLUENCY

4 Calculate the percentage profit or loss on the cost price for each part in question **1**. Round your answers to two decimal places.

5 Xiao pays $198 for her wireless headphones and sells them to a friend for $150 when a new model comes out.

a Did Xiao make a profit or a loss? State the amount of profit or loss.

b Write the amount in part **a** as a percentage of the cost price.

6 As Benjamin became more successful at his BMX racing, he chose to sell his bike to buy a better model. The bike, which had cost him $240, was sold to a fellow competitor at a percentage profit of 5%.

a How much did Benjamin sell the bike for?

b The new bike Benjamin plans to buy will cost him $900. Write this as a percentage of the cost of his original bike.

7 For each of these:

i state the value of the profit or loss

ii write the profit or loss as a percentage of the cost price (rounded to the nearest 1%).

a cost price: $48, selling price: $34

b cost price: $112.50, selling price: $240

c cost price: $35.90, selling price: $85.95

d cost price: $1649, selling price: $1238

e cost price: $29 895, selling price: $17 500

f cost price: $156 985, selling price: $425 850

8 John buys pears at the orchards for $2.95 per kilogram to sell at his market stall.

a How much does John mark up the cost of the pears per kilogram (see photo)?

b Write the mark-up amount as a percentage of the price John pays for the pears. State the mark-up as a percentage to the nearest 1%.

9 A wireless printer is initially priced at $249.95 and is offered for sale at a discounted price of $222.50.

a State the amount of the discount.

b Write the discount as a percentage of the initial price to the nearest 1%.

10 The revenue a company earns is the total amount of money received from sales.

Revenue = selling price × sales

Profit per sale is the difference in the selling price and cost price, which can be multiplied by the number of sales to determine the total profit.

For example, if the original price is $5, the selling price is $8, and 20 are sold, then:

Revenue = $8 × 20 = $160

Profit = ($8 − $5) × 20 = $3 × 20 = $60

For each of the following, calculate the:

i revenue

ii total profit each item makes.

a cost price $2, selling price $5, 50 sold

b cost price $8, selling price $20, 100 sold

c cost price $10, selling price $40, 25 sold

d cost price $1.50, selling price $4.75, 45 sold

e cost price $7.99, selling price $13.99, 16 sold

f cost price 24c, selling price $1.99, 247 sold

11 Joseph sells remote-controlled cars in his store. He knows that identical cars are being sold by a competitor for $65 and wants to beat their price. Joseph can purchase these cars from a wholesaler for $28 per car.

a Joseph aims to make a 150% profit on the sale of each car and must add 10% for GST. Do you think this profit margin is a suitable pricing strategy? Briefly explain.

b Using whole number values, what is the maximum percentage increase Joseph should apply to the wholesale price to beat the competitor's price? Remember to add the GST charge.

c Why is it necessary to consider a maximum percentage increase rather than any percentage increase Joseph wishes to apply?

12 A small share portfolio was purchased at a price of $1200 and sold 12 months later for $1500.

a Write the profit as a percentage of the original price.

b Write the final selling price as a percentage of the original purchase price.

c Compare the percentage increase in part **a** with the answer in part **b**. Briefly explain how they are related.

13 A car is bought for $20 000 and sold 6 months later for $16 000.

a Write the loss as a percentage of the original price paid for the car.

b Write the final selling price as a percentage of the original purchase price.

c Compare the percentage decrease in part **a** with the answer in part **b**. Briefly explain how they are related.

14 What percentage increases or decreases match the following profits or losses?

a doubling your money **b** tripling your money **c** breaking even

d halving your money **e** quadrupling your money **f** losing all your money

15 The finance report on the nightly news displays the daily movement in the cost of common commodities. If the values given in this table represent the end-of-day trading figures, what were the values at the start of the day's trading?

Commodity	Final price $	Movement %
Gold	1732.95	↓ 0.5
Silver	33.56	↓ 1.4
Oil	102.46	↑ 0.3
Copper	3.71	↑ 1.2

↓ represents a decrease in price

↑ represents an increase in price

16 Olivia purchases a large block of land on which she plans to build four townhouses. The land costs $645 000 and the cost for each house is $230 000 (including plans, permits and other related charges). The project takes 2 years to complete and Olivia is charged $2300 in council rates each year. The real estate agency earns a commission of 1.75% for the sale of each townhouse. The four house sales are $485 000, $490 000, $472 000 and $461 000.

a What were the total expenses accrued by Olivia before the sale of the townhouses?

b From the total sales, how much of the money:

i goes to Olivia

ii is earned as commission by the real estate agency?

c Does Olivia make a profit or a loss?

d Write your answer from part **c** as a percentage of Olivia's total expenses. Do not include the real estate agency's commission.

17 Mario runs a hairdressing business from his home and sells shampoos, hair treatment and styling products to his customers. On all product sales, he plans to make a profit of 80% of the wholesale price he pays for the goods. On top of this, he knows he must add an additional 10% for GST. Mario believes he can determine the selling price by simply adding 90% to the wholesale price.

a A jar of styling gel has a wholesale price of $8.50. What will the price be after Mario's profit mark-up?

b What will the selling price be after GST is added?

c Increase $8.50 by 90% and compare your answer with the answer from part **b**. Is Mario's method of calculating the selling price correct? Why or why not?

PROBLEM SOLVING AND REASONING

18 For each of the following, write, correct to two decimal places where appropriate:

i the profit as a percentage of the selling price

ii the selling price as a percentage of the profit

iii the selling price as a percentage of the cost price

iv the percentage mark-up from the cost price to the selling price.

a cost price \$2, selling price \$5

b cost price \$8, selling price \$20

c cost price \$10, selling price \$40

d cost price \$1.50, selling price \$4.75

e cost price \$7.99, selling price \$13.99

f cost price 24c, selling price \$1.99

19 Determine the missing values in the table.

Cost price	Selling price	Cost price as a percentage of selling price	Number of sales	Revenue	Total profit	Profit as a percentage of the revenue
\$3.60	\$12		35			
	\$15		20		\$200	
	\$21			\$777	\$296	
			150	\$375	\$210	
			25	\$875		26%
			125		\$320	32%
	\$99	51%	8			
\$0.28		25%	271			
	\$32				\$145.92	24%
	\$6		15			40%

20 **a** Explain why finding 70% of a value is equal to decreasing the value by 30%.

b Explain why finding 130% of a value is equal to increasing the value by 30%.

21 A small business said they earned \$10 000 this week. They spent \$12 000 to make the sales.

a If the \$10 000 is the profit the business made, how much revenue did they earn?

b If the \$10 000 is the revenue the business earned, how much profit or loss did they make?

CHALLENGE

22 **a** In question **17**, Mario likes to make a profit of 80% on his wholesale prices and then adds 10% for GST. What single calculation can Mario perform to determine his selling price for a jar of styling gel?

b A motorbike sells for \$1200 after a mark-up of 60% followed by GST being added. What single calculation can be performed to determine the wholesale price of the motorbike?

c GST is added to a price and then the item is discounted by 25% to sell for \$400. What single calculation will determine the original pre-GST price?

23 A company marks up their product's cost by 128% to its selling price. What percentage will the profit be of the revenue, correct to two decimal places?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Percentage profit and percentage loss

Topic quiz
3D

ANS p495

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

3A **1** Write each percentage as a fraction in its simplest form and as a decimal.

a 25% **b** 76% **c** 4% **d** 0.5% **e** 120% **f** 2.56%

3A **2** Write each decimal as a percentage and as a fraction in its simplest form.

a 0.63 **b** 0.12 **c** 0.85 **d** 3.3 **e** 0.006 **f** 7.5

3A **3** Write each fraction as a decimal and a percentage.

a $\frac{3}{8}$ **b** $\frac{13}{40}$ **c** $5\frac{2}{3}$ **d** $\frac{5}{16}$ **e** $\frac{11}{25}$ **f** $8\frac{1}{6}$

3B **4** Write each of these as a percentage.

a 40 g out of 200 g **b** 45 minutes out of 180 minutes **c** 6 months out of 4 years
d 3 hours out of one day **e** 60 cents out of $3.00 **f** 960 mm out of 72 m

3B **5** Calculate:

a 23% of 580 L **b** 120% of $1500 **c** 2.8% of 700 cm
d 18% of 1700 m **e** 61% of 800 beans **f** $5\frac{1}{2}$% of $3500

3B **6** Increase or decrease each amount by the given percentage. Where appropriate, round answers to two decimal places.

a Increase $9600 by 20%. **b** Decrease 800 km by 12.5%. **c** Increase $4500 by 9.5%.
d Decrease $45.80 by 7.5%. **e** Increase 8.3 km by 1.75%. **f** Decrease 120 minutes by 10%.

3C **7** Calculate the selling price of these (rounded to the nearest 5 cents).

a a 60% mark-up on $1800 **b** a 15% discount on $950 **c** a 20% discount on $9870
d a 25% mark-up on $765 **e** a 21.5% mark-up on $954 **f** a 28% discount on $1250

3C **8** For each item, calculate:

i the GST to be added
ii the price it will be sold for (round to the nearest five cents).

a novel $32 **b** jeans $85 **c** laptop $1250
d pen $3.50 **e** purse $26 **f** chair $35

3C **9** Calculate the original price, if:

a 40% of an amount is $180 **b** 35% of an amount is $3430
c 16% of an amount is $1376 **d** 12% of an amount is $1014.60
e 85% of an amount is $595 **f** 24% of an amount is $6072.60.

3D **10** For each of the following scenarios:

i state where a profit or loss has been made and determine the amount
ii write the profit or loss amount as a percentage of the cost price, correct to two decimal places where appropriate.

a A car is bought for $18 500 and sold a year later for $16 750.
b A vintage dress is bought for $15 and sold for $75.
c A trading card is purchased for $12 and sold for $100.
d A diamond ring is purchased for $3999 and is later sold for $2500.

3D **11** For each of the following, calculate the:

i revenue **ii** total profit each item makes.

a cost price $7, selling price $11, 70 sold **b** cost price $0.80, selling price $1.50, 330 sold
c cost price $5.50, selling price $12.50, 55 sold **d** cost price $18, selling price $27.50, 12 sold

3E Ratios

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify ratios in diagrams or written scenarios
- ✔ write ratios
- ✔ understand part-to-part and part-to-whole comparisons.

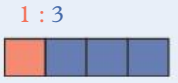
Inter-year links

Year 7	3H Ratios
Year 9	4E Direct variation

Ratios

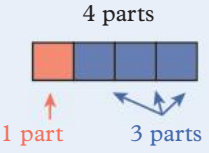
Key content video

- A **ratio** is a comparison of two or more quantities with the same units. A ratio must be written in the order of the given worded descriptions.
  For example, the ratio 1 : 3 reads '1 part compared to 3 parts' or '1 to 3'.



- Ratios can be written as fractions where the numerator is the number in the ratio and the denominator is the total number of parts.

$\frac{1}{4}$ of the squares are red



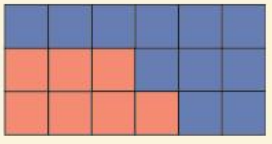
$\frac{3}{4}$ of the squares are blue

- Before writing a ratio, the numbers must be in the same unit of measurement.
- A part-to-part comparison is usually written as a ratio and a part-to-whole comparison is usually written as a fraction.

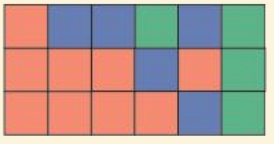
Example 3E.1 Determining ratios from a diagram

Write a ratio comparing the red to blue tiles in part **a** and the red to blue to green tiles in part **b**.

a



b



THINK

1. Count the number of each coloured tile.
2. Write the numbers as they appear in the question.
3. Write the ratio with a colon between each comparison.

WRITE

a There are 7 red tiles and 11 blue tiles.
red : blue
7 : 11

b There are 9 red tiles, 5 blue tiles and 4 green tiles.
red : blue : green
9 : 5 : 4

Example 3E.2 Writing ratios

Write each comparison as a ratio.

a 3 footballs to 8 soccer balls **b** 7 seconds to 3 minutes **c** 0.05 km to 23 cm to 6.73 m

THINK	WRITE
a Write the numbers as a ratio in the order they appear in the question.	**a** 3 : 8
b 1 Convert the quantities to the same unit. Converting the larger unit to the smaller unit is usually simpler. Convert minutes into seconds.	**b** 3 mins = 180 seconds
2 Write the numbers as a ratio in the order they appear in the question. Do not include units.	7 : 180
c 1 Convert the quantities to the same unit. Convert km to cm and m to cm.	**c** 0.05 km = 5000 cm 6.73 m = 673 cm
2 Write the numbers as a ratio in the order they appear in the question. Do not include units.	5000 : 23 : 673

Example 3E.3 Part-to-part versus part-to-whole comparisons

Purple paint is made by mixing 4 parts of blue paint with 5 parts of red paint.

a Write a ratio comparing the blue to red paint. **b** Write a ratio comparing the blue to purple paint.

c What fraction of the purple paint is blue? **d** What fraction of the purple paint is red?

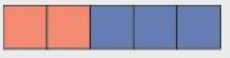
THINK	WRITE
a Write the numbers as a ratio in the order they appear in the question.	**a** blue : red 4 : 5
b Add up the total number of parts in the purple paint. Write the numbers as a ratio in the order they appear in the question.	**b** 4 + 5 = 9 blue : purple 4 : 9
c Write a fraction showing the numerator as the blue paint and the denominator as the total number of parts.	**c** blue paint = $\frac{4}{9}$
d Write a fraction showing the numerator as the red paint and the denominator as the total number of parts.	**d** red paint = $\frac{5}{9}$

Helpful hints

- ✔ Don't mix up the order of the numbers in a ratio. It tells you the order of the quantities you are comparing.
- ✔ Ratios are written in comparison order, not in size order.
- ✔ When writing ratios consider if you are comparing a part-to-part or a part-to-whole. For example, in the diagram shown, you can make both these comparisons:

  The ratio of red to blue tiles is 2 : 3.

  The ratio of blue tiles to the whole shape is 3 : 5.



ANS p495

Exercise 3E Ratios

1–4, 5–6(1st column), 7–13, 16, 18 | 1(e–h), 2, 5–6(2nd column), 9, 10, 13–15, 17, 19 | 2(b), 5–6(2nd column), 10, 14–17, 19, 20

UNDERSTANDING AND FLUENCY

1 Write each of these as a ratio.

a Last year Dominic won 11 tournaments and Stefanos won 7 tournaments.

b Last season Fedora kicked 89 goals while Sophie kicked 101 goals.

c Mark trains five times a week while Christian trains three times a week.

d Two-stroke petrol fuel is made by mixing 25 parts petrol to 1 part oil.

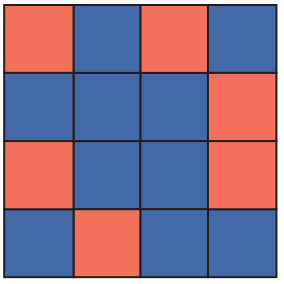
e A pancake mixture requires two cups of flour and three cups of milk.

f Concrete is made by mixing one part cement, two parts sand and three parts screenings.

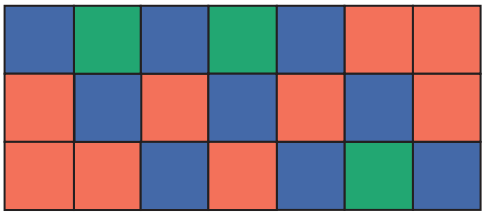
g A hair dye solution is made up in the ratio of three parts dye to seven parts water.

h Sarah works 8-hour shifts on the weekend and 3-hour shifts on a weekday.

3E.1 **2** **a** What is the ratio of red to blue tiles?



b What is the ratio of blue to red to green tiles?

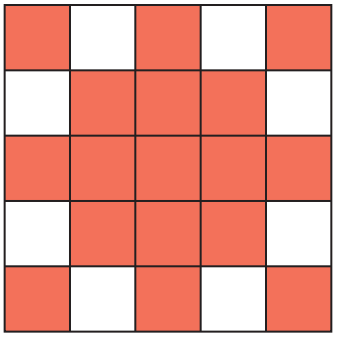


3 For each of these, write:

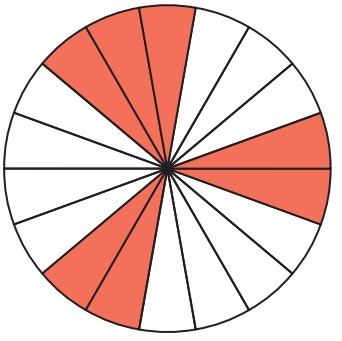
i the number of shaded sections to the number of non-shaded sections as a ratio

ii the number of non-shaded sections to the total number of sections as a fraction.

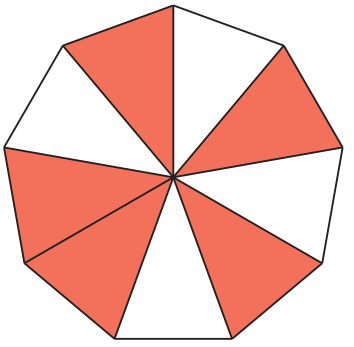
a



b



c



4 In question **3**, why is part **i** written as a ratio and part **ii** written as a fraction?

3E.2 **5** Write each comparison as a ratio in the given order.

a 20 seconds to 33 seconds

b 95 kg to 57 kg

c 113 m to 167 m

d 256 mL to 175 mL

e $499 to $575

f 23 days to 2 days

g 4 months to 13 months

h 19 cans to 21 cans

i 42 km to 117 km

j 16 L to 39 L

k 19 wins to 11 losses

l 23 losses to 7 wins to 3 draws

m 15 minutes to 17 minutes to 19 minutes

n 5 red to 7 blue to 11 yellow

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6 Write each comparison as a ratio.

a 13 days to 3 weeks
b 2 hours to 17 minutes
c 27 cents to $5
d 4000 mL to 3 L
e 1.7 km to 793 m
f 7 months to 3 years
g 509 g to 1.3 kg
h 472 kg to 2.371 tonnes
i 61 mm to 7.9 cm
j 38 minutes to 2 hours 11 minutes
k $4.20 to 97 cents
l 0.6 L to 53 mL
m 0.05 km to 23 cm to 6.73 m
n 5631 g to 1.753 kg to 1 tonne

7 **a** Write the number of pins in each comparison as a ratio.

i blue pins compared to yellow pins
ii white pins compared to green pins
iii yellow pins compared to total number of pins
iv red and green pins compared to blue pins

b Which of the ratios in part **a** could also be written as a fraction? Explain.

3E.3 **8** Green paint is made by mixing 3 parts blue paint with 4 parts yellow paint.

a Write a ratio comparing the blue to yellow paint.
b Write a ratio comparing the blue to green paint.
c What fraction of the green paint is blue?
d What fraction of the green paint is yellow?

9 A showbag contains 11 Cherry Ripes, 9 Flakes, 7 Picnics, 5 Time Outs and 8 Turkish Delights.

a Write the ratio comparing the number of:

i Cherry Ripes to Turkish Delights to Picnics
ii Flakes to the total number of chocolate bars.

b Write the number of:

i Time Outs and Turkish Delights as a fraction of the total number of chocolate bars
ii Flakes and Picnics as a percentage of the total number of chocolate bars.

10 Consider this group of animals.

a What is the ratio of the numbers of:

i dogs to cats **ii** mammals to birds **iii** rabbits to fish?

b What fraction of the total number of animals are lizards?

c Write the number of mammals and birds as a percentage (to the nearest per cent) of the total number of animals.

d Write the number of dogs and fish as a percentage (to the nearest per cent) of the total number of animals.

e Dane said that the ratio of dogs to cats was 1 : 2. Explain where Dane went wrong and what he should have written.

11 Sofia's win–loss ratio against Bianca is 4 : 5. What does this mean?

12 Eighty-seven per cent of an iceberg is submerged in water.

a What percentage of the iceberg is visible?

b Write as a ratio the percentage of the iceberg submerged to the percentage visible.

13 A manager finds that in a box of 50 nectarines, 18% have blemishes. Write the:

a ratio of the numbers of unblemished nectarines to blemished nectarines

b fraction of the total number of nectarines that have blemishes.

14 The mark-up on a pair of jeans purchased for $80 is 315%. What is the ratio of the cost price to the selling price?

PROBLEM SOLVING AND REASONING

15 Anzac biscuits date back at least 100 years. One of the earliest versions contained the ingredients listed on the right.

2 cups rolled oats
1 cup plain flour
1/2 cup sugar
125 g butter
1 tablespoon golden syrup
2 tablespoons boiling water

The 'dry ingredients' of oats, flour, and sugar are mixed together first.

a What is the ratio of oats to flour to sugar?

b What fraction of the dry ingredients is flour?

This recipe will make 12 biscuits. Sam decides to make Anzac biscuits for his year level.

c Sam combines the dry ingredients together and has 35 cups' worth. How many biscuits is he planning to make?

16 Consider the height of Sydney Tower (309 m) and Taiwan's Taipei 101 (1670 feet).

a Explain why the ratio for the heights of Sydney Tower and Taipei 101 cannot be written as 309 : 1670.

b What needs to be done to the measurements before a comparison can be made?

c Write the ratio of the Sydney Tower's height to Taipei 101's height to the nearest metre.
Hint: 1 foot = 0.3048 m

17 Whiter Than White bleach recommends three capfuls of bleach for each $2\frac{1}{2}$ L of water.

a Can the two quantities be written as a ratio as they are?

b If a capful is 19 mL, what is the ratio of the quantities of bleach to water?

c Sharon thinks the solution of bleach and water is quite strong. Suggest an alternative ratio which would give a weaker solution. Explain your logic.

18 One hundred people attended an auction on the weekend. Seven people expressed interest in bidding but on the day only three actually bid.

a Write a fraction for the number of bidders to the total number of people.

b Write a fraction for the number of non-bidders to the total number of people.

c Write a ratio for the number of bidders to the number of non-bidders.

d Write a sentence explaining when a ratio can be written as a fraction.

19 Lydia, Sylvia and Julia are three sisters, aged 16, 14 and 12, respectively. Their grandparents give them \$210 and tell them to divide it amongst the three of them. Lydia suggests the girls divide the money between them according to the ratio of their ages. Julia objects to this and suggests the money should be divided in thirds. Sylvia says she is happy either way. Why do you think Sylvia feels this way? Use evidence to support your answer.

CHALLENGE

20 The Golden Ratio is an irrational number that is not quite as famous as π, but it also uses a Greek letter for its symbol – Φ(phi). Many artists and architects believe that rectangles whose length and width are in the Golden Ratio make the most pleasing shape. We can define the Golden Ratio as the ratio of a shorter length, y, and a longer length, x, such that x divided by y is also equal to the sum of the two lengths divided by the longer length.

a Write out the two equations that define the Golden Ratio, Φ. When the shorter length, y, is equal to 1, what do you notice about the value of the Golden Ratio, Φ, and the longer length, x?

b Combine your equations from part **a** to create a new equation that does not contain x.

c The Golden Ratio, correct to 6 decimal places, is equal to 1.618 034. Substitute this value into your equation.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Visual representations of ratios

Interactive skillsheet
Units of measurement

Investigation
Aspects of framing

Topic quiz
3E

3F Equivalent ratios

Learning intentions

By the end of this topic you will be able to ...

✔ simplify ratios involving decimals and fractions

✔ use equivalent ratios to find unknown quantities.

Inter-year links

Support	Equivalent fractions
Year 7	3B Equivalent fractions
Year 9	4E Direct variation

Equivalent ratios

Key content video

- Numbers in a ratio can be multiplied or divided by the same value to create an **equivalent ratio**.

$$5 : 3 \overset{\times 4}{=} 20 : 12 \overset{\times 6}{=} 120 : 72$$

Simplifying ratios

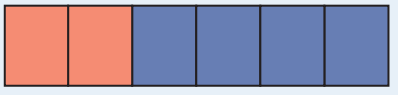
- A ratio is written in simplest form when the only common factor between the terms in the ratio is 1.

$$\overset{\div 24}{\downarrow}\ 120 : 72\ \overset{\div 24}{\downarrow} = 5 : 3 \quad \text{or} \quad \overset{\div 6}{\downarrow}\ 120 : 72\ \overset{\div 6}{\downarrow} = 20 : 12\ \overset{\div 4}{\downarrow}\ \overset{\div 4}{\downarrow} = 5 : 3$$

- Simplified ratios do not contain fractions or decimals.

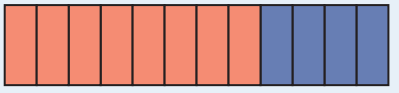
1 : 2

is equivalent to…



2 : 4

8 : 4



simplifies to…

2 : 1

Comparisons with decimals and fractions

- Ratios containing decimals can be written as whole number ratios by multiplying each part of the ratio by a power of 10 (that is, 10, 100, 1000, etc.) to form whole numbers.

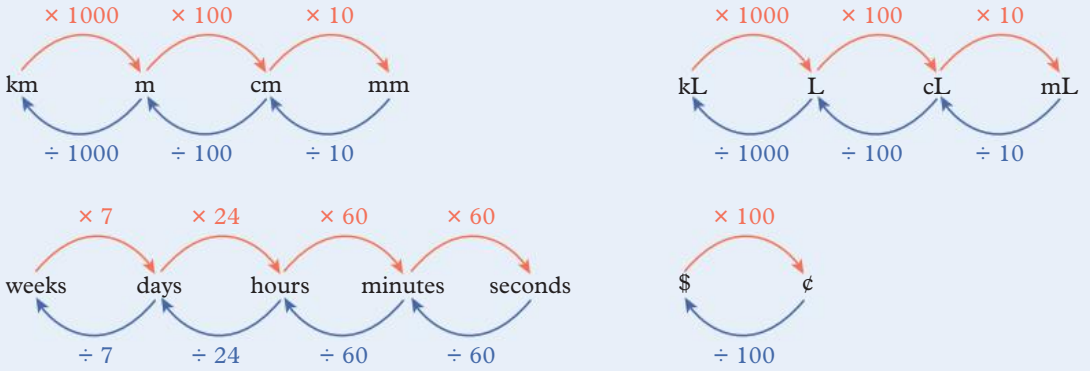
$$\overset{\times 10}{\downarrow}\ 3.4 : 1.3\ \overset{\times 10}{\downarrow} = 34 : 13$$

- Ratios containing fractions can be written as whole number ratios by first writing equivalent fractions with the lowest common denominator (LCD) and then multiplying each fraction by the value of the LCD to form whole numbers.

  → Mixed numbers must first be converted to improper fractions.

Converting units

- Converting between units of measurement is important when calculating ratios.



Example 3F.1 Simplifying ratios

Write each ratio in simplest form.

a 18 : 24 **b** 35 : 56

THINK

a **1** Find a common factor of 18 and 24. 2 is a common factor.
2 Divide the parts of the ratio by 2.
3 Find another common factor and repeat the process. Repeat until the only common factor is 1.

b **1** Find the highest common factor (HCF) of each number in the ratio.
2 Simplify by dividing each number in the ratio by the HCF.

WRITE

a 18 : 24 (÷ 2)
= 9 : 12 (÷ 3)
= 3 : 4

b The HCF of 35 and 56 is 7.
35 : 56 (÷ 7)
= 5 : 8

Example 3F.2 Simplifying ratios involving decimals and fractions

Write each comparison as a ratio in simplest form.

a 4.5 to 2.75 **b** $2\frac{1}{6}$ to $\frac{5}{8}$

THINK

a **1** Write the comparison as a ratio.
2 Multiply each number by 100 to obtain whole numbers.
3 Simplify the ratio by dividing each number in the ratio by the HCF of 25.

b **1** Convert the mixed number to an improper fraction. Write the comparison as a ratio.
2 Identify the lowest common denominator (LCD) of the two fractions. The LCD is 24. Write each fraction as an equivalent fraction with a denominator of 24.
3 Multiply each fraction by 24 to obtain whole numbers.
4 Check if it can be simplified.

WRITE

a 4.5 : 2.75 (× 100)
= 450 : 275 (÷ 25)
= 18 : 11

b $2\frac{1}{6} = \frac{13}{6}$
$\frac{13}{6} : \frac{5}{8}$ (× 4, × 3)
$= \frac{52}{24} : \frac{15}{24}$ (× 24, × 24)
= 52 : 15

Example 3F.3 Simplifying ratios involving different units

Write the comparison 20 seconds to 2 minutes 15 seconds as a ratio in simplest form.

THINK

1 Convert the quantities to the same unit.

2 Write the comparison as a ratio in the order given.

3 Simplify by dividing each number in the ratio by the HCF. The HCF of 20 and 135 is 5.

WRITE

2 minutes = 2 × 60 seconds
= 120 seconds
2 minutes 15 seconds = 120 + 15 seconds
= 135 seconds

÷ 5 20 : 135 ÷ 5
= 4 : 27

Example 3F.4 Working with equivalent ratios

Find the value of a if $3 : 5 = 21 : a$.

THINK

1 Compare the first number in each ratio, 3 and 21. What should 3 be multiplied by to give 21? 3 multiplied by 7 is 21.

2 As the ratios are equivalent, 5 multiplied by 7 is a.

3 Identify the value of a.

WRITE

× 7 3 : 5 × 7
= 21 : a

$5 \times 7 = a$

$a = 35$

Helpful hints

✔ It is okay to compare quantities that contain fractions or decimals in a ratio; however, ratios in their simplest form never contain fractions or decimals.

ANS p495

Exercise 3F Equivalent ratios

△ 1–6(1st, 2nd columns), 7–9, 12–15 | □ 1–6(2nd, 3rd columns), 7(2nd column), 8(f–i), 10–12, 15, 16, 18, 20, 21, 23 | ⬠ 1–3(3rd, 4th columns), 4(d, f, g, i), 7(2nd column), 8(f–i), 10, 15, 17, 19–22, 24, 25

3F.1 **1** Write each ratio in simplest form.

a 7 : 84 **b** 9 : 12 **c** 106 : 36 **d** 22 : 55
e 108 : 72 **f** 45 : 95 **g** 38 : 34 **h** 17 : 102
i 16 : 24 : 42 **j** 75 : 50 : 125 **k** 36 : 60 : 84 **l** 18 : 42 : 48

3F.2 **2** Write each comparison as a ratio in simplest form.

a 3.6 to 7.8 **b** 1.25 to 5.5 **c** 6.8 to 2.48 **d** 3.8 to 11.4
e 2.3 to 5.9 **f** 7.2 to 8.4 **g** 1.05 to 3.15 **h** 8.1 to 1.75
i 7.8 to 2.4 to 4.2 **j** 2.25 to 0.55 to 1.75 **k** 7.2 to 9.6 to 8.4 **l** 1.2 to 4.2 to 1.6

UNDERSTANDING AND FLUENCY

3 Write each comparison as a ratio in simplest form.

a $\frac{1}{6}$ to $\frac{1}{4}$ **b** $\frac{2}{3}$ to $\frac{3}{4}$ **c** $\frac{1}{3}$ to $\frac{2}{5}$ **d** $\frac{3}{7}$ to $\frac{2}{11}$

e $\frac{9}{22}$ to $\frac{5}{11}$ **f** $\frac{4}{7}$ to $\frac{6}{5}$ **g** $6\frac{1}{4}$ to $12\frac{1}{2}$ **h** $5\frac{2}{3}$ to $7\frac{5}{9}$

i $2\frac{1}{8}$ to $\frac{2}{5}$ **j** $3\frac{1}{5}$ to 4 **k** 0.25 to $\frac{1}{8}$ **l** $\frac{3}{4}$ to 0.125

3F.3 **4** Write each comparison as a ratio in simplest form.

a 65 mL to 3.2 L **b** 17 mm to 2.34 m **c** 92 cents to \$17.28

d $\frac{2}{3}$ hours to 0.8 hours **e** 5 days to 10 weeks **f** 3800 L to 4 kL

g 0.9 km to 550 m **h** 1.28 cm to 5 mm **i** $1\frac{3}{4}$ minutes to 420 seconds

5 Which pairs of ratios are equivalent? Explain how you know each pair is equivalent.

a 1 : 2 and 3 : 6 **b** 8 : 5 and 2 : 3 **c** 4 : 10 and 2 : 5 **d** 2 : 9 and 10 : 45

e 16 : 25 and 4 : 5 **f** 63 : 42 and 9 : 7 **g** 2 : 3 and 22 : 34 **h** 15 : 9 and 5 : 3

i 1 : 5 and 15 : 30 **j** 17 : 51 and 68 : 204 **k** 49 : 64 and 7 : 8 **l** 27 : 18 and 3 : 2

3F.4 **6** Find the value of each pronumeral.

a $5 : 9 = 25 : a$ **b** $15 : 50 = 3 : b$ **c** $9 : 2 = c : 16$ **d** $8 : 36 = d : 9$

e $e : 55 = 19 : 11$ **f** $f : 4 = 42 : 24$ **g** $2 : g = 22 : 66$ **h** $20 : h = 5 : 2$

i $i : 23 = 12 : 92$ **j** $j : 6 = 30 : 60$ **k** $15 : k = 3 : 17$ **l** $18 : l = 2 : 13$

m $7 : 15 = 49 : m$ **n** $72 : 168 = 6 : n$ **o** $68 : 140 = x : 35$ **p** $96 : 80 = p : 5$

7 Fill in the gaps to complete the equivalent ratios.

a 2 : 5 = _____ : 20 = 20 : _____ **b** 6 : 7 = _____ : 35 = 54 : _____

c _____ : 360 = 70 : 60 = 7 : _____ **d** 225 : _____ = 9 : 12 = _____ : 4

e _____ : 104 = 68 : _____ = 17 : 13 **f** _____: 48 = 75 : _____ = 15 : 4

g 20 : _____ = 40 : 96 = _____ : 240 **h** _____: 636 = 72 : 212 = 18 : _____

8 Find the value of the letter in each equivalent ratio statement. Hint: Simplify the complete ratio in the statement first.

a $a : 24 = 20 : 30$ **b** $b : 52 = 20 : 13$ **c** $13 : c = 65 : 5$

d $25 : d = 10 : 14$ **e** $48 : 32 = 18 : e$ **f** $33 : 44 = f : 12$

g $33 : 54 = g : 72$ **h** $15 : 20 = 36 : h$ **i** $40 : 36 = i : 27$

PROBLEM SOLVING AND REASONING

9 Write each quantity as a ratio in simplest form.

a The number of students in a school is 975 while the number of teachers is 75.

b Mix 25 mL of cordial with 225 mL of water.

c 8500 Diamonds fans and 4500 Silver Ferns fans watched a netball match between the two teams.

d I have 8 gold bracelets and 14 silver bracelets.

10 Eric paid a deposit of \$704.70 for his overseas trip, which cost \$4698. Write the ratio, in simplest form, of the deposit paid to the amount owing.

11 The ratio of rainy days to sunny days in a month was 2 : 3. If there were 18 sunny days, how many rainy days were there?

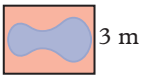
12 The ratio of girls to boys in a class is 5 : 4. If there are 15 girls in the class, find:

a how many boys are in the class

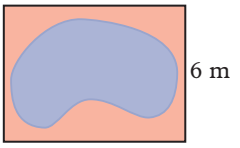
b how many students are there in total.

13 The scale on a map is given as 1 : 500 000, which means that each centimetre in the map represents 500 000 cm in real life. The distance between Narrabarba and Kiah on the map is 2.4 cm. What is the real-life distance between the two locations?

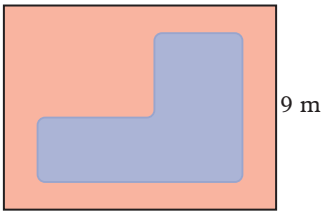
14 When you compare the ratio of a part to the whole, the ratio can be written as a fraction. Consider the four different pool enclosures pictured.



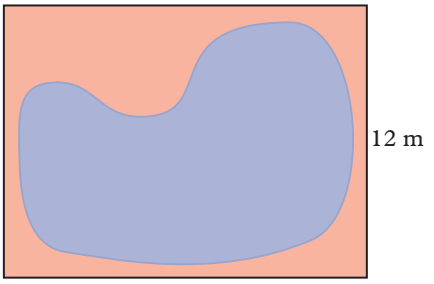
Enclosure A



Enclosure B



Enclosure C



Enclosure D

a Complete the table for each pool enclosure.

Pool enclosure	Length	Width	Perimeter
A			
B			
C			
D			

b For each enclosure, write the ratio of the length to the perimeter.

c Describe a pattern in your answers to part **b**.

d Use your answers to part **b** to write each ratio as a fraction in simplest form.

e Compare the fractions you have obtained. What is special about them?

f Describe what this fraction tells you about the length and perimeter of each pool enclosure.

15 The ratio of blue pens to black pens to red pens in a classroom is 7 : 4 : 1. If there are 24 black pens in the classroom, how many pens are there altogether?

16 For best results, the instructions on a new cleaning product recommend the ratio of detergent to water be 2 : 7.

a How many litres of water are required if 300 mL of detergent is used?

b How many millilitres of detergent are required if 28 L of water is used?

17 A two-stroke fuel mixture is made by mixing oil and petrol in the ratio of 1 : 25.

a How many litres of petrol should be added to 850 mL of oil?

b How many millilitres of oil should be added to 15 L of petrol?

18 Gordon bought shares in a Telco company for $8.20 and sold them for $40.80. Find the ratio, in simplest form, of:

a the cost price to the selling price

b the cost price to the difference between the selling price and the cost price.

PROBLEM SOLVING AND REASONING

19 Imelda purchased a pair of shoes that had been discounted by 60%. The shoes were originally priced at $850. Find the ratio, in simplest form, of:

a the cost price to the sale price

b the discount compared to the cost price

c the discount compared to the sale price.

20 Jasmine is making Anzac biscuits for her class. Some of the ingredients to create 16 biscuits are listed on the right.

Anzac biscuits (makes 16)

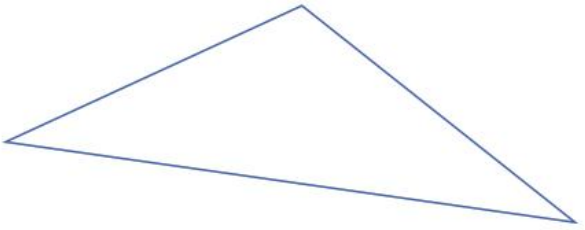
1 cup rolled oats
1 cup plain flour
1 cup desiccated coconut
0.5 cups sugar

a Explain why the listed ingredients can be expressed as a ratio.

b How much of each ingredient would be needed to make 64 cookies?

c How much of each ingredient would be needed to make 8 cookies?

21 The side lengths of a scalene triangle are in the ratio 7 : 8 : 5.



a If the shortest side length is 17.4 cm, find the length of the longest side.

b Find the perimeter of the scalene triangle.

CHALLENGE

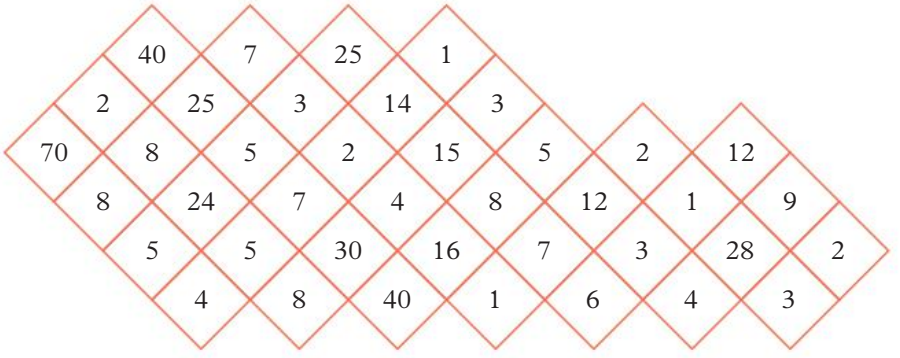
22 a Comment on how the part-to-whole ratio 22 : 27 compares with the part-to-whole ratios 8 : 9 and 12 : 15.

b Obtain three ratios that lie between the part-to-whole ratios 22 : 27 and 12 : 15.

23 Compare the part-to-whole ratios 4 : 5 and 10 : 13 and state which is larger.

24 Without using equivalence, compare the part-to-whole ratios 17 : 33 and 12 : 25 and state which is larger. Explain your answer.

25 Using only the numbers below, find 10 pairs of equivalent ratios. You must use all the numbers in the diagram and each number can only be used once. You may like to present your answers in a table like the one below.



1st ratio	2nd ratio

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Ratios in simplest form

Investigation
Motorbike dilemmas

Topic quiz
3F

3G Working with ratios

Learning intentions

By the end of this topic you will be able to ...

✔ divide a quantity into a given ratio

✔ use the unitary method to find a quantity from a given ratio.

Inter-year links

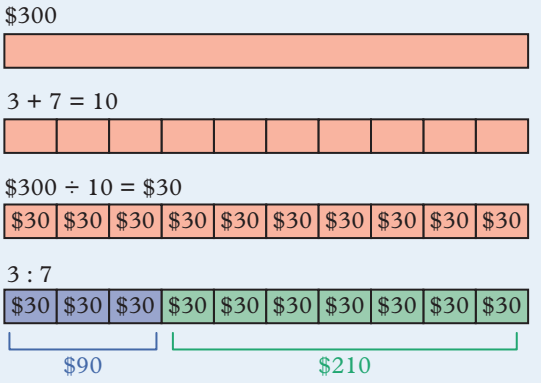
Support	Dividing a decimal by a whole number
Year 7	4E Dividing decimals

Dividing a quantity into a given ratio

Key content video

- When dividing a quantity in a given ratio, follow these steps:
  1. Find the total number of parts in the ratio.
  2. Divide the quantity by the total number of parts.
  3. Multiply the result of the previous step by each of the ratio.

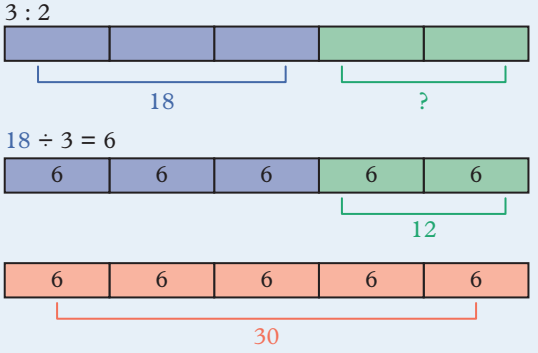
  For example, dividing \$300 into 3 : 7 gives \$90 and \$210.
- Remember to include the same units with your answer.
- One way to check that you have divided the quantity correctly is to add the individual amounts to see whether the result is the same as the original quantity.



Finding a quantity from a given ratio

- The unitary method can be used to find a quantity from a given ratio:
  1. Determine the value of 1 part of the ratio. Divide the given quantity by the number of parts in the corresponding side of the ratio.
  2. When the value of 1 part is found, any part of the ratio and the whole can be found.

  For example, if the ratio of rice to water in a recipe is 3 : 2 and a chef uses 18 cups of rice:
  - → 18 cups of rice is related to the 3 parts in the ratio.
  - → 18 divided by 3 gives 1 part equal to 6 cups.
  - → There are 2 parts water to 3 parts rice, so the water required to match the rice is 12 cups.
  - → 30 cups in total are required for this recipe.



Example 3G.1 Dividing a quantity into a ratio

Divide 4800 in the ratio 3 : 1.

THINK	WRITE
1 Find the total number of parts in the ratio.	Number of parts $= 3 + 1$ $= 4$
2 Divide the quantity by the total number of parts.	$4800 \div 4 = 1200$
3 Multiply 1200 by the number of parts in each section of the ratio.	$1200 \times 3 = 3600$ $1200 \times 1 = 1200$ The ratio 3 : 1 divides 4800 into 3600 and 1200.

Example 3G.2 Dividing a decimal quantity into a ratio

Divide \$20.25 into the ratio 4 : 5.

THINK	WRITE
1 Find the total number of parts in the ratio.	Total number of parts $= 4 + 5$ $= 9$
2 Divide by the total number of parts.	$20.25 \div 9 = 2.25$
3 Multiply 2.25 by the number of parts in each section of the ratio. Remember to give your answer in dollars.	$2.25 \times 4 = 9$ $2.25 \times 5 = 11.25$ The ratio 4 : 5 divides \$20.25 into \$9 and \$11.25.

Example 3G.3 Dividing a fraction quantity into a ratio

Divide $\frac{11}{45}$ into the ratio 2 : 3.

THINK	WRITE
1 Find the total number of parts in the ratio.	Total number of parts $= 2 + 3$ $= 5$
2 Divide the quantity by the total number of parts.	$\frac{11}{45} \div 5 = \frac{11}{45} \times \frac{1}{5}$ $= \frac{11}{225}$
3 Multiply $\frac{11}{225}$ by the number of parts in each section of the ratio. Remember to simplify your fractions where possible.	$\frac{11}{225} \times 2 = \frac{22}{225}$ $\frac{11}{225} \times 3 = \frac{33}{225}$ $= \frac{11}{75}$ The ratio 2 : 3 divides $\frac{11}{45}$ into $\frac{22}{225}$ and $\frac{11}{75}$.

Example 3G.4 Finding a quantity from a given ratio

The ratio of cats to dogs in a neighbourhood is 11 : 6. If there are 102 dogs in the neighbourhood:

a how many cats are there

b how many cats and dogs are there in total?

THINK	WRITE
a **1** Determine the total number of parts in the ratio.	**a** 11 + 6 = 17
2 Determine the number of pets in one part of the ratio by dividing the given number of pets (102) by the number of parts in this section (6).	102 ÷ 6 = 17
3 Multiply the value from step **2** by the number of parts the cats represent.	17 × 11 = 187 There are 187 cats in the neighbourhood.
b Add the number of cats to the number of dogs.	**b** 187 + 102 = 289 There are 289 cats and dogs in total.

Helpful hints

- ✔ Always add your ratio parts together to find out how many parts there are in total.
- ✔ If you need to refresh your skills on multiplying and dividing decimals and fractions refer to Chapter 2. There is some good stuff in there!
- ✔ You can consider the process of finding one part as simplifying. From **Example 3G.4**:

6 parts = 102 dogs

÷ 6 ↓ ↓ ÷ 6

1 part = 17 dogs

ANS p496

Exercise 3G Working with ratios

△ 1–10, 12–15, 17, 22 □ 4–7, 10, 11, 13, 15, 16, 18, 19, 21, 23(a–d) ⬠ 5–7, 13, 15, 18, 20–24

1 Divide 500 in the following ratios. Use the diagram to help you.

a 1 : 9 **b** 2 : 8 **c** 3 : 7 **d** 6 : 4 **e** 3 : 2 **f** 4 : 1

3G.1 **2** Divide $9200 in each ratio.

a 3 : 2 **b** 5 : 3 **c** 3 : 1 **d** 1 : 1 **e** 21 : 25 **f** 2 : 8

3 Divide 36 000 km in each ratio.

a 5 : 7 **b** 7 : 1 **c** 10 : 8 **d** 4 : 5 **e** 12 : 3 **f** 1 : 1

4 Divide $240 in each ratio.

a 1 : 2 **b** 9 : 1 **c** 2 : 10 **d** 10 : 6 **e** 3 : 1 : 4 **f** 7 : 2 : 1

5 Divide 300 kg in each ratio.

a 4 : 6 **b** 1 : 9 **c** 13 : 2 **d** 4 : 11 **e** 4 : 5 : 1 **f** 8 : 5 : 3 : 4

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

3G.2 **6** Divide \$3.60 in each ratio.

a 3 : 1 **b** 1 : 8 **c** 11 : 1 **d** 5 : 4 **e** 1 : 3 : 5 **f** 2 : 3 : 7

3G.3 **7** Divide $\frac{8}{15}$ in each ratio.

a 2 : 1 **b** 3 : 2 **c** 3 : 5 **d** 6 : 2 **e** 2 : 3 : 3 **f** 6 : 5 : 4

8 Use the diagram in Question **1** to divide 35 cups of water in each ratio.

a 3 : 2 **b** 4 : 1 **c** 7 : 3 **d** 9 : 1 **e** 3 : 5 : 2 **f** 3 : 4 : 3

3G.4 **9** The ratio of fiction to non-fiction books in a library is 7 : 8. If there are 3500 fiction books in the library:

a how many non-fiction books are there **b** how many books are there in total?

10 The ratio of tomato seedlings to chilli seedlings in Paul's garden is 5 : 7. If there are 21 chilli seedlings:

a how many tomato seedlings are there **b** how many seedlings are there in total?

11 The ratio of germinated seeds to non-germinated seeds in Rita's garden is 9 : 4. If there are 4500 germinated seeds:

a how many non-germinated seeds are there

b how many seeds are there in total?

PROBLEM SOLVING AND REASONING

12 The ratio of female to male fans at a football training session is 5 : 2. If there are 224 fans at the training session, how many are male?

13 Morning Sunshine fruit juice drink contains fruit juice and water in the ratio 1 : 3.

a How much water (in mL) is in a 2 L container of Morning Sunshine? Hint: 1 L = 1000 mL.

b How much fruit juice is in 600 mL of Morning Sunshine?

14 The size of the angles in a triangle are in the ratio 3 : 4 : 2. What is the size of each angle? Hint: The sum of angles in any triangle is 180°.

15 Complete the following steps to divide \$300 in the ratio 4 : 6.

a Calculate the total number of parts in the ratio.

b Write each number in the ratio as a fraction of the total number of parts.

c Multiply each fraction from part **b** by the quantity (\$300).

d Check that the amounts found in part **c** add to \$300.

16 Nadia divides her pocket money for April in the ratio 3 : 2 : 1 to cover buying a gift, entertainment expenses and savings. Calculate the amount she has allocated for entertainment expenses if her total pocket money for April is \$84.

17 Katie, Mike and Sam contributed different amounts to a \$5 raffle ticket. When they won, they distributed the winnings in the same ratio as they put in. If Katie won \$450, Mike won \$200 and Sam won \$350, find:

a the total amount won

b the ratio of their contribution

c how much they each put towards the ticket.

18 Sarah, Cameron and Hunter bought a \$5 raffle ticket. If Sarah contributed \$2.50, Cameron contributed \$1.50 and Hunter contributed \$1.00, find:

a the ratio of their contributions

b their individual winnings, if the ticket won:

i \$50 **ii** \$150 **iii** \$700 **iv** \$2000 **v** \$10 000 **vi** \$13 000.

PROBLEM SOLVING AND REASONING

19 Jia is the treasurer of her netball club which has \$200 in its budget. She has decided that $\frac{3}{20}$ of the budget will be spent on a present for the coach, $\frac{1}{4}$ of the budget will be spent on trophies for all the players and the rest of the budget will be spent on snacks at the games.

a What fraction of the budget will be spent on snacks?

b Write a ratio representing the money Jia will spend on the present, trophies and snacks.

c Find the percentage that is allocated to each part of the budget.

d Find how much money Jia will spend on each part of the budget.

20 Bella wants to organise her time in the holidays. She decides she will spend $\frac{2}{5}$ of her time catching up with friends, $\frac{1}{4}$ of her time doing outdoor activities, $\frac{1}{10}$ of her time doing her homework and chores, and the remainder of her time relaxing at home.

a Find the percentage of time that Bella will catch up with friends, do outdoor activities and do homework or chores. Hint: For each fraction find an equivalent fraction with a denominator of 100.

b From part **a**, find the percentage of time she will relax at home.

c Write these percentages as a ratio in simplest form.

d Assuming that for the two-week holiday she has 10 'usable' hours a day, find the number of hours Bella spends on each activity.

21 The perimeter of a rectangular computer screen is 228 cm, with a width-to-height ratio of 12 : 7.

a Calculate the width and height of the screen.

b Calculate the area of the screen.

22 How do the questions in this topic highlight the difference between a ratio that shows the relationship of part to part, and a fraction that shows the relationship of part to whole?

CHALLENGE

23 To create two distinct shades of pink paint, the following ratios of white to red paint are required: 4 : 1 and 7 : 1. A 4 L can in each shade is purchased.

a Calculate the amounts (in mL) of white and red paint in each can.

b The two cans are accidentally mixed together. What is the new ratio of white to red paint?

c Write the amount of red paint as a fraction, in simplest form, of the mixed paint.

d Write the amount of white paint as a percentage of the mixed paint.

e The mixed paint is divided evenly into two cans. How much red and while paint would have to go into each can for the shades to be exactly the same?

24 An aquarium has fish and amphibians. The amphibians are frogs and newts. There are 40 more fish than amphibians.

The ratio of frogs to newts to fish is 2 : 5 : 11. How many animals does the aquarium have?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Dividing a quantity into a ratio

Interactive skillsheet
Finding a quantity from a given ratio

Investigation
On the ropes

Topic quiz
3G

3H Rates

Learning intentions

By the end of this topic you will be able to ...

✔ write a rate in simplest form
✔ use a rate to solve a problem
✔ calculate the best buy.

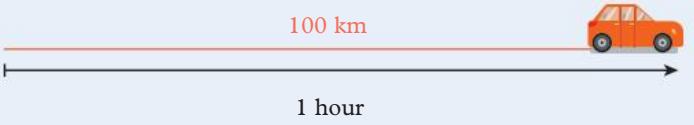
Inter-year links

Year 7	4E Dividing decimals
Year 9	4E Direct variation

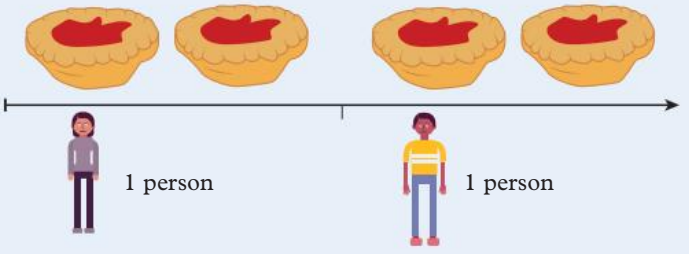
Rates

Key content video

- A **rate** is a comparison between two or more different quantities with different units. It is a measure of how a quantity changes for each unit of another quantity. The second quantity being compared is often a measure of time.
- The unit of a rate is two units separated by the word 'per' (meaning 'for each') or the symbol /. For example:
  → 100 km/h means the speed at which 100 km is covered every hour.

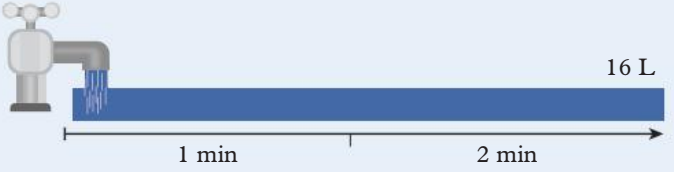


  → 2 meat pies per person means each person gets 2 meat pies.

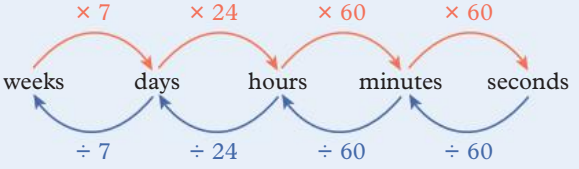


  → $500 per week means someone receives $500 each week.

  → 8 L/min means that for each minute, 8 litres flow.



- Order is important when writing a rate.
- Converting units of measurement, especially between units of time, is important when calculating rates.



Simplified rates

- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.
  1. Write the rate as a fraction including the units.
  2. Simplify the fraction so that the denominator is 1.
  3. Write your answer as a rate using '/' to indicate 'per'.

$$\frac{36\text{ km}}{12\text{ h}} \overset{\div 12}{=} \frac{3\text{ km}}{1\text{ h}} = 3\text{ km/h}$$

- When an item is available for sale at different rates, the unitary method can be used to determine the **best buy** of the item. The option which has the lowest unit rate will be the best buy.

$$\$25 \text{ for } 200\text{ g} = \frac{\$25}{200} \text{ for } \frac{200\text{ g}}{200}$$
$$= \$0.125 \text{ for } 1\text{ g}$$
$$\$55 \text{ for } 500\text{ g} = \frac{\$55}{500} \text{ for } \frac{500\text{ g}}{500}$$
$$= \$0.11 \text{ for } 1\text{ g}$$

Therefore, \$55 for 500 g is the best buy.

Example 3H.1 Writing statements as rates in simplest form

Write each statement as a rate in simplest form.

a 30 m in 5 seconds

b \$54 for 40 L of petrol

THINK

a 1 Write the two quantities as a fraction including the units.

2 Simplify the fraction so that the denominator is 1. For the rate to be in simplest form, the second quantity needs be 1. Divide both quantities by 5.

3 Write your answer as a rate using '/' to indicate 'per'.

b 1 Write the two quantities as a fraction including the units.

2 Simplify the fraction so that the denominator is 1. Divide both quantities by 40.

3 Write your answer as a rate using '/' to indicate 'per'. The dollar sign '\$' must be written before the number.

WRITE

a 30 m in 5 seconds $= \frac{30\text{ m}}{5\text{ s}}$

$$\frac{30\text{ m}}{5\text{ s}} \overset{\div 5}{=} \frac{6\text{ m}}{1\text{ s}}$$

$$\frac{6\text{ m}}{1\text{ s}} = 6\text{ m/s}$$

b \$54 for 40 L of petrol $= \frac{\$54}{40\text{ L}}$

$$\frac{\$54}{40\text{ L}} \overset{\div 40}{=} \frac{\$1.35}{1\text{ L}}$$

$$\frac{\$1.35}{1\text{ L}} = \$1.35/\text{L}$$

Example 3H.2 Solving problems using rates

Will loves to read. He can read 56 pages in half an hour.

a How many pages can he read in 2 hours?

b How many pages can he read in 3.5 hours?

c How long will it take him to read 1008 pages?

THINK

a **1** Write the two quantities as a fraction including the units. Simplify the fraction so that the denominator is 1.

2 Multiply the hourly rate by 2 to work out the new amount. The number of pages Will can read in 2 hours is found in the numerator of the equivalent rate.

b Multiply the hourly rate by 3.5 to work out the new amount. The number of pages Will can read in 3.5 hours is found in the numerator of the equivalent rate.

c Divide the number of pages by the number of pages Will can read in one hour.

WRITE

a 56 pages per half hour $= \dfrac{56 \text{ pages}}{0.5 \text{ hours}}$

$= \dfrac{112 \text{ pages}}{1 \text{ hour}}$

$$\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 2}{=} \frac{224 \text{ pages}}{2 \text{ hours}}$$

Will can read 224 pages in 2 hours.

b

$$\frac{112 \text{ pages}}{1 \text{ hour}} \overset{\times 3.5}{=} \frac{392 \text{ pages}}{3.5 \text{ hours}}$$

Will can read 392 pages in 3.5 hours.

c $1008 \div 112 = 9$

It takes Will 9 hours to read 1008 pages.

Example 3H.3 Determining the best buy

Determine which of these represents the best buy using the unitary method:
\$38.01 for 7 kg of oranges or \$28.50 for 4.5 kg of oranges.

THINK

1 For each purchasing option, write the two quantities as a rate.

2 Divide each price by the number of kilograms to determine the cost of 1 kg. Where necessary, round each amount to the nearest cent.

3 Compare the prices for 1 kg to determine which option is the best buy.

WRITE

Option 1: \$38.01 for 7 kg $= \dfrac{\$38.01}{7}$ for $\dfrac{7 \text{ kg}}{7}$

$=$ \$5.43 for 1 kg

Option 2: \$28.50 for 4.5 kg $= \dfrac{\$28.50}{4.5}$ for $\dfrac{4.5 \text{ kg}}{4.5}$

$\approx$ \$6.33 for 1 kg

\$38.01 for 7 kg or oranges is the best buy.

Helpful hints

- ✔ Make sure that you identify the units of the rate. It's important to understand the units because they determine how you deal with them.
- ✔ Choose multiplication or division to manipulate your rate to give your desired units.
- ✔ Don't forget to include units in your answer. Using units in your working will help ensure they also appear in your answer.
- ✔ When using the unitary method, you are often given the value of more than 1 unit. However, sometimes you are given less than 1 unit and you might have to multiply (or divide by a decimal) to find the value of 1 unit. For example, \$2 per 0.5 litre when doubled is \$4 per litre.

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Exercise 3H Rates

△ 1–2(1st column), 3–5, 7, 9, 11–13, 16 ▢ 2(2nd column), 3, 5–10, 14–17 ⬠ 3, 6, 7, 9, 10, 12, 15–19

1 Write each statement as a rate with the appropriate unit.

a 60 km in each hour
b 591 L in each minute
c four quarters in each game
d \$16 for each hour
e 45 cm each year
f five patients each hour
g \$1.49 for each day
h \$28.99 for each month
i \$5.80 for each box
j 0.5 kg for each week
k 290 mm each year
l 14 km for each litre

3H.1 **2** Write each statement as a rate in simplest form.

a 200 m in 4 minutes
b 6 L in three bottles
c 80 cents for 20 seconds
d 135 students with nine teachers
e 468 seats in 18 rows
f 100 newspapers in four bundles
g 68 runs in 10 overs
h \$20 for eight muffins
i 102 goals in 15 games
j 39 996 tickets sold in 9 minutes
k 77 goals over 22 games
l area of 33.6 m^2 requires 6.4 L of paint

3 Write a rate in its simplest form to represent each situation. Give your answers to two decimal places where appropriate.

a A car uses 18 L of petrol to travel 252 km.
b The cost of 15 mangoes is \$29.85.
c A bath fills up to 54 L in $4\frac{1}{2}$ minutes.
d A runner takes $2\frac{3}{4}$ hours to complete a 42.2 km marathon.
e New South Wales's population is 8 072 163 and its area is 801 150 km^2.
f Light travels 1080 million km in 60 minutes.

3H.2 **4** Lina can type 1200 words in half an hour.

a How many words can she type in 1 hour?
b How many words can she type in 20 minutes?
c How long does it take Lina to type 1800 words?

5 Boris earns \$182.40 for working 12 hours.

a How much does he earn for working 1 hour?

b How much does he earn for working 40 hours?

c How long does Boris have to work to earn \$1459.20?

6 Sound travels 23 160 m in 60 seconds.

a How far does sound travel in 1 second?

b How far does sound travel in 1 hour?

c How long does it take for sound to travel 772 000 m?

3H.3 **7** Determine which one of these represents the best buy using the unitary method.

	Option A	Option B
a	\$28.40 for 4 kg	\$34 for 5 kg
b	\$411.55 for 5 L	\$812.30 for 10 L
c	\$31.92 for 3.5 m	\$19.10 for 2.5 m
d	\$312.50 for 25 m^2	\$273.60 for 18 m^2
e	\$8.22 for 1.2 kg	\$5.76 for 900 g
f	\$9.99 for 1.25 L	\$4.95 for 600 mL
g	\$11.73 for 1235 cm	\$8.65 for 9.4 m
h	\$78 for 8000 cm^2	\$765.60 for 8 m^2

8 The best buy can also be determined by finding the amount you can buy per dollar.

For example:

Option A: 48 pencils for \$12 $= \frac{48 \text{ pencils}}{12}$ for $\frac{\$12}{12}$

$= 4$ pencils for \$1

Option B: 50 pencils for \$10 $= \frac{50 \text{ pencils}}{10}$ for $\frac{\$50}{10}$

$= 5$ pencils for \$1

\$10 for 50 pencils is better value than \$12 for 48 pencils as you get more pencils for each \$1 you spend.

Determine which of these represents the best buy using the unitary method per \$1.

	Option A	Option B
a	4 kg for \$2.50	3 kg for \$1.80
b	50 g for \$0.30	75 g for \$0.50
c	250 mL for \$3.20	750 mL for \$10.20
d	2.5 L for \$8.50	10 L for \$35.50
e	6.8 m for \$120	5.6 m for \$100
f	4520 mm for \$52	3780 mm for \$42
g	49 m^2 for \$200	62 m^2 for \$250
h	1.98 ha for \$365 000	12.14 ha for \$3 900 000

9 Max's team scored 230 runs in 40 overs. William's team scored 280 runs in 50 overs.

a Calculate each team's run rate per over.

b Which team had a better run rate?

10 Leandro's resting heart rate is 14 beats in 15 seconds.

a Heart rate is often expressed in beats per minute (bpm). Write Leandro's heart rate in bpm.

b How many minutes does it take for Leandro's heart to beat 1512 times?

11 Usain Bolt recorded a world-record time of 9.58 seconds for the 100 m sprint. Compare his rate with that of:

a Michael Johnson's time of 19.32 seconds for the 200 m sprint

b The US 4×100m relay team of 1992, who recorded a time of 37.4 seconds

c A cheetah that is able to cover a distance of 600 m in 37.5 seconds.

12 Victor takes 48 minutes to run 15 km.

a What distance does he cover per minute?

b If Victor is able to maintain this rate, what distance would he cover in these times?

i 55 min **ii** 60 min **iii** 1 hour 25 min **iv** 1.5 h

c At the same rate, how long would it take Victor to cover these distances?

i 625 m **ii** 31.25 km **iii** 25 km **iv** 2 km

13 Compare each pair of grocery items and work out which option is the better value for money.

a 5 kg of potatoes for \$4.95 or 7.5 kg for \$7.15

b a packet of four batteries for \$3.99 or a packet of ten for \$9.89

c a 500 g tin of coffee for \$9 or a 150 g jar for \$2.99

d a 440 g tin of peaches for \$1.85 or a 650 g tin for \$2.56

14 A friend tells you that the best buy is always the option with the lowest advertised price.

a Comment on the accuracy of this statement.

b When is the best buy the option with the lowest advertised price?

15 A phone company offers its customers two plans.

- Plan A: connection fee of 40 cents and then 40 cents per minute or part thereof
- Plan B: connection fee of 50 cents and then 38 cents per minute or part thereof

a Calculate the cost of a 1-minute call with each plan. Which offers the best deal?

b Calculate the cost of a 3-minute call with each plan. Which offers the best deal?

c Calculate the cost of a 5-minute call with each plan. Which offers the best deal?

d Calculate the cost of a 6-minute 23-second call with each plan. Which offers the best deal?

e Use your answers from parts **a–d** to explain when it would be best to use each plan.

16 When repaying a loan from a bank, you pay interest on top of the amount of the loan. The amount of interest, I, depends on the original amount you borrowed, P, the interest rate charged per year, r, and the time, T, it takes to repay the loan in years. Simple interest is calculated using the formula:

$$I = P \times r \times T$$

For each of the following sets of values:

i convert the interest rate to a fraction **ii** calculate the total simple interest charged

iii find the sum of the interest and the original loan amount, which is the total owed to the bank.

a $P = \$1000$, $r = 5\%$, $T = 2$ years

b $P = \$1200$, $r = 2.5\%$, $T = 3$ years

c $P = \$5000$, $r = 4\%$, $T = 5$ years

17 It takes 6 window washers 6 minutes to wash 6 windows. How long does it take 12 window washers to wash 12 windows?

18 Jack can paint a room in 3 hours and his friend can paint the same room in 4 hours. How quickly can they paint the room if they work together? Give your answer correct to the nearest minute.

19 Siv starts rowing across a 400 m wide river at a speed of $\frac{14}{9}$ m/s. At the same time Riley starts swimming from the other side of the river at a rate of 2 min per 100 metres. How far from Siv's starting point do they meet?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Rates

Investigation
Solution rates

Topic quiz
3H

PROBLEM SOLVING AND REASONING

CHALLENGE

3I Distance–time graphs

Learning intentions

By the end of this topic you will be able to ...

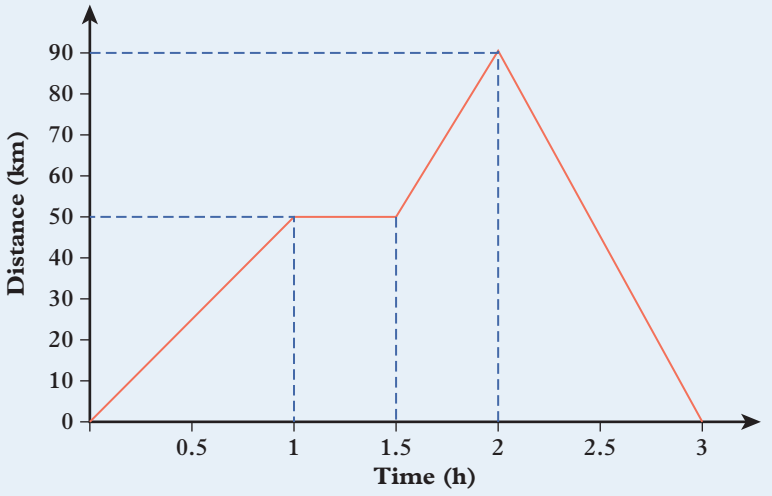
- ✔ interpret distance–time graphs
- ✔ calculate the speed for different parts of a journey from a distance–time graph
- ✔ create a distance–time graph from the details of a journey.

Inter-year links
Year 7 5E Line graphs

Distance–time graphs

Key content video

- A **distance–time graph** tracks the distance an object is from a fixed origin point (on the vertical axis) over time (on the horizontal axis). Distance–time graphs are constructed using connected straight-line segments that represent different parts of the object's journey.



- The **speed** the object is travelling at any time can be calculated by considering the **slope** of each line segment. The steeper the slope, the faster the object is travelling in that direction.
  - → A positive slope is a line segment pointing upwards (to the right) and represents movement away from the origin.
  - → A negative slope is line segment pointing downwards (to the right) and represents movement towards the origin.
  - → A horizontal slope represents no movement from or towards the origin (staying the same distance away).
- Speed is an example of a rate. The speed of an object can be calculated by dividing the distance travelled by the time taken.

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

  - → When calculating speed, only consider the distance travelled and time taken while travelling at that given speed.
- The average speed across an entire journey is the rate of the total distance travelled to the time taken to travel that distance.

Example 3I.1 Interpreting a distance–time graph

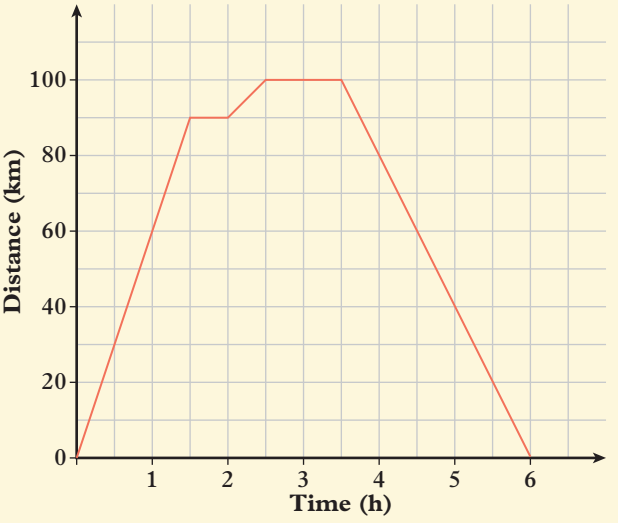
Jack went for a drive with his family. Their distance from home is shown in the distance–time graph on the right.

a How far from home was Jack after one hour?

b How long was it until Jack was first 30 km from home?

c What is the furthest distance Jack travelled from home?

d How long does it take Jack to get home from the point in part **c**?



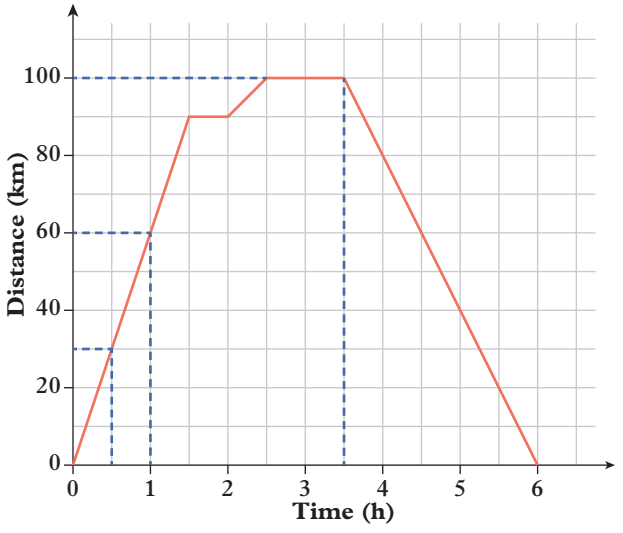
THINK

a Read the value on the distance axis where the time is 1 hour.

b Read the value on the time axis where the distance is 30 km.

c Read the value on the distance axis where the graph is first at the highest point.

d Determine the difference between the times from the end of the 100 km section to the intercept on the time axis (indicating a return home).



WRITE

a 60 km

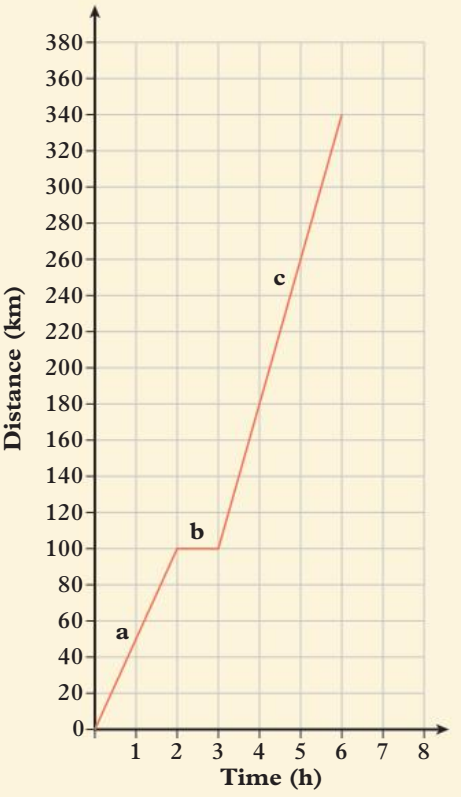
b 0.5 hour

c 100 km

d 6 – 3.5 = 2.5 hours

Example 3I.2 Calculating speeds from a distance–time graph

Determine the speed for each segment (**a**–**c**) of the distance–time graph on the right in kilometres per hour (km/h).



THINK

Calculate each speed by dividing the distance travelled by the time taken.

WRITE

a $\frac{100 - 0}{2} = \frac{100}{2}$

$= 50$ km/h

b $\frac{100 - 100}{3 - 2} = \frac{0}{1}$

$= 0$ km/h

c $\frac{340 - 100}{6 - 3} = \frac{240}{3}$

$= 80$ km/h

Example 3I.3 Constructing a distance–time graph

Tiaz went for a bike ride to the shops, which were located 8 km away, and then rode back home. Use the following descriptions to construct a distance–time graph of Tiaz's bike ride.

- rode at a constant speed for 30 minutes travelling 5 km
- took a 5-minute rest at the park
- rode for another 20 minutes at a constant speed in the same direction until reaching the shops
- took 10 minutes at the shops
- rode at a constant speed of 4 km/h all the way home

THINK

1 Calculate any unknown time durations. For the last segment Tiaz rode 8 km at a speed of 4 km/h, which would take 2 hours.

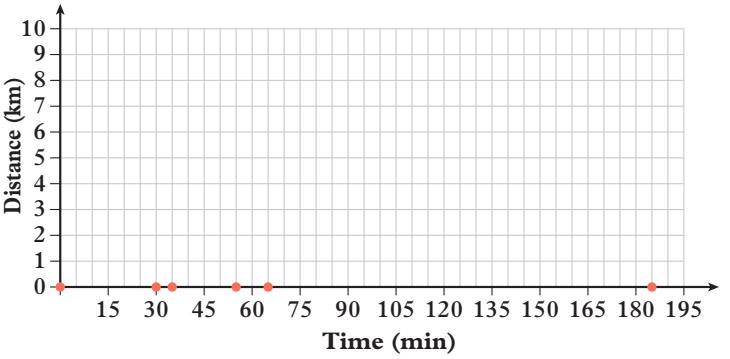
2 Draw axes, putting distance on the horizontal axis and time on the vertical axis. Use a scale to cover the maximum distance from home and the total time of the journey.

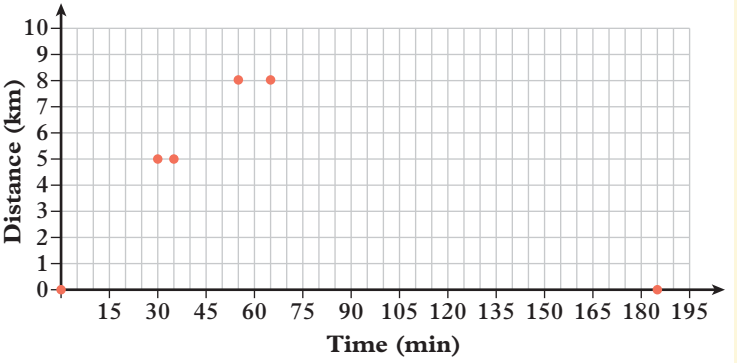
3 Mark the times where Tiaz changes speeds on the horizontal axis: 0, 30, 35, 55, 65 and 185 minutes.

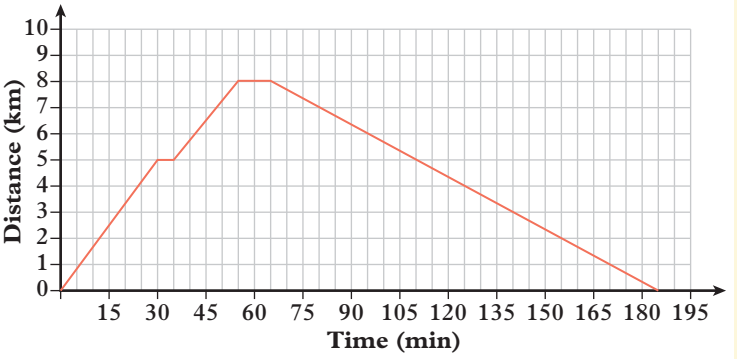
4 Determine the distance (in km) Tiaz is from home at each of the times from step **3**: 0, 5, 5, 8, 8, 0. Mark these points as coordinates on the graph.

5 Join the points from left to right using line segments.

WRITE







Helpful hints

- ✔ The units for distance and time must be written on the axes.
- ✔ Speed is a rate given by dividing the distance unit by the time unit. For example, km/h or m/s.
- ✔ The segments of a distance–time graph give the distance travelled in relation to the origin during each time period. They do not represent the shape of the paths (e.g. hills and valleys) travelled.

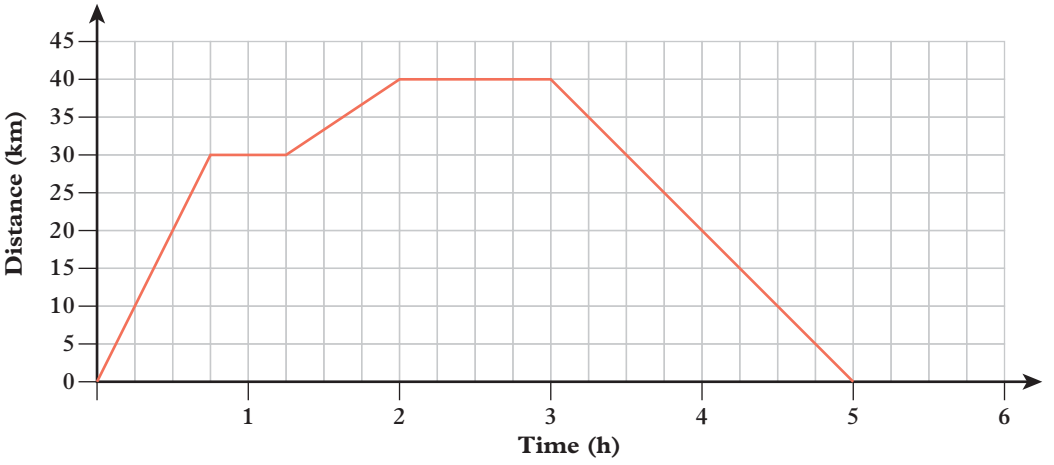
ANS p498

Exercise 3I Distance–time graphs

1–5, 7, 9, 12, 13 | 1, 3, 5, 7, 8, 10–15 | 2, 4, 6, 8–11, 14–17

UNDERSTANDING AND FLUENCY

3I.1 **1** Jacinta went for a drive with her family. Their distance from home is shown in the following distance–time graph.



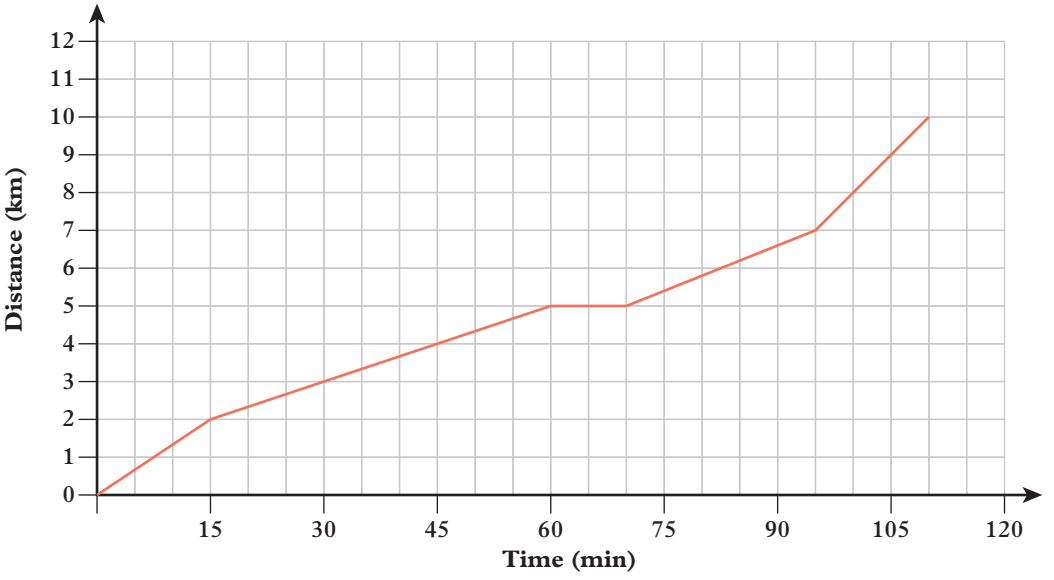
a How far from home was Jacinta after one hour?

b How long was it until Jacinta was first 20 km from home?

c What is the furthest distance Jacinta travelled from home?

d How long does it take Jacinta to get home from the point in part **c**?

2 Natalia went for a run. Her distance from home is shown in the following distance–time graph.



a How far from home was Natalia after 45 minutes?

b How long was it until Natalia was first 3 km from home?

c What is the furthest distance Natalia travelled from home?

d How long did Natalia rest for during her run?

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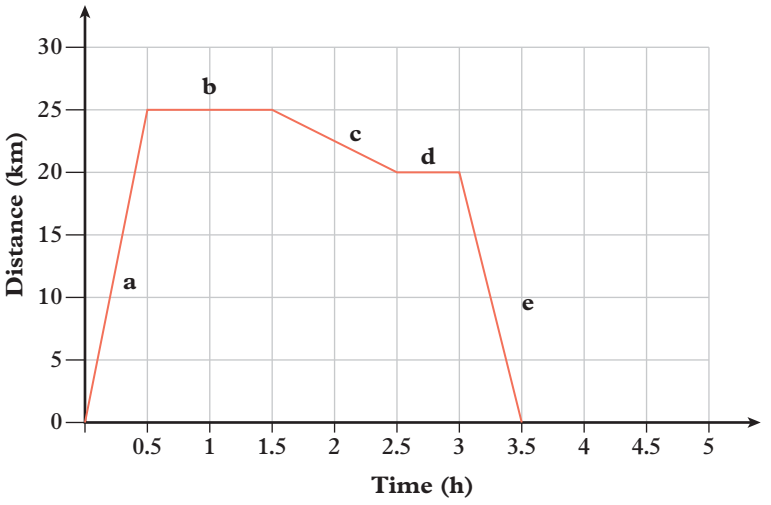
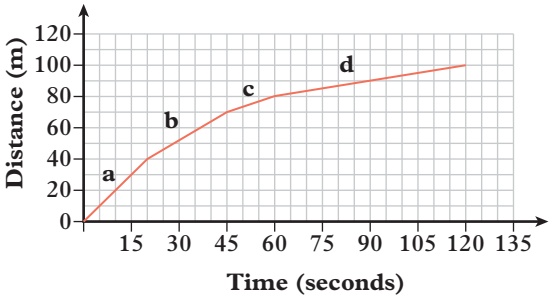
3I.2 **3** Determine the speed for each segment (**a**–**e**) of the following distance–time graph in kilometres per hour (km/h).



4 Determine the speed travelled for each segment (**a**–**d**) of the following distance–time graph in metres per second (m/s). Give your answers correct to two decimal places where appropriate.



3I.3 **5** Teresa went for a bike ride to the shops, which were located 12 km away, and then back home. Use the following descriptions to construct a distance–time graph of Teresa's bike ride.

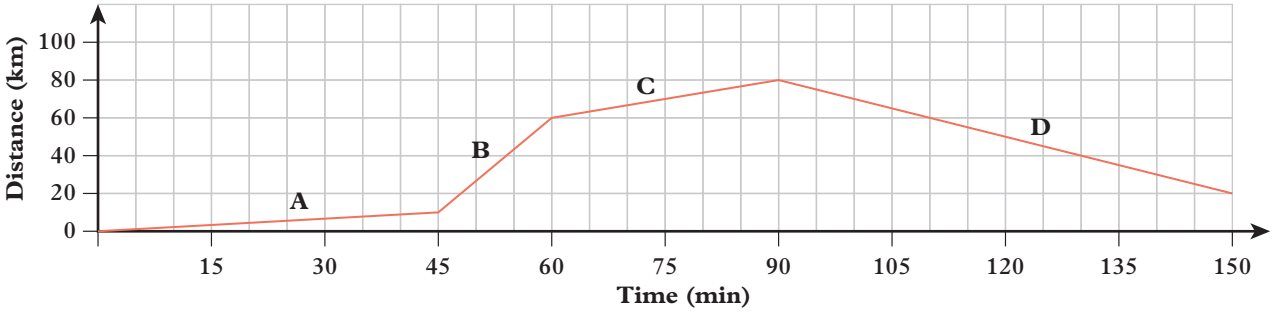
- rode at a constant speed for 20 minutes travelling 6 km
- took a 10-minute rest at the park
- rode for another 30 minutes at a constant speed in the same direction until reaching the shops
- took 20 minutes at the shops
- rode at a constant speed of 4.8 km/h all the way home

6 Matilda tracked an ant from its colony to a sugar cube and back again. Use the dot point descriptions to construct a distance–time graph of the ant's journey.

- walked at a constant rate for 5 minutes travelling 2 metres
- walked at a slower constant rate for 1 minute travelling 30 cm to the sugar cube
- took 30 seconds to collect some sugar
- walked at a constant rate for 8 minutes back to the colony

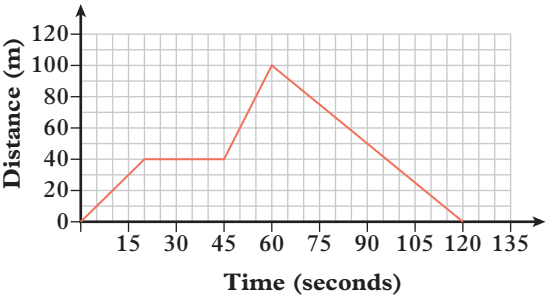
7 Order the stages (**A**–**D**) of the journey described by the following distance–time graph in ascending order of:

a distance travelled **b** time taken **c** speed.



8 Consider the distance–time graph on the right, which represents a student's walk, and another student's description of the journey.

- The student walks up a hill at 2 m/s.
- The student walks across flat ground for 25 seconds.
- The student walks up a steeper hill at 4 m/s.
- The student walks down a less steep hill at approximately 1.7 m/s.

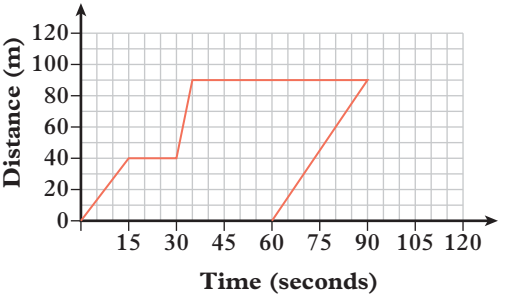


a Explain the errors in the description in relation to the slope of the line segments.

b Provide a bullet point description of the journey including information about each stage of the journey.

9 Consider the following description of a journey and the distance–time graph on the right which was drawn using the description.

- A student walks for 15 seconds travelling 40 m.
- The student stands still for 15 seconds.
- The student runs in the same direction at 10 m/s for 5 seconds.
- The student stands still for 55 seconds.
- The student walks at 3 m/s for 30 seconds back to the original position.



a Explain the error that has been made in the line segment drawn for the final stage of the journey.

b Construct a correct distance–time graph for this journey.

10 a Construct a distance–time graph in kilometres and minutes for the following journey, using Pedro's home as the origin.

- Pedro leaves the bowling alley, 18 km from home, at 2:08 pm.
- Pedro drives at a constant rate until 2:21 pm, when he is 12 km from home.
- Due to a detour, Pedro has to turn around for 1 km at 40 km/h and then takes a different road the rest of his way home.
- The last road has a speed limit of 60 km/h the rest of the way home.

b What is the earliest time Pedro can arrive home?

c What average speed was Pedro travelling between the bowling alley and the start of detour correct to one decimal place?

11 a Construct a distance–time graph for the following journey.

- Jonathan leaves home travelling at 60 km/h for 5 minutes.
- Jonathan realises he forgot his phone and makes a U-turn travelling at 40 km/h for 7.5 minutes through traffic to get back home.
- It takes 2 minutes for Jonathan to park, find his phone and get back to his car.
- Jonathan leaves again travelling at 60 km/h for 10 minutes, then at 80 km/h for 15 minutes and finally at 30 km/h for 5.5 minutes before arriving at his son's school.

b Determine how far Jonathan travelled to pick up his son.

c How far does Jonathan live from his son's school?

12 Explain why a distance–time graph will always only use the positive sections of the distance and time axes.

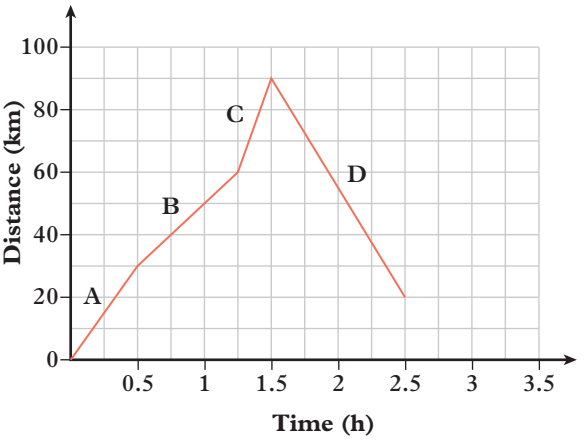
13 Consider the distance–time graph on the right.

a What distance was covered in total?

b How long did it take to travel this distance?

c Calculate the average speed for this journey.

d Calculate the speed of each section of the journey and enter it into the table below.



section	speed	time	speed × time
A			
B			
C			
D			
Total	N/A		

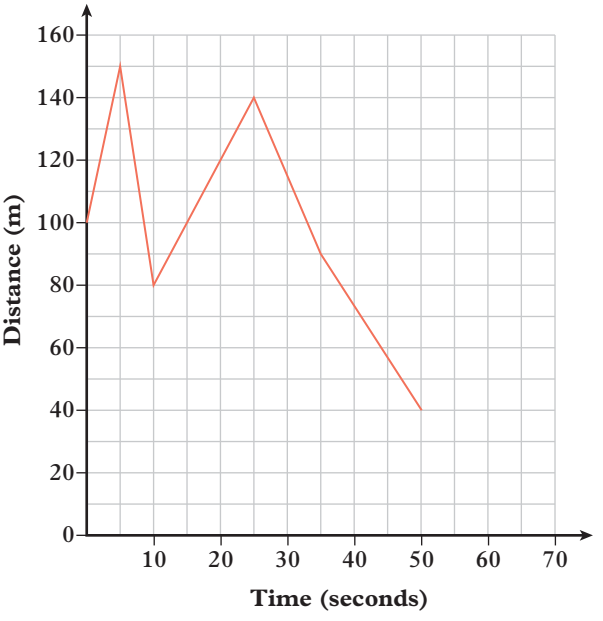
e Complete the rest of the table.

f Explain why the total of the 'speed × time' column is the same as the answer to part **a**.

g Use the table to calculate a weighted mean of the speeds travelled.

h Explain why the answer to part **g** is the same as the answer to part **c** by referring to the 'mean' and 'average'.

14 Consider the following distance–time graph describing the journey of an object. Note that the object does not start at the origin.



a Where does the object start in relation to the origin?

b Where does the object end up?

c Determine:

i the distance travelled in total

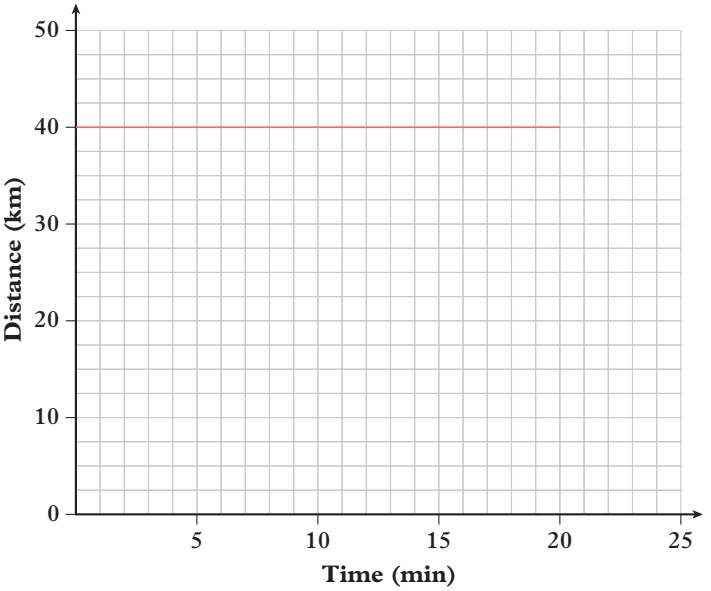
ii the distance between the starting and ending positions.

d Calculate the average speed using both distances in part **c**.

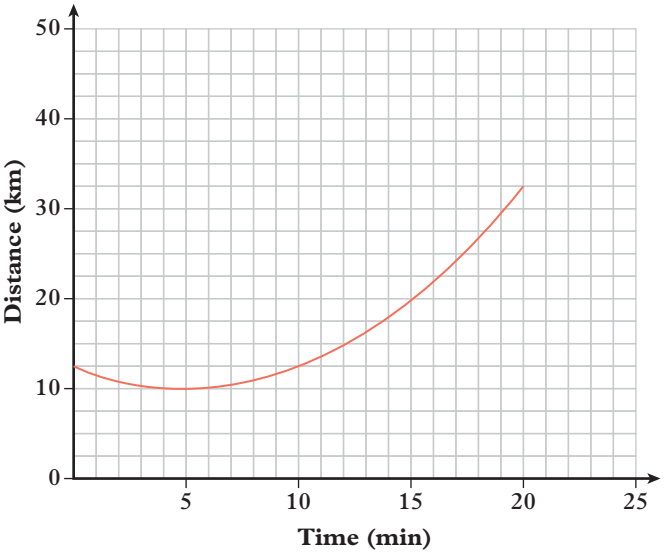
CHALLENGE

15 The following distance–time graphs follow the movement of an object moving at a constant speed but not in a straight line to or from the origin. Describe how the objects could be moving.

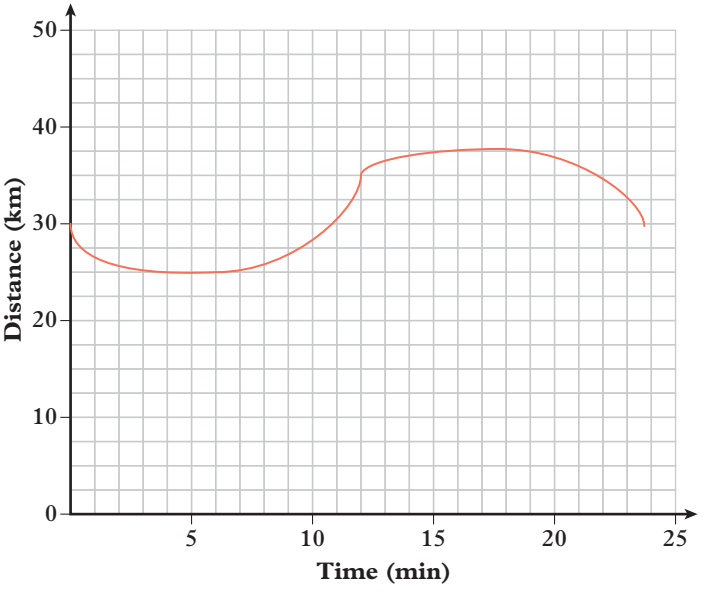
a



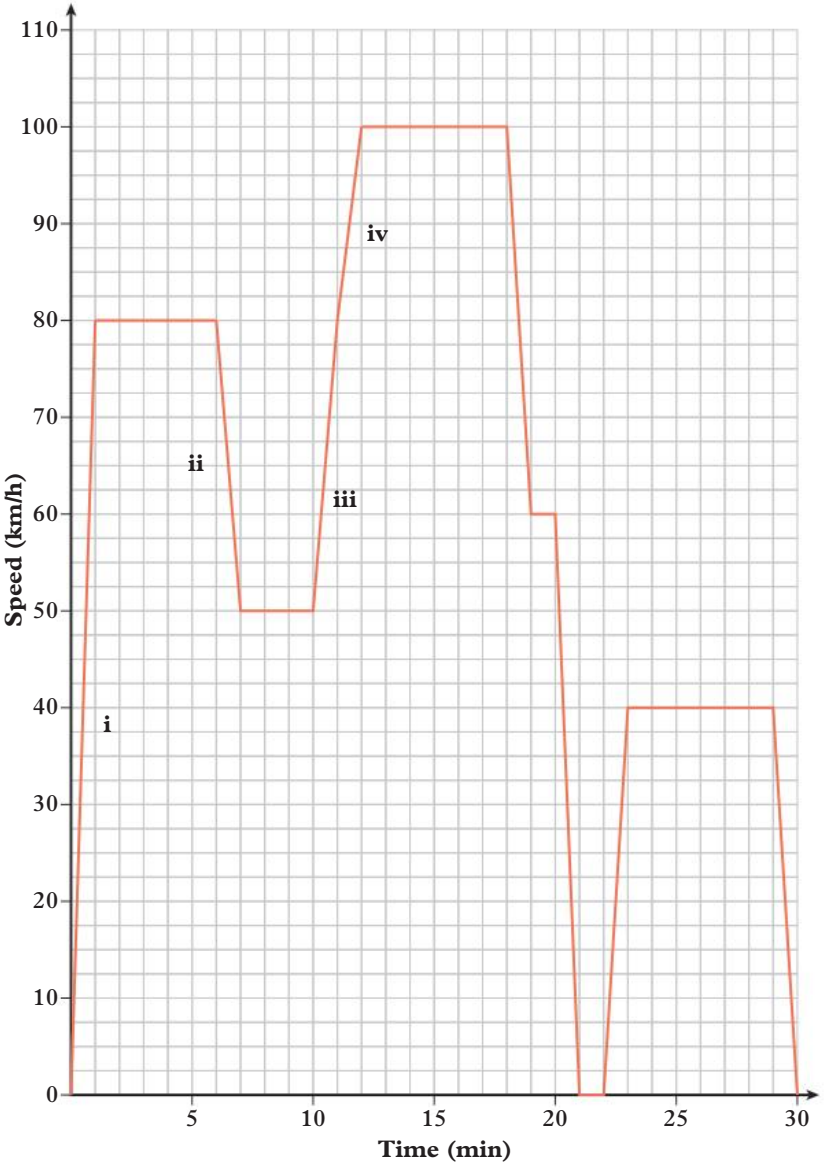
b



c



16 We can also plot the speed of an object against time. Consider the following speed–time graph.



a What is the speed of the object after 8 minutes?

b When is the object travelling at 60 km/h for the first time?

c At what time is the object first travelling the fastest?

d At what time is the object first not moving?

The area between the graph and the horizontal axis represents the distance the object travels. This is because the area is the sum of the products of the speed and the time.

e Calculate the area between the graph and the horizontal axis. Hint: Divide the shape into triangles and rectangles. Hence, state the total distance travelled by the object in kilometres.

The slope of the line segments of a speed–time graph describes how quickly the object is changing speed, that is, the acceleration of the object. Acceleration is the rate of the speed to the time spent travelling at that speed.

f Show that $\frac{\text{km}}{\text{h}} \div \text{h} = \frac{\text{km}}{\text{h}^2}$.

g Determine the acceleration of sections **i**–**iv** of the journey in km/h^2.

17 If a speed–time graph stays above the time axis, does the object ever change direction? Explain why or why not.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Distance–time graphs

Topic quiz
3I

Chapter summary

Percentage, fractions and decimals

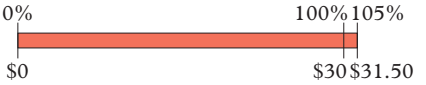
	Fraction	Decimal
Percentage to…	$15\% = \frac{15}{100} = \frac{3}{20}$	$23.4\% = \frac{23.4}{100} = 23.4 \div 100 = 0.234$

	Percentage	Decimal
Fraction to…	$\frac{1}{3} \times 100\% = \frac{100}{3}\% = 33\frac{1}{3}\%$	Align decimal points Dividend → $\frac{1}{8} = 8\overline{)1.0^{2}0^{4}0}$ = 0.125 Divisor → Trailing zeroes

	Percentage	Fraction
Decimal to…	$0.65 = 65\%$	$0.65 = \frac{65}{100}$

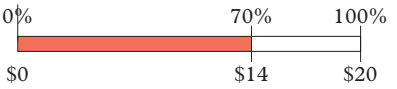
Mark-ups (percentage increase)

Increase \$30 by 5% = (100% + 5%) × \$30
= 105% × \$30
= 1.05 × \$30
= \$31.50



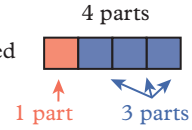
Discounts (percentage decrease)

Decreases \$20 by 30% = (100% – 30%) × \$20
= 70% × \$20
= 0.7 × \$20
= \$14



Ratios

$\frac{1}{4}$ of the squares are red



$\frac{3}{4}$ of the squares are blue

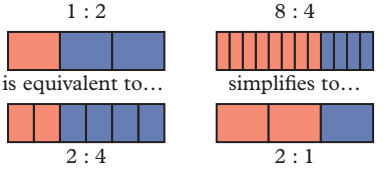
Simplifying ratios

3.4 : 1.3 (× 10) = 34 : 13

120 : 72 (÷ 24) = 5 : 3

Equivalent ratios

5 : 3 = 20 : 12 = 120 : 72 (× 4, × 6)



Rates

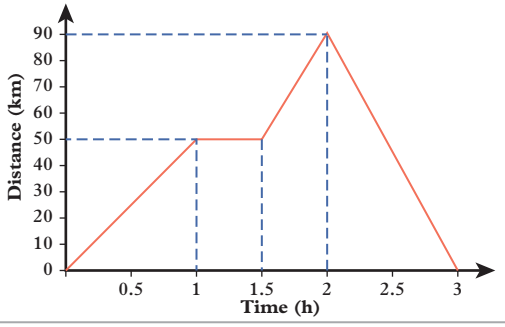
$$\frac{36 \text{ km}}{12 \text{ h}} = \frac{3 \text{ km}}{1 \text{ h}} = 3 \text{ km/h}$$

(÷ 12)

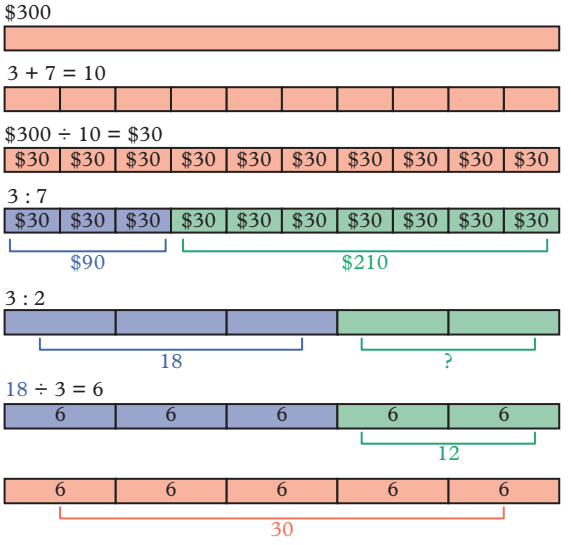
Profit and loss

- Profit = selling price – cost price
- Percentage profit = $\frac{\text{profit}}{\text{cost/selling price}} \times 100\%$
- Loss = cost price – selling price
- Percentage loss = $\frac{\text{loss}}{\text{cost/selling price}} \times 100\%$
- Revenue = selling price × number of items sold

Distance–time graphs



Dividing a quantity into a ratio



Speed

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

ANS p499

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Compete in teams to test your knowledge of legal definitions

Mathematical literacy review

The following key terms are used in this chapter:

- best buy
- cost price
- decimal
- discount
- distance–time graph
- equivalent ratio
- fraction
- GST
- line segment
- loss
- mark-up
- percentage
- percentage decrease
- percentage error
- percentage increase
- percentage loss
- percentage profit
- profit
- rate
- ratio
- revenue
- selling price
- simplest form
- slope
- speed
- unitary method

1 Which rate is calculated by dividing distance by time?

2 Explain the difference between a ratio and a rate.

3 Explain the difference between profit and loss.

4 Identify the key terms being referenced in each of these definitions.

a the option with the lowest unit cost

b the price paid to create or obtain a good or service before it is sold to a consumer

5 Using an example, provide a definition in your own words for the following key terms.

a mark-up

b revenue

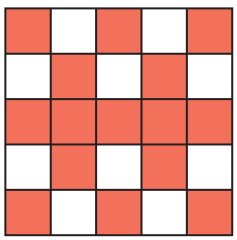
6 Complete the following sentences using words from the key term list.

a A ratio is written in _____ when the only common factor between the terms in the ratio is 1.

b A _____ is a proportion of a whole that has been divided into one hundred parts.

Multiple choice

3A **1** What percentage of the figure on the right is shaded?



A 10%
B 15%
C 40%
D 50%
E 60%

3A **2** What is $\frac{1}{5}$ written as a percentage?

A 10% **B** 15% **C** 20% **D** 25% **E** 95%

3A **3** Which value is *not* the same as 75%?

A 0.75 **B** 75 **C** $\frac{75}{100}$ **D** $\frac{3}{4}$ **E** $\frac{15}{20}$

3B **4** What is 45% of 360?

A 162 **B** 198 **C** 162% **D** 315 **E** 315%

3C **5** After a 32% discount, what is the selling price of an item originally marked $480?

A $153.60 **B** $326.40 **C** $448 **D** $633.60 **E** $1500

3D **6** Which of the following statements is correct?

A A profit occurs when the cost price is greater than the selling price.

B A discount increases the selling price of an item or service.

C A loss occurs when the cost price is less than the selling price.

D Revenue is found by multiplying the selling price by the number of items sold.

E The cost price will always be less than the selling price.

3E **7** Written as a ratio, what is 3 hours to 37 minutes?

A 3 : 37 **B** 37 : 3 **C** 37 : 180 **D** 33 : 7 **E** 180 : 37

3F **8** Which ratio is 16 : 28 : 12 written in simplest form?

A 4 : 7 : 3 **B** 3 : 7 : 4 **C** 8 : 14 : 6 **D** 6 : 18 : 2 **E** 32 : 56 : 24

3F **9** What is the value of a in $3 : 7 = a : 35$?

A 7 **B** 14 **C** 15 **D** 21 **E** 33

3G **10** When \$3600 is divided in the ratio 1 : 3, what are the two parts?

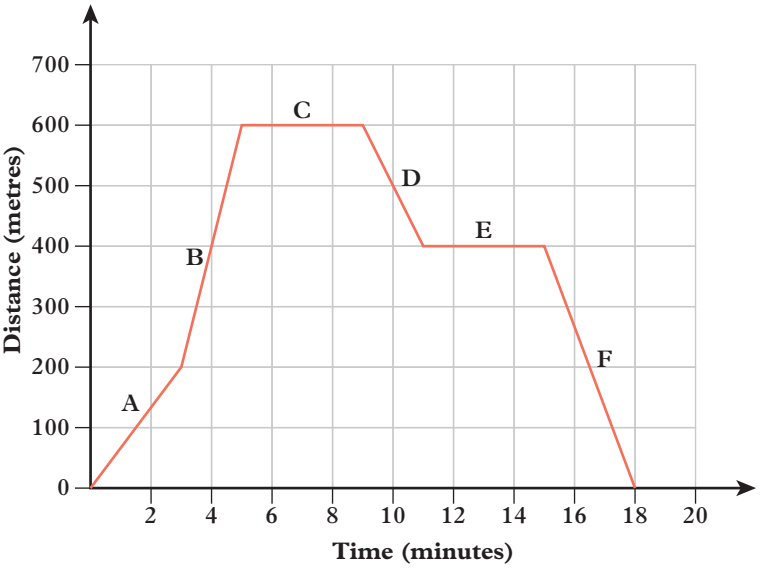
A \$1200 and \$2400 **B** \$900 and \$2700 **C** \$1500 and \$2100

D \$1200 and \$2700 **E** \$3600 and \$10 800

3H **11** What is 120 metres in 24 seconds written as a rate in its simplest form?

A $\frac{120}{24}$ m/s **B** $\frac{24}{120}$ m/s **C** $\frac{1}{5}$ m/s **D** 5 m/s **E** 6 m/s

3I **12** Consider the following distance–time graph of a bicycle journey. Which statement is not correct?



A The person is travelling fastest during section B.

B The person stopped twice during their bike ride.

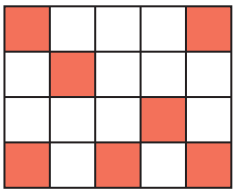
C The bike journey took 18 minutes in total.

D 600 m was the further distance reached from the start.

E The bike is travelling faster in section A than in section F.

Short answer

3A **1** Consider this figure.



a How many shaded sections are there?

b Write the number of shaded sections as a fraction of the total number of sections.

c Write the fraction in part **b** as an equivalent fraction with a denominator of 100.

d Write the shaded amount as a percentage.

3A **2** Write each percentage as a decimal.

a 23% **b** 62% **c** 16.52% **d** 291.6%

3A **3** Write each decimal as a percentage.

a 0.32 **b** 7.62 **c** 31.625 **d** 0.0003

3A **4** Write each fraction as a percentage.

a $\frac{3}{4}$ **b** $\frac{7}{10}$ **c** $\frac{17}{20}$ **d** $\frac{15}{32}$

3B **5** Write:

a 20 as a percentage of 50 **b** 24 as a percentage of 150 **c** 58 as a percentage of 125.

3B **6** Calculate:

a 10% of 340 **b** 25% of 62 **c** 150% of 1500 **d** 37% of 2400.

3C **7** Calculate the selling price after each mark-up or discount.

a a 10% mark-up on \$950 **b** a 44% mark-up on \$628 **c** a 28% discount on \$1255

3C **8** Calculate the percentage mark-up or discount.

a a radio with cost price \$40 and selling price \$55 **b** a book with cost price \$35 and selling price \$21

c shares with cost price \$2.98 and selling price \$4.52 **d** a TV with cost price \$1400 and selling price \$958

3D **9** Determine the amount of profit or loss in each of the following situations.

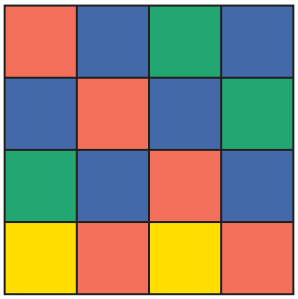
a cost price: \$7.35, selling price: \$12.99

b cost price: \$24.25, selling price: \$20.50

c cost price: \$45, selling price: \$78.20

3D **10** Write the profit or loss amount for each item in question **9** as a percentage of the cost price (rounded to the nearest %).

3E **11** Consider the following coloured grid.



a Write each comparison as a ratio.

i red sections to blue sections

ii yellow sections to green sections

b Write each comparison as a fraction.

i green sections to total sections

ii yellow sections to total sections

3E **12** Write each comparison as a ratio.

a 35 kg to 47 kg **b** 7 weeks to 31 weeks **c** 8 months to 3 years **d** 45 cents to \$1.25

3F **13** Simplify each ratio.

a 12 : 44 **b** 64 : 108 **c** 16 : 32 : 96 **d** 15 : 75 : 105

3F **14** Calculate the value of each pronumeral.

a $6 : 7 = a : 49$ **b** $13 : 4 = 52 : b$ **c** $c : 48 = 3 : 12$ **d** $15 : d = 3 : 7$

3G **15** Calculate the total number of parts for each ratio.

a 3 : 5 **b** 12 : 5 **c** 4 : 5 : 8 **d** 6 : 10 : 15

3G **16** Divide \$1500 in each ratio.

a 2 : 3 **b** 3 : 7 **c** 3 : 4 : 8 **d** 4 : 5 : 11

3H **17** Write each statement as a rate with an appropriate unit.

a 100 km per hour **b** 120 L in 40 seconds

c \$140 for 8 hours of work **d** 26 books read in 6 months

3H **18** Write each statement as a rate in simplest form.

a a football team scored 180 points in 120 minutes of play

b the cost of 45 L of petrol was $62.55

c a 375 mL can of drink costs $1.80

d a worker earned $218.40 for 13 hours of work

3I **19** Use the following descriptions to construct a distance–time graph with Mika's home as the origin.

- Mika walked 300 metres to the post box down the road in 3 minutes.
- Mika stopped for 1 minute to post some letters.
- Mika continued walking down the road for 4 minutes and reached a park, which was 0.8 km from her home.
- Mika stood still and chatted to a friend at the park for 3 minutes.
- Mika walked back home at a constant pace of 80 m/min.

Analysis

1 A market stall holder bought a variety of coloured 'hoodies' to sell. Each hoodie cost the stall holder $20 and she intends to sell them to the public at a mark-up of 65%.

a Write the percentage mark-up value as a:

i fraction in simplest form

ii decimal.

b Calculate the value of the mark-up on each hoodie.

c What price will the hoodies be?

d If the stall holder sells 35 hoodies on the first day, what is the difference between the total cost price of the hoodies and total amount they sell for?

To improve the sales of the hoodies for her second day at the market, the stall holder decides to offer a 10% discount on the selling price.

e What is the discount amount for each hoodie?

f What is the new selling price?

g What profit does the stall holder make on the sale of each hoodie?

h The stall holder sells 55 hoodies at the discounted amount. Write the profit from the sale of these as a percentage of the cost she paid for them.

2 Kris is doing some landscaping in her backyard.

Kris buys 3 cubic metres of Terracotta mulch at $75 per cubic metre and 5 cubic metres of Black mulch at $60 per cubic metre.

a What is the ratio of Terracotta mulch to Black mulch in simplest form?

b What price per cubic metre (to the nearest 5 cents) did Kris pay overall for mulch?

For her rock garden Kris decides she needs to buy River Rocks at $170 per cubic metre and Honeycomb Rocks at $100 per cubic metre. The ratio of River Rocks to Honeycomb Rocks will be 2 : 3 and Kris buys 1.6 cubic metres of River Rocks.

c How many cubic metres of Honeycomb Rocks does she buy?

d What is the total cost of the rocks?

Kris bought a total of 36 plants to put in her garden. The three types of plants were Grevillea, Callistemon and Banksia in the ratio 2 : 3 : 4.

e How many Callistemon plants did she buy?

f How many Banksia plants did she buy?

g Kris decides to increase the number of plants by 25%, while keeping the same ratio. How many extra of each plant does she buy?

3 My go-to Trail Mix recipe is shown on the right.

a Find the ratio of nuts to seeds in simplest form.

b Find the ratio of nuts to fruits in simplest form.

c How much chocolate would I need for 12 cups of trail mix?

d I decide to leave out the fruit and chocolate but want to have the same amount of trail mix as the original recipe. How much of each of the nuts and seeds do I need if I keep the same ratio?

$\frac{3}{4}$ cup pecans
$\frac{3}{4}$ cup cashews
$\frac{1}{2}$ cup sunflower seeds
$\frac{1}{2}$ cup pumpkin seeds
$\frac{1}{2}$ cup cherries
$\frac{1}{3}$ cup raisins
$\frac{2}{3}$ cup chocolate

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert between percentages, fractions and decimals ☐ Compare and order percentages, fractions and decimals	☐ Go back to Topic 3A Percentages, fractions and decimals
☐ Express a quantity as a percentage of another quantity ☐ Calculate a percentage of a quantity ☐ Increase and decrease a quantity by a percentage	☐ Go back to Topic 3B Calculating percentages
☐ Calculate mark-ups and discounts including GST ☐ Use the unitary method to solve financial problems	☐ Go back to Topic 3C Mark-ups and discounts
☐ Calculate the percentage profit or loss on the cost price ☐ Calculate the percentage profit or loss on the selling price ☐ Calculate the profit or loss in a given situation	☐ Go back to Topic 3D Profit and loss
☐ Identify ratios in diagrams or written scenarios ☐ Write ratios ☐ Understand part-to-part and part-to-whole comparisons	☐ Go back to Topic 3E Ratios
☐ Simplify ratios involving decimals and fractions ☐ Use equivalent ratios to find unknown quantities	☐ Go back to Topic 3F Equivalent ratios
☐ Divide a quantity into a given ratio ☐ Use the unitary method to find a quantity from a given ratio	☐ Go back to Topic 3G Working with ratios
☐ Write a rate in simplest form ☐ Use a rate to solve a problem ☐ Calculate the best buy	☐ Go back to Topic 3H Rates
☐ Interpret distance–time graphs ☐ Calculate the speed for different parts of a journey from a distance–time graph ☐ Create a distance–time graph from the details of a journey	☐ Go back to Topic 3I Distance–time graphs

4

Indices and roots

Index

4A Indices
4B Products and quotients of powers
4C Raising indices and the zero index
4D Combining the index laws
4E Roots

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Times tables
- ✔ Order of operations
- ✔ Prime and composite numbers
- ✔ Multiplying and dividing fractions
- ✔ Multiplying decimals

Curriculum links

- Operates with primes and roots, positive-integer and zero indices involving numerical bases and establishes the relevant index laws (MA4-IND-C-01)
  - Apply index notation to represent whole numbers as products of powers of prime numbers
  - Examine cube roots and square roots
  - Use index notation to establish the index laws with positive-integer indices and the zero index

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Materials

✔ Calculator

4A Indices

Learning intentions

By the end of this topic you will be able to ...

✔ convert numbers between index notation and expanded form
✔ calculate the value of numbers in index notation
✔ express integers as a product of prime factors.

Inter-year links

Year 7	1G Indices and square roots
Year 9	2A Indices
Year 10	2A Indices

Index notation

Key content video

- **Index notation** or **index form** is used to represent repeated multiplication.
  → 3^4 is read as '3 to the power of 4'.
- The base is the number that is repeatedly multiplied.
- The index (or exponent) indicates how many times the base appears in **expanded form**.
- If no index is shown, the implied index is 1.
  For example, $2 = 2^1$.
- The square of a number is the result of multiplying that number by itself.
  → 3^2 can be read as '3 to the power of 2', '3 squared' or 'the square of 3'.
- The cube of a number is the result of raising that number to the power of 3.
  → 4^3 can be read as '4 to the power of 3', '4 cubed' or 'the cube of 4'.

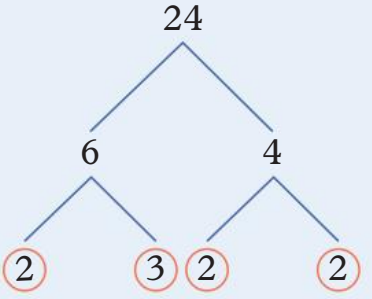
index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

index notation — expanded form — basic numeral

Prime factorisation

- The **prime factorisation** of an integer is the unique product of the **prime numbers** that make up that integer.
  → Prime factorisation is often expressed in index notation with the bases listed in ascending order.
  For example, the prime factorisation of $24 = 2^3 \times 3$.
- Prime factorisation can be performed using **factor trees**. In factor trees, **composite numbers** are broken down into pairs until only prime numbers remain.



Example 4A.1 Calculating the value of a number in index notation

Write 2^5 in expanded form and calculate its value.

THINK	WRITE
1 Identify the base and the index. Write the number in expanded form by multiplying five twos together.	$2^5 = 2 \times 2 \times 2 \times 2 \times 2$
2 Perform the multiplication.	$= 32$

Example 4A.2 Calculating squares and cubes

Calculate:

a 4^2 **b** the square of 6 **c** 3 cubed

THINK

1. Identify the base and the index. 'Square' refers to an index of 2 and 'cube' refers to an index of 3.
2. Perform the multiplication.

WRITE

a $4^2 = 4 \times 4$
$= 16$

b $6^2 = 6 \times 6$
$= 36$

c $3^3 = 3 \times 3 \times 3$
$= 27$

Example 4A.3 Prime factorisation using factor trees

Use factor trees to express each number as the product of its prime factors. Write your answer in index notation.

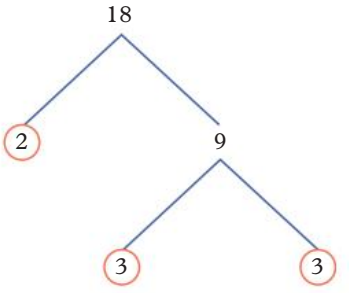
a 18 **b** 225

THINK

1. Divide the composite number by its lowest prime factor to identify a factor pair. The lowest prime factor of even numbers is always 2. Remember that if the sum of all the digits in a number is divisible by 3, that number is also divisible by 3.
2. Repeat step **1** until each branch ends in a prime number.
3. Write the initial composite number as a product of its prime factors. Write the answer in index notation and list the bases in ascending order.

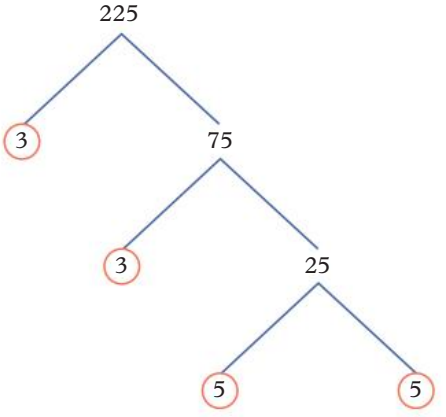
WRITE

a



$18 = 2 \times 3 \times 3$
$= 2 \times 3^2$

b



$225 = 3 \times 3 \times 5 \times 5$
$= 3^2 \times 5^2$

Helpful hints

- ✔ Take care when writing indices – they should be written smaller than the base and sit high up on the shoulder of the base to avoid confusion between 3^4 and 34.
- ✔ Remember that calculating indices and multiplication are different operations. For example, $3^2 \neq 3 \times 2$, $3^2 = 3 \times 3$.
- ✔ Read questions involving indices carefully. You may be asked to:
  - → write a number in expanded form; for example, $3^2 = 3 \times 3$
  - → calculate or evaluate; for example, $3^2 = 9$.

ANS p500

Exercise 4A Indices

△ 1–11, 12(a, c), 13, 14 □ 3–11, 12(c, d), 13, 15–18 ⬠ 5–7, 10, 12(c, d), 15–19

UNDERSTANDING AND FLUENCY

1 The number 5^3 is written in index notation.

a State the value of the base.

b State the value of the index.

c Write the number in expanded form, as a repeated multiplication.

d Perform the repeated multiplication to convert 5^3 into a basic numeral.

4A.1 **2** Write each number in expanded form and calculate its value.

a 2^5 **b** 4^3 **c** 1^9 **d** 3^3 **e** 10^7

f 2^8 **g** 5^4 **h** 7^2 **i** 8^3 **j** 6^4

4A.2 **3** Calculate:

a 8^2 **b** 7 squared **c** the square of 5

d 11^2 **e** 6 squared **f** the square of 1

g 60^2 **h** 100 squared **i** the square of 20

4 Calculate:

a 3^3 **b** 2 cubed **c** the cube of 4

d 100^3 **e** 10 cubed **f** the cube of 5

5 Complete the table below.

	Index notation	Base	Index	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	
b	2^5	2			32
c			2	8×8	
d				6×6	

6 Write each number in expanded form and calculate its value.

a $(0.8)^2$ **b** $(3.4)^2$ **c** $(0.5)^3$ **d** $(0.3)^4$ **e** $(0.1)^5$ **f** $(0.2)^7$

7 Write each number in expanded form and calculate its value.

a $\left(\frac{1}{4}\right)^3$ **b** $\left(\frac{7}{9}\right)^2$ **c** $\left(\frac{5}{6}\right)^3$ **d** $\left(\frac{3}{10}\right)^4$ **e** $\left(\frac{1}{2}\right)^5$ **f** $\left(3\frac{1}{3}\right)^6$

8 Use a calculator to evaluate the following numbers.

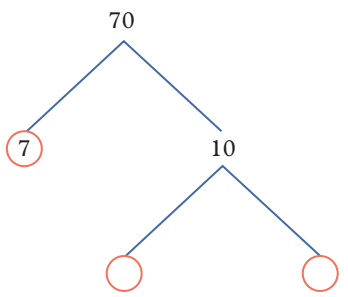
a 57^2 **b** 101^3 **c** 13^7

d $(6.85)^3$ **e** $(1.9)^4$ **f** $(12.4)^5$

g $\left(\frac{18}{23}\right)^3$ **h** $\left(\frac{7}{11}\right)^6$ **i** $\left(\frac{3}{5}\right)^9$

9 Complete each factor tree, then write the prime factorisation of each composite number in index notation.

a



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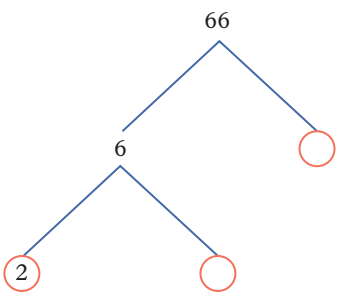

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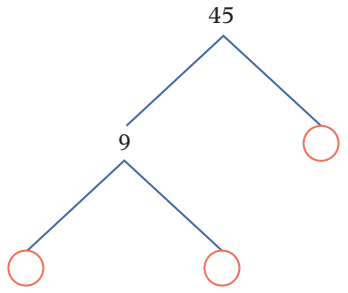
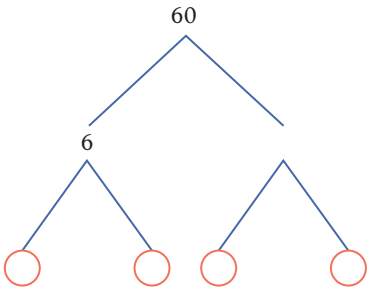


d



4A.3 **10** Use a factor tree to express each number as the product of its prime factors. Write your answer in index notation.

a 75 **b** 48 **c** 90

d 126 **e** 288 **f** 1092

11 A student incorrectly evaluates 4 squared as $4^2 = 8$. Identify the student's error and explain why $4^2 \neq 8$.

12 Use the correct order of operations to complete the following calculations.

a $2^3 + 8^2 \div 4$ **b** $5^2 \times 0.2 + 3^4$ **c** $5.8 - (1.2)^2 + (0.1)^4$ **d** $\frac{7}{10} + \left(\frac{2}{5}\right)^3 \times 6\frac{1}{4}$

13 a Find:

i 1^2 **ii** 11^2 **iii** 111^2 **iv** 1111^2

b Describe the pattern you see.

c Without using a calculator, use the pattern to write the results for:

i $11\,111^2$ **ii** $111\,111^2$ **iii** $1\,111\,111^2$ **iv** $11\,111\,111^2$

14 For the six weeks of the school holidays, Sarah's parents offer to pay her \$30 per week for doing some extra jobs around the house.

a How much would Sarah earn over the holidays if she agrees to this arrangement?

Instead, Sarah suggests that her parents pay her \$3 in the first week, then three times as much as the previous week in each of the following weeks.

b How much would Sarah earn in each week of the holidays under her suggested arrangement?

c Compare the total amount that Sarah could earn over the school holidays from each arrangement. Which plan do you think Sarah's parents will agree to?

d If Sarah's parents agree to her suggested arrangement over a longer period, express the amount that Sarah would earn in her eleventh week using index notation.

e Calculate how much Sarah would earn in the eleventh week of her arrangement.

PROBLEM SOLVING AND REASONING

15 The size of a colony of bacteria containing 1000 cells doubles every hour.

a How large is the colony after:

i one hour **ii** two hours **iii** three hours?

b Use your answers to part **a** to complete the second row of the following table.

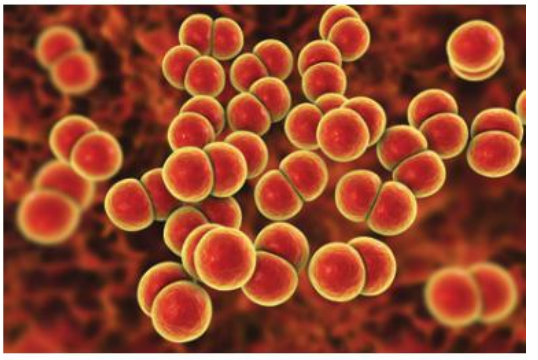
Number of hours	1	2	3	4	5
Size of colony					
Number of times larger than original amount		4		16	

c How many times larger than the original colony is the new colony after:

i one hour **ii** two hours **iii** three hours?

d Use your answers to part **c** to complete the third row of the table.

e Express each of the numbers in the third row of the table in index notation with a base of 2.



f In index notation, how many times larger than the original colony is the new colony after:

i 6 hours **ii** 10 hours

iii 24 hours?

g Use a calculator to work out how many times larger the colony will be after 24 hours.

h Use your answer to part **g** to determine the number of cells in the colony after 24 hours.

16 Prime factorisation can be used to find highest common factors (HCF).

a Write 144 as the product of its prime factors.

b Write 270 as the product of its prime factors.

c Identify the prime factors that are shared by both numbers, then multiply those factors together for the HCF.

d Use prime factorisation to find the HCF of the following pairs of numbers.

i 117 and 182 **ii** 420 and 132 **iii** 495 and 220 **iv** 432 and 405

CHALLENGE

17 The following pattern details the prime factorisation of powers of 20.

$20 = 2^2 \times 5$

$20^2 = 2^4 \times 5^2$

$20^3 = 2^6 \times 5^3$

a Use the pattern to represent the prime factorisation of 20^4 and 20^5.

b Find a general rule for the prime factorisation of 20^n.

18 A single parasite invades a host. The number of parasites doubles every 3 days. How many days until there are over 1 million parasites in the host?

19 The half-life of a chemical substance refers to the time it takes for the amount of that chemical to decrease by a half. The half-life of caffeine in your body is 6 hours. If a single cup of coffee contains c milligrams of caffeine and you drink one cup of coffee at 8 am, then two more cups at 2 pm, how much caffeine is still in your body at 8 pm in terms of c?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Indices

Interactive skillsheet Prime factorisation

Investigation What are the odds?

Topic quiz 4A

4B Products and quotients of powers

Learning intentions

By the end of this topic you will be able to ...

✔ simplify products of numbers with the same base

✔ simplify quotients of numbers with the same base.

Inter-year links

Support	Adding and subtracting whole numbers
Year 7	1B Adding whole numbers
Year 9	2B Products and quotients of powers
Year 10	2A Indices

The index laws

Key content video

- Simplification involves reducing the complexity of an expression.
- The **index laws** can be used to simplify expressions involving numbers in index notation.
- Index laws only apply to numbers with the *same base*.

Product of powers law

- When multiplying numbers in index notation with the same base, add the indices.
  → This can be generalised as $a^m \times a^n = a^{m+n}$.

$$\begin{aligned} 2^3 \times 2^5 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^{3+5} \\ &= 2^8 \end{aligned}$$

Quotient of powers law

- When dividing numbers in index notation with the same base, subtract the second index from the first index.
  → This can be generalised as $a^m \div a^n = a^{m-n}$.

$$\begin{aligned} 2^5 \div 2^3 &= (2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2) \\ &= 2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2} \div (\cancel{2} \times \cancel{2} \times \cancel{2}) \\ &= 2^{5-3} \\ &= 2^2 \end{aligned}$$

- Remember that quotients can be written as fractions. When simplifying fractional quotients, subtract the index of the number in the denominator from the index of the number in the numerator.

$$\begin{aligned} \frac{2^5}{2^3} &= \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\ &= \frac{2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} \\ &= 2^{5-3} \\ &= 2^2 \end{aligned}$$

Example 4B.1 Simplifying products using an index law

Use an index law to simplify the following products.

a $4^6 \times 4^4$ **b** 9×9^7

THINK

1. Check that the bases are the same.
2. Write the base and add the indices. Remember that $9 = 9^1$.

WRITE

a $4^6 \times 4^4 = 4^{(6+4)}$
$= 4^{10}$

b $9 \times 9^7 = 9^{(1+7)}$
$= 9^8$

Example 4B.2 Simplifying quotients using an index law

Use an index law to simplify the following quotients.

a $3^{11} \div 3^{10}$ **b** $\frac{8^5}{8^2}$

THINK

a **1** Check that the bases are the same.

2 Write the base and subtract the second index from the first index. Simplify the answer by leaving out the index of 1.

b **1** Check that the bases are the same.

2 Write the base and subtract the index of the number in the denominator from the index of the number in the numerator.

WRITE

a $3^{11} \div 3^{10} = 3^{(11-10)}$
$= 3^1$
$= 3$

b $\frac{8^5}{8^2} = 8^{(5-2)}$
$= 8^3$

Helpful hints

✔ 'Simplify' and 'evaluate' are two different command terms:

→ To simplify an expression in this chapter, use index laws to combine the terms, and hence reduce the complexity of the expression

→ To evaluate, or 'find the value' of an expression, convert that expression from index notation into a basic numeral.

ANS p501

Exercise 4B Products and quotients of powers

1–8, 9–10(a–c), 12, 13(a, b) | 5–8, 9–10(d–f), 11, 13, 14(a, b) | 5–7, 9–10(d–f), 11, 13, 14

UNDERSTANDING AND FLUENCY

4B.1 **1** Use an index law to simplify the following products:

a $3^3 \times 3^2$ **b** $4^5 \times 4^3$ **c** $6^2 \times 6^4$

d $5^3 \times 5^6$ **e** $8^2 \times 8$ **f** 2×2^4

2 Evaluate each of your answers to question **1** by converting each number into a basic numeral.

4B.2 **3** Use an index law to simplify the following quotients:

a $9^{14} \div 9^{11}$ **b** $4^6 \div 4^5$ **c** $6^4 \div 6$

d $\frac{4^7}{4^2}$ **e** $\frac{10^8}{10^5}$ **f** $\frac{6^{11}}{6^7}$

4 Evaluate each of your answers to question **3**.

5 Use an index law to first simplify, then evaluate the value of each of the following expressions.

a $2^5 \times 2^6$ **b** $5^2 \times 5^4$ **c** $3^4 \times 3^3$ **d** $4^6 \times 4^2$

e $5^7 \div 5^4$ **f** $\frac{3^9}{3^3}$ **g** $9^6 \div 9^3$ **h** $2^{12} \div 2^5$

i $6^3 \times 6^2$ **j** $\frac{8^5}{8^4}$ **k** $10^4 \times 10^5$ **l** $\frac{7^8}{7^6}$

6 The index laws also apply to non-integer bases. Use an index law to simplify each of the following expressions.

a $(1.5)^3 \times (1.5)^6$ **b** $(0.7)^4 \times (0.7)^2$ **c** $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4$ **d** $\left(\frac{7}{5}\right)^6 \times \left(\frac{7}{5}\right)^8$

e $(9.2)^8 \div (9.2)^5$ **f** $(0.63)^{11} \div (0.63)^4$ **g** $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^2$ **h** $\left(\frac{5}{2}\right)^9 \div \left(\frac{5}{2}\right)^7$

7 Determine the value of the missing index in the equations below.

a $7^5 \times 7^{\square} = 7^{15}$ **b** $6^{\square} \times 6 = 6^6$ **c** $2^{13} \div 2^{\square} = 2^5$

d $\frac{9^7}{9^{\square}} = 9^3$ **e** $\frac{9^{\square}}{9^7} = 9^{14}$ **f** $\frac{12^{\square}}{12^{15}} = 144$

8 Complete the following equations by filling in the missing indices.

a $5^2 \times 5^5 \times 5^4 = 5^7 \times 5^4$
$= 5^{\square}$

b $20^8 \times 20 \times 20^{10} = 20^{\square} \times 20^{10}$
$= 20^{\square}$

c $8^{32} \div 8^7 \div 8^{10} = 8^{25} \div 8^{\square}$
$= 8^{\square}$

d $7^{24} \div 7^6 \div 7 = 7^{\square} \div 7^{\square}$
$= 7^{\square}$

9 Use an index law to simplify these. Leave your answers in index notation.

a $2^5 \times 2^4 \times 2^3$ **b** $3^3 \times 3 \times 3^2$ **c** $10^3 \times 10^2 \times 10^5$

d $4^2 \times 4^4 \times 4^3$ **e** $5^7 \times 5^3 \times 5^6$ **f** $9 \times 9^5 \times 9^8$

10 Use an index law to simplify these. Leave your answers in index notation.

a $6^{10} \div 6^5 \div 6^3$ **b** $7^{12} \div 7^2 \div 7^6$ **c** $12^{16} \div 12^9 \div 12^6$

d $8^{10} \div 8^3 \div 8^3$ **e** $5^{15} \div 5^6 \div 5^8$ **f** $11^{11} \div 11 \div 11^5$

11 Light-years are a distance unit defined by the distance that light travels in one year. One light-year is approximately 10^{13} km. Icarus is one of the furthest stars from Earth to have been detected so far. If Icarus is 10^{23} km from Earth, how many light-years is Icarus from Earth? Express your answer in index notation.

12 Identify and explain the mistake made in each of the following calculations, then give the correct answer in each instance.

a $5^8 \div 5^2 = 5^4$ **b** $2^3 \times 2^3 = 2^9$ **c** $\frac{4^7}{4^7} = 4^1$ **d** $9^8 \div 9^{10} = 9^2$

13 **a** Given $(-3)^2 = (-3) \times (-3) = 9$, evaluate:

i $(-2)^2$ **ii** $(-2)^3$ **iii** $(-2)^4$ **iv** $(-2)^5$

b Describe the pattern in your answers to part **a**.

c Explain how you can determine the sign of your answer when evaluating a negative number raised to a given power.

d Evaluate:

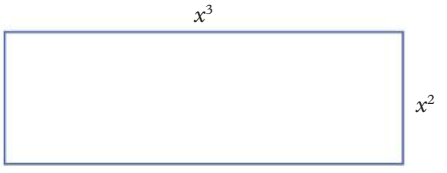
i $(-3)^3$ **ii** $(-2)^6$ **iii** $(-10)^4$ **iv** $(-10)^9$

14 The index laws can also be applied to terms containing pronumerals.

a Express the area of this rectangle as a product.

b Simplify your answer to part **a**.

c If the area of the rectangle is 7776 mm^2, what are the dimensions of the rectangle?



PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Products and quotients of powers

Worksheet Multiplying and dividing using index notation

Investigation Growing cubes

Topic quiz 4B

4C Raising indices and the zero index

Learning intentions

By the end of this topic you will be able to ...

✔ raise a number in index notation by another index

✔ evaluate expressions involving the zero index.

Inter-year links

Support	Multiplying and dividing whole numbers
Year 7	1D Multiplying whole numbers
Year 9	2C Raising indices and the zero index
Year 10	2A Indices

Power of a power law

Key content video

- When raising a number in index notation by another index, multiply the indices.
  → This can be generalised as $(a^m)^n = a^{m \times n}$.

$$(2^3)^5 = 2^{(3 \times 5)} = 2^{15}$$

Expressions involving brackets

- When a product or quotient is in brackets, every number inside the brackets should have its index multiplied by the index outside the brackets.

$$(2 \times 3)^5 = 2^5 \times 3^5$$

$$\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$$

The zero index

- Excluding 0, any number raised to the power of 0 is equal to 1.

$$2^0 = 1$$

Example 4C.1 Raising an index by another index

Use an index law to simplify $(3^4)^5$.

THINK

Write the base and multiply the indices.

WRITE

$$(3^4)^5 = 3^{(4 \times 5)} = 3^{20}$$

Example 4C.2 Distributing indices across brackets

Simplify:

a $(4 \times 7)^2$

b $\left(\frac{5}{2}\right)^4$

THINK

Multiply the index of every base inside the brackets by the index outside the brackets. Remember that base numbers that do not have a stated index have an index of 1, so $4 = 4^1$.

WRITE

a $(4 \times 7)^2 = 4^{(1 \times 2)} \times 7^{(1 \times 2)} = 4^2 \times 7^2$

b $\left(\frac{5}{2}\right)^4 = \frac{5^{(1 \times 4)}}{2^{(1 \times 4)}} = \frac{5^4}{2^4}$

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Example 4C.3 Evaluating calculations involving the zero index

Evaluate:

a 23^0 **b** $8^2 \div 8^2$

THINK

a Any number, excluding 0, to the power of 0 is equal to 1.

b When dividing numbers in index notation with the same base, subtract the second index from the first index.

WRITE

a $23^0 = 1$

b $8^2 \div 8^2 = 8^{(2-2)}$
$= 8^0$
$= 1$

Helpful hints

✔ Take care not to mix up the index laws.
→ across a multiplication sign, add indices
→ across a division sign, subtract indices
→ across brackets, multiply indices

✔ Remember that $2^0 = 1$, *not* 0.

Index law	Example
Product of powers law	$2^5 \times 2^3 = 2^{5+3}$
Quotient of powers law	$2^5 \div 2^3 = 2^{5-3}$
Power of a power law	$(2^5)^3 = 2^{5 \times 3}$

ANS p501

Exercise 4C Raising indices and the zero index

△ 1–7, 9(a–c), 12, 13 □ 1, 2, 4–6, 8, 9, 11–14 ⬠ 1, 2, 5, 8–10, 13–16

4C.1 **1** Use an index law to simplify:

a $(6^4)^5$ **b** $(2^6)^2$ **c** $(7^4)^3$
d $(11^2)^4$ **e** $(8^5)^3$ **f** $(3^7)^2$

4C.2 **2** Simplify:

a $(5 \times 9)^3$ **b** $\left(\frac{3}{4}\right)^6$ **c** $\left(\frac{9}{5}\right)^4$
d $(11 \times 4)^7$ **e** $\left(\frac{7}{10}\right)^3$ **f** $(3 \times 8)^9$

4C.3 **3** Evaluate:

a 4^0 **b** 15^0 **c** 144^0

4 Use an index law to simplify, then evaluate:

a 3.2^0 **b** $(2 \times 3)^0$ **c** $\left(\frac{7}{8}\right)^0$
d $9^3 \div 9^3$ **e** $\frac{8^5}{8^5}$ **f** $\frac{4}{4}$

5 Use an index law to simplify each of the following calculations.

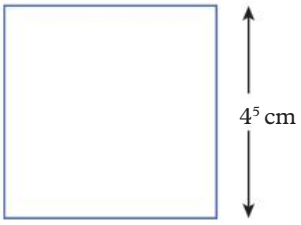
a $(0.5^4)^3$ **b** $(1.2^8)^0$ **c** $(3.8^5)^5$
d $(2^5 \times 2.5^2)^4$ **e** $(0.7^7 \times 5.4^3)^0$ **f** $(4.8^0 \times 6.6^6)^3$

UNDERSTANDING AND FLUENCY

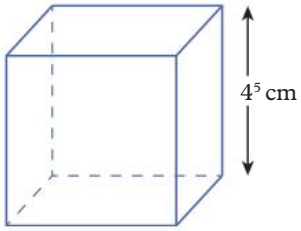
PROBLEM SOLVING AND REASONING

6 Writing your answer in index notation, determine the value of:

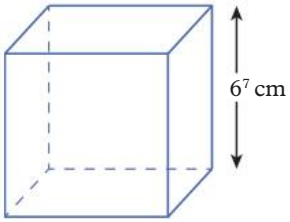
a the area of this square



b the volume of this cube



c the volume of this cube.



7 Determine the value of the missing index in the following equations.

a $(7^3)^{\square} = 7^{18}$ **b** $(9^{\square})^5 = 9^{20}$ **c** $(8^4)^{\square} = 1$

d $(6 \times 8)^{\square} = 6^5 \times 8^5$ **e** $\left(\frac{4}{15}\right)^{\square} = \frac{4^{10}}{15^{10}}$ **f** $\left(\frac{2}{5}\right)^{\square} = 1$

8 Complete the following calculations by filling in the missing numbers.

a $(2^3)^4 = \square^4 = \square^3 = 2^{\square}$

b $10^{5 \times 4} = 100\ 000^{\square} = 10\ 000^{\square} = 10^{\square}$

c $8^8 = \square^{24} = 4^{\square} = 16^{\square}$

9 Complete the following equations by filling in the missing indices.

a $(2^2)^{\square} = 32$ **b** $(2 \times 3)^{\square} = 36$ **c** $\left(\frac{1}{5}\right)^{\square} = \frac{1}{125}$

d $(4^{\square})^3 = 64$ **e** $(3 \times 3)^{\square} = 81$ **f** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

10 Without evaluating, list the following numbers in ascending order.

$3^{14}, (3^2)^5, (3^4)^3, (3^{15})^0, ((3^2)^3)^3$

11 Consider the expression $\frac{2^5}{2^5}$.

a Calculate the value of the expression.

b Use an index law to simplify the expression. Leave your answer in index notation.

c Use this method to write a calculation showing that $15^0 = 1$.

12 a Calculate the values of the numbers in the following pattern.

$3^5, 3^4, 3^3, 3^2, 3^1$

b Use this sequence to explain why $3^0 = 1$.

13 Decide whether the following numerical statements are true or false.

a $(5^3)^4 = (5^2)^6$ **b** $(2^4)^6 = (2^8)^3$

c $(4 \times 5)^6 = (20^3)^3$ **d** $(2 \times 3)^8 = (6^2)^4$

CHALLENGE

14 Find the value of x in the following equations.

a $(7^x)^5 = 7^{20}$ **b** $(7^x \times 7^x)^5 = 7^{20}$

15 Show that $(2^3)^6 = 4^9$.

16 If $x^y = y^x$ and $x \neq y$, what are the positive integer values of x and y?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Raising indices and the zero index

Investigation Bert and Ernie's index laws

Topic quiz 4C

ANS p502

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

4A 1 Write each number in expanded form and calculate its value.

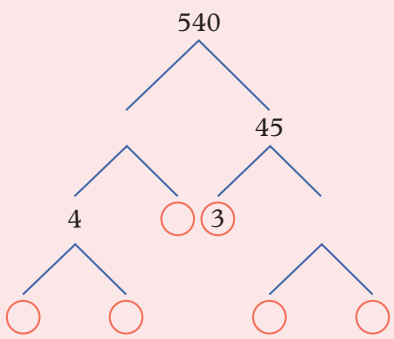
a 2^4 **b** 3^4 **c** 5^3 **d** 10^6

4A 2 Calculate:

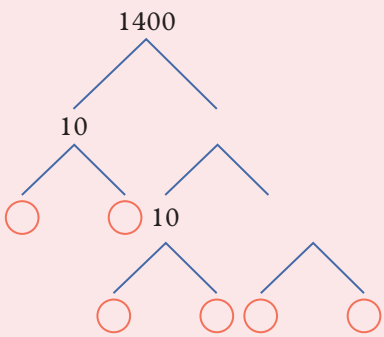
a 5 squared **b** the square of 9 **c** 4 cubed **d** the cube of 6

4A 3 Complete each factor tree, then write the prime factorisation of each number in index notation.

a



b



4A 4 Use a factor tree to express each number as the product of its prime factors. Write your answers in index notation.

a 24 **b** 72 **c** 195 **d** 350

4A 5 A scientist studies a colony of bacteria in a petri dish and notes the following information.

Number of minutes	0	1	2	3	4	5
Size of colony	10	30	90			
Number of times as large as original amount	1	3	9			

a Complete the table.

b Express each of the numbers in the third row of the table in index notation with a base of 3.

c In index notation, how many times larger than the original colony will the new colony be after:

i 10 minutes **ii** 1 hour **iii** 24 hours?

d Use a calculator to determine how many bacteria will be in the colony after 10 minutes.

4B 6 Use an index law to simplify the following products.

a $2^5 \times 2^3$ **b** $7^{13} \times 7^7$ **c** $11^5 \times 11$ **d** $9^9 \times 9^9$

4B 7 Use an index law to simplify the following quotients.

a $3^{10} \div 3^6$ **b** $10^{11} \div 10$ **c** $\frac{14^6}{14^2}$ **d** $\frac{8^{17}}{8^9}$

4B 8 Simplify then evaluate each of the following expressions.

a $2^2 \times 2^4$ **b** $5^3 \times 5$ **c** $6^7 \div 6^5$ **d** $\frac{10^{13}}{10^6}$

4C 9 Use an index law to simplify:

a $(4^3)^6$ **b** $(9^2)^3$ **c** $(13^{10})^4$ **d** $(10^{12})^8$

4C 10 Simplify:

a $(3 \times 5)^6$ **b** $\left(\frac{2}{7}\right)^4$ **c** $(9 \times 15)^5$ **d** $\left(\frac{9}{8}\right)^{10}$

4C 11 Evaluate:

a 16^0 **b** $(4 \times 5)^0$ **c** $8^4 \div 8^4$ **d** $\left(\frac{10}{3}\right)^0$

4D Combining the index laws

Learning intentions

By the end of this topic you will be able to ...

✔ use multiple index laws to simplify complex expressions.

Inter-year links

Support	Order of operations
Year 7	1H Order of operations
Year 9	2C Raising indices and the zero index
Year 10	2A Indices

Using multiple index laws to simplify complex expressions

Key content video

- Multiple index laws can be used to simplify complex expressions.
- The order of operations also applies to the simplification of expressions containing indices. Calculations contained in grouping symbols should be simplified first.

	Index law
Product of powers law	$a^m \times a^n = a^{m+n}$
Quotient of powers law	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$
Power of a power law	$(a^m)^n = a^{m \times n}$

$(a \times b)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Example 4D.1 Using index laws to simplify complex calculations

Use index laws to simplify:

a $(2^3)^3 \times (2^2)^{11}$

b $\frac{3^7 \times 3^2}{3^5}$

THINK

a 1 To raise the power, write each base and multiply the indices.

2 Check that the bases are the same. To simplify the product, write the base and add the indices.

b 1 The fraction line groups the terms in the numerator and the denominator, so the expressions in the numerator and denominator should be simplified before dividing the numerator by the denominator. The bases in the numerator are the same, so to simplify the numerator, write the base and add the indices.

2 The bases in the numerator and the denominator are the same, so to simplify the fraction, write the base and subtract the index of the number in the denominator from the index of the number in the numerator.

WRITE

a $(2^3)^3 \times (2^2)^{11} = 2^{(3 \times 3)} \times 2^{(2 \times 11)}$

$= 2^9 \times 2^{22}$

$= 2^{(9+22)}$

$= 2^{31}$

b $\frac{3^7 \times 3^2}{3^5} = \frac{3^{(7+2)}}{3^5}$

$= \frac{3^9}{3^5}$

$= 3^{(9-5)}$

$= 3^4$

Example 4D.2 Simplifying calculations by writing numbers in index notation

Consider the expression $\frac{1024}{32 \times 8}$.

a Write each of the numbers in the expression in index notation.

b Simplify the expression using the index laws.

c Evaluate the expression.

THINK

a Rewrite each number in index notation.

b Rewrite the fraction. All numbers are expressed in index notation with a base of 2, so the expression can be simplified by using the index laws. Simplify the denominator by adding the indices, then simplify the fraction by subtracting the index of the number in the denominator from the index of the number in the numerator.

c Calculate the value of the expression by performing the multiplication.

WRITE

a $8 = 2^3$, $32 = 2^5$, $1024 = 2^{10}$

b $\frac{1024}{32 \times 8} = \frac{2^{10}}{2^5 \times 2^3}$

$= \frac{2^{10}}{2^{(5+3)}}$

$= \frac{2^{10}}{2^8}$

$= 2^{(10-8)}$

$= 2^2$

c $2^2 = 2 \times 2$

$= 4$

Helpful hints

✔ Always check that the bases are the same before simplifying products and quotients of numbers in index notation!

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Exercise 4D Combining the index laws

1–5, 6(a, c), 7(a, d, e), 8 | 1–6, 7(c, e, f), 8(b, d, f, h), 9–12 | 3–6, 7(d, f), 8(b, d, f, h), 9–13

4D.1

1 Use index laws to simplify:

a $(3^2)^3 \times (3^4)^2$ **b** $(9^5)^4 \times (9^3)^7$ **c** $(12^3)^{12} \times (12^4)^0$ **d** $(8^0)^5 \times (8^0)^{21}$

2 Use index laws to simplify:

a $\frac{4^5 \times 4^3}{4^6}$ **b** $\frac{7^6 \times 7^6}{7^{10}}$ **c** $\frac{6^4 \times 6^7}{6^0}$ **d** $\frac{11^8 \times 11^3}{11^5}$

e $\frac{5^6 \times 5^{10}}{5^9 \times 5^3}$ **f** $\frac{8^7 \times 8^2}{8 \times 8^5}$ **g** $\frac{2^8 \times 2^6}{2^0 \times 2^4}$ **h** $\frac{4^{12} \times 4^6}{4^{12} \times 4^3}$

3 Simplify:

a $\frac{(3^2)^4 \times 3^5}{3^4 \times 3^3}$ **b** $\frac{3^5 \times 3^6 \times 3^7}{3^{11} \times 3^2}$ **c** $\frac{4^{14} \times 4^8}{4 \times (4^5)^0}$ **d** $\frac{2^{13} \times (2^4)^3 \times 2^5}{2^{12} \times 2^{10}}$

UNDERSTANDING AND FLUENCY

4 Complete this table.

Base 2	**Index notation**	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	Basic numeral	2	4	8							
Base 3	**Index notation**	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	Basic numeral	3	9	27							

4D.2 **5** **a** Use the table of values from question **4** to express each of the numbers in the following expressions in index notation.

i 32×16 **ii** $1024 \div 256$ **iii** $59\,049 \div 729$ **iv** $\dfrac{128 \times 32}{16}$

v $\dfrac{27 \times 243}{2187}$ **vi** $64 \times 256 \div 512$ **vii** $\dfrac{512 \times 64}{128 \times 16}$ **viii** $\dfrac{128 \times 1024}{256 \times 64}$

b Use the index laws to simplify each expression.

c Refer to the table to write the index notations from part **b** as basic numerals.

6 Evaluate each of the following expressions without a calculator.

a $512 \times 32 \div 64$ **b** $9 \times 243 \div 27$ **c** $\dfrac{32 \times 256}{8 \times 16}$ **d** $\dfrac{6561 \times 19\,683}{729 \times 81}$

PROBLEM SOLVING AND REASONING

7 Use index laws to simplify, then evaluate the following expressions.

a $\dfrac{3^8 \times 2^6}{3^6 \times 2^3}$ **b** $\dfrac{5^9 \times 3^6}{5^7 \times 3^5}$ **c** $\dfrac{6^4 \times 4^7}{6^3 \times 4^5}$

d $\dfrac{8^0 \times 9^8 \times 10^5}{9^7 \times 10^2}$ **e** $\dfrac{7^6 \times 5^{10}}{7^6 \times 5^7}$ **f** $\dfrac{10^7 \times 8^4}{10 \times 8^2 \times 7^0}$

8 Fill in the missing indices in the following equations.

a $(2^3)^4 \times (2^4)^{\square} = 2^{20}$ **b** $\dfrac{6^7 \times 6^7}{6^{\square}} = 6^2$

c $\dfrac{3^{\square} \times 3^2}{3^6} = 1$ **d** $(15^{\square})^8 \times (15^{10})^2 = 15^{20}$

e $\dfrac{(5^2)^4}{5^{\square}} = 5^3$ **f** $\dfrac{4 \times 4^{16}}{4^7 \times 4^{\square}} = 4$

g $(9^{\square} \times 9^4)^2 = 9^{14}$ **h** $\dfrac{(11^{\square})^6 \times 11^6}{11^8 \times 11^7} = 11^9$

9 Explain why $(-5)^2 \neq -(5)^2$, but $(-5)^3 = -(5)^3$.

10 Without evaluating, decide whether the following numerical statements are true or false.

a $(-9)^4 \neq -(9)^4$ **b** $(-11)^9 \neq -(11)^9$ **c** $(-8)^5 = -(2^3)^5$

d $(-100)^4 \neq -(10)^8$ **e** $(-2 \times 3)^9 \neq -(6^3)^3$ **f** $(-3 \times 5)^{16} = -(15^3)^8$

CHALLENGE

11 Use the order of operations to evaluate the following expressions.

a $(2^3 \times 2^2) + (4^3)^0$ **b** $(6^3 \div 6^2 \times 3^0)^2$ **c** $\dfrac{3^5 \div 3^3}{4^6}$

12 Use index laws to find the value of x in the following equation.

$$\frac{(9^x)^3 \times (9^4)^x}{(9^x)^2} = 9^{45}$$

13 Simplify then evaluate the following expression by first writing each base as the product of its prime factors.

$$\frac{96^4 \times 1296^3}{864^5}$$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Index laws

Investigation
Bases of 4 and 5

Topic quiz
4D

4E Roots

Learning intentions

By the end of this topic you will be able to ...

- ✔ calculate square and cube roots
- ✔ identify surds
- ✔ give solutions to problems involving surds as exact or approximate values.

Inter-year links

Year 7	1G Indices and square roots
Year 9	2F Rounding and estimating
Year 10	3A Rational and irrational numbers

Roots

Key content video

- Squaring and taking the square root are inverse (opposite) operations. Similarly, cubing and taking the cube root are inverse operations.
- The square root, $\sqrt{}$, of a number is the number that, when squared, results in the original number.

  For example, $\sqrt{16} = 4$ because $4^2 = 16$.
- The cube root, $\sqrt[3]{}$, of a number is the number that, when cubed, results in the original number.

  For example $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Squares			Cubes		
$1^2 = 1 \times 1 = 1$		$\sqrt{1} = 1$	$1^3 = 1 \times 1 \times 1 = 1$		$\sqrt[3]{1} = 1$
$2^2 = 2 \times 2 = 4$		$\sqrt{4} = 2$	$2^3 = 2 \times 2 \times 2 = 8$		$\sqrt[3]{8} = 2$
$3^2 = 3 \times 3 = 9$		$\sqrt{9} = 3$	$3^3 = 3 \times 3 \times 3 = 27$		$\sqrt[3]{27} = 3$

Surds

- A **perfect square** is the square of an integer and a **perfect cube** is the cube of an integer.

Bases, a	Perfect squares, a^2	Perfect cubes, a^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125

- A square or cube root of a number that produces an irrational number (a non-terminating, non-repeating decimal) is called a **surd**.

  For example, $\sqrt{2}$ is a surd because $\sqrt{2} = 1.414\,21...$

Exact and approximate solutions

- When solving problems involving square or cube roots, solutions that contain surds can be given as either an approximate value (by rounding) or as an exact value (by keeping the surd in the solution).

Example 4E.1 Calculating the value of square roots and cube roots

Calculate the value of:

a the square root of 49

b $\sqrt{121}$

c $\sqrt[3]{125}$

THINK

a Use the times tables to determine which number, multiplied by itself, is equal to 49.

b Use the times tables to determine which number, multiplied by itself, is equal to 121.

c The '3' above the root sign denotes 'the cube root' of 125. Determine which number, when cubed, is equal to 125.

WRITE

a $7 \times 7 = 49$
$\sqrt{49} = 7$

b $11 \times 11 = 121$
$\sqrt{121} = 11$

c $5 \times 5 \times 5 = 125$
$\sqrt[3]{125} = 5$

Example 4E.2 Identifying surds

Use a calculator to determine whether the following roots are surds or not.

a $\sqrt{0.36}$

b $\sqrt{1.25}$

THINK

a $\sqrt{0.36} = 0.6$, which is a terminating decimal, so $\sqrt{0.36}$ is a rational number.

b $\sqrt{1.25} = 1.118\,03...$, which is a non-terminating, non-recurring decimal, so $\sqrt{1.25}$ is an irrational number.

WRITE

a $\sqrt{0.36}$ is not a surd.

b $\sqrt{1.25}$ is a surd.

Helpful hints

- ✔ As with indices, take care when writing cube roots. In a cube root, the '3' should be small and sit high up on the shoulder of the root sign to avoid confusion between $\sqrt[3]{8}$ and $3\sqrt{8}$.
- ✔ Remember that if no number is specified to the left of the root sign, it stands for 'square root'.

  For example, $\sqrt{4} = 2$.
- ✔ Recall that a root sign is a grouping symbol, so operations contained within a root sign should be completed before applying the root.

  For example: $\sqrt{16 + 9} = \sqrt{25}$
  $= 5$.

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Exercise 4E Roots

1–7, 9–11, 12(a, b), 14(a, b) | 2, 3, 5–11, 14, 17 | 2, 3, 5–9, 12–17

UNDERSTANDING AND FLUENCY

1 Complete the table below by evaluating the numbers in index notation.

Index notation	Perfect square	Index notation	Perfect cube
2^2		2^3	
3^2		3^3	
4^2		4^3	
5^2		5^3	
6^2		6^3	
7^2		7^3	
8^2		8^3	
9^2		9^3	
10^2		10^3	

4E.1 **2** Calculate the value of:

a the square root of 16 **b** the square root of 9 **c** the square root of 25
d $\sqrt{1}$ **e** $\sqrt{4}$ **f** $\sqrt{100}$
g $\sqrt{81}$ **h** $\sqrt{225}$ **i** $\sqrt{144}$
j $\sqrt{400}$ **k** $\sqrt{169}$ **l** $\sqrt{900}$

3 Calculate the value of:

a the cube root of 8 **b** the cube root of 64 **c** the cube root of 125 **d** the cube root of 1
e $\sqrt[3]{27}$ **f** $\sqrt[3]{1000}$ **g** $\sqrt[3]{216}$ **h** $\sqrt[3]{8000}$

4 Use a calculator to evaluate the following square roots.

a $\sqrt{196}$ **b** $\sqrt{256}$ **c** $\sqrt{1089}$
d $\sqrt{26\,244}$ **e** $\sqrt{2.25}$ **f** $\sqrt{53.29}$
g $\sqrt{67.24}$ **h** $\sqrt{475.24}$ **i** $\sqrt{25.3009}$

4E.2 **5** Use a calculator to determine whether the following roots are surds or not.

a $\sqrt{144}$ **b** $\sqrt{20}$ **c** $\sqrt{54.76}$
d $\sqrt{331\,776}$ **e** $\sqrt{8.3}$ **f** $\sqrt{0.49}$

6 To evaluate the square root of a fraction, take the square root of the numerator and the denominator individually.

For example, $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$.

Evaluate the following fractional square roots.

a $\sqrt{\frac{1}{16}}$ **b** $\sqrt{\frac{1}{64}}$ **c** $\sqrt{\frac{4}{25}}$
d $\sqrt{\frac{9}{49}}$ **e** $\sqrt{\frac{81}{100}}$ **f** $\sqrt{\frac{36}{121}}$

7 Use the correct order of operations to determine the value of the following expressions. Hint: the root sign is a grouping symbol, so operations contained within the root sign should be completed before applying the root.

a $4(\sqrt{25} + 15)$ **b** $(\sqrt{36} - \sqrt{25})^4$ **c** $2\sqrt{5^2 - 4^2}$ **d** $3\left(\sqrt{13^2 - 12^2}\right)^2$

PROBLEM SOLVING AND REASONING

8 Evaluate the following expressions, giving your answers as:

i exact values **ii** approximate values correct to two decimal places.

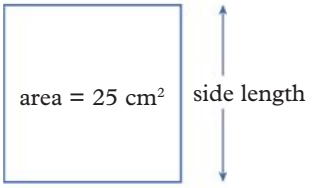
a $\sqrt{3^3 + 2^2}$ **b** $\sqrt{5 \times 4^3} - 2$ **c** $\sqrt[3]{21-3^3}$ **d** $\sqrt[3]{(3^2)^2}$

9 Identify and explain the errors in the following calculations. Then change the right-hand side to make the equation a true statement.

a $\sqrt{16} = 8$ **b** $\sqrt[3]{64} = 8$

10 Explain how finding the square and square root of a number are related.

11 a If a square has area 25 cm², explain how you can calculate its side length.



b Calculate the side length of a square with area:

i 169 cm² **ii** 200 m² **iii** 268.96 km².

c Which of your answers is a surd?

12 The fourth root of a number can be determined using a similar method to the one used to find the square root.

a If $5^4 = 625$, what is the fourth root of 625, $\sqrt[4]{625}$?

b Evaluate the following roots without using a calculator.

i $\sqrt[4]{16}$ **ii** $\sqrt[4]{81}$ **iii** $\sqrt[4]{10\,000}$

13 a Use a calculator to evaluate the following roots.

i $\sqrt[3]{3375}$ **ii** $\sqrt[3]{68.921}$ **iii** $\sqrt[3]{99}$

iv $\sqrt[4]{2401}$ **v** $\sqrt[4]{73}$ **vi** $\sqrt[4]{0.1}$

b Classify each root in part **a** as surds or not surds.

14 a If $(-3)^3 = -27$, what is $\sqrt[3]{-27}$?

b If $(-2)^5 = -32$, what is $\sqrt[5]{-32}$?

c Explain why it is not possible to calculate $\sqrt{-4}$.

15 a Calculate the values of the following pairs of expressions.

i $\sqrt{4} \times \sqrt{16}$ and $\sqrt{64}$ **ii** $\sqrt{9} \times \sqrt{36}$ and $\sqrt{324}$ **iii** $\sqrt{25} \times \sqrt{16}$ and $\sqrt{400}$

b Use your answers to part **a** to write an equation linking $\sqrt{a}$, $\sqrt{b}$ and $\sqrt{ab}$.

16 Use your answer from question **15b** to simplify and evaluate the following square roots without using a calculator.

a $\sqrt{324}$ **b** $\sqrt{676}$ **c** $\sqrt{1936}$ **d** $\sqrt{5184}$

CHALLENGE

17 Prime factorisation can be used to determine if a number is a perfect square.

For example, 196 is a perfect square because the prime factors of 196 can be divided into two identical groups.

$$\begin{aligned} 196 &= 2 \times 2 \times 7 \times 7 \\ &= 2 \times 7 \times 2 \times 7 \\ &= 14 \times 14 \\ &= 14^2 \end{aligned}$$

a Use prime factorisation to determine whether the following numbers are perfect squares, and find the square root of each perfect square.

i 168 **ii** 289 **iii** 576 **iv** 786

b Use prime factorisation to determine whether the following numbers are perfect cubes, and find the cube root of each perfect cube.

i 216 **ii** 512 **iii** 730 **iv** 1331

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Roots

Investigation
Estimating square roots and cube roots

Topic quiz
4E

Chapter summary

Terminology

index/exponent

base → $3^4 = 3 \times 3 \times 3 \times 3 = 81$

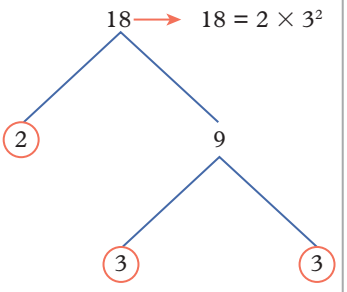
index notation — expanded form — basic numeral

3^2 can be read as '3 to the power of 2', '3 squared' or 'the square of 3'.

4^3 can be read as '4 to the power of 3', '4 cubed' or 'the cube of 4'.

Prime factorisation

- The prime factorisation of an integer is the product of the primes that make up that integer.



Index laws

	Index law	Example
Product of powers law	$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^{5+3} = 2^8$
Quotient of powers law	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$	$\frac{2^8}{2^5} = 2^8 \div 2^5 = 2^{8-5} = 2^3$
Power of a power law	$(a^m)^n = a^{m \times n}$	$(2^3)^5 = 2^{3 \times 5} = 2^{15}$

$(a \times b)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Combining index laws

BIDMAS

- Multiple index laws can be used to simplify complex calculations.

$$\frac{3^7 \times 3^2}{3^5} = \frac{3^{(7+2)}}{3^5} = \frac{3^9}{3^5} = 3^{(9-5)} = 3^4$$

$$(2^3)^3 \times (2^2)^{11} = 2^{(3 \times 3)} \times 2^{(2 \times 11)} = 2^9 \times 2^{22} = 2^{(9+22)} = 2^{31}$$

The zero index

- Excluding 0, any number to the power of 0 is equal to 1.

$a^0 = 1$

$$8^2 \div 8^2 = 8^{(2-2)} = 8^0 = 1$$

Surds

- Irrational numbers are non-recurring and non-terminating decimals.
- A square or cube root that produces an irrational number is a surd.

$\sqrt{2} = 1.414\,21\ldots$

Perfect squares and perfect cubes

Squares

$1^2 = 1 \times 1 = 1$ $\sqrt{1} = 1$

$2^2 = 2 \times 2 = 4$ $\sqrt{4} = 2$

$3^2 = 3 \times 3 = 9$ $\sqrt{9} = 3$

Cubes

$1^3 = 1 \times 1 \times 1 = 1$ $\sqrt[3]{1} = 1$

$2^3 = 2 \times 2 \times 2 = 8$ $\sqrt[3]{8} = 2$

$3^3 = 3 \times 3 \times 3 = 27$ $\sqrt[3]{27} = 3$

Bases, a	Perfect squares, a^2	Perfect cubes, a^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125

Exact and approximate solutions

- Solutions that contain surds can be given as exact or approximate values.

$\sqrt{3}$ or 1.73 (to two decimal places)

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Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- base
- composite number
- cube
- cube root
- expanded form
- factor trees
- index
- index form
- index laws
- index notation
- irrational number
- order of operations
- perfect cube
- perfect square
- power
- power of a power law
- prime factorisation
- prime number
- product of powers law
- quotient of powers law
- simplification
- square
- square root
- surd
- zero index

1 Which key term can be used instead of index notation?

2 What can be used to help determine the prime factorisation of an integer?

3 If a square or cube root is an irrational number, what is it also known as?

4 Identify the key terms being referenced in each of these definitions.

a the cube of an integer

b a non-terminating, non-repeating decimal

5 Using an example, provide a definition in your own words for the following key terms.

a quotient of powers law

b composite number

6 Complete the following sentences using words from the key term list.

a The _____ of a number is the result of multiplying a number by itself.

b _____ involves reducing the complexity of an expression.

Multiple choice

4A **1** 3^4 is equal to:

A 1 **B** 12 **C** 34 **D** 64 **E** 81

4A **2** The cube of 8 is:

A 800 **B** 512 **C** 64 **D** 24 **E** 2

4B **3** Which of the following is the simplest form of $3^5 \times 3^4$?

A $\frac{1}{3}$ **B** 3^0 **C** 3 **D** 3^9 **E** 3^{20}

4B **4** Simplify $\frac{12^6}{12^2}$.

A 12 **B** 12^3 **C** 12^4 **D** 12^8 **E** 12^{12}

4C **5** Evaluate $\left(\frac{2}{5}\right)^3$.

A $\frac{8}{5}$ **B** $\frac{6}{5}$ **C** $\frac{6}{15}$ **D** $\frac{8}{125}$ **E** $\frac{2}{125}$

4C **6** Which of the following expressions does not equal 1?

A 10^0 **B** $10^0 \times 10$ **C** $\frac{10^3}{10^3}$ **D** $(10^3)^0$ **E** $(10^3 \times 10)^0$

4D **7** Use index laws to simplify $(5^2)^7 \times (5^6)^4$.

A 54 **B** 5^7 **C** 5^{19} **D** 5^{38} **E** 5^{90}

4D **8** The missing index in the equation $\frac{12^3 \times 12^{\square}}{12^4} = 12^5$ is:

A 2 **B** 3 **C** 4 **D** 5 **E** 6

4E **9** The cube root of 64 is:

A 3 **B** 4 **C** 8 **D** 16 **E** 24

4E **10** Which of the following roots is a surd?

A $\sqrt{8}$ **B** $\sqrt{16}$ **C** $\sqrt{64}$ **D** $\sqrt[3]{27}$ **E** $\sqrt[3]{216}$

Short answer

4A **1** Write each number in expanded form, then calculate its value.

a 3^3 **b** 2^4 **c** 8^2 **d** 10^5

4A **2** Evaluate:

a 6^2 **b** 8 squared **c** the cube of 2 **d** 4 cubed

4A **3** Use a factor tree to find the prime factorisation of the following numbers. Write your answer in index notation.

a 45 **b** 124

4B **4** Use an index law to simplify:

a $3^5 \times 3^7$ **b** $2^7 \times 2^3$ **c** $12^3 \times 12$ **d** $4^6 \times 4^2 \times 4^3$

4B **5** Simplify the following expressions using an index law.

a $6^8 \div 6^2$ **b** $5^7 \div 5^7$ **c** $\frac{10^{15}}{10^8}$ **d** $\frac{7^2}{7}$

4C **6** Use an index law to simplify:

a $(9^5)^3$ **b** $(6 \times 3)^{10}$ **c** $\left(\frac{3}{10}\right)^3$ **d** $\left(\frac{2 \times 7}{9}\right)^8$

4C **7** Evaluate the following expressions.

a 4×4^0 **b** $(7 \times 5)^0$ **c** $\frac{20^{10}}{20^{10}}$ **d** $\left(\frac{5}{5^0}\right)^2$

4D **8** Simplify, then evaluate the following expressions.

a $\frac{7^6 \times 7^5}{7^9}$ **b** $\frac{2^8 \times (2^4)^2 \times 2^3}{2^{14}}$

4D **9** For each of the following expressions:

i express each number in index notation with the same base

ii simplify using the index laws

iii evaluate.

a $\frac{5^4}{25}$ **b** $\frac{3^2 \times 81}{27}$ **c** $\frac{64 \times 256}{4 \times 16}$ **d** $16 \times (4^2)^2$

4E **10** Calculate the value of:

a the square root of 4

b $\sqrt{81}$

c the cube root of 27

d $\sqrt[3]{125}$

4E **11** **a** Use a calculator to determine the value of the following to two decimal places where necessary.

i $\sqrt{529}$ **ii** $\sqrt{700}$ **iii** $\sqrt{156.25}$ **iv** $\sqrt[3]{42}$

b Classify each number in part **a** as a surd or not a surd.

4E **12** Evaluate:

a $\sqrt{4} + 2 \times \sqrt[3]{8}$ **b** $3^3 - 5 \times \sqrt{16}$ **c** $(4^3 + 2) \times \sqrt{25}$ **d** $\frac{(4^2 \times 4^2 + 2 \times \sqrt{9})}{(3 + 12^0)}$

Analysis

1 A radioactive metal has a half-life of 1 day. This means that each day, half the mass of the metal disintegrates into the atmosphere as a result of radioactive decay.

A physicist observes a 240 gram piece of the metal over a number of days and records the following data.

Number of days	Mass of metal (grams)	$\frac{\text{mass of metal}}{\text{original mass of metal}}$
0	240	
1		$\frac{1}{2}$
2		
3		

a Complete the table.

b Express each of the numbers in the third row in index notation with a base of $\frac{1}{2}$.

c Write a calculation in index notation for the amount of metal remaining after one week.

d Use an index law to simplify, then evaluate your answer to part **c**. Express your answer as a fraction.

e How much metal will be left after 1 week? Express your answer as a decimal.

f How long until there is less than 1 gram of metal remaining?

2 Consider the fraction $\frac{490 \times 36^2}{756 \times 507^0}$.

a Use prime factorisation to rewrite the fraction, expressing each number as the product of its prime factors.

b Simplify the fraction using index laws.

c Evaluate your answer to part **b**.

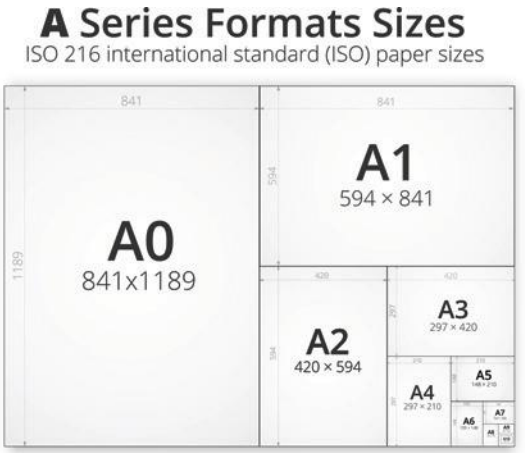
3 The thickness of a piece of A0 sized paper is 0.05 mm.

a When a piece of A0 paper is folded in half once, the size becomes A1. What is the thickness of the A1 paper?

b When a piece of A0 paper is folded in half twice, the size becomes A2. What is the thickness of the A2 paper?

c Determine the thickness of the A6 paper if the A0 paper is folded in half six times to become create the A6 size.

d If the thickness of a piece of paper is a, and the paper is folded in half n times, what is the thickness of the folded paper?



4 **a** Arrange the following numbers in ascending order using the less than sign '<':
1, 2, 4, 6, 9, 16, 20, 25

b Without using a calculator, determine which of the following roots are surds.

i $\sqrt{1}$ **ii** $\sqrt{2}$ **iii** $\sqrt{4}$ **iv** $\sqrt{6}$

v $\sqrt{9}$ **vi** $\sqrt{16}$ **vii** $\sqrt{20}$ **viii** $\sqrt{25}$

c Use a calculator to evaluate the roots in part **b** correct to two decimal places where necessary.

d Arrange your answers from part **c** in ascending order using the less than sign '<'.

e Compare your answers to parts **a** and **d**. What do you notice?

f Use your answer to part **e** to complete the following number statements.

i $\sqrt{30}$ _____ 6 **ii** $\sqrt{18}$ _____ 4 **iii** $\sqrt{12}$ _____ 4 **iv** $\sqrt{8}$ _____ 3

v $\sqrt{130}$ _____ 10 **vi** $\sqrt{60}$ _____ 8 **vii** $\sqrt{155}$ _____ 12 **viii** $\sqrt{300}$ _____ 20

g Arrange the following numbers in ascending order using the less than sign '<':
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \frac{1}{16}, \frac{1}{20}, \frac{1}{25}$

h Without using a calculator, determine which of the following roots are surds.

i $\sqrt{1}$ **ii** $\sqrt{\left(\frac{1}{2}\right)}$ **iii** $\sqrt{\left(\frac{1}{4}\right)}$ **iv** $\sqrt{\left(\frac{1}{6}\right)}$

v $\sqrt{\left(\frac{1}{9}\right)}$ **vi** $\sqrt{\left(\frac{1}{16}\right)}$ **vii** $\sqrt{\left(\frac{1}{20}\right)}$ **viii** $\sqrt{\left(\frac{1}{25}\right)}$

i Use a calculator to evaluate the roots in part **h** correct to two decimal places where necessary.

j Arrange your answers from part **i** in ascending order using the less than sign '<'.

k Compare your answers to parts **h** and **j**. Does your answer from part **e** still hold?

l Use your answer to part **k** to complete the following number statements.

i $\sqrt{\left(\frac{1}{30}\right)}$ _____ $\frac{1}{5}$ **ii** $\sqrt{\left(\frac{1}{70}\right)}$ _____ $\frac{1}{8}$ **iii** $\sqrt{\left(\frac{1}{10}\right)}$ _____ $\frac{1}{3}$ **iv** $\sqrt{\left(\frac{1}{17}\right)}$ _____ $\frac{1}{5}$

v $\sqrt{\left(\frac{7}{30}\right)}$ _____ $\frac{1}{3}$ **vi** $\sqrt{\left(\frac{3}{10}\right)}$ _____ $\frac{2}{5}$ **vii** $\sqrt{\left(\frac{7}{20}\right)}$ _____ $\frac{2}{5}$ **viii** $\sqrt{\left(\frac{4}{13}\right)}$ _____ $\frac{1}{4}$

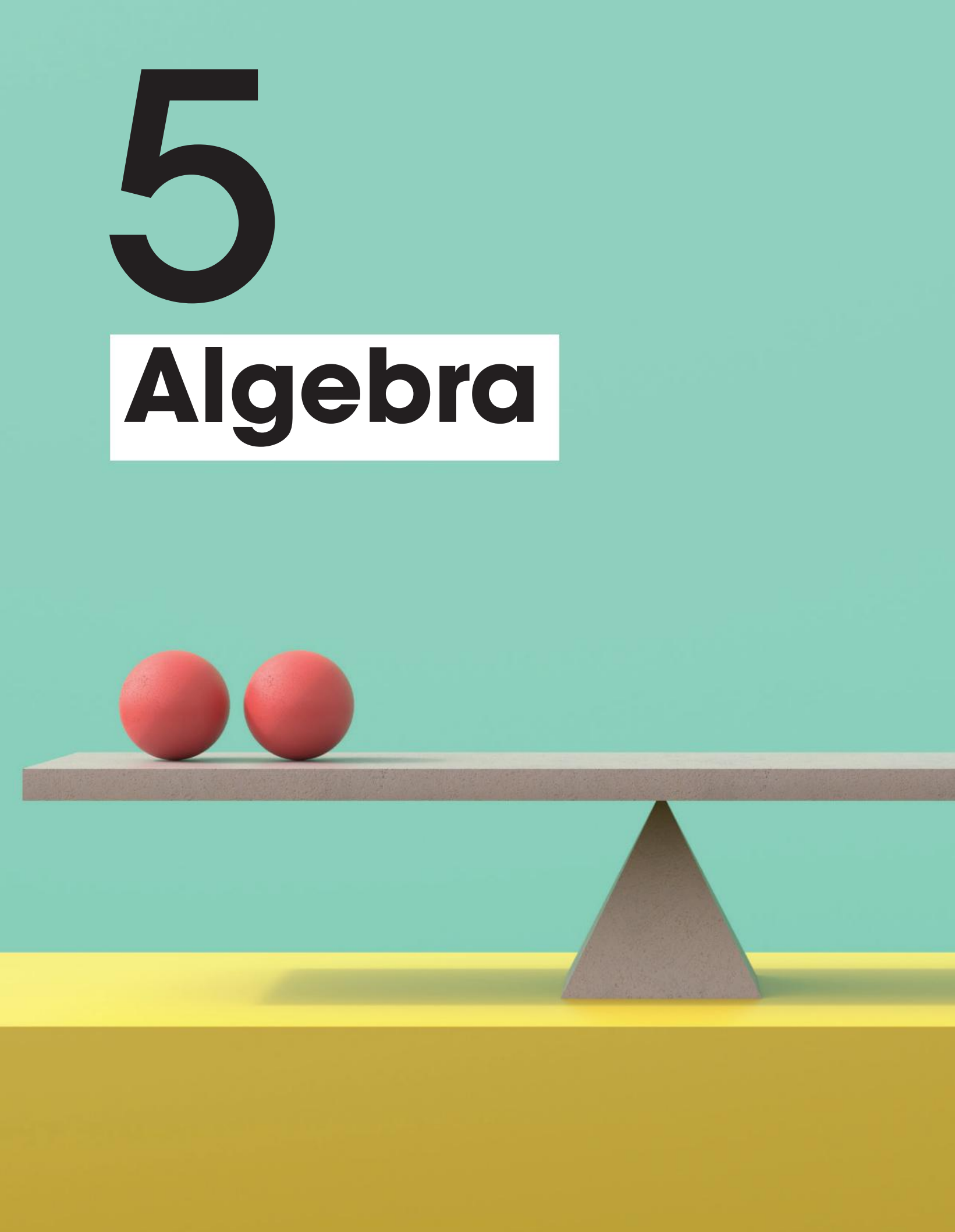
Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert numbers between index notation and expanded form ☐ Calculate the value of numbers in index notation ☐ Express integers as a product of prime factors	☐ Go back to Topic 4A Indices
☐ Simplify products of numbers with the same base ☐ Simplify quotients of numbers with the same base	☐ Go back to Topic 4B Products and quotients of powers
☐ Raise a number in index notation by another index ☐ Evaluate expressions involving the zero index	☐ Go back to Topic 4C Raising indices and the zero index
☐ Use multiple index laws to simplify complex expressions	☐ Go back to Topic 4D Combining the index laws
☐ Calculate square and cube roots ☐ Identify surds ☐ Give solutions to problems involving surds as exact or approximate values	☐ Go back to Topic 4E Roots

5

Algebra



Index

5A Expressions
5B Substitution
5C Adding and subtracting algebraic terms
5D Multiplying algebraic terms
5E Dividing algebraic terms
5F Expanding
5G Factorising

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Variables
- ✔ Writing formulas
- ✔ Grouping symbols
- ✔ Order of operations
- ✔ Highest common factor

Curriculum links

- Generalises number properties to operate with algebraic expressions including expansion and factorisation (MA4-ALG-C-01)
  - Examine the concept of pronumerals as a way of representing numbers
  - Create algebraic expressions and evaluate them by substitution
  - Extend and apply the laws and properties of arithmetic to algebraic terms and expressions
  - Extend and apply the distributive law to the expansion of algebraic expressions
  - Factorise algebraic expressions by identifying numerical and algebraic factors

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Materials

✔ Calculator

5A Expressions

Learning intentions

By the end of this topic you will be able to ...

✔ identify terms and expressions

✔ write expressions to represent worded problems.

Inter-year links

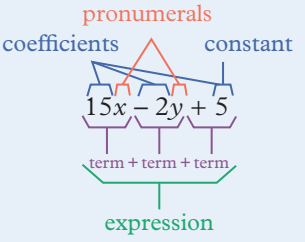
Support	Writing formulas
Year 7	6B Writing formulas

Pronumerals

Key content video

- A **pronumeral** is a letter or symbol that is used in place of a number. Pronumerals can be used to represent an unknown number or a variable.
  - → An **unknown number** has one value.
  - → A **variable** is a quantity that can have different values.

Terms and expressions



- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations.
- A **term** is a single pronumeral, a single number, or the product of a number and one or more pronumerals. For example, $3x$, $4xy$, $-2y$, xyz, 5. Terms in an expression are separated from each other by a plus or minus sign (as long as the plus or minus sign is not inside any brackets).

  Note: If the term is separated on the left by a minus sign, then the term is negative.
- A **coefficient** is the number acting as a multiplier in an algebraic term, usually written before the pronumeral. A pronumeral without a number preceding it has a coefficient of 1.
- A **constant** is a term without any pronumerals, it is also called the constant coefficient or the constant term.

Algebraic notation

- Products are simplified by leaving out the multiplication sign and placing the number first.

  $7 \times x = 7x$ and $7 \times (x + 2) = 7(x + 2)$
- When a pronumeral is multiplied by 1, the 1 is left out.

  $1 \times x = x$
- Quotients are represented using fractions.

  $x \div 7 = \frac{x}{7}$ and $(x + 2) \div 7 = \frac{(x + 2)}{7}$
- Terms with fractional coefficients can be written in two ways.

  $\frac{x}{7} = \frac{1}{7}x$
- **Grouping symbols** include brackets, the fraction line in a quotient and the root sign. Grouping symbols are used to preserve order of operations in algebraic expressions.

  $7(x + 2)$

  $\frac{x + 2}{7}$

  $7\sqrt{x + 2}$

Example 5A.1 Identifying terms, coefficients and constants

Consider the expression $4a + \frac{b}{2} - c - 11$.

a List the terms in the expression.

b List all variables in the expression.

c Identify the coefficient of each term that contains a variable.

d List any constant terms.

THINK	WRITE
a Identify the individual terms, which are separated by a + or – sign. A minus sign can be read as a negative sign and therefore 'belongs' to the term to the right of the sign.	**a** The terms are $4a$, $\frac{b}{2}$, $-c$ and -11.
b Variables are represented by pronumerals. List the pronumerals.	**b** The variables are a, b and c.
c Identify the number that comes before each pronumeral. Terms with fractional coefficients can be written in two ways: $\frac{b}{2} = \frac{1}{2}b$, so the coefficient of the b term is $\frac{1}{2}$. As the negative sign is a property of the c term, the coefficient of c is -1.	**c** The coefficient of a is 4. The coefficient of b is $\frac{1}{2}$. The coefficient of c is -1.
d List the terms that do not contain pronumerals.	**d** The constant term is -11.

Example 5A.2 Writing expressions

Write an expression for each statement.

a 5 less than a

b a quarter of b

THINK	WRITE
a Think about which operation is taking place. 'Less than' means subtract, so 5 less than a is a minus 5.	**a** $a - 5$
b 'A quarter' of b means b divided by 4. Simplify the quotient by rewriting it as a fraction. Terms with fractional coefficients can be written in two ways – either form is acceptable.	**b** $b \div 4$ $= \frac{b}{4}$ or $\frac{1}{4}b$

Example 5A.3 Writing expressions using grouping symbols

Write an expression for each statement using grouping symbols where appropriate.

a Add 7 to b, then multiply the result by 4.

b Divide the product of a and d by 2.

THINK	WRITE
a **1** Think about which operations are taking place. 'Add 7' means +7.	**a** $b + 7$
2 Look for words that refer to the order of operations. '*Then* multiply *the result* by 4' means the multiplication should be performed *after* the addition. Group $b + 7$ using brackets, then multiply the brackets by 4.	$(b + 7) \times 4$
3 Simplify the product by placing the number first and removing the multiplication sign.	$= 4(b + 7)$
b **1** Finding 'the product' means to multiply.	**b** $a \times d$
2 'Divide the product by 2' means the whole product should be divided by 2.	$(a \times d) \div 2$
3 Simplify the product and the quotient. The fraction line is a grouping symbol, so the brackets are no longer necessary.	$= \frac{ad}{2}$

Example 5A.4 Writing an expression from a worded problem

Lee works casually as a nurse. He earns \$35 an hour, plus a bonus of \$50 for each night shift. Write an expression to represent the amount of money Lee will earn for a night shift of t hours.

THINK	WRITE
1 Identify the variable in the problem.	t = number of hours in the shift
2 Think about which operations are taking place. If Lee earns \$35 per hour, he will earn 35 times the number of hours he works. 'Plus' means add 50 to Lee's total earnings for a given night shift.	$35 \times t + 50$
3 Write an expression to represent the amount of money Lee will earn for a night shift of t hours. Simplify the product.	$= 35t + 50$

Helpful hints

- ✔ In the expression $5 + x$, the coefficient of x is 1, not 0.
- ✔ Watch out for negative coefficients in expressions containing two or more terms. In the expression $5 - 3x$, for example, the coefficient of x is -3.
- ✔ The order of operations changes with the placement of grouping symbols. Take care when placing brackets and writing fractions and roots! For example:
  $\frac{x + 1}{2}$ is not the same as $\frac{x}{2} + 1$.

ANS p504

Exercise 5A Expressions

1–6, 7(b, c, f, h), 8–11, 13, 15 | 1, 3, 5, 6, 8, 11, 12, 14, 16, 17, 19 | 1, 3, 5, 8, 12, 16–20

UNDERSTANDING AND FLUENCY

5A.1 **1** For each of the following expressions:

i list the terms in the expression

ii list all variables in the expression

iii identify the coefficient of each term that contains a variable

iv list any constant terms.

a $6a + b - 1$ **b** $2xy - 3x$ **c** $11f - g + 15h - 10$ **d** $-\frac{9}{4}p$

e $\frac{3}{4}c + d + \frac{1}{3}$ **f** $\frac{x}{2} + \frac{y}{11} - \frac{7}{10} - \frac{z}{8}$ **g** $-\frac{m}{5} - \frac{n}{7} - 8p$ **h** $\frac{5x}{8} - 9y + \frac{z}{4} - \frac{2}{5}$

2 State whether each of these is an expression or an equation.

a $2a + 3$ **b** $x - 6 = 1$ **c** $7 = 5 + \frac{5}{6}h$

d $x + y + z = 9$ **e** $3a + b - 7$ **f** $k = \frac{4m + 1}{8}$

3 Answer true or false for each statement. For the statements that are false, provide a reason for your answer.

a $2x + \frac{1}{3} = 4$ is an expression.

b $5 - 12a + b$ has 3 terms.

c $7 - y$ is a term.

d $\frac{5}{7}$ is a term.

e The coefficient of $-d$ is 1.

f The coefficient of $\frac{b}{12}$ is 12.

g $-\frac{5f}{4}$ is an expression.

h The coefficient of $\frac{2x}{3}$ is $\frac{2}{3}$.

5A.2 **4** Write an expression for each statement.

a 5 more than a

b z divided by 3

c 23 less than y

d 50 times m

e the product of x and $\frac{2}{5}$

f twice the value of p

g half of s

h the sum of three-quarters of w and 5

5 Write an expression for each statement. Use x to represent the unknown number.

a A number is multiplied by 7 and then 2 is added.

b A number is divided by 3 and then 6 is subtracted.

c 10 less than three-quarters of a number

d the sum of 13 and a third of a number

e 28 less than the product of 15 and a number

f 5 less than half of a number

UNDERSTANDING AND FLUENCY

6 Write a number sentence to describe the order of operations in each of the following expressions.

a $c + 8$ **b** $11 - x$ **c** $\frac{x}{5}$

d $\frac{9}{10}q$ **e** $3x - 5$ **f** $\frac{10a}{3} + 16$

5A.3 **7** Write an expression for each statement using grouping symbols where appropriate.

a add 6 to x, then multiply the result by 3

b subtract 10 from n, then divide the result by 7

c the product of f and g, all multiplied by $\frac{3}{5}$

d 1 less than the square root of t

e the sum of b and 12, all divided by 15

f the square root of the sum of z and 9

g 7 less two-thirds of v

h triple the sum of 33 and y

8 Write an expression to represent each of the following number sentences. Use x for the unknown number.

a subtract 3 from a number, then multiply the result by 8

b the sum of a number and 20, all multiplied by 6

c 2 less than the product of $\frac{5}{6}$ and a number

d 12 more than four-fifths of a number

e double the sum of a number and 10

f the sum of a number and 4 is divided by 12, then 16 is added to the result

PROBLEM SOLVING AND REASONING

5A.4 **9** Amir has n songs on his phone. He downloads a further 18 songs. Write an expression to represent the number of songs he now has on his phone.

10 Four friends bought a lottery ticket together and agreed to share the prize evenly. If they win \$$c$ in total, how much does each person get?

11 Pritesh finds x old records in his shed and decides to sell them for \$8 each. Write an expression for the total price of the records.

12 Ash is running low on free space on his laptop. He deletes 10 GB (gigabytes) of data that he doesn't need anymore. Write an equation for the amount of data remaining on his laptop, y, compared to the original amount of data, x.

13 Hayden is y years old. Answer each of the following questions using an algebraic expression.

a How old was Hayden:

i 6 years ago **ii** 10 years ago?

b How old will Hayden be in:

i 5 years' time **ii** 23 years' time?

c How old is Hayden's sister if she is x years younger than him?

d How old is Hayden's uncle if he is twice as old as Hayden?

14 A kitchen hand is washing up c cups and p plates.

a How many items does he have to wash?

b If he breaks a plate, how many plates are left?

c After he breaks a plate, how many cups and plates does he now have to dry?

15 An art teacher orders n brushes from an online store that charges \$12 for delivery. The total cost of his order can be represented by the algebraic expression $8n + 12$.

a List the terms in the expression.

b What does the constant term represent?

c What does the coefficient of the term with n represent?

d What will the total cost of the order be if the art teacher orders 10 brushes?

16 Gillian hires a photobooth for a birthday party. The photobooth company charges \$130 per hour and an extra flat fee of \$25 for props and backdrops.

a Write an expression for the total cost of hiring the photobooth for t hours.

b How much will it cost Gillian to hire the photobooth, props and backdrops for 3 hours?

17 At the end of autumn, Ron has x chickens on his farm. 7 baby chickens hatch over winter. By the end of spring, more chickens have hatched: Ron now has double the number of chickens than he did at the end of winter.

a Write an expression for the total number of chickens on Ron's farm at the end of spring.

b If Ron had 13 chickens at the end of autumn, how many chickens did he have at the end of spring?

18 Naomi and Rupi are working on the following question:

'A container of x puzzle blocks is divided between three students. If each student has the same number of blocks and one block is left over, write an expression for the number of blocks each student has to work with.'

Naomi believes the answer is $\frac{x-1}{3}$ while Rupi thinks the answer should be $\frac{x+1}{3}$.

a Decide who is correct. Explain why their expression is correct.

b Rewrite the question to represent the other expression.

19 Consider the expression $\frac{4x-5}{9}$.

a Rewrite the expression as two separate fractions. Hint: consider $(a + b) \div c = a \div c + b \div c$.

b What is the coefficient of the term with variable x?

c What is the value of the constant term?

20 During a sale, a shop advertises 20% off the full price of t-shirts and 30% off the full price of pants. A customer buys x t-shirts that cost \$$y$ full price, and a pants that cost \$$b$ full price.

a Write a two-term expression for the total cost of the customer's purchase.

b If the customer bought twice as many t-shirts as pants, the t-shirts cost \$20 each at full price and the pants cost \$50 each at full price, rewrite the expression using one pronumeral only.

c How many t-shirts and pairs of pants did the customer buy if they spent a total of \$134?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Expressions and equations

Worksheet Writing algebraic expressions

Investigation Pronumerals in mark-ups and discounts

Topic quiz 5A

5B Substitution

Learning intentions

By the end of this topic you will be able to ...

✔ substitute values into and evaluate expressions

✔ substitute values into formulas to find unknown values.

Inter-year links

Year 7 6E Substitution

Substitution

Key content video

- **Substitution** involves replacing each pronumeral in an algebraic expression with a number.
- For clarity, the number being substituted into an expression is enclosed in brackets when:
  - → the number is negative
  - → the expression involves multiplication or indices.
- To **evaluate** an algebraic expression, substitute the known values of each pronumeral into the expression and complete the operations in the correct order.
- A **formula** is an algebraic statement that describes the relationship between two or more variables.

  For example, the formula for the area of a triangle is:

  $A = \frac{1}{2}bh$

  where A = area

  b = base length

  h = height.
- Substitution can be used to find the value of an unknown variable when the values of all the other variables in a formula are known.

Let $x = -2$

$5x^2 - x = 5(-2)^2 - (-2)$

B	**Brackets**
I	**Indices or exponents**
D	**Division**
M	**Multiplication**
A	**Addition**
S	**Subtraction**

Example 5B.1 Evaluating expressions

Evaluate each expression by substituting 3 for x.

a $\frac{x}{3} + 1$

b $4x^2 + 1$

THINK

a Replace x with 3 and complete the operations in the correct order. Division is performed before addition.

b Replace x with 3 and complete the operations. As the expression involves multiplication and indices, enclose the 3 in brackets for clarity. Indices are applied first, then multiplication, then addition.

WRITE

a $\frac{x}{3} + 1 = \frac{3}{3} + 1$

$= 1 + 1$

$= 2$

b $4x^2 + 1 = 4(3)^2 + 1$

$= 4(9) + 1$

$= 36 + 1$

$= 37$

Example 5B.2 Substituting negative values into expressions

Evaluate each expression by letting $m = 5$ and $n = -2$.

a $3m - n$

b $2mn^2$

THINK

a Replace m with 5 and n with -2 and complete the operations in the correct order. Multiplication is performed before subtraction. Remember that subtracting a negative integer is equivalent to adding a positive integer.

b Replace m with 5 and n with -2 and complete the operations. Indices are applied before multiplication. Remember that the product of two negative numbers is a positive number, so $(-2)^2 = -2 \times -2 = 4$.

WRITE

a $3m - n = 3(5) - (-2)$
$= 15 + 2$
$= 17$

b $2mn^2 = 2(5)(-2)^2$
$= 2(5)(4)$
$= 40$

Example 5B.3 Evaluating expressions containing grouping symbols

Substitute $a = 3$ and $b = 4$ into each expression and evaluate.

a $7(a - b)$

b $\frac{2a + b}{5}$

THINK

a Replace a with 3 and b with 4 and complete the operations in the correct order. Bracketed operations are performed first, so complete the subtraction in the brackets before the multiplication. Remember that the product of a positive number and a negative number is a negative number.

b Replace a with 3 and b with 4 and complete the operations. The fraction line is a grouping symbol, so the expression in the numerator of the fraction should be evaluated first.

WRITE

a $7(a - b) = 7(3 - 4)$
$= 7(-1)$
$= -7$

b $\frac{2a + b}{5} = \frac{2(3) + 4}{5}$
$= \frac{6 + 4}{5}$
$= \frac{10}{5}$
$= 2$

Example 5B.4 Using formulas to find unknown values

Use the formula $y = 2\sqrt{x + 9}$ to find the value of y if $x = 7$.

THINK

To find the value of y, substitute $x = 7$ into the expression on the right-hand side of the equation and evaluate. Remember that the root sign is a grouping symbol, so the expression contained within the root sign should be evaluated first.

WRITE

$y = 2\sqrt{x + 9}$
$= 2\sqrt{7 + 9}$
$= 2\sqrt{16}$
$= 2 \times 4$
$= 8$

Helpful hints

✔ Take care when substituting negative numbers into expressions.

→ Recall that adding a negative number is equivalent to subtracting a positive number and subtracting a negative number is equivalent to adding a positive number.

→ Writing calculations out in full will help you to simplify calculations involving negative numbers correctly.

$+(+) = +$	$+ \times + = +$
$+(-) = -$	$+ \times - = -$
$-(+) = -$	$- \times + = -$
$-(-) = +$	$- \times - = +$

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Exercise 5B Substitution

△ 1–8, 9(a, b, e, f), 10, 11, 13 ▢ 2, 3, 5, 6, 8, 9, 11, 12, 14, 16(a, b) ⬠ 5, 6, 8, 9, 12, 14, 15, 17

UNDERSTANDING AND FLUENCY

5B.1 **1** Evaluate each expression by substituting 4 for x.

a $x + 8$ **b** $\frac{x}{2}$ **c** $\sqrt{x}$ **d** $5x - 10$ **e** $\frac{2x}{5}$

f $5x^2$ **g** $4x^3 - 2$ **h** $2\sqrt{x} - 5$ **i** $5x^2 - x$ **j** $\frac{1}{2}x^2 - 8$

5B.2 **2** Evaluate each expression by letting $m = 4$ and $n = -3$.

a $m + n + 4$ **b** $2m + 5n$ **c** $7m - n$ **d** $\frac{1}{4}mn$

e $3m^2 + 6n^2$ **f** $\frac{1}{8}m^2 - 2n$ **g** $\frac{m^2n}{12}$ **h** $6mn + n^2$

3 Evaluate $3a + 5$ by substituting each value for a into the expression.

a $a = 2$ **b** $a = 7$ **c** $a = 1$ **d** $a = 0$

e $a = -1$ **f** $a = -3$ **g** $a = \frac{1}{3}$ **h** $a = -\frac{2}{3}$

4 Evaluate $6y^2 - y$ by substituting each value for y into the expression.

a $y = 1$ **b** $y = 3$ **c** $y = -1$ **d** $y = -4$

e $y = 0$ **f** $y = \frac{1}{2}$ **g** $y = \frac{1}{3}$ **h** $y = 2.5$

5B.3 **5** Substitute $a = 5$ and $b = 2$ into each expression and evaluate.

a $10(a - b)$ **b** $\frac{2a + 2b}{7}$ **c** $\frac{1}{7}(3a - 4b)$ **d** $2\sqrt{2a + 3b}$

e $\frac{3b + 1}{a}$ **f** $\sqrt{a + 10b} - 12$ **g** $\frac{ab - 5}{9}$ **h** $\frac{3(a - b^2)}{7}$

5B.4 **6** Use each of the following formulas to find the value of y if $x = 3$.

a $y = 2x + 15$ **b** $y = \frac{x}{3} + 5$ **c** $y = 5\sqrt{x + 6}$ **d** $y = \frac{4x - 15}{5}$

e $y = \frac{2}{9}(10 - x^2)$ **f** $y = \frac{1}{x} + \frac{5}{3}$ **g** $y = \frac{2}{5}x - 2$ **h** $y = \sqrt{12x} - 11$

7 Find the value of y for each of these values of x using the formula $y = 2x + 7$.

a $x = 3$ **b** $x = 8$ **c** $x = -6$ **d** $x = -1$

e $x = 4.5$ **f** $x = 0$ **g** $x = \frac{1}{2}$ **h** $x = -\frac{3}{2}$

8 Find the value of k for each of these values of m using the formula $k = 4(m^2 - 3)$.

a $m = 1$ **b** $m = 6$ **c** $m = -2$ **d** $m = 50$

e $m = 1.5$ **f** $m = \frac{1}{4}$ **g** $m = 0$ **h** $m = -\frac{3}{4}$

9 Recall the rules for carrying out the four operations with negative numbers and evaluate each expression by letting $p = -6$, $q = 2$ and $r = -3$.

a $2(q - r)$ **b** $\sqrt{14q - 6p}$ **c** $q(p + r)$ **d** $\frac{r - p}{q}$

e $\frac{pq}{r}$ **f** $\frac{p^2 - q^2}{r}$ **g** $4(p^2 - q) - 5r$ **h** $-\frac{3p}{q - r}$

10 The relationship between average speed, s, distance, d, and time, t, can be represented by the formula $s = \frac{d}{t}$.

a Find the average speed, in metres per second, of a bicycle that travels 120 m in 40 seconds.

b Find the average speed, in kilometres per hour, of a motorcyclist who travels 282 km in 3 hours.

11 The distance a skydiver falls through the air, d, in metres after a given amount of time, t, in seconds can be roughly calculated using the formula $d = 4.9t^2$.

a How far does the skydiver fall in the first second of his dive?

b How far does the skydiver fall:

i in the first 3 seconds of his dive

ii in the first 5 seconds of his dive?

c Does the skydiver's speed increase or decrease over the first 5 seconds of his dive? Explain.

12 Use the formulas in the table to answer the following questions.

Shape	Area formula, A	Dimensions
Square	$A = l^2$	l = side length
Rectangle	$A = lb$	l = length b = breadth
Triangle	$A = \frac{1}{2}bh$	b = base length h = height

a What is the area of a square that has a side length of 2 cm?

b What is the area of a rectangle that has a length of 10 cm and a breadth of 7 cm?

c What is the area of a triangle that has a base length of 5 cm and a height of $\frac{5}{2}$ cm?

d **i** Which has the greater area – a square with a side length of 13 cm, or a rectangle with a length of 5 cm and a breadth of 3 cm?

ii Which has the greater area – a rectangle with a length of 10 cm and a breadth of $\frac{3}{5}$ cm, or a triangle with a base length of $\frac{5}{4}$ cm and a height of 15 cm?

13 For each of the following:

i identify and explain the error made in each calculation

ii determine the correct result.

a $2x + 6$

Let $x = 5$

$2x + 6 = 25 + 6$
$= 31$

b $3x^2 + 6$

Let $x = 1$

$3x^2 + 6 = 3(1)^2 + 6$
$= 9 + 6$
$= 15$

c $2^x + 4x$

Let $x = 0$

$2^x + 4x = 2^{(0)} + 2(0)$
$= 0 + 0$
$= 0$

d $x^2 - 4x$

Let $x = -3$

$x^2 - 4x = -(3)^2 - 4(-3)$
$= -9 + 12$
$= 3$

PROBLEM SOLVING AND REASONING

14 If $\$P$ is kept in a bank account that earns interest at a rate of r% per year, the amount of money in the bank account, $\$A$, after n years is given by:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

a If \$1000 is kept in a bank account for 1 year at an interest rate of 5%, calculate how much money there will be in the account at the end of the year.

b \$2000 is invested in a bank account with an interest rate of 4% per year. Use a calculator to find the amount of money in the account after 5 years, correct to 2 decimal places.

15 Consider the expression $2x^2 - 16x$. There are two values of x that, when substituted into the expression, give 0. Fill in the following table to find the two values of x that reduce the expression to 0.

x	x^2	$2x^2$	$16x$	$2x^2 - 16x$
0				
2				
4				
6				
8				

CHALLENGE

16 In most countries of the world, temperature is measured in degrees Celsius. Some countries, such as the USA, use degrees Fahrenheit. A formula can be used to convert a temperature from one unit to the other. Let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit.

a Use the formula $C = \frac{5}{9}(F - 32)$ to convert these temperatures into degrees Celsius. Write each answer to the nearest degree.

i 77°F **ii** −4°F **iii** 41°F **iv** 100°F **v** −1°F **vi** 62°F

b Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures into degrees Fahrenheit. Write each answer to the nearest degree.

i 30°C **ii** −10°C **iii** 0°C **iv** 27°C **v** −8°C **vi** 42°C

c The temperature in Chicago is 23°F. What is it in degrees Celsius?

d Which temperature is lower: 86°F or 32°C? Justify your answer.

17 The salespeople at a certain car dealership are permitted to lower the selling price of different cars by different set amounts when bargaining with customers.

For a car marketed at $\$x$, the salespeople can lower the price by $\$y$. After this, they can take 5% off the reduced value. Finally, if the customer still insists on a lower price, they can take off a further $\$z$.

a Write a formula for the lowest possible selling price, $\$P$, of a car marketed at $\$x$.

b A customer haggles on the price of a hatchback marketed at \$19 990. The salesperson first reduces the price by \$1000, then reduces it by a further 5% and finally takes off \$500. What is the final price of the hatchback?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Substitution

Worksheet Substitution

Investigation Arithmetic and geometric sequences

Topic quiz 5B

5C Adding and subtracting algebraic terms

Learning intentions

By the end of this topic you will be able to ...

✔ identify like terms

✔ simplify expressions by collecting like terms.

Inter-year links

Year 9 — 3A Simplifying

Year 10 — 2C Simplifying

The laws of arithmetic

Key content video

- All the laws of arithmetic apply to algebraic terms and expressions.

	Arithmetic	Algebra
The associative law	$(3 + 5) + 2 = 3 + (5 + 2)$ $(3 \times 5) \times 2 = 3 \times (5 \times 2)$	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
The commutative law	$3 + 2 = 2 + 3$ $3 \times 2 = 2 \times 3$	$a + b = b + a$ $a \times b = b \times a$

Like terms

- **Like terms** are terms that contain exactly the same pronumerals.
  - → Following the associative law, the order of pronumerals can be different in two like terms.
  - → It can be helpful to write terms containing powers out in expanded form to determine whether they are like terms.

    For example, $a^2b = a \times a \times b$ and $ab^2 = a \times b \times b$ so a^2b and ab^2 are not like terms.

$$\underbrace{3a^2b}_{\text{index notation}} = \underbrace{3 \times a \times a \times b}_{\text{expanded form}}$$

Pronumeral(s)	Examples of like terms
x	$x,\ 3x,\ \frac{2}{3}x,\ 0.7x$ ...
ab	$ba,\ -2ab,\ \frac{ab}{5},\ 6.6ab$ ...
m^2n	$-m^2n,\ 18m^2n,\ \frac{9m^2n}{4},\ 0.05m^2n$ ...

Simplification by addition and subtraction of like terms

- Simplification involves reducing the complexity of an expression. This can be done by adding or subtracting like terms, multiplying terms or dividing terms.
- To **simplify** an algebraic expression by adding and subtracting like terms, or 'collecting like terms':
  1. Circle the like terms in the expression.
  2. Group the like terms together.
  3. Add and subtract the coefficients of each set of like terms.

$$6ab + \frac{3}{5}a + ab - \frac{2}{5}a = 6ab + ab + \frac{3}{5}a - \frac{2}{5}a$$

$$= 7ab + \frac{1}{5}a$$

Example 5C.1 Identifying like terms

Identify and list the like terms in the following groups of terms.

a $36x, -x, 5y, \frac{3x}{4}, 0.5y, \frac{1}{2}y$

b $7xy, \frac{5}{2}yx, -2x, -4x^2, \frac{x^2}{6}, 10x$

THINK

a Circle the like terms, or terms that contain exactly the same pronumerals. The values of the coefficients do not affect whether or not two or more terms are like terms.

b Circle the like terms. According to the commutative law, $xy = yx$, so $7xy$ and $\frac{5}{2}yx$ are like terms. Remember that $x^2 = x \times x$, so x and x^2 are *not* like terms.

WRITE

a $36x, -x, 5y, \frac{3x}{4}, 0.5y, \frac{1}{2}y$

The two sets of like terms in the list are:

$36x$, $-x$ and $\frac{3x}{4}$

$5y$, $0.5y$ and $\frac{1}{2}y$

b $7xy, \frac{5}{2}yx, -2x, -4x^2, \frac{x^2}{6}, 10x$

The three sets of like terms in the list are:

$7xy$ and $\frac{5}{2}yx$

$-2x$ and $10x$

$-4x^2$ and $\frac{x^2}{6}$

Example 5C.2 Adding and subtracting like terms

Simplify each expression where possible.

a $10x - 3x + 3 + x$

b $-6xy + 2x - 3yx - \frac{x}{2}$

c $2 + xy^2 - 5x^2y - 12xy^2 - 7 - 2y^2x$

THINK

a **1** Circle the like terms in the expression.

2 Group the like terms together.

3 Add and subtract the coefficients of like terms. Note that '3' is a constant and there are no other constants in the expression, so 3 cannot be combined with any other terms.

b **1** Circle the like terms.

2 Group the like terms.

3 Add and subtract the coefficients of each set of like terms. Remember that fractional coefficients can be written in two ways, so $-\frac{x}{2} = -\frac{1}{2}x$.

c **1** Circle the like terms.

2 Group the like terms.

3 Add and subtract the coefficients of each set of like terms. Note that $xy^2 = x \times y \times y$, but $x^2y = x \times x \times y$, so xy^2 and x^2y are *not* like terms and $-5x^2y$ cannot be combined with any other terms.

WRITE

a $10x - 3x + 3 + x = 10x - 3x + x + 3$
$= 8x + 3$

b $-6xy + 2x - 3yx - \frac{x}{2} = -6xy - 3yx + 2x - \frac{x}{2}$
$= -9xy + \frac{3}{2}x$

c $2 + xy^2 - 5x^2y - 12xy^2 - 7 - 2y^2x = 2 - 7 + xy^2 - 12xy^2 - 2y^2x - 5x^2y$
$= -5 - 13xy^2 - 5x^2y$

Helpful hints

✔ For two terms to be like terms, the pronumeral components of the terms must be *exactly* the same, but following the commutative law, the order of the pronumerals can vary.
For example, xy and yx are like terms.

✔ The plus and minus signs in an algebraic expression also indicate when a coefficient is positive or negative. When grouping like terms, remember that the signs 'belong' to the terms to the right of them, so minus signs should move with their terms.
For example, $x + 7y - 3x + 5y = x - 3x + 7y + 5y$.

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Exercise 5C Adding and subtracting algebraic terms

△ 1–4, 5(1st column), 6, 7(b, d, f, h), 8, 9(b, c, e), 10, 11(a–c)

□ 1(b, d, f, h), 3, 5, 7, 9(d–f), 11–13, 15–16(a–c)

⬠ 1(g, h), 3(g, h), 5, 7, 9, 14 15–16(c, d), 17–19

5C.1 **1** Identify and list the like terms in the following groups of terms.

a $20x, 8y, -15y, y, -12x, 3x$

b $5x, \frac{3}{7}x, \frac{5}{7}y, -25x, 6y, \frac{x}{7}$

c $4xy, -yx, 0.1x, \frac{1}{8}xy, -\frac{yx}{10}, \frac{y}{15}$

d $-a, 5a^2, \frac{4}{9}a^2, 12.5a, 2.3a^2, -\frac{9a^2}{10}$

e $\frac{11}{18}p^3, \frac{p^2}{5}, 5.5p^3, p^3, -11p^2, -0.05p^2$

f $\frac{9ab}{10}, 7abc, -cab, \frac{5}{6}ab, 13cab, 7.2ba$

g $-9fg, 120f^2g, \frac{20fg^2}{3}, -\frac{14}{3}f^2g, 0.33fg^2, \frac{4fg}{5}$

h $\frac{2}{9}xy^2z, 0.095zyx, -12y^2xz, 18.6xyz, \frac{x^2y^2z}{24}, 99zy^2x$

2 Answer true or false to each statement. Explain why each false statement is incorrect.

a $2x$ and $-2x$ are like terms.

b $15a$ and $15a^2$ are like terms.

c $8p$ and $\frac{p}{5}$ are like terms.

d $\frac{5}{8}def$ and $5fed$ are like terms.

e $4ab$ and $6a^2b$ are like terms.

f $-3a^2b$ and $5ba^2$ are like terms.

UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

3 Simplify each pair of like terms by adding or subtracting the coefficients of each term.

a $5x + 4x$

b $10kp - 2kp$

c $3a^2 - 7a^2$

d $5g - g$

e $yx - 3xy$

f $-5g^2h + 7g^2h$

g $\frac{3}{4}abc - \frac{1}{2}cab$

h $\frac{2xyz^2}{5} - \frac{2}{3}xz^2y$

4 Answer true or false to each statement. For the statements that are false, write the correct simplified expression.

a $8x + 6x$ simplifies to $14x$.

b $11k - k$ simplifies to 11.

c $3p + 6q$ simplifies to $9pq$.

d $5m^2 + 2n + m^2$ simplifies to $6m + 2n$.

e $2ab + 3a + 7ba$ simplifies to $12ab$.

f $\frac{1}{4}a + \frac{1}{3}c + \frac{1}{2}a + c$ simplifies to $\frac{3}{4}a + \frac{4}{3}c$.

g $x^2y + 2x + y$ cannot be simplified.

h $\frac{2}{9}p^2q - \frac{5}{6}p^2 + \frac{1}{6}qp^2$ cannot be simplified.

5C.2 **5** Simplify each expression where possible.

a $3b + 5 + 5b - 2b$

b $3n + 4mn - 9mn - 4n$

c $12 - 5c^2 - 8 - 3c^2$

d $4gh + 3hg - 3 - 6gh$

e $5pq - 6pq - 10q^2p + 8p^2q$

f $2x^2y + 10yx^2 - x^2y - 12xy^2$

g $\frac{5}{2}str + \frac{1}{3}st - \frac{1}{6}ts - \frac{2}{7}rst$

h $\frac{abc}{3} - \frac{2}{3}ab - \frac{5}{6}cab - 2 + \frac{ab}{6}$

i $\frac{g^3h}{5} - \frac{5}{3} + \frac{2}{10}g^3h + \frac{7}{2} + \frac{4g^2h}{5} - \frac{1}{15}gh$

j $\frac{5}{4}m^2n + \frac{1}{4}m^2n^4 + m^2n - \frac{3}{2} - m^2n^4 + \frac{5}{3}$

6 Consider the expression $7x^2 + 2xy + 9x^2 - yx$.

a Evaluate the expression by substituting $x = 3$ and $y = 7$ into the four terms.

b Simplify the expression by adding like terms.

c Check your answer to part **b** by substituting $x = 3$ and $y = 7$ into the simplified expression. Do you obtain the same answer as part **a**?

d Which expression was easier to evaluate? Explain.

7 Simplify then evaluate each expression by letting $a = 5$ and $b = 3$.

a $4a + 2a + 6b + 8b$

b $9a - 6a + 7b + 2b$

c $5ab + 4b - 2a - b$

d $6ab - ba - 7ba + a^2$

e $-4b^2 - b^2 + 8a^2 - a^2$

f $\frac{3}{4}a^2 - \frac{1}{2}a^2 + \frac{2}{3}b^2 - \frac{4}{3}b^2$

g $-ab + 3ab^2 - 10ba - 5b^2a$

h $7ab^2 + 2a^2b - 9ab^2 - 5ba^2$

i $\frac{3}{4}a^2b - 2ab^2 + \frac{a^2b}{5} - \frac{7}{6}ab^2$

PROBLEM SOLVING AND REASONING

8 When grouping the like terms in an expression, Tania writes:

$$3x - 2y + 7x - 4y = 3x - 7x + 2y - 4y$$

a Identify and explain Tania's mistake.

b Show the correct grouping of like terms in the expression $3x - 2y + 7x - 4y$.

9 Simplify each expression by collecting the like terms.

a $5x + 3y + 6y - 2x - 7y + 8x$

b $\frac{1}{4}y + \frac{1}{6} - \frac{1}{2}y - \frac{4}{3} + \frac{5}{4}y - y$

c $-2xy + 5yx + 4y - 8y + 2y - 3xy$

d $xy - x + 5yx - 3xy + 9x - 5yx$

e $8x^2y + 3yx^2 - 6xy^2 + 4 + 5yx^2 - 9$

f $\frac{3x^2y}{4} + 2xy^2 - 2x^2y - \frac{3}{5}yx^2 - \frac{xy^2}{2} + \frac{7}{4}y^2x - xy^2$

10 Paree is planning to build a rectangular garden bed.

a Draw a diagram of Paree's garden bed. If l = length and b = breadth, label each of the four sides of the diagram with the appropriate pronumeral.

b Write an expression for the perimeter of the garden bed by adding the lengths of each side.

c Simplify the expression by collecting the like terms.

d If P = perimeter, write a formula for the perimeter of the garden bed.

e Use the formula to calculate the total perimeter of three different garden beds with the following dimensions.

i $l = 4$ m, $b = 3$ m

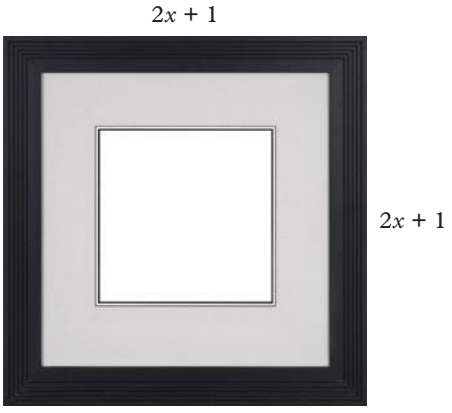
ii $l = 6.4$ m, $b = 2.8$ m

iii $l = 15\frac{1}{2}$ m, $b = 7\frac{1}{2}$ m

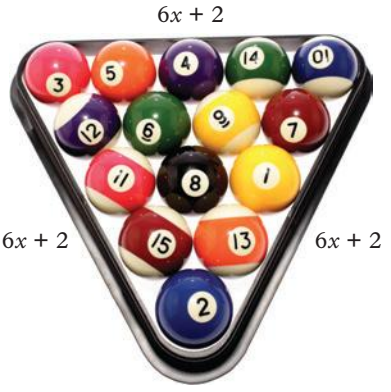
f Paree decides to plant a box hedge around the edge of her garden bed. She has enough plants to extend around a perimeter of 20 m. Suggest one possible set of length and breadth measurements for her garden bed.

11 Write an expression for the perimeter of each of the objects shown below.

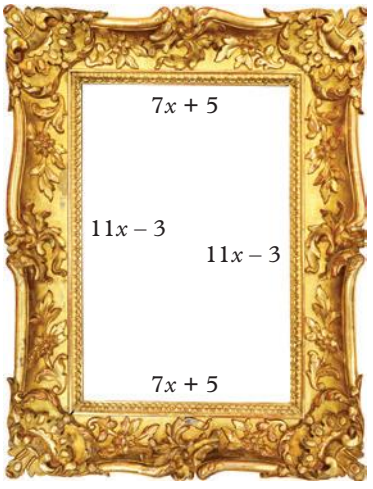
a length of outer edge of frame



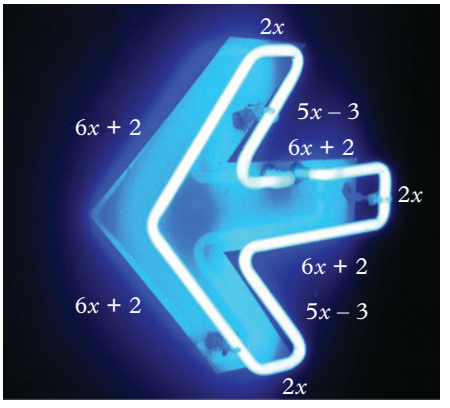
b perimeter of triangle



c perimeter of inner frame



d length of neon used



12 Calculate the perimeter for each object shown in question **11** if $x = 4$ cm.

13 Consider the following working out and explain the error made in each calculation. Then find the correct answer.

a $a + a + a + a = a^4$

b $2x + 3y = 5xy$

c $9x + 4x = 13x^2$

d $8x - x = 8$

PROBLEM SOLVING AND REASONING

14 The price of an adult ticket to a theme park is $\$x$. The price of a concession ticket is three-fifths of an adult ticket and the price of a child ticket is five-eighths of a concession ticket. Write an expression for the total cost for a group visiting the theme park with three adults, five concession card holders and two children.

15 For each of the following statements, substitute the given values into each expression and evaluate them.

a $\frac{4}{3x}$ and $\frac{4}{3}x$

i $x = 1$ **ii** $x = -1$ **iii** $x = 2$

b $(-x)^2$ and $-x^2$

i $x = 0$ **ii** $x = 1$ **iii** $x = -1$

c $(-x)^3$ and $-x^3$

i $x = 0$ **ii** $x = 1$ **iii** $x = -1$

d $\frac{-3x}{5}$, $\frac{3x}{-5}$, $-\frac{3x}{5}$, and $\frac{-3x}{-5}$

i $x = 1$ **ii** $x = -1$ **iii** $x = 2$

CHALLENGE

16 For each set of expressions, state whether the expressions are equivalent or not equivalent and provide reasons to support your claim.

a $\frac{4}{3x}$ and $\frac{4}{3}x$ **b** $(-x)^2$ and $-x^2$

c $(-x)^3$ and $-x^3$ **d** $\frac{-3x}{5}$, $\frac{3x}{-5}$, $-\frac{3x}{5}$ and $\frac{-3x}{-5}$

17 Simplify the following expression.

$2a^2b^7c^5d^5e^9f^{11}g^{14} + 4f^{11}g^{14}b^7d^5c^5a^2e^9 + 6a^2e^9g^{14}b^5f^{11}c^5d^7 + 5b^7f^{11}a^2c^5e^9g^{14}d^5 + 3f^{11}g^{14}d^7c^5b^5a^2e^9$

18 Simplify the expression $1.\dot{6}x + 0.1\dot{6}x + 0.41\dot{6}x$.

19 We can consider terms as like terms if they contain a factor of the exact same grouped term.

For example, $2(x+3)$, $5(x+3)$, $\frac{x+3}{2}$ are like terms because they all contain $(x+3)$ as a factor. However, $(x+3)$, $\frac{1}{x+3}$, $\sqrt{x+3}$ are not like terms. Simplify the following expressions.

a $3(2x+3) + 9(2x+3) + 4(2x+5) + 7(2x+5)$

b $7(x-3y+4) - 4(4-3y+x) + 9(x+4-3y) - 2(-3y+4+x)$

c $7(x+2) + 2(x-2) - 5(x+2) + 6(x-2)$

d $\frac{6}{3x+1} + \frac{7}{3x+1} + \frac{8}{3x-1} + \frac{4}{3x-1}$

e $5\sqrt{7-2x} + 4\sqrt{7+2x} - 8\sqrt{7+2x} + 2\sqrt{7-2x}$

f $-7\sqrt{a+b} + 8(a+b) + \frac{2}{a+b} + 6\sqrt{a+b} - \frac{5}{a+b} + (a+b)$

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Interactive skillsheet
Like terms

Investigation
Help the animals

Topic quiz
5C

5D Multiplying algebraic terms

Learning intentions

By the end of this topic you will be able to ...

✔ convert algebraic terms from expanded form to index notation

✔ simplify expressions by multiplying algebraic terms.

Inter-year links

Year 7	1G Indices and square roots
Year 9	3A Simplifying
Year 10	2C Simplifying

The index laws

Key content video

- Repeated multiplication of variables can be written in index notation.

$$\underbrace{3 \times a \times a \times a \times b}_{\text{expanded form}} = \underbrace{3a^3b}_{\text{index notation}}$$

Simplification by multiplication of algebraic terms

- To multiply algebraic terms:
  1. Write each term in expanded form.
  2. Group the coefficients and like variables.
  3. Convert the term back into index notation by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

$$\begin{aligned} 3a^3b \times (-2a^2b^2) &= 3 \times a \times a \times a \times b \times (-2) \times a \times a \times b \times b \\ &= 3 \times (-2) \times a \times a \times a \times a \times a \times b \times b \times b \\ &= -6a^5b^3 \end{aligned}$$

Example 5D.1 Writing algebraic terms in index notation

Simplify the following terms by expressing them in index notation.

a $2 \times a \times a \times a$

b $-9 \times b \times a \times a$

THINK	WRITE
a 1 Write any repeated multiplications in index notation. Identify the index of each base by counting the number of times that base is repeated in expanded form. a is repeated 3 times in expanded form, so $a \times a \times a = a^3$.	**a** $2 \times a \times a \times a = 2 \times a^3$
2 Leave out the multiplication signs.	$= 2a^3$
b 1 Write any repeated multiplications in exponent form.	**b** $-9 \times b \times a \times a = -9 \times b \times a^2$
2 Leave out the multiplication signs and list the variables in alphabetical order.	$= -9a^2b$

Example 5D.2 Multiplying algebraic terms

Simplify the following products.

a $4ab \times 3c$ **b** $5a^2 \times (-a^4)$ **c** $-2p^2q^3 \times 7qp^5$

THINK

1. Write each term in expanded form. Remember the coefficient of $-a^4$ is -1.
2. Group the coefficients and variables.
3. Convert the term back into index notation by counting the number of times each variable is multiplied. Write the coefficient first, followed by the variables and their corresponding index, listed in alphabetical order.

WRITE

a $4ab \times 3c = 4 \times a \times b \times 3 \times c$
$= 4 \times 3 \times a \times b \times c$
$= 12abc$

b $5a^2 \times (-a^4) = 5 \times a \times a \times (-1) \times a \times a \times a \times a$
$= 5 \times (-1) \times a \times a \times a \times a \times a \times a$
$= -5a^6$

c $-2p^2q^3 \times 7qp^5 = -2 \times p \times p \times q \times q \times q \times 7 \times q \times p \times p \times p \times p \times p$
$= -2 \times 7 \times p \times p \times p \times p \times p \times p \times p \times q \times q \times q \times q$
$= -14p^7q^4$

Helpful hints

✔ Indices only apply to the number or variable immediately to the left of the index.
For example, in the term $4gh^3$, the index of 3 only applies the variable h, hence: $4gh^3 = 4 \times g \times h \times h \times h$.

✔ Use brackets to indicate that the index applies to more than one number or variable.
For example, $(4gh)^3 = 4^3g^3h^3$
$= 64g^3h^3$.

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Exercise 5D Multiplying algebraic terms

△ 1–3, 4(1st column), 5, 6, 7(1st column), 8(b), 9–10(2nd, 3rd columns), 11, 12(a), 13

□ 2, 4(2nd column), 6, 7, 9–10(2nd, 3rd columns), 12, 14, 15

⬠ 4(g–j), 6(e, f), 7(2nd column), 9–10(3rd column), 12, 14, 15

1 Simplify the following terms by determining the sign of the product, placing the coefficient first, and leaving out the multiplication sign.

a $4 \times x$ **b** $6 \times k$ **c** $d \times 7$ **d** $m \times 5$
e $1 \times p$ **f** $-3 \times h$ **g** $a \times (-9)$ **h** $-y \times 10$
i $8 \times (-c)$ **j** $-1 \times e$ **k** $-2 \times (-f)$ **l** $-m \times (-3)$

2 Simplify:

a $3 \times a \times b$ **b** $4 \times m \times n$ **c** $p \times 2 \times t$ **d** $d \times (-8) \times c$ **e** $x \times y \times 9$
f $k \times f \times 5$ **g** $a \times b \times c$ **h** $h \times y \times e$ **i** $2 \times q \times 4$ **j** $m \times 3 \times (-6)$

3 Match each of the terms written in expanded form with its index notation.

Expanded form	**Index notation**
a $2 \times a \times a \times a \times b \times c$	**I** $2a^2bc$
b $2 \times a \times a \times b \times c$	**II** $2a^3bc$
c $2 \times a \times b \times b \times c \times c$	**III** $2ab^2c^2$

5D.1 **4** Simplify the following terms by expressing them in index notation.

a $5 \times m \times m$

b $-2 \times a$

c $-1 \times x \times y$

d $n \times 2 \times n$

e $-6 \times d \times d \times c \times c$

f $7 \times a \times a \times c \times b$

g $s \times s \times t \times t \times t \times (-2)$

h $-10 \times g \times g \times g \times f \times f$

i $1 \times q \times p \times q \times r \times q \times r \times r$

j $z \times x \times y \times (-11) \times x \times x \times y \times z$

5 Complete each of the multiplication problems below.

a $2^3 \times 2^4$

$= 2 \times 2 \times 2 \times$ ___ $\times$ ___ $\times$ ___ $\times$ ___

$= 2^7$

b $a^3 \times a^4$

$= a \times$ ___ $\times$ ___ $\times a \times$ ___ $\times$ ___ $\times$ ___

$=$ ___

6 Complete each of the multiplication problems below.

a $a^2 \times a^3$

b $k^4 \times k$

c $c^4 \times c \times c^2$

d $e \times e^5 \times f^2 \times f^3$

e $a^3 \times d^5 \times a^2 \times d$

f $m \times n \times n^3 \times m^4 \times m^5 \times n$

5D.2 **7** Simplify the following products.

a $6a \times 2bc$

b $-a^2 \times h^3$

c $3wz \times 7xy$

d $6x^4 \times 2y^5$

e $7a^5 \times 3a^2$

f $-5p \times p^{11}$

g $8n \times 3n$

h $5x^6y^5 \times 3x^2y^5$

i $a^5b^4 \times (-a^3b^2)$

j $9xy \times (-2xz)$

k $\frac{1}{6}w^4x^3y^5 \times \frac{6}{5}x^2yz$

l $\frac{3}{7}a^3b^3c \times (-6ca^2b^4)$

8 Complete the multiplication tables below by multiplying each term listed in the left-hand column by each of the terms in the top row.

a

×	$4a$	3	$6m$	$2abc$
$3d$				
ab^2				
$5mn$				
$2ac$				

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$				
xy^2				
$6xy$				
$-7yz$				

9 Simplify each of the following products.

a $-2gh \times 5b \times 7k$

b $3g^4 \times 5h^3 \times 2g^6$

c $4b^3 \times 3b \times (-b^5)$

d $mn \times 6m \times 8q$

e $7gh \times (-3gh) \times fgh$

f $\frac{1}{2}m^7 \times (-\frac{3}{4}m^3n) \times \frac{3}{5}mn^2$

g $4a^2b^5 \times 3ac \times 2a^3$

h $-2y^5wx^2 \times 5x^5w^2y \times (-xy^2)$

i $\frac{5}{4}ab^9d \times (-3a^5b^4c) \times \frac{2}{3}d^3c^6a^2$

10 Complete the following products by finding the missing factor in each of the factor pairs.

a $2 \times$ __ $= 6$

b $8 \times$ __ $= -16$

c __ $\times k = 6k$

d $3 \times$ __ $= 3x$

e $a \times$ __ $= ab$

f $x \times$ __ $= -7xy$

g __ $\times c = 5cd$

h $4a \times$ __ $= 8ab^2$

i __ $\times 3mn = 15mn^4$

j $6e \times$ __ $= -12e^3f$

k __ $\times 4l^2 = 20jk^2l^3$

l $5xy \times$ __ $= -5x^2yz^5$

PROBLEM SOLVING AND REASONING

11 There are a few conventions mathematicians follow when using fractional coefficients. Use your understanding of fractions and expanding expressions to answer the following.

a Which of the following is equal to $\frac{3}{2} \times 5$?

A $\frac{3 \times 5}{2 \times 5}$ **B** $\frac{3}{2 \times 5}$ **C** $\frac{3 \times 5}{2}$ **D** $\frac{3}{2} \times \frac{1}{5}$ **E** $\frac{2}{3} \times 5$

b Which of the following is equal to $\frac{3}{2}x$?

A $\frac{2}{3}x$ **B** $\frac{3}{2} \times \frac{1}{x}$ **C** $\frac{3}{2x}$ **D** $\frac{3x}{2}$ **E** $\frac{3x}{2x}$

c State the coefficient of x and y for $\frac{2 + 7x - y}{5}$.

d Which of the following is equal to $3\frac{1}{2}$?

A $3 - \frac{1}{2}$ **B** $3 \times \frac{1}{2}$ **C** $3 \div \frac{1}{2}$ **D** $3 + \frac{1}{2}$ **E** $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

e Which of the following is equal to 32?

A $30 - 2$ **B** 3×2 **C** $3 \div 2$ **D** $30 + 2$ **E** $2 \times 2 \times 2$

f Which of the following is equal to $3x$?

A $3 - x$ **B** $3 \times x$ **C** $3 \div x$ **D** $3 + x$ **E** $x \times x \times x$

g Using your answers from part **c** and **d**, explain why mathematicians would choose to not use a mixed number, where possible, as a coefficient, such as $3\frac{1}{2}x$.

12 a Complete the following calculations.

i $(3x^2)^4 = (3x^2) \times (___) \times (___) \times (___)$
$= 3 \times __ \times __ \times __ \times x^2 \times __ \times __ \times __$
$= ____$

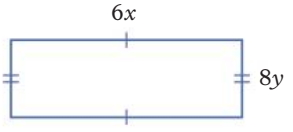
ii $\left(\frac{x^2}{3}\right)^4 = \left(\frac{x^2}{3}\right) \times (\equiv) \times (\equiv) \times (\equiv)$
$= \frac{x^2 \times __ \times __ \times __}{3 \times __ \times __ \times __}$
$= \equiv$

b Use the method in part **a** to simplify the following expressions by writing them without brackets.

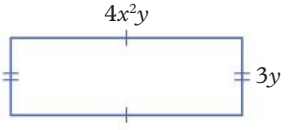
i $(2x^5)^6$ **ii** $\left(\frac{y^9}{2}\right)^5$ **iii** $(4a^2b^7)^3$ **iv** $\left(\frac{5u^6}{2v^5}\right)^4$

13 Write a simplified expression for the areas of the following shapes.

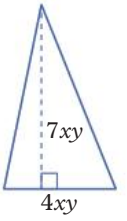
a



b



c



CHALLENGE

14 For each of the following:

i draw a labelled diagram

ii write a simplified expression for the area of the shape.

a A rectangle has a length three times its breadth.

b A triangle has a perpendicular height that is 4.3 times the breadth of the base.

c An isosceles triangle has a height, perpendicular to one of the two equal side lengths, that is half the length of the two equal side lengths.

15 Simplify $2^{5a+1} \times 3^{8+10a} \times 2^{7a} \times 2^4 \times 3^{3a+8}$.

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Interactive skillsheet
Multiplying terms

Topic quiz
5D

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

5A 1 For the following expressions, identify:

i the pronumerals
ii the number of terms
iii the coefficients
iv the constant
v the highest index of each pronumeral.

a $4x - 3 + 9y - 4xy^2$
b $-\frac{9}{13} + \frac{5m^2}{3} + nv^3$

5A 2 Write an expression for each of the following.

a the quotient of x and 8
b the sum of a, b and c multiplied by 7
c the product of y and three more than y
d the square root of the difference of c and a

5A 3 Write an expression for each of the following.

a A cruel telephone company charges \$20 per gigabyte after you exceed your data limit.
b The score an AFL team gets in a game is six times the number of goals plus the number of behinds.
c One-third of the total number of flowers in a garden containing tulips, roses and poppies must be red.
d Three more than 15% of the number of bread rolls in a bakery have seeds.

5B 4 Substitute the values into the expression and evaluate.

a $2x + 3y + 7z$ where $x = 5$, $y = 6$ and $z = 8$
b $3x^2 - 4x$ where $x = 2$
c $7(xy^2 + 1)$ where $x = 9$ and $y = 3$
d $\frac{4 + x^2}{5} + \frac{3}{10}y$ where $x = 4$ and $y = 15$

5B 5 Substitute the given values into the formula and evaluate.

a Find the value of y using the formula $y = mx + c$ where $m = 7$, $x = 5$ and $c = -4$.
b Find the value of s using the formula $s = \frac{1}{2}(u + v)t$ where $u = 4$, $v = 12$ and $t = 8$.

5B 6 Substitute the values into the expression and evaluate.

a $2x + 3y + 7z$ where $x = 0.5$, $y = \frac{1}{6}$ and $z = 8\frac{1}{2}$
b $3x^2 - 4x$ where $x = -2$

5C 7 Simplify each of the following expressions.

a $2x + x + 7y + 4x - 3y + 9 + 1$
b $7a + 9b - 3ab + b - 2a + 5ba$
c $-6pqr + 7rpq - 9qpr + 5prq + 11qrp$
d $14ikl + 16jkl + 17lki - 8klj - kli$

5C 8 Simplify each of the following expressions.

a $2x + x^2 - 5x + 3x^2$
b $7xy^2 + 8yx^2 - 6x^2y + 5y^2x$

5C 9 Simplify each of the following expressions.

a $1.72x + 4.24y - 9.18x - 3.98y$
b $\frac{5}{4}x^2 + \frac{3x^2}{8} - \frac{7}{2}x^2$

5D 10 Simplify each of the following expressions.

a $7x \times 9y$
b $-3ac \times (-9bd)$
c $2(5gh)(-11pq)$
d $4xy \times abc \times 9uv$

5D 11 Simplify each of the following expressions.

a $12x^2 \times 3x^3$
b $7xy^2 \times 8x^3y^2$

5D 12 Simplify each of the following expressions.

a $0.12x \times 3.1x$
b $\frac{25x^8}{16} \times \frac{24}{15}x^4$

5E Dividing algebraic terms

Learning intentions

By the end of this topic you will be able to ...

✔ simplify expressions by dividing algebraic terms.

Inter-year links

Support	Equivalent fractions
Year 7	3B Equivalent fractions
Year 9	3B Algebraic fractions with numerical denominators

Dividing algebraic terms

Key content video

- To divide algebraic terms:
  1. Write each term in expanded form.
  2. Divide the coefficients by the highest common factor (HCF).
  3. Cancel out repeated variables that appear in both the numerator and denominator.
  4. Convert the term back into index notation by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

$$\frac{6a^5b^3}{3a^3b} = \frac{\cancel{6}^2 \times \cancel{a} \times \cancel{a} \times \cancel{a} \times a \times a \times \cancel{b} \times b \times b}{\cancel{3}^1 \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b}}$$
$$= \frac{2 \times a \times a \times b \times b}{1}$$
$$= 2a^2b^2$$

- Remember that quotients can be expressed as fractions.

  $x \div 7 = \frac{x}{7}$

- Remember that a number divided by itself has a value of 1.

  $x \div x = \frac{x}{x} = 1$

Example 5E.1 Dividing algebraic terms

Simplify the following quotients.

a $\frac{5a^2}{a}$

b $\frac{8a^3b}{2a^2}$

THINK

1. Write each term in expanded form.
2. Divide the coefficients of the denominator and numerator by their highest common factor (HCF).
3. Cancel out repeated variables that appear in both the numerator and denominator. Remember that any term divided by itself is equal to 1, so $\frac{a}{a} = 1$ and $\frac{b}{b} = 1$.
4. Convert the term(s) back into index notation by counting the number of times each variable is repeated. Write the coefficient first, followed by the variables listed in alphabetical order.

WRITE

a $\frac{5a^2}{a} = \frac{5 \times a \times a}{a}$

$= \frac{5 \times \cancel{a}^1 \times a}{\cancel{a}^1}$

$= \frac{5 \times a}{1}$

$= 5a$

b $\frac{8a^3b}{2a^2} = \frac{8 \times a \times a \times a \times b}{2 \times a \times a}$

$= \frac{\cancel{8}^4 \times \cancel{a}^1 \times \cancel{a}^1 \times a \times b}{\cancel{2}^1 \times \cancel{a}^1 \times \cancel{a}^1}$

$= \frac{4 \times a \times b}{1}$

$= 4ab$

Helpful hints

- ✔ Take care when writing variables with a large index in expanded form.
- ✔ The divisibility rules can help you to find the HCF. Recall that a number is:
  - → divisible by 2 if the number is even
  - → divisible by 3 if the number's digits add to a multiple of three
  - → divisible by 5 if the number ends in zero or five.

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Exercise 5E Dividing algebraic terms

▲ 1(2nd, 3rd columns), 2, 3(1st, 3rd columns), 4–10 ■ 1(3rd, 4th columns), 3(1st, 4th columns), 4–8, 10, 11, 12 ⬟ 3, 6–8, 10, 11, 13–15

UNDERSTANDING AND FLUENCY

5E.1 **1** Simplify the following quotients.

a $\frac{5a}{5}$ **b** $\frac{15x}{3y}$ **c** $\frac{9d}{d}$ **d** $\frac{12w}{4w}$ **e** $\frac{3m}{9m}$

f $\frac{4ab}{a}$ **g** $\frac{18c^3}{6c}$ **h** $\frac{10b^5}{15b^2}$ **i** $\frac{4mnp}{24mn}$ **j** $\frac{2m^6n^7}{2m^5n^2}$

k $\frac{6a^4b^2}{ab^2}$ **l** $\frac{5m^3}{m^2}$ **m** $\frac{6m^3bc}{9mc}$ **n** $\frac{14s^3t^4}{7s^3t^2}$ **o** $\frac{8h^5k^2j}{10h^3k}$

2 Complete the following division problems.

a $xy \div y$ **b** $8k \div 4$ **c** $18ab \div 2b$

d $9x^2y \div xy$ **e** $3mn^2 \div 12mn$ **f** $ab^2c^3d \div abc^2$

3 Simplify each algebraic fraction where possible.

a $\frac{6x^6y^5}{6x^2y^5}$ **b** $\frac{2jk^5}{k^4}$ **c** $\frac{3xy^4}{y^4}$ **d** $\frac{m^7}{p^3}$ **e** $\frac{b^5c^4}{b^3c^2}$

f $\frac{4k^8}{2a^5}$ **g** $\frac{18x^4}{22y^4}$ **h** $\frac{6a^3b}{2c^3}$ **i** $\frac{x^2y}{4x}$ **j** $\frac{8mn^2}{2m}$

4 Simplify:

a $\frac{-9xy}{3x}$ **b** $30a^3b^2 \div (-6ab^2)$ **c** $\frac{-12uv^3}{-8uv}$

5 Simplify:

a $\frac{x^4 \times x^3}{x^2}$ **b** $\frac{m^7 \times m^6}{m^9}$ **c** $\frac{6a^2 \times a^8}{a^4}$ **d** $\frac{n^5 \times n^7}{n^3 \times n^4}$ **e** $\frac{5d^6 \times d^3}{d^9}$

6 Fill in the blanks to complete the equation.

a $16m \div$ ___ $= 2m$ **b** $\frac{27jk}{\quad} = -3k$ **c** ___ $\div 2xy = 8x^2y$ **d** $\frac{\quad}{10a^2b^3} = \frac{a}{5b}$

7 The area of a rectangle is $16xy$.

a If the length is $8x$, write an expression for the breadth of the rectangle.

b If the length of the rectangle is 16 m and y is 5 m, calculate the breadth and the area of the rectangle using the result from part **a**.

8 The area of a right-angled triangle is $6x^2$.

a If the base length is $4x$, write an expression for the height of the triangle.

b If the height of the rectangle is 12 m, calculate the area and base length of the triangle using the result from part **a**.

PROBLEM SOLVING AND REASONING

9 Fabian and his father are laying cement pavers in the backyard to create a smooth surface for skateboarding. The pavers come in different sizes but are rectangular in shape and have a length twice their breadth.

a Let x represent the breadth of a paver in centimetres. Write the length in terms of x and hence write an algebraic term for the area of one paver.

b The ground to be paved is square with a length represented by $10x$ cm. Write an algebraic term for the area of the square to be paved.

c How many pavers are required to pave the ground?

d To check whether your answer to part **c** is correct, substitute a value for x. For example, let $x = 40$, and evaluate each of the following using this value of x.

i breadth of paver **ii** length of paver

iii area of paver **iv** length of square to be paved

v area of square to be paved

e Use your answers to part **d iii** and **v** to calculate the number of pavers required. How does this compare to your answer to part **c**?

f Will the size of the pavers affect the number of pavers needed? Explain.

10 To divide two terms, we can multiply the first term by the reciprocal of the second term. Complete the following division problems.

a $\frac{3x}{8} \div \frac{5x}{4}$ **b** $\frac{9x^3}{4y} \div \frac{4x}{3y^2}$

11 The lowest common multiple (LCM) of two numbers is the common multiple with the lowest value. When written as the product of prime factors, the LCM is the product of the highest power of each factor.

For example, the LCM of $2^3 \times 3^2 \times 5^6$ and $2^4 \times 3^5 \times 5$ is $2^4 \times 3^5 \times 5^6$. Similarly, the LCM of $x^3y^2z^6$ and x^4y^5z is $x^4y^5z^6$. Determine the LCM of the following:

a $2^9 \times 13^5 \times d^{12}$ and $2^{14} \times 13^2 \times d^6$ **b** $3^2 \times 7^4 \times a^9 \times b^2$ and $3^4 \times 7^2 \times a^5 \times b^6$

c $p^6q^7r^{11}s^6$ and $p^9q^2r^{15}s^2$ **d** $r^{42}s^{51}v^{99}$ and $r^{45}t^{78}u^{84}$

12 Simplify the following quotients, then evaluate by substituting the given values.

a $\frac{42c^2d^6}{35c^5d^4}$ where $d = 5$ and $c = 6$ **b** $a^5z^3 \div az^2$ where $a = 2$ and $z = 4$

c $\frac{25pq}{5p}$ where $p = 3.5$ and $q = 8.1$ **d** $\frac{28x^4y^9}{4y^9x^4}$ where $x = 31$ and $y = 78$

CHALLENGE

13 Consider the expression $\frac{x}{y^4z^2} \div \frac{z}{xy^2} \times \frac{y^2z^3}{x^2}$.

a Evaluate the expression when:

i $x = 2$, $y = 3$ and $z = 5$ **ii** $x = 3$, $y = 5$ and $z = 2$ **iii** $x = 2$, $y = 2$ and $z = 2$

b Simplify the expression and use the simplified form to explain the answers to part **a**.

14 Simplify the following expression:

$$\frac{2x - 2x^2 \div 2x + 2x \times x - x^5 \div x^3}{3x + x^3 \div \left(5x \times \frac{x}{5}\right)}$$

15 a Use six of the digits 1–9 only once such that the quotient below has the largest sum of the indices of x and y possible. State this sum.

$x^{\square}y^{\square} \div x^{\square}y^{\square} = x^{\square}y^{\square}$

b How do you know this is the largest sum?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Dividing terms

Topic quiz
5E

5F Expanding

Learning intentions

By the end of this topic you will be able to ...

✔ expand algebraic expressions.

Inter-year links

Year 7	6D Order of operations in algebra
Year 9	3D Expanding
Year 10	2D Expanding

Expanding algebraic expressions

Key content video

- To **expand** an algebraic product containing brackets using the distributive law:
  1. Multiply each term inside the brackets by the term outside the brackets.
  2. Simplify the results by performing any multiplication, addition and subtraction.

$$a(b + c) = a \times b + a \times c$$
$$= ab + ac$$

The distributive law

- The distributive law applies to both numerical and algebraic expressions.

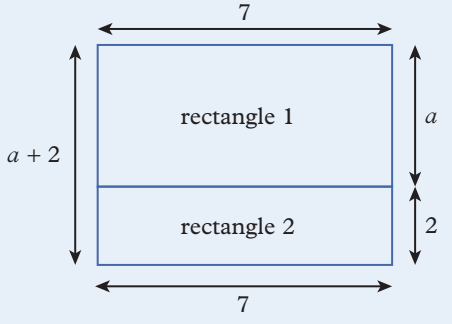
	Arithmetic	Algebra
The distributive law	$7(30 + 2) = 7 \times 30 + 7 \times 2$ $= 210 + 14$ $= 224$	$a(b + c) = ab + ac$

The area model

- The application of the distributive law to algebraic expressions can also be demonstrated using the area of a rectangle.

area of the large rectangle = area of rectangle 1 + area of rectangle 2

$$\text{length} \times \text{breadth} = \text{length} \times \text{breadth}_1 + \text{length} \times \text{breadth}_2$$
$$7 \times (a + 2) = 7 \times a + 7 \times 2$$
$$7(a + 2) = 7a + 14$$



Example 5F.1 Expanding by a whole number

Expand and simplify each expression using the distributive law.

a $6(a + 4)$ **b** $2(3x - y)$ **c** $3(2x + 4y + 5)$

THINK	WRITE
a 1 Multiply each term inside the brackets by the term outside the brackets.	**a** $6(a + 4) = 6 \times a + 6 \times 4$
2 Simplify the results by performing the multiplications.	$= 6a + 24$
b 1 Multiply each term inside the brackets by the term outside the brackets. Note that the second term inside the brackets is negative.	**b** $2(3x - y) = 2 \times 3x + 2 \times (-y)$
2 Simplify the results by performing the multiplications.	$= 6x - 2y$
c 1 Multiply each of the three terms inside the brackets by the term outside the brackets.	**c** $3(2x + 4y + 5) = 3 \times 2x + 3 \times 4y + 3 \times 5$
2 Simplify the results by performing the multiplications.	$= 6x + 12y + 15$

Example 5F.2 Expanding by a variable

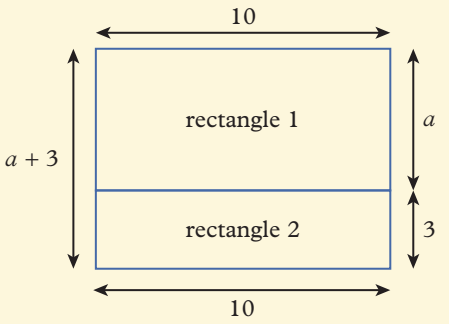
Expand and simplify each expression.

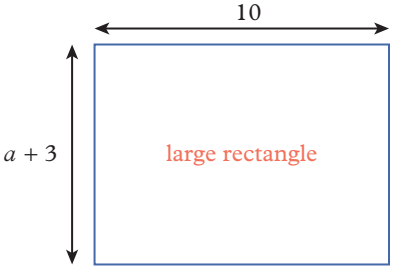
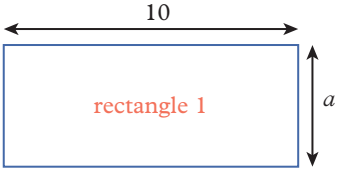
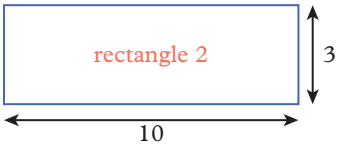
a $-7p(3q - 4)$ **b** $2m(5m - 4n)$ **c** $5a(a - 3b - 5)$

THINK	WRITE
a 1 Multiply each term inside the brackets by the term outside the brackets. Enclose the second term in brackets to separate the plus sign and the negative coefficient of the p term.	**a** $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$
2 Simplify the results by performing the multiplications. Remember the product of two negative numbers gives a positive number.	$= -21pq + 28p$
b 1 Multiply each term inside the brackets by the term outside the brackets.	**b** $2m(5m - 4n) = 2m \times 5m + 2m \times (-4n)$
2 Simplify the results by performing the multiplications. Remember that $m \times m = m^2$.	$= 10m^2 - 8mn$
c 1 Multiply each of the three terms inside the brackets by the term outside the brackets.	**c** $5a(a - 3b - 5) = 5a \times a + 5a \times (-3b) + 5a \times (-5)$
2 Simplify the results by performing the multiplications. Remember the product of a negative and positive number is a negative.	$= 5a^2 - 15ab - 25a$

Example 5F.3 Demonstrating the expansion of algebraic expressions using the area model

Demonstrate that $10(a + 3) = 10a + 30$ is true by finding the area of each rectangle in the diagram.



THINK	WRITE
1 Write an expression for the area of the large, composite rectangle.	area of large rectangle = length × breadth $= 10 \times (a + 3)$ $= 10(a + 3)$ 
2 Write an expression for the area of rectangle 1.	area of rectangle 1 $= 10 \times a$ $= 10a$ 
3 Write an expression for the area of rectangle 2.	area of rectangle 2 $= 10 \times 3$ $= 30$ 
4 Show that the area of the large rectangle is equal to the sum of the area of rectangle 1 and rectangle 2.	area of large rectangle = area of rectangle 1 + area of rectangle 2 Therefore, $10(a + 3) = 10a + 30$

Helpful hints

- ✔ When multiplying a negative term, use brackets for clarity.
  For example, $-7p(3q - 4) = -7p \times 3q + (-7p) \times (-4)$.
- ✔ Show all your working out to avoid making arithmetical errors and errors related to sign.
- ✔ After expanding expressions containing brackets, always simplify your results.
  Simplify, simplify, simplify!

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Exercise 5F Expanding

△ 1(a, b, g, h, i), 2–6, 7(a, b, e, h), 8, 10–12

□ 2, 3, 4(c, d), 5(b, d, f, h), 6, 7(c, f, g, h), 9, 12–14, 16

⬠ 2(2nd, 3rd columns), 3(e, f), 5(b, d, f, h), 7(c, f, g, h), 9(c, d), 13–18

UNDERSTANDING AND FLUENCY

5F.1 **1** Expand and simplify each expression using the distributive law.

a $3(a + 2)$ **b** $5(k - 7)$ **c** $2(5 + a)$
d $8(a + b)$ **e** $4(2x + 6)$ **f** $2(-1 + 8m)$
g $7(5a + 3b)$ **h** $6(8c - 5d)$ **i** $3(-5m - 2n)$

5F.2 **2** Expand and simplify each expression.

a $a(b + 4)$ **b** $x(3 + w)$ **c** $d(c - e)$
d $p(2p + 3)$ **e** $k(1 - 4k)$ **f** $2x(4y + 3w)$
g $3x(1 - 4x)$ **h** $\frac{1}{3}m(2m + 7n)$ **i** $-3a(2a - 4b)$
j $-2e^2(-4e - 5)$ **k** $5a^2\left(\frac{1}{10}a^2 - 3ab\right)$ **l** $-11y(x^2 + y^2)$

3 The distributive law can also be applied to the expansion of brackets containing more than two terms. Use the distributive law to expand and simplify the following expressions.

a $2(3x + y + 7)$ **b** $5(2ab + 4d - 3)$ **c** $7(2c - d + 5ef)$
d $-6(3g - 2h + gh)$ **e** $-4(5k + 2km - 9)$ **f** $-3(2xy + 4 - 7w)$

4 Apply the distributive law, then simplify by collecting any like terms to complete the following calculations.

a $5(a + 4) + 2(a + 7)$
$= 5 \times __ + 5 \times __ + 2 \times __ + 2 \times __$
$= 5a + __ + 2a + __$
$= _a + __$

b $3b(c + 2) + 2c(a + 5b)$
$= __ \times c + __ \times 2 + 2c \times __ + 2c \times __$
$= 3bc + __ + __ + __$
$= __ + __ + __$

c $d(d + 8) + 6(d - 3)$
$= d \times __ + d \times __ + __ \times d + __ \times (-3)$
$= __ + 8d + __ - __$
$= __ + __ - __$

d $4k(2k + 3) - 2(k - 5)$
$= 4k \times __ + 4k \times __ + (-2) \times __ + (-2) \times (-5)$
$= 8k^2 + __ - __ + __$
$= __ + __ + __$

5 Expand and simplify each expression.

a $2(a + 6) + 5(a + 3)$ **b** $4(b - 3) + 6(b + 2)$
c $5m(n + 4) + 3n(k + 2m)$ **d** $7x(3 + y) + \frac{2}{3}y(8 - 3x)$
e $\frac{k}{4}(k - 1) + 5(k + 4)$ **f** $c(c + 4) - 7(c - 5)$
g $rst(2 + 5t) + 9r(4t + st)$ **h** $h(h^2 - 2) - 4h(h + 3)$

6 Complete the following expansion.

$$\begin{aligned} -(a+2) &= -1 \times (a+2) \\ &= -1 \times ___ - 1 \times ___ \\ &= -___ - ___ \end{aligned}$$

7 Expand and simplify each expression.

a $6(p+3)+(p+2)$

b $3(d-2)-(d+8)$

c $x(x+2)+(x+5)$

d $k(k+4)-(3k+1)$

e $2e\left(e+\frac{1}{2}\right)-\left(e-\frac{3}{2}\right)$

f $4a(b+1)-(6+ab)$

g $\frac{1}{3}w(y-3)-(wy+2)$

h $mp(n-m)+(n-3m^2p)$

8 Consider the expression $5a(3a+10)$.

a Calculate the value of the expression if $a = 4$.

b Expand the expression.

c Substitute $a = 4$ into your answer to part **b**.

d Was it easier to evaluate the expression containing brackets, or its expanded form? Explain.

9 Expand and then evaluate each expression by letting $a = 2$.

a $2a(a+5)$

b $3a(10-2a)$

c $5a(3a-13)$

d $\frac{a}{3}(6a-9)$

10 In three years, Emily will be twice as old as Chantelle.

a If Chantelle is currently c years old, write an expression for Chantelle's age in three years' time.

b Use your answer to part **a** to write an expression for Emily's age in three years' time.

c Use the distributive law to expand the expression for Emily's age in three years' time.

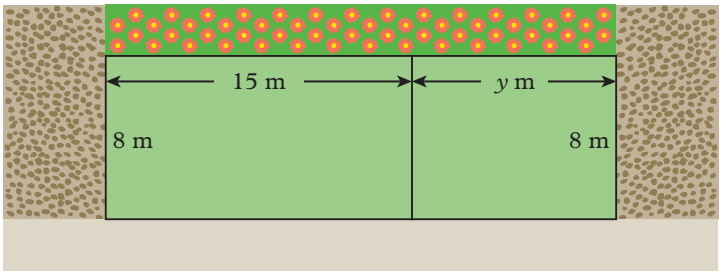
d Determine Emily's age in three years' time if Chantelle is currently 18 years old.

11 One of Andre's jobs is to mow the front lawn. The front lawn is rectangular and measures 15 m by 8 m.

a Calculate the area of lawn to be mown.

A neighbour notices Andre is about to start mowing and asks Andre to mow his front lawn as well.

b Let y be equal to the breadth of the neighbour's front lawn. Write an expression for the area of lawn in the neighbour's front yard.



c Add your answers to parts **a** and **b** to write an expression for the total area of lawn to be mown.

d Now consider both lawns as one large rectangle. Draw a diagram of this rectangle and label the length and breadth measurements.

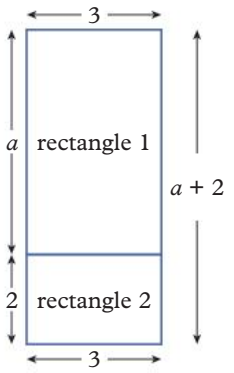
e Use brackets to write an expression for the total area of the large rectangle.

f Expand the expression and compare the expanded expression to your answer to part **c**. Are the expressions the same? Why or why not?

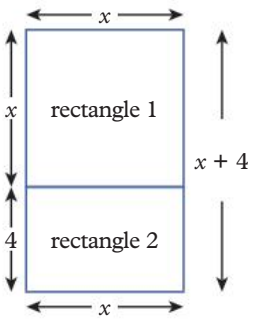
PROBLEM SOLVING AND REASONING

5F.3 **12** Demonstrate that each statement is true by finding the area of each rectangle in the diagrams below.

a $3(a + 2) = 3a + 6$



b $x(x + 4) = x^2 + 4x$



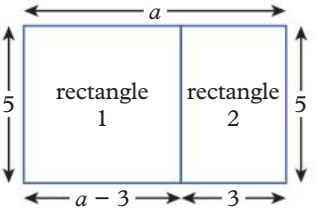
13 Draw diagrams that could be used to demonstrate that each of the following statements is true.

a $6(k + 8) = 6k + 48$

b $4(2d + 9) = 8d + 36$

c $2p(p + 3) = 2p^2 + 6p$

14 The diagram below can be used to demonstrate that $5(a - 3) = 5a - 15$.



a What is the length and breadth of rectangle 1?

b Use your answer to part **a** to write an expression for the area of rectangle 1 using brackets.

c Another way to calculate the area of rectangle 1 is to subtract the area of rectangle 2 from the total area of the large rectangle.

i State the length and breadth of the large rectangle.

ii Use these length and breadth measurements to write an expression for the area of the large rectangle.

iii State the length and breadth of rectangle 2.

iv Use these length and breadth measurements to write an expression for the area of rectangle 2.

v Use your answers to parts **ii** and **iv** to write an expression for the area of rectangle 1.

15 Draw a diagram and demonstrate $c(d - 5) = cd - 5c$.

16 If a and b are both positive integers, is $5a \times b + 3 < 5a(b + 3)$? Use expansion to justify your answer.

CHALLENGE

17 Expand and simplify each expression.

a $3x^2(4x^3 - 7) - 5x^4(x + 2)$

b $a^3(ab + c - 1) + a^2(3ac - a^2b + a)$

c $2m(3m^2np^2 + 5mn) - 6n(m^2 + m^2p^2 - 4)$

d $5w(4w^2x - 3y) + 2y(9w - 6xy) - 4x(5w^3 - 3y^2)$

18 Evaluate each expression in question **17** using these values:

$a = -3, b = 7, c = 4, m = 2, n = -4, p = -1, w = -5, x = 2, y = -6$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Expanding over one pair of brackets

Investigation Andrew's vegie patch

Topic quiz 5F

5G Factorising

Learning intentions

By the end of this topic you will be able to ...

✔ find the HCF of two or more algebraic terms

✔ factorise algebraic expressions by taking out the HCF.

Inter-year links

Support	Multiples and factors
Year 7	2D Factors and the highest common factor
Year 9	3E Factorising using the HCF
Year 10	2F Factorising

Highest common factor (HCF)

Key content video

- Numbers or algebraic expressions are factors of an algebraic expression if they multiply to give that expression. For example, 2, a, b, ab, $2a$, $2b$ are all factors of $2ab$.
- A common factor of two or more algebraic terms is a term that divides exactly into each of those terms.
- The highest common factor (HCF) of two or more algebraic terms is the 'highest' or 'largest' combination of coefficients, pronumerals and algebraic expressions that are factors of every term. The HCF of a set of algebraic terms can have both a numerical and a pronumeral component.

Set of algebraic terms	HCF
x, $7x$	x
$6m$, $12mn$, $8mp$	$2m$
$15a^3bc$, $9a^2c$	$3a^2c$

- To identify the HCF of a set of algebraic terms:
  1. Write each term as the product of its prime and algebraic factors.
  2. Circle the common factors.
  3. Multiply the common factors for the HCF.

$$15a^3bc = 3 \times 5 \times a \times a \times a \times b \times c$$
$$9a^2c = 3 \times 3 \times a \times a \times c$$
$$\text{HCF} = 3 \times a \times a \times c$$
$$= 3a^2c$$

Factorising algebraic expressions

- **Factorising** an algebraic expression is the opposite to expanding an algebraic expression.

expanding →

$$7(a + 2) = 7a + 14$$

← factorising

- To factorise an algebraic expression:
  1. Identify the HCF of all the terms in the expression.
  2. Write the HCF in front of the brackets.
  3. Inside the brackets, divide each term in the original expression by the HCF.
  4. Simplify the expression inside the brackets.

$$7a + 14 = 7\left(\frac{\cancel{7}^1 a}{\cancel{7}^1} + \frac{\cancel{14}^2}{\cancel{7}^1}\right)$$
$$= 7(a + 2)$$

Original expression: $7a + 14$

HCF: 7

Factorised expression: $7(a + 2)$

Example 5G.1 Identifying the highest common factor

Find the highest common factor (HCF) of each pair of terms.

a $2a$ and $5a$

b $12xy$ and $-18x^3$

THINK

a **1** Write each term as the product of its prime and algebraic factors.

2 Circle the common factors.

3 The only common factor is 'a', so the HCF is a.

b **1** Write each term as the product of its prime and algebraic factors. The negative sign can be represented using the factor '-1'.

2 Circle the common factors.

3 Multiply the common factors for the HCF. Simplify the product.

WRITE

a $2a = 2 \times a$

$5a = 5 \times a$

HCF $= a$

b $12xy = 2 \times 2 \times 3 \times x \times y$

$-18x^3 = -1 \times 2 \times 3 \times 3 \times x \times x \times x$

HCF $= 2 \times 3 \times x$

$= 6x$

Example 5G.2 Factorising algebraic expressions

Factorise the following expressions.

a $8x + 28$

b $9xyz - 12wxy$

THINK

a **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term in the original expression by the HCF.

4 Simplify the expression inside the brackets.

b **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term in the original expression by the HCF.

4 Simplify the expression.

WRITE

a HCF $= 4$

$$8x + 28 = 4\left(\frac{\cancel{8}^{2}x}{\cancel{4}^{1}} + \frac{\cancel{28}^{7}}{\cancel{4}^{1}}\right)$$

$$= 4(2x + 7)$$

b HCF $= 3xy$

$$9xyz - 12wxy = 3xy\left(\frac{\cancel{9}^{3}\cancel{x}^{1}\cancel{y}^{1}z}{\cancel{3}^{1}\cancel{x}^{1}\cancel{y}^{1}} - \frac{\cancel{12}^{4}w\cancel{x}^{1}\cancel{y}^{1}}{\cancel{3}^{1}\cancel{x}^{1}\cancel{y}^{1}}\right)$$

$$= 3xy(3z - 4w)$$

Example 5G.3 Factorising algebraic expressions containing indices

Factorise each expression.

a $a^2 - 6a$

b $10m^3n + 15m^2n^2$

THINK

a **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term by the HCF. It is helpful to write the pronumerals with indices in expanded form. Note that the minus sign remains from the original expression.

4 Simplify the bracketed expression.

b **1** Identify the HCF of all the terms in the expression.

2 Write the HCF in front of the brackets.

3 Inside the brackets, divide each term by the HCF. It is sometimes helpful to write the pronumerals with indices in expanded form. Alternatively, an index law can be used to simplify the indices.

4 Simplify the bracketed expression.

WRITE

a HCF = a

$$a^2 - 6a = a\left(\frac{a^1 \times a}{a^1} - \frac{6 \times a^1}{a^1}\right)$$
$$= a(a - 6)$$

b HCF = $5m^2n$

$$10m^3n + 15m^2n^2 = 5m^2n\left(\frac{10^2 \times m^1 \times m^1 \times m \times n^1}{5^1 \times m^1 \times m^1 \times n^1} + \frac{15^3 \times m^1 \times m^1 \times n^1 \times n}{5^1 \times m^1 \times m^1 \times n^1}\right)$$
$$= 5m^2n(2m + 3n)$$

Helpful hints

✔ When looking for the highest common factor, remember to consider any coefficients and all pronumerals.

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Exercise 5G Factorising

△ 1, 2(1st column), 3, 4(1st, 2nd columns), 5, 6(1st, 2nd columns), 7, 8–9(1st column), 10–13

□ 2(2nd column), 4(2nd, 3rd columns), 6(3rd, 4th columns), 7, 8–11(2nd column), 14, 15–16(a, c), 18

⬠ 2(h, j, l, n), 4(3rd column), 6(4th column), 7, 8–11(2nd column), 14, 15, 16(b, d), 17, 18

1 Match each pair of terms with their HCF.

Terms	**HCF**
a $15a$ and 10	**I** $15b$
b $5ab$ and $5a$	**II** $10ab$
c $20a^2b$ and $10ab$	**III** 5
d $15a^5b^3$ and $30bc^2$	**IV** $5a$

5G.1 **2** Find the highest common factor (HCF) of each pair of terms.

a $4x$ and 12
b $10d$ and $-5d$
c $6j$ and $8jk$
d $3ab$ and $-3ab$
e $-9cd$ and $9ef$
f $7xy$ and $14yz$
g abc and bcd
h $10abcd$ and $-14bcde$
i $6xy$ and $2y^2$
j $3c^2$ and $9b^2c$
k $30g^2h$ and $-24g^4$
l $4ab^3$ and $8bc^3$
m $2m^2n^2p^2$ and $3m^3p^2$
n $15r^2s^4t$ and $-20r^3s^4t^5$

3 Complete the following factorised expressions.

a $2x + 6 = 2\left(\frac{2x}{} + \frac{6}{}\right)$
$= 2(__+__)$

b $8a - 36 = __\left(\frac{8a}{4} - \frac{36}{4}\right)$
$= __(2a - 9)$

c $25mn + 30n = 5n\left(\frac{}{5n} + \frac{}{5n}\right)$
$= 5n(__+__)$

d $kmn - mn = __\left(\frac{kmn}{} - \frac{}{}\right)$
$= __(k - __)$

e $12ef - 18df = 6f\left(\frac{}{} - \frac{}{}\right)$
$= 6f(__-__)$

f $-26ab + 24cd = __\left(\frac{-26ab}{} + \frac{}{}\right)$
$= __(13ab + __)$

g $-abc + acd = -ac\left(\frac{}{} + \frac{}{}\right)$
$= -ac(__-__)$

h $-21xyz - 28wxy = __(\frac{}{} - \frac{}{})$
$= __(__+4w)$

5G.2 **4** Factorise each expression.

a $4a + 8$
b $7e + 21$
c $45 + 5k$
d $3 + 12y$
e $27 - 9p$
f $24ab + 18b$
g $12st - 14t$
h $5mn + 10n$
i $3g - 9gh$
j $22xy - 4xz$
k $3ab + 3bc$
l $16mn - 8mp$
m $27xyz - 18wxy$
n $12kmn + 15mn$
o $-21efg + 24fgh$

5 Complete the following factorised expressions.

a $h^2 + 7h = h\left(\frac{h^2}{} + \frac{7h}{}\right)$
$= h(__+__)$

b $10a + 5a^2 = __\left(\frac{10a}{} + \frac{5a^2}{}\right)$
$= __(__+a)$

c $6p^3 + 9p^5 = 3p^3(\frac{}{} + \frac{}{})$
$= 3p^3(__+__)$

d $6xy + 4x^2y = __\left(\frac{6xy}{} + \frac{4x^2y}{}\right)$
$= 2xy(__+__)$

e $mn^2 + 5mn = __\left(\frac{}{} + \frac{5mn}{}\right)$
$= __(__+5)$

f $24efg^2 + 32fg^7h^3 = __\left(\frac{24efg^2}{} + \frac{}{}\right)$
$= __(3e + __)$

5G.3 **6** Factorise each expression.

a $c^2 + 4c$
b $6k - k^2$
c $a^2 - 2a$
d $3m^2 + 3m^4$
e $10p^3 + 11p^5$
f $4h - 7h^4$
g $6x^2y + 3x^3y^6$
h $5b^2c^5 - 10bc^2$
i $8mn^2 + 36m^2n^3$
j $27xyz^2 - 15x^2yz$
k $14a^3bc^{10} + 16a^2b^7c^4$
l $12e^2fg^2 + 18ef^5g^2$

7 Check your answers to question **6** by expanding each expression.

8 In some cases, it can be best to factorise using a HCF with a negative coefficient. Find the HCF that has a negative coefficient for each pair of terms.

a $-5a$ and -10
b $-2pq$ and $-6p$
c $-9hk$ and $-15jk$
d $-14rst$ and $-7st$
e $-4x^2$ and $4x$
f $-12m^3$ and $20m^2n^7p^5$
g $-x^2y$ and $-xy$
h $-8b^2c^2$ and $28abc^4$

9 Factorise each expression by finding the HCF that has a negative coefficient.

a $-4a - 12$
b $-6 - 8k$
c $-7m + 14mn$
d $-15b + 35ab$
e $-3st - 9tr$
f $-18abc - 24bcd$
g $-12x^{12} + 21x^{21}$
h $-h^2 - 2h$
i $-16x^2y^7 + 8x^5y^{10}$
j $-45ab^6 - 10a^{15}b^{10}c^8$
k $-42xyz^2 - 36x^4y^{13}z^2$
l $-11a^2bc^3 + 9ab^2c^9$

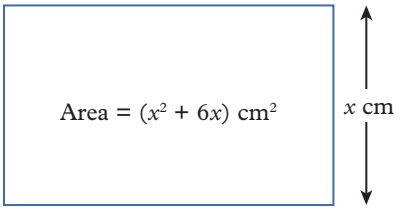
10 Find the HCF of the following sets of terms.

a $12a, 12b, 3a$
b $5abcd, 6bdfh, -7bdhp$
c $15rm, 30ms, 40nmq$
d $36ad, -42ab, -24ac, 18ae$
e $-8xyz, 4uxz, 16vzx, 32zwx$
f $21\,i^4j^2k, 49\,j^5i^2k^3, 28\,k^4i^3j^2$

11 Factorise:

a $15p + 15q - 5$
b $6jk - 3ik - 10kl$
c $-8mnpq + 4jknp - 2pmkn$
d $27fg - 54gf + 45fh - 36hf$
e $17uvwx + 34xvu - 51vzxu - 68uv$
f $56\,a^8b^{12}c^4 + 32\,a^9b^7c^9 - 80\,a^7b^{11}c^7$

12 Consider this rectangle, which has a breadth of x cm and an area of $(x^2 + 6x)$ cm^2.



a Factorise the expression for the area of the rectangle.
b List the two factors that multiply to give the expression for the area.
c What is the expression for the length of this rectangle?
d Demonstrate you have the correct expression for the length of this rectangle by multiplying the length by the breadth to obtain the expression for its area.
e Another way of showing that you have the correct expression for the length of this rectangle is to use substitution. Try this method by letting $x = 3$.

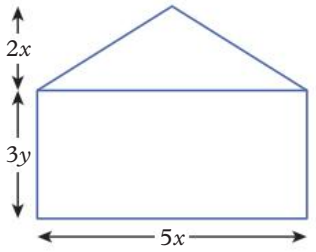
13 Since factorising is the opposite of expanding, we can check our answers by doing the opposite manipulations to our answers. That is, expand after we factorise or factorise after we expand.

i Perform the following manipulations (expand or factorise).
ii Use the opposite manipulation (factorise or expand) to check if your answer is correct.

a Expand $8(4 - 3x)$.
b Factorise $7y + 49$.
c Expand $3b^3(2b^2 - 5b^5)$.
d Factorise $25a^2 - 125ab$.

PROBLEM SOLVING AND REASONING

14 Consider the composite shape shown below.



a Write an expression for the area of the composite shape.

b Factorise your answer to part **a**.

15 It is possible to take any term or expression out as a factor, even if it's not the highest common factor or even a common factor.

For example, to take 7 out as a factor of $x + 5y + 11z$, write 7 in front of $(x + 5y + 11z)$ and divide each term in the brackets by 7: $7\left(\frac{x}{7} + \frac{5y}{7} + \frac{11z}{7}\right)$. If possible, simplify the fractions.

For each of the expressions, take out the following terms as a factor.

i 6 **ii** -1 **iii** $\frac{1}{2}$ **iv** x **v** $-3x$

a $x + 5y + 11z$

b $2x - 3y + 6z$

c $18x^2 + 36x^3y - 54x^4z$

d $4xy - 6x - 10xz$

16 Each of the following problems appeared on a class test. Students were asked to fully factorise the expressions. Below are some calculations where a student has made a mistake.

i Describe the mistake in each calculation.

ii Factorise the expression correctly.

a $-18p + 12q = -6\left(\frac{-18p}{-6} + \frac{12q}{-6}\right) = -6(3p + 2q)$

b $8r - 16r^3 = 4r\left(\frac{8r}{4r} - \frac{16r^3}{4r}\right) = 4r(2 - 4r^2)$

c $20y^4 + 5y^3 = 5y^3\left(\frac{20y^4}{5y^3} + \frac{5y^3}{5y^3}\right) = 5y^3(4y)$

d $6x^3y + 7xy^3 = 6x^3y\left(\frac{6x^3y}{6x^3y} + \frac{7xy^3}{6x^3y}\right) = 6x^3y\left(1 + \frac{7y^2}{6x^2}\right)$

CHALLENGE

17 a Determine the HCF of each expression.

b Factorise and simplify each expression.

i $y(3x + 2) + y(4x - 7)$

ii $(x + 2)(3x + 2) + (x + 2)(4x - 7)$.

18 Factorise the expression $\frac{5}{8}x^{12}y^{19} - \frac{25}{8}y^{20}x^{21} + \frac{15}{8}x^{17}y^{17} - \frac{10}{8}y^{40}x^{32}$.

Check your Student obook pro for these digital resources and more: pro

Interactive skillsheet
Factorising using the HCF

Topic quiz
5G

Chapter summary

Terms and expressions

pronumerals

coefficients

constant

$$\underbrace{\underbrace{15x}_{\text{term}} + \underbrace{2y}_{\text{term}} + \underbrace{5}_{\text{term}}}_{\text{expression}}$$

Algebraic notation

$7 \times x = 7x$

$7 \times (x + 2) = 7(x + 2)$

$1 \times x = x$

$x \div 7 = \frac{x}{7}$

$(x + 2) \div 7 = \frac{(x + 2)}{7}$

$\frac{x}{7} = \frac{1}{7}x$

Grouping symbols

$7(x + 2)$ $\quad \frac{x + 2}{7}$ $\quad 7\sqrt{x + 2}$

Substitution

Let $x = -2$

$5x^2 - x = 5(-2)^2 - (-2)$

Simplifying signs

$+(+) = +$ $\quad +\times + = +$

$+(-) = -$ $\quad +\times - = -$

$-(+) = -$ $\quad -\times + = -$

$-(-) = +$ $\quad -\times - = +$

Like terms

Pronumeral(s)	Examples of like terms
x	$x,\ 3x,\ \frac{2}{3}x,\ 0.7x...$
ab	$ba,\ -2ab,\ \frac{ab}{5},\ 6.6ab...$
m^2n	$-m^2n,\ 18m^2n,\ \frac{9m^2n}{4},\ 0.05m^2n...$

Adding and subtracting like terms

$6ab + \frac{3}{5}a + ab - \frac{2}{5}a = 6ab + ab + \frac{3}{5}a - \frac{2}{5}a$

$= 7ab + \frac{1}{5}a$

Index and expanded form

$$\underbrace{3a^2b}_{\text{index notation}} = \underbrace{3 \times a \times a \times b}_{\text{expanded form}}$$

Expanding and factorising

expanding →

$7(a + 2) = 7a + 14$

← factorising

Expanding

$3(2a - 6) = 3 \times 2a + 3 \times (-6)$

$= 6a + (-18)$

$= 6a - 18$

Multiplying algebraic terms

$3a^3b \times (-2a^2b^2) = 3 \times a \times a \times a \times b \times (-2) \times a \times a \times b \times b$

$= 3 \times (-2) \times a \times a \times a \times a \times a \times b \times b \times b$

$= -6a^5b^3$

Identifying the HCF

$15a^3bc = 3 \times 5 \times a \times a \times a \times b \times c$

$9a^2c = 3 \times 3 \times a \times a \times c$

HCF $= 3 \times a \times a \times c$

$= 3a^2c$

Dividing algebraic terms

$$\frac{6a^5b^3}{3a^3b} = \frac{\cancel{6}^2 \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times b \times b}{\cancel{3}^1 \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b}}$$

$$= \frac{2 \times a \times a \times b \times b}{1}$$

$$= 2a^2b^2$$

Factorising

$$7a + 14 = 7\left(\frac{\cancel{7}^1 a}{\cancel{7}^1} + \frac{\cancel{14}^2}{\cancel{7}^1}\right)$$

$$= 7(a + 2)$$

Original expression: $7a + 14$

HCF: 7

Factorised expression: $7(a + 2)$

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Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- algebraic expression
- associative law
- coefficient
- commutative property
- constant
- distributive law
- evaluate
- expand
- expanded form
- exponent form
- factor
- factorise
- formula
- grouping symbols
- index notation
- like terms
- operation
- pronumeral
- simplify
- substitution
- term
- unknown number
- variable

1 Describe the following calculations, pay attention to the order of operations.

a $2(a - b) - 4$ **b** $a - 2b - 4$ **c** $\frac{a - b}{2} - 4$

d $a - (\frac{b}{2} - 4)$ **e** $a - b^2$ **f** $(a - b)^2$

2 What is the singular form of 'indices'?

3 Demonstrate the associative, commutative and distributive laws using the pronumerals a, b and c.

Multiple choice

5A **1** What is the coefficient of the second term in the expression $10a^2 - 6a^2b + 13b - 15$?

A 6 **B** −6 **C** 13 **D** 15 **E** −15

5A **2** On a rainy day, the volume of water in a rainwater tank increases by 10 litres every hour. If there was originally 5 litres of water in the tank and it rains for t hours, this total volume of water in the tank is best represented by the expression:

A $15t$ **B** $10t$ **C** $10(t + 5)$

D $t + 5$ **E** $10t + 5$

5B **3** If $x = 4$ and $y = -3$, $2xy - 9y$ is equal to:

A 3 **B** 51 **C** −3

D −51 **E** −60

5C **4** Which list contains only like terms?

A $3a, ab, -5a$ **B** $4mn, mn, 2nm$ **C** $-8p, 6p^2, 5p^3$ **D** $2cd, de, -7ce$ **E** $5xy, 5x, 5y$

5C **5** Simplify $7m + 2n - 5m - 2n$.

A $2m$ **B** $12m + 4n$ **C** $2mn$ **D** $2m + 4n$ **E** $12mn$

5C **6** What does $2x^2 - 3x^2$ simplify to?

A $5x^2$ **B** 1 **C** $-x^2$ **D** x^2 **E** −1

5D **7** What is the expanded form of $-5abc^2$?

A $-5 \times a \times a \times b \times b \times c \times c$ **B** $-5 \times a \times b \times c$ **C** $-5 \times a \times a \times b \times c$

D $-5 \times a \times b \times b \times c$ **E** $-5 \times a \times b \times c \times c$

5D **8** Which of these is *not* equivalent to $24a^6b^3c^5$?

A $2a^2b^2 \times 3a^2c^3 \times 4a^2bc^2$ **B** $3a^4b \times 8b^2c \times a^2c^4$ **C** $3a^4c^5 \times 4a^2b \times 2ab^2$

D $bc^4 \times 6a^3 \times 4a^3b^2c$ **E** $4abc \times 3a^4bc^3 \times 2abc$

5E **9** What does $\frac{4ab^2}{8ab}$ simplify to?

A $2b$ **B** $\frac{1}{2b}$ **C** $\frac{2}{b}$ **D** $\frac{b}{2}$ **E** $\frac{1}{2}ab$

5F **10** When expanded, $5m(3m + 2n - 6)$ equals:

A $15m^2 + 2n - 6$ **B** $8m^2 + 7mn - 11m$ **C** $15m^2 + 10mn + 30m$

D $8m^2 + 10mn - 6$ **E** $15m^2 + 10mn - 30m$

5F **11** Which expression simplifies to $12mn + 5m$?

A $m(3n - 4) + 2m(5n + 4)$ **B** $4n(2m - 3) + 2m(2n + 4)$ **C** $m(2n - 3) + 2m(5n + 4)$

D $3m(3n - 3) + m(3n + 4)$ **E** $6m(n - 1) + 6m(n + 4)$

5G **12** What is the HCF of $24ab^2c$ and $18bc$?

A $2bc$ **B** $3c$ **C** $6bc$ **D** $8ab$ **E** $3ab$

Short answer

5A **1** Write an expression for each statement. Use m to represent the unknown number.

a a number is multiplied by 4 and then 3 is added

b half of the product of a number and 5

c add 3 to a number, then divide the result by 7

d 6 less than a number all multiplied by $\frac{3}{5}$

5A **2** Zoo entry is \$25 per adult, \$13 per child and \$20 per pensioner. Write an expression for each of the following, in dollars.

a the cost of entry for x adults

b the cost of entry for y children

c the cost of entry for p pensioners

d the total cost of entry for x adults, y children and p pensioners

5B **3** Evaluate each expression by substituting $a = 2$, $b = 5$ and $c = -3$.

a $7a + 2b$ **b** $5a^2 - 4bc$ **c** $-\frac{1}{3}(6ab - 12)$ **d** $\frac{b^2 - a^2}{c}$

5B **4** A cyclist is travelling with a constant acceleration, a m/s^2. Their final speed, v, is equal to the square root of the sum of the square of their initial speed, u, and twice the product of their acceleration and the distance they cover, s.

a Write a formula for the final speed of the cyclist.

b If the cyclist is travelling initially at 2 m/s and is accelerating at 3 m/s^2 for 42 m, what is the final speed?

5C **5** Identify and list the like terms in the following groups of terms.

a $6a, \frac{1}{7}a, b, -b, \frac{a}{10}, \frac{3}{8}b$

b $0.5x, 95y, \frac{1}{2}y, 1.7x, 10x, \frac{5}{6}x$

c $5de, -ed, \frac{2ef}{15}, 0.01de, 12fe, ef$

d $p^2, 2p^3, 5.1p^2, -15p^2, \frac{3p^3}{2}, \frac{3}{2}p^2$

e $\frac{9}{10}ab, 6.5abc, -6.5ba, 0.9cab, \frac{11}{20}bca, -ab$

f $\frac{4}{9}f^2g, \frac{gf^2}{3}, -23g^2f, -2.3g^2, \frac{6}{7}f^2, -3.14fg^2$

5C **6** Simplify:

a $4m + 5n + 8m + 3n$

b $3cd + 2c + 5dc + 4$

c $2k^2 + 6 - 9k^2 - 1$

d $5ab^2 - 2a^2b + 4a^2b - 3ab^2$

e $2xyz + 5xy - 7zyx - yx + \frac{3}{2}xy - \frac{2}{5}y$

f $\frac{3}{4}pq^2 - 7q^2 - \frac{1}{2}q^2p - q^2 - pq^2$

5D **7** Simplify the following products.

a $x^6 \times x^8$

b $m \times m$

c $h^7 \times h \times h^3$

d $q^5 \times p^{10} \times q \times p^2$

e $x^5 \times y^{12} \times x \times x^{19} \times y \times y^2$

f $a \times c^9 \times b^{21} \times a^5 \times b^9 \times c^3$

5D **8** Simplify each product.

a $-5 \times f \times g$ **b** $p^2 \times 2s$ **c** $7g \times 8g^9$

d $3ab \times 8bc$ **e** $2m^5n^8 \times (-3m^7n^4)$ **f** $-2ef^2 \times \frac{2}{5}efg^2 \times (-3f^7g^{16})$

5E **9** Simplify the following quotients.

a $k^{11} \div k^7$ **c** $x^9y^2 \div x^6y$ **e** $p^{20}q^8r^{20} \div p^9q^6r^2$

b $8c^9 \div c$ **d** $m^6n^7 \div m^3n^7$ **f** $a^3b^{24}c^{11} \div a^2b^{12}c$

5E **10** Simplify:

a $\frac{15a}{20}$ **b** $\frac{6cd}{2d}$ **c** $\frac{6abcd}{10bc}$

d $\frac{2e^2hk^2}{5ek}$ **e** $\frac{16x^{13}y^6}{28x^6y}$ **f** $\frac{2t^4 \times 4t^5}{8t^9}$

5F **11** Expand:

a $5(a + 3)$ **b** $4(7b - c)$ **c** $\frac{c}{2}(6a + 10d)$ **d** $-6m(4m^2 + n)$

5F **12** Expand and simplify:

a $3(k + 7) + -5(3k - 2)$ **b** $2y(y - 6) - (y + 4)$

5G **13** Find the highest common factor (HCF) of each pair of terms.

a 2 and $6a$ **b** $9p$ and $12pq$ **c** $10b^2$ and $-5bc$

d $16rst$ and $24st$ **e** $-a^4b^3c^{10}$ and $11a^2b$ **f** $-42mn^9p^8$ and $18m^6n^{12}p$

5G **14** Factorise:

a $14h - 18$ **b** $20ab + 10ac$ **c** $8c^2 + 12c$

d $-5efg - 5fgh$ **e** $15a^2b - 40ab$ **f** $18x^2y^5 - 24wx^{10}y^3$

Analysis

1 A roadside stall sells fresh, free-range eggs daily. The price varies every week depending on the time of year and the number of eggs that were laid. Johnny shops at the stall daily to buy fresh eggs to always have a fresh supply. Johnny buys two more eggs than the day before each day for the entire week to have enough for the different meals he cooks on different days.

Let x be the number of eggs Johnny buys on Monday and let y be the price the stall sells eggs at for the week.

a Write an expression for the total cost for Johnny after a week as the sum of the costs of each day. Do not expand or simplify the expression.

b If the price of an egg was \$0.43 and Johnny bought four eggs on Monday, what was the total cost for the week?

c To make Johnny's life easier with budgeting, simplify the total cost expression from part **a** by expanding and collecting like terms.

d The next week, the price of an egg is \$0.48 and Johnny buys three eggs on Monday. Use the expression from part **c** to determine the total cost for the second week.

e Johnny thinks the expression will be even simpler if it is factorised. Factorise the expression from part **c**.

f For the third week, the price of an egg is \$0.39 and Johnny buys five eggs on Monday. Use the expression from part **e** to determine the total cost for the third week.

2 A gardening supplier is designing a rectangular prism–shaped planter box that has a length and breadth that are square numbers, but not necessarily the same square number, and a height that is a cube number.

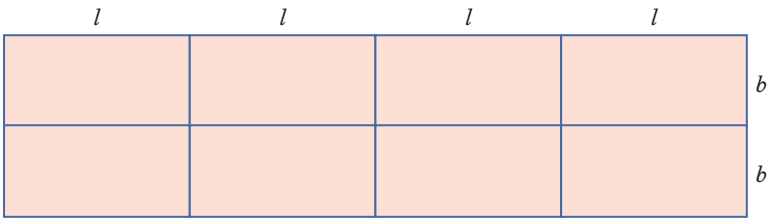
Let a be the number squared for the length, b be the number squared for the breadth, and c be the number cubed for the height.

a Write an expression for the perimeter of the base of one planter box.

b Write an expression for the area of the base of one planter box.

c Write an expression for the volume of one planter box.

A gardener buys eight of these planter boxes and arranges them into two rows and four columns as shown.



d Write an expression for the perimeter of the base of the array of eight planter boxes.

e Write an expression for the area of the base of the array of eight planter boxes.

f Write an expression for the volume of the array of eight planter boxes.

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify terms and expressions ☐ Write expressions to represent worded problems	☐ Go back to Topic 5A Expressions
☐ Substitute values into and evaluate expressions ☐ Substitute values into formulas to find unknown values	☐ Go back to Topic 5B Substitution
☐ Identify like terms ☐ Simplify expressions by collecting like terms	☐ Go back to Topic 5C Adding and subtracting algebraic terms
☐ Convert algebraic terms from expanded form to index notation ☐ Simplify expressions by multiplying algebraic terms	☐ Go back to Topic 5D Multiplying algebraic terms
☐ Simplify expressions by dividing algebraic terms	☐ Go back to Topic 5E Dividing algebraic terms
☐ Expand algebraic expressions	☐ Go back to Topic 5F Expanding
☐ Find the HCF of two or more algebraic terms ☐ Factorise algebraic expressions by taking out the HCF	☐ Go back to Topic 5G Factorising

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Semester 1 review

Short answer

1 Round the following numbers according to the description in the brackets.

a 569 238.472 940 (correct to the nearest thousand)

b 569 238.472 940 (correct to the leading digit)

c 208 492.175 207 (correct to the nearest thousandth)

d 208 492.175 207 (correct to two decimal places)

e 0.000 047 19 (correct to the leading digit)

f 28 174.201 (correct to the nearest multiple of 20)

2 Fill the gaps to produce equivalent fractions.

a $\frac{7}{8} = \frac{21}{}$ **b** $\frac{-8}{10} = \frac{}{15}$ **c** $2\frac{4}{5} = \frac{70}{}$ **d** $\frac{-1.2}{-0.07} = \frac{}{70}$

3 Simplify the following.

a $\frac{-90}{-120}$ **b** $\frac{300}{18}$ **c** $7\frac{45}{60}$ **d** $56 : 48$

e $36 : 24 : 72$ **f** $0.5 : 0.025$ **g** $\frac{1}{9} : \frac{2}{3}$ **h** 150 km per 2.5 hours

4 Evaluate the following sums.

a $579 + 643 + 87$ **b** $14 + (-52)$ **c** $\frac{5}{9} + \frac{5}{6}$

d $12.41 + 9.015$ **e** $2^4 + 2^3$ **f** $3\frac{4}{5} + 6\frac{2}{3}$

5 Evaluate the following differences.

a $1041 - 828$ **b** $(-16) - (-23)$ **c** $\frac{1}{7} - \frac{5}{8}$

d $48.029 - 9.994$ **e** $3^4 - 3^2$ **f** $6\frac{3}{8} - 2\frac{7}{12}$

6 Evaluate the following products.

a 245×18 **b** -5×12 **c** $\frac{12}{10} \times \frac{25}{18}$

d $-4.2 \times (-31.8)$ **e** $2^2 \times 2^3$ **f** $3\frac{1}{5} \times 4\frac{1}{6}$

7 Evaluate the following quotients.

a $5144 \div 8$ **b** $-255 \div (-3)$ **c** $\frac{12}{25} \div \left(-\frac{9}{10}\right)$

d $0.042 \div 0.6$ **e** $3^8 \div 3^5$ **f** $4\frac{1}{12} \div 2\frac{5}{6}$

8 Evaluate the following.

a 6^3 **b** 3^5 **c** $\left(\frac{5}{4}\right)^3$

d $(-0.2)^4$ **e** $\left(3\frac{1}{2}\right)^4$ **f** 7^0

9 Evaluate the following roots.

a $\sqrt{36}$ **b** $\sqrt[3]{8}$ **c** $\sqrt{\frac{81}{25}}$ **d** $\sqrt{0.49}$

10 Evaluate the following expressions.

a $15 - 3^2 \times 8 \div 6 + 2$ **b** $25 \div (-5) - (-9) \times 7$

c $\frac{5}{6} - \frac{2}{3} \div \frac{4}{7} - \frac{3}{4}$ **d** $3.4 \times (2.5 + 3.7) - 8 \times 1.1^3$

11 Simplify the following.

a $5y + (-8xy) + 4x + 3yx + x + (-2y)$ **b** $12b - 8a - (-3b) - 7b$ **c** $2^4 \times 2^3$

d $7x^2y \times 5xy^5$ **e** $3^{12} \div 3^4$ **f** $\frac{24a^5b^6}{3b^2}$

g $(3^6)^5$ **h** $2^8 \times 3^{10} \div 2^2 \div 3^5$ **i** $\frac{6x + 12}{3}$

12 For the following expressions:

i round each value to the leading digit (first non-zero digit) and estimate the calculation

ii evaluate the actual result of the calculation

iii determine the percentage error between the estimation and the actual result, correct to two decimal places.

a $710\,284 \times 0.021\,83$

b $9107.4029 \div 2.145$

13 Write these lists of numbers in ascending order.

a 842.01, 8.4201, 824.01, 24.01, 24.1, 842.1

b −5.31, 3.15, 5.13, −5.1, −5.031, 5.103

c $\frac{7}{8}, \frac{3}{4}, \frac{4}{5}, \frac{11}{12}, \frac{9}{10}, \frac{2}{3}$

d $-\frac{3}{2}, \frac{6}{-2}, \frac{-3}{4}, \frac{-3}{-4}, \frac{4}{3}, -\frac{4}{3}$

14 State if the following are:

i terminating or non-terminating

ii recurring or non-recurring, if they are non-terminating.

a 2.33333…

b 5.194 817 53…

c 2.849

d 0.001 212 121 2…

15 Complete the table below.

Fraction	Decimal / recurring decimal	Percentage
	1.45	
$\frac{7}{8}$		
	2.5	
$\frac{21}{5}$		
		$33.\dot{3}\% = 33\frac{1}{3}\%$
		108%
$\frac{4}{33}$		

16 Write the following.

a 12 games played as a fraction out of 15 games owned

b 16 quests completed as a percentage out of 40 possible quests

c 8 episodes watched as a percentage out of 25 episodes

d the ratio of connected to unconnected puzzle pieces when 840 pieces are connected in a 1000-piece puzzle

e 57 eggs cooked in three weeks as a rate per week

f the average speed of a car that travels 276.25 km in 3.25 hours

17 Calculate:

a $\frac{5}{8}$ of 72

b 0.8 of 55

c 110% of 99

d 58 increased by 12%

e 120 decreased by 20%

f \$2700 shared in the ratio 4 : 5

g the number of blue counters in a bag, if the bag has 35 red counters and the ratio of blue counters to red counters is 3 : 5

h the mass of a block if its density is 20 kg/m^3 and the volume of the block is 0.04 m^3

i the distance a bike will travel when ridden at 8 km/h for 30 minutes

j the time taken for someone walking at 2 m/s to walk 1 km.

18 Write an expression for the following:

a the difference of the squares of two numbers, p and q

b the sum of the product of 3 and a number, x, and the quotient of another number, y, and 12

c the quotient of 6 more than a number, x, and 6 less than the number x

d the square root of the number 7 times a number k.

19 Expand and simplify:

a $8(4 - 3x)$ **b** $5 - 3(8x - 11) + 2x$ **c** $2b(6a + 5b + 3)$ **d** $5x^2y(6xy - 9y^4)$

20 Factorise:

a $36x + 27$ **b** $12a + 36b - 18c$ **c** $\frac{10}{3}p^3 + \frac{25}{3}p^7$ **d** $-\frac{3}{8}u^2v + \frac{5}{4}uv^2$

21 Calculate the following:

a the cost price of a cupcake, if it was marked up by 280% to be sold for \$2.10

b the cost price of a book, if it was discounted by 40% to be sold for \$15.00

c the selling price of a bag of lollies that is bought at \$0.90 per bag and marked up by 120%

d the selling price of an indoor plant that was \$80 and is discounted by 20%

e the cost, including GST, of a meal that was \$45 not including GST.

22 Calculate the LCM and HCF of the following.

a 18 and 30

b 210 and 700

c $2^4 \times 3^6 \times 5^4$ and $2^3 \times 3^9 \times 5^4$

d $2^7 \times 5^8 \times 7^4$ and $2^3 \times 3^8 \times 7 \times 11^2$

23 Substitute the value in brackets into each expression, then evaluate.

a $8d^2 + 5d + 4$ $(d = 10)$ **b** $\frac{3}{4}x + x^2$ $(x = -2)$

c $6y - \frac{2}{y}$ $\left(y = \frac{5}{3}\right)$ **d** $\frac{1.03 - r^2}{r + 0.7}$ $(r = 0.5)$

24 State if the following are true or false.

a $(8 - 3) - 4 = 8 - (3 - 4)$ **b** $(120 \div 6) \div 2 = 120 \div (6 \times 2)$

c $(3^4)^5 = 3^{(4^5)}$ **d** $3\frac{1}{2} \times 8 = 3 \times 8 + \frac{1}{2} \times 8$

e $4 - (-3) = 4 + 3$ **f** $3^8 - 3^4 = 3^2$

25 Express the following numbers as the product of their prime factors. Write your answer in index notation.

a 120 **b** 875

Analysis

1 Barry owns a \$2 shop and uses a simple cash register where the prices of items must be entered manually. Part of the stock is fake flowers. He stocks three types of fake flowers.

- lower quality flowers that cost \$0.50 per flower
- medium quality flowers that cost \$1.25 per flower
- higher quality flowers that cost \$1.95 per flower

Barry marks up his stock 80% from the cost price to the selling price.

a Determine the selling price of the three types of flowers.

b A customer buys 5 lower quality flowers, 3 medium quality flowers, and 4 higher quality flowers. How much does the customer pay?

Barry uses a different colour price tag each year to keep track of how long the stock has been in the store. Any item that has been in the store for two or more years is sold at a discount of 30%.

c A customer buys 10 lower quality flowers, 8 medium quality flowers, and 5 higher quality flowers, all of which have been in the store for two or more years.

i How much does the customer pay?

ii How much more money would Barry have made if the flowers were sold at full price?

d Barry gets a delivery of 100 lower quality flowers, 75 medium quality flowers, and 50 higher quality flowers. The delivery cost was \$20. Calculate:

i the total cost of purchasing the flowers including delivery

ii the difference between the total selling price and the total cost price if Barry sells all the flowers within two years.

Another supplier sells their flowers for the following prices.

- lower quality flowers that cost \$0.35 per flower
- medium quality flowers that cost \$1.15 per flower
- higher quality flowers that cost \$2.10 per flower
- delivery cost of \$25 per shipment

e How much cheaper or more expensive would Barry's purchase in part **d** be from this supplier?

2 A teacher notices a number of mistakes on a recent revision quiz.

i $372 + 654 = 926$ **ii** $1025 - 856 = 231$ **iii** $632 \times 72 = 5688$

iv $2760 \div 8 = 37$ **v** $4^4 = 16$ **vi** $\sqrt{16} = 8$

vii $\sqrt[3]{27} = 9$ **viii** $2^3 \times 2^2 = 2^6$ **ix** $3^3 \div 3^3 = 0$

a Determine the correct answer for each calculation.

b Calculate the percentage error between the correct and incorrect answers, correct to two decimal places.

c Modify the first number in each calculation so that the answers are correct.

3 Harrison was investigating the index laws and how they apply with algebra. He sets up the following pairs of products of prime factors:

i $2^5 \times 3^2 \times 5$ and $2^3 \times 3^5 \times 5^3$

ii $3^5 \times 2^2 \times 7$ and $3^3 \times 2^5 \times 7^3$

iii $3^5 \times 5^2 \times 7$ and $3^3 \times 5^5 \times 7^3$

iv $a^5 b^2 c$ and $a^3 b^5 c^3$

For each of the following, leave your answer as the product of prime factors.

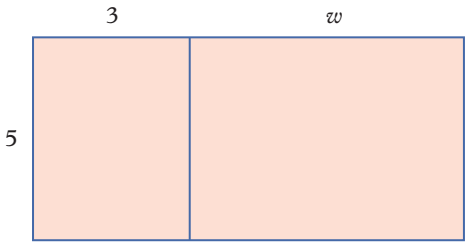
a Determine the product of each pair.

b Determine the quotient of each pair.

c Determine the highest common factor of each pair.

d Determine the lowest common multiple of each pair.

4 Gavin is extending his patio. Currently, his patio is 5 metres by 3 metres.



a If Gavin wants to extend his patio by w metres, write an expression, using brackets, for the area of his new patio in m^2.

b Expand your answer to part **a**.

c If Gavin can only extend his patio to a total breadth of 8 m, what is the maximum area of his extension?

d Gavin wants to check the total area of the patio if he extends the patio by 1 m, 2 m, 3 m and 4 m. Calculate the area of the patio for these extension lengths.

Gavin's partner wants to plant flowers along the two shorter sides and one of the longer sides of the patio.

e Write an expression for the length of the flowerbed that will surround three sides of the patio.

f Gavin digs a trench with a breadth of x metres for the flowerbed. When Gavin's partner sees the flowerbed, he asks Gavin to double the breadth of the flowerbed. Use brackets to write an expression for the final area of the flowerbed in m^2.

g Expand your answer to part **f**.

h If Gavin decides to extend the patio by 3 metres, and the flowerbed has a breadth of 0.5 metres, what will the total area of the flowerbed be?

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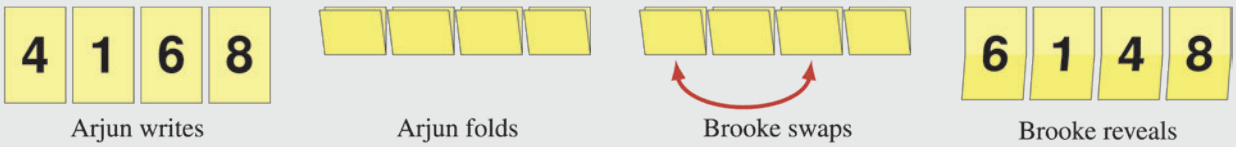
EXPLORATIONS 1

1 Note swap

Arjun and Brooke make up a game called 'note swap'. Arjun takes a number of sticky notes and places them in a line on the desk. While Brooke is not looking, he writes a single digit on each note to form a number, read left to right, that is divisible by the number of notes used. Arjun then folds each note in half so Brooke cannot see the digits. Brooke's job is to swap two sticky notes of her choosing and then reveal all the digits to form her own number, again reading left to right. If the new number is also divisible by the number of notes used, Brooke wins. If not, Arjun wins.



a Arjun uses 3 sticky notes. Explain why Brooke will always win, no matter how she swaps the notes.

b Arjun uses 4 sticky notes. What strategy should Brooke use to guarantee she wins?

c Arjun uses 7 sticky notes. Explain why swapping the first and last digit is a winning strategy for Brooke.

d From 2 to 12 inclusive, there is only one number of notes which allows Arjun to come up with a winning strategy. What is the number and what is his winning strategy?

e Show that Brooke has a winning strategy for all remaining cases from 2 to 12 inclusive.

f Do either Arjun or Brooke have a winning strategy for 14 notes?

2 Recurring themes

a By long or short division, show that $\frac{1}{7}$ as a decimal is $0.\overline{142857}$.

b Because every possible remainder from 1 to 6 occurs in the above calculation, it follows that the same pattern of recurring digits must also occur in the decimal forms of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ and $\frac{6}{7}$. Use this fact to write each of these fractions as a decimal, without doing any division.

c Convert $\frac{1}{14}$ to an equivalent (but unsimplified) fraction with a multiple of 10 in the denominator. Use this and part **b** to help you write $\frac{1}{14}$ as a decimal.

d What is $\frac{3}{14} - \frac{1}{5}$ as a simplified fraction? Use this to help you write 3 as a decimal.

In part **d**, it made sense to subtract $\frac{1}{3}$ for two reasons: it is close to but slightly less than $\frac{3}{14}$, and its own decimal form terminates because 5 is a factor of 10.

e Adapt the ideas above to write $\frac{5}{14}, \frac{9}{14}, \frac{11}{14}$ and $\frac{13}{14}$ as decimals.

f Find a clever way to work out $\frac{2}{63}$ as a decimal without doing the division.

3 Parcel packers

In the AMT warehouse, it takes 8 professional packers 15 hours to pack the parcels of competition papers to send to schools.

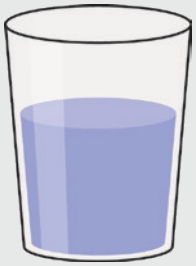
a Assuming that all professional packers work at the same constant rate and do not get in each others' way, how long would it take 12 professionals to pack all the parcels?

b The 12 professionals are joined by 8 trainee packers who can each work at only half the rate of a professional. How long would it take the professionals and trainees to pack all the parcels?

c Some professionals and 6 trainees started packing the parcels and would have taken 12 hours to complete the job. However, after $2\frac{1}{2}$ hours two professionals and all trainees had to leave. How many professionals were there? How much longer after the 8 packers left would the remaining packers take to finish the job?

d Suppose x professionals and y trainees require t hours to complete a job, but after T hours only X professionals and Y trainees continue. Find a formula for the number of hours needed by the remaining packers to finish the job.

4 Sharesies



Seven girls sit around a circular table. Each has a glass and some of the glasses contain water. Each glass can hold up to 720 mL. In turn, going clockwise around the table, each girl divides the water in her glass into six equal shares and adds these to the glasses of each of the other girls. No water is ever spilt.

a On one occasion, the first girl begins with a full glass. When all seven girls have completed their turns, the first girl ends up with a full glass again. How much water did the third girl have at the beginning?

b On another occasion, the total amount of water is 2.1 L. When all seven girls have completed their turns, each of them ends up with the same amount of water as they had originally. How much water did each girl have at the beginning?

c What is the answer to part **b** if each girl divides the water in her glass into seven equal parts and keeps one of the parts for herself?

5 Powerful stuff

a Without using a calculator, write the number 4096 as a:

i power of 2 **ii** power of 4 **iii** perfect square **iv** perfect cube.

b Explain why the number 1 has all of the properties in part **a**: it is a power of 2, a power of 4, a perfect square and a perfect cube. What is the next smallest number with all of these properties?

c Write the number $12^3 \times 18^5$ as the product of a perfect square and a perfect cube. Now find a second way to do this.

d Find all powers of 20 that *cannot* be written as the product of a perfect square and a perfect cube. What about powers of 24? Can you generalise your results?

e The sum $256^2 = 32^3 + 8^5$ shows a perfect square that can be written as the sum of a perfect cube and a fifth power. Another example is $3^2 = 2^3 + 1^5$, but we are not interested in the trivial case $0^2 = 0^3 + 0^5$.

i Find a perfect cube that can be written as the sum of a perfect square and a fifth power.

ii Find a fifth power that can be written as the sum of a perfect square and a perfect cube.

f Do some research to find out about *Pythagorean triples* and *Fermat's last theorem*.

i Can a fourth power be written as the sum of two perfect squares?

ii Can a sixth power be written as the sum of two perfect cubes?

6 House points

Maths teacher Mr Stan Deliver has his class practising mental arithmetic. Each student chooses a handful of consecutive numbers, none of which ends in zero. They add up the numbers and then divide the total by the last digit of each number. For each division which results in a whole number answer, the student earns a house point.

For example, Zeba chooses 23, 24 and 25 with total 72. She then calculates $72 \div 3$, $72 \div 4$ and $72 \div 5$. Two of these result in a whole number, so Zeba earns two house points.

a Andy chooses 15, 16 and 17. How many house points does he win? Is his score improved by adding one more number to his list?

b Basma chooses eight consecutive numbers. Explain why she always wins at least 2 house points. What is the smallest possible total of Basma's eight numbers if she only wins 2 house points?

c Cheren chooses six consecutive numbers. Explain why she cannot win more than 2 house points. What is the smallest number of house points Cheren can win?

d Dmitri chooses nine consecutive numbers. Explain why he cannot win more than 5 house points. What is the smallest possible total of Dmitri's numbers if he does win the maximum 5 house points?

e Eli chooses well and wins a house point for every number in his list. What is the largest possible number of consecutive numbers he could have chosen? What is the smallest possible total of Eli's numbers?

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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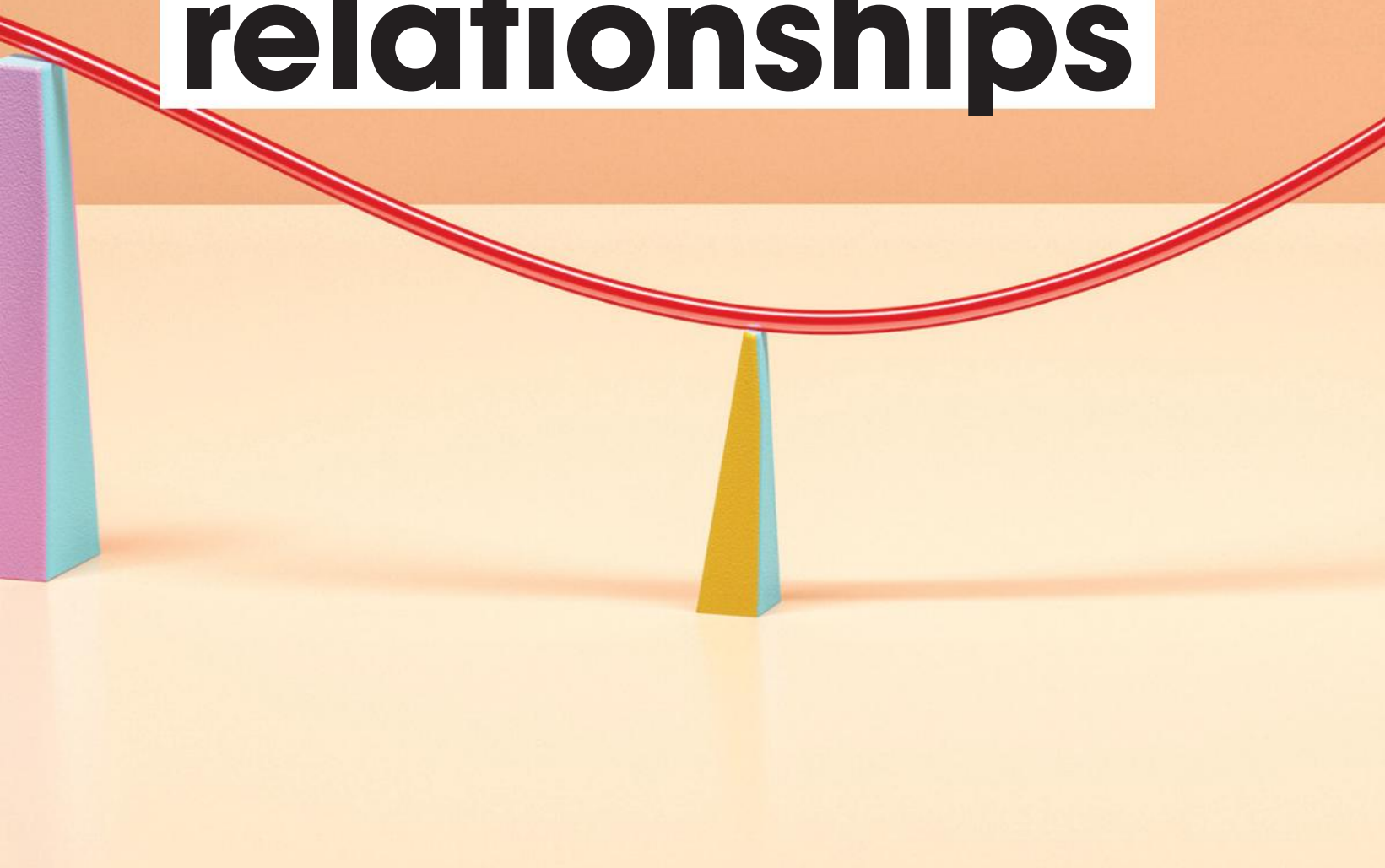
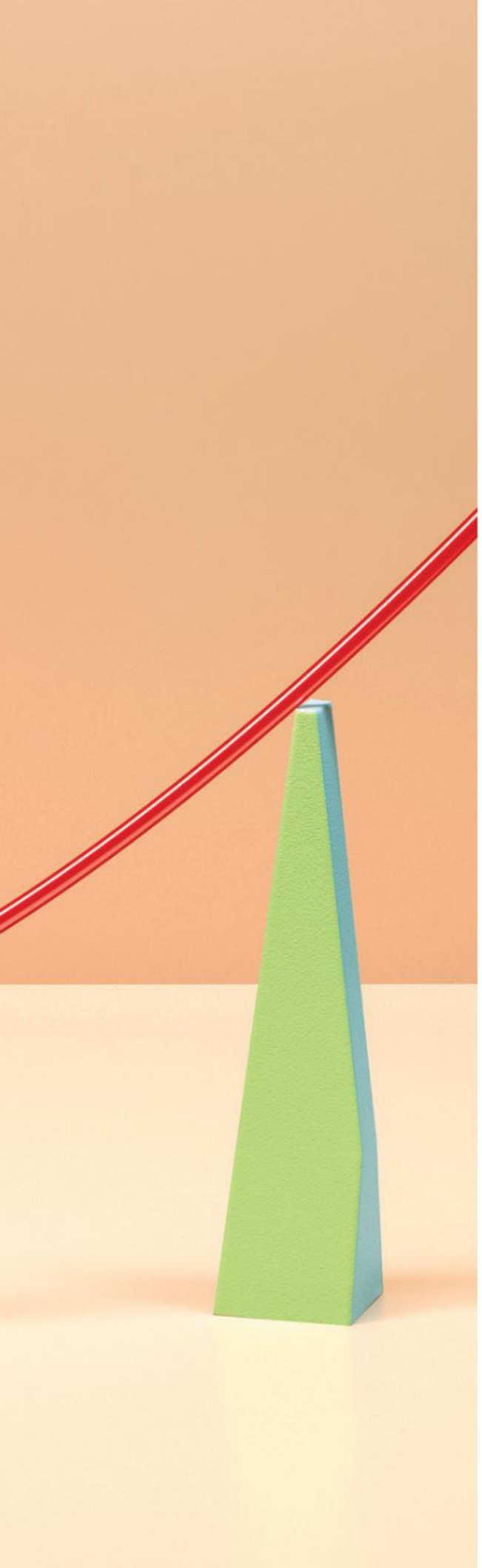
6

Equations and linear relationships





Index

6A Equations
6B Solving linear equations
6C Solving equations with the unknown on both sides
6D Changing the subject of a formula
6E Solving simple quadratic equations
6F Plotting linear relationships
6G Solving linear equations using graphs
6H Intersecting lines
6I Finding linear equations

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Order of operations
- ✔ Substitution
- ✔ Like terms
- ✔ Squares and square roots
- ✔ The Cartesian plane

Curriculum links

- Solves linear equations of up to 2 steps and quadratic equations of the form $ax^2 = c$ (MA4-EQU-C-01)
  - Solve linear equations up to 2 steps
  - Solve and verify linear equations by substitution
  - Solve quadratic equations
- Creates and displays number patterns and finds graphical solutions to problems involving linear relationships (MA4-LIN-C-01)
  - Plot and identify points on the Cartesian plane
  - Plot linear relationships on the Cartesian plane
  - Solve linear equations using graphical techniques

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6A Equations

Learning intentions

By the end of this topic you will be able to...

- ✔ check solutions using substitution
- ✔ solve simple linear equations by inspection
- ✔ write equations to represent worded problems.

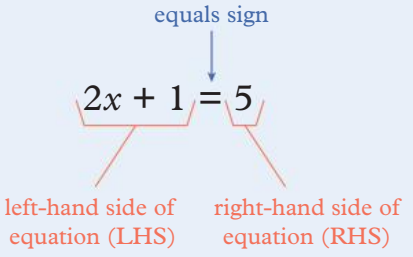
Inter-year links

Year 7	6C Terms, expressions and equations
Year 9	4A Solving linear equations

Equations

Key content video

- An **equation** is a mathematical statement that shows equivalence between the expression on the left-hand side (LHS) of the equation and the right-hand side (RHS) of the equation.
  → If two expressions are *not* equivalent, this is denoted using the 'not equal to' symbol, ≠.



Solving equations

- To find a **solution** to an equation, find a value which, when substituted for the unknown variable or pronumeral, makes the equation a true statement.
- Solutions are usually given with the pronumeral on the left- and the value on the right-hand side of an equals sign, for example $x = 3$.
- To **solve by inspection**, examine the equation and mentally identify the number that when substituted for the unknown pronumeral makes the equation a true statement.
- To check if a value is a solution to an equation, substitute that value into the equation to see whether it makes a true statement.

Example 6A.1 Checking a solution

Determine whether $x = -3$ is the solution to the equation $2x + 5 = 1$.

THINK

1. Substitute $x = -3$ into the expression on the left-hand side of the equation and evaluate. Remember that the product of a positive and a negative number gives a negative number.
2. Determine whether the statement is true or false by comparing the value of the left- and right-hand sides of the equation.

WRITE

$2x + 5 = 1$

$\text{LHS} = 2(-3) + 5$
$= -6 + 5$
$= -1$

$\text{RHS} = 1$

$\text{LHS} \neq \text{RHS}$

$x = -3$ is not the solution to $2x + 5 = 1$.

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Example 6A.2 Solving equations by inspection

Solve each equation by inspection.

a $a + 5 = 12$

b $7c = -35$

THINK	WRITE
a **1** Consider what number, when added to 5, gives 12.	**a** $a + 5 = 12$
2 Check the solution.	Let $a = 7$ LHS $= 7 + 5$ $= 12$ RHS $= 12$ LHS = RHS
3 State the solution to the equation.	$a = 7$
b **1** Consider what number, when multiplied by 7, gives -35. As the coefficient of $7c$ is positive and the number on the right-hand side of the equation is negative, c must be a negative value.	**b** $7c = -35$
2 Check the solution.	Let $c = -5$ LHS $= 7(-5)$ $= -35$ RHS $= -35$ LHS = RHS
3 State the solution to the equation.	$c = -5$

Example 6A.3 Writing equations from number sentences

Write an equation to represent each of these number sentences. Use n for the unknown number in each instance.

a 3 more than a number is equal to 8.

b 5 less than a number is equal to -2.

c Twice a number is equal to 14.

THINK	WRITE
a Read through the number sentence from left to right. The left-hand side of the equation is '3 more than a number' or $n + 3$. The right-hand side of the equation is 8.	**a** $n + 3 = 8$
b Read through the number sentence from left to right. The left-hand side of the equation is '5 less than a number' or $n - 5$. The right-hand side of the equation is -2.	**b** $n - 5 = -2$
c Read through the number sentence from left to right. The left-hand side of the equation is 'twice a number' or $2n$. The right-hand side of the equation is 14.	**c** $2n = 14$

Example 6A.4 Writing equations to represent worded problems

A rose bush grows three times as many roses as last year. Erin cuts 5 roses to fill a vase.

a If r roses grew on the bush last year, write an expression for the number of roses on the bush after Erin fills the vase.

b After Erin fills the vase, there are 13 roses left on the bush. Write an equation that can be used to determine how many roses grew on the bush last year.

c Solve the equation by inspection.

d How many roses grew on the bush last year?

THINK	WRITE
a Think about which operations are taking place. 'Three times' means multiply by 3. If Erin removes 5 roses to fill the vase, there will be 5 fewer roses on the bush.	**a** $3r - 5$
b If there are '13 roses left on the bush', the number of roses is equal to 13.	**b** $3r - 5 = 13$
c Consider what number makes the equation a true statement. Check the solution.	**c** Let $r = 6$ LHS $= 3r - 5$ $= 3(6) - 5$ $= 18 - 5$ $= 13$ RHS $= 13$ LHS = RHS $r = 6$
d Interpret the solution. As r = the number of roses that grew on the bush last year, 6 roses grew on the bush last year.	**d** 6 roses grew on the bush last year.

Helpful hints

- ✔ Remember to define your pronumerals before writing equations to represent a worded problem.
- ✔ If you have difficulty in solving equations by inspection, try using either the 'guess, check and improve' or 'balance model' methods from Year 7.

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Exercise 6A Equations

△ 1–4, 5(1st, 2nd columns), 6, 8, 10–14, 16, 17, 19 ▢ 4, 5(3rd, 4th columns), 7, 9–13, 15, 18, 19, 21, 23 ⬠ 4, 5(3rd, 4th columns), 11–13, 17, 20–24

1 **a** State whether each of these is an equation or an expression.

i $5x + 4$ **ii** $8k = 24$ **iii** $14 - 3p = 5$

iv $\frac{3k - 1}{2}$ **v** $2 = 9 - x$ **vi** $4(7d + 3) = 8$

b Explain how you can tell the difference between an equation and an expression.

2 Substitute $x = 6$ into each equation. Does this value of x make the equation a true statement?

a $x + 3 = 9$ **b** $x - 5 = -1$ **c** $2x = 8$
d $\frac{x}{3} = 2$ **e** $4x + 5 = 19$ **f** $3x - 7 = 11$

3 Substitute $y = -9$ into each equation. Does this value of y make the equation a true statement?

a $y - 1 = -8$ **b** $3y = 27$ **c** $\frac{18}{y} = -2$
d $-5y = 45$ **e** $4y + 1 = -35$ **f** $2y + 6 = 24$

6A.1 **4** Determine if the value given in brackets for each pronumeral is the solution to the equation.

a $a - 5 = 6$ $(a = 11)$ **b** $4k = -16$ $(k = -4)$ **c** $m + 3 = 11$ $(m = 7)$ **d** $\frac{h}{5} = 2$ $(h = -10)$
e $6p - 4 = 2$ $(p = 1)$ **f** $3n + 7 = -1$ $(n = -2)$ **g** $\frac{b+2}{3} = 2$ $(b = 7)$ **h** $15 - f = 16$ $(f = -1)$

6A.2 **5** Solve each equation by inspection.

a $a + 3 = 10$ **b** $b - 2 = 4$ **c** $3c = 24$ **d** $\frac{d}{4} = 3$
e $-4e = -8$ **f** $\frac{f}{3} = -1$ **g** $g - 5 = -4$ **h** $h + 6 = 6$
i $9 - i = 12$ **j** $8j = -40$ **k** $4 + k = 11$ **l** $l - 15 = 20$
m $\frac{24}{m} = 6$ **n** $-10n = 100$ **o** $\frac{x}{2} = -7$ **p** $p - 4 = -4$
q $8 = q + 2$ **r** $48 = -4r$ **s** $9 = 13 + s$ **t** $5 = \frac{t}{5}$

6 Which value of x makes $2x + 5 = 13$ a true statement?

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$ **E** $x = 5$

7 Which value of y makes $3y - 1 = -7$ a true statement?

A $y = -7$ **B** $y = 4$ **C** $y = -4$ **D** $y = 2$ **E** $y = -2$

8 Which value of m makes $\frac{m+3}{8} = 1$ a true statement?

A $m = 5$ **B** $m = 6$ **C** $m = 7$ **D** $m = 8$ **E** $m = 9$

9 Which value of k makes $\frac{k}{3} - 1 = 0$ a true statement?

A $k = 1$ **B** $k = 2$ **C** $k = 3$ **D** $k = 4$ **E** $k = 5$

10 What is the value of the unknown that makes each equation a true statement?

a $3a + 9 = 15$ **b** $5b - 3 = 2$ **c** $7c - 2 = 19$ **d** $2d + 10 = 10$
e $\frac{e+4}{5} = 2$ **f** $\frac{f-1}{3} = 3$ **g** $\frac{g}{2} + 6 = 8$ **h** $\frac{h}{7} - 1 = -2$

11 Solve each equation by inspection.

a $4x + 1 = 21$ **b** $3x - 2 = 10$ **c** $7x + 1 = -6$ **d** $2x - 5 = 13$
e $\frac{x}{5} - 6 = -8$ **f** $\frac{x}{3} + 11 = 11$ **g** $\frac{x+9}{2} = 3$ **h** $\frac{x-4}{4} = -2$

6A.3 **12 a** Write an equation to represent each of these number sentences. Use n for the unknown number.

i 4 added to a number gives 11.
ii 3 subtracted from a number gives −2.
iii 15 more than a number is equal to 21.
iv 8 less than a number is equal to −4.
v Twice a number is equal to 14.
vi One-quarter of a number is 9.
vii The sum of a number and −7 is equal to −20.
viii The product of a number and 11 is equal to 22.
ix When a number is multiplied by 5, the result is 55.
x When a number is divided by 6, the result is −3.

b Solve each equation by inspection.

13 **a** Write an equation to represent each of these number sentences. Use x for the unknown number.

i A number is multiplied by 3, then 8 is added to give 11.

ii A number is divided by 6, then 2 is subtracted to give 1.

iii The sum of a number and 7 all divided by 2 is equal to 5.

iv 9 less than a number all times 3 gives 3.

b Solve each equation.

PROBLEM SOLVING AND REASONING

6A.4 **14** Evie and her older brother Harry earn pocket money weekly. Harry receives $38 each week.

a Write an expression for the total amount of money they receive each week if Evie receives x dollars per week.

b Write an equation that can be used to find how much Evie receives each week if they receive a total of $62 per week.

c Solve the equation by inspection.

d How much pocket money does Evie receive each week?

15 Stella has 26 rolls of washi tape. A delivery containing more rolls of washi tape has just arrived.

a Write an expression for the total number of rolls of washi tape Stella will have if there are x rolls in the delivery.

b Write an equation that can be used to find how many rolls of washi tape are in the delivery if Stella has a total of 81 rolls of tape after the delivery.

c Solve the equation by inspection.

d How many rolls of washi tape were in the delivery?

16 Theo has invited some friends over for dinner. He has bought enough dumplings so that each person will have eight.

a If x represents the number of people at Theo's dinner, write an equation that can be used to find the number of people if Theo bought a total of 96 dumplings.

b Solve the equation by inspection.

c How many people were at the dinner at Theo's house?

17 A bag of macadamia nuts is divided amongst four students.

a Define a pronumeral to represent the variable in this problem.

b Write an expression for the number of macadamia nuts each student receives.

c Write an equation for finding the number of macadamia nuts in the bag if each person receives nine macadamia nuts.

d Solve the equation by inspection to find the value of the pronumeral.

e How many macadamia nuts are in the bag?

18 The selection of truffles shown to the right costs a total of $24.

a Assuming each truffle costs the same amount of money, write an equation that can be used to determine the cost of each truffle.

b Solve the equation by inspection to calculate the cost of one truffle.

19 Nakkiah takes half of the fruit in a fruit bowl to basketball training, then her dad puts 3 new pieces of fruit into the bowl.

a If there are now 7 pieces of fruit in the bowl, write an equation to represent this problem.

b Solve the equation to calculate how many pieces of fruit were originally in the fruit bowl.

20 The number of cars in a car park changes during the day. At 11 am, there are twice as many cars as there were at 9 am. At 4 pm, there are 7 more cars in the car park than at 11 am.

a If there are 11 cars in the car park at 4 pm, write an equation that can be used to calculate how many cars were in the car park at 9 am.

b Use the equation to determine how many cars were in the car park at 9 am.

21 Tickets to Taronga Zoo cost \$25 per child and \$40 per adult.

a Write an expression for the cost of c children's tickets.

b Write an expression for the cost of a adult tickets.

c Write an equation for the number of children's and adult tickets you could buy with \$155.

d Poh's family is made up of 2 adults and 3 children. He thinks it will cost \$155 to purchase tickets to Taronga Zoo for his whole family. Use your equation to check if Poh's calculations are correct.

22 Sia starts a fitness routine and records the number of minutes she spends exercising each week. In the third week, Sia exercises 30 minutes more than she did in the first week. In the fifth week she exercises double the number of minutes she did in the third week.

a If Sia exercises for 270 minutes in the fifth week, write an equation that can be used to calculate how long she exercised in the first week.

b Use the equation to find how long Sia exercised in the first week.

23 Arvan records the amount of time he spends practising the guitar. Unfortunately, he spills some water on his record and some of the information is lost.

Day	Time spent practising	Comments
Monday		Good start!
Tuesday		Same as Monday
Wednesday		Double Monday!
Thursday		20 minutes less than yesterday
Friday		Same as Monday
Total	190 minutes	

Arvan thinks he can write an equation using the information in his table to find the missing amounts. Arvan's equation is $x + x + x + x + x = 190$.

a Explain why Arvan's equation is incorrect.

b Write a new equation that will find the number of minutes Arvan practises on Monday.

c Fill in the table for each day.

d Arvan's equation does calculate a certain value that could be useful for him. What is it?

24 Lina visited the Milton Market one Sunday. At one stall she bought a handbag. At another stall she bought 3 t-shirts. At a food van she bought 3 donuts for \$10. Lina went to the market with \$100, came home with no money, and the handbag cost double the amount of all 3 t-shirts.

a Write an equation to solve the cost of the handbag.

b Solve the equation by inspection.

c Check your answer by substitution.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Writing formulas

Interactive skillsheet
Solving equations by inspection

Investigation
Circus antics

Topic quiz
6A

6B Solving linear equations

Learning intentions

By the end of this topic you will be able to ...

✔ solve linear equations using inverse operations.

Inter-year links

Year 7	6H Solving equations using inverse operations
Year 9	4A Solving linear equations
Year 10	4A Solving linear equations

Key content video

Equivalent equations

- **Equivalent equations** are equations that have the same solution.
- When the same operation is performed on both sides of an equation, the equality between the two sides of the equation is preserved.

Inverse operations

- **Inverse operations**, or opposite operations, reverse the effect of another operation.
- Inverse operations can be used to manipulate the expressions on the left- and right-hand side of an equation until we obtain an equation with the pronumeral by itself on one side of the equation, and therefore determine the solution to the equation.

  Each manipulation of the expressions produces an equivalent equation, with the simplest equivalent equation revealing the solution.

Operation	Inverse operation
$+ 3$	$- 3$
$- 3$	$+ 3$
$\times 3$	$\div 3$
$\div 3$	$\times 3$

- To solve equations using inverse operations:
  1. Identify the operation(s) acting on the pronumeral.
  2. Identify the inverse operation(s) required to reverse the operation(s) and get x by itself on one side of the equation.
  3. Apply the inverse operations to both sides of the equation or 'do the same to both sides' to maintain equivalence.

equivalent equations

$$2x + 1 = 5$$
$$2x + 1 - 1 = 5 - 1$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$x = 2$$

inverse operations

solution

- For equations involving more than one operation, inverse operations must be performed in the *reverse order* to BIDMAS.
  → For example, in the equation $2(x + 5) = 17$ the first inverse operation to perform would be that required to reverse the multiplication ($\div 2$), and the second would be that required to reverse the addition within the brackets ($- 5$).

BIDMAS

Example 6B.1 Solving one-step equations using inverse operations

Solve the following equations using inverse operations.

a $x + 3 = 11$

b $\frac{x}{5} = -8$

c $-4x = 16$

d $x - 12 = 7$

THINK	WRITE
a 1 Identify the operation acting on the pronumeral. In this case, 3 is being added to the x term.	**a** The operation acting on x is $+ 3$.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, subtract 3 from both sides of the equation.	The inverse operation to $+ 3$ is $- 3$. $x + 3 = 11$ $x + 3 - 3 = 11 - 3$
3 Simplify both sides of the equation by completing the inverse operation.	
4 Write the solution to the equation.	$x = 8$
b 1 Identify the operation acting on the pronumeral. In this case, the x term is being divided by 5.	**b** The operation acting on x is $\div 5$.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, multiply both sides of the equation by 5.	The inverse operation to $\div 5$ is $\times 5$. $\frac{x}{5} = -8$ $\frac{x}{5} \times 5 = -8 \times 5$
3 Simplify both sides of the equation by completing the inverse operation.	$\frac{\cancel{5}x}{\cancel{5}} = -40$
4 Write the solution to the equation.	$x = -40$
c 1 Identify the operation acting on the pronumerals. In this case, the x term is being multiplied by -4.	**c** The operation acting on x is $\times -4$.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, divide both sides of the equation by -4.	The inverse operation to $\times -4$ is $\div -4$. $-4x = 16$
3 Simplify both sides of the equation by completing the inverse operation.	$\frac{\cancel{-4}^{1}x}{\cancel{-4}^{1}} = \frac{\cancel{16}^{-4}}{\cancel{-4}^{1}}$
4 Write the solution to the equation.	$x = -4$
d 1 Identify the operation acting on the pronumerals. In this case, 12 is subtracted from the x term.	**d** The operation acting on x is -12.
2 Identify and apply the inverse operation to *both sides* of the equation. In this case, add 12 to both sides of the equation.	The inverse operation to -12 is $+12$. $x - 12 = 7$ $x - 12 + 12 = 7 + 12$
3 Simplify both sides of the operation by completing the inverse operation.	
4 Write the solution to the equation.	$x = 19$

Example 6B.2 Solving two-step equations using inverse operations

Use inverse operations to solve the following equations.

a $10 - 4x = 26$

b $\frac{x+3}{7} = 6$

THINK

a **1** Apply the inverse operations to both sides of the equation in the reverse order to BIDMAS. First, identify any addition/subtraction acting on the x term. In this case, reordering the terms so that the term containing the variable appears first clarifies that 10 is being added to the x term. Apply the inverse operation to both sides of the equation.

2 Second, identify any multiplication/division acting on x and apply the inverse operation to both sides of the equation to obtain the solution. Remember that dividing a positive number by a negative number gives a negative number.

b **1** Apply the inverse operations to both sides of the equation in the reverse order to BIDMAS. Note that the '+ 3' is contained within a fraction and the fraction line acts like a bracket. The '+ 3' is therefore considered a bracketed operation, not an addition/subtraction. As there is no addition/subtraction, begin by identifying any multiplication/division and apply the inverse operation.

2 The '+ 3' is no longer contained within a fraction, so apply the inverse operation to obtain the solution.

WRITE

a

$$10 - 4x = 26$$
$$-4x + 10 = 26$$
$$-4x + 10 - 10 = 26 - 10$$
$$-4x = 16$$
$$\frac{\cancel{-4}^{1}x}{\cancel{-4}^{1}} = \frac{\cancel{16}^{-4}}{\cancel{-4}^{1}}$$
$$x = -4$$

b

$$\frac{x+3}{7} = 6$$
$$\frac{x+3}{7} \times 7 = 6 \times 7$$
$$\frac{\cancel{7}^{1}(x+3)}{\cancel{7}^{1}} = 42$$
$$x + 3 = 42$$
$$x + 3 - 3 = 42 - 3$$
$$x = 39$$

Helpful hints

✔ Take care when solving equations containing grouping symbols. Inverse operations must be performed in the reverse order to BIDMAS, meaning operations contained within grouping symbols are often addressed towards the end of the calculations.

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Exercise 6B Solving linear equations

△ 1–11, 12(a–c), 13, 17(a–c) □ 4–9, 11, 12(c, e, f), 13(c, d), 14, 15, 17(c, e, g, h), 19, 20 ⬠ 5–9, 12, 13(c, d), 16–20

1 Identify the inverse operation for each of the operations listed below.

a $+ 6$ **b** $\times 4$ **c** $- 7$ **d** $\div 3$

e $\times (-2)$ **f** $\div (-8)$ **g** $- 11$ **h** $\times (-20)$

2 Apply the inverse operation given in the brackets to both sides of the equation to solve each equation.

a $x + 6 = 19$ $(- 6)$ **b** $x - 13 = 29$ $(+ 13)$ **c** $11x = 132$ $(\div 11)$ **d** $\frac{x}{3} = 9$ $(\times 3)$

e $-5x = -5$ $(\div -5)$ **f** $x - 1 = -2$ $(+ 1)$ **g** $\frac{x}{5} = -5$ $(\times 5)$ **h** $-12x = 36$ $(\div -12)$

3 Identify the inverse operation required to solve each of these equations.

a $x - 6 = 22$ **b** $x + 9 = 11$ **c** $3x = 21$ **d** $\frac{x}{4} = 6$

e $x - 2 = 9$ **f** $\frac{x}{5} = -9$ **g** $7x = -49$ **h** $x + 1 = 12$

4 Identify the two inverse operations required (in order) to solve each of these equations.

a $3x + 3 = 24$ **b** $2x - 8 = 16$ **c** $\frac{x}{2} + 5 = 11$ **d** $5(x + 5) = 35$

e $5x - 11 = -36$ **f** $\frac{2x}{5} = 8$ **g** $9(x + 4) = -63$ **h** $\frac{x + 4}{3} = 10$

6B.1 **5** Solve the following equations using inverse operations.

a $x + 4 = 16$ **b** $x - 11 = 3$ **c** $6x = 42$ **d** $\frac{x}{2} = 5$

e $x + 5 = 5$ **f** $\frac{x}{11} = -2$ **g** $9x = -45$ **h** $x + 2 = -1$

i $-2x = 4$ **j** $-5x = 0$ **k** $x - 7 = -10$ **l** $-x = 17$

6 Solve the following questions.

a $x + 4.5 = 7$ **b** $x - 0.6 = -5.6$ **c** $1.2x = -6$ **d** $-1.4x = 2.8$

6B.2 **7** Use inverse operations to solve the following equations.

a $2x + 7 = 23$ **b** $9x - 4 = 5$ **c** $\frac{x}{6} + 3 = 7$ **d** $8(x - 2) = 32$

e $\frac{x + 2}{5} = 4$ **f** $\frac{x}{3} - 9 = 2$ **g** $\frac{x - 8}{2} = 6$ **h** $3(x + 7) = 27$

i $7x - 4 = 45$ **j** $\frac{x + 9}{5} = 3$ **k** $10x + 5 = 15$ **l** $2(x - 6) = 14$

8 Solve the following equations for x.

a $-5x + 8 = 3$ **b** $-4x - 3 = 5$ **c** $11 + 2x = 19$ **d** $7 - 3x = 1$

e $-2(x - 6) = -18$ **f** $-8x + 1 = -15$ **g** $-x + 4 = 13$ **h** $6 - x = -4$

i $-x - 5 = 17$ **j** $-3(x + 8) = 21$ **k** $-(x + 3) = -2$ **l** $-(x - 7) = 1$

9 The solution to a given equation will not always be an integer. Leave your answers as fractions.

a $5x = 1$ **b** $6x = 27$ **c** $3x - 5 = 12$

d $-5x + 7 = 3$ **e** $\frac{x - 2}{3} = \frac{4}{5}$ **f** $-\frac{2}{3}(x + 4) = 5$

10 Caleb decided that a quick way to find the solution to $x - 37 = 85$ is to subtract 37 from both sides of the equation. Is he correct? Explain.

11 Yuhan and Connor take different approaches to solving $\frac{2x}{3} = 4$ for x. Their approaches are shown below.

Yuhan

$$\frac{2x}{3} = 4$$
$$\frac{2x}{3} \times 3 = 4 \times 3$$
$$2x = 12$$
$$\frac{2x}{2} = \frac{12}{2}$$
$$x = 6$$

Connor

$$\frac{2x}{3} = 4$$
$$\frac{2}{3}x = 4$$
$$x = 4 \div \frac{2}{3}$$
$$= 4 \times \frac{3}{2}$$
$$= \frac{12}{2}$$
$$= 6$$

a Check each student's working. Has each student solved the equation correctly?

b Describe one advantage and one disadvantage of each approach.

12 Solve the following questions:

a $\frac{4x}{7} = 8$ **b** $\frac{8x}{3} = -16$ **c** $-\frac{3}{5}x = 6$

d $\frac{2}{3}x = \frac{1}{2}$ **e** $\frac{2x}{9} = 0.4$ **f** $\frac{8x}{9} = \frac{5}{3}$

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

13 **i** Write an equation to represent each problem.

ii Solve the equation using inverse operations to answer each question.

a The temperature rises by 4°C in the first hour after sunrise and then doubles by midday. If it is 26°C at midday, what was the temperature at sunrise?

b Hiring a party venue costs \$280. There is a \$15 per person charge for finger food. If Marita has a budget of \$1600, how many people can attend a party at this venue?

c Ryan is saving to buy a sound mixing board that costs \$649. He saves \$56 per month. If he currently has \$257, in how many months will he be able to buy the mixing board?

d Reese cooks some donuts. While they are still hot, his friends eat six of them. Half of the donuts that are left are iced and the other half are filled with jam. If there are 11 donuts filled with jam, how many donuts did Reese make?

14 By the end of Tuesday Jin is up to page 256 in her book. On Monday Jin read $\frac{1}{3}$ the number of pages she read on Tuesday. If Jin started Monday on page 216, how many pages did she read on Tuesday? Write an equation and solve it using inverse operations.

15 Henry plants x seedlings each week, but $\frac{1}{4}$ of the seedlings he plants die. At the end of 5 weeks Henry has 75 seedlings. Write an equation and solve it to find x.

16 Jenny's mum awards her 10 points for doing a job and deducts 5 points if she forgets to do a job. At the end of one week, Jenny has forgotten to do $1\frac{1}{2}$ as many jobs as she has remembered to do.

a Write an expression to represent the total number of points Jenny will have at the end of the week.

b Explain why you do not have enough information to determine the number of jobs that Jenny completed that week.

c Explain why Jenny will not end up with negative points.

d If Jenny ends up with 20 points, find how many jobs she was expected to do that week.

17 Solve each equation.

a $4(x + 2) - 5 = 15$ **b** $\frac{2x - 6}{5} = 4$ **c** $\frac{2x}{3} + 8 = 14$ **d** $\frac{5x + 1}{6} = -4$

e $\frac{6 - x}{5} = 2$ **f** $-\frac{x}{4} + 9 = 7$ **g** $9(x - 4) + 3 = 57$ **h** $\frac{4x}{7} - 3 = 5$

CHALLENGE

18 **i** Identify the inverse operations (in order) required to solve these equations.

ii Find the positive solutions to the equations.

a $3x^2 + 5 = 32$ **b** $4(x^2 - 2) = 56$ **c** $\sqrt{2x + 5} = 5$

19 Sophie is $\frac{1}{4}$ the age she will be in 36 years' time. How old is she now?

20 Giovanni solved the following equation and got $b = 12$ as his solution: $\frac{b + 12}{4} = 15$.

a Prove by substitution that the solution is wrong. **b** Identify the mistake. **c** What is the correct solution?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Solving equations using inverse operations

Investigation
Household bills

Topic quiz
6B

6C Solving equations with the unknown on both sides

Learning intentions

By the end of this topic you will be able to ...

✔ solve linear equations with the unknown on both sides.

Inter-year links

Year 7	6H Solving equations using inverse operations
Year 9	4A Solving linear equations
Year 10	4A Solving linear equations

Solving equations with the unknown on both sides

Key content video

- As a pronumeral is just a letter standing in for a number, inverse operations can also be applied to terms containing a pronumeral.
- To solve an equation in which the unknown appears on both sides of the equation:
  1. Use inverse operations to remove the pronumeral term from one side of the equation.
  2. Solve the equation using inverse operations.
     For example, to remove $3x$ from the right-hand side of the equation to the right, subtract $3x$ from both sides of the equation.

$$\begin{aligned} 4x - 2 &= 3x + 1 \\ 4x - 3x - 2 &= 3x - 3x + 1 \\ x - 2 &= 1 \\ x - 2 + 2 &= 1 + 2 \\ x &= 3 \end{aligned}$$

- When using inverse operations, it is important to show your working. This can be done by explicitly applying the inverse operations to both sides or, alternatively, by writing each inverse operation in brackets to the right of each equivalent equation.

$$\begin{aligned} 4x - 2 &= 3x + 1 \\ x - 2 &= 1 && (-3x) \\ x &= 3 && (+2) \end{aligned}$$

Example 6C.1 Solving equations with the unknown on both sides – subtracting the pronumeral term

Solve each equation using inverse operations.

a $6x - 3 = 5x + 7$

b $8x - 5 = 10x + 3$

THINK

1. Remove the pronumeral term from one side of the equation by subtracting the pronumeral term with the smaller coefficient from both sides of the equation.
2. Solve the equation using inverse operations.

WRITE

a
$$\begin{aligned} 6x - 3 &= 5x + 7 \\ 6x - 5x - 3 &= 5x - 5x + 7 \\ x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x &= 10 \end{aligned}$$

b
$$\begin{aligned} 8x - 5 &= 10x + 3 \\ 8x - 8x - 5 &= 10x - 8x + 3 \\ -5 &= 2x + 3 \\ -5 - 3 &= 2x + 3 - 3 \\ -8 &= 2x \\ \frac{\cancel{-8}^{-4}}{\cancel{2}^{1}} &= \frac{\cancel{2}^{1}x}{\cancel{2}^{1}} \\ -4 &= x \text{ or } x = -4 \end{aligned}$$

Example 6C.2 Solving equations with the unknown on both sides – adding the pronumeral term

Find the value of x in the following equations.

a $3x + 7 = -3x - 23$

b $10 - 2x = 4x - 12$

THINK

1. Remove the negative pronumeral term from one side of the equation by adding the pronumeral term to both sides of the equation.
2. Solve the equation using inverse operations.

WRITE

a

$$\begin{aligned} 3x + 7 &= -3x - 23 \\ 3x + 3x + 7 &= -3x + 3x - 23 \\ 6x + 7 &= -23 \\ 6x + 7 - 7 &= -23 - 7 \\ 6x &= -30 \\ x &= -5 \end{aligned}$$

b

$$\begin{aligned} 10 - 2x &= 4x - 12 \\ 10 - 2x + 2x &= 4x + 2x - 12 \\ 10 &= 6x - 12 \\ 10 + 12 &= 6x - 12 + 12 \\ 22 &= 6x \\ \frac{\cancel{22}^{11}}{\cancel{6}_{3}} &= \frac{\cancel{6}^{1}x}{\cancel{6}_{1}} \\ \frac{11}{3} &= x \text{ or } x = \frac{11}{3} \end{aligned}$$

Helpful hints

- ✔ When solving an equation with the unknown on both sides, first remove the pronumeral with the smaller coefficient to avoid having to work with negative coefficients.
- ✔ To find the solution, the pronumeral does not have to appear on the left-hand side of the equation – as long as the pronumeral is by itself on one side of the equation, you have found the solution!

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Exercise 6C Solving equations with the unknown on both sides

△ 1–3(1st, 2nd columns), 4–8, 10–12, 15 ■ 1–3(3rd, 4th columns), 4, 5, 7, 9, 12, 13, 15, 16(a, d, f, g) ⬠ 2–3(3rd, 4th columns), 4, 7, 10, 12, 14–17

6C.1 **1** Solve each equation using the inverse operations.

a $9x - 2 = 8x + 11$ **b** $6x + 5 = 5x + 9$ **c** $2x + 7 = x + 12$ **d** $7x - 3 = 6x - 1$

e $4x - 9 = 5x - 3$ **f** $4x - 10 = 4 + 3x$ **g** $2x + 18 = 8 + 3x$ **h** $2 + 7x = 8x - 13$

i $3x + 9 = 4x - 7$ **j** $x - 12 = 1 + 2x$ **k** $11x - 5 = 10x + 3$ **l** $19x + 3 = 20x + 3$

2 Solve:

a $5a = 3a + 4$ **b** $4b = 2b + 8$ **c** $2c = 6c + 4$ **d** $3d + 8 = 7d$

e $11e - 2 = 6 + 13e$ **f** $5f + 9 = 7f - 2$ **g** $2g + 6 = 5g - 4$ **h** $10 + 5h = h - 2$

i $7i + 2 = 3i + 12$ **j** $12 + j = 6j - 13$ **k** $8k + 3 = k - 3$ **l** $8l - 9 = 12l + 9$

6C.2 **3** Find the value of x in the following equations.

a $3x - 11 = -2x + 9$ **b** $2x + 1 = -5x - 13$ **c** $8x + 3 = -2x - 7$ **d** $3x - 8 = -6x + 19$

e $5x + 6 = -3x + 21$ **f** $3x - 5 = 7 - 9x$ **g** $-4x + 9 = -x - 9$ **h** $6 - x = x - 4$

i $29 - 2x = 5 - 6x$ **j** $2x + 2 = 30 - 4x$ **k** $43 - x = x + 1$ **l** $-12 - 5x = 24 - 13x$

4 **i** Solve the following equations for x.

ii Check your solutions using substitution.

a $8x - 5 = 5x + 13$ **b** $2x - 5 = 15 - 3x$ **c** $4x - 15 = -4x + 17$ **d** $-2x + 12 = -x - 7$

e $4 + 7x = 5 + 12x$ **f** $-19 - 3x = 13 - x$ **g** $x - 6 = 9x + 10$ **h** $19 - 3x = 15 + 3x$

5 **a** Simplify $2x + 3 + 4x + 1 = 3x + 7 + x - 5$ by adding like terms on each side.

b Solve the simplified equation.

6 Three students produce different steps of working to solve $6x + 11 = 2x - 13$.

Andy

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 8x + 11 &= -13 \\ 8x + 11 - 11 &= -13 - 11 \\ 8x &= -24 \\ \frac{8x}{8} &= \frac{-24}{8} \\ x &= -3 \end{aligned}$$

Kwame

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 &= 2x - 13 - 2x \\ 6x + 11 &= -13 \\ 6x + 11 - 11 &= -13 - 11 \\ 6x &= -24 \\ \frac{6x}{6} &= \frac{-24}{6} \\ x &= -4 \end{aligned}$$

Hayley

$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= 13 \\ 4x + 11 - 11 &= 13 - 11 \\ 4x &= 2 \\ \frac{4x}{4} &= \frac{2}{4} \\ x &= \frac{1}{2} \end{aligned}$$

a Check each student's work. Identify and describe any mistakes.

b Solve the equation correctly. Show your working.

7 Solve each equation.

a $3p + 2p - 5 = p + 7$ **b** $5q + 2 + 3q + 4 = 13 + 2q + 5$

c $6r + 7 - 3r + 3 = r + 4$ **d** $4s + 5 - 2s + 1 = 26 - 3s$

e $6t + 2 + 4t + 9 = 2t + 3 + 3t - 2$ **f** $9 - 5u + 3u - 7 = 5 - 3u + 10$

8 Selena and Hayden have the same amount of money. Selena buys 5 kg of cherries and has \$2 left over. Hayden buys 2 kg of cherries and has \$20 left over.

a If x represents the cost of 1 kg of cherries, which equation best suits this situation?

A $5x + 20 = 2x + 2$ **B** $5x + 2 = 2x + 20$

C $5x - 2 = 2x - 20$ **D** $5x - 20 = 2x - 2$

E $5x - 20 = 2x + 2$

b Solve the equation to find the cost of 1 kg of cherries.

9 The number of biscuits in a box is unknown. However, Kayla can fill her biscuit tin with five more than the contents of three boxes or seven less than the contents of four boxes.

a If n represents the number of biscuits in a box, which equation best suits this situation?

A $3n - 5 = 4n - 7$ **B** $3n - 5 = 4n + 7$

C $3n + 5 = 4n + 7$ **D** $3n + 5 = 4n - 7$

E $5 - 3n = 7 - 4n$

b Solve the equation to find the number of biscuits in a box.

10 At the school canteen, the cost of three apples and five oranges is the same as the cost of seven apples and two oranges. Each orange costs 80 cents. Let a represent the cost of an apple in cents.

a Write an equation to represent this problem.

b Solve the equation to find the cost of an apple.

PROBLEM SOLVING AND REASONING

11 Sean poses this problem to his neighbour: 'In four years, I will be three times as old as my cousin is now. Two years ago, I was twice my cousin's current age. Can you work out how old I am?' If a represents the current age of Sean's cousin:

a One expression for Sean's current age is $3a - 4$. Write another expression for Sean's current age.

b Use the expressions in part **a** to write an equation.

c Solve the equation to determine the current age of Sean's cousin.

d How old is Sean?

12 Jada and Tom have an equal number of marbles. Jada has four small marbles and Tom has ten small marbles. Jada has three times as many large marbles as Tom. Let n represent the number of large marbles Tom has.

a Write an expression for how many marbles Tom has.

b Write an expression for how many marbles Jada has.

c Form an equation using these expressions.

d Solve the equation to find the value of n.

e How many large marbles does Tom have?

f How many large marbles does Jada have?

g If they agree to swap all their marbles, how many marbles will each person swap?

13 In an AFL match, each goal is worth 6 points and each behind is worth 1 point. An AFL match ends in a draw, with the home team scoring twice as many goals as the away team. The home team scored 3 behinds while the away team scored 27 behinds.

a Write an expression for the number of points scored by the away team using g to represent the number of goals scored by the away team.

b Write an expression for the number of points scored by the home team.

c Form an equation using these expressions.

d Solve the equation to find the value of g and hence list the final score of each team.

14 In rugby league, each try is worth 4 points and each penalty goal is worth 2 points. In a rugby league match, the away team scored twice as many points as the home team. The away team scored two penalty goals and three times as many tries as the home team. The home team scored four penalty goals.

a Write an expression for the number of points scored by the home team using t to represent the number of tries scored by the home team.

b Write an expression for the number of points scored by the away team.

c Form an equation using these expressions. Hint: Remember, the away team scored twice as many points.

d Solve the equation to find the value of t.

e List the number of tries scored by each team.

CHALLENGE

15 **a** Expand the expressions on the left- and right-hand sides of the equation $5(x - 2) = 3(x + 8)$.

b Solve the equation for x.

16 Solve each equation for x.

a $2(x - 5) = x + 19$ **b** $7(x + 2) = 3x + 2$ **c** $4(x - 1) = 2 - 2x$

d $3(x + 4) = 2(x + 9)$ **e** $5(x - 7) = 3(x + 5)$ **f** $6(x + 3) - x + 5 = 2(x + 1)$

g $4(2x - 3) = 3(7 - x)$ **h** $7(3x + 5) = 5(2x + 7)$ **i** $11(3x + 2) = 4(2x - 7)$

17 Find the positive solutions to the equations.

a $5(x^2 + 2) = 2(2x^2 + 7)$ **b** $3(x^2 - 4) = 2(x^2 - 6)$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Solving equations with the unknown on both sides

Investigation
Profit and loss

Topic quiz
6C

6D Changing the subject of a formula

Learning intentions

By the end of this topic you will be able to ...

✔ solve for unknown values by first substituting known values

✔ rearrange an equation with only the subject on one side.

Inter-year links

Year 7	6E Substitution
Year 9	4A Solving linear equations
Year 10	4A Solving linear equations

Changing the subject of a formula

Key content video

- Equations often contain several variables. As one or more of the variables change, the other variables must also change to maintain the equality of the relationship. For example, we can write the equation distance travelled = speed travelled × time taken as $d = s \times t$ or $d = st$.
- An unknown value in an equation with several variables can be determined using substitution if values for all other variables are known.
- A pronumeral is the **subject** of an equation if it is by itself on one side of the equation and does not appear on the other side of the equation.
- The subject of an equation is written 'in terms of' the pronumerals on the other side of the equation.
- Inverse operations can be used to rearrange an equation to make any pronumeral the subject.

  For example, $\frac{x}{a} - b = c \quad (+b)$

  $x = b + c \quad (\times a)$

  $x = a(b + c)$.

Example 6D.1 Solving for an unknown with substitution

Determine the value of x in the following equations using the given values of the other pronumerals.

a $y = mx$ where $y = 10$ and $m = 2$

b $y = abx + \frac{c}{d}$ where $y = 10$, $a = 2$, $b = 3$, $c = -32$, and $d = 4$

THINK	WRITE
a 1 Substitute the known values into the equation.	**a** $y = mx$
	$10 = 2x \quad (\div 2)$
2 Solve the equation using inverse operations.	$x = 5$
b 1 Substitute the known values into the equation.	**b** $y = abx + \frac{c}{d}$
	$10 = 2(3)x + \frac{-32}{4}$
2 Simplify the equation by evaluating any calculations.	$10 = 6x - 8 \quad (+8)$
	$18 = 6x \quad (\div 6)$
3 Solve the equation using inverse operations.	$x = 3$

Example 6D.2 Rearranging equations

Rearrange each equation to make x the subject on the left-hand side.

a $y = mx$ **b** $y = mx + c$

THINK

a 1 Identify the correct inverse operation to make x the subject. Divide both sides by m.

2 Rewrite the equation with x on the left-hand side.

b 1 Identify the correct inverse operations to make x the subject. Subtract c from both sides.

2 Divide both sides by m and rewrite the equation with x on the left-hand side.

WRITE

a $y = mx$ $(\div m)$

$\frac{y}{m} = x$

$x = \frac{y}{m}$

b $y = mx + c$ $(-c)$

$y - c = mx$

$x = \frac{y - c}{m}$

Helpful hints

- ✔ Writing the inverse operation beside the appropriate line of working is a great way to keep track of your calculations.

  $\frac{x}{3} - 5 = 7$ (+ 5)
- ✔ Writing the pronumeral on the LHS of your solution is only a convention. Once you have isolated the unknown on one side of the equation, you have the solution. For example, $3 = x$ is the same as $x = 3$.
- ✔ The same steps are used to solve an equation with substitution as are used for rearranging the equation for the same pronumeral. The result is the same if you either substitute known values and then solve for the required unknown or solve and then substitute.

ANS p514

Exercise 6D Changing the subject of a formula

1(a–f), 2–4, 6–10 | 1(d–h), 3–6, 8, 11–13 | 1(g, h), 3–5, 8, 11–13

UNDERSTANDING AND FLUENCY

6D.1 **1** Determine the value of x in the following equations using given values of the other pronumerals.

a $y = mx$ where $y = 12$ and $m = 3$

b $y = mx$ where $y = 3$ and $m = 12$

c $y = a + x$ where $y = 9$ and $a = 4$

d $y = a + x$ where $y = 4$ and $a = 9$

e $y = \frac{k}{x}$ where $y = 8$ and $k = 80$

f $y = \frac{k}{x}$ where $y = 80$ and $k = 8$

g $y = abx + \frac{c}{d}$ where $y = 40$, $a = -2$, $b = 6$, $c = 20$ and $d = 5$

h $y - k = m(x - h)$ where $y = 15$, $k = -3$, $m = 3$ and $h = -5$

2 Identify the pronumeral that is the subject of each of the following equations. If no pronumeral is the subject, write 'no subject'.

a $y = mx$ **b** $ab = c$ **c** $a^2 + b^2 = c^2$ **d** $v = u + at$

e $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ **f** $Ax + By = C$ **g** $y - k = m(x - h)$ **h** $pq + r = p$

6D.2 3 Rearrange each equation to make x the subject on the left-hand side.

a $y = mx$ **b** $y = a + x$ **c** $y = \frac{x}{k}$ **d** $y = \frac{k}{x}$

e $ax + b = c$ **f** $\frac{x}{a} - b = c$ **g** $\frac{x + b}{a} = c$ **h** $\frac{x}{a} + b = c$

4 Rearrange each equation from question **3** to make each pronumeral other than x the subject on the left-hand side. Write a separate equation for each such variable.

5 Consider the equation $x(2t + 1) = t(g - x)$.

a Show that when x is the subject, the equation is $x = \frac{tg}{3t + 1}$.

b Use your equation from part **a** to evaluate x when $g = 10$ and $t = 5$.

c Solve the original equation, $x(2t + 1) = t(g - x)$ for x where $g = 10$ and $t = 5$ and verify that you get the same answer as part **b**.

6 Equations in the form $y = kx$ are very common as they describe a multiplicative relationship between the variables x, y and k. For example: rectangle area = width × length, distance = speed × time, mass = density × volume, momentum = mass × velocity, force = mass × acceleration, voltage = restistance × currentand power = current × voltage.

a Rearrange $y = kx$ to make x the subject.

b Rearrange $y = kx$ to make k the subject.

c Write down four pairs of values for x and y such that $k = 4$.

d Write down four pairs of values for x and k such that $y = 36$.

7 We can extend the multiplicative relationship by introducing another variable. Rearrange each of the equations below to make:

i x the subject **ii** k the subject **iii** m the subject.

a $y = mkx$ **b** $y = \frac{kx}{m}$ **c** $y = kx + m$

8 Temperatures are commonly written in one of three different units:

- degrees Celsius, °C, calibrated originally on a 100-degree scale from water freezing at 0°C to water boiling at 100°C at sea level
- degrees Fahrenheit, °F, calibrated originally on a 180-degree scale from water freezing at 32°F to water boiling at 212°F at sea level
- kelvin, K, calibrated from absolute zero, 0 K, and such that a one kelvin increase in temperature is equivalent to a one-degree Celsius increase in temperature.

With multiple units, we need to be able to convert between the three units. Converting between degrees Celsius and kelvin is the simplest as absolute zero 0 K = −273.15°C, and both units increase at the same rate.

a Write an equation that converts the temperature in degrees Celsius, C, to the temperature in kelvin, K.

b Convert the following temperatures to kelvin.

i 0°C **ii** 100°C **iii** 26.85°C **iv** 180°C

c Rearrange this equation to make C the subject.

d Convert the following temperatures to degrees Celsius.

i 500 K **ii** 1000 K **iii** 20 K **iv** 180 K

PROBLEM SOLVING AND REASONING

9 To convert a temperature in degrees Fahrenheit, F, to the temperature in Celsius, C, we use the equation $C = \frac{5}{9}(F - 32)$.

a Convert the following temperatures to degrees Celsius. Round your answers to two decimal places where necessary.

i 0°F **ii** 100°F **iii** 50°F **iv** 86°F

b Rearrange this equation to make F the subject.

c Convert the following temperatures to degrees Fahrenheit.

i 30°C **ii** −10°C **iii** 20°C **iv** 180°C

10 The equation for converting the temperature in degrees Fahrenheit, F, to the temperature in kelvin, K, is $K = \frac{5}{9}(F - 32) + 273.15$. This can be found by converting from Fahrenheit to Celsius and then converting from Celsius to kelvin.

a Convert the following temperatures to kelvin. Round your answers to two decimal places where necessary.

i 10°F **ii** 150°F **iii** −20°F **iv** 400°F

b Rearrange this equation to make F the subject.

c Convert the following temperatures to degrees Fahrenheit.

i 200 K **ii** 1500 K **iii** 273.15 K **iv** 373.15 K

11 The equation of a line can be written in many ways. Two common forms are $y = mx + c$ and $Ax + By = D$.

a Rearrange $Ax + By = D$ to make y the subject.

b By comparing the coefficients of your answer in part **a** with $y = mx + c$, write two equations: one where m is in terms of A, B and D and another where c is in terms of A, B and D.

c Rearrange your equations from part **b** to make A and D the subjects.

d The form $Ax + By = D$ is more commonly used to avoid writing fractions when y is the subject. Write $y = \frac{p}{q}x + c$ in the form $Ax + By = D$ where A, B and D are integers.

Another form of the equation of a line is $m = \frac{y - k}{x - h}$.

e Rearrange $m = \frac{y - k}{x - h}$ to make y the subject. Fully expand your answer.

12 The equation $E = mgh + \frac{1}{2}mv^2$ gives the total energy of a body as the sum of the potential energy, mgh, and the kinetic energy, $\frac{1}{2}mv^2$.

a Determine the value of h where $E = 1500$, $m = 20$, $g = 10$ and $v = 9$.

b Rearrange the equation to make h the subject. Ensure that any fractions are simplified.

c Determine the value of m where $E = 8000$, $h = 12$, $g = 10$ and $v = 5$. Write your answer correct to two decimal places.

d Rearrange the equation to make m the subject. Ensure that any fractions are simplified.

CHALLENGE

13 Two tanks are being filled with water at different rates, m L/min and n L/min. The two tanks will have the same amount of water in their tanks after t minutes.

a If the first tank starts with a L and the second tank with b L, write an equation in terms of a, b, m, n and t that would give the time taken for the tanks to fill to the same amount.

b Determine the time it would take if the first tank starts will 50 L and fills at 5 L/min and the second tank starts with 35 L and fills at 8 L/min.

c Rearrange the equation from part **a** to make t the subject.

d Verify your answer to part **b** by substituting into your equation from part **c**.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Changing the subject of a formula

Topic quiz
6D

6E Solving simple quadratic equations

Learning intentions

By the end of this topic you will be able to ...

- ✔ explain why there are two solutions to simple quadratic equations
- ✔ solve simple quadratic equations
- ✔ solve simple quadratic equations after substituting known values into a formula.

Inter-year links

Year 7	1G Indices and square roots
Year 9	5A Solving quadratic equations
Year 10	4D Solving quadratic equations

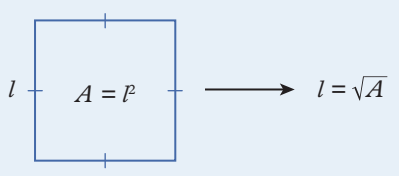
Solving simple quadratic equations

Key content video

- A **quadratic equation** is a single-variable equation in which the highest index of the variable is 2.
- A simple quadratic equation is a single-variable equation in which the only index of the variable is 2.

Quadratic equations		Not quadratic equations
Simple	**Not simple**	
$x^2 = 9$	$3x^2 + 2x + 1 = 8$	$x^3 = 9$
$5y^2 = 4$	$6h^2 = 216h$	$2y^5 = 4$
$25 - b^2 = 0$	$3a - 8a^2 = 0$	$4x^3 + 3x^2 + 2x + 1 = 8$
$9z^2 - 4 = -1$	$x(x - 2) = 6$	$2x - 1 = 5$

- The equation $x^2 = c$ has two solutions when c is greater than 0: a positive solution $x = \sqrt{c}$ and a negative solution $x = -\sqrt{c}$. This is because $(\sqrt{c})^2 = c$ and $(-\sqrt{c})^2 = c$. These solutions can be written as $x = \sqrt{c}$ or $x = -\sqrt{c}$ or together as $x = \pm\sqrt{c}$ (read 'x is equal to positive or negative $\sqrt{c}$').
- There are no solutions to $x^2 = c$ if c is negative because both positive and negative numbers square to give a positive number.
- The solutions to a quadratic equation can be written as the square root of the number if it is an irrational number (this is called exact form) or the decimal expansion can be rounded to a given accuracy to approximate the exact solution.
  - → For example, if $x^2 = 11$, then $x = \pm\sqrt{11} \approx \pm 3.32$.
  - → Fractional square roots can be simplified using the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- Many formulas include squared variables. Given values for any other variables, use substitution to solve for possible values of the squared variable. For example, to determine the side length of a square given the area, substitute the area, A, into the formula $A = l^2$ and solve for l.



  - → It is important to consider whether a negative solution is possible in a particular context.

  For example, the solutions to $A = l^2$ are $\pm\sqrt{A}$, but length is positive so $l = \sqrt{A}$.

Example 6E.1 Solving simple quadratic equations

Solve the following equations for x.

a $x^2 = 25$ **b** $121x^2 = 36$ **c** $x^2 - 0.09 = 0.25$

THINK	WRITE
a 1 Take the square root of both sides of the equation to eliminate the index, write your answer separately or together using ±.	**a** $x^2 = 25$ $x = \pm\sqrt{25}$
2 Evaluate the perfect square root.	$x = \pm 5$
b 1 Use inverse operations to make x^2 on one side of the equation.	**b** $121x^2 = 36$ $x^2 = \frac{36}{121}$
2 Take the square root of both sides of the equation to eliminate the index, write your answer separately or together using ±.	$x = \pm\sqrt{\frac{36}{121}}$
3 Evaluate the square root by taking the square root of the numerator and the denominator separately.	$x = \pm\frac{6}{11}$
c 1 Use inverse operations to make x^2 on one side of the equation.	**c** $x^2 - 0.09 = 0.25$ $x^2 = 0.16$
2 Take the square root of both sides of the equation to eliminate the index, write your answer separately or together using ±.	$x = \pm\sqrt{0.16}$
3 Evaluate the perfect square root.	$x = \pm 0.4$

Example 6E.2 Solving equations with exact and approximate solutions

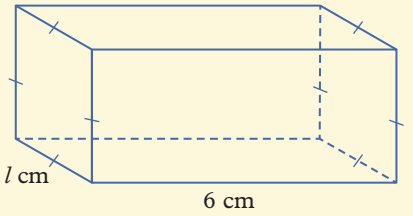
Solve $x^2 = 26$ for x. Write your answers:

a in exact form **b** correct to two decimal places.

THINK	WRITE
a Take the square root of both sides of the equation to eliminate the index. If the square root is irrational, leave the answer in square root form.	**a** $x = \pm\sqrt{26}$
b 1 Use a calculator to find a decimal approximation of the square root.	**b** $\sqrt{26} = 5.0990195\ldots$
2 Round the decimal approximation to two decimal places.	$\sqrt{26} \approx 5.10$
3 Write both the positive and negative approximations.	$x \approx \pm 5.10$

Example 6E.3 Substituting into quadratic equations

Solve for the unknown side length of the following square prism with volume $V = 24\ \text{cm}^3$. Recall that the volume of a rectangular prism is found using the formula $V = l \times b \times h$.



THINK	WRITE
1 Use the formula for the volume of a rectangular prism to find the formula for the volume of a square prism.	$V = l^2 \times h$
2 Identify the known values in the formula.	$V = 24\ \text{cm}^3$ and $h = 6$ cm
3 Substitute the known values into the formula and solve for the squared variable.	$24 = l^2 \times 6$ $24 = 6l^2$ $l^2 = 4$
4 Write down the form of the two possible square root solutions. Simplify if the value under the square root is a perfect square.	$l = \pm\sqrt{4}$ $l = \pm 2$
5 Keep only the positive solution because side lengths can't be negative. Remember to include the appropriate units.	$l = 2$ cm

Helpful hints

- ✔ Don't forget to consider the negative square root solution!
- ✔ Always give your answer in simplified, exact form unless otherwise specified.
- ✔ Remember to check whether a negative solution makes sense.
- ✔ For $x^2 = 0$, $x = 0$ is the only solution as 0 is neither positive nor negative.

ANS p515

Exercise 6E Solving simple quadratic equations

△ 1, 3–7, 9–13, 15 ▢ 2, 3, 6–9, 10–11(1st, 2nd columns), 12–16 ⬠ 2–5(e, f), 6, 8, 10–11(3rd column), 13–16

6E.1 **1** Solve the following equations for the unknown.

a $x^2 = 100$ **b** $y^2 = 9$ **c** $z^2 = 49$
d $a^2 = 81$ **e** $k^2 = 1$ **f** $h^2 = 0$

2 Solve the following equations for the unknown.

a $b^2 = \frac{25}{64}$ **b** $u^2 = \frac{144}{49}$ **c** $d^2 = 1\frac{9}{16}$
d $w^2 = 0.36$ **e** $i^2 = 0.0004$ **f** $r^2 = 1.002001$

UNDERSTANDING AND FLUENCY

3 Solve the following equations for the unknown.

a $x^2 - 5 = 116$ **b** $y^2 + 0.18 = 0.67$ **c** $z^2 - \frac{37}{20} = \frac{257}{80}$

d $5a^2 = 405$ **e** $\frac{b^2}{3} = 27$ **f** $\frac{4}{3}c^2 = \frac{3}{4}$

g $5m^2 + 13 = 333$ **h** $\frac{n^2 - 29}{5} = 23$ **i** $\frac{6}{p^2} - 5 = -4.76$

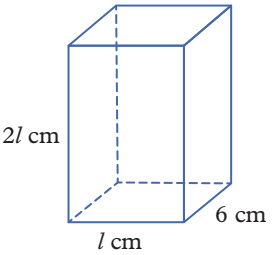
6E.2 **4** Solve the following equations for the unknown. Write your answers in exact form.

a $x^2 = 10$ **b** $y^2 = 91$ **c** $z^2 = 0.4$

d $i^2 = 1.6$ **e** $6j^2 = 13$ **f** $8k^2 - 7 = 49$

5 Write your answers to question **4** correct to two decimal places.

6E.3 **6** Solve for the unknown side lengths of the following rectangular prism with volume $V = 300$ cm³, length l cm and height $2l$ cm. Recall that the volume of a rectangular prism is found using the formula $V = l \times b \times h$.



7 Recall that the formula for the area of a square with side length l is $A = l^2$.

a What is the formula for the total surface area of all six faces of a cube with side lengths l?

b Solve for the unknown side length l if the cube has a total surface area of 216 cm².

8 Substitute the values given into the formula and solve for the unknown. Write your answers correct to two decimal places.

a $v^2 = 2as + u^2$ for v where $a = 9$, $s = 80$ and $u = 5$

b $a^2 + b^2 = c^2$ for c where $a = 10$ and $b = 7$

c $v^2 = 2as + u^2$ for u where $v = 30$, $a = -5$ and $s = 80$

d $K = \frac{1}{2}mv^2$ for v where $K = 100$ and $m = 80$

e $\Delta = b^2 - 4ac$ for b where $\Delta = 25$, $a = 6$ and $c = 3$

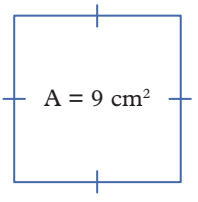
PROBLEM SOLVING AND REASONING

9 **a** The formula $a^2 + b^2 = c^2$ gives the square of the longest side, c, given the lengths of the two perpendicular sides, a and b in a right-angled triangle. How many solutions would there be for the longest side, c? Explain.

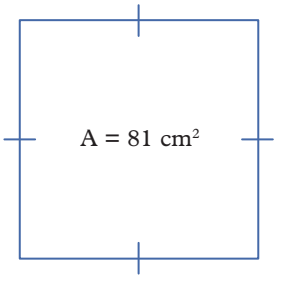
b The formula $A = \pi r^2$ gives the area, A, of a circle with a radius of length r. How many solutions would there be for the radius, r? Explain.

10 Determine the side lengths of the following squares using the area formula $A = l^2$.

a



b



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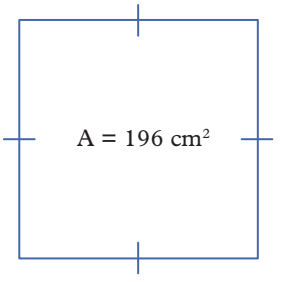

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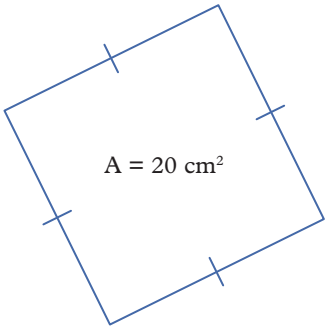

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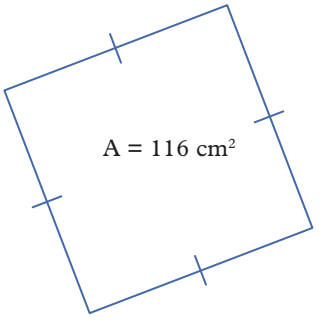

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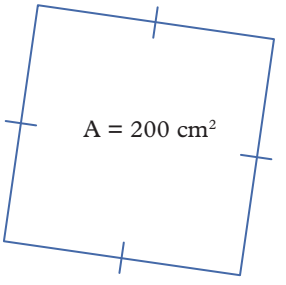
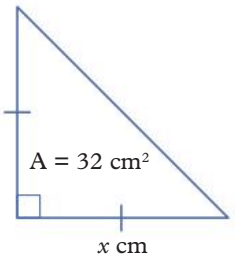


11 Determine the value of the unknown of the following triangles using the area formula $A = \frac{1}{2}bh$.

a



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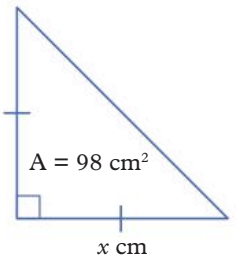

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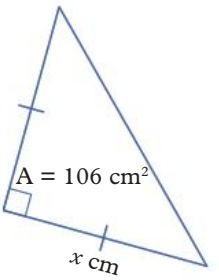

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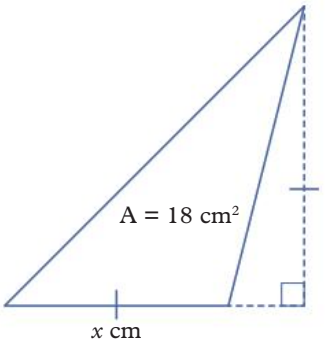

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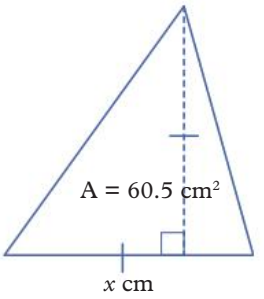

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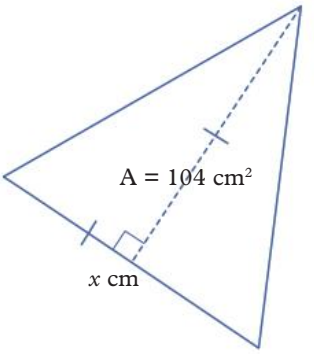


12 Antonina says that $x^2 + 25 = 0$ should have two solutions because 'the 25 is positive'.

a Explain why Antonina is incorrect.

b Explain why there are zero solutions to the equation $x^2 + 25 = 0$.

13 Solve the following equations for the unknown.

a $-8a^2 = -200$ **b** $-7b^2 = -305$ **c** $25 - c^2 = 9$

d $100 - d^2 = 25$ **e** $36 + 9e^2 = 81$ **f** $13 - 8f^2 = -1339$

14 The formula $L = \frac{gT^2}{4\pi^2}$ gives the length of a pendulum, L in metres, given the time, T, it takes for the pendulum to swing in seconds, and the force of gravity, $g = 9.8$ m/s² on Earth.

a Determine value(s) of T of a pendulum with a length of 25 m.

b Explain why you either accepted or rejected the negative solution.

15 A quadratic equation in the form $(x + h)^2 = k$ can be solved for x by first finding the positive and negative roots of k, then subtracting h from both sides. For example,

$(x + 3)^2 = 25$

$x + 3 = \sqrt{25}$ or $x + 3 = -\sqrt{25}$

$x + 3 = 5$ or $x + 3 = -5$

$x = 2$ or $x = -8$

Solve the following equations for x. Write your answers correct to two decimal places where required.

a $(x + 5)^2 = 36$ **b** $(x + 8)^2 = 64$ **c** $(x - 4)^2 = 100$

d $(x - 7)^2 = 49$ **e** $(x - 5)^2 = 20$ **f** $(x + 9)^2 = 90$

16 When solving equations like those in question **15**, we can write the exact form of both solutions as $x = -h \pm \sqrt{k}$. Solve the following equations for the unknown leaving your answer in exact form.

a $(x + 5)^2 = 35$ **b** $(x + 8)^2 = 63$ **c** $(x - 4)^2 = 99$

d $(x - 7)^2 = 50$ **e** $(x - 5)^2 = 20$ **f** $(x + 9)^2 = 90$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Solving simple quadratic equations

Topic quiz
6E

PROBLEM SOLVING AND REASONING

CHALLENGE

ANS p516

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

6A **1** Check if the given value of x is a solution to the following equations.

a $x + 2 = 4, x = 2$ **b** $6x = 30, x = 5$ **c** $\frac{1}{2}x = 4, x = 4$ **d** $\frac{1}{2}x + 2 = 4, x = 4$

6A **2** Solve the following equations by inspection.

a $3a = 12$ **b** $\frac{p}{3} = -7$ **c** $z + 8 = -9$

d $7a + 5 = 12$ **e** $\frac{2j}{5} = -6$ **f** $10y + 8 = 78$

6A **3** Write an equation to represent each of these number sentences. Use x for the unknown number.

a 6 added to a number gives 18.
b 10 less than a number is equal to −4.
c A number is multiplied by 4, then 9 is added to give 33.
d The sum of a number and 12 all divided by 3 is equal to 6.
e The sum of a number and 3 all multiplied by 8 is equal to 64.

6B **4** Solve the following equations using inverse operations.

a $x + 11 = 25$ **b** $\frac{x}{7} = 5$ **c** $-6x = 54$

6B **5** Use inverse operations to solve the following equations.

a $3x + 8 = 20$ **b** $6(x - 4) = 48$ **c** $\frac{x + 7}{2} = 10$

6B **6** Solve the following equations for x.

a $-6x + 4 = -20$ **b** $\frac{x - 8}{6} = -4$ **c** $\frac{x + 8}{3} = -11$

d $7x = 1$ **e** $5(x + 3) = 21$ **f** $\frac{x - 4}{5} = \frac{2}{3}$

6C **7** Solve the following equations for x.

a $2x + 7 = x + 15$ **b** $8x + 3 = 7x + 15$ **c** $9x + 4 = 6x + 13$

6C **8** Solve the following equations for x.

a $6x + 9 = 8x - 11$ **b** $2x - 21 = 9x + 13$ **c** $7(x - 5) = 3x + 6$ **d** $5(x + 3) - 4x + 6 = 3(x + 7)$

6D **9** Given the equation $y = kx + a$, solve for x when:

a $y = 0, k = -2, a = 6$ **b** $y = 1, k = 2, a = 2$.

6D **10** Rearrange the following equations to make x the subject.

a $y = kx + a$ **b** $xy + k = a$ **c** $2x - xy = k$

6E **11** Solve the following equations for x.

a $x^2 = 4$ **b** $3x^2 = 48$ **c** $9x^2 = 16$

6E **12** Solve the following equations for x.

a $x^2 + 9 = 10$ **b** $x^2 + 7 = 7$ **c** $2x^2 + \frac{1}{9} = 1$

6F Plotting linear relationships

Learning intentions

By the end of this topic you will be able to ...

✔ use equations to complete tables of values

✔ plot linear graphs from tables of values and equations.

Inter-year links

Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 9	4C Plotting linear relationships

The Cartesian plane and Cartesian coordinates

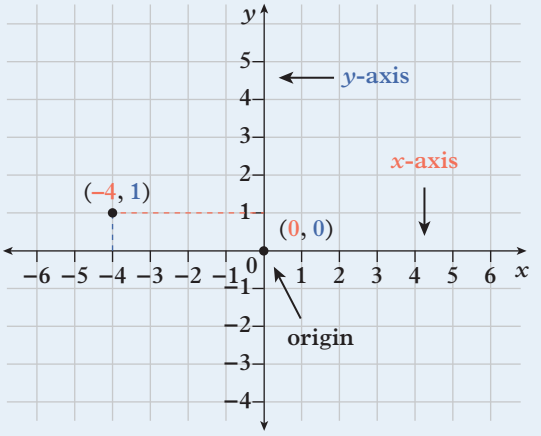
Key content video

- The **Cartesian plane** consists of a horizontal ***x*-axis** and a vertical ***y*-axis** that intersect at right angles at a point called the **origin**.
- **Coordinates** describe the position of a point on the Cartesian plane and consist of an ***x*-coordinate** and a ***y*-coordinate**.

(–4, 1)

x-coordinate – horizontal distance from the origin

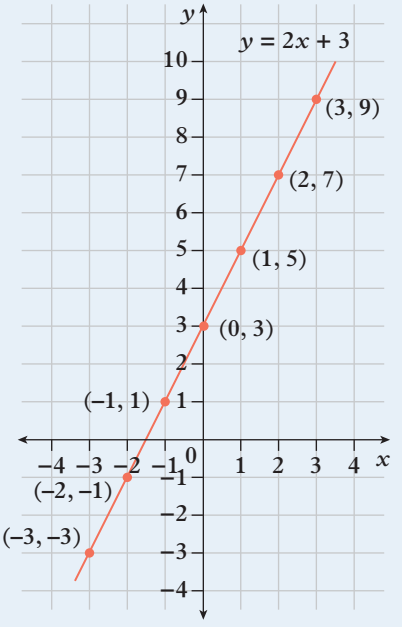
y-coordinate – vertical distance from the origin



Linear relationships

- The relationship between two variables can be represented by an algebraic equation, a table of values, a set of coordinates or a graph.
- **Linear relationships** produce straight line graphs called **linear graphs**.

  For example, a graph of the linear relationship $y = 2x + 3$ is shown on the right.
- Linear graphs on the same set of axes can be compared in many ways: by looking at how steep each graph is, whether they go up or down as you move from left to right, where they cross each axis, and if the two linear graphs cross (or intersect).



Drawing graphs

- Whereas a **plot** is composed of individual coordinate points, a **graph** is a continuous line made up of infinitely many coordinate points.
- Graphs can be drawn by plotting a series of coordinates. If there is a distinct pattern to the points, a straight line or smooth curve can be used to join the points together.
- To draw a graph from an equation:

1 Construct a table of x and y values by selecting values for x, then substituting each value of x into the equation to find the corresponding value of y.

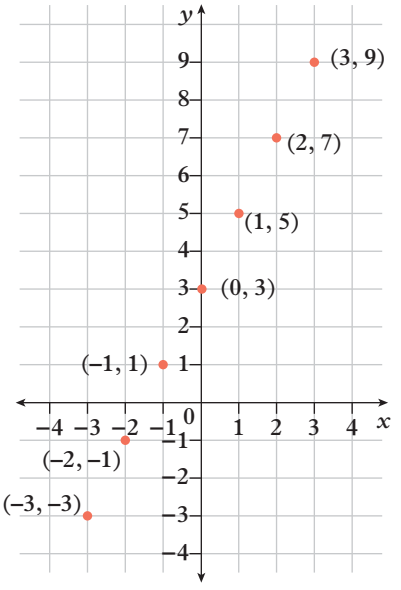
$y = 2x + 3$

x	−3	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7	9

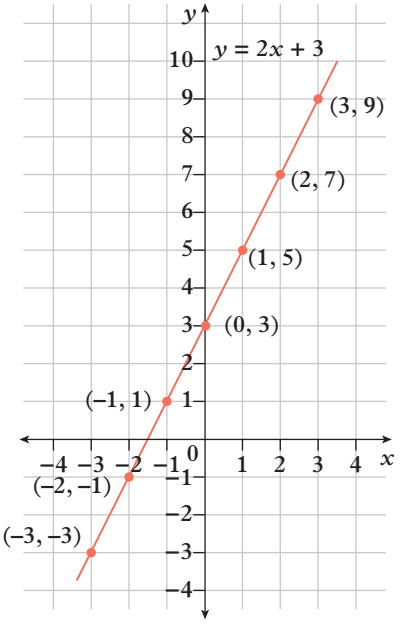
2 Write out the coordinate points listed in the table.

(−3, −3), (−2, −1), (−1, 1), (0, 3), (1, 5), (2, 7), (3, 9)

3 Plot the coordinate points on the Cartesian plane.



4 Join the points with a smooth line.



Example 6F.1 Plotting points on the Cartesian plane

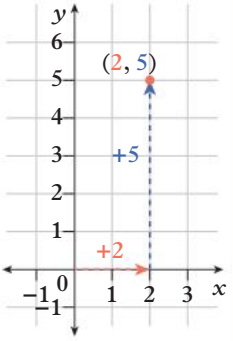
Plot the following points on the Cartesian plane and label each point with its coordinates.

a (2, 5) **b** (−5, −1)

THINK

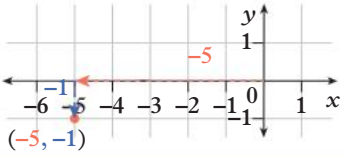
1 Draw the Cartesian plane. The x- and y-axes should extend far enough to accommodate the largest coordinates.

2 Mark each point with a dot and label the dot with its coordinates.

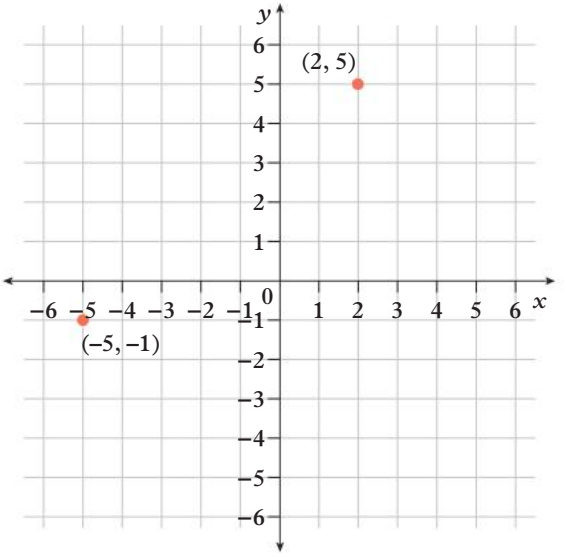


a The point (2, 5) has an x-coordinate of 2 and a y-coordinate of 5. Start at the origin, move 2 units to the right and 5 units up.

b The point (−5, −1) has an x-coordinate of −5 and a y-coordinate of −1. Start at the origin, move 5 units to the left and 1 unit down.



WRITE



Example 6F.2 Using an equation to complete a table of values

Complete the table of values for $y = 2x - 1$.

x	−2	−1	0	1	2
y					

THINK

1 Substitute each value of x into the equation to find the corresponding value of y.

2 Complete the table of values.

WRITE

Let $x = -2$

$y = 2(-2) - 1$
$= -4 - 1$
$= -5$

Let $x = -1$

$y = 2(-1) - 1$
$= -2 - 1$
$= -3$

Let $x = 0$

$y = 2(0) - 1$
$= 0 - 1$
$= -1$

Let $x = 1$

$y = 2(1) - 1$
$= 2 - 1$
$= 1$

Let $x = 2$

$y = 2(2) - 1$
$= 4 - 1$
$= 3$

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

Example 6F.3 Graphing linear relationships

Plot a graph of $y = 5 - x$ by first completing a table of values for integer values of x from -3 to 3.

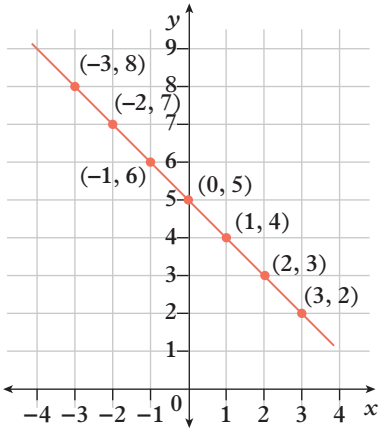
THINK

1. Construct a table of values for x from -3 to 3. Substitute each value of x into the equation to find the corresponding values of y.
2. Write out the coordinates of each point listed in the table.
3. Plot the points on the Cartesian plane.
4. Join the points with a smooth line.

WRITE

x	-3	-2	-1	0	1	2	3
y	8	7	6	5	4	3	2

$(-3, 8)$, $(-2, 7)$, $(-1, 6)$, $(0, 5)$, $(1, 4)$, $(2, 3)$, $(3, 2)$



Helpful hints

- ✔ When plotting points, locate the value of the x-coordinate first, then move up or down to the value on the y-coordinate.
- ✔ When plotting a linear graph, use a ruler when joining the points with a straight line.
- ✔ Vertical and horizontal straight lines are also linear relationships.

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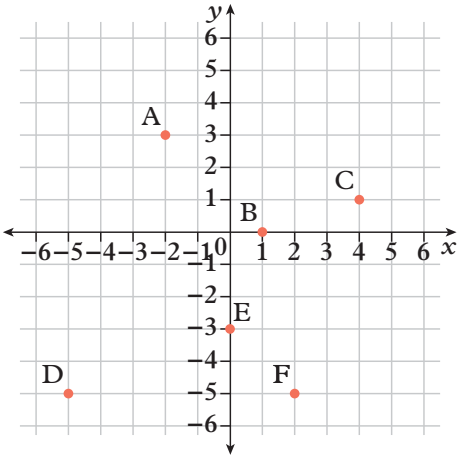
Exercise 6F Plotting linear relationships

△ 1–3, 4, 5(1st, 2nd columns), 6–11 ▢ 2–4, 5(2nd, 3rd columns), 6–12, 14 ⬠ 5(2nd, 3rd columns), 6–11, 13, 14

UNDERSTANDING AND FLUENCY

1 Match the coordinates to the points shown on the Cartesian plane.

a $(-2, 3)$ **b** $(4, 1)$ **c** $(2, -5)$
d $(1, 0)$ **e** $(0, -3)$ **f** $(-5, -5)$



6F.1 2 Plot the following points on the Cartesian plane and label each point with its coordinates.

a $(3, 5)$ **b** $(-4, -5)$ **c** $(2, -2)$
d $(6, 0)$ **e** $(-1, 4)$ **f** $(0, 3)$

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6F.2 **3** Complete the table of values for each of the following linear equations.

a $y = x + 1$

x	−2	−1	0	1	2
y					

b $y = 2x + 2$

x	−3	−2	−1	0	1
y					

c $y = 3 - x$

x	−1	0	1	2	3
y					

d $y = 3x - 4$

x	−1	0	1	2	3
y					

e $y = 4x - 1$

x	−2	−1	0	1	2
y					

f $y = -x + 1$

x	−3	−2	−1	0	1
y					

4 Plot a graph of each of the linear relationships in question **3**.

6F.3 **5** Plot a graph of each linear relationship by first completing a table of values for integer values of x from −3 to 3.

a $y = x + 4$ **b** $y = x - 3$ **c** $y = -x$
d $y = 2 - x$ **e** $y = 5 - x$ **f** $y = -x - 1$
g $y = 3x$ **h** $y = 2x + 3$ **i** $y = 4x - 2$
j $y = 3x + 1$ **k** $y = 6 - 3x$ **l** $y = 4 - 2x$

6 a Plot the graph of each relationship using the table of values.

i $y = 2$

x	−2	−1	0	1	2
y	2	2	2	2	2

ii $y = -1$

x	−2	−1	0	1	2
y	−1	−1	−1	−1	−1

iii $y = 8$

x	−2	−1	0	1	2
y	8	8	8	8	8

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

7 a Plot the graph of each relationship using the table of values.

i $x = 3$

x	3	3	3	3	3
y	−2	−1	0	1	2

ii $x = 0$

x	0	0	0	0	0
y	−2	−1	0	1	2

iii $x = -6$

x	−6	−6	−6	−6	−6
y	−2	−1	0	1	2

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

8 Without using a table of values, draw the graph of each relationship.

a $x = 2$ **b** $y = 4$ **c** $y = -3$
d $x = -1$ **e** $x = 5$ **f** $y = 0$

9 Two hot air balloons lift off at the same time. Their heights over the first 10 seconds of flight are recorded in these tables.

Red balloon

Time (s)	0	2	4	6	8	10
Height (m)	0	4	8	12	16	20

Blue balloon

Time (s)	0	2	4	6	8	10
Height (m)	0	8	14	18	19	20

a What are the two variables shown in each table?

When two variables are related, the variable that 'depends' on the other is called the dependent variable and the other variable is called the independent variable. Generally, the independent variable is shown in the first row in a table of values, and on the horizontal axis on the Cartesian plane.

b In the hot air balloon scenario, which is the:

i independent variable

ii dependent variable?

c Write the coordinates of the points listed in each table.

d Showing time on the horizontal axis and height on the vertical axis, plot the points on the Cartesian plane and join each set of points with a smooth line to produce two graphs. Label each graph.

e Use the graphs to estimate:

i the height of each balloon after 5 seconds

ii the time it takes each balloon to reach a height of 7 metres.

10 This table shows the distance Jamie covers in each hour of a road trip in her car.

Time (h)	0	1	2	3	4	5
Distance (km)	0	80	160	240	320	400

a What is the independent variable in this relationship? What is the dependent variable?

b Plot a graph of this relationship and label the axes appropriately.

c Use the graph to determine how far Jamie travelled in 2.5 hours.

d Use the graph to determine how long it took her to travel 260 km.

e If Jamie travelled for 7 hours in total, what total distance would she cover? What assumption do you need to make for this prediction?

f Write an equation to represent the relationship between the distance, d, Jamie travels and the time, t, it takes her to cover that distance. Explain your thinking.

g Use the equation to check your answer to part **e**.

11 The water consumption for a household can be described by the linear equation $w = 900d$, where w represents the amount of water (in L) used for the month and d represents the number of days from the start of the month.

a Identify the independent and dependent variables in this relationship.

b Plot a graph of this relationship for the first ten days of the month and label the axes appropriately.

c Is this relationship linear? Explain.

d Use the graph to determine the amount of water used in the first week of the month.

e A resident of the household wants to determine how many days it takes to consume 20 000 L of water.

i Should the resident use the equation or a graph to determine the time taken to consume 20 000 L of water? Justify your choice.

ii Determine how many days it will take to consume 20 000 L of water.

12 Gavin's water tank has sprung a leak and water is leaking out at a constant rate of 800 L/h. He used the following table to record how much water is left in the tank.

Time after the leakage started (h)	0	0.25	0.5	0.75	1
Water left in the tank (L)	2000	1800	1600	1400	1200

a How much water was in the tank before the leakage started?

b Write a formula to represent the relationship between the water left in the tank (w) and time (t).

c In this relationship, which is the:

i dependent variable **ii** independent variable?

d Plot a graph of this relationship and label the axes appropriately.

e Gavin called a plumber immediately after the leakage started. If it takes 3 hours for the plumber to travel to Gavin's place, how much water would be left in the tank by the time the plumber arrives?

13 The equation $y = kx$ gives a different linear relationship for each value of k.

a Plot the graphs of $y = kx$ for $k = 1$, $k = 2$ and $k = 3$ on the same set of axes.

b What do you notice about these lines? What can you say about their steepness? Where do they intercept the x- and y- axes?

c On the same set of axes used in part **b**, plot the graphs of $y = kx$ for $k = -1$, $k = -2$ and $k = -3$.

d What do you notice about the graphs of these lines? How do they compare to each other? How do they compare to the graphs drawn in part **a**?

e On a new set of axes, plot the graphs of $y = kx + 1$ for $k = 1$ and $k = 2$. What do you notice about how these graphs intercept the x- and y-axes? What do you notice about how they intersect each other?

f On a new set of axes, plot the graphs of $y = 2x$, $y = 2x + 1$ and $y = 2(x + 1)$. How do they compare to each other?

14 a Not all tables of values correspond to linear relationships. If a relationship is non-linear, the corresponding graph is not a straight line. For each of the following tables of values, plot the points on the Cartesian plane and join the points with a smooth line, where necessary, use $\sqrt{3} \approx 1.7$.

i

x	−2	−1	0	1	2
y	4	1	0	1	4

ii

x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

iii

x	−2	−1	0	1	2
y	6	3	2	3	6

iv

x	−2	−1	0	1	2
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

v

x	-2	$-\sqrt{3}$	$-\sqrt{3}$	-1	-1	0	0	1	1	$\sqrt{3}$	$\sqrt{3}$	2
y	0	1	−1	$-\sqrt{3}$	$\sqrt{3}$	−2	2	$-\sqrt{3}$	$\sqrt{3}$	1	−1	0

b Which of the tables of values in part **a** correspond to quadratic relationships? Hint: Recall that $c^2 = (-c)^2$ for any positive number c and a quadratic relationship must have a term with an index of 2.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Plotting graphs

Investigation
Graphing length and width of a rectangle of fixed area

Topic quiz
6F

6G Solving linear equations using graphs

Learning intentions

By the end of this topic you will be able to ...

✔ solve linear equations using a graph.

Inter-year links

Support	Interpreting graphs
Year 7	5E Line graphs
Year 9	4F Determining linear equations
Year 10	5A Linear relationships

Solving linear equations using graphs

Key content video

- Each point on a graph is made up of an x-coordinate and a corresponding y-coordinate. The equation of a graph describes the relationship between the x- and y-coordinates of all the points that make up the graph.

Graph

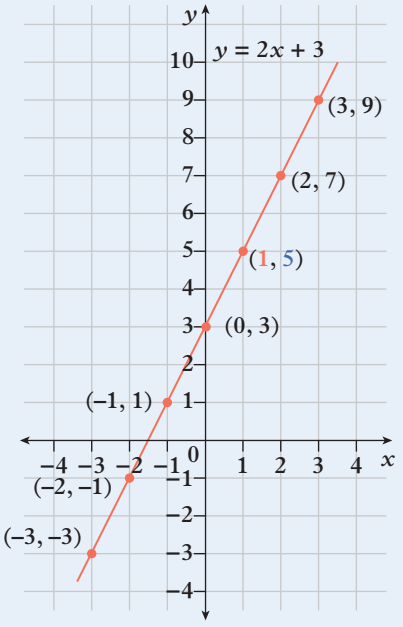


Table of values

x	–3	–2	–1	0	1	2	3
y	–3	–1	1	3	5	7	9

Equation

$y = 2x + 3$

Coordinate

(1, 5)

Solution

Let $y = 5$

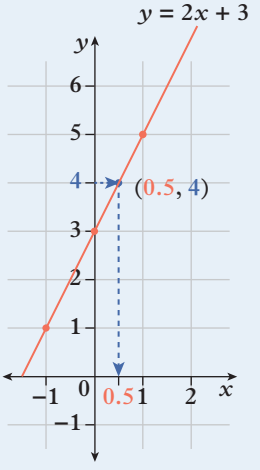
$5 = 2x + 3$

$2 = 2x$

$x = 1$

- The x-coordinate of each point on a graph is the solution to a specific equation related to the equation of the graph.

  For example, to solve the equation $2x + 3 = 4$ using the graph of $y = 2x + 3$, locate 4 on the y-axis, then trace across to the graph and down to the x-axis to find the corresponding value of x.



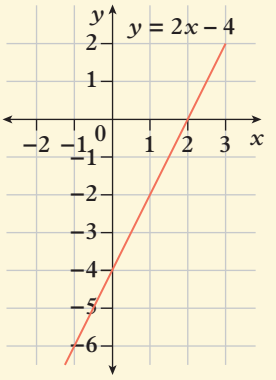
Example 6G.1 Solving linear equations using a graph

Solve the following equations using the graph of $y = 2x - 4$.

a $2x - 4 = 2$

b $2x - 4 = 0$

c $2x - 4 = 1$



THINK

a Use the graph to find the value of x when $y = 2$. Locate 2 on the y-axis, trace across to the equation line, then down to the x-axis to find the corresponding value of x.

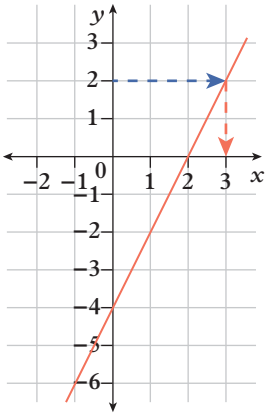
b Use the graph to find the value of x when $y = 0$. $y = 0$ is on the x-axis, so trace along the x-axis to the equation line to find the value of x when $y = 0$.

c Use the graph to find the value of x when $y = 1$. Locate 1 on the y-axis, trace across to the equation line, then down to the x-axis to find the corresponding value of x.

WRITE

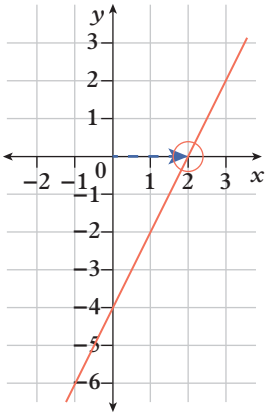
a When $y = 2$, $x = 3$.

The solution to $2x - 4 = 2$ is $x = 3$.



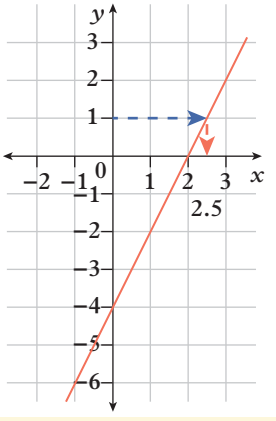
b When $y = 0$, $x = 2$.

The solution to $2x - 4 = 0$ is $x = 2$.



c When $y = 1$, $x = 2.5$ or $2\frac{1}{2}$.

The solution to $2x - 4 = 1$ is $x = 2.5$ or $2\frac{1}{2}$.



Example 6G.2 Solving linear equations by plotting graphs

Consider the following set of linear equations.

i $5 - x = 2$ **ii** $5 - x = -3$ **iii** $5 - x = 0$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y values that need to be included on the graph to solve each equation graphically.

c Use a table of values for x from -2 to 2 to plot the graph, and hence find the solution to each equation.

THINK

a Identify the expression that is shared by all the equations in the set. The expression on the left-hand side of every equation in the set is '$5 - x$', so a graph of $y = 5 - x$ is required to solve this set of equations graphically.

b Each equation in the set is written in the form $5 - x = y$. List the value of y in each equation.

c **1** Construct a table of values for x is equal to $-2, -1, 0, 1, 2$.

2 Plot the coordinates on the Cartesian plane and join the points with a smooth line to draw the graph.

3 Use the graph to find the value of x for each y value listed in step **b**.

4 Write the solution to each equation.

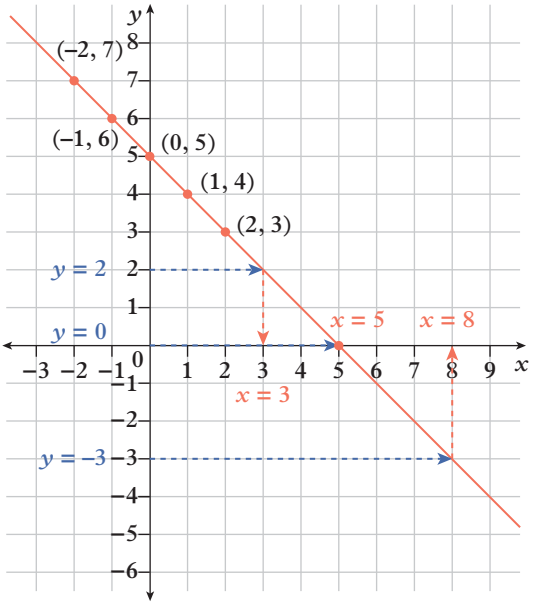
WRITE

a $y = 5 - x$

b $y = 2, -3, 0$

c

x	-2	-1	0	1	2
y	7	6	5	4	3



i $x = 3$ **ii** $x = 8$ **iii** $x = 5$

Helpful hints

- ✔ The coordinates of graphs do not have to be integer values. For example, in the graph $y = 2x + 3$, $x = 1.5$, $x = 1.2$ and $x = -11.37$ all have a corresponding y-value.
- ✔ Although x and y are commonly used to represent the independent and dependent variables when working on the Cartesian plane, remember that variables can be represented using any pronumeral.
- ✔ When the value of a solution read from a graph is not an exact value, the graph can still be used to provide an estimate for a solution, which can be calculated exactly by solving an equation.

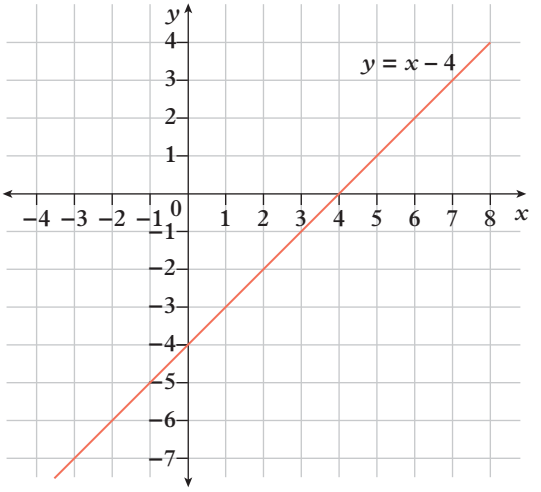
ANS p522

Exercise 6G Solving linear equations using graphs

1–6, 8, 10, 11 | 2–4, 6–9, 11–13 | 3, 5, 8, 9, 12–16

UNDERSTANDING AND FLUENCY

1 Consider the graph of $y = x - 4$ shown on the right.



a Use the graph to find the value of x that corresponds to each of the following values of y.

i $y = 2$ **ii** $y = -3$

iii $y = -6$ **iv** $y = -2$

b Use your answers to part **a** to solve each equation.

i $x - 4 = 2$ **ii** $x - 4 = -3$

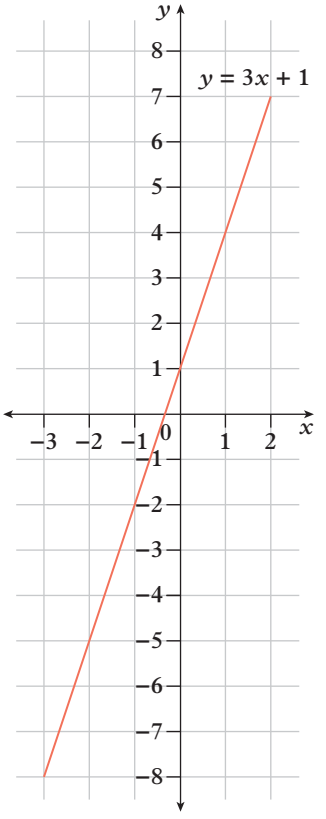
iii $x - 4 = -6$ **iv** $x - 4 = -2$

c Use the graph to solve the following equations.

i $x - 4 = 4$ **ii** $x - 4 = 0$

iii $x - 4 = -4$ **iv** $x - 4 = -6$

6G.1 **2** Solve the following equations using the graph of $y = 3x + 1$ shown on the right.



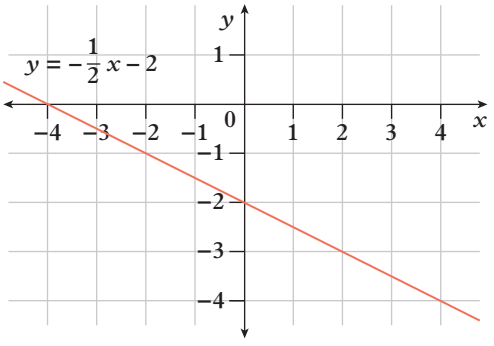
a $3x + 1 = 7$ **b** $3x + 1 = -2$ **c** $3x + 1 = 4$

d $3x + 1 = 1$ **e** $3x + 1 = -5$ **f** $3x + 1 = -8$

3 Solve the following equations using the graph of $y = -\frac{1}{2}x - 2$ shown below.

a $-\frac{1}{2}x - 2 = -3$ **b** $-\frac{1}{2}x - 2 = -1$ **c** $-\frac{1}{2}x - 2 = -2$

d $-\frac{1}{2}x - 2 = 0$ **e** $-\frac{1}{2}x - 2 = -\frac{1}{2}$ **f** $-\frac{1}{2}x - 2 = -\frac{5}{2}$



4 **a** Plot the graph of $y = 2 - x$ for values of x from -3 to 3.

b Use the graph to solve the following set of equations.

i $2 - x = 1$ **ii** $2 - x = -1$ **iii** $2 - x = 0$

iv $2 - x = 2$ **v** $2 - x = 1.5$ **vi** $2 - x = 0.5$

5 **a** Plot the graph of $y = 2x + 3$ for values of x from -3 to 3.

b Use the graph to solve the following set of equations.

i $2x + 3 = 1$ **ii** $2x + 3 = -3$ **iii** $2x + 3 = 6$

iv $2x + 3 = 3$ **v** $2x + 3 = -2$ **vi** $2x + 3 = 8$

UNDERSTANDING AND FLUENCY

6G.2 **6** Consider the following set of linear equations.

i $3x - 2 = 7$ **ii** $3x - 2 = -5$ **iii** $3x - 2 = -20$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y value that needs to be used to solve each equation.

c Plot the graph and hence find the solution to each equation.

7 Consider the following set of linear equations.

i $4x - 5 = 3$ **ii** $4x - 5 = -13$ **iii** $4x - 5 = -7$

a Write the equation of the graph required to solve this set of equations graphically.

b List the y values that need to be included on the graph to solve each equation graphically.

c Plot the graph and hence find the solution to each equation.

8 Solve each equation graphically.

a $1 - 2x = 3$ **b** $1 - 2x = -5$ **c** $1 - 2x = 7$

d $1 - 2x = -9$ **e** $1 - 2x = 9$ **f** $1 - 2x = -4$

9 Solve each equation graphically.

a $8x - 3 = 5$ **b** $8x - 3 = 13$ **c** $8x - 3 = 21$

d $8x - 3 = -3$ **e** $8x - 3 = -11$ **f** $8x - 3 = -7$

PROBLEM SOLVING AND REASONING

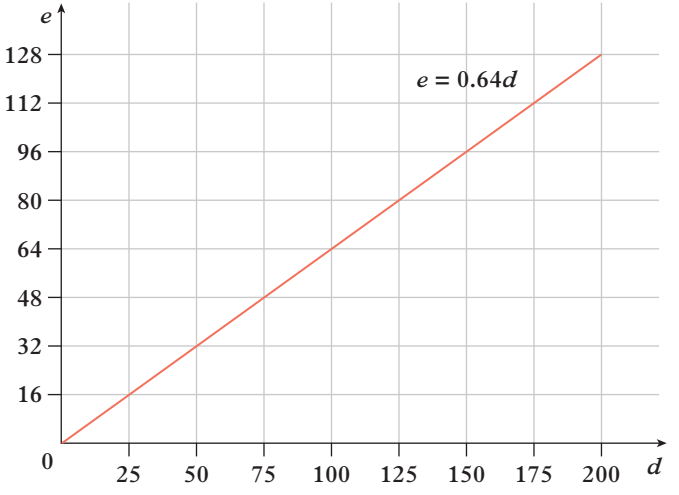
10 Consider the equation $-4x + 6 = -4$.

a Solve the equation using inverse operations.

b Solve the equation graphically.

c Describe one advantage and one disadvantage of the graphical solution method compared to the algebraic solution method.

11 Matteo has been comparing the prices of goods in Italy with those in Australia. To save time, he decides to plot a graph to convert Euros, e, to Australian dollars, d. He looks up the exchange rate and finds that \$1 = 0.64 Euros, or $e = 0.64d$.



a Use the graph to solve each of the following equations.

i $0.64d = 32$

ii $0.64d = 96$

iii $0.64d = 128$

b A designer belt costs 50 Euros.

i Write an equation that can be used to work out the cost of the belt in Australian dollars.

ii Use the graph to solve the equation. What is the equivalent cost of the belt in Australian dollars, to the nearest dollar?

c Matteo compiles a list of items for comparison.

shirt: 64 Euros

shoes: 100 Euros

jeans: 42 Euros

leather jacket: 140 Euros

i Write an equation that can be used to work out the equivalent cost of each item in Australian dollars.

ii Use the graph to solve each equation, stating the value of d to the nearest whole number.

iii Write the cost of each item in Australian dollars.

PROBLEM SOLVING AND REASONING

12 In Australia, temperature is measured in degrees Celsius. Some countries, such as the USA, measure temperature in degrees Fahrenheit. The formula $F = \frac{9}{5}C + 32$ can be used to convert temperatures in degrees Celsius, C, into degrees Fahrenheit, F.

a Use the formula to complete the table of values below.

C	−25	0	25	50	75
F					

b Use the table of values to plot a graph of the relationship between degrees Celsius, C, and degrees Fahrenheit, F.

c Use the graph to solve each equation to find the value of C, rounded to the nearest integer.

i $\frac{9}{5}C + 32 = 40$ **ii** $\frac{9}{5}C + 32 = 104$ **iii** $\frac{9}{5}C + 32 = 14$

Olivia is watching an American cooking show when the host gives the following advice on how to store food bought from the supermarket.

'Frozen meat should be stored in the freezer at −5°F, dairy products and vegetables should be stored in the fridge at about 37°F and tomatoes should be kept at room temperature, ideally around 68°F.'

d i Write three equations that can be used to calculate the equivalent temperatures in degrees Celsius.

ii Use the graph to solve each equation, rounded to the nearest integer.

iii Rewrite the advice for an Australian audience.

e The host recommends keeping hot, cooked food above 140°F. What temperature is this in degrees Celsius?

CHALLENGE

13 Equations with the unknown on both sides can also be solved graphically. Use the following to solve $4x - 3 = 2x + 1$ graphically.

a Plot the graph of $y_1 = 4x - 3$ for x in the range −1 to 3.

b Plot the graph of $y_2 = 2x + 1$ on the same set of axes.

c Find the point where these lines intersect and write the coordinates of the point where $y_1 = y_2$.

d Write the solution to $4x - 3 = 2x + 1$.

e Check your solution by substituting the value for x into both sides of the equation.

14 Use the following to solve each of the linear equations below graphically.

i Write the equations of the graphs to be plotted.

ii Plot each pair of graphs on the same set of axes.

iii Read the solution to the equation off the graph.

iv Use substitution to check your solution.

a $x - 2 = 7 - 2x$ **b** $2x + 4 = -3x - 6$

15 The number of lamingtons in a packet is unknown. However, Adam can fill a plastic container with two more than the contents of one packet or four less than the contents of two packets.

a Write an equation to represent this problem.

b Write the equations of the graphs that need to be plotted to solve the equation graphically.

c Solve the equation graphically.

d How many lamingtons are in a packet?

16 Solve each equation graphically:

a $3x - 4 = x - 1$ **b** $2x - 5 = -x - 5$

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Solving linear equations graphically

Investigation
Integer solutions to linear equations

Topic quiz
6G

6H Intersecting lines

Learning intentions

By the end of this topic you will be able to ...

- ✔ identify the intersection point of two lines graphically
- ✔ identify the intersection point of two lines using a table of values
- ✔ verify a point is the point of intersection of two lines.

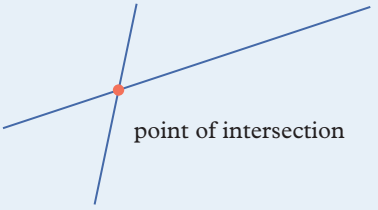
Inter-year links

Support	The Cartesian plane
Year 7	5E Line graphs
Year 10	4C Simultaneous linear equations

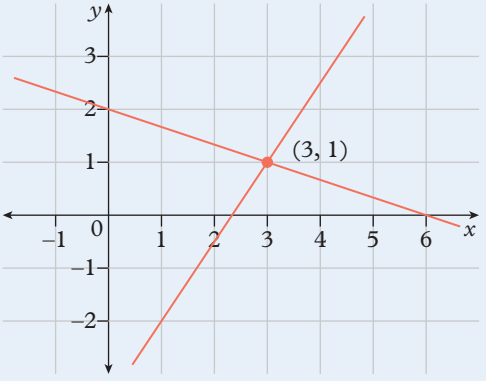
Intersecting lines

Key content video

- When two or more graphs are drawn, they often intersect each other. The point where two or more lines intersect is called a 'point of intersection'.



- A point of intersection can be identified visually on the Cartesian plane as the coordinates of the point where the lines intersect.



- A point is said to satisfy the equation of a line if the point lies on the line. To check if a point lies on the line, substitute the x- and y-coordinates into the equation of the line and verify that the left-hand side of the equation is equal to the right-hand side.
- A point of intersection satisfies the equations of all the intersecting lines.
- For example, the two intersecting lines above are $y = \frac{3}{2}x - \frac{7}{2}$ and $y = -\frac{1}{3}x + 2$. To verify the point of intersection is (3, 1), check that the point lies on both lines.

$$y = \frac{3}{2}x - \frac{7}{2}$$
$$\text{LHS} = 1$$
$$\text{RHS} = \frac{3}{2} \times 3 - \frac{7}{2}$$
$$= \frac{(9 - 7)}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

$\therefore$ (3, 1) lies on the line.

$$y = -\frac{1}{3}x + 2$$
$$\text{LHS} = 1$$
$$\text{RHS} = -\frac{1}{3} \times 3 + 2$$
$$= -1 + 2$$
$$= 1$$

$\therefore$ (3, 1) lies on the line.

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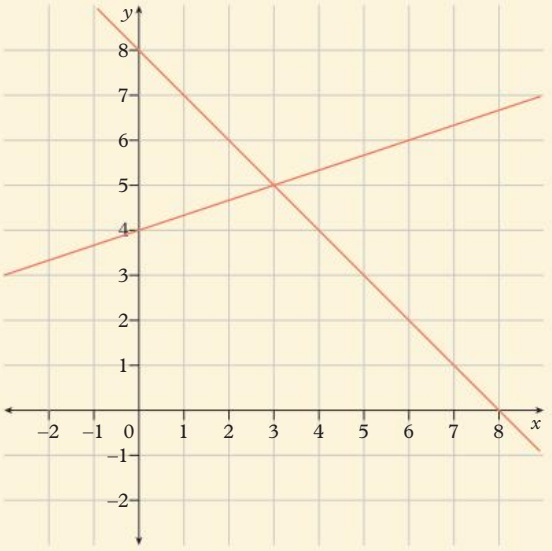
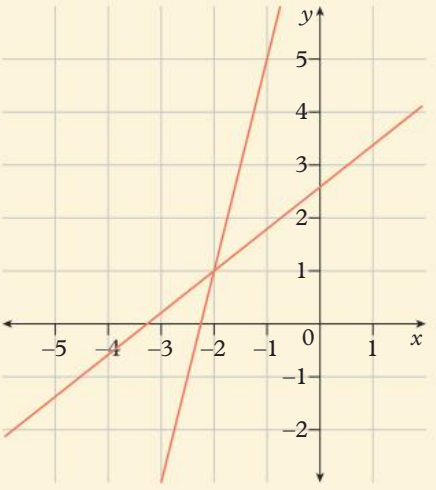
Example 6H.1 Identifying a point of intersection graphically

For each pair of lines given, identify the point of intersection by writing its coordinates.

a



b



THINK

1 Identify the point where the two graphs intersect.

2 Write down the coordinates of that point.

WRITE

a (3, 5)

b (−2, 1)

Example 6H.2 Identifying a point of intersection using a table of values

For each pair of equations, identify the point of intersection using the table of values given.

a $y = -x + 4$ and $y = 2x - 5$

x	0	1	2	3	4	5	6
$y = -x + 4$	4	3	2	1	0	−1	−2
$y = 2x - 5$	−5	−3	−1	1	3	5	7

b $y = -3x - 7$ and $y = -2x - 5$

x	−4	−3	−2	−1	0	1	2
$y = -3x - 7$							
$y = -2x - 5$							

THINK

a 1 Look for the column in the table where the two expressions have the same value.

2 State the point of intersection using the x-value and matching y-values.

b 1 Complete the table of values by substituting the x-values into each expression.

2 State the point of intersection using the x-value and matching y-values.

WRITE

a The column $x = 3$ contains the same value of y for each equation.

The point of intersection is (3, 1)

b

x	−4	−3	−2	−1	0	1	2
$y = -3x - 7$	5	2	−1	−4	−7	−10	−13
$y = -2x - 5$	3	1	−1	−3	−5	−7	−9

The column $x = -2$ contains the same value of y for each equation. The point of intersection is (−2, −1).

Example 6H.3 Verifying the point of intersection of two lines

For the following pairs of equations, check if the coordinate provided is a point of intersection of their graphs.

a $y = -3x + 1$ and $y = 5x - 7$ at (1, −2)

b $y = 5x + 2$ and $y = -x - 4$ at (−3, −1)

THINK

a **1** Substitute the x- and y-coordinates into both sides of both equations.

2 State if the point is a point of intersection (LHS = RHS for both equations) or not (LHS and RHS do not equal for one or both of the equations).

b **1** Substitute the x- and y-coordinates into both sides of both equations.

2 State if the point is a point of intersection (LHS = RHS for both equations) or not (LHS and RHS do note equal for one or both of the equations).

WRITE

a $y = -3x + 1$

LHS = −2

RHS = −3(1) + 1

= −2

$y = 5x - 7$

LHS = −2

RHS = 5(1) − 7

= −2

(1, −2) is a point of intersection of $y = -3x + 1$ and $y = 5x - 7$.

b $y = 5x + 2$

LHS = −1

RHS = 5(−3) + 2

= −13

$y = -x - 4$

LHS = −1

RHS = −(−3) − 4

= −1

(−3, −1) is not a point of intersection of $y = 5x + 2$ and $y = -x - 4$.

Helpful hints

- ✔ Intercepts and intersections sound similar. Make sure not to confuse the two! An intercept is an intersection with an axis rather than another graph.
- ✔ Use grid paper to ensure your graphs are drawn accurately so that points of intersection are easier to determine graphically.

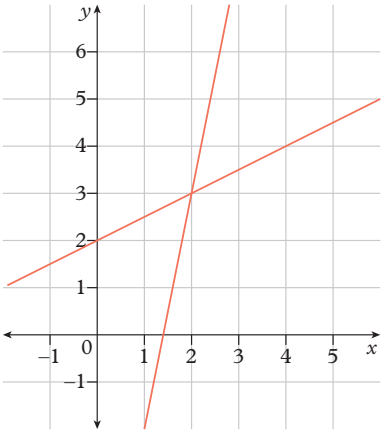
ANS p525

Exercise 6H Intersecting lines

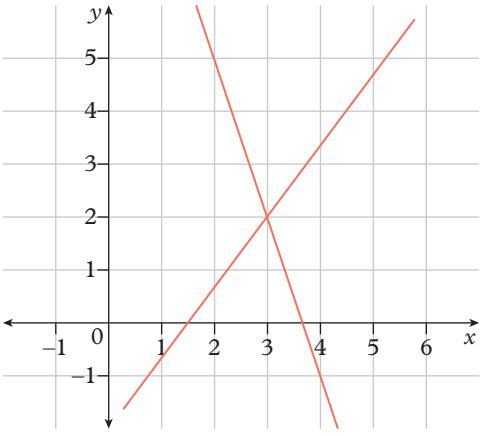
△ 1–7(1st column), 9, 10 ■ 1–7(2nd column), 8–14 ⬠ 1–7(c, d), 8, 11, 12, 14, 15

6H.1 **1** For each pair of lines given, identify the point of intersection by writing its coordinates.

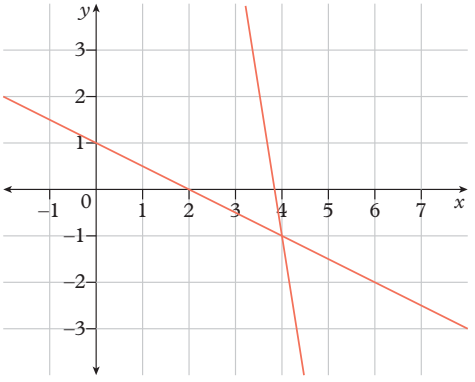
a



b



c



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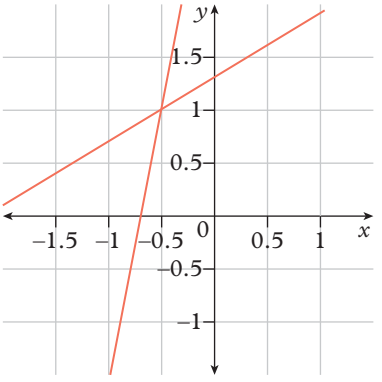

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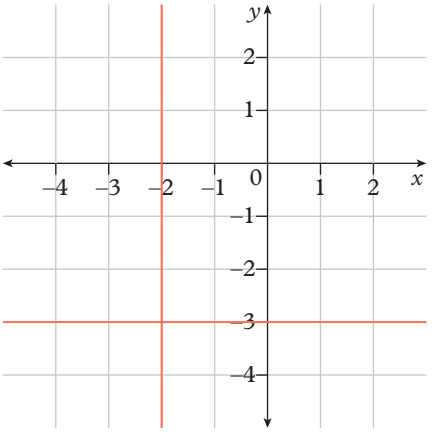

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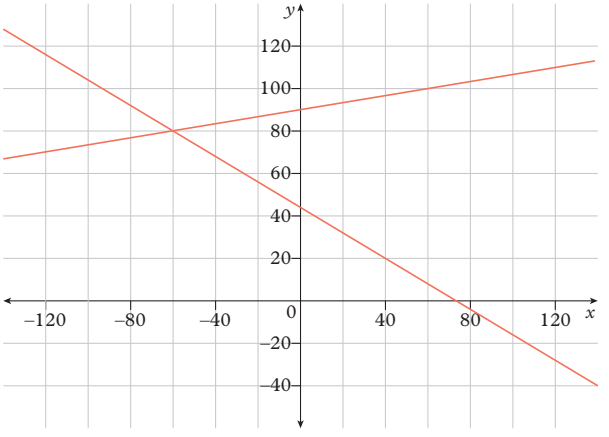


UNDERSTANDING AND FLUENCY

6H.2 **2** For each pair of equations, identify the point of intersection using the table of values given.

a $y = -2x + 11$ and $y = x - 1$

x	0	1	2	3	4
$y = -2x + 11$	11	9	7	5	3
$y = x - 1$	−1	0	1	2	3

b $y = 2x + 3$ and $y = -3x + 3$

x	−2	−1	0	1	2
$y = 2x + 3$	−1	1	3	5	7
$y = -3x + 3$	9	6	3	0	−3

c $y = 4x + 9$ and $y = 2x + 5$

x	−3	−2	−1	0	1
$y = 4x + 9$	−3	1	5	9	13
$y = 2x + 5$	−1	1	3	5	7

d $y = -7x - 9$ and $y = -2x - 4$

x	−3	−2	−1	0	1
$y = -7x - 9$	12	5	−2	−9	−16
$y = -2x - 4$	2	0	−2	−4	−6

e $y = 3x + 73$ and $y = -2x - 52$

x	−26	−25	−24	−23	−22
$y = 3x + 73$	−5	−2	1	4	7
$y = -2x - 52$	0	−2	−4	−6	−8

f $y = 3x + 16$ and $y = 2x + 21$

x	2	3	4	5	6
$y = 3x + 16$	22	25	28	31	34
$y = 2x + 21$	25	27	29	31	33

3 For each pair of equations, identify the point of intersection by first completing the table of values provided.

a $y = 3x - 2$ and $y = 2x - 1$

x	−1	0	1	2	3
$y = 3x - 2$					
$y = 2x - 1$					

b $y = 2x + 5$ and $y = -2x - 3$

x	−3	−2	−1	0	1
$y = 2x + 5$					
$y = -2x - 3$					

c $y = 7x - 16$ and $y = 4x - 10$

x	−2	−1	0	1	2
$y = 7x - 16$					
$y = 4x - 10$					

d $y = -2x - 6$ and $y = 6x + 2$

x	−3	−2	−1	0	1
$y = -2x - 6$					
$y = 6x + 2$					

e $y = -x - 23$ and $y = 2x + 55$

x	−27	−26	−25	−24	−23
$y = -x - 23$					
$y = 2x + 55$					

f $y = -3x - 43$ and $y = 4x - 78$

x	2	3	4	5	6
$y = -3x - 43$					
$y = 4x - 78$					

6H.3 **4** For each pair of equations, verify that the coordinate provided is a point of intersection of their graphs.

a $y = -10x - 4$ and $y = 5x + 11$ at $(-1, 6)$

b $y = -3x - 18$ and $y = x + 2$ at $(-5, -3)$

c $y = 5x - 27$ and $y = 7x - 37$ at $(5, -2)$

d $x - y = 14$ and $x + 2y = -10$ at $(6, -8)$

e $4x + 3y = 16$ and $5x + y = -2$ at $(-2, 8)$

f $x - y = -7$ and $-3x + 4y = 24$ at $(-4, 3)$

5 For each pair of equations, determine whether the coordinate provided is a point of intersection of their graphs.

a $y = x - 6$ and $y = -10x - 39$ at $(-3, -9)$

b $y = -3x + 29$ and $y = 4x - 38$ at $(9, -2)$

c $y = -5x + 23$ and $y = 10x - 67$ at $(6, -7)$

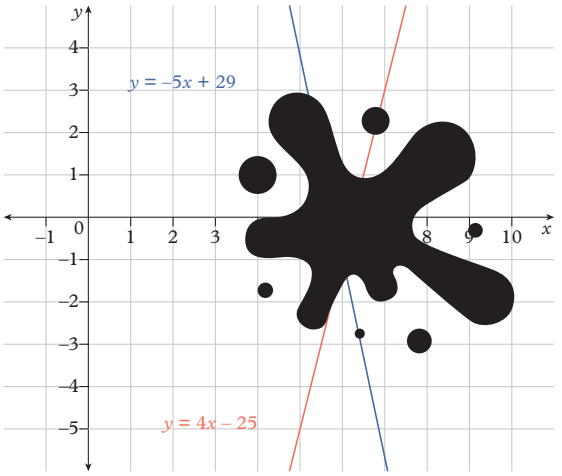
d $x + y = -1$ and $3x + 5y = -15$ at $(5, -6)$

e $-5x - 2y = 53$ and $4x + 5y = -17$ at $(7, -9)$

f $5x - y = -52$ and $x - 2y = 14$ at $(-10, 2)$

6 Ink has been spilled on the following graphs obscuring the point of intersection of the lines drawn on each. Determine the point of intersection for each graph.

a



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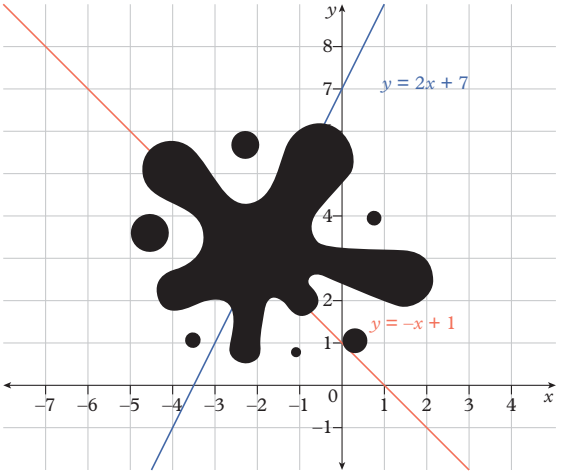

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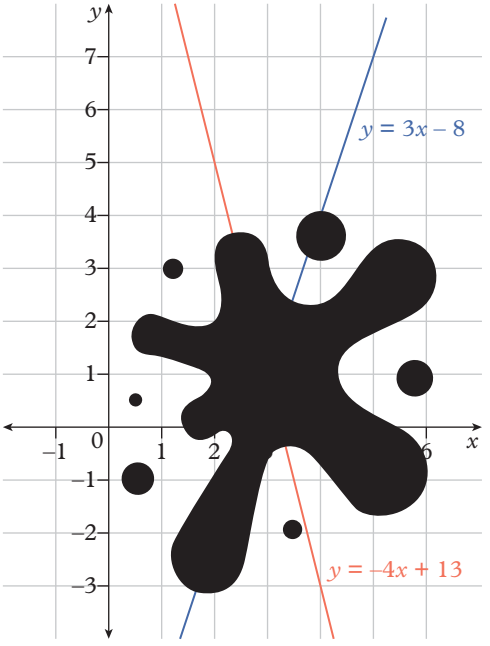

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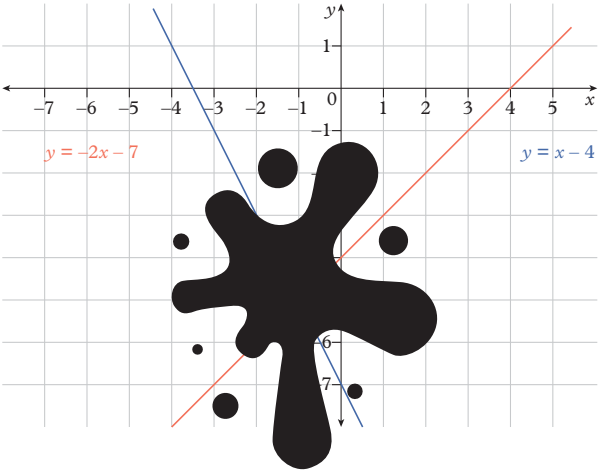


7 For each of the following pairs of equations:

i sketch the graph of both equations on the same pair of axes

ii identify the point of intersection.

a $y = 4x + 5$ and $y = -3x - 2$

b $y = 8x - 22$ and $y = -2x + 8$

c $y = 5x + 26$ and $y = -3x - 22$

d $y = 3x + 3$ and $y = -x + 3$

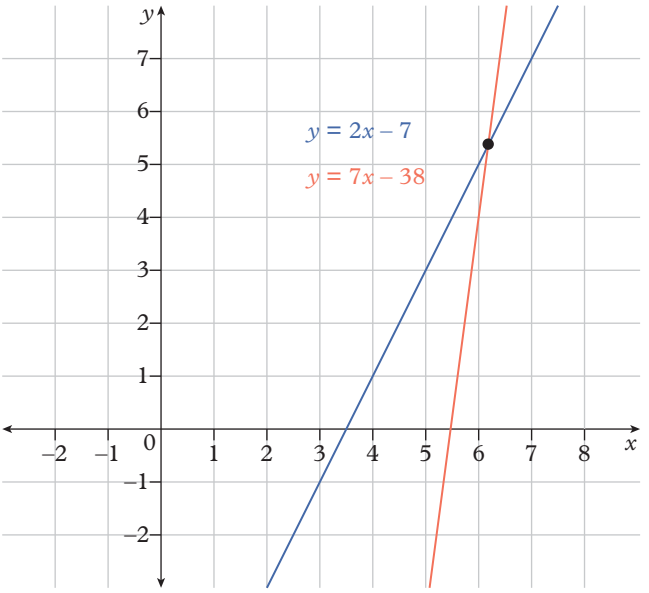
8 Graph each of the equations below on the same set of axes. Use the graph to match two equations that intersect at each point of intersection A to F.

$A(-3, 3)$, $B(0, 2)$, $C(1, 1)$, $D(-3, -1)$, $E(-2, 1)$, $F(0, 0)$

$y = -2x - 3$, $y = 2x + 5$, $y = -x$, $y = x$, $y = x + 2$, $y = -x + 2$

PROBLEM SOLVING AND REASONING

9 The coordinates of the intersection point of the graphs for $y = 2x - 7$ and $y = 7x - 38$ are not integers. Below is the graph of both equations.



a Complete the following sentences using the closest integer values.

i The x-coordinate of the intersection point lies between ____ and _____.

ii The y-coordinate of the intersection point lies between ____ and _____.

b Use a table of values, incrementing the x-coordinate by 0.1 each time, to determine the point of intersection of the two lines.

10 Use the steps from question **9** to find the intersection point of the following pairs of equations.

a $y = -4x + 29$ and $y = 6x - 44$

b $y = -5x + 29$ and $y = 5x - 22$

c $y = -3x + 22$ and $y = 7x - 69$

d $y = -5x + 45$ and $y = -10x + 84$

e $y = -2x - 8$ and $y = 8x + 47$

f $y = -x - 3$ and $y = 4x - 31$

11 a Draw an example of a pair of straight lines that do not intersect.

b What feature about these lines explains why they do not intersect?

12 Draw an example of a pair of straight lines that intersect at more than one point. What do you notice about this pair of lines?

CHALLENGE

13 The solution to the equation $4x - 8 = 8x - 20$ is the x-coordinate of the point of intersection of the graphs of $y = 4x - 8$ and $y = 8x - 20$. This is because at the point of intersection the graphs share the same y-coordinate, and therefore the expressions $4x - 8$ and $8x - 20$ are equal to each other.

a Sketch the graphs of $y = 4x - 8$ and $y = 8x - 20$ on the same pair of axes.

b Identify the point of intersection of the two graphs.

c State the solution to the equation $4x - 8 = 8x - 20$.

d Verify your solution in part **c**:

i by substitution

ii by solving the equation $4x - 8 = 8x - 20$ algebraically.

14 Use the steps from question **13** to solve the following equations graphically.

a $-5x + 8 = 7x - 16$

b $-5x - 8 = -x + 4$

c $x + 3 = 5x + 11$

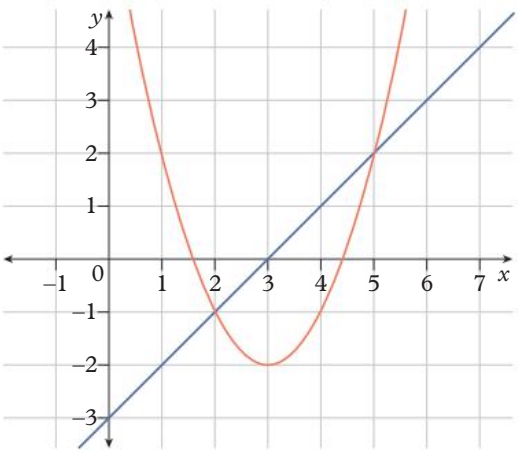
d $-5x - 24 = 2x + 4$

e $-x + 7 = 4x + 2$

f $-x - 4 = -3x - 16$

15 Graphs of non-linear relations can also intersect each other. Along with intersecting once or not at all, they can have more than one intersection with each other. Identify the intersection points of the following pairs of graphs.

a



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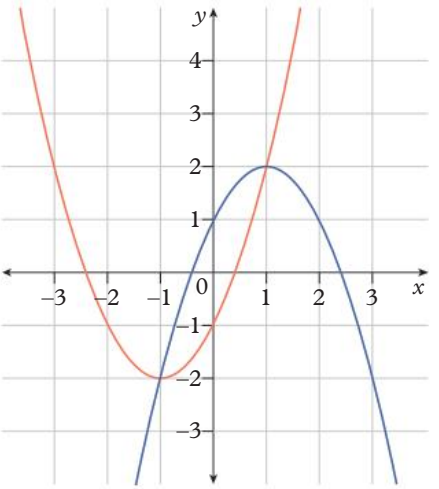

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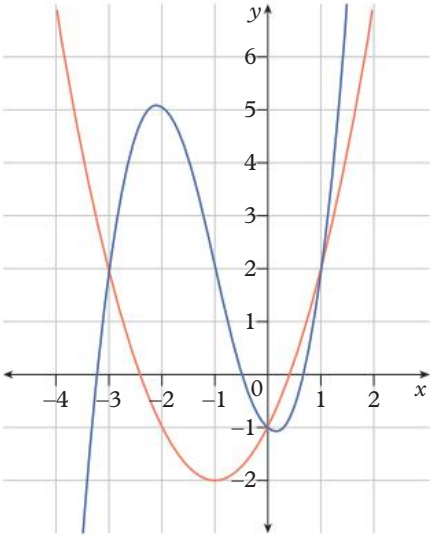

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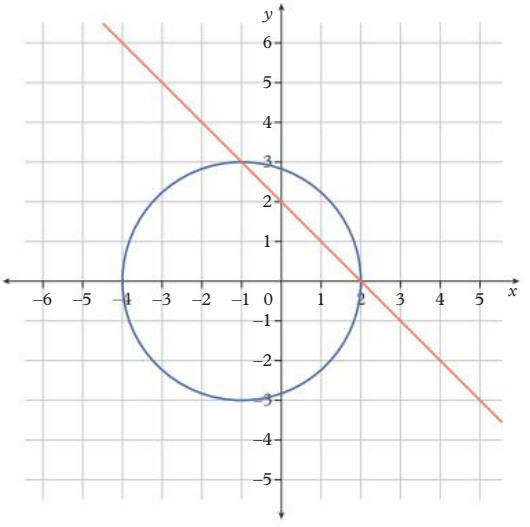

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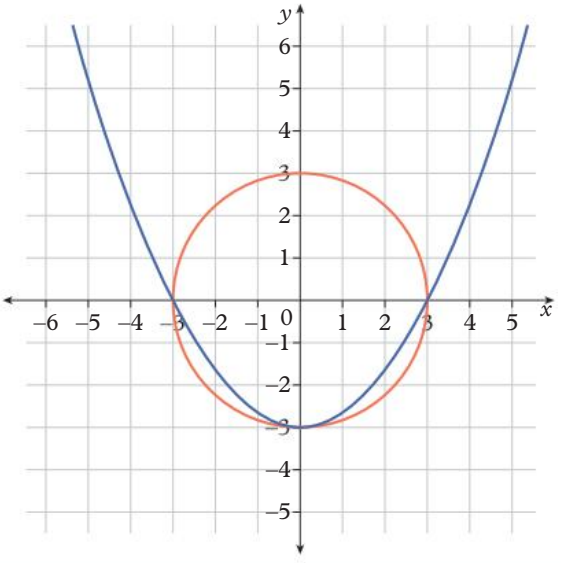

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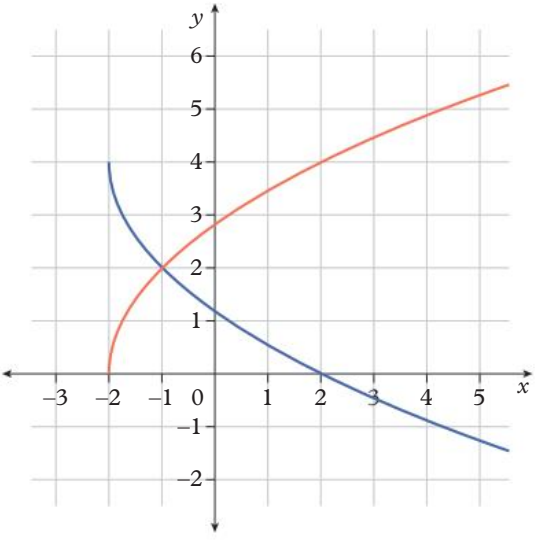


Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Intersecting lines

Topic quiz
6H

6I Finding linear equations

Learning intentions

By the end of this topic you will be able to ...

✔ find the linear equation for a table of values

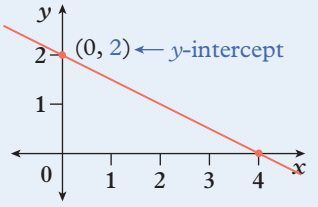
✔ find the linear equation for a graph when the coordinates are known.

Inter-year links

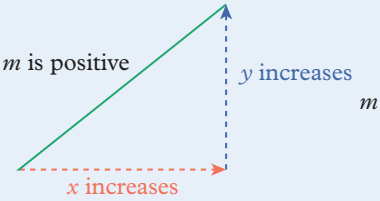
Support	The Cartesian plane
Year 7	5D The Cartesian plane
Year 9	4F Determining linear equations
Year 10	5A Linear relationships

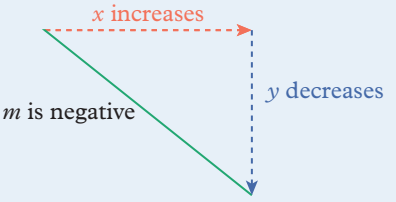
Finding the equation for a linear relationship

Key content video



- Linear equations can be written in the form: $y = mx + c$
- The constant, c, in $y = mx + c$ is equal to the value of y when $x = 0$. This is also known as the ***y*-intercept**.
- The coefficient, m, in $y = mx + c$ is the slope of the relationship, which is equal to the change in the value of y as x increases by 1.
  - → When y increases as x increases, m is positive.
  - → When y decreases as x increases, m is negative.
  - → The magnitude of m (the value ignoring the sign) indicates the steepness of the slope. A larger magnitude means a steeper slope.





For example, the equation for the table of values below is $y = 2x + 3$ because:

→ each time the value of x increases by 1, the value of y increases by 2

→ when $x = 0$, $y = 3$.

+1 +1 +1 +1

x	−2	−1	0	1	2
y	−1	1	3	5	7

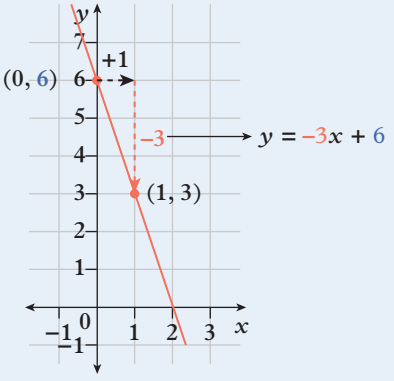
+2 +2 +2 +2

→ $y = 2x + 3$

The equation for the graph below is $y = -3x + 6$ because:

→ each time the value of x increases by 1, the value of y decreases by 3

→ when $x = 0$, $y = 6$.



Example 6I.1 Finding the linear equation for a table of values

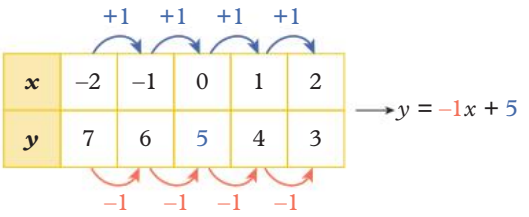
Use the table below to find the linear equation which represents the relationship between x and y.

x	−2	−1	0	1	2
y	7	6	5	4	3

THINK

1. The coefficient of x is equal to the change in y when x increases by 1. Each time the value of x increases by 1, the value of y decreases by 1 so the coefficient of x, $m = -1$.
2. The constant is equal to the value of y when $x = 0$. When $x = 0$, $y = 5$ so the constant $c = 5$.
3. Write the equation for the table.

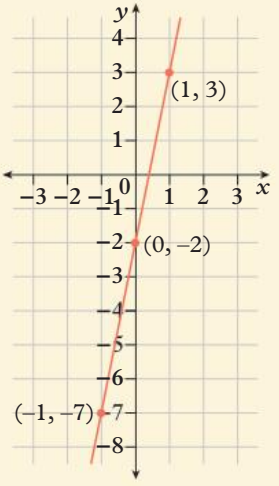
WRITE



The linear equation for the table is $y = -x + 5$.

Example 6I.2 Finding the linear equation of a graph

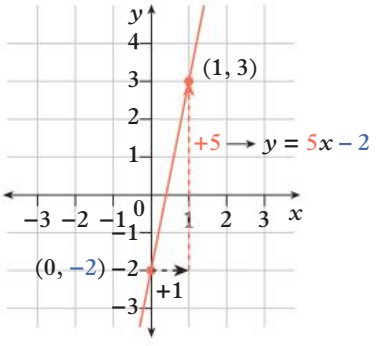
Find the equation of the graph below.



THINK

1. The coefficient of x is equal to the change in y when x increases by 1. Select two consecutive coordinate points and note the change in y when x increases by 1. Each time the value of x increases by 1, the value of y increases by 5, so the coefficient of x is $m = 5$.
2. The constant is equal to the value of y when $x = 0$. When $x = 0$, $y = -2$ so the constant $c = -2$.
3. Write the equation for the graph.

WRITE



The linear equation for the graph is $y = 5x - 2$.

Helpful hints

✔ If you're unsure about a question involving a table, plot the points and sketch the graph; this may help you find the equation!

ANS p526

Exercise 6I Finding linear equations

△ 1–9, 10(a, c) □ 1–6, 8–12 ⬠ 1(d–f), 2–4, 8–13

UNDERSTANDING AND FLUENCY

6I.1 **1** Use the following tables to find the linear equations which represent the relationships between x and y.

a

x	0	1	2	3	4
y	2	4	6	8	10

b

x	0	1	2	3	4
y	4	1	−2	−5	−8

c

x	−2	−1	0	1	2
y	−3	−1	1	3	5

d

x	−2	−1	0	1	2
y	1	−1	−3	−5	−7

e

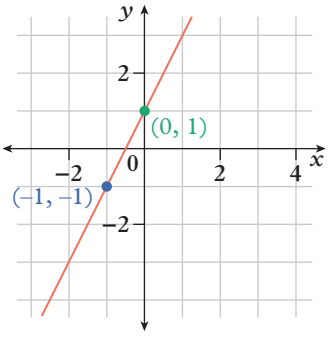
x	−4	−3	−2	−1	0
y	−4	−2	0	2	4

f

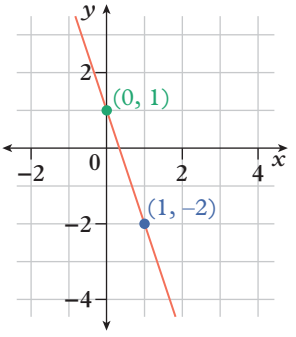
x	−4	−3	−2	−1	0
y	7	4	1	−2	−5

6I.2 **2** Find the equations for each of the graphs below.

a



b



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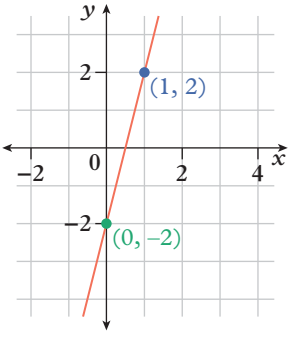

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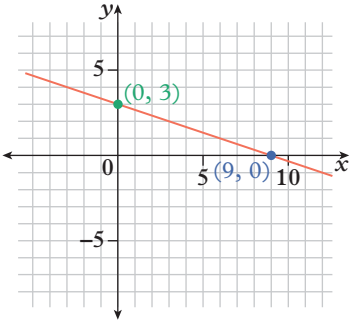

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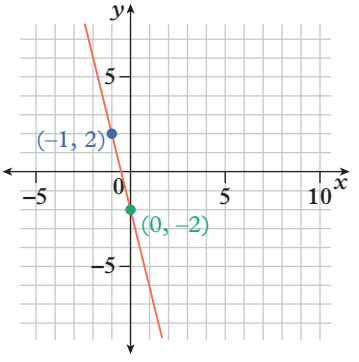

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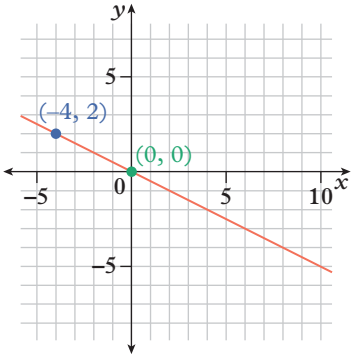


3 Draw the graph of $y = 2x - 3$ by plotting the y-intercept and one other point.

4 Draw the graph of $y = -3x + 2$ by plotting the y-intercept and one other point.

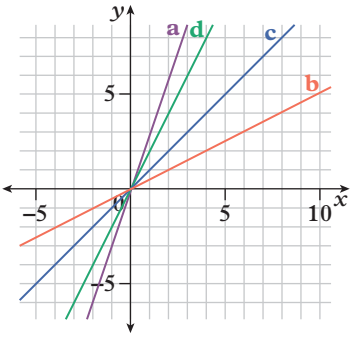
5 Match the graph with the equation.

I $y = 0.5x$

II $y = x$

III $y = 2x$

IV $y = 3x$



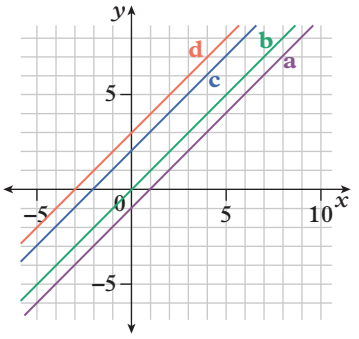
6 Match the graph with the equation.

I $y = x + 2$

II $y = x$

III $y = x - 1$

IV $y = x + 3$



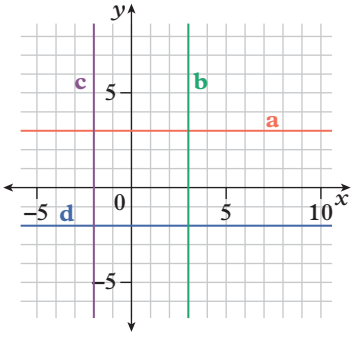
7 Match the graph with the equation.

I $y = 3$

II $x = 3$

III $y = -2$

IV $x = -2$



8 Find the linear equations which are represented in the following tables.

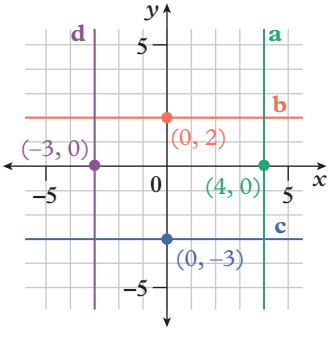
a

x	−4	−2	0	2	4
y	21	13	5	−3	−11

b

x	0	3	6	9	12
y	−5	−14	−23	−32	−41

9 Write the equations of these horizontal and vertical lines.



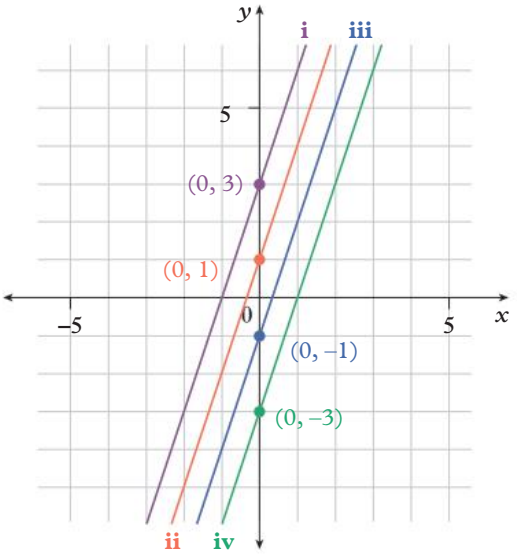
UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

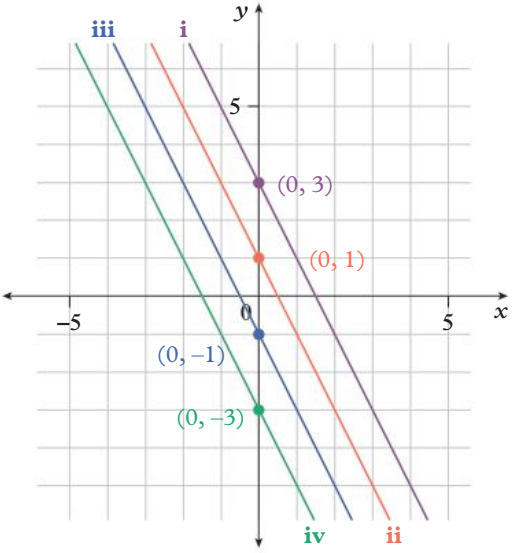
PROBLEM SOLVING AND REASONING

10 Write the equations of each of the linear graphs shown.

a



b



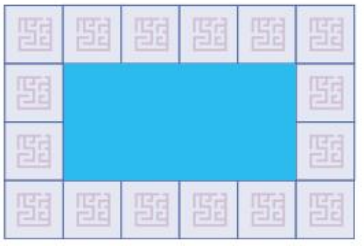
c What do the relationships in part **a** have in common?

d What do the relationships in part **b** have in common?

11 Consider your answers to question **10**. What can you say about linear graphs that have the same value of m in the equation $y = mx + c$?

CHALLENGE

12 Tori used 16 tiles to make one row of tiles around her pool as shown. Tori wants to add more rows to the path and needs to know how many tiles she will need for each additional row.

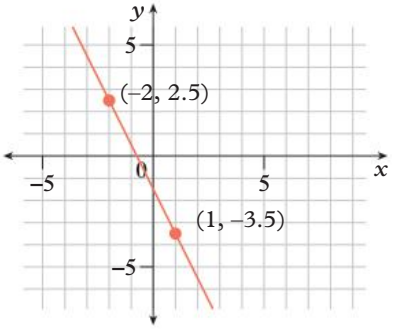


a Complete the following table.

Row	1	2	3	4
Tiles in the row	16			

b Use the table to write an equation that will give Tori the number of tiles she needs for each row.

13 Find the equation of this straight line.



Check your Student obook pro for these digital resources and more:

Interactive skillsheet Determining linear equations by inspection

Investigation Up, up and away!

Topic quiz 6I

Chapter summary

Equations

equals sign

$$2x + 1 = 5$$

left-hand side of equation (LHS) — right-hand side of equation (RHS)

- A solution to an equation makes the equation a true statement.
- To solve by inspection, identify the correct value of the pronumeral.

Changing the subject of a formula

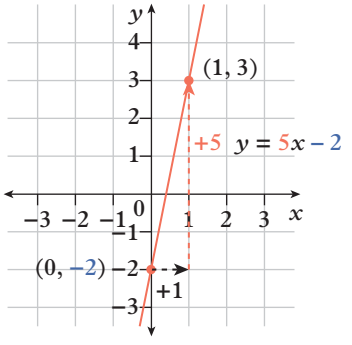
- An unknown value in an equation can be determined by substitution if the values of all other variables are known.
- Inverse operations can be used in equations with several variables to change which variable is the subject of the equation.

Finding linear equations

All linear equations can be written in the form

$$y = mx + c$$

- m is equal to the change in value of y as x increases by 1.
- c is equal to the value of y when $x = 0$.



Solving an equation using inverse operations

1. Identify the operation(s) acting on the pronumeral.
2. Identify the inverse operations.
3. Apply the inverse operations to both sides of the equation.

$x + 1 = 5$

$x + 1 - 1 = 5 - 1$ ← inverse operations

$x = 4$ ← solution

Solving equations with the unknown on both sides

1. Use inverse operations to remove the pronumeral term from one side.
2. Solve the equation using inverse operations.

$4x - 2 = 3x + 1$

$4x - 3x - 2 = 3x - 3x + 1$

$x - 2 = 1$

$x - 2 + 2 = 1 + 2$

$x = 3$

Solving linear equations using graphs

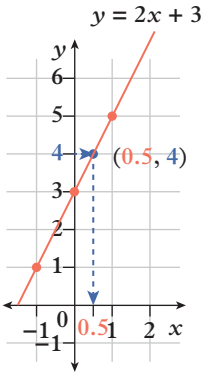
1. Make the side of the equation containing the unknown equal y.
2. Plot the graph of the equation from step 1.
3. Locate the value from the other side of the equation on the y-axis, and read off the corresponding value of x.

+1 +1 +1 +1

x	−2	−1	0	1	2
y	−1	1	3	5	7

+2 +2 +2 +2

→ $y = 2x + 3$



$2x + 3 = 4$

$x = 0.5$

Solving simple quadratic equations

- Quadratic equations are equations where the largest index of the variable is 2.
- The equation $x^2 = c$ has two solutions if c is positive, one solution if $c = 0$ and no solutions if c is negative.

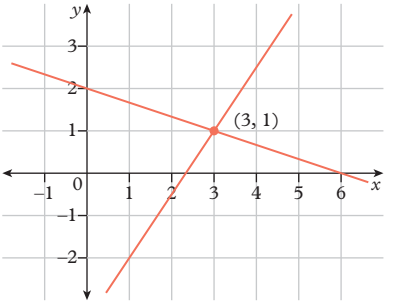
$ax^2 = c$ $(\div a)$

$x^2 = \frac{c}{a}$

$x^2 = \pm\sqrt{\frac{c}{a}}$ (take the square root on both sides)

Intersecting lines

- Points of intersection can be determined graphically by plotting the equations on the same set of axes.
- Points of intersection can also be determined using a table of values.
- A point of intersection satisfies the equation of both lines.



ANS p527

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- Cartesian plane
- coordinates
- equation
- equivalent equations
- graph
- inverse operations
- linear graph
- linear relationship
- origin
- plot
- point of intersection
- quadratic equation
- solution
- solve by inspection
- subject
- x-axis
- x-coordinate
- y-axis
- y-coordinate
- y-intercept

1 On the Cartesian plane, label the x-axis, y-axis and the origin.

2 Fill in the blanks using terms from the key word list above.

a The _____ of a point tells you how far above or below the origin a point is located.

b The _____ of a point tells you how far left or right of the origin a point is located.

3 Identify the key terms being referenced in each of these definitions.

a mentally identifying the number which when substituted makes an equation a true statement

b the point at which two lines meet

c the point at which a graph crosses the y-axis

4 Using an example, provide a definition in your own words for the following key terms.

a inverse operations **b** linear relationship **c** subject **d** quadratic equation

Multiple choice

6A **1** The solution to $x + 5 = 12$ is:

A $x = 17$ **B** $x = 12$ **C** $x = 7$ **D** $x = 5$ **E** $x = 1$

6A **2** $y = -4$ is the solution to which equation?

A $y + 4 = 0$ **B** $2y - 3 = 4$ **C** $3y - 12 = 0$ **D** $4 - y = 0$ **E** $8 - 2y = 0$

6A **3** A dance class has double the number of students that it had last week. Four of the students have to leave early, leaving 28 students in the class.

Which of the following equations can be used to determine how many students were in the class last week?

A $2s + 4 = 28$ **B** $2s - 4 = 28$ **C** $\frac{s}{2} + 4 = 28$ **D** $\frac{s}{2} - 4 = 28$ **E** $\frac{s}{4} = 28$

6B **4** Which equivalent equation is formed when 5 is subtracted from both sides of $2x - 6 = 14$?

A $2x - 1 = 14$ **B** $2x - 6 = 9$ **C** $2x - 11 = 9$ **D** $2x - 1 = 9$ **E** $x = 10$

6B **5** Which of the following operation(s) should be applied to both sides of the equation to solve $\frac{3x}{4} = 5$?

A subtract 5

B divide by $\frac{4}{3}$

C divide by 4, then multiply by 3

D multiply by $\frac{3}{4}$

E multiply by 4, then divide by 3

6C **6** What is the solution to $3x + 7 = 2x + 9$?

A $x = 16$ **B** $x = -16$ **C** $x = \frac{16}{5}$ **D** $x = 2$ **E** $x = -2$

6C **7** The value of x in the equation $8x + 15 = 3 - x$ is:

A $x = -3$ **B** $x = 2$ **C** $x = -2$ **D** $x = \frac{4}{3}$ **E** $x = -\frac{4}{3}$

6D **8** Which equation is equivalent to $y = 2x + 5$ where x is the subject?

A $2x = y - 5$ **B** $x = \frac{y - 5}{2}$ **C** $x = \frac{y + 5}{2}$ **D** $x = \frac{y}{2} - 5$ **E** $x = 2y + 5$

6E **9** What kind of solutions exist to the equation $x^2 + 9 = 0$?

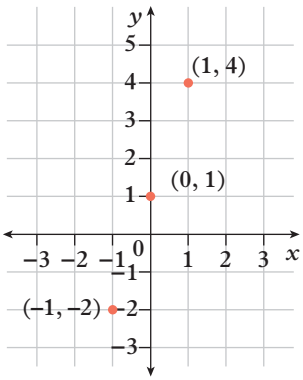
A no solutions
B one integer solution
C two integer solutions
D two non-integer solutions
E one non-integer solution

6E **10** What are the solution(s) for x in the equation $v = 2x^2 + a$ where $v = 90$ and $a = 40$?

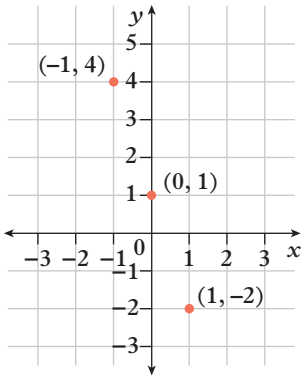
A 5 **B** ± 10 **C** ± 5 **D** 10 **E** ± 25

6F **11** Which of the following plot points satisfy the equation of $y = 1 - 3x$?

A



B



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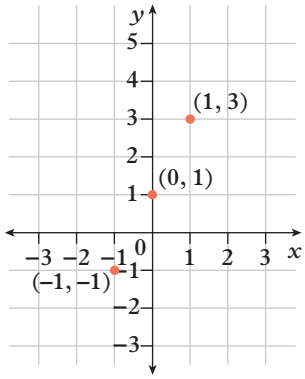
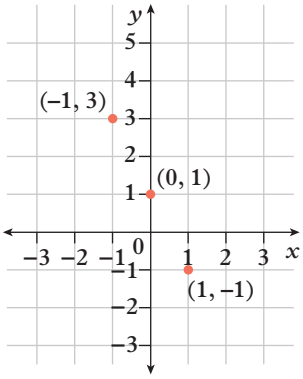
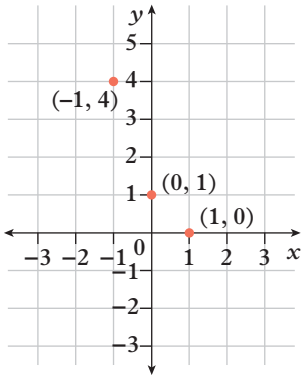


D



E



6G **12** Consider the set of related equations below.

$\frac{x}{2} + 5 = 7$ $\frac{x}{2} + 5 = 0$ $\frac{x}{2} + 5 = -1$

The equation of the graph that can be used to solve this set of equations is:

A $\frac{x}{2} + 5 = 7$ **B** $\frac{x}{2} + 5 = 0$ **C** $\frac{x}{2} + 5 = -1$ **D** $y = \frac{x}{2}$ **E** $y = \frac{x}{2} + 5$

6H **13** What is the point of intersection for the lines $y = x$ and $y = -2x + 3$?

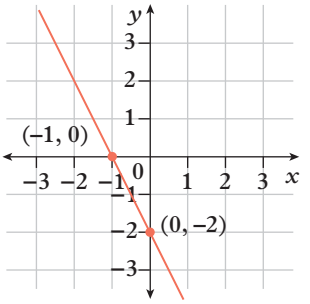
A $(1, 1)$ **B** $(1, -1)$ **C** $(3, 3)$ **D** $(3, 1)$ **E** $(-1, 1)$

6I **14** The equation for the table of values below is:

x	-2	-1	0	1	2
y	-5	-3	-1	1	3

A $y = x + 1$ **B** $y = 2x - 1$ **C** $y = 2x + 1$ **D** $y = -2x - 1$ **E** $y = -2x + 1$

6I **15** The equation of this graph is:



A $y = -2x - 2$
B $y = 2x - 2$
C $y = 2x + 2$
D $y = -2x + 2$
E $y = -x - 2$

Short answer

6A **1** Use substitution to determine whether $x = -3$ is the solution to the following equations.

a $x + 11 = 8$ **b** $5x = -15$ **c** $4x + 12 = 0$ **d** $2x - 3 = 9$

6A **2** Solve the following equations by inspection.

a $a + 7 = 16$ **b** $b - 6 = 8$ **c** $4c = 12$ **d** $\frac{d-7}{4} = 2$

6B **3** Use inverse operations to solve the equations.

a $5x - 2 = 13$ **b** $-\frac{1}{3}x + 8 = 11$ **c** $\frac{4c}{7} = 8$

d $4(3d + 5) = 44$ **e** $\frac{4x}{5} + 12 = 13$ **f** $\frac{6 - 8x}{7} = 10$

6B **4** A bookstore owner is shipping small books and large books in a box. Small books weigh half as much as large books. There are 12 of each type of book in the box.

a If a small book weighs s kg, write an equation to represent this situation given that a box weighs 36 kg.

b Solve the equation and find the weight of a large and small book.

6C **5** Solve:

a $5x + 6 = 3x - 4$ **b** $8x + 3 = 6x + 7$

c $-4x - 15 = 6x + 15$ **d** $x + 3 = -2 - x$

6C **6** Suzi's age in 11 years will be equal to five times her age five years ago. Write an equation and solve it to find how old Suzi is now.

6D **7** Rearrange $y = ax + by + c$ to make the subject of the equation:

a x **b** a **c** y.

6D **8** Determine the value of x in the equation $y = \frac{a}{b}(x - a) + c$ where $a = 4$, $b = 2$, $c = 5$ and $y = 17$.

6E **9** Solve the following equations for x.

a $x^2 = 36$ **b** $3x^2 = 108$ **c** $x^2 + 5 = 5$

6F **10** Plot a graph of each of the following relationships by first completing a table of values for integer values of x from -3 to 3.

a $y = x - 4$ **b** $y = 4x + 1$ **c** $y = 3 - x$ **d** $y = 6 - 3x$

6G **11** Write an equation for each of the tables below.

a

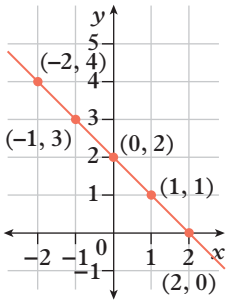
x	−2	−1	0	1	2
y	−13	−9	−5	−1	3

b

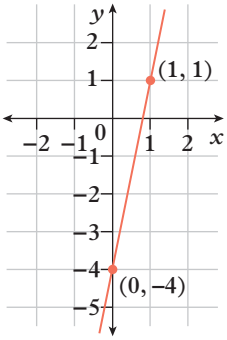
x	−2	−1	0	1	2
y	2	1	0	−1	−2

6G **12** Determine the equations of the graphs below.

a



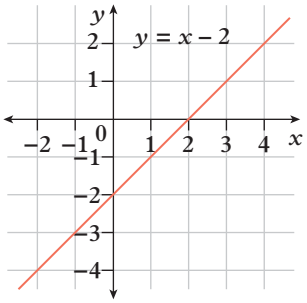
b



6H **13** Plot the graphs of the equations $y = 2x + 2$ and $y = x + 1$ on the same set of axes. Determine the point of intersection of these graphs visually and confirm it using a table of values.

6I **14** Refer to the graph of $y = x - 2$ on the right to find the value of x that corresponds to each of the following values of y.

a $y = 2$ **b** $y = -3$ **c** $y = 1$ **d** $y = 0$



6I **15** Refer to the graph of $y = x - 2$ on the right to solve each equation.

a $x - 2 = -4$ **b** $x - 2 = -1$ **c** $x - 2 = -2$

Analysis

1 Joss is the manager of a bookstore. He uses Crate Couriers to deliver his orders. Crate Couriers charge Joss \$5 per book plus \$3 flat fee per delivery.

a Write an equation to find the total cost, C, of delivering x books.

b How much does it cost Joss to have 10 books delivered to a customer?

c How many books can Joss have delivered for \$38?

d Complete the table below.

Number of books	0	1	2	3	4	5
Cost of delivery						

e Plot a graph of cost of delivery vs number of books for Crate Couriers.

A new courier approaches Joss. Crown Couriers don't have a flat fee but they charge \$6 per book delivery. At first Joss thinks Crown will be too expensive but he decides he should investigate further.

f Fill in the table for Crown Couriers below.

Number of books	0	1	2	3	4	5
Cost of delivery						

g On the same graph as part **e**, plot the graph of cost of delivery vs number of books for Crown Couriers.

h After looking at the two graphs Joss can see that the situation is not so simple. Explain when Joss should use Crate Couriers and when he should use Crown Couriers.

2 A suburban party venue costs \$150 to hire, plus \$20 per person for games and food.

a If p people attend the party, write an equation for the total cost, C, of the venue hire, games and food.

b Kacie's mother gives her \$450 for venue hire, games and food. Substitute this value into the equation to determine how many people Kacie can have at her party.

Kacie finds another party venue in the city that offers a similar service. Their costs consist of a hire fee of \$250 plus \$15 per person for games and food.

c Write a second equation for the total cost, C, of venue hire, games and food for a party attended by p people at the second venue.

d Complete the tables below for each venue.

Suburban venue

p	0	10	20	30	40	50
C						

City venue

p	0	10	20	30	40	50
C						

e Use the tables of values to plot a graph of each relationship on the same set of axes. Label each graph.

f Do these graphs show a linear relationship? Why or why not?

g Use the graphs to find:

i the cost of inviting 25 people to a party at each venue

ii how many people Kacie can invite to a party at each venue with a budget of \$450.

Although the city venue has a higher hire fee, if Kacie invites more than a certain number of people, the city venue will be cheaper than the suburban venue.

h Use your graph to determine which venue will be cheaper if Kacie invites:

i 10 people **ii** 50 people **iii** 20 people.

3 Sometimes collected data can be represented as a table of values and then graphed on the Cartesian plane.

a Plot the points in the following table of values on the Cartesian plane.

x	0	1	1	2	3	4	4	5	5	6	7	8
y	0	1	2	3	3	4	5	6	7	8	9	10

b Does the table of values in part **a** correspond to a linear relationship? Why or why not?

c Graph the following linear relationships on the same Cartesian plane as in part **a**.

A line of best fit is a line which best approximates data plotted on the Cartesian plane.

i $y = x$

ii $y = 2x$

d Give a reason why each of the linear relationships in part **c** are not good lines of best fit for the table of values in part **a**.

The linear relationship $y = \frac{5}{4}x$ is a good line of fit for the table of values in part **a**. To be a good line of best fit, there should be the same number of points above and below the line, and ideally, some points on the line.

e Plot the points in the table of values and the line $y = \frac{5}{4}x$ on a new set of axes.

f Explain why $y = \frac{5}{4}x$ is a better line of best fit compared to the lines in part **c**.

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Check solutions using substitution ☐ Solve simple linear equations by inspection ☐ Write equations to represent worded problems	☐ Go back to Topic 6A Equations
☐ Solve linear equations using inverse operations	☐ Go back to Topic 6B Solving linear equations
☐ Solve linear equations with the unknown on both sides	☐ Go back to Topic 6C Solving equations with the unknown on both sides
☐ Solve for unknown values by first substituting known values ☐ Rearrange an equation with only the subject on one side	☐ Go back to Topic 6D Changing the subject of a formula
☐ Explain why there are two solutions to simple quadratic equations ☐ Solve simple quadratic equations ☐ Solve simple quadratic equations after substituting known values into a formula	☐ Go back to Topic 6E Solving simple quadratic equations
☐ Use equations to complete tables of values ☐ Plot linear graphs from tables of values and equations	☐ Go back to Topic 6F Plotting linear relationships
☐ Solve linear equations using a graph	☐ Go back to Topic 6G Solving linear equations using graphs
☐ Identify the intersection point of two lines graphically ☐ Identify the intersection point of two lines using a table of values ☐ Verify a point is the point of intersection of two lines	☐ Go back to Topic 6H Intersecting lines
☐ Find the linear equation for a table of values ☐ Find the linear equation for a graph when the coordinates are known	☐ Go back to Topic 6I Finding linear equations

7 Triangles and quadrilaterals

Index

7A Angles
7B Triangles
7C Quadrilaterals
7D Pythagoras' theorem
7E Finding the length of a shorter side
7F Applications of Pythagoras' theorem

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Solving equations by inspection
- ✔ Like terms
- ✔ Solving equations using inverse operations

Curriculum links

- Applies Pythagoras' theorem to solve problems in various contexts (MA4-PYT-C-01)
  - Identify and define Pythagoras' theorem
  - Examine problems involving Pythagoras' theorem
- Identifies and applies the properties of triangles and quadrilaterals to solve problems (MA4-GEO-C-01)
  - Classify triangles according to their side and angle properties
  - Classify quadrilaterals and describe their properties
  - Apply the properties of triangles and quadrilaterals

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Materials

- ✔ Compass
- ✔ Ruler
- ✔ Protractor

7A Angles

Learning intentions

By the end of this topic you will be able to ...

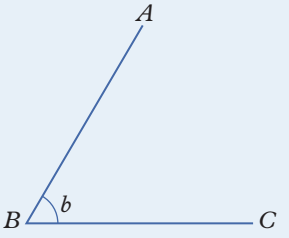
✔ calculate and use complementary and supplementary angles
✔ calculate and use angles at a point
✔ calculate and use related angles on parallel lines.

Inter-year links
Support — Understanding angles
Year 7 — 7B Angles at a point

Types of angles

Key content video

- Angles can be named using either a pronumeral, for example b, or an angle symbol followed by capital letters which represent the vertices of the angle, for example $\angle ABC$.
- Angles are classified according to their size:
  - → **acute**: between 0° and 90°
  - → **right**: 90°
  - → **obtuse**: between 90° and 180°
  - → **straight**: 180°
  - → **reflex**: between 180° and 360°
  - → **revolution**: 360°.

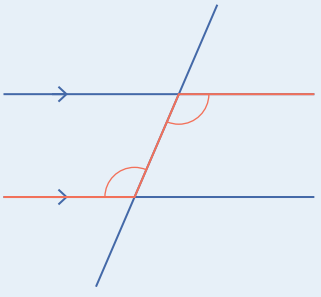


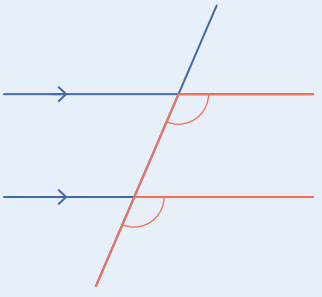
Angle properties

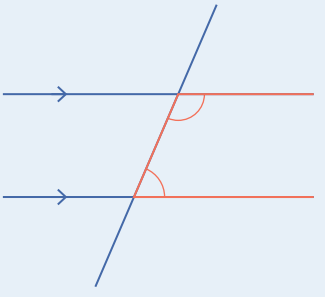
- Angles at a point that form a straight line add to 180°.
- Angles at a point that form a revolution add to 360°.
- Two angles that add to 90° are called **complementary angles**.
- Two angles that add to 180° are called **supplementary angles**.
- Angles at a point form **vertically opposite angles** that are equal in size.

Angles and parallel lines

- When two or more lines have arrowheads on them, this means that they are parallel. Multiple arrowheads may be used if there is more than one pair of parallel lines in a diagram.
- A **transversal** is a straight line that cuts across two or more other lines.
- When two or more **parallel lines** are intersected by a transversal:
  - → **Alternate angles** are equal.
  - → **Corresponding angles** are equal.
  - → **Co-interior angles** are supplementary.







- Related angles can be used to determine if two lines are parallel. If any of the relationships listed above are true, then the lines crossed by a transversal are parallel.

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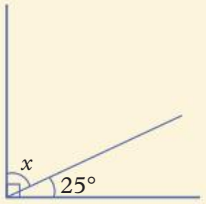
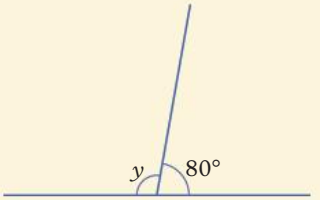
Example 7A.1 Using complementary and supplementary angles

Find the size of each labelled unknown angle.

a



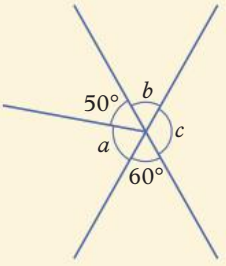
b



THINK	WRITE
a **1** Identify the relationship between the unknown and known angles. x and 25° are complementary angles.	**a** $x = 90° - 25°$
2 Calculate the size of the unknown angle. Complementary angles add to 90°, so $x = 90° - 25°$.	$= 65°$
b **1** Identify the relationship between the unknown and known angles. y and 80° are supplementary angles.	**b** $y = 180° - 80°$
2 Calculate the size of the unknown angle. Supplementary angles add to 180°, so $y = 180° - 80°$.	$= 100°$

Example 7A.2 Using angles at a point

Find the size of each labelled unknown angle.



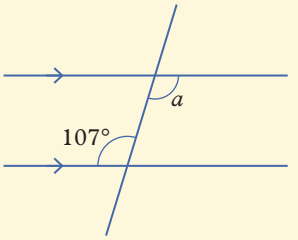
THINK	WRITE
1 Identify any relationships between unknown and known angles. b is vertically opposite to 60° and vertically opposite angles are equal in size.	$b = 60°$
2 c and 60° are supplementary angles, so $c = 180° - 60°$.	$c = 180° - 60°$ $= 120°$
3 50°, a and 60° form a straight angle, so $a = 180° - 50° - 60°$.	$a = 180° - (50° + 60°)$ $= 70°$

Example 7A.3 Finding unknown angles on parallel lines

For the following diagram:

a Identify the relationship between the unknown and known angles.

b Find the size of the angle a.



THINK

a a and 107° are the internal angles of a reverse 'Z' shape, so they are alternate angles.

b Alternate angles formed by parallel lines are equal in size.

WRITE

a a and 107° are alternate angles.

b $a = 107°$

Helpful hints

✔ Unless stated, you will not need to measure any of the angles in a diagram. All the information required to answer questions will be provided.

✔ Don't assume that two given lines are parallel if they look like they are parallel in a diagram! Pay attention to whether lines have parallel marks or if you are given that information in a question.

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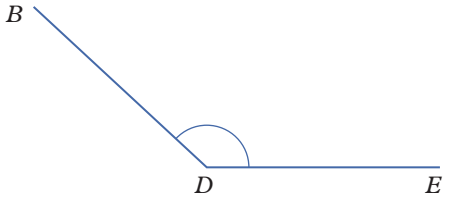
Exercise 7A Angles

△ 1–9, 11–13 | □ 1, 2(a, d, e), 3–5, 6(e–i), 7–9, 12–15, 17(a) | ⬠ 1(d–f), 3–5, 6(f–i), 7(d–f), 8, 10–12, 16–18

UNDERSTANDING AND FLUENCY

1 Name each angle using the vertices and state what type of angle it is.

a



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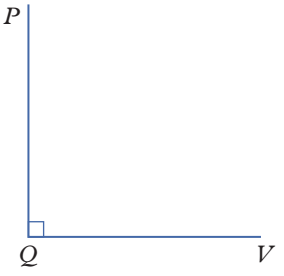

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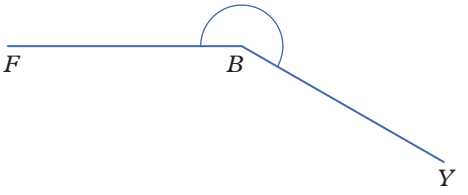

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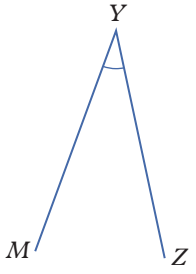

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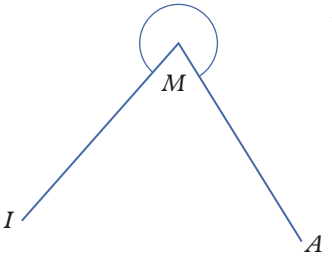

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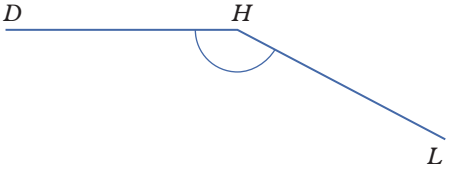
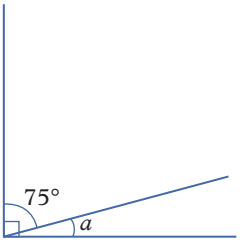


7A.1 **2** Find the size of each labelled unknown angle.

a



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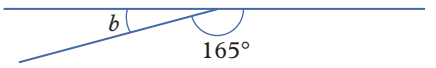

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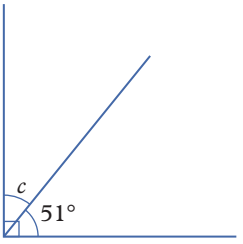

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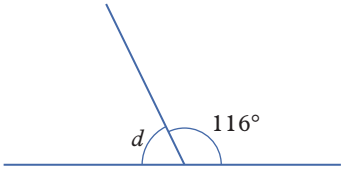

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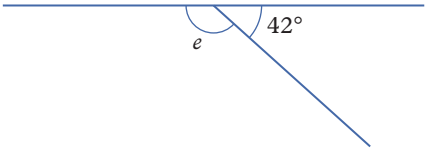

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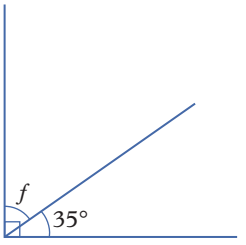
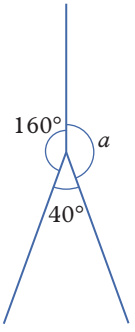


7A.2 **3** Find the size of each labelled unknown angle.

a



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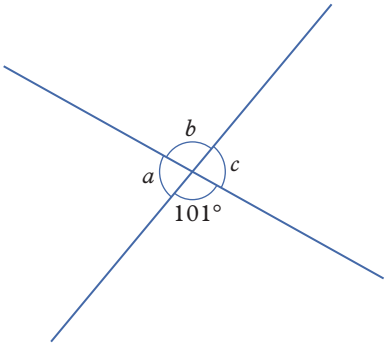

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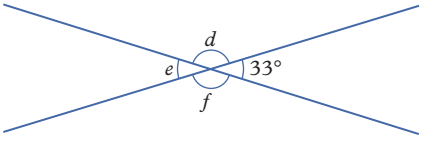

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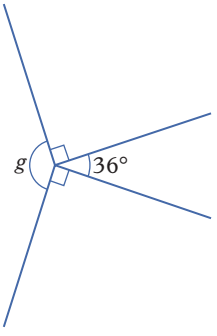

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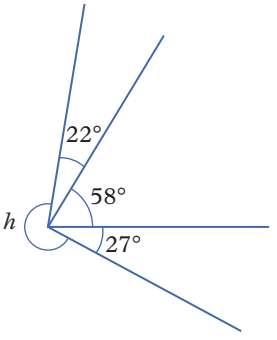

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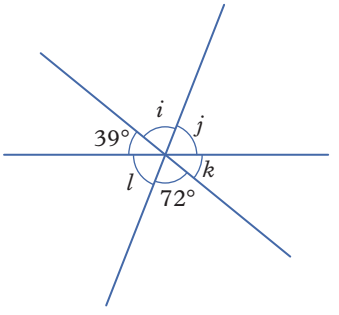


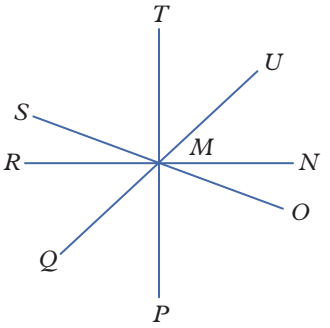
4 Identify the relationship (complementary, supplementary or vertically opposite) between each pair of angles in the following diagram.

a $\angle UMT$ and $\angle NMU$

b $\angle OMT$ and $\angle PMO$

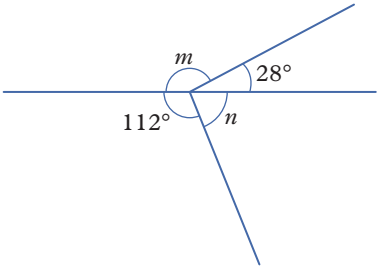
c $\angle SMR$ and $\angle NMO$

d $\angle TMQ$ and $\angle PMQ$



5 Find the value of each pronumeral without using a protractor.

a



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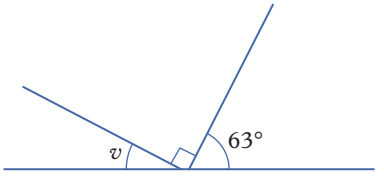

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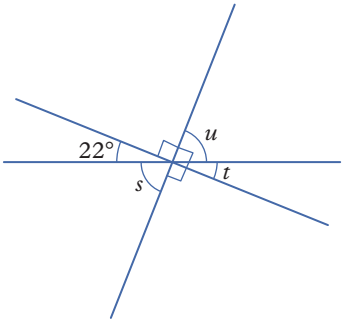

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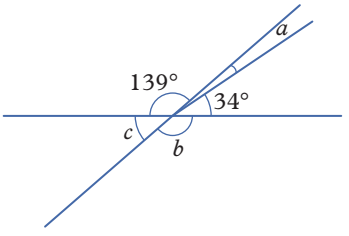

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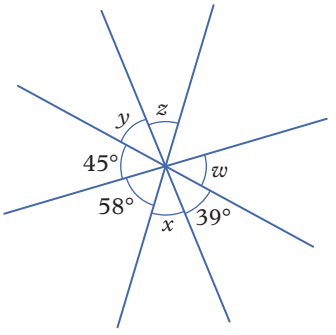

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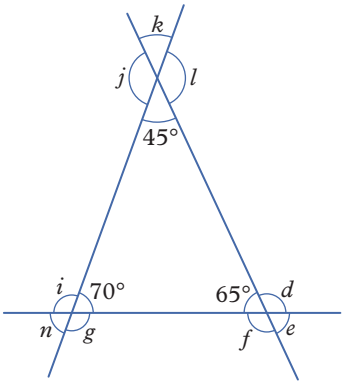
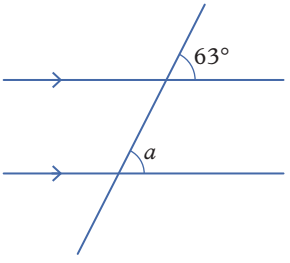


7A.3 **6** For each of the following diagrams:

i Identify the relationship between the unknown and known angles.

ii Find the size of the labelled unknown angle.

a



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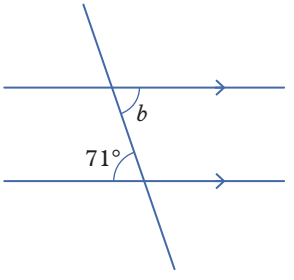

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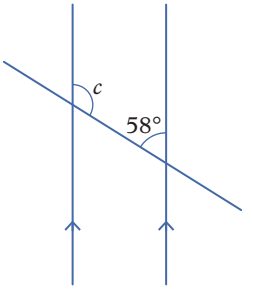

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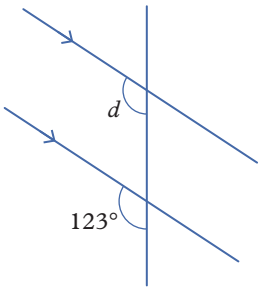

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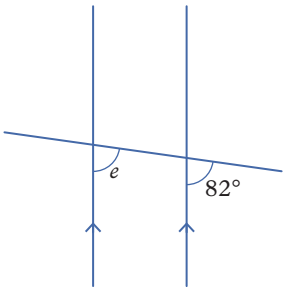

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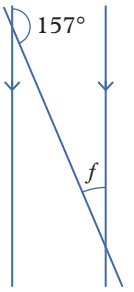

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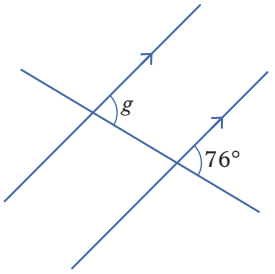

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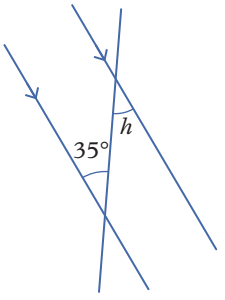

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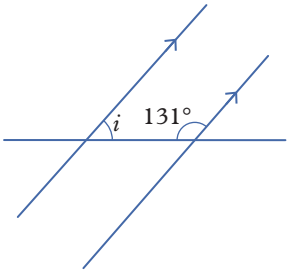
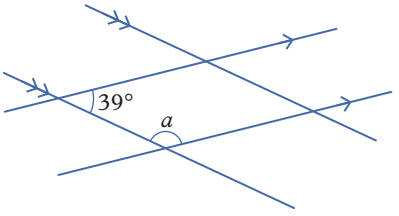


7 Find the size of each labelled unknown angle. In each case, state whether the angle is alternate, corresponding or co-interior to the given angles.

a



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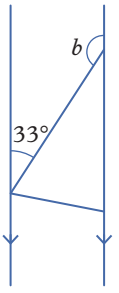
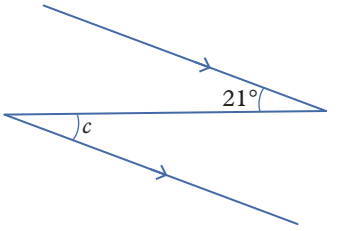


c



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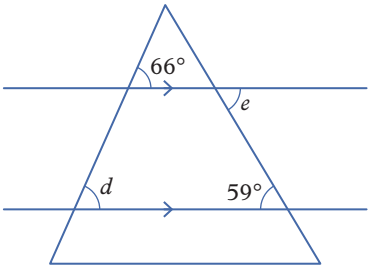

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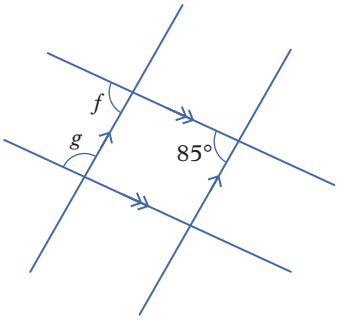

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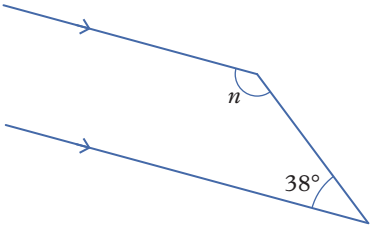
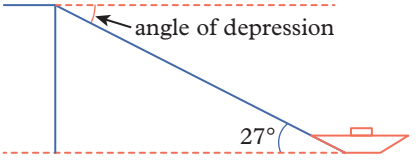
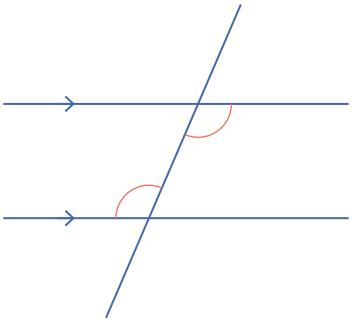


8 The angle of elevation (from a boat to a cliff top) is 27°. What is the angle of depression from the cliff top to the boat? Justify your answer.

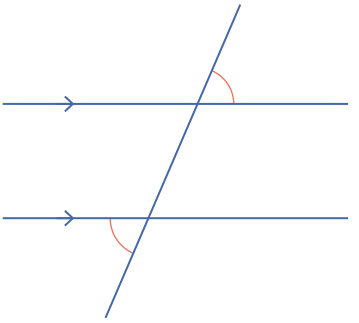


9 The Australian standard for staircases prefers that the angle that stairs make with the horizontal be between 30° and 38°, and states that it must be between 20° and 45°. The handrail is parallel to the stairs and all vertical support posts are parallel to each other. If one of the angles between the handrail and the support posts is 67°, determine if this staircase meets the Australian standard and if it is in the preferred range.

10 Alternate angles usually refer to the angles on opposite sides of the transversal between two lines; however, the angles on the outside of the lines on opposite sides of the transversal are also alternate angles. To distinguish between the two, we call the ones between the lines alternate interior angles and the ones outside of the lines alternate exterior angles. Explain why alternate exterior angles will be equal for parallel lines.
Note: You can assume the term 'alternate angles' always refers to alternate interior angles, unless specified otherwise.



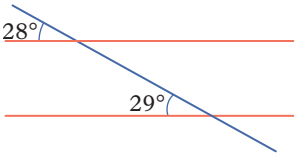
alternate interior angles



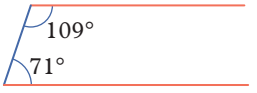
alternate exterior angles

11 Use your knowledge of co-interior, alternate and corresponding angles to check whether the pairs of red lines are parallel or not. Give a reason why each pair is or is not parallel.

a



b



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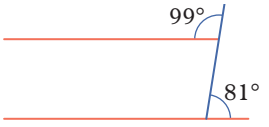
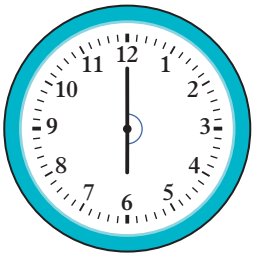


12 An understanding of angles around a point can help you to draw analogue clocks accurately.

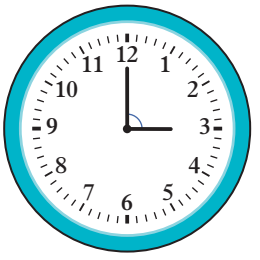
a How many degrees are in a circle?

b Find the size of the angle indicated on each clock.

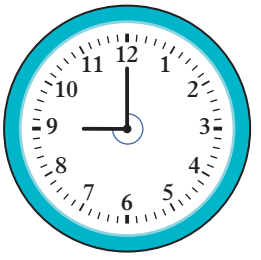
i



ii



iii

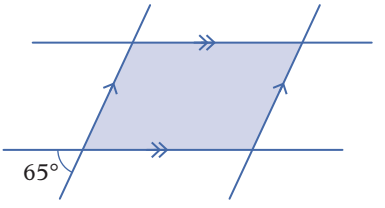


c What size angle would be between the hands of a clock displaying these times? Start from 12 o'clock and move clockwise to the other hand.

i 4:00 pm **ii** 11:00 am **iii** 8:00 pm **iv** 1:00 am

13 a What shape is formed in the diagram on the right?

b Use your knowledge of angles and parallel lines to state the size of all the internal angles of the shape.

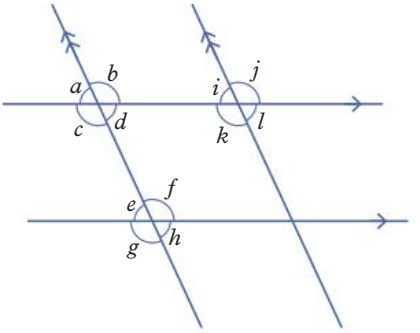


14 In the diagram on the right, identify all pairs of:

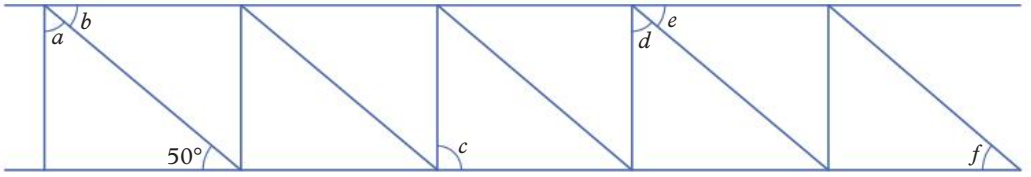
a alternate angles

b corresponding angles

c co-interior angles.



15 The diagram below shows a building's structural support beams. If you know that the main two horizontal beams are at right angles to the vertical beams, and the slanted beams are all parallel to each other, use the information provided to find the size of each angle labelled with a pronumeral, giving reasons for your answers.



16 a If a pair of parallel lines is cut by a single transversal, explain how knowing the size of one angle in the diagram means that you can find the size of every other angle.

b If a pair of parallel lines is cut by a number of transversals that are parallel to each other, explain how knowing the size of one angle in the diagram still means that you can find the size of every other angle.

17 a What is the size of the angle between two consecutive hour marks on a clock face?

b What is the size of:

i half this angle

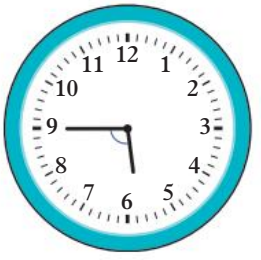
ii a quarter of this angle

iii three quarters of this angle?

c Use your answers to parts **a** and **b** to find the size of the angle indicated on these clocks.

i

ii



iii

18 Find the size of the smaller angle between the two hands of a clock displaying these times.

a 10:20 **b** 6:55 **c** 4:35 **d** 2:27 **e** 11:09

Use these hints to help you.

- How many minute marks are displayed on a clock?
- What is the size of the angle between two of the minute marks?
- What is $\frac{1}{60}$ of the size of the angle between two of the minute marks?
- How far does the small hand move every time the big hand moves 1 minute?

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Classifying angles

Interactive skillsheet Angles at a point

Interactive skillsheet Angles and parallel lines

Worksheet Identifying and drawing angles

Investigation Angles and parallel lines using technology

Topic quiz 7A

7B Triangles

Learning intentions

By the end of this topic you will be able to ...

✔ classify triangles

✔ calculate unknown interior and exterior angles of triangles.

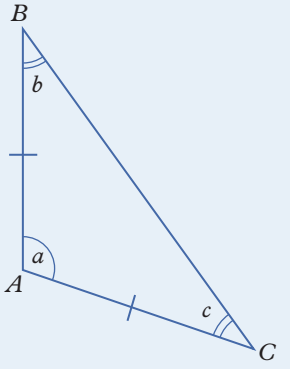
Inter-year links

Year 7 7D Classifying triangles

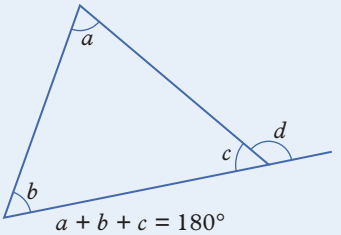
Triangle properties

Key content video

- A triangle is a two-dimensional (2D) shape with three straight sides.
- The **interior angles** of a triangle add to 180°.
- Triangles can be classified by their side lengths and interior angles.
  - → An **equilateral triangle** has three sides of equal length and three interior angles of 60°.
  - → An **isosceles triangle** has two sides of equal length and two interior angles of equal size. The equal angles are opposite the sides of equal length.
  - → A **scalene triangle** has three sides of different length and three different interior angles.
  - → An **acute-angled triangle** has three interior acute angles (less than 90°).
  - → A **right-angled triangle** has one interior right angle (90°) and two interior acute angles (less than 90°).
  - → An **obtuse-angled triangle** has one interior obtuse angle (more than 90°) and two interior acute angles (less than 90°).
- When drawing geometric shapes, sides of equal length are marked using hatch marks or dashes, and angles of equal size are marked using the same number of arcs.
  For example, in this triangle, sides AB and AC are of equal length and angles b and c are equal in size.



- An **exterior angle** is an angle between a side of a shape and a line extended from the adjacent side.
- The exterior angle of a triangle is supplementary to the adjacent interior angle; therefore, it is also equal to the sum of the two opposite interior angles.



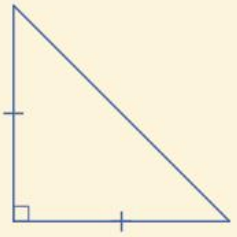
$a + b + c = 180°$

$c + d = 180°$

$d = a + b$

Example 7B.1 Classifying triangles

Classify the following triangle according to its sides and angles.



THINK

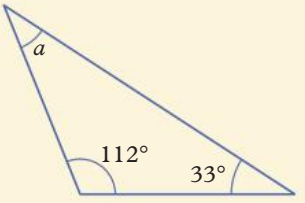
1 Determine how many sides of the triangle are equal in length. The triangle has two sides equal in length, so it is an isosceles triangle.
2 Look at the angles in the triangle. The triangle has a right angle, so it is a right-angled triangle.

WRITE

This is a right-angled isosceles triangle.

Example 7B.2 Finding an unknown angle in a triangle

Find the size of angle a.



THINK

The interior angles in a triangle add to 180°. Subtract each of the known angles from 180° to calculate the third angle, a.

WRITE

$a = 180° - 112° - 33°$
$= 35°$

Example 7B.3 Finding the exterior angle of a triangle

Find the size of angle e.

THINK

The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

WRITE

$e = 60° + 90°$
$= 150°$

Helpful hints

- ✔ Don't guess that a triangle is equilateral or isosceles based on how it looks! This should only be determined once you know the side lengths and/or angles of a triangle.
- ✔ Every triangle with exactly two equal angles will have two sides of equal length opposite those angles and is an isosceles triangle.

ANS p530

Exercise 7B Triangles

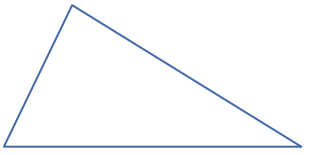
1–6, 7(a, b, d), 8, 9 | 2–6, 7(b, c, e), 10–13 | 2(d, f), 3(e, f), 4, 5, 7(e, f), 10, 11, 13, 14

UNDERSTANDING AND FLUENCY

7B.1 **1** Classify each triangle according to its sides and angles.

a

b



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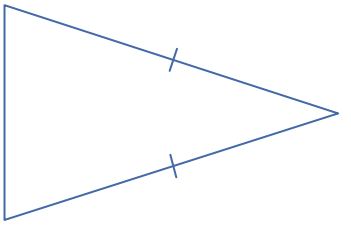

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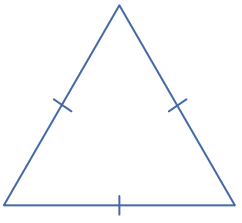

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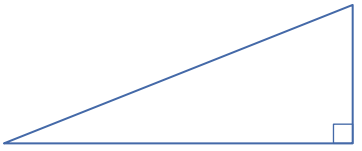
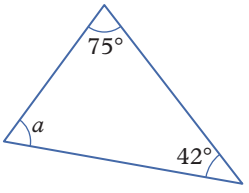


f

7B.2 **2** Find the size of each unknown angle.

a



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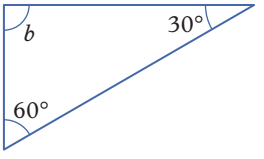

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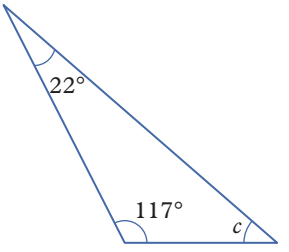

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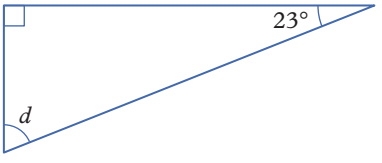

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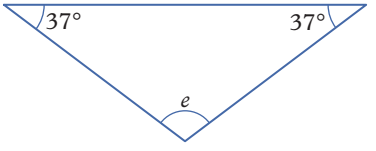

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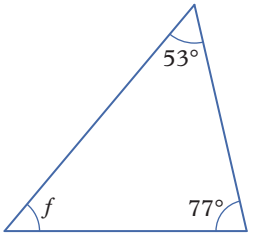
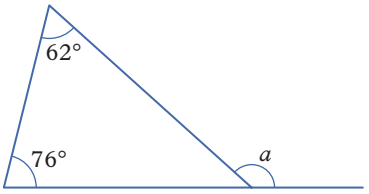


7B.3 **3** Find the size of each labelled unknown angle.

a



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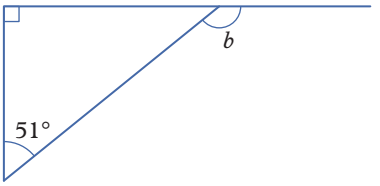

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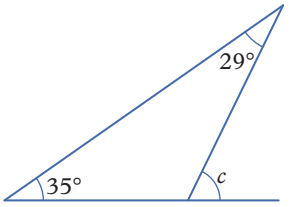

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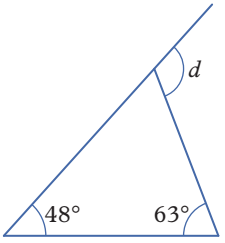

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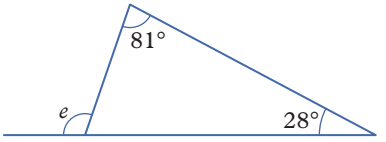

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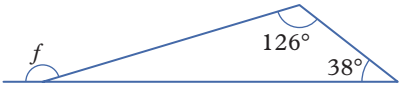
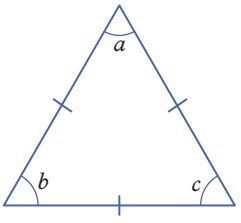


4 Use your knowledge of triangle properties to find the size of each labelled unknown angle.

a



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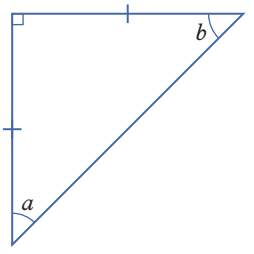

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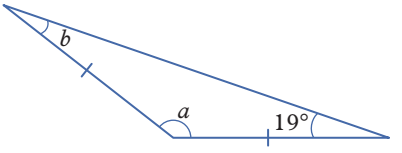

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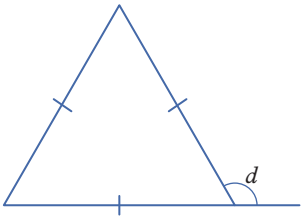

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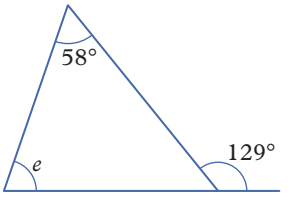

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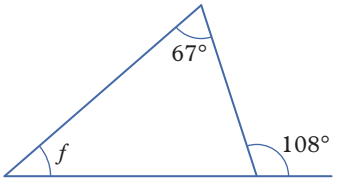

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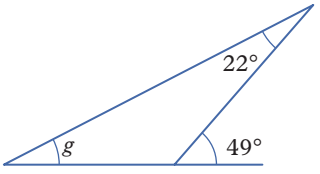

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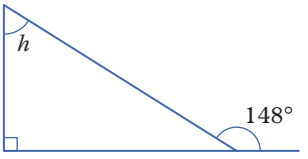
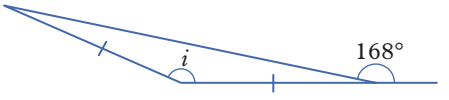


i



5 Classify each of the triangles from question **4** according to its sides and angles.

6 Answer true or false to the following statements, giving a reason for your answers.

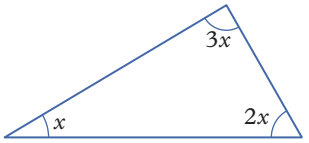
a An equilateral triangle can have a right angle.

b An isosceles triangle can have an obtuse angle.

c A scalene triangle can have a right angle.

7 Use your knowledge of triangle properties and algebra to find the value of x in each triangle.

a



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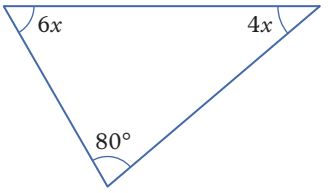

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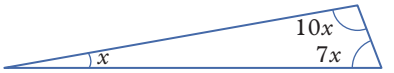

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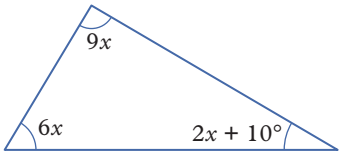

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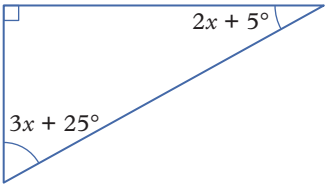

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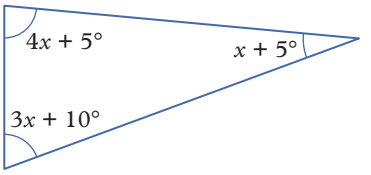
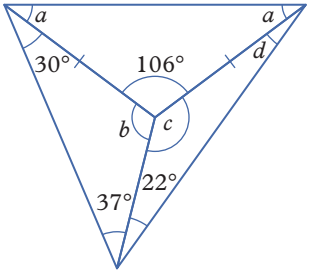


8 Rebecca measures the angles in a triangle and finds they are in a ratio of 1 : 2 : 3. That is, angle b is twice as big as angle a and angle c is three times as big as angle a.

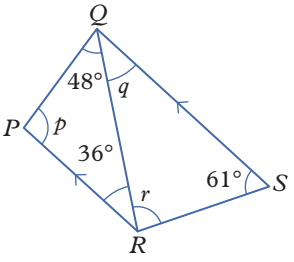
a Find the size of each angle in this triangle.

b Classify this triangle, justifying your answer.

9 Determine the values of the pronumerals a, b, c and d.



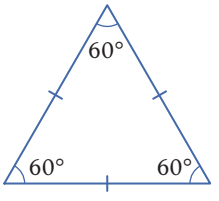
10 a Determine the values of the pronumerals p, q and r.



b What angle must $\angle RSQ$ be changed to such that PQ and RS would be parallel?

c Which other angle will change if PQ and RS become parallel by changing $\angle RSQ$, and to what size?

11 a Consider this equilateral triangle.

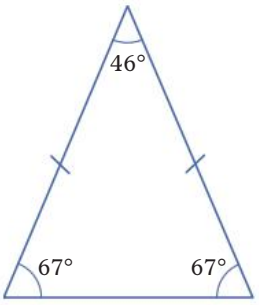


i Extend each side to create an exterior angle at each vertex.

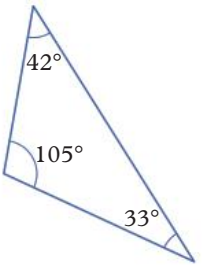
ii Find the size of each of the exterior angles and add them together. What do they add to?

Repeat **i** and **ii** for these two triangles.

b



c



d What can you say about the sum of the exterior angles of a triangle?

12 A classmate was given a number of triangles to construct, but is having trouble drawing them. For each set of instructions, draw a diagram of the triangle or provide a reason why it cannot be drawn.

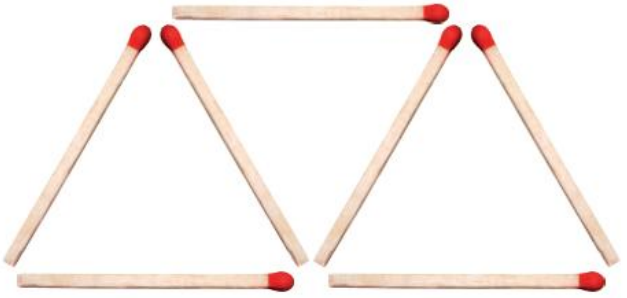
a a triangle with internal angles of 90°, 50° and 60°

b a triangle with two internal angles of 40°

c a triangle with side lengths of 10 cm, 3 cm and 5 cm

d a triangle with two obtuse angles

13 You can construct equilateral triangles using matches, as shown here.



a How many matches were used to construct these three triangles?

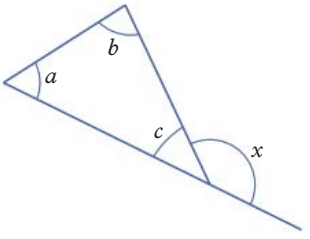
b Use:

i nine matches to construct four equilateral triangles

ii nine matches to construct five equilateral triangles

iii six matches to construct four equilateral triangles.

14 Use algebra to show that $x = a + b$.



Check your Student obook pro for these digital resources and more:

Interactive skillsheet Triangle properties

Investigation Stained glass triangles

Topic quiz 7B

PROBLEM SOLVING AND REASONING

CHALLENGE

7C Quadrilaterals

Learning intentions

By the end of this topic you will be able to ...

✔ classify quadrilaterals

✔ find missing angles in quadrilaterals.

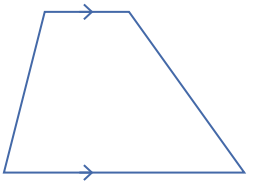
Inter-year links

Year 7 7E Classifying quadrilaterals

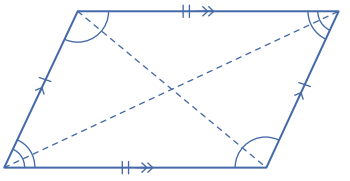
Properties of quadrilaterals

Key content video

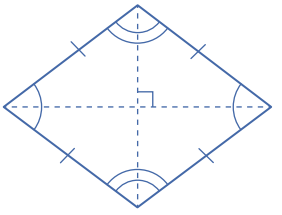
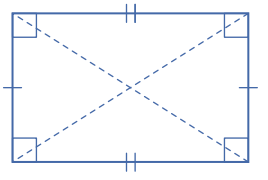
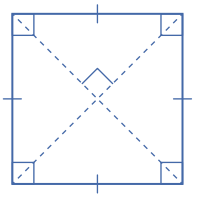
- A **quadrilateral** is a two-dimensional (2D) shape with four straight sides.
- The angles in a quadrilateral add to 360°.
- Quadrilaterals can be classified by their side lengths and interior angles.

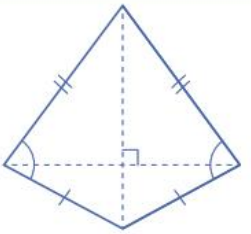
Type of quadrilateral	Properties	Example
Trapezium	• at least one pair of parallel sides	

A parallelogram is a special type of trapezium.

Parallelogram	• two pairs of parallel sides • two pairs of opposite angles that are equal in size • opposite sides are equal in length • diagonals bisect each other (divide each other into two equal parts)	

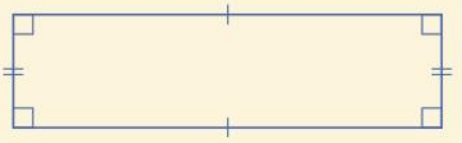
Three special types of parallelogram are the rhombus, rectangle and square.

Rhombus	• four sides of equal length • diagonals are perpendicular • diagonals bisect the vertex angles	
Rectangle	• four right angles, 90° • diagonals are equal in length	
Square	• four sides of equal length • four right angles, 90° • diagonals are equal in length • diagonals are perpendicular • diagonals bisect the vertex angles	

Type of quadrilateral	Properties	Example
Kite	• no parallel sides • two pairs of sides that are equal in length • one pair of opposite angles that are equal in size • diagonals are perpendicular • one diagonal bisects the other diagonal • one diagonal bisects a pair of vertex angles	
Irregular quadrilateral	• four different internal angles • no parallel sides	

Example 7C.1 Classifying quadrilaterals

Classify this quadrilateral.



THINK

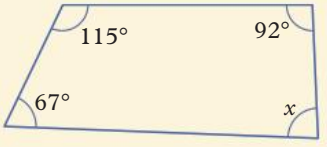
Examine the properties of this quadrilateral. The quadrilateral has opposite sides that are equal in length and it has four right angles, so it is a rectangle.

WRITE

The quadrilateral is a rectangle.

Example 7C.2 Finding an internal angle in a quadrilateral

Find the size of angle x.



THINK

The internal angles in a quadrilateral add to 360°. Subtract each of the known angles from 360° to determine the size of the fourth angle, x.

WRITE

$x = 360° - 115° - 92° - 67°$
$= 86°$

Helpful hints

- ✔ The diagonals of a non-square rectangle are not perpendicular (at right angles to each other)!
- ✔ As rhombuses, rectangles and squares are all parallelograms, they also have the properties of parallelograms.
- ✔ Every square is also a rhombus and a rectangle, as well as being a parallelogram.

ANS p532

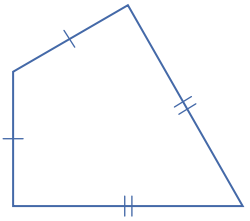
Exercise 7C Quadrilaterals

1–4, 5(a–c), 7–9, 12 | 1–4, 5(d–f), 6, 8, 9, 11, 13, 14 | 3–5, 10, 11, 13–15

UNDERSTANDING AND FLUENCY

7C.1 **1** Classify each quadrilateral. Give a reason for your answers.

a



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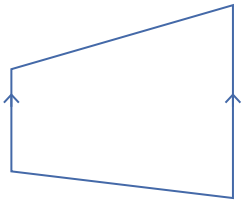

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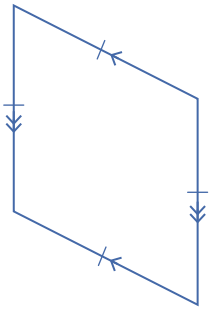

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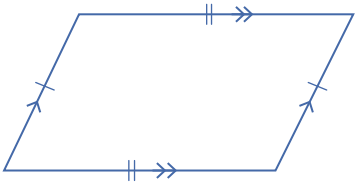

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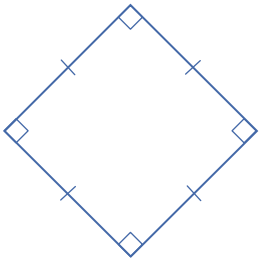

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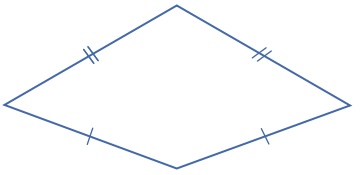
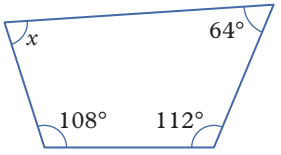


7C.2 **2** Find the size of angle x in each quadrilateral.

a



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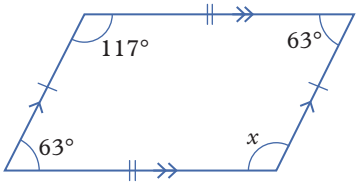

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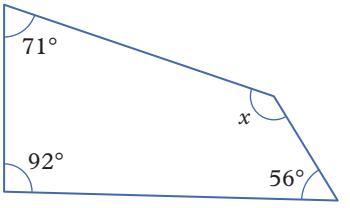

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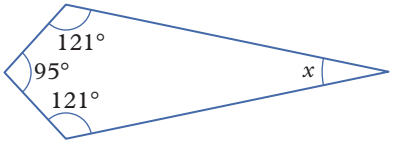

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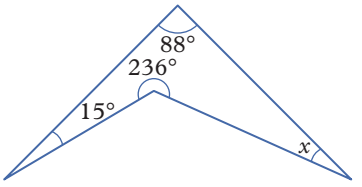

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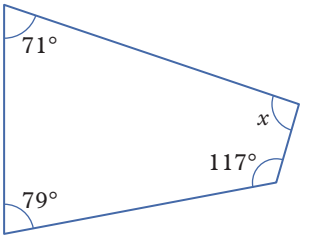
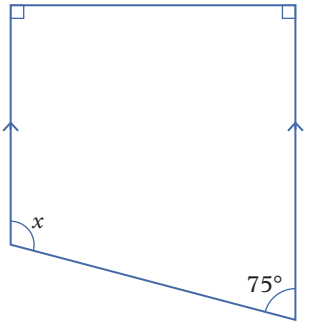


3 Classify each quadrilateral and find the size of angle x.

a



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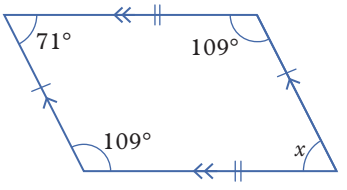

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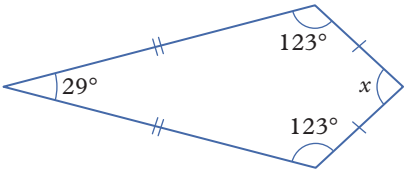
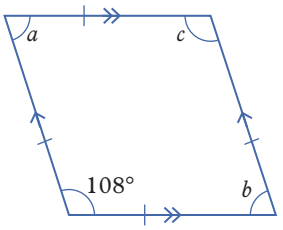


4 Use your knowledge of quadrilaterals to:

i name each figure **ii** find the size of the unknown angles.

a



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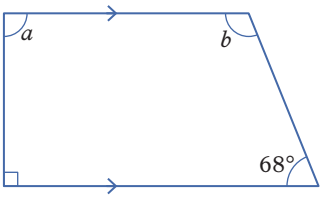

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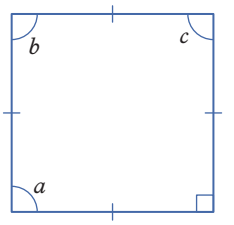

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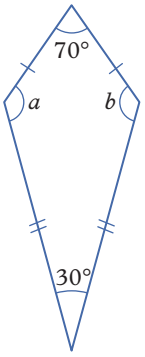

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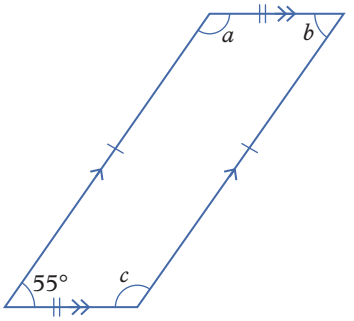

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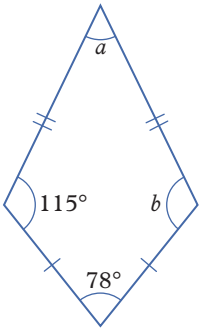
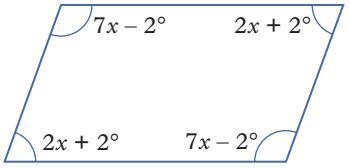


5 Use your understanding of quadrilateral properties and basic algebra to find the value of x in each quadrilateral.

a



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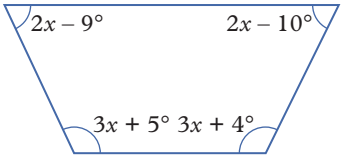

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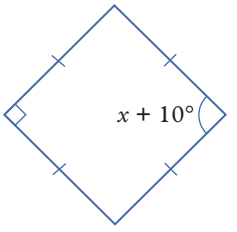

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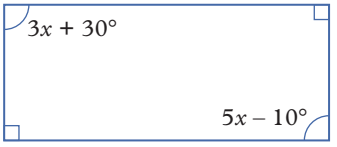

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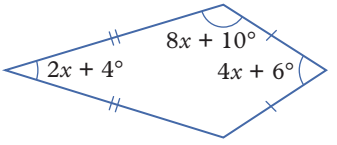

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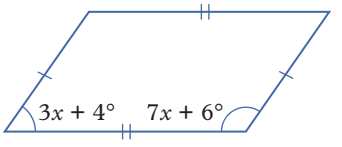
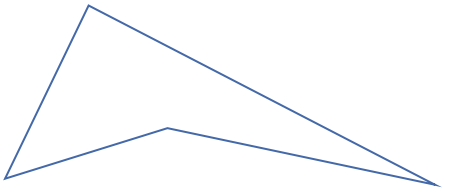


6 An irregular shape can be convex or concave. A convex shape has internal angles all smaller than 180°. A concave shape has at least one reflex angle (greater than 180°).



a Explain why a concave quadrilateral can have only one internal reflex angle.

b Find the size of the largest angle if the other three angles measure 20°, 20° and 50°.

7 What name is given to a parallelogram with interior angles of 90°?

8 Decide if each statement is true or false. If they are false, give a reason why.

a A square is a type of parallelogram.

b A kite has a pair of parallel sides.

c A parallelogram is a type of rectangle.

d A trapezium has exactly one pair of parallel sides.

e A rhombus always has four equal sides and four equal angles.

f A rectangle always has four equal sides and four equal angles.

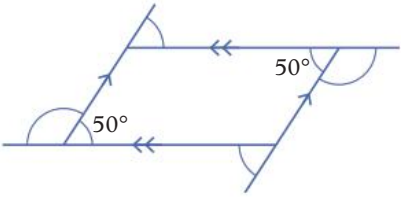
9 By considering that the angle sum of a triangle is 180°, explain why the angle sum of a quadrilateral is 360°.

PROBLEM SOLVING AND REASONING

10 Exterior angles can be found for quadrilaterals as well as for triangles, as shown in this parallelogram.

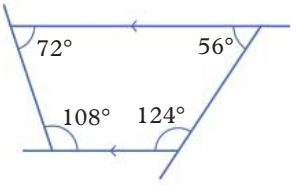
a **i** What is the size of each exterior angle in this parallelogram?

ii What is the sum of the exterior angles?



Repeat **i** and **ii** for these two quadrilaterals.

b



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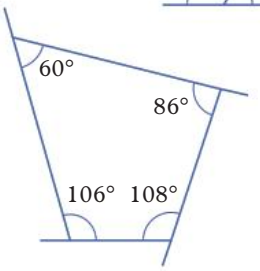
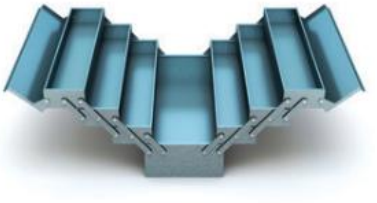


d What can you say about the sum of the exterior angles of a quadrilateral?

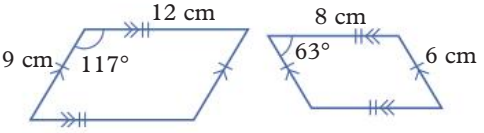
11 Many toolboxes are designed so that the trays of the boxes stay horizontal as they are opened. Use the properties of parallelograms to explain how these toolboxes work.



12 Quadrilaterals can be ordered into a hierarchy based on their properties. For instance, every rectangle is also a parallelogram. Create a diagram detailing the hierarchy of quadrilaterals.

13 Two figures are said to be similar if they have exactly the same shape. Use your understanding of quadrilateral properties to decide whether each pair of shapes is similar.

a



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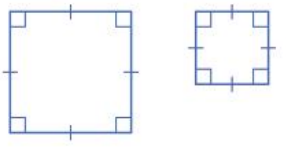

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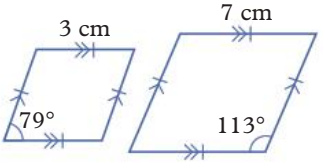

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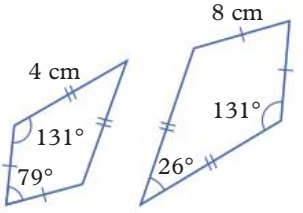
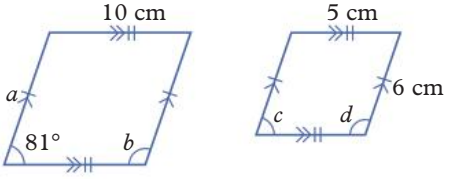


14 Use your understanding of quadrilateral properties to find the value of the pronumerals in each pair of similar figures.

a



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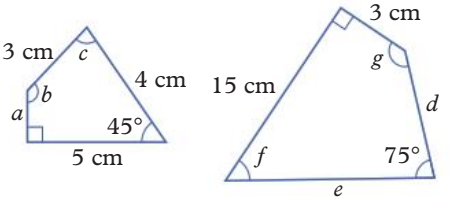


CHALLENGE

15 Each part below gives a clue about a different quadrilateral. Use the clues to:

i find the size of each angle **ii** name the quadrilateral.

a All angles are equal and all sides are equal.

b Two pairs of angles are equal and one pair of angles is twice the size of the other pair. Two pairs of side lengths are equal and one pair is twice the length of the other pair.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Quadrilateral properties

Investigation Stained glass quadrilaterals

Topic quiz 7C

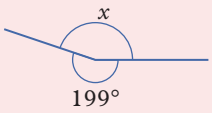
ANS p532

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

7A **1** Determine the value of each of the pronumerals.

a



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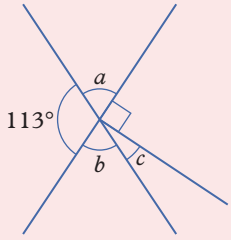
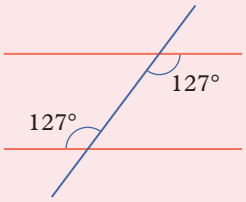


7A **2** Determine if the red lines are parallel or not parallel. State why or why not.

a



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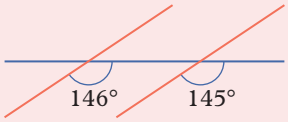

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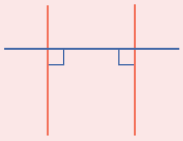
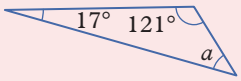


7B **3** Determine the size of the missing angles and classify the type of triangle according to its sides and angles.

a



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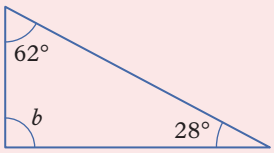

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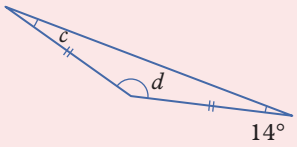

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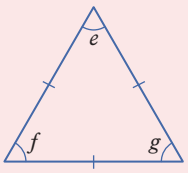
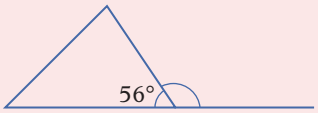


7B **4** Determine the size of the marked exterior angles.

a



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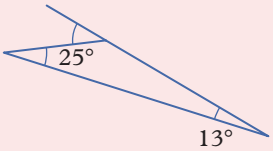

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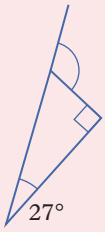

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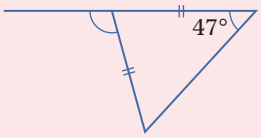
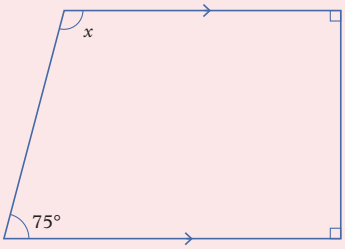


7C **5** Classify each quadrilateral and find the size of angle x.

a



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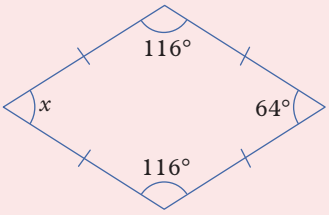

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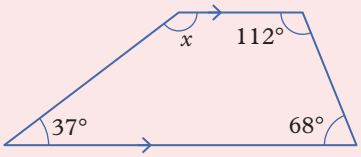


7C **6** Decide if the following statements are true or false, giving reasons why.

a A rectangle is a type of square.
b A trapezium is a type of kite.
c A rhombus is a type of trapezium.

7D Pythagoras' theorem

Learning intentions

By the end of this topic you will be able to ...

- ✔ use Pythagoras' theorem to determine whether a triangle is right-angled
- ✔ use Pythagoras' theorem to find the hypotenuse of a right-angled triangle.

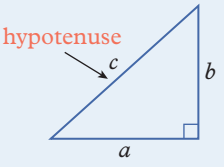
Inter-year links

Year 7	1G Indices and square roots
Year 10	9A Pythagoras' theorem

Key content video

Pythagoras' theorem in right-angled triangles

- Pythagoras of Samos was an ancient Greek philosopher and mathematician who lived from approximately 570 to 495 BCE. He has been credited with many mathematical and scientific discoveries, although the origins of his most famous theorem are disputed to this day.
- The **hypotenuse** is the longest side of a right-angled triangle. It is the side directly opposite the right angle.
- **Pythagoras' theorem** states the relationship between the three side lengths of a right-angled triangle. That is, the sum of the squares of the shorter sides is equal to the square of the hypotenuse. The relationship for the triangle shown is:



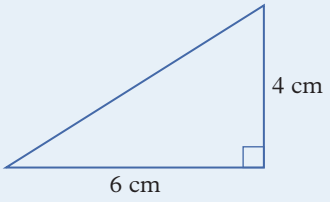
$$c^2 = a^2 + b^2$$

hypotenuse shorter sides

- A triangle is right-angled if it has side lengths that satisfy Pythagoras' theorem.
- Any set of three positive whole numbers that satisfy Pythagoras' theorem is called a **Pythagorean triad** (or Pythagorean triple).
  For example, 3, 4, 5 is a Pythagorean triad as $5^2 = 3^2 + 4^2$.

Finding the length of the hypotenuse

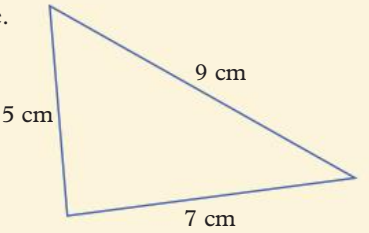
- To find the length of the hypotenuse using Pythagoras' theorem:
  1. State Pythagoras' theorem and define a, b and c.
  2. Substitute in the values for a, b and c and simplify the equation.
  3. Solve for c by finding the square root of both sides of the equation using inverse operations.
- The result for the length of the hypotenuse can be written as an exact value (whole number, terminating decimal or in surd form) or approximated to a specific number of decimal places.
  For example, the side length of the hypotenuse in this triangle can be written as $\sqrt{52}$ cm or 7.21 cm correct to two decimal places.



$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 6^2 + 4^2 \\ &= 36 + 16 \\ &= 52 \\ c &= \sqrt{52} \text{ cm} \\ &\approx 7.21 \text{ cm} \end{aligned}$$

Example 7D.1 Using Pythagoras' theorem to determine whether a triangle is right-angled

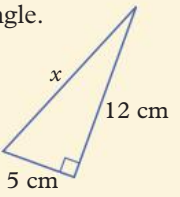
Use Pythagoras' theorem to determine whether this is a right-angled triangle.



THINK	WRITE
1 Identify the length of the longest side and define as c.	Let $c = 9$ cm.
2 Calculate the value of c^2.	$c^2 = 9^2$ $= 81$
3 Identify the lengths of the two shorter sides and define as a and b.	Let $a = 5$ cm and $b = 7$ cm.
4 Calculate the value of $a^2 + b^2$.	$a^2 + b^2 = 5^2 + 7^2$ $= 25 + 49$ $= 74$
5 Compare c^2 to $a^2 + b^2$. If the two values are equal, then the triangle is right-angled. If the two values are not equal, then it is not a right-angled triangle.	$c^2 \neq a^2 + b^2$ The triangle is not a right-angled triangle.

Example 7D.2 Calculating the length of the hypotenuse (whole number)

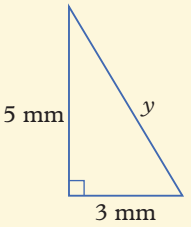
Use Pythagoras' theorem to calculate the length of the hypotenuse of this triangle.



THINK	WRITE
1 The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = 5$ cm, $b = 12$ cm and $c = x$.
2 Substitute in the values for a, b and c, and simplify the equation.	$x^2 = 5^2 + 12^2$ $= 25 + 144$ $= 169$
3 Solve for x by finding the square root of both sides of the equation using inverse operations.	$x = \sqrt{169}$ $= 13$ cm

Example 7D.3 Calculating the length of the hypotenuse (decimal value)

Calculate the length of the hypotenuse of this triangle correct to two decimal places.



THINK	WRITE
1 The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = 3$ mm, $b = 5$ mm and $c = y$.
2 Substitute in the values for a, b and c, and simplify the equation.	$y^2 = 3^2 + 5^2$ $= 9 + 25$ $= 34$
3 Solve for y by finding the square root of both sides of the equation.	$y = \sqrt{34}$
4 Use a calculator to find the approximate length. Round to two decimal places. Note: $\sqrt{34}$ mm is the exact length.	≈ 5.83 mm

Helpful hints

- ✔ Make sure you correctly identify the hypotenuse. It is the longest side and is opposite the right angle. It may not be obvious, so always look at the numbers carefully.
- ✔ Pythagoras' theorem is an excellent formula to memorise. You will use it many times in your mathematics studies. Writing the formula before substituting the values in will help you to memorise it quicker.
- ✔ It doesn't matter which shorter sides you assign to be a or b, but the hypotenuse must always be c!

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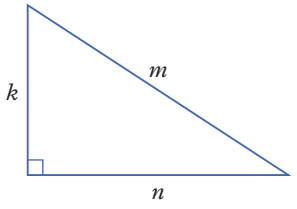
Exercise 7D Pythagoras' theorem

1–7, 9, 11, 12(a, b, f), 16 | 2, 3, 5, 7, 10, 12(c, e, f), 13, 14, 16, 17(a, b), 19 | 2, 6–8, 10, 13–16, 17(c, d), 18

1 Identify the pronumeral shown on the hypotenuse in each triangle.

a



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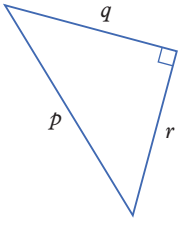
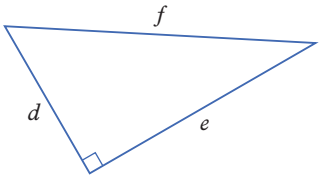
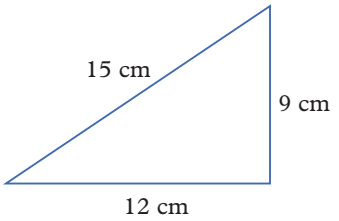


c



7D.1 **2** Use Pythagoras' theorem to determine whether these are right-angled triangles.

a



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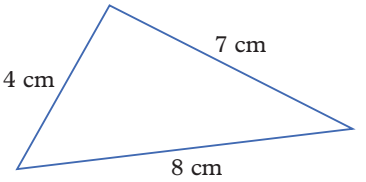

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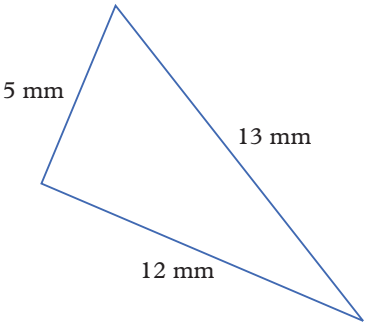

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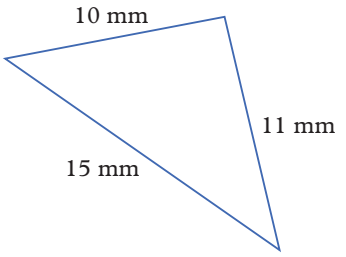
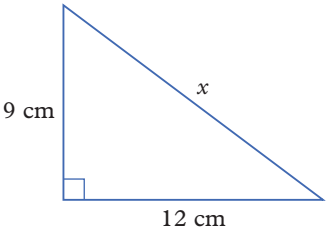


3 Determine whether each triangle with the given side lengths is right-angled or not.
Hint: The longest side in a right-angled triangle is the hypotenuse.

a 6 mm, 8 mm, 10 mm
b 7 cm, 24 cm, 25 cm
c 8 cm, 10 cm, 14 cm
d 11 mm, 15 mm, 19 mm
e 20 cm, 21 cm, 29 cm
f 10 mm, 18 mm, 20 mm

7D.2 **4** Use Pythagoras' theorem to calculate the length of the hypotenuse in each triangle.

a



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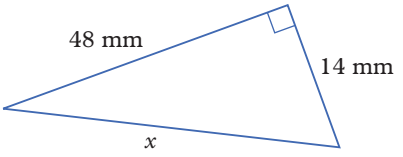

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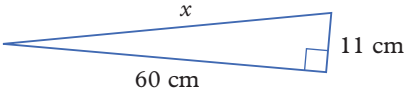

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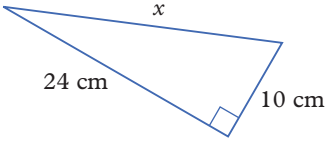
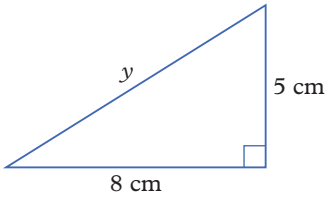


7D.3 **5** Calculate the length of the hypotenuse in each triangle correct to two decimal places.

a



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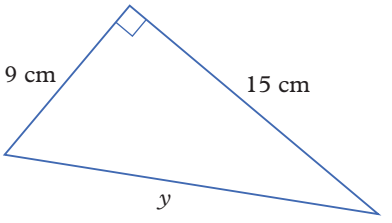

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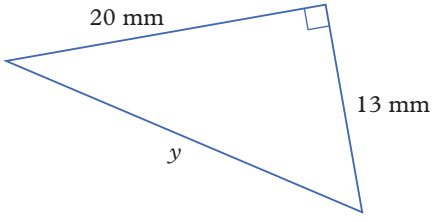

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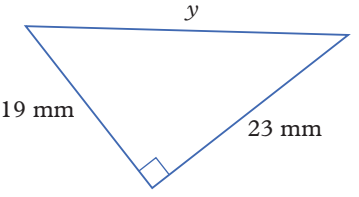
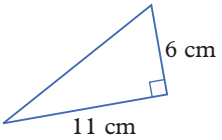


UNDERSTANDING AND FLUENCY

6 Find the unknown side length in each right-angled triangle. Give your answers **i** as exact values and **ii** correct to two decimal places.

a



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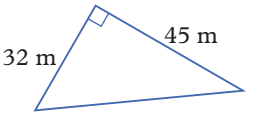

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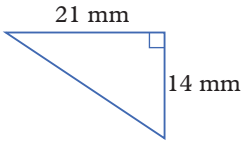

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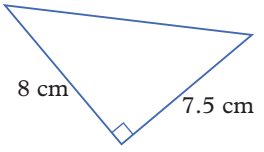
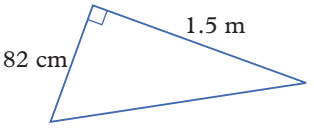


7 Find the length of the hypotenuse in each right-angled triangle, correct to the nearest centimetre.
Hint: Convert all length measurements to centimetres first.

a



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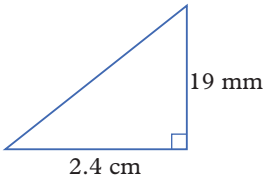

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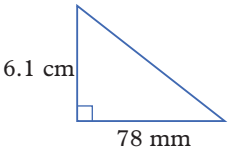

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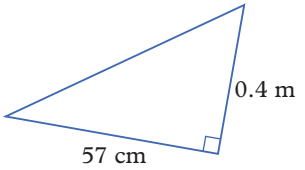


PROBLEM SOLVING AND REASONING

8 Both multiplication and addition are commutative. That is, when numbers or variables are multiplied or added, the order doesn't matter as the result will be the same.
For example: $2 \times 3 = 3 \times 2$ and $2 + 3 = 3 + 2$
In Pythagoras' theorem, a and b refer to the lengths of the two shorter sides of a right-angled triangle. Does it matter which of the shorter sides is labelled a and which is labelled b? Explain by referring to commutative properties.

9 Pythagorean triads can be generated using formulas. One method for finding Pythagorean triads uses three formulas and two known values, m and n where $m > n$, to generate the shorter side lengths a and b, and the hypotenuse c. The formulas are:

- $a = m^2 - n^2$
- $b = 2mn$
- $c = m^2 + n^2$.

Use the formulas to complete the table below to generate Pythagorean triads. The first row has been completed for you.

Values		**Pythagorean triad**		
m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	$2^2 - 1^2 = 3$	$2 \times 2 \times 1 = 4$	$2^2 + 1^2 = 5$
3	1			
3	2			
4	1			
4	2			
4	3			
5	1			
5	2			
5	3			
5	4			
6	1			

10 Pythagorean triads form the lengths of right-angled triangles. Some triads are multiples of another triad and they form similar triangles (triangles with the same shape but different sizes).

For example, (6, 8, 10) and (9, 12, 15) are multiples of (3, 4, 5).

A Pythagorean triad that does not have a common factor between the three sides is called a primitive Pythagorean triad.

a List the primitive Pythagorean triads from question **9**.

b The Pythagoreans triads below are all multiples of a primitive Pythagorean triad. State the primitive Pythagorean triad in each case.

i (50, 120, 130) **ii** (24, 45, 51) **iii** (27, 36, 45)

iv (36, 160, 164) **v** (390, 432, 582) **vi** (231, 2420, 2431)

11 Consider Triangle A shown.

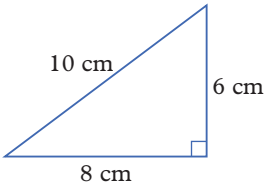
a Use Pythagoras' theorem to confirm that Triangle A is right-angled.

b A new triangle, Triangle B, is formed by doubling the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle B?

c Triangle C is formed by tripling the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle C?

d Triangle D is formed by halving the side lengths of Triangle A. Does Pythagoras' theorem still work for Triangle D?

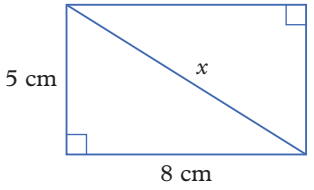
e Will Pythagoras' theorem still work if the side lengths of Triangle A are multiplied by 10?



Triangle A

12 Use Pythagoras' theorem to calculate the unknown length indicated by the pronumeral in each diagram, correct to two decimal places.

a



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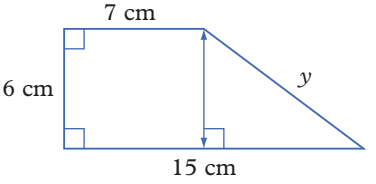

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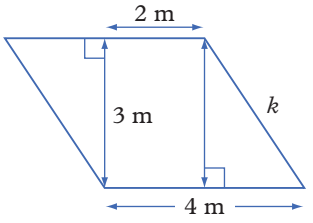

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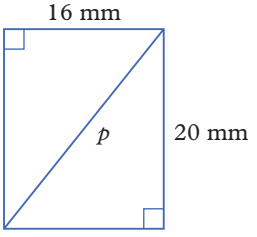

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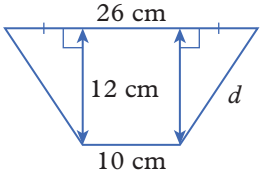

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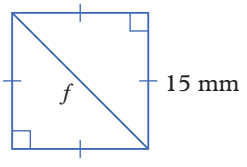
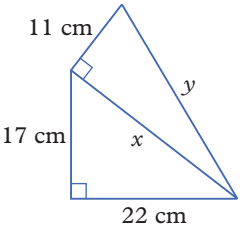


13 Find the lengths indicated by x and y in each diagram, correct to two decimal places.

a



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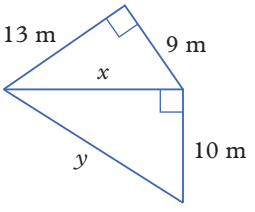
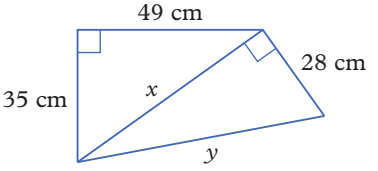


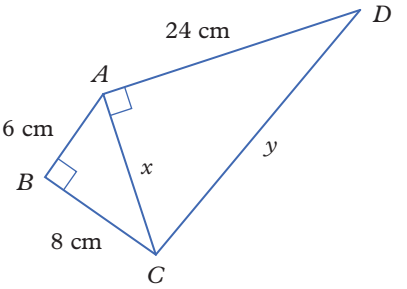
c



14 The square root of a positive number is sometimes given with a ± sign, for example $\sqrt{9} = \pm 3$. This is because the square of the negative value is positive: $(-3)^2 = 9$.

Explain why we only need to consider the positive solution of a square root when using Pythagoras' theorem.

15 Consider the shape *ABCD* shown on the right.



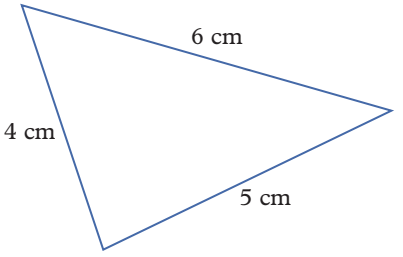
a How many right-angled triangles can you see? Name them.

b Calculate the length of *AC*. Which side lengths did you use in your calculation?

c Calculate the length of *CD*. Explain how you were able to do this.

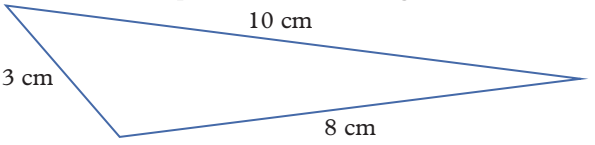
d Find the perimeter of the shape *ABCD*.

16 a Calculate the square of the side lengths of this acute-angled triangle.



b Complete the following sentence.
In an acute-angled triangle, the square of the longest side is _____ the sum of the squares of the two other sides.

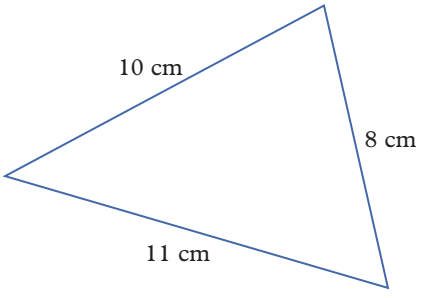
c Calculate the square of the side lengths of this obtuse-angled triangle.



d Complete the following sentence.
In an obtuse-angled triangle, the square of the longest side is _____ the sum of the squares of the two other sides.

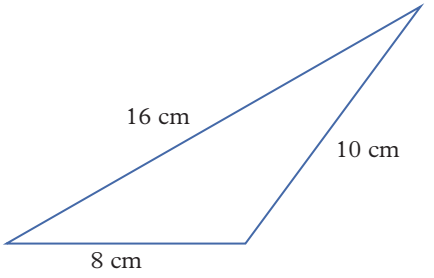
e Confirm that your sentences work for the following triangles.

i



Acute-angled triangle

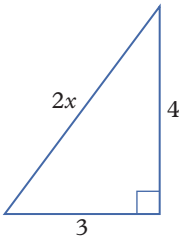
ii



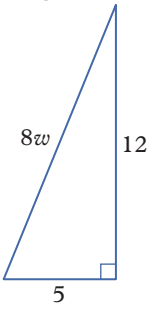
Obtuse-angled triangle

17 Determine the value of the pronumeral in each of the following. Write your answer as a fraction where appropriate.

a



b



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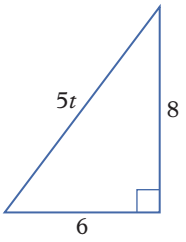

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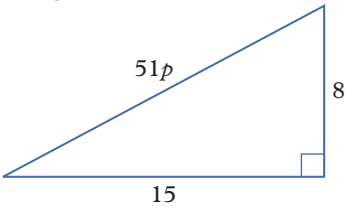


18 Pythagoras' theorem is one of the most thoroughly proved theorems with hundreds of unique proofs (mathematical explanations). One proof, which we will look at, is known as 'Proof by rearrangement'.

Consider Diagram 1 and Diagram 2. Both diagrams have the same four right-angled triangles with shorter side lengths of a and b, and a hypotenuse with length c.

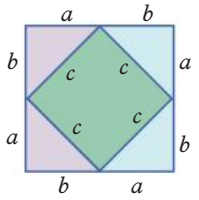


Diagram 1

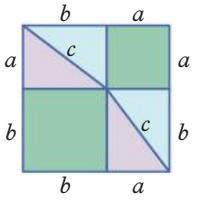


Diagram 2

a Explain why the two squares have the same area.

b Using your ideas from part **a**, explain how this pair of diagrams shows that $c^2 = a^2 + b^2$.

We can prove this algebraically by writing an equation equating the areas of each square.

c Write an equation for the total area in each diagram by adding the area of their smaller sections.

d The area in both diagrams is the same, so we can equate the equations from part **c.** Show that this equation is equivalent to $c^2 = a^2 + b^2$.

Another proof for Pythagoras' theorem can be shown using Diagram 3.

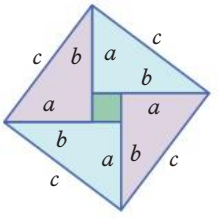


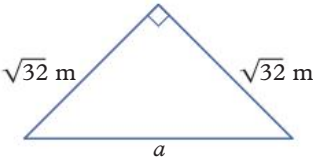
Diagram 3

e Using this diagram, explain why $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$ by considering the area of each part.

f Show that the equation from part **e** is equivalent to $c^2 = a^2 + b^2$.

19 Squaring and taking the square root of a number are inverse operations, so $(\sqrt{a})^2 = a$. Use this information to determine the values of the pronumerals in these diagrams, giving your answers correct to two decimal places where required.

a



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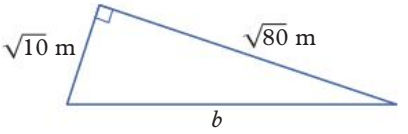

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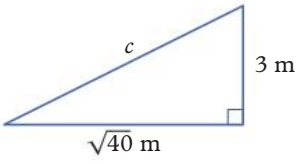

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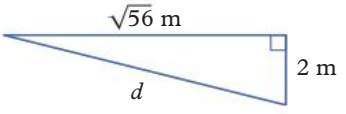


Check your Student obook pro for these digital resources and more:

- **Interactive skillsheet** Pythagoras' theorem
- **Interactive skillsheet** Finding the length of the hypotenuse
- **Investigation** Proving Pythagoras' theorem
- **Topic quiz** 7D

7E Finding the length of a shorter side

Learning intentions

By the end of this topic you will be able to ...

✔ use Pythagoras' theorem to determine the shorter side of a right-angled triangle.

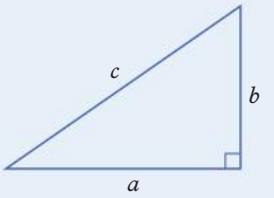
Inter-year links

Year 7	6H Solving equations using inverse operations
Year 10	9A Pythagoras' theorem

Finding the length of a shorter side

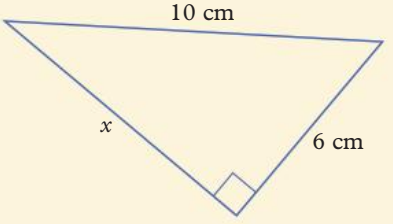
Key content video

- Pythagoras' theorem states that in a right-angled triangle with side lengths a, b and c, where c is the length of the hypotenuse, $c^2 = a^2 + b^2$.
- To find the length of one of the shorter sides using Pythagoras' theorem:
  1. State Pythagoras' theorem and define a, b and c.
  2. Substitute the values for a, b and c, and simplify the equation.
  3. Solve for a or b by finding the square root of both sides of the equation using inverse operations. Write the solution including the correct unit for the side length.



Example 7E.1 Calculating the length of a shorter side (whole number)

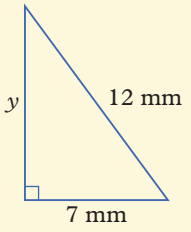
Use Pythagoras' theorem to calculate the unknown side length in this triangle.



THINK	WRITE
1 The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = x$, $b = 6$ cm and $c = 10$ cm.
2 Substitute the values for a, b and c, and simplify the equation.	$10^2 = x^2 + 6^2$
3 Solve for x by first subtracting 36 from both sides of the equation and then finding the square root.	$100 = x^2 + 36$ $64 = x^2$ $\sqrt{64} = x$ $8 = x$ $x = 8$ cm

Example 7E.2 Calculating the length of a shorter side (decimal value)

Calculate the unknown side length in this triangle, correct to two decimal places.



THINK	WRITE
1 The triangle is right-angled, so state Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = 7$ mm, $b = y$ and $c = 12$ mm.
2 Substitute the values for a, b and c, and simplify the equation.	$12^2 = 7^2 + y^2$
3 Solve for y by first subtracting 49 from both sides of the equation and then finding the square root.	$144 = 49 + y^2$ $95 = y^2$ $\sqrt{95} = y$
4 Use a calculator to find the approximate length correct to two decimal places. (Note that $\sqrt{95}$ mm is the exact length.)	$y \approx 9.75$ mm

Helpful hints

- ✔ A common mistake when using Pythagoras' theorem occurs in the second-last step. When looking at $y^2 = 95$, instead of taking the square root of both sides, some students will halve the result giving 47.5. The 2 is an index, not a factor!
- ✔ Remember that c represents the length of the hypotenuse in $c^2 = a^2 + b^2$, not the unknown side. So when finding the length of the shorter side, substitute the pronumeral into a or b.
- ✔ The solution of an equation $x^2 = 64$ has two solutions, $x = \pm 8$, but you only write the positive solution if x represents a length, as lengths can't be negative.

ANS p533

Exercise 7E Finding the length of a shorter side

▲ 1, 2–4(1st column), 5–7, 8(a, b), 9 ■ 2–4(2nd column), 5, 6–7(2nd column), 9, 10, 14, 15(a, b) ⬟ 4, 5, 6–7(2nd column), 9–12, 13(b), 14, 15(c, d), 16

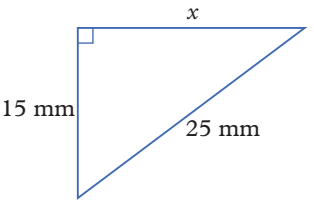
1 Solve the following equations for x. Write your answers correct to two decimal places.

a $x^2 + 25 = 36$ **b** $x^2 + 7^2 = 10^2$ **c** $16 + x^2 = 64$

d $81 = x^2 + 9$ **e** $12^2 = 5^2 + x^2$ **f** $36 = x^2 + x^2$

7E.1 **2** Use Pythagoras' theorem to calculate the unknown side length in each triangle.

a



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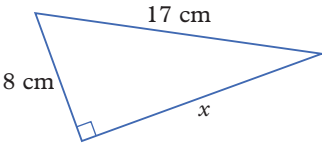

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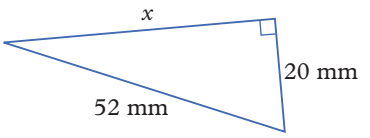

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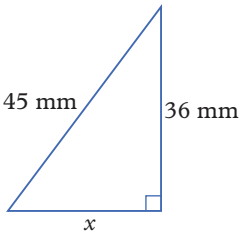

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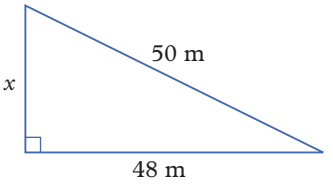

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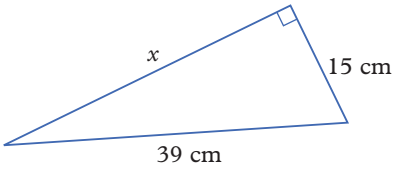
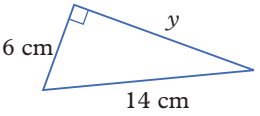


7E.2 **3** Calculate the unknown side length in each triangle, correct to two decimal places.

a



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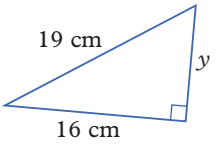

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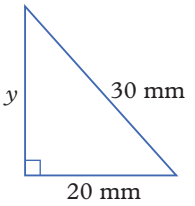

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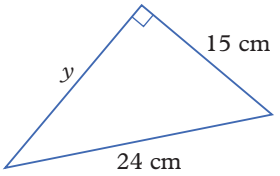

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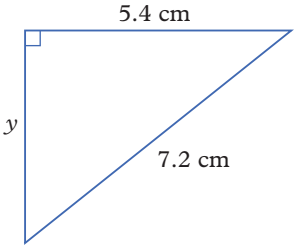

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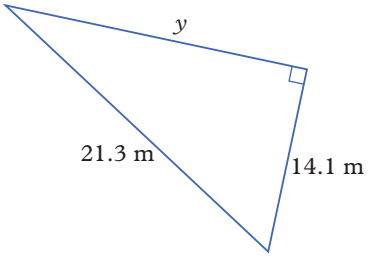
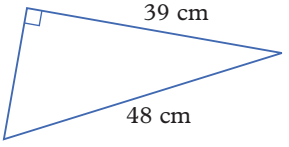


4 Find the unknown side length in each right-angled triangle. Give your answers **i** as exact values and **ii** correct to two decimal places.

a



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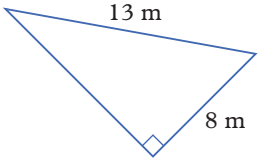

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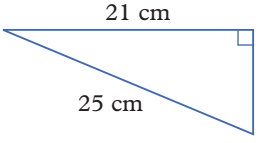

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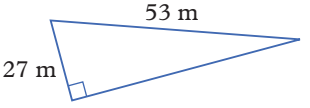
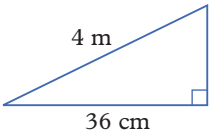


5 Find the unknown length in each right-angled triangle, correct to the nearest centimetre.
Hint: Convert all length measurements to centimetres first.

a



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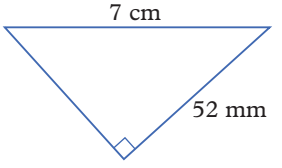

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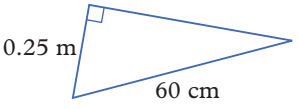

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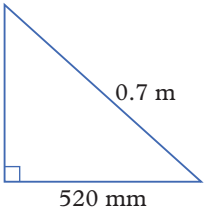
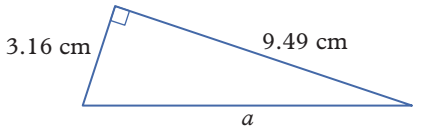


6 Calculate the unknown side length in each right-angled triangle, correct to two decimal places.

a



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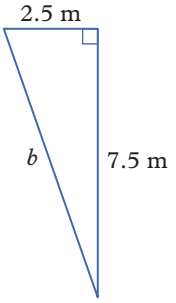

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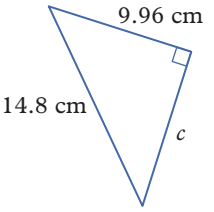

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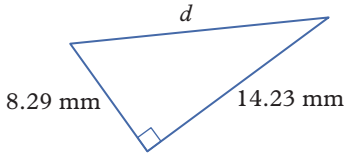

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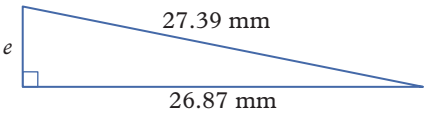

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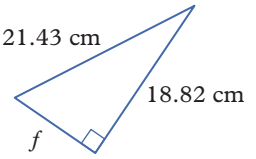
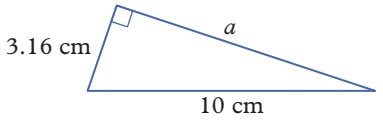


7 Calculate the area of the following right-angled triangles, correct to two decimal places.

a



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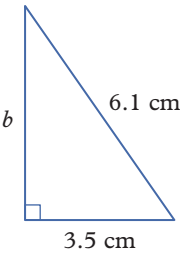

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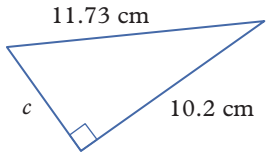

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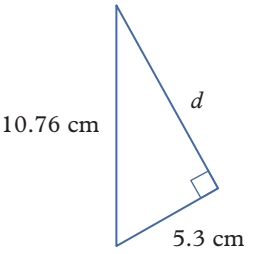
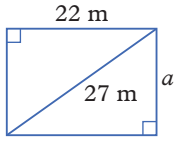
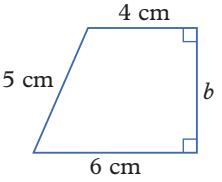


8 Use Pythagoras' theorem to calculate the unknown lengths indicated by the pronumerals in each diagram, correct to two decimal places.

a



b



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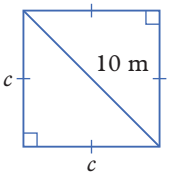
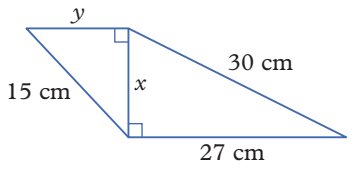


9 Find the lengths labelled x and y in each diagram, correct to two decimal places.

a



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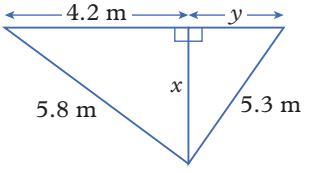

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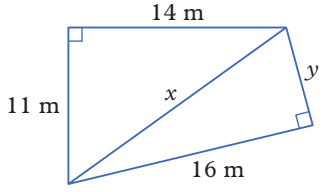

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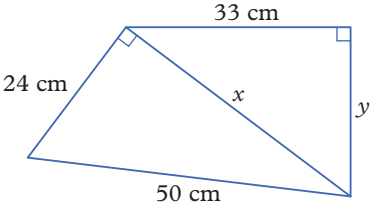


10 For each shape in question **9**, find correct to two decimal places:

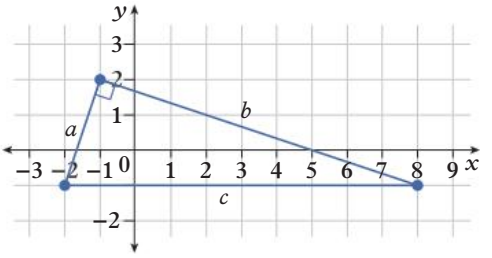
i the perimeter **ii** the area.

UNDERSTANDING AND FLUENCY

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

11 Consider the triangle shown on the Cartesian plane.



a State the length of c.

b Determine the square of the lengths of a and b.

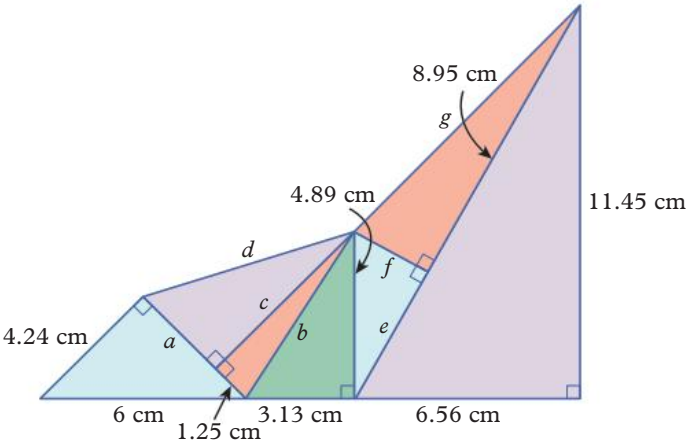
c Show that the triangle is right-angled.

d Determine the midpoint of each side length. Write your answer using decimals.

e Determine the squares of the lengths between each pair of midpoints.

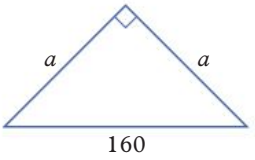
f Determine whether the triangle formed by the midpoints is right-angled.

12 Determine the value of all pronumerals in the following diagram, a–g, correct to two decimal places.

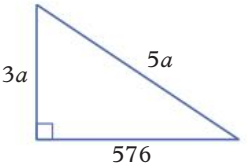


13 Determine the value of the pronumerals, correct to two decimal places where appropriate.

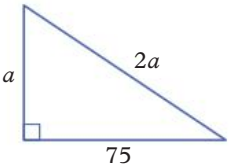
a



b



c



14 Joey is determining the unknown side length of a right-angled triangle with the two side lengths 7 cm and 15 cm. Joey writes down all possible calculations for the third length.

i $\sqrt{7^2 + 15^2}$ **ii** $\sqrt{15^2 + 7^2}$ **iii** $\sqrt{7^2 - 15^2}$ **iv** $\sqrt{15^2 - 7^2}$

a If the missing side length is the hypotenuse, which calculation(s) could be used?

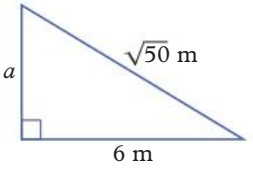
b If the missing side length is not the hypotenuse, which calculation(s) could be used?

c Is there a calculation listed that does not result in the side length of a right-angled triangle? Explain why or why not.

CHALLENGE

15 Determine the values of the pronumerals in these diagrams, giving your answers correct to two decimal places where required.

a



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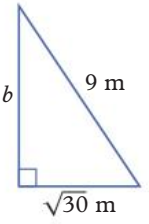

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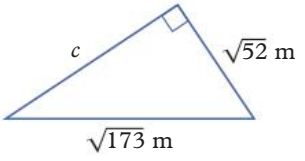

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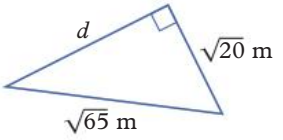


16 A square is drawn on each side of a right-angled isosceles triangle. The sum of the areas of the squares is 60 cm². What is the area of the triangle?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Finding the length of a shorter side

Topic quiz
7E

7F Applications of Pythagoras' theorem

Learning intentions

By the end of this topic you will be able to ...

✔ use Pythagoras' theorem to solve a range of different problems.

Inter-year links
Year 10 9A Pythagoras' theorem

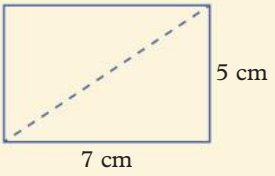
Applications of Pythagoras' theorem

Key content video

- Pythagoras' theorem allows you to calculate the third side length of any right-angled triangle, providing the other two side lengths are known. $a^2 + b^2 = c^2$
- Right angles are widely used in many industries such as architecture, engineering and construction. Right angles can also be found in nature; for example, a tree which stands vertically will form a right angle with the ground.
  → When a right angle is formed, a right-angled triangle can be drawn, and unknown distances can be calculated.

Example 7F.1 Using Pythagoras' theorem to calculate an unknown length

Calculate the length of the diagonal of this rectangle. Give your answer as an exact value.



THINK	WRITE
1 The shape is a rectangle, so the corners are right angles and the diagonal will be the hypotenuse of the triangle including the two side lengths. State Pythagoras' theorem and define a, b and c.	$c^2 = a^2 + b^2$, where $a = 5$ cm and $b = 7$ cm.
2 Substitute the values for a and b into the equation for Pythagoras' theorem. Let c be the unknown side and solve the equation for c.	$c^2 = 5^2 + 7^2$ $= 25 + 49$ $= 74$ $c = \sqrt{74}$

Helpful hints

✔ If you are not given a diagram in a question which requires Pythagoras' theorem, draw a diagram as your first step to ensure you correctly identify the hypotenuse and the two shorter sides.

ANS p534

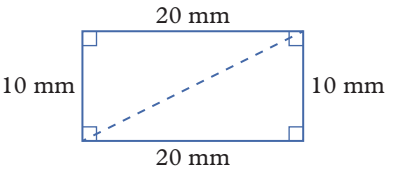
Exercise 7F Applications of Pythagoras' theorem

1–2(a–c), 5(a–c), 6–9, 11, 13 | 1–2(d–f), 5(d–f), 7, 10, 11, 13–15, 17 | 3, 4, 5(d–f), 7, 9, 12, 15–19

UNDERSTANDING AND FLUENCY

7F.1 **1** Calculate the length of the diagonal in each of these rectangles. Give your answers in exact form.

a



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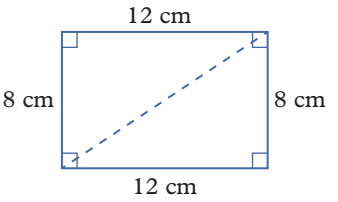

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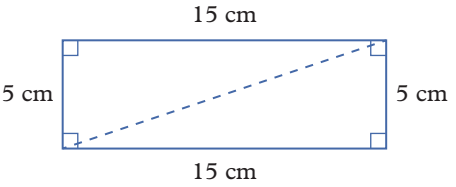

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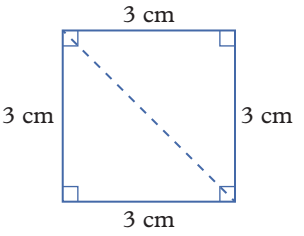

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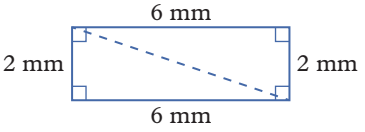

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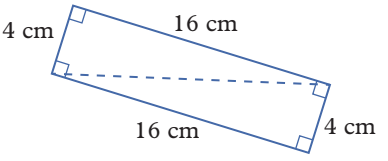


2 Rewrite the lengths of the diagonals of the rectangles in question **1**, correct to one decimal place.

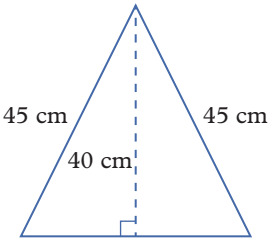
3 Calculate the length of the diagonal of a square with each of these side lengths. Give your answers as exact values in simplest form.

a 5 cm **b** 20 mm **c** 8 cm **d** 25 cm

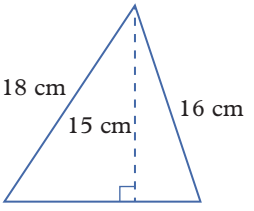
4 Rewrite the lengths of the diagonals of the squares in question **3**, correct to one decimal place.

5 Calculate the length of the unknown side for each of these triangles, using the given height of each triangle to help you. Give your answers as approximate values correct to one decimal place.

a



b



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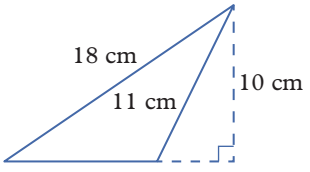

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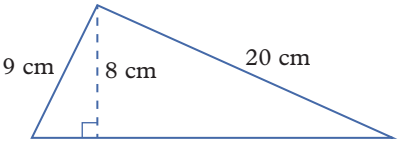

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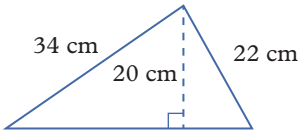

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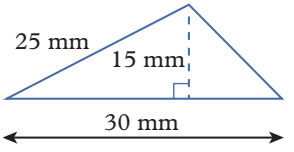
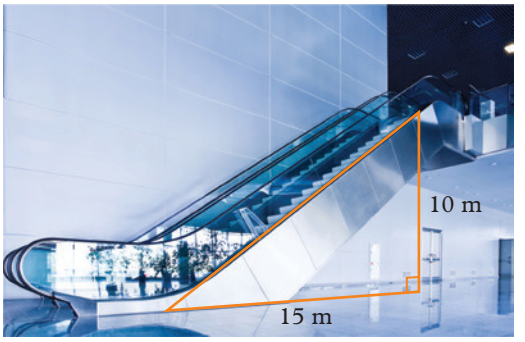
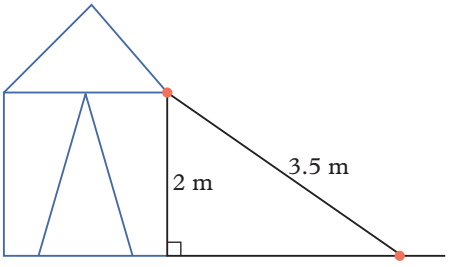


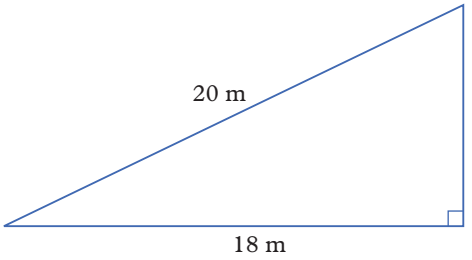
6 An escalator rises a vertical distance of 10 m for a horizontal distance of 15 m. What distance would you travel from the bottom to the top of the escalator? Give your answer correct to two decimal places.



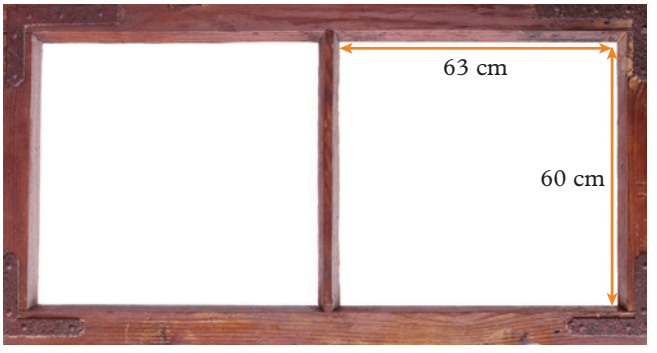
7 A guy rope on a tent is 3.5 m long and is attached to the tent at a height of 2 m. How far from the tent does the guy rope reach the ground? Give your answer correct to two decimal places.



8 A BMX rider uses the ramp shown in this diagram to perform stunts. How high is the ramp? Give your answer correct to two decimal places.



9 The timber frames in a window should meet at right angles. The diagonal length of the glass in one section of the window below is measured to be 87 cm. Use Pythagoras' theorem to check whether this window frame has been made correctly.



10 A rectangular picture frame measures 70 cm by 24 cm. The picture framer checks that the frame is 'square' (right-angled) by measuring the distance from the top left corner to the bottom right corner of the frame. The measured distance is 73 cm.

a Draw a diagram of the frame and label it with all the known information.

b Use Pythagoras' theorem to check whether the frame is 'square'. If it is not, should the diagonal distance be longer or shorter?

c Suggest a length for the diagonal distance.

11 Josh has an extension ladder that is 9.8 m long. He needs to clean the windows on the outside of a building. The lower part of the windows are 9.8 m from the base of the building, and the windows are 1.2 m tall. Unfortunately, bushes prevent Josh from placing his ladder any closer to the base of the building than 3 m. Josh is 1.7 m tall.

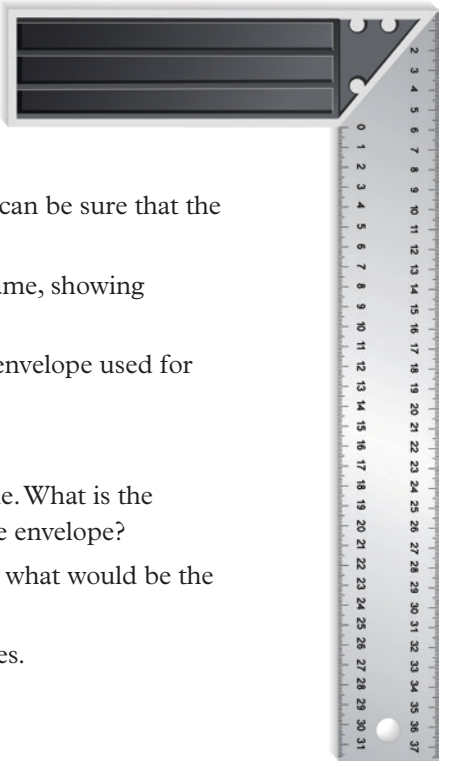
a Draw a diagram to show the situation.

b Calculate the height the ladder reaches up the building correct to one decimal place.

c Explain whether Josh will be able to clean the windows.

12 A cubby house is built on two levels in a tree. The lower level is 10 m above the ground, and the upper level is a further 5 m higher vertically. The ladder from the ground to the lower level is 12 m long. Sam wants to construct another ladder to go from the ground, 1 m further along the ground from the base of the tree than the first ladder, to the top level. How long should he make this ladder?

13 A squaring tool is used when making picture frames, to ensure that the corners of the frames are true right angles.



Imagine you are constructing a square picture frame with side lengths of 54 cm.

a One of the diagonals of the frame measures 78 cm. Explain how you can be sure that the frame is not truly square.

b The other diagonal measures 76 cm. Draw a labelled sketch of the frame, showing its true shape.

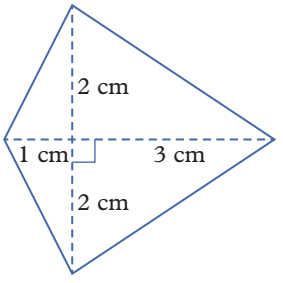
14 A sheet of writing paper is 30 cm long and has a breadth of 21 cm. The envelope used for this writing paper has a length of 22 cm and a diagonal of 24.6 cm.

a What is the breadth of the envelope?

b The writing paper is folded into equal-sized strips along its longer side. What is the minimum number of folds necessary to ensure the paper will fit in the envelope?

c If the paper was folded along its shorter side as well as its longer side, what would be the minimum number of folds needed for it to fit the envelope?

15 Calculate the perimeter of the following kite, correct to two decimal places.



16 An equilateral triangle has side lengths of 8 cm.

a Draw a sketch of the triangle with two vertices forming the base of the triangle. From the top vertex, draw a line that is perpendicular (at right angles) to the base. Label the lengths of all sides of the figure.

b Use Pythagoras' theorem to calculate the height of the triangle. Give your answer as:

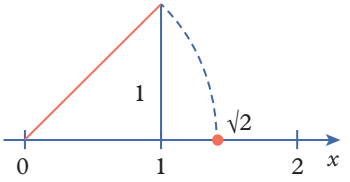
i an exact value

ii an approximate value, correct to one decimal place.

c Derive the formula for the area of any equilateral triangle with side lengths of x.

17 Builders in ancient Egypt used a piece of rope with 12 equally spaced knots as a tool. How did this show that a corner was 'square'? Explain using a diagram.

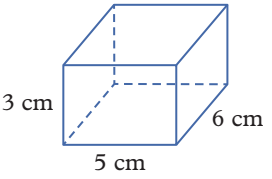
18 Pythagoras' theorem can be used to locate the position of surds on a number line. For example, to locate the position of $\sqrt{2}$ draw a perpendicular line to the number line at 1, with a height of 1 unit. This creates an equilateral triangle with a hypotenuse of length $\sqrt{2}$. Use a compass with one end placed at 0 on the number line and the other end placed at the end of the hypotenuse and draw an arc to the number to locate the position of $\sqrt{2}$.



Following this technique, locate the positions of the following surds on a number line.

a $\sqrt{5}$ **b** $\sqrt{10}$

19 Pythagoras' theorem can also be used to find unknown lengths within 3D objects. Consider the following rectangular prism.

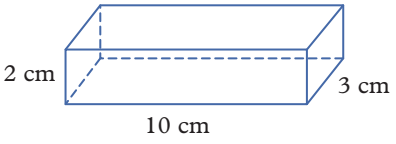


a Use Pythagoras' theorem to determine the length of the diagonal on the base of the prism. Give your answer as an exact value.

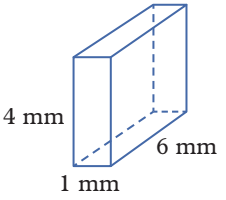
b Use your answer to part **a** and the height of the prism to determine the length of the diagonal within the prism correct to two decimal places.

20 Use the technique from question **19** to find the lengths of the diagonals within these rectangular prisms correct to two decimal places.

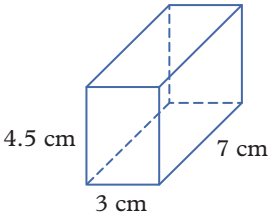
a



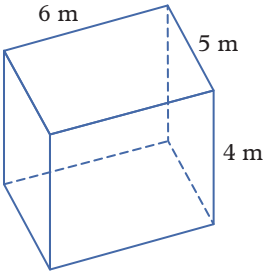
b



c



d



Check your Student obook pro for these digital resources and more:

Topic quiz
7F

CHALLENGE

Chapter summary

Types of angles

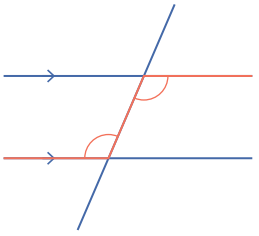
- Acute: between 0° and 90°
- Right: 90°
- Obtuse: between 90° and 180°
- Straight: 180°
- Reflex: between 180° and 360°
- Revolution: 360°

Angle properties

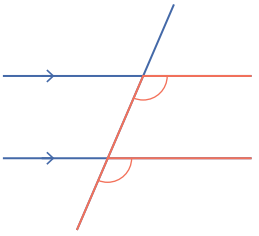
- Angles on a straight line: add to 180°
- Angles at a point: add to 360°
- Complementary angles: add to 90°
- Supplementary angles: add to 180°
- Vertically opposite angles: equal in size

Angles and parallel lines

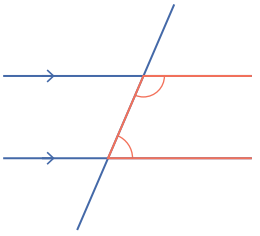
Alternate angles are equal.



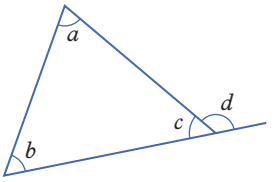
Corresponding angles are equal.



Co-interior angles are supplementary.



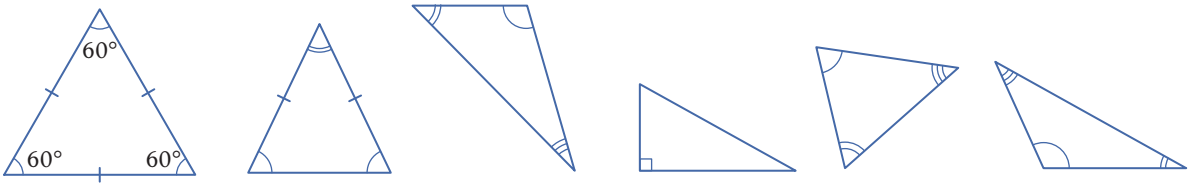
Exterior angle of a triangle



$a + b + c = 180°$
$c + d = 180°$
$d = a + b$

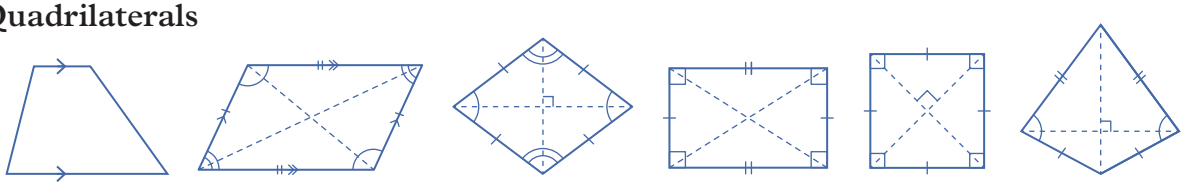
Triangles

$a + b + c = 180°$



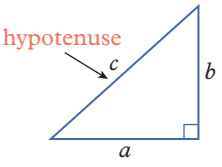
Equilateral | Isosceles | Scalene | Right-angled | Acute-angled | Obtuse-angled

Quadrilaterals



Trapezium | Parallelogram | Rhombus | Rectangle | Square | Kite

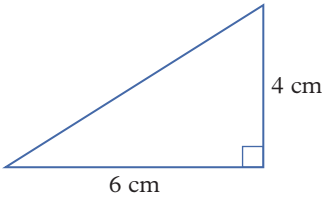
Pythagoras theorem



$c^2 = a^2 + b^2$

hypotenuse shorter sides

Length of the hypotenuse



$c^2 = a^3 + b^2$
$= 6^2 + 4^2$
$= 36 + 16$
$= 52$
$c = \sqrt{52}$ cm
≈ 7.21 cm

ANS p535

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- acute angle
- acute-angled triangle
- alternate angles
- co-interior angles
- complementary angles
- corresponding angles
- equilateral triangle
- exterior angle
- hypotenuse
- interior angle
- irregular quadrilateral
- isosceles triangle
- kite
- obtuse angle
- obtuse-angled triangle
- parallel lines
- parallelogram
- Pythagoras' theorem
- Pythagorean triad
- rectangle
- reflex angle
- revolution
- rhombus
- right angle
- right-angled triangle
- scalene triangle
- square
- straight angle
- supplementary angles
- transversal
- trapezium
- vertically opposite angles

1 Which type of triangle has exactly two interior angles of equal size?

2 Which type of quadrilateral is both a rectangle and a rhombus?

3 What angle type is between 90° and 180°?

4 Identify the key terms being referenced in each of these definitions.

a a straight line that cuts across two or more other lines

b a quadrilateral with at least one pair of parallel sides

5 Using an example, provide a definition in your own words for the following key terms.

a Pythagorean triad

b complementary angles

6 Complete the following sentences using words from the key term list.

a The _____ is the longest side of a right-angled triangle.

b An _____ is the angle between a side of a shape and a line extended from the adjacent side.

Multiple choice

7A **1** Which statement is *not* correct?

A Vertically opposite angles are equal in size.

B The sum of two obtuse angles will always be less than a revolution.

C Complementary angles add to 180°.

D Two supplementary angles make a straight angle.

E A reflex angle is always larger than an obtuse angle.

7A **2** What is the size of angle x?

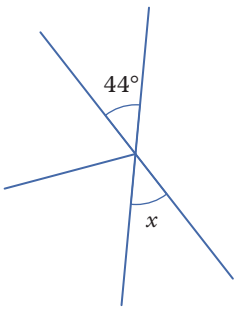
A 136°

B 132°

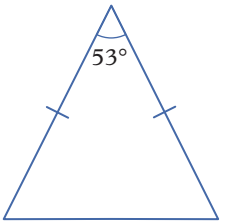
C 88°

D 44°

E 22°

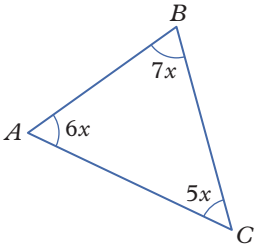


7B **3** What is the classification of this triangle?



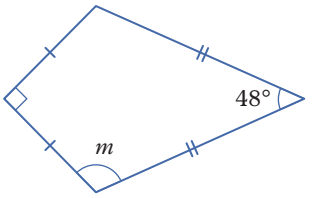
A acute-angled equilateral triangle
B acute-angled isosceles triangle
C acute-angled scalene triangle
D obtuse-angled isosceles triangle
E obtuse-angled scalene triangle

7B **4** What size is angle $\angle BAC$?



A 6°
B 10°
C 18°
D 60°
E 120°

7C **5** What is the size of angle m?

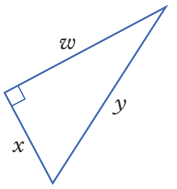


A 42°
B 84°
C 132°
D 222°
E 111°

7C **6** Which statement is true?

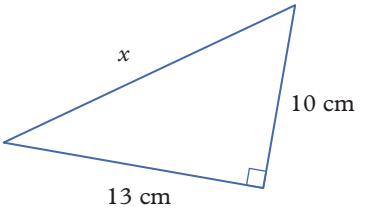
A A rhombus is a rectangle.
B A parallelogram is a square.
C A square is a rhombus.
D A trapezium cannot have a right angle.
E A kite has parallel sides.

7D **7** Using Pythagoras' theorem, which statement shows the relationship between the side lengths of this triangle?



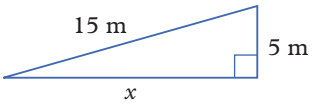
A $w^2 = x^2 + y^2$
B $x^2 = w^2 + y^2$
C $y^2 = x^2 + w^2$
D $y^2 = x^2 - w^2$
E $y^2 = w^2 - x^2$

7D **8** What is the value of the unknown side length x correct to one decimal place?



A 4.8 cm
B 8.3 cm
C 16.0 cm
D 16.4 cm
E 23.0 cm

7E **9** The unknown side length x in this triangle, correct to the nearest m, is:



A 3 m
B 10 m
C 14 m
D 16 m
E 20 m

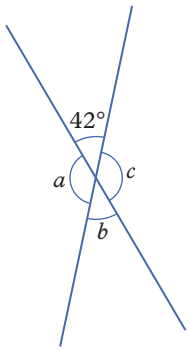
7F **10** Hitesh puts a 3-metre ladder against a wall. If the bottom of the ladder is 80 cm away from the base of the wall, how high up the wall is the top of the ladder?

A 2.89 m **B** 2.20 cm **C** 1.95 m **D** 3.10 m **E** 3.80 m

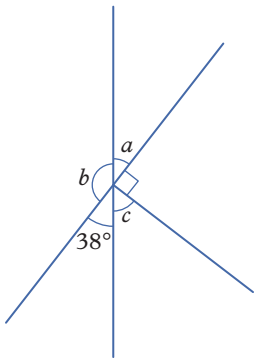
Short answer

7A **1** Without using a protractor find the size of each unknown angle.

a

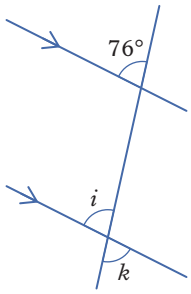


b



7A **2** Find the size of the unknown pronumerals.

a



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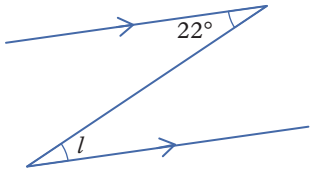
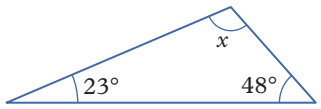


7B **3** Find the size of angles x and y.

a



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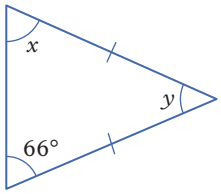
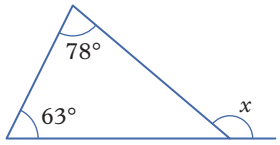
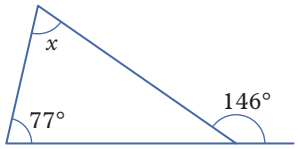


c



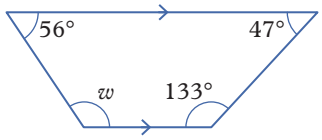
d



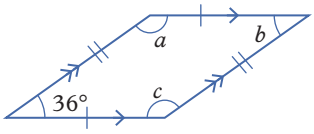
7B **4** For each triangle shown in question **3**, identify whether it is isosceles or scalene, and whether it is acute-angled or obtuse-angled.

7C **5** Find the size of the unknown angles in these quadrilaterals.

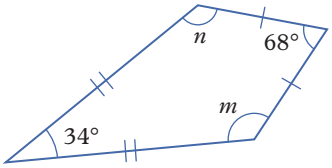
a



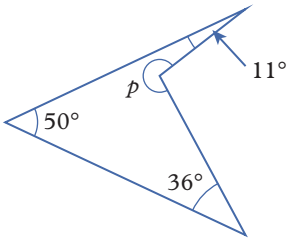
b



c



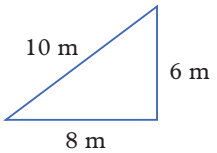
d



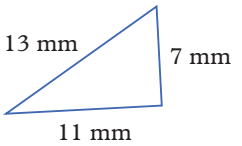
7C **6** Name the quadrilaterals shown in question **5**.

7D **7** Use Pythagoras' theorem to decide whether these are right-angled triangles.

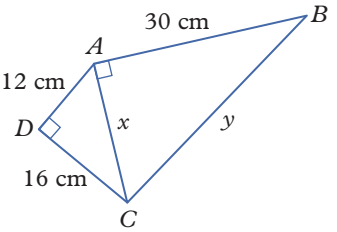
a



b

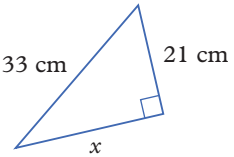


7D **8** Find the length of the sides represented by x and y in this diagram, correct to two decimal places.

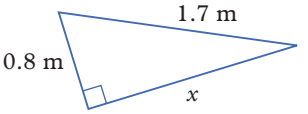


7E **9** Calculate the side length x in each triangle, correct to two decimal places.

a



b



7F **10** The size of a television screen is usually given in inches, where the given distance is the diagonal of the screen.

a What is the height of a 50" television screen with a width of 43.6"?

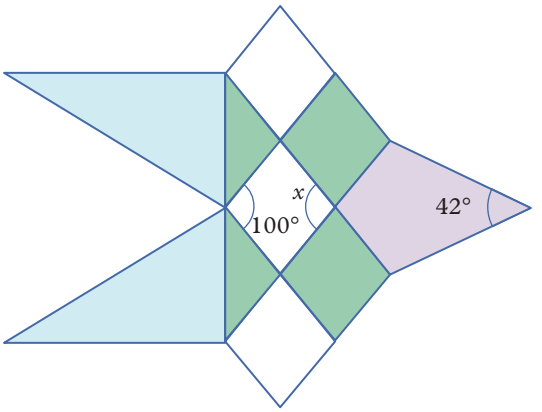
b What is the width of a 55" television screen with a height of 27"?

c What is the size of a television screen with a width of 65.4" and a height of 36.8"?

Give your answers correct to one decimal place.

Analysis

1 Consider the following piece of geometric artwork. The pairs of sides which form the green and white quadrilaterals are parallel and the light blue triangles are right-angled.



a What is the size of angle x? Give a reason with your answer.

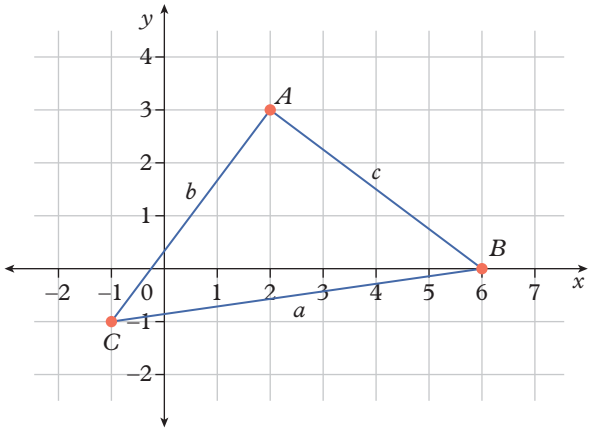
The purple quadrilateral is a kite.

b What is the size of the angle opposite 42° in the kite? Give a reason with your answer.

c What are the sizes of the other two angles in the kite?

d The light-blue triangles have a height of 3 cm and a width of 5.5 cm. What is the length of the sloping edges, correct to one decimal place?

2 Consider the triangle shown on this Cartesian plane.



a State the coordinates of points A, B and C.

b Calculate the length of the line segments a, b and c, giving your answers as exact values.

c Given that $\angle ABC$ is a right angle, classify the triangle according to its side and angles.

d What is the height of the triangle starting from point A? Give your answer as an exact value.

e Point A is moved 3 units up and 2 units left. What is the new coordinate of point A?

f Classify the new triangle according to its sides and angles.

g Calculate the perimeter of the new triangle, correct to two decimal points.

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Calculate and use complementary and supplementary angles ☐ Calculate and use angles at a point ☐ Calculate and use related angles on parallel lines	☐ Go back to Topic 7A Angles
☐ Classify triangles ☐ Calculate unknown interior and exterior angles of triangles	☐ Go back to Topic 7B Triangles
☐ Classify quadrilaterals ☐ Find missing angles in quadrilaterals	☐ Go back to Topic 7C Quadrilaterals
☐ Use Pythagoras' theorem to determine whether a triangle is right-angled ☐ Use Pythagoras' theorem to find the hypotenuse of a right-angled triangle	☐ Go back to Topic 7D Pythagoras' theorem
☐ Use Pythagoras' theorem to determine the shorter side of a right-angled triangle	☐ Go back to Topic 7E Finding the length of a shorter side
☐ Use Pythagoras' theorem to solve a range of different problems	☐ Go back to Topic 7F Applications of Pythagoras' theorem

8

Units of measurement

Index

8A Length and perimeter
8B Area of triangles and rectangles
8C Area of quadrilaterals
8D Area of a circle
8E Volume and capacity
8F Time

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Substitution
- ✔ Multiplying and dividing by powers of 10
- ✔ Adding and subtracting decimals
- ✔ The Cartesian plane
- ✔ Understanding time

Curriculum links

- Applies knowledge of the perimeter of plane shapes and the circumference of circles to solve problems (MA4-LEN-C-01)
  - Solve problems involving the perimeter ...
  - Describe the relationships between ...
- Applies knowledge of area and composite area involving triangles, quadrilaterals and circles to solve problems (MA4-ARE-C-01)
  - Develop and use formulas to find the area of rectangles, triangles and parallelograms ...
  - Develop and use the formula to find the area of circles and sectors to solve problems
  - Develop and use the formulas to find the area of trapeziums, rhombuses and kites ...
  - Choose appropriate units of measurement ...
- Applies knowledge of volume and capacity to solve problems involving right prisms and cylinders (MA4-VOL-C-01)
  - Describe the different views of prisms ...
  - Develop and apply the formula to find ...
  - Develop the formula for finding the volume ...
  - Choose appropriate units of measurement ...

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8A Length and perimeter

Learning intentions

By the end of this topic you will be able to ...

✔ convert between units of length

✔ find the circumference and arc length of a circle

✔ find the perimeter of two-dimensional shapes.

Inter-year links

Support — Length and perimeter

Year 7 — 8A Length and perimeter

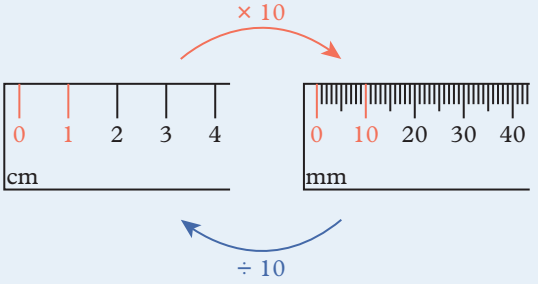
Units of length

Key content video

- To specify a **length**, you need a numerical value as well as a unit of length (for example, 5 km).

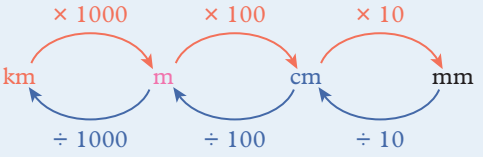
Metric units of length	
Name	**Abbreviation**
Millimetre	mm
Centimetre	cm
Metre	m
Kilometre	km

- To convert between units of length, multiply or divide by the appropriate **conversion factor**.
  For example, a length of 1 cm is the same as 10 mm; therefore, the conversion factor when converting from cm to mm is 10.



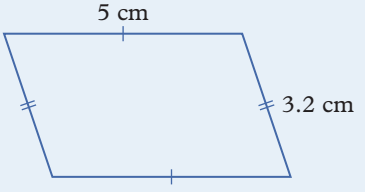
Conversion factors

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

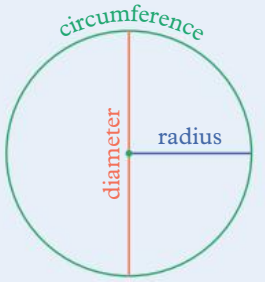


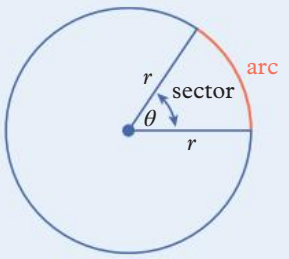
Perimeter

- The **perimeter** of a two-dimensional (2D) shape is the length of the shape's boundary.
- To calculate the perimeter of a shape, add the lengths of each side together.
  For example, the perimeter of the parallelogram is 5 cm + 3.2 cm + 5 cm + 3.2 cm = 16.4 cm.
  → All measurements must be in the same unit before being added together.
  → Sides of equal length are marked using hatch marks.



- The perimeter of a circle is called its **circumference** (C). To find the circumference of a circle, multiply the diameter by **pi** (π). The irrational number π has an infinite number of non-recurring decimal places and is approximately equal to 3.14.
  - → The **diameter** of a circle (d) is the length from one point on the circumference to another through the centre of the circle.
  - → The **radius** of a circle (r) is the length from the centre of the circle to any point on the circumference. The radius is half the length of the diameter.
  - → A part of the circumference of a circle is called an **arc**.
  - → The length of an arc (l) is proportional to the size of the angle (θ) that subtends it.
  - → θ is the angle formed between two radii, the arc length subtended by θ is $\frac{\theta}{360}$ of the circumference.
  - → $l = \frac{\theta}{360} \times C = \frac{\pi r \theta}{180}$
  - → $C = \pi d$ or $C = 2\pi r$





Example 8A.1 Converting between units of length

Convert 1.23 m into millimetres.

THINK	WRITE
1 Convert from metres to centimetres. To convert from a larger unit to a smaller unit, multiply by the conversion factor.	1 m = 100 cm 1.23 m = 1.23 × 100 cm = 123 cm
2 Convert from centimetres to millimetres.	1 cm = 10 mm 123 cm = 123 × 10 mm = 1230 mm

Example 8A.2 Calculating the circumference of a circle

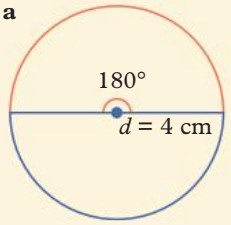
Calculate the circumference of a circle with diameter 8 m, correct to two decimal places.

THINK	WRITE
1 Identify which formula to use.	$C = \pi d$
2 Substitute the measurement for the diameter.	$= \pi \times 8$
3 Calculate using π on your calculator.	= 25.132 740 8 ...
4 Round to two decimal places and include the appropriate unit.	≈ 25.13 cm

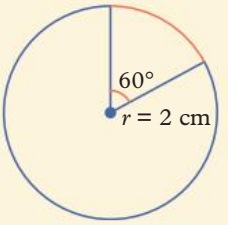
Example 8A.3 Finding the arc length

Calculate the length of the arc in the following circles, correct to two decimal places.

a



b



THINK

a **1** Calculate the circumference of the circle using the formula $C = \pi d$.

2 Identify the measurement for the diameter and substitute it into the formula.

3 180° subtends half a circle, so the length of the arc is half the circumference of the circle. Divide the circumference by 2 to get the arc length.

4 Calculate the result using the π button on your calculator, rounding to two decimal places.

b **1** Write the formula $l = \frac{\pi r \theta}{180}$.

2 Substitute the value of θ and r into the formula.

3 Calculate the result using the π button on your calculator, rounding to two decimal places.

WRITE

a $C = \pi d$

$= 4\pi$ cm

$l = \frac{4\pi}{2}$

≈ 6.28 cm

b $l = \frac{\pi r \theta}{180}$

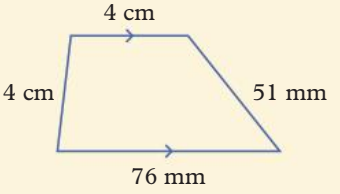
$= \frac{\pi \times 2 \times 60}{180}$

$= \frac{2\pi}{3}$

≈ 2.09 cm

Example 8A.4 Calculating perimeter

Calculate the perimeter of the following shape in centimetres.



THINK

1 Convert all measurements into the unit specified by the question.

2 Write out the sum of the side lengths. There are four sides so there should be four lengths in the sum.

3 Add all the lengths. Include the appropriate unit in the answer.

WRITE

51 mm = 51 ÷ 10 cm
= 5.1 cm

76 mm = 76 ÷ 10 cm
= 7.6 cm

perimeter = 4 cm + 51 mm + 76 mm + 4 cm
= 4 cm + 5.1 cm + 7.6 cm + 4 cm
= 20.7 cm

Helpful hints

- ✔ Once the units are the same, you can leave them out of calculations, but you must remember to include them in your final answer!
- ✔ Always estimate your answer first. If your calculated answer is significantly different than your estimate, check your working to ensure mistakes haven't been made.
- ✔ When converting from a larger unit to a smaller unit (e.g. cm to m) you multiply, and when converting from a smaller unit to a larger unit (e.g. m to km) you divide.
- ✔ Unless instructed otherwise, use π on your calculator instead of an approximation such as 3.14. You should only round your answers at the final step of the calculations.

ANS p535

Exercise 8A Length and perimeter

△ 1, 2, 3(a–e), 4, 5–6(a–e), 7, 8(a–e), 9, 10–11(1st column), 12, 14, 15, 20

□ 1(b, c, e, g), 2(b, d, f, h), 3(b, d, f), 4, 5–6(b, d, f), 8(b, d, f), 10–11(c–e), 13, 16–18, 21, 22, 24, 25

⬠ 1–2(d, f, h), 3(d–f), 5–6(d–f), 8(d–f), 10–11(e–f), 16, 19, 23, 24, 26

UNDERSTANDING AND FLUENCY

8A.1 **1** Convert these lengths to the units shown in brackets.

a 58 m (cm) **b** 2.7 km (m) **c** 521 cm (m) **d** 398 m (km)

e 1.2 cm (mm) **f** 3987 m (km) **g** 423 mm (cm) **h** 7 mm (cm)

2 Convert these lengths to the units shown in brackets.

a 5.34 m (mm) **b** 3.95 km (cm) **c** 235 000 mm (m) **d** 145 000 000 mm (km)

e 2368 cm (km) **f** 287 mm (m) **g** 1.934 km (cm) **h** 0.91 km (mm)

8A.2 **3** Find the circumference of a circle with:

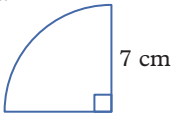
a diameter 1.2 m **b** radius 2.5 cm **c** diameter 46 mm

d radius 0.8 cm **e** radius 91 cm **f** radius 1.3 mm

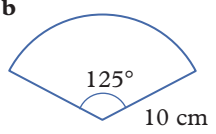
g diameter 1.75 m **h** diameter 2.6 cm **i** diameter 82 km.

8A.3 **4** Calculate the arc length, correct to two decimal places.

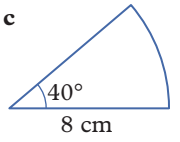
a



b

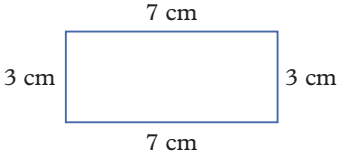


c



5 Calculate the perimeter of these quadrilaterals.

a



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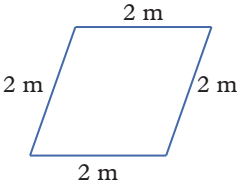
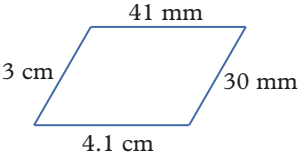
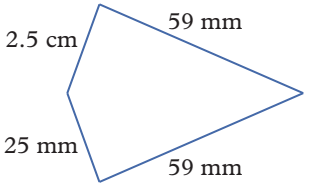


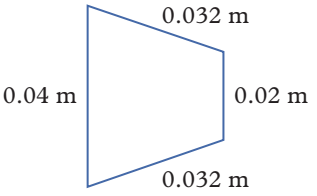
c



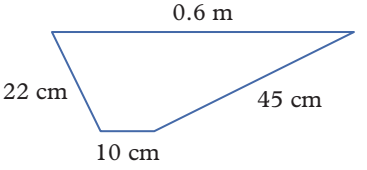
d



e

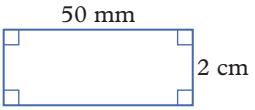


f



6 Calculate the perimeter of these quadrilaterals.

a



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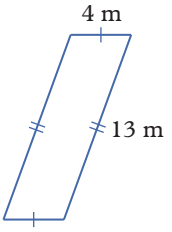

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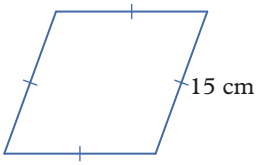

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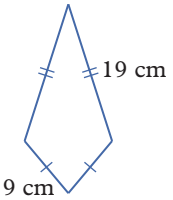

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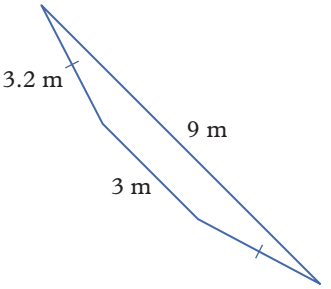
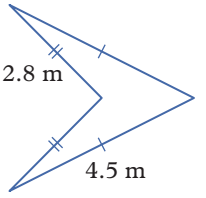


f



7 Calculate the perimeter of:

a a square with side length 4 cm

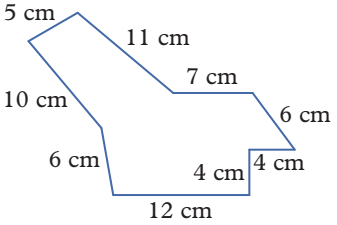
b a rhombus with base 15 mm

c a rectangle with length 10 mm and breadth 15 mm

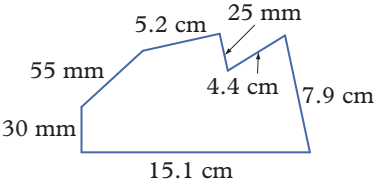
d a parallelogram with side lengths 18 cm and 12 cm.

8A.4 **8** Calculate the perimeter of these shapes.

a



b



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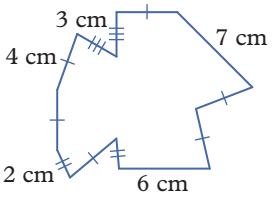

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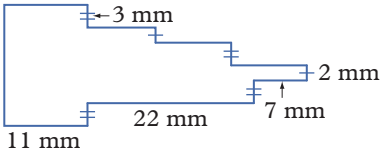

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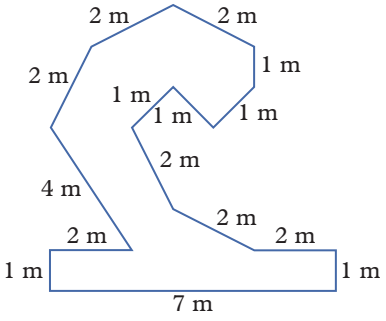

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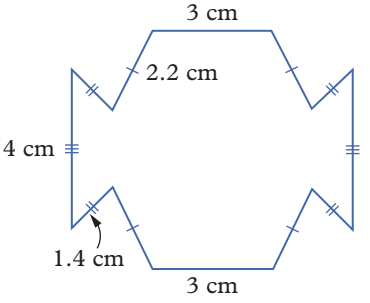


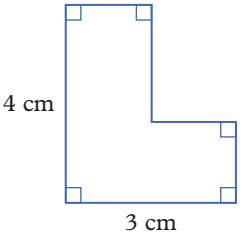
9 Calculate the perimeter of:

a an equilateral triangle with side length 2.5 cm

b a regular hexagon with side length 2.5 m

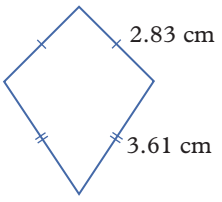
c an L-shaped hexagon with longer sides 3 cm and 4 cm

d a regular octagon with side length 7.7 cm.

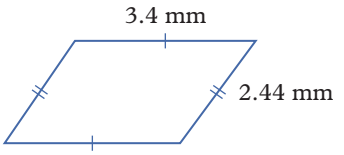


10 Calculate the perimeter of the following shapes, correct to two decimal places.

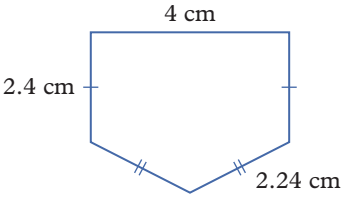
a



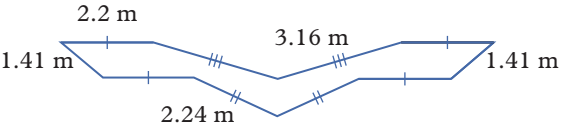
b



c



d



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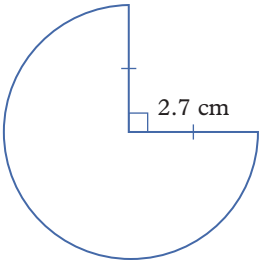
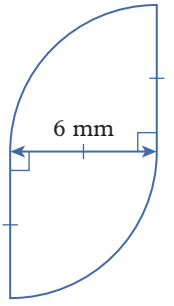
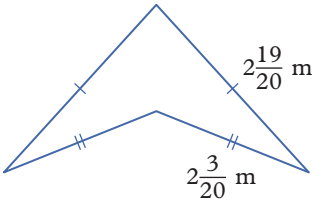


f

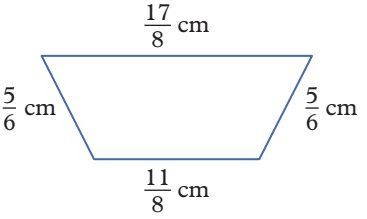


11 Calculate the perimeter of the following shapes. Leave your answers as exact values.

a



b



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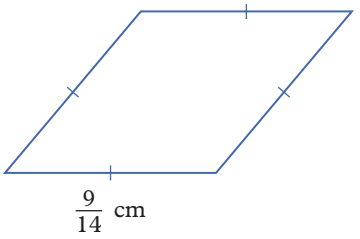

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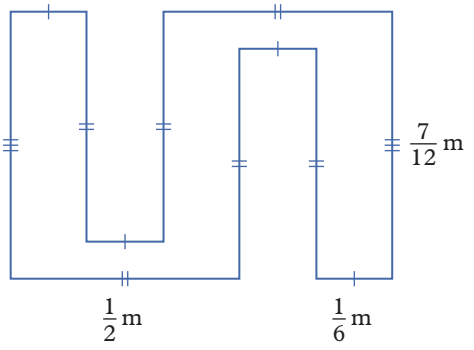

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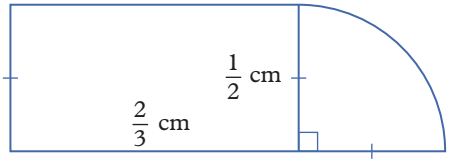

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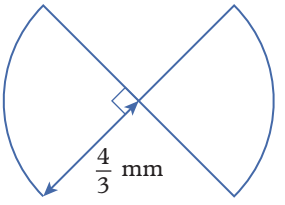
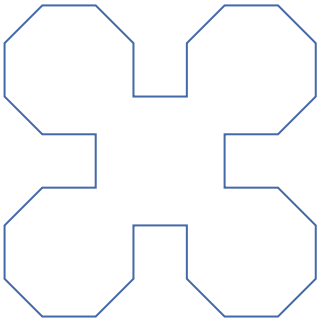


12 Trish makes earrings. One design is a rectangular jewel with wire wrapped around its perimeter twice. If the jewel is 4.5 cm long and 1.5 cm wide, how much wire (in metres) is used for 20 pairs of earrings?

13 A road has a white line painted on each edge, and a dashed line down the middle. If every 10 m section of the road has eight 650 mm long dashes down its centre, calculate the total length of painted line required (in kilometres) for a 10 km road.

14 The perimeter of each of the congruent regular octagons that make up this shape is 48 cm. Determine the perimeter of this shape, justifying your answer.



15 A square has a perimeter of 36 cm. What is its side length? Explain your answer.

16 A new television screen has a perimeter of 6.1 m. If it has a length of 1.9 m, find its breadth.

17 A circular flower bed has a circumference of 4.71 m. What is the diameter correct to two decimal places?

18 A rectangular pool is 12.5 m long and 4.7 m wide.

a What length of paving is required around the pool's edge if the pavers used are 50 cm wide?

b Jordan incorrectly measured the length of the pool as 13.4 m and the breadth as 5.2 m. What is the size of the error in each measurement?

c Find the percentage error in each measurement by calculating the size of the error as a percentage of the actual measurement. Give your answers correct to one decimal place.

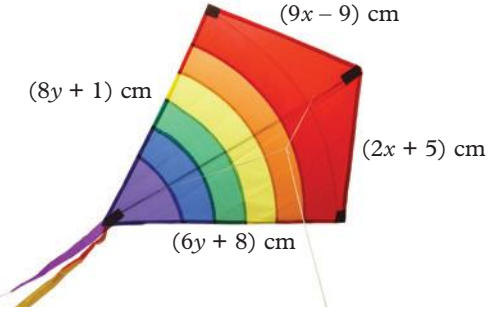
d Which measurement has a greater:

i error **ii** percentage error?

e Calculate the length of paving required if Jordan's measurements are used.

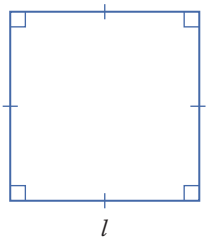
f Find the percentage error in the length of paving if using Jordan's measurements. Give your answer correct to one decimal place.

19 Determine the values of x and y and hence calculate the perimeter, P, of this kite.



20 Write an expression for the perimeter of the following quadrilaterals.

a



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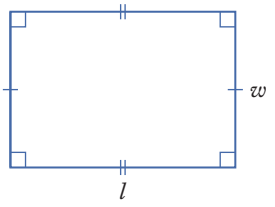

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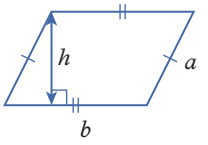

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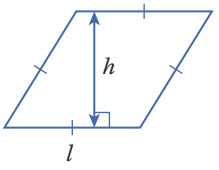

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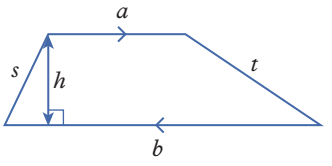

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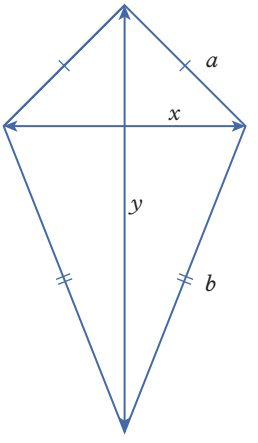

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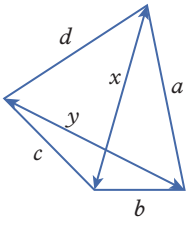
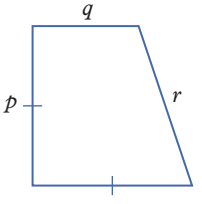
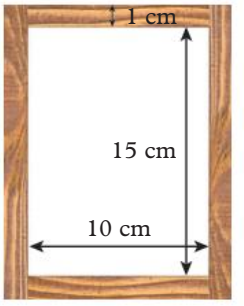


h



21 Alice wants to make some photo frames for a friend. She buys some wood to fit around the outside of a standard-sized photo.



a What is the perimeter of the photo that fits in this frame?

Alice wants the frame to be 1 cm wide, as shown in the image.

b Why is the length of wood needed greater than the perimeter of the photo?

Hint: Are the corner parts of the frame included in the photo perimeter?

c What length of wood is needed?

d What is the perimeter around the photo frame? Why is this different from part **c**?

e Investigate and explain how the results to parts **c** and **d** would change if:

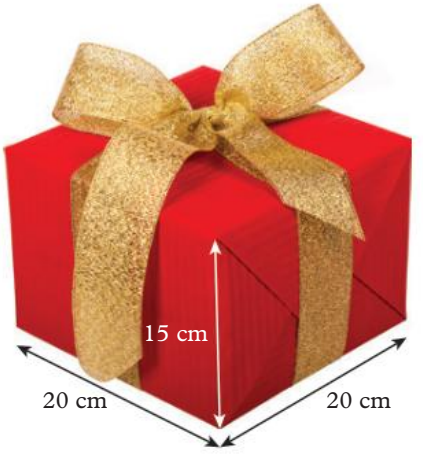
i the frame was 2 cm wide

ii the frame was 3 cm wide

iii the frame was for a 15 cm × 20 cm photo

iv the frame was for a 20 cm × 25 cm photo.

22 Alice packs the photo frames from question **21** into a gift box with the dimensions shown in the photo on the right and wants to tie a ribbon around the present. Assuming that she will need a minimum of 60 cm of ribbon for the bow, at least how much ribbon will she need to buy in order to wrap the present as shown?



23 A rectangle has a perimeter of 30 m. List all the possible whole number dimensions for the rectangle.

24 Jackson takes on a scrapbooking project. He decides to cover the internal perimeter of each page in ribbon that is 2 cm wide. If there are 56 square pages that measure 25 cm in the book, find the length of ribbon (in metres) required to place ribbon inside the perimeter of every page. Justify your answer.

PROBLEM SOLVING AND REASONING

CHALLENGE

25 A rectangular vegetable garden has a perimeter of 28 m.

a Write down a possible set of whole number dimensions for this garden.

A fence is to be placed 1 m from the edge around the vegetable garden to allow space for more vegetables.

b What length of fencing is required?

c What length of fencing would be required if the distance between the fencing and the edge of the garden was:

i 2 m

ii 3 m?

d Can you see a pattern or rule that you could follow in order to find the length of fencing required, no matter how close or far from the edge it is placed?

e Repeat parts **a–d** using a different set of dimensions. Do your findings match the rule you found in part **d**? Explain your answer.

26 An inch is 2.54 cm.

a A foot is equal to 12 inches. What is the conversion factor from feet to centimetres?

b A yard is equal to 3 feet. What is the conversion factor from yards to metres?

c One mile is equal to 1760 yards. What is the conversion factor from miles to kilometres?

d The official distance for a marathon is 26 miles and 385 yards. What is this distance in kilometres?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Converting between units of length

Interactive skillsheet Perimeter

Interactive skillsheet Circumference

Investigation In the extreme

Topic quiz 8A

8B Area of triangles and rectangles

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the area of rectangles and triangles

✔ convert between units of area.

Inter-year links

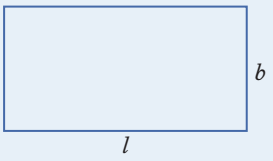
Support	Understanding area
Year 7	8C Area of a rectangle
Year 9	6A Area of composite shapes
Year 10	8A Area review

Key content video

Area

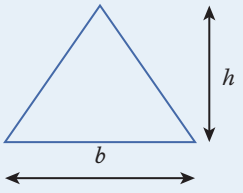
- The **area** of a 2D shape is the space occupied by that shape.
- To find the area of a rectangle, multiply its length (l) by its breadth (b).
- To find the area of a triangle, multiply its base (b) by its height (h) then divide by 2.
  → The **base** length and height must be perpendicular (at a right angles) to each other.

Rectangle



$A = lb$

Triangle



$A = \frac{1}{2}bh$

- Measurements must be in the same unit before they are substituted into area formulas.

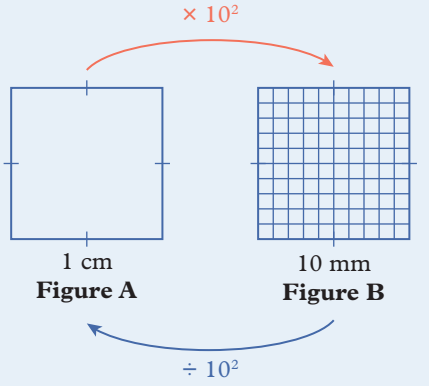
Metric units of length	
Name	**Abbreviation**
Square millimetre	mm^2
Square centimetre	cm^2
Square metre	m^2
Square kilometre	km^2

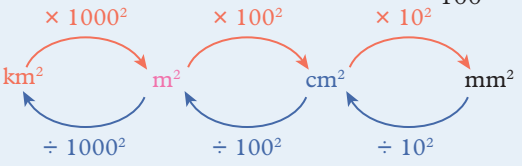
Conversion factors

$1\ cm^2 = 10^2\ mm^2$
$= 100\ mm^2$
$1\ m^2 = 100^2\ cm^2$
$= 10\ 000\ cm^2$
$1\ km^2 = 1000^2\ m^2$
$= 1\ 000\ 000\ m^2$

Converting between units of area

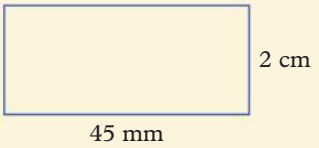
- To convert between square units of area, multiply or divide by the square of the conversion factor between the lengths.
  For example, a square centimetre ($1\ cm \times 1\ cm = 1\ cm^2$) is equivalent to 100 square millimetres ($10\ mm \times 10\ mm = 10^2\ mm^2 = 100\ mm^2$). That means we can multiply the number of cm^2 by 10^2 to determine the number of mm^2.
  Likewise we can divide the number of mm^2 by 10^2 to determine the number of cm^2, since each mm^2 is $\frac{1}{100}$ th of a cm^2.





Example 8B.1 Calculating the area of a rectangle

Calculate the area of this rectangle in cm^2.



THINK

1. Identify the measurements for l and b and convert the length into the unit specified by the question.
2. Substitute the values for l and b into the formula for the area of a rectangle.
3. Calculate the area of the rectangle. Remember to include the appropriate unit in the answer.

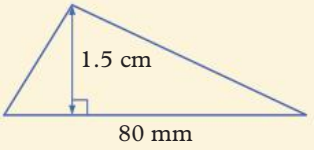
WRITE

$l = 45$ mm
$= 4.5$ cm
$b = 2$ cm

$A = l \times b$
$= 4.5 \times 2$
$= 9\ cm^2$

Example 8B.2 Calculating the area of a triangle

Calculate the area of this triangle in mm^2.



THINK

1. Identify the measurements for b and h and convert the height into the units specified by the question.
2. Substitute the values for b and h into the formula for the area of a triangle.
3. Calculate the area of the triangle. Remember to include the appropriate unit in the answer.

WRITE

$b = 80$ mm
$h = 1.5$ cm
$= 15$ mm

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 80 \times 15$
$= 600\ mm^2$

Example 8B.3 Converting to a smaller area unit

Convert 3.8 cm^2 into mm^2.

THINK	WRITE
1 Identify the appropriate conversion factor. To convert from a larger unit to a smaller unit, multiply by the conversion factor.	$1\ cm^2 = 10^2\ mm^2$ $= 100\ mm^2$
2 Complete the calculation.	$3.8\ cm^2 = 3.8 \times 100$ $= 380\ mm^2$

Example 8B.4 Converting to a larger area unit

Convert 120 000 mm^2 into m^2.

THINK	WRITE
1 First convert from mm^2 to cm^2. To convert from a smaller unit to a larger unit, divide by the conversion factor.	$1\ cm^2 = 10^2\ mm^2$ $= 100\ mm^2$ $1\ m^2 = 100^2\ cm^2$ $= 10\ 000\ cm^2$ $120\ 000\ mm^2 = 120\ 000 \div 100$ $= 1200\ cm^2$
2 Complete the calculation by converting from cm^2 to m^2.	$1200\ cm^2 = 1200 \div 10\ 000$ $= 0.12\ m^2$

Helpful hints

✔ When multiplying by a power of ten, move the decimal point to the *right*.

$5.23 \times 10 = 52.3$

$5.23 \times 100 = 523.$

$5.23 \times 1000 = 5230.$

✔ When dividing by a power of ten, move the decimal point to the *left*.

$5.23 \div 10 = 0.523$

$5.23 \div 100 = 0.0523$

$5.23 \div 1000 = 0.005\,23$

✔ Remember that larger units take up more area for each square unit, so there will be less of them than for smaller units. Likewise, smaller units take up less area for each square unit, so there will be more of them than for larger units.

ANS p536 Exercise 8B Area of triangles and rectangles

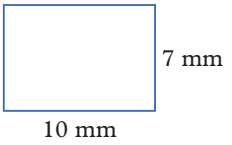
1, 2, 3(a–e), 4–7, 8–10, 11(a, b), 13, 17, 19

1, 2, 3(f–i), 4(b, c, f), 5–7, 10, 11, 14, 18, 20, 21(a, b)

3(f–i), 4–7, 11, 12, 15–16, 20, 21

8B.1 **1** Calculate the area of each rectangle. Where more than one unit is used, give your answer in terms of the smaller unit.

a



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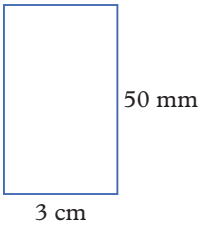

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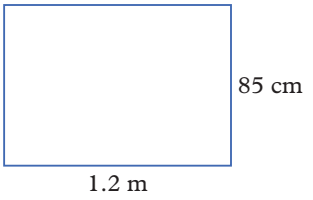

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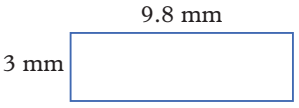

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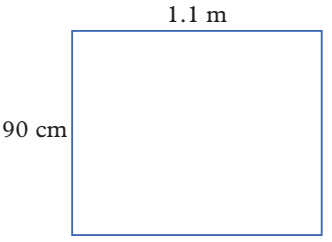

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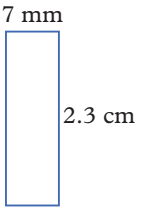

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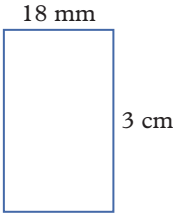
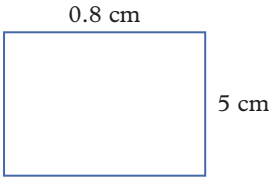
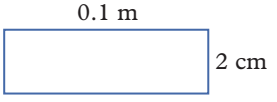


h

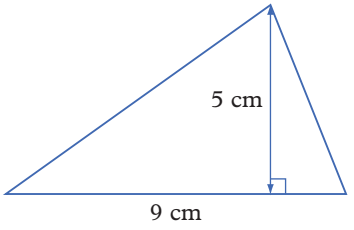


i

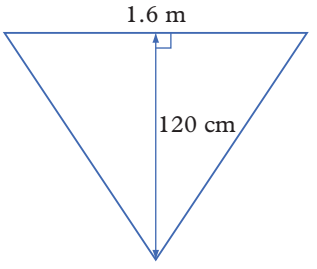


8B.2 **2** Calculate the area of each triangle. Where more than one unit is used, give your answer in terms of the larger unit.

a



b



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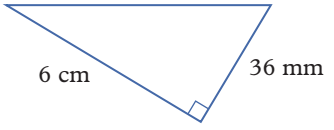

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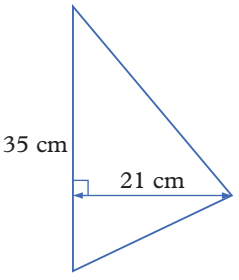

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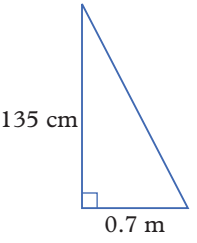

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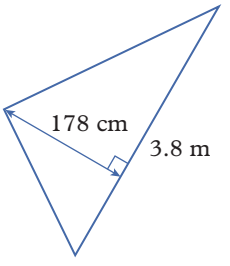
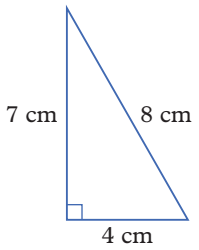


3 Calculate the area of each shape in the unit specified in brackets.

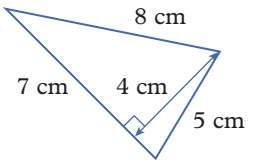
a a rectangle with length 35 mm, breadth 4 cm (mm^2)
b a rectangle with length 12 mm, breadth 17 cm (cm^2)
c a triangle with base 15 mm, height 2 cm (mm^2)
d a square with length 7.2 mm (mm^2)
e a triangle with base 25 cm, height 10 mm (cm^2)
f a triangle with base 8 cm, height 42 mm (cm^2)
g a square with base 19 cm (cm^2)
h a rectangle with base 4.2 cm, height 11 mm (mm^2)
i a triangle with base 0.7 m, height 49 cm (cm^2)

4 Calculate the area of each shape.

a



b



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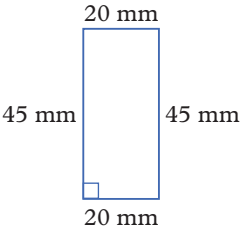

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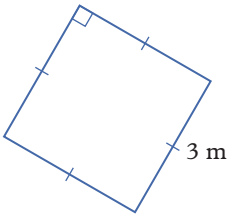

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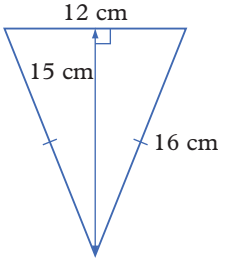

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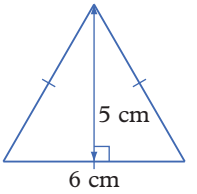


8B.3 **5** Convert each area measurement.

a 3 m^2 into cm^2
b 10 cm^2 into mm^2
c 6.5 m^2 into cm^2
d 19.7 cm^2 into mm^2
e 1.6 km^2 into m^2
f 0.75 km^2 into m^2

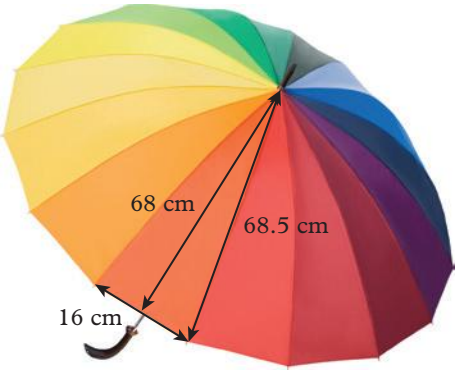
8B.4 **6** Convert each area measurement.

a 25 000 cm^2 into m^2
b 750 000 mm^2 into cm^2
c 400 mm^2 into cm^2
d 125 000 m^2 into km^2
e 675 cm^2 into m^2
f 500 000 000 m^2 into km^2

7 Convert each area measurement.

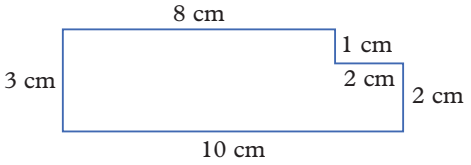
a 150 000 mm^2 into m^2
b 0.002 km^2 into cm^2
c 1.5 m^2 into mm^2
d 379 000 cm^2 into km^2

8 How much material is needed to make the multi-coloured umbrella shown?



9 A classmate attempted question **8** and after using 68.5 cm as the height of each triangle, obtained an answer of 8768 cm^2. Explain where they went wrong.

10 A composite shape is a shape that can be split up into two or more basic shapes. For example, the composite shape below can be split up into rectangles as shown in Figures A and B.



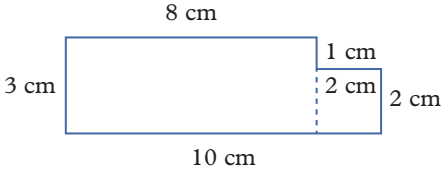


Figure A

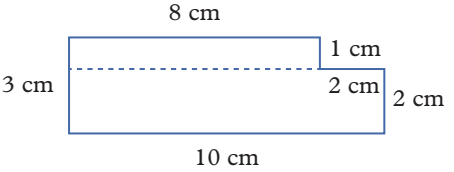


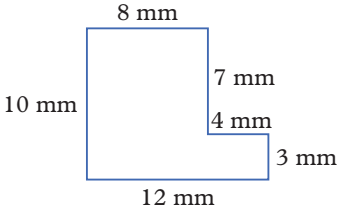
Figure B

a Find the area of each rectangle in Figure A and hence the area of the entire shape.

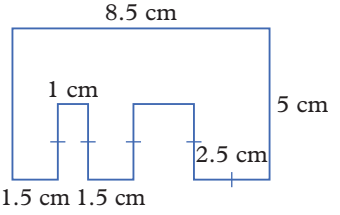
b Find the area of each rectangle in Figure B and hence the area of the entire shape.

11 Find the area of each of these composite shapes. Hint: For some parts it may be easier to subtract shapes.

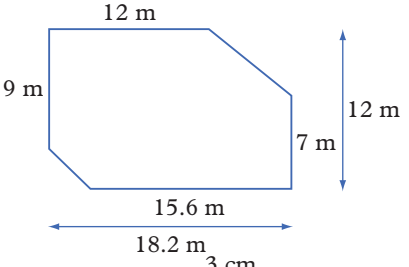
a



b



c



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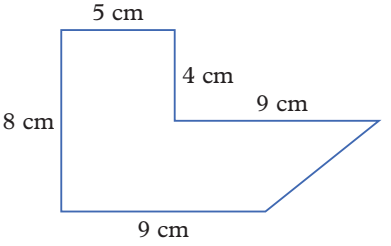

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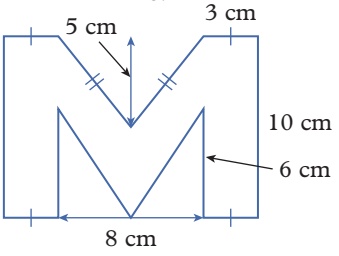

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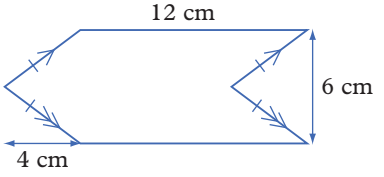


12 Which unit of area (mm^2, cm^2, m^2 or km^2) would be best for measuring the area of these? Justify your answers.

a basketball court **b** wristwatch face **c** national park

d five-dollar note **e** surface of a skateboard ramp **f** family-size pizza

13 Ahmed needed to convert 11 cm^2 into mm^2. He said that because there are 10 mm in 1 cm, then there are 110 mm^2 in 11 cm^2. Explain where he went wrong and provide the correct answer.

PROBLEM SOLVING AND REASONING

14 Alex was completing a worksheet on conversions and provided the following answer to a question:

$40\,000\ cm^2 = (40\,000 \div 100)\ m^2$
$= 400\ m^2$

Explain where Alex made a mistake and provide the correct answer.

15 **a** A hectare is 10 000 m^2. How many hectares are in 1 km^2?

b There are many other units of area that are still used today that are not metric, such as the acre. A hectare is roughly equal to 2.5 acres. Approximately how many square metres are there in an acre?

16 Aaron harvests wheat on his farm and gets a yield of about 3.2 tonnes (3200 kg) per hectare.

a Write this in:

i kilograms per hectare

ii kilograms per square metre

iii grams per square metre

iv grams per square centimetre.

b Which unit do you think makes the most sense to use? Explain your answer.

c If Aaron is paid 35 cents per kilogram of wheat, find how much money he will earn if he harvests 3000 hectares of wheat.

17 A rectangle has an area of 20 cm^2 and a length of 4 cm. Find its breadth and explain why there is only one rectangle that you can draw that fits this description.

18 A rectangle has a perimeter of 28 cm. If the dimensions are whole numbers, find three possible areas this rectangle could have.

19 A square has an area of 64 mm^2. Sketch this square, stating its dimensions, its perimeter and why there is only one square that matches this description.

20 A wooden frame is to go around a rectangular mirror with an area of 3600 cm^2.

a If the mirror has a length of 90 cm, what is its breadth?

b Calculate the area of the wooden surface if the frame is to be 5 cm wide? Hint: Draw a picture to help you with your calculations.

CHALLENGE

21 Mohammed is making a rectangular vegetable garden and has 24 m of fencing to put around the garden.

a List three possible sets of dimensions and corresponding areas for the garden.

b What is the maximum area that you can find using 24 m of fencing? What shape is this?

Mohammed realises he can enclose a bigger area if he puts the garden against the shed wall and only has to fence three sides of the garden.

c List another three possible sets of dimensions and corresponding areas for the veggie garden if one side is against the shed wall.

d What is the maximum area that you can find now using 24 m of fencing for three sides? What shape is this? How does this relate to your answer to part **b**?

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Area of a rectangle

Interactive skillsheet Area of a triangle

Interactive skillsheet Converting between units of area

Investigation Units of area and accuracy

Topic quiz 8B

8C Area of quadrilaterals

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the area of parallelograms, kites, rhombuses and trapeziums.

Inter-year links

Support	Understanding area
Year 7	8D Area of a parallelogram
Year 9	6A Area of composite shapes
Year 10	8A Area review

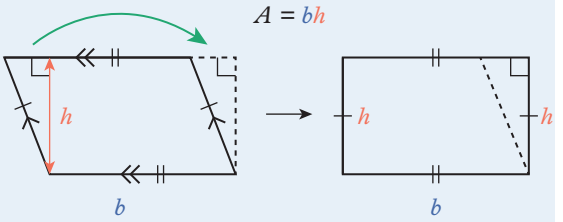
Area of quadrilaterals

Key content video

- Every parallelogram can be rearranged into a rectangle with the same base length and height; therefore, the area of a parallelogram can be calculated using the formula $A = bh$, where b is the base length and h is the perpendicular height.

Parallelogram

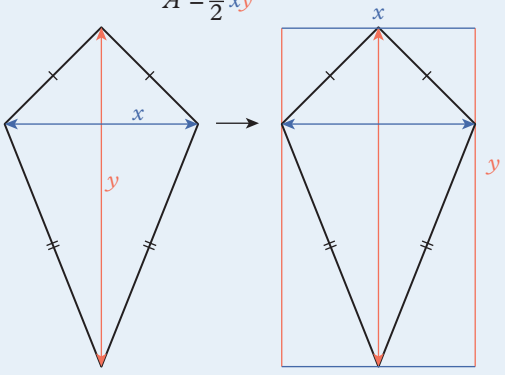
$A = bh$



- The area of a kite is exactly half the area of the rectangle that encloses it. Therefore, the area of a kite can be calculated using the formula $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals of the kite.

Kite

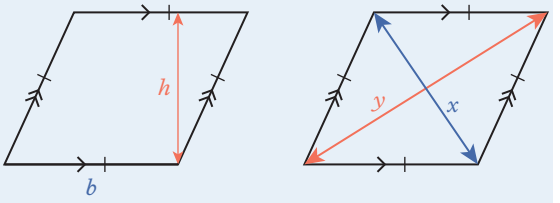
$A = \frac{1}{2}xy$



- As every rhombus is both a kite and a parallelogram, the area of a rhombus can be calculated using either the formula for the area of a parallelogram, $A = bh$, or the formula for the area of a kite, $A = \frac{1}{2}xy$, depending on the information provided.

Rhombus

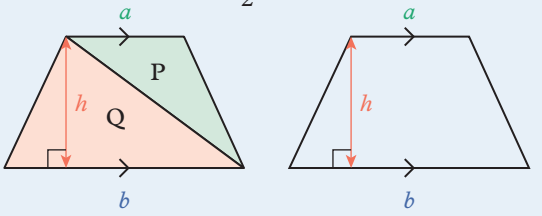
$A = bh$ $\quad$ $A = \frac{1}{2}xy$



- Every trapezium can be split into two triangles, both with the same perpendicular height. The area of triangle P is $\frac{1}{2}ah$ and the area of triangle Q is $\frac{1}{2}bh$; therefore, the area of the trapezium is $A = \frac{1}{2}ah + \frac{1}{2}bh$, or more concisely, $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height.

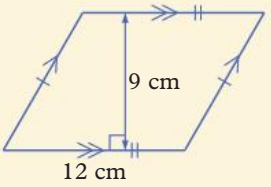
Trapezium

$A = \frac{h}{2}(a + b)$



Example 8C.1 Calculating the area of a parallelogram

Calculate the area of this parallelogram.



THINK

1. Identify the measurements for b and h.
2. Substitute the values for b and h into the formula for the area of a parallelogram.
3. Calculate the result. Remember to include the appropriate unit in the answer.

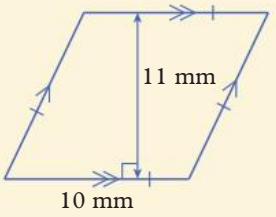
WRITE

$A = bh$
$= 12 \times 9$
$= 108 \text{ cm}^2$

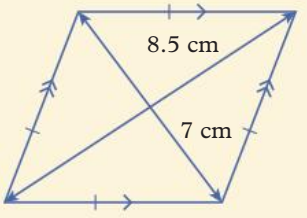
Example 8C.2 Calculating the area of a rhombus

Calculate the area of these rhombuses.

a



b



THINK

a
1. Determine which formula to use. b and h are given, so use the formula for the area of a parallelogram.
2. Identify the given measurements.
3. Substitute the values for b and h into the formula for the area of a parallelogram.
4. Calculate the result and include the appropriate unit.

b
1. Determine which formula to use. x and y are given, so use the formula for the area of a kite.
2. Identify the given measurements.
3. Substitute the values for x and y into the formula for the area of a kite.
4. Calculate the result and include the appropriate unit.

WRITE

a $b = 10$ mm, $h = 11$ mm
$A = bh$

$b = 10$ mm, $h = 11$ mm
$A = 10 \times 11$

$= 110 \text{ mm}^2$

b $x = 7$ cm, $y = 8.5$ cm
$A = \frac{1}{2}xy$

$x = 7$ cm, $y = 8.5$ cm
$A = \frac{1}{2} \times 7 \times 8.5$

$= 29.75 \text{ cm}^2$

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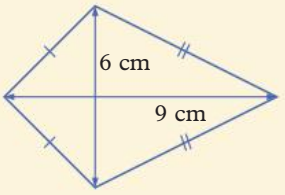
Example 8C.3 Calculating the area of a kite

Calculate the area of this kite.



THINK

1. Identify the measurements for x and y.
2. Substitute the values for x and y into the formula for the area of a kite.
3. Calculate the result. Remember to include the appropriate unit in the answer.

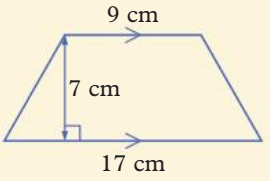
WRITE

$x = 6$ cm, $y = 9$ cm

$$A = \frac{1}{2}xy$$
$$= \frac{1}{2} \times 6 \times 9$$
$$= 27\,\text{cm}^2$$

Example 8C.4 Calculating the area of a trapezium

Calculate the area of this trapezium.



THINK

1. Identify the given measurements.
2. Substitute the values for a, b and h into the formula for the area of a trapezium.
3. Calculate the result and include the appropriate unit.

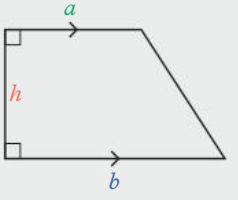
WRITE

$a = 9$ cm, $b = 17$ cm, $h = 7$ cm

$$A = \frac{h}{2}(a + b)$$
$$= \frac{7}{2} \times (9 + 17)$$
$$= \frac{7}{2} \times 26$$
$$= 91\ \text{cm}^2$$

Helpful hints

- ✔ All area formulas must include the multiplication of two lengths to create an area.
- ✔ In the formulas for the area of a parallelogram and trapezium, the height must be the perpendicular distance between parallel sides.
- ✔ In a right-angled trapezium, the height will be the length of the side joining the right angles.
- ✔ In the formula for the area of a kite, it doesn't matter which diagonal length you label x or y, since $xy = yx$! Likewise, in the formula for the area of a trapezium it doesn't matter which of the parallel sides you label a or b, since $a + b = b + a$!



ANS p537

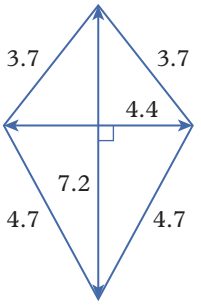
Exercise 8C Area of quadrilaterals

1–8, 9, 11, 12, 13 | 2–5(b, c), 6–8, 10, 13, 14, 16 | 6–8, 14, 15, 17, 18

UNDERSTANDING AND FLUENCY

1 Which of the lengths in the following shapes are needed to determine the area?

a



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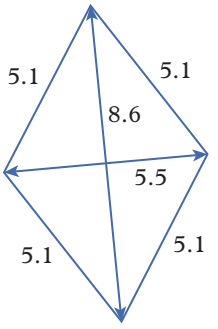
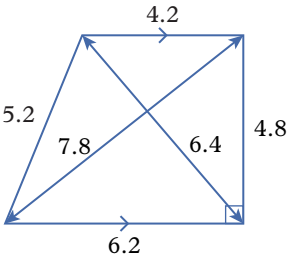


c



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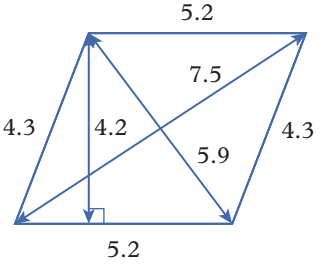

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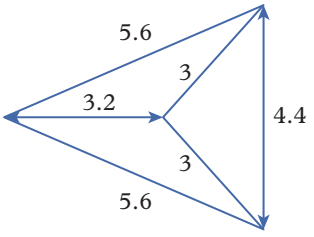

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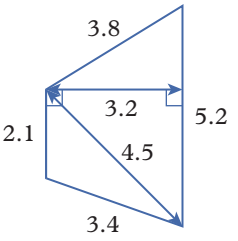
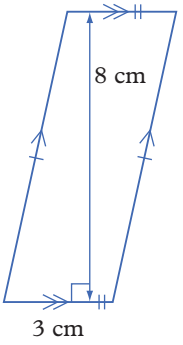


8C.1 **2** Calculate the area of each parallelogram.

a



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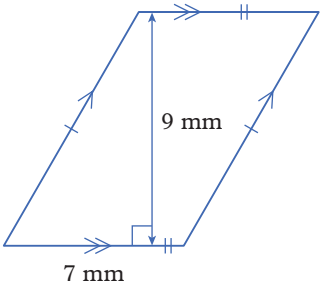

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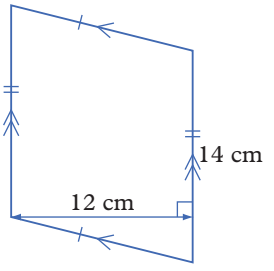
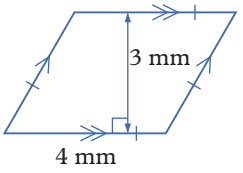


8C.2 **3** Calculate the area of each rhombus.

a



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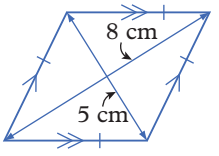

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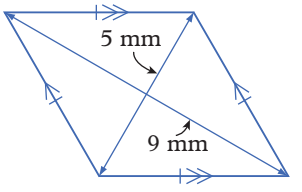
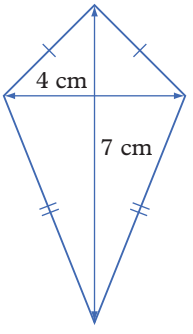


8C.3 **4** Calculate the area of each kite.

a



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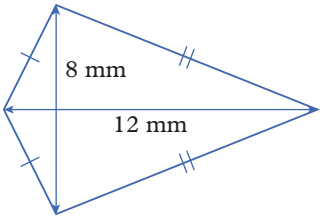

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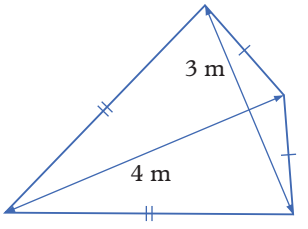
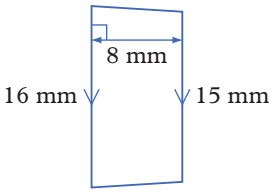


8C.4 **5** Calculate the area of each trapezium.

a



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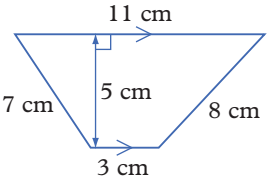

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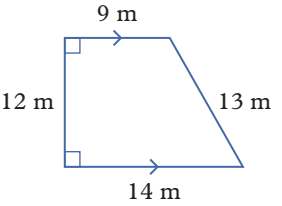


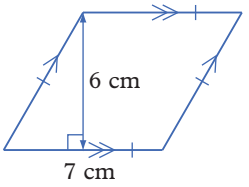
6 For each shape:

i identify if it is a parallelogram, kite, rhombus or trapezium

ii write the formula to find its area

iii calculate its area.

a



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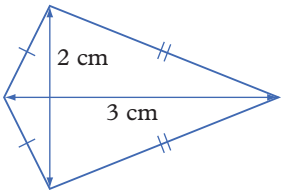

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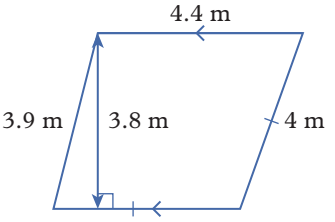

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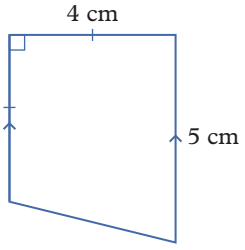

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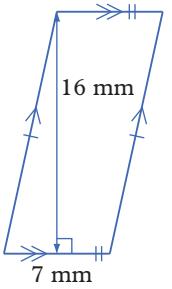

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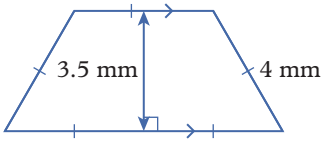
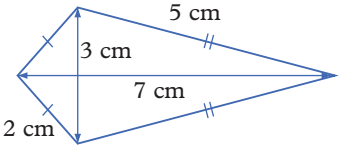


7 Calculate the area of each quadrilateral.

a



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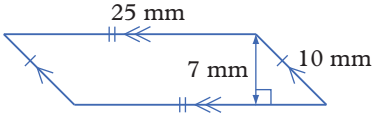

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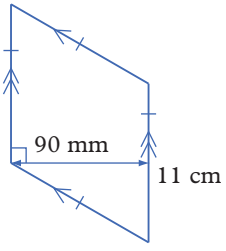

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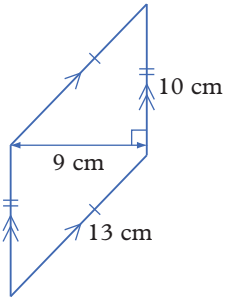

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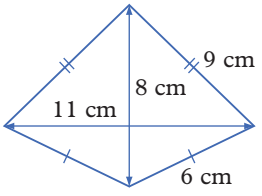

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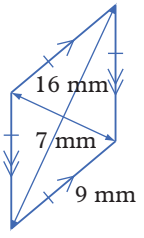
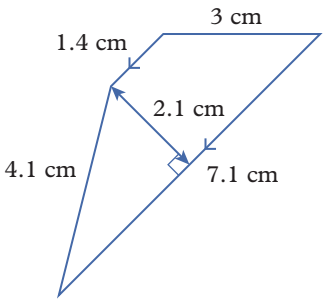


8 Evaluate the areas of the following shapes.

a



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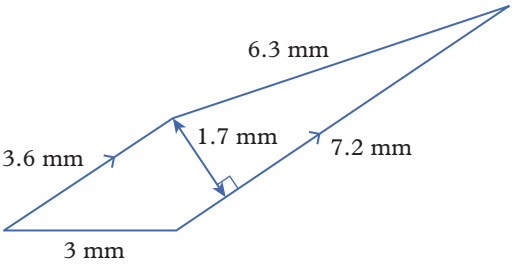


PROBLEM SOLVING AND REASONING

9 Consider this building in Hamburg.

a What shape is it?

b If the glass section is 21 m tall and the base is 86 m long, what is the area of this entire section?

c Is every window equal in area? Explain your answer.

d Which area formulas would you need to calculate the area of the windows?

10 Use the information provided to determine the missing lengths.

a The area of a kite is 72 cm^2 and the length of one of the diagonals is 16 cm. Determine the length of the other diagonal.

b The area of a rhombus is 17 cm^2 and the base length is 4 cm. Determine the height of the rhombus.

c The area of a trapezium is 22.5 cm^2 and the sum of the lengths of the parallel sides is 9 cm. Determine the height of the trapezium.

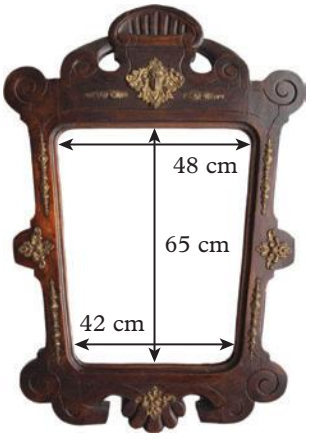
d The area of a trapezium is 48 mm^2, the height is 3 mm, and the length of one of the parallel sides is 1.3 cm. Determine the length of the other parallel side.

11 Amanda wants to make some kites. She decides to make two types, a large blue kite and a smaller red kite.

a The blue kite is 95 cm long and 65 cm wide. What is its area?

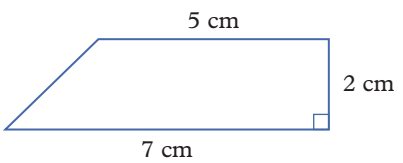
b The red kite has an area of 400 cm^2. If the kite is 40 cm long, how wide must it be?

12 Find the cost of fitting the frame on the right with a mirror if your local hardware store charges \$0.25 per cm^2 for mirror glass.



13 If no other side lengths are determined, is there enough information to calculate the area of the following shapes? Give reasons with your answers.

a



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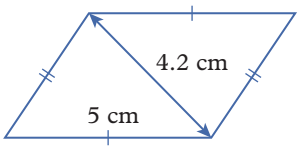

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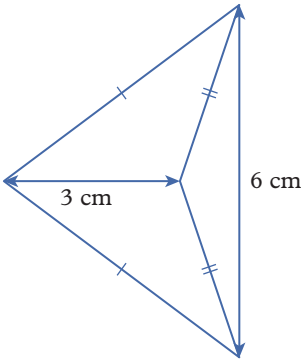

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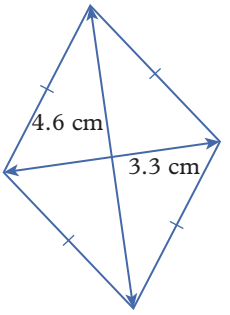

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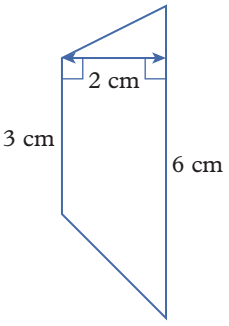

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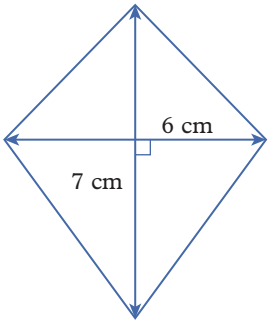
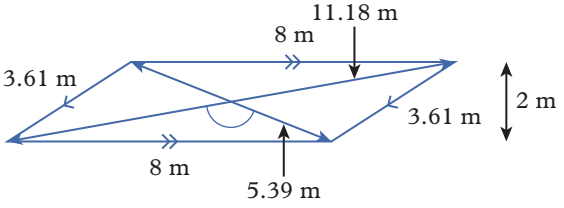


14 Some quadrilaterals can be named in multiple ways; for example, every square is also a rectangle. For each of the following quadrilaterals:

i state if the shape is a parallelogram, kite, rhombus and/or trapezium

ii determine the area of the quadrilateral, correct to one decimal place where necessary.

a



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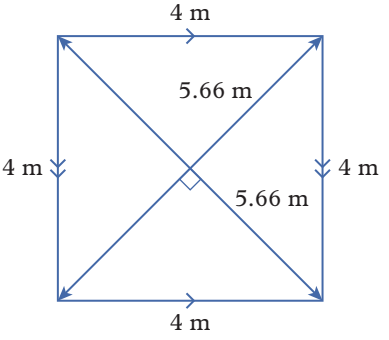

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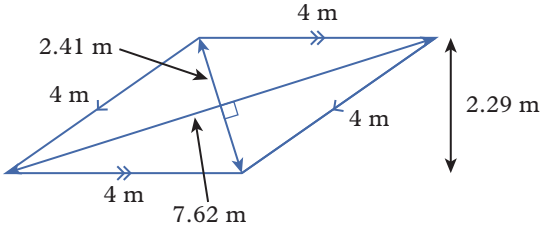

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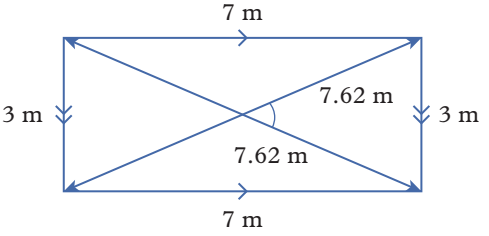
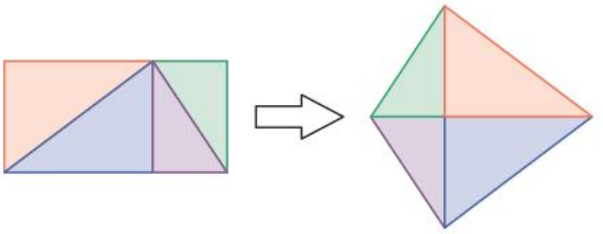


PROBLEM SOLVING AND REASONING

15 A rectangle has an area of 48 cm^2. The rectangle is cut into two pairs of congruent triangles which are rearranged to form a kite. If the lengths of the diagonals are integers, what possible dimensions (lengths of the diagonals) can the kite have?

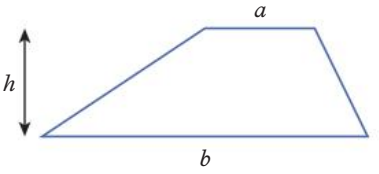


16 a The perimeter of a square is 36 cm. Determine the area of the square.

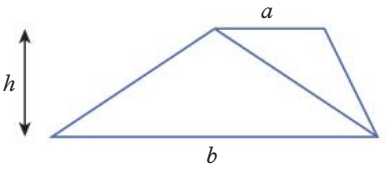
b The area of a square is 36 cm^2. Determine the perimeter of the square.

c Determine the side length of a square (in cm) such that the value of the area and perimeter are the same.

17 The formula for the area of a trapezium can be derived in multiple ways. Describe how each diagram has been constructed from the trapezium shown, and how it could be used to derive the formula for the area of the trapezium.



a



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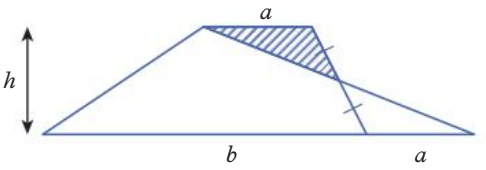

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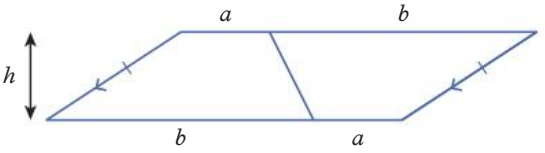

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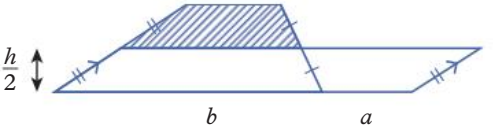

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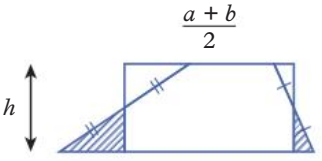
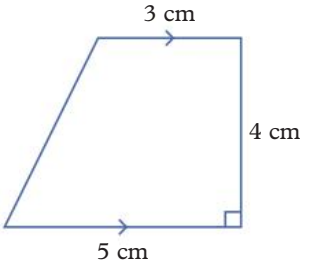


CHALLENGE

18 Consider the trapezium shown. The shorter parallel side is increased by 150%, the longer parallel side is increased by 20%, and the height is decreased by 75%.



a Determine the area of:

i the original trapezium

ii the altered trapezium.

b Determine the percentage change of the area, stating whether it is an increase or decrease.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet Area of quadrilaterals

Investigation Land to value

Topic quiz 8C

ANS p537

Checkpoint

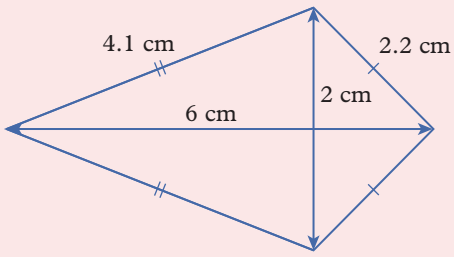
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter

8A **1** Convert the following.

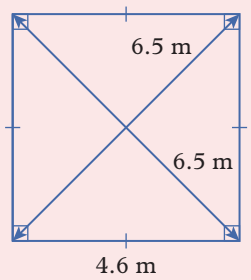
a 45 cm to m **b** 5.9 km to m **c** 351 mm² to cm² **d** 0.2 m² to cm²

8A **2** Calculate the perimeter of the following shapes.

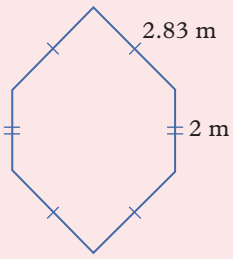
a



b



c



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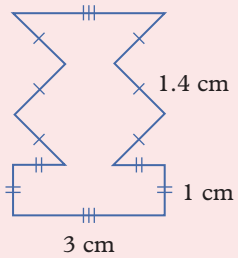
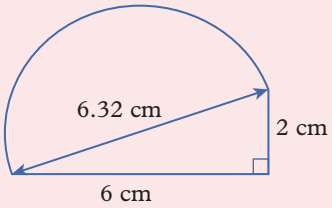
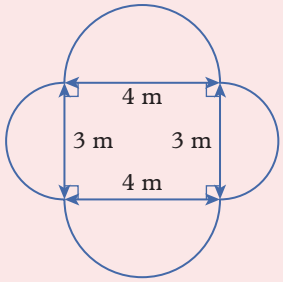


8A **3** Calculate the perimeter of the following composite shapes correct to two decimal places.

a



b

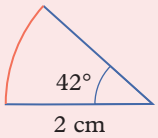


8A **4** Calculate the circumference of the following circles. Give your answers correct to two decimal places.

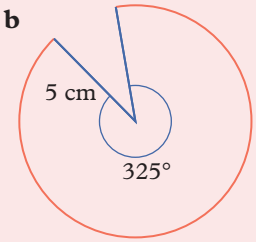
a circle with diameter 31 m **b** circle with radius 7 cm

8A **5** Calculate the arc length in the following shapes, round to two decimal places.

a



b



8C **6** Use the given information to find the missing length.

a The perimeter of a square is 22 m. Determine the side length of the square.

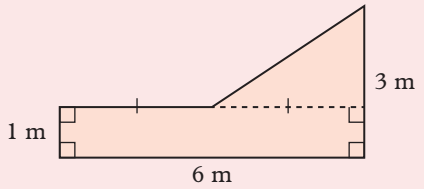
b The area of a rectangle with a breadth of 4 m is 28.8 m². Determine the length of the rectangle.

c The area of a kite is 31.5 cm². If one of the diagonals is 7 cm long, determine the length of the other diagonal.

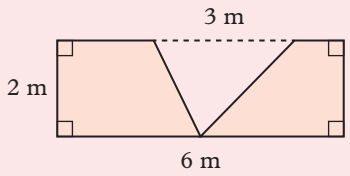
d The area of a trapezium is 37.8 cm². If one of the parallel sides is 9.5 cm long and the perpendicular height is 6 cm, determine the length of the other parallel side.

8B **7** Calculate the shaded area of the following composite shapes.

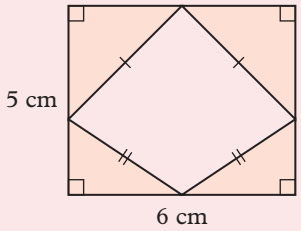
a



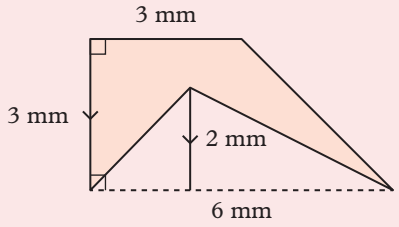
b



c



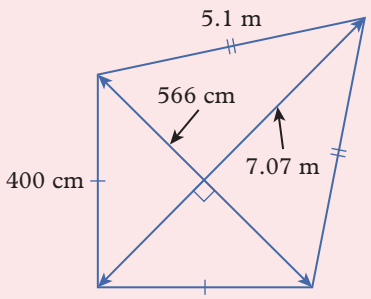
d



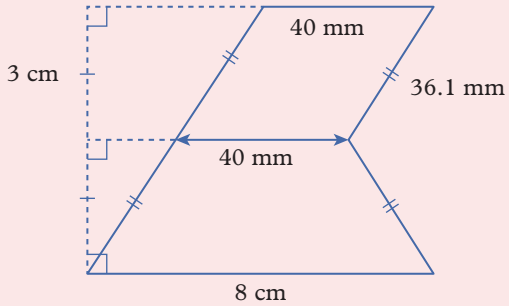
8C **8** For each of the following shapes, calculate the:

i perimeter **ii** area.

a

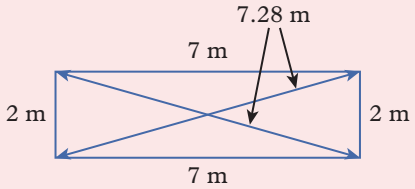


b

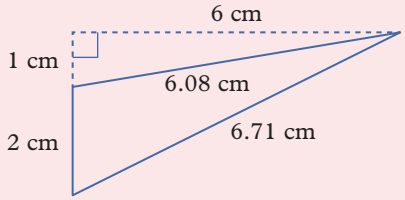


8C **9** Calculate the area of the following shapes, giving your answers correct to two decimal places where necessary.

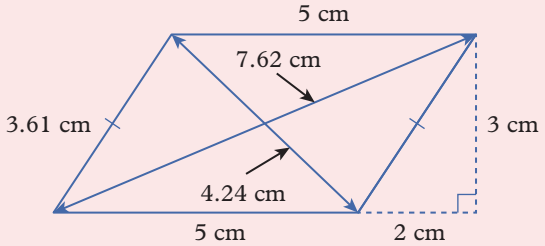
a



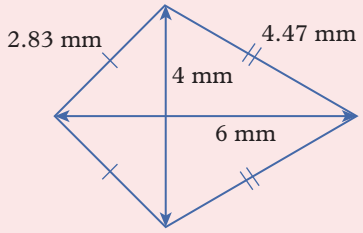
b



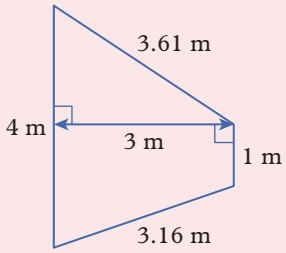
c



d



e



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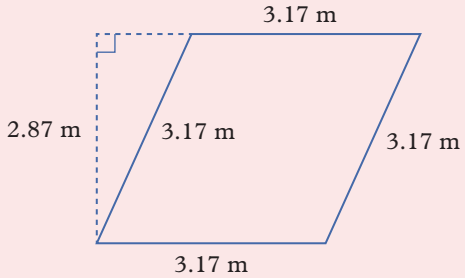


8D Area of a circle

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the area of a circle or a sector

✔ calculate the radius, diameter or circumference, given the area of a circle.

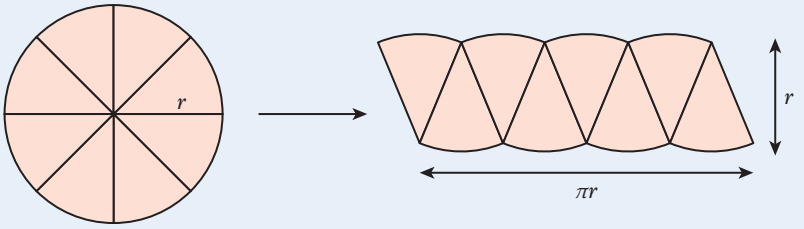
Inter-year links

Support	Understanding area
Year 7	8B Circumference of a circle
Year 9	6A Area of composite shapes
Year 10	8A Area review

Area of a circle

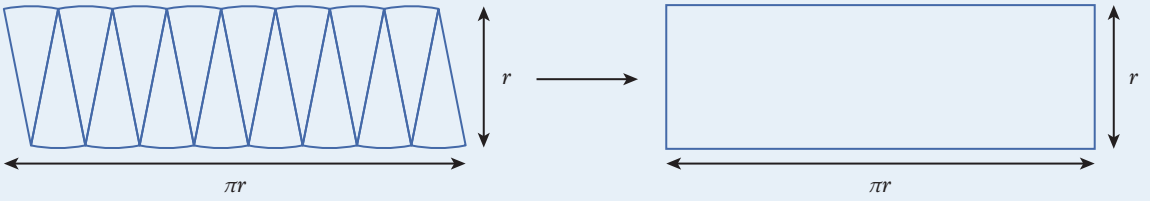
Key content video

- By dividing a circle into equal parts and rearranging these parts as shown, we can find the area of a circle.



→ The base of the rearranged shape is approximately equal to half of the circumference (πr).

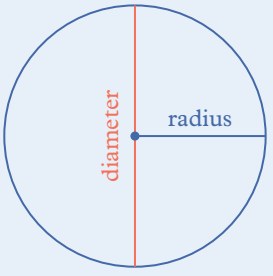
- As the number of parts increases, the resulting figure becomes rectangular in shape.

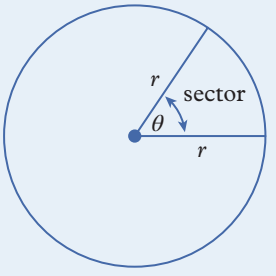


- Therefore, the formula for the area of a circle with radius r is: $A = \pi r^2$

→ If you are given the diameter, find the radius before using the formula.

→ This formula can be rearranged to find the radius of a circle, given the area.





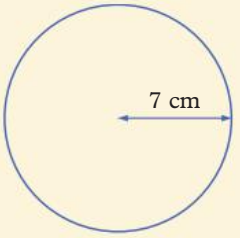
- A **sector** is a portion of a circle formed by two radii and part of the circumference (an arc). You can use knowledge of angles to calculate the area of a sector.
- The area of a sector is proportional to the size of the angle that subtends it.
- θ is the angle formed between two radii, the area of the section subtended by θ is $\frac{\theta}{360}$ of the area of the circle.
- $A_{\text{sector}} = \frac{\theta}{360} \times A_{\text{circle}}$

  $= \frac{\pi r^2 \theta}{360}$

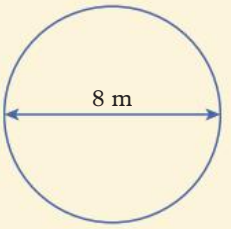
Example 8D.1 Calculating the area of a circle

Calculate the area of these circles correct to two decimal places.

a



b



THINK

a 1 Identify the value of the radius, r.

2 Substitute the value for r into the formula for the area of a circle.

3 Calculate the result using π on your calculator.

4 Round to two decimal places and include the appropriate unit.

b 1 Identify the value of the radius, r. Divide the diameter by 2 to find the radius.

2 Substitute the value for r into the formula for the area of a circle.

3 Calculate the result using π on your calculator.

4 Round to two decimal places and include the appropriate unit.

WRITE

a $r = 7$ cm

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 7^2 \\ &= 153.938... \\ &\approx 153.94 \text{ cm}^2 \end{aligned}$$

b $r = d \div 2$

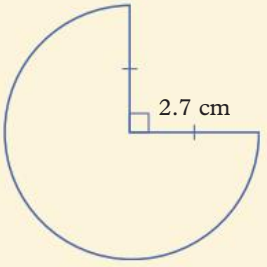
$$\begin{aligned} &= 8 \div 2 \\ &= 4 \text{ m} \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 4^2 \\ &= 50.265... \\ &\approx 50.27 \text{ m}^2 \end{aligned}$$

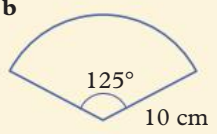
Example 8D.2 Finding the area of a sector

Calculate the area of the sectors in the following circles, correct to two decimal places.

a



b



THINK

a 1 Calculate the area of the circle using the formula $A = \pi r^2$.

2 Identify the measurement for the radius and substitute it into the formula.

3 The area of the sector is three quarters of the circle. Multiply the area of the circle by $\frac{3}{4}$ to get the area of the sector.

4 Calculate the result using the π button on your calculator, rounding to two decimal places.

WRITE

a $A_{\text{circle}} = \pi r^2$

$$= 2.7 \times 2.7 \times \pi \text{ cm}^2$$

$$\begin{aligned} A_{\text{sector}} &= \frac{3}{4} \times A_{\text{circle}} \\ &= \frac{3}{4} \times 2.7 \times 2.7 \times \pi \\ &\approx 17.18 \text{ cm} \end{aligned}$$

b **1** Write the formula $A_{\text{sector}} = \frac{\theta}{360} \times \pi r^2$.

2 Substitute the values of θ and r into the formula.

3 Calculate the result using the π button on your calculator, rounding to two decimal places.

b $A_{\text{sector}} = \frac{\theta}{360} \times \pi r^2$

$= \frac{125}{360} \times 10^2 \times \pi$

$\approx 109.08 \text{ cm}^2$

Example 8D.3 Calculating the radius of a circle, given the area

Find the radius of a circle which has an area of 184 cm², correct to two decimal places.

THINK

1. Identify both the given value and the required value.
2. Substitute the value for A into the formula for the area of a circle.
3. Rearrange the formula to calculate the value of the radius. Remember to follow the order of operations.
4. Round to two decimal places and include the appropriate unit.

WRITE

$A = 184 \text{ cm}^2$, $r = ?$

$A = \pi r^2$

$184 = \pi r^2$

$\frac{184}{\pi} = r^2$

$\sqrt{\frac{184}{\pi}} = r$

$r = 7.653...$

$r \approx 7.65 \text{ cm}$

Helpful hints

✔ The formulas for the circumference of a circle ($2\pi r$) and the area of a circle (πr^2) are similar. To tell them apart look at where the 2 is: when the 2 is an exponent, it's an area formula; when the 2 is a coefficient, it's a circumference formula.

ANS p538

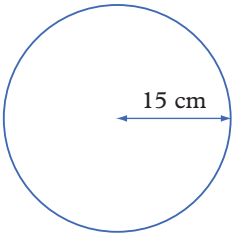
Exercise 8D Area of a circle

△ 1–8, 10, 15, 16, 19(a, b) | □ 1, 3–7, 9, 11, 12, 14, 19(a, c, e), 22 | ⬠ 1(c), 4(d–f), 5–7, 11–14, 17, 18, 19(d–f), 20–22

For all relevant questions in this topic, use π on your calculator and unless stated otherwise round answers to two decimal places.

8D.1 **1** Calculate the area of each circle.

a



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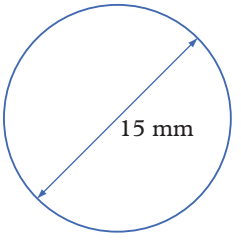

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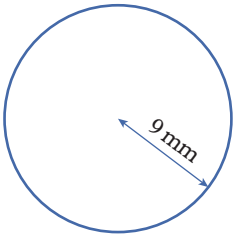
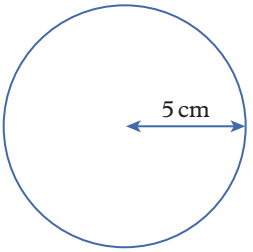


UNDERSTANDING AND FLUENCY

UNDERSTANDING AND FLUENCY

d



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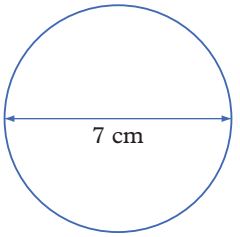

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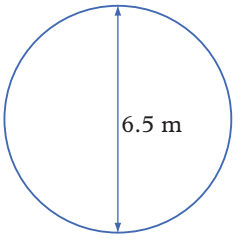
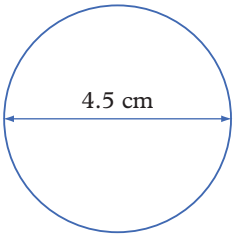


2 Calculate the exact area of each circle by leaving π in the answer.

a



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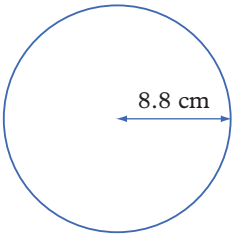

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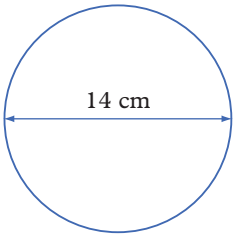


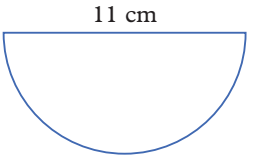
8D.1 **3** Calculate the area and perimeter of a circle with:

a radius 4.4 cm **b** diameter 7.5 m **c** diameter 15 mm **d** radius 29 mm.

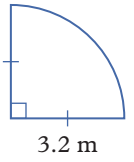
4 A semicircle is exactly half of a circle, and a quadrant is exactly a quarter of a circle. To find the area of semicircles and quadrants, find the area of the full circle and then divide by 2 or 4, respectively.

Find the area of these sections of circles.

a

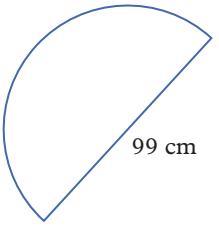


b

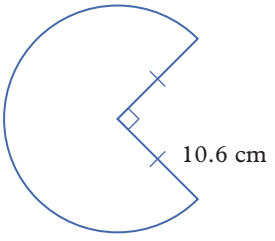


c

d



e



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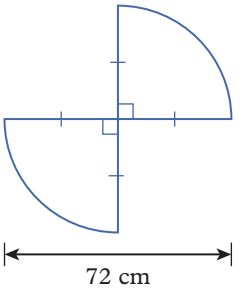
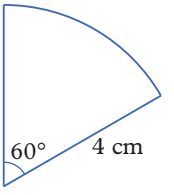


PROBLEM SOLVING AND REASONING

5 A sector is a portion of a circle formed by two radii and part of the circumference (an arc). You can use knowledge of angles to calculate the area of a sector. Look at the sector shown here.



a What is the angle at the vertex of the sector (where the two radii meet)?

b How many degrees are in a circle?

c Use your answers to parts **a** and **b** to write the angle for this sector as a fraction of the degrees in a circle.

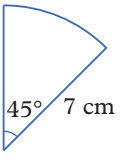
d What is the radius of this sector?

e What would be the area of the circle if it was complete?

f Use your answers to parts **c** and **e** to find the area of the sector.

8D.2 **6** Calculate the area of each sector.

a



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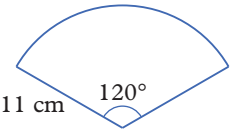

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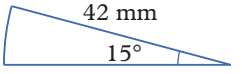


8D.3 **7** Find the radius of a circle which has an area of:

a 12 cm^2 **b** 25 mm^2 **c** 100 cm^2 **d** 68 cm^2.

8 Find the diameter of a circle which has an area of:

a 80 cm^2 **b** 49 cm^2 **c** 20 mm^2 **d** 135 cm^2.

9 Anika wants to make a circular placemat which has a diameter of 55 cm. What is the area of the placemat?

10 What is the total area covered by a sprinkler which has a range of 7.5 m?

11 A pizzeria sells two sizes of pizza: medium with a diameter of 12 inches and large with a diameter of 18 inches. Explain why you get more pizza by ordering one large rather than two medium pizzas.

12 a The area of a circle can be written as $A = \frac{1}{4}\pi d^2$, where d is the diameter of the circle. Show that $\frac{1}{4}\pi d^2 = \pi r^2$.

b The area of a circle can be written as $A = \frac{1}{2}Cr$, where C is the circumference of the circle and r is the radius. Show that $\frac{1}{2}Cr = \pi r^2$.

13 Use $A = \pi r^2$ or $A = \frac{1}{4}\pi d^2$ to solve the following problems.

a Find A when $d = 8$ m.

b Find A when $r = 3$ cm.

c Find A when $C = 40$ mm.

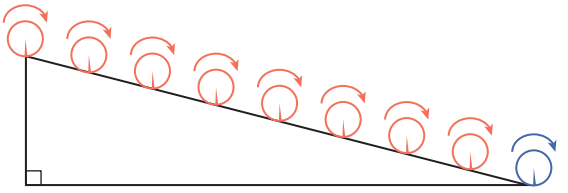
d Find A when $C = 50$ cm.

e Find d when $A = 125$ cm^2.

f Find C when $A = 90$ mm^2.

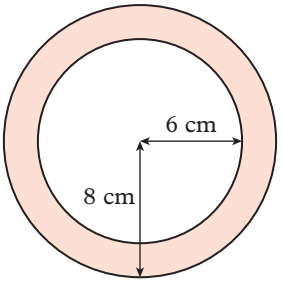
g Find C when $A = 20$ cm^2.

14 A circular object with an area of 58 cm^2 rolls down an inclined ramp after exactly 8 revolutions. How long is the ramp correct to the nearest integer?



15 You can subtract the area of one circle from another to find the area of a ring. Another name for a ring is an annulus. Find the area for each annulus.

a



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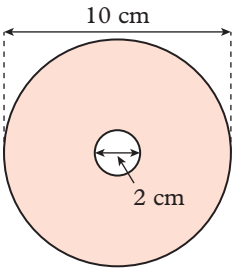

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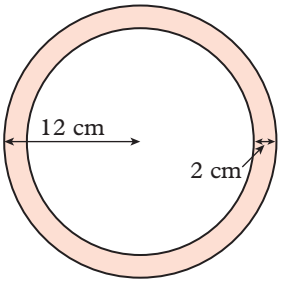
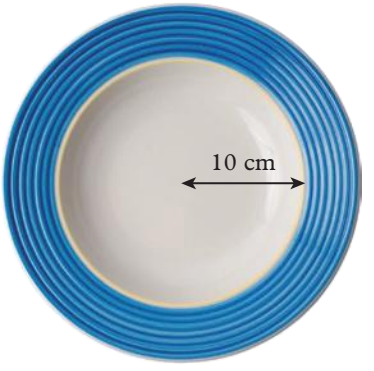


16 What is the largest area of pancake that could be cooked in a frying pan with a base diameter of 24 cm?

17 Adrian wants to cook a pancake using the frying pan from question **16** and place it on this plate.



a What is the maximum area a pancake could have if it had to fit on the white section of this plate, and the white section has a radius of 10 cm?

b To make a pancake the size you found in part **a**, what distance should be between the edges of the pancake and the side of the frying pan?

c What is the entire area of the plate if it has a diameter of 30 cm?

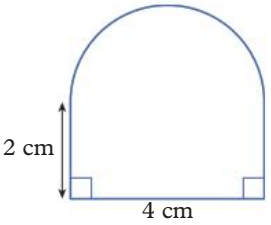
d What is the area of the painted section of the plate?

PROBLEM SOLVING AND REASONING

18 What is the total area that this button covers if it has a diameter of 20 mm and each hole has a radius of 1.5 mm?

19 Use your understanding of the area of a circle to find the area of each composite shape.

a



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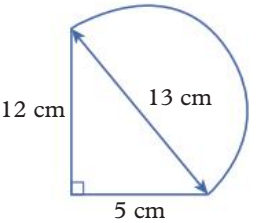

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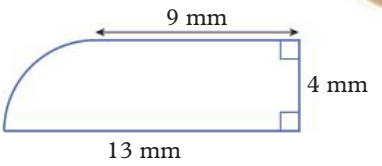
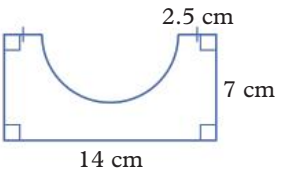
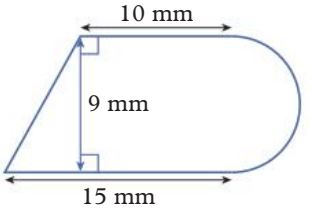


d



e



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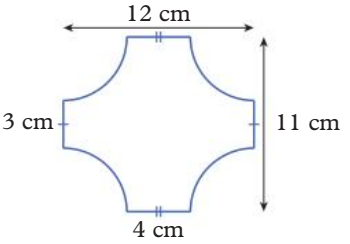


20 Explain why you can only find the exact length of the radius of a circle if the area is given with π still included in the value. Support your answer by finding the radius of a circle with an area of:

a 16 mm^2 **b** 16π mm^2.

21 **a** A circle has an area of 49π cm^2. Find its:

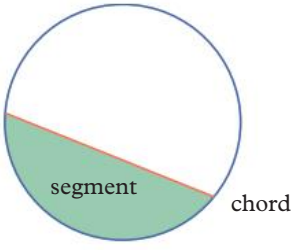
i diameter **ii** circumference (keeping π in the answer).

b A circle has a circumference of 8π m. Find its:

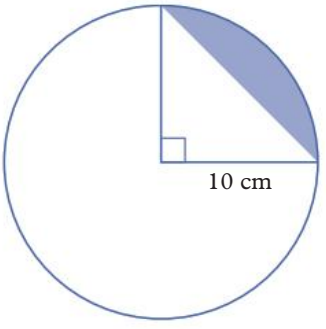
i radius **ii** area (keeping π in the answer).

CHALLENGE

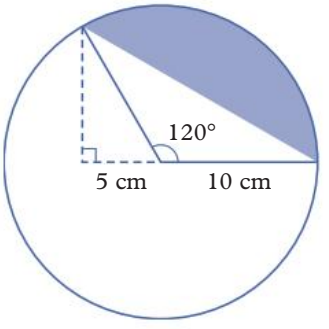
22 A chord in a circle is a line segment connecting two points on a circle's circumference. A segment is a region of a circle enclosed by a chord and an arc on the circumference. The area of a segment can be calculated by subtracting the area of the triangle formed between the two radii and the chord from the area of the sector. Calculate the area of the shaded segments in the following shapes, correct to two decimal places.



a



b



Check your Student obook pro for these digital resources and more:

Interactive skillsheet Area of a circle

Investigation Using arrays and rectangles to approximate areas

Topic quiz 8D

8E Volume and capacity

Learning intentions

By the end of this topic you will be able to ...

✔ calculate the volume and capacity of right prisms and cylinders

✔ convert units of volume and capacity.

Inter-year links

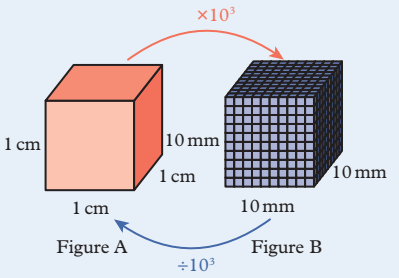
Support	Volume and capacity
Year 7	8F Volume and capacity
Year 9	6D Volume of composite solids
Year 10	8C Volume review

Volume of prisms

Key content video

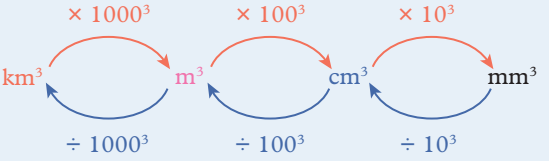
- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.

Metric units of volume	
Name	**Abbreviation**
Cubic millimetre	mm^3
Cubic centimetre	cm^3
Cubic metre	m^3

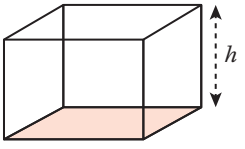
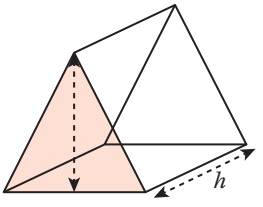


Conversion factors

$1\ cm^3 = 10^3\ mm^3$
$= 1000\ mm^3$
$1\ m^3 = 100^3\ cm^3$
$= 1\ 000\ 000\ cm^3$
$1\ km^3 = 1000^3\ m^3$
$= 1\ 000\ 000\ 000\ m^3$

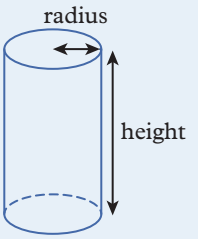


- To convert between units of volume, multiply or divide by the cube of the conversion factor.
- A **prism** is a 3D object with a consistent cross section. The cross section has the same shape as the **base** of the prism.
  → The shape of the cross section must be a **polygon**.
- The volume (V) of any prism can be found by multiplying the area of the base (A) by the height (h), $V = Ah$.

Shape	Diagram	Formula
Rectangular prism		V = (area of rectangle) × h
Triangular prism		V = (area of triangle) × h

- The base and height of a prism must be perpendicular (at right angles) to each other.

Volume of cylinders



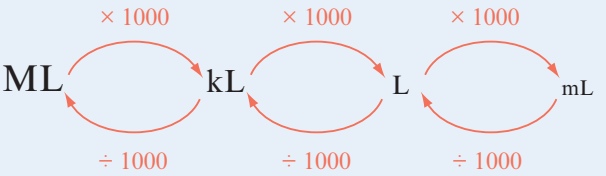
Cylinder

- A **cylinder** is a 3D object with a circular base and a constant cross-section.
- The volume of a cylinder can be found by multiplying the base area by the height. As the base area is $A = \pi r^2$, this gives the following formula:
  $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

Capacity

- The **capacity** of a 3D object is a measure of how much that object can hold.

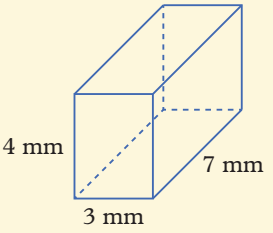
Metric units of capacity	
Name	**Abbreviation**
Millilitres	mL
Litres	L
Kilolitres	kL
Megalitres	ML



→ A container with an inside volume of 1 cm^3 holds 1 mL of liquid.
→ A container with an inside volume of 1000 cm^3 holds 1 L of liquid.
→ A container with an inside volume of 1 m^3 holds 1 kL of liquid.

Example 8E.1 Calculating the volume of a rectangular prism

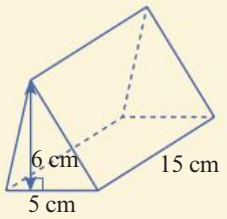
Calculate the volume of this rectangular prism.



THINK	WRITE
1 Identify the base of the prism. In this example we will designate the 3 mm by 4 mm side as the base.	$l = 3$ mm, $b = 4$ mm
2 Use the formula for the area of a rectangle, $A = lb$, to calculate the area of the base, A.	$A = lb$ $= 3 \times 4$ $= 12$ mm^2
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the volume of a prism.	$h = 7$ mm $V = Ah$ $= 12 \times 7$ $= 84$ mm^3
4 Calculate the volume of the rectangular prism. Remember to include the appropriate unit in the answer.	

Example 8E.2 Calculating the volume of a triangular prism

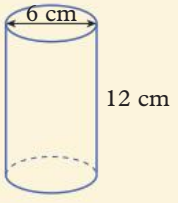
Calculate the volume of this triangular prism.



THINK	WRITE
1 Identify the base of the prism. In this example the base is the triangular face. As we will be dealing with two heights, we will refer to them as h_1 and h_2.	$b = 5$ cm, $h_1 = 6$ cm
2 Calculate the area of the base, A. In this example use the formula for the area of a triangle, $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh_1$ $= \frac{1}{2} \times 5 \times 6$ $= 15\ \text{cm}^2$
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the area of a prism.	$h_2 = 15$ cm $V = Ah_2$ $= 15 \times 15$
4 Calculate the volume of the triangular prism. Remember to include the appropriate unit in the answer.	$= 225\ \text{cm}^3$

Example 8E.3 Calculating the volume of a cylinder

Calculate the volume of this cylinder correct to two decimal places.



THINK	WRITE
1 Identify the values for the radius of the base (r) and the height of the cylinder (h).	$r = d \div 2$ $= 6 \div 2$ $= 3$ cm $h = 12$ cm
2 Substitute the values for r and h into the formula for the volume of the cylinder.	$V = \pi r^2 h$ $= \pi \times 3^2 \times 12$ $= 108\pi$
3 Calculate the result using the π button on your calculator and rounding your answer to two decimal places. Remember to include the appropriate unit.	$= 333.292...$ $\approx 333.29\ \text{cm}^3$

Example 8E.4 Converting units of volume

a Convert 2 m^3 into cm^3.

b Convert 216 mm^3 into cm^3.

THINK

a **1** Identify the appropriate conversion factor. To convert from a larger unit to a smaller unit, multiply by the conversion factor.

2 Complete the calculation.

b **1** Identify the appropriate conversion factor. To convert from a smaller unit to a larger unit, divide by the conversion factor.

2 Complete the calculation.

WRITE

a $1\ m^3 = 100^3\ cm^3$
$= 1\,000\,000\ cm^3$

$2\ m^3 = 2 \times 1\,000\,000$
$= 2\,000\,000\ cm^3$

b $1\ cm^3 = 10^3\ mm^3$
$= 1000\ mm^3$

$216\ mm^3 = 216 \div 1000$
$= 0.216\ cm^3$

Example 8E.5 Converting units of capacity

a Convert 3 L into mL.

b Convert 500 L into kL.

THINK

a **1** Identify the appropriate conversion factor. To convert from a larger to a smaller unit, multiply by the conversion factor.

2 Complete the calculation.

b **1** Identify the appropriate conversion factor. To convert from a smaller to a larger unit, divide by the conversion factor.

2 Complete the calculation.

WRITE

a 1 L = 1000 mL
3 L = 3 × 1000
= 3000 mL

b 1 kL = 1000 L
500 L = 500 ÷ 1000
= 0.5 kL

Helpful hints

- ✔ For rectangular prisms, any of the sides can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.
- ✔ Recall the following formulas for area:
  - → Rectangle: $A = lb$
  - → Triangle: $A = \frac{1}{2}bh$
  - → Parallelogram: $A = bh$
  - → Kite: $A = \frac{1}{2}xy$
  - → Rhombus: $A = bh$ or $A = \frac{1}{2}xy$
  - → Trapezium: $A = \frac{1}{2}(a + b)h$
- ✔ Volume and capacity are not the same! Volume is a measure of how much space an object occupies (expressed in mm^3, cm^3 or m^3), whereas capacity is a measure of how much an object can hold (expressed in mL, L, kL or ML).

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Exercise 8E Volume and capacity

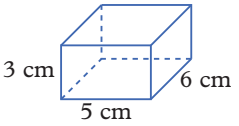
1–3, 4(a, e, f), 5(1st column), 6, 7(a, e, f), 8, 9, 11–13, 16, 19, 23, 25(a, b)

1(c), 3(c), 4(b, c, d), 5, 7(b, c, d), 8–10, 13, 17, 19, 22, 25, 27, 28, 29

1–7(2nd column), 8, 10–12, 15, 16, 18, 20, 21, 24–29

8E.1 **1** Calculate the volume of each rectangular prism.

a



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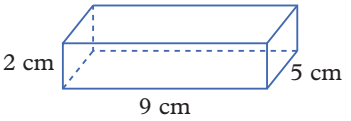
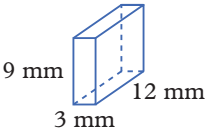
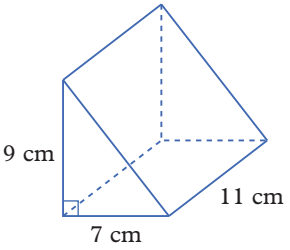


c



8E.2 **2** Calculate the volume of each triangular prism.

a



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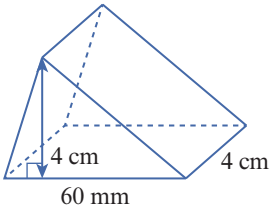

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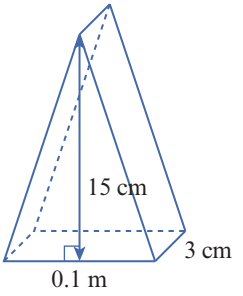
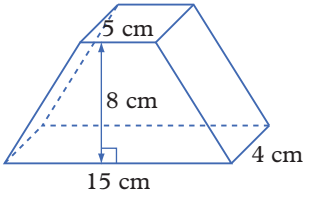


3 Use the formula $V = Ah$ to calculate the volume of each object.

a



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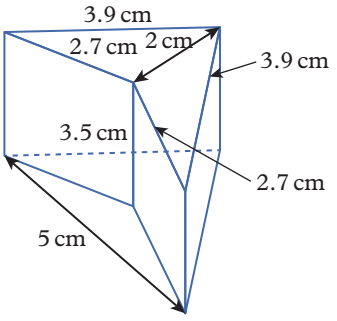
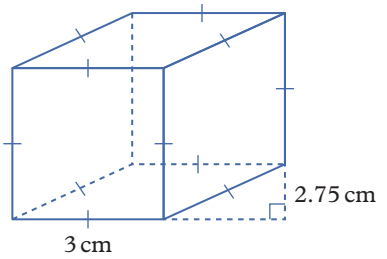
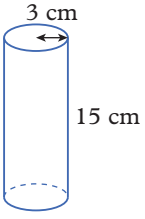


c



8E.3 **4** Calculate the volume of each cylinder. Give your answers to two decimal places.

a



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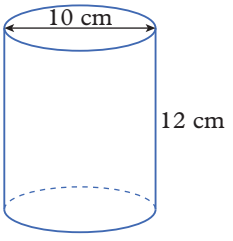

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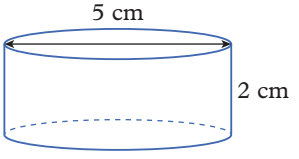
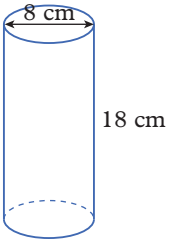
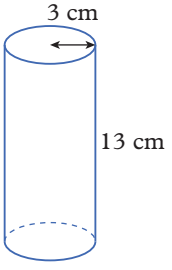


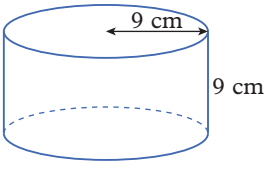
d



e



f



UNDERSTANDING AND FLUENCY

8E.4 **5** Convert each measurement.

a 9 cm^3 into mm^3 **b** 18 m^3 into cm^3 **c** 16 000 mm^3 into cm^3

d 1 750 000 m^3 into km^3 **e** 4.2 km^3 into m^3 **f** 4 000 000 cm^3 into m^3

g 4.56 m^3 into cm^3 **h** 845 000 cm^3 into m^3 **i** 0.15 km^3 into m^3

j 6.1 cm^3 into mm^3 **k** 28 mm^3 into cm^3 **l** 1 700 000 000 000 m^3 into km^3

8E.5 **6** Use the conversion factors to complete these statements.

a 8 L = _____ mL **b** 15 kL = _____ L **c** 70 mL = _____ L

d 6.4 ML = _____ kL **e** 2500 L = _____ kL **f** 3 kL = _____ mL

7 Calculate the volume of each cylinder. Give your answers to two decimal places.

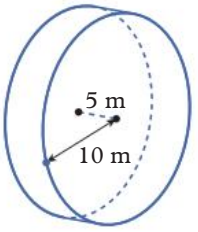
a $h = 8$ cm, $r = 10$ cm **b** $h = 2.1$ m, $r = 3$ m

c $h = 3.2$ m, $d = 7.3$ m **d** $h = 23$ cm, $d = 31$ cm

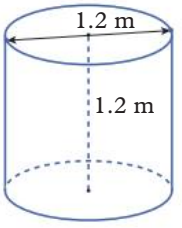
8 Calculate the capacity of each cylinder in question **7**, correct to the nearest litre.

9 Calculate the capacity of each cylinder, correct to the nearest litre.

a



b



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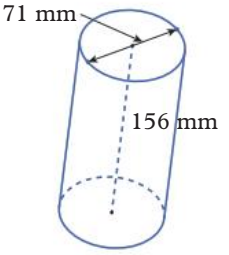
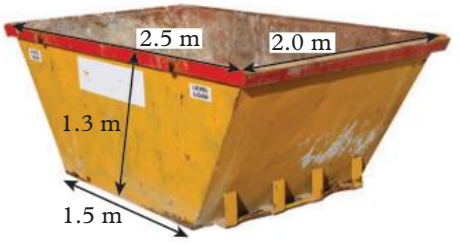


10 A skip has the dimensions shown. Calculate its volume in cubic metres.



PROBLEM SOLVING AND REASONING

11 A tissue box is 21 cm long, 11.4 cm wide and 12.3 cm tall.

a What is its volume?

b If each tissue laid flat in the box is 1 mm high, how many tissues would it fit?

12 A rectangular prism has a square base with sides that measure 5 cm. If the prism has a volume of 100 cm^3, what is its height?

13 The volume of a rectangular prism can also be given by using the formula $V = lbh$, where l, b and h represent the length, breadth and height, respectively. Explain why it doesn't matter which of the measurements you designate as the length, breadth or height.

14 Priya was completing some volume conversions and wrote the following answers.

i Explain the mistake Priya made in each instance.

ii Provide the correct working for the conversion.

a

$7\ m^3 = (7 \times 100^3)\ cm^3$
$= 2100\ cm^3$

b

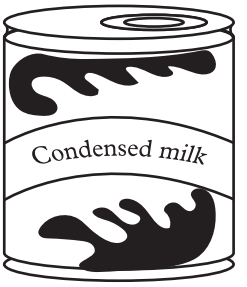
$3000\ mm^3 = (3000 \div 100^3)\ cm^3$
$= 30\ cm^3$

15 A cylinder has a volume of 150 cm^3.

a If it has a radius of 6 cm, find its height.

b If it has a height of 6 cm, find its radius.

16 A can of condensed milk has an internal diameter of 7 cm and a height of 12 cm.



a What is the volume of condensed milk in the can? Give your answer to the nearest cm^3.

b If condensed milk has a density of 1.3 g/cm^3, what is the weight of the milk if the can is filled? Give your answer to the nearest gram. (Weight = volume × density)

17 A $1 coin has a diameter of 2.5 cm and a thickness of 3 mm.

a How much metal is in a stack of thirty $1 coins? Write your answer in both cubic millimetres and cubic centimetres, correct to two decimal places.

b How many $1 coins could be made from 1000 cm^3 of metal?

18 The capacity of a cylinder with diameter 7 mm and height 15 mm correct to the nearest litre is 0 L. Explain why rounding to this degree of accuracy does not make sense.

19 Oliver has two cylindrical glasses. The first glass has a diameter of 6.5 cm and a height of 22 cm. The second glass has a radius of 5 cm and a height of 10 cm. Which glass has the greatest capacity and by how much? Give your answer in millilitres, correct to two decimal places.

20 What is the capacity of a cylindrical bottle cap if it has an internal diameter of 2.8 cm and a height of 1.1 cm? Give your answer in mL, correct to two decimal places.

21 Identify the most appropriate units to measure the capacity of the following objects.

a a coffee cup **b** a swimming pool **c** a rubbish bin

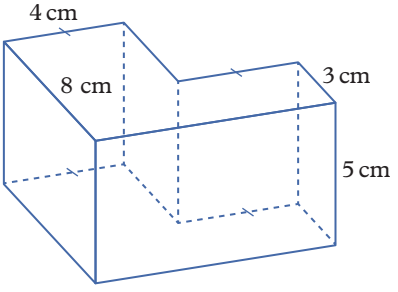
22 **a** Calculate the volume of this perfume bottle in cubic centimetres.

b Convert the volume from part **a** into cubic millimetres.

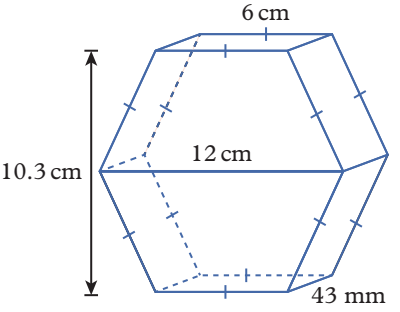
c Another way to do this is to first convert all the dimensions into millimetres and then find the volume. Show how doing the conversion first gives you the same answer as in part **b**.

23 Calculate the volume of these hexagonal prisms.

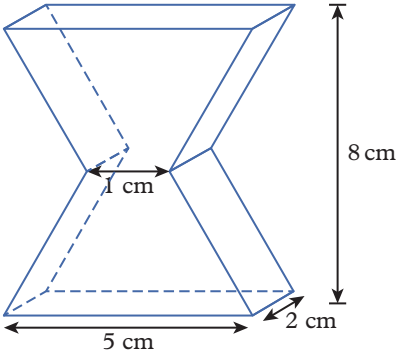
a



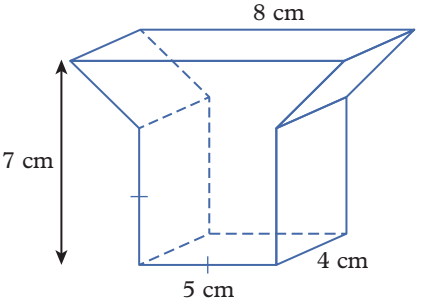
b



c



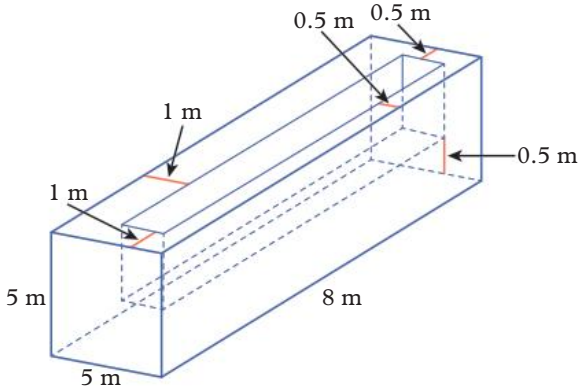
d



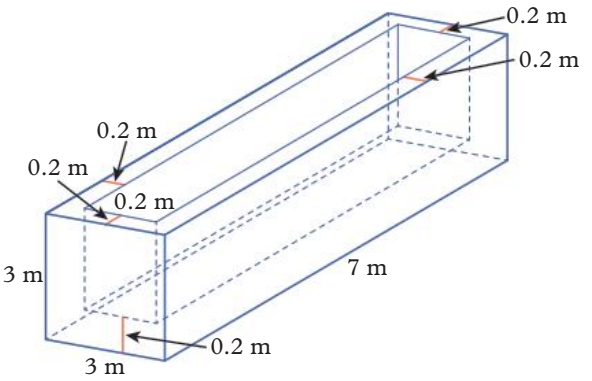
PROBLEM SOLVING AND REASONING

24 Calculate the capacity of each of the following troughs in litres.

a

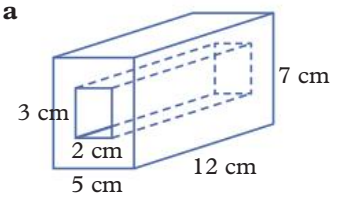


b

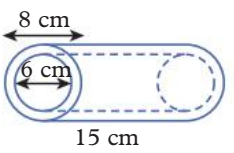


25 Find the volume of these hollow objects. Where necessary, give your answers to two decimal places.

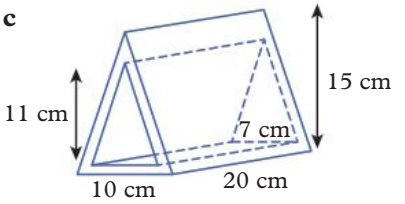
a



b



c

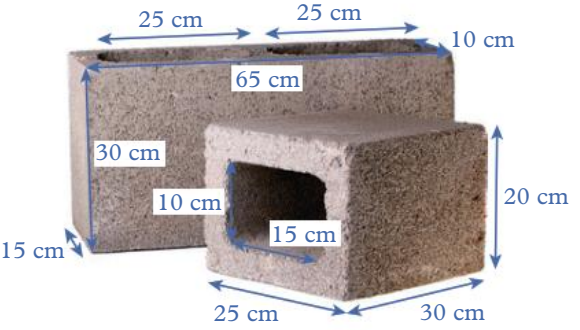


26 A candle has a diameter of 6 cm and a height of 16 cm. If the wick can be thought of as a cylinder with a diameter of 3 mm, what is the amount of wax contained within a single candle? Give your answer in both cubic centimetres and in millilitres correct to two decimal places.

CHALLENGE

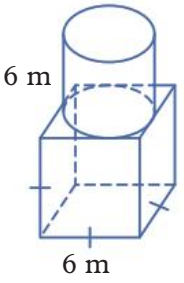
27 Explain why if the radius of a cylinder is doubled it will have twice the volume of the same cylinder with its height doubled.

28 Find the volume of concrete needed to make these two blocks.

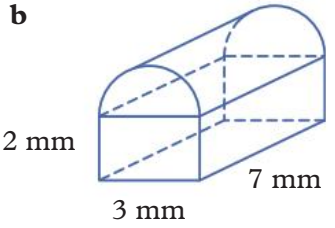


29 Find the volume of the following composite solids.

a



b



Check your Student obook pro for these digital resources and more:

- Interactive skillsheet — Volume of prisms
- Interactive skillsheet — Converting between units of volume
- Investigation — A bit of trash
- Topic quiz — 8E

8F Time

Learning intentions

By the end of this topic you will be able to ...

✔ convert between units of time

✔ calculate the duration between two given times

✔ use time zones to calculate the time in different locations.

Inter-year links
Support Understanding time

Units of time

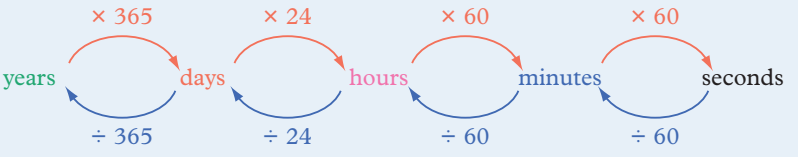
Key content video

- To convert between units of time, multiply or divide by the appropriate conversion factor.

  For example, 5 hours = 5 × 60
  = 300 minutes.

Conversion factors

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 day

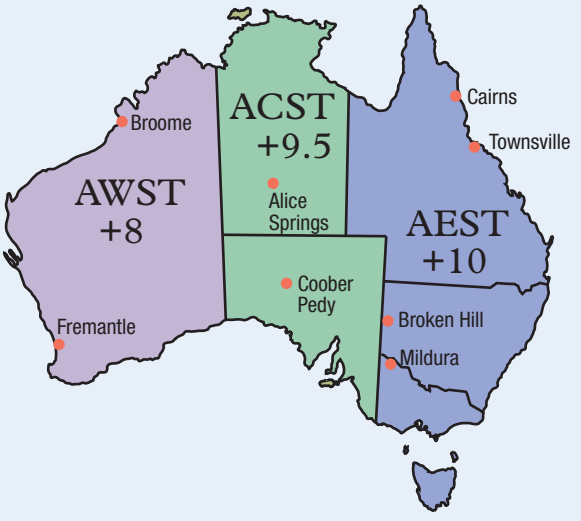


12- and 24-hour time

- In **12-hour time**, am and pm are used to describe whether a time is before or after midday. For example, 7 am represents 7 o'clock in the morning.
- In **24-hour time**, the time is given as a four-digit number. Each day starts at 0000 (midnight) and finishes at 2359. Times after midday have 12 hours added to them. For example, 2 pm is 1400 hours.
- A **duration** is the length between two given times.

Time zones

- The Earth is divided into 24 major **time zones**. A time zone map shows how many hours ahead or behind a time zone is from **Coordinated Universal Time (UTC)**, which is the time in Greenwich (in the UK). For example, UTC+10 means that the time is 10 hours ahead of UTC time.
- Australia covers three time zones:
  - → Australian Western Standard Time (AWST) is UTC+8.
  - → Australian Central Standard Time (ACST) is UTC+9.5.
  - → Australian Eastern Standard Time (AEST) is UTC+10.



Example 8F.1 Converting between units of time

Convert 2 hours into seconds.

THINK	WRITE
1 Convert from hours to minutes. To convert from a larger unit to a smaller unit, multiply by the conversion factor.	1 hour = 60 minutes 2 hours = 2 × 60 = 120 minutes
2 Convert from minutes to seconds.	1 minute = 60 seconds 120 minutes = 120 × 60 = 7200 seconds

Example 8F.2 Converting between 12- and 24-hour time

Convert:

a 9:20 pm into 24-hour time

b 1345 into 12-hour time.

THINK	WRITE
a As the time is after midday (pm), add 1200 to this value to express the time in 24-hour time.	**a** 9:20 pm = 0920 + 1200 = 2120
b As the given time is greater than 1200, subtract 1200 from 1345 and include pm to express the time in 12-hour time.	**b** 1345 = 1345 – 1200 = 1:45 pm

Example 8F.3 Calculating durations of elapsed time

A car leaves home at 11:07 am and arrives at its destination at 4:12 pm. How long did it take to get there?

THINK	WRITE
1 Count the number of minutes from the departure time to the next whole hour.	Time from 11:07 am to 12:00 pm = 53 minutes
2 Count the number of hours from the time at the end of the previous step to the whole hour of the arrival time.	Time from 12:00 pm to 4:00 pm = 4 hours
3 Count the number of minutes from the time at the end of the previous step to the arrival time.	Time from 4:00 pm to 4:12 pm = 12 minutes
4 Calculate the total duration by adding the individual time intervals. Remember that 60 minutes = 1 hour.	53 mins + 4 hrs + 12 mins = 4 hrs + 65 mins = 5 hrs and 5 mins

Example 8F.4 Calculating times in different locations

The time is 8:35 am in Perth (AWST). Calculate the time in Sydney (AEST).

THINK	WRITE
1 Calculate the difference between the two time zones. Eastern Standard Time (AEST) is UTC+10 and Western Standard Time (AWST) is UTC+8, so AEST is 2 hours ahead of WST.	AEST is 2 hours ahead of AWST.
2 As AEST is 2 hours ahead of AWST, add 2 hours to the time in Perth to determine the time in Sydney.	8:35 am + 2 hours = 10:35 am The time in Sydney is 10:35 am.

Helpful hints

- ✔ All times expressed in 24-hour time must be given as four-digit numerals. For example, 7:35 am is 0735, not 735!
- ✔ When calculating the duration of elapsed time remember that there are only 60 minutes in 1 hour; for example, 73 minutes = 1 hour and 13 minutes.

ANS p539

Exercise 8F Time

△ 1–7, 8(1st, 2nd columns), 9(a, c, e), 10–12, 14, 17 | □ 1–7, 8(2nd, 3rd columns), 9(b, d, f), 10, 13, 15, 19(a–d) | ⬠ 4–7, 8(3rd column), 9(b, d, f), 16, 18–20

8F.1 **1** Convert each of these into minutes.

a 3 hours **b** 5 hours **c** 10 hours
d 120 seconds **e** 600 seconds **f** 6000 seconds

2 Convert each of these.

a 10 minutes into seconds **b** 5 minutes into seconds **c** 60 minutes into seconds
d 120 minutes into hours **e** 1800 minutes into hours **f** 240 minutes into hours

3 Convert each of these.

a 30 minutes into hours **b** 45 minutes into hours **c** 20 minutes into hours
d 15 seconds into minutes **e** 6 seconds into minutes **f** 48 seconds into minutes

4 Convert each of these.

a 3 hours into seconds **b** 10 hours into seconds **c** 6 hours into seconds
d 1.5 hours into seconds **e** 14 400 seconds into hours **f** 9000 seconds into hours

5 How many:

a minutes in a day **b** hours in a week **c** seconds in a day
d hours in a leap year **e** seconds in a week **f** hours in June?

8F.2 **6** Convert these times into 24-hour time.

a 2:30 am **b** 4:15 pm **c** 1:48 pm **d** 11:11 pm

7 Convert these times into 12-hour time.

a 1925 **b** 1200 **c** 2218 **d** 0958

8F.3 **8** Calculate the duration between each pair of times.

a 4:00 pm and 9:00 pm **b** 8:00 am and 11:00 pm **c** 11:30 pm and 2:30 am
d 0900 and 1400 **e** 1330 and 2230 **f** 1600 and 2130
g 5:27 pm and 7:45 pm **h** 12:52 pm and 6:05 pm **i** 6:43 pm and 2:37 am
j 0004 and 1931 **k** 1919 and 2222 **l** 0636 and 1421

8F.4 **9** Find the time in the given time zone.

a 3:45 pm (UTC+8) in UTC+10 **b** 3:45 pm (UTC+10) in UTC+8
c 11:05 am (UTC+9.5) in UTC+10 **d** 11:05 am (UTC+9.5) in UTC+8
e 2345 (UTC+10) in UTC+9.5 **f** 2345 (UTC+8) in UTC+9.5

10 The time is given in one of Australia's time zones. Determine what time it is in the other places.

a The time is 1:49 pm AEST. What is the time in:
i Broome, WA **ii** Alice Springs, NT **iii** Cairns, QLD?

b The time is 2:28 am ACST. What is the time in:
i Coober Pedy, SA **ii** Broken Hill, NSW **iii** Fremantle, WA?

c The time is 1417 AWST. What is the time in:
i Geelong, VIC **ii** Perth, WA **iii** Adelaide, SA?

d The time is 0040 AEST. What is the time in:
i Darwin, NT **ii** Townsville, QLD **iii** Geraldton, WA?

11 a Convert 15 minutes into a fraction of an hour.

b Write this fraction as a decimal.

c Explain why 1.25 hours is not equivalent to 1 hour and 25 minutes.

12 A family takes a road trip from Sydney to Cairns. Their trip is plotted in the table below, showing when they leave, arrive and when they stop for lunch.

	Day 1	**Day 2**	**Day 3**	**Day 4**
Departure time	8:30 am	8:00 am	8:15 am	8:20 am
Start lunch	12:30 pm	12:30 pm	12:55 pm	1:05 pm
Finish lunch	1:00 pm	1:00 pm	1:40 pm	1:30 pm
Arrival time	7:30 pm	6:00 pm	7:45 pm	7:15 pm

a How long did they take from departure to arrival on their first day?

b How long was their lunch break on the first day?

c Use your answers to parts **a** and **b** to find how long they drove for on the first day.

d Find how long they drove for on the second, third and fourth days.

e How many hours driving did they do for the whole trip?

13 **i** Determine the error each person made when calculating the following durations.

ii Determine the correct duration.

a 2:45 pm to 6:25 pm

2:45 + 00:15 = 2:00
2:00 + 00:25 = 2:25
15 + 25 = 40 min
2:25 + 4:00 = 6:25
4 h 40 min

b 2:45 am to 6:25 pm

2:45 + 00:15 = 3:00
3:00 + 00:25 = 3:25
15 + 25 = 40 min
3:25 + 3:00 = 6:25
3 h 40 min

c 2:45 pm to 6:25 am

2:45 + 00:15 = 3:00
3:00 + 00:25 = 3:25
15 + 25 = 40 min
3:25 pm + 16:00 = 6:25 am
16 h 40 min

14 Andrew was studying for an upcoming test. He spent 85 minutes making flash cards, had a 10 minute break, spent 30 minutes quizzing himself using the flash cards, had a 30 minute lunch break, spent another 30 minutes quizzing himself using the flash cards, then sat a practice test for 50 minutes. If it was 11:10 am when he started, at what time did he finish the practice test?

15 Jaime has a bike with an odometer on it that tells him how many metres he has travelled. When he leaves for a bike ride along the Hanging Rock Trail in Blue Mountains, the odometer is at 1391 and the time is 1104. Upon arriving at the Hanging Rock, the odometer reads 5403 and the time is 1334.

a How far did Jaime ride?

b How long did it take him to get to the Hanging Rock?

c What is Jaime's average speed in m/s correct to two decimal places?

16 Harry and family are driving from Cairns, QLD to Eucla, WA overnight. They leave Cairns at 8 pm and Harry falls asleep during the drive. He is awoken by his parents when they arrive at the hotel in Eucla. He sees the clock in the lobby of the hotel that says it is 10 am. Harry assumes it took them 14 hours to get there.

a Why is Harry wrong?

b How long did it actually take Harry and family to get to the hotel?

c If the car travelled at an average speed of 80 km/h over the whole trip, approximately how many kilometres did they drive?

d Harry checks the route on his phone and sees it was 1490 km. Calculate the error between the approximation in part **c** and the actual distance as a percent of the actual distance, correct to the whole number.

17 Use the provided time zone map to decide which place is not in the same time zone as the others.

a **A** Coral Sea Islands **B** Papua New Guinea **C** Guam
D Solomon Islands **E** Tasmania

b **A** Jakarta, Indonesia **B** Makassar, Indonesia **C** Manila, Philippines
D Shanghai, China **E** Perth, Australia

18 Calculate the duration between the following times. Write all answers in hours and minutes.

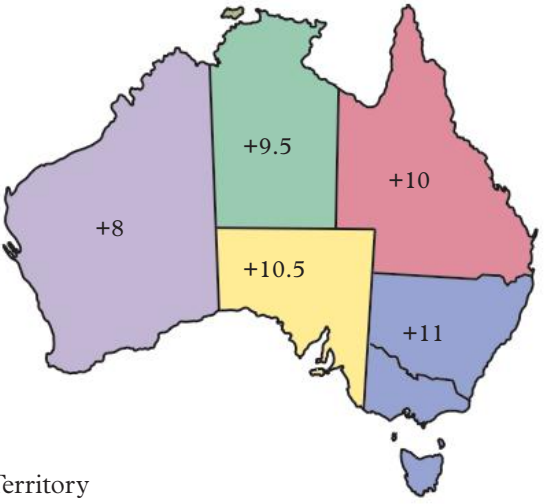
a 2:45 pm today and 11:10 am tomorrow

b 10:15 am on 22/08 and 11:00 am on 24/08

c 3:30 pm on 19/10 and 9:00 am on 22/10

d 1114 on 14/11/2022 and 1530 on 14/11/2023. Hint: 2023 is not a leap year.

19 Daylight saving time is when people put their clocks forward 1 hour on the first Sunday in October to have an 'extra' hour of daylight in the evenings. The clocks then move backwards 1 hour on the first Sunday in April. In Australia, the Australian Capital Territory, New South Wales, South Australia, Tasmania and Victoria observe daylight saving time. The Northern Territory, Queensland and Western Australia do not. This diagram shows the different time zones in summer when daylight saving time is observed.



a In summer, how many hours is:

i New South Wales ahead of Queensland

ii South Australia ahead of Western Australia

iii the Australian Capital Territory ahead of the Northern Territory

iv Tasmania ahead of Western Australia?

b In winter, if it is 6 pm in South Australia, what time is it in:

i Western Australia

ii the Northern Territory

iii New South Wales

iv Queensland?

c Repeat part **b**, but for summer.

d What is special about the time difference between South Australia and Queensland in summer and winter?

e Find the time in Darwin when it is:

i 4 pm on 27 January in Melbourne

ii 3 am on 16 July in Cairns

iii midday on 4 December in Perth

iv 9:30 am on 25 December in Sydney

v 5:45 pm on 30 June in Hobart

vi 11 am on 30 November in Adelaide.

20 Zoë and her friend Wes want to take a holiday in Thailand (UTC+7) during July.

a Zoë's flight leaves Melbourne (UTC+10) at 1:00 pm and arrives in Thailand at 7:00 pm. How long will the flight be?

b Wes has to take a different flight. His flight leaves at 1445. If his flight takes the same time, what time will he land in Thailand?

c On the way home they plan to stop in Bali (UTC+8). If they leave Thailand at 1000 and arrive in Bali at 1530, how long is this flight?

d What time will Zoë and Wes land home if they leave Bali at 1330 and their flight is 8 hours long?

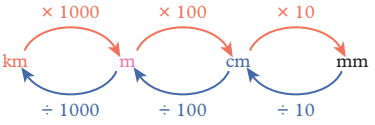
CHALLENGE

Check your Student obook pro for these digital resources and more:

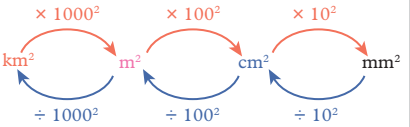
- Interactive skillsheet: Converting between units of time
- Interactive skillsheet: Durations of time
- Interactive skillsheet: Time zones
- Investigation: Daylight saving time
- Topic quiz: 8F

Chapter summary

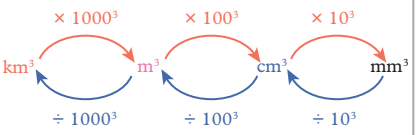
Units of length



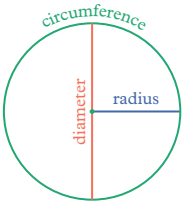
Units of area



Units of volume



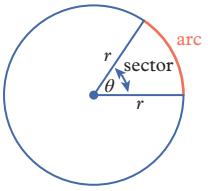
Circle, arc, section



$C = 2\pi r$

$C = \pi d$

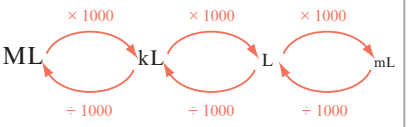
$A_{circle} = \pi r^2$



$l = \frac{\theta}{360} \times 2\pi r$

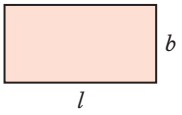
$A_{sector} = \frac{\theta}{360} \times \pi r^2$

Units of capacity



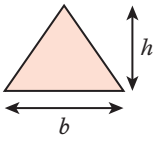
Area

Rectangle



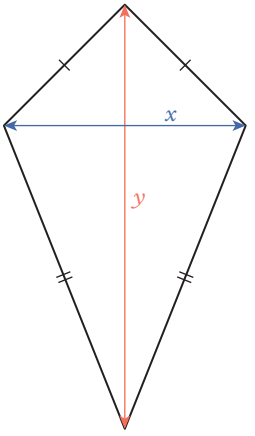
$A = lb$

Triangle



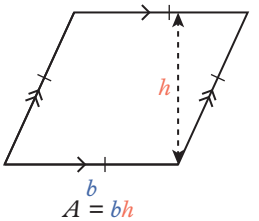
$A = \frac{1}{2}bh$

Kite

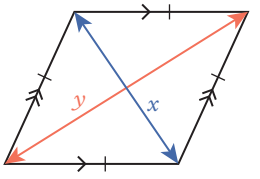


$A = \frac{1}{2}xy$

Rhombus

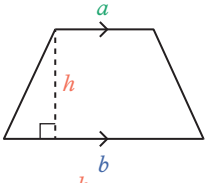


$A = bh$



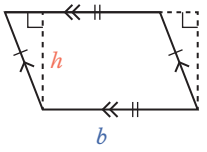
$A = \frac{1}{2}xy$

Trapezium



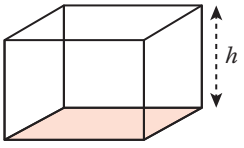
$A = \frac{h}{2}(a + b)$

Parallelogram

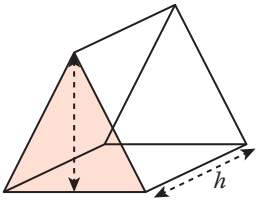


$A = bh$

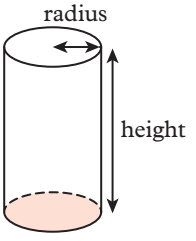
Volume of prisms and cylinders



V = (area of rectangle) × h



V = (area of triangle) × h



V = (area of circle) × h

Units of time

1 minute = 60 seconds

1 hour = 60 minutes

1 day = 24 hours

1 year = 365 days

12- and 24-hour time

7 am = 0700

2 pm = 1400

Time zones

Australian Western Standard Time (AWST) is UTC+8.

Australian Central Standard Time (ACST) is UTC+9.5.

Australian Eastern Standard Time (AEST) is UTC+10.

ANS p539

Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter

Quizlet
Test your knowledge of this topic by working individually or in teams

Mathematical literacy review

The following key terms are used in this chapter:

- 12-hour time
- 24-hour time
- angle
- arc
- area
- base
- capacity
- circle
- circumference
- composite
- conversion factor
- cylinder
- Coordinated Universal Time (UTC)
- diameter
- hectare
- kite
- length
- parallelogram
- perimeter
- pi
- prism
- polygon
- quadrant
- quadrilateral
- radius
- rectangular prism
- rhombus
- sector
- semicircle
- subtend
- time zone
- trapezium
- triangular prism
- volume

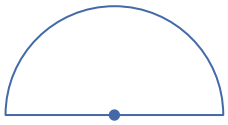
1 What are the following acronyms short for?

a ACST **b** AEST **c** AWST

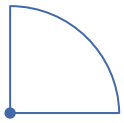
2 Research the meaning of am and pm.

3 Name the following shapes using key terms from the list.

a



b



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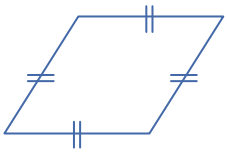
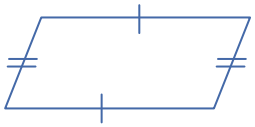


d



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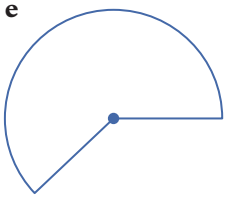

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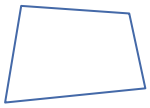


Multiple choice

8A **1** What is 7.84 m equal to?

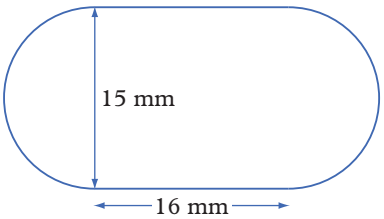
A 0.784 km **B** 7840 mm **C** 78.4 cm **D** 0.0784 km **E** 7840 cm

8B **2** A circle has a diameter of 6 cm. What is its radius?

A 12 cm **B** 6π cm **C** 36π cm **D** 3 cm **E** 3π cm

8A **3** Which of these is closest to the total perimeter of this shape?

A 31 mm
B 47 mm
C 62 mm
D 63.1 mm
E 79.1 mm



8B **4** A triangle has a base length of 10 cm and an area of 20 cm^2. What is its height?

A 1 cm **B** 2 cm **C** 4 cm **D** 12 cm **E** 200 cm

8B **5** 2000 m^2 is not equal to which of the following?

A 40 m × 50 m **B** 2 000 000 000 mm^2 **C** 20 000 000 cm^2

D 2 km^2 **E** 0.2 hectares

8B **6** What is the surface area of a rectangular prism with length 15 cm, breadth 4 cm and height 9 cm?

A 231 cm^2 **B** 192 cm^2 **C** 390 cm^2 **D** 462 cm^2 **E** 540 cm^2

8C **7** A rhombus has diagonals of lengths 5 cm and 6 cm. What is its area?

A 30 cm^2 **B** 11 cm^2 **C** 56 cm^2 **D** 15 cm^2 **E** 60 cm^2

8D **8** Which of these is closest to the area of the shape in question **3**?

A 946.9 mm^2 **B** 754 mm^2 **C** 480 mm^2 **D** 416.7 mm^2 **E** 240 mm^2

8E **9** What is the volume of an irregular prism with base area 2.5 cm^2 and length 10 mm?

A 250 mm^3 **B** 25 mm^3 **C** 2.5 cm^3 **D** 25 cm^3 **E** 125 mm^3

8E **10** What is 3 cm^3 equal to?

A 30 mm^3 **B** 300 mm^3 **C** 300 m^3 **D** 3000 mm^3 **E** 30 000 mm^3

8E **11** Which pair of volume and capacity measurements are equivalent?

A 40 mL and 4 cm^3 **B** 1 m^3 and 10 L **C** 250 mL and 2.5 L

D 500 cm^3 and 0.5 L **E** 150 mL and 1.5 mm^2

8E **12** What is the capacity of a cylinder with height 2.5 cm and radius 35 cm?

A 9.6 L **B** 6.7 L **C** 1.7 L **D** 38 L **E** 2.4 L

8F **13** What is the time duration between 1156 and 2014?

A 7 hours, 18 minutes **B** 8 hours, 42 minutes **C** 8 hours, 18 minutes

D 9 hours, 18 minutes **E** 7 hours, 42 minutes

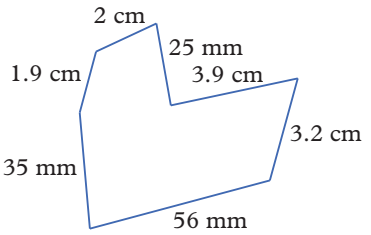
8F **14** If the time is 4:45 pm in Melbourne (UTC+10), what is the time in Perth (UTC+8)?

A 2:45 pm **B** 4:15 pm **C** 4:45 pm **D** 5:15 pm **E** 6:45 pm

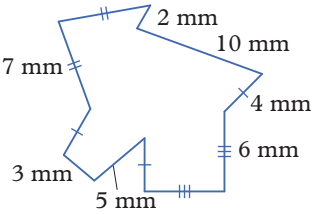
Short answer

8A **1** Calculate the perimeter of each shape.

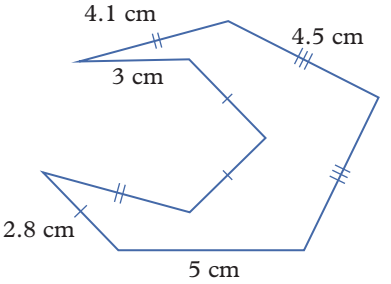
a



b

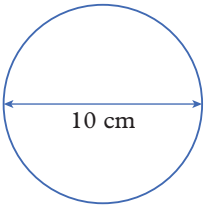


c

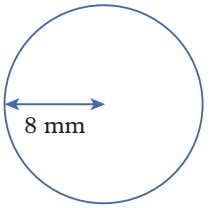


8A **2** Calculate the circumference of each circle correct to two decimal places.

a

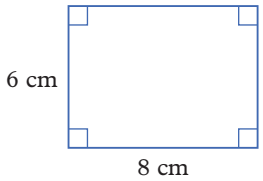


b

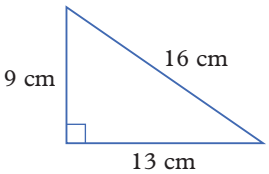


8B **3** Calculate the area of each shape.

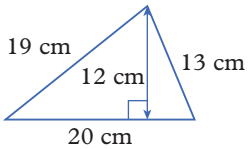
a



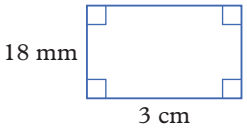
b



c



d



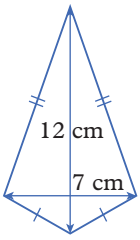
8B **4** Convert each area measurement.

a 5 m^2 into cm^2

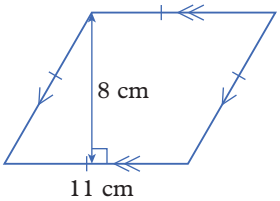
b 50 mm^2 into cm^2

8C **5** Calculate the area of each shape.

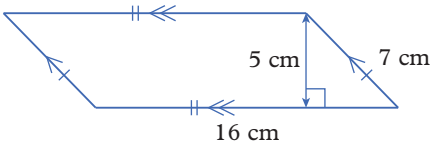
a



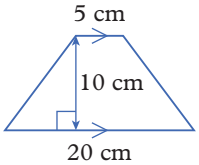
b



c



d



8D **6** Calculate the area of each circle in question **2**. Give your answers correct to two decimal places.

8E **7** Convert each measurement.

a 3.5 cm^3 into mm^3

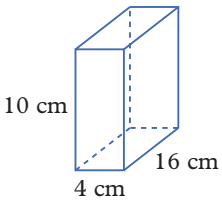
b 400 cm^3 into m^3

c 35 000 mL to L

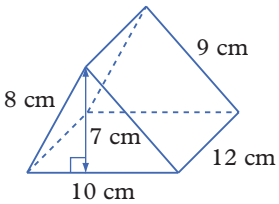
d 0.124 L to mL

8E **8** Calculate the volume of each object.

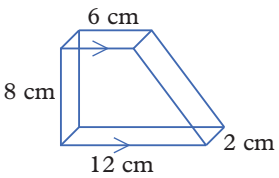
a



b



c



8E **9** The volume of a cylindrical water tank with radius 10 m is 4000 m^3.

a What is the capacity of the water tank?

b Calculate the height of the tank to the nearest metre.

8F **10** Calculate the duration between the following times.

a 3:45 pm to 7:30 pm

b 10:05 pm to 6:37 am

c 0712 to 1546

d 2055 to 0322

8F **11** Convert the following times.

a 7:23 am UTC+10 to UTC+8

b 2:49 pm UTC+9.5 to UTC+10

c 9:10 am AWST to AEST

d 1:20 pm AEST to ACST

Analysis

1 A friend wants to have a ball pit at their birthday party. They have set aside a rectangular space in their backyard that is 3 m long and 2 m wide.

a What area would this cover?

b How much fencing would be needed to go around the ball pit?

c Your friend can only buy the wood that they need for the fencing in planks that are 50 cm wide. How many wood planks are needed?

They also want to paint a diamond on the front fence of the ball pit. From point to point (that is, its diagonals) it measures 55 cm vertically and 30 cm horizontally.

d What is the area of this diamond?

Around the diamond will be six circles representing the coloured balls in the ball pit. Each circle has a diameter of 6.6 cm.

e What is the circumference of each circle correct to two decimal places?

f Find the total area to be painted for the six circles correct to two decimal places.

g The walls of the pit need to stand 1 m tall. What is the volume of the pit?

h If each ball to go into the pit has a volume of 150 cm^3, how many could theoretically fit into the pit? Be careful with unit conversions!

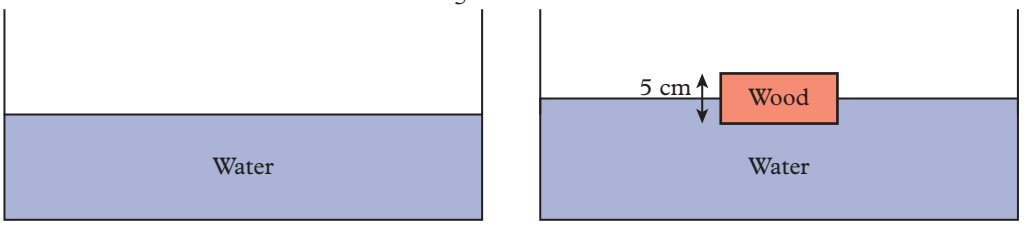
i Your friend finds that only about 25 000 balls fit into the pit. What explanation can you offer for this difference?

2 Archimedes knows that when an object is submerged under water, the water level will rise as the water becomes *displaced* by the object. He also knows that the volume of the submerged object is equal to the volume of the displaced water.

Archimedes has a rectangular glass container with base dimensions 30 cm × 40 cm. He fills it with 12 L of water and finds that the water level is exactly halfway up the total height of the glass container.

a Determine the height of Archimedes' container.

Archimedes then floats a rectangular wooden block on the surface of the water. The wooden block has dimensions 10 cm × 8 cm × 5 cm. He measures $\frac{3}{5}$ of the wooden block is submerged under water.



b Determine the volume of the displaced water by first finding the volume of the submerged object.

c Find the new water level by considering the relationship: volume of water in the container remains the same = base of the tank × water level − volume of the block submerged under water.

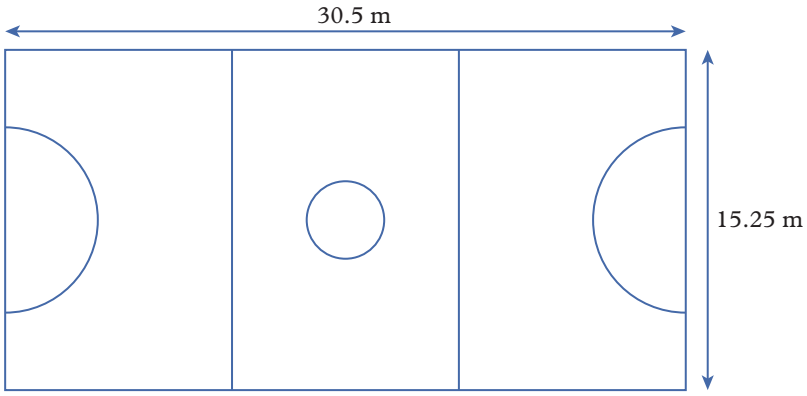
d Determine the difference in the water level in cm.

e Archimedes keeps adding water to the container until the surface of the wooden block is at the same level as the edge of the container, what's the water level now?

f How much water has been added?

g Archimedes added the water at a constant rate of 5 L/minute. How long did it take for the surface of the block to be at the same level as the edge of the container?

3 To warm up before a netball match, Jordan has the option of either running six laps around the netball court or doing four shuttle runs.



a How far is one lap around the netball court?

b How far is six laps around the netball court?

A shuttle run involves starting at the left of the court and running hard to the first third-line then back to the start, running to the second third-line and back to the start, and finally running up to the end of the court and back.

c How far is the first part of the shuttle run (to the first third-line and back)? Give your answer correct to two decimal places.

d What distance does a single shuttle run cover?

e How far is four shuttle runs? Explain how you calculated the answer.

f Which warm-up option is shorter?

g What is the area of the netball court?

h The centre circle in the middle of the court is 0.9 m in diameter, the semicircles (shooting circles) at either end of the court are 9.8 m in diameter. If the centre circle is going to be painted blue and the shooting circles are going to be painted red, what is the ratio of the blue paint and red paint needed?

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert between units of length ☐ Find the circumference and arc length of a circle ☐ Find the perimeter of two-dimensional shapes	☐ Go back to Topic 8A Length and perimeter
☐ Calculate the area of rectangles and triangles ☐ Convert between units of area	☐ Go back to Topic 8B Area of triangles and rectangles
☐ Calculate the area of parallelograms, kites, rhombuses and trapeziums	☐ Go back to Topic 8C Area of quadrilaterals
☐ Calculate the area of a circle or a sector ☐ Calculate the radius, diameter or circumference, given the area of a circle	☐ Go back to Topic 8D Area of a circle
☐ Calculate the volume and capacity of right prisms and cylinders ☐ Convert units of volume and capacity	☐ Go back to Topic 8E Volume and capacity
☐ Convert between units of time ☐ Calculate the duration between two given times ☐ Use time zones to calculate the time in different locations	☐ Go back to Topic 8F Time

9

Statistics and probability

Index

9A Summary statistics
9B Constructing graphs
9C Grouped data and histograms
9D Describing data
9E Comparing data
9F Theoretical probability

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Angles at a point
- ✔ Fractions, decimals and percentages
- ✔ Interpreting graphs
- ✔ Dot plots
- ✔ Stem-and-leaf plots

Curriculum links

- Classifies and displays data using a variety of graphical representations (MA4-DAT-C-01)
  - Display data using graphical representations relevant to the purpose of the data
  - Interpret data in graphical representations
- Analyses simple datasets using measures of centre, range and shape of the data (MA4-DAT-C-02)
  - Calculate and compare the mean, median, mode and range for simple datasets
  - Interpret the effect individual data points have on measures of centre and range
  - Analyse datasets presented in various ways and draw conclusions
- Solves problems involving the probabilities of simple chance experiments (MA4-PRO-C-01)
  - Determine probabilities for chance experiments
  - Determine probabilities for complementary events

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Materials

✔ Ruler ✔ Protractor ✔ Calculator

9A Summary statistics

Learning intentions

By the end of this topic you will be able to ...

✔ calculate summary statistics from frequency tables
✔ calculate summary statistics from displays
✔ use summary statistics to describe key features of data.

Inter-year links

Support	Understanding data
Year 7	9B Summary statistics
Year 9	8A Five-number summary and interquartile range

Measures of centre and spread

Key content video

- The **mean**, **median** and **mode** are **measures of centre**, as they each represent a central point in a dataset.

  $$\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$$

  Median = middle value when data is listed in order (divides a dataset in half)

  Mode = most common value (there can be more than one mode)
- The **range** is a **measure of spread**, as it gives an indication of the spread of the data.

  Range = maximum value – minimum value

Describing data using summary statistics

- Summary statistics can be used to describe data.
- When deciding which measure of centre should be used to describe data, the presence of outliers should be taken into consideration. An **outlier** is an extreme data point that is much higher or lower than the other values in a dataset. For example, 50 is an outlier in the dataset 1, 2, 3, 2, 50.
  - → Outliers can skew the value of the mean but will not impact the median or mode.
  - → A small sample size with the mean being significantly higher or lower than the median may indicate an outlier skewing the value of the mean.
- When describing a set of data, people use words such as:
  - → 'average', which usually refers to the mean, but sometimes median
  - → 'most common', 'most popular' and 'most often', which refer to the mode
  - → 'middle' and 'centre', which refer to the median
  - → 'spread' or 'span', which usually refer to the range.

Calculating summary statistics from data displays

- The **frequency** of a data value is a measure of how often a piece of data occurs in a sample.
- A **frequency table** can be used to tally frequencies in a dataset.
- To calculate the mean from a frequency table, insert a column multiplying each individual score by its frequency, and divide the total of this column by the total of the frequencies.
- To calculate the median from a frequency table, insert a column for the **cumulative frequency** (a running total of the frequencies) to help identify where the middle value lies.
- To calculate summary statistics from a display that details individual scores (e.g. stem-and-leaf plots, dot plots), the data can be treated as either a raw list or as a frequency table.

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Example 9A.1 Finding summary statistics from a frequency table

For the data shown in the following frequency table, find the:

a mean **b** median **c** mode **d** range.

Score (x)	Frequency (f)
1	4
2	7
3	8
4	8
5	2
6	1

THINK

a **1** Insert a 'Score × frequency' column and a 'Total' row and then complete the table. The total of the 'Frequency' column is the number of data values in the table.

2 To find the mean, divide the 'Score × frequency' total by the 'Frequency' total.

b **1** Insert a 'Cumulative frequency' column in the table. Check that the uppermost number matches the number of data values.

2 Divide the number of data values by 2 to find out how many values lie below the median. Then identify the row that contains the median score by using the cumulative frequency column.

c Identify the score with the highest frequency.

d Identify the minimum and maximum values in the score column of the dataset. Subtract the minimum value from the maximum value.

WRITE

Score (x)	Frequency (f)	Score × frequency ($x \times f$)	Cumulative frequency
1	4	$1 \times 4 = 4$	4
2	7	$2 \times 7 = 14$	$4 + 7 = 11$
3	8	$3 \times 8 = 24$	$11 + 8 = 19$
4	8	$4 \times 8 = 32$	$19 + 8 = 27$
5	2	$5 \times 2 = 10$	$27 + 2 = 29$
6	1	$6 \times 1 = 6$	$29 + 1 = 30$
Total	**30**	**90**	

a Mean $= \frac{90}{30}$

$= 3$

b There are 30 values in the dataset. $\frac{30}{2} = 15$, so 15 data values lie below the median.

The number of data values is even so the median is between the 15th and 16th value.

The 12th to 19th scores are 3, so both the 15th and 16th score are 3.

Median = 3

c The highest frequency is 8 for both scores 3 and 4.

Modes = 3, 4

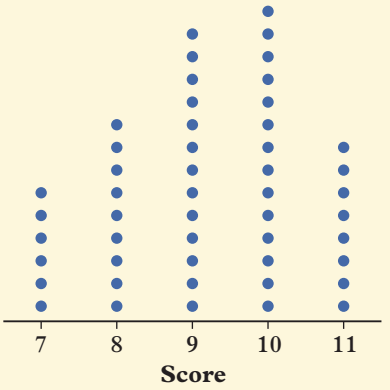
d Range $= 6 - 1$

$= 5$

Example 9A.2 Finding summary statistics from a dot plot

For the data shown in the following dot plot, find the:

a mean **b** median **c** mode **d** range.



THINK

a **1** Create a frequency table by counting the number of dots in each column.

2 Insert a 'Total' row, a 'Score × frequency' column and a 'Cumulative frequency' column in the table.

3 Find the mean by dividing the 'Score × frequency' total by the 'Frequency' total.

b Divide the number of values by 2 to find out how many values lie below the median.

c Find the mode by identifying the score with the highest frequency.

d Find the range by subtracting the minimum score from the maximum score.

WRITE

Score (x)	Frequency (f)	Score × frequency ($x \times f$)	Cumulative frequency
7	6	$7 \times 6 = 42$	6
8	9	$8 \times 9 = 72$	15
9	13	$9 \times 13 = 117$	28
10	14	$10 \times 14 = 140$	42
11	8	$11 \times 8 = 88$	50
Total	**50**	**459**	

a Mean $= \frac{459}{50}$

$= 9.18$

b $\frac{50}{2} = 25$, so 25 data values lie below the median.

The number of data values is even so the median is between the 25th and 26th score.

The 16th to 28th scores are 9, so both the 25th and 26th score are 9.

Median = 9

c The highest frequency is 14 for the score 10.

Mode = 10

d Range $= 11 - 7$

$= 4$

Example 9A.3 Finding summary statistics from a stem-and-leaf plot

For the data shown in the following stem-and-leaf plot, find the:

a mean **b** median **c** mode **d** range.

Stem	Leaf
1	5 9
2	3 7 8 8 9
3	0 1 2 2 6 9 9
4	0 2 3 3 6 6 6 7 8
5	1 5

Key 2 | 3 = 23

THINK

1. Find the mean by dividing the sum of scores by the number of scores.
2. Divide the number of values by 2 to find out how many values lie below the median.
3. Find the mode by identifying the most common scores.
4. Find the range by calculating the difference between the minimum score and the maximum score.

WRITE

a Mean $= \frac{915}{25}$
$= 36.6$

b $\frac{25}{2} = 12.5$, so 12 data values lie below the median.
The number of data values is odd so the median is the 13th score, which is 39.
Median = 39

c The highest frequency is 3 for the score 46.
Mode = 46

d Range = 55 − 15
= 40

Example 9A.4 Identifying summary statistics

Identify which summary statistics are being used in these sentences.

a The most popular team was the Parramatta Eels.

b The levels of a chemical in the water varied by up to 20 g.

THINK

a The term 'most popular' refers to the mode.

b The word 'varied' refers to the spread of the data, which is the range.

WRITE

a mode

b range

Helpful hints

- ✔ When calculating summary statistics from a stem-and-leaf plot make sure that you use the full data values by combining the stem and leaf and using the key.
- ✔ Remember that if the median falls between two different values, the median will be the average (mean) of these values.
- ✔ A dataset can have more than one mode.

ANS p540

Exercise 9A Summary statistics

△ 1(a, c, e), 2, 4, 5(a, b), 6(a, b), 7, 9, 11(a, c–e), 13 ▢ 1(b, d, f), 3, 5, 6(b, c), 7, 10, 13–15 ⬠ 1(d, f), 3, 5–6(b, c), 10, 12–14, 16

UNDERSTANDING AND FLUENCY

9A.1 **1** For the data shown in each frequency table, find the **i** mean, **ii** median, **iii** mode and **iv** range. Give your answers to two decimal places where necessary.

a

Score (x)	Frequency (f)
1	6
2	7
3	5
4	3
5	1

b

Score (x)	Frequency (f)
10	8
20	6
30	8
40	2

c

Score (x)	Frequency (f)
13	3
14	4
15	8
16	11
17	12
18	4

d

Score (x)	Frequency (f)
0	11
1	13
2	6
3	3
4	1

e

Score (x)	Frequency (f)
15	29
20	41
25	58
30	72

f

Score (x)	Frequency (f)
1	6
2	11
3	9
4	4
5	3
19	1

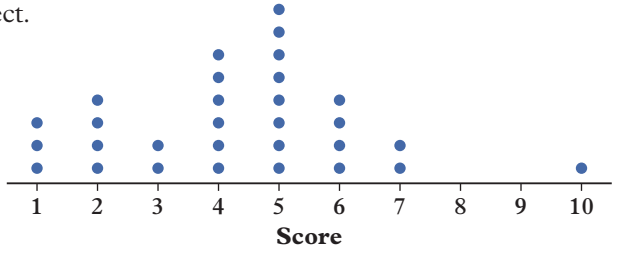
2 This dot plot shows the time in hours to complete a project.

a How many people were surveyed?

b State the mode and range.

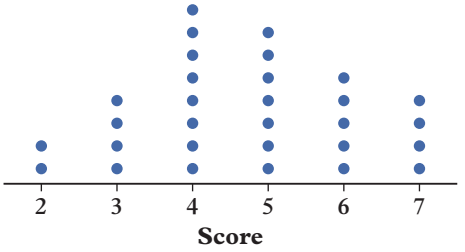
c Create a frequency table.

d Use the frequency table to calculate the median and mean of project completion time.

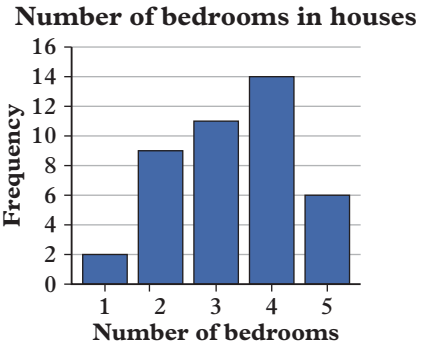


9A.2 **3** For the data shown in the following dot plot, find the:

a mean **b** median **c** mode **d** range.



4 This column graph shows the number of bedrooms in a survey of houses.



a What is the most common number of bedrooms in the houses surveyed?

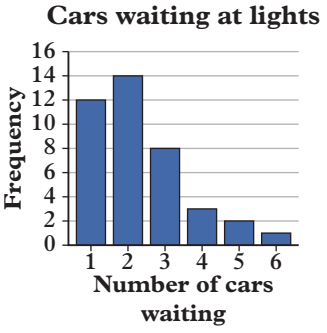
b What is the range of the number of bedrooms?

c Create a frequency table to represent the data shown in the column graph.

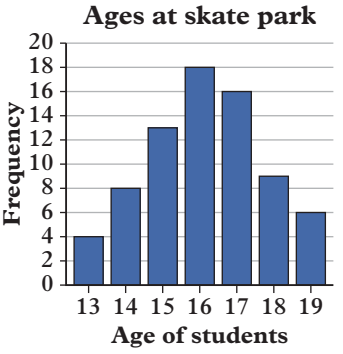
d Use the frequency table to calculate the median and the mean number of bedrooms. Give your answers to two decimal places where necessary.

5 Find the mean, median, mode and range for the data shown in each column graph. Give your answers to two decimal places where necessary.

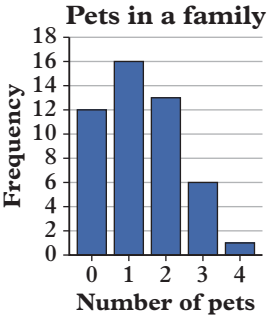
a



b



c



9A.3 **6** Find the mean, median, mode and range for the data displayed in each stem-and-leaf plot. Give your answers to two decimal places where necessary.

a

Stem	Leaf
3	2 4 7
4	2 2 2 6 9
5	3 6 8 9 9
6	1 1 4 6 7 7 8 8
7	0 4

Key 3 | 2 = 32

b

Stem	Leaf
1	2 3 8 8 8 9
2	0 0 1 2 3 6 7 8
3	2 3 7 9
4	4 6
5	2

Key 1 | 3 = 1.3

c

Stem	Leaf
1	0 1 2 4 5 6 7 7 9
2	2 3 4 4 6 8 8
3	0 1 1 1 3
4	3 7 9
5	1
6	
7	7

Key 1 | 2 = 120

9A.4 **7** Identify which summary statistics are being used in these sentences.

a The average height for a 14 year old is 162 cm.

b The temperature covered a span of 10°C.

c The most popular music in the class was hip-hop.

d The times varied by up to 40 minutes.

e The train is most often 2 minutes late.

f Andrew is in the middle of his height range.

8 A number of people were surveyed on how many pairs of shoes they bought in a year. Use the results shown in the table to find the mean, median, mode and range. Give your answers to two decimal places where necessary.

Score (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	9	13	21	18	13	11	7	4	1	3

9 Explain why the only summary statistic that you can find for categorical data is the mode.

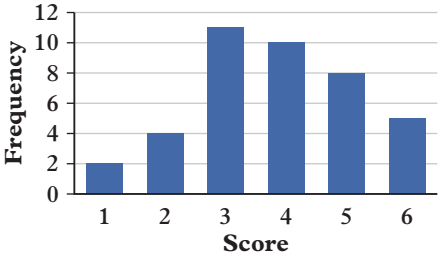
10 Match these summary statistics with these graphs.

a mean = 3.3, median = 3, mode = 3, range = 5

b mean = 3.825, median = 4, mode = 3, range = 5

c mean = 40.74, median = 36.5, mode = 74, range = 97

I



II

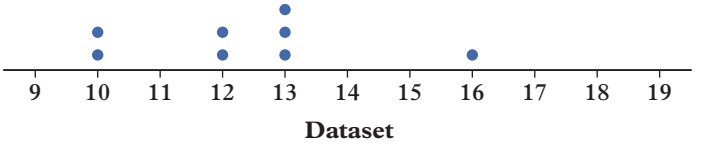
Stem	Leaf
0	1 2 6
1	2 6 7 7 8 9
2	0 1 1 2 4 6 7 7 8 9
3	0 3 4 4 5 6 7 9
4	0 1 2 2 5 9
5	0 1 2 3 5
6	0 1 4 8 9
7	4 4 4 4
8	1 9
9	8

Key 2 | 1 = 21

III

11 The dot plot on the right is missing one dot. Find a possible value of the missing dot for each of the following scenarios.

In each case, state whether your answer is unique or if it is one answer out of several that are possible.



a The maximum value is 17.

b The mean is 12.

c The median is 13.

d The modes are 10 and 13.

e The range is 7.

f The mean is a whole number.

12 A large dataset contains 457 data points that are listed in a spreadsheet. The numbers are ordered from smallest to largest and are listed in rows of 30. This means the number in column 1 of row 1 is the smallest number in the set, the number in column 1 of row 2 is the 31st smallest number in the set, and so on.

a Find the row number and column number of the greatest number in the dataset.

b Find the row number and column number of the median of the dataset.

13 Draw a dot plot on a number line as shown below using a different dataset consisting of exactly nine numbers for each of the following scenarios. In each case, write a sentence explaining your reasoning for placing the dots in the positions you chose.

a The minimum value is 2 and the range is 7.

b The median and the maximum value are both 6.

c The mode includes four different numbers.

14 The marketing manager of a drug company claims that the company's new pain relief drug blocks pain for up to 18 hours. This is based on a sample of 15 people recording the number of hours they were pain-free. Here is that data:

2	18	3	2	2
1	2.5	2	1.5	2.5
2	1.5	2	1.5	1.5

a Why is the company's marketing manager making this claim? Explain why it is misleading.

b Calculate the three measures of centre (mean, median and mode) for the given data.

c Calculate the three measures of centre excluding the outlier.

d Which measure of centre has been affected by the outlier?

e The marketing manager changes tactics and decides to claim that the pain relief drug blocks pain for an average of 3 hours. Comment on this claim. Is the claim now fair? Explain.

15 Elena has completed four Maths tests this year with an average score of 78%.

a What is the maximum value her average could increase to after completing the next test?

b What is the minimum value her average could drop to after completing the next test?

c What percentage score must she achieve on her next test to increase her average to 80%?

16 Another measure of spread is the standard deviation. It measures the average spread of each data value from the mean. A small standard deviation means that most values are close to the mean and a large standard deviation means that the values are spread far from the mean. Standard deviation is best found using a calculator with the appropriate function. It can also be found using the formula:

$$s = \sqrt{\frac{\sum(\bar{x} - x)^2}{n - 1}}$$

where s represents standard deviation, $\sum$ means 'the sum of', $\bar{x}$ represents the mean, x represents an individual score and n represents the number of scores.

a Use your calculator or the standard deviation formula to find the standard deviation for each dataset, correct to two decimal places.

i 4, 8, 2, 6, 4, 9, 7, 7, 4, 1, 2, 4, 6, 9, 7, 4, 6, 7, 8, 1, 2, 3, 1, 6, 2, 6

ii 22, 27, 35, 64, 12, 74, 37, 93, 27, 33, 11, 71, 64, 42, 81, 37, 13, 19, 88, 50

iii 103, 118, 109, 111, 117, 116, 117, 117, 105, 107, 116, 113, 108, 109, 112, 113

b Use your results to state whether each dataset from part **a** has a large or small spread from the mean.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Summary statistics from frequency tables

Interactive skillsheet
Summary statistics from displays

Investigation
Estimating population means and medians

Topic quiz
9A

9B Constructing graphs

Learning intentions

By the end of this topic you will be able to ...

✔ construct column graphs, bar charts, line graphs and sector graphs.

Inter-year links

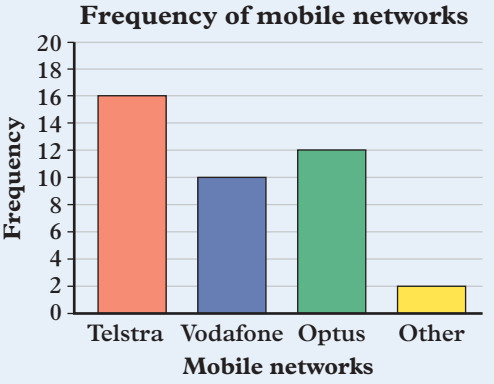
Support	Interpreting graphs
Year 7	9C Interpreting graphs
Year 10	11A Bivariate data and scatter plots

Key content video

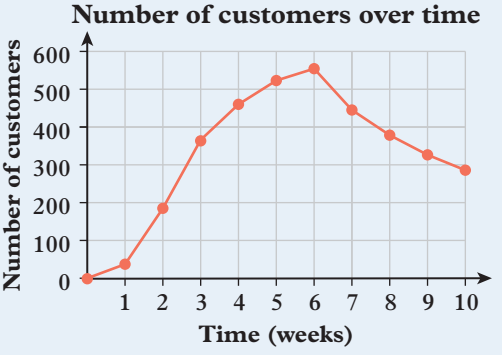
Types of graphs

- **Bar charts** and **column graphs** can be used to display data involving one or two variables.
  - → One axis will display either a discrete numerical or categorical variable.
  - → The other axis will display either the frequency (for univariate data) or a numerical variable (for bivariate data).

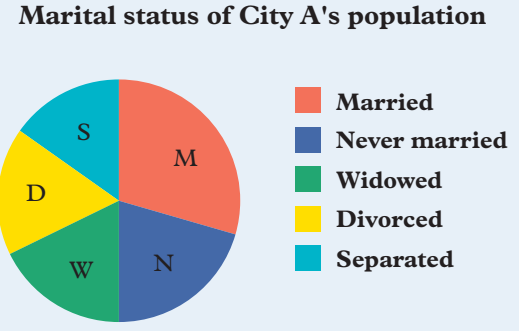
  For example, the column graph on the right is displaying one categorical variable, with the frequency detailed in the vertical axis.



- **Line graphs** can be used to display data involving two numerical variables.
  - → The horizontal axis must be a continuous variable and the vertical axis displays the variable that is being measured or observed. For example, when graphing the changes of a variable over time, time is always on the horizontal axis.
  - → Distance–time graphs are an example of line graphs.
  - → If necessary, a break in the scale on the vertical axis, indicated by a zigzag, can be used to show the focus of the graph is a distance away from the origin.



- **Sector graphs** show the relative size of each category but does not display frequency.
  - → Sector graphs represent different categories of data as sectors of a circle.



Source: Bureau of Statistics 2021 Census

Constructing graphs

- A good graph is easy to read and displays data clearly. When displaying data, choose a graph that is clear and informative.
- All graphs should include:
  - → title axes that are clearly labelled (including any units)
  - → a regular scale on both axes
  - → a legend or key if different parts of the graph are in different colours.
  - → source of the data where applicable
- To draw a column graph and bar chart, draw the axes first. Long category labels are better suited to bar charts rather than column graphs.
- To draw a sector graph you must either have data in a frequency table or as percentages. Aways start with the largest sector, multiply percentage (or fraction of the total) by 360°. Sector graphs are easily drawn with software such as Microsoft Excel.

Example 9B.1 Drawing a column graph

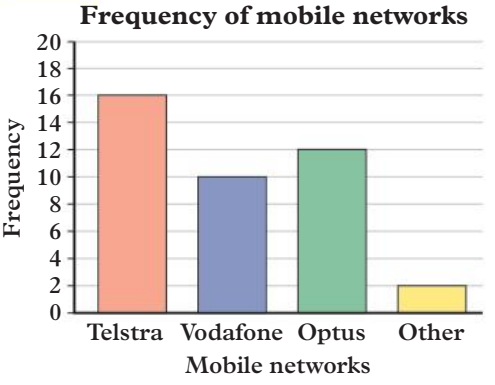
Sanam conducted a survey to find out which mobile phone network students in her school used and recorded the following results in a frequency table. Create a column graph from the data.

Mobile network	Frequency
Telstra	16
Vodafone	10
Optus	12
Other	2

THINK

1. Identify the categories with the smallest and largest frequencies.
2. Draw the vertical axes. The top of the vertical scale should be a number that is at least equal to the largest frequency.
3. Choose an interval so that the frequency of each category is clear.
4. Draw the horizontal scale, providing enough space to fit the categories and allowing for space between each column.
5. Use a ruler to draw each column accurately.
6. Include a title and axis labels.

WRITE



Example 9B.2 Drawing a line graph

The data in the following table records the weight of a puppy over 10 days.

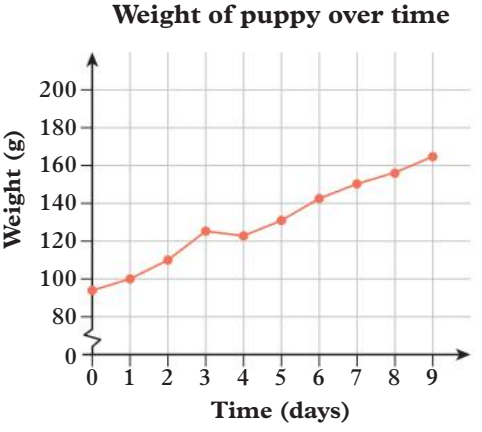
Day	0	1	2	3	4	5	6	7	8	9
Weight of puppy (g)	92	100	110	125	123	131	142	150	156	164

Draw a line graph to represent this data.

THINK

1. The weight of the puppy is being measured over time. Place time on the horizontal axis and weight on the vertical axis.
2. Draw the vertical axis. As the values start above 80, use a break in the axis to focus the graph above here.
3. Draw the horizontal scale, providing enough space to fit in all the independent values.
4. Plot the points from the table and join subsequent points with straight lines.
5. Include a title and axis labels.

WRITE



Example 9B.3 Drawing a sector graph by hand

Sammy conducted a survey to find out which ice-cream store is most popular in her class and recorded the following results in a frequency table.

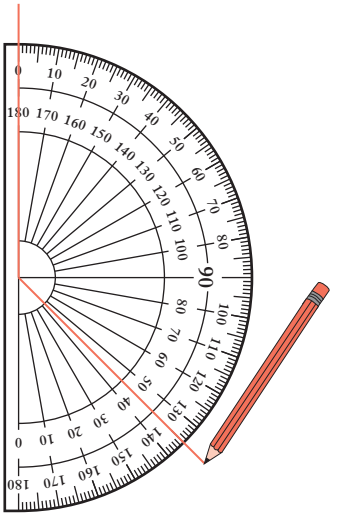
Favourite ice-cream store	
Store	**Frequency**
Cold Rock	15
Trampoline	12
Gelatissimo	7
Baskin Robbins	4
Wendy's	2

THINK

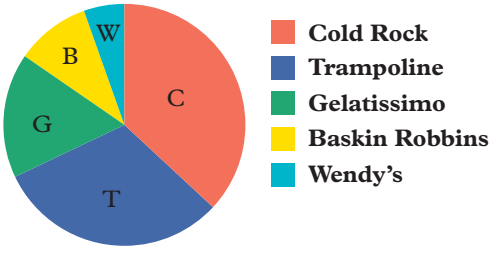
1. Add two columns to the table headed Fraction and Angle of sector.
2. Find the total of the Frequency column and use this to complete the Fraction column.
3. Multiply each fraction by 360° to find the angle of each sector.
4. Draw a vertical line for the start of your sector graph. Using a protractor, measure the first angle (135°) in a clockwise direction.
5. Continue to add sectors to the sector graph until it is complete.
6. Include a title and a legend to complete the sector graph.

WRITE

Store	Frequency	Fraction	Angle of sector
Cold Rock	15	$\frac{15}{40}$	$\frac{15}{40} \times 360° = 135°$
Trampoline	12	$\frac{12}{40}$	$\frac{12}{40} \times 360° = 105°$
Gelatissimo	7	$\frac{7}{40}$	$\frac{7}{40} \times 360° = 63°$
Baskin Robbins	4	$\frac{4}{40}$	$\frac{4}{40} \times 360° = 36°$
Wendy's	2	$\frac{2}{40}$	$\frac{2}{40} \times 360° = 18°$



Favourite ice-cream stores



Example 9B.4 Drawing a sector graph using software

Create a sector graph of this data.

Book title	Frequency
Harry Potter	8
Deltora Quest	4
Chronicles of Narnia	2
Spiderwick Chronicles	6

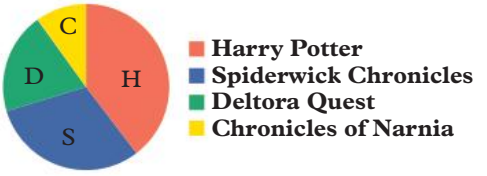
THINK

1. Open a new spreadsheet and enter the categories into column A and the frequency into column B in order from largest to smallest.
2. Highlight all data and create a sector graph using the appropriate tool bars. See your teacher if you need help with this.
3. Add a Chart title and any other information you need to finish your sector graph.

WRITE

	A	B
1	Harry Potter	8
2	Spiderwick Chronicles	6
3	Deltora Quest	4
4	Chronicles of Narnia	2

Favourite book series



Example 9B.5 Displaying data in an appropriate graph

Use the data in the table to draw a graph. Explain your choice.

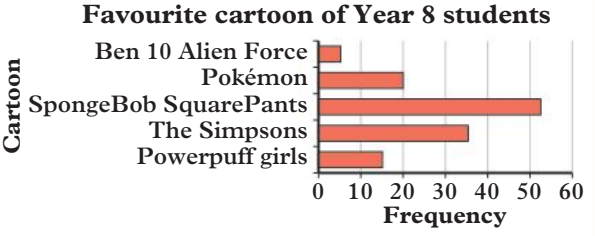
Favourite cartoon of Year 8 students	Frequency
Powerpuff girls	15
The Simpsons	35
SpongeBob SquarePants	54
Pokemon	20
Ben 10 Alien Force	5

THINK

1. Choose a graph. Since there are categories and the names are long, it is best to use a bar chart.
2. Look at the frequency column of the table. The highest number is 54 and the lowest is 5. Choose a scale that allows you to show all numbers. One option is from 0 to 60 with intervals of 10.
3. Draw the vertical axis long enough to fit the five categories and the horizontal axis long enough for the scale.
4. Using a ruler, draw each bar accurately. Remember to include a title and axis labels.

WRITE

A bar chart is best because the category names are long.



Helpful hints

✔ Univariate (single variable) data is usually displayed in a column graph with the frequency of the categories on the vertical axis. Bivariate data consists of two distinct variables, for example, there are two variables in a distance-time graph.

ANS p541

Exercise 9B Constructing graphs

1–5, 7, 8, 12, 14, 16 | 2–4, 6, 7, 9–11, 15, 17, 18 | 3, 4, 6, 8, 11, 13, 15, 17, 18

UNDERSTANDING AND FLUENCY

1 Sanjeev conducted a survey on people's favourite clothing store and recorded the results in a frequency table.

Clothing store	Frequency
Rebel Sport	18
Ghanda	6
Osmosis	27
Sportsgirl	9
Cotton On	12

a How many categories are there?

b What is the minimum value?

c What is the maximum value?

d Create a column graph from the data.

9B.1 **2** Present the data in this table in a bar chart.

State/Territory	Number of people on tour
Australian Capital Territory	4
New South Wales	11
Northern Territory	9
Queensland	4
South Australia	8
Tasmania	13
Victoria	10
Western Australia	7

3 The result of a survey on the make of the family car are shown below.

Holden	Toyota	Ford	Audi	Ford
Ford	Volvo	Holden	Holden	Hyundai
BMW	Holden	Toyota	Audi	Ford
Ferrari	Toyota	Ford	Ford	BMW
Hyundai	Volvo	Volvo	Toyota	Holden
Ford	Hyundai	Ford	Holden	Hyundai

a Create a bar chart to show the data.

b What is the most common type of car?

c What is the least common type of car?

9B.2 **4** The data in the following table shows the estimated market value of a gaming console for 10 months after its purchase. Draw a line graph to represent this data.

Time since purchase (months)	0	1	2	3	4	5	6	7	8	9	10
Market value ($)	600	400	350	320	300	270	250	250	220	220	220

OXFORD UNIVERSITY PRESS

5 The following data shows the number of kilometres driven by Antoine over a period of 9 days.

Day	1	2	3	4	5	6	7	8	9
Distance driven (km)	18	11	0	18	15	8	10	6	13

a Draw a line graph to represent this data.

b On which day(s) did Antoine drive the most kilometres?

c On how many days did Antoine drive more than 10 km?

9B.3 9B.4 **6** Use this frequency table to create a sector graph by hand or using technology.

Candidate	Will	Caleb	Teagan	Kyle	Emma	Zac	Andy
Number of votes	15	25	20	40	30	35	35

7 The ingredient of Sanitarium's Berry and Yoghurt Coated Flaked Muesli are listed below. Create a sector graph to represent the percentage of each ingredient in this muesli.

68% cereals, 11% raisins, 8% berry fruit pieces, 3.5% yoghurt-coated cornflakes, 0.5% freeze-dried fruit, 9% others (including flavours, colours, vitamins)

8 A group of Year 8 students were surveyed on the brand of their mobile phone. The study found that 41% had iPhones, 17% had Googles, 15% had Oppos, 10% Samsungs, 9% Motorolas and the remainder had Nokias.

a What percentage of students had a Nokia?

b Create a sector graph showing the mobile phone brands that Year 8 students own.

9B.5 **9** Present the data in the table using an appropriate graph. Explain your choice of graph.

Hair colour	Black	Blonde	Brown	Red	Dyed
Frequency	13	8	15	4	7

10 Present the data in the table below using an appropriate graph. Explain your choice of graph.

Birth month	**Frequency**	**Birth month**	**Frequency**
January	17	July	18
February	31	August	33
March	28	September	54
April	20	October	42
May	16	November	21
June	25	December	26

11 The following data is the monthly profit (rounded to the nearest $100) of a small business over the course of a year.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Profit ($)	4000	5300	3800	4100	4300	4000	3800	3400	5100	5400	4900	4600

a Draw a line graph to represent this data, using an appropriate scale.

b In which month was the greatest profit?

c In which month was the least profit?

d In which month was the greatest increase in profit over the previous month? How much was this increase?

e In which month was the greatest decrease in profit over the previous month? How much was this decrease?

12 Year 8 students were surveyed on their favourite drink. The results are shown in the box below.

Coke, Pepsi, Sprite, Big M, Coke, Coke, Fanta, Sprite, fruit juice, Sprite, Pepsi, Coke, fruit juice, Coke, Big M, Pepsi, Sprite, Big M, Coke, iced tea, Coke, Big M, iced tea, Big M

a Create a frequency table of this data.

b Create a sector graph. Remember to put them in order of highest to lowest frequency.

c What drink is most popular?

d What drink is least popular?

13 A specialty store sells Christmas items all year round. The table below shows the average number of customers per day, rounded to the nearest 10 customers, in each month over the course of 12 months.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Average daily customers	10	20	20	30	60	170	300	40	20	10	0	10

a Draw a line graph to represent this data.

b It is safe to assume that, in this dataset, month 12 does not represent December? Which month number do you think represents December? Explain your answer.

c Is it accurate to say that the store had no customers at all in month 11? Explain your answer.

14 Eleonora asked her classmates their favourite genre of film. She made a tally in the table below.

Genre	**Tally**	**Frequency**	**Percentage frequency**
Action	𝍸 𝍸 ||		
Drama	||||		
Romance	|		
Comedy	𝍸 |		
Other	||		

a Copy the table and complete the frequency and percentage frequency column.

b Construct a column graph for the data using percentage frequency on the vertical scale.

c Construct a sector graph using the same data.

d State the most common category. What percentage of Eleonora's classmates chose this category?

e State the least common category. What percentage of Eleonora's classmates chose this category?

15 The table below shows the monthly profit, rounded to the nearest $1000, of a small ice-cream store in Newcastle. In this case, January is represented by month 1.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Profit ($)	9000	8000	6000	3000	1000	−1000	−2000	0	3000	4000	7000	9000

a Draw a line graph to represent this data. Remember to make space below the horizontal axis for the negative values.

b Explain what the negative values in month 6 and month 7 mean.

c Explain what the 0 in month 8 means.

d What was the total profit of the ice-cream store over the course of the year?

e From month 8 to month 12, the profit is increasing each month. Do you expect this trend to continue forever? Explain your answer.

f How would you expect the monthly profit of an ice-cream store to be different for a similarly sized store in the northern hemisphere (for example, in a small town in Spain)? Explain your answer.

16 The following table shows the height (in cm) of a person over the first 15 years of their life.

Age (years)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Height (cm)	52	75	85	92	100	108	112	120	128	135	140	147	154	160	165	172

a Draw a line graph to represent this data.

b The person had a height of 52 cm when they were born and 172 cm 15 years later. Using these two points, find the average height they gained per year.

c Assuming the person continues growing at this average rate per year, how tall would they be at 25 years old?

d Instead, suggest a reasonable height that the person might be at 25 years old.

e Explain what you would expect the rest of the graph to look like if it continued to when the person was 25 years old.

17 Students in years 8, 9 and 10 were asked to choose their preferred ice-cream flavour from three choices given. The data is shown below.

	Year 8	Year 9	Year 10
Chocolate	140	135	130
Strawberry	100	90	70
Vanilla	60	75	60

a Create a stacked bar chart for the set of data if the category on the horizontal axis is year level.

b Create a stacked bar chart for the set of data if the category on the horizontal axis is ice-cream flavour.

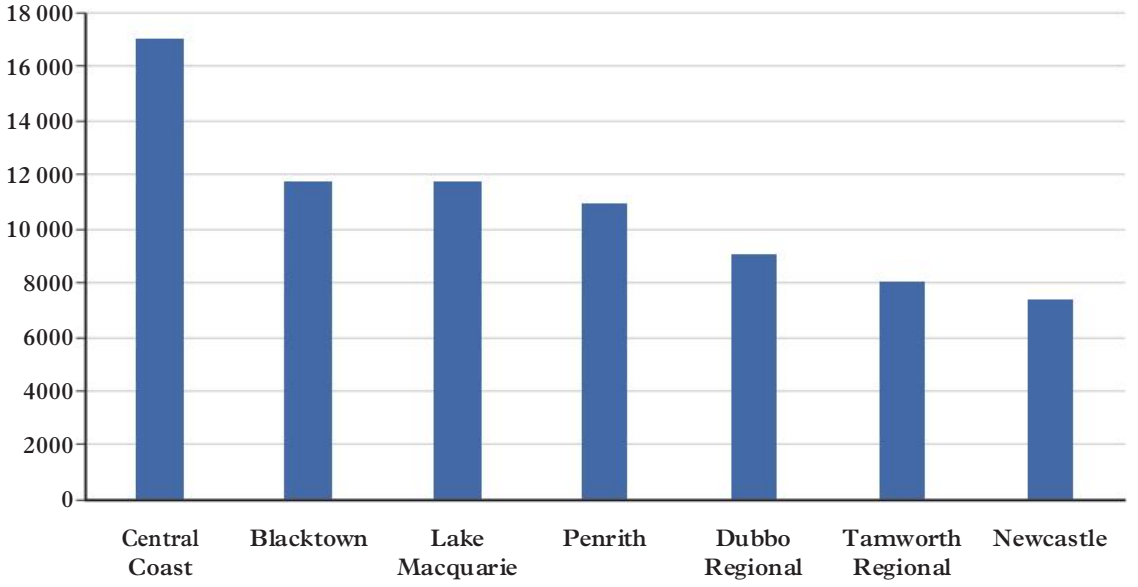
c Which of the two graphs is better for showing which ice-cream flavour is the most popular with Year 9 students?

d Is it accurate to say that, in general, Year 9 students prefer chocolate ice-cream more than Year 10 students? Why or why not?

18 The following graph shows the Local Government Area (LGA) with the most Aboriginal and Torres Strait Islander population in New South Wales.

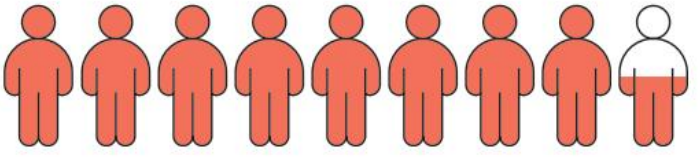
a Construct a sector graph from the column graph.

LGAs with the largest Aboriginal and Torres Strait Islander population in NSW



Source: ABS 2021 census

b Lauren is creating a pictogram to represent the same information. She used the following pictures to represent the Aboriginal and Torres Strait Islander population in Central Coast. How many people does a fully shaded figure represent?



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Column graphs

Interactive skillsheet
Stacked bar charts

Topic quiz
9B

9C Grouped data and histograms

Learning intentions

By the end of this topic you will be able to ...

✔ group numerical data into class intervals
✔ interpret data in a histogram
✔ represent grouped numerical data in a histogram.

Inter-year links

Support	Data displays
Year 7	9A Collecting and classifying data
Year 9	8C Distributions of data

Grouping data into class intervals

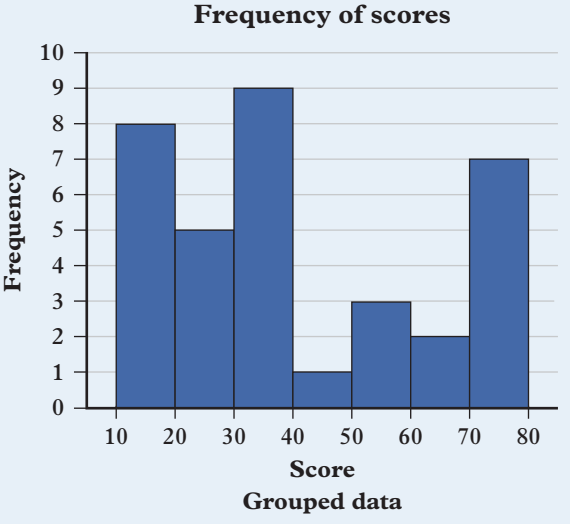
Key content video

- **Numerical data** can be counted or measured. There are two types of numerical data: discrete and continuous.
  - → **Discrete data** can be counted; for example, the number of fish in an aquarium.
  - → **Continuous data** can be measured; for example, the time it takes to listen to a song.
- **Grouped data** is numerical data that has been sorted into groups or **class intervals**. **Ungrouped data** is raw data that has not been placed into class intervals.
- When grouping data, class intervals should be chosen so that a frequency table contains 5 to 10 classes.
  - → Class intervals should start at a round number if possible, for example 0 or 10.
- Before constructing a frequency table with class intervals, identify whether the numerical data is discrete or continuous.
  - → Discrete data uses class intervals such as 0–9, which means any value from 0 to 9, including both 0 and 9.
  - → Continuous data uses class intervals such as 0–<10, which means any value from 0 to 10, including 0 but not including 10.

Discrete interval	Continuous interval
0–9	0–<10
10–19	10–<20

Histograms

- A **histogram** is a special kind of column graph that can be used to represent grouped numerical data.
  - → Ungrouped discrete numerical data is usually represented by using a standard column graph.
- Histograms have no gap between columns, but there is a small gap between the vertical axis and the first column when the intervals do not start at 0.
- When displaying grouped data in a histogram, the **lower bound** of the class interval is shown on the left of each column and the **upper bound** of the class interval is shown on the right of each column.
  - → For discrete data, where the upper and lower bounds of subsequent classes are not equal, the lower bound of the higher class interval should be shown. For example, for the discrete intervals 0–9, 10–19, 20–29, the intervals on the horizontal axis are labelled 0, 10, 20.



Example 9C.1 Understanding histograms

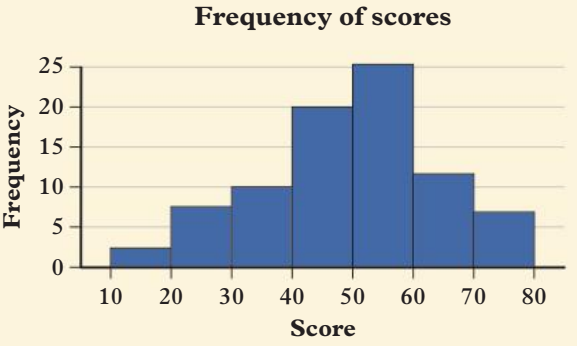
Consider this histogram representing continuous data.

a What is the width of each class interval?

b What is the frequency of the class interval 30–<40?

c What is the most common class interval? What is its frequency?

d Which class has a frequency of 20?



THINK

a Look at the horizontal axis. The width of the class intervals is the width of each column.

b Look at the column covering the marks 30 to 40 and read off the frequency from the vertical axis. The question mentioned the graph is representing continuous data, use –< to connect the upper and lower bound in your answer.

c Identify the tallest column. Use the horizontal axis to identify the class interval and read off the frequency from the vertical axis.

d Identify the column with a frequency of 20 on the vertical axis, and then use the horizontal axis to identify the class interval.

WRITE

a The width of each class interval is 10.

b The class interval 30–<40 has a frequency of 10.

c The most common class interval is 50–<60 and it has a frequency of 25.

d The class interval with a frequency of 20 is 40–<50.

Example 9C.2 Drawing a frequency table to represent grouped data

Use the frequency table below with class intervals of width 10 to represent this continuous data.

4.5, 11.6, 67.3, 33.7, 28.1, 36.4, 22.6, 54.8, 1.4, 66.8, 36.4, 29.3, 37.8, 42.3, 52.1, 38.3

Class							
Frequency							

THINK

1 Identify the minimum and maximum scores in the dataset. The minimum score is 1.4 and the maximum score is 67.3.

2 The data is continuous so the class intervals must be in the form of 0–<10, etc.

3 Draw the frequency table and group the raw data into each class interval.

WRITE

Class	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60	60–<70
Frequency	2	1	3	5	1	2	2

Example 9C.3 Drawing a histogram

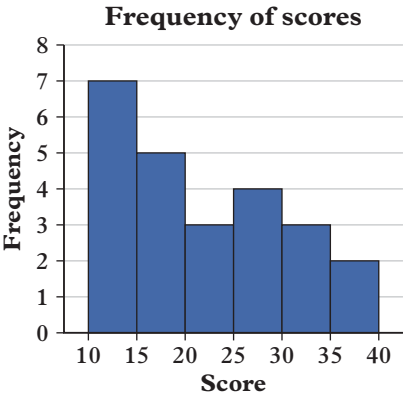
Draw a histogram to represent the data in this frequency table.

Class	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40
Frequency	7	5	3	4	3	2

THINK

1. Draw the axes with an even scale that allows the minimum and maximum values to be shown. Because the lower bound of the first class interval is not 0, ensure that there is a half space between the vertical axis and first class interval.
2. Use the frequency table to draw the columns of the histogram.
3. Label both axes and provide a title.

WRITE



Helpful hints

- ✔ If your data is discrete, use – to connect the lower and upper bounds; if your data is continuous, use –< instead.
- ✔ The class interval 0–9 contains 10 values. If this doesn't make sense to you, try counting the values using your hands: 0, 1, 2, … 8, 9.
- ✔ When grouping raw data, a tally column can help to ensure you don't miss any scores.
- ✔ You don't need to order the values in a dataset before grouping data. Avoiding this unnecessary step can save you time when answering questions.

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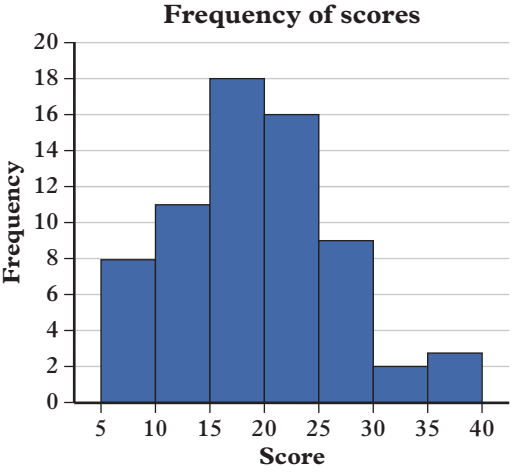
Exercise 9C Grouped data and histograms

1–3, 4(a, c), 5(a, b), 6(a, b), 8, 9, 12, 15 | 2, 4(b, c), 5(a, c), 6, 7, 10, 13, 15, 16 | 4(d), 5(a, c), 6(a), 11, 14, 16–18

UNDERSTANDING AND FLUENCY

9C.1 **1** Consider this histogram representing continuous data.



- **a** What is the width of each class interval?
- **b** What is the frequency of class interval 20–<25?
- **c** What is the most common class interval? What frequency does it have?
- **d** Which class interval has a frequency of 9?
- **e** What is the total frequency of scores that are larger than 25?
- **f** What is the total frequency of scores that are less than or equal to 15?
- **g** What percentage of the scores are more than 35?

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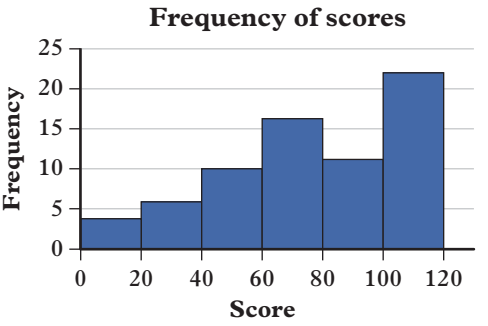
2 Consider the histogram on the right representing continuous data.



a How many class intervals are there?

b What is the least common class interval? What frequency does it have?

c How many people were surveyed?

3 Joseph collected the following data on the heights of the people in his class.

145, 183, 167, 172, 161, 158, 153, 168, 165, 174, 157, 152, 173, 166, 158, 159, 160, 171, 171, 161, 165, 172, 165, 158, 154

a Identify the minimum value in the dataset.

b Identify the maximum value in the dataset.

c Calculate the range of the data.

d Group the data into class intervals of 5 (e.g. 150–<155) in a frequency table.

9C.2 4 Draw a frequency table with the given class interval widths to represent each of these continuous datasets.

a 5, 16, 28, 24, 31, 39, 3, 18, 13, 11, 25, 33, 8, 12, 19, 21, 31, 28 [class interval width = 5]

b 14.5, 73.2, 22.1, 43.9, 42.0, 58.4, 19.8, 37.6, 62.1, 29.4, 34.5, 72.1, 59.1, 52.3, 63.1, 26.3, 34.0, 41.9, 48.5, 16.4, 31.2, 52.9 [class interval width = 10]

c 1.2, 5.4, 9.3, 11.4, 3.3, 4.7, 3.3, 3.9, 4.8, 6.6, 2.9, 1.9, 10.6, 9.7, 10.8, 3.6, 4.8, 2.7, 2.1, 1.7, 1.9, 11.9, 6.7, 5.4, 5.1, 1.6, 1.8 [class interval width = 2]

d 42, 79, 56, 49, 77, 50, 51, 46, 48, 72, 61, 78, 63, 45, 58, 53, 73, 58, 49, 61, 68, 67, 43, 49, 75, 77, 58, 54, 67, 72, 51, 56, 53, 48, 76, 78, 72, 42, 48, 53 [class interval width = 5]

9C.3 5 Use each frequency table to draw a histogram.

a

Class	5–<10	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40	40–<45	45–<50
Frequency	8	6	7	2	3	1	5	6	9

b

Class	0–<20	20–<40	40–<60	60–<80	80–<100	100–<120
Frequency	14	21	18	13	8	2

c

Class	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60	60–<70
Frequency	5	8	12	3	11	9	6

6 Use each dataset to draw an appropriate histogram with the given class interval widths.

a 13, 46, 13, 17, 35, 9, 22, 15, 8, 2, 35, 42, 42, 17, 16, 22, 29, 31, 47, 29, 13, 20, 36, 47, 28, 23, 30, 38 [class interval width = 10]

b 18.1, 24.5, 32.1, 15.6, 22.5, 29.1, 34.6, 16.7, 19.4, 17.5, 21.8, 27.5, 29.2, 30.1, 20.0, 33.1, 32.8, 31.9, 33.8, 14.3 [class interval width = 5]

c 64, 18, 120, 7, 29, 40, 145, 38, 72, 38, 18, 29, 2, 56, 49, 87, 99, 104, 59, 5, 29, 112, 118, 34, 59, 29, 19, 13 [class interval width = 20]

7 Data was collected on the time spent listening to music in hours per week, as shown below.

7	10	2	4	24	3	7	9	5	15
17	16	19	20	5	3.5	5	10	14	7
7	5	9	10	17	7	4	10	11	16
4	5.5	12	14	6	7	12	14	16	3

a Create a frequency table with class intervals of width 5 to collate the data.

b Draw a histogram to represent this data.

8 Data was collected on the weights (in kg) of various dogs, as shown below.

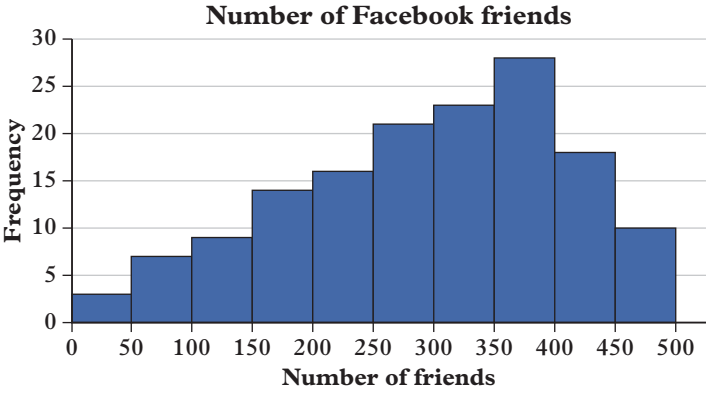
11.2	14.8	6.4	19.7	22.1	4.8	5.4	12.6	18.2	9.2
13.1	20.1	8.6	4.9	8.4	17	6.2	14.3	26.3	16.6
18.6	19.2	14.5	28.3	16.2	9	11.4	14.9	11.3	13.2

a Fill in the following frequency table.

Class	Frequency
0–<5	
5–<10	
10–<15	
15–<20	
20–<25	
25–<30	

b Based on your frequency table, draw a histogram to represent this data.

9 Consider the following histogram.



a What does it show?

b What is the size of its class intervals?

c State the most common class interval and its frequency.

d If you were to add a piece of data with a value of 140 to the histogram, to which class interval would you add it?

10 Data collected on the ages of customers in a clothes store in a day is shown in the table below.

a Why is this table difficult to read?

b Redraw the table with class intervals of width 10 so that it is easier to read.

Class	Frequency
<10	2
10–<12	6
12–<14	12
14–<16	16
16–<18	14
18–<20	13
20–<22	11
22–<24	9
24–<26	10
26–<28	8
28–<30	6

Class	Frequency
30–<32	4
32–<34	6
34–<36	5
36–<38	3
38–<40	2
40–<42	3
42–<44	1
44–<46	1
46–<48	0
48–<50	1
≥50	6

11 Explain why you can't accurately decrease the size of the class intervals in this table.

Class	Frequency
0–<20	16
20–<40	48
40–<60	42

12 The following ordered data shows the number of kilometres travelled by different people to attend a conference.

4, 4, 11, 12, 12, 13, 19, 19, 20, 21, 21, 25, 28, 31, 33, 34, 42, 44, 52, 55, 57, 58

a Create a histogram with class intervals of width 20 to represent this data.

b Create a histogram with class intervals of width 10 to represent this data.

c Create a histogram with class intervals of width 5 to represent this data.

d Which of the three histograms do you think displays the data most appropriately? Explain your answer.

13 This data was collected on how many thousands of people were present at NRL matches at Allianz Stadium, Sydney.

10	51	35	22	25	46	34	21	8	28	11	19
22	31	38	42	22	22	48	30	25	26	15	21
18	18	41	39	33	33	38	25	28	10	17	13

a Create a histogram with class intervals of width 5 to represent this data.

b Create a histogram with class intervals of width 10 to represent this data.

c What is similar and what is different about your graphs for parts **a** and **b**?

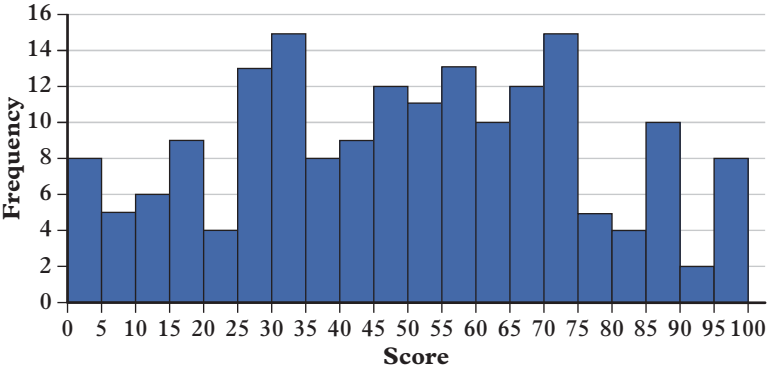
14 Consider the histogram on the right.

a What is the width of the class intervals?

b What can you say about the number of classes? Is it easy to read? Can you see a pattern?

c Redraw the histogram with class intervals of width 10.

d What patterns can you see now?



15 Stem-and-leaf plots can be used to display discrete data, where each piece of data is split into two parts (a stem and a leaf). The key indicates the value of the data. Consider this stem-and-leaf plot.

Stem	Leaf
1	2 6 7 8 8 9
2	0 1 5 6 7
3	3 6 8
4	0 2
5	1

Key: 1 | 2 = 12

a Which part of the plot represents the class intervals?

b What is the size of the class intervals?

c How many people were surveyed?

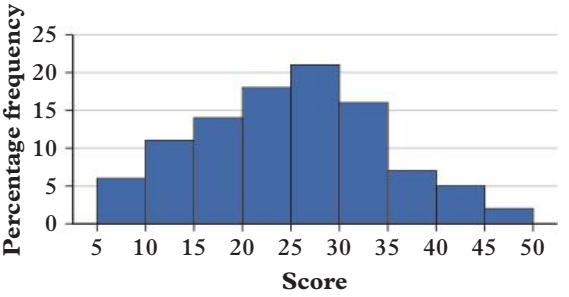
d Use the stem-and-leaf plot to draw a histogram.

e What advantage does a stem-and-leaf plot have over a histogram?

f Explain why you can't accurately construct a stem-and-leaf plot from a histogram.

CHALLENGE

16 Consider this percentage frequency histogram.



a How is it different from a normal histogram?

b What percentage of scores are between 30 and 35?

c What percentage of scores are greater than 35?

d Without performing a calculation, state the sum of the percentage frequency columns. Explain how you know the answer.

e If there were 400 scores in total, calculate:

i the number of scores between 15 and 20

ii the number of scores less than 25

iii the number of scores between 20 and 40.

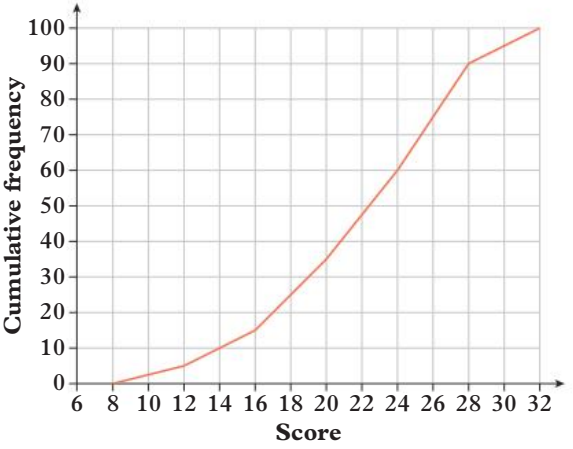
17 Create a percentage frequency histogram with class intervals of width 0.2 to represent this dataset.

Weights of newborn babies at a particular hospital in one week (in kg):

3.25, 4.15, 2.75, 3.60, 3.95, 3.05, 2.85, 4.20, 1.95, 3.50, 3.65, 3.15, 3.70, 3.95, 4.10, 4.85, 2.90, 3.10, 3.30, 3.25, 3.50, 4.05, 3.45, 3.85, 3.75, 3.15, 3.45, 3.20, 3.25, 4.25, 2.55, 2.95, 3.40, 3.85, 3.80, 3.55, 3.20, 3.00, 3.20, 3.75, 4.00, 4.15, 3.80, 3.75, 3.40, 3.25, 3.15, 3.05, 3.85, 2.95

18 A cumulative frequency polygon, also called an ogive (pronounced 'oh-jive'), is a graph plotting the cumulative frequency of the scores in a dataset (on the vertical axis) against those scores (on the horizontal axis).

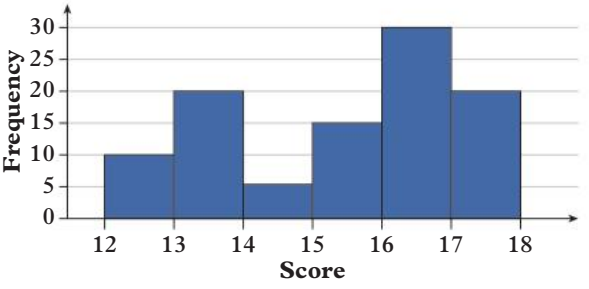
a Read the following cumulative frequency polygon to complete the frequency table.



Score	Frequency	Cumulative frequency
8–<12		
12–<16		
16–<20		
20–<24		
24–<28		
28–<32		

b Draw a histogram to represent the data in part **a**.

c Each section of the cumulative frequency polygon should start at the lowest value of a class interval and end at the highest value of the class interval. Draw a cumulative frequency polygon for the following histogram.



Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Histograms

Investigation
Pocket money

Topic quiz
9C

ANS p546

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this chapter.

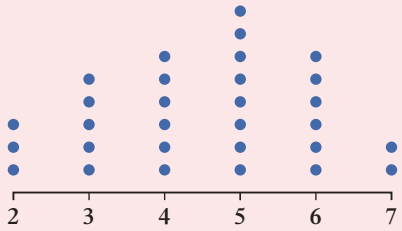
9A **1** Find the **i** mean, **ii** median, **iii** mode and **iv** range for the following datasets:

a 8, 4, 1, 2, 9, 2, 4, 13, 2

b

Score (x)	Frequency (f)
0	4
1	6
2	8
3	7
Total	**25**

c



d

Stem	Leaf
1	1 5 9
2	2 4 6 8
3	0 1 3 3 8 8 9
4	1 5 5 5 6 9
5	2

Key: 1 | 1 = 11

9B **2** A class of 30 students wrote down their birth month. The data was collected and put into the frequency table below. Construct a bar chart for this data.

Birth month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of students	4	4	2	2	3	5	0	2	4	0	2	2

9B **3** Consider the following table of data. Draw a line graph to represent this data.

Hours since midnight	0	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°C)	8	9	9	11	10	12	13	15	17	18	17	20	22

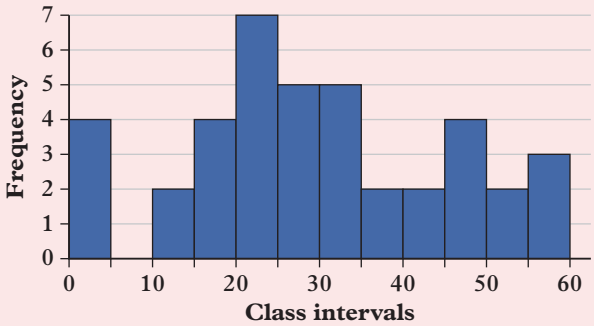
9C **4** A researcher asked 24 people how many minutes they spend in the shower every day on average. The results are shown below.

7	12	14	8	6	4	4	9	8	7	6	11
16	18	21	3	4	9	8	10	12	13	14	20

a Draw a frequency table with class intervals of 5 to represent this data.

b Use the frequency table to draw a histogram for this data.

9C **5** Redraw the following histogram with class intervals of 10, starting at 0.

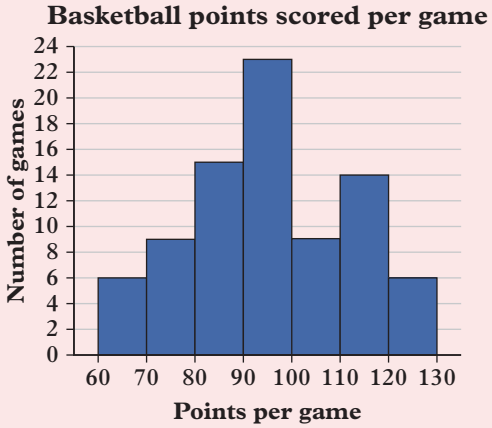


9C **6** Teams in the NBA play 82 games of basketball in a regular season. The number of points scored per game for a certain team is displayed in the histogram.

a Which interval contains the most values?

b In how many games did the team score at least 120 points?

c In how many games did the team score less than 80 points?



9D Describing data

Learning intentions

By the end of this topic you will be able to ...

- ✔ determine whether a dataset is uniform, unimodal, bimodal or multimodal
- ✔ identify whether a dataset is symmetric or skewed
- ✔ determine which measure of centre best describes a dataset.

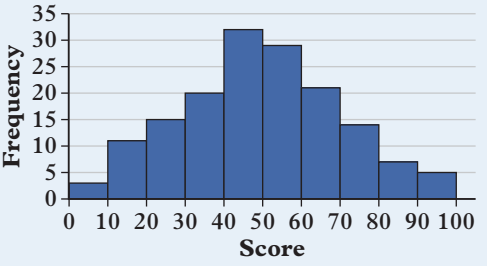
Inter-year links

Year 7	9D Dot plots
Year 9	8C Distributions of data

Describing data

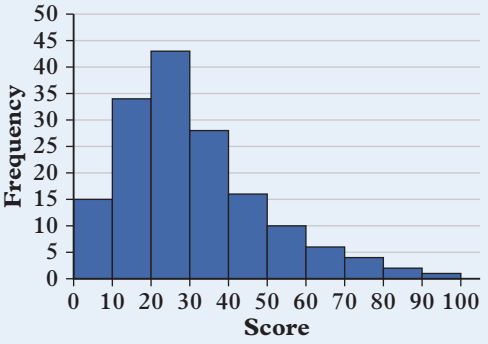
Key content video

- By displaying numerical data, you can identify the shape of the distribution of the data and use this to describe datasets.
- The mean, median and mode can all be used to represent a central point of a dataset, but the shape of the distribution should be considered before deciding which measure of centre to use for this purpose.
- **Symmetric** distributions have a peak in the middle and are roughly evenly spread on either side.
  - → When describing symmetric distributions, either the mean or the median are appropriate representations of the centre. In perfect symmetric distributions, the mean, mode and median are equal.

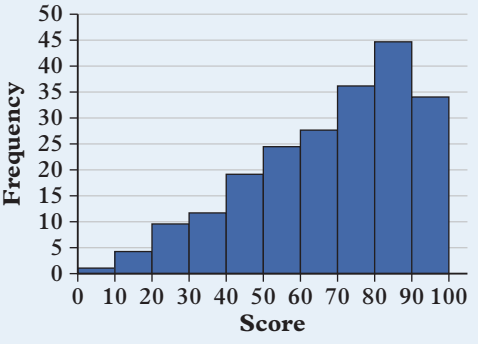


Symmetric

- A distribution is **skewed** when the data points are clustered on one side of the distribution. There is a tail on the other side of the distribution.
  - → **Positively skewed** distributions are skewed towards the vertical axis with a tail on the right of the distribution.
  - → **Negatively skewed** distributions are skewed away from the vertical axis with a tail on the left of the distribution.

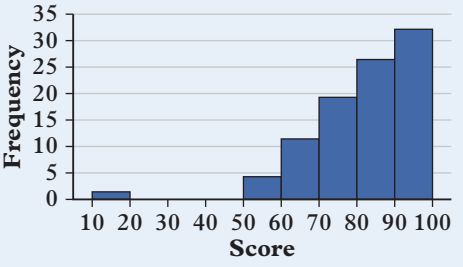


Positively skewed

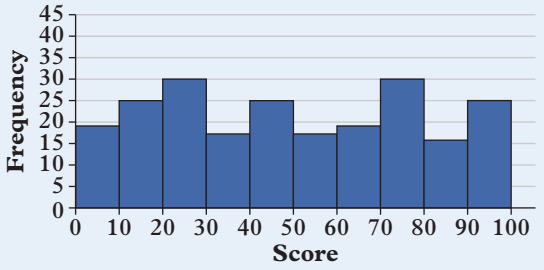


Negatively skewed

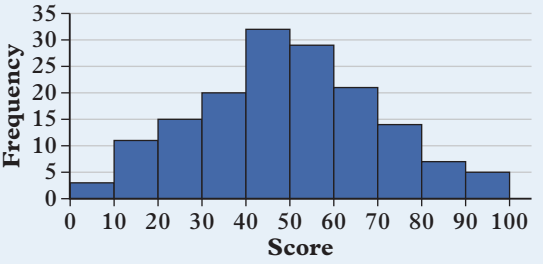
  - → When describing skewed distributions, the median is a better representation of the centre than the mean, as the mean is skewed by the values in the tail.
- An outlier is a data point that is significantly higher or lower than the other values in a dataset. There is often a gap between the outlier and the rest of the dataset on a histogram.
  - → The presence of outliers can have a significant impact on the mean, so distributions with outliers are best described by using the median.



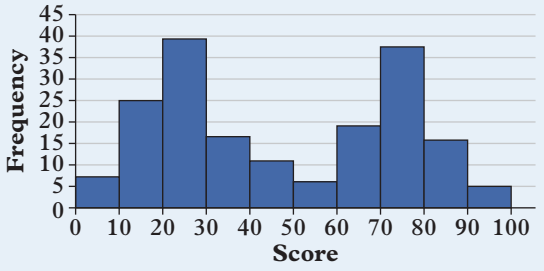
- A distribution can have one clear peak, two peaks, multiple peaks or no peaks.
  - → A distribution is **unimodal** if there is one clear peak in the data. A unimodal distribution can be symmetric, positively skewed or negative skewed.
  - → A distribution is **bimodal** if there are two clear peaks in the data. Bimodal distributions are best described using the mode of each peak.
  - → A distribution is **multimodal** if there are more than two clear peaks in the data.
  - → A distribution is **uniform** if there are no clear peaks in the data.



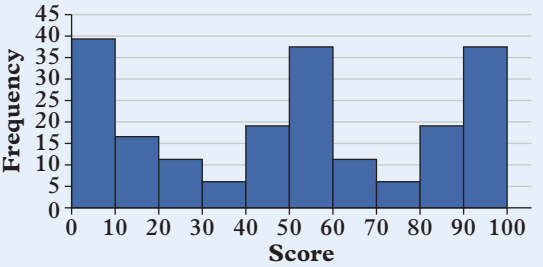
Uniform



Unimodal



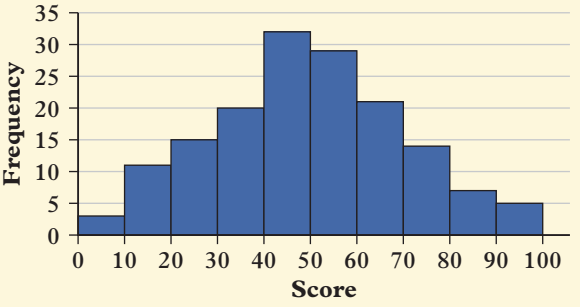
Bimodal



Multimodal

Example 9D.1 Describing a distribution

Describe the distribution and decide which measure(s) of centre would best describe this distribution.



THINK

1. Identify whether the distribution is uniform, unimodal, bimodal or multimodal.
2. Identify whether the distribution is symmetric or skewed.
3. Identify how the shape of the distribution determines which measure of centre should be used to describe the distribution.
4. Write your answer.

WRITE

The distribution is symmetric so the centre of the distribution could be described by the mean or the median.

Example 9D.2 Describing a dataset using histogram

Use the data below to:

a draw a histogram with class intervals of width 10

b describe its distribution

c state the best measure of centre to describe the data, providing a reason.

49, 3, 16, 12, 20, 49, 22, 37, 32, 18, 34, 13, 4, 7, 17, 9, 13, 59, 25, 1, 15, 30, 23, 27, 4, 17, 26, 3, 10, 5, 8, 2, 31, 11, 8, 20, 27, 13, 16, 11

THINK

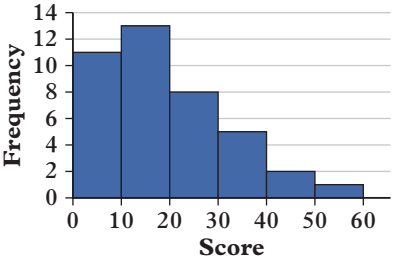
a Use a frequency table to draw a histogram with an even scale and labels on both axes.

b Decide whether the data is skewed.

c Identify the best measure of centre to describe the data. The data is positively skewed, so the median should be used.

WRITE

a



b The distribution is positively skewed.

c The median should be used to describe the centre, because the distribution is skewed, which affects the value of the mean.

Helpful hints

✔ The peaks in a bimodal or multimodal distribution do not need to be of equal height.

ANS p547

Exercise 9D Describing data

1–3, 5(a, b), 6, 8, 9 | 1, 2, 4, 5(b, c), 6, 9–11, 12 | 2, 3, 5(b, c), 7, 9, 11–13

UNDERSTANDING AND FLUENCY

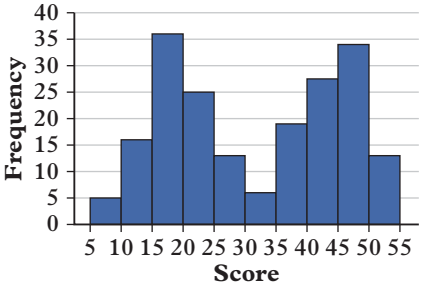
1 For each histogram:

a Determine if the data has any outlier.

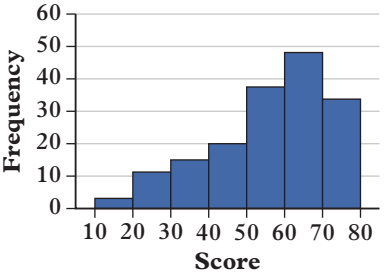
b Determine if the distribution is uniform, unimodal, bimodal or multimodal.

c If a distribution is unimodal, determine if it is symmetric, positively skewed or negatively-skewed.

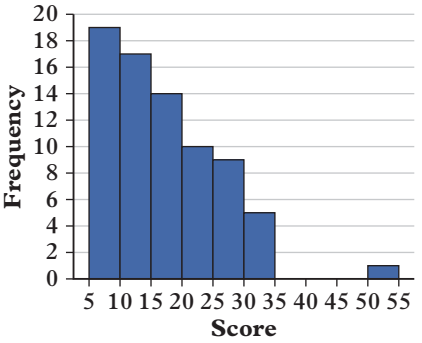
i



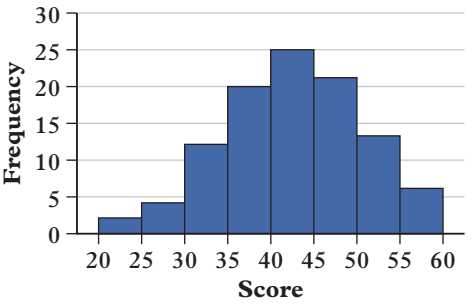
ii



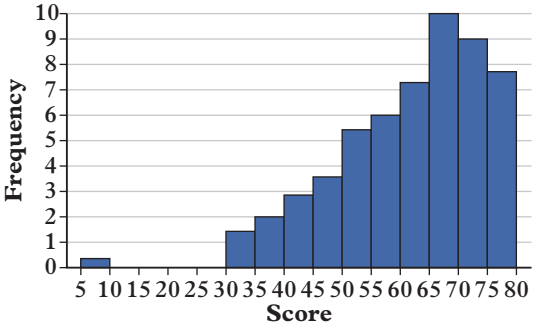
iii



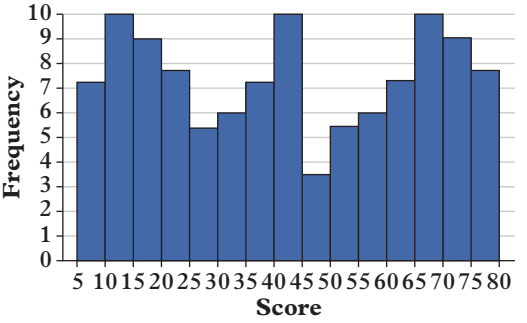
iv



v



vi



2 Describe the distribution of each stem-and-leaf plot.

a

Stem	Leaf
1	0 2
2	1 1 4
3	0 4 5 6
4	4 5
5	3

Key 1 | 2 = 12

b

Stem	Leaf
0	1 4 6 8
1	3 4 5 5 6
2	1 2 4 4
3	4 8 9
4	3
5	1

Key 1 | 3 = 13

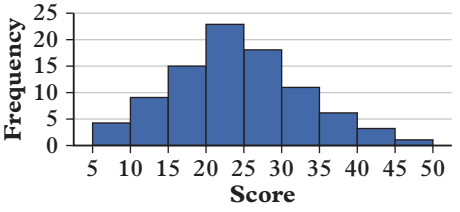
c

Stem	Leaf
0	9
1	
2	
3	1 9
4	4 5 6
5	4 5 5 6
6	0 1 1 2 4

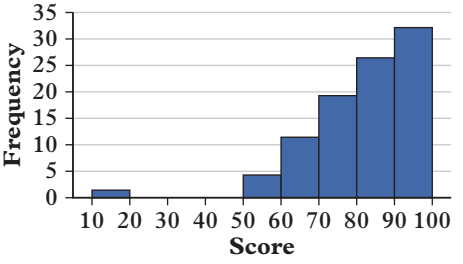
Key 3 | 9 = 39

9D.1 **3** Describe the distribution and decide which measure(s) of centre would best describe each distribution.

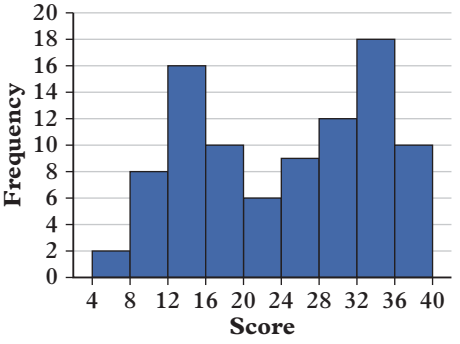
a



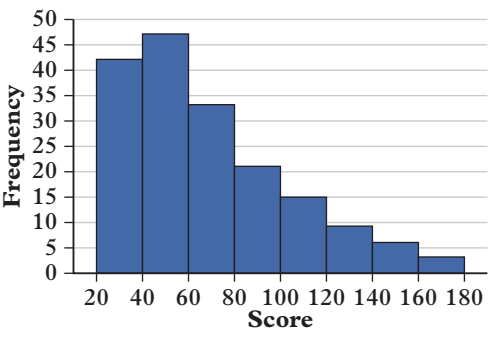
b



c



d



4 Decide which measure(s) of centre would best describe each distribution in question **1**.

9D.2 **5** For each dataset:

i draw a histogram with the given class interval width

ii describe its distribution

iii state the best measure of centre to describe the data, providing a reason.

a 35, 33, 42, 99, 54, 68, 4, 91, 97, 55, 99, 86, 40, 58, 41, 95, 38, 62, 35, 88, 82, 98, 77, 69, 78, 78, 82, 81, 98, 57, 88, 41, 60, 85, 82, 85, 91, 90, 80, 49, 58, 66, 97, 95, 82, 84, 78, 91, 62, 42 [width = 10]

b 9, 2, 3, 9, 43, 8, 2, 15, 12, 17, 10, 10, 14, 9, 34, 7, 12, 18, 18, 47, 2, 12, 24, 34, 19, 1, 12, 18, 35, 47, 6, 14, 8, 35, 7, 9, 4, 17, 2, 20, 8, 12, 21, 24, 48, 6, 7, 8, 17, 41 [width = 5]

c 49, 23, 45, 23, 31, 77, 62, 21, 52, 51, 60, 54, 46, 69, 27, 60, 142, 41, 32, 80, 52, 21, 80, 65, 37, 33, 74, 45, 48, 78, 70, 21, 55, 64, 33, 42, 59, 67, 32, 79, 30 [width = 10]

PROBLEM SOLVING AND REASONING

6 This stem-and-leaf plot does not seem to have a noticeable pattern. However, it is possible to redraw this stem-and-leaf plot with more stems, just like histograms can be redrawn with different class sizes. For example, the number 1 can be the stem for two leaves, with the numbers 10–<15 listed in the first stem labelled 1 and the numbers 15–<20 listed in the second stem labelled 1*.

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 3 3 5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3 9

Key 1 | 3 = 13

a Redraw it as a split stem-and-leaf plot with:

i four stems **ii** ten stems.

b Describe the distributions that you see from the split stem-and-leaf plots.

7 The following tables show the data that Ava collected on the weight of dogs (in kilograms) in her community.

Class	0–<2	2–<4	4–<6	6–<8	8–<10	10–<12	12–<14	14–<16	16–<18	18–<20
Frequency	1	6	3	5	6	5	8	6	7	3

Class	20–<22	22–<24	24–<26	26–<28	28–<30	30–<32	32–<34	34–<36	36–<38	38–<40
Frequency	7	2	5	3	4	5	4	2	1	4

a Draw a histogram to represent the data.

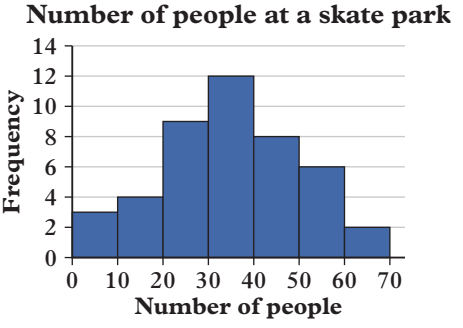
b What pattern can you see?

c Reorder the data into class intervals of width 4 and redraw your histogram.

d What pattern can you now see?

e Write a sentence describing the weight distribution of dogs as shown in your second histogram.

8 The histogram on the right shows the number of people at a skate park recorded at various times.



a Describe the shape of the distribution.

b What measure(s) of centre would be the most appropriate to use?

c Create a frequency table that represents the histogram.

d Use the frequency table to determine in which class interval the median lies.

e Write a sentence that describes the distribution in terms of the number of people at the skate park.

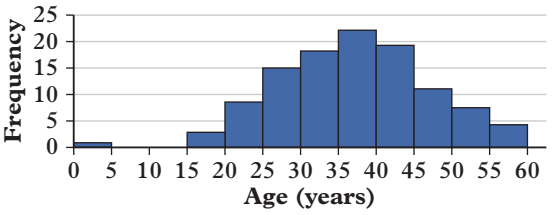
9 For each data distribution:

i describe its shape

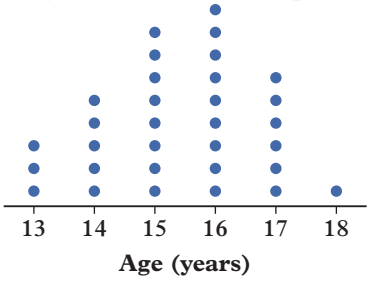
ii calculate the appropriate measure(s) of centre

iii write a sentence that summarises the graph in its context.

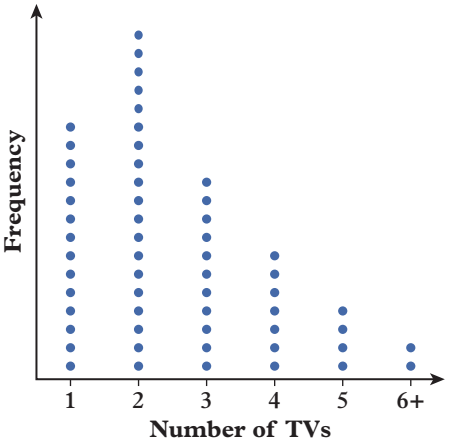
a **Ages of people at a hairdresser in a week**



b **Ages of students at a pool**



c **Number of TVs in a home**



d **Length of hair in Year 8 students (cm)**

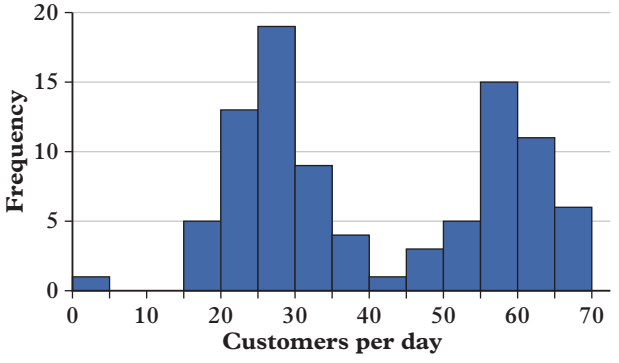
Stem	Leaf
0	1 2 2 2 3 3 3 4 4 6 6 6 6 7 8 8 8 9
1	0 2 4 6 8
2	0 1 5 8 8 9 9
3	0 1 2 2 2 2 2 4 5 5 5 5 6 7 8 9 9
4	1 2 2 4 4 5 6 6 8 9
5	0 4 5 6 8 8 8
6	2 9

Key 1 | 2 = 12

10 A mini-golf course is open every day of the year except Christmas day. The number of customers who visited the course each day in the months of October, November and December has been placed in the frequency table on the right. The histogram for this data is also shown.

Class	Frequency
0–4	1
5–9	0
10–14	0
15–19	5
20–24	13
25–29	19
30–34	9
35–39	4
40–44	1
45–49	3
50–54	5
55–59	15
60–64	11
65–69	6

Number of mini-golf customers



a Suggest a reason why the class interval 0–4 has a frequency of 1.

b Suggest a reason why the number of daily customers at the mini-golf course has a bimodal distribution.

c Give examples of other businesses that will most likely have a bimodal distribution for the number of daily customers.

11 A boat tour company offers ocean cruises where customers have a chance to see different wildlife, including orcas (killer whales). The following dataset is the number of orcas that were spotted during tours for each day of February.

0	0	0	0	10	0	0	8	0	0	0	26	0	0
0	13	0	41	0	0	7	7	0	0	0	0	0	0

The whale watching company decides to run an advertisement during March on social media with the claim 'Last month we saw an average of 4 killer whales per tour!'

a Explain why this claim, while technically true, is misleading.

b Why would the company not want to advertise the median number of whales they saw per tour?

c Create a histogram with class intervals of size 10 to display the data.

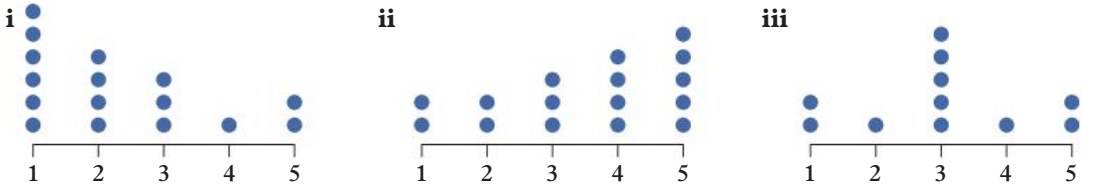
d Explain why the class interval 0–9 is misleading in the histogram.

e The company decides to change the advertisement to be more honest. The new advertisement will instead highlight the percentage of tours on which killer whales were spotted. Complete the sentence below to help the company write their new advertisement.

'Last month we saw killer whales on ___% of tours!'

CHALLENGE

12 Consider the following dot plots.



a Calculate the mean, median and mode of each dataset.

b Determine if each dataset is symmetric, positively skewed or negatively skewed.

c Use your conclusion from parts **a** and **b** to complete the following sentences:

If mean < median < mode, then the distribution is _____.

If mean = median = mode, then the distribution is _____.

If mean > median > mode, then the distribution is _____.

d If a dataset has the summary statistics mean = 7, mode = 12 and median = 10, is the dataset positively skewed or negatively skewed?

13 The Benford's law, or the first-digit law, is an observation that in many real-life sets of numerical data, the leading digit is more likely to be a smaller digit than a larger one. The following table represents the expected distribution of the first digit according to Benford's law.

First digit	1	2	3	4	5	6	7	8	9
Relative frequency	30%	18%	13%	10%	8%	7%	6%	5%	5%

a Describe the shape of the distribution.

b The following list shows the length of twelve major rivers in Australia, complete the following table to check if the distribution obeys the Benford's law.

Murray River 2508 km	Cooper Creek 1300 km	Flinders River 960 km
Darling River 1472 km	Warrego River 1380 km	Gascoyne River 865 km
Murrumbidgee River 1485 km	Fitzroy River 1000 km	Victoria River 800 km
Lachlan River 1440 km	Macquarie River 960 km	Goulburn River 654 km

First digit	1	2	3	4	5	6	7	8	9
Frequency									
Relative frequency									

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Describing data

Topic quiz
9D

9E Comparing data

Learning intentions

By the end of this topic you will be able to ...

✔ compare datasets in different graphical displays

✔ compare datasets using summary statistics.

Inter-year links

Year 7 9E Stem-and-leaf plots

Comparing datasets

Key content video

- Datasets are easier to compare when displayed in the same format; for example, when two or more datasets are both displayed in histograms.
- By placing the data in graphical displays, the range and shape of the distributions can be compared.
- To make a more thorough comparison of two or more datasets, summary statistics should be calculated so the centre and spread of each dataset can be compared.

Example 9E.1 Comparing datasets in graphical displays

Make a brief comparison of the two datasets detailing the ages of people at a pool in different seasons.

Summer

Stem	Leaf
0	8
1	1 3 4 4 5 5 6 6 7 7 7 8
2	0 1 3 4
3	1 6
4	
5	6

Key: 1 | 4 = 14

Winter

Stem	Leaf
0	9
1	3 4 5
2	3 7 7
3	1 6 6 8
4	4 4 5 5 8 8
5	2 6 7

Key: 1 | 4 = 14

THINK

Look at the centre and spread of the two datasets. Where does the centre appear to be for each dataset? What does this indicate about the ages at different times of year?

WRITE

The two datasets have a similar range of age brackets, but they are skewed in different directions and have different measures of centre. The data suggests that the pool attracts younger people in summer and older people in winter.

Example 9E.2 Comparing datasets using summary statistics

Rachel investigated the average age of customers in two different cafes in her local area. Use her results to compare the ages of the customers in cafes.

Cafe	Mean	Median	Range
Gumtree	21	22	17
Jinx	27	21	45

THINK

1. Compare the given measure of spread (range).
2. Compare the given measures of centre (mean and median). The medians are similar, but Jinx has a higher mean than Gumtree. This difference combined with Jinx's large range suggests that Jinx's distribution may be skewed or affected by an outlier.
3. Summarise your findings.

WRITE

Jinx (45) covers a much wider range of ages than Gumtree (17).

The average ages are similar but Jinx (27) has a slightly higher mean than Gumtree (21). Jinx may have a positively skewed distribution or a data value for an older customer that is an outlier.

The mean customer age at the cafes is similar but Jinx has a larger range of ages.

Helpful hints

- ✔ When dealing with information from samples, the conclusions you draw will not be certain, so use words to show this, such as, 'The data indicates that …' or 'It appears that …'
- ✔ If you want to compare datasets but the data you're given is not in the same format, put the datasets in the same format before you begin the comparison.

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Exercise 9E Comparing data

1, 2, 3(a, b), 4(a–c), 5–8, 12 | 2, 3(b, c), 4(c, e), 6, 9, 10, 13, 15 | 3(b, c), 4(c, e), 10, 11, 13, 14, 16

UNDERSTANDING AND FLUENCY

1 Use the following stem-and-leaf plots to answer these questions.

a What is the maximum score:

i in group A **ii** in group B **iii** overall?

Group A

Stem	Leaf
0	1 3 4 7 9
1	1 2 3 4 6 8 8 9
2	0 3 5 7
3	3 8
4	1

Key: 1 | 4 = 14

Group B

Stem	Leaf
0	7 9
1	3 4 6 8
2	0 1 1 4 4 4 5 8 9
3	2 2 3 4 7 9 9
4	2 3 5

Key: 1 | 4 = 14

b What is the most common score:

i in group A **ii** in group B **iii** overall?

c How would you describe the distribution of:

i group A **ii** group B?

9E.1 **2** Make a brief comparison of the two datasets below.

a **Group A**

Stem	Leaf
1	1
2	2 7 8
3	1 3 6 7 9
4	1 1 3 5 6 7 7 8 9
5	0 1 3 4 6 7 9

Key: 1 | 9 = 19

Group B

Stem	Leaf
1	0 1 3 5 8 8
2	2 4 5 6 7 8 9 9
3	4 6 7 8
4	0 1 2
5	1 3

Key: 1 | 9 = 19

b **Group A**

Stem	Leaf
4	4 4 4 5 6 7 8 9
5	0 1 1 3 4 5 6 6 7 8
6	0 3 3 4 5 6 7 8

Key: 4 | 5 = 45

Group B

Stem	Leaf
3	0 0 1 2 4 5 6
4	1 2 5 5 6 7 8 9 9
5	0 3 4 5 5 6 8
6	
7	
8	1

Key: 4 | 5 = 45

3 Make a brief comparison of the two datasets.

a Number of people at a cinema over 6 months of weekends:

Friday: 65, 48, 67, 55, 32, 92, 64, 51, 49, 57, 76, 61, 29, 46, 61, 59, 53, 67, 72, 88, 71, 58, 54, 57, 56, 42, 30

Saturday: 67, 78, 84, 26, 37, 42, 99, 84, 75, 68, 64, 55, 75, 85, 59, 66, 77, 78, 83, 81, 92, 94, 77, 76, 79, 89

b Daily maximum temperature in February:

Darwin: 32.3, 32.1, 32.9, 33.2, 33.5, 33.8, 33.4, 32.8, 32.5, 33.3, 32.5, 29.5, 32.8, 33.4, 33.1, 32.7, 33.4, 31.4, 33.0, 31.3, 29.9, 30.8, 30.0, 32.2, 32.1, 32.6, 31.8, 30.4

Adelaide: 20.6, 23.7, 25.4, 29.4, 34.2, 38.4, 27.0, 28.3, 28.0, 26.3, 28.3, 31.2, 33.8, 34.3, 36.0, 37.1, 39.2, 40.5, 26.6, 28.4, 32.1, 33.8, 35.6, 38.2, 28.7, 33.4, 21.5, 22.7

c Mass of dogs of two breeds (to nearest hundred grams):

Boston Terriers: 5100, 6800, 7800, 10 500, 9200, 5500, 4900, 11 200, 8600, 9200, 10 500, 4900, 5500, 7600, 8400, 6800, 9200, 8600, 7500, 4600, 8300, 10 500, 11 000, 8000, 7600, 5100, 6700, 8300, 9200, 10 000, 4900, 7500, 7900, 6200, 8200, 4600, 7800, 8100, 6400, 9900

French Bulldogs: 9800, 12 900, 8600, 9900, 9800, 12 500, 11 200, 9100, 12 900, 11 200, 9500, 9200, 10 500, 11 900, 12 500, 11 800, 9400, 9800, 10 800, 10 600, 11 500, 9600, 12 300, 13 000, 11 000, 12 000, 10 600, 11 100, 12 700, 10 900, 9500, 11 500, 12 900, 10 100, 11 100, 9200, 11 500, 10 900, 9800, 11 300

9E.2 **4** For each table, use the given statistics to make a comparison of the two datasets.

a

Heights in class	Mean	Median	Range
8A	172 cm	168 cm	46 cm
8B	166 cm	167 cm	22 cm

b

Jeans price	Mean	Median	Range
Store A	$120	$122	$35
Store B	$124	$122	$30

c

Internet usage per week	Mean	Median	Range
Primary school students	4.5 hours	4 hours	8 hours
Secondary school students	11.2 hours	12 hours	17 hours

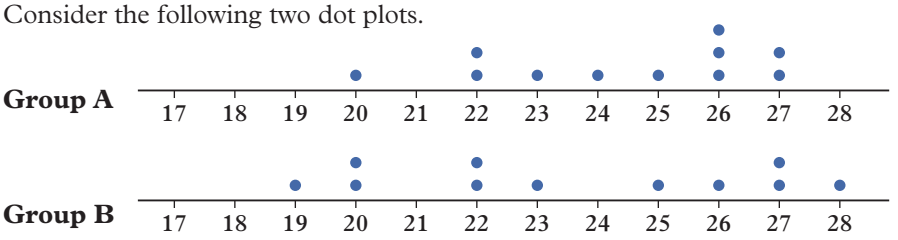
d

SD card capacity	Mean	Median	Range
Store A	20 GB	16 GB	60 GB
Store B	16 GB	16 GB	60 GB

e

Goals per game	Mean	Median	Range
Player A	4.2	4	9
Player B	4.8	4	7

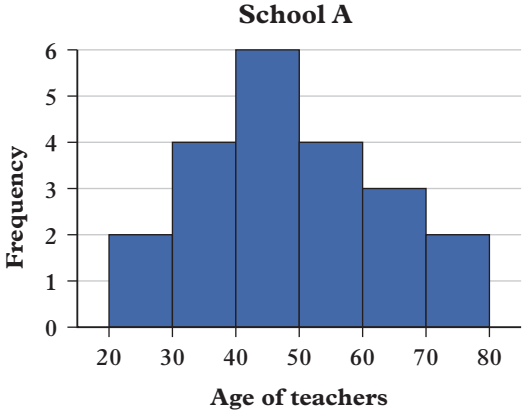
5 Consider the following two dot plots.

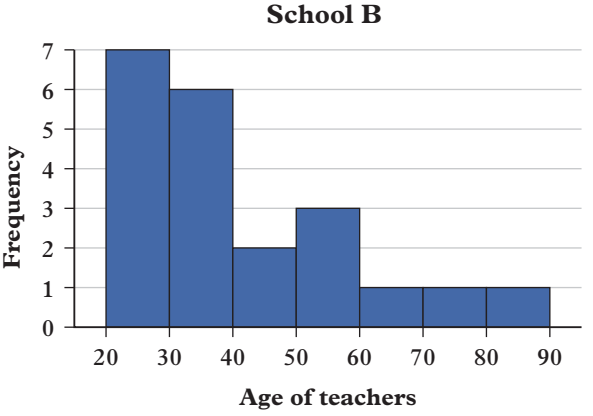


a Which group has the greater range? Justify your answer with calculations.

b Which group has the greater median? Justify your answer.

6 Consider the following two histograms that show the ages of teachers from two different schools.





a Which school has the oldest teacher?

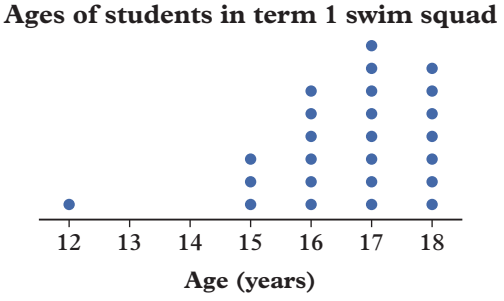
b Which school has the most teachers below the age of 40?

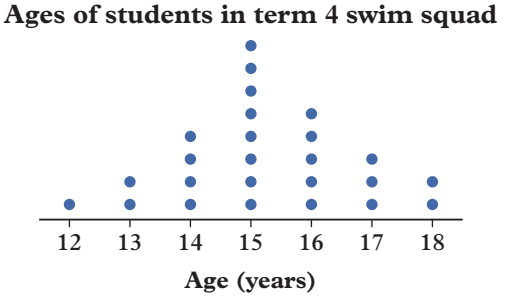
c Make a brief comparison of the two datasets.

7 **a** Compare the shape of the two dot plots below.

b Find the mean, median and range of each dot plot.

c Write a couple of sentences comparing the two datasets with reference to their summary statistics. Be sure to explain the meaning of the range and centre in terms of the context of the datasets.





8 A class of Year 7 students and a class of Year 8 students were each asked to calculate the average number of minutes they spend studying per school night. The data is shown in the stem-and-leaf plots on the right.

a Calculate the range of each group.

b Calculate the median of each group.

c Is it accurate to say that, in general, the Year 8 group studies more than the Year 7 group? Justify your answer.

Year 7

Stem	Leaf
0	0 5
1	4
2	3 5 7 7
3	0 0 2 3 8
4	1 4 4 9
5	5 8 9
6	0 5
7	1 4
8	3 6

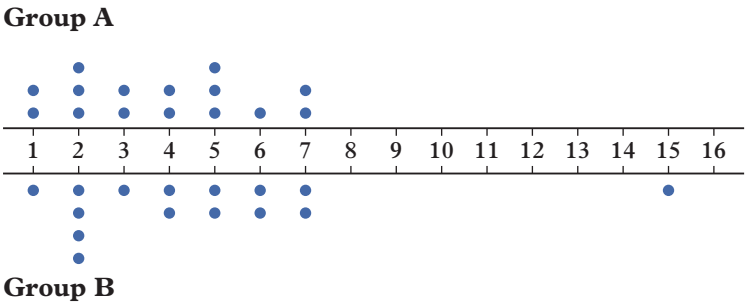
Key: 4 | 1 = 41

Year 8

Stem	Leaf
1	8 9
2	0 6
3	5 7
4	5 8 9
5	1 2 4 5
6	0 5
7	1 4 6 7 7
8	3 6
9	0 2 7

Key: 5 | 2 = 52

9 Two groups of 15 people were asked how many times they go to the hairdressers each year. Their responses are recorded in the back-to-back dot plot below. The data for Group A is above the number line and the data for Group B is below the number line.



a Find the median of each group.

b Complete the table below. Round the mean of each dataset to 1 decimal place.

	Mean	Range
A		
B		

c Explain why the summary statistics in part **b** may be misleading to a person who has not seen the back-to-back dot plot.

10 Consider the following stem-and-leaf plot and histogram.

a Explain why you cannot redraw the histogram as a stem-and-leaf plot.

b Redraw the Group A data in the stem-and-leaf plot as a histogram with class intervals of 10 starting at 0.

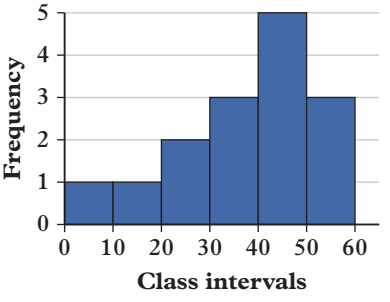
c Use the two histograms to make a brief comparison of the two datasets.

Group A

Stem	Leaf
0	3 8 9
1	2 4 6 7
2	3 7 7
3	1 5
4	6 7
5	1

Key: 3 | 5 = 35

Group B

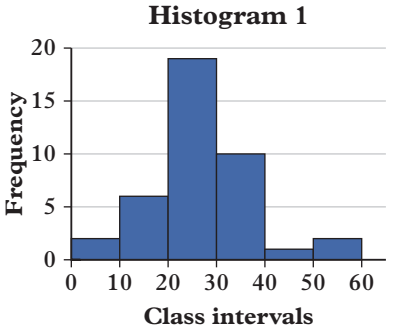


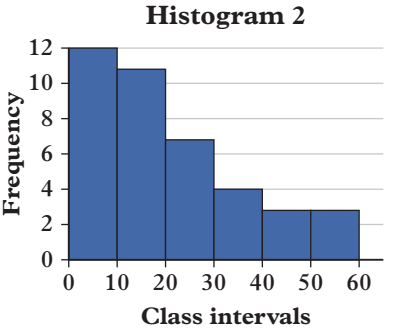
11 Consider the following summary statistics for two different datasets.

	Mean	Median	Range
A	21	17	57
B	27	25	56

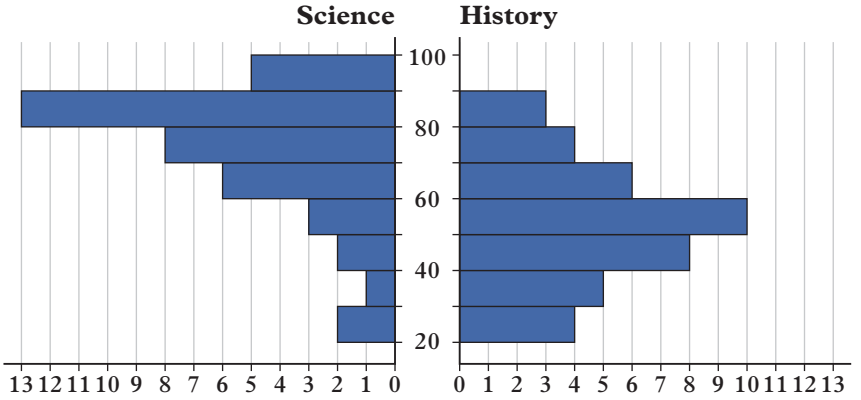
These datasets have been drawn as histograms, but they were mistakenly drawn without labels.

Which dataset matches with which histogram? Explain your answer.





12 Two sets of grouped data can be displayed in one back-to-back histogram. The following back-to-back histogram shows the exam scores (out of 100) for the same 40 students in two different exams: Science and History.

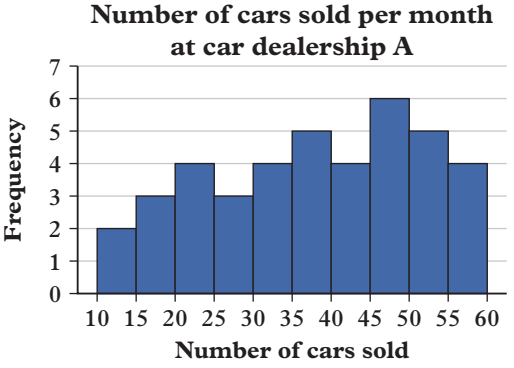


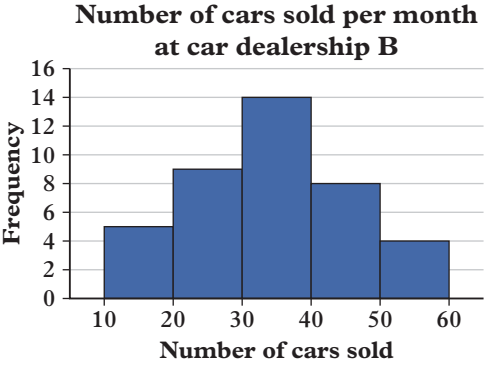
a Describe the distribution of the Science exam scores.

b Describe the distribution of the History exam scores.

c How many exam scores overall were in the interval 80–<90?

13 Consider these two histograms.





a Why is it difficult to make a quick comparison of the two histograms as presented?

b Redraw the first histogram so that its class intervals match those of the second.

c Use your answer to part **b** to make a comparison of the two datasets.

d Why can't you redraw the second histogram to match the first histogram?

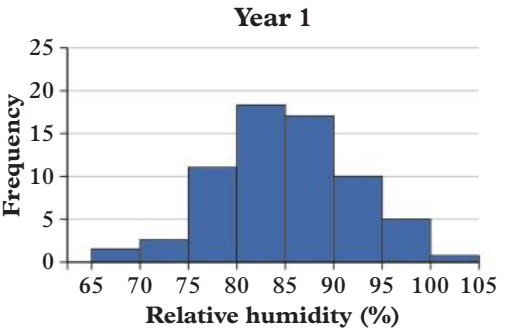
PROBLEM SOLVING AND REASONING

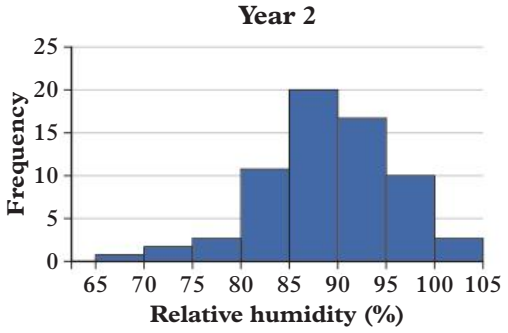
14 Relative humidity is an indicator of how much water vapor is in the air. As part of his research on climate change, Patrick recorded the relative humidity of this hometown every day for three consecutive years. The data he collected has been drawn as histograms shown on the right and below.

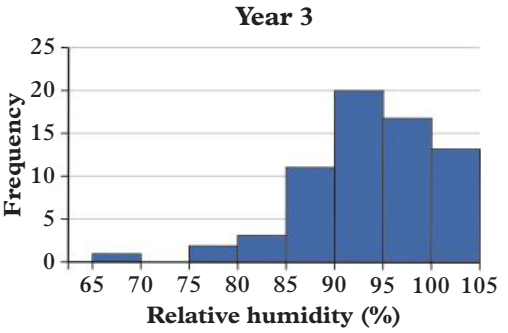
a What is the most common class interval in Year 1, Year 2, and Year 3?

b Which year has the greater median?

c What would you expect the histogram to look like in Year 4, assuming Patrick continues his research?







CHALLENGE

15 A study on the gaming habits of Year 8 students focused on three separate classes, each with exactly 25 students. Each student was asked how many hours they spent per week playing video games. The study found the following results.

	Mean	Median	Range
Class A	5.5	5	12.5
Class B	7	6	28
Class C	6.5	8.5	14.5

a One class had an outlier. Which class had the outlier and how can you tell?

b The median for class C is much higher than the mean. Does this mean the data for class C is positively skewed or negatively skewed? Explain your answer.

c In general, which class plays video games the most? Justify your answer.

16 In a school there are 250 students in Year 7 and 200 students in Year 8. The following statistics have been calculated about the students' heights (in cm).

	Mean	Median	Minimum	Maximum
Year 7	160	161	142	173
Year 8	166	164	144	179

a Find the range for the heights of each year level.

b Find the range for the heights of all students in two year levels combined.

c Find the average height of all students in the two year levels combined, rounded to 1 decimal place. Hint: The answer is not 163 cm.

d Explain why it is not possible to find the median height of all students in the two year levels combined.

Check your Student obook pro for these digital resources and more:

Interactive skillsheet
Comparing data

Investigation
Closing the Gap

Topic quiz
9E

9F Theoretical probability

Learning intentions

By the end of this topic you will be able to ...

✔ describe events using the language of probability
✔ identify complementary events
✔ calculate the probability of an event and its complement.

Inter-year links

Support	Understanding probability
Year 7	9G Theoretical probability
Year 9	8E Two-step chance experiments
Year 10	11D Theoretical probability

Theoretical probability

Key content video

- The **sample space** of an **experiment** is a list of the possible outcomes in the experiment.
  → An **event** is a collection of one or more outcomes from the sample space of an experiment.
- The **probability** of an event is how likely that event is to happen. A probability must have a value in the range of 0 (impossible) to 1 (certain).
- The probabilities of all outcomes in a sample space add up to 1.
- A **favourable outcome** is an outcome that you want.
- If all outcomes in the sample space are equally likely to occur, the **theoretical probability** of an event is calculated by:

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes in event}}{\text{total number of possible outcomes}}$$

  → If outcomes within the sample space are not equally likely to occur, you can write them as a list of outcomes that reflect their chance of occurring.
  For example, for selecting a letter at random from the word ENERGY, the list can be written as E, N, E, R, G, Y, where each outcome is equally likely to occur.

Describing events

- 'at least' means equal to or above the given value. For example, 'rolling at least 3' means 3, 4, 5, …
- 'at most' means equal to or below the given value. For example, 'rolling at most 5' means 5, 4, 3, …
- 'more than' means above the given value. For example, 'more than 3' means 4, 5, 6, …
- 'fewer than' means below the given value. For example, 'fewer than 5' means 4, 3, 2, …
- 'and' means both outcomes must be met. For example, 'selecting a red card and a king' means selecting a red king.
- 'or' means either of the given outcomes.
  → 'or' can be inclusive, meaning if both outcomes are met the outcome is favourable. For example, 'rolling a prime number or more than 4' when rolling a standard six-sided die means 2, 3, 5 or 6.
  → 'or' can also be exclusive, meaning if both outcomes are met the outcome is not favourable. For example, 'rolling a prime number or more than 4 but not both' when rolling a standard six-sided die means 2, 3 or 6 (5 is not included).

Complementary events

- The complement of event A is the event that A does not occur. For example, the complement of the event 'rolling a 1 on a standard six-sided die' is 'not rolling a 1 on a standard six-sided die.'
  → If the probability of event A is Pr(A), then the probability of the complement of event A is Pr(A').
- The probabilities of **complementary events** always add to 1. This means that Pr(A) + Pr(A') = 1.

Example 9F.1 Interpreting the language of probability

Identify the favourable outcomes in each of the following situations.

a rolling at least 5 on a standard six-sided die

b landing on a 1 or a prime number when this spinner is spun

c landing on a multiple of 2 or a multiple of 3, but not both, when this spinner is spun

d getting a multiple of 2 and a multiple of 3 when randomly picking a number from 1–12

THINK

a 'at least' means any outcome equal to or above the value is favourable.

b 'or' means any section which satisfies either condition is favourable.

c 'not both' means that any section which satisfies both conditions is not favourable.

d 'and' means that both conditions must be met for an outcome to be favourable.

WRITE

a $S = \{5, 6\}$

b $S = \{1, 2, 3, 5, 7\}$

c $S = \{2, 3, 4, 8\}$

d $S = \{6, 12\}$

Example 9F.2 Calculating theoretical probability

The letters from the word FRANCE are placed in a bag and a letter is selected at random from the bag.

a State the number of possible outcomes.

b What is the probability of selecting a C?

c What is the probability of selecting a vowel?

THINK

a Count the total number of outcomes, with each letter counting as a separate outcome.

b Count the number of favourable outcomes and substitute into the formula with the total number of outcomes.

c Count the number of favourable outcomes (A and E) and substitute into the formula with the total number of outcomes.

WRITE

a total number of outcomes = 6

b $\Pr(\text{C}) = \frac{1}{6}$

c $\Pr(\text{vowel}) = \frac{2}{6}$

$= \frac{1}{3}$

Example 9F.3 Finding the complement of probabilities

Find Pr(A') if Pr(A) is equal to:

a 0.3

b $\frac{5}{7}$.

THINK

Complementary events add to 1. Subtract the given probability from 1.

WRITE

a $1 - 0.3 = 0.7$

b $1 - \frac{5}{7} = \frac{2}{7}$

Example 9F.4 Using complementary events

A standard six-sided die is rolled and the number on the upper face is noted.

a List the sample space.

b What is the probability of 'rolling at most 2'?

c What is the complementary event to 'rolling at most 2'?

d What is the probability of this complementary event in part **c**?

THINK	WRITE
a List all the possible outcomes within curly brackets.	**a** $S = \{1, 2, 3, 4, 5, 6\}$
b The favourable outcomes are rolling a 1 and rolling a 2.	**b** $\Pr(\text{at most } 2) = \frac{2}{6}$ $= \frac{1}{3}$
c The complement to 'at most' a given number is 'more than' that number.	**c** More than 2
d Complementary events add to 1. Subtract the original probability from 1.	**d** $\Pr(\text{more than } 2) = 1 - \frac{1}{3}$ $= \frac{2}{3}$

Helpful hints

✔ When using 'at least', 'at most', 'more than' or 'fewer than', make sure you identify whether or not the given value is included in the favourable outcome.

ANS p550

Exercise 9F Theoretical probability

1, 2, 4–9, 11, 14–16 | 1, 3–7, 9–13, 16, 19, 20, 22(a, b) | 1(c–f), 5, 6, 7(c–f), 11, 13, 14, 17, 18, 20–22

UNDERSTANDING AND FLUENCY

9F.1 **1** A 10-sided die numbered from 1 to 10 is rolled. Identify the favourable outcomes in each of the following situations.

a getting at least a 5

b getting at most a 5

c getting an odd number and a prime number

d getting an odd number and not a prime number

e getting an odd number or a prime number or both

f getting an odd number or a prime number, but not both

9F.2 **2** A letter is selected at random from the word LEMON. State:

a the number of possible outcomes

b the probability of selecting an L

c the probability of selecting a vowel.

3 Calculate the theoretical probability of:

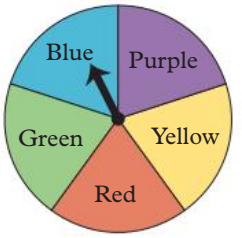
a rolling a standard six-sided die and obtaining a 6

b selecting the letter Z from the word CRAZY

c selecting a blue marble from a bag containing three red, two blue, one yellow and four green marbles

d randomly guessing the correct answer to a multiple-choice question with options A–E.

4 Consider this spinner if spun once.



a State the total number of possible outcomes.

b What is the probability of landing on blue?

c What is the probability of landing on yellow or green?

d What is the probability of landing on anything except red?

5 Calculate the theoretical probability of:

a selecting a consonant from the word CRAZY

b selecting a green or yellow marble from a bag containing four blue, three yellow, two red and six green marbles

c rolling a standard six-sided die and obtaining a number greater than 4

d selecting a C, E or S from the word SUPERCALIFRAGILISTICEXPIALIDOCIOUS

e rolling a standard six-sided die and obtaining any number except 6.

9F.3 **6** Find Pr(A') if Pr(A) is equal to:

a 0.4 **b** $\frac{3}{4}$ **c** 0.1 **d** $\frac{1}{2}$ **e** $\frac{1}{8}$ **f** 0.7 **g** $\frac{2}{3}$ **h** 0.25.

7 A 10-sided die numbered from 1 to 10 is rolled. State, in words (without writing 'not event') and as a set of values, the complementary event of each of the following.

a getting at least a 5

b getting at most a 5

c getting an odd number and a prime number

d getting an odd number and not a prime number

e getting an odd number or a prime number or both

f getting an odd number or a prime number, but not both

9F.4 **8** A standard six-sided die is rolled. State:

a the sample space

b the probability of rolling at least a 5

c the complementary event to rolling at least a 5

d the probability of the complementary event.

9 A letter is selected at random from the word EXCITEMENT.

a What is the probability of selecting a consonant?

b What is the complementary event to selecting a consonant?

c What is the probability of the complementary event?

10 In the image shown there are 5 blue, 5 red, 4 yellow and 1 pink button. A button is selected at random from the collection. Assume that every button has an equally likely chance of being selected.

a How many buttons are there?

b What is the probability of selecting a blue button?

c State the complementary event to selecting a blue button and find its probability.

d What is the probability of selecting a white button?

e What is the probability of selecting a button with four holes?

f State the complementary event to part **e** and find its probability.

11 A number of households were surveyed about the number of eggs they used in total on Sunday morning. The data collected is summarised in the table below.

Number of eggs	0	1	2	3	4	5	6	7	8
Frequency	5	2	4	3	5	2	6	3	4

Calculate the probability that a household selected at random will have used:

a at least one egg **b** at most four eggs **c** at least four eggs **d** an even number of eggs.

e Calculate the probability of the complementary event of part **a**. State how many eggs were used in these households.

f Are parts **b** and **c** complementary events? Explain why or why not. If they are not complementary, describe the event that is complementary to part **b** and to part **c**.

12 Explain why, when calculating theoretical probability, the outcomes listed in the sample space must be equally likely to occur for the number of elements to be used as the total number of possible outcomes.

13 A standard deck of cards has four suits with 13 cards each: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king.

a How many cards are there in total?

b What is the probability of selecting an ace?

c What is the probability of the complementary event to 'selecting an ace'?

d What is the probability of selecting a picture card (jack, queen or king)?

e Are you more likely to select a queen of hearts or an ace? Explain.

f What is the probability of selecting a picture card that is not a heart?

g What is the probability of the complementary event to part **f**?

h What is the probability of selecting an even number from the deck?

14 Consider the phrase YEAR EIGHT IS BETTER THAN YEAR SEVEN.

a How many letters are there in total? **b** What is the probability of selecting an E?

c What is the probability of the complementary event to part **b**?

d What is the probability of selecting an A or a B? **e** What is the complementary event to part **d**?

Now consider the letters as vowels and consonants.

f How many vowels are in the phrase? **g** State the probability of selecting a vowel.

h State the complement of this event and its probability.

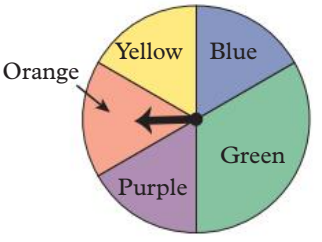
15 Consider this spinner, where the green section is twice the size of every other section. If it is spun once:

a What is the probability that you will land on red or blue?

b Explain why the answer to part **a** is not $\frac{2}{5}$, even though there are five different colours.

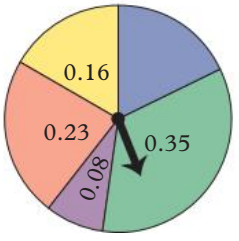
c What is the probability that you will land on yellow?

d State the complementary event to 'landing on yellow' and state its probability.

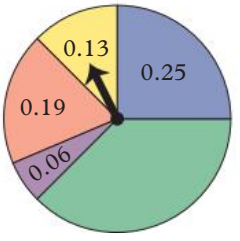


16 Use your knowledge of the sum of probabilities to find the probability of landing on the unlabelled sector in each spinner.

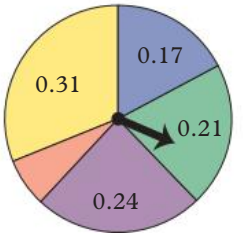
a



b



c



17 A spinner has Pr(blue) $= \frac{1}{2}$, Pr(red) $= \frac{1}{4}$, Pr(green) $= \frac{1}{8}$ and the remaining portion is yellow. State the probability of spinning yellow.

18 At a sporting match there are 200 spectators. Ninety-five people are there to support Team A, eighty-six people are there to support Team B and the remainder don't have a preference.

a Find the probability that a randomly chosen spectator doesn't have a preference who wins.

b If you are there to support Team B, what is the probability that a person who sits next to you will also support Team B?

19 Maynard flipped two coins and got two tails. He said that the complementary event of 'flipping two tails' is 'flipping two heads'. Can you explain where he may have gone wrong?

20 A packet of M&M's contains 55 individual pieces. You count out 13 blue, 7 brown, 9 green, 11 orange, 7 red and the rest yellow.

a What is the probability of selecting a yellow M&M?

b What is the probability of selecting a blue or green M&M?

c Imagine you select a yellow M&M.

i How many M&M's are left in the packet now?

ii How many of these are yellow?

iii What is the probability of selecting a yellow M&M from the packet now?

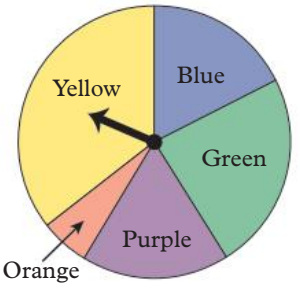
d Imagine that the second M&M you select is blue.

i How many M&M's are left in the packet now?

ii How many of these are blue?

iii What is the probability of selecting a blue or green M&M now?

21 Use a protractor and your knowledge of angles in a circle to find the probability of spinning the spinner shown below and landing in each of the sectors.



22 A bag contains a number of marbles coloured blue, yellow, green and red. You have a $\frac{1}{4}$ chance of selecting a blue marble, $\frac{1}{5}$ chance of selecting green and $\frac{3}{5}$ chance of not selecting yellow.

a What is the probability of selecting a yellow marble?

b What is the probability of selecting a red marble?

c Hence find the smallest number of marbles that could be in the bag.

PROBLEM SOLVING AND REASONING

CHALLENGE

Check your Student obook pro for these digital resources and more:

Interactive skillsheet The language of probability

Interactive skillsheet Calculating theoretical probability

Interactive skillsheet Complementary events

Investigation Scrabble probabilities

Topic quiz 9F

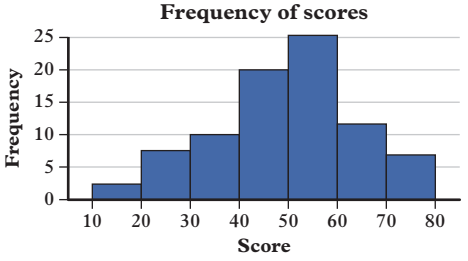
Chapter summary

Measure of centre and spread

- Mean = $\dfrac{\text{sum of all data points}}{\text{total number of data points}}$
- Median = middle value when data is listed in order
- Mode = most common value
- Range = maximum value – minimum value
- To calculate the mean and median from a frequency table, add 'score × frequency' and 'cumulative frequency' columns to the table.
- When describing a population or sample, people use words such as:
  - → 'average' usually refers to the mean, but sometimes median
  - → 'most common', 'most popular' and 'most often' refer to the mode
  - → 'middle' and 'centre' refer to the median
  - → 'spread' or 'span' refer to the range.

Grouped data and histogram

Discrete interval	Continuous interval
0–9	0–<10
10–19	10–<20



Describing distribution

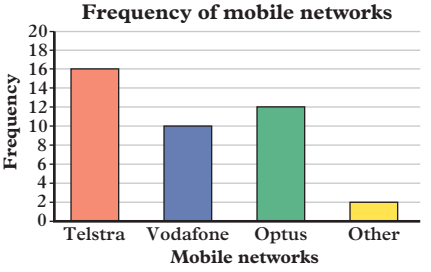
- Symmetric: use mean or median
- Skewed: use median
- Bimodal/multimodal: use mode of peaks
- Outliers: use median

Comparing datasets

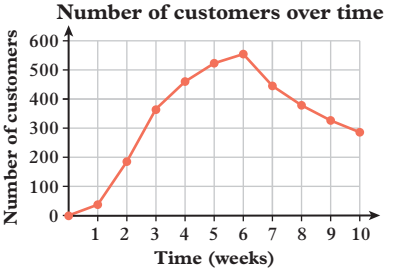
- Put datasets in same format
- Compare centre and spread

Constructing graphs

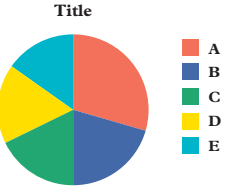
- All graphs should include:
  - → title axes that are clearly labelled (including any units)
  - → a regular scale on both axes
  - → a legend or key if different parts of the graph are in different colours.
  - → source of the data where applicable
- Bar chart or column graph: numerical variable or frequency on one axis, categorical data on the other axis.



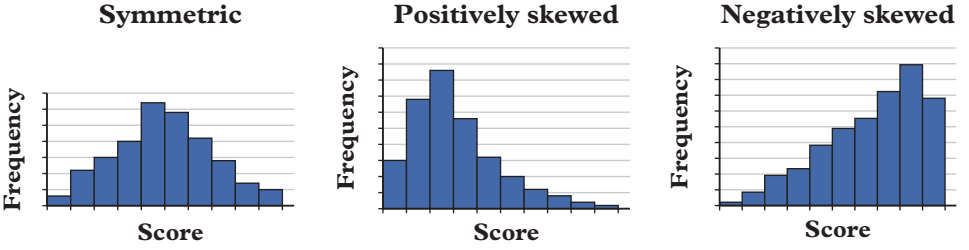
- Line graph: two numerical variables



- Sector graph: categorical data in a frequency table or as percentages



Data distributions



Theoretical probability

Assuming all outcomes are equally likely:

- the complement of an event is the probability that the event does not occur
- $\Pr(A) + \Pr(A') = 1$
- $\Pr(event) = \dfrac{\text{number of outcomes in event}}{\text{total number of possible outcomes}}$

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Chapter review

Chapter review quiz
Take the chapter review quiz to assess your knowledge of this chapter.

Quizlet
Test your knowledge of this topic by working individually or in teams.

Mathematical literacy review

The following key terms are used in this chapter:

- average
- bar chart
- bias
- bimodal
- chance
- class interval
- column graph
- complementary event
- continuous data
- cumulative frequency
- data
- dataset
- discrete data
- dot plot
- equally likely outcome
- event
- experiment
- favourable outcome
- frequency
- frequency table
- grouped data
- histogram
- line graph
- lower bound
- mean
- measure of spread
- measures of centre
- median
- mode
- multimodal
- negatived skewed
- numerical data
- outlier
- pie graph
- positively skewed
- probability
- range
- sample space
- sector graph
- skewed
- skewness
- stem-and-leaf plot
- symmetric
- ungrouped data
- uniform
- unimodal
- upper bound

1 Briefly explain the different between a column graph and a histogram.

2 Fill in the blanks using terms from the key word list above.

a Grouped data is _____ that has been sorted into grouped or _____. Grouped data can be displayed in _____.

b A distribution is _____ if it has a peak in the middle and the data points are roughly evenly spread on either side. A distribution is _____ if the data points are clustered on one side of the distribution.

c An _____ is a data point that is significantly higher or lower than the other values in a dataset.

3 Identify the summary statistics being referenced in each of these words.

a most often, most popular **b** middle value **c** spread, span

4 Use a standard six-sided die as an example to provide a definition in your own words for the following key terms.

a event **b** favourable outcome **c** complementary event

Multiple choice

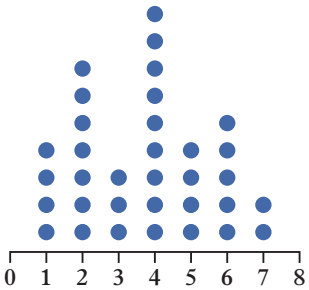
9A **1** What is the range of the data below?

1.4, 2.3, 2.8, 3.8, 3.9, 1.7, 4.1, 11.6, 1.8, 11.2, 10.4, 10.1, 8.9, 7.6, 6.0, 7.6, 6.5

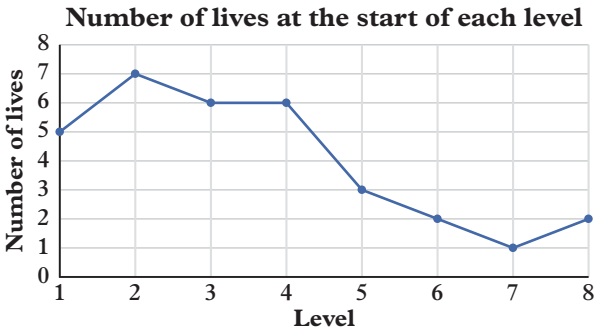
A 60 **B** 7.5 **C** 10.2 **D** 6 **E** 102

9A **2** What is the most common number in this dot plot?

A 2 **B** 3 **C** 4 **D** 5 **E** 6



9B **3** The line graph below tracks the number of lives a gamer has when they begin each level of a video game. The gamer gains two lives during each level. During which level did they lose the most lives?

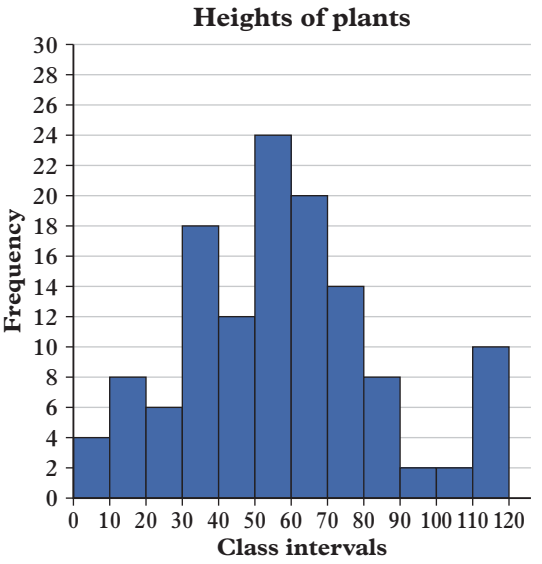


A 1 **B** 2 **C** 3 **D** 4 **E** 5

9B **4** Students who prefer the colour blue make up 25% of the class. In a sector graph showing favourite colours, what would be the angle for the blue sector?

A 25° **B** 90° **C** 180° **D** 50° **E** 100°

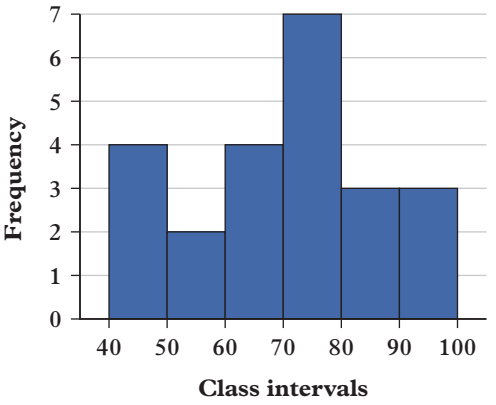
9C **5** The histogram below displays the results of research where the heights of plants (in cm) were measured. How many plants are less than 70 cm tall?



A 14 **B** 20 **C** 36 **D** 44 **E** 92

9C **6** In the histogram below, which class interval contains the median?

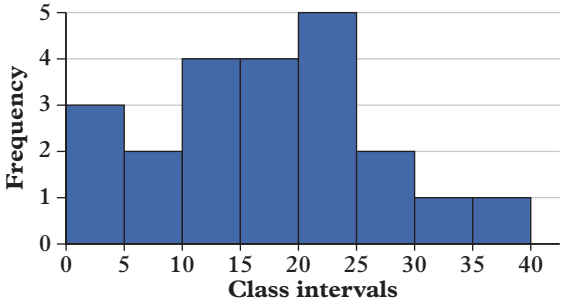
A 40–<50 **B** 50–<60 **C** 60–<70 **D** 70–<80 **E** 80–<90



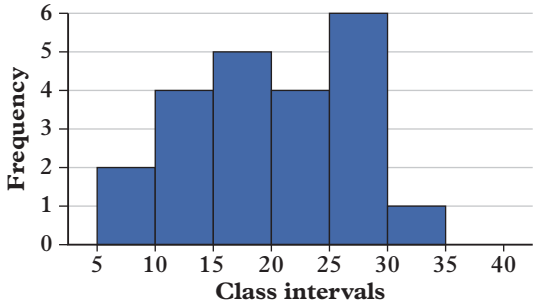
9C **7** The table of data below is represented in which histogram?

Class	0–<5	5–<10	10–<15	15–<20	20–<25	25–<30	30–<35	35–<40
Frequency	3	2	4	4	5	2	1	1

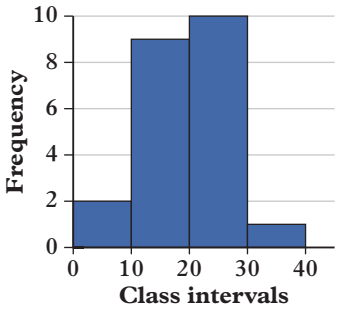
A



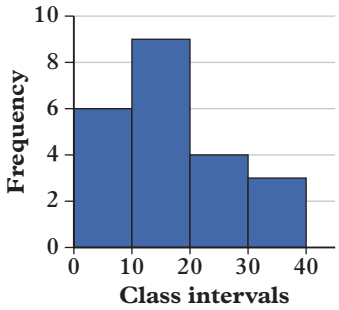
B



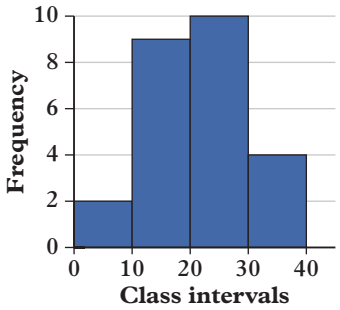
C



D



E



9E **8** Which description of the distribution of datasets is true?

A Symmetric distributions have a centre closer to the right of the distribution, away from the *y*-axis.

B Any distribution that is skewed or has an outlier should have its centre described using the median rather than the mean.

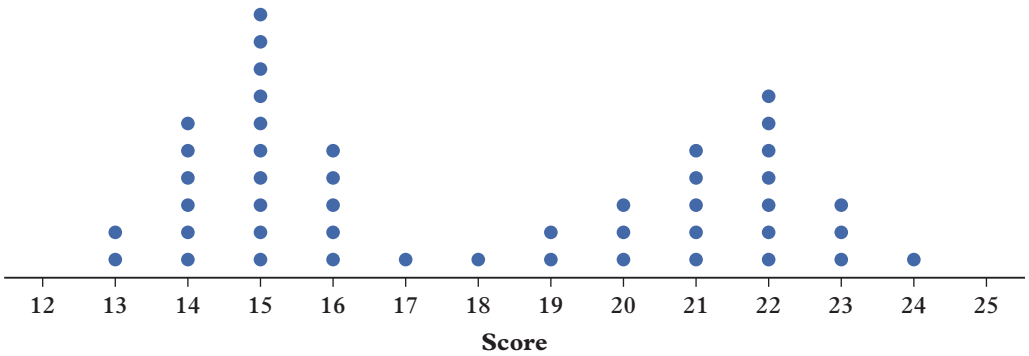
C Positively skewed distributions have a middle peak and a roughly even spread on either side of this peak.

D Negatively skewed distributions have a centre closer to the left of the distribution, towards the vertical axis.

E The peaks in a bimodal distribution must be exactly the same height.

9D **9** Which of the following is an appropriate measure of centre for the dot plot shown?

A Mean **B** Range **C** Median **D** Mode(s) **E** No centre



9E **10** How is each dataset distributed in the stem-and-leaf plots on the right?

A Both groups are symmetric.

B Group A is negatively skewed and Group B is positively skewed.

C Group A is positively skewed and Group B is negatively skewed.

D Both groups are negatively skewed.

E Both groups are positively skewed.

Group A

Stem	Leaf
0	2 3 4 4 6 6 8 9
1	1 3 4 5 7 8
2	2 7 7
3	5
4	4
5	1

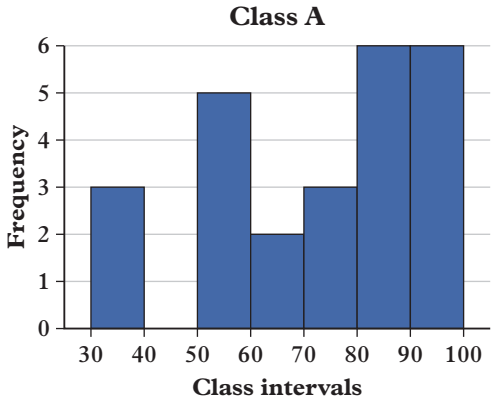
Key: 1 | 4 = 14

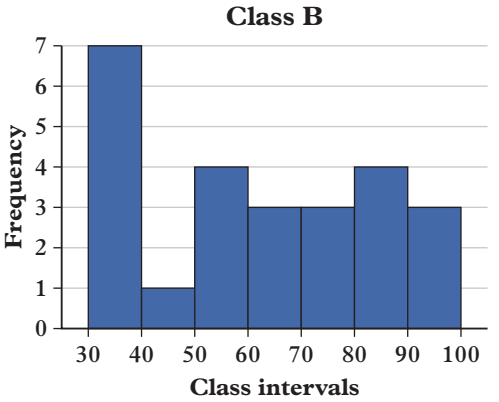
Group B

Stem	Leaf
0	9
1	1
2	3 4
3	2 4 7 9
4	2 3 3 4 6 8 9 9
5	1 5 6 6 8

Key: 1 | 4 = 14

9E **11** The two histograms below show the Geography exam scores of two different Year 8 classes. Which of the following statements is definitely true?





A The range of each class is at least 60.

B There are more students in Class A than in Class B.

C The range of Class A is greater than the range of Class B.

D The median of Class A is greater than the median of Class B.

E Class B performed better on the exam than Class A.

9F **12** What is the complementary probability to rolling a 1 or 2 on a standard die?

A $\frac{1}{6}$ **B** $\frac{2}{6}$ **C** $\frac{3}{6}$ **D** $\frac{4}{6}$ **E** $\frac{5}{6}$

Short answer

9A **1** Find the mean, median, mode and range for this dataset, correct to two decimal places where appropriate.

Score	Frequency
3	10
4	12
5	23
6	5

9A **2** Calculate the mean, mode, median and range for the data represented in this stem-and-leaf plot, correct to two decimal places where appropriate.

Stem	Leaf
0	2 4 5 7 7 7 7
1	5 8 8 8 9
2	1 1 3 4 7 9
3	7 8 8
4	1 2 5
5	8

Key 2|4 = 24

9B **3** Marina has started a new job selling cars at a car dealership. The number of cars she sold during each of her first 9 months is shown in the table below.

Month	1	2	3	4	5	6	7	8	9
Number of cars sold	2	4	8	14	12	15	19	21	22

a Draw a line graph to represent this data.

b In which month(s) did Marina sell fewer cars than in the previous month?

c At the car dealership, Marina is paid a fixed salary of \$4000 per month plus an extra \$200 for each car she sells. What was her total income from this job during month 6?

9C **4** Data was collected on the number of hours Year 8 students spent using various forms of social media over the course of a week.

45 21 37 21 20 17 31 32 11 15 17 18 20 31 48 32 21 5 11 7 19 18 27 42

40 21 32 23 24 19 38 37 14 15 19 28 22 35 41 33 27 2 10 5 18 18 25 43

a Construct a frequency table with class intervals of width 10.

b How many people were surveyed?

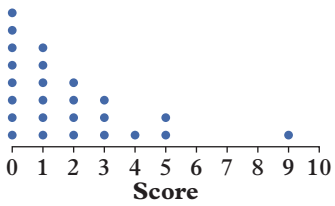
c What is the most common class interval?

d Draw a histogram to represent this data.

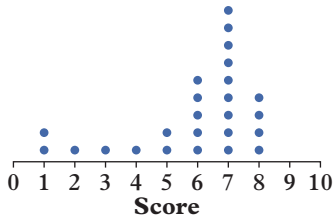
e Which interval is the most common?

9D **5** Consider the following three dot plots.

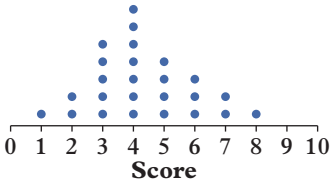
Dot plot 1



Dot plot 2



Dot plot 3



a Which dot plot has the greatest range?

b Which dot plot is positively skewed?

c Which dot plot has a roughly symmetric distribution?

d Is the mean or median the most appropriate measure of centre for dot plot 2? Explain your reasoning.

e One of the dot plots represents the number of pets owned by each student in a Year 8 class. Which dot plot do you think represents this data? Explain your reasoning.

9D **6** The ages of 53 people are shown in the stem-and-leaf plot below.

Stem	Leaf
0	4 4 5 5 7 8
1	0 1 1 1 3 4 4 5 6 6 6 6 7 9 9 9
2	0 0 1 3 4 4 4 4 5 6 6 6 7 7 7 8 9 9
3	0 0 0 0 1 1 1 2 2 3 3 3 3

Key: 1 | 3 = 13

a Find the median age of the group.

b Find the range of the dataset.

c Draw a histogram to represent this data. Use class intervals of 5 with the first interval starting at 0.

d Describe the distribution of ages in the dataset.

9E **7** The results achieved in Maths tests by two different classes are recorded in two stem-and-leaf plots.

Class 8A

Stem	Leaf
4	3 5 5 8
5	3 7 9
6	3 4 5 6 7
7	0
8	1 3 5 7 9
9	0

Key: 4 | 6 = 46

Class 8B

Stem	Leaf
4	6 7 8 8 9
5	1 3 6 7
6	3 4 7
7	2 4 8
8	2 5 6 7
9	8 9

Key: 4 | 6 = 46

a Find the mean, median, mode and range for each class.

b Compare the Maths results of the two classes.

9E **8** The following data comparing the number of hours spent at football training per fortnight were recorded for Year 9 boys from two different secondary schools.

School 1: 35 11 27 11 10 7 21 22 1 5 7 8 10 21 38 22 11 5 1 7 9 8 17 32

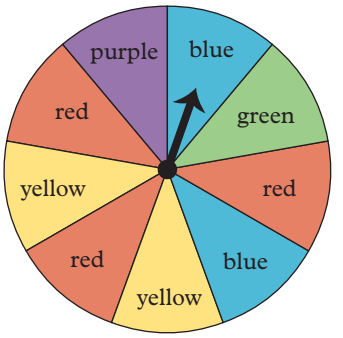
School 2: 12 14 16 17 20 11 22 11 11 16 19 14 2 4 10 19 17 6 4 6 2 7 16 31

a Collate the data in two histograms using class intervals of width 5.

b Calculate the mean, median and range for each school, correct to one decimal place.

c Which measure of centre would you use to discuss the results for each school?

9F **9** Consider this spinner.



a What is the theoretical probability of spinning blue?

b What is the theoretical probability of spinning green or red?

c Find the theoretical probability of the complementary event to part **b**.

9F **10** Calculate the probability of:

a rolling a standard six-sided die and getting a 3

b flipping a coin twice and getting tails both times

c selecting a consonant from the word REVISION.

9F **11** Consider the following heights of 10 students in a class:

159 cm, 181 cm, 159 cm, 161 cm, 168 cm, 147 cm, 166 cm, 167 cm, 158 cm, 155 cm.

Find the probability that a randomly selected student from the class:

a is at least 166 cm tall

b is at most 166 cm tall

c is less than the mean height.

Analysis

1 The following stem-and-leaf plots show the measurements of shrubs taken at different nurseries.

Sydney

Stem	Leaf
11	4 4 4 5 7 8 9
12	1 3 6 7
13	2 3 9
14	1 4 6 7 8 9
15	1 2 5 5

Key: 11 | 4 = 11.4 cm

Brisbane

Stem	Leaf
11	4 4 4 7
12	2 2 3 4 6 7
13	1 2 6 7 8
14	0 1
15	2 3 5 8 9

Key: 11 | 4 = 11.4 cm

a Compare the number of shrubs measured at each location.

b Calculate the mean, median, mode and range of shrub heights at each location, correct to two decimal places where necessary.

c Write a short comparison of the height of shrubs at the two locations.

The manager of the nursery company wanted to collate the data for all of his businesses.

d Generate lists of raw data for Brisbane and Sydney. Using this data and the lists below, create a frequency table using suitable class intervals and collate all of the data about shrub height.

Victoria: 12.1, 11.7, 18.3, 11.4, 11.4, 14.5, 15.6, 17.1, 16.5, 18.6, 13.0, 12.6, 12.4, 10.9, 11.4, 14.0, 16.9, 17.1, 16.5, 18.6

Western Australia: 10.2, 10.6, 19.3, 11.4, 11.4, 15.9, 14.7, 18.3, 17.7, 13.4, 10.0, 12.5

e How many plants were measured in total?

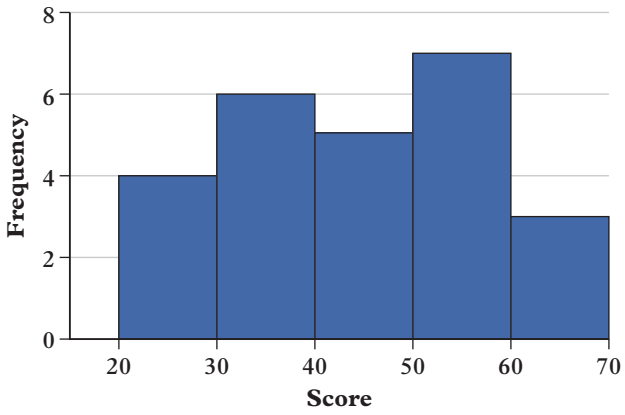
f Find the probability that a randomly selected value in the whole dataset is at least 14.1 cm.

g Represent this data as a histogram and comment on the shape of the distribution.

h Calculate the mean shrub height for Victoria and Western Australia correct to two decimal places.

i Write a statement that could be used in a marketing campaign, which includes information about the smallest and largest shrubs and the average shrub height.

2 A researcher has collected discrete data and arranged it into the following histogram. The data is discrete, so the first interval is 20–29, the second interval is 30–39, etc. The raw data (i.e. the list of numbers that make up the dataset) is not available.



a What is the maximum possible value of the range? Explain your reasoning.

b What is the minimum possible value of the range? Explain your reasoning.

c Explain why it is not possible to find the median.

d In which interval is the median contained?

e Explain why it is not possible to calculate the mean or even to solve for the interval in which it is contained.

f It is possible to find the minimum and maximum possible values of the mean, find the minimum possible value of the mean by assuming every data point occurs at the lowest possible value of its interval.

g Find the maximum possible value of the mean by assuming every data point occurs at the highest possible value of its interval.

h Find the average value of the minimum possible mean and the maximum possible mean. Is this a good way to estimate the mean for data presented in a histogram? Explain your answer.

Chapter checklist

Now that you have completed this chapter, reflect on your ability to do the following.

I can do this	I need to review this
☐ Calculate summary statistics from frequency tables ☐ Calculate summary statistics from displays ☐ Use summary statistics to describe key features of data	☐ Go back to Topic 9A Summary statistics
☐ Construct column graphs, bar charts, line graphs and sector graphs	☐ Go back to Topic 9B Constructing graphs
☐ Group numerical data into class intervals ☐ Interpret data in a histogram ☐ Represent grouped numerical data in a histogram	☐ Go back to Topic 9C Grouped data and histograms
☐ Determine whether a dataset is uniform, unimodal, bimodal or multimodal ☐ Identify whether a dataset is symmetric or skewed ☐ Determine which measure of centre best describes a dataset	☐ Go back to Topic 9D Describing data
☐ Compare datasets in different graphical displays ☐ Compare datasets using summary statistics	☐ Go back to Topic 9E Comparing data
☐ Describe events using the language of probability ☐ Identify complementary events ☐ Calculate the probability of an event and its complement	☐ Go back to Topic 9F Theoretical probability

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Semester 2 review

Short answer

1 Check if the bracketed values are the solution to the following equations.

a $3x - 5 = 19$ $(x = 8)$

b $3 - 8x = -1$ $\left(y = \frac{1}{2}\right)$

c $-7x + 9 = 37$ $(x = -3)$

d $3w + 8 = 5w - 2$ $(w = 5)$

2 Solve the following equations for x.

a $5x - 3 = 47$

b $8(x + 3) = 48$

c $8x + 8 = 7x - 3$

d $4x + 7 = 9x - 2$

e $x^2 - 25 = 0$

f $4x^2 = 121$

3 Rearrange each equation to make x the subject on the left-hand side.

a $y = ax + b$

b $y = \frac{x + c}{d}$

c $cx + a = -m(x - b)$

d $x(y - 2) = n(x + 1)$

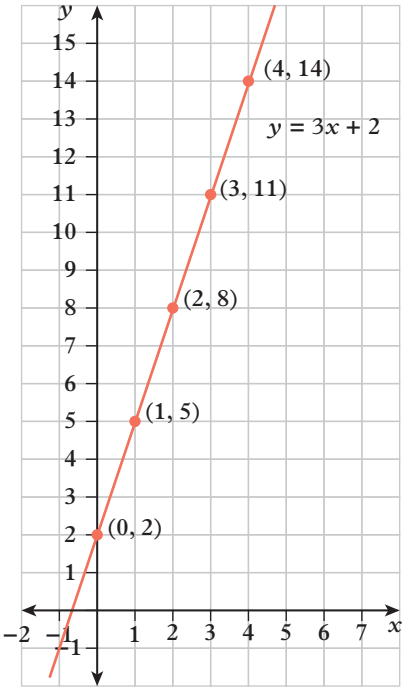
4 Solve the equations using the graphs below.

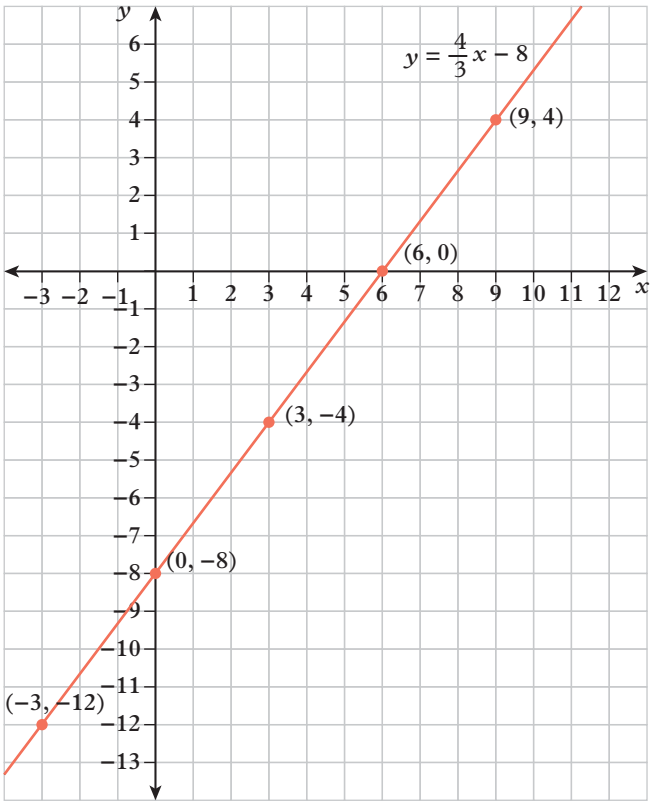
a i $3x + 2 = 11$

ii $3x + 2 = 2$

b i $\frac{4}{3}x - 8 = -12$

ii $\frac{4}{3}x - 8 = 0$





5 a i Create a table of values for $y = 2x + 3$ for integer values of x from 0 to 3.

ii Plot the graph on the Cartesian plane.

b i Create a table of values for $y = 3 - x$ for integer values of x from 0 to 3.

ii Plot the graph on the Cartesian plane.

6 Determine the equation of each of the following linear relationships.

a

x	0	1	2	3	4
y	7	5	3	1	−1

b

x	−2	−1	0	1	2
y	−5	−1	3	7	11

c

x	0	3	6	9	12
y	−2	−5	−8	−11	−14

d

x	−2	0	2	4	6
y	5	6	7	8	9

7 For each pair of lines, determine whether the coordinate provided is a point of intersection of their graphs.

a $y = 3x + 4$ and $y = 2x + 8$ at (4, 16)

b $y = 4 - 2x$ and $y = 5x - 13$ at (3, −2)

8 State if the following angles are complementary, supplementary or equal.

a vertically opposite angles

b the non-right angles in a right-angled triangle

c corresponding angles on parallel lines cut by a transversal

d co-interior angles on parallel lines cut by a transversal

e the exterior angle and the interior angle at the same vertex

f the included angles between unequal sides in a kite

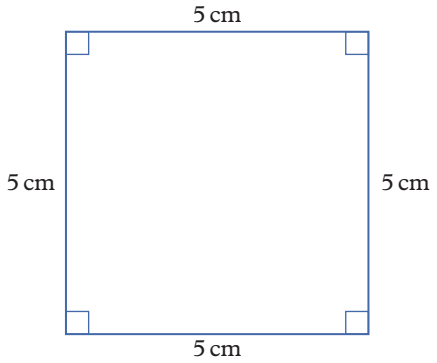
g adjacent angles in parallelograms

9 For each of the following quadrilaterals:

i determine the type of quadrilateral, giving the most specific name

ii determine the area.

a



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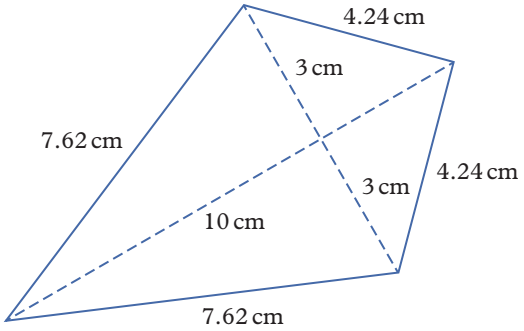

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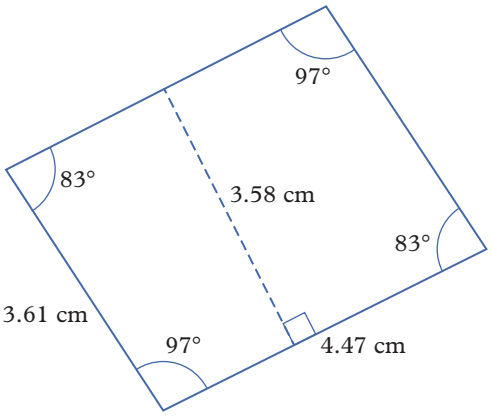

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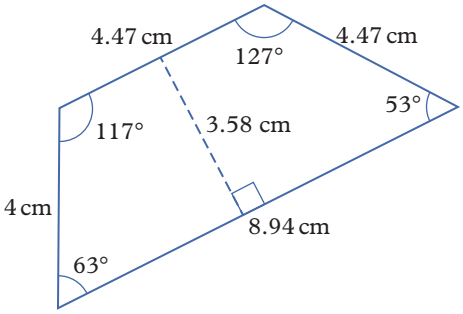

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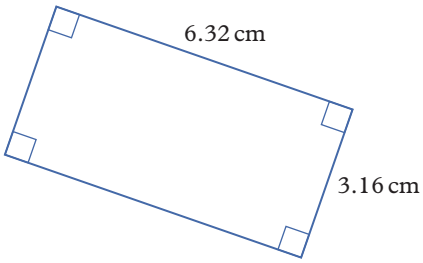

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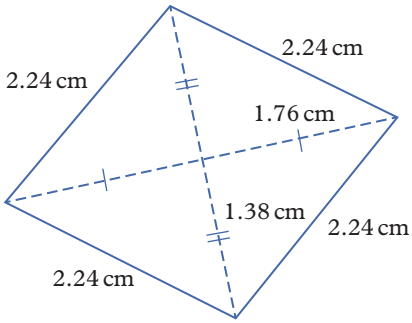
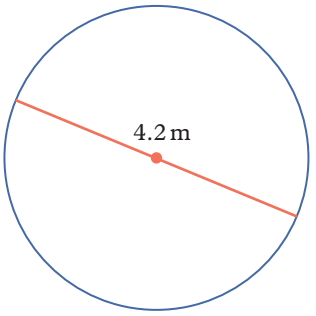


10 Determine the area and perimeter of the following shapes, correct to two decimal places.

a



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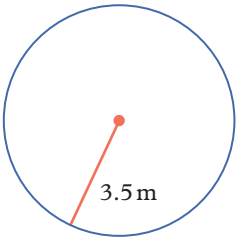

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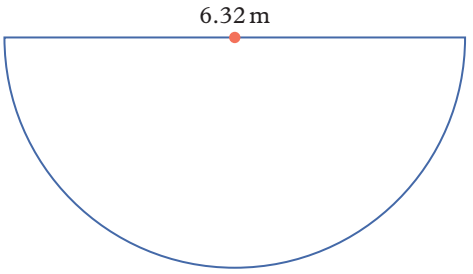

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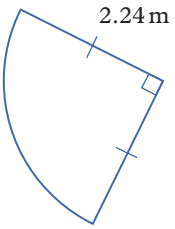

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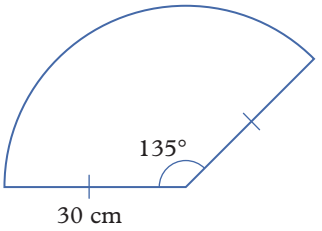

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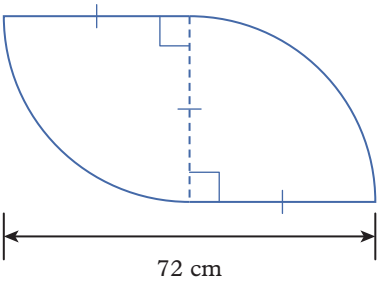


11 For each of the following, write and then solve an equation using the given information.

a 5 more than 3 times a number is equal to 29.

b The perimeter of a rectangle with a length of 3.4 m is 10.1 m. Find the breadth of the rectangle.

c The circumference of a circle is 30 cm. Find the length of the diameter of the circle, correct to two decimal places.

d The area of a triangle with a side length of 1.5 cm is 2.25 cm^2. Find the perpendicular height to the 1.5 cm side length.

e The area of a parallelogram with a base length of 7.2 cm is 24.48 cm^2. Find the perpendicular height to the base.

f The area of a circle is 30 cm^2. Find the length of the radius of the circle, correct to two decimal places.

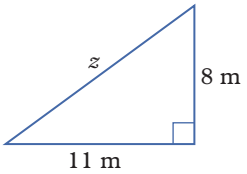
g Two of the interior angles of a triangle are 34° and 57°. Find the size of the third interior angle.

h The mean of 5, 8, 13, 2 and one other number is 6.2. Find the last number.

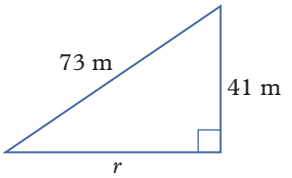
i The probability of three out of four outcomes from a sample space are 0.3, 0.12, 0.44. Find the probability of the fourth outcome.

12 Determine the length of the unknown side lengths, correct to two decimal places.

a



b



13 Determine whether these side lengths can form right-angled triangles.

a 7.7 cm, 41.7 cm, 42.7 cm

b 29 mm, 128 mm, 131 mm

c 20 cm, 48 cm, 52 cm

d 4 m, 4.2 m, 5.8 m

14 Find the probability of the following.

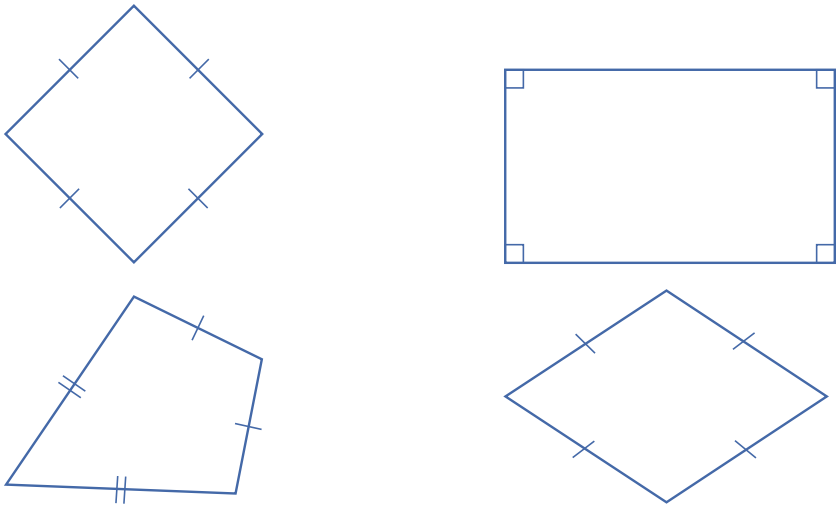
a selecting a number with a recurring decimal from the following list

$\frac{2}{3}, \frac{5}{7}, \frac{3}{10}, \frac{3}{6}, \frac{7}{25}, \frac{5}{12}, \frac{11}{20}, \frac{18}{120}, \pi$

b selecting an expression from the following list

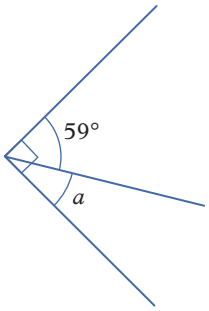
$2x + 4y, -\frac{4}{3}, 8x + 1 = 15, y = 4 - 3x, 3 \times 4 + 5 = 17, 0.2z$

c selecting a quadrilateral from the shapes below that has perpendicular diagonals



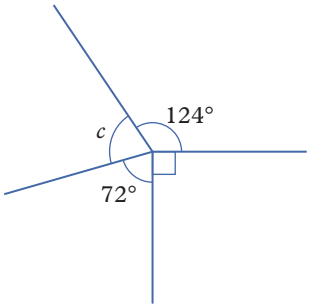
15 Find the value of the pronumerals.

a



b

c



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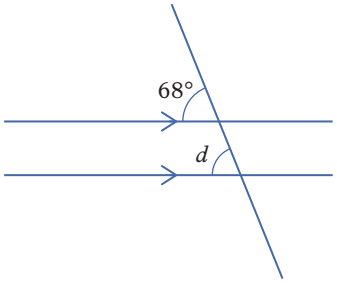

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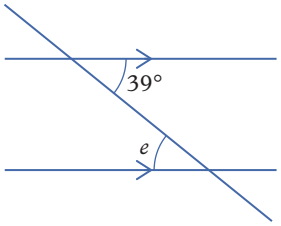

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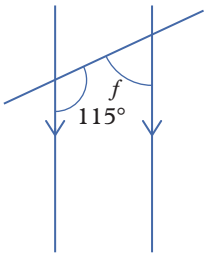


16 Convert the following into the unit in brackets.

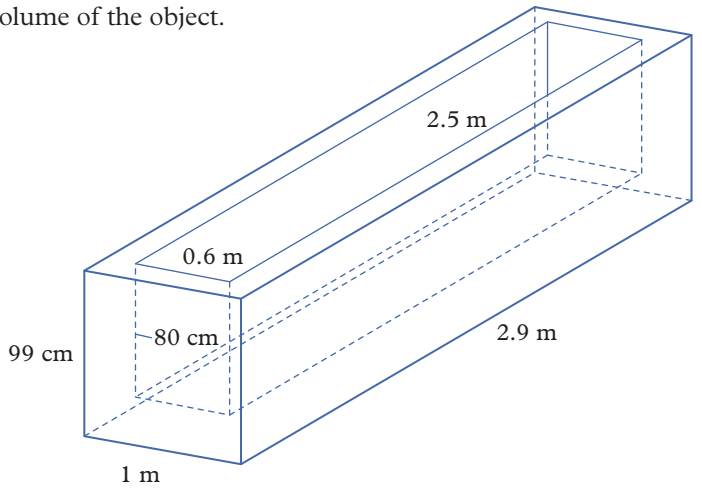
a 24 mm (m)
b 5 m^3 (cm^3)
c 8001 km^2 (m^2)
d 4240 mm^3 (cm^3)
e 0.2 cm^2 (m^2)
f 30 km (cm)
g 4320 seconds (hours)
h 2.5 hours (minutes)
i 1823 (12-hour time)
j 12:30 am (24-hour time)
k 1240 mL (L)
l 35L (mL)

17 A rectangular prism has side lengths 1 m, 2.9 m and 99 cm.

a Calculate the volume of the rectangular prism.

b A rectangular prism with side lengths 0.6 m, 2.5 m and 80 cm is carved out of the original rectangular prism. Calculate the capacity of the carved out prism.

c Calculate the new volume of the object.



18 Convert the following times into the time zone in brackets.

a 2:03 pm AEST (ACST)
b 5:41 am AWST (AEST)
c 9:00 pm UTC+10 (UTC+1)
d 8:31 am UTC−2 (UTC+5)

19 Determine the length of time elapsed when these times are in the same time zone.

a 0847 to 1423
b 9:45 pm to 1:10 am

20 Determine the length of time elapsed for these pairs of times.

a 6:38 pm AEST to 6:12 pm AWST
b 0555 UTC+1 to 2320 UTC+10

21 Calculate the mean of the following dataset.

Score	Frequency
3	2
5	4
7	6
9	8

22 A number of households were asked how many bathrooms they have. The data collected is shown below.

3 2 1 1 2 1 8 2 2 1.5 2 3 2.5 1 1.5 2

a Calculate the mean (to two decimal places), median, mode and range.

b Recalculate the mean, median, mode and range excluding the outlier. Which summary statistics change and which stay the same?

23 Write a sentence to describe the following statistics.

a survey of shoppers about how long they spent at the shops, mean = 1 h 20 m

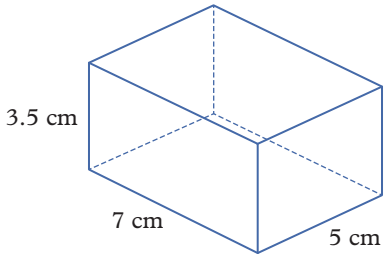
b survey about the number of board games people owned, median = 4

c survey about people's favourite colour, mode = blue

d sample of the length of a number of chess matches, range = 34 turns

24 Calculate the volume of the objects, correct to one decimal place where required.

a



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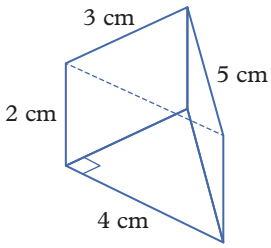

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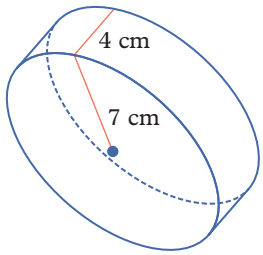

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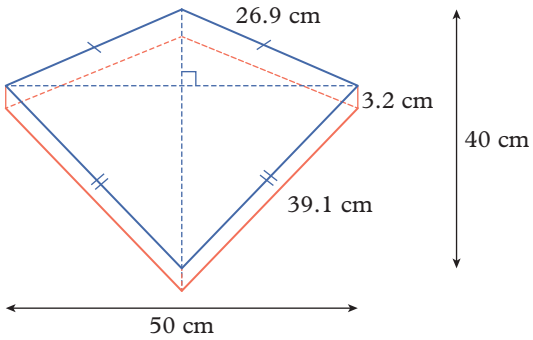


25 Construct the specified graph for each of the following datasets.

a Sector graph

Favourite	Frequency
Board games	10
Card games	45
Video games	50
Physical games	25

b Line graph

Day	1	2	3	4	5	6	7	8	9
Number	12	20	16	18	24	22	24	24	26

c Bar chart

Value	Frequency
8	10
9	26
10	34
11	38
12	30

d Histogram

Class	Frequency
0–<10	30
10–<20	60
20–<30	10
30–<40	90
40–<50	30

26 Determine each of the following values for each set of data. Round to two decimal places if required.

i mean **ii** median **iii** mode **iv** range

a 21, 19, 19, 19, 17, 16, 18, 26, 24, 19

b

Score (x)	10	11	12	13	14
Frequency (f)	6	12	18	20	14

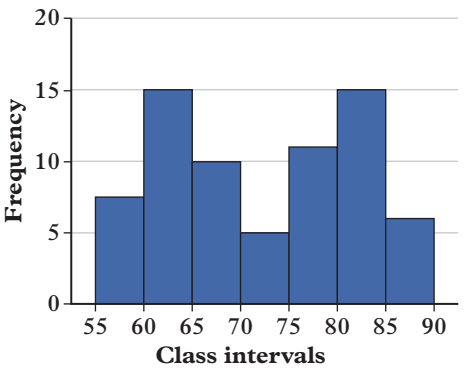
c

Stem	Leaf
2	0 3 4 6 7
3	1 2 3 3 4 4 4 5 7
4	1 2 5
5	1 3 3 6 8
6	0 1 1 2 4 6 7

Key 2 | 1 = 2.1

27 Describe the shape of the distribution in each of the following graphs.

a



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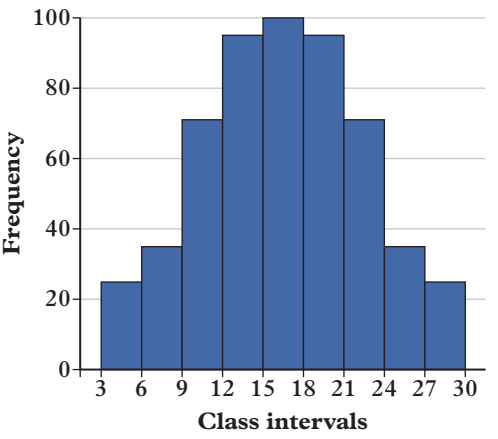

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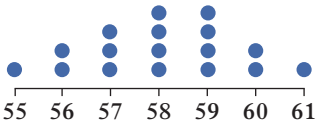

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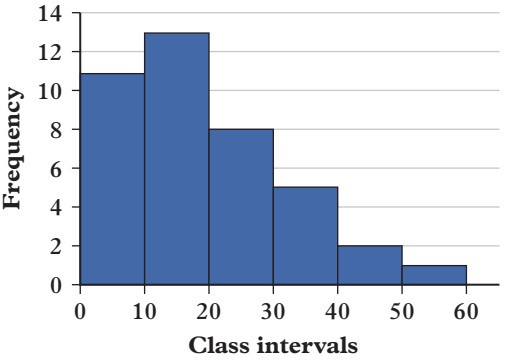

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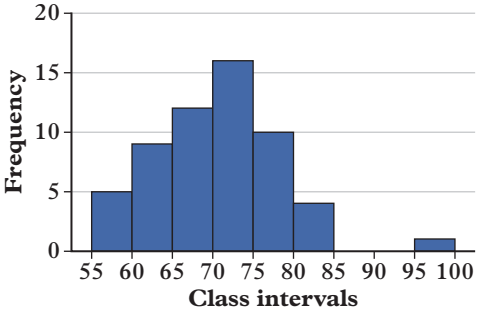

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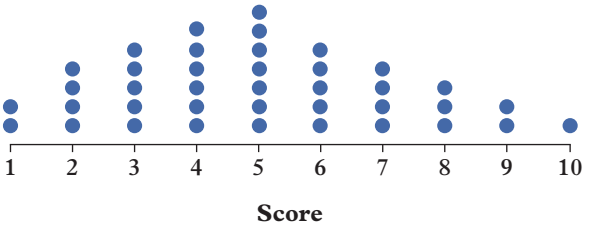
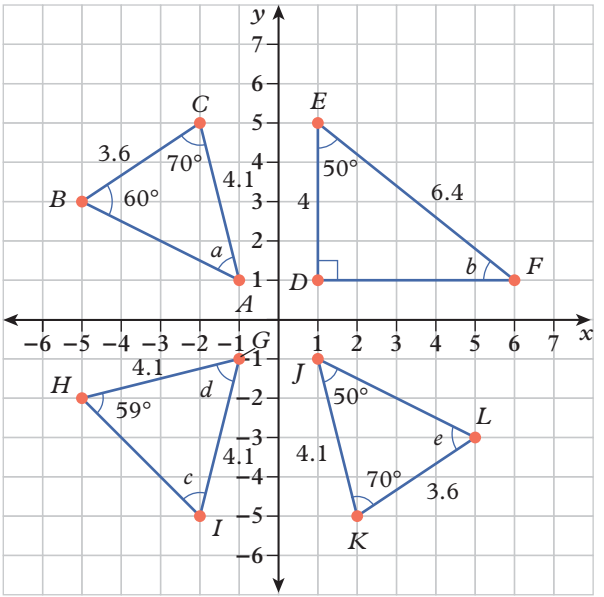


28 State which measures of centre are most appropriate to use to describe each of the distributions in question **27**.

Analysis

1 Consider the triangles to the right.

- **a** Determine the value of all pronumerals.
- **b** Determine the mean, median, mode and range of the interior angles of the triangles.
- **c** What type of data are the angles in the triangles?
- **d** If one interior angle is selected at random from any of the triangles, what is the probability that it is acute?
- **e** Which triangle has the largest area?



2 Kate works at a toy store which specialises in crocheted children's toys. She earns \$20 per hour. Each of her shifts lasts eight hours and she works five shifts a week.

a What is the base rate that Kate earns per week?

Kate earns a bonus of \$15 if she opens the store and a bonus of \$20 if she is responsible for closing the store. She opens the store twice a week and closes the store three times a week.

b How much does Kate earn per week including bonuses?

The regional manager offers Kate a promotion where she can choose between earning an extra 15% per hour or increasing the store opening and closing bonuses by \$5 each. The number of times she opens and closes the store would stay the same. Kate wants to use her knowledge of linear relationships to compare the amount she would earn in one week with each option.

c Write the equation of the linear relationship between Kate's pay, P_1, and hours worked, h, if she chooses the first option.

d Write the equation of the linear relationship between Kate's pay, P_2, and hours worked, h, if she chooses the second option.

e Sketch both linear relationships on the same set of axes for values of h between 0 and 40.

f Based on your graphs, which option should Kate choose?

Kate is offered a job by a rival toy company where she would earn \$30 per hour but there would be no bonuses for opening and closing the store.

g Write the equation representing the linear relationship between Kate's potential pay, P_3, and hours worked, h.

h Graph the linear relationship you found in part **g** on the same set of axes as the other two linear relationships for values of h between 0 and 40.

i Assume Kate will continue to work the same number of hours each week. Will she earn more per week if she accepts the new job?

3 Arielle and Brenna have started to track their sleeping habits. They recorded how many minutes each slept in February, correct to the nearest 10 minutes.

Arielle: 550, 530, 480, 480, 470, 410, 480,
510, 520, 490, 430, 480, 480, 370,
460, 440, 490, 480, 500, 520, 470,
470, 410, 480, 490, 500, 430, 470

Brenna: 570, 420, 560, 460, 530, 480, 510,
410, 450, 510, 430, 540, 600, 570,
600, 440, 530, 460, 630, 370, 580,
500, 500, 550, 510, 400, 510, 460

a Construct two histograms using the data above and intervals of 50.

b Compare the shape of the two datasets.

c Determine the median and range for Arielle's and Brenna's minutes of sleeping.

d What is the probability of selecting a value from the whole dataset above that is:

i at least 440

ii no more than 510

iii less than 410?

e What are the complementary events to each of the events in part **d**?

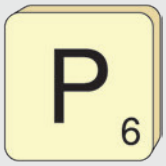
f Calculate the probability of the events in part **e**.

ANS p555

EXPLORATIONS 2

1 Foodle

Foodle is a word game in which players place letter tiles in a row to make the name of a food. A player's score is the sum of the numbers on the tiles that make up the word. Each letter of the alphabet scores a different number of points from 1 to 26. The letter P scores 6 points.



a The word PEA scores 10 points, PEAR scores 18 points, PEPPER scores 28 points and PAELLA scores 23 points. How many points is the letter L worth?

b The word PEACH scores 19 points and CELERY scores 27 points. How many points does CHERRY score?

c The word CARROT scores 44 points. How many points does POTATO score?

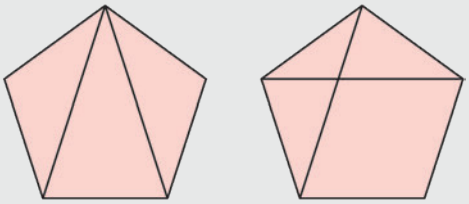
d The word GRAPE scores 22 points, GARLIC scores 31 points, ONION scores 57 points and CREAM scores 29 points. How many points does MELON score?

e Find the lowest possible score for COCONUT.

f The word OCTOPUS scores 89 points and BURGER scores 69 points. Find the lowest and highest possible scores for BRUSSELS SPROUT.

g Explain why the score for BEANS must be even.

2 Pentagon wedges



a What is the angle sum of any pentagon? Hint: What is the smallest number of triangles into which a pentagon can be divided?

b What is the angle at each corner of a regular pentagon?

c The diagram shows the two cases when a pair of diagonals is drawn on a regular pentagon. What special types of triangles and quadrilaterals are formed by the diagonals? Find the angles in each shape.

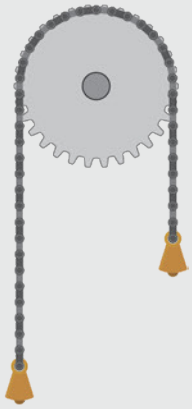
Given an isosceles triangle, we call it *narrow* if it has the same angles as the acute-angled triangles in the diagrams above. We call it *wide* if it has the same angles as the obtuse-angled triangles.

d Show how to divide a narrow triangle into two smaller triangles, one of which is narrow and the other wide.

e Show that a regular pentagon can be divided into 33 narrow triangles and 67 wide triangles. Can you generalise this result to other pairs of numbers?

f There are four types of isosceles triangle that can be divided into two smaller isosceles triangles. Two of these are the narrow and wide triangles described above. Find the other two types.

3 Bicycle chimes



Max pulls apart an old bicycle and builds a machine using the bicycle chain and one of the sprocket wheels. He breaks the chain, hangs it over the wheel and attaches a bell to each end, as shown. He then attaches a motor that turns the wheel clockwise for exactly one hour and then anticlockwise for exactly one hour, both at the same constant rate.

Max positions the centre of the wheel on the wall so that at 12 noon the bell on the left hits the floor and rings. The wheel turns clockwise for one hour so that at exactly 1 pm the right bell hits the floor and rings. At 2 pm the left bell rings again, and this process continues alternately every hour.

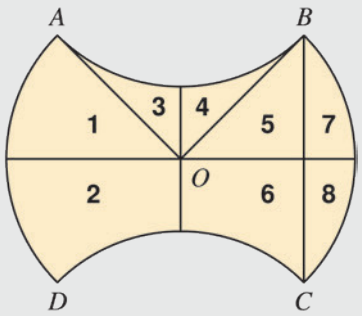
a At 2:10 pm the difference in height between the two bells is 80 cm. How far off the floor is the left bell at 3 pm?

b The diameter of the sprocket wheel is 10 cm. To two decimal places, how many revolutions does the wheel make per hour?

c As the years go by, the sprockets on the wheel wear out and the chain starts to slip. Max replaces the old wheel with a 14 cm–diameter sprocket wheel from the old bicycle. The motor still turns the wheel clockwise and anticlockwise at the same constant rate as before. To the nearest millimetre, by how much does Max need to raise or lower the centre of the new wheel so that the bells still ring every hour?

4 Dogbones

In the dogbone shown, points A, B, C and D are the vertices of a square with centre O. The four circular arcs which join the vertices all have radius 4 units.



a Find the exact area of each numbered piece, in square units. Hint: Find the areas of shapes 1 and 5 first, then shapes 7 and 4.

b Show how to rearrange the 8 pieces to make the following shapes:

i two identical rectangles

ii a right-angled isosceles triangle

iii a symmetrical hexagon

5 Freaky frequencies

Sebastian is practising converting lists of data into frequency distribution tables. For example, starting with the list {1, 2, 2, 3, 3, 2, 4, 1}, he puts all possible scores in ascending order in the first column, namely 1, 2, 3 and 4. He then records the frequency of each score (that is, the number of times it occurs in the list) in the second column, as shown.

Score	Freq.
1	2
2	3
3	2
4	1

Sebastian notices something unusual about this example: reading the entries in the table from left to right then top to bottom produces the list {1, 2, 2, 3, 3, 2, 4, 1}, which exactly matches the one he started with! He calls such a collection of data a *freaky-frequency list*.

a Find, with reasons, the only freaky-frequency list of length 2.

b Show that there are exactly two freaky-frequency lists that involve only the numbers 1, 2, 3 and 4.

Given any list of data which is *not* a freaky-frequency list, we can still read off the entries in its table to produce another list. For example, starting with {1}, the table's only row is $\boxed{1}\boxed{1}$ which produces the list {1, 1}. This in turn has only the row $\boxed{1}\boxed{2}$ in its table, producing the list {1, 2}. Continuing in this way, we get the following sequence of lists:

$$\{1\} \rightarrow \{1,1\} \rightarrow \{1,2\} \rightarrow \{1,1,2,1\} \rightarrow \{1,3,2,1\} \rightarrow \ldots$$

c Find the 100th list in the above sequence.

d Find the 100th list in the sequence which starts with the list {1, 2, 3, 4, 5}.

e The fourth list in a sequence is {1, 1, 2, 3, 3, 1, 5, 1}. Find the length of the first list.

6 Robot designs

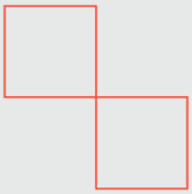
My drawing robot accepts the following commands:

- ► Fx : move forward x units.
- ► Lx : turn x degrees to the left.
- ► Rn [...] : repeat the instructions in the brackets n times.

For example, the instructions R3 [F30 L120] tell the robot to draw an equilateral triangle with a side length 30 units.

I tried to program my robot to draw the three designs shown below, but each set of instructions I came up with had a single mistake. Find and correct each mistake.

a R2 [R4 [F50 L90] L90]



b R3 [R3 [F50 L120] L60]

c R6 [R6 [F50 L60] L120]

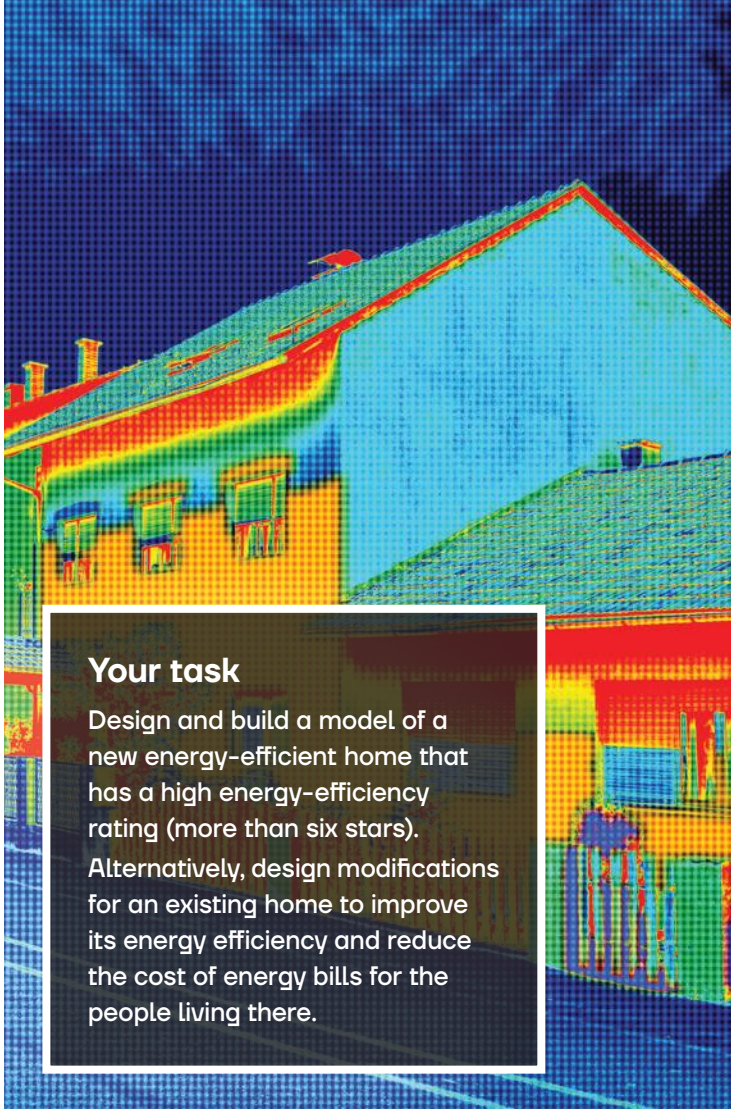
[STEAM project 1]

How can we build more energy-efficient homes so that we live more sustainably?

Have you ever considered how much energy it takes to heat or cool your home? A study by the Australian Government found that homes contribute 11 per cent of Australia's total carbon emissions. Reducing the amount of energy needed to heat or cool a home will not only save people money on their gas and electricity bills, it will make housing more sustainable.

NatHERS

The Nationwide House Energy Rating Scheme (NatHERS) is a rating system that is used across Australia to identify the energy efficiency of homes (where ten stars is the most energy efficient). Computer modelling software uses the local climate, the orientation of the home and materials used in construction to estimate the amount of energy needed to heat or cool the home. In most states and territories of Australia, new homes must reach a minimum of six stars to be approved for construction. There are many ways to improve the energy efficiency of your home. For example, if you insulate the walls, floors and roof, and use awnings (covers that extend over windows or doors), you can reduce the amount of thermal energy needed to cool the home in summer. Or in cooler climates, if you design the home so that living areas are on the northern side, you can maximise the sunlight that is available to heat the home in winter.



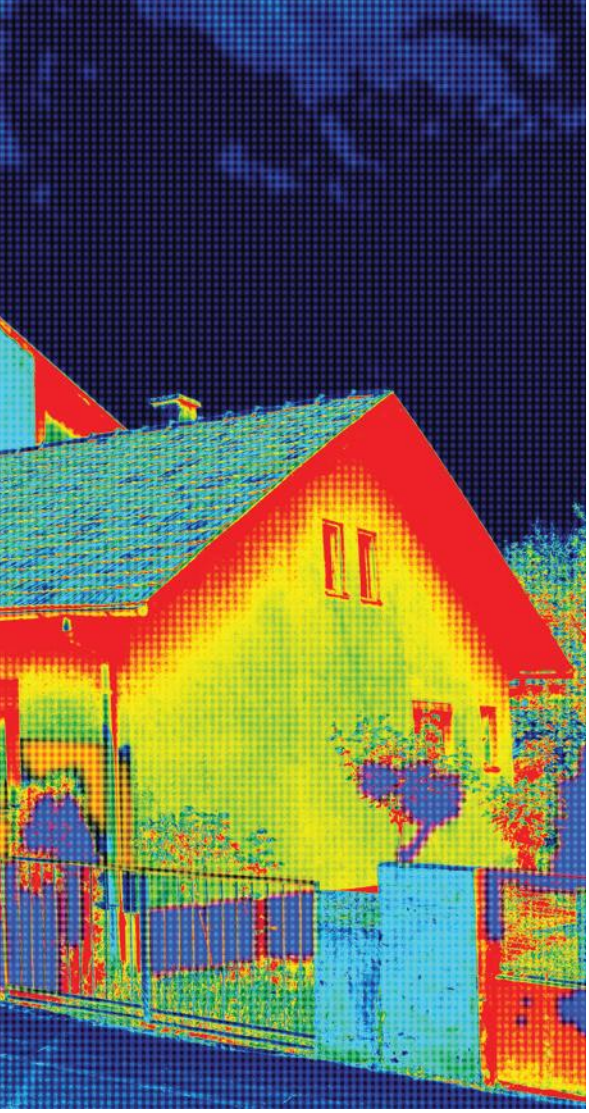
Your task

Design and build a model of a new energy-efficient home that has a high energy-efficiency rating (more than six stars). Alternatively, design modifications for an existing home to improve its energy efficiency and reduce the cost of energy bills for the people living there.

Figure 1 Infrared imaging shows warmer temperatures as red and cooler temperatures as blue. A building with insulation will lose less thermal energy to the outside environment in winter and will gain less thermal energy from the outside environment in summer.

Retrofitting homes

Not everyone wants to or can afford to build a new home. People who rent sometimes have little choice when it comes to the energy efficiency of their home, but this doesn't mean that they need to have large gas and electricity bills. There are things that they can do to reduce the movement of thermal energy in summer or winter. Covering the floor with rugs, preventing heat from moving in or out of windows, and controlling windows to manage airflow are all ways to make a home more energy-efficient and liveable.



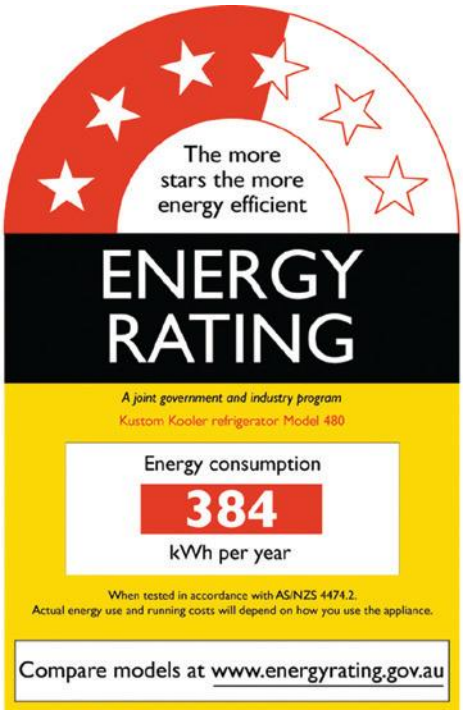


Figure 2 Energy rating schemes help consumers to understand the energy efficiency of products or appliances. The more stars, the more energy efficient the product or appliance is.

HSIE

In Stage 4 Geography, you will learn about the importance of sustainable housing in Australia's urban centres and major cities. You will explore how the forecasted growth of Australia's cities could impact their liveability, and the environmental issues resulting from urbanisation.

In Commerce, you will study the housing market, the interaction between businesses and consumers in meeting consumer demands and when the government intervenes in markets.

To complete this task successfully, you will need to study seasonal weather patterns at the location of your energy-efficient home, to improve energy use.

MATHS

In Stage 4 Maths, you will learn how to determine the area and volume of different shapes, using and converting between appropriate units. You will also learn skills for dealing with percentage changes in financial contexts. These skills will help you to quantify the costs and benefits of design features and predict the popularity of market incentives. You will perform calculations with and without digital technology.

To complete this task successfully, you will need to perform calculations for your model home, and then scale up to estimate the potential benefits of your design at a national level.

You will find help for applying these maths skills in sections 3B 'Calculating percentages', 3C 'Mark-ups and discounts', 8C 'Area of quadrilaterals' and 8D 'Area of a circle' of *Oxford Maths 8 NSW Curriculum*.

SCIENCE

In Stage 4 Science, you will learn about how energy can be transferred between objects and transformed (such as from the Sun's light energy to electrical energy). You will also examine earthquake resistant materials used in construction.

To complete this task successfully, you will need to identify how each element of a home's design can affect its heating and cooling needs, and how construction materials can affect a home's stability.

You will also need to identify the elements that can be changed in a new home design, and compare these to the elements that can be changed in an established home.

[STEAM project 1]

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for? How big is their family and home?
- What factors might affect the liveability of the home?
- What do they need? What do they not need?
- What does it feel like to face these limitations on liveability? What words would you use to describe how it would feel to face these limitations?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your home modifications, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1. Describe the orientation of the land where the home will be built or modified (include the location of sunrise and sunset, the angle of sun in the middle of summer and winter and any existing shadows).
2. Describe any existing energy-efficient features of the current home or design.
3. Describe how you will measure any improvement in energy-efficient features of the home.

Ideate

Once you know who you're designing for, and what the criteria are, it's time to get creative!

- Outline the criteria or requirements your design must fulfil (for example, improving heating or cooling of the home).
- Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Draw each individual design.

Include in the individual designs:

a labels for each part of the design

b the materials you will use for the home's construction

c a description of how the modifications will improve the energy star rating of the home

d an estimation of the cost of applying the modifications.

Present your design to your group.

Build the prototype

Choose and build two or three model homes for your group design (scale: 2 cm = 1 m).

Use the following questions as a guideline for your prototype.

- What materials will you need to build your model home?
- What skills will you need to build your model home?
- How will you test the effectiveness of your model home? (What will you compare it to?)
- How will you identify each energy-efficient feature of your model home?
- How will you collect data that supports your claims about energy efficiency?

Test

Use the scientific method to design and test the energy efficiency of your model home. You will test more than one design so that you can compare them, so you will need to control your variables between tests.

What criteria will you use to determine the success of your model home?

Conduct your tests and record your results in an appropriate table.

Communicate

Present your home design as though you are trying to get your peers to invest in it.

In your presentation, you will need to:

- construct a labelled diagram of your model home in its orientation, including the location of sunrise and sunset, the angle of sun in the middle of summer and winter, and any existing shadows
- describe the key energy-efficient features of your model home
- explain how each energy-efficient feature affects the liveability of your model home
- explain the principles that support your design (the importance of energy efficiency and how the existing landscape affected the design of the home)
- estimate the cost of implementing your design
- explain and quantify the benefits on a national scale if all new homes were to include your design features.

Check your Student obook pro for the following digital resources to help you with this STEAM project:

Student booklet
This helpful booklet will guide you step-by-step through the project.

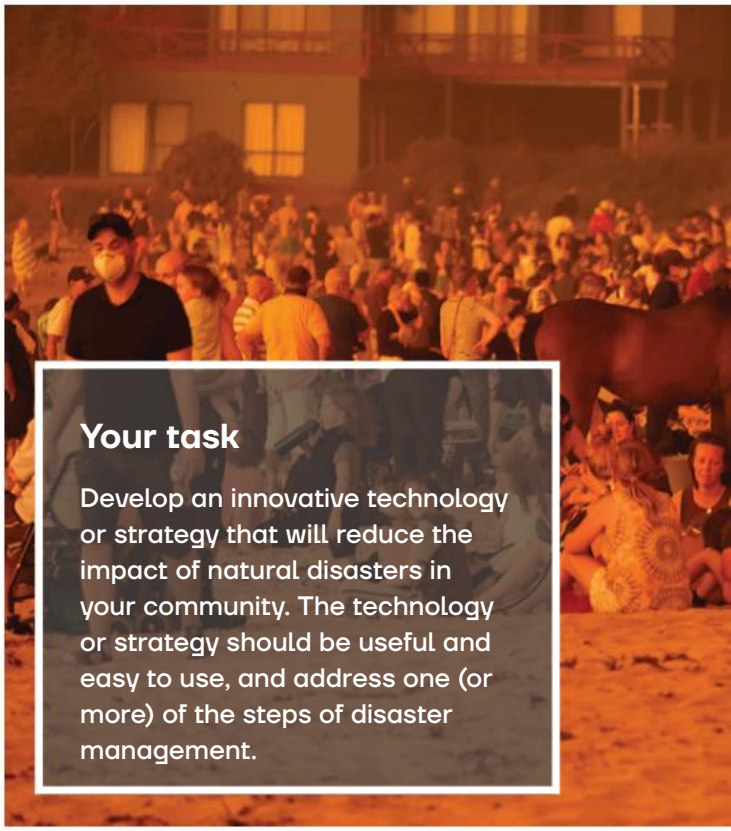
What is the design cycle?
This video will help you to better understand each phase of the design cycle.

How to manage your project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

[STEAM project 2]

How can we use technology so that the impact of natural disasters is reduced?



Your task

Develop an innovative technology or strategy that will reduce the impact of natural disasters in your community. The technology or strategy should be useful and easy to use, and address one (or more) of the steps of disaster management.

Figure 1 Hundreds of people (and animals) huddled on Malua Bay beach in NSW for almost 24 hours, as bushfires tore through communities in the nearby Batemans Bay area.

Across the world, climate patterns are changing and weather events are becoming more extreme. In 2020, 20 per cent of Australia's forests were burnt, a record-breaking 30 named hurricanes developed over the Atlantic Ocean, and an extreme monsoon season in Asia caused flooding across a quarter of Bangladesh. Australia is particularly prone to natural disasters. Bushfires, flooding, drought periods and cyclones dramatically affect people's lives, and alter natural landscapes.

Natural disasters can be devastating for communities affected by them. Recovering from natural disasters also comes with huge economic costs. This can sometimes lead to a harmful cycle in which a nation gets stuck between experiencing disaster and responding to disaster.

Disaster management

Climate action is one of the 17 Sustainable Development Goals agreed to in 2015 by world leaders. One of the identified targets in combating climate change is to strengthen the resilience and adaptive capacity of all countries to natural disasters and hazards related to climate.

It is essential that communities prepare for disasters so as to reduce their impact. The United Nations Office for Disaster Risk Reduction (UNDRR) recommends using technology as part of disaster management, for example:

- warning systems in Japan that trigger emergency breaks in bullet trains if earthquakes are detected
- systems in sub-Saharan Africa that monitor rainfall data and analyse trends to forecast and build resilience to drought
- atlases in China that record the risk of natural disasters, using location data from spatial technologies such as GPS.

After the 2019–20 Australian bushfires, it was argued that satellite technology, drones and mobile phone apps would have been extremely helpful for fighting the fires and communicating with the people affected.

Disaster management should include four steps:

- prevention – reducing hazards before a disaster takes place, so its impact is not as severe (e.g. building schools that are earthquake resistant)
- preparation – training people so they know how to act when a disaster happens (e.g. running evacuation drills)
- response – taking action during a disaster (e.g. emergency crews and volunteers taking on emergency operations)
- recovery – taking action to help people rebuild their lives (e.g. restoring services in a community).

Disaster management is an ongoing process that can occur like a cycle between the phases of prevention, preparation, response and recovery.

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Figure 2 Floodwaters near Sydney

HSIE

In Stage 4 Geography, you will learn about landscapes and landforms and how they can be degraded by both human and natural causes. You will study a geomorphic hazard, its impacts on a place and various ways of responding to it.

In History, you might also study how past societies have dealt with disasters and managed their responses; for example, how Rapa Nui (Easter Island) inhabitants adapted to life without trees, how Shogunate Japan developed policies to sustain its forests, or how an unstable climate (including drought and heavy monsoons) affected the Khmer Empire.

To complete this task successfully, you will need to investigate a potential natural hazard in your local area. You will research any current disaster-response plans related to managing such an event, and gain an understanding of how various people and businesses in the community would be impacted.

MATHS

In Stage 4 Maths, you will consolidate and extend your skills in describing and interpreting. This will include creating and analysing plots of non-linear data, investigating the use of sampling methods, and broadening your understanding of measures of spread. You will also perform calculations with percentage changes in financial contexts. You will analyse and represent data, both with and without digital technology.

To complete this task successfully, you will need to weigh up the costs of your disaster-management technology or strategy against its potential benefits, including by estimating the likelihood of natural disasters and the severity of their effects.

You will find help for applying these maths skills and statistics in sections 3B 'Calculating percentages', 3C 'Mark-ups and discounts' and 6F 'Plotting linear relationships' of *Oxford Maths 8 NSW Curriculum*.

SCIENCE

In Stage 4 Science, you will learn about how the energy of the Earth over long periods of time generates forces that can melt rocks, produce volcanoes and make diamonds. You will also learn how the kinetic energy in the air and waves can cause damage to the surrounding environments during cyclones and tsunamis. The Australian bush also contains large amounts of chemical energy that is transformed into thermal energy during the summer fire season.

To complete this task successfully, you will need to consider how energy is transferred and transformed during natural disasters. This understanding will allow you to predict and potentially reduce the impact of the disaster in your local community.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What natural disaster could they face?
- How often could this disaster occur, and on what scale?
- What do they need? What do they not need?
- What does it feel like to face these problems? What words would you use to describe these feelings?
- What could the cost of such a disaster be in terms of lives lost, income lost, damage to private property and public infrastructure, and environmental impact?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your innovative strategy or technology, you need to define the parameters you are working towards.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and why it is important. Use the following phrase as a guide.

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1. Define the term 'innovation'. Describe how much a strategy needs to differ from current practice to be considered innovative.
2. Describe how the natural disaster will affect the people in the community.
3. Describe how you could measure whether your solution will reduce the impact of the natural disaster.
4. Explain how you could determine what cost (price) should be acceptable for implementing your solution.

Ideate

Once you know who you're designing for, and you know what the criteria are, it's time to get creative!

- Outline the criteria or requirements your designed strategy or technology must fulfil (for example, how many people will be helped, how much they will be helped, how long they will need the help).
- Brainstorm at least one idea per person that fulfils the criteria.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Draw each design strategy or technological idea. Label each stage of the strategy or part of the technology and explain how it will be used by the community.

Include in the individual strategy or technology:

a the timeline for the activation of the strategy or technology (i.e. how long after the disaster will the idea be ready to be used?)

b the number of people and materials that will be needed for the strategy or technology to reduce the impact of the natural disaster

c a description of how the community will benefit from the strategy or technology.

Present your strategy or technology to your group.

Build the prototype

Choose and build two or three versions of the prototype strategy or technology for your group design.

Use the following questions as a guideline for your prototype.

- What equipment do you have access to?
- What skills do you have, or will you need, to make your prototype strategy or technology?
- How could you model your strategy or technology if equipment is not available?
- How could you test the effectiveness of the prototype strategy or technology with the community?

Test

Use the scientific method to design an experiment that will test the effectiveness of your prototype strategy or technology. Alternatively, conduct a survey of the community to determine their opinion of the usefulness of the prototype.

Conduct your tests or survey your community, and record your results in an appropriate table.

Consider how you could use the results of the experiment or survey to modify your design.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your strategy or technology.

In your presentation, you will need to:

- describe how the natural disaster will affect your community
- describe your prototype strategy or technology
- describe how the prototype will reduce the impact of the natural disaster on the community
- describe the materials, people and money that will be required to have the required effect on the natural disaster.

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How to manage your project
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How to define a problem
This 'how-to' video will help you to narrow your ideas down and define a specific problem.

Answers

CHAPTER 1 Integers

EX p6 1A Rounding and estimating

1 **a** 400 **b** 500 **c** 500 **d** 500
e 400 **f** 400

2 **a** 7000 **b** 6000 **c** 6000 **d** 7000
e 7000 **f** 6000

3 **a** **i** 480 **ii** 6380 **iii** 56 030
iv 738 490 **v** 8080 **vi** 904 510
b **i** 500 **ii** 6400 **iii** 56 000
iv 738 500 **v** 8100 **vi** 904 500

4 **a** **i** 36 000 **ii** 8000 **iii** 184 000
iv 51 000 **v** 6000 **vi** 1000
b **i** 36 400 **ii** 7500 **iii** 183 900
iv 50 700 **v** 6100 **vi** 800

5 **a** 80 **b** 40 **c** 80 **d** 300
e 200 **f** 600 **g** 900 **h** 2000
i 7000 **j** 20 000

6 **a** 90 **b** 100 **c** 1000
d 90 000 **e** 9000 **f** 1000

7 Sample answers:
a 37, 42, 35 **b** 692, 705, 744
c 2995, 3125, 3451 **d** 52 168, 46 854, 50 369
e 795 002, 755 036, 827 039
f 1 758 364, 2 498 567, 2 215 000

8 **a** 1600 **b** 290 **c** 600
d 20 **e** 20 000 **f** 3800
g 40 **h** 5500 **i** 2000
j 2 100 000 **k** 11 000 **l** 15 000

9 **a** 1200 **b** 50 **c** 1500
d 40 **e** 12 000 **f** 6200
g 125 **h** 50

10 **a** 10 **b** 1 800 000 **c** 6200 **d** 7000
e 560 000 000 **f** 18 000 **g** 150 **h** 19 200

11 **a** 40 000 **b** 35 800 **c** 36 000
d 35 780 **e** 40 000

12 **a** 20 000 000 km^2 **b** 17 098 200 km^2
c 17 098 000 km^2 **d** 17 100 000 km^2
e 17 100 000 km^2 **f** 17 000 000 km^2

13 **a** \$300
b overestimation

14 Tina is correct. Since the leading digit is 9 and the second digit is 6, the first digit increases by one and the digits that follow are replaced with zero. Increasing 9 by one gives 10 so there is a total of five digits in the answer.

15 **a** 30 **b** \$1800 **c** no
d 20 months

16 **a** 400 **b** \$2800
c \$2695; estimate is \$105 more

17 **a** 40
b The estimate of 40 would be lower than the actual number of boxes as the approximate price (\$50) is higher than the actual price (\$47). There would be money left over.

18 **a** Australia: 8 000 000 km^2
Indonesia: 2 000 000 km^2
b **i** 6 000 000 km^2
ii Australia is approximately 4 times the area of Indonesia.
c **i** 5 787 455 km^2
ii Australia is 4.039 times the area of Indonesia.

19 **a** 48 000 **b** 52 196

20 **a** WA: 3 000 000 km^2, Qld: 2 000 000 km^2,
NT: 1 000 000 km^2, SA: 1 000 000 km^2,
NSW: 800 000 km^2, Vic: 200 000 km^2,
Tas: 70 000 km^2, ACT: 2000 km^2
b 8 072 000 km^2
c WA: 2 530 000 km^2, Qld: 1 731 000 km^2,
NT: 1 349 000 km^2, SA: 983 000 km^2,
NSW: 801 000 km^2, Vic: 227 000 km^2,
Tas: 68 000 km^2, ACT: 2000 km^2
d 7 691 000 km^2
e Values rounded to the nearest thousand produce an estimate much closer to the actual values.

21 **a** $\frac{8}{30} = \frac{4}{15} \approx 0.27$ km^2 per person
b New Zealand has approximately $\frac{3}{50} \approx 0.06$ km^2 per person. So, Australia has approximately 4.5 times more area per person than New Zealand.

22 Record yourself speaking for a minute and count the number of words you say. Consider how many minutes of the last hour you have spoken for. Multiplying by 12, is this a good estimate of your day?

EX p12 1B Adding and subtracting whole numbers

1 **a** **i** 97 **ii** 161 **iii** 979 **iv** 10 897
b **i** 719 **ii** 819 **iii** 584 **iv** 3591
c **i** 123 **ii** 528 **iii** 1123 **iv** 8104
d **i** 177 **ii** 1139 **iii** 8824 **iv** 6570

2 **a** 958 **b** 692 **c** 1219 **d** 1502

3 **a** 5241 **b** 10 475 **c** 18 872 **d** 122 530

4 **a** 1223 **b** 1857 **c** 4141 **d** 4918
e 7366 **f** 20 350

5 **a** 91 852 **b** 128 077 **c** 24 921 **d** 2460

6 **a** **i** 63 **ii** 87 **iii** 94 **iv** 693
b **i** 33 **ii** 695 **iii** 256 **iv** 366
c **i** 60 **ii** 5190 **iii** 210 **iv** 112
d **i** 62 **ii** 890 **iii** 523 **iv** 1136

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7 **a** 39 **b** 276 **c** 1380 **d** 3378
8 **a** 29 **b** 3489 **c** 4418 **d** 6320
9 **a** 7166 **b** 998 **c** 74 949 **d** 43 642
10 **a** 476 **b** 5074 **c** 38 150 **d** 8473
11 2514
12 **a** 2 841 016 **b** 768 853 **c** 5 258 147
13 **a** 81 593 **b** 73 248
14 **a** 1177 km **b** $1088 **c** $734 **d** $178
15 **a** 6199 **b** 4637
16 **a** 44 km **b** 189 m
17 85 pairs of shoes
18 **a** 5 **b** 2000
19 S=4, E=5, D=1, W=6, A=8, T=3, R=2, F=7, U=9, I=0
S=5, E=8, D=9, W=3, A=6, T=1, R=2, F=4, U=0, I=7
S=6, E=8, D=9, W=3, A=5, T=1, R=2, F=4, U=0, I=7
S=7, E=1, D=3, W=4, A=8, T=9, R=6, F=5, U=0, I=2
S=8, E=1, D=3, W=4, A=8, T=9, R=6, F=5, U=0, I=2
S=8, E=5, D=1, W=6, A=4, T=3, R=2, F=7, U=9, I=0

EX p18

1C Multiplying and dividing whole numbers

1 **a** 126 **b** 612 **c** 1421 **d** 2745
e 1666 **f** 8217 **g** 2280 **h** 779
i 6262 **j** 29 029 **k** 1067 **l** 14 985
2 **a** 408 **b** 630 **c** 444 **d** 5280
e 600 **f** 92 100 **g** 700 **h** 29 000
i 186 000
3 **a** 630 **b** 9600 **c** 7700 **d** 6300
e 320 400 **f** 8 295 000
g 24 867 000 **h** 83 700 000
4 **a** 720 000 **b** 448 000 000
c 9 600 000 **d** 375 000 000
5 **a** 44 764 **b** 24 402 **c** 105 754 **d** 147 936
e 665 640 **f** 4 903 600
6 **a** 332 820 **b** 226 044
c 316 448 **d** 1 726 202
e 1 509 024 **f** 29 792 690
7 **a** **i** 800 000 **ii** 270 000 **iii** 60 000 000
iv 640 000 000
b **i** 803 890 **ii** 287 013 **iii** 59 804 375
iv 654 565 110
8 **a** 57 remainder 7 **b** 58
c 22 remainder 6 **d** 75
e 76 remainder 5 **f** 1420 remainder 2
9 **a** 1456 **b** 1399 remainder 5
c 1403 remainder 1 **d** 16 855 remainder 1
e 6254 remainder 3 **f** 212 remainder 5
10 **a** 23 remainder 5 **b** 81 remainder 5
c 35 remainder 1 **d** 163
11 **a** 92
b 23
12 **a** **i** 9 **ii** 9 **b** no
c **i** 3
ii −3; subtraction does not obey the commutative law.
d **i** 18
ii 18; multiplication obeys the commutative law.
e **i** 2
ii 0.5; division does not obey the commutative law.
13 **a** **i** 18 **ii** 18 **iii** 18
b no
c Multiplication obeys the associative law, but subtraction and division do not.
For example, multiplication:
$5 \times 4 \times 2 = 4 \times 5 \times 2 = 2 \times 5 \times 4 = 40$;
subtraction: $(7 - 2) - 3 = 2$ and $7 - (2 - 3) = 8$ (does not obey the associative law);
division: $(8 \div 4) \div 2 = 1$ and $8 \div (4 \div 2) = 4$ (does not obey the associative law).
14 **a** **i** 54 + 16, since the result (70) is a multiple of 10 and easier to add
ii 188
b **i** 25 × 4, since the result (100) is easier to multiply by
ii 8700
c As order is not important, any two of the three numbers can be added or multiplied first. If this results in a number that is a multiple of 10, it can make the final operation easier.
15 **a** **i** 44 **ii** 20 + 24 = 44; the answers are the same
b **i** 48 **ii** 60 − 12 = 48; the answers are the same
c Each number inside the brackets is multiplied by the number at the front of the brackets.
d Sample answer:
$10 \times (5 + 4) = 10 \times 5 + 10 \times 4 = 50 + 40 = 90$
$10 \times (5 + 4) = 10 \times 9 = 90$
16 **a** **i** 320 **ii** 64 **iii** 384
b **i** 45 000 **ii** 90 **iii** 44 910
c Write one of the numbers to be multiplied as either a sum or difference that involves a multiple of 10 or power of 10 and a single-digit number.
17 **a** $12 **b** 3 **c** $10 **d** $400
18 **a** 65 **b** 11 **c** $195
19 **a** 346 L **b** 7 **c** $700
20 23
21 **a** 60 g **b** 32

EX p24

1D Multiples, factors, indices and roots

1 **a** 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
b 18, 36, 54, 72, 90
c 18
2 **a** 24 **b** 36 **c** 150 **d** 60
e 140 **f** 45

3 **a** 210 **b** 315 **c** 72 **d** 120
e 360 **f** 54

4 **a** 1, 3, 5, 9, 15, 45
b 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
c 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
d 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
e 1, 2, 4, 5, 10, 20, 25, 50, 100
f 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

5 **a** 18 **b** 3 **c** 6

6 **a** 14 **b** 12 **c** 9 **d** 4
e 120 **f** 25

7 **a** LCM = 360, HCF = 3
b LCM = 144, HCF = 4
c LCM = 60, HCF = 1
d LCM = 924, HCF = 1

8 **a** 9, 27 **b** 4, 8 **c** 100, 1000
d 49, 343 **e** 36, 216 **f** 144, 1728
g 64, 512 **h** 16, 64 **i** 1, 1
j 81, 729 **k** 121, 1331 **l** 196, 2744

9 **a** 3 **b** 2 **c** 5 **d** 8
e 9 **f** 6 **g** 10 **h** 7
i 8 **j** 13 **k** 14 **l** 15

10 **a** 24 **b** 33 **c** 49 000 **d** 16
e 4096 **f** 100 000

11 **a** 972 **b** 36 **c** 6 **d** 2048
e 256 **f** 3600

12 **a** 21 **b** 60 **c** 15 **d** 35

13 **a** 4 **b** 24
c $\sqrt[3]{64}$ is the cube root of 64, whereas $3\sqrt{64}$ is 3 multiplied by the square root of 64.

14 60

15 **a** 120 **b** 8

16 90

17 24

18 8

19 Pontiac, Desiree, Sebago

20 500

EX p26 Checkpoint

1 **a** 600 **b** 900 **c** 7900 **d** 2800
e 45 600 **f** 76 600

2 **a** 1200 **b** 40 **c** 2800 **d** 30
e 15 000 **f** 9800 **g** 100 **h** 500

3 **a** 162 **b** 1577 **c** 12 158 **d** 7058
e 6717 **f** 15 368

4 **a** 18 **b** 486 **c** 89 **d** 5655
e 8369 **f** 3826

5 **a** 321 **b** 577 **c** 6471 **d** 108 206

6 **a** 33 024 **b** 64 296 **c** 354 198 **d** 768 746
e 594 336 **f** 4 207 980

7 **a** 98 remainder 3 **b** 92 remainder 2
c 1665 remainder 3 **d** 8488 remainder 6
e 6212 remainder 9 **f** 108 964 remainder 5

8 **a** 42 **b** 36 **c** 60 **d** 90
e 72 **f** 108

9 **a** 6 **b** 9 **c** 15 **d** 5
e 150 **f** 144

10 **a** 100 **b** 9 **c** 144 **d** 8
e 148 **f** 253

EX p29 1E Negative integers

1 **a** negative integers: −1, −2, −3, −4, −5
b positive integers: 1, 2, 3, 4, 5
c 0

2 **a** 9 **b** 5 **c** 1 **d** 5
e 0 **f** −4

3 **a** 5 **b** 0 **c** −2 **d** −5
e −1 **f** −9

4 **a** 0 **b** 24 **c** 14 **d** 125
e −25 **f** 0

5 **a** −70 **b** −21 **c** −68 **d** −50
e −354 **f** −450

6 **a** −3 < 2 **b** −8 < −4 **c** 0 > −1
d 9 > −9 **e** −3 > −5 **f** −7 < −2
g 4 > −6 **h** 5 > −12 **i** 6 > −4
j −18 < −8 **k** −6 > −7 **l** −8 < 0

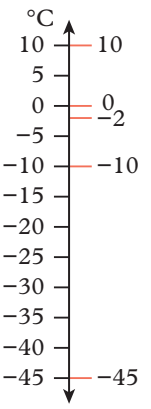
7 **a** −20, −12, −11, −7, 8, 10, 14
b −41, −33, −19, 0, 6, 29, 42
c −88, −48, −28, −18, −8, 8, 68
d −140, −126, −82, 3, 73, 104, 145

8 **a** 71, 4, −9, −10, −15, −27
b 10, 5, −1, −2, −5, −11
c 21, 5, 0, −9, −14, −19
d 71, 51, 12, −11, −62, −71, −72

9 −6, −5, −4, −3, −2, −1, 0, 1, 2, 3

10 **a** −3 **b** 2174 **c** −18 **d** −408
e 5895 **f** 500 **g** −46 **h** −40

11 **a**



b higher
c −45°C, −10°C, −2°C, 0°C, 10°C
d warmer

12 **a** +86 **b** $14 **c** −14

13 **a** long sighted
b short sighted
c Elle has weaker vision.
d normal vision; that is, neither long nor short sighted

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14 **a** **i** \$34 **ii** \$51

b **i** \$13 **ii** \$35

c Account is overdrawn; the person owes money to the bank.

15 **a** three

b **i**

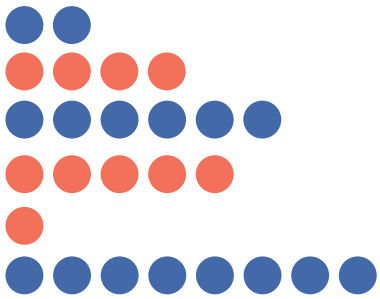
ii

iii

iv

v

vi



c The reference point for opposite integers is always zero as both integers are the same number of units away from 0. Moving forward to the positive integer and then moving back as indicated by the opposite integer will always result in zero. When combined, opposite integers 'cancel' each other to result in zero. This can be modelled by equal numbers of blue and red dots.

d Two blue dots and two red dots produce two zero pairs; the remaining three red dots represent the integer −3.

e **i** 5 **ii** −4 **iii** −4

16 12

17 −9, −8, −7, −6, −5

EX p34 1F Adding and subtracting integers

1 **a** −5 + 3 = −2 **b** 6 − 2 = 4

c −4 − 5 = −9 **d** −2 + 5 = 3

2 **a** III **b** I **c** II

3 **a** −15 **b** −9 **c** 9

4 **a** 3 **b** 6 **c** 5 **d** −8

e −4 **f** −3 **g** −6 **h** 5

i −4

5 **a** 3 **b** −6 **c** −4 **d** 16

e −19 **f** −10 **g** −17 **h** −2

i −13 **j** 0 **k** −20 **l** 2

6 **a** negative **b** positive **c** negative

d zero **e** positive **f** positive

g negative **h** negative **i** negative

7 **a** −3 − 7 **b** 1 + 6 **c** −4 − 5

d 2 + 4 **e** 5 − 9 **f** −6 + 8

8 **a** −1 + 3 **b** 8 − 4 **c** −5 − 2

d 6 + 1 **e** 4 + 3 **f** 3 − 9

g −5 + 8 **h** −3 + 1 **i** −7 − 3

9 **a** 3 **b** 6 **c** −4 **d** 0

e −1 **f** 5 **g** 1 **h** −5

i 9

10 **a** positive **b** negative **c** zero

d negative **e** negative **f** negative

g positive **h** negative

11 **a**

+	−5	−3	0	1	4
−3	−8	−6	−3	−2	1
−2	−7	−5	−2	−1	2
−1	−6	−4	−1	0	3
3	−2	0	3	4	7
6	1	3	6	7	10

b

+	−10	−14	−7	13	19
11	1	−3	4	24	30
14	4	0	7	27	33
−10	−20	−24	−17	3	9
−22	−32	−36	−29	−9	−3
−18	−28	−32	−25	−5	1

12 **a** −5 **b** −99 **c** −32 **d** 9

e −40 **f** −275 **g** 242 **h** 118 **i** 0

13 **a** 10 **b** −4 **c** −3 **d** −13 **e** 0

f −33 **g** −10 **h** −26 **i** −30 **j** −17

14 **a** −6 **b** 15 **c** 0 **d** 12 **e** −14

f −31 **g** 8 **h** 2

15 four levels below ground; 15 − 19 = −4

16 first level below ground; 3 − 9 + 5 = −1

17 51°C; 33 − (−18) = 51

18 **a** −60 **b** −60 + (+35) **c** \$25

19 **a** A: 50, B: −2, C: 32

b **i** 50 − (−2) = 50 + 2

ii 32 − (−2) = 32 + 2

iii 50 − (+32) = 50 − 32

c **i** 52 m **ii** 34 m **iii** 18 m

d 52 m; it shows the greatest distance the bungee rope extends to before rebounding a lesser distance each time after.

20 **a** −900 − 15 = −915; 915 m down

b −2500 − (−900) = −1600; 1600 m down

c 15 − (−2500) = 2515; 2515 m up

d 9 m above the sea level

21 **a** 45 **b** −30 **c** 1261 **d** −367

e −1020 **f** −3260

22 **a** 33 **b** 77 **c** −125 **d** −40

23 **a** \$63; (−247) − (−310) = 63

b Initially, account is overdrawn by \$310. During the month, \$63 is added to the account and the overdrawn amount is reduced from \$310 to \$247.

EX p39 1G Multiplying and dividing integers

1 **a** 5 × 9 **b** 6 × −7 **c** −10 × 6 **d** −4 × −3

2 **a** −12 **b** 35 **c** −12 **d** 27 **e** −8 **f** −24

g 72 **h** 20 **i** −44 **j** −7 **k** 8 **l** 45

3 **a** −5 **b** 4 **c** 6 **d** −7 **e** 17 **f** −8

g −7 **h** 9 **i** −6 **j** −12 **k** 10 **l** −9

4 **a** −8 **b** −12 **c** 17 **d** 15 **e** 8 **f** −3

5 **a** −35 **b** −3 **c** 7 **d** −60 **e** −4 **f** 60

g −5 **h** −36 **i** 90 **j** −1 **k** −38 **l** 45

6 **a** −3 **b** −7 **c** −9
d 100 **e** −70 **f** −3

7 **a**

×	−2	−1	0	1	2
−4	8	4	0	−4	−8
−2	4	2	0	−2	−4
0	0	0	0	0	0
2	−4	−2	0	2	4
4	−8	−4	0	4	8
6	−12	−6	0	6	12

b

×	−25	−20	−10	10	20
7	−175	−140	−70	70	140
5	−125	−100	−50	50	100
0	0	0	0	0	0
−2	50	40	20	−20	−40
−4	100	80	40	−40	−80
−6	150	120	60	−60	−120

8 **a** $-1 \times (-7) = 7$ **b** $-1 \times +12 = -12$
c $-1 \times (-88) = 88$ **d** $-1 \times 25 = -25$
e $-1 \times (8 - 5) = -1 \times 3 = -3$
f $-1 \times (-3 + 2) = -1 \times (-1) = 1$

9 **a** 60 **b** −42 **c** −54 **d** −48 **e** −130
f 84 **g** −160 **h** 110 **i** −180

10 **a** 36 **b** −5 **c** −24 **d** −6 **e** 30
f −42

11 E

12 **a** negative **b** positive **c** positive
d positive

13 **a** $-9 \times (-9) = 81$
b $8 \times 8 = 64$
c $-6 \times (-6) \times (-6) = -216$
d $7 \times 7 \times 7 = 343$
e $-5 \times (-5) \times (-5) \times (-5) = 625$
f $1 \times 1 \times 1 \times 1 = 1$
g $4 \times 4 \times 4 \times 4 \times 4 = 1024$
h $-10 \times (-10) \times (-10) \times (-10) \times (-10) = -100\,000$

14 **a** $(-5)^3 \times (-9)^4$ **b** $(-4)^6 \times 3^3$
c $7^2 \times (-6)^6$ **d** $(-8)^4 \times (-10)^2$

15 **a** $-2 \times (-2) \times (-2) \times (-4) \times (-4) = -128$
b $-5 \times (-5) \times 3 \times 3 \times 3 \times 3 = 2025$
c $-10 \times (-10) \times (-10) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 32\,000$
d $-3 \times (-3) \times (-3) \times (-3) \times (-3) \times (-1) \times (-1) = -243$
e $-6 \times (-6) \times (-2) \times (-2) \times (-2) = -288$
f $-1 \times (-1) \times (-1) \times (-1) \times 3 \times 3 = 9$
g $-7 \times (-7) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -49$
h $-3 \times (-3) \times (-3) \times (-2) \times (-2) = -108$
i $-1 \times (-1) \times (-1) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 32$

16 **a** 10, 8, 6, 4, 2, 0, −2, −4, −6, −8, −10; the terms decrease by 2, or you get the next number by subtracting 2 (or adding −2)
b −15, −12, −9, −6, −3, 0, 3, 6, 9, 12, 15; the terms increase by 3, or you get the next number by adding 3 (or subtracting −3)
c 20, 16, 12, 8, 4, 0, −4, −8, −12, −16, −20; the terms decrease by 4, or you get the next number by subtracting 4 (or adding −4)
d −25, −20, −15, −10, −5, 0, 5, 10, 15, 20, 25; the terms increase by 5, or you get the next number by adding 5 (or subtracting −5)

17 **a** $2 \times 3 = 6$ so $6 \div 2 = 3$ or $6 \div 3 = 2$
b $2 \times (-3) = -6$ so $-6 \div 2 = -3$ or $-6 \div (-3) = 2$
c $-2 \times 3 = -6$ so $-6 \div (-2) = 3$ or $-6 \div 3 = -2$
d $-2 \times (-3) = 6$ so $6 \div (-2) = -3$ or $6 \div (-3) = -2$

18 **a** **i** −1 **ii** 1 **iii** −1
iv 1 **v** −1 **vi** 1
b those with even exponent or power (**ii**, **iv**, **vi**)
c those with odd exponent or power (**i**, **iii**, **v**)
d Negative numbers raised to an even power will give a positive result and negative numbers raised to an odd power will give a negative result.
e **i** −1 **ii** 1 **iii** 1
iv −1 **v** 1 **vi** −1

19 **a** **i** 0 **ii** 0 **iii** 0 **iv** 0
b The answer is always zero.
c **i** 0 **ii** 0 **iii** 0 **iv** 0
d The answer is always zero.

20 **a** −6, −3 **b** 9, −5 **c** −11, 6
d 4, 6 or −6, −4 **e** −9, −1 or 1, 9 **f** −16, 2 or −2, 16

21 **a** 13, 17, 29 and 53
b −53, −29, −17 and −13
c −49, −25, −23, −13, −7, −5, −1, 1, 5, 7, 13, 23, 25 and 49

22 **a** −120 **b** 80 **c** 200 m **d** 110 **e** 40
f **i** 40 m **ii** 230 m
g −160 **h** −40
i Hayden. He needs to walk 20 m west.
j Natalie: 320 m; Tyler: 240 m; Hayden: 120 m; Rhys: 320 m; Sophie: 160 m; Imogen: 220 m

23 **a** 5 and −5; both numbers give the result of 25 when squared.
b No; to square a number is to multiply the number by itself. Either a negative number is squared ($- \times - \rightarrow +$) or a positive number is squared ($+ \times + \rightarrow +$). In both cases the result is positive.
c only one number (2)
d only one number (−2)
e Two; raising both 2 and −2 to the power of 4 gives the result of 16. No number can be raised to the power of 4 to give a negative result.

f There will be two numbers that when raised to an even power (such as 2, 4, 6, 8, etc.) give the same positive result. For example, $2^6 = 64$ and $(-2)^6 = 64$. No number raised to an even power gives a negative result. There will only be one number that when raised to an odd power (such as 3, 5, 7, etc.) results in a given answer. For example, $2^5 = 32$ and $(-2)^5 = -32$.

24 a 7 and −7 **b** 9 and −9 **c** 1 and −1
d 2 and −2 **e** 8 and −8 **f** 10 and −10

25 a 3 **b** −5 **c** 4 **d** −1 **e** −4 **f** −10

26 a $(-3)^2$ and 3^2

b $(-3)^2 = -3 \times (-3) = +9$
while $-3^2 = -(3^2) = -(3 \times 3) = -9$

c i yes **ii** no **iii** no **iv** yes

d A number of the form $-x^a$ will always produce a negative result regardless of whether the power is odd or even. A negative base raised to an odd power will also produce a negative result.

EX p45 1H Order of operations

1 a multiplication, subtraction, addition
b subtraction (within brackets), division, addition
c square root, multiplication, addition, subtraction
d squaring, multiplication, subtraction, addition
e addition (within inner brackets), subtraction (within outer brackets), addition
f addition (within brackets), squaring, division, addition

2 a 23 **b** 40 **c** 75 **d** 38 **e** 59 **f** 47

3 a −9 **b** 0 **c** −5 **d** 7 **e** −16 **f** 14

4 a 2 **b** 390 **c** 22 **d** 5 **e** 123
f 36 **g** 51 **h** 555

5 a 6 **b** −207 **c** −4 **d** −16 **e** 25 **f** 0

6 a 66 – 7 **b** 415 + 322 **c** 365 – 109
d 1750 + 3487 **e** 4376 – 237 **f** 6548 – 2399

7 a A negative has been added to the answer.
b Addition has been calculated before the multiplication.
c Addition has been calculated before the division.
d A negative divided by a negative is a positive.
e Negative 4 to the power of 3 is −64, so the answer is also negative.
f Ignored the negative in front of the 5.

8 a 64 **b** 10 **c** 0 **d** 4 **e** −8 **f** −21

9 C

10 a 195 – 3 × 58
b 200 – 3 × 60 = 20; \$20
c \$21; the estimate is \$1 less.

11 14 × 7 + 11 × 6 + 9 × 4 + 16 × 5 + 25 × 1 = \$305

12 a 13 **b** −6
c 13 – (−6) **d** 19 m

13 a

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Difference	6	7	2	4	5	3	1

b Monday
c i 1°C **ii** −3°C
d 4°C

14 a deposit **b** withdrawal
c i He has money in the bank.
ii He owes the bank money.
d −\$96 **e** \$134
f −\$27, a withdrawal of \$27
g Yes, from 21 May to 24 May. A fee and/or interest is charged.

15 Negative. For the sum of two numbers to be zero, they must have different signs. As one number is positive and the other negative, the product will be negative (+ × − → −), unless they were both zero.

16 −6, −4, 5

17 a 10 **b** 12

18 a Sample answer:

Number	Equation
0	4 ÷ 4 × 4 – 4
1	4 ÷ 4 + 4 – 4
2	4 – (4 + 4) ÷ 4
3	(4 × 4 – 4) ÷ 4
4	4 + 4 × (4 – 4)
5	(4 × 4 + 4) ÷ 4
6	(4 + 4) ÷ 4 + 4
7	4 + 4 – 4 ÷ 4
8	4 ÷ 4 × 4 + 4
9	4 ÷ 4 + 4 + 4
10	Not possible

Number	Equation
11	Not possible
12	4 × (4 – 4 ÷ 4)
13	Not possible
14	Not possible
15	4 × 4 – 4 ÷ 4
16	4 × 4 + 4 – 4
17	4 × 4 + 4 ÷ 4
18	Not possible
19	Not possible
20	4 × (4 ÷ 4 + 4)

b Sample answer (considering the numbers not possible in part **a**):

Number	Equation
10	$(4 + 4 + 4) - \sqrt{4}$
11	Not possible
13	Not possible

Number	Equation
14	$4 \times 4 - 4 \div \sqrt{4}$
18	$4 \times 4 + 4 - \sqrt{4}$
19	Not possible

c Sample answer (considering the numbers not possible in part **a** or **b**):

Number	Equation
11	$(4! \times \sqrt{4} - 4) \div 4$
13	$(4! \times \sqrt{4} + 4) \div 4$
19	4! – (4 + 4 ÷ 4)

d Place brackets around each calculation and add a negative sign in front. This will keep the value of the calculation the same but give a negative result.

EX p49

CHAPTER 1 review

Mathematical literacy review

1 compensation method

2 I (indices)

3 the whole-number result of the division

4 **a** integers
b estimate
c product

5 **a** The multiples of a whole number are the numbers produced by multiplying that number by another whole number. For example, the multiples of 5 are 5, 10, 15, 20, …
b The square root of a number is the number that when squared results in the original number. For example, the square root of 49 is 7, as $7 \times 7 = 49$.
c A list of numbers are written in ascending order when they are in order from smallest to largest. For example, the list 23, 27, 29, 35, 40 is written in ascending order.

6 **a** The commutative law of addition states that the order in which numbers are added does not change the result.
b The factors of a whole number are the whole numbers which divide exactly into the number.

Multiple choice

1 B	**2** D	**3** E	**4** D	**5** A	**6** D
7 C	**8** E	**9** D	**10** A	**11** E	**12** A
13 E	**14** C	**15** D	**16** B	**17** E	**18** A
19 D	**20** A				

Short answer

1 **a** 200 **b** 70 000 **c** 4000
d 100 000

2 **a** 14 000 **b** 60 000 **c** 120 000 **d** 50

3 \$2 296 240

4 312 398

5 168

6 11

7 18

8 **a** 216 **b** 128 **c** 64 **d** 121

9 **a** 2 **b** 9 **c** 3 **d** 5

10 **a** $-5 < 2$ **b** $-5 < -2$
c $7 > -4$ **d** $4 > -7$

11 **a** −40, −24, −22, −14, 16, 20, 24
b −35, −27, −17, 0, 9, 21, 63
c −66, −46, −26, −16, −6, 6, 86
d −409, −300, −87, 3, 45, 65, 190

12 **a** −9 **b** +9 **c** −12 **d** +19

13 \$122

14 21°C

15 **a** −1 **b** −10 **c** −8 **d** −10
e 0 **f** −100

16 **a** −56 **b** +45 **c** −9 **d** +5
e 0 **f** −60 **g** −3 **h** +14

17 **a** 180 **b** −35 **c** 12 **d** −40

18 **a** −32 **b** 9 **c** −1 **d** 10 000

19 **a** 21 **b** 22 **c** −130 **d** −10

20 **a** −180 **b** 43 **c** −11 **d** −8

Analysis

1 **a** 2700 kg, 1800 kg **b** 900 kg **c** 4500 kg **d** 900 kg
e 450 kg **f** 270 kg and 180 kg
g 1795 kg, 1775 kg, 1675 kg
h The elephant should be losing 5 kg each day, but Jez's expression suggests the elephant lost 20 kg on the second day and 100 kg on the third day.
i $2 \times 3^2 \times 10^2 - n \times 5$

2 **a** 1 469 200
b 8 000 000, 7 000 000, 5 000 000, 3 000 000, 2 000 000, 500 000, 400 000, 200 000
c 1 000 000; the difference between the estimated and the actual value is 469 200 people
d 26 100 000

3 **a** **i** Wednesday **ii** Friday
b Monday: 7°C, Tuesday: 7°C, Wednesday: 11°C, Thursday: 7°C, Friday: 4°C, Saturday: 7°C, Sunday: 6°C
c Wednesday
d average minimum temperature: −2°C, average maximum temperature: 5°C
e 7°C

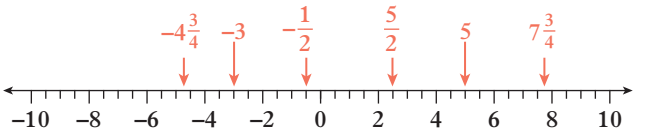
CHAPTER 2 Fractions and decimals

EX p59

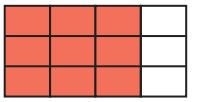
2A Fractions

1 **a** $\frac{1}{-2} = -\frac{1}{2} = \frac{-1}{2}$ **b** $\frac{-10}{3} = -\frac{10}{3} = \frac{10}{-3}$
c $\frac{-5}{-6} = \frac{5}{6}$ **d** $\frac{-2}{-3} = \frac{2}{3}$
e $-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$ **f** $-\frac{-3}{-7} = -\frac{3}{7} = \frac{-3}{7} = \frac{3}{-7}$

2



3 **a** $\frac{3}{4}$
b Sample answer:



c $\frac{9}{12}$

4 **a** $\frac{3}{10} = \frac{6}{20}$ **b** $\frac{2}{7} = \frac{10}{35}$ **c** $\frac{18}{33} = \frac{6}{11}$ **d** $\frac{16}{48} = \frac{2}{6}$
e $-\frac{4}{5} = -\frac{24}{30}$ **f** $\frac{-8}{36} = \frac{-2}{9}$ **g** $-1\frac{3}{8} = -\frac{55}{40}$
h $3\frac{3}{4} = \frac{75}{20}$ **i** $\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = \frac{8}{32} = \frac{20}{80} = \frac{25}{100}$
j $-\frac{3}{7} = \frac{-15}{35} = \frac{-21}{49} = \frac{-30}{70} = \frac{-300}{700} = -\frac{90}{210}$

k $1\frac{1}{5} = \frac{12}{10} = \frac{60}{50} = \frac{300}{250} = \frac{54}{45} = \frac{72}{60}$

l $-2\frac{1}{3} = -\frac{14}{6} = \frac{-35}{15} = \frac{-49}{21} = -\frac{21}{9} = -\frac{700}{300}$

5 **a** $\frac{8}{20}, \frac{22}{55}, \frac{4}{10}, \frac{10}{25}, \frac{18}{45}$ **b** $\frac{20}{15}, \frac{28}{21}, 1\frac{4}{12}$

6 **a** $\frac{3}{8} < \frac{1}{2}$ **b** $\frac{4}{7} > \frac{5}{14}$ **c** $\frac{-9}{10} < \frac{-4}{5}$

d $-\frac{5}{12} > -\frac{2}{3}$ **e** $1\frac{5}{6} > 1\frac{19}{24}$ **f** $\frac{17}{15} < \frac{7}{3}$

g $-\frac{2}{3} > -\frac{3}{4}$ **h** $\frac{5}{6} > \frac{4}{5}$ **i** $-\frac{11}{8} > -\frac{5}{3}$

j $2\frac{3}{4} < \frac{17}{6}$ **k** $-\frac{29}{3} < -9\frac{1}{2}$ **l** $3\frac{5}{12} > \frac{25}{8}$

7 **a** $\frac{7}{9}$ **b** $\frac{11}{4}$ **c** $-\frac{12}{5}$

d $-\frac{43}{6}$ **e** $3\frac{7}{11}$ **f** $-8\frac{1}{2}$

g $\frac{16}{3}$ **h** $-\frac{55}{9}$ **i** $3\frac{5}{7}$

8 **a** $\frac{4}{5}$ **b** $\frac{2}{3}$ **c** $\frac{7}{9}$

d $\frac{5}{8}$ **e** $\frac{10}{3}\left(\text{or } 3\frac{1}{3}\right)$ **f** $\frac{11}{6}\left(\text{or } 1\frac{5}{6}\right)$

9 **a** $\frac{2}{5}$ **b** $-\frac{3}{7}$ **c** $-\frac{2}{5}$

d $-\frac{5}{3}\left(\text{or } -1\frac{2}{3}\right)$ **e** $\frac{8}{9}$ **f** $\frac{3}{7}$

g $3\frac{3}{4}$ **h** $-5\frac{5}{9}$ **i** $4\frac{3}{11}$

j $-1\frac{4}{7}$ **k** $2\frac{3}{8}$ **l** $-7\frac{5}{6}$

10 **a** $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{7}{5}$

b $\frac{3}{8}, \frac{9}{8}, 1\frac{1}{4}, \frac{7}{4}, 2\frac{5}{8}, 2\frac{3}{4}$

c $-\frac{9}{2}, -1\frac{9}{10}, -\frac{9}{5}, -1\frac{1}{2}, -\frac{2}{5}, -\frac{3}{10}$

11 **a** $\frac{19}{11}, 1\frac{5}{11}, \frac{13}{11}, \frac{9}{11}, \frac{5}{11}, \frac{3}{11}$

b $-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -1\frac{1}{4}, -\frac{4}{3}, -1\frac{1}{2}$

c $1\frac{1}{3}, \frac{5}{6}, \frac{1}{6}, -\frac{5}{12}, -\frac{11}{6}, -2\frac{1}{6}$

12 **a** $\frac{3}{5}$ **b** $\frac{2}{9}$ **c** $\frac{7}{24}$ **d** $\frac{3}{10}$

e $\frac{1}{6}$ **f** $\frac{2}{5}$ **g** $\frac{2}{7}$ **h** $\frac{3}{20}$

i $\frac{7}{15}$ **j** $\frac{3}{17}$ **k** $\frac{1}{30}$ **l** $\frac{2}{15}$

13 **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{5}{8}$ **d** $\frac{2}{3}$

14 $\frac{1}{4}$

15 **a** $\frac{3}{4}, \frac{6}{8} = \frac{3}{4}, \frac{9}{12} = \frac{3}{4}, \frac{12}{16} = \frac{3}{4}, \frac{15}{20} = \frac{3}{4}, \frac{18}{24} = \frac{3}{4}, \frac{21}{28} = \frac{3}{4}, \frac{24}{32} = \frac{3}{4}, \frac{27}{36} = \frac{3}{4}$

b All of the fractions made by dividing a multiple of 3 by the corresponding multiple of 4 will simplify to $\frac{3}{4}$ as they are equivalent fractions.

c $\frac{7}{5}, \frac{14}{10} = \frac{7}{5}, \frac{21}{15} = \frac{7}{5}, \frac{28}{20} = \frac{7}{5}, \frac{35}{25} = \frac{7}{5}, \frac{42}{30} = \frac{7}{5}, \frac{49}{35} = \frac{7}{5}, \frac{56}{40} = \frac{7}{5}, \frac{63}{45} = \frac{7}{5}$

d All of the fractions made by dividing a multiple of 7 by the corresponding multiple of 5 will simplify to $\frac{7}{5}$ as they are equivalent fractions.

16 **a** $\frac{4}{7}$ **b** $-\frac{7}{11}$ **c** $\frac{1}{2}$ **d** $-\frac{4}{9}$

e $\frac{1}{3}$ **f** $-\frac{1}{8}$ **g** $\frac{1}{24}$ **h** $2\frac{14}{15}$

17 The first test

18 15

19 **a** **i** Since $\frac{4}{7} > \frac{1}{2}$ and $\frac{7}{15} < \frac{1}{2}$, we can compare the fractions using the benchmark $\frac{1}{2}$.

ii $\frac{4}{7} > \frac{7}{15}$

b **i** Since the numerators are the same, we can compare the denominators.

ii $\frac{15}{17} < \frac{15}{4}$

c **i** Both fractions are one piece away from 1; we can compare the difference between the fractions and the benchmark 1.

ii $\frac{99}{100} < \frac{100}{101}$

d **i** Both mixed numbers have a whole number part of 1, so we only need to compare the fraction parts. Since the numerators are the same, we can compare the denominators.

ii $1\frac{1}{8} > 1\frac{1}{11}$

e **i** Since the numerators and denominators are quite small, finding any equivalent fractions will be simple.

ii $1\frac{3}{5} < 1\frac{2}{3}$

f **i** Since 10 is a multiple of 5, we can compare using an equivalent fraction of $\frac{39}{5}$ with a denominator of 10.

ii $\frac{77}{10} < \frac{39}{5}$

20 **a** Ash (lower value is better rate in serving)

b 10 **c** 153

21 **a** D

b Answers will vary but may include some of the strategies from question **19**.

EX p66 2B Adding and subtracting fractions

1 **a** The first number is written more simply as $\frac{1}{7}$ and the operations + (−) can be replaced with −.

b $-\frac{2}{7}$

2 **a** $\frac{13}{17}$ **b** $\frac{2}{5}$ **c** 1 **d** $\frac{1}{3}$

3 **a** 15 **b** 14 **c** 30 **d** 24

e 15 **f** 20 **g** 12 **h** 18

4 **a** $\frac{21}{22}$ **b** $\frac{7}{20}$ **c** $1\frac{17}{30}$ **d** $\frac{7}{24}$

e $\frac{1}{30}$ **f** $1\frac{7}{12}$ **g** $\frac{7}{18}$ **h** $\frac{31}{40}$

5 **a** $\frac{4}{7}$ **b** $-2\frac{1}{3}$ **c** $-\frac{2}{8} = -\frac{1}{4}$ **d** $-\frac{2}{5}$

e $-1\frac{2}{3}$ **f** $-1\frac{1}{4}$ **g** $\frac{2}{9}$ **h** $-1\frac{1}{10}$

6 **a** $-\frac{1}{5}$ **b** $-\frac{3}{7}$ **c** $-\frac{9}{11}$

d $-1\frac{1}{6}$ **e** $-\frac{3}{14}$ **f** $\frac{1}{8}$

7 **a** $\frac{3}{8}$ **b** $3\frac{1}{4}$ **c** $4\frac{7}{15}$ **d** $2\frac{3}{22}$

e $7\frac{5}{12}$ **f** $2\frac{7}{10}$ **g** $-\frac{2}{3}$ **h** $-5\frac{7}{10}$

8 **a** $-\frac{1}{3}$ **b** 0 **c** $-\frac{2}{3}$ **d** $-\frac{3}{20}$

e $\frac{7}{18}$ **f** $-2\frac{6}{7}$ **g** $1\frac{11}{24}$ **h** $-\frac{1}{20}$

9 **a** **i** $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$

b **i** $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$

c Each diagonal adds to $1\frac{1}{2}$.

d They are all the same. **e** $1\frac{1}{2}$

10 **a**

$\frac{2}{15}$	$\frac{7}{15}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
$\frac{4}{15}$	$\frac{1}{5}$	$\frac{8}{15}$

b

$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	$\frac{7}{24}$
$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{4}$

c

$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$
0	$-\frac{5}{6}$	$\frac{1}{3}$

11 **a** positive **b** negative **c** positive **d** zero

e negative **f** negative **g** zero **h** negative

12 **a** $1\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $-\frac{1}{2}$ **d** $-1\frac{1}{2}$

e $10\frac{1}{4}$ **f** $-10\frac{1}{4}$

13 It is better to convert to improper fractions first when the fraction part of the mixed number you are subtracting is greater than the fraction part of the mixed number you are subtracting from.

14 $\frac{1}{3} + 1\frac{3}{4} = 2\frac{1}{12}$ L; the container is not large enough.

15 $4\frac{7}{12}$ hours

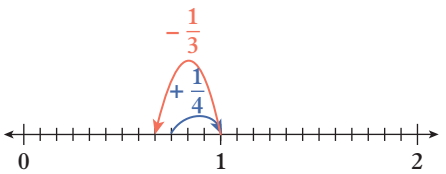
16 **a** $\frac{3}{10}$ **b** $\frac{29}{30}$ **c** $\frac{1}{30}$

17 $\frac{1}{6}$

18 **a** $\frac{13}{125}$ inches **b** The $\frac{41}{250}$ inch screw

19 **a** **i** B **ii** D **iv** A **v** C

E



iii $\frac{3}{2} + \frac{1}{6} = \left(\frac{3}{2} + \frac{3}{6}\right) - \frac{2}{6} = 2 - \frac{1}{3} = 1\frac{2}{3}$

b Each calculation partitioned the second fraction into the part that would make a whole number and the remainder to avoid changing denominators multiple times. This is the compensation method for fractions.

20 D

EX p72 2C Multiplying and dividing fractions

1 **a** $\frac{9}{20} \times \frac{5}{3}$ **b** $\frac{9}{20} \times \frac{15}{7}$ **c** $\frac{20}{9} \times \frac{7}{15}$ **d** $\frac{20}{9} \times \frac{15}{7}$

e $\frac{2}{9} \times \frac{4}{7}$ **f** $\frac{2}{9} \times \frac{21}{5}$ **g** $\frac{9}{2} \times \frac{5}{21}$ **h** $\frac{9}{2} \times \frac{21}{5}$

2 **a** $\frac{3}{20}$ **b** $\frac{6}{5}$ $\left(\text{or } 1\frac{1}{5}\right)$ **c** $\frac{7}{10}$ **d** $\frac{12}{35}$

e $\frac{1}{50}$ **f** 9 **g** $\frac{20}{21}$ **h** 1

3 **a** $\frac{9}{8}$ or $1\frac{1}{8}$ **b** 4 **c** 10

d $\frac{3}{4}$ **e** $12\frac{4}{5}$ or $\frac{64}{5}$ **f** 6

g $2\frac{5}{8}$ or $\frac{21}{8}$ **h** $32\frac{1}{2}$ or $\frac{65}{2}$ **i** 144

j $60\frac{1}{5}$ or $\frac{301}{5}$ **k** $\frac{40}{23}$ **l** $\frac{125}{39}$

4 **a** $\frac{10}{21}$ **b** $\frac{5}{8}$ **c** $\frac{3}{8}$ **d** $4\frac{1}{2}$ or $\frac{9}{2}$

e 7 **f** $\frac{1}{7}$ **g** $1\frac{4}{5}$ or $\frac{9}{5}$ **h** 40

5 **a** $6\frac{3}{5}$ or $\frac{33}{5}$ **b** $\frac{9}{16}$ **c** $\frac{1}{2}$ **d** $\frac{2}{9}$

e $1\frac{1}{2}$ or $\frac{3}{2}$ **f** $\frac{2}{7}$ **g** $\frac{17}{28}$ **h** 14

i $\frac{28}{17}$ or $1\frac{11}{17}$ **j** $4\frac{5}{27}$ or $\frac{113}{27}$ **k** $\frac{3}{26}$ **l** $\frac{31}{10}$

6 **a** 7 hours **b** \$36 **c** 14 kg

d 32 cm **e** 6 days **f** 12 mm

7 **a** $\frac{4}{11}$ **b** $-\frac{1}{3}$ **c** $-1\frac{1}{5}$

d $\frac{108}{245}$ **e** $-\frac{7}{12}$ **f** $-\frac{5}{27}$

g -15 **h** $\frac{2}{3}$ **i** $-1\frac{1}{2}$

8 **a** $1\frac{5}{11}$ **b** $\frac{3}{2}$ $\left(\text{or } 1\frac{1}{2}\right)$ **c** $-\frac{1}{12}$ **d** $8\frac{11}{12}$

9 **a** -1 **b** $\frac{3}{7}$ **c** $-\frac{13}{14}$

d -1 **e** $\frac{1}{3}$ **f** $-\frac{3}{14}$

10 66

11 **a** **i** 200 g butter, 1 cup sugar, 4 large eggs, 3 cups flour, 2 tsp baking powder, 1 cup milk, $1\frac{1}{2}$ cups cocoa powder, 2 cups boiling water

ii 50 g butter, $\frac{1}{4}$ cup sugar, 1 large egg, $\frac{3}{4}$ cup flour, $\frac{1}{2}$ tsp baking powder, $\frac{1}{4}$ cup milk, $\frac{3}{8}$ cup cocoa powder, $\frac{1}{2}$ cup boiling water

iii $66\frac{2}{3}$ g butter, $\frac{1}{3}$ cup sugar, $1\frac{1}{3}$ large eggs, 1 cup flour, $\frac{2}{3}$ tsp baking powder, $\frac{1}{3}$ cup milk, $\frac{1}{2}$ cup cocoa powder, $\frac{2}{3}$ cup boiling water

b $1\frac{1}{8}$ cups icing sugar, $112\frac{1}{2}$ g butter, 90 g chocolate, $\frac{3}{8}$ cup thickened cream

12 **a** 8 **b** 29 kg

13 **a** 24 buttons **b** 25 red buttons **c** $\frac{5}{4}$

14 **a** 36 **b** 147 **c** $42\frac{2}{3}$ **d** $36\frac{4}{5}$

15 **a** $5\frac{2}{19}$ **b** 59 **c** $15\frac{2}{5}$ **d** $4\frac{6}{65}$

16 **a** $\frac{8}{125}$ **b** $\frac{16}{81}$ **c** $-\frac{8}{27}$ **d** $\frac{81}{49}$ **e** $\frac{1000}{27}$

17 **a** $\frac{4}{5}$ **b** $\frac{7}{3}$ **c** $\frac{9}{10}$ **d** $\frac{5}{3}$ **e** $\frac{11}{6}$

18 **a** $18\frac{17}{18}$ **b** $78\frac{8}{15}$ **c** $39\frac{1}{5}$ **d** $18\frac{115}{168}$

19 By writing a division calculation as a multiplication by the reciprocal, we can apply the distributive law to the dividend (left fraction) by multiplying the whole part and the fraction part of the sum by the reciprocal of the divisor (right fraction). We can then confirm this is equivalent to dividing each part by the original divisor by writing the multiplications as divisions by the reciprocal. However, we cannot apply the distributive law to the divisor as it is no longer the sum of 5 and $\frac{2}{3}$ after we get its reciprocal.

$$3\frac{9}{10} \div 5\frac{2}{3} = 3\frac{9}{10} \div \frac{17}{3} = 3\frac{9}{10} \times \frac{3}{17}$$
$$= \left(3 \times \frac{3}{17}\right) + \left(\frac{9}{10} \times \frac{3}{17}\right)$$
$$= \left(3 \div 5\frac{2}{3}\right) + \left(\frac{9}{10} \div 5\frac{2}{3}\right)$$

20 a C and E

b Multiplying fractions that do not have a common denominator already will generally have larger numerators and denominators to then multiply after finding the equivalent fractions with common denominators. The quotient will also then need to be simplified. The same applies to raising a fraction to a power.

c Addition and subtraction need the same-sized pieces (denominator) to calculate the sum or difference. When dividing fractions, if the divisor has a common denominator with the dividend, then they will divide to 1: $\frac{5}{10} \div \frac{3}{10} = \frac{5}{\cancel{10}^1} \times \frac{\cancel{10}^1}{3} = \frac{5}{3}$.

EX p76 Checkpoint

1 a $\frac{6}{10} = \frac{48}{80}$ **b** $\frac{55}{20} = 2\frac{3}{4}$ **c** $\frac{63}{18} = \frac{28}{8}$ **d** $\frac{6}{5} = \frac{-18}{-15}$

2 a $\frac{4}{5}$ **b** $\frac{4}{3}$ **c** $8\frac{1}{4}$ **d** $-\frac{5}{9}$

3 a $\frac{2}{3} < \frac{3}{4}$ **b** $\frac{7}{9} > -\frac{8}{9}$ **c** $-\frac{17}{10} > -\frac{9}{5}$ **d** $\frac{30}{36} > \frac{36}{54}$

4 a 12 **b** 18 **c** 24 **d** 33

5 a $\frac{16}{15} = 1\frac{1}{15}$ **b** $\frac{35}{9} = 3\frac{8}{9}$

c $\frac{56}{25} = 2\frac{6}{25}$ **d** $-\frac{99}{28} = -3\frac{15}{28}$

6 a $-\frac{1}{12}$ **b** $-\frac{101}{15} = -6\frac{11}{15}$

c $\frac{43}{48}$ **d** $-\frac{25}{21} = -1\frac{4}{21}$

7 a 10 **b** $-\frac{32}{27} = -1\frac{5}{27}$

c $-2\frac{1}{5}$ **d** $\frac{40}{9} = 4\frac{4}{9}$

8 a $\frac{5}{3}$ **b** 7 **c** $\frac{1}{9}$ **d** $\frac{3}{17}$

9 a $\frac{3}{5}$ **b** -7 **c** $-\frac{16}{35}$ **d** $\frac{8}{9}$

10 a $\frac{27}{25} = 1\frac{2}{25}$ **b** $-\frac{7}{10}$ **c** $-\frac{65}{36} = -1\frac{29}{36}$ **d** $\frac{27}{175}$

11 a 65 GB **b** $7\frac{1}{2}$ L **c** 750 g **d** \$4.25

EX p80 2D Decimals

1

	Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
a		0	.	0	0	6	5
b		0	.	6	5	0	0
c		6	.	0	0	5	
d	6	0	.	5			

2 a 5 thousandths or $\frac{5}{1000}$ **b** 5 tens or 50

c 5 ones or 5 **d** 5 tenths or $\frac{5}{10}$

e 5 ten thousandths or $\frac{5}{10\,000}$

f 5 hundredths or $\frac{5}{100}$

3 a 3 **b** 3 **c** 1 **d** 2 **e** 4 **f** 5

4 a 0.104, 0.14, 0.142, 0.2, 0.214, 0.41

b 8.072, 8.2, 8.702, 8.97, 9.782, 9.87

c –15.40, –10.45, 10.54, 15.04, 105.4, 150.4

d –800.8, –800.08, –800.07, –799.8, 799.08, 800.7

5 a 13.713, 13.7, 13.31, 13.173, 13.137, 13.07

b 0.685, 0.658, 0.6508, 0.65, 0.6085, 0.6058

c 555.55, 5.5555, 5.55000, 5.00500, –50.0505, –50.5050

d 999.90, 999.099, 100, –99.009, –99.90, –100

6 Comparing each place value from left to right, 9.46 contains 6 hundredths whereas 9.452 only contains 5 hundredths. (The first two place values are identical.)

7 a 25.68 **b** −673.82 **c** 0.78

d −101.02 **e** 8148.26 **f** −54.26

g 330.41 **h** −4.85 **i** 16.00

8 a 57.23 **b** 8.914 **c** 92.0871

d −0.757 **e** −135.2 **f** 65.071 23

g 7.510 **h** −208.0099 **i** 1.727 73

j 845.8 **k** −0.1235 **l** −0.123 456 8

9 a $1\frac{3}{100}$ **b** $2\frac{9}{10}$ **c** $3\frac{7}{1000}$ **d** $-\frac{1}{10\,000}$

e $10\frac{7}{10}$ **f** $-\frac{3}{100\,000}$ **g** $12\frac{9}{1\,000\,000}$

h $-\frac{7}{10\,000\,000}$

10 a $\frac{3}{25}$ **b** $-\frac{27}{50}$ **c** $\frac{803}{1000}$ **d** $-\frac{1}{25}$ **e** $\frac{1}{40}$

f $-\frac{3}{5000}$ **g** $-\frac{37}{50}$ **h** $\frac{63}{250}$ **i** $-\frac{3}{2000}$

j $-\frac{6}{125}$ **k** $\frac{17}{100\,000}$ **l** $-\frac{17}{20}$

11 a 0.7 **b** −0.29 **c** 0.451 **d** −0.2047

e 0.04 **f** −0.009 **g** 0.0005 **h** −0.063

12 a 0.8 **b** 0.625 **c** −0.5 **d** 1.25

e 0.125 **f** −0.4 **g** 0.35 **h** −0.36

i 0.025 **j** −0.6 **k** 0.22 **l** −0.3125

13 Rosa; the digit in the fourth decimal place is 5 so you must round up.

14 Jayden; the digit in the third decimal place is 6 so you must round up. Adding one to the digit in the second decimal place gives 10 which means you write 0 in the

second decimal place and add one to the first decimal place (7 + 1 = 8).

15 Kota; Peter correctly rounded to 16 but did not write the two decimal places that were asked for, so it appears he rounded to the nearest whole number.

16 Sabine has not rounded directly to one decimal place. For this number, rounding to two decimal places meant that the digit in the hundredths place changed from 4 to 5 which results in rounding up rather than rounding down.

17 0.7 cm

18 **a** 1.568 m **b** 1.657 m
c 1.568 m, 1.58 m, 1.615 m, 1.63 m, 1.651 m, 1.657 m
d 1.65 m, 1.63 m, 1.57 m, 1.62 m, 1.58 m, 1.66 m
e 1.57 m, 1.58 m, 1.62 m, 1.63 m, 1.65 m, 1.66 m; the measurements are still all different and there is no change to the order.
f 1.7 m, 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m
g 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m, 1.7 m; there are only two different measurements.

19 **a** 0.02 mm **b** 0.018 mm **c** 0.0184 mm

20 **a** **i** 0.3 mm **ii** 0.28 mm **iii** 0.285 mm
b $\frac{569}{2000}$ mm

21 **a** \$42.35 **b** \$207.60 **c** \$7.25
d \$15.05 **e** \$60.70 **f** \$100.00

22 Sample answer: 0.918273645

EX 2E Adding and subtracting decimals

p86

1 **a** 10.1 **b** 26.25
c 2.052 **d** 29.38

2 **a** 247.96 **b** 33.7686 **c** 690.984
d 106.5953 **e** 0.2776 **f** 119.0154

3 **a** 82.83 **b** 785.654 **c** 0.1344 **d** 26.251

4 **a** 2.7312 **b** 177.196 **c** 3794.295
d 64668.44 **e** 8999.445 **f** 1.9999

5 **a** −2.6 **b** −9 **c** 2.6
d 48.266 **e** −3.9078 **f** −495.238

6 **a** −3.7 **b** −13.26 **c** 3.7
d −61.783 **e** −273.7973 **f** −1.209

7 **a** −0.9 **b** −8.74
c −82.85 **d** −0.2488

8 **a** −7.6 **b** 5.21
c 1.1 **d** −88.189 34

9 Estimate: 12 km, exact: 12.025 km

10 **a** **i** 368 cm **ii** 382.9 cm
b Sample answer: Left hand; Lee may be right-handed and use her left hand less, so there is less wear on the nails.
c 750.9 cm

11 **a** D
b When you expand the brackets using the distributive law, the calculation becomes 7.2 – 9.4 – 2.3; however, the original calculation calls for + 2.3. A similar expression that is equivalent to the original calculation is 7.2 + (−9.4 + 2.3).
c Not when one subtraction follows another. For example, the correct answer to 10 – 5 – 3 is 2, but if the second subtraction (5 – 3) is performed first the result is 10 – 2 = 8, which is incorrect.

12 **a** −13.3168 **b** −40.911
c 0.0472 **d** −163.0612

13 **a** Yes, \$20.61 extra **b** 3 weeks

14 The sum will have at most x decimal places.

15 **a**

$$-(3.12 + 4.2) = -3.12 + (-4.2)$$
$$= -3.12 - 4.2$$

If you find the sum of 3.12 and 4.2, then multiply by negative 1, you get the same answer as −3.12 – 4.2.

b

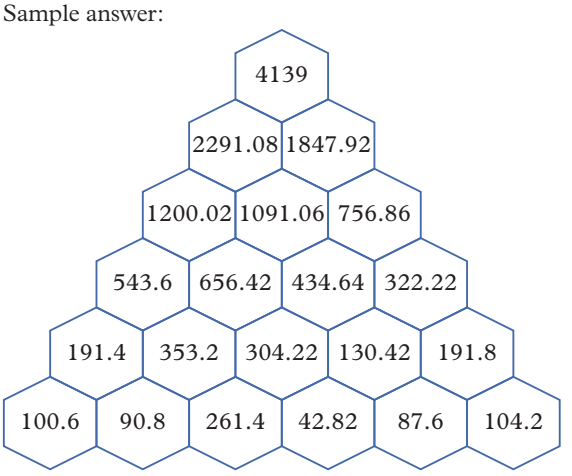
$$-(4.2 - 3.12) = -4.2 - (-3.12)$$
$$= -4.2 + 3.12$$
$$= 3.12 + (-4.2)$$
$$= 3.12 - 4.2$$

If you subtract 3.12 from 4.2, then multiply by negative 1, you get the same answer as 3.12 – 4.2.

16 **a** 21 – 13.9 = 7.1
b 161.67 – 100.28 = 61.39
c 9.319 – 14.78 = −5.461
d 92.08 – 181.88 = −89.8

17 **a** 81.178° clockwise **b** 519.628°

18 Sample answer:



EX 2F Multiplying and dividing decimals

p92

1 **a** 0.052 **b** 1040 **c** 9.21
d 0.034 **e** 0.000 57 **f** 0.045 78

2 **a** 194.8 **b** 1222.4 **c** 52 058.1
d 2.947 59

3 **a** 43.831 **b** −30.864 **c** 2281.44
d 112.5234 **e** −4.232 **f** −2.024 204
g 7.890 54 **h** 1124.4616

4 **a** 18.149 **b** −1.3448 **c** −8.3249
d 21.6985 **e** 0.591 25 **f** −0.156 75
g 0.0256 **h** −27.614 28

5 **a** 10 **b** 35.68
c To keep an equivalent division problem
d 8.92

6 **a** 72.4 ÷ 2 **b** 49.02 ÷ (−6) **c** 1976.4 ÷ 3
d −172.8 ÷ (−8) **e** 1436.8 ÷ 4 **f** −749.1 ÷ 6
g 48.72 ÷ (−15) **h** 280.8 ÷ 12 **i** −163.8 ÷ (−9)

7 **a** 36.2 **b** −8.17 **c** 658.8
d 21.6 **e** 359.2 **f** −124.85
g −3.248 **h** 23.4 **i** 18.2

8 **a** 1.875 **b** 11.552 **c** 10.8
d 19.322 **e** −30.568 **f** −49.998 078

9 **a** \$29.40 **b** \$5.85 **c** \$3.37
d \$9.88 **e** \$4.94 **f** \$11.99

10 **a** \$65 **b** \$65.45 **c** \$34.55

11 **a** −0.000 544 **b** −59.094 09
c 0.0004 **d** −8000

12 **a** **i** \$0.60 **ii** \$2.89 **iii** \$18.95
b **i** \$4.65 **ii** \$6.85 **iii** \$13.61
c **i** \$12.95 **ii** \$1.59 **iii** \$1.95

13 **a** box: \$0.75, individual: \$0.72
b Buying individual pencils, as the cost per pencil is less

14 **a** 400 g pack: \$0.92, loose: \$0.99
b William, as the cost is cheaper per 100 g
c Someone may need more or less than the 400 g in a package.

15 **a** −0.064 **b** −0.000 000 001
c 0.0081 **d** 97.656 25

16 **a** $6.3 \times 2 = 12.6$ **b** $9.2 \times 5 = 46$
c $0.49 \times 4 = 1.96$ **d** $7.4 \times \frac{4}{5} = 5.92$
e $0.041 \times \frac{2}{5} = 0.0164$ **f** $10.21 \times \frac{4}{25} = 1.6336$

17 perimeter = 20.27 cm, area = 18.5097 cm^2

18 Sample answer: $5.074 - 3.96 \times 1.28 = 0.0052$

EX p96

2G Terminating, non-terminating and recurring decimals

1 **a** N **b** T **c** N **d** T **e** N **f** N
g N **h** T **i** N **j** T **k** N **l** T

2 **c**, **f**, **g**, **k**

3 Sample answer: A terminating decimal 'terminates' or stops after a number of decimal places. A non-terminating decimal has an endless (infinite) number of decimal places. A recurring decimal is a special type of non-terminating decimal in which some or all of the digits after the decimal point form a recurring (repeating) pattern.

4 **a** $0.\dot{9}$ (or $0.\overline{9}$) **b** $-4.\overline{36}$ (or $-4.\dot{3}\dot{6}$)
c $87.2\dot{5}$ **d** $3.\overline{476}$ (or $3.\dot{4}7\dot{6}$)
e $-0.738\dot{2}$ **f** $16.19\dot{6}$
g $483.\dot{8}$ **h** $-6.72\overline{91}$ (or $6.72\dot{9}\dot{1}$)
i $-0.18\overline{367}$ (or $-0.18\dot{3}6\dot{7}$)
j $1.\overline{35628}$ (or $1.\dot{3}56\dot{2}\dot{8}$)
k $28.\overline{70}$ (or $28.\dot{7}\dot{0}$)
l $-9.23071\dot{4}$

5 **a** 0.111 111 111 111…
b 5.377 777 777 777…
c −14.828 282 828 282…
d −21.569 696 969 696…
e 8.104 104 104 104…
f 0.651 365 136 513…
g −232.684 444 444 444…
h 44.761 359 359 359…
i −94.570 202 020 202…
j −1.739 247 392 473…
k 11.812 111 111 111…
l −6.904 040 404 040…
m 14.241 341 341 341…
n 0.562 062 062 062…
o −139.274 527 452 745…

6 **a** 0.7 **b** −0.5 **c** $0.\dot{3}$ **d** 1.25
e $0.\overline{18}$ **f** −0.625 **g** $2.\dot{6}$ **h** −0.75

7 **a** −1.125 **b** 1.4
c $2.8\dot{3}$ **d** $-1.\overline{142857}$
e 3.6 **f** $-4.\dot{1}$ **g** $1.\overline{72}$ **h** $-2.41\dot{6}$

8 **a** **i** **a**, **b**, **d**, **f**, **h** **ii** **c**, **e**, **g**
b **c**, **e**, **g** **c** No

9 **a** $\frac{1}{2}$ **b** $\frac{4}{1}$ **c** $6\frac{1}{5}$ $\left(\text{or } \frac{31}{5}\right)$
d $\frac{37}{100}$ **e** $\frac{18}{1}$ **f** $2\frac{509}{1000}$ $\left(\text{or } \frac{2509}{100}\right)$

10 **a** They can be expressed as fractions.
b Yes, because they can be expressed as fractions.

11 **a** rational **b** rational **c** rational
d irrational **e** rational **f** irrational
g rational **h** rational **i** rational
j rational **k** irrational **l** rational

12 **a** Sample answer: 3.141 592 654
b **i** 3.14 **ii** 3.1416
iii 3.141 59 **iv** 3.141 593

13 **a**

Unit fraction	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{11}$
Decimal	$0.\dot{1}$	0.1	0.2	0.5	$0.1\dot{6}$	$0.\overline{09}$
Denominator's prime factorisation	3^2	2×5	5	2	2×3	11
Is 2 or 5 a factor of the denominator?	No	Yes	Yes	Yes	Yes	No
Any prime factors other than 2 and/or 5?	Yes	No	No	No	Yes	Yes
Is the decimal terminating?	No	Yes	Yes	Yes	No	No

Unit fraction	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{8}$	$\frac{1}{15}$
Decimal	0.25	0.05	$0.0\dot{6}$	0.125	$0.08\dot{3}$
Denominator's prime factorisation	2^2	$2^2 \times 5$	3×5	2^3	$2^2 \times 3$
Is 2 or 5 a factor of the denominator?	Yes	Yes	Yes	Yes	Yes
Any prime factors other than 2 and/or 5?	No	No	Yes	No	Yes
Is the decimal terminating?	Yes	Yes	No	Yes	No

b The fraction has a terminating decimal when the only factors of the denominator are powers of 2 or 5.

c **i** terminating, 0.04
ii non-terminating, $0.041\dot{6}$
iii non-terminating, $0.0\dot{5}$
iv terminating, 0.025
v terminating, 0.008

d **i** non-terminating, $3.\overline{428\,571}$
ii terminating, 0.21875
iii terminating, 0.7
iv non-terminating, $0.\overline{153\,846}$
v terminating, 2.8

14 **a** Sample answer: 1.414 213 562
b **i** 1.41 **ii** 1.4142
iii 1.414 214 **iv** 1.414 213 6
c One possible answer is 1.732 050 807.
d yes
e **i** 1.73 **ii** 1.7321
iii 1.732 05 **iv** 1.732 051

15 **a** **i** $0.\dot{1}$ **ii** $0.\dot{2}$ **iii** $0.\dot{3}$ **iv** $0.\dot{4}$
b recurring decimal
c **i** $0.\dot{5}$ **ii** $0.\dot{6}$ **iii** $0.\dot{7}$ **iv** $0.\dot{8}$

16 **a** 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
b **i** $0.\overline{09}$ **ii** $0.\overline{18}$ **iii** $0.\overline{27}$ **iv** $0.\overline{36}$
c The recurring digits are the multiples of 9. $\frac{9}{11} = 0.\overline{81}$

17 **a** 0.333 33…, 0.666 66…
b 0.999 99… **c** 1. 0.999 99… is equivalent to 1.

18 **a** 3.160 494, 3.140 845, 3.142 857, 3.141 593, 3.141 600, 3.141 057, 3.142 732
b π will sit at 3.14
c $\frac{355}{113}$
d Sample answer: It is an irrational number (non-terminating but not recurring decimal), so it cannot be written as an exact fraction or decimal.

19 **a** 1 IV, 2 III, 3 V, 4 VI, 5 II, 6 I
b To find another number with the same recurring part, so that we could subtract to eliminate the decimal part.
c **i** $\frac{2}{9}$ **ii** $\frac{7}{9}$ **iii** $\frac{7}{3}$ **iv** $\frac{17}{3}$

EX p102

CHAPTER 2 review

Mathematical literacy review

1 **a** lowest **b** lowest **c** highest

2 Sample answer: In a mixed number, the whole number part represents the total number of units or objects, while the fraction part represents a portion of a unit or object. The whole number part of the mixed number does not change during the simplifying process because it represents a quantity that cannot be further simplified and is separate from the fractional part of the mixed number.

3 Keep the first number the same (the dividend).
Change the division symbol to a multiplication symbol.
Flip the second number (the divisor) and write it as its reciprocal.

4 **a** True
b False. Multiplying by a fraction can make a number either larger or smaller, depending on the value of the fraction. For example, multiplying a number by $\frac{3}{2}$ makes a number larger.
c False. Non-terminating, non-repeating decimals cannot be written as fractions.
d True

5 Sample answer:
a A digit is a single numerical symbol used to represent a value; a decimal place refers to the position of a digit within a decimal number. For example, in the number 12.345, 1, 2, 3, 4 and 5 are digits and the number has three decimal places.
b A placeholder zero is a zero used to hold the place of a digit with a value of zero in a number. A trailing zero is a zero at the end of a number, after all the significant digits have been written. For example, in the number 1.030, the zero between 1 and 3 is a placeholder zero, the zero on the right of 3 is a trailing zero.
c A rational number is a number that can be expressed as a ratio of two integers, irrational numbers are non-terminating, non-repeating decimals that cannot be written as fractions.

Multiple choice

1 B **2** C **3** C **4** B **5** E **6** B
7 D **8** C **9** E **10** C **11** C

Short answer

1 **a** $\frac{7}{12}$ **b** $-1\frac{7}{8}\left(\text{or } -\frac{15}{8}\right)$
c $-3\frac{9}{16}$ **d** $5\frac{2}{3}$

2 **a** $\frac{29}{35}$ **b** $\frac{13}{30}$ **c** $6\frac{1}{33}$ **d** $2\frac{29}{35}$

3 **a** $\frac{6}{35}$ **b** $\frac{1}{2}$ **c** $21\frac{1}{4}$ **d** $\frac{9}{4}$ or $2\frac{1}{4}$

4 **a** $\frac{6}{7}$ **b** $-\frac{2}{3}$ **c** $-\frac{299}{9} = -33\frac{2}{9}$ **d** $\frac{18}{35}$

5 a $-\frac{1}{4}$ b $\frac{1}{15}$ c $-\frac{6}{7}$
d $4\frac{1}{2}$ e $-\frac{4}{35}$ f $-\frac{7}{8}$

6 a 24 km/h b $\frac{1}{3}$ of a cup
c 70 metres d 3 minutes

7 a 0.0012, 0.120, 0.123, 0.2, 0.213, 0.231
b 8.16, 8.9, 8.916, 9.16, 9.6, 9.916
c −5, −0.5, −0.05, 0.05, 0.50, 5
d −14.2, −12.04, −10.42, −2.41, −2.14, −0.421

8 a 0.717, 0.71, 0.7, 0.107, 0.0701, 0.017
b 2.721, 2.72, 2.7, 2.27, 2.127, 2.1
c 90, 89, 80.9, −0.89, −0.899, −9.8
d −0.01, −0.099, −0.1, −0.9, −0.99, −1

9 a 13.22 b −0.0012 c 54.546 d −181.2

10 a 143.67 b 23.48
c 265.7842 d 989.5993

11 a −3.83 b 4.182 c −2.313 d 114.78

12 a 25.2512 b −10.9315
c −19.722 d 416.758 52

13 a i terminating ii non-terminating
iii terminating iv non-terminating
b only part **ii**

14 a $0.\dot{3}$ or $0.\overline{3}$ b $5.\overline{385}$ or $5.\dot{3}8\dot{5}$
c $-45.68\overline{57}$ or $45.68\dot{5}\dot{7}$
d $-0.092\overline{451}$ or $-0.092\dot{4}5\dot{1}$

15 a −0.233 333 333 333… b 3.272 727 272 727…
c 46.358 787 878 787…
d −17.364 854 854 854…

16 a $0.0\dot{3}$ or $0.0\overline{3}$ b $-0.\dot{5}7142\dot{8}$ or $-0.\overline{571428}$
c $-9.\dot{5}$ or $-9.\overline{5}$ d $1.\dot{5}\dot{4}$ or $1.\overline{54}$

Analysis

1 a i Wednesday ii Friday
b 5.8°C
c Tuesday: 6.3°C, Wednesday: 8.7°C, Thursday: 5.9°C, Friday: 4.5°C, Saturday: 5.7°C, Sunday: 5.1°C
d Wednesday
e i −1.8°C ii 4.2°C
f 6.0°C

2 a \$39.30 b 13 c \$0.43
d \$35.88 e \$26.91
f yes, because it is a 33.3% discount
g i $\$1.99\dot{3}$ ii \$1.99

3 a 399 students
b i 378 students ii 198 students
c i $\frac{67}{266}$ ii 3 students above one-quarter
d i 417 students ii $\frac{139}{532}$
e 1600 students

CHAPTER 3 Percentages, ratios and rates

3A Percentages, fractions and decimals

EX p111

1 a 60% b 40% c 60% d 52%

2 a 40% b 60% c 40% d 48%

3 a 24% b 77% c 83% d 58%
e 7% f 46%

4 a $\frac{17}{100}$, 0.17 b $\frac{21}{100}$, 0.21
c $\frac{7}{20}$, 0.35 d $\frac{2}{25}$, 0.08
e $2\frac{3}{20}$, 2.15 f $\frac{49}{50}$, 0.98
g $1\frac{37}{50}$, 1.74 h $\frac{7}{25}$, 0.28

5 a $\frac{7}{100}$, 0.07 b $\frac{31}{1000}$, 0.031
c $1\frac{31}{50}$, 1.62 d $\frac{721}{1000}$, 0.721
e $3\frac{47}{100}$, 3.47 f $\frac{8109}{10\,000}$, 0.8109
g $\frac{1057}{10\,000}$, 0.1057 h $\frac{339}{5000}$, 0.0678

6 a 71%, $\frac{71}{100}$ b 28%, $\frac{7}{25}$
c 5%, $\frac{1}{20}$ d 8%, $\frac{2}{25}$
e 90%, $\frac{9}{10}$ f 31%, $\frac{31}{100}$
g 54%, $\frac{27}{50}$ h 95%, $\frac{19}{20}$

7 a 36.9%, $\frac{369}{1000}$ b 24.8%, $\frac{31}{125}$
c 1006%, $10\frac{3}{50}$ d 81%, $\frac{81}{100}$
e 705.7%, $7\frac{57}{1000}$ f 185.9%, $1\frac{859}{1000}$
g 1210%, $12\frac{1}{10}$ h 24.46%, $\frac{1223}{5000}$

8 a i $\frac{16}{20}$ ii $\frac{80}{100}$ iii 80%
b i $\frac{15}{20}$ ii $\frac{75}{100}$ iii 75%
c i $\frac{7}{10}$ ii $\frac{70}{100}$ iii 70%
d i $\frac{6}{10}$ ii $\frac{60}{100}$ iii 60%

9 a 0.6, 60% b 0.7, 70%
c 0.085, 8.5% d 0.92, 92%
e 1.3, 130% f 1.5, 150%
g 0.0275, 2.75% h 3.5, 350%

10 a 225% b 600.5% c 730% d 375%
e 428% f 580% g 102.4% h 1326%

11 a 62.5% or 0.625 b 67.5% or 0.675
c $533\frac{1}{3}$% or $5.\dot{3}$ d 18.75% or 0.1875
e 8.8% or 0.088 f $322\frac{2}{9}$% or $3.\dot{2}$
g $73\frac{1}{3}$% or $0.7\dot{3}$ h $142\frac{6}{7}$% or $1.\overline{428\,571}$

12 a $\frac{1}{400}$, 0.0025 b $\frac{11}{300}$, $0.03\dot{6}$
c $\frac{1}{160}$, 0.00625 d $\frac{1}{40}$, 0.025
e $\frac{11}{125}$, 0.088 f $\frac{47}{300}$, $0.15\dot{6}$
g $\frac{8}{225}$, $0.03\dot{5}$ h $\frac{79}{600}$, $0.131\dot{6}$

13 a $0.46 < \frac{1}{2}$ b $\frac{3}{5} > 57\%$
c $0.95 > 71\%$ d $59\% > \frac{4}{7}$

e $\frac{5}{4} > 122\%$ **f** $252\% < 21.2$

g $13.4 > 1\frac{3}{4}$ **h** $1\frac{1}{6} > 206\%$

14 a $\frac{3}{5} = \frac{60}{100} = 60\%$ **b** $\frac{9}{20} = \frac{45}{100} = 45\%$

c $\frac{17}{25} = \frac{68}{100} = 68\%$ **d** $\frac{46}{200} = \frac{23}{100} = 23\%$

e $\frac{17}{20} = \frac{85}{100} = 85\%$ **f** $\frac{1}{5} = \frac{20}{100} = 20\%$

g $\frac{144}{600} = \frac{24}{100} = 24\%$ **h** $\frac{75}{250} = \frac{3}{10} = \frac{30}{100} = 30\%$

15 a 80% **b** 81% **c** 92%

d 88% **e** 71.5% **f** $66\frac{2}{3}\%$

16 a Both are correct.

b Sample answers include: shade 30 out of 200, shade 60 out of 400.

17 As the capacity of each container is different, 50% (or 50 mL out of every 100 mL) would be a different volume of liquid for each.

18 All percentages can be converted to fractions with the numerator and denominator both being integers, therefore all percentages are rational numbers.

19 a $\frac{3}{4}, \frac{4}{5}, 0.89, 1.3, \frac{7}{5}, 200\%$

b $4\frac{3}{10}\ 4.9, \frac{22}{4}, 560\%, \frac{13}{2}, 7.2$

c $0.28, 65\%, \frac{3}{2}, 1\frac{4}{5}, \frac{16}{5}, 340\%$

20 a 66% **b** $55\frac{1}{2}\%$ **c** $79\frac{2}{3}\%$ **d** 83.1%

e 24% **f** $4\frac{2}{5}\%$ **g** 74.7% **h** 20.44%

21 a $\frac{3}{13}$ **b** 0.2308 **c** 23.08%

d i 30.77% **ii** 7.69%

iii 38.46% **iv** 84.62%

22 $\frac{21}{400}$

23 Sample answer: $0, \frac{10}{11}\%, 0.52, \frac{7}{11}, 0.76, \frac{88}{115}, 100\%$

24 a 10 cm = 100 mm, so a length of 1 mm would represent 1%. For 80%, shade 80 mm (or 8 cm); for 92%, shade 92 mm (or 9.2 cm); for 25%, shade 25 mm (or 2.5 cm); for 17%, shade 17 mm (or 1.7 cm); for 43%, shade 43 mm (or 4.3 cm); for 66%, shade 66 mm (or 6.6 cm).

b Double the percentage amount. For example, 80% means 80 out of 100 or 160 out of 200. Shade 160 of the 200 squares in the grid to represent 80%.

EX p118

3B Calculating percentages

1 a 25% **b** 90% **c** 76% **d** 75%

e 120% **f** 125%

2 a i 80% **ii** 20%

b i 70% **ii** 30%

c i 30% **ii** 70%

d i 90% **ii** 10%

3 a i $17\frac{1}{2}\%$ **ii** 17.5%

b i $\frac{1}{5}\%$ **ii** 0.2%

c i $36\frac{4}{5}\%$ **ii** 36.8%

d i $28\frac{1}{2}\%$ **ii** 28.5%

e i $72\frac{1}{2}\%$ **ii** 72.5%

f i $587\frac{1}{2}\%$ **ii** 587.5%

4 a 23% **b** 25% **c** 35%

d 70% **e** 7.5%

5 a $1\frac{1}{4}\%$ or 1.25% **b** 10%

c 25% **d** 75%

e 10% **f** $6\frac{1}{4}\%$ or 6.25%

g 3% **h** 25%

6 a 84 **b** 756 **c** 9 **d** 1152

e 725 **f** 112.5

7 a 453.6 m **b** 135 seconds

c 231 cm **d** 100.8 L

e 76 buttons **f** 11.76 mL

8 a \$360 **b** \$1250 **c** \$100 **d** \$1575

e \$16 **f** \$576.45 **g** \$110.50 **h** \$448.50

i \$69.30 **j** \$129.15 **k** \$41.29 **l** \$3690

9 a \$4480 **b** \$435 **c** 72 kg

d 1.56 km **e** 45 minutes **f** 4.9 L

10 a 510 **b** 640 **c** 1820 **d** 160

11 a $\frac{89}{420}$ **b** 21.19%

c i 26.97% **ii** 74.17% **iii** 127.27%

d Sample answer: Big Jake may be 252 cm and Thumbelina may be 53 cm.

e Assuming the proportions of the horses remain the same, using the heights in part **d**, the percentage from part **b** changes to 21.03% (not much difference!).

12 a 10 **b** 40 **c** 8

d Part **c** calculates 20% of the remaining cupcakes not the original amount.

e 32

13 a \$81 200 **b** 13.79%

14 a 0.75% **b** 0.65% **c** 1.14%

15 a 4 kg **b** 0.2 kg **c** 24%

EX p122

3C Mark-ups and discounts

1 a \$36.05 **b** \$1060.50 **c** \$110.15 **d** \$480

e \$1615 **f** \$70.08 **g** \$233.28 **h** \$3337.61

2 a \$120 **b** \$255 **c** \$630

d \$408 **e** \$619 **f** \$404.80

g \$470.25 **h** \$87.15 **i** \$622.50

3 a \$504 **b** \$1002 **c** \$198

d \$1090.32 **e** \$2520.76 **f** \$3520

4 a \$1560 **b** \$336.75 **c** \$1350 **d** \$260.75

e \$948.15 **f** \$465 **g** \$552 **h** \$787.50

i \$323.75 **j** \$458.85

5 a \$142.10 **b** \$36.25 **c** \$94.25 **d** \$147.90

6 a \$1413.50 **b** \$264 **c** \$661.60

d \$107.25 **e** \$346.12 **f** \$80.25

7 a i $194.40 ii $74.40
b i $778.70 ii $419.30
c i $42.42 ii $7.49

8 a $300 b $31.25 c $13.95
d $86.65 e $11 422.85 f $624.60
g $49.90 h $50 i $342 857.15

9 a $141.50 b $214.50 c $165.45
d $109.55

10 a 87.5% b $138.20

11 a i $440 ii $60
b i $152.58 ii $26.93
c i $229.08 ii $19.92
d i $860.11 ii $35.84
e i $537.16 ii $87.44
f i $28 345.28 ii $1649.73
g i $11 095 ii $1585
h i $1234.19 ii $261.80

12 a $820 b $600 c $1.41 d $1120

13 a $2500 b $150 c $33 d $255

14 a $44.60 b $37.91 c $82.51

15 a 210% b $188.80

16 a i $768.80 ii $148.80
b i $130.43 ii $40.48
c i $2284.20 ii $1015.20
d i $835.68 ii $385.18
e i $15 000 ii $8750
f i $789.73 ii $438.74
g i $31 078.13 ii $16 453.13
h i $5450.63 ii $3155.63

17 Jane is finding 20% of the discount price, not the original price. Her calculations will give: 0.2 × $198 = $39.60
Original price = $198 + $39.60 = $237.60
Find 20% of $237.60 and see if it gives a reduced price of $198.
A 20% discount means you pay 80% or 0.8 of the original price of the bike:
$198 ÷ 0.8 = $247.50
20% of $247.50 gives a reduced price of $198.
Hence, Tim is correct.

18 $247.50

19 70% of $699 = $489.30; 80% of $699 is $559.20, then 90% of $559.20 is $503.28; the customer is correct.

20 a $2800 b $980

21 a i $5.55 ii $1.77 iii $2.85
iv $25.73 v $3.56 vi $6.56
b i 43.04% ii 42.74% iii 43.22%
iv 43.02% v 42.97% vi 42.92%
c When you increase the price by 40%, you find 140%, or 1.4 times the original. When you increase the price by 30% then 10%, you first find 130% of, or 1.3 times, the original price, then you find 110% of, or 1.1 times, this second price. Increasing by 30% then 10% means multiplying by 1.3, then 1.1, which is equivalent to multiplying by 1.3 × 1.1 = 1.43 = 143% = 100% + 43%. Increasing by 30%, then 10% is the same as increasing by 43%. The percentage is not exactly 43% because of errors introduced by rounding.

22 'All stock discounted by 45%' means all stock will receive a discount of 45%. 'Stock discounted by up to 45%' means the discount can be any amount from 0% to 45%; for example, 10%, 20%, 35%, 45%.

23 a $46 000 is between $45 001 and $120 000, so tax paid is $5092 plus 32.5 cents per $1 over $45 000, which works out to be 32.5% of $46 000 – $45 000 = $1000.
b i $2242 ii $11 592 iii $25 892
iv $35 017 v $67 917 vi $420 667
c They are the amount of tax paid on earnings below the corresponding row in the table.
5092 = 19% × (45 000 – 18 200)
29 467 = 5092 + 32.5% × (120 000 – 45 000)
51 667 = 29 467 + 37% × (180 000 – 120 000)
d i 7.5% ii 17.8% iii 23.8%
iv 25.9% v 30.7% vi 42.1%
e They only pay 45% tax on income over $180 000, not on their entire income. The more they earn, the closer the percentage paid in tax is to 45%.

24 a $1161.62 b 1085.10 c 1277.78
d It does not matter as multiplication is commutative. That is, a 45% discount then a 10% discount is the same as a 10% discount then a 45% discount:
0.55 × 0.90 = 0.90 × 0.55

EX p127 3D Profit and loss

1 a profit of $10 b loss of $14
c loss of $4.55 d loss of $154.48

2 a i loss of $60 ii 25%
b i profit of $7.30 ii 292%
c i loss of $0.40 ii 7.69%
d i loss of $5390 ii 21.88%
e i profit of $230 ii 191.67%
f i loss of $21.95 ii 73.29%

3 a i 33.33% ii 74.49%
iii 8.33% iv 28%
v 65.71% vi 274.38%
b If a loss occurred the percentages will be greater, but if a profit occurred the percentages will be less.

4 a 28.57% b 17.07% c 4.92% d 51.50%

5 a loss of $48 b 24.24%

6 a $252 b 375%

7 a i loss of $14 ii 29%
b i profit of $127.50 ii 113%

c **i** profit of \$50.05 **ii** 139%

d **i** loss of \$411 **ii** 25%

e **i** loss of \$12 395 **ii** 41%

f **i** profit of \$268 865 **ii** 171%

8 **a** \$2.55 per kg **b** 86%

9 **a** \$27.45 **b** 11%

10 **a** **i** \$250 **ii** \$150

b **i** \$2000 **ii** \$1200

c **i** \$1000 **ii** \$750

d **i** \$213.75 **ii** \$146.25

e **i** \$223.84 **ii** \$96

f **i** \$491.53 **ii** \$432.25

11 **a** No, as it makes Joseph's cars more expensive than his competitor's.

b 111% profit

c A maximum percentage increase enables Joseph to keep his selling price below the selling price of his competitor's cars while maximising the profit.

12 **a** 25% **b** 125%

c 125% = 100% (the purchase price)
+ 25% (the price increase)

13 **a** 20% **b** 80%

c 20% + 80% = 100%
If percentage decrease is subtracted from 100%, the remaining percentage value gives the percentage of the original price of car.

14 **a** 100% increase **b** 200% increase

c 0% increase or decrease **d** 50% decrease

e 300% increase **f** 100% decrease

15

Commodity	Starting price \$	Final price \$	Movement %
Gold	1741.66	1732.95	↓ 0.5
Silver	34.04	33.56	↓ 1.4
Oil	102.15	102.46	↓ 0.3
Copper	3.67	3.71	↓ 1.2

16 **a** \$1 569 600

b **i** \$1 874 610 **ii** \$33 390

c profit of \$305 010 **d** 19.43%

17 **a** \$15.30 **b** \$16.83

c \$16.15; less than price in part b, because it doesn't take into account that GST is 10% of the marked-up price, not the wholesale price.

18 **a** **i** 60% **ii** 166.67%

iii 250% **iv** 150%

b **i** 60% **ii** 166.67%

iii 250% **iv** 150%

c **i** 75% **ii** 133.33%

iii 400% **iv** 300%

d **i** 68.42% **ii** 146.15%

iii 316.67% **iv** 216.67%

e **i** 42.89% **ii** 233.17%

iii 175.09% **iv** 75.09%

f **i** 87.94% **ii** 113.71%

iii 829.17% **iv** 729.17%

19

Cost price	Selling price	Cost price as a percentage of selling price	Number of sales
\$3.60	\$12	30%	35
\$5	\$15	33.33%	20
\$13	\$21	61.90%	37
\$1.10	\$2.50	44%	150
\$25.90	\$35	74%	25
\$5.44	\$8	68%	125
\$50.49	\$99	51%	8
\$0.28	\$1.12	25%	271
\$24.32	\$32	76%	19
\$3.60	\$6	60%	15

Revenue	Total profit	Profit as a percentage of the revenue
\$420	\$294	70%
\$300	\$200	66.67%
\$777	\$296	38.10%
\$375	\$210	56%
\$875	\$227.50	26%
\$1000	\$320	32%
\$792	\$388.08	49%
\$303.52	\$227.64	75%
\$608	\$145.92	24%
\$90	\$36	40%

20 **a** To decrease by 30%, we can multiply by (100% − 30%) = 70%, which is the same percentage we would multiply by to find 70% of its value.

b To increase by 30%, we can multiply by (100% + 30%) = 130%, which is the same percentage we would multiply by to find 130% of its value.

21 **a** \$22 000 **b** \$2000 loss

22 **a** selling price = wholesale price × 1.8 × 1.1
= wholesale price × 1.98
He can add 98% to the wholesale price or multiply the wholesale price by 1.98.

b \$1200 = wholesale price × 1.6 × 1.1
= wholesale price × 1.76
So wholesale price = \$1200 ÷ 1.76 = \$681.82
The single calculation is dividing by 1.76.

c \$400 = original price × 1.1 × 0.75
= original price × 0.825
So original price = \$400 ÷ 0.825 = \$484.85
The single calculation is dividing by 0.825.

23 56.14%

EX p131 Checkpoint

1 **a** $\frac{1}{4}$, 0.25 **b** $\frac{19}{25}$, 0.76 **c** $\frac{1}{25}$, 0.04
d $\frac{1}{200}$, 0.005 **e** $1\frac{1}{5}$, 1.2 **f** $\frac{16}{625}$, 0.0256

2 **a** 63%, $\frac{63}{100}$ **b** 12%, $\frac{3}{25}$ **c** 85%, $\frac{17}{20}$
d 330%, $3\frac{3}{10}$ **e** 0.6%, $\frac{3}{500}$ **f** 750%, $7\frac{1}{2}$

3 **a** 0.375, 37.5% **b** 0.325, 32.5%
c $5.\dot{6}$, $566\frac{2}{3}$% **d** 0.3125, 31.25%
e 0.44, 44% **f** $8.1\dot{6}$, $816\frac{2}{3}$%

4 **a** 20% **b** 25% **c** 12.5% **d** 12.5%
e 20% **f** $1\frac{1}{3}$%

5 **a** 133.4 L **b** \$1800 **c** 19.6 cm **d** 306 m
e 488 beans **f** \$192.50

6 **a** \$11 520 **b** 700 km **c** \$4927.50 **d** \$42.37
e 8.45 km **f** 108 minutes

7 **a** \$2880 **b** \$807.50 **c** \$7896
d \$956.25 **e** \$1159.10 **f** \$900

8 **a** **i** \$3.20 **ii** \$35.20
b **i** \$8.50 **ii** \$93.50
c **i** \$125 **ii** \$1375
d **i** \$0.35 **ii** \$3.85
e **i** \$2.60 **ii** \$28.60
f **i** \$3.50 **ii** \$38.50

9 **a** \$450 **b** \$9800 **c** \$8600 **d** \$8455
e \$700 **f** \$25 302.50

10 **a** **i** loss of \$1750 **ii** 9.46%
b **i** profit of \$60 **ii** 400%
c **i** profit of \$88 **ii** 733.33%
d **i** loss of \$1499 **ii** 37.48%

11 **a** **i** \$770 **ii** \$280
b **i** \$495 **ii** \$231
c **i** \$687.50 **ii** \$385
d **i** \$330 **ii** \$114

EX p134 3E Ratios

1 **a** 11 : 7 **b** 89 : 101 **c** 5 : 3 **d** 25 : 1
e 2 : 3 **f** 1 : 2 : 3 **g** 3 : 7 **h** 8 : 3

2 **a** 6 : 10 **b** 9 : 9 : 3

3 **a** **i** 17 : 8 **ii** $\frac{8}{25}$
b **i** 7 : 11 **ii** $\frac{11}{18}$
c **i** 5 : 4 **ii** $\frac{4}{9}$

4 Part-to-part comparisons are best shown as ratios and part-to-whole comparisons are best shown as fractions.

5 **a** 20 : 33 **b** 95 : 57 **c** 113 : 167
d 256 : 175 **e** 499 : 575 **f** 23 : 2
g 4 : 13 **h** 19 : 21 **i** 42 : 117
j 16 : 39 **k** 19 : 11 **l** 23 : 7 : 3
m 15 :17 : 19 **n** 5 : 7 : 11

6 **a** 13 : 21 **b** 120 : 17 **c** 27 : 500
d 4 : 3 **e** 1700 : 793 **f** 7 : 36
g 509 : 1300 **h** 472 : 2371 **i** 61 : 79
j 38 : 131 **k** 420 : 97 **l** 600 : 53
m 5000 : 23 : 673
n 5631 : 1753 : 1 000 000

7 **a** **i** 11 : 9 **ii** 2 : 7 **iii** 9 : 40 **iv** 18 : 11
b The ratio 9 : 40 could be written as the fraction $\frac{9}{40}$ as it represents how a part compares to the whole.

8 **a** 3 : 4 **b** 3 : 7 **c** $\frac{3}{7}$ **d** $\frac{4}{7}$

9 **a** **i** 11 : 8 : 7 **ii** 9 : 40
b **i** $\frac{13}{40}$ **ii** 40%

10 **a** **i** 2 : 1 **ii** 5 : 1 **iii** 1 : 2
b $\frac{1}{9}$ **c** 67% **d** 44%
e They are in the incorrect order; 2 : 1.

11 For every four games she wins, she loses five; or out of every nine games she plays, she wins four and loses five.

12 **a** 13% **b** 87 : 13

13 **a** 41 : 9 **b** $\frac{9}{50}$

14 80 : 332 (or 20 : 83)

15 **a** 4 : 2 : 1 **b** $\frac{2}{7}$ **c** 120 biscuits

16 **a** Measurements must be put into same units.
b Convert one to same unit as the other measurement.
c 1670 feet = (1670 × 0.3048) m ≈ 509 m; 309 : 509

17 **a** No, units are not the same.
b 57 : 2500
c More water will result in a weaker solution so increase the second value in the ratio. Less bleach will also result in a weaker solution so decrease the first value in the ratio. Both options could apply.

18 **a** $\frac{3}{100}$ **b** $\frac{97}{100}$ **c** 3 : 97
d A ratio can be written as a fraction when comparing a part to the whole.

19 The ratio of the ages simplifies to 8 : 7 : 6. Adding them up and dividing \$210 by the total gives one part equal to \$10. The money divided by the girls' ages would be \$80 to Lydia, \$70 to Sylvia and \$60 to Julia. Dividing the money into thirds gives each girl \$70 each. In both cases, Sylvia gets the same amount of money.

20 **a** $\Phi = \frac{x}{y}, \Phi = \frac{x+y}{x}$
When $y = 1$, $\Phi = x$, $\Phi = \frac{x+1}{x}$
When $y = 1$, the value of the Golden Ratio $\Phi = x$, and the longer length $x = \frac{(x+1)}{x}$.
b $\Phi^2 = \Phi + 1$

EX p139 3F Equivalent ratios

1 **a** 1 : 12 **b** 3 : 4 **c** 53 : 18 **d** 2 : 5
e 3 : 2 **f** 9 : 19 **g** 19 : 17 **h** 1 : 6
i 8 : 12 : 21 **j** 3 : 2 : 5 **k** 3 : 5 : 7 **l** 3 : 7 : 8

2 **a** 6 : 13 **b** 5 : 22 **c** 85 : 31 **d** 1 : 3
e 23 : 59 **f** 6 : 7 **g** 1 : 3 **h** 162 : 35
i 13 : 4 : 7 **j** 45 : 11 : 35 **k** 6 : 8 : 7 **l** 6 : 21 : 8

3 **a** 2 : 3 **b** 8 : 9 **c** 5 : 6
d 33 : 14 **e** 9 : 10 **f** 10 : 21 (20 : 42)
g 1 : 2 **h** 3 : 4 (51 : 68) **i** 85 : 16
j 4 : 5 (16 : 20) **k** 2 : 1 **l** 6 : 1

4 **a** 13 : 640 **b** 17 : 2340 **c** 23 : 432
d 5 : 6 **e** 1 : 14 **f** 19 : 20
g 18 : 11 **h** 64 : 25 **i** 1 : 4

5 **a** Yes. The second ratio is obtained by multiplying the first ratio by 3.
b No
c Yes. The second ratio is obtained by dividing the first ratio by 2.
d Yes. The second ratio is obtained by multiplying the first ratio by 5.
e No **f** No **g** No
h Yes. The second ratio is obtained by dividing the first ratio by 3.
i No
j Yes. The second ratio is obtained by multiplying the first ratio by 4.
k No
l Yes. The second ratio is obtained by dividing the first ratio by 9.

6 **a** 45 **b** 10 **c** 72 **d** 2
e 95 **f** 7 **g** 6 **h** 8
i 3 **j** 3 **k** 85 **l** 117
m 105 **n** 14 **o** 17 **p** 6

7 **a** 2 : 5 = 8 : 20 = 20 : 50
b 6 : 7 = 30 : 35 = 54 : 63
c 420 : 360 = 70 : 60 = 7 : 6
d 225 : 300 = 9 : 12 = 3 : 4
e 136 : 104 = 68 : 52 = 17 : 13
f 180 : 48 = 75 : 20 = 15 : 4
g 20 : 48 = 40 : 96 = 100 : 240
h 216 : 636 = 72 : 212 = 18 : 53

8 **a** 16 **b** 80 **c** 1
d 35 **e** 12 **f** 9
g 44 **h** 48 **i** 30

9 **a** 13 : 1 **b** 1 : 9 **c** 17 : 9 **d** 4 : 7

10 3 : 17

11 12

12 **a** 12 **b** 27

13 12 km

14 **a**

Pool enclosure	Length	Width	Perimeter
A	4 m	3 m	14 m
B	8 m	6 m	28 m
C	12 m	9 m	42 m
D	16 m	12 m	56 m

b A 4 : 14, B 8 : 28, C 12 : 42, D 16 : 56
c The ratios are multiples of each other.
d $\frac{4}{14} = \frac{2}{7}, \frac{8}{28} = \frac{2}{7}, \frac{12}{42} = \frac{2}{7}, \frac{16}{56} = \frac{2}{7}$
e Each ratio simplifies to $\frac{2}{7}$.
f The length is always $\frac{2}{7}$ of the perimeter.

15 72 pens

16 **a** 1.05 L **b** 8000 mL

17 **a** 21.25 L **b** 600 mL

18 **a** 41 : 204 **b** 41 : 163

19 **a** 5 : 2 **b** 3 : 5 **c** 3 : 2

20 **a** The listed ingredients are all in the same unit, so they can be expressed as a ratio.
b 4 cups rolled oats, 4 cups plain flour, 4 cups desiccated coconut, 2 cups sugar
c 0.5 cups rolled oats, 0.5 cups plain flour, 0.5 cups desiccated coconut, 0.25 cups sugar

21 **a** 27.84 cm **b** 69.6 cm

22 **a** The ratio 22 : 27 is larger than 12 : 15 but smaller than 8 : 9.
b Sample answers include: 81 : 100, 161 : 200, 101 : 125.

23 4 : 5 is larger.

24 17 : 33 is the larger ratio, as 17 is more than half of 33 and 12 is less than half of 25.

25 Sample answer

1st ratio	2nd ratio
3 : 5	9 : 15
7 : 20	14 : 40
4 : 7	40 : 70
5 : 25	1 : 5
25 : 30	5 : 6
3 : 7	12 : 28
8 : 16	4 : 8
8 : 12	2 : 3
8 : 24	1 : 3
2 : 4	1 : 2

EX p145

3G Working with ratios

1 **a** 50 and 450 **b** 100 and 400
c 150 and 350 **d** 300 and 200
e 300 and 200 **f** 400 and 100

2 **a** \$5520 and \$3680 **b** \$5750 and \$3450
c \$6900 and \$2300 **d** \$4600 and \$4600
e \$4200 and \$5000 **f** \$1840 and \$7360

3 **a** 15 000 km and 21 000 km
b 31 500 km and 4500 km
c 20 000 km and 16 000 km
d 16 000 km and 20 000 km
e 28 800 km and 7200 km
f 18 000 km and 18 000 km

4 **a** \$80 and \$160 **b** \$216 and \$24
c \$40 and \$200 **d** \$150 and \$90
e \$90, \$30 and \$120 **f** \$168, \$48 and \$24

5 **a** 120 kg and 180 kg **b** 30 kg and 270 kg
c 260 kg and 40 kg **d** 80 kg and 220 kg

e 120 kg, 150 kg and 30 kg

f 120 kg, 75 kg, 45 kg and 60 kg

6 **a** \$2.70 and \$0.90 **b** \$0.40 and \$3.20

c \$3.30 and \$0.30 **d** \$2.00 and \$1.60

e \$0.40 and \$1.20 and \$2.00

f \$0.60 and \$0.90 and \$2.10

7 **a** $\frac{16}{45}$ and $\frac{8}{45}$ **b** $\frac{8}{25}$ and $\frac{16}{75}$

c $\frac{1}{5}$ and $\frac{1}{3}$ **d** $\frac{2}{5}$ and $\frac{2}{15}$

e $\frac{2}{15}$ and $\frac{1}{5}$ and $\frac{1}{5}$ **f** $\frac{16}{75}$ and $\frac{8}{45}$ and $\frac{32}{225}$

8 **a** 21 cups and 14 cups **b** 28 cups and 7 cups

c $24\frac{1}{2}$ cups and $10\frac{1}{2}$ cups

d $31\frac{1}{2}$ cups and $3\frac{1}{2}$ cups

e $10\frac{1}{2}$ cups and $17\frac{1}{2}$ cups and 7 cups

f $10\frac{1}{2}$ cups and 14 cups and $10\frac{1}{2}$ cups

9 **a** 4000 non-fiction books **b** 7500 books

10 **a** 15 tomato seedlings **b** 36 seedlings

11 **a** 2000 non-germinated **b** 6500 seeds

12 64

13 **a** 1500 mL **b** 150 mL

14 60°, 80°, 40°

15 **a** 10

b $\frac{4}{10}$ and $\frac{6}{10}$ or $\frac{2}{5}$ and $\frac{3}{5}$

c \$120 and \$180 **d** Yes

16 \$28

17 **a** \$1000 **b** 9 : 4 : 7

c Katie \$2.25; Mike \$1.00; Sam \$1.75

18 **a** 5 : 3 : 2

b **i** \$25, \$15, \$10 **ii** \$75, \$45, \$30

iii \$350, \$210, \$140 **iv** \$1000, \$600, \$400

v \$5000, \$3000, \$2000 **vi** \$6500, \$3900, \$2600

19 **a** $\frac{3}{5}$ **b** 3 : 5 : 12

c 15% to a present, 25% to trophies, 60% to snacks

d \$30 on a present, \$50 on trophies, \$120 on snacks

20 **a** Catching up with friends 40%; outdoor activities 25%; doing homework/chores 10%

b 25% **c** 8 : 5 : 2 : 5

d 56 hours, 35 hours, 14 hours, 35 hours

21 **a** width 72 cm, height 42 cm **b** 3024 cm^2

22 The ratio that shows the relationship of part to part compares one part to another. For example, in 2 : 3 there is a total of 5 parts. The first number represents $\frac{2}{5}$ of the total and the second number represents $\frac{3}{5}$ of the total. The ratio that shows the relationship of a part to a whole is better expressed as a fraction as it compares one number to the total.

23 **a** Can A has 3200 mL of white paint and 800 mL of red paint. Can B has 3500 mL of white paint and 500 mL of red paint.

b 67 : 13 **c** $\frac{13}{80}$ **d** $83\frac{3}{4}$% or 83.75%

e To make 4 : 1 from 67 : 13, one option could be to add 1 part red and 3 parts white. To make 7 : 1 from 67 : 13, one option could be to add 24 parts white.

24 180

EX p151 3H Rates

1 **a** 60 km/h **b** 591 L/min

c four quarters/game **d** \$16/h

e 45 cm/year **f** five patients/h

g \$1.49/day **h** \$28.99/month

i \$5.80/box **j** 0.5 kg/week

k 290 mm/year **l** 14 km/L

2 **a** 50 m/min **b** 2 L/bottle

c 4 cents per second **d** 15 students/teacher

e 26 seats/row **f** 25 newspapers/bundle

g 6.8 runs/over **h** \$2.50/muffin

i 6.8 goals/game **j** 4444 tickets/min

k 3.5 goals/game **l** 5.25 m^2/L

3 **a** 14 km/L **b** \$1.99/mango

c 12 L/min **d** 15.3 km/h

e 10.08 people/km^2 **f** 18 million km/min

4 **a** 2400 words **b** 800 words **c** 45 minutes

5 **a** \$15.20 **b** \$608 **c** 96 hours

6 **a** 386 m **b** 1 389 600 m **c** $33\frac{1}{3}$ minutes

7 **a** B **b** B **c** B **d** A

e B **f** A **g** B **h** A

8 **a** B **b** A **c** A **d** A

e A **f** B **g** B **h** A

9 **a** Max 5.75 runs/over, William 5.6 runs/over

b Max's team

10 **a** 56 bpm **b** 27 min

11 Usain Bolt's rate is 10.4384 m/s.

a 10.35 m/s **b** 10.70 m/s **c** 16 m/s

12 **a** 0.3125 km (312.5 m)

b **i** 17.1875 km **ii** 18.75 km

iii 26.5625 km **iv** 28.125 km

c **i** 2 min **ii** 1 h 40 min

iii 1 h 20 min **iv** 6 min 24 s

13 **a** 7.5 kg of potatoes (\$0.95/kg compared to \$0.99/kg)

b packet of 10 batteries (\$0.989/battery compared to \$0.9975/battery)

c 500 g tin of coffee (\$0.018/g compared to \$0.01993/g)

d 650 g tin of peaches (\$0.0039/g compared to \$0.0042/g)

14 **a** The statement is not accurate. The best buy is the option that has the lowest price per unit or lowest price when written as rates with the same units.

b It is the best buy when the amount of the cheaper option is the same or more than the other option.

15 **a** Plan A: 80c, Plan B: 88c; Plan A

b Plan A: \$1.60, Plan B: \$1.64; Plan A

c Plan A: \$2.40, Plan B: \$2.40; both plans are the same.

d Plan A: \$3.20, Plan B: \$3.16; Plan B

e Use Plan A if most calls are shorter than 5 min, use Plan B if most calls are longer than 5 min, use either plan for calls of exactly 5 min.

16 **a** **i** $\frac{1}{20}$ (or equivalent) **ii** \$100 **iii** \$1100

b **i** $\frac{1}{40}$ (or equivalent) **ii** \$90 **iii** \$1290

c **i** $\frac{1}{25}$ (or equivalent) **ii** \$1000 **iii** \$6000

17 6 minutes. When doubling the number of window washers and the number of windows, the minutes taken will remain the same.

18 1 hour, 43 minutes

19 260.5 m

EX 3I Distance–time graphs

p158

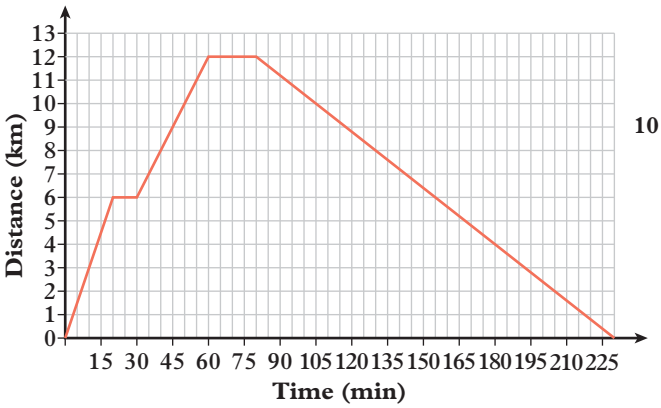
1 **a** 30 km **b** 0.5 hours **c** 40 km **d** 2 hours

2 **a** 4 km **b** 30 min **c** 10 km **d** 10 min

3 **a** 50 km/h **b** 0 km/h **c** 5 km/h **d** 0 km/h **e** 40 km/h

4 **a** 2 m/s **b** 1.2 m/s **c** 0.67 m/s **d** 0.33 m/s

5



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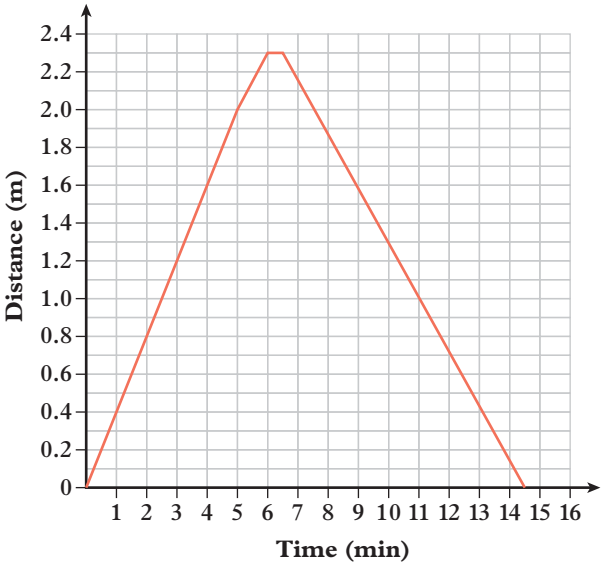


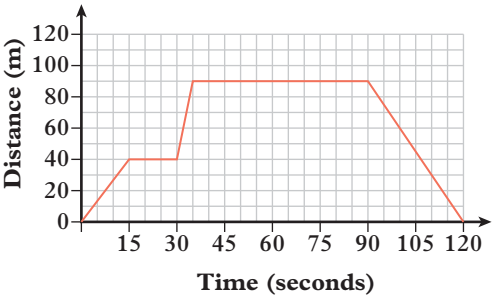
7 **a** A, C, B, D **b** B, C, A, D **c** A, C, D, B

8 **a** The slopes of the line segments of the graph do not correspond to the slopes of the (possible) hills the student is walking up and down. The student would probably be slower up a steeper hill and faster going downhill than uphill. The student also stands still for 25 seconds instead of walking across flat ground.

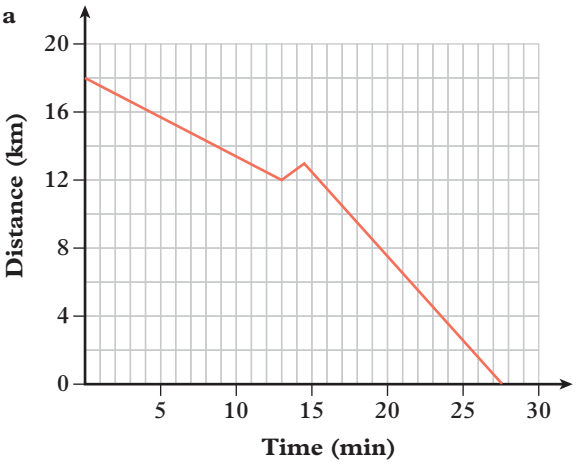
b
- The student walks at 2 m/s for 20 seconds.
- The student stands still for 25 seconds.
- The student walks in the same direction at 4 m/s for 15 seconds.
- The student walks in the opposite direction for 60 seconds, travelling 100 m back to the starting position.

9 **a** The segment is drawn backwards to show the backwards movement, but that would imply that two segments of the journey are happening at the same time which is not possible.

b

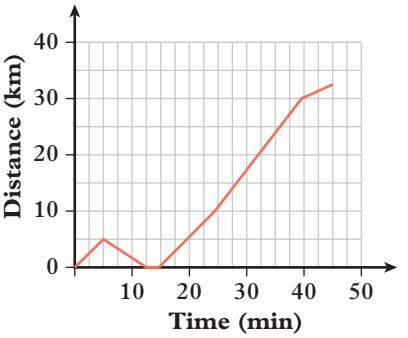


10 **a**



b 2:35:30 pm **c** 27.7 km/h

11 **a**



b 42.75 km **c** 32.75 km

12 Since we cannot measure a negative length (distance) or time elapsed, we cannot plot points on a distance–time graph with negative distance or time.

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13 **a** 160 km **b** 2.5 hours **c** 64 km/h

d, e

Section	Speed	Time	Speed × Time
A	60	0.5	30
B	40	0.75	30
C	120	0.25	30
D	70	1	70
Total	N/A	2.5	160

f Speed × time calculates the distance travelled at that speed for that time, so adding up all the partial distances travelled will equal the same as the total distance travelled.

g 64 km/h

h The weighted mean divides the total distance, 160 km, by the total time, 2.5 hours, as the average speed does. That is, the 'average' in average speed is referring to the 'mean' average.

14 **a** 100 m away from the origin

b 40 m away from the origin

c **i** 280 m **ii** 60 m

d **i** 5.6 m/s **ii** 1.2 m/s

15 **a** moving in a circular arc with a radius of 40 km from the origin, centred at the origin

b moving in a straight line that is at a perpendicular distance of 10 km away from the origin

c moving along a circular path that is not centred at the origin

16 **a** 50 km/h **b** 45 seconds **c** 12 minutes

d 21 minutes **e** 1860 square units, 31 km

f $\frac{\text{km}}{\text{h}} \div \text{h} = \frac{\text{km}}{\text{h}} \times \frac{1}{\text{h}}$

$= \frac{\text{km}}{\text{h}^2}$

g **i** 4800 km/h^2 **ii** –1800 km/h^2

iii 1800 km/h^2 **iv** 1200 km/h^2

17 The object does not change direction as it would have to come to a stop (touching the time axis) and then keep going. Even then, we wouldn't be able to tell if that meant turning around or stopping and starting unless we graphed the speed with direction, so that a negative speed indicates moving backwards.

EX p165

CHAPTER 3 review

Mathematical literacy review

1 speed

2 A ratio is a comparison between two or more quantities with the same unit, whereas a rate is a comparison between two or more quantities with different units.

3 A profit is when the selling price is greater than the cost price, and a loss is when the cost price is greater than the selling price.

4 **a** best buy **b** cost price

5 **a** an amount added to the cost price in order to obtain a profit

b income generated by the sale of goods or services

6 **a** A ratio is written in simplest form when the only common factor between the terms in the ratio is 1.

b A percentage is a proportion of a whole that has been divided into one hundred parts.

Multiple choice

1 E **2** C **3** B **4** A **5** B **6** D

7 E **8** A **9** C **10** B **11** D **12** E

Short answer

1 **a** 7 **b** $\frac{7}{20}$ **c** $\frac{35}{100}$ **d** 35%

2 **a** 0.23 **b** 0.62 **c** 0.1652 **d** 2.916

3 **a** 32% **b** 762% **c** 3162.5% **d** 0.03%

4 **a** 75% **b** 70% **c** 85% **d** 46.875%

5 **a** 40% **b** 16% **c** 46.4%

6 **a** 34 **b** 15.5 **c** 2250 **d** 888

7 **a** \$1045 **b** \$904.32 **c** \$903.60

8 **a** 37.5% profit **b** 40% loss

c 51.68% profit **d** 31.57% loss

9 **a** profit of \$5.64 **b** loss of \$3.75

c profit of \$33.20

10 **a** 76% **b** 15% **c** 74%

11 **a** **i** 5 : 6 **ii** 2 : 3

b **i** $\frac{3}{16}$ **ii** $\frac{1}{8}$

12 **a** 35 : 47 **b** 7 : 31 **c** 2 : 9 **d** 9 : 25

13 **a** 3 : 11 **b** 16 : 27 **c** 1 : 2 : 6 **d** 1 : 5 : 7

14 **a** 42 **b** 16 **c** 12 **d** 35

15 **a** 8 **b** 17 **c** 17 **d** 31

16 **a** \$600, \$900 **b** \$450, \$1050

c \$300, \$400, \$800 **d** \$300, \$375, \$825

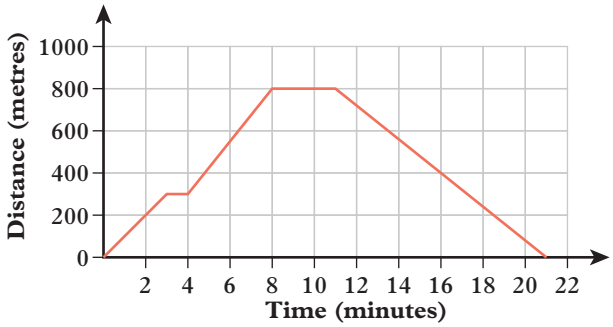
17 **a** 100 km/h **b** 3 L/s

c \$17.50/h **d** 4.333 books/month

18 **a** 1.5 points/min **b** \$1.39/L

c \$0.0048/mL or 0.48 c/mL **d** \$16.80/h

19



Analysis

1 **a** **i** $\frac{13}{20}$ **ii** 0.65

b \$13 **c** \$33 **d** \$455 **e** \$3.30

f \$29.70 **g** \$9.70 **h** 48.5%

2 **a** 3 : 5 **b** \$65.60 **c** 2.4 cubic metres

d \$512 **e** 12 **f** 16

g 2 Grevilleas, 3 Callistemons and 4 Banksias

3 **a** 3 : 2 **b** 9 : 5 **c** 2 cups

d $1\frac{1}{5}$ cups of pecans, $1\frac{1}{5}$ cups of cashews, $\frac{4}{5}$ cup of sunflower seeds and $\frac{4}{5}$ cup of pumpkin seeds.

CHAPTER 4 Indices and roots

EX 4A Indices

p174

1 **a** 5 **b** 3 **c** $5 \times 5 \times 5$ **d** 125

2 **a** $2 \times 2 \times 2 \times 2 \times 2 = 32$ b $4 \times 4 \times 4 = 64$

c $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

d $3 \times 3 \times 3 = 27$

e $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10\,000\,000$

f $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

g $5 \times 5 \times 5 \times 5 = 625$

h $7 \times 7 = 49$

i $8 \times 8 \times 8 = 512$

j $6 \times 6 \times 6 \times 6 = 1296$

3 **a** 64 **b** 49 **c** 25 **d** 121 **e** 36

f 1 **g** 3600 **h** 10 000 **i** 400

4 **a** 27 **b** 8 **c** 64 **d** 1 000 000

e 1000 **f** 125

5

	Index notation	Base	Index	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	81
b	2^5	2	5	$2 \times 2 \times 2 \times 2 \times 2$	32
c	8^2	8	2	8×8	64
d	6^2	6	2	6×6	36

6 **a** $0.8 \times 0.8 = 0.64$

b $3.4 \times 3.4 = 11.56$

c $0.5 \times 0.5 \times 0.5 = 0.125$

d $0.3 \times 0.3 \times 0.3 \times 0.3 = 0.0081$

e $0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.000\,01$

f $0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = 0.0000128$

7 **a** $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

b $\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$

c $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

d $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{81}{10\,000}$

e $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$

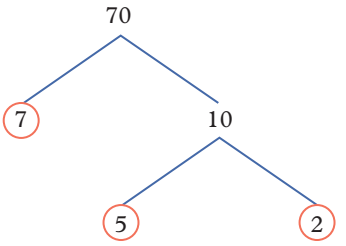
f $3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} = \frac{1\,000\,000}{729}$

8 **a** 3249 **b** 1 030 301 **c** 62 748 517

d 321. 419 125 **e** 13.0321 **f** 293 162.50624

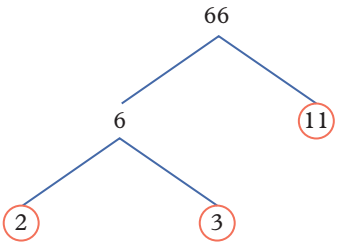
g $\frac{5832}{12\,167}$ **h** $\frac{117\,649}{1\,771\,561}$ **i** $\frac{19\,683}{1\,953\,125}$

9 **a**



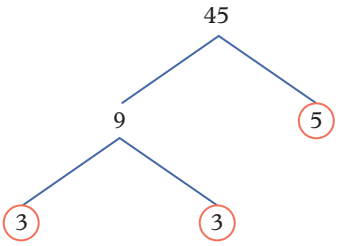
$70 = 2 \times 5 \times 7$

b



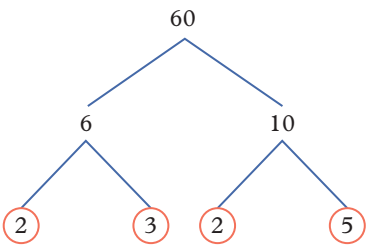
$66 = 2 \times 3 \times 11$

c



$45 = 3^2 \times 5$

d



$60 = 2^2 \times 3 \times 5$

10 **a** $75 = 3 \times 5^2$

b $48 = 2^4 \times 3$

c $90 = 2 \times 3^2 \times 5$

d $126 = 2 \times 3^2 \times 7$

e $288 = 2^5 \times 3^2$

f $1092 = 2^2 \times 3 \times 7 \times 13$

11 The student calculated 4^2 as $4 \times 2 = 8$, but $4^2 = 4 \times 4$.

12 **a** 24 **b** 86

c 4.3601 **d** $\frac{11}{10}$ $\left(\text{or } 1\frac{1}{10}\right)$

13 **a** **i** 1 **ii** 121

iii 12 321 **iv** 1 234 321

b Sample answer: The resulting number displays digits in a pattern with 1 as the first digit, 2 as the second digit, 3 as the third digit, etc, increasing by one each time until it reaches a value that is the same as the number of digits in the base. The digits that follow then decrease by one to finish the pattern at 1. For example, the base

of 111^2 contains three digits so the first three digits in the number are 123 followed by 21 to give 12 321.

c **i** 123 454 321
ii 12 345 654 321
iii 1 234 567 654 321
iv 123 456 787 654 321

14 **a** \$180 **b** \$3, \$9, \$27, \$81, \$243, \$729
c \$30 per week **d** $\$3^{11}$
e \$177 147

15 **a** **i** 2000 cells **ii** 4000 cells **iii** 8000 cells
b 2000, 4000, 8000, 16 000, 32 000
c **i** 2 **ii** 4 **iii** 8
d 2, 4, 8, 16, 32 **e** $2^1, 2^2, 2^3, 2^4, 2^5$
f **i** 2^6 **ii** 2^{10} **iii** 2^{24}
g 16 777 216 **h** 16 777 216 000

16 **a** $144 = 2^4 \times 3^2$
b $270 = 2 \times 3^3 \times 5$
c HCF $= 2 \times 3^2 = 18$
d **i** 13 **ii** 12
iii 55 **iv** 27

17 **a** $20^4 = 2^8 \times 5^4$, $20^5 = 2^{10} \times 5^5$
b $20^n = 2^{2n} \times 5^n$

18 60 days

19 $\frac{5}{4}c$

EX p178 4B Products and quotients of powers

1 **a** 3^5 **b** 4^8 **c** 6^6 **d** 5^9 **e** 8^3 **f** 2^5

2 **a** 243 **b** 65 536 **c** 46 656
d 1 953 125 **e** 512 **f** 32

3 **a** 9^3 **b** 4 **c** 6^3
d 4^5 **e** 10^3 **f** 6^4

4 **a** 729 **b** 4 **c** 216
d 1024 **e** 1000 **f** 1296

5 **a** $2^{11} = 2048$ **b** $5^6 = 15\,625$
c $3^7 = 2187$ **d** $4^8 = 65\,536$
e $5^3 = 125$ **f** $3^6 = 729$
g $9^3 = 729$ **h** $2^7 = 128$
i $6^5 = 7776$ **j** $8^1 = 8$
k $10^9 = 1\,000\,000\,000$ **l** $7^2 = 49$

6 **a** $(1.5)^9$ **b** $(0.7)^6$ **c** $\left(\frac{1}{2}\right)^7$ **d** $\left(\frac{7}{5}\right)^{14}$
e $(9.2)^3$ **f** $(0.63)^7$ **g** $\left(\frac{3}{4}\right)^4$ **h** $\left(\frac{5}{2}\right)^2$

7 **a** 10 **b** 5 **c** 8 **d** 4 **e** 21 **f** 17

8 **a** $5^2 \times 5^5 \times 5^4 = 5^7 \times 5^4$
$= 5^{11}$
b $20^8 \times 20 \times 20^{10} = 20^9 \times 20^{10}$
$= 20^{19}$
c $8^{32} \div 8^7 \div 8^{10} = 8^{25} \div 8^{10}$
$= 8^{15}$
d $7^{24} \div 7^6 \div 7 = 7^{18} \div 7^1$
$= 7^{17}$

9 **a** 2^{12} **b** 3^6 **c** 10^{10} **d** 4^9 **e** 5^{16} **f** 9^{14}

10 **a** 6^2 **b** 7^4 **c** 12 **d** 8^4 **e** 5 **f** 11^5

11 10^{10} light-years

12 **a** Indices have been divided instead of subtracted; $5^8 \div 5^2 = 5^6$.
b Indices have been multiplied instead of added; $2^3 \times 2^3 = 2^6$.
c Indices have been divided instead of subtracted; $\frac{4^7}{4^7} = 4^0$.
d The first index has been subtracted from the second, instead of the other way round; $9^8 \div 9^{10} = 9^{-2}$.

13 **a** **i** 4 **ii** −8 **iii** 16 **iv** −32
b Sample answer: The sign of the answers alternates between positive and negative and the number doubles in value.
c Sample answer: When a negative number is raised to a power, if the index is even the answer will be positive, and if the index is odd the answer will be negative.
d **i** −27 **ii** 64 **iii** 10 000
iv −1 000 000 000

14 **a** $x^2 \times x^3$ **b** x^5 **c** 36 × 216 mm

EX p181 4C Raising indices and the zero index

1 **a** 6^{20} **b** 2^{12} **c** 7^{12} **d** 11^8 **e** 8^{15} **f** 3^{14}

2 **a** $5^3 \times 9^3$ **b** $\frac{3^6}{4^6}$ **c** $\frac{9^4}{5^4}$
d $11^7 \times 4^7$ **e** $\frac{7^3}{10^3}$ **f** $3^9 \times 8^9$

3 **a** 1 **b** 1 **c** 1

4 **a** 1 **b** 1 **c** 1 **d** 1 **e** 1 **f** 1

5 **a** 0.5^{12} **b** 1 **c** 3.8^{25}
d $2^{20} \times 2.5^8$ **e** 1 **f** 6.6^{18}

6 **a** 4^{10} cm^2 **b** 4^{15} cm^3 **c** 6^{21} cm^3

7 **a** 6 **b** 4 **c** 0 **d** 5 **e** 10 **f** 0

8 **a** $(2^3)^4 = 8^4 = 16^3 = 2^{12}$
b $10^{5 \times 4} = 100\,000^4 = 10\,000^5 = 10^{20}$
c $8^8 = 2^{24} = 4^{12} = 16^6$

9 **a** 2 **b** 2 **c** 3 **d** 1 **e** 2 **f** 4

10 $(3^{15})^0$, $(3^2)^5$, $(3^4)^3$, 3^{14}, $((3^2)^3)^3$

11 **a** 1 **b** 2^0
c Sample answer: $\frac{15^5}{15^5} = 15^{(5-5)}$
$= 15^0$
$= 1$

12 **a** 243, 81, 27, 9, 3
b Each term in the sequence has an index of one less than the previous term, so the next term is 3^0. The value of each term in the sequence is the value of the previous term in the sequence divided by 3, so the value of the next term is $3 \div 3 = 1$. Therefore, $3^0 = 1$.

13 **a** true **b** true **c** false **d** true

14 **a** $x = 4$ **b** $x = 2$

15 Sample answer: $(2^3)^6 = 2^{18}$
$= (2^2)^9$
$= 4^9$

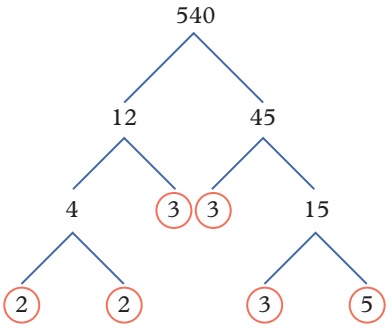
16 $x = 2, y = 4$, or $x = 4, y = 2$

EX Checkpoint
p183

1 **a** 16 **b** 81
c 125 **d** 1 000 000

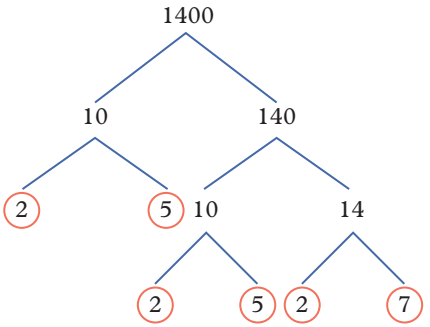
2 **a** 25 **b** 81 **c** 64 **d** 216

3 **a**



$540 = 2^2 \times 3^3 \times 5$

b



$1400 = 2^3 \times 5^2 \times 7$

4 **a** $2^3 \times 3$ **b** $2^3 \times 3^2$
c $3 \times 5 \times 13$ **d** $2 \times 5^2 \times 7$

5 **a**

Number of minutes	0	1	2	3	4	5
Size of colony	10	30	90	270	810	2430
Number of times as large as original amount	1	3	9	27	81	243

b $3^0, 3^1, 3^2, 3^3, 3^4, 3^5$
c **i** 3^{10} **ii** 3^{60} **iii** 3^{1440}
d 590 490

6 **a** 2^8 **b** 7^{20} **c** 11^6 **d** 9^{18}

7 **a** 3^4 **b** 10^{10} **c** 14^4 **d** 8^8

8 **a** 64 **b** 625
c 36 **d** 10 000 000

9 **a** 4^{18} **b** 9^6 **c** 13^{40} **d** 10^{96}

10 **a** $3^6 \times 5^6$ **b** $\frac{2^4}{7^4}$ **c** $9^5 \times 15^5$ **d** $\frac{9^{10}}{8^{10}}$

11 **a** 1 **b** 1 **c** 1 **d** 1

EX 4D Combining the index laws
p185

1 **a** 3^{14} **b** 9^{41} **c** 12^{36} **d** 8^0

2 **a** 4^2 **b** 7^2 **c** 6^{11} **d** 11^6
e 5^4 **f** 8^3 **g** 2^{10} **h** 4^3

3 **a** 3^6 **b** 3^5 **c** 4^{21} **d** 2^8

4

Base 2	Index notation	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
	Basic numeral	16	32	64	128	256	512	1024
Base 3	Index notation	3^4	3^5	3^6	3^7	3^8	3^9	3^{10}
	Basic numeral	81	243	729	2187	6561	19 683	59 049

5 **a** **i** $2^5 \times 2^4$ **ii** $2^{10} \div 2^8$ **iii** $3^{10} \div 3^6$
iv $\frac{2^7 \times 2^5}{2^4}$ **v** $\frac{3^3 \times 3^5}{3^7}$ **vi** $2^6 \times 2^8 \div 2^9$
vii $\frac{2^9 \times 2^6}{2^7 \times 2^4}$ **viii** $\frac{2^7 \times 2^{10}}{2^8 \times 2^6}$
b **i** 2^9 **ii** 2^2 **iii** 3^4 **iv** 2^8
v 3^1 **vi** 2^5 **vii** 2^4 **viii** 2^3
c **i** 512 **ii** 4 **iii** 81 **iv** 256
v 3 **vi** 32 **vii** 16 **viii** 8

6 **a** 256 **b** 81 **c** 64 **d** 2187

7 **a** 72 **b** 75 **c** 96 **d** 9000
e 125 **f** 64 000 000

8 **a** 2 **b** 12 **c** 4 **d** 0
e 5 **f** 9 **g** 3 **h** 3

9 Sample answer: The number on the right-hand side of each equation is negative because the negative sign is located outside the brackets. When a negative number is raised to a power, if the index is even the answer will be positive, and if the index is odd the answer will be negative, so $(-5)^2$ is positive, but $(-5)^3$ is negative.

10 **a** true **b** true **c** true **d** true
e false **f** false

11 **a** 33 **b** 36 **c** 3

12 $x = 9$

13 384

EX 4E Roots
p189

1

Index notation	Perfect square	Index notation	Perfect cube
2^2	4	2^3	8
3^2	9	3^3	27
4^2	16	4^3	64
5^2	25	5^3	125
6^2	36	6^3	216
7^2	49	7^3	343
8^2	64	8^3	512
9^2	81	9^3	729
10^2	100	10^3	1000

2 **a** 4 **b** 3 **c** 5 **d** 1
e 2 **f** 10 **g** 9 **h** 15
i 12 **j** 20 **k** 13 **l** 30

3 **a** 2 **b** 4 **c** 5 **d** 1
e 3 **f** 10 **g** 6 **h** 20

4 **a** 14 **b** 16 **c** 33 **d** 162 **e** 1.5
f 7.3 **g** 8.2 **h** 21.8 **i** 5.03

5 **a** not a surd **b** surd **c** not a surd
d not a surd **e** surd **f** not a surd

6 **a** $\frac{1}{4}$ **b** $\frac{1}{8}$ **c** $\frac{2}{5}$ **d** $\frac{3}{7}$ **e** $\frac{9}{10}$ **f** $\frac{6}{11}$

7 **a** 80 **b** 1 **c** 6 **d** 75

8 **a** **i** $\sqrt{31}$ **ii** 5.57
b **i** $\sqrt{78}$ **ii** 8.83
c **i** $\sqrt[3]{12}$ **ii** 2.29
d **i** $\sqrt[3]{81}$ **ii** 4.33

9 **a** The value has been divided by 2 instead of finding the square root; $\sqrt{16} = 4$.
b The square root has been found instead of the cube root; $\sqrt[3]{64} = 4$.

10 Sample answer: Finding the square root is the inverse or opposite of squaring a number.

11 **a** Sample answer: The area of a square is found by squaring the side length, so the side length can be found by taking the square root of the area.
b **i** 13 cm
ii 14.142 135 623… m
iii 16.4 km
c **ii** is a surd

12 **a** 5
b **i** 2 **ii** 3 **iii** 10

13 **a** **i** 15 **ii** 4.1
iii 4.626 065 009… **iv** 7
v 2.923 012 786... **vi** 0.562 341 325…
b **i** not a surd **ii** not a surd **iii** surd
iv not a surd **v** surd **vi** surd

14 **a** −3
b −2
c It is not possible to take the square root of a negative number as the square of any negative number is positive.

15 **a** **i** 8 and 8 **ii** 18 and 18 **iii** 20 and 20
b $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

16 **a** 18 **b** 26 **c** 44 **d** 72

17 **a** **i** not a perfect square
ii perfect square; $\sqrt{289} = 17$
iii perfect square; $\sqrt{576} = 24$
iv not a perfect square
b **i** perfect cube; $\sqrt[3]{216} = 6$
ii perfect cube; $\sqrt[3]{512} = 8$
iii not a perfect cube
iv perfect cube; $\sqrt[3]{1331} = 11$

EX p192 CHAPTER 4 review

Mathematical literacy review

1 index form
2 factor trees
3 a surd
4 **a** perfect cube **b** irrational number
5 **a** when dividing numbers in index form with the same base, subtract the second index from the first index
b a positive whole number that is divisible by at least one whole number other than itself and 1
6 **a** The square of a number is the result of multiplying a number by itself.
b Simplification involves reducing the complexity of an expression.

Multiple choice

1 E **2** B **3** D **4** C **5** D
6 B **7** D **8** E **9** B **10** A

Short answer

1 **a** $3 \times 3 \times 3 = 27$
b $2 \times 2 \times 2 \times 2 = 16$
c $8 \times 8 = 64$
d $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

2 **a** 36 **b** 64 **c** 8 **d** 64

3 **a** $45 = 3^2 \times 5$
b $124 = 2^2 \times 31$

4 **a** 3^{12} **b** 2^{10} **c** 12^4 **d** 4^{11}

5 **a** 6^6 **b** 5^0 or 1 **c** 10^7 **d** 7^1

6 **a** 9^{15} **b** $6^{10} \times 3^{10}$ **c** $\frac{3^3}{10^3}$ **d** $\frac{2^8 \times 7^8}{9^8}$

7 **a** 4 **b** 1 **c** 1 **d** 25

8 **a** $7^2 = 49$ **b** $2^5 = 32$

9 **a** **i** $\frac{5^4}{5^2}$ **ii** 5^2 **iii** 25
b **i** $\frac{3^2 \times 3^4}{3^3}$ **ii** 3^3 **iii** 27
c **i** Sample answer: $\frac{2^6 \times 2^8}{2^2 \times 2^4}$
ii 2^8 **iii** 256
d **i** Sample answer: $2^4 \times ((2^2)^2)^2$
ii 2^{12} **iii** 4096

10 **a** 2 **b** 9 **c** 3 **d** 5

11 **a** **i** 23 **ii** 26.46
iii 12.5 **iv** 3.48
b **i** not a surd **ii** surd
iii not a surd **iv** surd

12 **a** 6 **b** −11 **c** 330 **d** 65.5

Analysis

1 a

Number of days	Mass of metal (grams)	$\frac{\text{mass of metal}}{\text{original mass of metal}}$
0	240	1
1	120	$\frac{1}{2}$
2	60	$\frac{1}{4}$
3	30	$\frac{1}{8}$

b $\left(\frac{1}{2}\right)^0, \left(\frac{1}{2}\right)^1, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3$ c $240 \times \left(\frac{1}{2}\right)^7$

d $\frac{15}{8}$ e 1.875 grams f 8 days

2 a $\frac{(2 \times 5 \times 7^2) \times (2^2 \times 3^2)^2}{(2^2 \times 3^3 \times 7) \times (3 \times 13^2)^0}$ b $2^3 \times 3 \times 5 \times 7$

c 840

3 a 0.1 mm b 0.2 mm c 3.2 mm d $a \times 2^n$

4 a $1 < 2 < 4 < 6 < 9 < 16 < 20 < 25$

b **ii, iv, vii**

c **i** 1 **ii** 1.41 **iii** 2 **iv** 2.45
v 3 **vi** 4 **vii** 4.47 **viii** 5

d $1 < 1.41 < 2 < 2.45 < 3 < 4 < 4.47 < 5$

e The bigger a number is, the bigger the square root of the number is.

f **i** < **ii** > **iii** < **iv** <
v > **vi** < **vii** > **viii** <

g $\frac{1}{25} < \frac{1}{20} < \frac{1}{16} < \frac{1}{9} < \frac{1}{6} < \frac{1}{4} < \frac{1}{2} < 1$

h **ii, iv, vii**

i **i** 1 **ii** 0.71 **iii** $\frac{1}{2}$ **iv** 0.41
v $\frac{1}{3}$ **vi** $\frac{1}{4}$ **vii** 0.22 **viii** $\frac{1}{5}$

j $\frac{1}{5} < 0.22 < \frac{1}{4} < \frac{1}{3} < 0.41 < \frac{1}{2} < 0.71 < 1$

k Yes, the bigger a number is, the bigger the square root of the number is.

l **i** < **ii** < **iii** < **iv** >
v > **vi** > **vii** > **viii** >

CHAPTER 5 Algebra

5A Expressions

EX p201

1 a **i** $6a$, b and -1
ii a and b
iii The coefficient of $6a$ is 6. The coefficient of b is 1.
iv -1

b **i** $2xy$ and $-3x$
ii x and y
iii The coefficient of $2xy$ is 2. The coefficient of $-3x$ is -3.
iv No constant terms

c **i** $11f$, $-g$, $15h$ and -10
ii f, g and h
iii The coefficient of $11f$ is 11. The coefficient of $-g$ is -1. The coefficient of $15h$ is 15.
iv -10

d **i** $-\frac{9}{4}p$
ii p
iii The coefficient of $-\frac{9}{4}p$ is $-\frac{9}{4}$.
iv No constant terms

e **i** $\frac{3}{4}c$, d and $\frac{1}{3}$
ii c and d
iii The coefficient of $\frac{3}{4}c$ is $\frac{3}{4}$. The coefficient of d is 1.
iv $\frac{1}{3}$

f **i** $\frac{x}{2}$, $\frac{y}{11}$, $-\frac{7}{10}$ and $-\frac{z}{8}$
ii x, y and z
iii The coefficient of $\frac{x}{2}$ is $\frac{1}{2}$. The coefficient of $\frac{y}{11}$ is $\frac{1}{11}$. The coefficient of $-\frac{z}{8}$ is $-\frac{1}{8}$.
iv $-\frac{7}{10}$

g **i** $-\frac{m}{5}$, $-\frac{n}{7}$ and $-8p$
ii m, n and p
iii The coefficient of $-\frac{m}{5}$ is $-\frac{1}{5}$. The coefficient of $-\frac{n}{7}$ is $-\frac{1}{7}$. The coefficient of $-8p$ is -8.
iv No constant terms

h **i** $\frac{5x}{8}$, $-9y$, $\frac{z}{4}$ and $-\frac{2}{5}$
ii x, y and z
iii The coefficient of $\frac{5x}{8}$ is $\frac{5}{8}$. The coefficient of $-9y$ is -9. The coefficient of $\frac{z}{4}$ is $\frac{1}{4}$.
iv $-\frac{2}{5}$

2 a expression b equation c equation
d equation e expression f equation

3 a false, it is an equation
b true
c false, there are 2 terms
d true
e false, the coefficient of $-d$ is -1
f false, the coefficient of $\frac{b}{12}$ is $\frac{1}{12}$
g true h true

4 a $a + 5$ b $\frac{z}{3}$ c $y - 23$ d $50m$
e $\frac{2}{5}x$ f $2p$ g $\frac{s}{2}$ h $\frac{3}{4}w + 5$

5 a $7x + 2$ b $\frac{x}{3} - 6$ c $\frac{3}{4}x - 10$
d $13 + \frac{1}{3}x$ e $15x - 28$ f $\frac{1}{2}x - 5$

6 a The sum of c and 8
b x less than 11
c x divided by 5
d the product of nine-tenths and q
e 5 less than the product of 3 and x
f 16 more than the product of ten-thirds and a

7 a $3(x+6)$ b $\frac{n-10}{7}$ c $\frac{3}{5}fg$ d $\sqrt{t}-1$
e $\frac{b+12}{15}$ f $\sqrt{z+9}$
g $\frac{2}{3}v-7$ h $3(33+y)$

8 a $8(x-3)$ b $6(x+20)$ c $\frac{5}{6}x-2$
d $\frac{4}{5}x+12$ e $2(x+10)$ f $\frac{x+4}{12}+16$

9 $n+18$

10 $\frac{c}{4}$

11 $8x$

12 $y=x-10$

13 a i $y-6$ ii $y-10$
b i $y+5$ ii $y+23$
c $y-x$ d $2y$

14 a $c+p$ b $p-1$ c $c+p-1$

15 a $8n$ and 12
b the delivery fee
c the price of each brush
d \$92

16 a $130t+25$ b \$415

17 a $2(x+7)$ b 40 chickens

18 a Naomi is correct. If one block is left over, then $x-1$ blocks are divided between the three students.
b Sample answer: An extra block is added to the container so that the blocks can be divided equally between three students.

19 a $\frac{4x}{9}-\frac{5}{9}$ b $\frac{4}{9}$ c $-\frac{5}{9}$

20 a $0.8xy+0.7ab$ b Sample answer: $67a$
c 4 t-shirts, 2 pairs of pants

EX p206 5B Substitution

1 a 12 b 2 c 2 d 10 e $1\frac{3}{5}\left(\text{or } \frac{8}{5}\right)$
f 80 g 254 h −1 i 76 j 0

2 a 5 b −7 c 31 d −3 e 102
f 8 g −4 h −63

3 a 11 b 26 c 8 d 5 e 2
f −4 g 6 h 3

4 a 5 b 51 c 7 d 100 e 0
f 1 g $\frac{1}{3}$ h 35

5 a 30 b 2 c 1 d 8 e $1\frac{2}{5}\left(\text{or } \frac{7}{5}\right)$
f −7 g $\frac{5}{9}$ h $\frac{3}{7}$

6 a 21 b 6 c 15 d $-\frac{3}{5}$ e $\frac{2}{9}$
f 2 g $-\frac{4}{5}$ h −5

7 a 13 b 23 c −5 d 5 e 16
f 7 g 8 h 4

8 a −8 b 132
c 4 d 9988
e −3 f $-11\frac{3}{4}\left(\text{or } -\frac{47}{4}\right)$
g −12 h $-9\frac{3}{4}\left(\text{or } -\frac{39}{4}\right)$

9 a 10 b 8
c −18 d $1\frac{1}{2}\left(\text{or } \frac{3}{2}\right)$
e 4 f $-10\frac{2}{3}\left(\text{or } -\frac{32}{3}\right)$
g 151 h $3\frac{3}{5}\left(\text{or } \frac{18}{5}\right)$

10 a 3 m/s b 94 km/h

11 a 4.9 m
b i 44.1 m ii 122.5 m
c increase; average speed in first second is 4.9 m/s, in first 3 seconds is $\frac{44.1}{3}$ or 14.7 m/s, in first 5 seconds is $\frac{122.5}{5}$ or 24.5 m/s.

12 a 4 cm² b 70 cm² c $6\frac{1}{4}$ cm² $\left(\text{or } \frac{25}{4}\text{ cm}^2\right)$
d i the square ii the triangle

13 a i Instead of substituting $x=5$ into $2x$ and multiplying 2 by 5, x has been replaced with 5 in the expression $2x$ without multiplying.
ii 16
b i Incorrect order of operations. Multiplication has been performed before applying the exponent.
ii 9
c i 2^0 does not equal 0, $2^0=1$. ii 1
d i The negative sign has been left outside the brackets.
ii 21

14 a \$1050 b \$2433.31

15

x	x^2	$2x^2$	$16x$	$2x^2-16x$
0	0	0	0	0
2	4	8	32	−24
4	16	32	64	−32
6	36	72	96	−24
8	64	128	128	0

$x=0$ and $x=8$

16 a i 25°C ii −20°C iii 5°C iv 38°C
v −18°C vi 17°C
b i 86°F ii 14°F iii 32°F iv 81°F
v 18°F vi 108°F
c −5°C
d 86°F, as 32°C = 89.6°F

17 a $P=0.95(x-y)-z$ b \$17 540.50

EX p211 5C Adding and subtracting algebraic terms

1 a $20x$, $-12x$ and $3x$
$8y$, $-15y$ and y
b $5x$, $\frac{3}{7}x$, $-25x$ and $\frac{x}{7}$
$\frac{5}{7}y$ and $6y$
c $4xy$, $-yx$, $\frac{1}{8}xy$ and $-\frac{yx}{10}$
d $-a$ and $12.5a$
$5a^2$, $\frac{4}{9}a^2$, $2.3a^2$ and $-\frac{9a^2}{10}$
e $\frac{11}{18}p^3$, $5.5p^3$ and p^3
$\frac{p^2}{5}$, $-11p^2$ and $-0.05p^2$
f $\frac{9ab}{10}$, $\frac{5}{6}ab$ and $7.2ba$
$7abc$, $-cab$ and $13cab$

g $-9fg$ and $\frac{4fg}{5}$
$120f^2g$ and $-\frac{14}{3}f^2g$
$\frac{20fg^2}{3}$ and $0.33fg^2$

h $\frac{2}{9}xy^2z$, $-12y^2xz$ and $99zy^2x$
$0.095zyx$ and $18.6xyz$

2 **a** true
b false; the powers of the pronumerals are not the same
c true **d** true
e false; the powers of the pronumerals are not the same
f true

3 **a** $9x$ **b** $8kp$ **c** $-4a^2$ **d** $4g$
e $-2xy$ **f** $2g^2h$ **g** $\frac{1}{4}abc$ **h** $-\frac{4}{15}xyz^2$

4 **a** true b false; $10k$
c false; $3p + 6q$ (does not simplify further)
d false, $6m^2 + 2n$
e false, $9ab + 3a$
f true g true
h false; $\frac{7}{18}p^2q - \frac{5}{6}p^2$

5 **a** $6b + 5$ **b** $-n - 5mn$ **c** $4 - 8c^2$ **d** $gh - 3$
e $-pq - 10q^2p + 8p^2q$
f $11x^2y - 12xy^2$
g $\frac{31}{14}rst + \frac{1}{6}st$
h $-\frac{1}{2}abc - \frac{1}{2}ab - 2$
i $\frac{2}{5}g^3h + \frac{11}{6} + \frac{4g^2h}{5} - \frac{1}{15}gh$
j $\frac{9}{4}m^2n - \frac{3}{4}m^2n^4 + \frac{1}{6}$

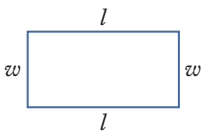
6 **a** 165 **b** $16x^2 + xy$ **c** yes
d simplified expression; only had to substitute values into two terms

7 **a** $6a + 14b$; 72 **b** $3a + 9b$; 42
c $5ab + 3b - 2a$; 74 **d** $-2ab + a^2$; -5
e $-5b^2 + 7a^2$; 130 **f** $\frac{1}{4}a^2 - \frac{2}{3}b^2$; $\frac{1}{4}$
g $-11ab - 2ab^2$; -255 **h** $-2ab^2 - 3a^2b$; -315
i $\frac{19}{20}a^2b - \frac{19}{6}ab^2$; $-\frac{285}{4}$

8 **a** Tania didn't keep the minus sign with $2y$ and the addition sign with $7x$.
b $3x + 7x - 2y - 4y$

9 **a** $11x + 2y$ **b** $-\frac{7}{6}$
c $-2y$ **d** $-2xy + 8x$
e $16x^2y - 6xy^2 - 5$ **f** $-\frac{37}{20}x^2y + \frac{9}{4}xy^2$

10 **a**



b $l + w + l + w$ c $2l + 2w$ d $P = 2l + 2w$
e **i** 14 m **ii** 18.4 m **iii** 46 m
f Sample answer: length 8 m, width 2 m

11 **a** $8x + 4$ **b** $18x + 6$ **c** $36x + 4$ **d** $40x + 2$

12 **a** 36 cm **b** 78 cm **c** 148 cm **d** 162 cm

13 **a** The a has been multiplied four times instead of added four times. The expression simplifies to $4a$.
b The terms have been treated as like terms and added together instead of as unlike terms. The expression $2x + 3y$ is already in its simplest form.
c The x in each term has been multiplied together instead of treated as like terms. The expression simplifies to $13x$.
d The $-x$ term has been interpreted as dividing by x to cancel out the x in the $8x$ term. Instead, $-x$ is the same as $-1x$ so the expression simplifies to $7x$.

14 $\frac{27}{4}x$

15 **a** **i** $\frac{4}{3}$ and $\frac{4}{3}$ **ii** $-\frac{4}{3}$ and $-\frac{4}{3}$ **iii** $\frac{2}{3}$ and $\frac{8}{3}$
b **i** 0 and 0 **ii** 1 and -1 **iii** 1 and -1
c **i** 0 and 0 **ii** -1 and -1 **iii** 1 and 1
d **i** $-\frac{3}{5}$, $-\frac{3}{5}$, $-\frac{3}{5}$, and $\frac{3}{5}$
ii $\frac{3}{5}$, $\frac{3}{5}$, $\frac{3}{5}$, and $-\frac{3}{5}$
iii $-\frac{6}{5}$, $-\frac{6}{5}$, $-\frac{6}{5}$, and $\frac{6}{5}$

16 **a** The expressions are not equivalent. The expressions are equal for $x = 1$ and $x = -1$, $\frac{4}{3x}$ is dividing x and $\frac{4}{3}x$ is multiplying x, hence why the expressions are different for $x = 2$.
b The expressions are not equivalent. Despite the fact that the expressions are equal for $x = 0$, squaring a positive or negative number always results in a positive number whereas taking the negative of a squared number always results in a negative: $(-x)^2 = (-x)(-x) = xx = x^2$ but $-x^2 = -xx$.
c The expressions are equivalent. Cubing the negative of a number always results in the negative of the cube of that number: $(-x)^3 = (-x)(-x)(-x) = -xxx = -x^3$.
d The first three expressions are equivalent but $\frac{-3x}{-5}$ is not equivalent. Looking at the coefficients of the first three expressions, they all simplify to the same negative fraction. The coefficient of the final expression simplifies to a positive.

17 $11a^2b^7c^5d^5e^9f^{11}g^{14} + 9a^2b^5c^5d^7e^9f^{11}g^{14}$

18 $2.25x$ or $\frac{9}{4}x$

19 **a** $12(2x + 3) + 11(2x + 5)$ **b** $10(x - 3y + 4)$
c $2(x + 2) + 8(x - 2)$ **d** $\frac{13}{3x + 1} + \frac{12}{3x - 1}$
e $7\sqrt{7 - 2x} - 4\sqrt{7 + 2x}$ **f** $9(a + b) - \frac{3}{a + b} - \sqrt{a + b}$

EX p216 5D Multiplying algebraic terms

1 **a** $4x$ **b** $6k$ **c** $7d$ **d** $5m$
e p **f** $-3h$ **g** $-9a$ **h** $-10y$
i $-8c$ **j** $-e$ **k** $2f$ **l** $3m$

2 **a** $3ab$ **b** $4mn$ **c** $2pt$ **d** $-8cd$
e $9xy$ **f** $5fk$ **g** abc **h** ehy
i $8q$ **j** $-18m$

3 **a** II **b** I **c** III

4 **a** $5m^2$ **b** $-2a$ **c** $-xy$ **d** $2n^2$
e $-6c^2d^2$ **f** $7a^2bc$ **g** $-2s^2t^3$
h $-10f^2g^3$ **i** pq^3r^3 **j** $-11x^3y^2z^2$

5 **a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$
b $a \times a \times a \times a \times a \times a \times a = a^7$

6 **a** a^5 **b** k^5 **c** c^7 **d** e^6f^5 **e** a^5d^6 **f** $m^{10}n^5$

7 **a** $12abc$ **b** $-a^2h^3$ **c** $21wxyz$
d $12x^4y^5$ **e** $21a^7$ **f** $-5p^{12}$
g $24n^2$ **h** $15x^8y^{10}$ **i** $-a^8b^6$
j $-18x^2yz$ **k** $\frac{1}{5}w^4x^5y^6z$ **l** $-\frac{18}{7}a^5b^7c^2$

8 **a**

×	$4a$	3	$6m$	$2abc$
$3d$	$12ad$	$9d$	$18dm$	$6abcd$
ab^2	$4a^2b^2$	$3ab^2$	$6ab^2m$	$2a^2b^3c$
$5mn$	$20amn$	$15mn$	$30m^2n$	$10abcmn$
$2ac$	$8a^2c$	$6ac$	$12acm$	$4a^2bc^2$

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$	$-6x^2$	$-8x^2yz$	$10xy$	$-4x^3$
xy^2	$3x^2y^2$	$4x^2y^3z$	$-5xy^3$	$2x^3y^2$
$6xy$	$18x^2y$	$24x^2y^2z$	$-30xy^2$	$12x^3y$
$-7yz$	$-21xyz$	$-28xy^2z^2$	$35y^2z$	$-14x^2yz$

9 **a** $-70bghk$ **b** $30g^{10}h^3$ **c** $-12b^9$ **d** $48m^2nq$
e $-21fg^3h^3$ **f** $-\frac{9}{40}m^{11}n^3$ **g** $24a^6b^5c$
h $10w^3x^8y^8$ **i** $-\frac{5}{2}a^8b^{13}c^7d^4$

10 **a** $2 \times 3 = 6$ **b** $8 \times -2 = -16$
c $6 \times k = 6k$ **d** $3 \times x = 3x$
e $a \times b = ab$ **f** $x \times -7y = -7xy$
g $5d \times c = 5cd$ **h** $4a \times 2b^2 = 8ab^2$
i $5n^3 \times 3mn = 15mn^4$ **j** $6e \times -2e^2f = -12e^3f$
k $5jk^2l \times 4l^2 = 20jk^2l^3$ **l** $5xy \times -xz^5 = -5x^2yz^5$

11 **a** C **b** D
c The coefficient of x is $\frac{7}{5}$; the coefficient of y is $-\frac{1}{5}$.
d D **e** D **f** B
g When using algebraic terms, placing pronumerals or a numeral and a pronumeral next to each other generally implies multiplication. However, for mixed numbers, placing a numeral in front of a fraction implies addition. Putting the algebraic and mixed numbers together leads to conflicting conventions presented simultaneously. For clarity, we do not generally use mixed numbers as coefficients.

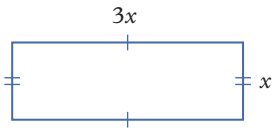
12 **a** **i** $(3x^2)^4 = (3x^2) \times (3x^2) \times (3x^2) \times (3x^2)$
$= 3 \times 3 \times 3 \times 3 \times x^2 \times x^2 \times x^2 \times x^2$
$= 81x^8$

ii $\left(\frac{x^2}{3}\right)^4 = \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right) \times \left(\frac{x^2}{3}\right)$
$= \frac{x^2 \times x^2 \times x^2 \times x^2}{3 \times 3 \times 3 \times 3}$
$= \frac{x^8}{81}$

b **i** $64x^{30}$ **ii** $\frac{y^{45}}{32}$
iii $64a^6b^{21}$ **iv** $\frac{625u^{24}}{16v^{20}}$

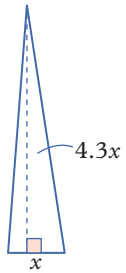
13 **a** $48xy$ **b** $12x^2y^2$ **c** $14x^2y^2$

14 **a** **i**



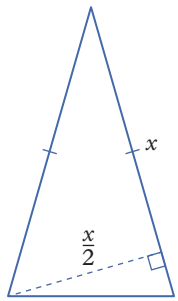
ii $3x^2$

b **i**



ii $2.15x^2$

c **i**



ii $\frac{x^2}{4}$

15 $2^{12a+5} \times 3^{13a+16}$

EX p219

Checkpoint

1 **a** **i** x and y **ii** 4 **iii** 4, 9, −4
iv −3 **v** x: 1, y: 2
b **i** m, n, and v **ii** 3 **iii** $\frac{5}{3}$, 1
iv $-\frac{9}{13}$ **v** m: 2, n: 1, v: 3

2 **a** $\frac{x}{8}$ **b** $7(a+b+c)$ **c** $y(y+3)$
d $\sqrt{c+a}$ or $\sqrt{a+c}$

3 **a** $20g$ **b** $6g+b$ **c** $\frac{1}{3}(t+r+p)$
d $0.15b+3$ or $\frac{3b}{20}+3$

4 **a** 84 **b** 4 **c** 574 **d** $\frac{17}{2}$

5 **a** $y = 31$ **b** $s = 64$

6 **a** 61 **b** 20

7 **a** $7x+4y+10$ **b** $2ab+5a+10b$
c $8pqr$ **d** $30ikl+8jkl$

8 **a** $-3x+4x^2$ **b** $12xy^2+2x^2y$

9 **a** $-7.46x+0.26y$ **b** $-\frac{15}{8}x^2$

10 **a** $63xy$ **b** $27abcd$
c $-110ghpq$ **d** $36abcuvxy$

11 **a** $36x^5$ **b** $56x^4y^4$

12 **a** $0.372x^2$ **b** $\frac{5}{2}x^{12}$ or $\frac{5x^{12}}{2}$

EX p221 5E Dividing algebraic terms

1 **a** a **b** $\frac{5x}{y}$ **c** 9 **d** 3 **e** $\frac{1}{3}$
f $4b$ **g** $3c^2$ **h** $\frac{2b^3}{3}$ **i** $\frac{p}{6}$ **j** mn^5
k $6a^3$ **l** $5m$ **m** $\frac{2m^2b}{3}$ **n** $2t^2$ **o** $\frac{4h^2kj}{5}$

2 **a** x **b** $2k$ **c** $9a$ **d** $9x$ **e** $\frac{n}{4}$ **f** bcd

3 **a** x^4 **b** $2jk$ **c** $3x$ **d** $\frac{m^7}{p^3}$ (can't be simplified)
e b^2c^2 **f** $\frac{2k^8}{a^5}$ **g** $\frac{9x^4}{11y^4}$ **h** $\frac{3a^3b}{c^3}$ **i** $\frac{xy}{4}$ **j** $4n^2$

4 **a** $-3y$ **b** $-5a^2$ **c** $\frac{3v^2}{2}$

5 **a** x^5 **b** m^4 **c** $6a^6$ **d** n^5 **e** 5

6 **a** $16m \div 8 = 2m$ **b** $\frac{27jk}{-9j} = -3k$
c $16x^3y^2 \div 2xy = 8x^2y$ **d** $\frac{2a^3b^2}{10a^2b^3} = \frac{a}{5b}$

7 **a** $2y$ **b** 10 m, 160 m^2

8 **a** $3x$ **b** 16 m, 96 m^2

9 **a** length = $2x$ cm; area = $2x^2$ cm^2
b $100x^2$ cm^2 **c** 50 pavers
d **i** 40 cm **ii** 80 cm **iii** 3200 cm^2
iv 400 cm **v** 160 000 cm^2
e 50; same
f No; the expression used to calculate the number of pavers simplifies to 50 in each case as the pronumeral x is cancelled in the fraction.

10 **a** $\frac{3}{10}$ **b** $\frac{27x^2y}{16}$

11 **a** $2^{14} \times 13^5 \times d^{12}$ **b** $3^4 \times 7^4 \times a^9 \times b^6$
c $p^9q^7r^{15}s^6$ **d** $r^{45}s^{51}t^{78}u^{84}v^{99}$

12 **a** $\frac{5}{36}$ **b** 64 **c** 40.5 **d** 7

13 **a** **i** 1 **ii** 1 **iii** 1
b The expression simplifies to 1. It doesn't matter what the values of x, y and z are, the expression will always simplify to 1.

14 $\frac{x+1}{4}$

15 **a** Sample answer: $x^9y^7 \div x^1y^2 = x^8y^5$, sum is $8 + 5 = 13$
b 9 cannot be an exponent of the divisor (second term) otherwise a subtraction would result in a negative exponent. The largest exponent for the quotient that can be created using a 9 is 8 by subtracting 1. Using the remaining digits, the three largest possible sums are $15 = 8 + 7$, $14 = 8 + 6$ and $13 = 8 + 5$. Since 9 and 8 are already in use, it is not possible to have a difference of 7 with the remaining digits: $7 = \cancel{9} - 2 = \cancel{8} - \cancel{1}$. Similarly, since 9, 8 and 1 are in use, it is not possible to have a difference of 6 with the remaining digits: $6 = \cancel{9} - 3 = \cancel{8} - 2 = 7 - \cancel{1}$. However, it is possible to have a difference of 5 as $5 = 7 - 2$. If instead we started with $9 - 2 = 7$, the largest sum is still 13 since $15 = 7 + 8$ but $8 = \cancel{9} - 1$ and $14 = 7 + \cancel{7}$.

EX p226 5F Expanding

1 **a** $3a + 6$ **b** $5k - 35$
c $10 + 2a$ **d** $8a + 8b$
e $8x + 24$ **f** $-2 + 16m$
g $35a + 21b$ **h** $48c - 30d$
i $-15m - 6n$

2 **a** $ab + 4a$ **b** $3x + wx$
c $cd - de$ **d** $2p^2 + 3p$
e $k - 4k^2$ **f** $8xy + 6wx$
g $3x - 12x^2$ **h** $\frac{2}{3}m^2 + \frac{7}{3}mn$
i $-6a^2 + 12ab$ **j** $8e^3 + 10e^2$
k $\frac{1}{2}a^4 - 15a^3b$ **l** $-11x^2y - 11y^3$

3 **a** $6x + 2y + 14$ **b** $10ab + 20d - 15$
c $14c - 7d + 35ef$ **d** $-18g + 12h - 6gh$
e $-20k - 8km + 36$ **f** $-6xy - 12 + 21w$

4 **a** $5 \times a + 5 \times 4 + 2 \times a + 2 \times 7$
$= 5a + 20 + 2a + 14 = 7a + 34$
b $3b \times c + 3b \times 2 + 2c \times a + 2c \times 5b$
$= 3bc + 6b + 2ac + 10bc = 13bc + 6b + 2ac$
c $d \times d + d \times 8 + 6 \times d + 6 \times (-3)$
$= d^2 + 8d + 6d - 18 = d^2 + 14d - 18$
d $4k \times 2k + 4k \times 3 + (-2) \times k + (-2) \times (-5)$
$= 8k^2 + 12k - 2k + 10$
$= 8k^2 + 10k + 10$

5 **a** $7a + 27$ **b** $10b$
c $11mn + 20m + 3kn$ **d** $21x + 5xy + \frac{16}{3}y$
e $\frac{k^2}{4} + \frac{19k}{4} + 20$ **f** $c^2 - 3c + 35$
g $11rst + 5rst^2 + 36rt$ **h** $h^3 - 4h^2 - 14h$

6 $-(a + 2) = -1 \times (a + 2)$
$= -1 \times a - 1 \times 2$
$= -a - 2$

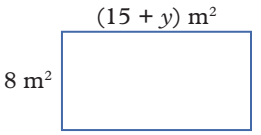
7 **a** $7p + 20$ **b** $2d - 14$
c $x^2 + 3x + 5$ **d** $k^2 + k - 1$
e $2e^2 + \frac{3}{2}$ **f** $3ab + 4a - 6$
g $-\frac{2}{3}wy - w - 2$ **h** $mnp - 4m^2p + n$

8 **a** 440 **b** $15a^2 + 50a$ **c** 440
d Sample answer: It was easier to evaluate the expression in expanded form as there are just two terms to evaluate.

9 **a** $2a^2 + 10a$; 28 **b** $30a - 6a^2$; 36
c $15a^2 - 65a$; -70 **d** $2a^2 - 3a$; 2

10 **a** $c + 3$ **b** $2(c + 3)$ **c** $2c + 6$ **d** 42

11 **a** 120 m^2 **b** $8y$ m^2 **c** $(120 + 8y)$ m^2
d



e $8(15 + y)$ m^2

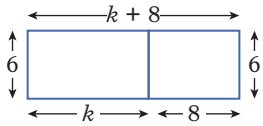
f $(120 + 8y)$ m², the expressions are the same as they represent the same area of lawn.

12 area of large rectangle = sum of areas of two smaller rectangles

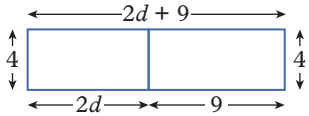
a area of large rectangle = $(a + 2) \times 3$ or $3(a + 2)$
sum of areas of smaller rectangles =
$3 \times a + 3 \times 2 = 3a + 6$

b area of large rectangle = $(x + 4) \times x$ or $x(x + 4)$
sum of areas of smaller rectangles =
$x \times x + x \times 4 = x^2 + 4x$

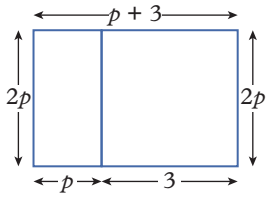
13 a



b



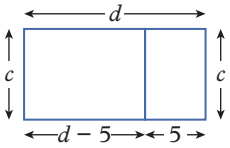
c



14 a $a - 3, 5$ **b** $5(a - 3)$

c **i** $a, 5$ **ii** $5a$ **iii** $3, 5$ **iv** 15
v $5a - 15$

15



16 $5a(b + 3) = 5ab + 15a$, so if a and b are both positive integers, $5a \times b + 3$ will always be less than $5a(b + 3)$.

17 a $7x^5 - 10x^4 - 21x^2$ **b** $4a^3c$

c $6m^3np^2 + 4m^2n - 6m^2np^2 + 24n$ **d** $3wy$

18 a -20 **b** -432 **c** -256 **d** 90

EX p231 5G Factorising

1 a III **b** IV **c** II **d** I

2 a 4 **b** $5d$ **c** $2j$ **d** $3ab$ **e** 9
f $7y$ **g** bc **h** $2bcd$ **i** $2y$ **j** $3c$
k $6g^2$ **l** $4b$ **m** m^2p^2 **n** $5r^2s^4t$

3 a $2x + 6 = 2\left(\frac{2x}{2} + \frac{6}{2}\right) = 2(x + 3)$

b $8a - 36 = 4\left(\frac{8a}{4} - \frac{36}{4}\right) = 4(2a - 9)$

c $25mn + 30n = 5n\left(\frac{25mn}{5n} + \frac{30n}{5n}\right) = 5n(5m + 6)$

d $kmn - mn = mn\left(\frac{kmn}{mn} - \frac{mn}{mn}\right) = mn(k - 1)$

e $12ef - 18df = 6f\left(\frac{12ef}{6f} - \frac{18df}{6f}\right) = 6f(2e - 3d)$

f $-26ab + 24cd = -2\left(\frac{-26ab}{-2} + \frac{24cd}{-2}\right)$
$= -2(13ab - 12cd)$

g $-abc + acd = -ac\left(\frac{-abc}{-ac} + \frac{acd}{-ac}\right) = -ac(b - d)$

h $-21xyz - 28wxy = -7xy\left(\frac{-21xyz}{-7xy} - \frac{28wxy}{-7xy}\right)$
$= -7xy(3z + 4w)$

4 a $4(a + 2)$ **b** $7(e + 3)$ **c** $5(9 + k)$
d $3(1 + 4y)$ **e** $9(3 - p)$ **f** $6b(4a + 3)$
g $2t(6s - 7)$ **h** $5n(m + 2)$ **i** $3g(1 - 3h)$
j $2x(11y - 2z)$ **k** $3b(a + c)$ **l** $8m(2n - p)$
m $9xy(3z - 2w)$ **n** $3mn(4k + 5)$ **o** $-3fg(7e - 8h)$

5 a $h^2 + 7h = h\left(\frac{h^2}{h} + \frac{7h}{h}\right) = h(h + 7)$

b $10a + 5a^2 = 5a\left(\frac{10a}{5a} + \frac{5a^2}{5a}\right) = 5a(2 + a)$

c $6p^3 + 9p^5 = 3p^3\left(\frac{6p^3}{3p^3} + \frac{9p^5}{3p^3}\right) = 3p^3(2 + 3p^2)$

d $6xy + 4x^2y = 2xy\left(\frac{6xy}{2xy} + \frac{4x^2y}{2xy}\right) = 2xy(3 + 2x)$

e $mn^2 + 5mn = mn\left(\frac{mn^2}{mn} + \frac{5mn}{mn}\right) = mn(n + 5)$

f $24efg^2 + 32fg^7h^3 = 8fg^2\left(\frac{24efg^2}{8fg^2} + \frac{32fg^7h^3}{8fg^2}\right)$
$= 8fg^2(3e + 4g^5h^3)$

6 a $c(c + 4)$ **b** $k(6 - k)$ **c** $a(a - 2)$
d $3m^2(1 + m^2)$ **e** $p^3(10 + 11p^2)$ **f** $h(4 - 7h^3)$
g $3x^2y(2 + xy^5)$ **h** $5bc^2(bc^3 - 2)$ **i** $4mn^2(2 + 9mn)$
j $3xyz(9z - 5x)$ **k** $2a^2bc^4(7ac^6 + 8b^6)$
l $6efg^2(2e + 3f^4)$

7 See answers for question **6**

8 a -5 **b** $-2p$ **c** $-3k$ **d** $-7st$
e $-4x$ **f** $-4m^2$ **g** $-xy$ **h** $-4bc^2$

9 a $-4(a + 3)$ **b** $-2(3 + 4k)$ **c** $-7m(1 - 2n)$
d $-5b(3 - 7a)$ **e** $-3t(s + 3r)$ **f** $-6bc(3a + 4d)$
g $-3x^{12}(4 - 7x^9)$ **h** $-h(h + 2)$
i $-8x^2y^7(2 - x^3y^3)$ **j** $-5ab^6(9 + 2a^{14}b^4c^8)$
k $-6xyz^2(7 + 6x^3y^{12})$ **l** $-abc^3(11a - 9bc^6)$

10 a 3 **b** bd **c** $5m$ **d** $6a$
e $4xz$ **f** $7i^2j^2k$

11 a $5(3p + 3q - 1)$
b $k(6j - 3i - 10l)$
c $-2np(4mq - 2jk + km)$
d $-9f(3g - h)$
e $17uv(wx + 2x - 3xz - 4)$
f $8a^7b^7c^4(7ab^5 + 4a^2c^5 - 10b^4c^3)$

12 a $x(x + 6)$ **b** x and $x + 6$ **c** $x + 6$
d $(x + 6) \times x = x \times (x + 6) = x \times x + x \times 6 = x^2 + 6x$
e length = 3 + 6 = 9 cm; width = 3 cm so
area = length × width = 9 × 3 = 27 cm²
using $x^2 + 6x$, area = $3^2 + 6 \times 3 = 9 + 18 = 27$ cm²

13 a **i** $32 - 24x$ **ii** $8(4 - 3x)$
b **i** $7(y + 7)$ **ii** $7y + 49$
c **i** $6b^5 - 15b^8$ **ii** $3b^3(2b^2 - 5b^5)$
d **i** $25a(a - 5b)$ **ii** $25a^2 - 125ab$

14 a $5x \times 3y + \frac{1}{2} \times 5x \times 2x = 15xy + 5x^2$
b $5x(3y + x)$

15 a i $6\left(\frac{x}{6}+\frac{5y}{6}+\frac{11z}{6}\right)$ **ii** $-(-x-5y-11z)$

iii $\frac{1}{2}(2x+10y+22z)$ **iv** $x\left(1+\frac{5y}{x}+\frac{11z}{x}\right)$

v $-3x\left(-\frac{1}{3}-\frac{5y}{3x}-\frac{11z}{3x}\right)$

b i $6\left(\frac{x}{3}-\frac{y}{2}+z\right)$ **ii** $-(-2x+3y-6z)$

iii $\frac{1}{2}(4x-6y+12z)$ **iv** $x\left(2-\frac{3y}{x}+\frac{6z}{x}\right)$

v $-3x\left(-\frac{2}{3}+\frac{y}{x}-\frac{2z}{x}\right)$

c i $6(3x^2+6x^3y-9x^4z)$

ii $-(-18x^2-36x^3y+54x^4z)$

iii $\frac{1}{2}(36x^2+72x^3y-108x^4z)$

iv $x(18x+36x^2y-54x^3z)$

v $-3x(-6x-12x^2y+18x^3z)$

d i $6\left(\frac{2xy}{3}-x-\frac{5xz}{3}\right)$ **ii** $-(-4xy+6x+10xz)$

iii $\frac{1}{2}(8xy-12x-20xz)$ **iv** $x(4y-6-10z)$

v $-3x\left(-\frac{4y}{3}+2+\frac{10z}{3}\right)$

16 a i When evaluating $\frac{12q}{-6}$, the student got $2q$, not $-2q$, and used a positive sign instead of a negative sign.

ii $-6(3p-2q)$

b i The student did not take out the HCF, $8r$, as a factor so the expression is not fully factorised.

ii $8r-16r^3=8r\left(\frac{8r}{8r}-\frac{16r^3}{8r}\right)=8r(1-2r^2)$

c i The student simplified $\frac{5y^3}{5y^3}$ to 0, not 1, as the numerator and denominator are the same.

ii $20y^4+5y^3=5y^3\left(\frac{20y^4}{5y^3}+\frac{5y^3}{5y^3}\right)=5y^3(4y+1)$

d i The student took out the first term as the factor, not the HCF.

ii $6x^3y+7xy^3=xy\left(\frac{6x^3y}{xy}+\frac{7xy^3}{xy}\right)=xy(6x^2+7y^2)$

17 a i y **ii** $(x+2)$

b i $y(7x-5)$ **ii** $(x+2)(7x-5)$

18 $\frac{5}{8}x^{12}y^{17}(y^2-5x^9y^3+3x^5-2x^{20}y^{23})$

EX p236

CHAPTER 5 review

Mathematical literacy review

1 Sample answer:

a Multiply the difference of a and b by 2, then subtract the result by 4

b Subtract $2b$ from a, then subtract the result by 4

c Divide the difference of a and b by 2, then subtract the result by 4

d Subtract the difference of half of b and 4 from a

e The difference between a and the square of b

f The square of the difference between a and b

2 index

3 Sample answer:

associative law $a+(b+c)=(a+b)+c$

commutative law $a+b+c=b+c+a$

distributive law $a(b+c)=ab+ac$

Multiple choice

1 B	**2** E	**3** A	**4** B	**5** A	**6** C
7 E	**8** C	**9** D	**10** E	**11** C	**12** C

Short answer

1 a $4m+3$ **b** $\frac{5m}{2}$ **c** $\frac{m+3}{7}$

d $\frac{3}{5}(m-6)$

2 a $25x$ **b** $13y$ **c** $20p$

d $25x+13y+20p$

3 a 24 **b** 80 **c** -16 **d** -7

4 a $v=\sqrt{u^2+2as}$ **b** 16 m/s

5 a $6a, \frac{1}{7}a, \frac{a}{10}$ and $b, -b, \frac{3}{8}b$

b $0.5x, 1.7x, 10x, \frac{5}{6}x$ and $95y, \frac{1}{2}y$

c $5de, -ed, 0.01de$ and $\frac{2ef}{15}, 12fe, ef$

d $p^2, 5.1p^2, -15p^2, \frac{3}{2}p^2$ and $2p^3, \frac{3p^3}{2}$

e $\frac{9}{10}ab, -6.5ba, -ab$ and $6.5abc, 0.9cab, \frac{11}{20}bca$

f $\frac{4}{9}f^2g, \frac{gf^2}{3}$ and $-23g^2f, -3.14fg^2$

6 a $12m+8n$ **b** $8cd+2c+4$

c $-7k^2+5$ **d** $2ab^2+2a^2b$

e $-5xyz+\frac{11}{2}xy-\frac{2}{5}y$ **f** $-\frac{3}{4}pq^2-8q^2$

7 a x^{14} **b** m^2 **c** h^{11}

d $p^{12}q^6$ **e** $x^{25}y^{15}$ **f** $a^6b^{30}c^{12}$

8 a $-5fg$ **b** $2p^2s$ **c** $56g^{10}$

d $24ab^2c$ **e** $-6m^{12}n^{12}$ **f** $\frac{12}{5}e^2f^{10}g^{18}$

9 a k^4 **b** $8c^8$ **c** x^3y

d m^3 **e** $p^{11}q^2r^{18}$ **f** $ab^{12}c^{10}$

10 a $\frac{3a}{4}$ **b** $3c$ **c** $\frac{3ad}{5}$

d $\frac{2ehk}{5}$ **e** $\frac{4x^7y^5}{7}$ **f** 1

11 a $5a+15$ **b** $28b-4c$

c $3ac+5cd$ **d** $-24m^3-6mn$

12 a $-12k+31$ **b** $2y^2-13y-4$

13 a 2 **b** $3p$ **c** $5b$

d $8st$ **e** a^2b **f** $6mn^9p$

14 a $2(7h-9)$ **b** $10a(2b+c)$ **c** $4c(2c+3)$

d $-5fg(e+h)$ **e** $5ab(3a-8)$ **f** $6x^2y^3(3y^2-4wx^8)$

Analysis

1 a $yx+y(x+2)+y(x+4)+y(x+6)+y(x+8)+y(x+10)+y(x+12)$

b \$30.10 **c** $7xy+42y$

d \$30.24 **e** $7y(x+6)$

f \$30.03

2 **a** $2a^2 + 2b^2$ **b** a^2b^2
c $a^2b^2c^3$ **d** $8a^2 + 4b^2$
e $8a^2b^2$ **f** $8a^2b^2c^3$

EX p240

SEMESTER 1 review

Short answer

1 **a** 569 000 **b** 600 000 **c** 208 492.175
d 208 492.18 **e** 0.000 05 **f** 28 180

2 **a** $\frac{7}{8} = \frac{21}{24}$ **b** $\frac{-8}{10} = \frac{-12}{15}$
c $2\frac{4}{5} = \frac{70}{25}$ **d** $\frac{-1.2}{-0.07} = \frac{1200}{70}$

3 **a** $\frac{3}{4}$ **b** $\frac{50}{3} = 16\frac{2}{3}$ **c** $7\frac{3}{4}$ **d** 7 : 6
e 3 : 2 : 6 **f** 20 : 1 **g** 1 : 6 **h** 60 km/h

4 **a** 1309 **b** −38 **c** $\frac{25}{18} = 1\frac{7}{18}$
d 21.425 **e** 24 **f** $10\frac{7}{15} = \frac{157}{15}$

5 **a** 213 **b** 7 **c** $-\frac{27}{56}$
d 38.035 **e** 72 **f** $\frac{91}{24} = 3\frac{19}{24}$

6 **a** 4410 **b** −60 **c** $\frac{5}{3} = 1\frac{2}{3}$
d 133.56 **e** 32 **f** $\frac{40}{3} = 13\frac{1}{3}$

7 **a** 643 **b** 85 **c** $-\frac{8}{15}$
d 0.07 **e** 27 **f** $\frac{49}{34} = 1\frac{15}{34}$

8 **a** 216 **b** 243
c $\frac{125}{64} = 1\frac{61}{64}$ **d** 0.0016
e $\frac{2401}{16} = 150\frac{1}{16}$ **f** 1

9 **a** 6 **b** 2
c $\frac{9}{5} = 1\frac{4}{5}$ **d** 0.7

10 **a** 5 **b** 58
c $-\frac{13}{12} = -1\frac{1}{12}$ **d** 10.432

11 **a** $3y - 5xy + 5x$ **b** $8b - 8a$
c 2^7 **d** $35x^3y^6$
e 3^8 **f** $8a^5b^4$
g 3^{30} **h** $2^6 \times 3^5$
i $2x + 4$

12 **a** **i** 14 000 **ii** 15 505.499 72
iii 9.71%
b **i** 4500 **ii** 4245.875 477...
iii 5.99%

13 **a** 8.4201, 24.01, 24.1, 824.01, 842.01, 842.1
b −5.31, −5.1, −5.031, 3.15, 5.103, 5.13
c $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}, \frac{11}{12}$ d $\frac{6}{-2}, -\frac{3}{2}, -\frac{4}{3}, \frac{-3}{4}, \frac{-3}{-4}, \frac{4}{3}$

14 **a** **i** non-terminating **ii** recurring
b **i** non-terminating **ii** non-recurring
c **i** terminating
d **i** non-terminating **ii** recurring

15

Fraction	Decimal / recurring decimal	Percentage
$\frac{29}{20} = 1\frac{9}{20}$	1.45	145%
$\frac{7}{8}$	0.875	87.5%
$\frac{5}{2} = 2\frac{1}{2}$	2.5	250%
$\frac{21}{5}$	4.2	420%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\% = 33\frac{1}{3}\%$
$\frac{27}{25} = 1\frac{2}{25}$	1.08	108%
$\frac{4}{33}$	$0.\overline{12}$	$12.\overline{12}\% = 12\frac{4}{33}\%$

16 **a** $\frac{4}{5}$ **b** 40% **c** 32%
d 21 : 4 **e** 19 eggs/week **f** 85 km/h

17 **a** 45 **b** 44 **c** 108.9
d 64.96 **e** 96 **f** \$1200 and \$1500
g 21 blue **h** 0.8 kg **i** 4 km
j 8 min 20 seconds

18 **a** $p^2 - q^2$ **b** $3x + \frac{y}{12}$ **c** $\frac{x+6}{x-6}$ **d** $\sqrt{7k}$

19 **a** $32 - 24x$ **b** $-22x + 38$
c $12ab + 10b^2 + 6b$ **d** $30x^3y^2 - 45x^2y^5$

20 **a** $9(4x + 3)$ **b** $6(2a + 6b - 3c)$
c $\frac{5}{3}p^3(2 + 5p^4)$
d $\frac{uv}{4}\left(-\frac{3u}{2} + 5v\right) = -\frac{uv}{4}\left(\frac{3u}{2} - 5v\right)$

21 **a** 55c or \$0.55 **b** \$25
c \$1.98 **d** \$64
e \$49.50

22 **a** LCM = 90, HCF = 6
b LCM = 2100, HCF = 70
c LCM = $2^4 \times 3^9 \times 5^4$, HCF = $2^3 \times 3^6 \times 5^4$
d LCM = $2^7 \times 3^8 \times 5^8 \times 7^4 \times 11^2$, HCF = $2^3 \times 7$

23 **a** 854 **b** $2\frac{1}{2} = \frac{5}{2}$
c $8\frac{4}{5} = \frac{44}{5}$ **d** 0.65

24 **a** false **b** true **c** false
d true **e** true **f** false

25 **a** $2^3 \times 3 \times 5$ **b** $5^3 \times 7$

Analysis

1 **a** lower quality flowers \$0.90
medium quality flowers \$2.25
higher quality flowers \$3.51
b \$25.29
c **i** \$31.19 **ii** \$13.36
d **i** \$261.25 **ii** \$173
e \$10 cheaper

2 **a** **i** 372 + 654 = 1026 **ii** 1025 − 856 = 169
iii 632 × 72 = 45 504 **iv** 2760 ÷ 8 = 345

v $4^4 = 256$ **vi** $\sqrt{16} = 4$

vii $\sqrt[3]{27} = 3$ **viii** $2^3 \times 2^2 = 2^5$

ix $3^3 \div 3^3 = 1$

b **i** 9.75% **ii** 36.69% **iii** 87.5% **iv** 89.28%

v 93.75% **vi** 100% **vii** 200% **viii** 100%

ix 100%

c **i** $272 + 654 = 926$ **ii** $1087 - 856 = 231$

iii $79 \times 72 = 5688$ **iv** $296 \div 8 = 37$

v $2^4 = 16$ **vi** $\sqrt{64} = 8$

vii $\sqrt[3]{729} = 9$ **viii** $2^4 \times 2^2 = 2^6$

ix $0^3 \div 3^3 = 0$

3 **a** **i** $2^8 \times 3^7 \times 5^4$ **ii** $3^8 \times 2^7 \times 7^4$

iii $3^8 \times 5^7 \times 7^4$ **iv** $a^8 b^7 c^4$

b **i** $\dfrac{2^2}{3^3 \times 5^2}$ **ii** $\dfrac{3^2}{2^3 \times 7^2}$

iii $\dfrac{3^2}{5^3 \times 7^2}$ **iv** $\dfrac{a^2}{b^3 c^2}$

c **i** $2^3 \times 3^2 \times 5$ **ii** $3^3 \times 2^2 \times 7$

iii $3^3 \times 5^2 \times 7$ **iv** $a^3 b^2 c$

d **i** $2^5 \times 3^5 \times 5^3$ **ii** $3^5 \times 2^5 \times 7^3$

iii $3^5 \times 5^5 \times 7^3$ **iv** $a^5 b^5 c^3$

4 **a** $5(w + 3)$ **b** $5w + 15$

c 25 m^2

d $w = 1$: $A = 20$ m^2
$w = 2$: $A = 25$ m^2
$w = 3$: $A = 30$ m^2
$w = 4$: $A = 35$ m^2

e $w + 13$ **f** $2x(w + 13)$

g $2xw + 26x$ **h** 16 m^2

EX EXPLORATIONS 1

p244

1 **a** The sum of digits is always a multiple of 3.

b Swap the first two digits.

c The value changes by a multiple of 999 999, which is itself a multiple of 7.

d With 2 notes, put an odd digit first and an even digit second.

e For 5, 6, 8, 9, 10 and 12 notes, swap the first two digits, and for 11 notes, swap the 1st and 3rd digits (among other possibilities).

f Brooke can always win by swapping the 1st and 7th digits (among other possibilities).

2 **a** $7\overline{)1.{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0}$ (quotient: 0.1428 57...)

b $\frac{2}{7} = 0.\overline{285714}$, $\frac{3}{7} = 0.\overline{428571}$, $\frac{4}{7} = 0.\overline{571428}$, $\frac{5}{7} = 0.\overline{714285}$, $\frac{6}{7} = 0.\overline{857142}$

c $\frac{1}{14} = \frac{5}{70} = 0.07\overline{14285}$

d $\frac{1}{70}$, so $\frac{3}{14} = 0.2\overline{142857}$

e $\frac{5}{14} = 0.3\overline{571428}$, $\frac{9}{14} = 0.6\overline{428571}$, $\frac{11}{14} = 0.7\overline{857142}$ and $\frac{13}{14} = 0.9\overline{285714}$

f $\frac{2}{63} = \frac{1}{7} - \frac{1}{9} = 0.\overline{031746}$

3 **a** 10 hours **b** 71 hours

c 7 professionals and 19 hours

d $\dfrac{(x + y/2)(t - T)}{X + Y/2} = \dfrac{(2x + y)(t - T)}{2X + T}$

4 **a** 480 mL

b 600 mL, 500 mL, 400 mL, 300 mL, 200 mL, 100 mL and 0 mL

c 525 mL, 450 mL, 375 mL, 300 mL, 225 mL, 150 mL and 75 mL

5 **a** **i** 2^{12} **ii** 4^6 **iii** 64^2

iv 16^3

b $1 = 2^0 = 4^0 = 1^2 = 1^3$ and $64 = 2^6 = 4^3 = 8^2 = 4^3$

c $18^2 \times 216^3$, $3888^2 \times 6^3$, $144^2 \times 54^3$ or 486×24^3

d Only 20^1 and 24^1 cannot be written this way (1 is both a square and a cube). For any whole number n, the only power which may not be able to be written this way is n^1, since all higher powers always can.

e There are many solutions. Two are given for each part: one which can be constructed from powers of 2 and a second which is essentially a coincidence.

i $128^3 = 1024^2 + 16^5$ (and $7^3 = 10^2 + 3^5$)

ii $32^5 = 4096^2 + 256^3$ (and $8^5 = 104^2 + 28^3$)

f **i** Yes. For example, 7-24-25 is a Pythagorean triad, so $5^4 = 25^2 = 7^2 + 24^2$.

ii No (aside from $0^6 = 0^3 + 0^3$). The sixth power is also a perfect cube, but Fermat's last theorem says that no cube is the sum of two smaller cubes.

6 **a** 1 house point. No, adding 14 or 18 will not improve his score.

b If the first number is n, the total is $4(2n + 7)$ which is divisible by at least 2 and 4. The smallest total with only 2 house points is 44, when $n = 2$.

c If the first number ends in 1, the total is of the form $60n+21$, which is only divisible by last digits 1 and 3; similarly for the other cases. The smallest number of house points is 0 (for example, from 4, 5, 6, 7, 8, 9).

d The total is always odd, so cannot be divided by 2, 4, 6 or 8. The smallest total is 315.

e 5 consecutive numbers, with smallest total 120.

CHAPTER 6 Equations and linear relationships

EX 6A Equations

p250

1 **a** **i** expression **ii** equation **iii** equation

iv expression **v** equation **vi** equation

b Equations contain an equals sign (=).

2 **a** yes **b** no **c** no **d** yes **e** no **f** yes

3 **a** no **b** no **c** yes **d** yes **e** yes **f** no

4 **a** yes **b** yes **c** no **d** no **e** yes **f** no

g no **h** yes

5 **a** $a = 7$ **b** $b = 6$ **c** $c = 8$ **d** $d = 12$

e $e = 2$ **f** $f = -3$ **g** $g = 1$ **h** $h = 0$
i $i = -3$ **j** $j = -5$ **k** $k = 7$ **l** $l = 35$
m $m = 4$ **n** $n = -10$ **o** $x = -14$ **p** $p = 0$
q $q = 6$ **r** $r = -12$ **s** $s = -4$ **t** $t = 25$

6 D

7 E

8 A

9 C

10 a $a = 2$ **b** $b = 1$ **c** $c = 3$ **d** $d = 0$
e $e = 6$ **f** $f = 10$ **g** $g = 4$ **h** $h = -7$

11 a $x = 5$ **b** $x = 4$ **c** $x = -1$ **d** $x = 9$
e $x = -10$ **f** $x = 0$ **g** $x = -3$ **h** $x = -4$

12 a i $n + 4 = 11$ **ii** $n - 3 = -2$
iii $n + 15 = 21$ **iv** $n - 8 = -4$
v $2n = 14$ **vi** $\frac{n}{4} = 9$
vii $n + (-7) = -20$ or $n - 7 = -20$
viii $11n = 22$ **ix** $5n = 55$
x $\frac{n}{6} = -3$
b i $n = 7$ **ii** $n = 1$
iii $n = 6$ **iv** $n = 4$
v $n = 7$ **vi** $n = 36$
vii $n = -13$ **viii** $n = 2$
ix $n = 11$ **x** $n = -18$

13 a i $3x + 8 = 11$ **ii** $\frac{x}{6} - 2 = 1$
iii $\frac{x+7}{2} = 5$ **iv** $3(x - 9) = 3$
b i $x = 1$ **ii** $x = 18$
iii $x = 3$ **iv** $x = 10$

14 a $x + 38$ **b** $x + 38 = 62$
c $x = 24$ **d** \$24

15 a $26 + x$ **b** $26 + x = 81$
c $x = 55$ **d** 55 rolls

16 a $8x = 96$ **b** $x = 12$ **c** 12 people

17 a m = number of macadamia nuts
b $\frac{m}{4}$ **c** $\frac{m}{4} = 9$
d $m = 36$ **e** 36 macadamia nuts

18 a $8t = 24$, where t = cost of each truffle
b \$3

19 a $\frac{f}{2} + 3 = 7$, where f = number of pieces of fruit initially in the bowl
b 8 pieces of fruit

20 a $2c + 7 = 11$, where c = number of cars in the car park at 9 am
b 2 cars

21 a $25c$ **b** $40a$ **c** $25c + 40a = 155$
d Substituting $a = 2$ and $c = 3$ into the equation shows Poh's calculations are correct.

22 a $2(x + 30) = 270$ **b** 105 minutes

23 a The time Arvan spends practising every day is not the same every day.
b $x + x + 2x + 2x - 20 + x = 190$ (or $7x - 20 = 190$)
c

Day	Time spent practising	Comments
Monday	30	Good start!
Tuesday	30	Same as Monday
Wednesday	60	Double Monday!
Thursday	40	20 minutes less than yesterday
Friday	30	Same as Monday
Total	190	

d The average (mean) time Arvan spends practising

24 a $h + \frac{h}{2} + 10 = 100$
b $h = 60$ **c** $60 + \frac{60}{2} + 10 = 100$

EX p256 6B Solving linear equations

1 a -6 **b** $\div 4$ **c** $+7$ **d** $\times 3$
e $\div -2$ **f** $\times -8$ **g** $+11$ **h** $\div -20$

2 a $x = 13$ **b** $x = 42$ **c** $x = 12$ **d** $x = 27$
e $x = 1$ **f** $x = -1$ **g** $x = -25$ **h** $x = -3$

3 a $+6$ **b** -9 **c** $\div 3$ **d** $\times 4$
e $+2$ **f** $\times 5$ **g** $\div 7$ **h** -1

4 a $-3, \div 3$ **b** $+8, \div 2$ **c** $-5, \times 2$ **d** $\div 5, -5$
e $+11, \div 5$ **f** $\times 5, \div 2$ or $\div 2, \times 5$
g $\div 9, -4$ **h** $\times 3, -4$

5 a $x = 12$ **b** $x = 14$ **c** $x = 7$ **d** $x = 10$
e $x = 0$ **f** $x = -22$ **g** $x = -5$ **h** $x = -3$
i $x = -2$ **j** $x = 0$ **k** $x = -3$ **l** $x = -17$

6 a $x = 2.5$ **b** $x = -5$ **c** $x = -5$ **d** $x = -2$

7 a $x = 8$ **b** $x = 1$ **c** $x = 24$ **d** $x = 6$
e $x = 18$ **f** $x = 33$ **g** $x = 20$ **h** $x = 2$
i $x = 7$ **j** $x = 6$ **k** $x = 1$ **l** $x = 13$

8 a $x = 1$ **b** $x = -2$ **c** $x = 4$ **d** $x = 2$
e $x = 15$ **f** $x = 2$ **g** $x = -9$ **h** $x = 10$
i $x = -22$ **j** $x = -15$ **k** $x = -1$ **l** $x = 6$

9 a $x = \frac{1}{5}$ **b** $x = 4\frac{1}{2}$ $\left(\text{or } x = \frac{9}{2}\right)$
c $x = 5\frac{2}{3}$ $\left(\text{or } x = \frac{17}{3}\right)$ **d** $x = \frac{4}{5}$
e $x = 4\frac{2}{5}$ $\left(\text{or } x = \frac{22}{5}\right)$
f $x = -11\frac{1}{2}$ $\left(\text{or } x = -\frac{23}{2}\right)$

10 No, as Caleb has not performed the inverse operation on both sides of the equation. He needs to add 37 to obtain the solution.

11 a Both students have solved the equation correctly.
b Yuhan's method: Eliminating the fraction first means you don't need to work with fractions after that step; however, you need to consistently work on both sides of the equation.
Connor's method: By completing one calculation at each step, the solution is easy to follow; however, you need to be confident working with fractions.

12 a $x = 14$ **b** $x = -6$ **c** $x = -10$
d $x = \frac{3}{4}$ **e** $x = 1.8$ **f** $x = 1\frac{7}{8}$ $\left(\text{or } x = \frac{15}{8}\right)$

13 **a** **i** $2(x + 4) = 26$ **ii** $x = 9$

b **i** $280 + 15x = 1600$ **ii** $x = 88$

c **i** $257 + 56x = 649$ **ii** $x = 7$

d **i** $\frac{x-6}{2} = 11$ **ii** $x = 28$

14 $216 + \frac{t}{3} + t = 256,\ t = 30$

15 $5\left(x \times \frac{3}{4}\right) = 75,\ x = 20$

16 **a** $10x - 5\left(\frac{3}{2}x\right)$

b You do not know how many points Jenny had at the end of the week.

c The expression simplifies to $\frac{5}{2}x$, which will always be a positive value as the number of jobs completed will be positive (or zero).

d 20

17 **a** $x = 3$ **b** $x = 13$ **c** $x = 9$ **d** $x = -5$

e $x = -4$ **f** $x = 8$ **g** $x = 10$ **h** $x = 14$

18 **a** **i** $-5, \div 3, \sqrt{}$ **ii** 3

b **i** $\div 4, +2, \sqrt{}$ **ii** 4

c **i** square, $-5, \div 2$ **ii** 10

19 12 years old

20 **a** $\frac{12+12}{4} = \frac{24}{4}$
$= 6$

b Giovanni performed the inverse operations in the incorrect order. He subtracted 12 from each side first, before multiplying by 4, whereas he should've multiplied each side by 4 first.

c 48

EX p260 6C Solving equations with the unknown on both sides

1 **a** $x = 13$ **b** $x = 4$ **c** $x = 5$

d $x = 2$ **e** $x = -6$ **f** $x = 14$

g $x = 10$ **h** $x = 15$ **i** $x = 16$

j $x = -13$ **k** $x = 8$ **l** $x = 0$

2 **a** $a = 2$ **b** $b = 4$ **c** $c = -1$ **d** $d = 2$

e $e = -4$ **f** $f = 5\frac{1}{2}\ \left(\text{or } f = \frac{11}{2}\right)$

g $g = 3\frac{1}{3}\ \left(\text{or } g = \frac{10}{3}\right)$ **h** $h = -3$

i $i = 2\frac{1}{2}\ \left(\text{or } i = \frac{5}{2}\right)$ **j** $j = 5$

k $k = -\frac{6}{7}$

l $l = -4\frac{1}{2}\ \left(\text{or } l = -\frac{9}{2}\right)$

3 **a** $x = 4$ **b** $x = -2$

c $x = -1$ **d** $x = 3$

e $x = 1\frac{7}{8}\ \left(\text{or } x = \frac{15}{8}\right)$ **f** $x = 1$

g $x = 6$ **h** $x = 5$

i $x = -6$ **j** $x = 4\frac{2}{3}\ \left(\text{or } x = \frac{14}{3}\right)$

k $x = 21$ **l** $x = 4\frac{1}{2}\ \left(\text{or } x = \frac{9}{2}\right)$

4 **a** $x = 6$ **b** $x = 4$ **c** $x = 4$

d $x = 19$ **e** $x = -\frac{1}{5}$ **f** $x = -16$

g $x = -2$ **h** $x = \frac{2}{3}$

5 **a** $6x + 4 = 4x + 2$ **b** $x = -1$

6 **a** Andy: added $2x$ to left side instead of subtracting; Kwame: did not subtract $2x$ from both sides of equation; Hayley: simplified right side incorrectly (-13, not $+13$)

b
$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= -13 \\ 4x + 11 - 11 &= -13 - 11 \\ 4x &= -24 \\ \frac{4x}{4} &= \frac{-24}{4} \\ x &= -6 \end{aligned}$$

7 **a** $p = 3$ **b** $q = 2$ **c** $r = -3$

d $s = 4$ **e** $t = -2$ **f** $u = 13$

8 **a** B **b** \$6

9 **a** D **b** 12

10 **a** $3a + 400 = 7a + 160$ **b** 60 cents

11 **a** $2a + 2$ **b** $3a - 4 = 2a + 2$

c $a = 6$; Sean's cousin is 6 years old.

d Sean is 14 years old.

12 **a** $n + 10$ **b** $3n + 4$ **c** $3n + 4 = n + 10$

d $n = 3$ **e** 3 **f** 9 **g** 13

13 **a** $6g + 27$ **b** $12g + 3$ **c** $12g + 3 = 6g + 27$

d $g = 4$; home team: 8 goals 3 behinds; away team: 4 goals 27 behinds; each team scored 51 points

14 **a** $4t + 8$ **b** $12t + 4$

c $12t + 4 = 2(4t + 8)$ or $12t + 4 = 8t + 16$

d $t = 3$

e home team: 3 tries; away team: 9 tries

15 **a** $5x - 10 = 3x + 24$ **b** $x = 17$

16 **a** $x = 29$ **b** $x = -3$ **c** $x = 1$

d $x = 6$ **e** $x = 25$ **f** $x = -7$

g $x = 3$ **h** $x = 0$ **i** $x = -2$

17 **a** $x = 2$ **b** $x = 0$

EX p264 6D Changing the subject of a formula

1 **a** $x = 4$ **b** $x = \frac{1}{4}$ **c** $x = 5$ **d** $x = -5$

e $x = 10$ **f** $x = \frac{1}{10}$ **g** $x = -3$ **h** $x = 1$

2 **a** y **b** c **c** no subject

d v **e** no subject **f** C

g no subject **h** no subject

3 **a** $x = \frac{y}{m}$ **b** $x = y - a$ **c** $x = ky$

d $x = \frac{k}{y}$ **e** $x = \frac{c-b}{a}$

f $x = a(b + c)$ or $x = ab + ac$ **g** $x = ac - b$

h $x = a(c - b)$ or $x = ac - ab$

4 **a** $y = mx$, $m = \frac{y}{x}$

b $y = a + x$, $a = y - x$

c $y = \frac{x}{k}$, $k = \frac{x}{y}$

d $y = \frac{k}{x}$, $k = xy$

e $c = ax + b$, $b = c - ax$, $a = \frac{c - b}{x}$

f $c = \frac{x}{a} - b$, $b = \frac{x}{a} - c$, $a = \frac{x}{b + c}$

g $c = \frac{x + b}{a}$, $b = ac - x$, $a = \frac{x + b}{c}$

h $a = \frac{x}{(c - b)}$, $b = \frac{x}{a} - c$, $c = \frac{x}{a} + b$,

5 **a** $x(2t + 1) = t(g - x)$

$2tx + x = tg - tx$

$x(3t + 1) = tg$

$x = \frac{tg}{3t + 1}$

b $x = \frac{25}{8} = 3.125$

c $x(2(5) + 1) = 5(10 - x)$

$11x = 50 - 5x$

$16x = 50$

$x = \frac{50}{16} = \frac{25}{8} = 3.125$

6 **a** $x = \frac{y}{k}$

b $k = \frac{y}{x}$

c any four pairs where $\frac{y}{x} = 4$

d any four pairs where $kx = 36$

7 **a** **i** $x = \frac{y}{mk}$ **ii** $k = \frac{y}{mx}$ **iii** $m = \frac{y}{kx}$

b **i** $x = \frac{ym}{k}$ **ii** $k = \frac{ym}{x}$ **iii** $m = \frac{kx}{y}$

c **i** $x = \frac{y - m}{k}$ **ii** $k = \frac{y - m}{x}$

iii $m = y - kx$

8 **a** $K = C + 273.15$

b **i** 273.15 K **ii** 373.15 K

iii 300 K **iv** 453.15 K

c $C = K - 273.15$

d **i** 226.85°C **ii** 726.85°C

iii −253.15°C **iv** −93.15°C

9 **a** **i** −17.78°C **ii** 37.78°C

iii 10°C **iv** 30°C

b $F = \frac{9}{5}C + 32$ or $F = 1.8C + 32$

c **i** 86°F **ii** 14°F

iii 68°F **iv** 356°F

10 **a** **i** 260.93 K **ii** 338.71 K

iii 244.26 K **iv** 477.59

b $F = \frac{9}{5}(K - 273.15) + 32$

c **i** −99.67°F **ii** 2240.33°F

iii 32°F **iv** 212°F

11 **a** $y = -\frac{A}{B}x + \frac{D}{B}$

b $m = -\frac{A}{B}$ and $c = \frac{D}{B}$

c $A = -mB$ and $D = cB$

d $-px + qy = qc$ or $px - qy = -qc$

e $y = mx - mh + k$

12 **a** $h = 3.45$

b $h = \frac{2E - mv^2}{2mg}$ or $h = \frac{E}{mg} - \frac{v^2}{2g}$

c $m = 60.38$ **d** $m = \frac{2E}{2gh + v^2}$

13 **a** $mt + a = nt + b$ **b** 5 minutes

c $t = \frac{b - a}{m - n}$ or $t = \frac{a - b}{n - m}$ **d** 5 minutes

EX p269 6E Solving simple quadratic equations

1 **a** $x = \pm 10$ **b** $y = \pm 3$ **c** $z = \pm 7$

d $a = \pm 9$ **e** $k = \pm 1$ **f** $h = 0$

2 **a** $b = \pm\frac{5}{8}$ **b** $u = \pm\frac{12}{7}$ **c** $d = \pm\frac{5}{4} = \pm 1\frac{1}{4}$

d $w = \pm 0.6$ **e** $i = \pm 0.02$ **f** $r = \pm 1.001$

3 **a** $x = \pm 11$ **b** $y = \pm 0.7$ **c** $z = \pm\frac{9}{4}$

d $a = \pm 9$ **e** $b = \pm 9$ **f** $c = \pm\frac{3}{4}$

g $m = \pm 8$ **h** $n = \pm 12$ **i** $p = \pm 5$

4 **a** $x = \pm\sqrt{10}$ **b** $y = \pm\sqrt{91}$ **c** $z = \pm\sqrt{0.4}$

d $i = \pm\sqrt{1.6}$ **e** $j = \pm\sqrt{\frac{13}{6}}$ **f** $k = \pm\sqrt{7}$

5 **a** $x \approx \pm 3.16$ **b** $y \approx \pm 9.54$ **c** $z \approx \pm 0.63$

d $i \approx \pm 1.26$ **e** $j \approx \pm 1.47$ **f** $k \approx \pm 2.65$

6 $l = 5$ cm and $h = 10$ cm

7 **a** $A = 6l^2$ **b** $l = 6$ cm

8 **a** $v = \pm 38.28$ **b** $c = \pm 12.21$ **c** $u = \pm 41.23$

d $v = \pm 1.58$ **e** $b = \pm 9.85$

9 **a** 1, a length cannot be negative

b 1, radius is a length and cannot be negative

10 **a** $l = 3$ cm **b** $l = 9$ cm **c** $l = 14$ cm

d $l = \sqrt{20}$ cm **e** $l = \sqrt{116}$ cm **f** $l = \sqrt{200}$ cm

11 **a** $x = 8$ cm **b** $x = 14$ cm **c** $x = \sqrt{212}$ cm

d $x = 6$ cm **e** $x = 11$ cm **f** $x = \sqrt{208}$ cm

12 **a** The equation is not in the form $x^2 = c$ so it does not matter if 'the 25 is positive' yet.

b $x^2 + 25 = 0$ is equivalent to $x^2 = -25$, which has zero real solutions as no real number (positive, negative or zero) squared is negative.

13 **a** $a = \pm 5$ **b** $b = \pm\sqrt{\frac{305}{7}}$ **c** $c = \pm 4$

d $d = \pm\sqrt{75}$ **e** $e = \pm\sqrt{5}$ **f** $f = \pm 13$

14 **a** $T = 10.04$ seconds

b The period is a length of elapsed time so it cannot be negative.

15 **a** $x = -11$ or $x = 1$ **b** $x = -16$ or $x = 0$

c $x = -6$ or $x = 14$ **d** $x = 0$ or $x = 14$

e $x = 0.53$ or $x = 9.47$

f $x = -18.49$ or $x = 0.49$

16 **a** $x = -5 \pm \sqrt{35}$ **b** $x = -8 \pm \sqrt{63}$

c $x = 4 \pm \sqrt{99}$ **d** $x = 7 \pm \sqrt{50}$

e $x = 5 \pm \sqrt{20}$ **f** $x = -9 \pm \sqrt{90}$

EX Checkpoint
p272

1 **a** yes **b** yes **c** no **d** yes

2 **a** $a = 4$ **b** $p = -21$ **c** $z = -17$
d $a = 1$ **e** $j = -15$ **f** $y = 7$

3 **a** $x + 6 = 18$ **b** $x - 10 = -4$ **c** $4x + 9 = 33$
d $\frac{x + 12}{3} = 6$ **e** $8(x + 3) = 64$

4 **a** $x = 14$ **b** $x = 35$ **c** $x = -9$

5 **a** $x = 4$ **b** $x = 12$ **c** $x = 13$

6 **a** $x = 4$ **b** $x = -16$ **c** $x = -41$
d $x = \frac{1}{7}$ **e** $x = \frac{6}{5}$ **f** $x = \frac{22}{3}$

7 **a** 8 **b** 12 **c** 3

8 **a** $x = 10$ **b** $x = -\frac{34}{7}$ **c** $x = \frac{41}{4}$ **d** $x = 0$

9 **a** 3 **b** $-\frac{1}{2}$

10 **a** $x = \frac{y - a}{k}$ **b** $x = \frac{a - k}{y}$ **c** $x = \frac{k}{2 - y}$

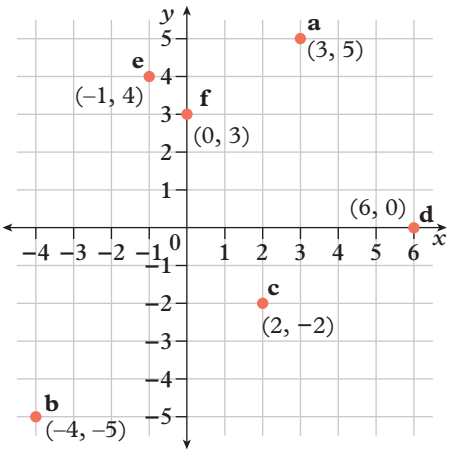
11 **a** $x = \pm 2$ **b** $x = \pm 4$ **c** $x = \pm\frac{4}{3}$

12 **a** $x = \pm 1$ **b** $x = 0$ **c** $x = \pm\frac{2}{3}$

EX 6F Plotting linear relationships
p276

1 **a** A **b** C **c** F **d** B **e** E **f** D

2



3 **a**

x	−2	−1	0	1	2
y	−1	0	1	2	3

b

x	−3	−2	−1	0	1
y	−4	−2	0	2	4

c

x	−1	0	1	2	3
y	4	3	2	1	0

d

x	−1	0	1	2	3
y	−7	−4	−1	2	5

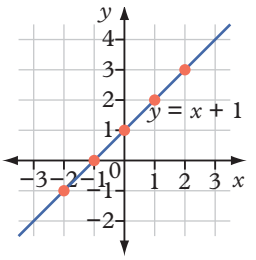
e

x	−2	−1	0	1	2
y	−9	−5	−1	3	7

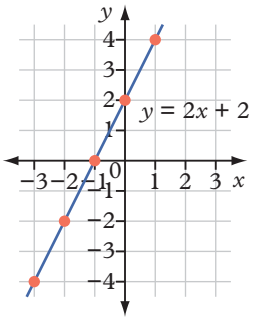
f

x	−3	−2	−1	0	1
y	4	3	2	1	0

4 **a**



b



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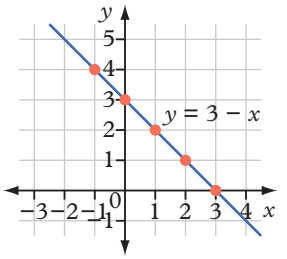

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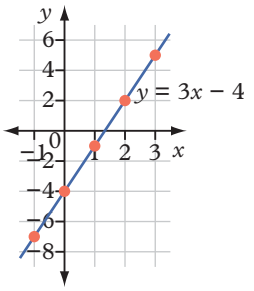

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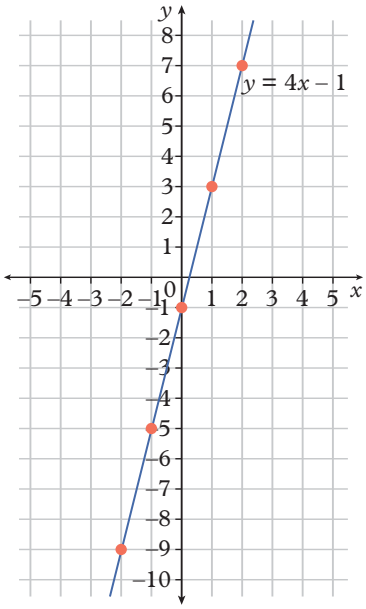

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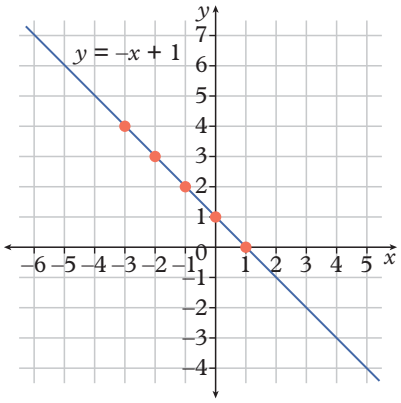
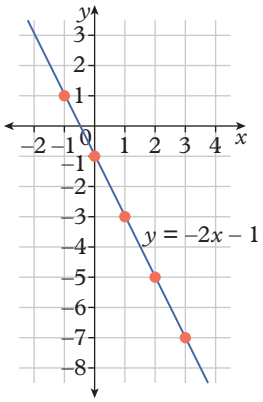
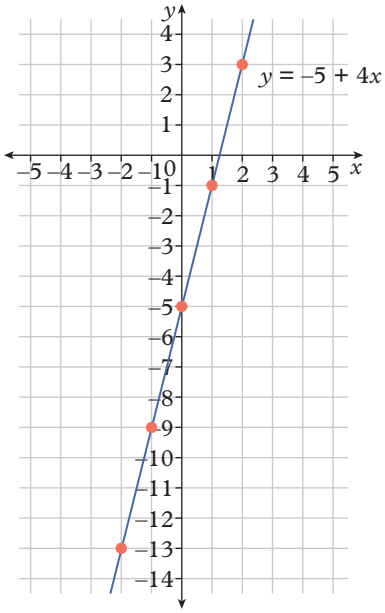


g

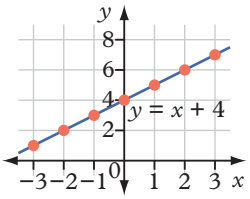


h



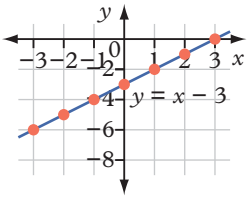
5 a

x	–3	–2	–1	0	1	2	3
y	1	2	3	4	5	6	7



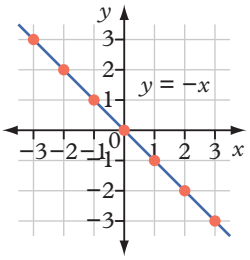
b

x	–3	–2	–1	0	1	2	3
y	–6	–5	–4	–3	–2	–1	0



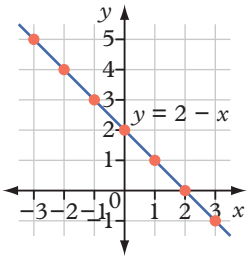
c

x	–3	–2	–1	0	1	2	3
y	3	2	1	0	–1	–2	–3



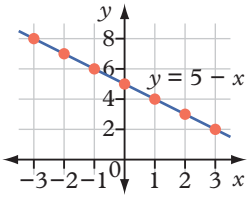
d

x	−3	−2	−1	0	1	2	3
y	5	4	3	2	1	0	−1



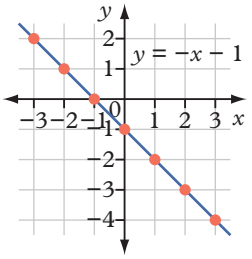
e

x	−3	−2	−1	0	1	2	3
y	8	7	6	5	4	3	2



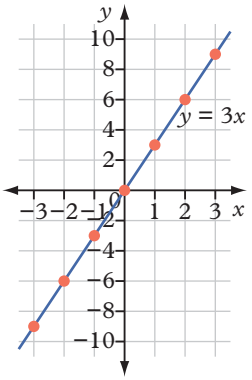
f

x	−3	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3	−4



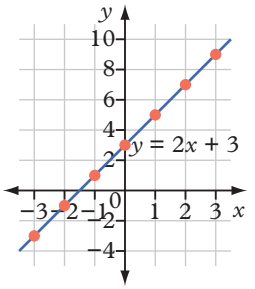
g

x	−3	−2	−1	0	1	2	3
y	−9	−6	−3	0	3	6	9



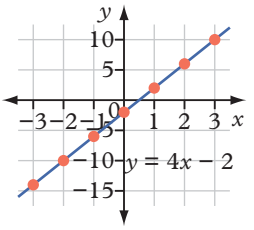
h

x	−3	−2	−1	0	1	2	3
y	−3	−1	1	3	5	7	9



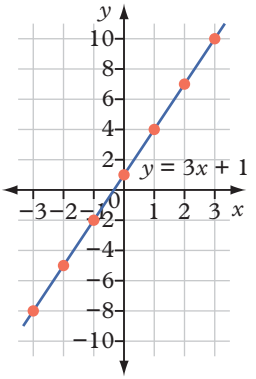
i

x	−3	−2	−1	0	1	2	3
y	−14	−10	−6	−2	2	6	10



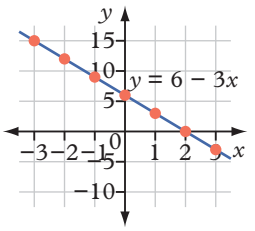
j

x	−3	−2	−1	0	1	2	3
y	−8	−5	−2	1	4	7	10



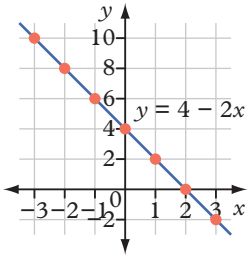
k

x	−3	−2	−1	0	1	2	3
y	15	12	9	6	3	0	−3

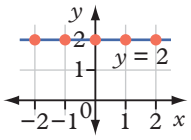


1

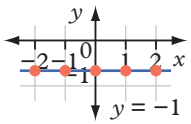
x	−3	−2	−1	0	1	2	3
y	10	8	6	4	2	0	−2



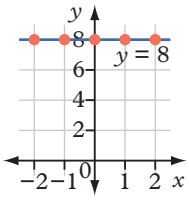
6 a i



ii



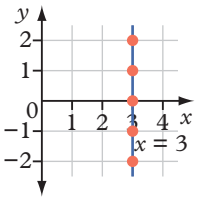
iii



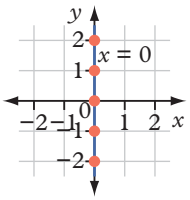
b yes, they are straight lines

c The horizontal line passes through the y-value described by the equation.

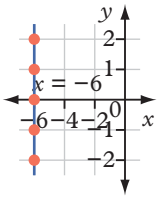
7 a i



ii



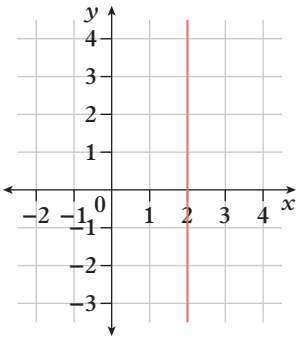
iii



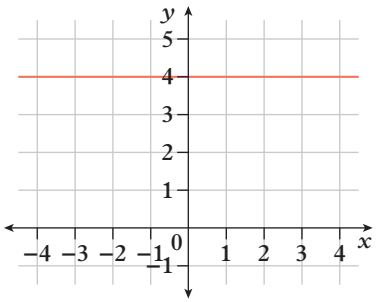
b yes, they are straight lines

c The vertical line passes through the x-value described by the equation.

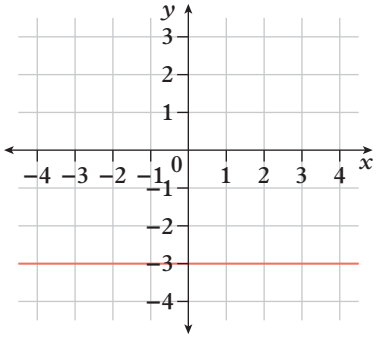
8 a



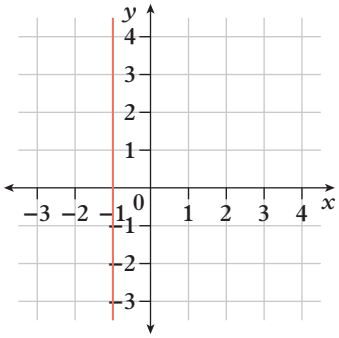
b



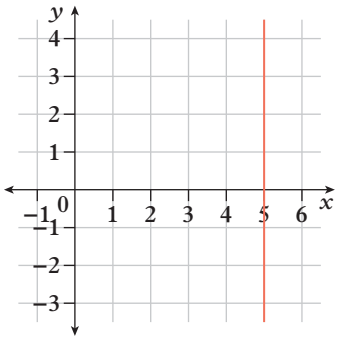
c



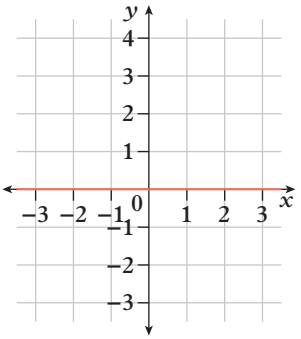
d



e



f



9 **a** time and height

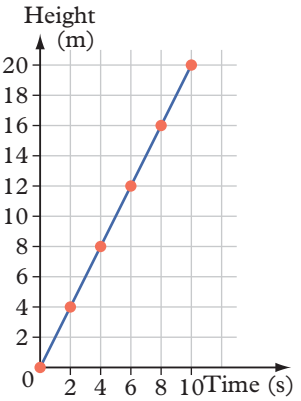
b **i** time; shown first in table of values

ii height

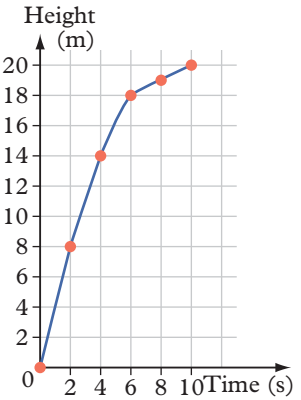
c Red balloon: (0, 0), (2, 4), (4, 8), (6, 12), (8, 16), (10, 20)

Blue balloon: (0, 0), (2, 8), (4, 14), (6, 18), (8, 19), (10, 20)

d Red balloon:



Blue balloon:

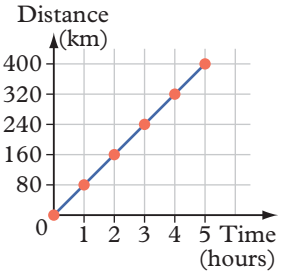


e **i** red balloon: 10 m; blue balloon: 16 m

ii 3.5 seconds for red balloon, 1.75 seconds for the blue balloon

10 **a** independent = time, dependent = distance

b



c 200 km

d $3\frac{1}{4}$ hours

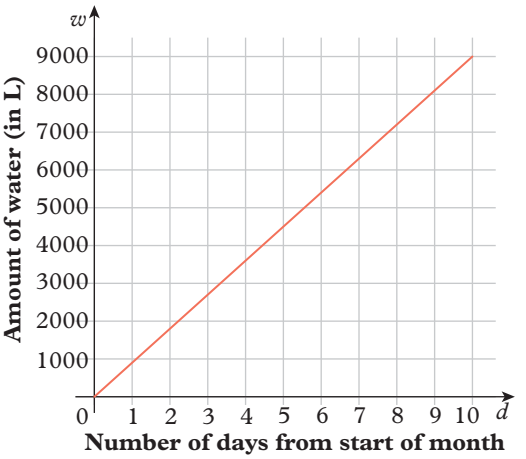
e 560 km; assume Jamie maintains the same speed

f $d = 80t$; multiply the number of hours by 80 to determine the distance Jamie travels.

g $d = 80 \times 7 = 560$

11 **a** independent = number of days, dependent = amount of water

b



c yes, as the points form a straight line

d 6300 L

e **i** the equation, as the result can be calculated exactly and because the graph doesn't cover the range needed

ii approx. 22 days $\left(22\frac{2}{9}\text{ days}\right)$

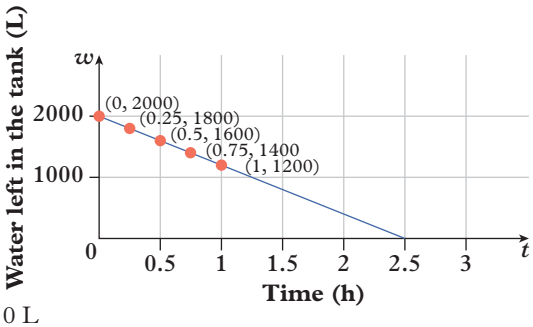
12 a 2000 L

b $w = 2000 - 800t$

c **i** water left (w)

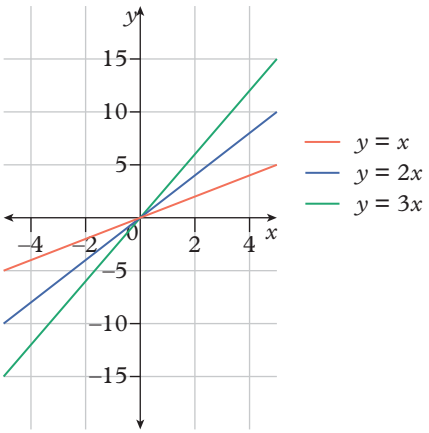
ii time passed (t)

d



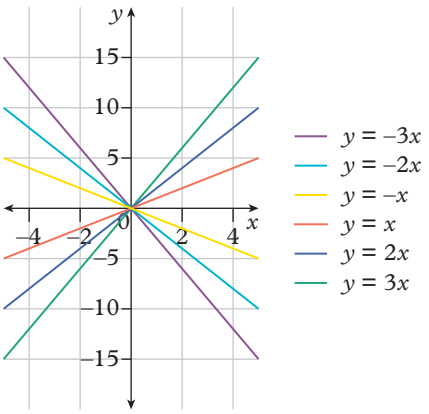
e 0 L

13 a



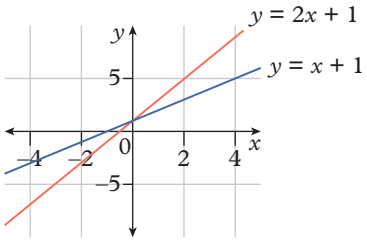
b The graphs all intercept the x- and y-axes at the same point, the origin (0, 0). This point is also the point of intersection of these graphs. The y-value of these line graphs all increase as you move from left to right. The y-values of $y = 3x$ increase more quickly than the y-values of $y = 2x$, which increase more quickly than the y-values of $y = x$.

c



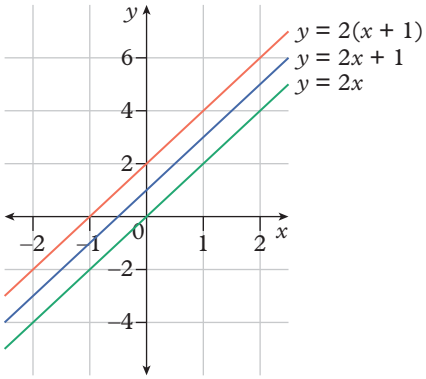
d The graphs all intercept the x- and y-axes at the same point, the origin (0, 0). This point is also the point of intersection of these graphs. Unlike the graphs drawn in part **a**, the y-values of the graphs drawn in part **c** all decrease as you move from left to right. The y-values of $y = -3x$ decrease more quickly than the y-values of $y = -2x$, which decrease more quickly than the y-value of $y = -x$.

e



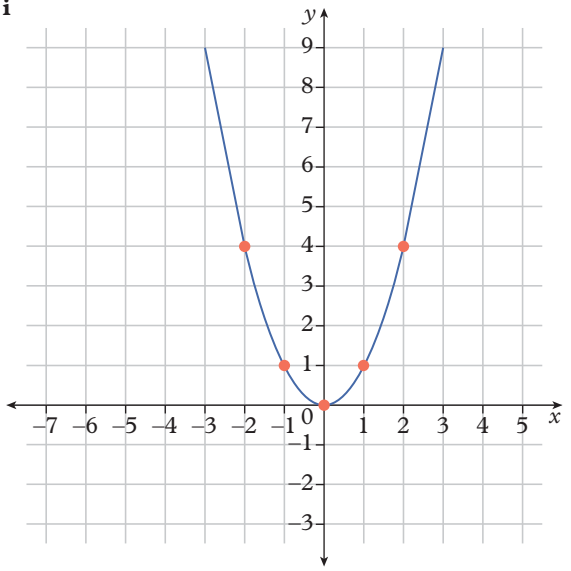
The graphs both intersect the y-axis at $y = 1$, the point (0, 1). This point is also the point of intersection of these graphs. These graphs do not intersect the x-axis at the same point. The y-values of both graphs increase moving from left to right. The y-values of $y = 2x + 1$ increase more quickly.

f

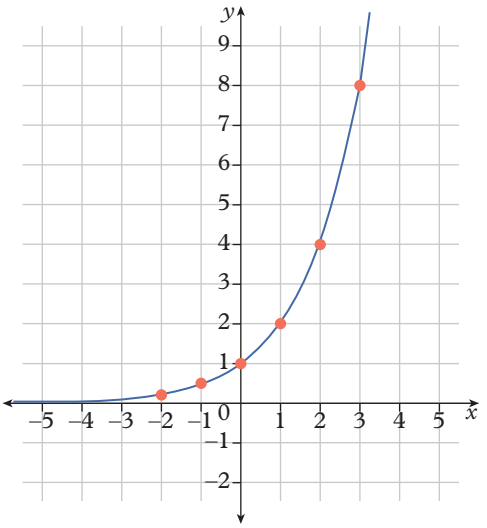


The graphs all intersect the x- and y-axes at different points. The y-values of each graph increases moving from left to right, and they all increase by 2 for each increase of 1 in the x-value. The graph $y = 2x + 1$ is the same as the graph of $y = 2x$ shifted one unit up, and the graph of $y = 2(x + 1)$ is the same as the graph of $y = 2x$ shifted two units up. The graph of $y = 2(x + 1)$ is also the same as the graph of $y = 2x$ shifted one unit left. The graph of $y = 2(x + 1)$ is also the same as the graph of $y = 2x$ shifted 0.5 units left.

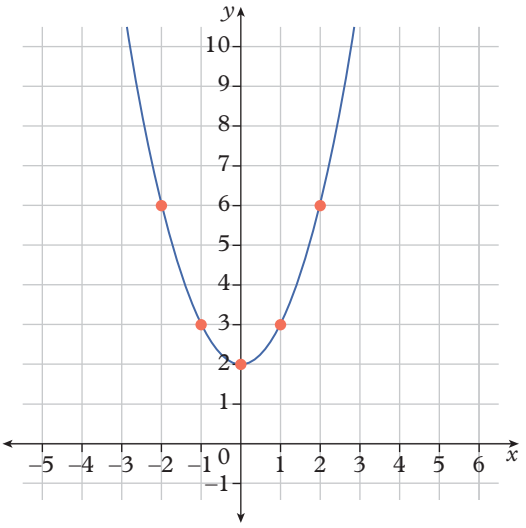
14 a i



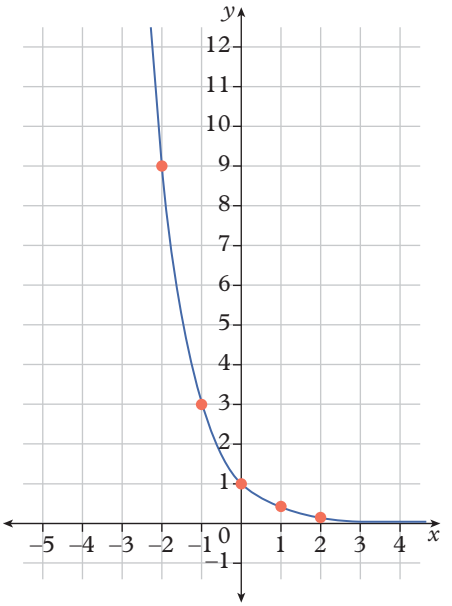
ii



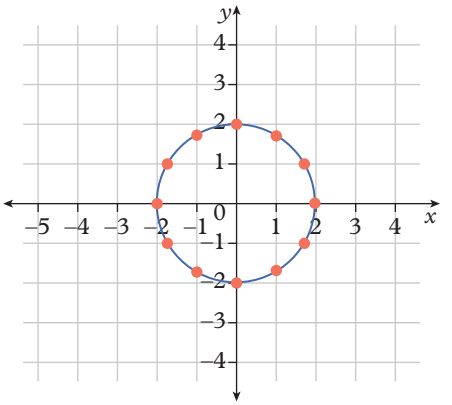
iii



iv



v



b **i** and **iii** are quadratic relationships.

EX 6G Solving linear equations using graphs

p283

1 **a** **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$ **iv** $x = 2$

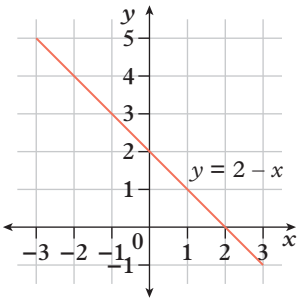
b **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$ **iv** $x = 2$

c **i** $x = 8$ **ii** $x = 4$ **iii** $x = 0$ **iv** $x = -2$

2 **a** $x = 2$ **b** $x = -1$ **c** $x = 1$ **d** $x = 0$ **e** $x = -2$ **f** $x = -3$

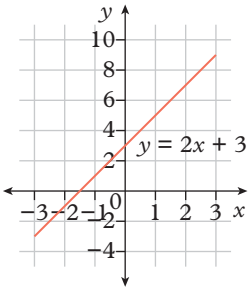
3 **a** $x = 2$ **b** $x = -2$ **c** $x = 0$ **d** $x = -4$ **e** $x = -3$ **f** $x = 1$

4 **a**



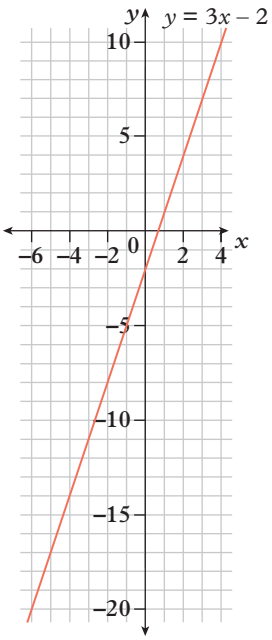
b **i** $x = 1$ **ii** $x = 3$ **iii** $x = 2$
iv $x = 0$ **v** $x = 0.5$ **vi** $x = 1.5$

5 **a**



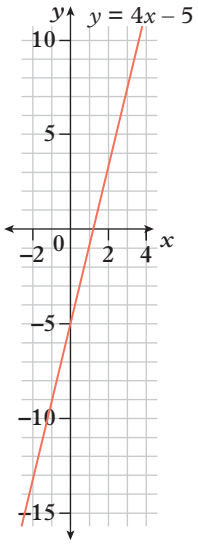
b **i** $x = -1$ **ii** $x = -3$ **iii** $x = 1.5$
iv $x = 0$ **v** $x = -2.5$ **vi** $x = 2.5$

6 **a** $y = 3x - 2$
b **i** $y = 7$ **ii** $y = -5$ **iii** $y = -20$
c



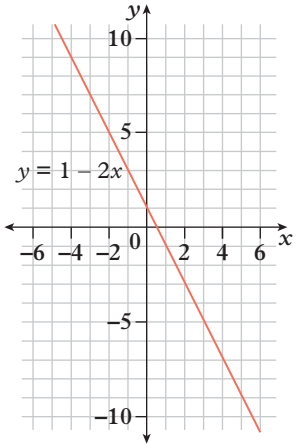
i $x = 3$ **ii** $x = -1$ **iii** $x = -6$

7 **a** $y = 4x - 5$
b **i** $y = 3$ **ii** $y = -13$ **iii** $y = -7$
c



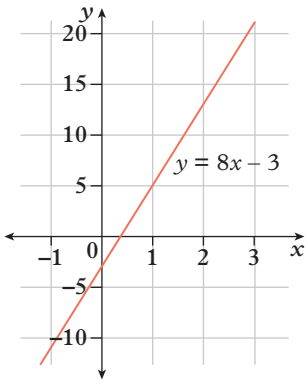
i $x = 2$ **ii** $x = -2$ **iii** $x = -0.5$

8



a $x = -1$ **b** $x = 3$ **c** $x = -3$
d $x = 5$ **e** $x = -4$ **f** $x = 2.5$

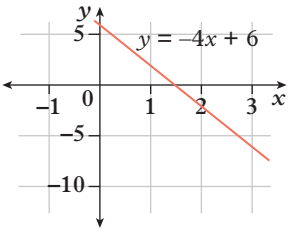
9



a $x = 1$ **b** $x = 2$ **c** $x = 3$
d $x = 0$ **e** $x = -1$ **f** $x = -0.5$

10 a $x = 2.5$

b



$x = 2.5$

c Sample answer: The graphical solution method can be used to solve many similar equations quickly; however, it takes longer to draw a graph and it is difficult to read non-integer values accurately from a graph.

11 a **i** $d = 50$ **ii** $d = 150$ **iii** $d = 200$

b **i** $0.64d = 50$ **ii** $d = 78.125$, \$78

c shirt:

i $0.64d = 64$ **ii** $d = 100$ **iii** \$100

shoes:

i $0.64d = 100$ **ii** $d = 156$ **iii** \$156

jeans:

i $0.64d = 42$ **ii** $d = 66$ **iii** \$66

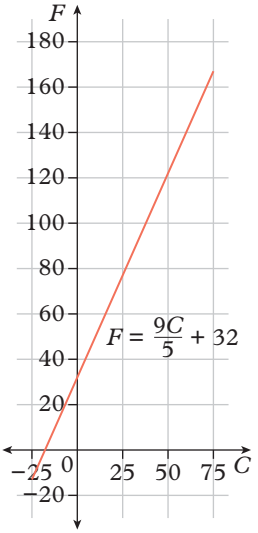
leather jacket:

i $0.64d = 140$ **ii** $d = 219$ **iii** \$219

12 a

C	−25	0	25	50	75
F	−13	32	77	122	167

b



c **i** $C \approx 4$ **ii** $C = 40$ **iii** $C = -10$

d meat:

i $\frac{9}{5}C + 32 = -5$ **ii** $C \approx -21$

dairy products and vegetables:

i $\frac{9}{5}C + 32 = 37$ **ii** $C \approx 3$

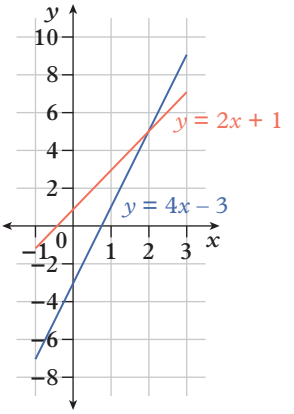
tomatoes:

i $\frac{9}{5}C + 32 = 68$ **ii** $C = 20$

iii 'Meat to be frozen should be stored in the freezer at −21°C, dairy products and vegetables stored in the fridge at about 3°C and tomatoes kept at room temperature, ideally around 20°C.'

e 60°C

13 a, b



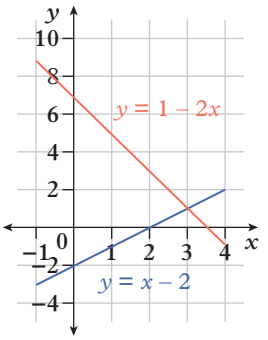
c (2, 5)

d $x = 2$

e
$$\begin{aligned} 4x - 3 &= 2x + 1 \\ 4(2) - 3 &= 2(2) + 1 \\ 8 - 3 &= 4 + 1 \\ 5 &= 5 \end{aligned}$$

14 a **i** $y_1 = x - 2$ and $y_2 = 7 - 2x$

ii

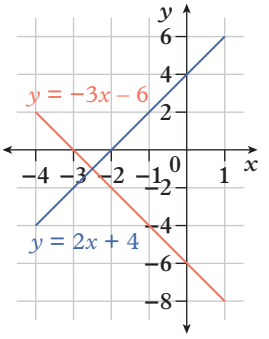


iii $x = 3$

iv
$$\begin{aligned} x - 2 &= 7 - 2x \\ 3 - 2 &= 7 - 2(3) \\ 1 &= 7 - 6 \\ 1 &= 1 \end{aligned}$$

b **i** $y_1 = 2x + 4$ and $y_2 = -3x - 6$

ii



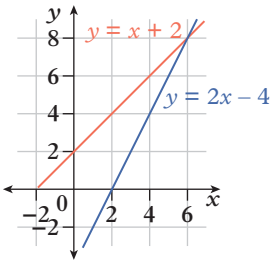
iii $x = -2$

iv
$$\begin{aligned} 2x + 4 &= -3x - 6 \\ 2(-2) + 4 &= -3(-2) - 6 \\ -4 + 4 &= 6 - 6 \\ 0 &= 0 \end{aligned}$$

15 a $x + 2 = 2x - 4$

b $y_1 = x + 2$ and $y_2 = 2x - 4$

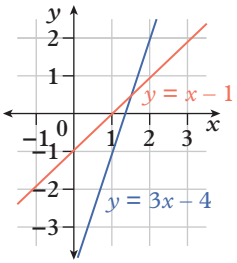
c



$x = 6$

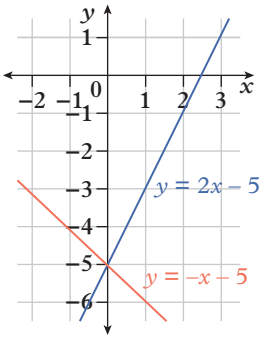
d six lamingtons

16 a



$x = 1.5$

b



$x = 0$

EX p289

6H Intersecting lines

1 a (2, 3) **b** (3, 2) **c** (4, −1)
d (−0.5, 1) **e** (−2, 3) **f** (−60, 80)

2 a (4, 3) **b** (0, 3) **c** (−2, 1)
d (−1, −2) **e** (−25, −2) **f** (5, 31)

3 a (1, 1)

x	−1	0	1	2	3
$y = 3x - 2$	−5	−2	1	4	7
$y = 2x - 1$	−3	−1	1	3	5

b (−2, 1)

x	−3	−2	−1	0	1
$y = 2x + 5$	−1	1	3	5	7
$y = -2x - 3$	3	1	−1	−3	−5

c (2, −2)

x	−2	−1	0	1	2
$y = 7x - 16$	−30	−23	−16	−9	−2
$y = 4x - 10$	−18	−14	−10	−6	−2

d (−1, −4)

x	−3	−2	−1	0	1
$y = -2x - 6$	0	−2	−4	−6	−8
$y = 6x + 2$	−16	−10	−4	2	8

e (−26, 3)

x	−27	−26	−25	−24	−23
$y = -x - 23$	4	3	2	1	0
$y = 2x + 55$	1	3	5	7	9

f (5, −58)

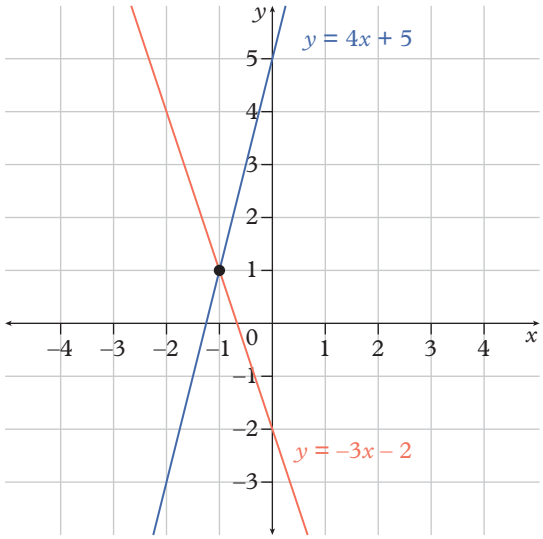
x	2	3	4	5	6
$y = -3x - 43$	−49	−52	−55	−58	−61
$y = 4x - 78$	−70	−66	−62	−58	−54

4 a-f Substituting the point into both lines shows that the point lies on both lines and so is a point of intersection.

5 a is a point of intersection
b is not a point of intersection
c is a point of intersection
d is a point of intersection
e is not a point of intersection
f is not a point of intersection

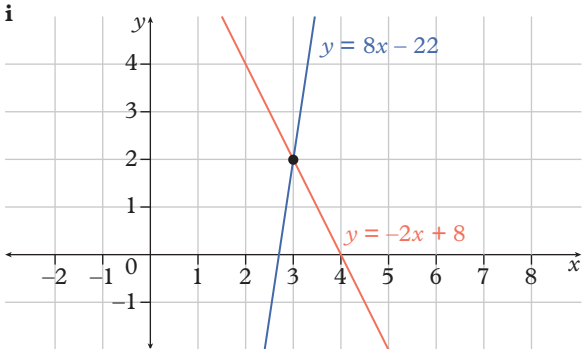
6 a (6, −1) **b** (−2, 3) **c** (3, 1)
d (−5, −2) **e** (−1, −5)

7 a i



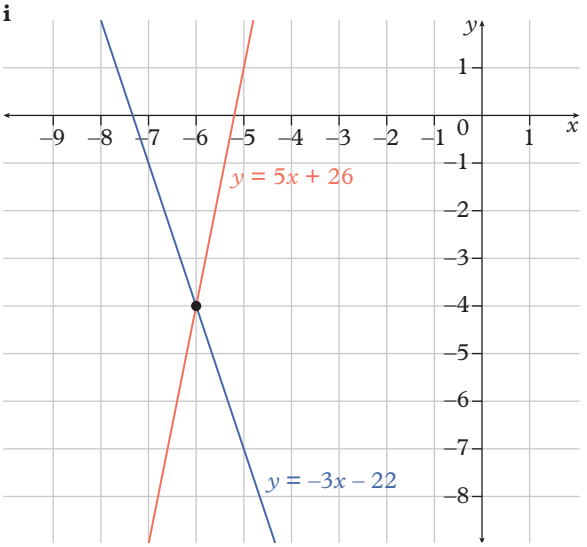
ii (−1, 1)

b i



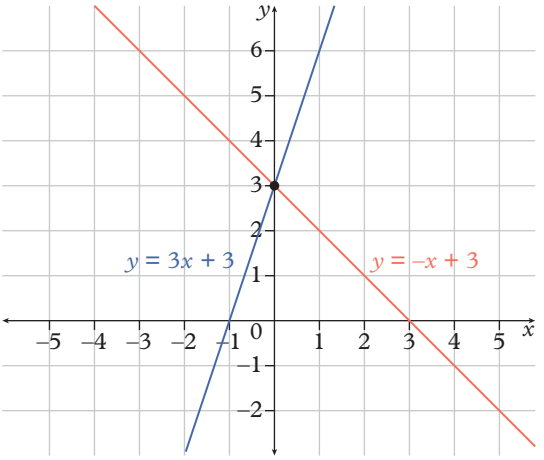
ii (3, 2)

c **i**



ii (−6, −4)

d **i**



ii (0, 3)

8 $A(-3,3)$: $y = -2x - 3$ and $y = -x$
$B(0,2)$: $y = x + 2$ and $y = -x + 2$
$C(1,1)$: $y = x$ and $y = -x + 2$
$D(-3, -1)$: $y = 2x + 5$ and $y = x + 2$
$E(-2,1)$: $y = -2x - 3$ and $y = 2x + 5$
$F(0,0)$: $y = -x$ and $y = x$

9 **a** **i** The x-coordinate of the intersection point lies between 6 and 7.

ii The y-coordinate of the intersection point lies between 5 and 6.

b (6.2,5.4)

x	6	6.1	6.2	6.3	6.4	6.5
$y = 2x - 7$	5	5.2	5.4	5.6	5.8	6
$y = 7x - 38$	4	4.7	5.4	6.1	6.8	7.5

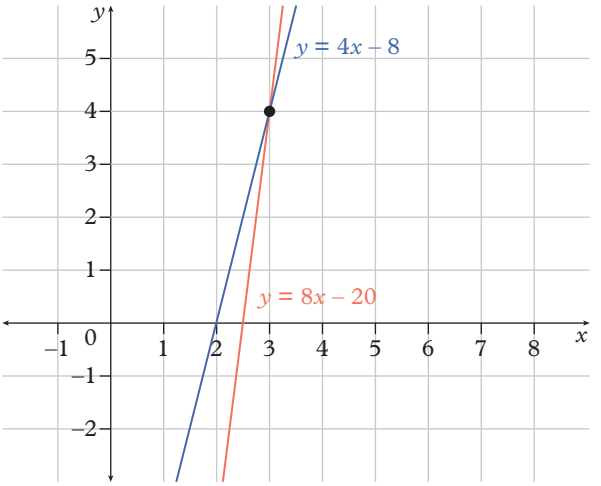
10 **a** (7.3, −0.2) **b** (5.1,3.5) **c** (9.1, −5.3)
d (7.8,6) **e** (−5.5,3) **f** (5.6, −8.6)

11 **a** Any pair of parallel lines will not intersect.

b The lines are parallel so the values of y change at the same rate. This means the lines will always stay the same distance (difference in y-values for each x-value) apart. The lines are parallel. If two lines are not parallel, then there must be some point on the Cartesian plane where they intersect.

12 If two line graphs intersect in more than one point, then they are the same graph and they have the same equation.

13 **a**



b (3,4)

c $x = 3$

d **i** $y = 4x - 8$
LHS = 4
RHS = 4(3) − 8 = 4

$y = 8x - 20$
LHS = 4
RHS = 8(3) − 20 = 4

ii $4x - 8 = 8x - 20$
$4x + 12 = 8x$
$12 = 4x$
$3 = x$

14 **a** $x = 2$ **b** $x = -3$ **c** $x = -2$
d $x = -4$ **e** $x = 1$ **f** $x = -6$

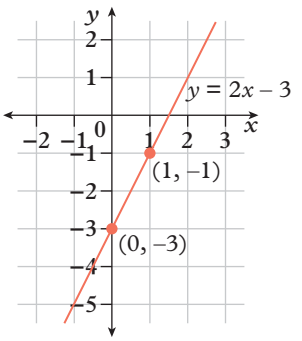
15 **a** (2, −1) and (5, 2) **b** (−1, −2) and (1, 2)
c (−3, 2), (0, −1) and (1, 2) **d** (−1, 3) and (2, 0)
e (−3, 0), (3, 0) and (0, −3) **f** (−1, 2)

EX p296 6I Finding linear equations

1 **a** $y = 2x + 2$ **b** $y = -3x + 4$ **c** $y = 2x + 1$
d $y = -2x - 3$ **e** $y = 2x + 4$ **f** $y = -3x - 5$

2 **a** $y = 2x + 1$ **b** $y = -3x + 1$ **c** $y = 4x - 2$
d $y = -\frac{1}{3}x + 3$ **e** $y = -4x - 2$ **f** $y = -\frac{x}{2}$

3



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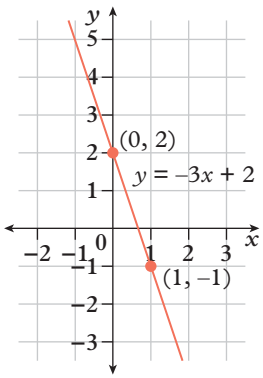


5 **a** IV **b** I **c** II **d** III

6 **a** III **b** II **c** I **d** IV

7 **a** I **b** II **c** IV **d** III

8 **a** $y = -4x + 5$ **b** $y = -3x - 5$

9 **a** $x = 4$ **b** $y = 2$ **c** $y = -3$ **d** $x = -3$

10 **a** **i** $y = 3x + 3$ **ii** $y = 3x + 1$ **iii** $y = 3x - 1$
iv $y = 3x - 3$

b **i** $y = -2x + 3$ **ii** $y = -2x + 1$ **iii** $y = -2x - 1$
iv $y = -2x - 3$

c The equations all have the same x coefficient

d The equations all have the same x coefficient

11 Linear graphs that have the same value for m in the equation $y = mx + c$ are parallel.

12 **a**

Rows	Tiles in the row
1	16
2	24
3	32
4	40

b $t = 8r + 8$

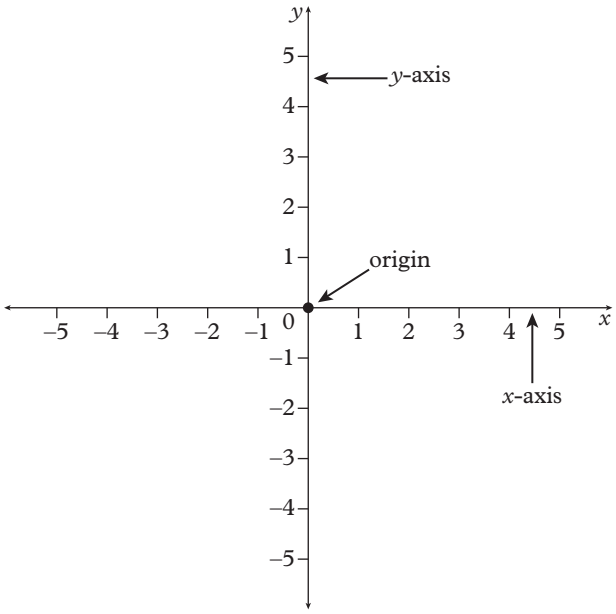
13 $y = -2x - 1.5$

EX p300

CHAPTER 6 review

Mathematical literacy review

1



2 **a** The y-coordinate of a point tells you how far above or below the origin a point is located.

b The x-coordinate of a point tells you how far left or right of the origin a point is located.

3 **a** solve by inspection

b point of intersection

c y-intercept

4 **a** Inverse operations reverse the effect of another operation. For example, the inverse operation of adding three (+3) is subtracting three (−3).

b A linear relationship is a relationship between two variables which produces a straight line graph. For example, $y = 2x - 3$ is a linear relationship.

c The subject of an equation is a pronumeral written by itself on one side of the equation and which does not appear on the other side of the equation. For example, y is the subject in the equation $y = mx + c$.

d A quadratic equation is an equation in which the highest index of any variable is 2. For example, $y = 2x^2 + x + 1$ is a quadratic equation.

Multiple choice

1 C 2 A 3 B 4 C 5 E

6 D 7 E 8 B 9 C 10 C

11 B 12 E 13 A 14 B 15 A

Short answer

1 **a** yes **b** yes **c** yes **d** no

2 **a** $a = 9$ **b** $b = 14$ **c** $c = 3$ **d** $d = 15$

3 **a** $x = 3$ **b** $x = -9$ **c** $c = 14$ **d** $d = 2$

e $x = 1\frac{1}{4}$ $\left(\text{or } x = \frac{5}{4}\right)$ **f** $x = -8$

4 **a** $12s + 24s = 36$

b $s = 1$; small book = 1 kg, large book = 2 kg

5 **a** $x = -5$ **b** $x = 2$ **c** $x = -3$

d $x = -2\frac{1}{2}$ $\left(\text{or } x = -\frac{5}{2}\right)$

6 $s + 11 = 5(s - 5)$, Suzi is 9 years old.

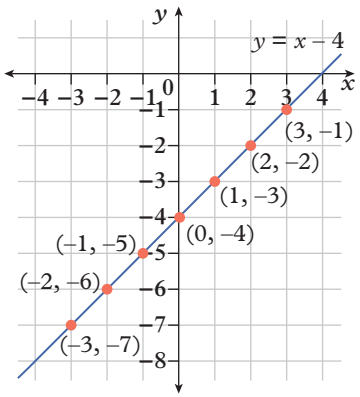
7 **a** $x = \frac{y - by - c}{a}$ **b** $a = \frac{y - by - c}{x}$ **c** $y = \frac{ax + c}{1 - b}$

8 $x = 10$

9 **a** ±6 **b** ±6 **c** 0

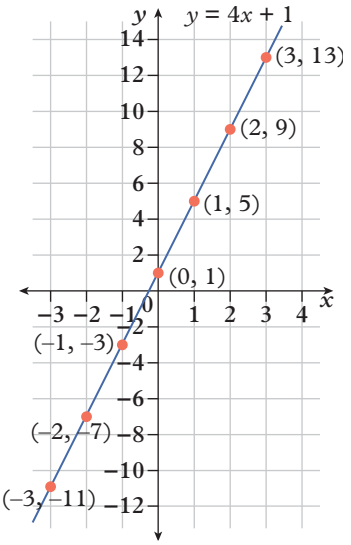
10 **a**

x	−3	−2	−1	0	1	2	3
y	−7	−6	−5	−4	−3	−2	−1



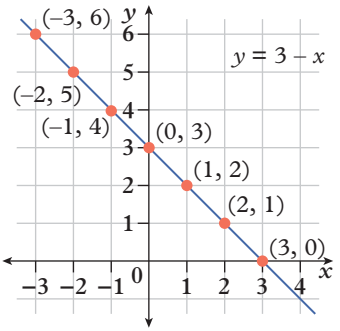
b

x	−3	−2	−1	0	1	2	3
y	−11	−7	−3	1	5	9	13



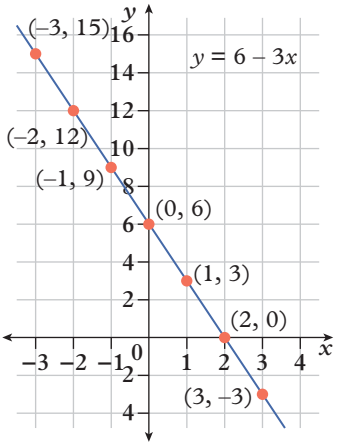
c

x	−3	−2	−1	0	1	2	3
y	6	5	4	3	2	1	0



d

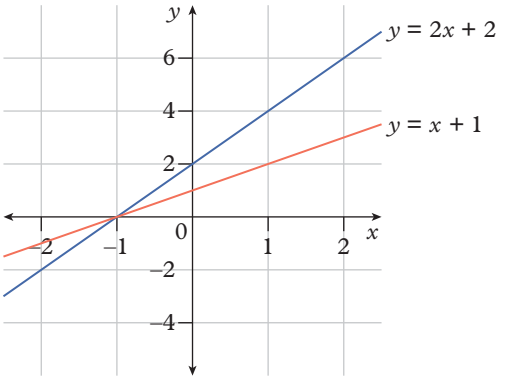
x	−3	−2	−1	0	1	2	3
y	15	12	9	6	3	0	−3



11 **a** $y = 4x - 5$ **b** $y = -x$

12 **a** $y = -x + 2$ **b** $y = 5x - 4$

13



x-value	−2	−1	0	1	2
$y = x + 1$	−1	0	1	2	3
$y = 2x + 2$	−2	0	2	4	6

Point of intersection is (−1, 0).

14 **a** $x = 4$ **b** $x = -1$ **c** $x = 3$ **d** $x = 2$

15 **a** $x = -2$ **b** $x = 1$ **c** $x = 0$

Analysis

1 **a** $C = 150 + 20p$

b 15 people can attend the party.

c $C = 250 + 15p$

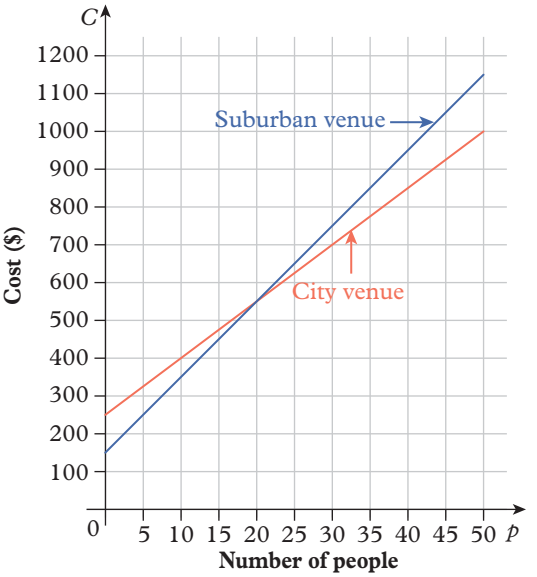
d Suburban venue

p	0	10	20	30	40	50
C	150	350	550	750	950	1150

City venue

p	0	10	20	30	40	50
C	250	400	550	700	850	1000

e



f yes, as the points form a straight line

g **i** suburban = \$650, city = \$625

ii suburban = 15, city = 13

h **i** suburban **ii** city **iii** same cost

2 **a** $C = 5x + 3$

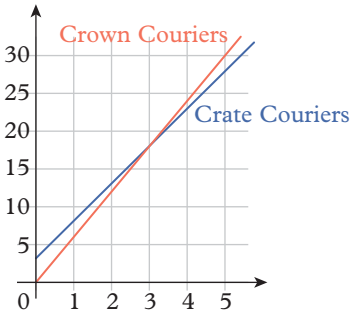
b \$53

c 7

d

Number of books	0	1	2	3	4	5
Cost of delivery	\$3	\$8	\$13	\$18	\$23	\$28

e, g

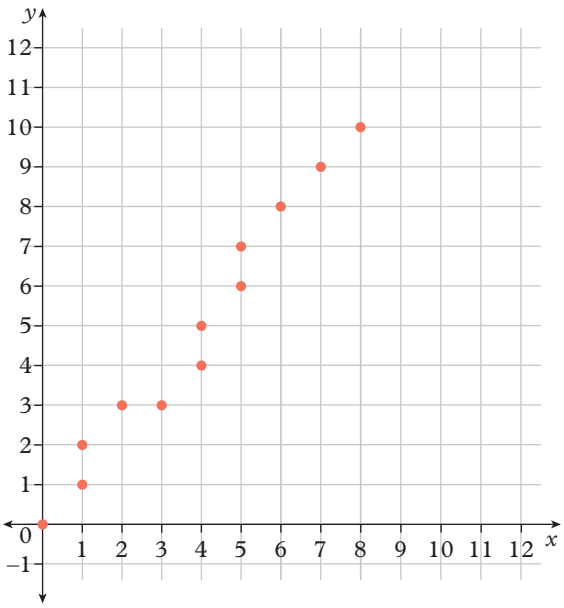


f

Number of books	0	1	2	3	4	5
Cost of delivery	\$0	\$6	\$12	\$18	\$24	\$30

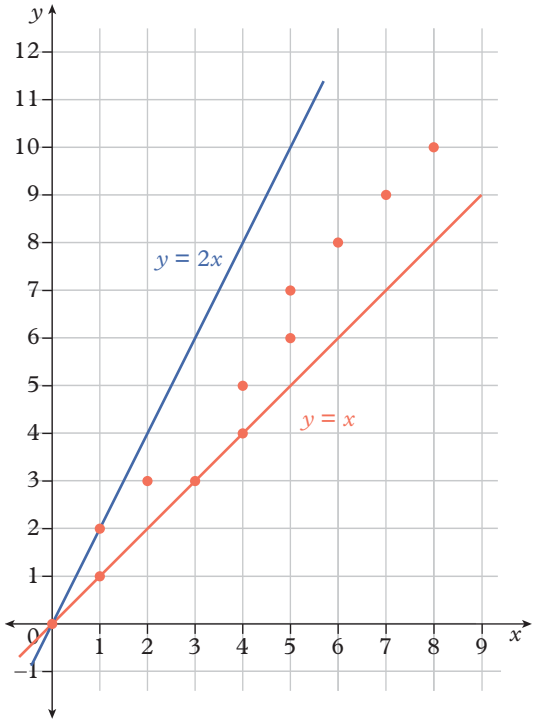
h Joss should use Crate Couriers for delivering more than 3 books and he should use Crown Couriers for fewer than 3 books. For 3 books it makes no difference.

3 **a**



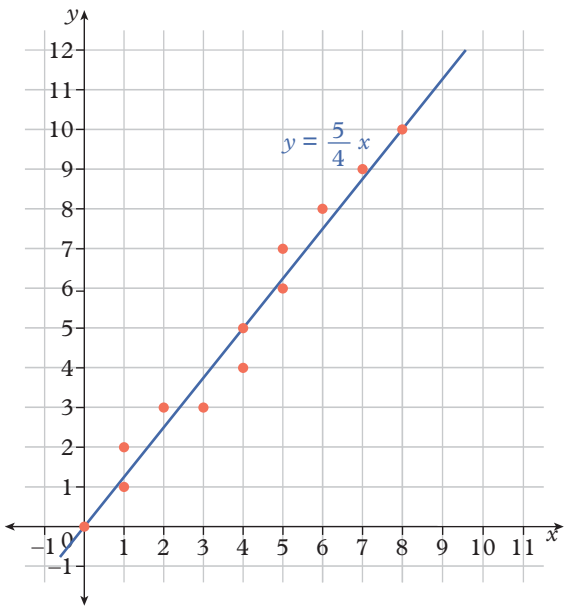
b No. The points can't be joined by a straight line.

c



d Most of the points are above the graph of $y = x$ and most of the points are below the graph of $y = 2x$.

e



f The graph of $y = \frac{5}{4}x$ passes through some of the points and the other points are split above and below the line.

EX p310 CHAPTER 7 Triangles and quadrilaterals

7A Angles

1 **a** $\angle BDE$ or $\angle EDB$, obtuse angle

b $\angle PQV$ or $\angle VQP$, right angle

c $\angle FBY$ or $\angle YBF$, reflex angle

d $\angle MYZ$ or $\angle ZYM$, acute angle

e $\angle IMA$ or $\angle AMI$, reflex angle

f $\angle DHL$ or $\angle LHD$, obtuse angle

2 **a** $a = 15°$ **b** $b = 15°$ **c** $c = 39°$
d $d = 64°$ **e** $e = 138°$ **f** $f = 55°$

3 **a** $a = 160°$
b $a = 79°$, $b = 101°$, $c = 79°$
c $d = 147°$, $e = 33°$, $f = 147°$
d $g = 144°$
e $h = 253°$
f $i = 72°$, $j = 69°$, $k = 39°$, $l = 69°$

4 **a** complementary **b** supplementary
c vertically opposite **d** supplementary

5 **a** $m = 152°$, $n = 68°$
b $v = 27°$
c $s = 68°$, $t = 22°$, $u = 68°$
d $a = 7°$, $b = 139°$, $c = 41°$
e $w = 45°$, $x = 38°$, $y = 39°$, $z = 38°$
f $d = 115°$, $e = 65°$, $f = 115°$, $g = 110°$, $i = 110°$, $j = 135°$, $k = 45°$, $l = 135°$, $n = 70°$

6 **a** **i** corresponding **ii** $a = 63°$
b **i** alternate **ii** $b = 71°$
c **i** co-interior **ii** $c = 122°$
d **i** corresponding **ii** $d = 123°$
e **i** corresponding **ii** $e = 82°$
f **i** co-interior **ii** $f = 23°$
g **i** corresponding **ii** $g = 76°$
h **i** alternate **ii** $h = 35°$
i **i** co-interior **ii** $i = 49°$

7 **a** $a = 141°$, co-interior
b $b = 147°$, co-interior
c $c = 21°$, alternate
d $d = 66°$, corresponding; $e = 59°$, alternate
e $f = 85°$, corresponding; $g = 95°$, co-interior
f $n = 142°$, co-interior

8 27°, angles are alternate and equal

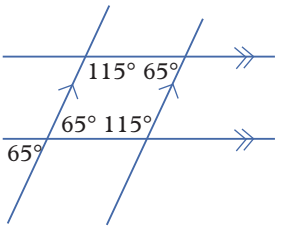
9 The stairs have an angle of 23° with respect to the horizontal and meet the standards but are not in the preferred range.

10 Alternate exterior angles are supplementary to the alternate interior angles (which are equal). The supplements of equal angles will be equal to each other, so alternate exterior angles are equal for parallel lines.

11 **a** Angles are corresponding but not equal, so the lines are not parallel.
b Angles are co-interior and supplementary, so the lines are parallel.
c 99° is supplementary to the angle alternate to 81°, so the lines are parallel.

12 **a** 360°
b **i** 180° **ii** 90° **iii** 270°
c **i** 120° **ii** 330° **iii** 240°
iv 30°

13 **a** parallelogram
b



14 **a** c and f; d and e; i and d; b and k
b a and e; b and f; c and g; d and h; j and b; i and a; l and d; k and c
c c and e; d and f; i and b; d and k

15 $a = 40°$ (angles in a triangle), $b = 50°$ (alternate angles), $c = 90°$ (right angle), $d = 40°$ (corresponding angle), $e = 50°$ (corresponding angle), $f = 50°$ (corresponding angle)

16 **a** The transversal creates two intersections. Around each intersection there will be two pairs of vertically opposite equal angles and they are related to each other by being supplementary. The pairs of angles from one intersection and the other are related and either equal or supplementary by being co-interior, corresponding or alternate. So all angles are either equal or supplementary to a given angle.
b From part **a**, the angles formed by one transversal are all related and either equal or supplementary to a given angle. For each new parallel transversal, all of its angles correspond to an angle from the previous transversal. So no matter how many parallel transversals are added, all angles are equal or supplementary to a given angle.

17 **a** 30°
b **i** 15° **ii** 7.5° **iii** 22.5°
c **i** 165° **ii** 97.5° **iii** 52.5°

18 **a** 170° **b** 122.5° **c** 72.5° **d** 88.5° **e** 79.5°

EX p318

7B Triangles

1 **a** obtuse-angled isosceles
b acute-angled scalene
c acute-angled isosceles
d acute-angled equilateral
e right-angled scalene
f obtuse-angled scalene

2 **a** 63° **b** 90° **c** 41°
d 67° **e** 106° **f** 50°

3 **a** 138° **b** 141° **c** 64°
d 111° **e** 109° **f** 164°

4 **a** $a = 60°$, $b = 60°$, $c = 60°$
b $a = 45°$, $b = 45°$ **c** $a = 142°$, $b = 19°$
d $d = 120°$ **e** $e = 71°$ **f** $f = 41°$
g $g = 27°$ **h** $h = 58°$ **i** $i = 156°$

5 **a** acute-angled equilateral
b right-angled isosceles

c obtuse-angled isosceles
d acute-angled equilateral
e acute-angled scalene
f acute-angled scalene
g obtuse-angled scalene
h right-angled scalene
i obtuse-angled isosceles

6 **a** False; the angles in an equilateral must be 60° (acute).
b True; the other angles in an obtuse-angled isosceles triangle will both be less than 45°.
c True; the other angles must sum to 90° and not be equal.

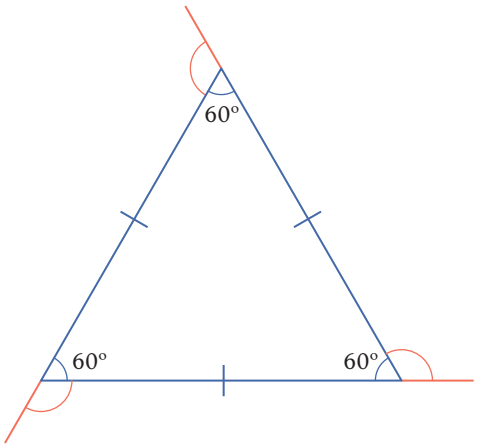
7 **a** $x = 30°$ **b** $x = 10°$ **c** $x = 10°$
d $x = 10°$ **e** $x = 12°$ **f** $x = 20°$

8 **a** $a = 30°$, $b = 60°$, $c = 90°$
b Right-angled scalene; the triangle has three different interior angles including a right angle.

9 $a = 37°$, $b = 113°$, $c = 141°$, $d = 17°$

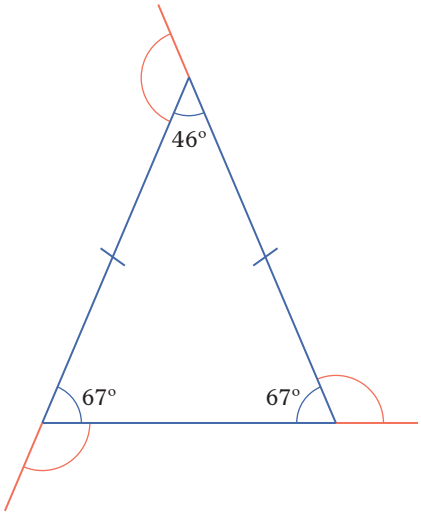
10 **a** $p = 96°$, $q = 36°$, $r = 83°$
b 96° **c** $\angle QRS = 48°$

11 **a** **i**



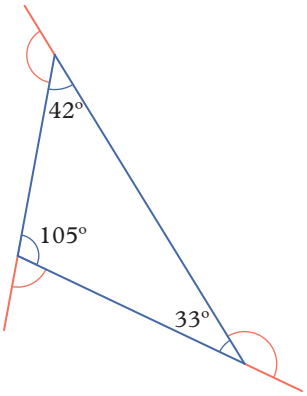
ii Each exterior angle is 120°; they add to 360°.

b **i**



ii Exterior angles are 134°, 113° and 113°; they add to 360°.

c **i**

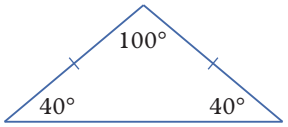


ii Exterior angles are 138°, 147° and 75°; they add to 360°.

d The exterior angles of any triangle add to 360°.

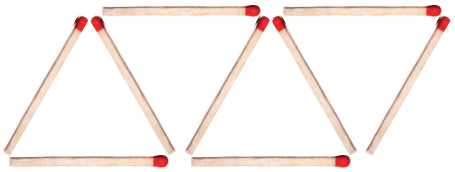
12 **a** Not possible. The angle sum of a triangle is 180°. These angles add to 200°.

b

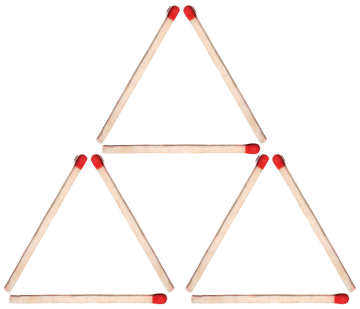


c Not possible. If the base length is 10 cm, the other two sides are not long enough to meet.
d Not possible: The angle sum of a triangle is 180°. Two obtuse angles add to more than 180°.

13 **a** seven matches
b **i**

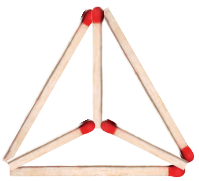


ii



(The large triangle is the fifth triangle.)

iii



(triangular pyramid)

14 $x + c = 180°$
$a + b + c = 180°$
$x + c = a + b + c$
$x = a + b$

EX p324

7C Quadrilaterals

1 **a** kite; two pairs of adjacent sides equal in length
b trapezium; one pair of opposite sides that are parallel
c rhombus; all four sides equal in length and opposite sides parallel, opposite angles equal, but not 90°
d parallelogram; opposite sides are parallel and equal in length, opposite angles are equal
e square; all four sides equal in length and all angles are 90°
f kite; two pairs of adjacent sides equal in length

2 **a** $x = 76°$ **b** $x = 117°$ **c** $x = 141°$
d $x = 23°$ **e** $x = 21°$ **f** $x = 93°$

3 **a** parallelogram; $x = 71°$
b rectangle; $x = 90°$ **c** kite; $x = 85°$

4 **a** **i** rhombus
ii $a = 72°$, $b = 72°$ and $c = 108°$
b **i** trapezium **ii** $a = 90°$, $b = 112°$
c **i** square **ii** $a = 90°$, $b = 90°$, $c = 90°$
d **i** kite **ii** $a = 130°$, $b = 130°$
e **i** parallelogram **ii** $a = 125°$, $b = 55°$, $c = 125°$
f **i** kite **ii** $a = 52°$, $b = 115°$

5 **a** $x = 20°$ **b** $x = 37°$ **c** $x = 80°$
d $x = 20°$ **e** $x = 15°$ **f** $x = 17°$

6 **a** Having two internal reflex angles would make the angle sum exceed 360°.
b 270°

7 rectangle

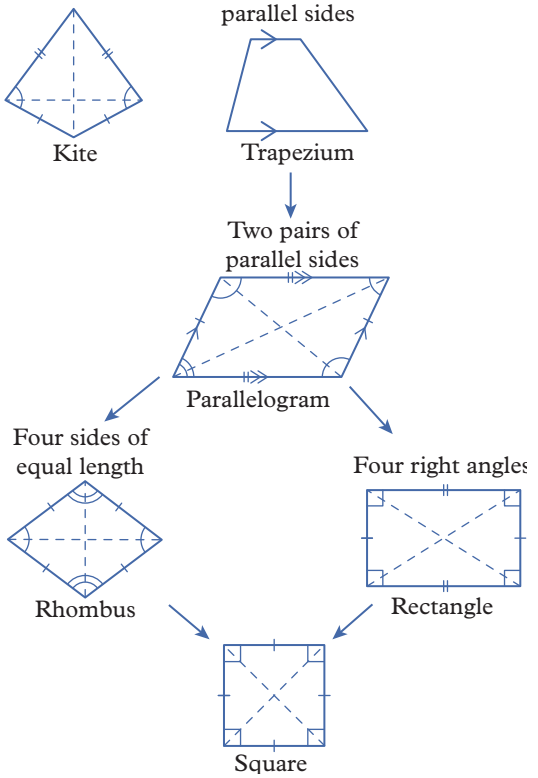
8 **a** True
b False. A kite has no parallel sides. It contains two pairs of adjacent sides equal in length, but not parallel.
c False. A rectangle is a type of parallelogram. The reverse is not true as a parallelogram does not necessarily have four angles equal to 90°.
d False. A trapezium has at least one pair of parallel sides. It can have two pairs of parallel sides.
e False. A rhombus has four equal sides and two pairs of angles equal in size. When all four angles are equal in size it is called a square.
f False. A rectangle has two pairs of opposite equal sides and four equal angles.

9 Every quadrilateral can be split into two triangles by drawing in one of the diagonals. The angle sum of a quadrilateral is equal to the angle sum of the two triangles added together: 180° + 180° = 360°.

10 **a** **i** 130°, 50°, 130°, 50° **ii** 360°
b **i** 108°, 124°, 56°, 72° **ii** 360°
c **i** 74°, 120°, 94°, 72° **ii** 360°
d The exterior angles of any quadrilateral add to 360°.

11 The hinges of the toolbox create a parallelogram with the trays being the parallel sides. These sides stay parallel as the toolbox opens, meaning that the trays stay horizontal.

12 No parallel sides At least one pair of parallel sides



13 **a** similar **b** similar **c** not similar
d not similar

14 **a** $a = 12$ cm, $b = 99°$, $c = 81°$, $d = 99°$
b $a = 1$ cm, $b = 150°$, $c = 75°$, $d = 9$ cm, $e = 12$ cm, $f = 45°$, $g = 150°$

15 **a** **i** 90° **ii** square
b **i** 60°, 60°, 120°, 120°
ii parallelogram

EX p327

Checkpoint

1 **a** $x = 161°$
b $a = 67°$, $b = 67°$, $c = 23°$

2 **a** parallel; the alternate angles are equal
b not parallel; the corresponding angles are not equal
c parallel; the co-interior angles are supplementary

3 **a** $a = 42°$, obtuse-angled scalene
b $b = 90°$, right-angled scalene
c $c = 14°$, $d = 152°$, obtuse-angled isosceles
d $e = 60°$, $f = 60°$, $g = 60°$, acute-angled equilateral

4 **a** 124° **b** 38° **c** 117° **d** 94°

5 **a** trapezium, $x = 105°$
b rhombus, $x = 64°$
c trapezium, $x = 143°$

6 **a** False, a rectangle does not need to have four sides of equal length. A square is a type of rectangle.
b False, a kite does has no pairs of parallel sides, whereas a trapezium has at least one pair of parallel sides.

c True, a rhombus is a type of parallelogram, which in turn is a type of trapezium.

EX p330 7D Pythagoras' theorem

1 **a** m **b** p **c** f

2 **a** right-angled **b** not right-angled
c right-angled **d** not right-angled

3 **a** right-angled **b** right-angled
c not right-angled **d** not right-angled
e right-angled **f** not right-angled

4 **a** 15 cm **b** 50 mm **c** 61 cm **d** 26 cm

5 **a** 9.43 cm **b** 17.49 cm
c 23.85 mm **d** 29.83 mm

6 **a** **i** $\sqrt{157}$ cm **ii** 12.53 cm
b **i** $\sqrt{3049}$ m **ii** 55.22 m
c **i** $\sqrt{637}$ mm **ii** 25.24 mm
d **i** $\sqrt{120.25}$ cm **ii** 10.97 cm

7 **a** 171 cm **b** 3 cm **c** 10 cm **d** 70 cm

8 No, because $a^2 + b^2$ is the same as $b^2 + a^2$.

9

Values		Pythagorean triad		
m	n	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$
2	1	$2^2 - 1^2 = 3$	$2 \times 2 \times 1 = 4$	$2^2 + 1^2 = 5$
3	1	8	6	10
3	2	5	12	13
4	1	15	8	17
4	2	12	16	20
4	3	7	24	25
5	1	24	10	26
5	2	21	20	29
5	3	16	30	34
5	4	9	40	41
6	1	35	12	37

10 **a** (3, 4, 5), (5, 12, 13), (15, 8, 17), (7, 24, 25), (21, 20, 29), (9, 40, 41), (35, 12, 37)
b **i** (5, 12, 13) **ii** (8, 15, 17)
iii (3, 4, 5) **iv** (9, 40, 41)
v (65, 72, 97) **vi** (21, 220, 221)

11 **a** $10^2 = 6^2 + 8^2$, $100 = 36 + 64$
b yes; $20^2 = 12^2 + 16^2$
c yes; $30^2 = 18^2 + 24^2$
d yes; $5^2 = 3^2 + 4^2$
e yes; $100^2 = 60^2 + 80^2$

12 **a** 9.43 cm **b** 10 cm **c** 3.61 m
d 25.61 mm **e** 14.42 cm **f** 21.21 mm

13 **a** $x = 27.80$ cm, $y = 29.90$ cm
b $x = 15.81$ m, $y = 18.71$ m
c $x = 60.22$ cm, $y = 66.41$ cm

14 Pythagoras' theorem calculates the length of a side, and a length is always a positive value.

15 **a** Triangle ABC and Triangle ACD
b $AC = 10$ cm; $AB = 6$ cm and $BC = 8$ cm
c $CD = 26$ cm; the hypotenuse in triangle ABC becomes one of the smaller sides in triangle ACD.
d 64 cm

16 **a** 16, 25, 36
b In an acute triangle, the square of the longest side is less than the sum of the squares of the two other sides.
c 9, 64, 100
d In an obtuse triangle, the square of the longest side is greater than than the sum of the squares of the two other sides.
e **i** $121 < 164$ $(64 + 100)$
ii $169 > 100$ $(36 + 64)$

17 **a** $x = \frac{5}{2}$ **b** $w = \frac{13}{8}$ **c** $t = 2$ **d** $p = \frac{1}{3}$

18 **a** Both squares have side lengths of $a + b$, so they both have an area of $(a + b)^2$.
b Both squares have the same area and include four identical triangles. So, the remaining space must be equal. The remaining area (shaded green) in Diagram 1 is a square with side lengths of c so it has an area of c^2.
The remaining area of Diagram 2 is two squares, one with side length a and the other with side length b. So their areas are a^2 and b^2 respectively. Therefore, equating the remaining areas, $c^2 = a^2 + b^2$.
c Diagram 1: $c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$
Diagram 2: $a^2 + b^2 + 4 \times \frac{1}{2}ab = a^2 + b^2 + 2ab$
d $c^2 + 2ab = a^2 + b^2 + 2ab$
$c^2 = a^2 + b^2$
e The total area of Diagram 3 with side lengths of c is c^2. The square is made up of four right-angled triangles with an area of $\frac{1}{2}ab$ and a square with side lengths of $b - a$ and an area of $(b - a)^2$.
f $c^2 = (b - a)^2 + 4 \times \frac{1}{2}ab$
$c^2 = b^2 - 2ab + a^2 + 2ab$
$c^2 = a^2 + b^2$

19 **a** $a = 8$ m **b** $b = 9.49$ m
c $c = 7$ m **d** $d = 7.75$ m

EX p337 7E Finding the length of a shorter side

1 **a** $x = \pm 3.32$ **b** $x = \pm 7.14$ **c** $x = \pm 6.93$
d $x = \pm 8.49$ **e** $x = \pm 10.91$ **f** $x = \pm 4.24$

2 **a** 20 mm **b** 15 cm **c** 48 mm
d 27 mm **e** 14 m **f** 36 cm

3 **a** 12.65 cm **b** 10.25 cm **c** 22.36 mm
d 18.73 cm **e** 4.76 cm **f** 15.96 m

4 **a** **i** $\sqrt{783}$ cm **ii** 27.98 cm
b **i** $\sqrt{105}$ m **ii** 10.25 m
c **i** $\sqrt{184}$ cm **ii** 13.56 cm
d **i** $\sqrt{2080}$ m **ii** 45.61 m

5 **a** 398 cm **b** 5 cm **c** 55 cm **d** 47 cm

6 **a** 10.00 cm **b** 7.91 m **c** 10.95 cm
d 16.47 mm **e** 5.31 mm **f** 10.25 cm

7 **a** 14.99 cm^2 **b** 8.74 cm^2 **c** 29.54 cm^2
d 24.82 cm^2

8 **a** 15.65 m **b** 4.58 cm **c** 7.07 m

9 **a** $x = 13.08$ cm, $y = 7.35$ cm
b $x = 4$ m, $y = 3.48$ m
c $x = 17.80$ m, $y = 7.81$ m
d $x = 43.86$ cm, $y = 28.90$ cm

10 **a** **i** 79.35 cm **ii** 224.65 cm^2
b **i** 18.78 m **ii** 15.36 m^2
c **i** 48.81 m **ii** 139.48 m^2
d **i** 135.90 cm **ii** 1003.17 cm^2

11 **a** $c = 10$ **b** $a^2 = 10$ and $b^2 = 90$
c A triangle is right-angled if its side lengths satisfy Pythagoras' theorem. $c^2 = 10^2 = 100$ and $a^2 + b^2 = 10 + 90 = 100$.
Therefore, the triangle is right-angled.
d $(-1.5, 0.5)$, $(3.5, 0.5)$, $(3, -1)$
e 22.5, 2.5, 25
f $c^2 = 25$ and $a^2 + b^2 = 22.5 + 2.5 = 25$. So, the triangle is right-angled.

12 $a = 3.00$ cm, $b = 5.81$ cm, $c = 5.67$ cm, $d = 6.41$ cm, $e = 4.25$ cm, $f = 2.43$ cm, $g = 9.27$ cm

13 **a** $a = 8.94$ **b** $a = 144$ **c** $a = 43.30$

14 **a** **i** and **ii** **b** **iv**
c **iii**; The calculation subtracts the square of the shorter side from the square of the hypotenuse. This is the wrong way around. The calculation is the square root of a negative number, which is not a real number as needed for the length of a right-angled triangle.

15 **a** $a = 3.74$ m
b $b = 7.14$ m
c $c = 11$ m
d $d = 6.71$ m

16 7.5 cm^2

EX 7F Applications of Pythagoras' theorem

p342

1 **a** $\sqrt{500}$ mm **b** $\sqrt{208}$ cm **c** $\sqrt{250}$ cm
d $\sqrt{18}$ cm **e** $\sqrt{40}$ mm **f** $\sqrt{272}$ cm

2 **a** 22.4 mm **b** 14.4 cm **c** 15.8 cm
d 4.2 cm **e** 6.3 mm **f** 16.5 cm

3 **a** $\sqrt{50}$ cm **b** $\sqrt{800}$ mm
c $\sqrt{128}$ cm **d** $\sqrt{1250}$ cm

4 **a** 7.1 cm **b** 28.3 mm **c** 11.3 cm **d** 35.4 cm

5 **a** 41.2 cm **b** 15.5 cm **c** 10.4 cm
d 22.5 cm **e** 36.7 cm **f** 18.0 mm

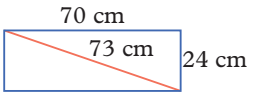
6 18.03 m

7 2.87 m

8 8.72 m

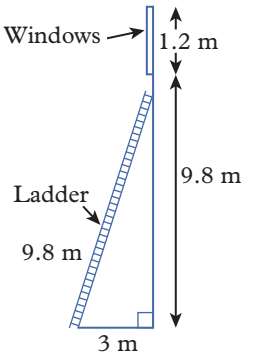
9 yes, because $60^2 + 63^2 = 87^2$

10 **a**



b The frame is not square. The diagonal distance should be longer.
c 74 cm

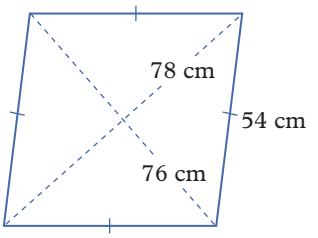
11 **a**



b 9.3 m
c The top of the window is 9.8 + 1.2 = 11 m above the ground. If Josh stands on the very top of the ladder, he is the same height above the ground as the top of the window (9.3 m + 1.7 m = 11 m). He will be able to clean the windows.

12 16.83 m

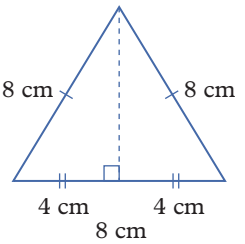
13 **a** $78^2 > 54^2 + 54^2$, so the frame is not truly square. The two sides meet at an obtuse angle.
b



14 **a** 11 cm **b** 2 folds **c** 2 folds

15 11.68 cm

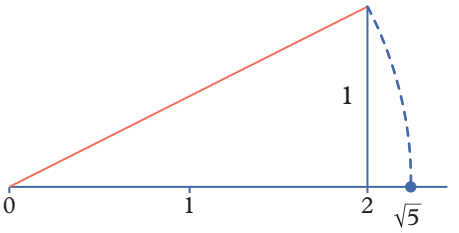
16 **a**



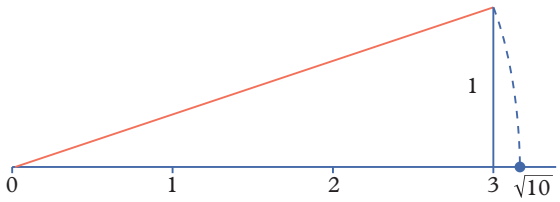
b **i** $4\sqrt{3}$ cm **ii** 6.9 cm
c $A = \frac{\sqrt{3}x^2}{4}$

17 The rope with 12 equally spaced knots forms a right-angled triangle with sides of 3 intervals, 4 intervals and 5 intervals. Pythagoras' theorem applies ($3^2 + 4^2 = 5^2$).

18 **a**



b



19 **a** $\sqrt{61}$ cm **b** 8.37 cm

20 **a** 10.63 cm **b** 7.28 mm **c** 8.85 cm **d** 8.77 cm

EX p347

CHAPTER 7 review

Mathematical literacy review

1 isosceles triangle

2 square

3 obtuse angle

4 **a** transversal **b** trapezium

5 **a** any set of three positive whole numbers that satisfies Pythagoras' theorem

b angles that add to 90° (a right angle)

6 **a** The hypotenuse is the longest side of a right-angled triangle.

b An exterior angle is the angle between a side of a shape and a line extended from the adjacent side.

Multiple choice

1 C **2** D **3** B **4** D **5** E

6 C **7** C **8** D **9** C **10** A

Short answer

1 **a** $a = 138°$, $b = 42°$, $c = 138°$

b $a = 38°$, $b = 142°$, $c = 52°$

2 **a** $j = 76°$, $k = 76°$ **b** $l = 22°$

3 **a** $x = 109°$ **b** $x = 66°$, $y = 48°$

c $x = 141°$ **d** $x = 69°$

4 **a** obtuse-angled scalene

b acute-angled isosceles

c acute-angled scalene

d acute-angled scalene

5 **a** $w = 124°$

b $a = 144°$, $b = 36°$, $c = 144°$

c $m = 129°$, $n = 129°$

d $p = 263°$

6 **a** trapezium **b** parallelogram

c kite **d** irregular quadrilateral

7 **a** right-angled

b not right-angled

8 $x = 20$ cm, $y = 36.06$ cm

9 **a** 25.46 cm **b** 1.50 m

10 **a** 24.5" **b** 47.9" **c** 75.0"

Analysis

1 **a** $x = 100°$; opposite angles in a parallelogram are equal

b 100°; vertically opposite angles with x

c Both angles are 109°.

d 6.3 cm

2 **a** $A(2, 3)$, $B(6, 0)$, $C(-1, -1)$

b $a = \sqrt{50}$, $b = 5$, $c = 5$

c The triangle is a right-angled isosceles triangle.

d $\frac{\sqrt{50}}{2}$

e $(0, 6)$

f The new triangle is an acute-angled isosceles triangle.

g 22.63

CHAPTER 8 Units of measurement

EX p357

8A Length and perimeter

1 **a** 5800 cm **b** 2700 m **c** 5.21 m

d 0.398 km **e** 12 mm **f** 3.987 km

g 42.3 cm **h** 0.7 cm

2 **a** 5340 mm **b** 395 000 cm **c** 235 m

d 145 km **e** 0.023 68 km **f** 0.287 m

g 193 400 cm **h** 910 000 mm

3 **a** 3.77 m **b** 15.71 cm **c** 144.51 mm

d 5.03 cm **e** 571.77 cm **f** 8.17 mm

g 5.50 m **h** 8.17 cm **i** 257.61 km

4 **a** 11.00 cm **b** 21.82 cm **c** 5.59 cm

5 **a** 20 cm **b** 8 m

c 142 mm (14.2 cm) **d** 168 mm (16.8 cm)

e 0.124 m **f** 137 cm (1.37 m)

6 **a** 14 cm (or 140 mm) **b** 34 m

c 60 cm **d** 56 cm

e 18.4 m **f** 14.6 m

7 **a** 16 cm **b** 60 mm **c** 50 mm **d** 60 cm

8 **a** 65 cm **b** 43.6 cm (436 mm)

c 47 cm **d** 112 mm

e 31 m **f** 28.4 cm

9 **a** 7.5 cm **b** 15 m **c** 14 cm **d** 61.6 cm

10 **a** 12.88 cm **b** 11.68 mm **c** 13.28 cm **d** 22.42 m

e 18.12 cm **f** 30.85 mm

11 **a** $10\frac{1}{5}$ m **b** $5\frac{1}{6}$ cm **c** $2\frac{4}{7}$ cm

d $4\frac{5}{6}$ m or $\frac{29}{6}$ m **e** $\frac{\pi}{4} + \frac{7}{3}$ cm **f** $\frac{16 + 4\pi}{3}$ mm

12 480 cm = 4.8 m

13 25.2 km

14 192 cm, the shape is made up of 32 edges of the octagons, each with a side length of 6 cm.

15 9 cm, the side length of a square is a quarter of the perimeter.

16 1.15 m

17 1.50 m

18 **a** 36.4 m

b length: 0.9 m, width 0.5 m

c length: 7.2%, width: 10.6%

d **i** length **ii** width

e 39.2 m

f 7.7%

19 $x = 2$, $y = 3.5$, $P = 76$ cm

20 **a** $4l$ **b** $2w + 2l = 2(w + l)$
c $2a + 2b = 2(a + b)$ **d** $4l$
e $a + b + s + t$ **f** $2a + 2b = 2(a + b)$
g $a + b + c + d$ **h** $2p + q + r$

21 **a** 50 cm
b The corners of the frame are where the frame is longer than the width and length of the photo. Because of this, the sides of the frame must be longer than the sides of the photo.
c 54 cm
d 58 cm; the perimeter is longer because the corner lengths are counted twice, horizontally and vertically.
e **i** length of wood = 58 cm, perimeter = 66 cm
ii length of wood = 62 cm, perimeter = 74 cm
iii length of wood = 74 cm, perimeter = 78 cm
iv length of wood = 94 cm, perimeter = 98 cm

22 20 × 4 + 15 × 4 + 60 = 200 cm

23 1 cm × 14 cm, 2 cm × 13 cm, 3 cm × 12 cm, 4 cm × 11 cm, 5 cm × 10 cm, 6 cm × 9 cm, 7 cm × 8 cm

24 51.52 m; there are 56 pages, each of which will require 92 cm of ribbon.

25 **a** Some possible answers are: 6 m × 8 m, 10 m × 4 m, 10.2 m × 3.8 m.
b 8 m longer than garden perimeter: 36 m
c **i** 44 m **ii** 52 m
d Multiply the distance by 8 and add the answer to the initial perimeter (28 m).
e The rule works for all cases. Start with the initial rectangle or the inside rectangle. Once you've found this perimeter, the outer perimeter can be found by adding 8 × the distance to the fence to the inside perimeter.

26 **a** 1 foot = 30.48 cm
b 1 yard = 0.9144 m
c 1 mile = 1.609 344 km
d 42.194 988 km

EX 8B Area of triangles and rectangles

p365

1 **a** 70 mm² **b** 1500 mm² **c** 10 200 cm²
d 29.4 mm² **e** 9900 cm² **f** 161 mm²
g 540 mm² **h** 4 cm² **i** 20 cm²

2 **a** 22.5 cm² **b** 0.96 m² **c** 10.8 cm²
d 367.5 cm² **e** 0.4725 m² **f** 3.382 m²

3 **a** 1400 mm² **b** 20.4 cm² **c** 150 mm²
d 51.84 mm² **e** 12.5 cm² **f** 16.8 cm²
g 361 cm² **h** 462 mm² **i** 1715 cm²

4 **a** 14 cm² **b** 14 cm² **c** 900 mm²
d 9 m² **e** 90 cm² **f** 15 cm²

5 **a** 30 000 cm² **b** 1000 mm² **c** 65 000 cm²
d 1970 mm² **e** 1 600 000 m² **f** 750 000 m²

6 **a** 2.5 m² **b** 7500 cm² **c** 4 cm²
d 0.125 km² **e** 0.0675 m² **f** 500 km²

7 **a** 0.15 m² **b** 20 000 000 cm²
c 1 500 000 mm² **d** 0.000 037 9 km²

8 8704 cm²

9 They used the slant height (68.5 cm) as the height of each trianglular section rather than the perpendicular height (68 cm).

10 **a** 24 cm² and 4 cm²; 28 cm²
b 8 cm² and 20 cm²; 28 cm²

11 **a** 92 mm² **b** 35 cm² **c** 199 m²
d 66 cm² **e** 96 cm² **f** 72 cm²

12 The area units should give numerical values in the units, tens and hundreds which people can easily understand.
a m² **b** mm² **c** km² **d** cm² **e** m² **f** cm²

13 11 cm² = (11 × 10²) mm² = 1100 mm²; Ahmed forgot to square the conversion factor.

14 Alex divided by the conversion factor instead of the square of the conversion factor.

40 000 cm² = (40 000 cm ÷ 100²) m²
= 4 m²

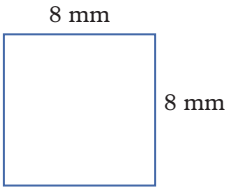
15 **a** 100 **b** 4000 m²

16 **a** **i** 3200 kg/hectare **ii** 0.32 kg/m²
iii 320 g/m² **iv** 0.032 g/cm²
b Grams per square metre will make the most sense for most people, as they are units people are more familiar with. For farmers, tonnes per hectare makes the most sense, as a farmer can have thousands of hectares of land.
c $3 360 000

17 Width is 5 cm. There is only one answer because only one number gives 20 when multiplied by 4.

18 Possible answers include: 40 cm² (10 cm × 4 cm), 48 cm² (6 cm × 8 cm), 24 cm² (12 cm × 2 cm).

19 A square has equal side lengths and its area is the square of its side length. There is only one positive number that when multiplied by itself gives 64: 8 × 8 = 64, so the dimensions must be 8 mm × 8 mm. The perimeter is 32 mm.



20 **a** 40 cm **b** 1400 cm²

21 **a** Possible answers include: 8 m × 4 m (32 m²), 7 m × 5 m (35 m²), 6 m × 6 m (36 m²), 2.4 m × 9.6 m (23.04 m²)
b 36 m²; the maximum area corresponds to a square with side lengths 6 m.
c Possible answers include: 2 m × 20 m (20 m side on shed); 40 m², 4 m × 16 m (16 m side on shed); 64 m², 5 m × 14 m (14 m side on shed); 70 m².

d 72 m² is the maximum area for a rectangle with dimensions 12 m × 6 m. Its shape is a half square. Its area is twice the area of the square in part **b**.

EX p372

8C Area of quadrilaterals

1 **a** 4.4 and 7.2 **b** 8.6 and 5.5 **c** 4.2, 4.8 and 6.2
d 4.2 and 5.2 **e** 3.2 and 4.4 **f** 2.1, 3.2, and 5.2

2 **a** 24 cm² **b** 63 mm² **c** 168 cm²

3 **a** 12 mm² **b** 20 cm² **c** 22.5 mm²

4 **a** 14 cm² **b** 48 mm² **c** 6 m²

5 **a** 124 mm² **b** 35 cm² **c** 138 m²

6 **a** **i** rhombus **ii** $A = bh$ **iii** 42 cm²
b **i** kite **ii** $A = \frac{1}{2}xy$ **iii** 3 cm²
c **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 15.96 m²
d **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 18 cm²
e **i** parallelogram **ii** $A = bh$ **iii** 112 mm²
f **i** trapezium **ii** $A = \frac{1}{2}(a + b)h$
iii 21 mm²

7 **a** 10.5 cm² **b** 175 mm²
c 99 cm² (or 9900 mm²) **d** 90 cm²
e 44 cm² **f** 56 mm²

8 **a** 8.925 cm² **b** 9.18 mm²

9 **a** parallelogram **b** 1806 m²
c No, as the windows on the far left or far right sides are not similar to others away from the side edges.
d For the rectangular windows, $A = lw$. For the trapezium windows, $A = \frac{1}{2}(a + b)h$. For the triangular windows, $A = \frac{1}{2}bh$.

10 **a** 9 cm **b** 4.25 cm **c** 5 cm
d 19 mm (1.9 cm)

11 **a** 3087.5 cm² **b** 20 cm

12 $731.25

13 **a** No, there is not enough information to confirm it is a trapezium, as we do not know the two sides that appear horizontal are parallel.
b No, there is enough information to confirm it is a parallelogram, but not enough information to determine its area as the perpendicular height is not known.
c Yes, there is enough information to confirm it is a kite, and the length of the perpendicular diagonals are both known.
d Yes, there is enough information to confirm it is a rhombus and both diagonal lengths are known (which are perpendicular since it is a rhombus).
e Yes, there is enough information to confirm it is a trapezium as the two vertical sides are both at right angles to the 2 cm length, which acts as a transversal indicating the vertical sides are parallel.
f No, there is not enough information to confirm it is a kite even though the diagonals are perpendicular.

14 **a** **i** parallelogram, trapezium
ii $8 \times 2 = 16$ m², $\frac{1}{2}(8 + 8) \times 2 = 16$ m²
b **i** parallelogram, kite, rhombus, trapezium
ii $4 \times 4 = 16$ m², $\frac{1}{2} \times 5.66 \times 5.66 \approx 16.0$ m²
(kite and rhombus), $\frac{1}{2}(4 + 4) \times 4 = 16$ m²
c **i** parallelogram, kite, rhombus, trapezium
ii $4 \times 2.29 = 9.2$ m², $\frac{1}{2} \times 2.41 \times 7.62 = 9.2$ m²
(kite and rhombus), $\frac{1}{2}(4 + 4) \times 2.29 = 9.2$ m²
d **i** parallelogram, trapezium
ii $3 \times 7 = 21$ m², $\frac{1}{2}(7 + 7) \times 3 = 21$ m²

15 1 cm × 96 cm, 2 cm × 48 cm, 3 cm × 32 cm, 4 cm × 24 cm, 6 cm × 16 cm, 8 cm × 12 cm

16 **a** 81 cm² **b** 24 cm **c** 4 cm

17 **a** The trapezium has been cut into two triangles, one with base length a, the other with base length b, and both with height h. The area of the trapezium is the sum of the area of the two triangles $\frac{1}{2}ah + \frac{1}{2}bh$, which is equal to $\frac{1}{2}(a + b)h$.
b Part of the trapezium has been cut off and rotated to form a triangle with a base length of $(a + b)$ and height h. The area of this triangle is equal to the area of the trapezium, which is $\frac{1}{2}(a + b)h$.
c The trapezium has been duplicated and rotated to form a parallelogram with a base length of $(a + b)$ and height h. The area of the trapezium is half of the area of the parallelogram, which is $\frac{1}{2}(a + b)h$.
d The top half of the trapezium has been cut off and rotated to form a parallelogram with a base length of $(a + b)$ and height $\frac{h}{2}$. The area of the trapezium is equal to the area of the parallelogram, which is $\frac{1}{2}(a + b)h$.
e Two triangles have been cut off and rotated to form a rectangle with a length of the average of the parallel sides, $\frac{a + b}{2}$, and width h. The area of the trapezium is equal to the area of the rectangle, which is $\frac{1}{2}(a + b)h$.

18 **a** **i** 16 cm² **ii** 6.75 cm²
b 57.8125% decrease

EX p377

Checkpoint

1 **a** 0.45 m **b** 5900 m
c 3.51 cm² **d** 2000 cm²

2 **a** 12.6 cm **b** 18.4 m **c** 15.32 m **d** 18.4 cm

3 **a** 17.93 cm **b** 21.99 m

4 **a** 97.39 m **b** 43.98 cm

5 **a** 1.47 cm **b** 28.36 cm

6 **a** 5.5 m **b** 7.2 m **c** 9 cm **d** 3.1 cm

7 **a** 9 m² **b** 9 m² **c** 15 cm² **d** 7.5 mm²

8 **a** **i** 1820 cm = 18.2 m
ii 200 081 cm² = 20.0081 m²
b **i** 264.4 mm = 26.44 cm
ii 3000 mm² = 30 cm²

9 a 14 m^2 b 6 cm^2 c 15 cm^2
d 12 mm^2 e 7.5 m^2 f 9.10 m^2

EX p381

8D Area of a circle

1 a 706.86 cm^2 b 176.71 mm^2 c 254.47 mm^2
d 78.54 cm^2 e 38.48 cm^2 f 33.18 m^2
2 a 5.0625π cm^2 b 77.44π cm^2 c 49π cm^2
3 a area = 60.82 cm^2, perimeter = 27.65 cm
b area = 44.18 m^2, perimeter = 23.56 m
c area = 176.71 mm^2, perimeter = 47.12 mm
d area = 2642.08 mm^2, perimeter = 182.21 mm
4 a 47.52 cm^2 b 8.04 m^2 c 38.48 cm^2
d 3848.84 cm^2 e 264.74 mm^2 f 2035.75 cm^2
5 a 60° b 360° c $\frac{60}{360}\left(\text{or } \frac{1}{6}\right)$
d 4 cm e 50.27 cm^2 f 8.38 cm^2
6 a 19.24 cm^2 b 126.71 cm^2 c 230.91 mm^2
7 a 1.95 cm b 2.82 mm
c 5.64 cm d 4.65 cm
8 a 10.09 cm b 7.90 cm
c 5.05 mm d 13.11 cm
9 2375.83 cm^2
10 176.71 m^2
11 A medium pizza has an area of 113.10 $inch^2$, so two medium pizzas have an area of 226.19 $inch^2$, whereas a large pizza has an area of 254.47 $inch^2$ which is greater than the area of two medium pizzas.
12 a $\frac{1}{4}\pi d^2 = \frac{1}{4}\pi(2r)^2 = \frac{1}{4}\pi \times 4r^2 = \pi r^2$
b $\frac{1}{2}Cr = \frac{1}{2}(2\pi r)r = \pi r^2$
13 a 50.27 m^2 b 28.27 cm^2 c 127.32 mm^2
d 198.94 cm^2 e 12.62 cm f 33.63 mm
g 15.85 cm
14 216 cm
15 a 87.96 cm^2 b 75.40 cm^2 c 138.23 cm^2
16 452.39 cm^2
17 a 314.16 cm^2 b 2 cm c 706.86 cm^2
d 392.70 cm^2
18 285.88 mm^2
19 a 14.28 cm^2 b 96.37 cm^2 c 48.57 mm^2
d 66.19 cm^2 e 144.31 mm^2 f 81.73 cm^2
20 To find the length of the radius, you first need to divide A by π. Unless the value of A contains π, the result of this calculation will be irrational; therefore, the exact length of the radius cannot be found.
a $\sqrt{\frac{16}{\pi}} \approx 2.26$ mm b $\sqrt{16} = 4$ mm
21 a i 14 cm ii 14π cm
b i 4 m ii 16π m^2
22 a 28.54 cm^2 b 61.42 cm^2

EX p389

8E Volume and capacity

1 a 90 cm^3 b 90 cm^3 c 324 mm^3
2 a 346.5 cm^3 b 48 cm^3 c 225 cm^3
3 a 320 cm^3 b 17.5 cm^3 c 24.75 cm^3
4 a 424.12 cm^3 b 942.48 mm^3 c 39.27 cm^3
d 904.78 cm^3 e 367.57 cm^3 f 2290.22 cm^3
5 a 9000 mm^3 b 18 000 000 cm^3
c 16 cm^3 d 0.00175 km^3
e 4 200 000 000 m^3 f 4 m^3
g 4 560 000 cm^3 h 0.845 m^3
i 150 000 000 m^3 j 6100 mm^3
k 0.028 cm^3 l 1700 km^3
6 a 8000 mL b 15 000 L c 0.07 L
d 6400 kL e 2.5 kL f 3 000 000 mL
7 a 2513.27 cm^3 b 59.38 m^3 c 133.93 m^3
d 17 359.66 cm^3
8 a 3 L b 59 380 L c 133 930 L
d 18 L
9 a 1 570 796 L b 1357 L c 1 L (618 mL)
10 5.2 m^3
11 a 2944.62 cm^3 b 123
12 4 cm
13 Multiplication is commutative, so it doesn't matter in which order you multiply the length, width and height; therefore, it doesn't matter which of the measurements you designate as the length, width or height.
14 a i Priya multiplied by the power (3) instead of mutliplying by 100 three times.
ii 7 m^3 = (7×100^3) cm^3 = 7 000 000 cm^3
b i Priya used the incorrect conversion factor.
ii 3000 mm^3 = $(3000 \div 10^3)$ cm^3 = 3 cm^3
15 a 1.33 cm b 2.82 cm
16 a 462 cm^3 b 600 g
17 a 44 178.65 mm^3 or 44.18 cm^3 b 679 coins
18 The cylinder has size and capacity, it's just small and rounds to 0 when measured in litres. Saying that its capacity is 0 L is misleading.
19 Second glass holds 55.37 mL more.
20 6.77 mL
21 a mL b kL c L
22 a 82.5 cm^3 b 82 500 mm^3
c $25 \times 30 \times 110 = 82\,500$ mm^3
23 a 220 cm^3 b 398.61 cm^3
c 48 cm^3 d 152 cm^3
24 a 102 375 L b 48 048 L
25 a 348 cm^3 b 329.87 cm^3 c 730 cm^3
26 451.25 cm^3 or 451.25 mL
27 In the calculation for the volume of a cylinder the radius is squared, so if the radius is doubled, the volume will be multiplied by 4 ($2^2 = 4$). The height is only used once in the calculation for the volume, so if the height is doubled, the volume will also be doubled.

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28 small block: 10 500 cm^3, large block: 14 250 cm^3

29 **a** 385.65 m^3 **b** 66.74 mm^3

EX p395

8F Time

1 **a** 180 minutes **b** 300 minutes **c** 600 minutes
d 2 minutes **e** 10 minutes **f** 100 minutes

2 **a** 600 seconds **b** 300 seconds **c** 3600 seconds
d 2 hours **e** 30 hours **f** 4 hours

3 **a** 0.5 or $\frac{1}{2}$ hour **b** 0.75 or $\frac{3}{4}$ hour
c $\frac{1}{3}$ hour **d** 0.25 or $\frac{1}{4}$ minute
e 0.1 or $\frac{1}{10}$ minute **f** 0.8 or $\frac{4}{5}$ minute

4 **a** 10 800 seconds **b** 36 000 seconds
c 21 600 seconds **d** 5400 seconds
e 4 hours **f** 2.5 hours

5 **a** 1440 minutes **b** 168 hours
c 86 400 seconds **d** 8784 hours
e 604 800 seconds **f** 720 hours

6 **a** 0230 **b** 1615 **c** 1348 **d** 2311

7 **a** 7:25 pm **b** 12:00 pm **c** 10:18 pm **d** 9:58 am

8 **a** 5 hours **b** 15 hours
c 3 hours **d** 5 hours
e 9 hours **f** 5 hours 30 minutes
g 2 hours 18 minutes **h** 5 hours 13 minutes
i 7 hours 54 minutes **j** 19 hours 27 minutes
k 3 hours 3 minutes **l** 7 hours 45 minutes

9 **a** 5:45 pm **b** 1:45 pm
c 11:35 am **d** 9:35 am
e 2315 **f** 0115 (next day)

10 **a** **i** 11:49 am **ii** 1:19 pm **iii** 1:49 pm
b **i** 2:28 am **ii** 2:58 am **iii** 12:58 am
c **i** 1617 **ii** 1417 **iii** 1547
d **i** 0010 **ii** 0040 **iii** 2240

11 **a** $\frac{1}{4}$ hour **b** 0.25 hour
c 1.25 hours = 1 hour 15 minutes. Decimals can be written as fractions out of 100, but minutes are written as fractions of 60 (of an hour).

12 **a** 11 hours **b** $\frac{1}{2}$ hour **c** 10.5 hours
d 9.5 hours, 10.75 hours, 10.5 hours
e 41 hours 15 minutes

13 **a** **i** Didn't change the hour after adding 15 mins to 2:45.
ii 3 hours 40 minutes
b **i** Didn't consider am/pm difference.
ii Duration = 15 h 40 min
c **i** Miscounted from 3:25 pm to 6:25 am.
ii Duration = 15 h 40 min

14 3:05 pm

15 **a** 4012 m **b** 2 hours 30 minutes **c** 0.45 m/s

16 **a** 14 hours after 8 pm AEST is 10 am AEST, not 10 am AWST, which is the time zone the clock is likely set to.
b 16 hours **c** 1280 km **d** 14%

17 **a** D **b** A

18 **a** 20 hours 25 minutes **b** 48 hours 45 minutes
c 65 hours 30 minutes **d** 8762 hours 57 minutes

19 **a** **i** 1 hour **ii** 2.5 hours **iii** 1.5 hours
iv 3 hours
b **i** 4:30 pm **ii** 6 pm **iii** 6:30 pm
iv 6:30 pm
c **i** 3:30 pm **ii** 5 pm **iii** 6:30 pm
iv 5:30 pm
d In winter, Queensland is ahead, but in summer, South Australia is ahead.
e **i** 2:30 pm **ii** 2:30 am **iii** 1:30 pm
iv 8 am **v** 5:15 pm **vi** 10 am

20 **a** 9 hours **b** 2045
c 4 hours 30 minutes **d** 2330

EX p401

CHAPTER 8 review

Mathematical literacy review

1 **a** Australian Central Standard Time
b Australian Eastern Standard Time
c Australian Western Standard Time

2 Sample answer: A.M. stands for the Latin ante meridiem, translating to 'before midday'. P.M. stands for post meridiem or 'after midday'.

3 **a** semicircle **b** quadrant
c rhombus **d** parallelogram
e sector **f** quadrilateral

Multiple choice

1 B **2** D **3** E **4** C **5** D **6** D
7 D **8** D **9** C **10** D **11** D **12** A
13 C **14** A

Short answer

1 **a** 22.6 cm (or 226 mm) **b** 58 mm **c** 33.6 cm

2 **a** 31.42 cm **b** 50.27 mm

3 **a** 48 cm^2 **b** 58.5 cm^2 **c** 120 cm^2
d 5.4 cm^2 (540 mm^2)

4 **a** 50 000 cm^2 **b** 0.5 cm^2

5 **a** 42 cm^2 **b** 88 cm^2 **c** 80 cm^2 **d** 125 cm^2

6 **a** 78.54 cm^2 **b** 201.06 mm^2

7 **a** 3500 mm^3 **b** 0.0004 m^3 **c** 35 L **d** 124 mL

8 **a** 640 cm^3 **b** 420 cm^3 **c** 144 cm^3

9 **a** 4000 kL **b** 12.73 m

10 **a** 3 hours 45 minutes **b** 8 hours 32 minutes
c 8 hours 34 minutes **d** 6 hours 27 minutes

11 **a** 5:23 am **b** 3:19 pm
c 11:10 am **d** 12:50 pm

Analysis

1 **a** 6 m^2 **b** 10 m **c** 20
d 825 cm^2 **e** 20.73 cm **f** 205.27 cm^2
g 6 m^3 **h** 40 000

i The balls will not take up the entire space as there will be space between them. The theoretical calculation does not take into account this space.

2 **a** 20 cm **b** 240 cm^3 **c** 10.2 cm
d 0.2 cm **e** 18 cm **f** 9.36 L
g 1.87 minutes

3 **a** 91.5 m **b** 549 m **c** 20.33 m
d 122 m
e 488 m, four times the distance of one shuttle run (calculated in part **d**)
f four shuttle runs **g** 465.125 m^2 **h** 81 : 9604

CHAPTER 9 Statistics and probability

EX 9A Summary statistics

p412

1 **a** **i** 2.36 **ii** 2 **iii** 2 **iv** 4
b **i** 21.67 **ii** 20 **iii** 10, 30 **iv** 30
c **i** 15.88 **ii** 16 **iii** 17 **iv** 5
d **i** 1.12 **ii** 1 **iii** 1 **iv** 4
e **i** 24.33 **ii** 25 **iii** 30 **iv** 15
f **i** 3.09 **ii** 2.5 **iii** 2 **iv** 18

2 **a** 30 people **b** mode = 5, range = 9
c

Hours	Frequency
1	3
2	4
3	2
4	6
5	8
6	4
7	2
8	0
9	0
10	1
Total	30

d mean = 4.3, median = 4.5

3 **a** mean = 4.7
b median = 5
c mode = 4
d range = 5

4 **a** four bedrooms **b** 4
c

Number of bedrooms	Frequency
1	2
2	9
3	11
4	14
5	6
Total	42

d mean = 3.31, median = 3

5 **a** mean = 2.3, mode = 2, median = 2, range = 5
b mean = 16.15, mode = 16, median = 16, range = 6
c mean = 1.33, mode = 1, median = 1, range = 4

6 **a** mean = 55.43, median = 59, mode = 42, range = 42
b mean = 2.70, median = 2.3, mode = 1.8, range = 4
c mean = 280.38, median = 250, mode = 310, range = 670

7 **a** mean **b** range **c** mode
d range **e** mode **f** median

8 mean = 4.21, median = 4, mode = 3, range = 9

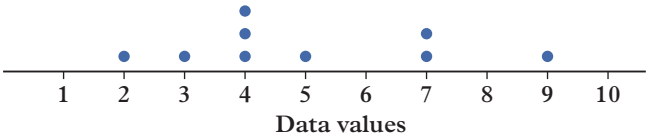
9 Categorical data does not contain any numerical scores. You can only find the most popular/common category, which is the mode.

10 **a** III **b** I **c** II

11 **a** 17 (unique)
b 9 (unique)
c Any number greater than or equal to 13 (not unique)
d 10 (unique)
e 9 or 17 (not unique)
f Any multiple of 9 (not unique)

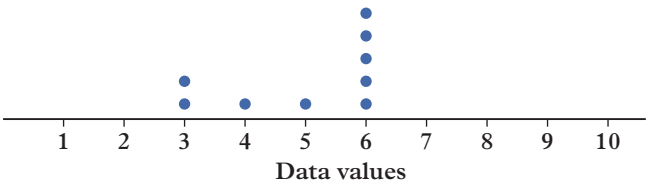
12 **a** 16th row, 7th column
b 8th row, 19th column

13 **a** Answers will vary.



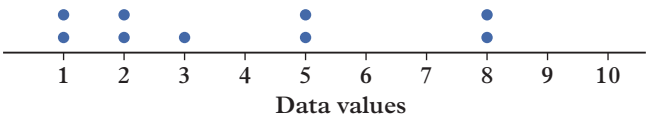
The lowest value must be 2, the highest value must be 9, and the other dots can be anywhere from 2 to 9.

b Answers will vary.



There must be at least five dots on 6 and all other dots must be below 6.

c Answers will vary.



There must be four values with exactly two dots each and one value with exactly one dot.

14 **a** Only one person claimed 18 hours pain free. Most of the claims are far less than 18 hours but, because one person said 18 hours, the claim of 'up to 18 hours' has been made.
b mean = 3, median = 2, mode = 2
c mean = 3, median = 2, mode = 2
d mean

e But still uses the outlier but it still uses the outlier in the calculation. However, the median or the mode would be a better representation of the data than the mean.

15 a 82.4% **b** 62.4% **c** 88%

16 a i 2.56 **ii** 26.37 **iii** 4.73

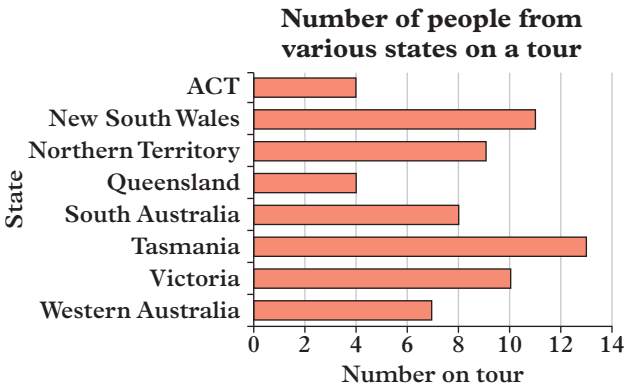
b The second set of data has a large spread of data from the mean (standard deviation = 26.37), but the first and third data sets have a small spread of data from the mean (standard deviations = 2.56 and 4.73, respectively).

EX p420 9B Constructing graphs

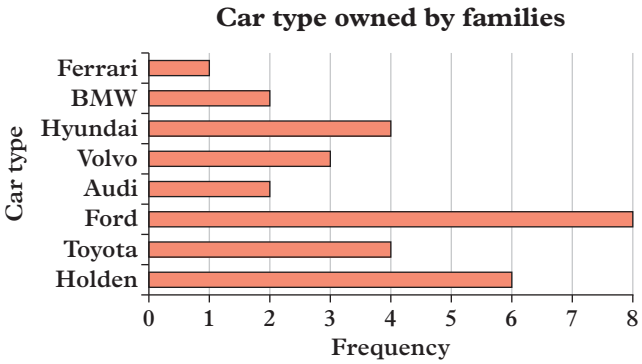
1 a 5 **b** 6 **c** 27

d

2

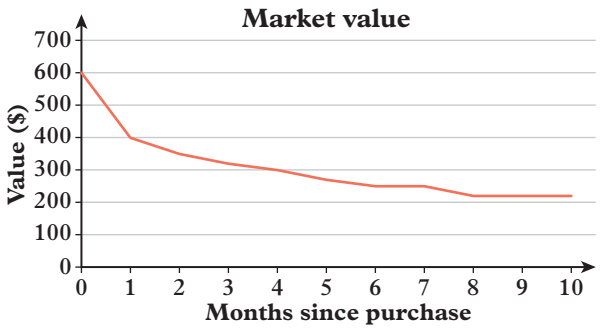


3 a

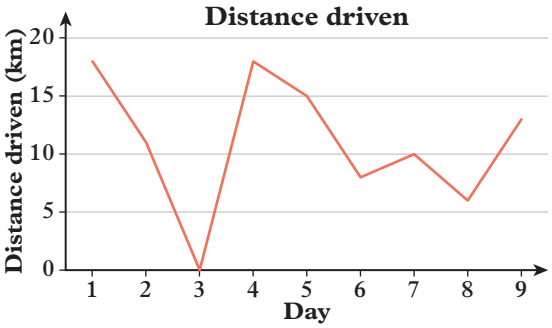


b Ford **c** Ferrari

4



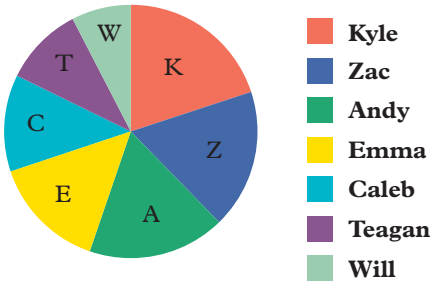
5 a



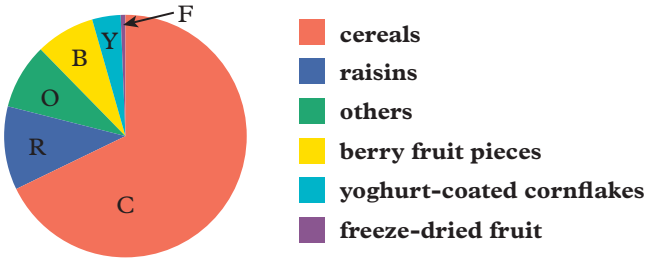
b day 1 and day 4

c 5

6 **Votes for candidate**

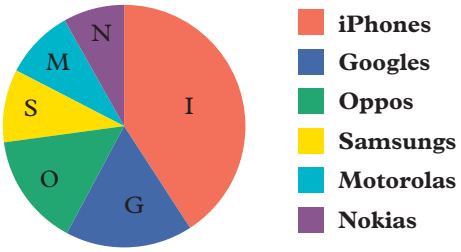


7 **Muesli ingredients**

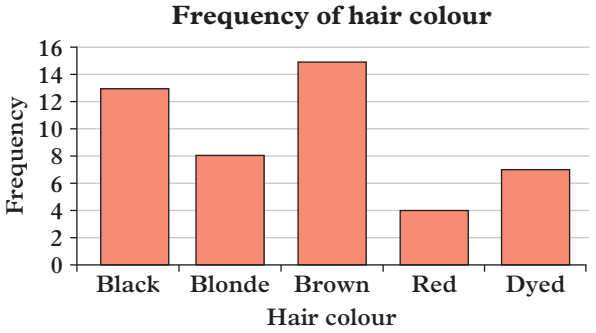


8 a 8%

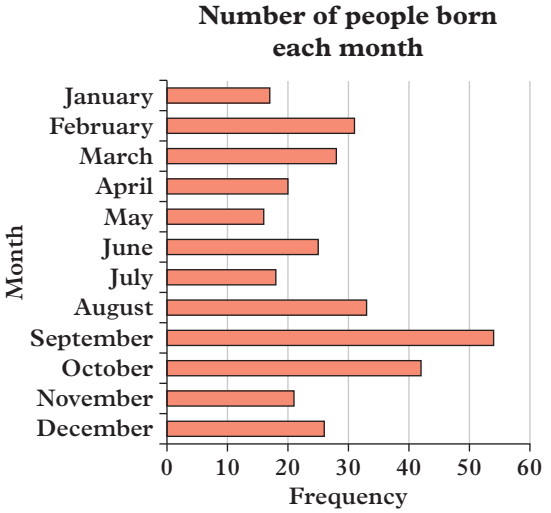
b **Mobile phone brands owned by year 8 students**



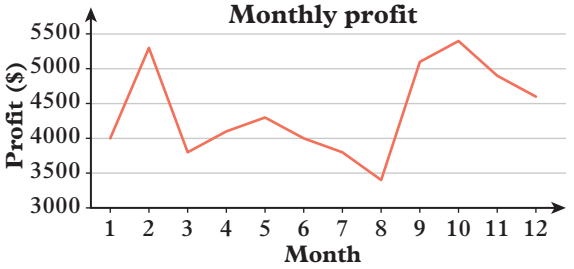
9 Sample answer: a column graph is appropriate because it makes the data easy to interpret. A bar graph could also be used, but there are too many data points for a dot plot.



10 Sample answer: a bar chart is appropriate as all the categories can be easily read.



11 a



b month 10

c month 8

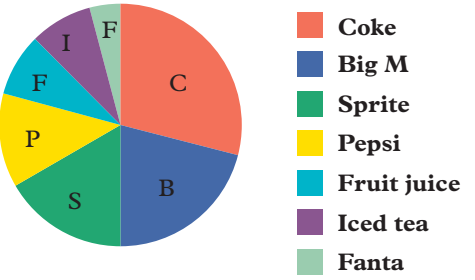
d month 9, $1700

e month 3, $1500

12 a

Favourite soft drink	Frequency
Coke	7
Pepsi	3
Sprite	4
Big M	5
Fanta	1
Fruit juice	2
Iced tea	2

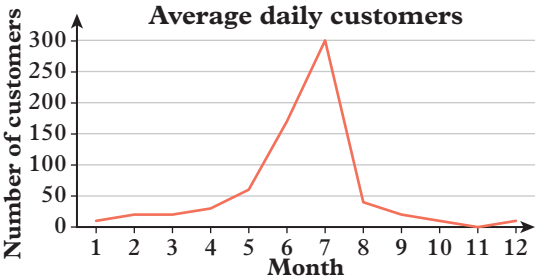
b **Favourite soft drink**



c Coke

d Fanta

13 a



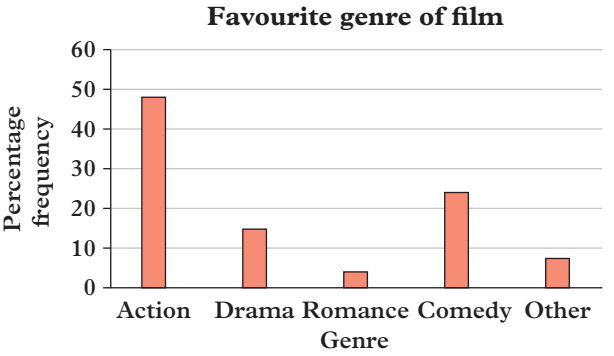
b Month 7. It is the most popular month, and it is followed by a large drop off, which makes sense because January would not be busy.

c No. The data is rounded to the nearest 10 customers and it is the average number of daily customers for the month, not the total number of customers in the month. The average could have been just below 5 customers per day, which would be around 150 customers total for the month.

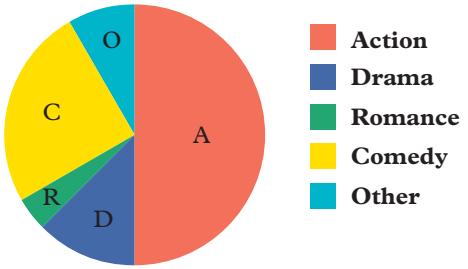
14 a

Genre	Tally	Frequency	Percentage frequency
Action	𝍸 𝍸 ||	12	48%
Drama	||||	4	16%
Romance	|	1	4%
Comedy	𝍸 |	6	24%
Other	||	2	8%

b



c **Favourite genre of film**



d Action, 48% **e** Romance, 4%

15 a

b Negative profit means the store lost money.

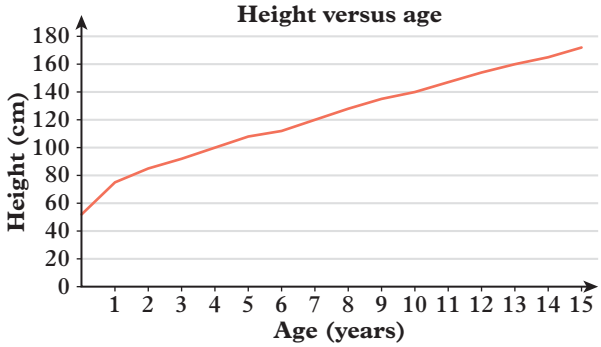
c $0 profit means the store broke even (did not make money or lose money).

d $47 000

e No. The profit is most likely associated with the time of year so it will be back down again in the middle of next year.

f Summer in the northern hemisphere occurs in the middle of the year so the months with the highest profit would be the months in the middle of the year, unlike the Geelong store, which has the highest profits at the start and end of the year.

16



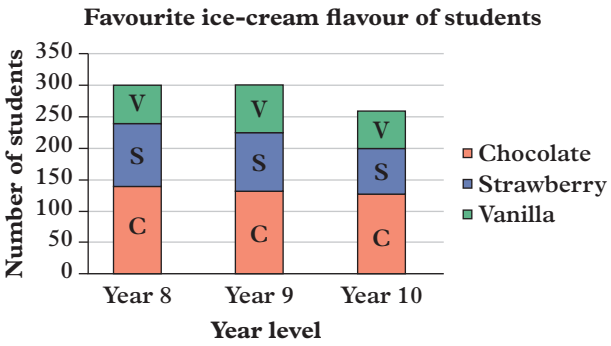
b 8 cm

c 252 cm

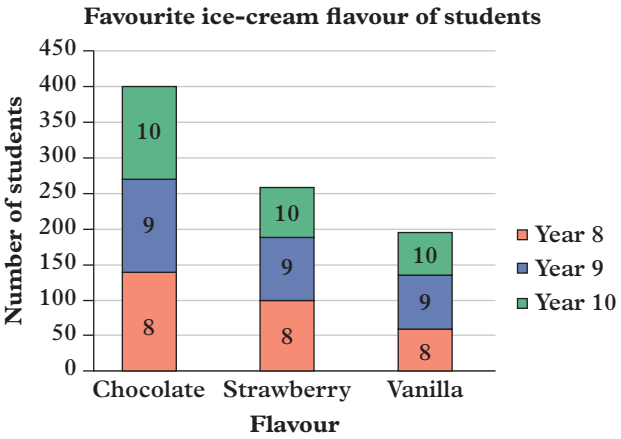
d 185 cm (answers will vary)

e Continue increasing until the age of 18 or 19, then remain constant.

17 a



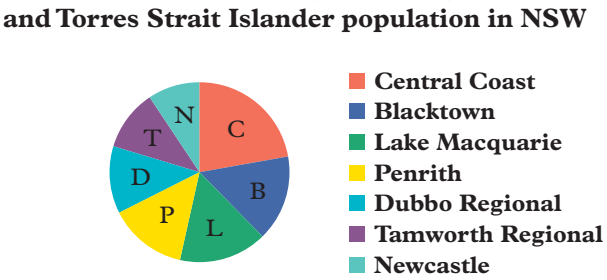
b



c The chart in part **a** with the year level on the horizontal axis

d No. Although slightly more Year 9 students preferred chocolate ice-cream than Year 10 students, there were more Year 9 students in the data.

18 a



b 2000

EX 9C Grouped data and histograms

p426

1 **a** 5 **b** 16
c 15–<20, 18 **d** 25–<30
e 14 **f** 19
g 4.5%

2 **a** 6 **b** 0–<20, 4 **c** 69

3 **a** 145 **b** 183 **c** 38

d

Class	Frequency
145–<150	1
150–<155	3
155–<160	5
160–<165	3
165–<170	6
170–<175	6
175–<180	0
180–<185	1
Total	25

4 **a**

Class	Frequency
0–<5	1
5–<10	2
10–<15	3
15–<20	3
20–<25	2
25–<30	3
30–<35	3
35–<40	1

b

Class	Frequency
10–<20	3
20–<30	3
30–<40	4
40–<50	4
50–<60	4
60–<70	2
70–<80	2

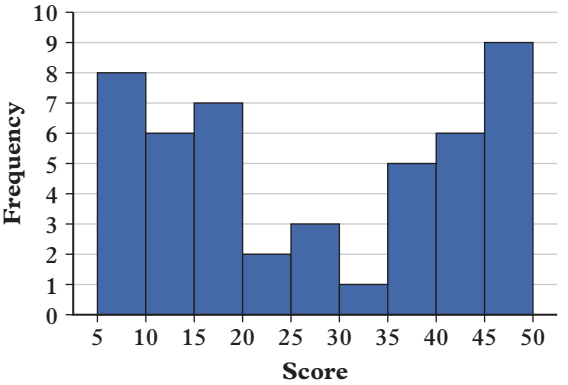
c

Class	Frequency
0–<2	6
2–<4	7
4–<6	6
6–<8	2
8–<10	2
10–<12	4

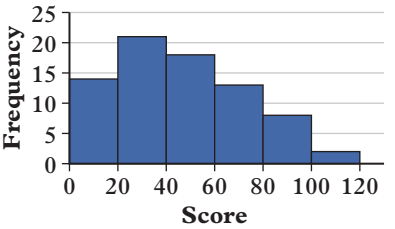
d

Class	Frequency
40–<45	3
45–<50	8
50–<55	7
55–<60	5
60–<65	3
65–<70	3
70–<75	4
75–<80	7

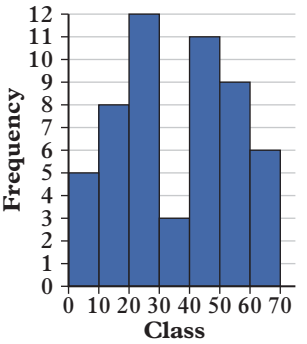
5 **a**



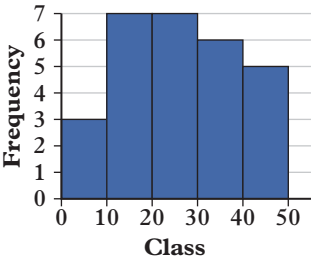
b



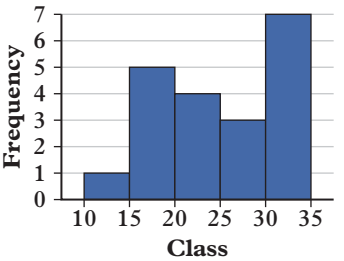
c



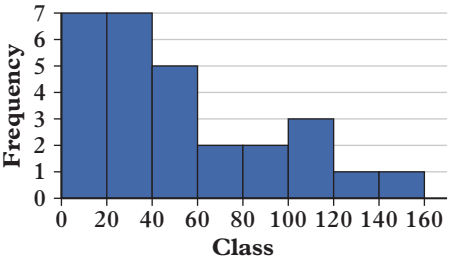
6 **a**



b



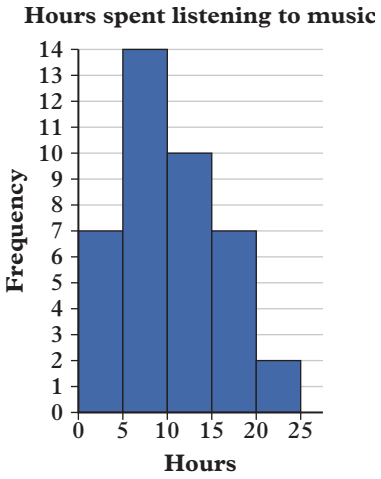
c



7 **a**

Hours	Frequency
0–<5	7
5–<10	14
10–<15	10
15–<20	7
20–<25	2

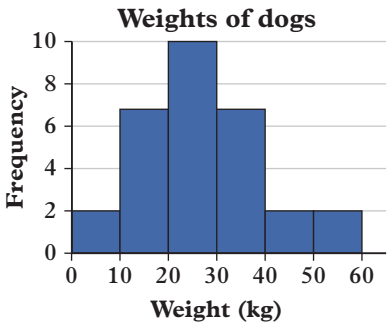
b



8 **a**

Class	Frequency
0–<5	2
5–<10	7
10–<15	10
15–<20	7
20–<25	2
25–<30	2

b



9 **a** number of Facebook friends

b 50 **c** 350–399; 28 **d** 100–149

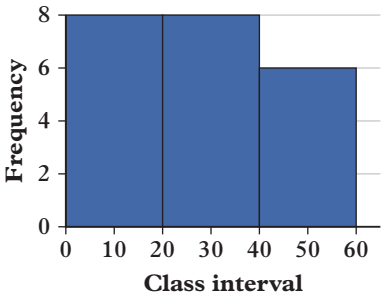
10 **a** There are too many class intervals, which makes the table long and difficult to read.

b

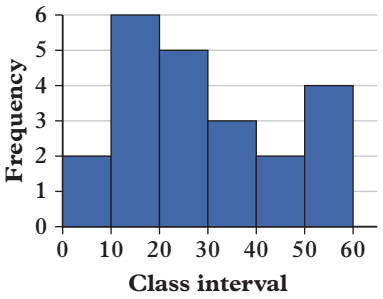
Ages of customers	Frequency
0–<10	2
10–<20	61
20–<30	44
30–<40	20
40–<50	6
≥50	6

11 Without the raw data it is not possible to determine the number in each of the smaller class intervals.

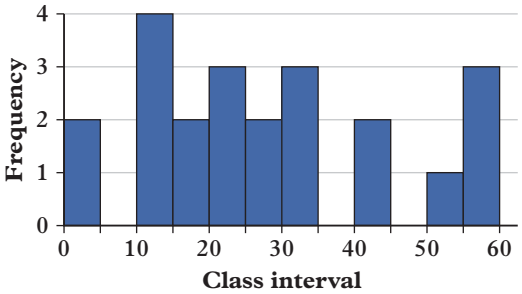
12 **a**



b

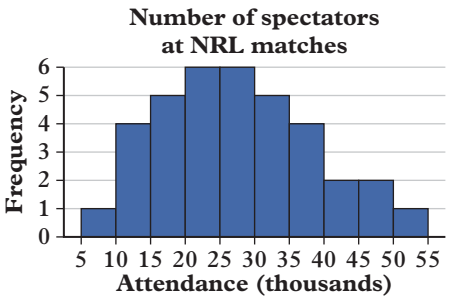


c

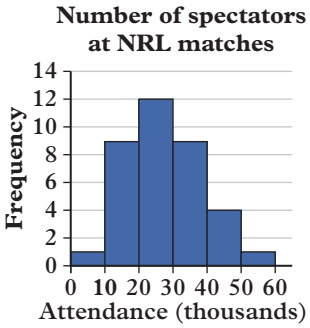


d Sample answer: The histogram in part **b** is most appropriate. The intervals in part **a** are too wide to give an accurate representation of the spread of the data. Both parts **b** and **c** display the data effectively, but the histogram in part **b** looks the neatest as it does not have intervals with frequency of 0 like part **c** does.

13 **a**



b

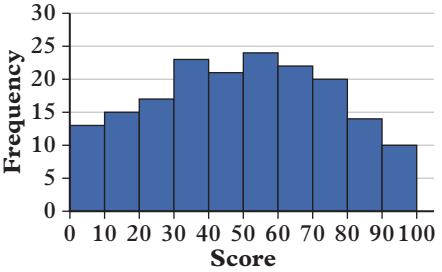


c Similar shape for each histogram (roughly symmetrical). Using smaller class intervals has resulted in smaller frequency values. That is, largest frequency is 6 compared with 12.

14 a 5

b There appears to be no pattern. Too many intervals make it harder to read.

c



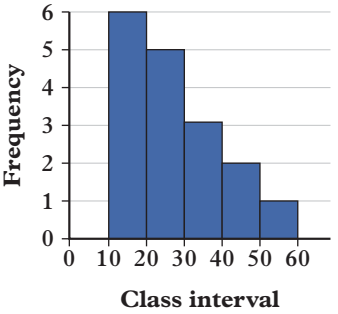
d The distribution is symmetrical, with lower frequencies at either end and higher frequencies in the middle.

15 a The stems represent the class intervals.

b 10; e.g. 10–19, 20–29 etc.

c 17

d



e In a stem-and-leaf plot, each individual piece of data can be seen.

f Histograms do not provide specific data within each class interval; they only provide the frequency of data for each class interval.

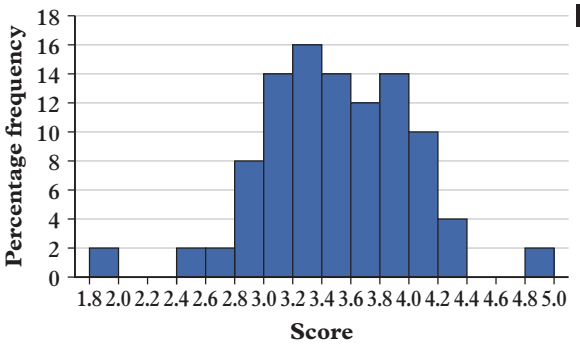
16 a The vertical axis represents percentage frequency rather than frequency.

b 16% **c** 14%

d 100%; because percentages of a whole add to 100%

e **i** 56 **ii** 196 **iii** 248

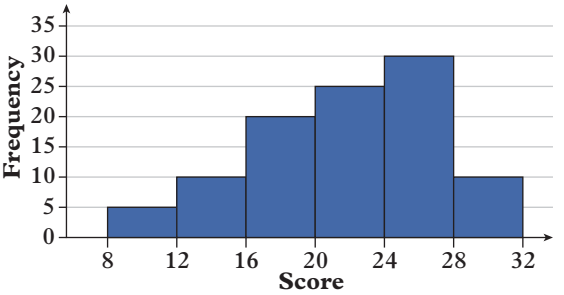
17



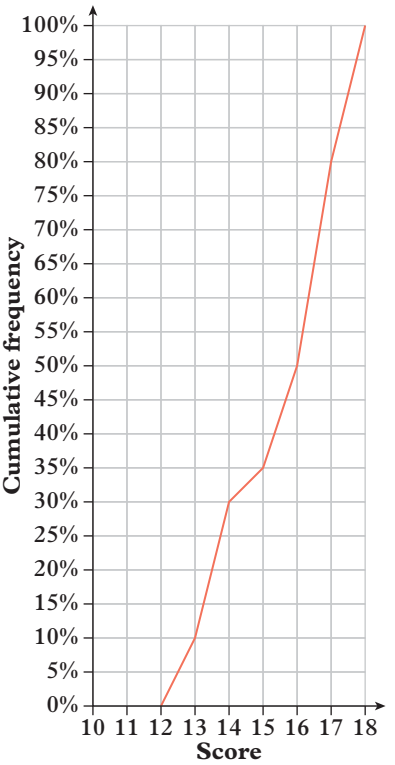
18 a

Score	Frequency	Cumulative frequency
8–<12	5	5
12–<16	10	15
16–<20	20	35
20–<24	25	60
24–<28	30	90
28–<32	10	100

b



c



EX Checkpoint

p431

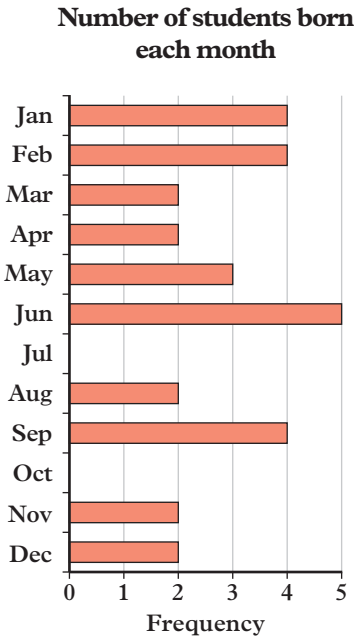
1 a **i** 5 **ii** 4 **iii** 2 **iv** 12

b **i** 1.72 **ii** 2 **iii** 2 **iv** 3

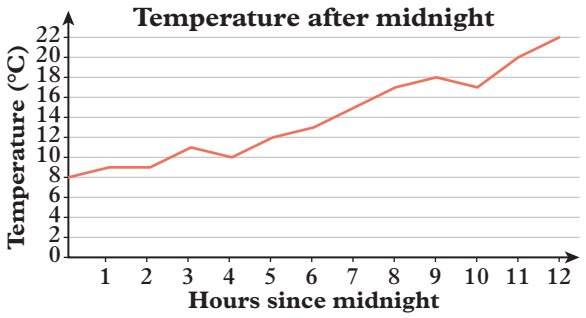
c **i** 4.5 **ii** 5 **iii** 5 **iv** 5

d **i** 33.81 **ii** 33 **iii** 45 **iv** 41

2



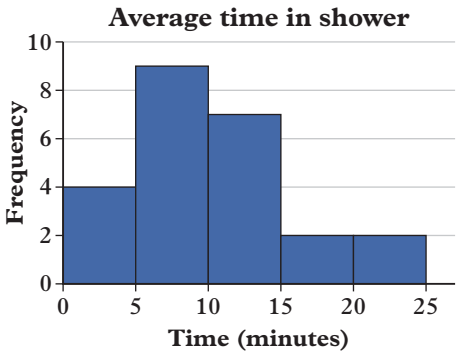
3



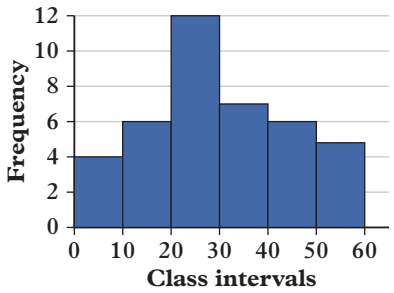
4 a

Class	Frequency
0–<5	4
5–<10	9
10–<15	7
15–<20	2
20–<25	2

b



5



6 a 90–99 b 6 c 15

EX 9D Describing data

p434

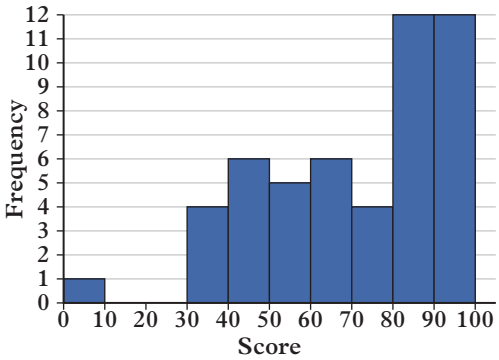
1 a i no outlier ii no outlier iii one outlier
iv no outlier v one outlier vi no outlier
b i bimodal ii unimodal iii unimodal
iv unimodal v unimodal vi multimodal
c ii negatively skewed iii positively skewed
iv symmetric v negatively skewed

2 a symmetric b positively skewed
c negatively skewed with an outlier

3 a symmetric, mean or median
b negatively skewed with an outlier, median
c bimodal, mode of each peak
d positively skewed, median

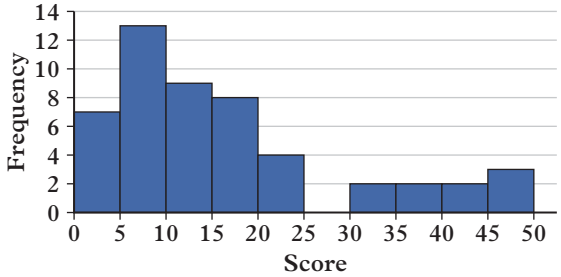
4 i mode of each peak ii median iii median
iv mean or median v median vi mean

5 a i



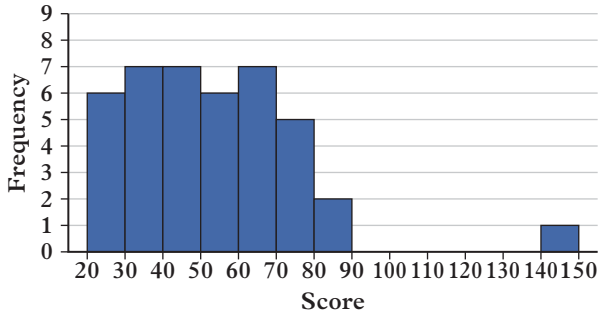
ii negatively skewed
iii Median. The graph is skewed and there is an outlier present, therefore the mean may not be an accurate measure of centre.

b i



ii positively skewed
iii Median. The graph is skewed, therefore the mean may not be an accurate measure of centre.

c i



ii symmetric or positively skewed with an outlier
iii Median. An outlier is present, therefore the mean may not be an accurate measure of centre.

6 a i

Stem	Leaf
1	0 0 0 1 2 2 2 2 2 3 3
1*	5 5 5 5 5 6 6 7 8 9
2	0 0 1 2 2 2 3 3 3
2*	9

Key: 1|3 = 13

ii

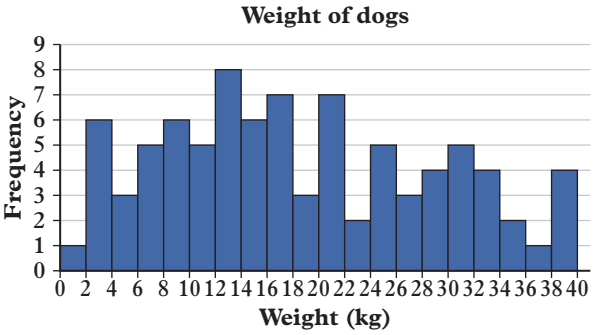
Stem	Leaf
1	0 0 0 1
1*	2 2 2 2 2 3 3
1^	5 5 5 5 5
1#	6 6 7
1+	8 9
2	0 0 1
2*	2 2 2 3 3 3
2^	
2#	
2+	9

Key: 1|3 = 13

b i positively skewed

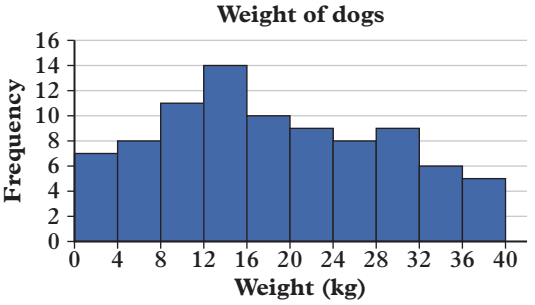
ii bimodal with an outlier

7 a



b no pattern visible

c



d very slightly positively skewed

e Weight of dogs ranges from below 2 kg up to 40 kg. The distribution is fairly even and roughly symmetric with a peak (modal class) around 12–16 kg.

8 a symmetric

b mean or median

c

Number of people	Frequency
0–<10	3
10–<20	4
20–<30	9
30–<40	12
40–<50	8
50–<60	6
60–<70	2
Total	44

d 30–<40

e Sample answer: The average number of people in the skate park is between 30 and 40.

9 a i symmetrical with an outlier

ii median is 35–<40

iii Ages of people at a hairdresser's in a week usually ranges from 15 to 60 years, with the median age being between 35 and 40 years.

b i negatively skewed

ii mean = 15.41, median = 15.5

iii Ages of students at a pool ranges from 13 to 18 years, with median age 15.5 years.

c i positively skewed **ii** median = 2

iii The majority of homes surveyed have one or two TVs, with the most common number of TVs in each home being two.

d i bimodal

ii mode of each peak = 0–<10 cm and 30–<40 cm

iii Of the Year 9 students surveyed, most have either short hair ranging from 1 to 10 cm in length or longer hair ranging from 30 to 40 cm in length.

10 a There were 0 customers on Christmas day because the course was closed.

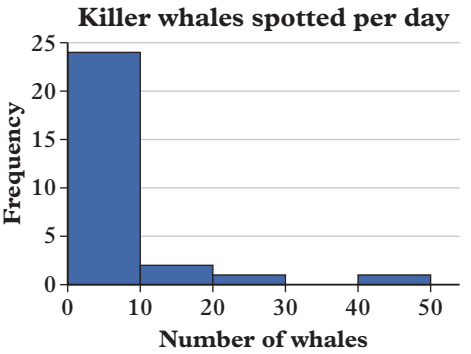
b A mini golf course is likely to have few customers during the week and do the majority of its business on Friday, Saturday, and Sunday.

c Any business that is much busier on weekends: cinemas, golf clubs, paintball, amusement parks, bars, hotel, etc.

11 a It could lead people to believe that customers usually see about 4 whales per tour, which is not true.

b The median number of whales spotted is 0, which would not be a statistic worth advertising.

c



d The 24 numbers in this interval are not evenly spread out. 21 of them are 0.

e 25

12 a **i** mean = 2.31, median = 2, mode = 1

ii mean = 3.5, median = 4, mode = 5

iii mean = 3, median = 3, mode = 3

b **i** positively skewed

ii negatively skewed

iii symmetric

c negatively skewed, symmetric, positively skewed

d negatively skewed

13 a positively skewed

b The dataset obeys Benford's law.

First digit	1	2	3	4	5	6	7	8	9
Frequency	6	1	0	0	0	1	0	2	2
Relative frequency	50%	8%	0%	0%	0%	8%	0%	16%	16%

EX 9E Comparing data

p440

1 a **i** 41 **ii** 45 **iii** 45

b **i** 18 **ii** 24 **iii** 18 and 24

c **i** positively skewed **ii** symmetrical

2 a Group A and Group B have a similar spread of data but are skewed in different directions. Group A is negatively skewed, whereas Group B is positively skewed. Group A has a larger centre than group B.

b Both groups A and B are symmetric, but group B has a larger range due to the presence of an outlier. The median should be used as a measure of centre for group B, which has a smaller centre than group A.

3 a Data sets are spread roughly over the same range but data set for Friday night is symmetrical, whereas the data set for Saturday night is negatively skewed. More people seem to attend the cinema on Saturday night than on Friday night.

b Adelaide has a much larger range of temperatures than Darwin. Adelaide seems to have a roughly symmetric distribution with a lower average temperature than Darwin. Darwin's temperatures cover a small range and appear to be negatively skewed, with a higher average temperature than Adelaide.

c Both dog breeds appear to have roughly symmetric distributions, with Boston terriers covering a larger range of masses. French bulldogs appear to be heavier on average than Boston terriers.

4 a Median heights of students in both classes are similar but 9A has a larger range of heights. The larger range and higher mean values suggests that 9A heights may be affected by presence of an outlier. Median should be used as measure of centre.

b The median and mean price of jeans and range of prices in all stores are similar. Distributions seem to be symmetrical. Mean should be used as measure of centre.

c Both the mean and median indicate that the use of the internet in secondary schools is greater than its use in primary schools. Range of hours is also greater in secondary schools. Median should be used as measure of centre.

d Both stores have the same range and median of storage capacity in their cards, however store A has a larger mean than store B (and larger than its median), which suggests that store A has a skewed distribution and the median is a better representation of centre.

e Players A and B have the same median but player B has a larger mean. Combined with the smaller range, it appears that player B is a more consistent goal kicker with a positively skewed distribution, which would make them a better goal kicker.

5 a B. The range of B is 9, whereas the range of A is 7.

b A. The median of A is 25, whereas the median of B is 23.

6 a School B

b School B (13 at school B and 6 at school A)

c The two data sets are spread out over roughly the same age brackets, but distributed differently. School A has a roughly symmetric distribution, whereas school B is positively skewed.

7 a dot plot 1: negatively skewed;
dot plot 2: symmetrical

b dot plot 1: mean = 16.6, median = 17, range = 6
dot plot 2: mean = 15.24, median = 15, range = 6

c Scores in both data sets are spread over the same range, 6. The dataset in the first dot plot has a higher mean, median and mode than the data set in the second dot plot. Therefore, students participating in the swim squad in term 1 tended to be older than students who participated in the school play in term 4.

8 a Year 7: 86, Year 8: 79

b Year 7: 41, Year 8: 55

c Yes, it is true in general because the median for Year 8 students is much higher.

9 a Group A: 4, Group B: 4

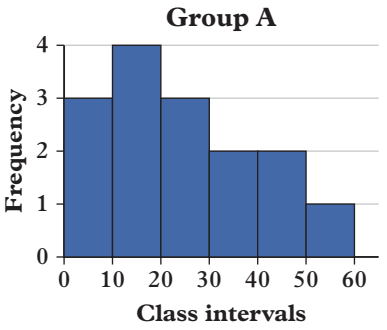
b

	Mean	**Range**
A	3.8	6
B	4.7	14

c The two data sets are very similar apart from the outlier in Group B.

10 a We do not have access to the raw data for the histogram. All that is known is how many numbers are in each interval, but not the actual values of those numbers.

b



c The two data sets are spread out over roughly the same values but distributed differently. Group A is positively skewed, whereas Group B is negatively skewed. Group B would have a higher centre than group A.

11 A matches with histogram 2 and B matches with histogram 1.

Reasons may vary. Any of the following are acceptable:

The median for histogram 2 is in the interval 10–<20 which matches with data set A.

The median for histogram 1 is in the interval 20–<30, which matches with data set B. Histogram 2 will have a lower mean than histogram 1 because so many more values are stacked in the first two bins.

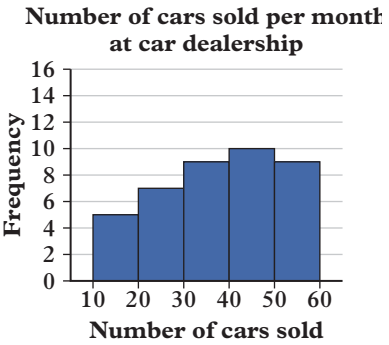
12 **a** negatively skewed

b roughly symmetric

c 16

13 **a** Class intervals used are not same size.

b



c Both histograms display data spread over the same range. The first histogram displays data that is negatively skewed. The median class for the number of cars sold each month by car dealership A was 30–40, with the most frequent number sold in any month being 40–50 cars. The second histogram displays data that is symmetrical. Car dealership B had the same median class, which is also the modal class for this data.

d We do not know how the data is spread over the class intervals to enable us to draw the histogram with smaller class intervals.

14 **a** Year 1: 80–85; Year 2: 85–90; Year 3: 90–95

b Year 3

c If the trend continues, in Year 4 the graph will be negatively skewed with a modal class of 95–100.

15 **a** Class B; it has a range of 28.

b Negatively skewed. A higher median than mean indicates that the centre of the distribution is a high value but there is a long tail of lower values to bring the mean down.

c Class C, because its median is the highest by far. Class B has the highest mean but was affected by one outlier, which does not indicate that class B plays video games more in general.

16 **a** Year 7: 31; Year 8: 35

b 37

c 162.7 cm

d Summary statistics cannot help us to find the middle value of all 450 student heights; the raw data is needed.

EX 9F Theoretical probability

p448

1 **a** $S = \{5, 6, 7, 8, 9, 10\}$ **b** $S = \{1, 2, 3, 4, 5\}$

c $S = \{3, 5, 7\}$ **d** $S = \{1, 9\}$

e $S = \{1, 2, 3, 5, 7, 9\}$ **f** $S = \{1, 2, 9\}$

2 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$

3 **a** $\frac{1}{6}$ **b** $\frac{1}{5}$ **c** $\frac{2}{10} = \frac{1}{5}$ **d** $\frac{1}{5}$

4 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{4}{5}$

5 **a** $\frac{4}{5}$ **b** $\frac{9}{15} = \frac{3}{5}$ **c** $\frac{2}{6} = \frac{1}{3}$

d $\frac{8}{34} = \frac{4}{17}$ **e** $\frac{5}{6}$

6 **a** 0.6 **b** $\frac{1}{4}$ **c** 0.9 **d** $\frac{1}{2}$

e $\frac{7}{8}$ **f** 0.3 **g** $\frac{1}{3}$ **h** 0.75

7 **a** Getting at most 4 or getting less than 5 $S = \{1, 2, 3, 4\}$

b Getting at least 6 or getting less than 5 $S = \{6, 7, 8, 9, 10\}$

c Getting an odd but not a prime, a prime but not an odd, or neither an odd nor a prime $S = \{1, 2, 4, 6, 8, 9, 10\}$

d Getting an odd and a prime, a prime and not an odd, or neither an odd nor a prime; alternatively, getting either an even number or a prime number $S = \{2, 3, 4, 5, 6, 7, 8, 10\}$

e Getting neither an odd nor a prime $S = \{4, 6, 8, 10\}$

f Getting an odd and a prime or neither an odd nor a prime $S = \{3, 4, 5, 6, 7, 8, 10\}$

8 **a** $S = \{1, 2, 3, 4, 5, 6\}$ **b** $\frac{1}{3}$

c probability of rolling less than 5

d $\frac{2}{3}$

9 **a** $\frac{6}{10}$ or $\frac{3}{5}$ **b** selecting a vowel **c** $\frac{4}{10}$ or $\frac{2}{5}$

10 **a** 15 **b** $\frac{5}{15}$ or $\frac{1}{3}$

c selecting a non-blue button; $\frac{10}{15}$ or $\frac{2}{3}$

d 0 **e** $\frac{7}{15}$

f not selecting a button with four holes; $\frac{8}{15}$

11 **a** $\frac{29}{34}$ **b** $\frac{19}{34}$ **c** $\frac{10}{17}$ **d** $\frac{12}{17}$

e $\frac{5}{34}$, 0 eggs

f At most four and no less than four are not complementary as the sum of their probabilities is not equal to one (they both include four). More than four eggs (or at least five eggs) is complementary to at most four eggs. Less than four eggs is complementary to at least four eggs.

12 The sample space usually lists every different outcome rather than every possible outcome; these must be equally likely or an incorrect answer will be obtained.

13 **a** 52 **b** $\frac{1}{13}$ **c** $\frac{12}{13}$ **d** $\frac{3}{13}$

e ace; there are four aces to choose from whereas there is only one queen of hearts.

f $\frac{9}{52}$ **g** $\frac{43}{52}$ **h** $\frac{5}{13}$

14 **a** 30 **b** $\frac{7}{30}$ **c** $\frac{23}{30}$ **d** $\frac{4}{30}$ or $\frac{2}{15}$

e selecting a Y, E, R, I, G, H, T, S, H, N or V

f 12 **g** $\frac{12}{30}$ or $\frac{2}{5}$

h selecting a consonant; $\frac{18}{30}$ or $\frac{3}{5}$

15 **a** $\frac{2}{6}$ or $\frac{1}{3}$

b The green sector is twice the size of other sectors, so the five colours are not all equally likely. It is like having six outcomes but two of these are green.

c $\frac{1}{6}$ **d** not spinning yellow; $\frac{5}{6}$

16 **a** 0.18 **b** 0.37 **c** 0.07

17 $\frac{1}{8}$

18 **a** $\frac{19}{200}$ **b** $\frac{85}{199}$

19 He forgot the two outcomes of one head, one tail.

20 **a** $\frac{8}{55}$ **b** $\frac{22}{55}$ or $\frac{2}{5}$

c **i** 54 **ii** 7 **iii** $\frac{7}{54}$

d **i** 53 **ii** 12 **iii** $\frac{21}{53}$

21 blue: $\frac{1}{6}$, green: $\frac{17}{72}$, purple: $\frac{13}{72}$, orange: $\frac{1}{18}$, yellow: $\frac{13}{36}$

22 **a** $\frac{2}{5}$ **b** $\frac{3}{20}$ **c** 20

EX p453

CHAPTER 9 review

Mathematical literacy review

1 Column graphs can be used to display both categorical data and numerical data, while histograms are best used for displaying numerical data.

2 **a** numerical data, class intervals, histograms

b symmetric, skewed

c outlier

3 **a** mode **b** median **c** range

4 Sample answer:

a An event is a collection of possible outcomes. For example, the event of rolling an even number.

b A favourable outcome refers to any outcome of an event that meets a specific condition or criteria. For example, if we define an event as rolling an even number on a six-sided die, the favourable outcomes would be 2, 4 and 6.

c The complementary event of an event refers to all possible outcomes that are not included in the original event. For example, the complementary event of rolling an even number on a six-sided die would be rolling an odd number.

Multiple choice

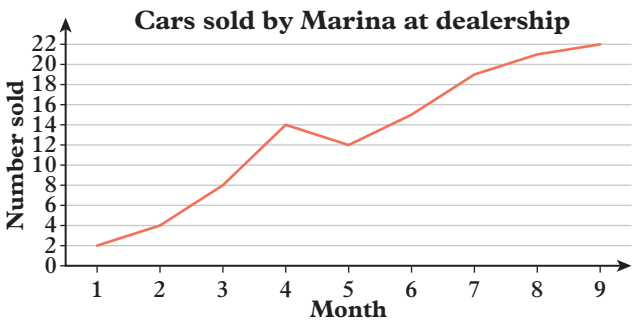
1	C	**2**	C	**3**	D	**4**	B	**5**	E	**6**	D
7	A	**8**	B	**9**	D	**10**	C	**11**	D	**12**	D

Short answer

1 mean = 4.46, median = 5, mode = 5, range = 3

2 mean = 22.84, median = 21, mode = 7, range = 56

3 **a**



b month 5

c $7000

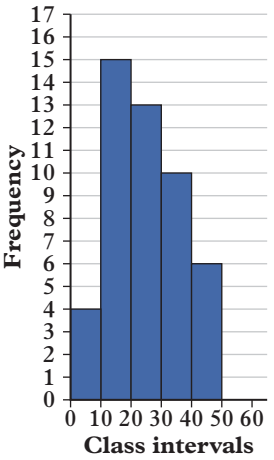
4 **a**

Class interval	Frequency
0–<10	4
10–<20	15
20–<30	13
30–<40	10
40–<50	6
Total	48

b 48

c 10–<20

d



e 10–<20

5 **a** dot plot 1 **b** dot plot 1 **c** dot plot 3

d Median. The mean is not an appropriate measure of centre because the data is skewed.

e Dot plot 1. Reasons may vary. Any of the following are appropriate reasons.

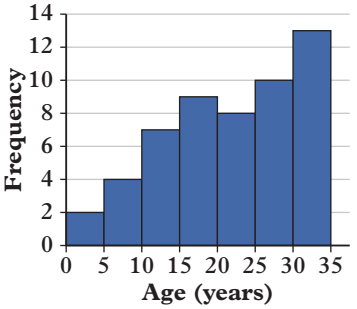
None of the other dot plots have zeroes but we would expect some students to have no pets.

Most values are 0, 1 and 2, which fits the context.

The values in dot plots 2 and 3 are too large.

6 **a** 24 **b** 29

c



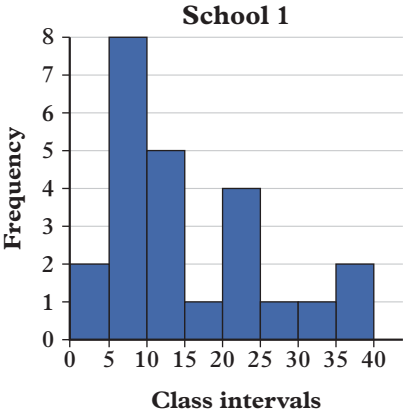
d negatively skewed

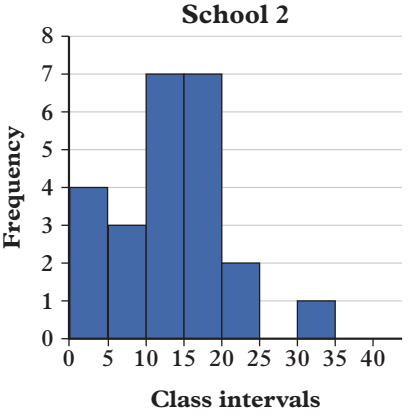
7 **a** Class 8A: mean = 66.32, median = 65, mode = 45, range = 47

Class 8B: mean = 67.14, median = 64, mode = 48, range = 53

b The means and medians are very similar despite the stem-and-leaf plots looking quite different, but Class 8B has a slightly larger range.

8 **a**





b School 1: mean = 14.4, median = 10.5, range = 37

School 2: mean = 12.8, median = 13, range = 29

c In discussing the results, the median will give the best measure of centre, since in school 1 the mean is affected by the positive skew.

9 **a** $\frac{2}{9}$ **b** $\frac{4}{9}$ **c** $\frac{5}{9}$

10 **a** $\frac{1}{6}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

11 **a** $\frac{2}{5}$ **b** $\frac{7}{10}$ **c** $\frac{3}{5}$

Analysis

1 **a** Brisbane: 22 shrubs were measured, Sydney: 24 shrubs were measured

b Brisbane: mean = 13.34, median = 13.15, mode = 11.4, range = 4.5

Sydney: mean = 13.33, median = 13.25, mode = 11.4, range = 4.1

c The data for the two locations is very similar: 22 shrubs were measured in Brisbane and 24 shrubs were measured in Sydney, and the mean and mode for both locations is almost identical. The median is slightly higher for Sydney. It could be said that the average shrub height was 13.3 cm.

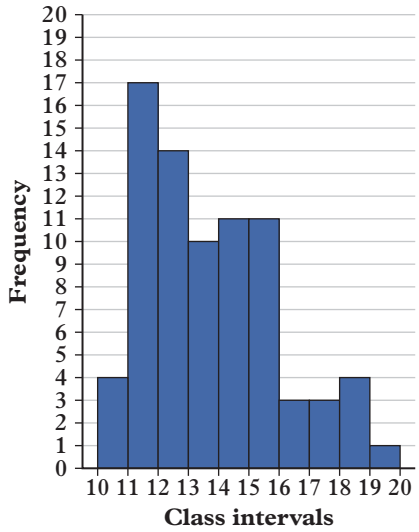
d

Class interval	Frequency
10–<11	4
11–<12	17
12–<13	14
13–<14	10
14–<15	11
15–<16	11
16–<17	3
17–<18	3
18–<19	4
19–<20	1
Total	78

e 78 plants

f $\frac{16}{39}$

g



The data displayed in the histogram is positively skewed.

h Victoria = 14.53, WA = 13.78

i If purchasing a shrub from this nursery chain, it will most likely be in the height range of 10–20 cm, with the average shrub height being approximately 14 cm.

2 a 49, because the highest possible value is 69 and the lowest possible value is 20.

b 31, because the minimum value of the highest number is 60 and the maximum value of the lowest number is 29.

c The raw data is not available.

d 40–<49

e It is not possible to calculate the mean because the raw data is not available. It is not possible to know which interval contains the mean since, unlike the median, the mean is not found based on position.

f 39.6

g 48.6

h 44.1. Yes, it is reasonable to assume that the values in each interval are evenly spread out rather than all clustered at the top or all clustered at the bottom.

EX SEMESTER 2 review

p460

Short answer

1 a solution **b** solution

c not the solution **d** solution

2 a $x = 10$ **b** $x = 3$

c $x = -11$ **d** $x = \frac{9}{5} = 1\frac{4}{5}$

e $x = \pm 5$ **f** $x = \pm\frac{11}{2} = \pm 5.5$

3 a $x = \frac{y - b}{a}$ **b** $x = dy - c$

c $x = \frac{mb - a}{c + m}$ **d** $x = \frac{n}{y - n - 2}$

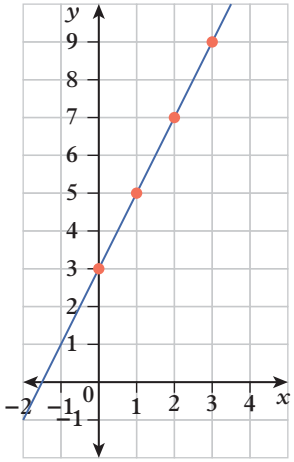
4 a i $x = 3$ **ii** $x = 0$

b i $x = -3$ **ii** $x = 6$

5 a i

x	0	1	2	3
y	3	5	7	9

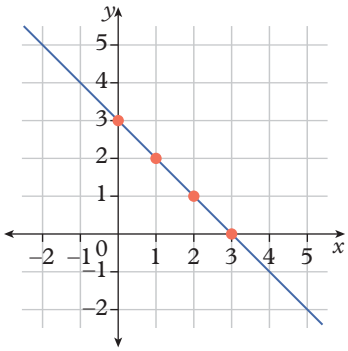
ii



b i

x	0	1	2	3
y	3	2	1	0

ii



6 a $y = -2x + 7$ **b** $y = 4x + 3$

c $y = -x - 2$ **d** $y = \frac{1}{2}x + 6$

7 a is a point of intersection

b is not a point of intersection

8 a equal **b** complementary

c equal **d** supplementary

e supplementary **f** equal

g supplementary

9 a i square **ii** $A = 25\ \text{cm}^2$

b i kite **ii** $A = 30\ \text{cm}^2$

c i parallelogram **ii** $A = 16.0026\ \text{cm}^2$

d i trapezium **ii** $A = 24.0039\ \text{cm}^2$

e i rectangle **ii** $A = 19.9712\ \text{cm}^2$

f i rhombus **ii** $A = 4.8576\ \text{cm}^2$

10 a $A = 13.85\ \text{m}^2$, $P = 13.19$ m

b $A = 38.48\ \text{m}^2$, $P = 21.99$ m

c $A = 15.69\ \text{m}^2$, $P = 16.25$ m

d $A = 3.94\ \text{m}^2$, $P = 8.00$ m

e $A = 1060.29\ \text{m}^2$, $P = 130.69$ m

f $A = 2035.75\ \text{m}^2$, $P = 185.10$ m

11 **a** $3x + 5 = 29$
$x = 8$

b $2 \times 3.4 + 2w = 10.1$
$w = 1.65$ m

c $\pi D = 30$
$D = 9.55$ cm

d $\frac{1}{2} \times 1.5 \times h = 2.25$
$h = 3$ cm

e $7.2h = 24.48$
$h = 3.4$ cm

f $\pi r^2 = 30$
$r = 3.09$ cm

g $34 + 57 + x = 180°$
$x = 89°$

h $\frac{5 + 8 + 13 + 2 + x}{5} = 6.2$
$x = 3$

i $0.3 + 0.12 + 0.44 + x = 1$
$x = 0.14$

12 **a** $z = 13.60$ m **b** $r = 60.40$ m

13 **a** no **b** no **c** yes **d** yes

14 **a** $\frac{3}{9} = \frac{1}{3}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$

15 **a** $a = 31°$ **b** $b = 154°$
c $c = 74°$ **d** $d = 68°$
e $e = 39°$ **f** $f = 65°$

16 **a** 0.024 m
b 5 000 000 cm^3
c 8 001 000 000 m^2
d 4.24 cm^3
e 0.000 02 m^2
f 3 000 000 cm
g 1.2 hours
h 150 minutes
i 6:23 pm
j 0030
k 1.24 L
l 35 000 mL

17 **a** 2.871 m^3 or 2 871 000 cm^3
b 1 200 000 mL or 1200 L
c 1.671 m^3 or 1 671 000 cm^3

18 **a** 1:33 pm ACST
b 7:41 am AEST
c 12:00 pm UTC+1
d 3:31 pm UTC+5

19 **a** 5 h 36 m **b** 3 h 25 m

20 **a** 1 hour 34 minutes **b** 8 hours 25 minutes

21 mean = 7

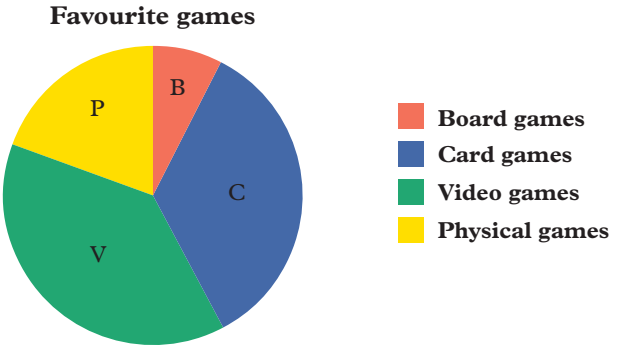
22 **a** mean = 2.22, median = 2, mode = 2, range = 7
b mean = 1.83, median = 2, mode = 2, range = 2. The median and range change. The mode and median stay the same.

23 **a** The average length of time shoppers spent at the shops was 1 h 20 m.
b The average number of board games people owned was 4.
c The most common favourite colour was blue.
d The length of a number of chess matches spanned 34 turns.

24 **a** 122.5 cm^3 **b** 12 cm^3
c 615.8 cm^3 **d** 3200 cm^3

25

a



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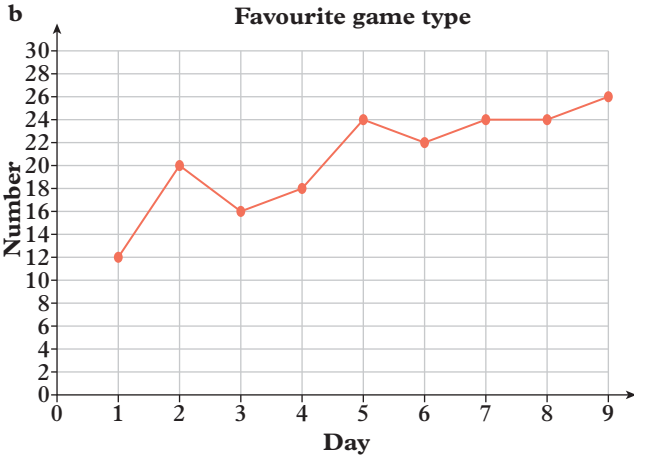

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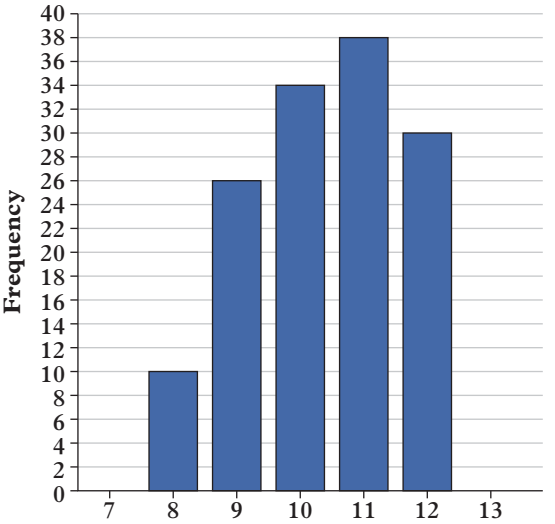
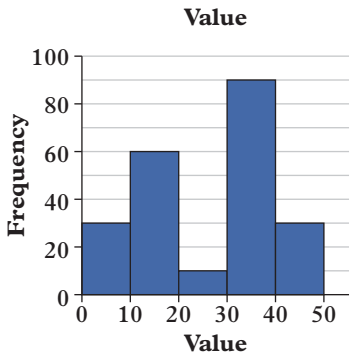


d



26 **a** **i** 19.8 **ii** 19 **iii** 19 **iv** 10

b **i** 12.34 **ii** 12 **iii** 13 **iv** 4

c **i** 4.36 **ii** 4.1 **iii** 3.4 **iv** 4.7

27 **a** bimodal symmetric **b** symmetric

c symmetric **d** positively skewed

e negatively skewed with outlier

f symmetric

28 **a** mode of each peak

b mean, median or mode

c median **d** median

e median **f** median

Analysis

1 **a** $a = 50°$, $b = 40°$, $c = 59°$, $d = 62°$, $e = 60°$

b mean = 60°, median = 59.5°, mode = 50°, range = 50°

c continuous numerical data

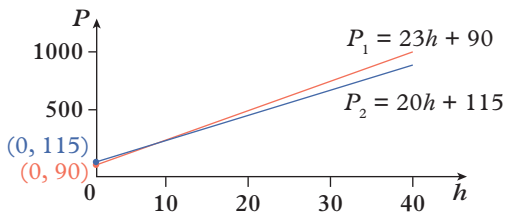
d $\frac{11}{12}$

e $\triangle DEF$

2 **a** \$800 **b** \$890 **c** $P1 = 23h + 90$

d $P_2 = 20h + 115$

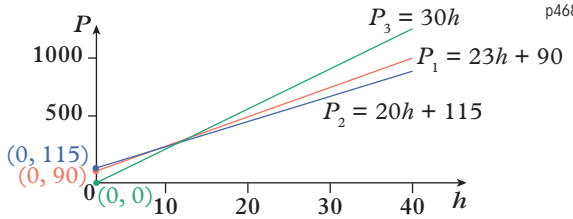
e



f Option 1

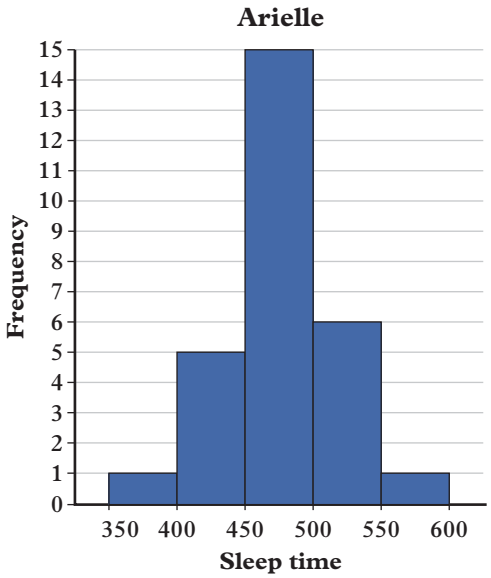
g $P_3 = 30h$

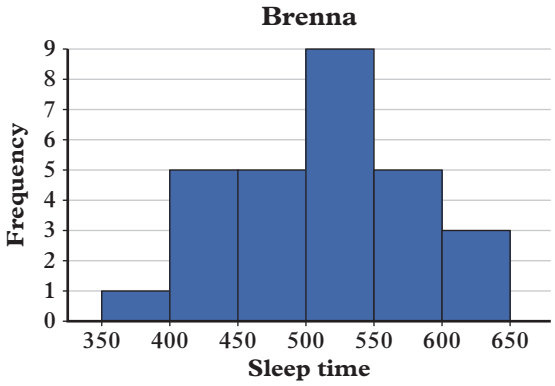
h



i Yes

3 **a**





b Brenna's minutes of sleeping is slightly negatively skewed, whereas Arielle's minutes of sleeping is approximately symmetric.

c Arielle: median = 480 minutes, range = 180 minutes

Brenna: median = 510 minutes, range = 260 minutes

d **i** $\frac{23}{28}$ **ii** $\frac{41}{56}$ **iii** $\frac{1}{28}$

e **i** less than 440

ii more than 510

iii at least 410

f **i** $\frac{5}{28}$ **ii** $\frac{15}{56}$ **iii** $\frac{27}{28}$

EX p468 EXPLORATIONS 2

1 **a** 5

b 36

c 55

d 45

e 65

f 209 and 221

g The word BEANS scores

$b + 1 + 3 + 13 + s = b + s + 17$ points.

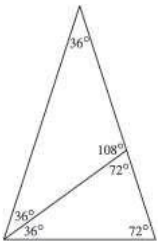
$s = b + 1$; therefore, $b + s$ is always odd, and $b + s + 17$ is always even.

2 **a** 540°

b 108°

c Isosceles triangles with apex angle 36° and base angles 72°, isosceles triangles with apex angle 108° and base angles 36°, and a rhombus with angles 72° and 108°.

d



e Show that a wide triangle can also be divided into one narrow and one wide triangle. It generalises to any pair of numbers as long as there is at least one narrow triangle and two wide triangles.

f Apex angle 90° and base angles 45° (it divides into two more of the same type) and apex angle $25\frac{5}{7}$° and base angles $77\frac{1}{7}$° (it divides into two different types).

3 **a** 120 cm

b 3.82 revolutions per hour

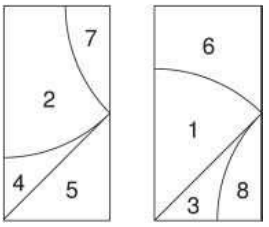
c Raise 209 mm

4 **a**

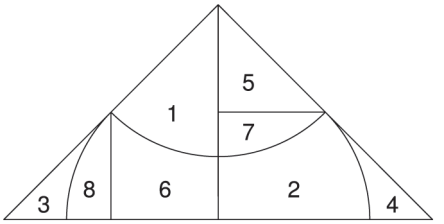
Number	1	2	3 and 4	5	6	7 and 8
Area	2π	8	$8 - 2\pi$	4	$12 - 2\pi$	$2\pi - 4$

b One possible arrangement is shown for each shape. There are many others.

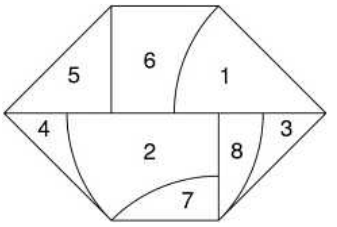
i



ii



iii



5 **a** {2, 2}

b The other freaky frequency list involving only the numbers 1, 2, 3 and 4 is {1, 3, 2, 1, 3, 3, 4, 1}

c {1, 2, 2, 3, 3, 2, 4, 1}

d {1, 4, 2, 2, 3, 2, 4, 2, 5, 1, 6, 1}

e 9

6 Corrections are underlined:

a R2 [R4 [F50 L90] <u>L180</u>]

b <u>R6</u> [R3 [F50 L120] L60]

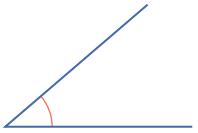
c R6 [R6 [F50 L60] <u>L60</u>]

Glossary

12-hour time
time measured over a 12-hour period from midnight and then again from midday; am and pm are used to describe whether a time is before or after midday

24-hour time
time measured over a 24-hour period from midnight. Usually shown with four digits to indicate the number of hours and minutes from midnight; for example, 1530 means 15 hours and 30 minutes after midnight – or 3:30 pm

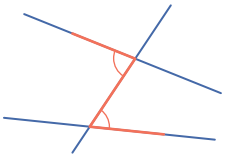
acute angle
angle greater than 0° but less than 90°



acute-angled triangle
triangle with three interior acute angles (less than 90°)

addition algorithm
an algorithm in which two or more numbers are added by first adding the ones values, then the tens values, then the hundreds values, and so on

alternate angles
pair of angles not adjacent to (next to) each other on opposite sides of a transversal between two lines, sometimes called 'Z angles'



arc
a proportion of the circumference of a circle. Angles are marked using an arc

area
amount of space enclosed by a two-dimensional (2D) shape. Common units are mm^2, cm^2, m^2, km^2, ha

ascending order
where a list of numbers is written from smallest to largest

associative law
regardless of how three or more numbers are grouped when added or multiplied, the result is the same; for example,

$$(4 + 5) + 6 = 9 + 6 = 15$$
$$4 + (5 + 6) = 4 + 11 = 15$$
$$(4 \times 5) \times 6 = 20 \times 6 = 120$$
$$4 \times (5 \times 6) = 4 \times 30 = 120$$

bar chart
graph where the frequency of categorical data is presented in horizontal bars

base
1. for a value expressed in index form, the base is the number which is repeatedly multiplied; for example, 2^4 has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$
2. in a 2D shape, it is a nominated side of the shape. The base and height are perpendicular to each other
3. in a 3D object, it is a nominated face of the object. The base and height are perpendicular to each other

best buy
the option with the lowest unit cost

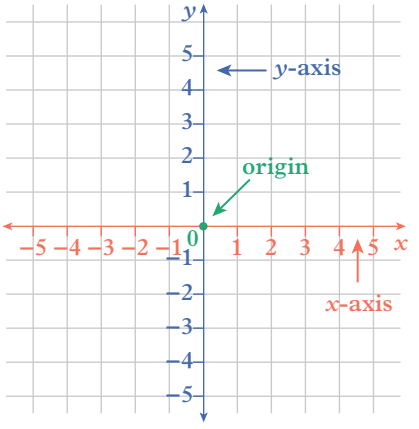
BIDMAS
acronym used to remember the correct order of operations: Brackets, Indices (and roots), Division and Multiplication, and Addition and Subtraction

bimodal
describes a distribution with two clear peaks

by-parts method
a way to add two or more numbers where the digits in each place value are added separately

capacity
amount of liquid that a three-dimensional object (container) can hold

Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point on the plane to be described. It consists of a horizontal x-axis and a vertical y-axis that intersect at right angles at a point called the origin

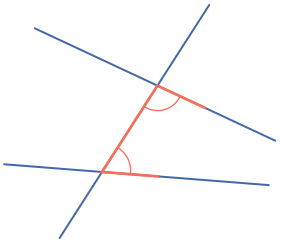


circumference
perimeter of a circle

class interval
a range of values into which numerical data can be grouped

coefficient
the number by which a pronumeral is multiplied; for example, the coefficient of $5m$ is 5 and the coefficient of $-3a$ is -3

co-interior angles
pair of angles on the same side of a transversal between two lines, sometimes called 'C angles'



column graph
graph where the frequency of categorical data is presented in vertical columns

common factor
factor that divides exactly into two or more whole numbers; for example, 12 and 9 have a common factor of 3

commutative law
the order in which two numbers are added or multiplied is not important; for example, $3 + 4 = 7$ and $4 + 3 = 7$; $3 \times 4 = 12$ and $4 \times 3 = 12$

compensation method
a way to add or subtract numbers whereby one number is rounded to make the calculation easier, and then the amount by which the numbers was rounded by is added to or subtracted from the result

complementary angles
angles that add to make a right angle (90°)

complementary events
a pair of events A and A' (not A) where only one of the two events can take place at a single time and their probabilities sum to 1

composite number
a positive whole number that is divisible by at least one whole number other than itself and 1; for example, 8 has factors of 1, 2, 4 and 8, and so is composite

constant
a term without any pronumeral component; for example, the constant in the expression $3b - 4c + 10$ is 10

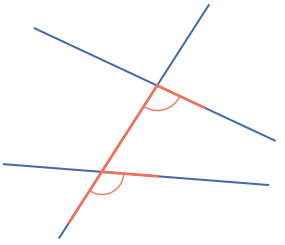
continuous data
numerical data that can be measured; for example, the height of a plant

conversion factor
a multiplier for converting a quantity expressed in one set of units into another quantity expressed in a different set of units; for example, multiply by 10 to convert from cm to mm

Coordinated Universal Time (UTC)
time zone referenced from Greenwich in London at 0° longitude (also known as Greenwich Mean Time). Parts of the world to the east of Greenwich are ahead of UTC, and parts of the world to the west are behind UTC. For example, Sydney is in the UTC+10 time zone. This means it is 10 hours ahead of UTC

coordinates
pair of numbers that describe the position of a point on a plane or graph

corresponding angles
pair of angles on the same side of a transversal and in corresponding positions on the two lines (both above a line or both below a line), sometimes called 'F angles'



cost price (or wholesale price)
the cost paid to create or obtain a good or service before it is sold to consumers

cube
to raise to the power of three; for example, $2^3 = 8$

cube root
number that when cubed results in the original number; for example, the cube root of 8 (or $\sqrt[3]{8}$) is 2

cumulative frequency
a running total of frequencies in a frequency table

cylinder
3D object with a uniform circular cross-section

decimal
number with a whole number component and a decimal component. The digits to the left of the decimal point indicate the whole number and the digits to the right of the decimal point indicate a value less than one; for example, 3.456

denominator
bottom number of a fraction. For example, the denominator of $\frac{2}{3}$ is 3

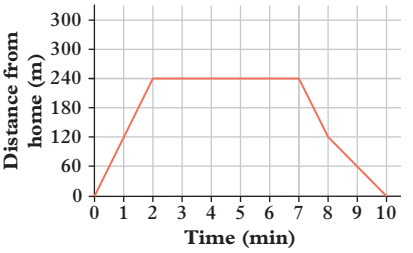
descending order
where a list of numbers is written from largest to smallest

diameter
line segment from one side of a circle to the other through the centre of the circle. Also defined as the length of this line segment

discount
an amount off the original price; the difference between the regular price and a lower price

discrete data
numerical data that can be counted; for example, number of siblings

distance–time graph
a graph which tracks the distance of an object from a fixed point over a period of time



distributive law
the same result is obtained whether you add the numbers inside a pair of brackets first before multiplying by another number or multiply each number inside the brackets by the number outside the brackets before adding, $a(b + c) = ab + ac$

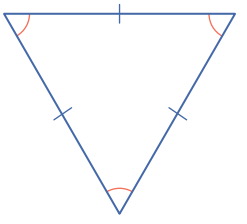
dividend
number that is being divided by another number (divisor); for example, 6.84 is the dividend, 2 is the divisor and 3.42 is the quotient in $6.84 \div 2 = 3.42$

divisor
number that divides into another number (dividend); for example, in $15 \div 3 = 5$, 3 is the divisor

duration
the length between two given times

equation
a collection of two or more algebraic terms separated by operation symbols and an equals sign

equilateral triangle
triangle with all three sides equal in length. All three angles are also equal in size



equivalent equations
equations that have the same solution

equivalent fractions
different fractions that have the same value. An equivalent fraction is formed by multiplying or dividing the numerator and the denominator of a fraction by the same number. For example, $\frac{24}{36} = \frac{4}{6} = \frac{2}{3}$

equivalent ratios
different ratios that have the same value. An equivalent ratio can be produced by multiplying or dividing all numbers in a ratio by the same value

estimation (estimate)
an approximation of a value or number

evaluate
substitute the known values of each pronumeral into an expression and complete operations in the correct order

event
a specified result that may be obtained in a probability experiment

expand
remove the brackets in an expression by using the distributive law

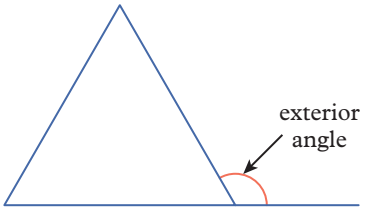
expanded form
1. power written as a repeated multiplication; for example, 4^3 written in expanded form is $4 \times 4 \times 4$
2. algebraic expression written without brackets, as opposed to factorised form

experiment
a repeatable procedure that has a clearly defined set of possible results

exponent
see index

expression
either a single term or the sum or difference of two or more terms; for example, $3b - 4c + 10$ is an expression

exterior angle
an angle between any side of a shape and a line extended from the adjacent side



factor tree
tree diagram where composite numbers are broken down into factor pairs until only prime factors remain

factorise
to write an algebraic expression as the product of its factors; for example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$

factors
whole numbers that divide exactly into a given whole number; for example, the factors of 8 are 1, 2, 4 and 8

favourable outcome
the outcome within a sample space that we are interested in

formula
relationship or rule between two or more variables, usually represented using pronumerals; for example, $s = \frac{d}{t}$ is the formula for speed equals distance divided by time

frequency
the number of times a particular value occurs in a data set

frequency table
a table organised to show data by recording the frequency of each value in the data set

GST (Goods and Services Tax)
introduced by the Australian Government in 1999. A 10% tax on the selling price of the products or service

graph
a continuous line made up of infinitely many coordinate points

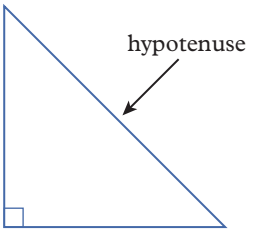
grouped data
numerical data that has been sorted into groups or class intervals

grouping symbols
symbols including brackets, roots and fraction lines which show where a group starts and ends, and help to establish the order to apply mathematical operations

highest common factor (HCF)
largest factor that is common to two or more given numbers; for example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so the HCF of 8 and 12 is 4

histogram
type of bar chart that can be used to display grouped continuous or discrete numerical data where the bounds of each class interval are located at the edges of the columns

hypotenuse
the side opposite the right angle of a right-angled triangle



improper fraction
fraction where the numerator is greater than (or equal to) the denominator; for example, $\frac{9}{7}$ or $\frac{6}{6}$

index
for a value expressed in index form, the index indicates the number of times the base is written as a repeated multiplication; for example, 2^4 has an index of 4 and can be written as $2 \times 2 \times 2 \times 2$. We read 2^4 as "2 to the power of 4"

index laws

a set of rules that can be used to simplify calculations involving numbers in index notation

	Index law
Product of powers	$a^m \times a^n = a^{m+n}$
Quotient of powers	$\frac{a^m}{a^n} = a^m + a^n = a^{m-n}$
Power of a power	$(a^m)^n = a^{m \times n}$

index notation (index form)

shorter form of writing a repeated multiplication. A number in index notation is written with a base and an index; for example, 2^4 is written in index notation

integer

a whole number that is positive, negative or zero

interior angle

an angle inside a shape or object

inverse operation

operation that reverses the effect of a previous operation; for example, multiplication and division are inverse operations and addition and subtraction are inverse operations

irrational number

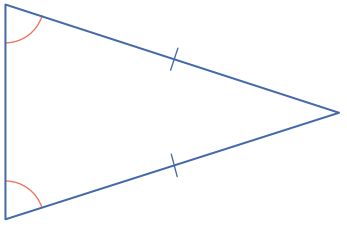
number that cannot be written exactly as a fraction. These are non-terminating decimals that are not recurring decimals; for example, π, $\sqrt{2}$, $\sqrt{3}$

irregular quadrilateral

quadrilateral with four different internal angles and no parallel sides

isosceles triangle

triangle with two sides equal in length. The angles opposite the equal sides are also equal

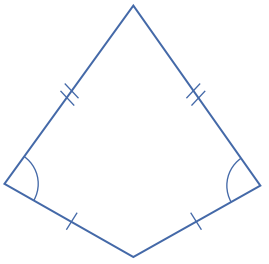


jump method

a way to add or subtract numbers whereby one of the given numbers is broken down, then each part of the number is added or subtracted in stages

kite

quadrilateral with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides



leading digit

first non-zero digit of a number; for example, the leading digit of 328 is 3

length

measurement from one end to the other of a shape or object. In a rectangle, it is the side measurement that is not defined as the breadth. Common units are mm, cm, m, km

like terms

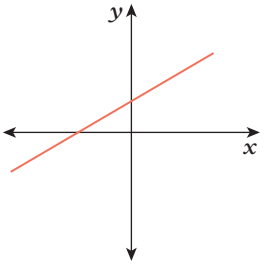
algebraic terms that contain exactly the same pronumerals in any order

line graph

graph that displays the relationship between two variables where points are plotted and joined by a straight line. A continuous variable must be displayed on the horizontal axis; for example, time

linear graph

a graph that forms a straight line on the Cartesian plane



linear relationship

relationship between two variables that produces a linear graph

loss

amount lost when the selling price is less than the cost price

lower bound

abbreviation of lower boundary; the lowest value in a class interval

lowest common denominator (LCD)

lowest common multiple of the denominators in two or more fractions. For example, $\frac{1}{3}$ and $\frac{1}{2}$ have an LCD of 6

lowest common multiple (LCM)

lowest (first) multiple that is common in the multiples list of two or more numbers; for example, multiples of 3 are 3, 6, 9, 12, 15, 18, … and multiples of 4 are 4, 8, 12, 16, 20, 24, … so the LCM of 3 and 4 is 12

magnitude

the value of a number ignoring the sign; for example the magnitude of −3 is 3

mark-up

amount added to a cost price; for example, an original price has a 25% mark-up added to obtain the selling price

mean

a measure of the centre of a data set (numerical average) calculated by adding all of the data values and then dividing by the number of values

measure of centre

number used to represent the central point in a data set. Measures of centre include the mean, median and mode

measure of spread

number used to indicate how spread out a data set is. The range is a measure of spread

median

a measure of the centre of a data set. For an ordered set of data, it is the middle number if there is an odd number of values, and the average of the two middle numbers if there is an even number of values

mixed number
number made up of a whole number component and a proper fraction component; for example, $3\frac{4}{5}$

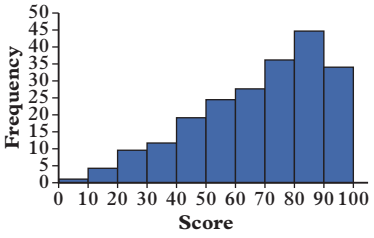
mode
a measure of the centre of a data set determined by finding the value, or values, that occurs most frequently in the data set

multimodal
describes a distribution with multiple clear peaks

multiples
result of multiplying a given number by whole numbers; for example, multiples of 4 are 4, 8, 12, 16, … (4 × 1, 4 × 2, 4 × 3, 4 × 4, …)

negative integers
whole numbers that are less than zero in value and are always preceded by a negative sign, –, placed directly before the first digit

negatively skewed
describes a distribution that is skewed away from the vertical axis with a tail to the left

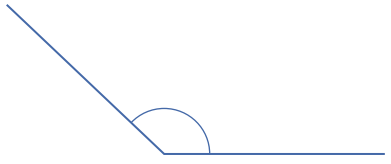


non-terminating decimal
decimals with an endless (infinite) number of decimal places; for example, 5.761 238… or 4.266 666…

numerator
top number of a fraction; for example, the numerator of $\frac{2}{3}$ is 2

numerical data
data that can be counted or measured

obtuse angle
angle greater than 90° but less than 180°



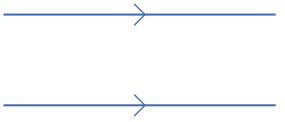
obtuse-angled triangle
triangle with one interior obtuse angle (more than 90°)

order of operations
the correct sequence of conducting mathematical operations

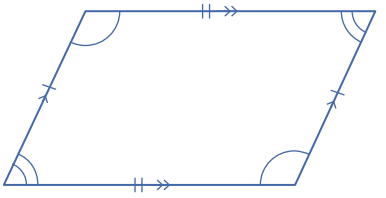
origin
point where the *x*- and *y*-axes cross on a Cartesian plane. Coordinates are (0, 0)

outlier
extreme piece of data that is much higher or lower than the rest of the data set

parallel lines
lines (or rays or segments) that are drawn side by side and are the same distance apart so they never touch are parallel. Matching arrowheads are used to indicate that lines are parallel



parallelogram
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size. Rectangles, squares and rhombuses are also classified as parallelograms



percentage
a portion of a whole divided into one hundred parts. The symbol for percentage is %

percentage decrease
occurs when a percentage amount is subtracted from the original amount

percentage error
the error of a measurement written as a percentage of the true value of the quantity

percentage increase
occurs when a percentage amount is added to the original amount

percentage loss
the amount of loss expressed as a percentage of the cost price (or selling price)

percentage profit
the amount of profit expressed as a percentage of the cost price (or selling price)

perfect cube
the product of three equal whole numbers; for example, 2 × 2 × 2 = 8, so 8 is a perfect cube

perfect square
the product of two equal whole numbers; for example, 3 × 3 = 9, so 9 is a perfect square

perimeter
length of the boundary of a two-dimensional (2D) shape. Common units are mm, cm, m, km

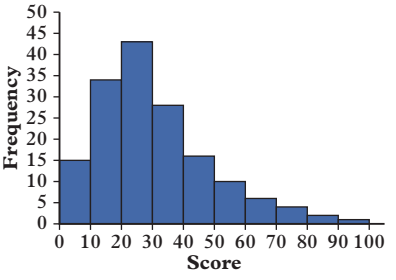
pi
irrational number (3.141 592 653 …) used in calculations involving the circumference and area of a circle. It is the ratio of the circumference of a circle to its diameter. The symbol for pi is π

plot
a Cartesian plane composed of individual coordinate points

polygon
general name for a two-dimensional shape with straight sides

positive integers
whole numbers that are greater than zero in value and can be represented using a positive sign, +. The positive sign can also be left out

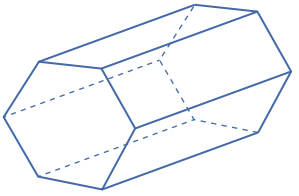
positively skewed
describes a distribution that is skewed towards the vertical axis with a tail to the right



prime factorisation
the unique product of the primes that make up an integer

prime number
a positive whole number greater than 1 that has exactly two factors, itself and 1; for example, the only factors of 7 are 1 and 7, so 7 is a prime number

prism
object with two ends that are identical polygons and joined by straight edges



probability
chance of an event occurring. The event can be very likely and have a high probability or not likely and have a low probability. Words or numbers can be used to describe the chance of an event occurring

product
result of a multiplication; for example, $10 \times 3 = 30$, where 30 is the product

profit
amount made when the selling price is greater than the cost price

pronumeral
letter or symbol that takes the place of a number

proper fraction
fraction where the numerator is smaller than the denominator; for example, $\frac{4}{5}$

Pythagoras' theorem
theorem that states that the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the other side lengths

Pythagorean triad (Pythagorean triple)
any set of three positive whole numbers that satisfy Pythagoras' theorem

quadrilateral
two-dimensional shape with four straight sides

quadratic equation
an equation that contains a pronumeral term or terms with the highest power of 2; for example, $x^2 + 3x - 2 = 0$

quotient
result of a division. For example, $28 \div 4 = 7$ where 7 is the quotient

radius (plural: radii)
line segment from the centre of a circle to the edge of the circle. Also defined as the length of this line segment and half the diameter of the circle

range
a measure of spread determined by finding the difference between the highest and lowest values in a data set

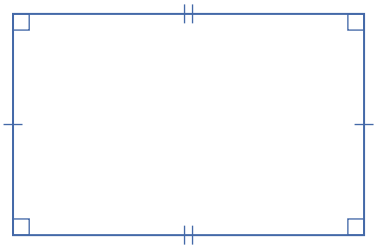
rate
a comparison between two or more different quantities. It is a measure of how one quantity changes for each increase in the unit of another quantity

ratio
a comparison of two or more quantities of the same kind; for example, when comparing the number of red jellybeans to blue jellybeans, the ratio is 2 : 3

rational number
number that can be written exactly as a fraction. This includes whole numbers, fractions, terminating decimals and recurring decimals

reciprocal
the multiplicative inverse of a number which can be found by write the number as a fraction and swapping the numerator and denominator. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. When a number is multiplied by its reciprocal, the result is 1

rectangle
quadrilateral with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°. A rectangle is also classified as a parallelogram



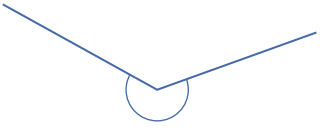
rectangular prism
prism with three pairs of identical rectangular faces

recurring decimal
a special type of non-terminating decimal. The digits after the decimal point form a recurring or repeating pattern. For example,

$0.333\,33\ldots = 0.\dot{3}$. and
$5.476\,767\,6\ldots = 5.\overline{476}$

reference point
a point or number used to define the location of another point relative to the reference point. This point is often zero on a number line or Cartesian Plane

reflex angle
angle that is greater than 180° but less than 360°

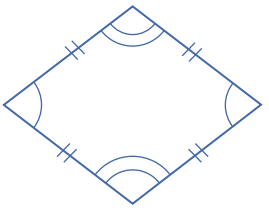


remainder
the part of the dividend that is left over when the number is not exactly divisible by the divisor. For example, 22 divided by 7 is 3 remainder 1

revenue
income generated by the sale of goods or services

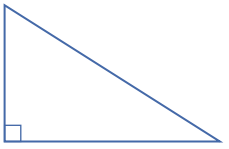
revolution
angle that is exactly 360°. Also known as a perigon

rhombus
quadrilateral with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size. A rhombus can also be classified as a parallelogram



right angle
angle that is exactly 90°. Often represented by a small box drawn where the lines meet

right-angled triangle
triangle where the size of one interior angle is 90°



round down
to find an approximation that is less than the original number

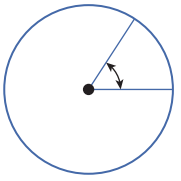
rounding
involves replacing the number with an approximation that is simpler and easier to use in calculations, but which makes any calculations less accurate. For example, 123 343 ≈ 120 000

round up
to find an approximation that is greater than the original number

sample space
list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a six-sided die is {1, 2, 3, 4, 5, 6}

scalene triangle
triangle with three sides of different lengths. All three angles are different in size

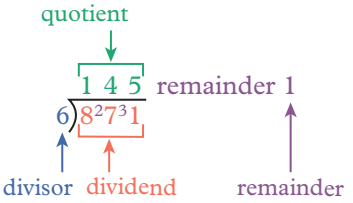
sector
portion of a circle formed by two radii and part of the circumference



sector graph
graph that displays the frequency of categorical data (size of sector) as a fraction of the total pieces of data (circle)

selling price
the price that a product or service is sold at by the seller

short division
an algorithm used to simplify a division problem by breaking it down into easy steps



sign (of a number)
whether a number is positive or negative

simplification
process of reducing the complexity of a calculation by adding, subtracting, multiplying and dividing terms

skewed
describes a data set where data points are clustered to the left or right of the distribution

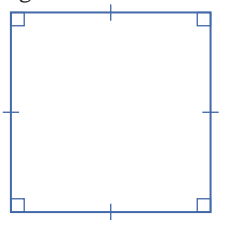
slope
the steepness of a line segment; a positive slope is pointing upwards (to the right) and a negative slope is pointing downwards (to the right)

solution
value of the pronumeral that makes an equation a true statement

solve by inspection
a mental strategy to solve simple equations, whereby the solution is identified without performing any calculations

speed
a rate which is calculated by dividing the distance travelled by the time taken

square
1 quadrilateral with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°



2 to raise to the power of 2; for example, $3^2 = 9$

square root
the square root of a number is the number that when multiplied by itself produces the original number; for example, the square root of 9 (or $\sqrt{9}$) is 3

straight angle
angle that is exactly 180°

subject
a pronumeral written by itself on one side of an equation and which does not appear on the other side of the equation; for example, y is the subject in the equation $y = mx + c$

substitution
replacement of a pronumeral with a given number; for example, substituting $x = 2$ into $3x + 4$ gives $3 \times 2 + 4$

subtraction algorithm
an algorithm in which two or more numbers are subtracted by first subtracting the ones values, then the tens values, then the hundreds values, and so on

supplementary angles
angles that add to make a straight angle (180°)

surd
a root of a number that results in an irrational solution

symmetrical (symmetric)
describes a data set with a peak in the middle, where data points are roughly evenly spread on either side of the peak

term
a single pronumeral, single number, or the product of a number and one or more pronumerals. For example, the expression $3b - 4c + 10$ has three terms

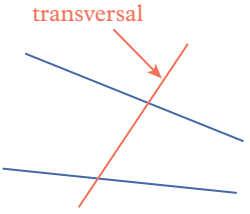
terminating decimal
decimal that 'terminates' or stops after a set number of decimal places; for example, 0.625

theoretical probability
chance of something happening based on mathematical reasoning shown on a scale from 0 and 1 (inclusive)

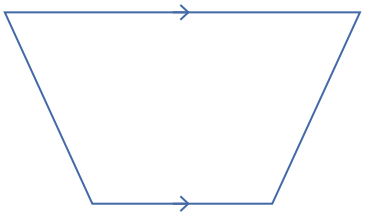
time zones
bands on the surface of the Earth where countries experience the same time. Each time zone usually spans a difference in longitude of 15°

trailing zeroes
zeroes to the right of the last non-zero digit after the decimal point in a decimals; for example, 3.900 has two trailing zeroes

transversal
line that crosses (intersects) a pair or set of lines



trapezium
quadrilateral with at least one pair of opposite sides that are parallel



triangular prism
3D object with five faces including two identical triangular faces whose respective vertices are joined by parallel lines

ungrouped data
raw data that has not been placed into class intervals

uniform
describes a distribution with no clear peaks

unimodal
describes a distribution with one clear peak

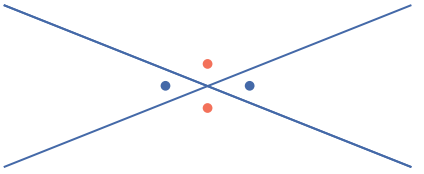
unitary method
a method for solving various financial problems by first calculating the value of a single unit, for example, 1% of a quantity

unknown number
a number that is represented using a pronumeral; for example, the x in $x + 3 = 5$

upper bound
abbreviation for upper boundary; the largest value in a class interval

variable
quantity that can have different values and can be represented with a word, symbol or pronumeral

vertically opposite angles
pair of equal angles not adjacent to (next to) each other, formed where two lines intersect



volume
amount of space that a three-dimensional (3D) object occupies

x-axis
horizontal number line that forms part of the Cartesian plane

x-coordinate
first of a pair of numbers (coordinates) that describes the point's horizontal distance from the origin on the Cartesian plane

$(-4, 1)$

x-coordinate – horizontal distance from the origin

y-axis
vertical number line that forms part of the Cartesian plane

y-coordinate
second of a pair of numbers (coordinates) that describes the point's vertical distance from the origin on the Cartesian plane

$(-4, 1)$

y-coordinate – vertical distance from the origin

y-intercept
the value of the y-coordinate where a line meets the y-axis (when $x = 0$). In a linear equation of the form $y = mx + c$, the y-intercept is equal to the constant, c

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